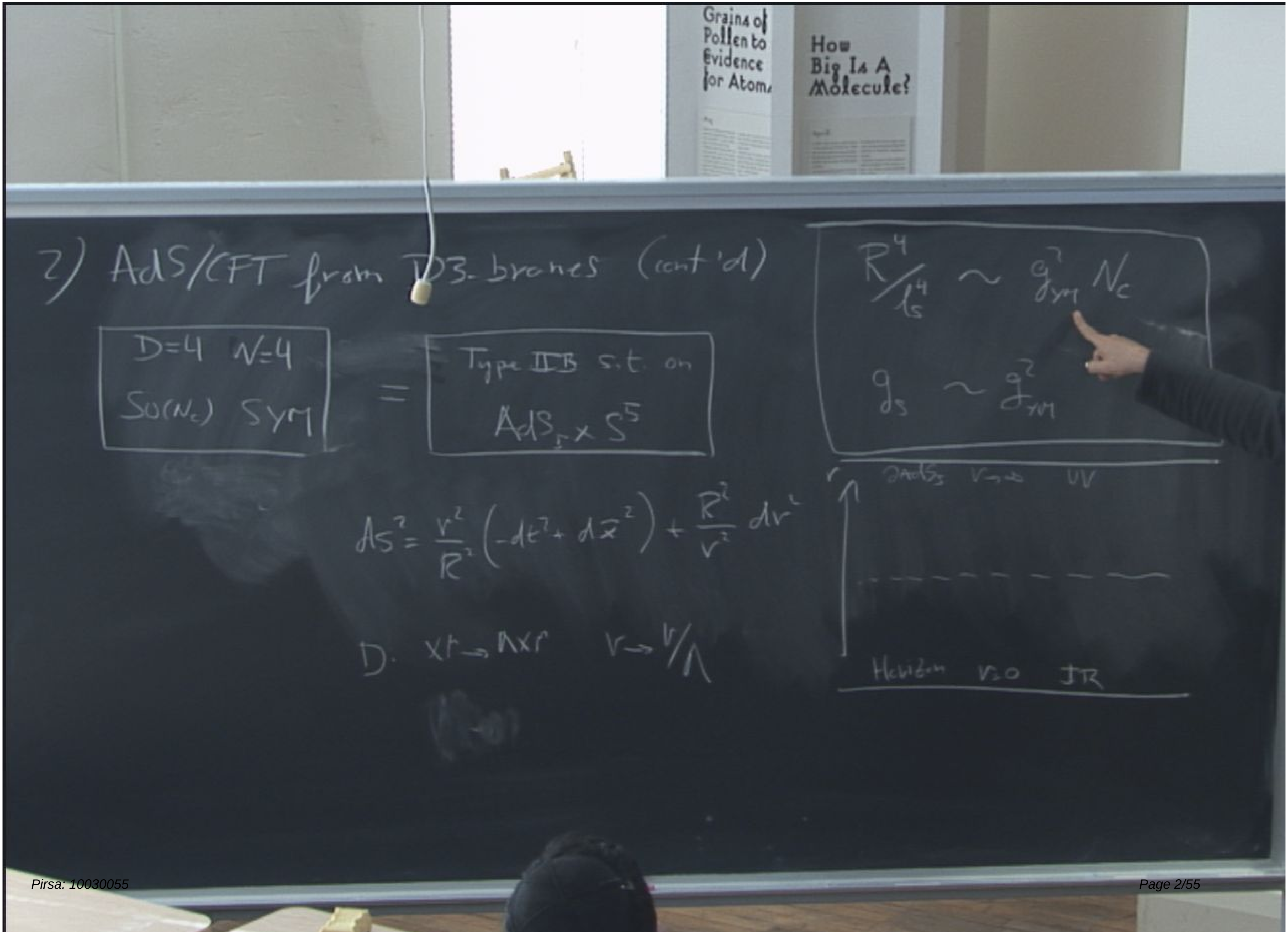
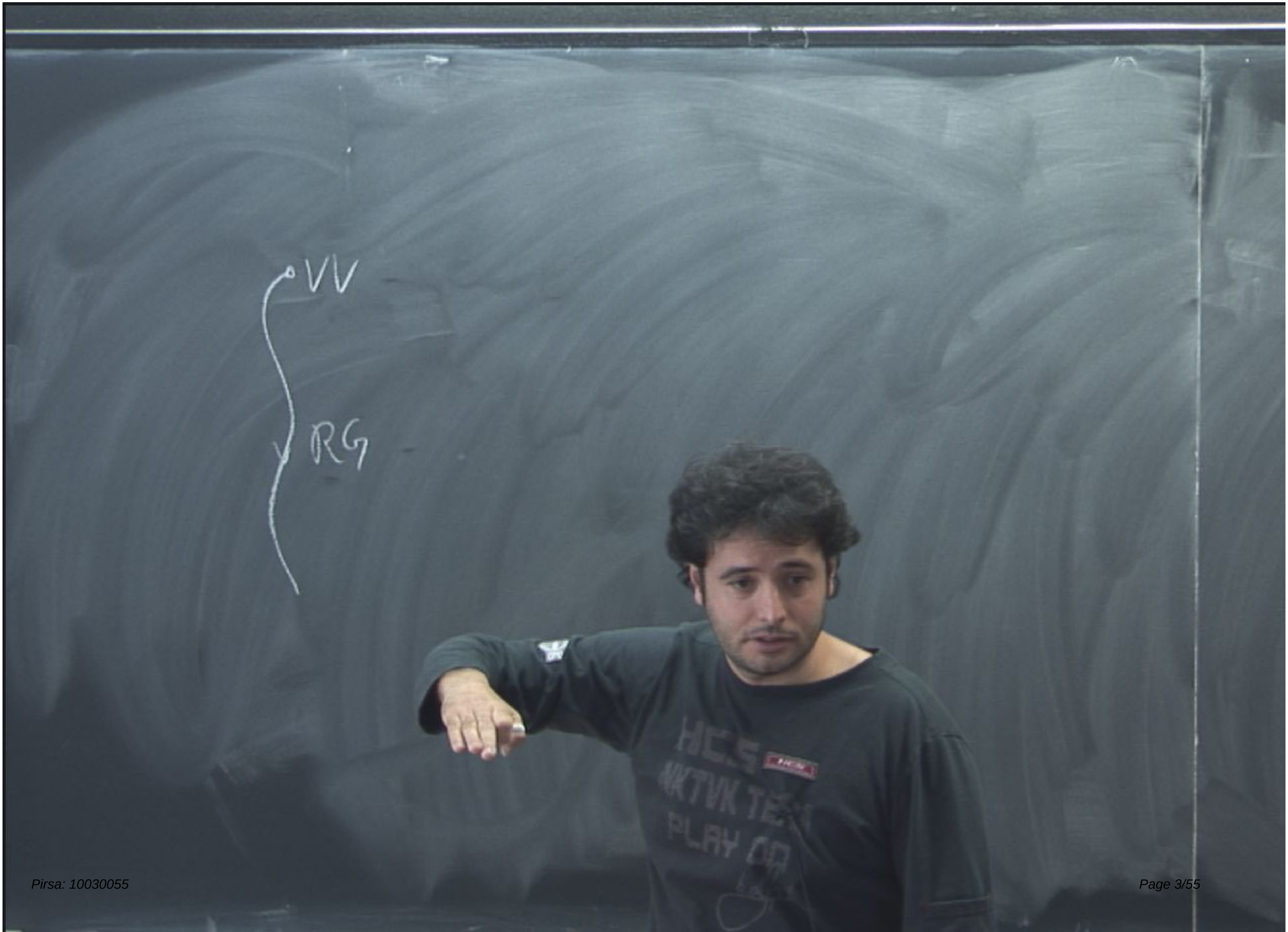
Title: Explorations in String Theory (PHYS 647) - Lecture 13

Date: Mar 03, 2010 11:20 AM

URL: http://pirsa.org/10030055

Abstract:


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
2) AdS/CFT from D3-branes (cont'd)
D=4 N=4 SO(N_c) SYM
=
Type IIB s.t. on AdS_5 x S^5
R^4/l_s^4 ~ g_YM^2 N_c
g_s ~ g_YM^2
ds^2 = r^2/R^2 (-dt^2 + dx^2) + R^2/r^2 dr^2
D. x^μ → Λx^μ   r → r/Λ
r
∂AdS_5   r→∞   UV
Horizon   r=0   IR


UV
RG

AdS/CFT from D3-branes (cont'd)

$$\boxed{\begin{matrix} D=4 \quad \mathcal{N}=4 \\ SU(N_c) \ \text{SYM} \end{matrix}} = \boxed{\begin{matrix} \text{Type IIB s.t. on} \\ AdS_5 \times S^5 \end{matrix}}$$

$$\boxed{ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^{\,2}\right) + \frac{R^2}{r^2}\, dr^2}$$

