Title: Enroute to a Quantum Dynamics for Causal Sets.

Date: Mar 10, 2010 04:00 PM

URL: http://pirsa.org/10030066

Abstract: In causal set quantum gravity, spacetime is assumed to have a fundamental atomicity or discreteness, and is replaced by a locally finite poset, the causal set. After giving a brief review of causal sets, I will discuss two distinct approaches to constructing a quantum dynamics for causal sets. In the first approach one borrows heavily from the continuum to construct a partition function for causal sets.
This is illustrated in a 2-d model of causal sets, in which typicality replaces quantum probabilities. The second approach is intrinsic, and uses the quantum measure formulation in which dynamics is described in the language of measure theory and observables are measurable sets in an event algebra. Using the example of complex percolation dynamics I will show that naive attempts to carry out this process for the quantum measure may not work. I will end by discussing possible ways to address this question.

EN ROUTE TO A QUANTUM DYNAMICS FOR CAUSAL SETS

SUMATI SURYA
(RAMAN RESEARCH INSTITUTE)

w/

- JOE HENSON & GRAHAM BRIGHTWELL
- FAY DOWKER & STEVEN JOHNSTON.

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OUTLINE:

- REVIEW OF CAUSAL SET THEORY
- A 2D MODEL FOR CAUSAL SETS
- A QUANTUM MEASURE FORMULATION
- COMPLEX PERCOLATION
- OPEN QUESTIONS

②

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- REVIEW OF CAUSAL SET THEORY
- A 2D MODEL FOR CAUSAL SETS
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- COMPLEX PERCOLATION
- OPEN QUESTIONS

CAUSAL SET THEORY

$$(M, g) \longrightarrow (M, <)$$

CAUSAL SPACETIME — CAUSAL STRUCTURE

$(M,<)$ IS A POSET:

(i) $x \nless x$: IRREFLEXIVE

(ii) $x < y,\ y < z \Rightarrow x < z$: TRANSITIVE.

• $(M,<)^{*}$ DETERMINES CONFORMAL CLASS $[g]$.

*(GIVEN CERTAIN CAUSALITY CONDS) — McCARTHY, KING, HAWKING — MALAMENT.

CAUSAL STRUCTURE + VOL. ELEMENT = GEOMETRY.

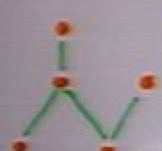
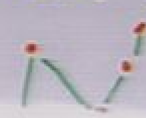
CAUSAL SET HYPOTHESIS: SPACETIME IS REPLACED BY A LOCALLY FINITE POSET, THE CAUSAL SET

(iii) $|I(x,y)| < \infty$, $I(x,y) \equiv \{z \mid x < z < y\}$

LOCAL FINITENESS

HYPOTHESIS OF FUNDAMENTAL DISCRETENESS

ORDER + CARDINALITY $\sim$ GEOMETRY.


CAUS

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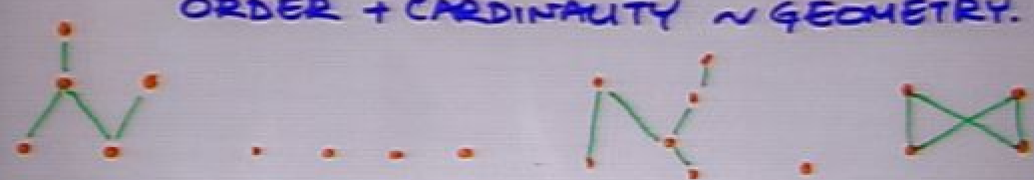

