Title: CauCats: the backbone of a quantum relativistic universe of interacting processes

Date: Mar 30, 2010 04:00 PM

URL: http://pirsa.org/10030058

Abstract: Our starting point is a particular `canvas' aimed to `draw' theories of physics, which has symmetric monoidal categories as its mathematical backbone. With very little structural effort (i.e. in very abstract terms) and in a very short time this categorical quantum mechanics research program has reproduced a surprisingly large fragment of quantum theory. Philosophically speaking, this framework shifts the conceptual focus from `material carriers' such as particles, fields, or other
`material stuff', to `logical flows of information', by mainly encoding how things stand in relation to each other. These relations could, for example, be induced by operations. Composition of these relations is the carrier of all structure.

Thus far the causal structure has been treated somewhat informally within this approach. In joint work with my student Raymond Lal, by restricting the capabilities to compose, we were able to formally encode causal connections. We call the resulting mathematical structure a CauCat, since it combines the symmetric monoidal stricture with Sorkin's CauSets within a single mathematical concept. The relations which now respect causal structure are referred to as processes, which make up the actual `happenings'. As a proof of concept, we show that if in a quantum teleportation protocol one omits classical communication, no information
is transfered. We also characterize Galilean theories.

Classicality is an attribute of certain processes, and measurements are special kinds of processes, defined in terms of their capabilities to correlate other processes to these classical attributes. So rather than quantization, what we do is classicization within our universe of processes. We show how classicality and the causal structure are tightly intertwined.

All of this is still very much work in progress!

Causal categories: a universe of interacting processes

Bob Coecke and Ray Lal

Oxford University Computing Laboratory

QUANTUM TELEPORTATION
(a graphical account)

Lemma 0. $(f \otimes 1) \circ (1 \otimes g) = (1 \otimes g) \circ (f \otimes 1)$.

Lemma 1. $\forall \, |\Psi\rangle \, , \, \exists \, f : |\Psi\rangle = (1 \otimes f) \circ |Bell\rangle$.

Lemma 2. $(f \otimes 1) \circ |Bell\rangle = (1 \otimes f^T) \circ |Bell\rangle$.

Lemma 3. $(\langle Bell| \otimes 1) \circ (1 \otimes |Bell\rangle) = 1$.

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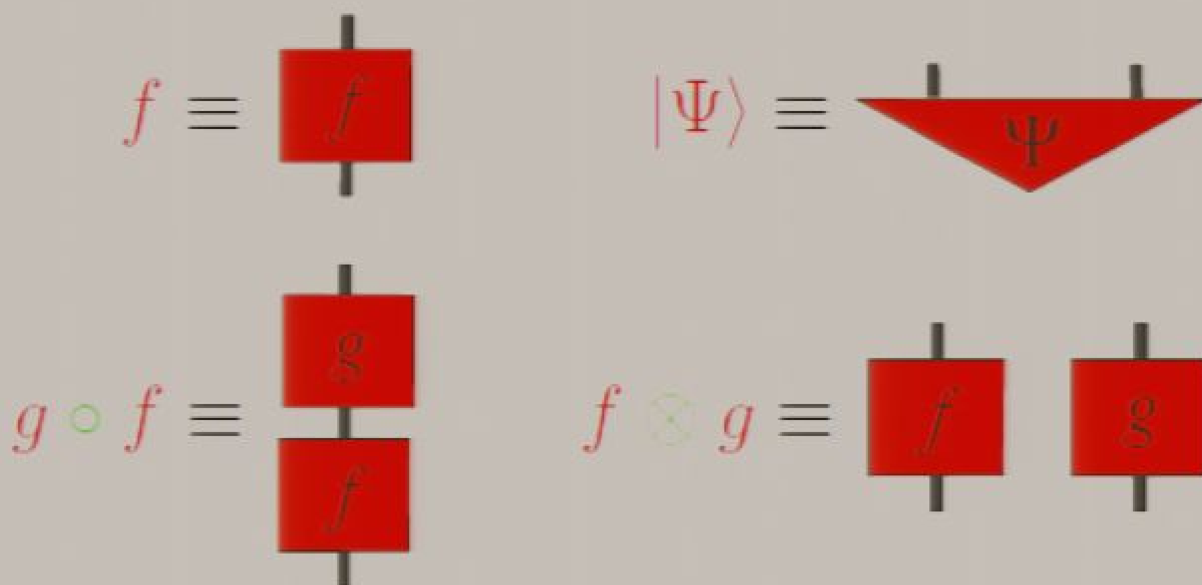
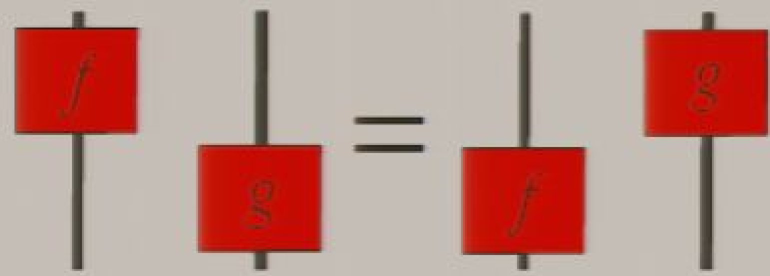
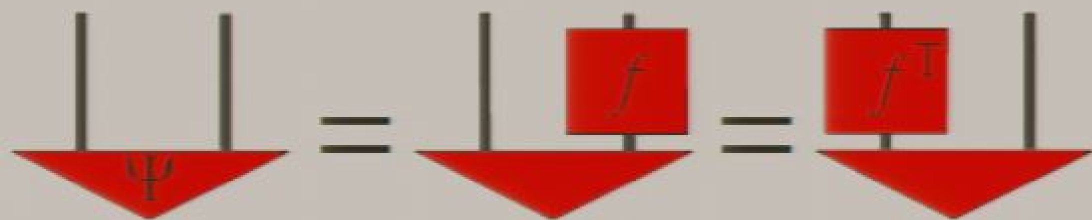


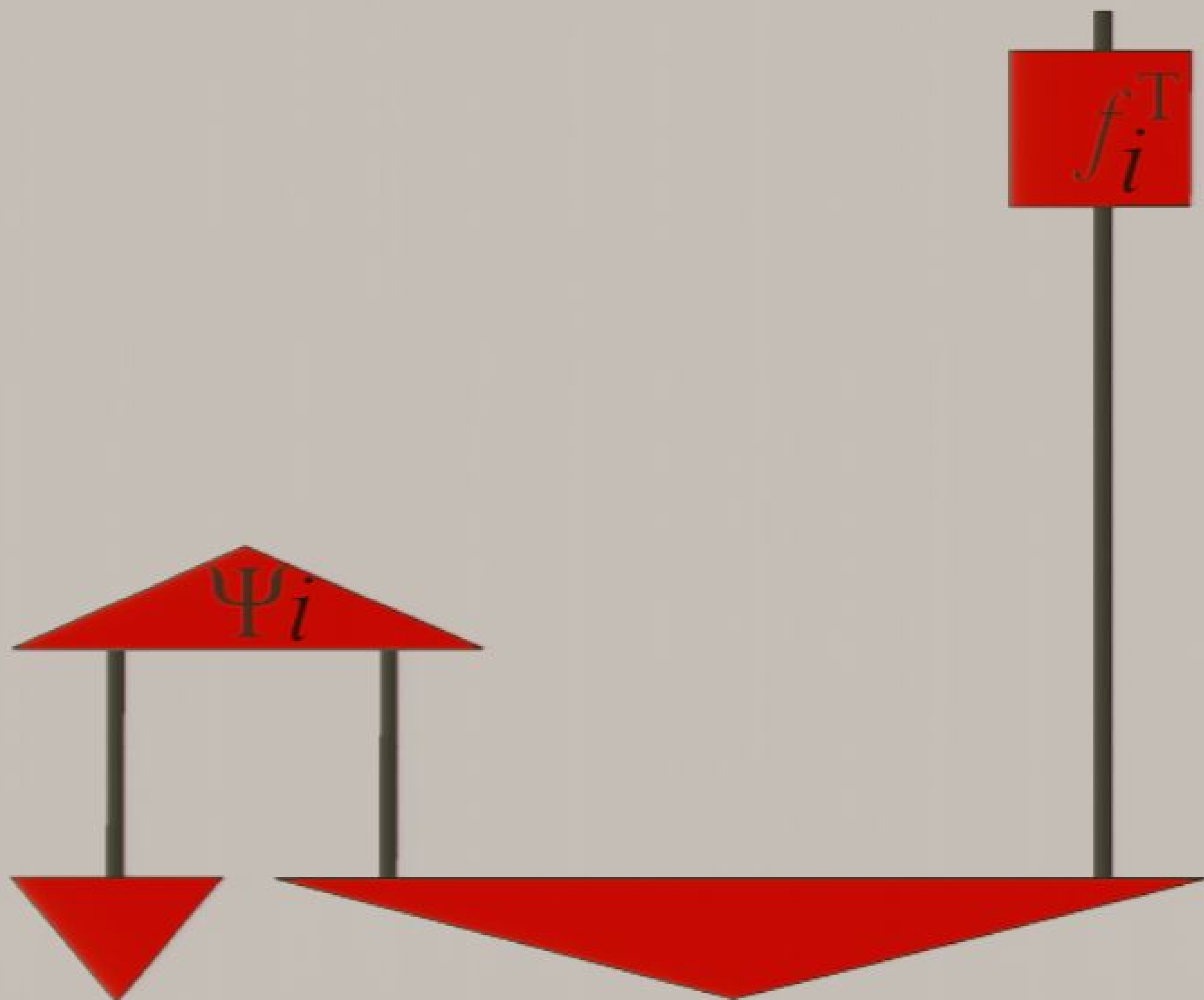
Lemma 0:

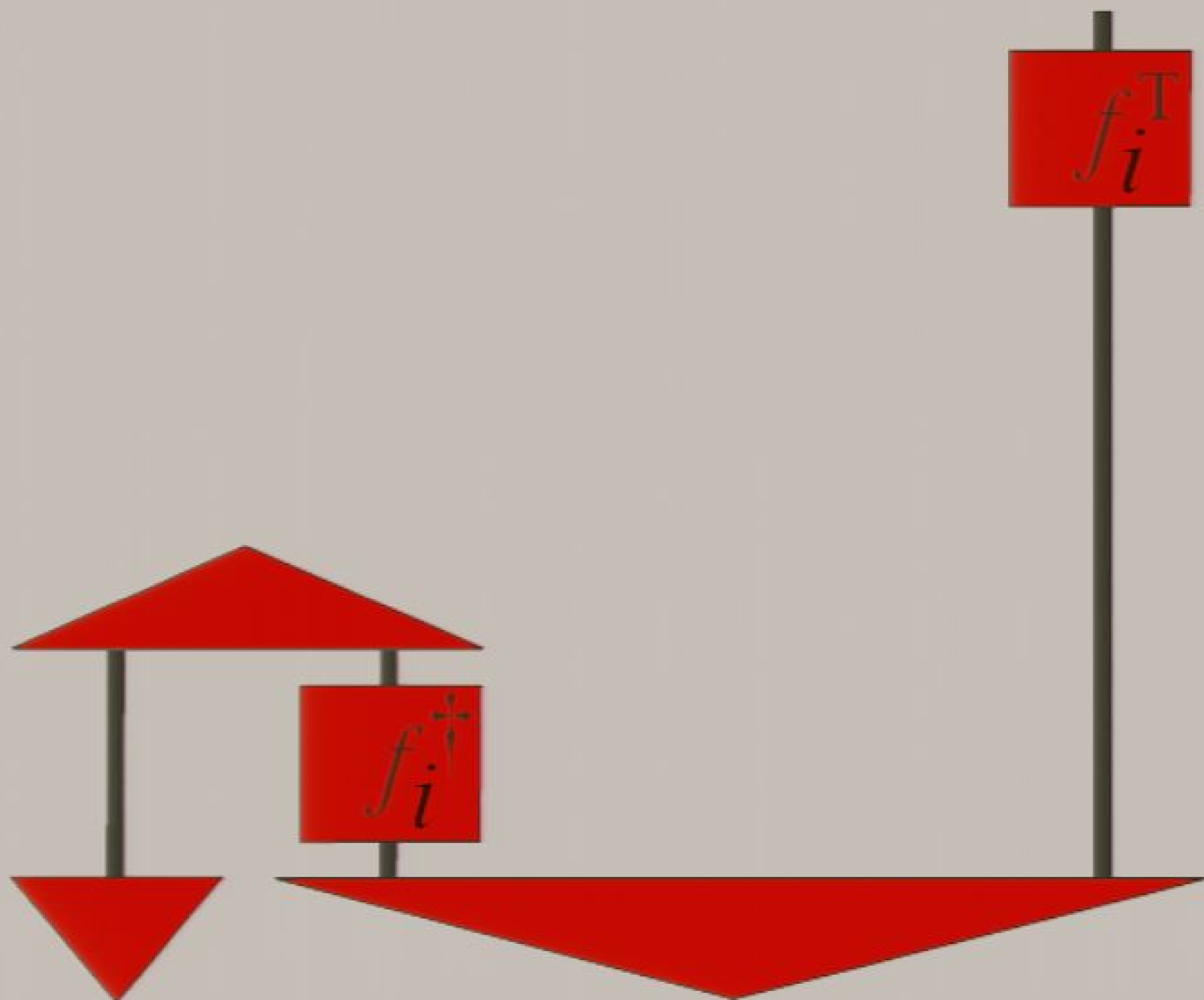


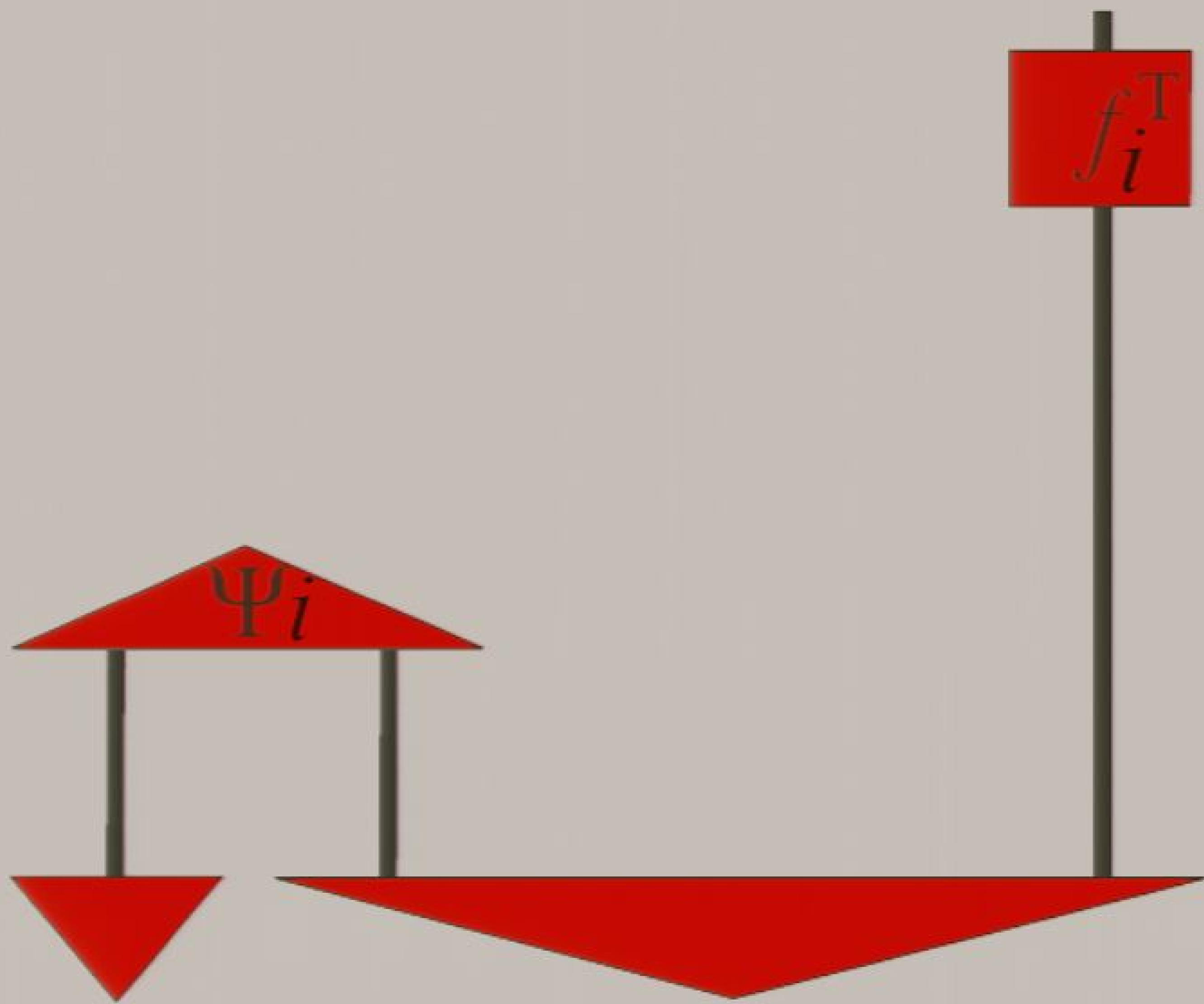
Lemma 1 & Lemma 2:



Lemma 3:


f_i^{T}
Ψ_i


f_i^{T}
$f_i^{\dagger}$



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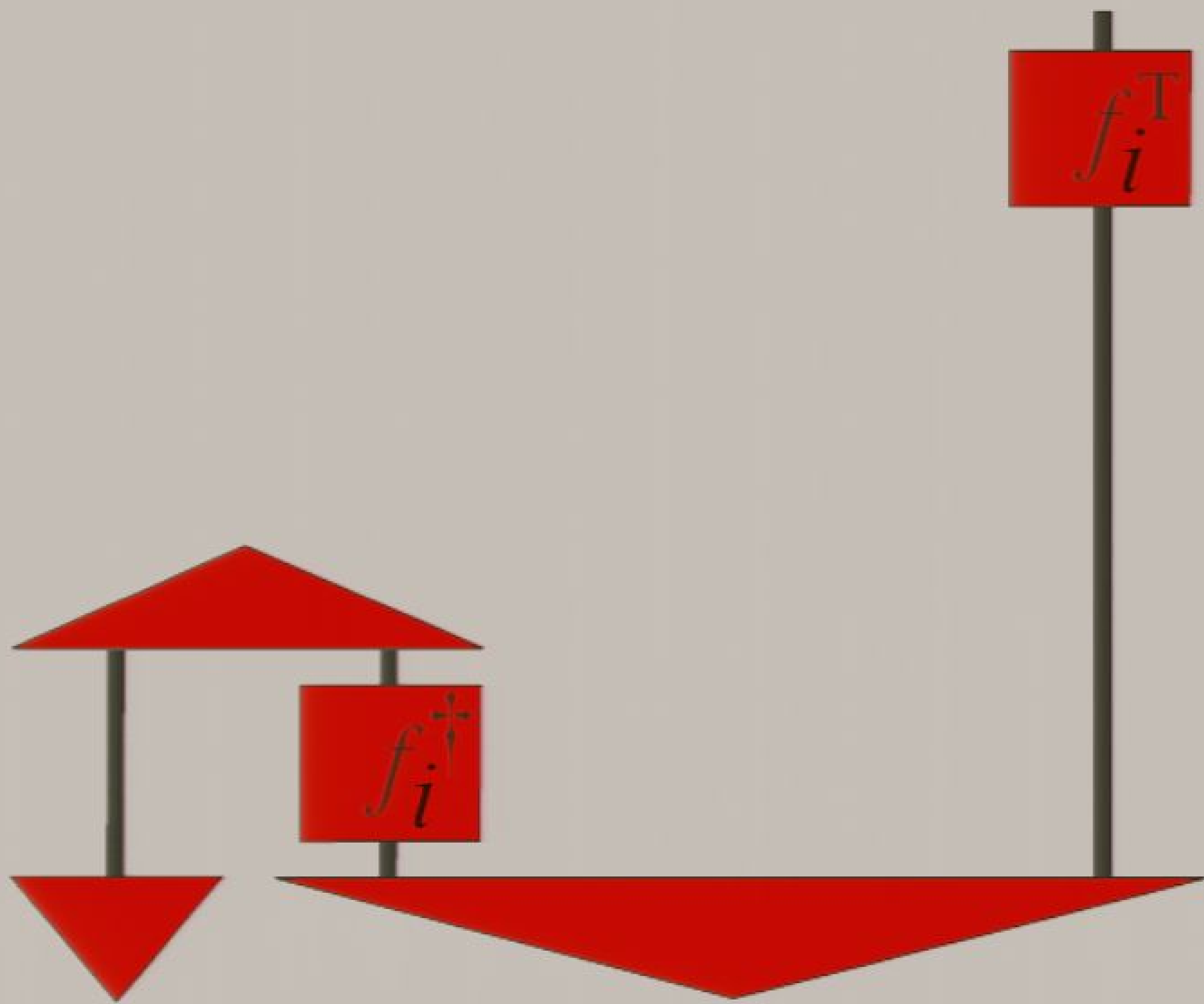
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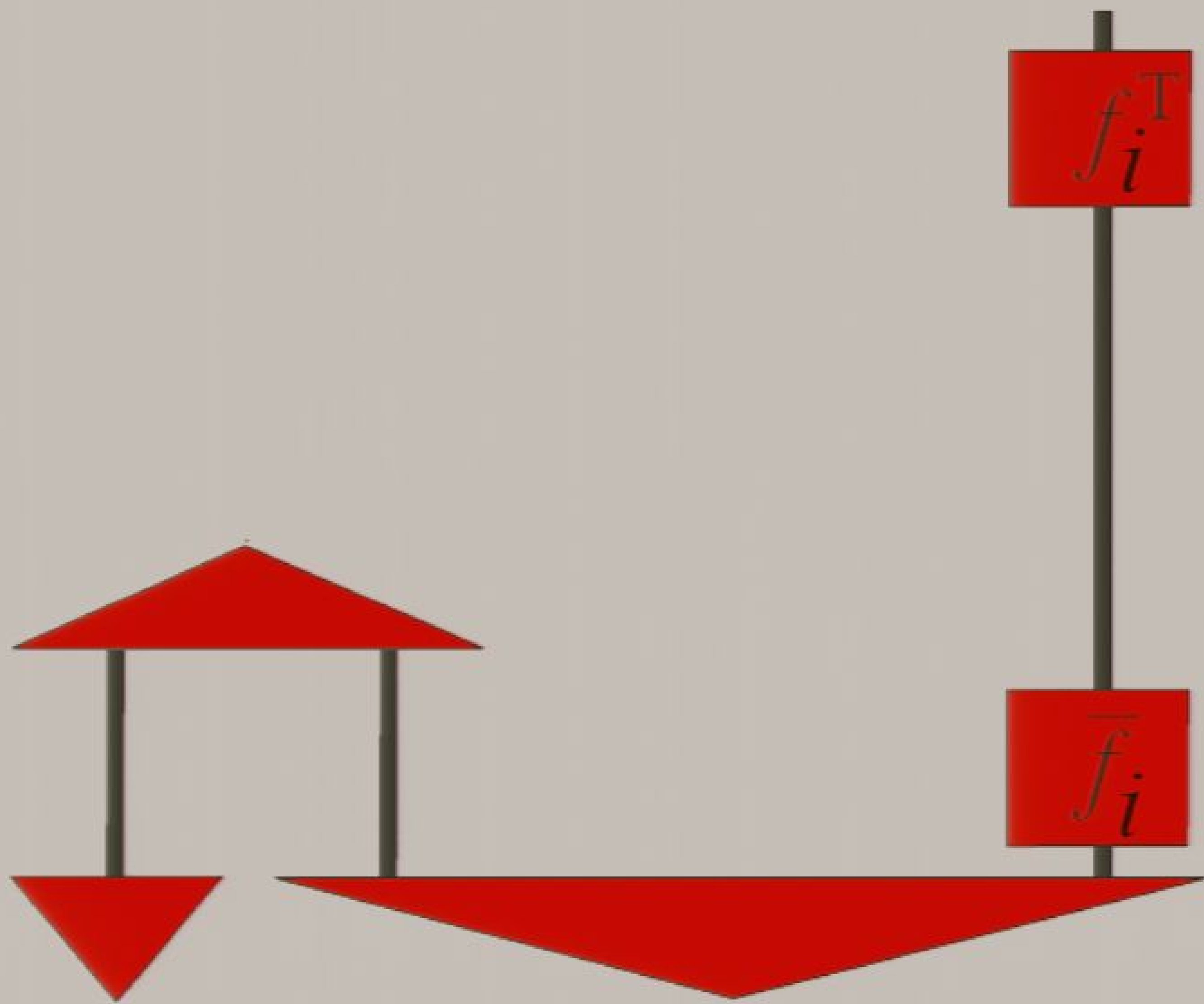
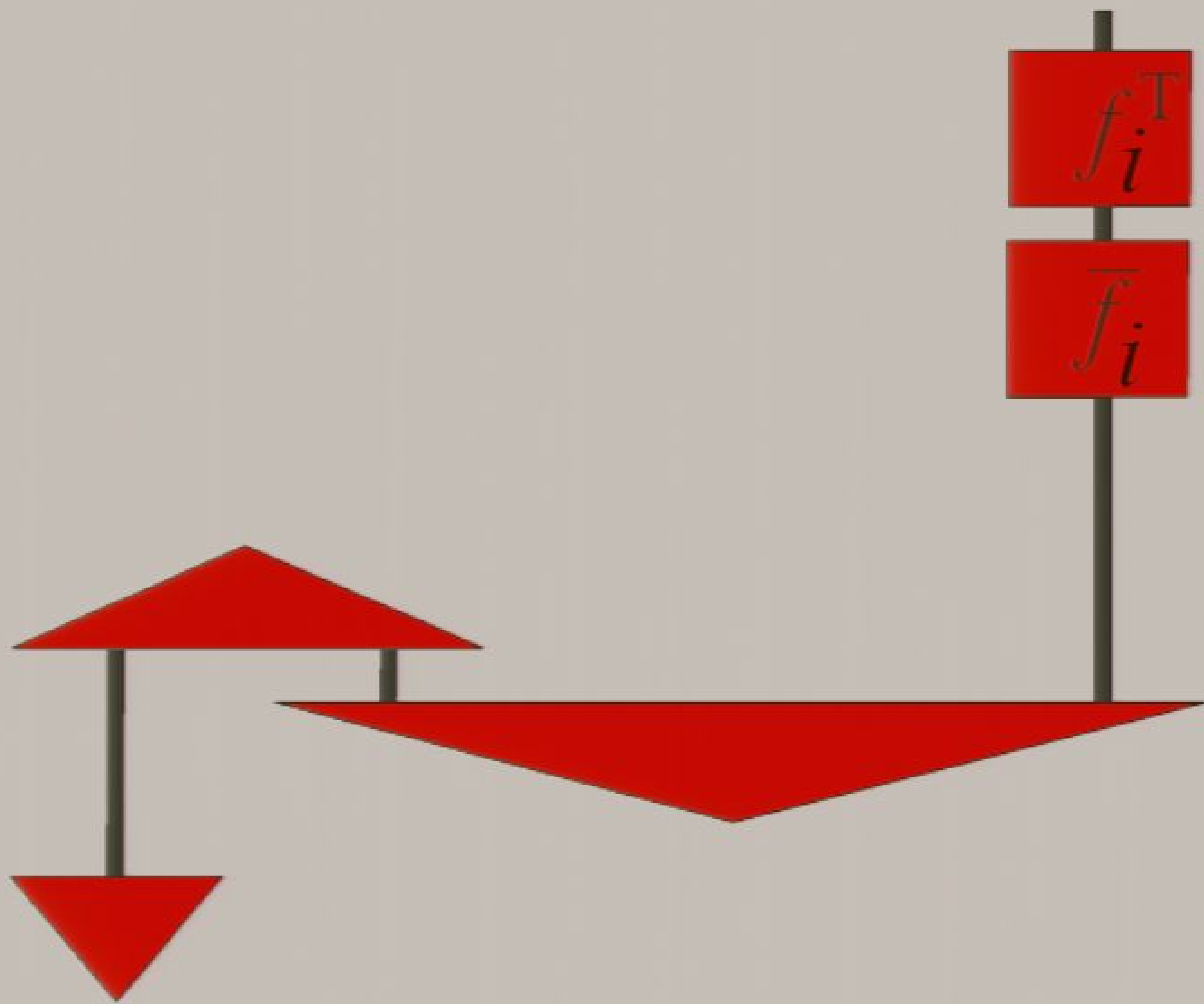

