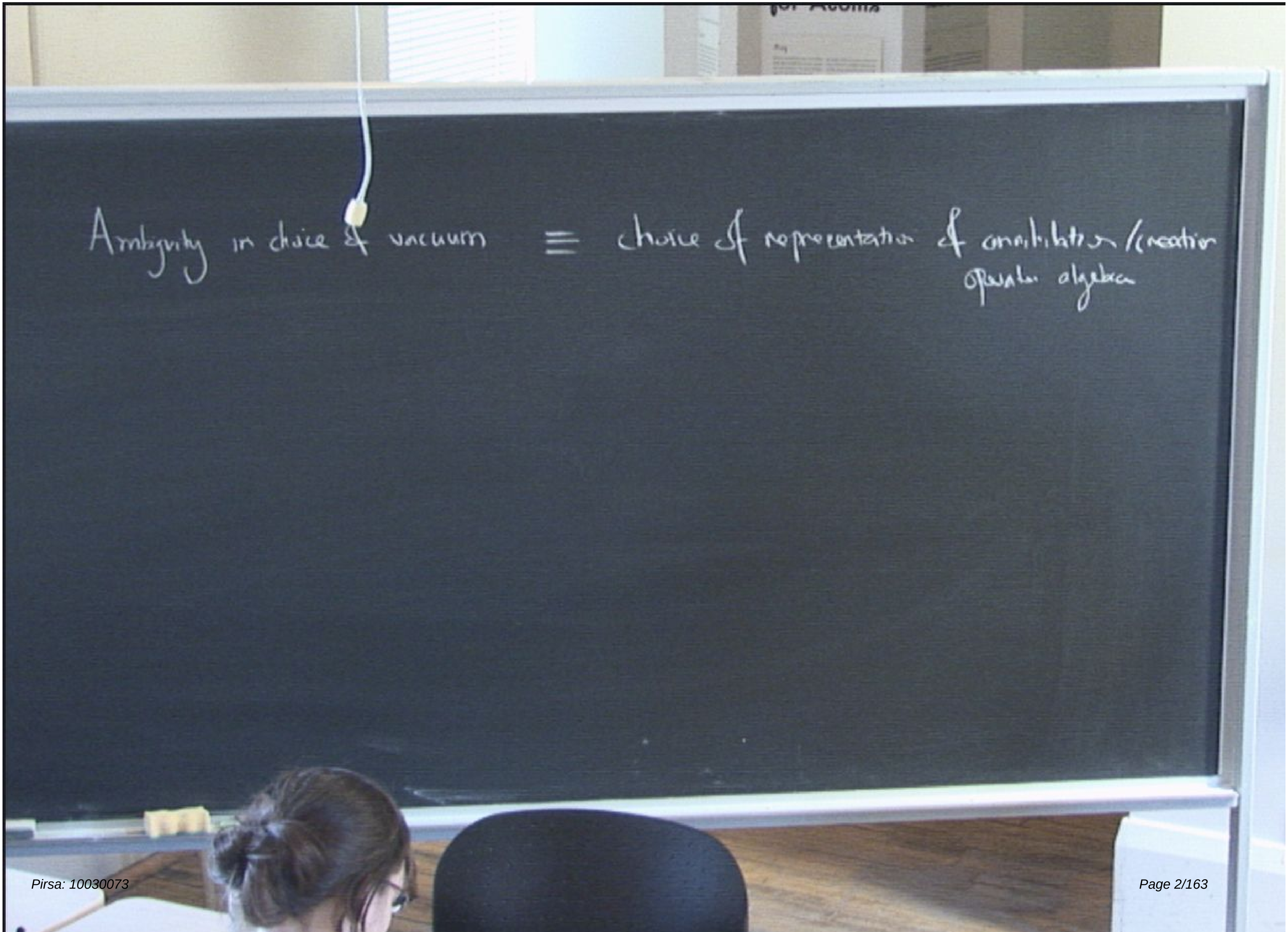
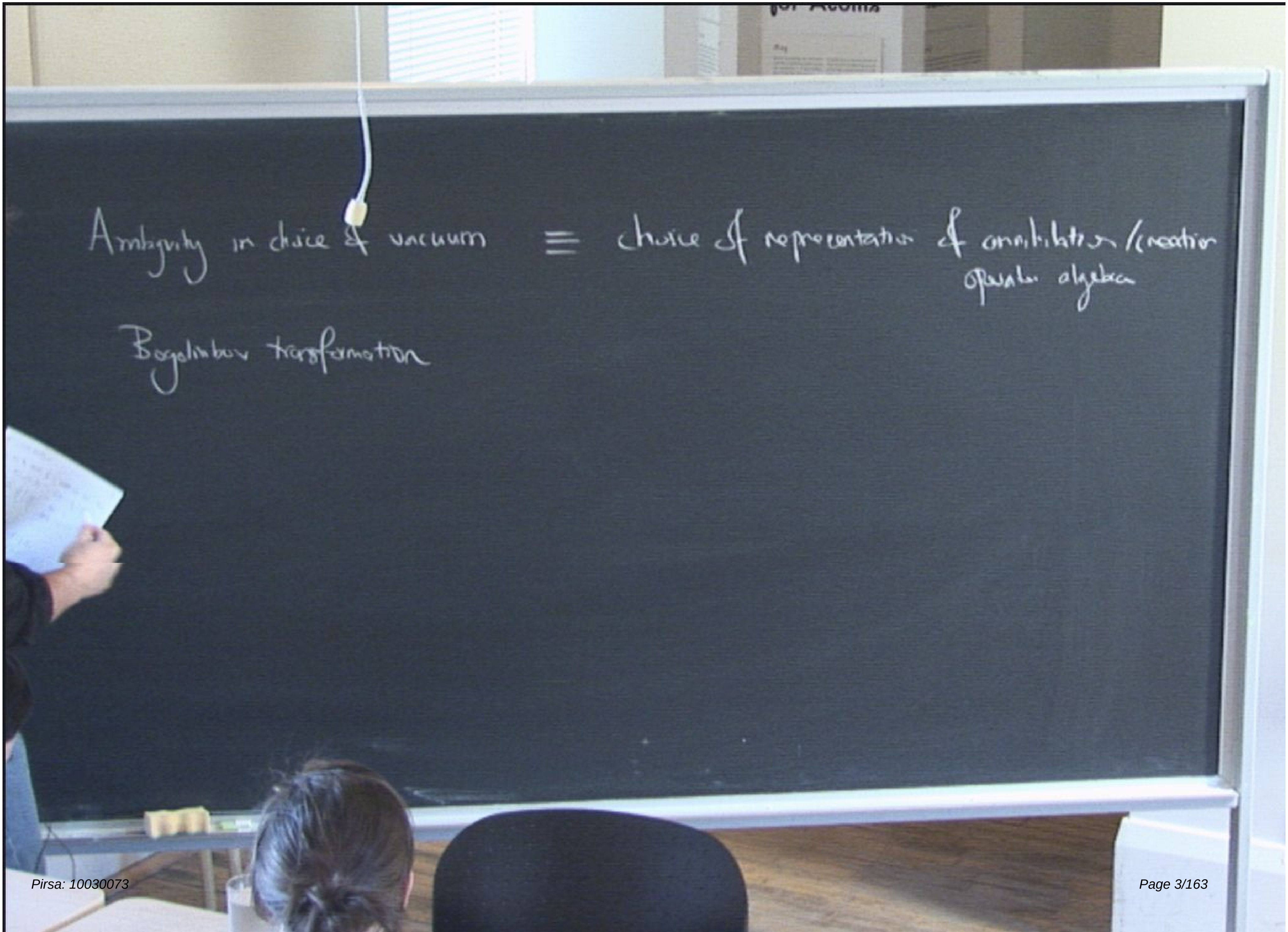
Title: Explorations in Cosmology (PHYS 649) - Lecture 4

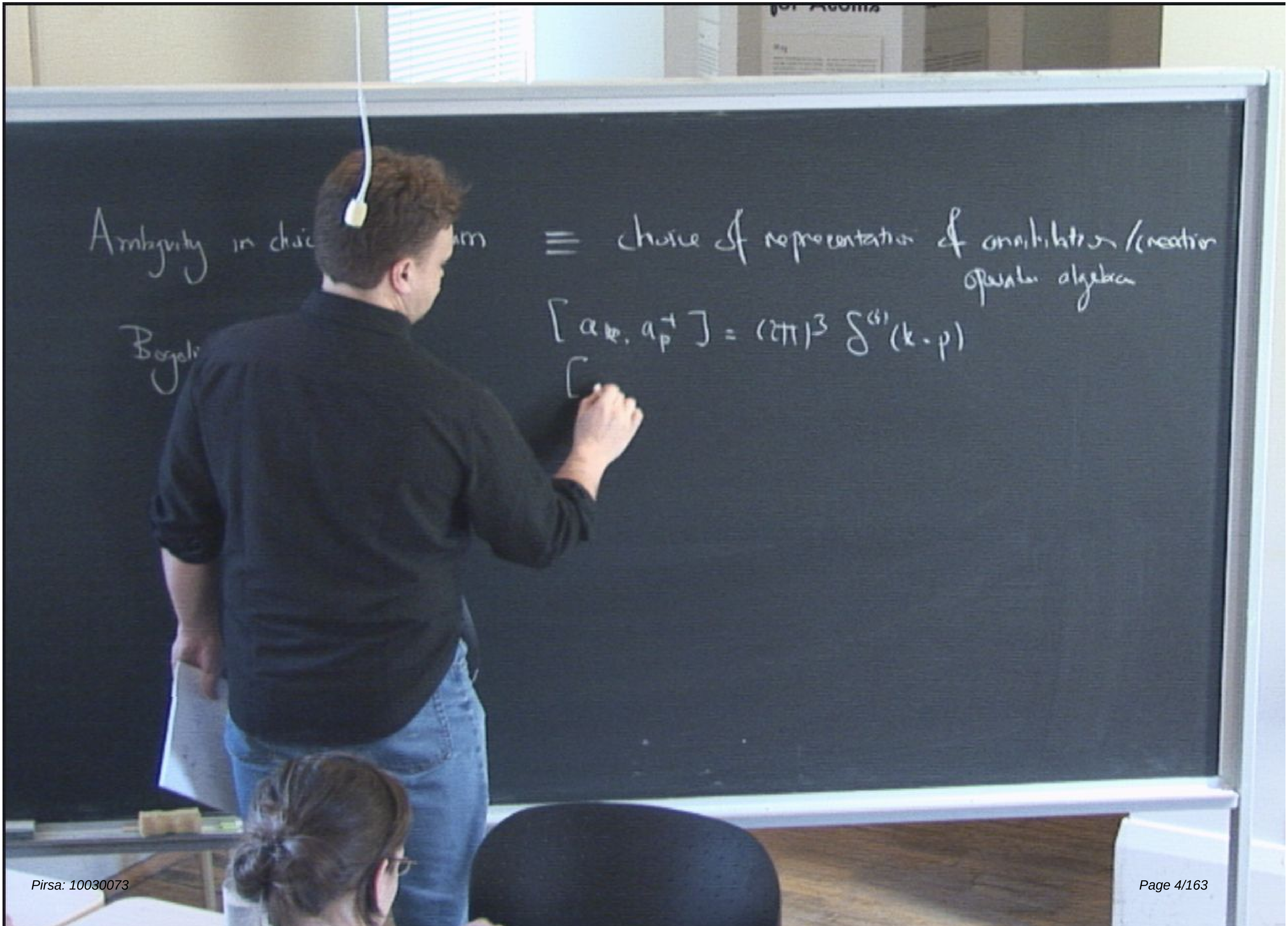
Date: Mar 18, 2010 09:00 AM

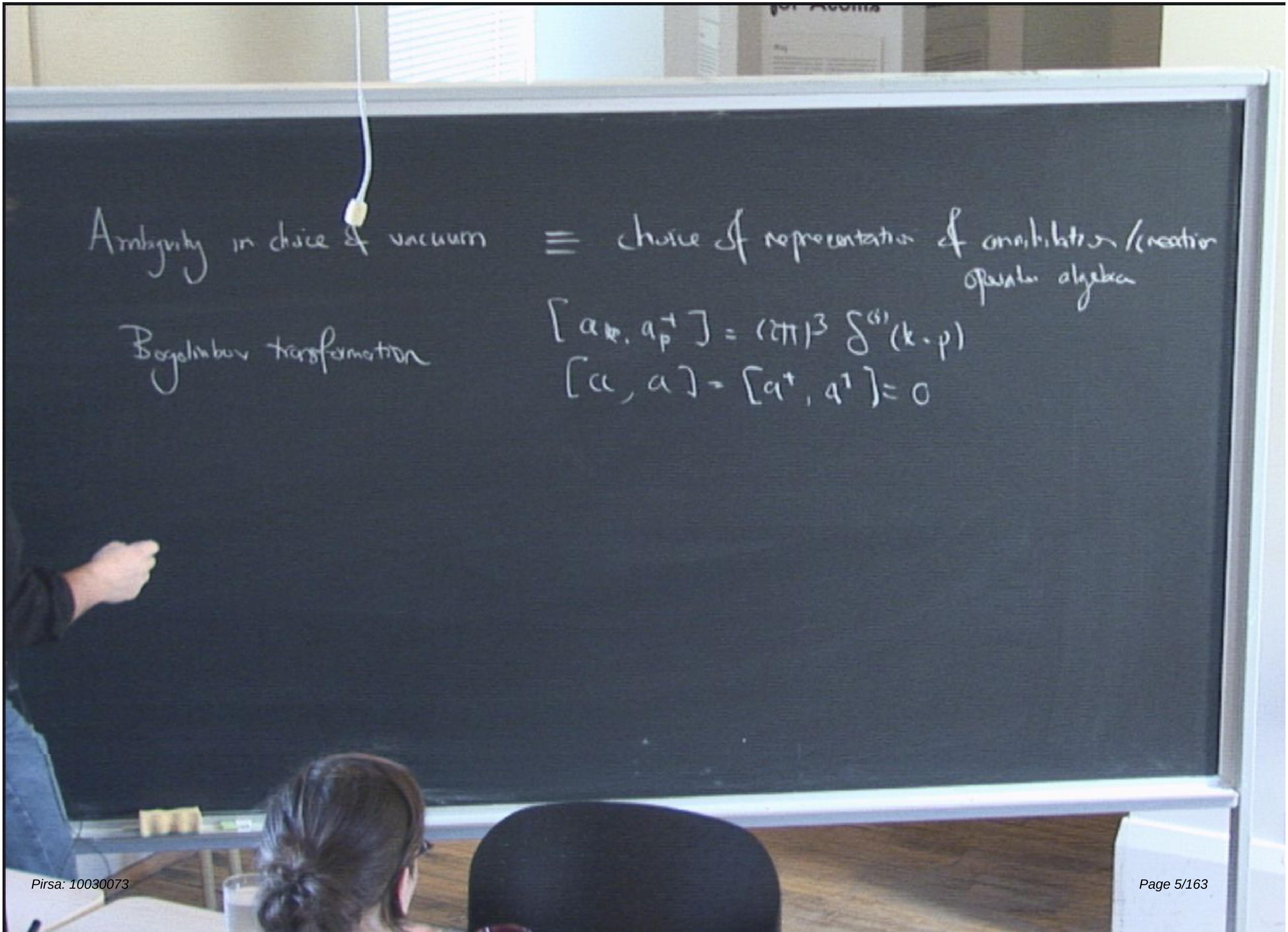
URL: http://pirsa.org/10030073

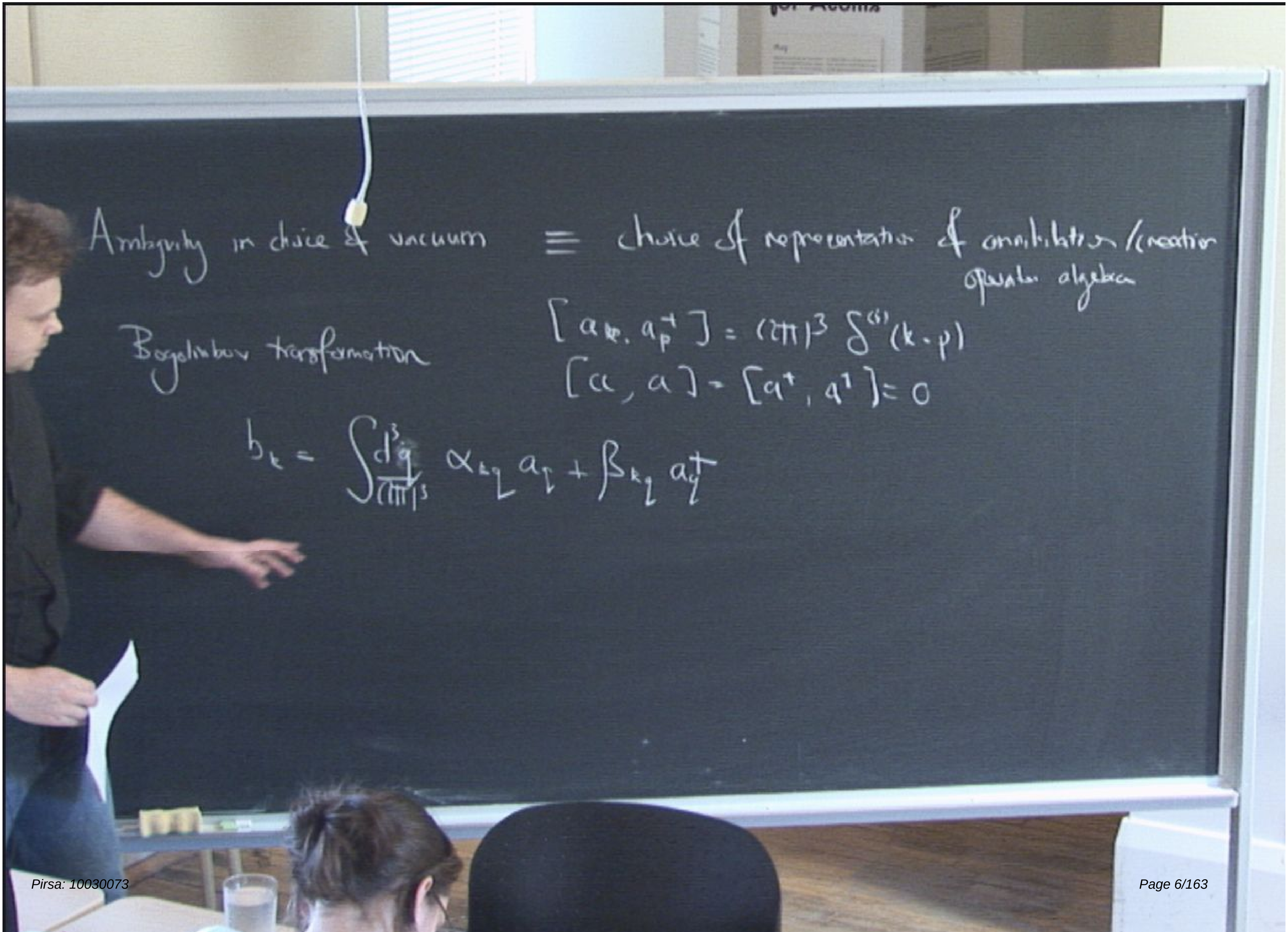
Abstract:

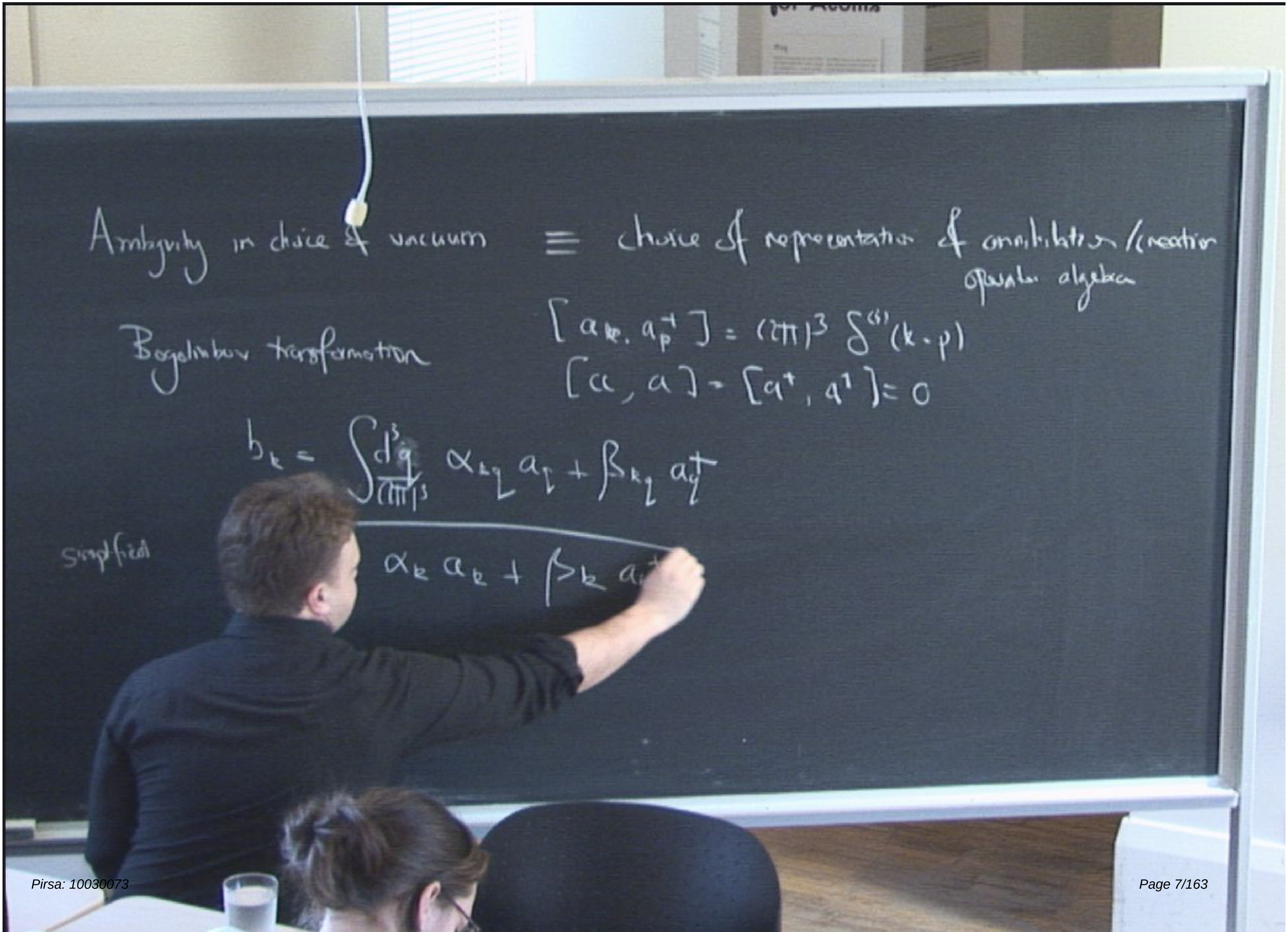

Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation
operator algebra

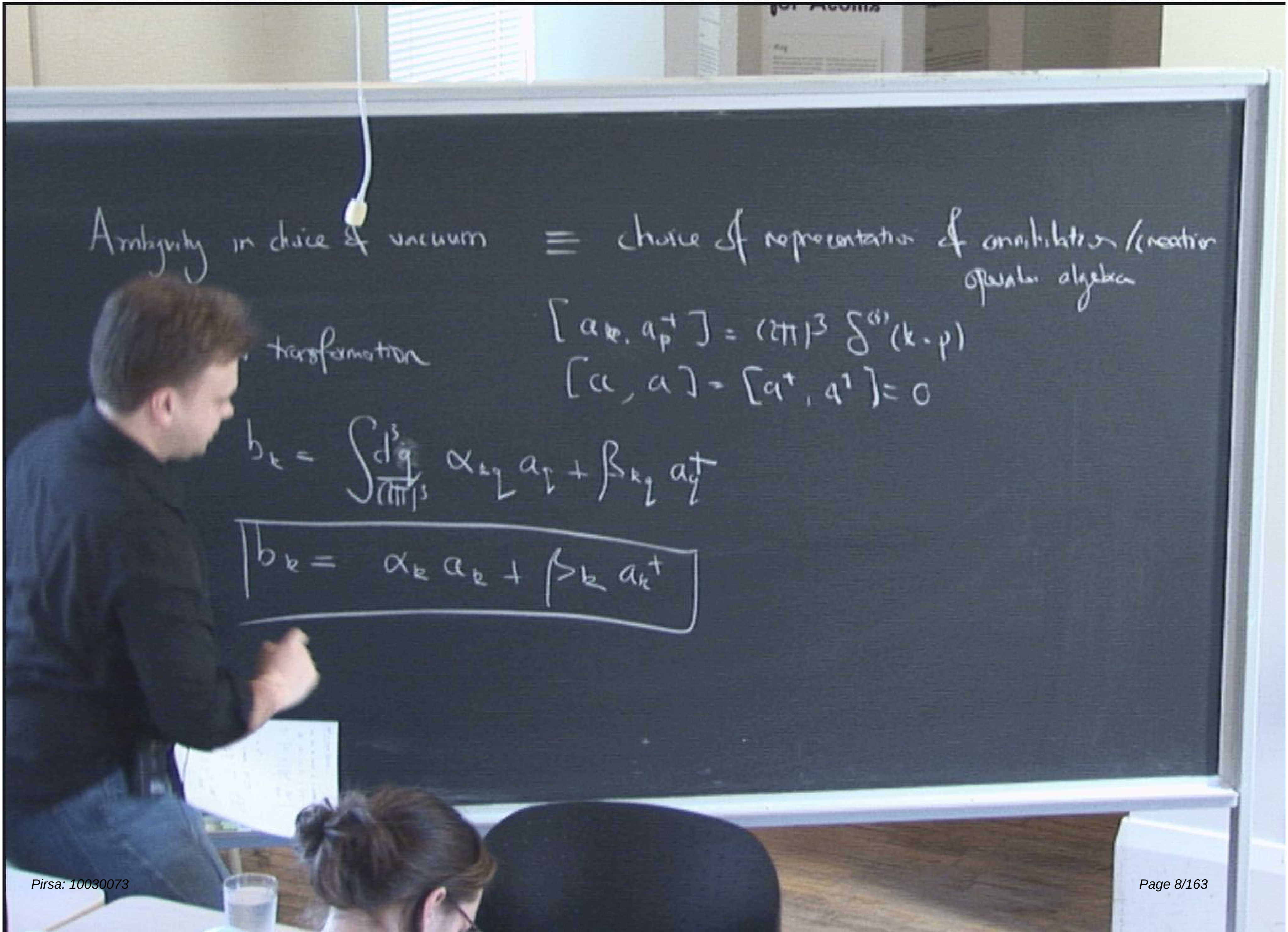

Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation operator algebra
Bogoliubov transformation

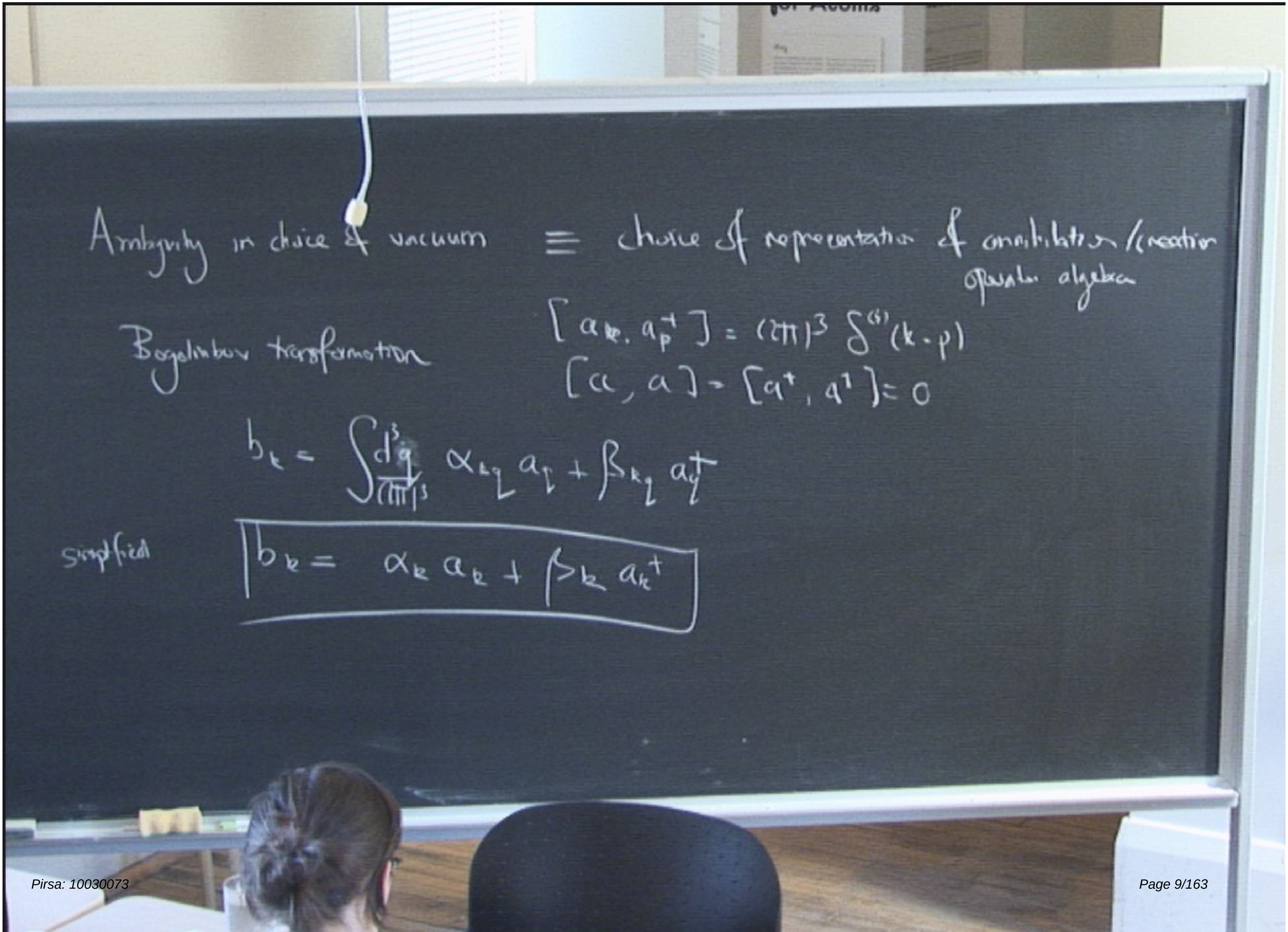

Ambiguity in choi
m ≡ choice of representation of annihilation/creation
operator algebra
$[a_k, a_p^\dagger] = (2\pi)^3 \delta^{(3)}(k-p)$

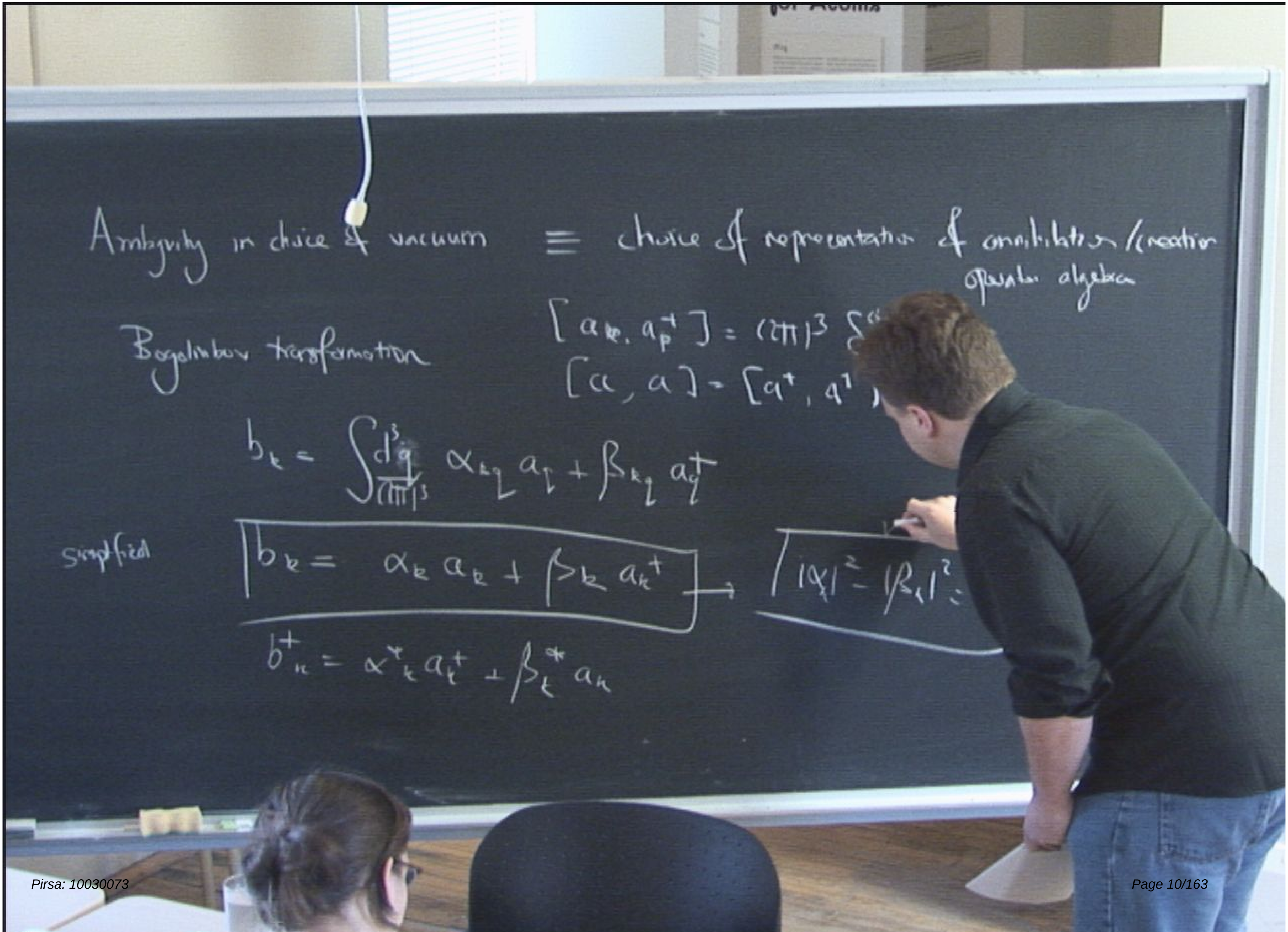

Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation operator algebra
Bogoliubov transformation
$[a_k, a_p^\dagger] = (2\pi)^3 \delta^{(3)}(k-p)$
$[a, a] = [a^\dagger, a^\dagger] = 0$

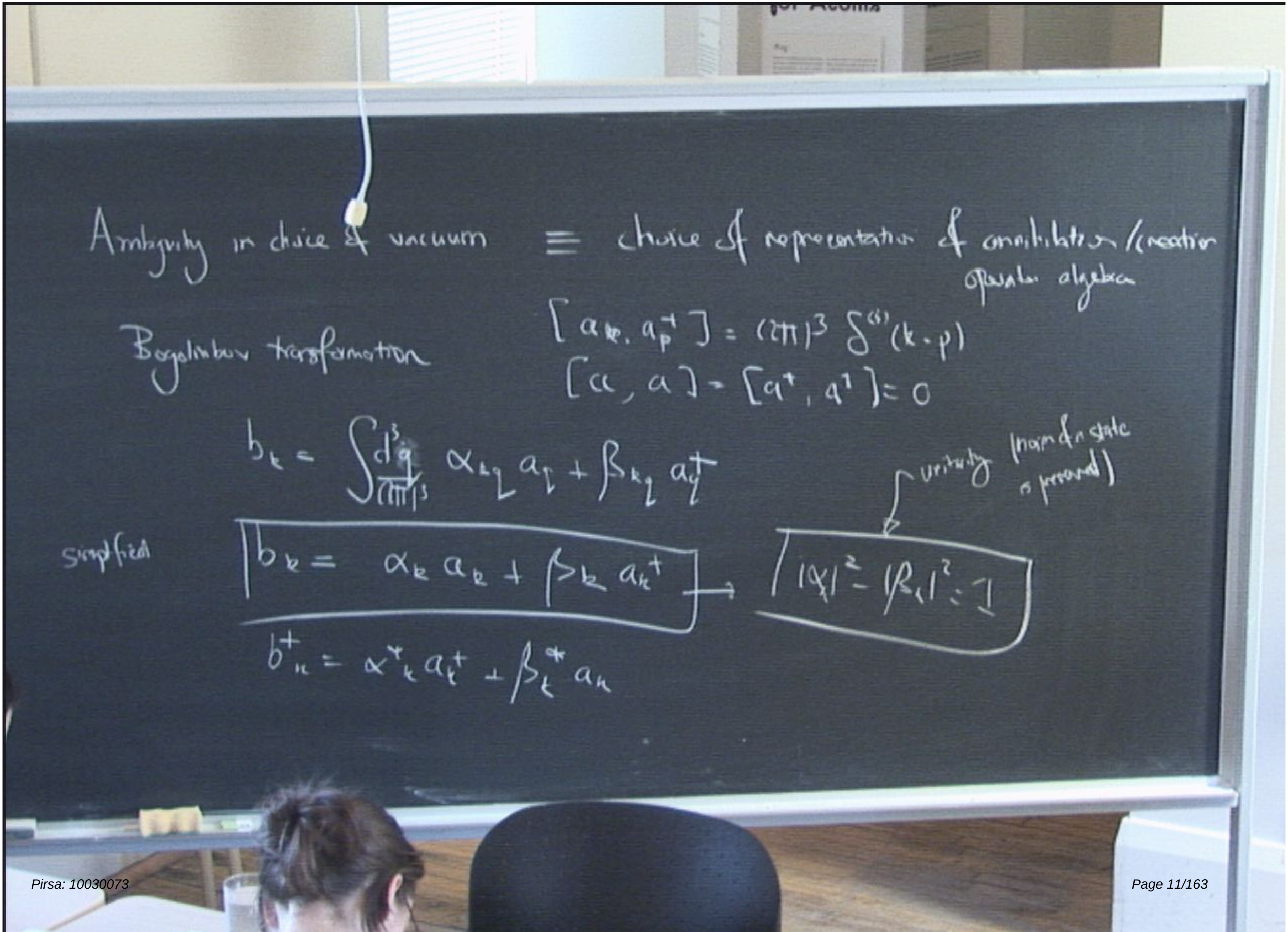

Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation operator algebra
[a_k, a_p^†] = (2π)^3 δ^(3)(k-p)
[a, a] = [a^†, a^†] = 0
Bogoliubov transformation
b_k = ∫ d^3q/(2π)^3 α_kq a_q + β_kq a_q^†


Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation
operator algebra
$[a_k, a_p^\dagger] = (2\pi)^3 \delta^{(3)}(k-p)$
$[a, a] = [a^\dagger, a^\dagger] = 0$
Bogoliubov transformation
$b_k = \int \frac{d^3q}{(2\pi)^3} \alpha_{kq} a_q + \beta_{kq} a_q^\dagger$
simplified
$\alpha_k a_k + \beta_k$


Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation operator algebra
transformation
$[a_k, a_p^\dagger] = (2\pi)^3 \delta^{(3)}(k-p)$
$[a, a] = [a^\dagger, a^\dagger] = 0$
$b_k = \int \frac{d^3q}{(2\pi)^3} \alpha_{kq} a_q + \beta_{kq} a_q^\dagger$
$b_k = \alpha_k a_k + \beta_k a_k^\dagger$


Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation operator algebra
$[a_k, a_p^\dagger] = (2\pi)^3 \delta^{(3)}(k-p)$
$[a, a] = [a^\dagger, a^\dagger] = 0$
Bogoliubov transformation
$b_k = \int \frac{d^3q}{(2\pi)^3} \alpha_{kq} a_q + \beta_{kq} a_q^\dagger$
simplified
$b_k = \alpha_k a_k + \beta_k a_k^\dagger$


Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation operator algebra
Bogoliubov transformation
$[a_k, a_p^\dagger] = (2\pi)^3 \delta$
$[a, a] = [a^\dagger, a^\dagger]$
$b_k = \int \frac{d^3q}{(2\pi)^3} \alpha_{kq} a_q + \beta_{kq} a_q^\dagger$
simplified
$b_k = \alpha_k a_k + \beta_k a_k^\dagger$
$|\alpha_k|^2 - |\beta_k|^2 =$
$b_k^\dagger = \alpha_k^* a_k^\dagger + \beta_k^* a_k$


Ambiguity in choice of vacuum ≡ choice of representation of annihilation/creation operator algebra
$[a_k, a_p^\dagger] = (2\pi)^3 \delta^{(3)}(k-p)$
$[a, a] = [a^\dagger, a^\dagger] = 0$
Bogoliubov transformation
$b_k = \int \frac{d^3q}{(2\pi)^3} \alpha_{kq} a_q + \beta_{kq} a_q^\dagger$
unitarity (norm of a state is preserved)
simplified
$b_k = \alpha_k a_k + \beta_k a_k^\dagger$
$|\alpha_k|^2 - |\beta_k|^2 = 1$
$b_k^\dagger = \alpha_k^* a_k^\dagger + \beta_k^* a_k$

$$a_k |0_a\rangle = 0$$

$$a_k |0, a\rangle = 0$$

$$b_k |0, b\rangle = 0$$

$$a_k |0, a\rangle = 0$$

$$b_k |0, b\rangle = 0$$

$$|k, b\rangle = b_k^{\dagger} |0, b\rangle$$

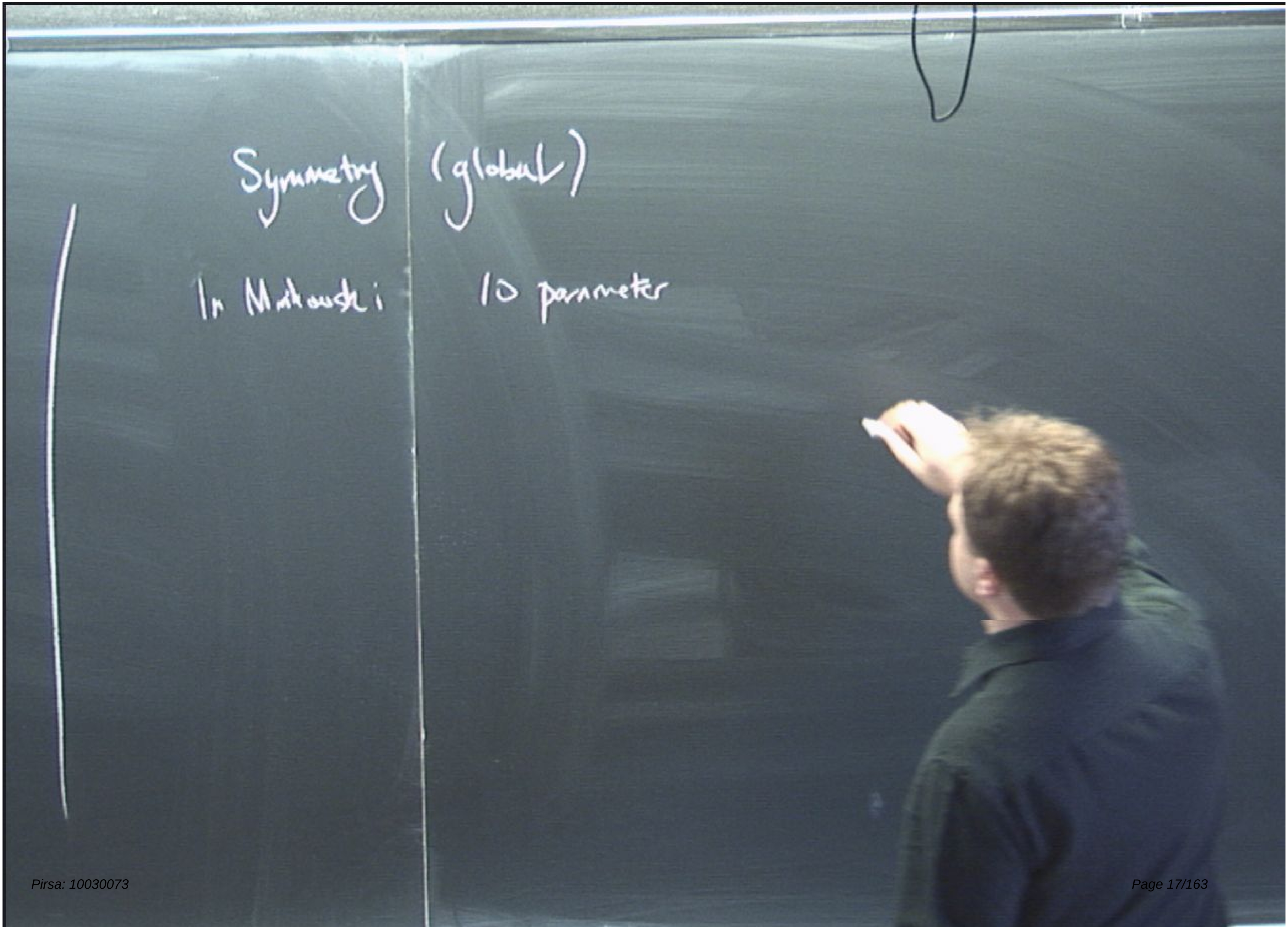
$$a_k |0, a\rangle = 0$$

$$b_k |0, b\rangle = 0$$

$$|k, b\rangle = b_k^\dagger |0, b\rangle$$

Vacuum $\longleftrightarrow$ fixed by physics

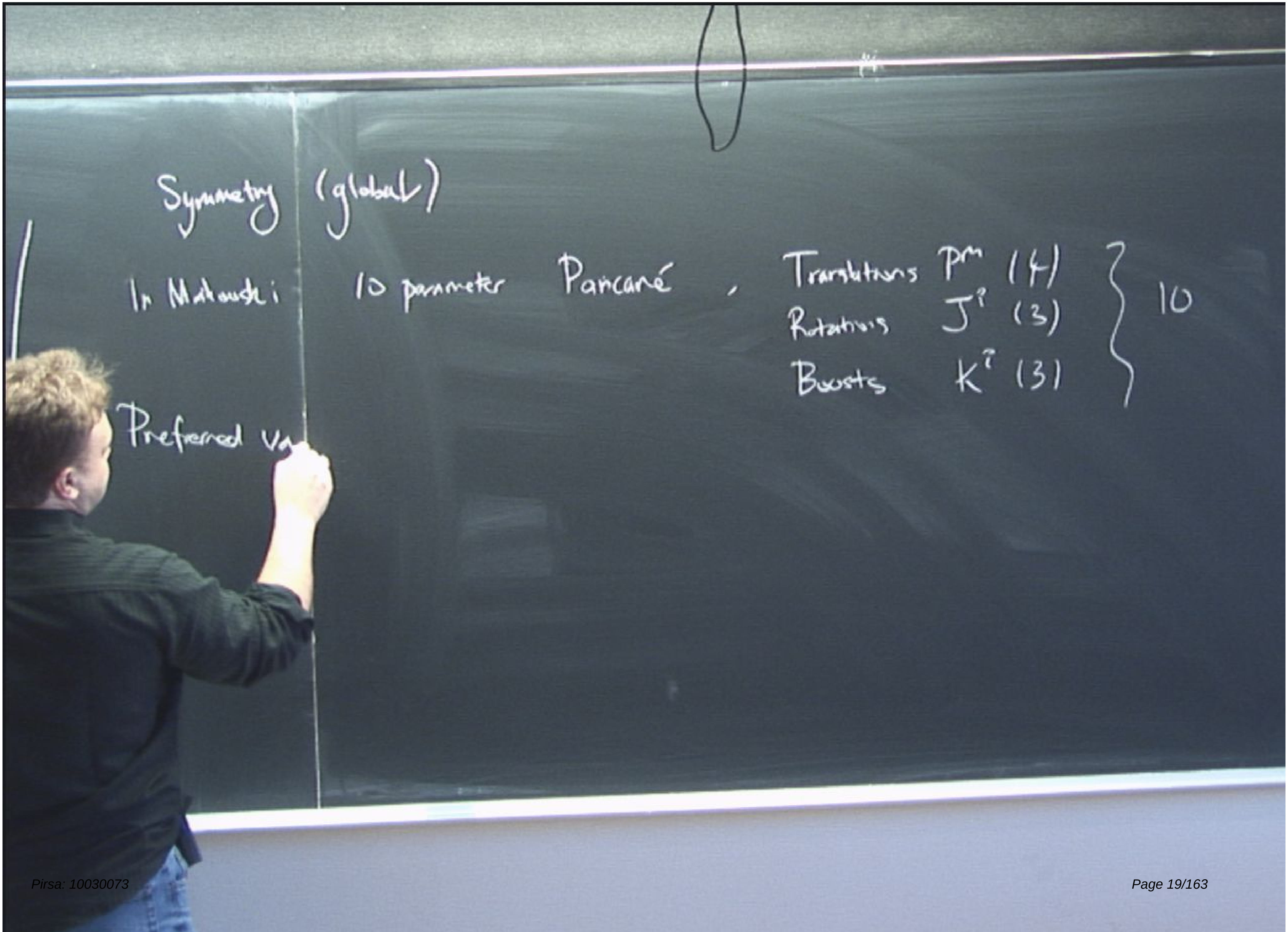
Symmetry (global)


Symmetry (global)
In Minkowski
10 parameter

Symmetry (global)

In Minkowski: 10 parameter Poincaré, Translations P^μ (4)

Rotations J^i (3)

