Title: Explorations in Cosmology (PHYS 649) - Lecture 3

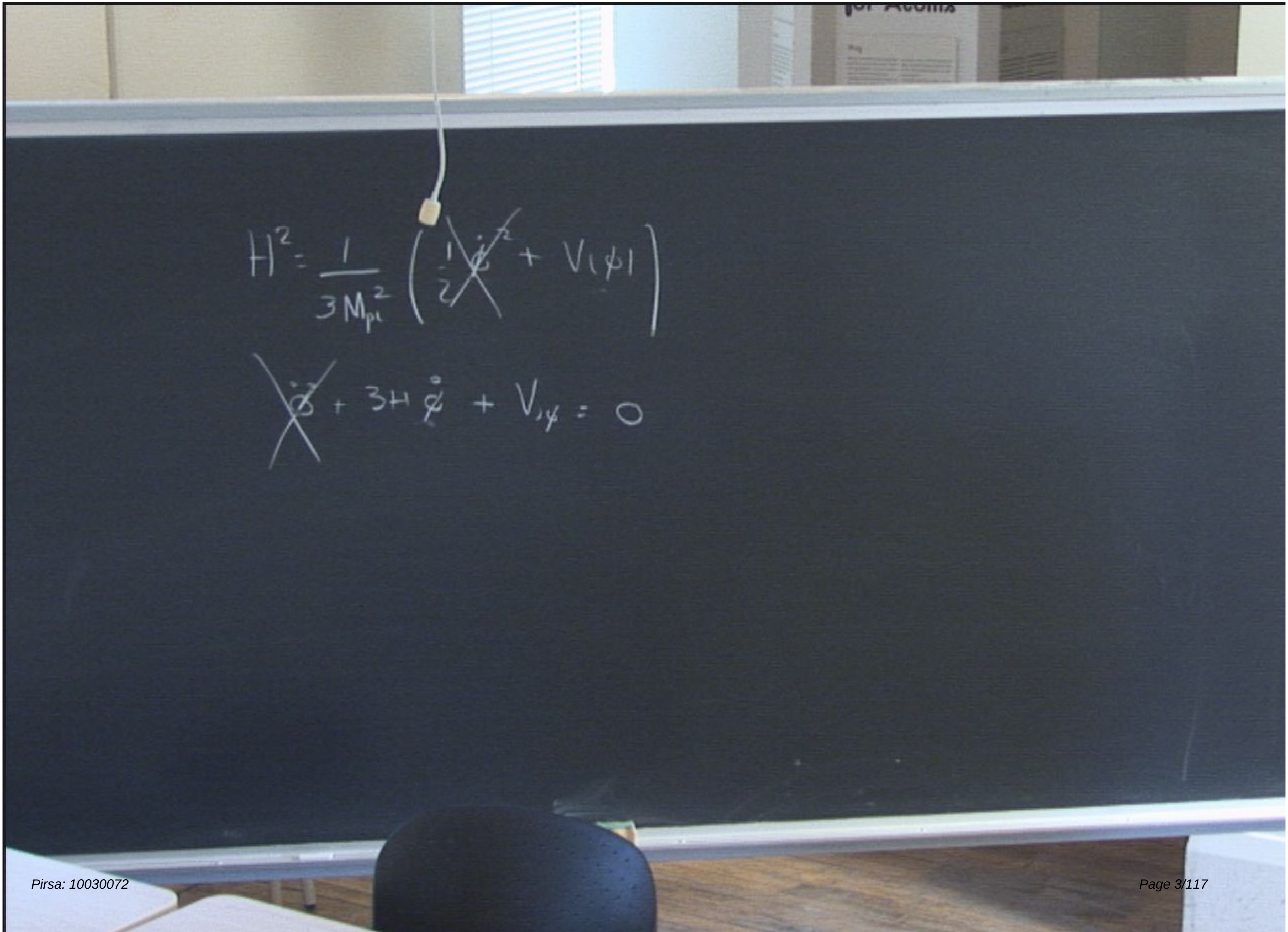
Date: Mar 17, 2010 09:00 AM

URL: http://pirsa.org/10030072

Abstract:

$$H^2 = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$


$H^2 = \frac{1}{3M_{pl}^2}\left(\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)\right)$
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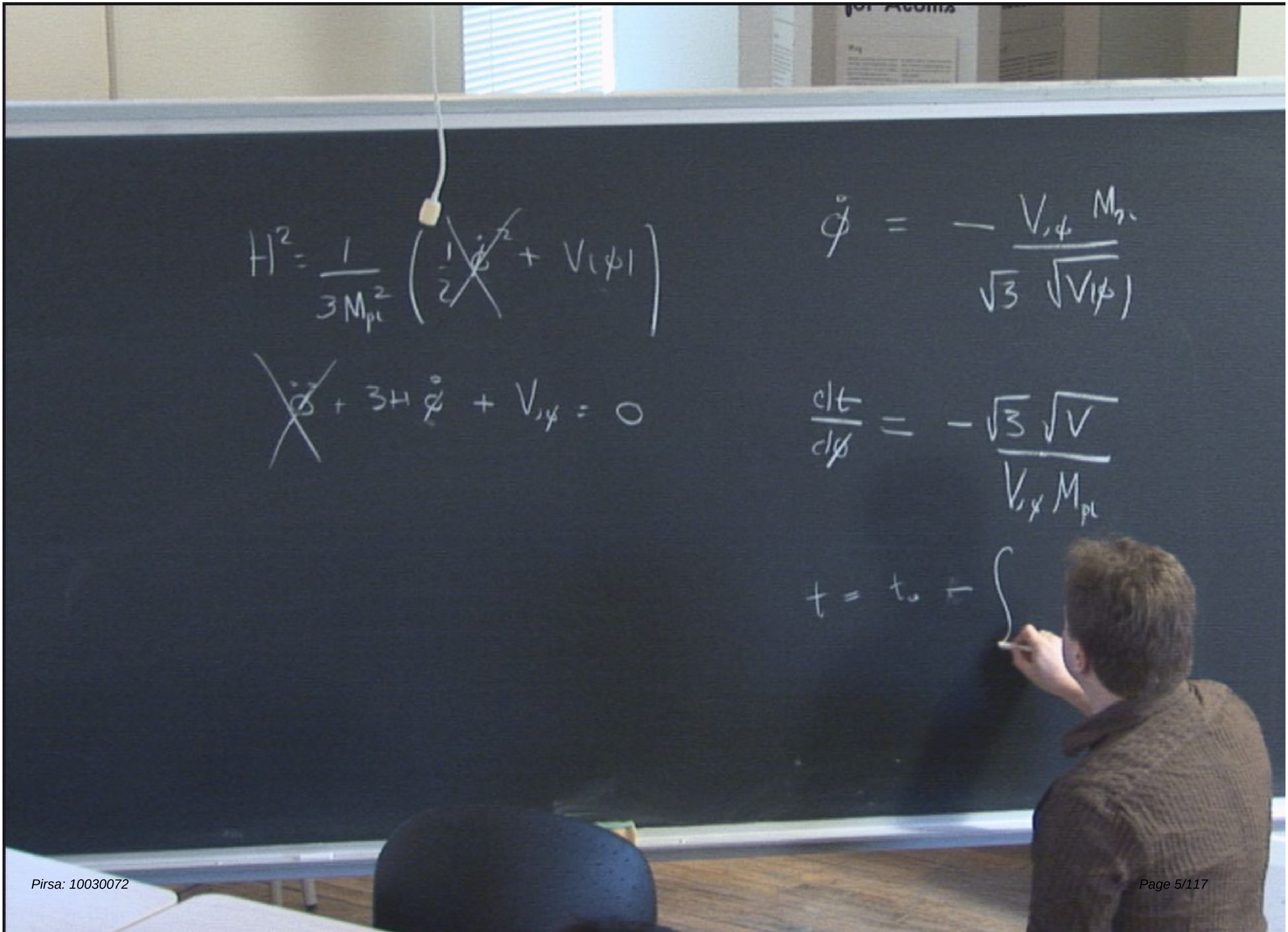
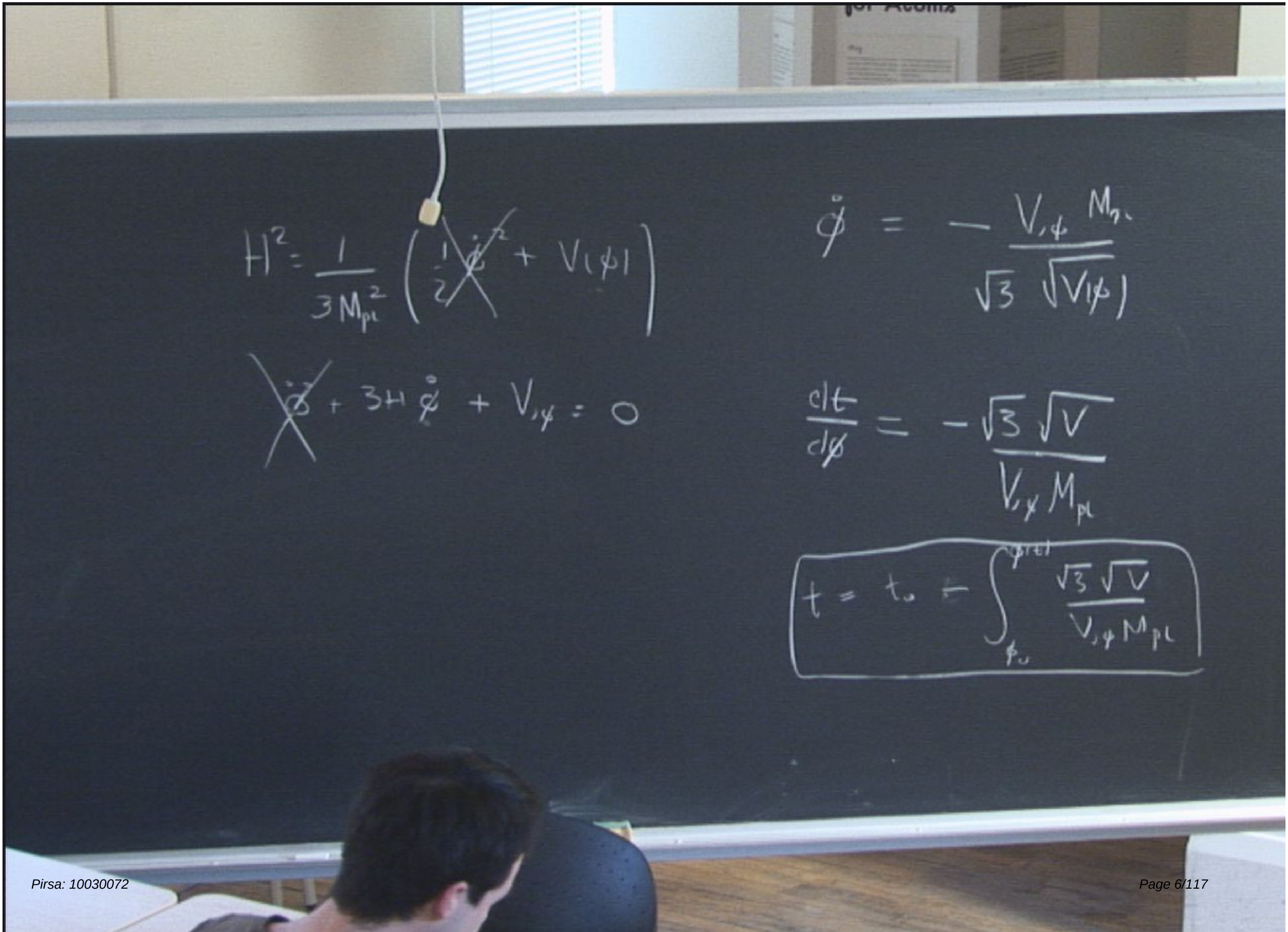
$$\dot\phi = -\frac{V_{,\phi}\, M_{pl}}{\sqrt{3}\,\sqrt{V(\phi)}}$$


$H^2 = \frac{1}{3M_{pl}^2}\left(\cancel{\frac{1}{2}\dot\phi^2} + V(\phi)\right)$
$\cancel{\ddot\phi} + 3H\dot\phi + V_{,\phi} = 0$
$\dot\phi = -\frac{V_{,\phi} M_{pl}}{\sqrt{3}\sqrt{V(\phi)}}$
$\frac{dt}{d\phi} = -\frac{\sqrt{3}\sqrt{V}}{V_{,\phi} M_{pl}}$
$t = t_0 + \int$


$H^2 = \frac{1}{3M_{pl}^2}\left(\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)\right)$
$\cancel{\ddot{\phi}} + 3H\dot{\phi} + V_{,\phi} = 0$
$\dot{\phi} = -\frac{V_{,\phi} M_{pl}}{\sqrt{3}\sqrt{V(\phi)}}$
$\frac{dt}{d\phi} = -\frac{\sqrt{3}\sqrt{V}}{V_{,\phi} M_{pl}}$
$\boxed{t = t_0 - \int_{\phi_0}^{\phi(t)} \frac{\sqrt{3}\sqrt{V}}{V_{,\phi} M_{pl}}}$

Chaotic inflation

$$V = \frac{\lambda \phi^n}{n!}$$

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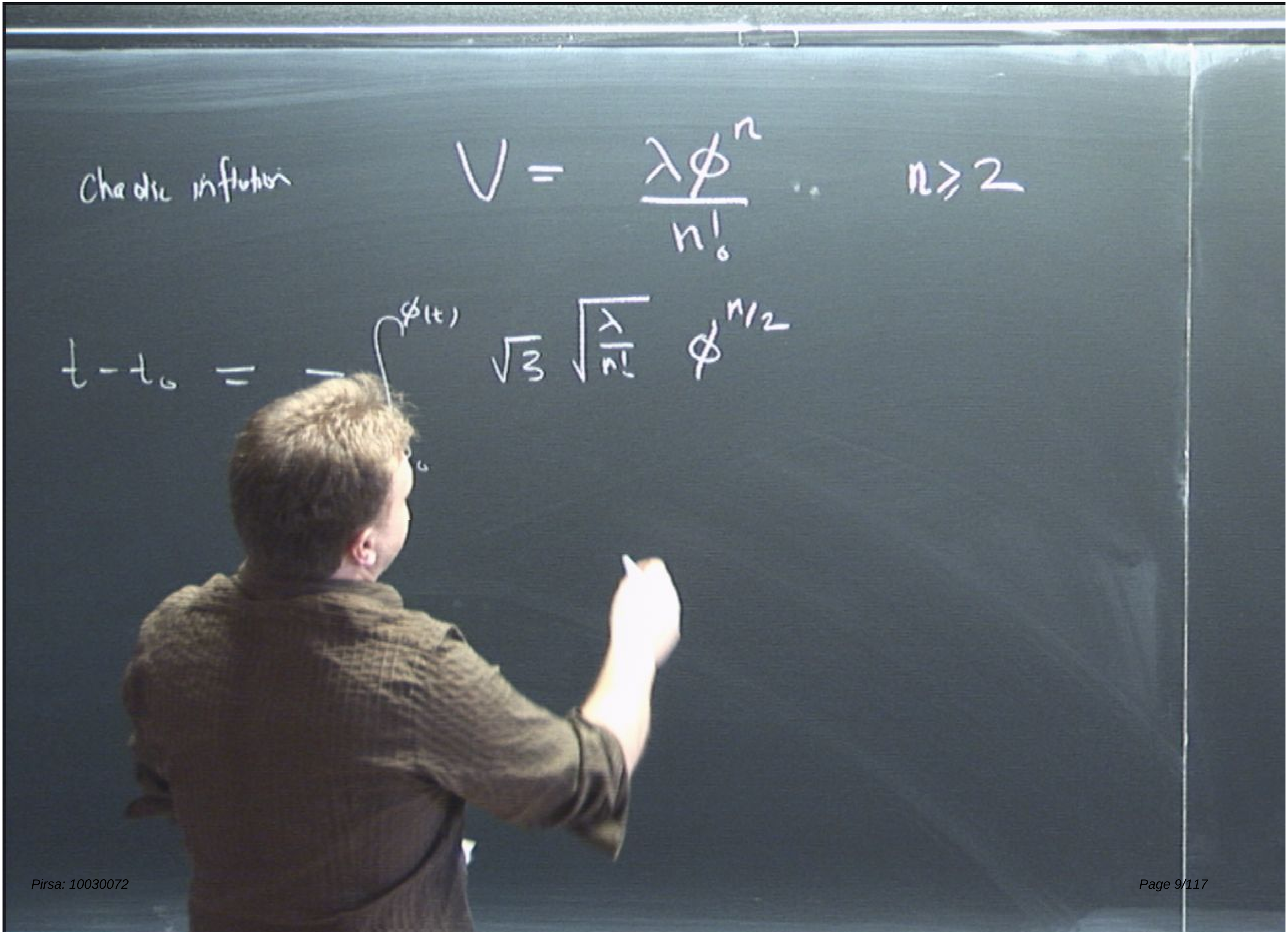
Chaotic inflation
$V = \frac{\lambda \phi^n}{n!}$
$n \geq 2$
$t - t_0 = - \int_{\phi_0}^{\phi(t)} \sqrt{3} \sqrt{\frac{\lambda}{n!}} \phi^{n/2}$

Chaotic inflation

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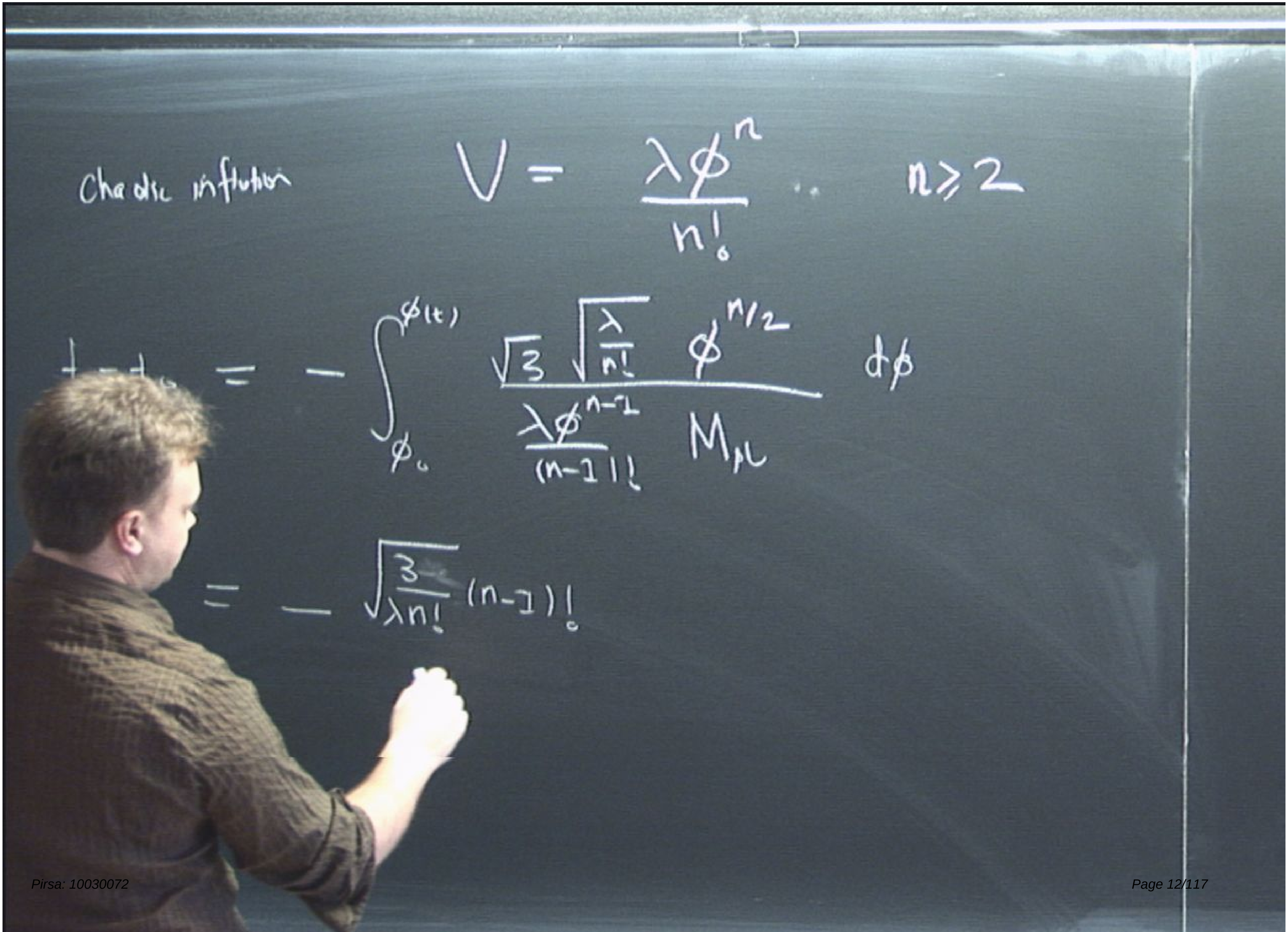
Chaotic inflation
$V = \frac{\lambda \phi^n}{n!}$
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$t - t_0 = - \int_{\phi_0}^{\phi(t)} \frac{\sqrt{3} \sqrt{\frac{\lambda}{n!}} \phi^{n/2}}{\frac{\lambda \phi^{n-1}}{(n-1)!} M_{pl}} d\phi$
$= - \sqrt{\frac{3}{\lambda n!}} (n-1)!$

