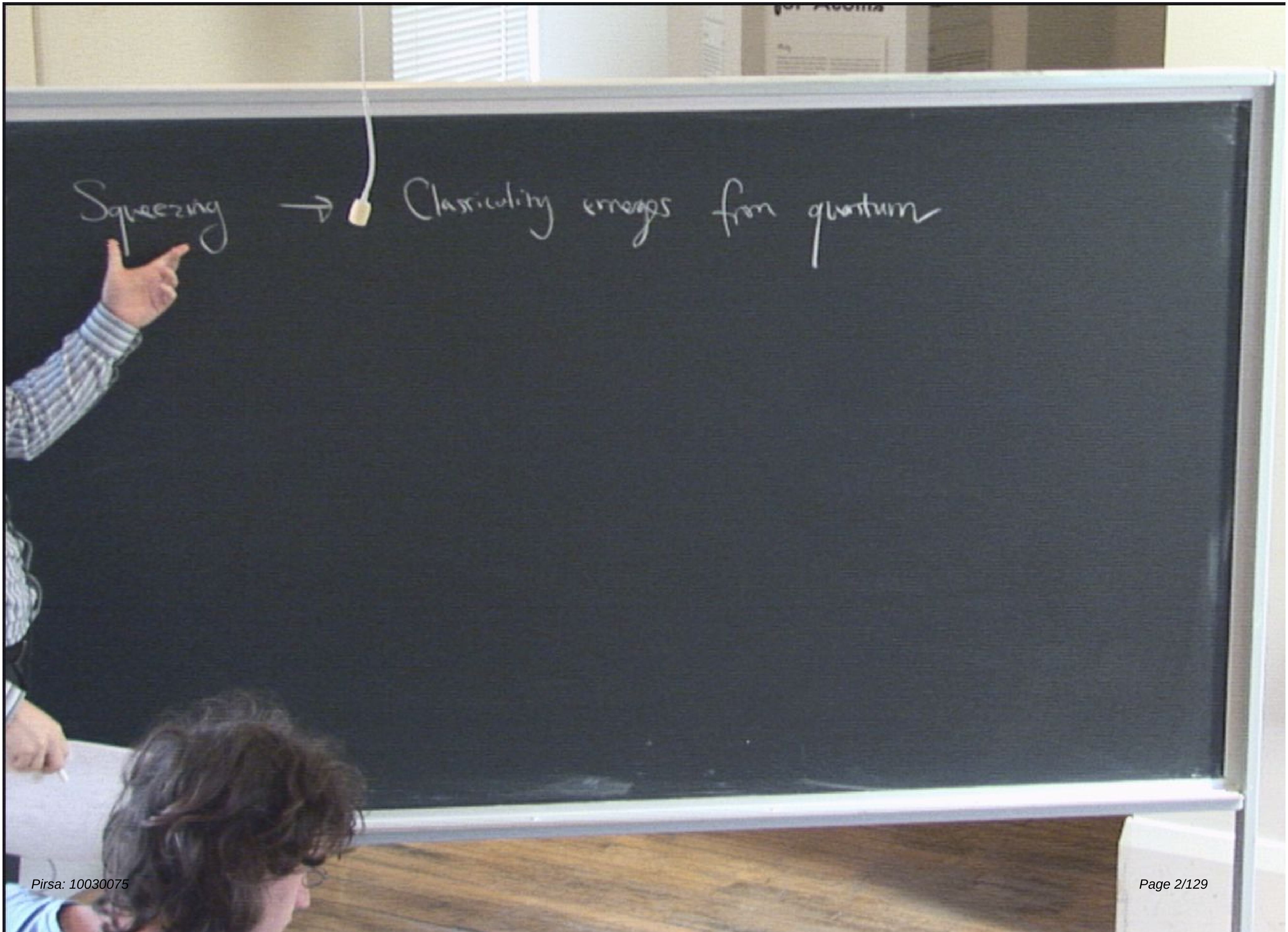
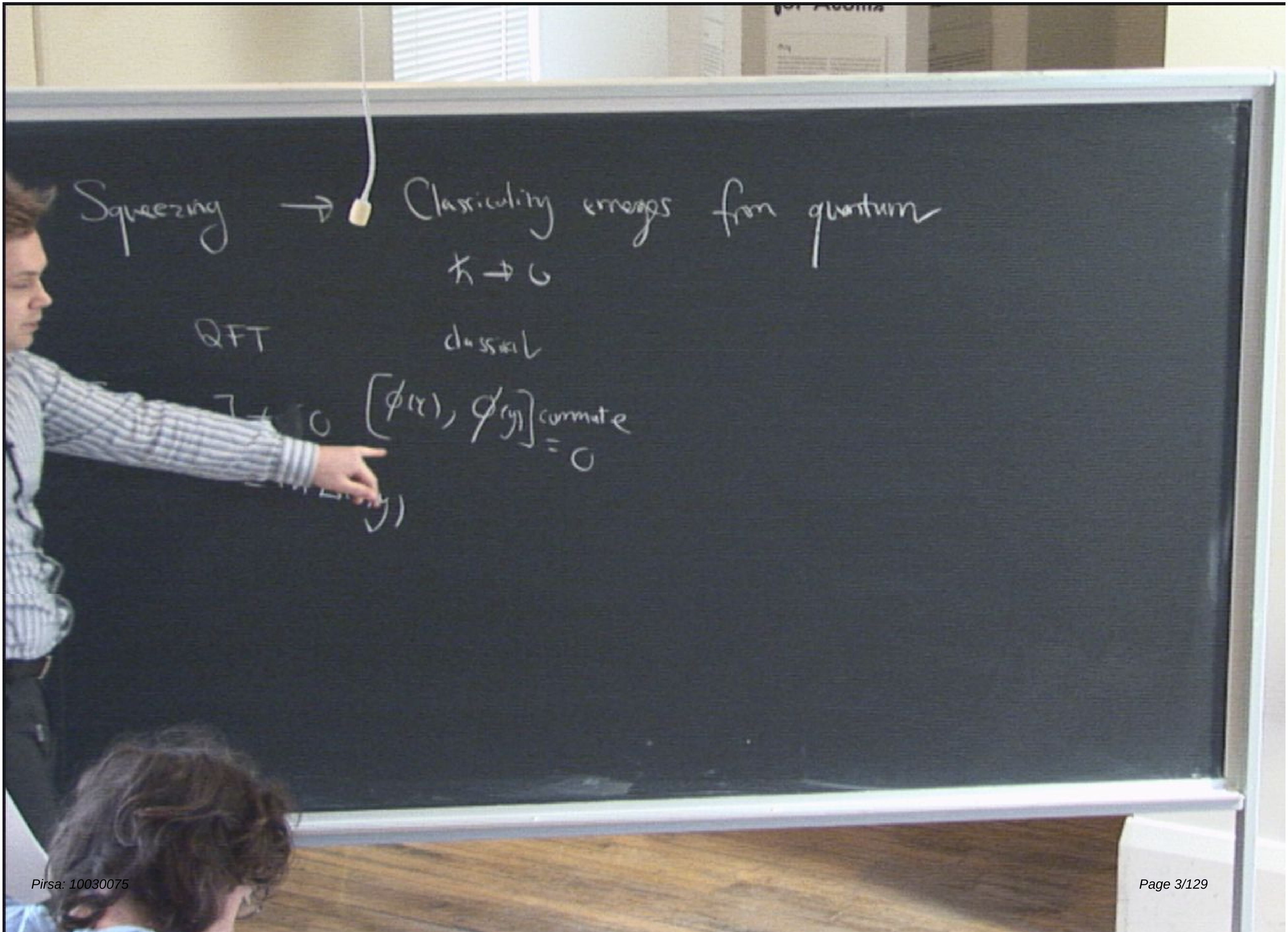
Title: Explorations in Cosmology (PHYS 649) - Lecture 6

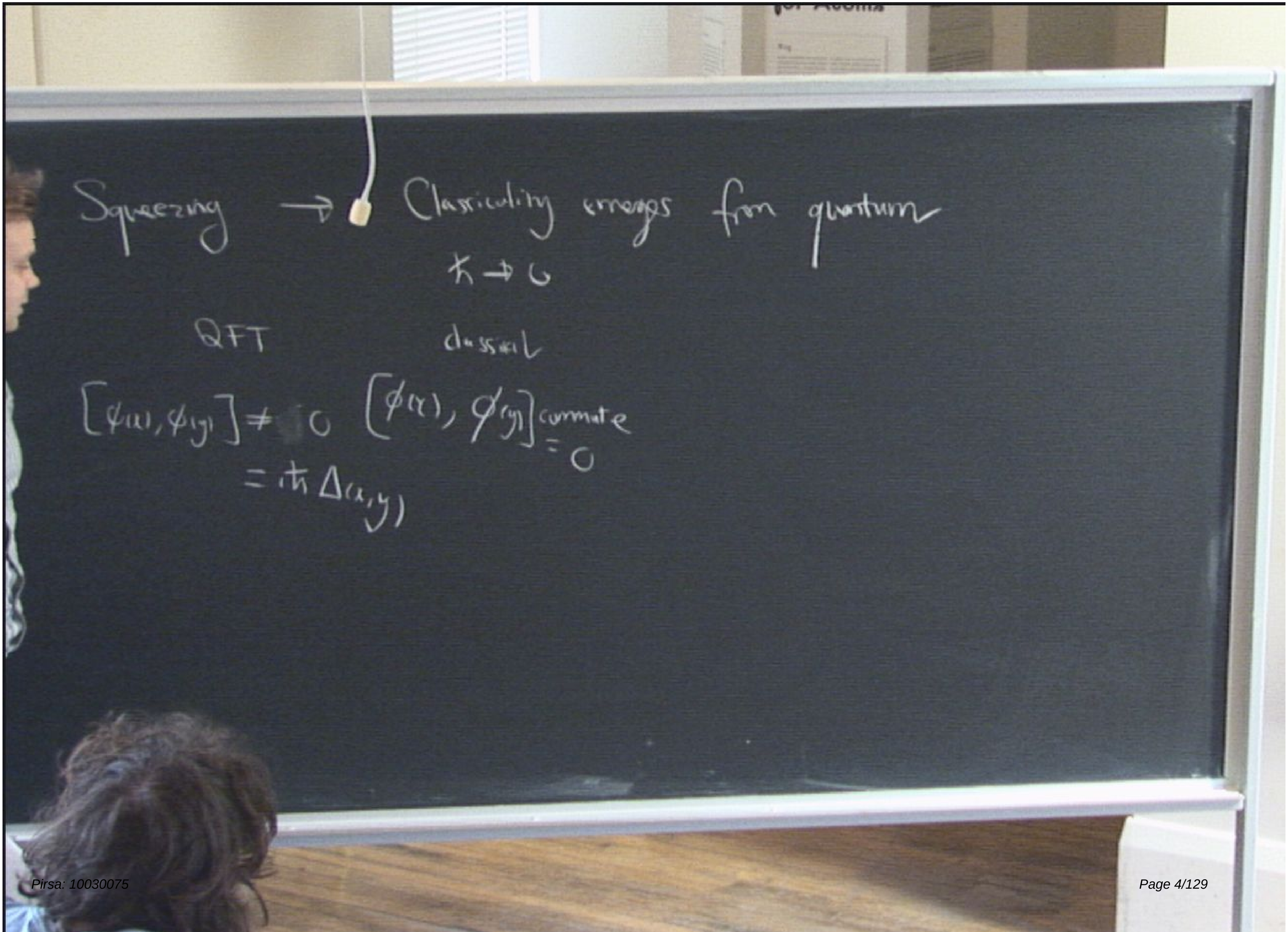
Date: Mar 22, 2010 09:00 AM

URL: http://pirsa.org/10030075

Abstract:


Squeezing → Classicality emerges from quantum


Squeezing → Classicality emerges from quantum
ħ → 0
QFT
classical
[φ(x), φ(y)] commute
= 0


Squeezing → Classicality emerges from quantum
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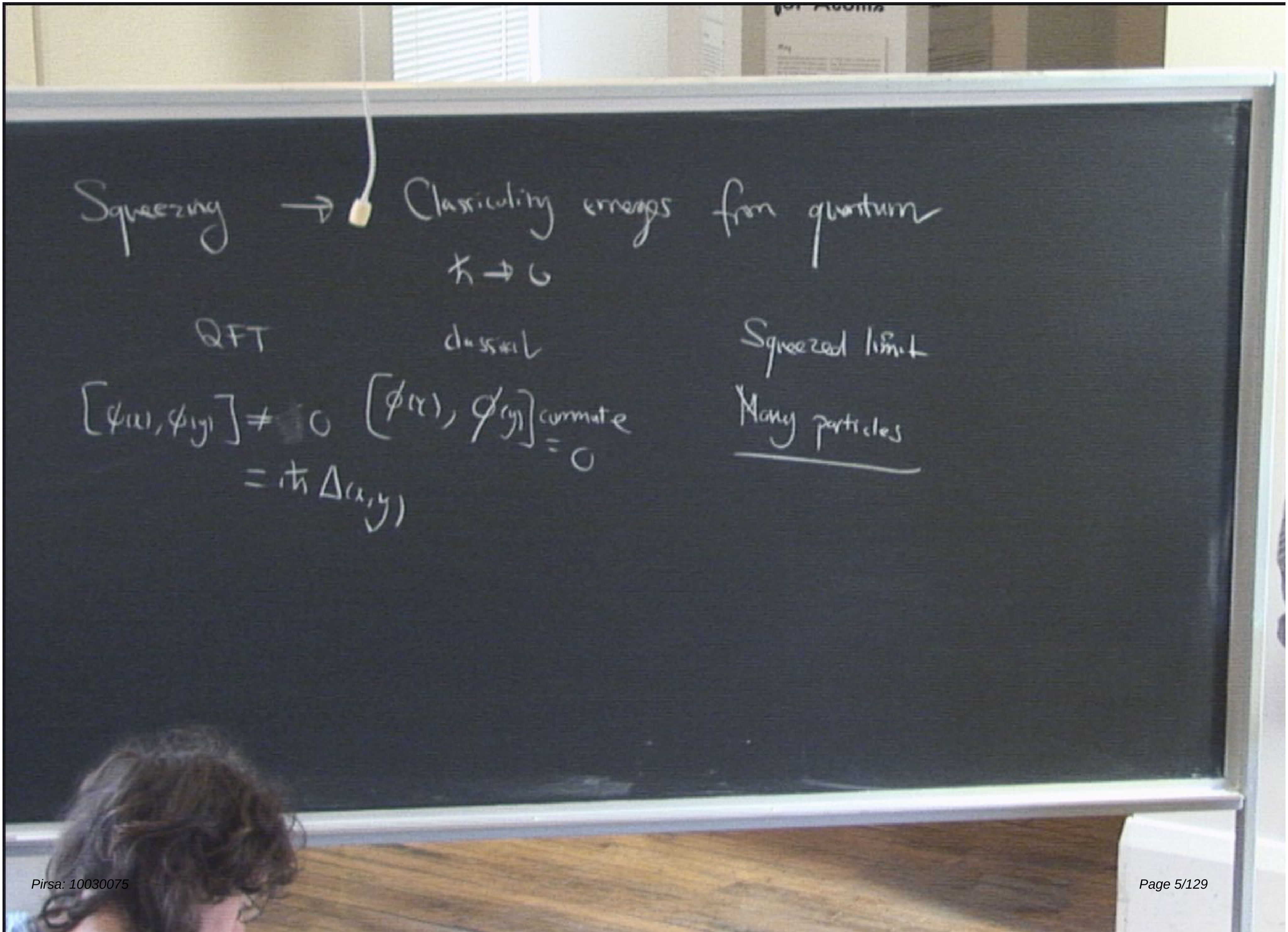
Squeezing → Classicality emerges from quantum
ħ → 0
QFT
classical
Squeezed limit
[φ(x), φ(y)] ≠ 0
= iħ Δ(x,y)
[φ(x), φ(y)] commute
= 0
Many particles

Minkowski:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[a_k u_k + a_k^\dagger u_k^* \right]$$

Minkowski

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^{\dagger} u_k^{*} \right]$$

$$u_k = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$$

Minkowski

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$a_k|0\rangle = 0$

$$u_k = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$$

number density $\quad |\beta_k|^2$

Minkowski

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$a_k|0\rangle = 0$

$$u_k = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$$

number density $|\beta_k|^2$

$$R(x,y) = \frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle}$$

Minkowski

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Minkowski:

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order parameter

$$R(x,y) = \frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \propto \hbar$$

$R \sim 1$ quantum regime

$R \sim 0$ classical regime

Minkowski

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^\dagger u_k^* \right]$$

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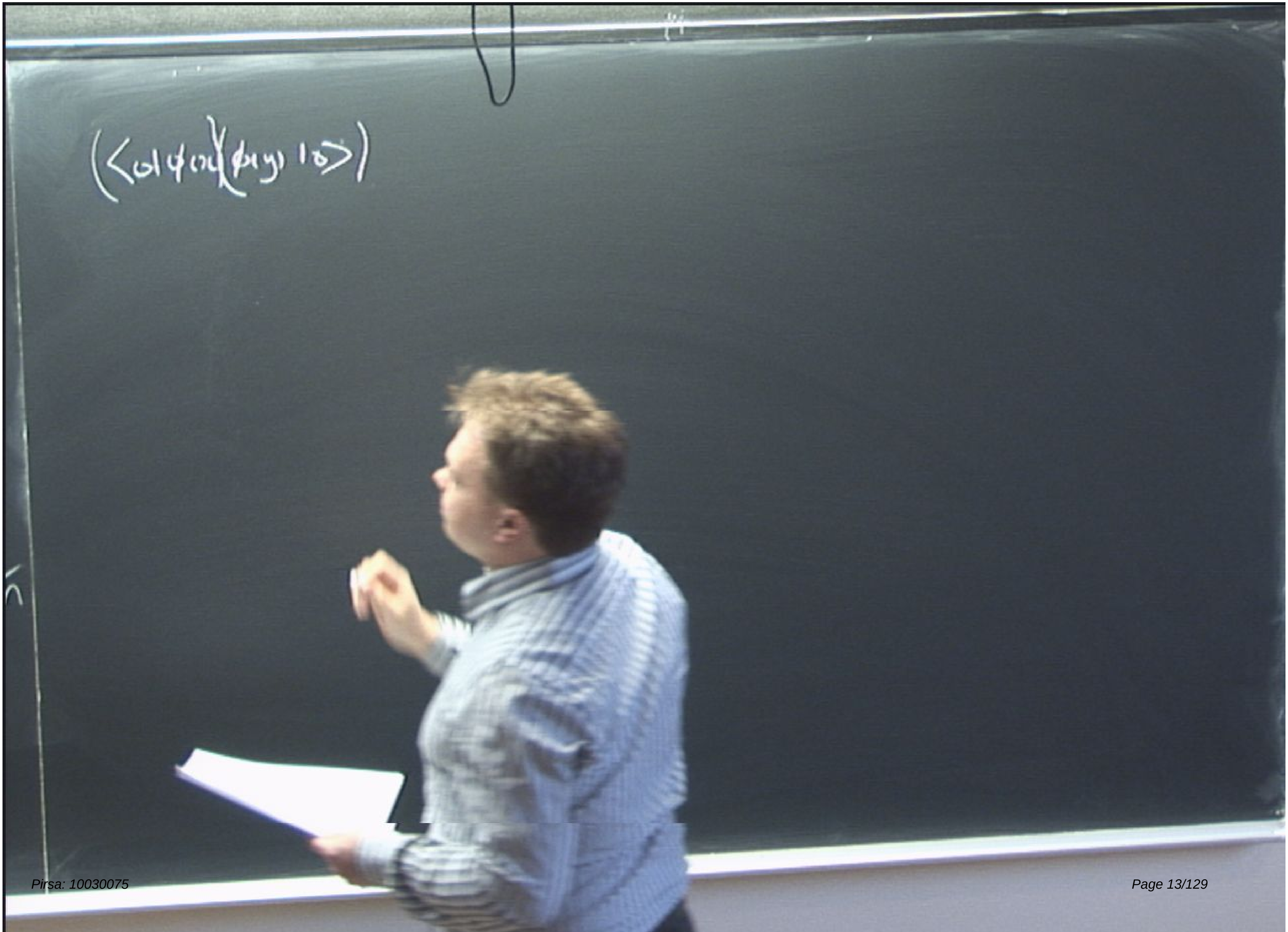

(<0| φ(x) φ(y) |0>)

$$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int\frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\; e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\; \langle 0|\, a_{k_1}\, a^{\dagger}_{k_2}\, |0\rangle$$

$$\left(\langle 0| \phi(x) \phi(y) |0\rangle \right) = \int \frac{d^3k_1}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{k_1}}} \int \frac{d^3k_2}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{k_2}}} \left(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0) \langle 0| a_{k_1} a^\dagger_{k_2} |0\rangle \times e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}(y_0) \right)$$

Minkowski $\quad \phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^\dagger u_k^* \right]$

$a_k|0\rangle = 0 \qquad u_k(t) = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$

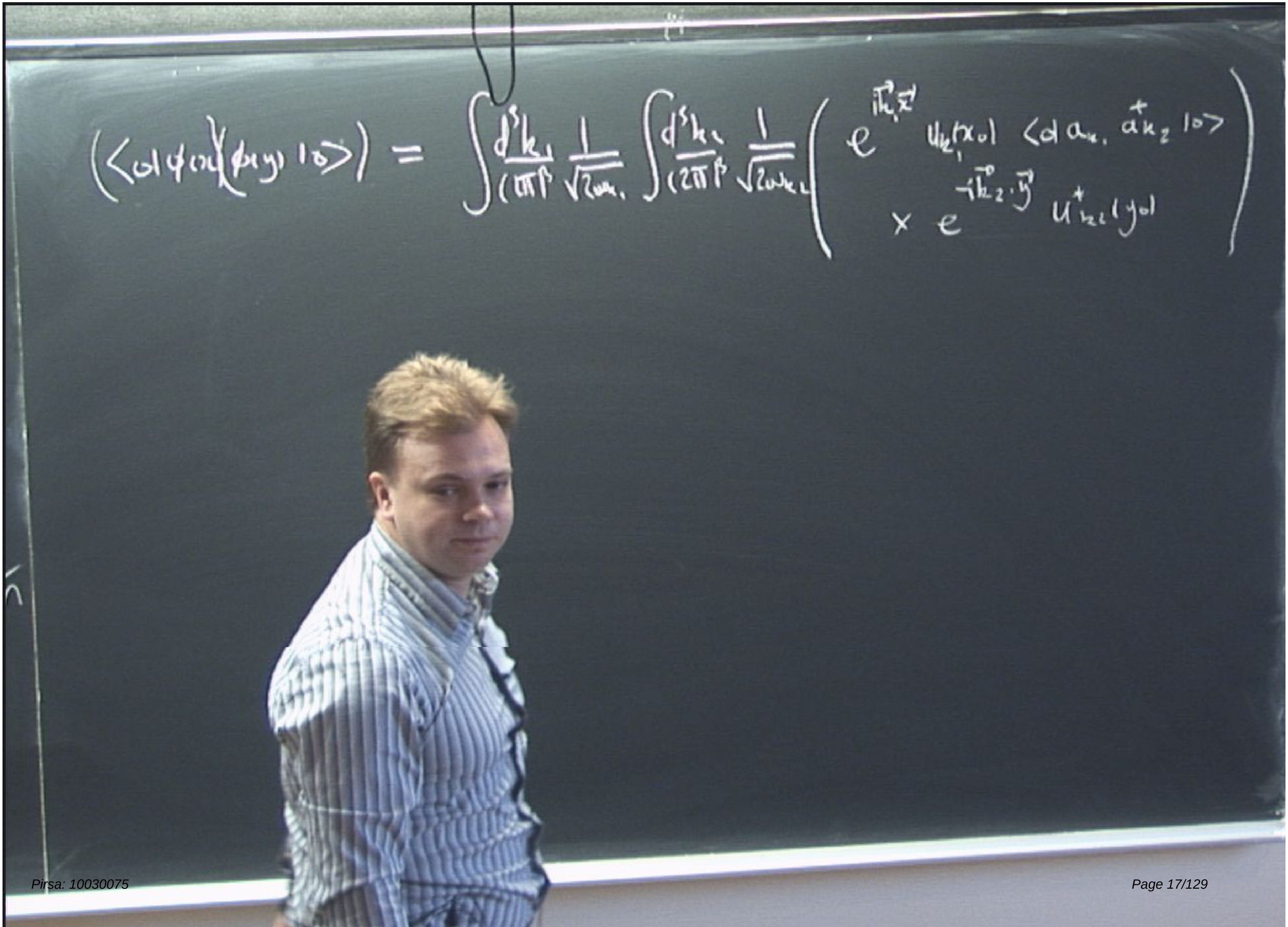
number density $\quad |\beta_k|^2$

order parameter

$$R(x,y) = \frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \quad \propto \hbar$$

$R \sim 1$ quantum regime

$R \sim 0$ classical regime


$(\langle 0|\phi(x)\phi(y)|0\rangle) = \int\frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\Big(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\ \langle 0|\, a_{k_1}\, a^\dagger_{k_2}\,|0\rangle$
$\times\, e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}(y_0) \Big)$

$$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\left(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\, \langle 0|a_{k_1}\, a^\dagger_{k_2}|0\rangle \times e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}(y_0)\right)$$

$x^\mu = (\vec{x}, x^0)$

$$= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$$

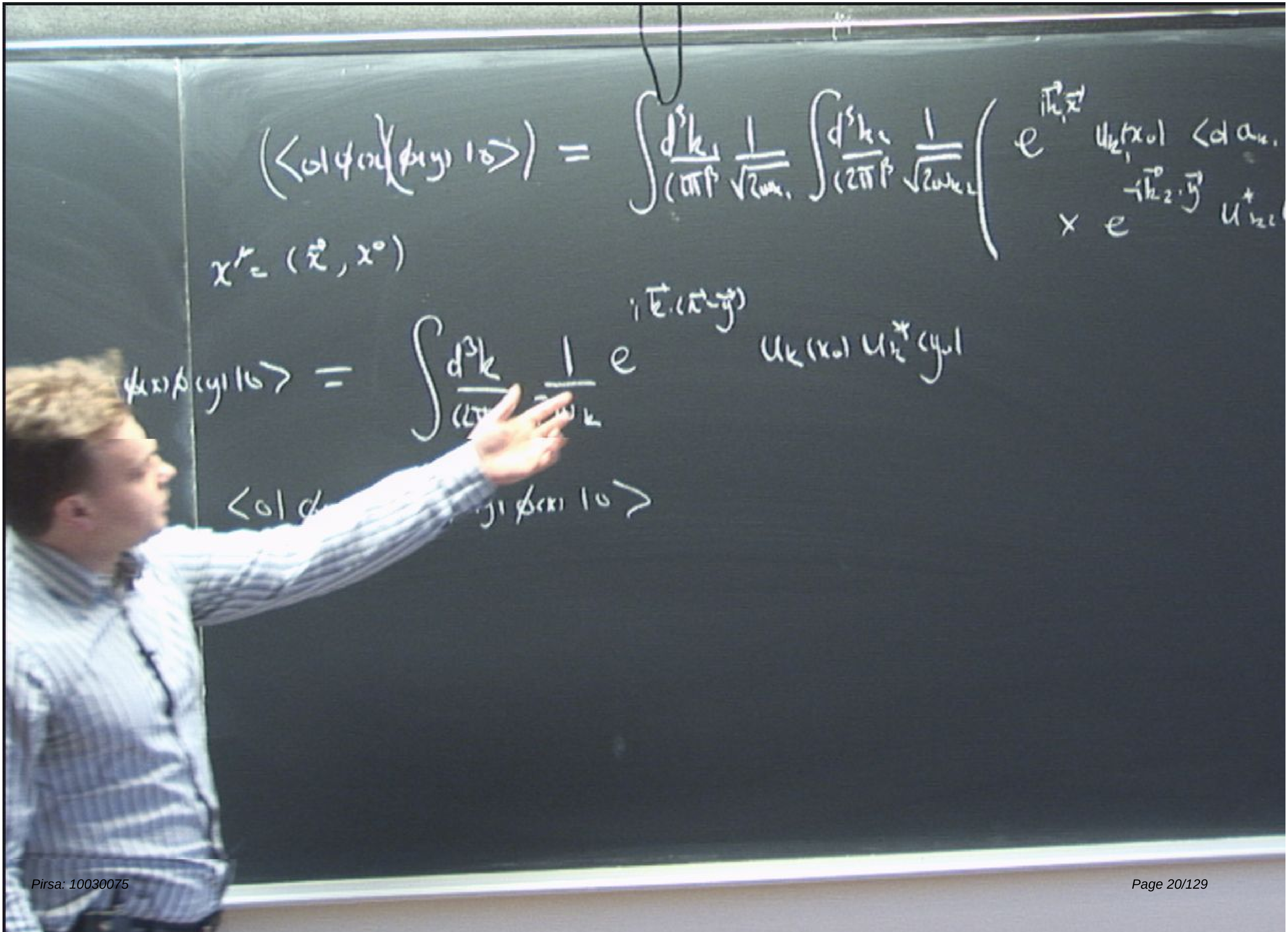
$$\left(\langle 0|\phi(x)\right)\left(\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\Big(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\, \langle 0\, a_{k_1}$$

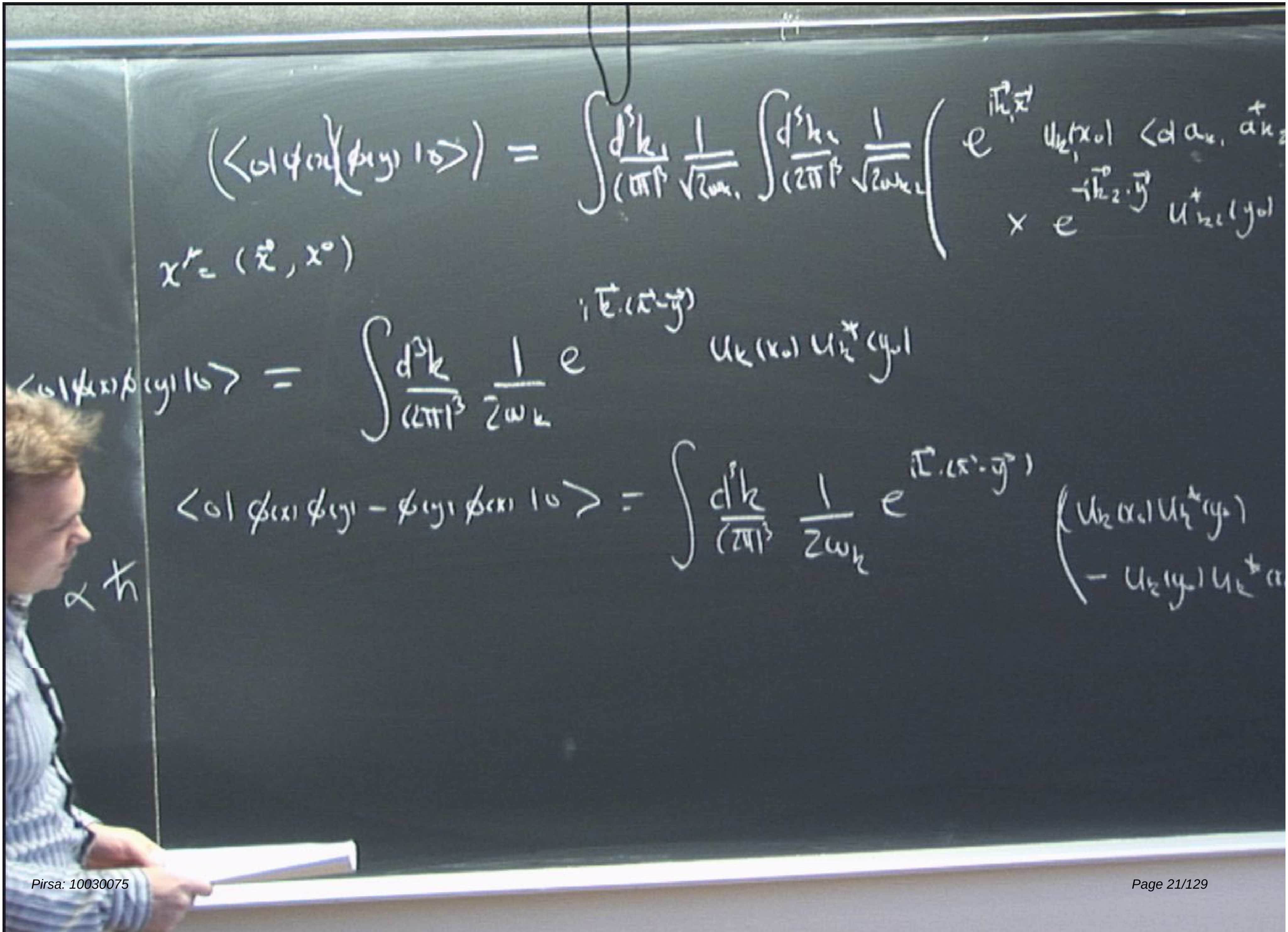
$$\times\, e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}($$