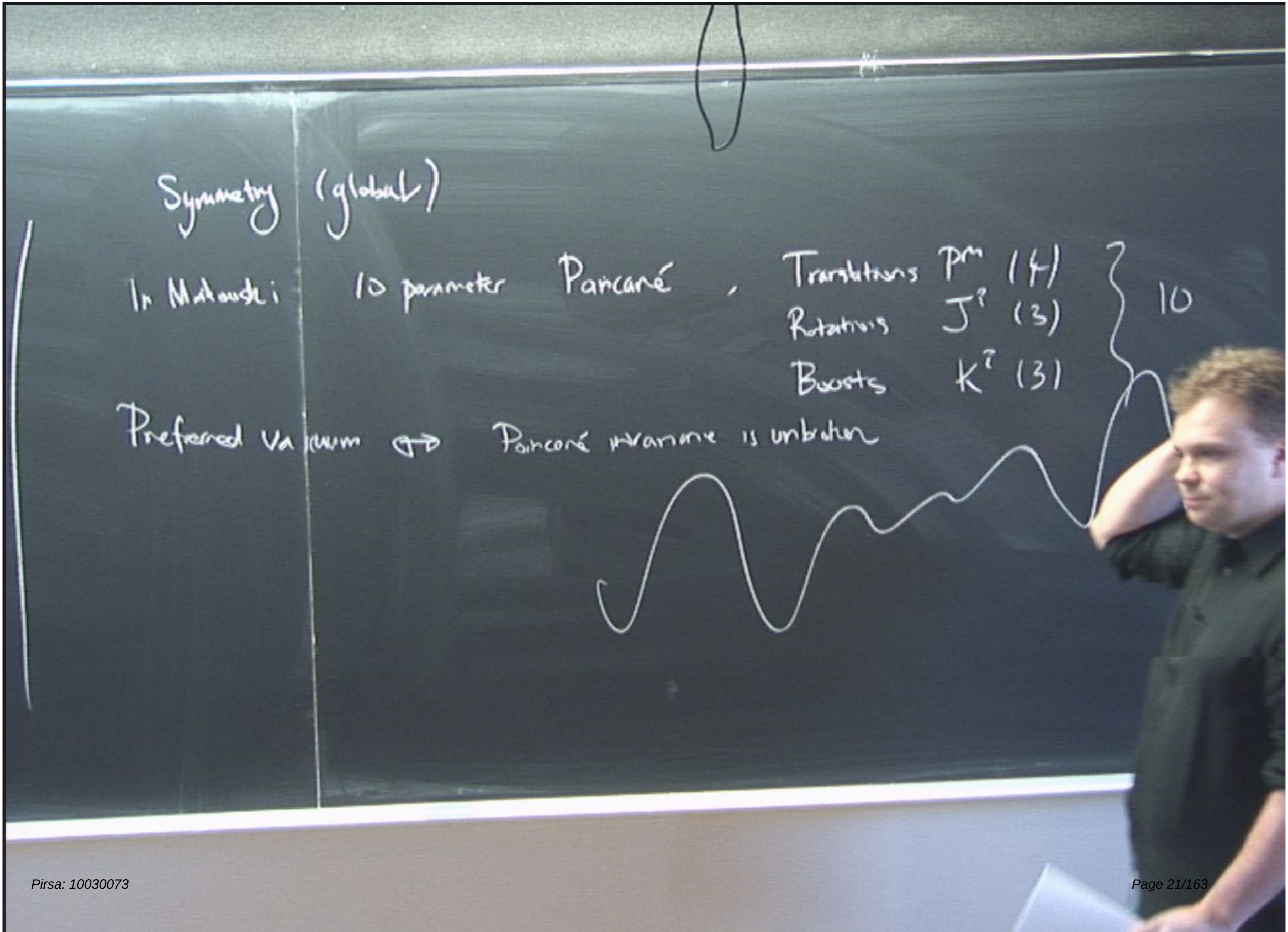

Symmetry (global)
In Minkowski
10 parameter Poincaré
Translations P^m (4)
Rotations J^i (3)
Boosts K^i (3)
10
Preferred va

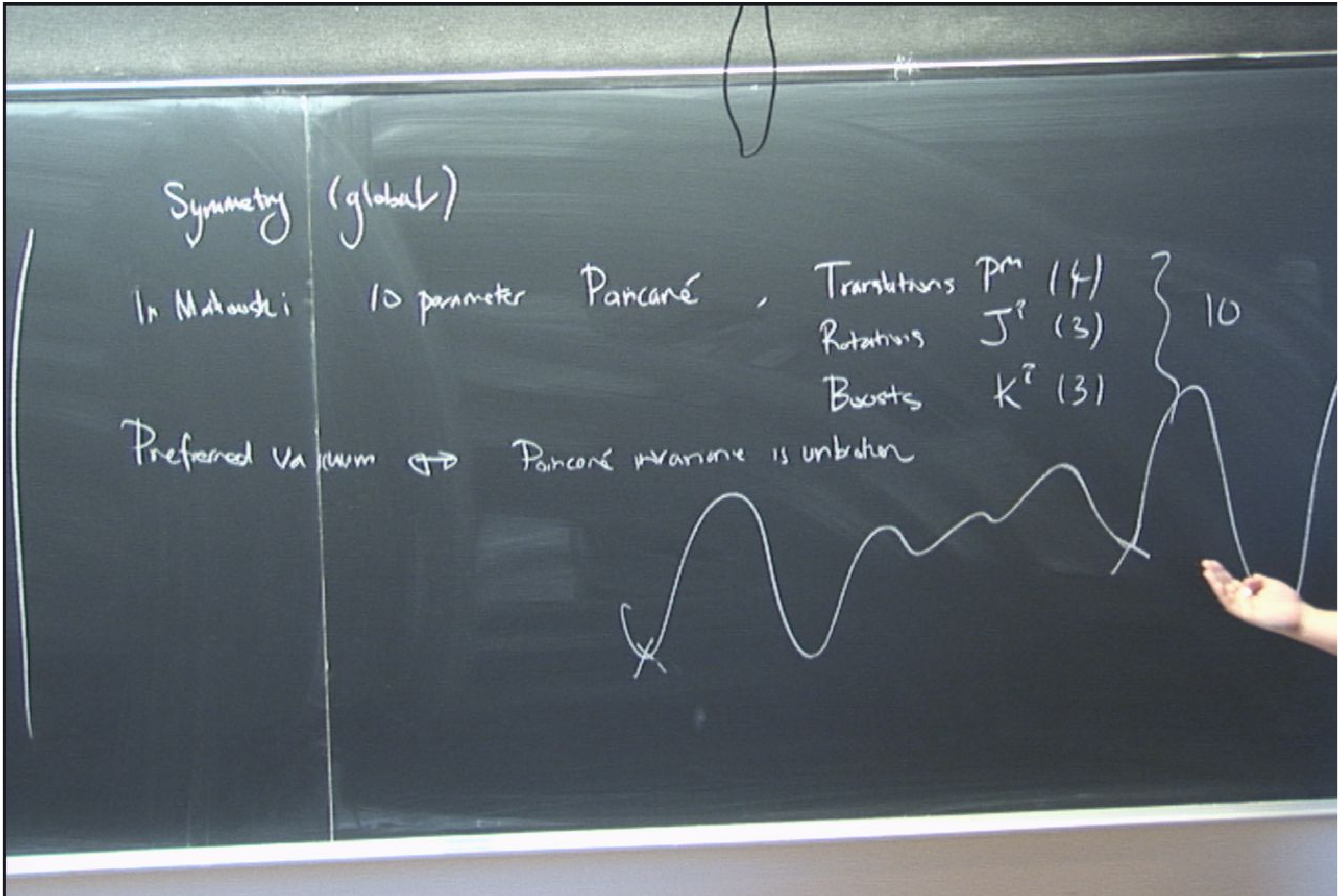
Symmetry (global)

In Minkowski: 10 parameter Pancaré,

Translations	P^μ	(4)	
Rotations	J^i	(3)	} 10
Boosts	K^i	(3)	

Preferred vacuum $\Leftrightarrow$ Poincaré invariance is unbroken


Symmetry (global)
In Minkowski
10 parameter Poincaré
Translations P^μ (4)
Rotations J^i (3)
Boosts K^i (3)
10
Preferred vacuum ⟺ Poincaré invariance is unbroken


Symmetry (global)
In Minkowski 10 parameter Poincaré ,
Translations P^μ (4)
Rotations J^i (3)
Boosts K^i (3)
10
Preferred vacuum ⟺ Poincaré invariance is unbroken

Symmetry (global)

In Minkowski: 10 parameter Poincaré, Translations P^μ (4)

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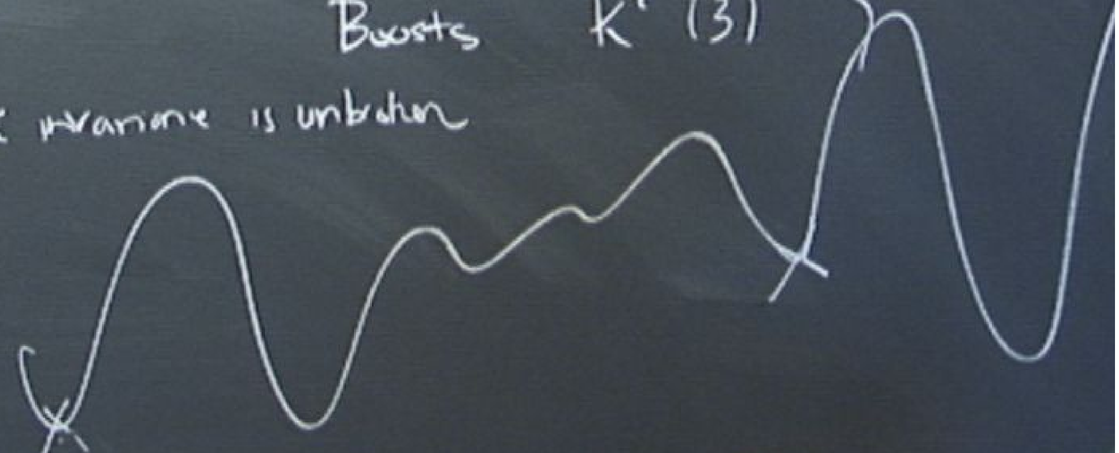
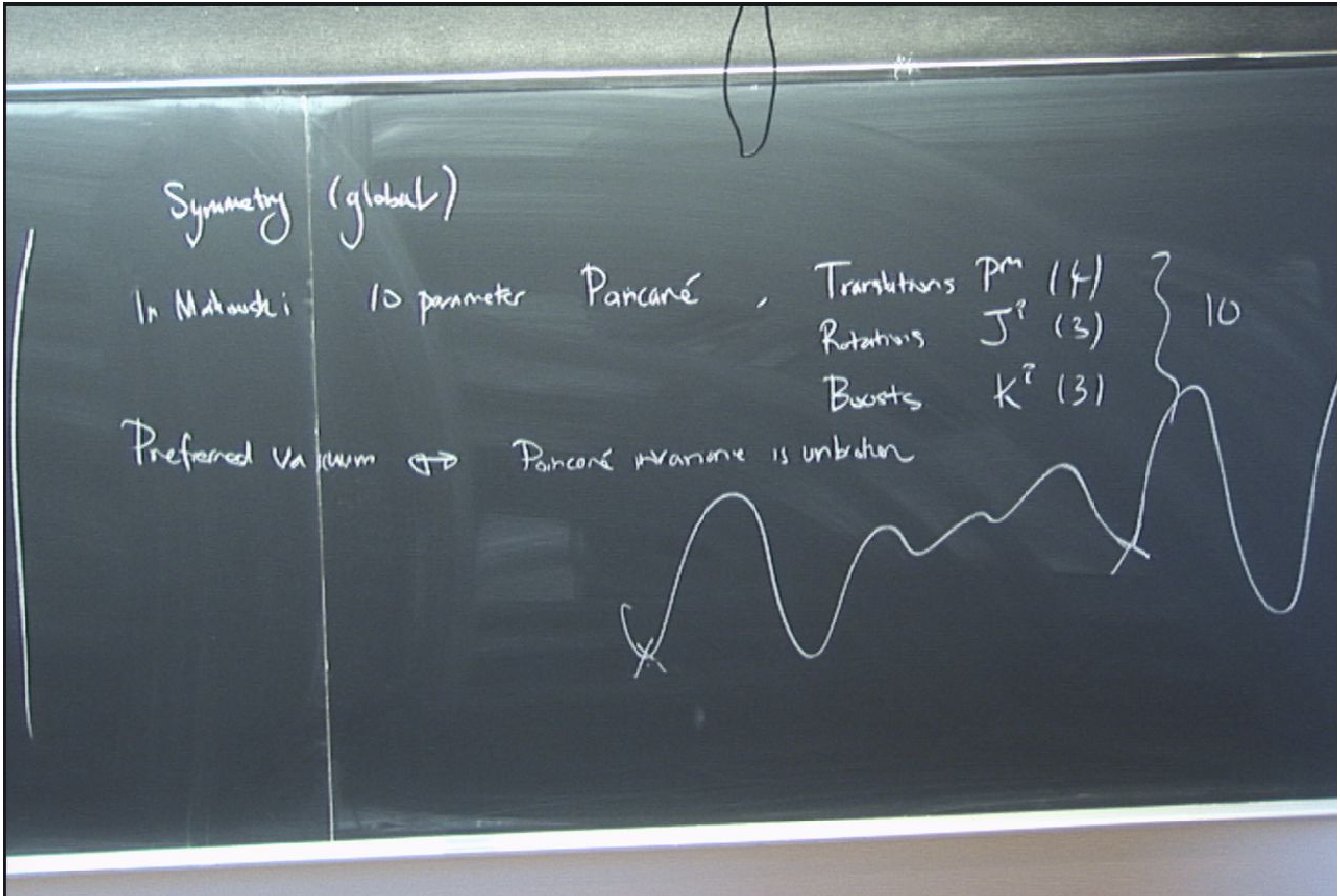
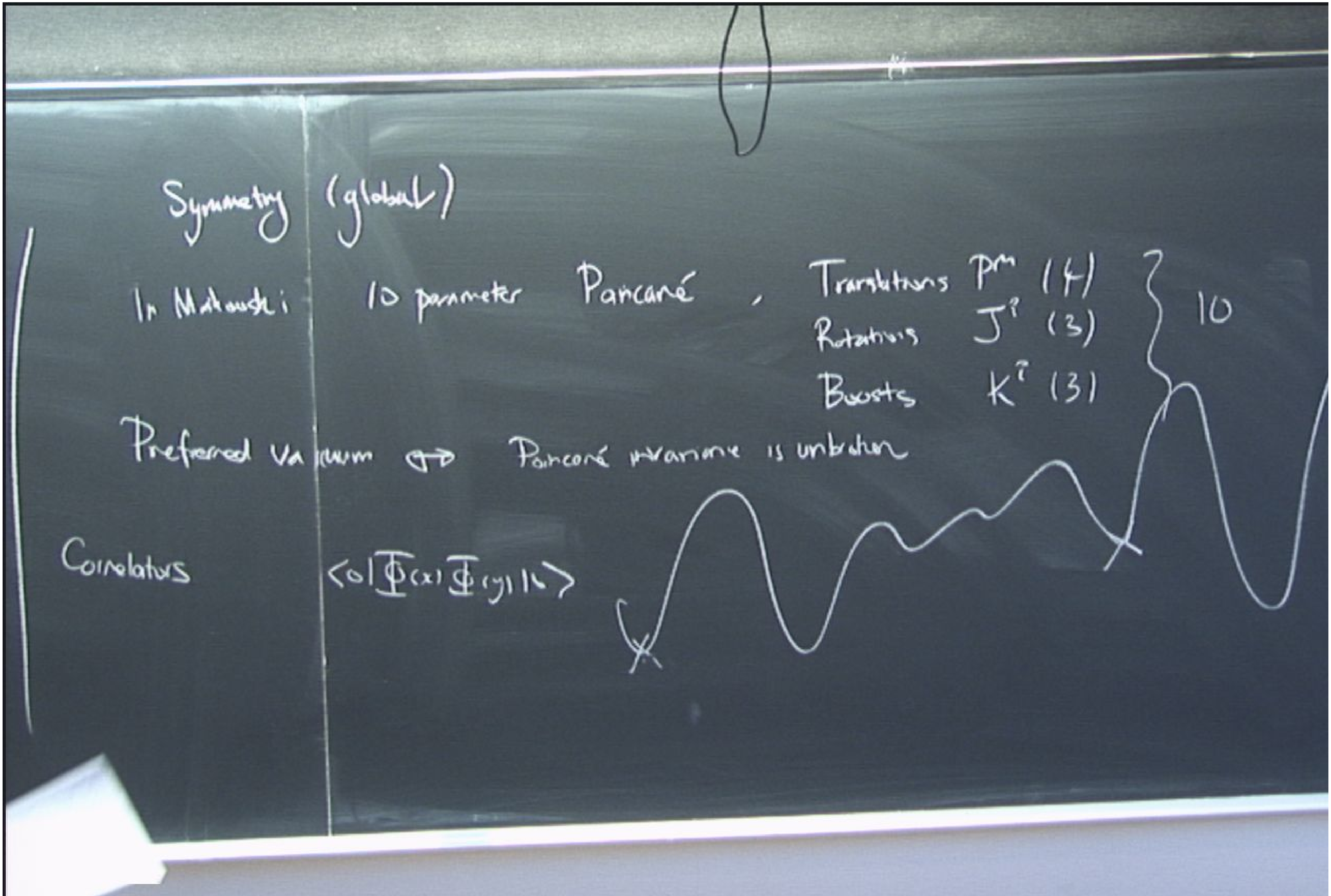
Boosts K^i (3)

} 10

Preferred vacuum $\Leftrightarrow$ Poincaré invariance is unbroken




Symmetry (global)
In Minkowski: 10 parameter Poincaré
Translations P^μ (4)
Rotations J^i (3)
Boosts K^i (3)
10
Preferred vacuum ⇔ Poincaré invariance is unbroken


Symmetry (global)
In Minkowski 10 parameter Poincaré ,
Translations P^μ (4)
Rotations J^i (3)
Boosts K^i (3)
10
Preferred vacuum ⟺ Poincaré invariance is unbroken
Correlators
⟨0|Φ(x) Φ(y)|0⟩

Symmetry (global)

In Minkowski: 10 parameter Poincaré,

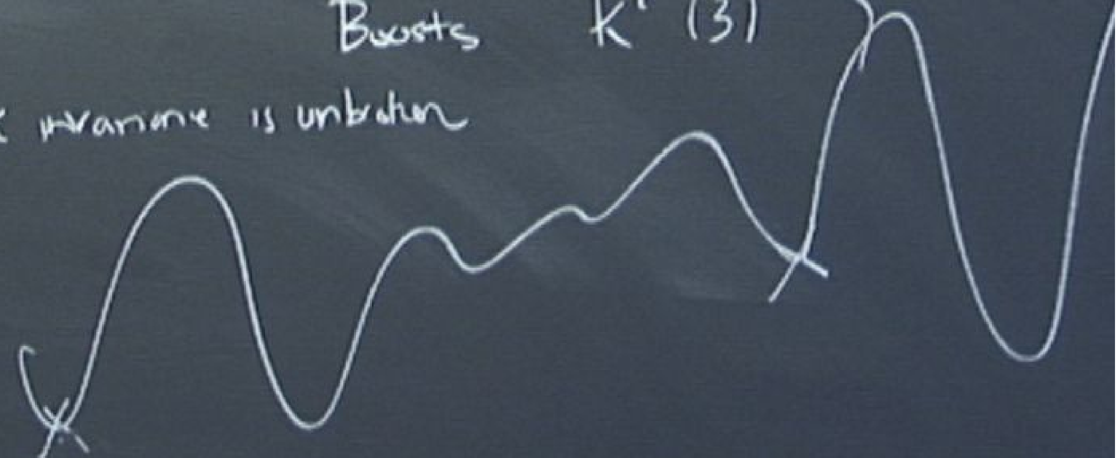
- Translations P^μ (4)
- Rotations J^i (3)
- Boosts K^i (3)

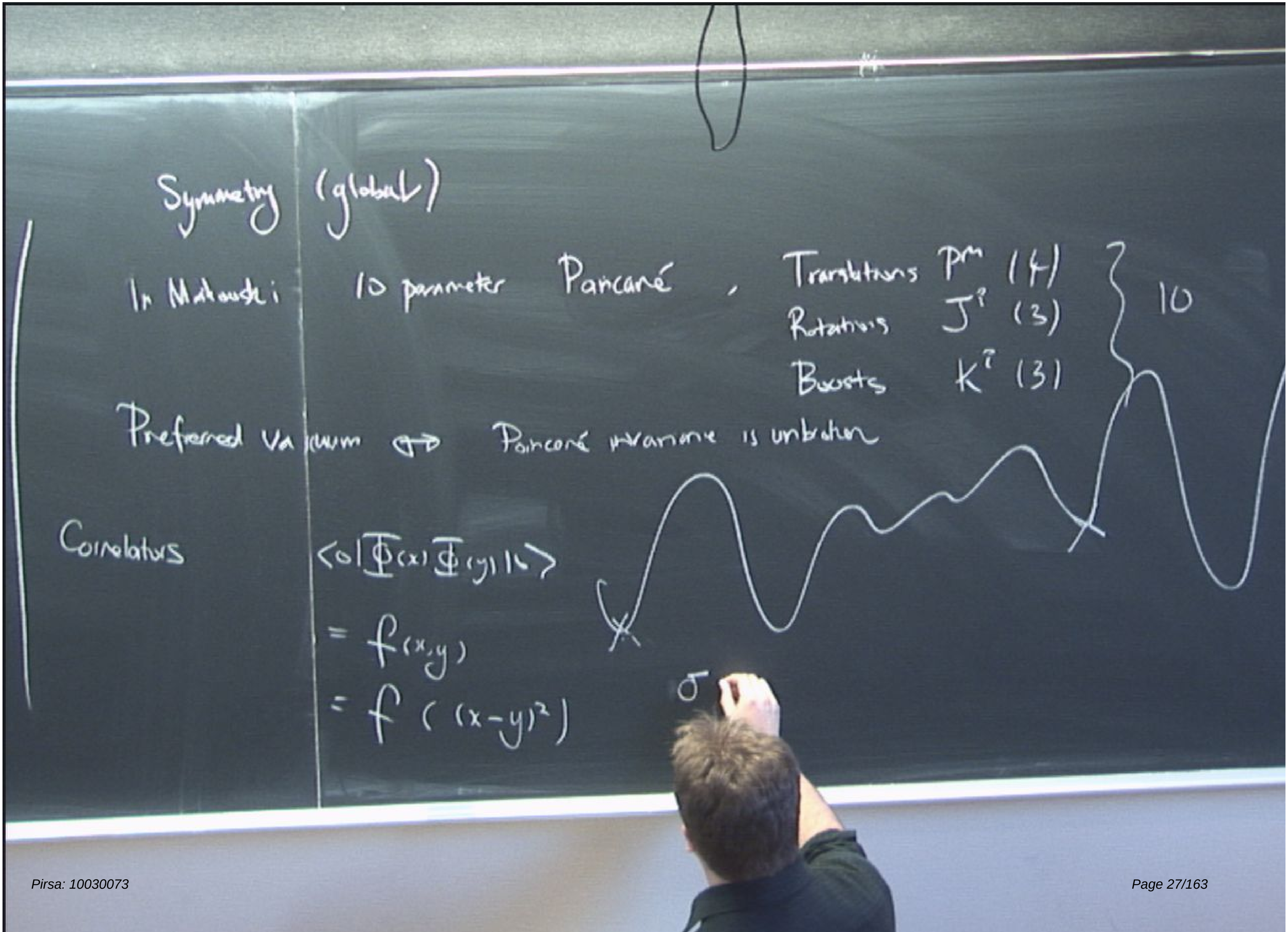
} 10

Preferred vacuum $\Leftrightarrow$ Poincaré invariance is unbroken

Correlators $\langle 0 | \Phi(x) \Phi(y) | 0 \rangle$

$= f(x,y)$




Symmetry (global)
In Minkowski
10 parameter Poincaré, Translations P^μ (4)
Rotations J^i (3)
Boosts K^i (3)
10
Preferred vacuum ⟺ Poincaré invariance is unbroken
Correlators
$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle$
$= f(x,y)$
$= f((x-y)^2)$
σ

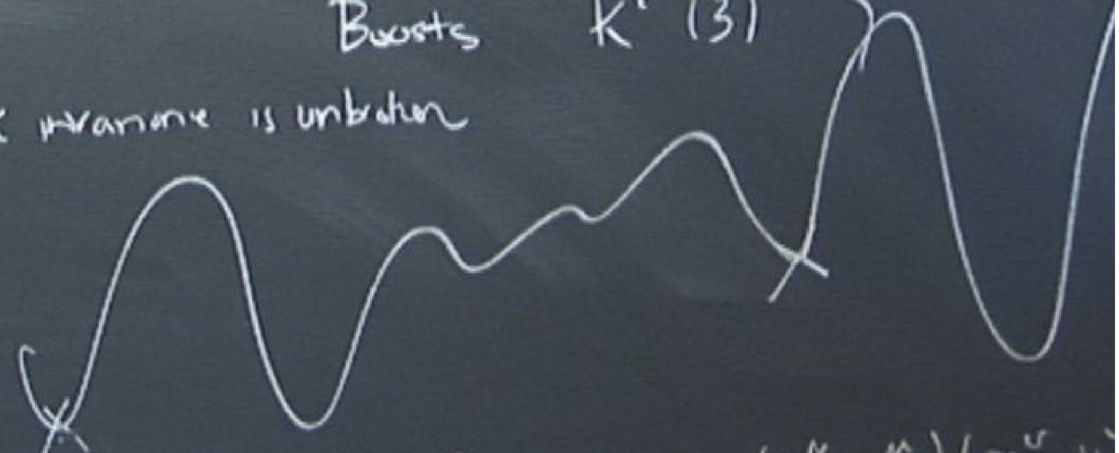
Symmetry (global)

In Minkowski: 10 parameter Poincaré, Translations P^μ (4), Rotations J^i (3), Boosts K^i (3) } 10

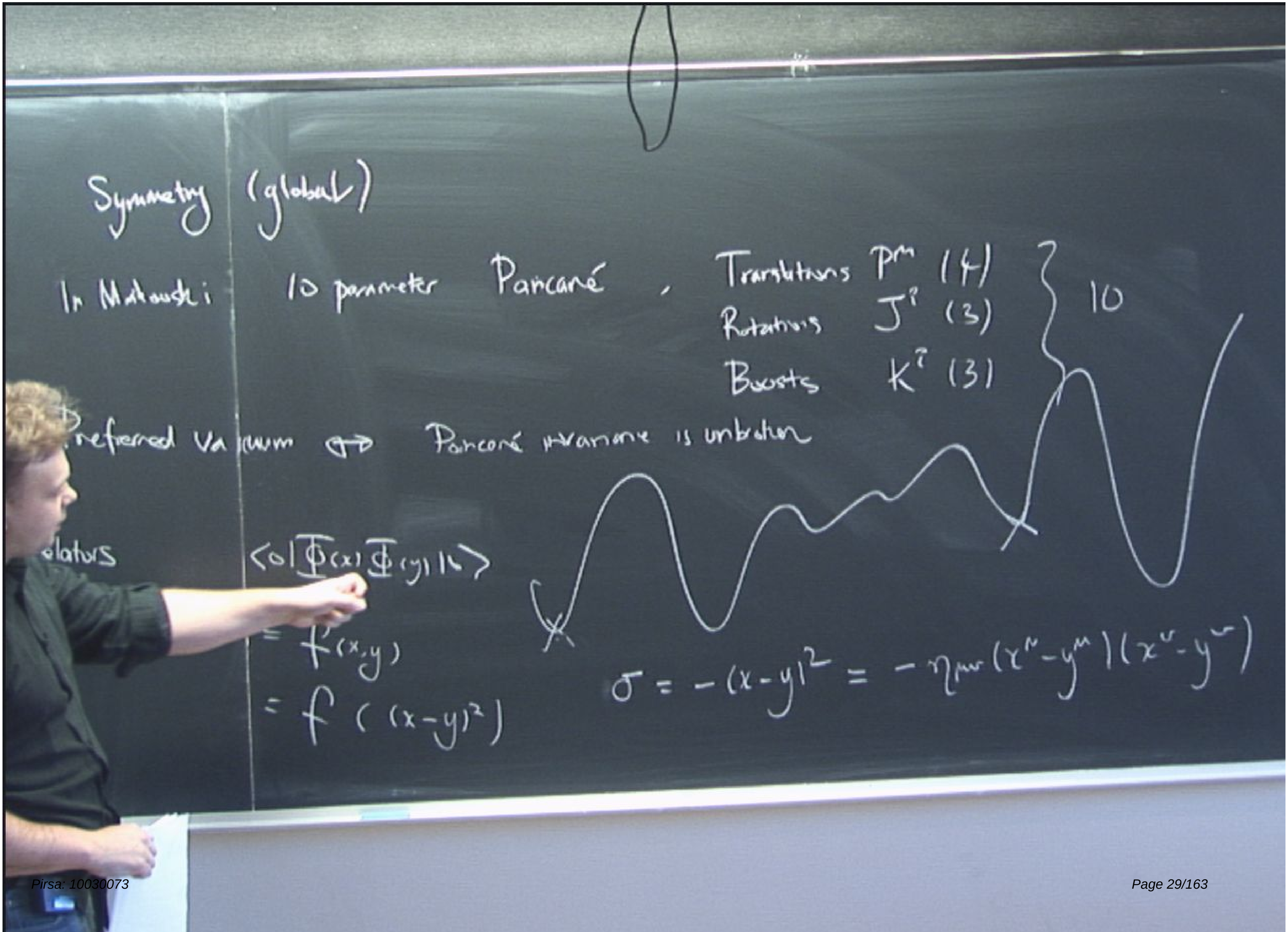
Preferred vacuum $\Leftrightarrow$ Poincaré invariance is unbroken

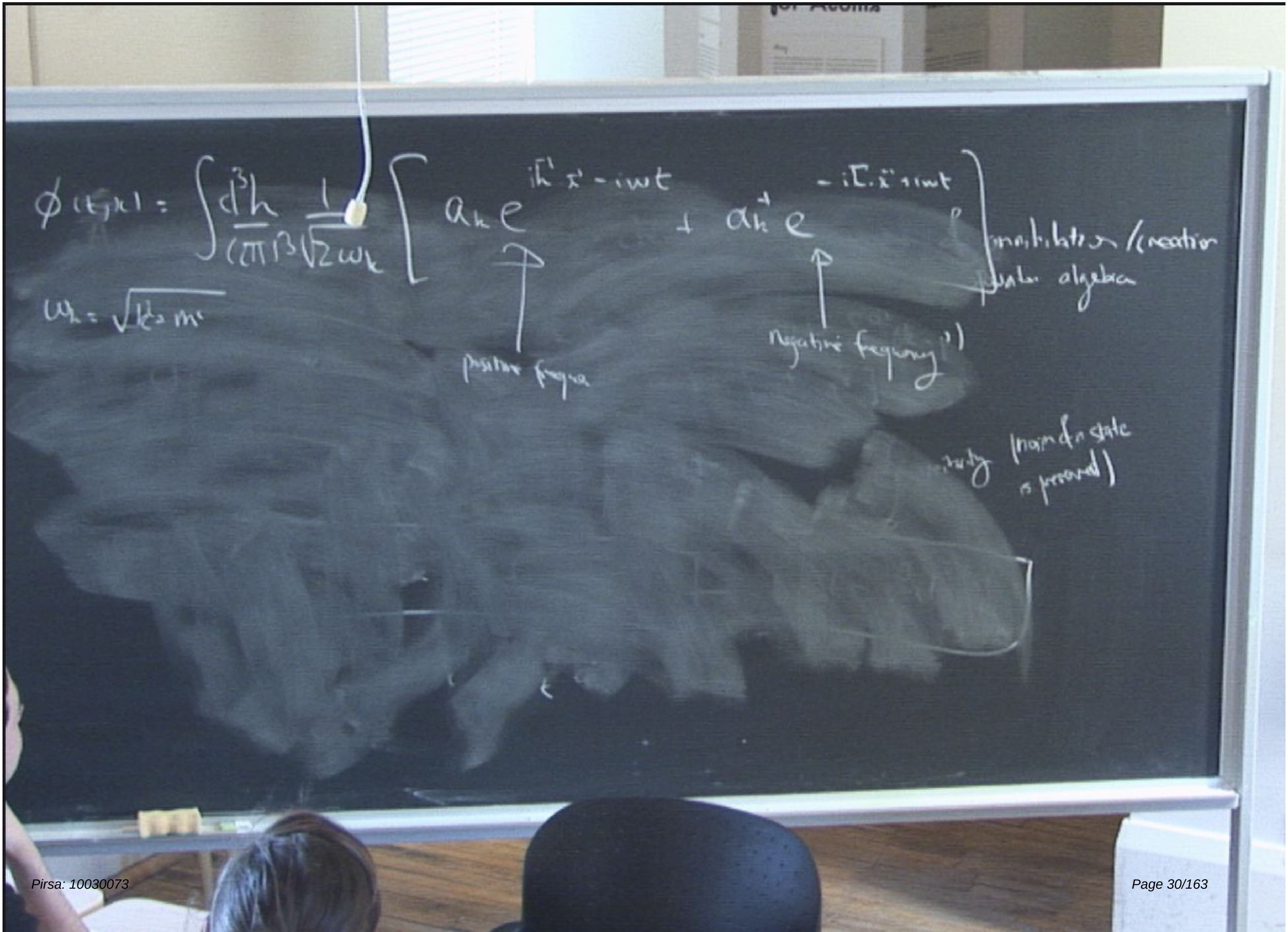
Correlators

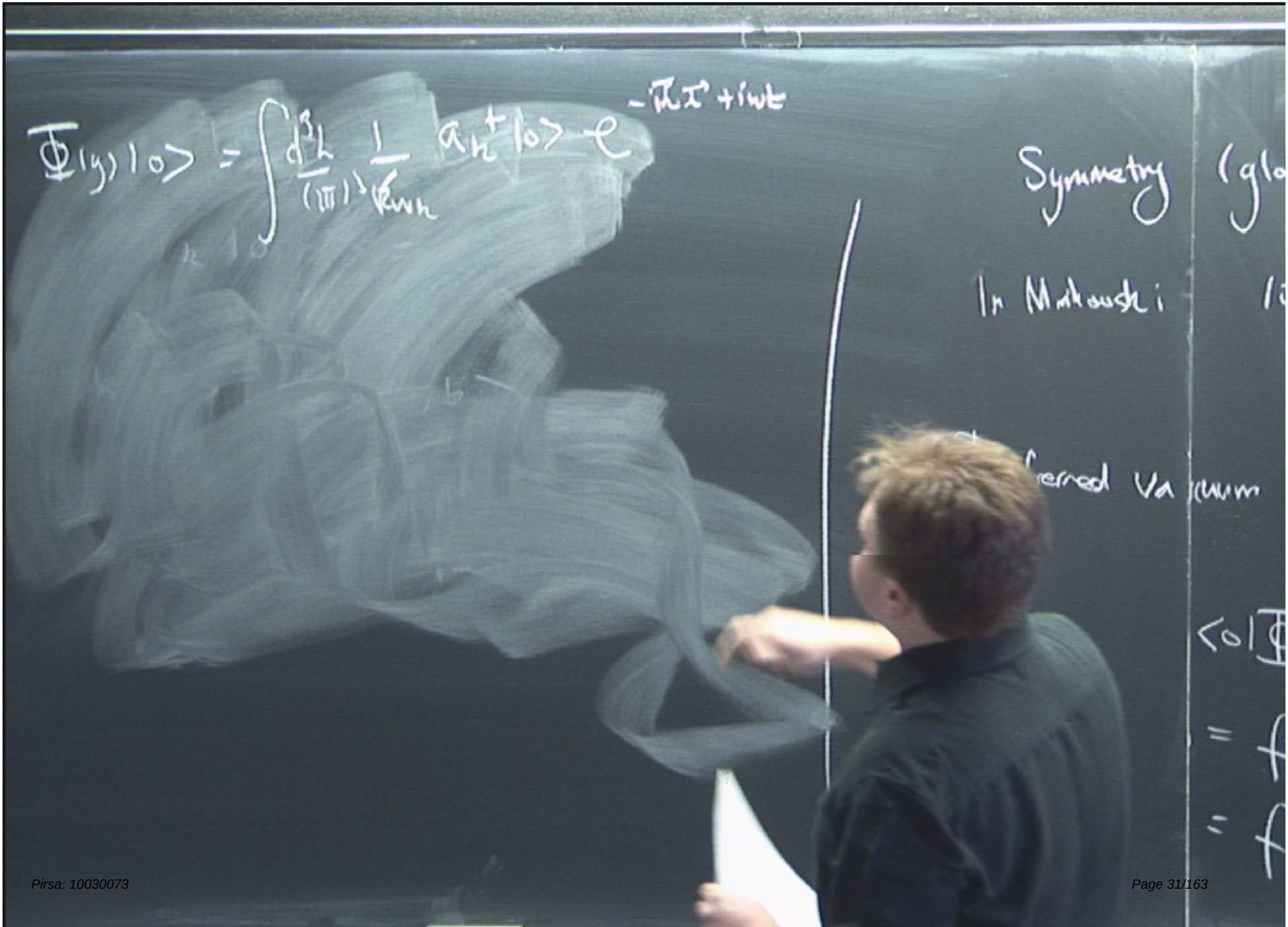
$$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle = f(x,y) = f((x-y)^2)$$

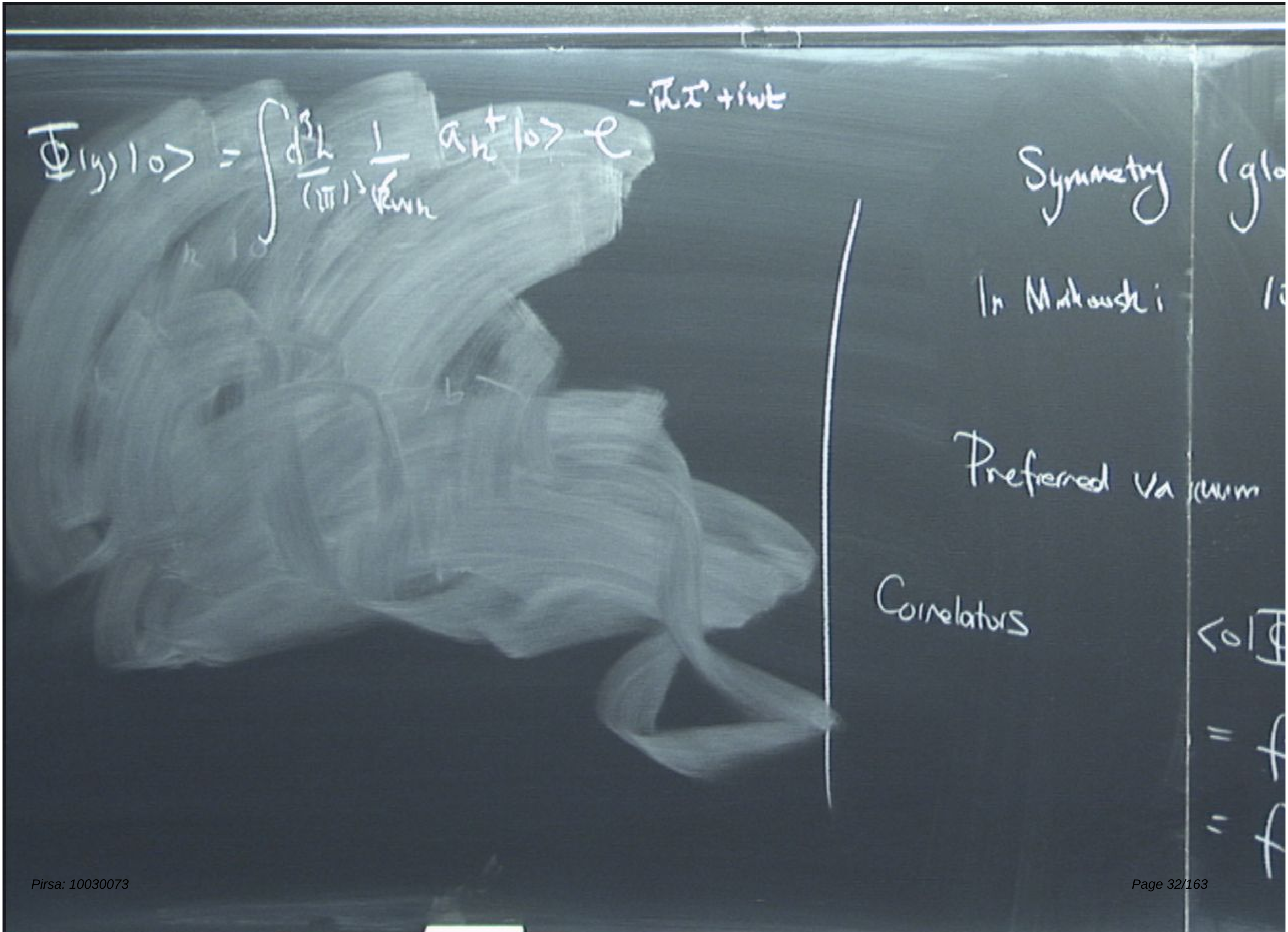


$$\sigma = -(x-y)^2 = -\eta_{\mu\nu}(x^\mu - y^\mu)(x^\nu - y$$


Symmetry (global)
In Minkowski
10 parameter Poincaré ,
Translations P^μ (4)
Rotations J^i (3)
Boosts K^i (3)
10
referred vacuum ⟺ Poincaré invariance is unbroken
elators
$\langle 0|\Phi(x)\Phi(y)|0\rangle$
$= f(x,y)$
$= f((x-y)^2)$
$\sigma = -(x-y)^2 = -\eta_{\mu\nu}(x^\mu-y^\mu)(x^\nu-y^\nu)$


$\phi(t,x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[a_k e^{i\vec{k}\cdot\vec{x} - i\omega t} + a_k^\dagger e^{-i\vec{k}\cdot\vec{x} + i\omega t} \right]$
$\omega_k = \sqrt{k^2 + m^2}$
positive frequency
negative frequency
annihilation / creation
algebra
(norm of a state is preserved)


Φ(y) |0> = ∫ d³k/(2π)³ 1/√(2ω_k) a_k† |0> e^{-ik·x + iωt}
Symmetry (glo
In Minkowski
ferred Vacuum
<0|Φ
= f
= f


Symmetry
In Minkowski
Preferred vacuum
Correlators

$$\Phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} a_k^\dagger |0\rangle e^{-i\vec{k}\cdot\vec{x} + i\omega t}$$

$$\Phi^\dagger = \Phi$$

$$\langle 0|\Phi(x) = \left(\Phi(x)|0\rangle\right)^\dagger$$

Symmetry

In Minkowski

Preferred Va

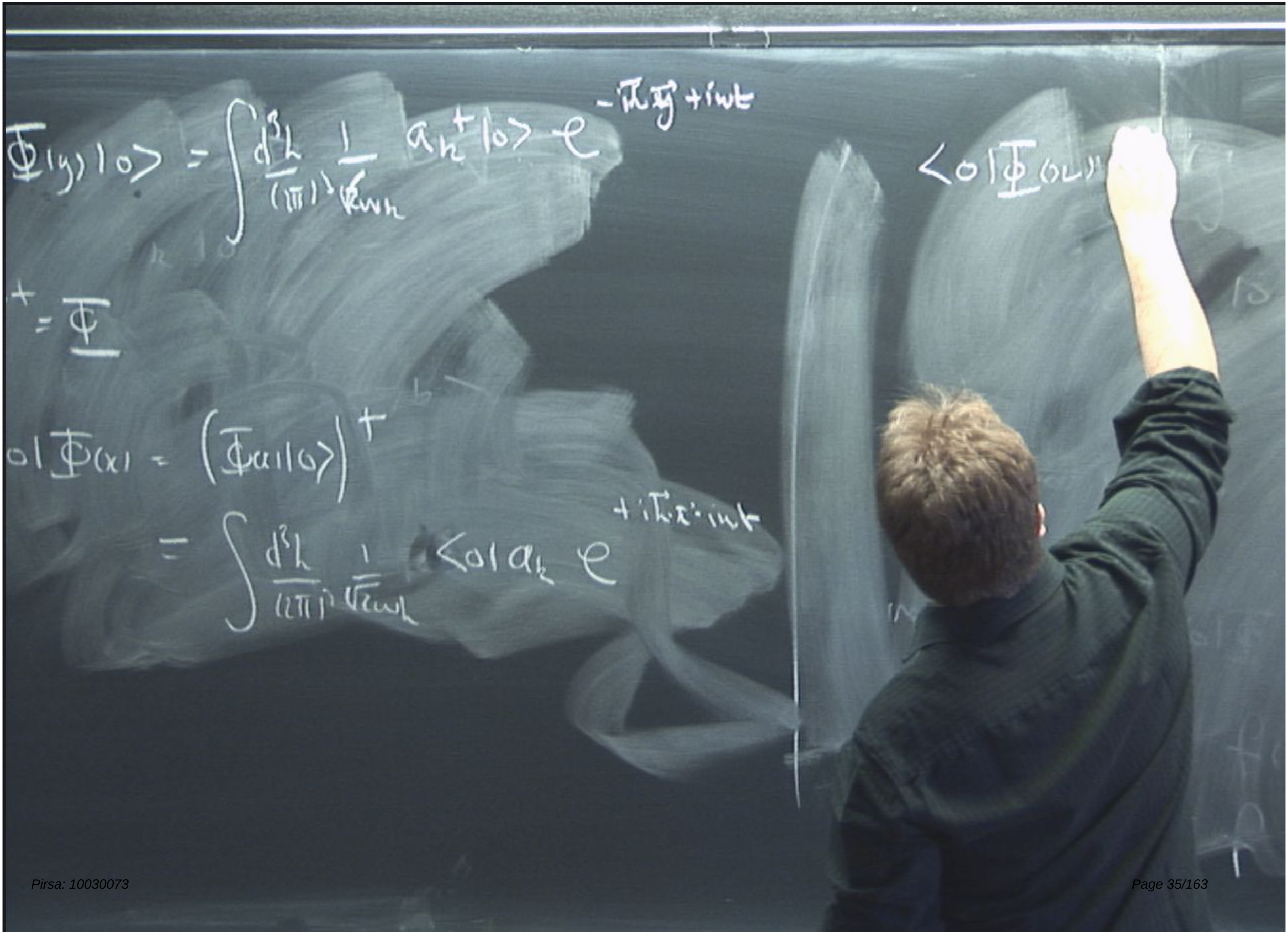
Correlators

$$\Phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \, a_k^\dagger |0\rangle \, e^{-i\vec{k}\cdot\vec{x} + i\omega t}$$