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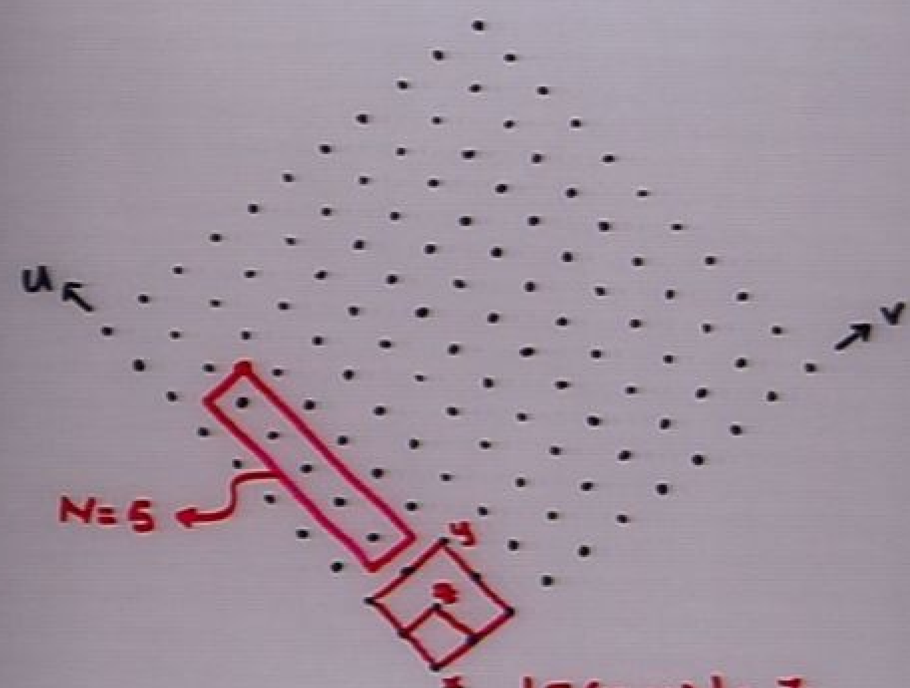
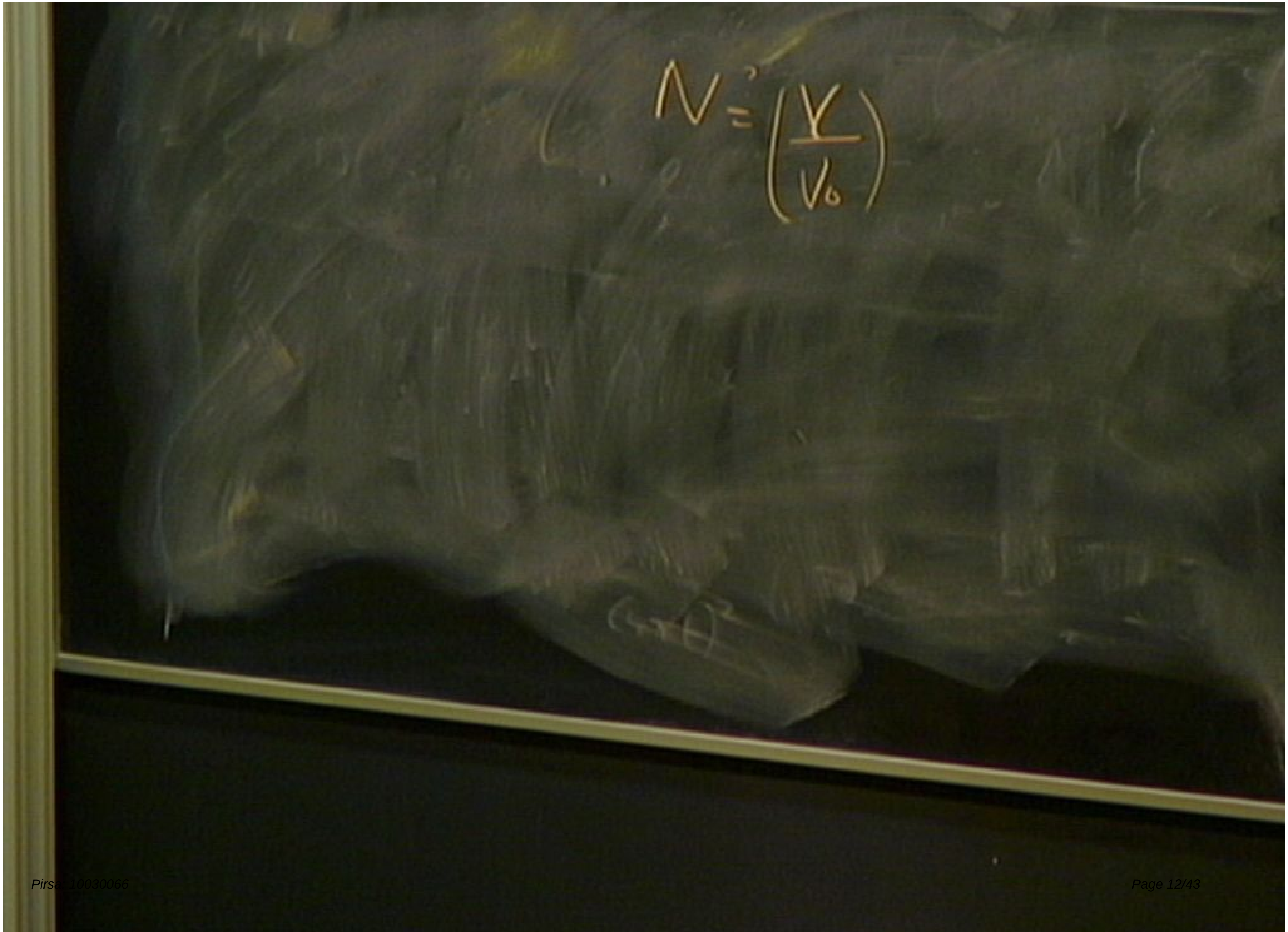
REGULAR DISCRETISATION



$|I(x,y)| = 7$

$|I(x,z)| = 2$

$N = \frac{V}{V_0}$ → FUNDAMENTAL VOLUME CUT-OFF


$N = \left(\frac{V}{V_0}\right)$

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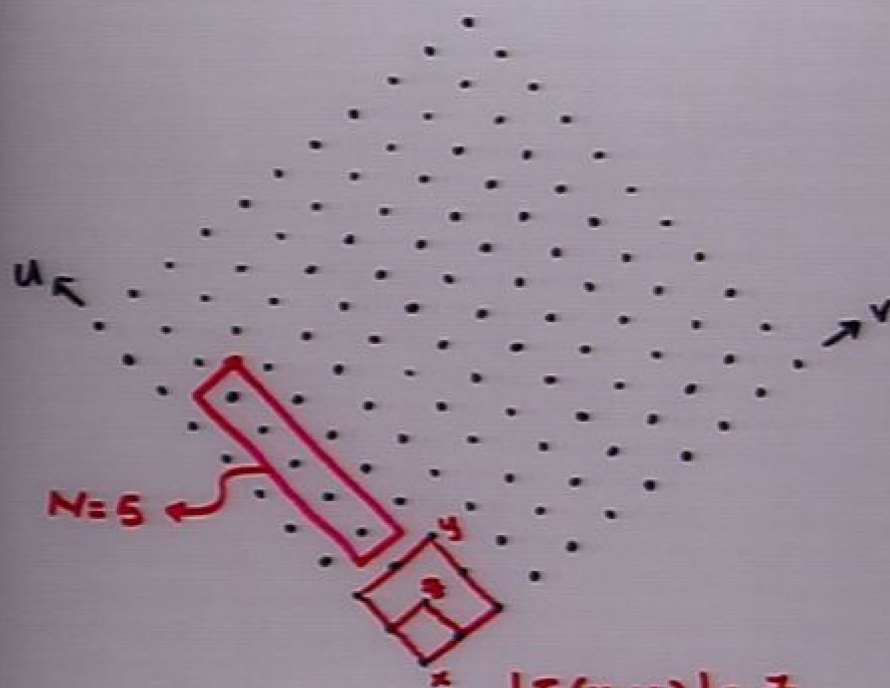