Chaotic inflation

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$$t - t_0 = -\int_{\phi_0}^{\phi(t)} \frac{\sqrt{3}\sqrt{\frac{\lambda}{n!}}\ \phi^{n/2}}{\frac{\lambda \phi^{n-1}}{(n-1)!}\ M_{pl}}\, d\phi$$

$$= -\sqrt{\frac{3}{\lambda n!}}\ \frac{(n-1)!}{M_{pl}} \int_{\phi_0}^{\phi(t)} \phi^{1-\frac{n}{2}}\, d\phi$$

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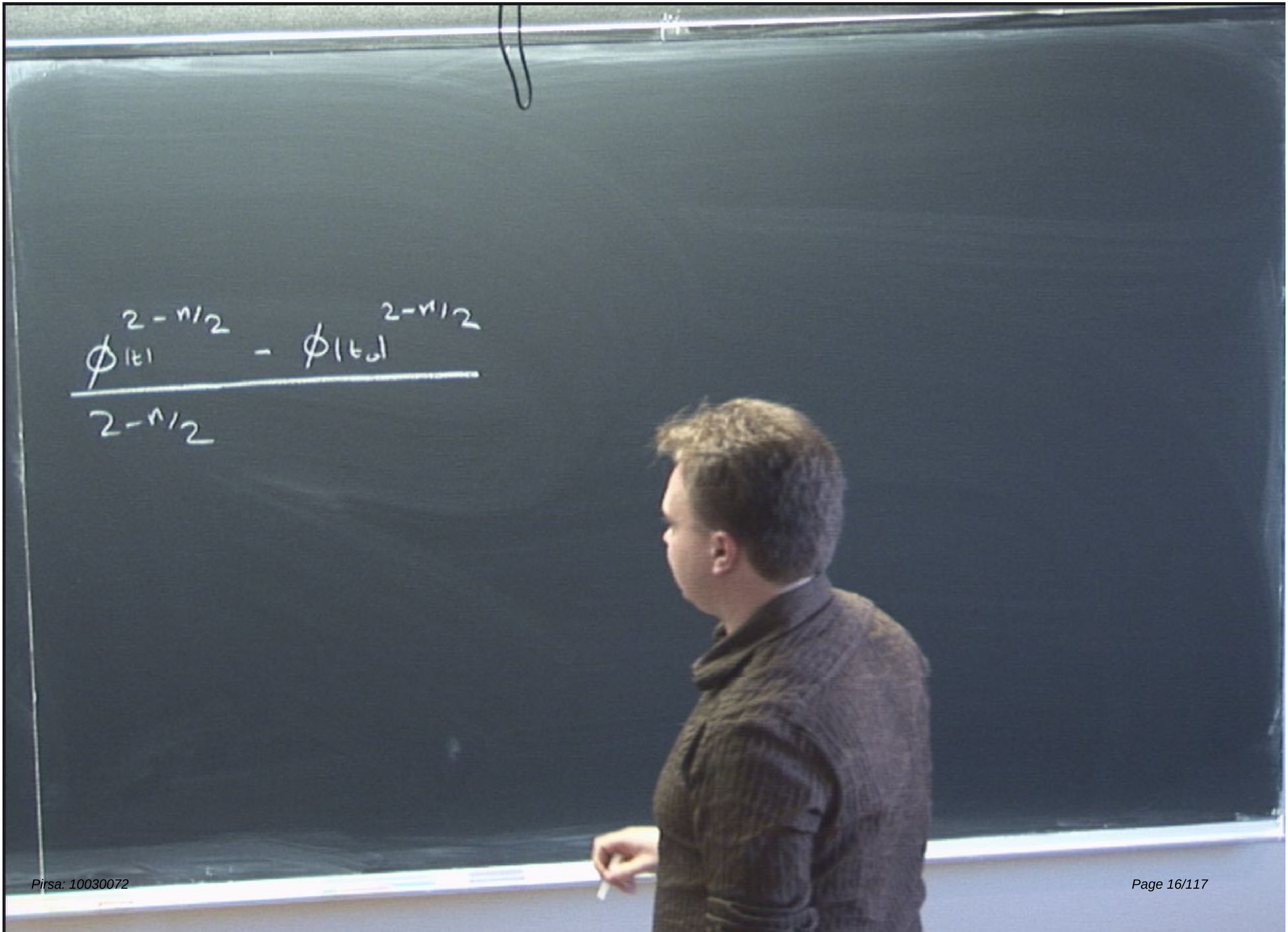
$$- \sqrt{\frac{3}{\lambda n!}}\ \frac{(n-1)!}{M_{pl}} \int_{\phi_0}^{\phi(t)} \phi^{1-\frac{n}{2}}\ d\phi$$

$$\frac{\phi^{2-n/2}}{2-n/2}$$


$\frac{\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2}}{2-n/2}$

$$\frac{\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2}}{2-n/2} = - \frac{M_{pl}}{(n-1)!} \sqrt{\frac{\lambda n!}{3}} \, (t - t_0)$$

$$\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2} = -\frac{(2-\frac{n}{2})M_{pl}}{(n-1)!!}\sqrt{\frac{\lambda n!}{3}}\,(t-t_0)$$

$$\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2} = -\frac{\left(2-\frac{n}{2}\right) M_{pl}}{(n-1)!} \sqrt{\frac{\lambda n!}{3}} \, (t-t_0)$$

$$M_{pl}^2 \left(\frac{V_{,\phi}}{V}\right)^2 = M_{pl}^2 \Big($$

$$\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2} = -\frac{(2-\frac{n}{2})M_{pl}}{(n-1)!}\sqrt{\frac{\lambda n!}{3}}\,(t-t_0)$$

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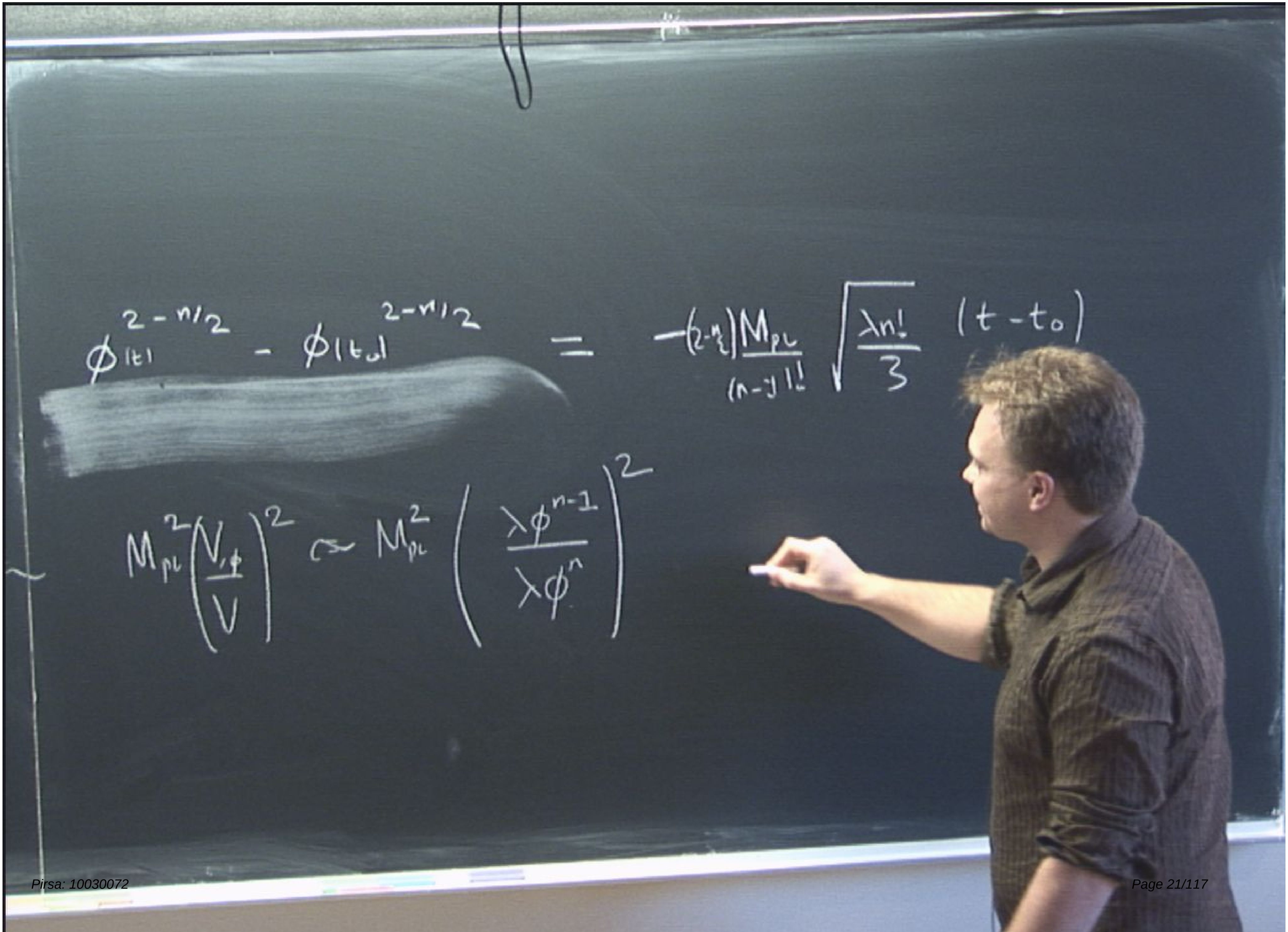
$\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2} = -\frac{(2-\frac{n}{2})M_{pl}}{(n-1)!}\sqrt{\frac{\lambda n!}{3}}\,(t-t_0)$
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$$\phi \gg M_{pl}$$

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$$\sim M_{pl}^2\left(\frac{V_{,\phi\phi}}{V}\right)$$

$$\lambda\phi^4$$

$$\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2} = -\frac{(2-\frac{n}{2})M_{pl}}{(n-1)!}\sqrt{\frac{\lambda n!}{3}}\,(t-t_0)$$

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$$R \quad - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda\phi^4$$

$$\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2} = -\frac{(2-\frac{n}{2}) M_{pl}}{(n-1)!} \sqrt{\frac{\lambda n!}{3}} \, (t - t_0)$$

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$$\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2} = -\frac{(2-\frac{n}{2}) M_{pl}}{(n-?)\,!!} \sqrt{\frac{\lambda n!}{3}} \, (t - t_0)$$

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$$M_{pl}^2 \left|\frac{V_{,\phi\phi}}{V}\right| \sim \frac{M_{pl}^2}{\phi^2} \qquad F_{\mu\nu} \sim m_{pl}^2$$

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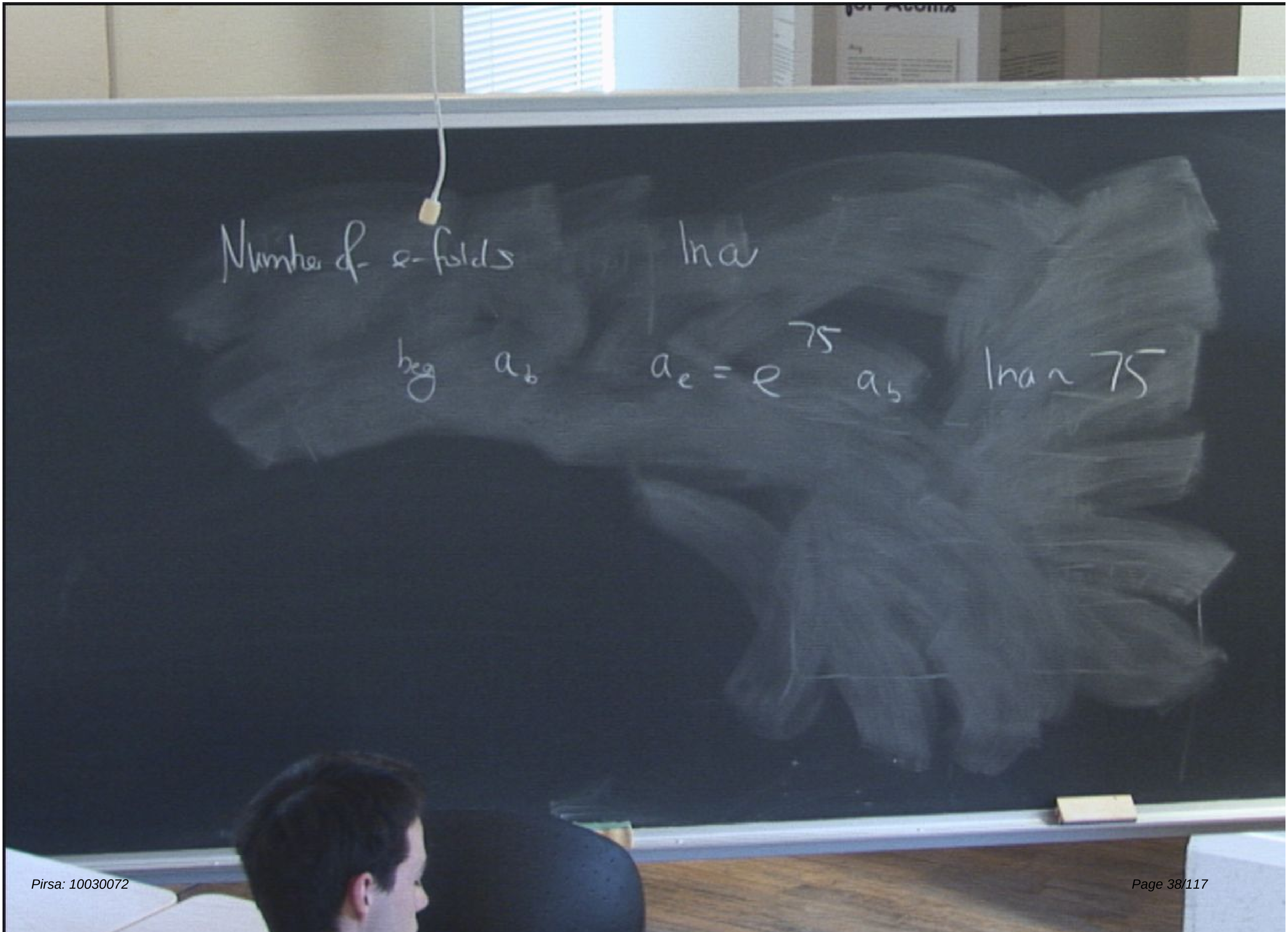
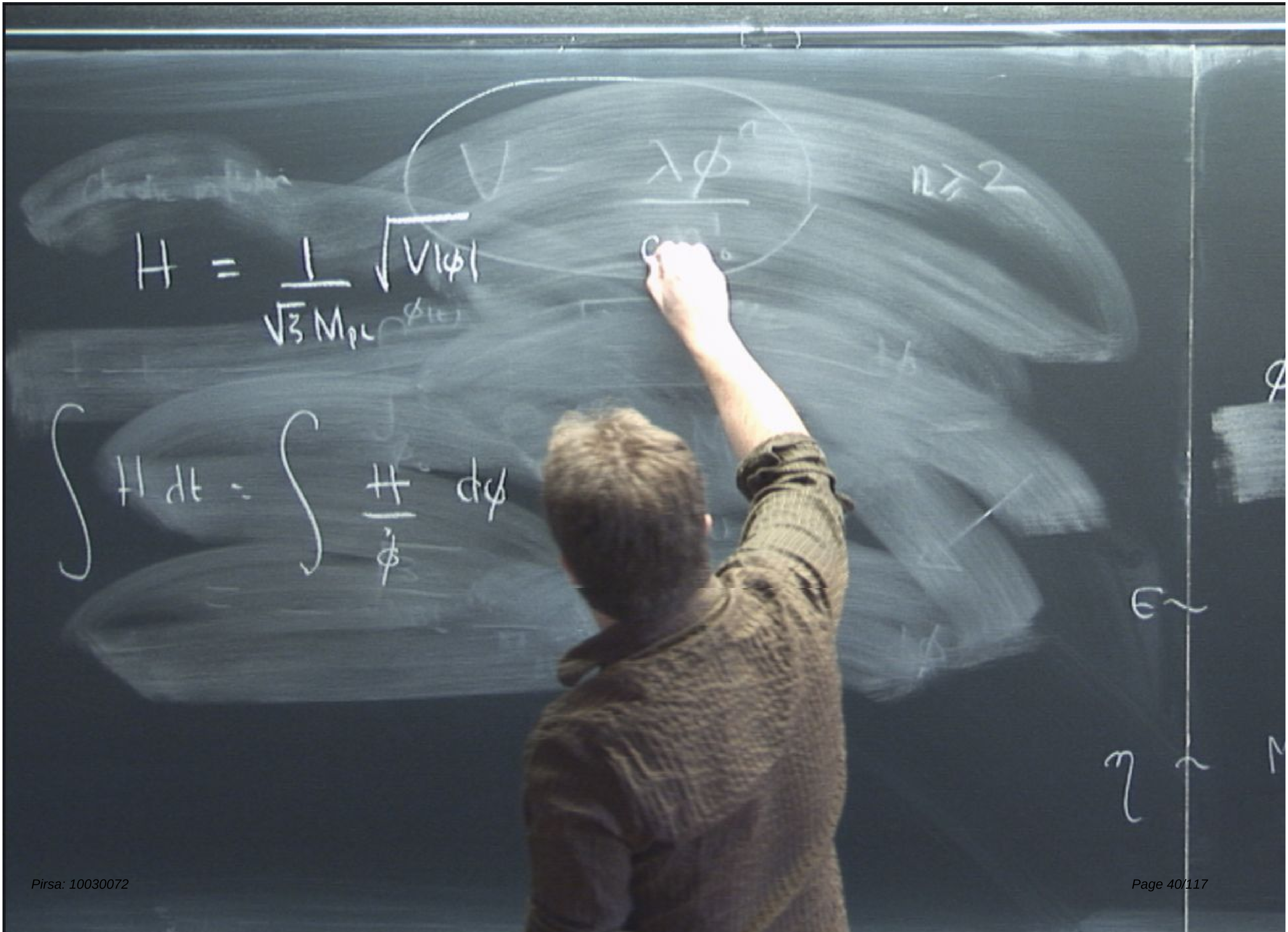
Number of e-folds
ln a


Number of e-folds
ln a
beg $a_b$
$a_e = e^{75} a_b$
$\ln a \sim 75$

Number of e-folds $\quad \ln a$

beg $\quad a_b \qquad a_e = e^{75}\, a_b \qquad \Delta \ln a \sim 75 \quad$ (75 e-folds)

$$N = \int_{t_b}^{t_e} (\dot{\ln a})\, dt = \int_{t_b}^{t_e} \frac{\dot a}{a}\, dt = \int_{t_b}^{t_e} H\, dt$$



$H = \frac{1}{\sqrt{3} M_{pl}} \sqrt{V(\phi)}$
$V = \lambda\phi^n$
$n \geq 2$
$\int H\, dt = \int \frac{H}{\dot{\phi}}\, d\phi$
$\epsilon \sim$
$\eta \sim$

