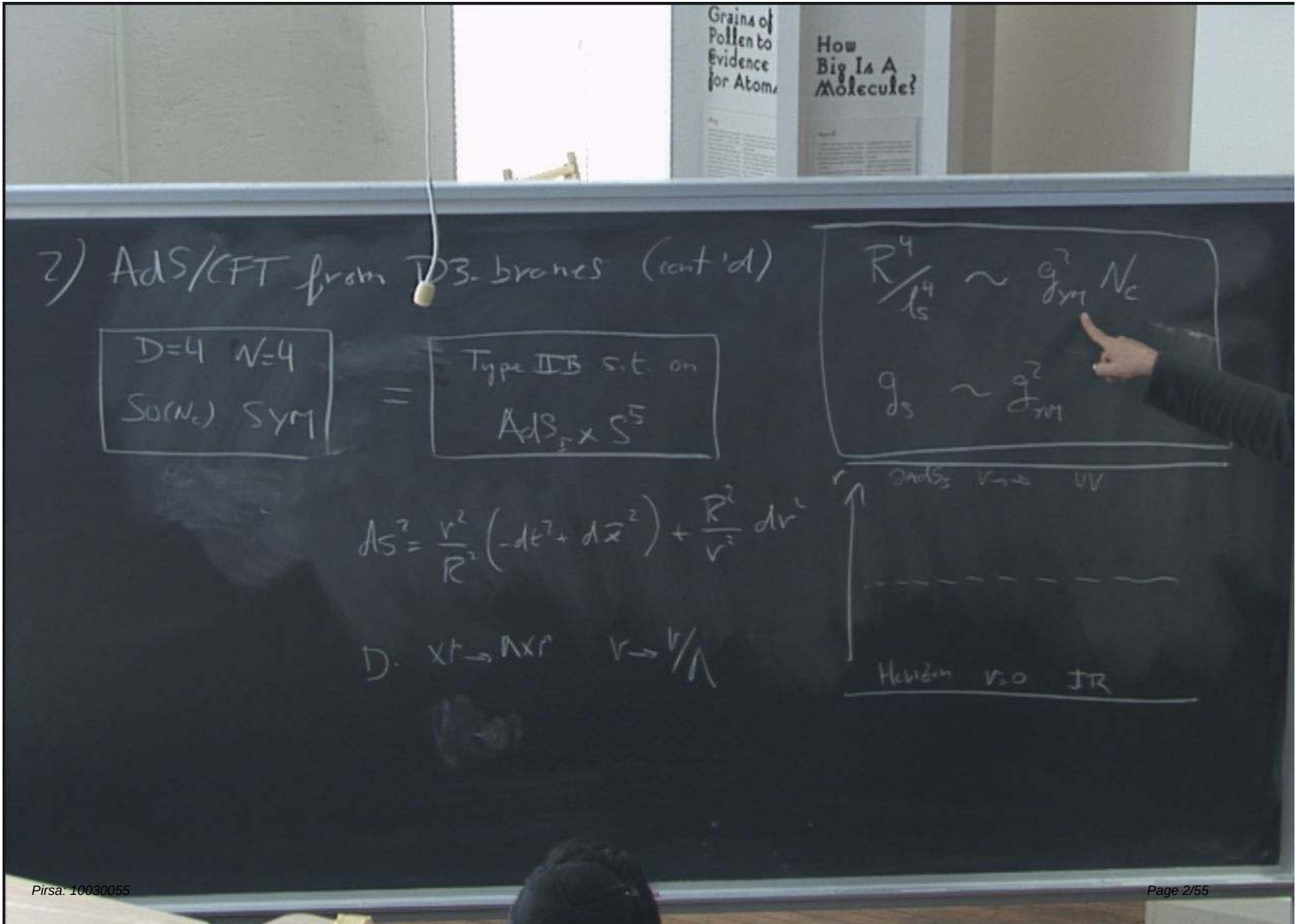
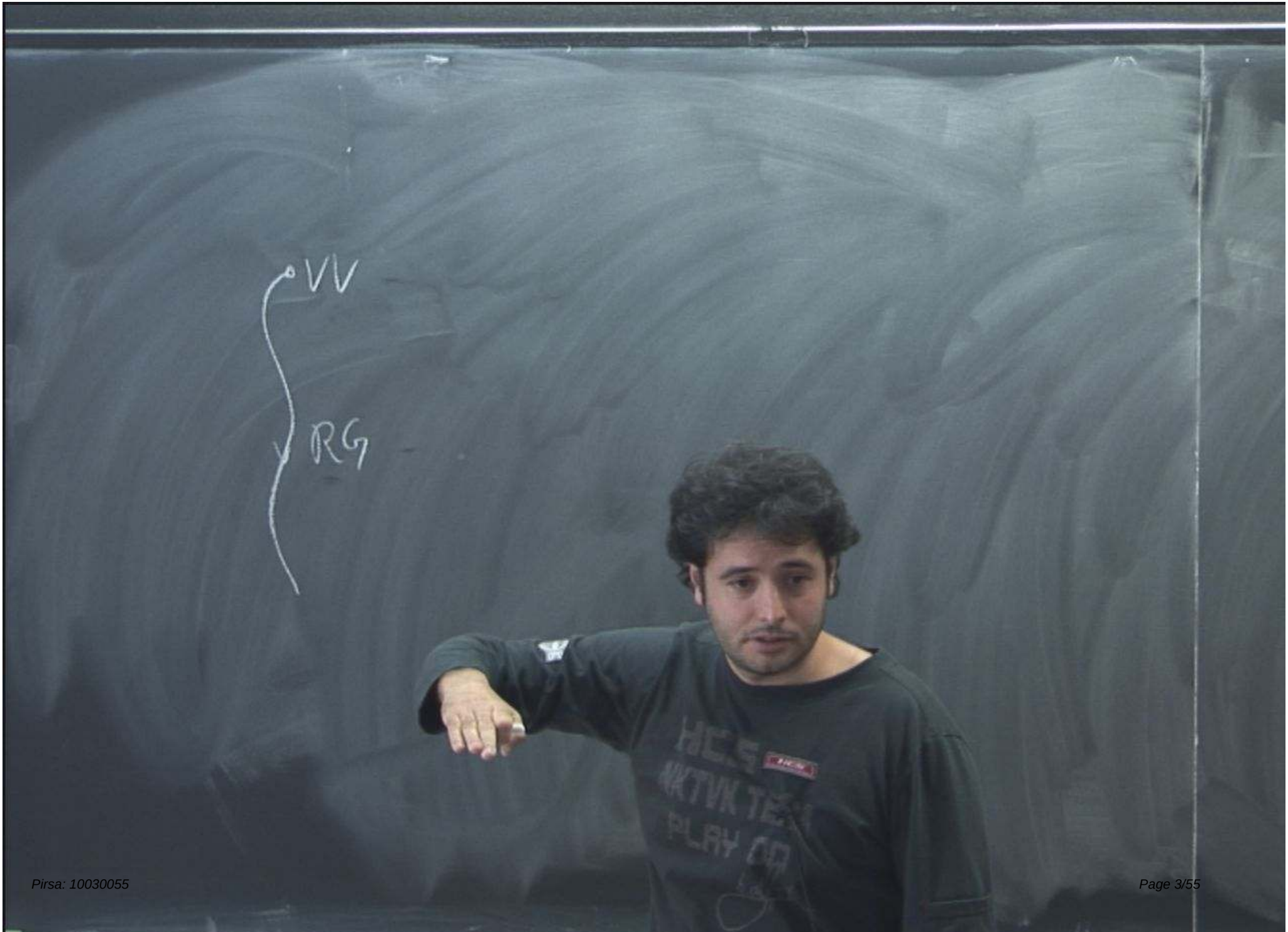
Title: Explorations in String Theory (PHYS 647) - Lecture 13

Date: Mar 03, 2010 11:20 AM

URL: http://pirsa.org/10030055

Abstract:


Grains of Pollen to Evidence for Atom
How Big Is A Molecule?
2) AdS/CFT from D3-branes (cont'd)
D=4 N=4 SU(N_c) SYM
=
Type IIB s.t. on AdS_5 × S^5
R^4/l_s^4 ~ g_YM^2 N_c
g_s ~ g_YM^2
ds^2 = r^2/R^2 (-dt^2 + dx^2) + R^2/r^2 dr^2
D: x^μ → λx^μ r → r/λ
r
∂AdS_5 r→∞ UV
Horizon r=0 IR


VV
RG

Evidence for Atom
Big Is A Molecule?

AdS/CFT from D3-branes (cont'd)

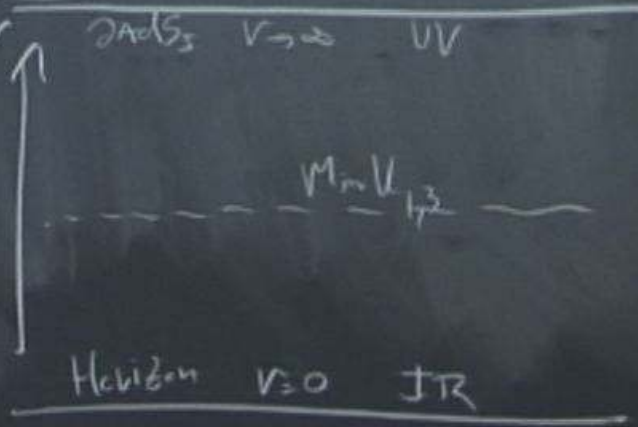
$D=4$ $\mathcal{N}=4$ $SU(N_c)$ SYM $=$ Type IIB s.t. on $AdS_5 \times S^5$

$$\frac{R^4}{l_s^4} \sim g_{YM}^2 N_c$$

$$g_s \sim g_{YM}^2$$

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2} dr^2$$

D: $x^\mu \to \Lambda x^\mu$ $\quad r \to r/\Lambda$



Precise formulation

$$g_s = e^{\Phi_\infty}$$

Precise formulation

$$g_s = e^{\mathbb{E}_{\alpha\beta}}$$

Precise formulation

$$g_{\mu\nu} = e^{\mp \Phi}$$

$$\int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \mp R)\, d\sigma}$$

Precise formulation

$$g_{\mu\nu} = e^{\Phi}$$

$$\int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \Phi R)\, d\sigma}$$

$$\Phi \int_\Sigma R \sim \Phi \chi$$

Precise formulation

$$g_{\mu\nu} = e^{\mp}$$

$$\sum_{\chi} \int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \underline{\mp} R)\, d^2\sigma} = e^{-\mp \chi} \int \mathcal{D}[X$$

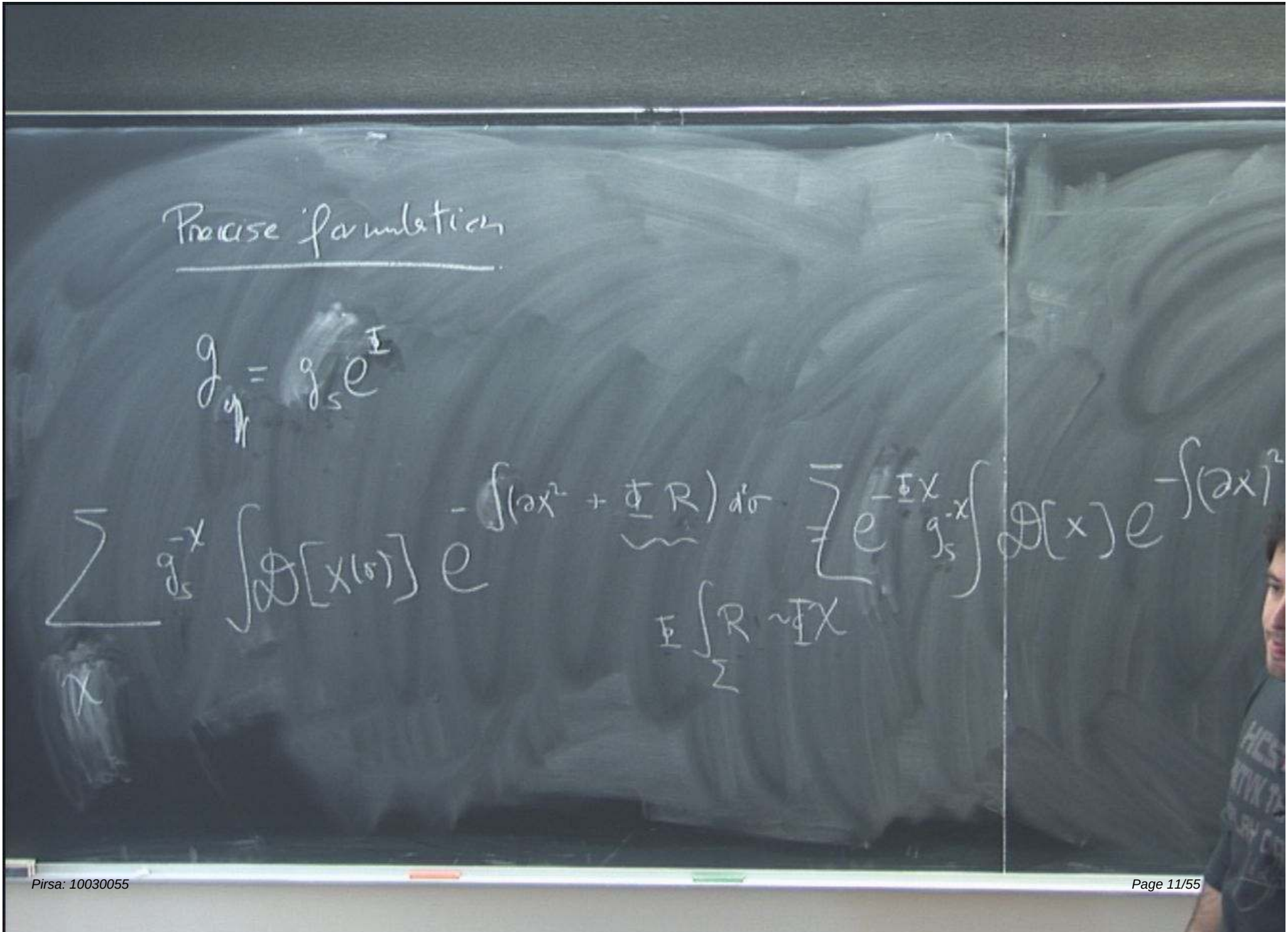
$$\mp \int_{\Sigma} R \sim \mp \chi$$

Precise formulation

$$g_{eff} = g_s e^{\Phi}$$

$$\sum_{\chi} g_s^{-\chi} \int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \Phi R)\, d^2\sigma} \sum e^{-\Phi \chi} g_s^{-\chi} \int \mathcal{D}[X$$

$$\Phi \int_{\Sigma} R \sim \Phi \chi$$


Precise formulation
$g_{\text{eff}} = g_s e^{\Phi}$
$\sum_{\chi} g_s^{-\chi} \int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X)^2 + \Phi R)\, d^2\sigma}$
$\sum_{\chi} e^{-\Phi \chi} g_s^{-\chi} \int \mathcal{D}(x)\, e^{-\int (\partial x)^2}$
$\Phi \int_{\Sigma} R \sim \Phi \chi$

