

Title: Explorations in String Theory (PHYS 647) - Lecture 11

Date: Mar 01, 2010 04:00 PM

URL: <http://pirsa.org/10030053>

Abstract:

String theory & QCD

- 1) Why QCD $\sim$ S.T.?
- 2) AdS/CFT from D3s
- 3) Finite T and RHIC
- 4) Conf / Decong
- 5) Adding fundamental

String theory & QCD

- 1) Why QCD $\sim$ S.T.?
- 2) AdS/CFT from D3s
- 3) Finite T and RHIC
- 4) Conf/Deconf phase trans.
- 5) Adding fundamental matter

Flux rules



0709.1523 [hep-th]

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Flux tubes



Regge behavior

$$M^2 \sim T J$$

Flux tubes



Regge behavior

$$M^2 \sim T J$$



Large N_c limit

QCD is $SU(3)_c$ gauge theory

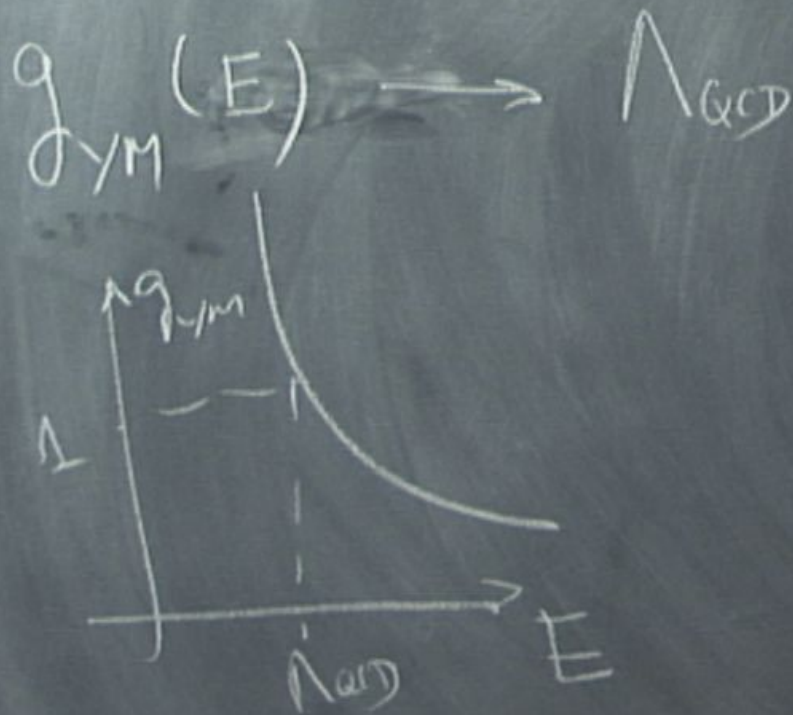
Large N_c limit

QCD is $SU(3)_C$ gauge theory

g_{YM}

Large N_c limit

QCD is $SU(3)_C$ gauge theory



$1/N_c \text{ trace} \rightarrow \text{SU}(N_c)$
Exponent is $1/N_c$

If $H_{eff} \rightarrow SU(N_c)$

Exponent is $1/N_c$

$N_c \rightarrow \infty$

D. of.

$U(N_c) \rightarrow SU(N_c)$

Exptl is $1/N_c$

$N_c \rightarrow \infty$

D. of. Gluons

Quarks

color
 i, j
 $i, j = 1, \dots, N_c$
 A_{ij}
Lev.

q_i
a

$U(N_c) \rightarrow SU(N_c)$

Exptl is $1/N_c$

$N_c \rightarrow \infty$

D. of. Gluons

Quarks

color
 A_{ij}
 Lorentz

$i, j = 1, \dots, N_c$

(N_c)

(N_c)

$q_i \leftarrow \dots \leftarrow N_c$
 $\bar{q}_a \leftarrow \dots \leftarrow N_c$

$U(N_c) \rightarrow SU(N_c)$

Exptl is $1/N_c$

$N_c \rightarrow \infty$

D. of.

color
 $\downarrow$
 A_{ij}
 $\uparrow$
Land.

$i, j = 1 \dots N_c$

N_c^2

N_c^2

$t_{a=1 \dots N_c^2}$

$1/N_c \text{ limit} \rightarrow \text{SU}(N_c)$

Expand in $1/N_c$

$N_c \rightarrow \infty$

D.o.f. Gluons

Quarks

color
 A_{ij}
 Limit

$i, j = 1 \dots N_c$

N_c^2

$q_i \leftarrow 1 \dots N_c$
 $\bar{q}_a \leftarrow 1 \dots N_c$

$N_c N_c$

$U(N_c) \rightarrow SU(N_c)$

Expt in $1/N_c$

$N_c \rightarrow \infty$

D. of. Gluons

Quarks

color
↓
 A_{ij}
↑
flavor

$i, j = 1 \dots N_c$

N_c^2

$q_i \leftarrow 1 \dots N_c$
flavor

$N_c N_c$

$$= \mathcal{L} = -\frac{1}{4} \text{Tr} F_{\mu\nu}^2 + \sum_{a=1}^{N_F} \bar{q}_a i \not{D} q_a \quad \text{QCD}$$

$$\mathcal{L} = -\frac{1}{4} \text{Tr} F_{\mu\nu}^2 + \sum_{a=1}^{N_F} \bar{q}_a i \not{D} q_a \quad \text{QCD}$$

$$q^i \rightarrow U^i_j q^j$$

$$\Phi$$

$$\mathcal{L} = -\frac{1}{4} \text{Tr} F_{\mu\nu}^2 + \sum_{a=1}^{N_F} \bar{q}_a i \not{D} q_a \quad \text{QCD}$$

$$q_i \rightarrow U_{ij} q_j$$

$$\Phi \rightarrow U \Phi U^\dagger$$

Large N_c limit

$$\mathcal{L} = -\frac{1}{4} \text{Tr} F_{\mu\nu}^2 + \sum_{a=1}^{N_c} \bar{q}_a i \not{D} q_a \quad \text{QCD} = \text{SU(3) gauge theory}$$

$i = 1, \dots, N_c$

N_c^2

N_c

$$q^i \rightarrow U_j^i q^j$$

$$\Phi_i \rightarrow U_i^\mu \Phi_\mu U^{-1}$$



Large N_c limit

$$-\frac{1}{4} \text{Tr} F_{\mu\nu}^2 + \sum_{a=1}^{N_c} \bar{q}_a i \not{D} q_a$$

QCD = SU(3) gauge theory

$$q_i \rightarrow U_{ij} q_j$$

$$A_j^i = \sum_{\alpha} A_{\alpha} (T^{\alpha})_j^i$$

$$F_j^i \rightarrow U_{\alpha}^i F_{\alpha} U^{-1j}$$



A Feynman diagram on a chalkboard. It features two external wavy lines, each labeled with a double vertical bar $||$. These lines are connected to a central cloud-like loop. The top of the loop is labeled iX and the bottom is labeled $|K$. To the right of the diagram is an approximation symbol $\sim$ followed by the expression g^2_{-11} .

$$|| \text{---} \text{cloud} \text{---} || \sim g^2_{-11}$$



$$N_c \rightarrow \infty, \quad g_{YM}^2 \rightarrow 0$$



A Feynman diagram showing a gluon loop. Two external wavy lines, labeled with indices i, j and k, l , enter and exit a loop. The loop is represented by a cloud-like shape with two vertices. The diagram is followed by an approximation symbol and the expression $g_{YM}^2 N_c$.

$$\sim g_{YM}^2 N_c$$

$$N_c \rightarrow \infty, \quad g_{YM}^2 \rightarrow 0$$

$$\lambda = g_{YM}^2 N_c \text{ fixed}$$

↑
t Hooft

A Feynman diagram showing a gluon loop (cloud) with four external lines labeled i, j, k, l . To the right of the diagram is the expression $\sim g_{YM}^2 N_c$.

$N_c \rightarrow \infty$ $g_{YM}^2 \rightarrow 0$

$\lambda = g_{YM}^2 N_c$ fixed

 't Hooft comp.