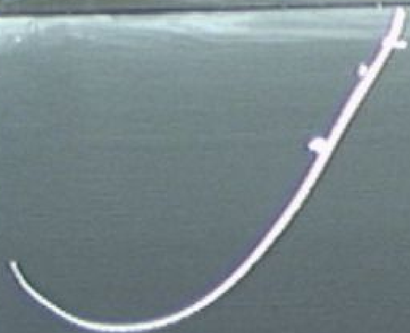
$$V = \lambda \phi^n \qquad n \geq 2$$

$$H = \frac{1}{\sqrt{3} M_{pl}} \sqrt{V(\phi)} \qquad \dot{\phi} = -\frac{1}{3H} V_{,\phi}$$

$$\int H \, dt = \int \frac{H}{\dot{\phi}} \, d\phi$$

$\epsilon \sim$

$\eta \sim$ M



$$M_{pl}^2 R \quad - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4!}\lambda\phi^4 + \frac{\phi^5}{M_{pl}} \quad \frac{\phi^6}{M_{pl}^2} \; \vdots \; \cdot \quad g \sim \frac{1}{M_{pl}}$$

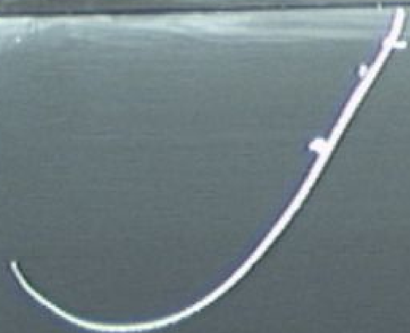
$$\phi(t)^{2-n/2} - \phi(t_0)^{2-n/2} = -\frac{(2-\frac{n}{2})M_{pl}}{(n-1)!} \sqrt{\frac{\lambda n!}{3}} \; (t - t_0)$$

$$\frac{V_{,\phi}}{V} \sim \frac{1}{\phi}$$

$$\sim \; M_{pl}^2 \left(\frac{V_{,\phi}}{V}\right)^2 \simeq M_{pl}^2 \left(\frac{\cancel{X}\phi^{n-1}}{\cancel{X}\phi^n}\right)^2 \sim \frac{M_{pl}^2}{\phi^2} \ll 1 \qquad \frac{V_{,\phi\phi}}{V} \sim \frac{1}{\phi^2}$$

$$\boxed{\phi \gg M_{pl}}$$

$$\sim \; M_{pl}^2 \left|\frac{V_{,\phi\phi}}{V}\right| \sim \frac{M_{pl}^2}{\phi^2} \qquad F_{\mu\nu} \sim M_{pl}^2$$



$$M_{pl}^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4!}\lambda\phi^4 + \frac{\phi^5}{M_{pl}} \; \frac{\phi^6}{M_{pl}^2} \; \dots \qquad g \sim \frac{1}{M_{pl}}$$

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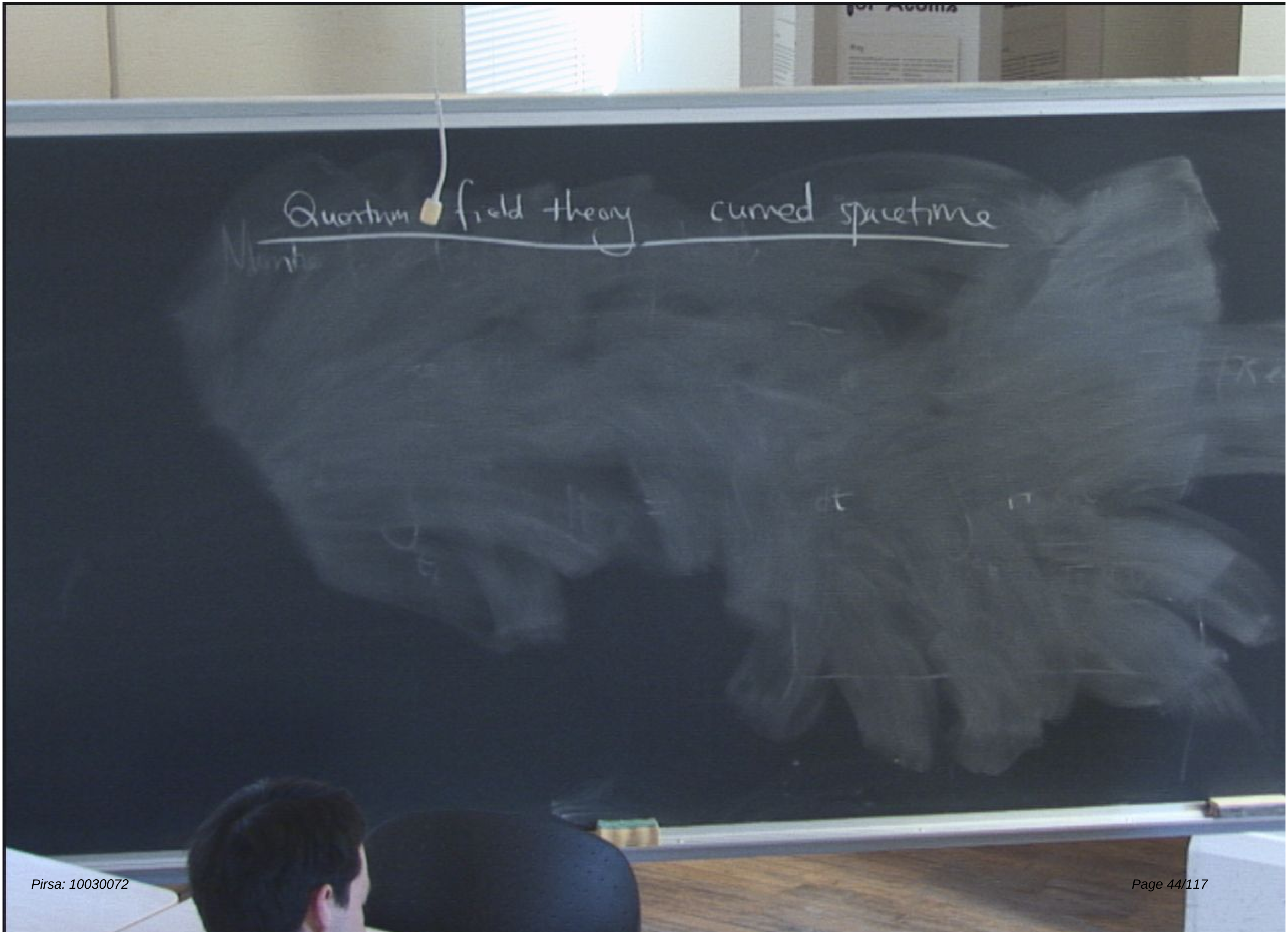
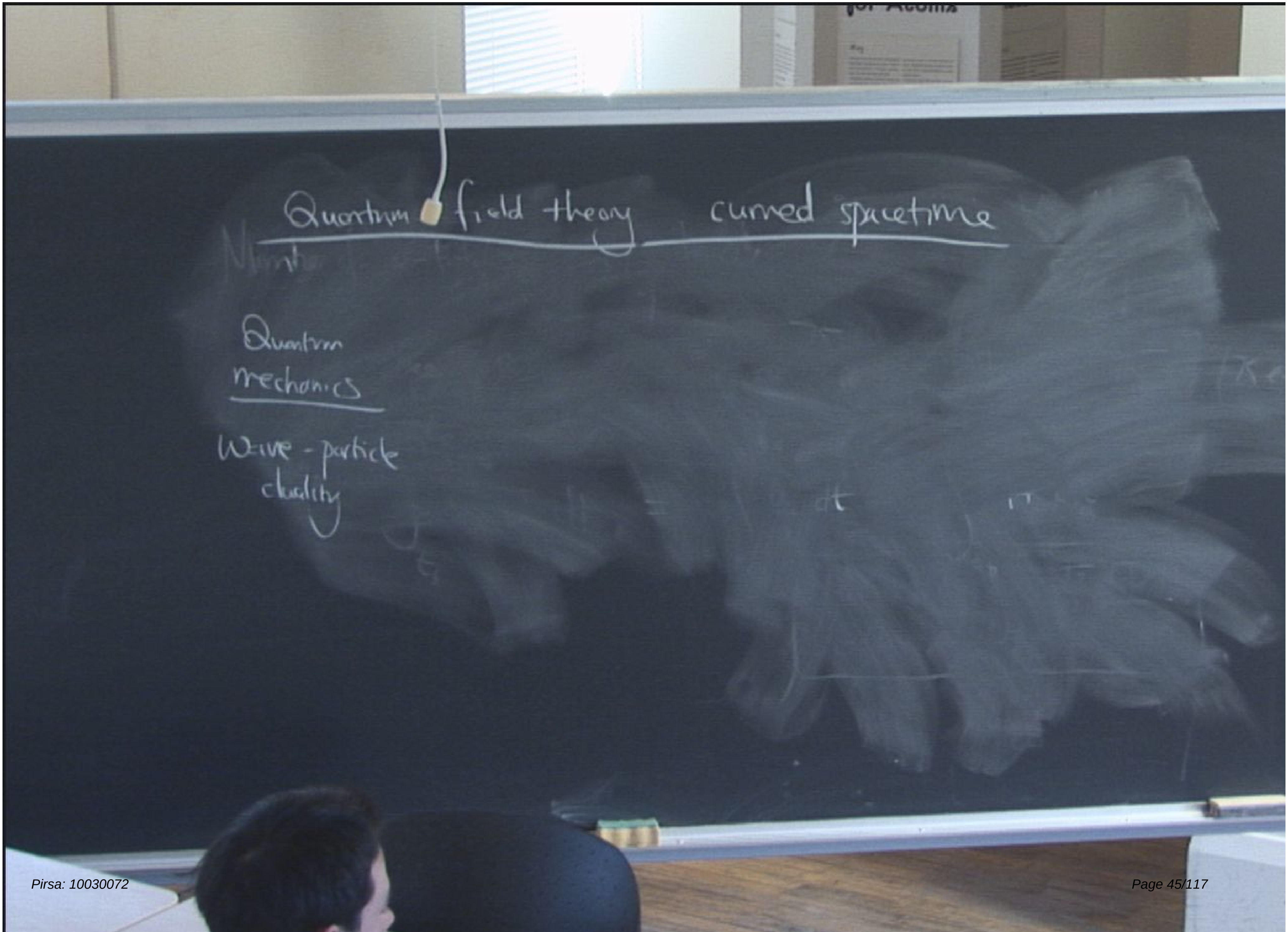
$$\frac{V_{,\phi}}{V} \sim \frac{1}{\phi}$$

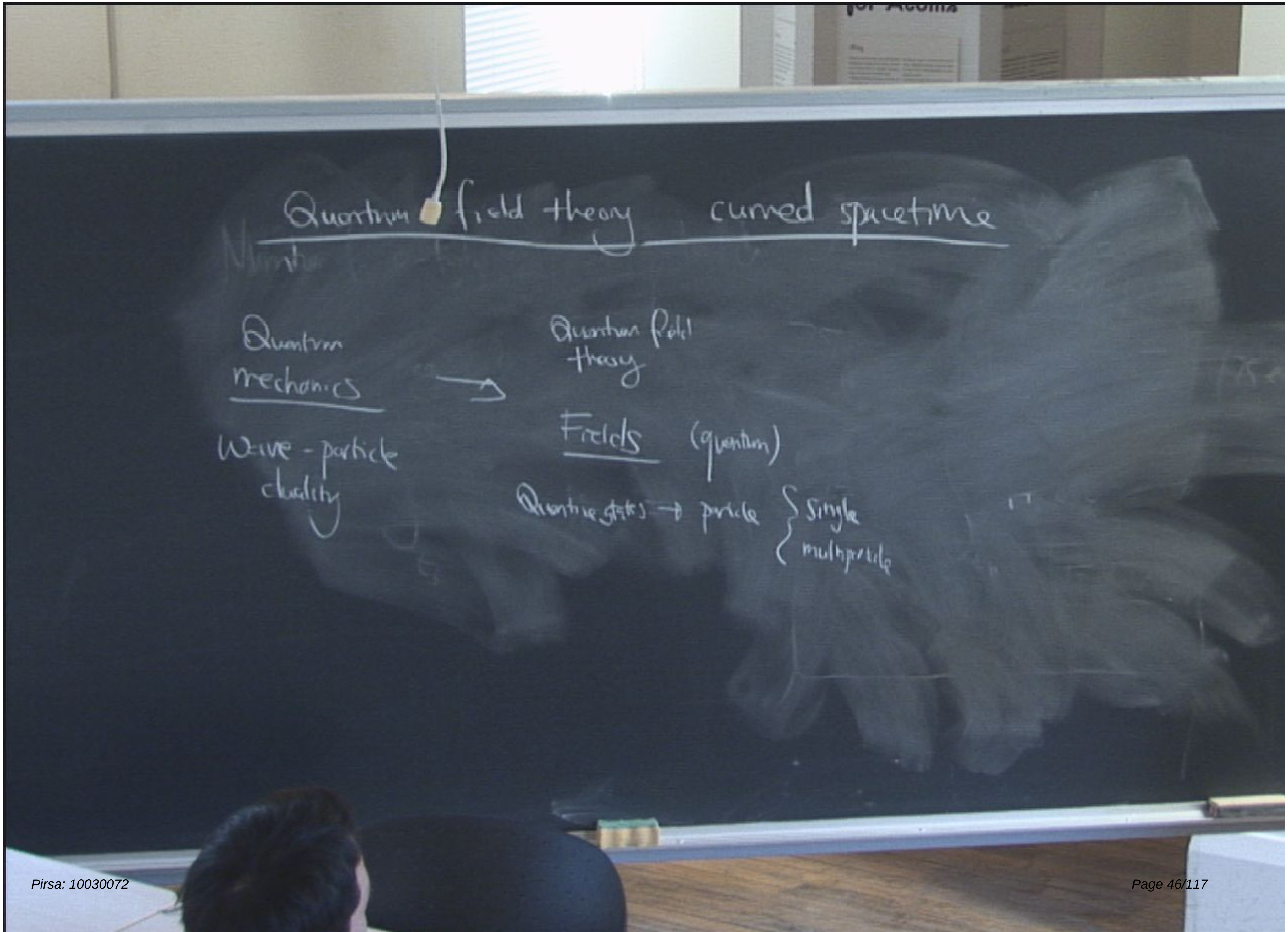
$$M_{pl}^2\left(\frac{V_{,\phi}}{V}\right)^2 \sim M_{pl}^2\left(\frac{\cancel{\lambda}\phi^{n-1}}{\cancel{\lambda}\phi^n}\right)^2 \sim \frac{M_{pl}^2}{\phi^2} \ll 1 \qquad \frac{V_{,\phi\phi}}{V} \sim \frac{1}{\phi^2}$$

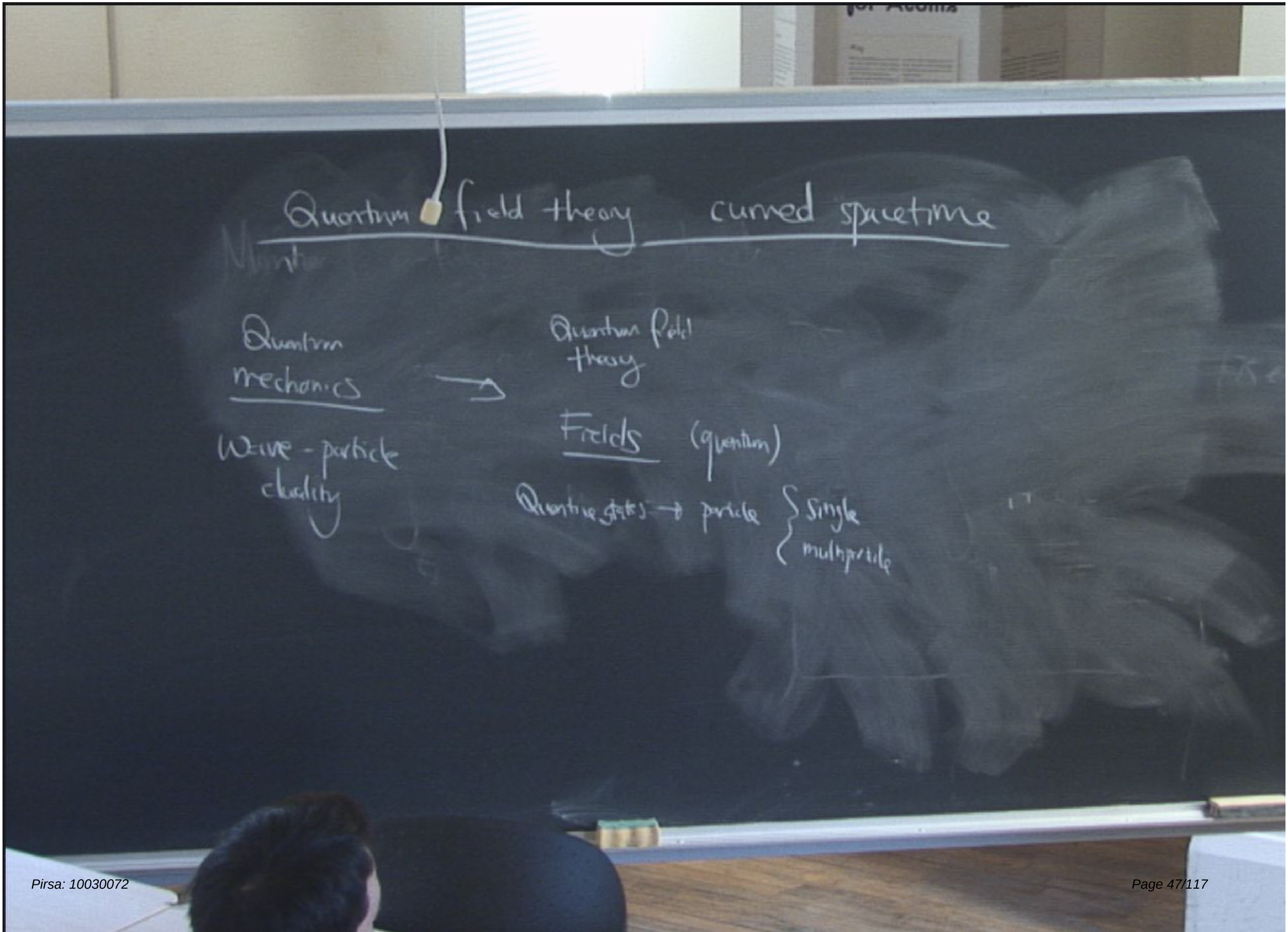
$$\boxed{\phi \gg M_{pl}}$$

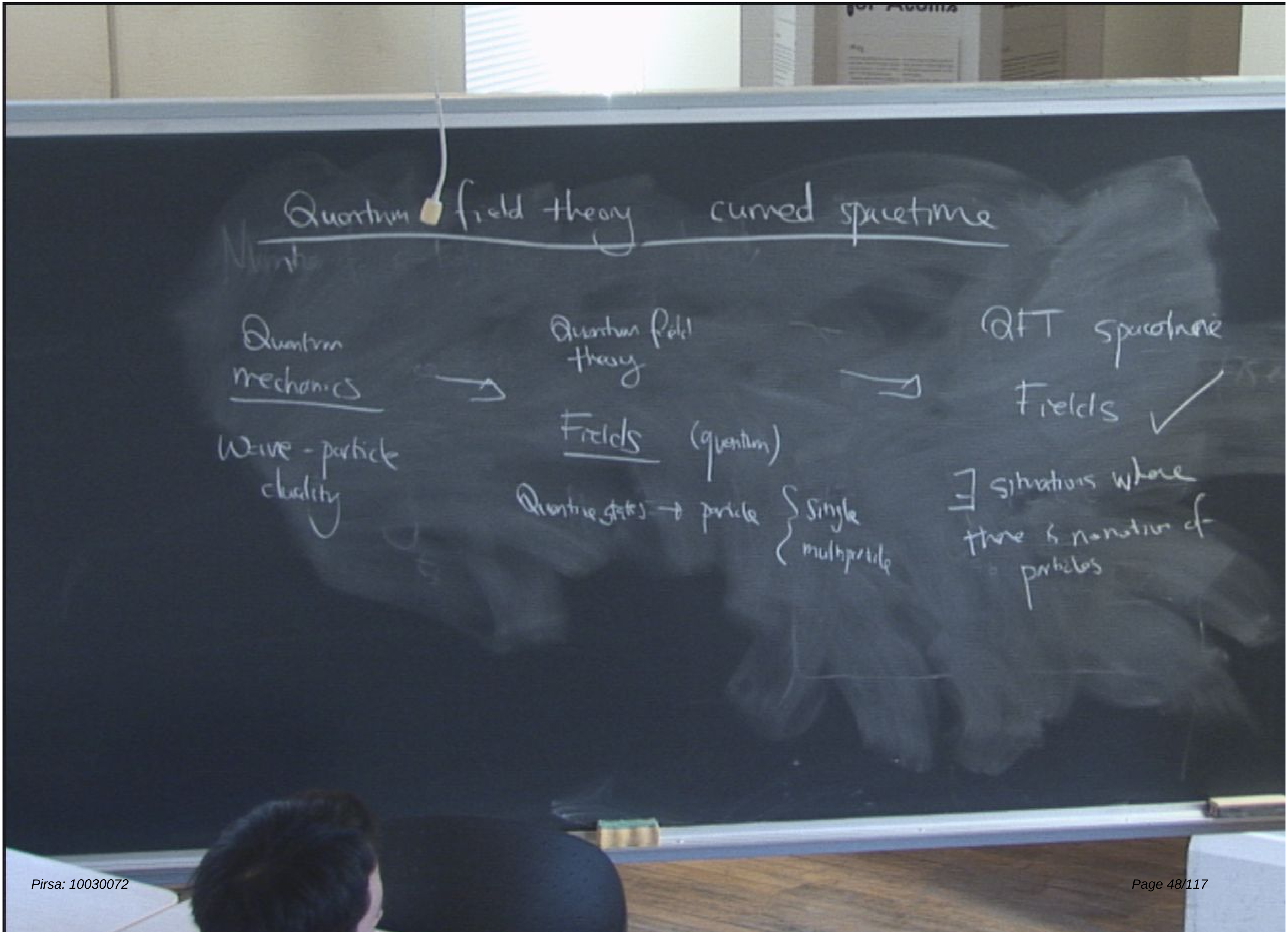
$$M_{pl}^2\left|\frac{V_{,\phi\phi}}{V}\right| \sim \frac{M_{pl}^2}{\phi^2} \qquad F_{\mu\nu} \sim M_{pl}^2$$