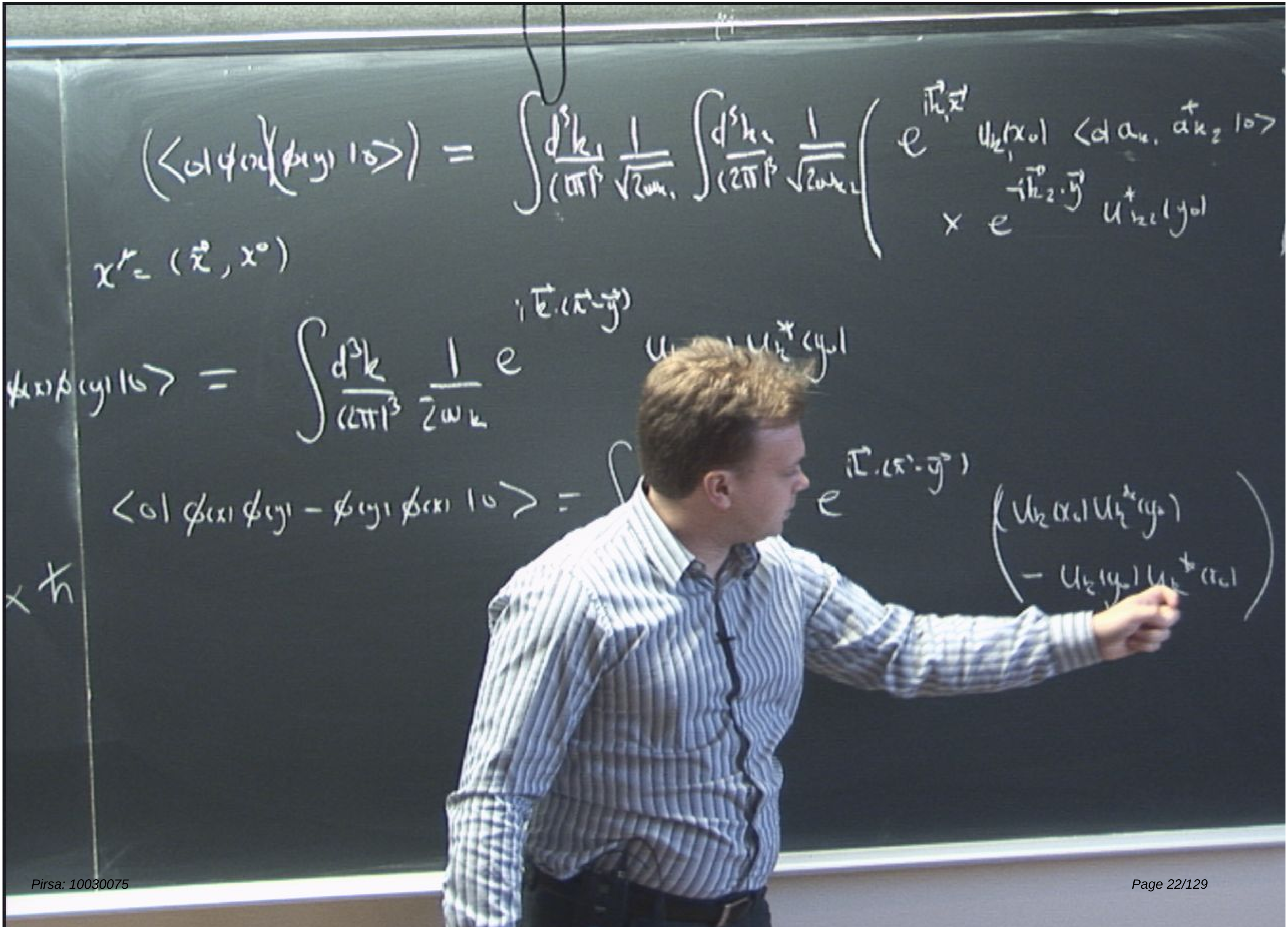
$$x^\mu = (\vec{x}, x^0)$$

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$$

$\propto \hbar$


$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\Big(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\ \langle 0|a_{k_1}$
$\times\, e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}$
$x^\mu = (\vec{x}, x^0)$
$\phi(x)\phi(y)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\, e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$
$\langle 0|\phi(y)\phi(x)|0\rangle$


$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\Big(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\, \langle 0|a_{k_1} a^\dagger_{k_2}$
$\times\, e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}(y_0)$
$x^\mu = (\vec{x}, x^0)$
$\langle 0|\phi(x)\phi(y)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$
$\langle 0|\phi(x)\phi(y) - \phi(y)\phi(x)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\Big(u_k(x_0)u_k^*(y_0) - u_k(y_0)u_k^*(x_0)$
$\propto \hbar$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$x^\mu = (\vec{x}, x^0)$
$\phi(x)\phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$
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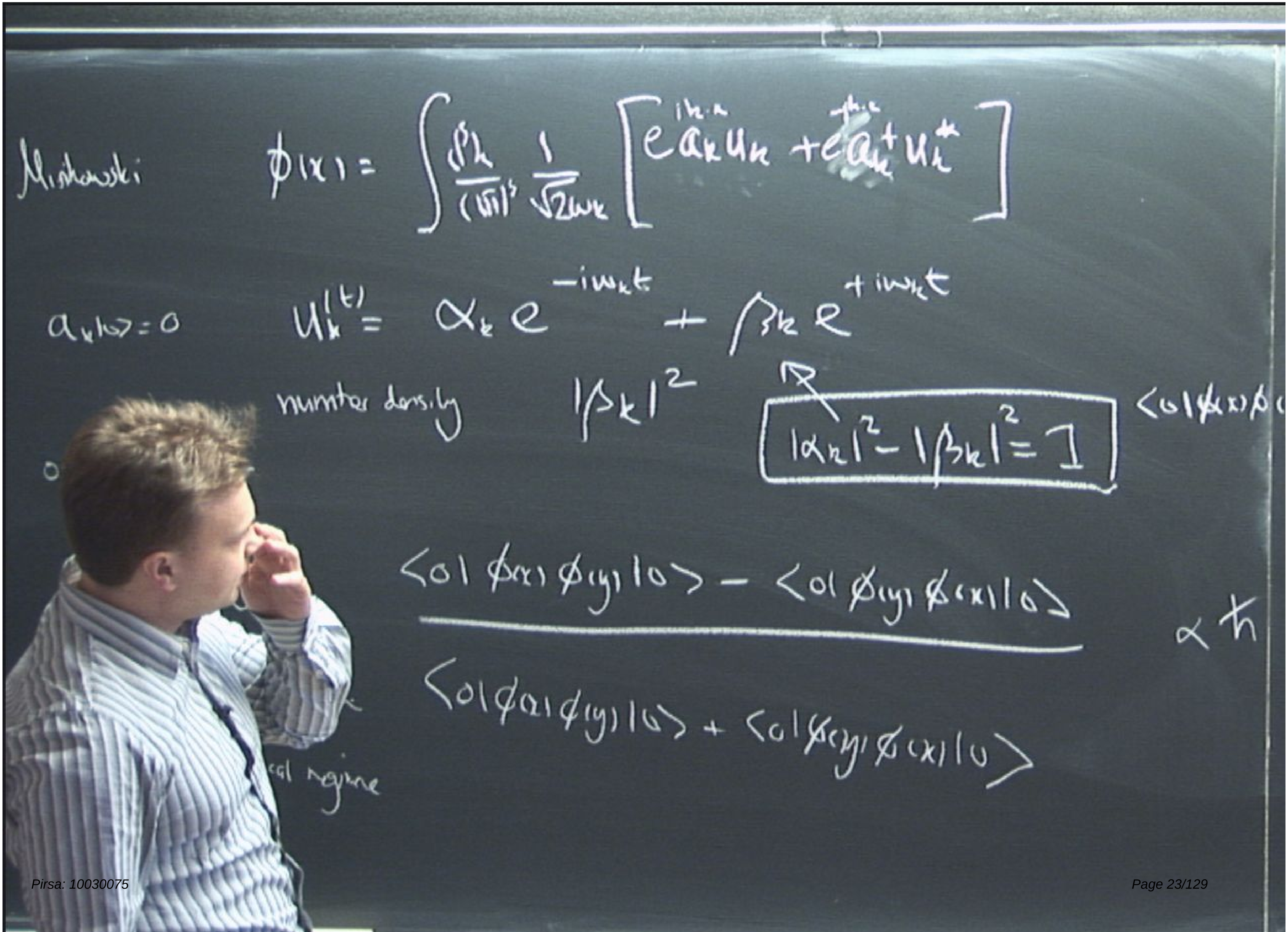
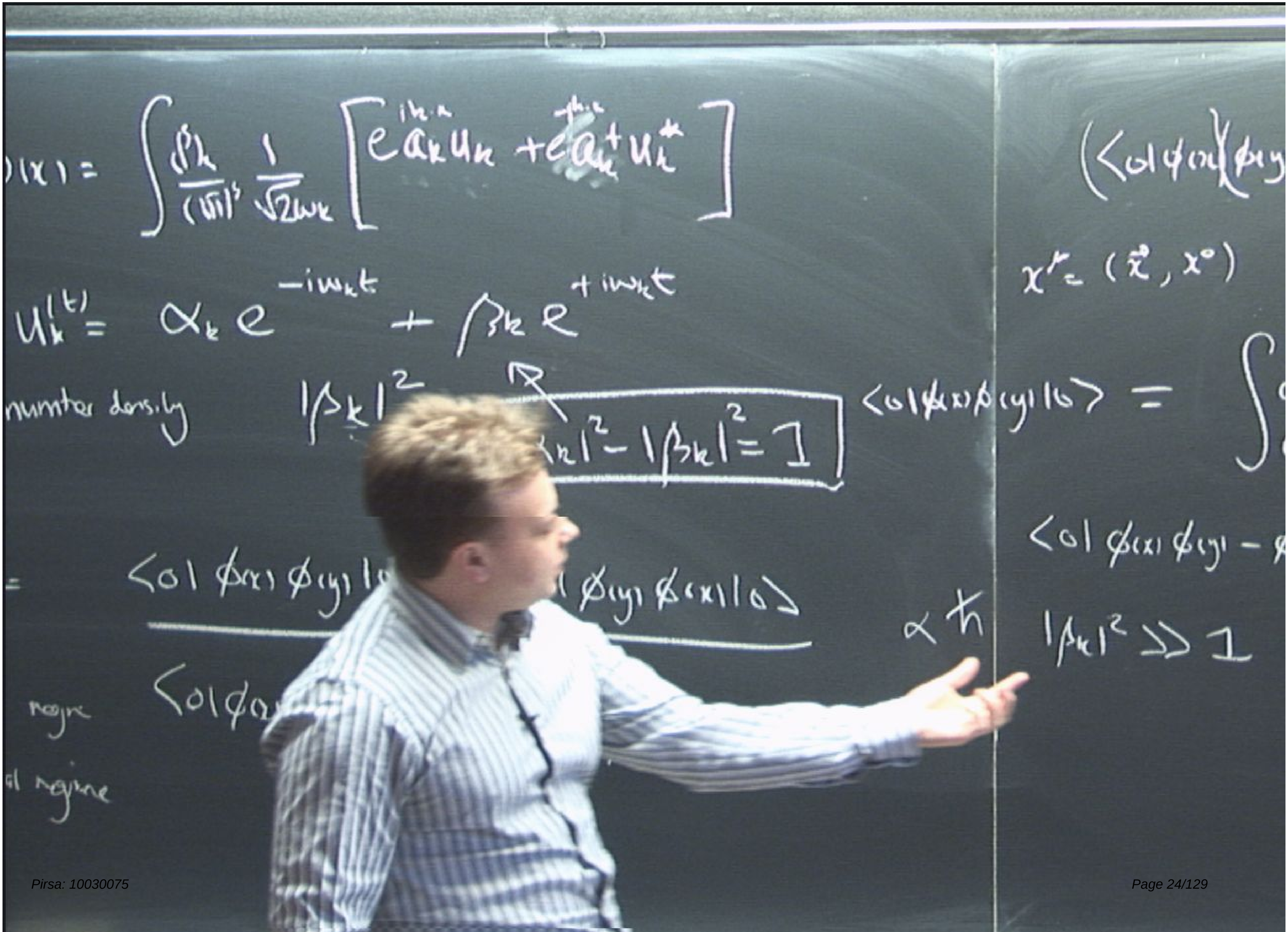
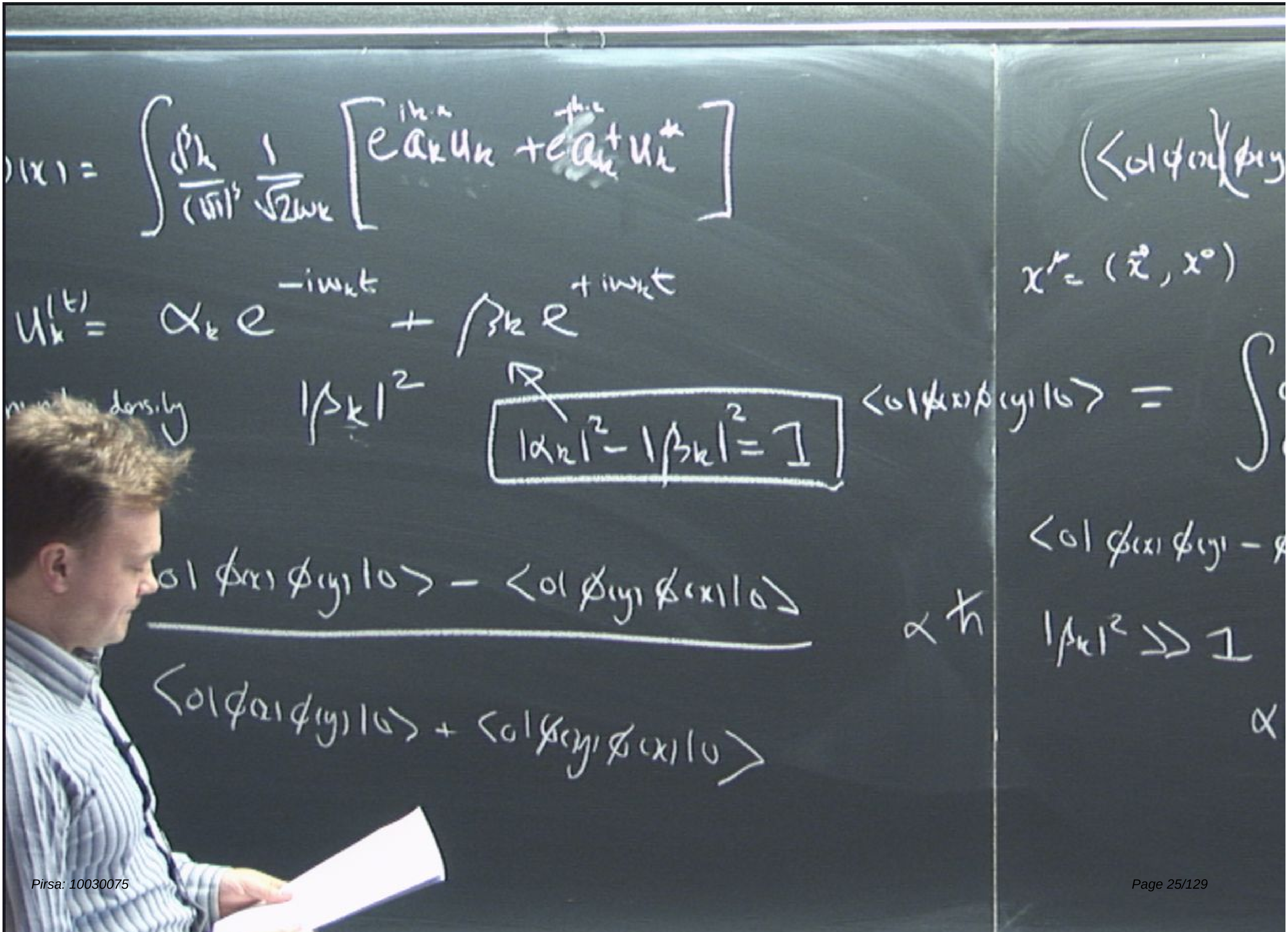
$\times\hbar$


Minkowski:
$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^{+} u_k^{*} \right]$
$a_k|0\rangle = 0$
$u_k(t) = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$
number density $|\beta_k|^2$
$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$
$\langle 0|\phi(x)\phi($
$\frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \propto \hbar$
cal regime



u_k(t) = α_k e^{-iω_k t} + β_k e^{+iω_k t}
number density
|β_k|^2
|α_k|^2 − |β_k|^2 = 1
x^μ = (x⃗, x^0)
⟨0|ϕ(x)ϕ(y)|0⟩ =
∝ ħ
|β_k|^2 >> 1
regime


$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^\dagger u_k^* \right]$
$u_k(t) = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$
density $|\beta_k|^2$
$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$
$\frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \propto \hbar$
$x^\mu = (\vec{x}, x^0)$
$\langle 0|\phi(x)\phi(y)|0\rangle =$
$|\beta_k|^2 \gg 1$

$$\ldots a_k u_k + e^{\ldots} a_k^\dagger u_k^* \Big]$$

$$\big(\langle 0| \phi(x) \phi(y) |0\rangle \big) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \ldots$$

$$x^\mu = (\vec{x}, x^0)$$

$$\ldots - \beta_k e^{+i\omega_k t}$$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$$\langle 0| \phi(x) \phi(y) |0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \quad u_k \ldots$$

$$\langle 0| \phi(x)\phi(y) - \phi(y)\phi(x) |0\rangle = \int \frac{d^3k}{(2\pi)^3} \ldots$$

$$\ldots\rangle - \langle 0| \phi(y) \phi(x) |0\rangle \qquad \propto \hbar$$

$$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto |\beta_k|^2$$

$$\alpha_k = e^{i\delta} \beta_k = \beta_k$$

$$\ldots\rangle + \langle 0| \phi(y) \phi(x) |0\rangle$$

$$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\Big(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\ \langle 0 \ldots \times e^{-i\vec{k}_2\cdot\vec{y}} \ldots$$

$$x^\mu = (\vec{x}, x^0)$$

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$$

$$\langle 0|\phi(x)\phi(y) - \phi(y)\phi(x)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\Big(u_k(x_0)u \ldots - u_k \ldots$$

$\propto \hbar$

$$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto |\beta_k|^2$$

$$\alpha_k = e^{i\delta}\beta_k = \beta_k \quad \text{(squeezed limit)}$$

$$\left(\langle 0 | \phi(x) \phi(y) | 0 \rangle \right) = \int \frac{d^3k_1}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{k_1}}} \int \frac{d^3k_2}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{k_2}}} \Big(e^{i\vec{k}_1 \cdot \vec{x}} u_{k_1}(x_0) \; \langle 0 \ldots$$

$$\times e^{-i\vec{k}_2 \cdot \vec{y}}$$

$$x^\mu = (\vec{x}, x^0)$$

$$\langle 0 | \phi(x) \phi(y) | 0 \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} u_k(x_0) u_k^*(y_0)$$

$$\langle 0 | \phi(x)\phi(y) - \phi(y)\phi(x) | 0 \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \Big(u_k(x_0) u \ldots$$

$$- u_k \ldots$$

$\propto \hbar$

$$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto |\beta_k|^2$$

$$\alpha_k = e^{i\delta} \beta_k = \beta_k \quad \text{(squeezed limit)}$$

$$u_k(x) \approx \beta_k \left(e^{-i\omega t} + e^{+i\omega t} \right)$$

$$\left(\langle 0|\phi(x)\right)\left(\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\Big(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\ \langle 0 \ \ \times e^{-i\vec{k}_2\cdot\vec{y}}$$

$x^\mu = (\vec{x}, x^0)$

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$$

$$\langle 0|\phi(x)\phi(y) - \phi(y)\phi(x)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\Big(u_k(x_0)\, u \ \ - u_k$$

$\propto \hbar$

$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto |\beta_k|^2$

$\alpha_k = e^{i\delta}\beta_k = \beta_k$ (squeezed limit)

$u_k(x) \approx \beta_k\left(e^{-i\omega t} + e^{+i\omega t}\right) = 2\beta_k\cos(\omega t)$

$$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\Big(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\ \langle 0 \ \ldots$$

$$\times\, e^{-i\vec{k}_2\cdot\vec{y}} \ \ldots$$

$x^\mu = (\vec{x}, x^0)$

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$$

$$\langle 0|\phi(x)\phi(y) - \phi(y)\phi(x)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \Big(u_k(x_0)\, u \ldots$$
$$- u_k \ldots$$

$\propto \hbar$

$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto |\beta_k|^2$

$$\alpha_k = e^{i\delta}\beta_k = \beta_k \quad \text{(squeezed limit)}$$

$$u_k(x) \approx \beta_k\left(e^{-i\omega t} + e^{+i\omega t}\right) = 2\beta_k\cos(\omega t) \qquad u^*_k(t) = 2\beta^*_k \ldots$$

$$\left(\langle 0| \phi(x) \phi(y) |0\rangle \right) = \int \frac{d^3k_1}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{k_1}}} \int \frac{d^3k_2}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{k_2}}} \left(e^{i\vec{k}_1 \cdot \vec{x}} u_{k_1}(x_0) \langle 0| a_{k_1} a^{\dagger}_{k_2} |0\rangle \times e^{-i\vec{k}_2 \cdot \vec{y}} u^*_{k_2}(y_0) \right)$$

$$x^\mu = (\vec{x}, x^0)$$

$$\phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} u_k(x_0) u_k^*(y_0)$$

$$\langle 0| \phi(x)\phi(y) - \phi(y)\phi(x) |0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \left(u_k(x_0) u_k^*(y_0) - u_k(y_0) u_k^*(x_0) \right)$$

$$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto |\beta_k|^2$$

$$\alpha_k = e^{i\delta} \beta_k = \beta_k \quad \text{(squeezed limit)}$$

$$u_k(x) \approx \beta_k \left(e^{-i\omega t} + e^{+i\omega t} \right) = 2\beta_k \cos(\omega t) \qquad u^*_k(x) = 2\beta^*_k \cos(\omega t)$$

$$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\left(e^{i\vec{k}_1\cdot\vec{x}} u_{k_1}(x_0)\, \langle 0|a_{k_1} a^\dagger_{k_2}|0\rangle \times e^{-i\vec{k}_2\cdot\vec{y}} u^*_{k_2}(y_0)\right)$$

$x^\mu = (\vec{x}, x^0)$

$$\phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} u_k(x_0)\, u_k^*(y_0)$$

$$4|\beta_k|^2 \cos(\omega_k x_0)\cos(\omega_k y_0)$$
$$-\,4|\beta_k|^2 \cos(\omega_k y_0)\cos(\omega_k x_0)$$

$$\langle 0|\phi(x)\phi(y) - \phi(y)\phi(x)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\left(u_k(x_0)u_k^*(y_0) - u_k(y_0)u_k^*(x_0)\right)$$

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$$\left(\langle 0|\phi(x)\phi(y)|0\rangle \right) = \int \frac{d^3k_1}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{k_1}}} \int \frac{d^3k_2}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{k_2}}} \left(e^{i\vec{k}_1\cdot\vec{x}} u_{k_1}(x_0) \langle 0| a_{k_1} a^\dagger_{k_2} |0\rangle \times e^{-i\vec{k}_2\cdot\vec{y}} u^*_{k_2}(y_0) \right)$$

$x^\mu = (\vec{x}, x^0)$

$$\phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} u_k(x_0) u_k^*(y_0)$$

$4|\beta_k|^2 \cos(\omega_k x_0)\cos(\omega_k y_0)$
$- 4|\beta_k|^2 \cos(\omega_k y_0)\cos(\omega_k x_0)$
$= 0$

$$\langle 0| \phi(x)\phi(y) - \phi(y)\phi(x) |0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \begin{pmatrix} u_k(x_0)u_k^*(y_0) \\ - u_k(y_0) u_k^*(x_0) \end{pmatrix}$$

$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto |\beta_k|^2$

$\alpha_k = e^{i\delta}\beta_k = \beta_k$ (squeezed limit)

$u_k(x) \approx \beta_k (e^{-i\omega t} + e^{+i\omega t}) = 2\beta_k \cos(\omega t) \qquad u^*_k(x) = 2\beta^*_k \cos(\omega t)$

Minkowski

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^\dagger u_k^* \right]$$

$a_k|0\rangle = 0$

$$u_k(t) = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$$

number density $|\beta_k|^2$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$\langle 0|\phi(x)\phi($

parameter

$$R(x,y) = \frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \quad \propto \hbar$$

1 quantum regime

classical regime

$R(x,y) \to 0 \quad |\beta_k| \gg 1$

$$\boxed{u \propto u^*}$$

Minkowski: $$\phi(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^+ u_k^*\right]$$

$a_k|0\rangle = 0$ $$u_k^{(t)} = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$$

number density $|\beta_k|^2$ $$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$ $\langle 0|\phi(x)\phi($

order parameter

$$R(x,y) = \frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \quad \propto \hbar$$

$R \sim 1$ quantum regime

$R \sim 0$ classical regime

$R(x,y) \to 0 \quad |\beta_k| \gg 1$ $$\boxed{u \propto u^*}$$

$$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_1}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\left(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\, \langle 0|a_{k_1} a^\dagger_{k_2}|0\rangle \times e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}(y_0)\right)$$

$x^\mu = (\vec{x}, x^0)$

$$\phi(y)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$$

$$4|\beta_k|^2\cos(\omega_k x_0)\cos(\omega_k y_0) - 4|\beta_k|^2\cos(\omega_k y_0)\cos(\omega_k x_0) = 0$$

$$\langle 0|\phi(x)\phi(y) - \phi(y)\phi(x)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\left(u_k(x_0)u_k^*(y_0) - u_k(y_0)u_k^*(x_0)\right)$$

$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto |\beta_k|^2$

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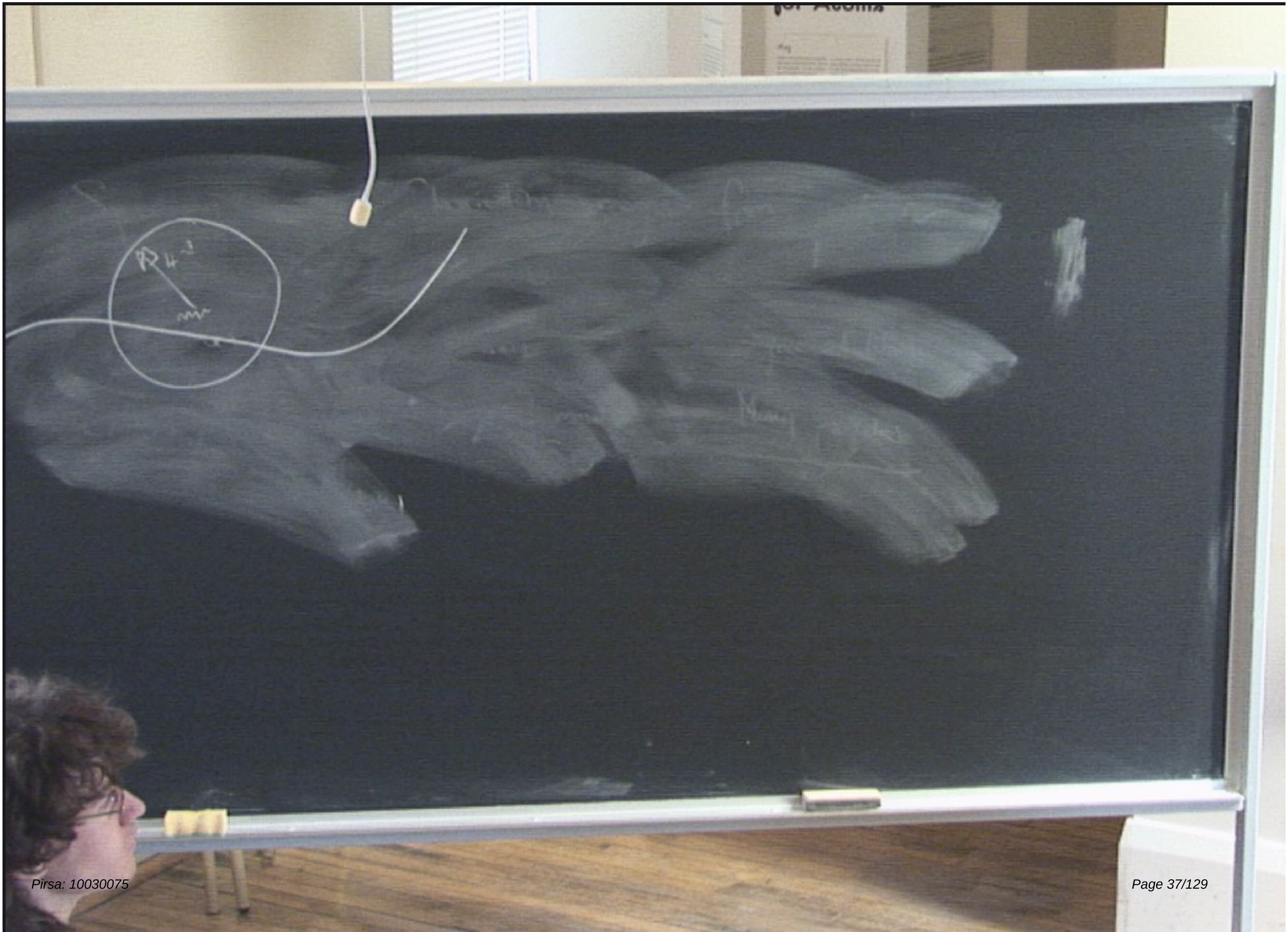
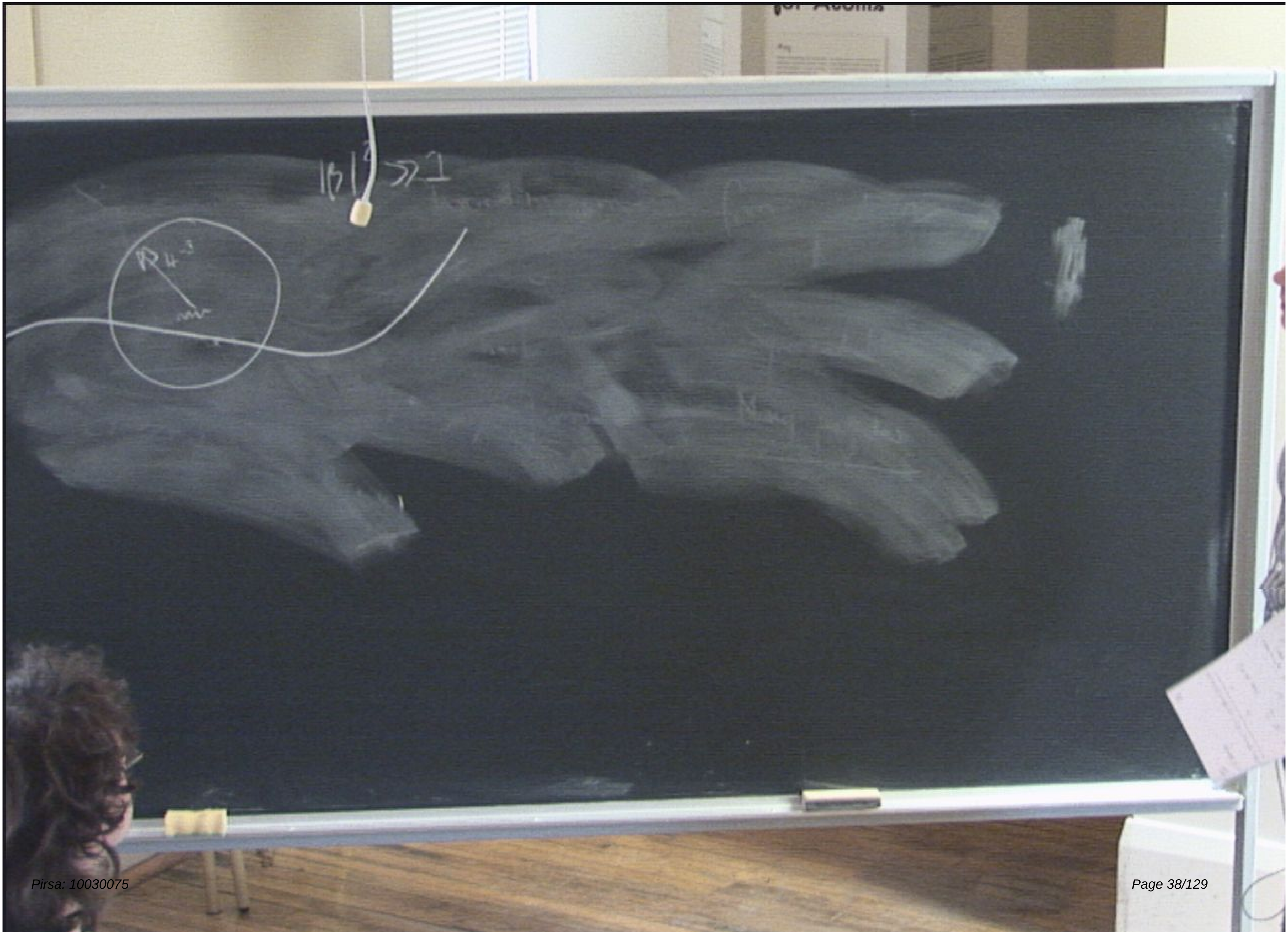
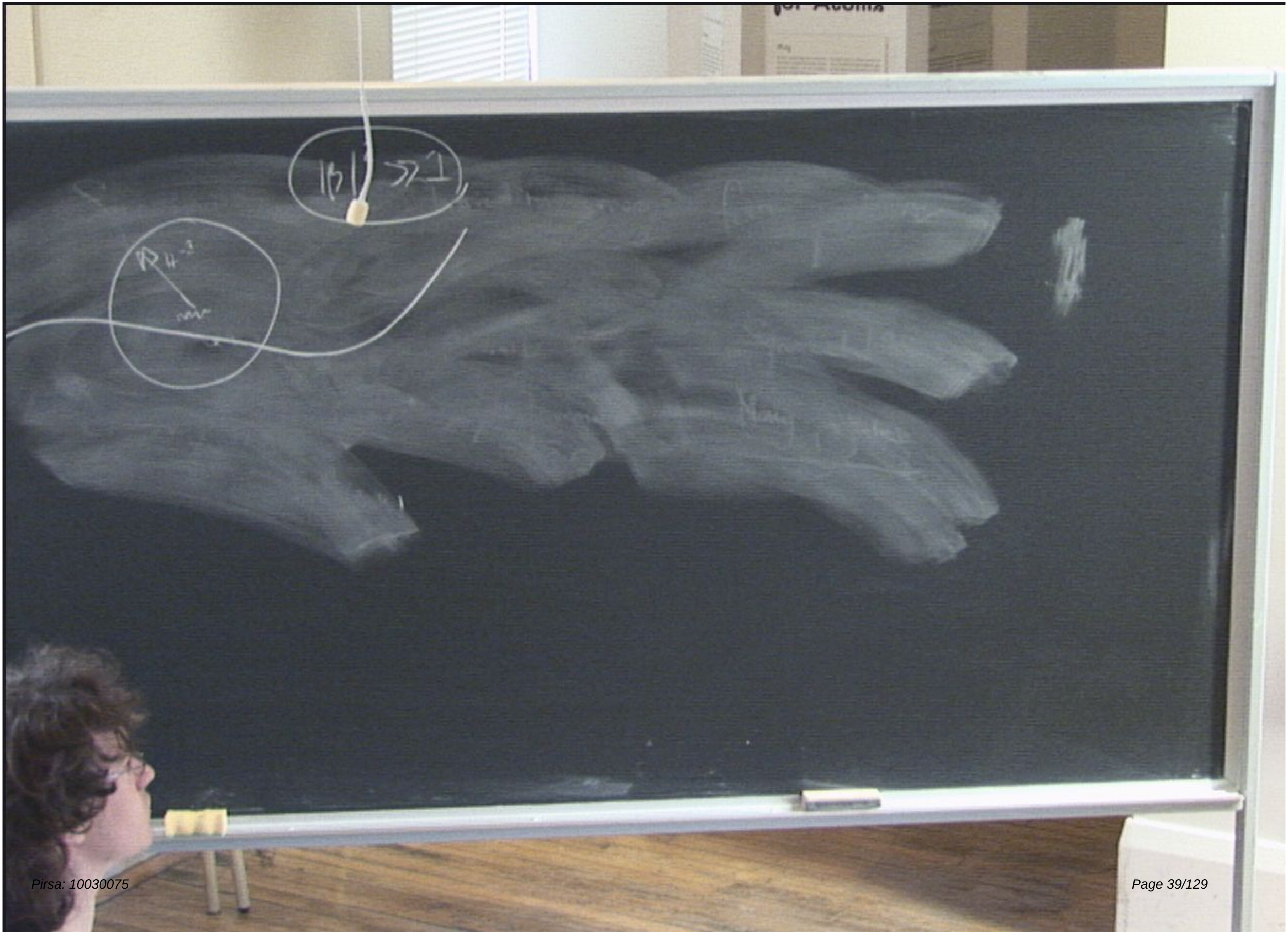