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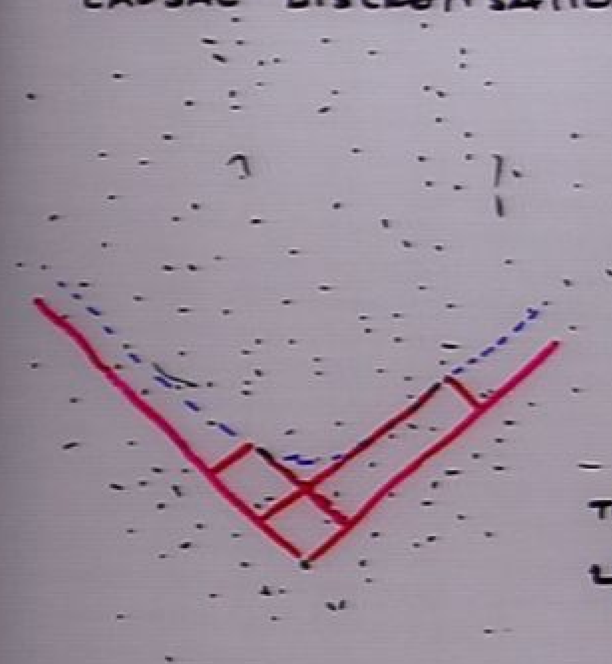
⑥



RANDOM LATTICE

$$P_n(V) = \left(\frac{V}{V_0}\right)^n \frac{e^{-(V/V_0)}}{n!}$$

$$\langle N \rangle = \frac{V}{V_0}$$

THIS PRESERVES LORENTZ INVARIANCE

—BOMBELLI, HENSON & SORKIN.

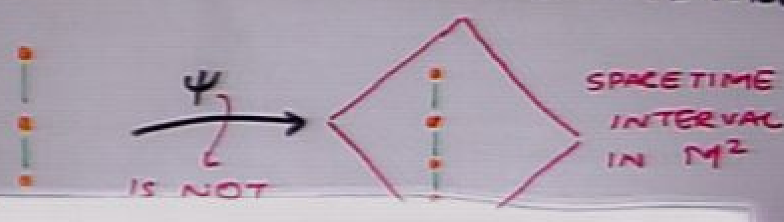
FAITHFUL EMBEDDING

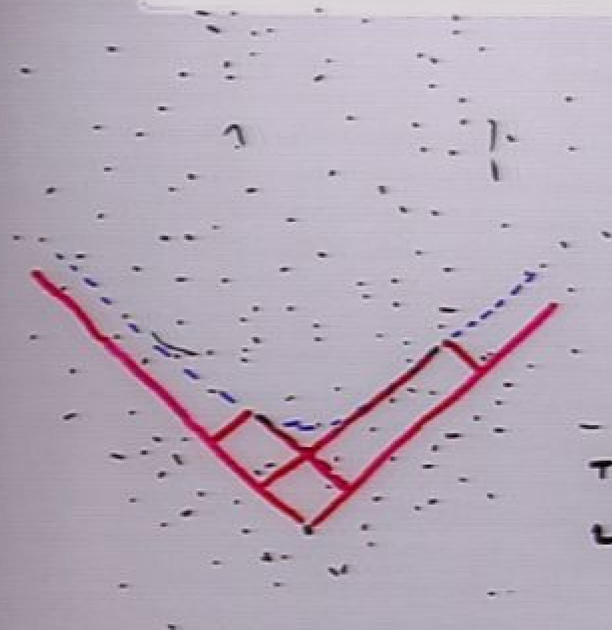
$$\Phi : (C, <) \longrightarrow (M, g)$$

(FUNDAMENTAL) (APPROXIMATION)

(i) $a < b \iff \Phi(a) < \Phi(b)$ [ORDER PRESERVING]

(ii) $\Phi(C) \subset M$ IS A POISSON SPRINKLING





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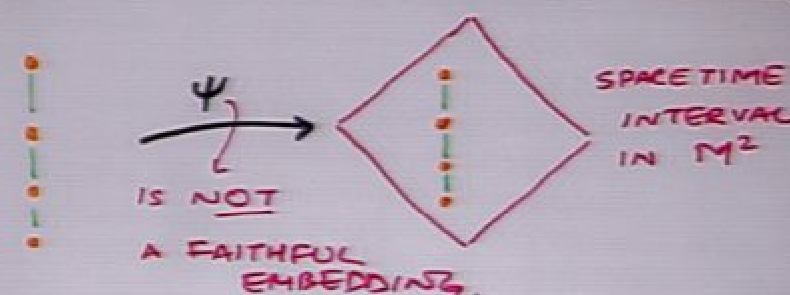