Quantum field theory curved spacetime


Quantum field theory curved spacetime
Quantum mechanics
Wave - particle duality


Quantum field theory curved spacetime
Quantum mechanics
Quantum field theory
Wave-particle duality
Fields (quantum)
particle
single
multiparticle


Quantum field theory curved spacetime
Quantum mechanics
Quantum field theory
Wave - particle duality
Fields (quantum)
Quantum states → particle
single
multiparticle

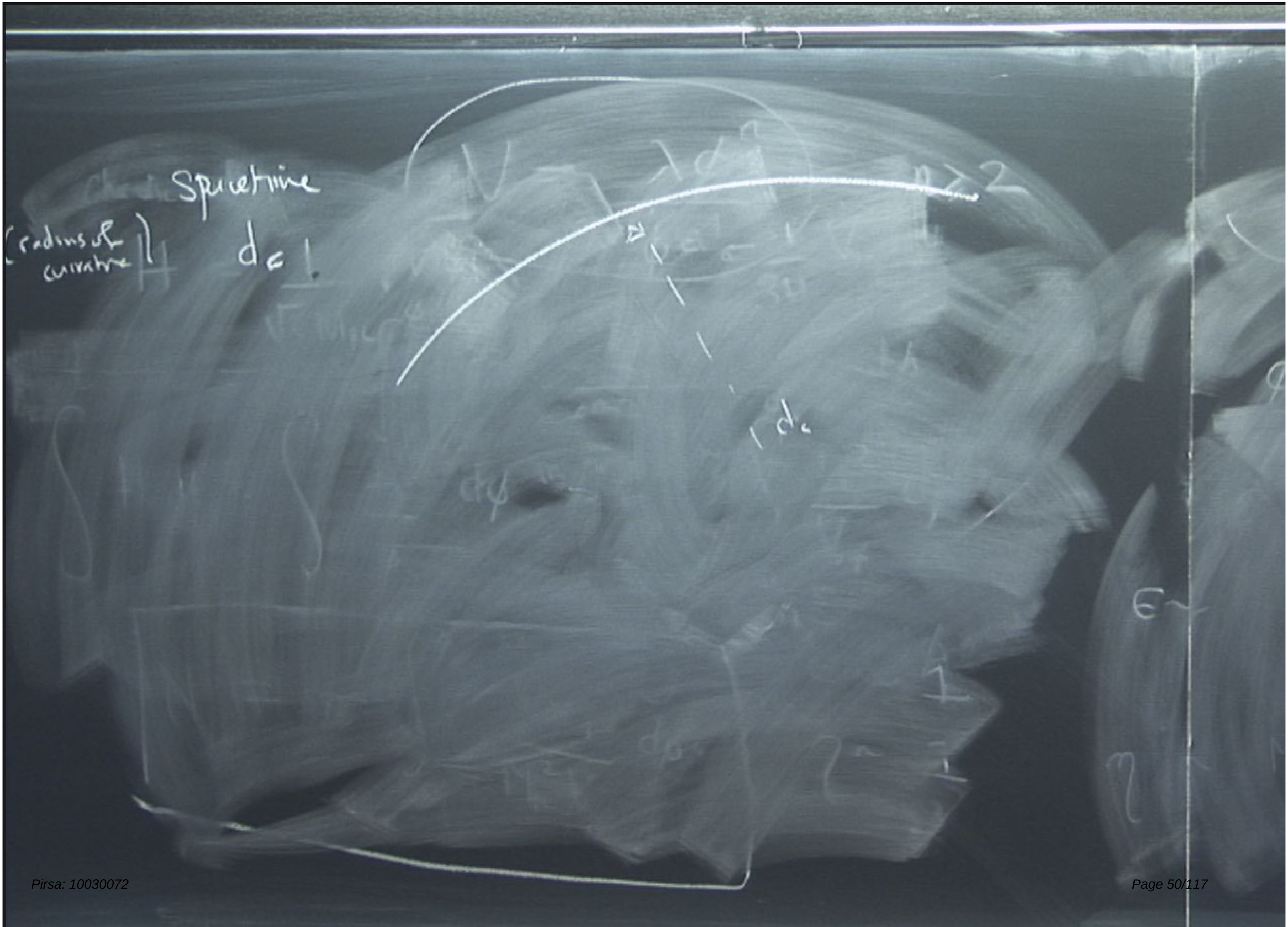

Quantum field theory curved spacetime
Quantum mechanics
Wave - particle duality
Quantum field theory
Fields (quantum)
Quantum states → particle
single
multiparticle
QFT spacetime
Fields
∃ situations where there is no notion of particles

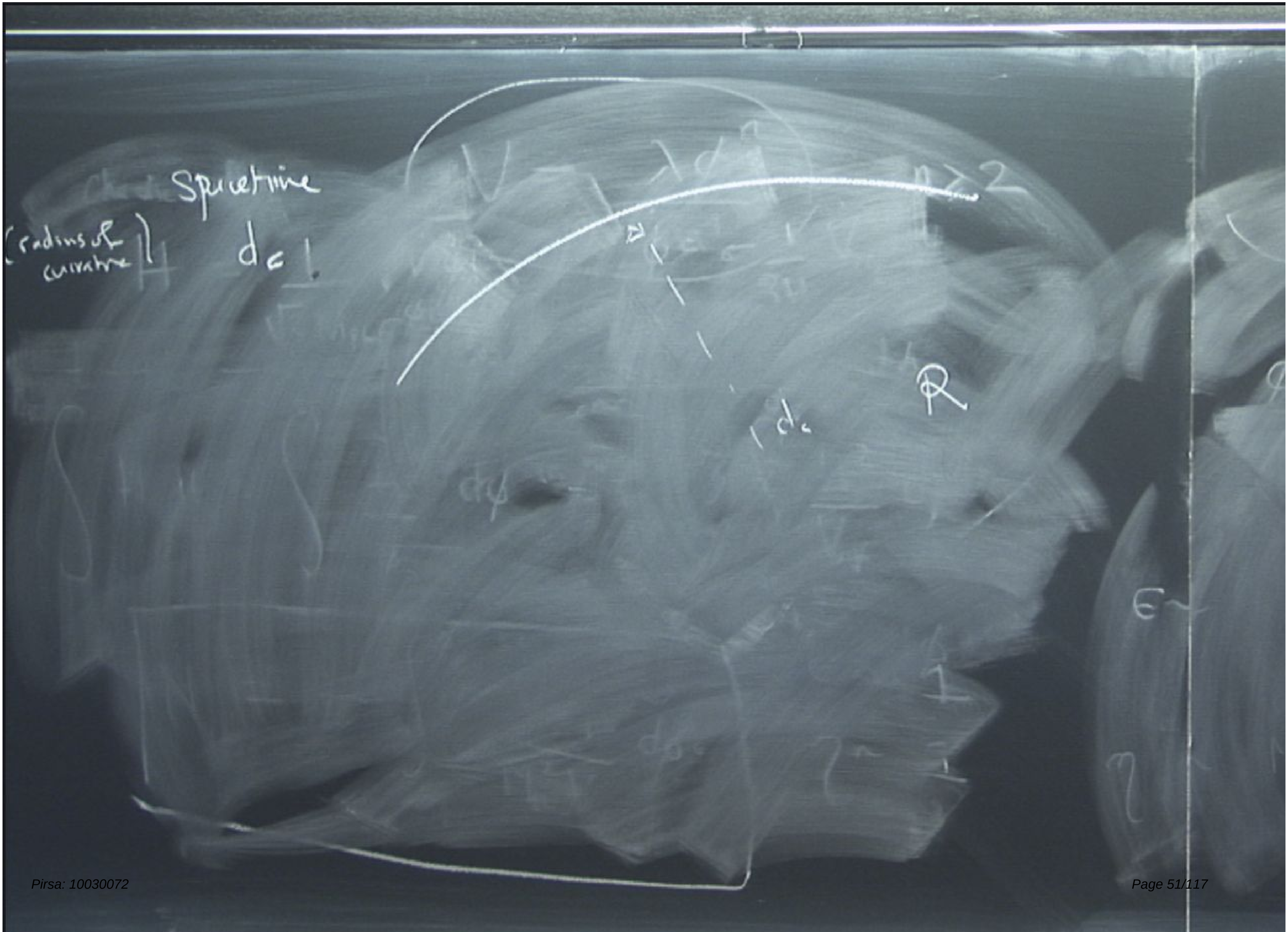
Spacetime

(radius of curvature)

$d = 1$

$n \geq 2$


Spacetime
(radius of curvature)
n ≥ 2


Spacetime
(radius of curvature)
$d = 1$
$n \geq 2$
R

Spacetime

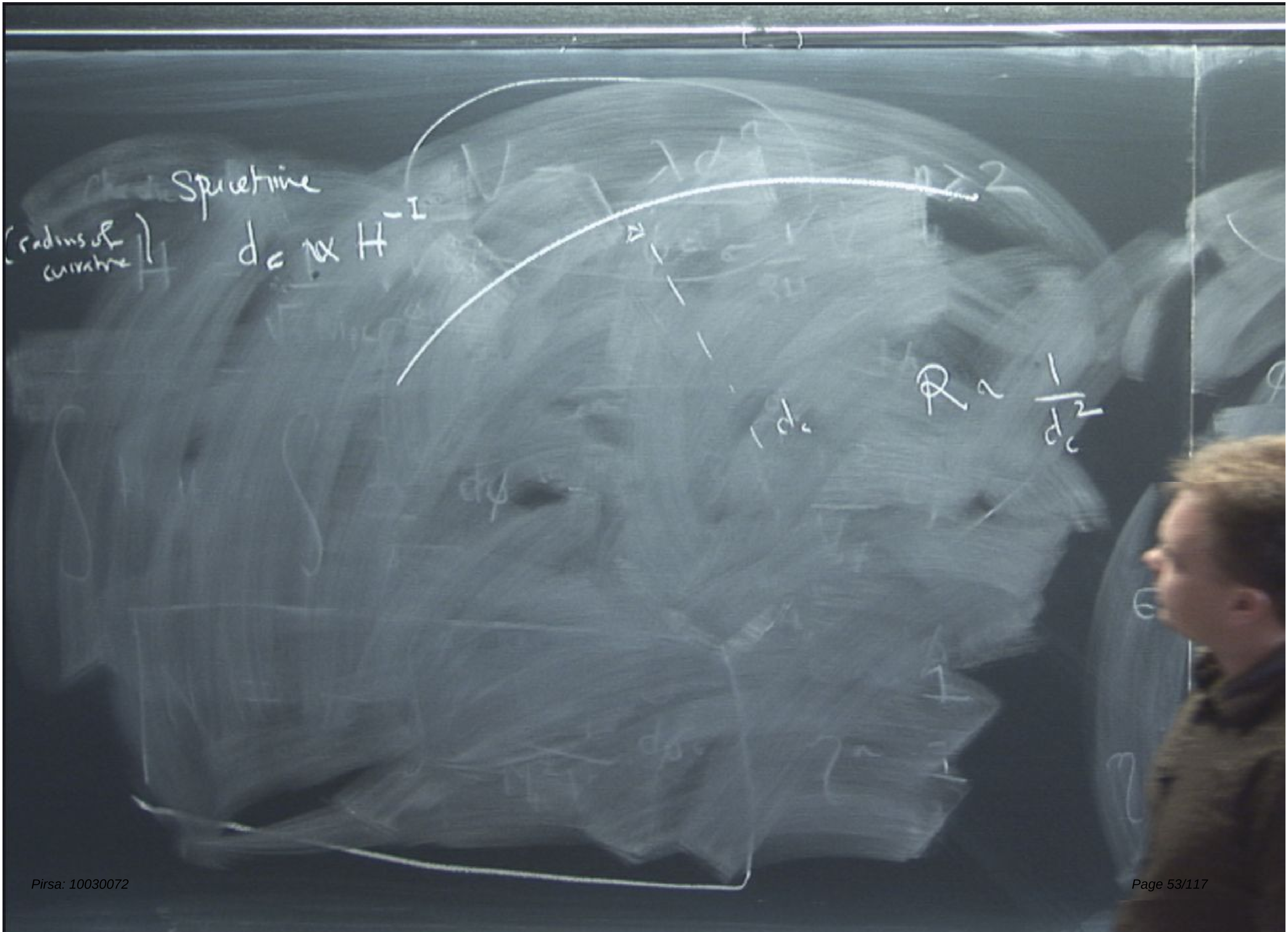
(radius of curvature) $d_c \propto H^{-1}$

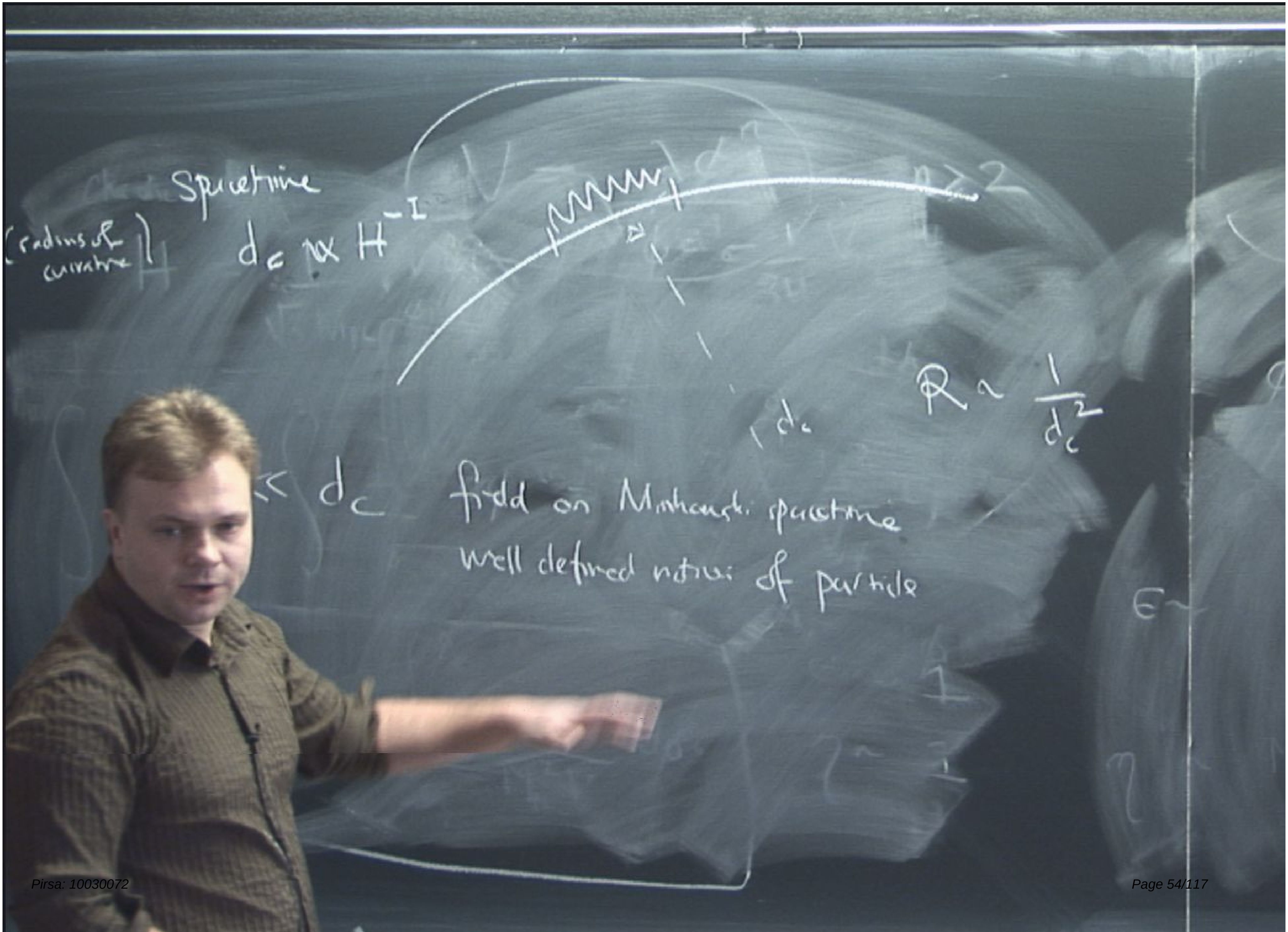
$n \geq 2$

$d_c$

$R \sim \frac{1}{d_c^2}$

$E \sim$


Spacetime
(radius of curvature)
d_c ∝ H^-1
n ≥ 2
d_c
R ~ 1/d_c^2


Spacetime
(radius of curvature)
d_c ∝ H^-1
n ≥ 2
d_c
R ~ 1/d_c^2
≪ d_c
field on Minkowski spacetime
well defined notion of particle

