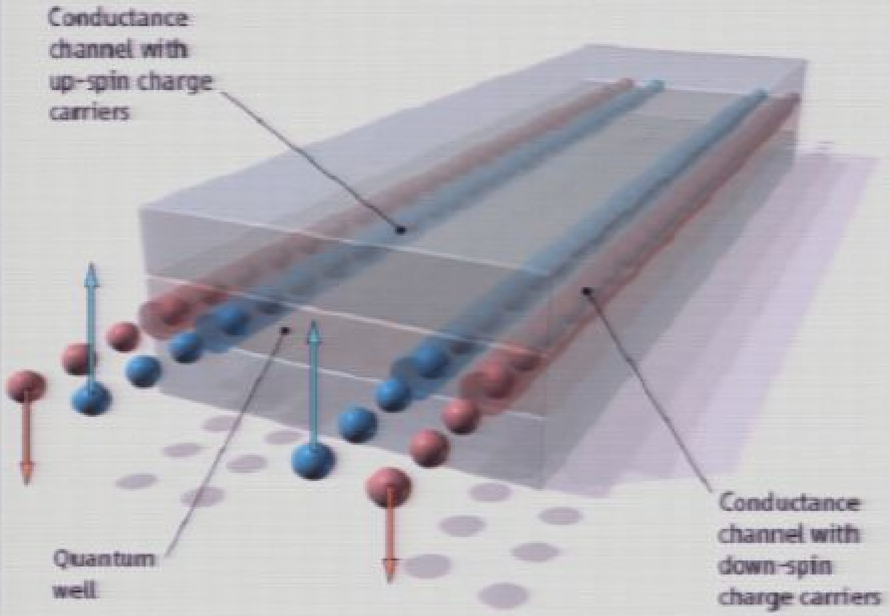
Title: Topological insulators and topological superconductors

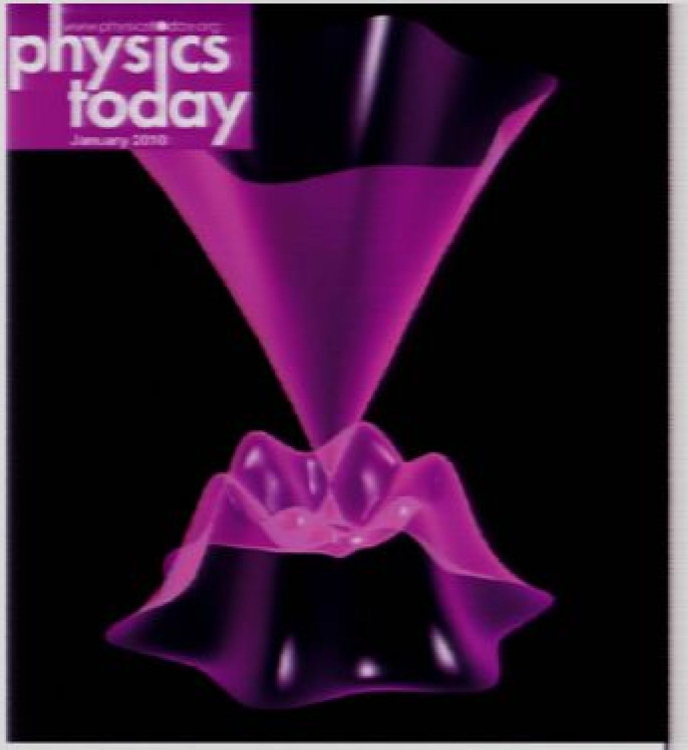
Date: Mar 31, 2010 02:00 PM

URL: http://pirsa.org/10030063

Abstract: Recently, a new class of topological states has been theoretically predicted and experimentally realized. The topological insulators have an insulating gap in the bulk, but have topologically protected edge or surface states due to the time reversal symmetry. In two dimensions the edge states give rise to the quantum spin Hall (QSH) effect, in the absence of any external magnetic field. I shall review the theoretical prediction[1] of the QSH state in HgTe/CdTe semiconductor quantum wells, and its recent experimental observation[2]. The edge states of the QSH state supports fractionally charged excitations[3]. The QSH effect can be generalized to three dimensions as the topological magneto-electric effect (TME) of the topological insulators[4]. Bi2Te3, Bi2Se3 and Sb2Te3 are theoretically predicted to be topological insulators with a single Dirac cone on the surface[5]. I shall present a realistic experimental proposals to observe the magnetic monopoles on the surface of topological insulators[6]. Topological superconductors and superfluid have been theoretically proposed recently [7], in both two and three dimensions. They have a full pairing gap in the bulk, and their mean field Hamiltonian look identical to that of the topological insulators. However, the gapless surface states consists of a single Majorana cone, containing only half the degree of freedom compared to the single Dirac cone on the surface of a topological insulators. I shall discuss their physics properties and the search for these novel states in real materials. [1] A. Bernevig, T. Hughes and S. C. Zhang, Science, 314, 1757, (2006) [2] M. Koenig et al, Science 318, 766, (2007) [3] J. Maciejko, Chaoxing Liu, Yuval Oreg, Xiao-Liang Qi, Congjun Wu, and Shou-Cheng Zhang, , Phys. Rev. Lett. {\bf 102}, 256803 (2009). [4] Xiao-Liang Qi, Taylor Hughes and Shou-Cheng Zhang, Phys. Rev B. 78, 195424 (2008) [5] Haijun Zhang, Chao-Xing Liu, Xiao-Liang Qi, Xi Dai, Zhong Fang, and Shou-Cheng Zhang, Nature Physics 5, 438 (2009). [6] Xiao-Liang Qi, Run-Dong Li, Jiadong Zang and Shou-Cheng Zhang, Science 323, 1184 (2009). [7] Xiao-Liang Qi, Taylor L. Hughes, Srinivas Raghu and Shou-Cheng Zhang, Phys. Rev. Lett. 102, 187001 (2009)

Topological insulators and superconductors





Topological insulators

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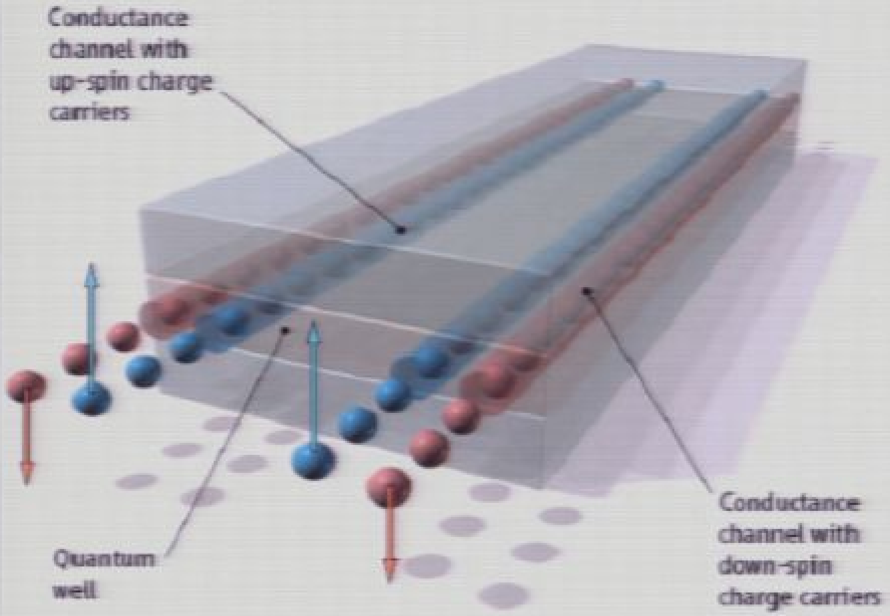

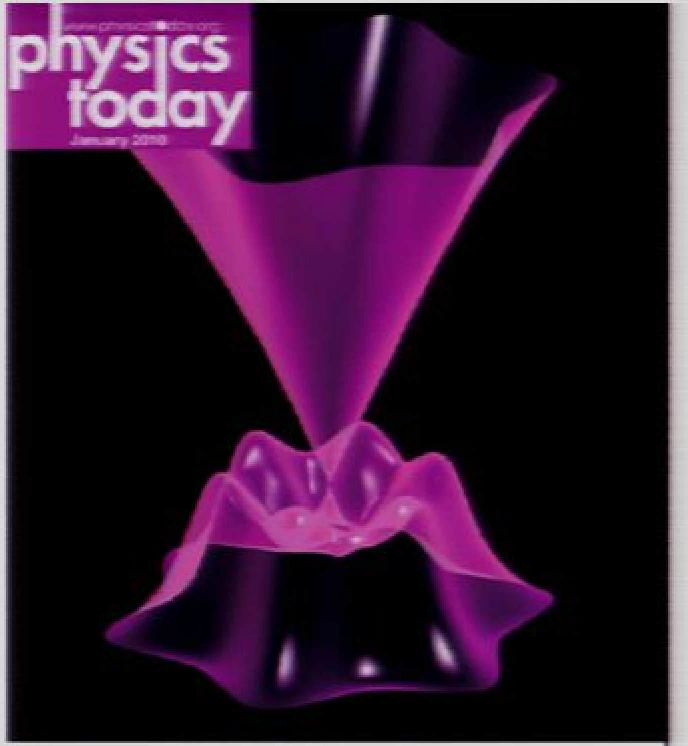

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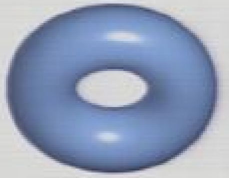

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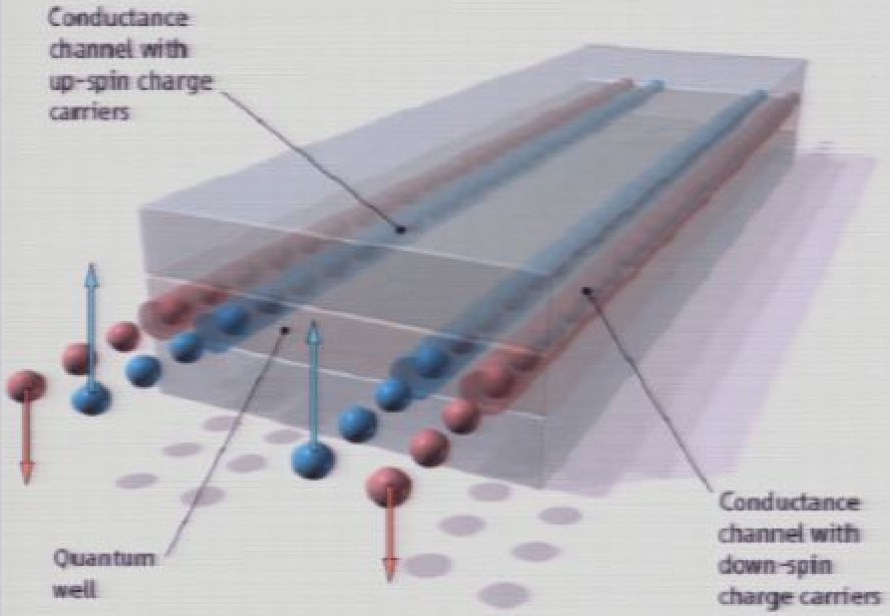

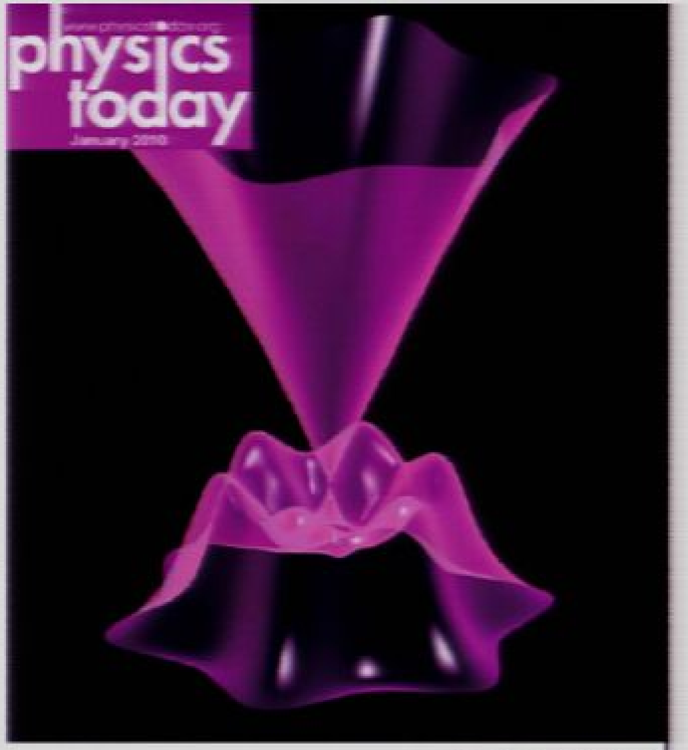

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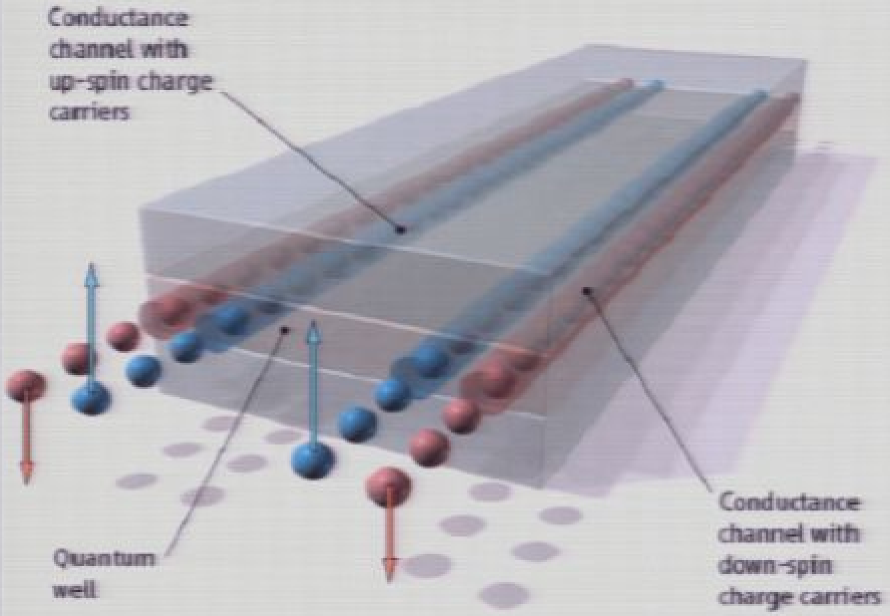

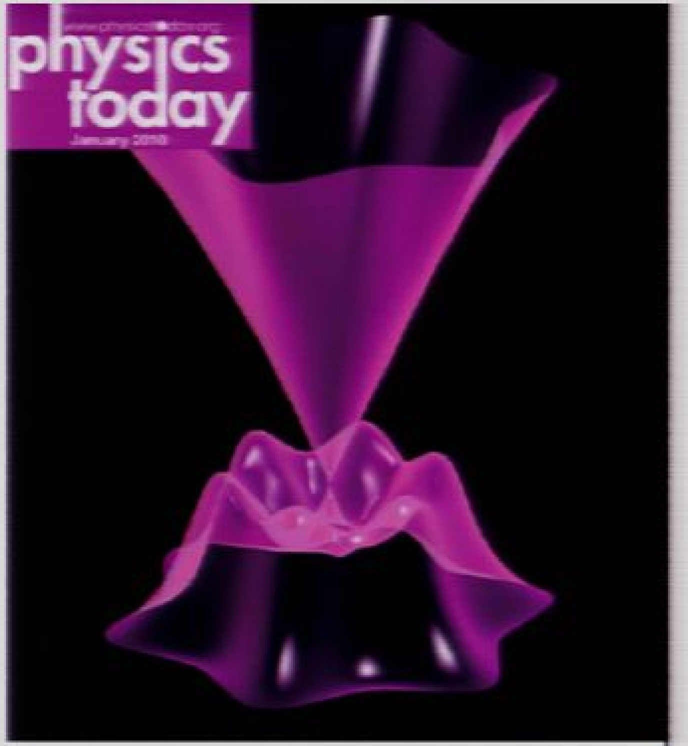

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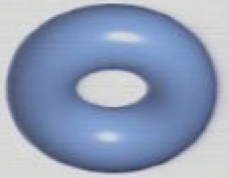

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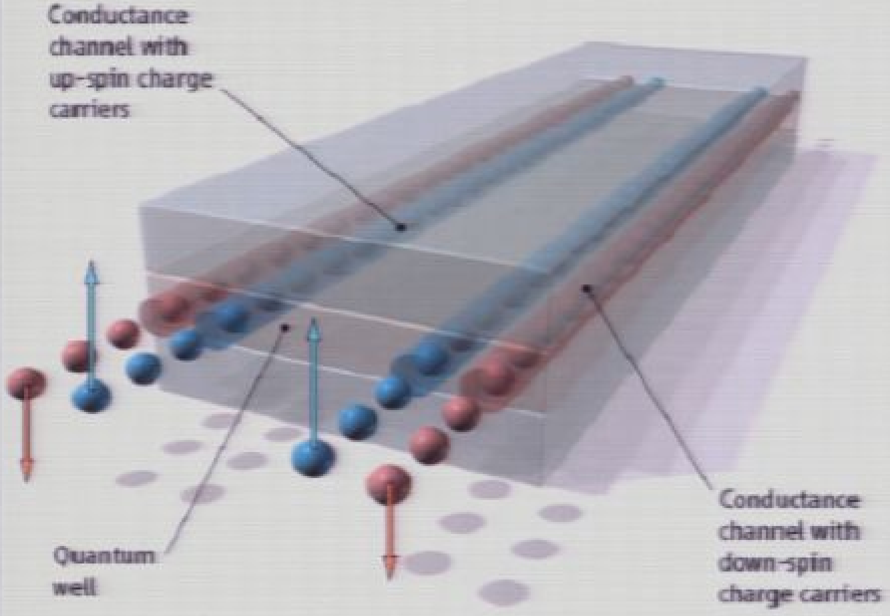

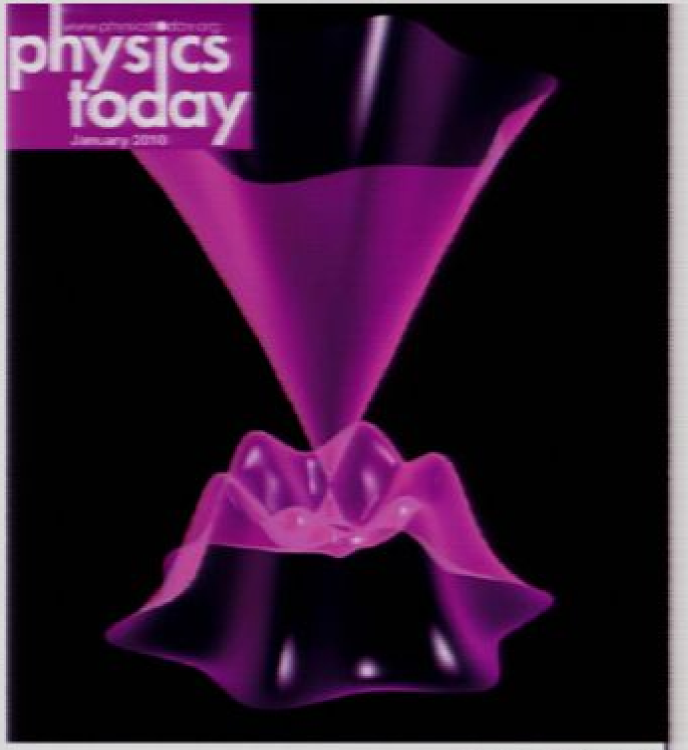

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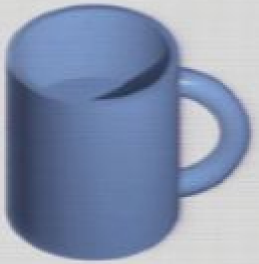

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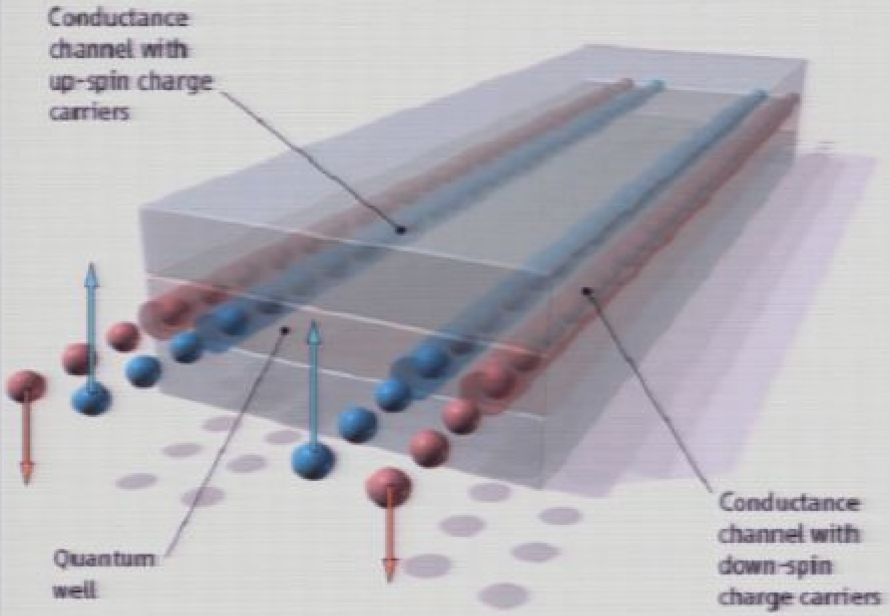

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Colloborators

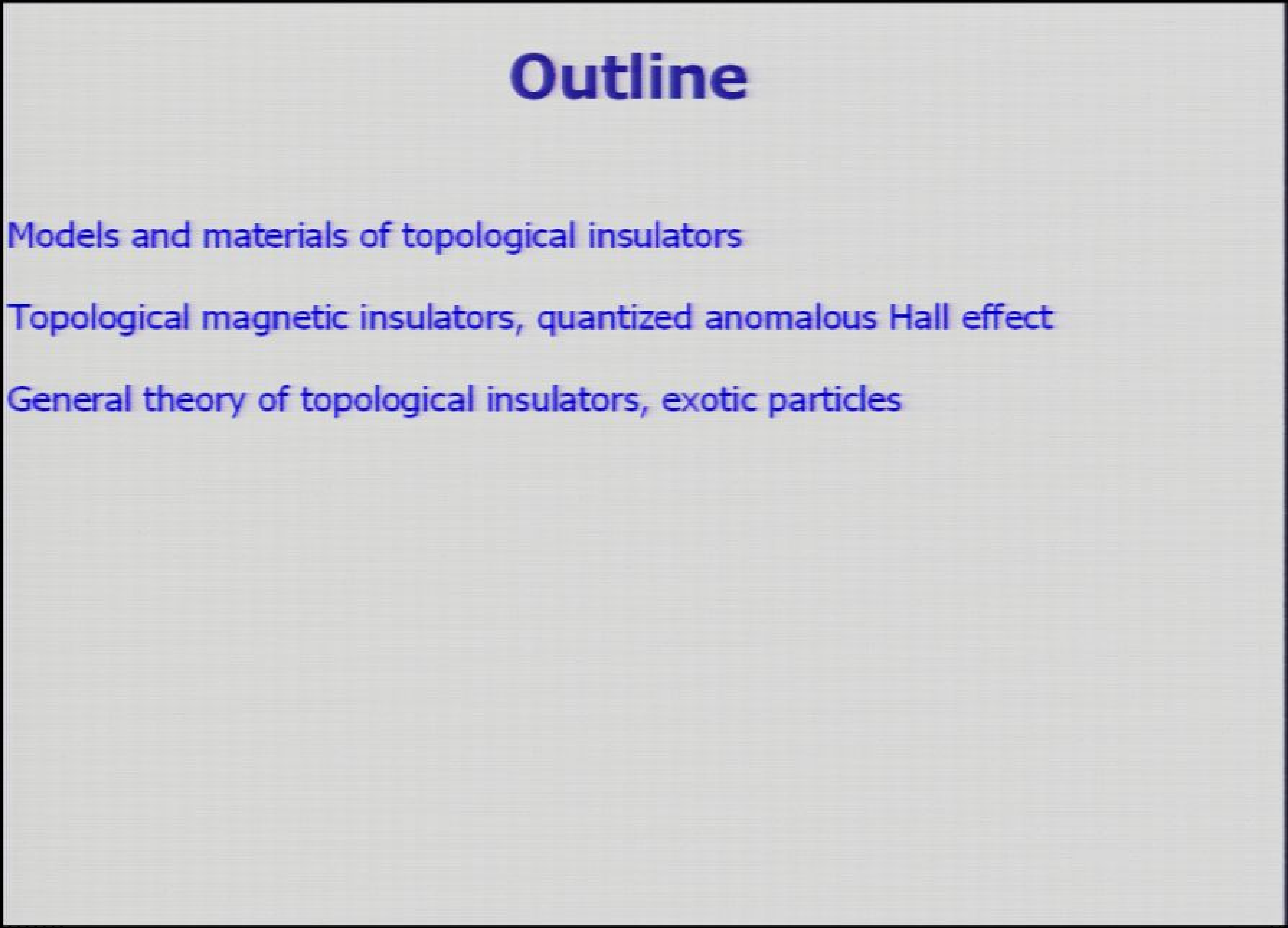
Stanford group: Xiaoliang Qi, Andrei Bernevig, Congjun Wu, Chaoxing Liu, Taylor Hughes, Sri Raghu, Suk-bum Chung

Stanford experimentalists: Yulin Chen, Ian Fisher, ZX Shen, Yi Cui, Aharon Kapitulnik, ...

Wuerzburg colleagues: Laurens Molenkamp, Hartmut Buhmann, Markus Koenig Ewelina Hankiewicz, Bjoern Trauzettle

IOP colleagues: Zhong Fang, Xi Dai, Haijun Zhang, ...

Tsinghua colleagues: Qikun Xue, Jinfeng Jia, Xi Chen,...



Outline

Models and materials of topological insulators

Topological magnetic insulators, quantized anomalous Hall effect

General theory of topological insulators, exotic particles

The search for new states of matter

The search for new elements led to a golden age of chemistry.

The search for new particles led to the golden age of particle physics.

In condensed matter physics, we ask what are the fundamental states of matter?

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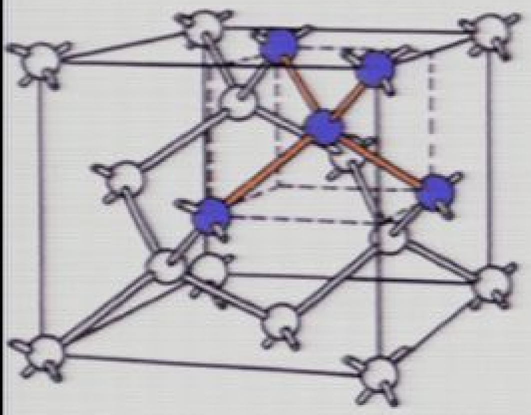
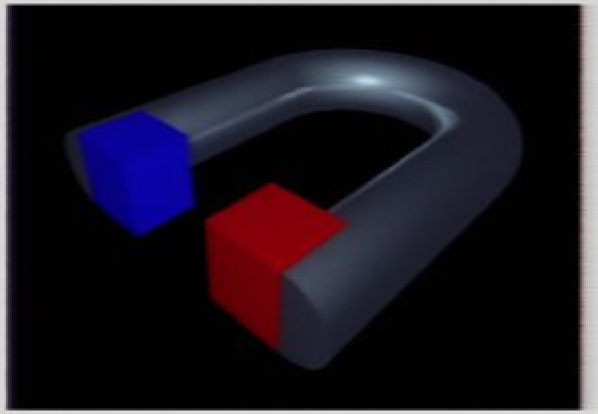


Crystal: Broken translational symmetry



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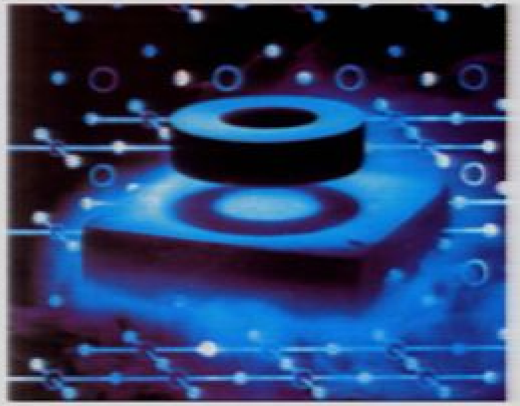

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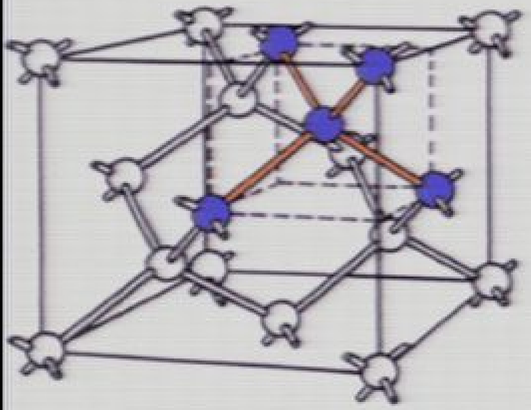

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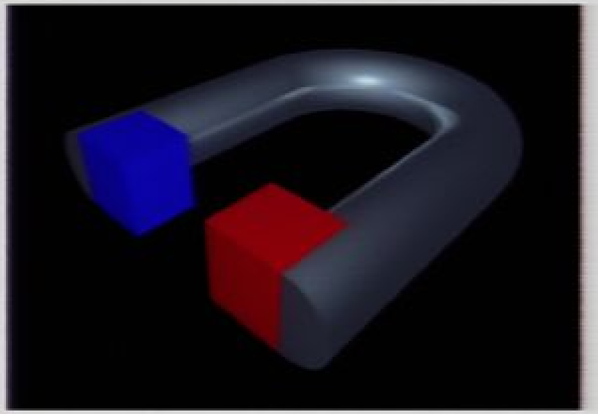

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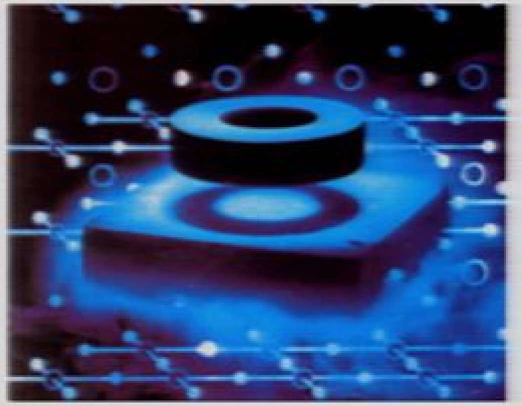

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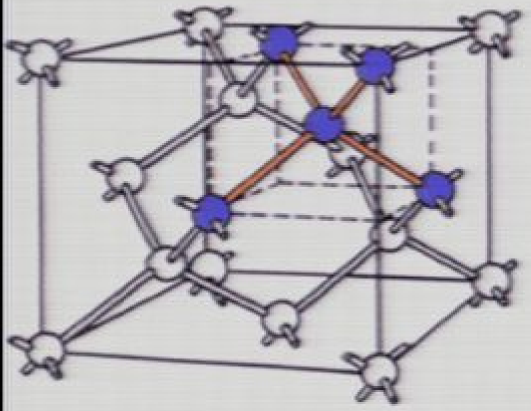

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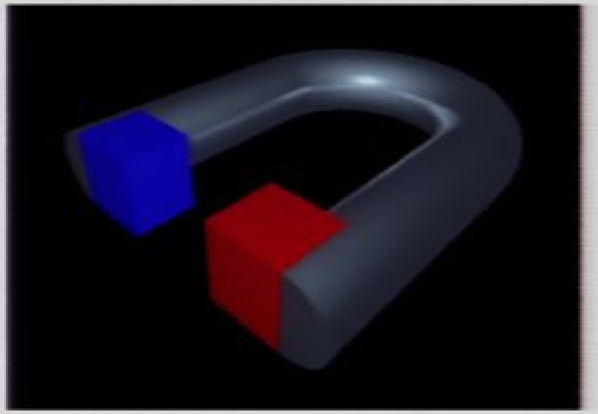

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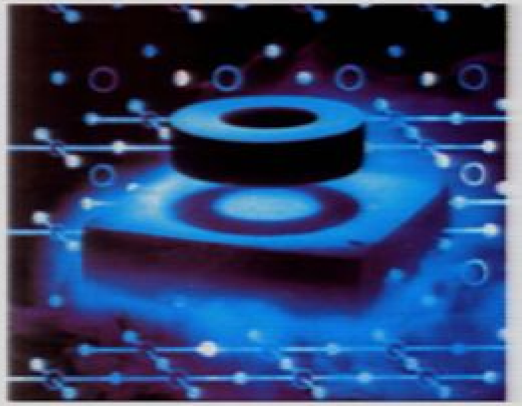

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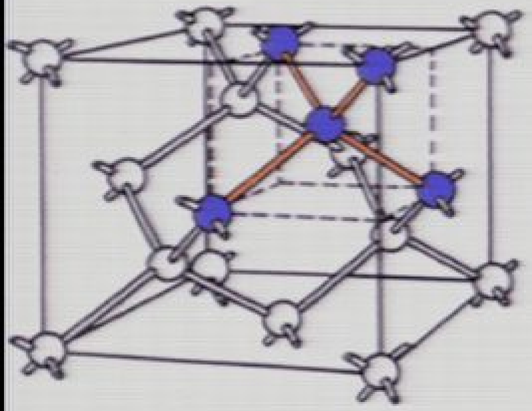

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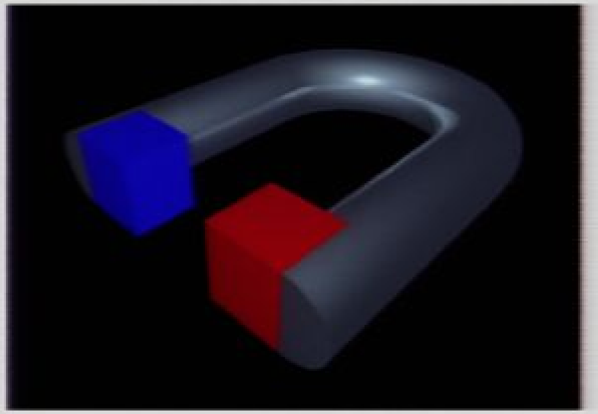

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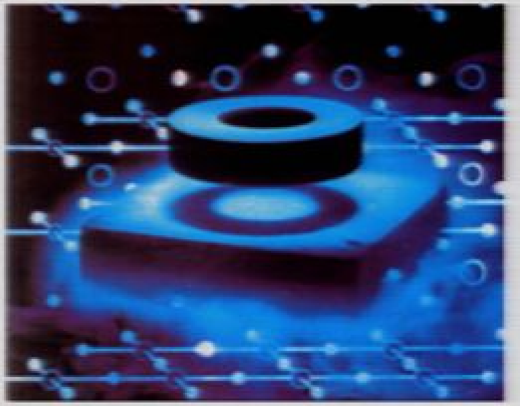

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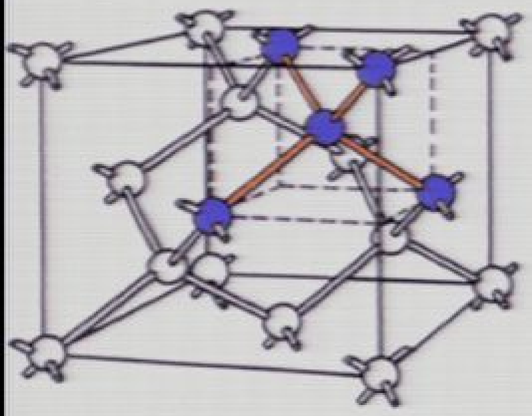

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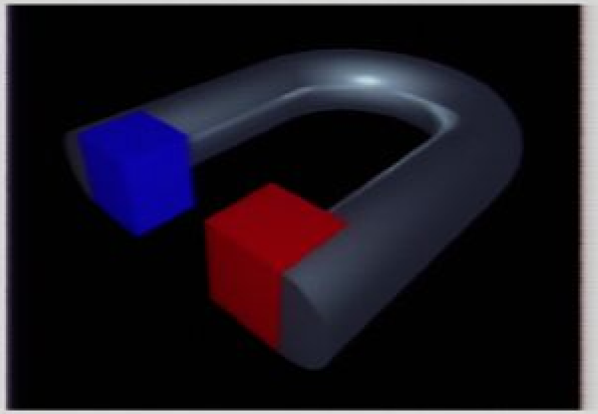

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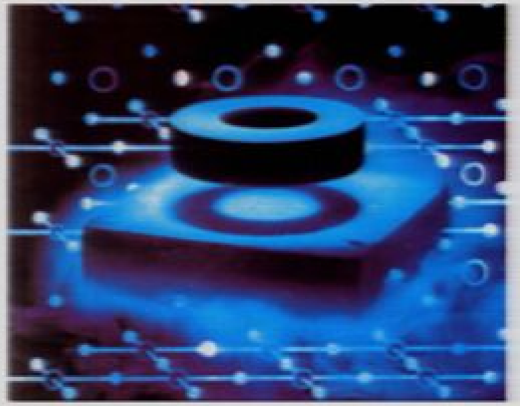

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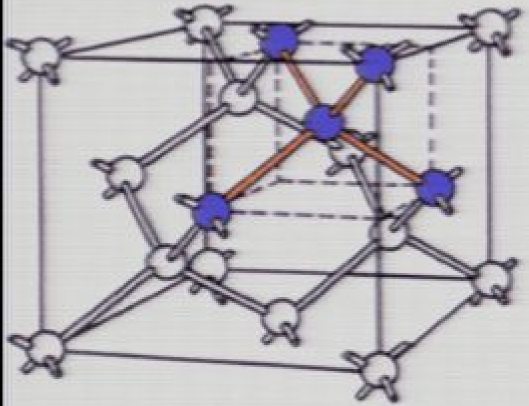

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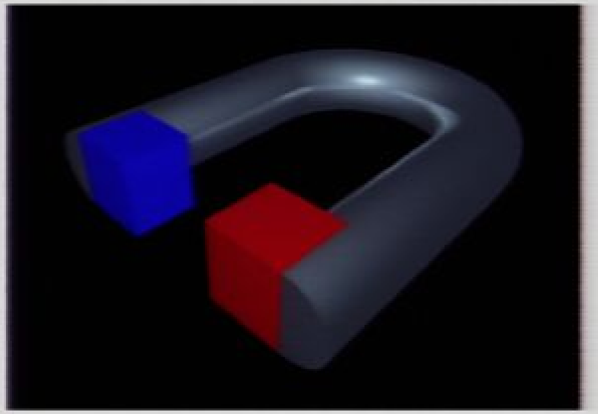

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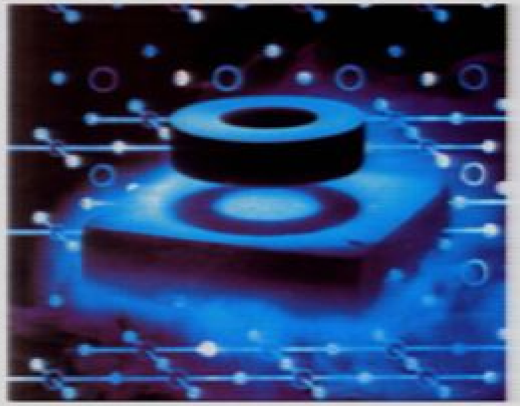

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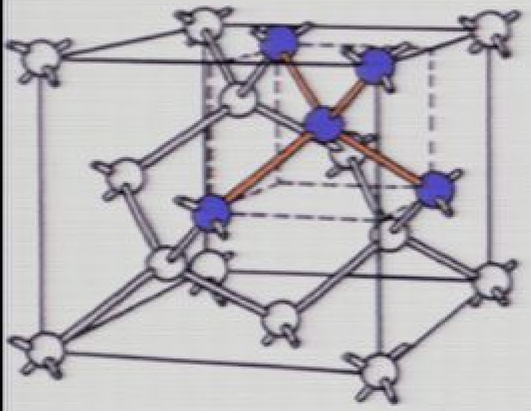

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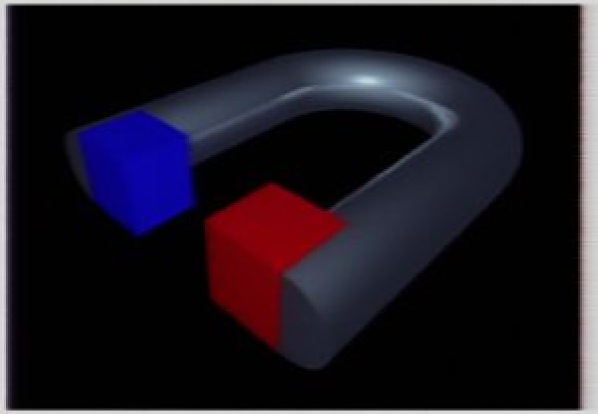

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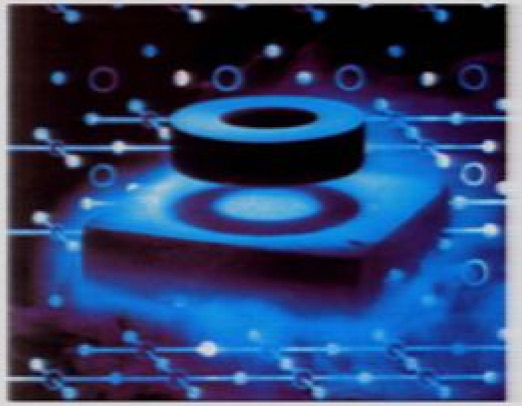

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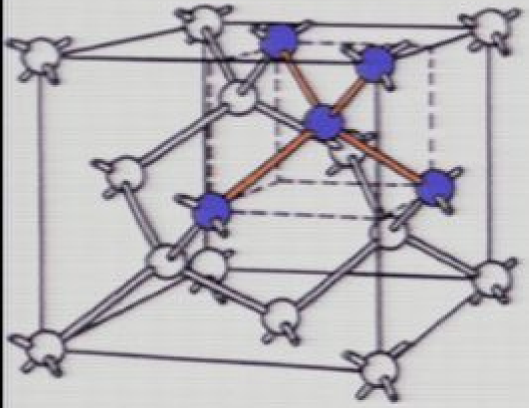

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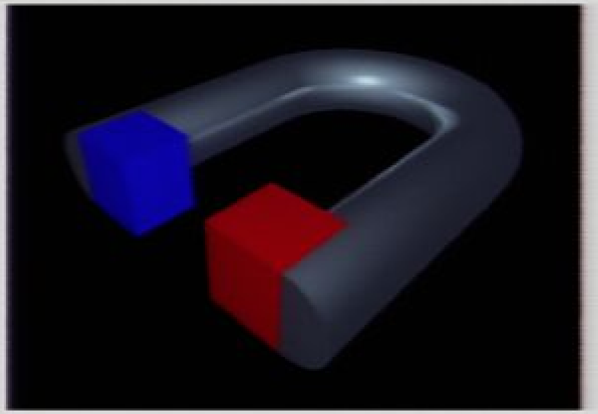

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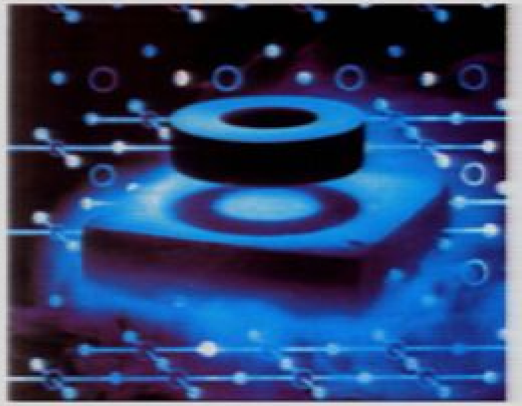

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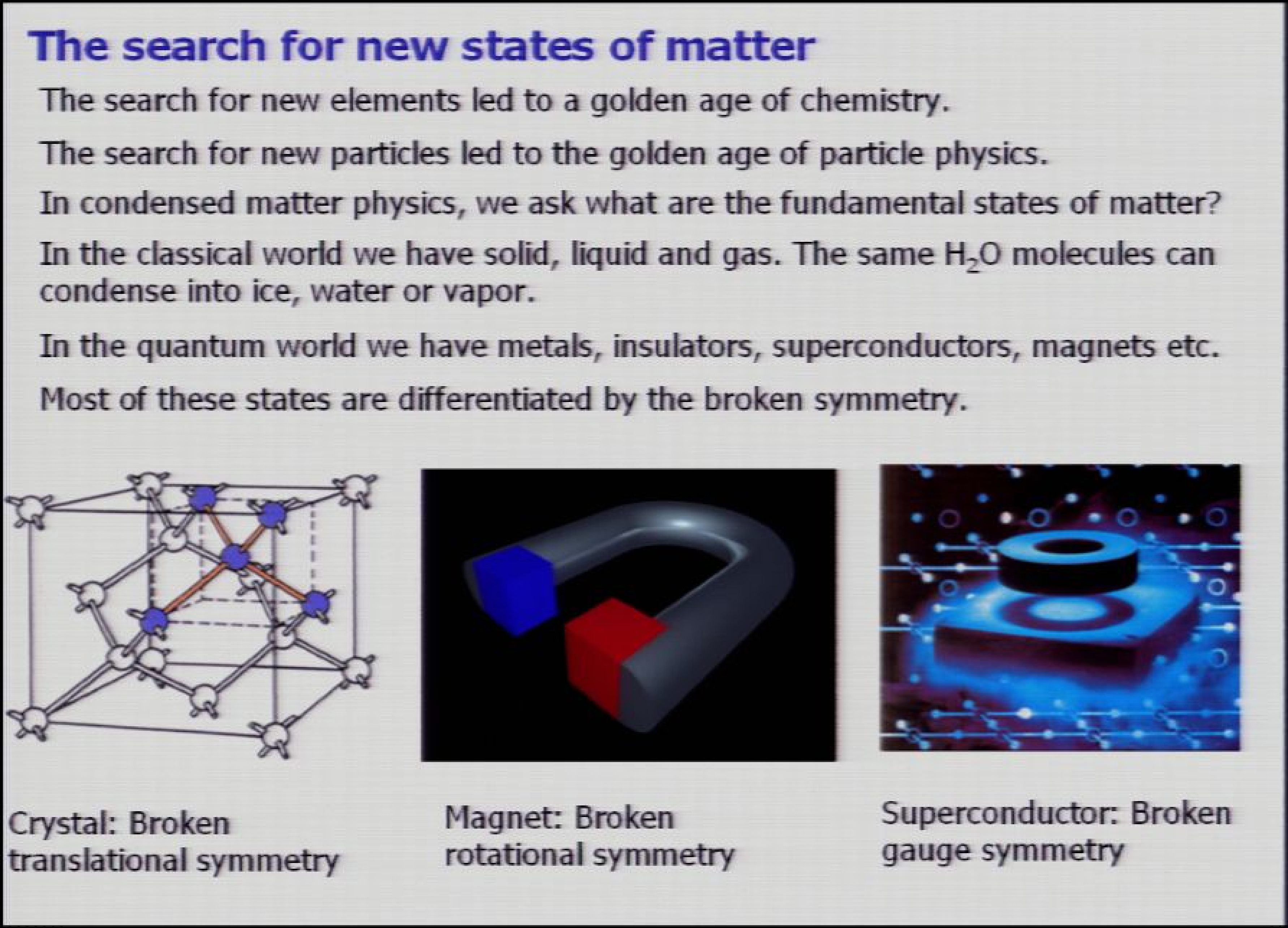

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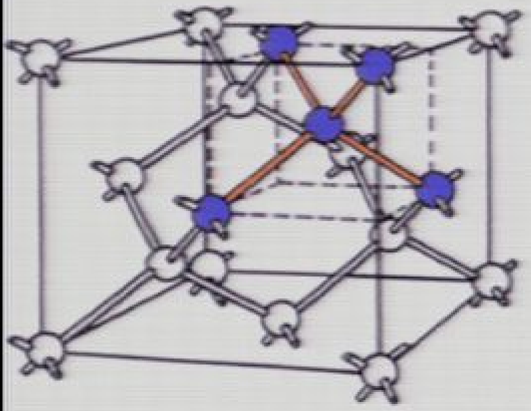

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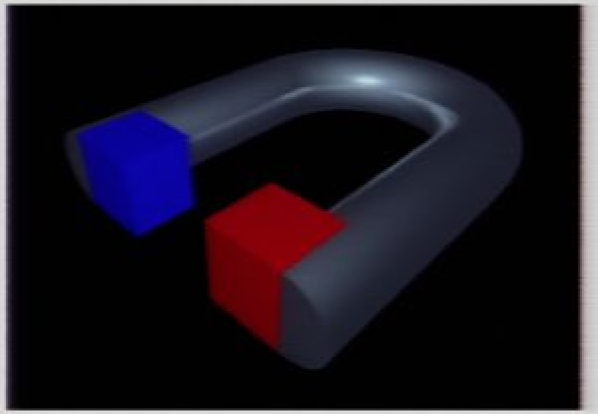

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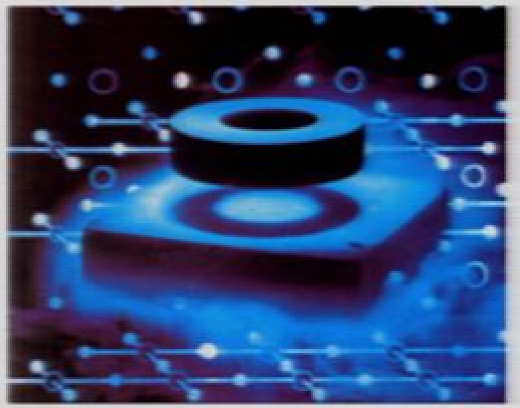

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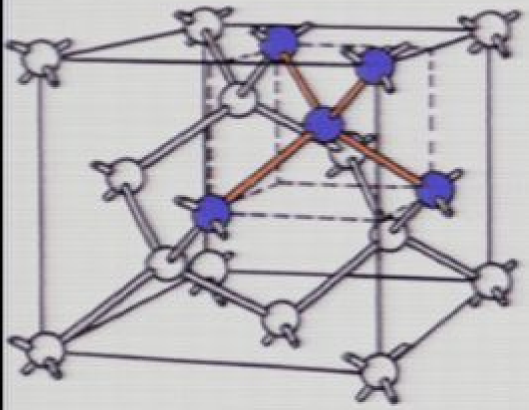

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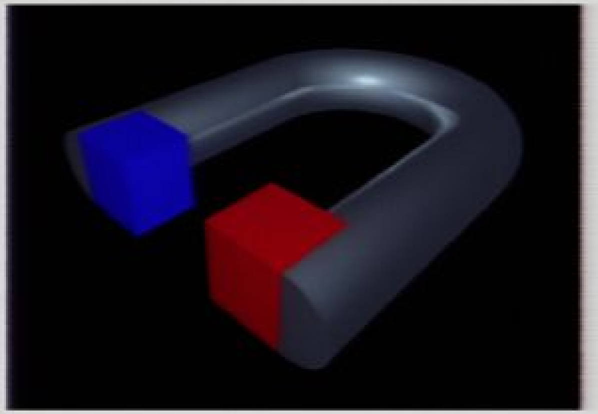

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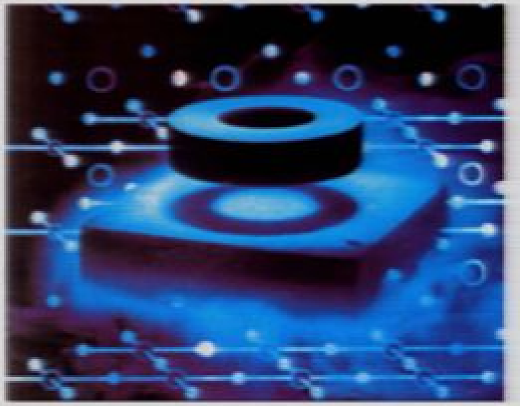