D. $x^\mu \to \Lambda x^\mu \quad r \to r/\Lambda$

$$\boxed{\begin{matrix} R^4/l_s^4 \sim g_{YM}^2 N_c \\ g_s \sim g_{YM}^2 \end{matrix}}$$

r

$\partial AdS_5 \quad r \to \infty \quad$ UV

$M \sim V_{1,3}$

Horizon $\quad r = 0 \quad$ IR

Precise formulation

$$g_s = e^{\Phi_\infty}$$

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Precise formulation

$$g_{eff} = e^{\Phi}$$

$$\int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \Phi R)\, d^2\sigma}$$

Precise formulation

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$$\int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \Phi R)\, d^2\sigma}$$

$$\Phi \int_{\Sigma} R \sim \Phi \chi$$

Precise formulation

$$g_{eff} = e^{\Phi}$$

$$\sum_{\chi} \int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \underbrace{\Phi R}_{})\, d^2\sigma} = e^{-\Phi \chi} \int \mathcal{D}[X$$

$$\Phi \int_{\Sigma} R \sim \Phi \chi$$

Precise formulation

$$g_{\text{eff}} = g_s e^{\Phi}$$

$$\sum_{\chi} g_s^{-\chi} \int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \underbrace{\Phi R})\, d^2\sigma} = \sum e^{-\Phi \chi} g_s^{-\chi} \int \mathcal{D}[X$$

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Precise formulation

$$g_{eff} = g_s e^{\Phi}$$

$$\sum_{\chi} g_s^{-\chi} \int \mathcal{D}[X(\sigma)]\, e^{-\int (\partial X^2 + \underbrace{\Phi R})\, d^2\sigma} = \sum e^{-\Phi \chi} g_s^{-\chi} \int \mathcal{D}[X]\, e^{-\int (\partial X)^2}$$

$$\Phi \int_{\Sigma} R \sim \Phi \chi$$

Precise formulation

$$g_s = e^{\Phi_\infty}$$

$$\left\{ \begin{array}{l} \text{Coupling} \\ \text{const. in g.t.} \end{array} \right\} \longleftrightarrow \Big\{$$

Precise formulation

$$g_s = e^{\Phi_\infty}$$

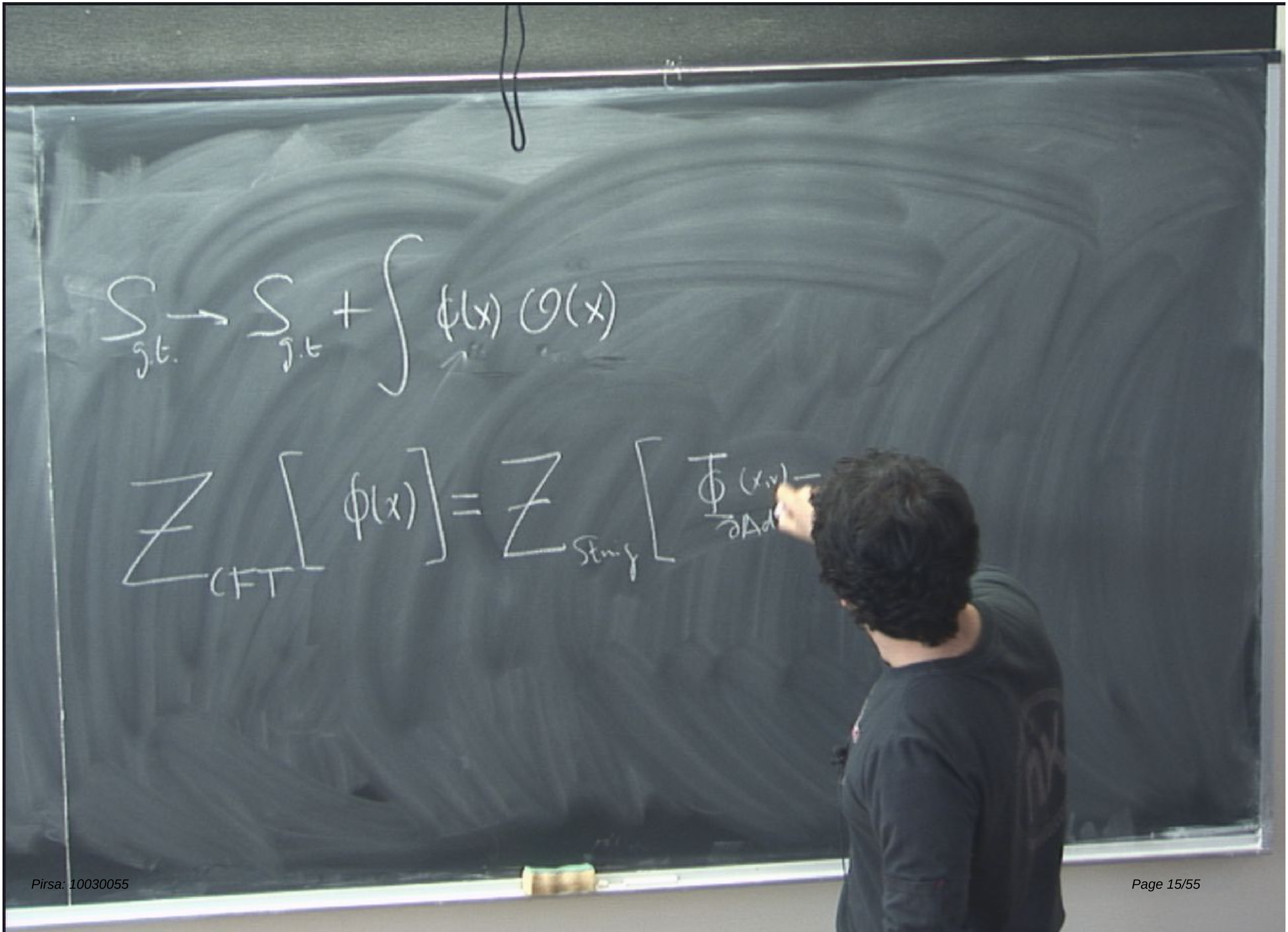
$$\left\{ \begin{array}{l} \text{Coupling} \\ \text{const. in g.t.} \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} \text{Fields at} \\ \infty \end{array} \right\}$$

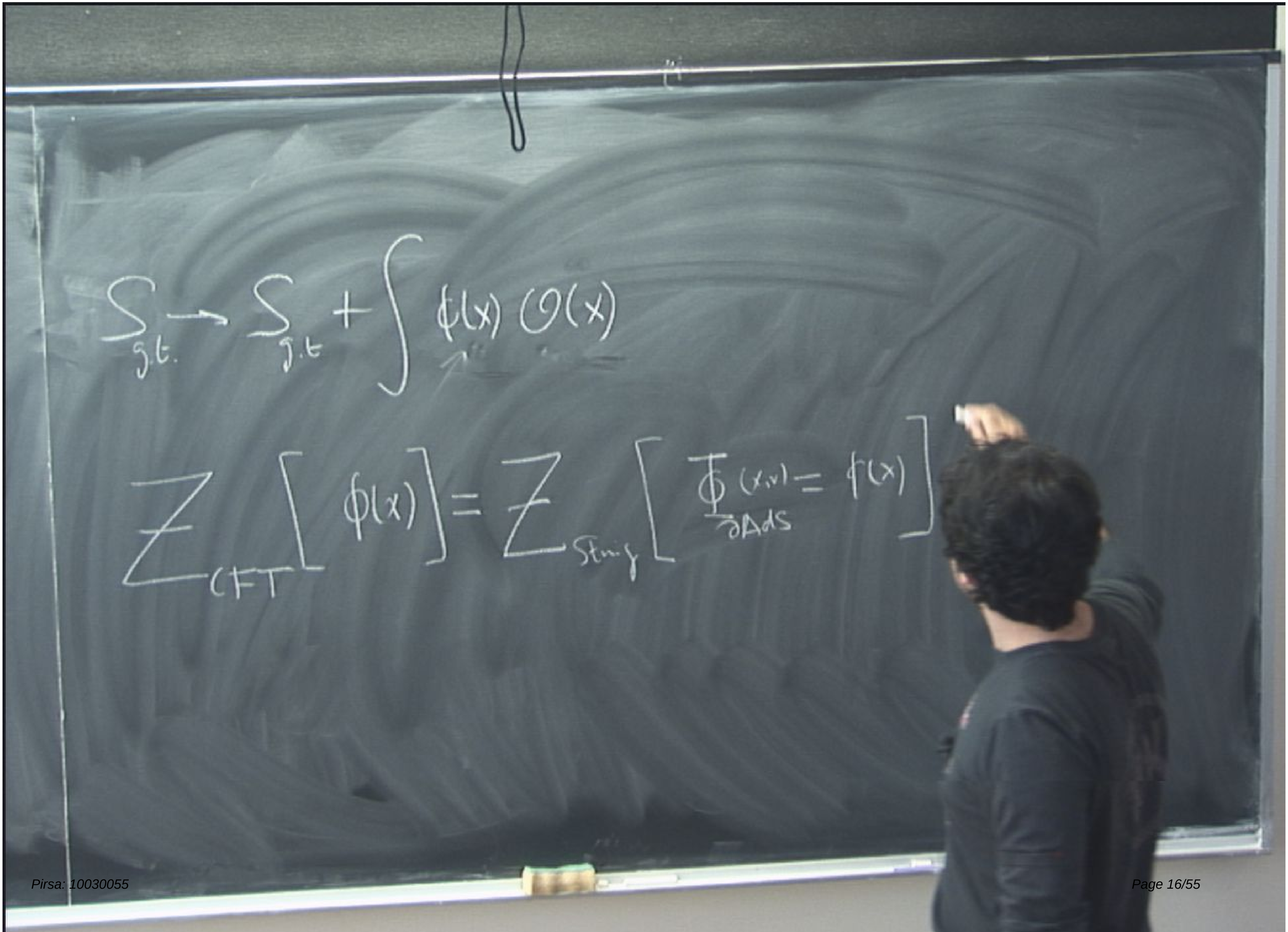
$$\phi(x) = \lim_{r\to\infty} \Phi(x, r)$$

