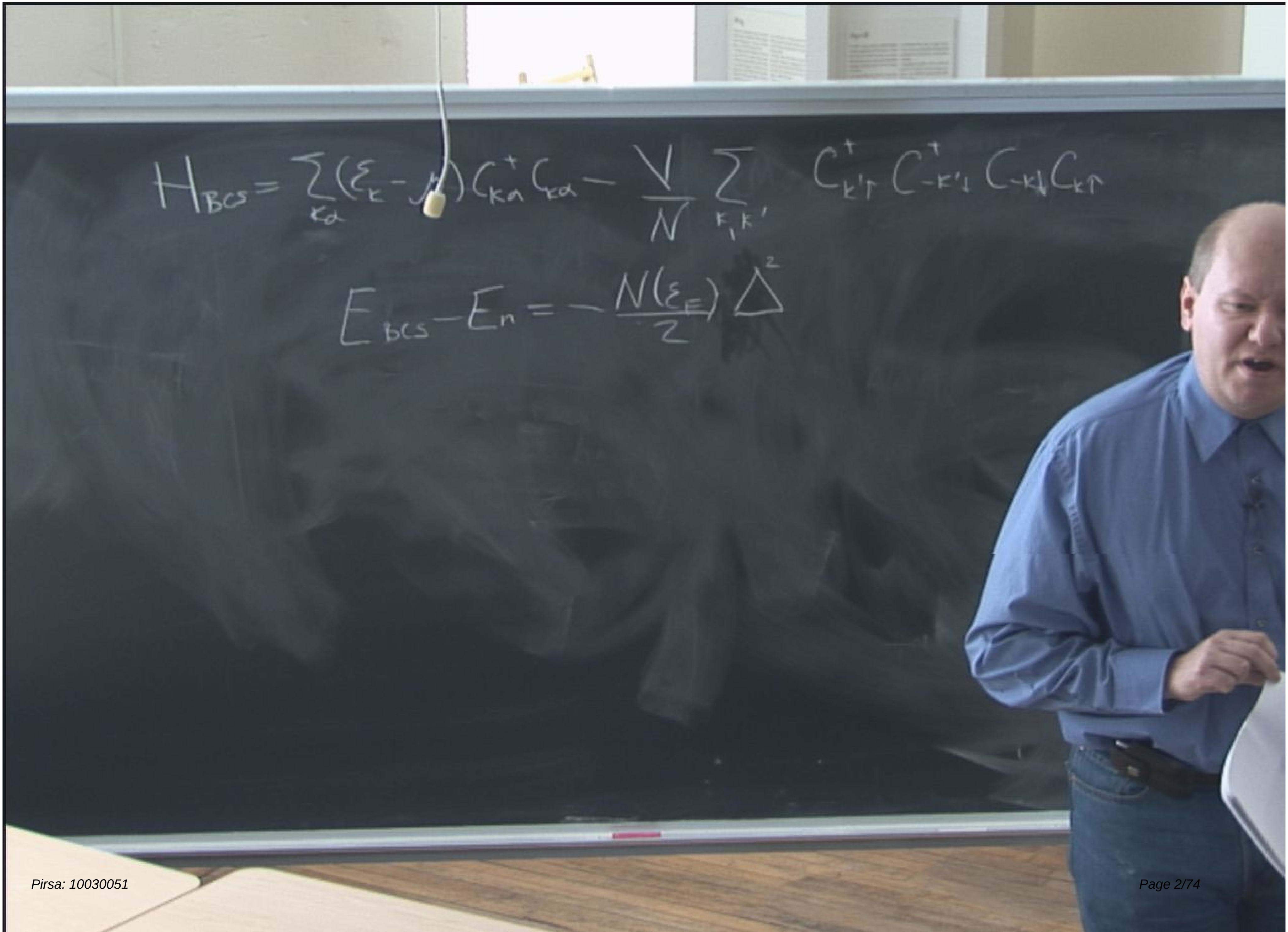
Title: Condensed Matter II - Lecture 13

Date: Mar 04, 2010 10:10 AM

URL: http://pirsa.org/10030051

Abstract:

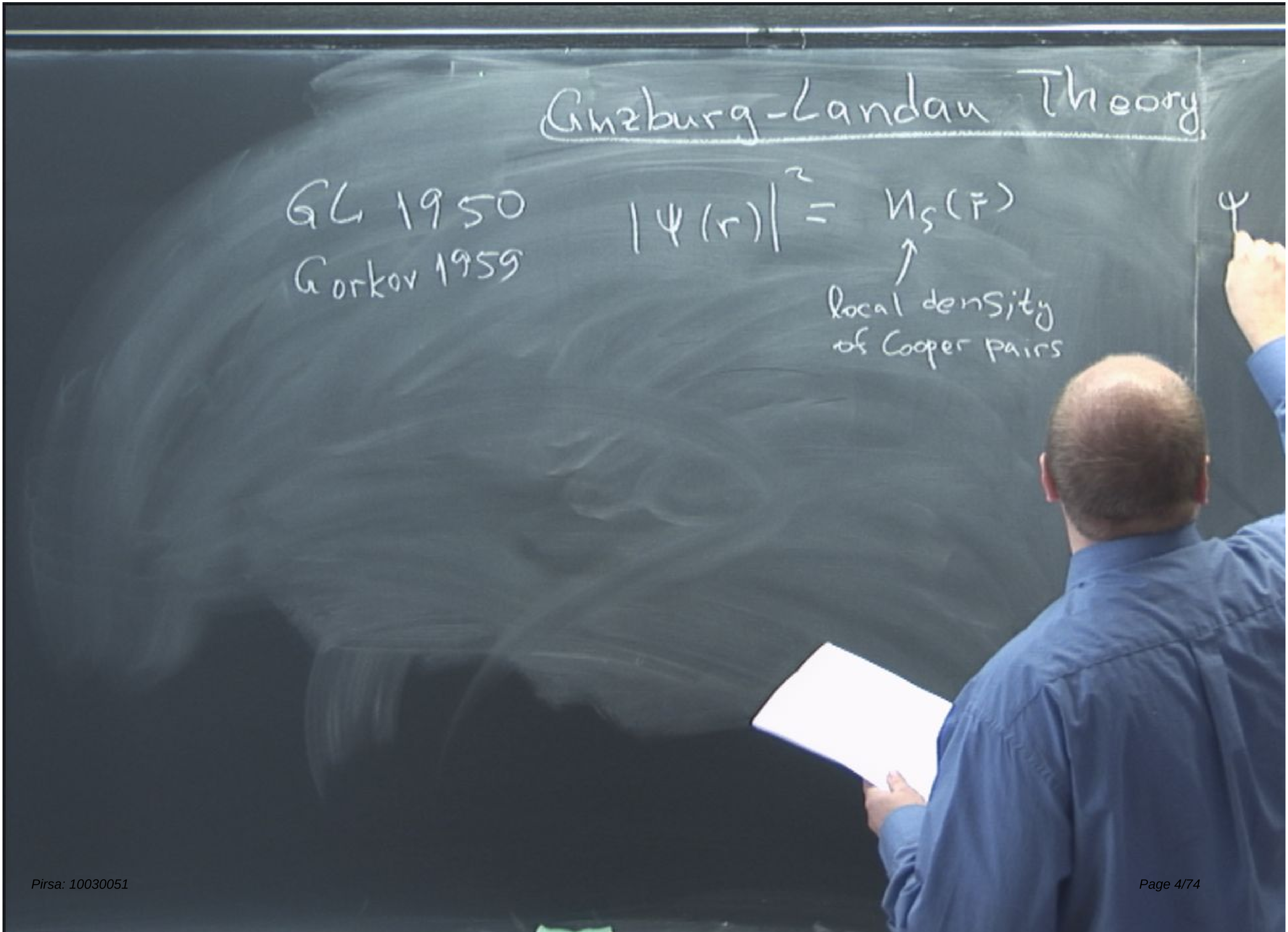


$$H_{BCS} = \sum_{k\alpha} (\varepsilon_k - \mu) C^{+}_{k\alpha} C_{k\alpha} - \frac{V}{N} \sum_{k,k'} C^{+}_{k'\uparrow} C^{+}_{-k'\downarrow} C_{-k\downarrow} C_{k\uparrow}$$

$$E_{BCS} - E_n = -\frac{N(\varepsilon_F)}{2} \Delta^2$$

Ginzburg-Landau Theory

GL 1950



Ginzburg-Landau Theory

GL 1950

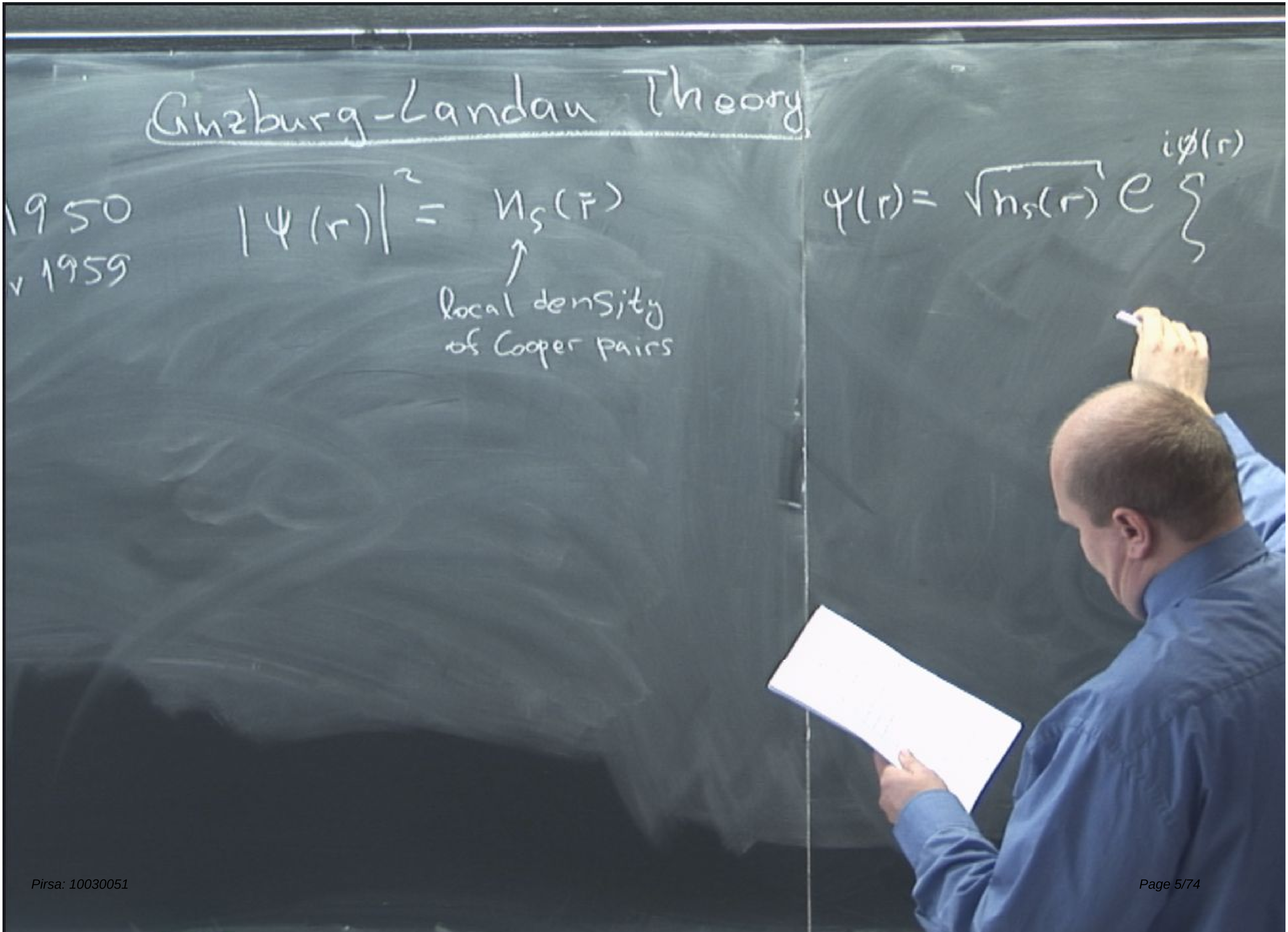
Gorkov 1959

$$|\psi(r)|^2 = n_s(\vec{r})$$

↑

local density of Cooper pairs

ψ



Ginzburg-Landau Theory

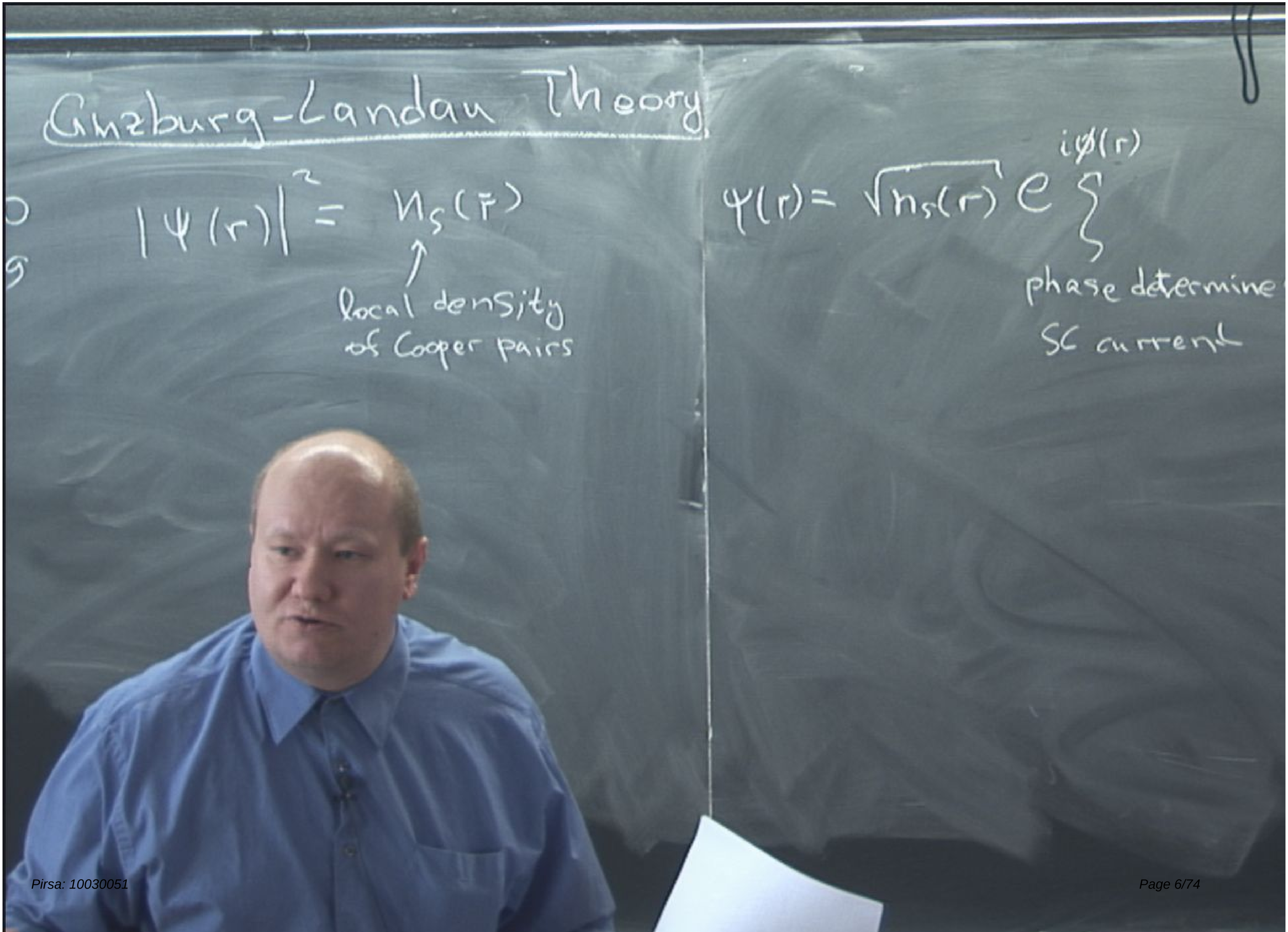
1950

v 1959

$$|\psi(r)|^2 = n_s(\vec{r})$$

local density of Cooper pairs

$$\psi(r) = \sqrt{n_s(r)}\, e^{i\phi(r)}\, \{$$



Ginzburg-Landau Theory

$$|\psi(r)|^2 = \underset{\substack{\uparrow \\ \text{local density} \\ \text{of Cooper pairs}}}{n_s}(\bar{r})$$

$$\psi(r) = \sqrt{n_s(r)}\, e^{\underbrace{i\phi(r)}_{\substack{\text{phase determine} \\ \text{SC current}}}}$$

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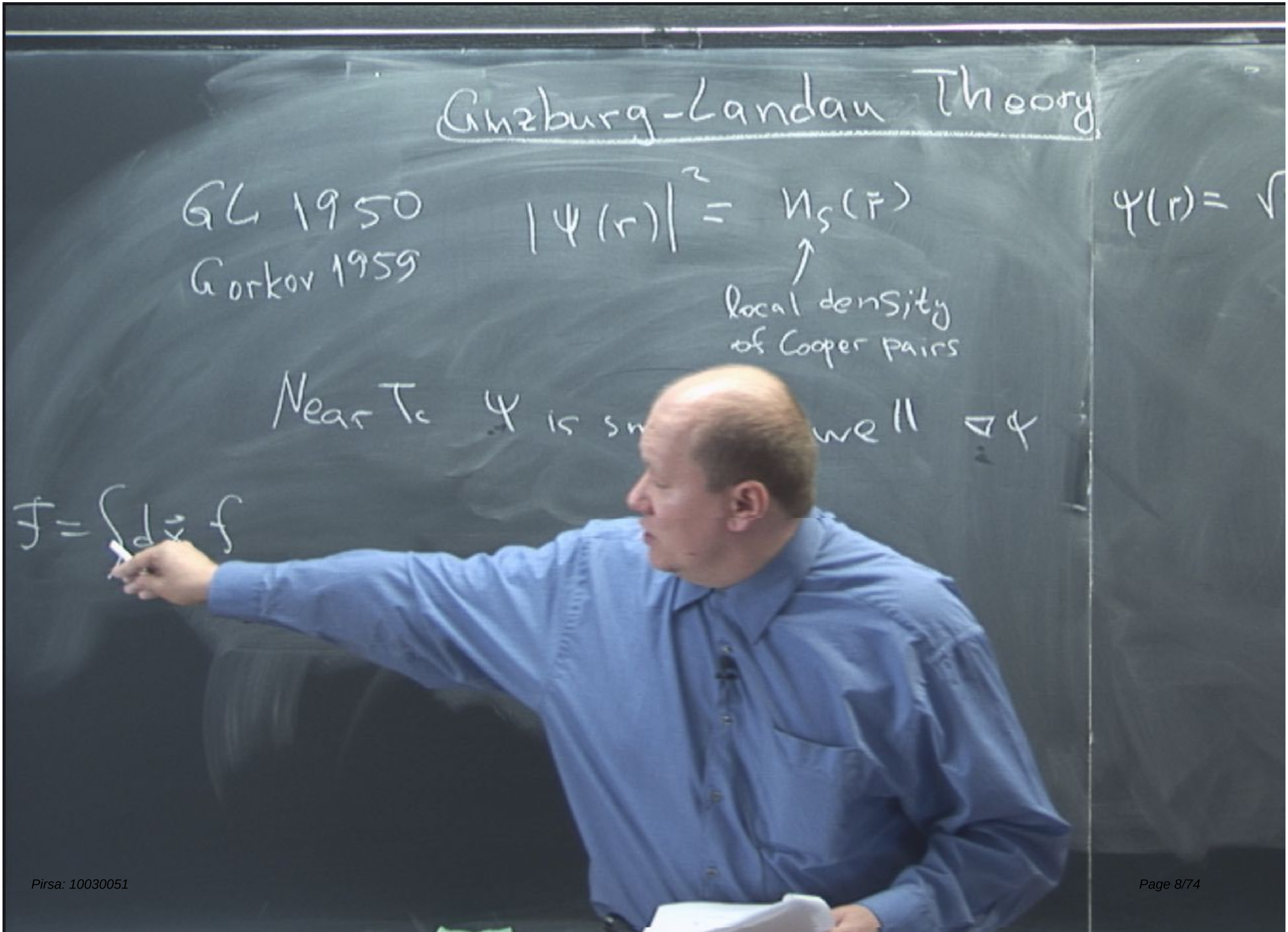
local density of Cooper pairs

$$\psi(r) = \sqrt{n_s(r)}\, e^{i\phi(r)}$$

phase determine
SC current



Ginzburg-Landau Theory

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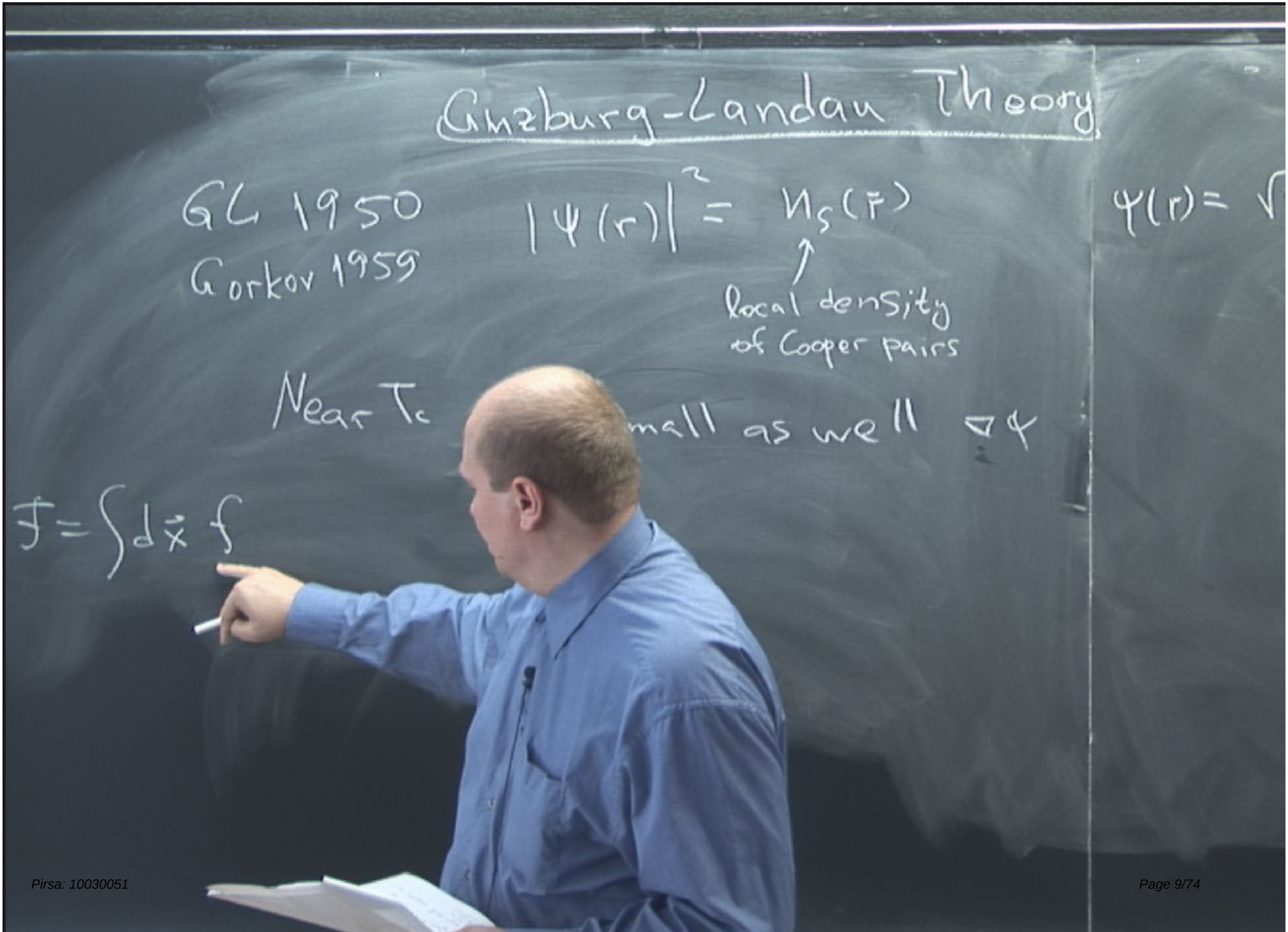
$$|\psi(r)|^2 = n_s(\vec{r})$$