f_i^{T}
$\overline{f}_i$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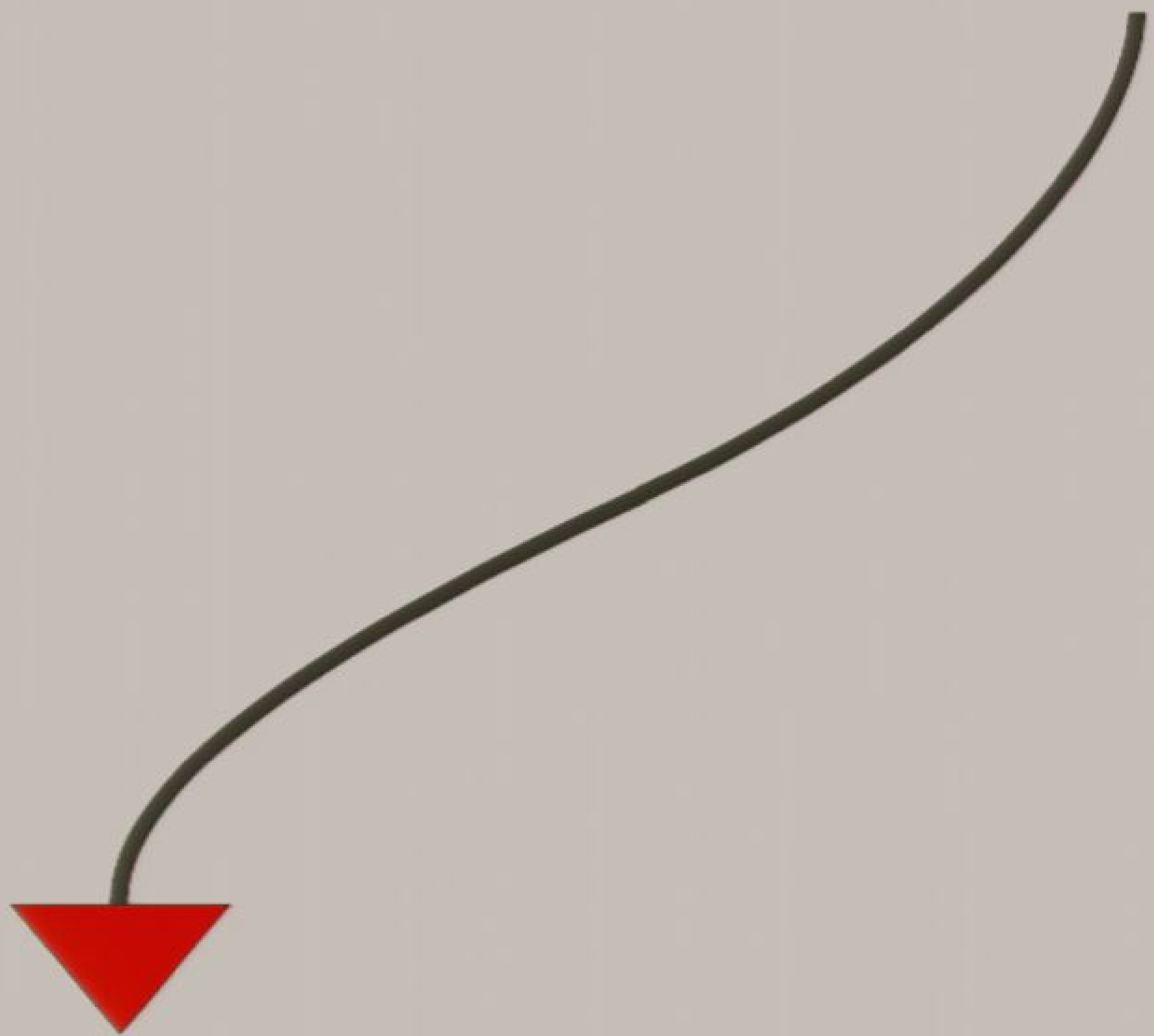

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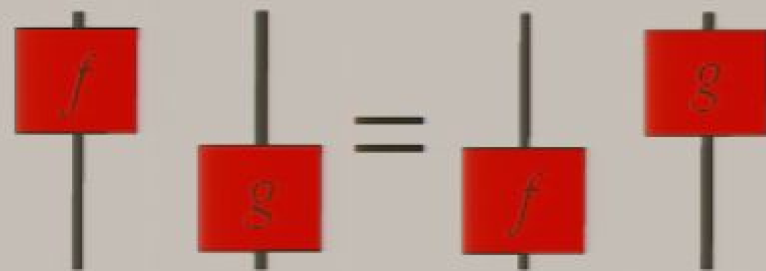

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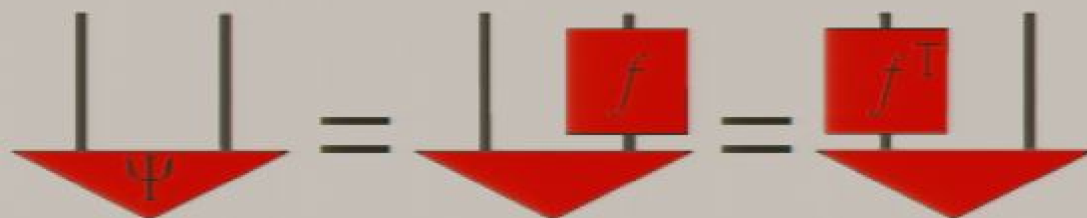
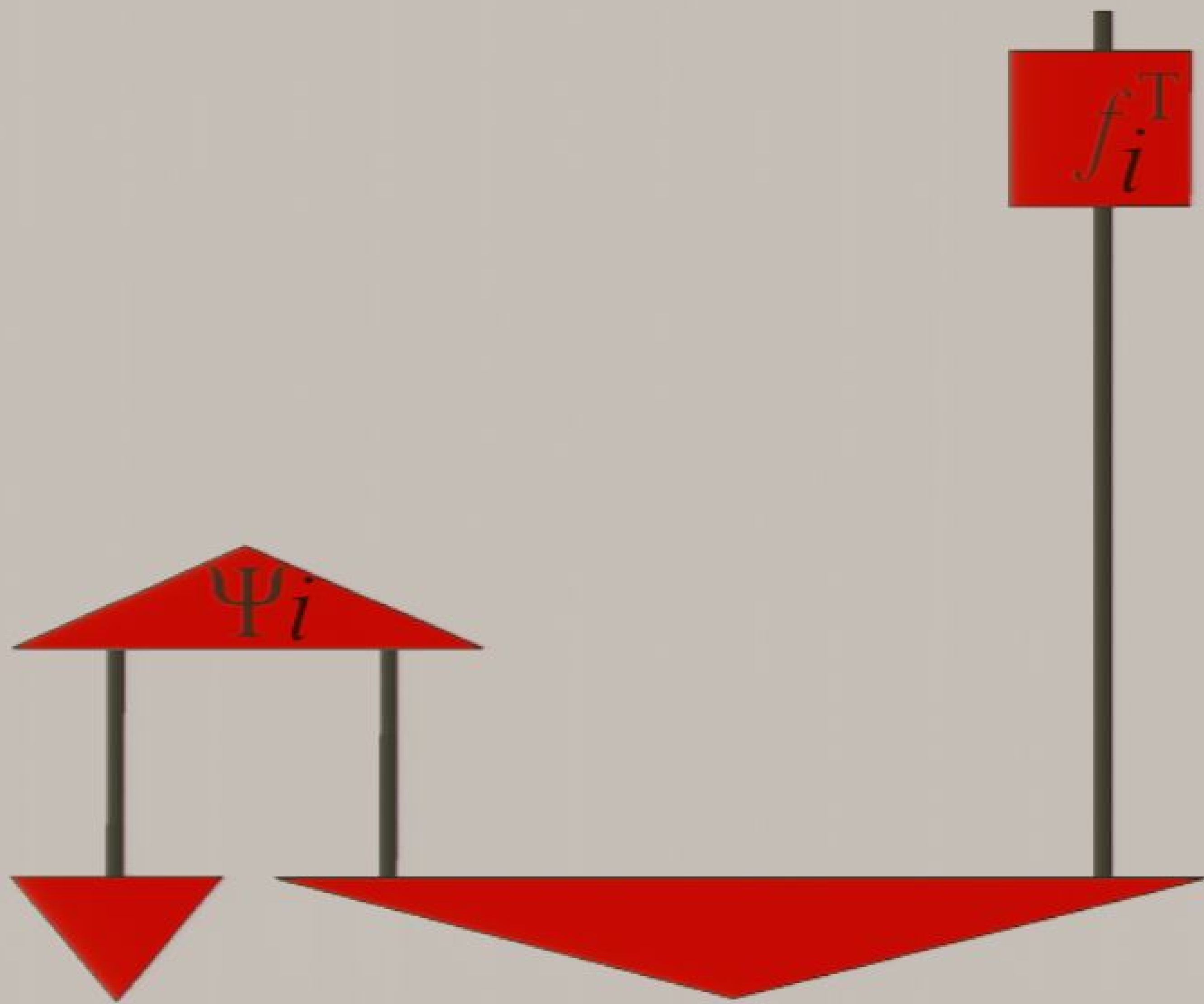


Lemma 3:

CATEGORICAL QUANTUM MECHANICS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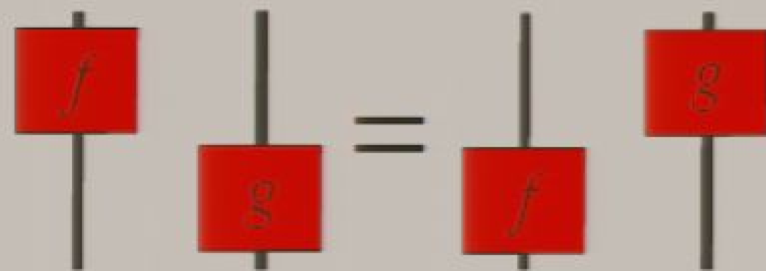

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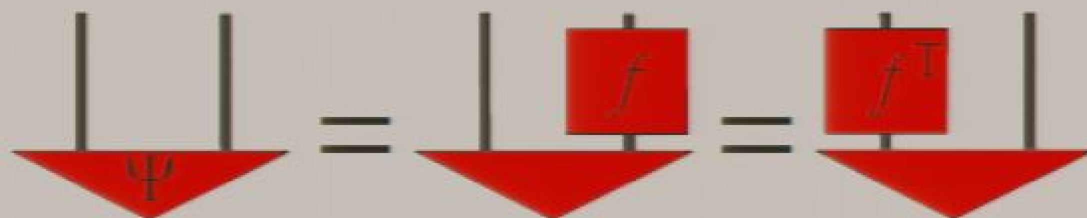

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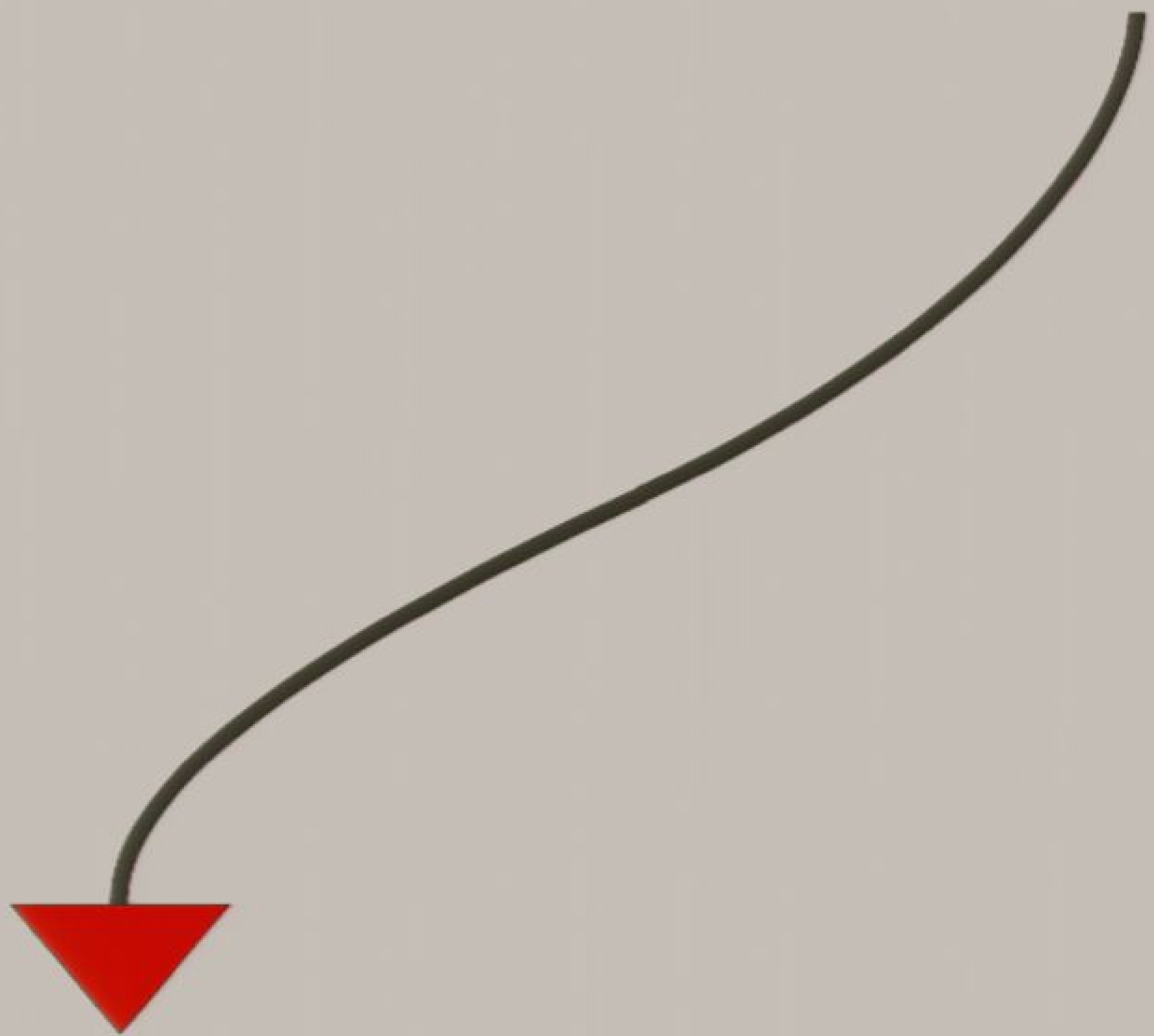

CATEGORICAL QUANTUM MECHANICS

— *merely a new notation?* —

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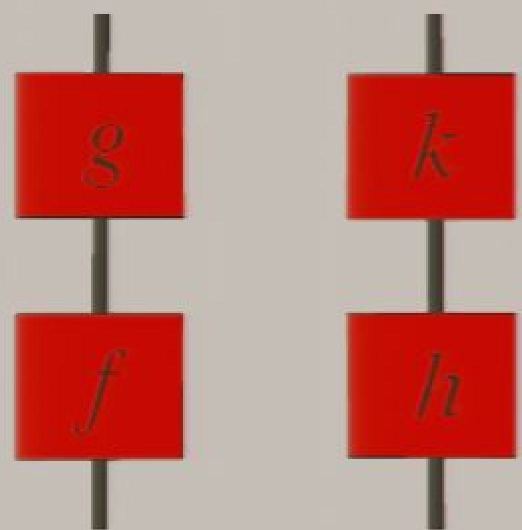
$$(g \circ f) \otimes (k \circ h)$$



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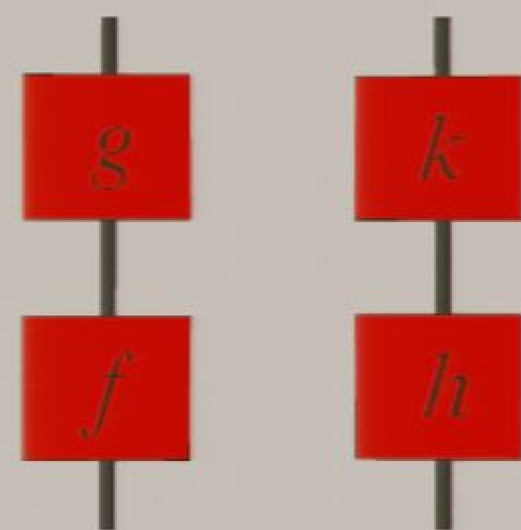

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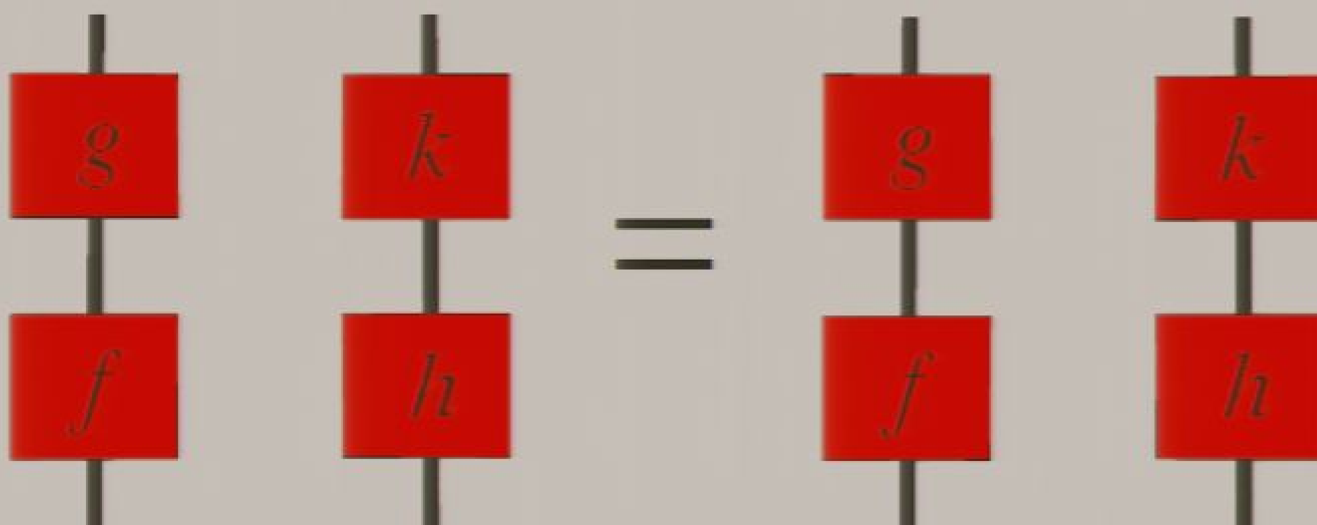

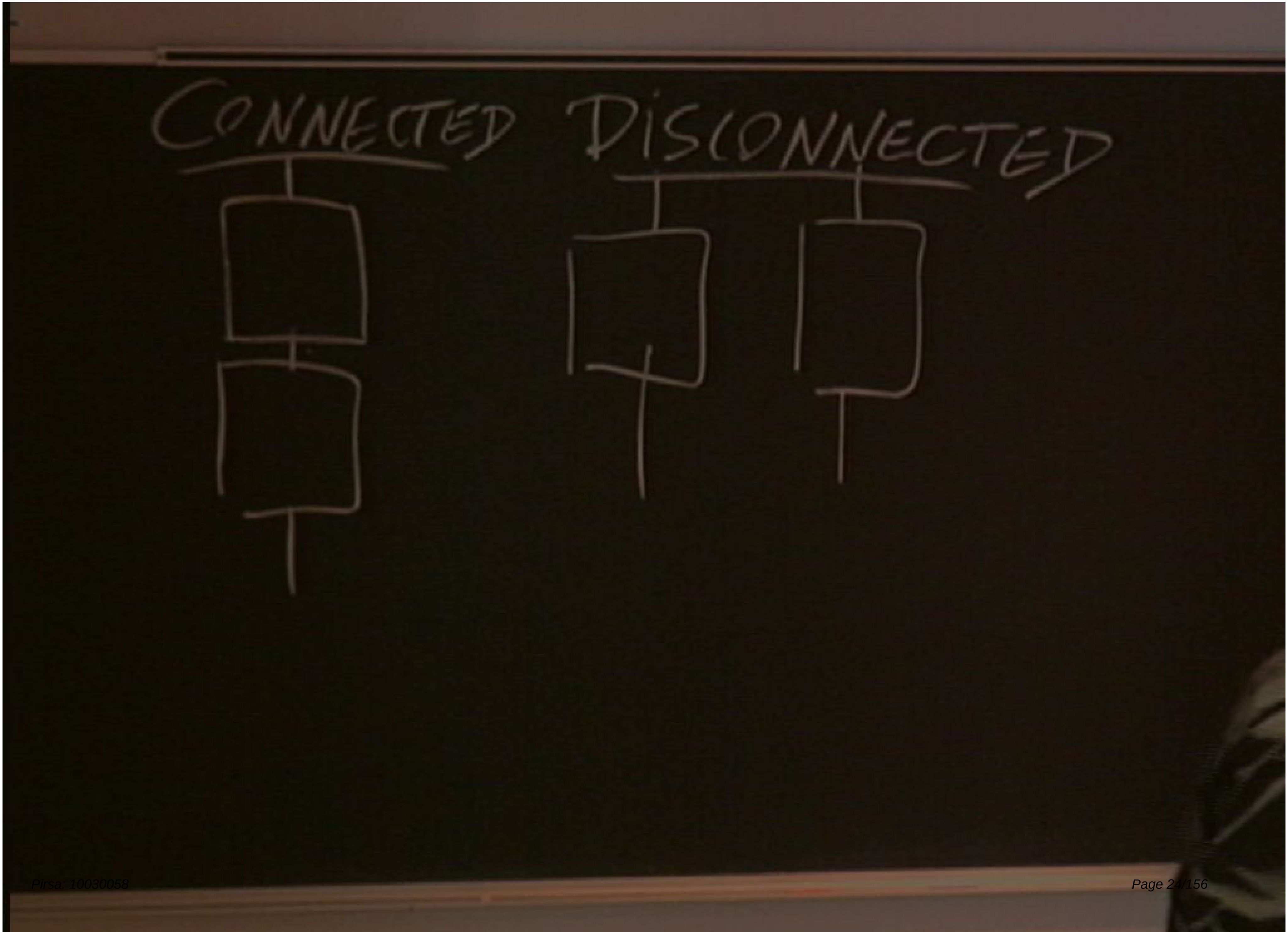
CONNECTED
DISCONNECTED

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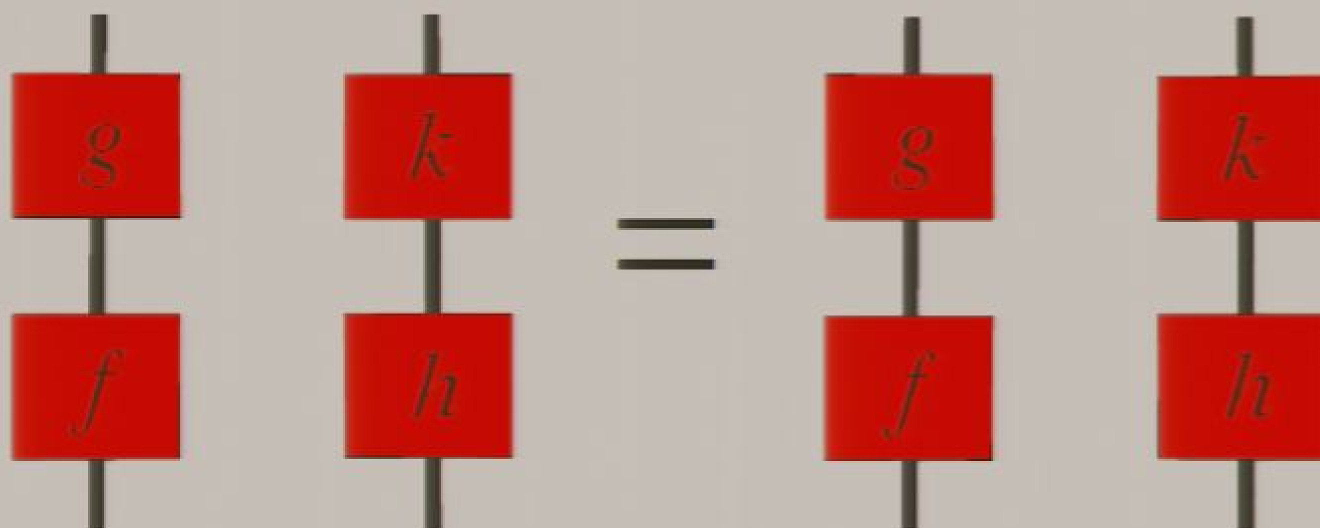