spacetime

(radius of curvature) $d_c \propto H^{-1}$

$n \geq 2$

$d_c$

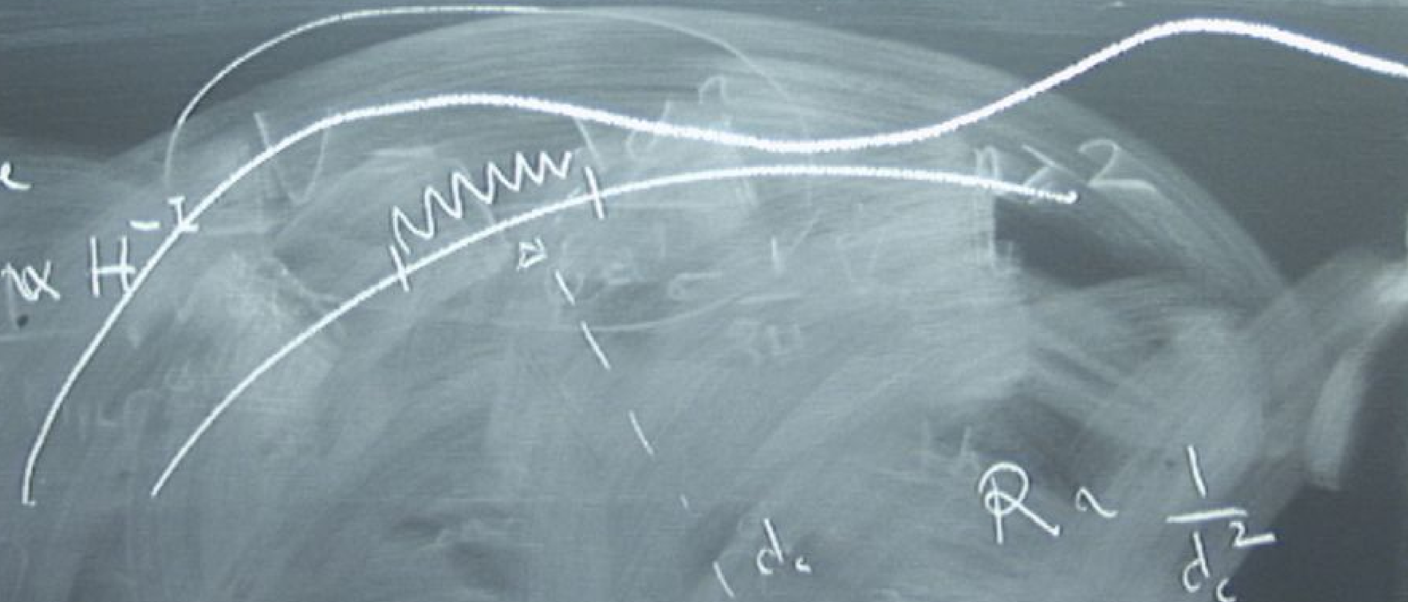
$R \sim \frac{1}{d_c^2}$

$\lambda \ll d_c$ field on Minkowski spacetime
well defined notion of particle

$\lambda \gg d_c$

Spacetime

(radius of curvature) $d_c \propto H^{-1}$



$R \sim \frac{1}{d_c^2}$

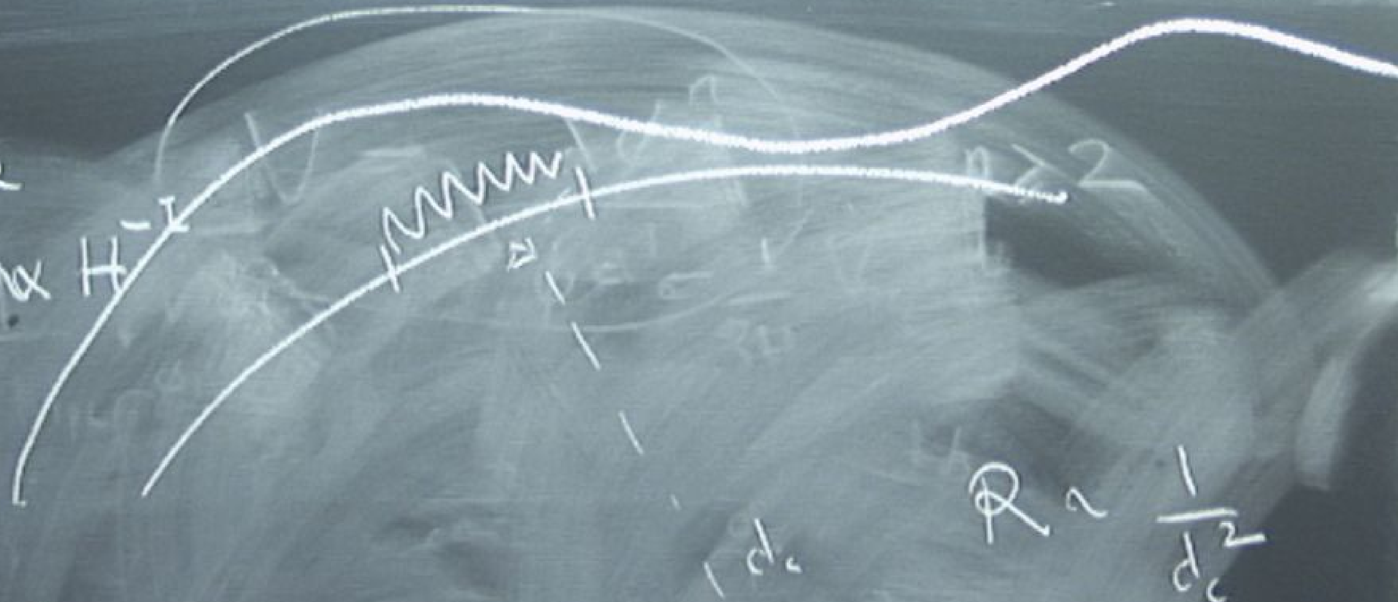
$\lambda \ll d_c$ field on Minkowski spacetime

well defined notion of particle

$\lambda \gg d_c$ no well defined notion of particles

Spacetime

(radius of curvature) $d_c \propto H^{-1}$




$R \sim \frac{1}{d_c^2}$

$\lambda \ll d_c$ field on Minkowski spacetime
well defined notion of particle

$\lambda \gg d_c$ no well defined notion of particles

During inflation $d_c \propto H^{-1} \simeq$ constant

$\lambda_{phys} = a(t)\,\lambda_{com}$

During inflation $d_c \propto H^{-1} \approx$ constant

$\lambda_{phys} = a(t)\,\lambda_{com}$ $\quad \lambda \uparrow$ until $\lambda_{phy} \gg H^{-1}$ (notion of particles breaks down)

During inflation $d_c \propto H^{-1} \simeq$ constant

$\lambda_{phys} = a(t)\,\lambda_{com}$ $\qquad \lambda \uparrow$ until $\lambda_{phy} \gg H^{-1}$ (notion of particles breaks down)

Inflation ends

During inflation $d_c \propto H^{-1} \simeq$ constant

$\lambda_{phys} = a(t)\,\lambda_{com}$ $\quad \lambda \uparrow$ until $\lambda_{phy} \gg H^{-1}$ (notion of particles breaks down)

Inflation ends

$$\frac{d_c}{a} \sim \frac{H^{-1}}{a} \uparrow$$

During inflation $d_c \propto H^{-1} \simeq$ constant

$\lambda_{phys} = a(t)\, \lambda_{com}$ $\quad \lambda \uparrow$ until $\lambda_{phys} \gg H^{-1}$ (notion of particles breaks down)

Inflation ends $\quad d_c \gg \lambda_{phys}$ (well defined notion of particles)

$$\frac{d_c}{a} \sim \frac{H^{-1}}{a} \uparrow$$

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Inflation ends $\quad d_c \gg \lambda_{phys}$ (well defined notion of particles)

$\frac{d_c}{a} \sim \frac{H^{-1}}{a} \uparrow$

10 particles $\longrightarrow$ ? particles $\longrightarrow$ N particles

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10 particles $\longrightarrow$ ? particles $\longrightarrow$ N particles

particle creation

During inflation $d_c \propto H^{-1} \simeq$ constant

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10 particles $\longrightarrow$ ? particles $\longrightarrow$ N particles

FT on time-dependent $\Longrightarrow$ particle creation

During inflation $d_c \propto H^{-1} \approx$ constant

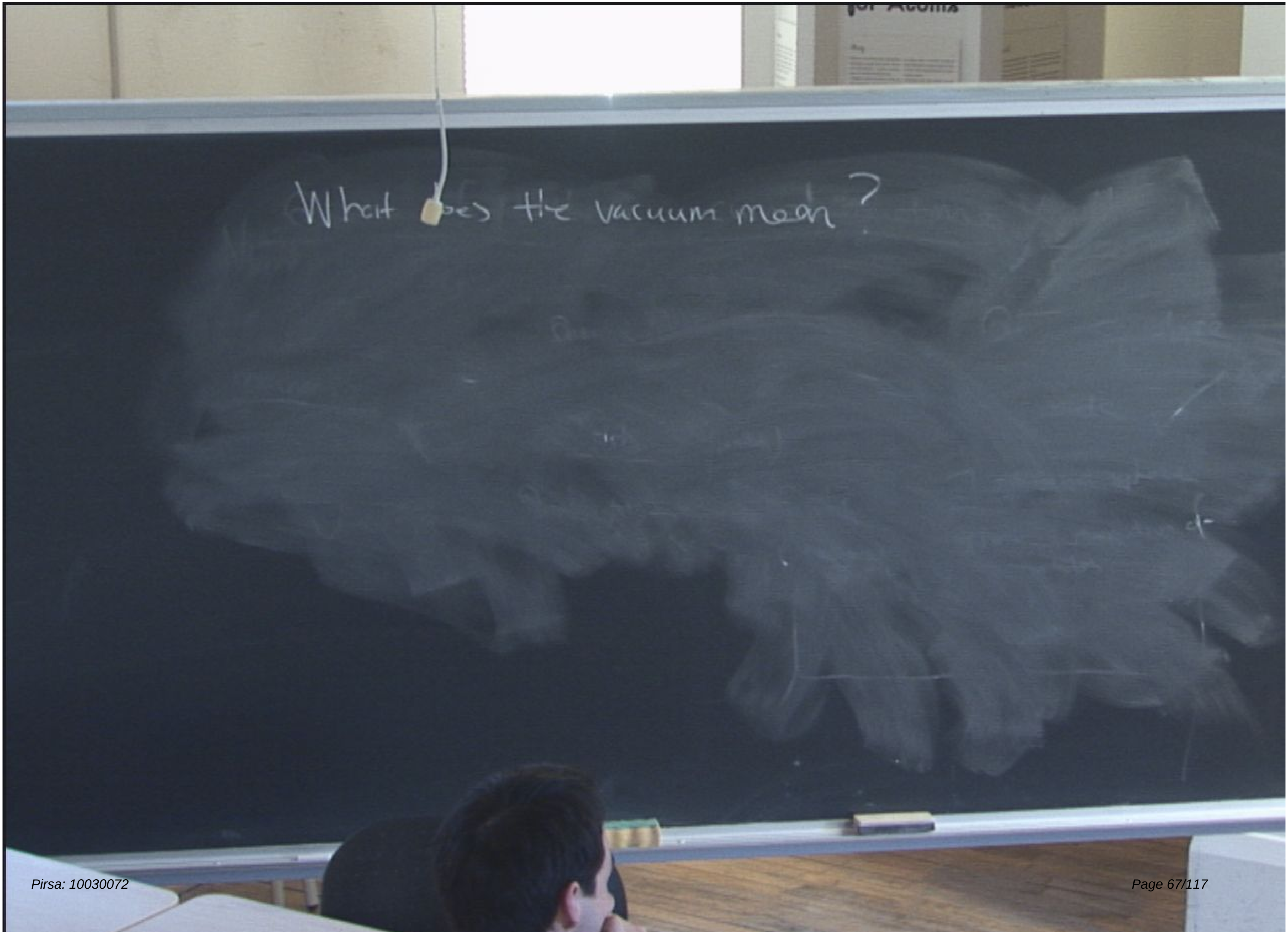
$\lambda_{phys} = a(t)\,\lambda_{com}$ $\quad \lambda \uparrow$ until $\lambda_{phys} \gg H^{-1}$ (notion of particles breaks down)

Inflation ends $\quad d_c \gg \lambda_{phys}$ (well defined notion of particles)

$\frac{d_c}{a} \sim \frac{H^{-1}}{a} \uparrow$

10 particles $\longrightarrow$ ? particles $\longrightarrow$ N particles

FT on time-dependent $\Longrightarrow$ particle creation


What does the vacuum mean?

During inflation $d_c \propto H^{-1} \simeq$ constant

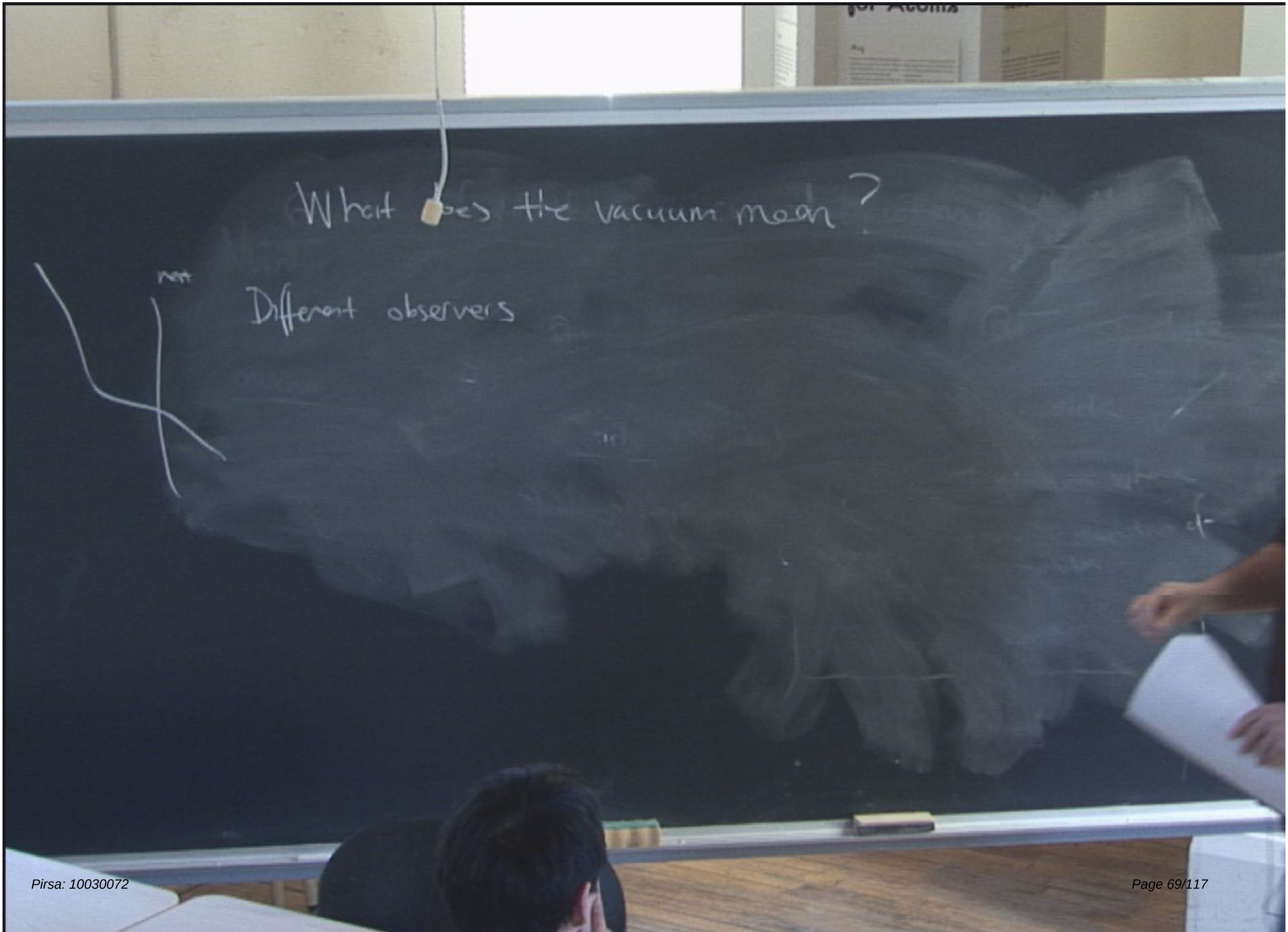
$\lambda_{phys} = a(t)\,\lambda_{com}$ $\quad \lambda \uparrow$ until $\lambda_{phy} \gg H^{-1}$ (notion of particles breaks down)

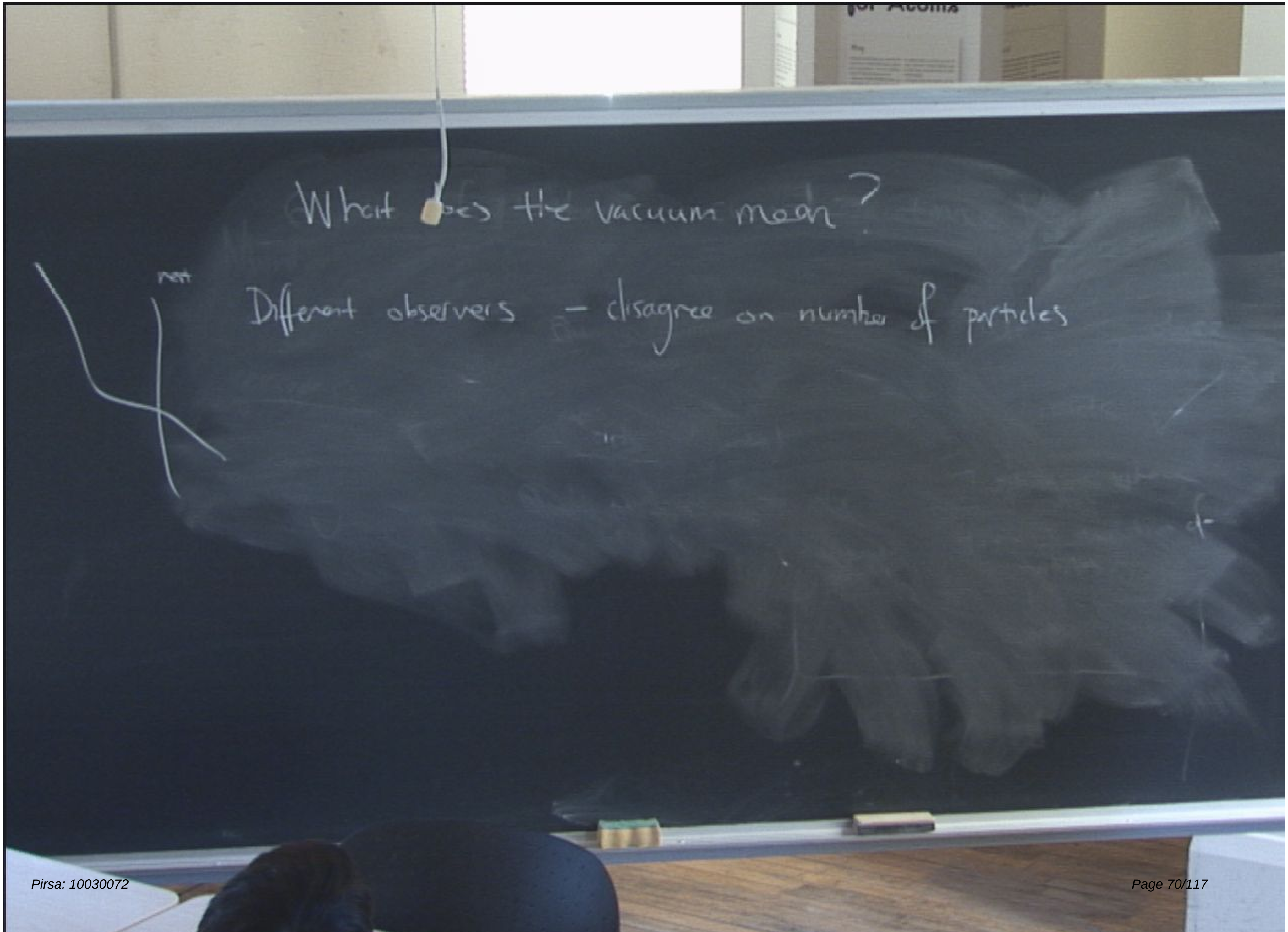
Inflation ends $\quad d_c \gg \lambda_{phys}$ (well defined notion of particles)

