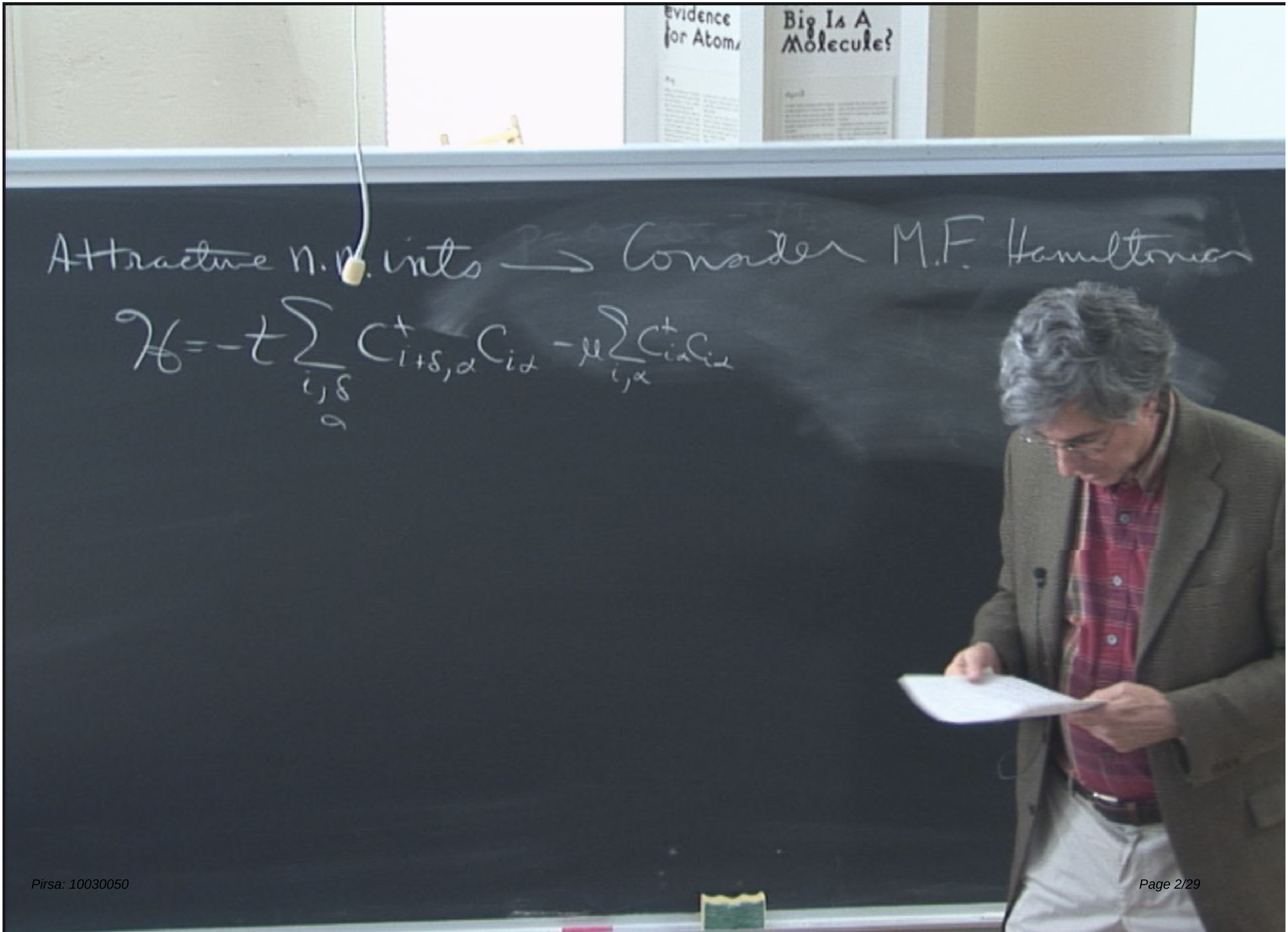
Title: Condensed Matter II - Lecture 12

Date: Mar 03, 2010 10:10 AM

URL: http://pirsa.org/10030050

Abstract:

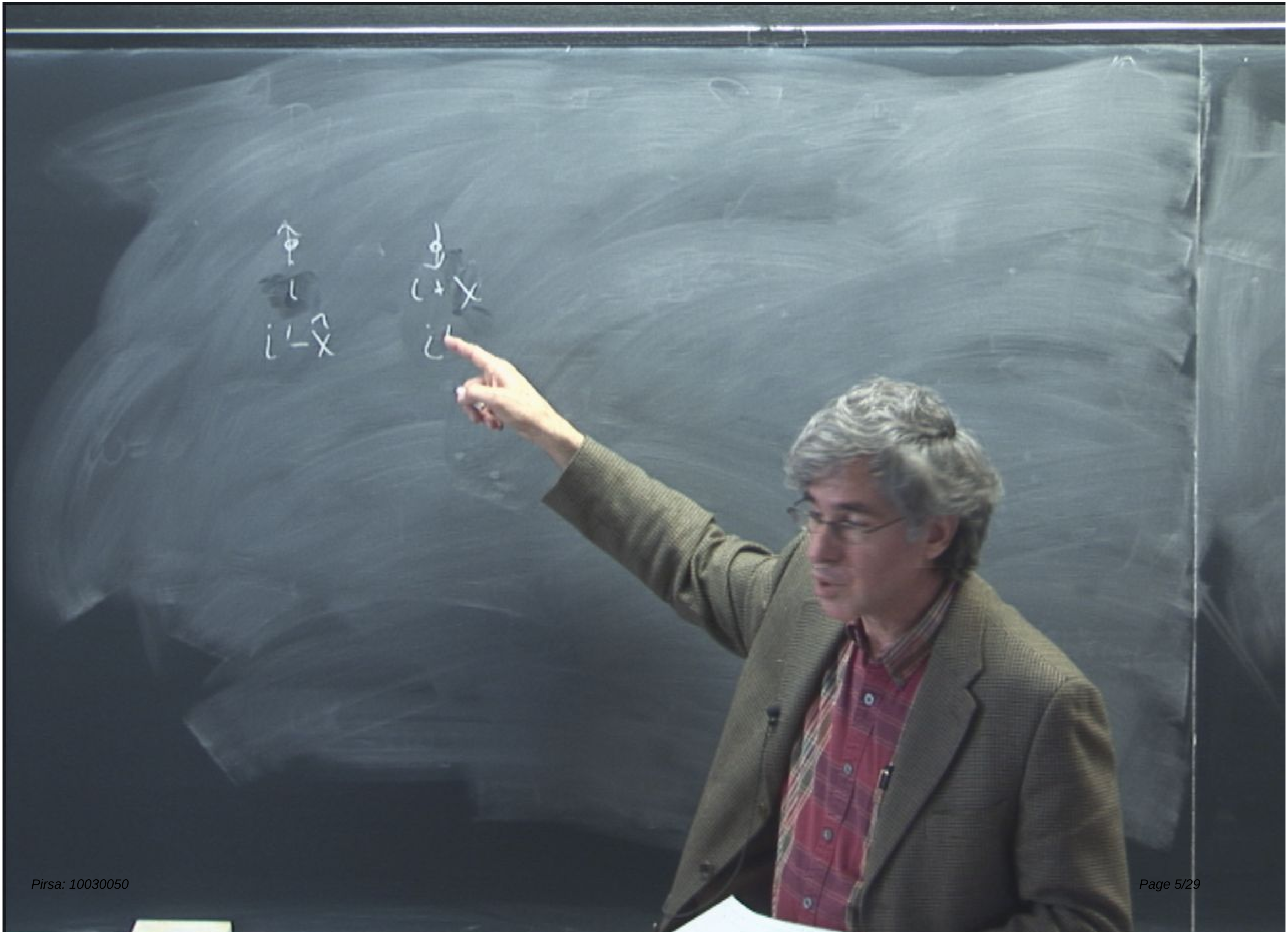

Big Is A Molecule?
Attractive n.n. ints → Consider M.F. Hamiltonian
$\mathcal{H} = -t \sum_{i,\delta,\alpha} c^{+}_{i+\delta,\alpha} c_{i\alpha} - \mu \sum_{i,\alpha} c^{+}_{i\alpha} c_{i\alpha}$