BIFUNCTORIALITY

— *physical processes as a symm. monoidal category* —

Systems:

$$A \qquad B \qquad C$$

Processes:

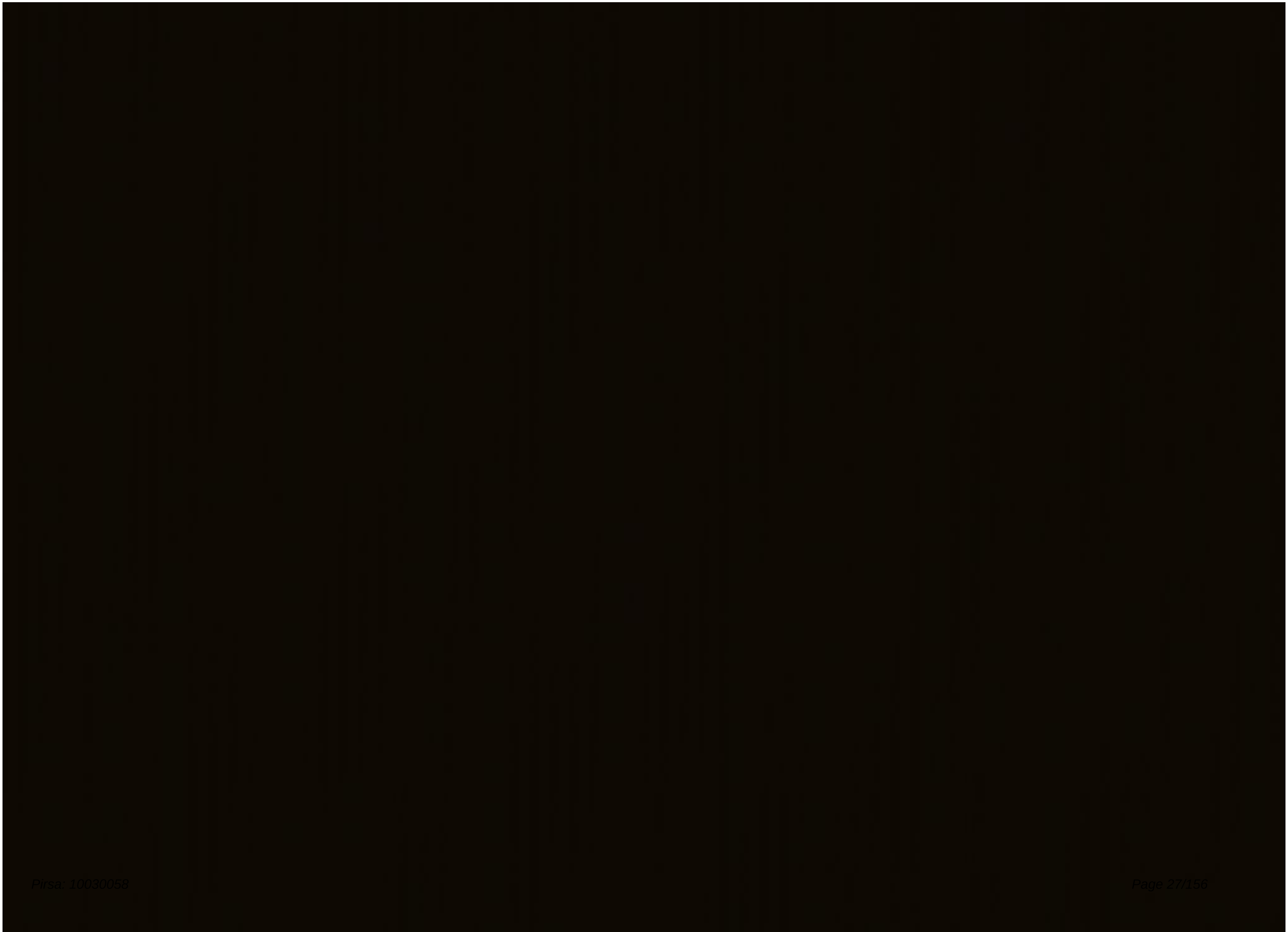
$$A \xrightarrow{f} A \qquad A \xrightarrow{g} B \qquad B \xrightarrow{h} C$$

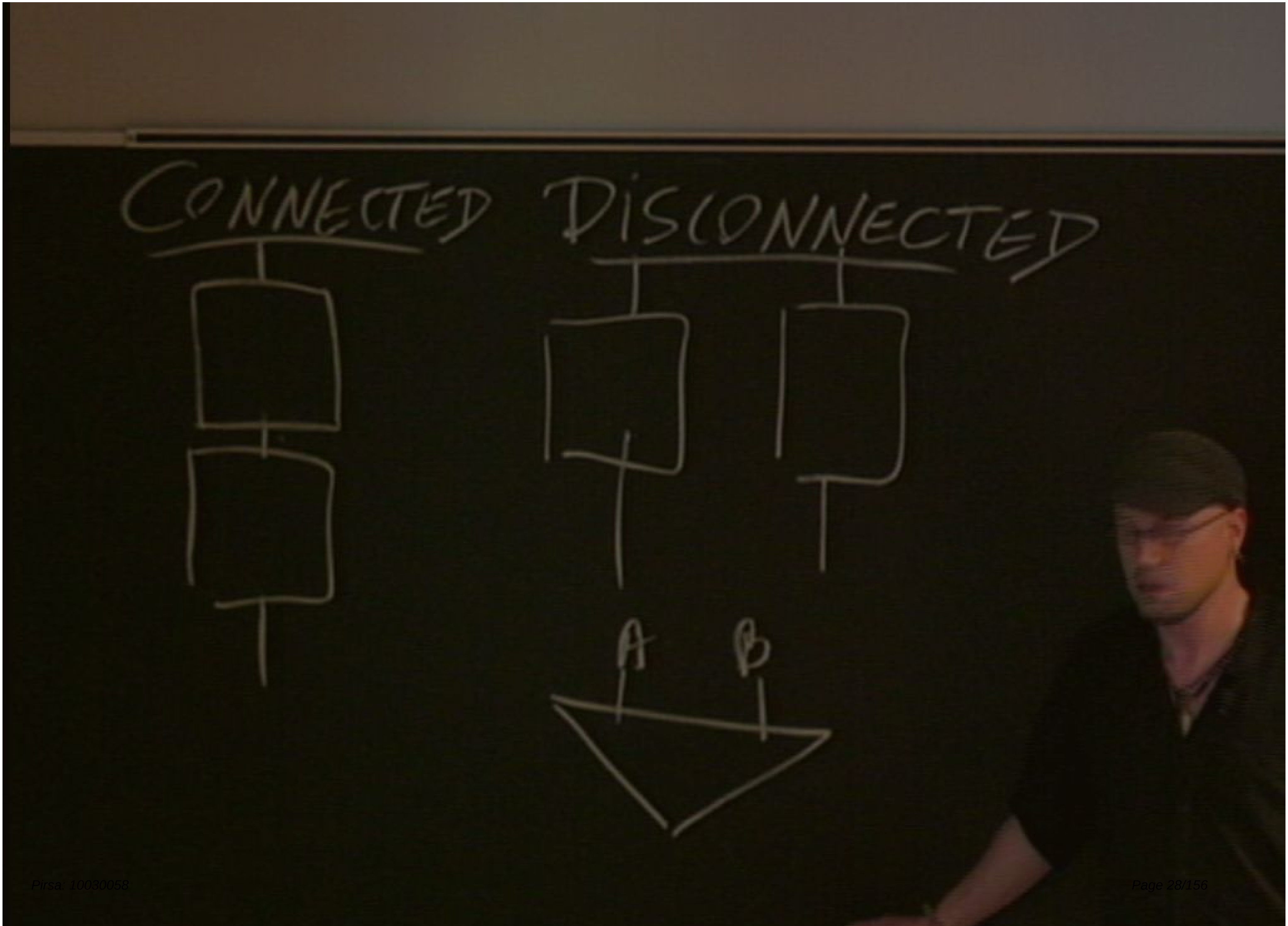
Compound systems:

$$A \otimes B \qquad \mathrm{I} \qquad A \otimes C \xrightarrow{f \otimes g} B \otimes D$$

Temporal composition:

$$A \xrightarrow{h \circ g} C \;:=\; A \xrightarrow{g} B \xrightarrow{h} C \qquad A \xrightarrow{1_A} A$$

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A
B

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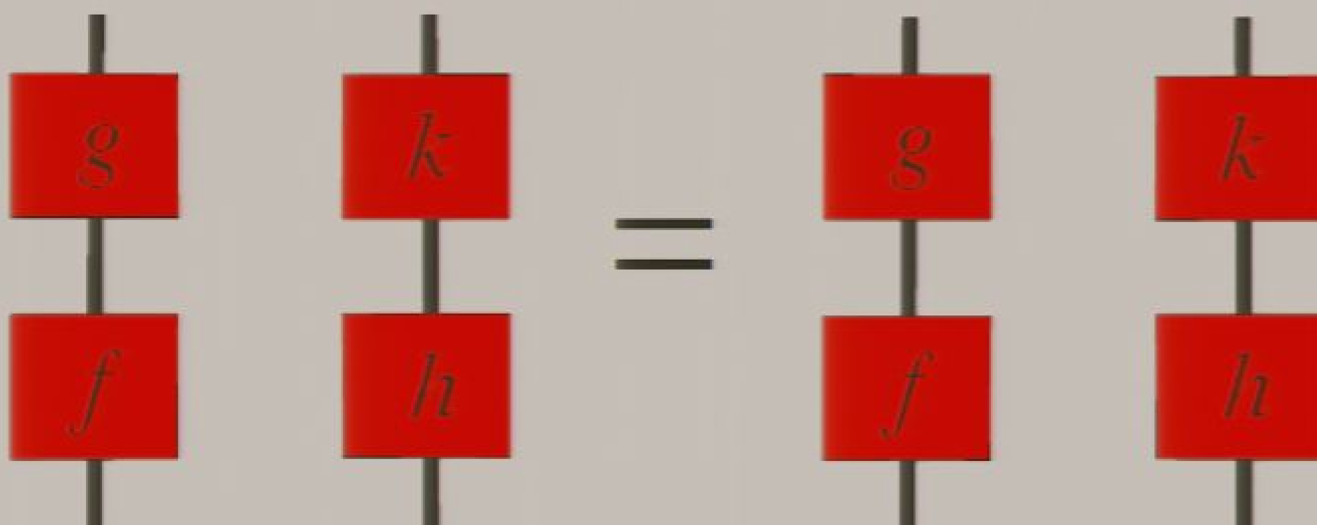

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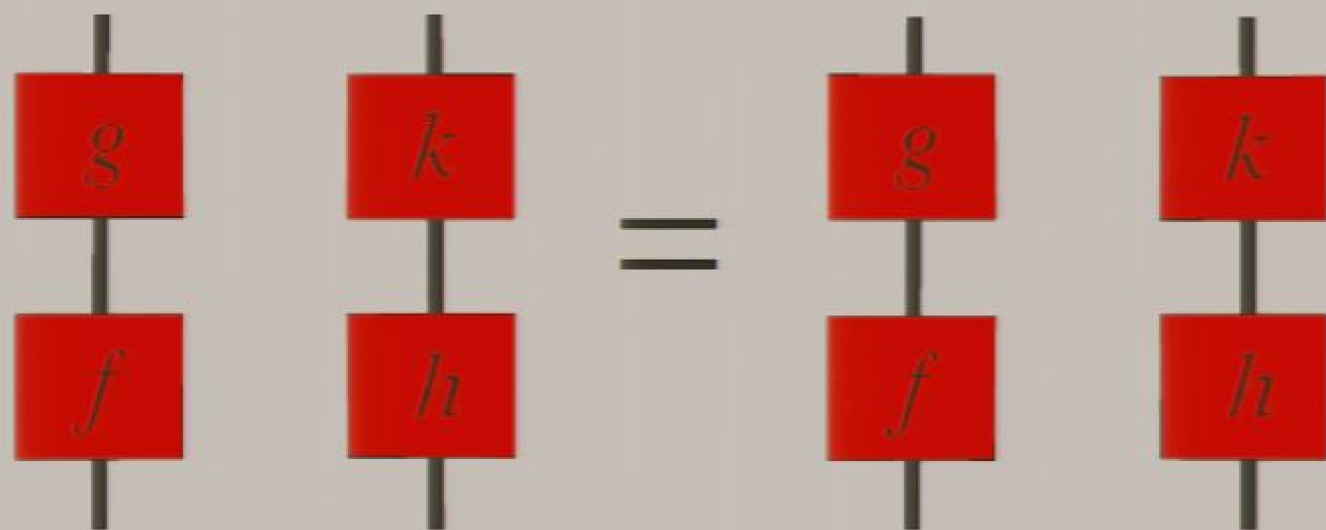

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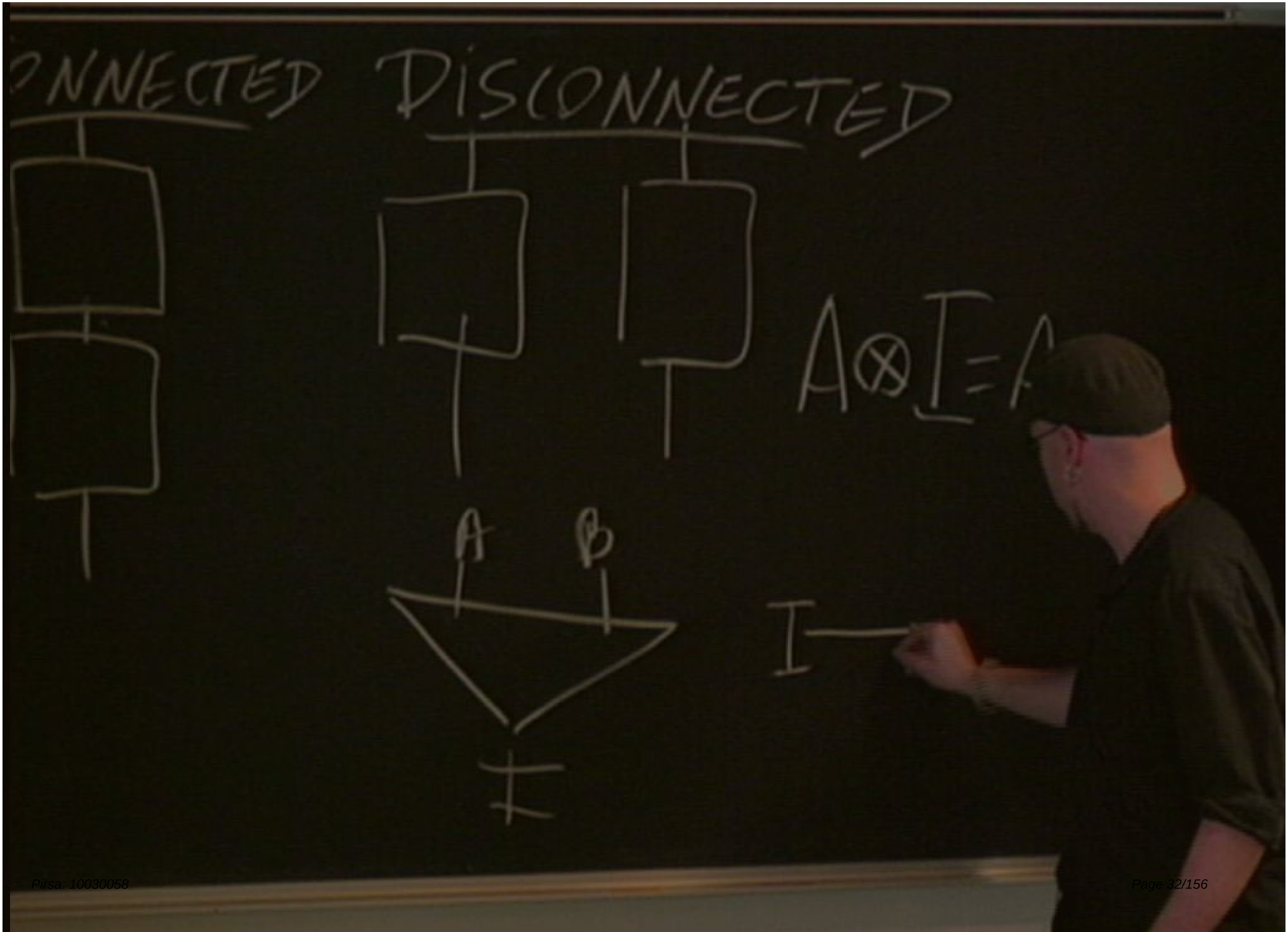
ONNECTED
DISCONNECTED
A⊗I=A
A
B
I
I

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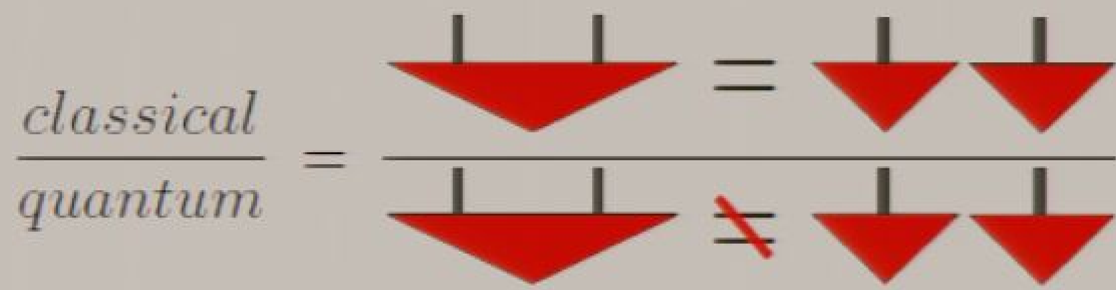
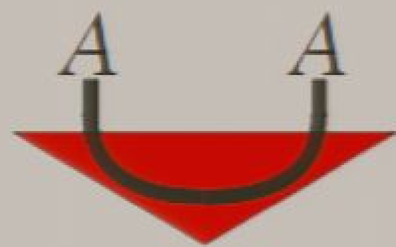
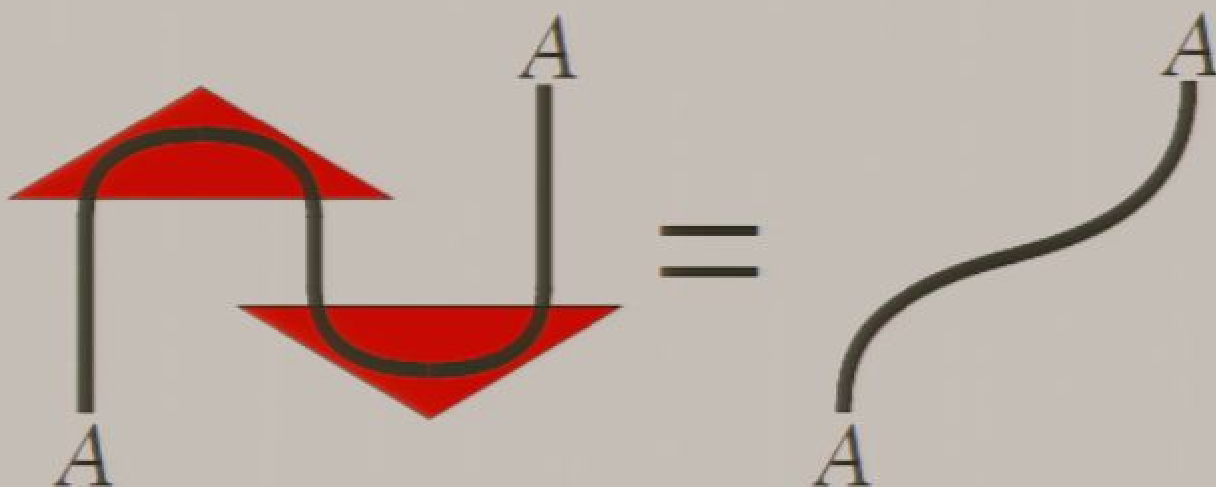
— (pure) Classical vs. Quantum —



— *quantum-like* —





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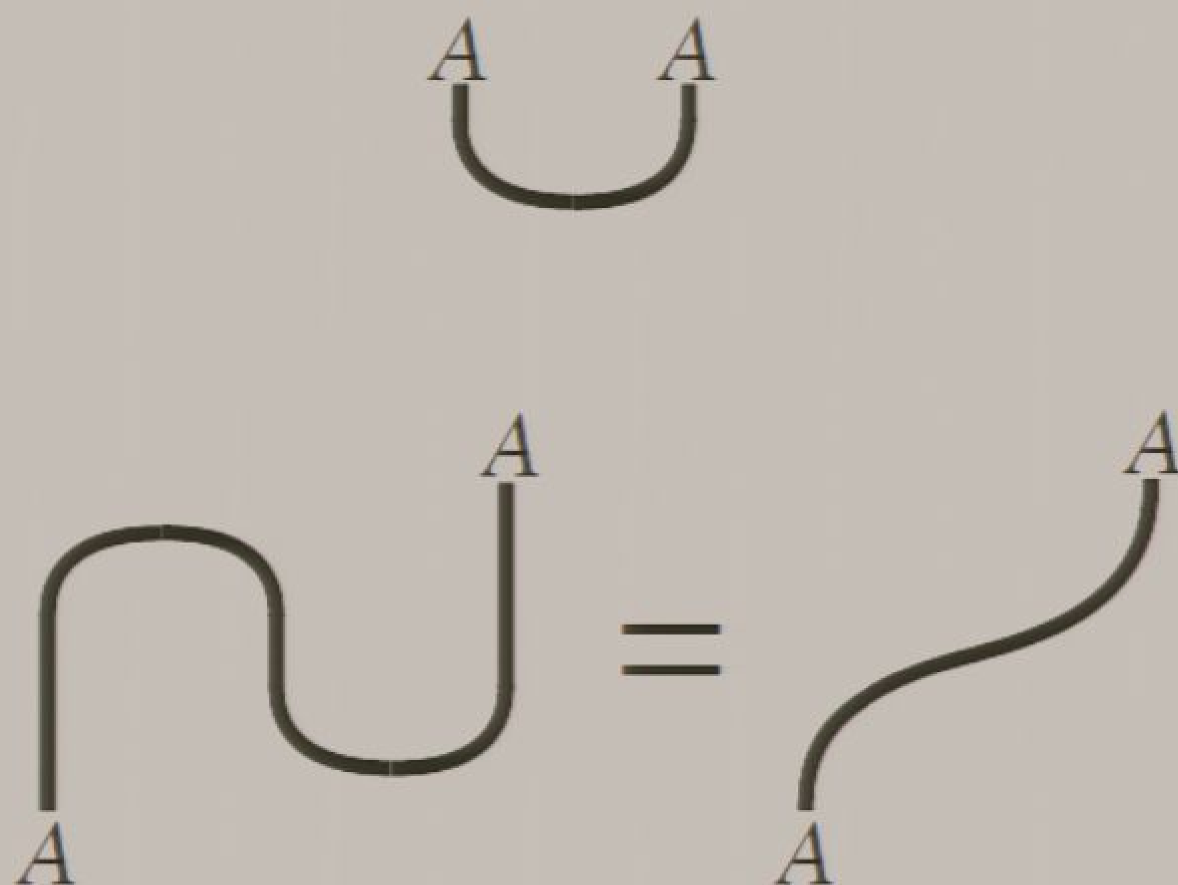