Minkowski:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^\dagger u_k^* \right]$$

$a_k|0\rangle = 0$

$$u_k^{(t)} = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$$

number density $|\beta_k|^2$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$\langle 0|\phi(x)\phi($

order parameter

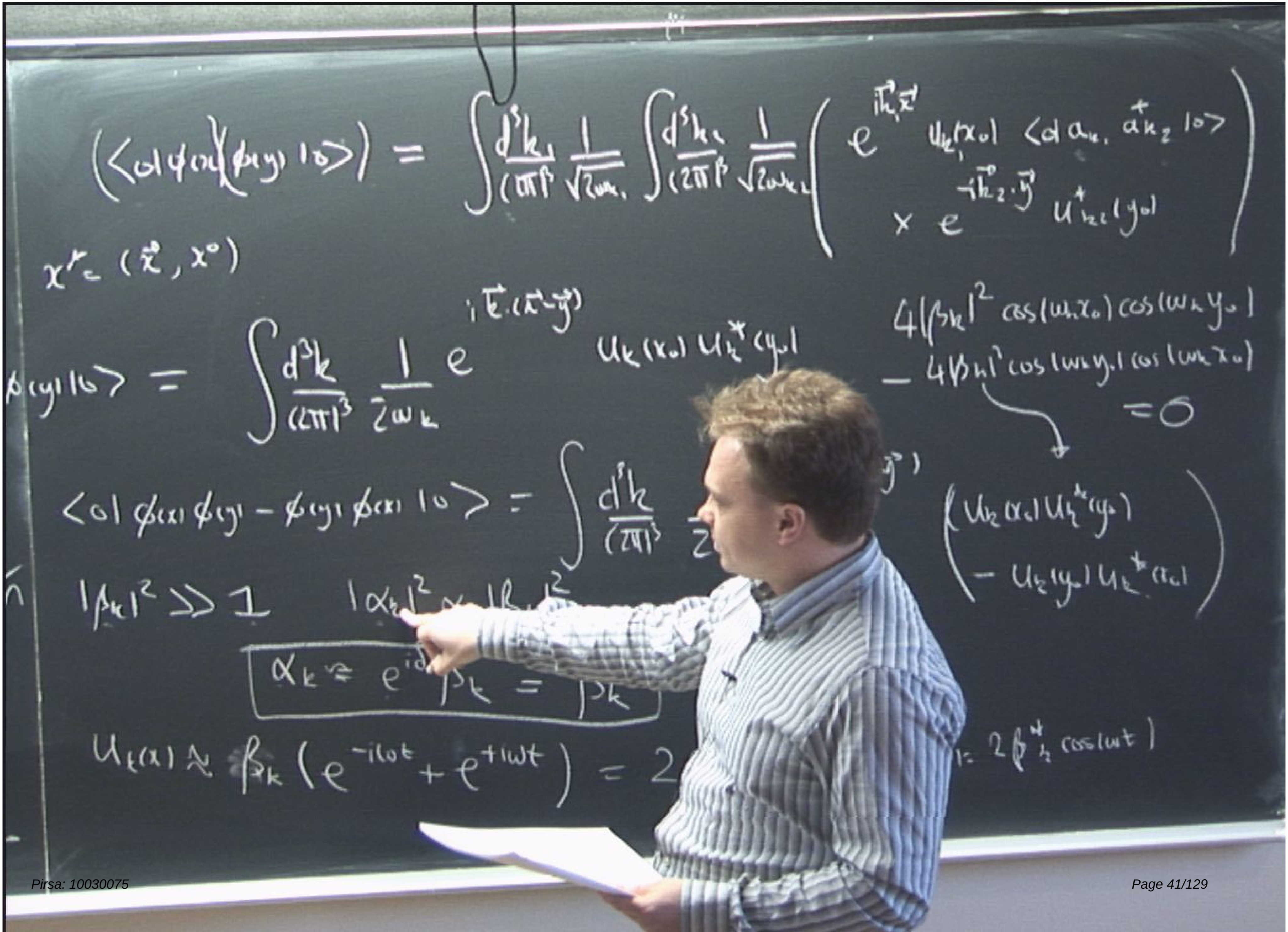
$$R(x,y) = \frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \quad \propto \hbar$$

$R \sim 1$ quantum regime

$R \sim 0$ classical regime

$R(x,y) \to 0 \quad |\beta_k| \gg 1$

$$\boxed{u \propto u^*}$$



$(\langle 0|\phi(x)\phi(y)|0\rangle) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\left(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\,\langle 0|a_{k_1} a^\dagger_{k_2}|0\rangle \times e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}(y_0)\right)$
$x^\mu = (\vec{x}, x^0)$
$\phi(y)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\, u_k(x_0)\, u_k^*(y_0)$
$4|\beta_k|^2\cos(\omega_k x_0)\cos(\omega_k y_0)$
$-4|\beta_k|^2\cos(\omega_k y_0)\cos(\omega_k x_0)$
$=0$
$\langle 0|\phi(x)\phi(y)-\phi(y)\phi(x)|0\rangle = \int\frac{d^3k}{(2\pi)^3}$
$\left(u_k(x_0)u_k^*(y_0) - u_k(y_0)u_k^*(x_0)\right)$
$|\beta_k|^2 \gg 1$
$|\alpha_k|^2 \approx |\beta_k|^2$
$\alpha_k \approx e^{i\delta}\beta_k = |\beta_k|$
$u_k(x) \approx \beta_k(e^{-i\omega t}+e^{+i\omega t}) = 2$
$= 2\beta_k^*\cos(\omega t)$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^\dagger u_k^* \right]$$

$$= \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$$

er density $\quad |\beta_k|^2$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$$\frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \propto \hbar$$

ine $\quad R(x,y) \to 0 \quad |\beta_k| \gg 1$

$$\boxed{u \propto u^*}$$

$$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) =$$

$$x^\mu = (\vec{x}, x^0)$$

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e$$

$$\langle 0|\phi(x)\phi(y) - \phi(y)\phi(x)|0\rangle$$

$$|\beta_k|^2 \gg 1 \qquad |\alpha_k|^2 \propto$$

$$\boxed{\alpha_k \approx e^{i\delta}\beta_k}$$

$$u_k(x) \approx \beta_k \left(e^{-i\omega t} + e\right.$$

Minkowski $\quad \phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^\dagger u_k^* \right]$

$a_k|0\rangle = 0 \qquad u_k^{(t)} = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$

number density $\quad |\beta_k|^2 \qquad \boxed{|\alpha_k|^2 - |\beta_k|^2 = \hbar} \qquad \langle 0|\phi(x)\phi($

order parameter

$$R(x,y) = \frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle} \qquad \propto \hbar$$

$R \sim 1$ quantum regime

$R \sim 0$ classical regime $\qquad R(x,y) \to 0 \quad |\beta_k| \gg 1 \qquad \boxed{u \propto u^*}$

$$\left(\langle 0|\phi(x)\phi(y)|0\rangle\right) = \int \frac{d^3k_1}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_1}}}\int\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k_2}}}\left(e^{i\vec{k}_1\cdot\vec{x}}\, u_{k_1}(x_0)\,\langle 0|a_{k_1}\, a^\dagger_{k_2}|0\rangle \times e^{-i\vec{k}_2\cdot\vec{y}}\, u^*_{k_2}(y_0)\right)$$

$x^\mu = (\vec{x}, x^0)$

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$$u_k^{(t)} = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$$

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$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$\langle 0|\phi(x)\phi($

order parameter

$$R(x,y) = \frac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle}$$

$\propto \hbar$

$\sim e^{-Ht}$

$R \sim 1$ quantum regime

$R \sim 0$ classical regime

$R(x,y) \to 0 \quad |\beta_k| \gg 1$

$$\boxed{u \propto u^*}$$

Minkowski: $\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{ik\cdot x} a_k u_k + e^{-ik\cdot x} a_k^\dagger u_k^* \right]$

$a_k|0\rangle = 0$ $\quad u_k(t) = \alpha_k e^{-i\omega_k t} + \beta_k e^{+i\omega_k t}$

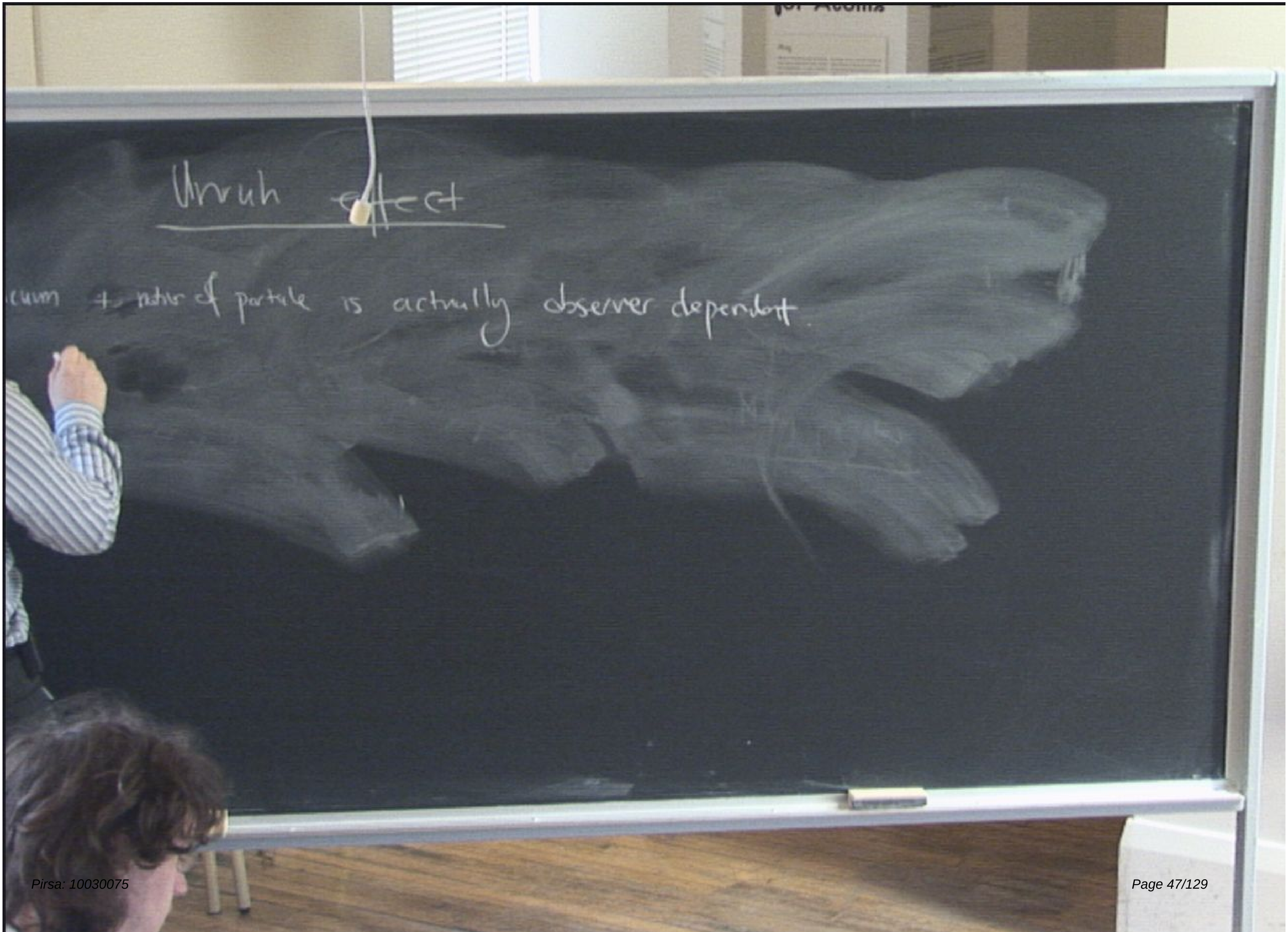
number density $|\beta_k|^2$ $\quad \boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$ $\quad \langle 0|\phi(x)\phi($

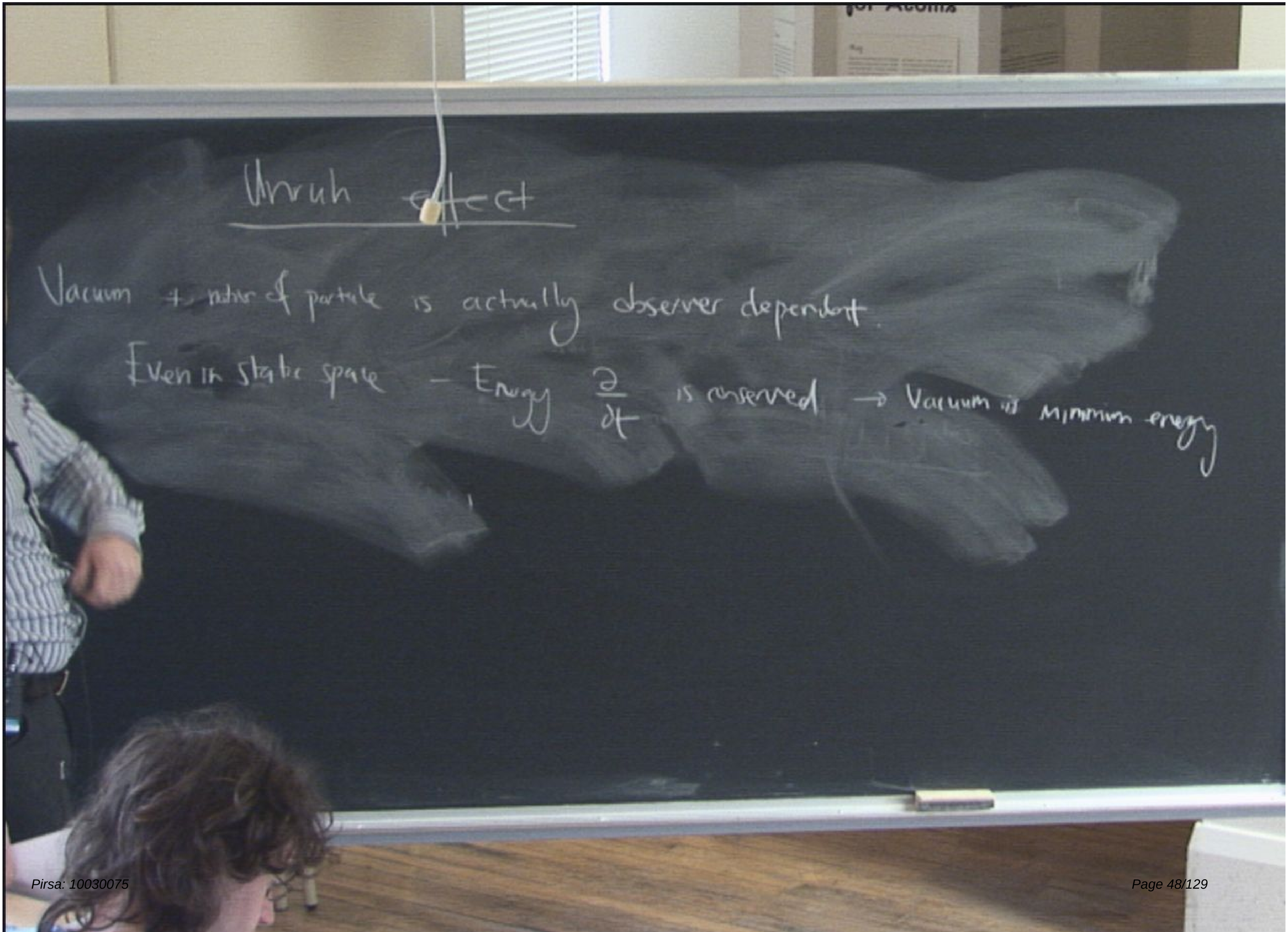
order parameter

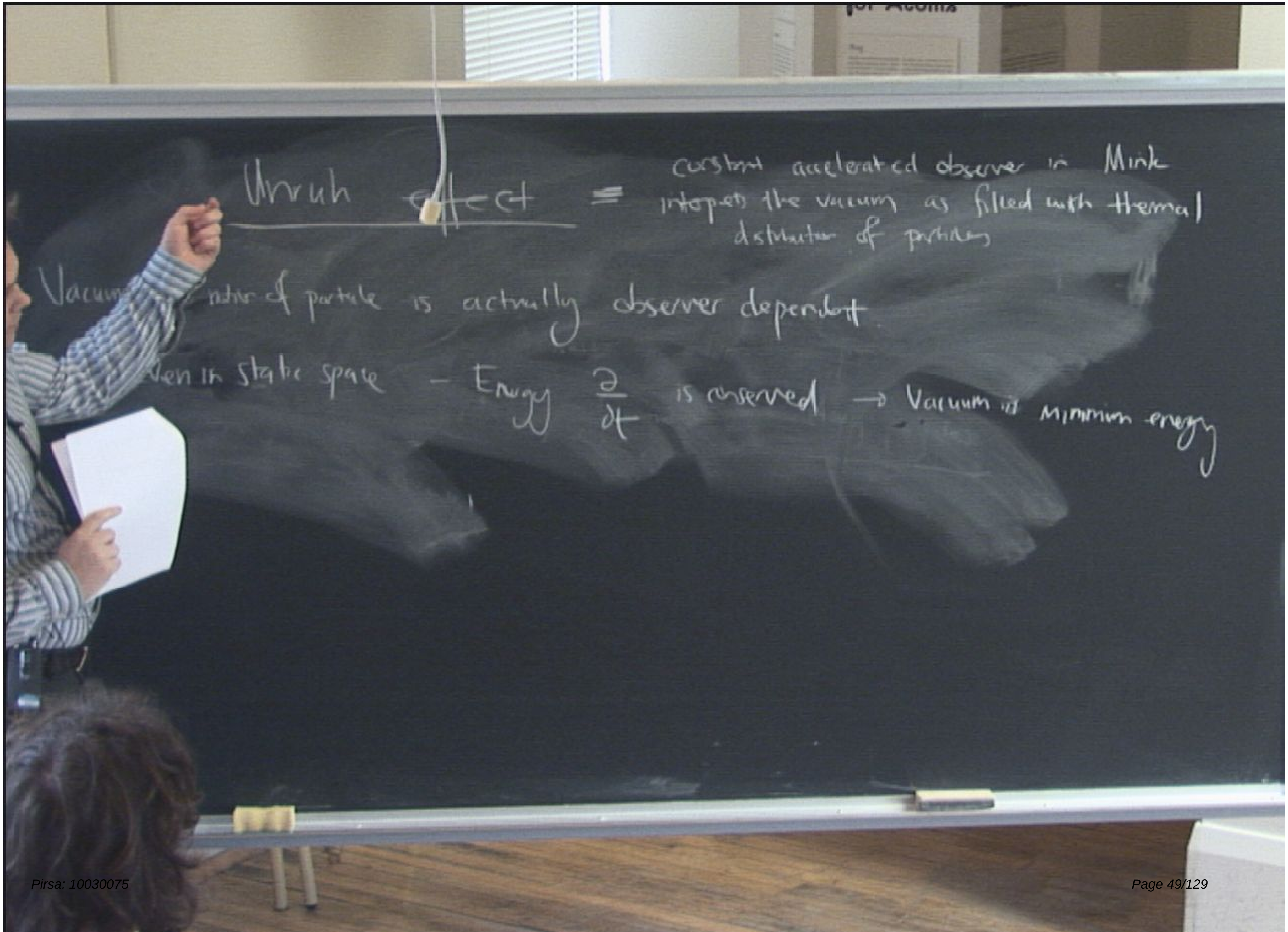
$R(x,y) = \dfrac{\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle}{\langle 0|\phi(x)\phi(y)|0\rangle + \langle 0|\phi(y)\phi(x)|0\rangle}$ $\quad \propto \hbar$ $\quad \sim e^{-Ht}$

1 quantum regime

$\to 0$ classical regime $\quad R(x,y) \to 0 \quad |\beta_k| \gg 1$ $\quad \boxed{u \propto u^*}$


Unruh effect
cuum + notion of particle is actually observer dependent.


Unruh effect
Vacuum + notion of particle is actually observer dependent.
Even in static space - Energy $\frac{\partial}{\partial t}$ is conserved → Vacuum is minimum energy


Unruh effect ≡ constant accelerated observer in Mink
interprets the vacuum as filled with thermal
distribution of particles
Vacuum notion of particle is actually observer dependent
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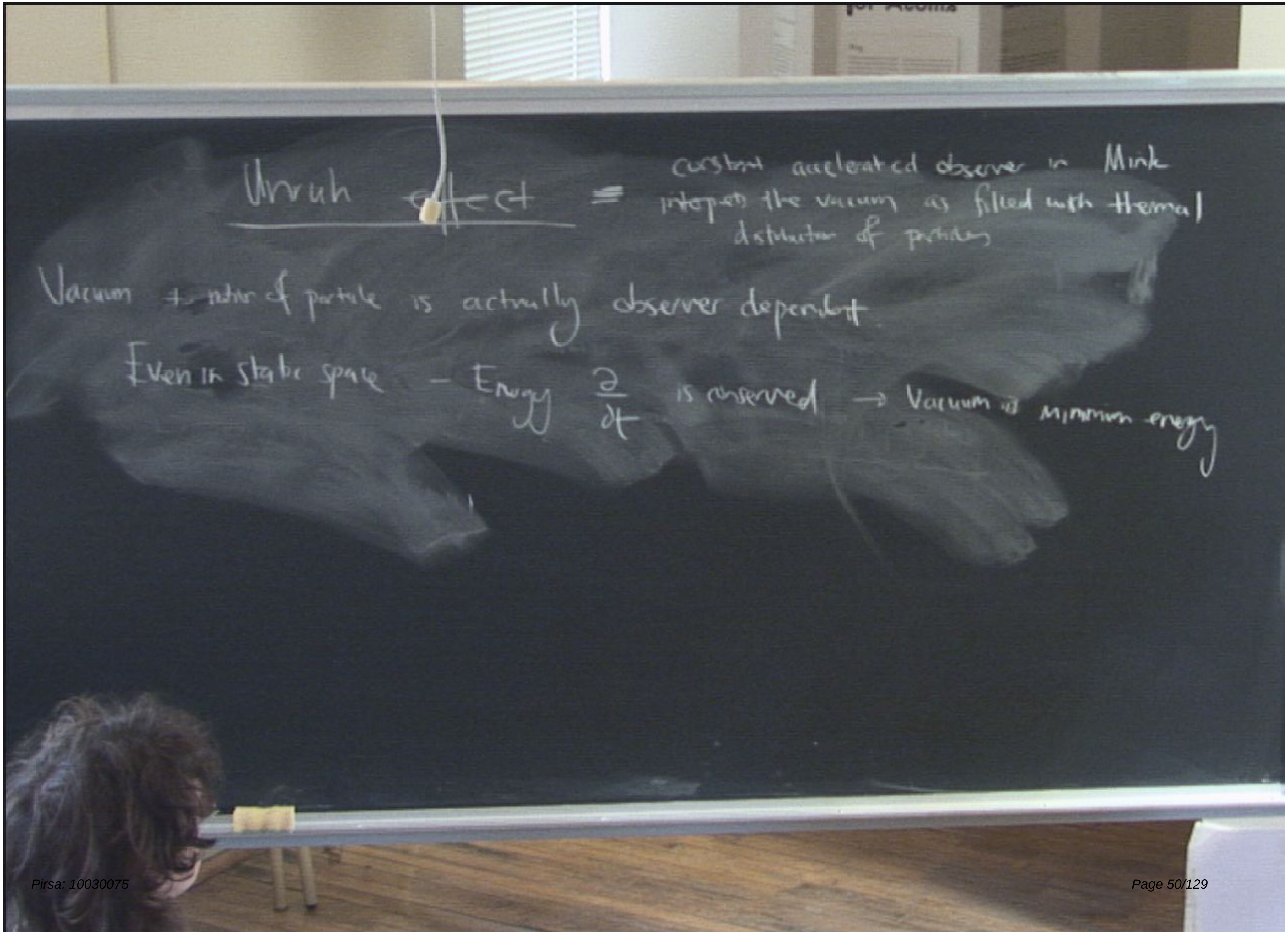
Unruh effect ≡ constant accelerated observer in Mink
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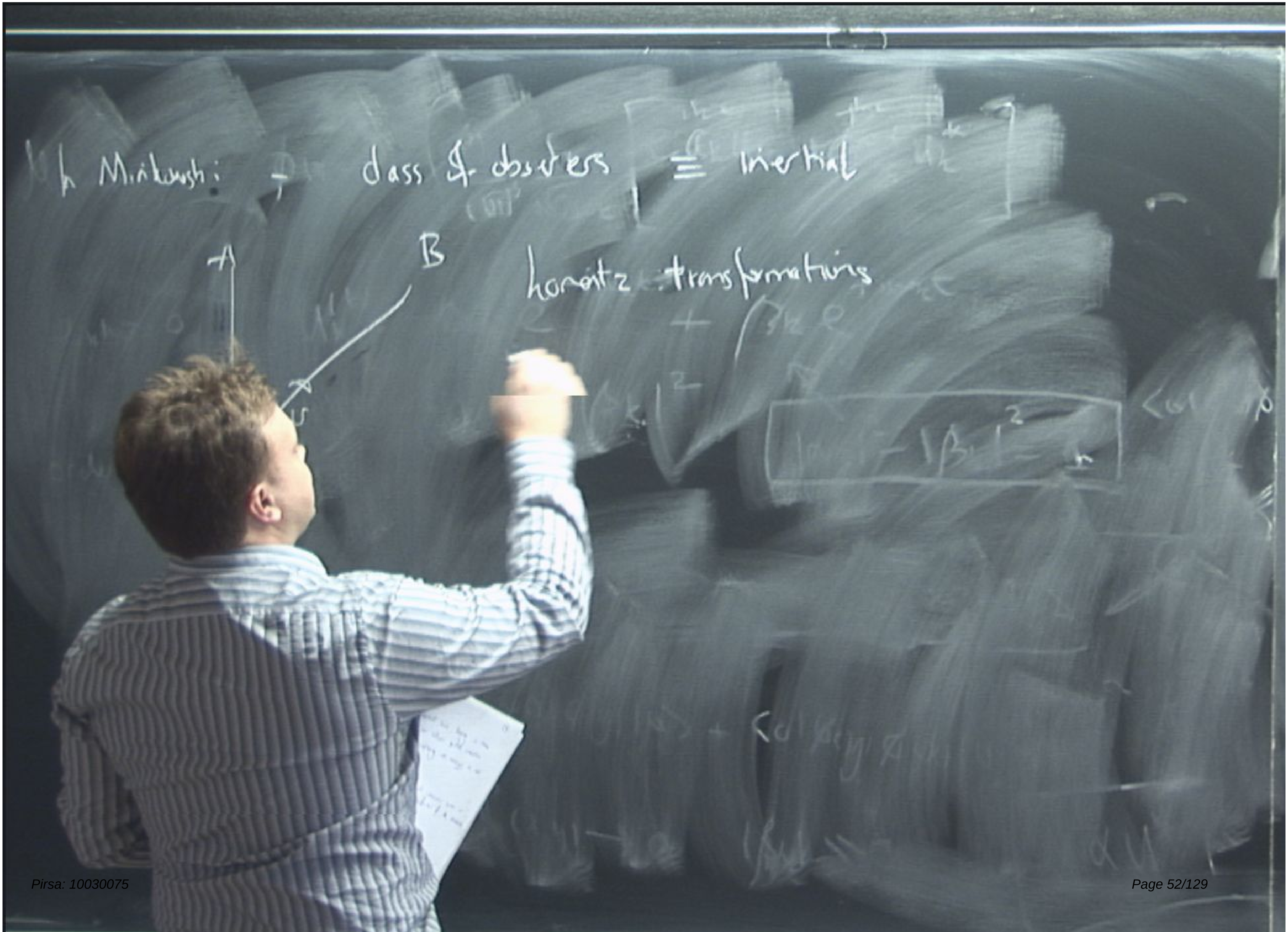
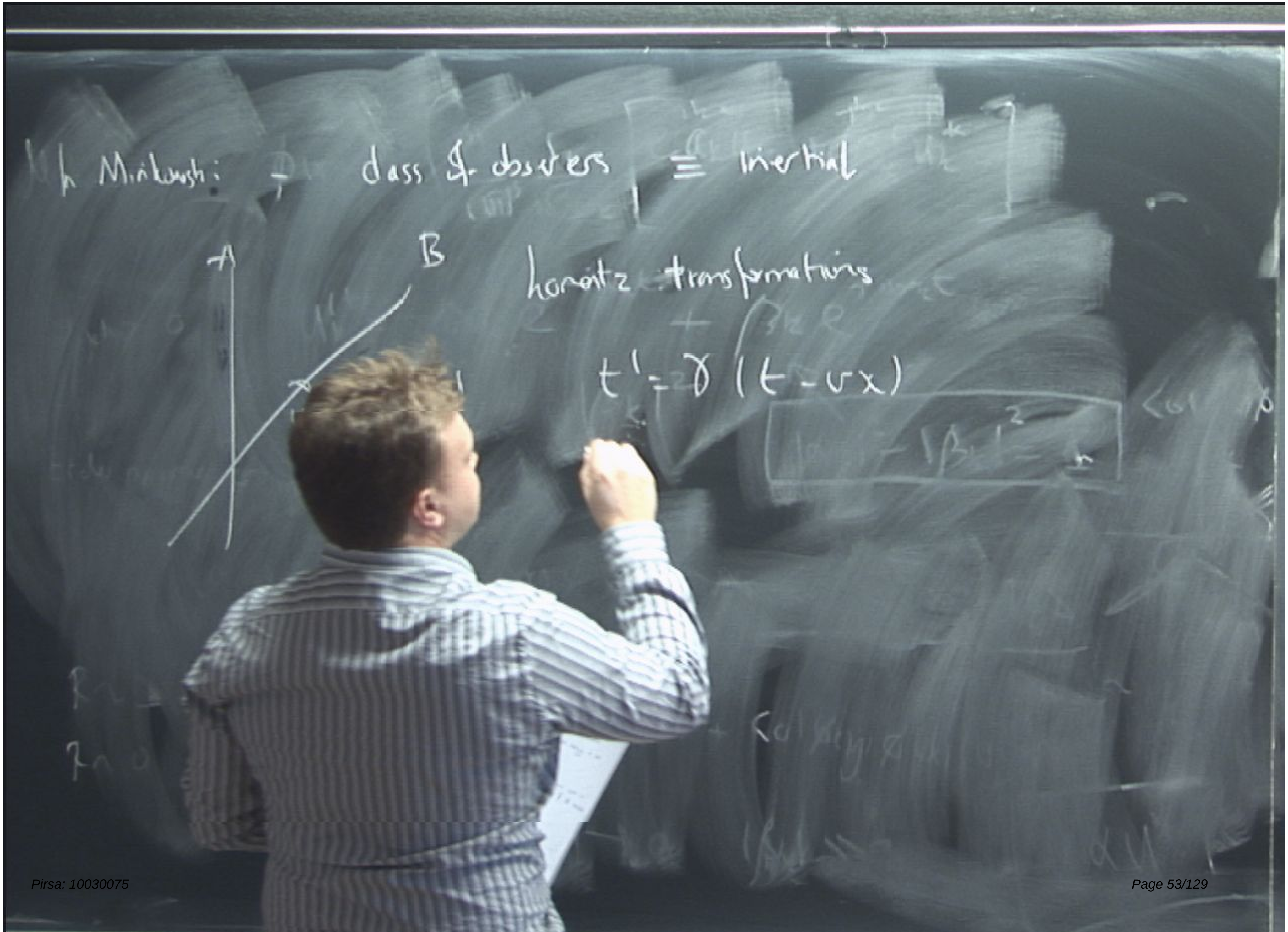
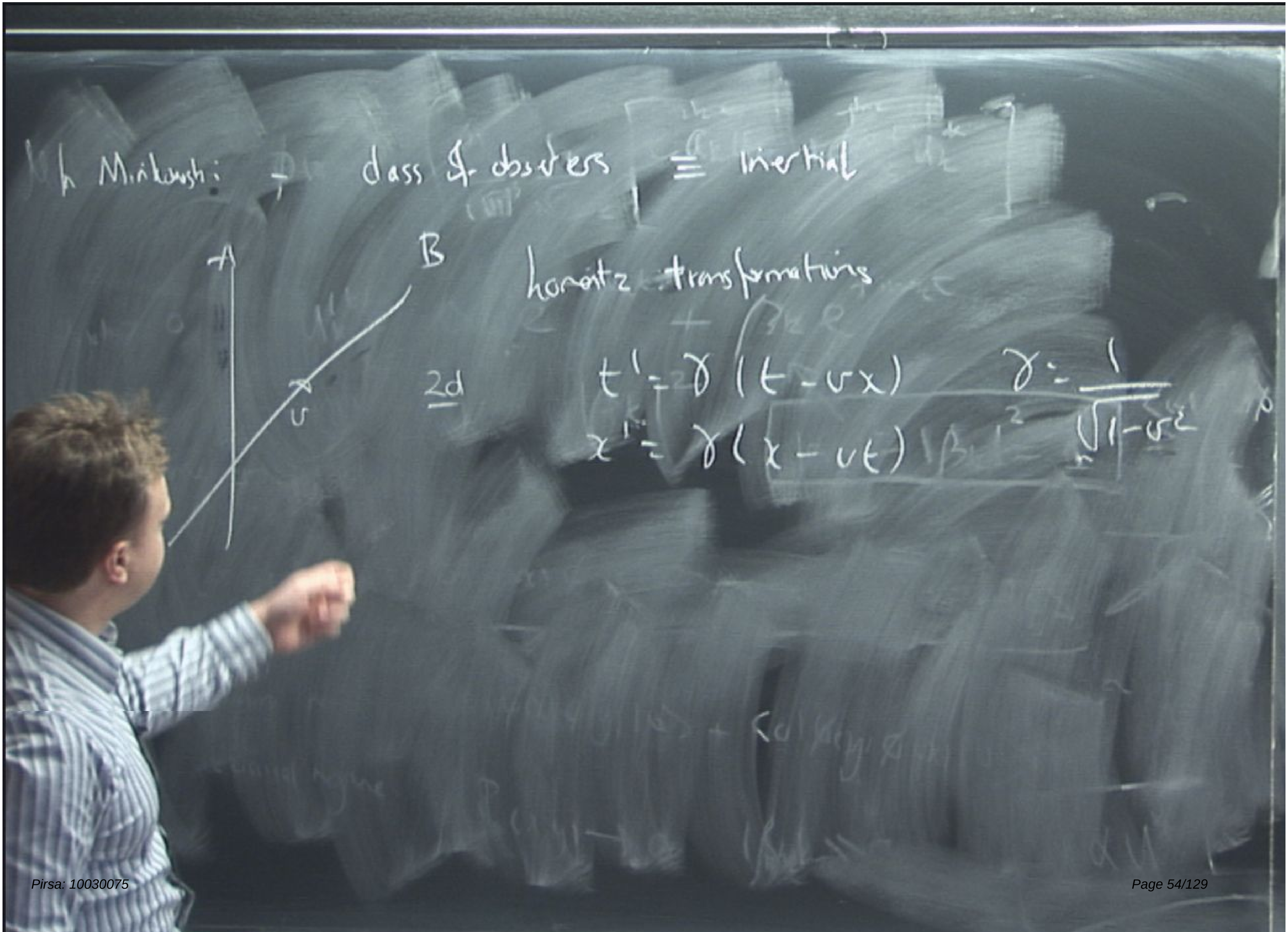
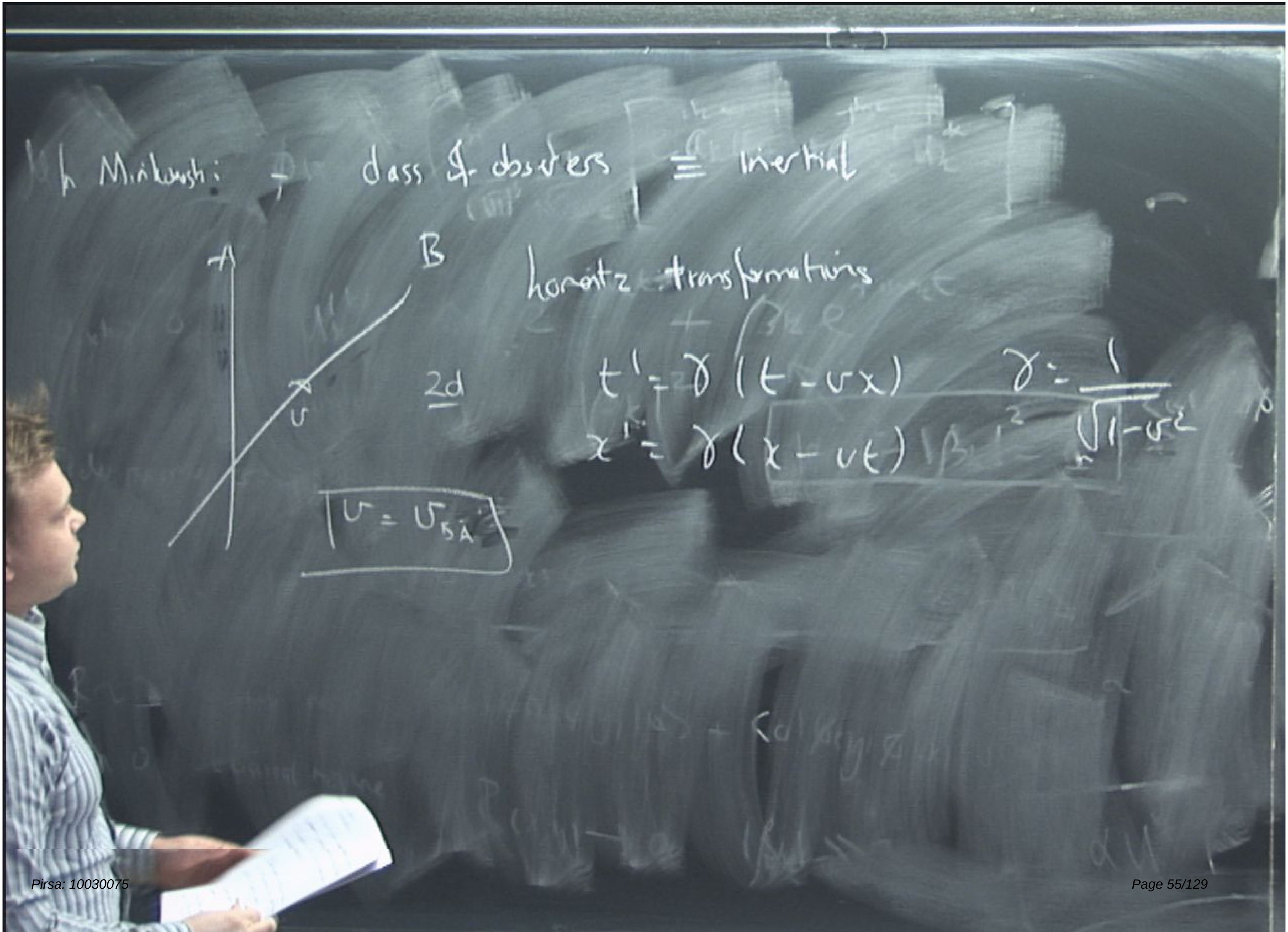
Vacuum + notion of particle is actually observer dependent
Even in static space – Energy ∂/∂t is conserved → Vacuum is minimum energy