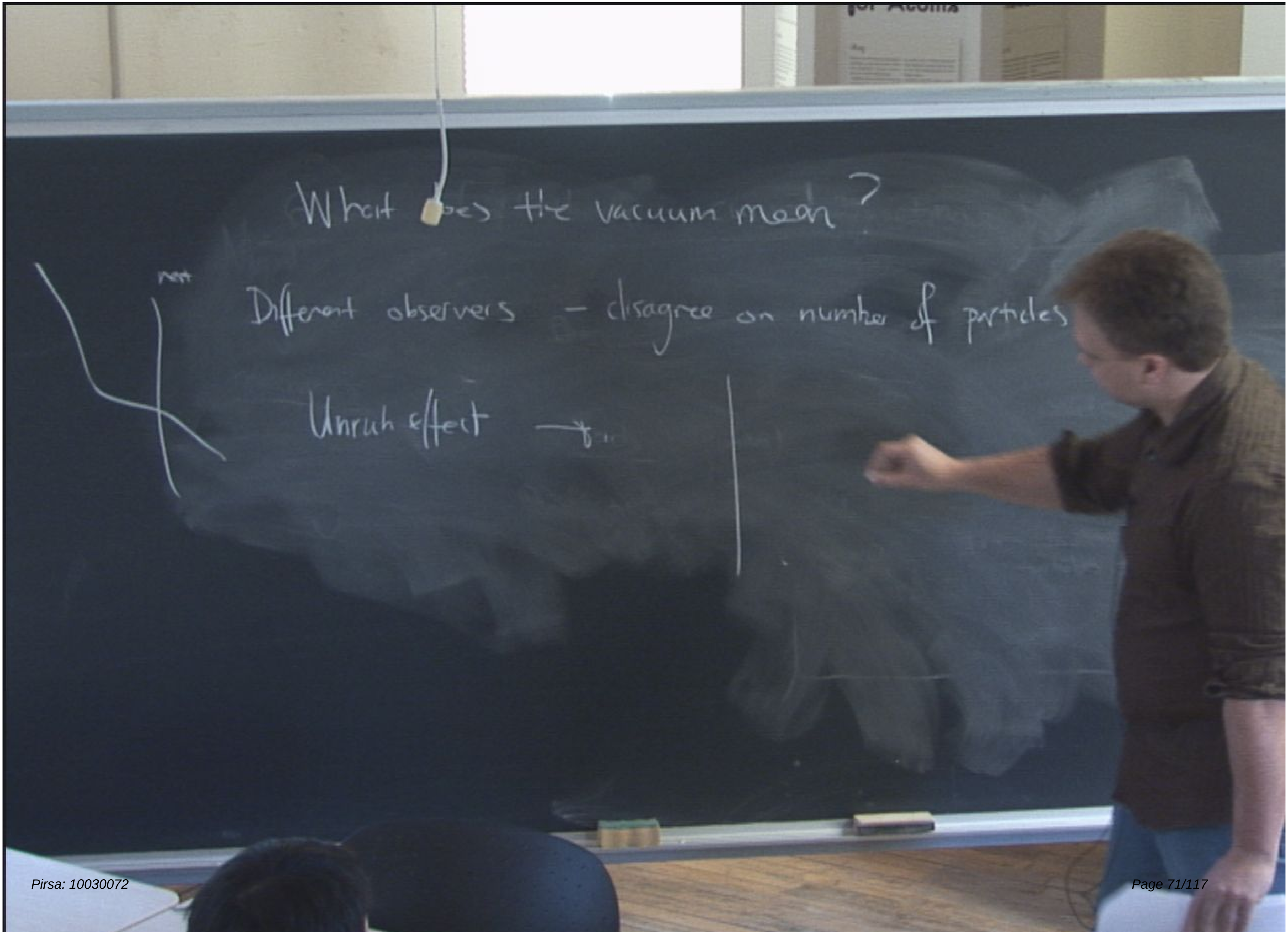
$$\frac{d_c}{a} \sim \frac{H^{-1}}{a} \uparrow$$

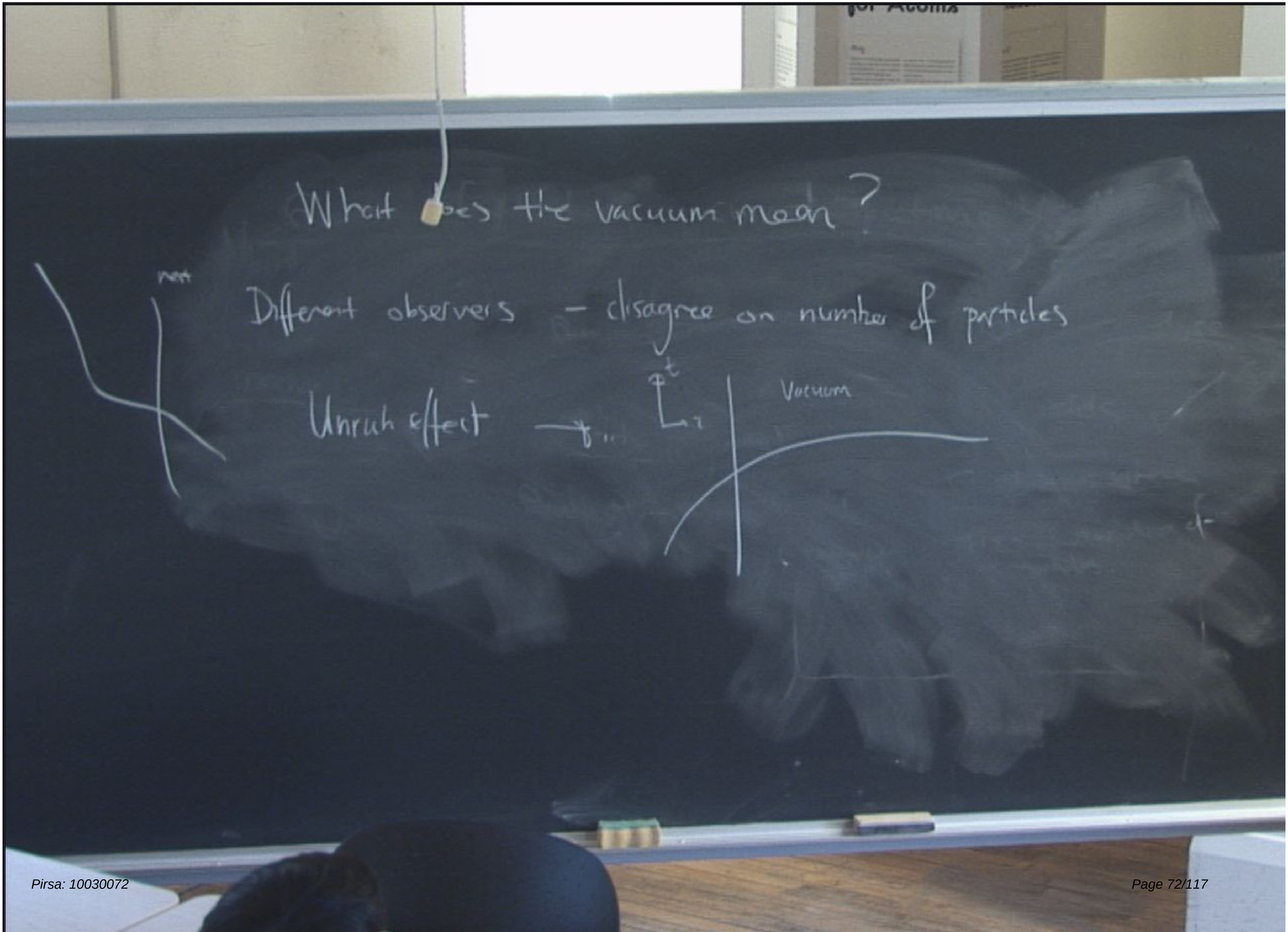
10 particles $\longrightarrow$ ? particles $\longrightarrow$ N particles

FT on time-dependent $\Longrightarrow$ particle creation


What oes the vacuum mean ?
not
Different observers


What does the vacuum mean?
Different observers – disagree on number of particles


What does the vacuum mean ?
Different observers – disagree on number of particles
Unruh effect →


What oes the vacuum mean?
Different observers – disagree on number of particles
Unruh effect
Vacuum

What does the vacuum mean?

Different observers – disagree on number of particles

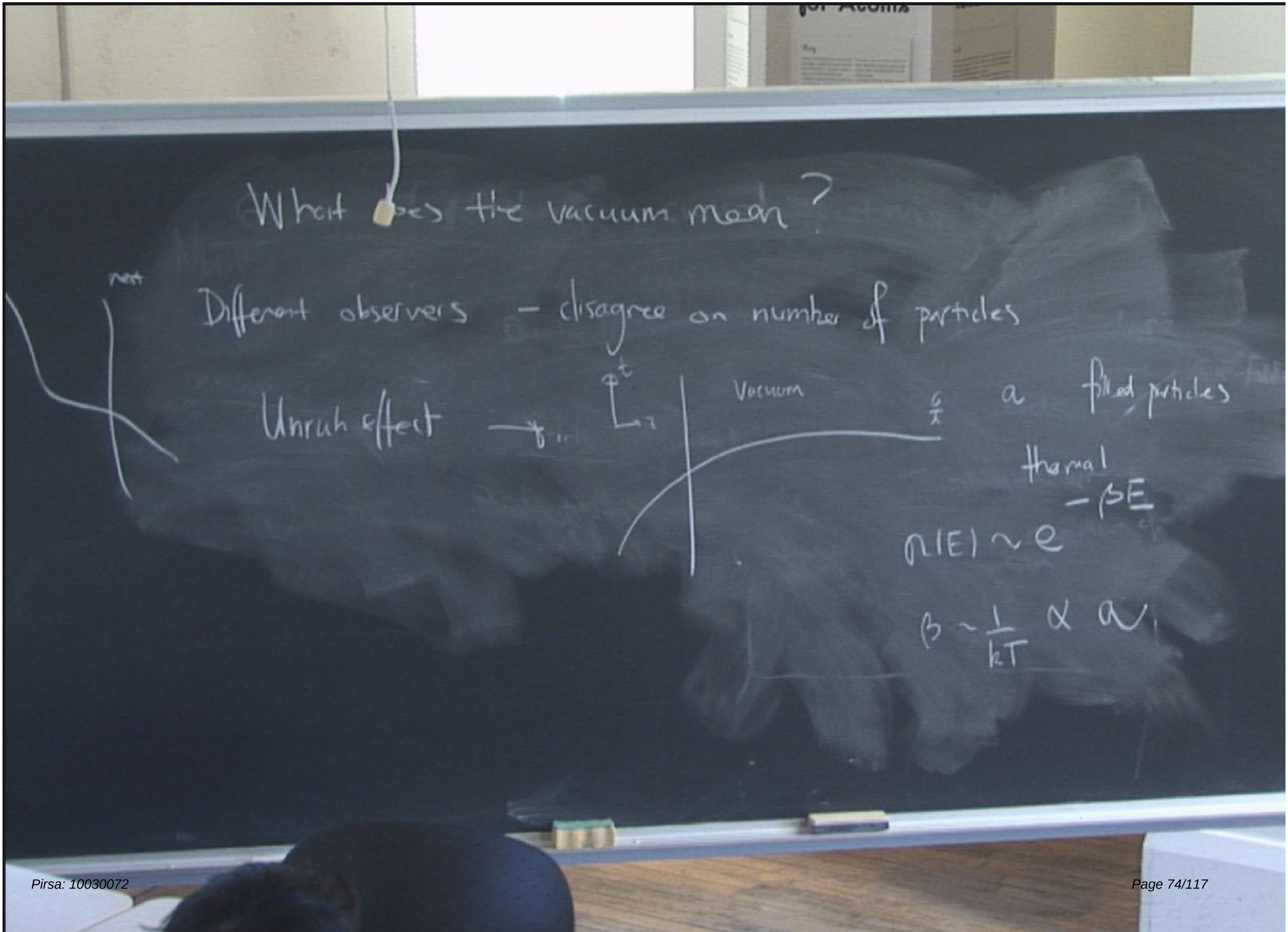
Unruh effect

vacuum

a filled particles

thermal

$n(E) \sim e^{-\beta E}$


What does the vacuum mean?
Different observers - disagree on number of particles
Unruh effect
Vacuum
a
filled particles
thermal
$n(E) \sim e^{-\beta E}$
$\beta \sim \frac{1}{kT} \propto a$

During inflation $d_c \propto H^{-1} \approx$ constant

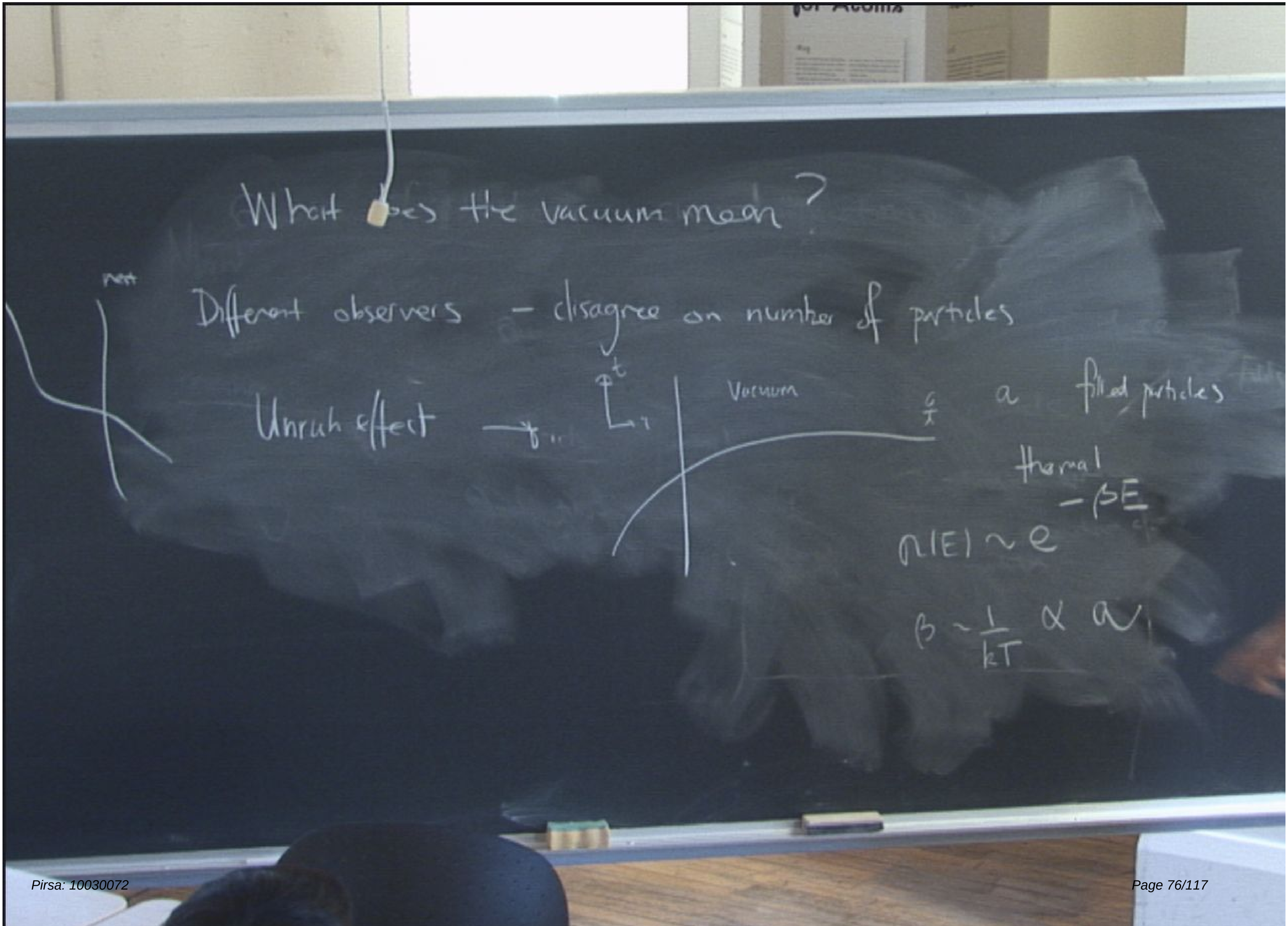
$\lambda_{phys} = a(t)\, \lambda_{com}$ $\quad \lambda \uparrow$ until $\lambda_{phy} \gg H^{-1}$ (notion of particles breaks down)

Inflation ends $\quad d_c \gg \lambda_{phys}$ (well defined notion of particles)

$$\frac{d_c}{a} \sim \frac{H^{-1}}{a} \uparrow$$

10 particles $\longrightarrow$ ? particles $\longrightarrow$ N particles

FT on time dependent $\Longrightarrow$ particle creation


What does the vacuum mean?
Different observers – disagree on number of particles
Unruh effect
Vacuum
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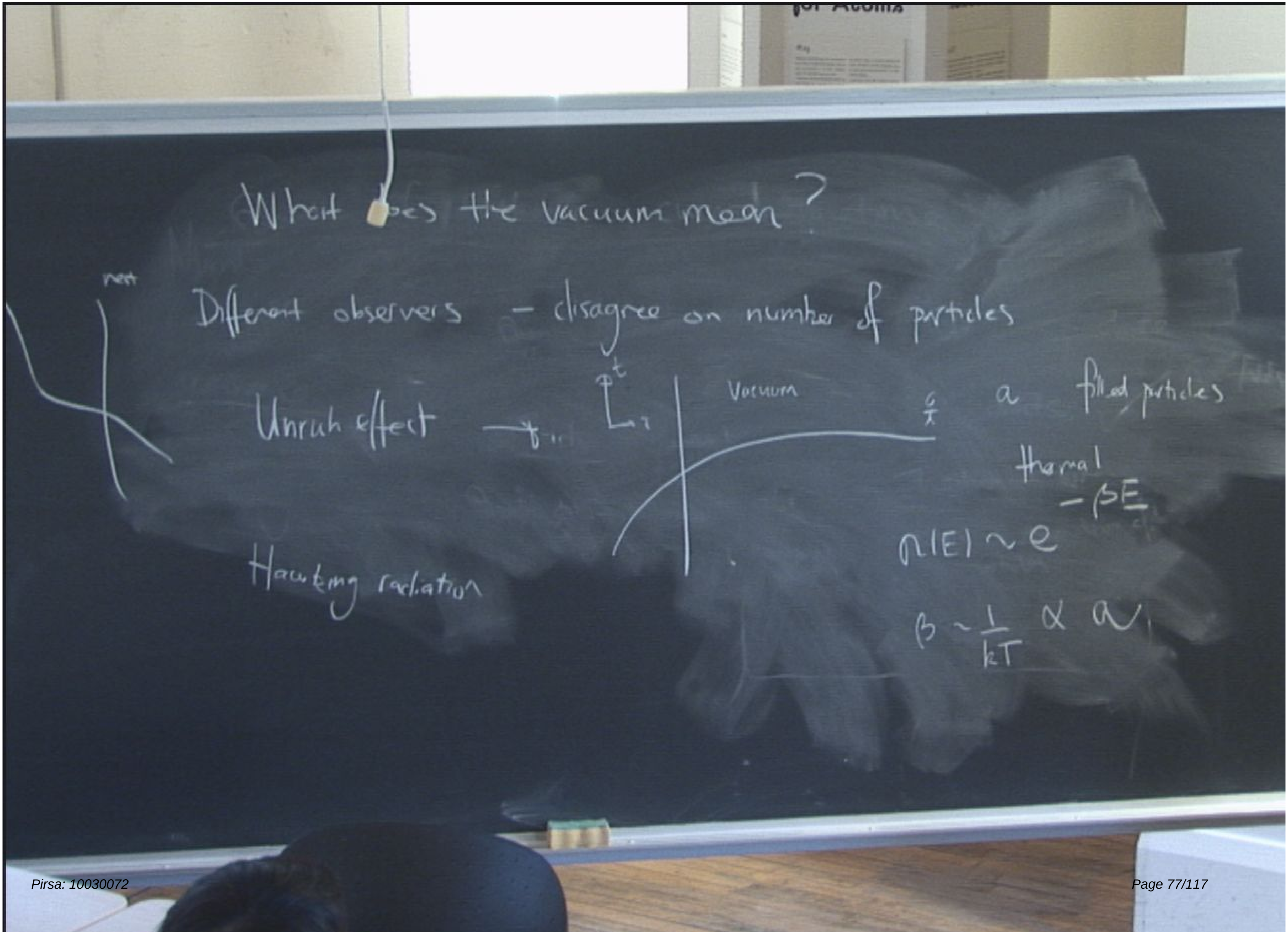
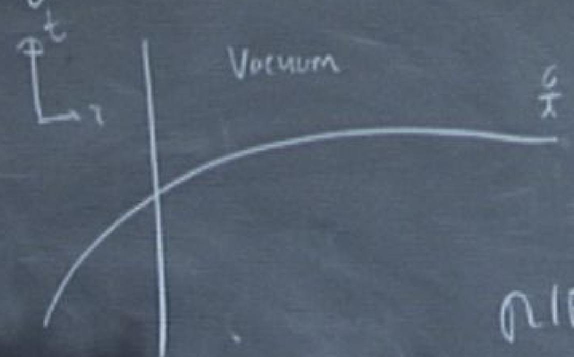

What does the vacuum mean?
Different observers – disagree on number of particles
Unruh effect
Vacuum
a filled particles
thermal
$n(E) \sim e^{-\beta E}$
$\beta \sim \frac{1}{kT} \propto a$
Hawking radiation

What does the vacuum mean?