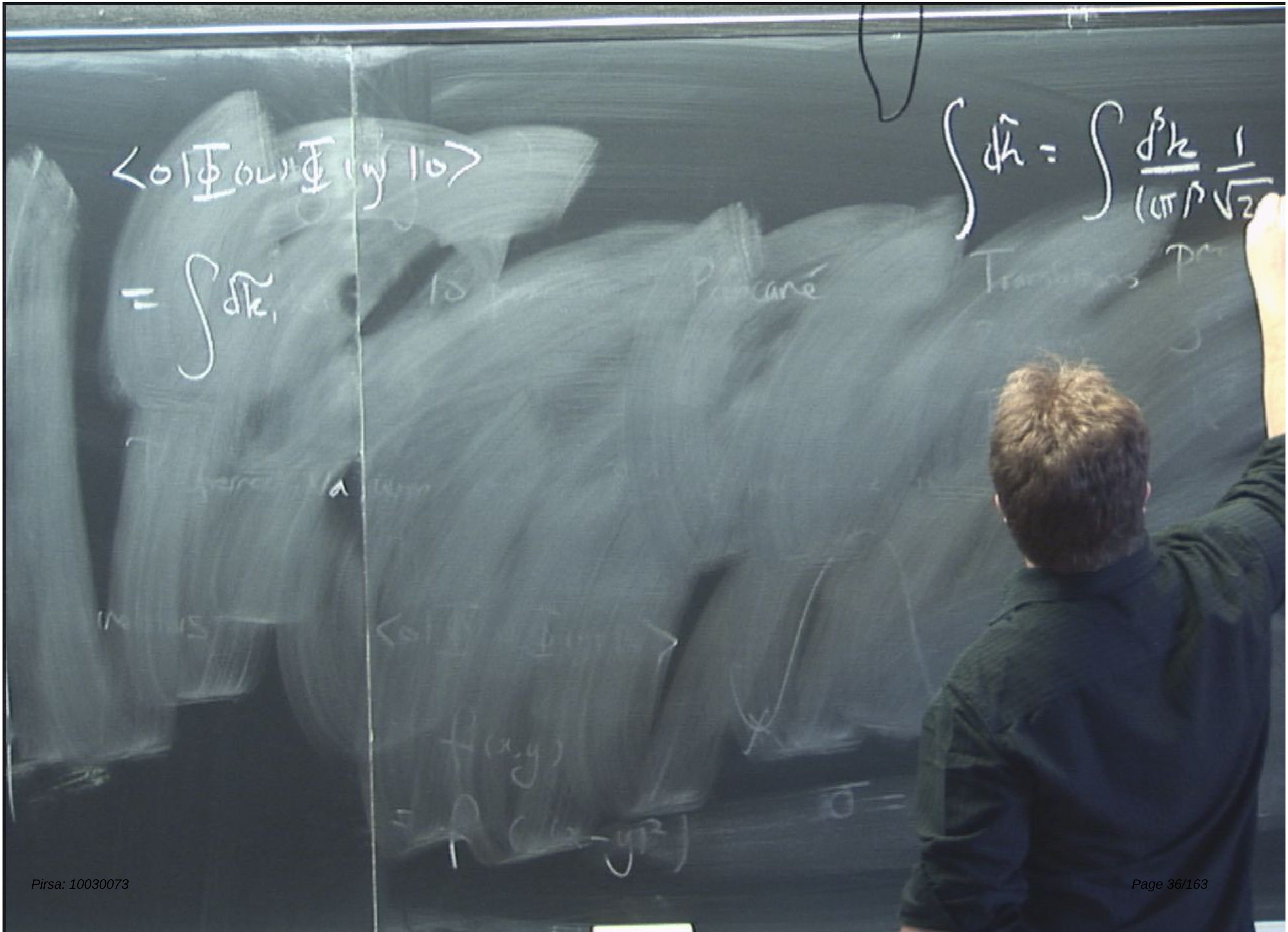
$$\Phi^\dagger = \Phi$$

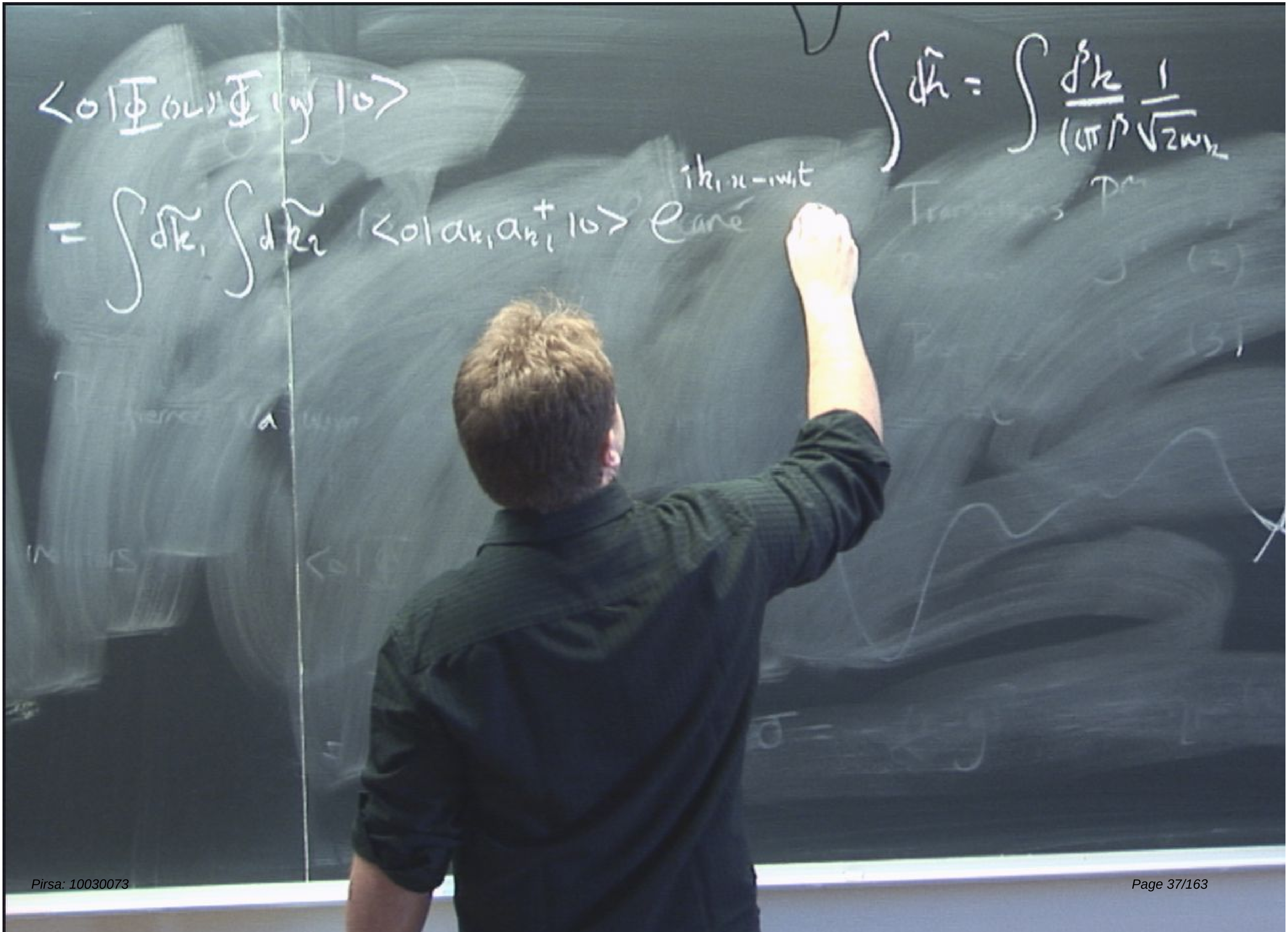
$$\langle 0|\Phi(x) = \left(\Phi(x)|0\rangle\right)^\dagger$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \, \langle 0| a_k \, e^{+i\vec{k}\cdot\vec{x} - i\omega t}$$



Φ(y)|0> = ∫ d³k/(2π)³ 1/√2ω_k a_k^+|0> e^{-ik·y + iωt}
= Φ
<0|Φ(x) = (Φ(x)|0>)^+
= ∫ d³k/(2π)³ 1/√2ω_k <0|a_k e^{+ik·x - iωt}
<0|Φ(x)


⟨0|Φ(x)Φ(y)|0⟩
= ∫ dk̃
∫ dk̃ = ∫ d³k/(2π)³ 1/√2
Poincaré
Translations
f(x,y)


⟨0|Φ(x)Φ(y)|0⟩
= ∫dk̃₁ ∫dk̃₂ ⟨0|a_{k₁} a†_{k₁}|0⟩ e^{ik₁x − iω₁t}
∫dk̃ = ∫ d³k/(2π)³ 1/√(2ω_k)

$$\langle 0|\Phi(x)\Phi(y)|0\rangle$$

$$= \int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0|a_{k_1} a^{\dagger}_{k_2}|0\rangle \, e^{ik_1\cdot x - i\omega_1 x^0} \, e^{-ik_2\cdot y + i\omega_2 y^0}$$

$$\int d\tilde{k} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}$$

$$\int d\tilde{k} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}}$$

$$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle$$

$$= \int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0 | a_{k_1} a^\dagger_{k_2} | 0 \rangle \, e^{i k_1 \cdot x - i\omega_1 x^0} \, e^{-i k_2 \cdot y + i \omega_2 y^0}$$

$$\Downarrow$$

$$\int d\tilde{k} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}$$

$$\langle 0|\Phi(x)\Phi(y)|0\rangle$$

$$= \int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0|a_{k_1} a^\dagger_{k_2}|0\rangle \, e^{ik_1\cdot x - i\omega_1 x^0} \, e^{-ik_2\cdot y + i\omega_2 y^0}$$

$$\downarrow$$

$$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$$

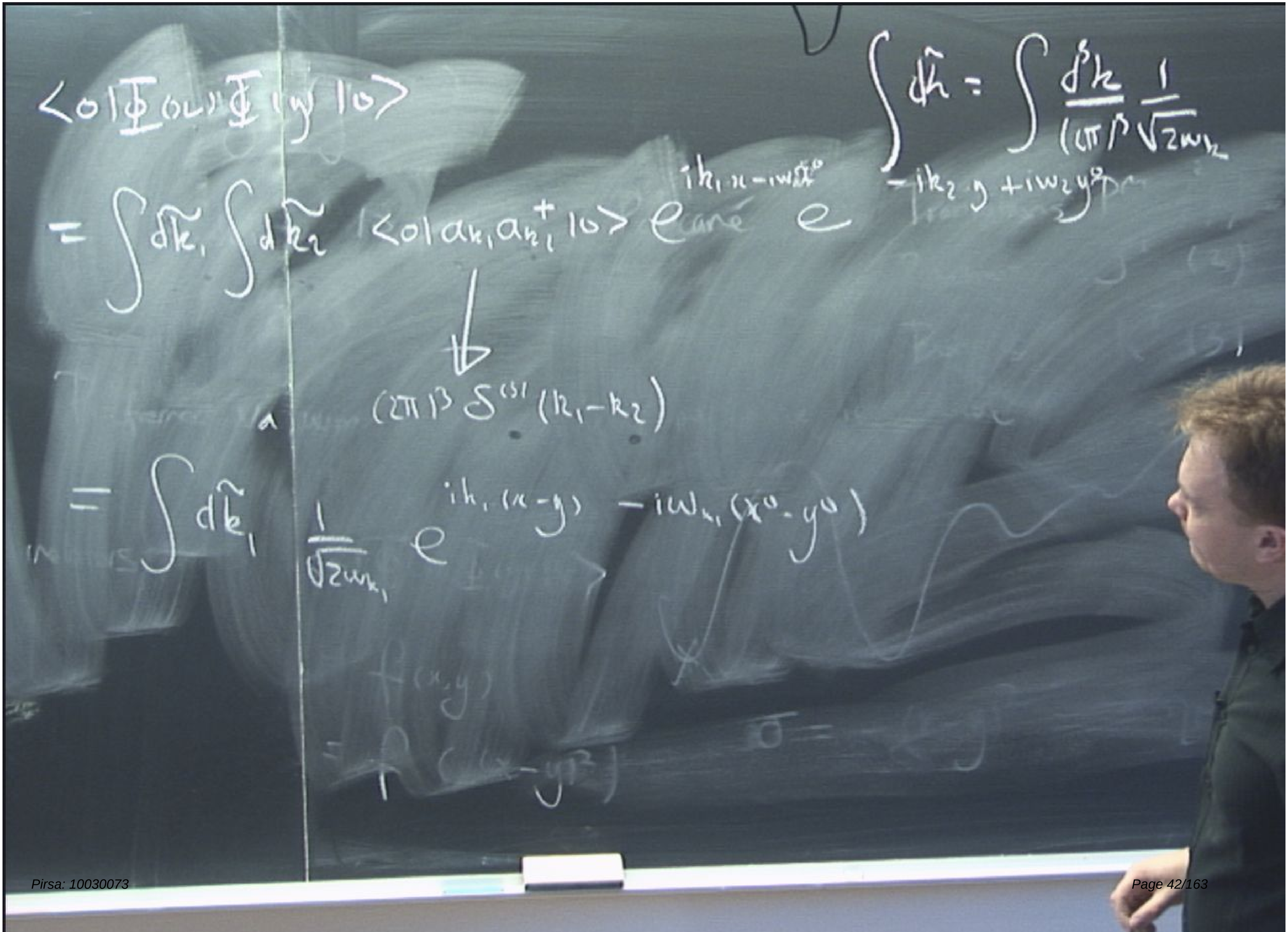
$$\int d\tilde{k} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}$$

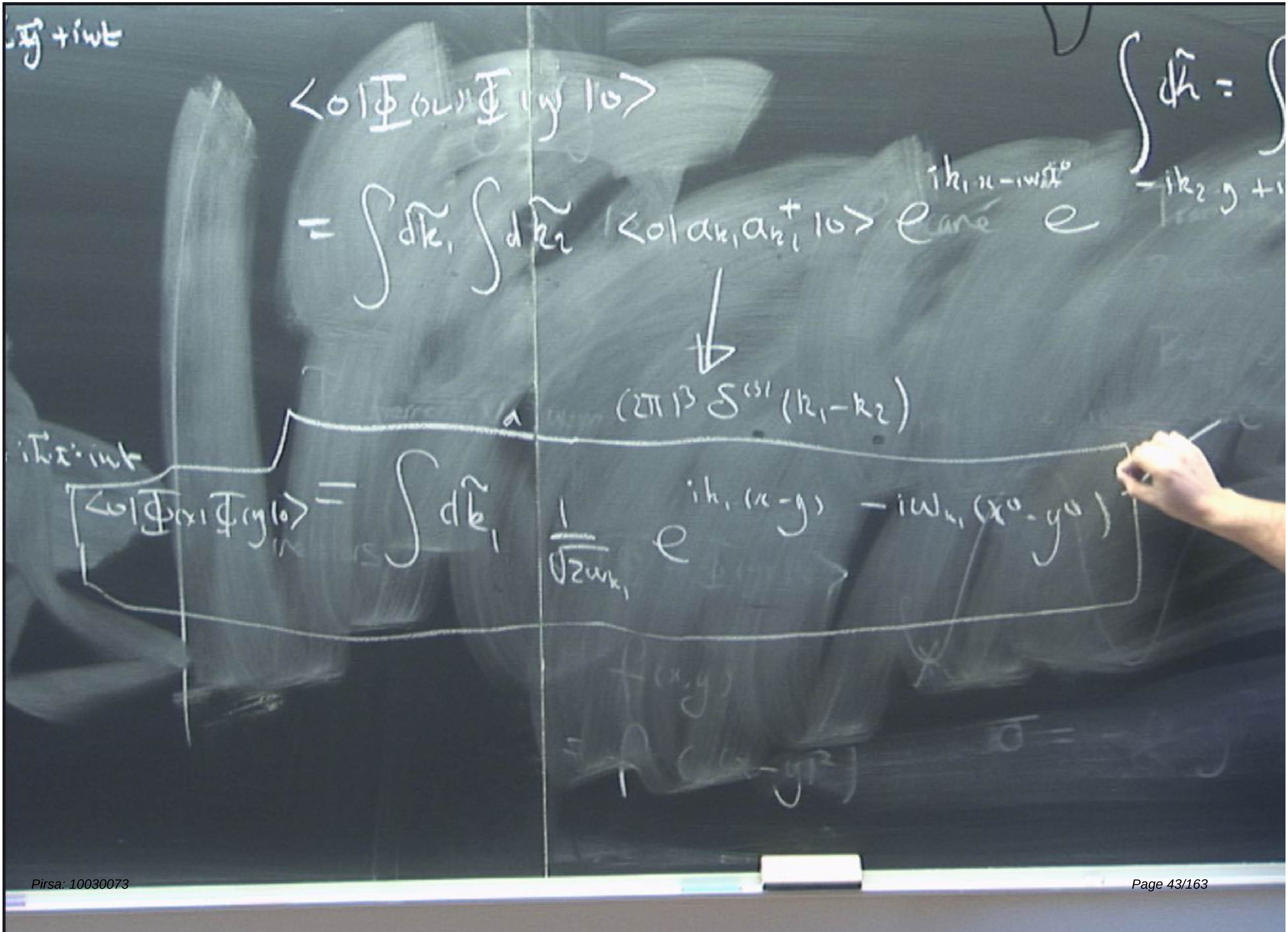
$$\langle 0|\Phi(x)\Phi(y)|0\rangle$$

$$= \int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0|a_{k_1} a^\dagger_{k_2}|0\rangle \, e^{ik_1\cdot x - i\omega_1 x^0} \, e^{-ik_2\cdot y + i\omega_2 y^0}$$

$$\langle 0|a_{k_1} a^\dagger_{k_2}|0\rangle \;\Downarrow\; (2\pi)^3 \delta^{(3)}(k_1-k_2)$$

$$= \int d\tilde{k}_1 \, \frac{1}{\sqrt{2\omega_{k_1}}}$$


⟨0|Φ(x)Φ(y)|0⟩
∫dk̃ = ∫ d³k/(2π)³ 1/√(2ω_k)
= ∫dk̃₁ ∫dk̃₂ ⟨0|a_{k₁} a⁺_{k₂}|0⟩ e^{ik₁·x − iω₁x⁰} e^{−ik₂·y + iω₂y⁰}
(2π)³ δ⁽³⁾(k₁−k₂)
= ∫dk̃₁ 1/√(2ω_{k₁}) e^{ik₁·(x−y) − iω_{k₁}(x⁰−y⁰)}


$\langle 0|\Phi(x)\Phi(y)|0\rangle$
$= \int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0|a_{k_1} a^\dagger_{k_1}|0\rangle \, e^{ik_1 \cdot x - i\omega x^0} \, e^{-ik_2 \cdot y + i}$
$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$
$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \, \frac{1}{\sqrt{2\omega_{k_1}}} \, e^{ik_1 \cdot (x-y) - i\omega_{k_1}(x^0 - y^0)}$
$\int d\hat{k} =$

$$\int d\hat{k} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2 \ldots}}$$

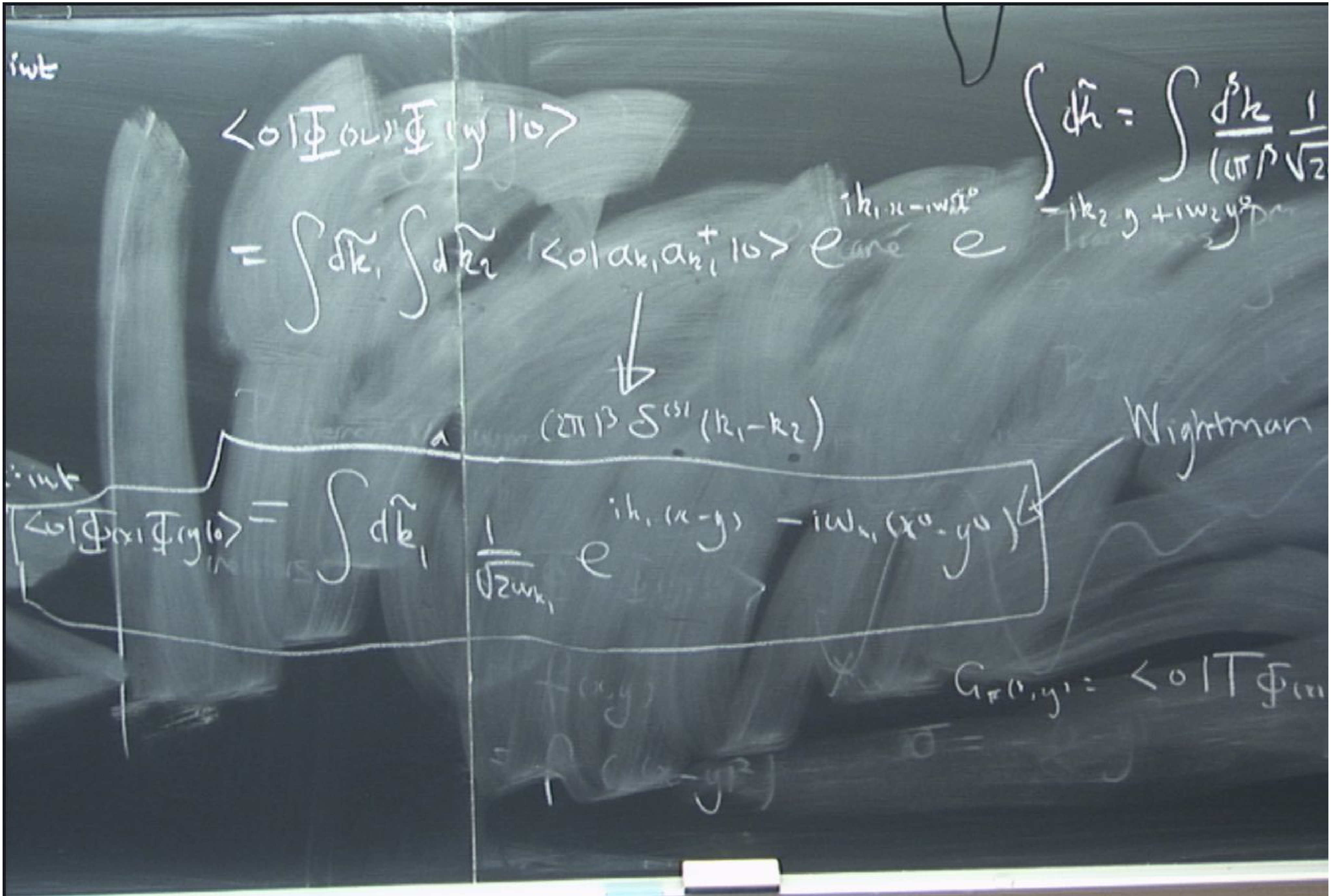
$$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle$$

$$= \int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0 | a_{k_1} a^{+}_{k_2} | 0 \rangle \, e^{ik_1 \cdot x - i\omega_1 x^0} \, e^{-ik_2 \cdot y + i\omega_2 y^0}$$

$$\langle 0 | a_{k_1} a^{+}_{k_2} | 0 \rangle \to (2\pi)^3 \delta^{(3)}(k_1 - k_2)$$

$$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}} e^{ik_1 \cdot (x-y) - i\omega_{k_1}(x^0 - y^0)}$$

Wightman


$\langle 0|\Phi(x)\Phi(y)|0\rangle$
$= \int d\tilde{k}_1 \int d\tilde{k}_2 \langle 0|a_{k_1} a^{+}_{k_2}|0\rangle e^{ik_1\cdot x - i\omega_1 x^0} e^{-ik_2\cdot y + i\omega_2 y^0}$
$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$
$\int d\tilde{k} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2}}$
$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}} e^{ik_1\cdot(x-y) - i\omega_{k_1}(x^0 - y^0)}$
Wightman
$G_F(x,y) = \langle 0|T\Phi(x)$

$\Phi(x)\Phi(y)|0\rangle$

$$\int d\tilde{k} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}$$

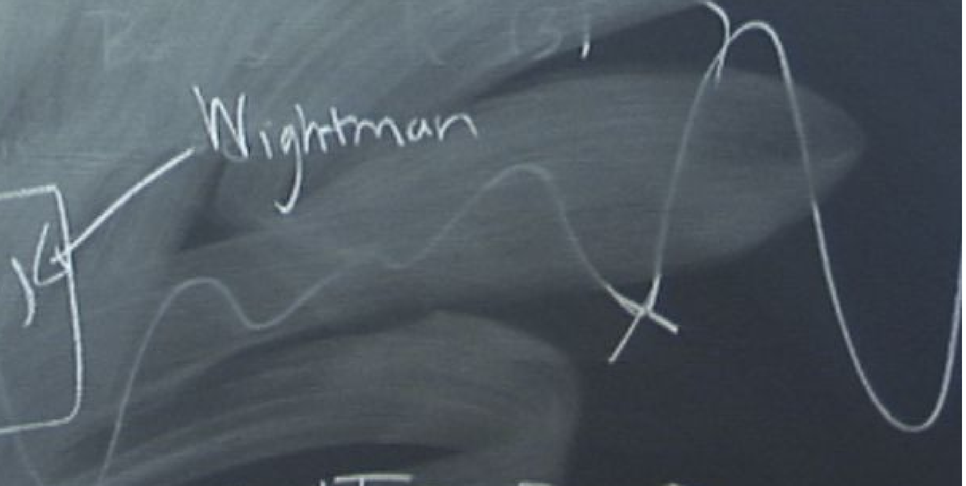
$$\int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0|a_{k_1} a^{\dagger}_{k_2}|0\rangle \, e^{ik_1\cdot x - i\omega_1 x^0} \, e^{-ik_2\cdot y + i\omega_2 y^0}$$

$$\downarrow$$

$$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$$

$$\int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}} e^{ik_1\cdot(x-y) - i\omega_{k_1}(x^0 - y^0)}$$

Wightman



$$G_F(x,y) = \langle 0|T\,\Phi(x)\Phi(y)|0\rangle$$

$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle$

$$\int d\hat{k} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}}$$

$$\int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0 | a_{k_1} a^\dagger_{k_2} | 0 \rangle \, e^{ik_1 \cdot x - i\omega_1 x^0} \, e^{-ik_2 \cdot y + i\omega_2 y^0}$$

$$\Downarrow$$

$$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$$

$$\int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}} e^{ik_1 \cdot (x-y) - i\omega_{k_1}(x^0 - y^0)}$$

← Wightman

$G_F(x,y) = \langle 0 | T \Phi(x) \Phi(y) | 0 \rangle$

$$\langle 0|\Phi(x)\Phi(y)|0\rangle$$

$$\int d\hat{k} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}$$

$$\int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0|a_{k_1} a^{\dagger}_{k_2}|0\rangle \, e^{ik_1\cdot x - i\omega_1 x^0} \, e^{-ik_2\cdot y + i\omega_2 y^0}$$

$$\downarrow$$

$$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$$

$$\int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}} e^{ik_1\cdot(x-y) - i\omega_{k_1}(x^0-y^0)}$$

Wightman

$$G_F(x,y) = \langle 0|T\,\Phi(x)\Phi(y)|0\rangle$$

$-ik\cdot y + i\omega t$

$\langle 0|\Phi(x)\Phi(y)|0\rangle$

$$= \int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0| a_{k_1} a^\dagger_{k_2} |0\rangle \, e^{ik_1\cdot x - i\omega_1 x^0} \, e^{-ik_2\cdot y + i\omega_2 y^0}$$

$\downarrow$

$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$

$\int d\hat{k} = \int \frac{d^3k}{(2\pi)}$

$+ ik\cdot x - i\omega t$

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \, \frac{1}{\sqrt{2\omega_{k_1}}} \, e^{ik_1\cdot(x-y) - i\omega_{k_1}(x^0 - y^0)}$$

Wightm

$G_F(x,y) = \langle 0|T$

$$\Phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} a_k^+ |0\rangle e^{-i\vec{k}\cdot\vec{y}+i\omega t}$$

$$\Phi^+ = \Phi$$

$$\langle 0|\Phi(x) = \left(\Phi(x)|0\rangle\right)^+$$

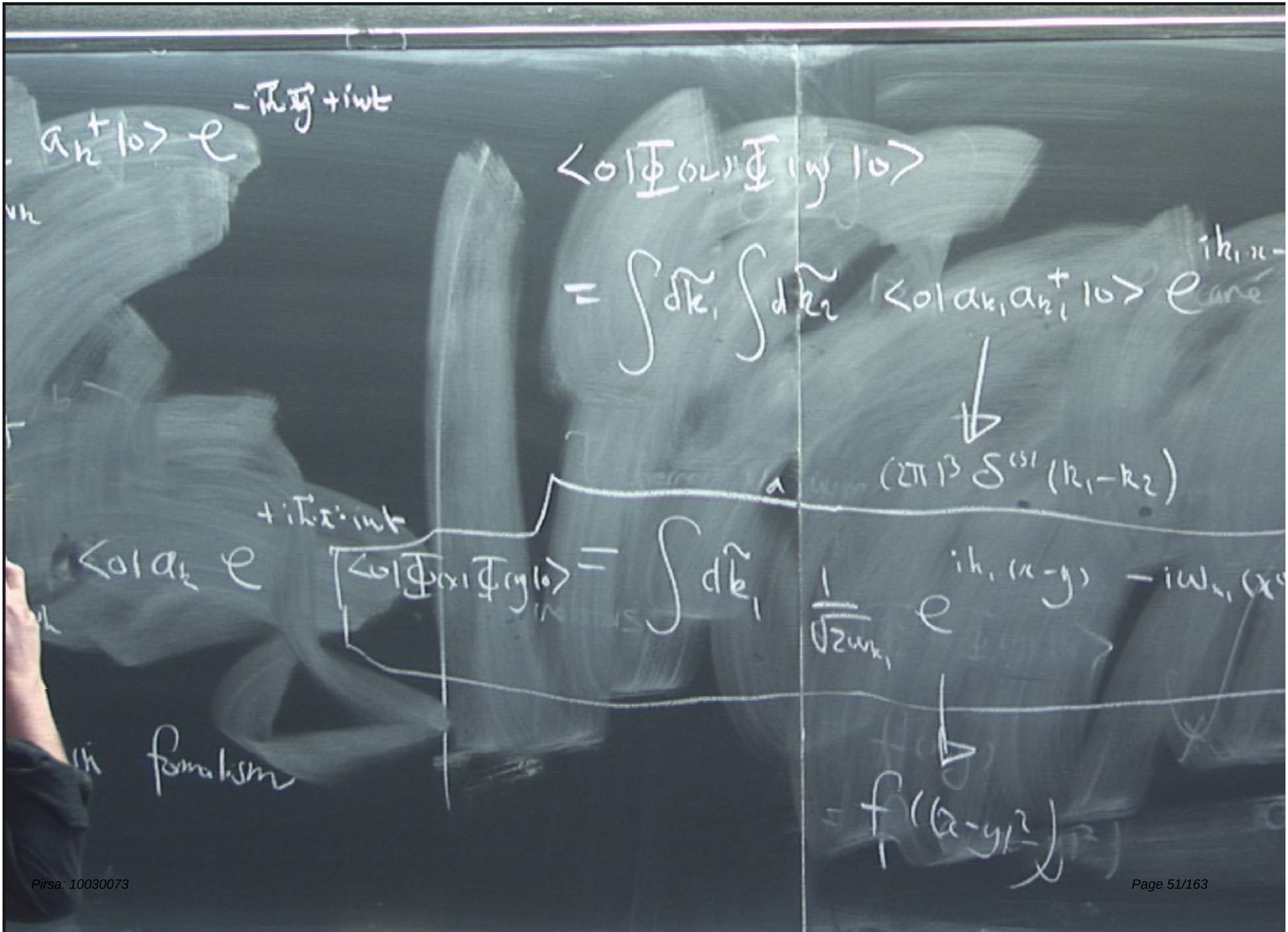
$$= \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \langle 0| a_k e^{+i\vec{k}\cdot\vec{x}-i\omega t}$$

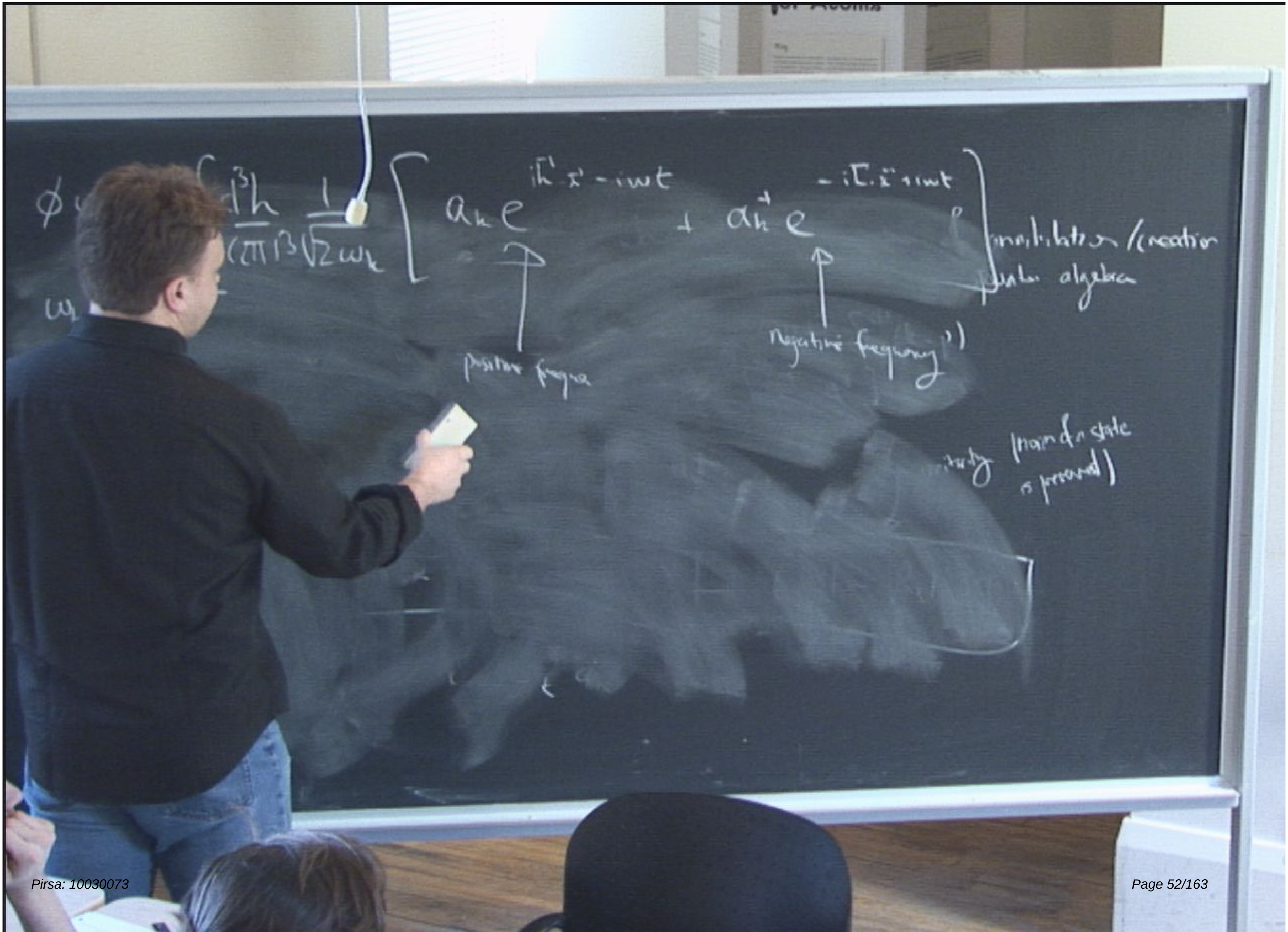
in-in formalism

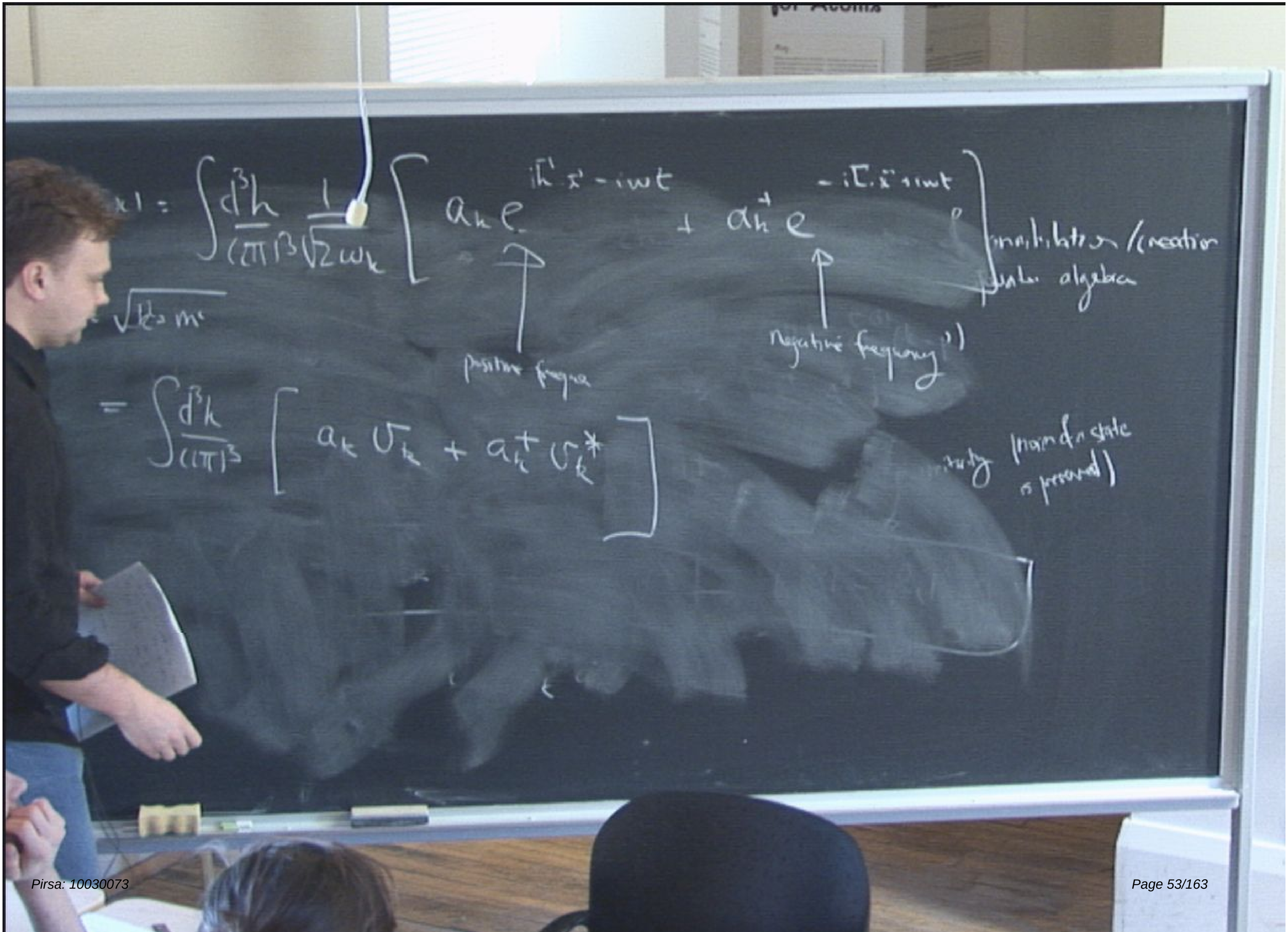
$$\langle 0|\Phi(x)\Phi(y)|0\rangle$$

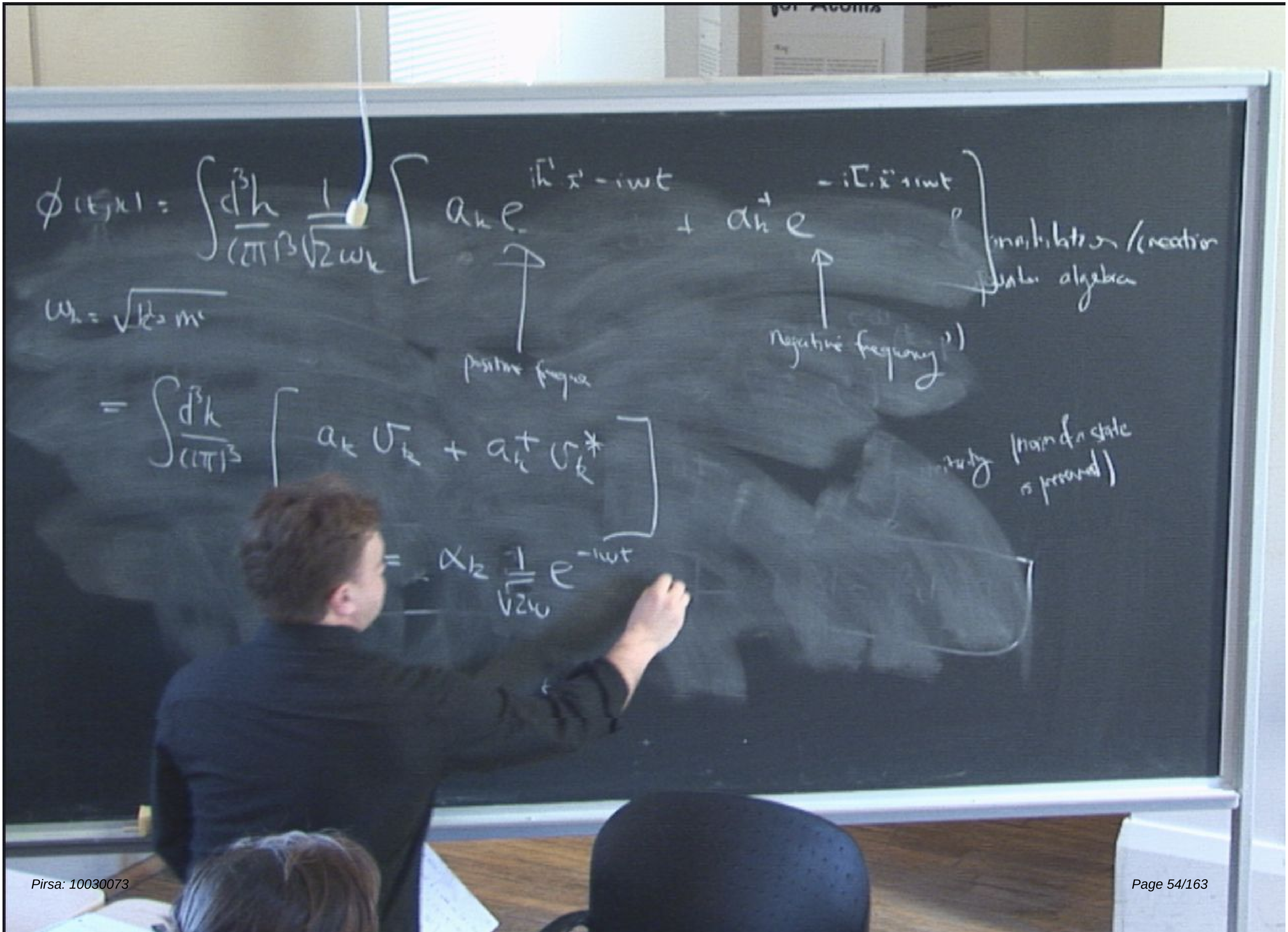
$$= \int d\tilde{k}_1 \int d\tilde{k}_2$$

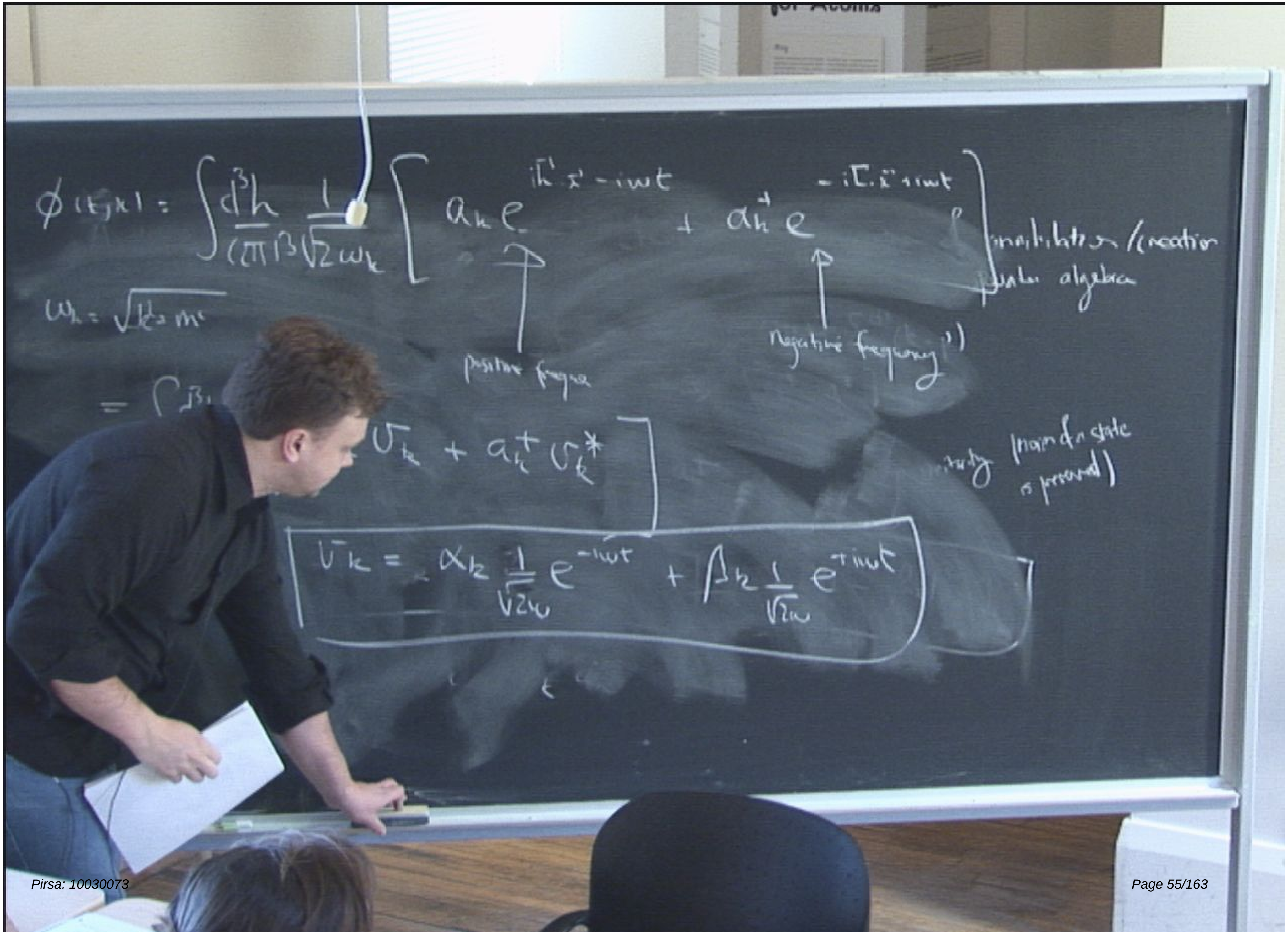
$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}}$$

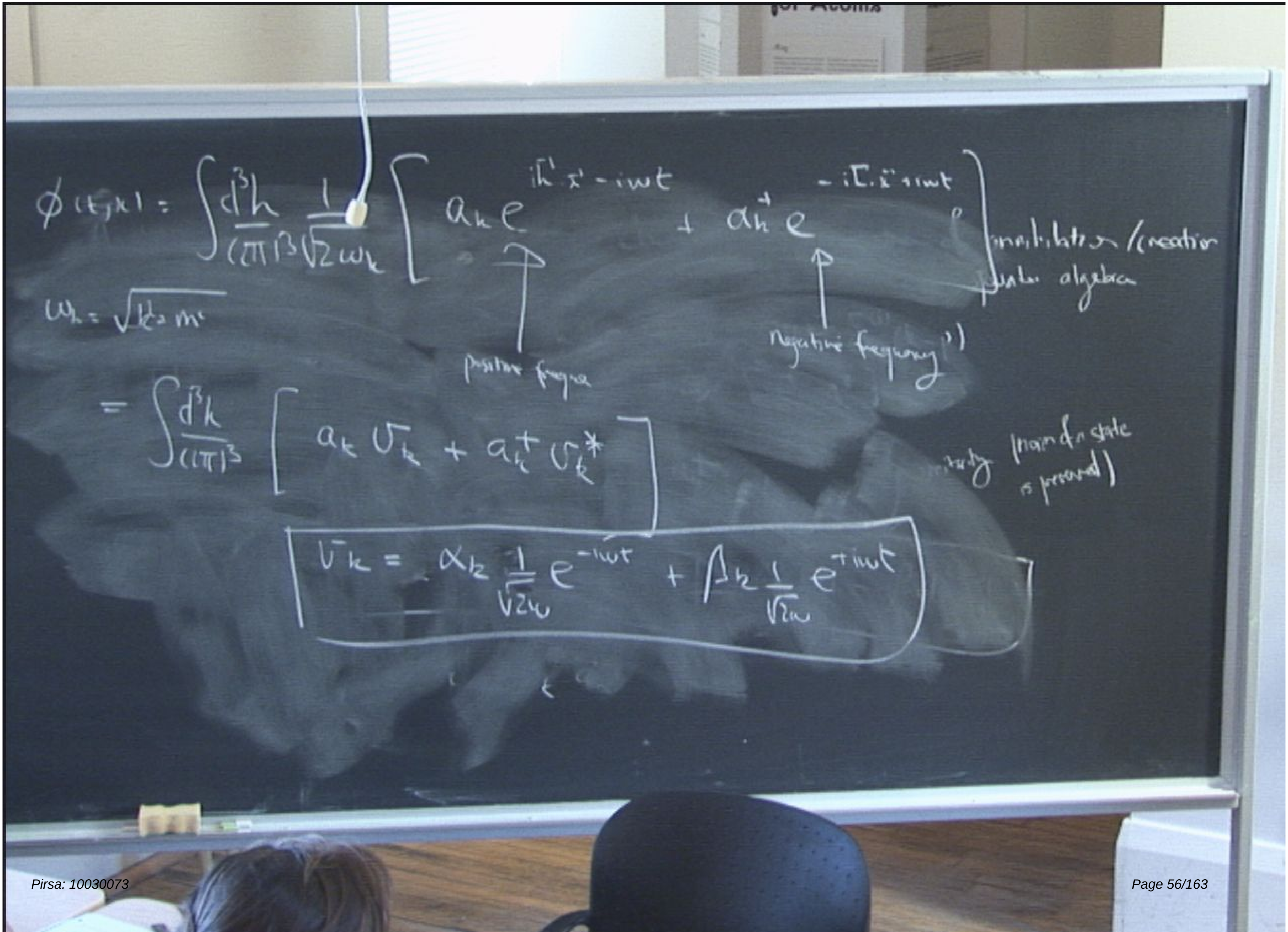

⟨0|Φ(x) Φ(y)|0⟩
= ∫dk̃₁ ∫dk̃₂ ⟨0|a_{k₁} a†_{k₁}|0⟩ e^{ik₁x}
(2π)³ δ⁽³⁾(k₁ − k₂)
⟨0|Φ(x) Φ(y)|0⟩ = ∫dk̃₁ (1/√(2ω_{k₁})) e^{ik₁(x−y) − iω_{k₁}(x
f((x−y)²)
formalism