KINEMATICS - WHAT WE KNOW

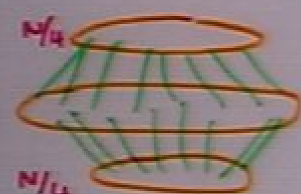
- DIMENSION : D.A. MYERS, J. MYRHEIM
- TOPOLOGY : S. MAJOR, D. RIDEOUT & S. SURYA
- DISTANCE : G. BRIGHTWELL, R. GREGORY & D. RIDEOUT & P. WALLDEN
- D'ALEMBERTIAN : R.D. SORKIN
- QUANTUM FIELD THEORY : S. JOHNSTON
- ACTION : D. BENINCASA & F. DOWKER

CAUSAL SET DYNAMICS

- HISTORIES FRAMEWORK MOST SUITABLE

$$\underbrace{\{(M, g)\}}_{\text{COLLECTION OF ALL CONTINUUM SPACETIMES}} \longrightarrow \underbrace{\{(C, <)\}}_{\text{COLLECTION OF ALL CAUSAL SETS.}}$$

MOST CAUSAL SETS ARE NOT MANIFOLD-LIKE:



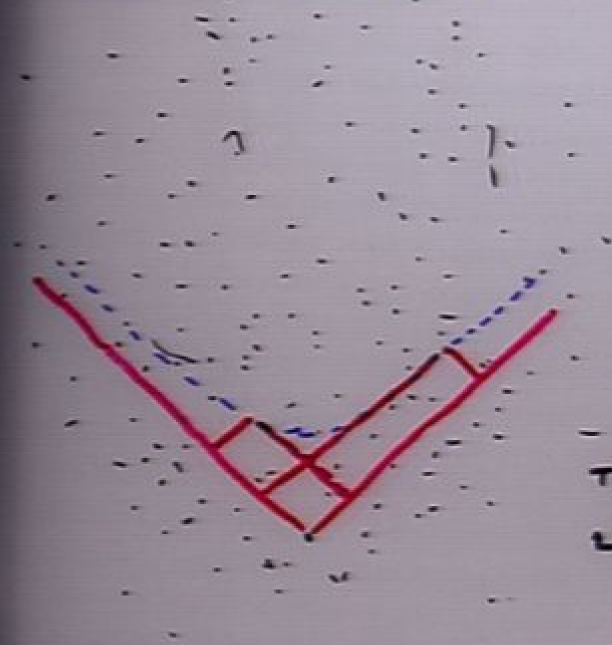
SUCH CAUSAL SETS DOMINATE $\{(C, <)\}$ AS $N \to \infty$.

DYNAMICS SHOULD OVERCOME THIS "ENTROPY"

CLASSICAL STOCHASTIC GROWTH MODELS OF CAUSAL SETS DO OVERCOME THIS ENTROPY.

– RIDEOUT & SORKIN.

CAUSAL DISCRETISATION



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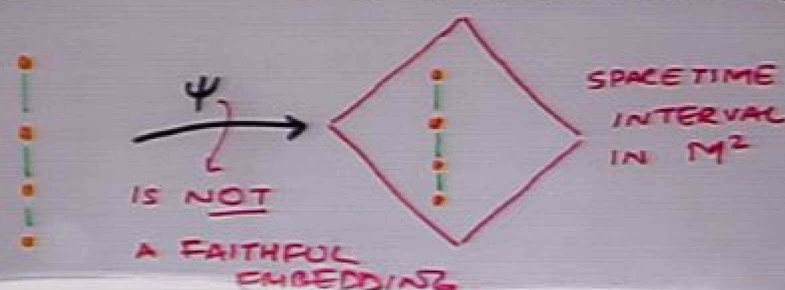
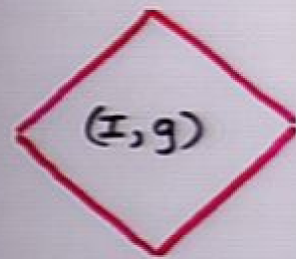


A 2D TOY MODEL — ...RTWELL, HENSON & SURYA



CLASS OF 2D INTERVAL SPACETIMES

(I,g) , g IS CONFORMALLY FLAT.

EA (LORENTZIAN ANALOGUE OF THE DISK.)

$$S[g] = \underbrace{\frac{1}{16\pi G}\int_I R\,dV - \frac{1}{8\pi G}\int_{\partial I} k\,dS - \sum_j \frac{1}{8\pi G}\theta_j}_{\text{CONST. (LORENTZIAN GAUSS-BONNET)}} - \underbrace{\frac{1}{8\pi G}\Lambda V_I}_{\text{UNIMODULAR THEORY FIXES } V_I.}$$

$$\therefore\ Z_{V_I} = (\ )\int 1\,[dg]$$

$\big\downarrow$ "DISCRETISE"

$$Z_N = (\ )\underbrace{\sum_{?}}_{} 1 .$$

WHAT CAUSAL SETS CORRESPOND TO CONFORMALLY FLAT 2D GEOMETRIES?