A Feynman diagram showing a gluon loop. Two wavy lines enter from the left, labeled with indices i and j . Two wavy lines exit to the right, labeled with indices k and l . The loop itself is a closed loop of wavy lines.

$$\sim g_{\text{YM}}^2 N_c$$

$$N_c \rightarrow \infty, \quad g_{\text{YM}}^2 \rightarrow 0$$

$$\lambda = g_{\text{YM}}^2 N_c \text{ fixed}$$

↳ 't Hooft comply

$q \rightarrow i$

$\bar{q} \leftarrow \bar{i}$

$\rightarrow$

$q \rightarrow i$

$\overline{q} \leftarrow \overline{i}$

$A' \begin{matrix} \rightarrow i \\ \leftarrow \end{matrix}$

$q \rightarrow i$

$\overline{q} \leftarrow \overline{i}$

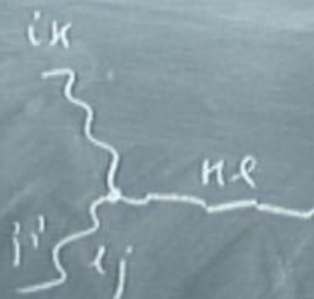
$A \begin{matrix} i \\ \vdots \\ i \end{matrix} \begin{matrix} \rightarrow \\ \leftarrow \end{matrix}$

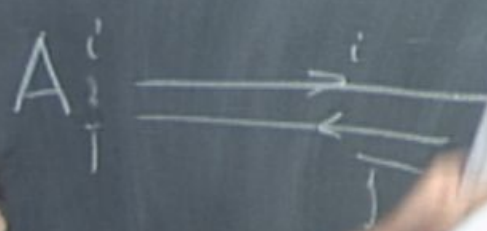
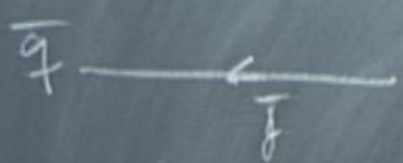
$q \rightarrow i$

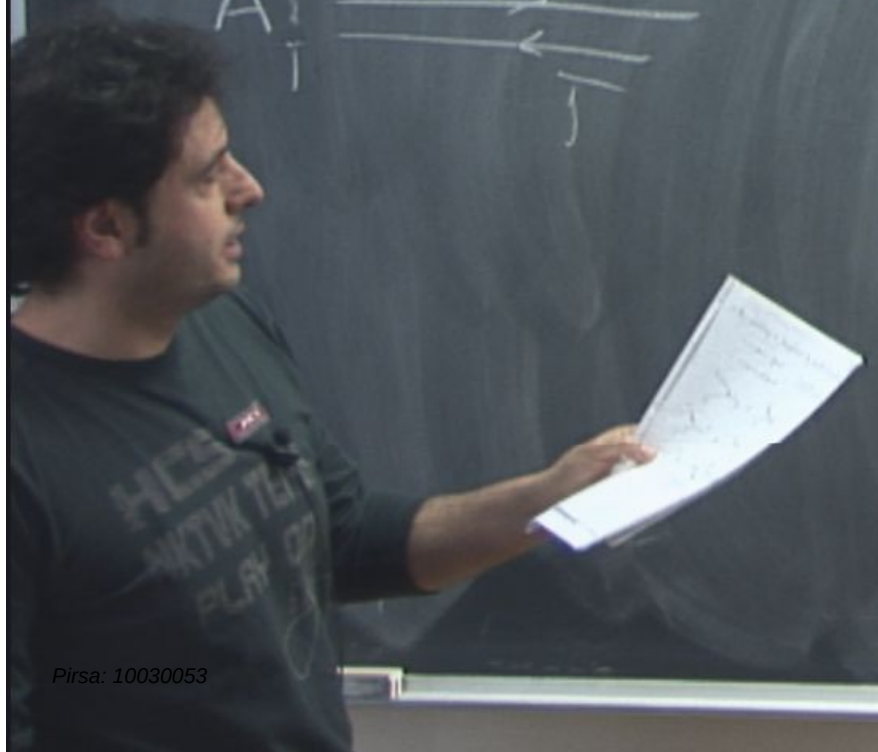
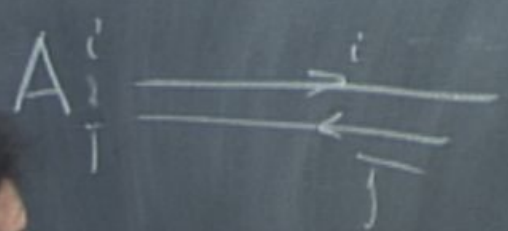
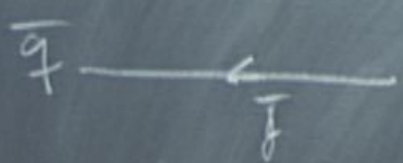
$\bar{q} \leftarrow \bar{j}$

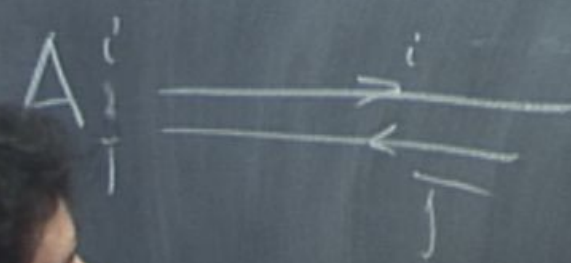
$A \begin{matrix} i \\ j \end{matrix} \rightarrow \begin{matrix} i \\ j \end{matrix}$

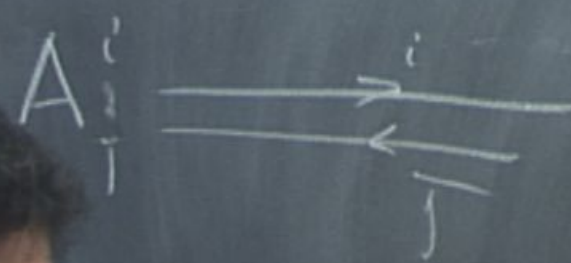














$$\sim g_{YM}^2 N_c$$

$$N_c \rightarrow \infty, \quad g_{YM}^2 \rightarrow 0$$



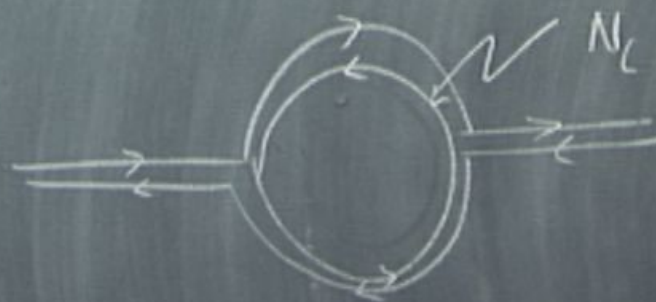
$$\lambda = g_{YM}^2 N_c$$

↑
4f H₀



$$\sim g_{YM}^2 N_c$$

$$N_c \rightarrow \infty, \quad g_{YM}^2 \rightarrow 0$$



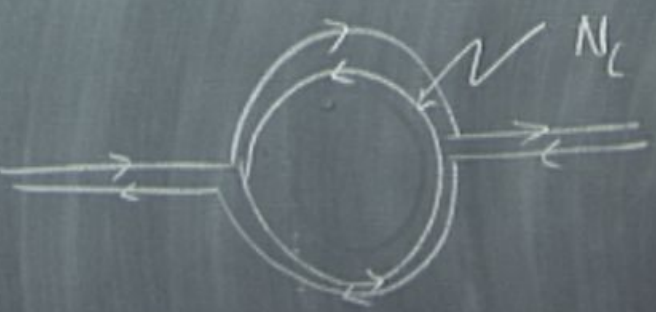
$$\lambda = g_{YM}^2 N_c$$

↑
H



$$\sim g_{YM}^2 N_c$$

$$N_c \rightarrow \infty, \quad g_{YM}^2 \rightarrow 0$$



$$\lambda = g_{YM}^2 N_c$$

1-loop



1-loop



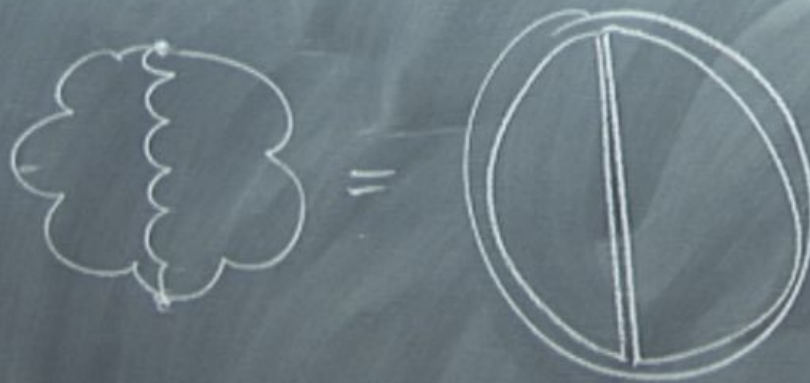
2-loop

1-loop



A Feynman diagram equation. On the left is a cloud-like shape with a scalloped edge. This is followed by an equals sign and a diagram of two concentric circles. The inner circle has a small arrow pointing clockwise. To the right of the circles is the expression $\sim N_c^2$.