↑ local density of Cooper pairs

$\psi(r) = \sqrt{}$

Near T_c ψ is sm [illegible] well $\nabla\psi$

$$\mathcal{F} = \int d\vec{x}\, f$$



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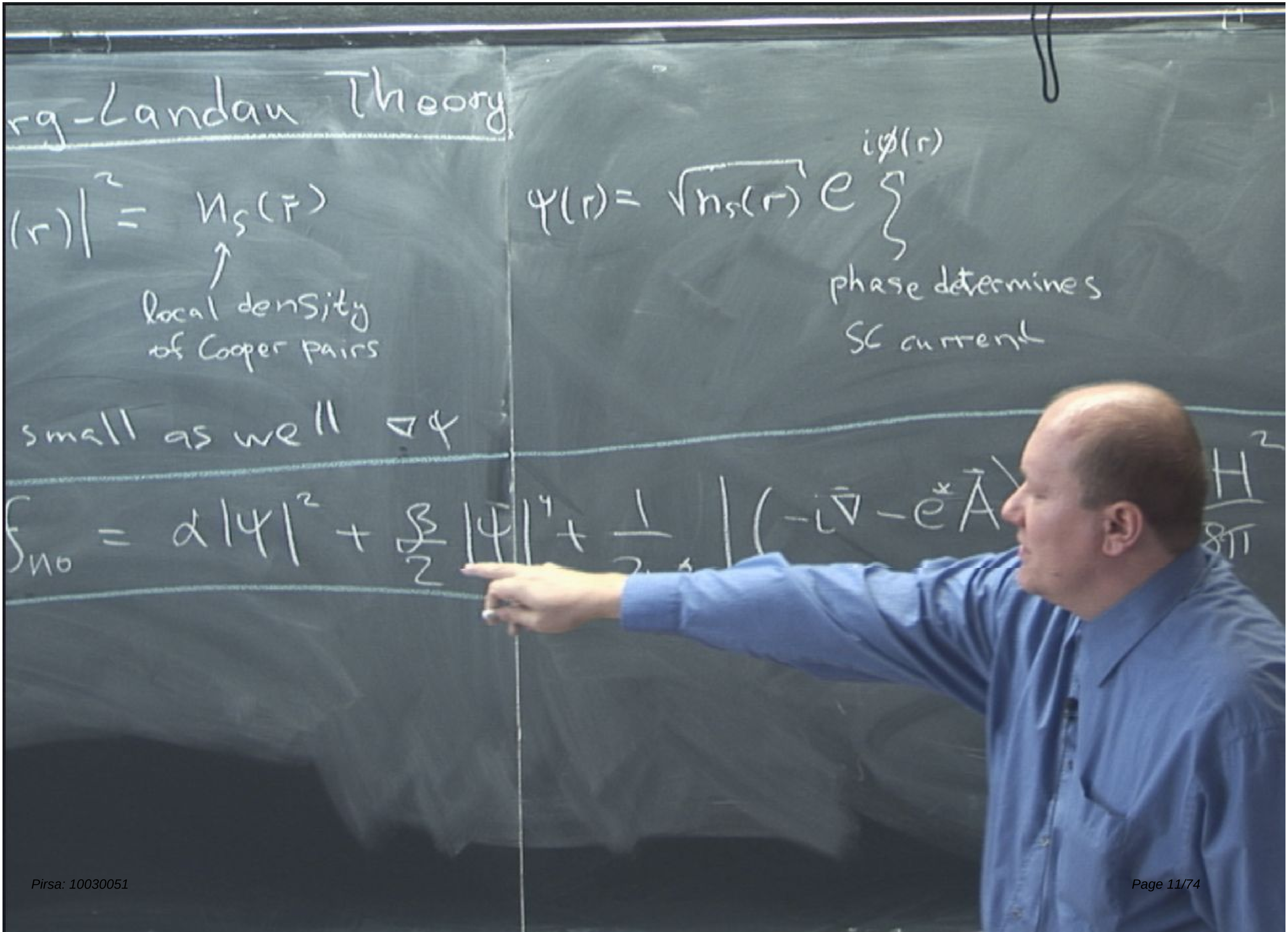
$\psi(r) = \sqrt{}$

Near T_c ψ is small as well $\nabla \psi$

$$\mathcal{F} = \int d\vec{x}\, f \,, \qquad f - f_{n0} = \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{1}{2m^*}$$



rg-Landau Theory

$(r)|^2 = n_s(\vec{r})$

local density of Cooper pairs

$\psi(r) = \sqrt{n_s(r)}\, e^{i\phi(r)}$

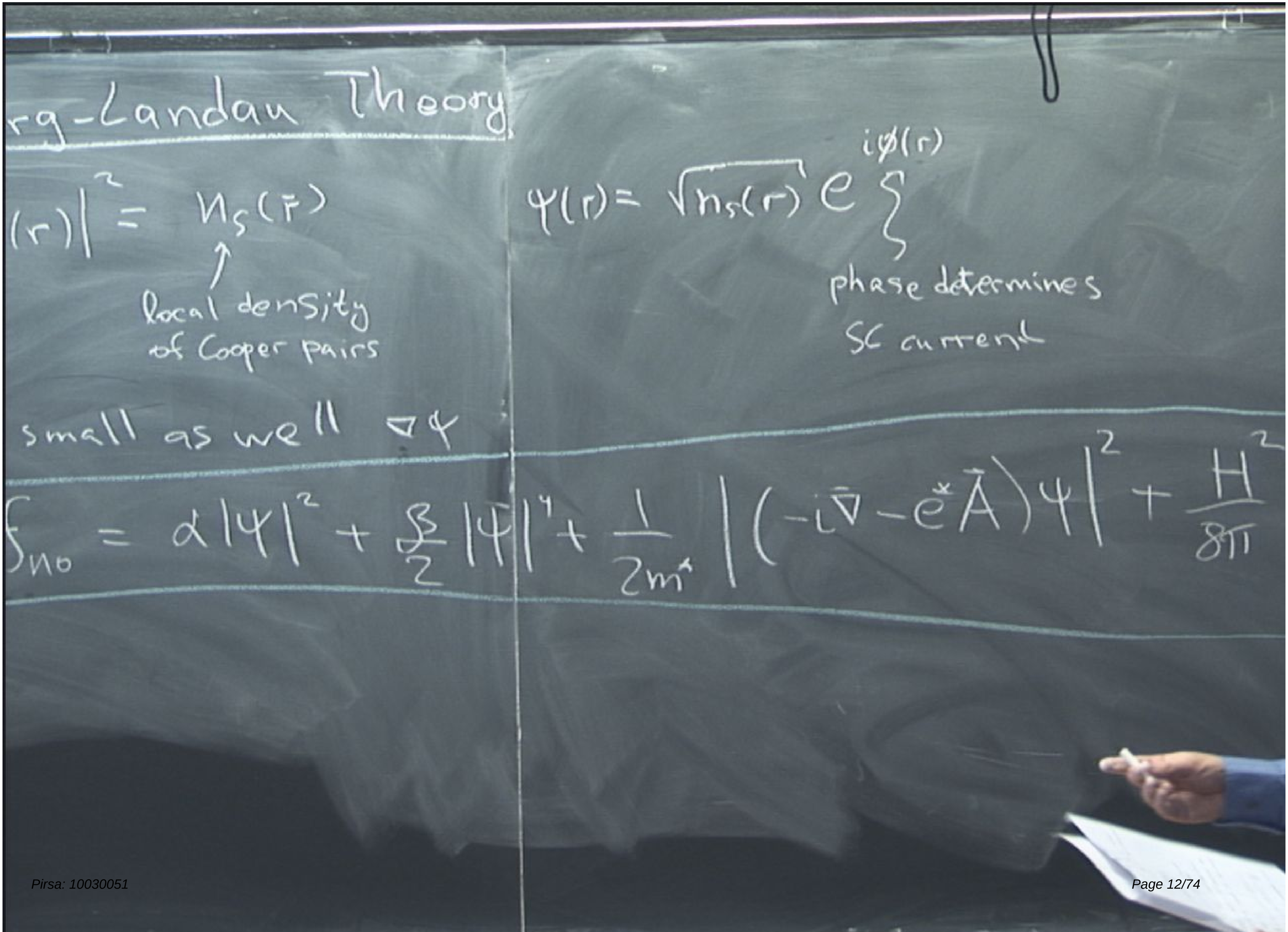
phase determines SC current

small as well $\nabla\psi$

$f_{no} = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2}\Big|\Big(-i\vec{\nabla} - e^*\vec{A}$

H^2

8π



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$(r)|^2 = n_s(\bar{r})$

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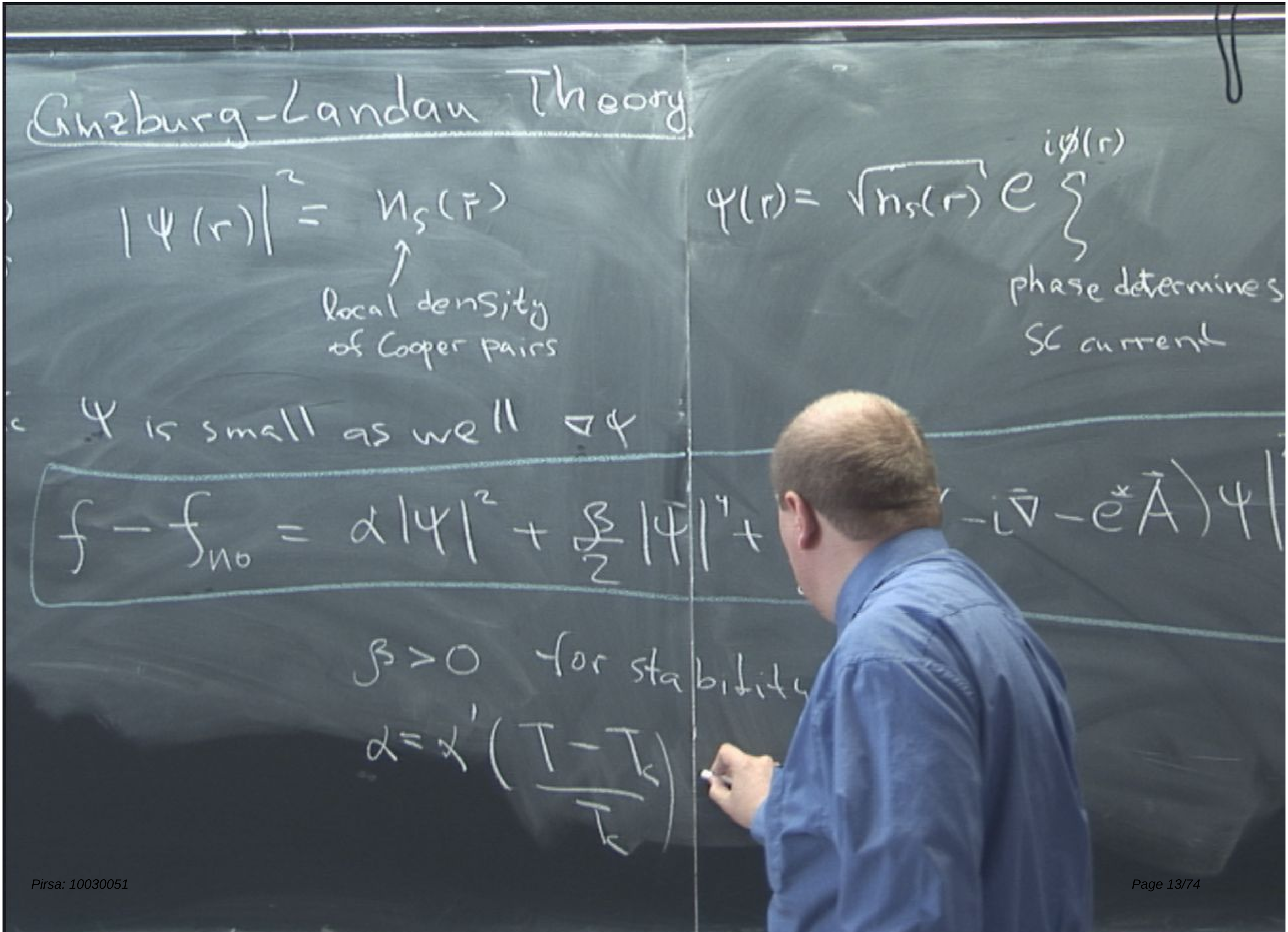

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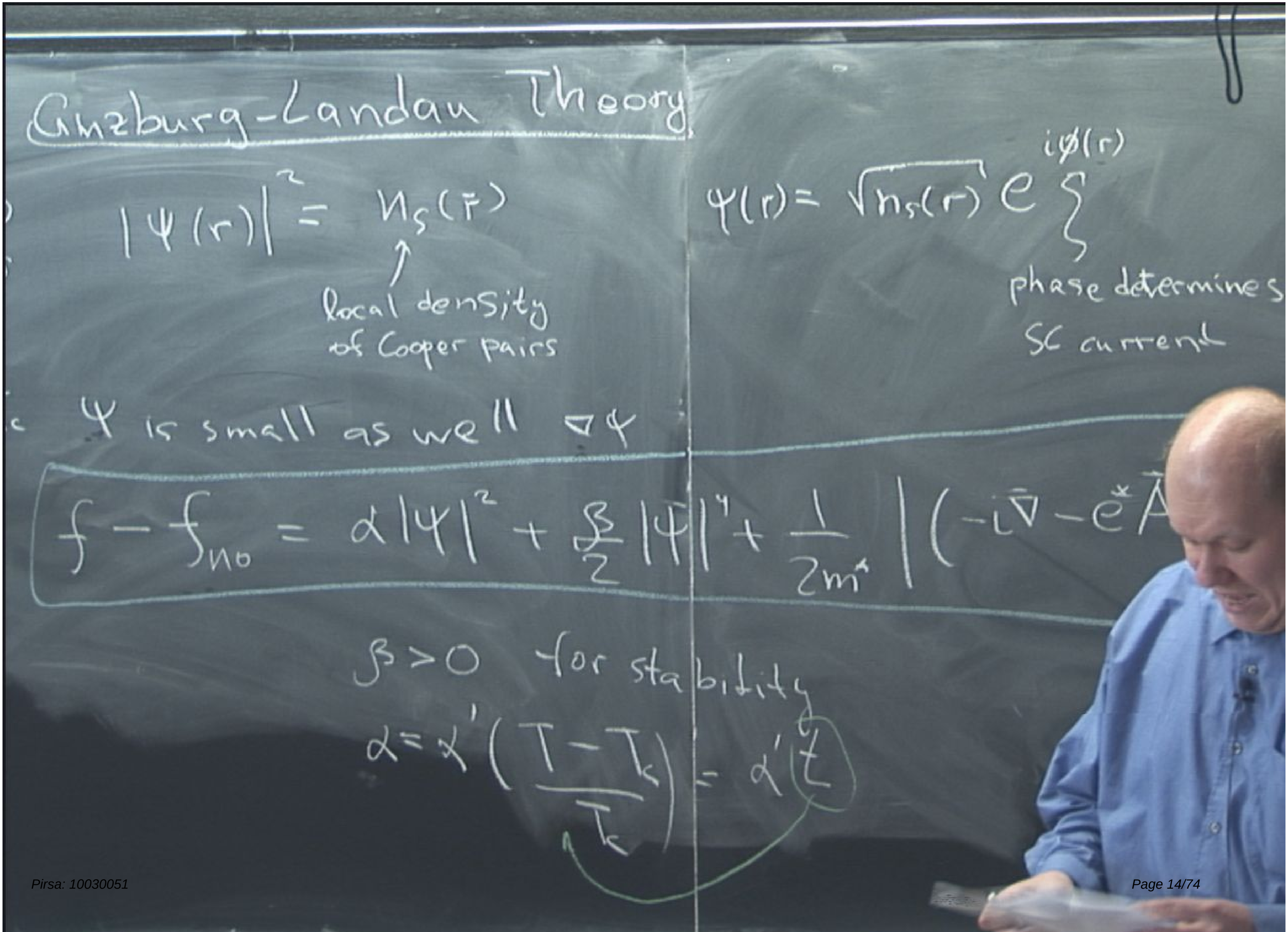
$$\boxed{f - f_{no} = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \quad -i\bar{\nabla} - e^*\vec{A})\psi|}$$

$\beta > 0$ for stability

$\alpha = \alpha'\left(\frac{T - T_c}{T_c}\right)$



Ginzburg-Landau Theory

$$|\psi(r)|^2 = \underset{\substack{\uparrow \\ \text{local density} \\ \text{of Cooper pairs}}}{n_s(\bar{r})}$$

$$\psi(r) = \sqrt{n_s(r)}\; e^{\overbrace{i\phi(r)}}$$

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$$\boxed{f - f_{no} = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\bar{\nabla} - e^*\bar{A}\right.\right.}$$

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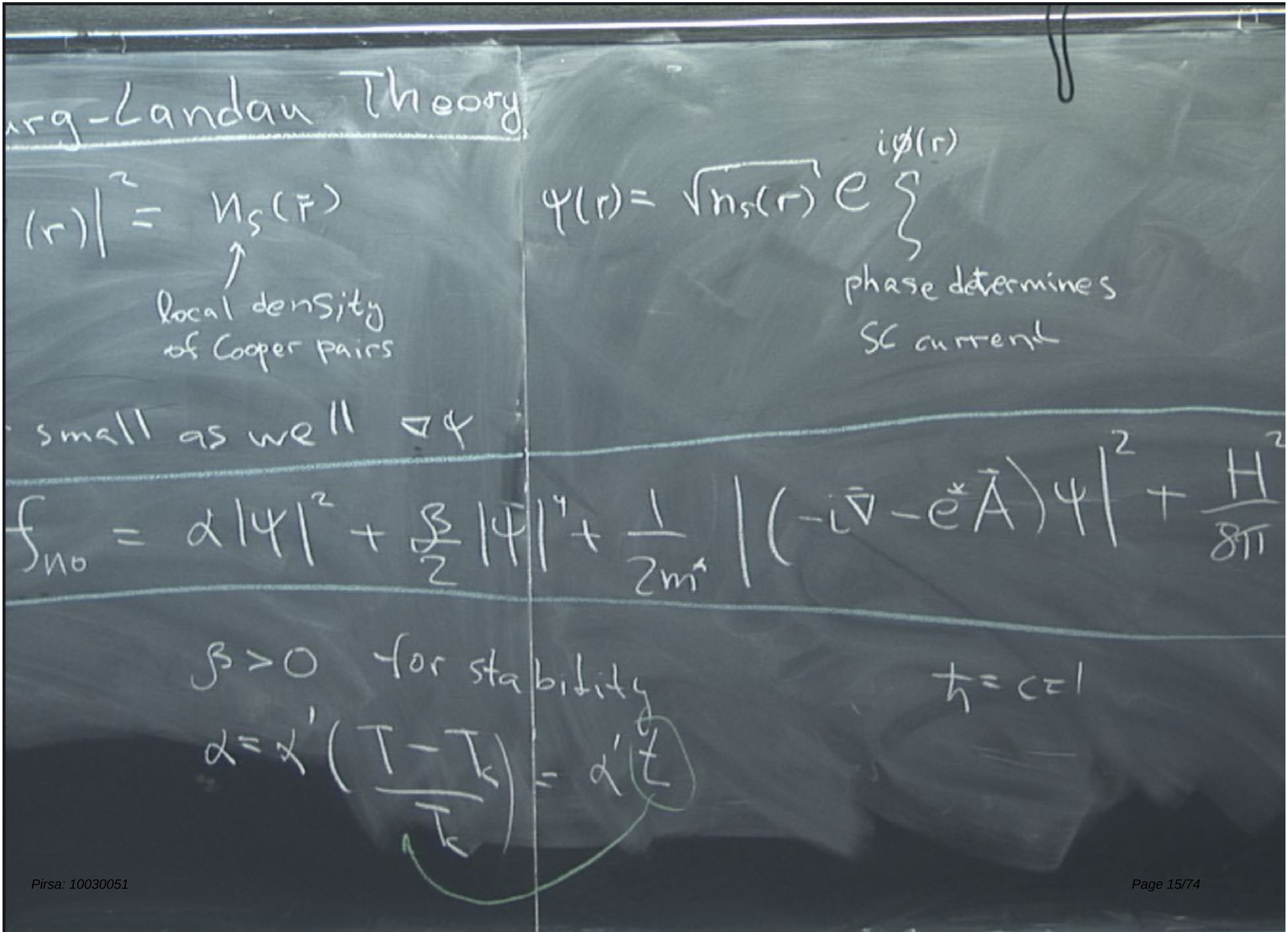
$$\alpha = \alpha'\left(\frac{T - T_c}{T_c}\right) = \alpha' t$$



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$$f_{no} = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\bar{\nabla} - e^*\bar{A}\right)\psi\right|^2 + \frac{H^2}{8\pi}$$