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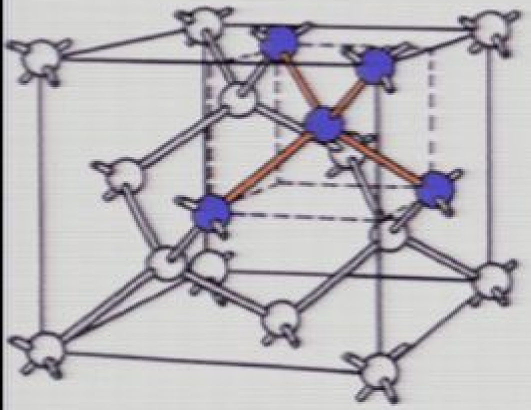

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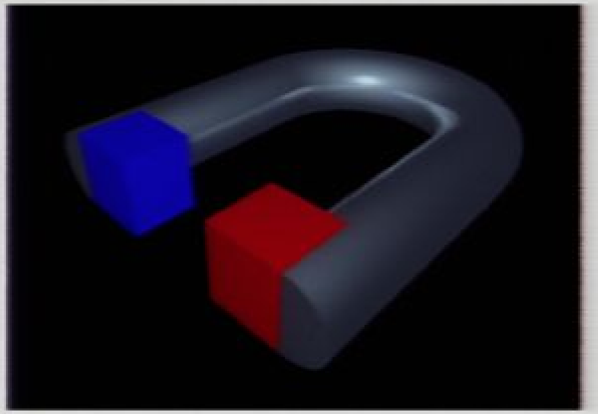

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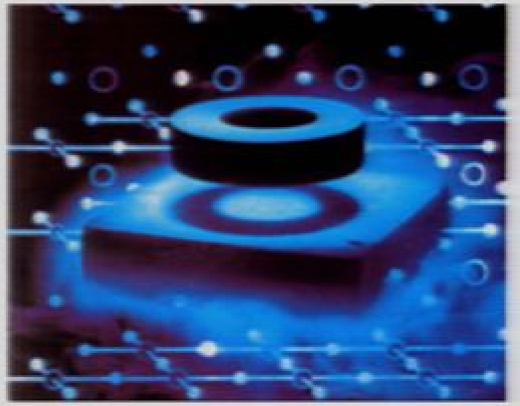

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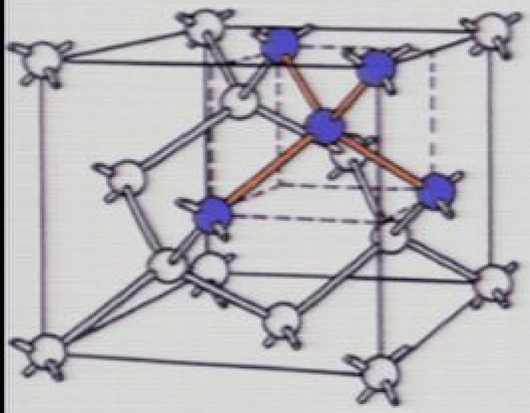

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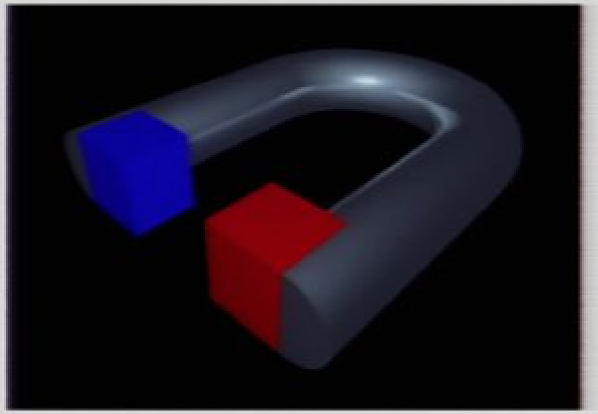

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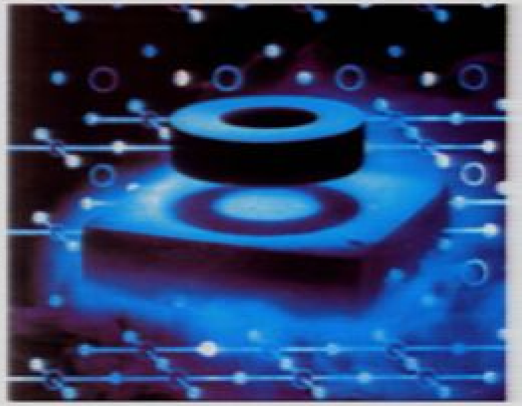

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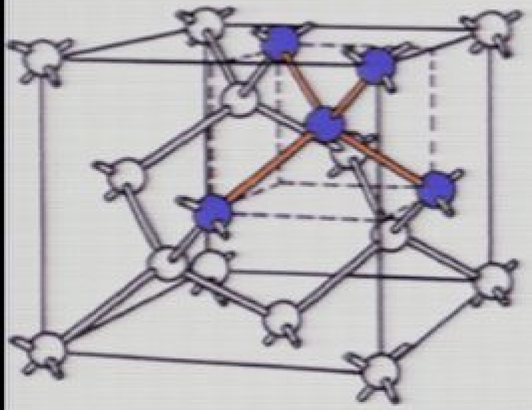

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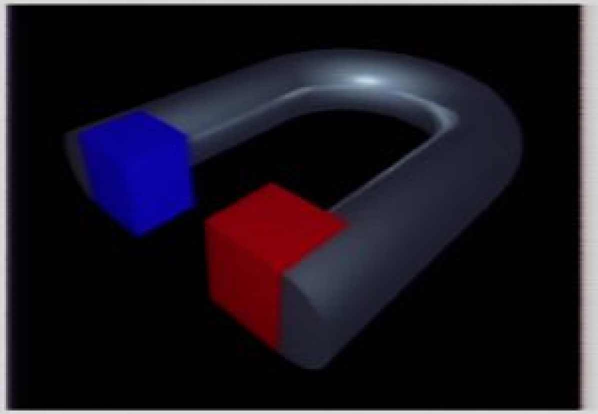

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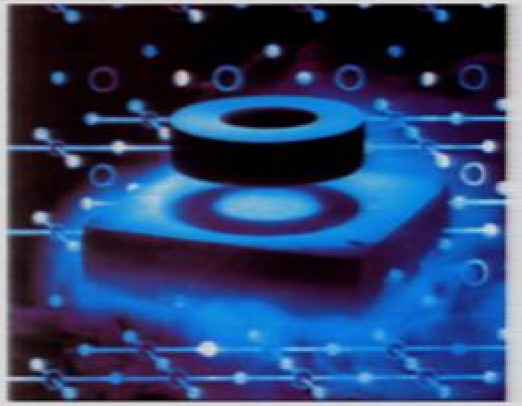

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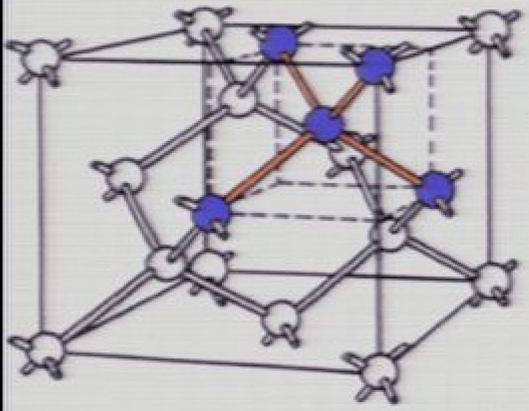

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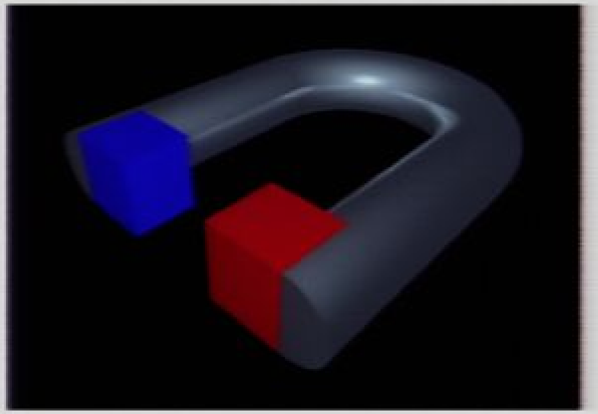

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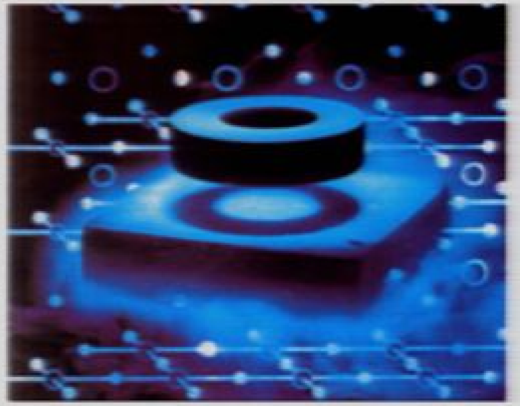

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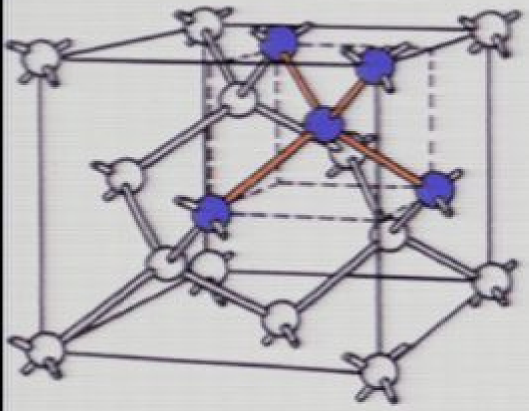

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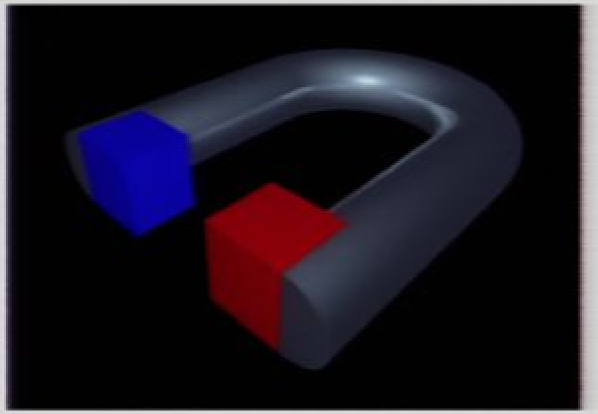

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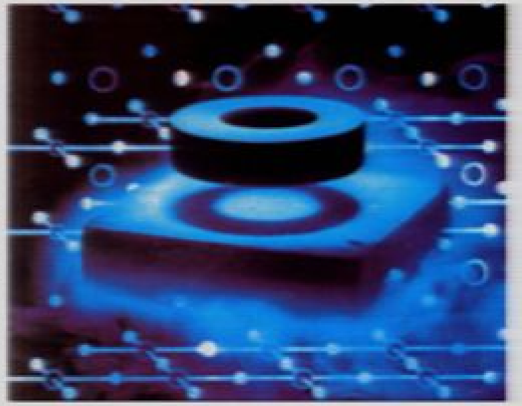

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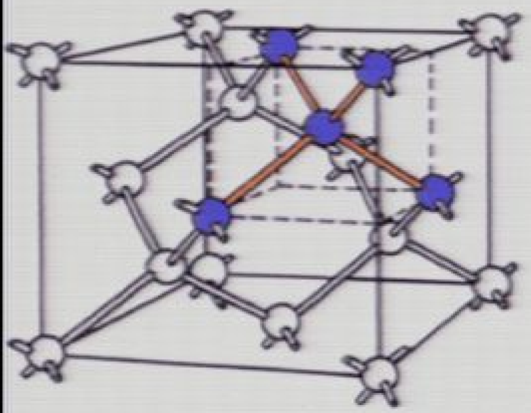

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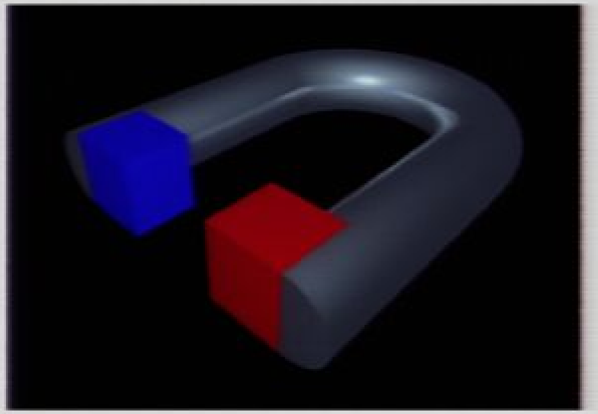

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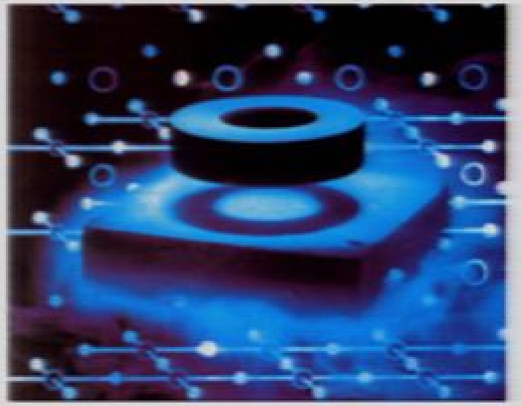

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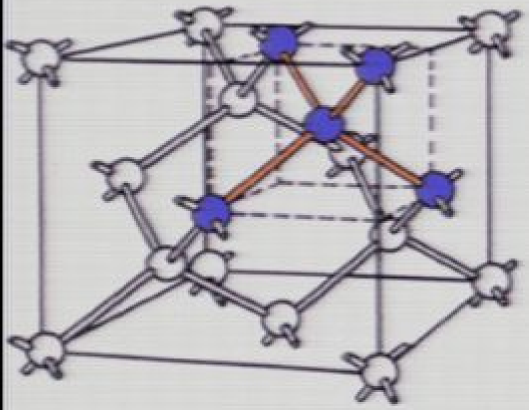

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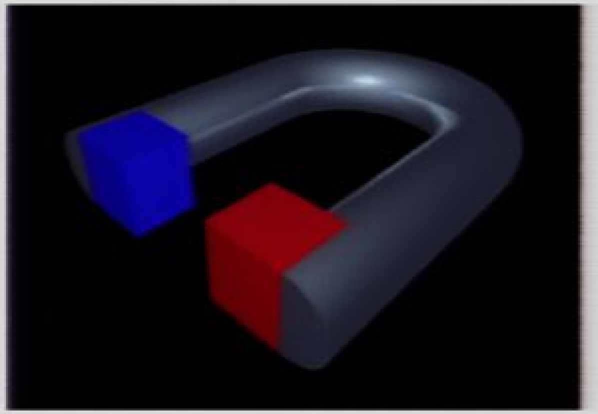

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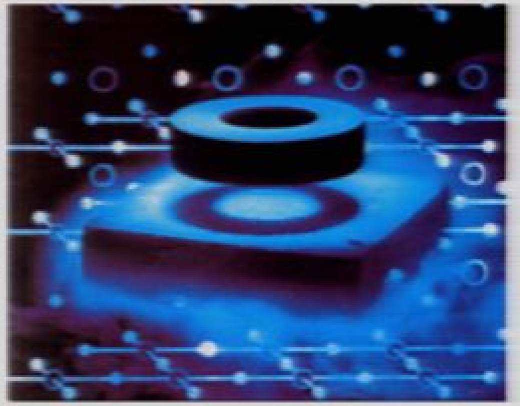

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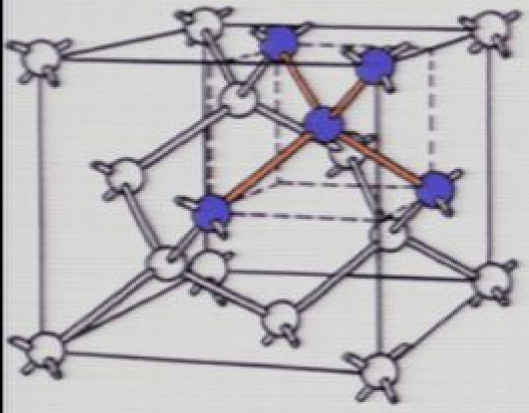

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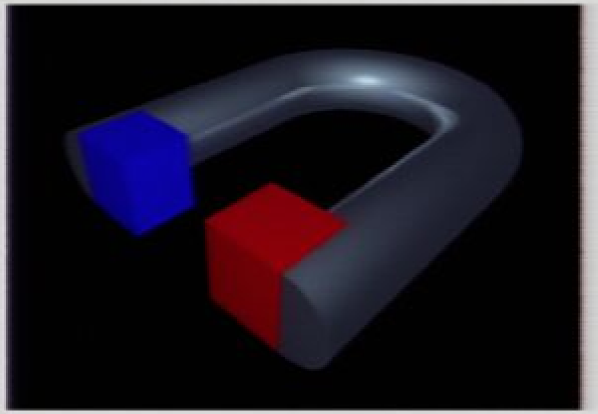

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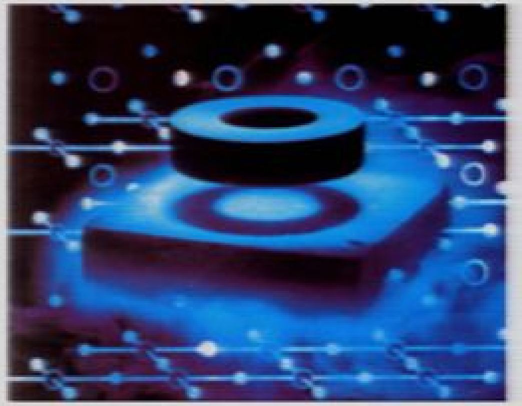

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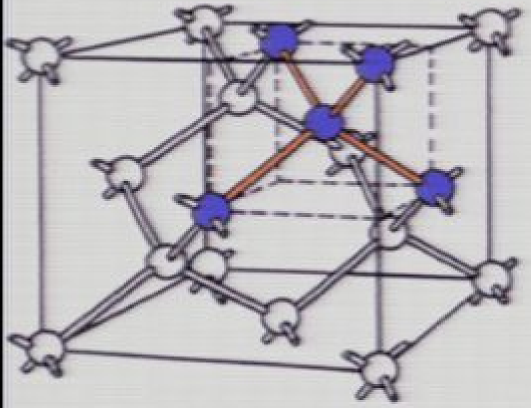

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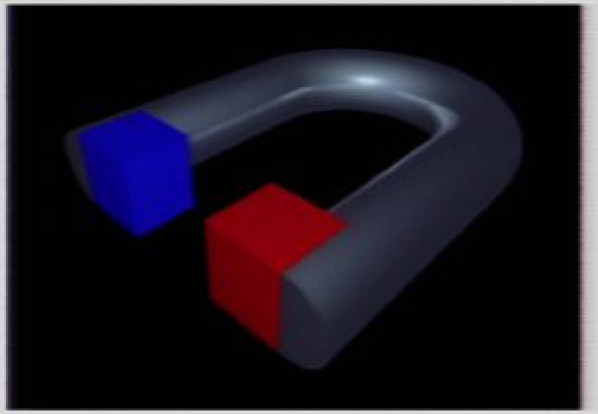

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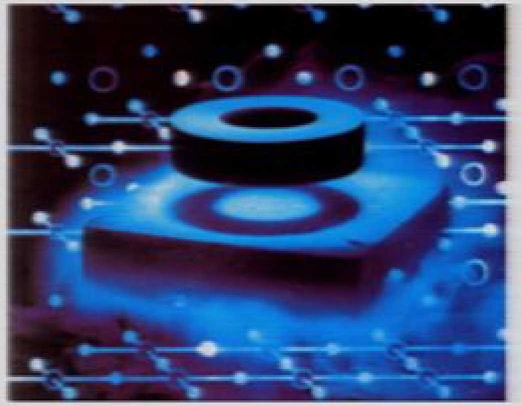

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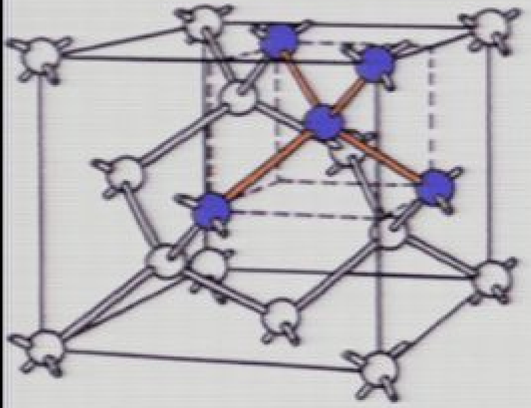

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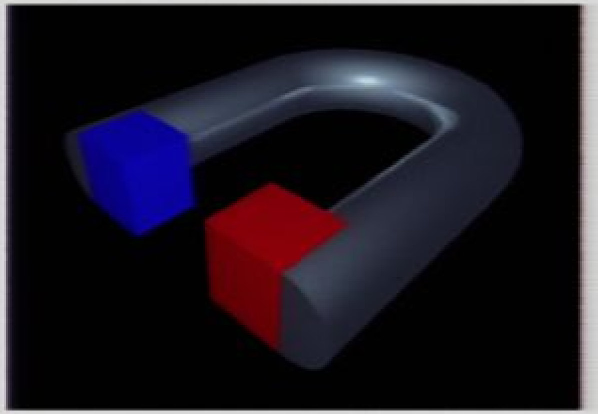

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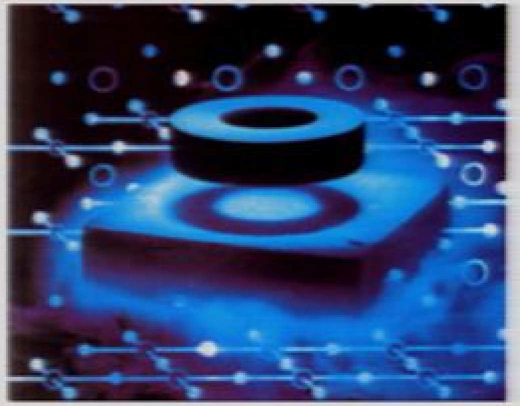

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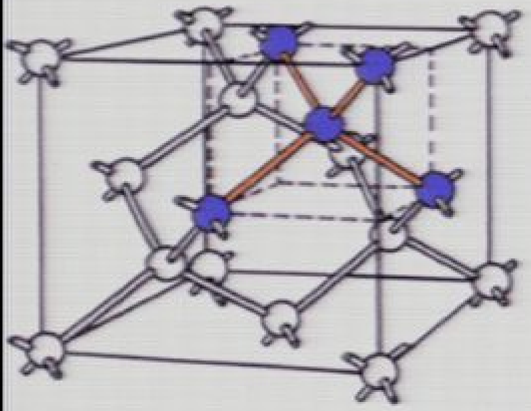

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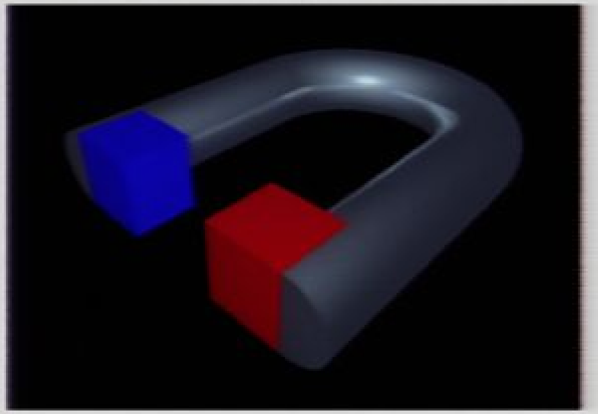

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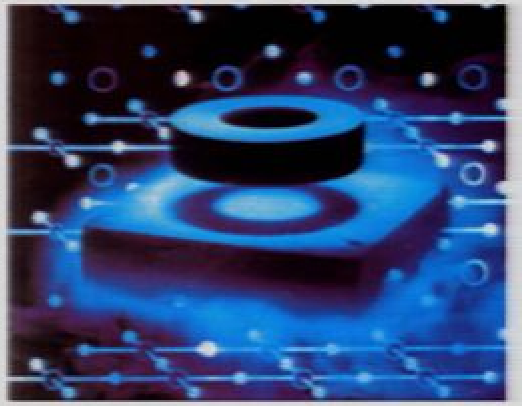

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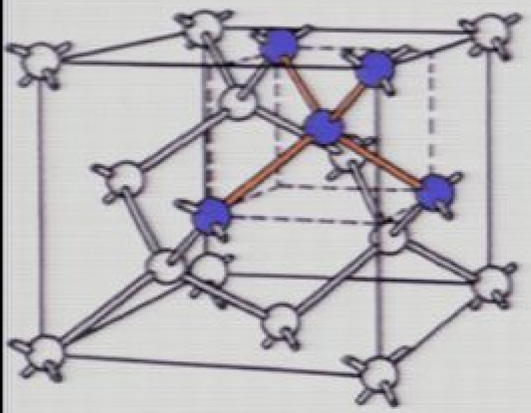

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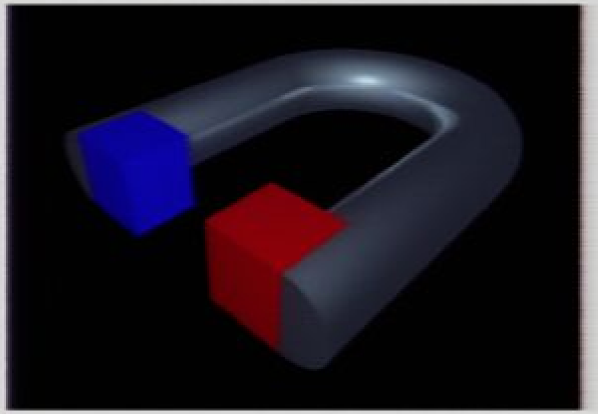

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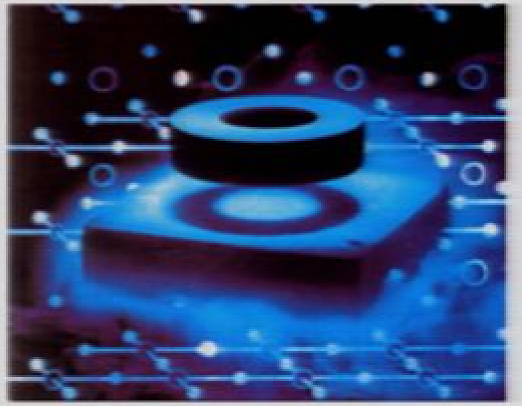

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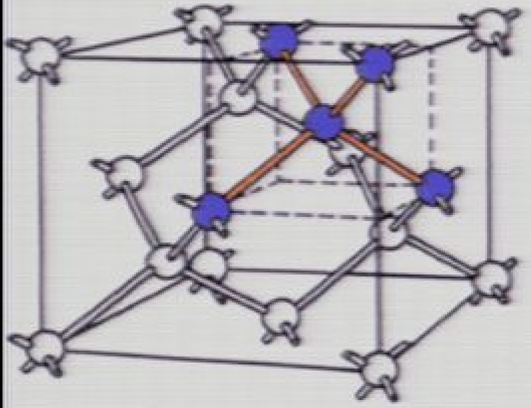

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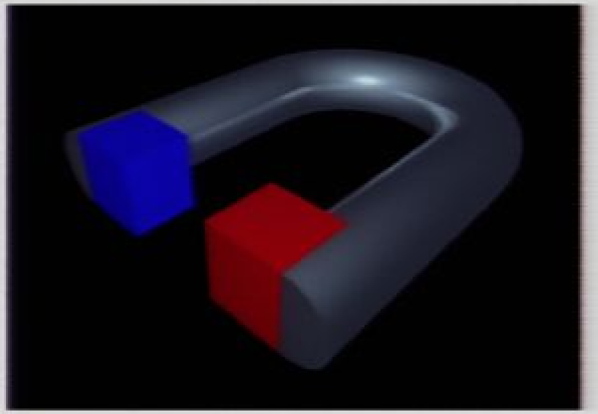

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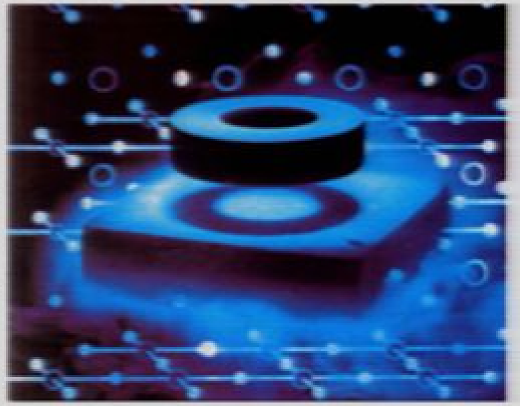

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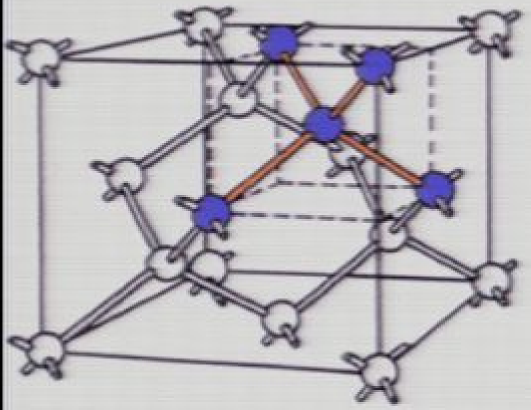

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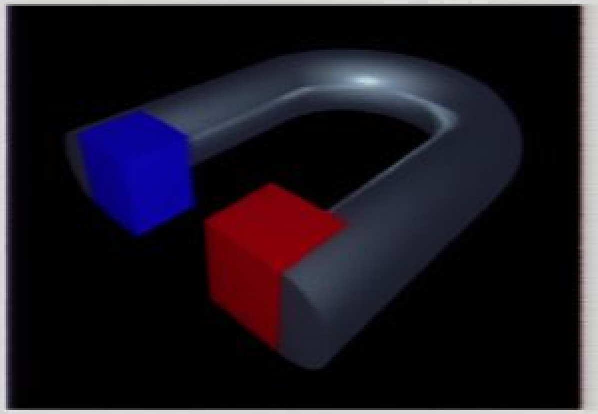

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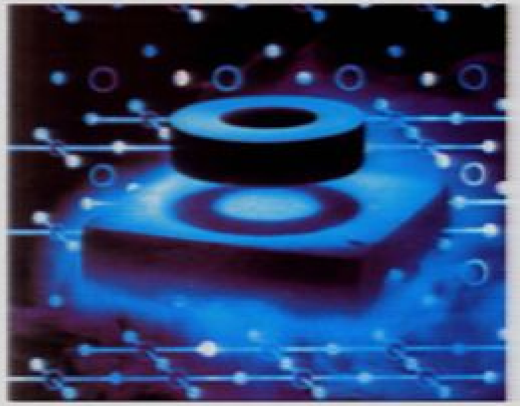

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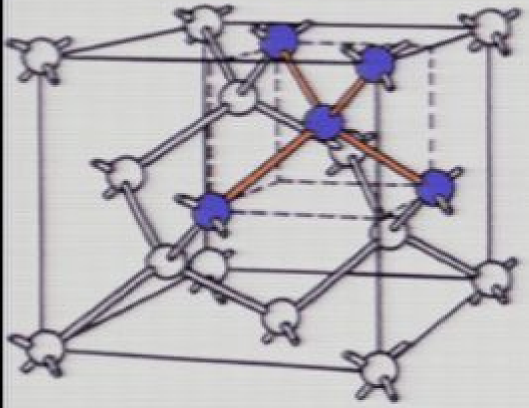

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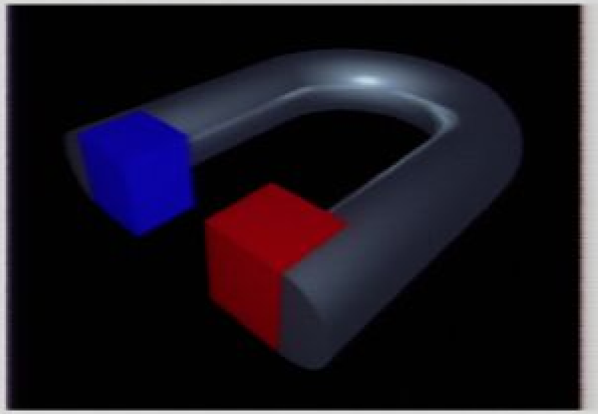

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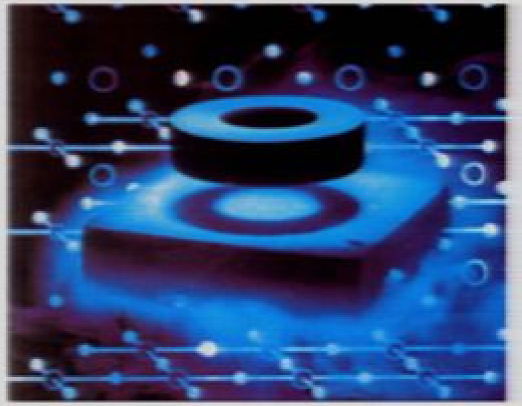

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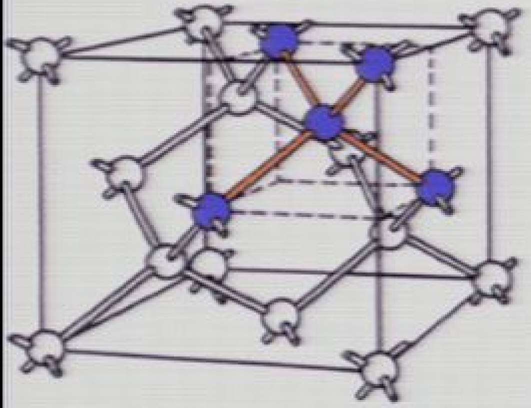

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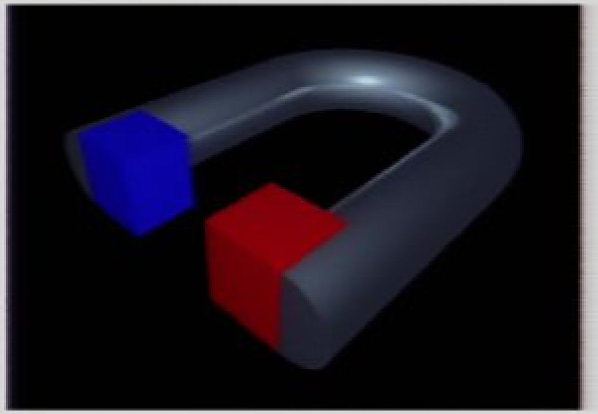

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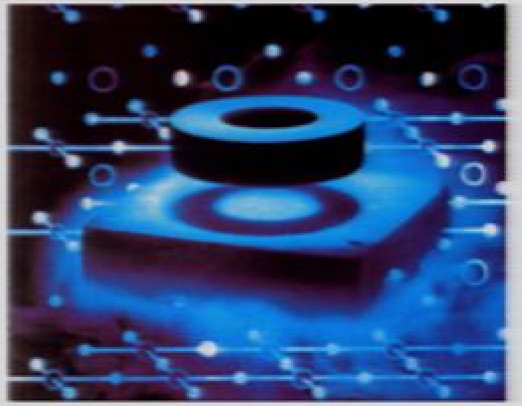

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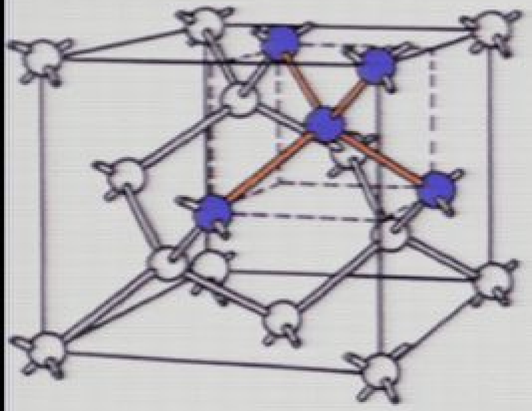

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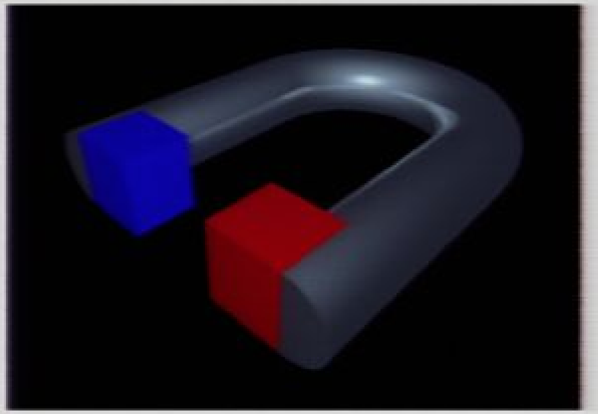

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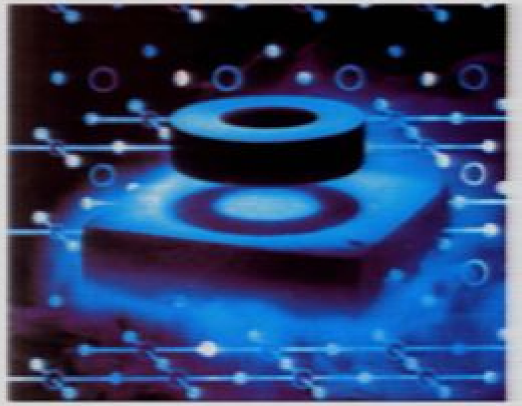

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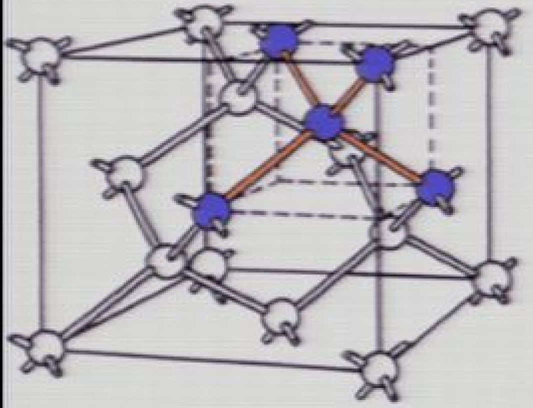

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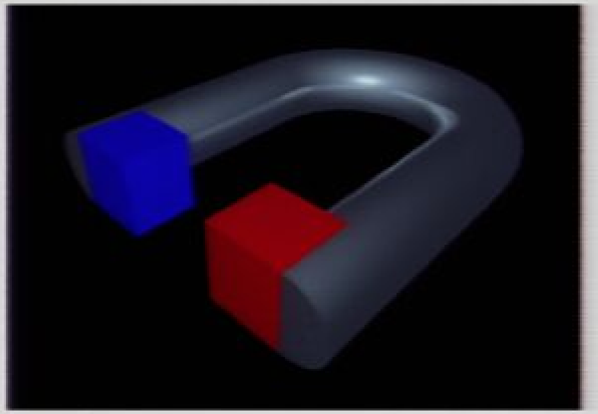

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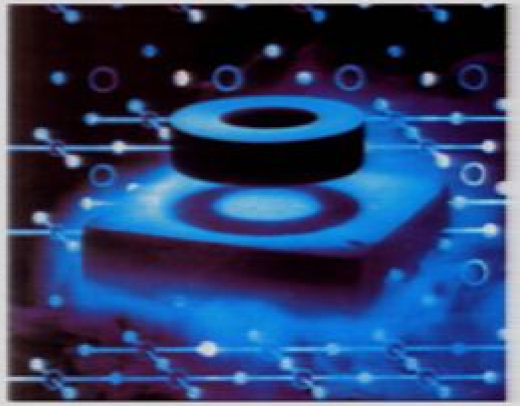

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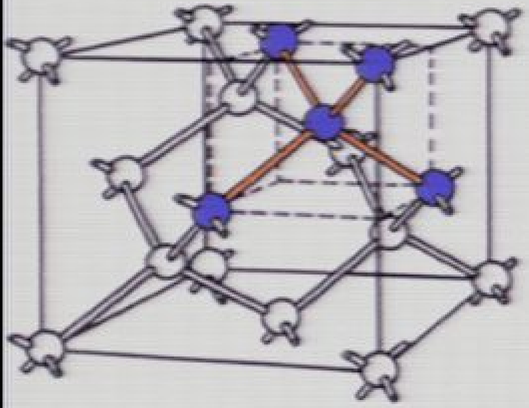

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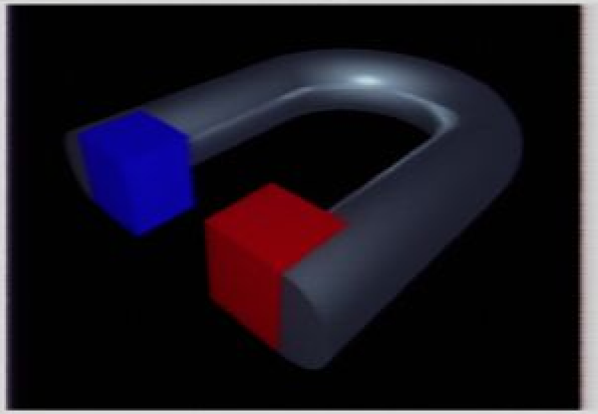

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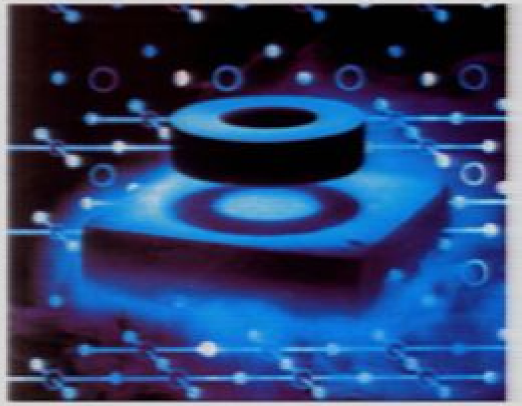

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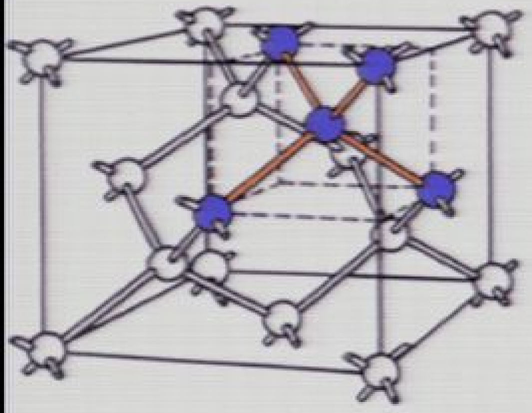

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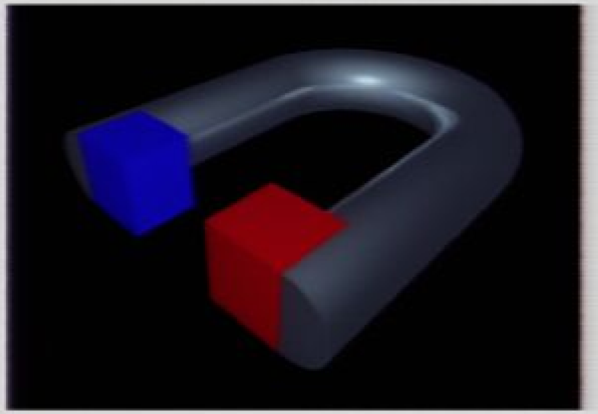

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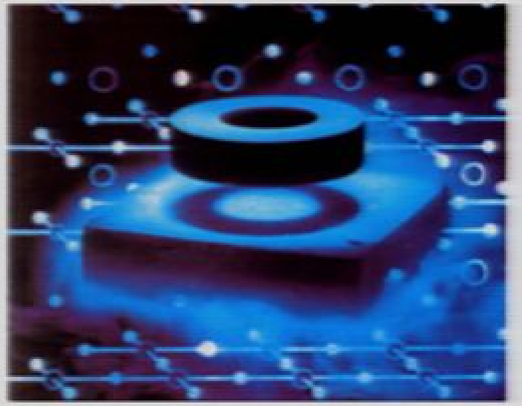

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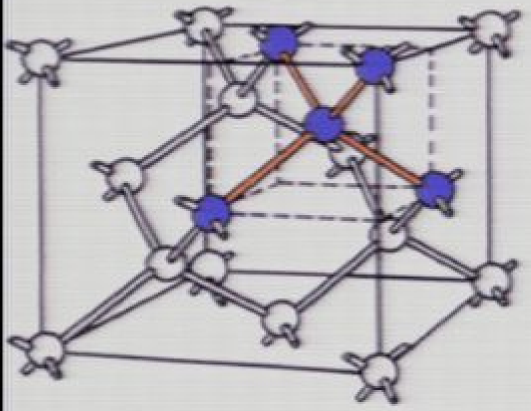

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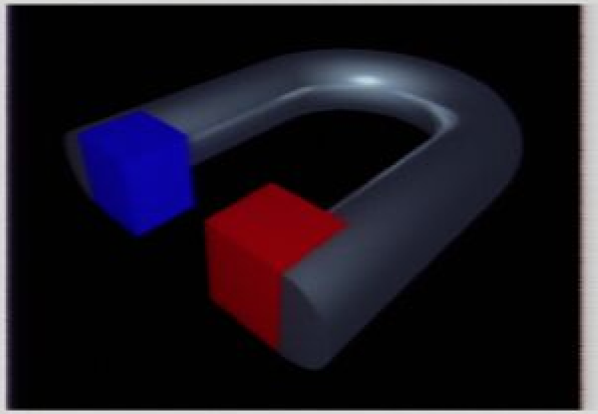

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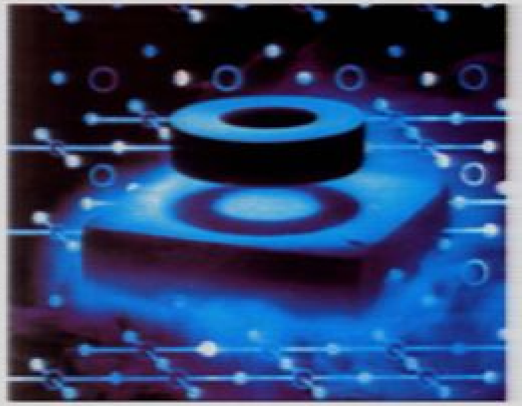

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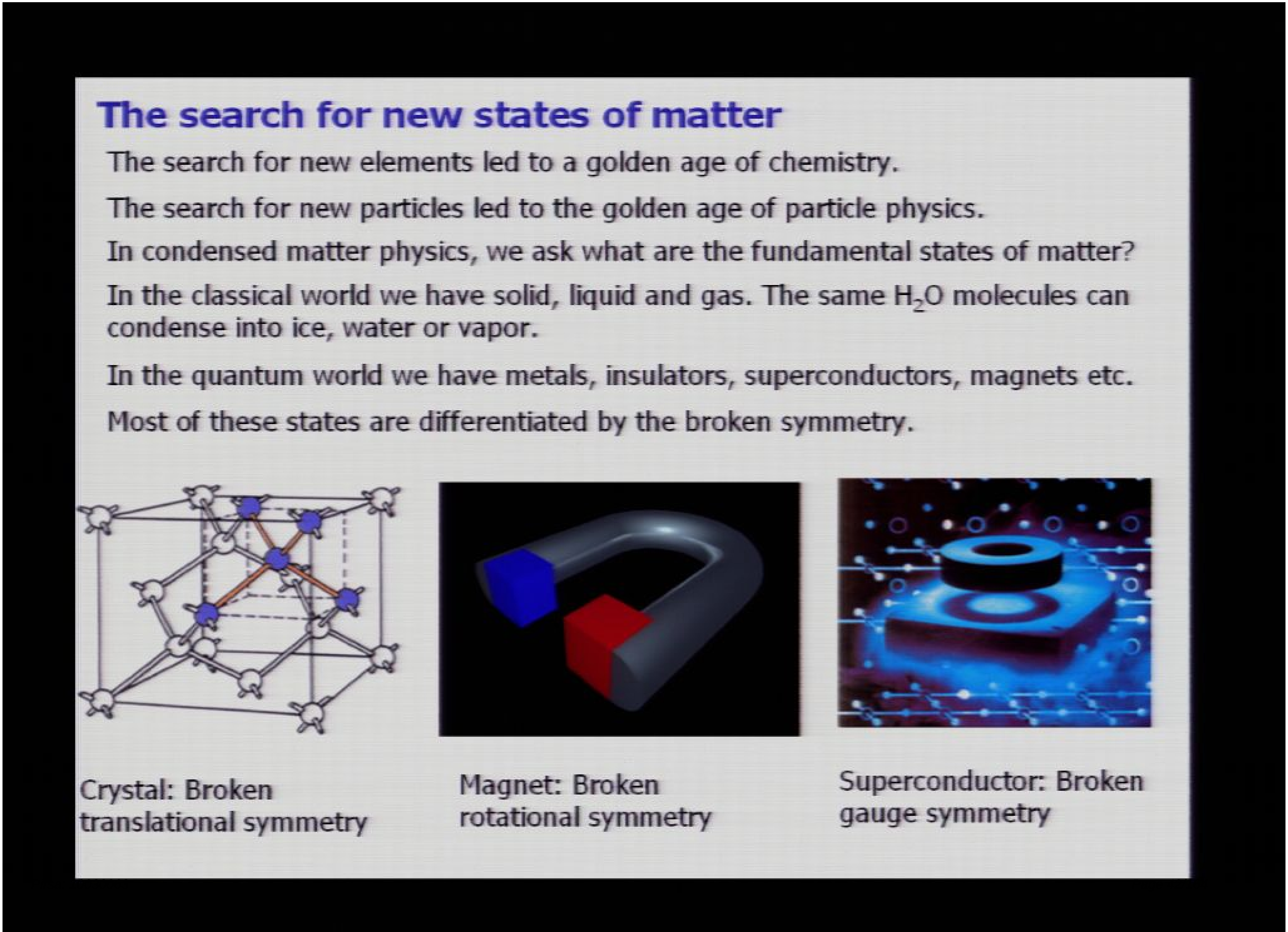

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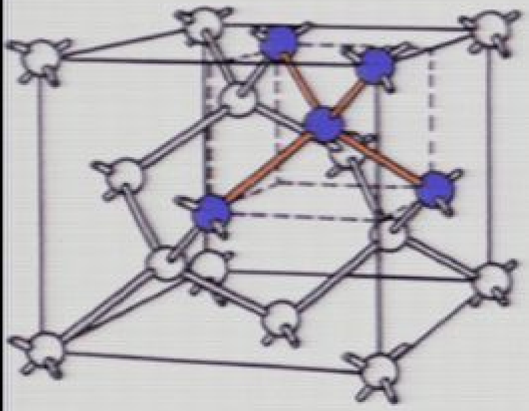

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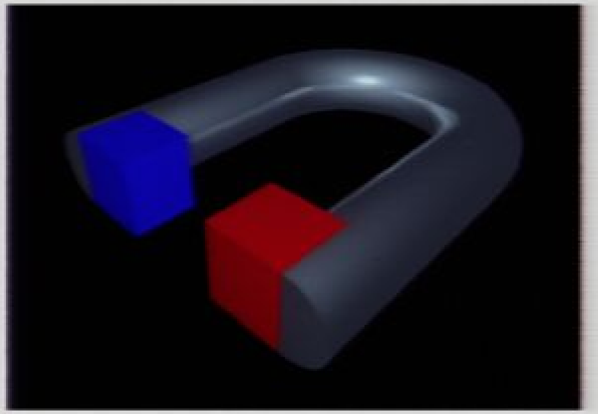

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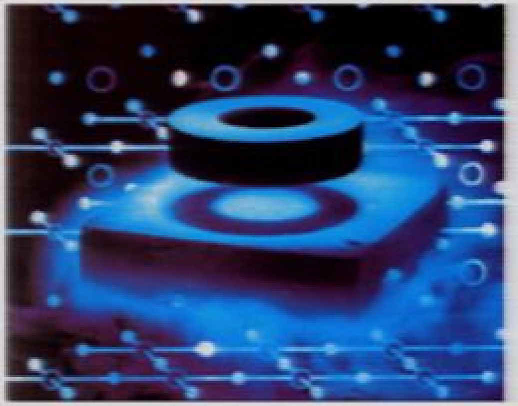