2-loop



A Feynman diagram equation. On the left is a cloud-like shape with a scalloped edge, featuring a vertical line with a dot at the top and a wavy line on its right side. This is followed by an equals sign and a diagram of two concentric circles with a vertical line segment connecting the top and bottom of the inner circle.

1-loop



A Feynman diagram showing a quark line (represented by a cloud-like shape) with a 1-loop gluon self-energy correction. The correction is represented by a circle with two internal lines, one of which is a gluon line (represented by a wavy line). The diagram is equated to a similar diagram with a single internal line, and the result is proportional to N_c^2 .

$$\sim N_c^2$$

2-loop



A Feynman diagram showing a quark line (represented by a cloud-like shape) with a 2-loop gluon self-energy correction. The correction is represented by a circle with two internal lines, one of which is a gluon line (represented by a wavy line). The diagram is equated to a similar diagram with a single internal line, and the result is proportional to $\int \frac{d^4k}{(2\pi)^4} N_c^3$.

$$\sim \int \frac{d^4k}{(2\pi)^4} N_c^3$$

1-loop



A Feynman diagram for a 1-loop process. On the left is a cloud-like shape representing a multi-point function. This is set equal to a diagram consisting of two concentric circles with a vertical line connecting them at the top, representing a loop. To the right of this diagram is the expression $\sim N_c^2$.

2-loop



A Feynman diagram for a 2-loop process. On the left is a cloud-like shape with a vertical line through its center, representing a multi-point function. This is set equal to a diagram consisting of two concentric circles with a vertical line connecting them at the top, representing a loop. To the right of this diagram is the expression $\sim \int \frac{d^4 p}{(2\pi)^4} N_c^3 \sim \lambda N_c^2$.

1-loop



$\sim N_c^2$

2-loop



$\sim g^2 N_c^3 \sim \lambda N_c^2$

3-loop

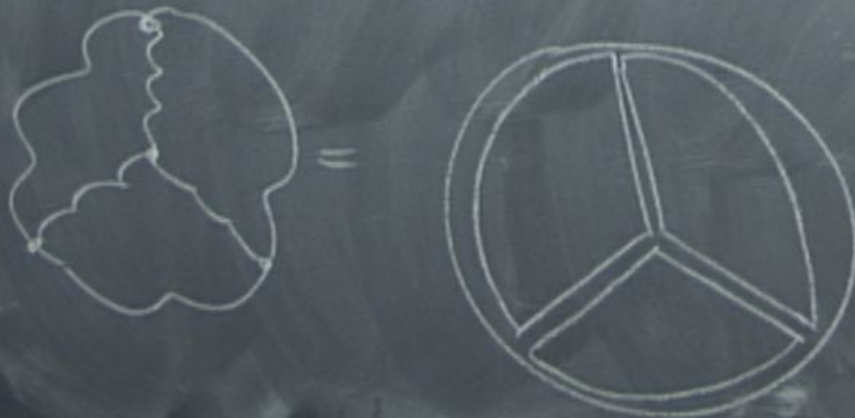
1-loop



2-loop



3-loop



1-loop



2-loop



3-loop



1-loop



2-loop



3-loop



1-loop



2-loop



3-loop



1-loop



2-loop



3-loop



1-loop



2-loop



3-loop



1-loop


$$\text{cloud} = \text{circle with double line and arrow} \sim N_c^2$$

2-loop


$$\text{cloud with vertical line} = \text{circle with vertical line} \sim \frac{g^2}{m} N_c^3 \sim \lambda N_c^2$$

3-loop


$$\text{cloud with vertical line and loop} = \text{circle with Y-shaped line} \sim \frac{g^4}{m} N_c^4 \sim \lambda^2 N_c^2$$

3. loop



3. loop



~



~ $g^4_{\gamma m}$

3. loop



$\sim$



$$\sim g_{YM}^4 N_c^2 \sim \lambda^2$$

3. loop



$\sim$



$$\sim g_{YM}^4 N_c^2 \sim \lambda^2$$

$$g_{YM}^2(\mu) N_c = \lambda(\mu)$$

3. loop



$\sim$



$$\sim g_{-m}^4 N_c^2 \sim \lambda^2$$

5. loop



$\sim$

3. loop



~



$$\sim g_{sm}^4 N_c^2 \sim \lambda^2$$

5. loop



~



~

3. loop



$\sim$



$$\sim g_{YM}^4 N_c^2 \sim \lambda^2$$

5. loop



$\sim$



$$\sim g_{YM}^8 N_c^2 \sim \frac{\lambda^4}{N_c^2}$$



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Diagrams classif. by topology