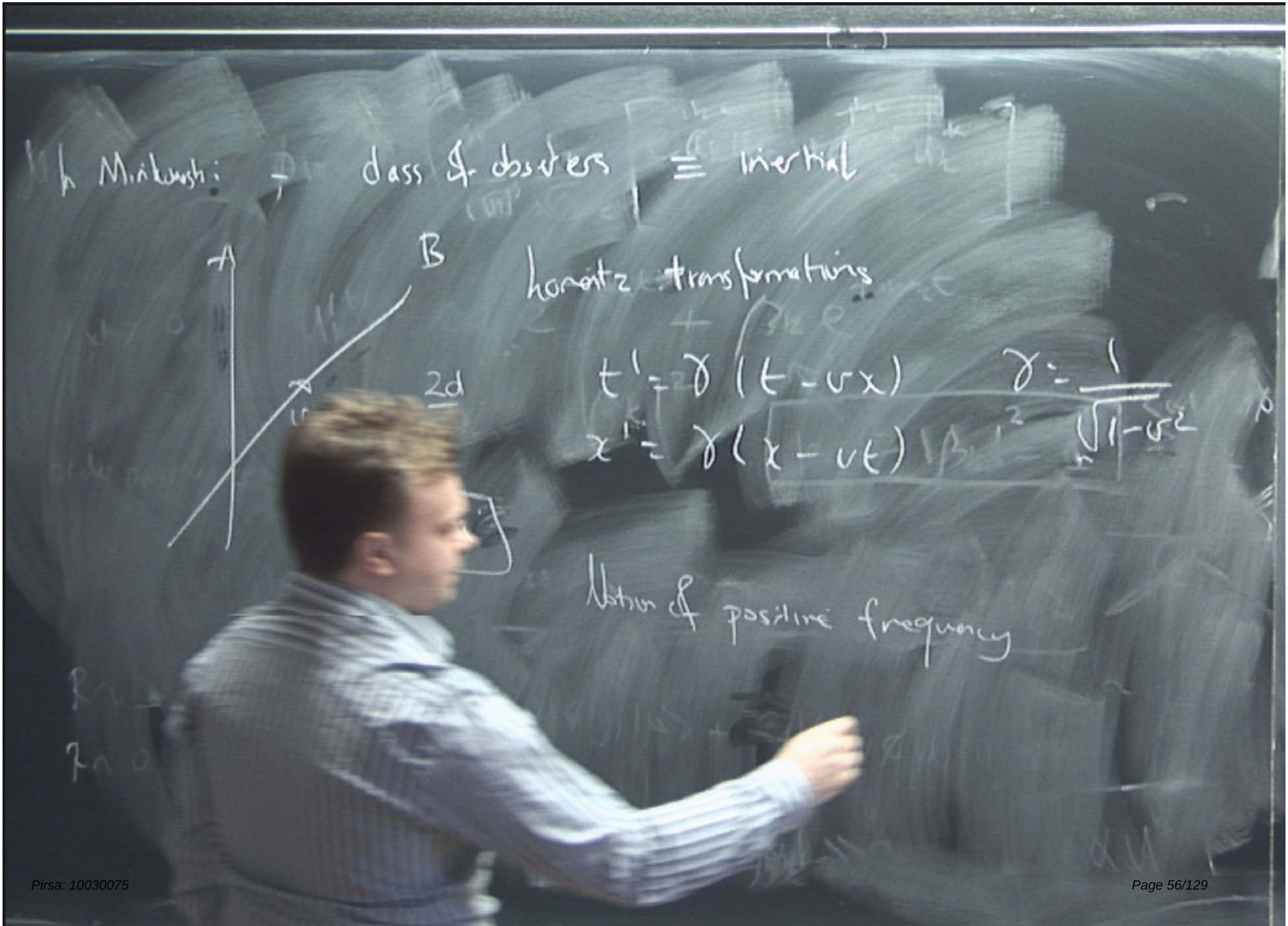
In Minkowski: $\exists$ class of observers $\equiv$ inertial

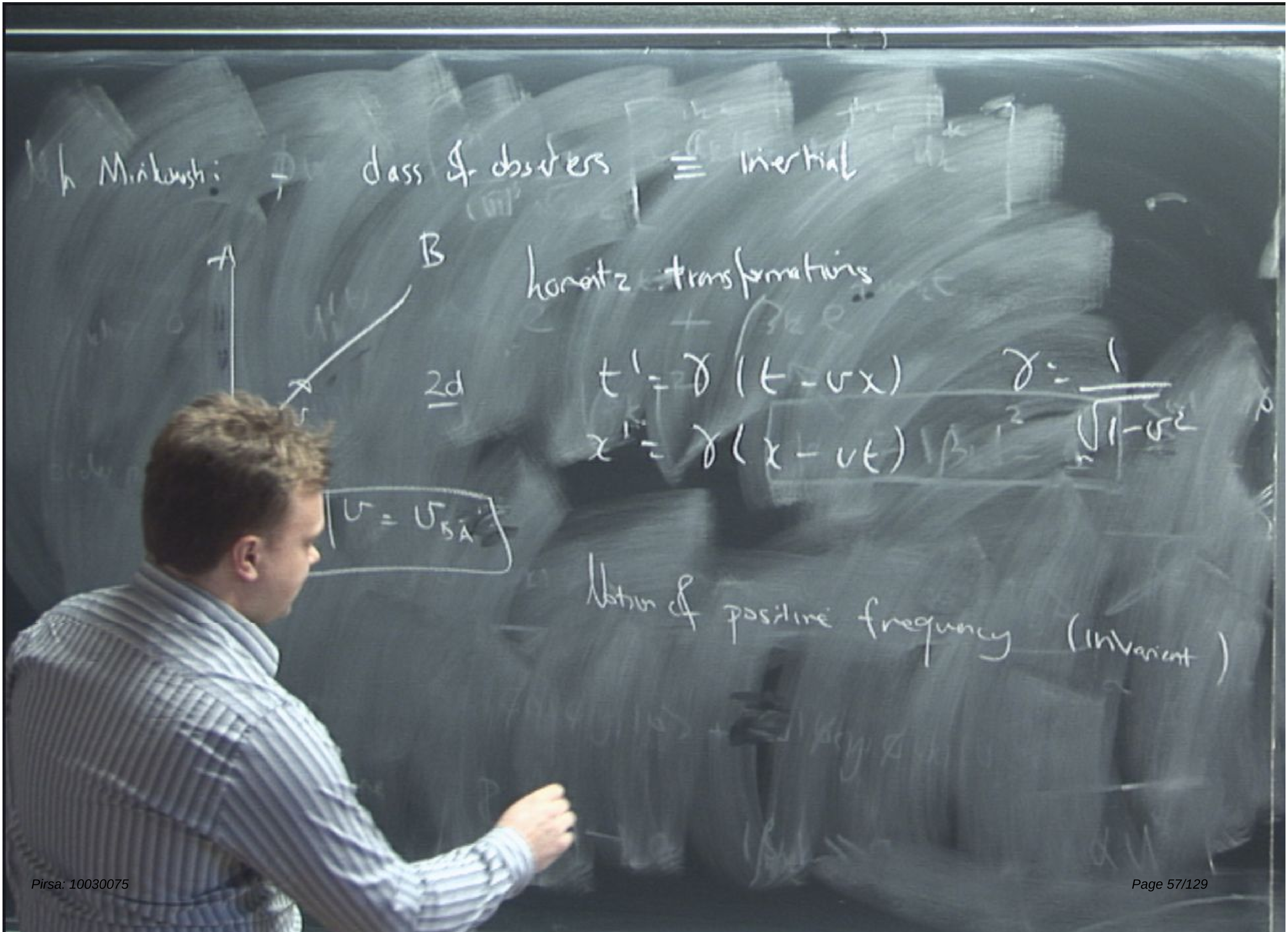

In Minkowski: class of observers ≡ inertial
A
B
Lorentz transformations

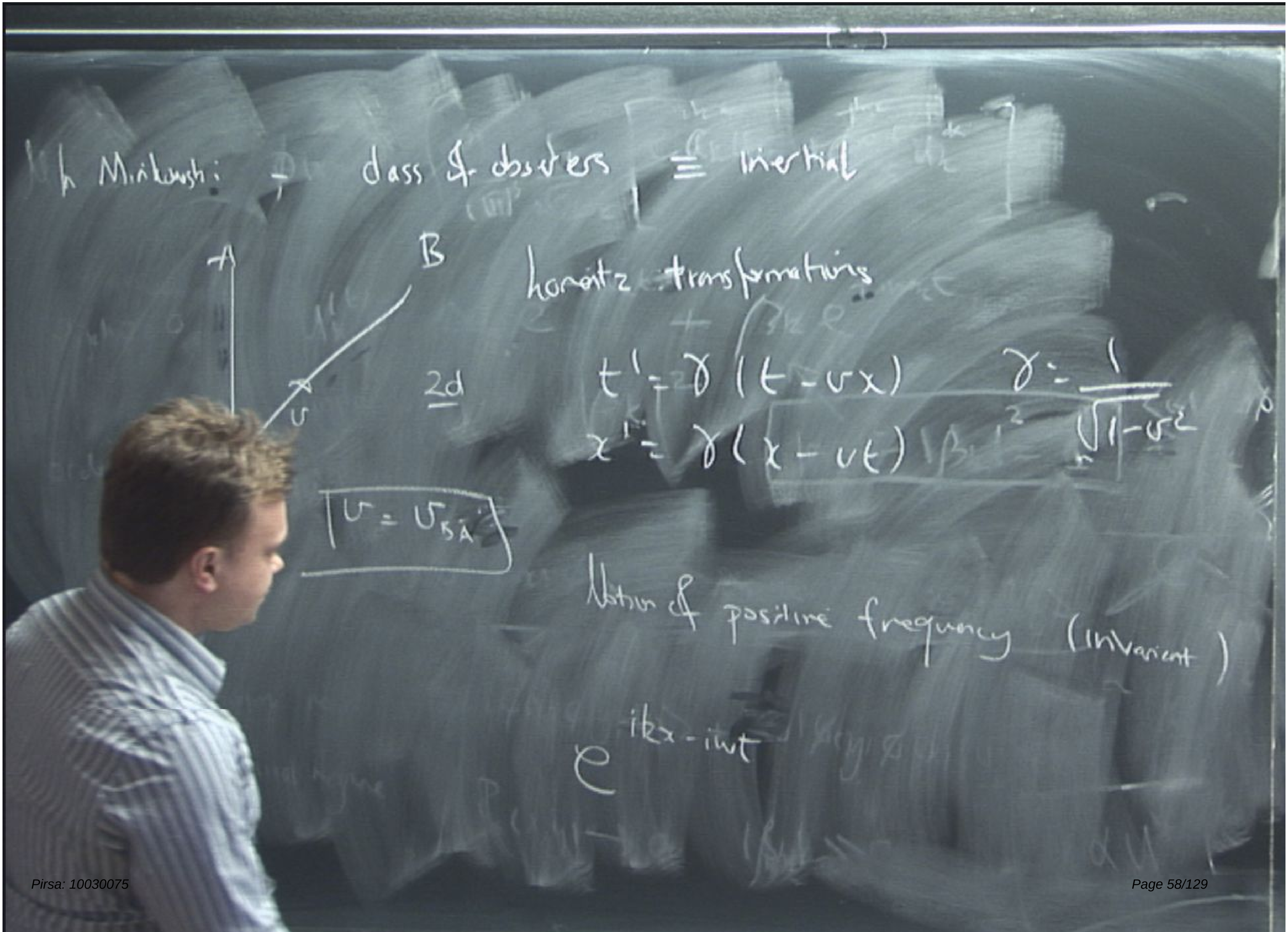

In Minkowski: class of observers ≡ inertial
A
B
Lorentz transformations
t' = γ (t - vx)


In Minkowski: class of observers ≡ inertial
A
B
v
Lorentz transformations
2d
t' = γ (t − vx)
x' = γ (x − vt)
γ = 1/√(1−v²)


In Minkowski: class of observers ≡ inertial
A
B
Lorentz transformations
2d
$t' = \gamma (t - vx)$
$x' = \gamma (x - vt)$
$\gamma = \frac{1}{\sqrt{1-v^2}}$
v
$\boxed{v = v_{BA}}$


In Minkowski: class of observers ≡ inertial
B
2d
Lorentz transformations
$t' = \gamma(t - vx)$
$x' = \gamma(x - vt)$
$\gamma = \frac{1}{\sqrt{1-v^2}}$
Notion of positive frequency


In Minkowski: class of observers ≡ inertial
A
B
Lorentz transformations
2d
t' = γ (t − vx)
x' = γ (x − vt)
γ = 1/√(1 − v²)
v = v_BA
Notion of positive frequency (invariant)


In Minkowski: class of observers ≡ inertial
A
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$t' = \gamma (t - vx)$
$\gamma = \frac{1}{\sqrt{1-v^2}}$
$x' = \gamma (x - vt)$
v
$\boxed{v = v_{BA}}$
Notion of positive frequency (invariant)
$e^{ikx - i\omega t}$

In Minkowski: $\exists$ class of observers $\equiv$ inertial

$$\boxed{v = v_{BA}}$$

Lorentz transformations

2d

$$t' = \gamma\,(t - vx) \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$

$$x' = \gamma\,(x - vt)$$

Notion of positive freq

$$e^{ikx - i\omega t}$$

bservers $\equiv$ inertial

Lorentz transformations

$$t' = \gamma(t - v x) \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$
$$x' = \gamma(x - v t)$$

Notion of positive frequency (invariant)

$$e^{ikx - i\omega t} \longrightarrow e^{ik\gamma(x' + v t') - i\omega\gamma(t' + v x')} = e^{ik'x' -}$$

tial

ations

$-vx)$ $\qquad \gamma = \dfrac{1}{\sqrt{1-v^2}}$

$-vt)$

line frequency (invariant)

$$\rightarrow e^{ik\gamma(x'+vt') - i\omega\gamma(t'+vx')} = e^{ik'x' - i\omega' t'}$$

$\sqrt{1-v^2}$

invariant)

$+vt') - i\omega\gamma(t'+vx') = e^{ik'x' - i\omega' t'}$

$\boxed{\omega' = \gamma\omega - vk}$

$$\frac{1}{\sqrt{1-v^2}}$$

invariant)

positive frequ. → positive frequency

$$+vt') - i\omega\gamma(t'+vx') = e^{ik'x' - i\omega' t'}$$

$$\boxed{\omega' = \gamma\omega - vk}$$

In Minkowski: class of observers $\equiv$ inertial

$\boxed{v = v_{BA}}$

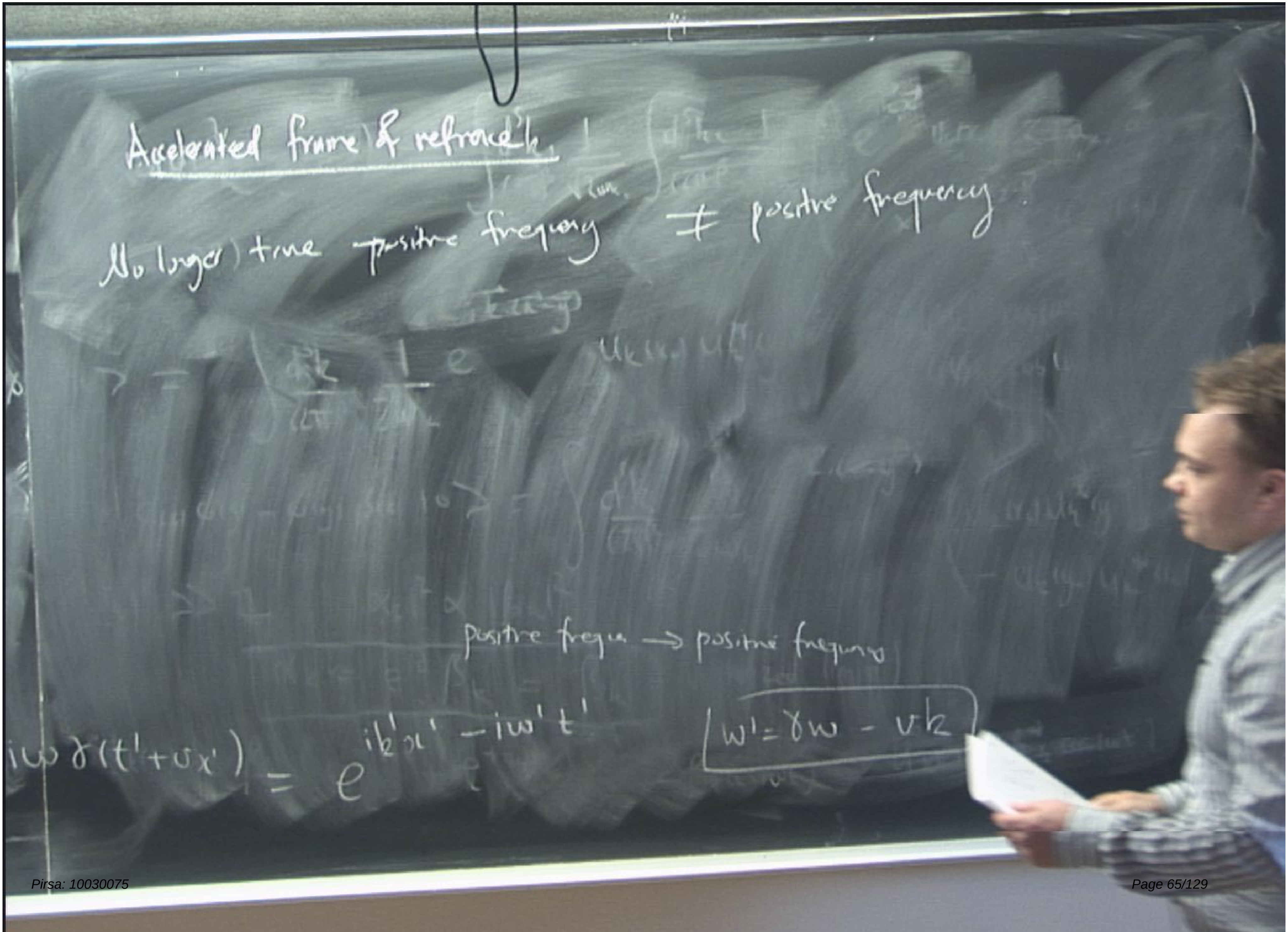
Lorentz transformations

2d

$$t' = \gamma (t - vx) \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$
$$x' = \gamma (x - vt)$$

Notion of positive frequency (invariant)

$$e^{ikx - i\omega t} \longrightarrow e^{ik\gamma(x' + vt') - i\omega \ldots}$$


Accelerated frame & refraction
No longer) time positive frequency ≠ positive frequency
positive frequ → positive frequency
$e^{i\omega\gamma(t'+vx')} = e^{ik'x' - i\omega' t'}$
$\boxed{\omega' = \gamma\omega - vk}$

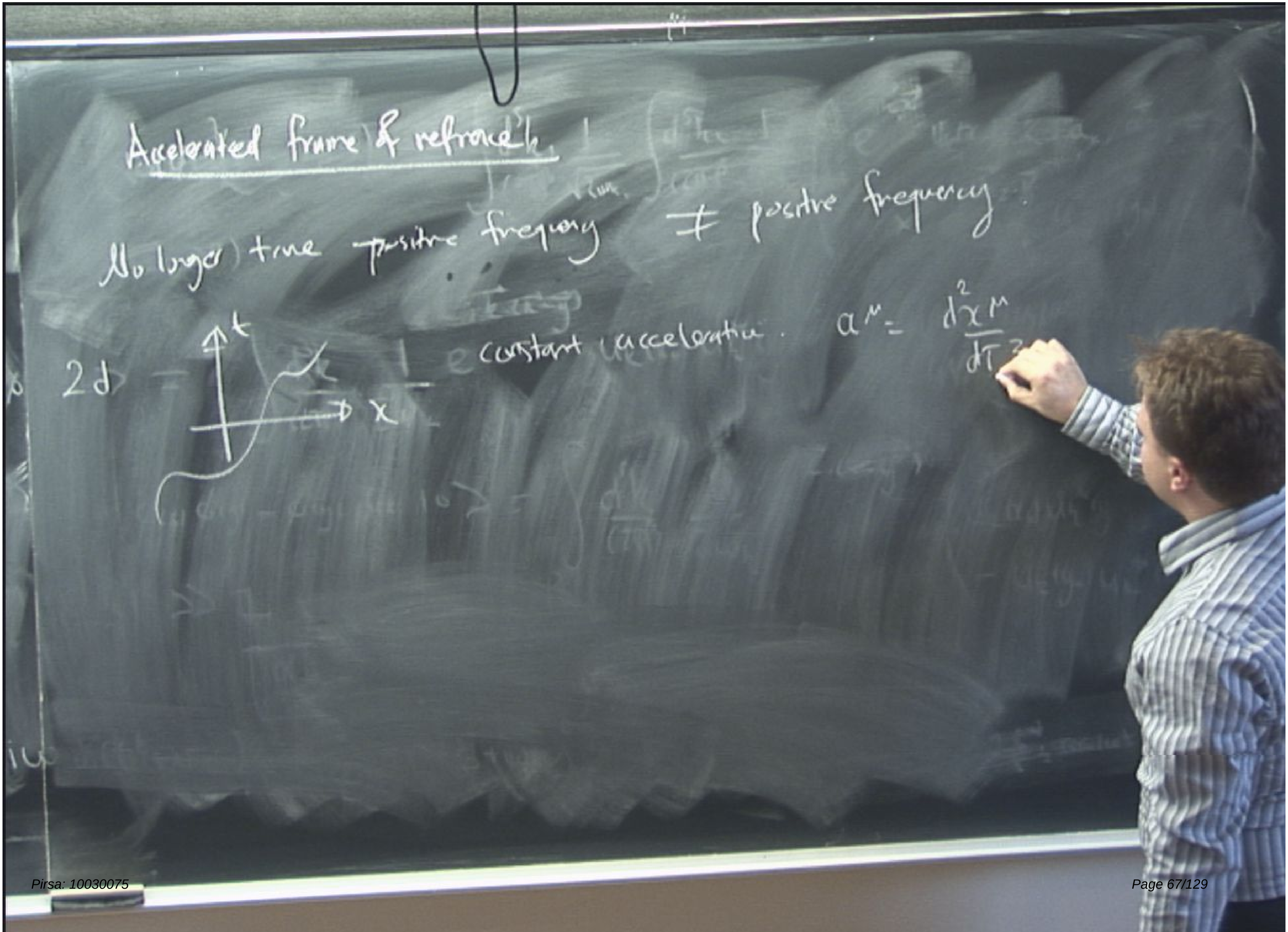
Accelerated frame & refraction

No longer true positive frequency $\neq$ positive frequency

positive frequ $\rightarrow$ positive frequency

$$i\omega\gamma(t'+vx') = e^{ik'x' - i\omega' t'}$$

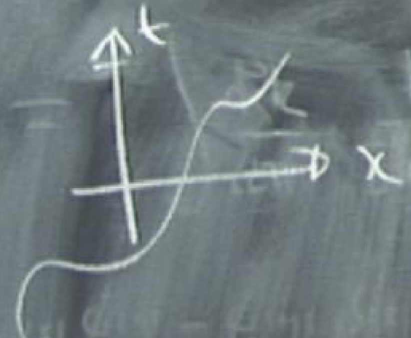
$$\boxed{\omega' = \gamma\omega - vk}$$


Accelerated frame & refraction
No longer time positive frequency ≠ positive frequency
2d
t
x
constant acceleration. $a^\mu = \frac{d^2 x^\mu}{d\tau^2}$

Accelerated frame & reference

(No longer) time positive frequency $\neq$ positive frequency

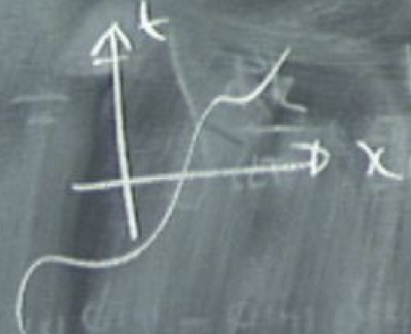
2d



constant acceleration. $a^\mu = \frac{d^2 x^\mu}{d\tau^2}$

Accelerated frame & reference

No longer true positive frequency $\neq$ positive frequency



constant acceleration. $a^\mu = \frac{d^2 x^\mu}{d\tau^2}$

τ = proper time

$d\tau = \sqrt{dt^2 - dx^2}$

Accelerated frame & reference

No longer true positive frequency $\neq$ positive frequency

2d

constant acceleration. $a^\mu = \dfrac{d^2 x^\mu}{d\tau^2}$

τ = proper time

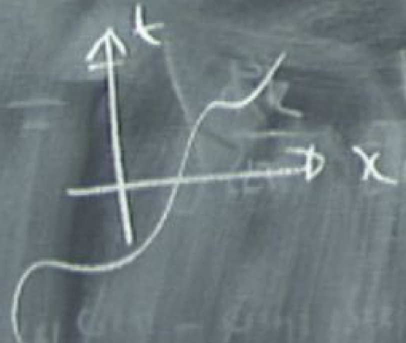
$$a^2 = -a^\mu a_\mu$$

$$d\tau = \sqrt{dt^2 - dx^2}$$

Accelerated frame & refrac'h

No longer) time positive frequency $\neq$ positive frequency

2d



constant acceleration. $a^\mu = \frac{d^2 x^\mu}{d\tau^2}$

τ = proper time

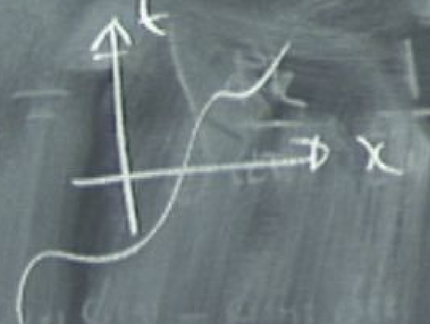
$a^2 = -a^\mu a_\mu$

$d\tau = \sqrt{dt^2 - dx^2}$

Accelerated frame & refrance

No longer) true positive frequency $\neq$ positive frequency

2d

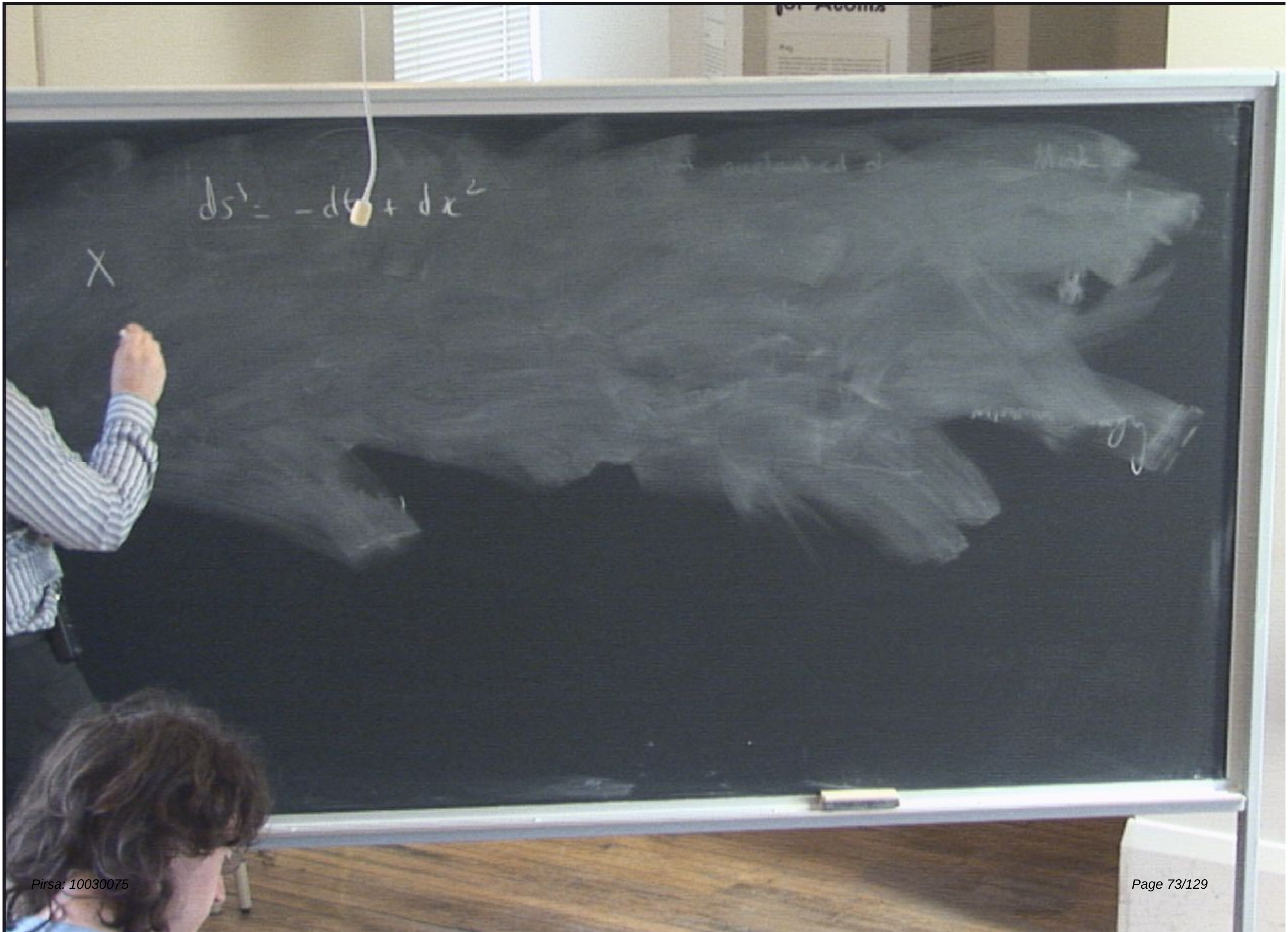


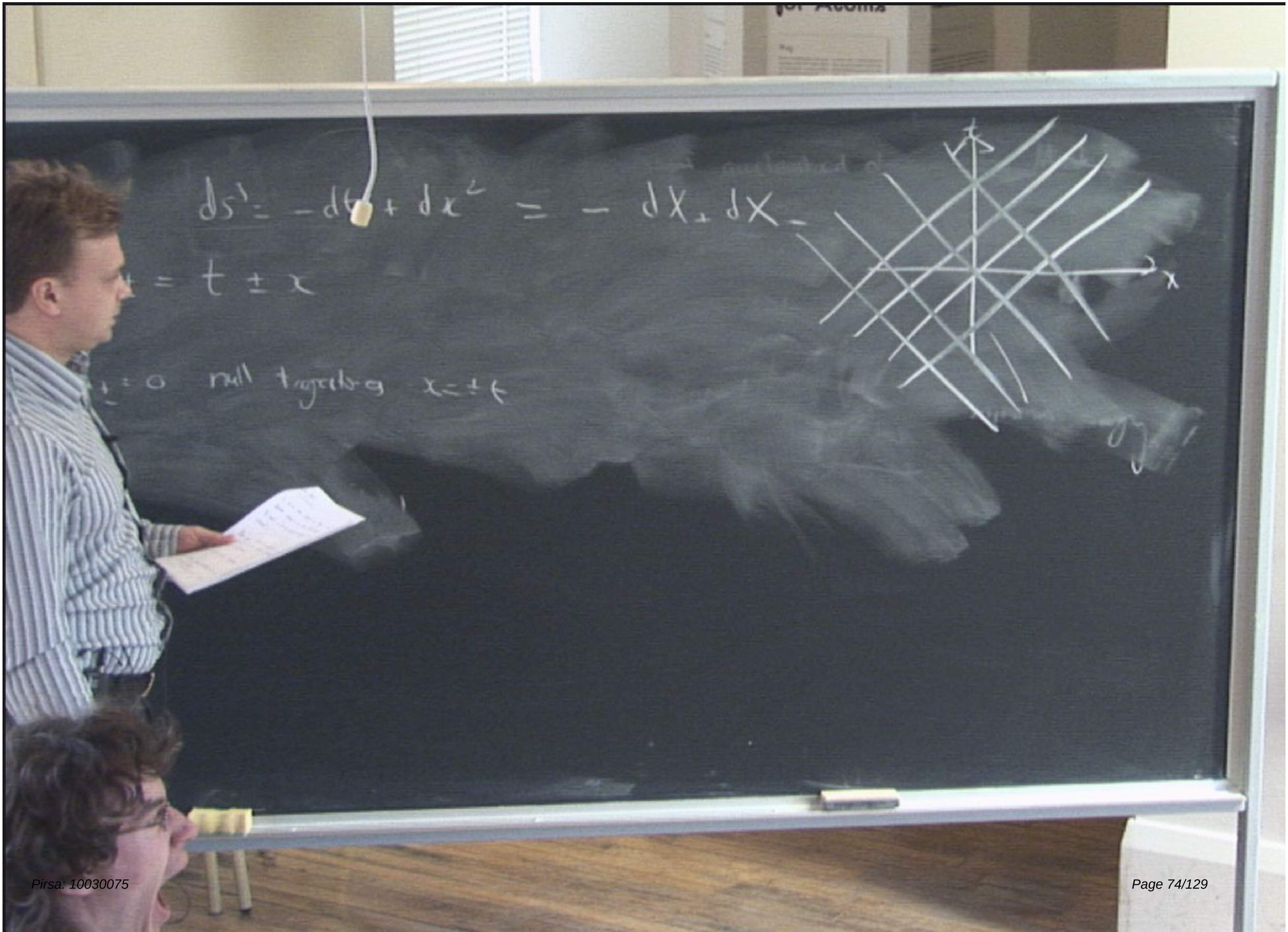
constant acceleration. $a^\mu = \frac{d^2x^\mu}{d\tau^2}$