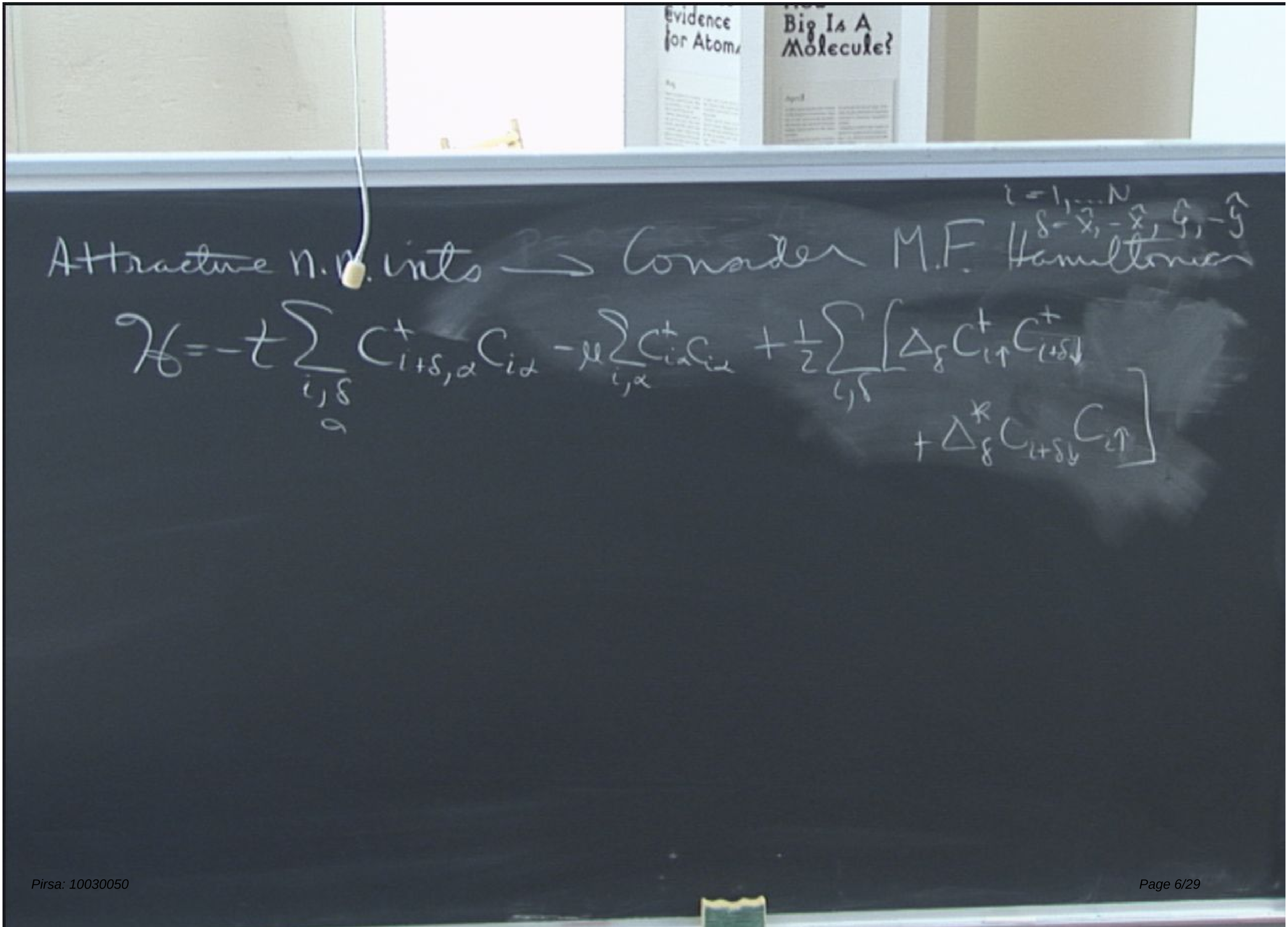
Attractive n.n. ints $\longrightarrow$ Consider M.F. Hamiltonian

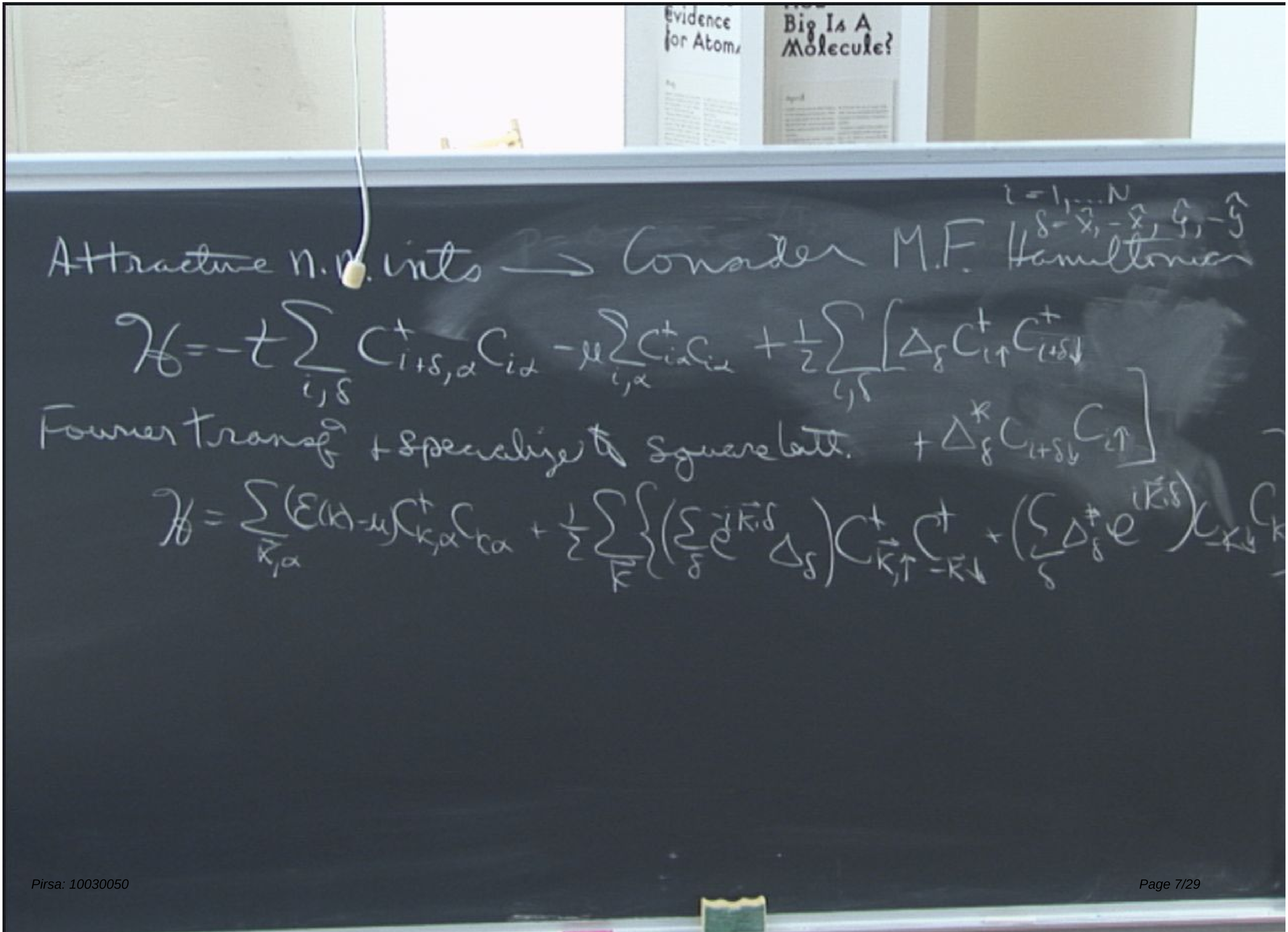
$$\mathcal{H} = -t\sum_{i,\delta,\alpha} c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} - \mu \sum_{i,\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} + \frac{1}{2}\sum_{i,\delta}\left[\Delta_{\delta} c^{\dagger}_{i\uparrow} c^{\dagger}_{i+\delta\downarrow} + \Delta^{*}_{\delta} c_{i+\delta\downarrow} c_{i\uparrow}\right]$$