Precise formulation

$$g_s = e^{I_\infty}$$

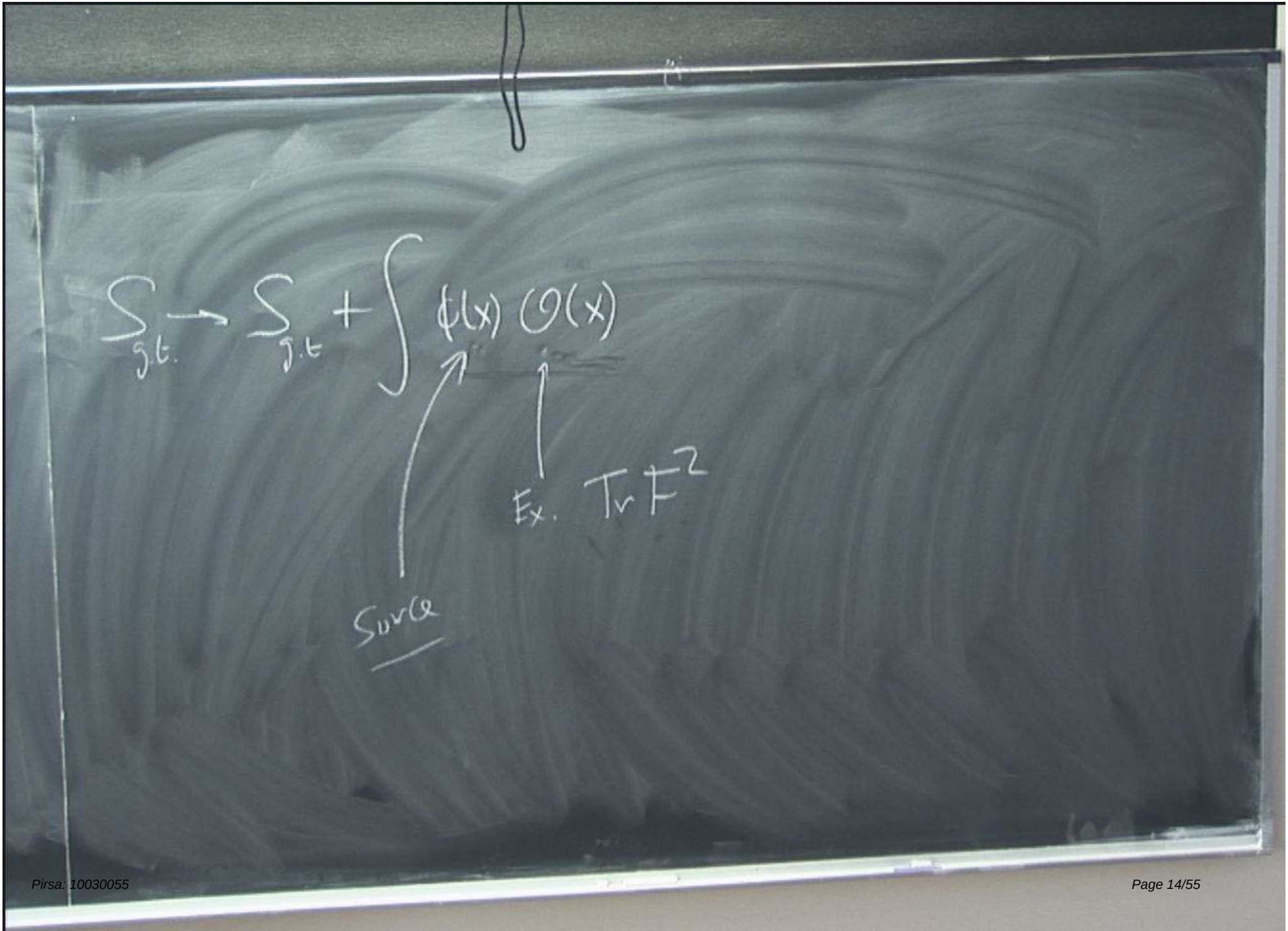
{ Couplings const. in g.t. } $\longleftrightarrow$ {