$\beta > 0$ for stability

$\hbar = c = 1$

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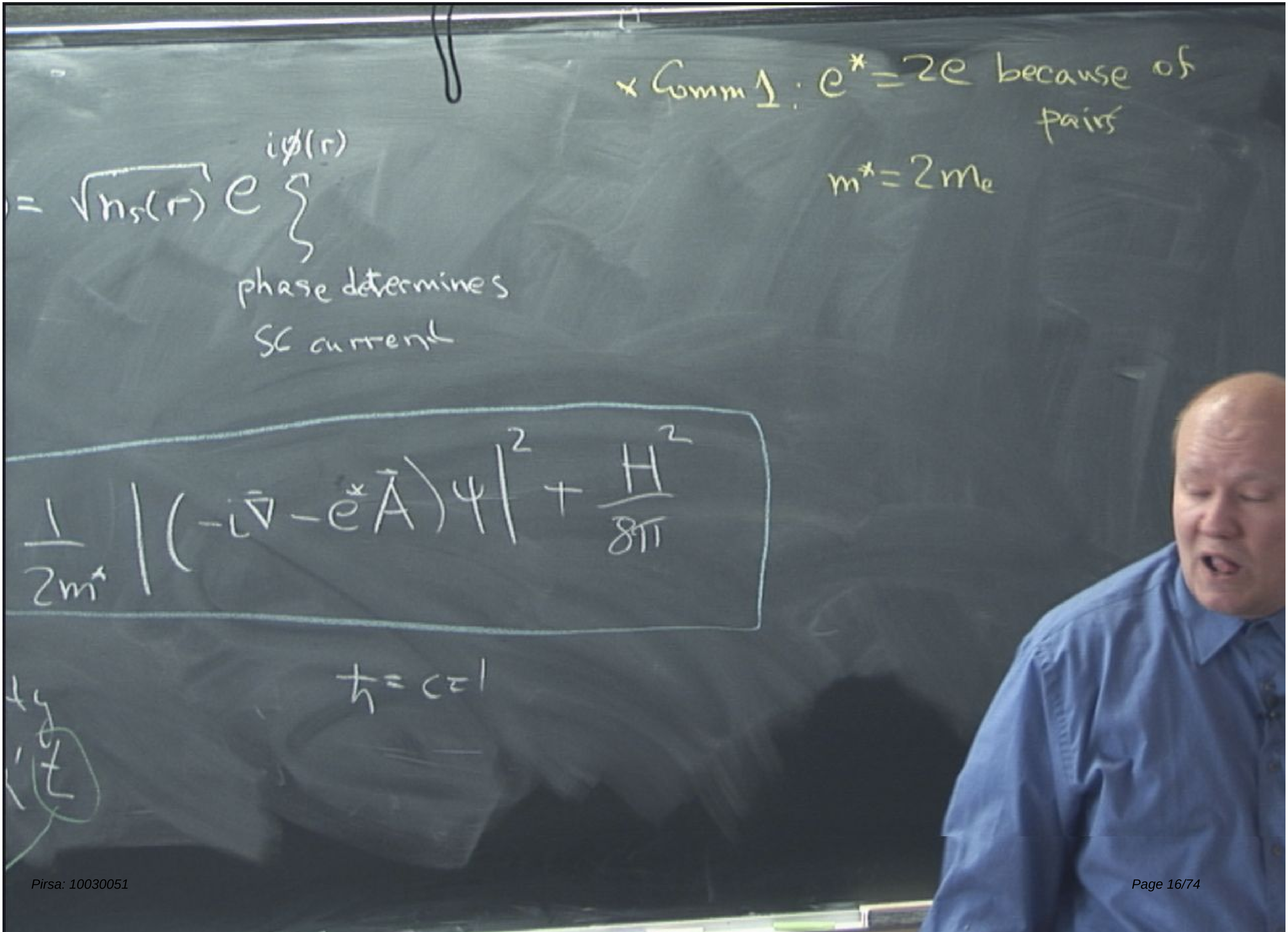


× Comm 1: $e^* = 2e$ because of pairs

$m^* = 2m_e$

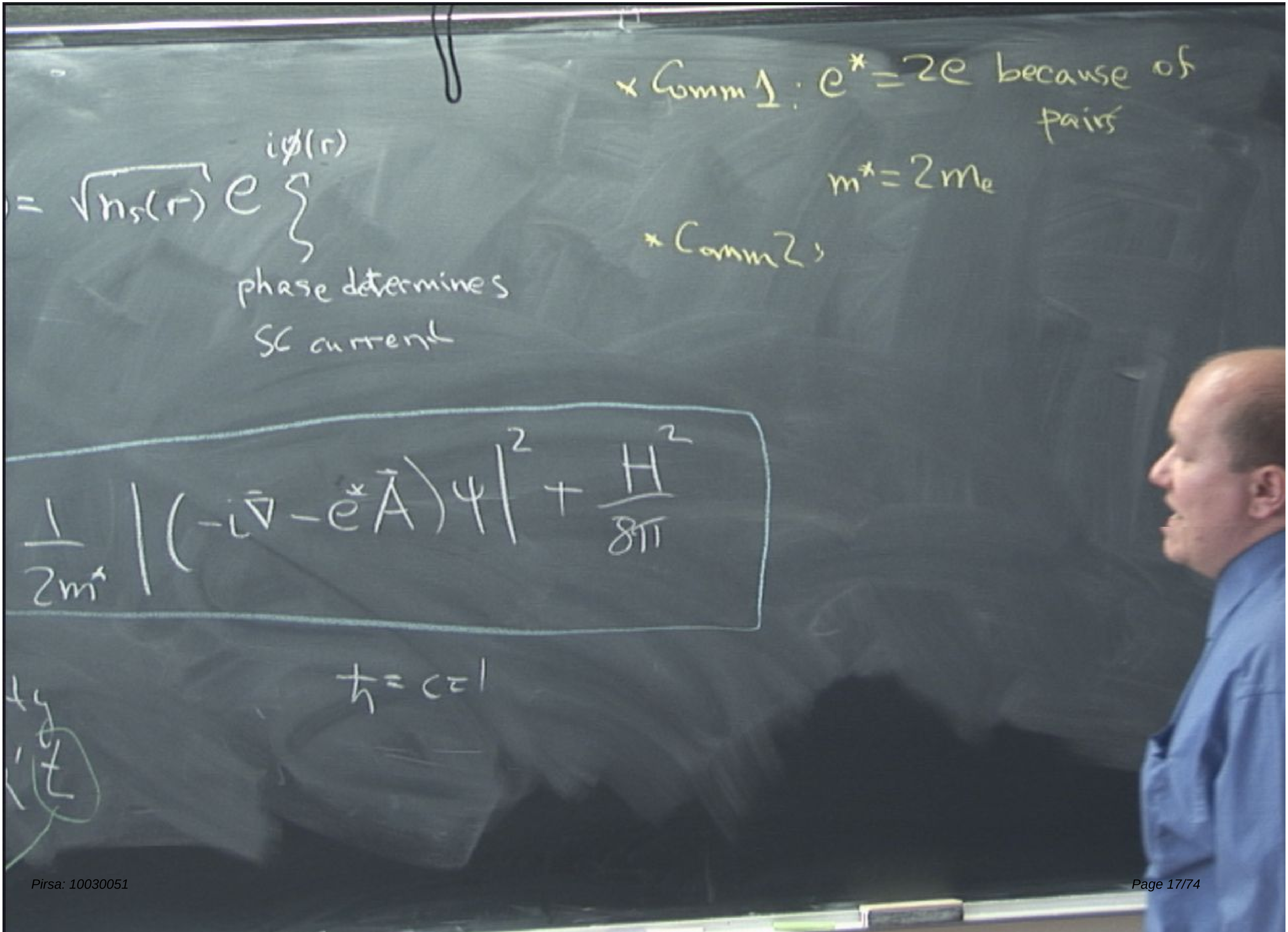
$$= \sqrt{n_s(r)}\, e^{\underbrace{i\phi(r)}_{\text{phase determines SC current}}}$$

$$\boxed{\frac{1}{2m^*}\left|\left(-i\bar{\nabla} - e^*\bar{A}\right)\psi\right|^2 + \frac{H^2}{8\pi}}$$

$\hbar = c = 1$

ty

(t)



$$= \sqrt{n_s(r)}\, \underbrace{e^{i\phi(r)}}_{\text{phase determines SC current}}$$

* Comm 1: $e^* = 2e$ because of pairs

$m^* = 2m_e$

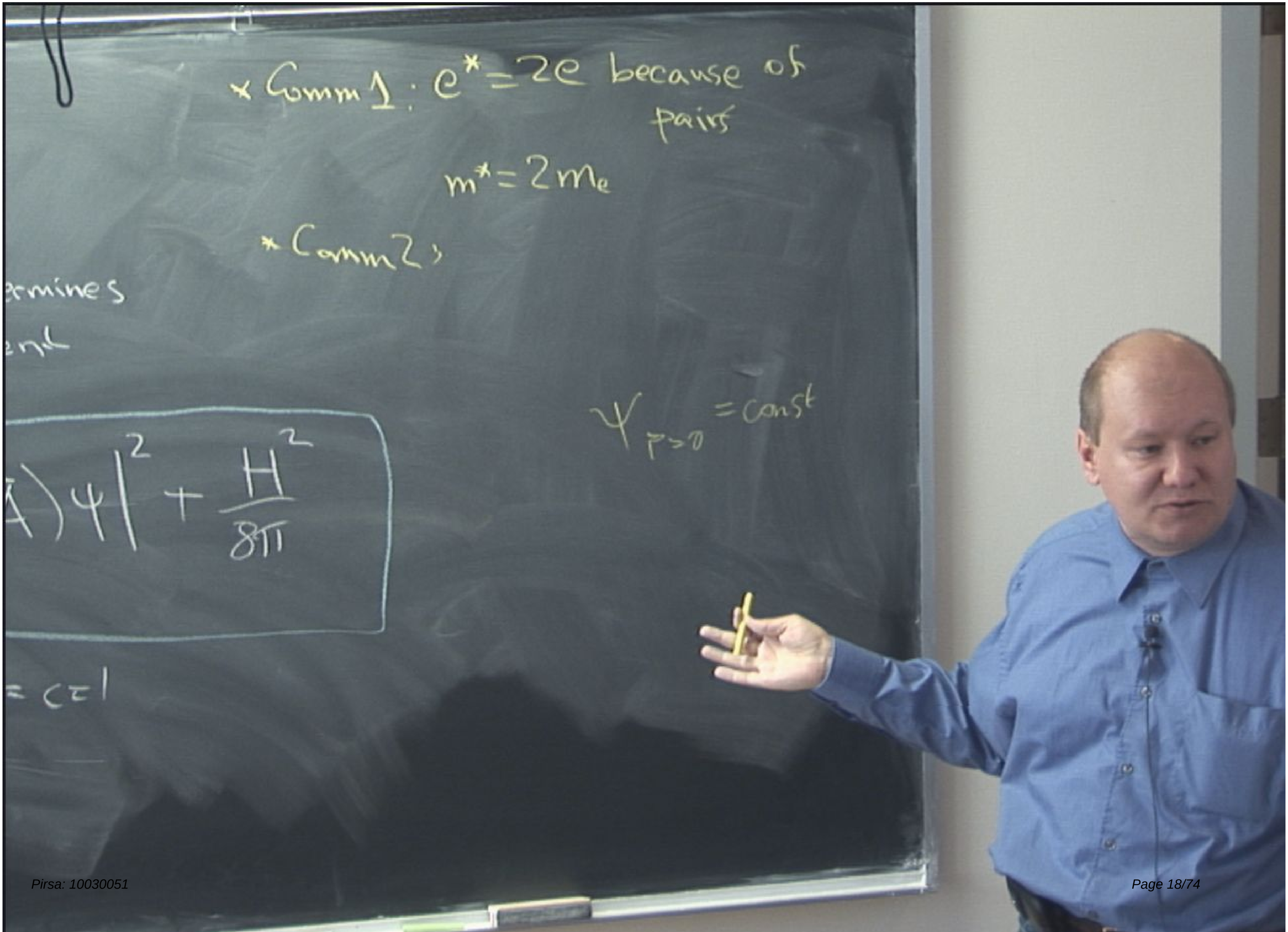
* Comm 2:

$$\boxed{\frac{1}{2m^*}\left|\left(-i\bar{\nabla} - e^*\bar{A}\right)\psi\right|^2 + \frac{H^2}{8\pi}}$$

$\hbar = c = 1$

ty

t



* Comm 1: $e^* = 2e$ because of pairs

$m^* = 2m_e$

* Comm 2:

ermines

end

$$\boxed{\bar{A})\psi|^2 + \frac{H^2}{8\pi}}$$

$\Psi_{p>0} = const$

$= c\tau'$

$$H_{BCS} = \sum_{k\alpha} (\varepsilon_k - \mu) C^{+}_{k\alpha} C_{k\alpha} - \frac{V}{N} \sum_{k,k'} C^{+}_{k'\uparrow} C^{+}_{-k'\downarrow} C_{-k\downarrow} C_{k\uparrow}$$

$$E_{bcs} - E_n = -\frac{N(\varepsilon_F)}{2} \Delta^2$$

* Comm 1: $e^* = 2e$ because of pairs

$m^* = 2m_e$

$e^{i\phi(r)}$

phase determines SC current

* Comm 2: $\alpha' = N(\varepsilon_F)$

$|\psi|^2 = |\Delta|^2$

$\beta = 0.1066 \frac{N(\varepsilon_F)}{T_c^2}$

$$-i\bar{\nabla} - e^*\bar{A})\psi\Big|^2 + \frac{H^2}{8\pi}$$

$\hbar = c = 1$

* Comm 1: $e^* = 2e$ because of pairs

$m^* = 2m_e$

* Comm 2; $\alpha' = N(\mathcal{E}_F)$

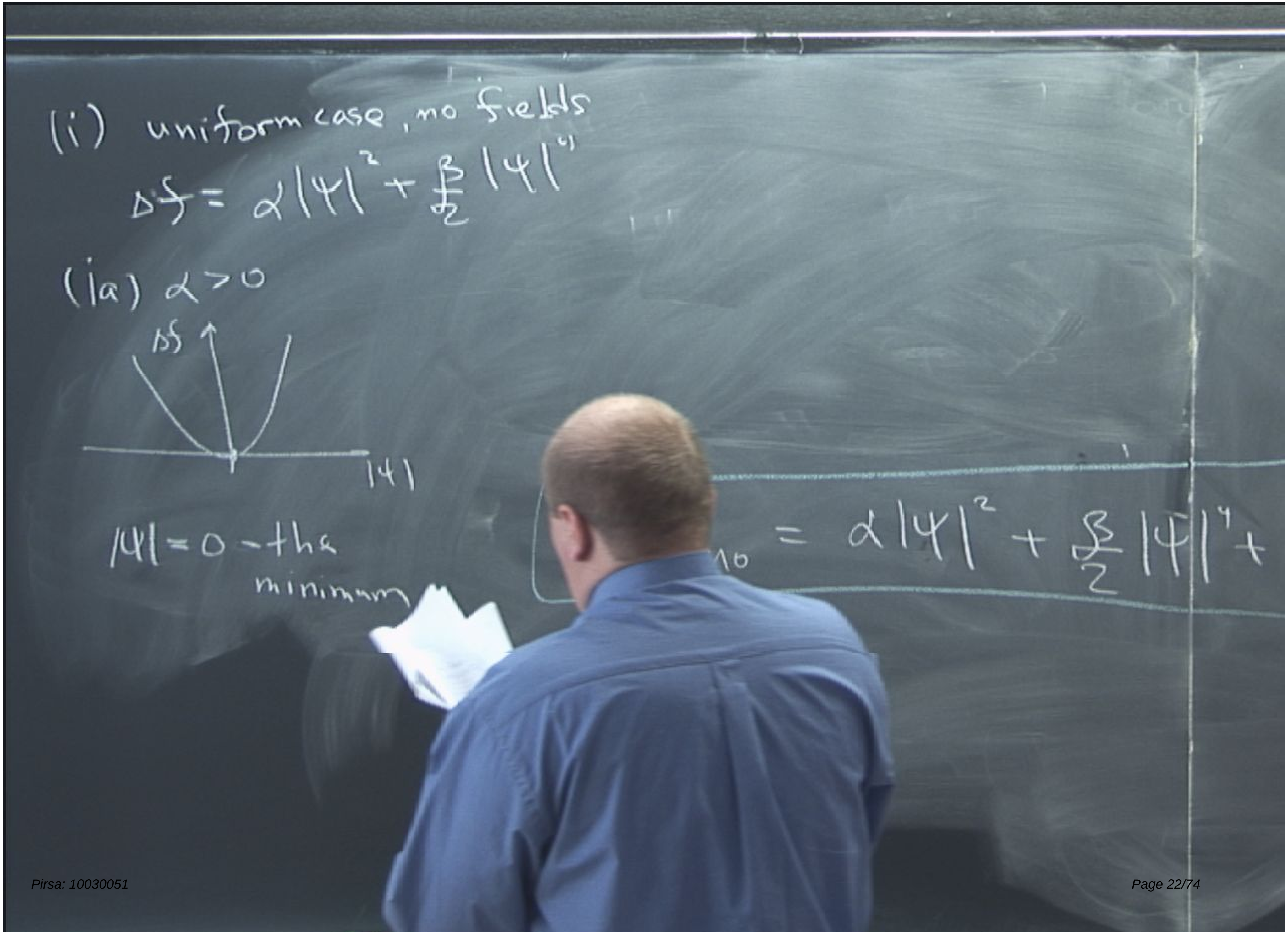
$|\psi|^2 = |\Delta|^2$

$$\underbrace{\beta = 0.1066 \frac{N(\mathcal{E}_F)}{T_c^2}}_{\text{from weak coupling BCS theory}}$$