Attractive n.n. ints $\Rightarrow$ Consider M.F. Hamiltonian

$$\mathcal{H} = -t\sum_{i,\delta,\alpha} c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} - \mu\sum_{i,\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} + \frac{1}{2}\sum_{i,\delta}\Big[\Delta_{\delta} c^{\dagger}_{i\uparrow} c^{\dagger}_{i+\delta\downarrow} + \Delta^{*}_{\delta} c_{i+\delta\downarrow} c_{i\uparrow}\Big]$$




Evidence
for Atom
Big Is A
Molecule?
$i = 1, \ldots N$
$\delta = \hat{x}, -\hat{x}, \hat{y}, -\hat{y}$
Attractive n.n. ints → Consider M.F. Hamiltonian
$\mathcal{H} = -t \sum_{i,\delta,\alpha} c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} - \mu \sum_{i,\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} + \frac{1}{2} \sum_{i,\delta} \left[\Delta_\delta c^{\dagger}_{i\uparrow} c^{\dagger}_{i+\delta\downarrow} + \Delta^{*}_\delta c_{i+\delta\downarrow} c_{i\uparrow} \right]$


Evidence
Big Is A
Molecule?
Attractive n.n. ints → Consider M.F. Hamiltonian
$i = 1, \ldots N$
$\delta = \hat{x}, -\hat{x}, \hat{y}, -\hat{y}$
$\mathcal{H} = -t \sum_{i,\delta} c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} - \mu \sum_{i,\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} + \frac{1}{2} \sum_{i,\delta} \left[\Delta_{\delta} c^{\dagger}_{i\uparrow} c^{\dagger}_{i+\delta\downarrow} + \Delta^{*}_{\delta} c_{i+\delta\downarrow} c_{i\uparrow} \right]$
Fourier Transf + specialize to square latt.
$\mathcal{H} = \sum_{\vec{k},\alpha} (\varepsilon(k) - \mu) c^{\dagger}_{k,\alpha} c_{k\alpha} + \frac{1}{2} \sum_{\vec{k}} \left\{ \left(\sum_{\delta} e^{-i\vec{k}\cdot\delta} \Delta_{\delta} \right) c^{\dagger}_{\vec{k},\uparrow} c^{\dagger}_{-\vec{k}\downarrow} + \left(\sum_{\delta} \Delta^{*}_{\delta} e^{i\vec{k}\cdot\delta} \right) c_{-\vec{k}\downarrow} c_{k}\right.$

$$\varepsilon(k) = -2t(\cos k_x + \cos k_y)$$

$$\Delta(k) = \Delta_x e^{-ik_x} + \Delta_y e^{-ik_y} + \Delta_{-x} e^{ik_x} + \Delta_{-y} e^{ik_y}$$

$$\mathcal{E}(k) = -2t(\cos k_x + \cos k_y)$$

$$\Delta(\vec{k}) = \Delta_x e^{-ik_x} + \Delta_y e^{-ik_y} + \Delta_{-x} e^{ik_x} + \Delta_{-y} e^{ik_y}$$

Consider 2 Possibilities

1.) $\Delta_x = \Delta_{-x} = \Delta_y = \Delta_{-y}$ (real)

Then $\Delta_s(k) = 2\Delta(\cos k_x + \cos k_y)$

$$\mathcal{E}(k) = -2t(\cos k_x + \cos k_y)$$

$$\Delta(\vec{k}) = \Delta_x e^{-ikx} + \Delta_y e^{-iky} + \Delta_{-x} e^{ikx} + \Delta_{-y} e^{iky}$$