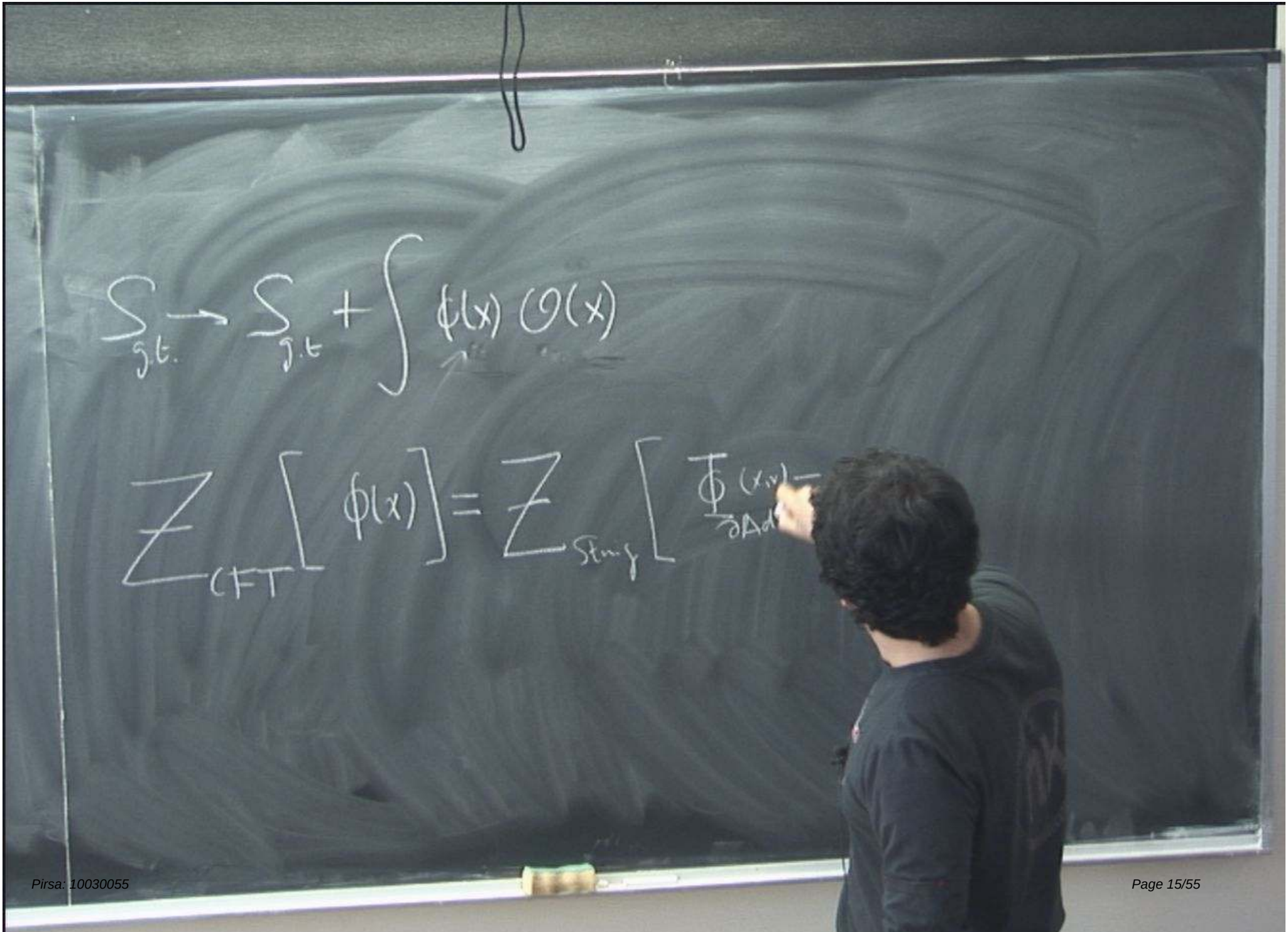
Precise formulation

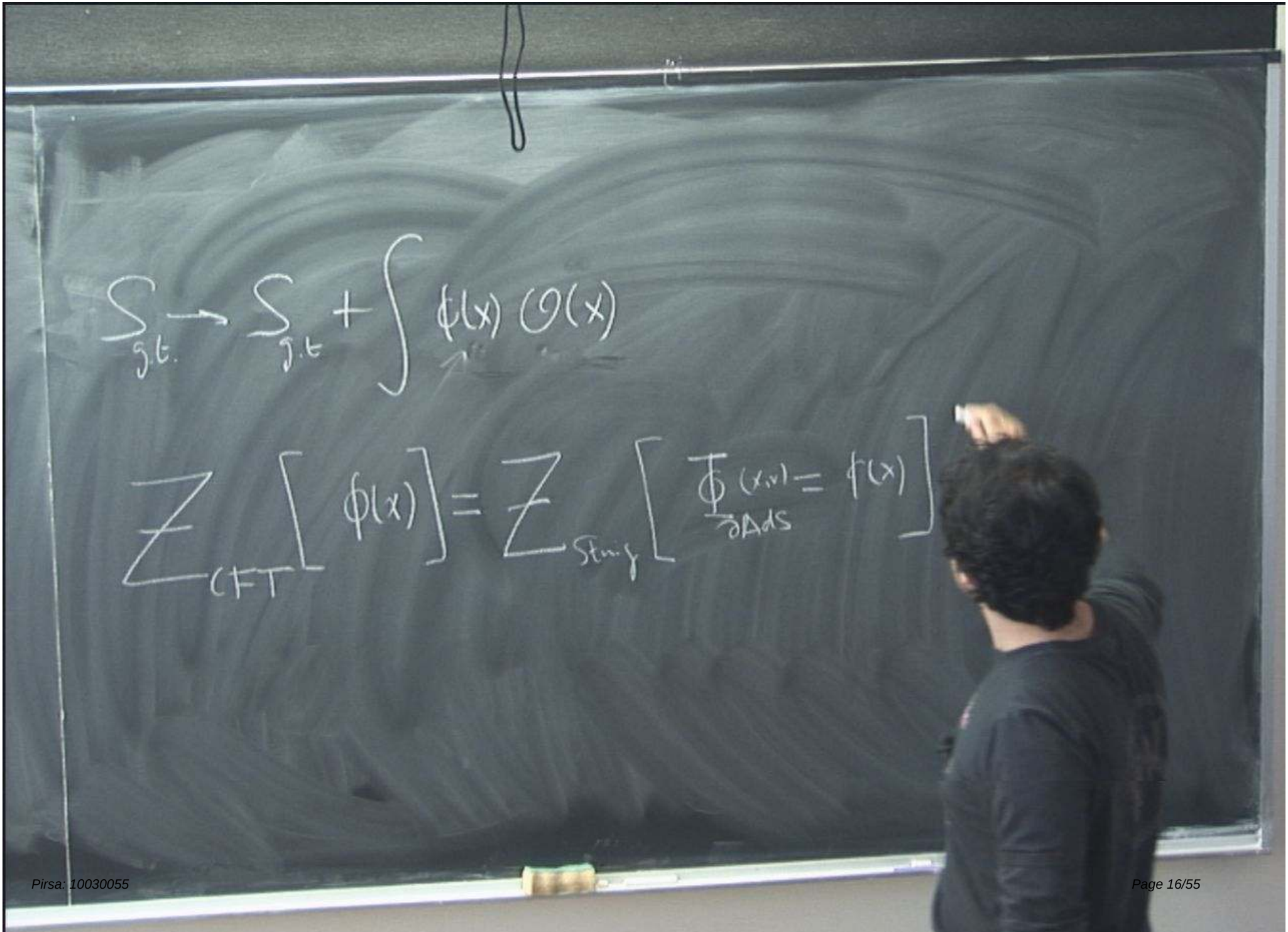
$$g_s = e^{I_\infty}$$

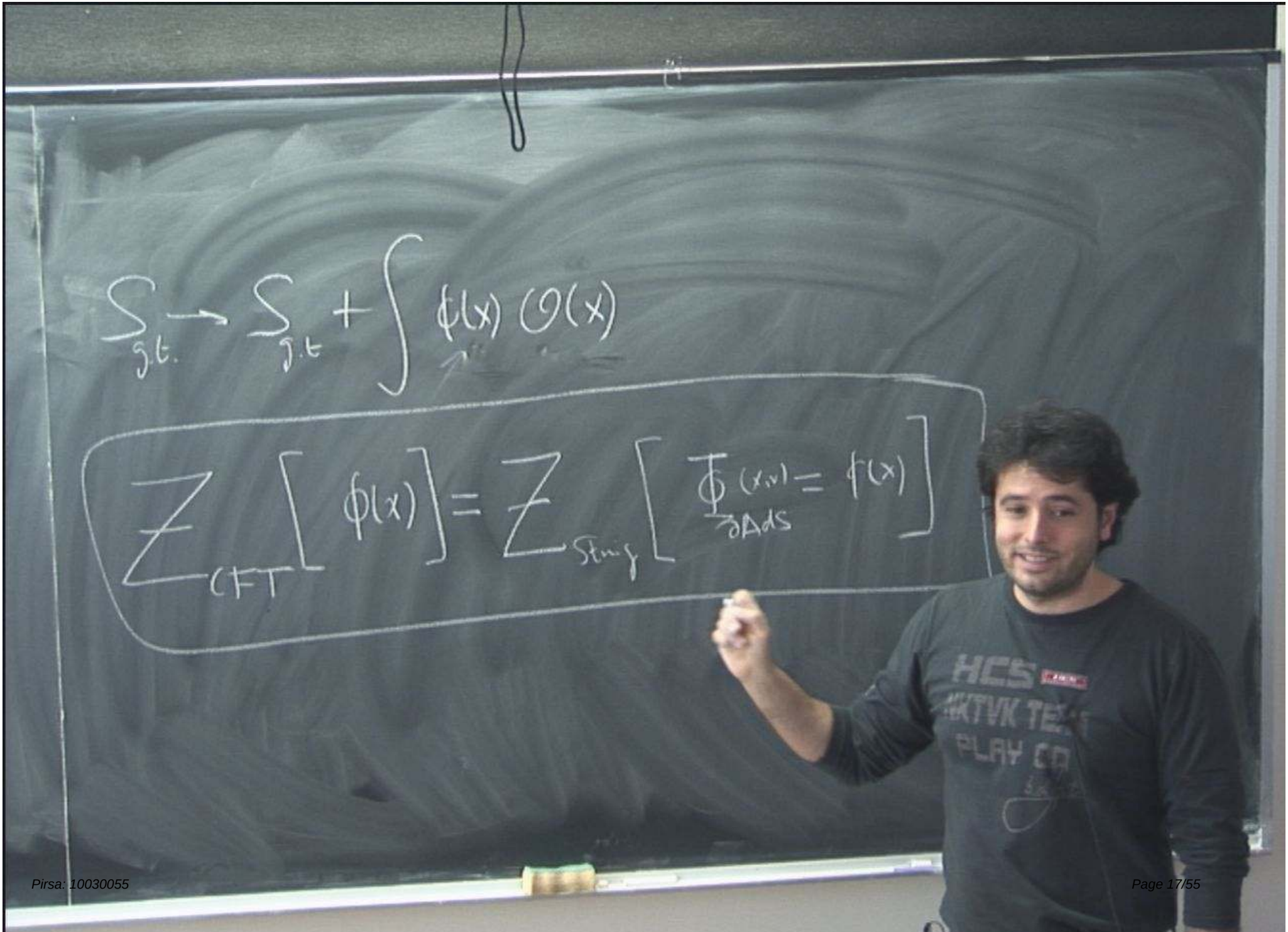
$$\left\{ \begin{array}{c} \text{Couplings} \\ \text{const. in g.t.} \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} \text{Fields on} \\ \infty \end{array} \right\}$$

$$\phi(x) = \lim_{r \to \infty} \Phi(x, r)$$


$S_{g.t.} \to S_{g.t.} + \int \phi(x)\, \mathcal{O}(x)$
Ex. $\mathrm{Tr}\, F^2$
Source


$S_{g.t.} \to S_{g.t} + \int \phi(x)\, \mathcal{O}(x)$
$Z_{CFT}[\phi(x)] = Z_{String}[$


$S_{g.t.} \to S_{g.t} + \int \phi(x)\, \mathcal{O}(x)$
$Z_{CFT}[\phi(x)] = Z_{String}[\Phi_{\partial AdS}(x,v) = f(x)]$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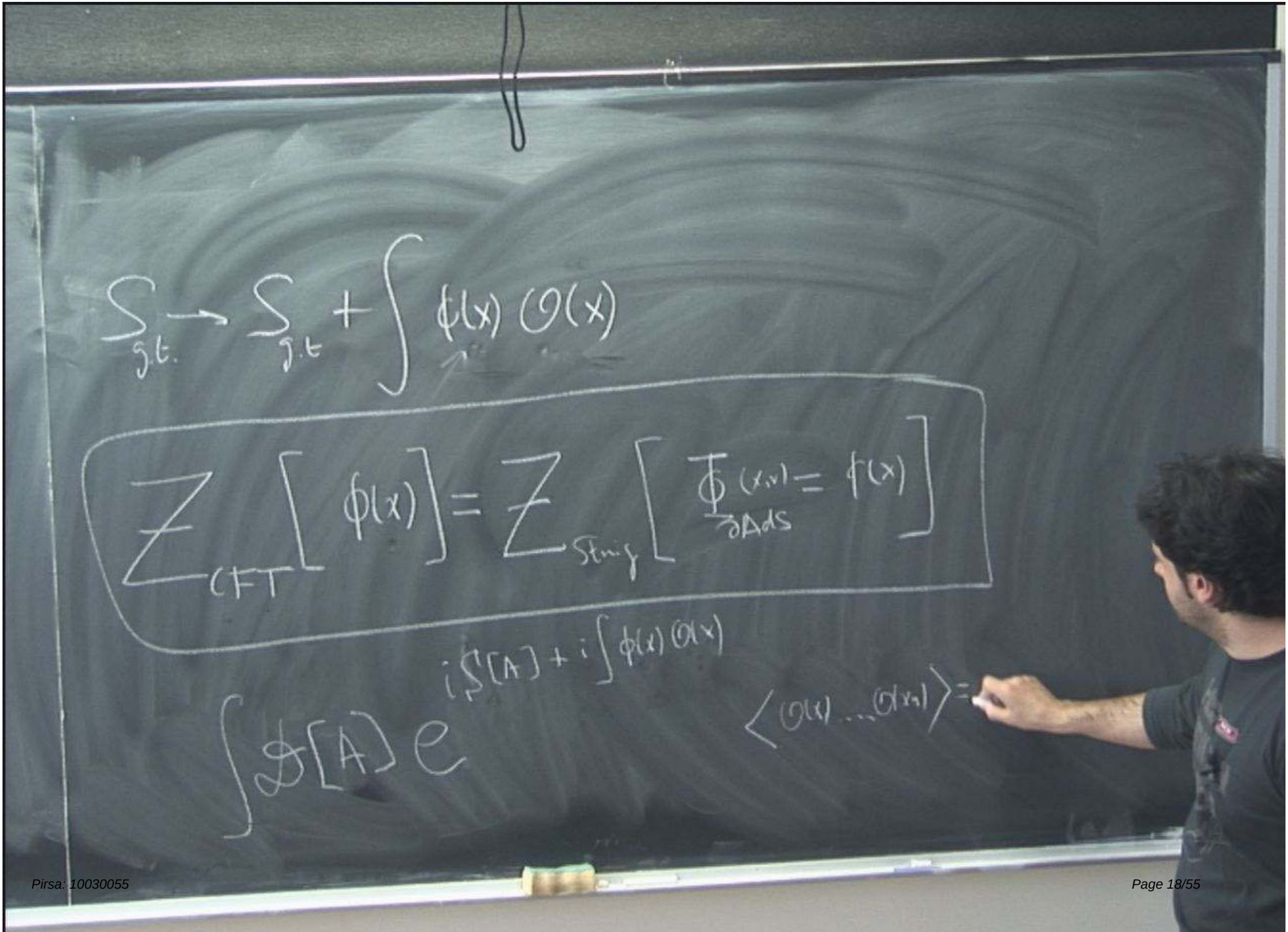

$S_{g.t.} \to S_{g.t} + \int \phi(x)\, \mathcal{O}(x)$
$Z_{CFT}[\phi(x)] = Z_{String}[\Phi_{\partial AdS}(x,r) = \phi(x)]$
$\int \mathcal{D}[A]\, e^{iS[A] + i\int \phi(x)\mathcal{O}(x)}$
$\langle \mathcal{O}(x) \ldots \mathcal{O}(x_n) \rangle =$

$$S_{g.t.} \to S_{g.t} + \int \phi(x)\, \mathcal{O}(x)$$

$$\boxed{Z_{CFT}\left[\phi(x)\right] = Z_{String}\left[\Phi_{\partial AdS}(x,r) = \phi(x)\right]}$$

$$\lambda = g^2_{YM} N_c \to \infty \quad \Rightarrow \quad S.t. \sim \text{Classical SUGRA.}$$