determines

rrent

$$\boxed{\;|^2 + \frac{H^2}{8\pi}}$$



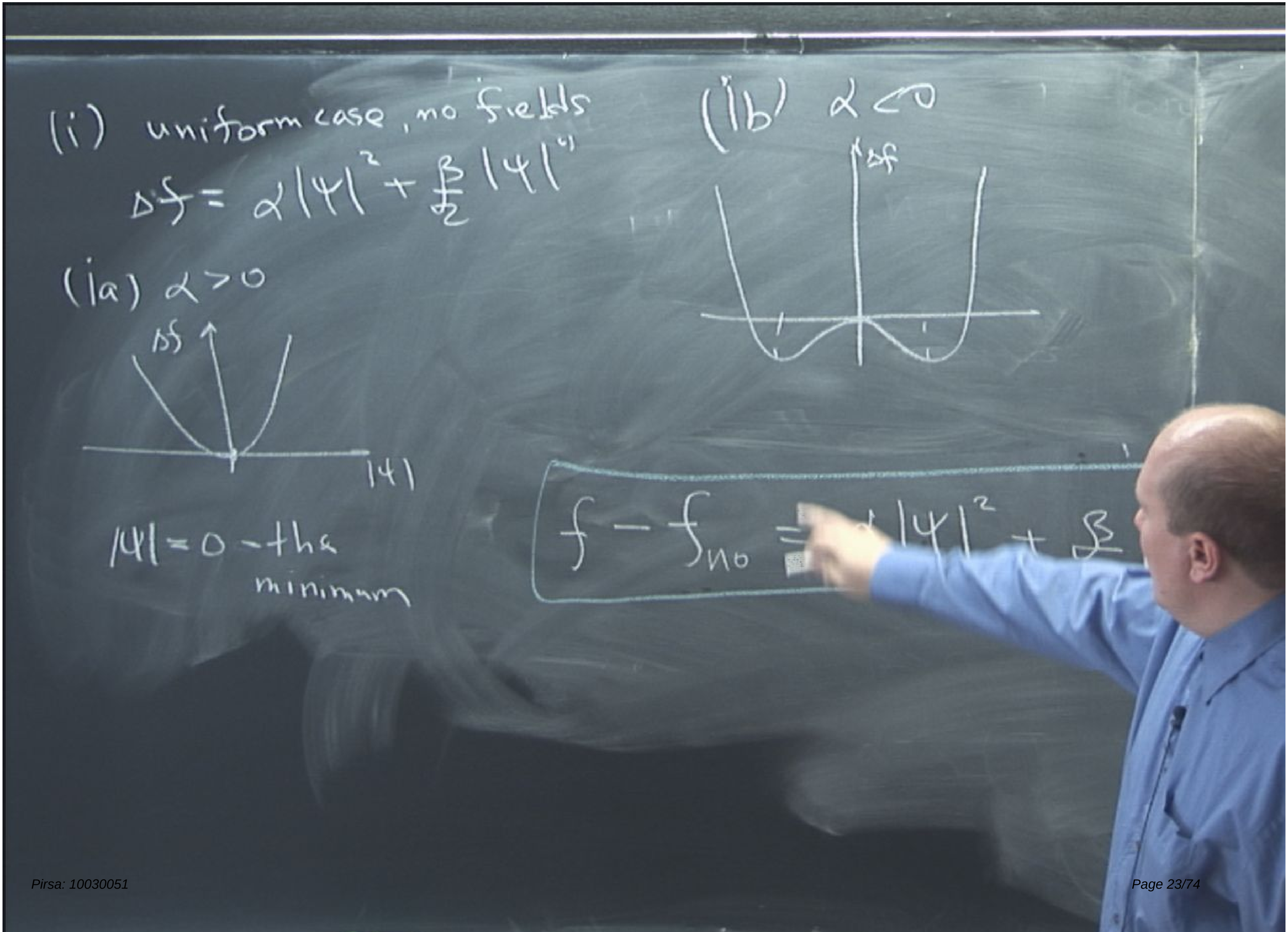
(i) uniform case, no fields

$$\Delta f = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$

(ia) $\alpha > 0$

$|\psi| = 0$ - the minimum

$${}_{0} = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 +$$



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(ib) $\alpha < 0$

$$\boxed{f - f_{no} = \text{[illegible]}|\psi|^2 + \beta}$$

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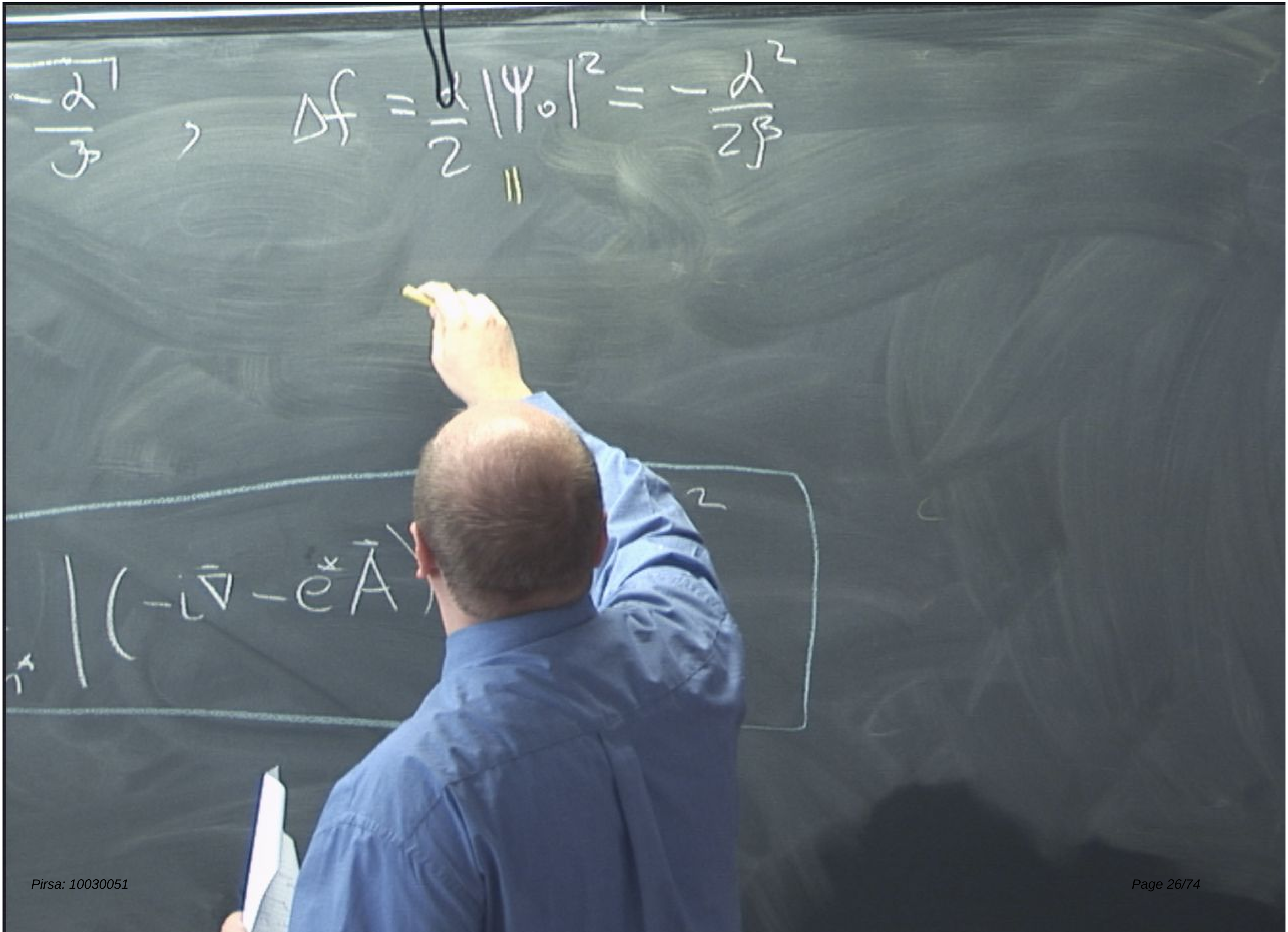
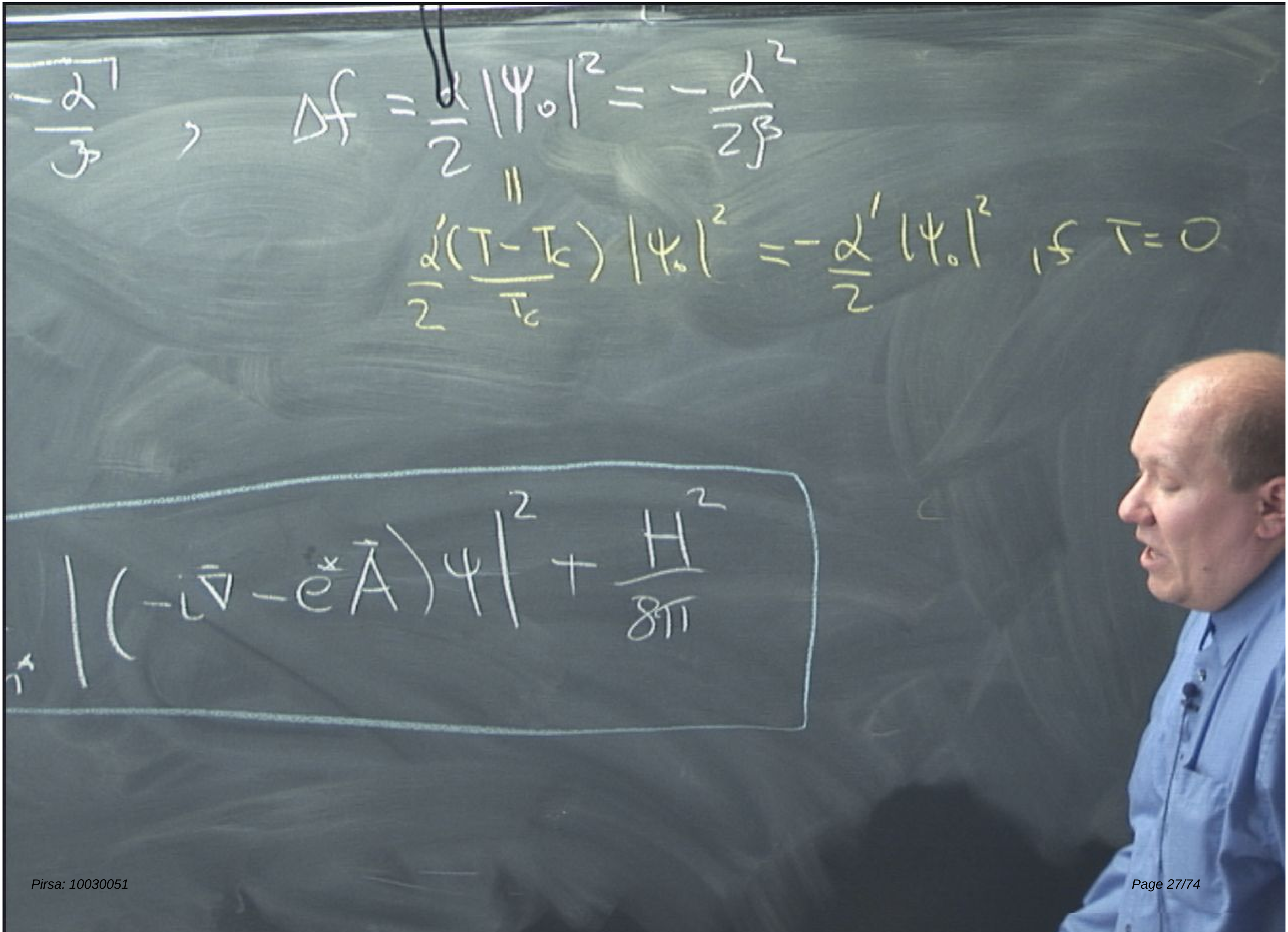


$$-\frac{\alpha}{\beta} \quad , \quad \Delta f = \frac{\alpha}{2}|\Psi_0|^2 = -\frac{\alpha^2}{2\beta}$$

$$\parallel$$

$$|(-i\vec{\nabla} - e^*\vec{A})$$

$$2$$

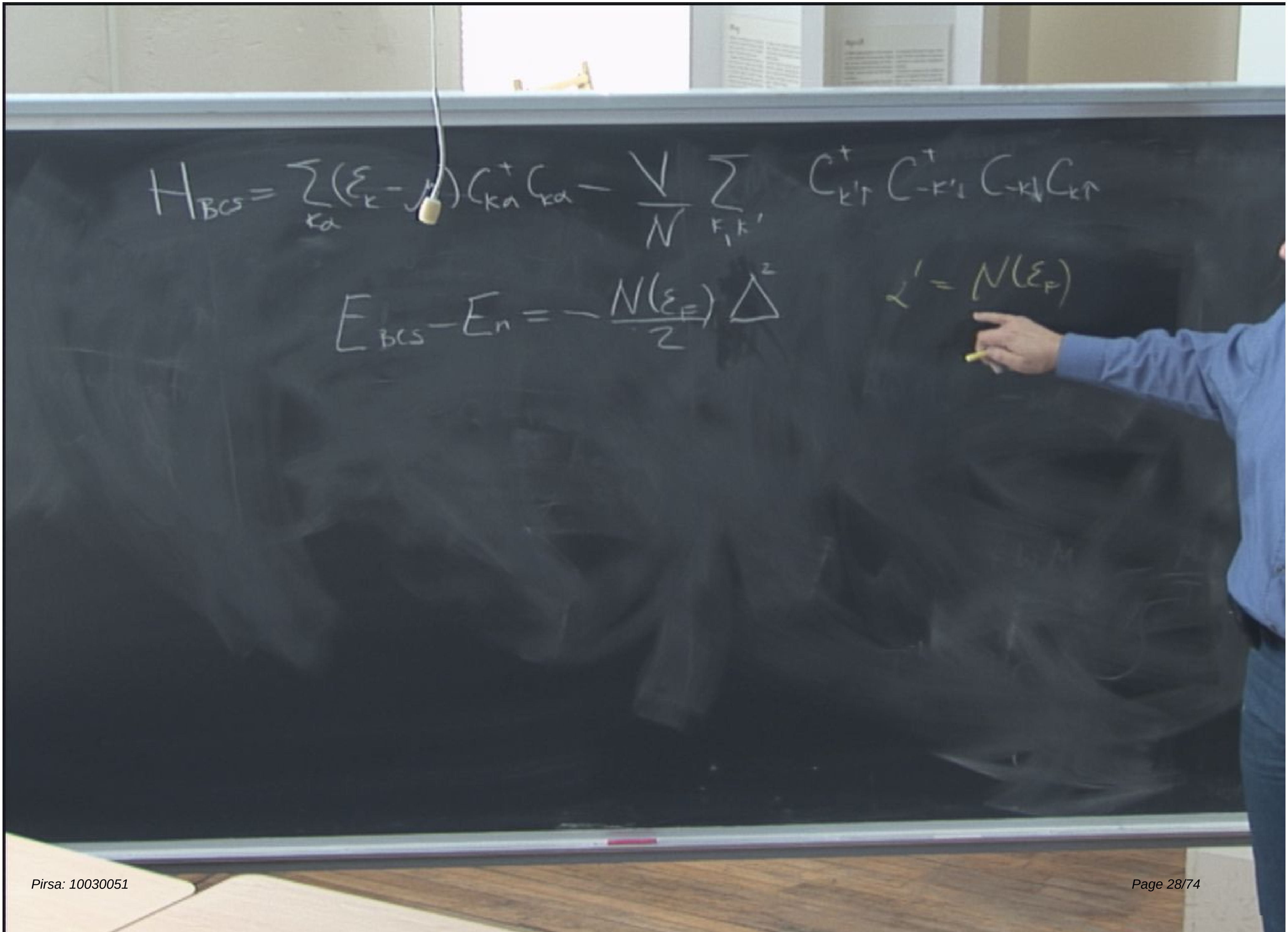


$$-\frac{\alpha}{\beta} \quad , \quad \Delta f = \frac{\alpha}{2}|\Psi_0|^2 = -\frac{\alpha^2}{2\beta}$$

$$\|$$

$$\frac{\alpha'}{2}\frac{(T-T_c)}{T_c}|\Psi_0|^2 = -\frac{\alpha'}{2}|\Psi_0|^2 \text{ if } T=0$$

$$\boxed{|(-i\vec{\nabla}-e^*\vec{A})\psi|^2+\frac{H^2}{8\pi}}$$



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$$E_{BCS} - E_n = -\frac{N(\varepsilon_F)}{2} \Delta^2$$

$$\lambda' = N(\varepsilon_F)$$

$$, \quad \Delta f = \frac{\alpha}{2}|\Psi_0|^2 = -\frac{\alpha^2}{2\beta}$$

$$\|$$

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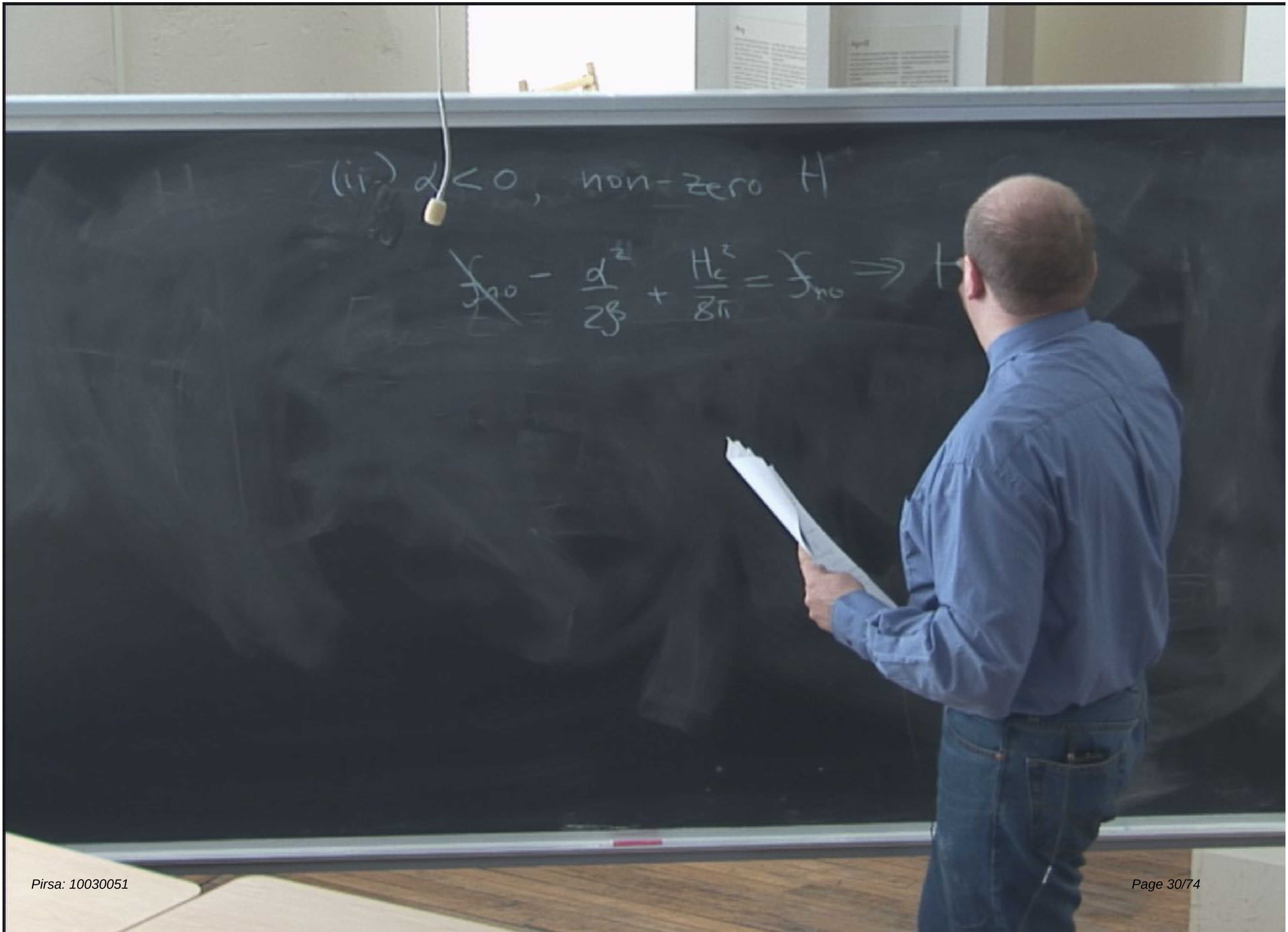
$$\boxed{-i\bar{\nabla} - e^*\bar{A})\psi\Big|^2 + \frac{H^2}{8\pi}}$$



(ii) $\alpha < 0$, non-zero H

$$\cancel{f_{n0}} - \frac{\alpha^2}{2\beta} + \frac{H_c^2}{8\pi} = f_{n0} \Rightarrow H$$

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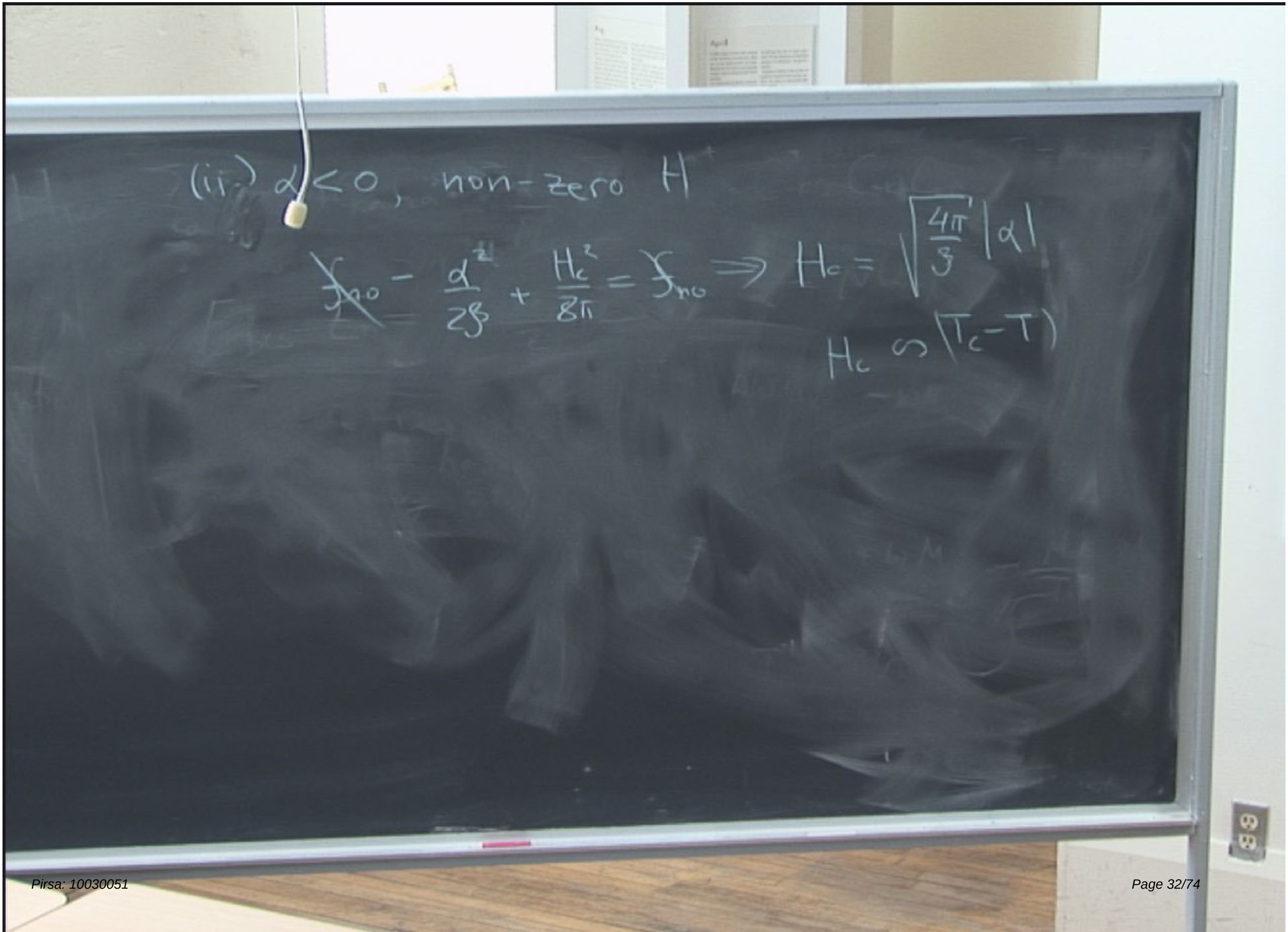
$$\cancel{f_{n0}} - \frac{\alpha^2}{2\beta} + \frac{H_c^2}{8\pi} = f_{n0} \Rightarrow H_c = \sqrt{\frac{4\pi}{\beta}}\,|\alpha|$$

$$H_c \sim (T_c - T)$$



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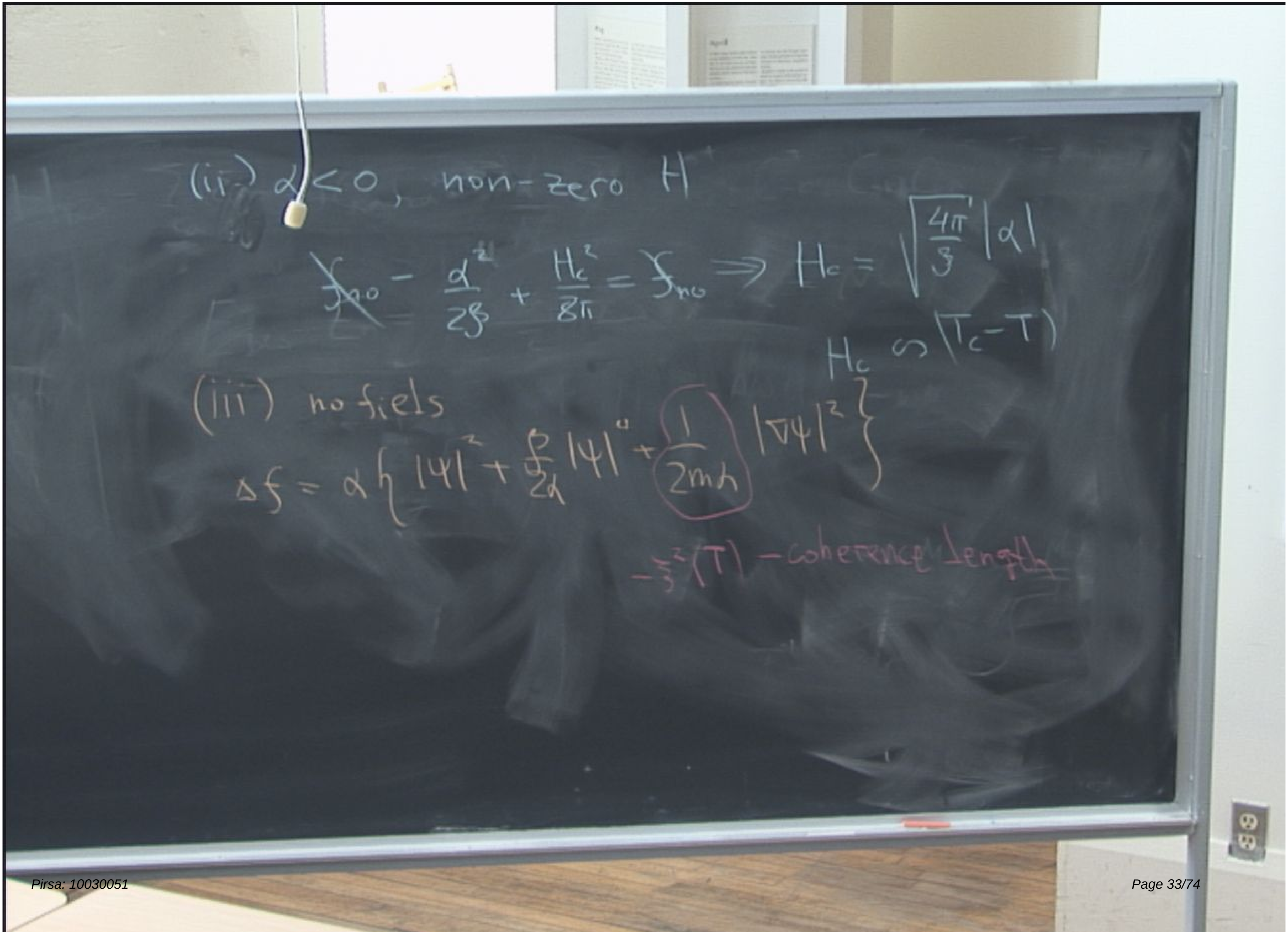

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$$H_c \backsim (T_c - T)$$

(iii) no fiels

$$\Delta f = \alpha\left\{ |\psi|^2 + \frac{\beta}{2\alpha}|\psi|^4 + \boxed{\frac{1}{2m\alpha}}\,|\nabla\psi|^2 \right\}$$