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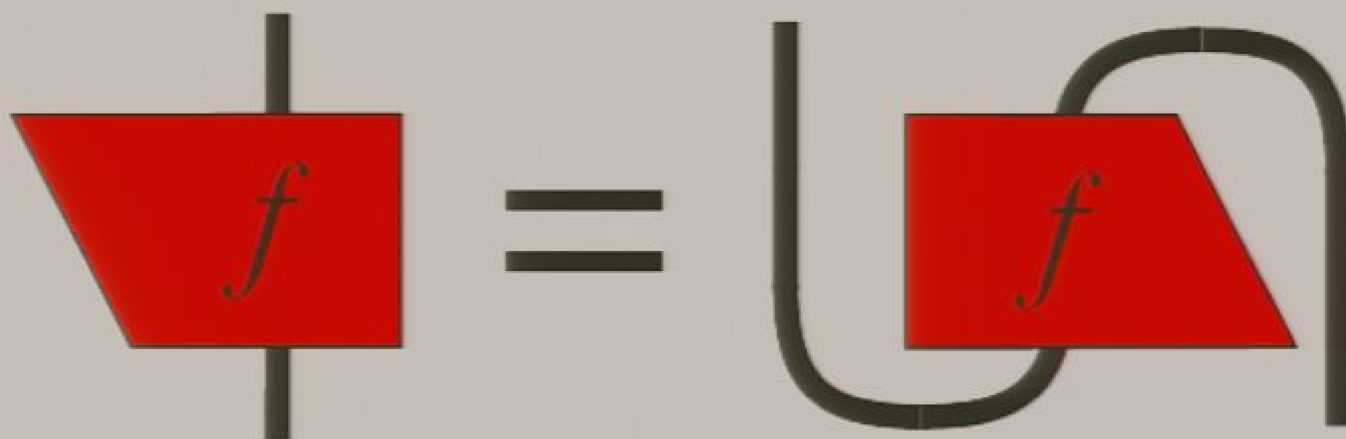

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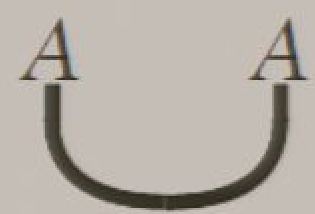

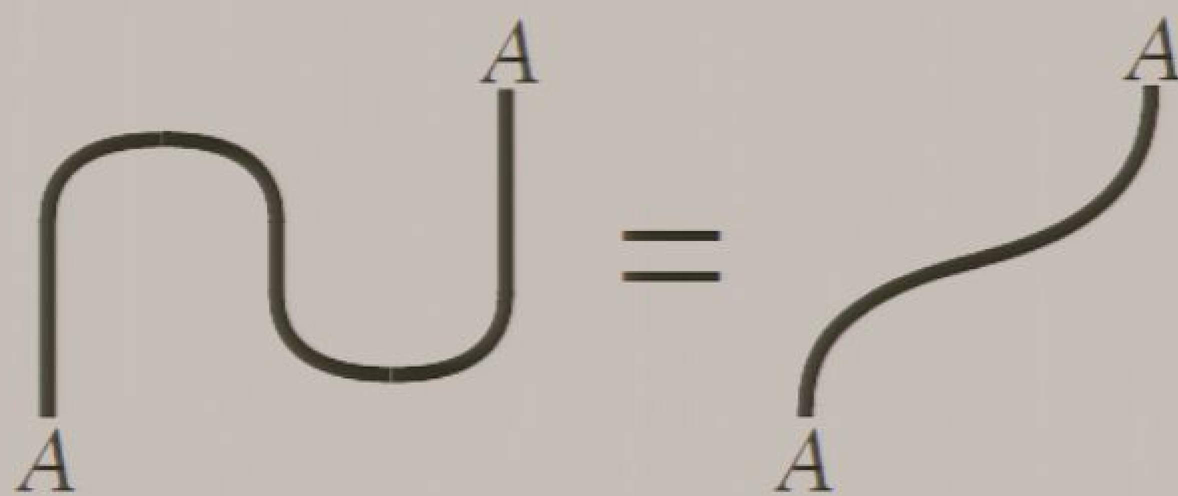

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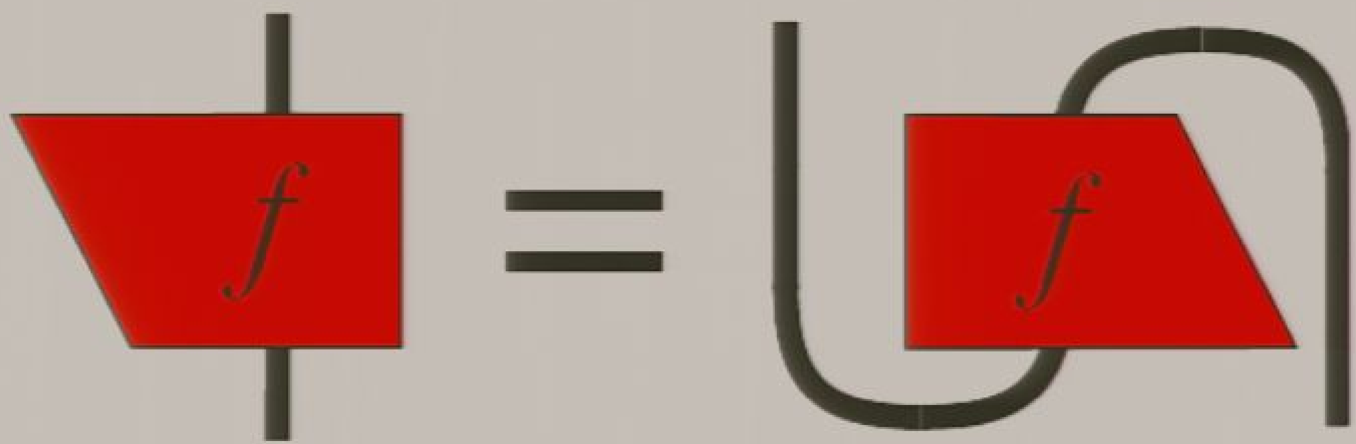
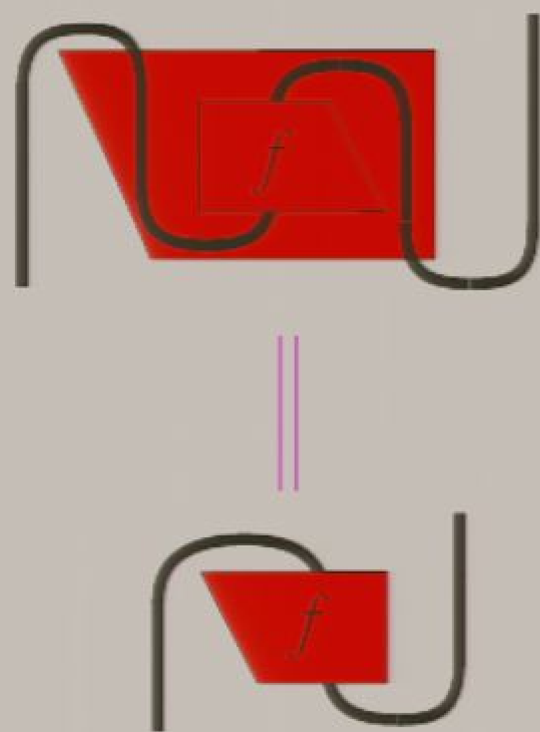
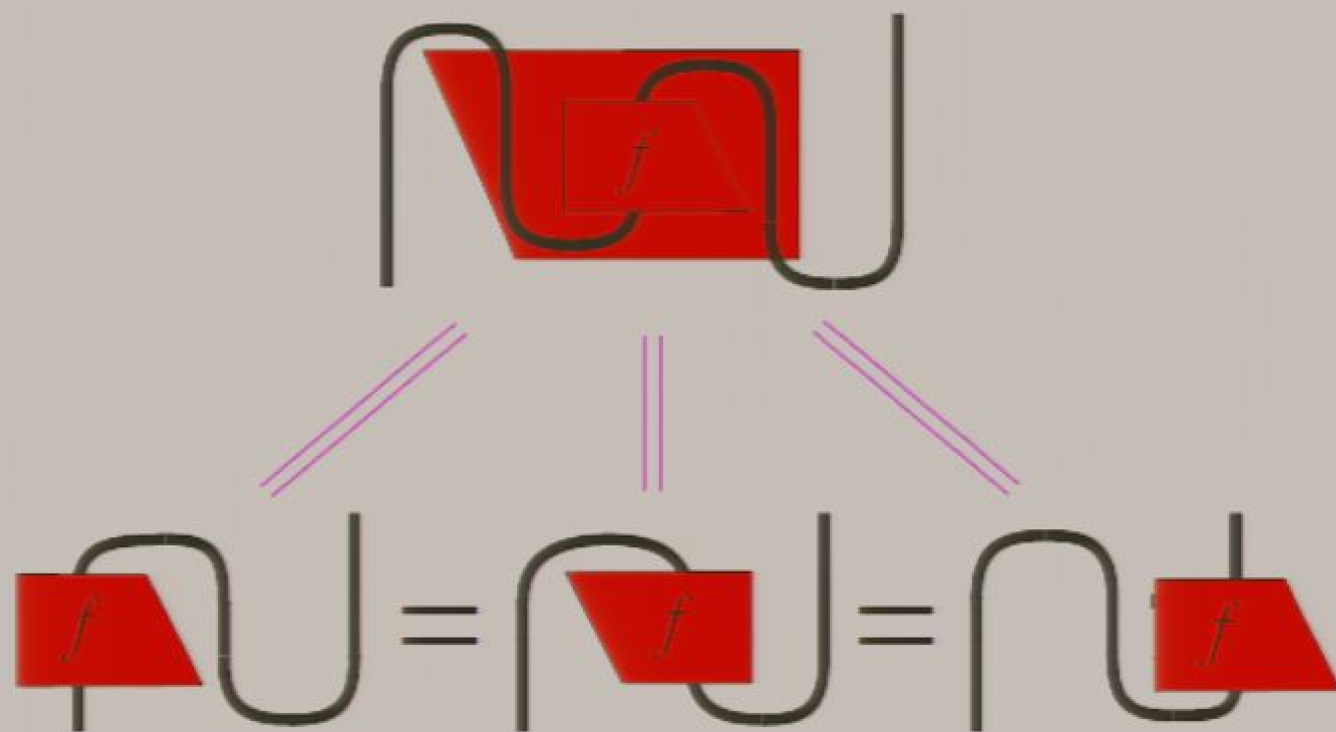