$$S_{g.t.} \to S_{g.t.} + \int \phi(x)\, \mathcal{O}(x)$$

Ex. $\mathrm{Tr}\, F^2$

source


$S_{g.t.} \to S_{g.t} + \int \phi(x)\,\mathcal{O}(x)$
$Z_{CFT}[\phi(x)] = Z_{String}[\Phi_{\partial Ad}(x,r) -$


$S_{g.t.} \to S_{g.t} + \int \phi(x)\, \mathcal{O}(x)$
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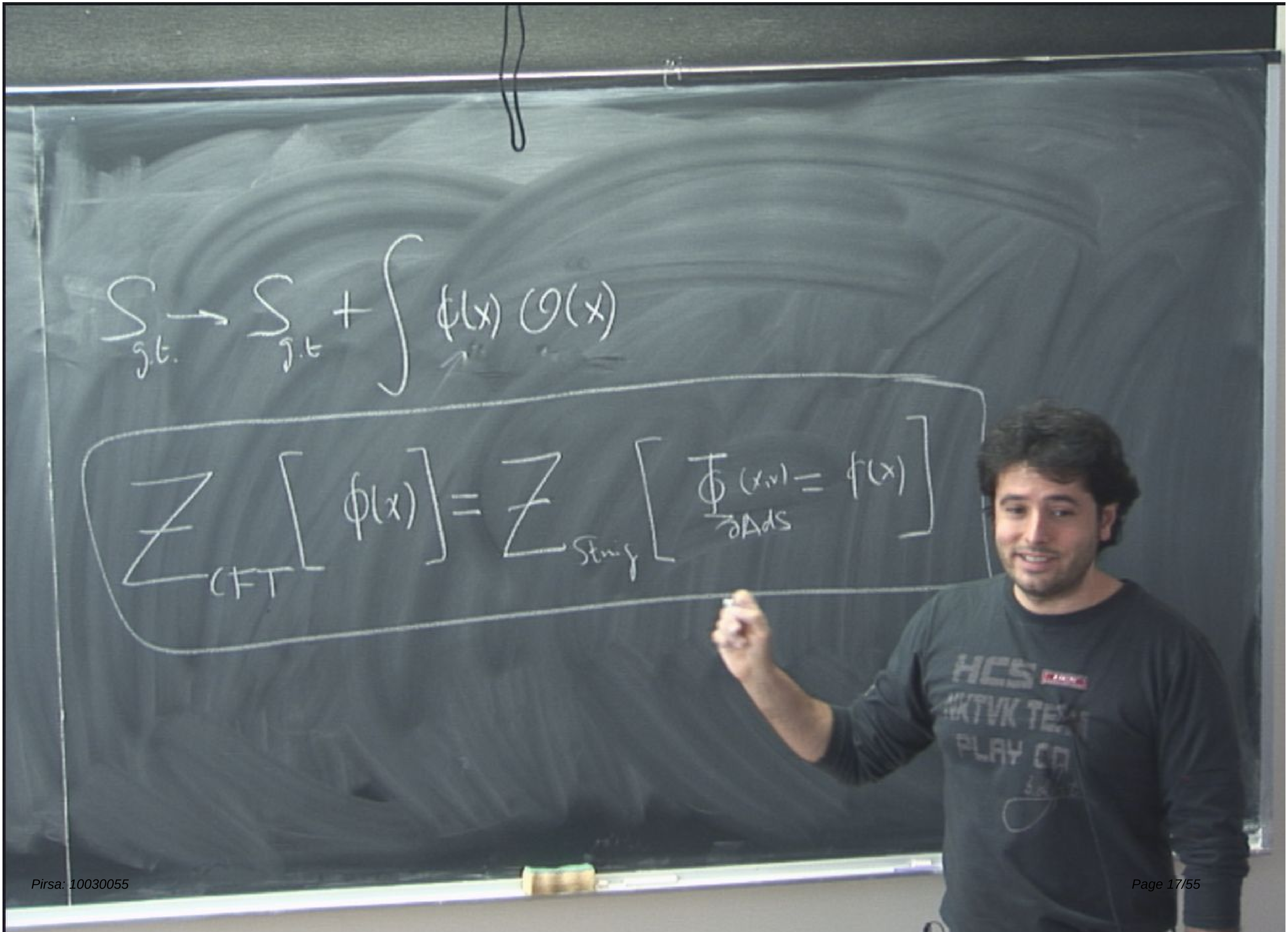
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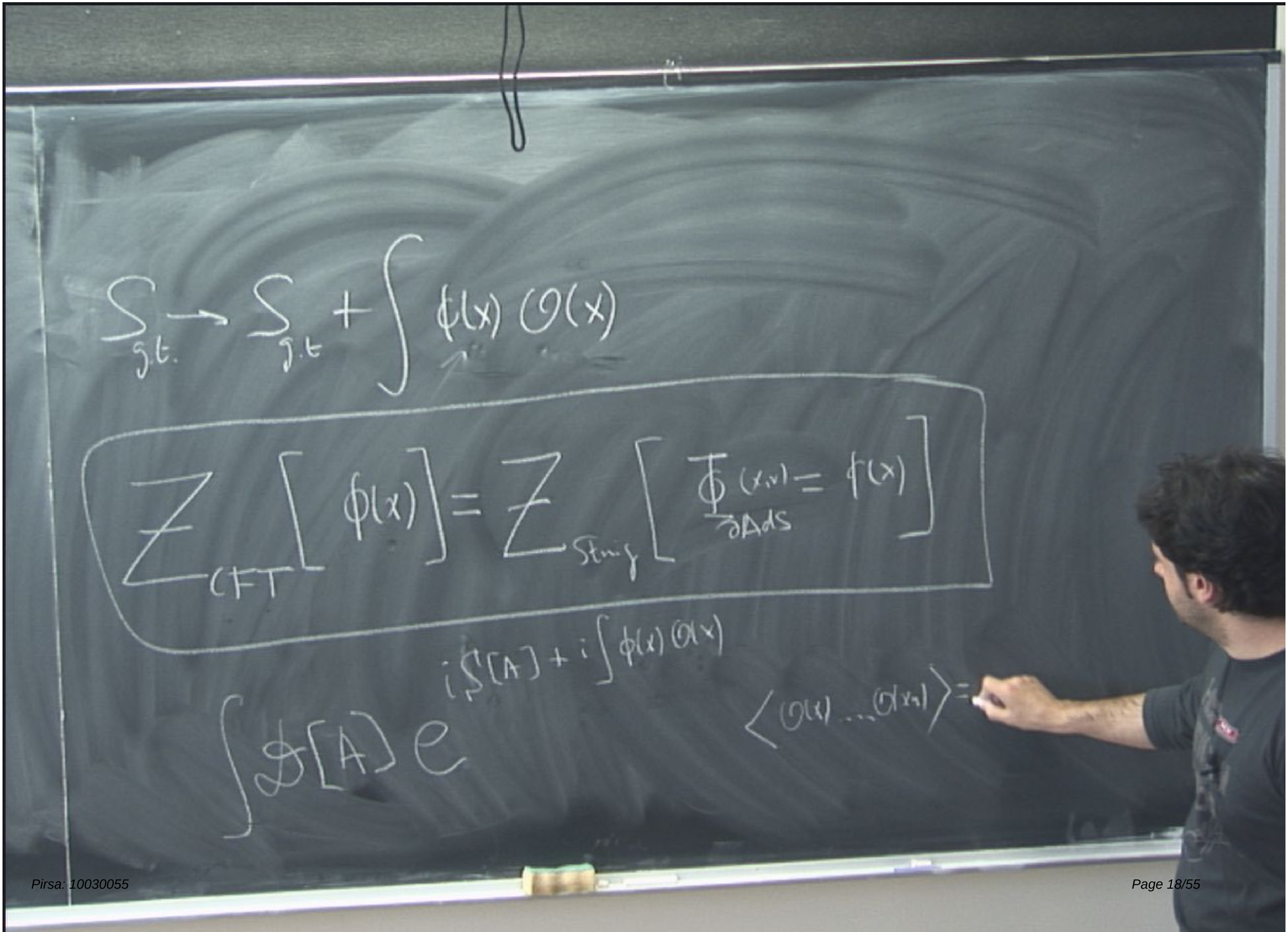