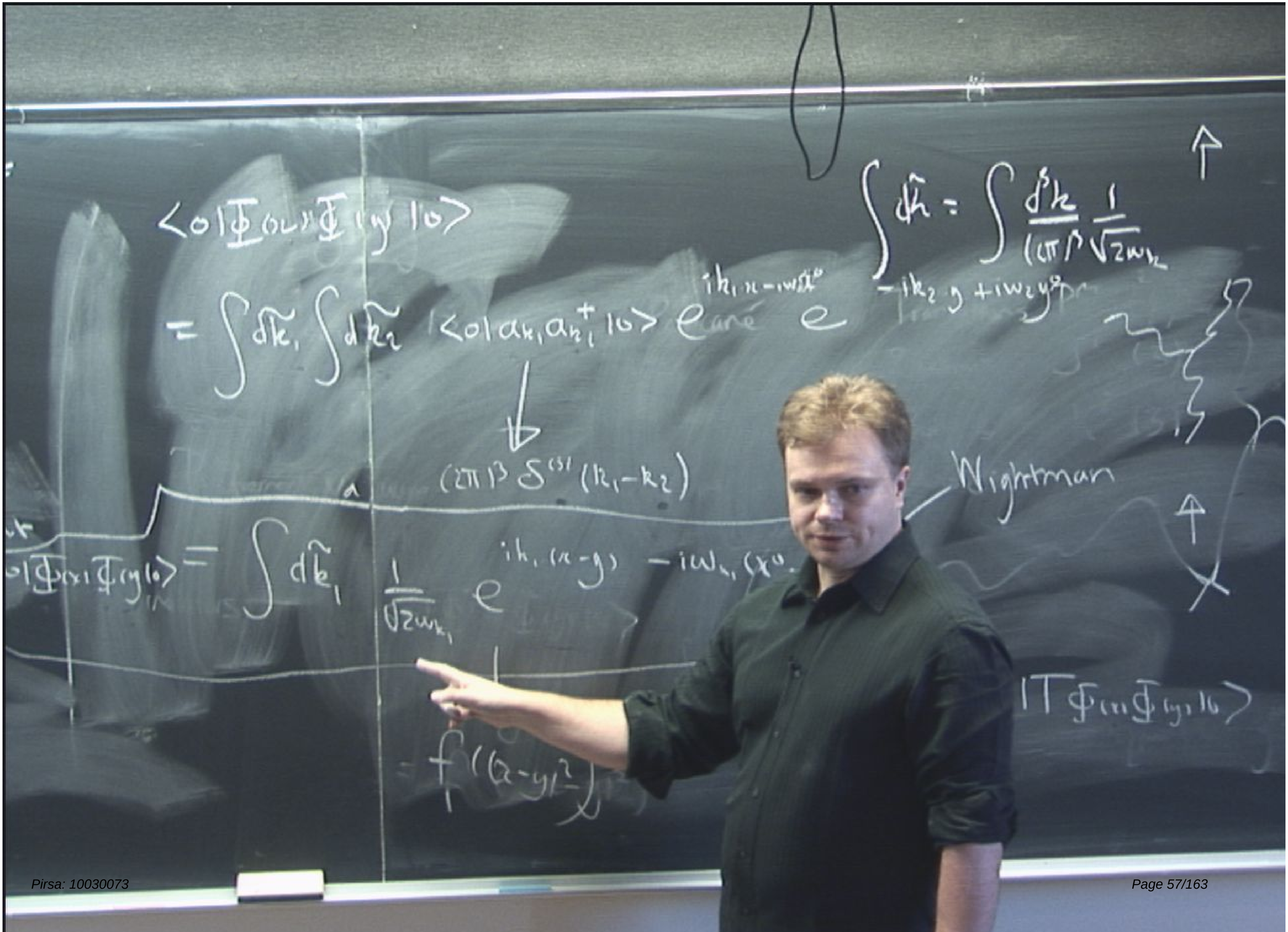

negative frequency
algebra

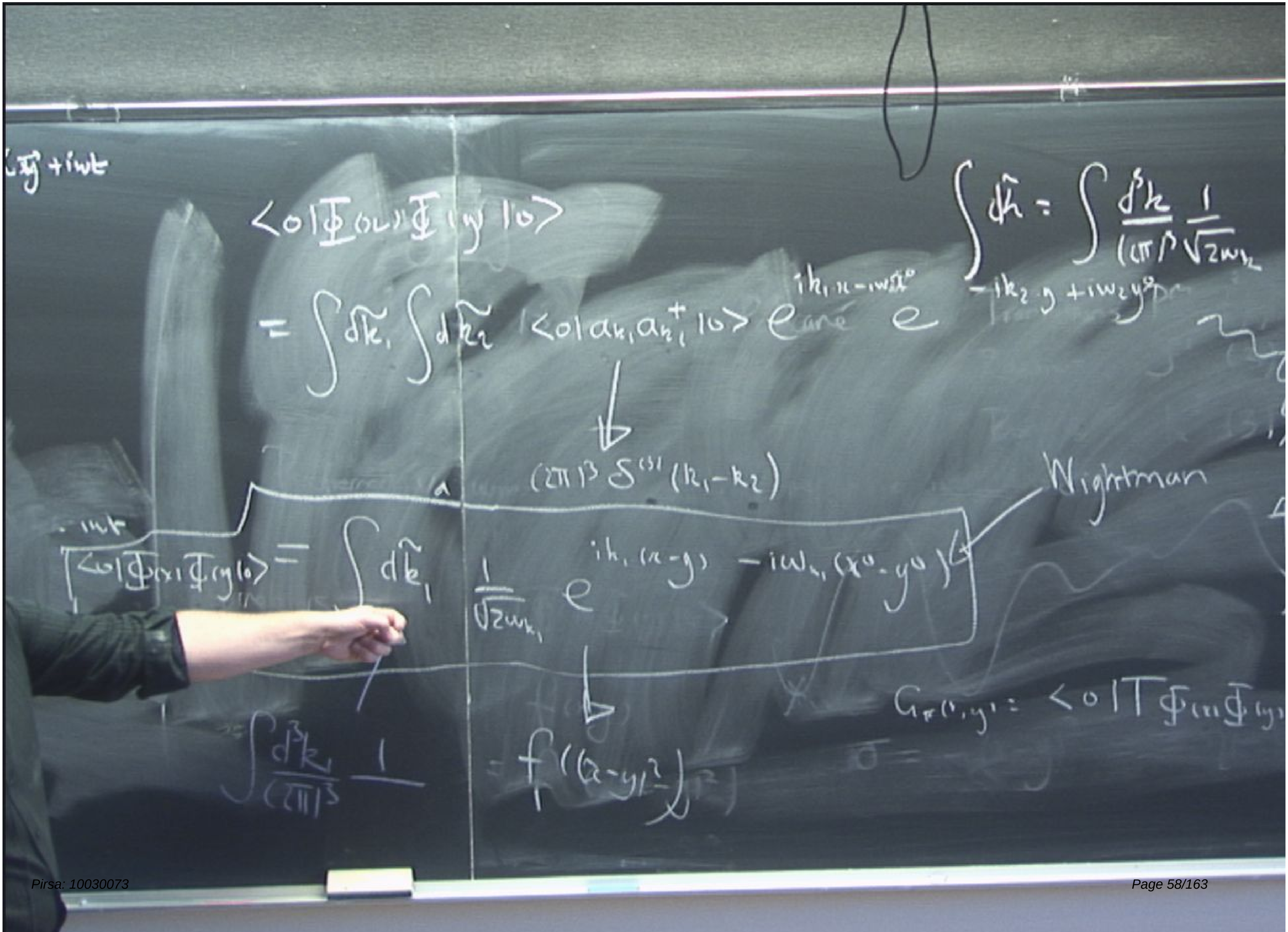

positive frequen
negative frequency
algebra

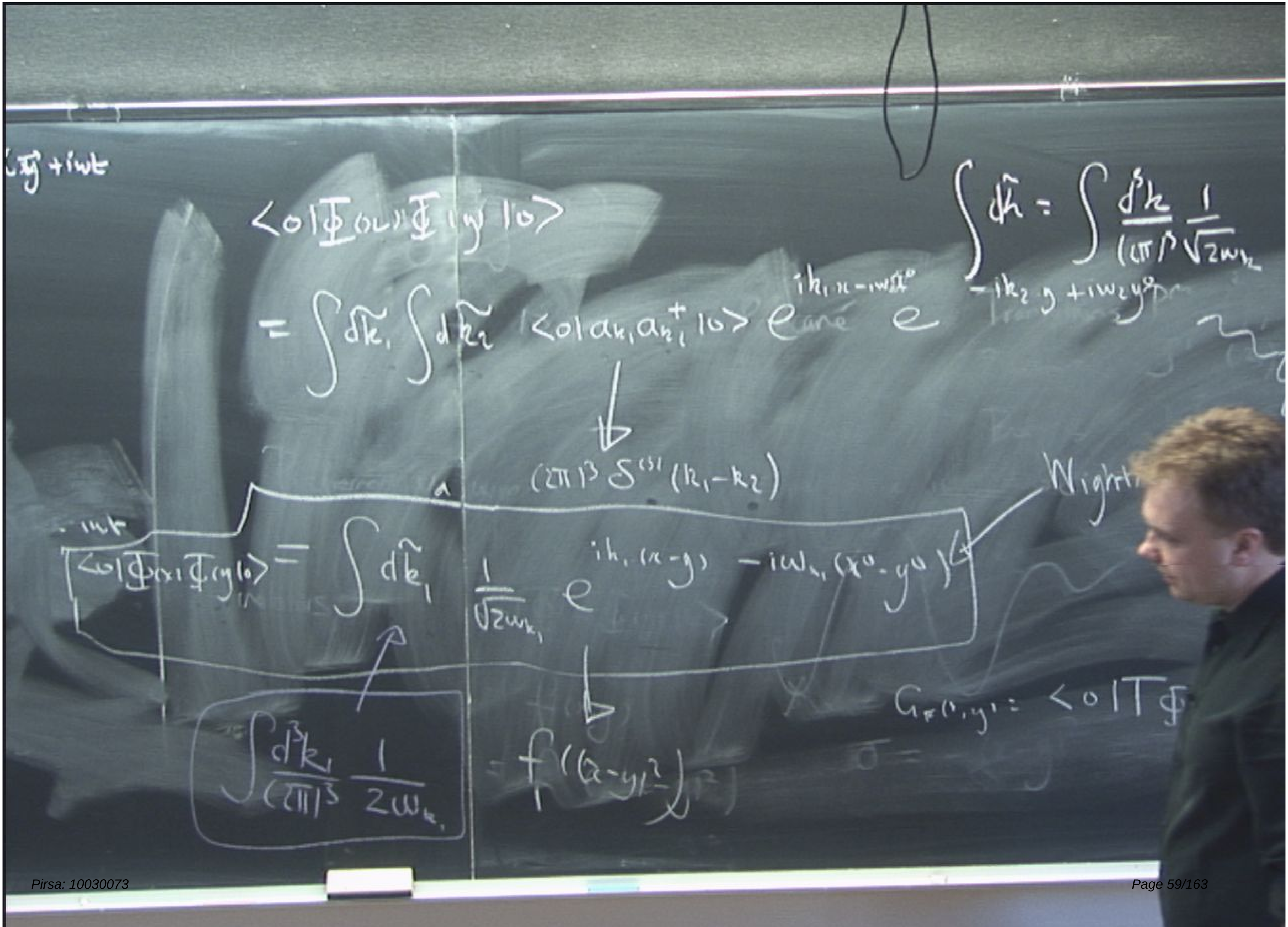

Negative frequency
algebra
is preserved)

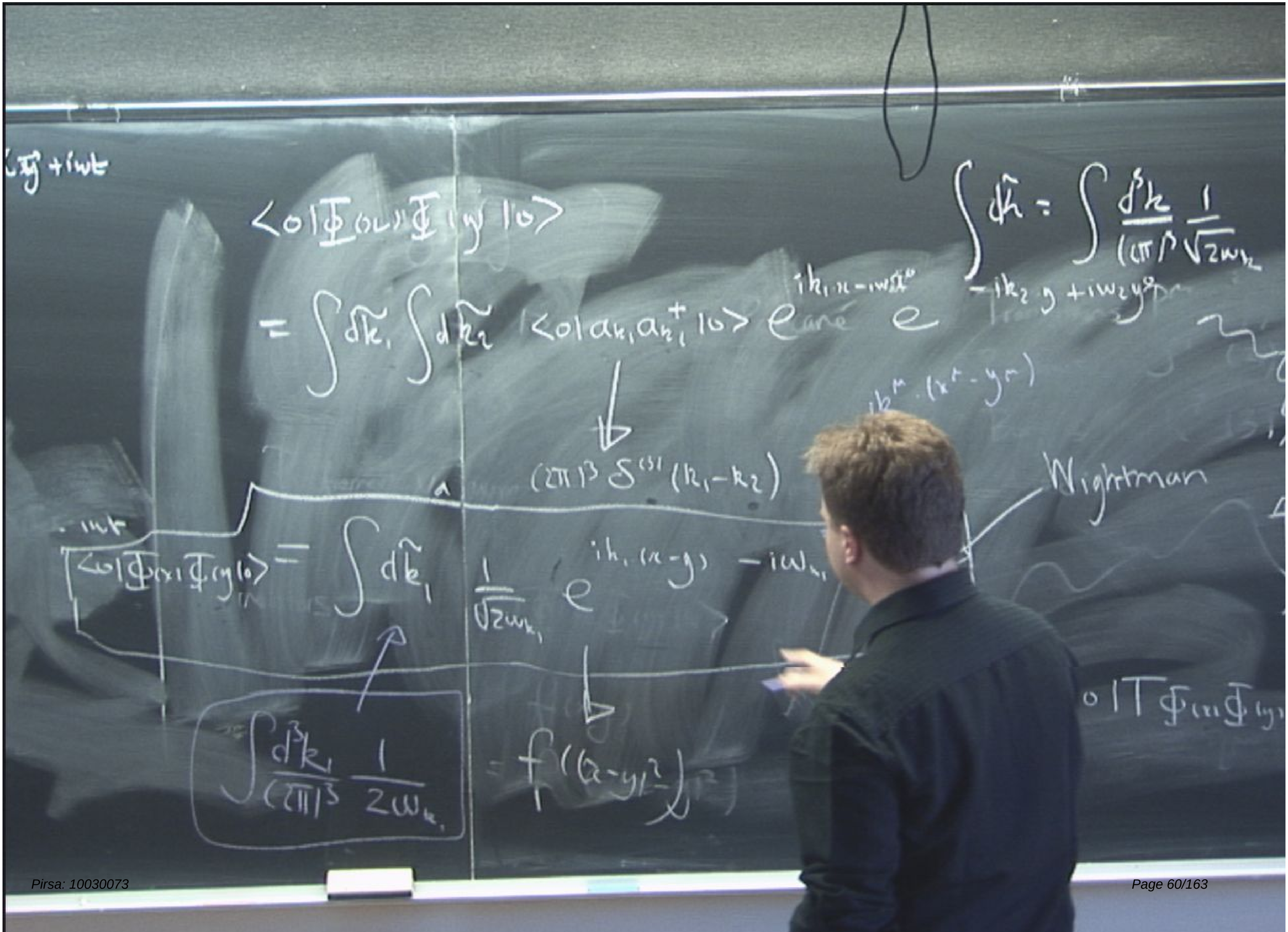

positive frequen
Negative frequency
annihilation/creation
algebra
(norm of a state
is preserved)

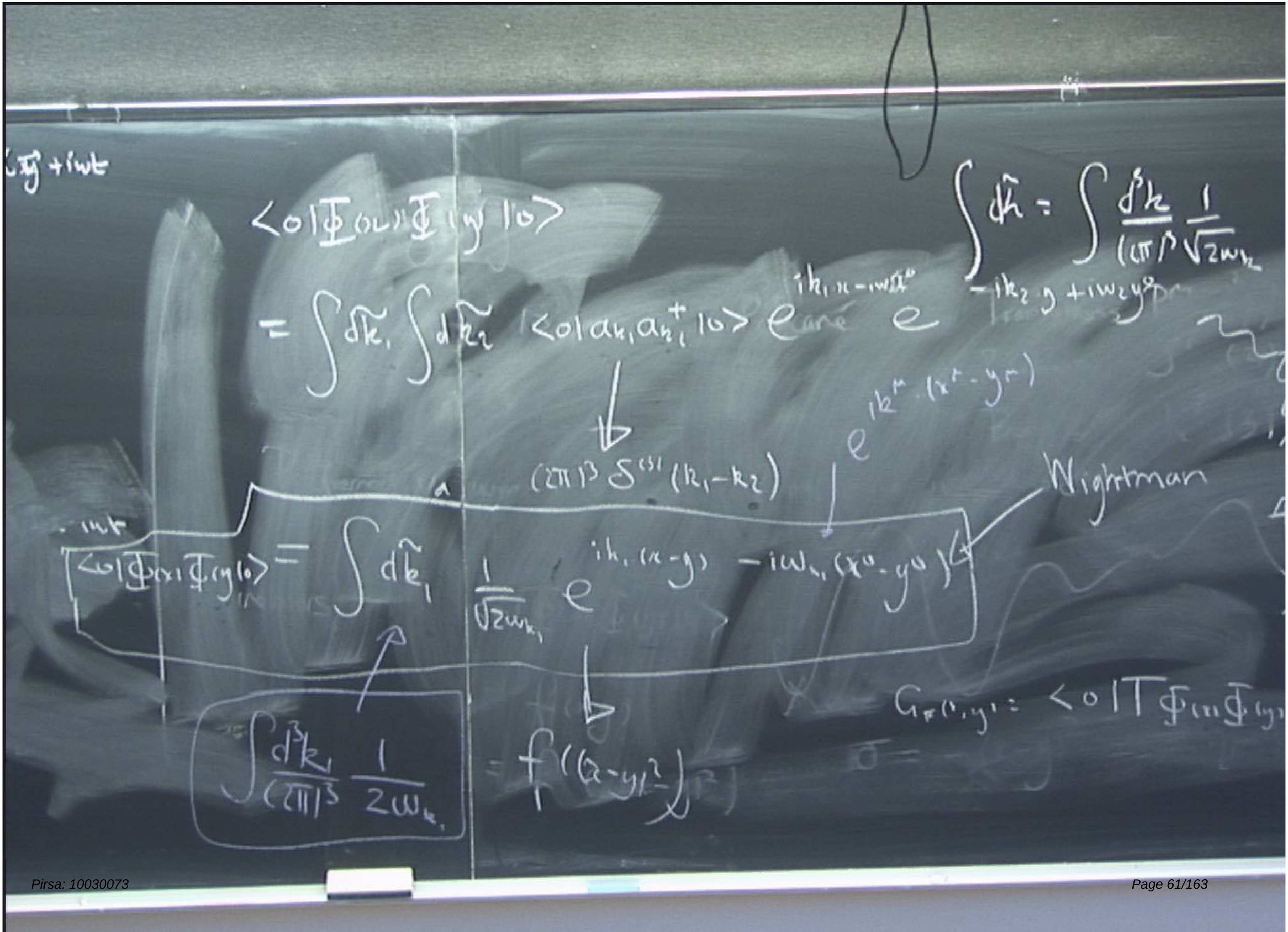

$\phi(t,x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[a_k e^{i\vec{k}\cdot\vec{x} - i\omega t} + a_k^\dagger e^{-i\vec{k}\cdot\vec{x} + i\omega t} \right]$
annihilation / creation
algebra
$\omega_k = \sqrt{k^2 + m^2}$
positive frequency
negative frequency
$= \int \frac{d^3k}{(2\pi)^3} \left[a_k v_k + a_k^\dagger v_k^* \right]$
(norm of a state is preserved)
$v_k = \alpha_k \frac{1}{\sqrt{2\omega}} e^{-i\omega t} + \beta_k \frac{1}{\sqrt{2\omega}} e^{+i\omega t}$


Wightman

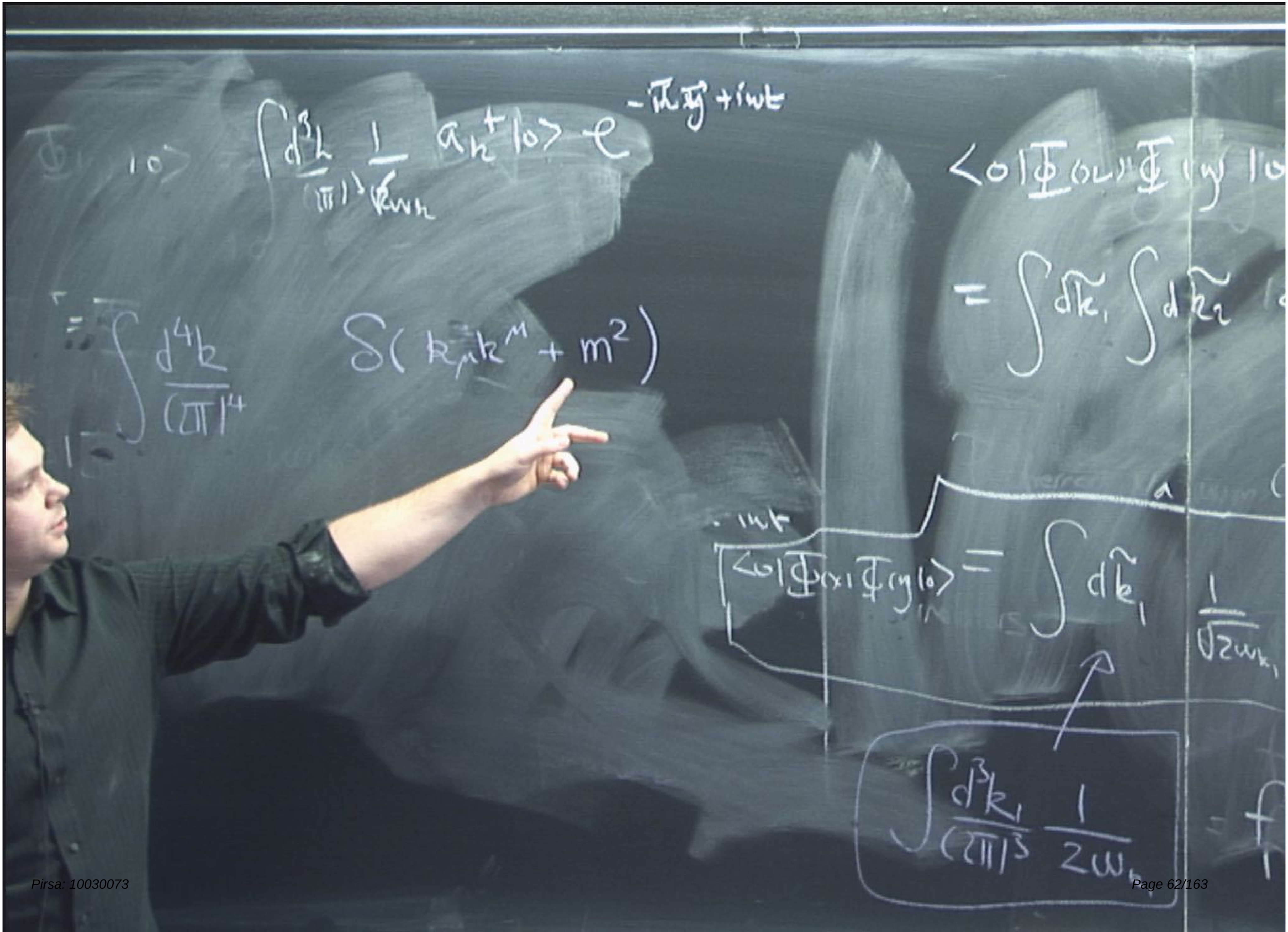

Wightman
$\langle 0|\Phi(x)\Phi(y)|0\rangle$
$\int d\tilde{k} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}$
$= \int d\tilde{k}_1 \int d\tilde{k}_2 \langle 0|a_{k_1}a^{+}_{k_2}|0\rangle e^{ik_1 x - i\omega_1 x^0} e^{-ik_2 y + i\omega_2 y^0}$
$(2\pi)^3\delta^{(3)}(k_1-k_2)$
$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}} e^{ik_1(x-y) - i\omega_{k_1}(x^0-y^0)}$
$G_F(x,y) = \langle 0|T\Phi(x)\Phi(y)$
$\int \frac{d^3k}{(2\pi)^3}\frac{1}{\ }$
$= f((x-y)^2)$

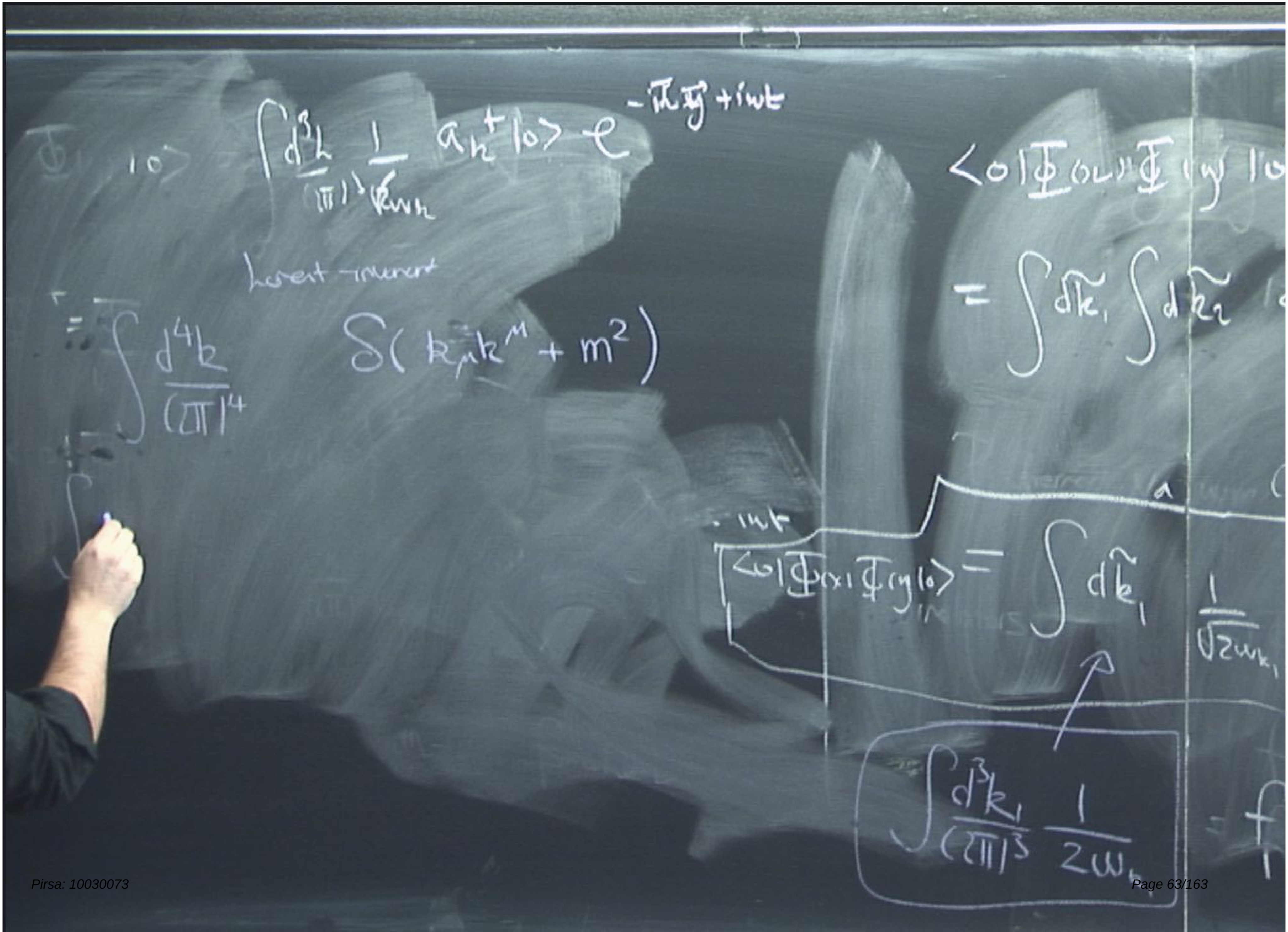

⟨0|Φ(x)Φ(y)|0⟩
(2π)³ δ⁽³⁾(k₁−k₂)
Wight
G_F(x,y) = ⟨0|TΦ


$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle$
$= \int d\tilde{k}_1 \int d\tilde{k}_2 \, \langle 0 | a_{k_1} a^\dagger_{k_2} | 0 \rangle \, e^{ik_1 x - i\omega t} \, e^{-ik_2 y + i\omega_2 y^0}$
$\int d\tilde{k} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}}$
$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$
$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}} e^{ik_1(x-y) - i\omega_{k_1}}$
Wightman
$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}}$
$= f((x-y)^2)$



⟨0|Φ(x)Φ(y)|0⟩
∫dk̃ = ∫ d³k/(2π)³ 1/√(2ω_k)
(2π)³ δ⁽³⁾(k₁−k₂)
Wightman
⟨0|Φ(x)Φ(y)|0⟩ = ∫dk̃₁ 1/√(2ω_k₁) e^{ik₁·(x−y) − iω_k₁(x⁰−y⁰)}
∫ d³k₁/(2π)³ 1/(2ω_k₁)
= f((x−y)²)
G_F(x,y) = ⟨0|T Φ(x)Φ(y)


$\int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} a_k^\dagger |0\rangle e^{-ik \cdot y + i\omega t}$
$= \int \frac{d^4k}{(2\pi)^4} \delta(k_\mu k^\mu + m^2)$
$\langle 0|\Phi(x)\Phi(y)|0\rangle$
$= \int d\tilde{k}_1 \int d\tilde{k}_2$
$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}}$
$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}}$


Lorentz invariant
<0| Φ(x) Φ(y) |0>
= ∫ dk̃₁ ∫ dk̃₂
<0| Φ(x) Φ(y) |0> = ∫ dk̃₁ 1/√(2ω_k₁)
∫ d³k₁/(2π)³ 1/(2ω_k₁)

$\Phi \, |0\rangle \; \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} a_k^\dagger |0\rangle e^{-i\vec{k}\cdot\vec{y}+i\omega t}$

Lorentz invariant

$= \int \frac{d^4k}{(2\pi)^4} \quad \delta(k_\mu k^\mu + m^2)$

$\int \frac{d^3k}{(2\pi)^3} \int \frac{dk_0}{(2\pi)} \; \delta($

$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle$

$= \int d\tilde{k}_1 \int d\tilde{k}_2$

$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle = \int d\tilde{k}_1 \; \frac{1}{\sqrt{2\omega_{k_1}}}$

$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}}$

$$\Phi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\, a_k^\dagger|0\rangle\, e^{-i\vec{k}\cdot\vec{y}+i\omega t}$$

Lorentz invariant

$$= \int \frac{d^4k}{(2\pi)^4}\,\delta(k_\mu k^\mu + m^2)$$

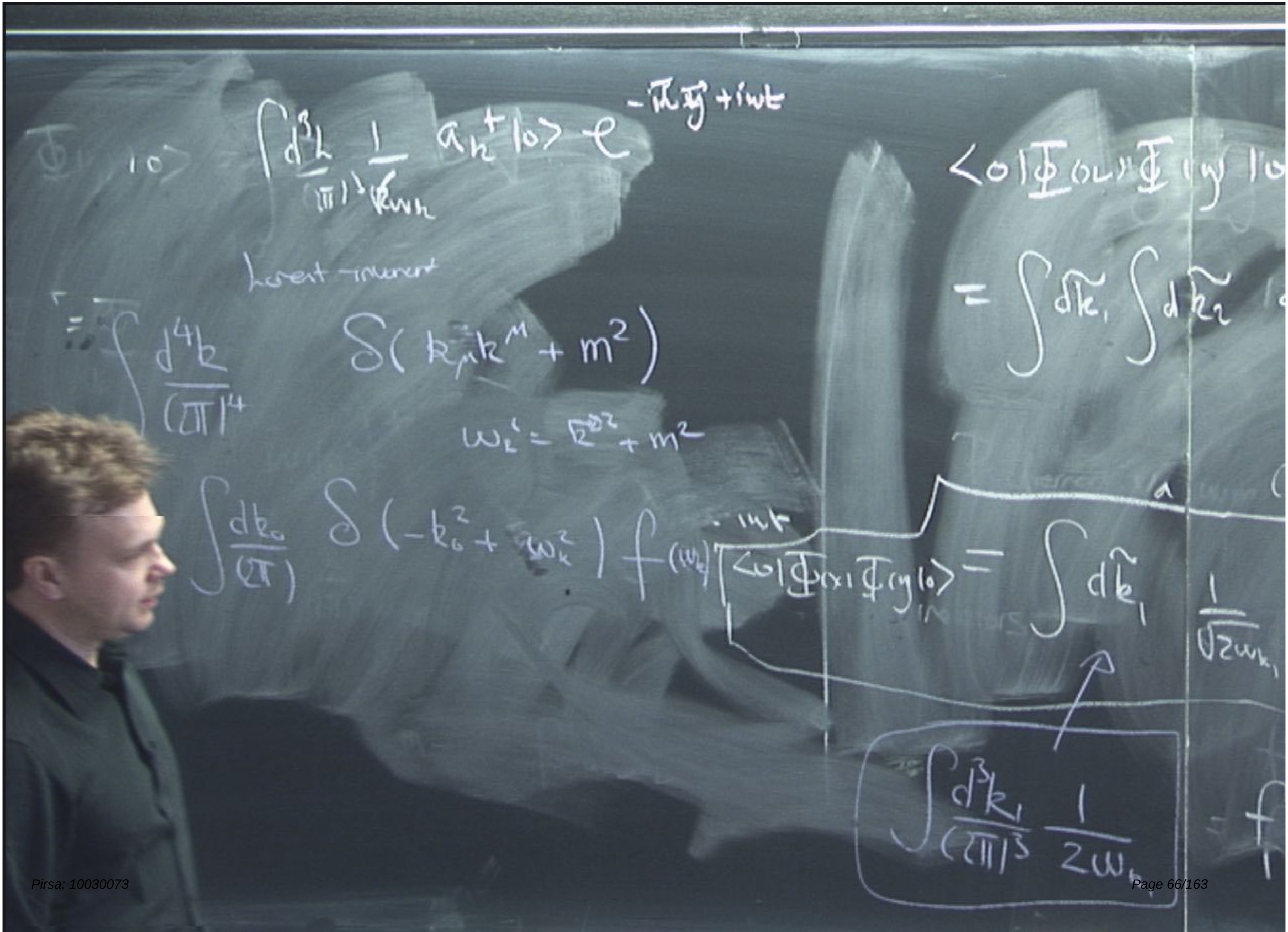
$$\omega_k^2 = \vec{k}^2 + m^2$$

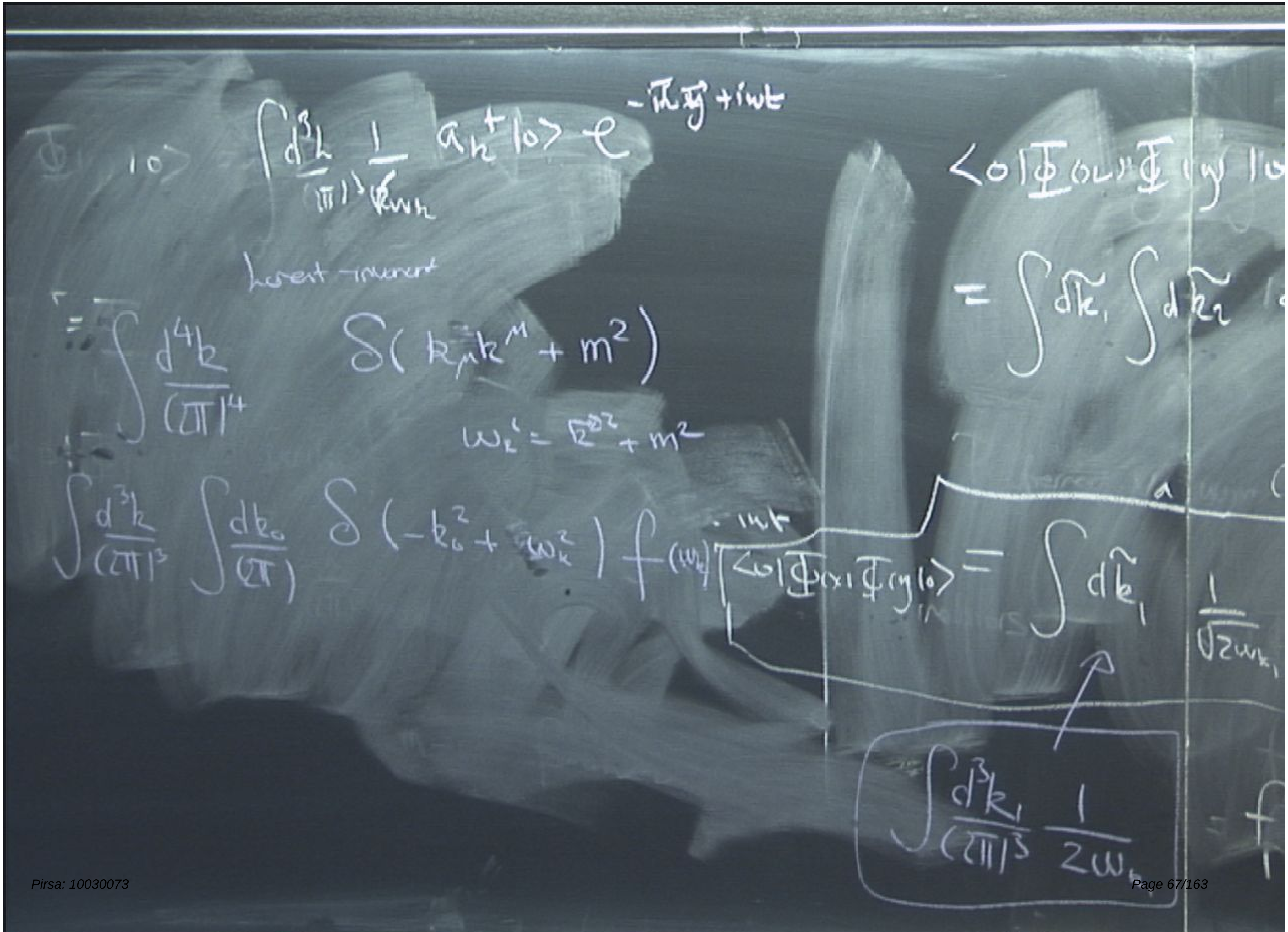
$$\int \frac{d^3k}{(2\pi)^3}\int \frac{dk_0}{(2\pi)}\,\delta(-k_0^2 + \omega_k^2)$$

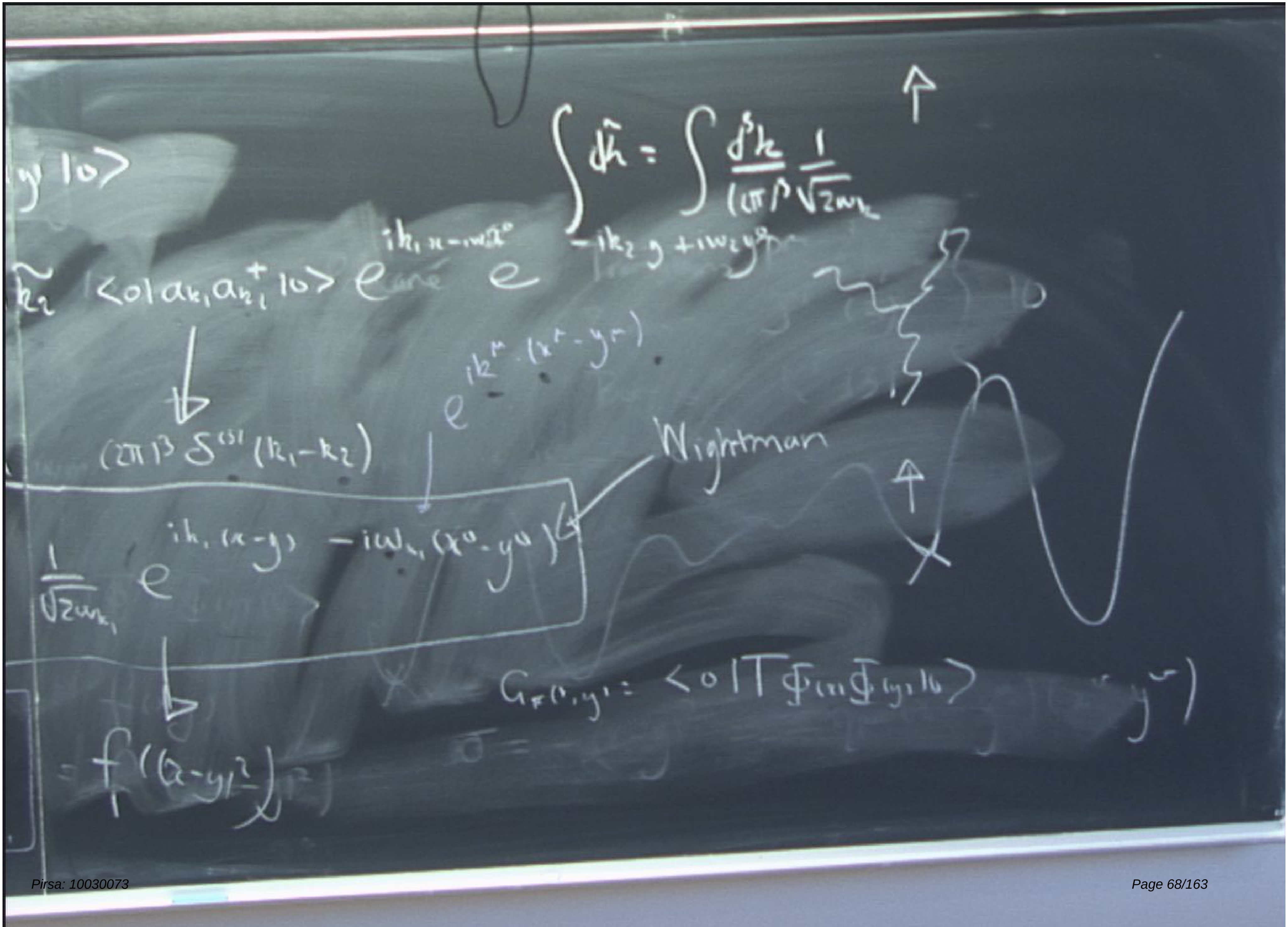
$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \int d\tilde{k}_2$$

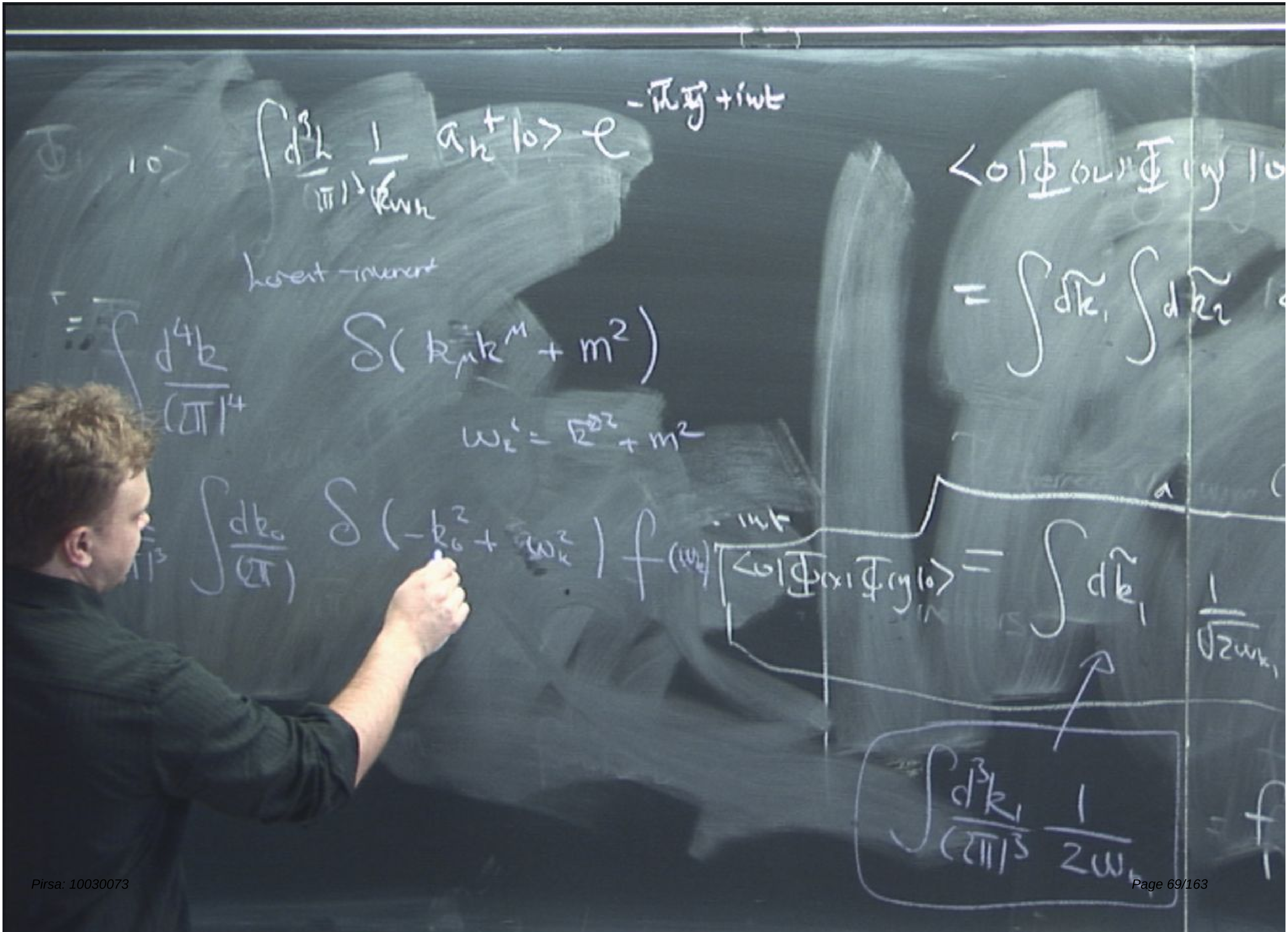
$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}}$$

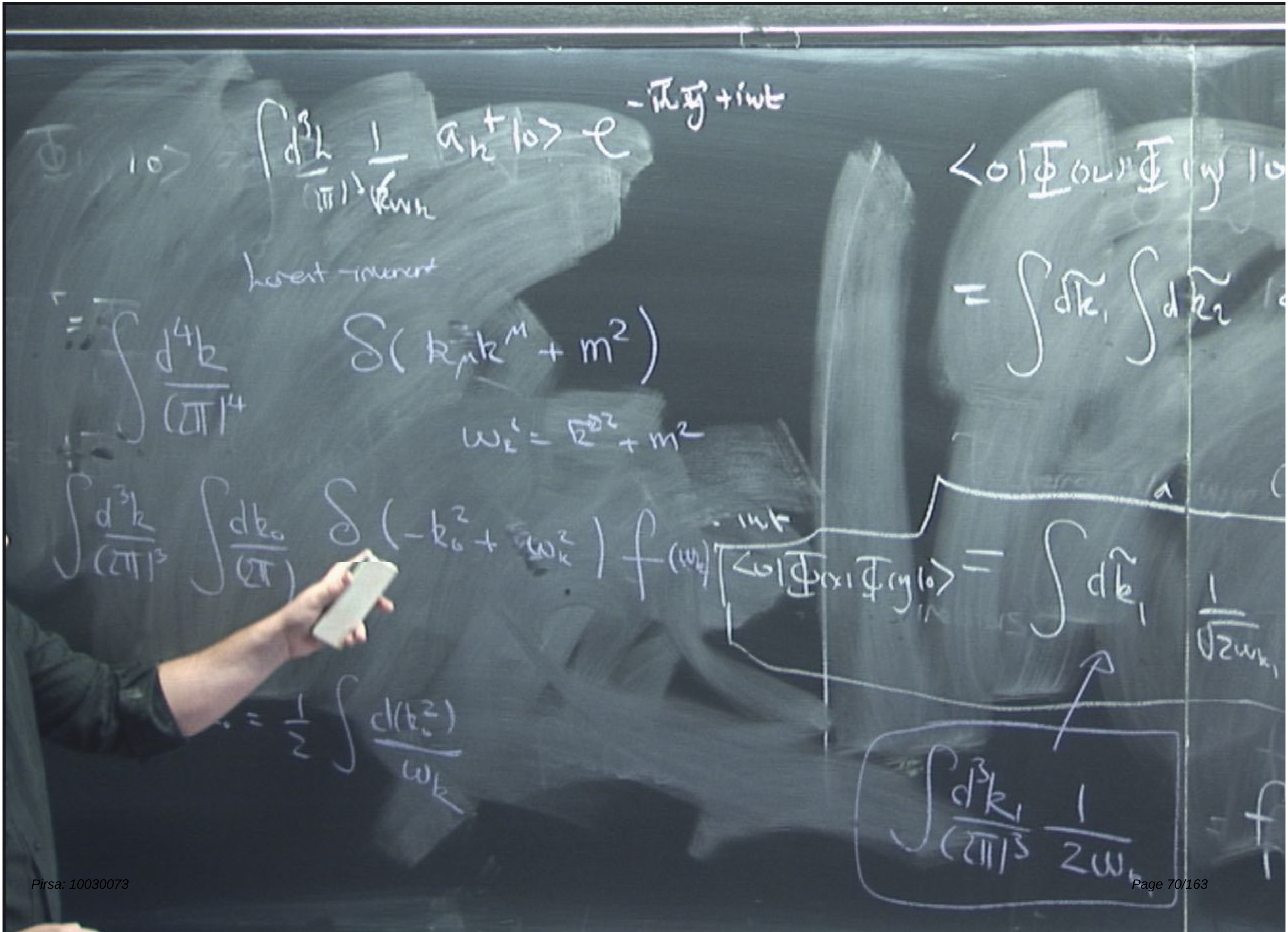
$$\int \frac{d^3k_1}{(2\pi)^3}\frac{1}{2\omega_{k_1}}$$