— *sliding* —

— *sliding* —

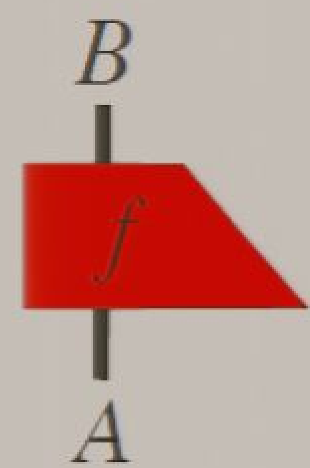


— *sliding* —

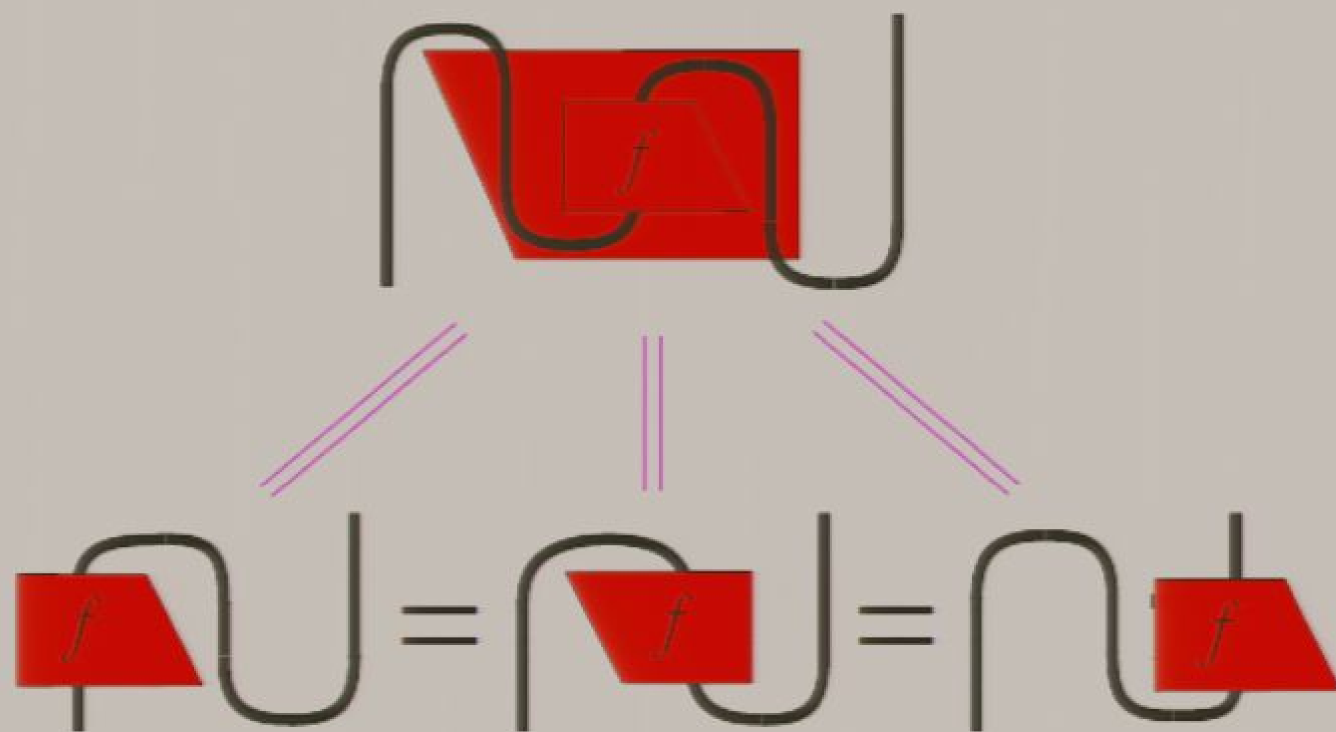


— dagger —

$$f : A \to B$$



— *sliding* —



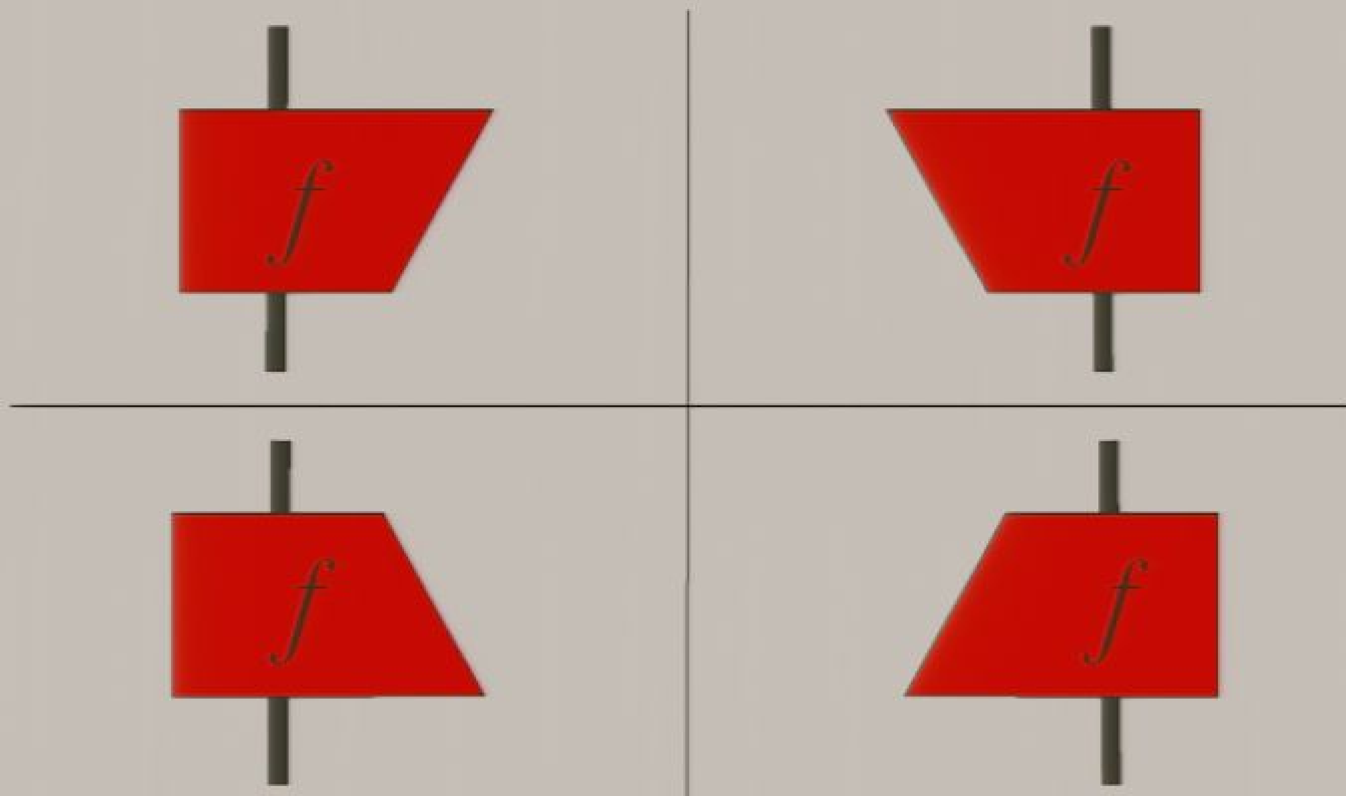
— dagger —

$$f : A \to B$$

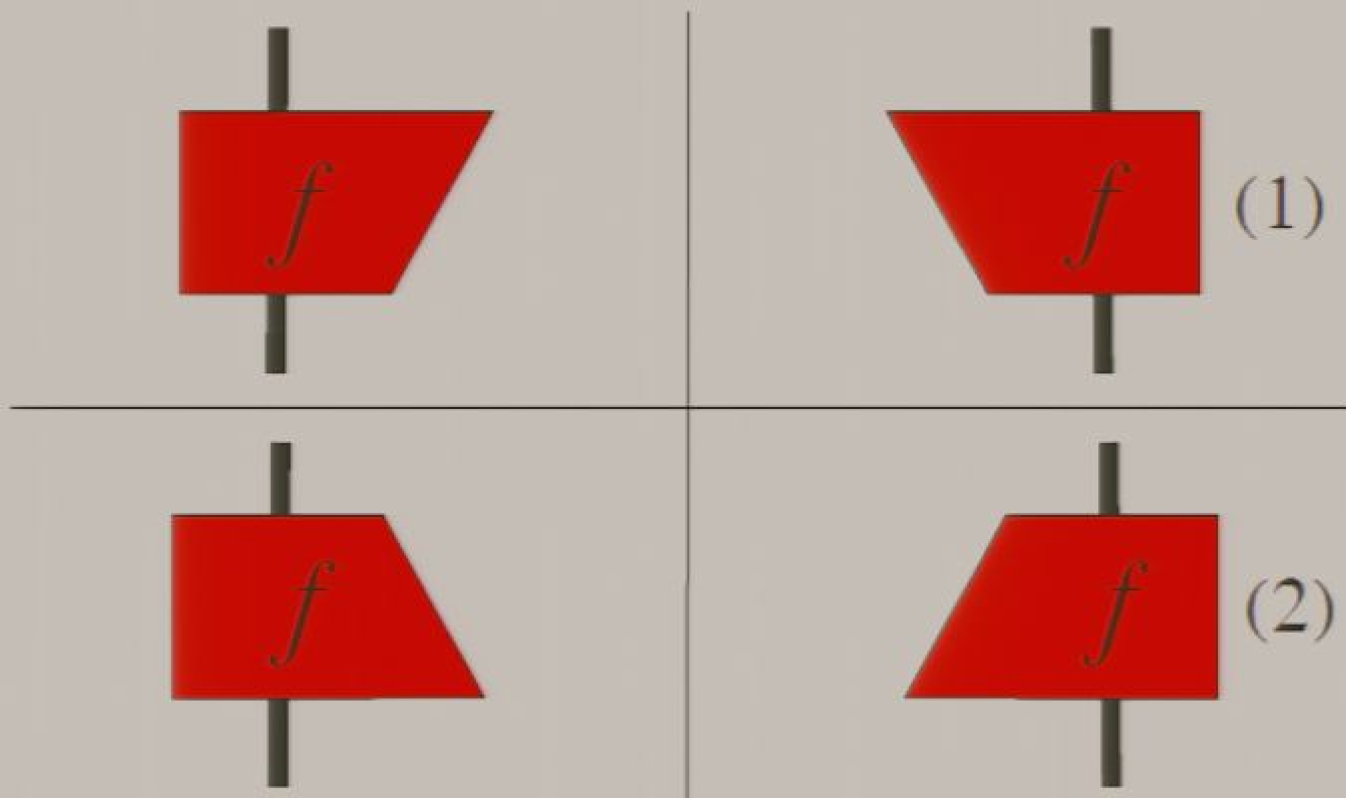
— dgger —

$$f^{\dagger} : B \to A$$

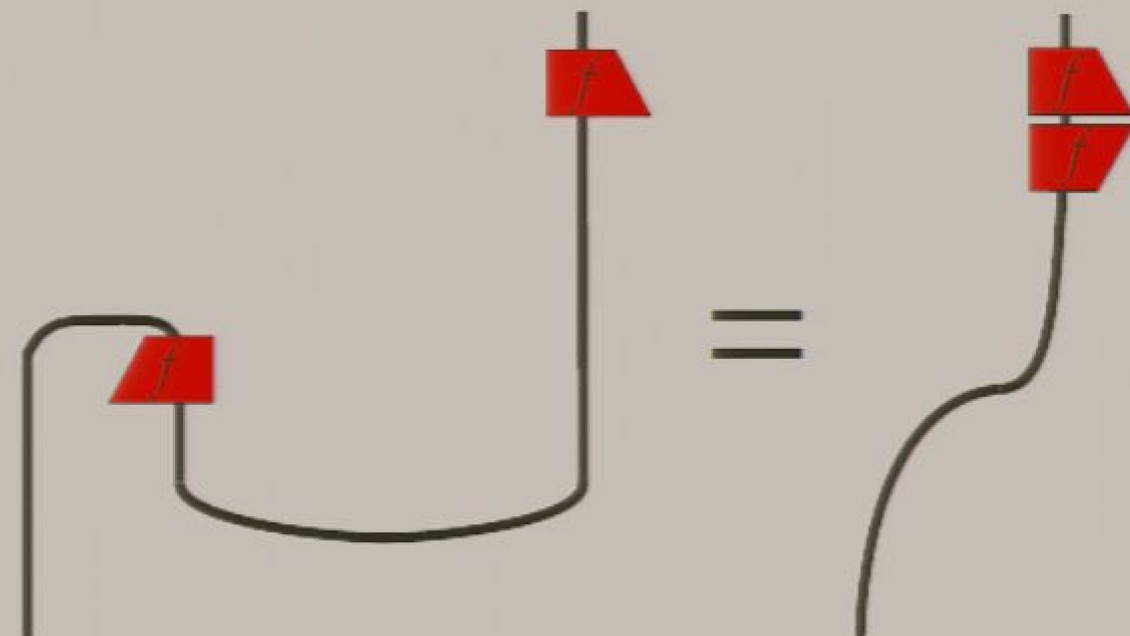
— *dagger* —

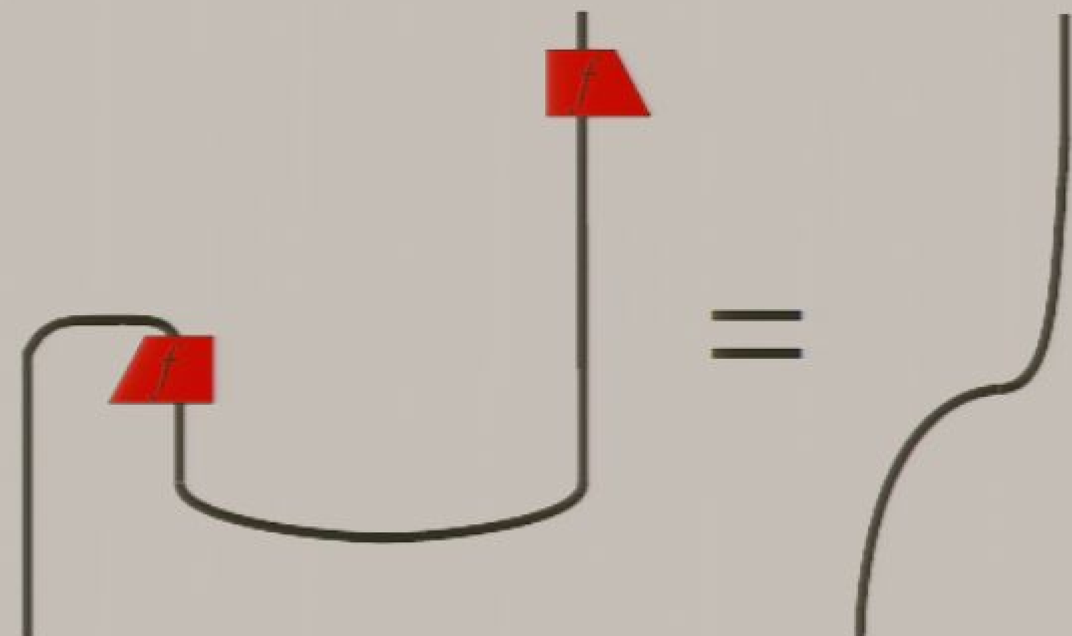


— *dagger* —



In <u>FHilb</u>: (1) ~ transpose & (2) ~ conjugate


f
f
f
=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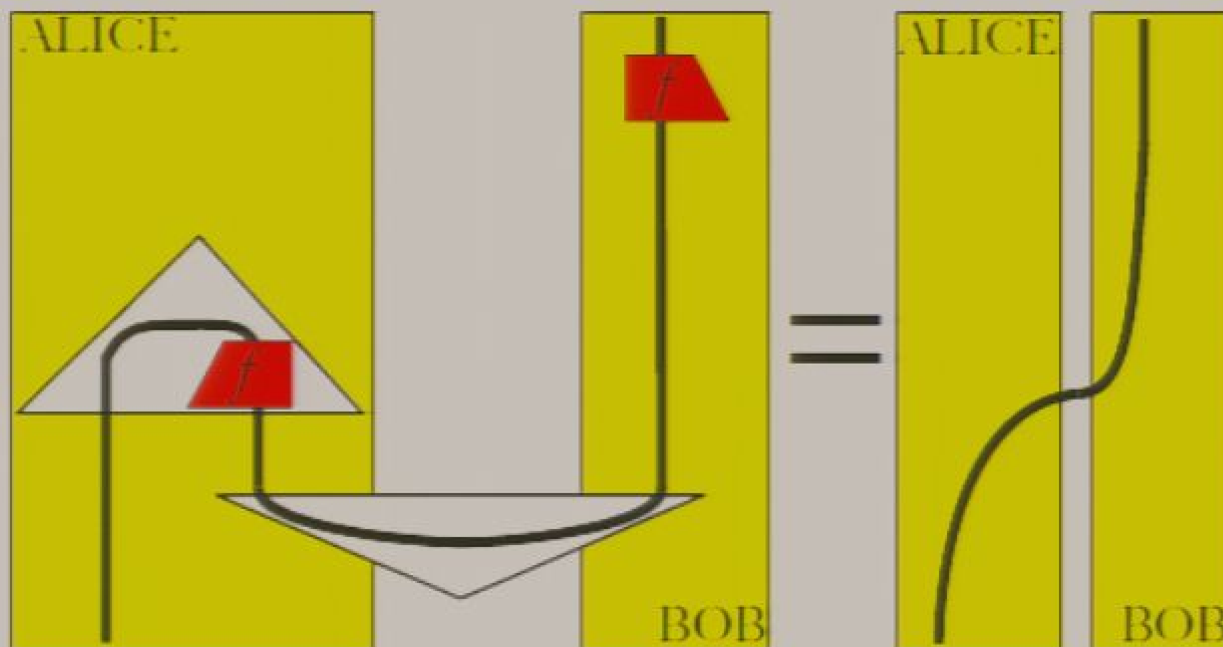
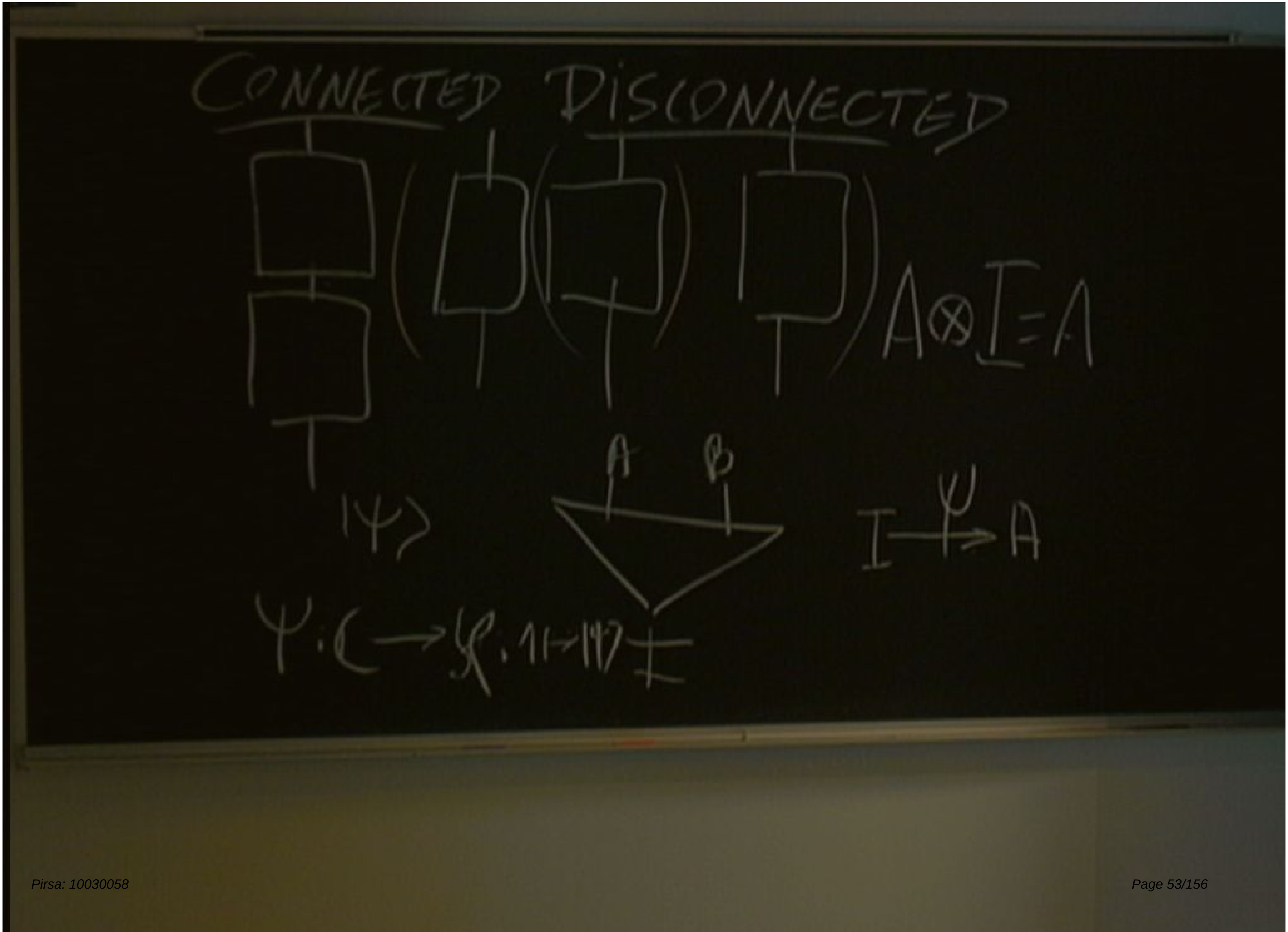


$\Rightarrow$ **quantum teleportation**

— *completeness* —

Thm. [Selinger '05] *An equational statement between expressions in dagger compact symmetric monoidal categorical language holds if and only if it is derivable in the graphical notation via homotopy.*


CONNECTED DISCONNECTED
A ⊗ I = A
|ψ⟩
A B
I → A
ψ

— *completeness* —

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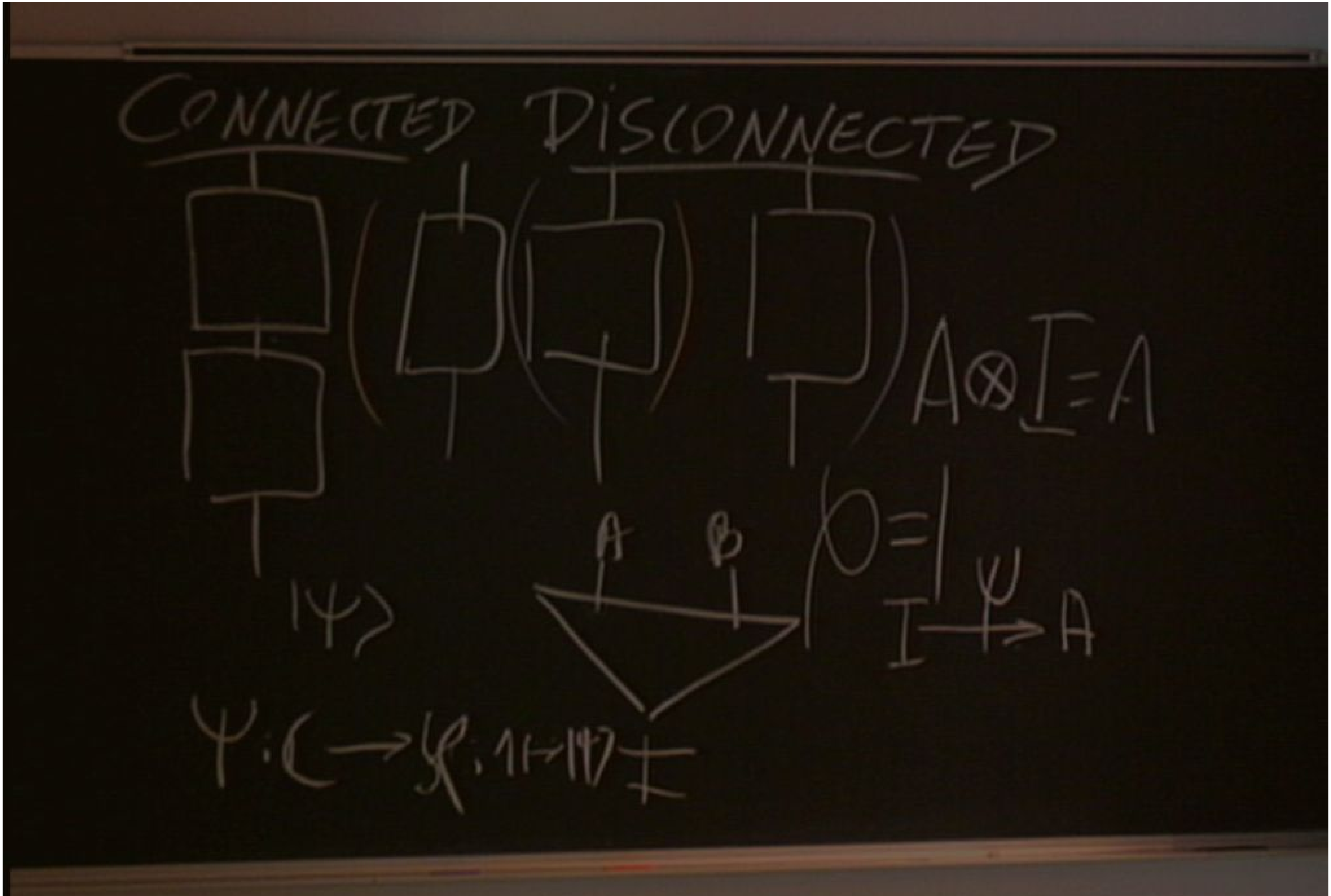
Thm. [Selinger '08] *An equational statement between expressions in dagger compact symmetric monoidal categorical language holds if and only if it is derivable for Hilbert spaces, linear maps, composition thereoff, Bell-states, tensor product, and adjoints.*


CONNECTED
DISCONNECTED
A⊗I=A
A
B
|ψ⟩
I
ψ
A

$$\text{CQM} \Rightarrow \text{FAME}$$

— *and Rock ’n Roll alike stardom* —

— and Rock 'n Roll alike stardom —

Gödel's Lost Letter and P=NP

a personal view of the theory of computation

Home About Me About P=NP and SAT Conventional Wisdom and P=NP My Wordle The Gödel Letter Cook's Paper Thank You Page

Logic Meets Complexity Theory

MARCH 21, 2010

by rjlipton

tags: Algorithms, circuits, Factoring, graphs, P=NP Problems, Proofs, SAT, time, Turing

A summary of the 2010 ASL conference

Bob Coecke is a lecturer from Oxford, who gave a series of three invited talks on quantum information theory at the **2010 ASL meeting.** He is a terrific speaker, and has a way of using pictures to give the best explanations I have ever seen—or heard—of complex quantum concepts such as teleportation.

MOST USED TAGS

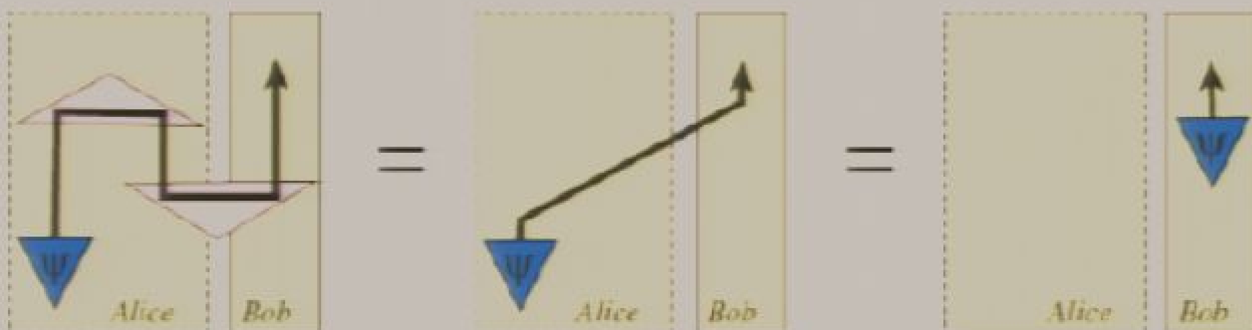
Algorithms approximation award boolean breaking crypto-systems circuits complexity crypto-systems decision procedure deterministic diagonalize Factoring FOCS formula Hilbert language lower bounds Machine matrix nondeterminism P=NP polynomial Problems Proof random randomness SAT Simulation space

... and the privileges thereof ...

— *and Rock 'n Roll alike stardom* —

A Picture is Worth a Thousand Qubits

Coecke uses pictures of a special kind to explain quantum information processing. This, in his hands, is in my opinion extremely insightful, and helped me really "get it." I thought I would try to give an over view of what he does with his wonderful pictures. If you want to see the real thing go **here** for the full story. He uses pictures like these to explain quantum teleportation and other operations:



... and the privileges thereof ...

— *and Rock ’n Roll alike stardom* —

If any of you discover a caveman drawing looking like this: we have a problem.

... and the privileges thereof ...

— and Rock 'n Roll alike stardom —

... and the privileges thereof ...

— !!! BUT !!! —

A Picture is Worth a Thousand Qubits

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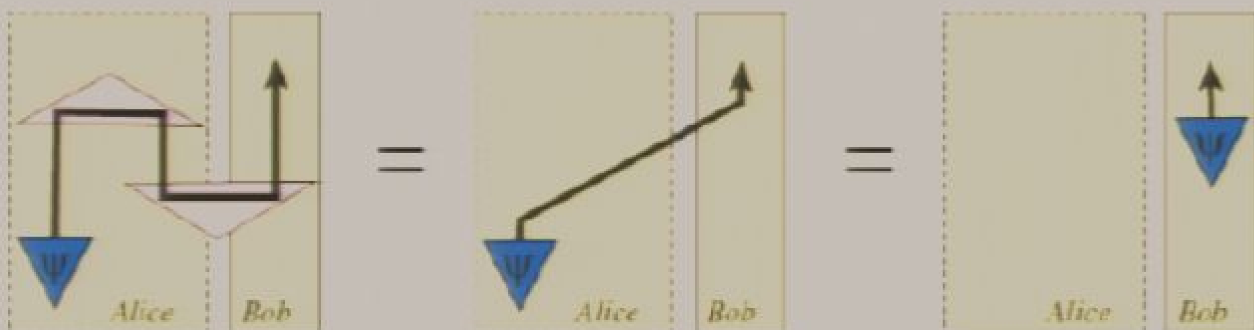

— !!! BUT !!! —

An 'ill-founded' lesson from categorical quantum stuff:

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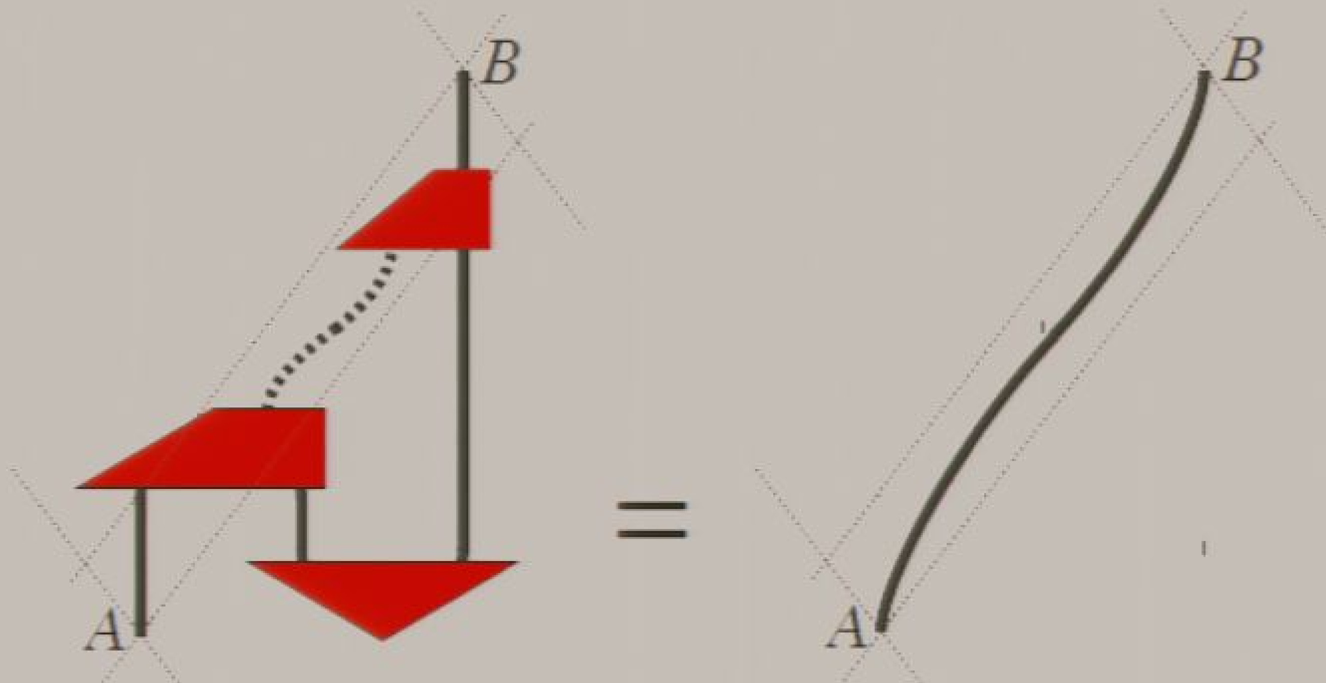
An 'ill-founded' lesson from categorical quantum stuff:

Reason: Postselection**!**

— !!! BUT !!! —

QM-relativity compatibility requires 'total' processes:



— !!! BUT !!! —

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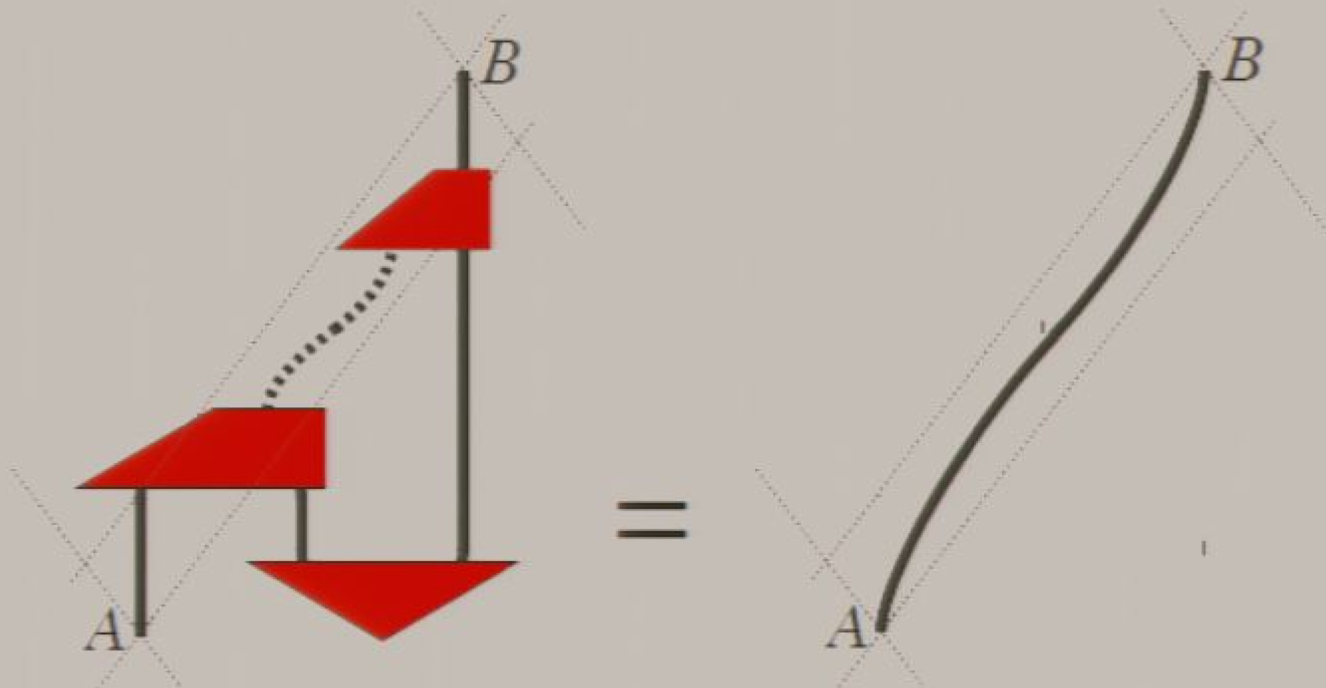

⇒ 'indexing' over dSCFAs (which represent bases)

— DISCLAIMER —

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The physical processes universe is NOT a dagger compact symm. monoidal category!