$S_{g.t.} \to S_{g.t} + \int \phi(x)\, \mathcal{O}(x)$
$Z_{CFT}[\phi(x)] = Z_{String}[\Phi_{\partial AdS}(x,r) = \phi(x)]$
$\int \mathcal{D}[A]\, e^{iS[A] + i\int \phi(x)\mathcal{O}(x)}$
$\langle \mathcal{O}(x_1) \ldots \mathcal{O}(x_n) \rangle =$

$$S_{g.t.} \to S_{g.t.} + \int \phi(x)\,\mathcal{O}(x)$$

$$\boxed{Z_{CFT}\left[\phi(x)\right] = Z_{String}\left[\Phi(x,r)\big|_{\partial AdS} = \phi(x)\right]}$$

$\lambda = g^2_{YM} N_c \to \infty \Rightarrow$ S.t. ~ Classical SUGRA. $\quad Z_{St} \simeq e^{-S_{SUGRA}[\Phi_{class}]}$

$g^2_{YM} \to 0,\ N_c \to \infty$

Precise formulation

$$\left\langle \mathrm{Tr}\, F^2(x)\, \mathrm{Tr}\, F^2(y) \right\rangle_{CFT}$$

Precise formulation

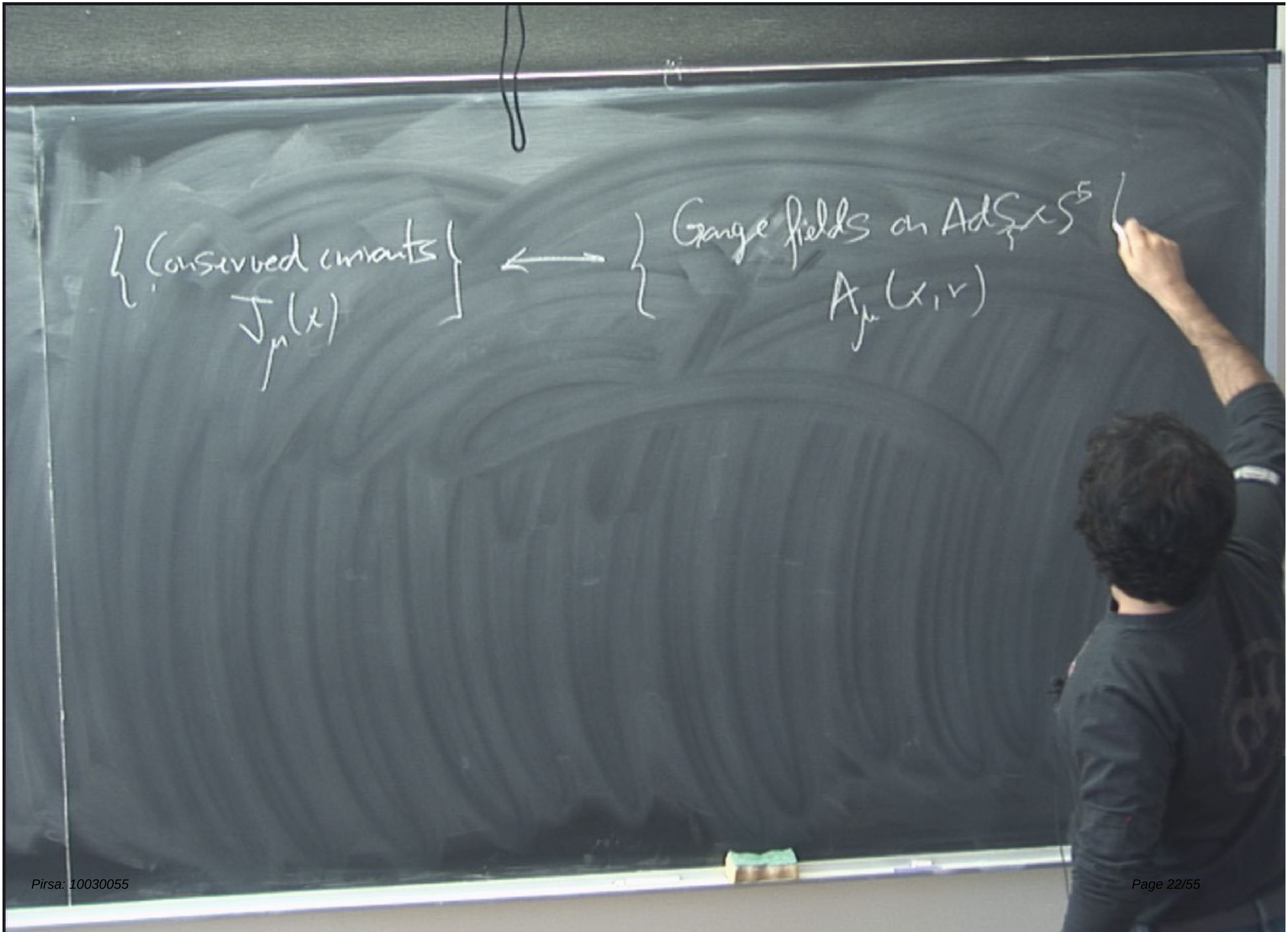
$$\left\langle \underbrace{\mathrm{Tr}\, F^2(x)}_{\mathcal{O}(x)}\ \mathrm{Tr}\, F^2(y) \right\rangle_{CFT}$$

$$\mathcal{O}(x)$$

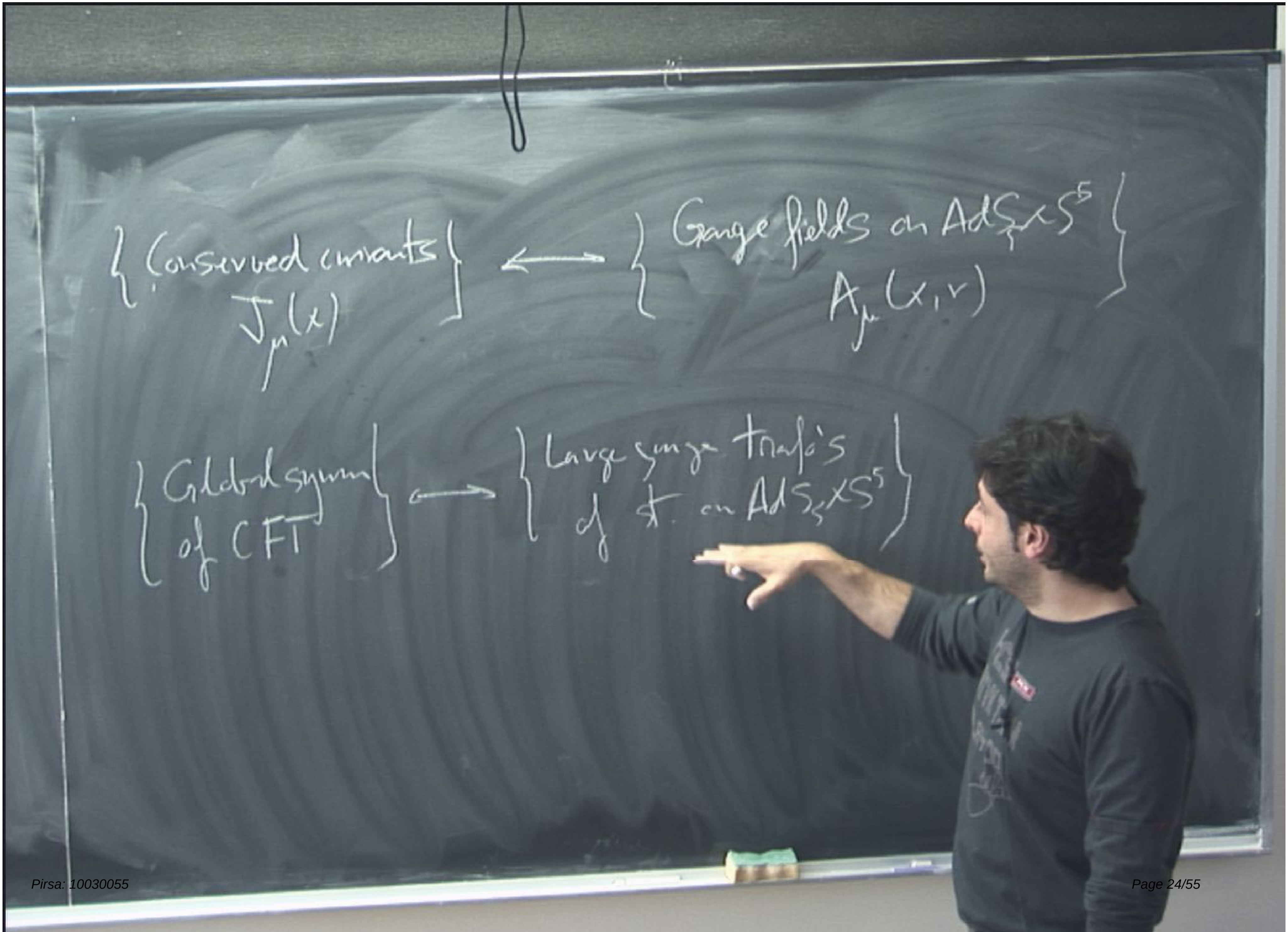
$$\updownarrow$$

$$\Phi(x, r)$$

Dilaton


{Conserved currents
$J_\mu(x)$} $\longleftrightarrow$ {Gauge fields on $AdS_5 \times S^5$
$A_\mu(x,r)$

$$\left\{ \begin{matrix} \text{Conserved currents} \\ J_\mu(x) \end{matrix} \right\} \longleftrightarrow \left\{ \begin{matrix} \text{Gauge fields on } AdS_5 \times S^5 \\ A_\mu(x,r) \end{matrix} \right\}$$