A 2D ... URYA

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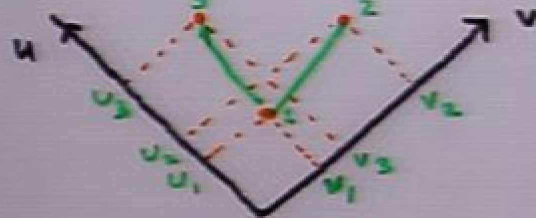
... NFORMALLY FLAT 2D GEOMETRIES?

2D ORDERS

$$\Phi : (C, <) \longrightarrow (I, g)$$

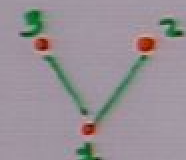


$v_1 < v_3 < v_2$

$u_1 < u_2 < u_3$.

$$\left.\begin{aligned} V &= (v_1, v_3, v_2) \\ U &= (u_1, u_2, u_3) \end{aligned}\right\}$$ each is linearly ordered.

$\Phi(C)$

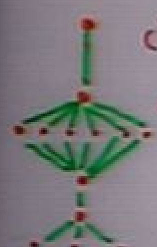


$= V \cap U$

• IF Φ IS FAITHFUL THEN $\Phi(C)$ IS A 2D ORDER

$$\therefore \; Z_N = (\;) \sum_{\text{2D-ORDERS.}} 1$$

(i) NOT ALL 2D-ORDERS ARE MANIFOLD LIKE



(ii) 2D-ORDERS ARE "SPATIALLY" TOPOLOGICALLY TRIVIAL.

IF $\tilde{U}(N)$ IS THE UNIFORM DISTRIBUTION OVER 2D ORDERS, THEN $\tilde{U}(N) \overset{*}{\longrightarrow}$ "$P(N)$ THE $\underbrace{\text{RANDOM ORDER}}_{?}$, AS $N \to \infty$

– EL-ZAHAR, SAUER, WINKLER

$S = \{e_1, e_2 \ldots\ldots e_N\}$ ADMITS $N!$ LINEAR ORDERINGS

U, V: ARE CHOSEN INDEPENDENTLY & RANDOMLY FROM THESE $N!$ ORDERINGS

- POISSON SPRINKLING INTO M^2 IS A RANDOM ORDER!

DOMINANT CONTRIBUTION TO Z_N AS $N \to \infty$ IS FLAT SPACETIME

"TYPICAL" 2D ORDER IS FAITHFULLY EMBEDDABLE INTO THE FLAT INTERVAL AS $N \to \infty$.

THE BENINCASA - DOWKER ACTION

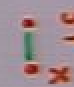
- arXiv:1001.2725

$$\frac{S^{(2)}[C]}{\hbar} = N - 2N_1 + 4N_2 - 2N_3$$

$$\frac{S^{(4)}[C]}{\hbar} = N - N_1 + 9N_2 - 16N_3 + 8N_4$$

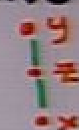
N = # OF ELEMENTS

N_1 = # OF LINKS



$\exists$ NO z S.THAT $x < z < y$.

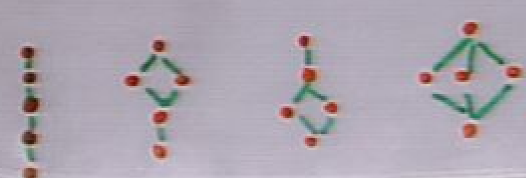
N_2 = # OF 3-ELEMENT INCLUSIVE ORDERS



$\exists$ NO $w \neq z$ S.T. $x < w < y$.

N_3 = # OF 4-ELEMENT INCLUSIVE ORDERS.

N_4 =



THE QUANTUM MEASURE FORMULATION

(13)

- VIEW QUANTUM THEORY AS A GENERALISED STOCHASTIC THEORY

- QUANTUM DYNAMICS IS DESCRIBED AS A $\underbrace{\text{QUANTUM MEASURE}}_{?}$ SPACE.

$\mu(A) \geq 0$

$\mu(A \sqcup B) \neq \mu(A) + \mu(B)$.

$\mu(A \sqcup B \sqcup C) = \mu(A \sqcup B) + \mu(A \sqcup C) + \mu(B \sqcup C) - \mu(A) - \mu(B) - \mu(C)$.

—R. SORKIN.

$(\Omega, \mathcal{A}, \mu)$: QUANTUM MEASURE SPACE

Ω → SPACE OF HISTORIES

$\mathcal{A}$ → EVENT ALGEBRA

μ → QUANTUM MEASURE

$\mu : \mathcal{A} \longrightarrow \mathbb{R}^{+} \cup \{0\}$

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• VIEW … USED STOCHASTIC THEORY

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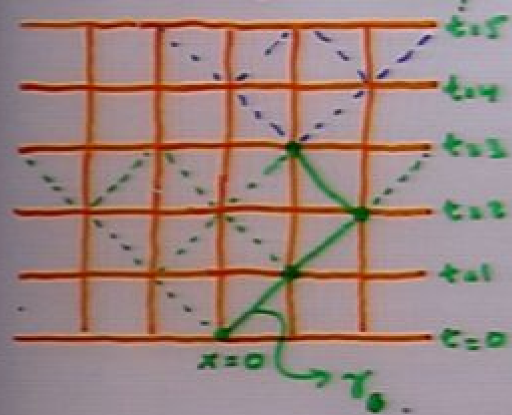
THE RANDOM WALK



$P(\gamma) = \frac{1}{8}$.

Ω = SET OF ∞ TIME PATHS.