Non-planar diag. are $\frac{1}{N_c^2}$ - suppressed

1-loop



2-loop



$\sim \frac{g^2}{4\pi}$

3-loop



1-loop

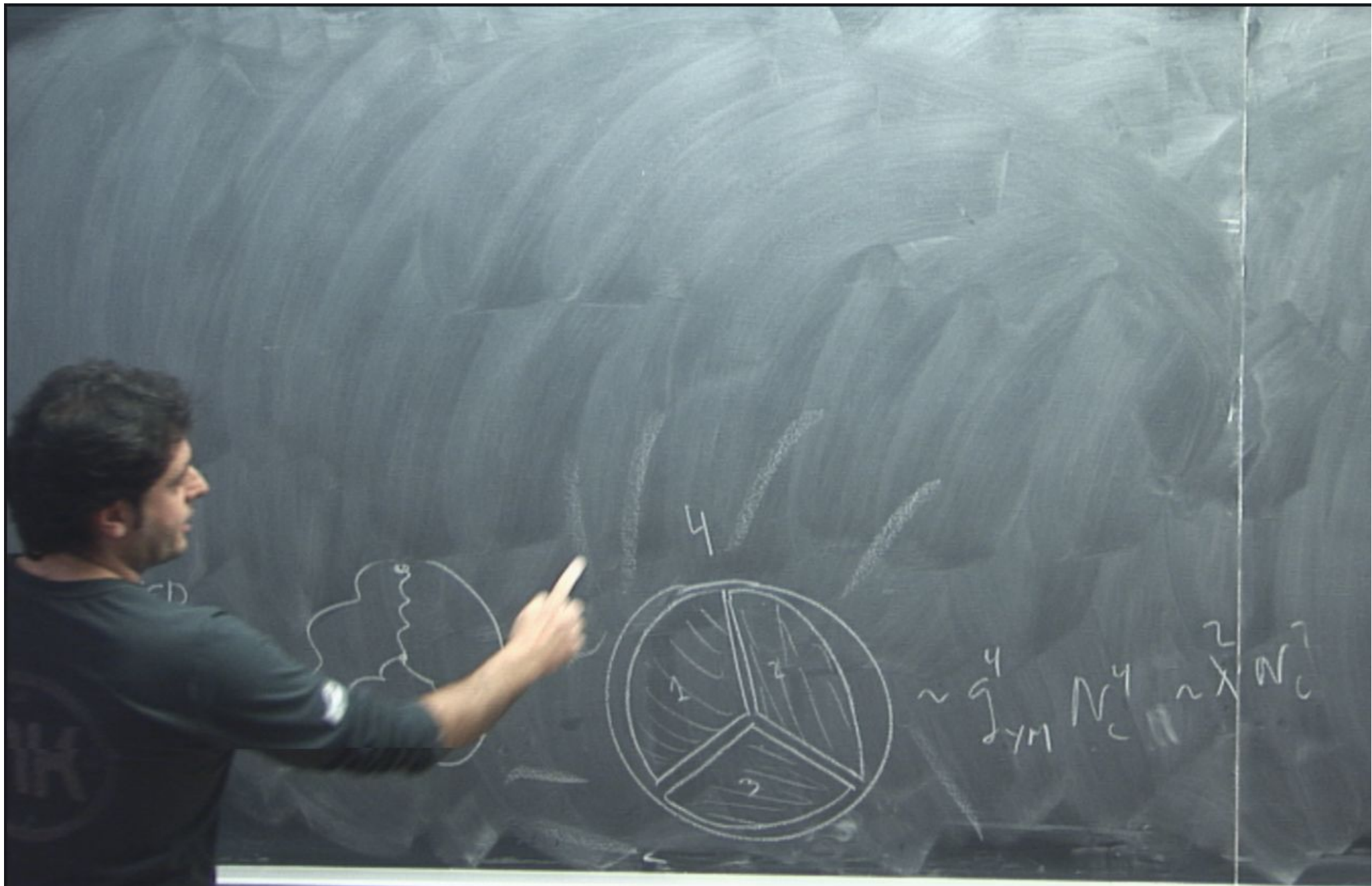


2-loop

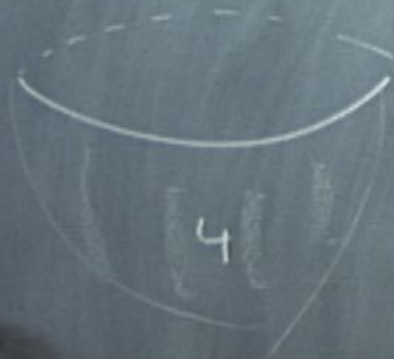


3-loop









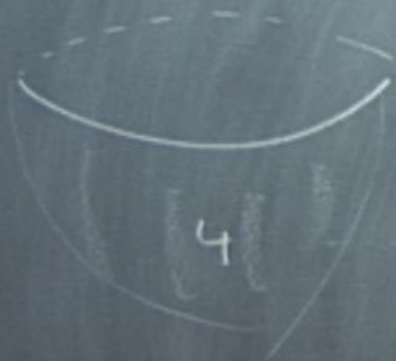
3-loop



=



$$\sim g_{YM}^4 N_c^4 \sim \chi^2 N_c^2$$



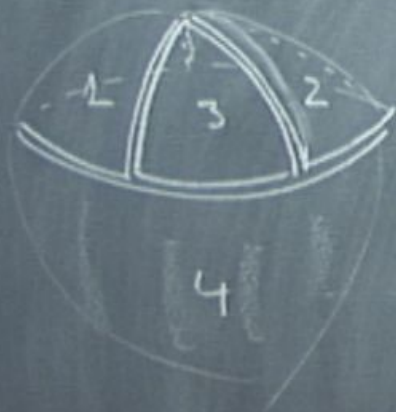
3-loop



=



$$\sim g_{YM}^4 N_c^4 \sim \chi^2 N_c^2$$



3-loop

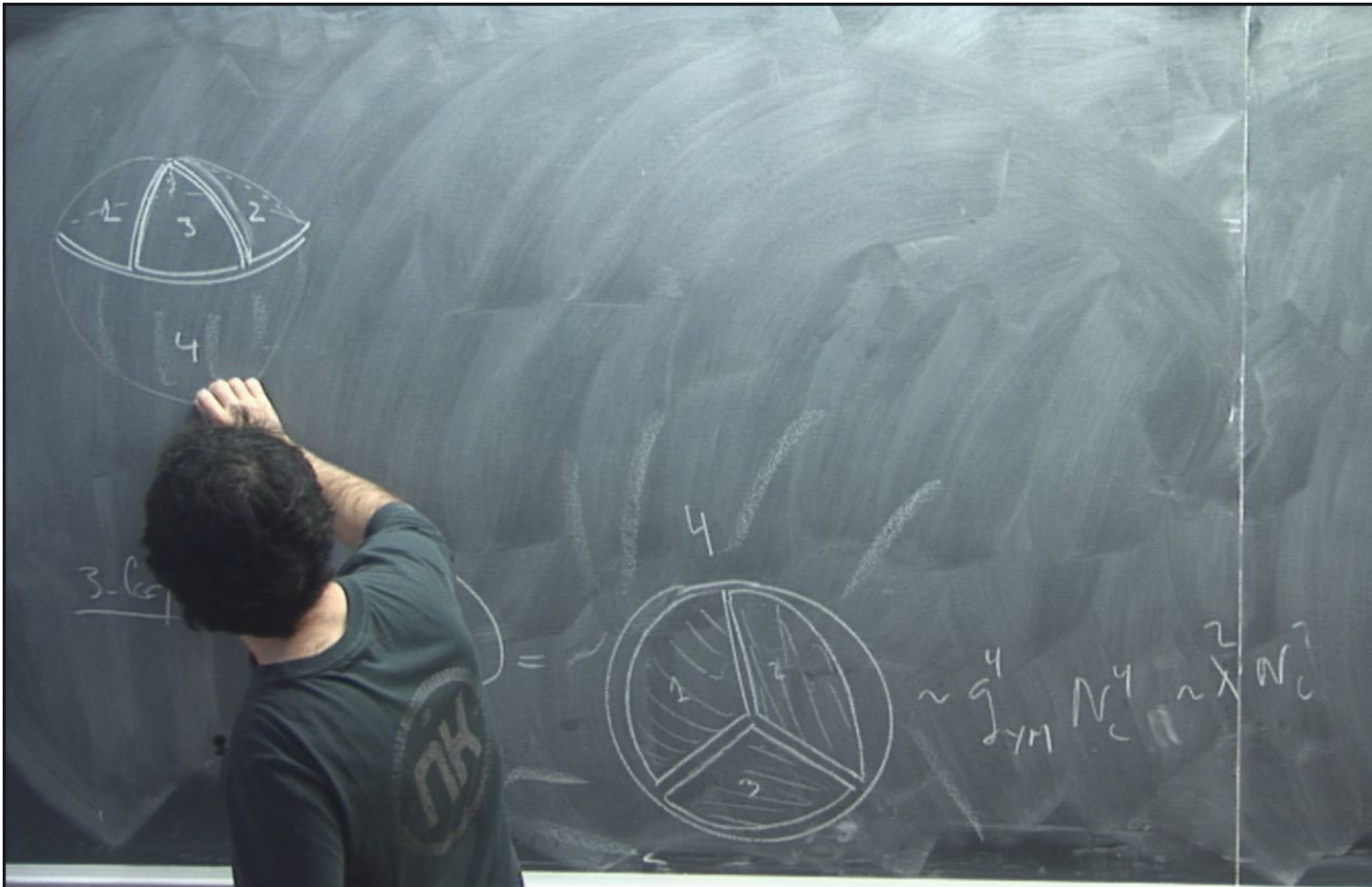


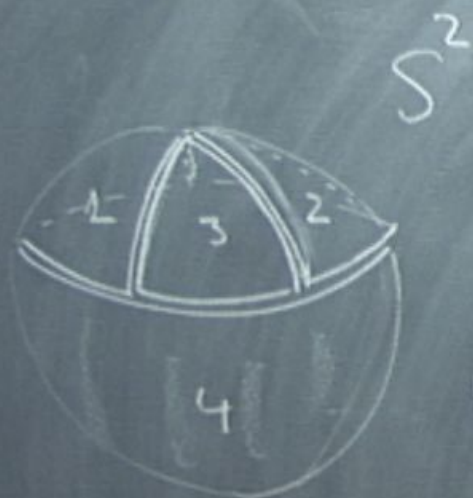
=



~

$\sim \chi^2 N_c$





3-loop



=



$$\sim g_{YM}^4 N_c^4 \sim \chi^2 N_c^2$$

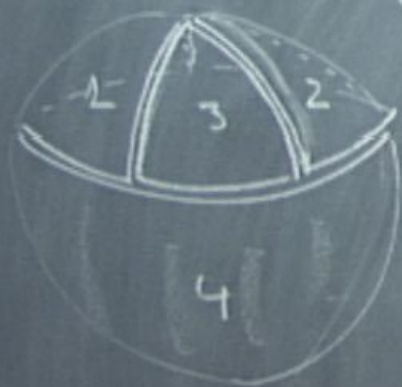