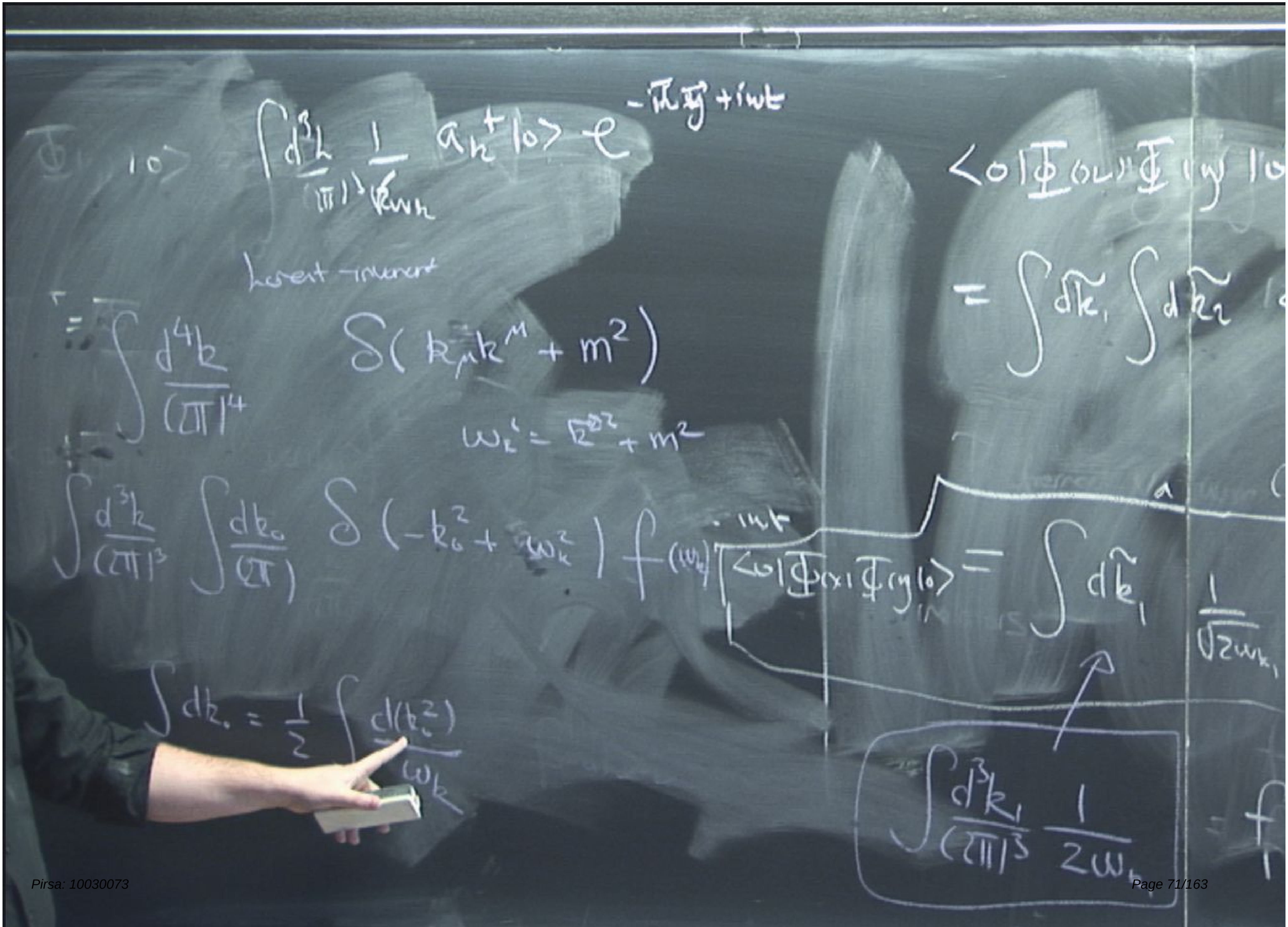

$\langle 0|\Phi(x)\Phi(y)|0\rangle$
$= \int d\tilde{k}_1 \int d\tilde{k}_2$
$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}}$
$\int \frac{d^3k_1}{(2\pi)^3}\frac{1}{2\omega_{k_1}}$
$\int \frac{d^4k}{(2\pi)^4}\,\delta(k_\mu k^\mu + m^2)$
$\omega_k^2 = \vec{k}^2 + m^2$
$\int \frac{d^3k}{(2\pi)^3}\int \frac{dk_0}{(2\pi)}\,\delta(-k_0^2 + \omega_k^2)\, f(\omega_k)$

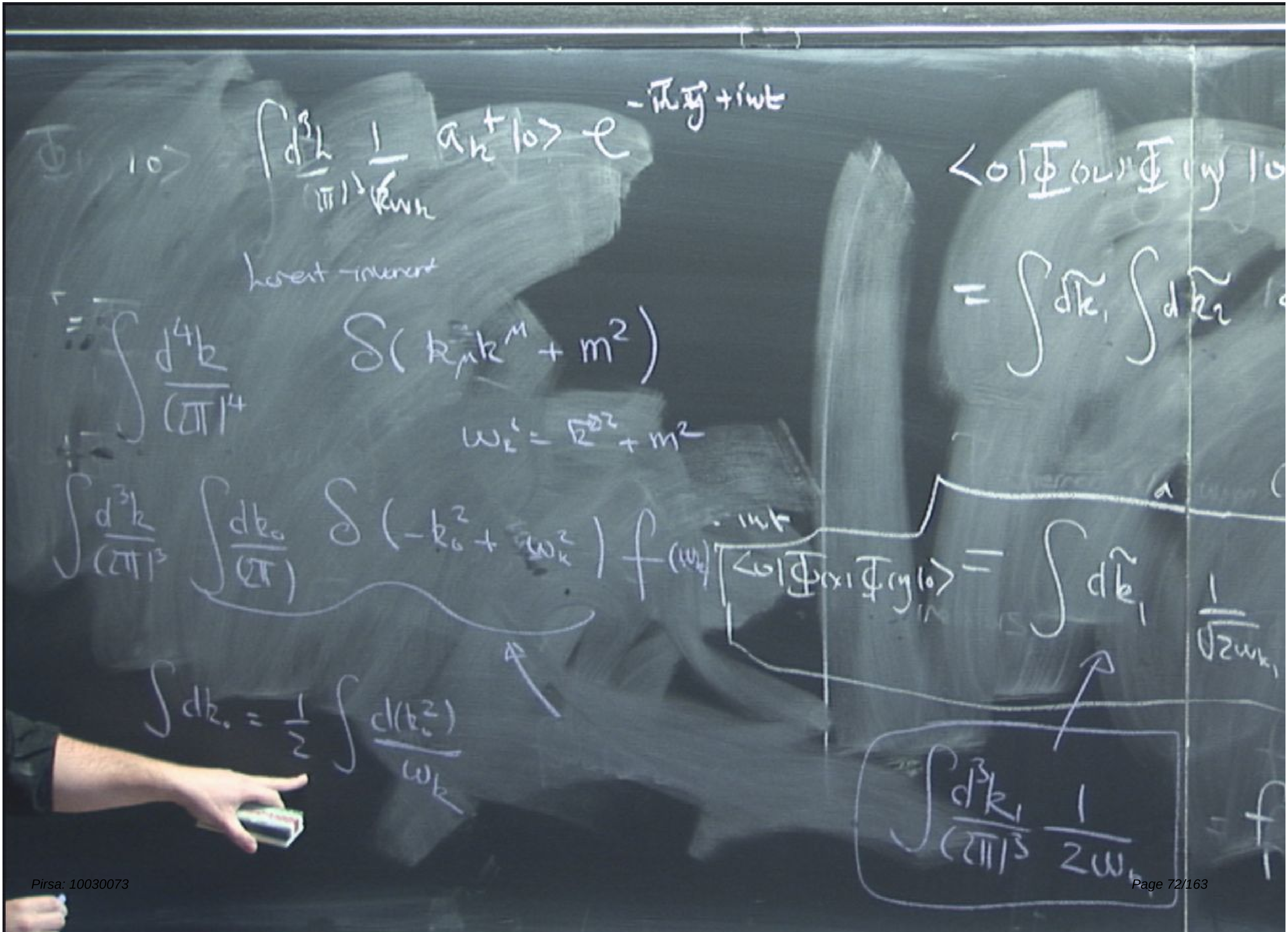

∫ dk̃ = ∫ d³k/(2π)³ 1/√(2ω_k)
⟨0| a_k1 a†_k1 |0⟩
(2π)³ δ⁽³⁾(k₁ − k₂)
Wightman
G_F(x, y) = ⟨0| T Φ(x) Φ(y) |0⟩





Lorentz invariant
δ(k_μ k^μ + m²)
ω_k² = k⃗² + m²
⟨0| Φ(x) Φ(y) |0⟩




$\int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} a_k^\dagger |0\rangle e^{-i\vec{k}\cdot\vec{y}+i\omega t}$
Lorentz invariant
$= \int \frac{d^4k}{(2\pi)^4} \delta(k_\mu k^\mu + m^2)$
$\omega_k^2 = \vec{k}^2 + m^2$
$\int \frac{d^3k}{(2\pi)^3} \int \frac{dk_0}{(2\pi)} \delta(-k_0^2 + \omega_k^2) f(\omega_k)$
$\int dk_0 = \frac{1}{2}\int \frac{d(k_0^2)}{\omega_k}$
$\langle 0|\Phi(x)\Phi(y)|0\rangle$
$= \int d\tilde{k}_1 \int d\tilde{k}_2$
$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}}$
$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}}$

$\Phi \; |0\rangle \quad \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} a_k^\dagger |0\rangle e^{-i\vec{k}\cdot\vec{y} + i\omega t}$

Lorentz invariant

$$= \int \frac{d^4k}{(2\pi)^4} \; \delta(k_\mu k^\mu + m^2)$$

$$\omega_k^2 = \vec{k}^2 + m^2$$

$$\int \frac{d^3k}{(2\pi)^3} \int \frac{dk_0}{(2\pi)} \; \delta(-k_0^2 + \omega_k^2) \; f(\omega_k) e^{-i\omega t}$$

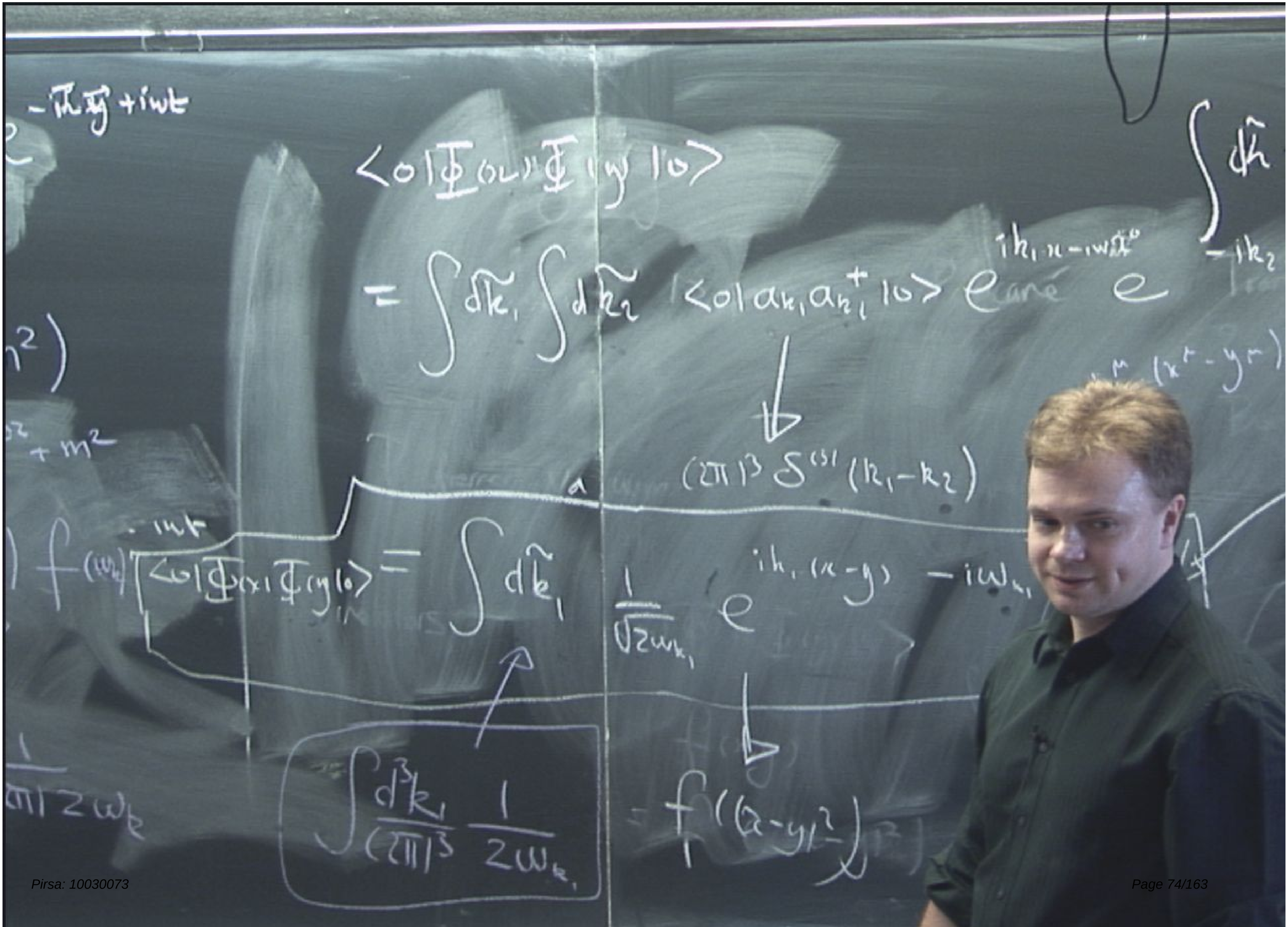
$$\int dk_0 = \frac{1}{2} \int \frac{d(k_0^2)}{\omega_k} \qquad \frac{1}{(2\pi)\, 2\omega_k}$$

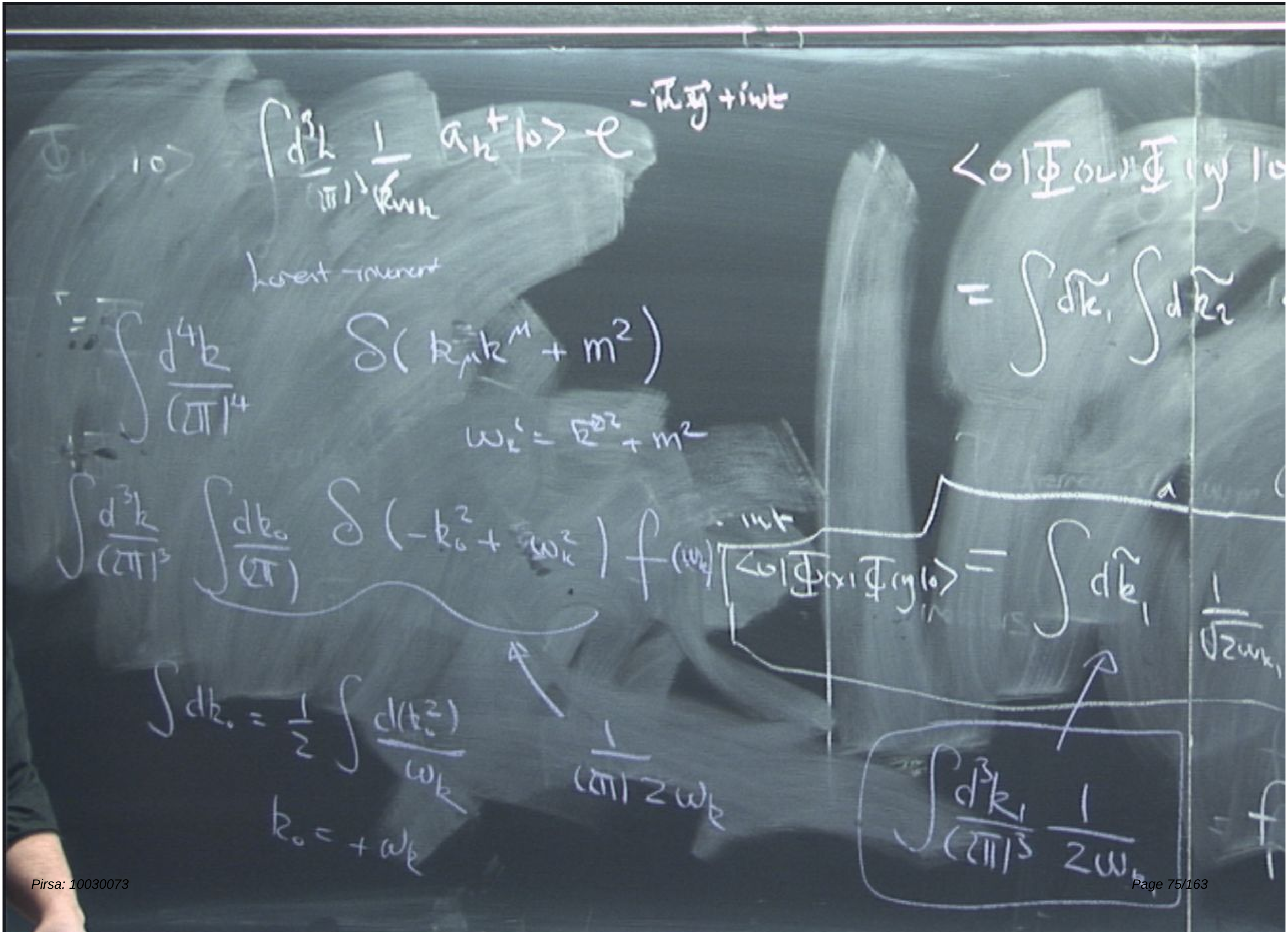
$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle$

$$= \int d\tilde{k}_1 \int d\tilde{k}_2$$

$$\langle 0 | \Phi(x) \Phi(y) | 0 \rangle = \int d\tilde{k}_1 \; \frac{1}{\sqrt{2\omega_{k_1}}}$$

$$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}}$$


$\langle 0|\Phi(x)\Phi(y)|0\rangle$
$= \int d\tilde{k}_1 \int d\tilde{k}_2 \langle 0|a_{k_1} a^\dagger_{k_2}|0\rangle e^{ik_1 x - i\omega x^0} e^{-ik_2}$
$(2\pi)^3 \delta^{(3)}(k_1 - k_2)$
$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}} e^{ik_1(x-y) - i\omega_{k_1}}$
$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}}$
$f((x-y)^2)$


Lorentz invariant
$\omega_k^2 = \vec{k}^2 + m^2$
$\int dk_0 = \frac{1}{2} \int \frac{d(k_0^2)}{\omega_k}$
$k_0 = +\omega_k$
$\frac{1}{(2\pi)\, 2\omega_k}$
$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}}$

$\Phi(\;)|0\rangle \quad \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}}\, a_k^\dagger |0\rangle\, e^{-i\vec{k}\cdot\vec{y} + i\omega t}$

Lorentz invariant

$$= \int \frac{d^4k}{(2\pi)^4}\, \theta(k_0)\, \delta(k_\mu k^\mu + m^2)$$

$$\omega_k^2 = \vec{k}^2 + m^2$$

$$\int \frac{d^3k}{(2\pi)^3} \int \frac{dk_0}{(2\pi)}\, \delta(-k_0^2 + \omega_k^2)\, f(\omega_k)\, e^{-i\omega t}$$

$$\int dk_0 = \frac{1}{2} \int \frac{d(k_0^2)}{\omega_k} \qquad \frac{1}{(2\pi)\, 2\omega_k}$$

$$k_0 = +\omega_k$$

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \int d\tilde{k}_2$$

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}}$$

$$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}} \quad = f$$

$\Phi \;|0\rangle \quad \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} a_k^\dagger |0\rangle e^{-i\vec{k}\cdot\vec{y} + i\omega t}$

Lorentz invariant

$$= \int \frac{d^4k}{(2\pi)^4} \theta(k_0)\, \delta(k_\mu k^\mu + m^2)$$

$\omega_k^2 = \vec{k}^2 + m^2$

$$\int \frac{d^3k}{(2\pi)^3} \int \frac{dk_0}{(2\pi)} \delta(-k_0^2 + \omega_k^2) f(\omega_k) e^{-i\omega t}$$

$$\int dk_0 = \frac{1}{2} \int \frac{d(k_0^2)}{\omega_k} \qquad \frac{1}{(2\pi)\, 2\omega_k}$$

$k_0 = +\omega_k$

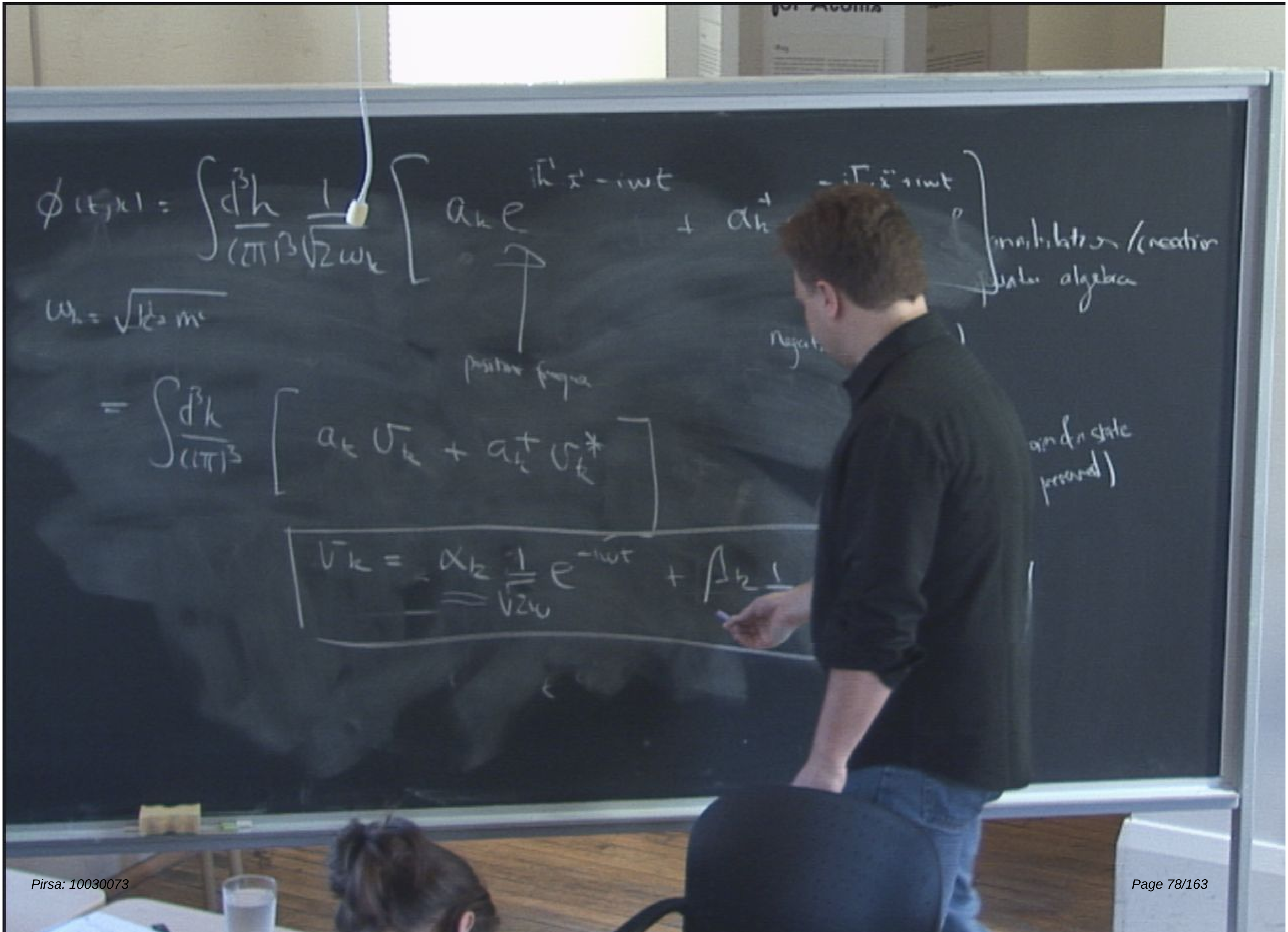
$$\langle 0 | \Phi(x) \Phi(y) | 0\rangle$$

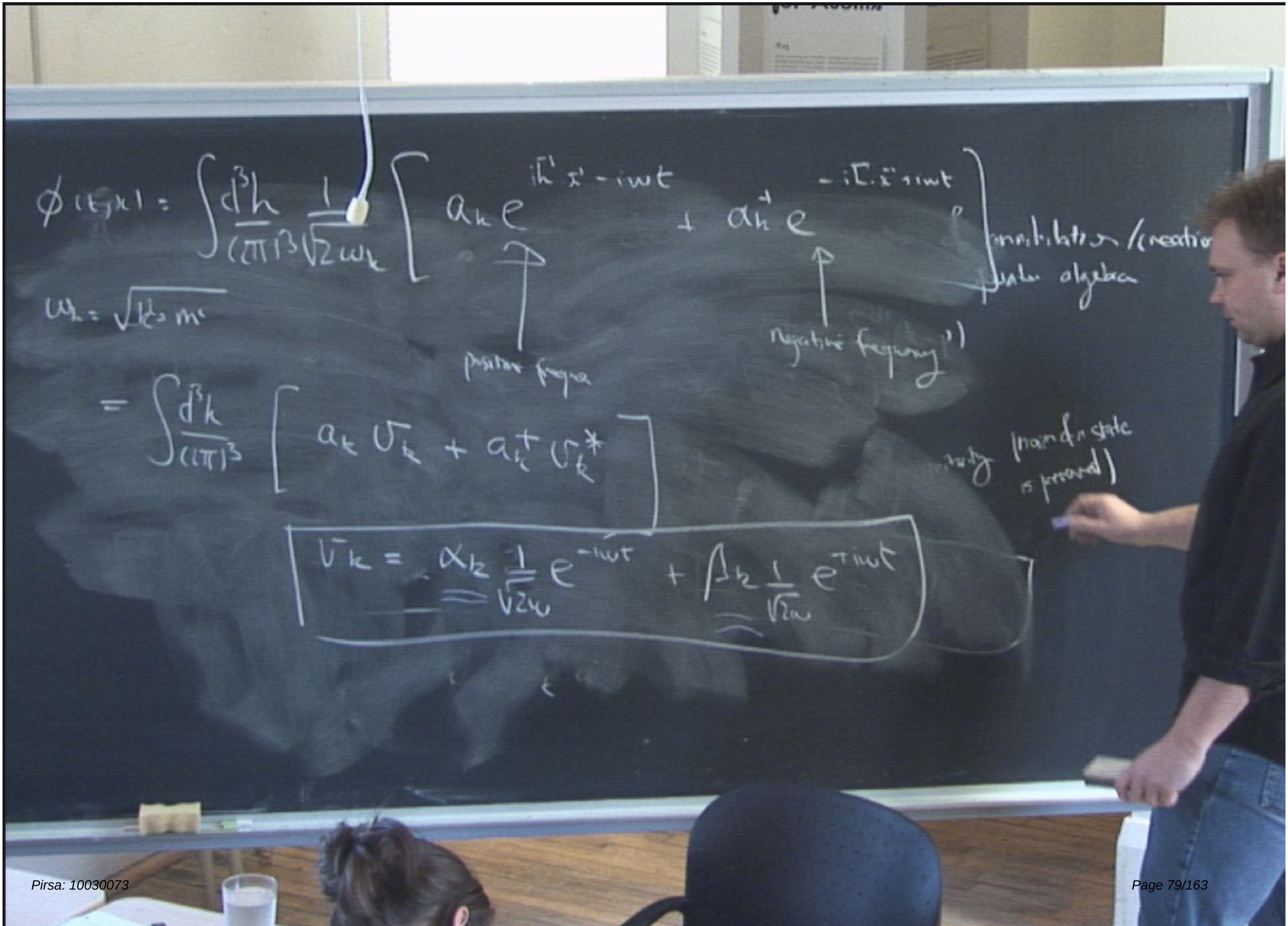
$$= \int d\tilde{k}_1 \int d\tilde{k}_2$$

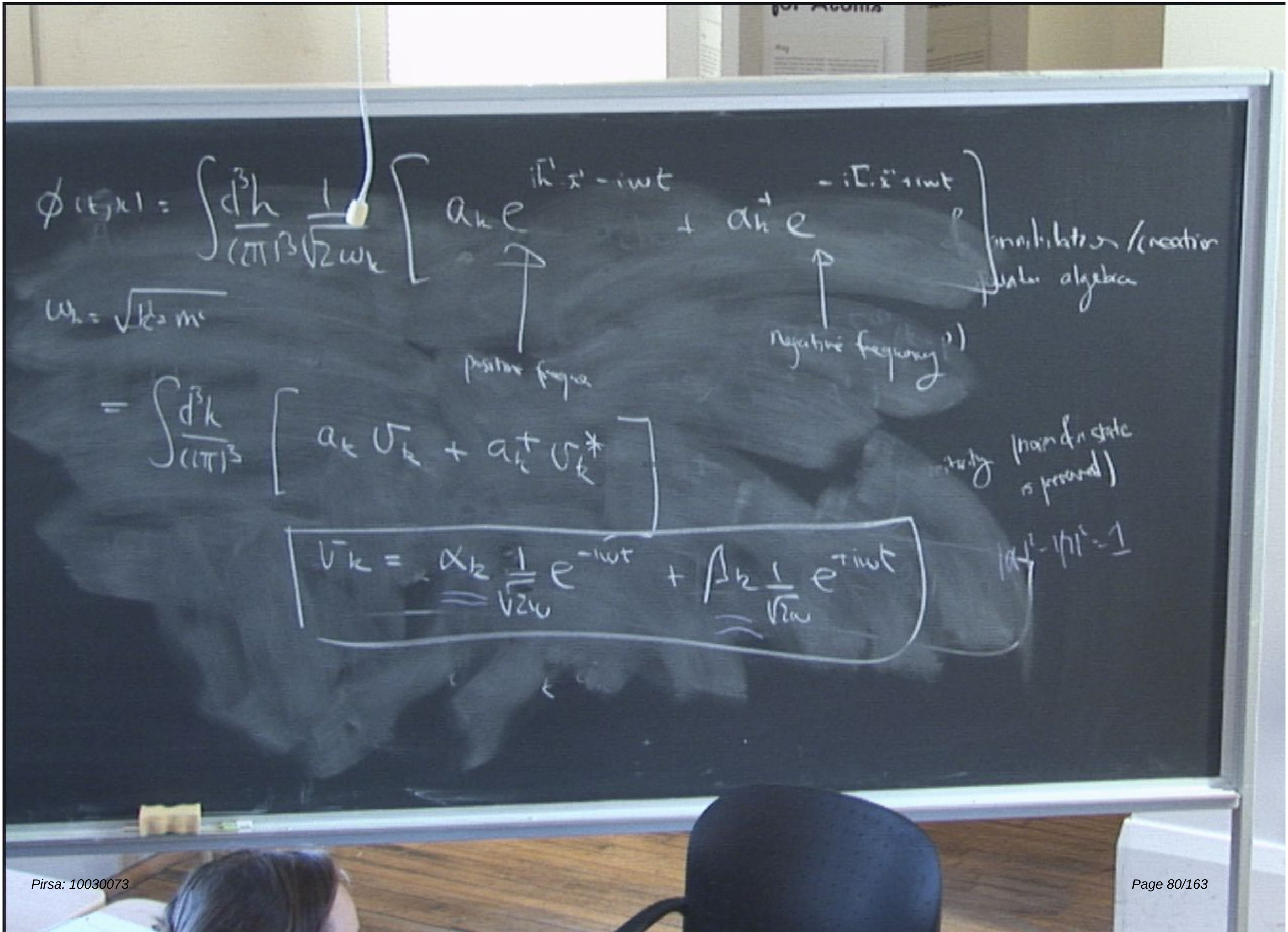
$a_k |0\rangle = 0$

$$\langle 0 | \Phi(x) \Phi(y) |0\rangle = \int d\tilde{k}_1 \frac{1}{\sqrt{2\omega_{k_1}}}$$

$$\int \frac{d^3k_1}{(2\pi)^3} \frac{1}{2\omega_{k_1}}$$


algebra
positive


positive freque
Negative frequency
annihilation / creation
algebra
(norm of in state
is preserved)

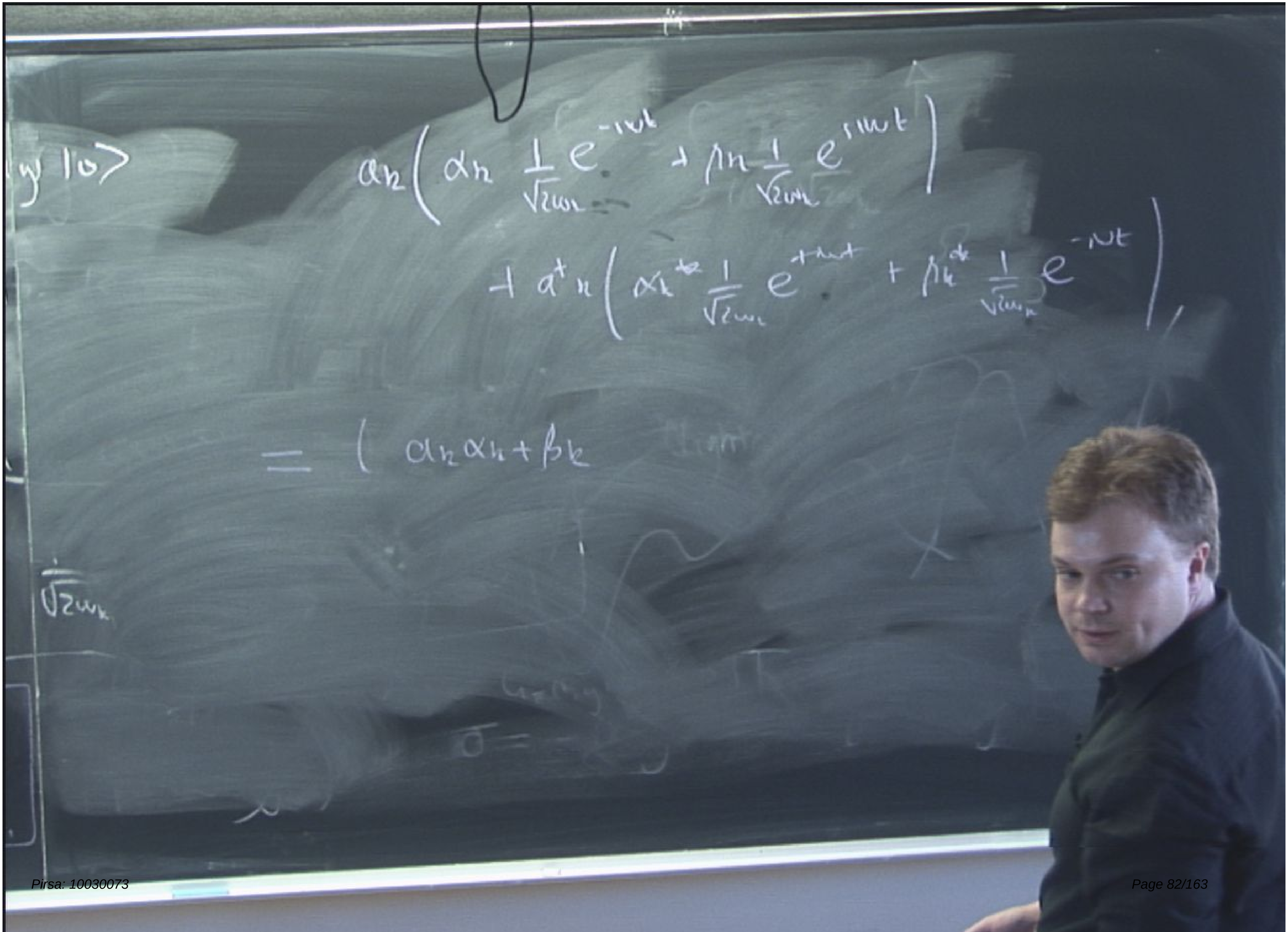

φ(t,x) = ∫ d³k/(2π)³ 1/√(2ω_k) [a_k e^{ik·x − iωt} + a_k† e^{−ik·x + iωt}]
annihilation/creation
algebra
ω_k = √(k² + m²)
positive frequency
negative frequency
= ∫ d³k/(2π)³ [a_k v_k + a_k† v_k*]
v_k = α_k 1/√(2ω) e^{−iωt} + β_k 1/√(2ω) e^{+iωt}
(norm of the state is preserved)
|α|² − |β|² = 1

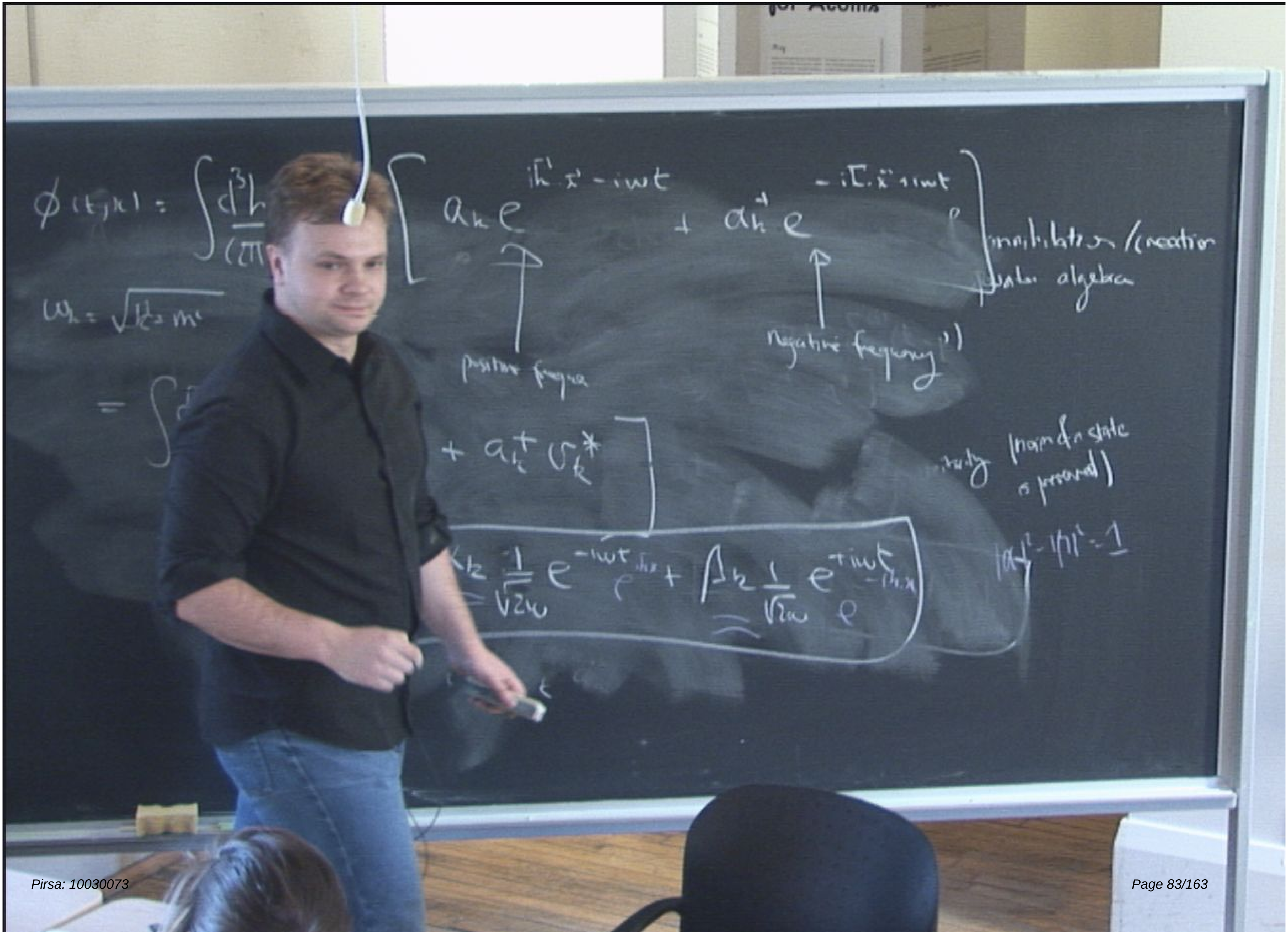
$y\,|0\rangle$

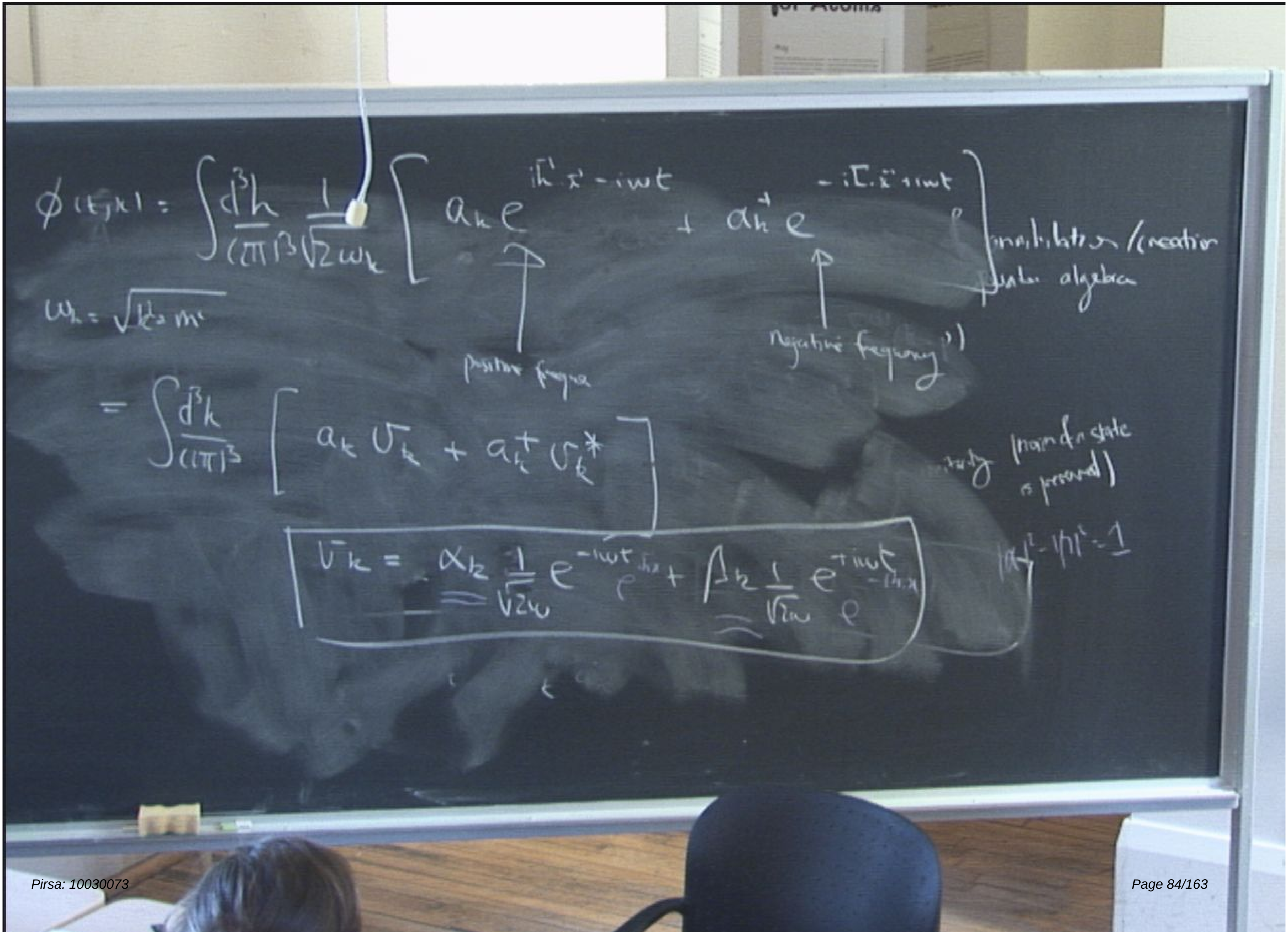
$$a_k \left(\alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t} + \beta_k \frac{1}{\sqrt{2\omega_k}} e^{i\omega t} \right)$$

$$+ a^\dagger_k \left(\alpha_k^* \frac{1}{\sqrt{2\omega_k}} e^{+i\omega t} + \beta_k^* \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t} \right)$$

$\sqrt{2\omega_k}$


|0>
$a_k \left(\alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t} + \beta_k \frac{1}{\sqrt{2\omega_k}} e^{i\omega t} \right)$
$+ a^\dagger_k \left(\alpha_k^* \frac{1}{\sqrt{2\omega_k}} e^{+i\omega t} + \beta_k^* \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t} \right)$
$= (a_k \alpha_k + \beta_k$
$\sqrt{2\omega_k}$


algebra
positive frequency
negative frequency
(norm of a state
is preserved)


$\phi(t,x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[a_k e^{i\vec{k}\cdot\vec{x} - i\omega t} + a_k^\dagger e^{-i\vec{k}\cdot\vec{x} + i\omega t} \right]$
annihilation / creation
algebra
$\omega_k = \sqrt{k^2 + m^2}$
positive frequen
Negative frequency
$= \int \frac{d^3k}{(2\pi)^3} \left[a_k v_k + a_k^\dagger v_k^* \right]$
(norm of state is preserved)
$v_k = \alpha_k \frac{1}{\sqrt{2\omega}} e^{-i\omega t} e^{ikx} + \beta_k \frac{1}{\sqrt{2\omega}} e^{+i\omega t} e^{-ikx}$
$|\alpha|^2 - |\beta|^2 = 1$

$|0\rangle$

$$a_k\left(\alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t} + \beta_k \frac{1}{\sqrt{2\omega_k}} e^{i\omega t}\right)$$

$$+ a^\dagger_k\left(\alpha_k^* \frac{1}{\sqrt{2\omega_k}} e^{+i\omega t} + \beta_k^* \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t}\right)$$

$$= \left(a_k \alpha_k + \beta_k^* a^\dagger_k\right) \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t}$$

$\sqrt{2\omega_k}$

$y|0\rangle$

$$a_k\left(\alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t} + \beta_k \frac{1}{\sqrt{2\omega_k}} e^{+i\omega t}\right)$$

$$+ a^+_k\left(\alpha_k^* \frac{1}{\sqrt{2\omega_k}} e^{+i\omega t} + \beta_k^* \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t}\right)$$

$$= \left(a_k \alpha_k + \beta_k^* a^+_k\right) \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t}$$

$$+ \left(\alpha_k^* a_k^+ + \beta_k a_k\right) \frac{1}{\sqrt{2\omega_k}} e^{+i\omega t}$$

$\frac{1}{\sqrt{2\omega_k}}$

$|0\rangle$

$$a_k\left(\alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t} + \beta_k \frac{1}{\sqrt{2\omega_k}} e^{i\omega t}\right)$$

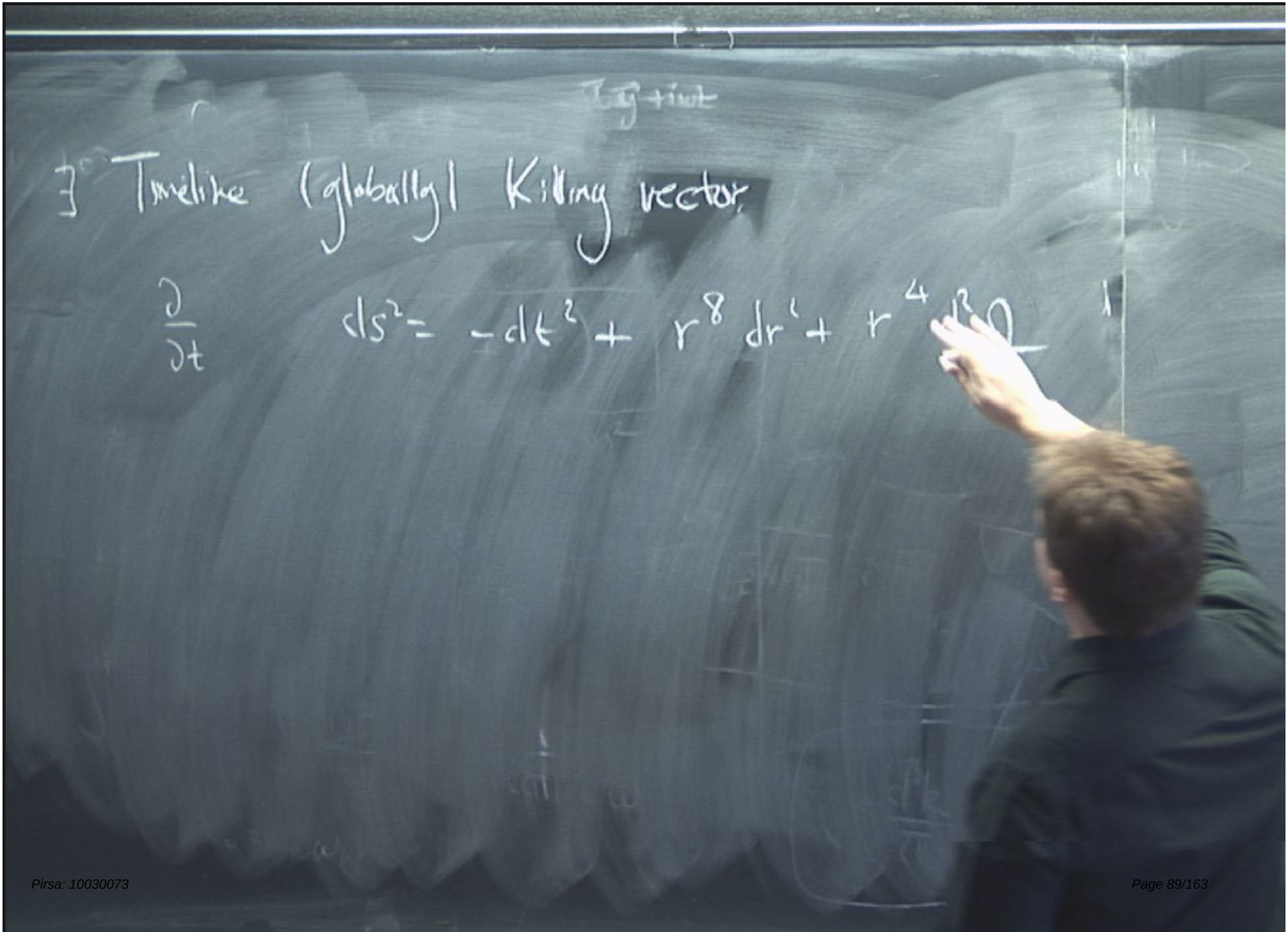
$$+ a^{\dagger}_k\left(\alpha_k^{*} \frac{1}{\sqrt{2\omega_k}} e^{+i\omega t} + \beta_k^{*} \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t}\right)$$

$$= \left(a_k\alpha_k + \beta_k^{*} a^{\dagger}_k\right) \frac{1}{\sqrt{2\omega_k}} e^{-i\omega t}$$

$$+ \left(\alpha_k^{*} a_k^{\dagger} + \beta_k a_k\right) \frac{1}{\sqrt{2\omega_k}} e^{+i\omega t}$$

$\sqrt{2\omega_k}$

3° Timelike (globally) Killing vector.