{Conserved currents $J_\mu(x)$} ⟷ {Gauge fields on $AdS_5 \times S^5$ $A_\mu(x,r)$}
{Global symm of CFT} ⟶ {Large gauge transf's of st. on $AdS_5 \times S^5$}

Precise formulation

$$\langle \underbrace{\mathrm{Tr}\, F^2(x)}_{\mathcal{O}(x)}\ \mathrm{Tr}\, F^2(y) \rangle_{CFT}$$

$$\mathcal{O}(x)$$

$$\updownarrow$$

$$\Phi(x, r) \xrightarrow{r \to \infty} \phi(x)$$

Dilaton

$$x \to (1 + \text{[illegible]})\, x$$

$$r \to r$$

Precise formulation

$$\Big\langle \underbrace{\mathrm{Tr}\,F^2(x)}_{\mathcal{O}(x)}\ \mathrm{Tr}\,F^2(y) \Big\rangle_{CFT}$$

$$\mathcal{O}(x)$$

$$\updownarrow$$

$$\Phi(x, r) \xrightarrow{r \to \infty} \phi(x)$$

Dilaton

$$x \to x + \frac{r}{r^2+1}\, a$$

$$r \to r$$

$$\left\{ \begin{array}{c} \text{Conserved currents} \\ J_\mu(x) \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} \text{Gauge fields on } AdS_5\times S^5 \\ A_\mu(x,r) \end{array} \right\}$$

$$\left\{ \begin{array}{c} \text{Global symm} \\ \text{of CFT} \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} \text{Large gauge transf's} \\ \text{of st. on } AdS_5\times S^5 \end{array} \right\}$$

Precise formulation

$$S \to S + \underbrace{\int \phi(x)\, \mathcal{O}(x)}$$

$$\int A_\mu(x)\, J^\mu(x)$$

$$A_\mu(x) = \lim_{r \to} A_\mu(x \mid r)$$

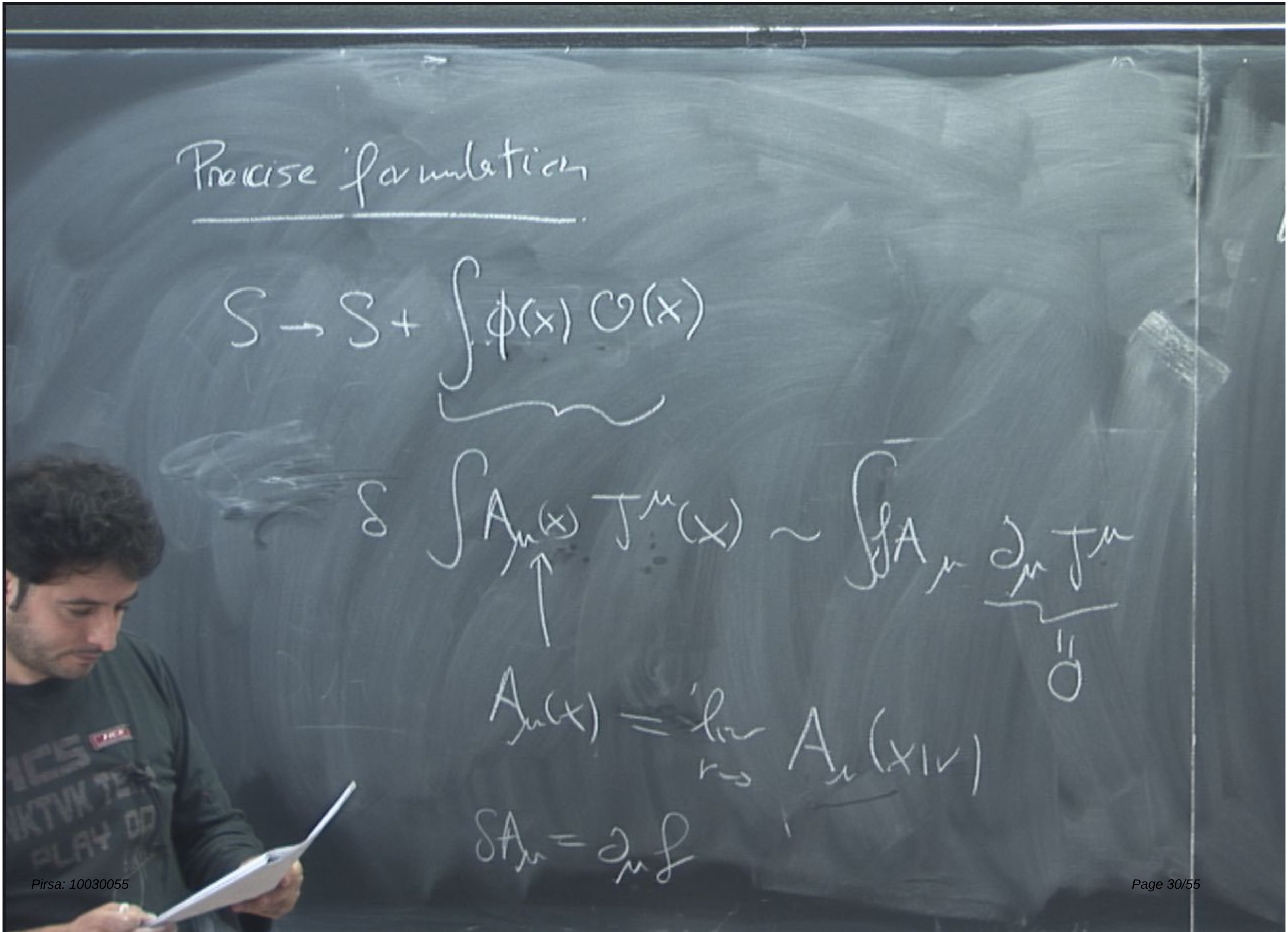
Precise formulation

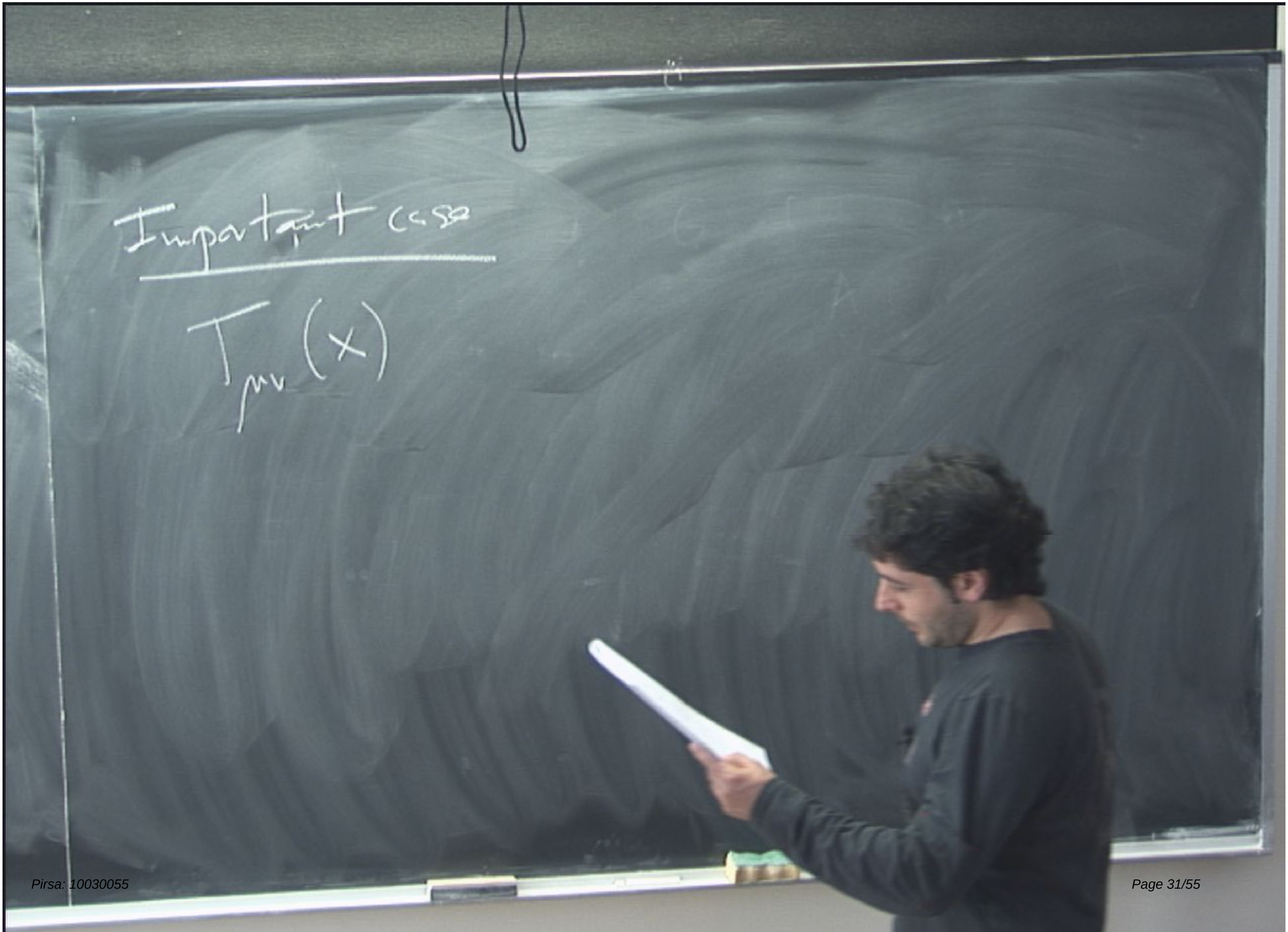
$$S \to S + \underbrace{\int \phi(x)\, \mathcal{O}(x)}$$