$$\boxed{\chi = 2 - 2g}$$

S^2

$g=0$

$g=1$

$g=2$



3-loop



=



$$\sim g^4 N_c^4 \sim \chi^2 N_c^2$$

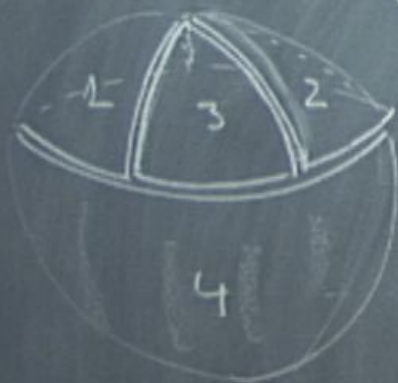
$$\chi = 2 - 2g$$

S^2

$g=0$

$g=1$

$g=2$



3-Rep



4



N_c^X

$$\sim g_{YM}^4 N_c^4 \sim \chi^2 N_c^2$$



=

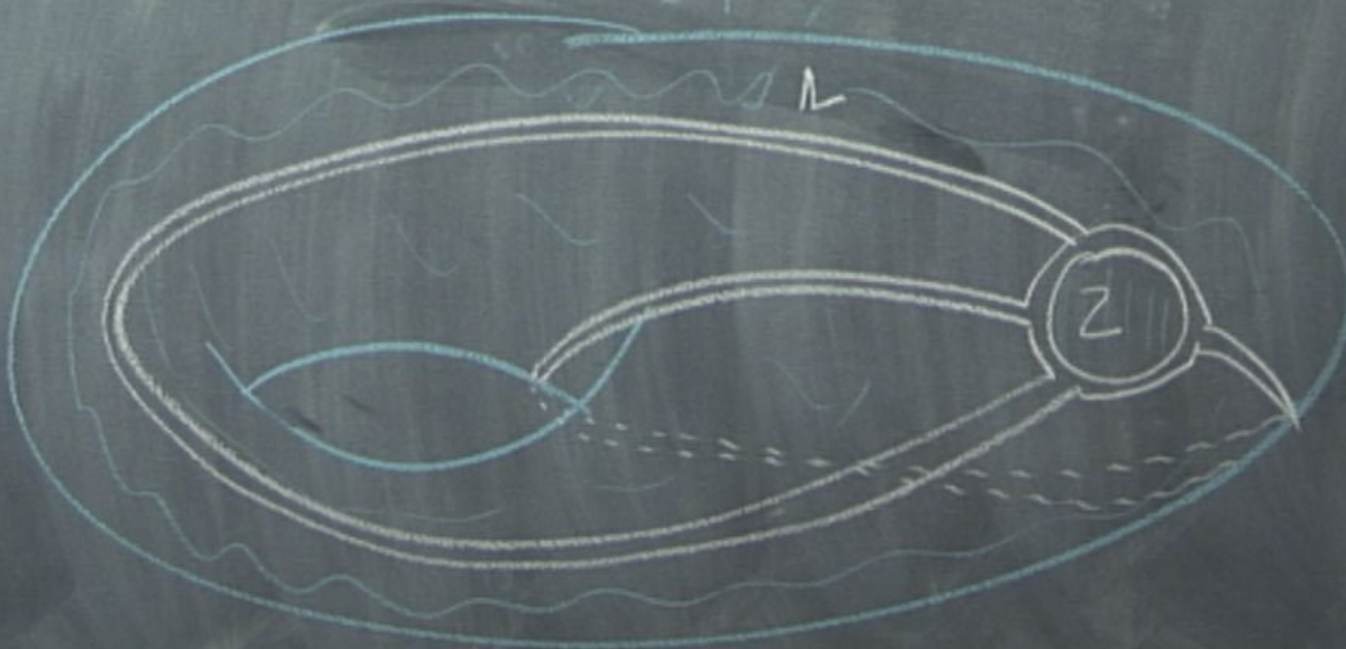








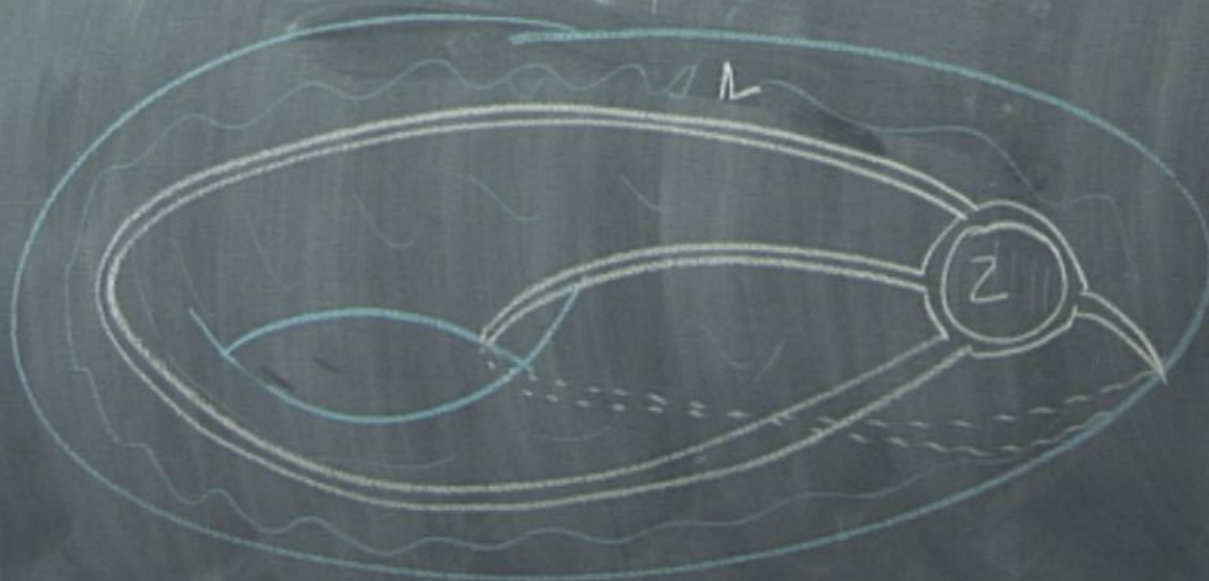






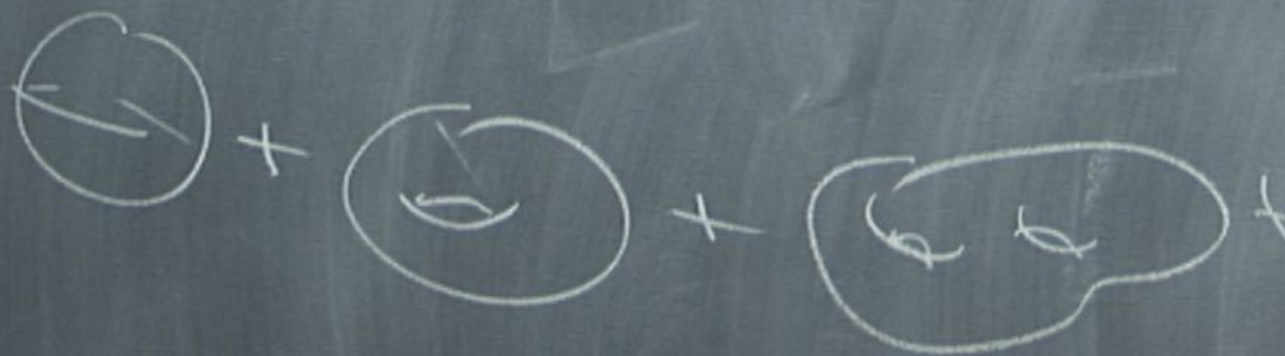
$$\chi = 2 - 2g = 2 - 2 = 0$$

$$N_c^x = N_c^0$$



$$A = \sum_{g=0}^{\infty} N_c^g \sum$$

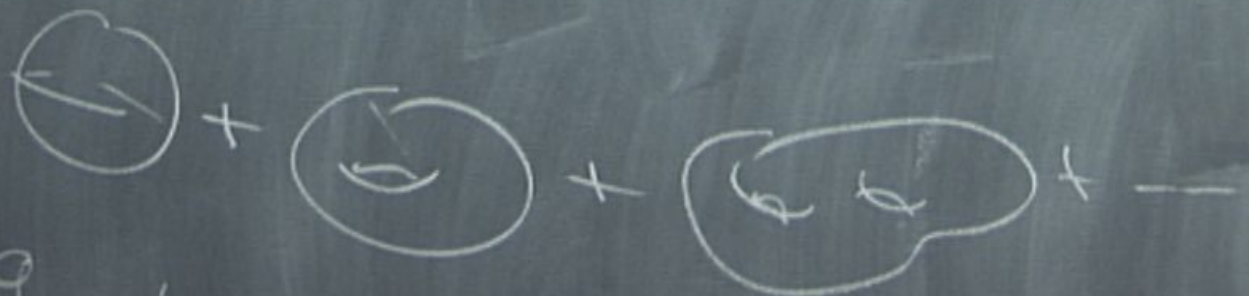
$$A = \sum_{g=0}^{\infty} N_c^g \sum_{n=0}^{\infty} c_{g,n} \lambda^n$$