∃° Timelike (globally) Killing vector.
$\frac{\partial}{\partial t}$
$ds^2 = -dt^2 + r^8 dr^2 + r^4 d^3\Omega$

$\exists$ Timelike (globally) Killing vector

$$\frac{\partial}{\partial t} \qquad ds^2 = -dt^2 + (r^2+a)dr^2 + r^2 d^2\Omega$$

$\exists$ Timelike (globally) Killing vector

$$\frac{\partial}{\partial t} \qquad ds^2 = -dt^2 + (r^2+a)dr^2 + r^2 d^2\Omega$$

$t \to t + \text{constant}$

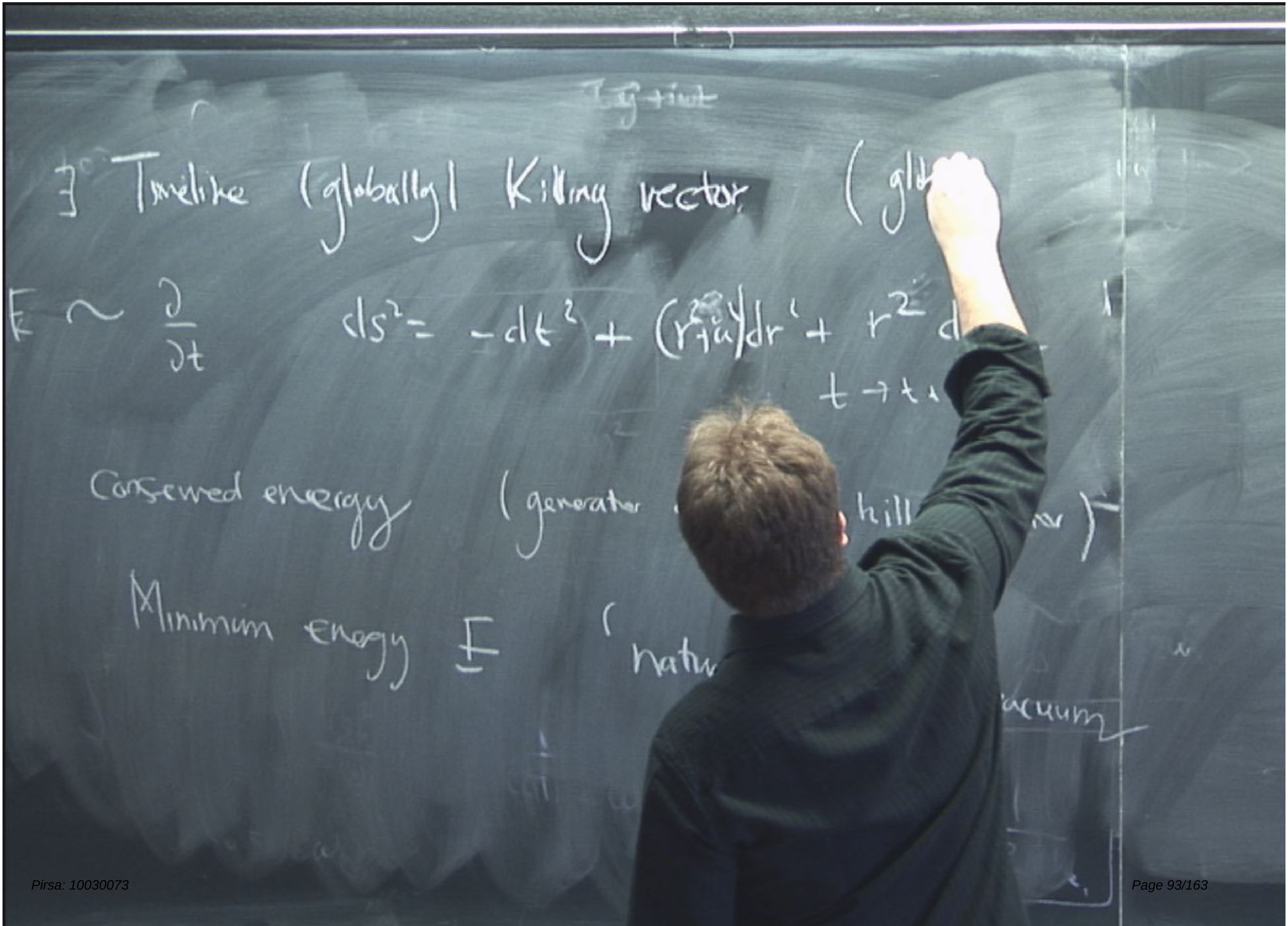
Conse

$\exists$ Timelike (globally) Killing vector

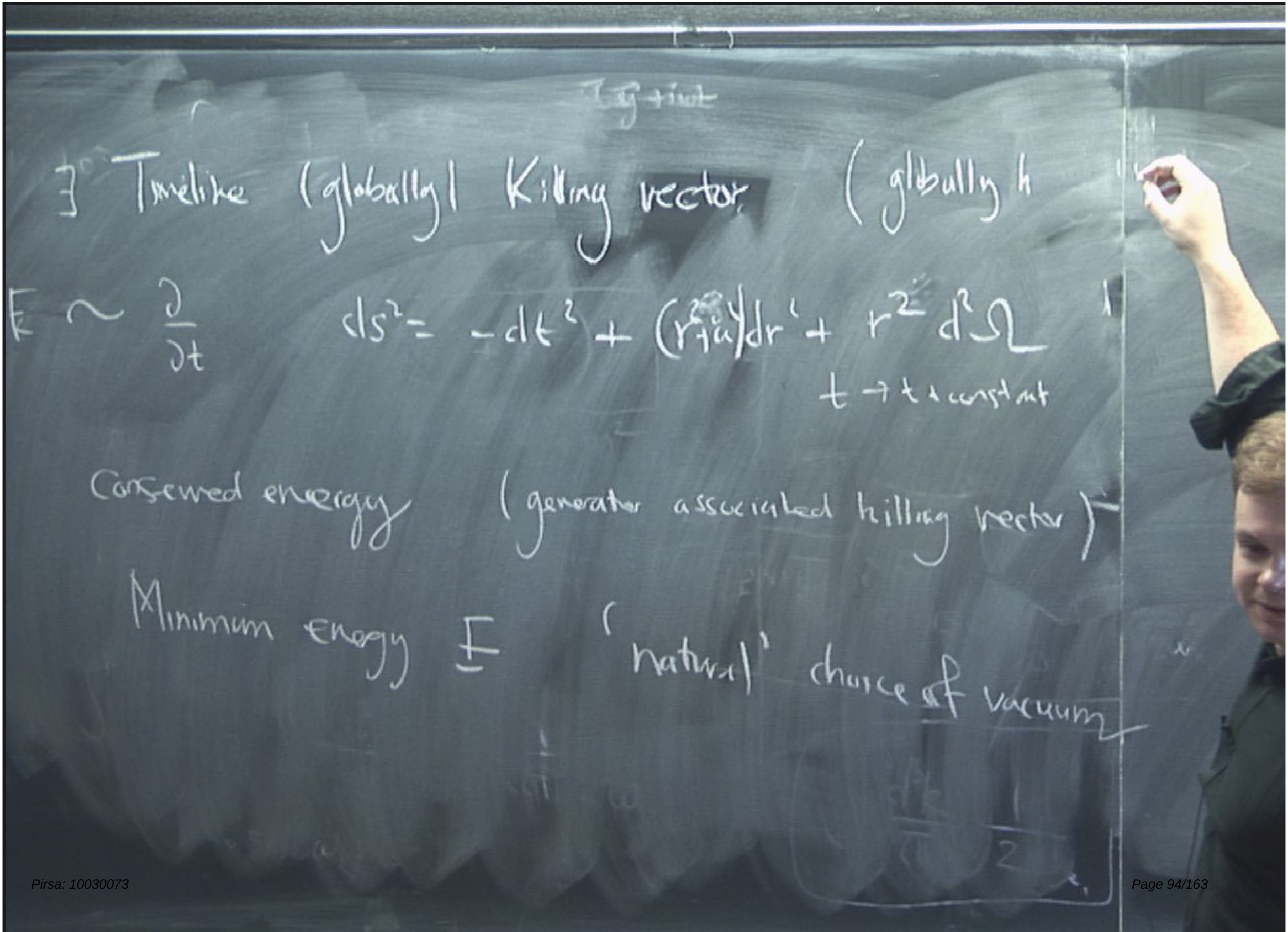
$\frac{\partial}{\partial t}$ $ds^2 = -dt^2 + (r^2+a)dr^2 + r^2 d^2\Omega$

$t \to t + \text{constant}$

Conserved energy (generator associated killing vector)



∃ Timelike (globally) Killing vector
ds² = −dt² +
t → t +
Conserved energy (generator
Minimum energy E
vacuum


∃ Timelike (globally) Killing vector (globally h
E ~ ∂/∂t
ds² = −dt² + (r²+a²) dr² + r² d²Ω
t → t + constant
Conserved energy (generator associated killing vector)
Minimum energy E 'natural' choice of vacuum

$\exists$ Timelike (globally) Killing vector ("globally hyperbolic")

$E \sim \frac{\partial}{\partial t}$ $\qquad ds^2 = -dt^2 + (r^2+a^2)dr^2 + r^2 d^2\Omega$

$t \to t + \text{constant}$

Conserved energy (generator associated killing vector)

Minimum energy E 'natural' choice of vacuum

$\exists$° Timelike (globally) Killing vector (globally hyperbolic)

$E \sim \frac{\partial}{\partial t}$ $ds^2 = -dt^2 + (r^2+a)dr^2 + r^2 d^2\Omega$

$t \to t + \text{constant}$

Conserved energy (generator associated Killing vector)

Minimum energy E 'natural' choice of vacuum

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x$$

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x$$

No killing vector

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x$$

timelike

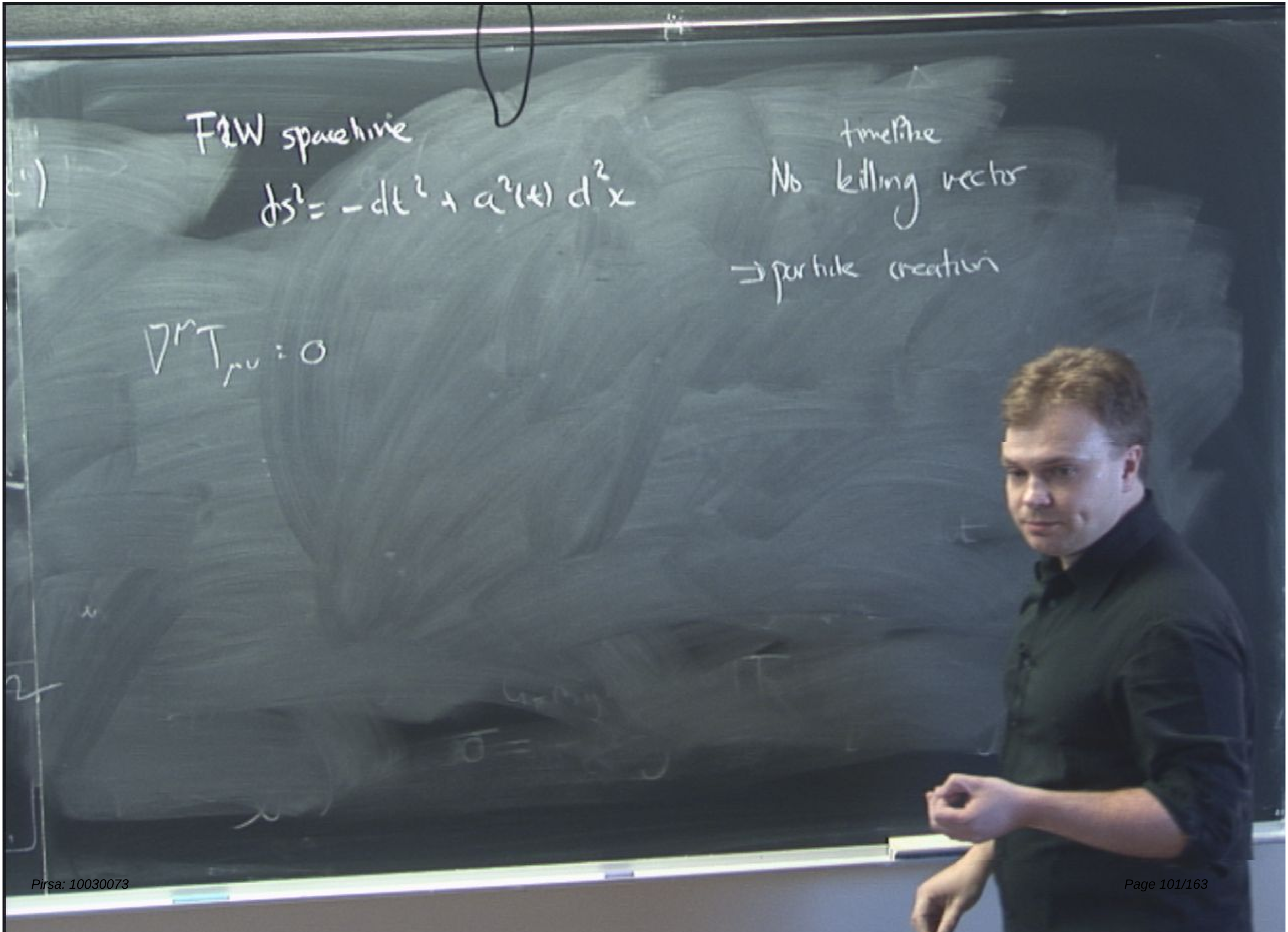
No killing vector

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x$$

No killing vector (timelike)

$\Rightarrow$ particle creation


FRW spacetime
$ds^2 = -dt^2 + a^2(t)\, d^2x$
timelike
No killing vector
⇒ particle creation
$\nabla^\mu T_{\mu\nu} = 0$

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x$$

timelike

No killing vector

$\Rightarrow$ particle creation

$$\nabla^\mu T_{\mu\nu} = 0$$

$$E = \int d^3x\, T_{00}$$

$$\dot{E} = 0$$

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x$$

timelike
No killing vector

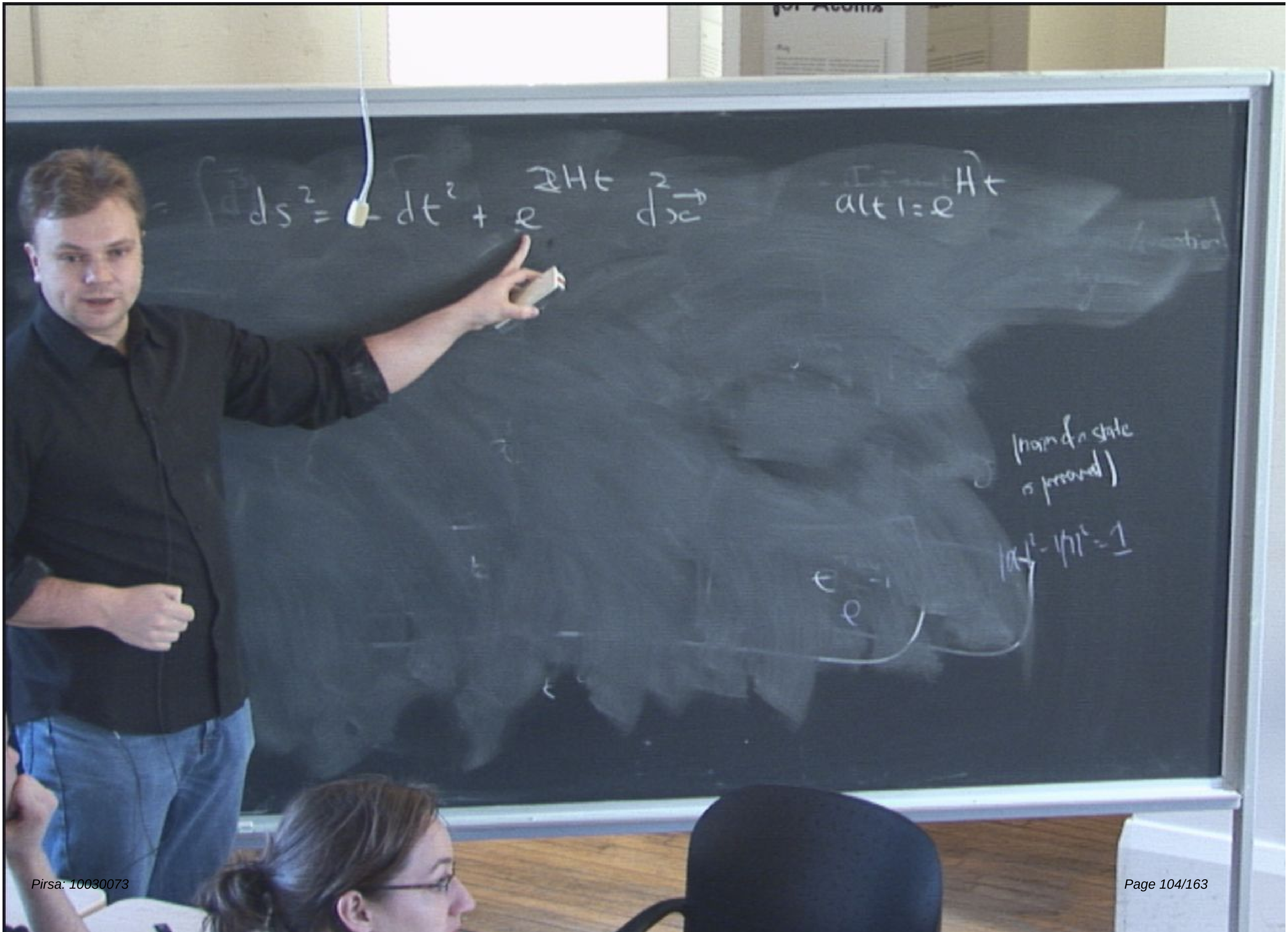
$\Rightarrow$ particle creation

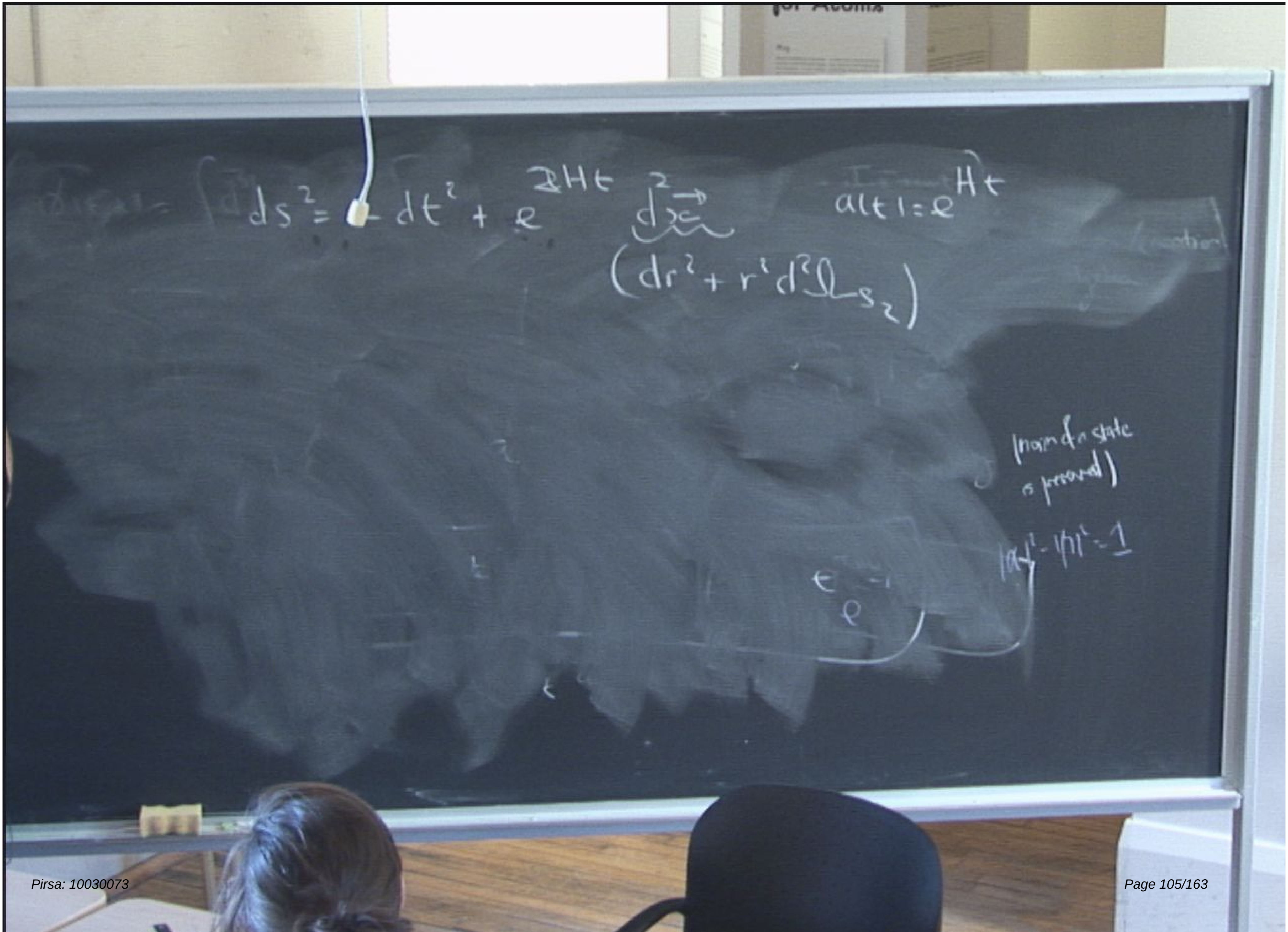
$$\nabla^\mu T_{\mu\nu} = 0$$

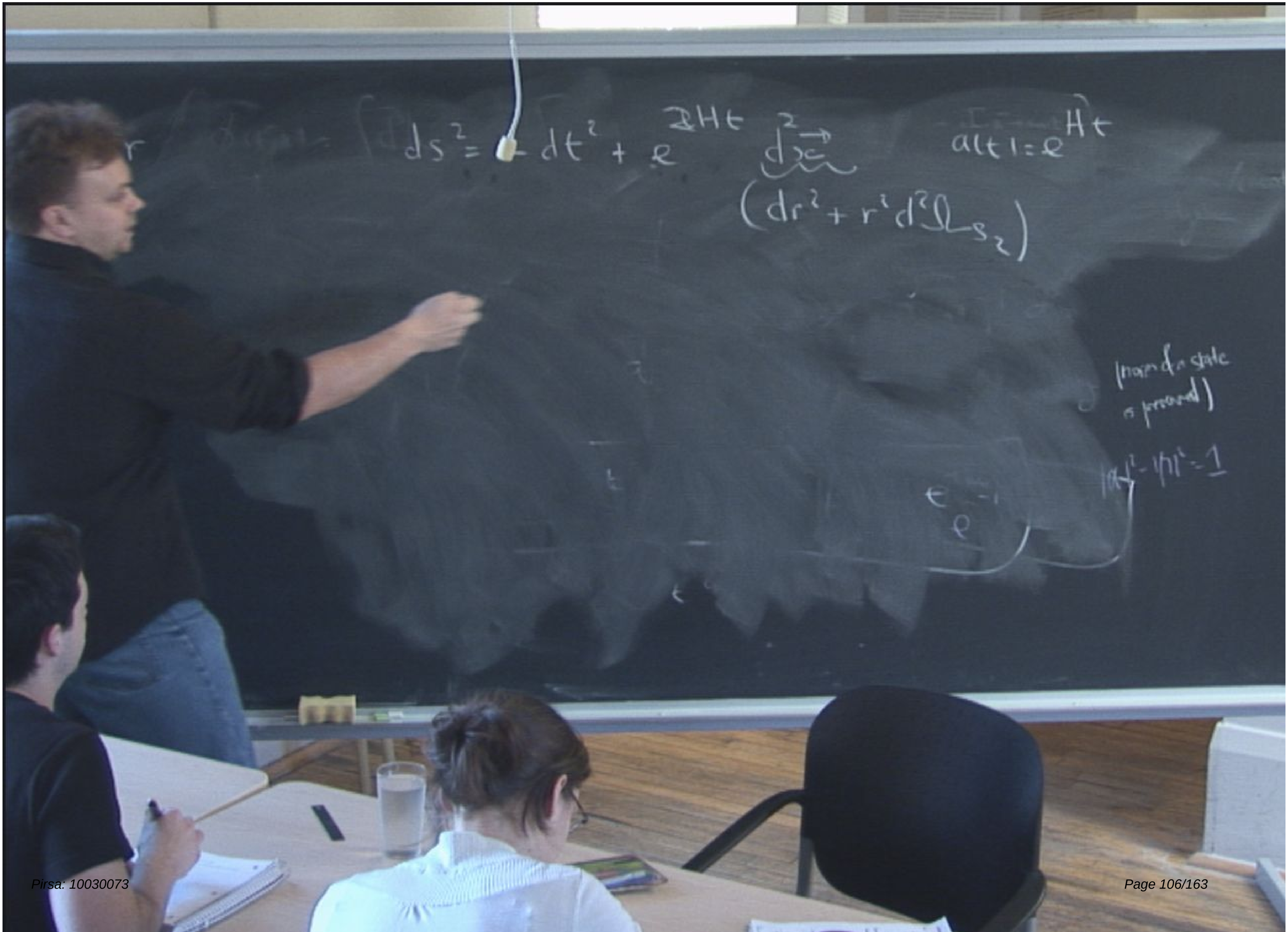
$$E = \int d^3x\, T_{00}$$

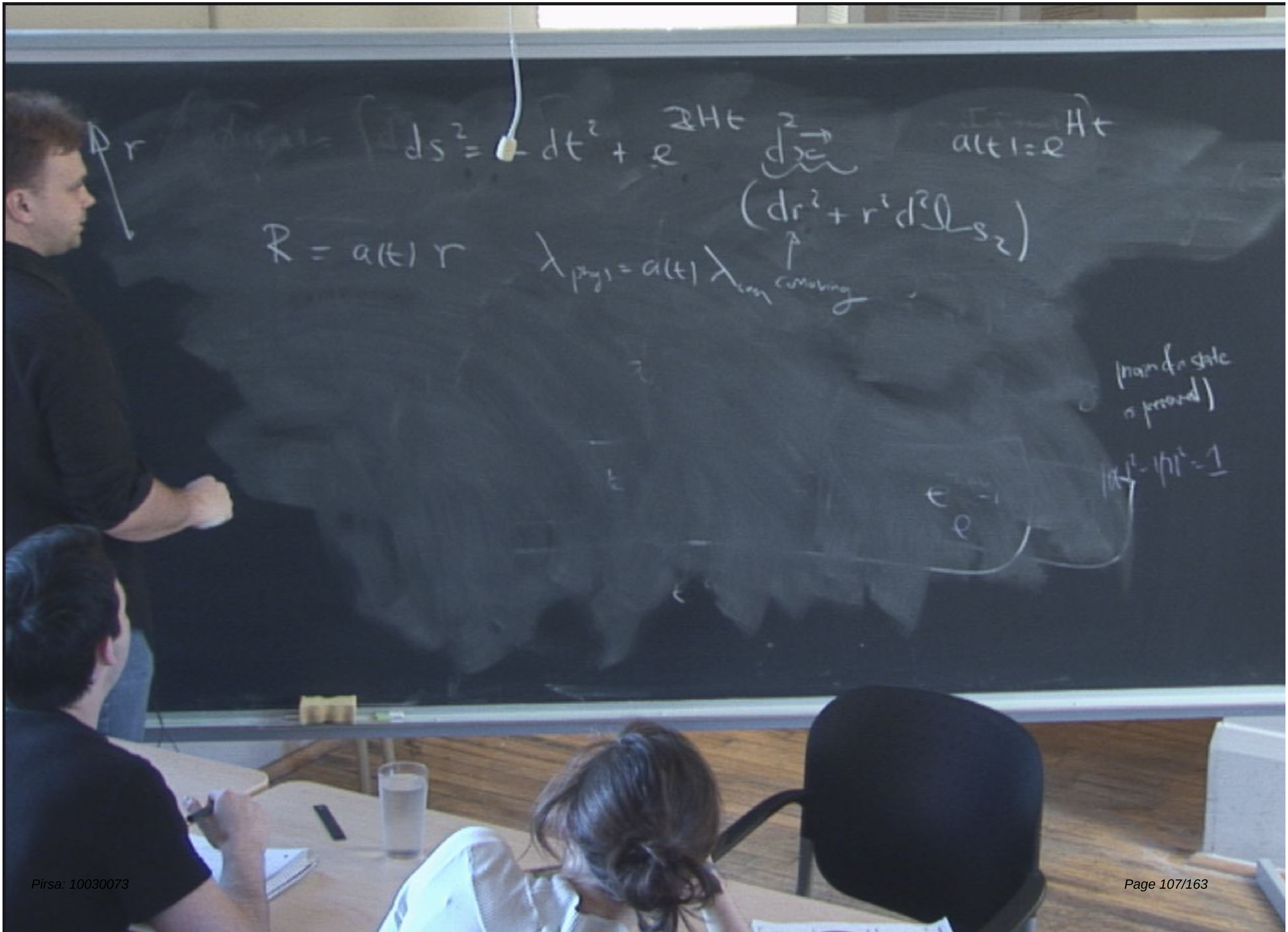
$$\dot{E} = 0$$

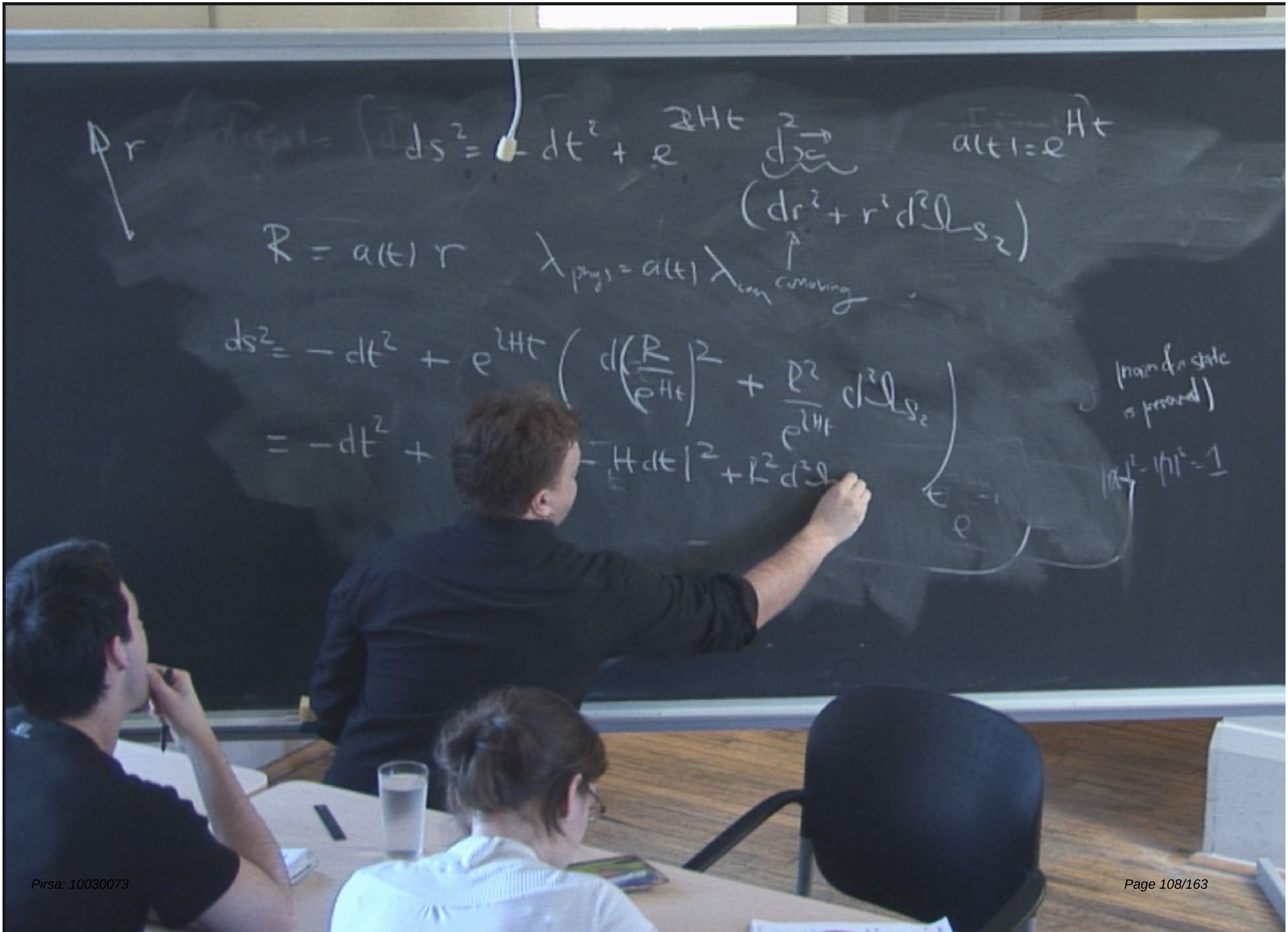
de Sitter spacetime — locally defined timelike killing vector

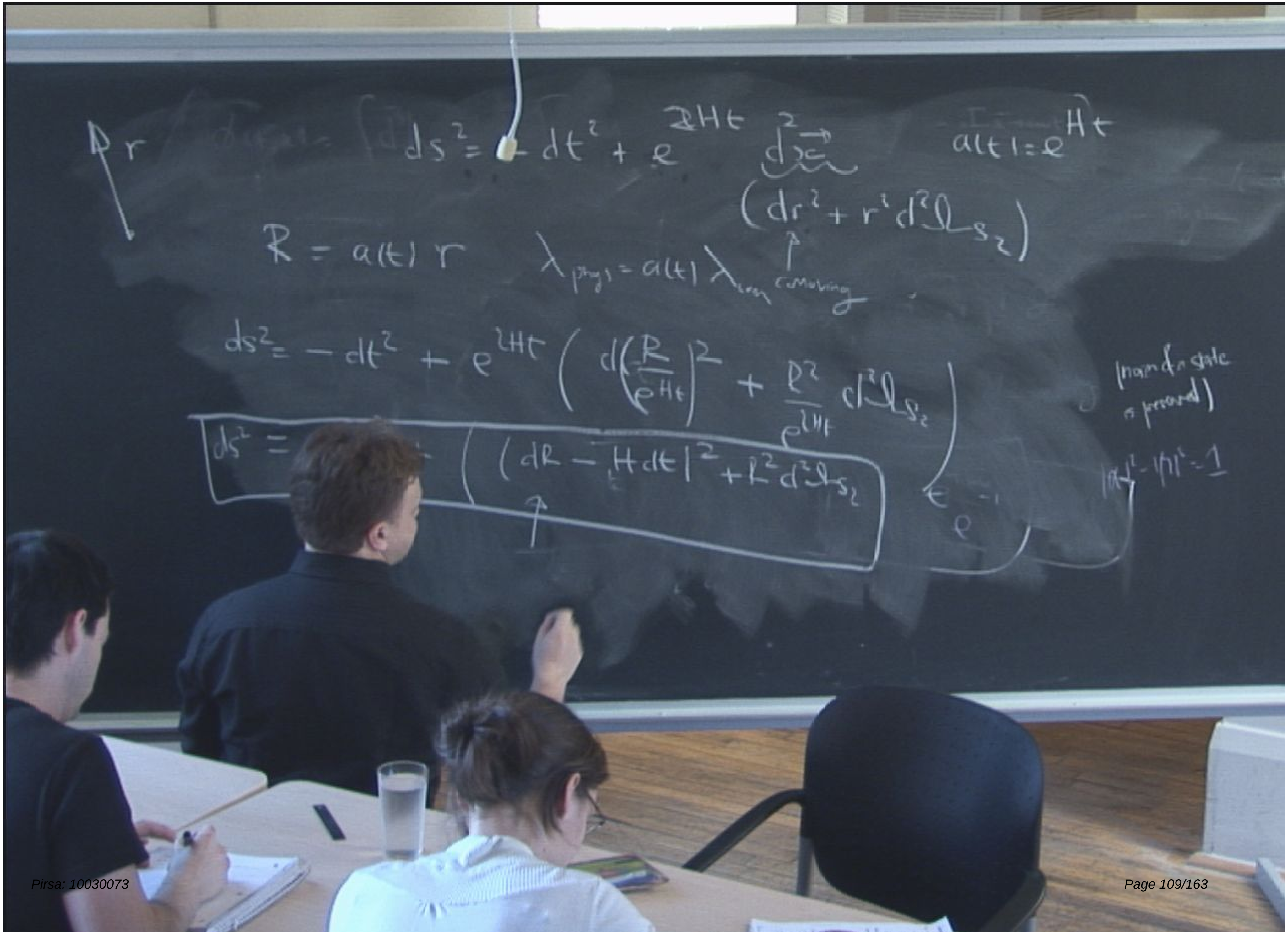

ds^2 = -dt^2 + e^2Ht dx^2
a(t) = e^Ht
(norm of a state is preserved)
|α|^2 - |β|^2 = 1

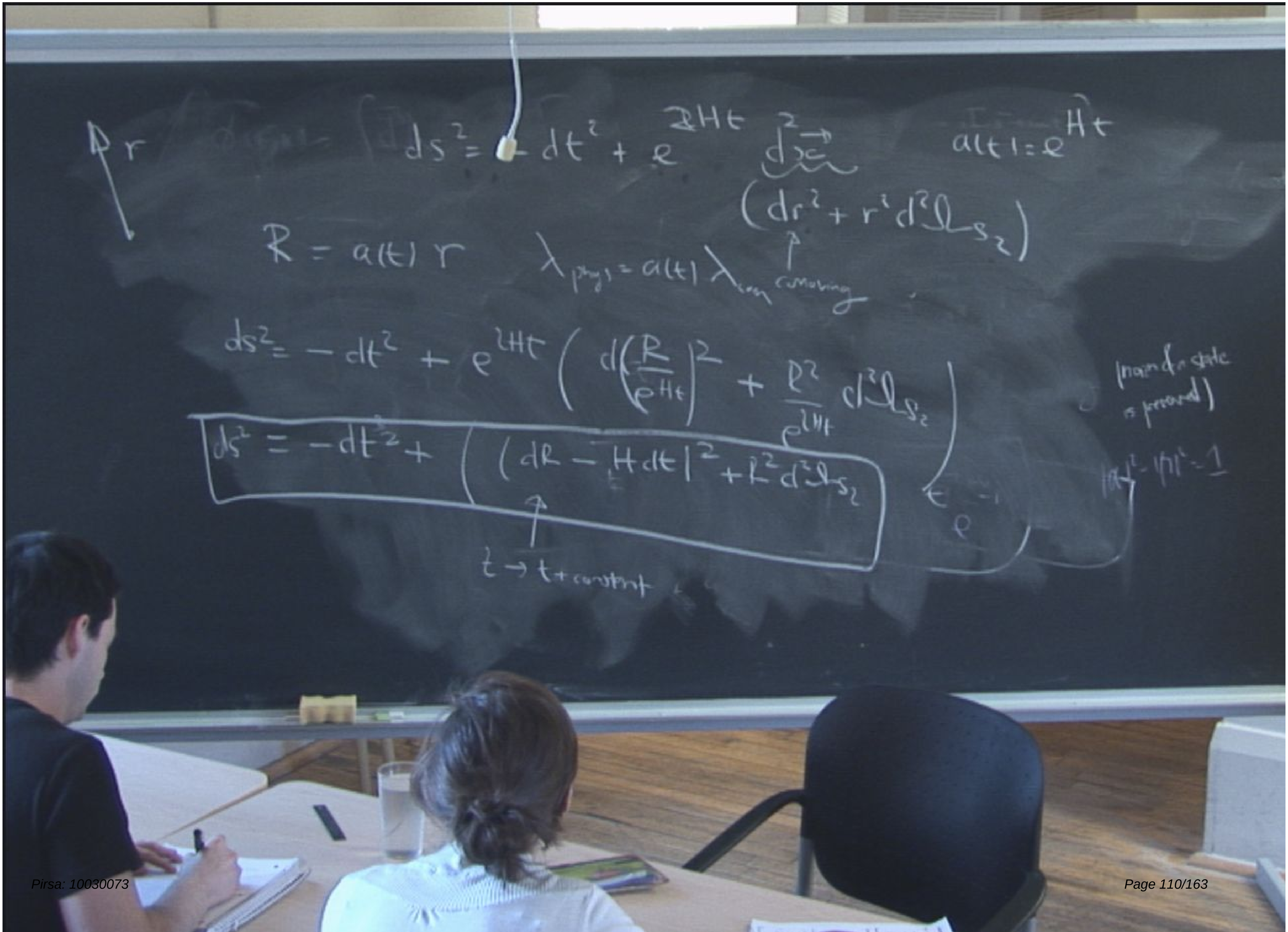

ds² = -dt² + e^2Ht dx²
(dr² + r² d²Ω_S2)
a(t) = e^Ht
(norm of a state is preserved)
|α|² - |β|² = 1


ds² = -dt² + e^{2Ht} dx²
a(t) = e^{Ht}
(dr² + r² dΩ_{S_2})

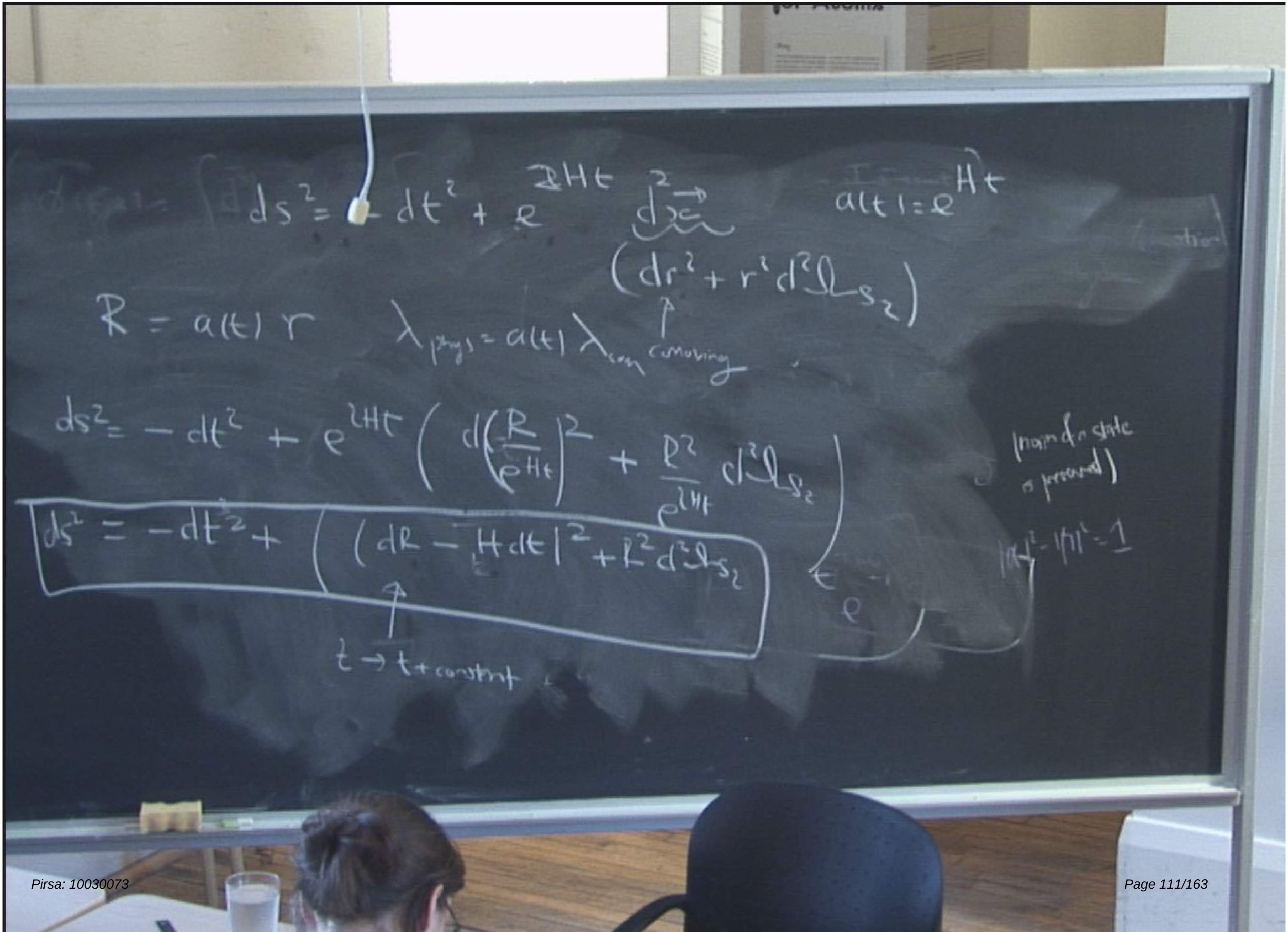

ds² = -dt² + e^{2Ht} dx²
(dr² + r² dΩ_{S2})
a(t) = e^{Ht}
R = a(t) r
λ_phys = a(t) λ_com
comoving

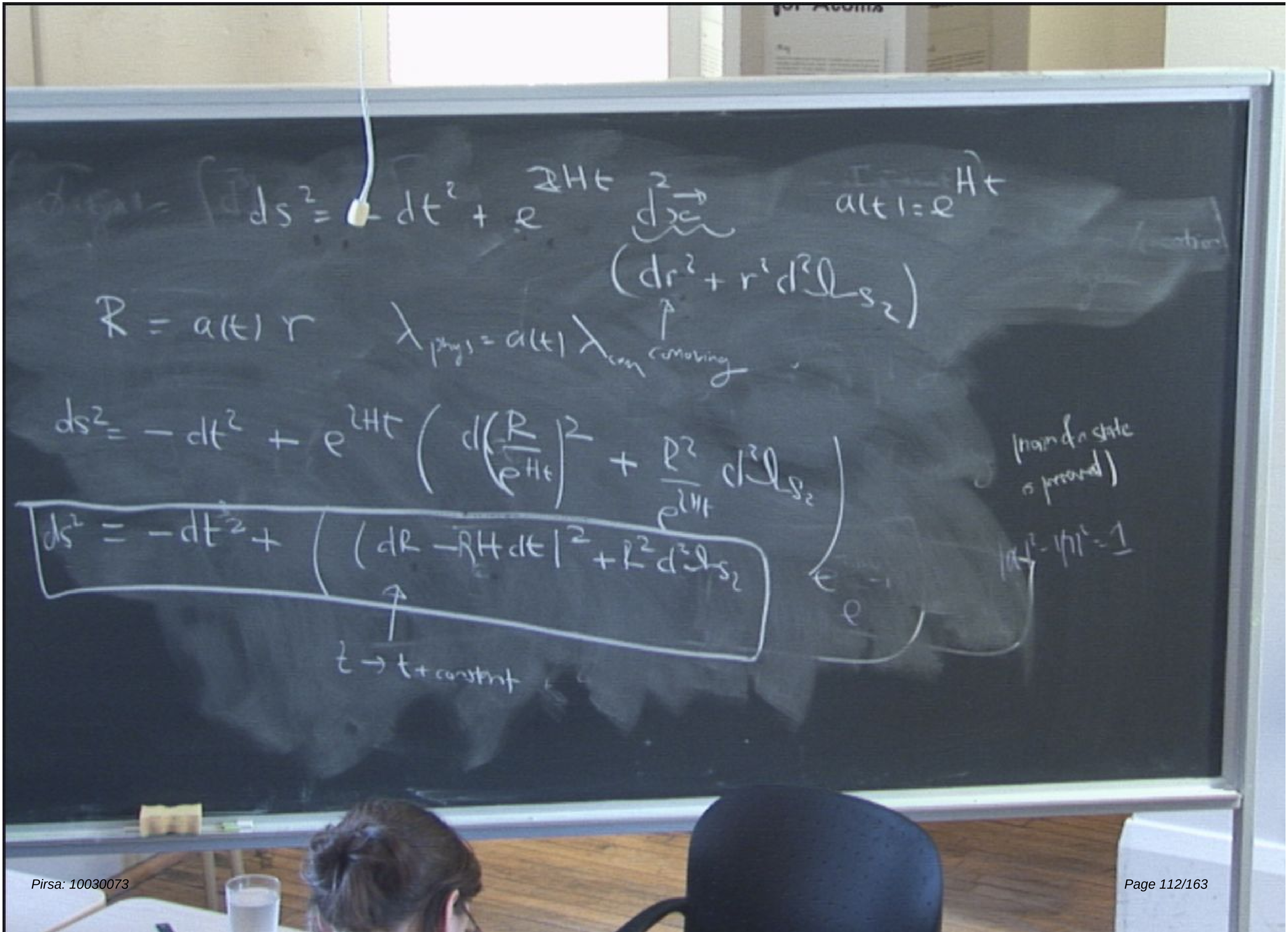

ds² = −dt² + e^{2Ht} dx²
a(t) = e^{Ht}
(dr² + r² d²Ω_{S2})
R = a(t) r
λ_{phys} = a(t) λ_{com}
comoving
ds² = −dt² + e^{2Ht} (d(R/e^{Ht})² + R²/e^{2Ht} d²Ω_{S2})
= −dt² +
H dt |² + R² d²Ω
(norm of a state is preserved)


ds² = −dt² + e^2Ht dx²
a(t) = e^Ht
(dr² + r² d²Ω S2)
R = a(t) r
λ phys = a(t) λ com
comoving
ds² = −dt² + e^2Ht (d(R/e^Ht)² + R²/e^2Ht d²Ω S2)
(norm. for state is preserved)
ds² =
(dR − H dt)² + R² d²Ω S2