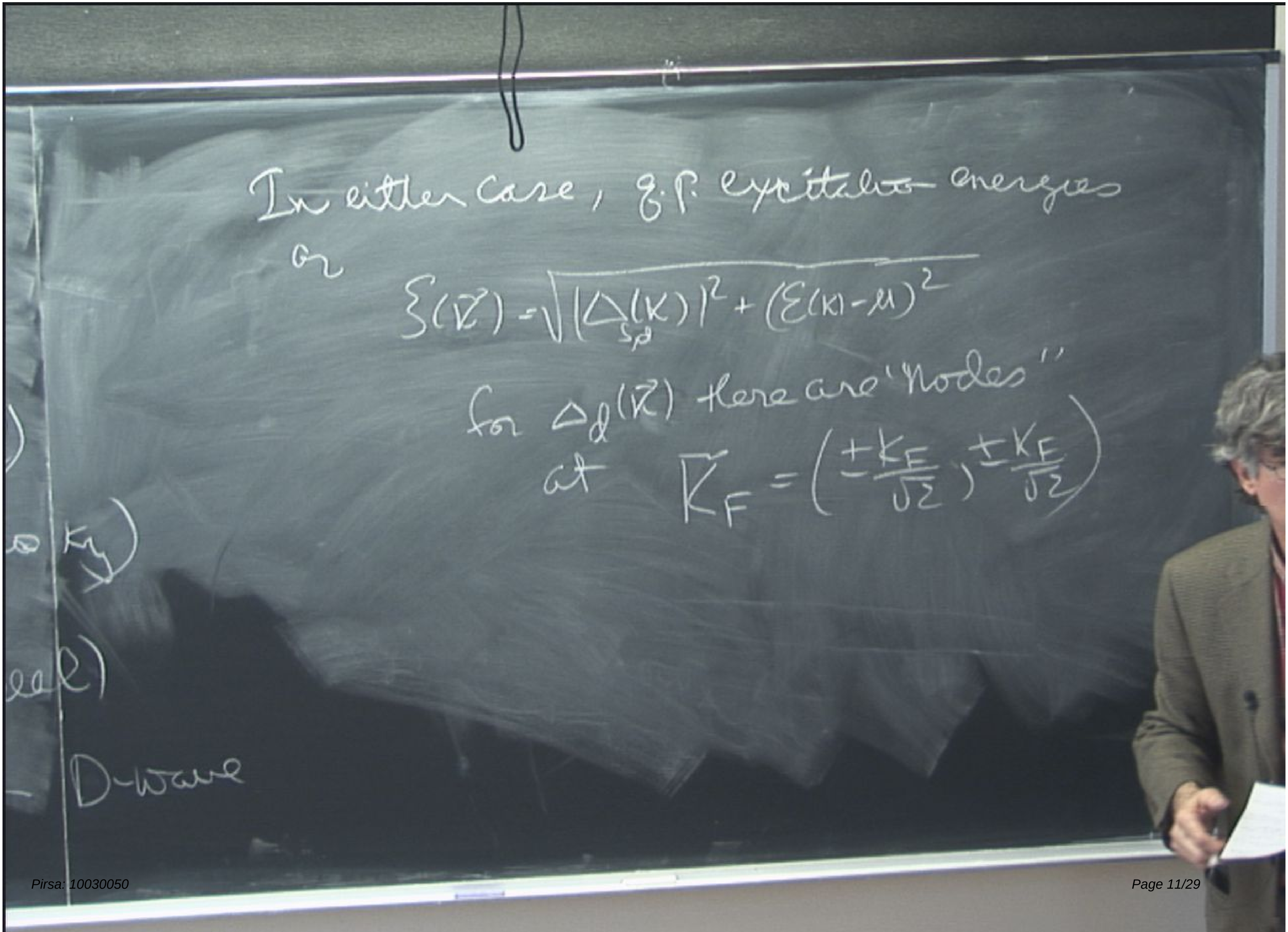
Consider 2 Possibilities

1.) $\Delta_x = \Delta_{-x} = \Delta_y = \Delta_{-y}$ (real)