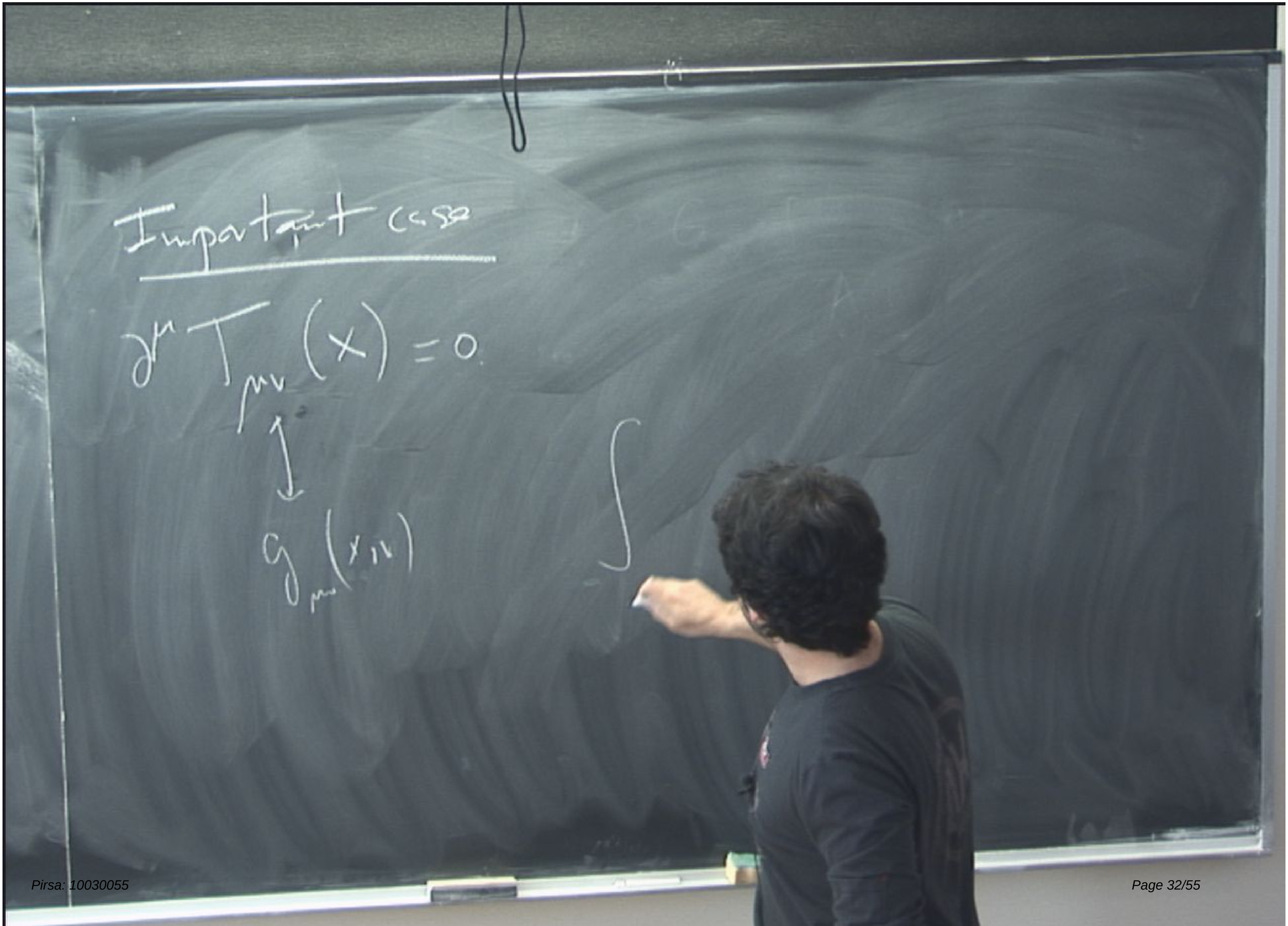
$$\int A_\mu(x)\, J^\mu(x)$$

$$A_\mu(x) = \lim_{r\to} A_\mu(x|r)$$

$$\delta A_\mu = \partial_\mu f$$


Precise formulation
$S \to S + \int \phi(x)\, \mathcal{O}(x)$
$\delta \int A_\mu(x) J^\mu(x) \sim \int \delta A_\mu \underbrace{\partial_\mu J^\mu}_{=0}$
$A_\mu(x) = \lim_{r \to} A_\mu(x|r)$
$\delta A_\mu = \partial_\mu f$


Important case
$T_{\mu\nu}(x)$


Important case
$\partial^\mu T_{\mu\nu}(x) = 0$
$g_{\mu\nu}(x, \nu)$

Important case

$$\partial^\mu T_{\mu\nu}(x) = 0$$

$$\updownarrow$$

$$g_{\mu\nu}(x, \nu)$$

$$\int g^{\mu\nu}(x)\, T_{\mu\nu}(x)$$

Naive: $\phi(x) = \lim_{r \to \infty} \Phi(x,r)$

$$\Phi(x,r) = \frac{\phi(x)}{r^{\alpha}} + \frac{\tilde{\phi}(x)}{r^{\beta}} + \dots$$

Naive:

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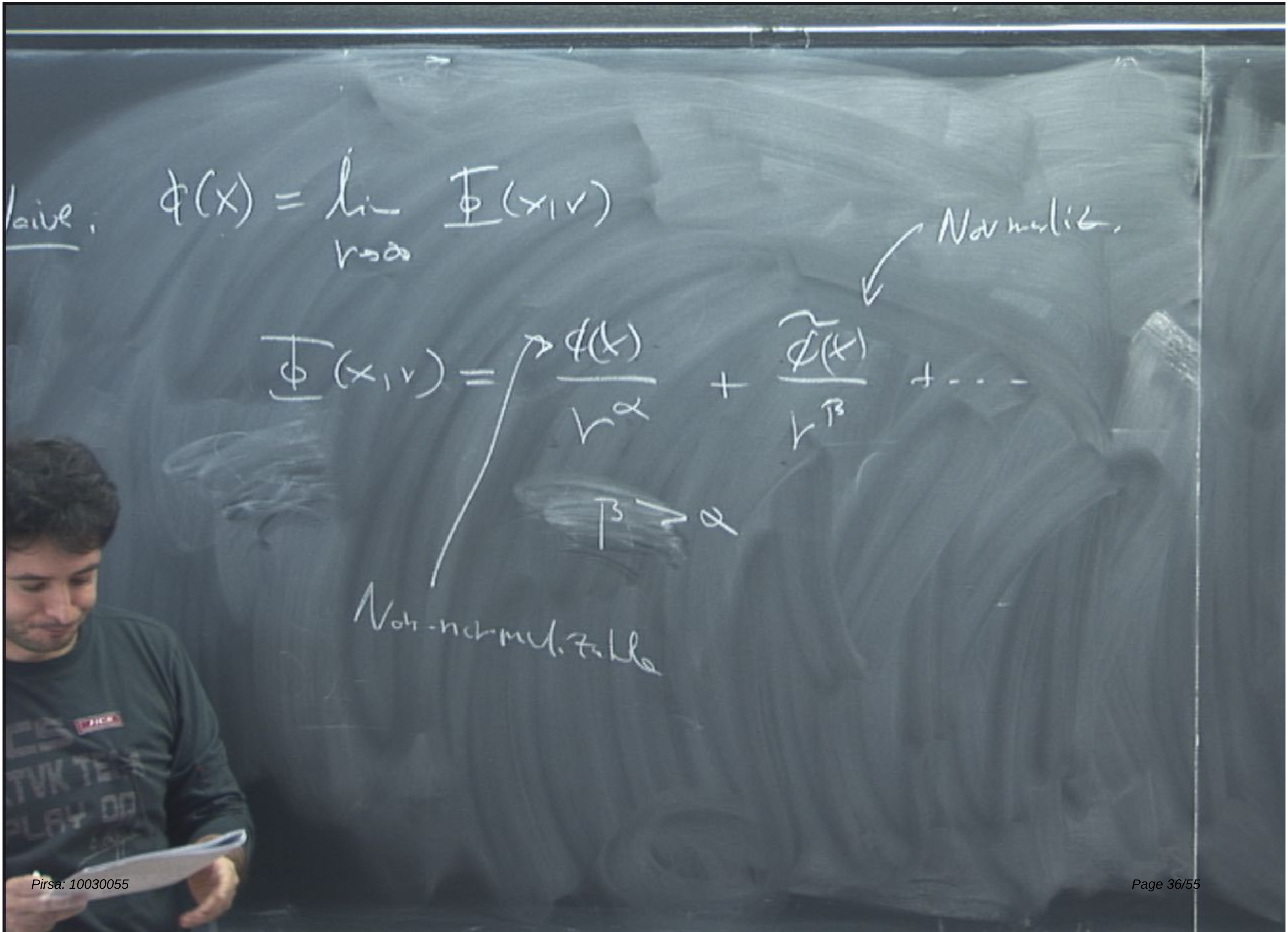
$$\alpha > \beta$$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$\beta > \alpha$
Normalizable
Non-normalizable

Ex. Scalar field of mass m

$$\Phi(x, r) = e^{ikx} \, r^{\gamma}$$

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$$\Phi(x, r) = e^{ikx} \, r^{\gamma} \qquad \gamma(\gamma+4) = m^2 R^2$$

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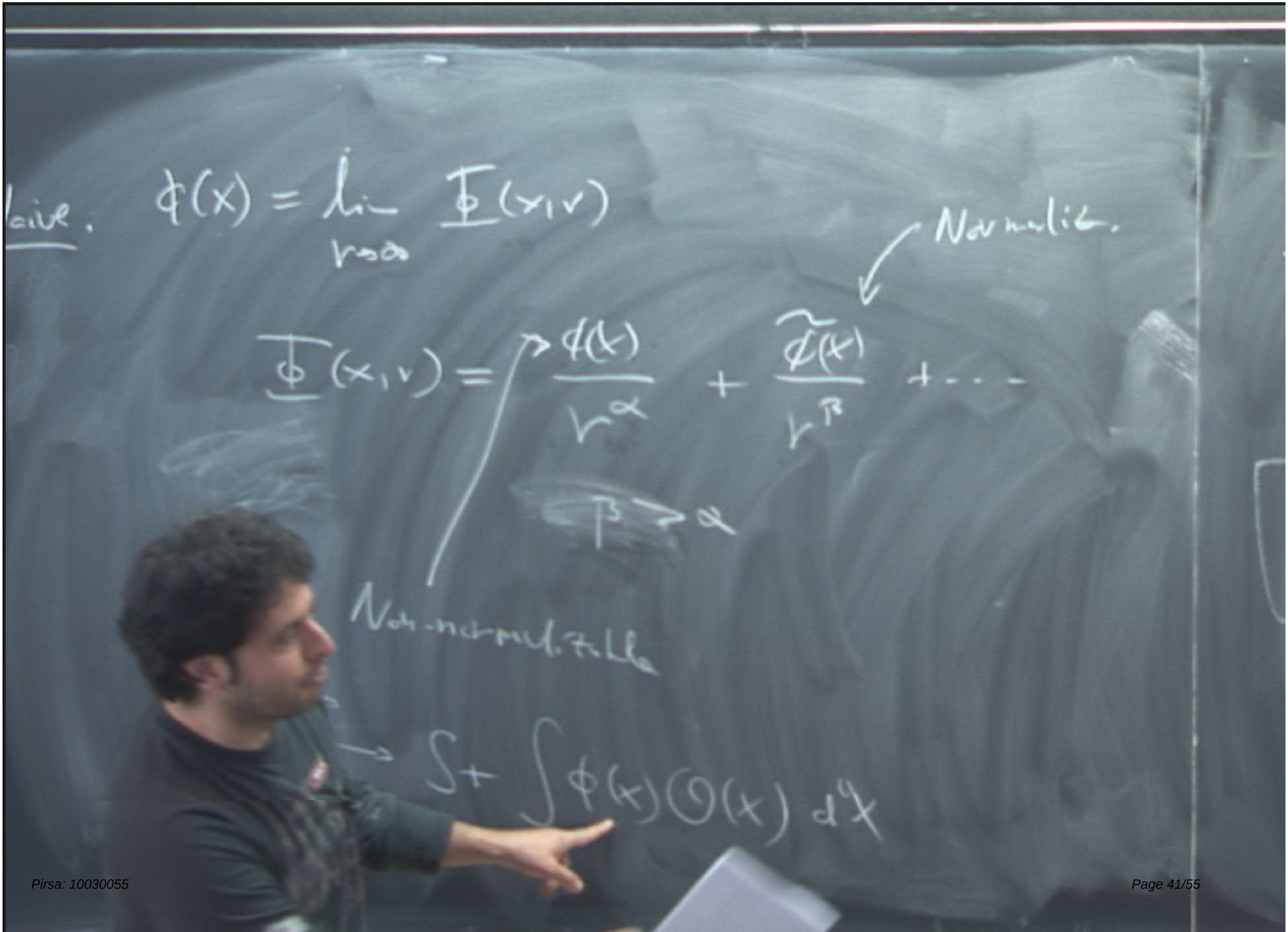
$$\boxed{\phi(x) = \lim_{r\to\infty} r^{\alpha}\, \Phi(x,r)}$$

$$\phi \to R^{-\alpha}\, \phi$$

$$[\phi] = L^{-\alpha} = M^{\alpha}$$

$$[\hat{\phi}] = L^{-\beta} = M^{\beta}$$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Normaliz.
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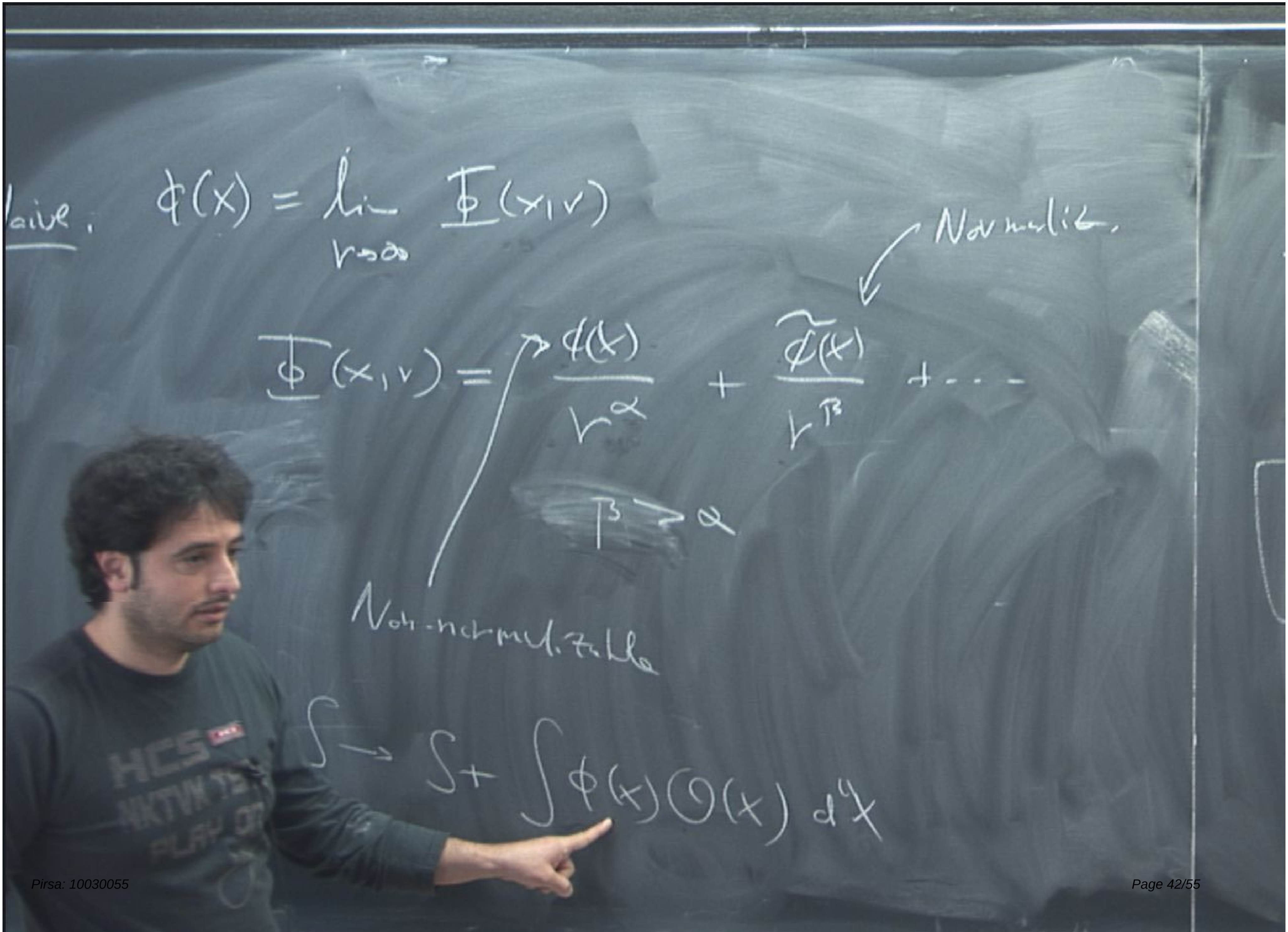
$\beta > \alpha$
Non-normalizable
$S + \int \phi(x)\, O(x)\, d^4x$