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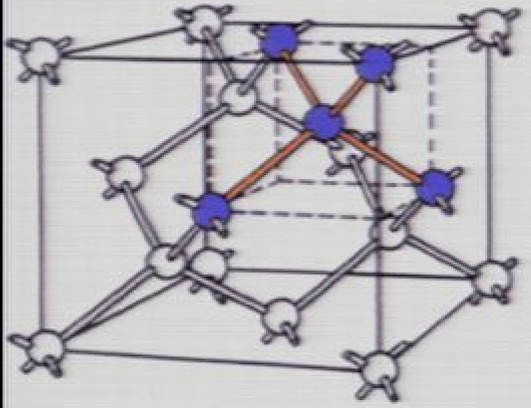

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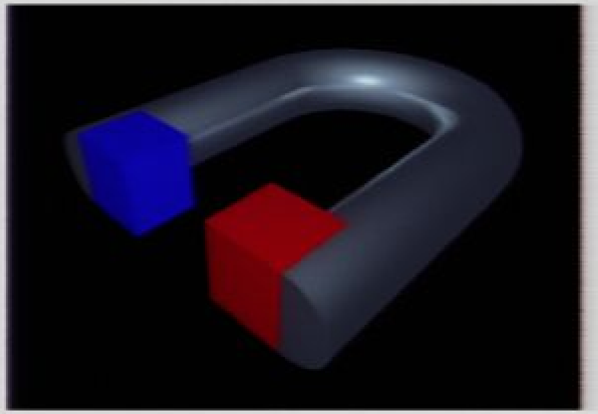

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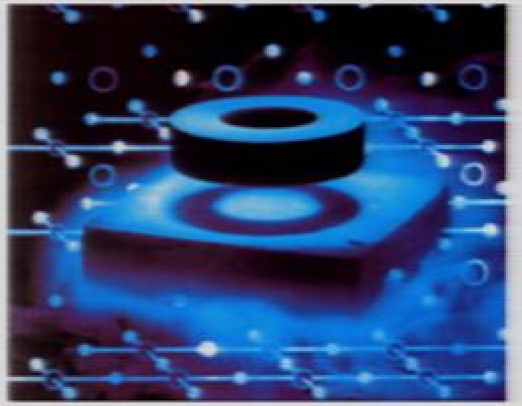

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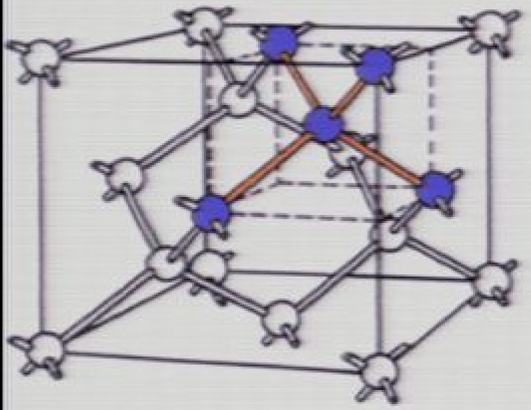

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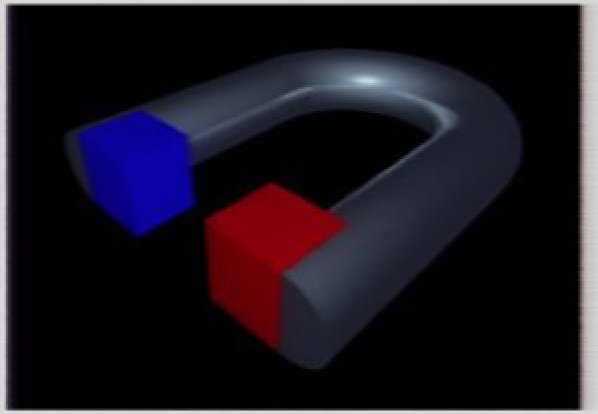

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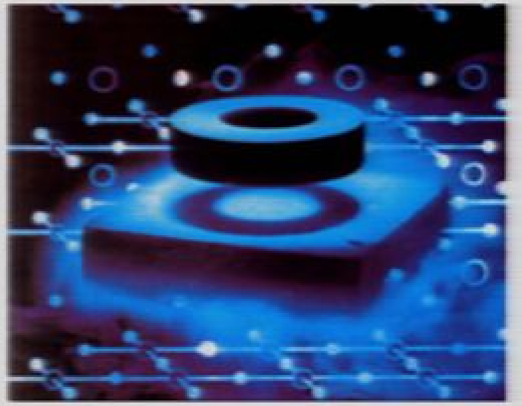

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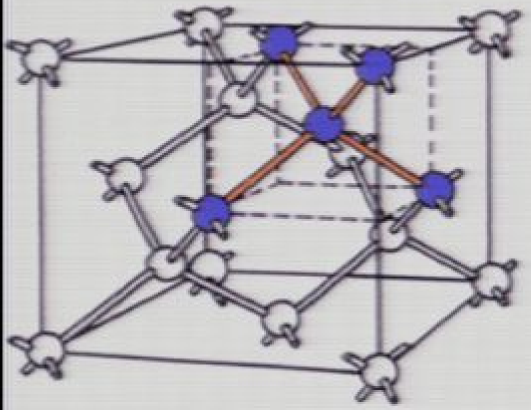

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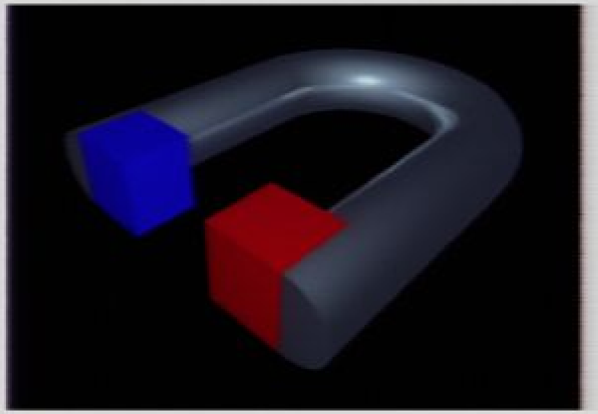

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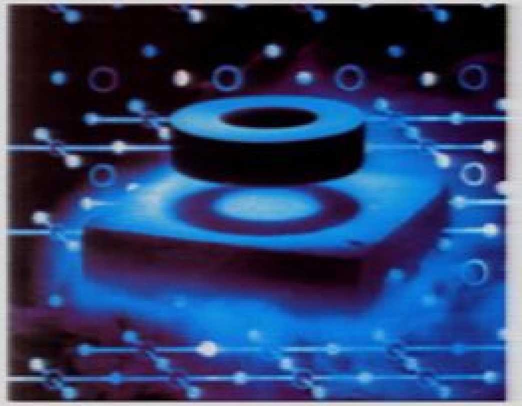

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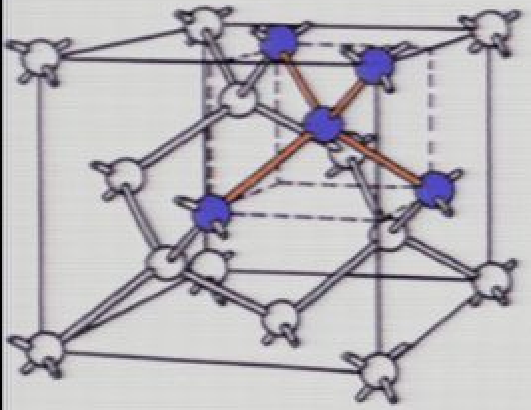

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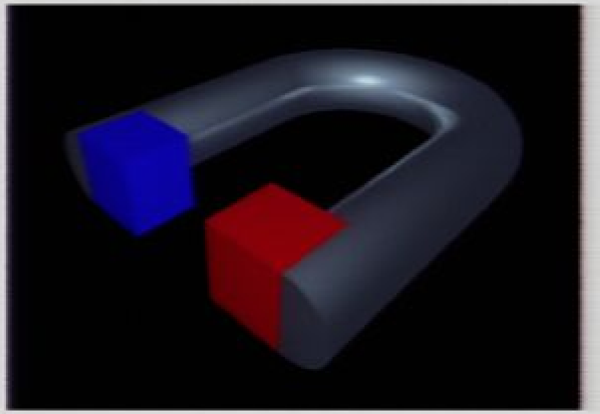

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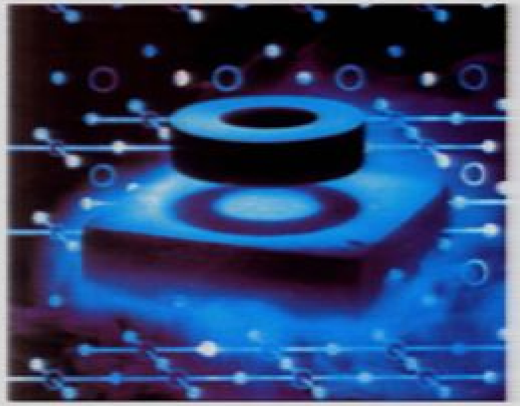

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The quantum Hall state, a topologically non-trivial state of matter

$$\sigma_{xy} = n\frac{e^2}{h}$$

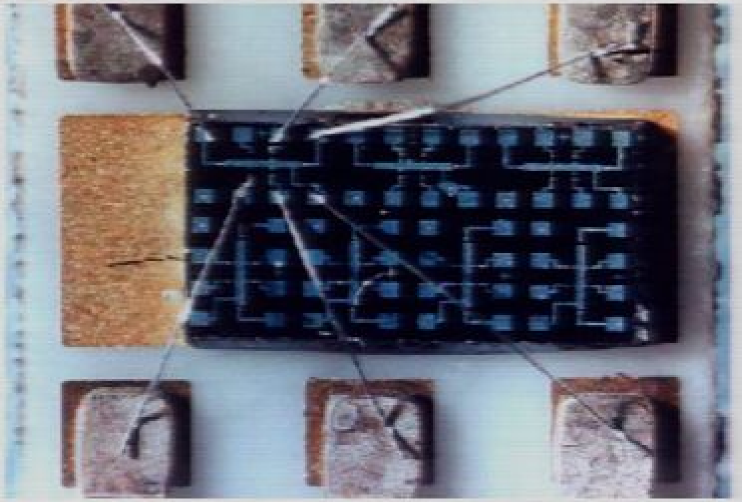
- TKNN integer=the first Chern number.

$$n = \int \frac{d^2k}{(2\pi)^2}\varepsilon^{\mu\nu}F_{\mu\nu}(k)$$

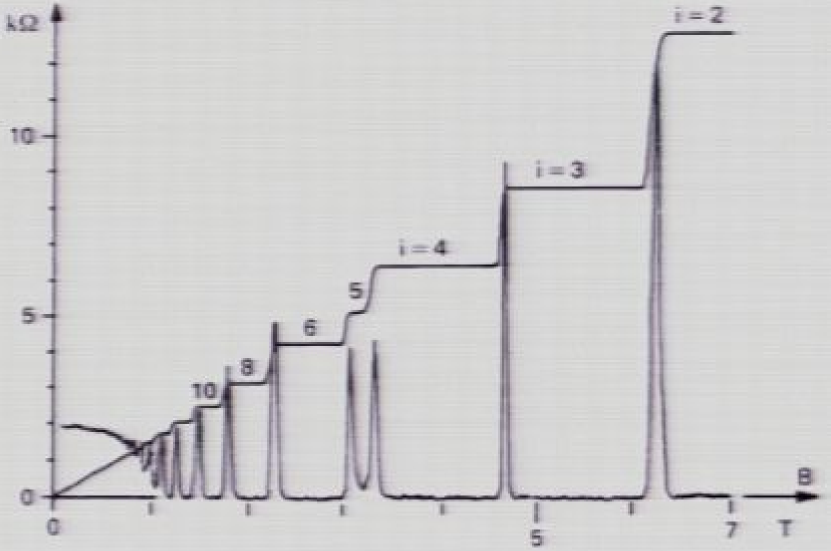
- Topological states of matter are defined and described by topological field theory:

$$S_{eff} = \frac{\sigma_{xy}}{2}\int d^2x dt \varepsilon^{\mu\nu\rho}A_\mu \partial_\nu A_\rho$$

- Physically measurable topological properties are all contained in the topological field theory, e.g. QHE, fractional charge, fractional statistics etc...



- von Klitzing, 1980



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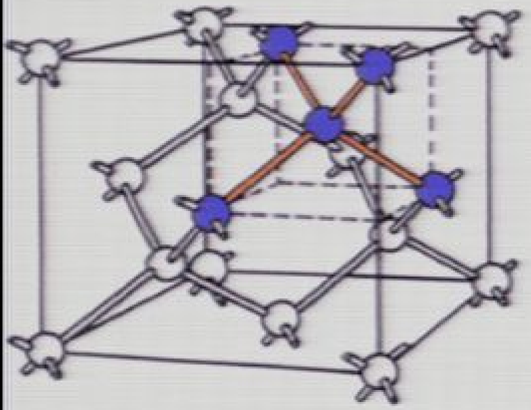

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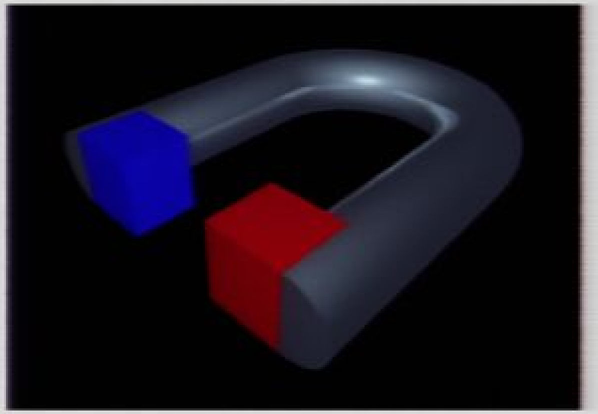

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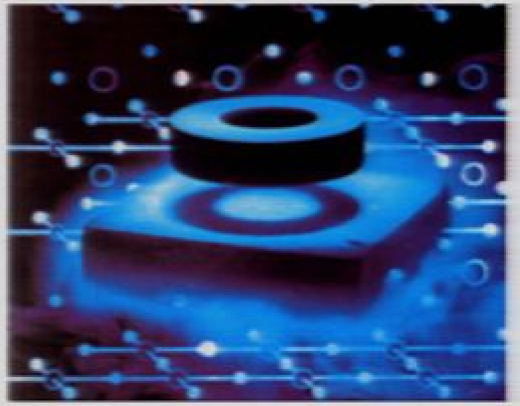

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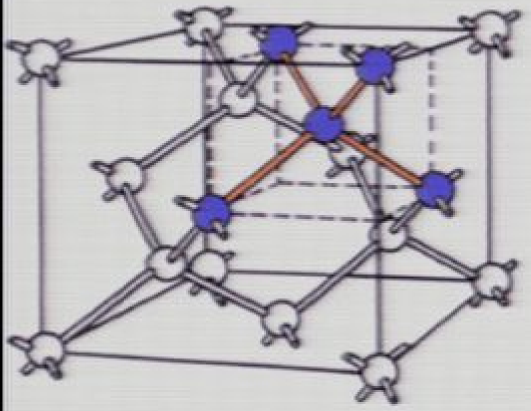

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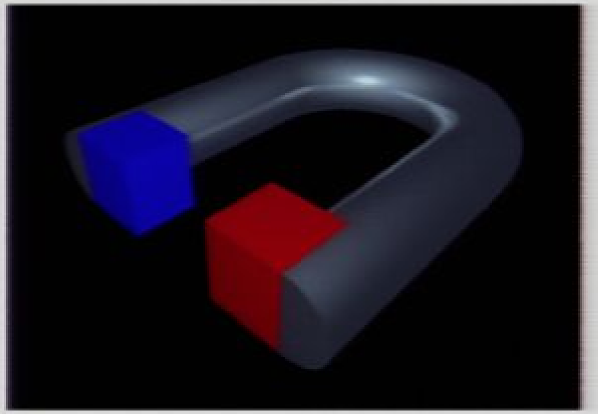

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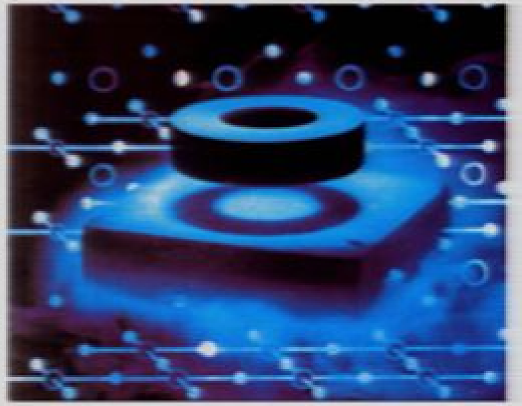

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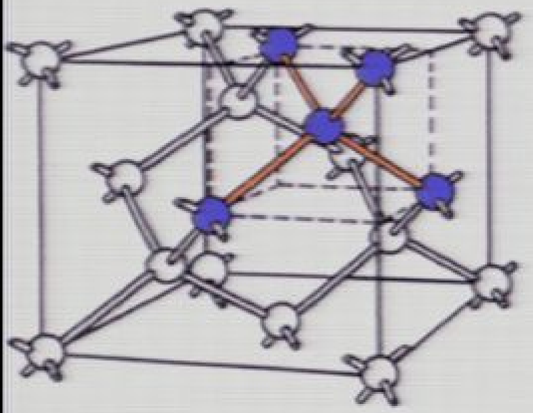

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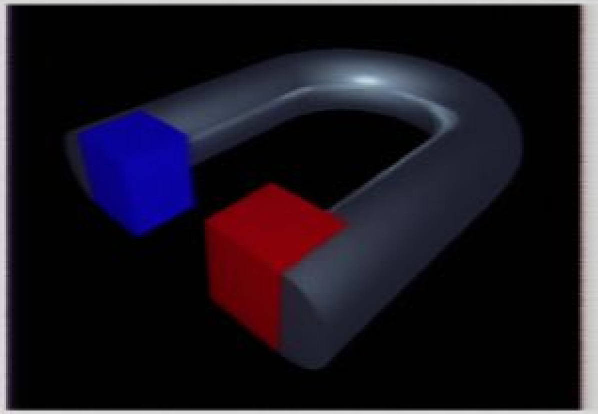

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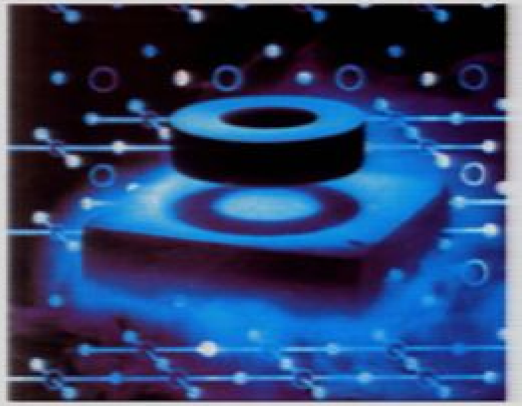

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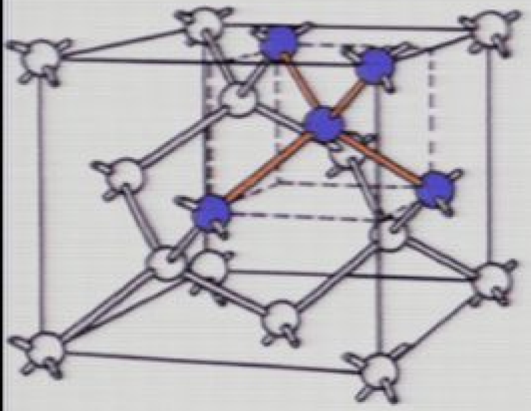

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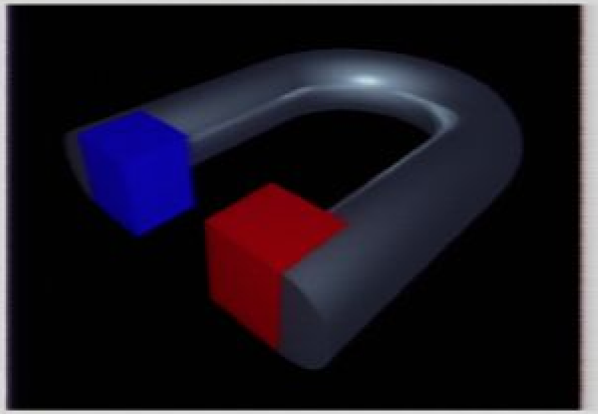

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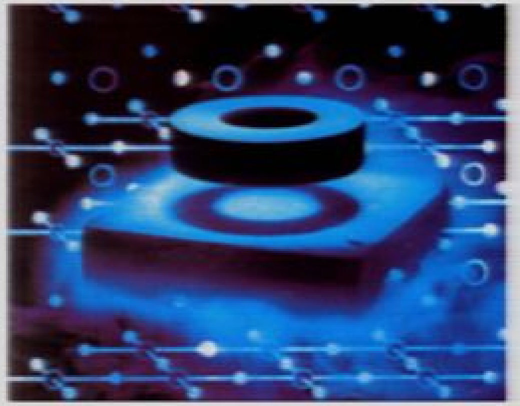

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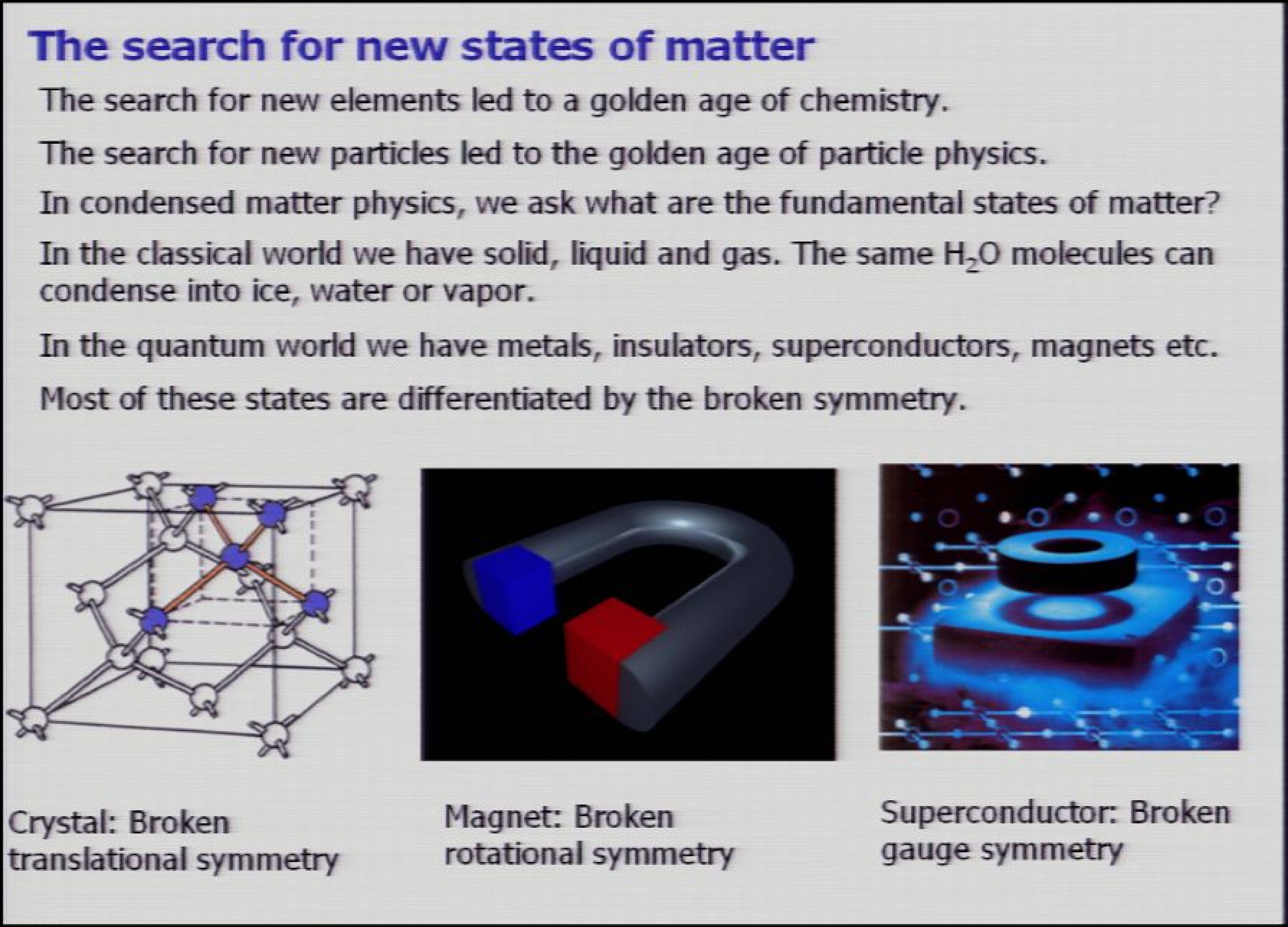

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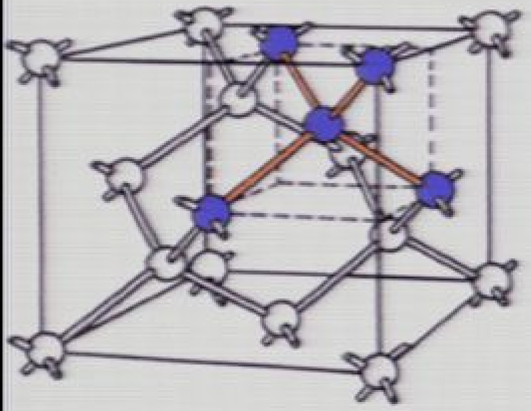

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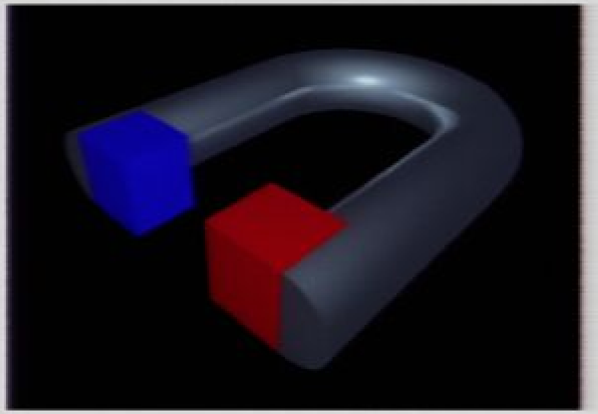

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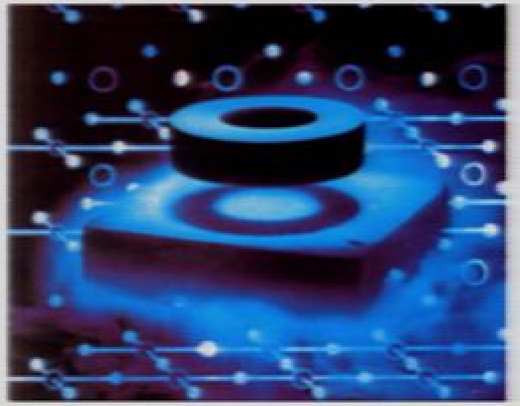

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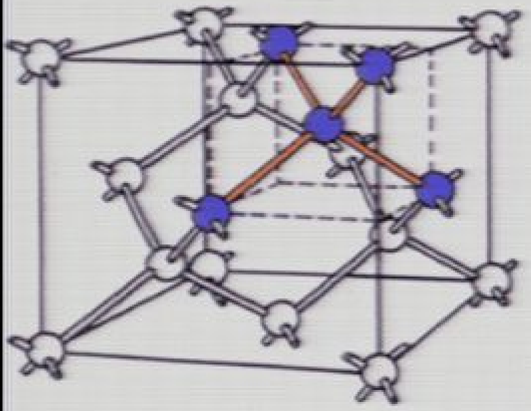

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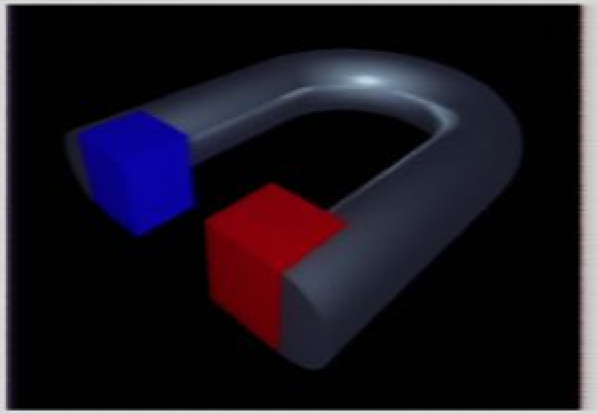

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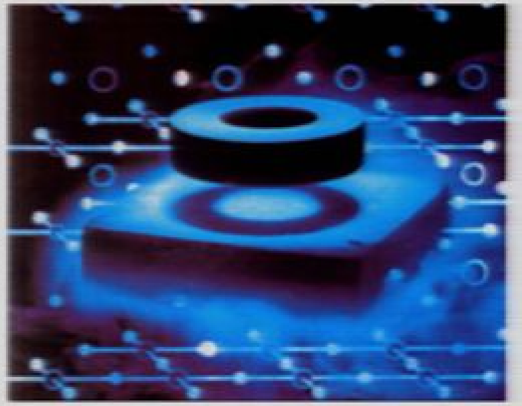

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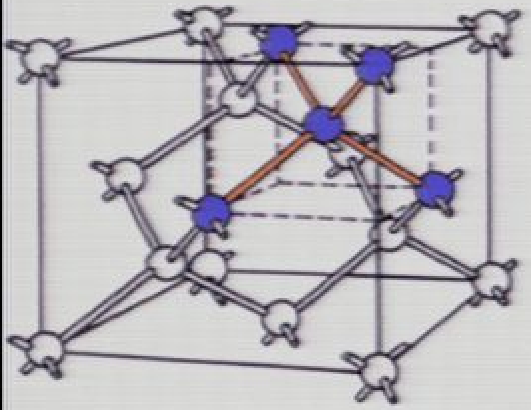

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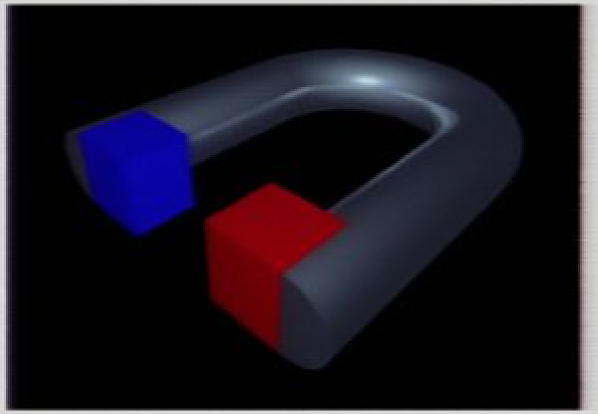

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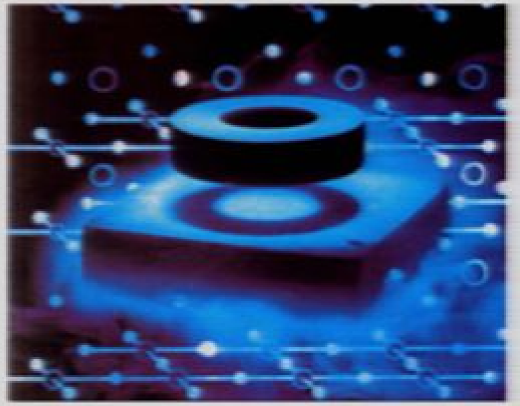

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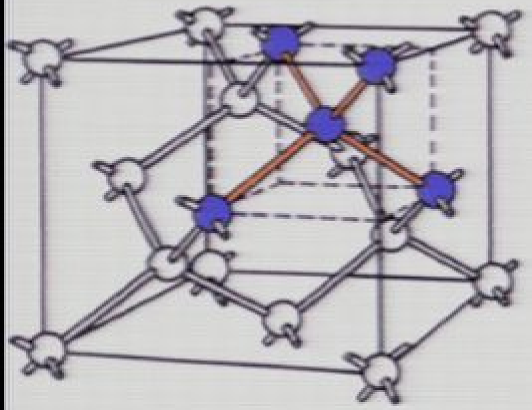

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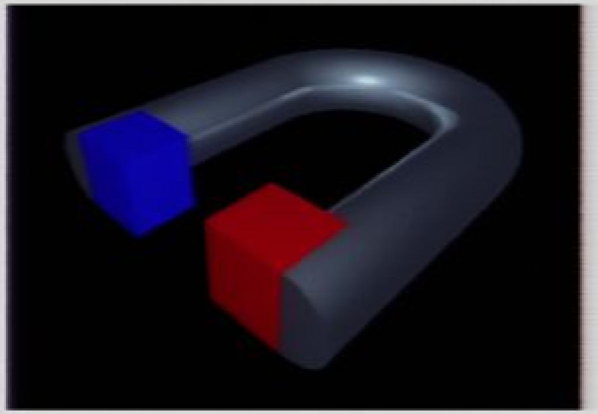

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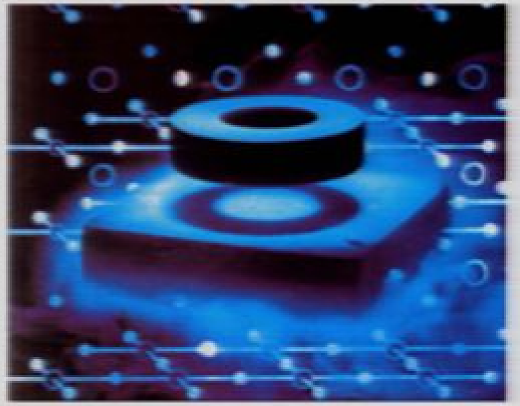

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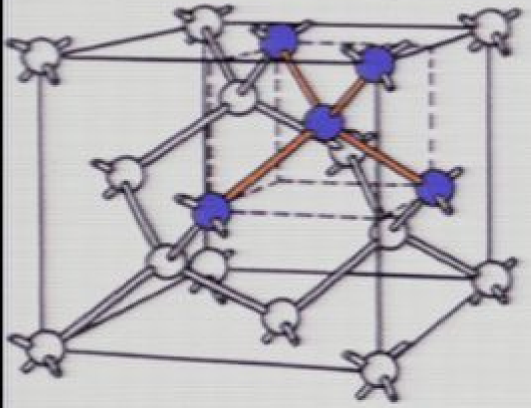
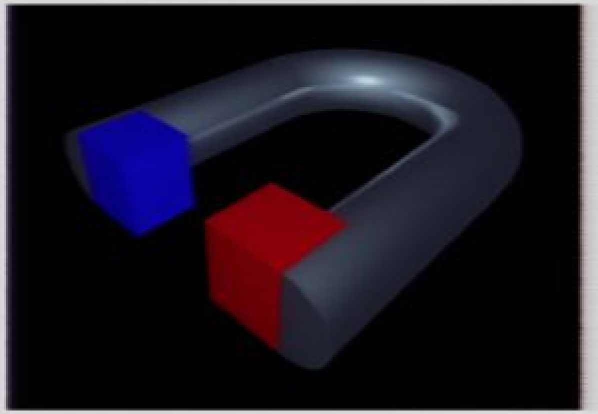


Crystal: Broken translational symmetry



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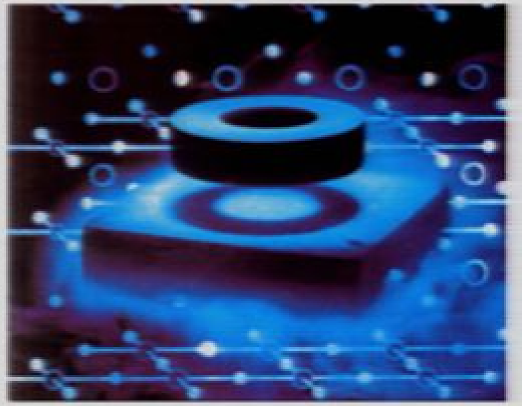

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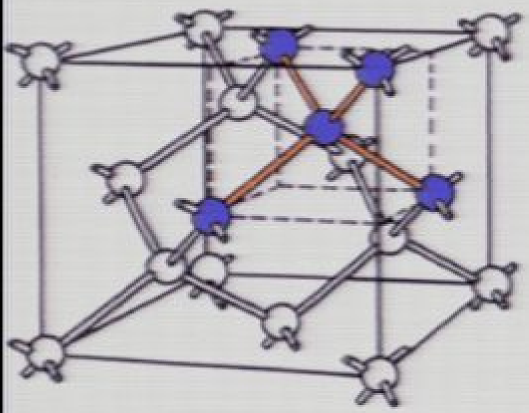

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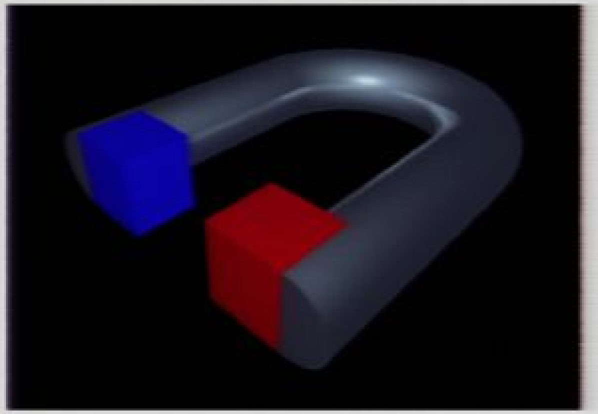

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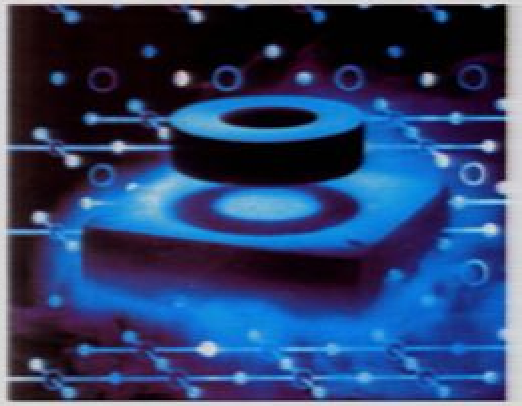

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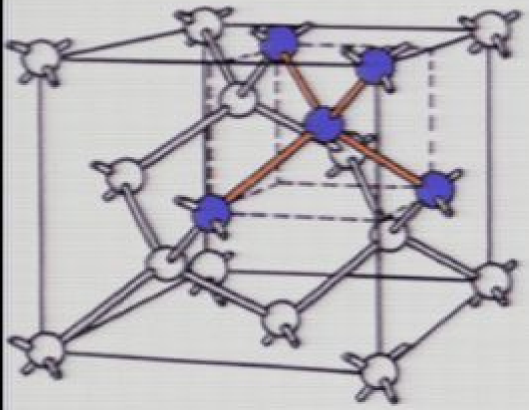

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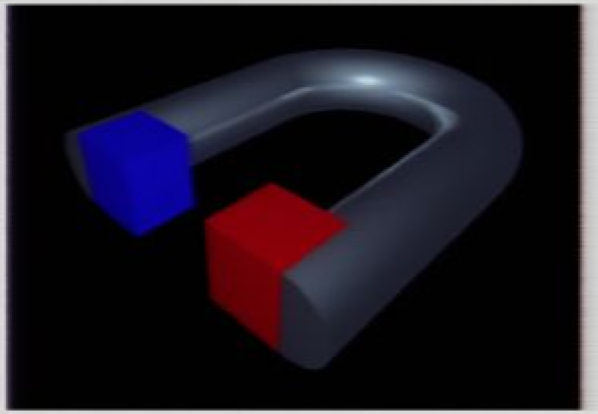

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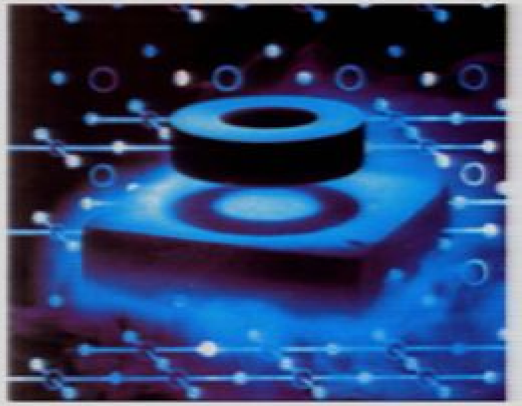

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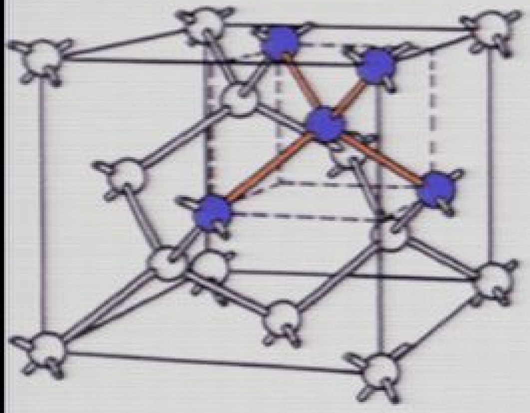

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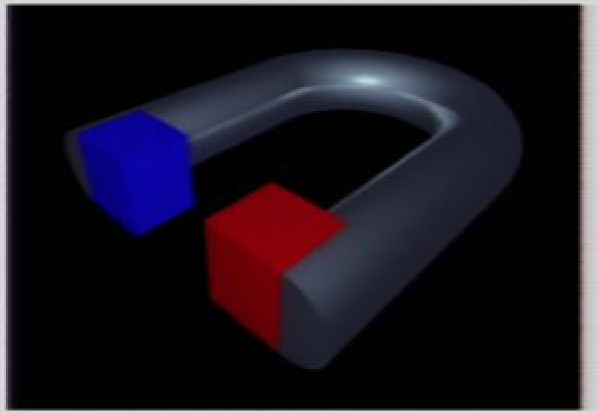

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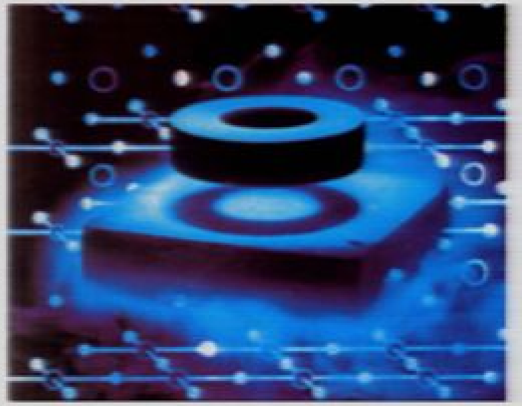

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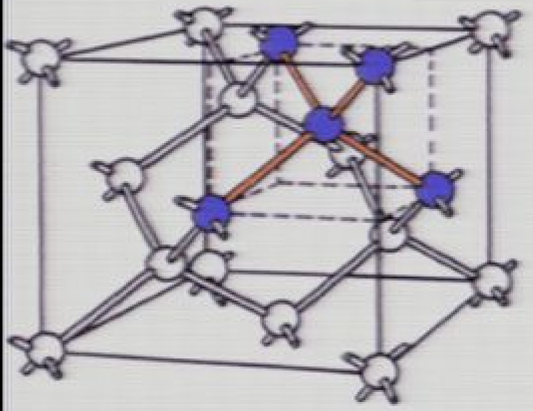

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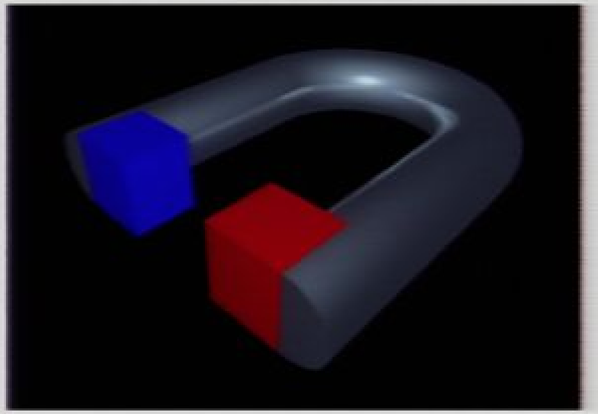

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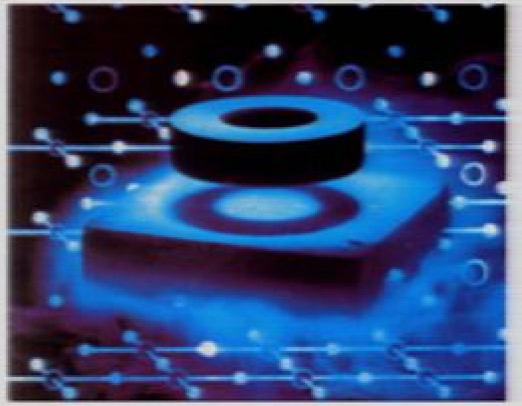

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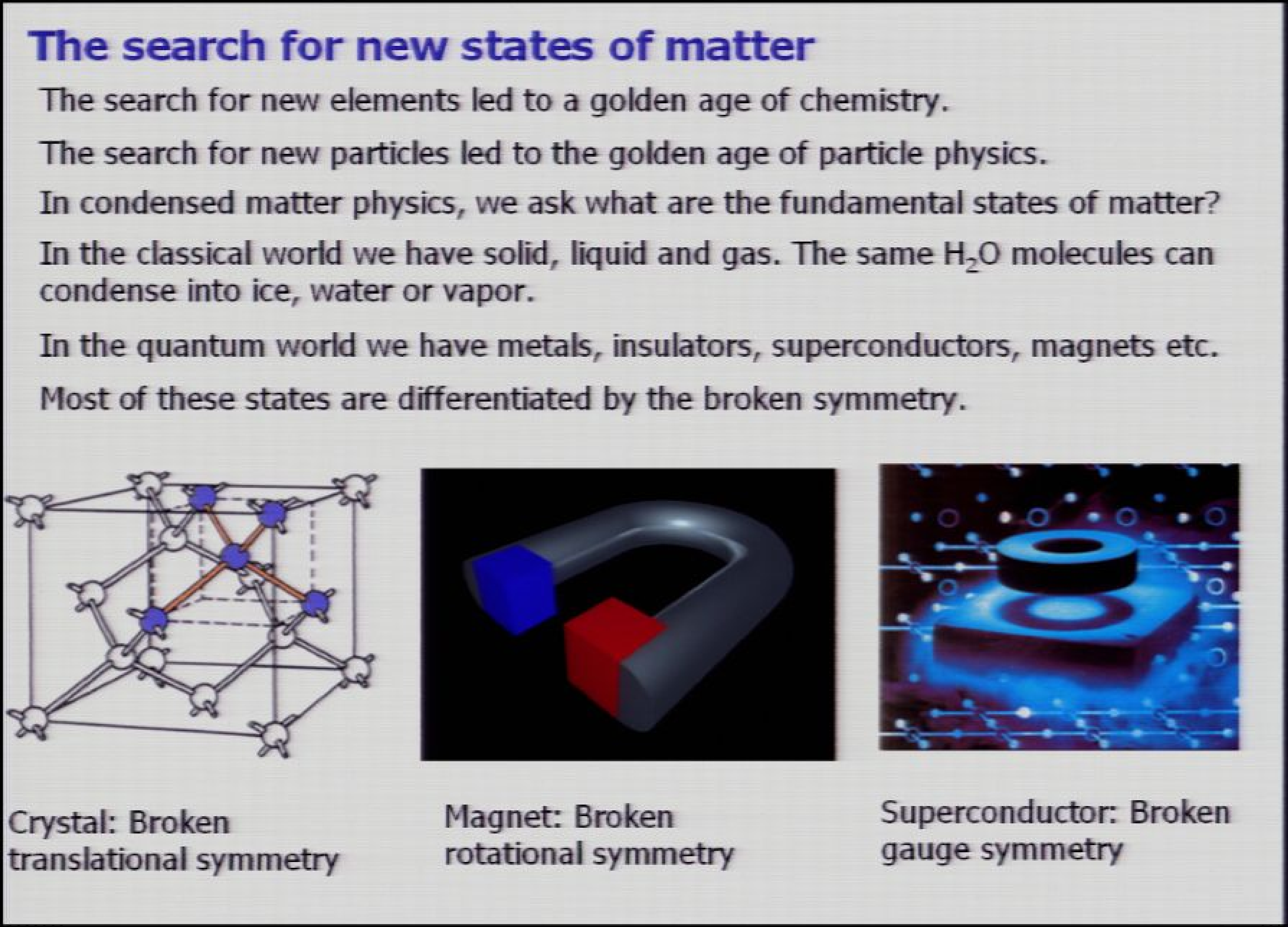

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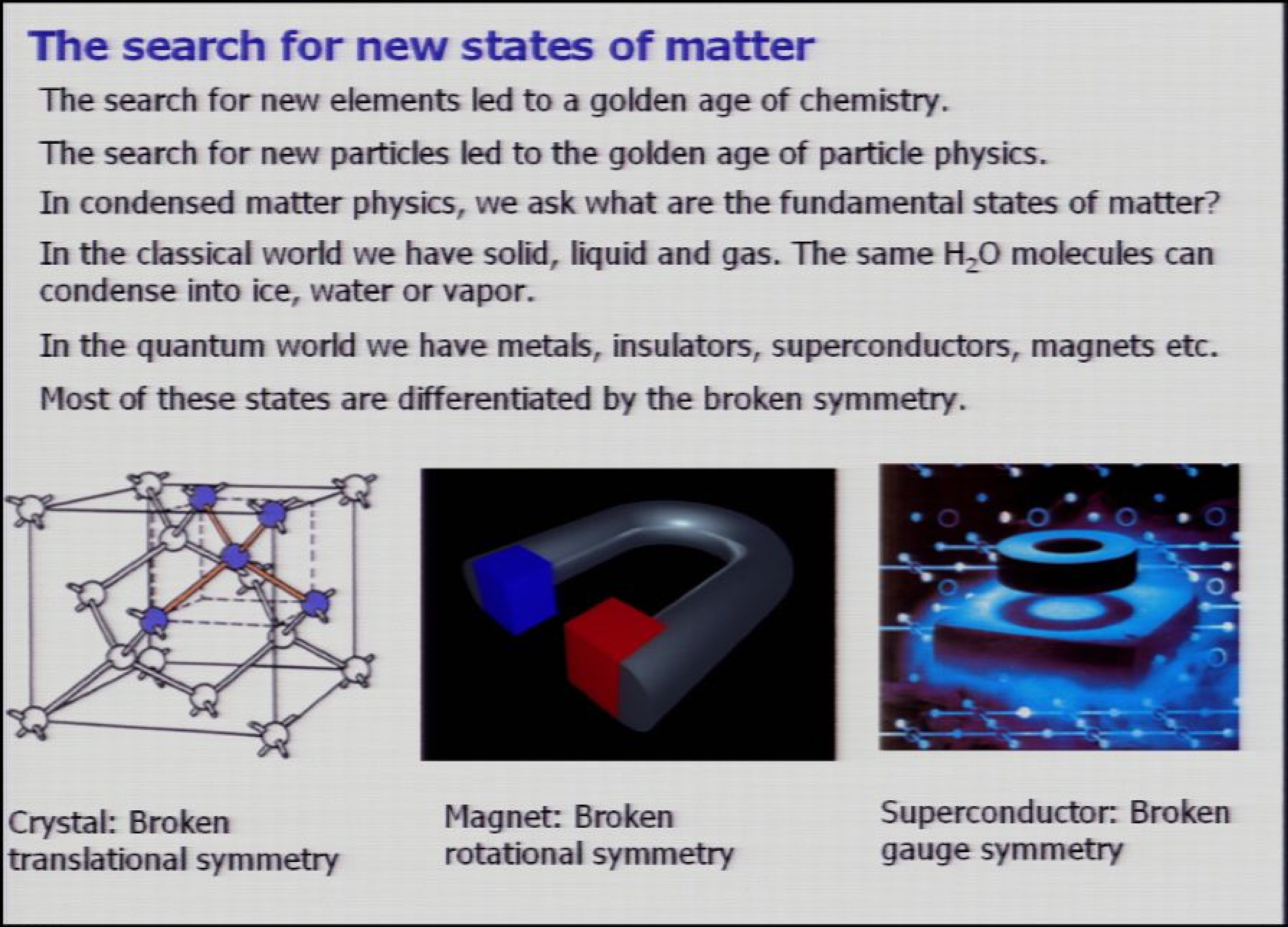