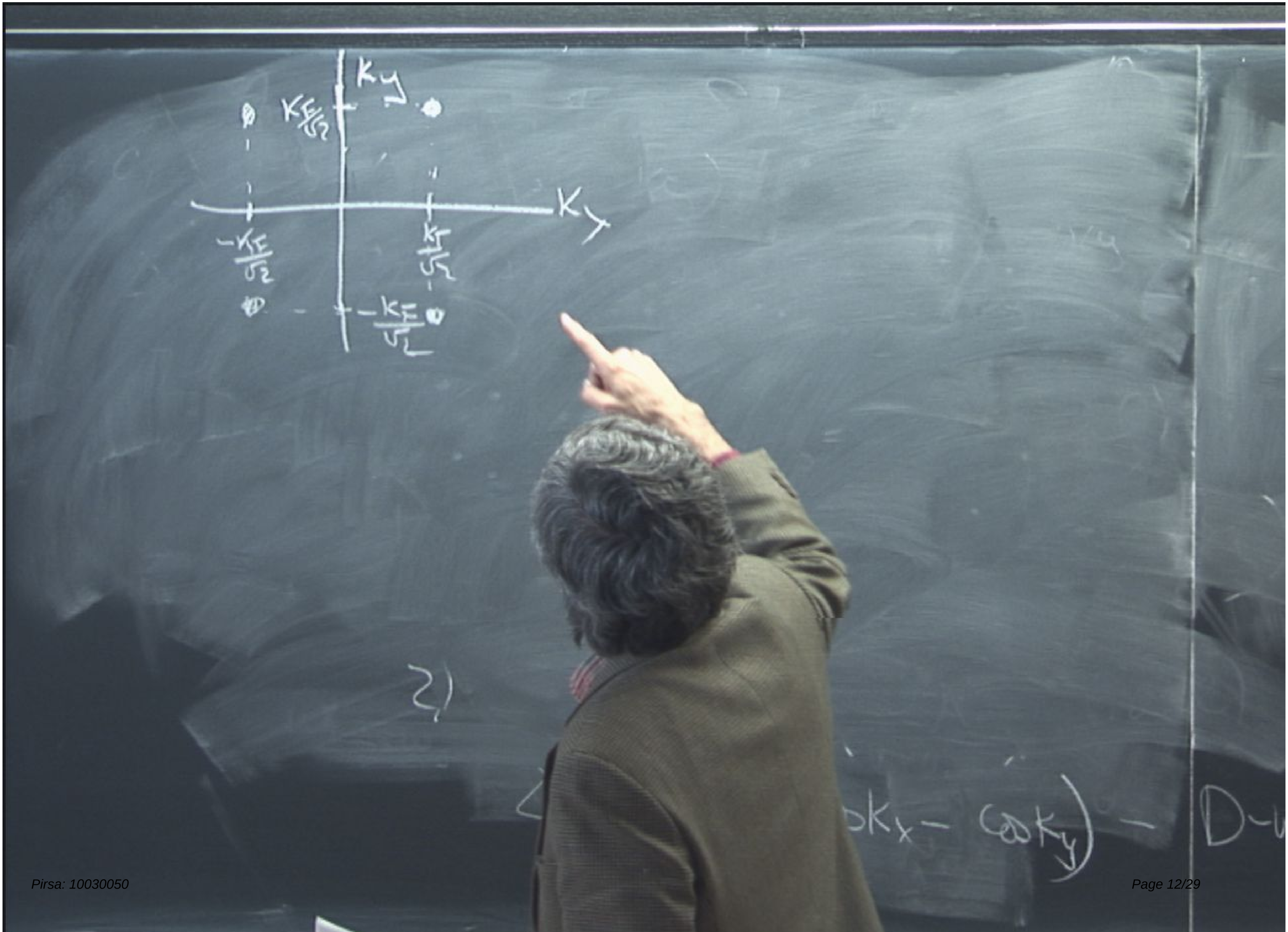
Then $\Delta_s(k) = 2\Delta(\cos k_x + \cos k_y)$