$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$
$a(t) = e^{Ht}$
$(dr^2 + r^2 d^2\Omega_{S_2})$
$R = a(t) r$
$\lambda_{phys} = a(t) \lambda_{com}$
comoving
$ds^2 = -dt^2 + e^{2Ht} \left(d\left(\frac{R}{e^{Ht}}\right)^2 + \frac{R^2}{e^{2Ht}} d^2\Omega_{S_2} \right)$
$ds^2 = -dt^2 + (dR - H dt)^2 + R^2 d^2\Omega_{S_2}$
$t \to t + \text{constant}$


ds^2 = -dt^2 + e^{2Ht} dx^2
a(t) = e^{Ht}
(dr^2 + r^2 d^2Ω_{S_2})
R = a(t) r
λ_phys = a(t) λ_com
comoving
ds^2 = -dt^2 + e^{2Ht} (d(R/e^{Ht})^2 + R^2/e^{2Ht} d^2Ω_{S_2})
ds^2 = -dt^2 + (dR - H dt)^2 + R^2 d^2Ω_{S_2}
t → t + constant
(norm of a state is preserved)


ds² = -dt² + e^2Ht dx²
a(t) = e^Ht
(dr² + r²d²Ω_S2)
R = a(t) r
λ_phys = a(t) λ_com
comoving
ds² = -dt² + e^2Ht (d(R/e^Ht)² + R²/e^2Ht d²Ω_S2)
(norm of a state is preserved)
ds² = -dt² + ((dR - RH dt)² + R² d²Ω_S2)
t → t + constant

$$ds^2 = -(1-H^2R^2)dt^2 - 2RdRHdt + dR^2 + R^2 d^2\Omega_{S_2}$$

$$ds^2 = -(1 - H^2R^2)\,dt^2 - 2R\,dR\,H\,dt + dR^2 + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR\,dR}{(1 - H^2R^2)}$$

$$ds^2 = -(1 - H^2R^2)\,dt^2 - 2R\,dR\,H\,dt + dR^2 + R^2\,d^2\Omega_{S_2}$$

$$T = t \;[illegible]\; \int \frac{H R\,dR}{(1 - H^2R^2)}$$

$$dT = dt + \frac{H R\,dR}{(1 - H^2R^2)}$$

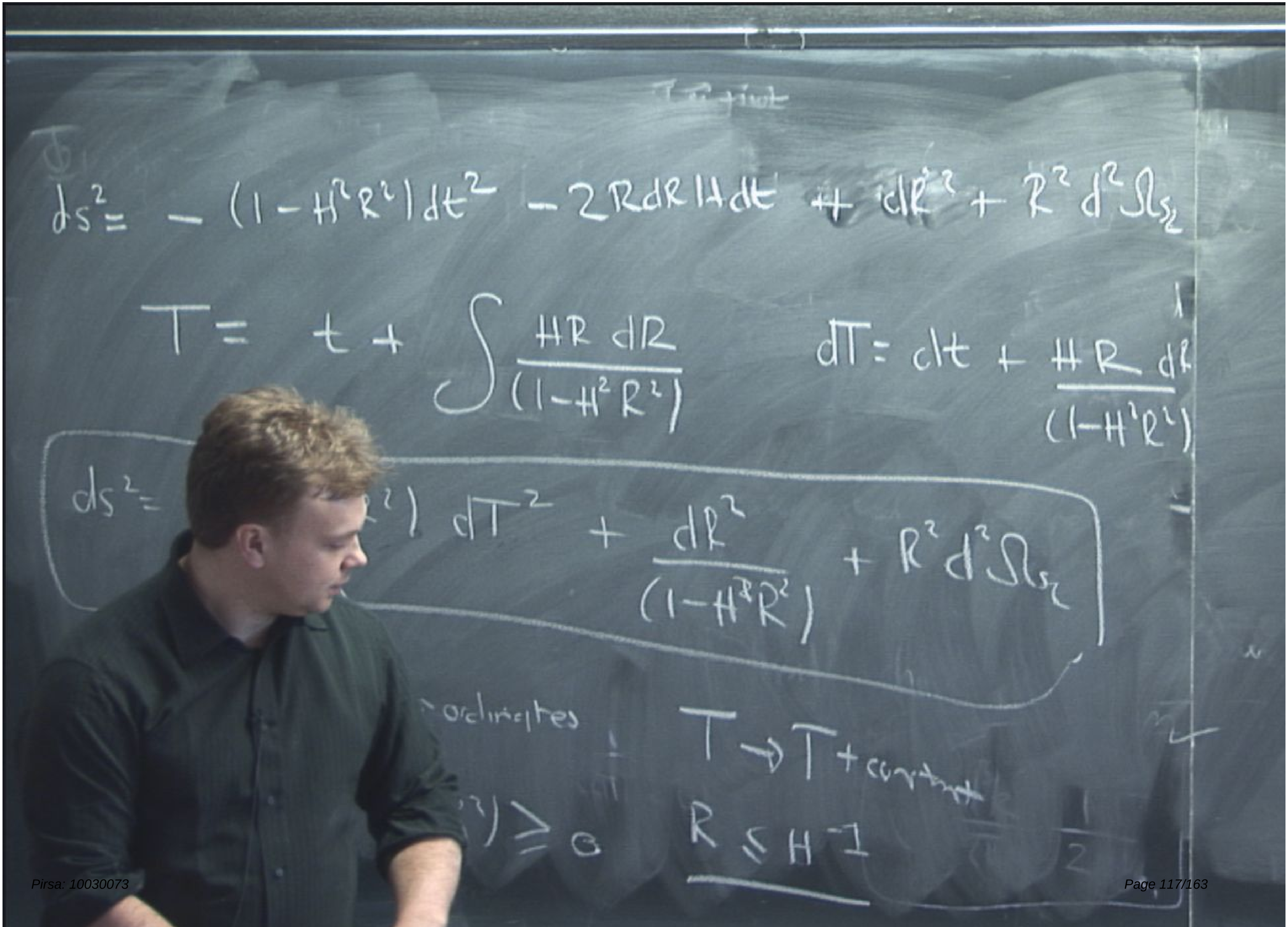
$$ds^2 = -(1-H^2R^2)\,dt^2 - 2R\,dR\,H\,dt + dR^2 + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR\,dR}{(1-H^2R^2)} \qquad dT = dt + \frac{HR\,dR}{(1-H^2R^2)}$$

$$ds^2 = -(1-H^2R^2)\,dT^2 + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

Static co-ordinates

$T \to T + \text{constant}$


$ds^2 = -(1-H^2R^2)dt^2 - 2RdRHdt + dR^2 + R^2 d^2\Omega_{S_2}$
$T = t + \int \frac{HR\,dR}{(1-H^2R^2)}$
$dT = dt + \frac{HR\,dR}{(1-H^2R^2)}$
$ds^2 =$
$) dT^2 + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$
ordinates
$T \to T +$ constant
$) \geq 0$
$R \leq H^{-1}$

$$ds^2 = -(1-H^2R^2)\,dt^2 - 2R\,dR\,H\,dt + dR^2 + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR\,dR}{(1-H^2R^2)} \qquad dT = dt + \frac{HR\,dR}{(1-H^2R^2)}$$

$$ds^2 = -(1-H^2R^2)\,dT^2 + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

Static co-ordinates

$$T \to T + \text{constant}$$

$$(1-H^2R^2) \geq 0 \qquad R \leq H^{-1}$$

$$ds^2 = -(1-H^2R^2)\,dt^2 - 2R\,dR\,H\,dt + dR^2 + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR\,dR}{(1-H^2R^2)} \qquad dT = dt + \frac{HR\,dR}{(1-H^2R^2)}$$

$$ds^2 = -(1-H^2R^2)\,dT^2 + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

Static co-ordinates

$T \to T + \text{constant}$

$(1-H^2R^2) \geq 0$ $\quad$ $R \leq H^{-1}$

$$ds^2 = -(1-H^2R^2)\,dt^2 - 2R\,dR\,H\,dt + dR^2 + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{H R\, dR}{(1-H^2R^2)} \qquad dT = dt + \frac{H R\, dR}{(1-H^2R^2)}$$

$$\boxed{ds^2 = -(1-H^2R^2)\,dT^2 + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}}$$

Static co-ordinates

$T \to T + \text{constant}$

$(1-H^2R^2) \geq 0 \qquad R \leq H^{-1}$

$$ds^2 = -(1 - H^2R^2)\,dt^2 - 2R\,dR\,H\,dt + dR^2 + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR\,dR}{(1 - H^2R^2)} \qquad dT = dt + \frac{HR\,dR}{(1 - H^2R^2)}$$

$$ds^2 = -(1 - H^2R^2)\,dT^2 + \frac{dR^2}{(1 - H^2R^2)} + R^2 d^2\Omega_{S_2}$$

Static coordinates

No well defined notion of energy

$T \to T + \text{constant}$

$(1 - H^2R^2) \geq 0 \qquad R \leq H^{-1}$

$$ds^2 = -(1-H^2R^2)\,dt^2 - 2R\,dR\,H\,dt + dR^2 + R^2 d^2\Omega_{S_2}$$

$$T = t + \int \frac{HR\,dR}{(1-H^2R^2)} \qquad dT = dt + \frac{HR\,dR}{(1-H^2R^2)}$$

$$ds^2 = -(1-H^2R^2)\,dT^2 + \frac{dR^2}{(1-H^2R^2)} + R^2 d^2\Omega_{S_2}$$

Static co-ordinates

No well defined notion of energy

$T \to T + \text{constant}$

$$(1-H^2R^2) \geq 0 \qquad R \leq H^{-1}$$

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x$$

Massive scalar field

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{ab} \partial_a \phi \, \partial_b \phi - \frac{1}{2} m^2 \phi^2 \right]$$

rell
…med natu…
energy

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$

(η: conformal time)

Massive scalar field

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{ab}\partial_a\phi\,\partial_b\phi - \frac{1}{2}m^2\phi^2\right]$$

FRW spacetime

conformal time

$$ds^2 = -dt^2 + a^2(t)\, d^2x = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$

$$\sqrt{-g} = a^4$$

Massive scalar field

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{ab}\partial_a\phi\,\partial_b\phi - \frac{1}{2}m^2\phi^2\right]$$

FRW spacetime

conformal time

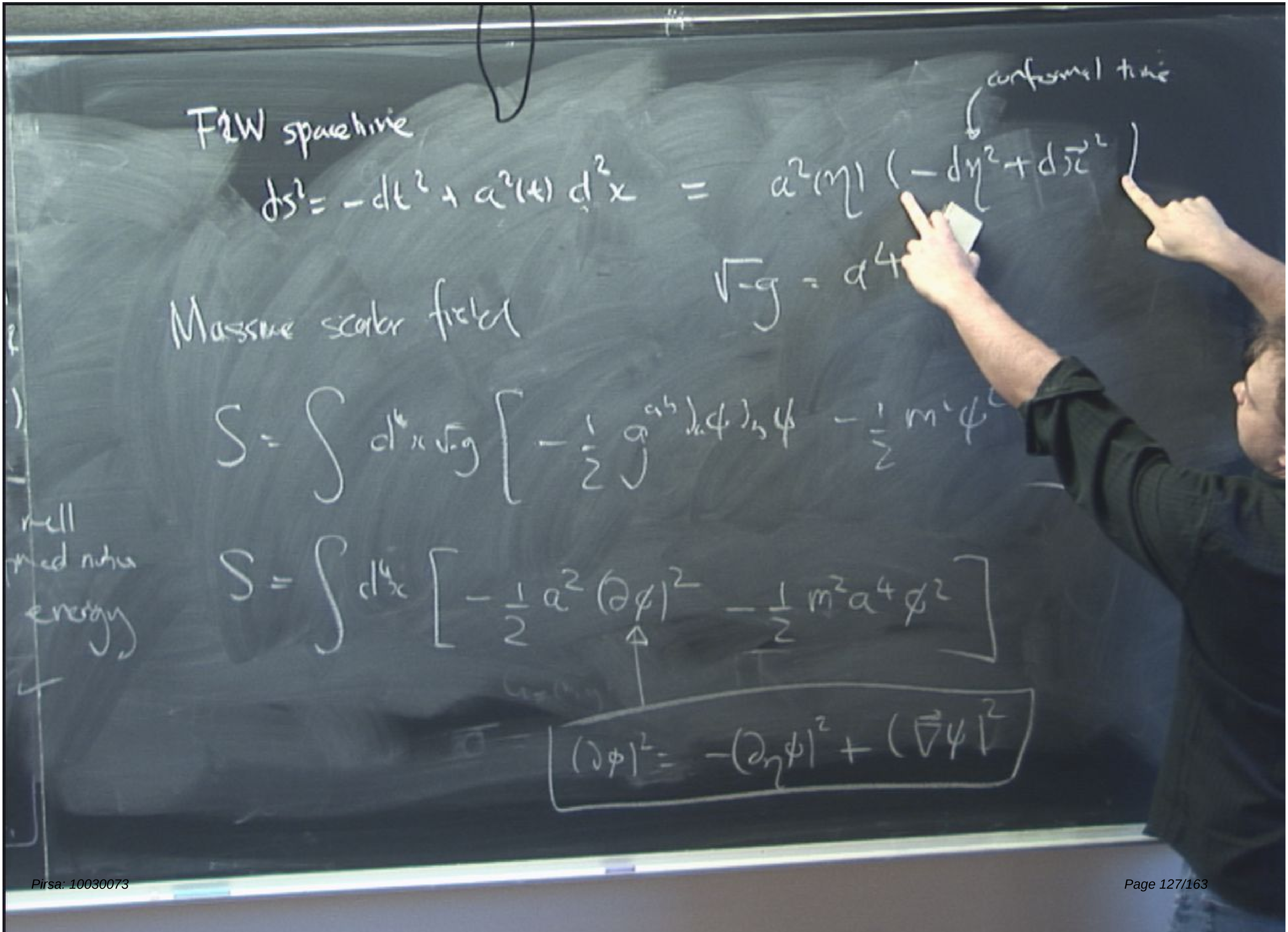
$$ds^2 = -dt^2 + a^2(t)\, d^2x = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$

$$\sqrt{-g} = a^4$$

Massive scalar field

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{ab}\partial_a\phi\,\partial_b\phi - \frac{1}{2} m^2\phi^2\right]$$

$$S = \int d^4x \left[-\frac{1}{2} a^2 (\partial\phi)^2 - \frac{1}{2} m^2 a^4 \phi^2\right]$$


FLW spacetime
$ds^2 = -dt^2 + a^2(t)\, d^2x = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$
conformal time
$\sqrt{-g} = a^4$
Massive scalar field
$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{ab} \partial_a\phi \partial_b\phi - \frac{1}{2} m^2 \phi^2 \right.$
$S = \int d^4x \left[-\frac{1}{2} a^2 (\partial\phi)^2 - \frac{1}{2} m^2 a^4 \phi^2 \right]$
$(\partial\phi)^2 = -(\partial_\eta\phi)^2 + (\vec{\nabla}\phi)^2$

FRW spacetime

conformal time

$$ds^2 = -dt^2 + a^2(t)\, d^3x = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$

$$\sqrt{-g} = a^4$$

Massive scalar field

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{ab}\partial_a\phi\, \partial_b\phi - \frac{1}{2} m^2\phi^2\right]$$

$$S = \int d^4x \left[-\frac{1}{2} a^2 (\partial\phi)^2 - \frac{1}{2} m^2 a^4 \phi^2\right]$$

$$(\partial\phi)^2 = -(\partial_\eta\phi)^2 + (\vec{\nabla}\phi)^2$$

conformal time

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^3x = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$

$$\sqrt{-g} = a^4$$

Massive scalar field

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{ab}\partial_a\phi\,\partial_b\phi - \frac{1}{2}m^2\phi^2\right]$$

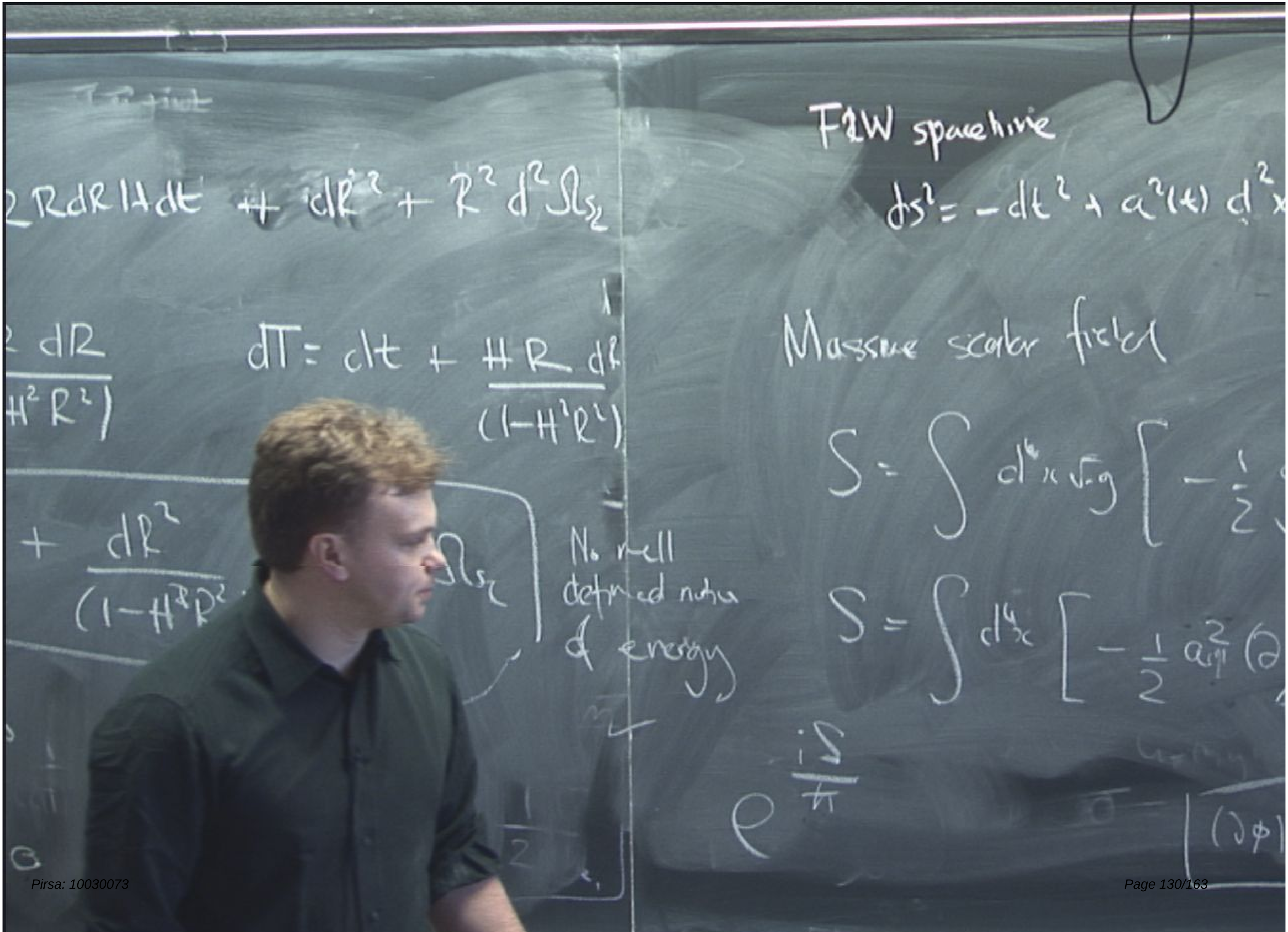
$$S = \int d^4x \left[-\frac{1}{2}a^2_{(\eta)}(\partial\phi)^2 - \frac{1}{2}m^2 a^4_{(\eta)}\phi^2\right]$$

$$(\partial\phi)^2 = -(\partial_\eta\phi)^2 + (\vec{\nabla}\phi)^2$$

well

...ed natu...

energy


FRW spacetime
ds² = − dt² + a²(t) d²x
Massive scalar field
S = ∫ d⁴x √-g [− 1/2
S = ∫ d⁴x [− 1/2 a²(η) (∂
No well defined notion of energy
dT = dt + HR dR / (1−H²R²)
+ dR² + R² d²Ω_{S2}

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x = a^2(\eta)\left(-d\eta^2 \ldots\right.$$

$$\sqrt{-g} = a^4$$

Massive scalar field

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{ab} \partial_a \phi \, \partial_b \phi - \frac{1}{2} m \ldots \right.$$

$$\frac{S}{\hbar} = \int d^4x \left[-\frac{1}{2} a_{(\eta)}^2 (\partial\phi)^2 - \frac{1}{2} m^2 a_{(\eta)}^4 \phi^2 \right]$$

$$(\partial\phi)^2 = -(\partial_\eta \phi)^2 + (\vec{\nabla}\phi)^2$$

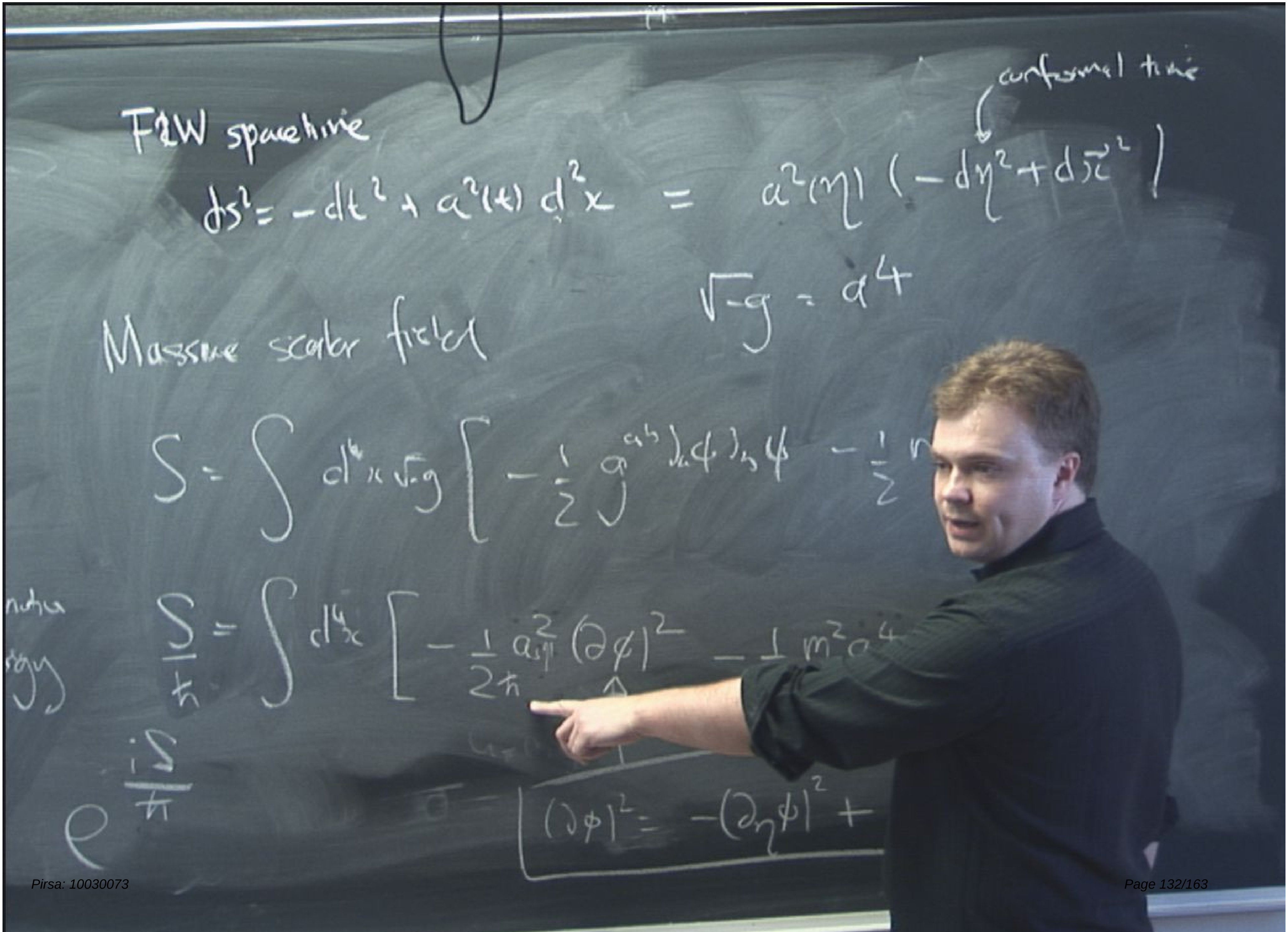
$$e^{\frac{iS}{\hbar}}$$

$R^2 d^2\Omega_{S_2}$

$+ \frac{HR\, dR}{(1-H^2R^2)}$

Ω_{S_2}

No well defined notion of energy


conformal time
FRW spacetime
$ds^2 = -dt^2 + a^2(t)\, d^2x = a^2(\eta)\,(-d\eta^2 + d\vec{x}^2)$
$\sqrt{-g} = a^4$
Massive scalar field
$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi - \frac{1}{2}\right.$
$\frac{S}{\hbar} = \int d^4x \left[-\frac{1}{2\hbar} a^2(\eta) (\partial\phi)^2 - \frac{1}{2} m^2 a^4\right.$
$e^{\frac{iS}{\hbar}}$
$(\partial\phi)^2 = -(\partial_\eta \phi)^2 +$

conformal time

FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\, d^2x \quad = \quad a^2(\eta)\,(-d\eta^2 + d\vec{x}^2)$$

$$\sqrt{-g} = a^4$$

Massive scalar field

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{ab}\partial_a\phi\,\partial_b\phi - \frac{1}{2} m^2\phi^2\right]$$

$$\frac{S}{\hbar} = \int d^4x \left[-\frac{1}{2\hbar} a^2_{(\eta)} (\partial\phi)^2 - \frac{1}{2}\frac{m^2}{\hbar} a^4_{(\eta)}\phi^2\right]$$

$$e^{\frac{iS}{\hbar}}$$

$$(\partial\phi)^2 = -(\partial_\eta\phi)^2 + (\vec{\nabla}\phi)^2$$

FRW spacetime

conformal time

$$ds^2 = -dt^2 + a^2(t)\, d^2x \;=\; a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$

$$\sqrt{-g} = a^4$$

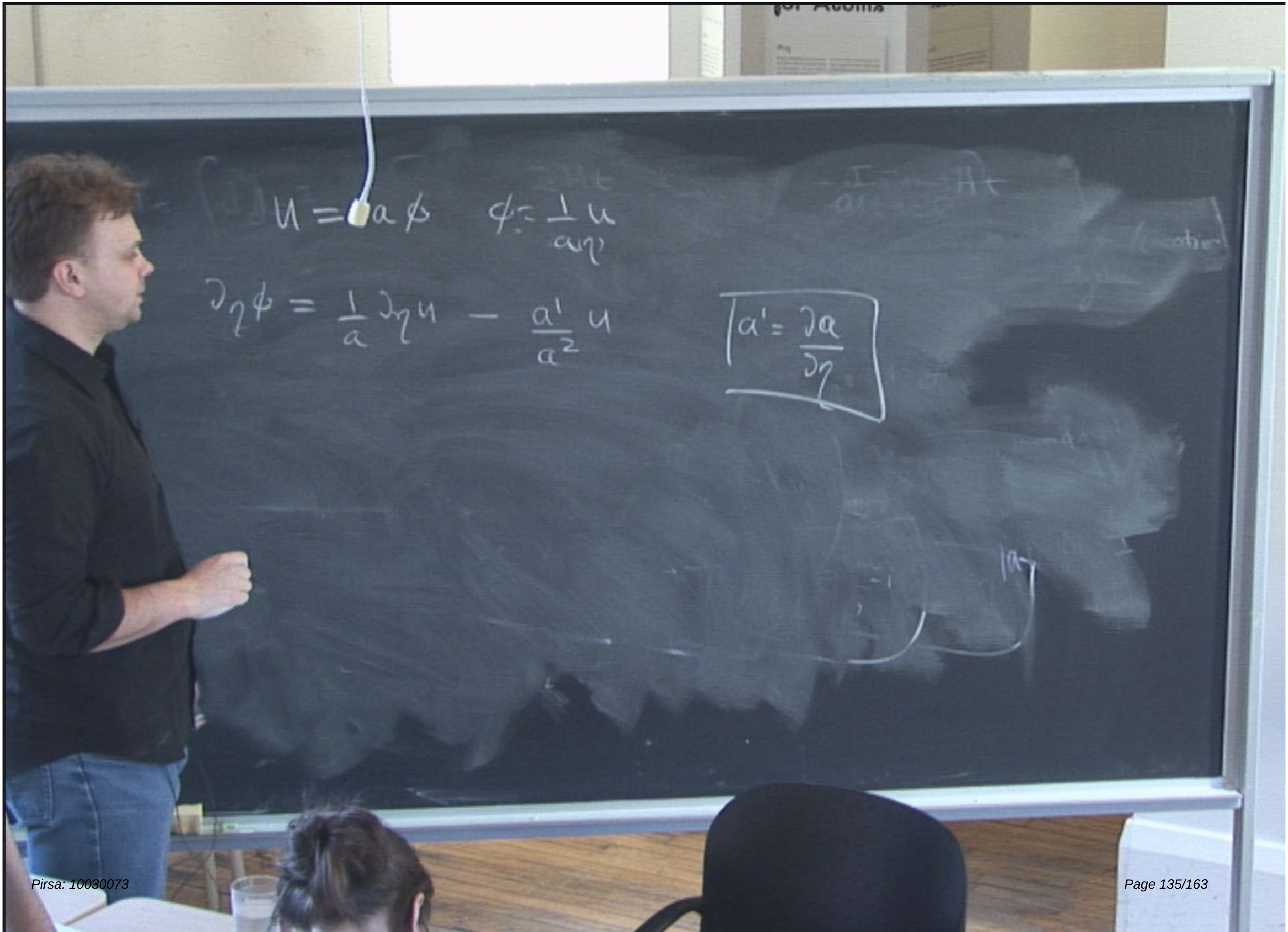
Massive scalar field

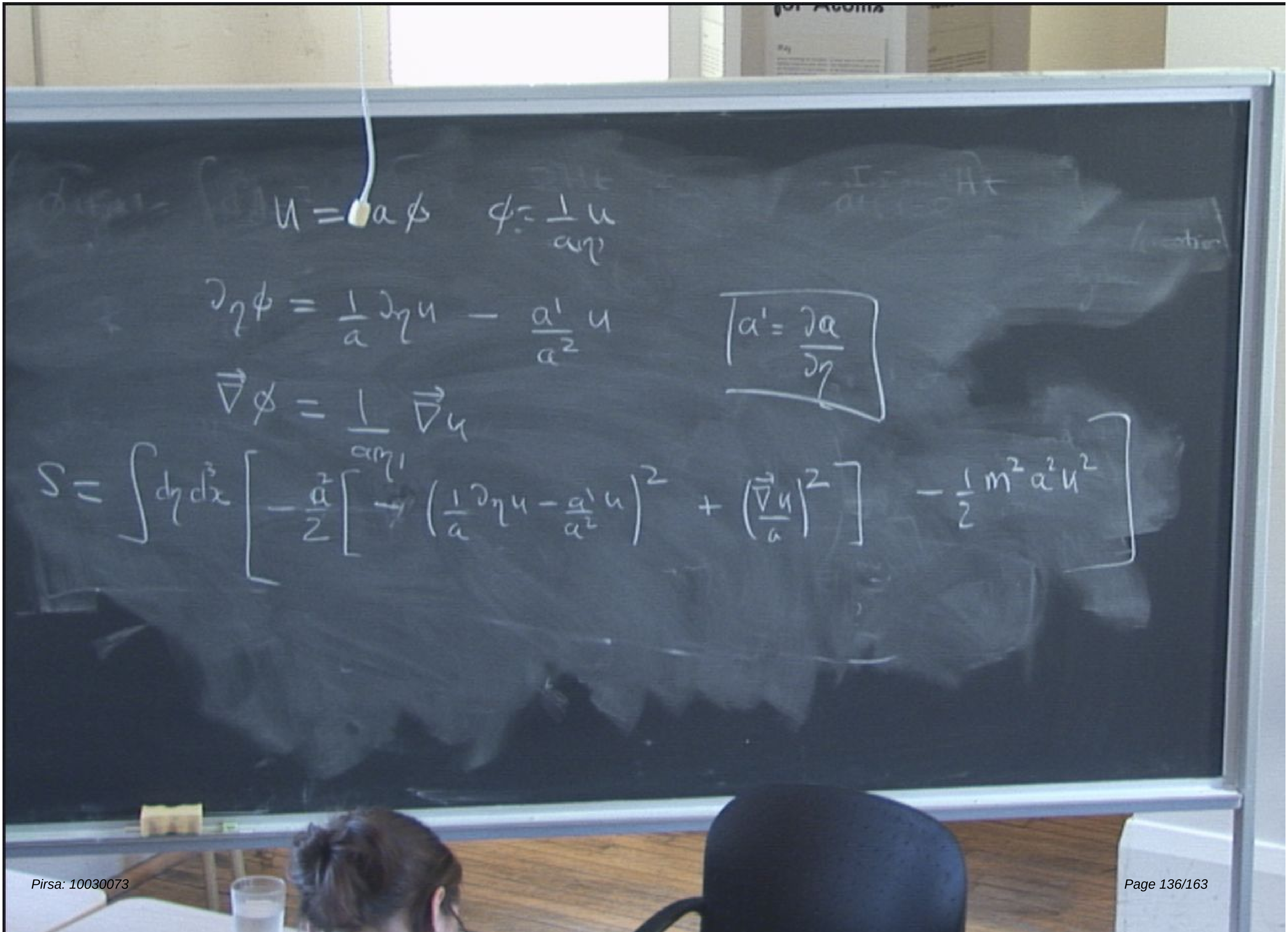
$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{ab}\partial_a\phi\,\partial_b\phi - \frac{1}{2} m^2\phi^2\right]$$

$$\frac{S}{\hbar} = \int d^4x \left[-\frac{1}{2\hbar} a^2_{(\eta)} (\partial\phi)^2 - \frac{1}{2}\frac{m^2}{\hbar} a^4_{(\eta)}\phi^2\right]$$

$$(\partial\phi)^2 = -(\partial_\eta\phi)^2 + (\vec{\nabla}\phi)^2$$

$$e^{\frac{iS}{\hbar}}$$


u = a φ
φ = 1/a u
∂η φ = 1/a ∂η u − a'/a² u
a' = ∂a/∂η


$u = a\phi \quad \phi = \frac{1}{a\eta} u$
$\partial_\eta \phi = \frac{1}{a}\partial_\eta u - \frac{a'}{a^2} u$
$a' = \frac{\partial a}{\partial \eta}$
$\vec{\nabla}\phi = \frac{1}{a\eta}\vec{\nabla} u$
$S = \int d\eta\, d^3x \left[-\frac{a^2}{2}\left[-\left(\frac{1}{a}\partial_\eta u - \frac{a'}{a^2}u\right)^2 + \left(\frac{\vec{\nabla} u}{a}\right)^2 \right] - \frac{1}{2} m^2 a^2 u^2 \right]$

FRW spacetime

conformal time

$$ds^2 = -dt^2 + a^2(t)\, d^3x = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$

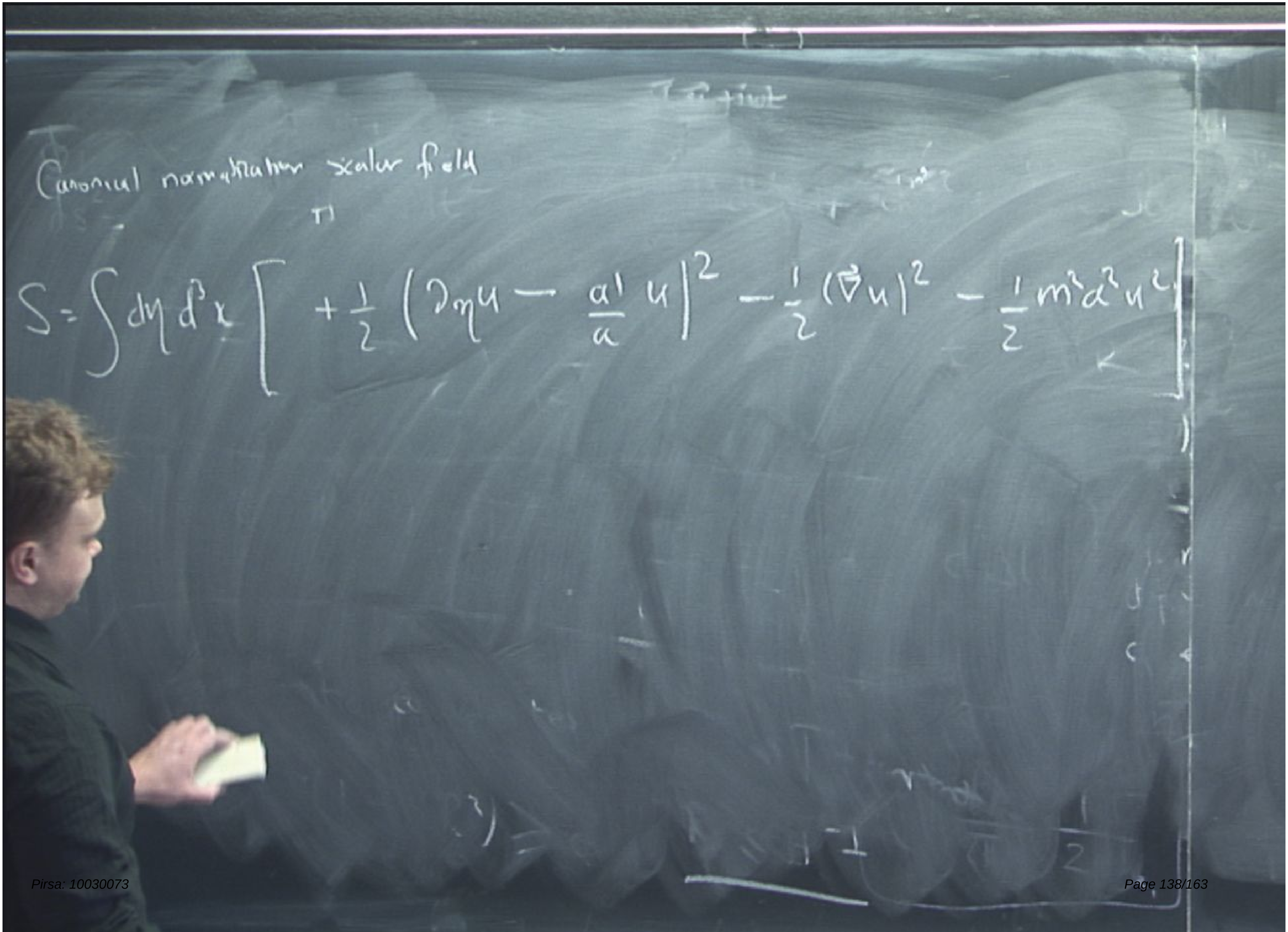
$$\sqrt{-g} = a^4$$

Massive scalar field

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{ab}\partial_a\phi\,\partial_b\phi - \frac{1}{2} m^2\phi^2\right]$$

$$\frac{S}{\hbar} = \int d^4x \left[-\frac{1}{2\hbar} a^2(\eta)(\partial\phi)^2 - \frac{1}{2}\frac{m^2}{\hbar} a^4(\eta)\phi^2\right]$$

$$(\partial\phi)^2 = -(\partial_\eta\phi)^2 + (\vec{\nabla}\phi)^2$$


Canonical normalization scalar field
$S = \int d\eta\, d^3x \left[+\frac{1}{2}\left(\partial_\eta u - \frac{a'}{a} u\right)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$

Canonical normalization scalar field

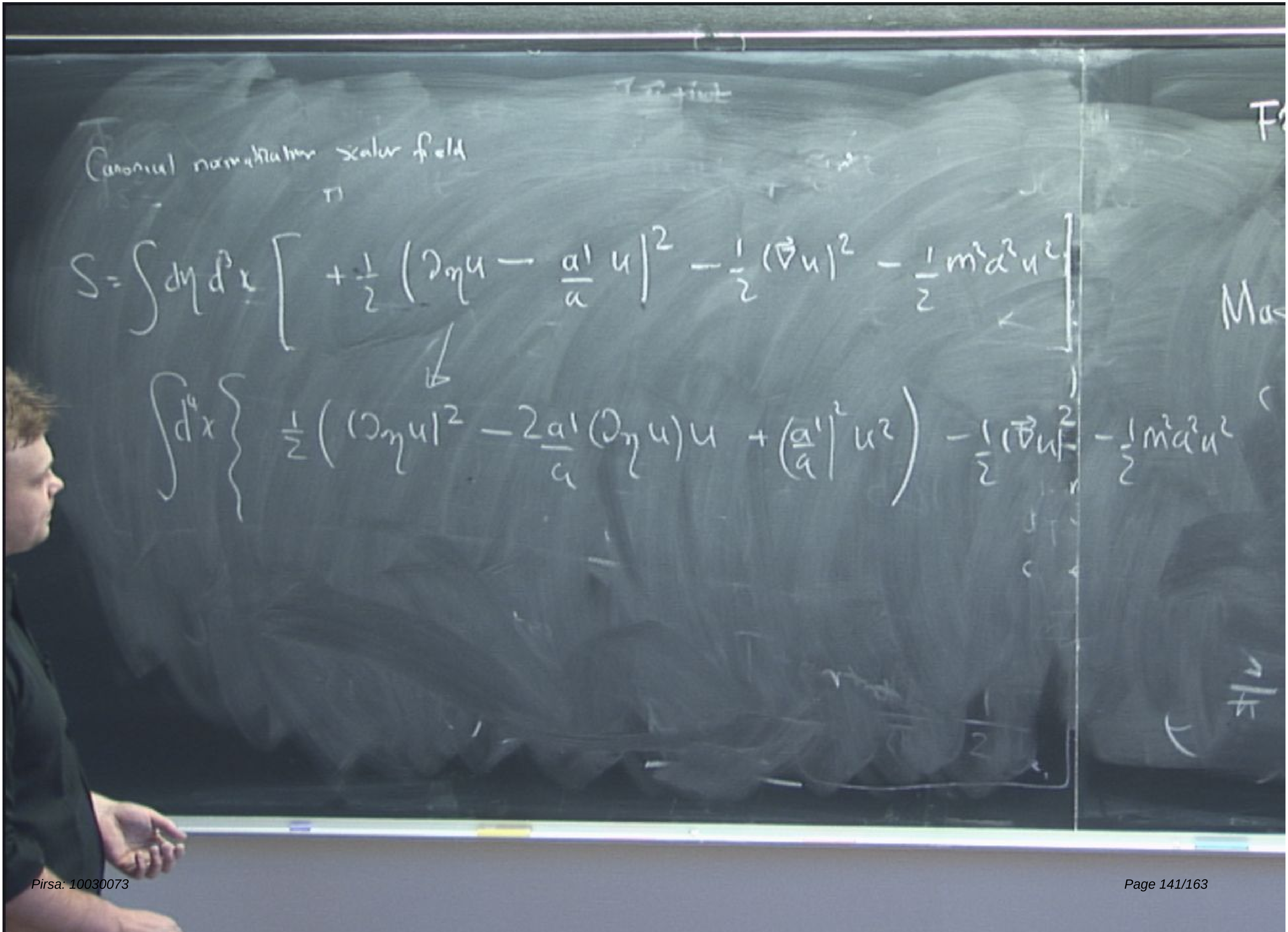
$$S = \int d\eta\, d^3x \left[+\frac{1}{2}\left(\partial_\eta u - \frac{a'}{a} u\right)^2 - \frac{1}{2}(\vec\nabla u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$$

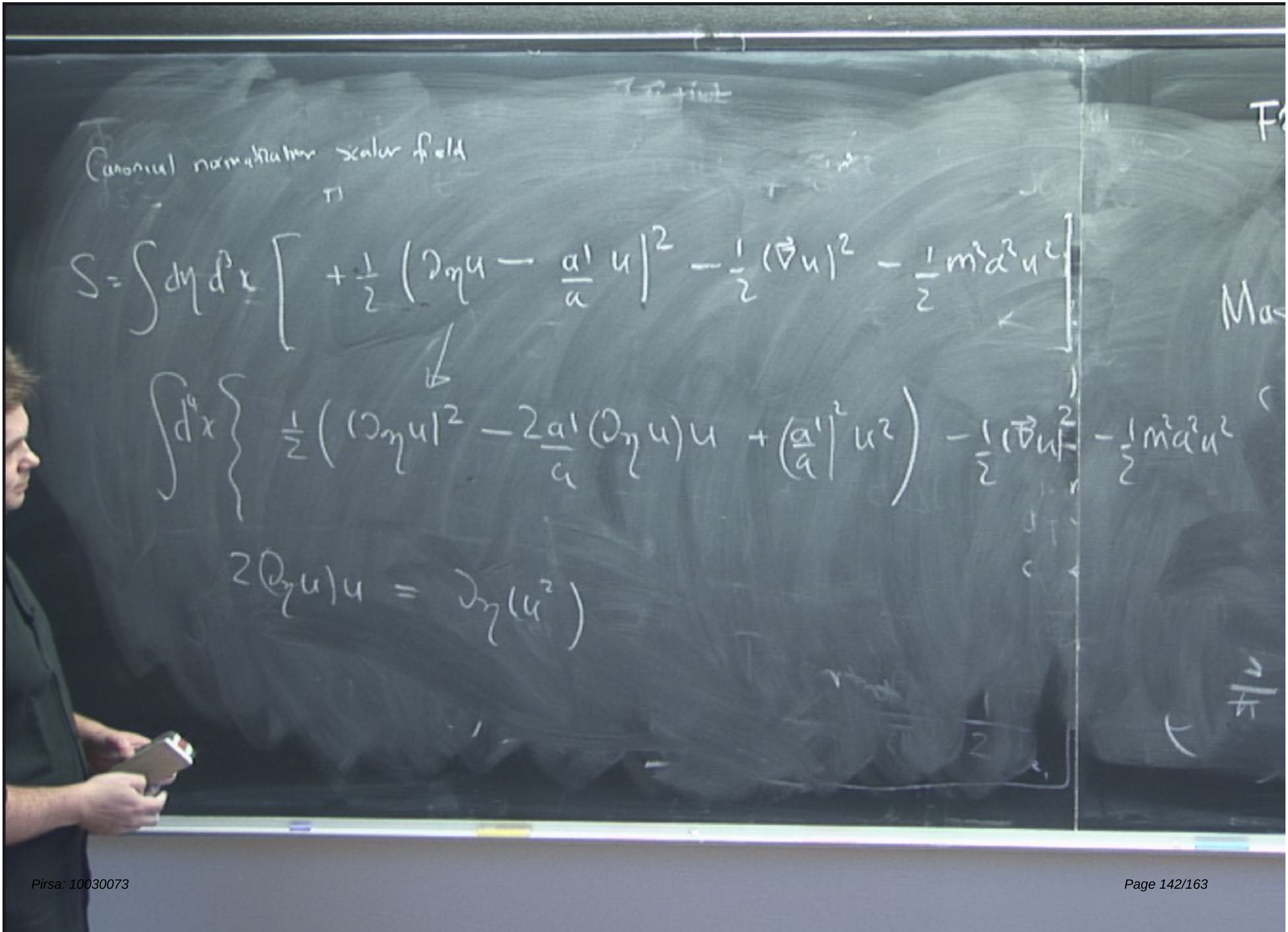
$$\frac{1}{2}\left((\partial_\eta u)^2 - 2\frac{a'}{a}(\partial_\eta u) u + \left(\frac{a'}{a}\right)^2 u^2 \right) - \frac{1}{2}(\vec\nabla u)^2 - \frac{1}{2} m^2 a^2 u^2$$

FRW

Massive


Canonical normalization scalar field
S = ∫dη d³x [+ ½ (∂ηu − a'/a u)² − ½ (∇u)² − ½ m²a²u²]
∫d⁴x { ½ ((∂ηu)² − 2 a'/a (∂η ... (a'/a)² u²) − ½ (∇u)² − ½ m²a²u²


Canonical normalization scalar field
$S = \int d\eta\, d^3x \left[+\frac{1}{2}\left(\partial_\eta u - \frac{a'}{a} u\right)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$
$\int d^4x \left\{ \frac{1}{2}\left((\partial_\eta u)^2 - 2\frac{a'}{a}(\partial_\eta u)u + \left(\frac{a'}{a}\right)^2 u^2 \right) - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right.$