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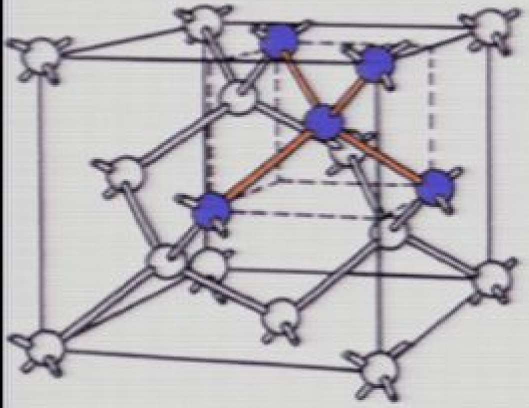

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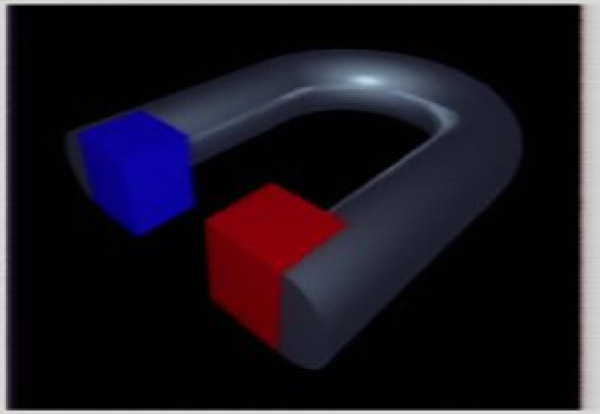

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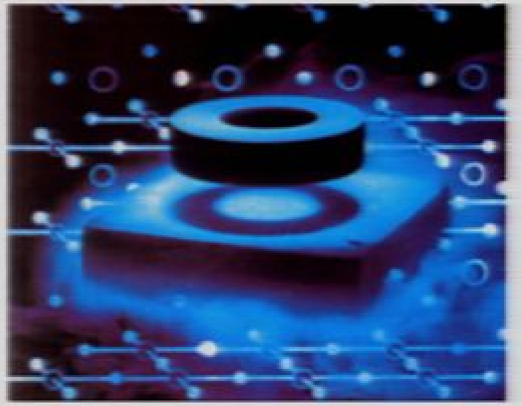

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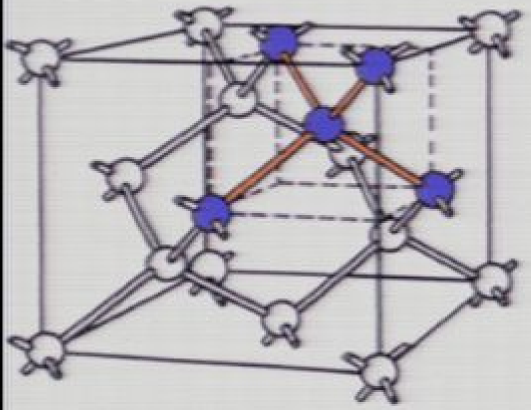

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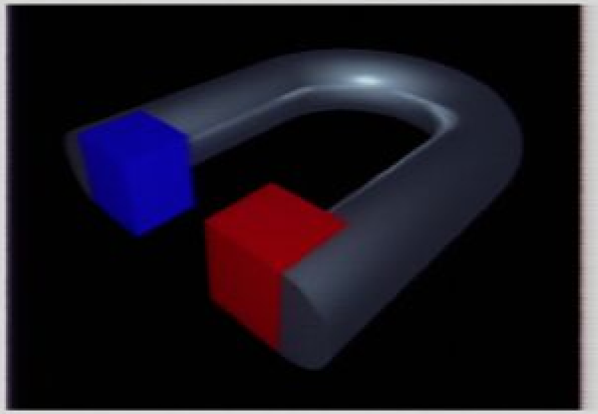

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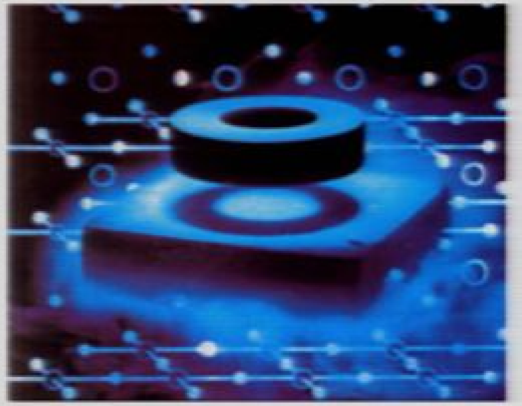

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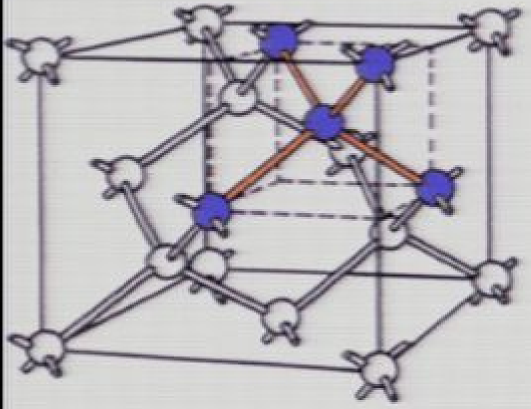

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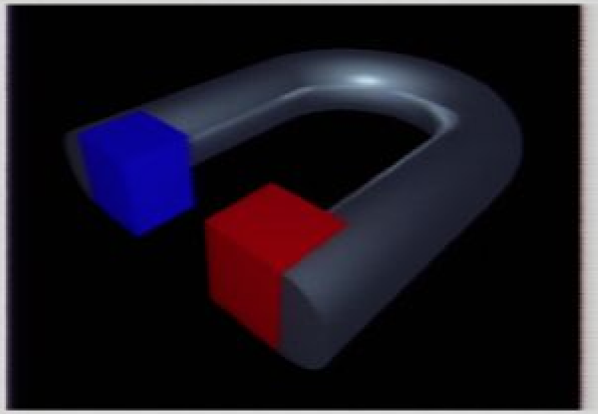

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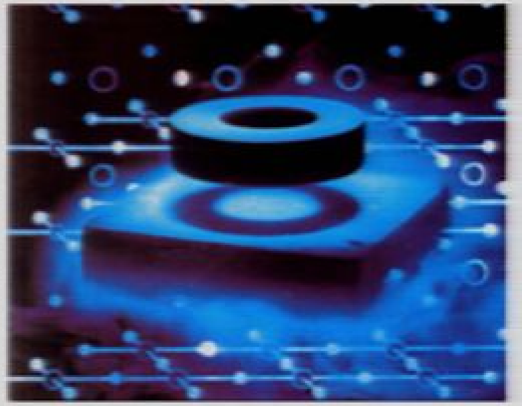

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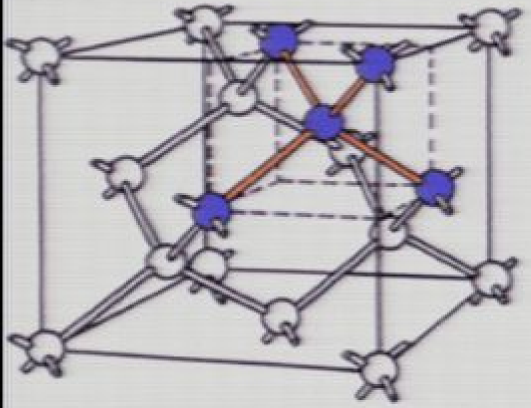

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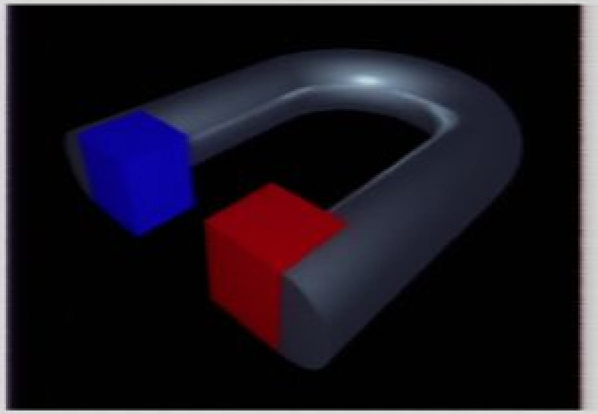

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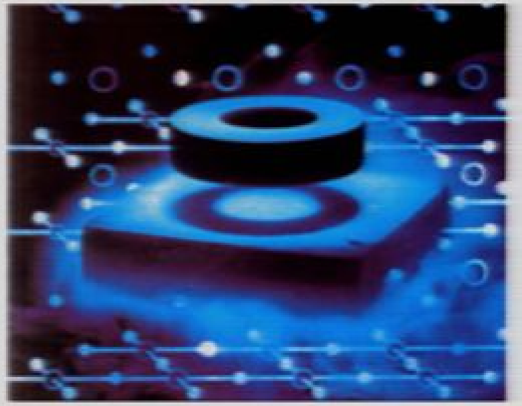

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The quantum Hall state, a topologically non-trivial state of matter

$$\sigma_{xy} = n\frac{e^2}{h}$$

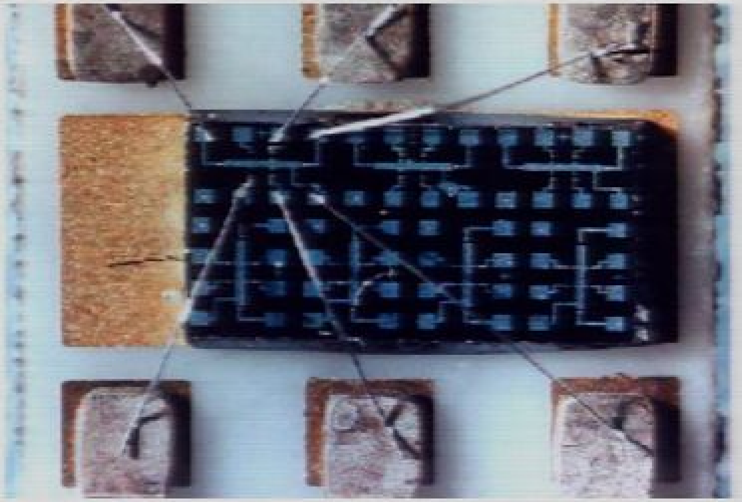
- TKNN integer=the first Chern number.

$$n = \int \frac{d^2k}{(2\pi)^2}\varepsilon^{\mu\nu}F_{\mu\nu}(k)$$

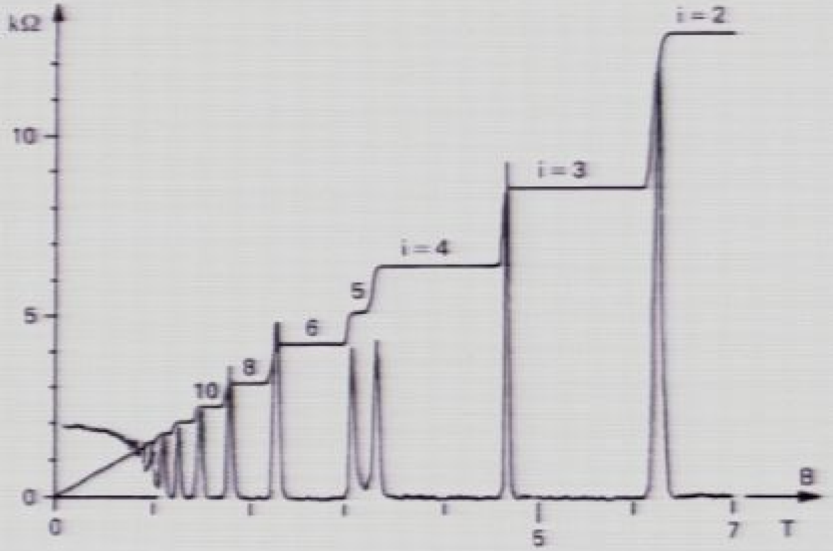
- Topological states of matter are defined and described by topological field theory:

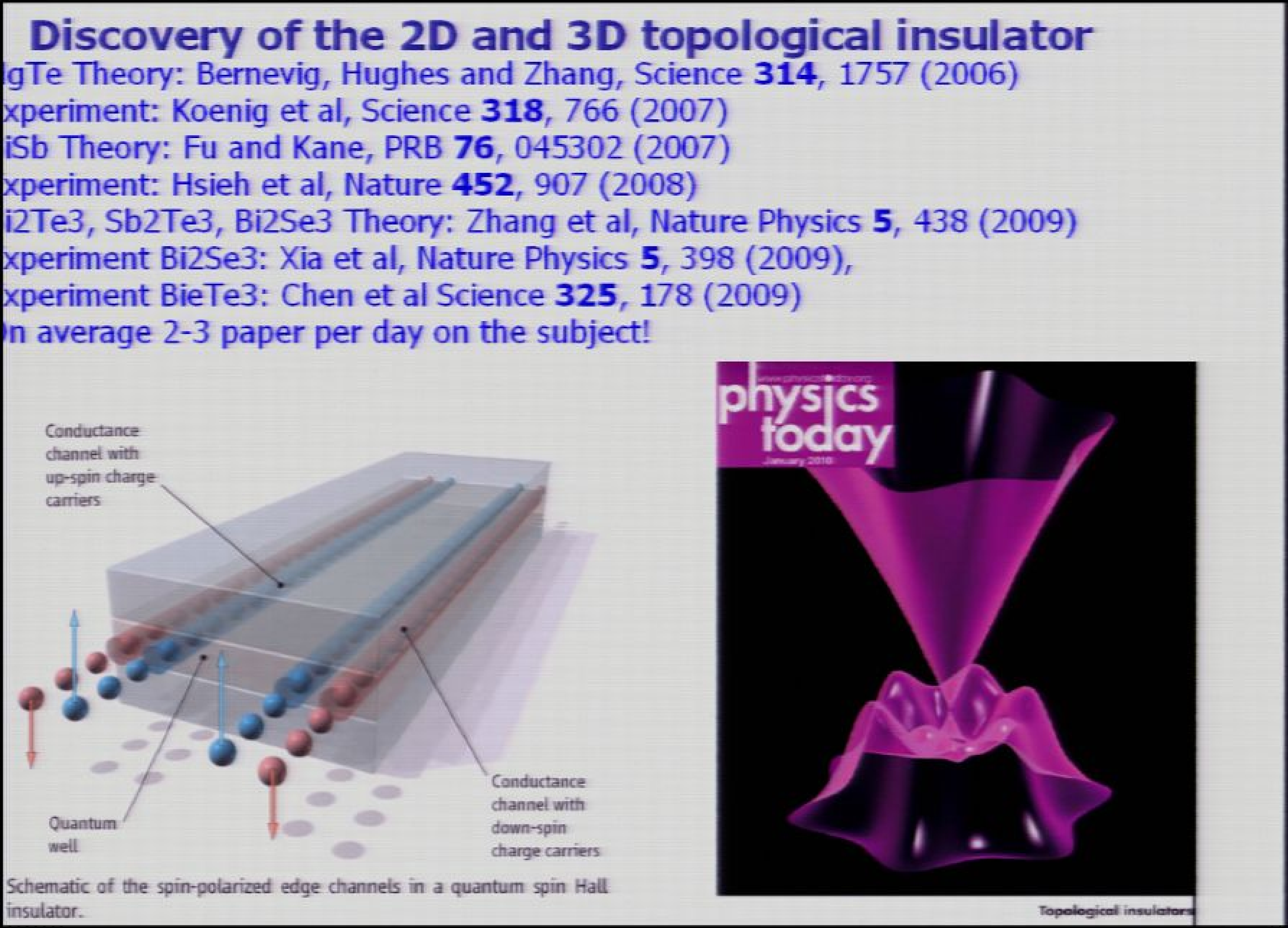
$$S_{eff} = \frac{\sigma_{xy}}{2}\int d^2x dt \varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$$

- Physically measurable topological properties are all contained in the topological field theory, e.g. QHE, fractional charge, fractional statistics etc...



- von Klitzing, 1980





Discovery of the 2D and 3D topological insulator

gTe Theory: Bernevig, Hughes and Zhang, Science **314**, 1757 (2006)
xperiment: Koenig et al, Science **318**, 766 (2007)
iSb Theory: Fu and Kane, PRB **76**, 045302 (2007)
xperiment: Hsieh et al, Nature **452**, 907 (2008)
i2Te3, Sb2Te3, Bi2Se3 Theory: Zhang et al, Nature Physics **5**, 438 (2009)
xperiment Bi2Se3: Xia et al, Nature Physics **5**, 398 (2009),
xperiment BieTe3: Chen et al Science **325**, 178 (2009)
n average 2-3 paper per day on the subject!

Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

Topological Insulator is a New State of Quantum Matter

reakthrough of the Year

6 ELECTRONS TAKE A NEW SPIN. Chalk one up for the theorists. Theoretical physicists in California recently predicted that semiconductor sandwiches with thin layers of mercury telluride (HgTe) in the middle should exhibit an unusual behavior of their electrons called the quantum spin Hall effect (QSHE). This year, they teamed up with experimental physicists in Germany and found just what they were looking for.

HYSICS

A New State of Quantum Matter

aoto Nagaosa

Experiments show that electron spins can flow without dissipation in a novel electrical insulator.

earch
discovery

Quantum spin Hall effect shows up in a quantum well insulator, just as predicted

The effect, which occurs without a magnetic field, is a new and topologically distinct electronic state.

From traffic jam to info-superhighway on chip

From traffic jam to info-superhighway on chip

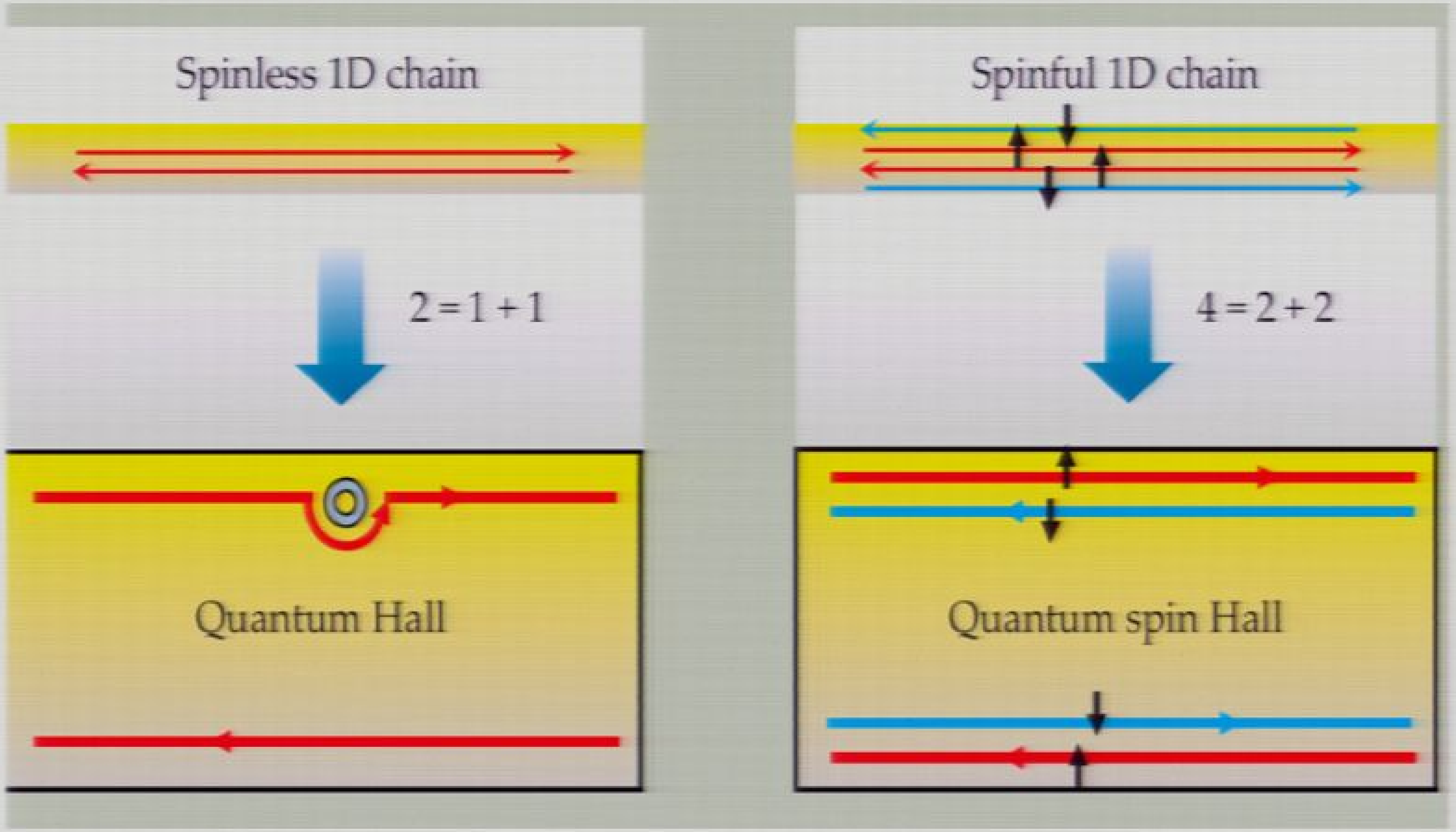
Traffic jam inside chips today

From traffic jam to info-superhighway on chip

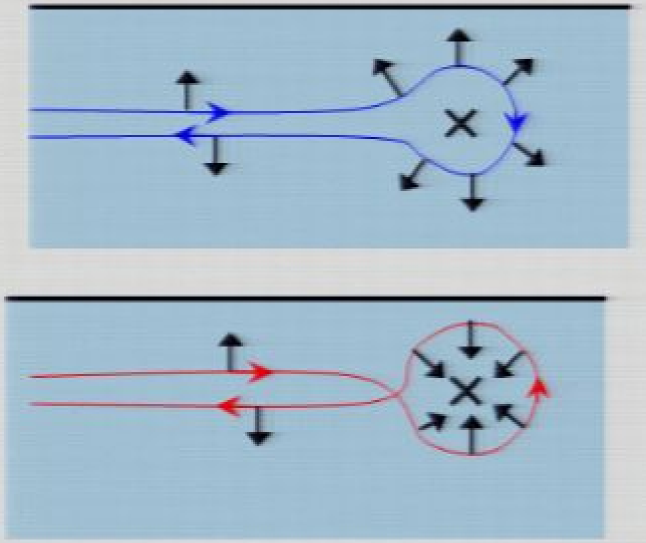
Traffic jam inside chips today

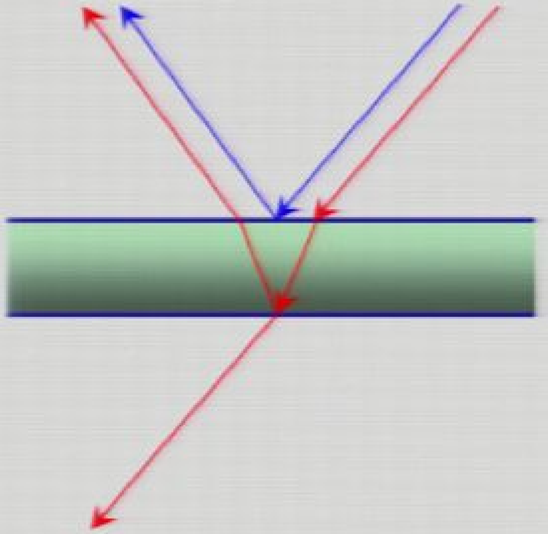
Info highways for the chips in the future

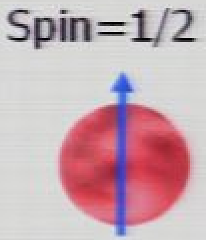
Quantum Hall effect and quantum spin Hall effect



Topological protection (Qi and Zhang, Phys Today, Jan, 2010)







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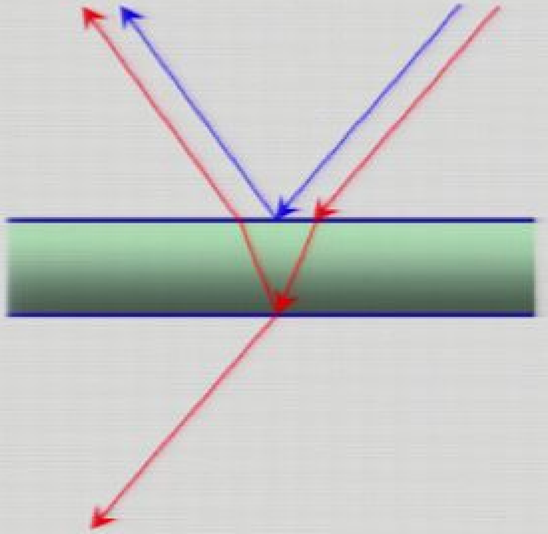
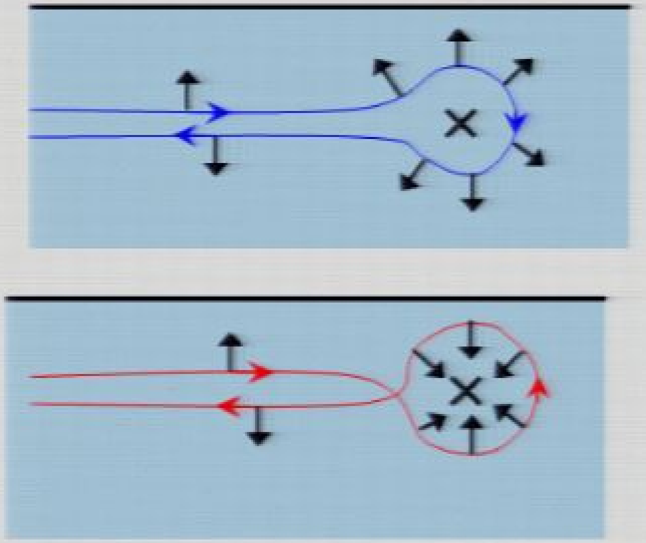
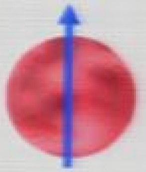




Spin=1/2



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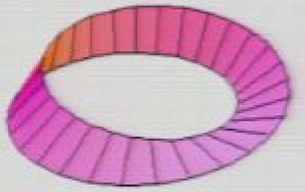
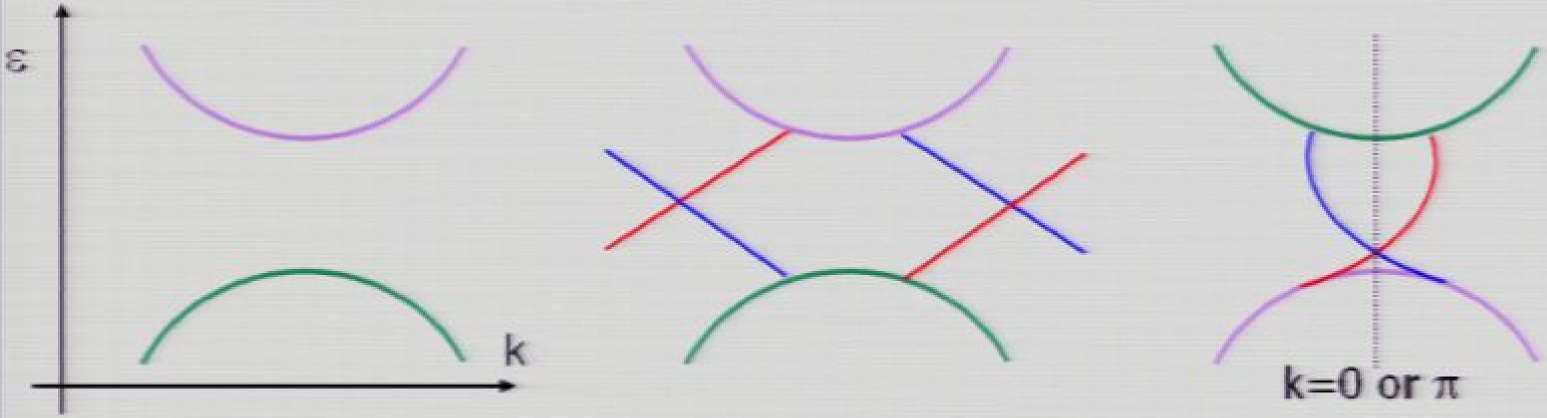


The topological distinction between a conventional insulator and a QSH insulator

Wu, Bernevig and Zhang; Xu and Moore, Kane and Mele

- Band diagram of a conventional insulator, a conventional insulator with accidental surface states (with animation), a QSH insulator (with animation). Blue and red color code for up and down spins.



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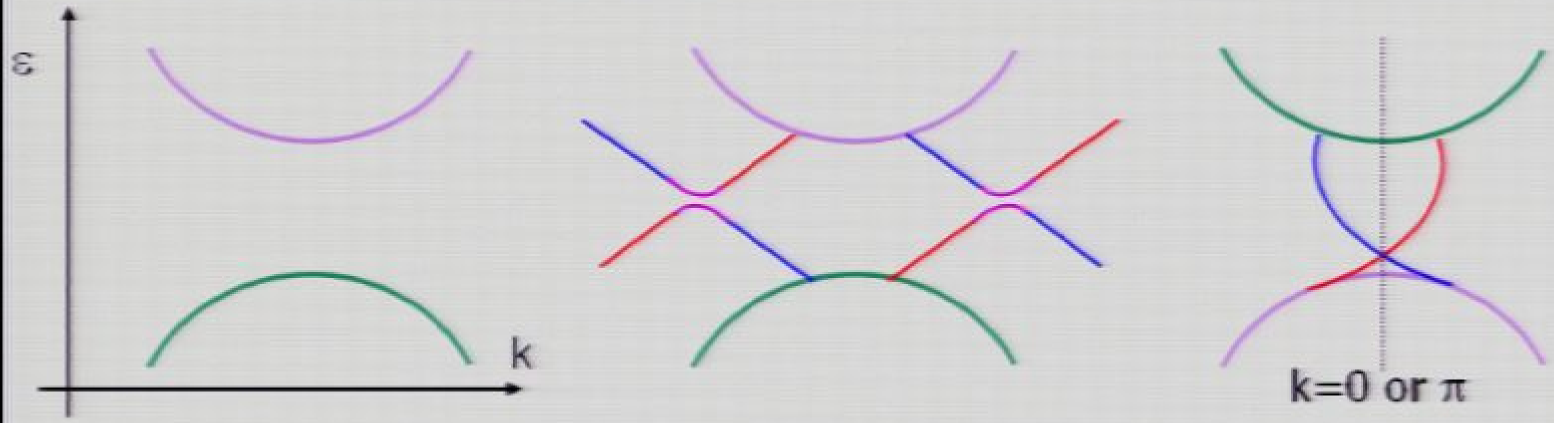

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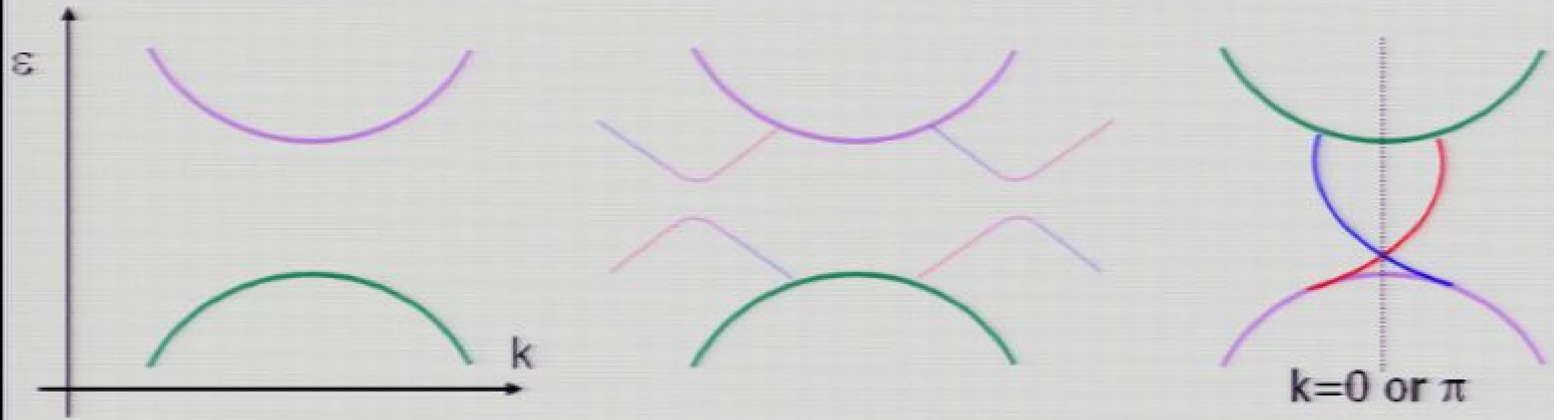

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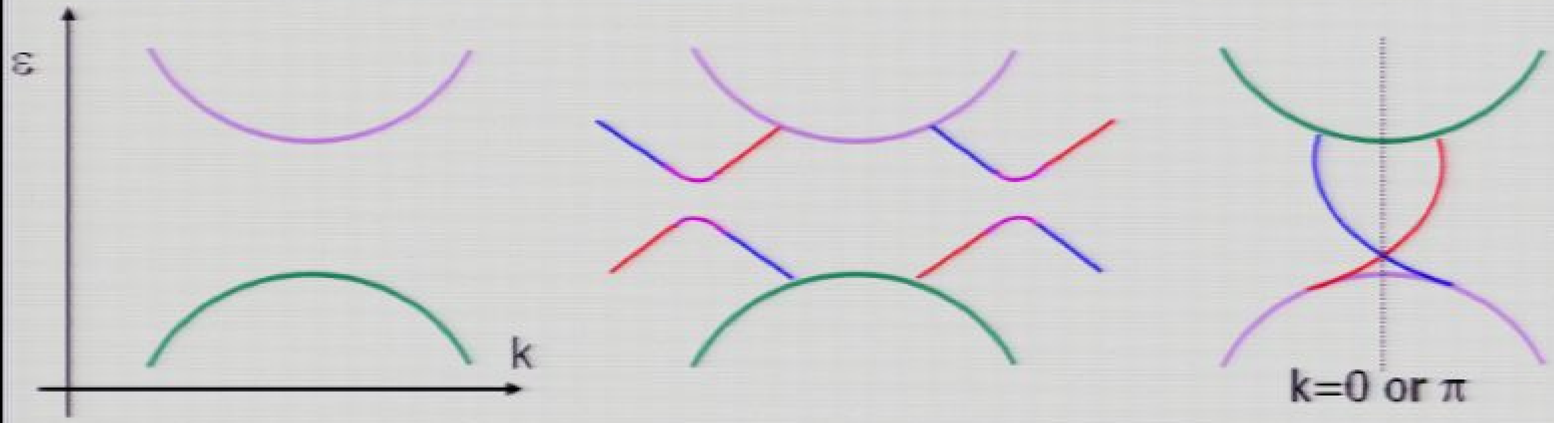

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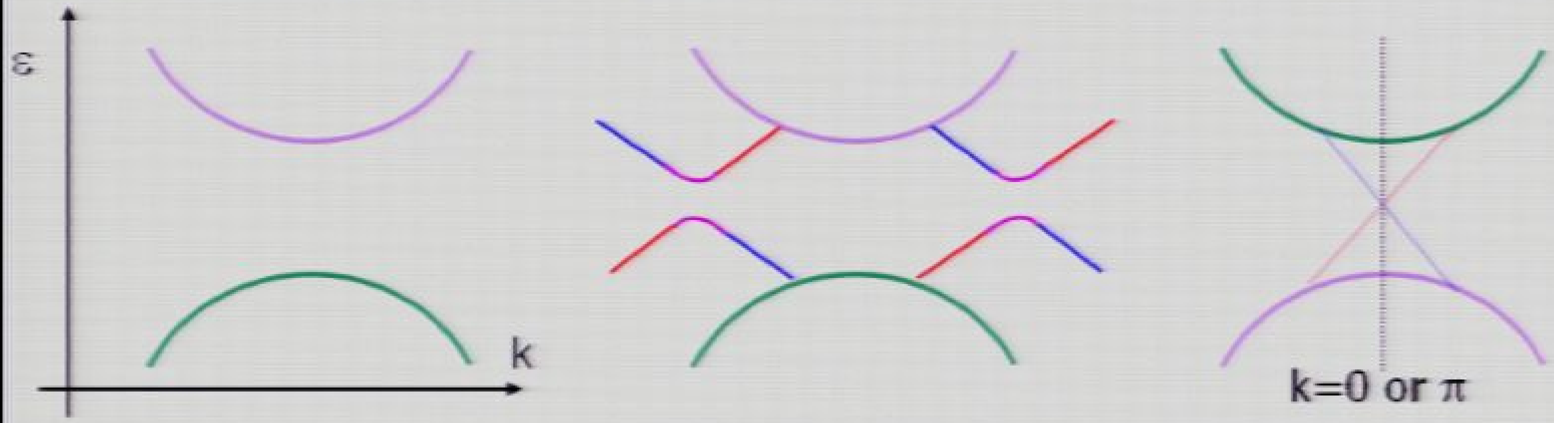

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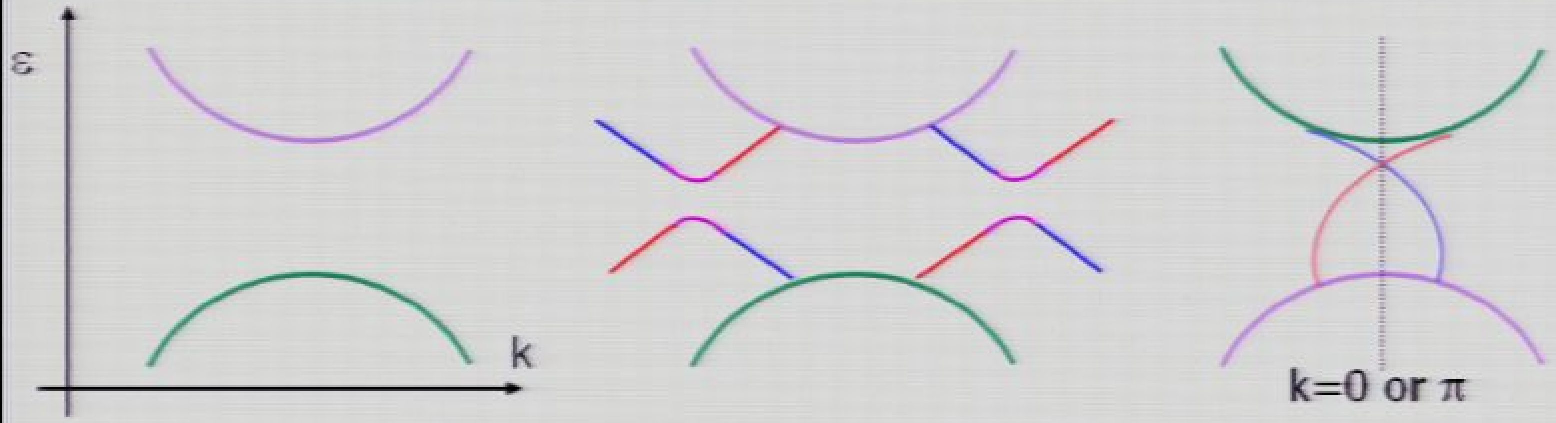

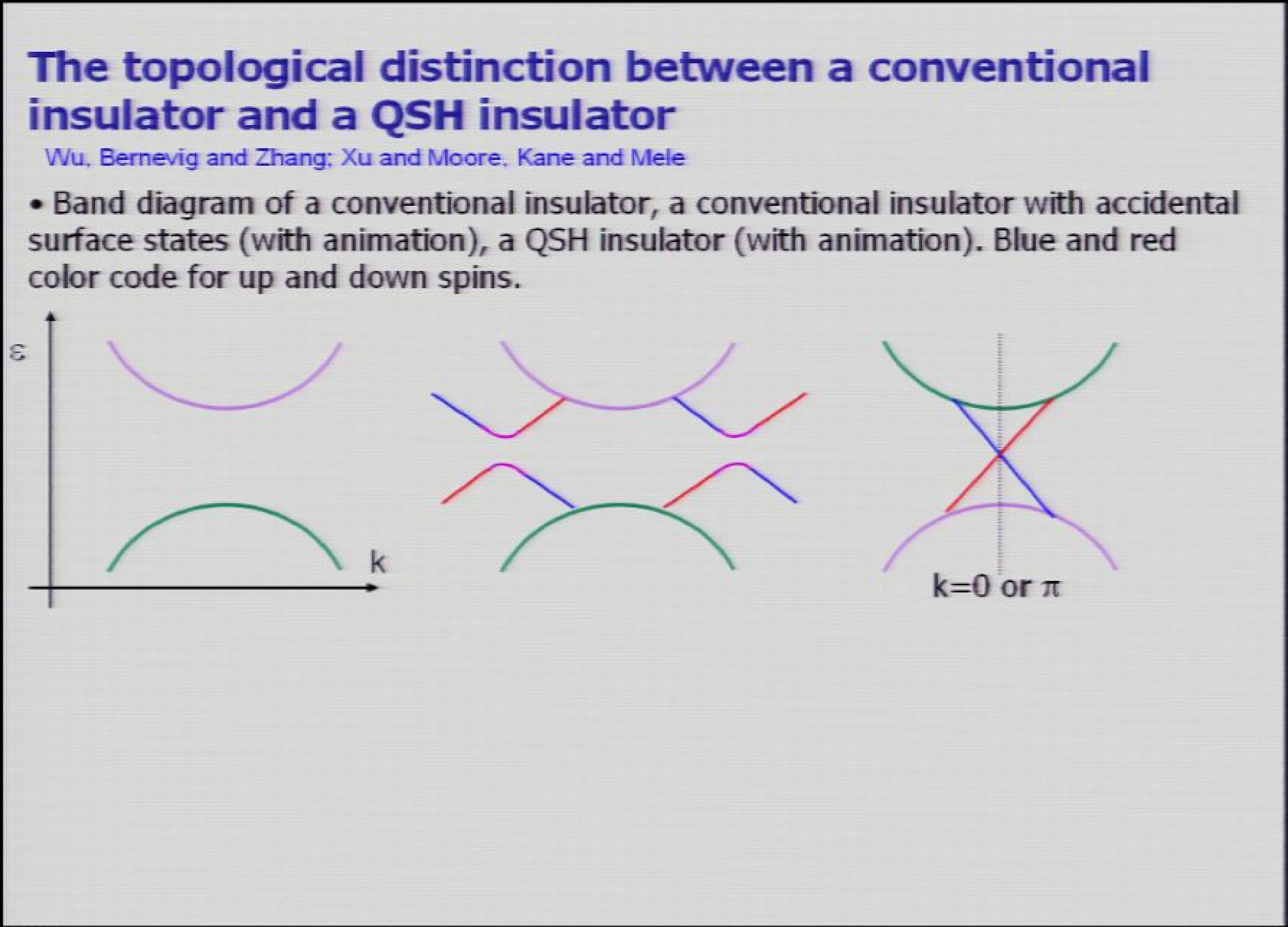

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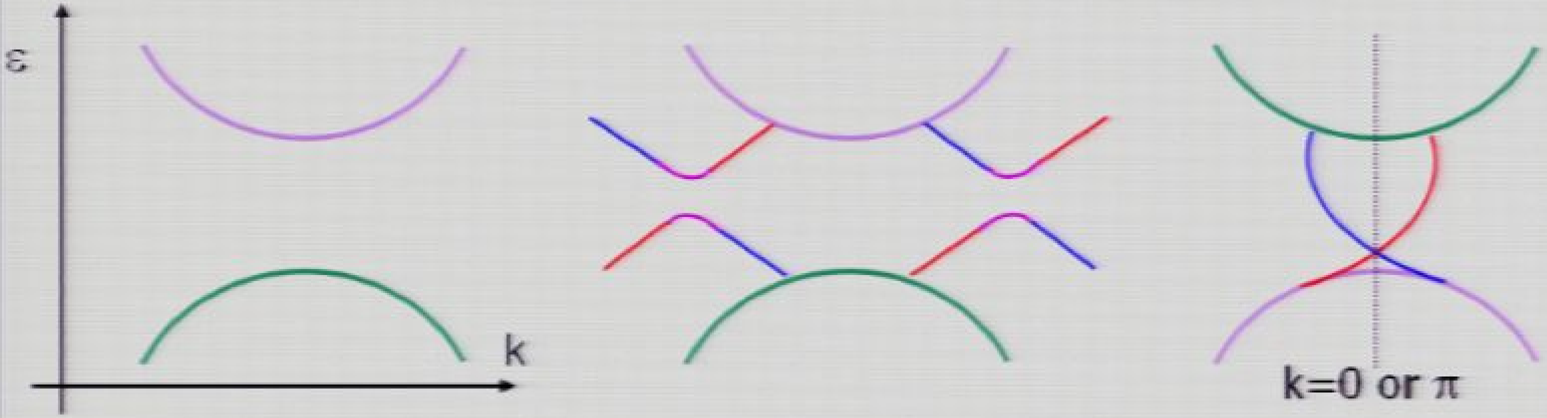

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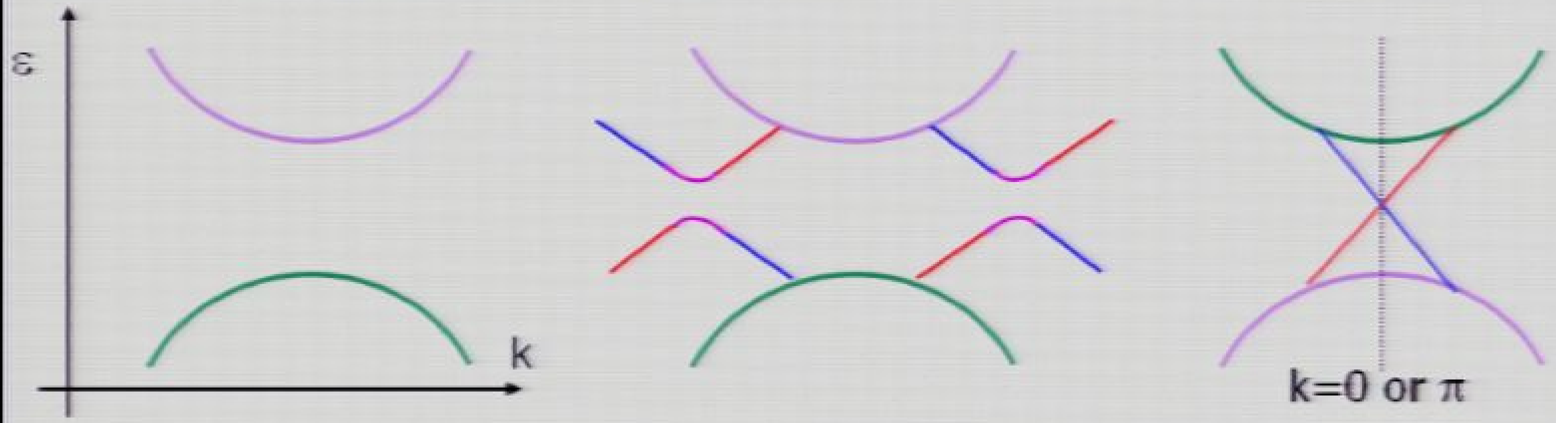
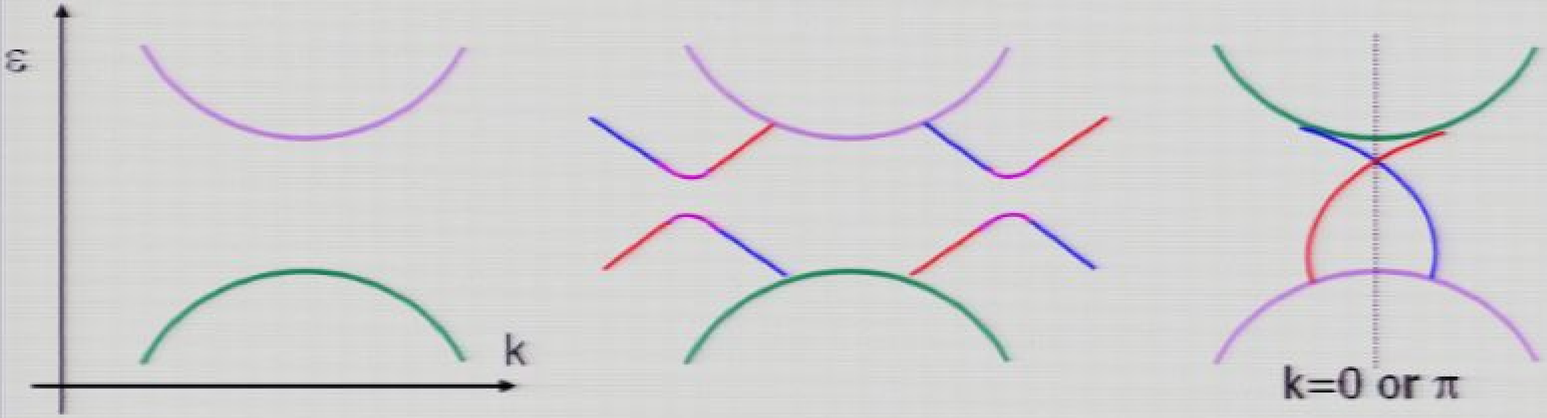


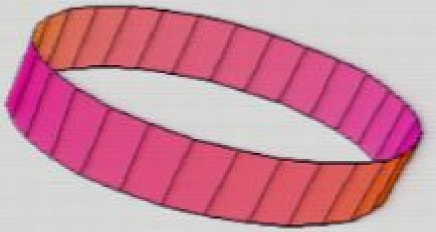
The topological distinction between a conventional insulator and a QSH insulator

Wu, Bernevig and Zhang; Xu and Moore, Kane and Mele

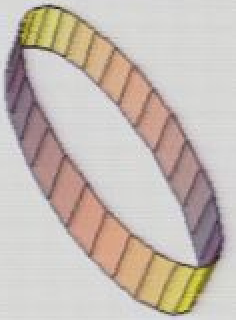
- Band diagram of a conventional insulator, a conventional insulator with accidental surface states (with animation), a QSH insulator (with animation). Blue and red color code for up and down spins.