$-\xi^2(T)$ – coherence length

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(ii) $\alpha < 0$, non-zero H

$$\cancel{f_{no}} - \frac{\alpha^2}{2\beta} + \frac{H_c^2}{8\pi} = f_{no} \Rightarrow H_c = \sqrt{\frac{4\pi}{\beta}}\,|\alpha|$$

$$H_c \sim (T_c - T)$$

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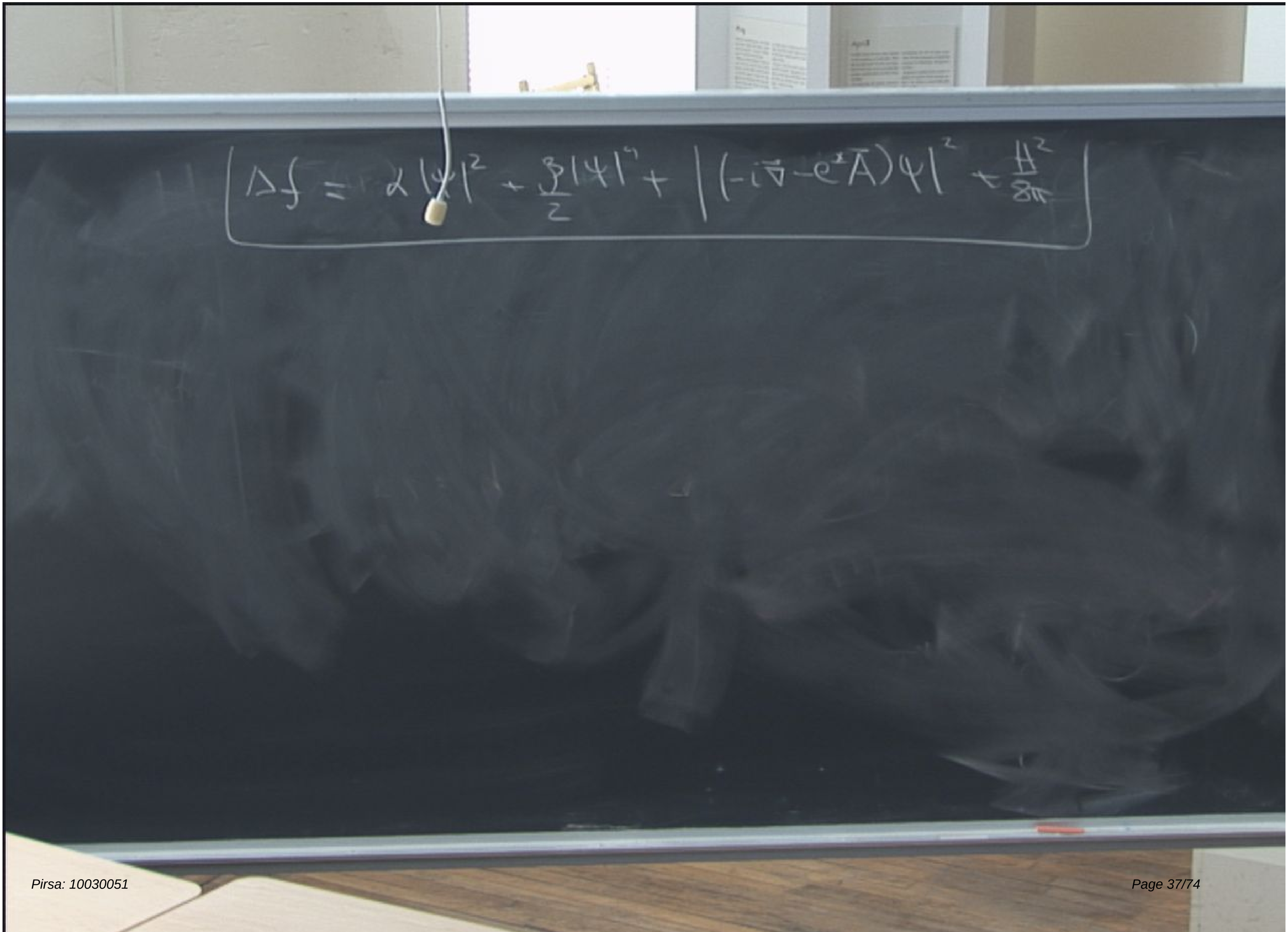
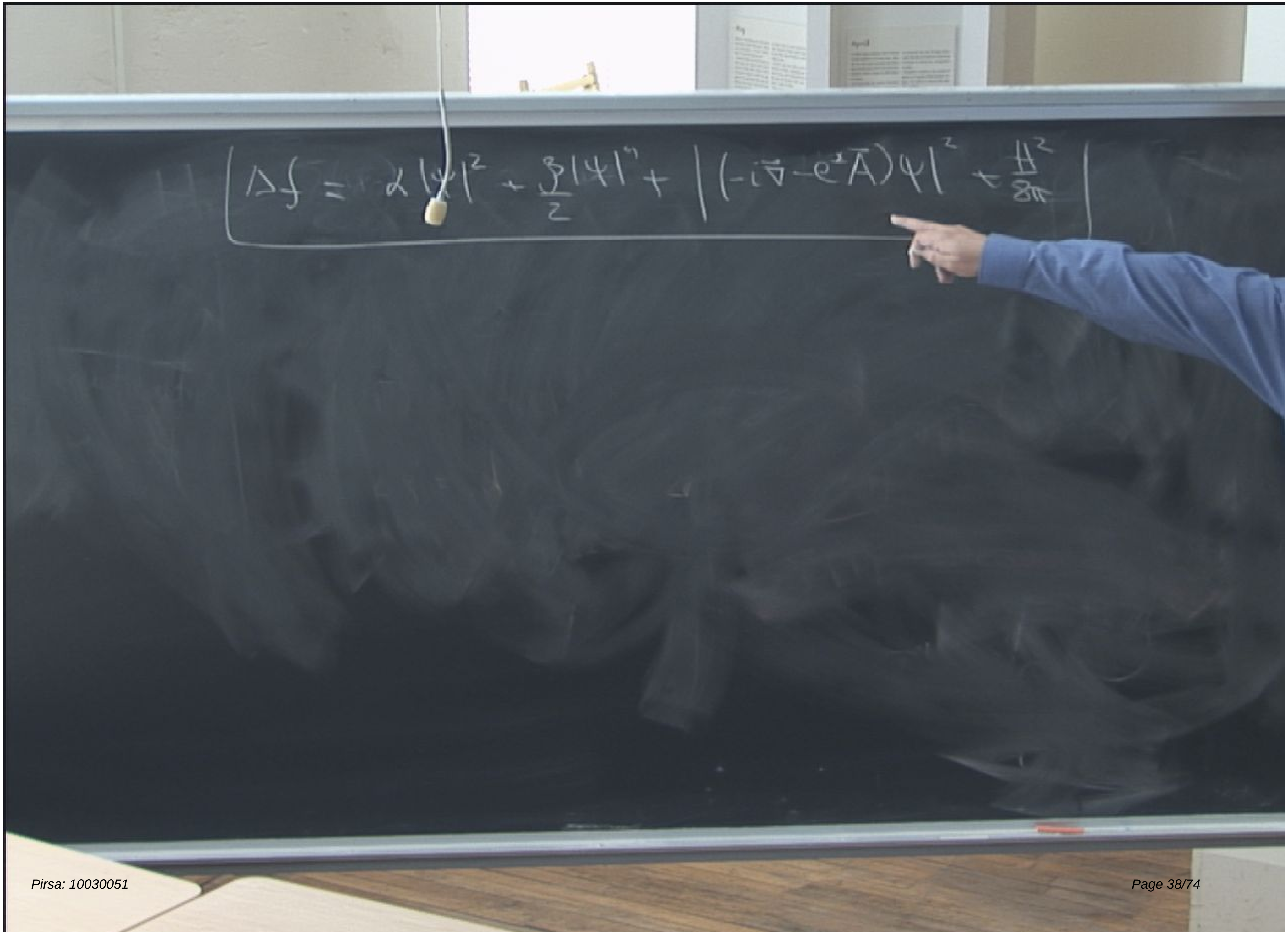
$-\xi^2(T)$ – coherence length

Ginzburg-Landau Equations



$$\boxed{\Delta f = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \left|(-i\vec{\nabla} - e^*\overline{A})\psi\right|^2 + \frac{H^2}{8\pi}}$$



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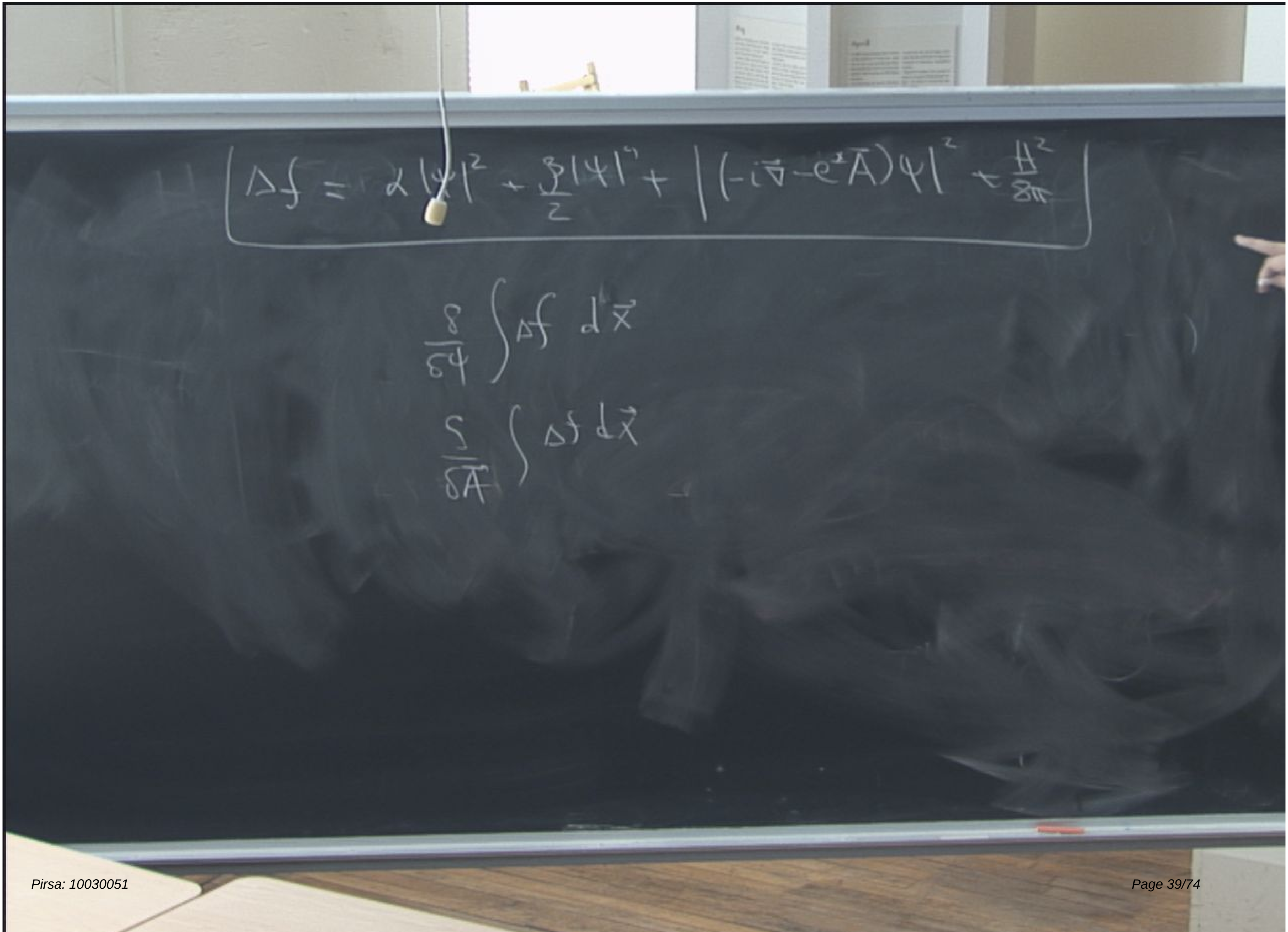


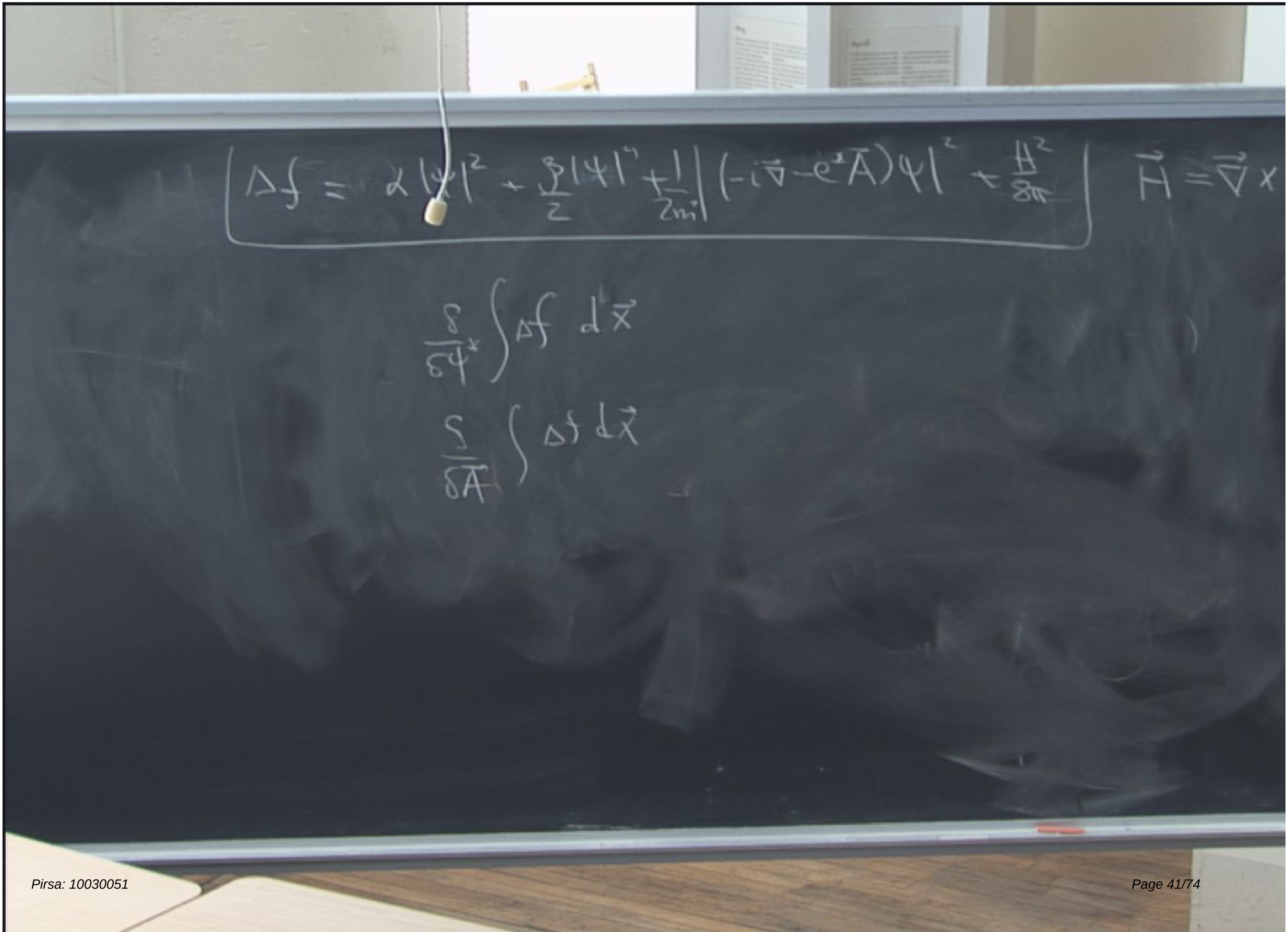
$$\boxed{\Delta f = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \left|\left(-i\vec{\nabla} - e^*\vec{A}\right)\psi\right|^2 + \frac{H^2}{8\pi}}$$

$$\frac{\delta}{\delta\psi}\int \Delta f \, d\vec{x}$$

$$\frac{\delta}{\delta\vec{A}}\int \Delta f \, d\vec{x}$$

Ginzburg-Landau Equat

(I) $$\frac{1}{2m^*}\left|-i\vec{\nabla}-e^*\vec{A}\right|^2\psi+\alpha\psi+\beta|\psi|^2\psi=0$$

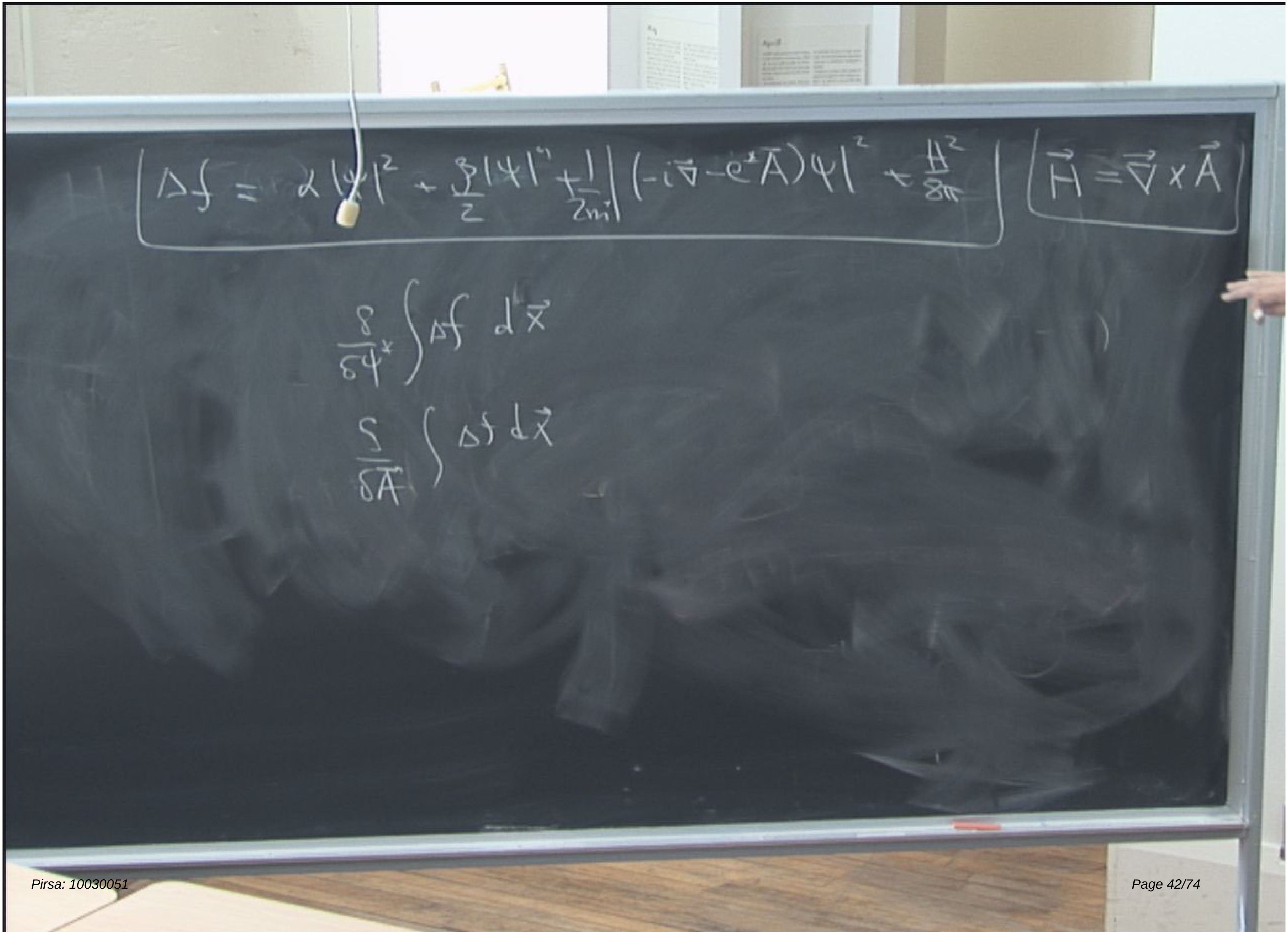


$$\boxed{\Delta f = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}|(-i\vec{\nabla} - e^*\vec{A})\psi|^2 + \frac{H^2}{8\pi}}$$

$$\vec{H} = \vec{\nabla} \times$$

$$\frac{\delta}{\delta\psi^*}\int \Delta f \, d\vec{x}$$

$$\frac{\delta}{\delta\vec{A}}\int \Delta f \, d\vec{x}$$



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$$\boxed{\vec{H} = \vec{\nabla} \times \vec{A}}$$

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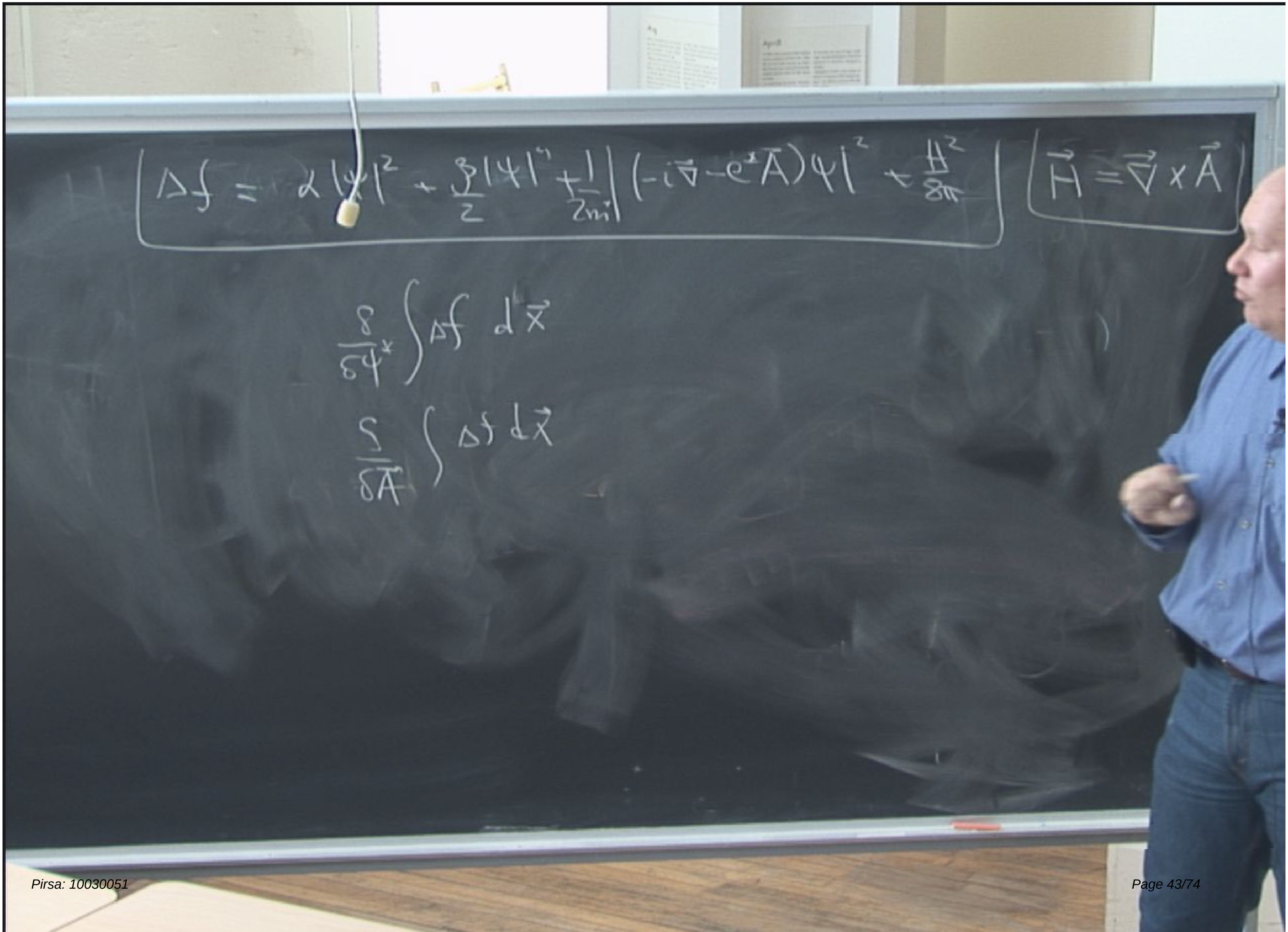

$$\boxed{\Delta f = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|(-i\vec{\nabla} - e^*\vec{A})\psi\right|^2 + \frac{H^2}{8\pi}} \quad \boxed{\vec{H} = \vec{\nabla} \times \vec{A}}$$