$$g^2_{YM} \to 0,\ N_c \to \infty$$

$$Z_{st} \simeq e^{-S^{\dagger}_{SUGRA}\left[\Phi_{class}\right]}$$

Precise formulation

$$\langle T_{\mu\nu} F^2(x)\, T_{\mu\nu} F^2(y) \rangle_{CFT}$$

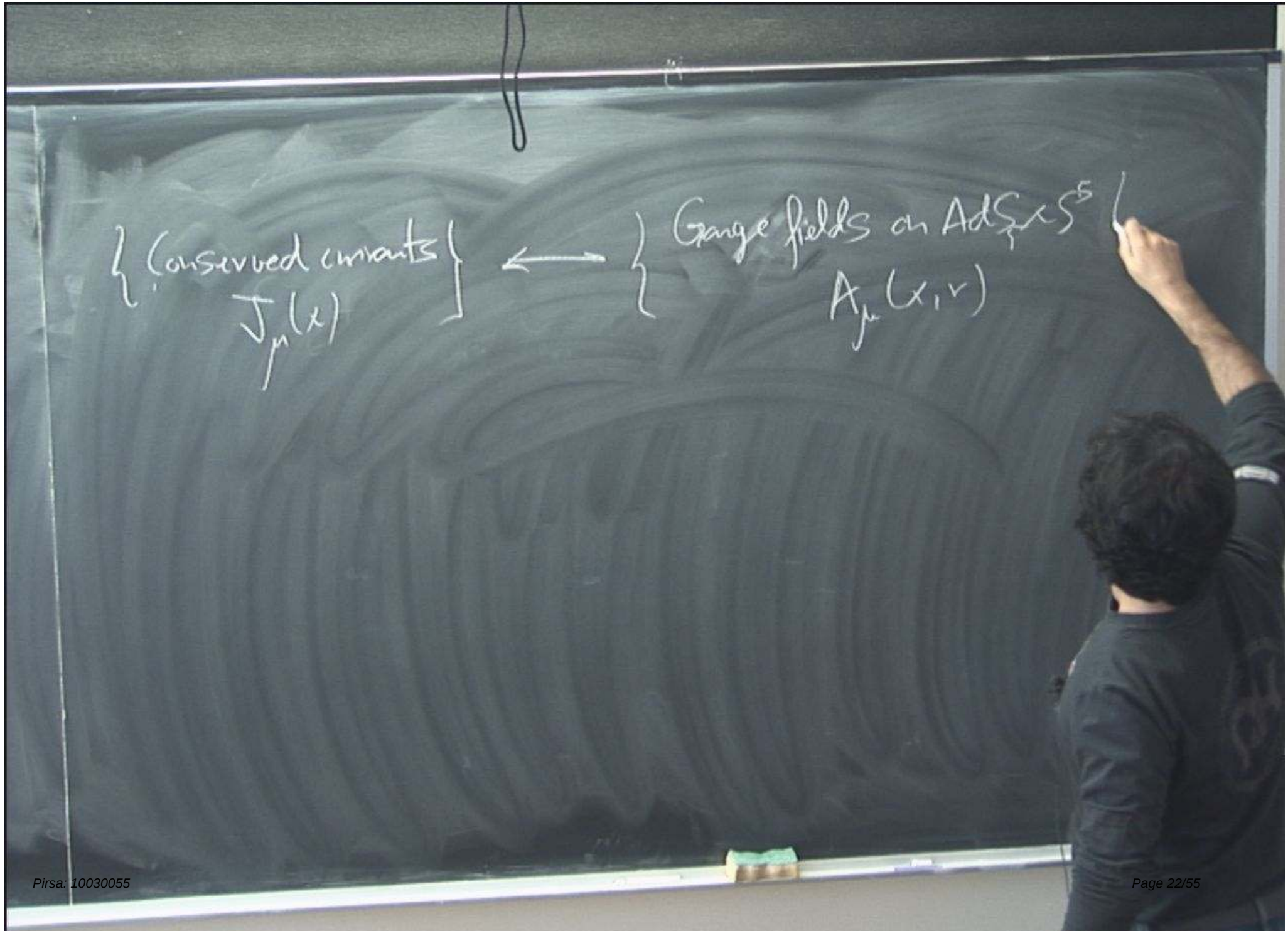
Precise formulation

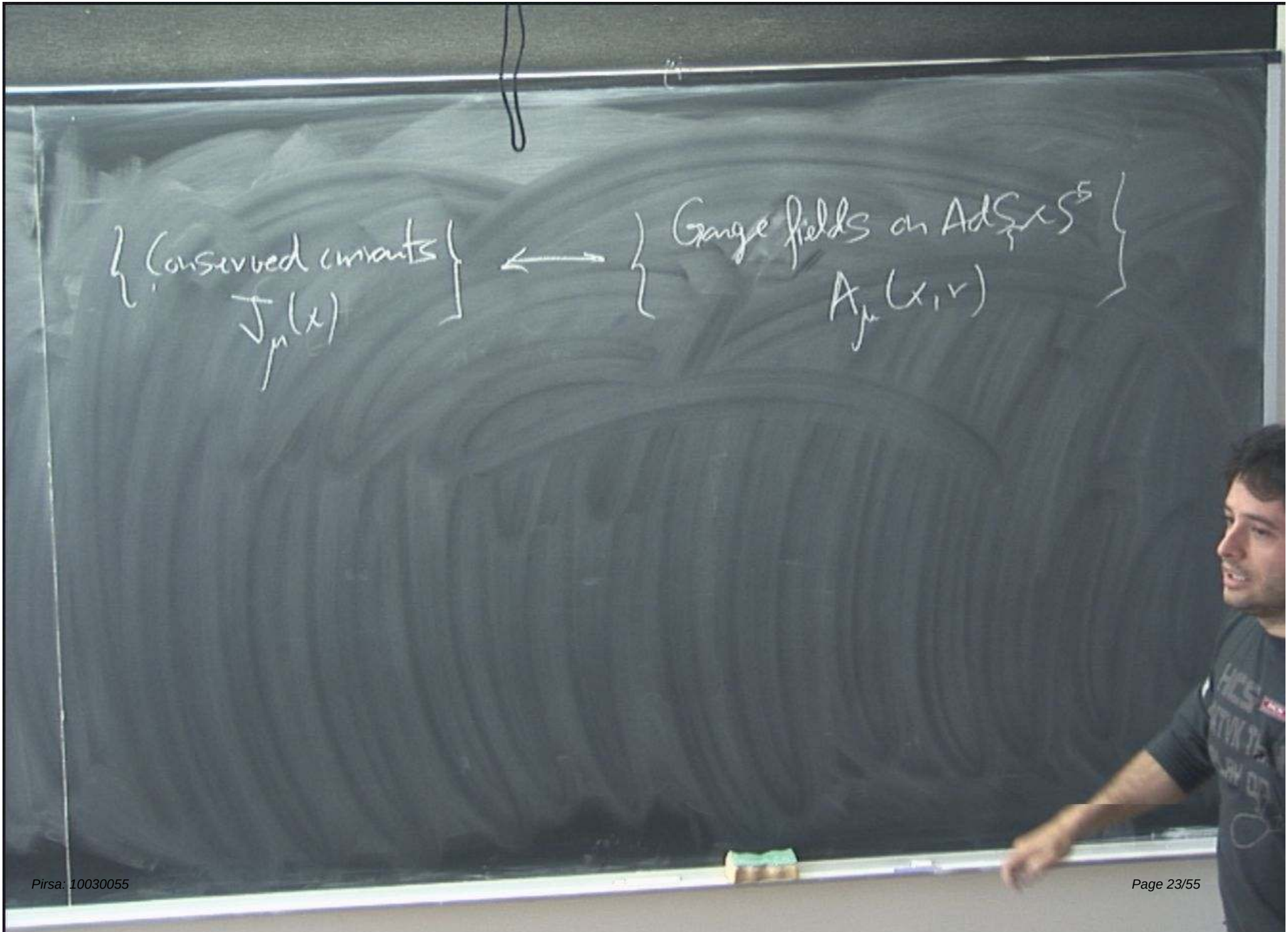
$$\langle T_\nu F^2(x)\; T_\nu F^2(y) \rangle_{CFT}$$

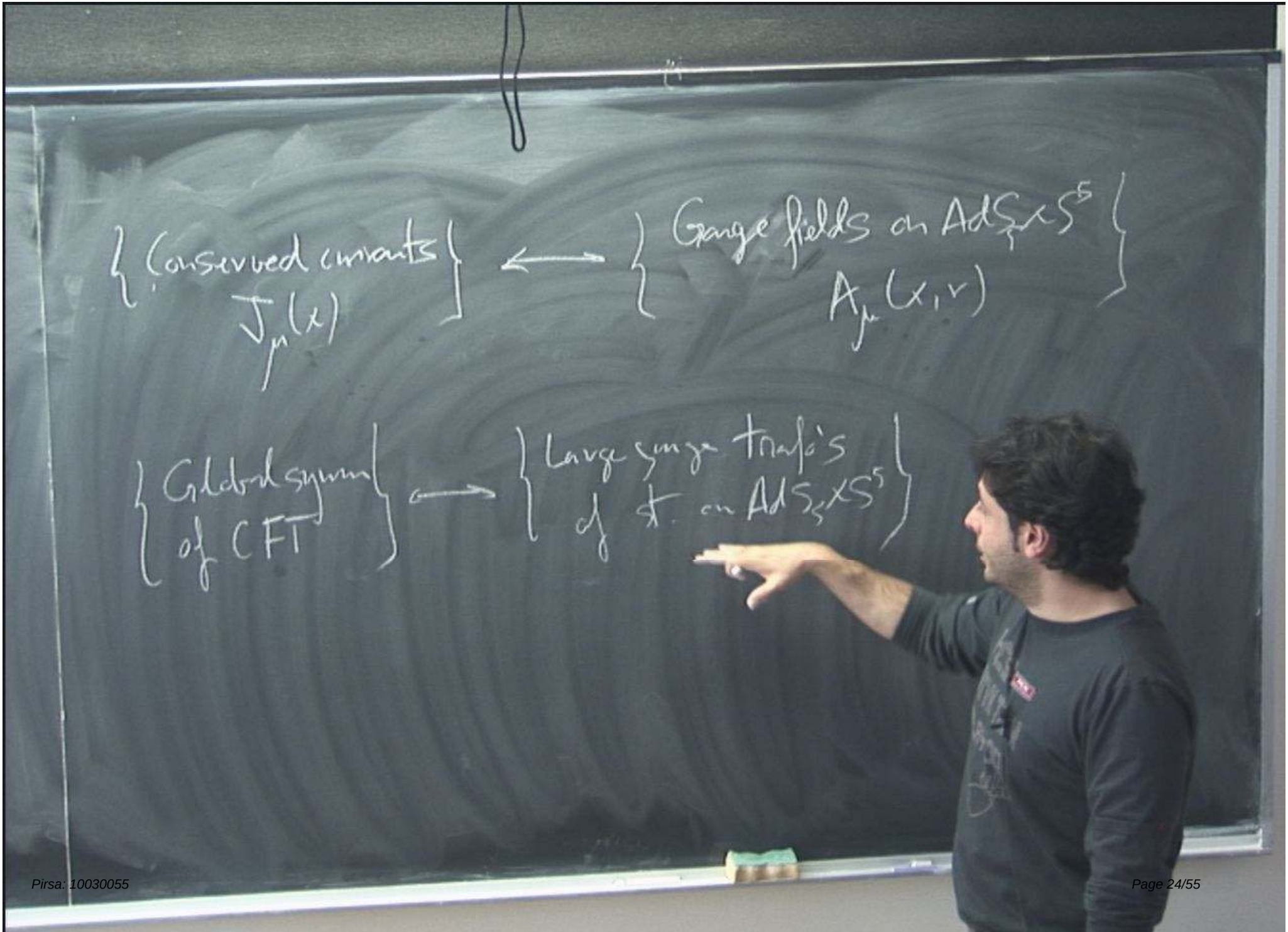
$\mathcal{O}(x)$

$\Phi(x,r)$

Dilaton


{ Conserved currents } ⟷ { Gauge fields on AdS
$J_\mu(x)$
$A_\mu(x,r)$


{ Conserved currents $J_\mu(x)$ } ⟷ { Gauge fields on $AdS_{x} \times S^5$ $A_\mu(x,r)$ }


{ Conserved currents $J_\mu(x)$ } ⟷ { Gauge fields on $AdS_5 \times S^5$ $A_\mu(x,r)$ }
{ Global symm of CFT } ⟶ { Large gauge trafo's of st. on $AdS_5 \times S^5$ }

Precise formulation

$$\langle T_{\mu\nu} F^2(x)\, T_{\mu\nu} F^2(y) \rangle_{CFT}$$

$\mathcal{O}(x)$

$\Phi(x,r) \xrightarrow{r\to 0} \phi(x)$

Dilaton

$x \to (1+\alpha)x$

$r \to r$

Precise formulation

$$\langle T_{\mu\nu} F^2(x)\, T_{\mu\nu} F^2(y) \rangle_{CFT}$$

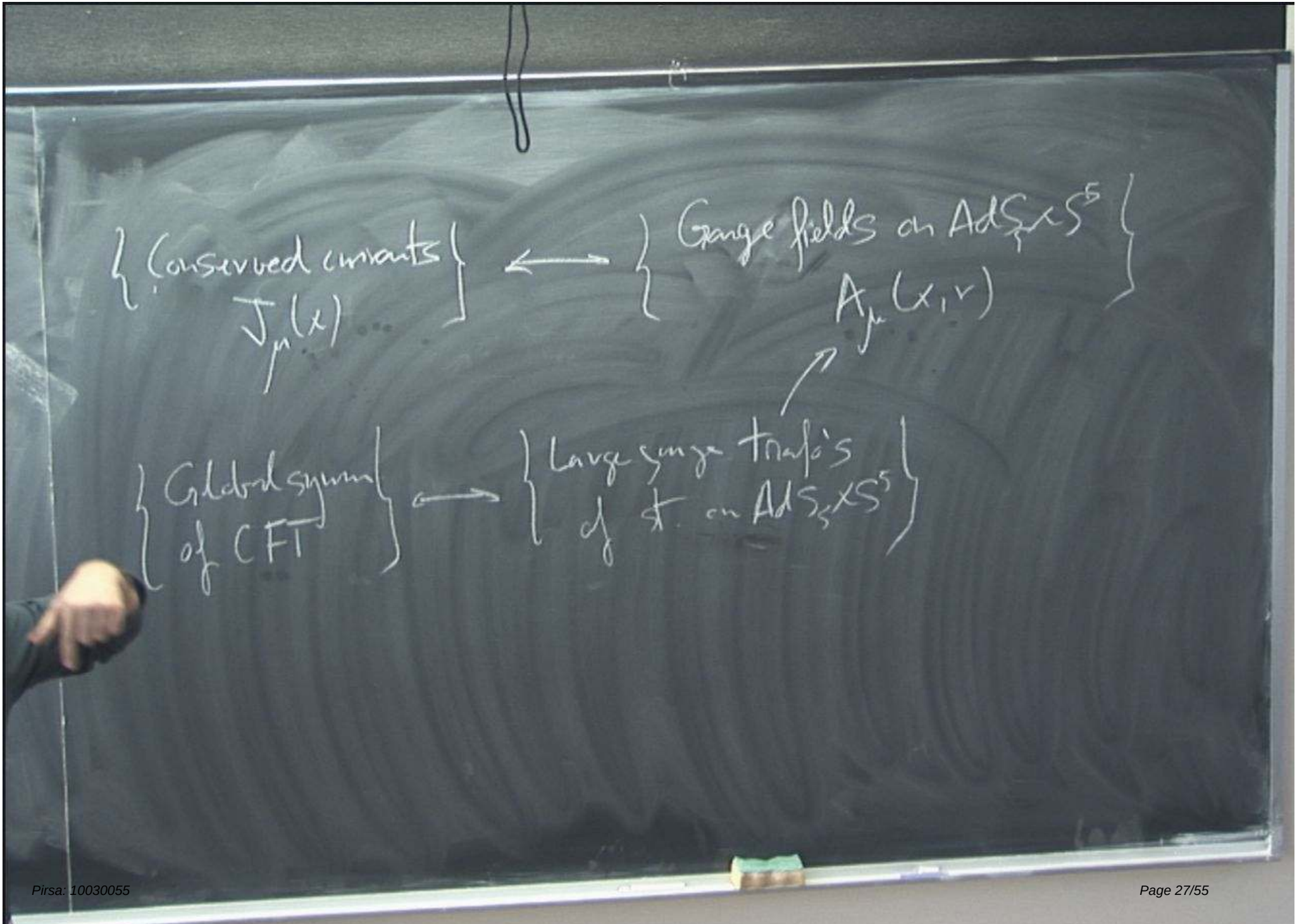
$\mathcal{O}(x)$

$\Phi(x, r) \xrightarrow{r \to 0} \phi(x)$

Dilaton

$x \to x + \frac{r}{r+1}\, a$

$r \to r$


{ Conserved currents $J_\mu(x)$ } ⟷ { Gauge fields on $AdS_5 \times S^5$ $A_\mu(x,r)$ }
{ Global symm of CFT } ⟶ { Large gauge trafo's of st. on $AdS_5 \times S^5$ }