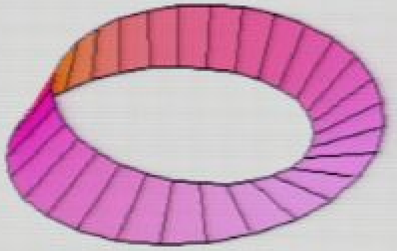
Trivial



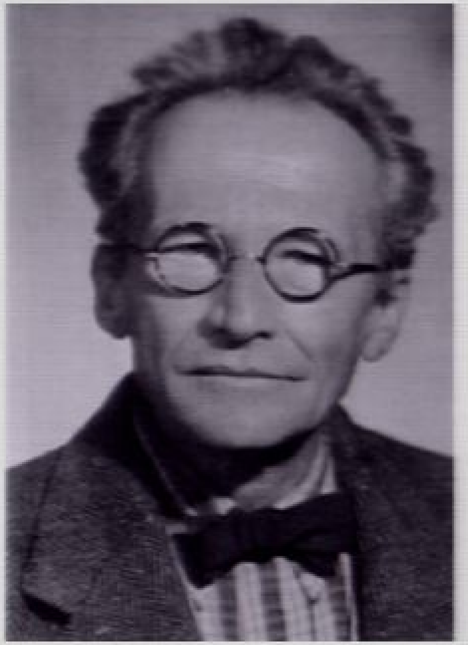
Trivial

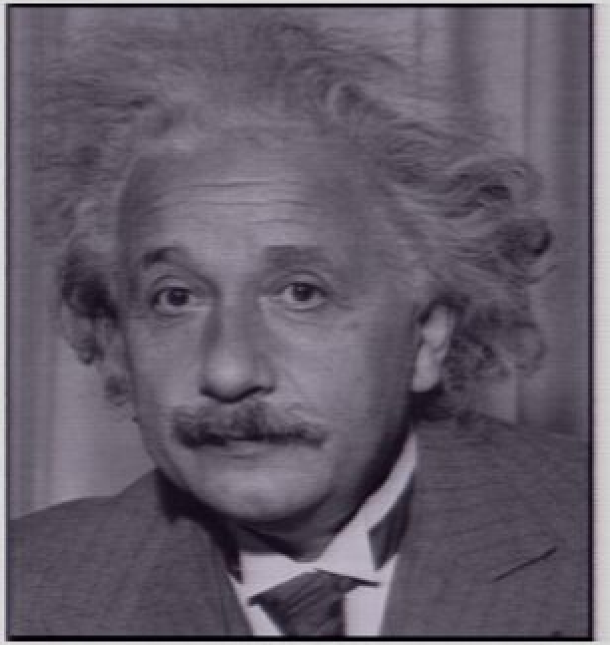


Non-trivial

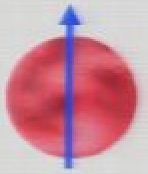


Quantum mechanics and special relativity:





Spin=1/2



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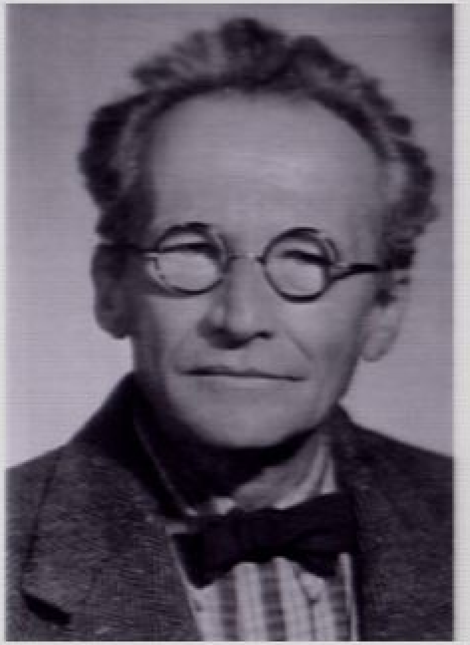

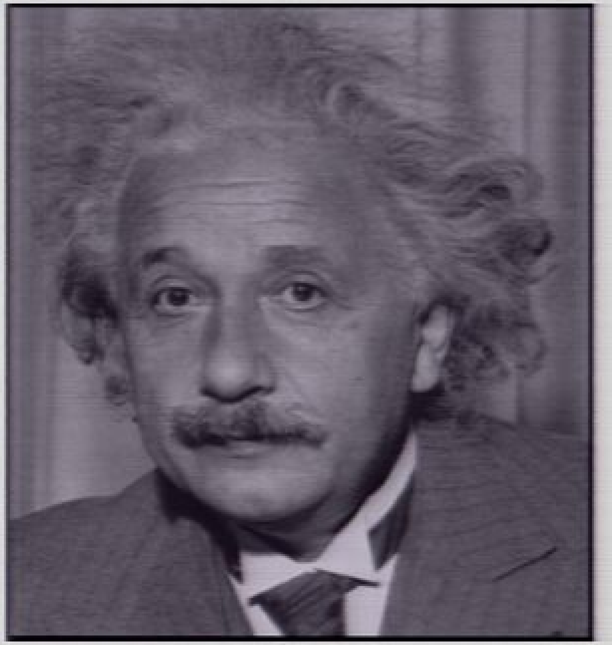

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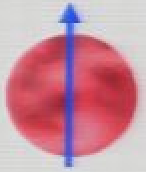

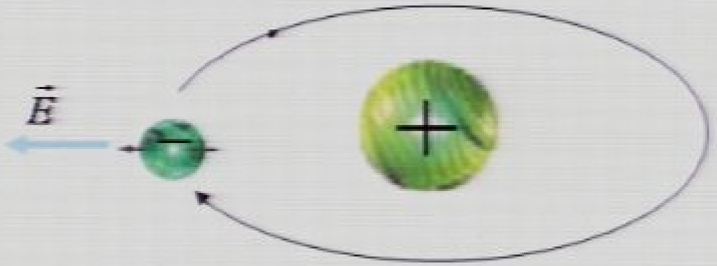

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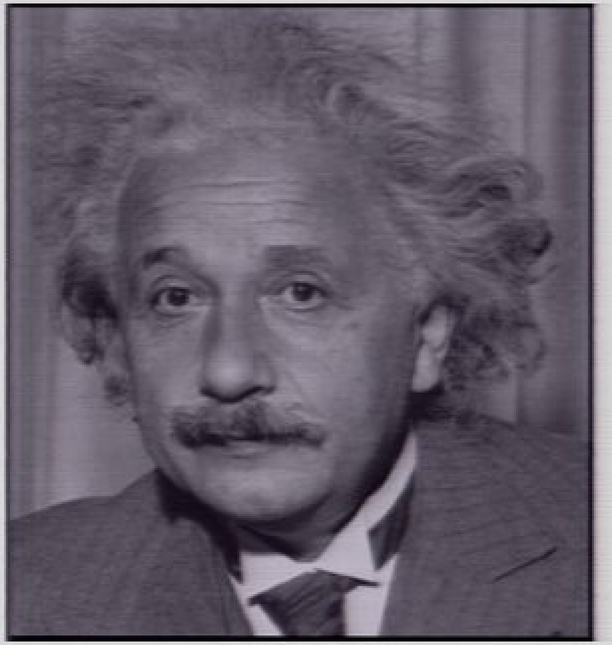
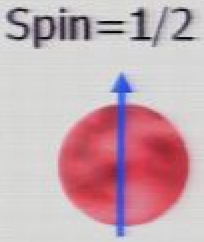
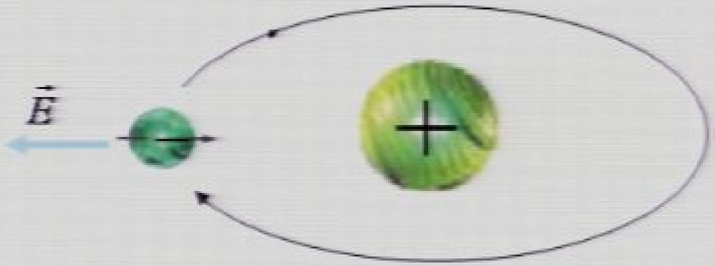
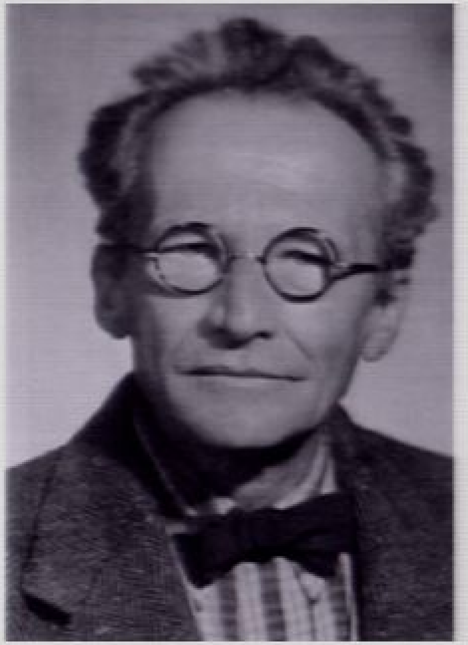


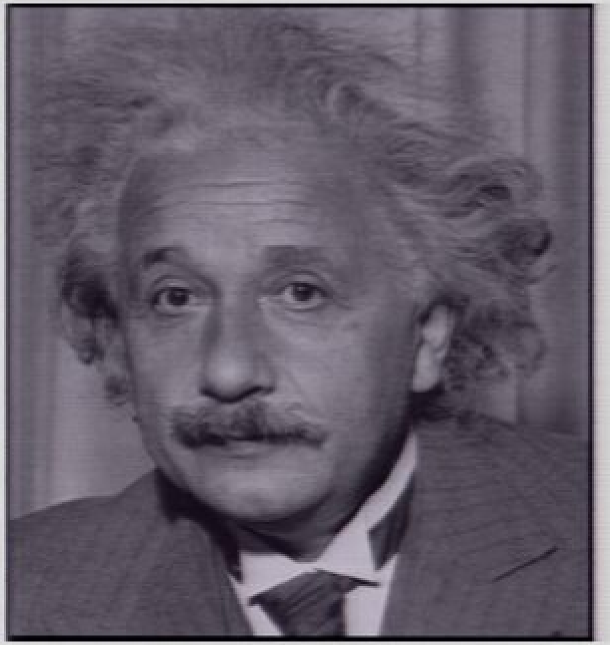


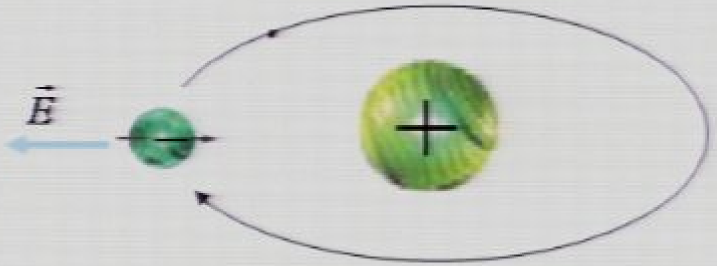


Quantum mechanics predicts spin
½ particles

Quantum mechanics and special relativity:







Quantum mechanics predicts spin ½ particles

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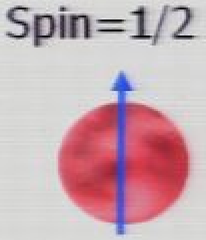
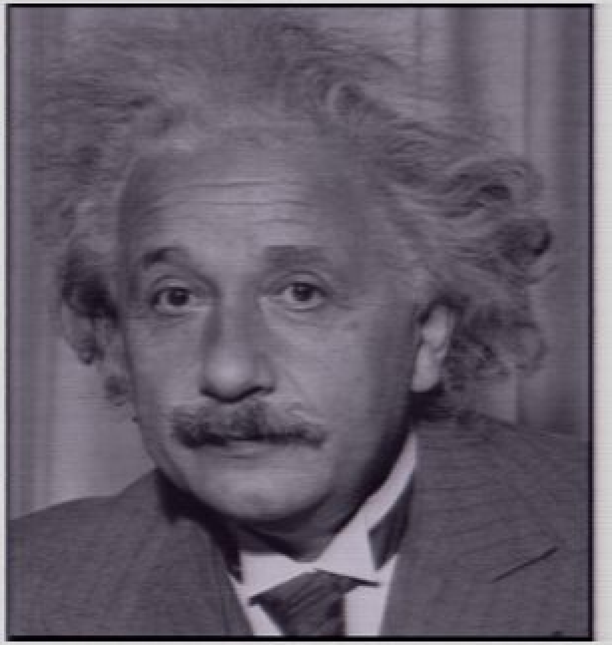
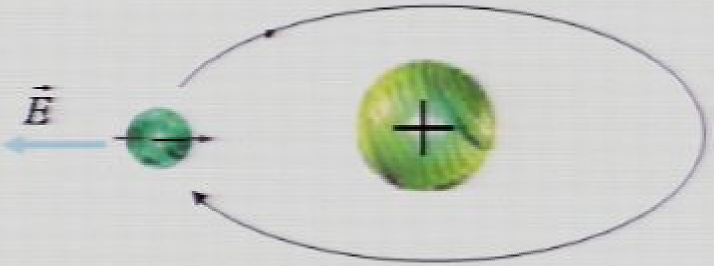
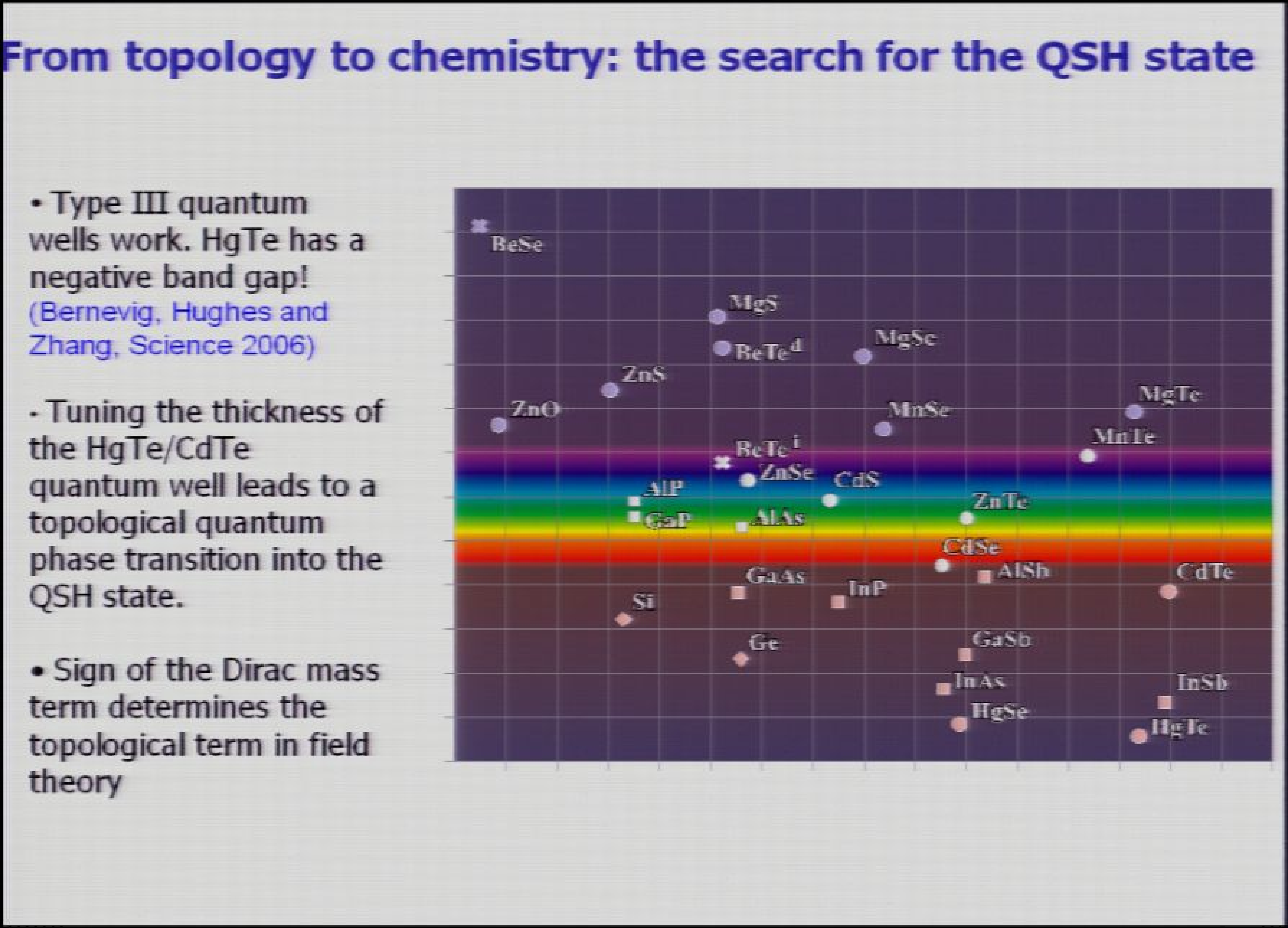


Quantum mechanics predicts spin ½ particles





Relativity predicts spin-orbit coupling



From topology to chemistry: the search for the QSH state

- Type III quantum wells work. HgTe has a negative band gap! (Bernevig, Hughes and Zhang, Science 2006)
- Tuning the thickness of the HgTe/CdTe quantum well leads to a topological quantum phase transition into the QSH state.
- Sign of the Dirac mass term determines the topological term in field theory

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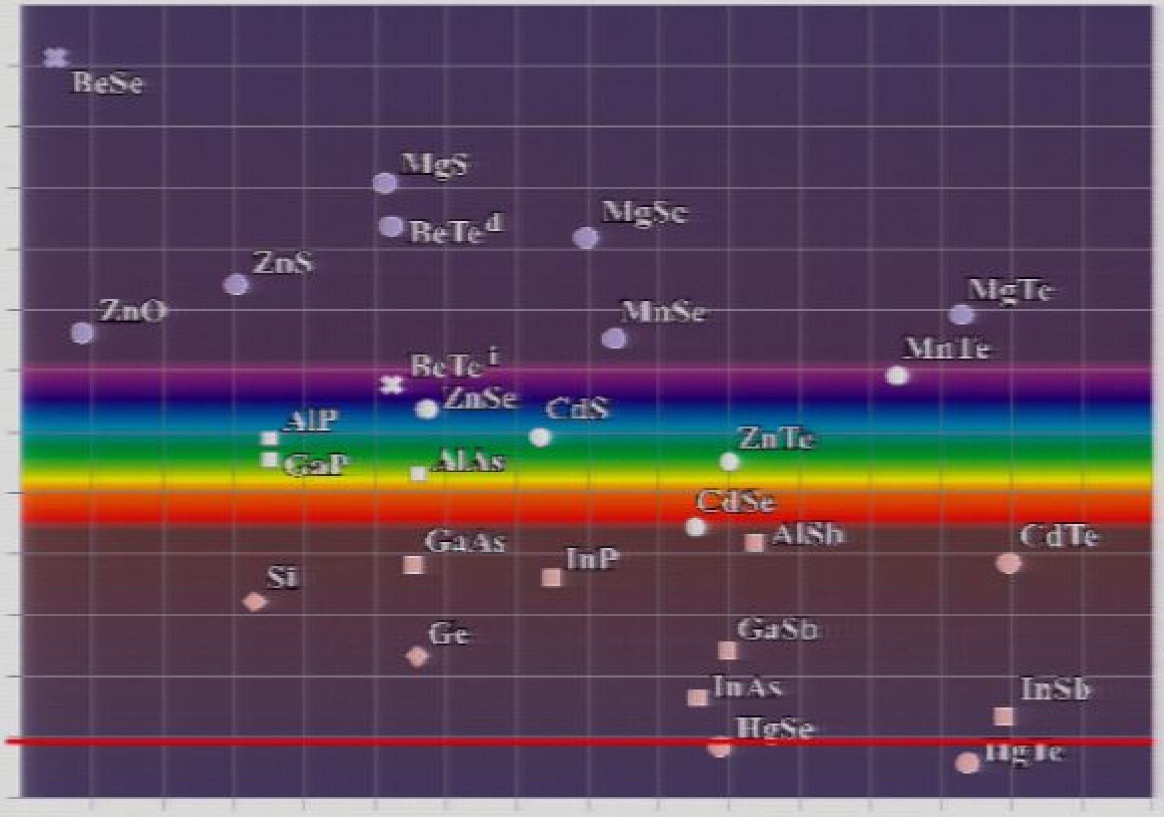
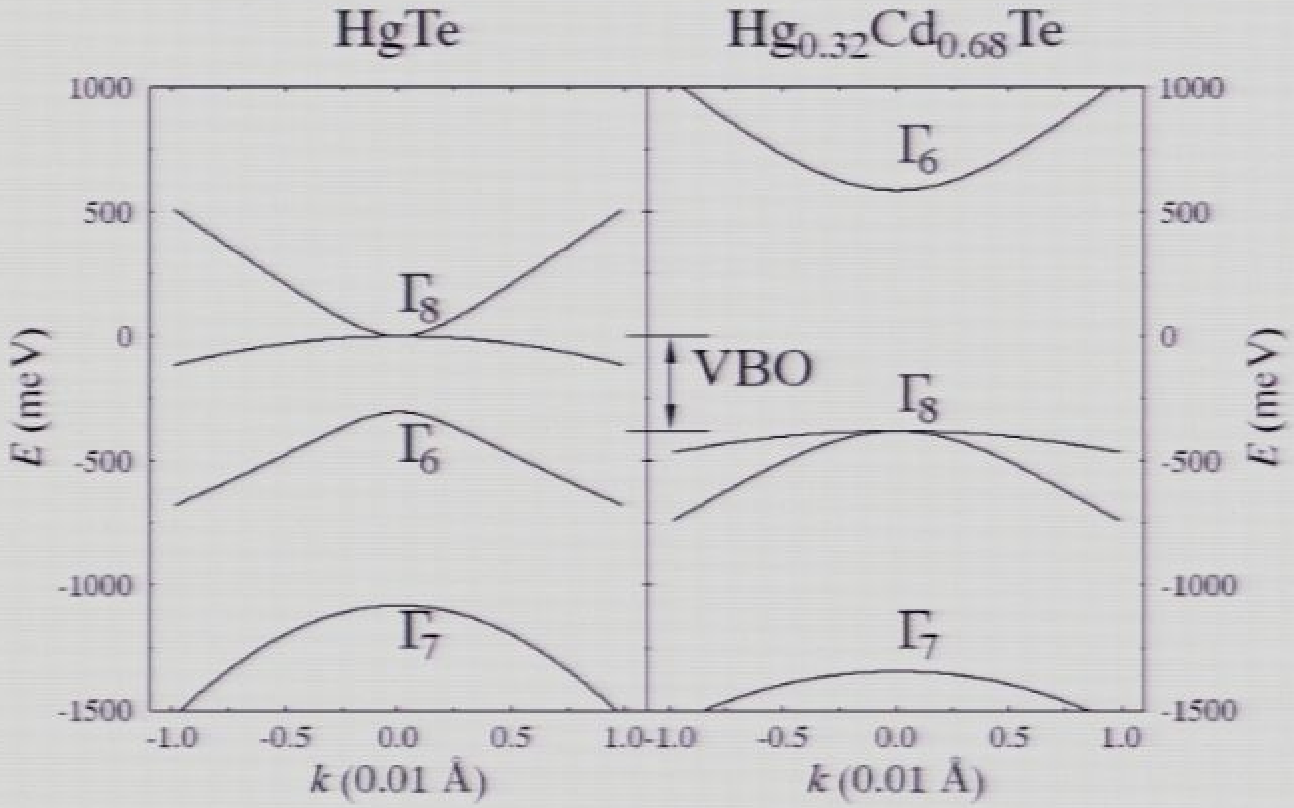
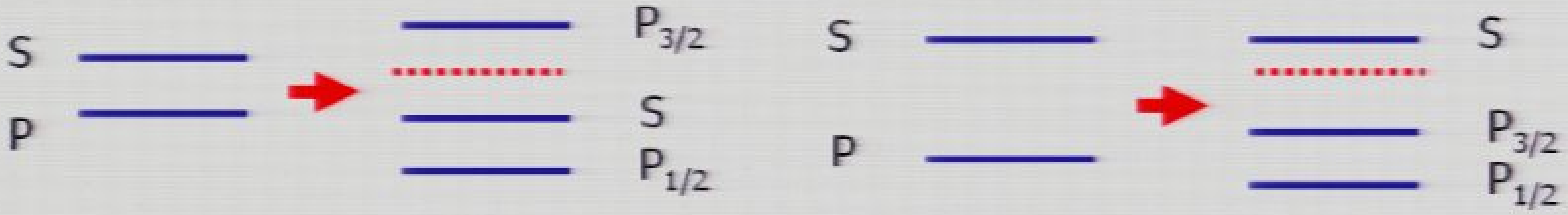


Band Structure of HgTe




S

P

$P_{3/2}$

S

$P_{1/2}$

S

P

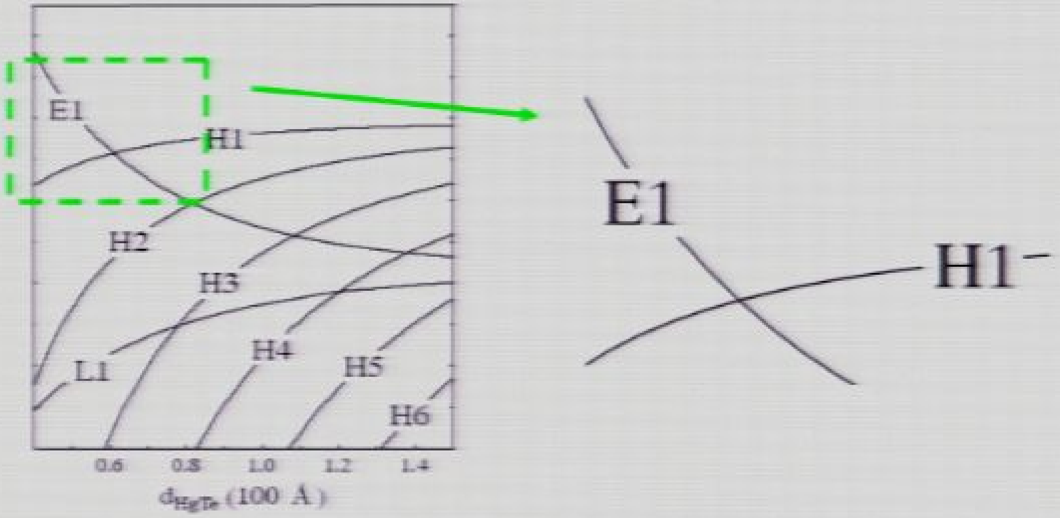
S

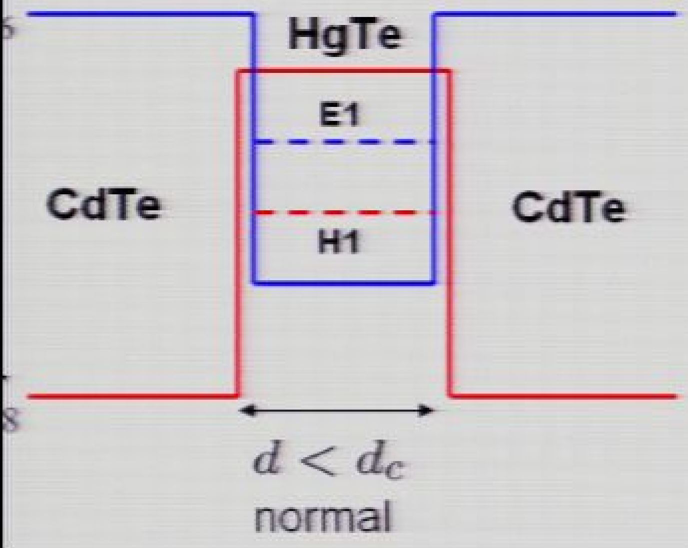
$P_{3/2}$

$P_{1/2}$

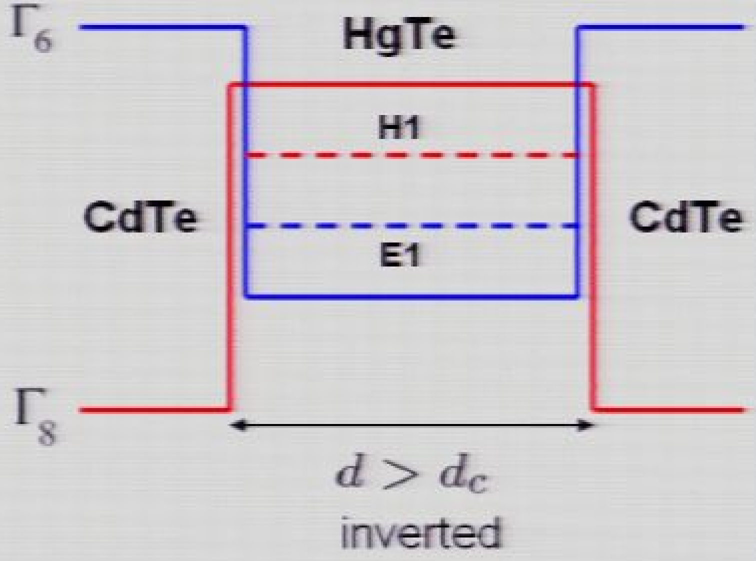
Band inversion in HgTe leads to a topological quantum phase transition

Let us focus on E1, H1 bands close to crossing point





normal

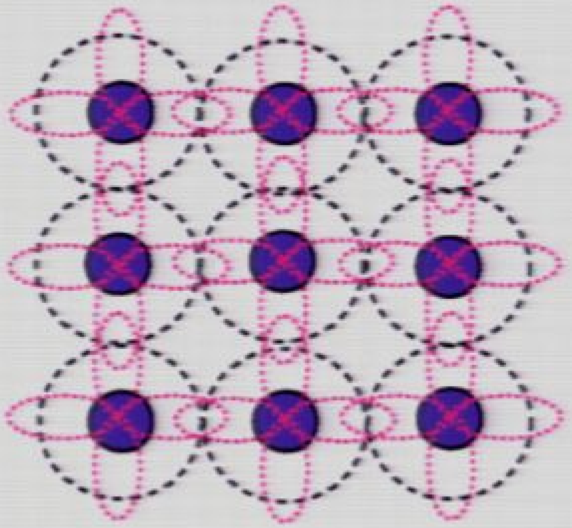


inverted

The model of the 2D topological insulator (BHZ, Science 2006)

Square lattice with 4-orbitals per site:

$$|s,\uparrow\rangle, |s,\downarrow\rangle, |(p_x+ip_y,\uparrow\rangle, |-(p_x-ip_y),\downarrow\rangle$$



Nearest neighbor hopping integrals. Mixing matrix elements between the s and the p states must be odd in k.

$$H_{eff}(k_x,k_y)=\begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix}$$

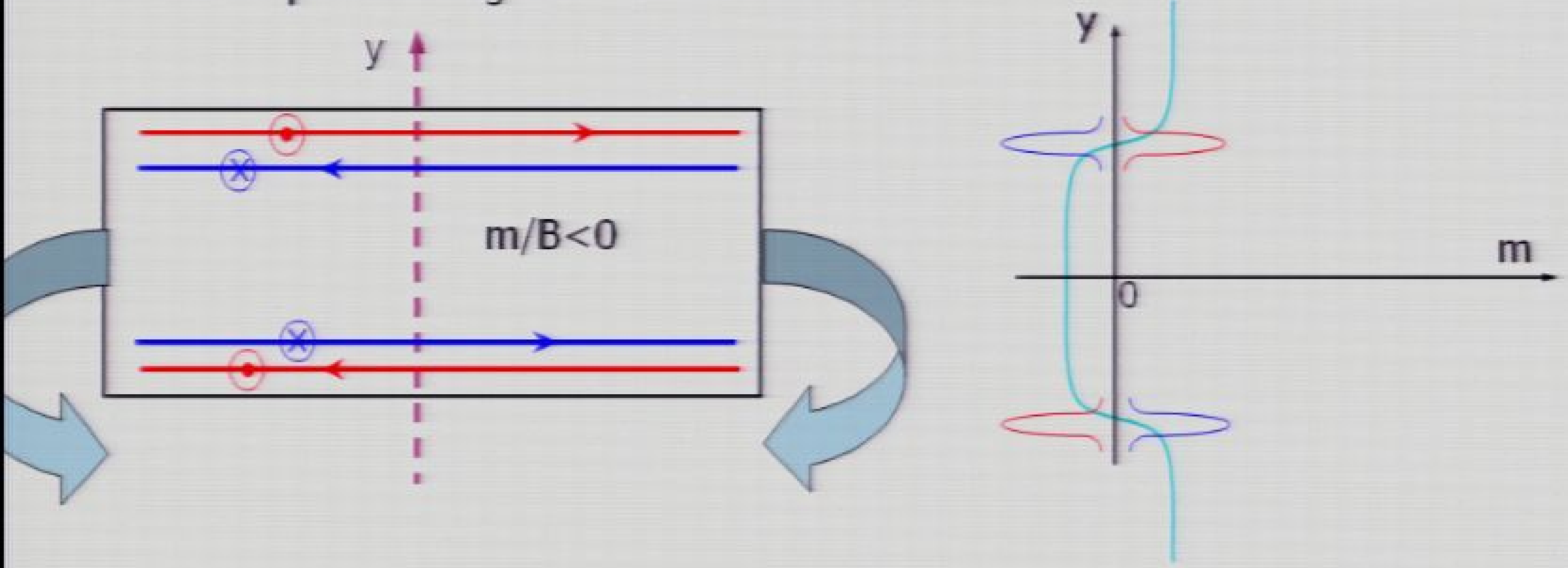
$$h(k)=\begin{pmatrix} m(k) & A(\sin k_x - i\sin k_y) \\ A(\sin k_x + i\sin k_y) & -m(k) \end{pmatrix} \equiv d_a(k)\tau^a$$

$$\Rightarrow \begin{pmatrix} m+Bk^2 & A(k_x-ik_y) \\ A(k_x+ik_y) & -m-Bk^2 \end{pmatrix}$$

Similar to relativistic Dirac equation in 2+1 dimensions, with a mass term tunable by the sample thickness d! $m/B<0$ for $d>d_c$.

Mass domain wall

Cutting the Hall bar along the y-direction we see a domain-wall structure in the band structure mass term. This leads to states localized on the domain wall which still disperse along the x-direction.



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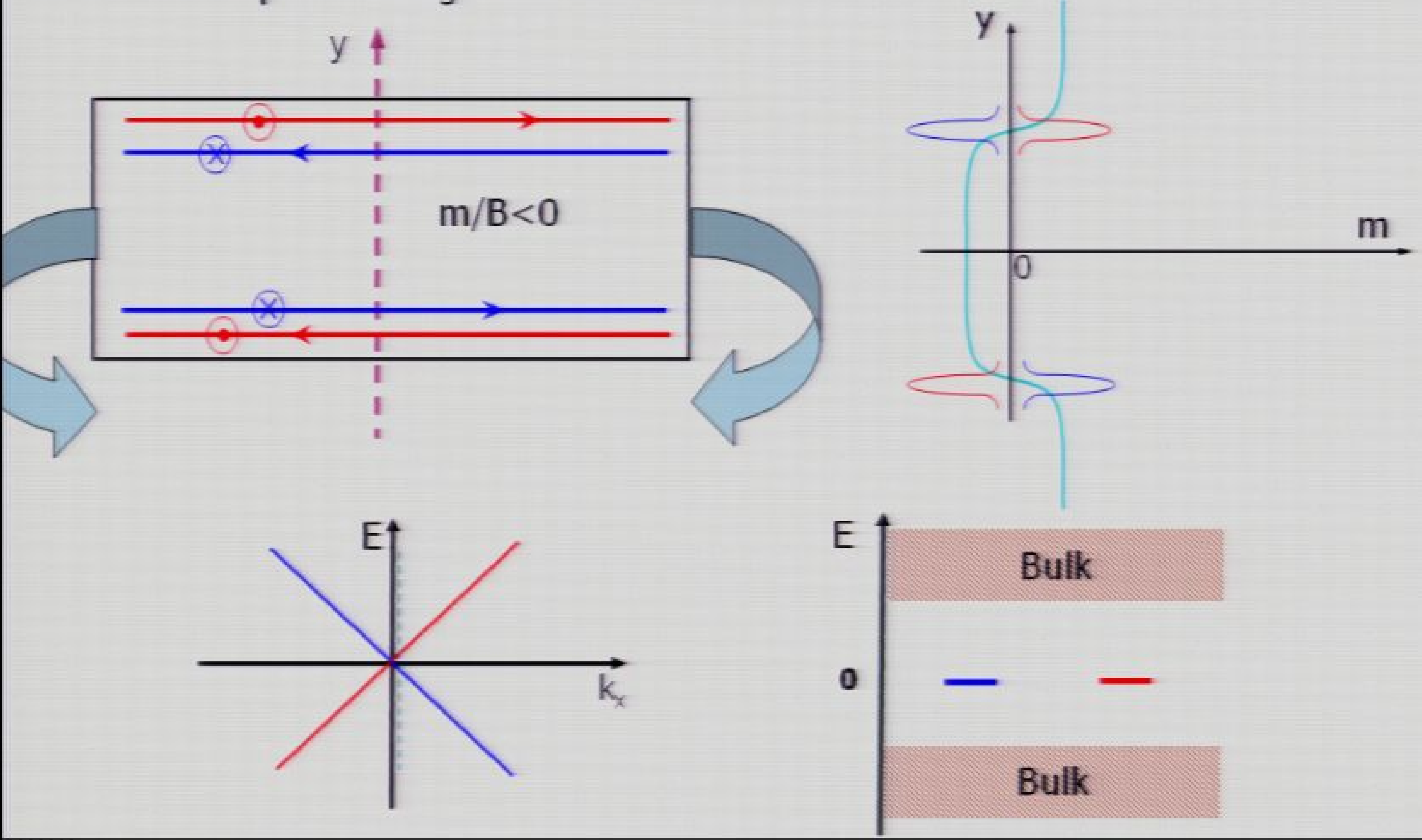

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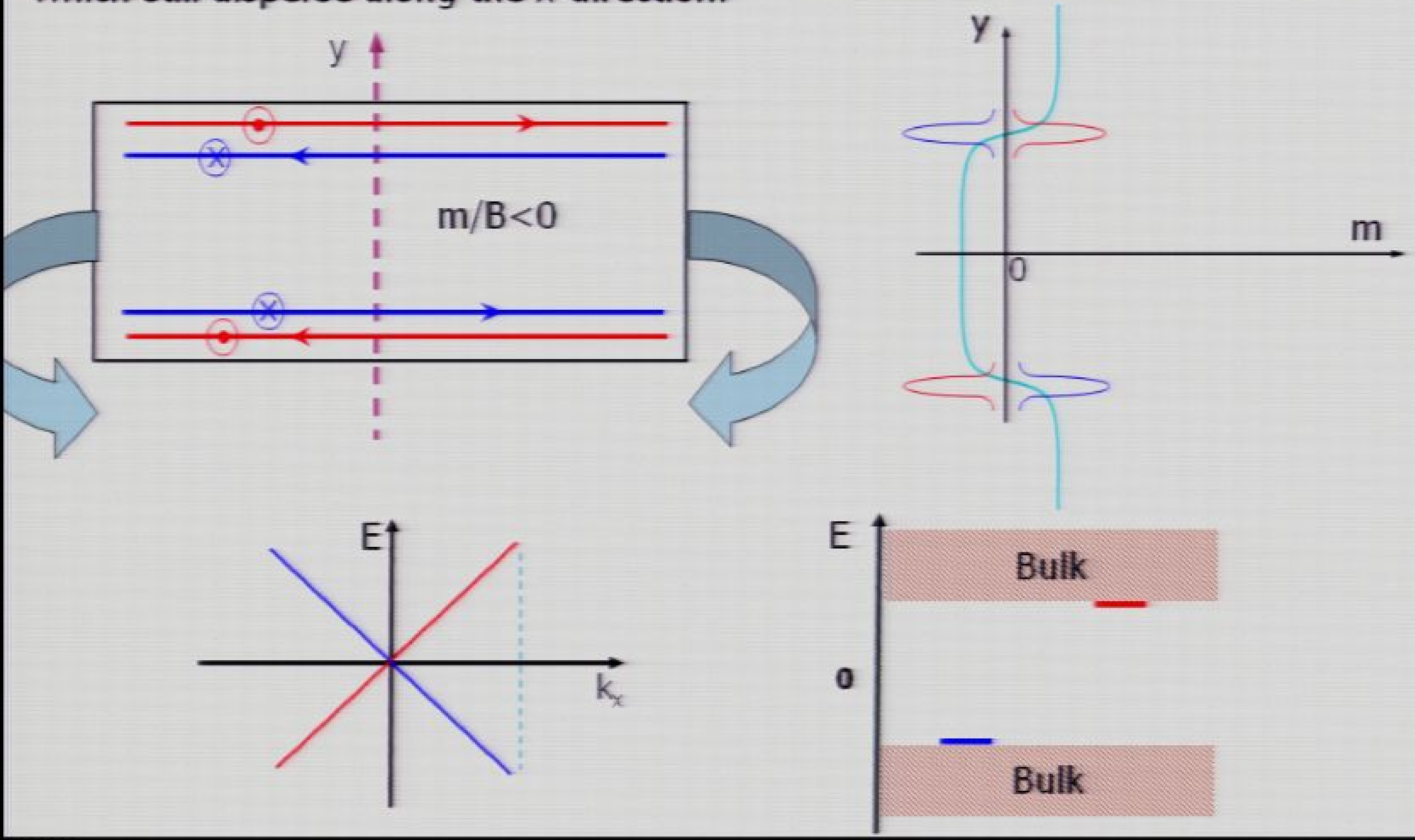

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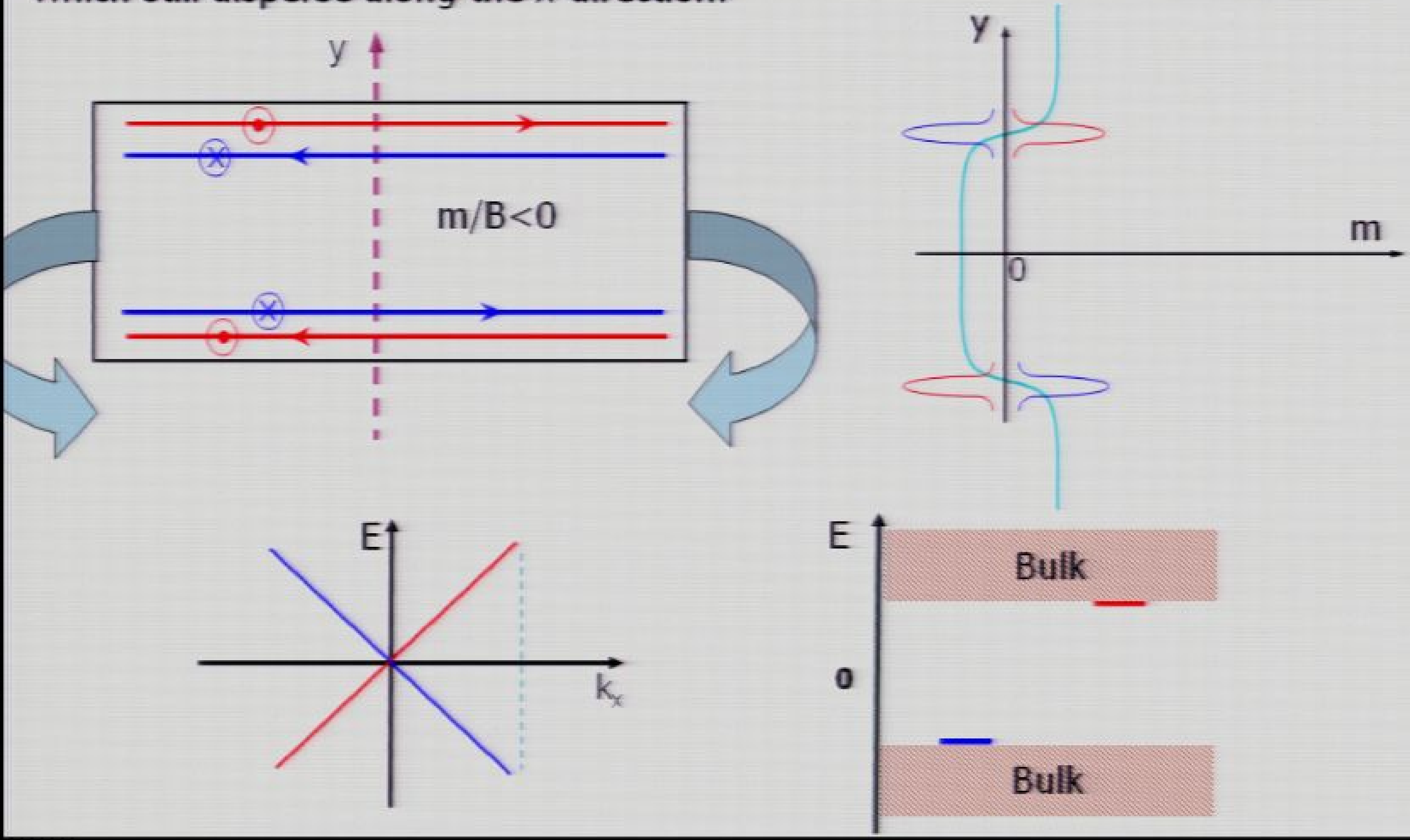
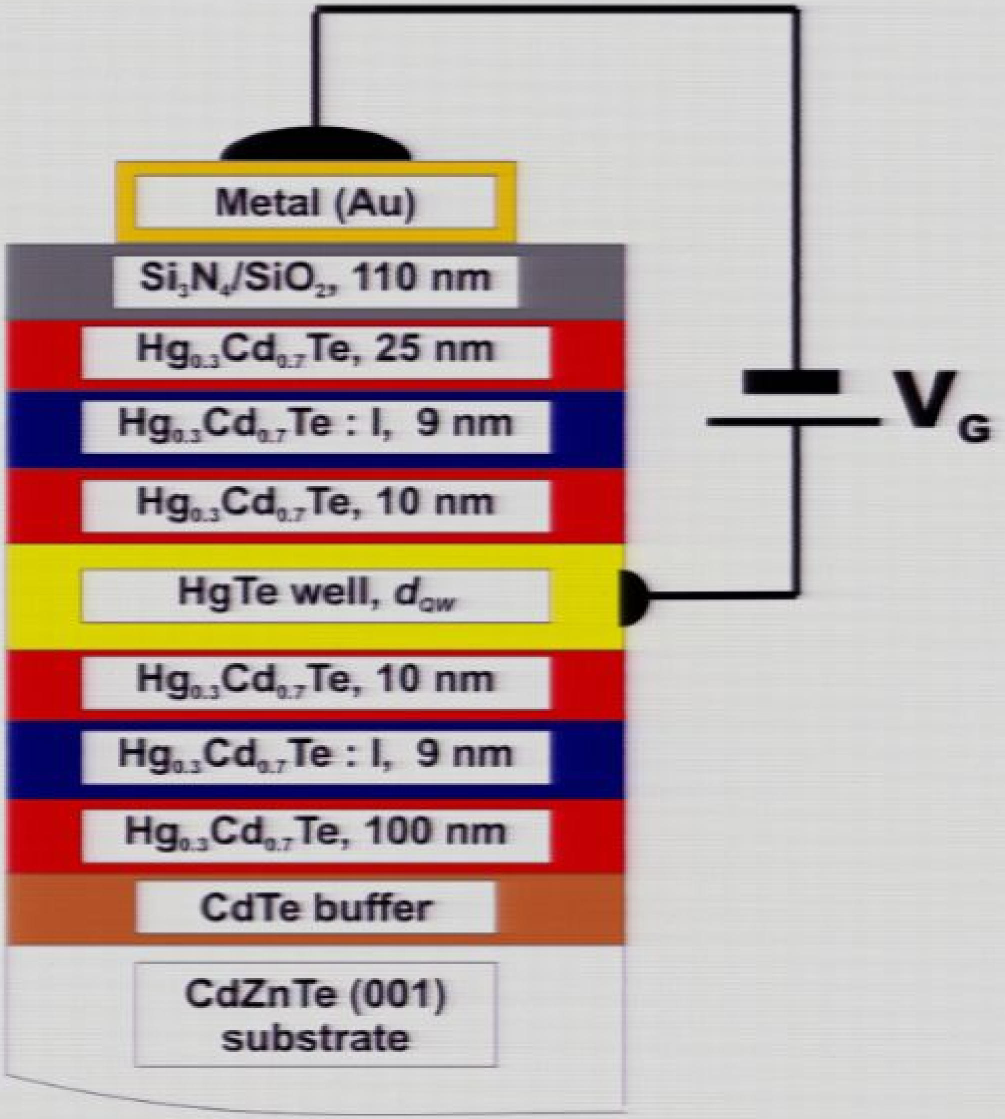


Experimental setup

- High mobility samples of HgTe/CdTe quantum wells have been fabricated.
- Because of the small band gap, about several meV, one can gate dope this system from n to p doped regimes.
- Two tuning parameters, the thickness d of the quantum well, and the gate voltage.