$cyl(\gamma)$ = SET OF PATHS IN Ω WITH γ AS FIRST 3 STEPS.

$\therefore P(cyl(\gamma)) = P(\gamma) = \frac{1}{8}$.

$\{cyl(\gamma)\}$ GENERATES AN EVENT ALGEBRA $\Downarrow$ $\mathcal{A}$.

FINITE UNIONS, FINITE INTERSECTIONS & COMPLEMENTATION.

$P: \underbrace{\mathcal{A}}_{\text{ALL FINITE TIME EVENTS.}} \longrightarrow [0,1]$. & $(\Omega, \mathcal{A}, P)$ IS A PROB. SPACE

INFINITE TIME QN: PROBABILITY FOR RETURN TO ORIGIN.

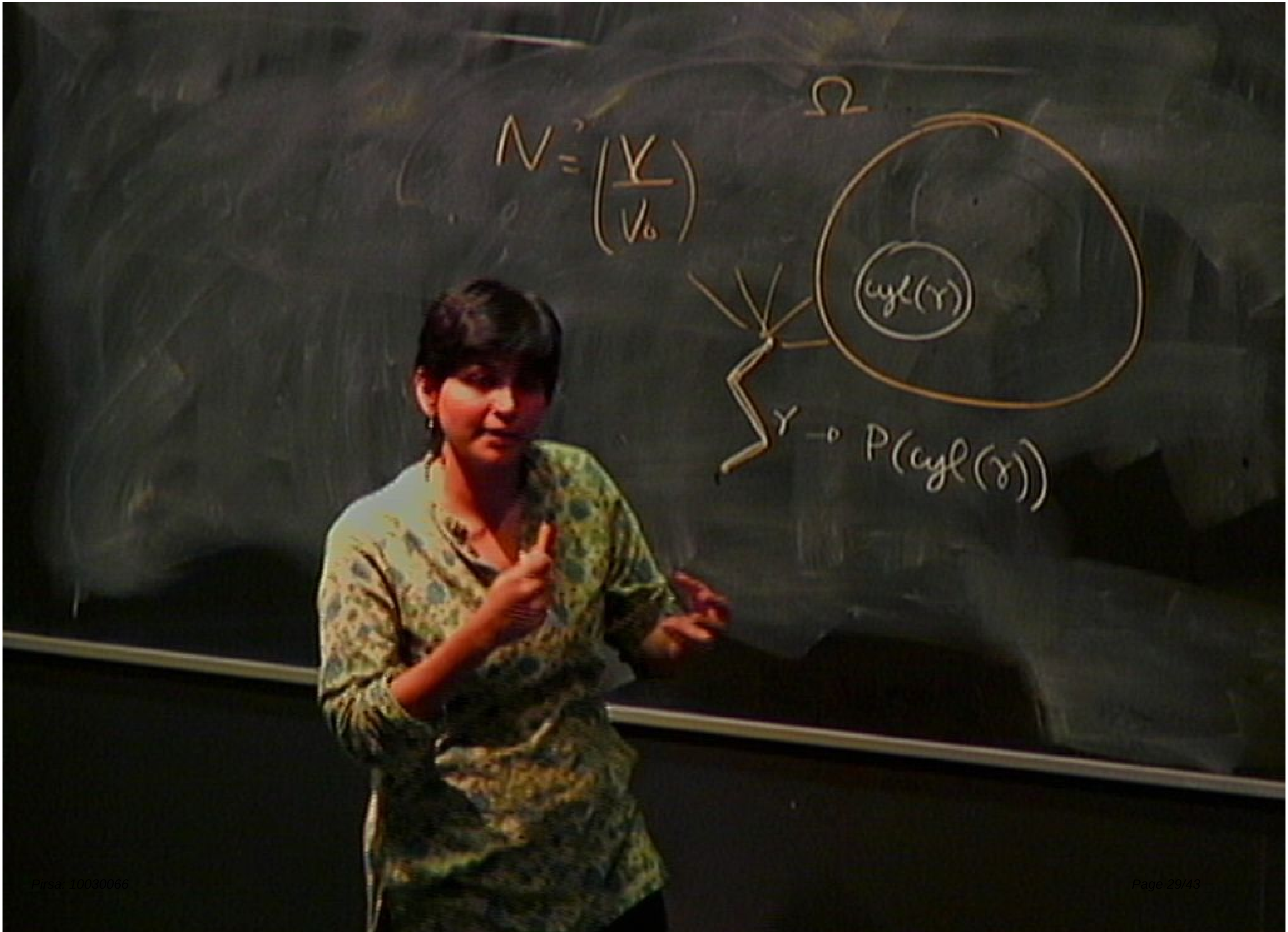
$\bigcup_{n=1}^{\infty} S[n]$. → FIRST RETURN AT TIME n.

$\notin \mathcal{A}$. $\in \Sigma$

EXTEND: $(\Omega, \mathcal{A}, P) \longrightarrow (\Omega, \Sigma, \bar{P})$ CARATHEO-DARY EXTN. THM

Σ → SIGMA ALGEBRA

OBSERVABLES ARE MEASURABLE SETS.


Ω
$N = \left(\frac{V}{V_0}\right)$
cyl(γ)
γ → P(cyl(γ))

EXTENSION OF A MEASURE.

$(\Omega, \mathcal{A}, \mu)$

$\mathcal{A}$: COLLECTION OF SUBSETS OF Ω CLOSED UNDER FINITE UNIONS, INTERSECTIONS & COMPLEMENTATION.