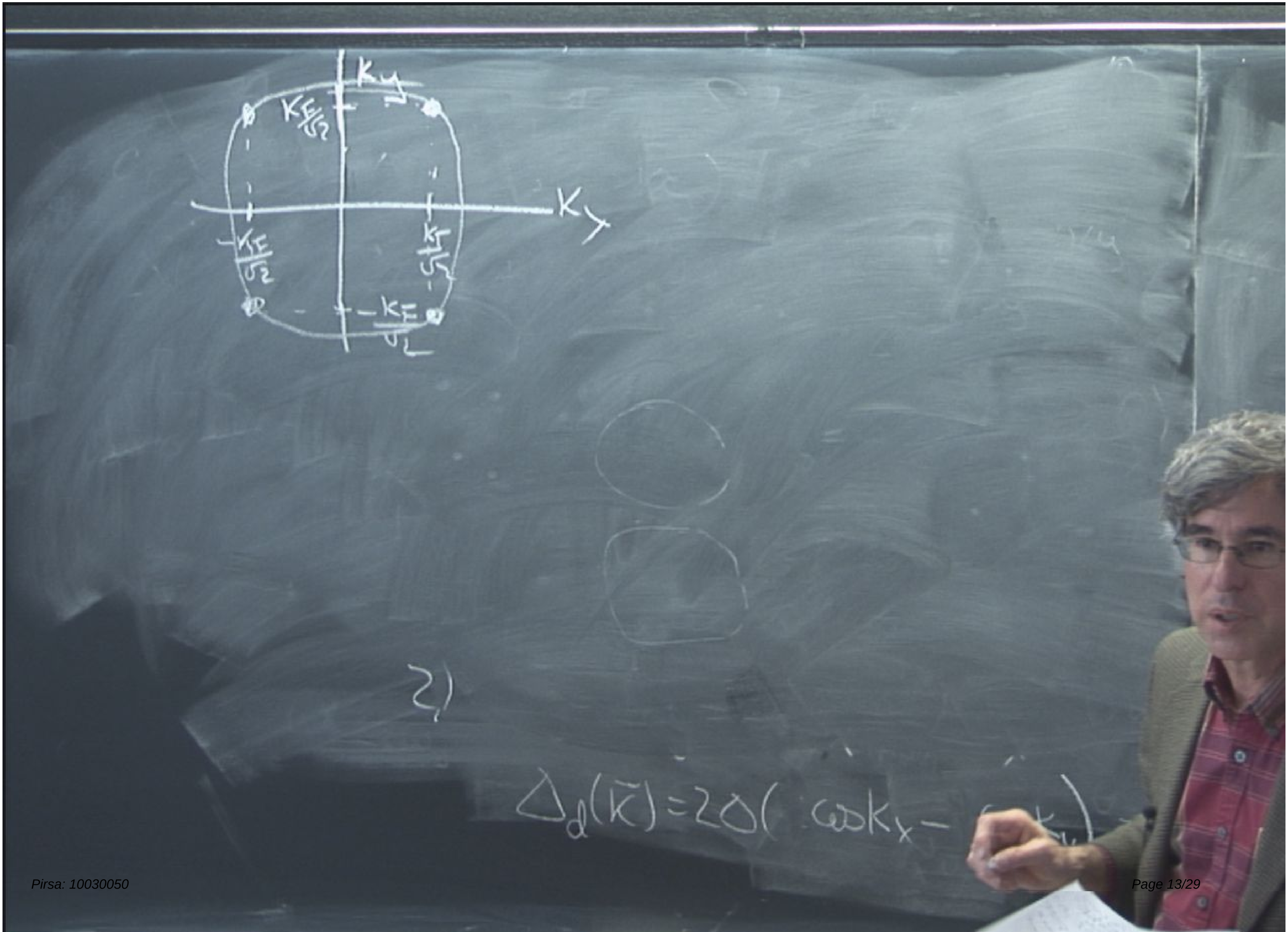
extended s-wave

2) $\Delta_y = \Delta_{-y} = -\Delta_x = -\Delta_{-x}$ (real)