Precise formulation

$$S \to S + \int \phi(x)\, \mathcal{O}(x)$$

$$\int A_\mu(x)\, J^\mu(x)$$

$$A_\mu(x) = \lim_{r \to \infty} A_\mu(x|r)$$

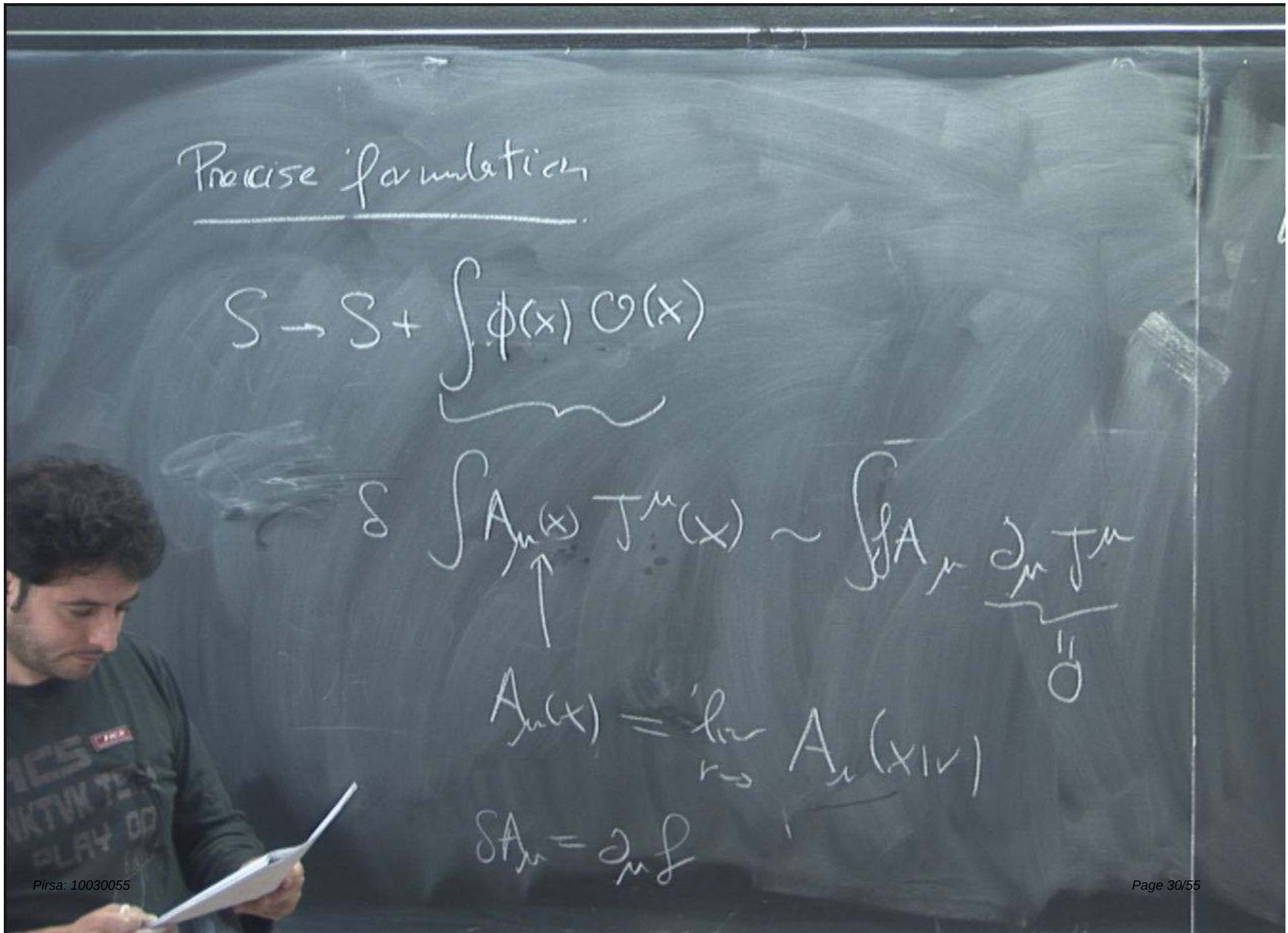
Precise formulation

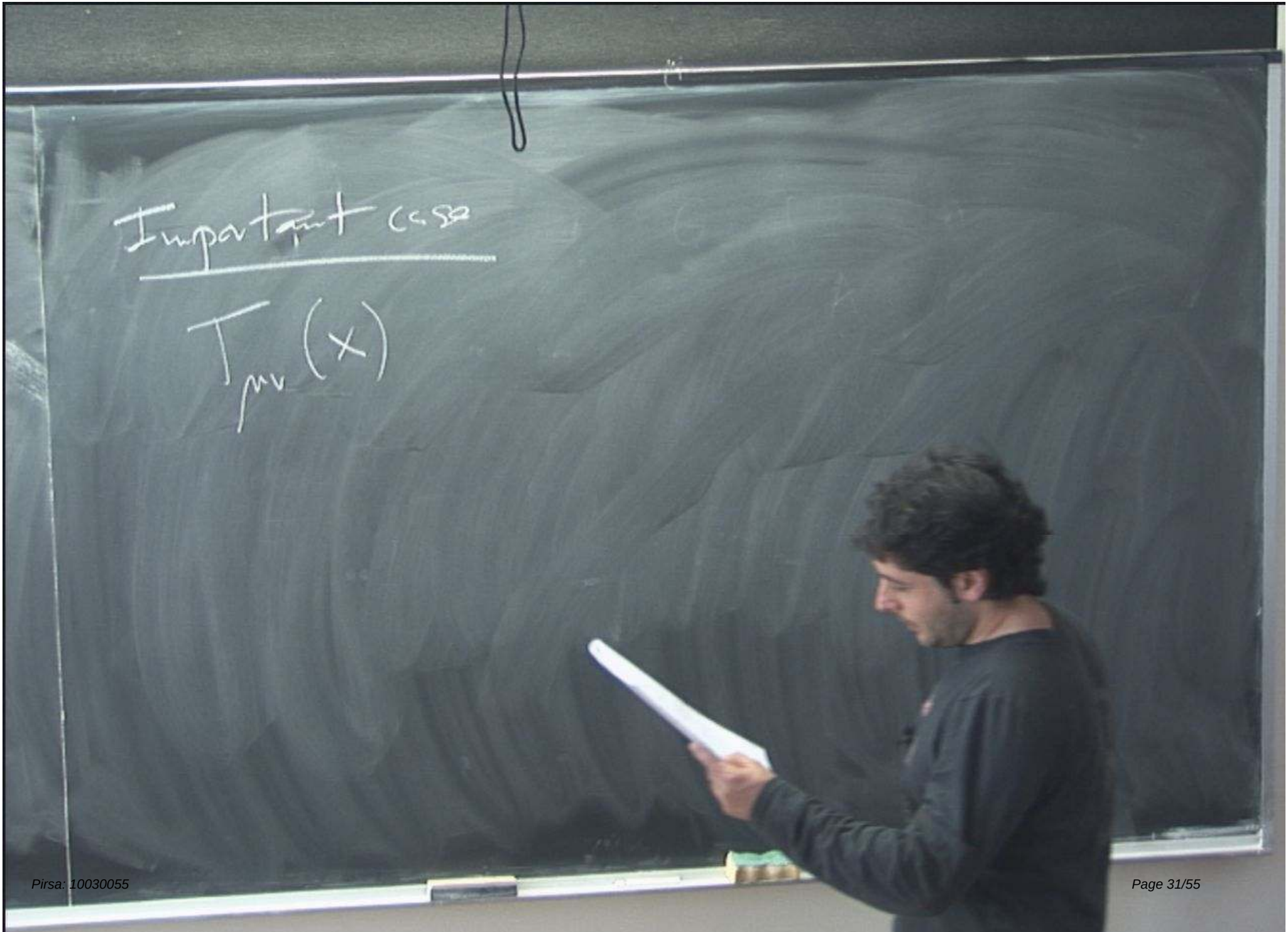
$$S \to S + \int \phi(x)\, \mathcal{O}(x)$$

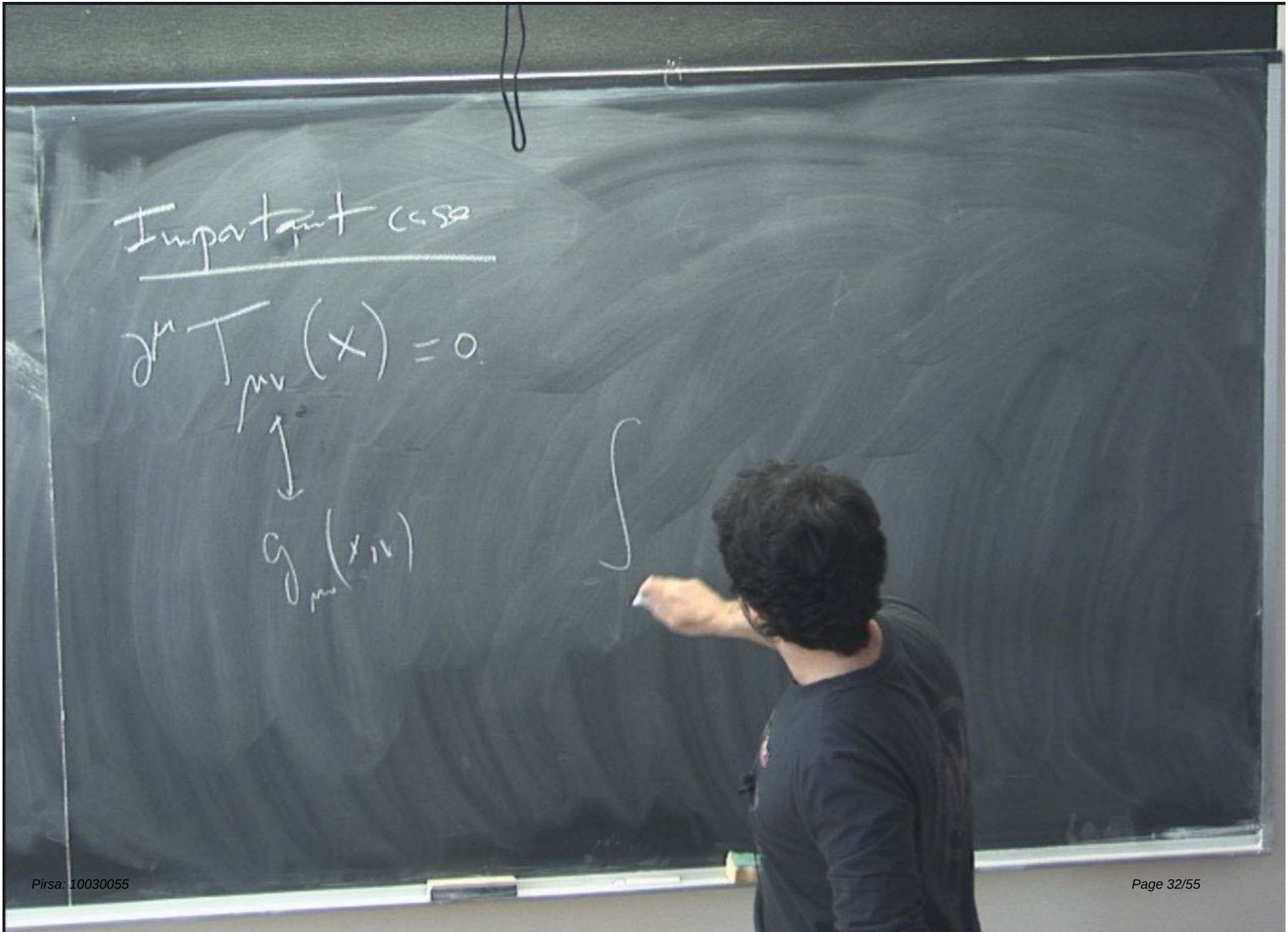
$$\int A_\mu(x)\, J^\mu(x)$$

$$A_\mu(x) = \lim_{r \to} A_\mu(x|r)$$

$$\delta A_\mu = \partial_\mu f$$


Precise formulation
$S \to S + \int \phi(x)\, \mathcal{O}(x)$
$\delta \int A_\mu(x)\, J^\mu(x) \sim \int \delta A_\mu\, \partial_\mu J^\mu$
$A_\mu(x) = \lim_{r \to \infty} A_\mu(x|r)$
$\delta A_\mu = \partial_\mu f$