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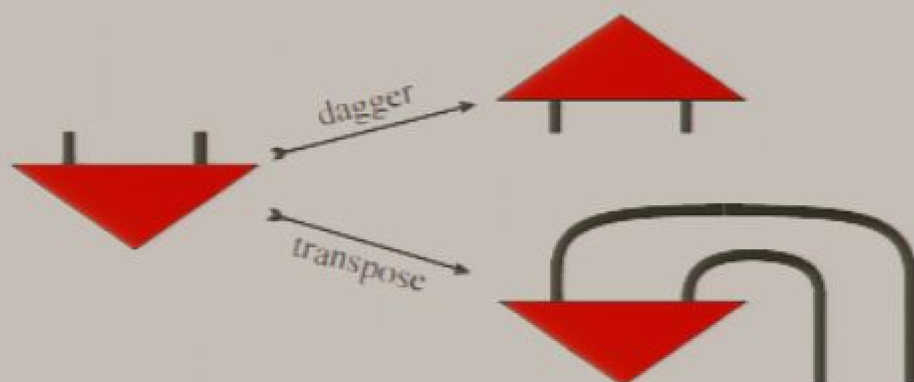
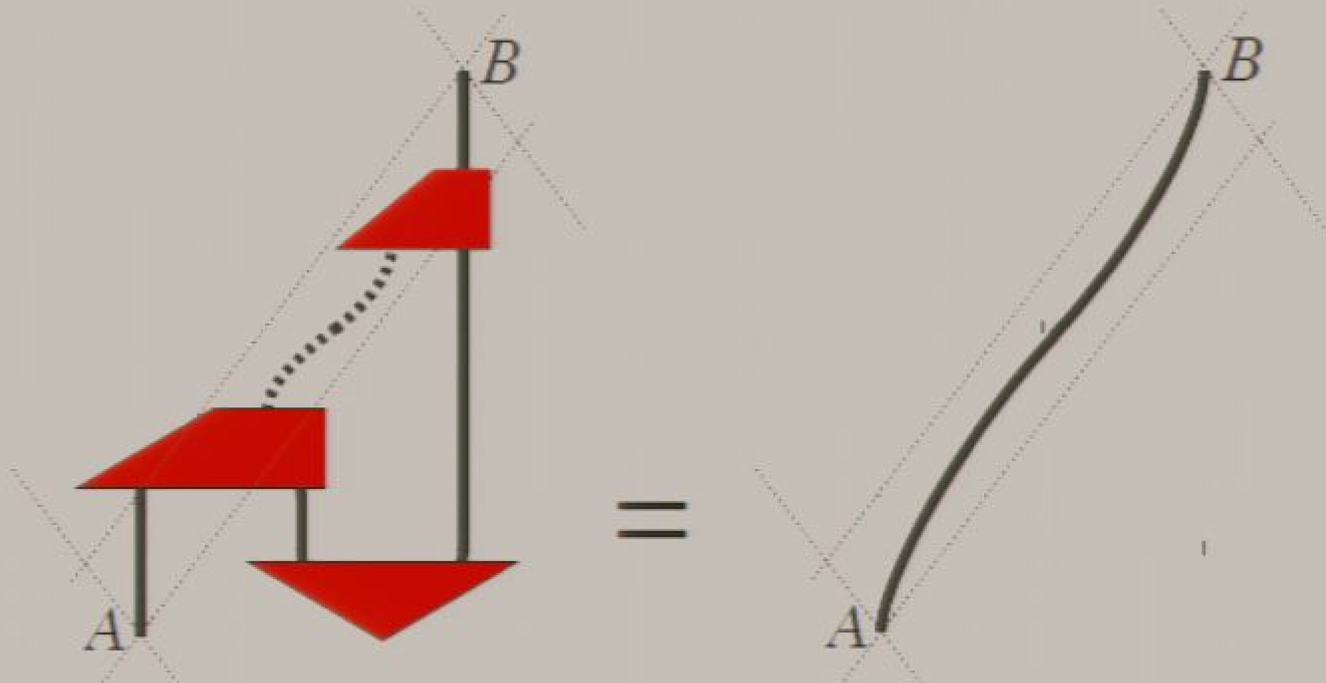
Evidence:

- Neither the dagger nor the transpose of a 'total' physical process is a physical process e.g.:



— !!! BUT !!! —

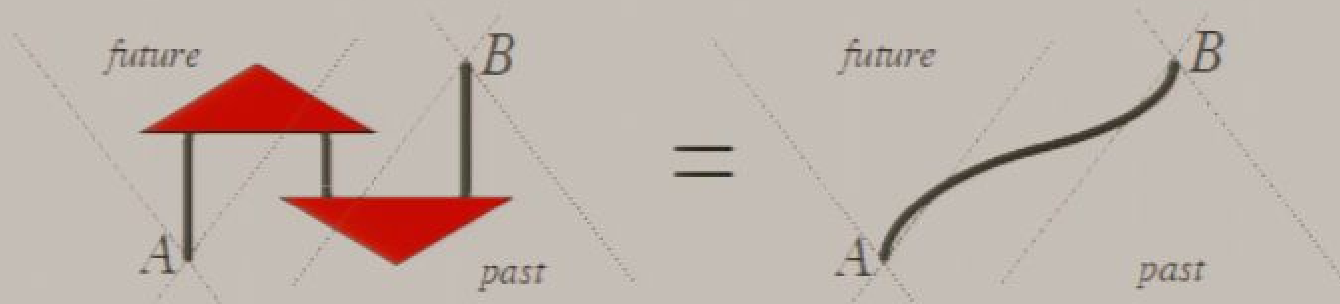
QM-relativity compatibility requires 'total' processes:



⇒ 'indexing' over dSCFAs (which represent bases)

— !!! BUT !!! —

An 'ill-founded' lesson from categorical quantum stuff:



Reason: Postselection**!**

— DISCLAIMER —

The physical processes universe is NOT a dagger compact symm. monoidal category!

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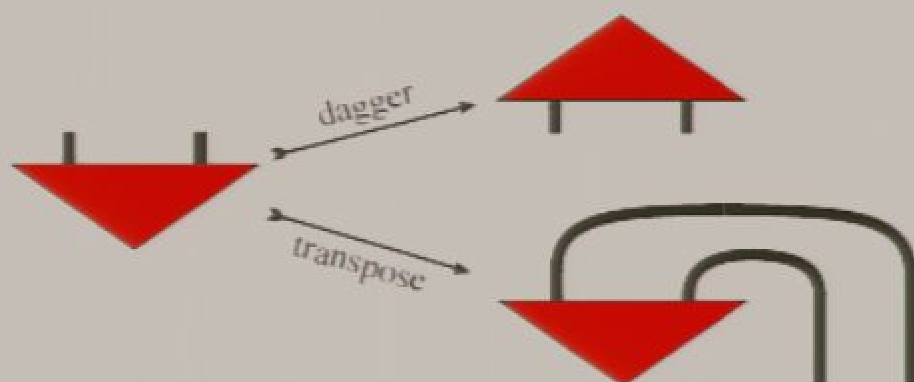

CAUSAL CATEGORIES

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- Strategy: encode 'causal set'-structure within category, yielding the concept of a **causal category**.

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$$\text{from } A \text{ we can derive } B \iff A \leq B\,.$$

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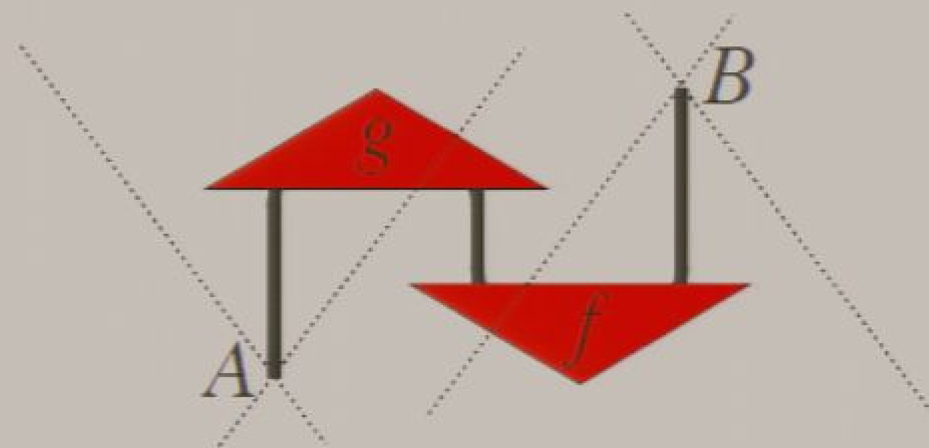

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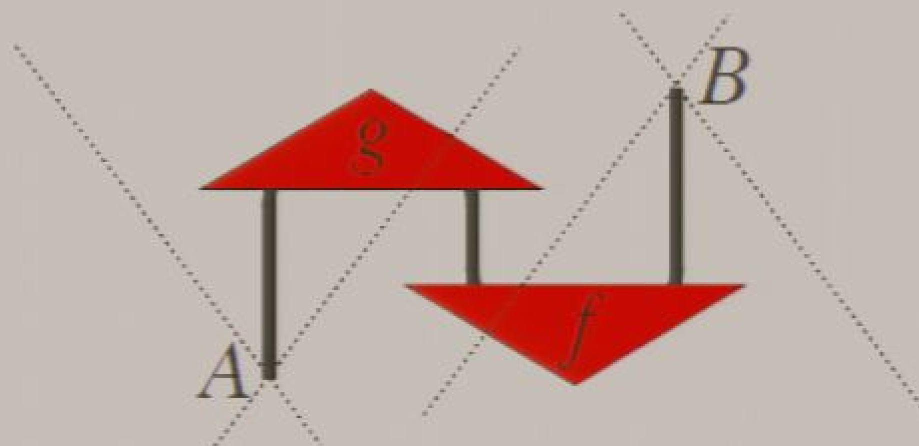

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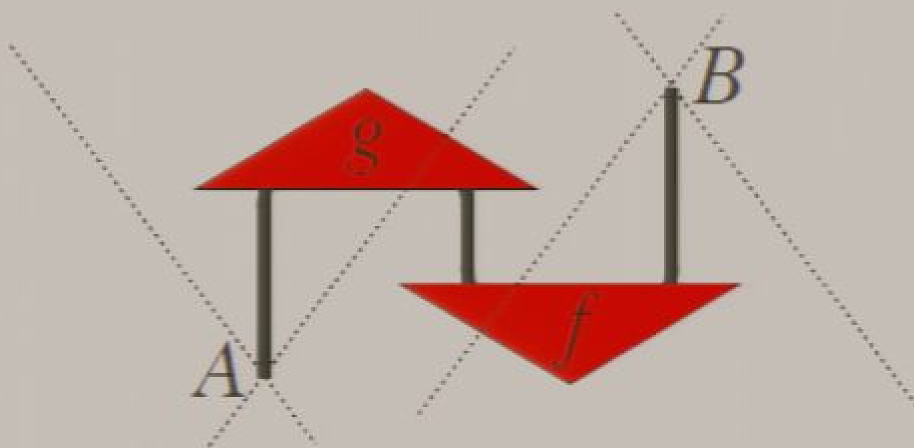

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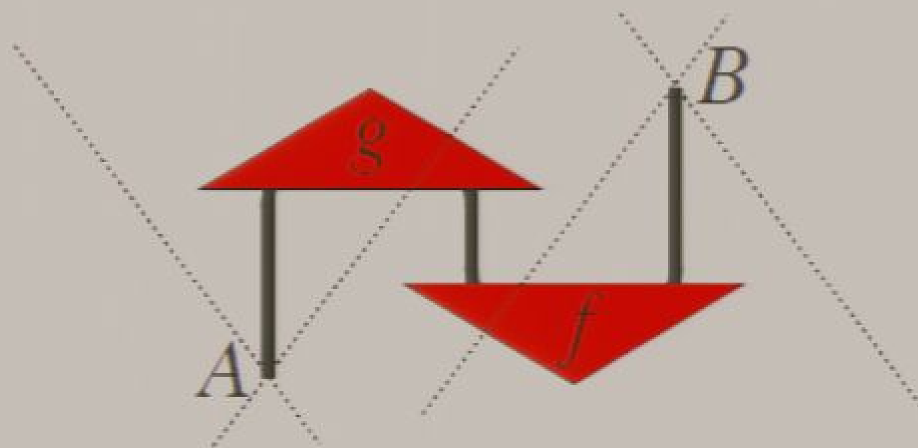

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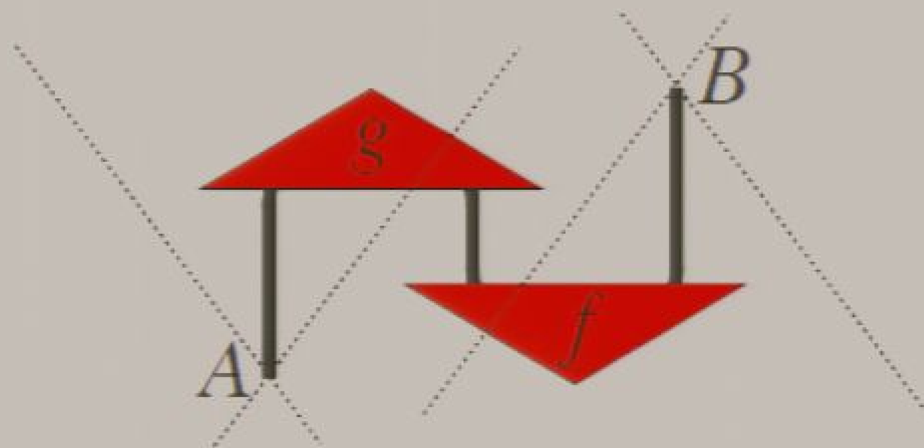

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Not existence but information flow is the key!

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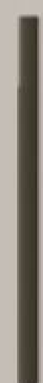	

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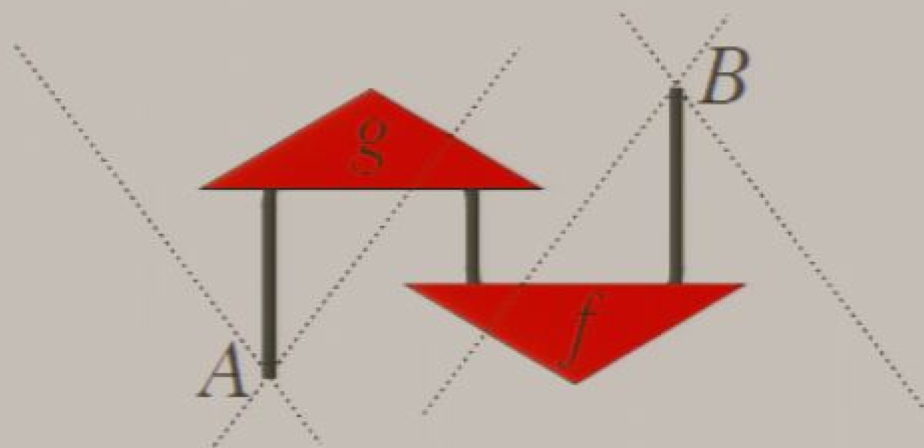

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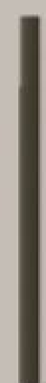	

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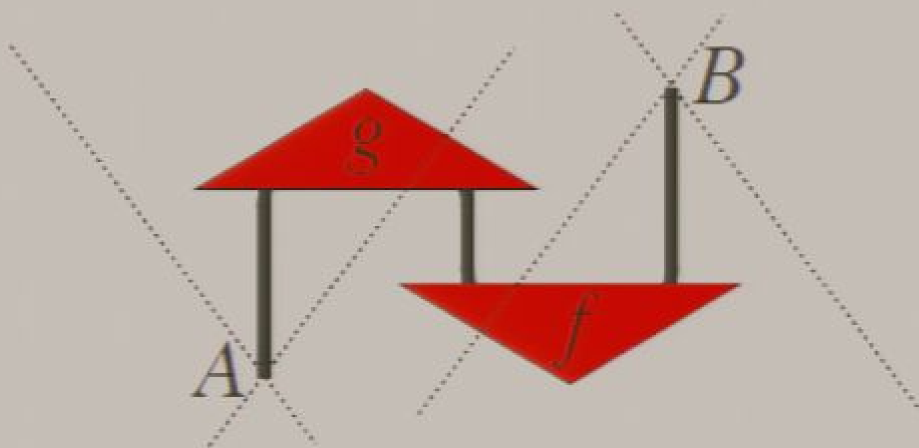

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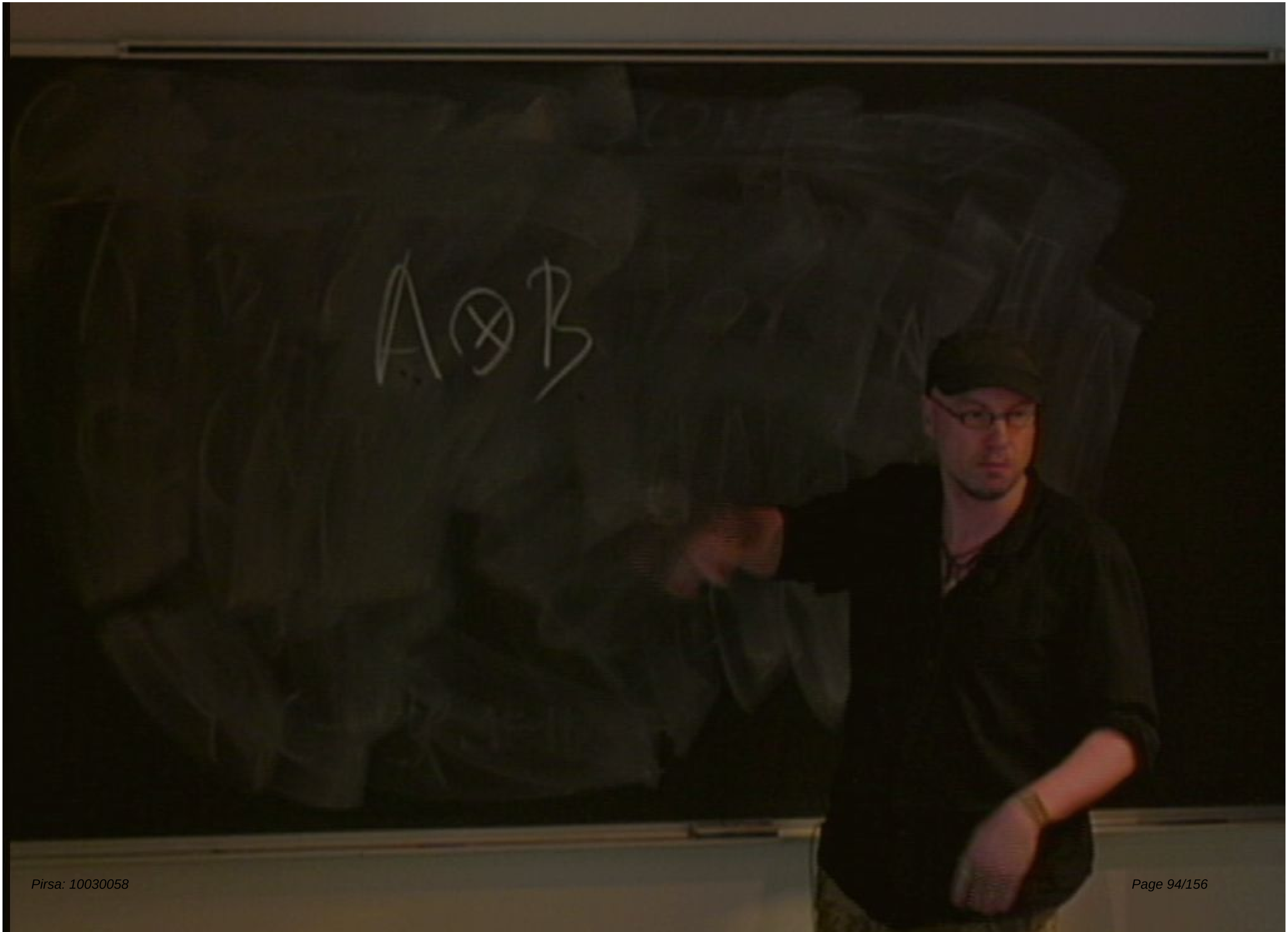
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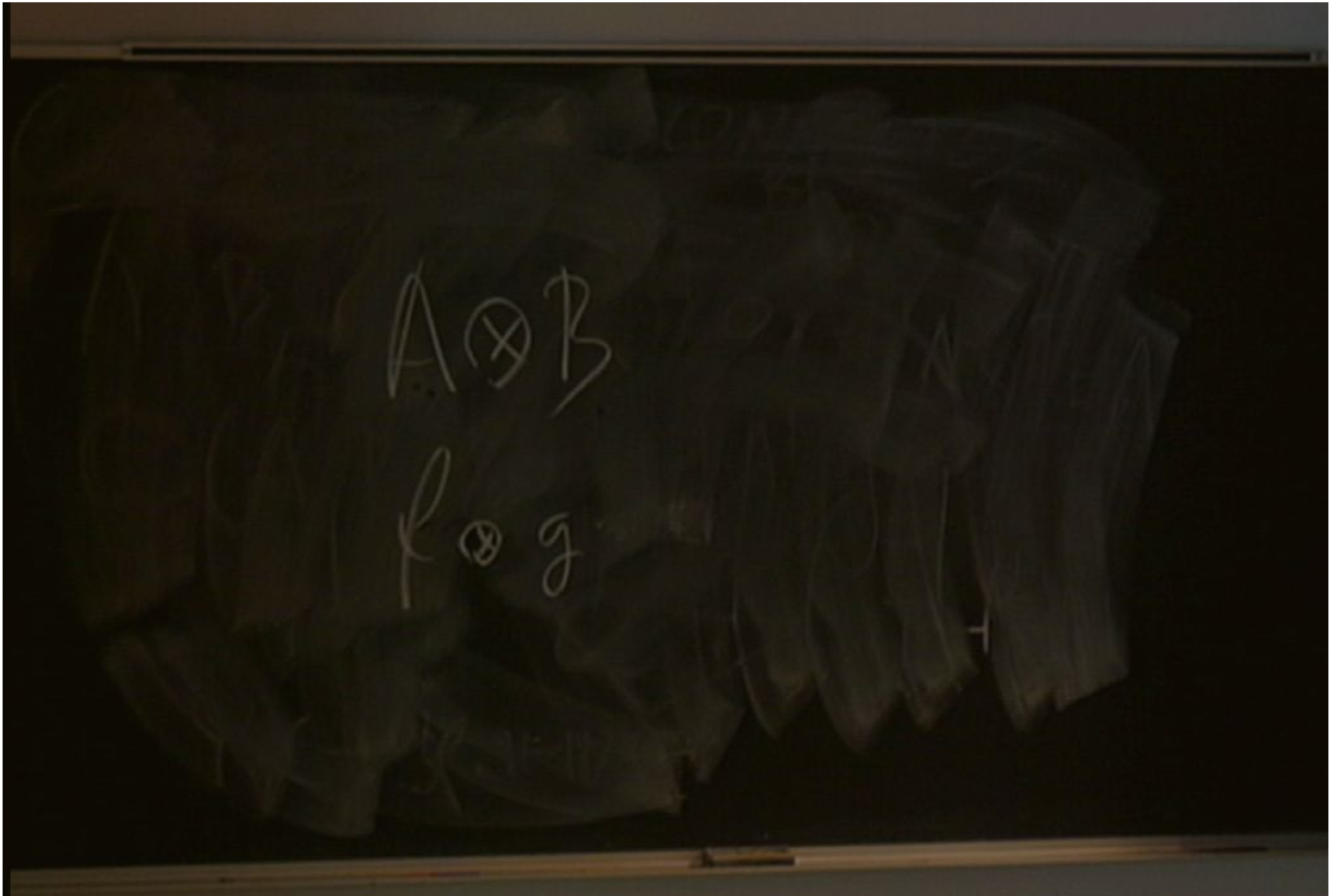
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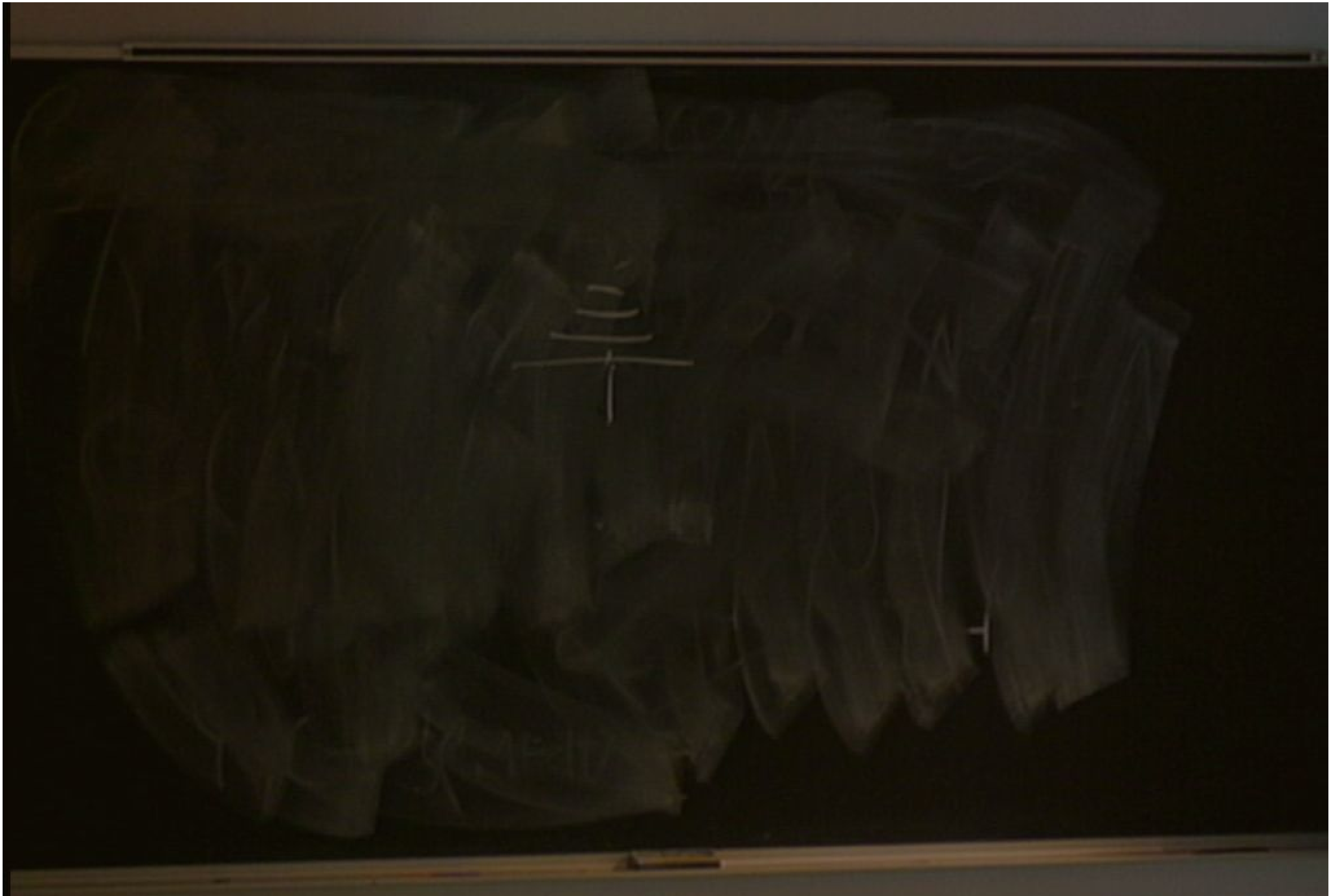