$$\frac{\delta}{\delta\psi^*}\int \Delta f \, d\vec{x}$$

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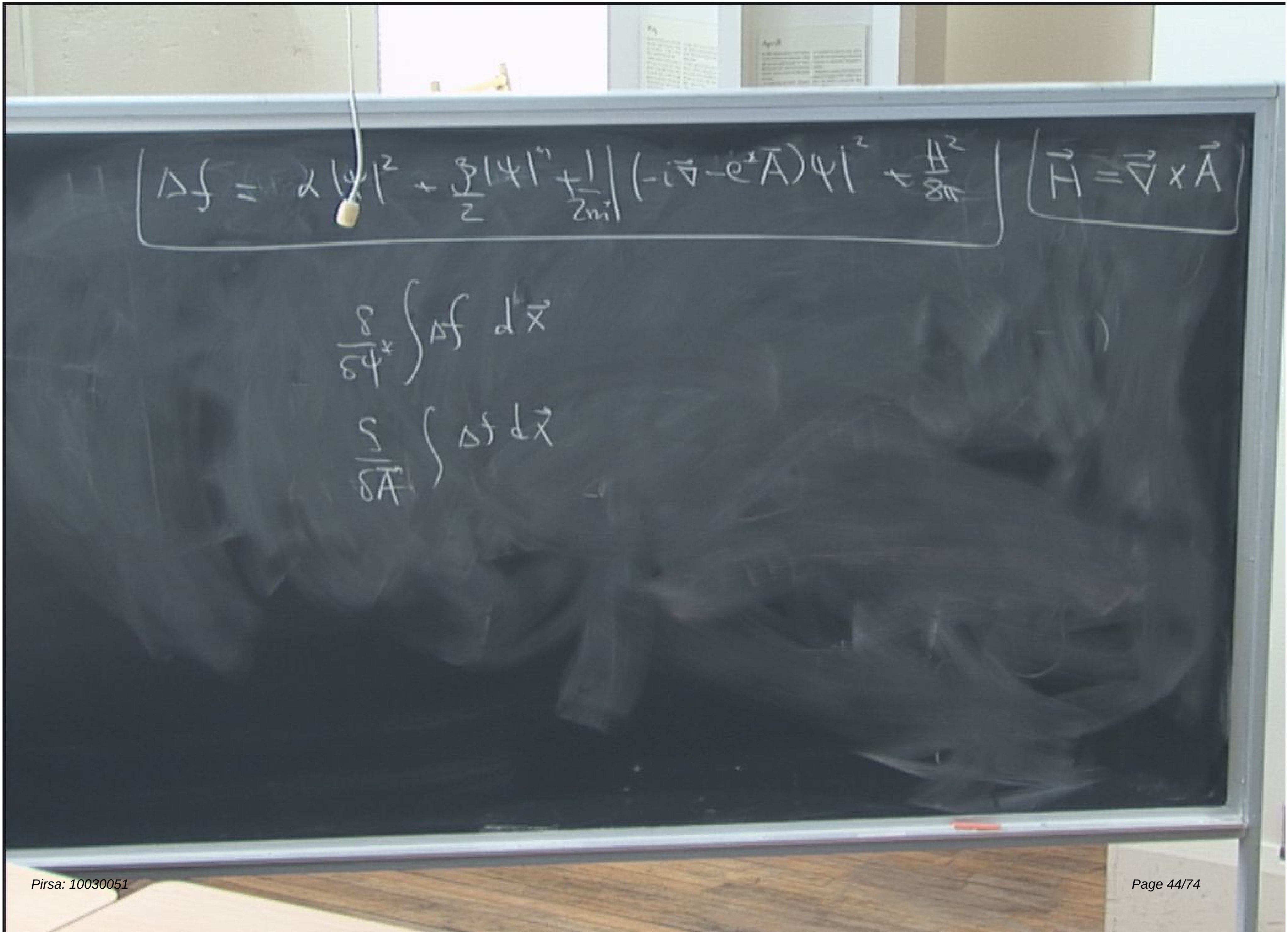

$$\boxed{\Delta f = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|(-i\vec{\nabla} - e^*\vec{A})\psi\right|^2 + \frac{H^2}{8\pi}} \qquad \boxed{\vec{H} = \vec{\nabla} \times \vec{A}}$$

$$\frac{\delta}{\delta\psi^*}\int \Delta f \, d^3\vec{x}$$

$$\frac{\delta}{\delta\vec{A}}\int \Delta f \, d\vec{x}$$

$$\delta \int d\vec{x}\, (\vec{\nabla} \times \vec{A})^2 = 2 \int (\vec{\nabla} \times \vec{A}) \cdot (\vec{\nabla} \times \delta \vec{A})\, d\vec{x}$$

$$(\vec{\nabla} \times \vec{A}) \cdot (\vec{\nabla} \times \delta \vec{A}) = \vec{\nabla} \cdot ((\vec{\nabla} \times \vec{A}) \times \delta \vec{A})$$

$$+ (\vec{\nabla} \times (\vec{\nabla} \times \vec{A}))\, \delta \vec{A}$$

$$\delta \int d\vec{x}\, (\vec{\nabla} \times \vec{A})^2 = 2 \int (\vec{\nabla} \times \vec{A}) \cdot (\vec{\nabla} \times \delta \vec{A})\, d\vec{x}$$

$$(\vec{\nabla} \times \vec{A}) \cdot (\vec{\nabla} \times \delta \vec{A}) = \vec{\nabla} \cdot ((\vec{\nabla} \times \vec{A}) \times \delta \vec{A})$$

$$+ (\vec{\nabla} \times (\vec{\nabla} \times \vec{A}))\, \delta \vec{A}$$

$$\delta \int d\vec{x}\, (\vec{\nabla}\times\vec{A})^2 = 2\int (\vec{\nabla}\times\vec{A})\cdot(\vec{\nabla}\times\delta\vec{A})\, d\vec{x}$$

$$(\vec{\nabla}\times\vec{A})\cdot(\vec{\nabla}\times\delta\vec{A}) = \vec{\nabla}\cdot\left((\vec{\nabla}\times\vec{A})\times\delta\vec{A}\right)$$

$$+\left(\vec{\nabla}\times(\vec{\nabla}\times\vec{A})\right)\delta\vec{A}$$

$$\delta \int d\vec{x}\, (\vec{\nabla}\times\vec{A})^2 = 2\int (\vec{\nabla}\times\vec{A})\cdot(\vec{\nabla}\times\delta\vec{A})\, d\vec{x}$$

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$$2\int (\vec{\nabla}\times(\vec{\nabla}\times\vec{A}))\delta\vec{A}\; dx$$

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$$2\int(\vec{\nabla}\times(\vec{\nabla}\times\vec{A}))\delta\vec{A}\, dx$$

$$\vec{J} = -\frac{ie^*}{2m^*}\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right] - \frac{(e^*)^2}{m^*}A|\psi|^2$$

Ginzburg-Landau Equat

(I) $\frac{1}{2m^*}\left|-i\vec{\nabla} - e^*\vec{A}\right|^2 \Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0$

(II) $\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = 4\pi\vec{J}$

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$$(I)\quad \frac{1}{2m^*}\left|-i\vec{\nabla}-e^*\vec{A}\right|^2\psi+\alpha\psi+\beta|\psi|^2\psi=0$$

$$(II)\quad \bar{\nabla}\times(\bar{\nabla}\times\vec{A})=4\pi\vec{J}$$

$$\bar{\nabla}\times(\bar{\nabla}\times\bar{A})+\frac{2\pi i e^*}{m^*}\left[\psi^*\nabla\psi-\psi\nabla\psi^*\right]-$$

$$-\frac{4\pi(e^*)^2}{m^*}\bar{A}|\psi|^2=0$$

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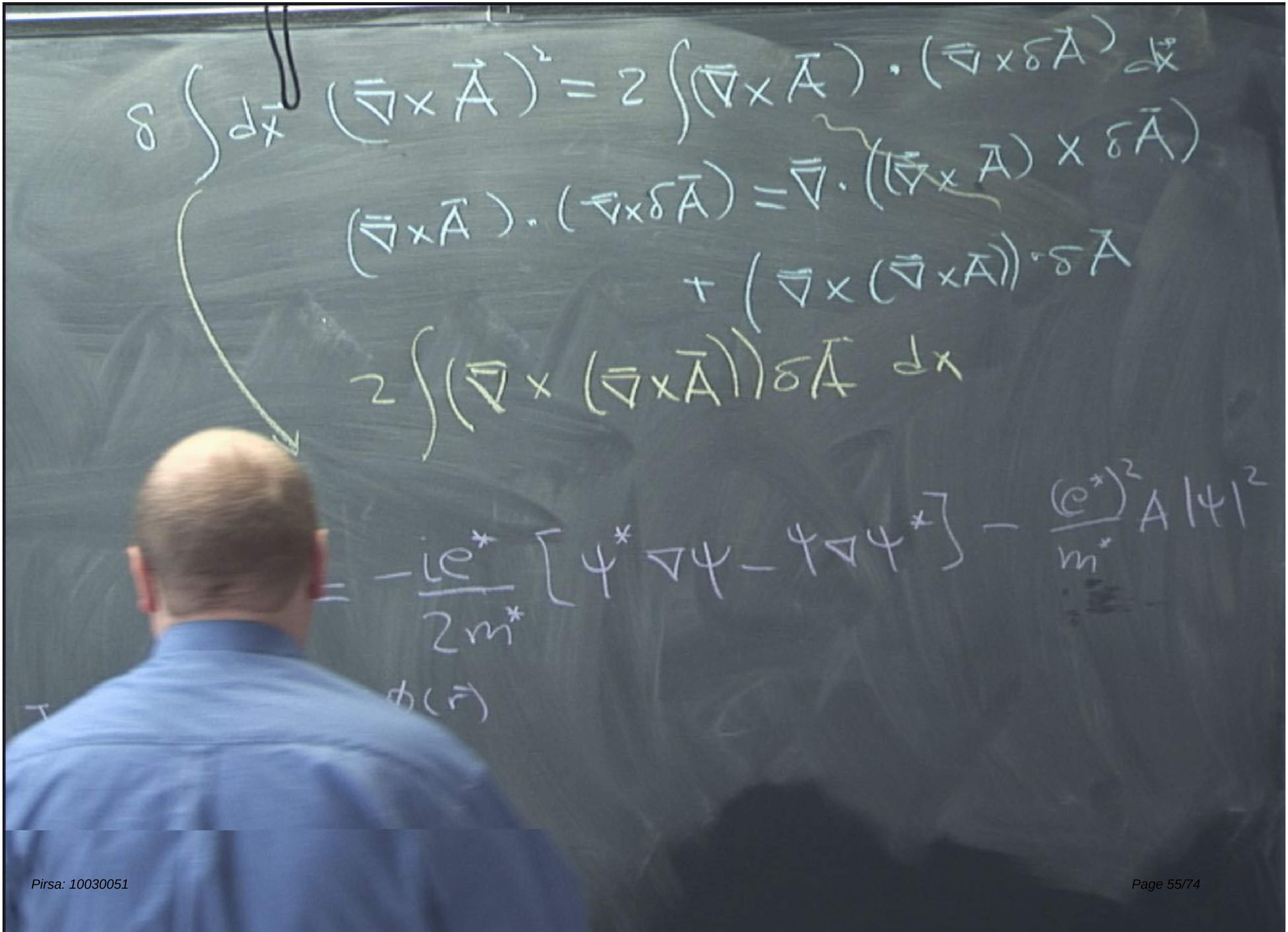

$$\delta \int d\vec{x}\, (\vec{\nabla}\times\vec{A})^2 = 2\int(\vec{\nabla}\times\vec{A})\cdot(\vec{\nabla}\times\delta\vec{A})\, d\vec{x}$$

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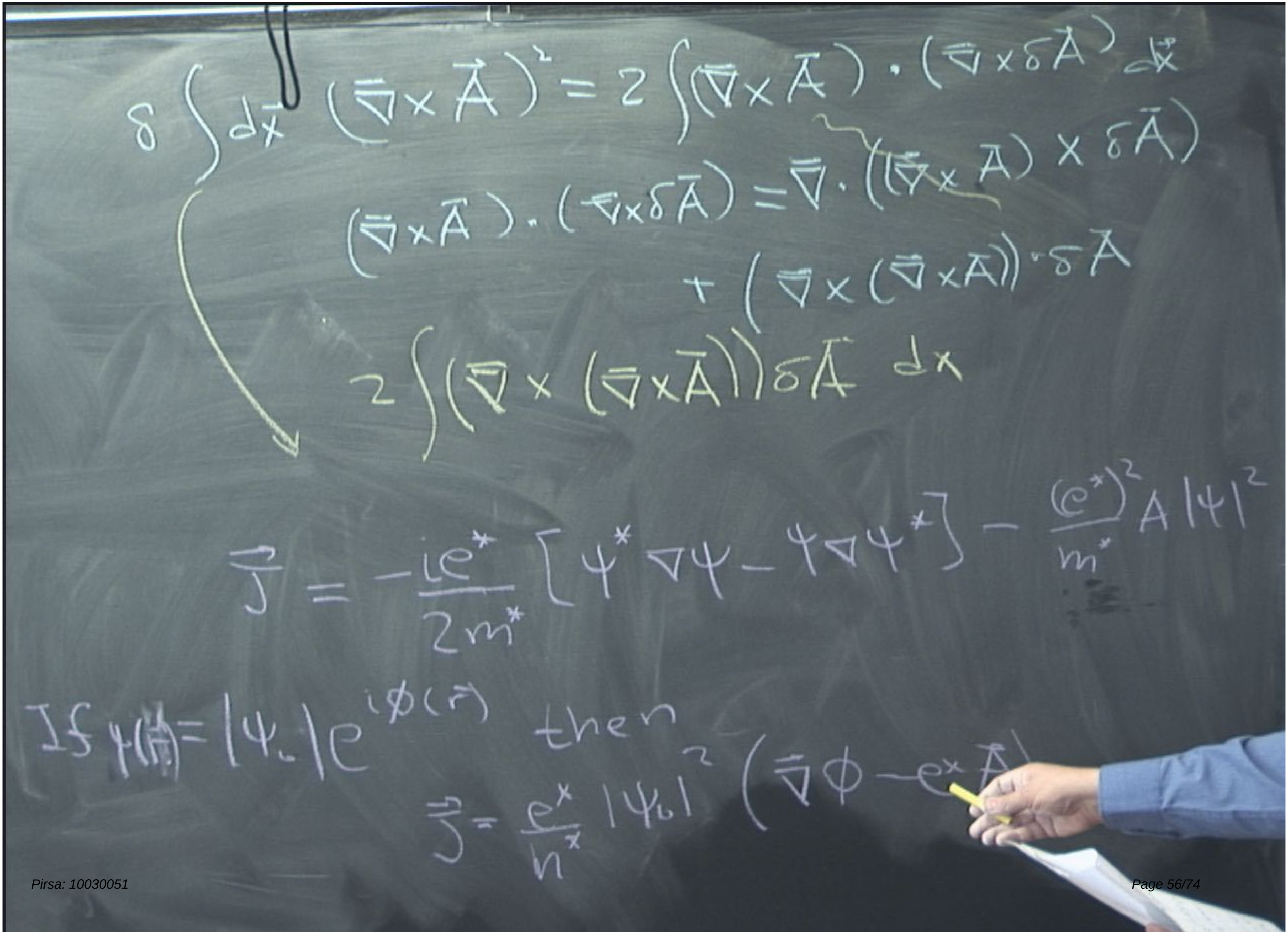
J $\phi(\vec{r})$



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$$\vec{J} = -\frac{ie^*}{2m^*}\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right] - \frac{(e^*)^2}{m^*}A|\psi|^2$$

If $\psi(\vec{r}) = |\psi_0|e^{i\phi(\vec{r})}$ then

$$\vec{J} = \frac{e^*}{n^*}|\psi_0|^2(\vec{\nabla}\phi - e^*\vec{A})$$

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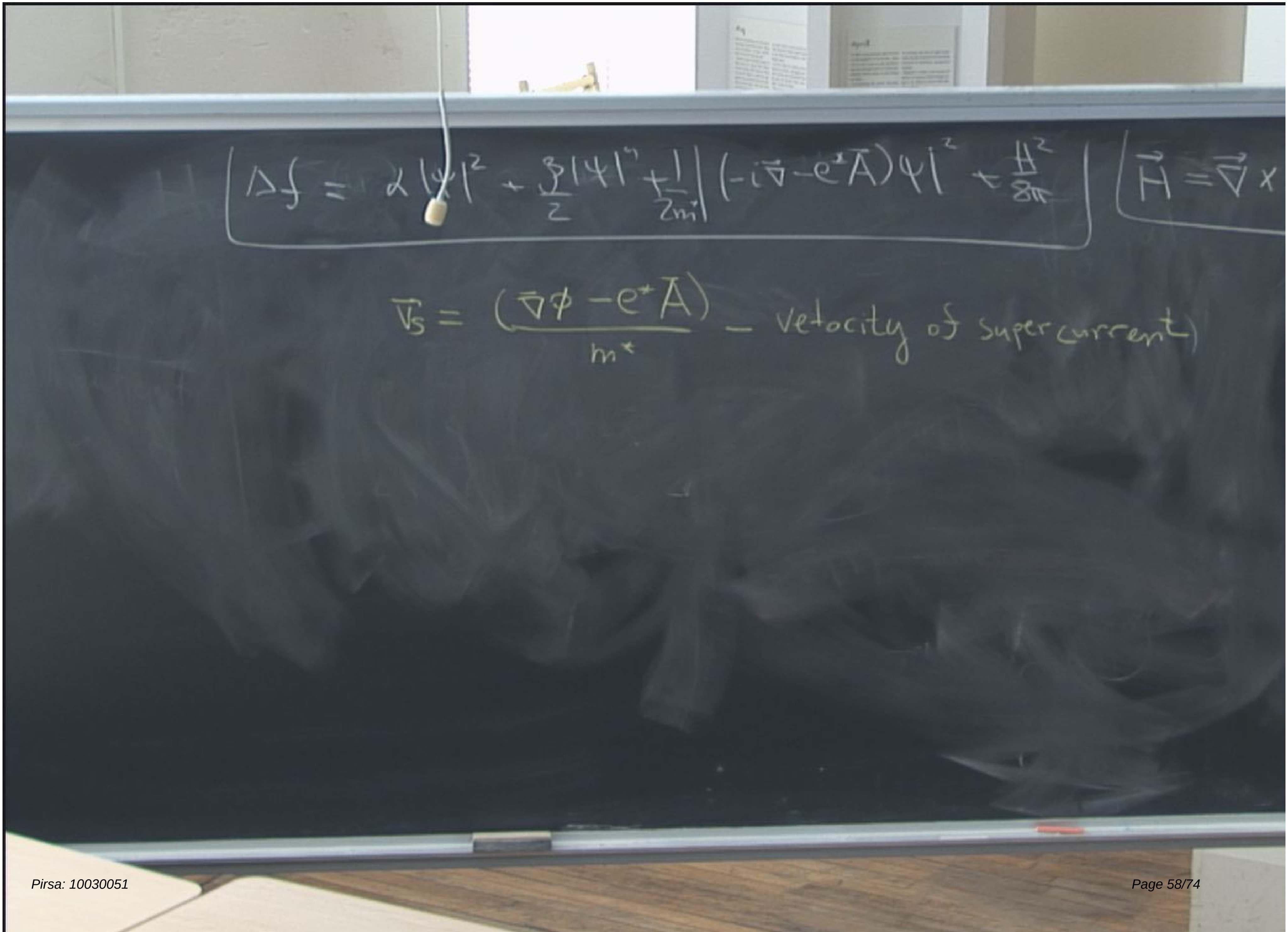

$$\boxed{\Delta f = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|(-i\vec{\nabla} - e^*\vec{A})\psi\right|^2 + \frac{H^2}{8\pi}}$$

$\vec{H} = \vec{\nabla} \times$

$$\vec{v}_s = \frac{(\vec{\nabla}\phi - e^*\vec{A})}{m^*}$$ – vetocity of supercurrent)

$$\delta \int d\vec{x}\, (\vec{\nabla} \times \vec{A})^2 = 2 \int (\vec{\nabla} \times \vec{A}) \cdot (\vec{\nabla} \times \delta \vec{A})\, d\vec{x}$$

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$$= e^* n_s \vec{V}_s$$

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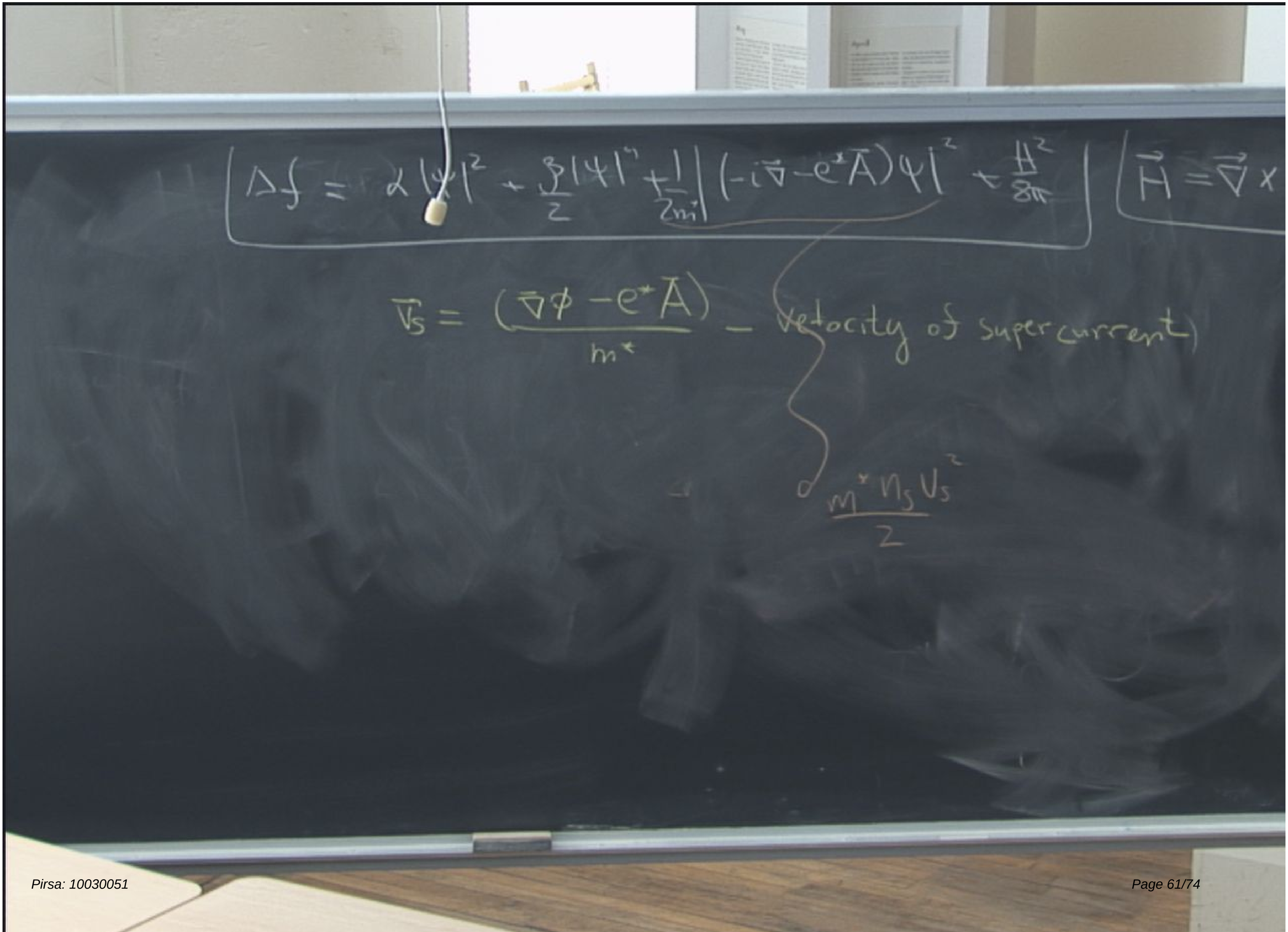