Important case
$T_{\mu\nu}(x)$

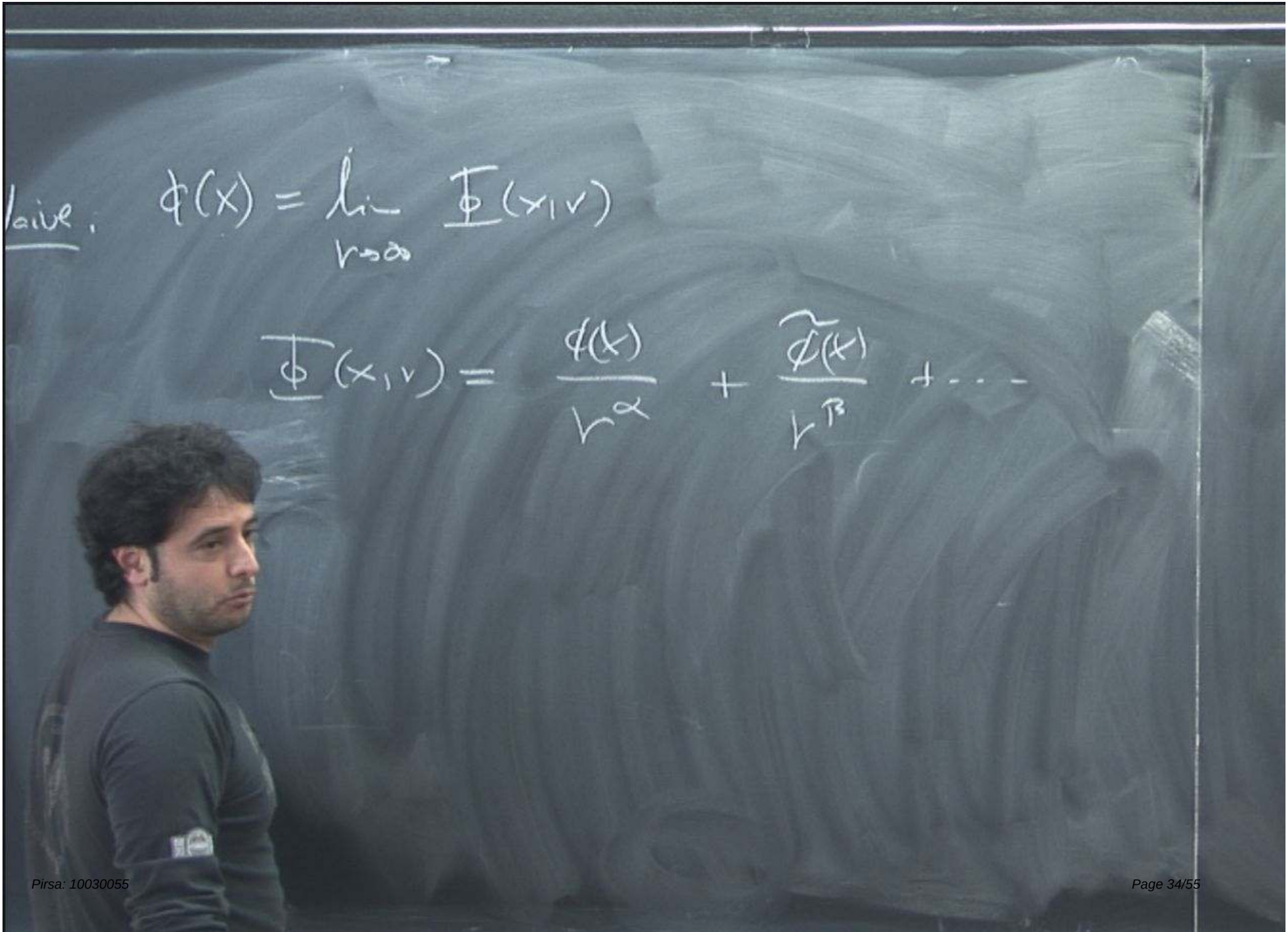

Important case
$\partial^\mu T_{\mu\nu}(x) = 0$
$g_{\mu\nu}(x_{1\nu})$
$\int$

Important case

$$\partial^\mu T_{\mu\nu}(x) = 0$$

$$T_{\mu\nu} \longleftrightarrow g_{\mu\nu}(x)$$

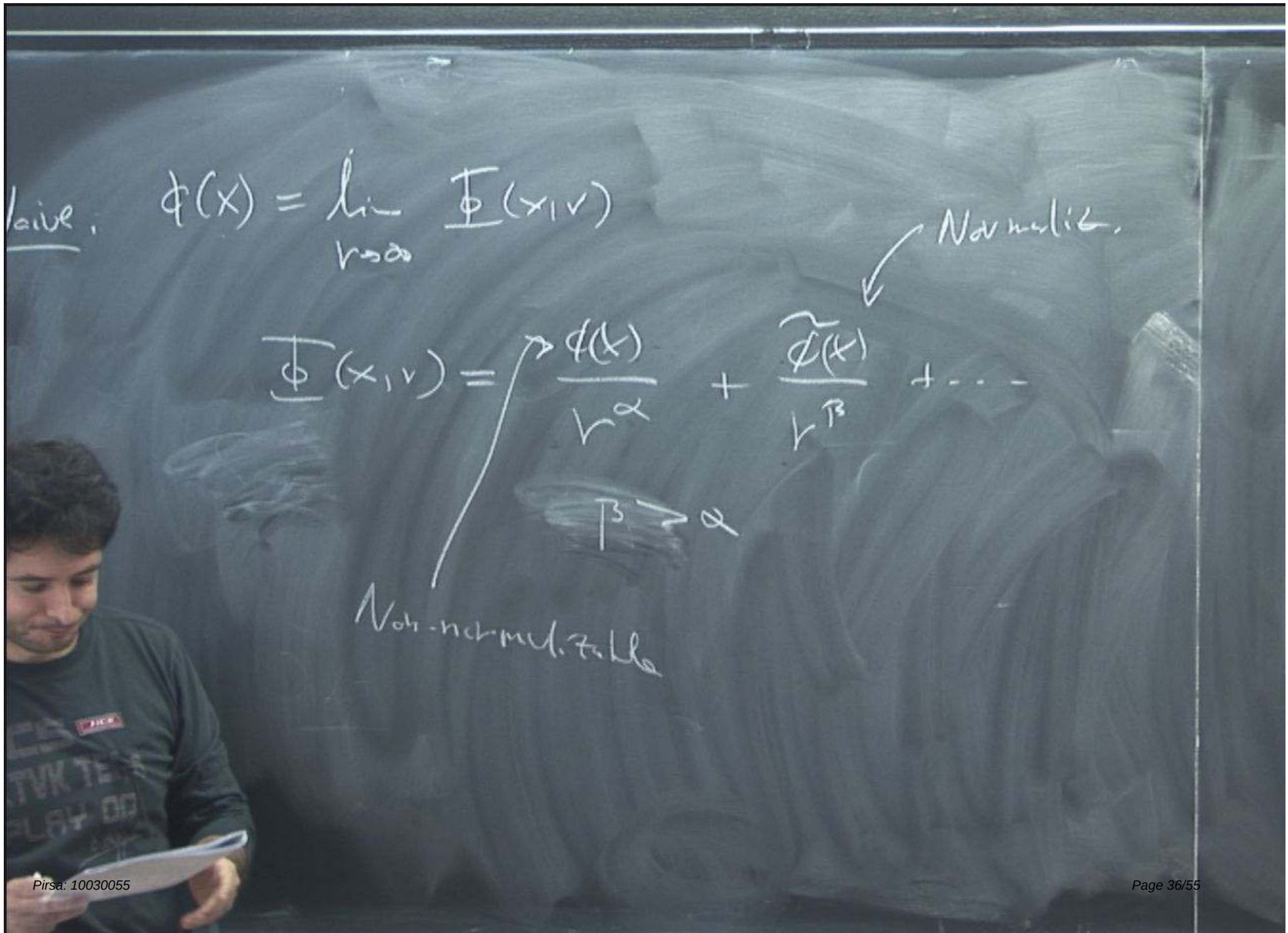
$$\int g^{\mu\nu}(x)\, T_{\mu\nu}(x)$$

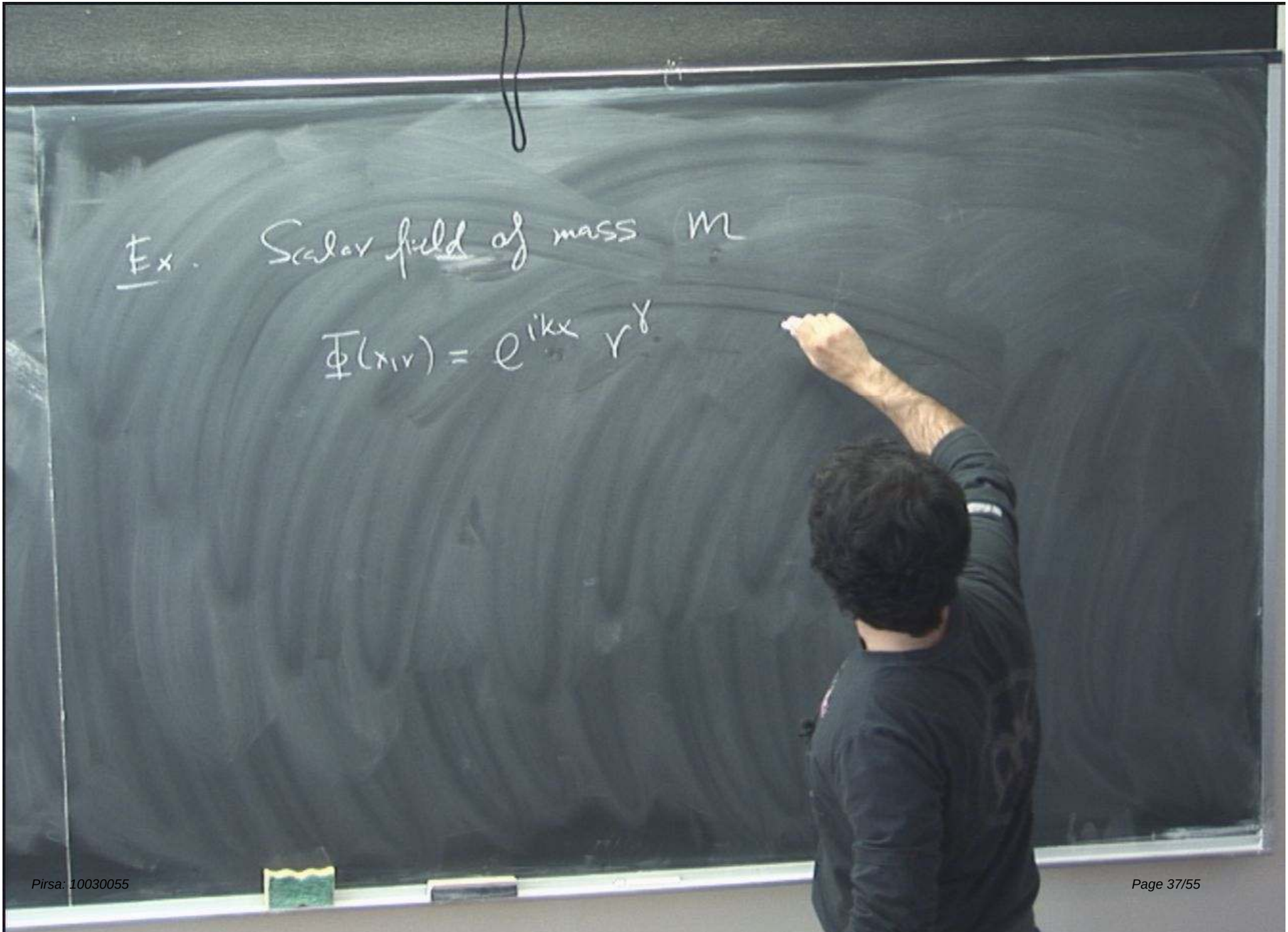

laive. $\phi(x) = \lim_{r\to\infty} \underline{\Phi}(x,v)$
$\underline{\Phi}(x,v) = \frac{\phi(x)}{r^{\alpha}} + \frac{\tilde{\phi}(x)}{r^{\beta}} + \dots$

Naive: $\phi(x) = \lim_{r\to\infty} \Phi(x,r)$

$$\Phi(x,r) = \frac{\phi(x)}{r^{\alpha}} + \frac{\tilde{\phi}(x)}{r^{\beta}} + \dots$$

$\alpha > \beta$


Naive. $\phi(x) = \lim_{r\to\infty} \Phi(x,r)$
Normalizable.
$\Phi(x,r) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \dots$
$\beta > \alpha$
Non-normalizable


Ex. Scalar field of mass m
$\Phi(x,r) = e^{ikx}\, r^{\gamma}$

Ex. Scalar field of mass m

$$\Phi(x,r) = e^{ikx}\, r^{\gamma} \qquad \gamma(\gamma+4) = m^2 R^2$$

Ex. Scalar field of mass m

$$\Phi(x,v) = e^{ikx}\, v^{\gamma} \qquad \gamma(\gamma+4) = m^2 R^2$$

$$\phi(x) = \lim_{v\to\infty} r^{\alpha}\, \Phi(x,v)$$

Ex. Scalar field of mass m

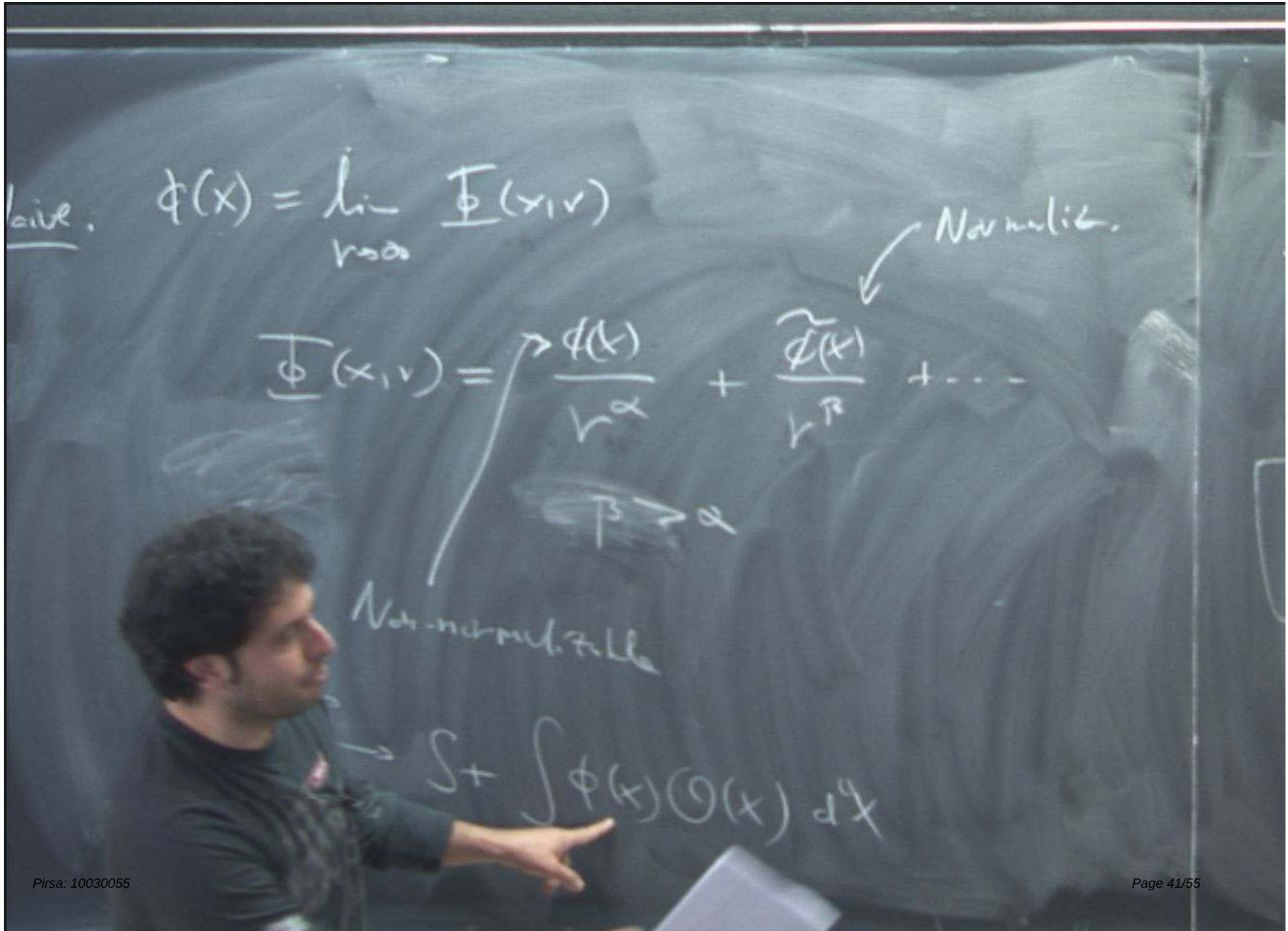
$$\Phi(x,v) = e^{ikx}\, v^{\gamma} \qquad \gamma(\gamma+4) = m^2 R^2$$