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Non-normalizable
$S \to S + \int \phi(x)\, \mathcal{O}(x)\, d^4x$

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Normaliz.

$$\Phi(x,r) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \cdots$$

$\beta > \alpha$

Non-normalizable

$$S \to S + \int \phi(x)\, \mathcal{O}(x)\, d^4x \Rightarrow [\mathcal{O}] = M^{4-\alpha}$$

Ex. Scalar field of mass $m=0$

$$\mathcal{O} = \mathrm{Tr}\, F^2 \longrightarrow \Phi(x,r) = e^{ikx}\, r^{\gamma} \qquad \gamma(\gamma+4) = 0 \Rightarrow \begin{cases} \alpha = 0 \\ \beta = 4 \end{cases}$$

$$\boxed{\phi(x) = \lim_{r\to\infty} r^{\alpha}\, \Phi(x,r)}$$

$$\phi \longrightarrow \lambda^{-\alpha}\phi$$

$$[\phi] = L^{-\alpha} = M^{\alpha}$$

$$[\hat{\phi}] = L^{-\beta} = M^{\beta}$$

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$$\boxed{\phi(x) = \lim_{r \to \infty} r^{\alpha}\, \Phi(x,r)}$$

$$S = -\int \frac{1}{4} \mathrm{Tr}\, F^2\, d^4x$$

$M^{4-\alpha}$

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∂M

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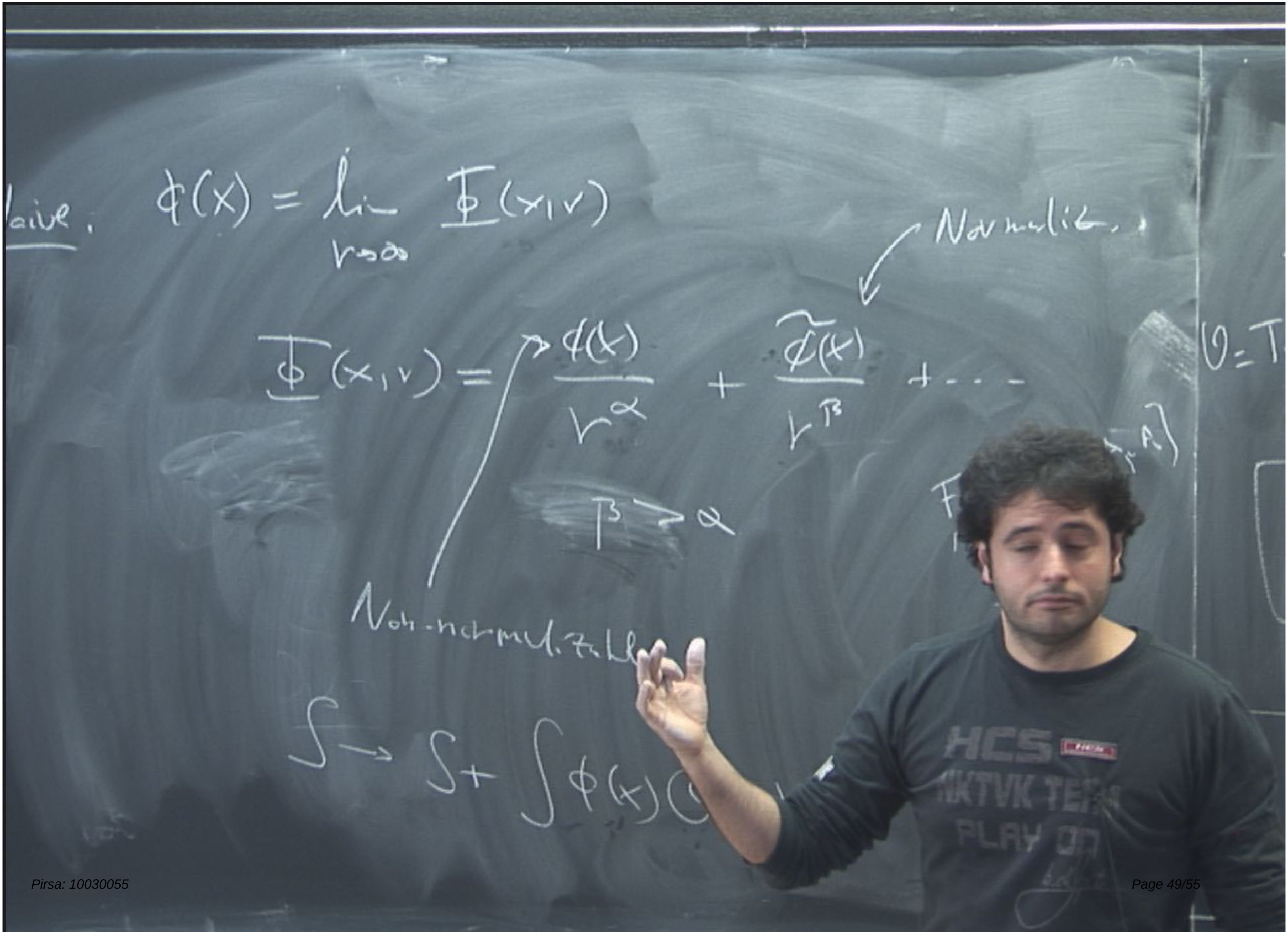
Naive.
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Normalizab.
$\Phi(x,r) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \ldots$
$\beta > \alpha$
Non-normalizable
$S \to S + \int \phi(x)\,O$

Naive: $\phi(x) = \lim_{r\to\infty} \Phi(x, r)$

Normaliz.

$$\Phi(x,r) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \cdots$$

Non-normalizable

$\beta > \alpha$

$F_{\mu\nu} = \partial_\mu A_\nu + [A_\mu, A_\nu]$

$\mathcal{O} = T$

$$S \to S + \int \phi(x)\,\mathcal{O}(x)\, d^4x \Rightarrow \boxed{[\mathcal{O}] = M^{4-\alpha}}$$

Naive: $\phi(x) = \lim_{r\to\infty} \Phi(x,r)$

Normaliz.

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$F_{\mu\nu} = \partial_\mu A_\nu + [A_\mu, A_\nu]$

$$\langle O \rangle = \frac{\delta}{\delta \phi(x)} S_{SUGRA} \Big|_{\phi=0}$$

$O = T$

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$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \ldots + [A_{\mu}, A_{\nu}]$

$$\langle \mathcal{O} \rangle = \frac{\delta}{\delta \phi(x)} S_{SUGRA} \Big|_{\phi=0}$$

$$S_{SUGRA} \sim \int_{AdS_5} (\partial \Phi)^2 \sim \int_{\partial AdS} \Phi \, \partial \Phi \sim$$

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Naive. $\phi(x) = \lim_{r\to\infty} \Phi(x,r)$

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$$\Phi(x,r) = \frac{\phi(x)}{r^\alpha} + \frac{\tilde{\phi}(x)}{r^\beta} + \cdots$$

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$$\langle \mathcal{O} \rangle = \frac{\delta}{\delta \phi(x)} S_{SUGRA} \Big|_{\phi=0} \sim \hat{\phi}(x)$$

$$S_{SUGRA} \sim \int_{AdS_5} (\partial\Phi)^2 \sim \int_{\partial AdS} \Phi\, \partial\Phi \sim \phi^2 + \phi\hat{\phi}$$

$\theta = T$