τ = proper time

$$a^2 = -a^\mu a_\mu = -\left(\frac{d^2x^\mu}{d\tau^2}\right)\left(\frac{d^2x_\mu}{d\tau^2}\right)$$

$$d\tau = \sqrt{dt^2 - dx^2}$$

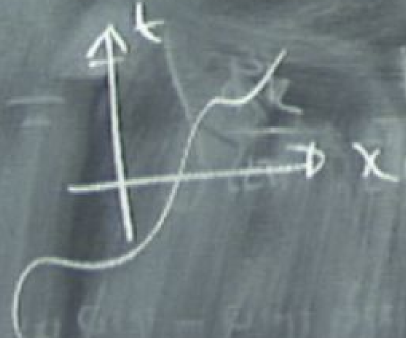

$ds^2 = -dt^2 + dx^2$
X



ds² = −dt² + dx² = − dX₊ dX₋
= t ± x
= 0 null trajectory x = ±t
x

Accelerated frame & refraction

No longer true positive frequency $\neq$ positive frequency



constant acceleration. $a^\mu = \frac{d^2x^\mu}{d\tau^2}$

τ = proper time

$$a^2 = -a^\mu a_\mu = -\left(\frac{d^2x^\mu}{d\tau^2}\right)\left(\frac{d^2x_\mu}{d\tau^2}\right)$$

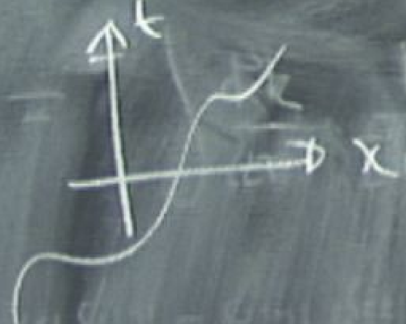
$$d\tau = \sqrt{dt^2 - dx^2}$$

$$\left(\frac{dx^\mu}{d\tau}\right)\frac{dx_\mu}{d\tau} = -1$$

Accelerated frame & refraction

No longer) time positive frequency $\neq$ positive frequency

2d



constant acceleration. $a^\mu = \frac{d^2 x^\mu}{d\tau^2}$

τ = proper time

$$a^2 = -a^\mu a_\mu = -\left(\frac{d^2x^\mu}{d\tau^2}\right)\left(\frac{d^2x_\mu}{d\tau^2}\right)$$

$$d\tau = \sqrt{dt^2 - dx^2}$$

$$\left(\frac{dx^\mu}{d\tau}\right)\frac{dx_\mu}{d\tau} = -1$$

$$\boxed{\frac{dX_+}{d\tau}\frac{dX_-}{d\tau} = 1}$$

Accelerated frame & refrac'h

No longer true positive frequency $\neq$ positive frequency

constant acceleration. $a^\mu = \frac{d^2 x^\mu}{d\tau^2}$

τ = proper time

$$a^2 = -a^\mu a_\mu = -\left(\frac{d^2x^\mu}{d\tau^2}\right)\left(\frac{d^2x_\mu}{d\tau^2}\right)$$

$$d\tau = \sqrt{dt^2 - dx^2}$$

$$\left(\frac{dx^\mu}{d\tau}\right)\frac{dx_\mu}{d\tau} = -1$$

$$\boxed{\frac{dX_+}{d\tau}\frac{dX_-}{d\tau} = 1}$$

$$\frac{dX_-}{d\tau} = \frac{1}{\left(\frac{dX_+}{d\tau}\right)}$$

Accelerated frame & refract'n

No longer true positive frequency ≠ positive frequency

2d

constant acceleration. $a^\mu = \frac{d^2x^\mu}{d\tau^2}$

τ = proper time

$$a^2 = -a^\mu a_\mu = -\left(\frac{d^2x^\mu}{d\tau^2}\right)\left(\frac{d^2x_\mu}{d\tau^2}\right) \qquad d\tau = \sqrt{dt^2 - dx^2}$$

$$\left(\frac{dx^\mu}{d\tau}\right)\frac{dx_\mu}{d\tau} = -1$$

$$\boxed{\frac{dX_+}{d\tau}\frac{dX_-}{d\tau} = 1}$$

$$\frac{dX_-}{d\tau} = \frac{1}{\left(\frac{dX_+}{d\tau}\right)}$$

Accelerated frame & refraction

No longer true positive frequency $\neq$ positive frequency

constant acceleration. $a^\mu = \dfrac{d^2 x^\mu}{d\tau^2}$

τ = proper time

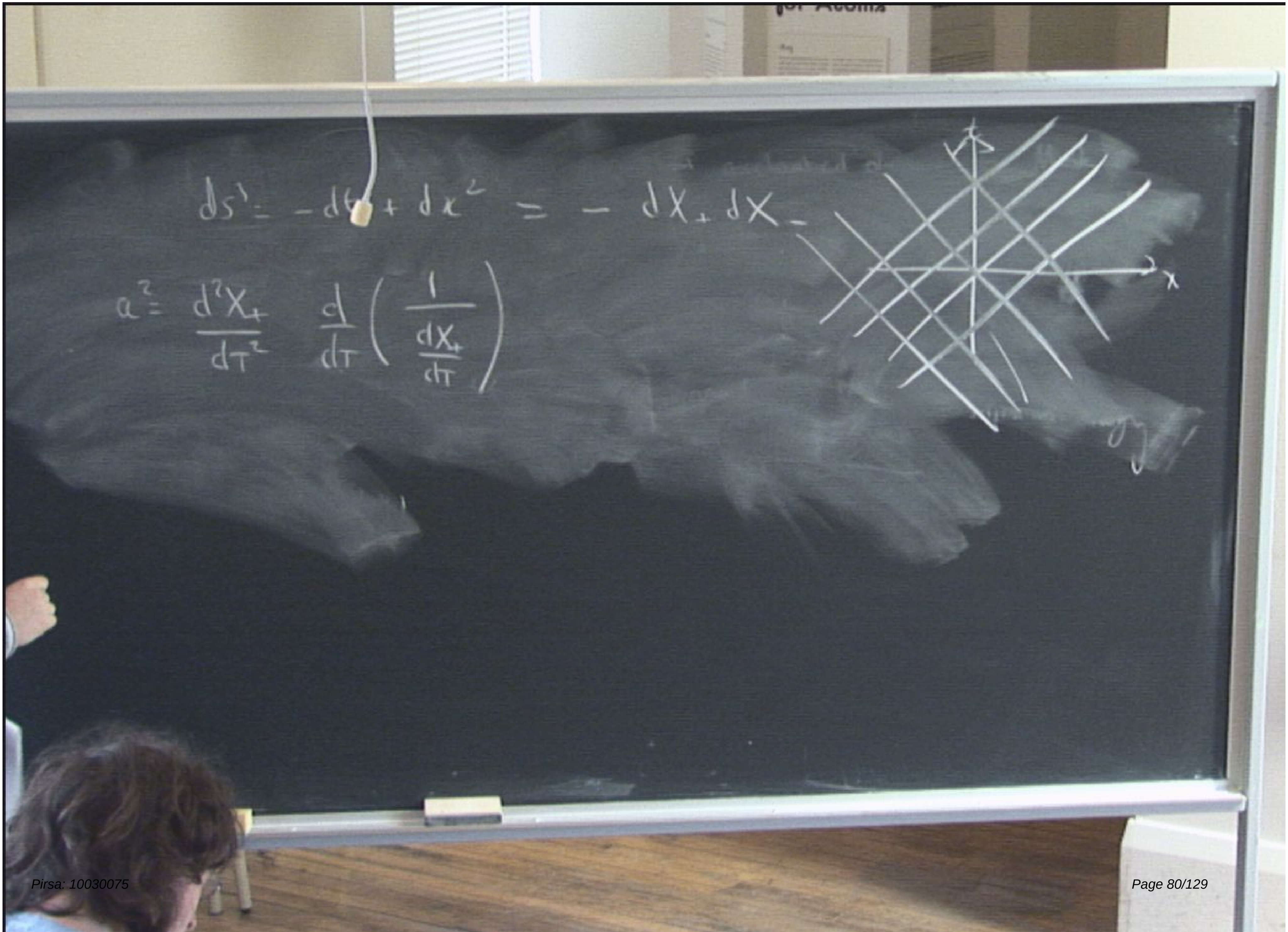
$$a^2 = -a^\mu a_\mu = -\left(\frac{d^2 x^\mu}{d\tau^2}\right)\left(\frac{d^2 x_\mu}{d\tau^2}\right) \qquad d\tau = \sqrt{dt^2 - dx^2}$$

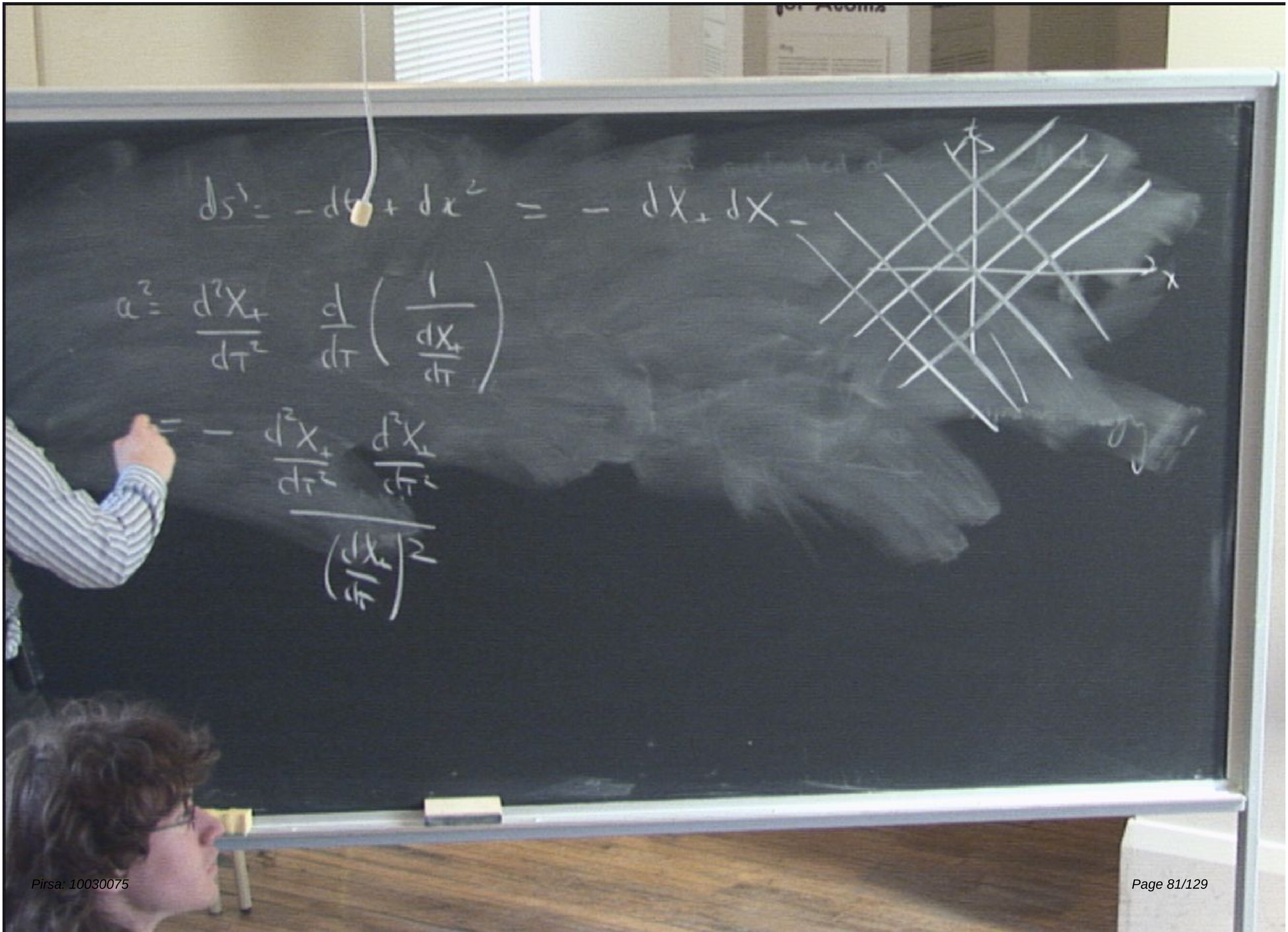
$$\left(\frac{dx^\mu}{d\tau}\right)\frac{dx_\mu}{d\tau} = -1$$

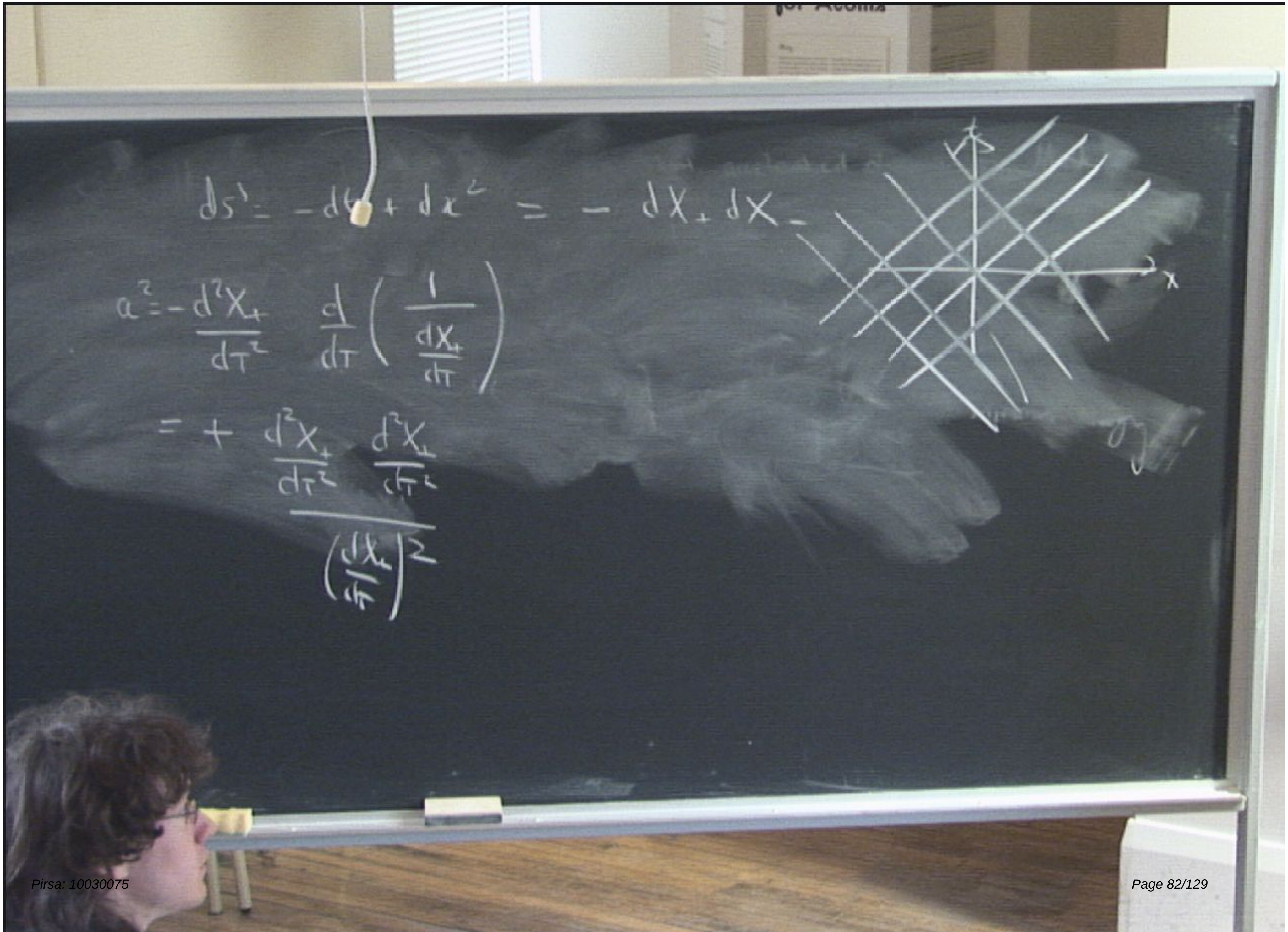
$$\boxed{\frac{dX_+}{d\tau}\frac{dX_-}{d\tau} = 1}$$

$$\frac{dX_-}{d\tau} = \frac{1}{\left(\frac{dX_+}{d\tau}\right)}$$

$$\boxed{a^2 = \frac{d^2X_+}{d\tau^2}\frac{d^2X_-}{d\tau^2}}$$


$ds^2 = -dt^2 + dx^2 = -dX_+ dX_-$
$a^2 = \frac{d^2X_+}{d\tau^2} \quad \frac{d}{d\tau}\left(\frac{1}{\frac{dX_+}{d\tau}}\right)$
t
x


$ds^2 = -dt^2 + dx^2 = -dX_+ dX_-$
$a^2 = \frac{d^2X_+}{d\tau^2} \frac{d}{d\tau}\left(\frac{1}{\frac{dX_+}{d\tau}}\right)$
$= -\frac{\frac{d^2X_+}{d\tau^2}\frac{d^2X_+}{d\tau^2}}{\left(\frac{dX_+}{d\tau}\right)^2}$
t
x



$ds^2 = -dt^2 + dx^2 = -dX_+ dX_-$
$a^2 = -\frac{d^2X_+}{d\tau^2} \quad \frac{d}{d\tau}\left(\frac{1}{\frac{dX_+}{d\tau}}\right)$
$= + \frac{\frac{d^2X_+}{d\tau^2} \frac{d^2X_+}{d\tau^2}}{\left(\frac{dX_+}{d\tau}\right)^2}$
t
x

Accelerated frame & refraction

No longer time positive frequency $\neq$ positive frequency

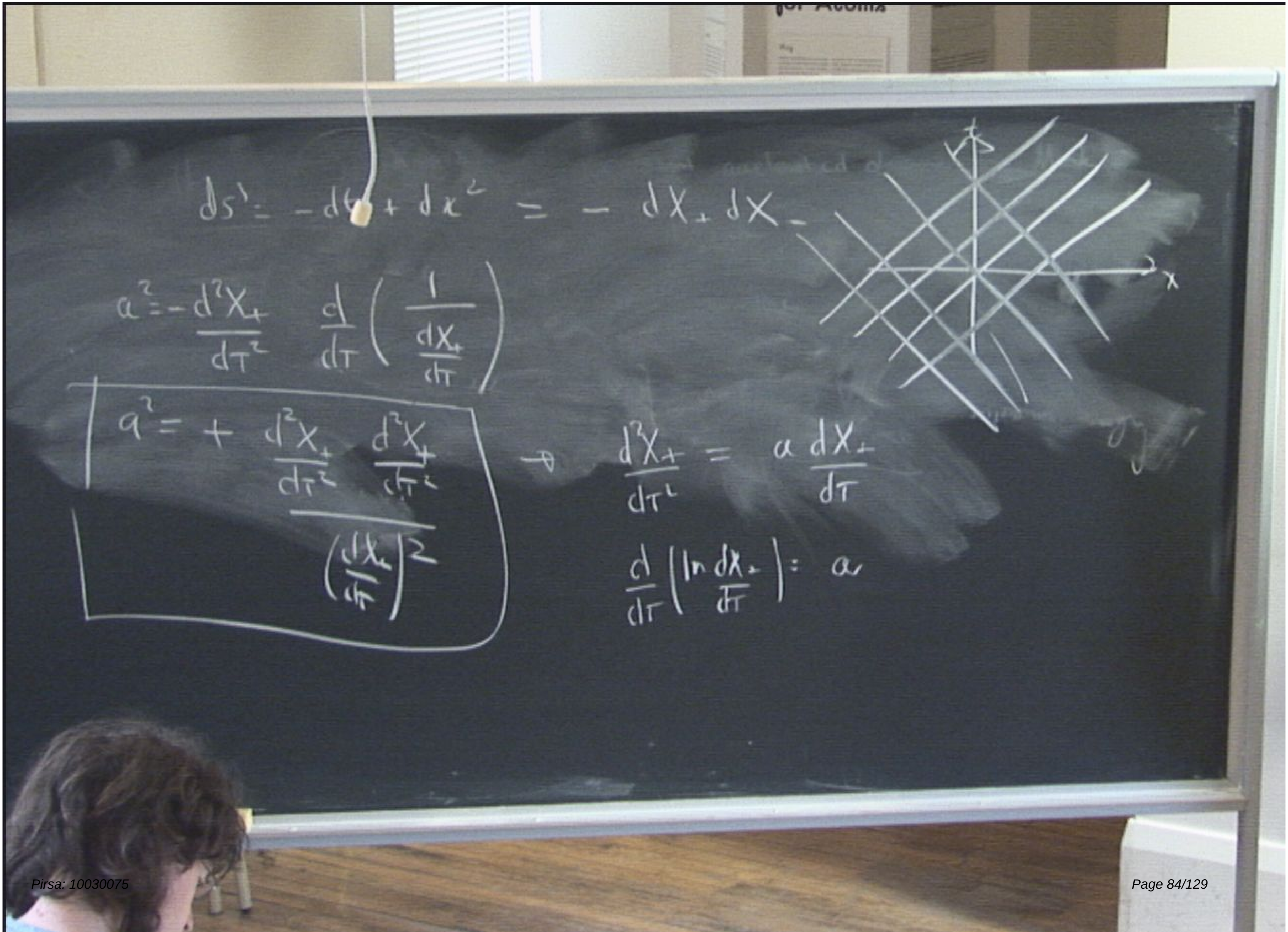
constant acceleration. $a^\mu = \frac{d^2x^\mu}{d\tau^2}$

τ = proper time

$$a^2 = + a^\mu a_\mu = +\left(\frac{d^2x^\mu}{d\tau^2}\right)\left(\frac{d^2x_\mu}{d\tau^2}\right) \qquad d\tau = \sqrt{dt^2 - dx^2}$$

$$\left(\frac{dx^\mu}{d\tau}\right)\frac{dx_\mu}{d\tau} = -1 \qquad \boxed{\frac{dX_+}{d\tau}\frac{dX_-}{d\tau} = 1} \qquad \frac{dX_-}{d\tau} = \frac{1}{\left(\frac{dX_+}{d\tau}\right)}$$

$$\boxed{a^2 = -\frac{d^2X_+}{d\tau^2}\frac{d^2X_-}{d\tau^2}}$$



$ds^2 = -dt^2 + dx^2 = -dX_+ dX_-$
$a^2 = -\frac{d^2X_+}{d\tau^2} \quad \frac{d}{d\tau}\left(\frac{1}{\frac{dX_+}{d\tau}}\right)$
$\boxed{a^2 = + \frac{\frac{d^2X_+}{d\tau^2}\frac{d^2X_+}{d\tau^2}}{\left(\frac{dX_+}{d\tau}\right)^2}}$
$\rightarrow \frac{d^2X_+}{d\tau^2} = a\frac{dX_+}{d\tau}$
$\frac{d}{d\tau}\left(\ln\frac{dX_+}{d\tau}\right) = a$
t
x

$$\ln \frac{dX_+}{d\tau} = A + a\tau$$

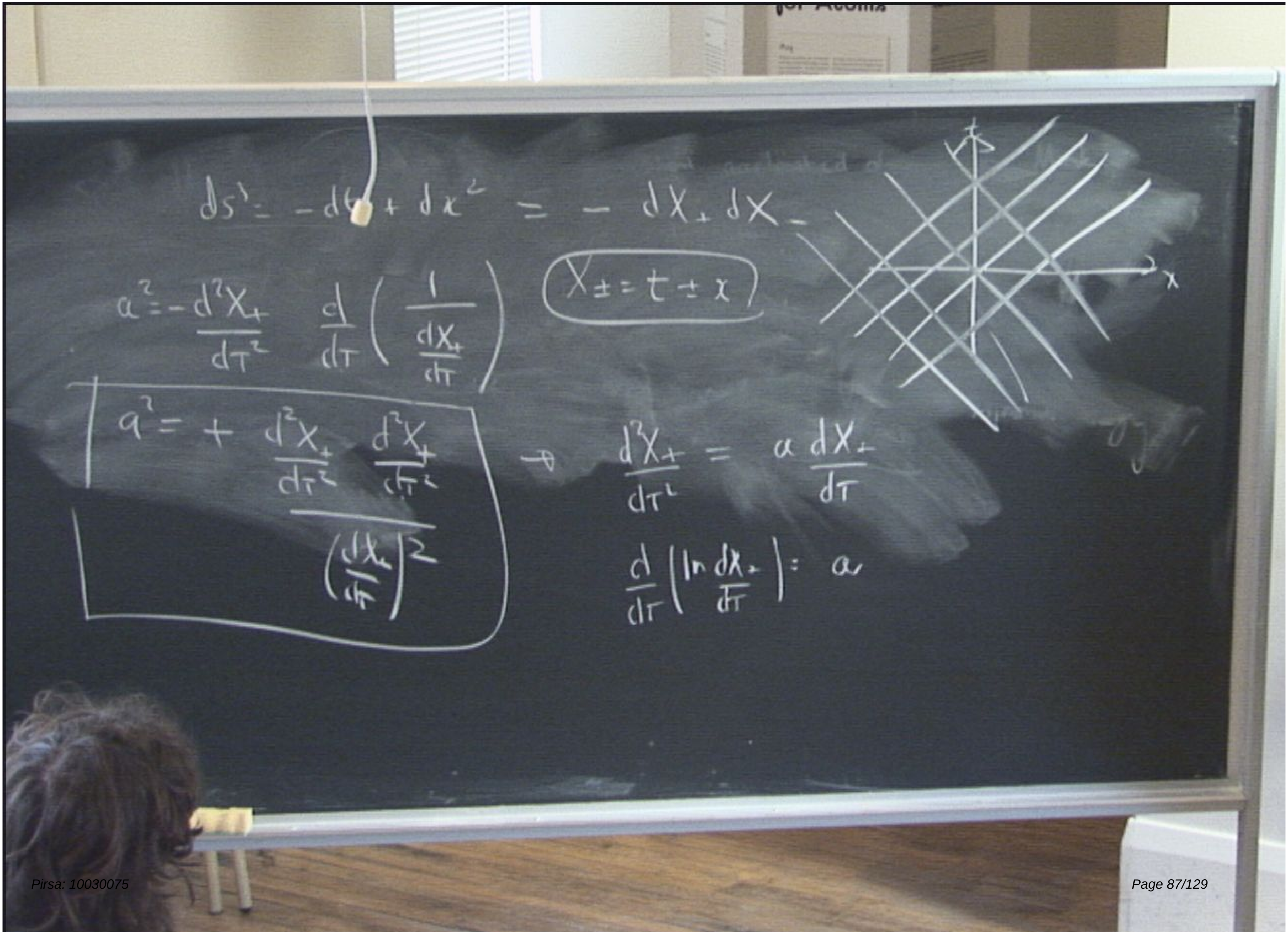
$$\frac{dX_+}{d\tau} = e^{A} e^{a\tau}$$

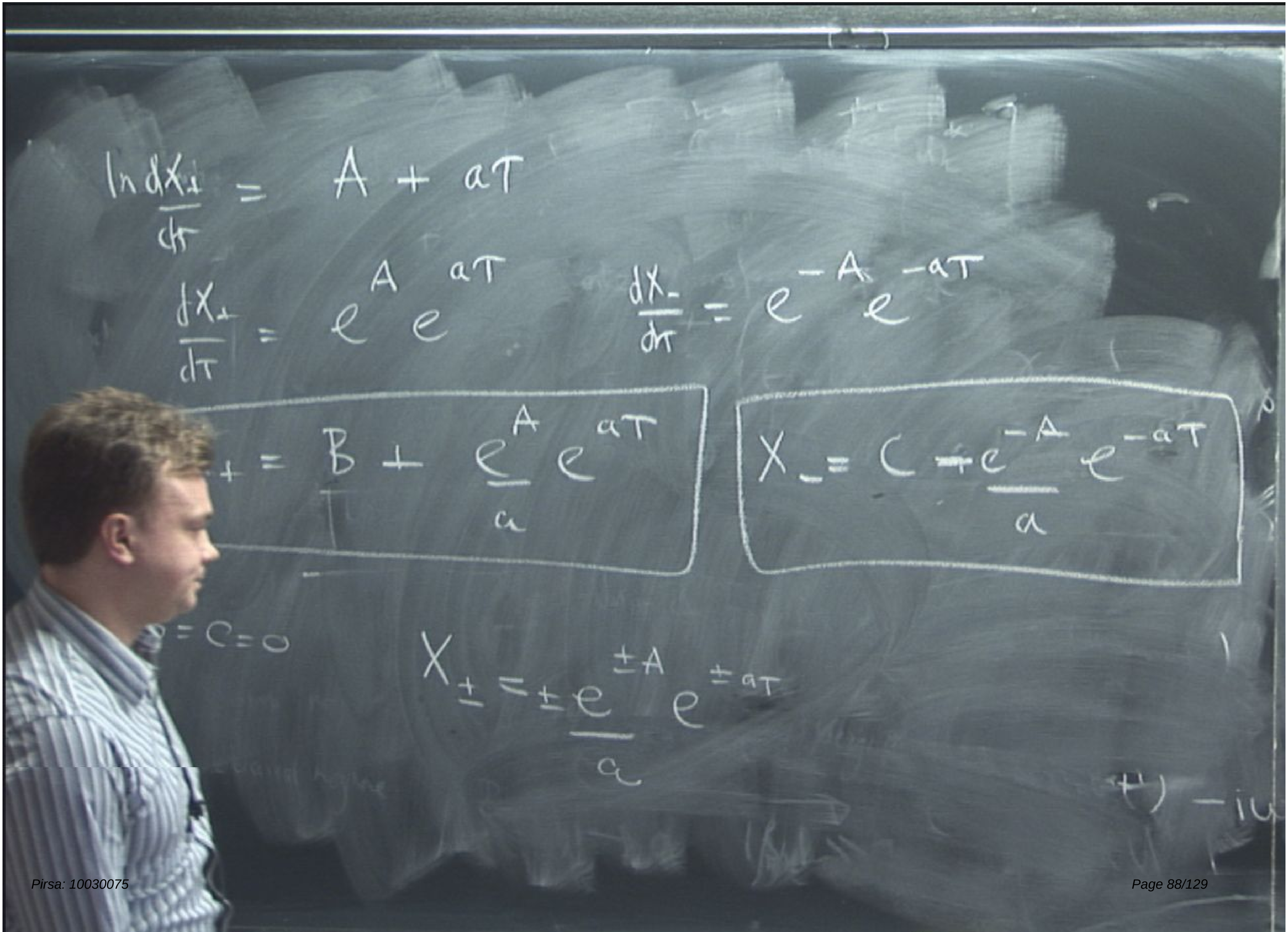
$$\boxed{X_+ = B + \frac{e^{A}}{a} e^{a\tau}}$$

$$\ln \frac{dX_+}{d\tau} = A + a\tau$$

$$\frac{dX_+}{d\tau} = e^{A} e^{a\tau} \qquad \frac{dX_-}{d\tau} = e^{-A} e^{-a\tau}$$

$$\boxed{X_+ = B + \frac{e^{A}}{a} e^{a\tau}} \qquad \boxed{X_- = C - \frac{e^{-A}}{a} e^{-a\tau}}$$