Different observers – disagree on number of particles


Unruh effect →



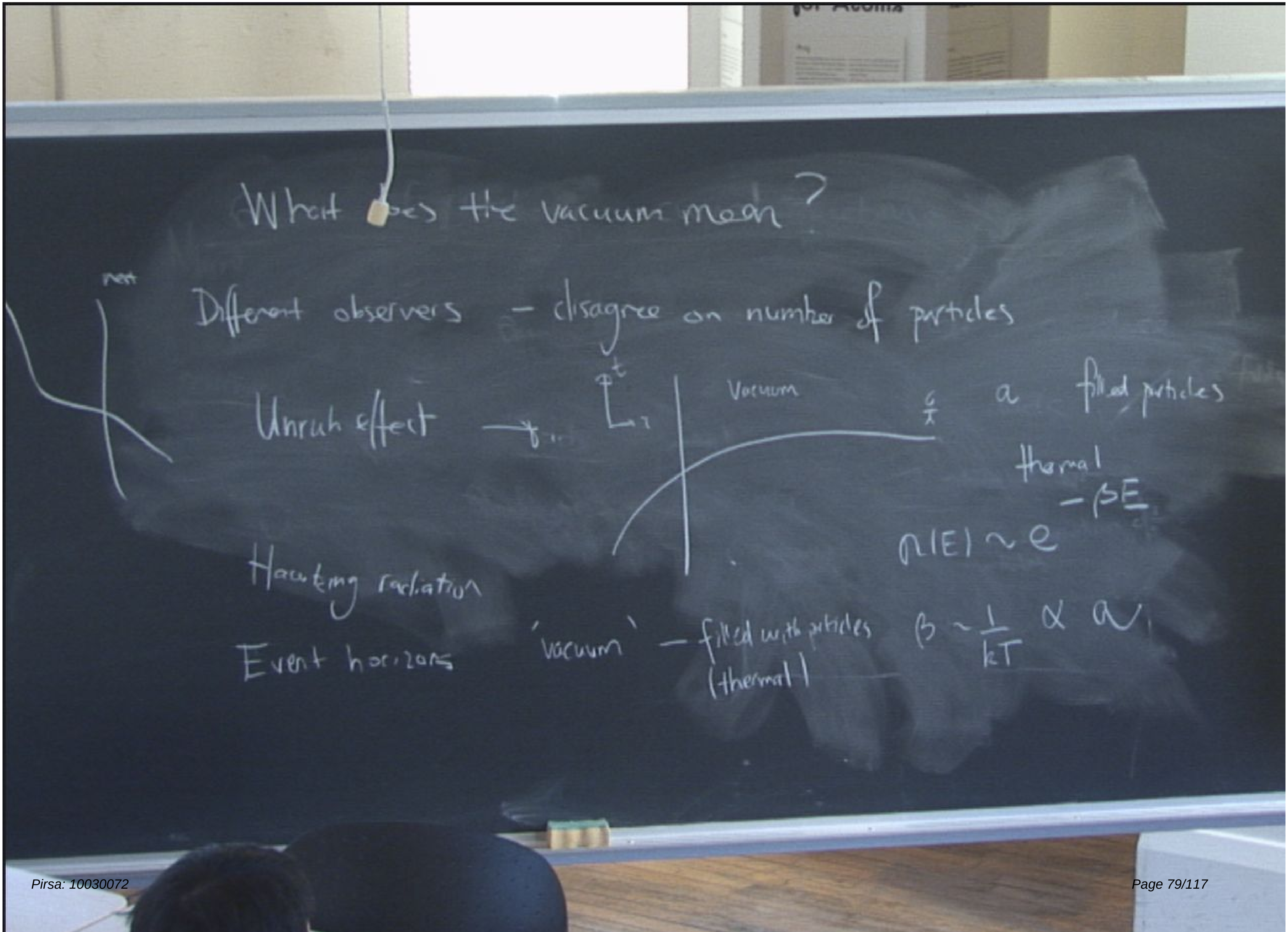

a filled particles

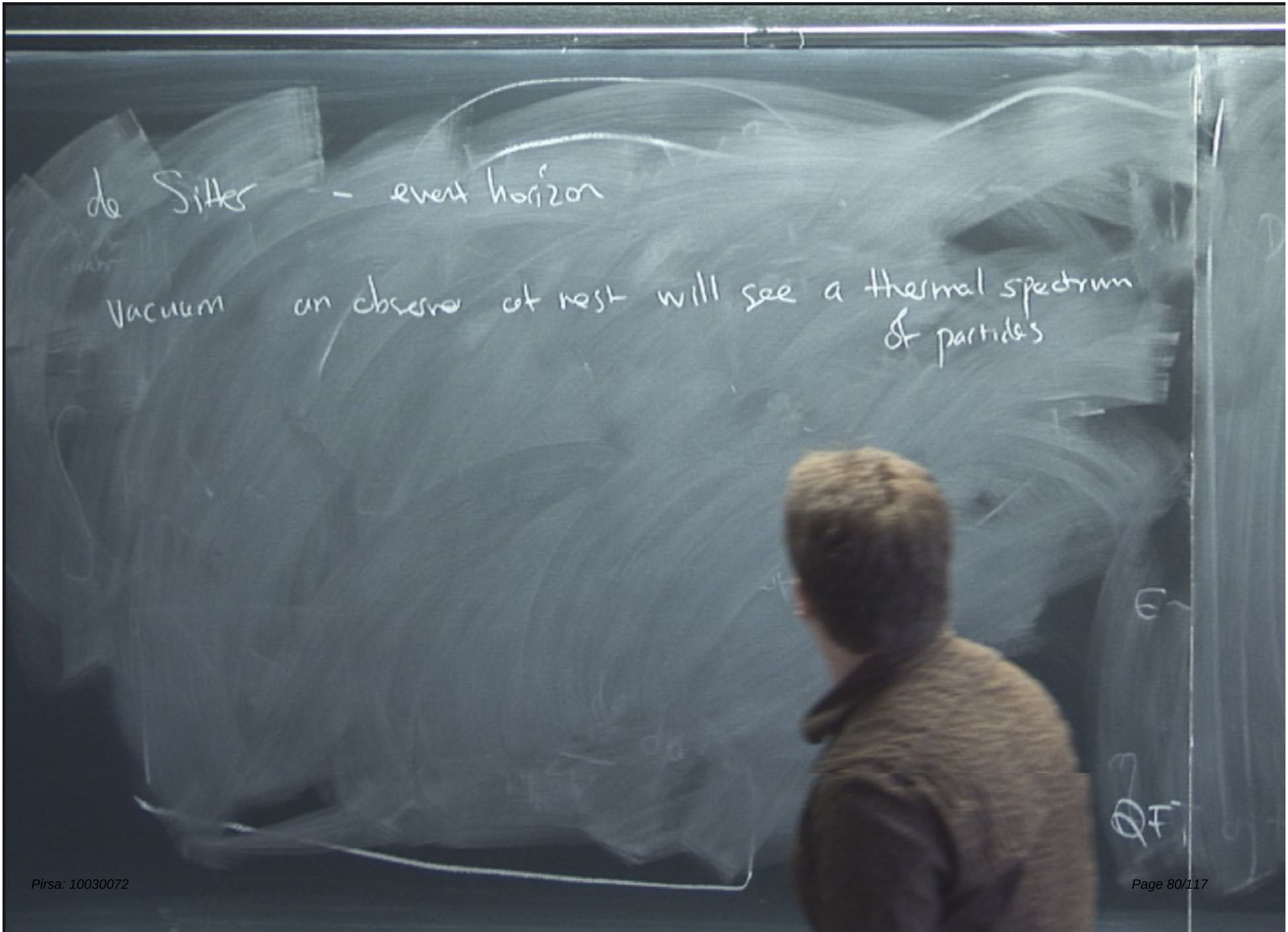
thermal

$$n(E) \sim e^{-\beta E}$$

$$\beta \sim \frac{1}{kT} \propto a$$

Hawking radiation

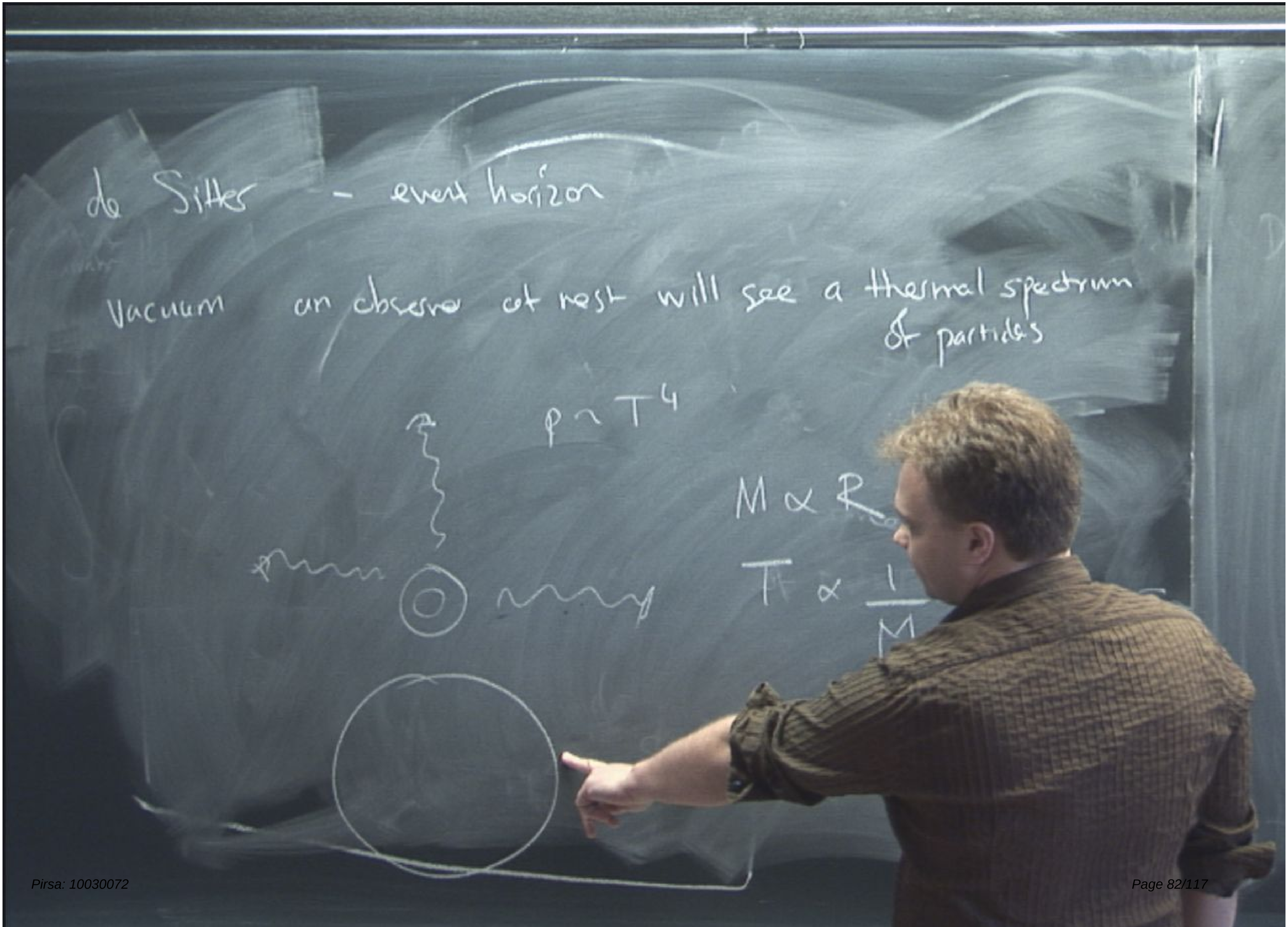

What does the vacuum mean?
Different observers - disagree on number of particles
Unruh effect
t
x
vacuum
a
filled particles
thermal
n(E) ~ e^{-βE}
Hawking radiation
Event horizons
'vacuum' - filled with particles (thermal)
β ~ 1/kT ∝ a


de Sitter - event horizon
vacuum an observer at rest will see a thermal spectrum
of particles

de Sitter – event horizon

vacuum an observer at rest will see a thermal spectrum of particles

$\rho \sim T^4$


de Sitter - event horizon
vacuum an observer at rest will see a thermal spectrum of particles
ρ ~ T^4
M ∝ R
T ∝ 1/M

de Sitter - event horizon

Vacuum an observer at rest will see a thermal spectrum of particles

$\rho \sim T^4$

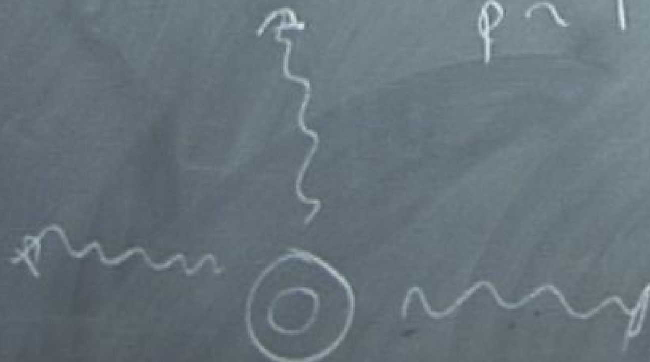
$M \propto R$

$T \propto \frac{1}{M}$

de Sitter - event horizon

Vacuum an observer at rest will see a thermal spectrum of particles

$\rho \sim T^4$



$M \propto R$

$T \propto \frac{1}{M}$

E~

QF

de Sitter – event horizon

vacuum   an observer at rest will see a thermal spectrum of particles

$\rho \sim T^4$

$M \propto R$

$T \propto \frac{1}{M}$

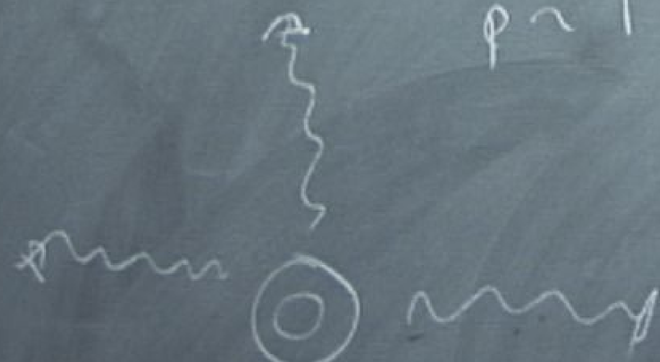
$E \sim$

QF

de Sitter - event horizon

Vacuum an observer at rest will see a thermal spectrum of particles

$\rho \sim T^4$



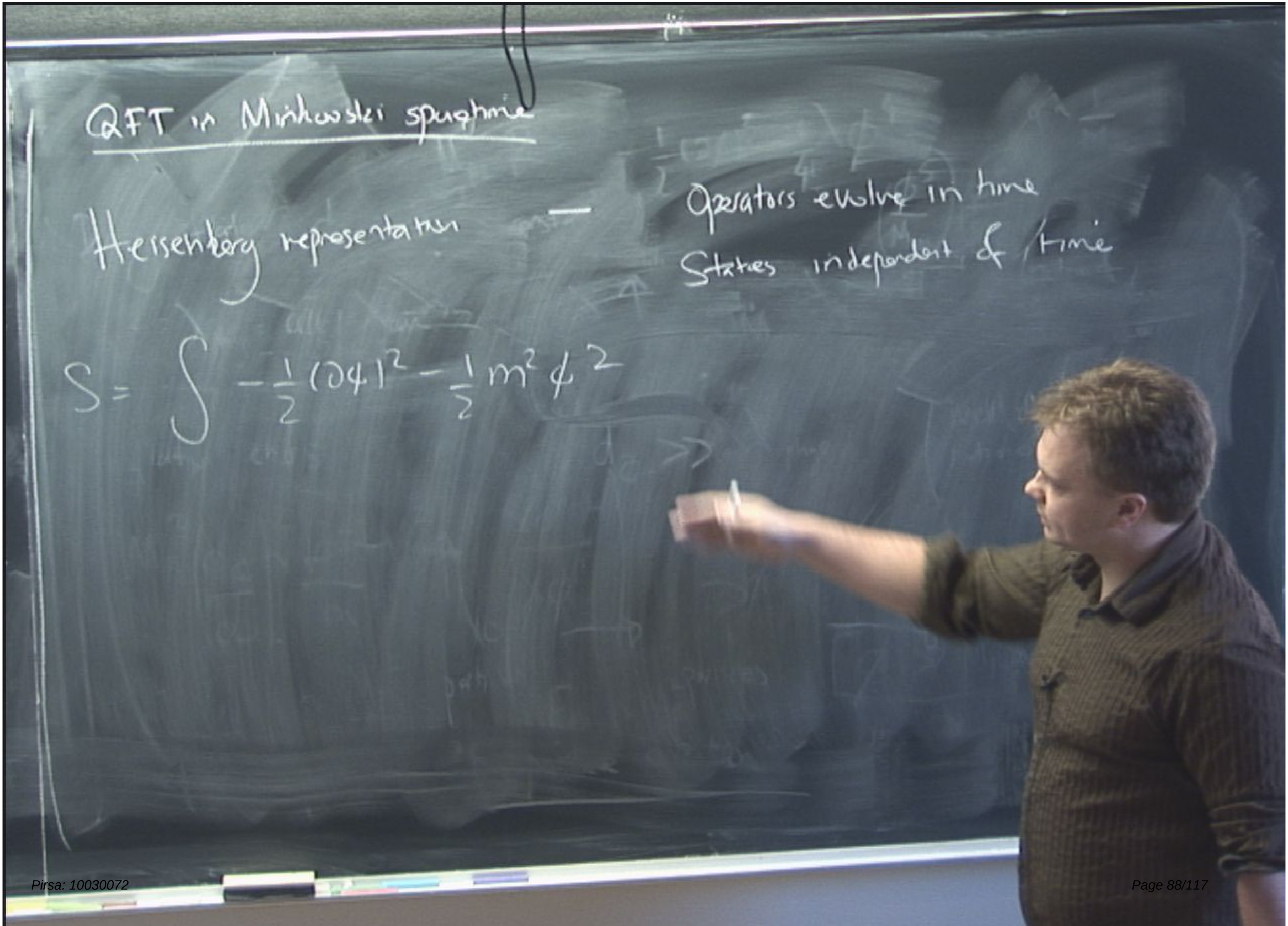
$M \propto R$

$T \propto \frac{1}{M}$

QFT in Minkowski spacetime

Heisenberg representation — Operators evolve in time

States independent of time


QFT in Minkowski spacetime
Heisenberg representation
Operators evolve in time
States independent of time
$S = \int -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$

QFT in Minkowski spacetime

Heisenberg representation — Operators evolve in time
States independent of time

$$\hat{S} = \int -\frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2$$

Fundame

## QFT in Minkowski spacetime

Heisenberg representation — Operators evolve in time
States independent of time

$$\hat{S} = \int -\frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2$$

Fundamental commutation relation

$$[\phi(t,\vec{x}), \Pi(t,\vec{y})] = i\,\delta^{(3)}(\vec{x}-\vec{y})$$

## QFT in Minkowski spacetime

Heisenberg representation

Operators evolve in time

States independent of time

$$\hat{S} = \int -\frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2$$

$$\pi = \frac{\partial \mathcal{L}}{\partial(\dot{\phi})} = \dot{\phi}$$

Fundamental commutation relation

$$[\phi(t,\vec{x}), \pi(t,\vec{y})] = i\,\delta^{(3)}(\vec{x}-\vec{y})$$

QFT in Minkowski spacetime

Heisenberg representation — Operators evolve in time
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Fundamental commutation relation

$$[\phi(t,\vec{x}), \Pi(t,\vec{y})] = i\,\delta^{(3)}(\vec{x}-\vec{y})$$

QFT in Minkowski spacetime

Heisenberg representation — Operators evolve in time
States independent of time

$$\hat{S} = \int -\frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2 \qquad \Pi = \frac{\partial \mathcal{L}}{\partial(\dot{\phi})} = \dot{\phi}$$