$$A = \sum_{g=0}^{\infty} N_c \chi \sum_{n=0}^{\infty} c_{g,n} \lambda^n$$

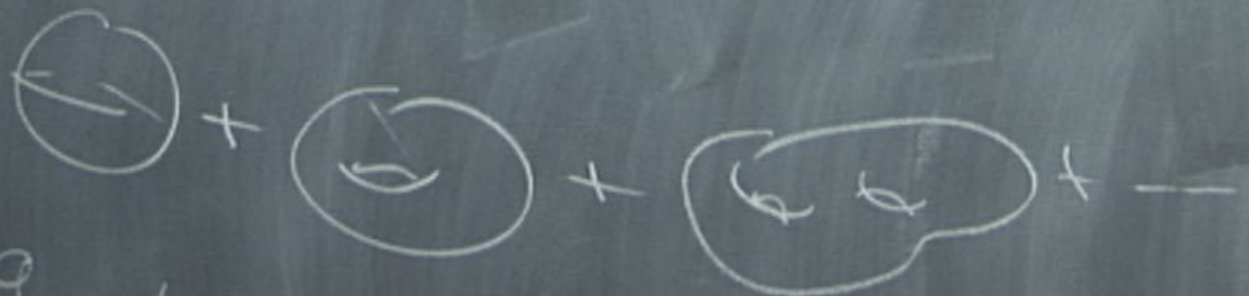


$$A = \sum_{g=0}^{\infty} N_c^g \sum_{n=0}^{\infty} c_{g,n} \lambda^n$$



$$g = \frac{1}{N_c}$$

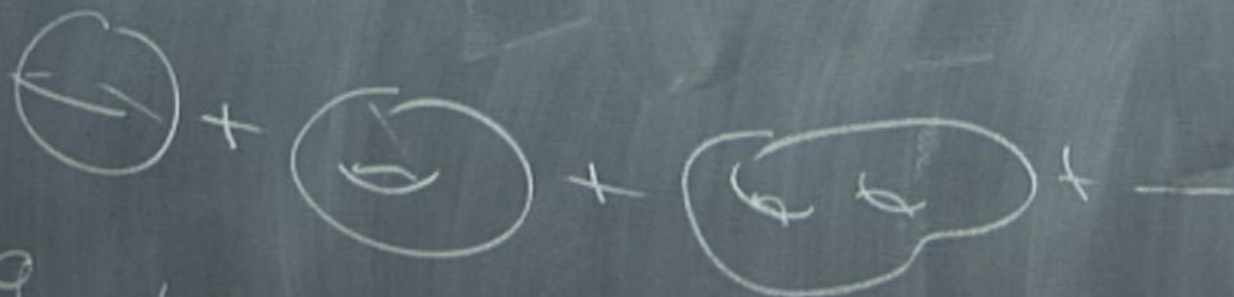
$$A = \sum_{g=0}^{\infty} N_c^g \sum_{n=0}^{\infty} c_{g,n} \lambda^n$$