$ds^2 = -dt^2 + dx^2 = -dX_+ dX_-$
$X_\pm = t \pm x$
$a^2 = -\frac{d^2X_+}{d\tau^2} \frac{d}{d\tau}\left(\frac{1}{\frac{dX_+}{d\tau}}\right)$
$a^2 = + \frac{\frac{d^2X_+}{d\tau^2} \frac{d^2X_+}{d\tau^2}}{\left(\frac{dX_+}{d\tau}\right)^2}$
$\frac{d^2X_+}{d\tau^2} = a \frac{dX_+}{d\tau}$
$\frac{d}{d\tau}\left(\ln \frac{dX_+}{d\tau}\right) = a$
t
x



$\ln \frac{dX_+}{d\tau} = A + a\tau$
$\frac{dX_+}{d\tau} = e^{A} e^{a\tau}$
$\frac{dX_-}{d\tau} = e^{-A} e^{-a\tau}$
$X_+ = B + \frac{e^{A}}{a} e^{a\tau}$
$X_- = C - \frac{e^{-A}}{a} e^{-a\tau}$
$= C = 0$
$X_\pm = \pm \frac{e^{\pm A}}{a} e^{\pm a\tau}$

$$\ln \frac{dX_+}{d\tau} = A + a\tau$$

$$\frac{dX_+}{d\tau} = e^{A} e^{a\tau} \qquad \frac{dX_-}{d\tau} = e^{-A} e^{-a\tau}$$

$$\boxed{X_+ = B + \frac{e^{A}}{a} e^{a\tau}} \qquad \boxed{X_- = C - \frac{e^{-A}}{a} e^{-a\tau}}$$

$B = C = 0$

$$X_\pm = \pm \frac{e^{\pm A}}{a} e^{\pm a\tau}$$

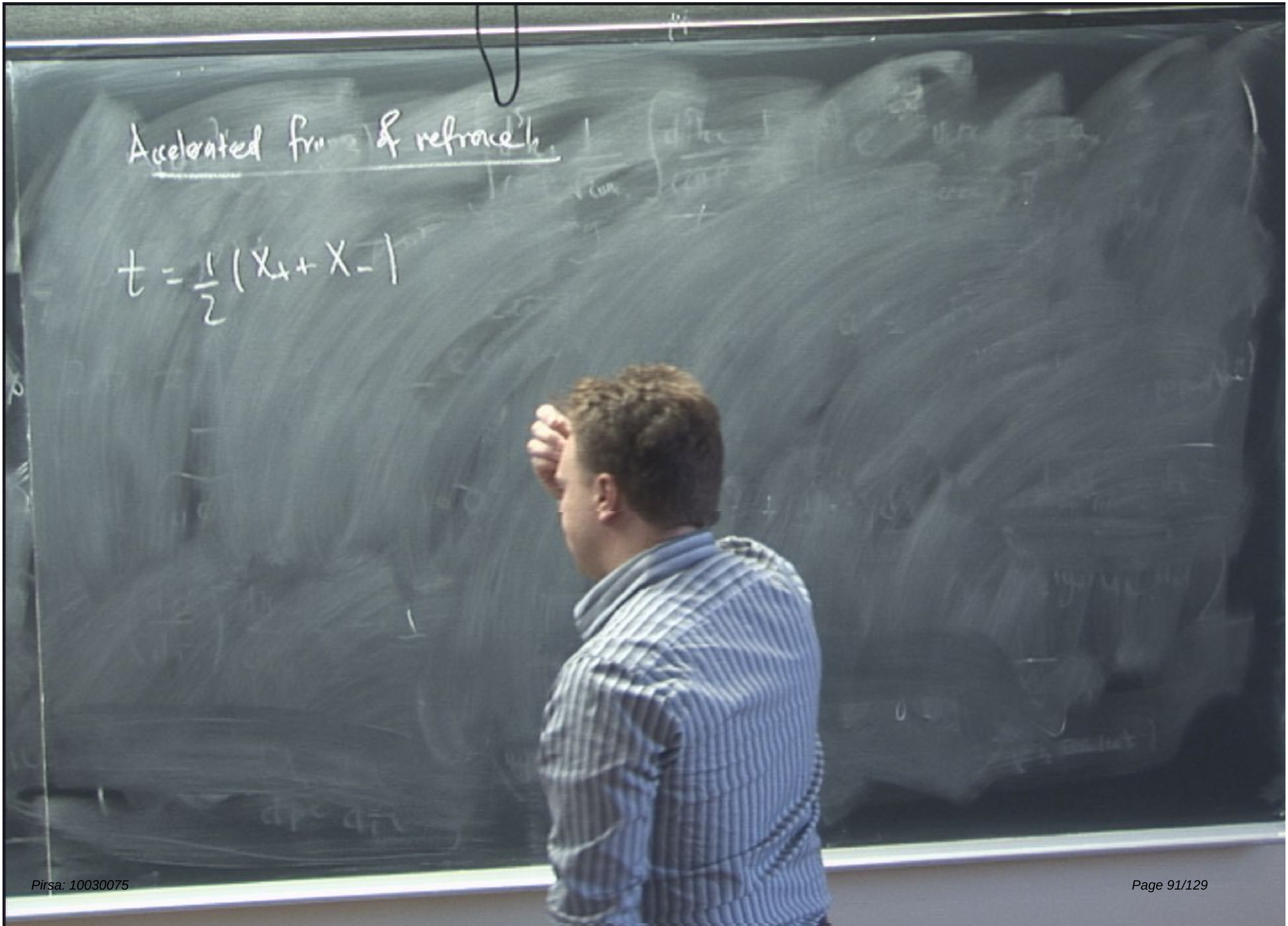
$$\ln \frac{dX_+}{d\tau} = A + a\tau$$

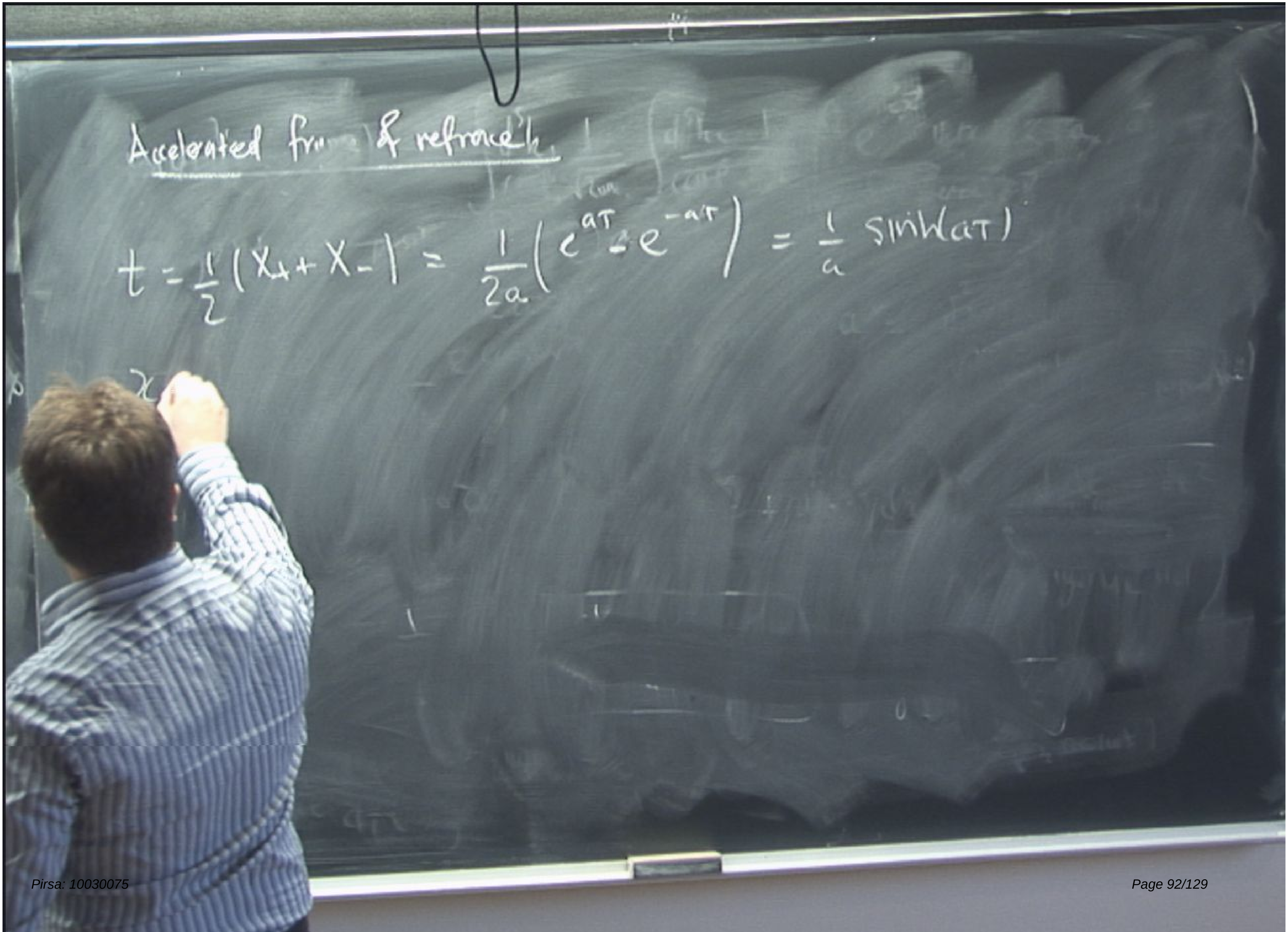
$$\frac{dX_+}{d\tau} = e^{A} e^{a\tau} \qquad \frac{dX_-}{d\tau} = e^{-A} e^{-a\tau}$$

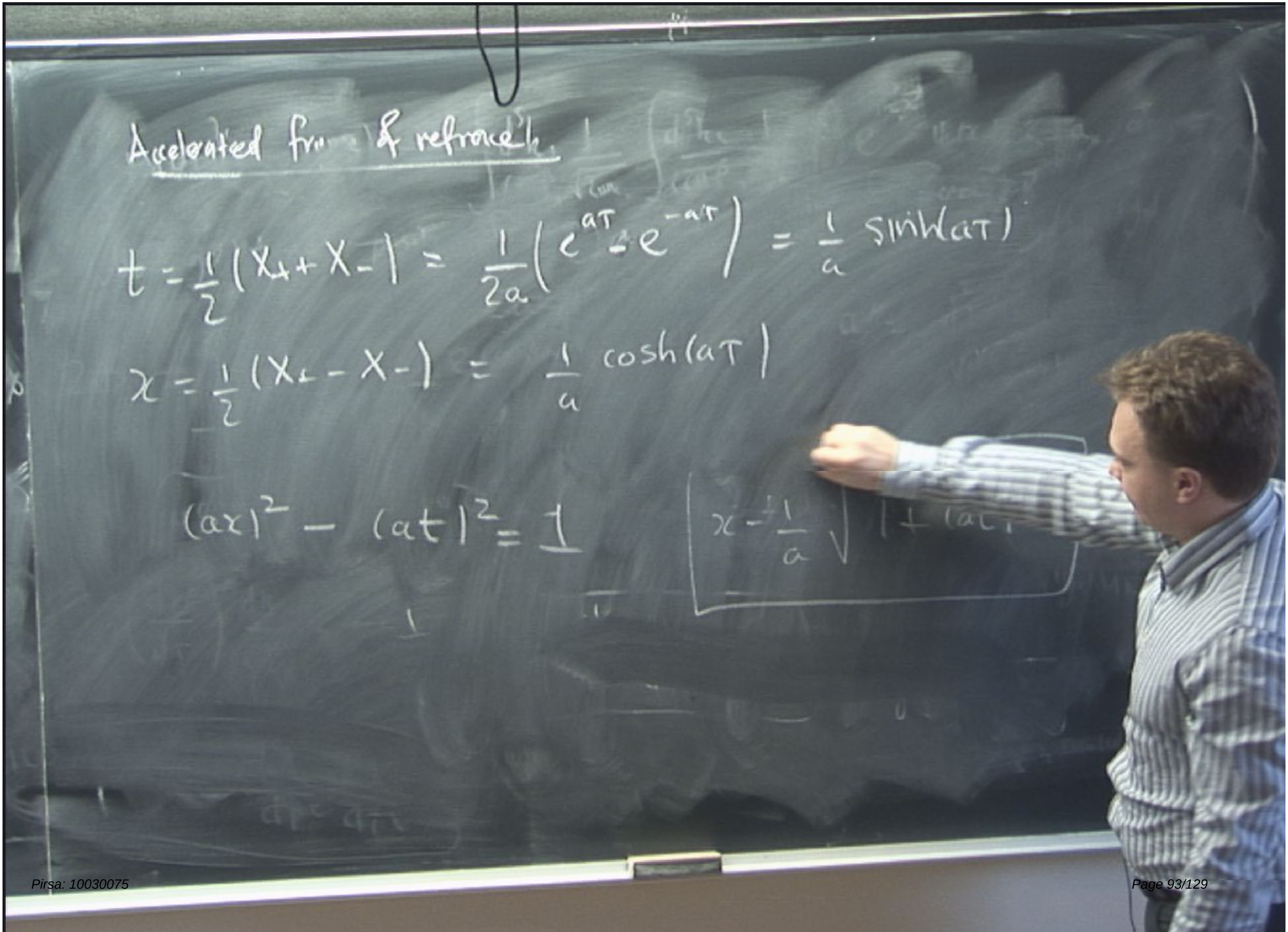
$$\boxed{X_+ = B + \frac{e^{A}}{a} e^{a\tau}} \qquad \boxed{X_- = C - \frac{e^{-A}}{a} e^{-a\tau}}$$

$B = C = 0$

$$X_\pm = \pm \frac{e^{\pm A}}{a} e^{\pm a\tau} \qquad A = 0$$


Accelerated
t = 1/2 (X+ + X-)


Accelerated fr... & refrac'h
$t = \frac{1}{2}(X_+ + X_-) = \frac{1}{2a}\left(e^{aT} - e^{-aT}\right) = \frac{1}{a}\sinh(aT)$
x


Accelerated fr & refrac
$t = \frac{1}{2}(X_+ + X_-) = \frac{1}{2a}\left(e^{aT} - e^{-aT}\right) = \frac{1}{a}\sinh(aT)$
$x = \frac{1}{2}(X_+ - X_-) = \frac{1}{a}\cosh(aT)$
$(ax)^2 - (at)^2 = 1$
$x = \frac{1}{a}\sqrt{1+(at}$

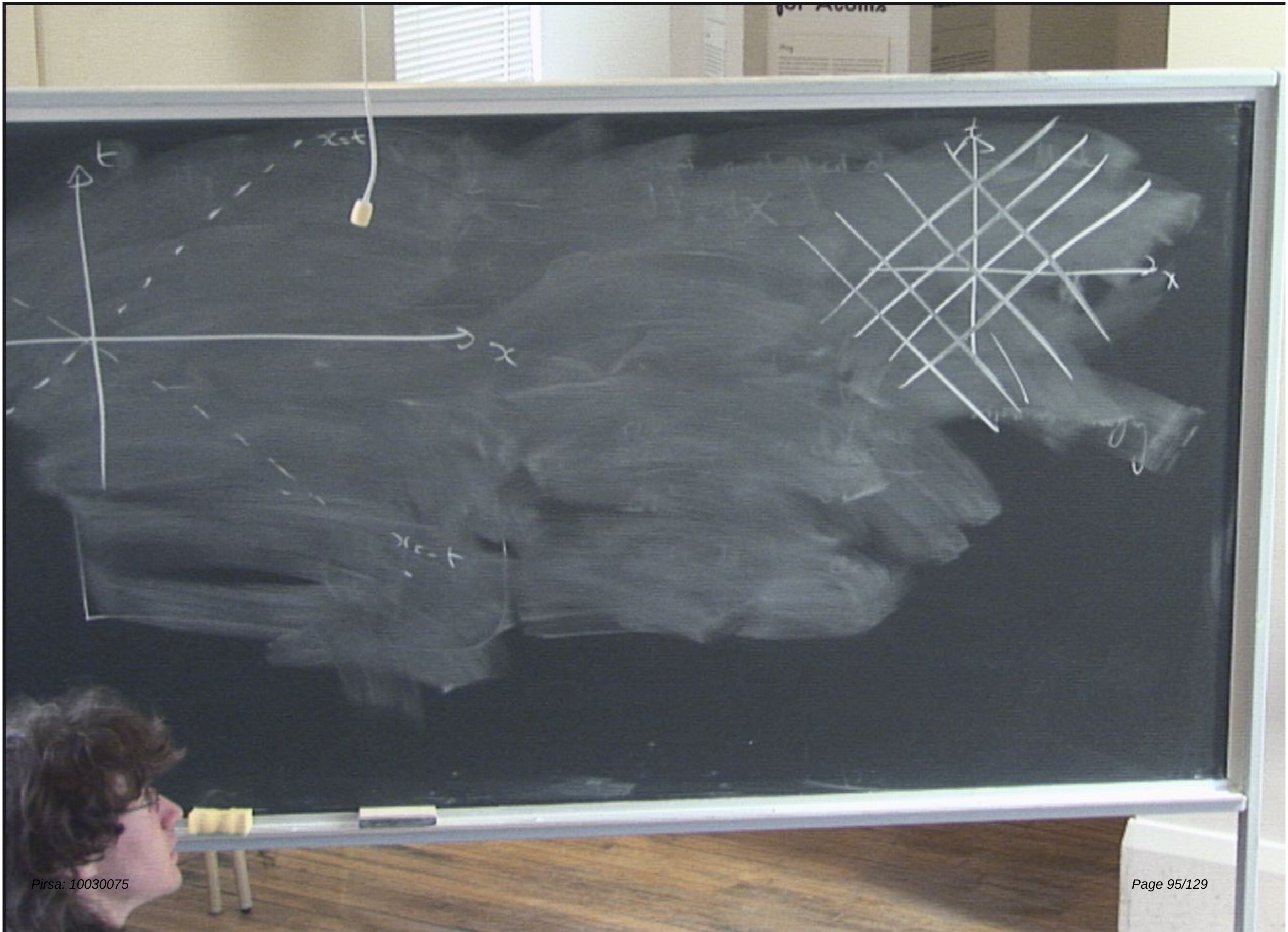
Accelerated fr & refrace'h

$$t = \frac{1}{2}(X_+ + X_-) = \frac{1}{2a}\left(e^{aT} - e^{-aT}\right) = \frac{1}{a}\sinh(aT)$$

$$x = \frac{1}{2}(X_+ - X_-) = \frac{1}{a}\cosh(aT)$$

$$(ax)^2 - (at)^2 = 1$$

$$\boxed{x = \frac{1}{a}\sqrt{1+(at)^2}}$$

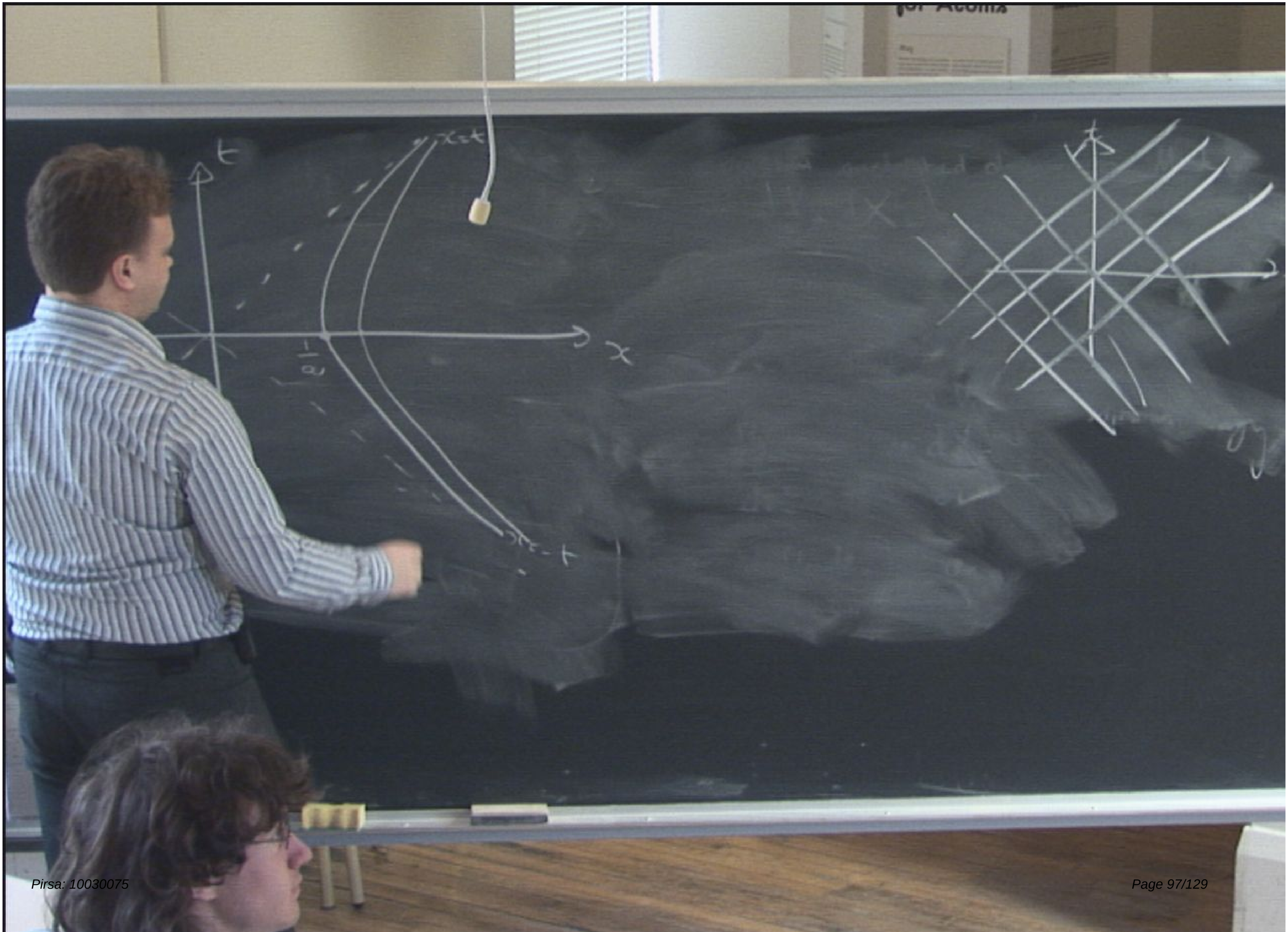

t
x=t
x
x=-t
t
x

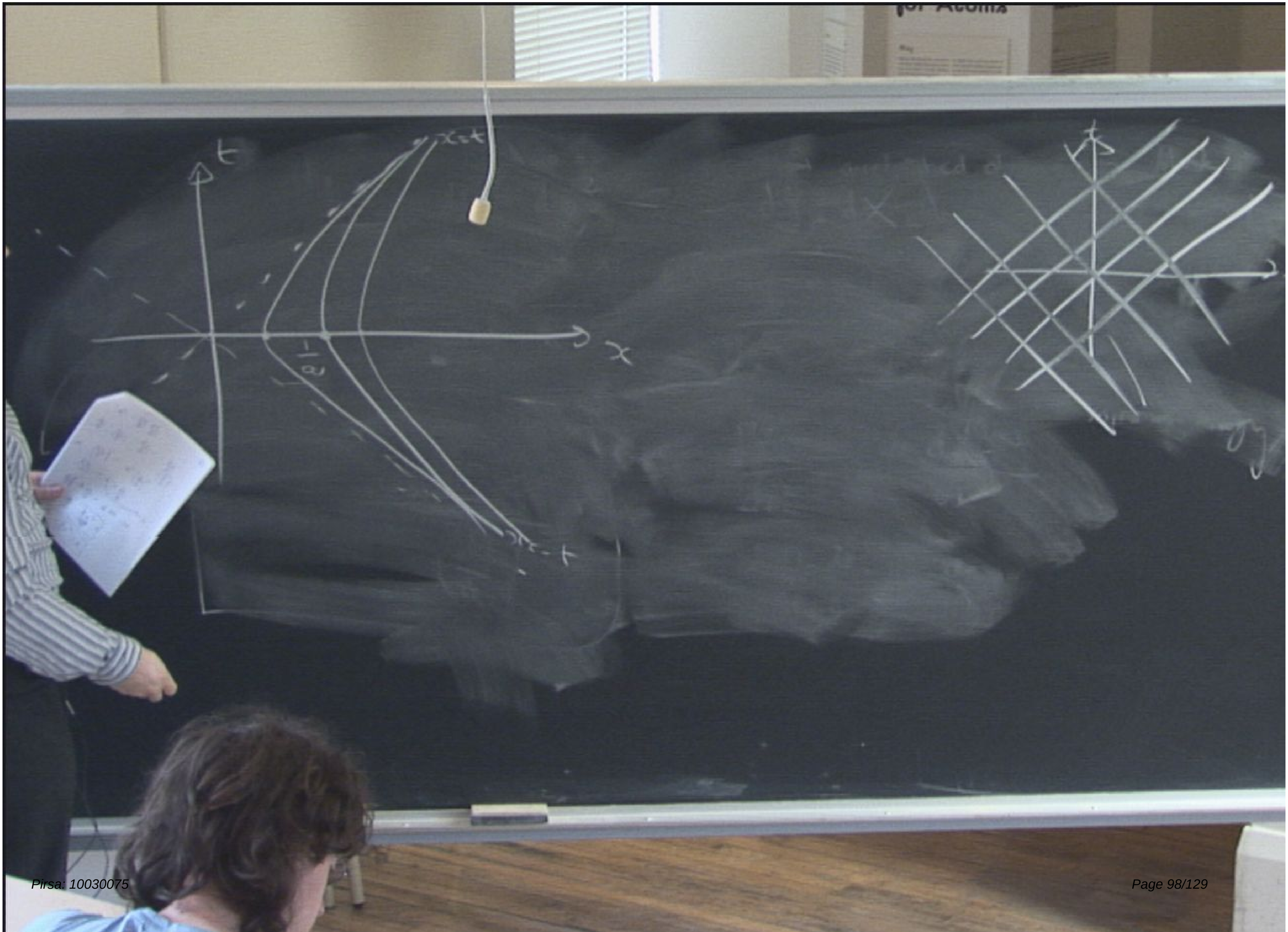
Accelerated fr... & refraction

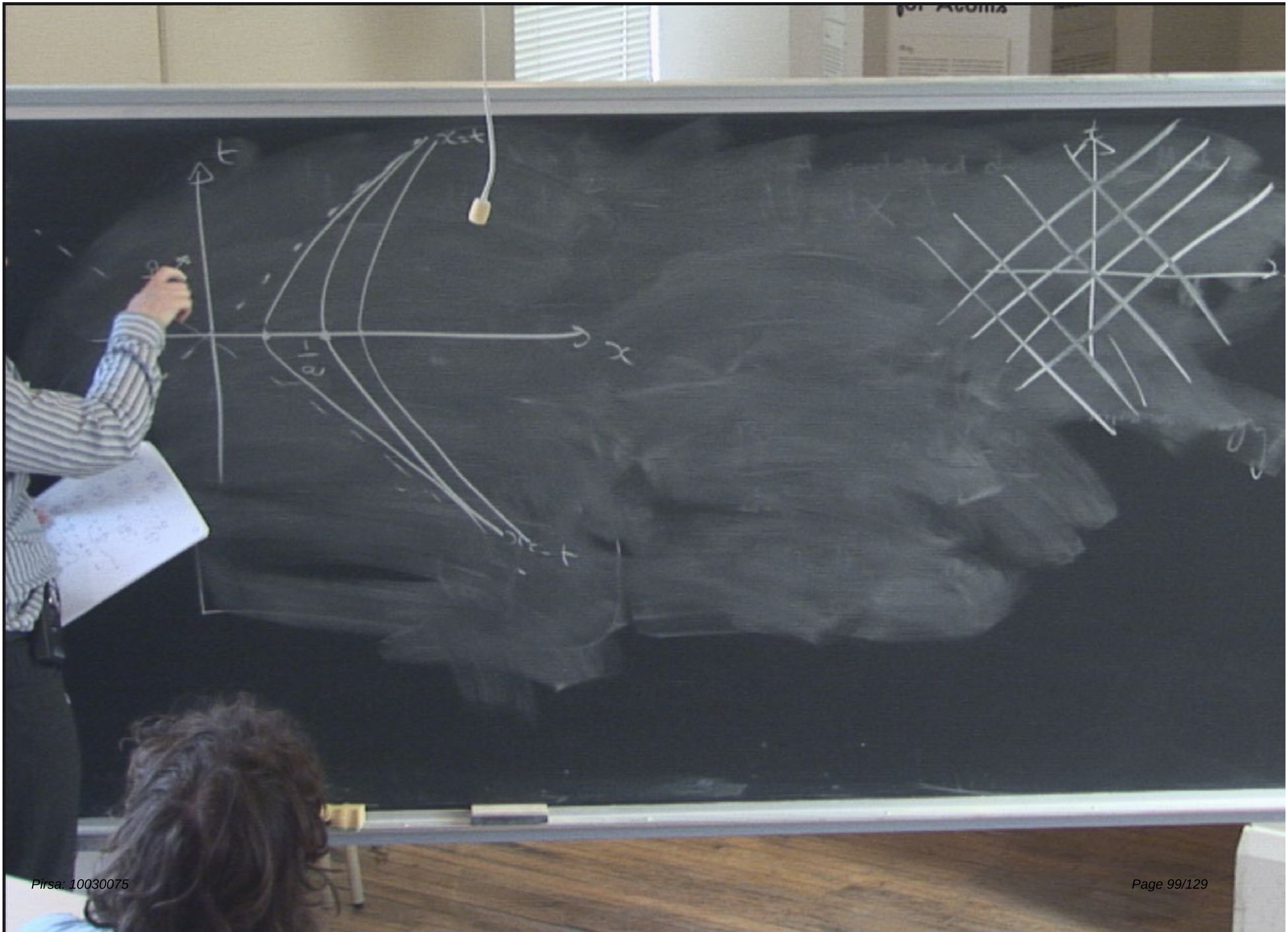
$$t = \frac{1}{2}(X_+ + X_-) = \frac{1}{2a}\left(e^{a\tau} - e^{-a\tau}\right) = \frac{1}{a}\sinh(a\tau)$$

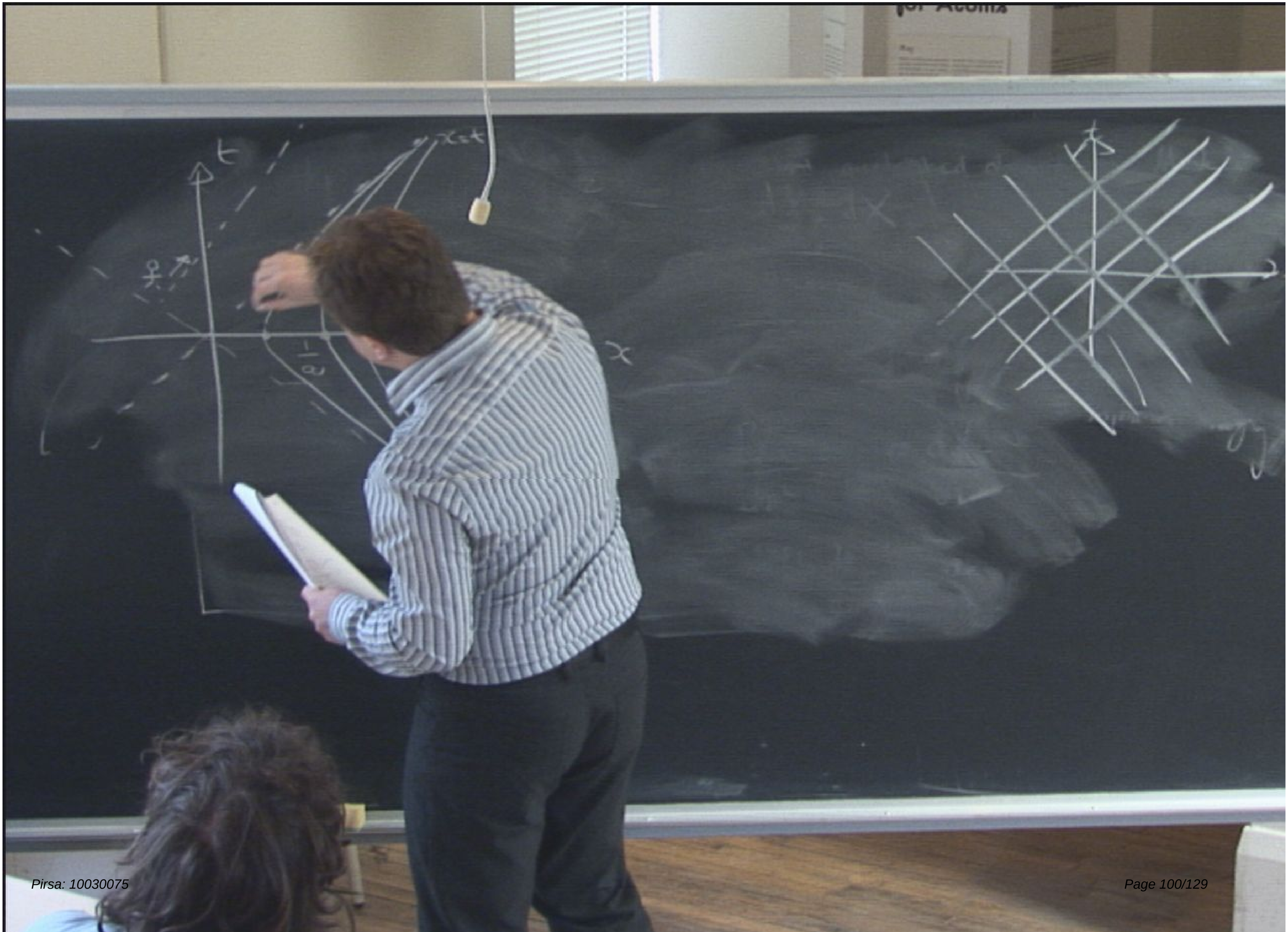
$$x = \frac{1}{2}(X_+ - X_-) = \frac{1}{a}\cosh(a\tau)$$