•(Koenig et al, Science 2007)



Metal (Au)

Si_3N_4/SiO_2, 110 nm

$Hg_{0.3}Cd_{0.7}Te$, 25 nm

$Hg_{0.3}Cd_{0.7}Te$: I, 9 nm

$Hg_{0.3}Cd_{0.7}Te$, 10 nm

HgTe well, d_{QW}

$Hg_{0.3}Cd_{0.7}Te$, 10 nm

$Hg_{0.3}Cd_{0.7}Te$: I, 9 nm

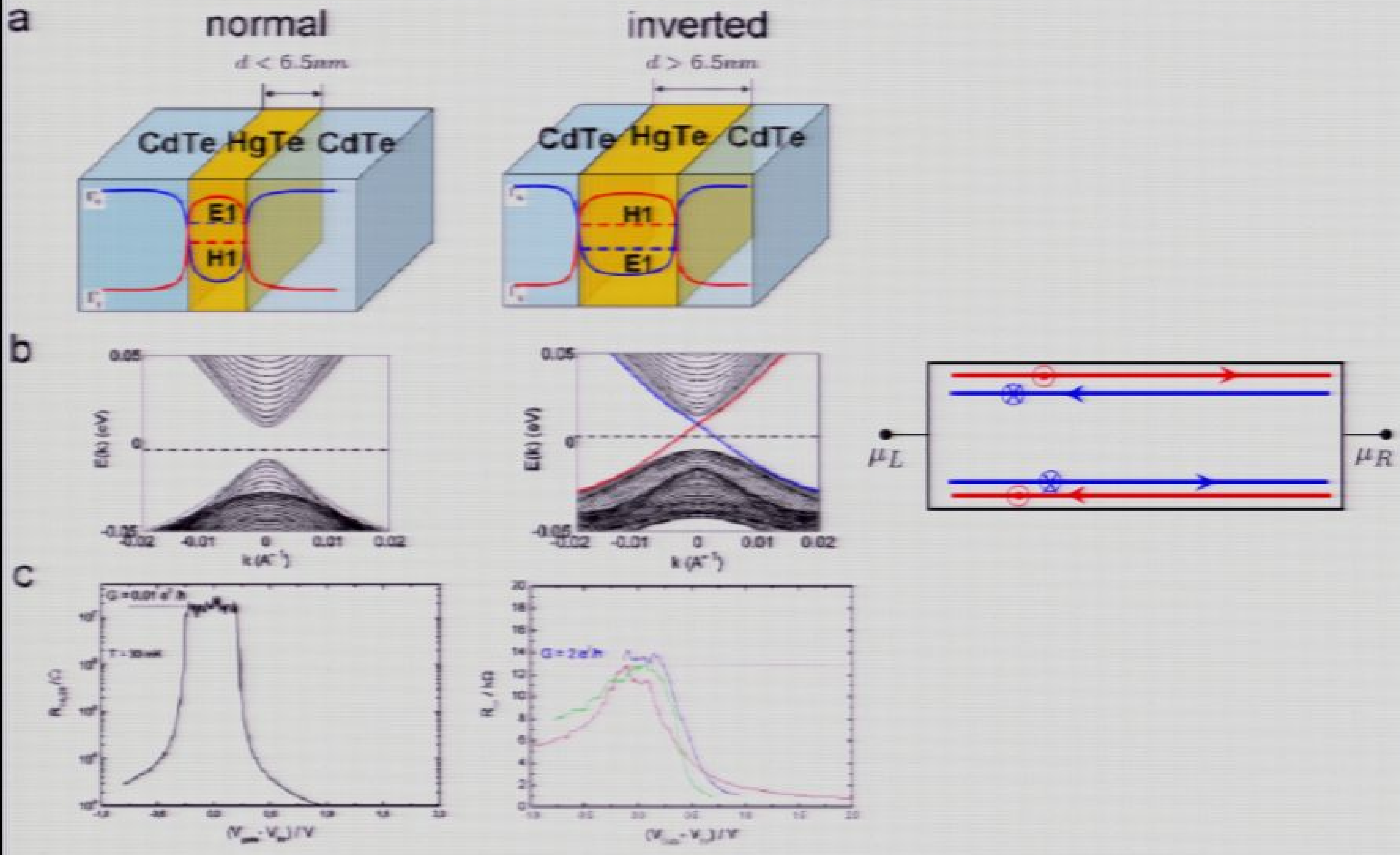
$Hg_{0.3}Cd_{0.7}Te$, 100 nm

CdTe buffer

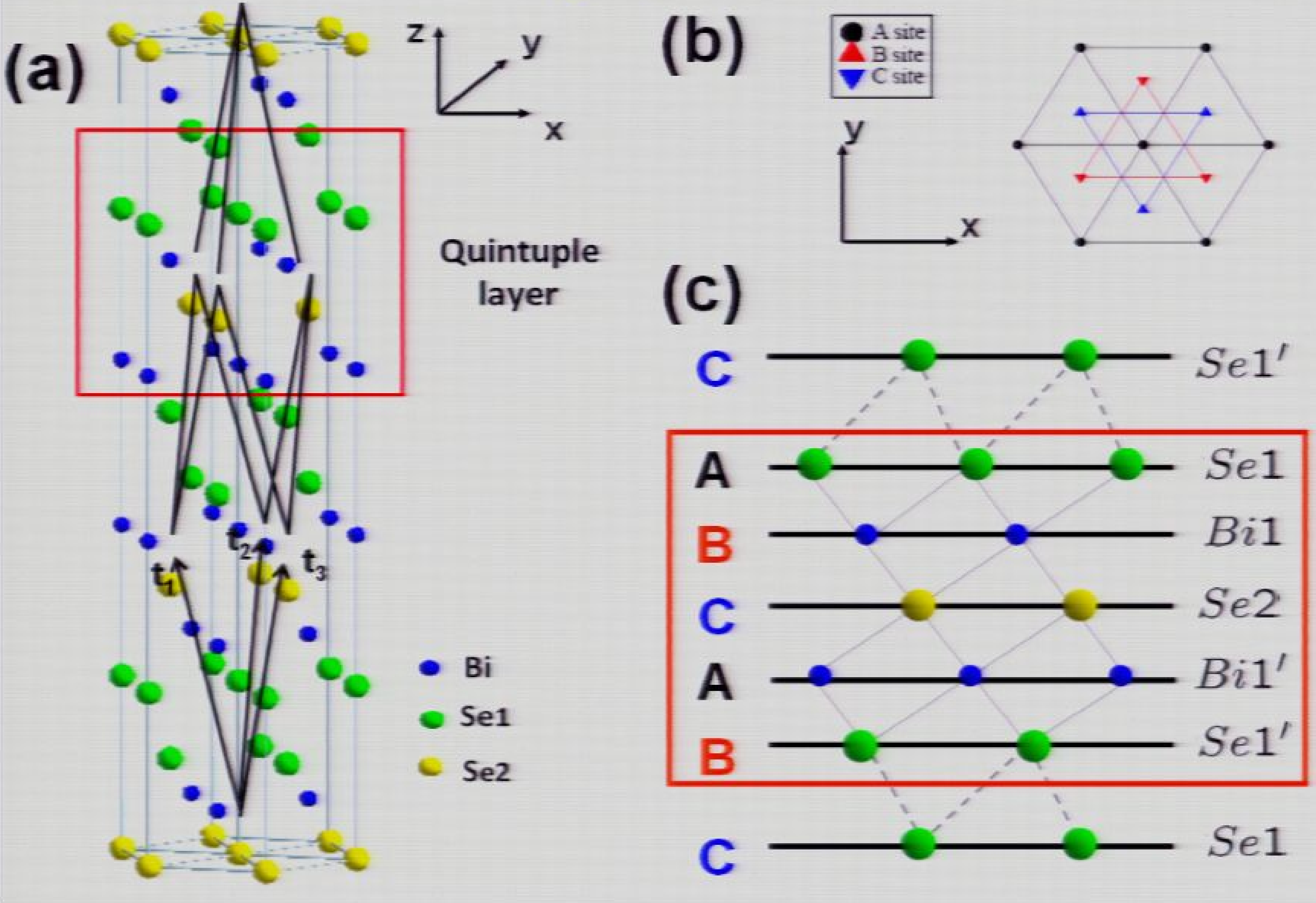
CdZnTe (001) substrate

Experimental observation of the QSH edge state

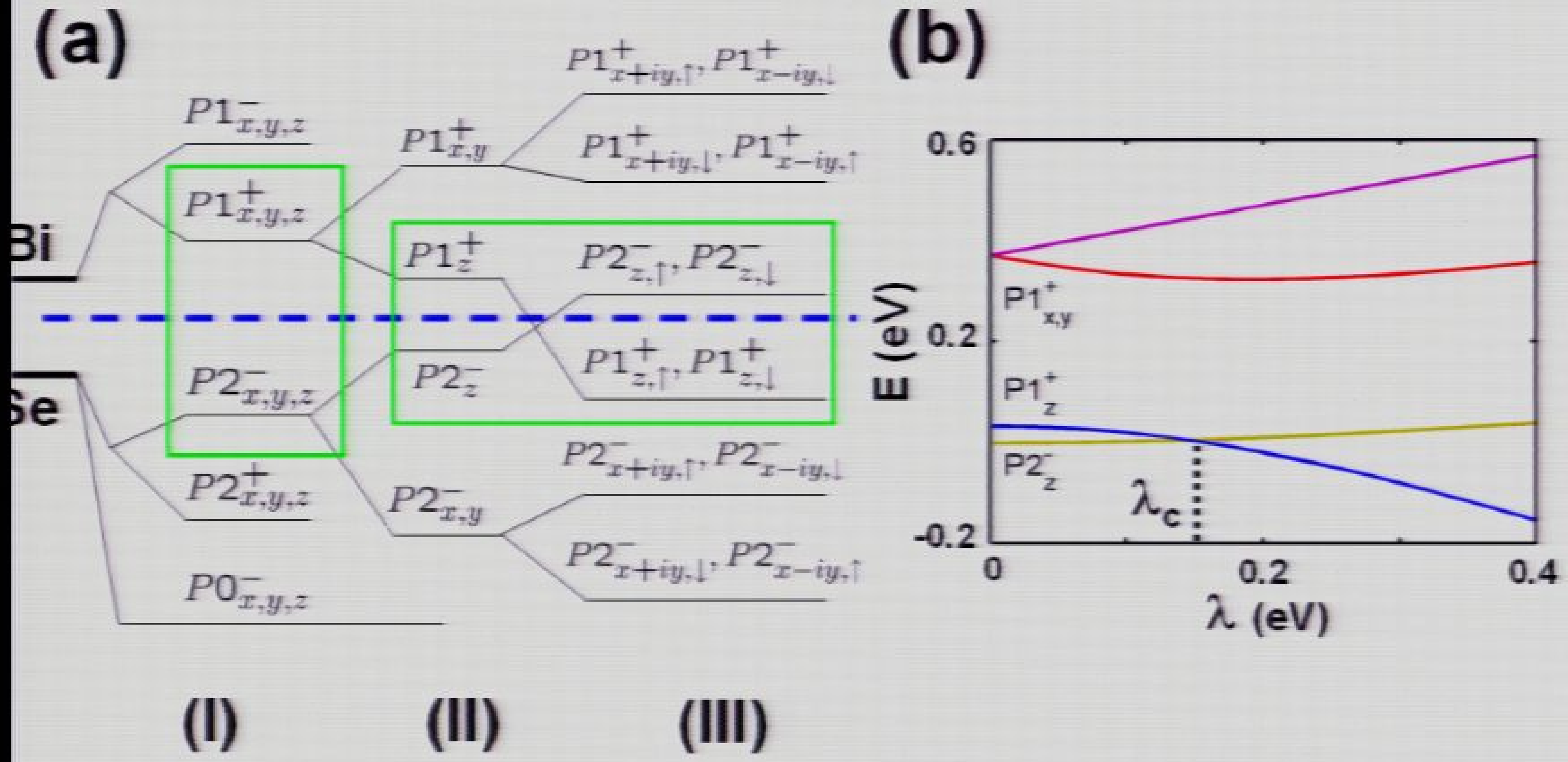
(Konig et al, Science 2007)



3D insulators with a single Dirac cone on the surface



Relevant orbitals of Bi2Se3 and the band inversion



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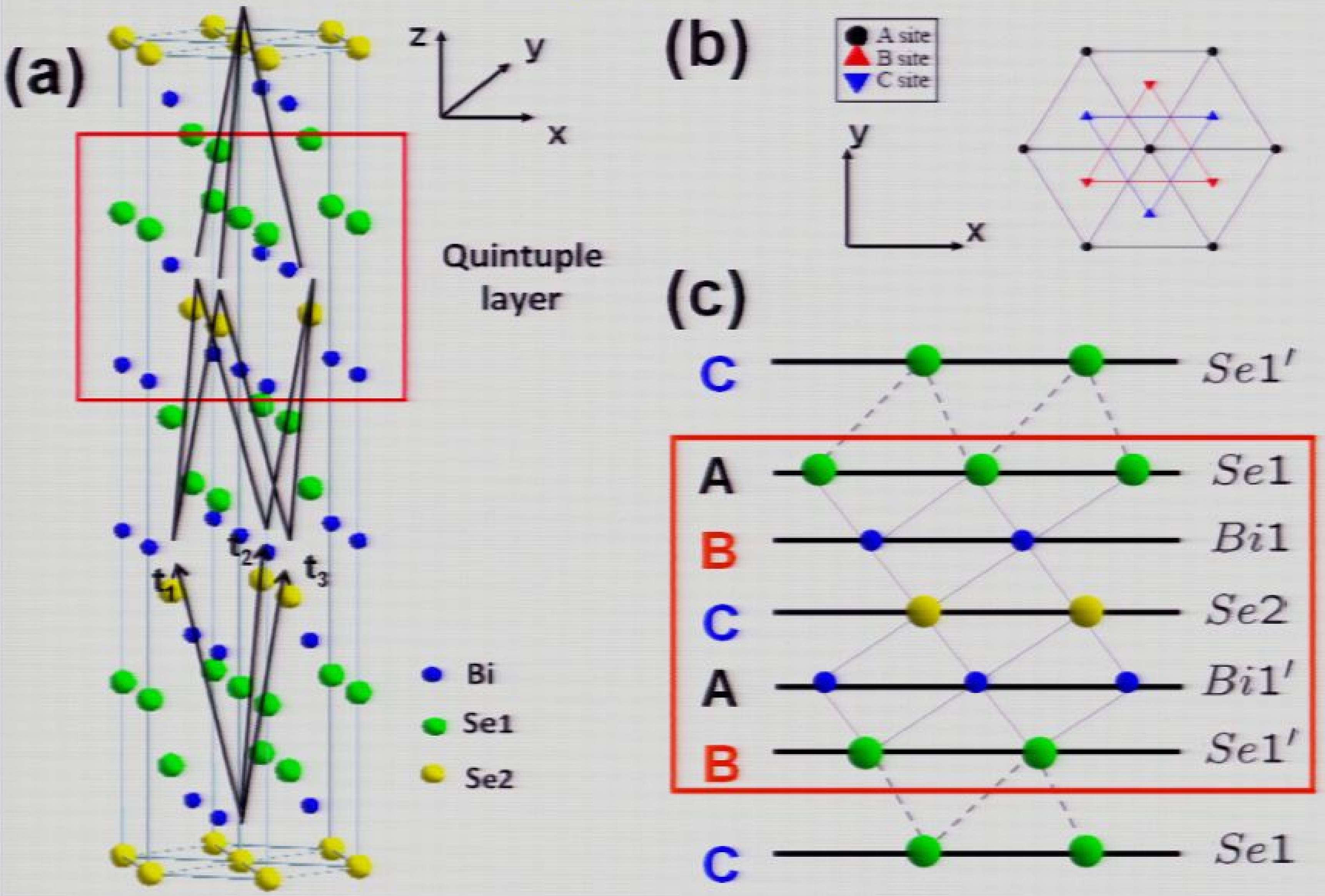

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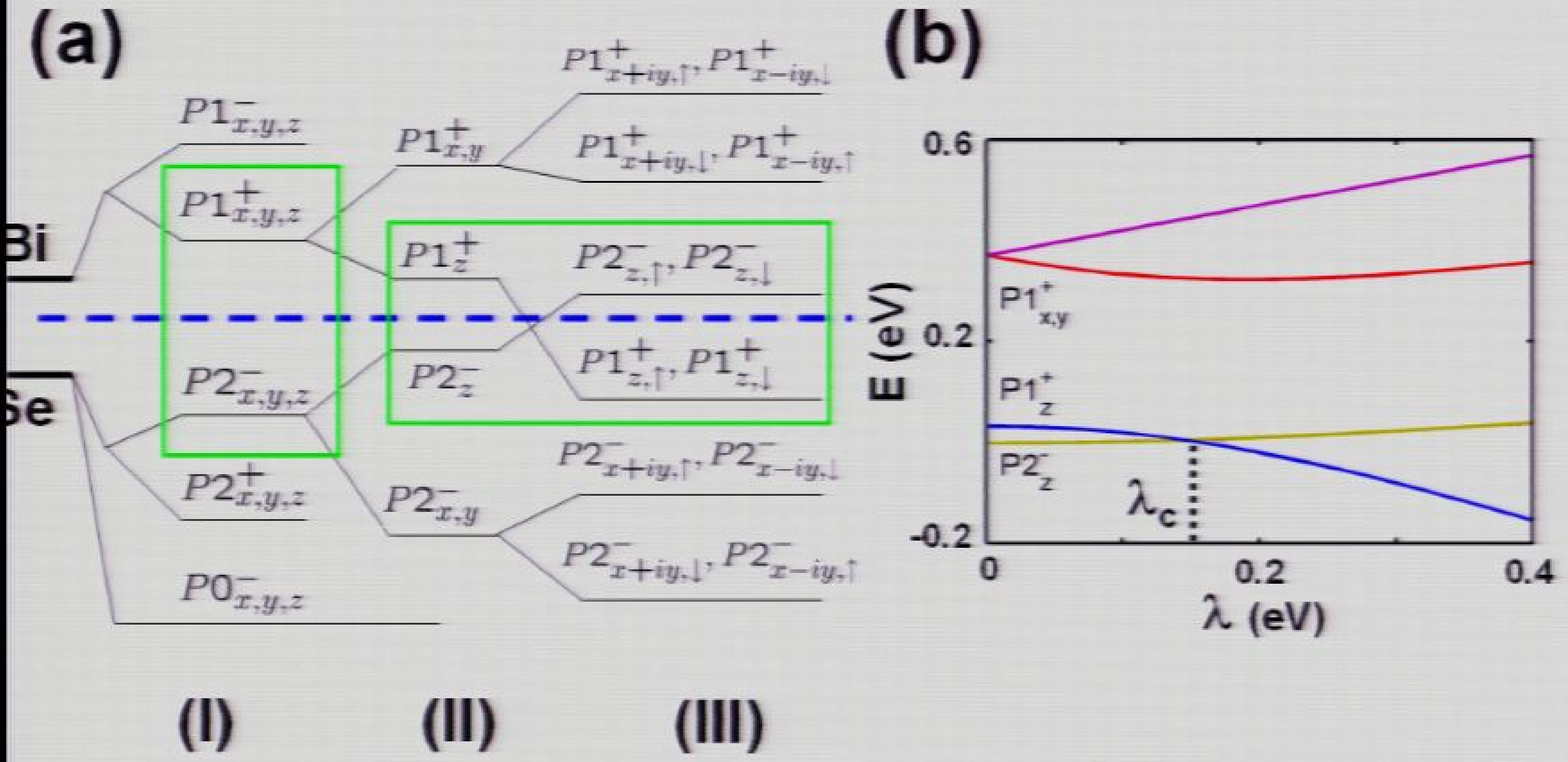
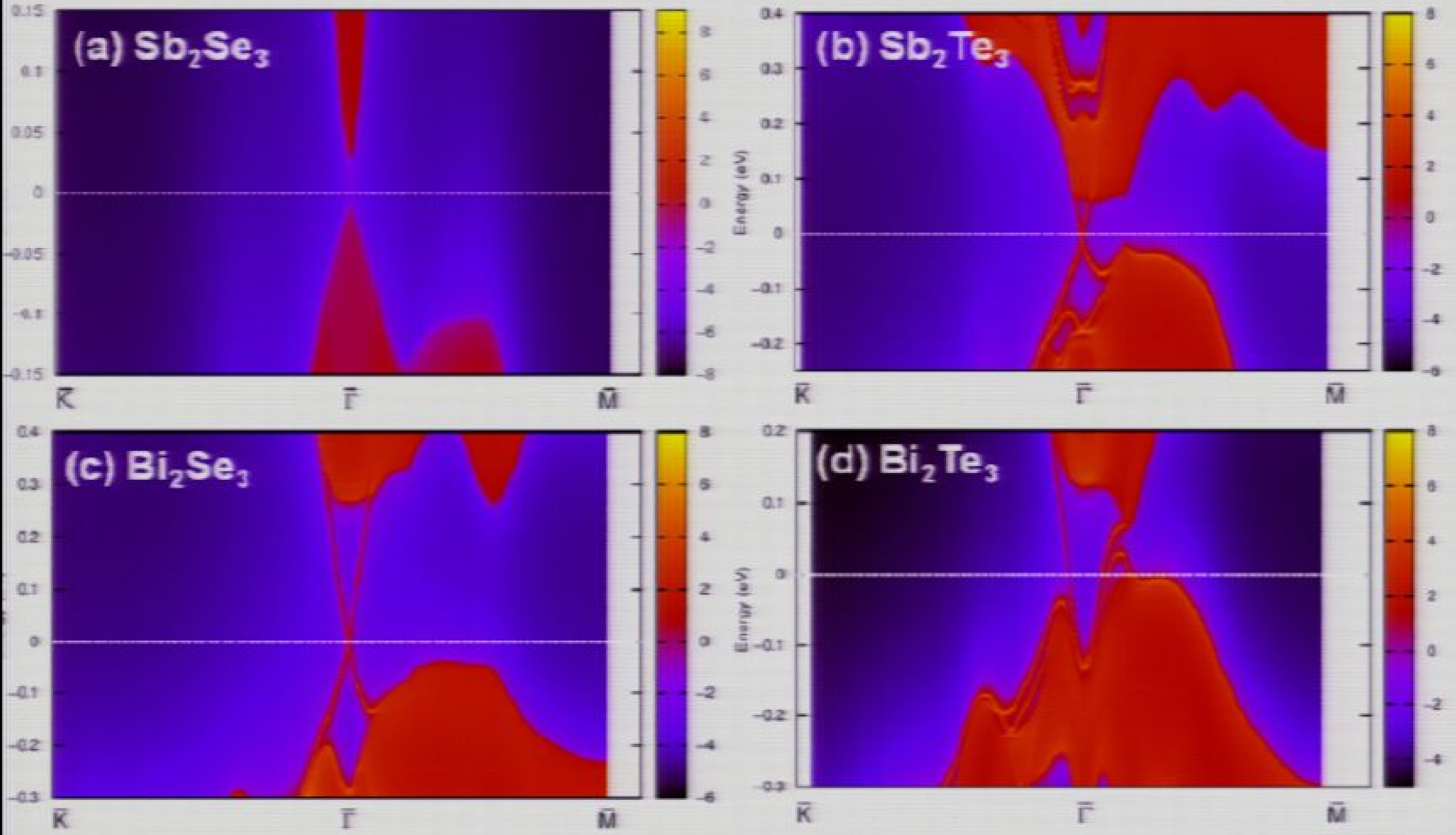


Bulk and surface states from first principle calculations



Model for topological insulator Bi2Te3, (Zhang et al, 2009)

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k})\mathrm{I}_{4\times 4} + \begin{array}{c} \begin{pmatrix} \mathcal{M}(\mathbf{k}) & A_1 k_z & 0 & A_2 k_- \\ A_1 k_z & -\mathcal{M}(\mathbf{k}) & A_2 k_- & 0 \\ 0 & A_2 k_+ & \mathcal{M}(\mathbf{k}) & -A_1 k_z \\ A_2 k_+ & 0 & -A_1 k_z & -\mathcal{M}(\mathbf{k}) \end{pmatrix} \\ \text{Pz+, up,} \quad \text{Pz-, up,} \quad \text{Pz+, down,} \quad \text{Pz-, down} \end{array} + o(\mathbf{k}^2)$$

Single Dirac cone on the surface of Bi2Te3

$$H = \int d^2\mathbf{x}\psi^\dagger(\mathbf{x})\left[\hbar v_f(\hat{\mathbf{z}} \times (-i\nabla)) \cdot \boldsymbol{\sigma} - \mu\right]\psi(\mathbf{x}),$$

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Pz+, up, Pz-, up, Pz+, down, Pz-, down

Single Dirac cone on the surface of Bi2Te3

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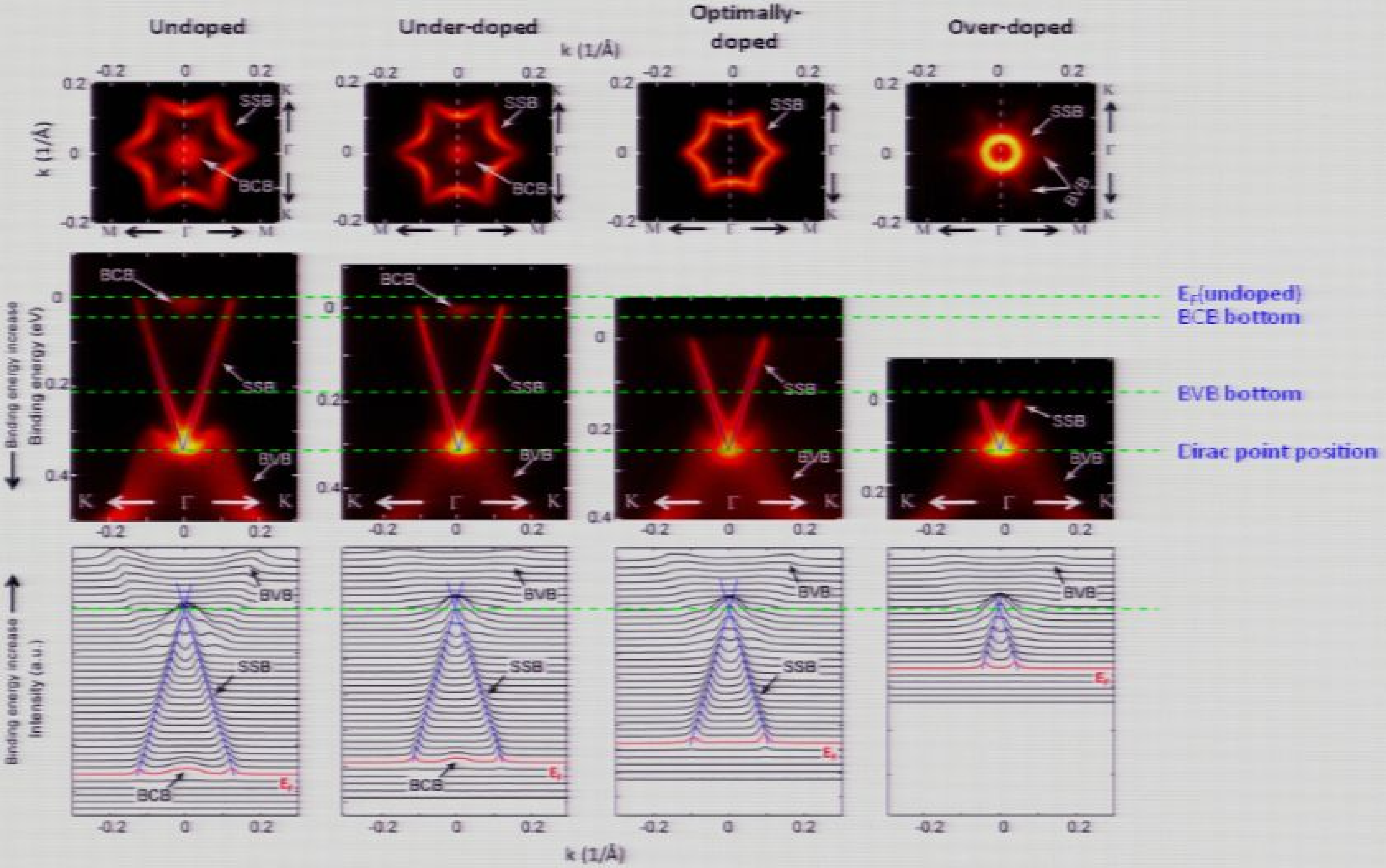
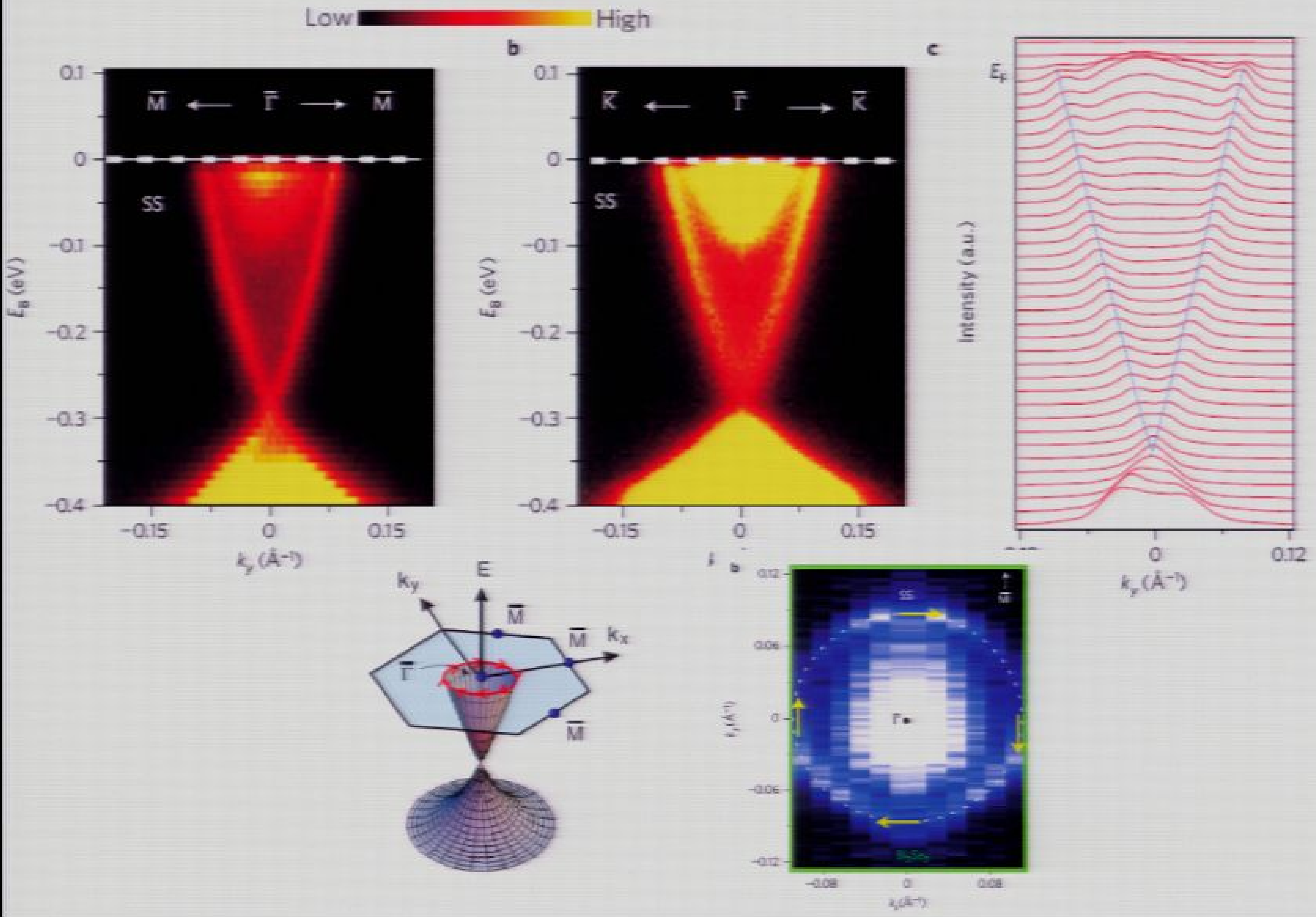
Surface of Bi2Te3 = ¼ Graphene !

Arpes experiment on Bi2Te3 surface states, Shen group

Doping evolution of the FS and band structure



Arpes experiment on Bi2Se3 surface states, Hasan group



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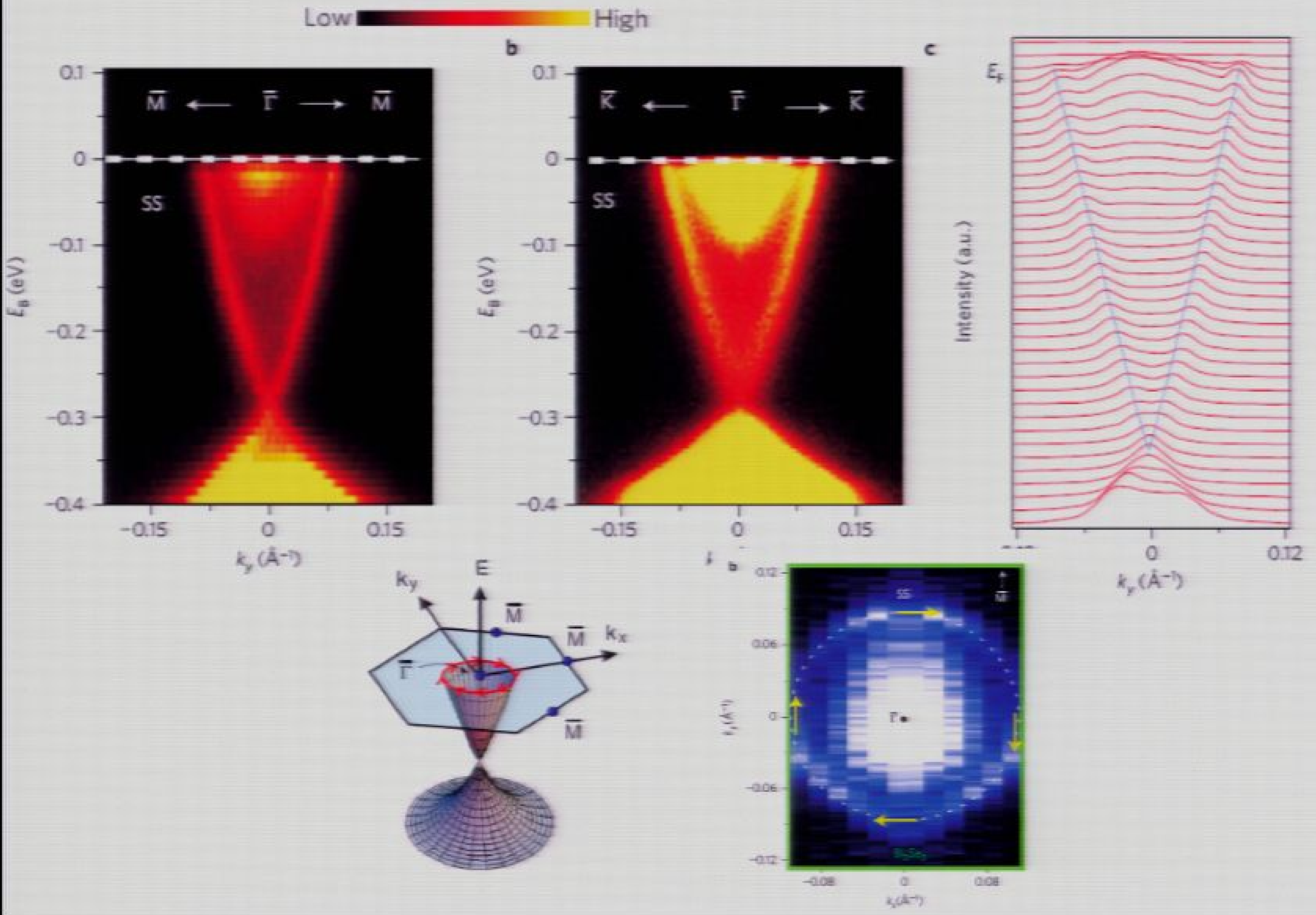
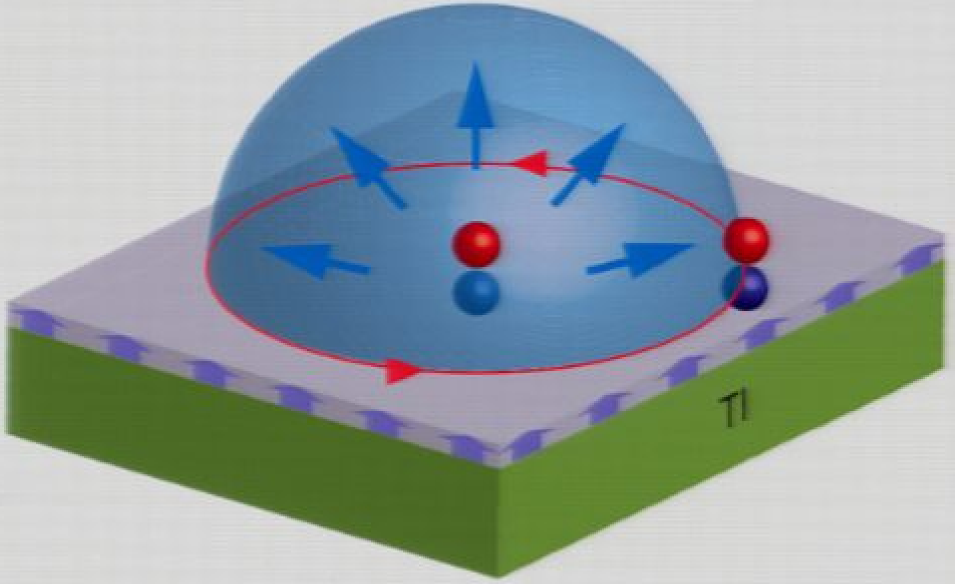


General theory of topological insulators

- Topological field theory of topological insulators. Generally valid for interacting and disordered systems. Directly measurable physically. Relates to axion physics! (Qi, Hughes and Zhang)

$$S_0 = \frac{1}{8\pi}\int d^3x\,dt\left(\varepsilon\mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right)$$



- For a periodic system, the system is time reversal symmetric only when
$\theta=0$ => trivial insulator
$\theta=\pi$ => non-trivial insulator

$$S_\theta = \left(\frac{\theta}{2\pi}\right)\left(\frac{\alpha}{2\pi}\right)\int d^3x\,dt\,\mathbf{E}\cdot\mathbf{B}$$

$$\alpha = \frac{e^2}{\hbar c}$$

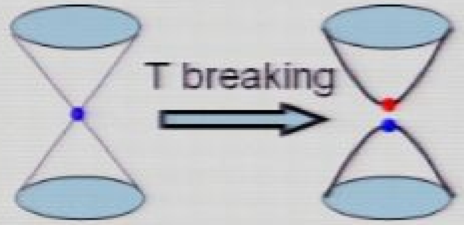
- Topological band theory based on Z2 topological band invariant of single particle states.
(Fu, Kane and Mele, Moore and Balents, Roy)

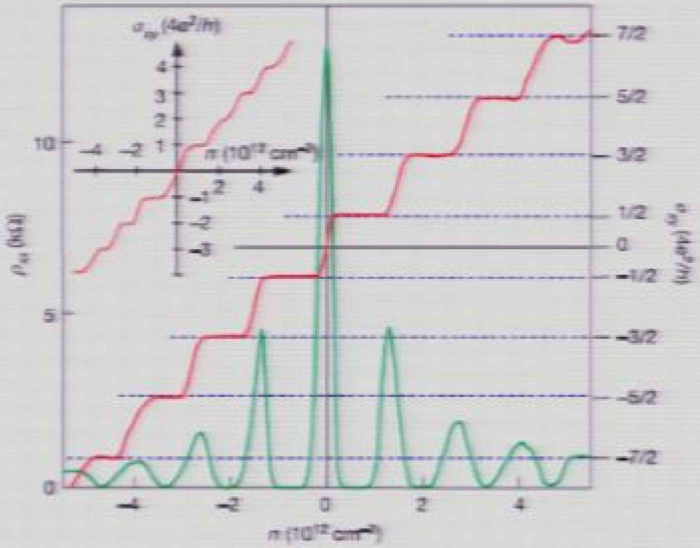
θ term with open boundaries

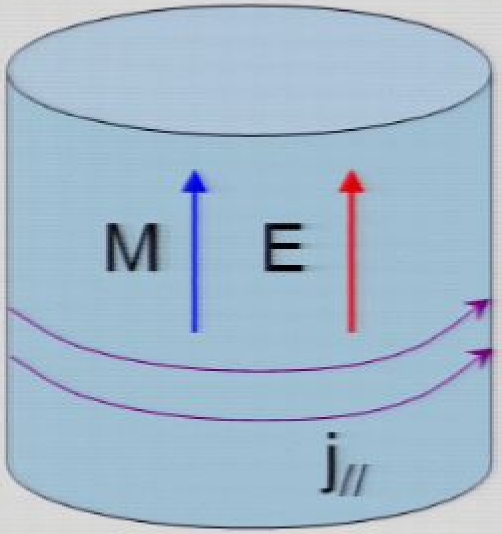
- θ=π implies QHE on the boundary with $\sigma_{xy} = \frac{1}{2}\frac{e^2}{h}$

$$S_\theta = \frac{\theta}{2\pi}\frac{\alpha}{16\pi}\int d^3x dt \epsilon_{\mu\nu\rho\tau} F^{\mu\nu} F^{\rho\tau} = \frac{\theta}{2\pi}\frac{\alpha}{4\pi}\int d^3x dt \partial^\mu (\epsilon_{\mu\nu\rho\sigma} A^\nu \partial^\rho A^\tau)$$

- For a sample with boundary, it is only insulating when a small T-breaking field is applied to the boundary. The surface theory is a CS term, describing the half QH.
- Each Dirac cone contributes σ_{xy}=1/2e^2/h to the QH. Therefore, θ=π implies an odd number of Dirac cones on the surface!

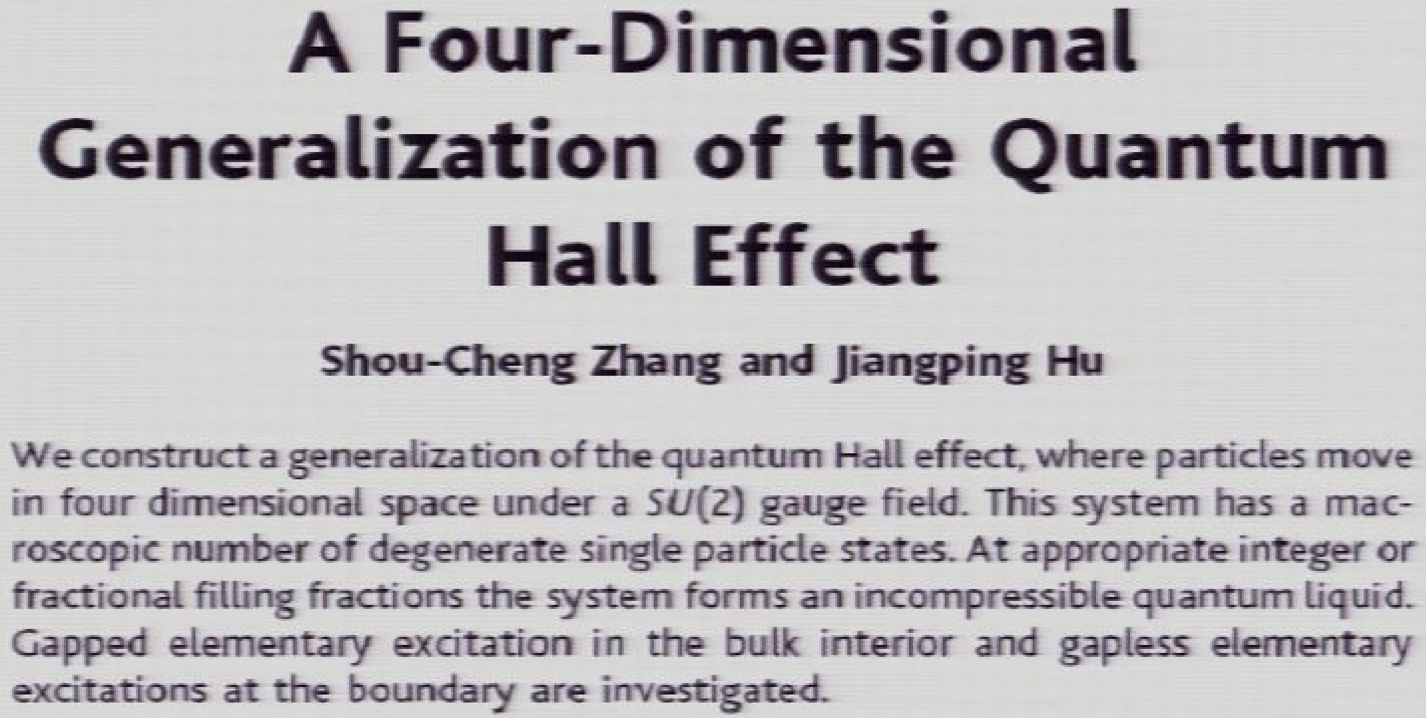






- Surface of a TI = ¼ graphene

Generalization of the QH topology state in d=2 to time reversal invariant topological state in d>2, in Science 2001



A Four-Dimensional Generalization of the Quantum Hall Effect

Shou-Cheng Zhang and Jiangping Hu

We construct a generalization of the quantum Hall effect, where particles move in four dimensional space under a $SU(2)$ gauge field. This system has a macroscopic number of degenerate single particle states. At appropriate integer or fractional filling fractions the system forms an incompressible quantum liquid. Gapped elementary excitation in the bulk interior and gapless elementary excitations at the boundary are investigated.

The periodic table of topological states:

(Qi, Hughes and Zhang, Ludwig et al, Kitaev)

Symmetry				d							
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

The TRI topological insulators form a dimensional chain:
4D=>3D=>2D

TRB topological insulators in d=2

- Chern-Simons topological field theory, odd under TR: $A_0 \rightarrow A_0,\ A_i \rightarrow -A_i$

$$S_{\text{eff}} = \frac{C_1}{4\pi}\int d^2x dt \epsilon^{\mu\nu\tau} A_\mu \partial_\nu A_\tau ,$$

- 1st Chern number

$$C_1 = \frac{e^2}{h}\frac{1}{2\pi}\int dk_x \int dk_y f_{xy}(\mathbf{k})$$

$$\begin{aligned} f_{xy}(\mathbf{k}) &= \frac{\partial a_y(\mathbf{k})}{\partial k_x} - \frac{\partial a_x(\mathbf{k})}{\partial k_y} \\ a_i(\mathbf{k}) &= -i \sum_{\alpha \in \text{ occ}} \langle \alpha \mathbf{k} | \frac{\partial}{\partial k_i} | \alpha \mathbf{k} \rangle ,\ i = x, y. \end{aligned}$$

TRI topological insulators in d=4

- Chern-Simons topological field theory, even under TR: $A_0 \rightarrow A_0,\ A_i \rightarrow -A_i$

$$S_{\text{eff}} = \frac{C_2}{24\pi^2}\int d^4x dt\, \epsilon^{\mu\nu\rho\sigma\tau} A_\mu \partial_\nu A_\rho \partial_\sigma A_\tau$$

- If we perform Kaluza-Klein compactification, we obtain the effective field theory of the d=3 topological insulator!

- 2st Chern number

$$C_2 = \frac{1}{32\pi^2}\int d^4k\, \epsilon^{ijk\ell}\mathrm{tr}\left[f_{ij} f_{k\ell}\right]$$

- If we replace k4 by an adiabatic parameter, we obtain the quantized cyclic change of the magneto-electric polarization in d=3!

The best way to understand TRI TI is D=4 => D=3 => D=2

The best way to understand critical phenomenon is D=4 => D=4-ε

Dimensional reduction

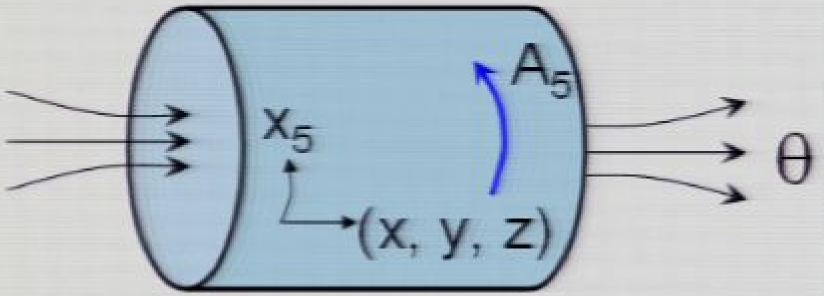
- From 4D QHE to the 3D topological insulator

Zhang & Hu, Qi, Hughes & Zhang

$$S_{4DQH} = \int d^4x dt \varepsilon^{\mu\nu\rho\sigma\tau} A_\mu F_{\nu\rho} F_{\sigma\tau}$$

$$\Rightarrow \int d^3x dt \left(\int dx_5 A_5(x,t)\right) \varepsilon^{\nu\rho\sigma\tau} F_{\nu\rho} F_{\sigma\tau}$$

$$\Rightarrow S_{3D} = \int d^3x dt \theta(x,t) \varepsilon^{\nu\rho\sigma\tau} F_{\nu\rho} F_{\sigma\tau}$$



- From 3D axion action to the 2D QSH

$$S_{3D} = \int d^3x dt \varepsilon^{\nu\rho\sigma\tau} A_\nu \partial_\rho \theta \partial_\sigma A_\tau$$

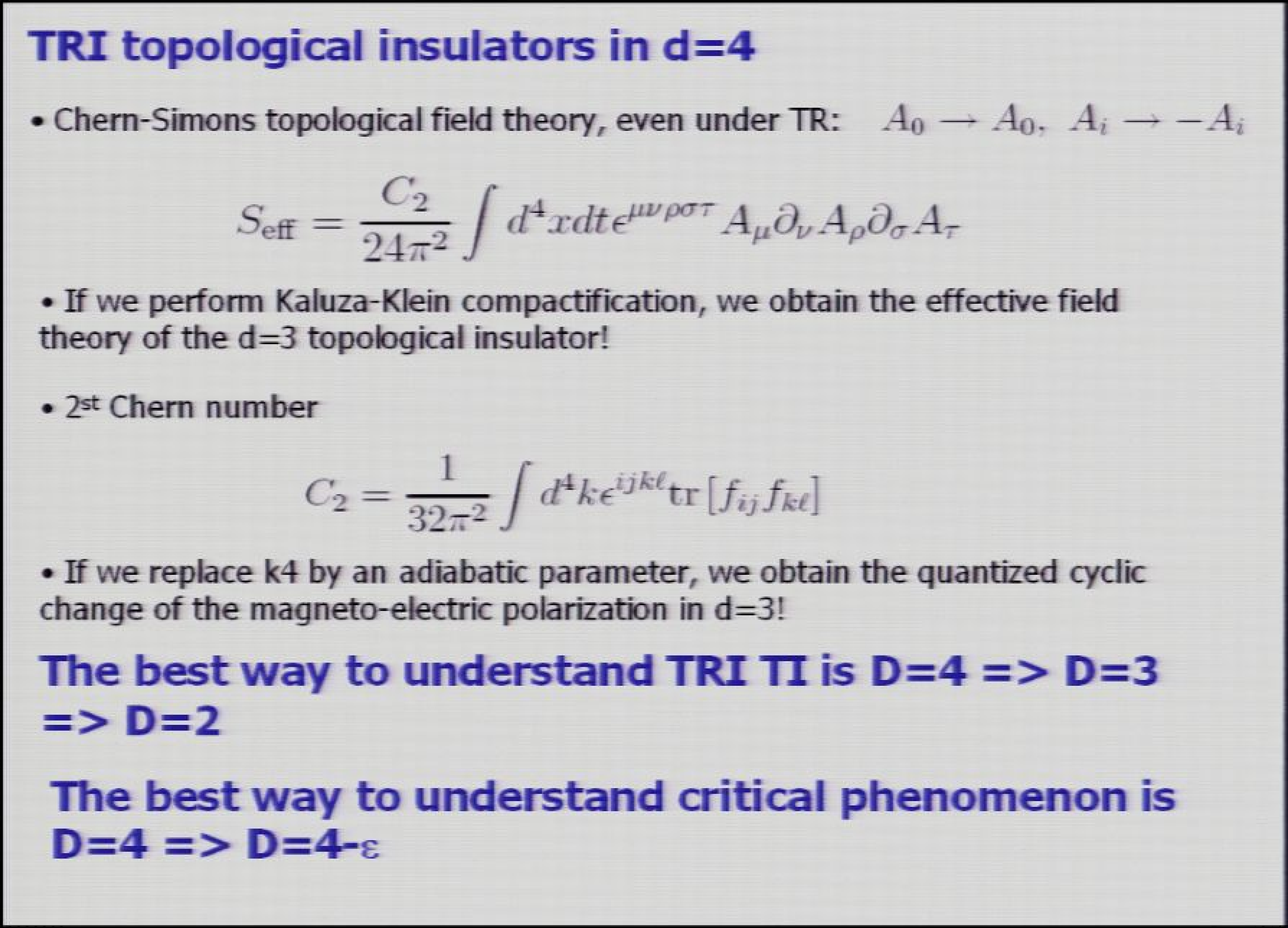
$$\Rightarrow \int d^2x dt \varepsilon^{\rho\sigma\tau} \left(\int dz A_z(x,t)\right) \partial_\rho \theta \partial_\sigma A_\tau$$

$$\Rightarrow S_{2D} = \int d^2x dt \varepsilon^{\rho\sigma\tau} \partial_\sigma \varphi \partial_\rho \theta A_\tau$$

$$J_{2D}^{\ \mu} = \frac{e}{2\pi^2} \varepsilon^{\mu\rho\sigma} \partial_\sigma \varphi \partial_\rho \theta$$

$$\Leftrightarrow J_{1D}^{\ \mu} = \frac{e}{2\pi} \varepsilon^{\mu\sigma} \partial_\sigma \varphi$$

Goldstone & Wilzcek



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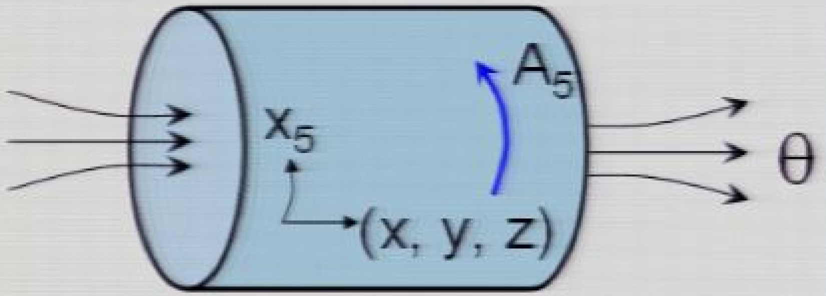

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Goldstone & Wilzcek

TRI topological insulators in d=3

- Axion field theory, even under TR only when θ=0, π: $A_0 \to A_0,\ A_i \to -A_i$

$$S_{\text{eff}} = S_{\text{Maxwell}} + S_{\text{topo}} = \int d^3x dt \left[\frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} + \frac{\theta\alpha}{32\pi^2}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau}\right]$$

- Magneto-electric polarization (QHZ 2008):

$$\theta \equiv 2\pi P_3(\theta) = \frac{1}{16\pi^2}\int d^3\mathbf{k}\epsilon^{ijk}\mathrm{Tr}\{[f_{ij}(\mathbf{k}) - \frac{2}{3}ia_i(\mathbf{k})\cdot a_j(\mathbf{k})]\cdot a_k(\mathbf{k})\}$$

$$\begin{aligned} f_{ij}^{\alpha\beta} &= \partial_i a_j^{\alpha\beta} - \partial_j a_i^{\alpha\beta} + i\left[a_i, a_j\right]^{\alpha\beta}, \\ a_i^{\alpha\beta}(\mathbf{k}) &= -i\left\langle \alpha, \mathbf{k}\right| \frac{\partial}{\partial k_i}\left|\beta, \mathbf{k}\right\rangle \end{aligned}$$

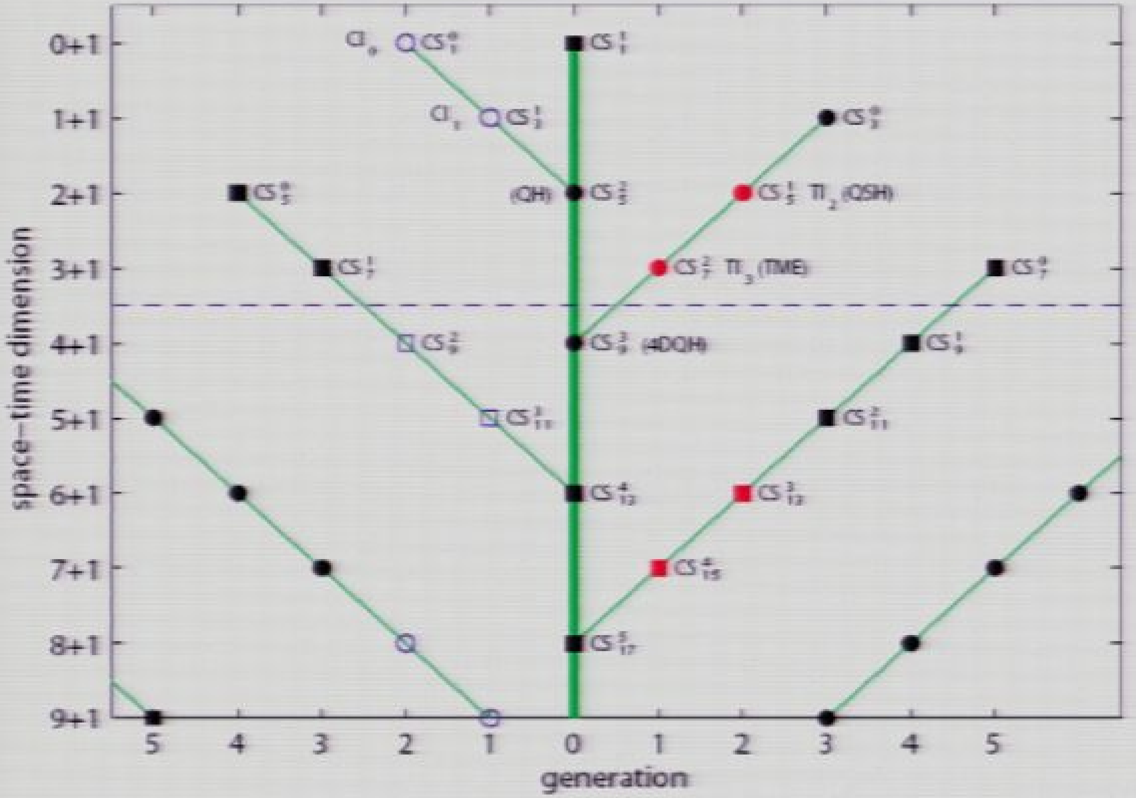
Topological field theory and the family tree

- Topological field theory of the QHE: (Thouless et al, Zhang, Hansson and Kivelson)

$$S = \int d^2k\, da(k) \int d^3x\, A(x) \wedge dA(x)$$

- Topological field theory of the TI: (Qi, Hughes and Zhang, 2008)

$$S = \int d^3k (a(k) \wedge da(k) + ..) \int d^4x\, dA(x) \wedge dA(x)$$



- More extensive and general classification soon followed (Kitaev, Ludwig et al)

TRI topological insulator d=2, characterized by the discrete Z2 topological number in 2005

Z_2 Topological Order and the Quantum Spin Hall Effect

C. L. Kane and E. J. Mele
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

(Received 22 June 2005; published 28 September 2005)

The quantum spin Hall (QSH) phase is a time reversal invariant electronic state with a bulk electronic band gap that supports the transport of charge and spin in gapless edge states. We show that this phase is associated with a novel Z_2 topological invariant, which distinguishes it from an ordinary insulator. The Z_2 classification, which is defined for time reversal invariant Hamiltonians, is analogous to the Chern number classification of the quantum Hall effect. We establish the Z_2 order of the QSH phase in the two band model of graphene and propose a generalization of the formalism applicable to multiband and interacting systems.