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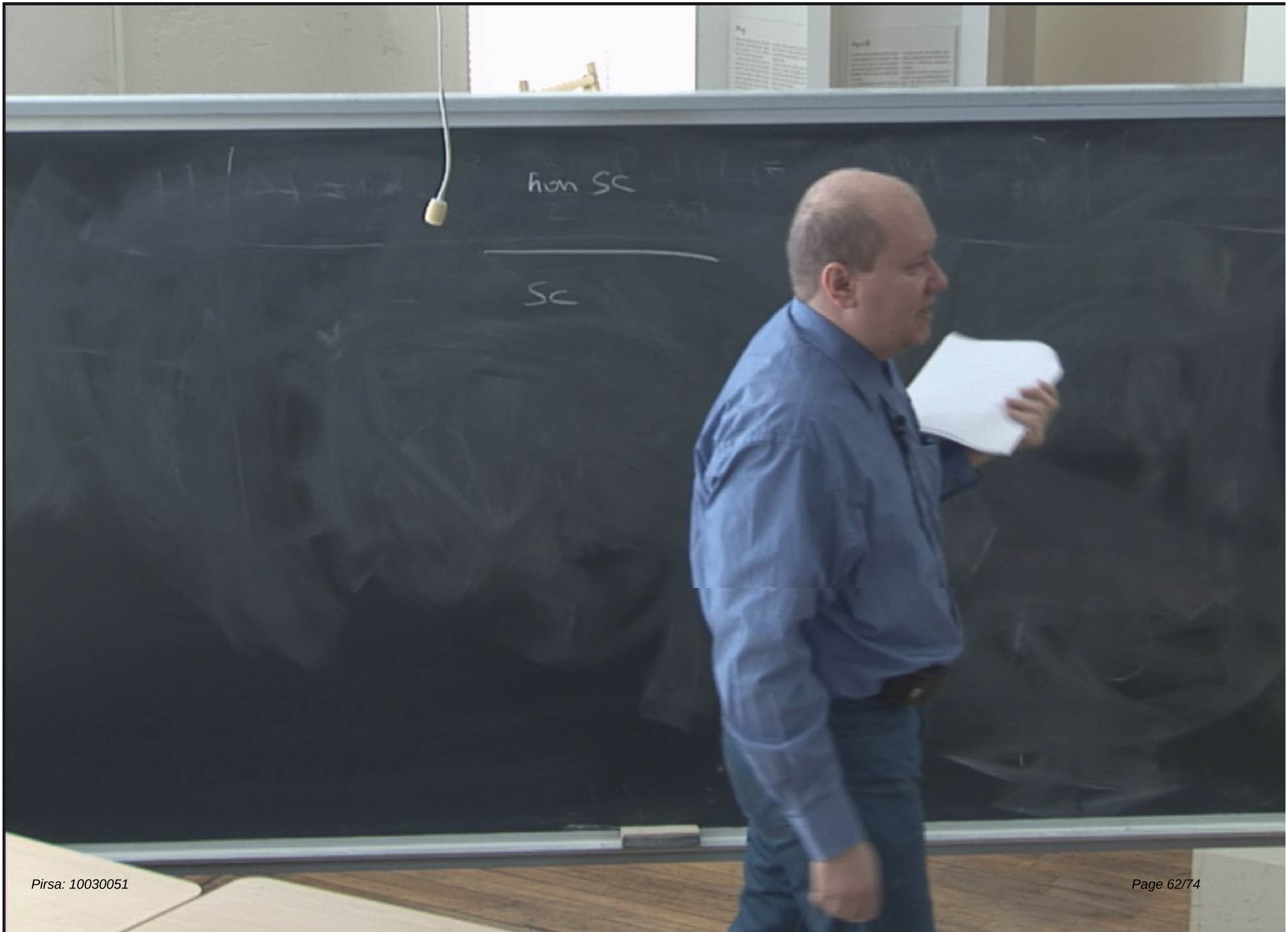
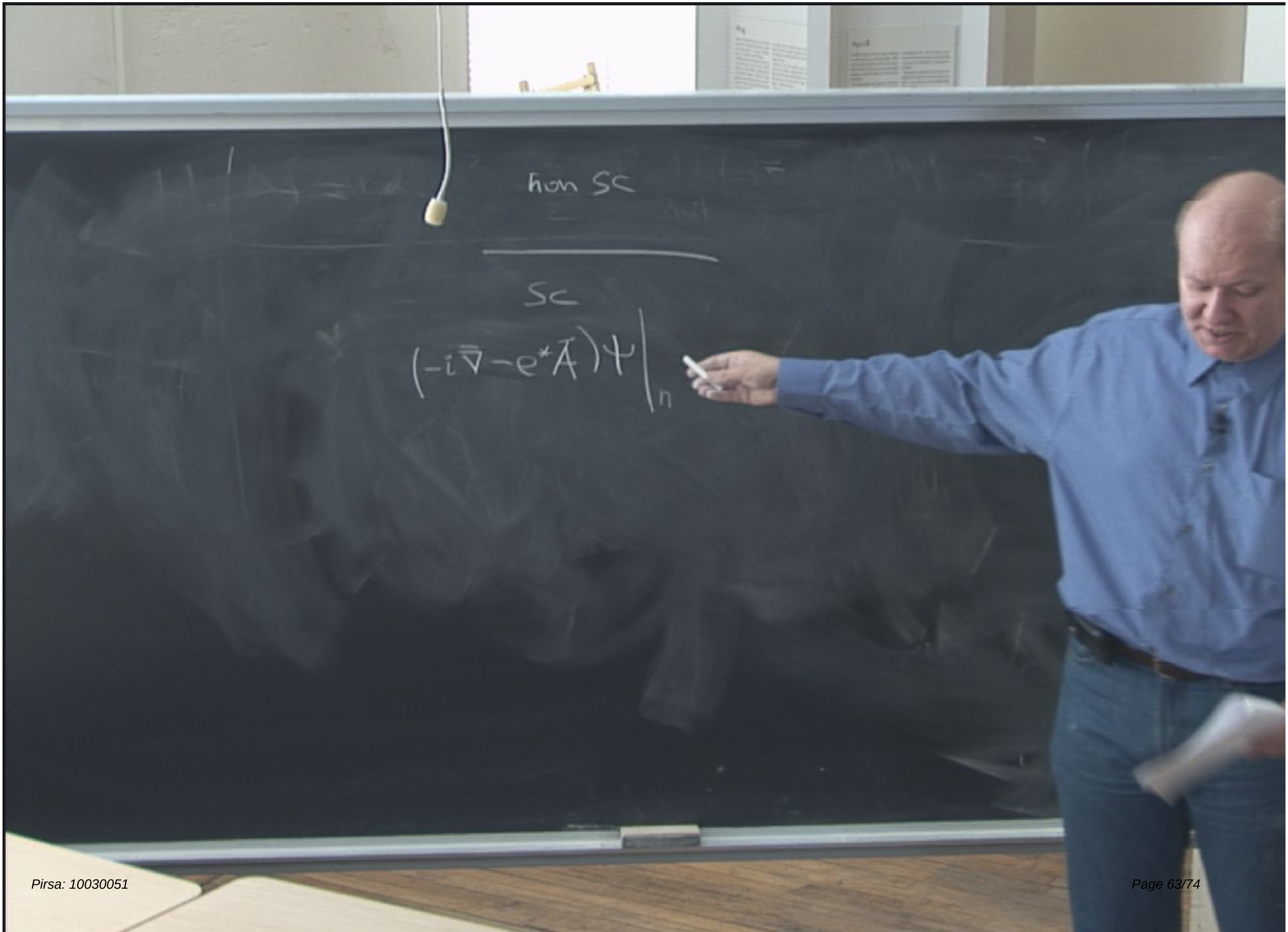
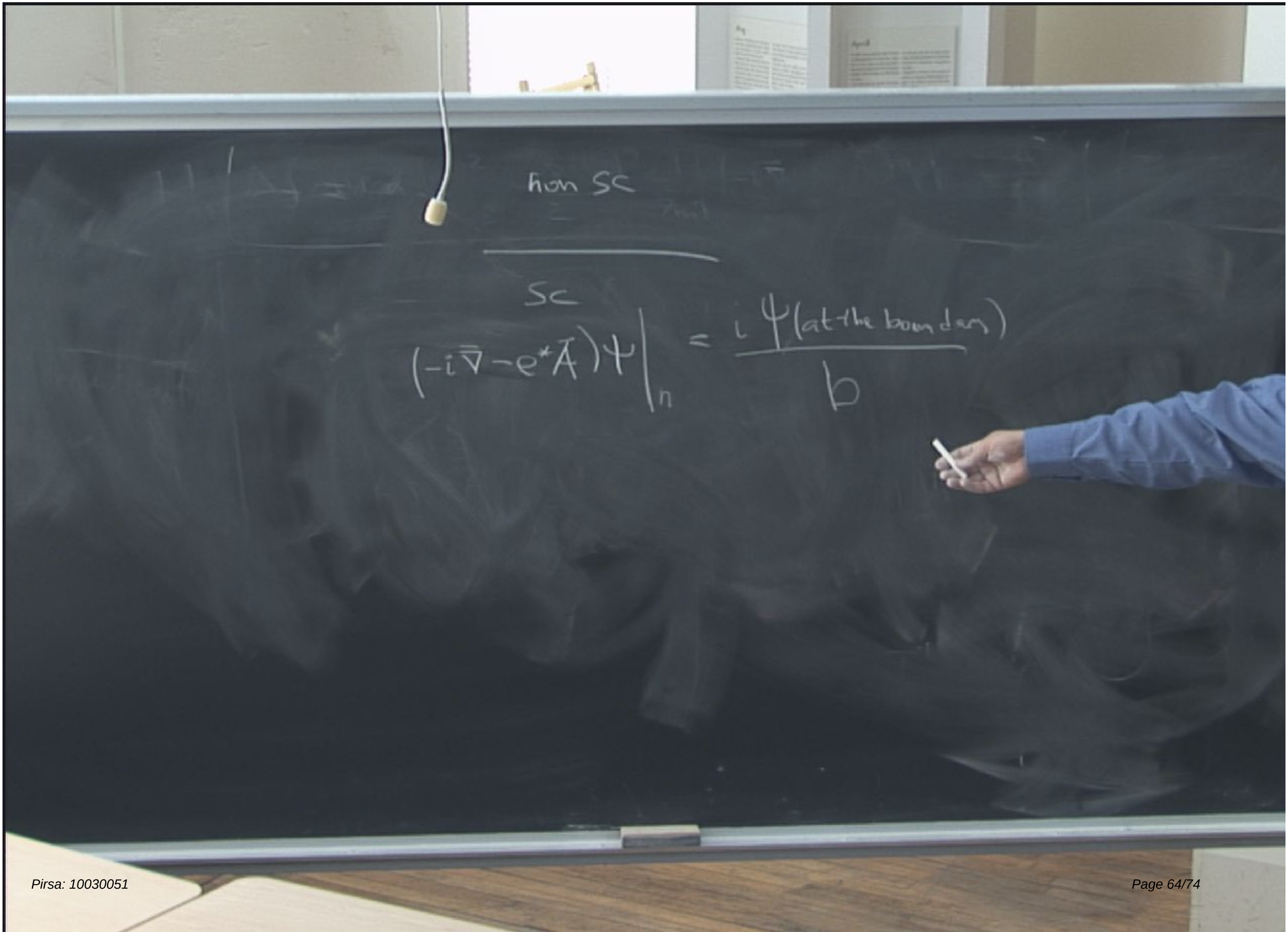

non SC
SC



non SC

SC

$$(-i\vec{\nabla}-e^{*}\vec{A})\psi\Big|_{n}$$



non SC

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$$\left.(-i\vec{\nabla}-e^{*}\vec{A})\psi\right|_{n} = \frac{i\,\psi(\text{at the boundary})}{b}$$

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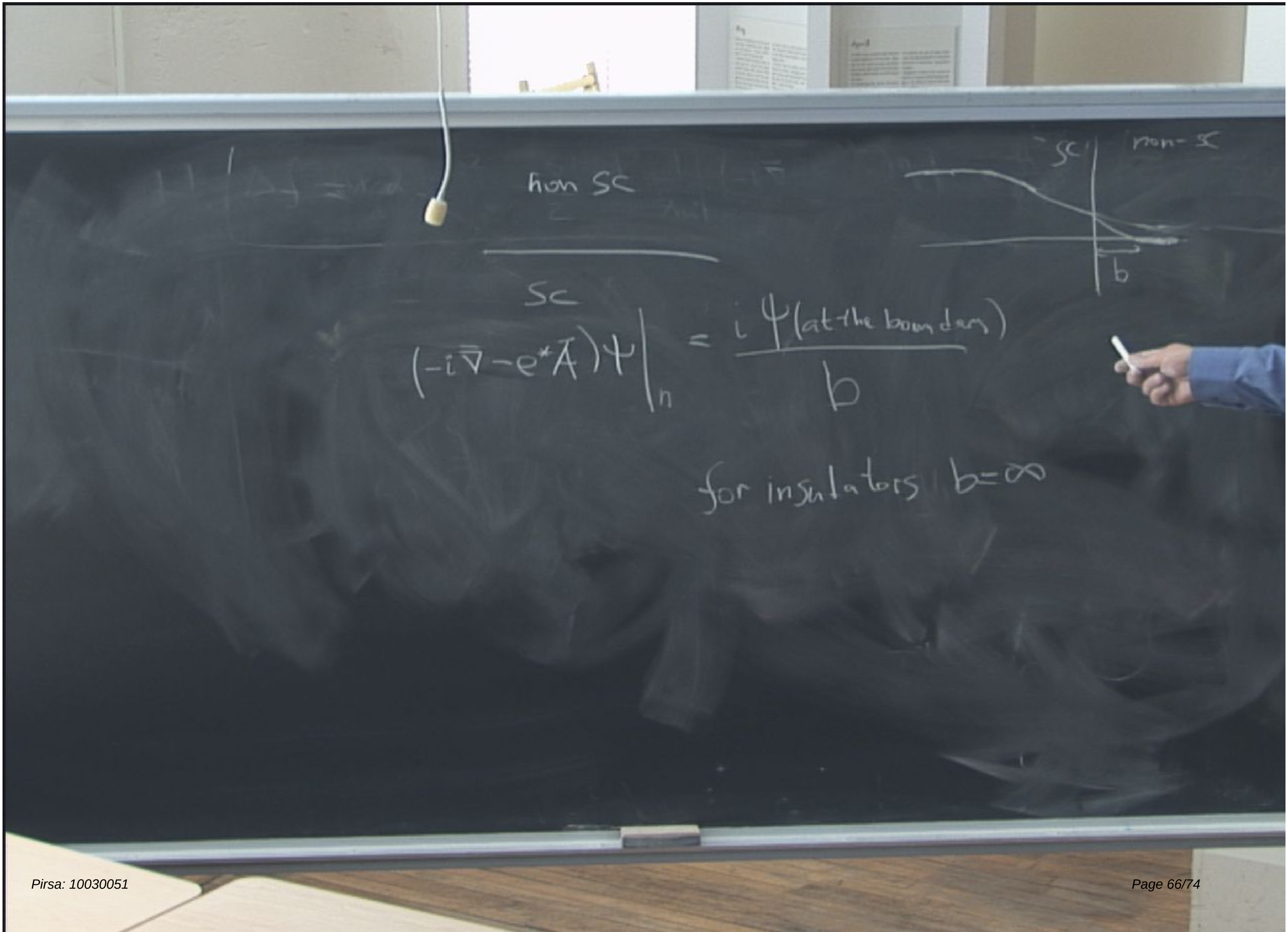
$$\left.(-i\bar{\nabla}-e^*\bar{A})\psi\right|_n = \frac{i\,\psi(\text{at the boundary})}{b}$$



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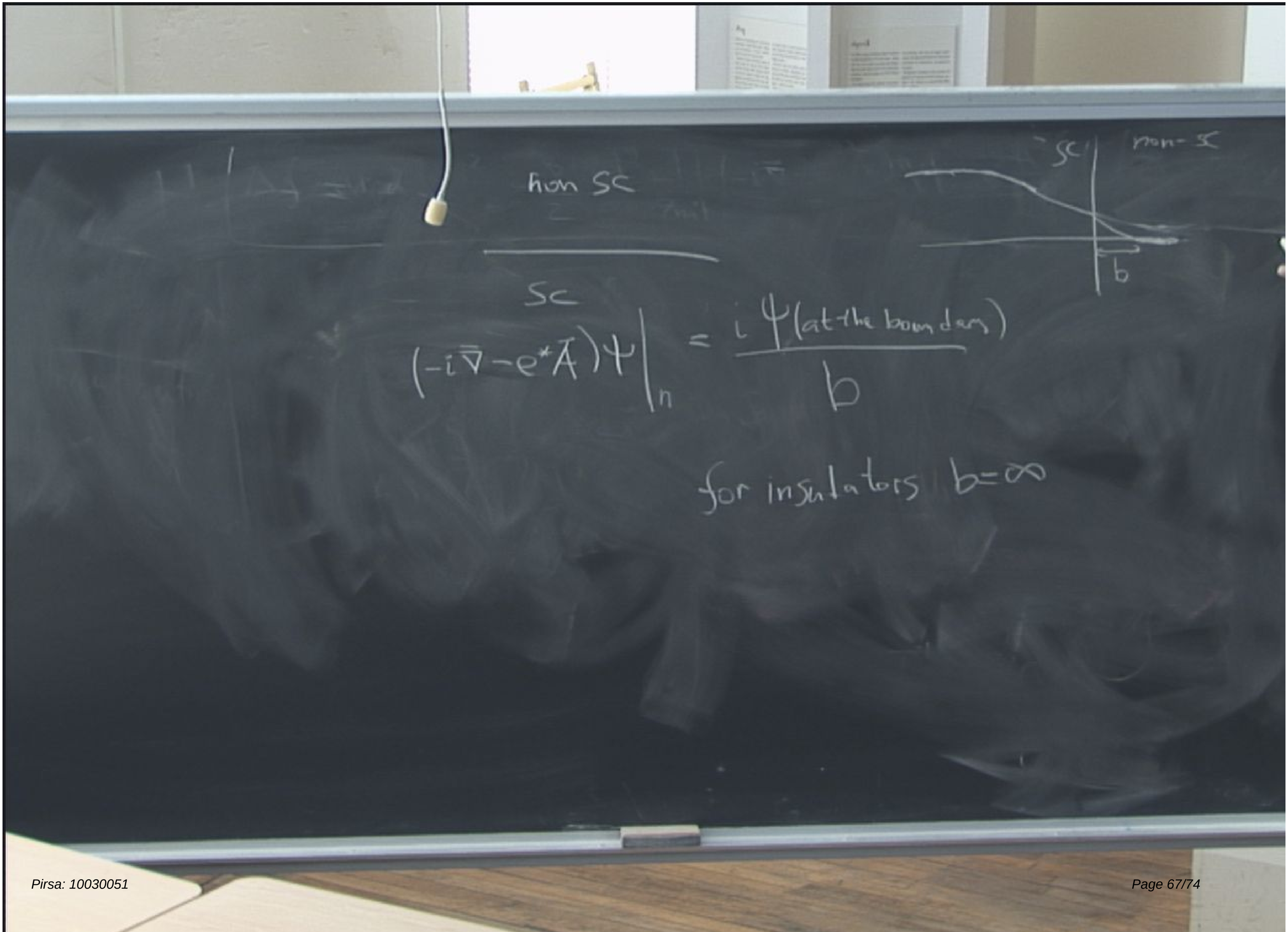
for insulators $b=\infty$



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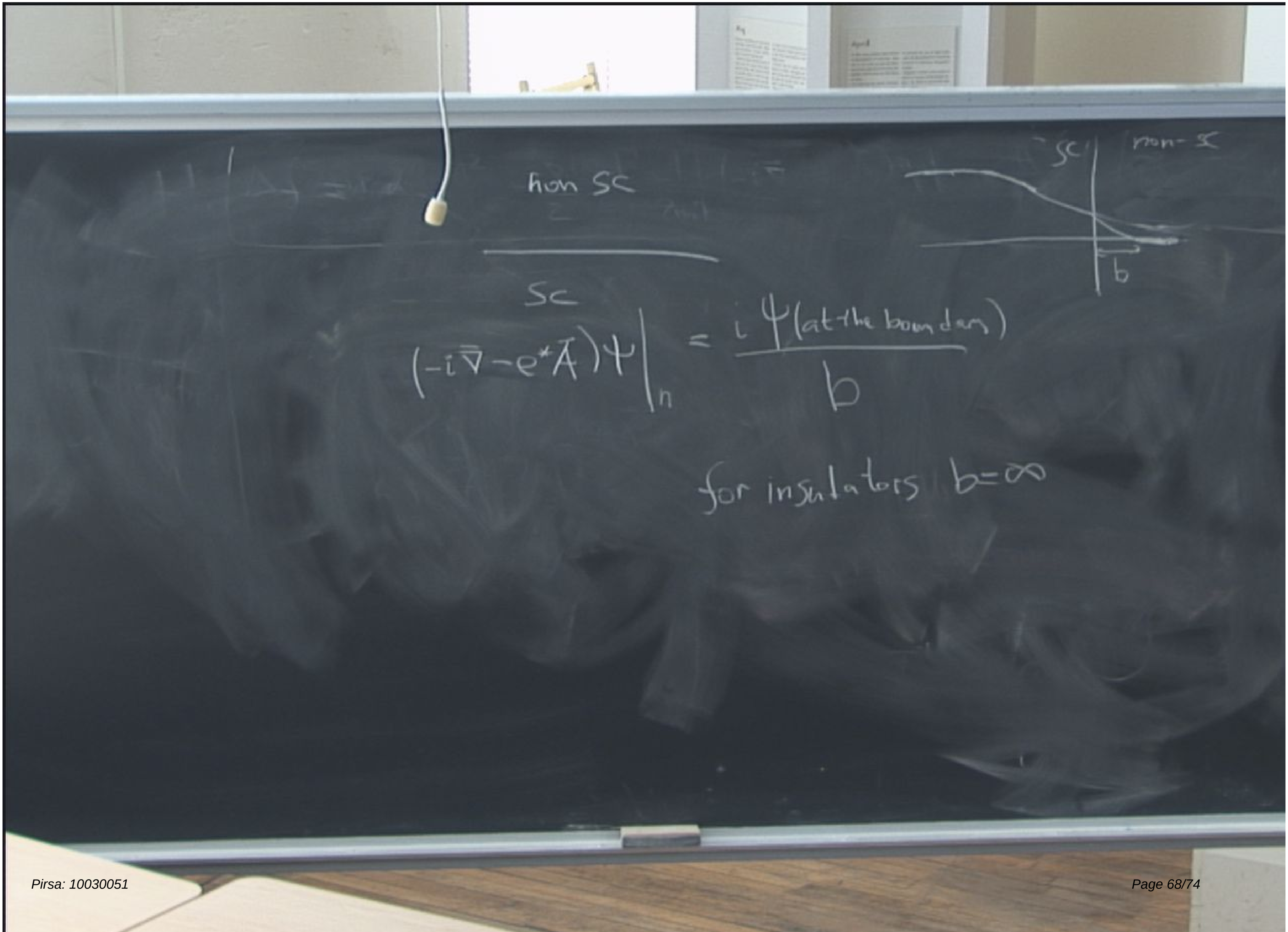