$\Sigma(\mathcal{A})$: SMALLEST Σ-ALGEBRA CONTAINING $\mathcal{A}$. IT IS CLOSED UNDER COUNTABLE SET OPERATIONS, $\cap, \cup, c$.

IF $(\Omega, \Sigma(\mathcal{A}), \bar{\mu})$ IS AN EXTENSION OF $(\Omega, \mathcal{A}, \mu)$ THEN

$\bar{\mu}|_{\mathcal{A}} = \mu$.

IF $\bar{\mu}$ IS UNIQUE THEN $(\Omega, \mathcal{A}, \mu)$ HAS A UNIQUE EXTENSION $(\Omega, \Sigma, \bar{\mu})$

EXTENSION OF A MEASURE.

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QUANTU

$$\underbrace{(\Omega, \mathcal{A},}_{\text{SHARED WITH CLASSICAL SYSTEMS}} \mu_v)$$

$\mu_v \longrightarrow$ QUANTUM MEASURE

EG: DISCRETE TIME EVOLUTION FOR FINITE UNITARY SYSTEMS

$\gamma = (\gamma_1, \gamma_2, \ldots \gamma_n) \quad \gamma_i \in \{0, 1, \ldots N\}.$

$\{Cyl(\gamma)\}$ GENERATES $\mathcal{A}$.

$$\mu_v(\gamma) \equiv (U^{\dagger})^n \hat{C}_\gamma |\psi_0\rangle$$

$\hat{C}_\gamma \longrightarrow$ CLASS OPERATOR

$$\hat{C}_\gamma = \hat{P}_{\gamma_n} \hat{U} \hat{P}_{\gamma_{n-1}} \hat{U} \ldots \hat{P}_{\gamma_1}$$

FINITE TIME QUESTIONS CAN $\therefore$ BE ANSWERED

BUT WHAT ABOUT INFINITE TIME?

IN OTHER WORDS DOES $(\Omega, \mathcal{A}, \mu_v)$ <u>EXTEND</u> TO $(\Omega, \Sigma, \overline{\mu_v})$?

DECOHERENCE FUNCTIONAL:

(i) $D(\alpha \sqcup \beta, \gamma \sqcup \delta) = D(\alpha, \gamma) + D(\beta, \gamma) + D(\alpha, \delta) + D(\beta, \delta)$

(ii) $D(\alpha, \beta) = D^*(\beta, \alpha)$

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μ_v: QUANTUM MEASURE

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(i) $D(\alpha \sqcup \beta, \gamma \sqcup \delta) = D(\alpha, \gamma) + D(\beta, \gamma) + D(\alpha, \delta) + D(\beta, \delta)$

(ii) [illegible]$, \beta) = D^*(\beta, \alpha)$

THE DECOHERENCE FUNCTIONAL

16

$$D : \mathcal{A} \times \mathcal{A} \longrightarrow \mathbb{C}.$$

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(iii) STRONGLY POSITIVE – $M_{ij} = D(\alpha_i, \alpha_j)$
HAS ~~≢~~ NON-NEGATIVE E-VALUES FOR ANY
FINITE COLLECTION $\{\alpha_i\}$

STD. QM:

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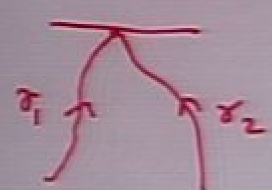

$$\mu(\alpha) = D(\alpha, \alpha) \geq 0.$$

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THE QUANTUM VECTOR MEASURE

$$\mu_V : \mathcal{A} \longrightarrow \underbrace{\mathcal{H}_{\mathcal{A}}}_{}$$

HISTORIES HILBERT SPACE

– DOWKER, JOHNSTON & SORKIN

V : VECTOR SPACE OF FNS. $u : \mathcal{A} \to \mathbb{C}$

S.T. $u(\alpha) \neq 0$ ONLY FOR FINITE # of α.

$$\langle u, v \rangle = \sum_{\alpha} \sum_{\beta} D(\alpha, \beta)\, u^*(\alpha)\, v(\beta).$$

↳ $\mathcal{H}_{\mathcal{A}}$ OBTAINED AFTER QUOTIENTING BY ZERO NORM VECTORS & CAUCHY COMPLETING THE SPACE

$$\mu_V(\alpha) \equiv [\chi_\alpha] \in \mathcal{H}_{\mathcal{A}}$$

↳ CHARACTERISTIC FUNCTION.

$$\|\mu_V(\alpha)\|^2 = D(\alpha, \alpha).$$

<u>μ_V IS ADDITIVE</u>.

<u>AND</u>...

$$(\Omega, \mathcal{A}, \mu_V) \xrightarrow{\text{EXTENDS}} (\Omega, \Sigma, \overline{\mu_V})$$

<u>IF</u> μ_V SATISFIES CERTAIN CONVERGENCE CONDNS.