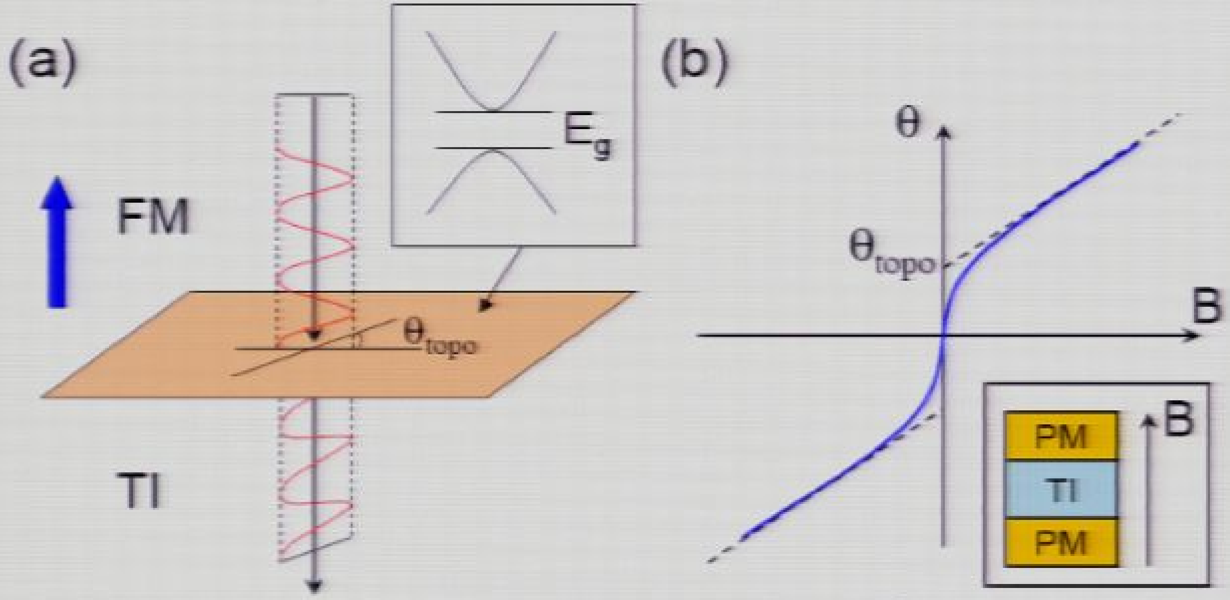
$$(-1)^{\nu_0} = \prod_{i=1}^{8} \delta_i$$

$$\delta_i = \frac{\sqrt{\det[B(\Gamma_i)]}}{\mathrm{Pf}[B(\Gamma_i)]} = \pm 1 \qquad B_{\alpha\beta}(\mathbf{k}) = \langle -\mathbf{k}, \alpha|\Theta|\mathbf{k}, \beta\rangle$$

Topological band theory

Low frequency Faraday/Kerr rotation

(Qi, Hughes and Zhang, PRB78, 195424, 2008)



Adiabatic

Requirement:

$$\hbar\omega \ll E_g$$

(surface gap)

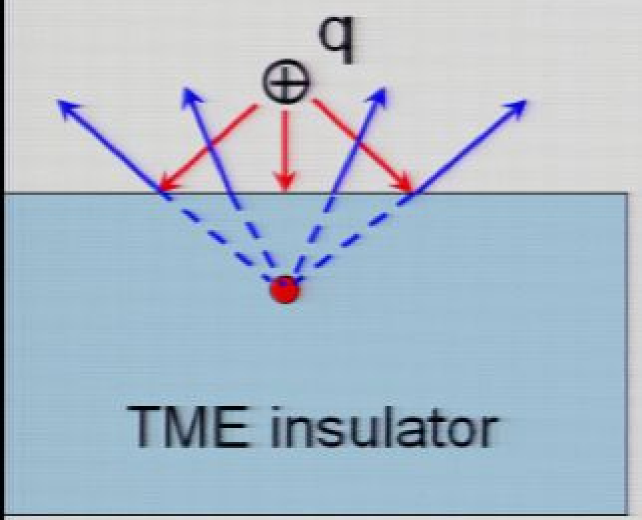
$$\theta(B) = uB + \mathrm{sgn}(B)\arctan\left(\frac{(2n-1)\alpha}{\sqrt{\epsilon/\mu}+\sqrt{\epsilon'/\mu'}}\right)$$

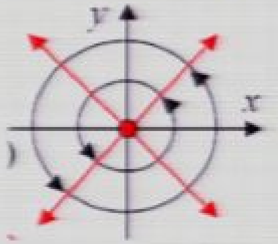
normal contribution

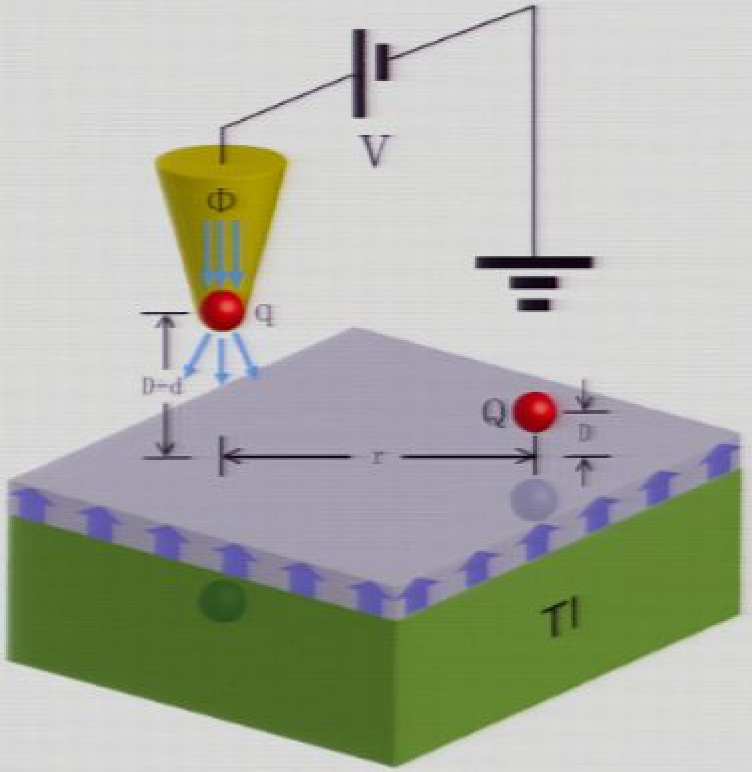
Topological contribution

θ_{topo}» 3.6x 10^{-3} rad

Seeing the magnetic monopole thru the mirror of a TME insulator, (Qi et al, Science 323, 1184, 2009)







$$g = \frac{\alpha P_3}{1+\alpha^2 P_3^2} q$$

higher order feed back

(for μ=μ', ε=ε')

similar to Witten's dyon effect

Magnitude of B:
$10^6 V/m \to 0.25G$

Dynamic axions in topological magnetic insulators

(Li et al, Nature Physics 2010)

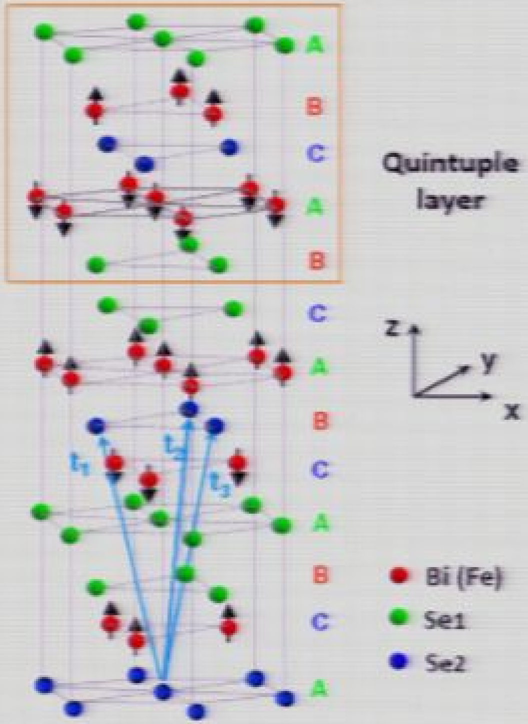
$$\theta = \frac{1}{4\pi}\int d^3k\epsilon^{ijk}Tr\left[A_i\partial_j A_k + \frac{2}{3}A_iA_jA_k\right]$$

- Hubbard interactions leads to anti-ferromagnetic order

$$H = H_0 + U\sum_i (n_{iA\uparrow}n_{iA\downarrow} + n_{iB\uparrow}n_{iB\downarrow}) + V\sum_i n_{iA}n_{iB}$$

- Effective action for dynamical axion

$$\begin{aligned}\mathcal{S}_{\rm tot} &= \mathcal{S}_{\rm Maxwell} + \mathcal{S}_{\rm topo} + \mathcal{S}_{\rm axion}\\ &= \frac{1}{8\pi}\int d^3x dt(\epsilon\mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2)\\ &+ \frac{\alpha}{4\pi^2}\int d^3x dt\,(\theta_0 + \delta\theta)\,\mathbf{E}\cdot\mathbf{B}\\ &+ g^2 J\int d^3x dt\,[(\partial_t\delta\theta)^2 - (v_i\partial_i\delta\theta)^2 - m^2\delta\theta^2]\end{aligned}$$



Axions and dark matter on your desktop? (Zhang group, Nature hysics 2010)

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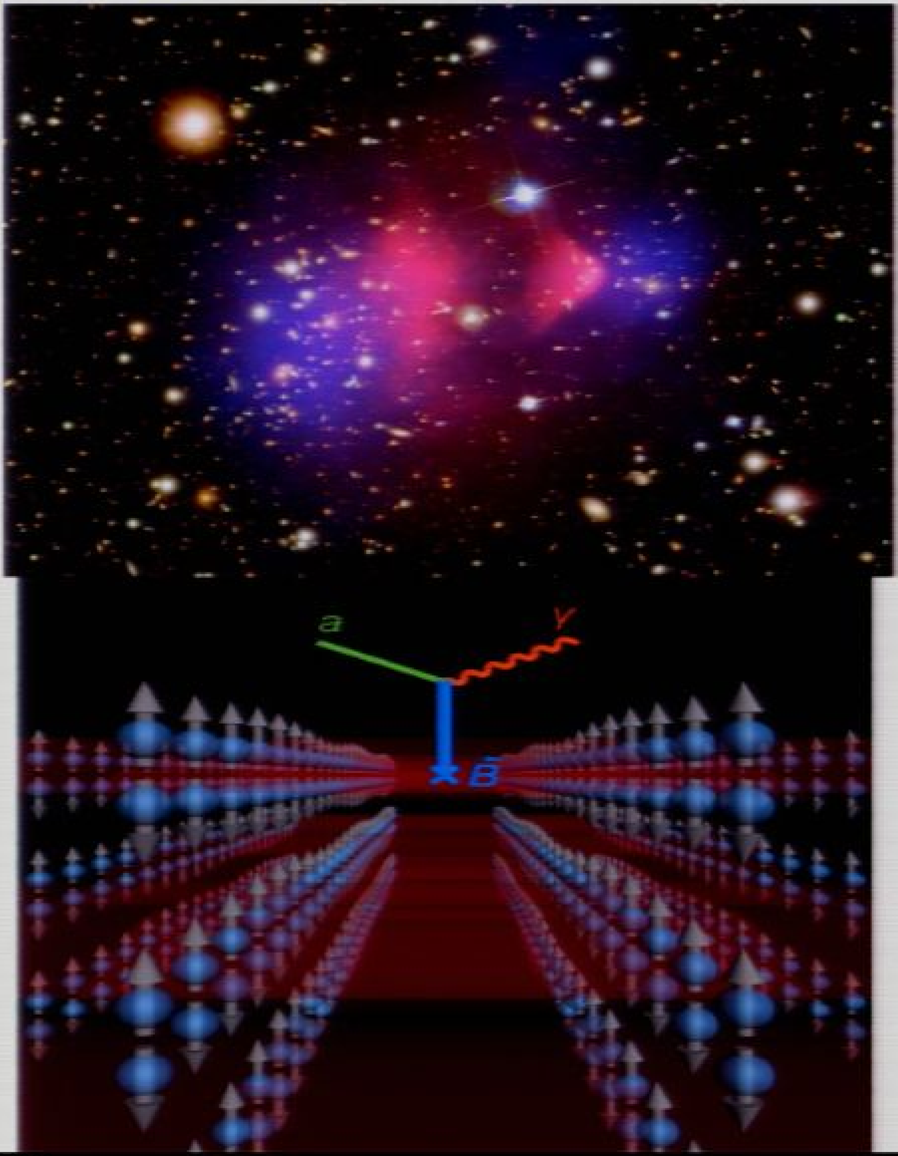

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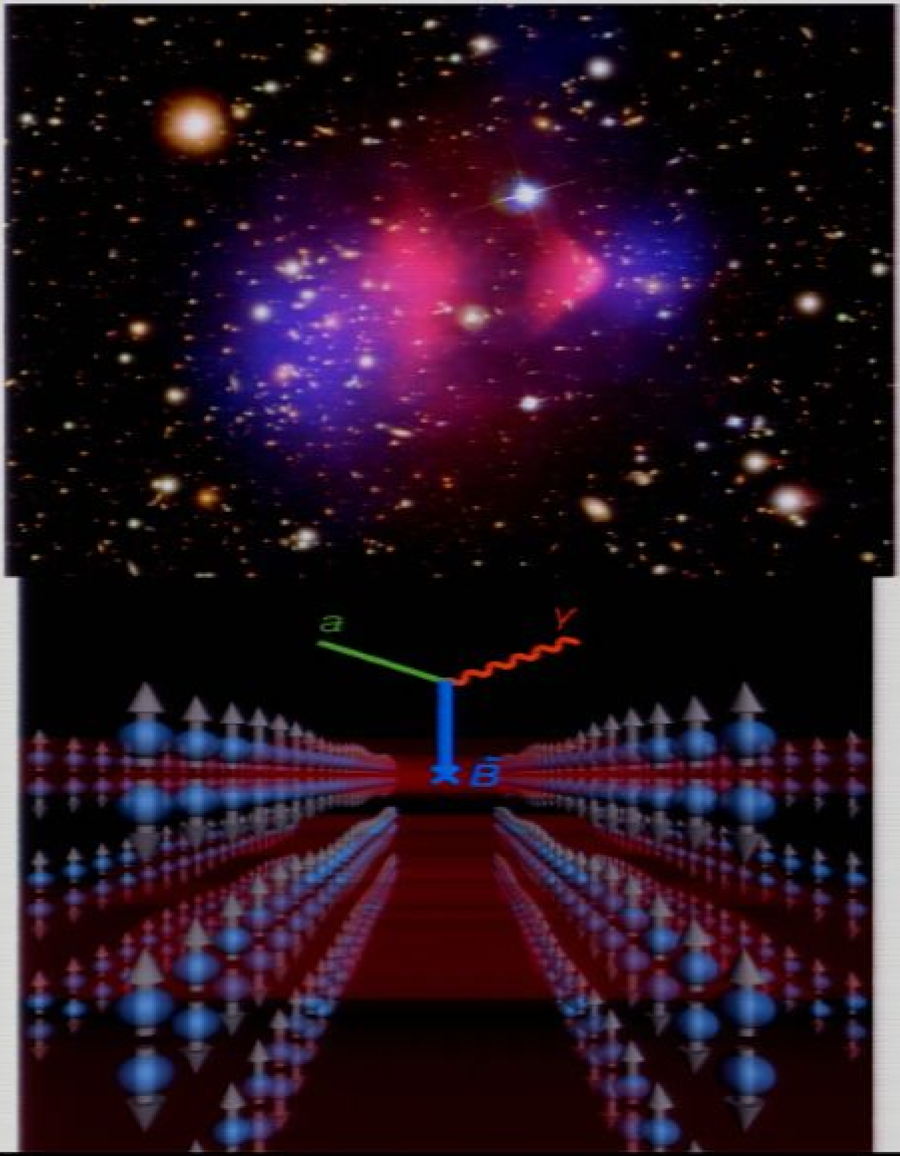
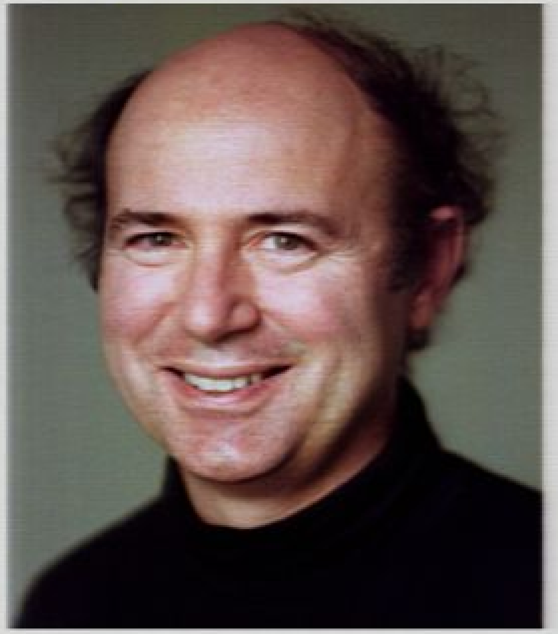
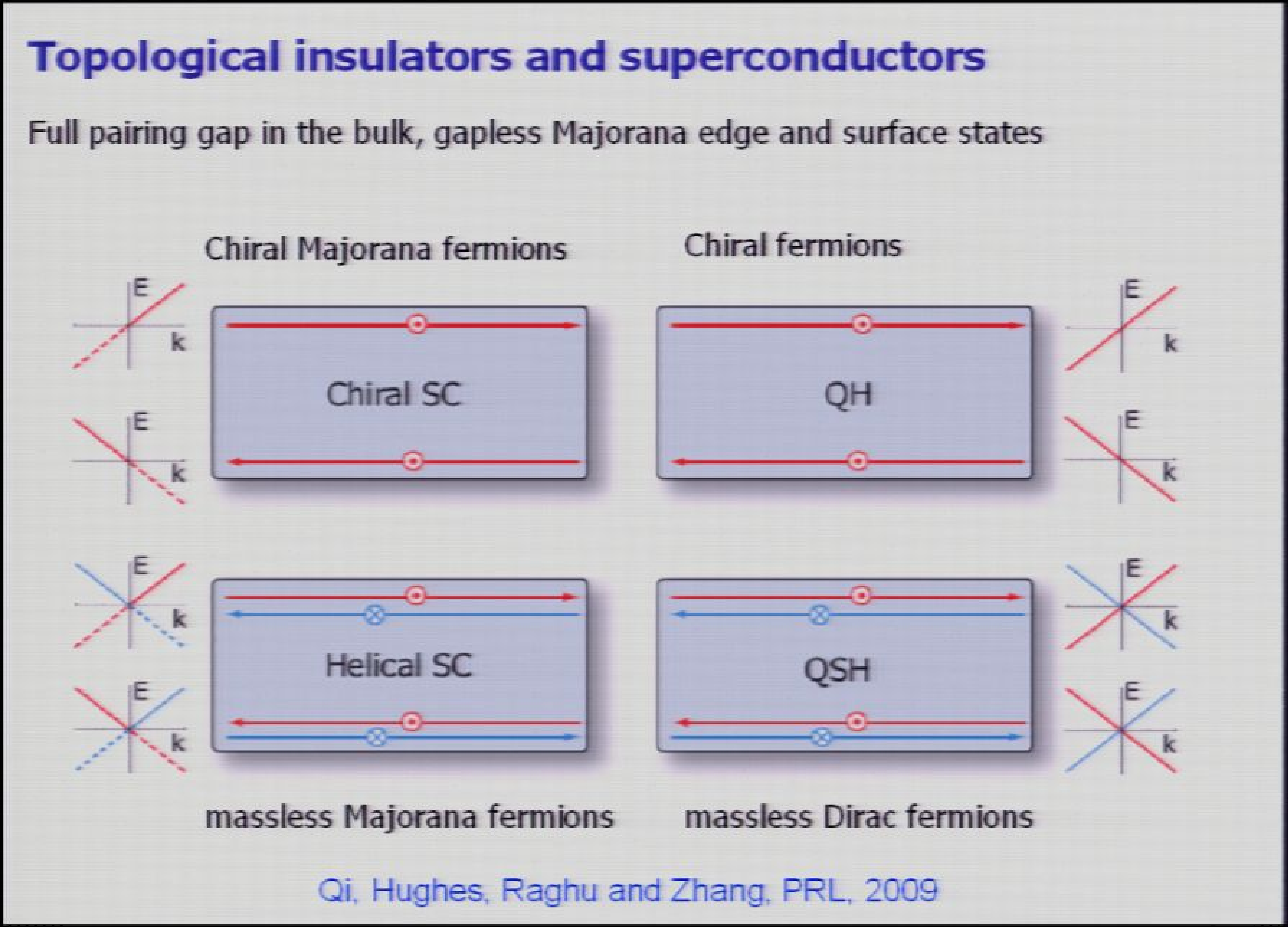




Now Shou-Cheng Zhang and his colleagues inform us that, all along, axions have been lurking unrecognized on surfaces of topological insulators.

Frank Wilzcek, NATURE 458, 129 (2009)



Topological insulators and superconductors

Full pairing gap in the bulk, gapless Majorana edge and surface states

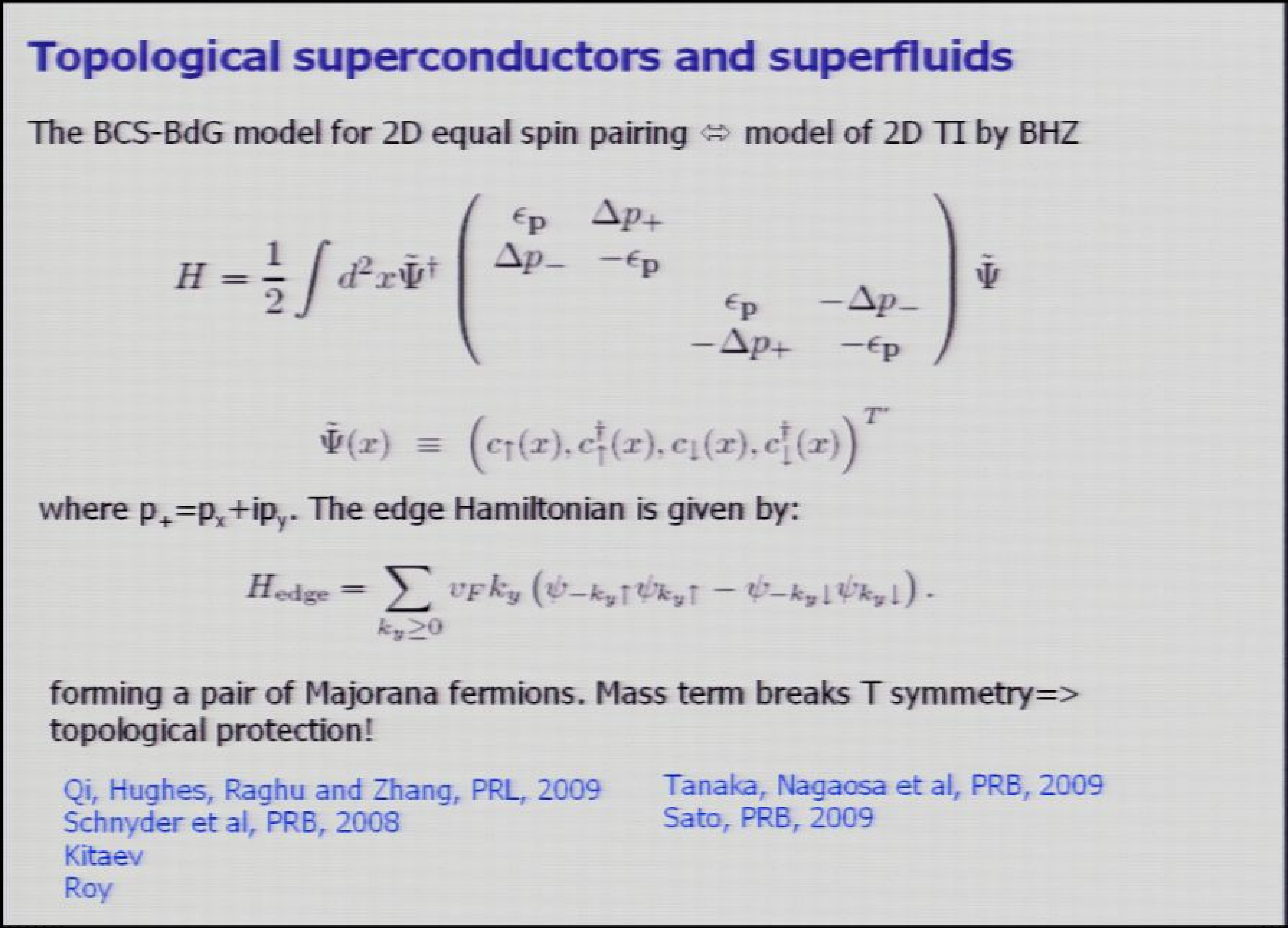
Chiral Majorana fermions

Chiral fermions

massless Majorana fermions

massless Dirac fermions

Qi, Hughes, Raghu and Zhang, PRL, 2009



Topological superconductors and superfluids

The BCS-BdG model for 2D equal spin pairing ⇔ model of 2D TI by BHZ

$$H = \frac{1}{2}\int d^2x \tilde{\Psi}^\dagger \begin{pmatrix} \epsilon_{\mathbf{p}} & \Delta p_+ & & \\ \Delta p_- & -\epsilon_{\mathbf{p}} & & \\ & & \epsilon_{\mathbf{p}} & -\Delta p_- \\ & & -\Delta p_+ & -\epsilon_{\mathbf{p}} \end{pmatrix} \tilde{\Psi}$$

$$\tilde{\Psi}(x) \equiv \left(c_\uparrow(x), c_\uparrow^\dagger(x), c_\downarrow(x), c_\downarrow^\dagger(x)\right)^T$$

where $p_+=p_x+ip_y$. The edge Hamiltonian is given by:

$$H_{\mathrm{edge}} = \sum_{k_y \geq 0} v_F k_y \left(\psi_{-k_y\uparrow}\psi_{k_y\uparrow} - \psi_{-k_y\downarrow}\psi_{k_y\downarrow}\right).$$

forming a pair of Majorana fermions. Mass term breaks T symmetry=> topological protection!

Qi, Hughes, Raghu and Zhang, PRL, 2009
Schnyder et al, PRB, 2008
Kitaev
Roy

Tanaka, Nagaosa et al, PRB, 2009
Sato, PRB, 2009

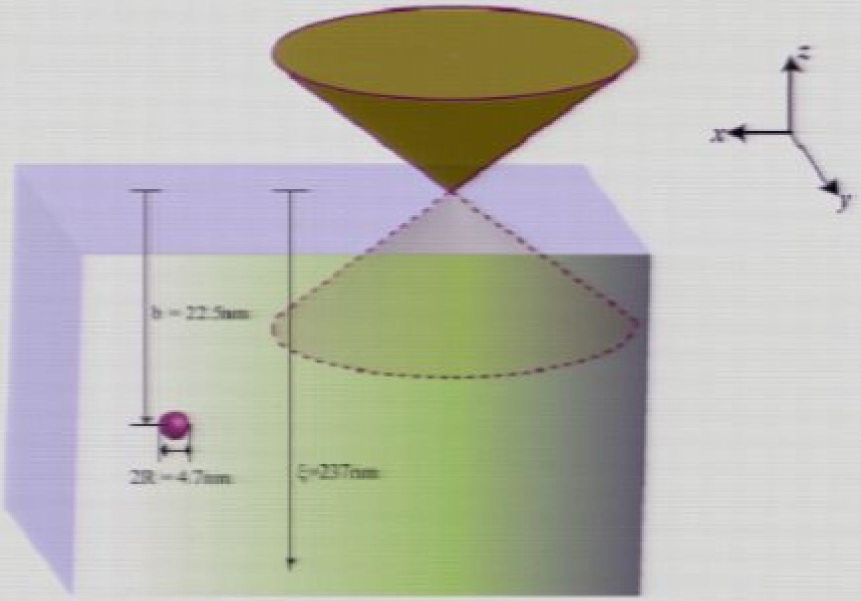
Probing He3B as a topological superfluid (Chung and Zhang, 2009)

The BCS-BdG model for He3B ⇔ Model of the 3D TI by Zhang et al

$$\hat{\mathcal{H}}_{BdG} = \begin{bmatrix} \epsilon_{\mathbf{p}} - E_F & 0 & -\frac{\Delta}{p_F}\hat{p}_- & \frac{\Delta}{p_F}\hat{p}_z \\ 0 & \epsilon_{\mathbf{p}} - E_F & \frac{\Delta}{p_F}\hat{p}_z & \frac{\Delta}{p_F}\hat{p}_+ \\ -\frac{\Delta}{p_F}\hat{p}_+ & \frac{\Delta}{p_F}\hat{p}_z & -\epsilon_{\mathbf{p}} + E_F & 0 \\ \frac{\Delta}{p_F}\hat{p}_z & \frac{\Delta}{p_F}\hat{p}_- & 0 & -\epsilon_{\mathbf{p}} + E_F \end{bmatrix}$$

Surface Majorana state:

$$\mathcal{H}_{surf} = v_F \boldsymbol{\sigma} \cdot (\hat{\mathbf{z}} \times \mathbf{p})$$



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From geometrical to topological laws of physics

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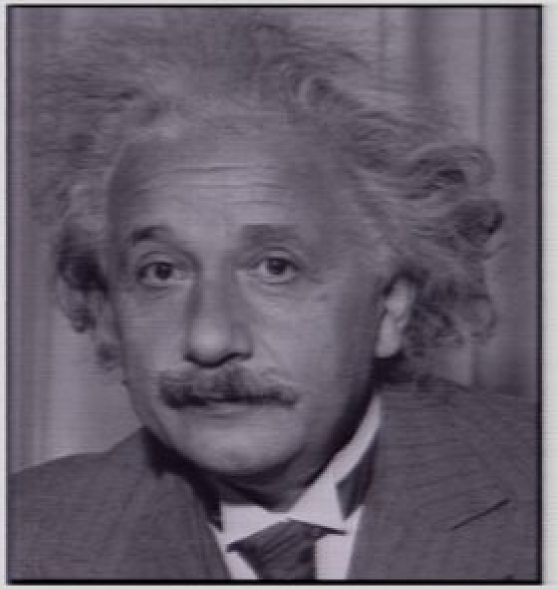

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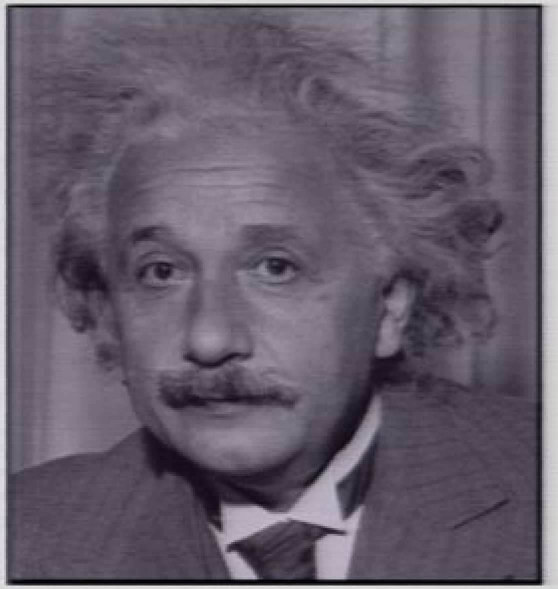

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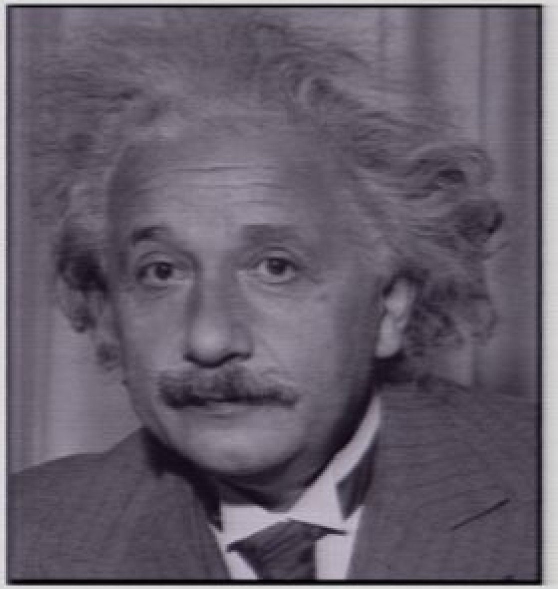

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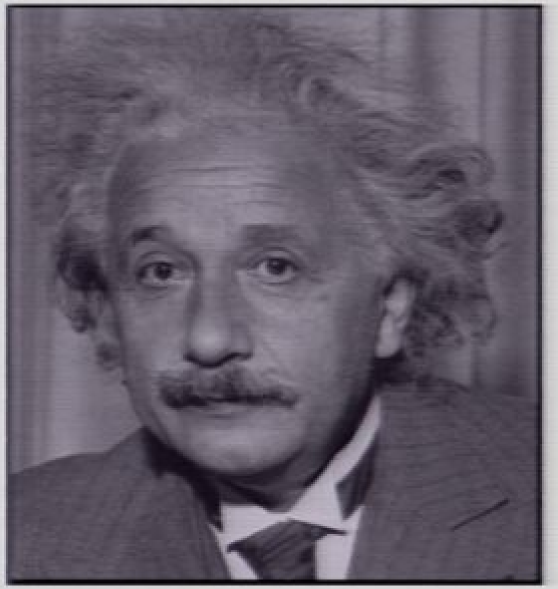

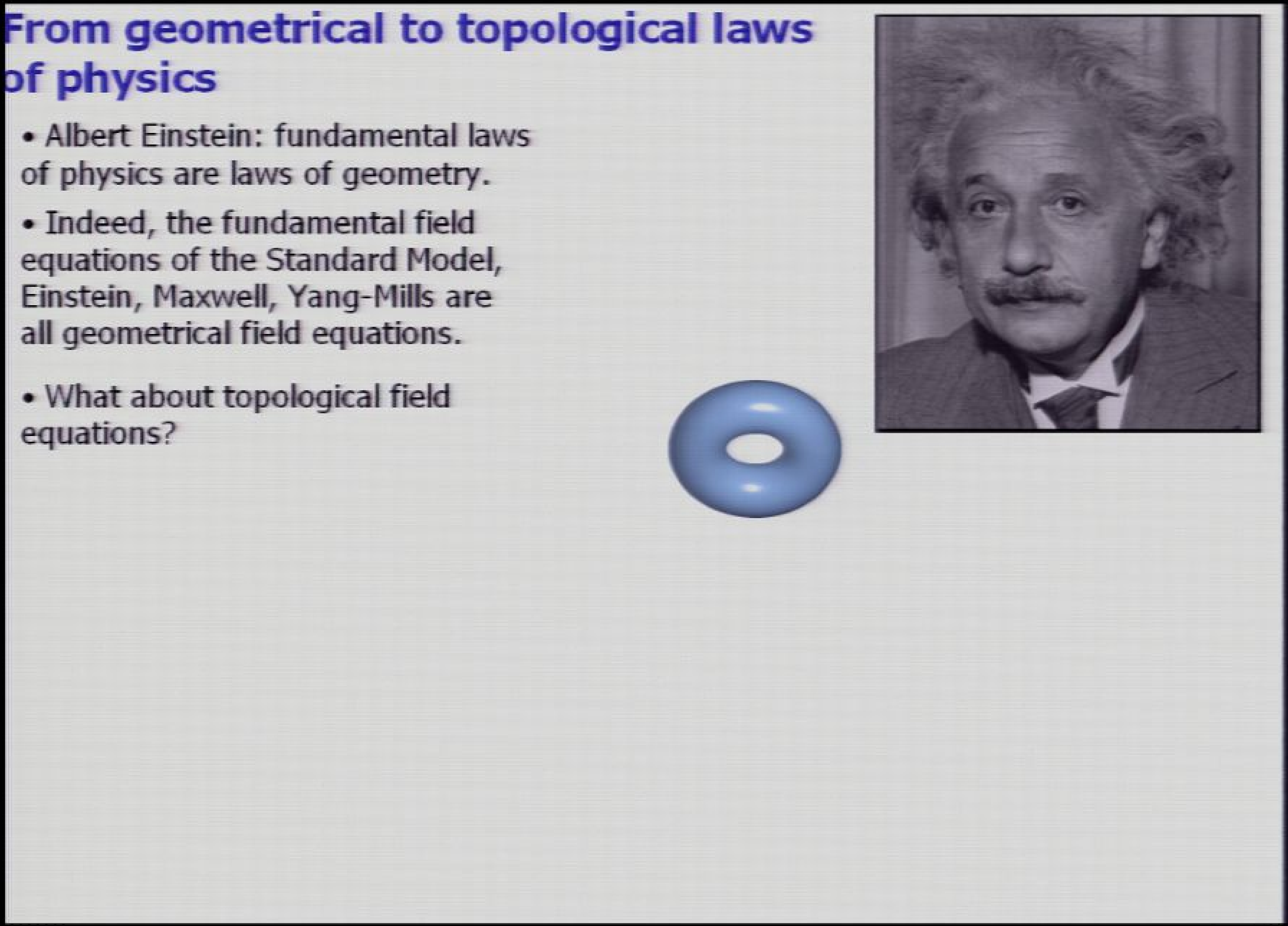

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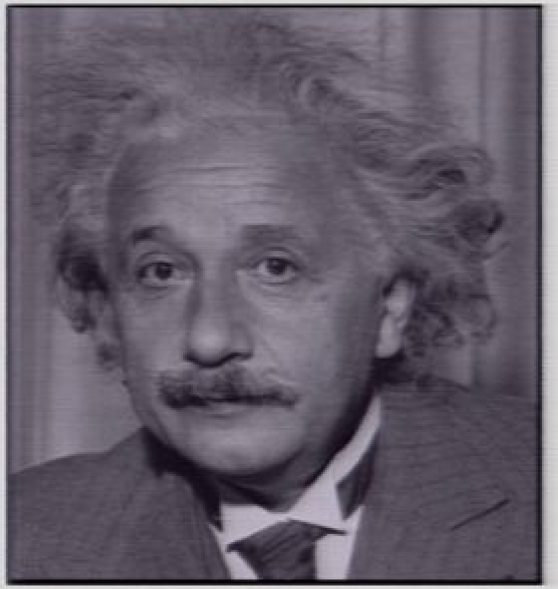

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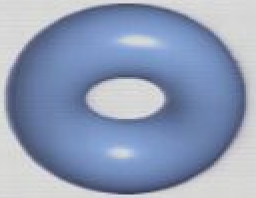

- The only topological term within the Standard Model:

$$S_\theta = \frac{\theta}{2\pi}\frac{\alpha}{16\pi}\int d^3x\,dt\,\varepsilon_{\mu\nu\rho\tau}F^{\mu\nu}F^{\rho\tau}$$

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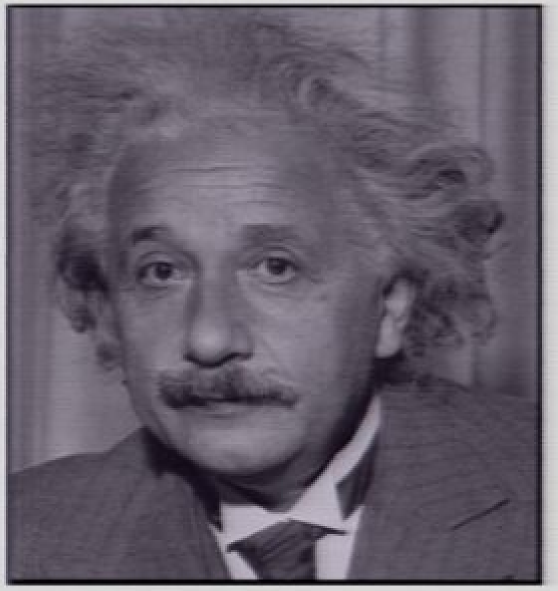

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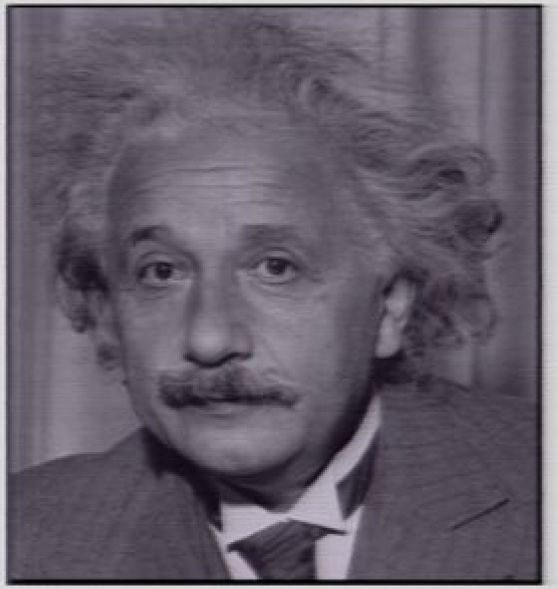

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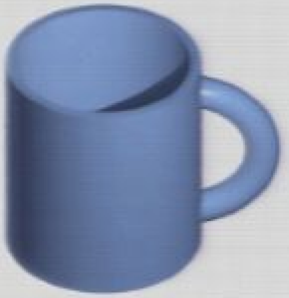

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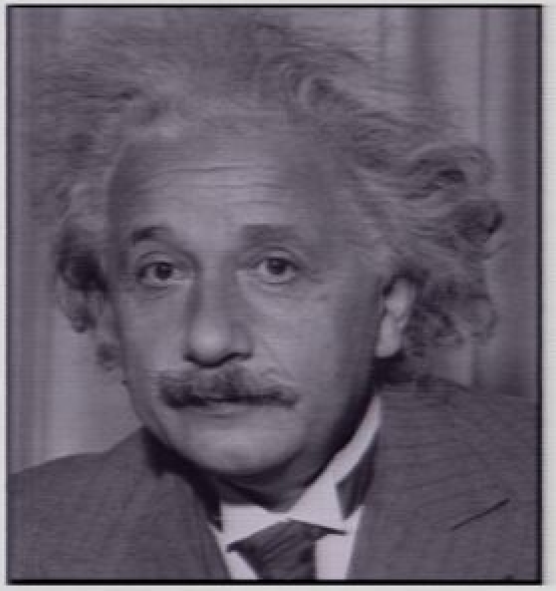

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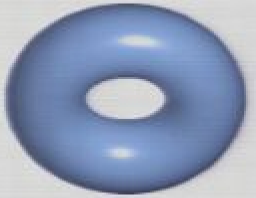

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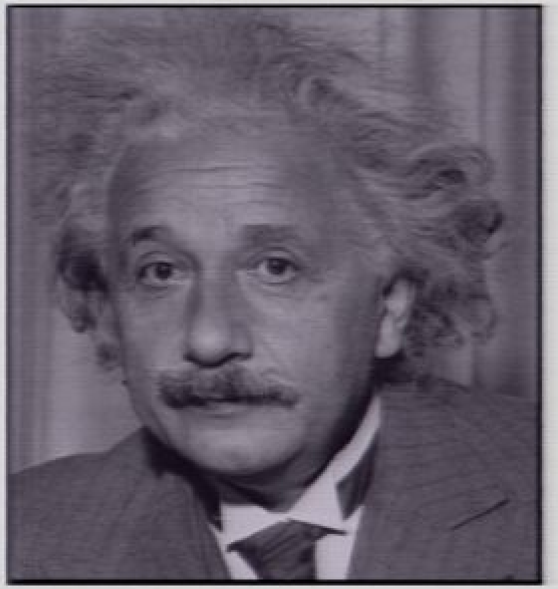

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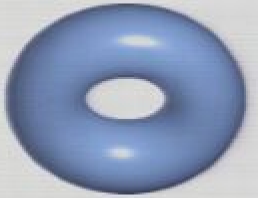

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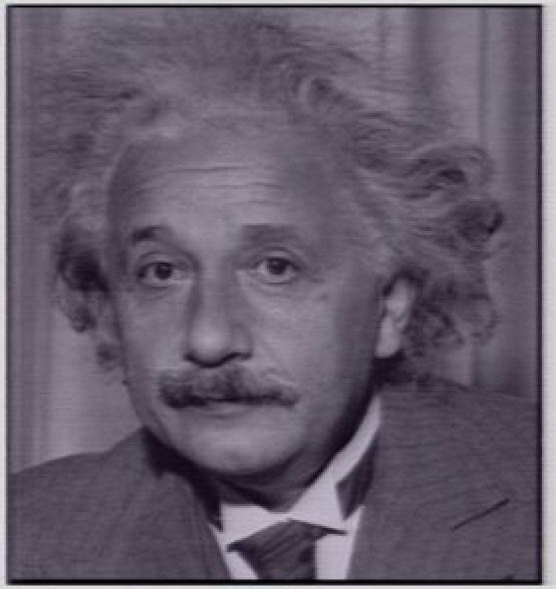

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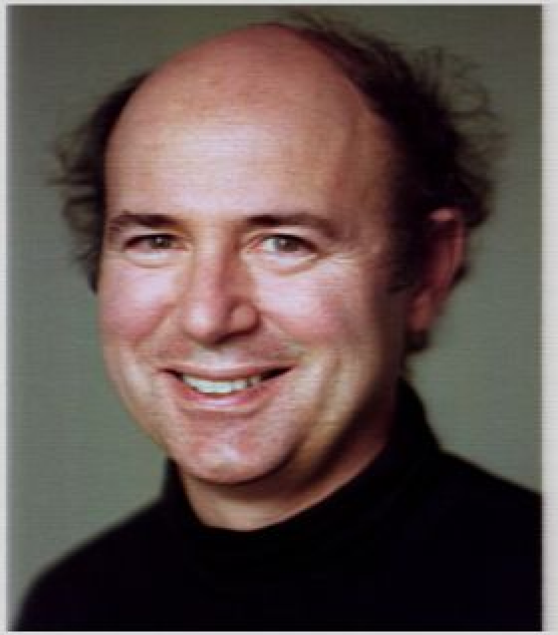

- Frank Wilzcek: Topological insulator is a window into the universe! (Nature **458**, 129, 2009)

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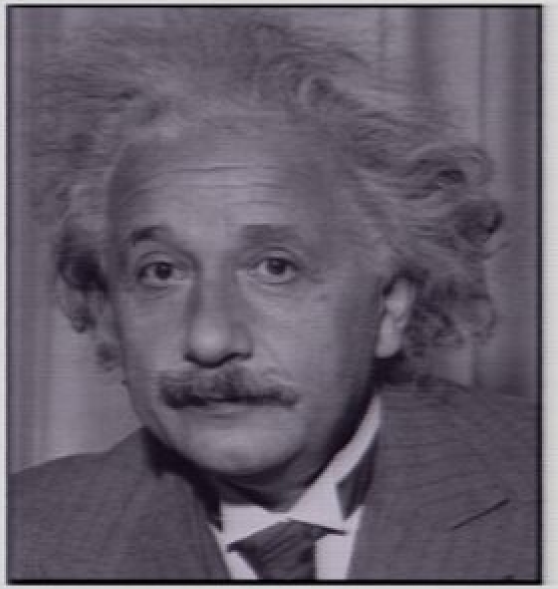
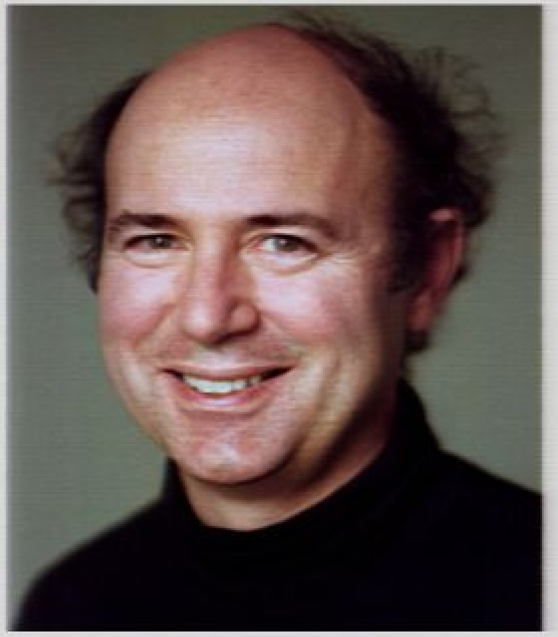
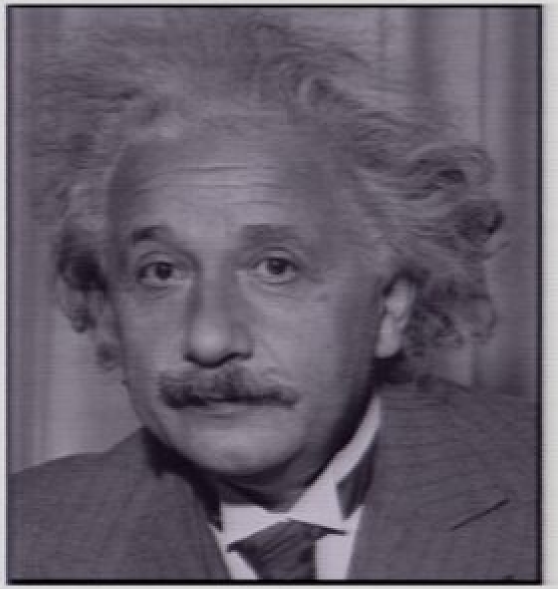
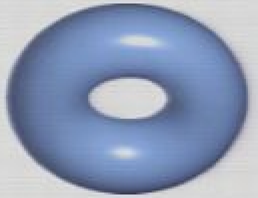
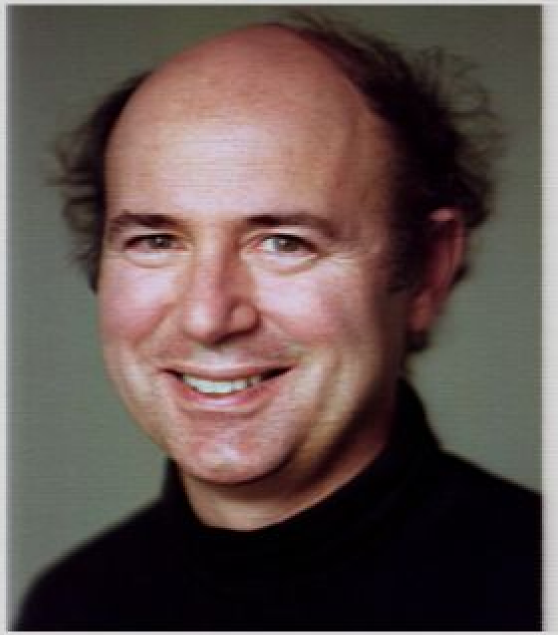