$$\boxed{\phi(x) = \lim_{v\to\infty} v^{\alpha}\, \Phi(x,v)}$$

$$\phi \to K^{-\alpha}\phi$$

$$[\phi] = L^{-\alpha} = M^{\alpha}$$

$$[\hat{\phi}] = L^{-\beta} = M^{\beta}$$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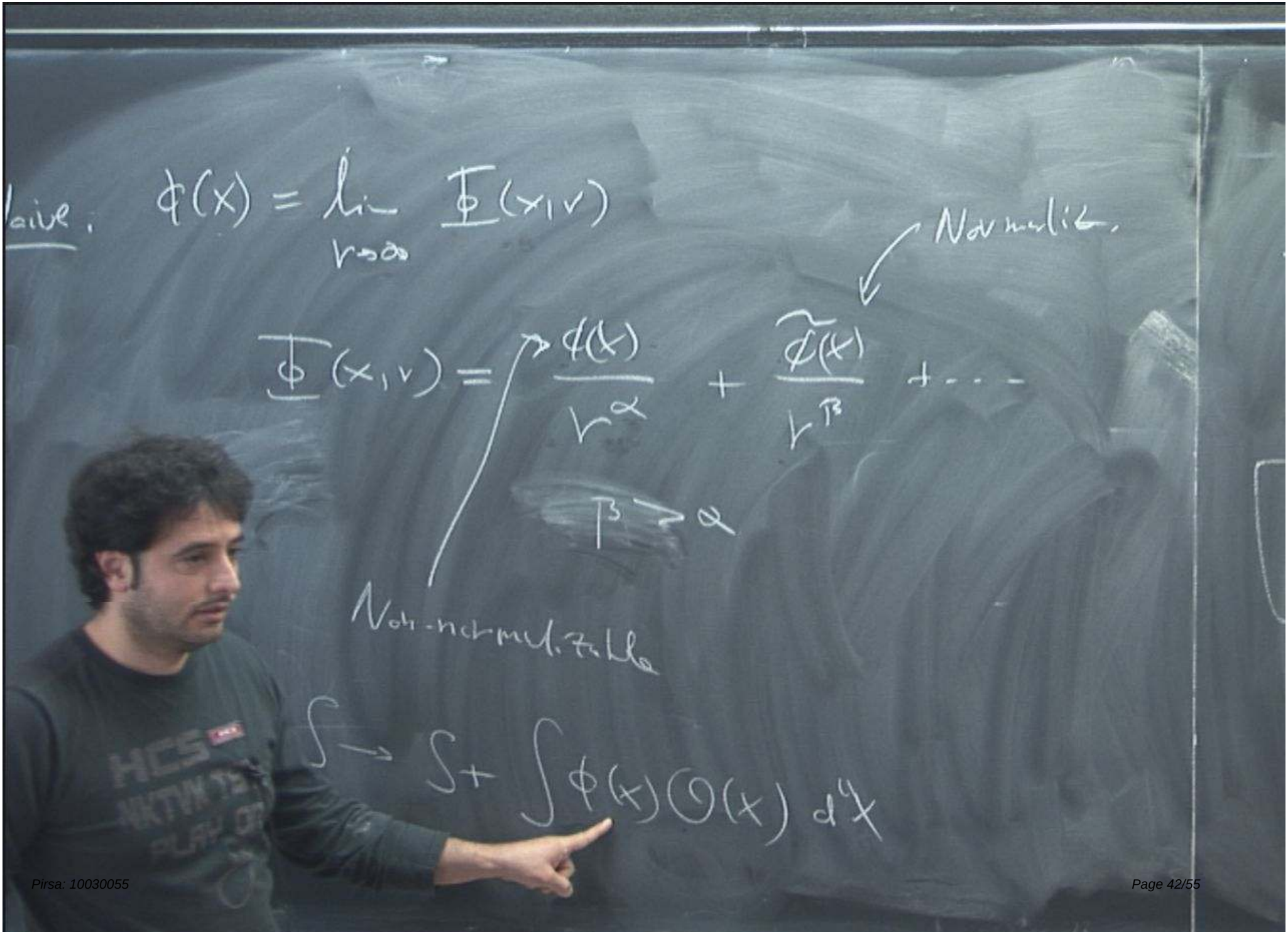
Normaliz.
$\Phi(x,v) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \dots$
$\beta > \alpha$
Non-normalizable
$\to S + \int \phi(x) \mathcal{O}(x)\, dx$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$\Phi(x,v) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \dots$
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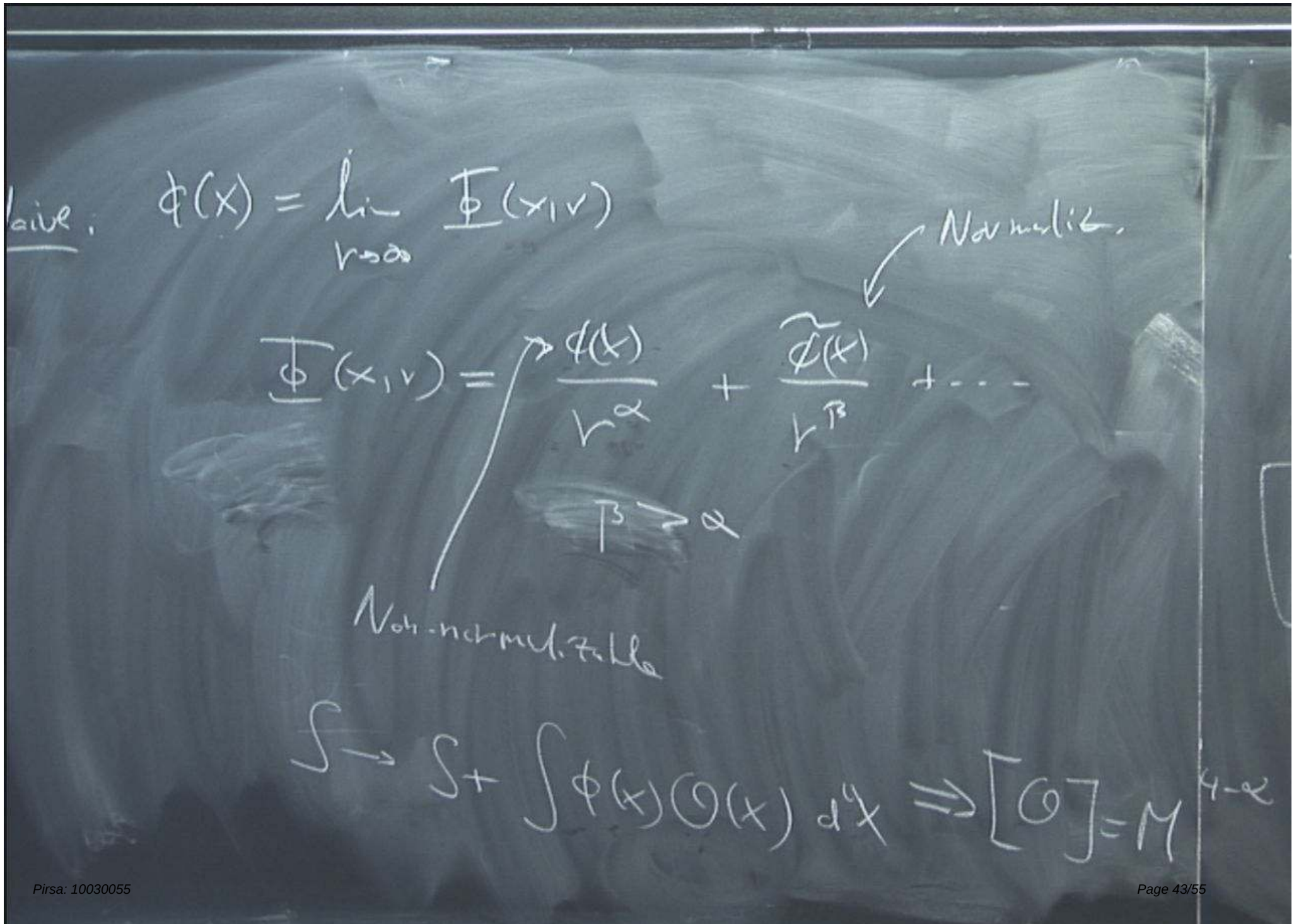
Non-normalizable
$S \to S + \int \phi(x)\,\mathcal{O}(x)\,d^4x$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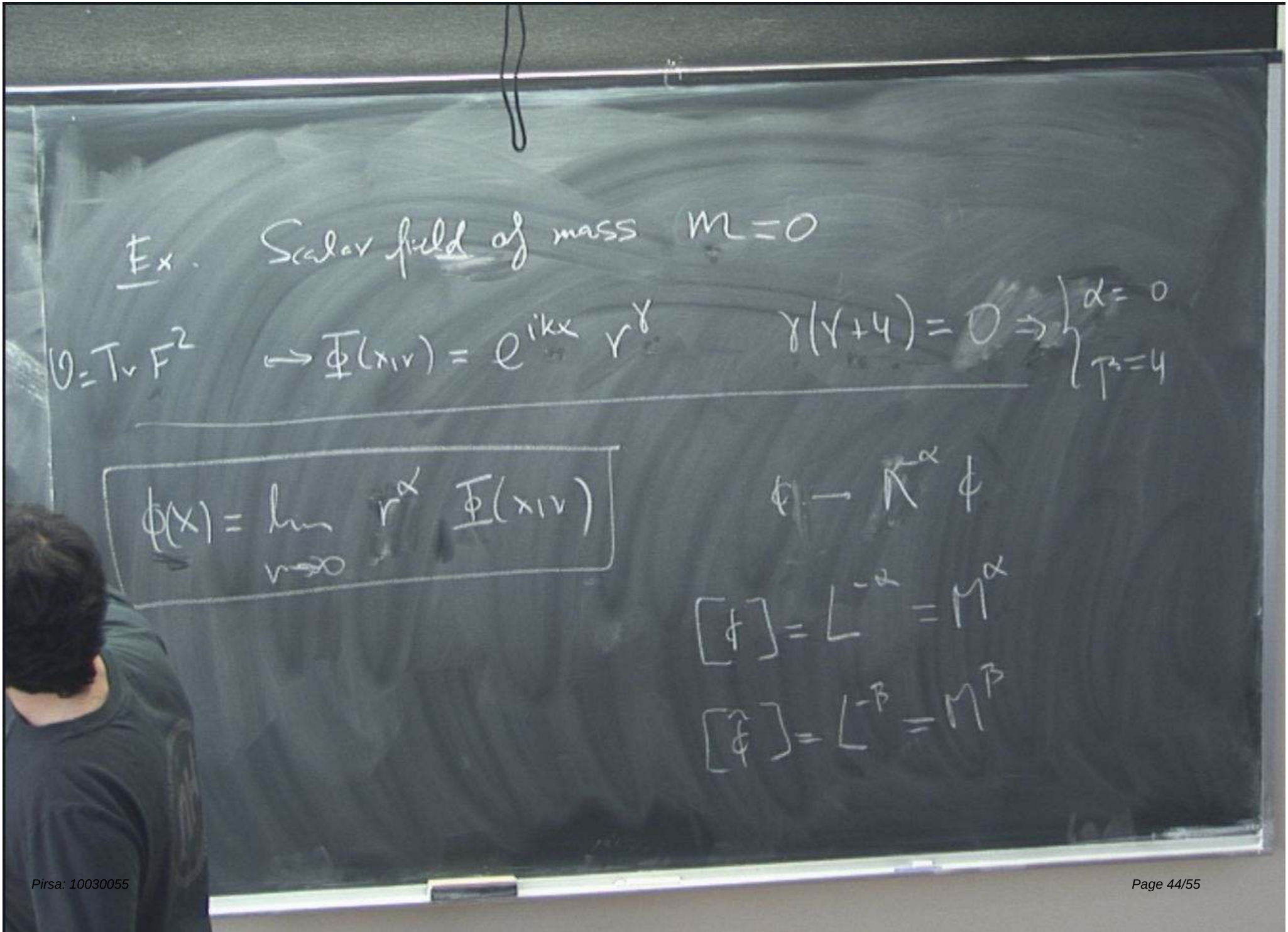
Non-normalizable
$S \to S + \int \phi(x)\mathcal{O}(x)\,dx \Rightarrow [\mathcal{O}] = M^{4-\alpha}$


Ex. Scalar field of mass m=0
$\mathcal{O} = T_\nu F^2 \hookrightarrow \Phi(x,v) = e^{ikx} v^\gamma$
$\gamma(\gamma+4)=0 \Rightarrow \begin{cases} \alpha = 0 \\ \gamma = -4 \end{cases}$
$\phi(x) = \lim_{v\to\infty} v^\alpha \Phi(x,v)$
$\phi \to K^\alpha \phi$
$[\phi] = L^{-\alpha} = M^\alpha$
$[\tilde{\phi}] = L^{-\beta} = M^\beta$