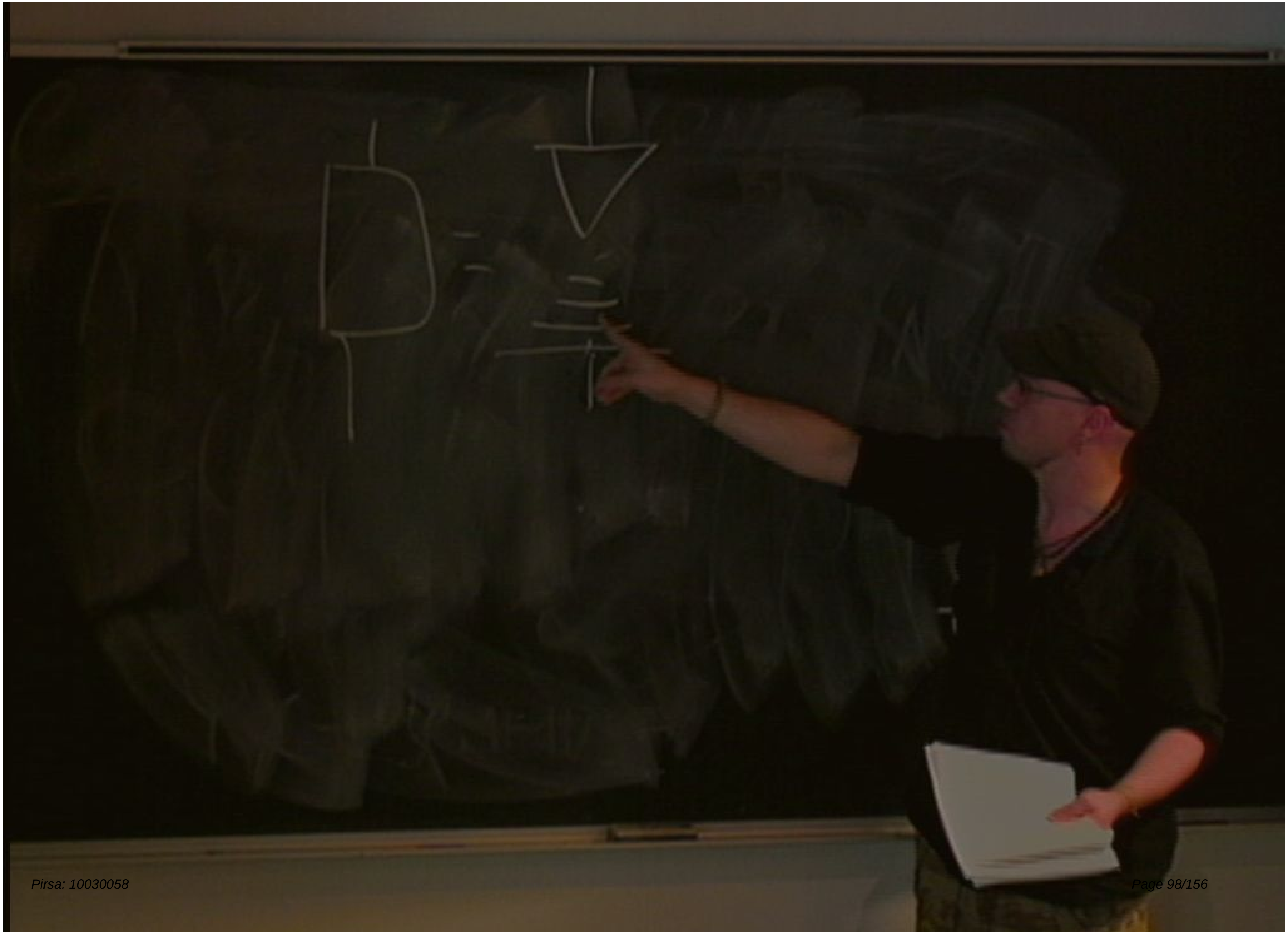

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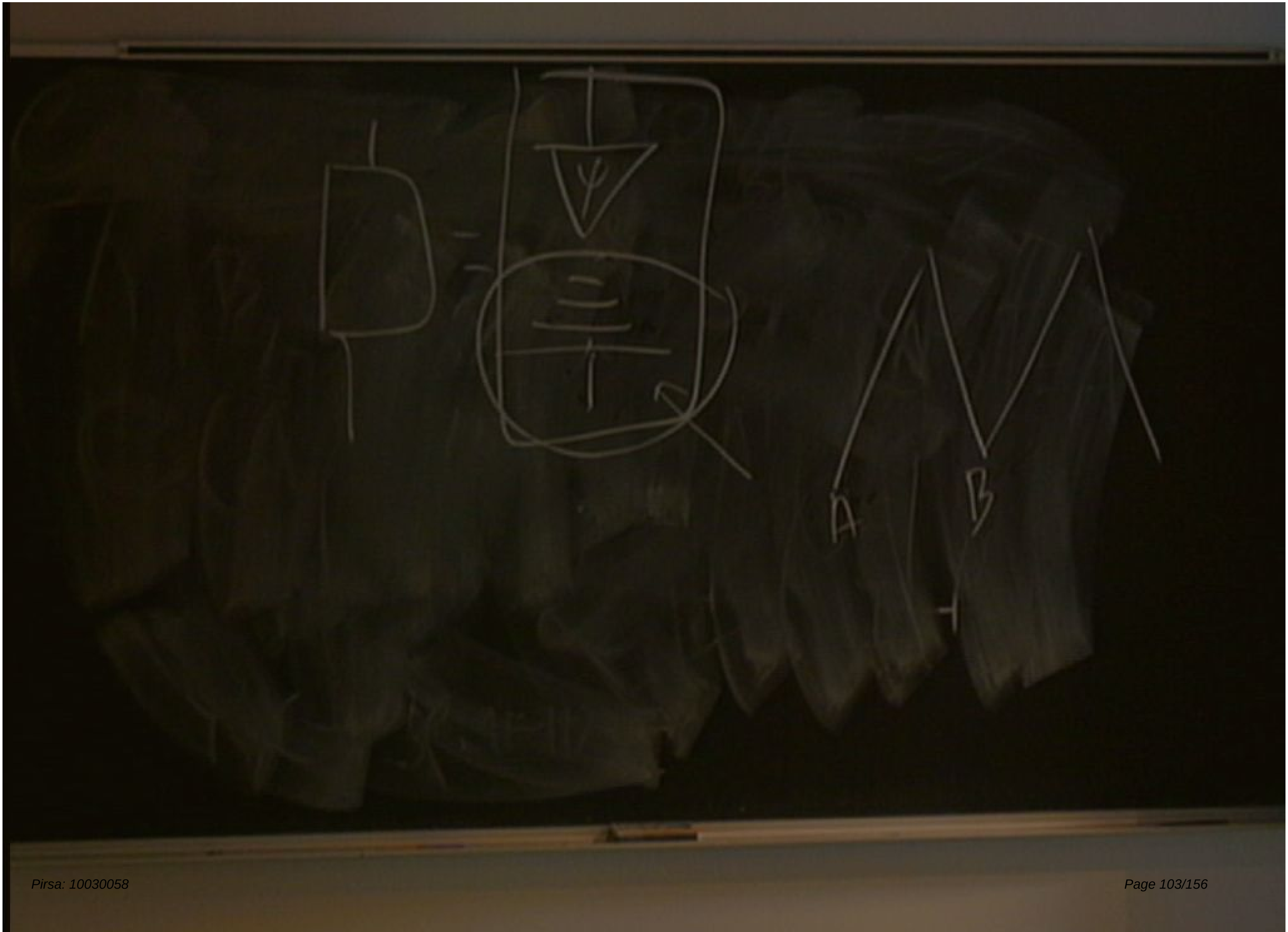
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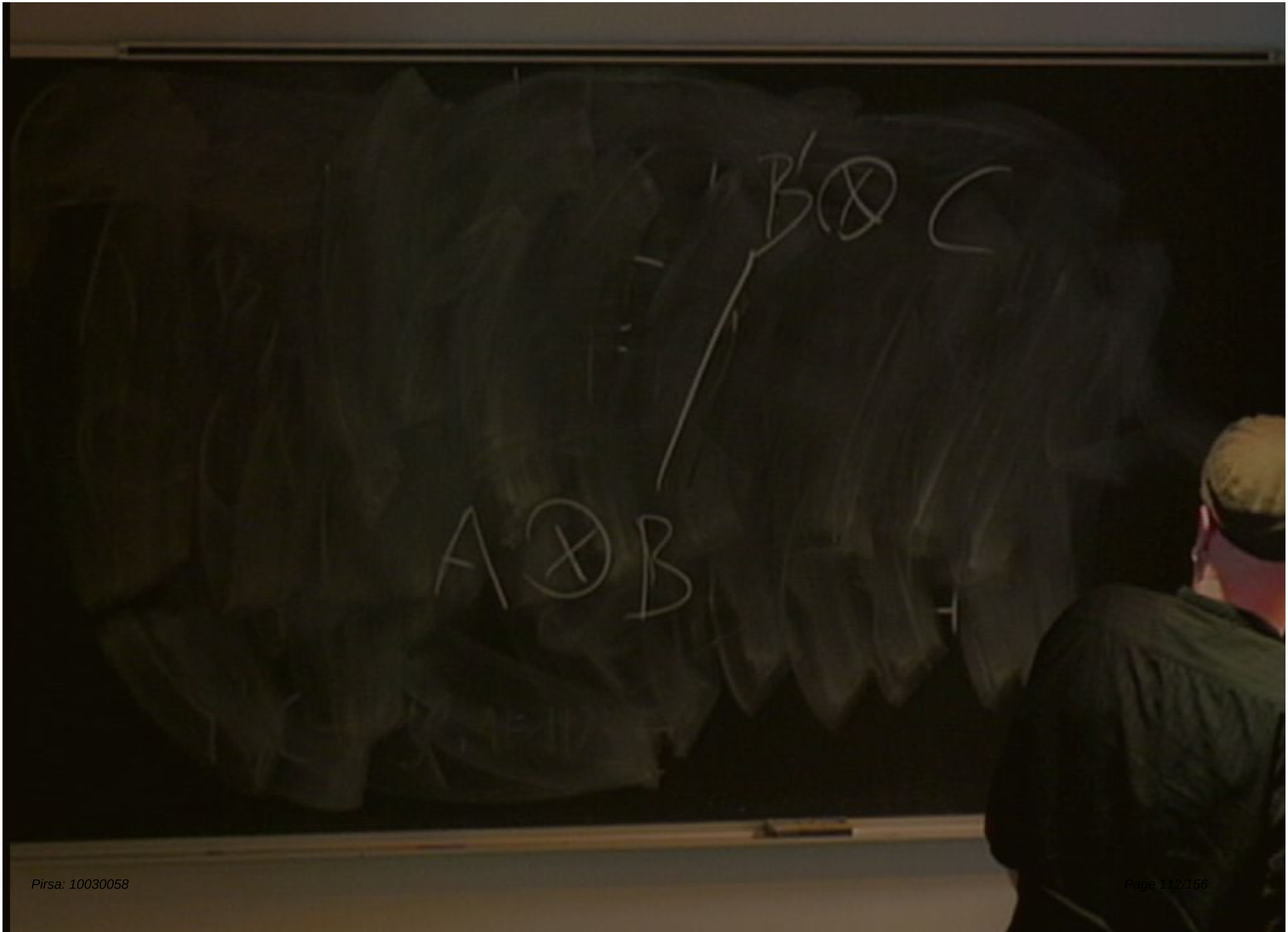
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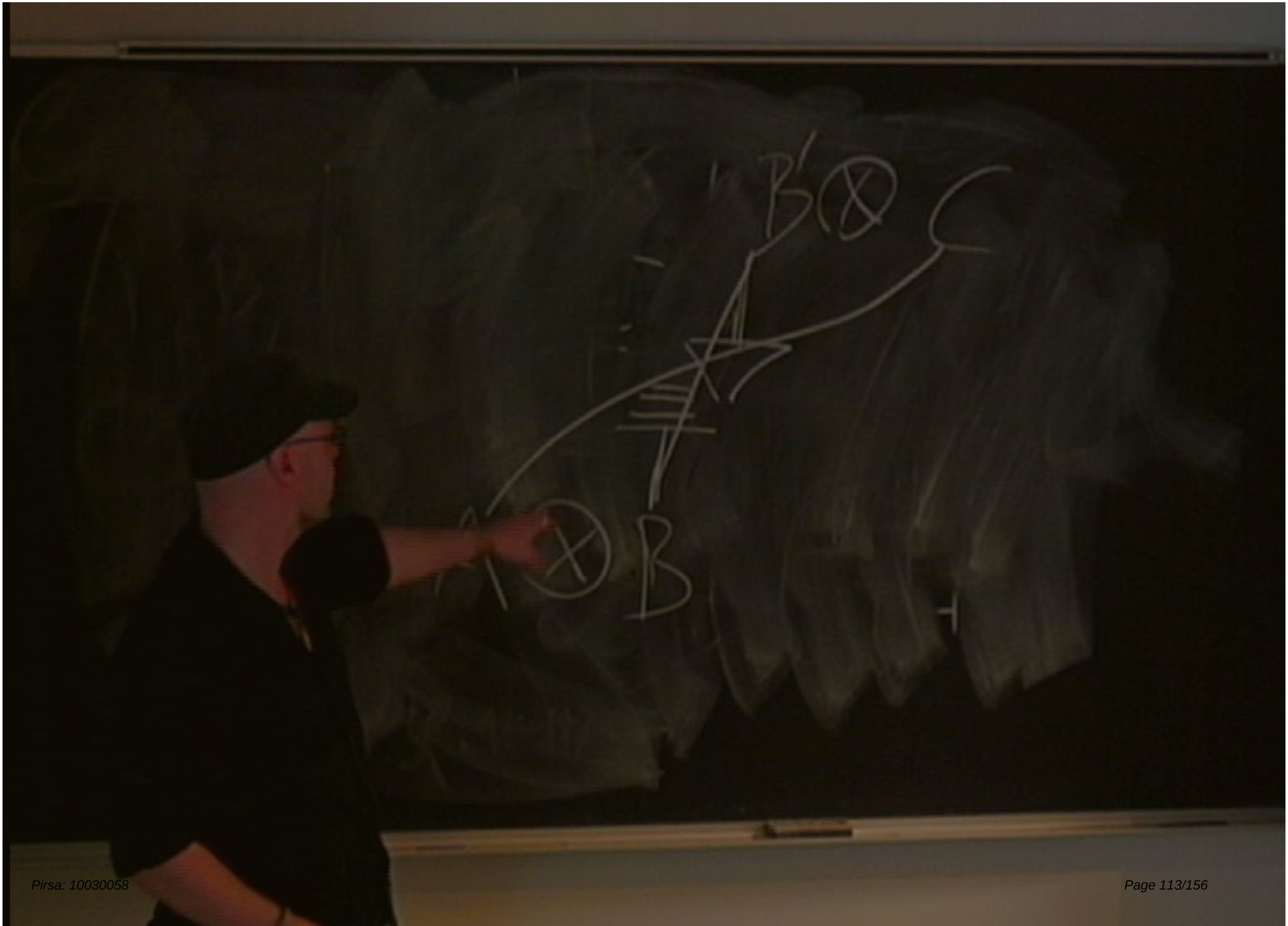
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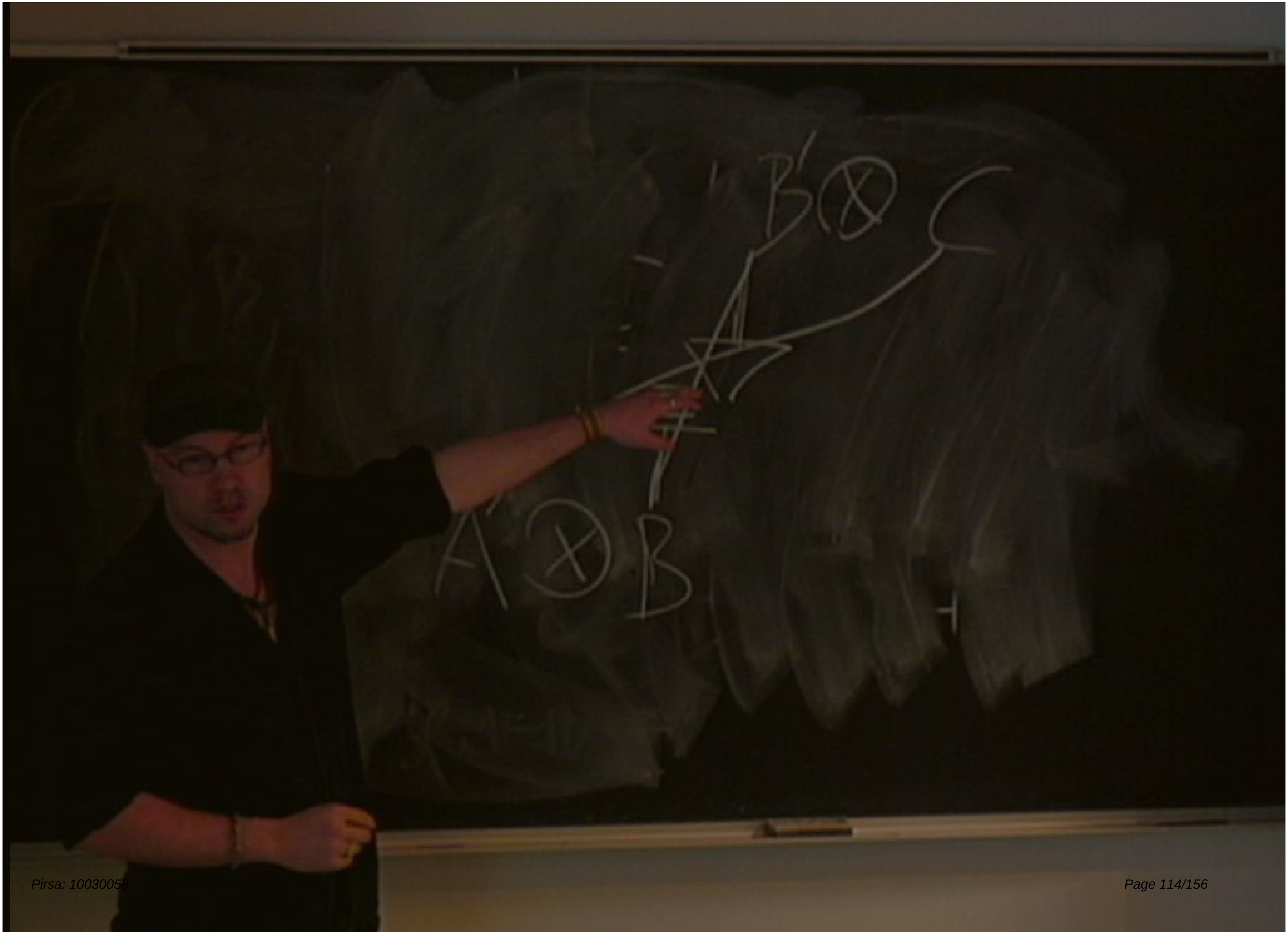
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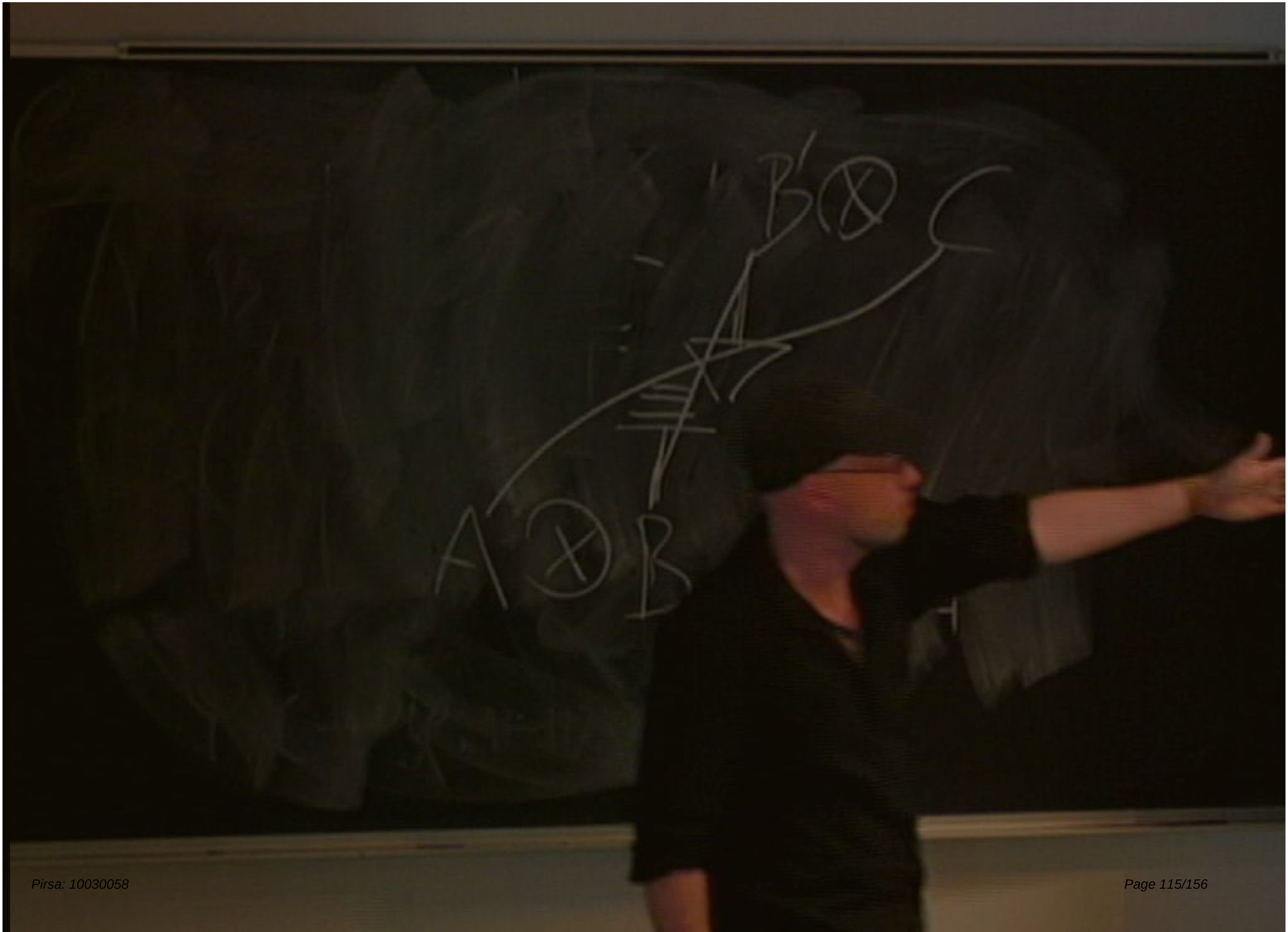
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Directed graph $\mathcal{G}$ with:

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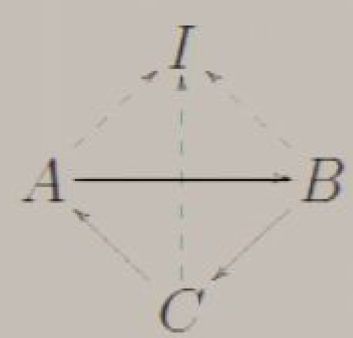
E.g. 1: $R(P)$

E.g. 2:



— *Galilean caucats* —

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Prop. A cau<u>cat</u> is Galilean if $\mathcal{G}$ is a Galilean cau<u>set</u>.

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Prop. F preserves normalization, G only embeds normalized processes, both preserve (dis)connectedness.