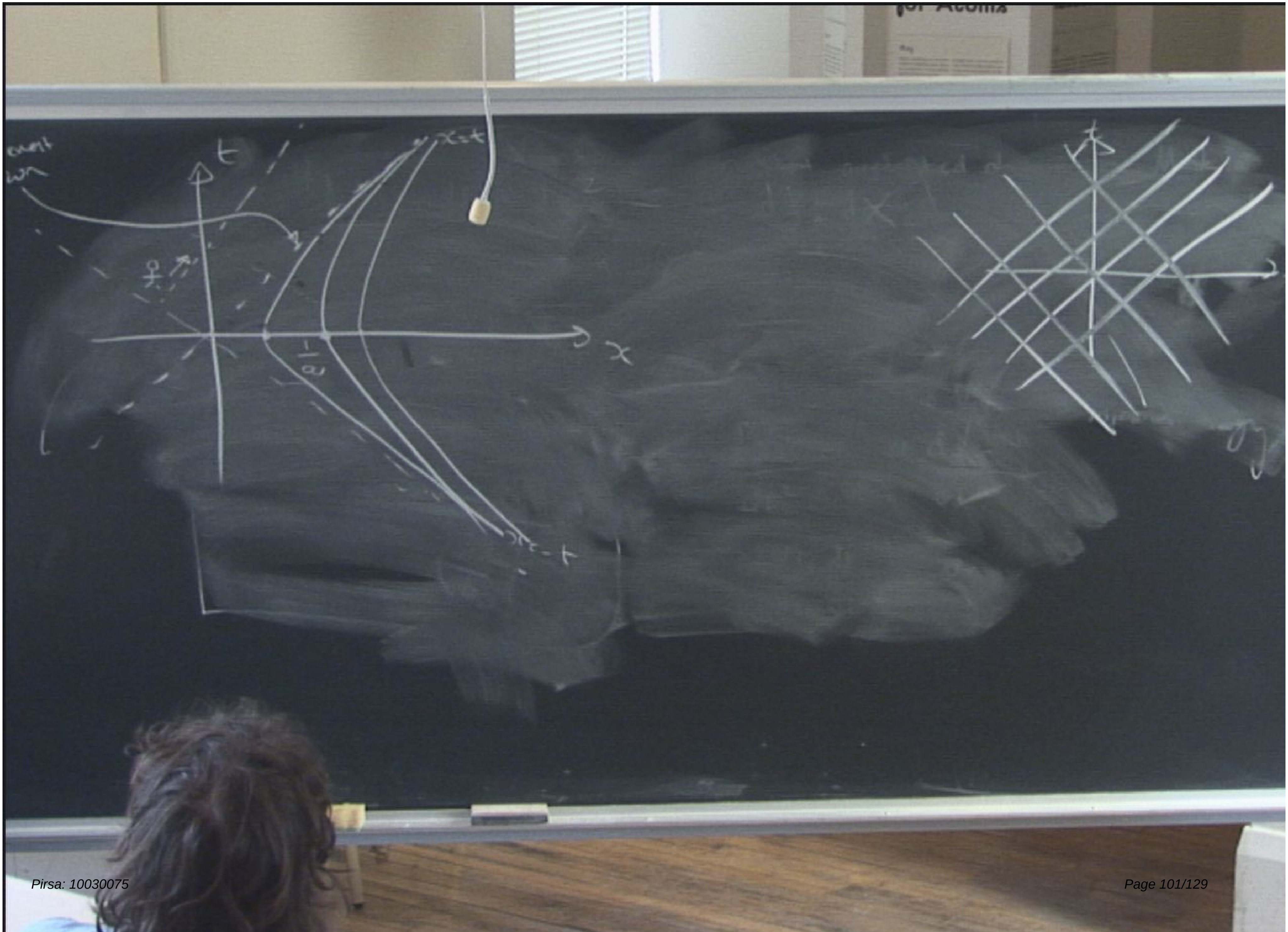
$$(ax)^2 - (at)^2 = 1 \qquad \boxed{x = \pm\frac{1}{a}\sqrt{1 + (at)^2}}$$

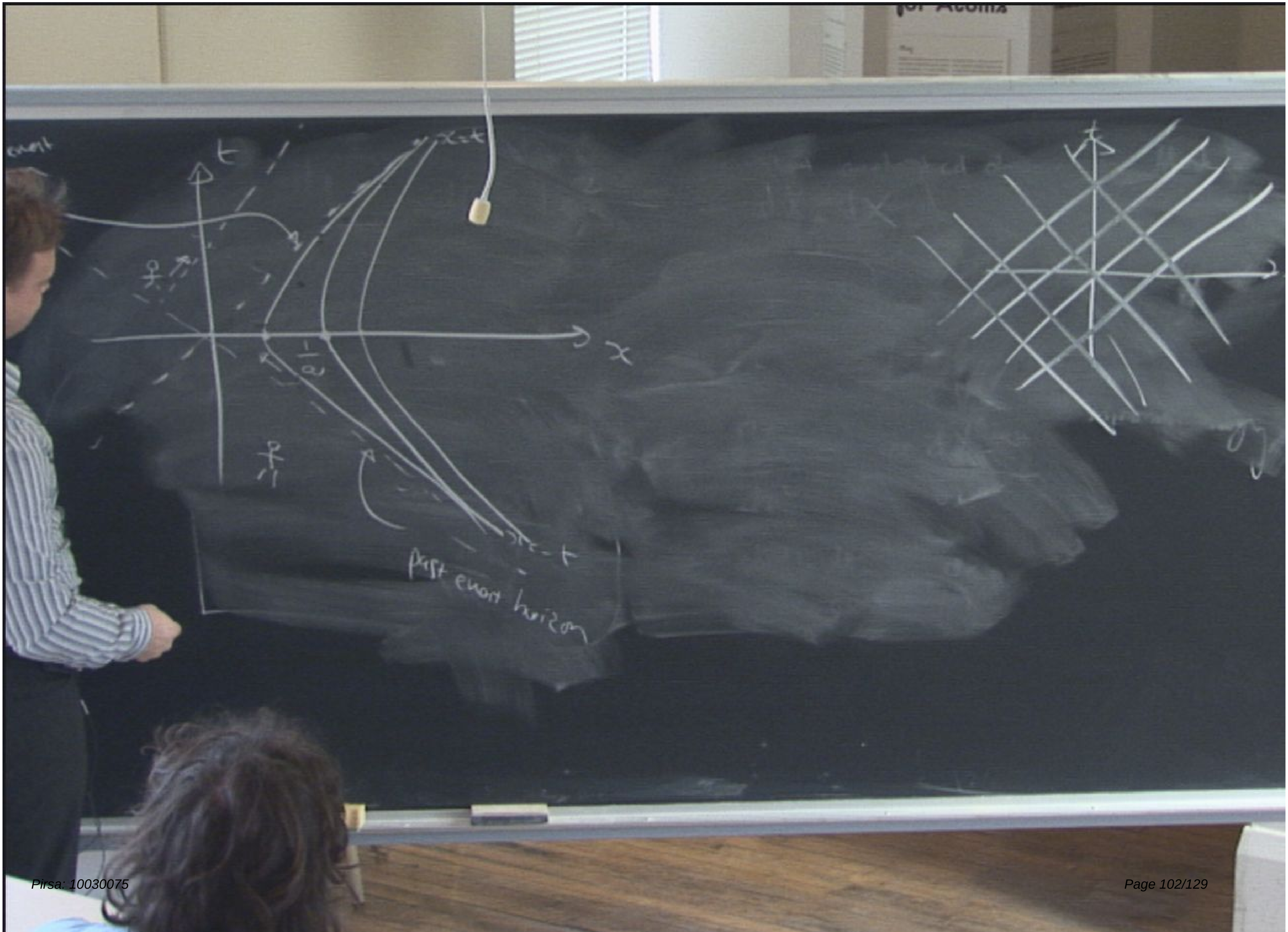

t
x=t
x
x=-t

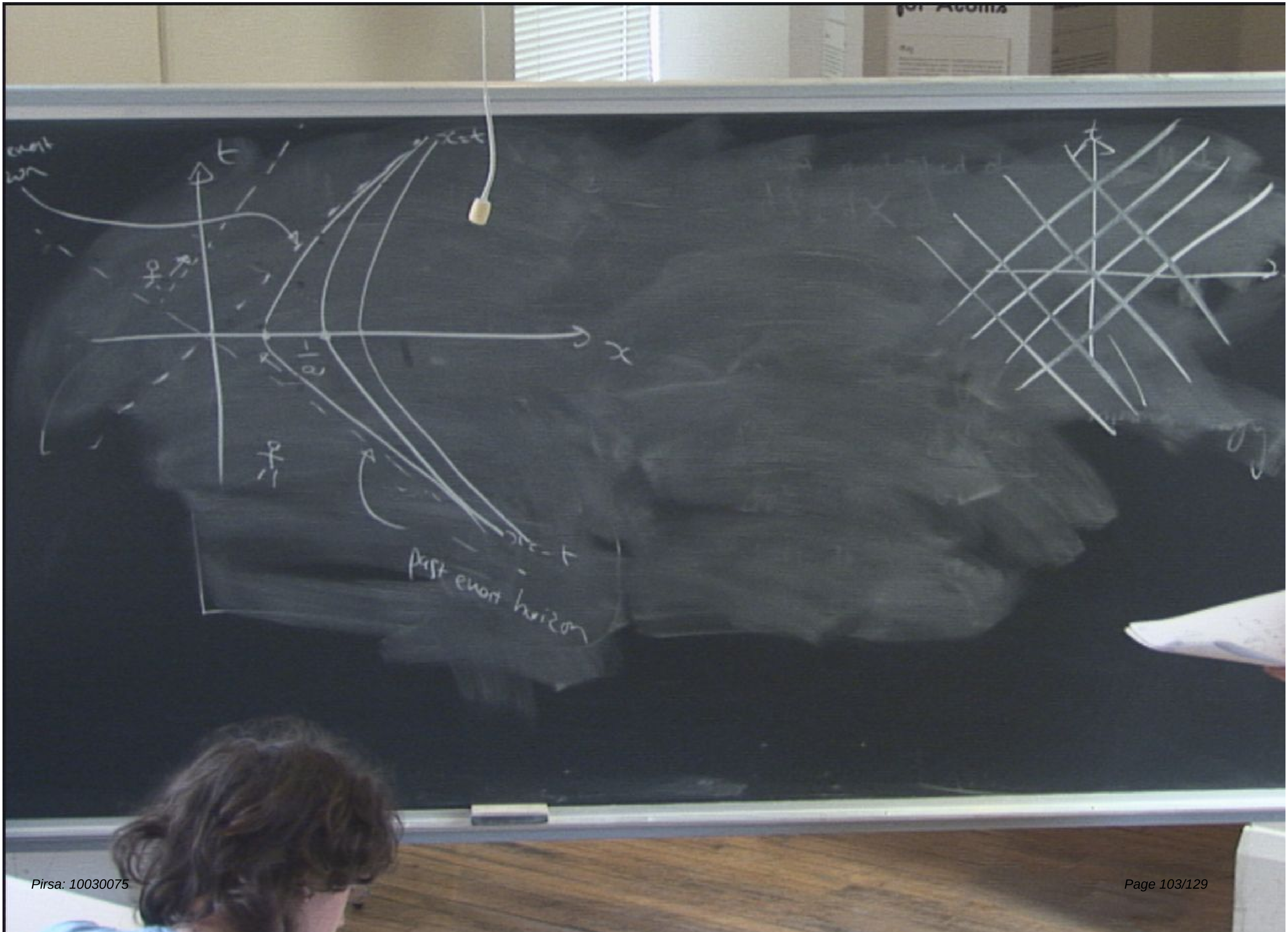

t
x=t
x
x=-t

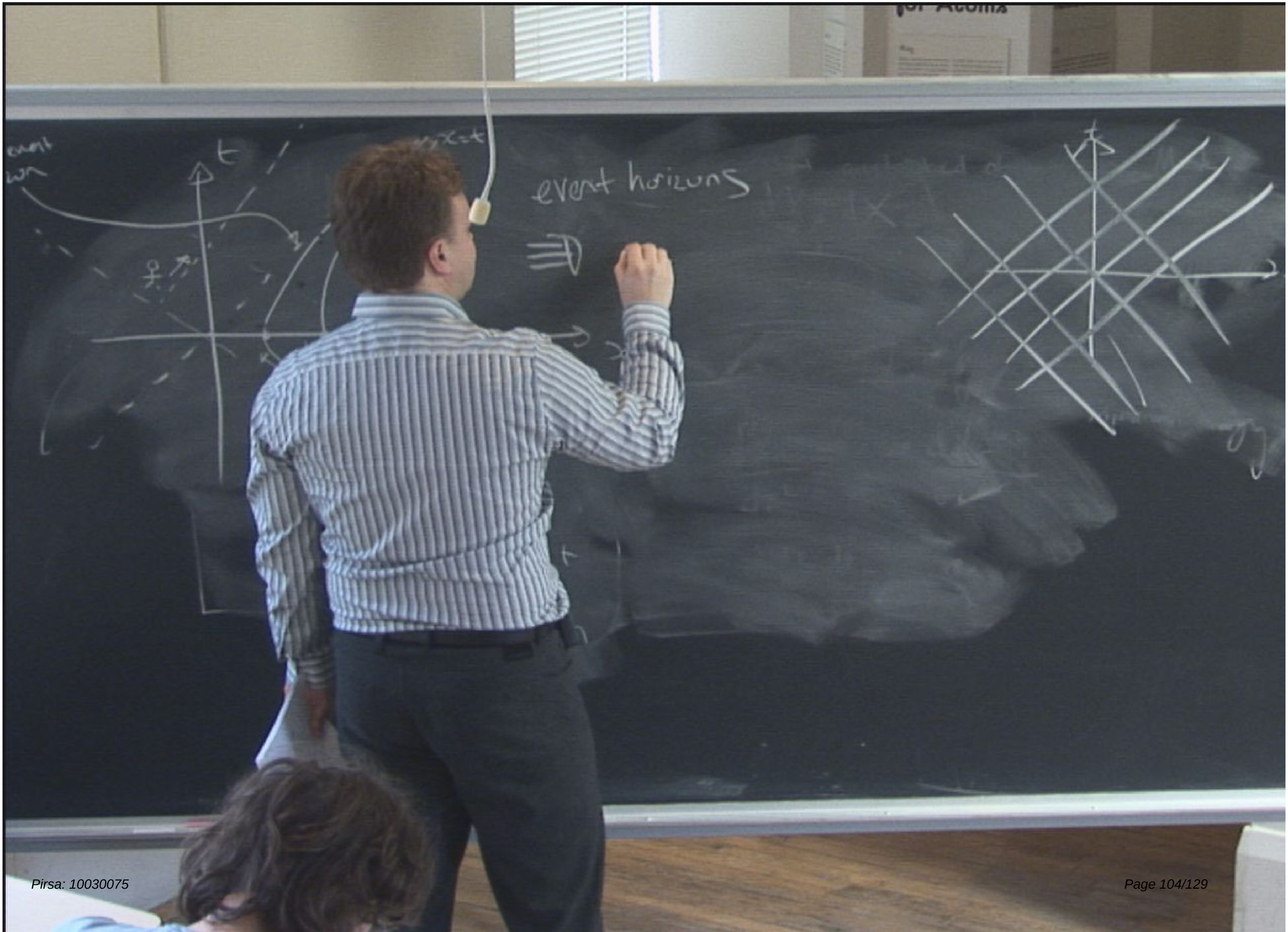

t
x=t
x
x=-t
t

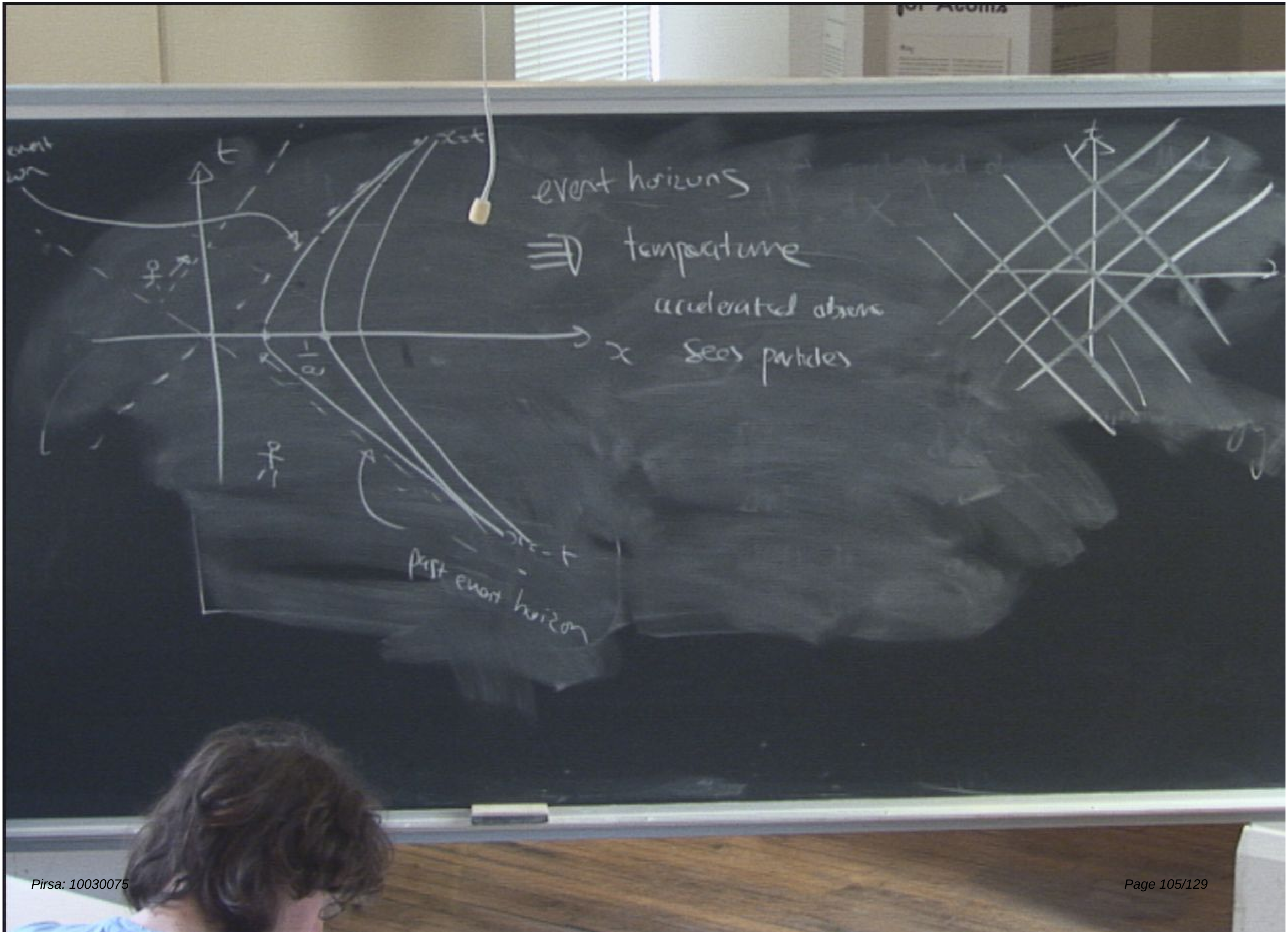


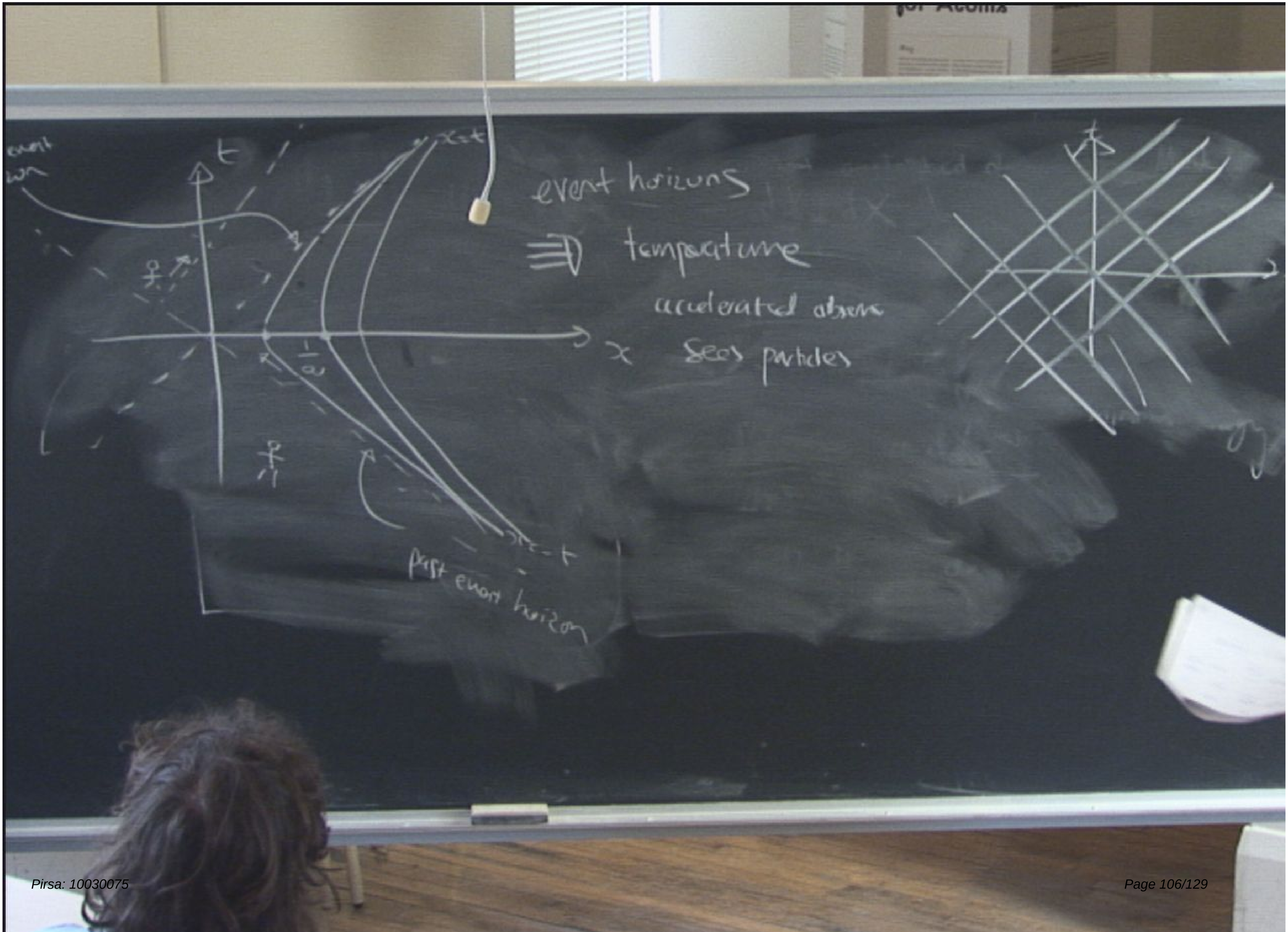

t
x=t
x
x=-t

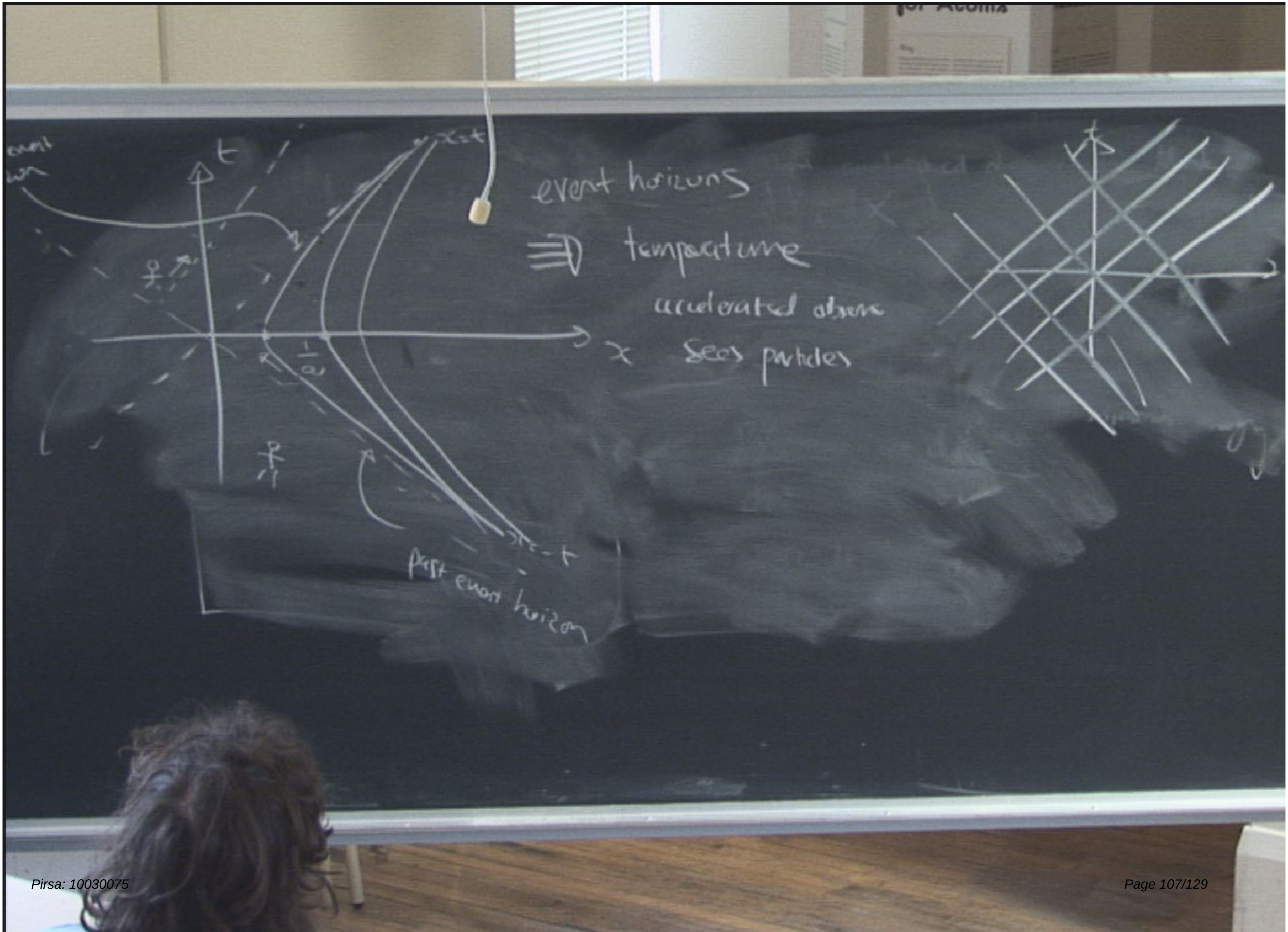

t
x=t
x
1/a
x=-t
past event horizon

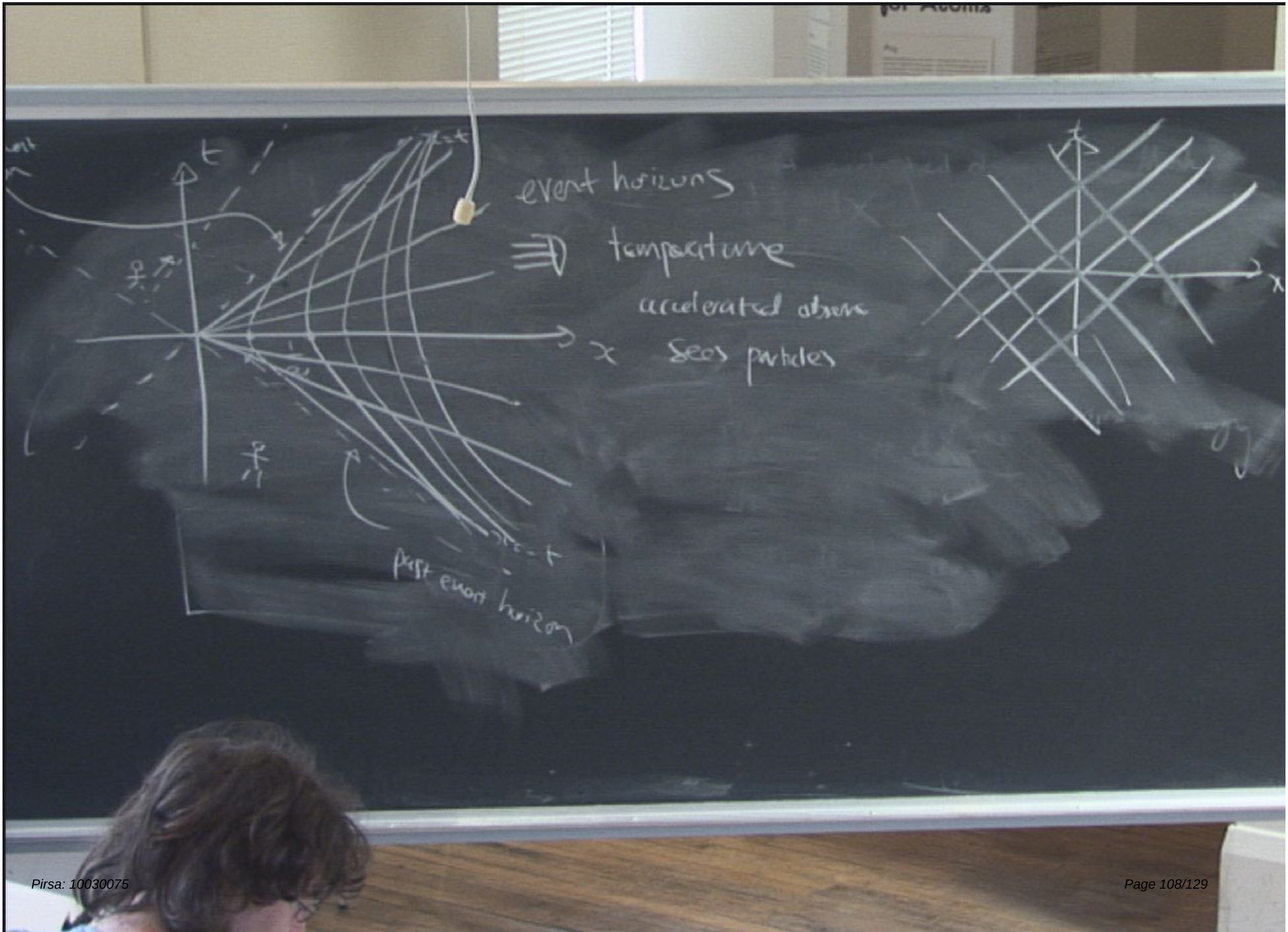

t
x
x=t
x=-t
past event horizon


event horizons


t
x=t
event horizons
⇒ temperature
accelerated observers
x
sees particles
1/a
x=-t
past event horizon


event horizons
temperature
accelerated obsers
sees particles
x=t
t
x
x=-t
past event horizon


t
x
x=t
x=-t
event horizons
temperature
accelerated observers
sees particles
past event horizon


event horizons
temperature
accelerated observer
sees particles
past event horizon
t
x

$$ds^2 = -\, dX_+ \, dX_-$$

$$X_+ = f_+(x_+) \qquad X_- = f_-(x_-)$$

$$ds^2 = -\, f_+' f_-' \, dx_+ \, dx_- = \Omega^2(x_+, x_-)\left(-\, dx_+ \, dx_-\right)$$

$$ds^2 = -\, dX_+ \, dX_-$$

$$X_+ = f_+(x_+) \qquad X_- = f_-(x_-)$$

$$ds^2 = -\, f_+' f_-' \, dx_+ \, dx_- = \Omega^2(x_+, x_-)\left(-\, dx_+ \, dx_-\right)$$