Ex. Scalar field of mass $m=0$

$\mathcal{O} = T_{\nu} F^2 \hookrightarrow \Phi(x,\nu) = e^{ikx}\, r^{\gamma}$ $\quad \gamma(\gamma+4)=0 \Rightarrow \begin{cases} \alpha = 0 \\ \gamma = -4 \end{cases}$

$$\phi(x) = \lim_{r\to\infty} r^{\alpha}\, \Phi(x,r)$$

$$S = -\int \frac{1}{4} T_{\nu} F^2 \, d^4x$$

$M^{4-\alpha}$

Ex. Scalar field of mass $m=0$

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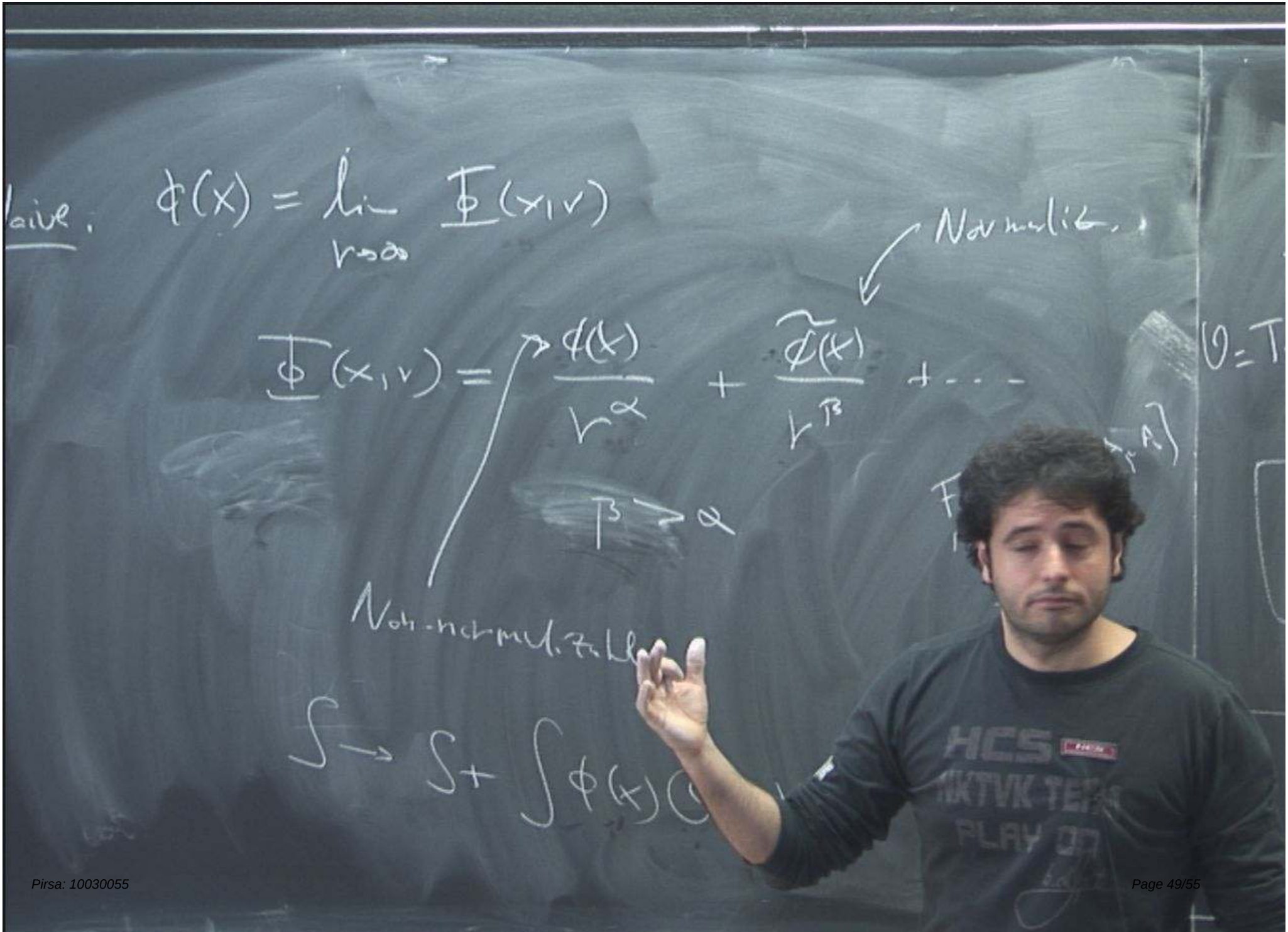
$M^{4-\alpha}$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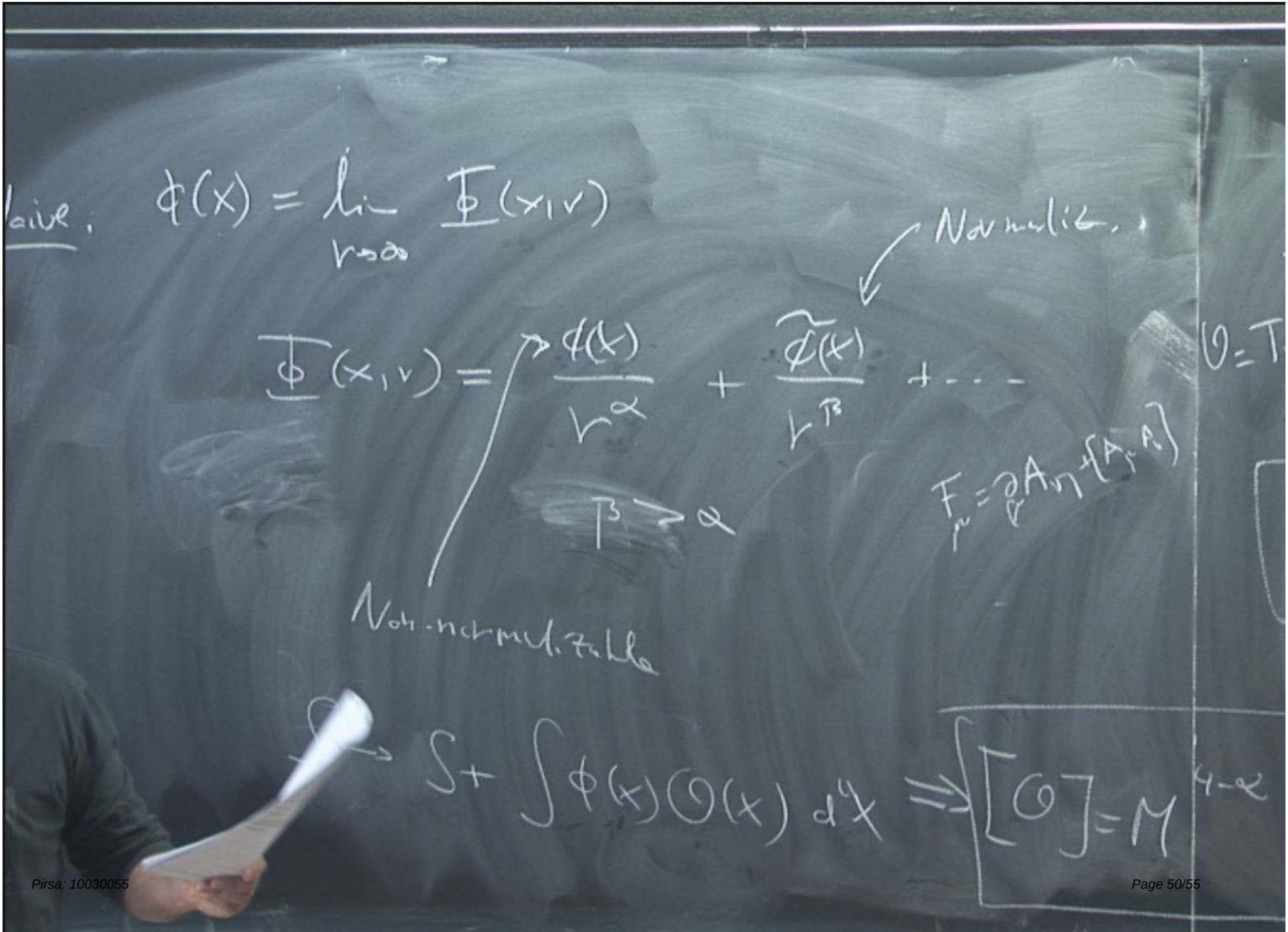
Normaliz.
$\Phi(x,v) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \dots$
$\beta > \alpha$
Non-normaliz.
$S \to S + \int \phi(x)$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$\Phi(x,v) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \dots$
$\beta > \alpha$
$F_{\mu\nu} = \partial_\mu A_\nu + [A_\mu, A_\nu]$
Non-normalizable
$S \to S + \int \phi(x) \mathcal{O}(x)\, d^4x \Rightarrow [\mathcal{O}] = M^{4-\alpha}$

Naive. $\phi(x) = \lim_{r\to\infty} \Phi(x,v)$

Normalizz.

$$\Phi(x,v) = \frac{\phi(x)}{r^{\alpha}} + \frac{\tilde{\phi}(x)}{r^{\beta}} + \dots$$

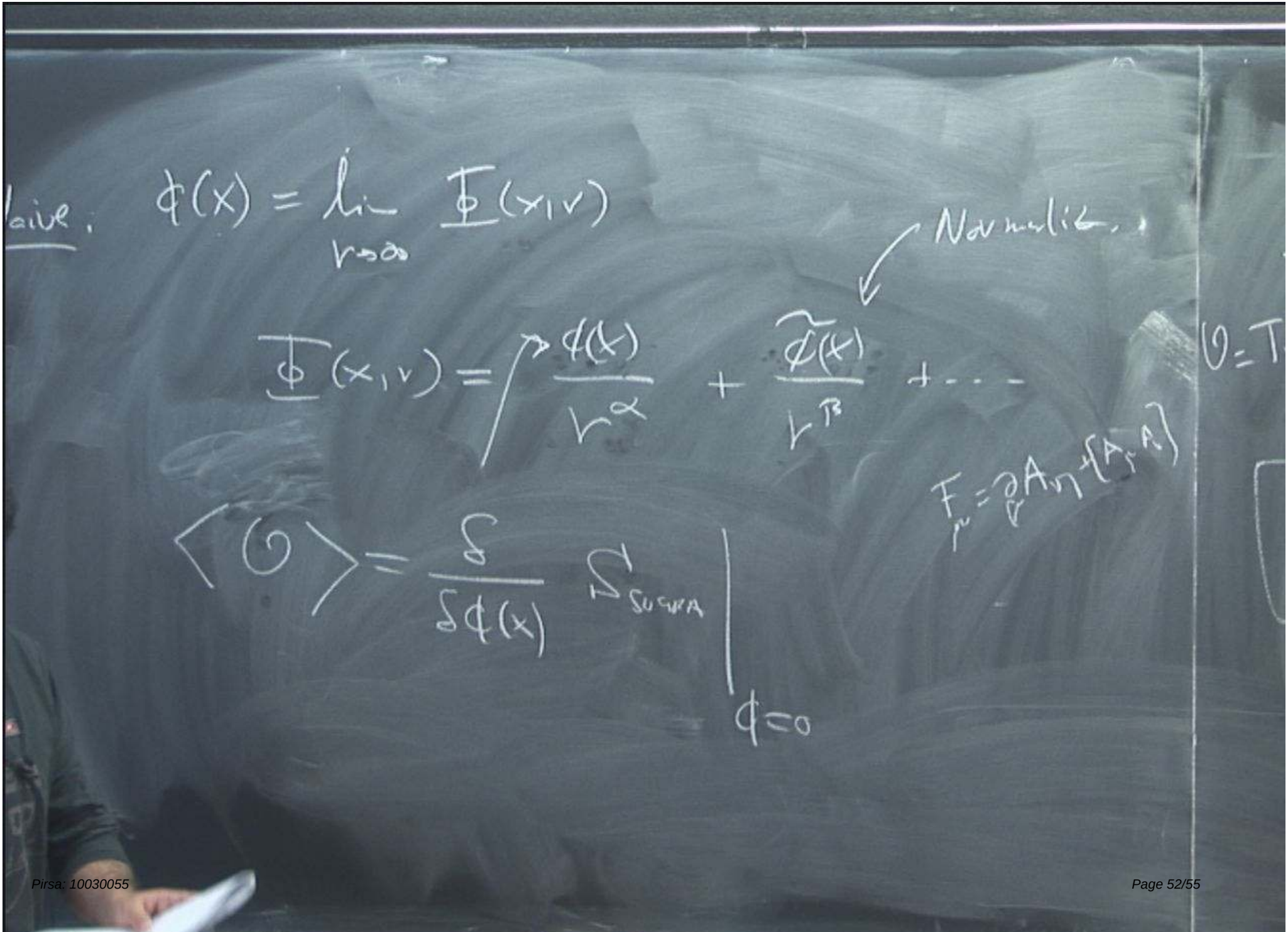
$\beta > \alpha$

Non-normalizzabile

$$S \to S + \int \phi(x)\mathcal{O}(x)\,d^4x \Rightarrow [\mathcal{O}] = 4-\alpha$$

$F_{\mu\nu} = \partial_\mu A_\nu + [A_\mu, A_\nu]$

$\mathcal{O} = T$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Normalize.
$\Phi(x,v) = \int \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \dots$
$\theta = T$
$F_{\mu\nu} = \partial A_{\nu\eta} + [A, A]$
$\langle \mathcal{O} \rangle = \frac{\delta}{\delta\phi(x)} S_{\text{SUGRA}} \Big|_{\phi=0}$

laive. $\phi(x) = \lim_{r\to\infty} \Phi(x,v)$

Normaliz.

$$\Phi(x,v) = \frac{\phi(x)}{r^{\alpha}} + \frac{\tilde{\phi}(x)}{r^{\beta}} + \dots$$

$$\langle \mathcal{O} \rangle = \frac{\delta}{\delta \phi(x)} S_{SUGRA}\Big|_{\phi=0}$$

$$S_{SUGRA} \sim \int_{AdS_5} (\partial\Phi)^2 \sim \int_{\partial AdS} \Phi\, \partial\Phi \sim$$

$$F_{\mu\nu} = \partial_\mu A_\nu + [A_\mu, A_\nu]$$

$\theta = T$

laive. $\phi(x) = \lim_{r\to\infty} \Phi(x,v)$

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$$S_{SUGRA} \sim \int_{AdS_5} (\partial\Phi)^2 \sim \int_{\partial AdS} \Phi\, \partial\Phi \sim$$

Naive. $\phi(x) = \lim_{r\to\infty} \Phi(x,r)$

Normalizable

$$\Phi(x,r) = \frac{\phi(x)}{r^{\alpha}} + \frac{\tilde{\phi}(x)}{r^{\beta}} + \dots$$

$$\langle \mathcal{O} \rangle = \frac{\delta}{\delta \phi(x)} S_{SUGRA}\Big|_{\phi=0} \sim \tilde{\phi}(x)$$

$F_{\mu\nu} = \partial_\mu A_\nu + [A_\mu, A_\nu]$

$$S_{SUGRA} \sim \int_{AdS_5} (\partial\Phi)^2 \sim \int_{\partial AdS} \Phi\, \partial\Phi \sim \phi^2 + \phi\tilde{\phi}$$

∂AdS

$\Theta = T$