Canonical normalization scalar field
$S = \int d\eta\, d^3x \left[+\frac{1}{2}\left(\partial_\eta u - \frac{a'}{a} u\right)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$
$\int d^4x \left\{ \frac{1}{2}\left((\partial_\eta u)^2 - 2\frac{a'}{a}(\partial_\eta u)u + \left(\frac{a'}{a}\right)^2 u^2 \right) - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right.$
$2(\partial_\eta u)u = \partial_\eta(u^2)$
Ma

Canonical normalization scalar field

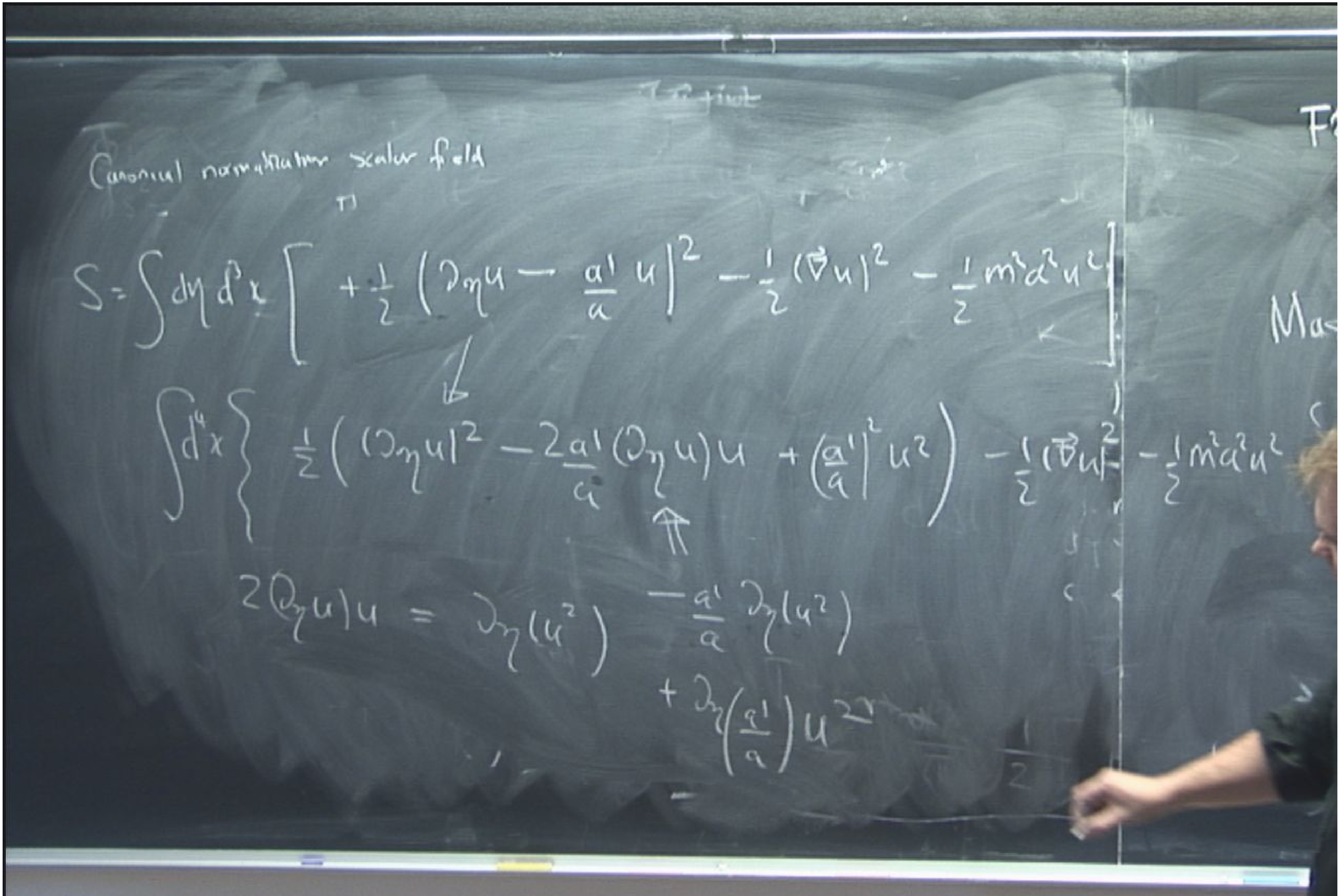
$$S = \int d\eta\, d^3x \left[+\frac{1}{2}\left(\partial_\eta u - \frac{a'}{a} u\right)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$$

$$\int d^4x \left\{ \frac{1}{2}\left((\partial_\eta u)^2 - 2\frac{a'}{a}(\partial_\eta u)u + \left(\frac{a'}{a}\right)^2 u^2 \right) - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right.$$

$$2(\partial_\eta u)u = \partial_\eta(u^2) \qquad -\frac{a'}{a}\partial_\eta(u^2)$$

F

Ma


Canonical normalization scalar field
$S = \int d\eta\, d^3x \left[+\frac{1}{2}\left(\partial_\eta u - \frac{a'}{a}u\right)^2 - \frac{1}{2}(\vec\nabla u)^2 - \frac{1}{2}m^2a^2u^2 \right]$
$\int d^4x \left\{ \frac{1}{2}\left((\partial_\eta u)^2 - 2\frac{a'}{a}(\partial_\eta u)u + \left(\frac{a'}{a}\right)^2 u^2 \right) - \frac{1}{2}(\vec\nabla u)^2 - \frac{1}{2}m^2a^2u^2 \right.$
$2(\partial_\eta u)u = \partial_\eta(u^2)$
$-\frac{a'}{a}\partial_\eta(u^2)$
$+\partial_\eta\left(\frac{a'}{a}\right)u^2$

Canonical normalization scalar field

$$S = \int d\eta\, d^3x \left[+ \frac{1}{2}\left(\partial_\eta u - \frac{a'}{a} u\right)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$$

$$\int d^4x \left\{ \frac{1}{2}\left((\partial_\eta u)^2 - 2\frac{a'}{a}(\partial_\eta u) u + \left(\frac{a'}{a}\right)^2 u^2 \right) - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right.$$

$$2(\partial_\eta u) u = \partial_\eta (u^2) \qquad - \frac{a'}{a} \partial_\eta (u^2)$$

$$+ \partial_\eta \left(\frac{a'}{a}\right) u^2$$

Canonical normalization scalar field

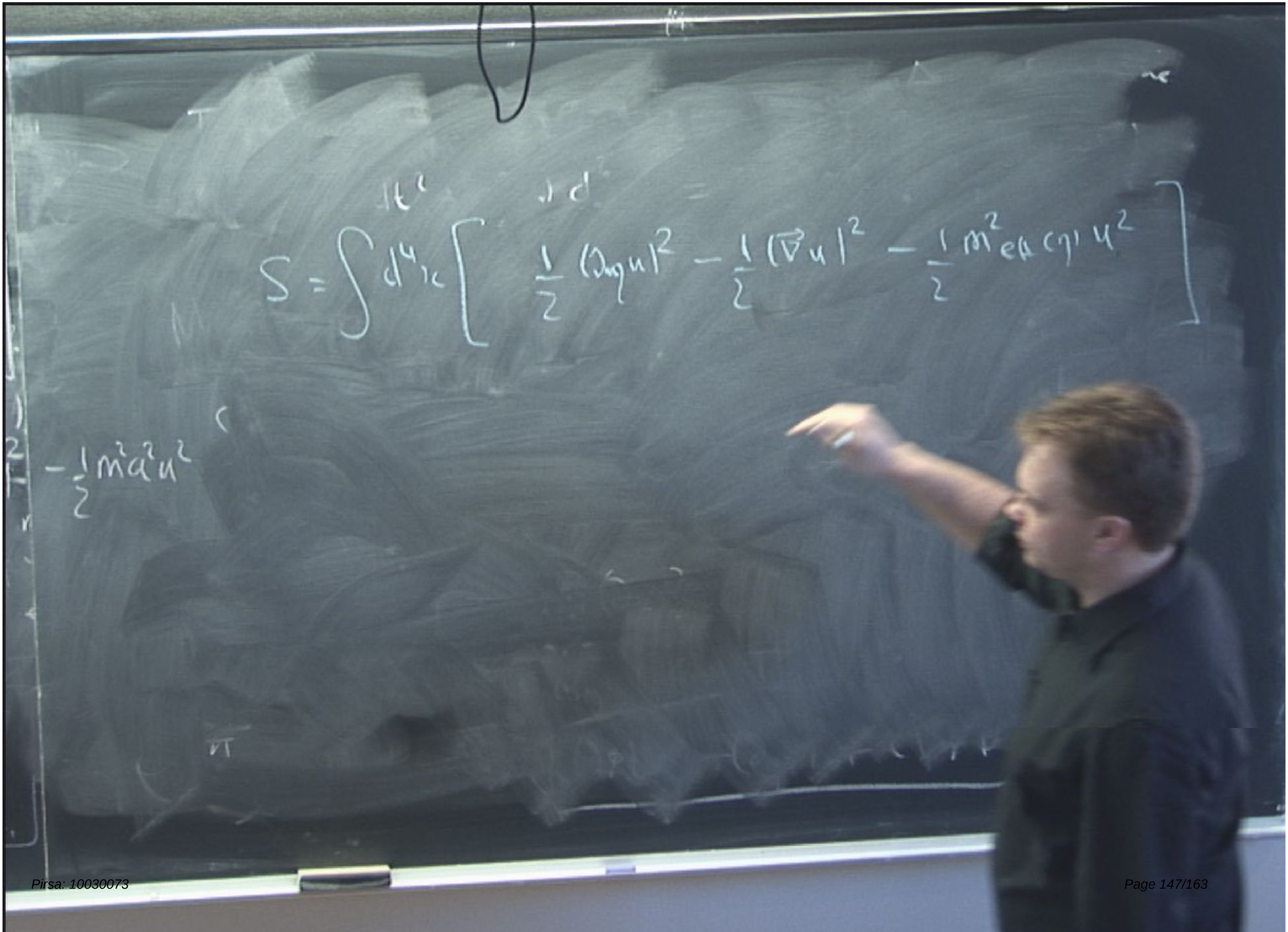
$$S = \int d\eta\, d^3x \left[+\frac{1}{2}\left(\partial_\eta u - \frac{a'}{a} u\right)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$$

$$\int d^4x \left\{ \frac{1}{2}\left((\partial_\eta u)^2 - 2\frac{a'}{a}(\partial_\eta u) u + \left(\frac{a'}{a}\right)^2 u^2 \right) - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right.$$

$$2(\partial_\eta u) u = \partial_\eta (u^2) \qquad -\frac{a'}{a}\partial_\eta(u^2)$$

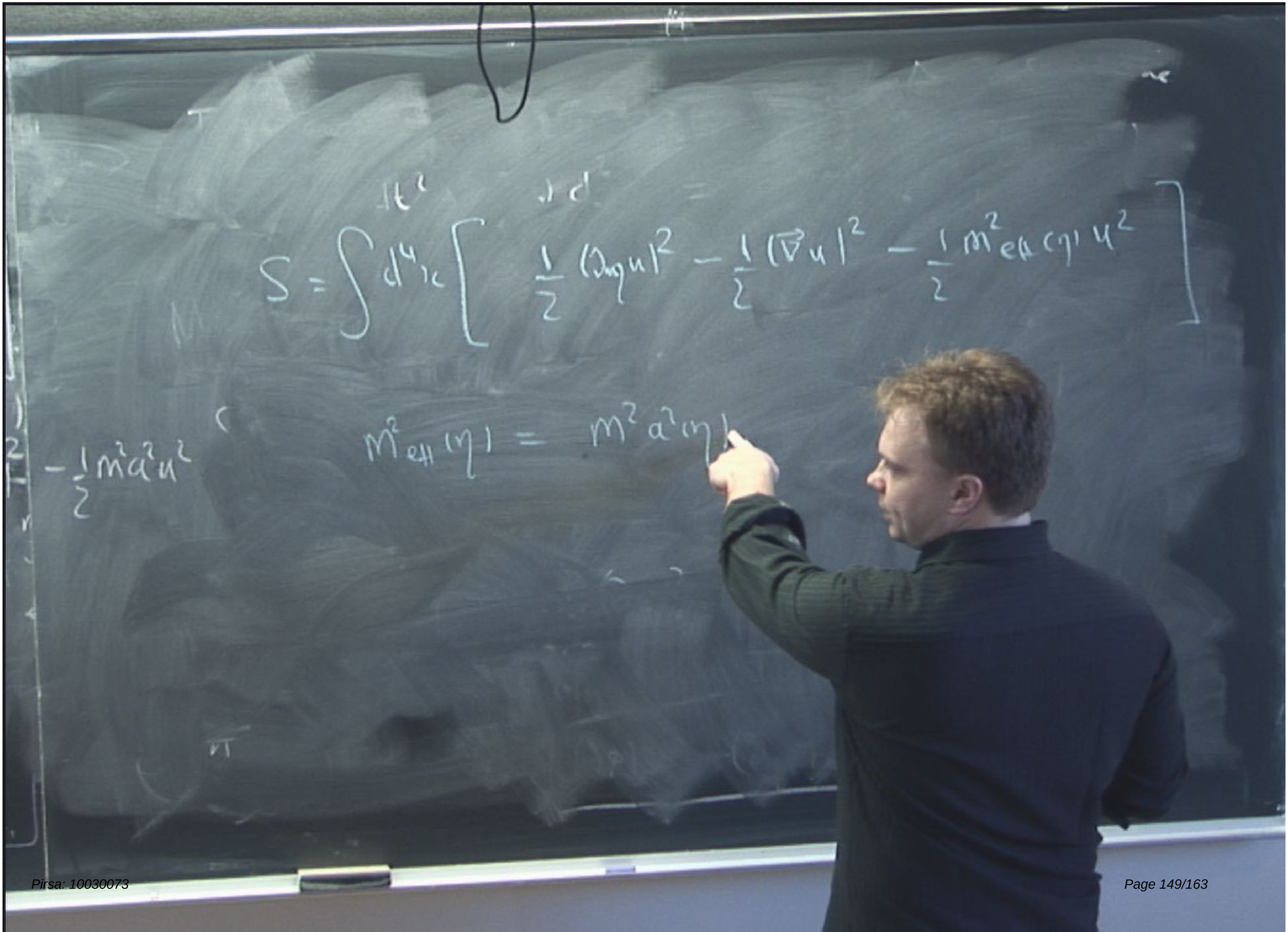
$$+\,\partial_\eta\left(\frac{a'}{a}\right) u^2$$

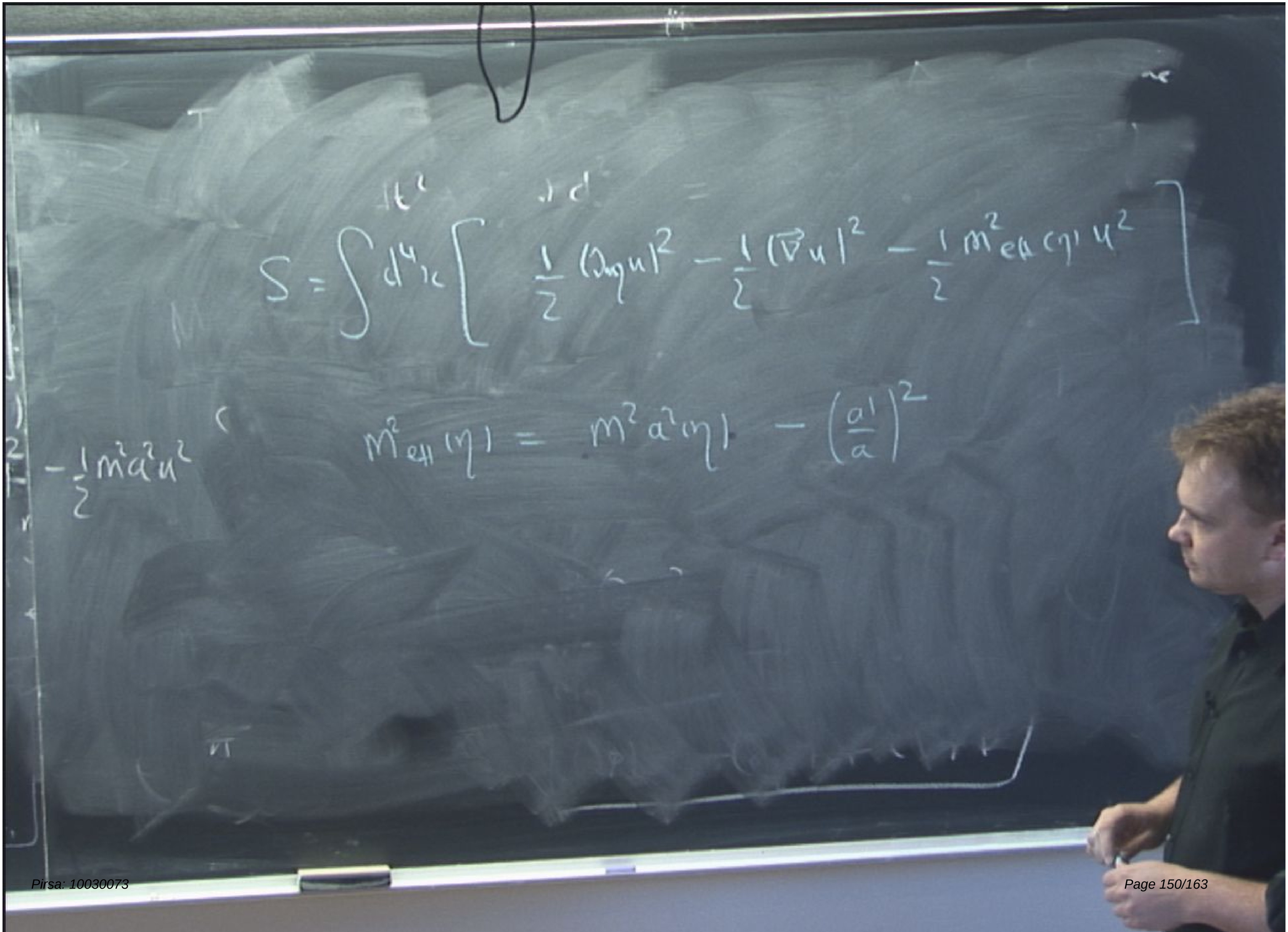
Ma


$S = \int d^4x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2(\eta) u^2 \right]$
$-\frac{1}{2} m^2 a^2 u^2$

$$S = \int d^4x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta)\, u^2 \right]$$

$$-\frac{1}{2} m^2 a^2 u^2$$


$S = \int d^4x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta)\, u^2 \right]$
$m^2_{eff}(\eta) = m^2 a^2(\eta)$
$-\frac{1}{2} m^2 a^2 u^2$



$S = \int d^4x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta)\, u^2 \right]$
$-\frac{1}{2} m^2 a^2 u^2$
$m^2_{eff}(\eta) = m^2 a^2(\eta) - \left(\frac{a'}{a}\right)^2$

$$S = \int d^4x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta)\, u^2 \right]$$

$$m^2_{eff}(\eta) = m^2 a^2(\eta) - \left(\frac{a'}{a}\right)^2 - \partial_\eta\left(\frac{a'}{a}\right)$$

$$-\frac{1}{2} m^2 a^2 u^2$$

$$S = \int d^4x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta)\, u^2 \right]$$

$-\frac{1}{2} m^2 a^2 u^2$

$$m^2_{eff}(\eta) = m^2 a^2(\eta) - \left(\frac{a'}{a}\right)^2 - \partial_\eta\left(\frac{a'}{a}\right)$$

$$\boxed{m^2_{eff}(\eta) = m^2 a^2(\eta) - \frac{a''}{a}}$$

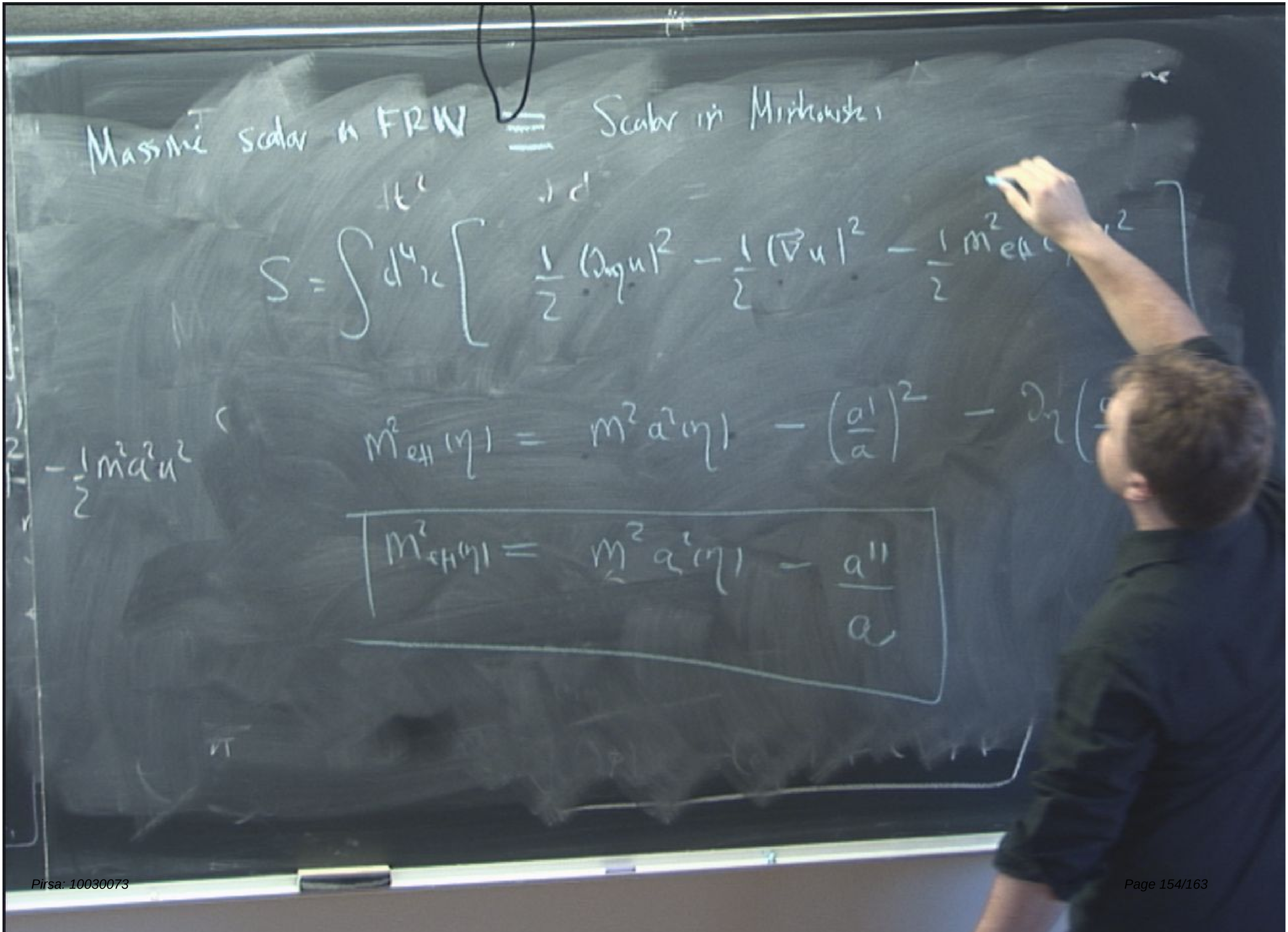
Massive scalar in FRW

$$S = \int d^4x \left[\frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta)\, u^2 \right]$$

$-\frac{1}{2} m^2 a^2 u^2$

$$m^2_{eff}(\eta) = m^2 a^2(\eta) - \left(\frac{a'}{a}\right)^2 - \partial_\eta \left(\frac{a'}{a}\right)$$

$$\boxed{m^2_{eff}(\eta) = m^2 a^2(\eta) - \frac{a''}{a}}$$


Massive scalar in FRW = Scalar in Minkowski
S = ∫ d⁴x [1/2 (∂η u)² − 1/2 (∇u)² − 1/2 m²eff
m²eff (η) = m² a²(η) − (a'/a)² − ∂η (
m²eff (η) = m² a²(η) − a''/a
−1/2 m²a²u²

Massive scalar in FRW $\equiv$ Scalar in Minkowski + time-dependent mass

$$S = \int d^4x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta)\, u^2 \right]$$

$-\frac{1}{2} m^2 a^2 u^2$

$$m^2_{eff}(\eta) = m^2 a^2(\eta) - \left(\frac{a'}{a}\right)^2 - \partial_\eta\left(\frac{a'}{a}\right)$$

$$\boxed{m^2_{eff}(\eta) = m^2 a^2(\eta) - \frac{a''}{a}}$$

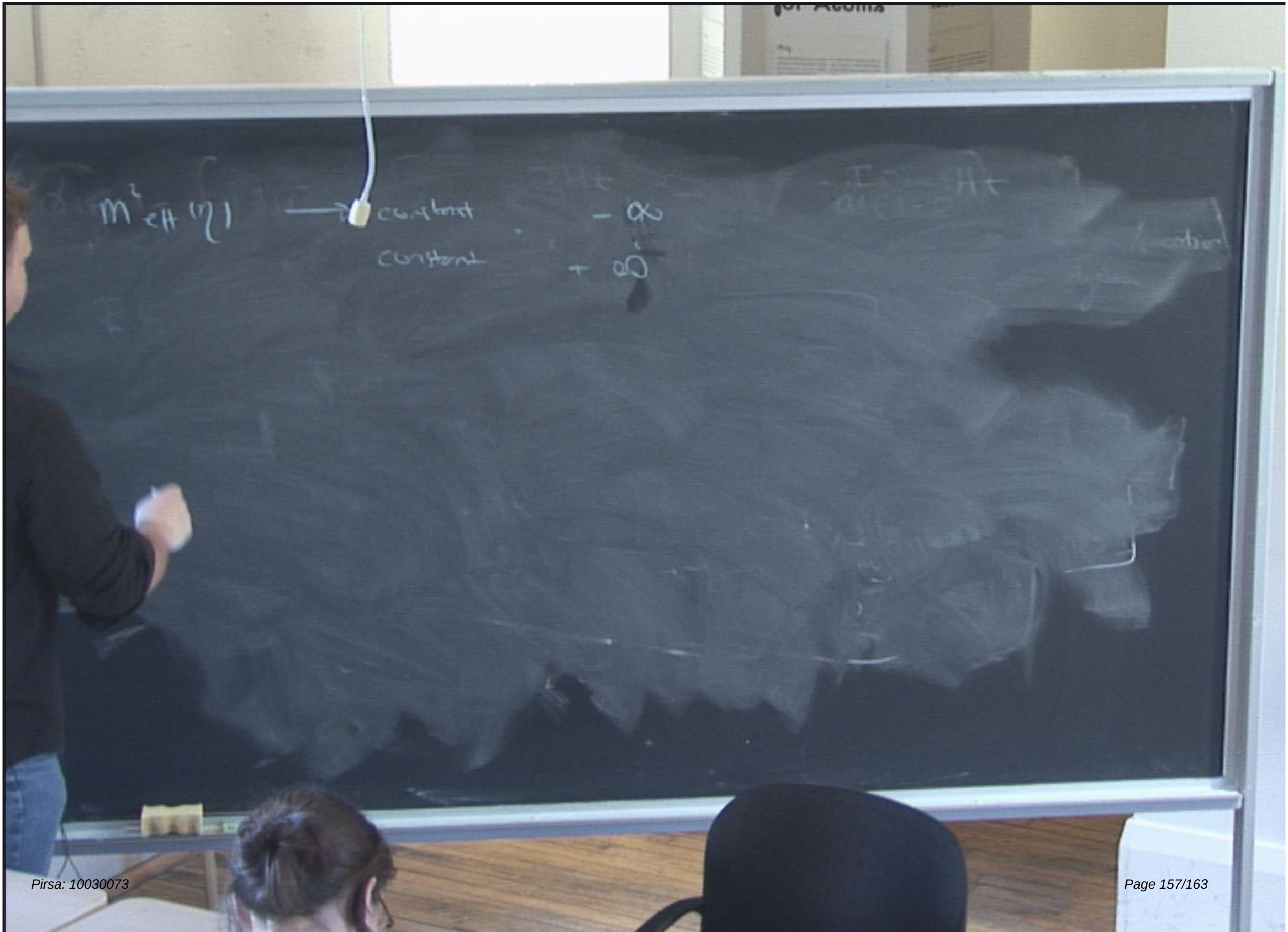
Massive scalar in FRW $\equiv$ Scalar in Minkowski + time-dependent mass

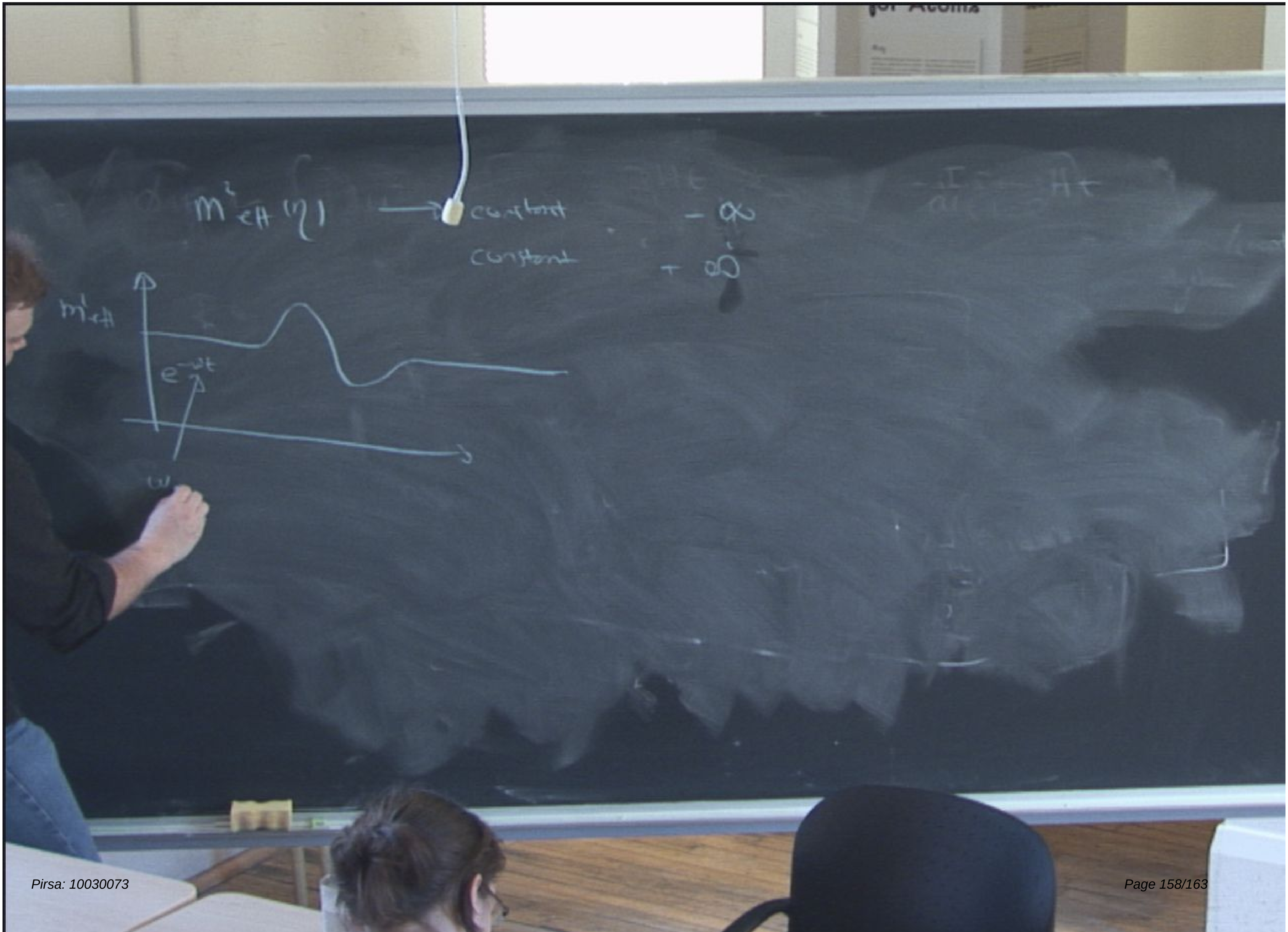
$$S = \int d^4x \left[\frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2_{\text{eff}}(\eta)\, u^2 \right]$$

$-\frac{1}{2} m^2 a^2 u^2$

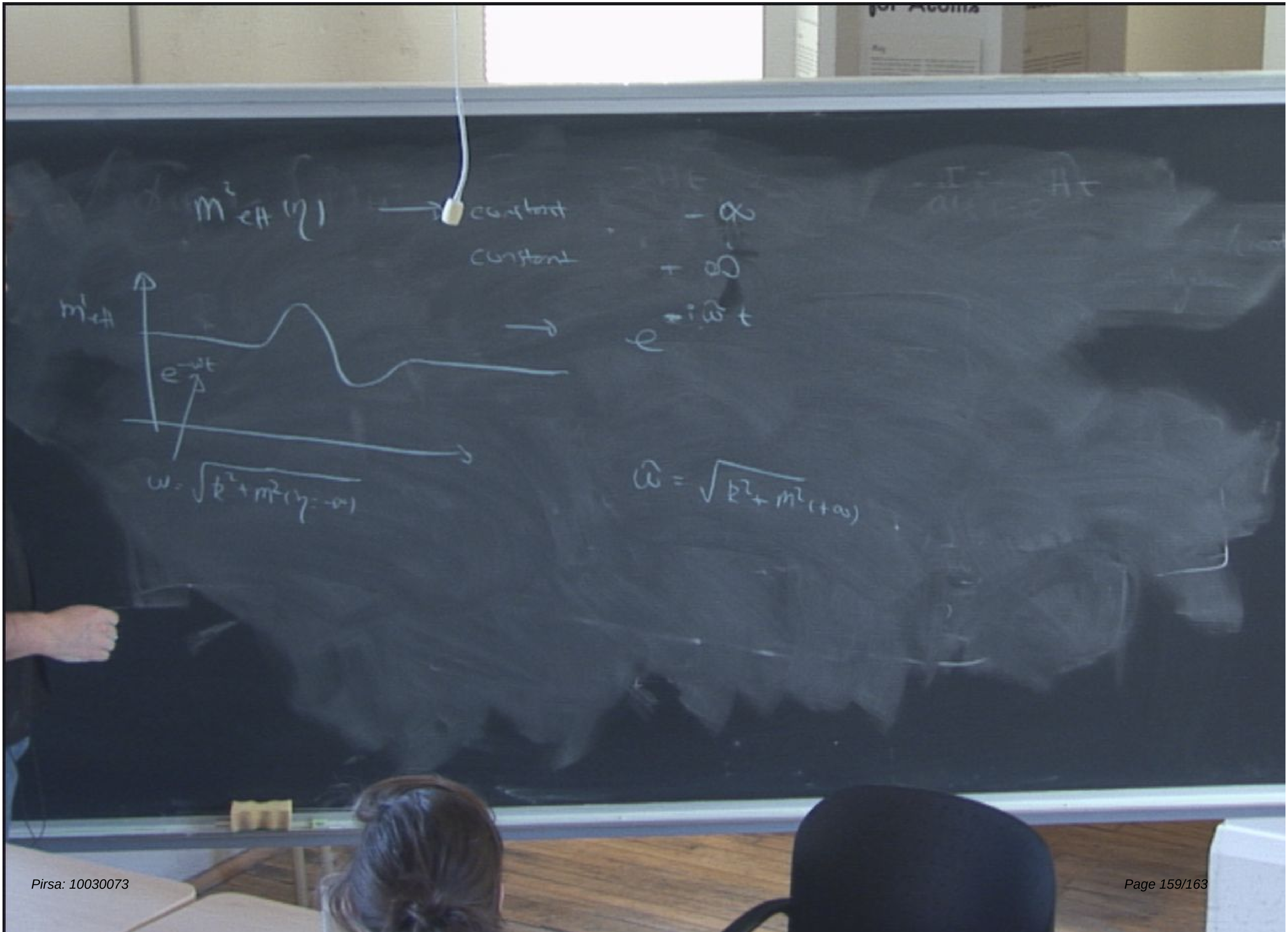
$$m^2_{\text{eff}}(\eta) = m^2 a^2(\eta) - \left(\frac{a'}{a}\right)^2 - \partial_\eta\left(\frac{a'}{a}\right)$$

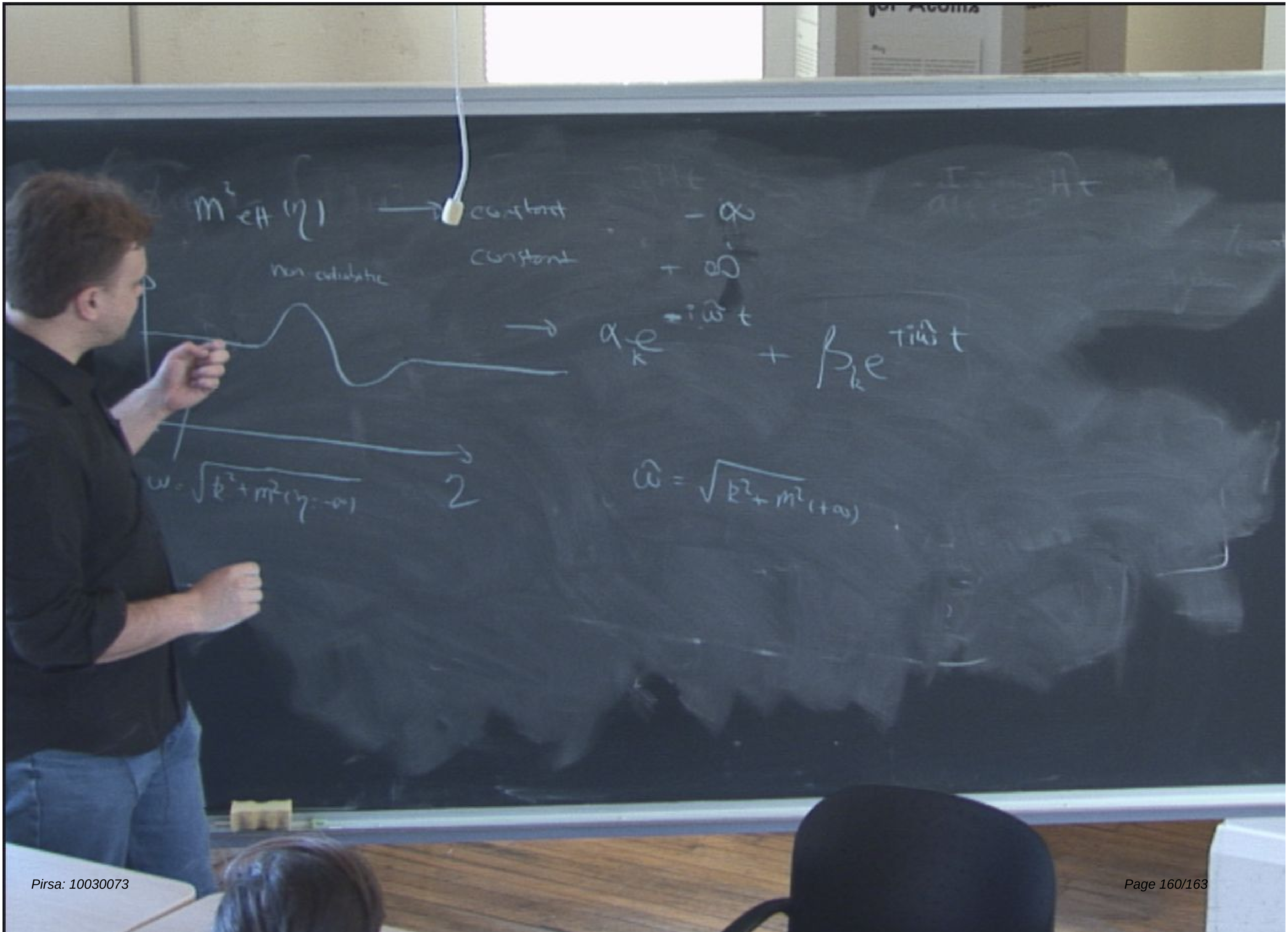
$$\boxed{m^2_{\text{eff}}(\eta) = m^2 a^2(\eta) - \frac{a''}{a}}$$

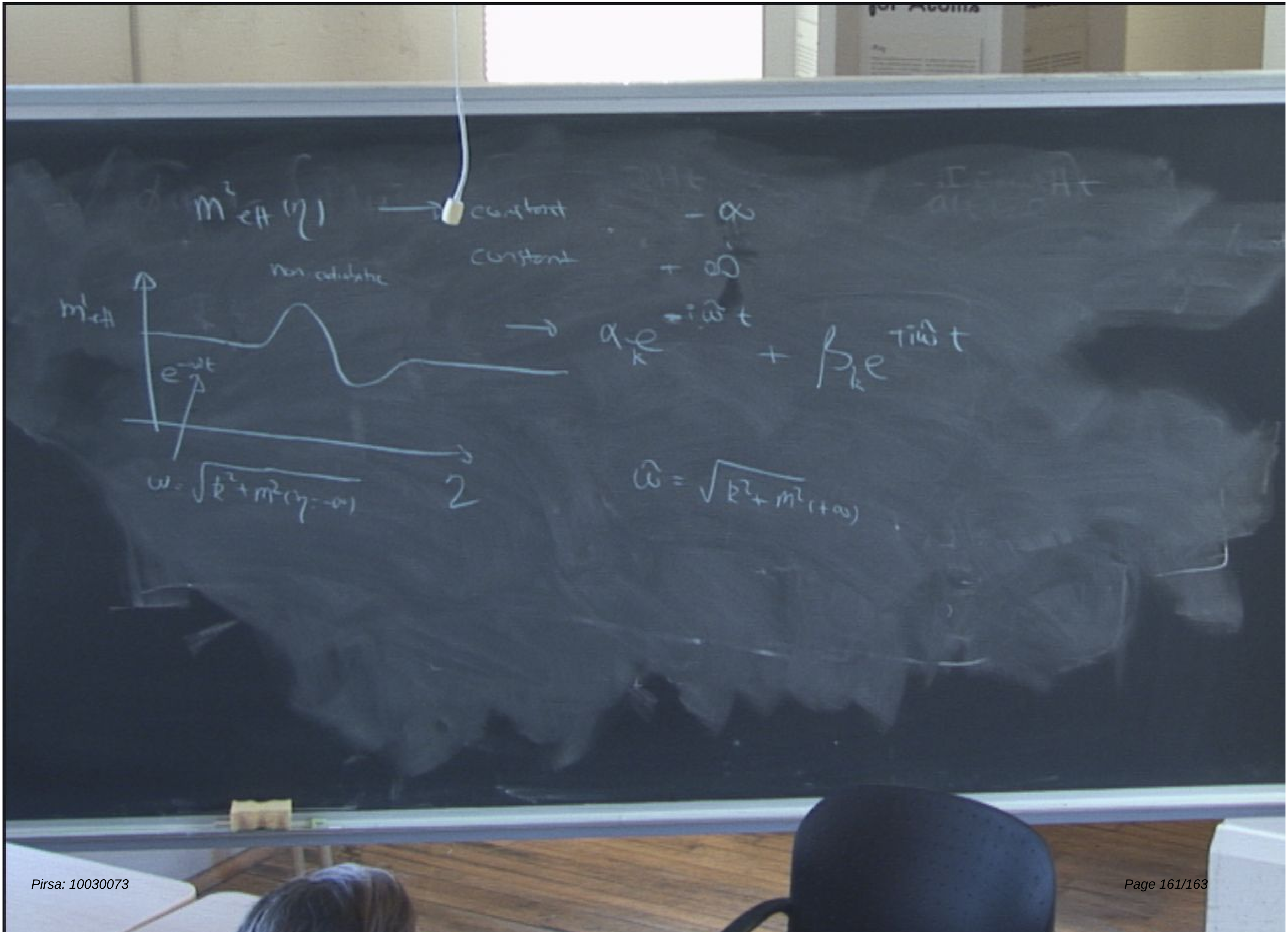

m²eff(η) → constant − ∞
constant + ∞

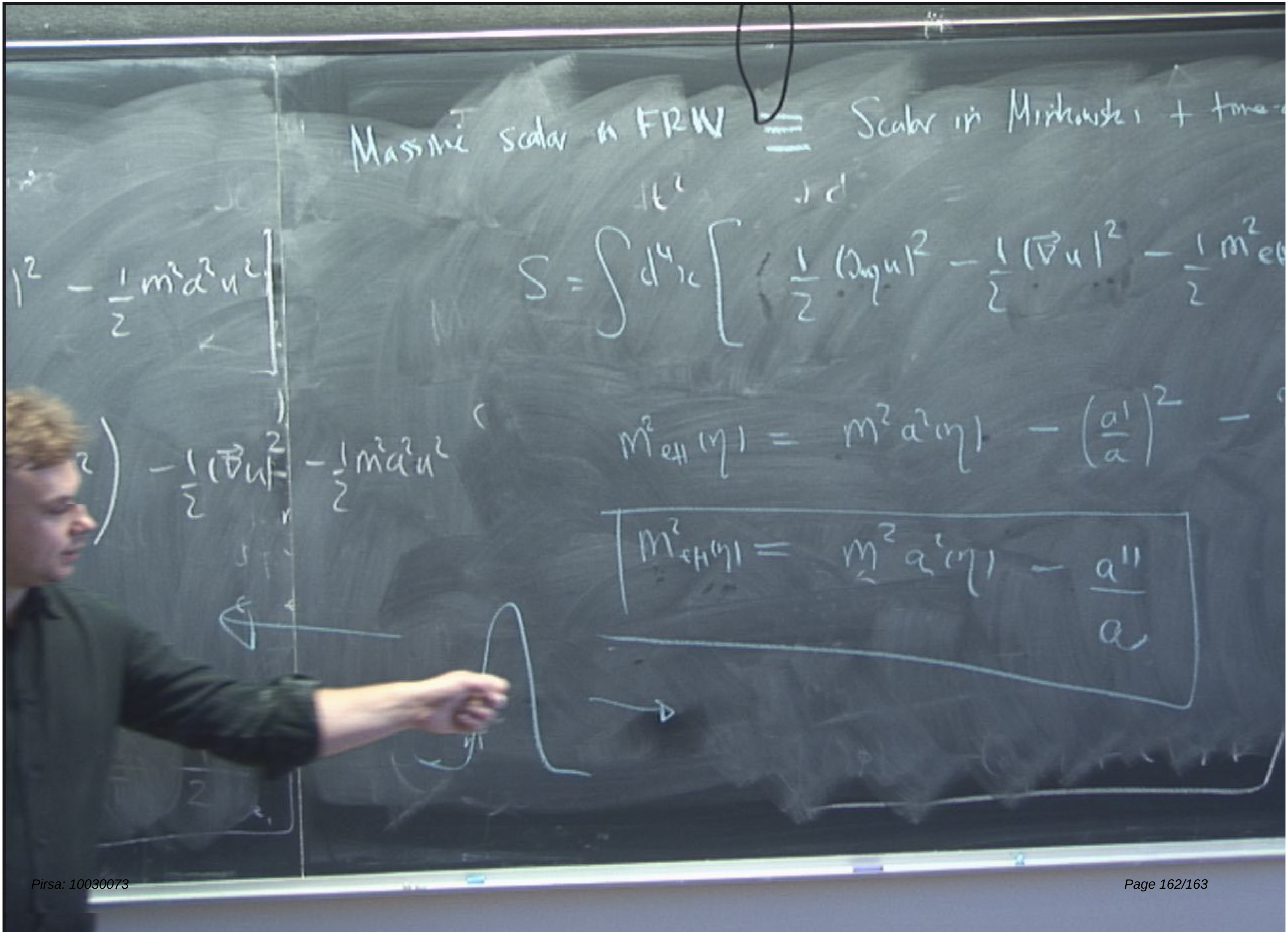


m²eff(η) → constant − ∞
constant + ∞
m²eff
e^{-ωt}


m²eff(η) → constant − ∞
constant + ∞
m²eff
→ e^{-iω̂t}
e^{-iωt}
ω = √(k² + m²(η = −∞))
ω̂ = √(k² + m²(+∞))


constant
constant
non adiabatic


m²eff(η) → constant − ∞
constant + ∞
non-adiabatic
m²eff
e^{−iωt}
→ α_k e^{−iω̂t} + β_k e^{+iω̂t}
ω = √(k² + m²(η = −∞))
2
ω̂ = √(k² + m²(+∞))


Massive scalar in FRW ≡ Scalar in Minkowski + time-
$S = \int d^4x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}\right.$
$-\frac{1}{2}m^2a^2u^2$
$-\frac{1}{2}(\vec{\nabla}u)^2 - \frac{1}{2}m^2a^2u^2$
$m^2_{eff}(\eta) = m^2 a^2(\eta) - \left(\frac{a'}{a}\right)^2 -$
$m^2_{eff}(\eta) = m^2 a^2(\eta) - \frac{a''}{a}$

Massive scalar in FRW $\equiv$ Scalar in Minkowski + time-

$$\ldots)^2 - \frac{1}{2} m^2 a^2 u^2$$

$$S = \int d^4x \left[\frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2_{e\ldots} \right.$$

$$\ldots)^2 u^2 \Big) - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2$$

$$m^2_{eff}(\eta) = m^2 a^2(\eta) - \left(\frac{a'}{a}\right)^2 - \ldots$$

$$\boxed{m^2_{eff}(\eta) = m^2 a^2(\eta) - \frac{a''}{a}}$$