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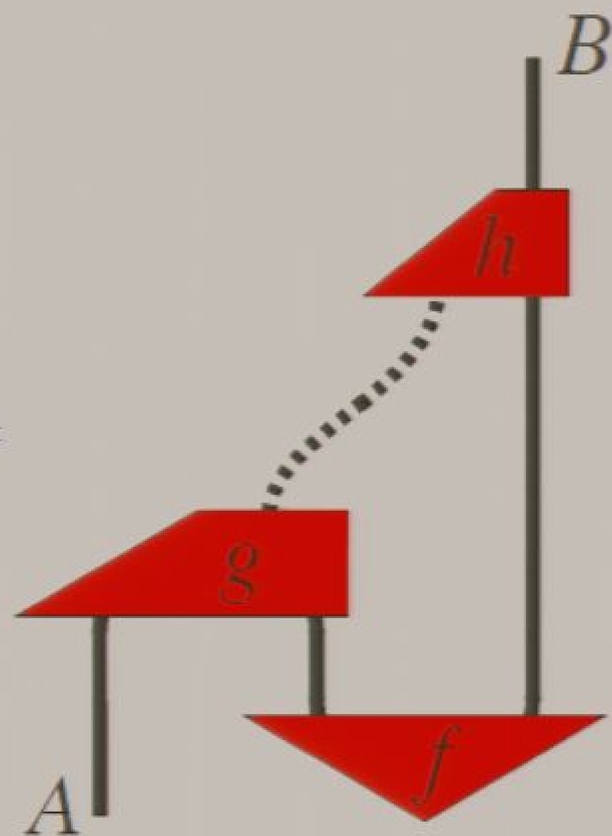
teleportation configuration:

$\mathcal{G}_1 =$ 

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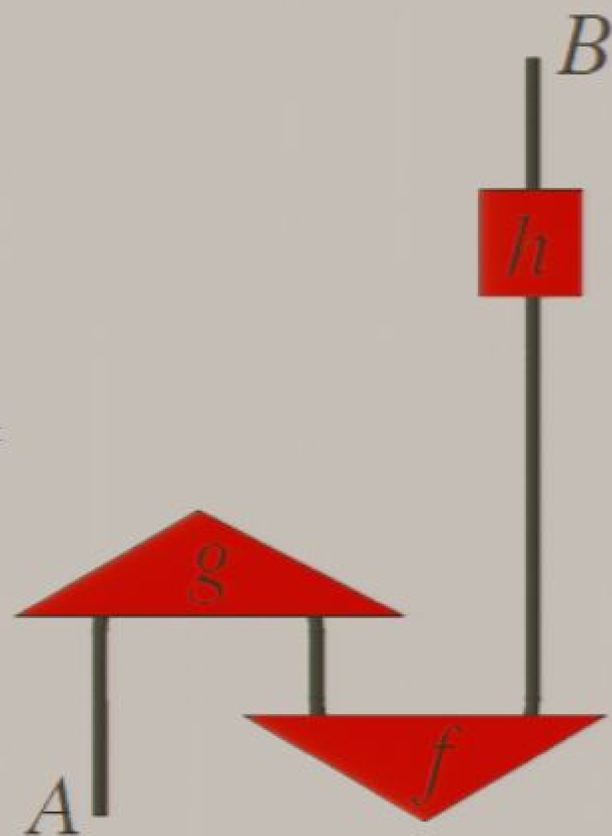

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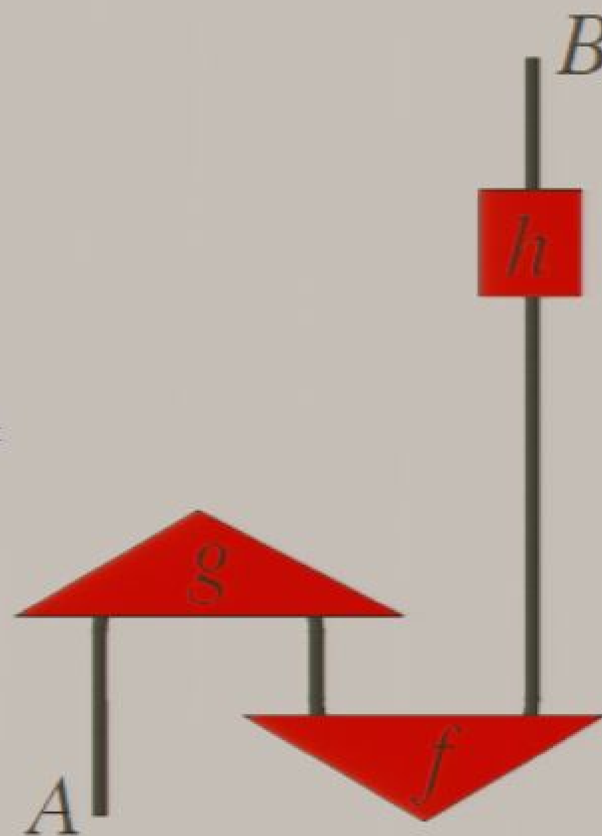

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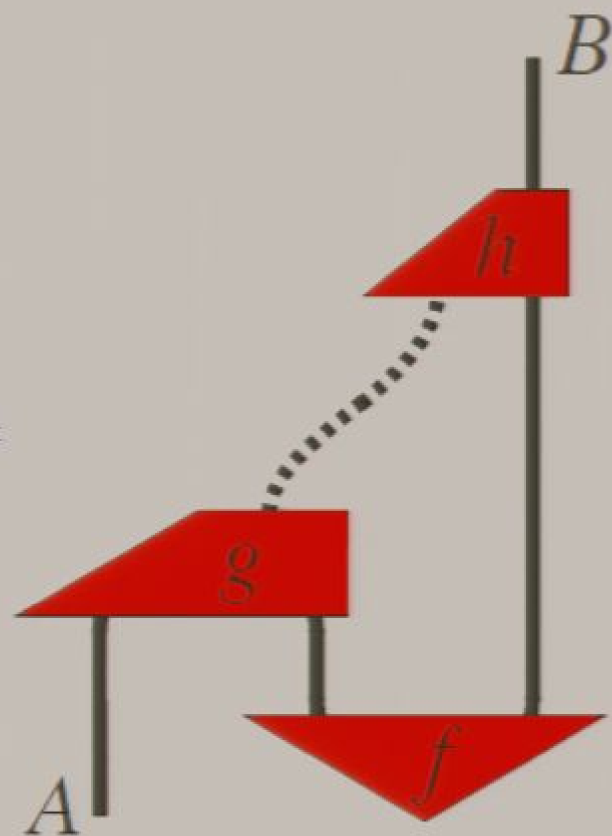
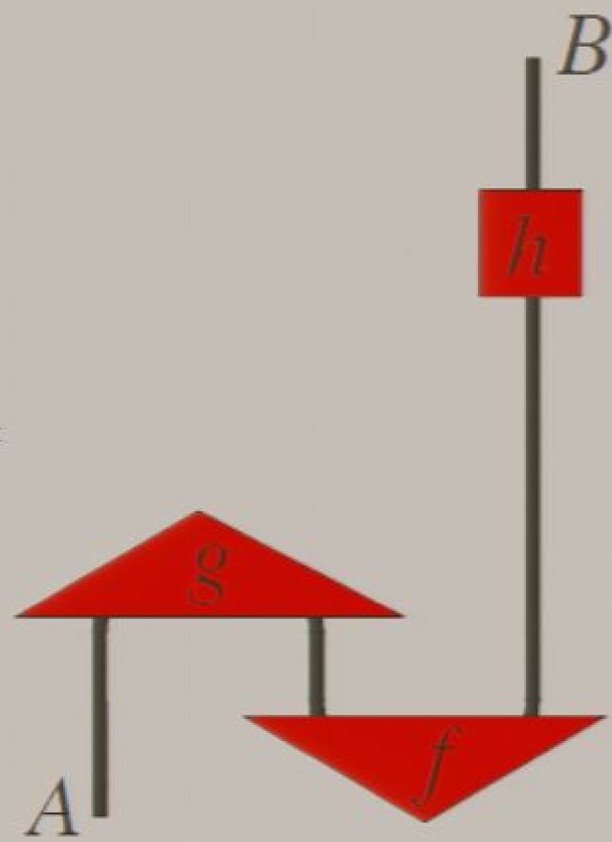


— *teleportation requires classical communication* —

teleportation minus classical communication conf.:

$\mathcal{G}_2 =$



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Thm. **For N-containing caucats $\mathcal{G}_2$ is disconnected.**

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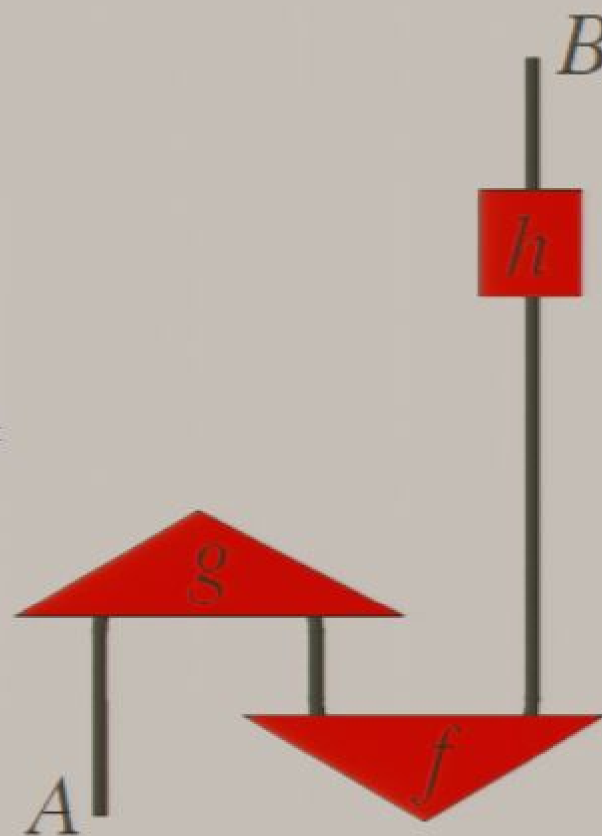

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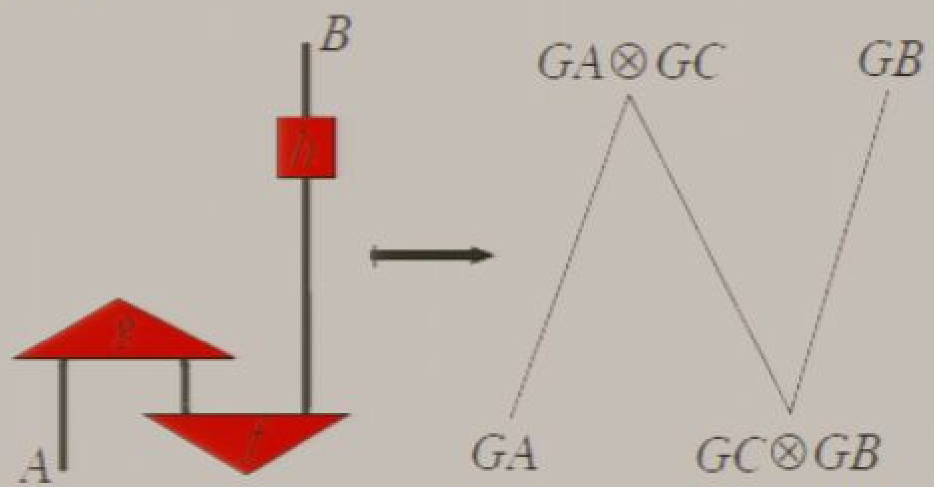

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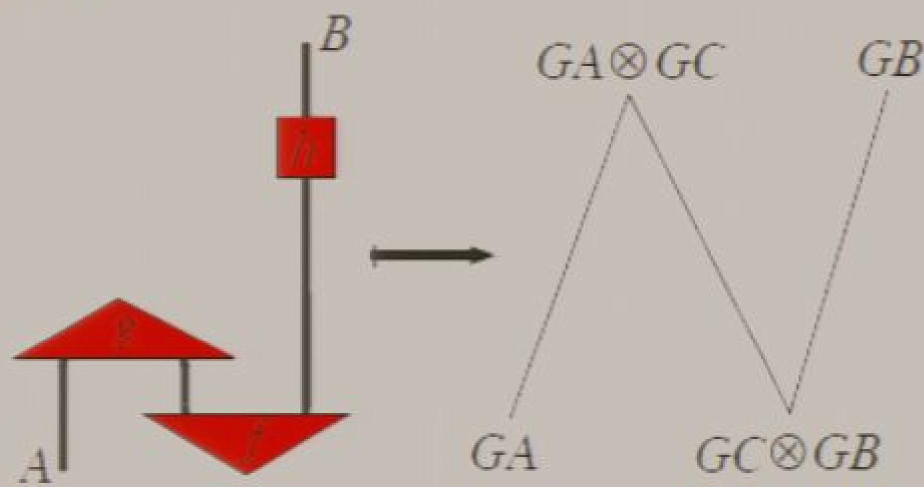

Then $G\mathcal{G}_2$ must be disconnected since $GA \otimes GB$.

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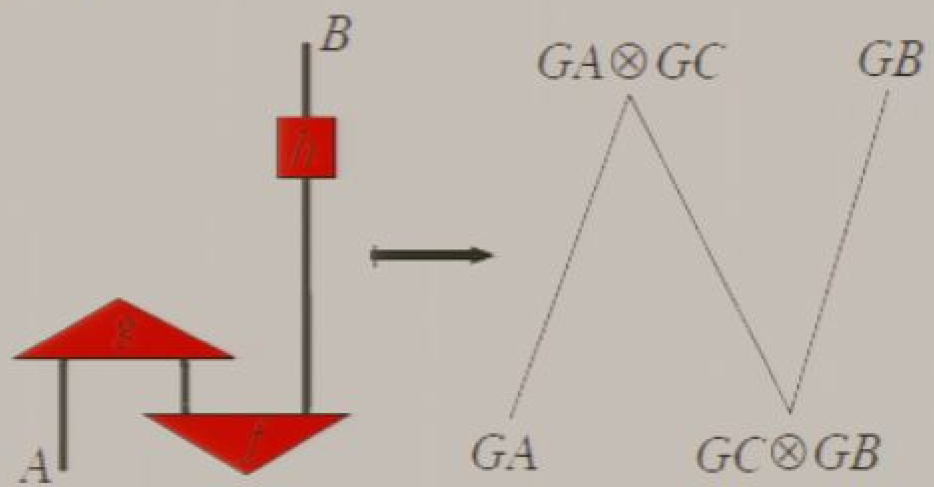

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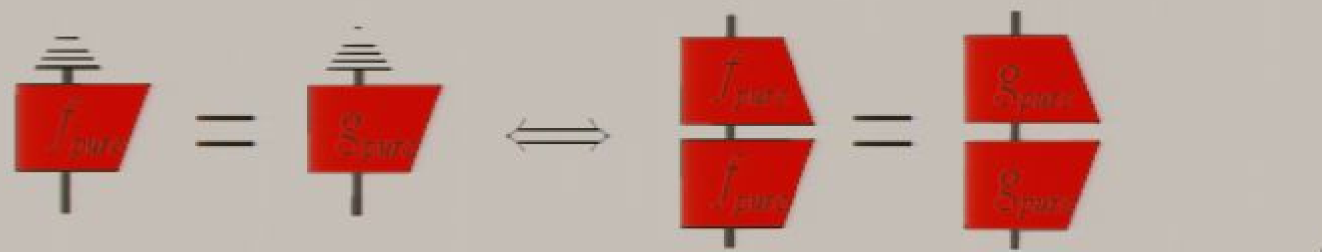
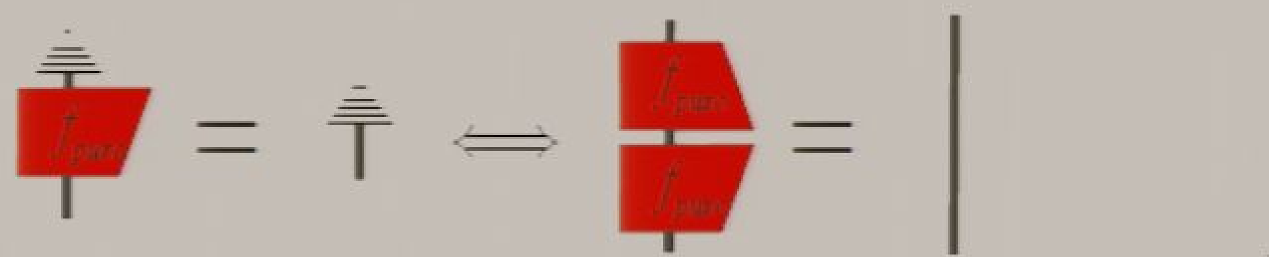
Hence $G_2 = FG\mathcal{G}_2$ and $G'\mathcal{G}_2$ must be disconnected.

THE BROADER PICTURE

Connecting environment and dagger:



Normalization then follows:



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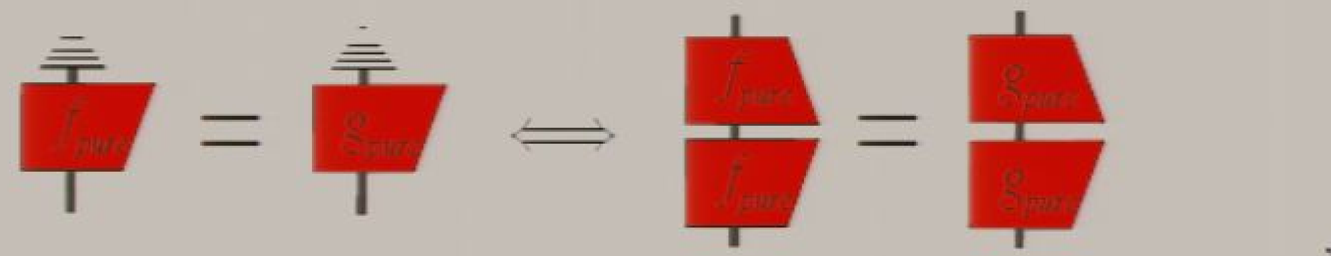

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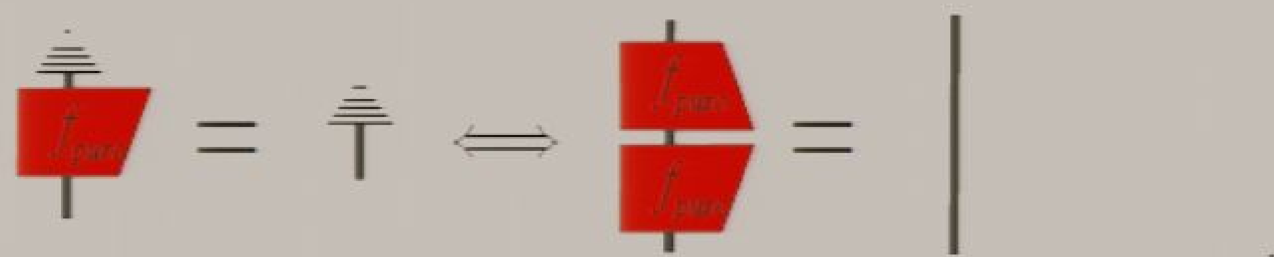

Then purifiable processes form Selinger's CPM-categories.

Connects up with Chiribella-D'Ariano-Perinotti.

OUTRAGEOUS

WAY BEHIND WILD

RANDOM GUESSES

(OWBWRGs)

ON QUANTUM GRAVITY

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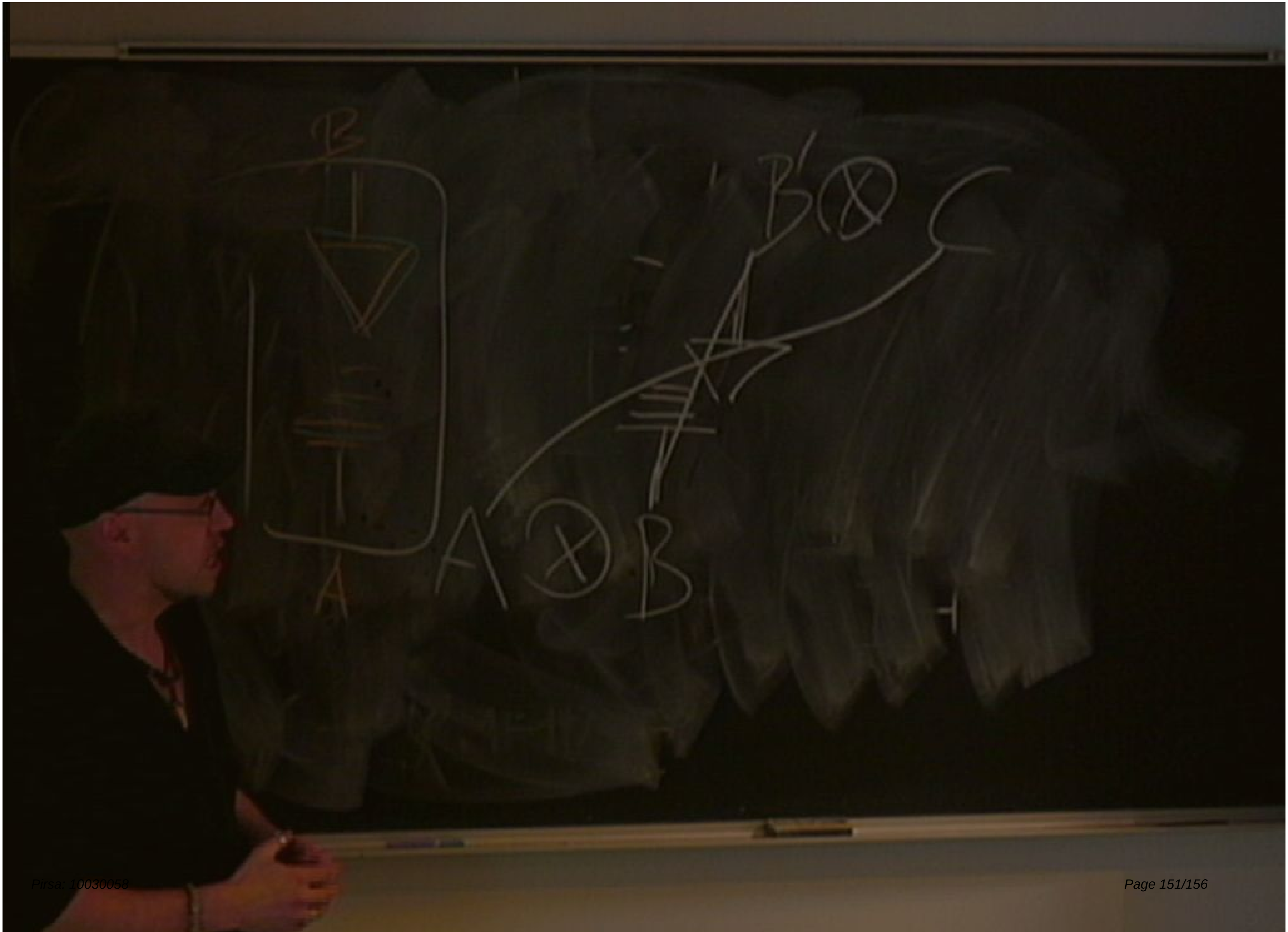
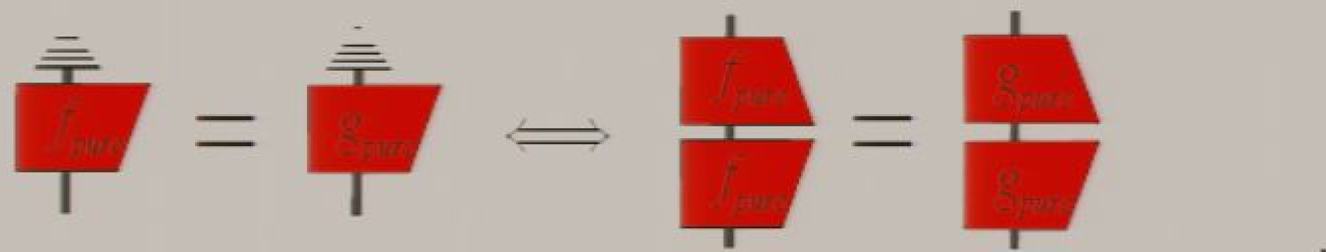

B
A
A ⊗ B
B' ⊗ C
A'

Connecting environment and dagger:



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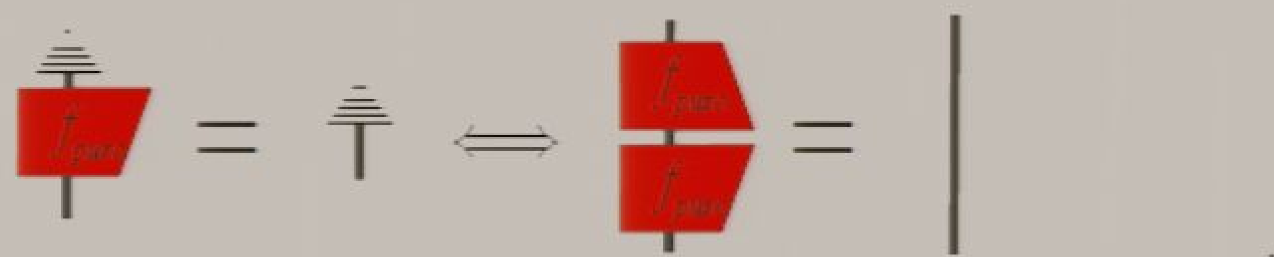

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— *OWBWRGs on QG* —

- Connectedness as a fundamental principle
- GHZ & W as generating modes of interaction
  - (cf. Borsten-Dahanayakea-Duff–Rubens on STU Black Holes in supergravity)

Alternatively:

- Varying (quantizing) the connectedness condition.

Example. Given:

- Posetal causal structure P
- Process category $CPM(\mathbf{C})$

Then define $\mathbf{CC}(CPM(\mathbf{C}), P)$:

- $(A \in |CPM(\mathbf{C})| \setminus \{I\}, a \in R(P) \setminus \emptyset)$ or $(I, \emptyset)$.
- $\mathbf{CC}(\mathbf{C}, P)((A, a), (B, b)) :=$

$$\begin{cases} CPM_{\perp}(\mathbf{C})(A, B) & a \leq b \\ [CPM_{\perp}(\mathbf{C})(I, B)] \circ \top_A & a \not\leq b \end{cases}$$

- If both $a \not\leq b$ and $b \not\leq a$, or if a or $b = \emptyset$:

$$(A, a) \otimes (B, b) := (A \otimes B, a \cup b) \, .$$

— *definition* —

A *causal category* (or *caucat*) CC is:

1. symmetric partial monoidal category

2. unit object I is terminal, i.e. for each $A \in |\mathbf{CC}|$ there is a unique morphism $\top_A : A \to I$

3. $A \otimes B$ exists iff

$$\begin{cases} \mathbf{CC}(A, B) = [\mathbf{CC}(I, B)] \circ \top_A \\ \mathbf{CC}(B, A) = [\mathbf{CC}(I, A)] \circ \top_B \end{cases}$$

Also, if $A \otimes B$, $A \otimes C$, $B \otimes C$ exist then $A \otimes (B \otimes C)$ exists, and $\mathbf{CC}(I, A) \neq \emptyset$ for all $A \in |\mathbf{CC}|$.

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