$$g_s = \frac{1}{N_c}$$

$$g_s \propto \lambda$$

$$iA = \sum_{g=0}^{\infty} N_c^g \sum_{n=0}^{\infty} c_{g,n} \lambda^n$$

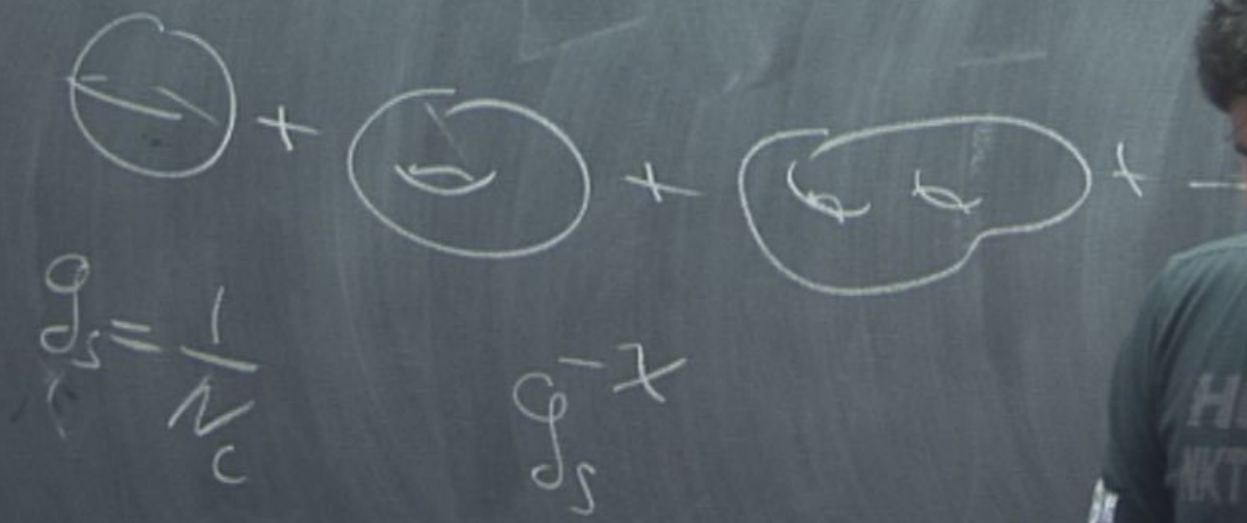


$$g_s = \frac{1}{A_c}$$

$$g_s \propto \lambda$$

$$A = \sum_{g=0}^{\infty} N_c^g \sum_{n=0}^{\infty} c_{g,n} \lambda^n$$

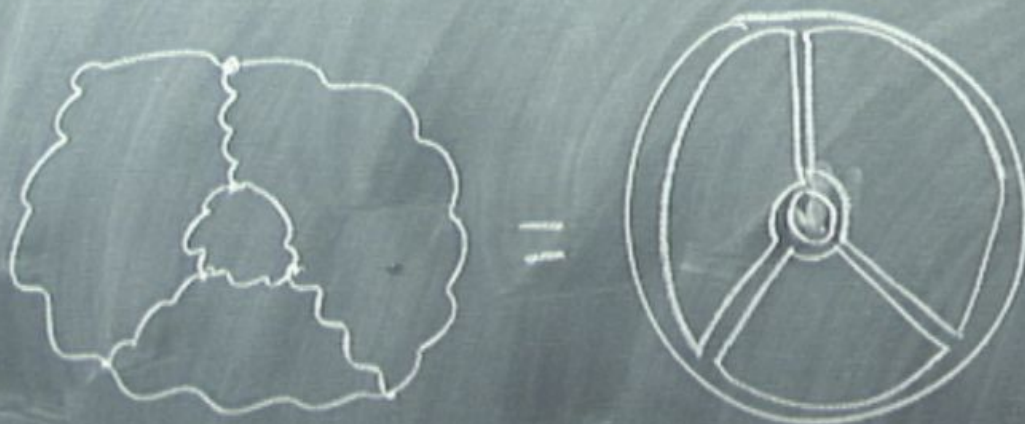
λ' expression



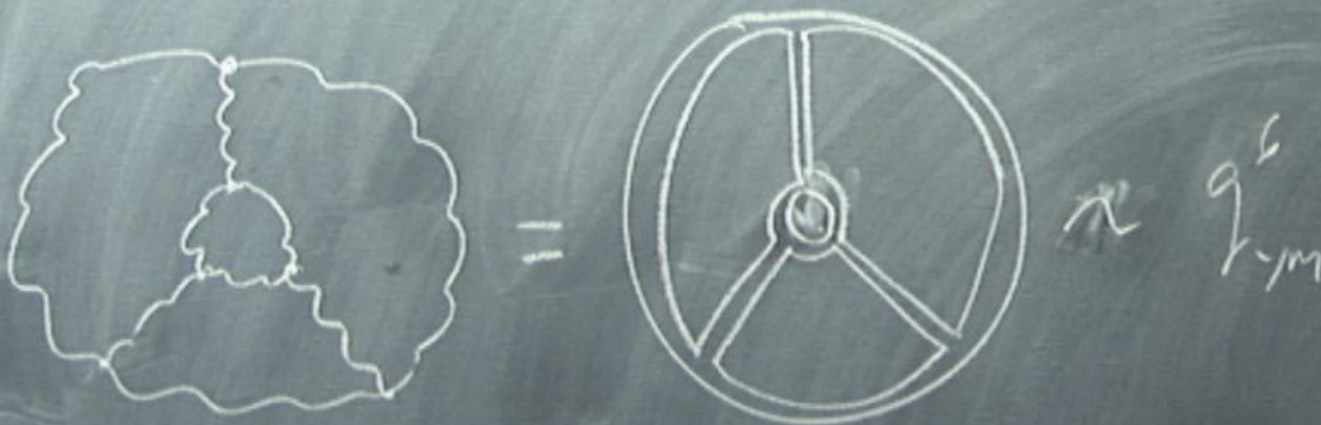
$$g_s = \frac{1}{N_c}$$

$$g_s^{-1}$$

Introduce quarks



Introduce quarks



Introduce quarks



Introduce quarks



$$\approx g_{YM}^6 N_c^5 \sim \lambda^3 N_c^2$$

Introduce quarks



$$\alpha \frac{g^6}{4\pi} N_c^5 \sim \lambda^3 N_c^2$$



Introduce quarks



A Feynman diagram showing a quark loop (represented by a cloud-like shape) with a gluon exchange (represented by a vertical line) between two vertices.

$$=$$


A Feynman diagram showing a quark loop (represented by a cloud-like shape) with a gluon exchange (represented by a vertical line) between two vertices.

$$\sim g_{YM}^6 N_c^5 \sim \lambda^3 N_c^2$$



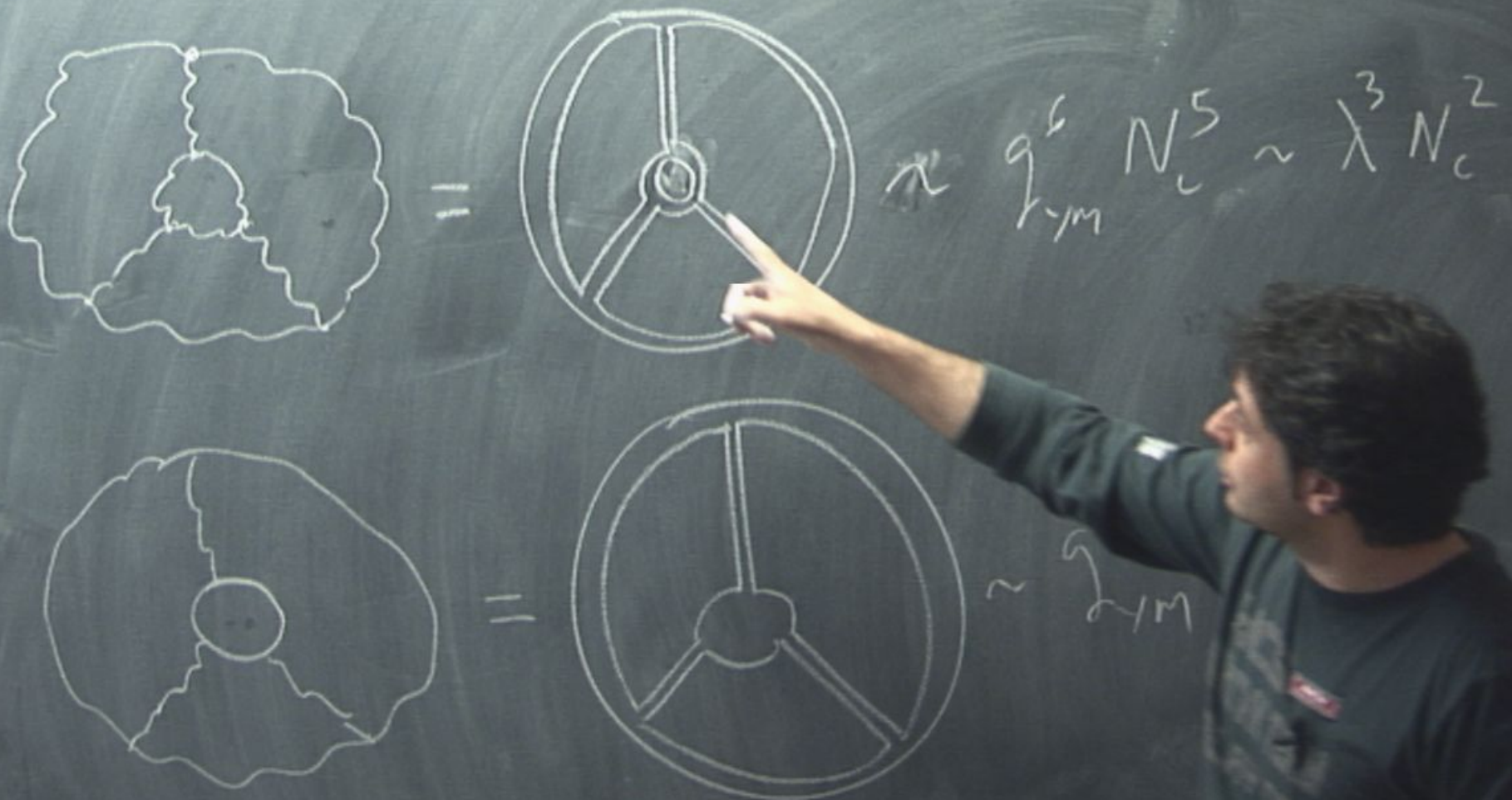
A Feynman diagram showing a quark loop (represented by a cloud-like shape) with a gluon exchange (represented by a vertical line) between two vertices.

$$=$$


A Feynman diagram showing a quark loop (represented by a cloud-like shape) with a gluon exchange (represented by a vertical line) between two vertices.

$$\sim \lambda^2 N_c$$

Introduce quarks



Introduce quarks



A Feynman diagram showing a quark loop (represented by a cloud-like shape) with a gluon exchange (represented by a circle with three internal lines meeting at a central point).

$$= \sim g_{YM}^6 N_c^5 \sim \lambda^3 N_c^2$$



A Feynman diagram showing a quark loop (represented by a cloud-like shape) with a gluon exchange (represented by a circle with three internal lines meeting at a central point).

$$= \sim g_{YM}^6 N_c^4$$

Introduce quarks



A Feynman diagram showing a quark loop (represented by a cloud-like shape) with a gluon exchange (represented by a circle with three internal lines meeting at a central point). The diagram is equated to a similar diagram with a different internal structure.

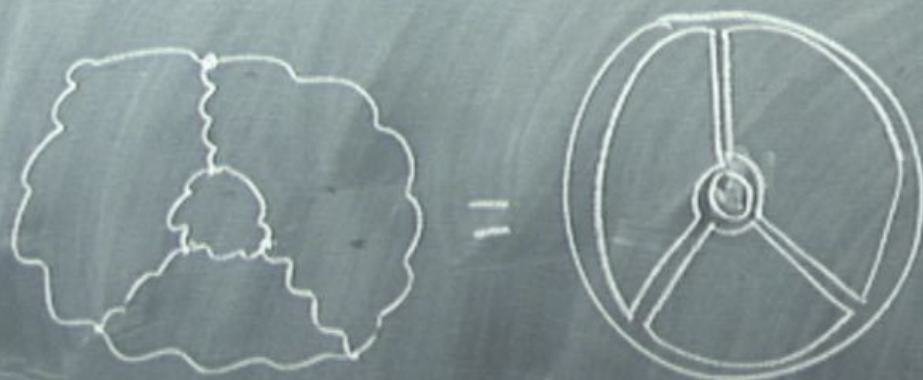
$$\sim g_{YM}^6 N_c^5 \sim \lambda^3 N_c^2$$



A Feynman diagram showing a quark loop (represented by a cloud-like shape) with a gluon exchange (represented by a circle with three internal lines meeting at a central point). The diagram is equated to a similar diagram with a different internal structure.

$$\sim g_{YM}^6 N_c^4 \sim \lambda^3 N_c$$

Introduce quarks



A Feynman diagram showing a quark-antiquark annihilation into a gluon, which then splits into two gluons. The quark and antiquark lines meet at a vertex, producing a single gluon line. This gluon line then splits into two gluon lines at a second vertex. The diagram is equated to a sum of terms.