$$x_\pm = \tau \pm \rho$$

$ds^2 = - dX_+ dX_-$

$X_+ = f_+(x_+) \quad X_- = f_-(x_-)$

$ds^2 = - f_+' f_-' \, dx_+ dx_- = \Omega^2(x_+, x_-) \left(- dx_+ dx_- \right)$

$x_\pm = \tau \pm \rho$

time accelerated

space

$$ds^2 = -\,dX_+ dX_-$$

$$X_+ = f_+(x_+) \qquad X_- = f_-(x_-)$$

$$ds^2 = -\,f_+' f_-' \, dx_+ dx_- = \Omega^2(x_+, x_-)\,(-\,dx_+ dx_-)$$

$$X_\pm = \pm \frac{1}{a} e^{\pm a x_\pm}$$

$$x_\pm = \tau \pm \rho$$

time accelerated

space

$$ds^2 = -\,dX_+ dX_-$$

$$X_+ = f_+(x_+) \qquad X_- = f_-(x_-)$$

$$ds^2 = -f_+' f_-' \, dx_+ dx_- = \Omega^2(x_+, x_-)\left(-dx_+ dx_-\right)$$

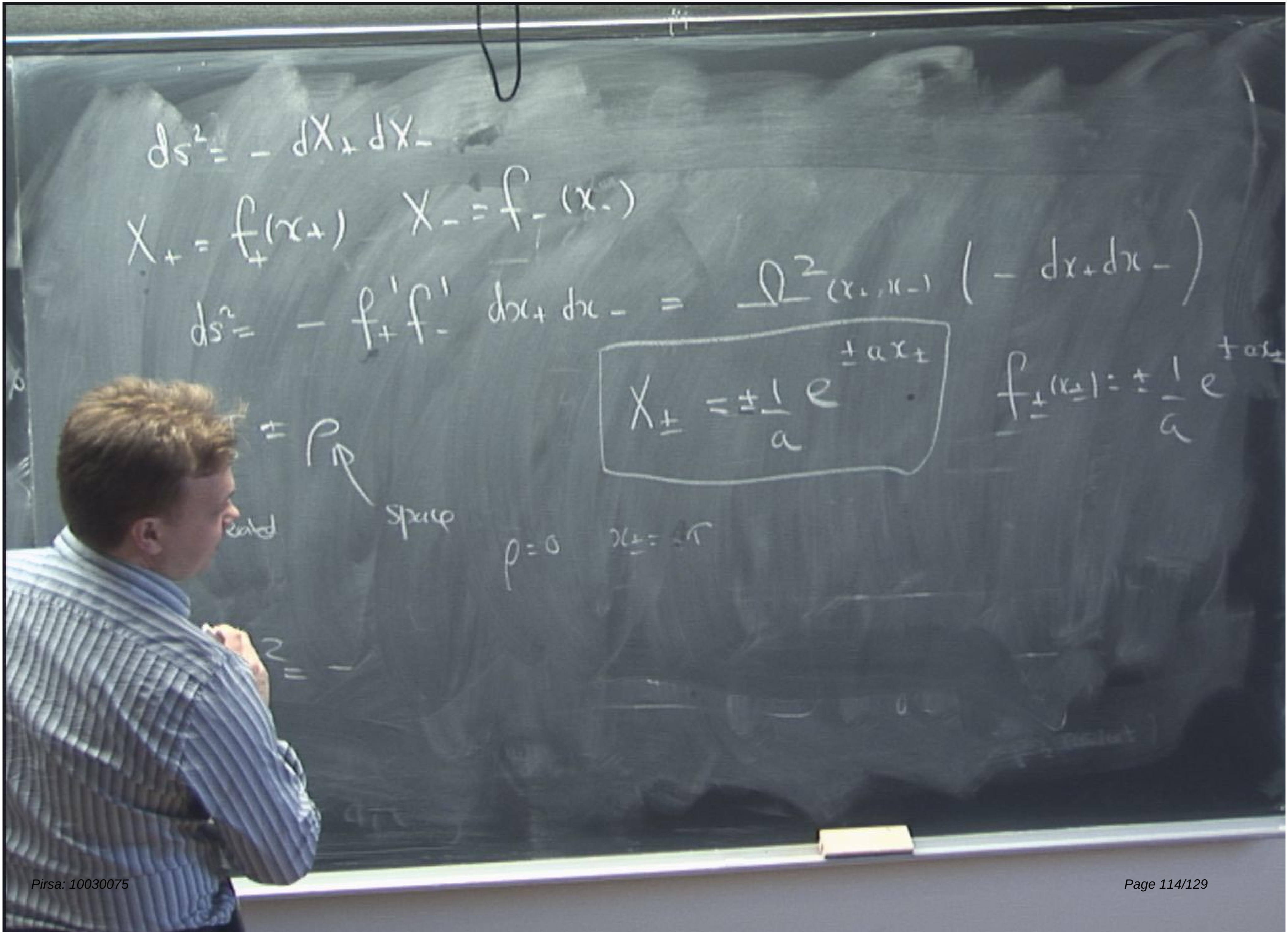
$$\boxed{X_\pm = \pm \frac{1}{a} e^{\pm a x_\pm}}$$

$$x_\pm = \tau \pm \rho$$

time accelerated

space

$\rho = 0 \quad x_\pm = \tau$


$ds^2 = - dX_+ dX_-$
$X_+ = f_+(x_+) \quad X_- = f_-(x_-)$
$ds^2 = - f_+' f_-' \, dx_+ dx_- = \Omega^2(x_+, x_-)(- dx_+ dx_-)$
$\boxed{X_\pm = \pm \frac{1}{a} e^{\pm a x_\pm}}$
$f_\pm(x_\pm) = \pm \frac{1}{a} e^{\pm a x_\pm}$
$\pm \rho$
space
$\rho = 0$

$$ds^2 = -\,dX_+\,dX_-$$

$$X_+ = f_+(x_+) \qquad X_- = f_-(x_-)$$

$$ds^2 = -\,f_+' f_-'\,dx_+\,dx_- = \Omega^2(x_+, x_-)\left(-\,dx_+\,dx_-\right)$$

$$\boxed{X_\pm = \pm\frac{1}{a}\,e^{\pm a x_\pm}} \qquad f_\pm(x_\pm) = \pm\frac{1}{a}\,e^{\pm a x}$$

$$x_\pm = \tau \pm \rho$$

time accelerated

space

$$\rho = 0 \qquad x_\pm = \tau$$

$$ds^2 = -\,e^{+ax_+}\,e^{ax_-}\left(dx_+\,dx_-\right) = -\,e^{a(x_+ + x_-)}\left(dx_+\,dx_-\right)$$

$$ds^2 = -\,dX_+\,dX_-$$

$$X_+ = f_+(x_+) \qquad X_- = f_-(x_-)$$

$$ds^2 = -\,f_+' f_-'\,dx_+\,dx_- = \Omega^2(x_+, x_-)\,(-\,dx_+\,dx_-)$$

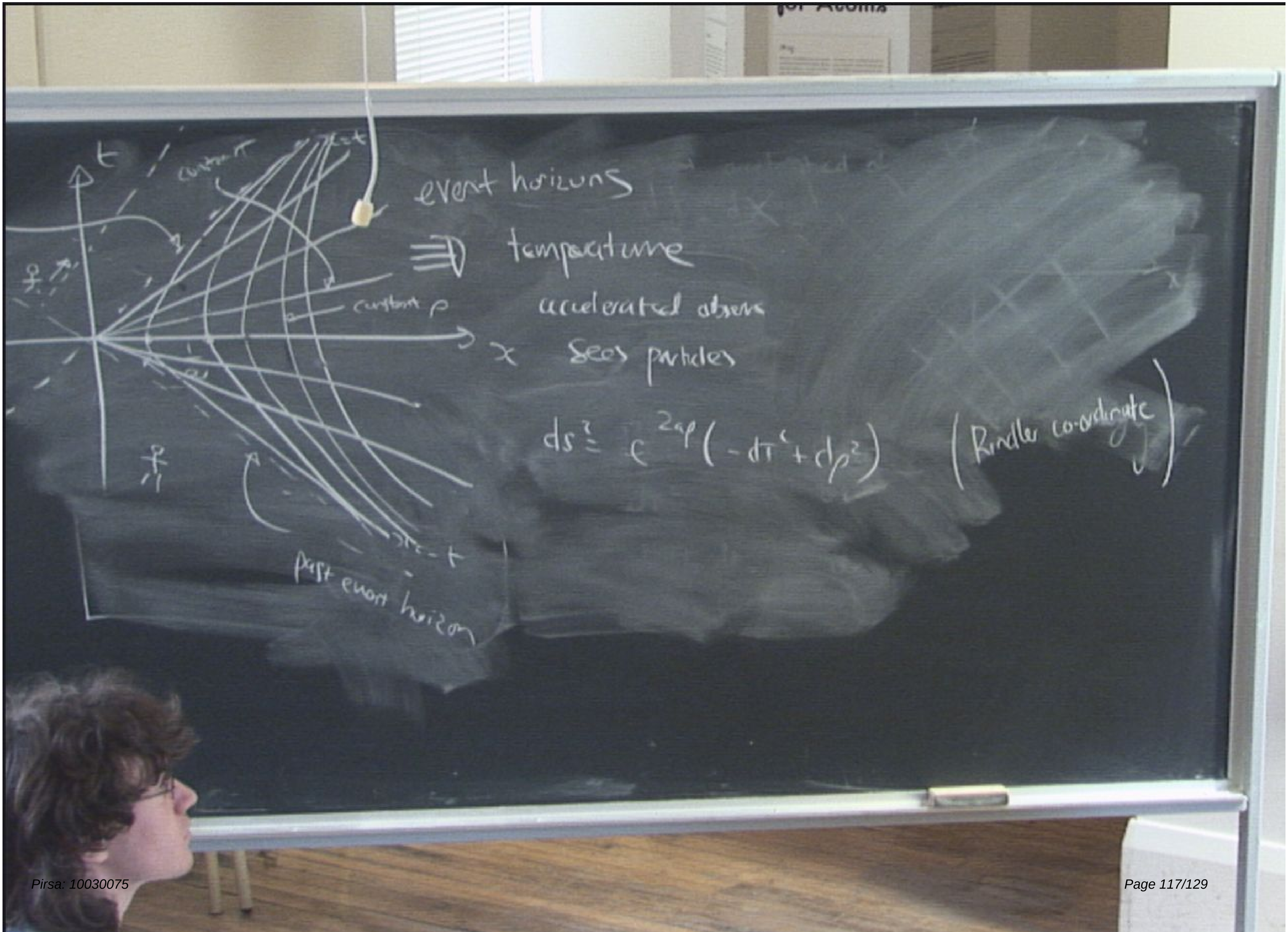
$$\boxed{X_\pm = \pm\frac{1}{a}e^{\pm a x_\pm}} \qquad f_\pm(x_\pm) = \pm\frac{1}{a}e^{\pm a x_\pm}$$

$$x_\pm = \tau \pm \rho$$

time accelerated   space

$$\rho = 0 \qquad x_\pm = \tau$$

$$ds^2 = -e^{+ax_+}e^{ax_-}(dx_+ dx_-) = -e^{a(x_+ + x_-)}(dx_+ dx_-) = \boxed{-e^{2a\rho}(d\tau^2 - d\rho^2)}$$


t
x
constant ρ
past event horizon
event horizons
temperature
accelerated observers
sees particles
ds² = e^{2aρ}(-dτ² + dρ²)
(Rindler co-ordinate)

$$ds^2 = -\,dX_+\,dX_-$$

$$X_+ = f_+(x_+) \qquad X_- = f_-(x_-)$$

$$ds^2 = -\,f_+'f_-'\,dx_+\,dx_- = \Omega^2(x_+,x_-)\left(-\,dx_+\,dx_-\right)$$

$$\boxed{X_\pm = \pm\frac{1}{a}e^{\pm a x_\pm}} \qquad f_\pm(x_\pm) = \pm\frac{1}{a}e^{\pm a x_\pm}$$

$$x_\pm = \tau \pm \rho$$

time accelerated

space

$$\rho = 0 \qquad x_\pm = \tau$$

$$ds^2 = -\,e^{+ax_+}e^{-ax_-}\left(dx_+\,dx_-\right) = -\,e^{a(x_+-x_-)}\left(dx_+dx_-\right) = \boxed{-\,e^{2a\rho}\left(d\tau^2 - d\rho^2\right)}$$

$$ds^2 = -\, dX_+ \, dX_-$$

$$X_+ = f_+(x_+) \qquad X_- = f_-(x_-)$$

$$ds^2 = -\, f_+' f_-' \, dx_+ \, dx_- = \underline{\Omega^2}(x_+, x_-) \left(-\, dx_+ dx_- \right)$$

$$\boxed{X_\pm = \pm \frac{1}{a} e^{\pm a x_\pm}} \qquad f_\pm(x_\pm) = \pm \frac{1}{a} e^{\pm a x_\pm}$$

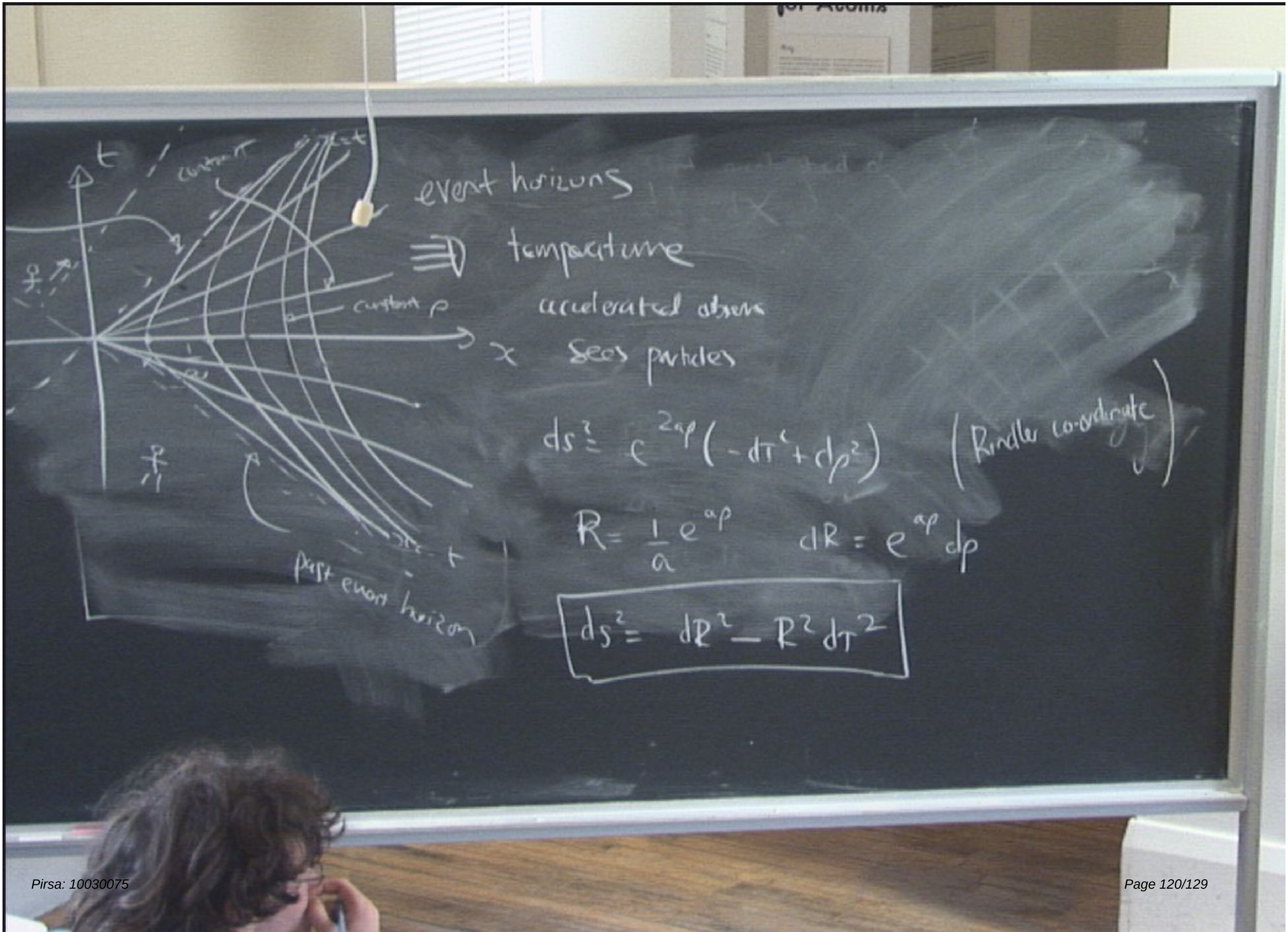
$$x_\pm = \tau \pm \rho$$

time accelerated — τ

space — ρ

$$\rho = 0 \qquad x_\pm = \pm\tau \qquad x_+ - x_- = \tau + \rho - (\tau - \rho) = 2\rho$$

$$ds^2 = -\, e^{+a x_+} e^{-a x_-} \left(dx_+ dx_- \right) = -\, e^{a(x_+ - x_-)} (dx_+ dx_-) = \boxed{-\, e^{2a\rho} (d\tau^2 - d\rho^2)}$$


t
x
constant ρ
event horizons
temperature
accelerated observers
sees particles
ds² = e^{2aρ}(-dτ² + dρ²)
(Rindler co-ordinate)
R = 1/a e^{aρ}
dR = e^{aρ} dρ
ds² = dR² − R² dτ²
past event horizon

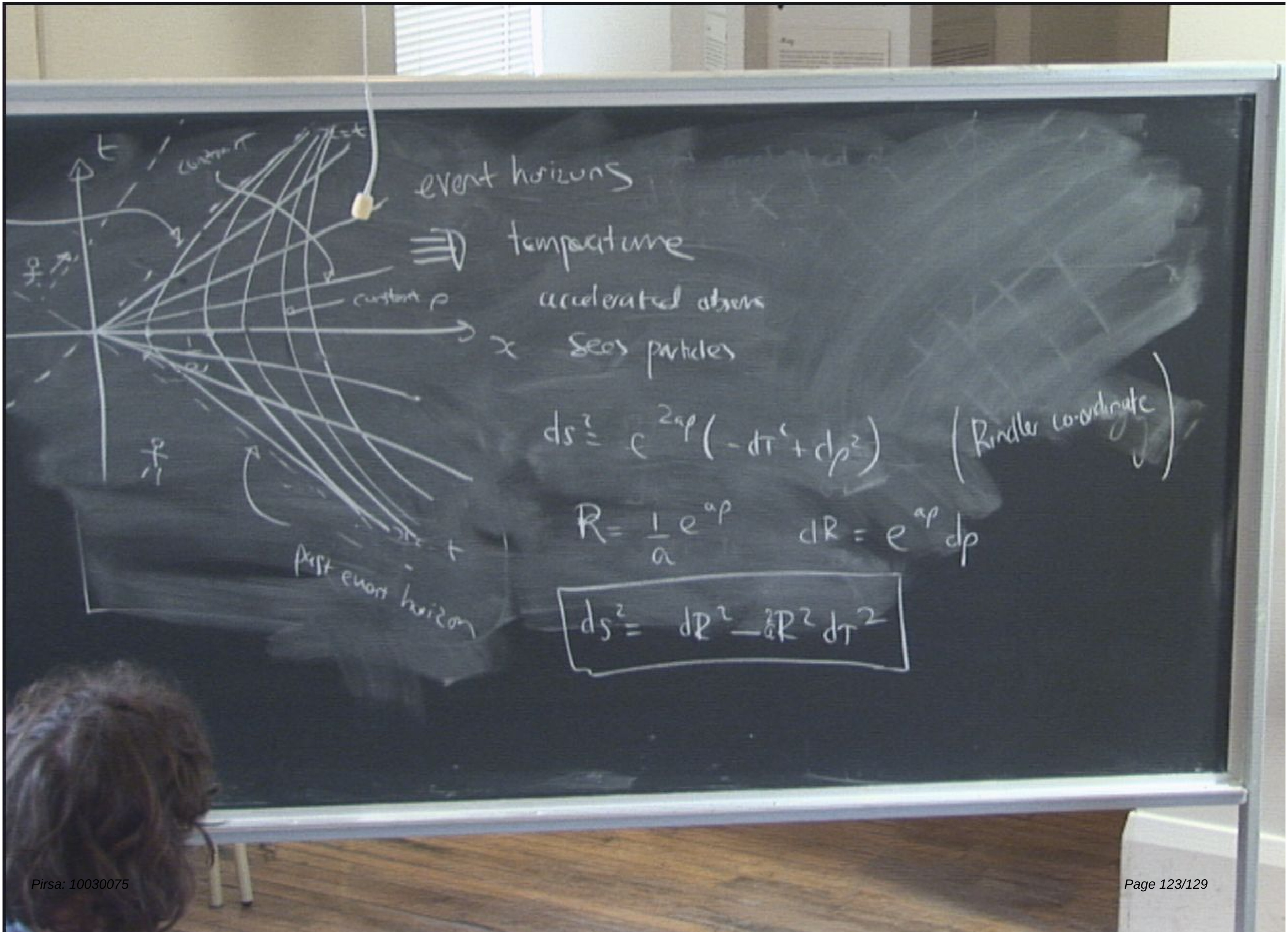
$T = i T_E$

$ds^2 = dR^2 + R^2 \, dT_E^2$

$T = i\tau_E$

$$\boxed{ds^2 = dR^2 + R^2 d\tau_E^2}$$

τ_E —

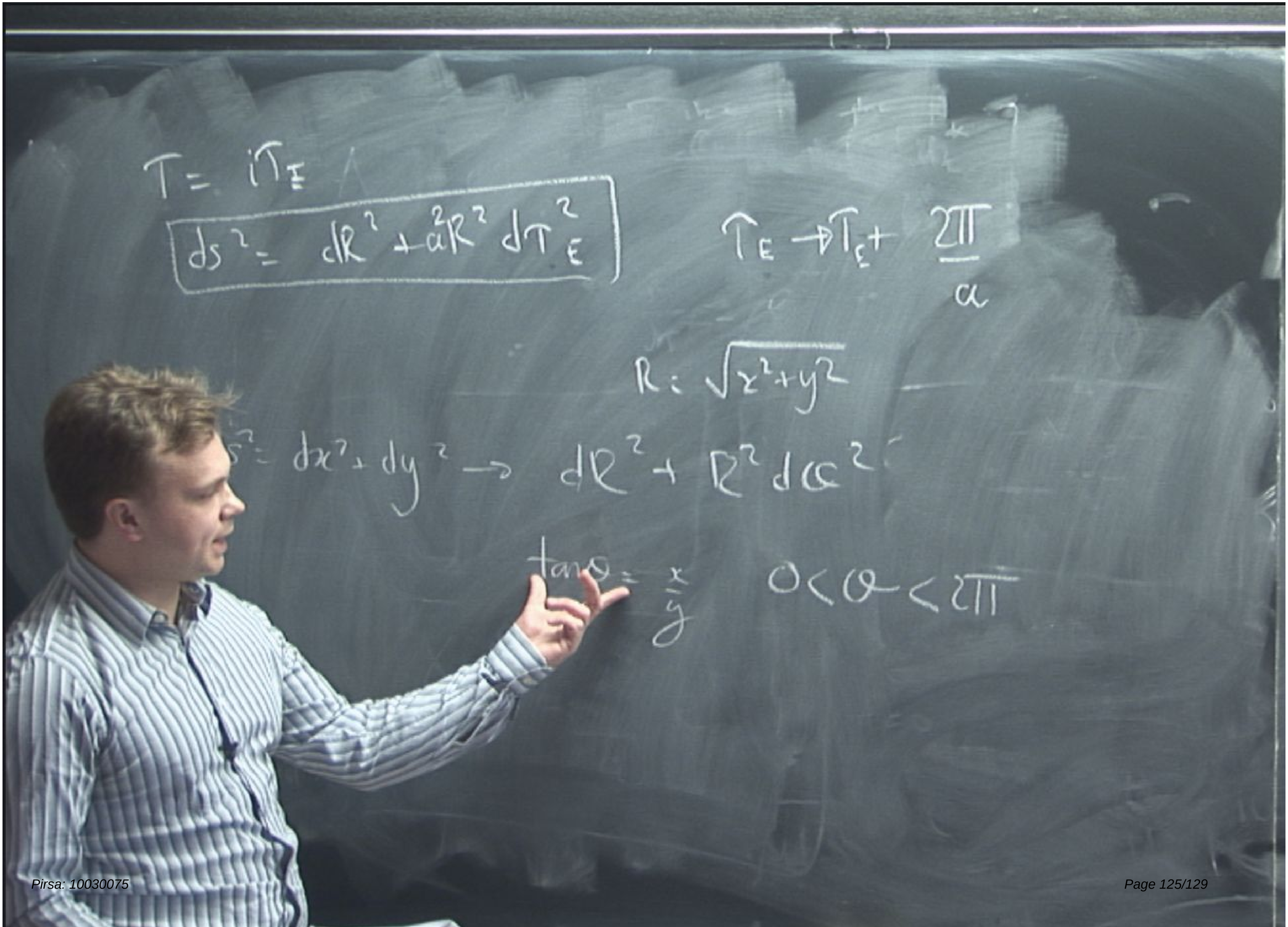


t
constant T
x = t
event horizons
⇒ temperature
constant ρ
accelerated observers
x
sees particles
$ds^2 = e^{2a\rho}(-d\tau^2 + d\rho^2)$
(Rindler co-ordinate)
$R = \frac{1}{a} e^{a\rho}$
$dR = e^{a\rho} d\rho$
$\boxed{ds^2 = dR^2 - a^2R^2 d\tau^2}$
x = -t
past event horizon

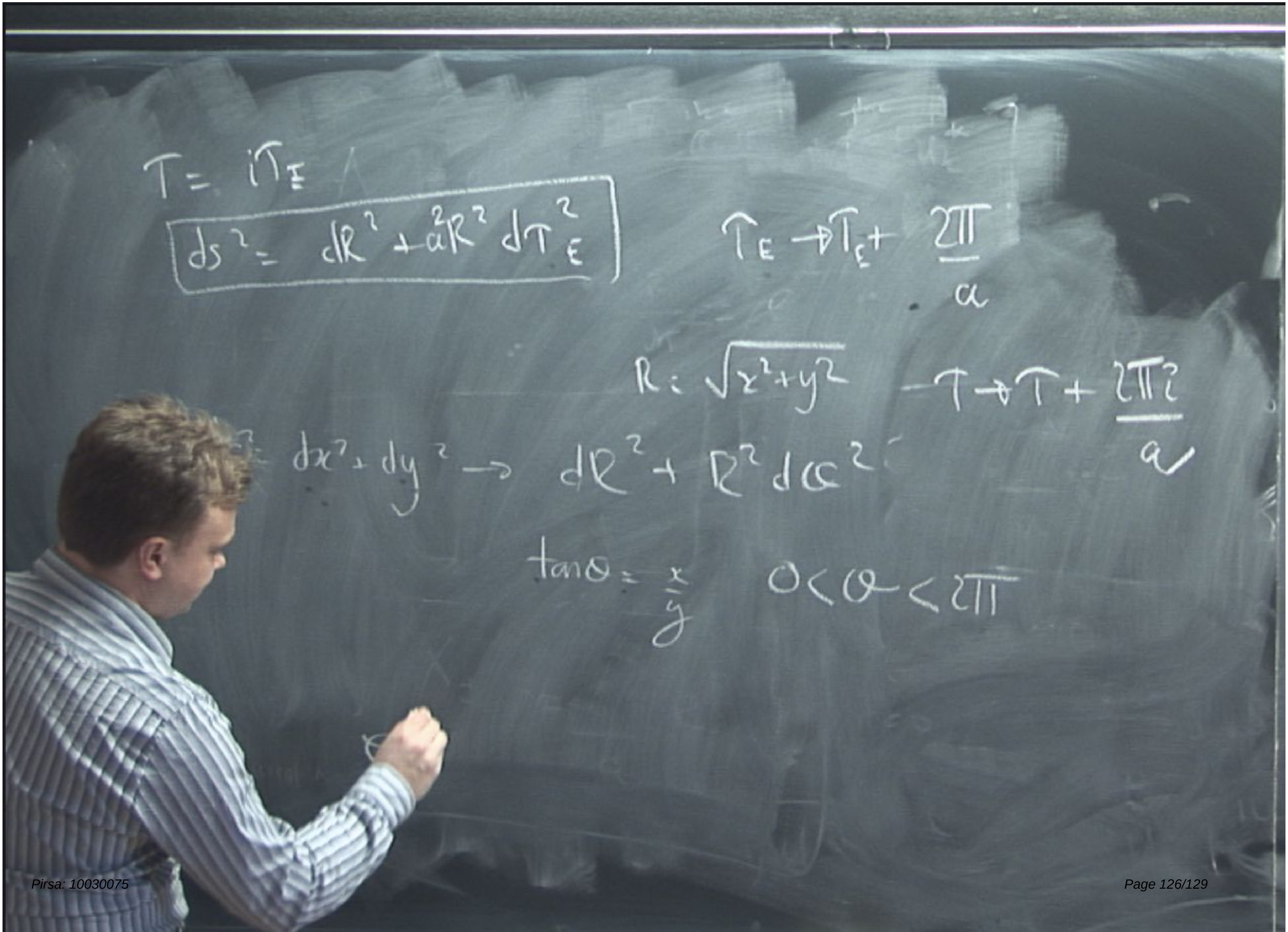
$T = i\tau_E$

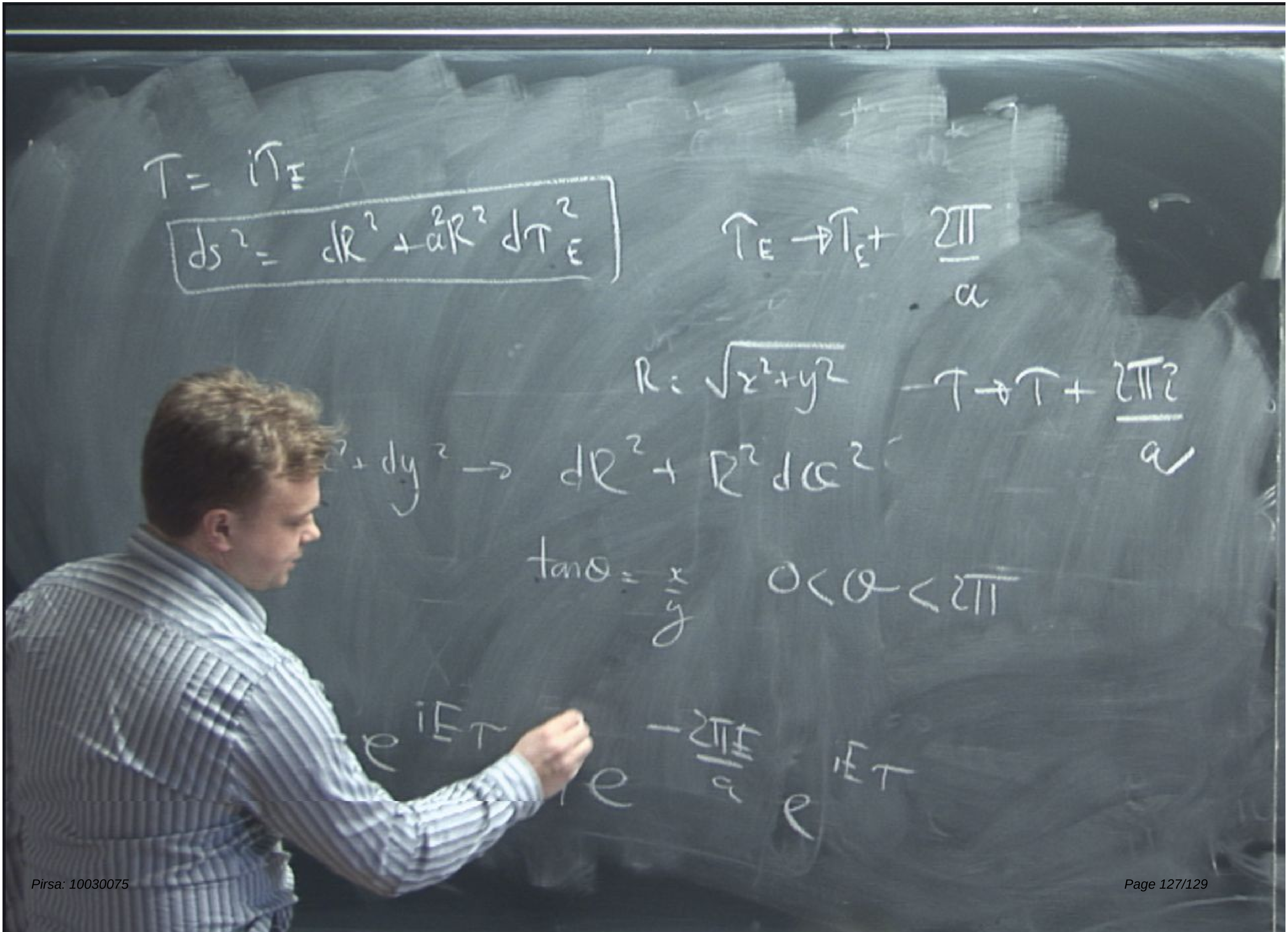
$$\boxed{ds^2 = dR^2 + a^2R^2\, d\tau_E^2}$$

$$\tau_E \to \tau_E + \frac{2\pi}{a}$$



$T = i\tau_E$
$\boxed{ds^2 = dR^2 + a^2R^2 d\tau_E^2}$
$\tau_E \to \tau_E + \frac{2\pi}{a}$
$R = \sqrt{x^2+y^2}$
$s^2 = dx^2 + dy^2 \to dR^2 + R^2 d\theta^2$
$\tan\theta = \frac{x}{y}$ $0 < \theta < 2\pi$


$T = iT_E$
$\boxed{ds^2 = dR^2 + a^2R^2\,dT_E^2}$
$T_E \to T_E + \frac{2\pi}{a}$
$R = \sqrt{x^2+y^2}$
$T \to T + \frac{2\pi i}{a}$
$dx^2 + dy^2 \to dR^2 + R^2\,d\theta^2$
$\tan\theta = \frac{x}{y}$
$0 < \theta < 2\pi$


T = iT_E
ds^2 = dR^2 + a^2R^2 dT_E^2
T_E → T_E + 2π/a
R = √(x^2+y^2)
T → T + 2πi/a
dy^2 → dR^2 + R^2 dθ^2
tan θ = x/y
0 < θ < 2π
e^{iET}
e^{-2πE/a} e^{iET}

$\tau = i\tau_E$

$$\boxed{ds^2 = dR^2 + a^2R^2\, d\tau_E^2}$$

$$\tau_E \to \tau_E + \frac{2\pi}{a}$$

$$R = \sqrt{x^2+y^2} \qquad \tau \to \tau + \frac{2\pi i}{a}$$

$$ds^2 = dx^2 + dy^2 \to dR^2 + R^2\, d\theta^2$$

$$\tan\theta = \frac{x}{y} \qquad 0 < \theta < 2\pi$$

$$e^{iE\tau} = \boxed{e^{-\frac{2\pi E}{a}}}\, e^{iE\tau}$$

$+\ \frac{2\pi}{a}$

$\tau \to \tau + \frac{2\pi i}{a}$

$\vartheta < 2\pi$

$E\tau \quad e^{-\beta E}$

$$ds^2 = -\,dX_+\,dX_-$$

$$X_+ = f_+(x_+) \quad X_- = f_-(x_-)$$

$$ds^2 = -\,f_+'f_-'\,dx_+\,dx_- = \Omega^2$$

$$x_\pm = \tau \pm \rho$$

time accelerated

space

$\rho = 0 \quad x_\pm =$

$$ds^2 = -e^{+ax_+}e^{-ax_-}(dx_+dx_-)$$

$$\beta = \frac{1}{kT} = \frac{2\pi}{a}$$

$$\boxed{T = \frac{a}{2\pi k}}$$

$\boxed{X_+ \ldots}$