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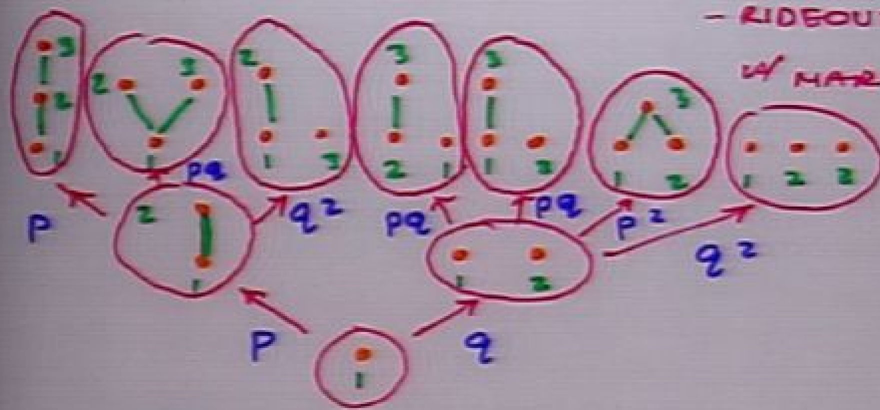
GROWTH MODELS FOR CAUSAL SETS

– RIDEOUT & SORKIN
W/ MARTIN & O'CONNOR



$p + q = 1$.

PROCESS GENERATES LABELLED CAUSAL SETS.

$\{cyl(C_n)\}$ GENERATES $\mathcal{A}$.

& IF μ IS A PROBABILITY MEASURE, THEN

$(\Omega, \Sigma, \bar{\mu})$ IS THE EXTENSION.

↳ SET OF LABELLED CAUSAL SETS

COVARIANT SETS OBTAINED FROM A QUOTIENT* OF Σ, NOT $\mathcal{A}$.

EXTENSION OF μ ON $\mathcal{A}$ TO A $\bar{\mu}$ ON Σ CRUCIAL FOR COVARIANT OBSERVABLES.

COMPLEX PERCOLATION

P & Q COMPLEX $\quad P+Q=1.$

IF $D(C_1, C_2) = A^*(C_1)\,A(C_2)$ THEN $\mathcal{H}_A \cong \mathbb{C}$.

EXTENSION OF μ_v ON $\mathcal{A}$ TO $\bar{\mu}_v$ ON Σ REQUIRES THAT

$$|\mu_v|(\alpha) = \sup_{\pi(\alpha)} \sum_i \|\mu_v(\alpha_i)\| < \infty$$

BOUNDED VARIATION.

EG: $|p| > 1$: $\quad \Omega = (n\text{-chain}) \sqcup (n\text{-chain})^c.$

$$\mu_v(\,\vdots\,) = p^{n-1}$$

$\therefore |\mu_v|(\Omega) > |p|^{n-1}.$

OR: $|q| > 1 \quad \Omega = (n\text{-antichain}) \sqcup (n\text{-antichain})^c$

$$\mu_v(\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot) = q^{n(n-1)}$$

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FOR $|p|, |q| < 1$ AS WELL μ_v IS NOT OF BOUNDED VARIATION. EXCEPT FOR P, Q REAL.

DOES THIS MEAN THAT COMPLEX PERCOLATION CANNOT BE MADE COVARIANT ??

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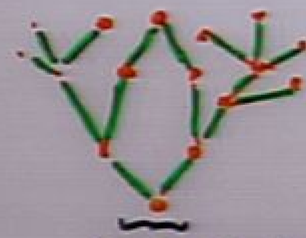
SOME COVARIANT OBSERVABLES CAN BE FOUND, EVEN THOUGH THERE IS NO EXTENSION



SINGLE BOTTOM ELEMENT. : ORIGINARY.

α = SET OF CAUSAL SETS IN Ω WHICH ARE ORIGINARY

$$\mu_v(\alpha) = \prod_{n=1}^{\infty}(1-q^n)$$: CONVERGES FOR $|q|<1$.

β = SET OF CAUSAL SETS IN Ω WITH A "STEM"

STEM OR PAST SET. S.THAT THE MAXIMAL ELEMENT OF THE STEM IS A POST.

$$\mu_v(\beta) = p^2 q \times \prod_{n=1}^{\infty}(1-q^n)$$

⋮

- PERHAPS BOUNDED VARIATION IS TOO STRONG ?
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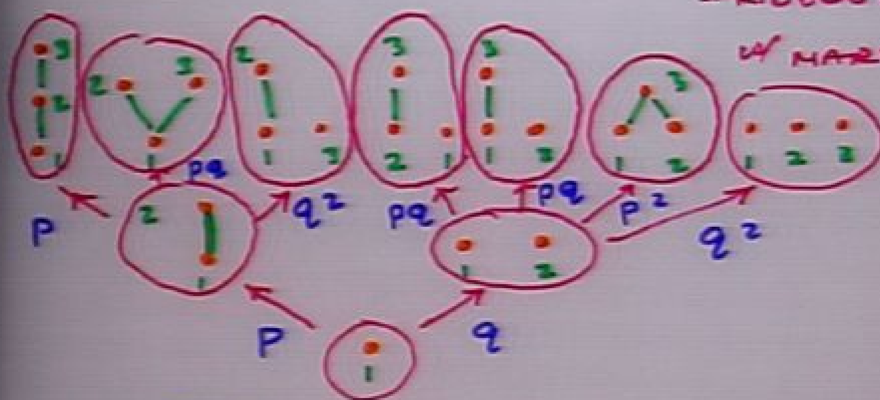

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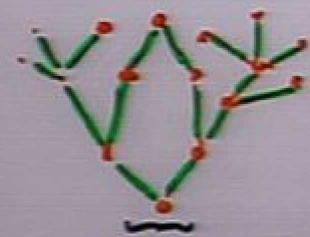

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$\vdots$

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2
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