From geometrical to topological laws of physics







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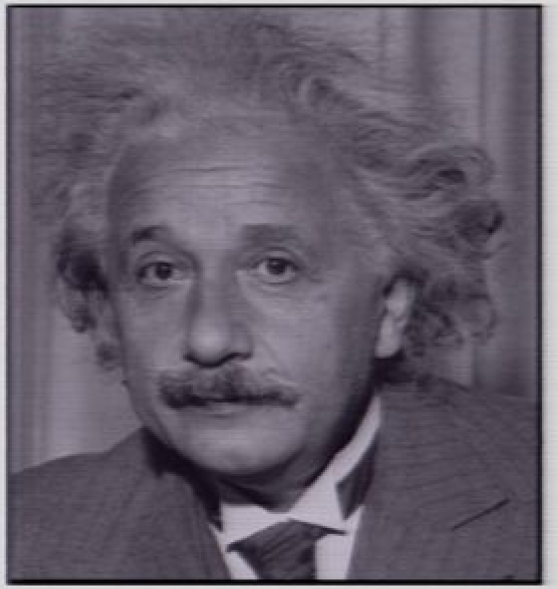

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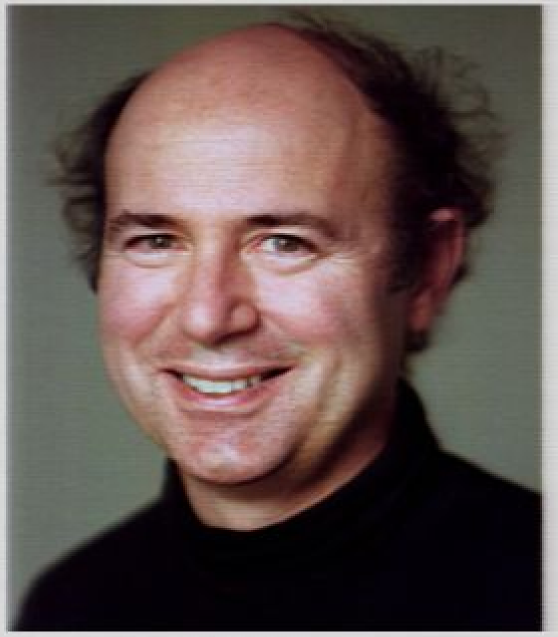

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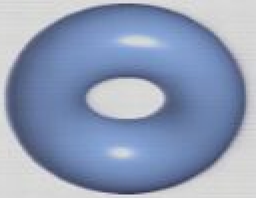

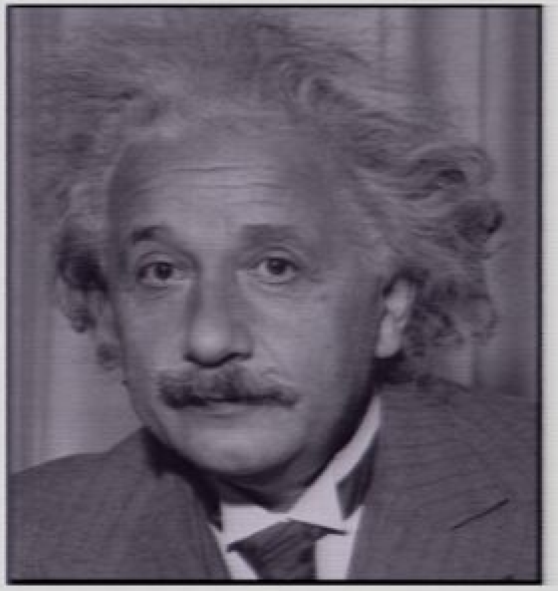

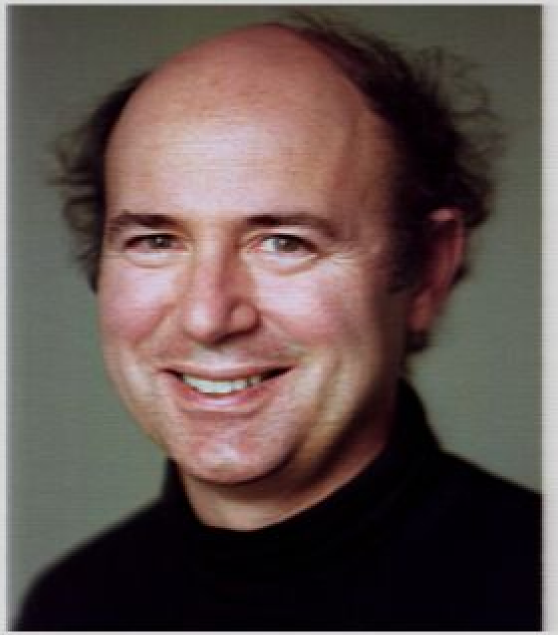

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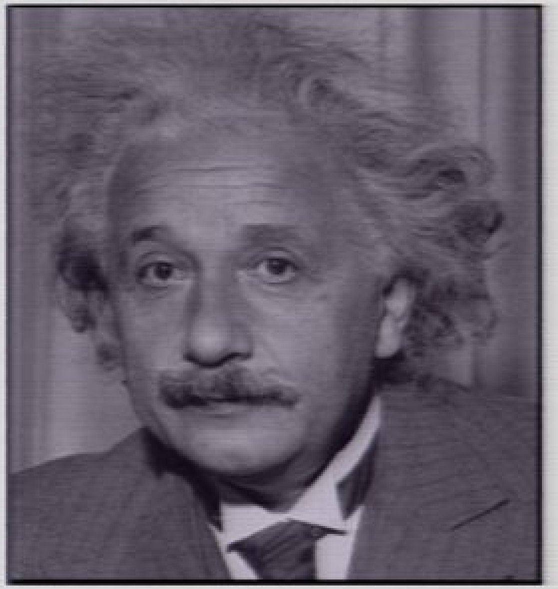
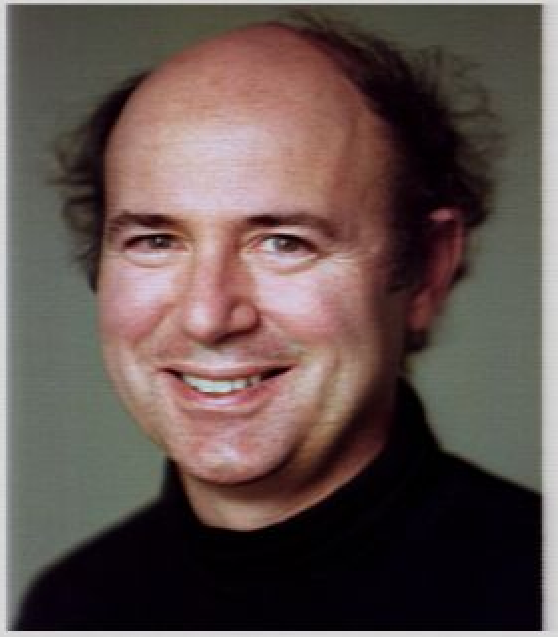
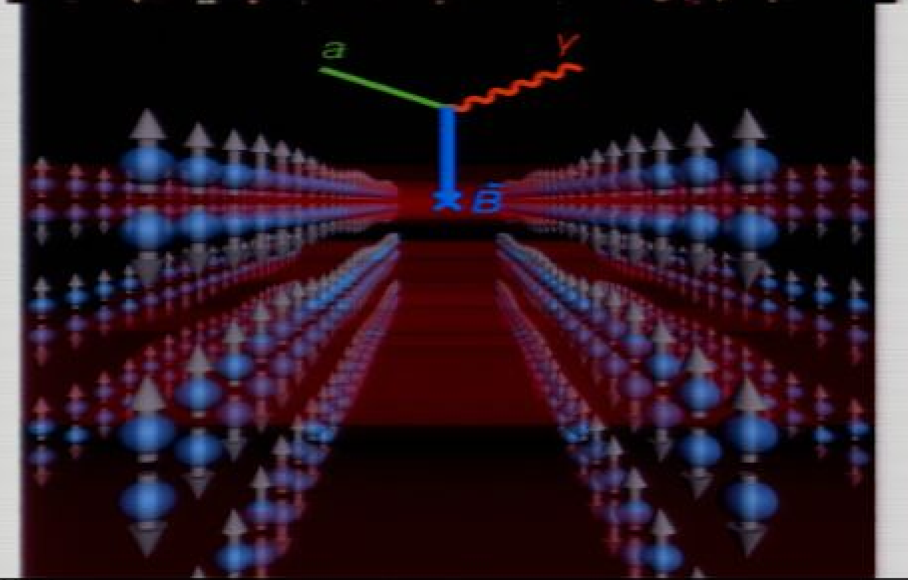
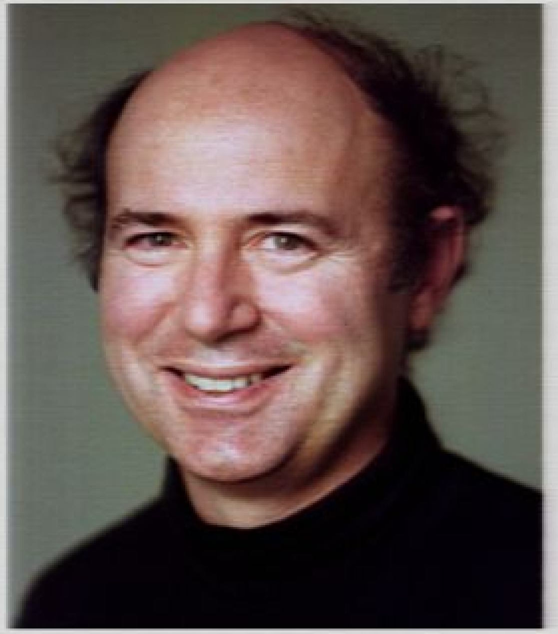




From geometrical to topological laws of physics

Axions and dark matter on your desktop? (Zhang group, Nature hysics 2010)

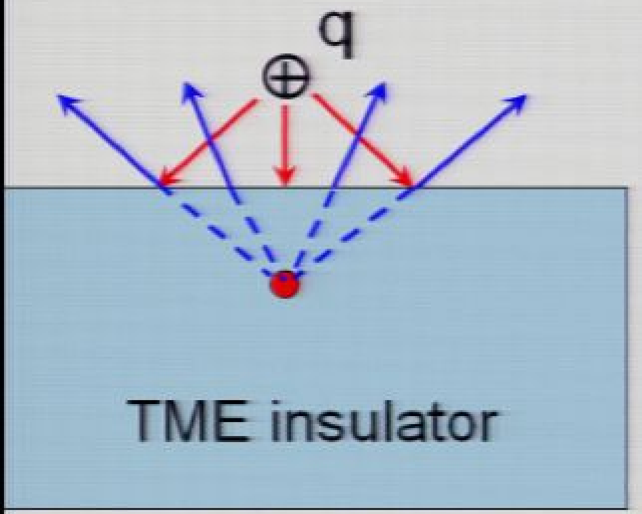


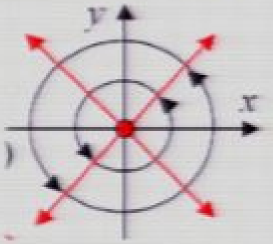


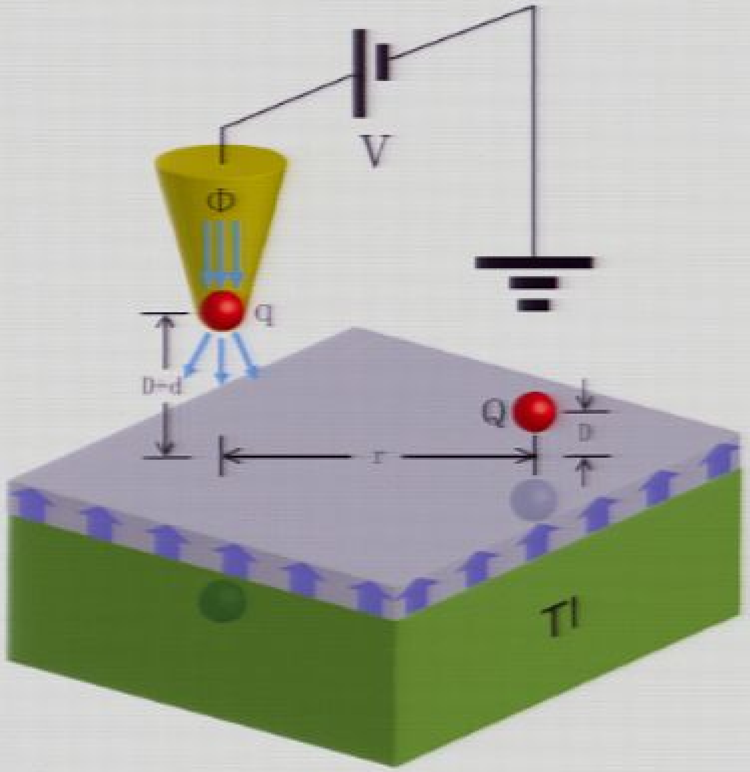
Now Shou-Cheng Zhang and his
colleagues inform us that, all along,
axions have been lurking
unrecognized on surfaces
of topological insulators.

Frank Wilzcek, NATURE 458, 129
(2009)

Seeing the magnetic monopole thru the mirror of a TME insulator, (Qi et al, Science 323, 1184, 2009)







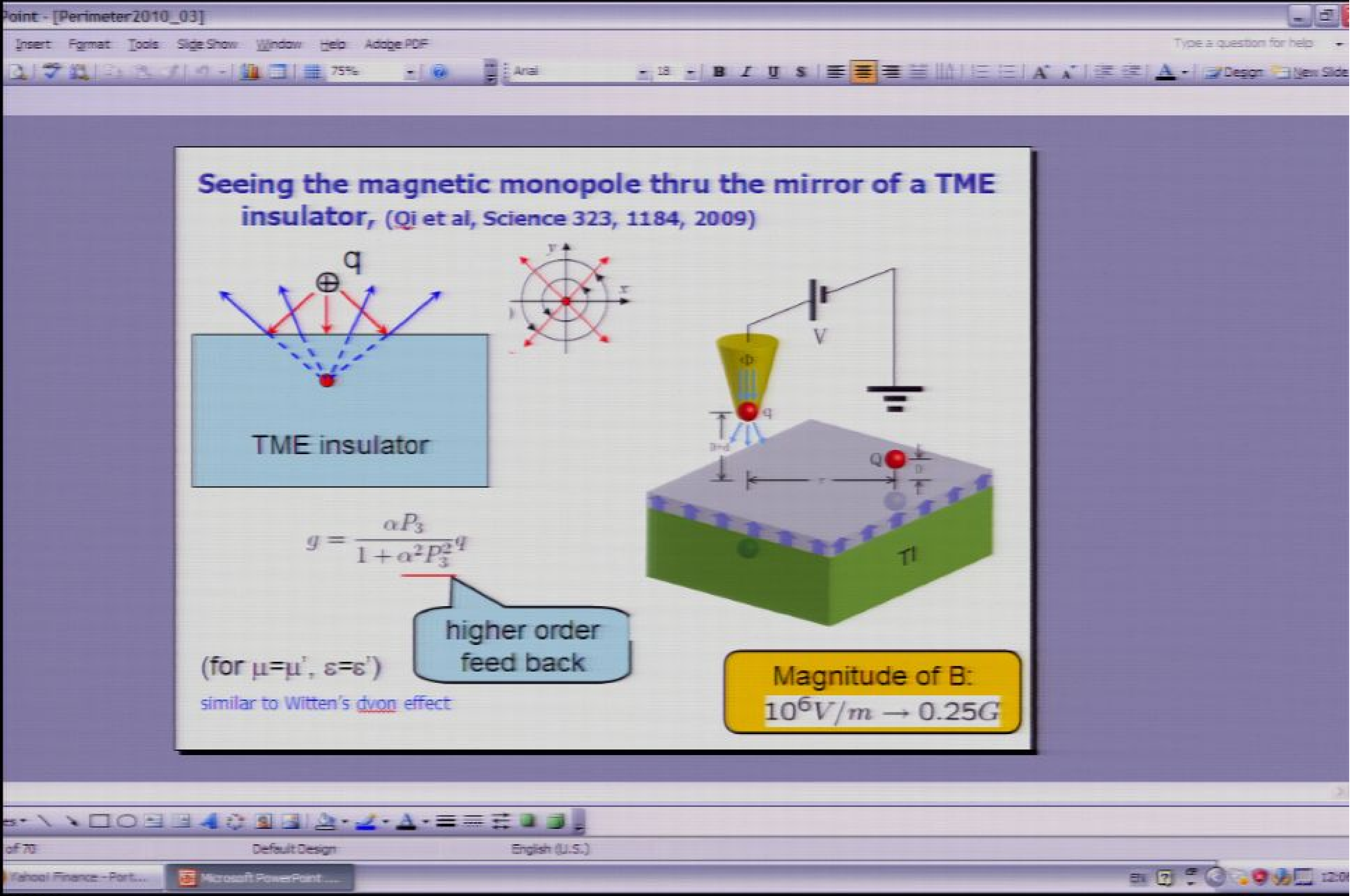
$$g = \frac{\alpha P_3}{1+\alpha^2 P_3^2} q$$

higher order feed back

(for μ=μ', ε=ε')

similar to Witten's dyon effect

Magnitude of B:
$10^6 V/m \rightarrow 0.25G$



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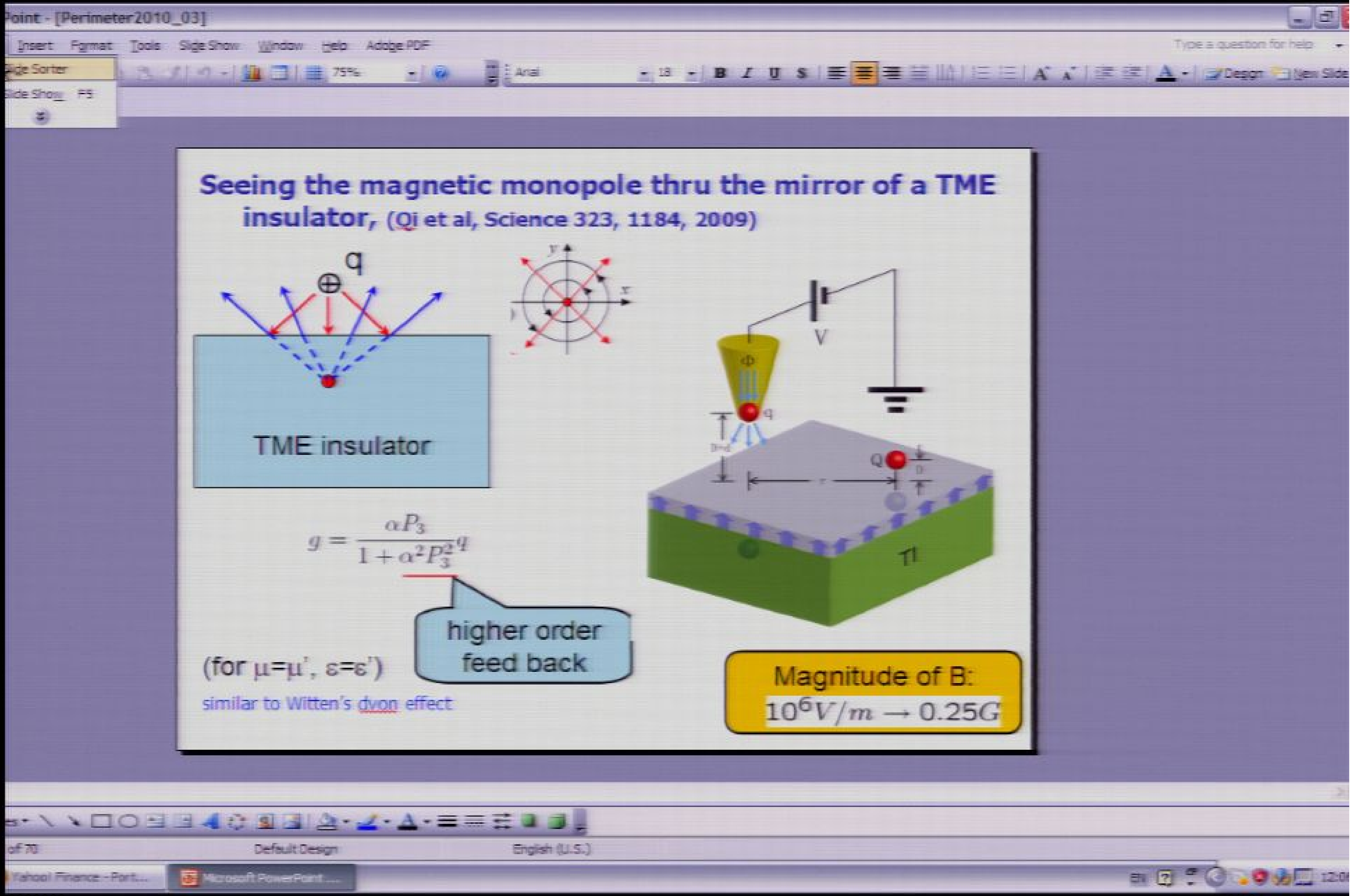

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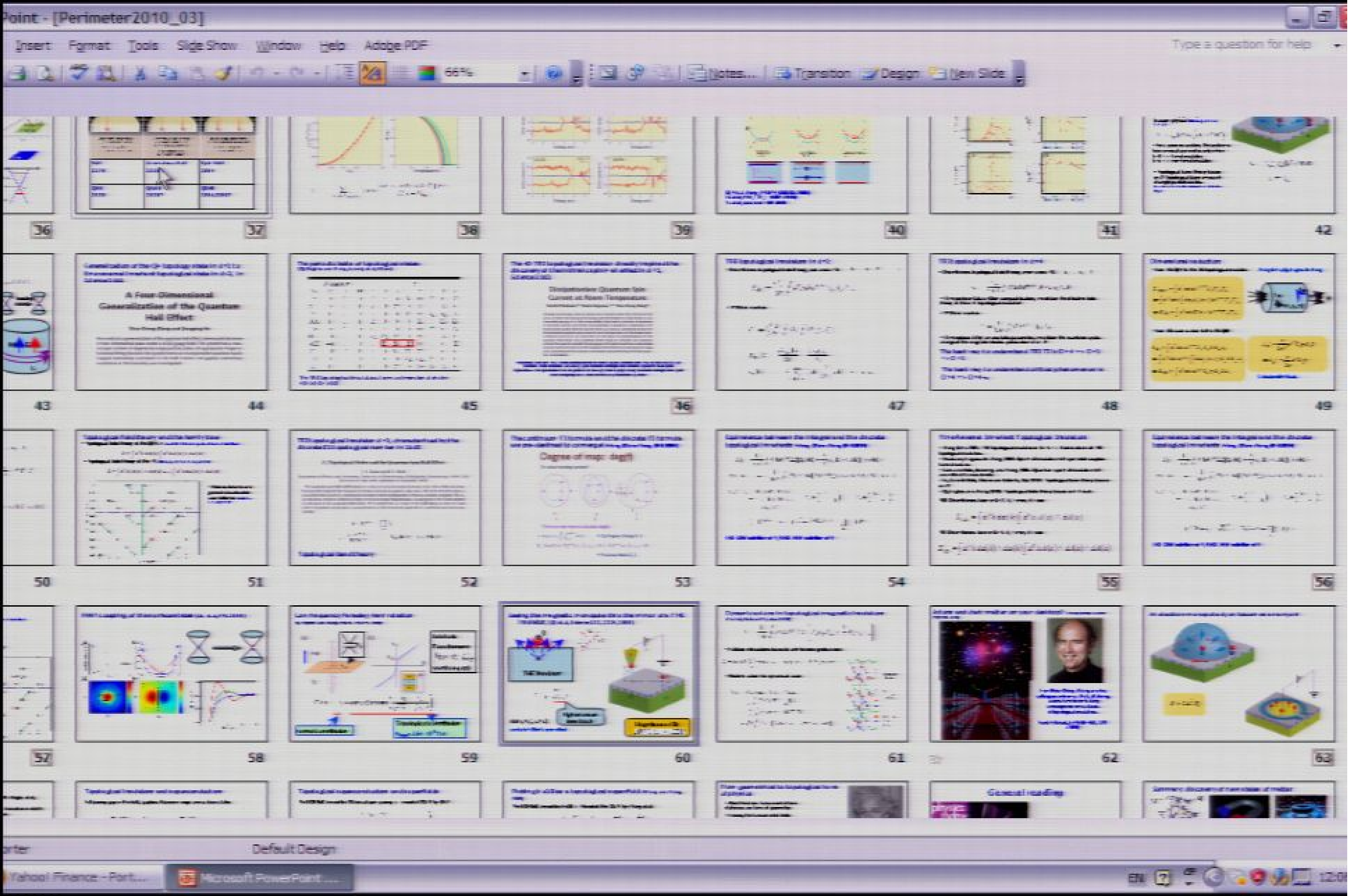
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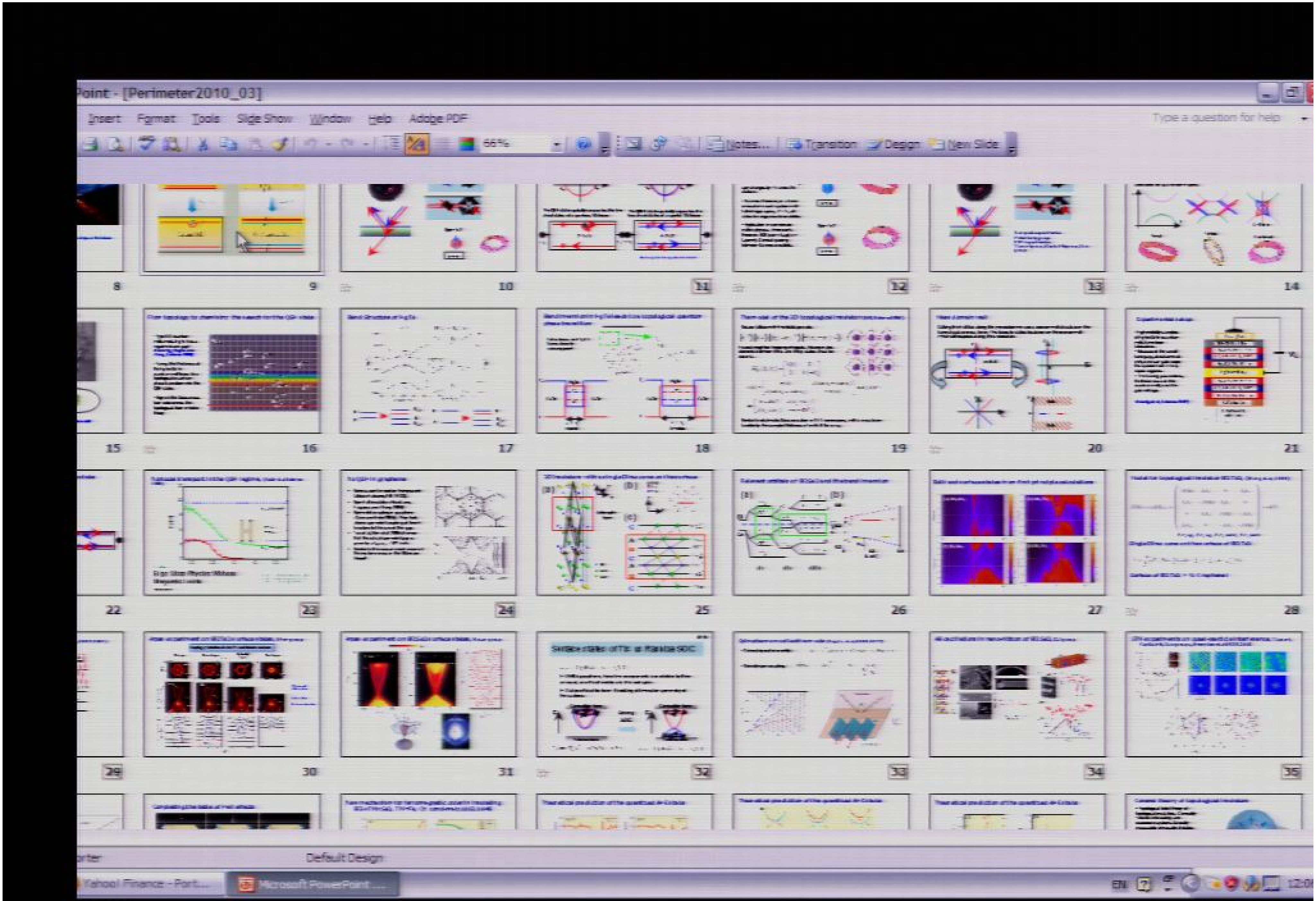
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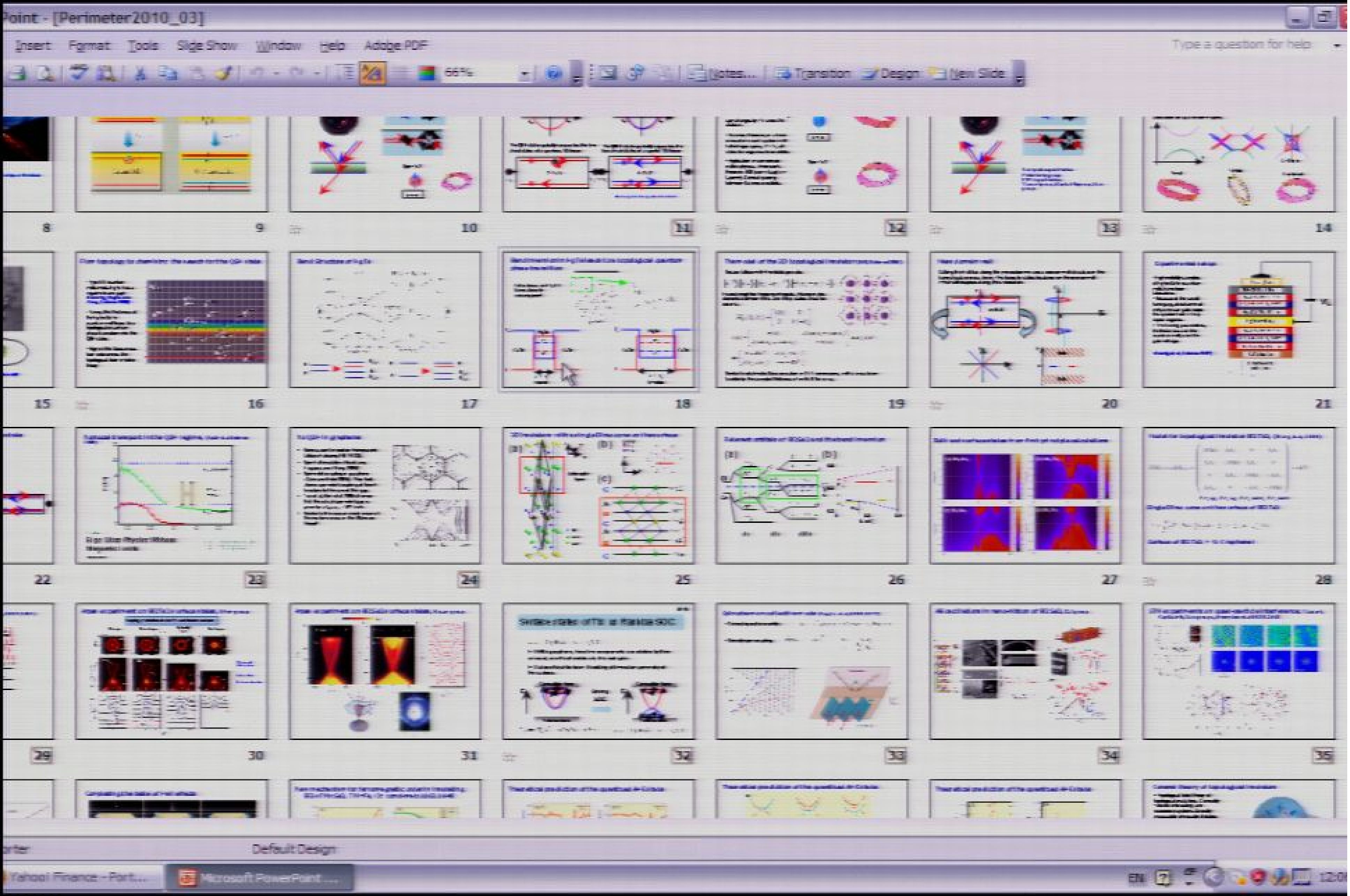
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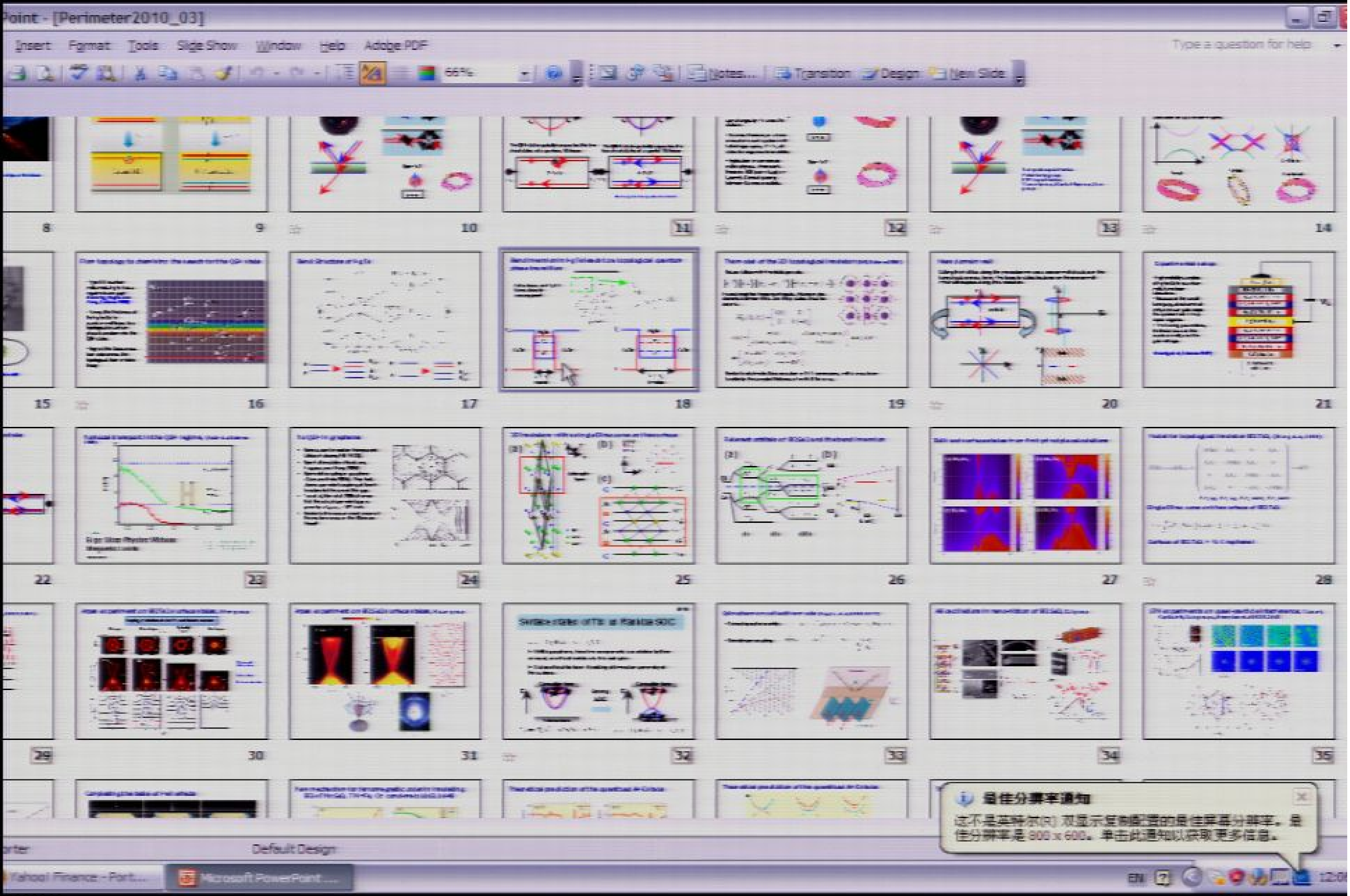

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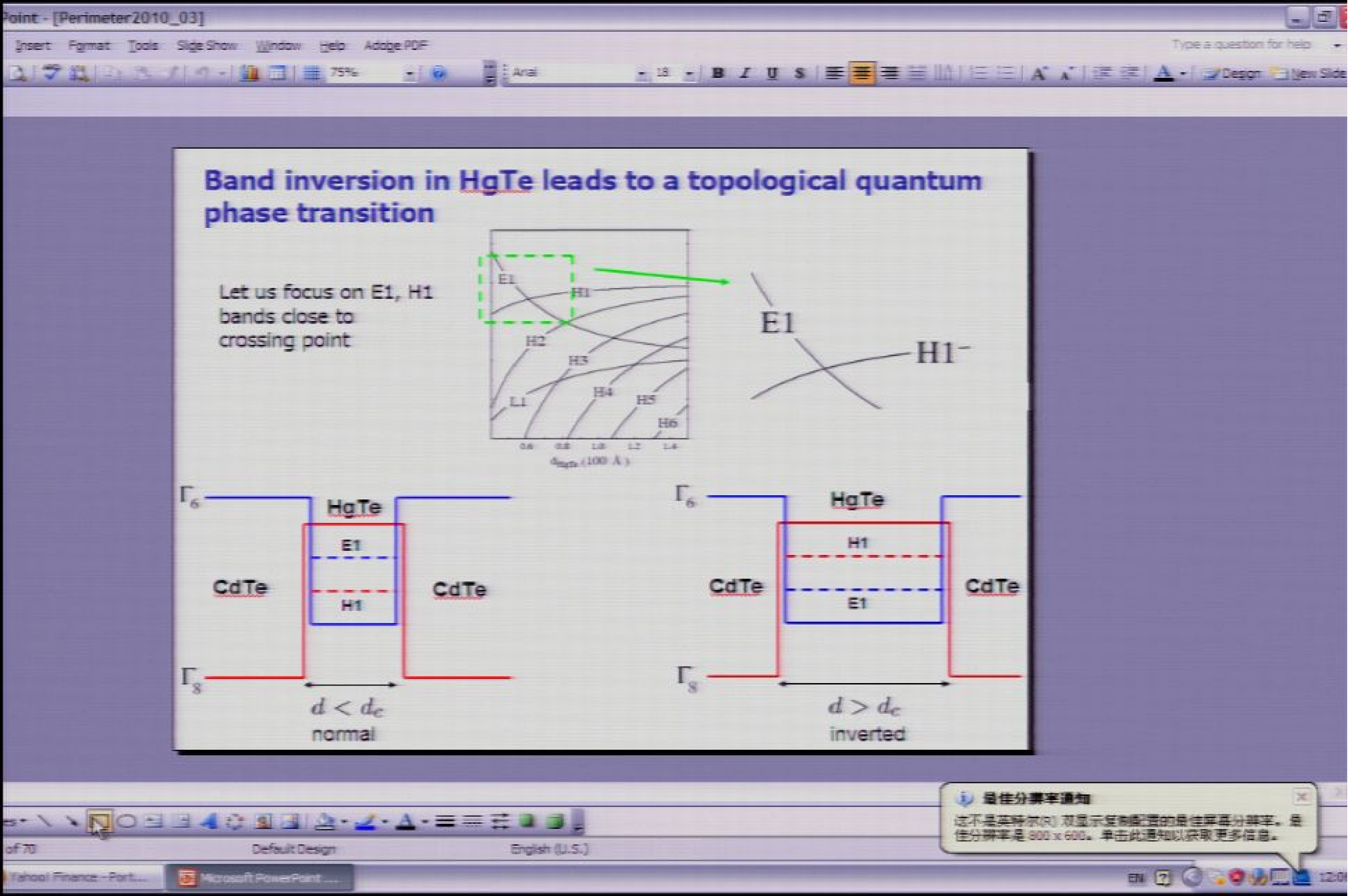

Band inversion in HgTe leads to a topological quantum phase transition

Let us focus on E1, H1 bands close to crossing point

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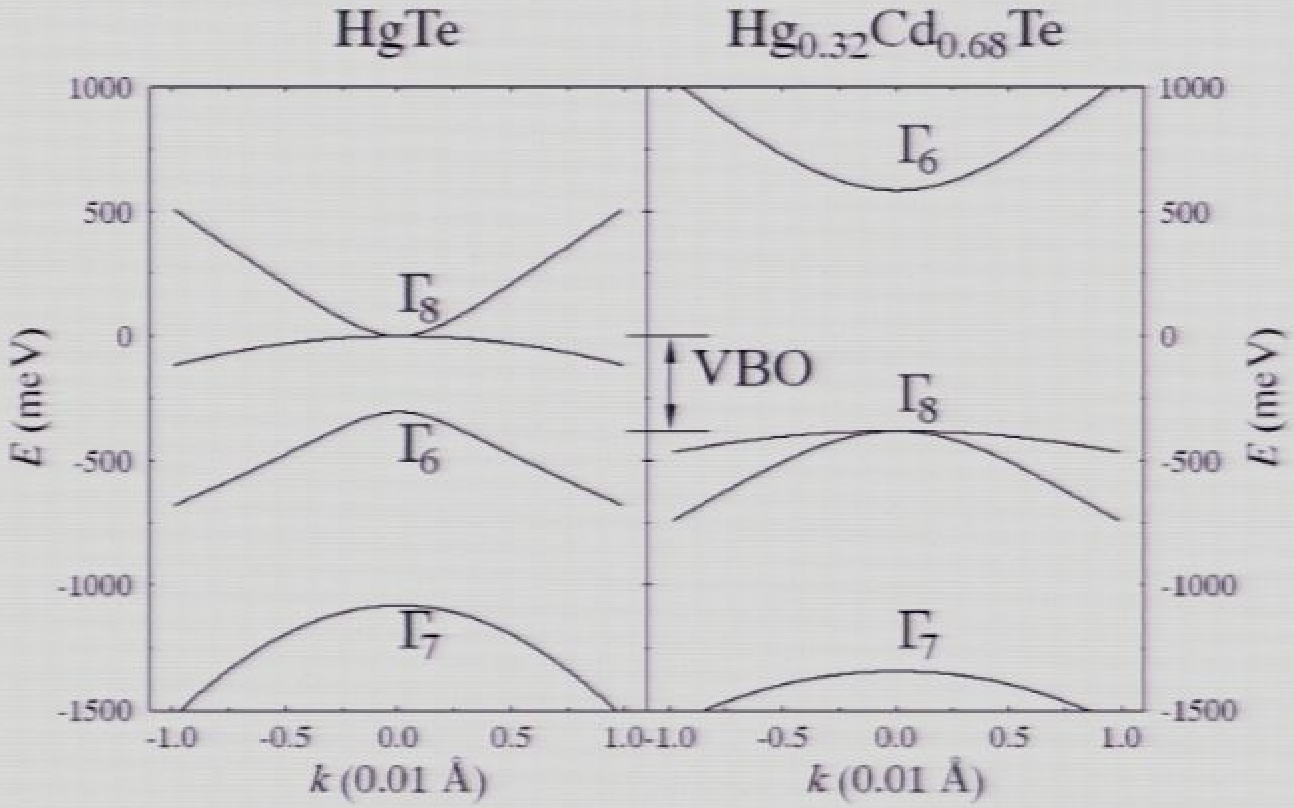
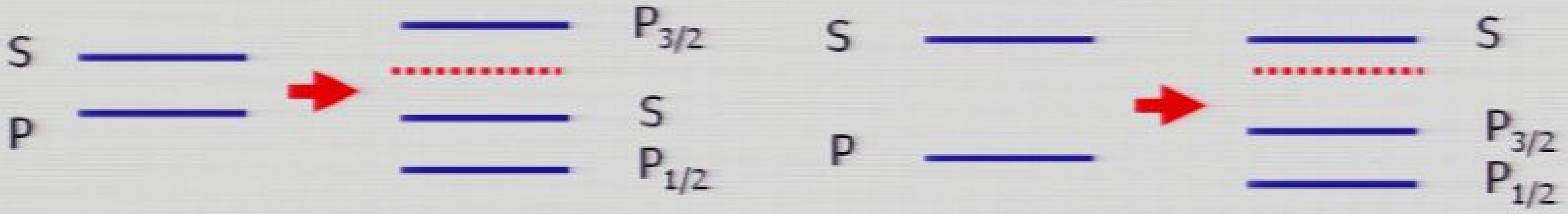
Band Structure of HgTe

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P

$P_{3/2}$

S

$P_{1/2}$

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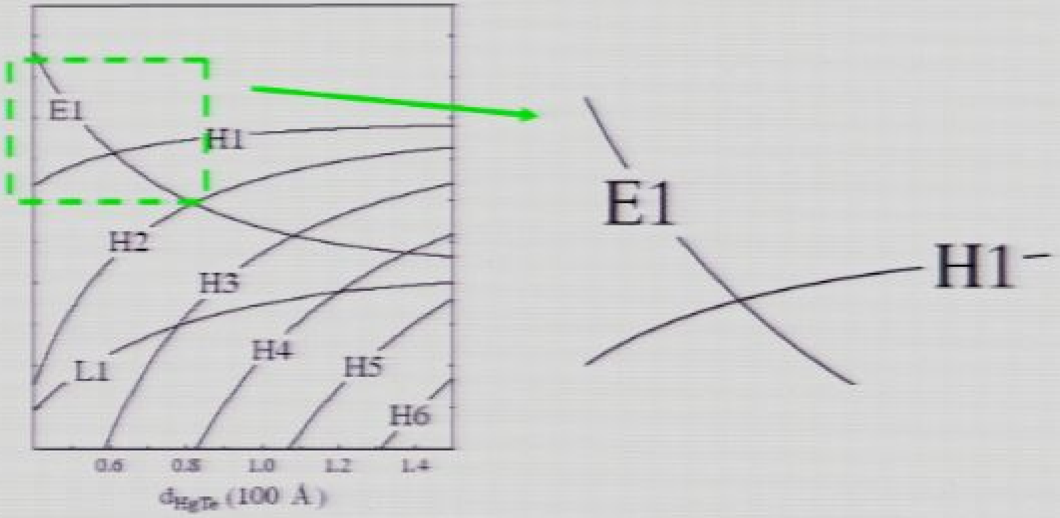
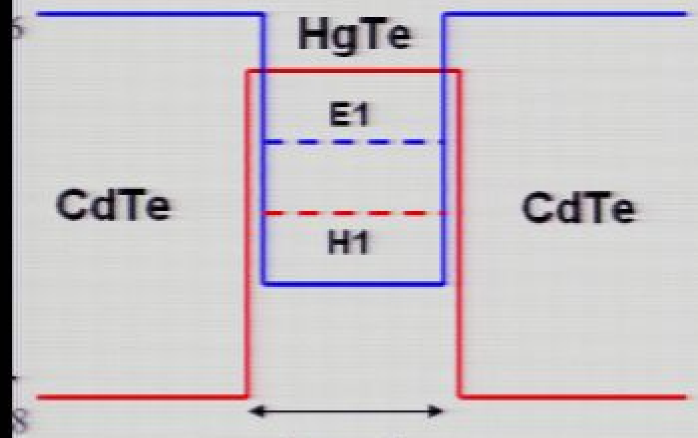
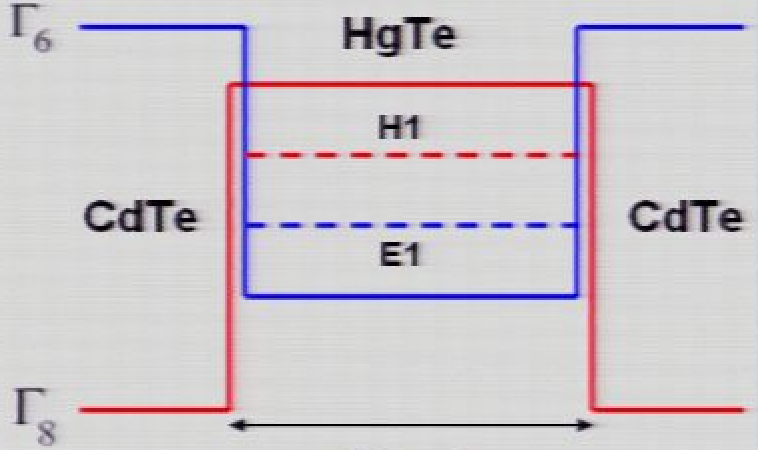




$d < d_c$

normal



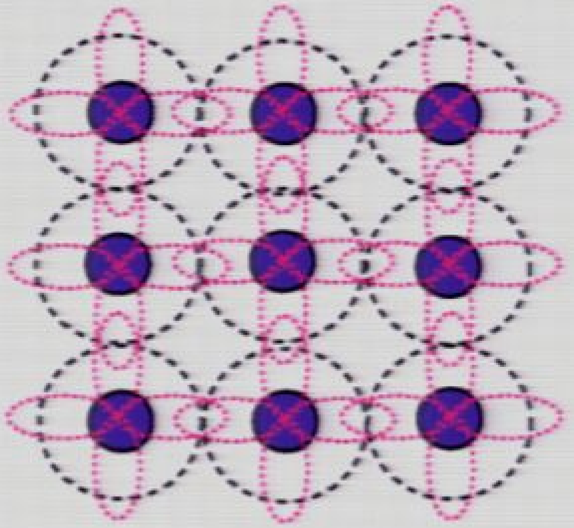
$d > d_c$

inverted

The model of the 2D topological insulator (BHZ, Science 2006)

Square lattice with 4-orbitals per site:

$$|s,\uparrow\rangle, |s,\downarrow\rangle, |(p_x+ip_y,\uparrow\rangle, |-(p_x-ip_y),\downarrow\rangle$$



Nearest neighbor hopping integrals. Mixing matrix elements between the s and the p states must be odd in k.

$$H_{eff}(k_x,k_y)=\begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix}$$

$$h(k)=\begin{pmatrix} m(k) & A(\sin k_x - i\sin k_y) \\ A(\sin k_x + i\sin k_y) & -m(k) \end{pmatrix} \equiv d_a(k)\tau^a$$

$$\Rightarrow \begin{pmatrix} m+Bk^2 & A(k_x-ik_y) \\ A(k_x+ik_y) & -m-Bk^2 \end{pmatrix}$$

Similar to relativistic Dirac equation in 2+1 dimensions, with a mass term tunable by the sample thickness d! $m/B<0$ for $d>d_c$.