Fundamental commutation relation

$$[\phi(t,\vec{x}), \dot{\Pi}(t,\vec{y})] = i\,\delta^{(3)}(\vec{x}-\vec{y})$$

$$\phi = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[ \hat{a}_k e^{-i\omega_k t - i\vec{k}\cdot\vec{x}} + a_k^{\dagger} e^{+i\omega_k t - i\vec{k}\cdot\vec{x}} \right.$$

$$\omega_k = \sqrt{\vec{k}^2 + m^2}$$

$$\phi = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[ \hat{a}_k e^{-i\omega_k t + i\vec{k}\cdot\vec{x}} + a_k^\dagger e^{+i\omega_k t - i\vec{k}\cdot\vec{x}} \right]$$

$$\omega_k = \sqrt{\vec{k}^2 + m^2}$$

$$[a_k, a_{k'}] = 0 \qquad [a_k^\dagger, a_{k'}^\dagger] = 0$$

$$\boxed{[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^{(3)}(k - k')}$$

$$\phi = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[ \hat{a}_k e^{-i\omega_k t + i\vec{k}\cdot\vec{x}} + a_k^\dagger e^{+i\omega_k t - i\vec{k}\cdot\vec{x}} \right]$$

$$\omega_k = \sqrt{\vec{k}^2 + m^2}$$

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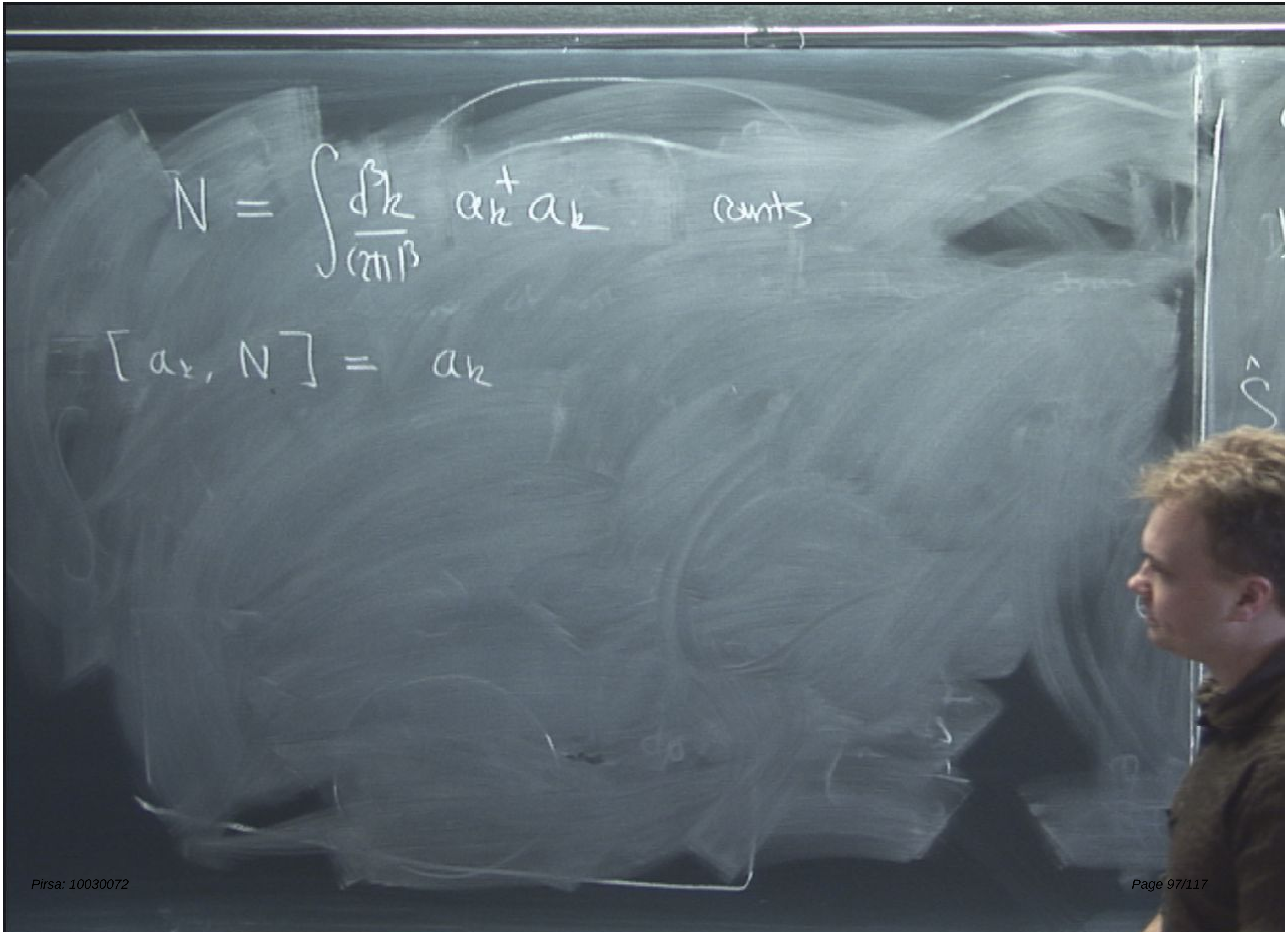
$$\boxed{[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^{(3)}(k - k')}$$


$N = \int \frac{d^3k}{(2\pi)^3} a_k^\dagger a_k$ counts
$[a_k, N] = a_k$

$$N = \int \frac{d^3k}{(2\pi)^3}\, a_k^{\dagger} a_k \quad \text{counts}$$

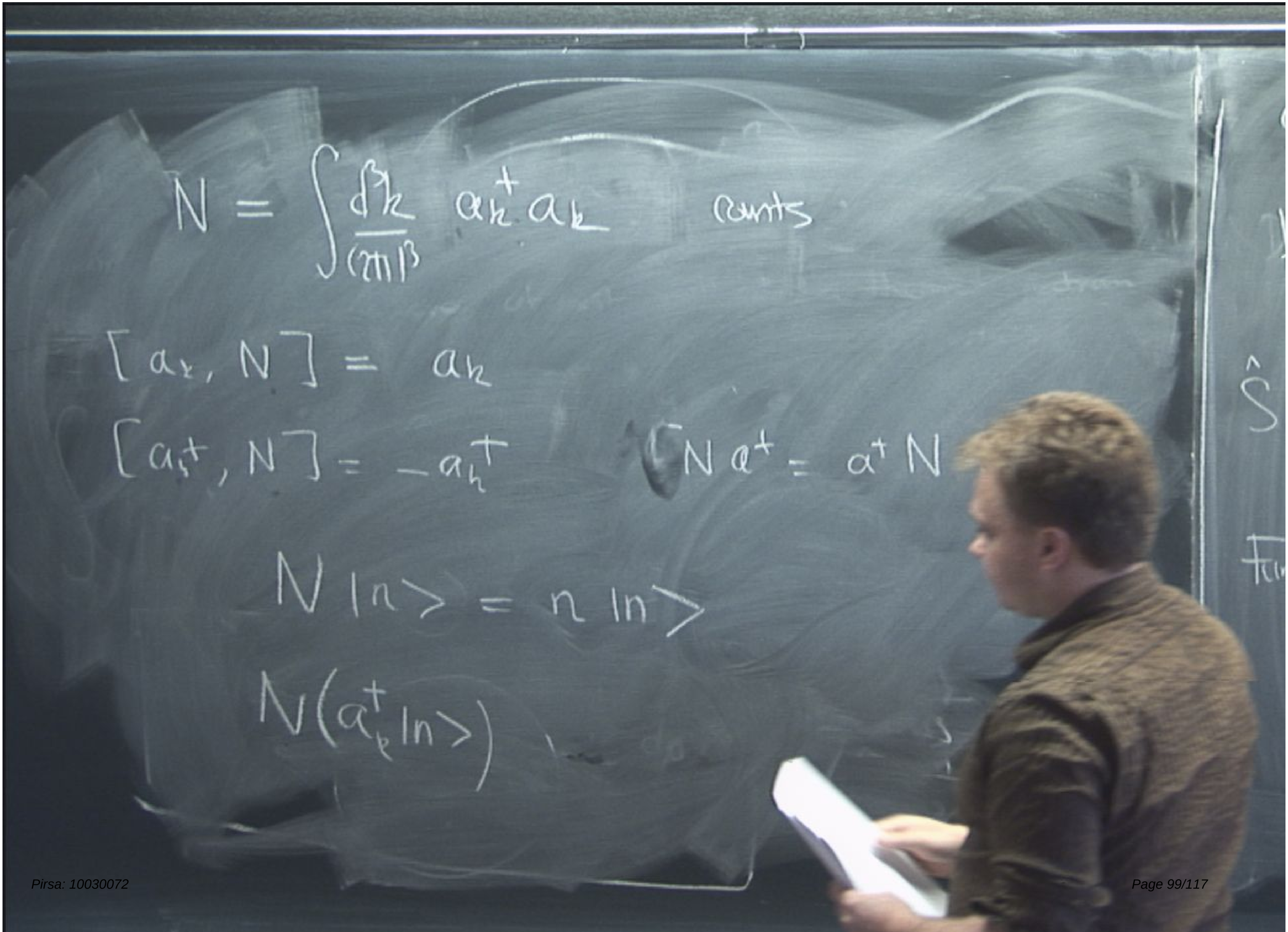
$$[a_k, N] = a_k$$

$$[a_k^{\dagger}, N] =$$

$$N|n\rangle = n|n\rangle$$

$\hat{S}$

$\epsilon \sim$


$N = \int \frac{d^3k}{(2\pi)^3} a_k^\dagger a_k$ counts
$[a_k, N] = a_k$
$[a_k^\dagger, N] = -a_k^\dagger$
$N a^\dagger = a^\dagger N$
$N|n\rangle = n|n\rangle$
$N(a_k^\dagger|n\rangle)$

$$N = \int \frac{d^3k}{(2\pi)^3}\, a_k^\dagger a_k \quad \text{counts}$$

$$[a_k, N] = a_k$$

$$[a_k^\dagger, N] = -a_k^\dagger \qquad N a^\dagger = a^\dagger N + a^\dagger$$

$$N|n\rangle = n|n\rangle$$

$$N(a_k^\dagger|n\rangle) = (a^\dagger N + a^\dagger)|n\rangle = (n+1)(a_k^\dagger|n\rangle)$$

$\hat{S}$

$\epsilon \sim$

$$N = \int \frac{d^3k}{(2\pi)^3} a_k^\dagger a_k \quad \text{counts}$$

$$[a_k, N] = a_k$$

$$[a_k^\dagger, N] = -a_k^\dagger \qquad N a^\dagger = a^\dagger N + a^\dagger$$

$$N|n\rangle = n|n\rangle$$

$$N(a_k^\dagger|n\rangle) = (a^\dagger N + a^\dagger)|n\rangle = (n+1)(a_k^\dagger|n\rangle$$

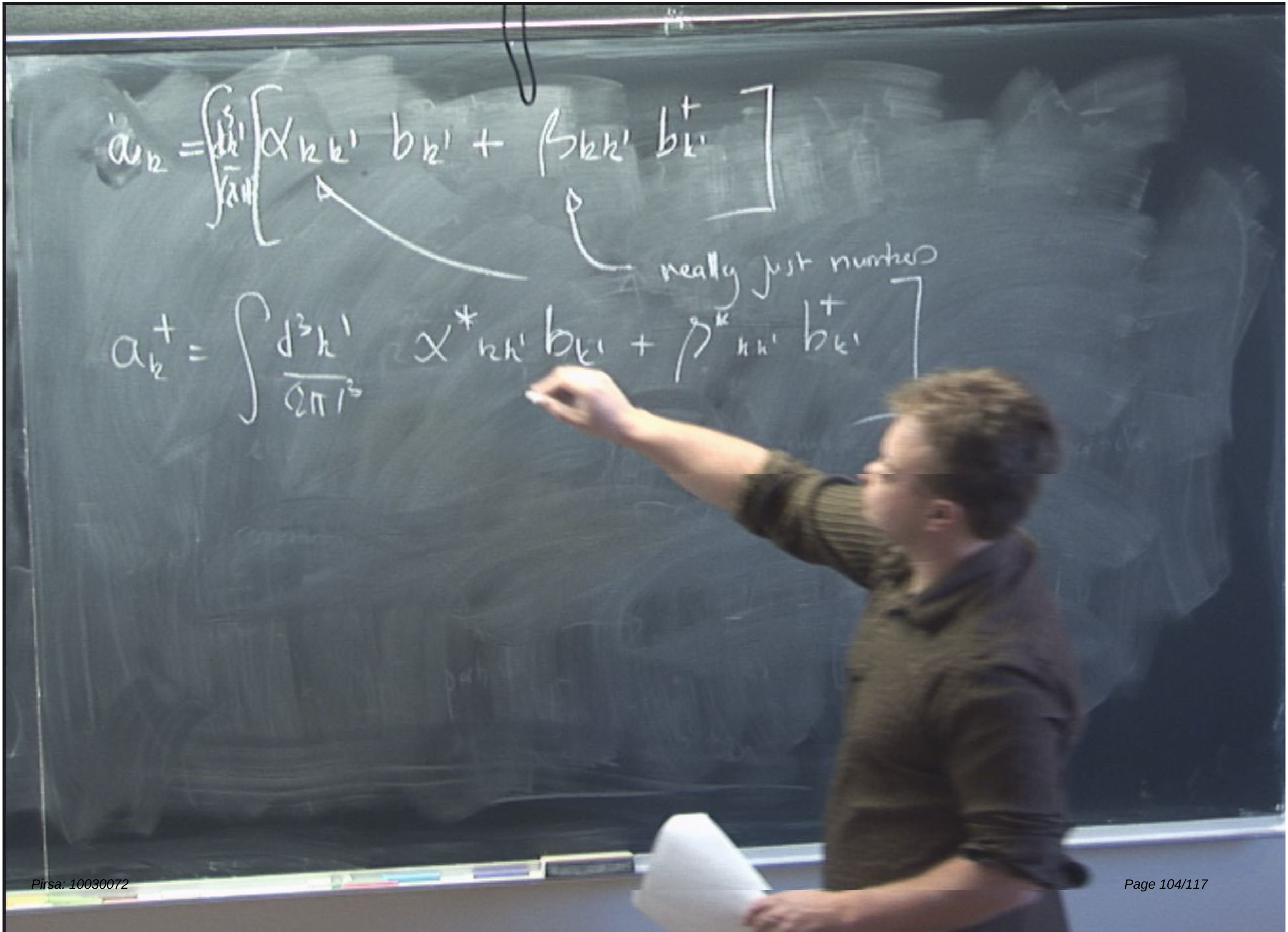
$$\phi = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[ \hat{a}_k e^{-i\omega_k t + i\vec{k}\cdot\vec{x}} + a_k^\dagger e^{+i\omega_k t - i\vec{k}\cdot\vec{x}} \right]$$

$$\omega_k = \sqrt{\vec{k}^2 + m^2}$$

$$\boxed{[a_k, a_{k'}] = 0 \qquad [a_k^\dagger, a_{k'}^\dagger] = 0 \qquad \boxed{[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^{(3)}(k - k')}}$$

$$a_k = \int \frac{d^3k'}{(2\pi)^3} \left[ \alpha_{kk'}\, b_{k'} + \beta_{kk'}\, b^{\dagger}_{k'} \right]$$

really just numbers


$a_k = \int \frac{d^3k'}{(2\pi)^3} \left[ \alpha_{kk'} b_{k'} + \beta_{kk'} b^+_{k'} \right]$
really just numbers
$a_k^+ = \int \frac{d^3k'}{(2\pi)^3} \; \alpha^*_{kk'} b_{k'} + \beta^*_{kk'} b^+_{k'} \,]$

$$a_k = \int \frac{d^3k'}{(2\pi)^3} \left[ \alpha_{kk'} \, b_{k'} + \beta_{kk'} \, b^+_{k'} \right]$$

really just numbers

$$a^+_k = \int \frac{d^3k'}{(2\pi)^3} \left[ \alpha^*_{kk'} \, b_{k'} + \beta^*_{kk'} \, b^+_{k'} \right]$$

$$[b_k, b_{k'}] = 0 \qquad [b^+_k, b^+_{k'}] = 0 \qquad [b_k, b^+_{k'}] = (2\pi)^3 \delta^{(3)}(k - k')$$

$$a_k = \int \frac{d^3k'}{(2\pi)^3}\left[\alpha_{kk'}\, b_{k'} + \beta_{kk'}\, b^{+}_{k'}\right]$$

really just numbers

$$a^{+}_{k} = \int \frac{d^3k'}{(2\pi)^3}\left[\alpha^{*}_{kk'}\, b_{k'} + \beta^{*}_{kk'}\, b^{+}_{k'}\right]$$

$$[b_k, b_{k'}] = 0 \qquad [b^{+}_{k}, b^{+}_{k'}] = 0 \qquad [b_k, b^{+}_{k'}] = (2\pi)^3\delta^{(3)}(k-k')$$

$$\boxed{\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\; \alpha_{kk_1}\alpha^{*}_{k'k_2} - \beta_{kk_1}\beta^{*}_{k'k_2} = (2\pi)^3\delta^{(3)}(k_1-k_2)}$$

$$\alpha_{kk'} = (2\pi)^3 \, \delta^{(3)}(k-k') \, \alpha_k$$

$$\alpha_{kk'} = (2\pi)^3 \delta^{(3)}(k-k')\,\alpha_k$$

$$\beta_{kk'} = (2\pi)^3 \delta^{(3)}(k-k')\,\beta_k$$

$$a_k = \alpha_k b_k + \beta_k b_k^\dagger$$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$$\alpha_{kk'} = (2\pi)^3\, \delta^{(3)}(k-k')\, \alpha_k$$

$$\beta_{kk'} = (2\pi)^3\, \delta^{(3)}(k-k')\, \beta_k$$

$$a_k = \alpha_k b_k + \beta_k b_k^\dagger$$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$$N_b = \int \frac{d^3k}{(2\pi)^3}\, b_k^\dagger b_k$$

$$\alpha_{kk'} = (2\pi)^3 \delta^{(3)}(k-k')\,\alpha_k$$

$$\beta_{kk'} = (2\pi)^3 \delta^{(3)}(k-k')\,\beta_k$$

$$a_k = \alpha_k b_k + \beta_k b_k^\dagger$$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$$N_b = \int \frac{d^3k}{(2\pi)^3}\, b_k^\dagger b_k \neq N_a$$

$$[N_b, N_a] \neq 0$$

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$$N|0\rangle = 0$$

$$\Downarrow$$

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$$\boxed{b_k|0,b\rangle = 0}$$

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$$N_b = \int \frac{d^3k}{(2\pi)^3}\, b_k^\dagger b_k \neq N_a$$

$$[N_b, N_a] \neq 0$$

$$a_k = \int \frac{d^3k'}{(2\pi)^3} \left[ \alpha_{kk'}\, b_{k'} + \beta_{kk'}\, b^+_{k'} \right]$$

Bogoliubov transformation

$\alpha(t), \beta(t)$

really just numbers

$$a^+_k = \int \frac{d^3k'}{(2\pi)^3} \left[ \alpha^*_{kk'}\, b_{k'} + \beta^*_{kk'}\, b^+_{k'} \right]$$

$$[b_k, b_{k'}] = 0 \qquad [b^+_k, b^+_{k'}] = 0 \qquad [b_k, b^+_{k'}] = (2\pi)^3 \delta^{(3)}(k-k')$$

$$\boxed{\int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3}\, \alpha_{kk_1} \alpha^*_{k'k_2} - \beta_{kk_1} \beta^*_{k'k_2} = (2\pi)^3 \delta^{(3)}(k_1 - k_2)}$$

$$a_k = \int \frac{d^3k'}{(2\pi)^3}\left[\alpha_{kk'}\, b_{k'} + \beta_{kk'}\, b^{+}_{k'}\right]$$ Bogoliubov transformation.

really just numbers

$\alpha(t), \beta(t)$

$$a_k^{+} = \int \frac{d^3k'}{(2\pi)^3}\left[\alpha^{*}_{kk'}\, b_{k'} + \beta^{*}_{kk'}\, b^{+}_{k'}\right]$$

$$[b_k, b_{k'}] = 0 \qquad [b^{+}_k, b^{+}_{k'}] = 0 \qquad [b_k, b^{+}_{k'}] = (2\pi)^3\delta^{(3)}(k-k')$$

$$\boxed{\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\; \alpha_{kk_1}\alpha^{*}_{k'k_2} - \beta_{kk_1}\beta^{*}_{k'k_2} = (2\pi)^3\delta^{(3)}(k_1-k_2)}$$