$$\sim g_{YM}^6 N_c^5 \sim \lambda^3 N_c^2$$



A Feynman diagram showing a quark-antiquark annihilation into a gluon, which then splits into a quark and an antiquark. The quark and antiquark lines meet at a vertex, producing a single gluon line. This gluon line then splits into a quark and an antiquark line at a second vertex. The diagram is equated to a sum of terms.

$$\sim g_{YM}^6 N_c^4 \sim \lambda^3 N_c N_f$$

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Diagrams classif. by topology

Non-planar diag. are $\frac{1}{N_c^2}$ - suppressed

(Extr. quark loops $\Rightarrow \frac{N_f}{N_c}$ suppress)

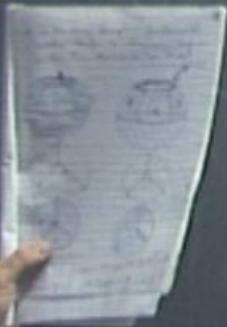
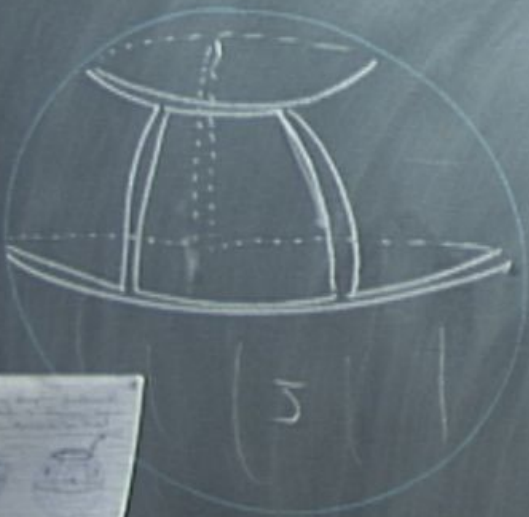
Introduce quarks



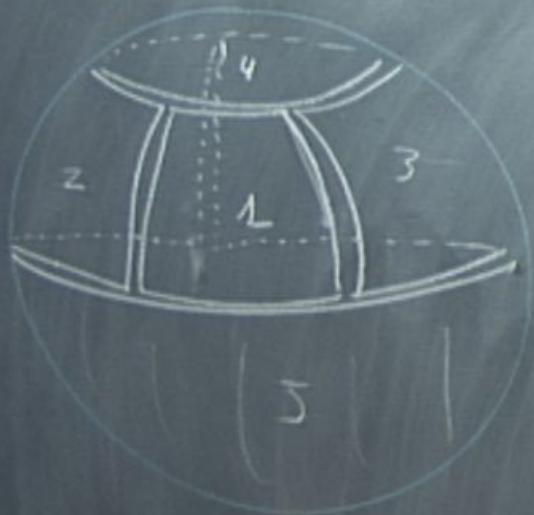
$$\sim g_{YM}^6 N_c^5 \sim \lambda^3 N_c^2$$

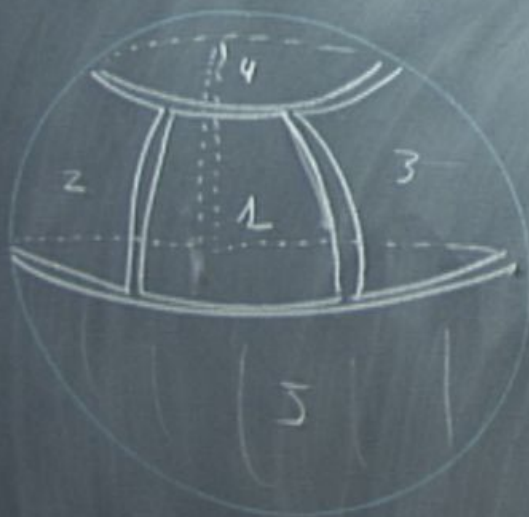


$$\sim g_{YM}^6 N_c^4 \sim \lambda^3 N_c N_f$$

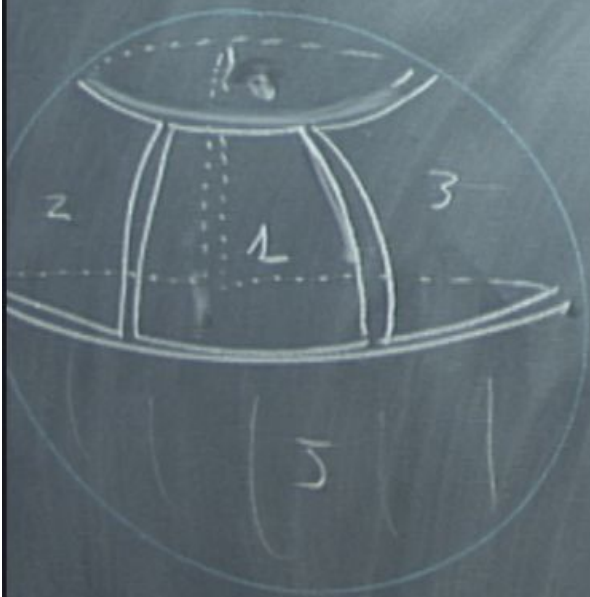








$$S^2 \quad g=0 \quad \chi=2 \quad N_c^2$$



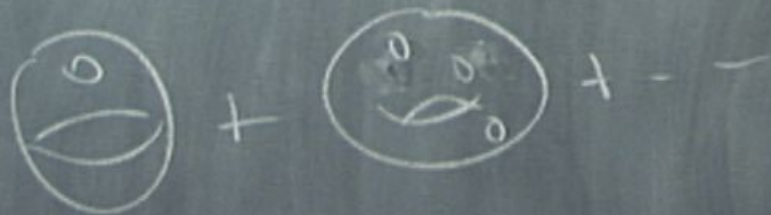
$$\chi = 2 - 2g - b$$

$$\chi = 2 - 2 \cdot 0 - 1 = 1$$

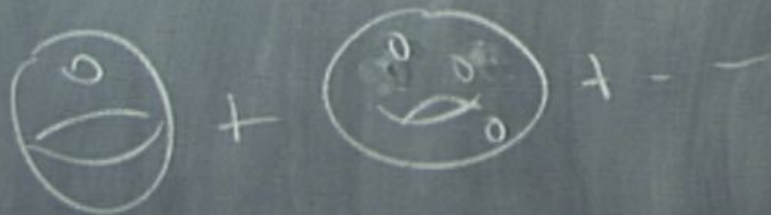
$$N_c^1$$

$$S^2 \quad g=0 \quad \chi=2 \quad N_c^2$$

$$V = \sum_{g, h} N_c^x \sum_n c_{g, h, n} x^n$$



$$A = \sum_{g, h} N_c^x \sum_n c_{g, h, n} x^n$$



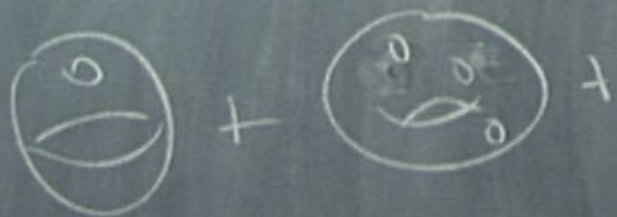
Gauge th. w/o quarks

Closed strings

~~Ends.~~

$$g_{\text{os}}^2 = \frac{1}{N_c^2}$$

$$A = \sum_{g, h} N_c^x \sum_n c_{g, h, n} x^n$$



Gauge th. w/o quarks

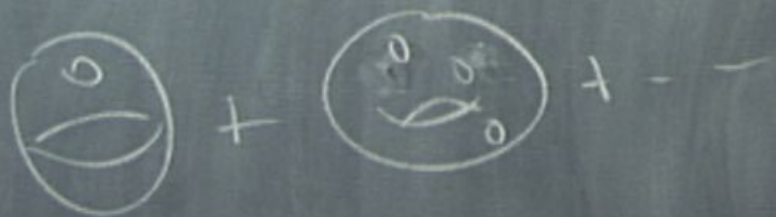
↓
Closed loops ~~≠~~ endgs.

$$g_s^2 = \frac{1}{N_c^2}$$

w/ quarks

↓
Closed + Open

$$A = \sum_{g, h} N_c^x \sum_n c_{g, h, n} x^n$$



Gluon th. w/o quarks

Closed loops ~~endgs.~~

$$g_s^2 = \frac{1}{N_c^2}$$

w/ quarks

Closed + open $1/N_c$