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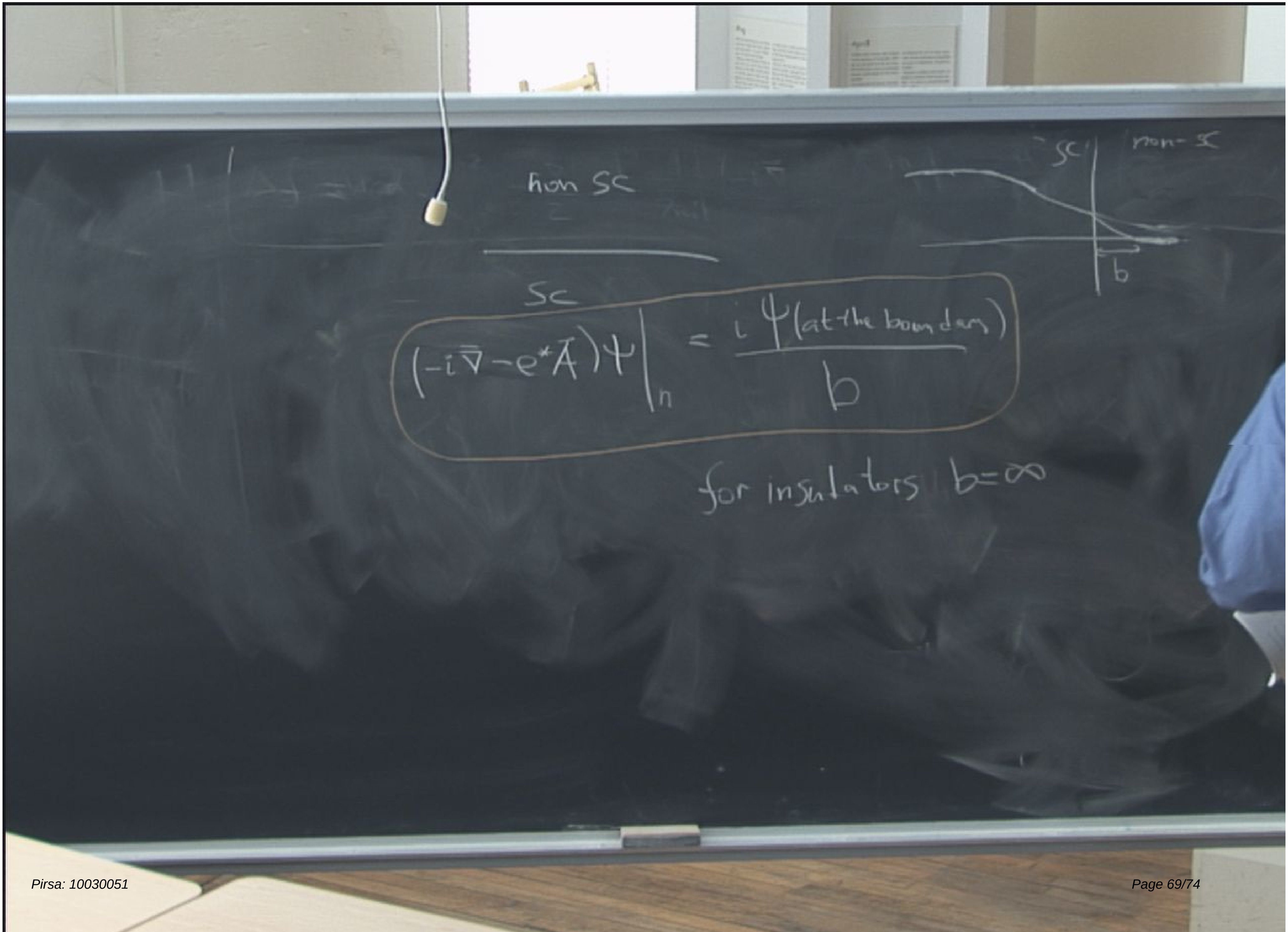

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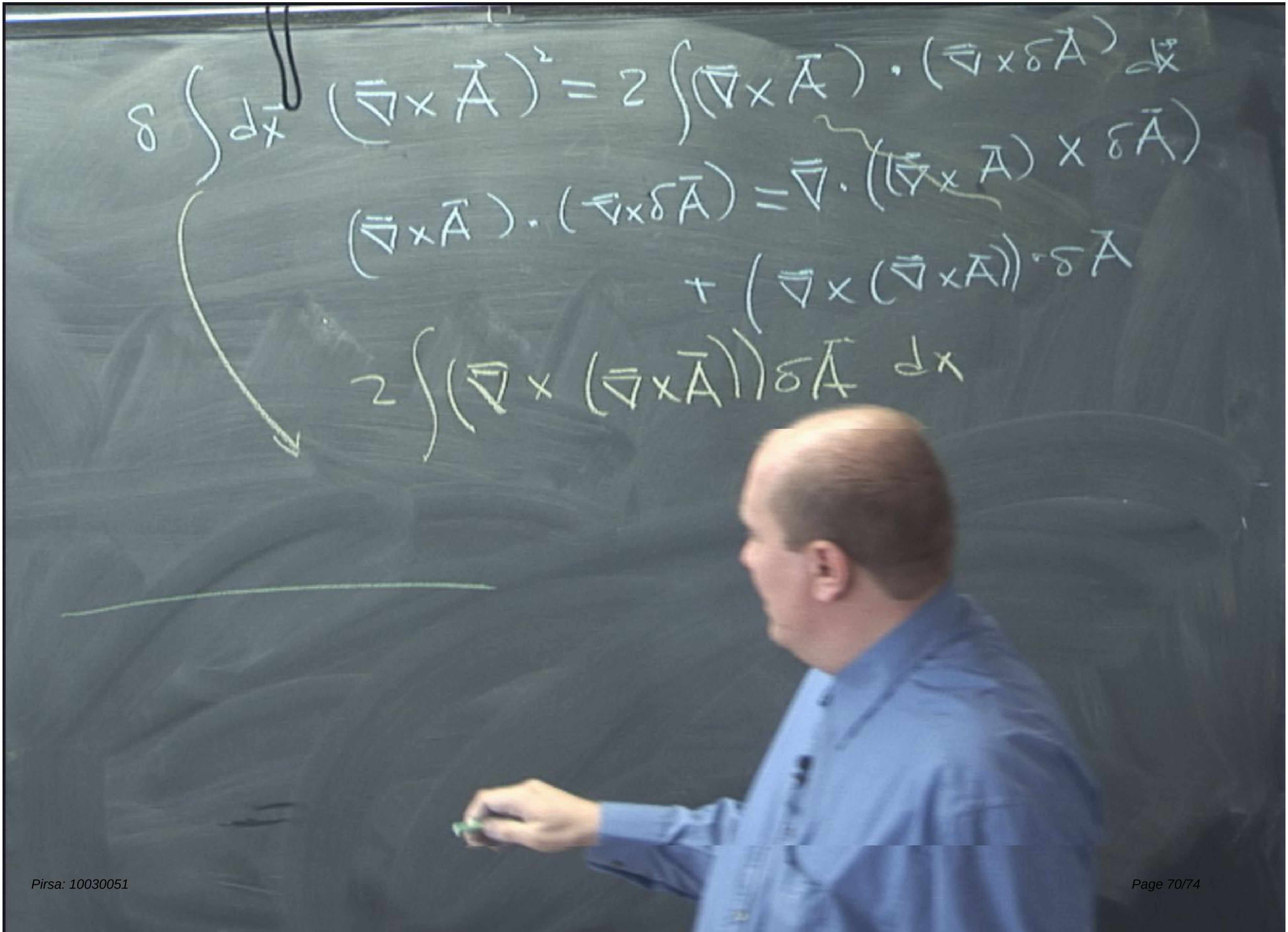

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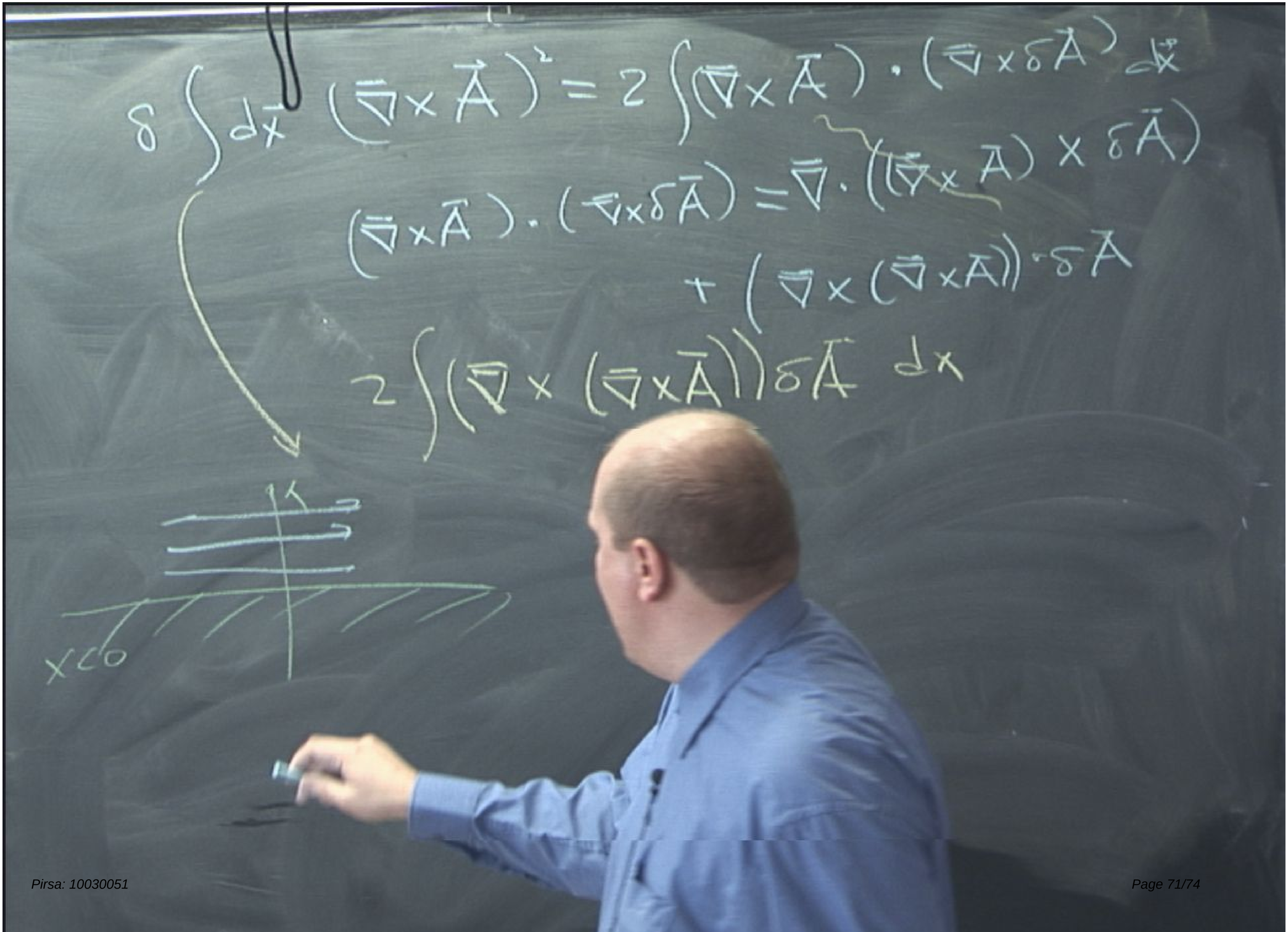

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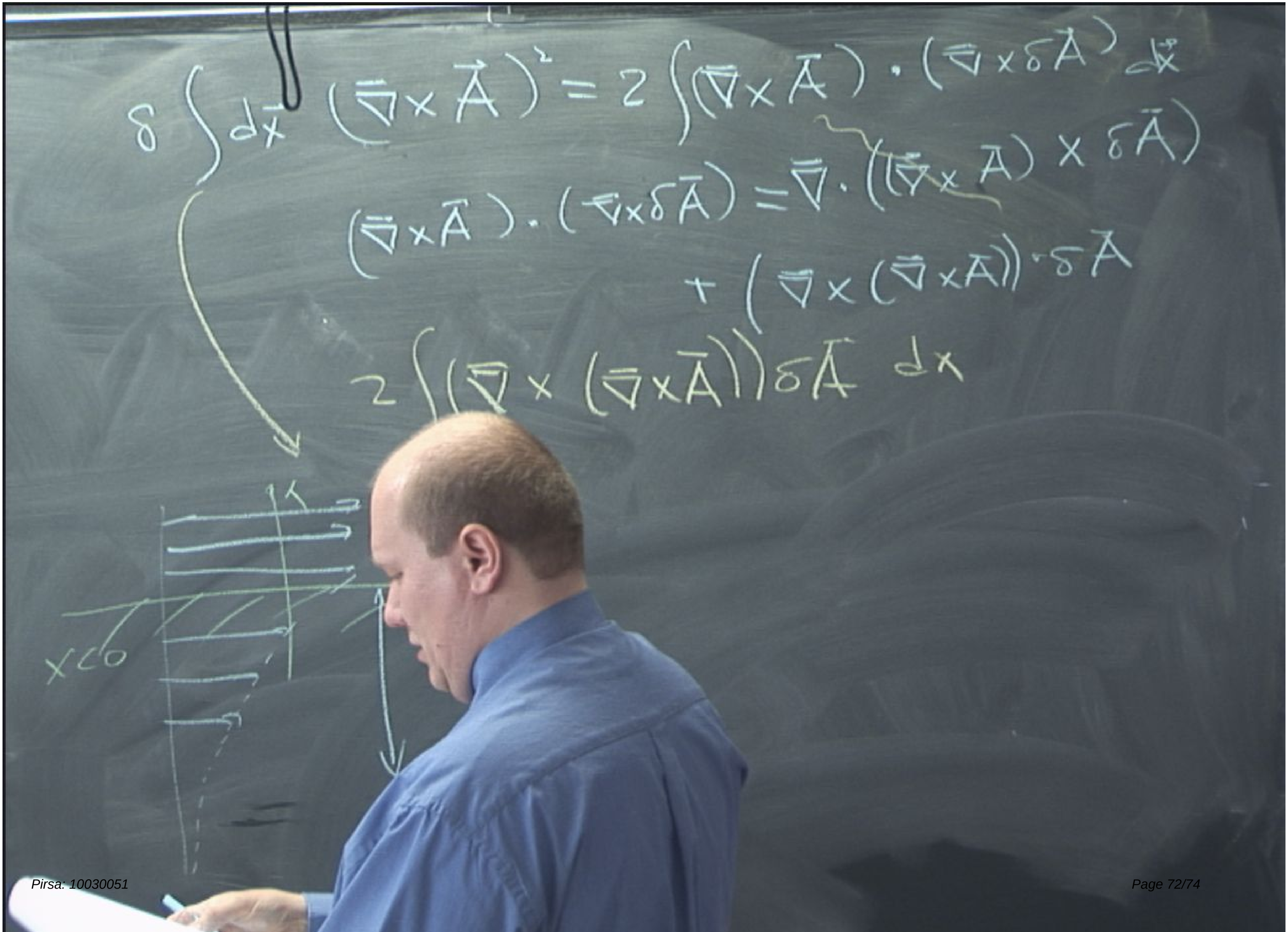

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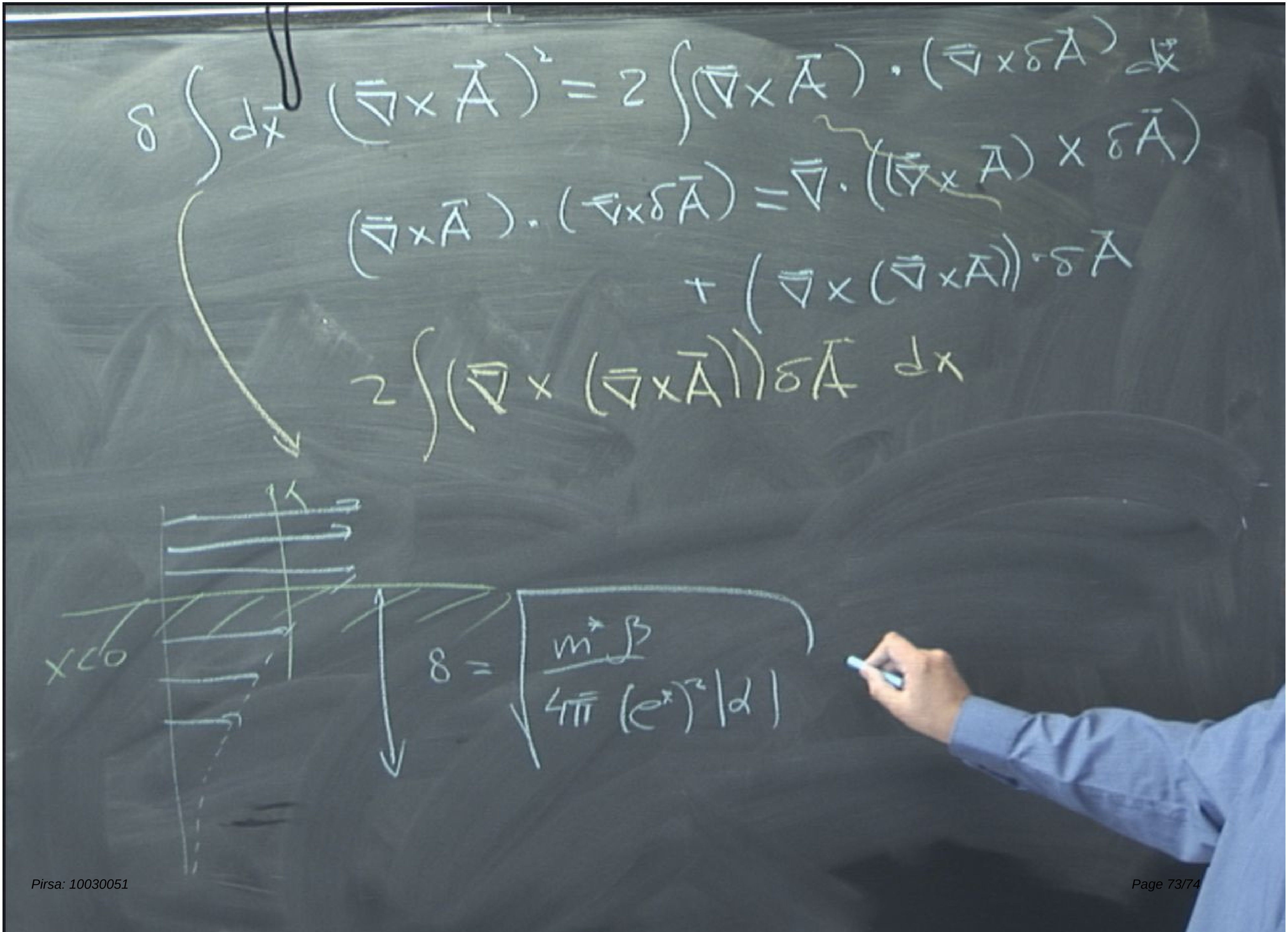

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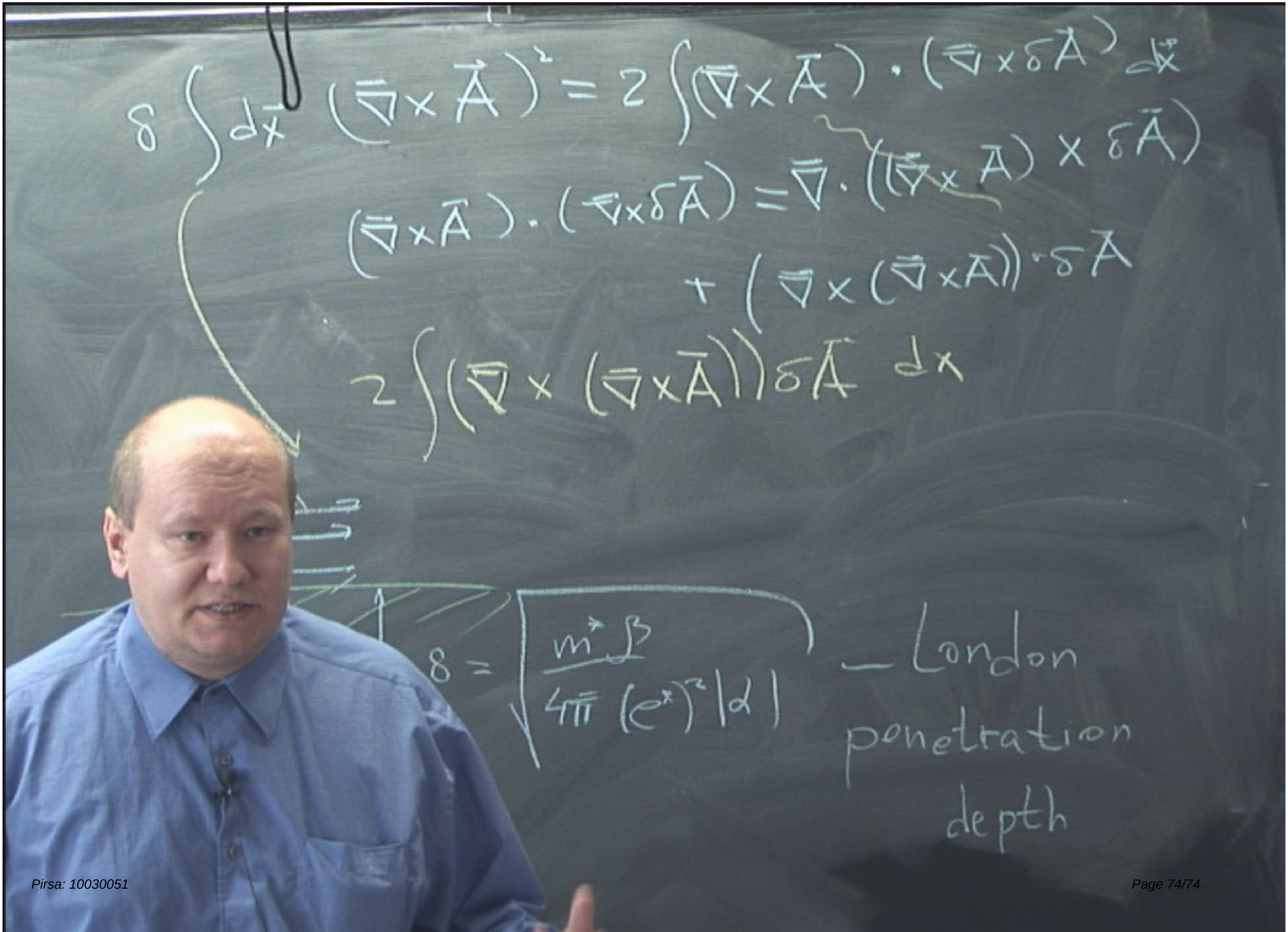

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