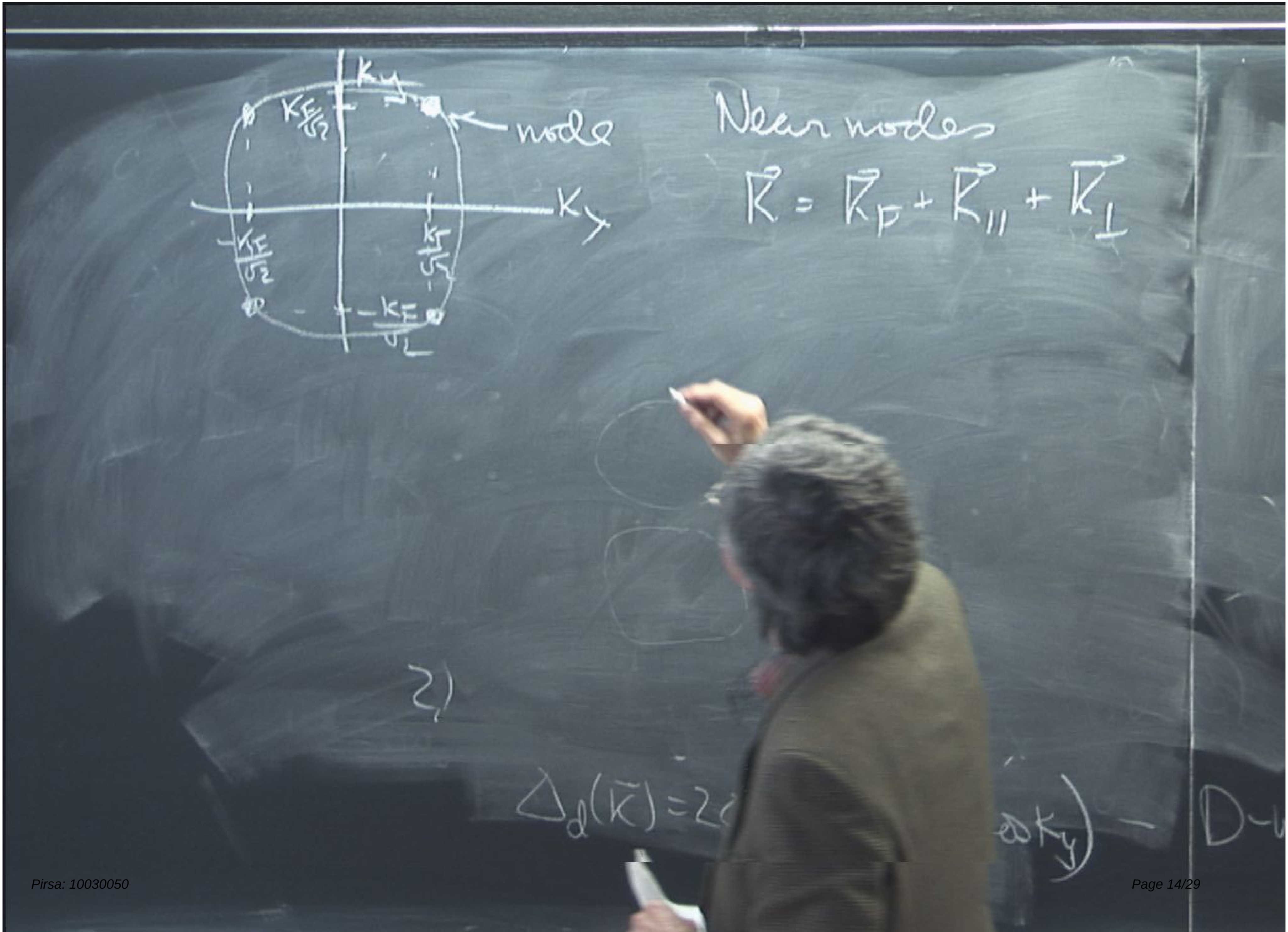


In either case, q.p. excitation energies are
$\xi(\vec{k}) = \sqrt{|\Delta_{s,d}(k)|^2 + (\varepsilon(k) - \mu)^2}$
for $\Delta_d(\vec{k})$ there are "nodes"
at $\vec{K}_F = \left(\pm\frac{K_F}{\sqrt{2}}, \pm\frac{K_F}{\sqrt{2}}\right)$
D-wave


ky
kF/√2
kx
-kF/√2
kF/√2
-kF/√2
2)



k_y
k_x
k_F/√2
−k_F/√2
2)
Δ_d(k) = 2Δ(cos k_x −



node
Near nodes
$\vec{K} = \vec{K}_F + \vec{K}_{\parallel} + \vec{K}_{\perp}$
k_y
k_x
$\frac{k_F}{\sqrt{2}}$
$-\frac{k_F}{\sqrt{2}}$
$\frac{k_F}{\sqrt{2}}$
$-\frac{k_F}{\sqrt{2}}$
2)

Near nodes

$$\vec{K} = \vec{K}_F + \vec{K}_{\parallel} + \vec{K}_{\perp}$$

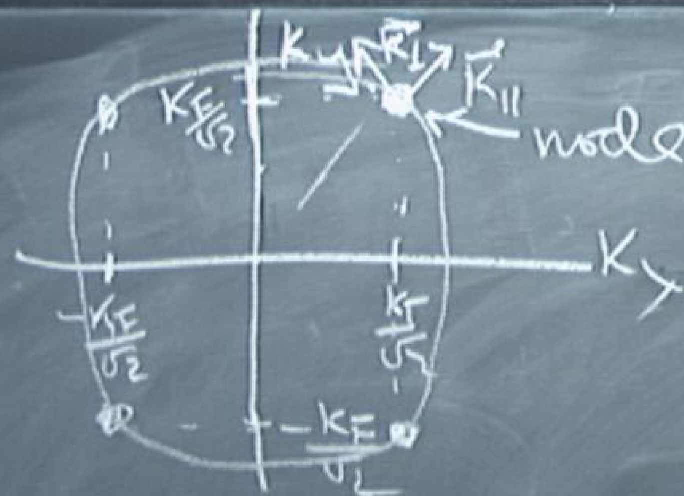
$$\vec{K}_F = K_F\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$

$$\vec{K}_{\parallel} = K_1\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$

$$\vec{K}_{\perp} = K_2\left(-\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$

2)

$$\Delta_d(\vec{K}) = 2\Delta(\cos K_x - \cos K_y)$$ — D-w



Near nodes

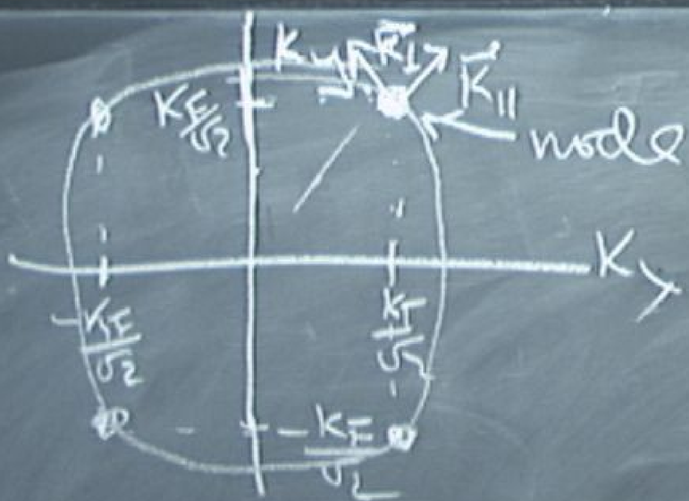
$$\vec{K} = \vec{K}_F + \vec{K}_{\parallel} + \vec{K}_{\perp}$$

$$\vec{K}_F = K_F\left(\frac{1}{\sqrt2}, \frac{1}{\sqrt2}\right)$$

$$\vec{K}_{\parallel} = K_1\left(\frac{1}{\sqrt2}, \frac{1}{\sqrt2}\right)$$

$$\vec{K}_{\perp} = K_2\left(-\frac{1}{\sqrt2}, \frac{1}{\sqrt2}\right)$$

$$\Delta(\vec{K}) = 2\Delta\left[\cos\left(\frac{K_F + K_1 - K_2}{\sqrt2}\right) - \cos\left(\frac{K_F + K_1 + K_2}{\sqrt2}\right)\right]$$



Near nodes

$$\vec{K} = \vec{K}_F + \vec{K}_{\parallel} + \vec{K}_{\perp}$$

$$\vec{K}_F = K_F\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$

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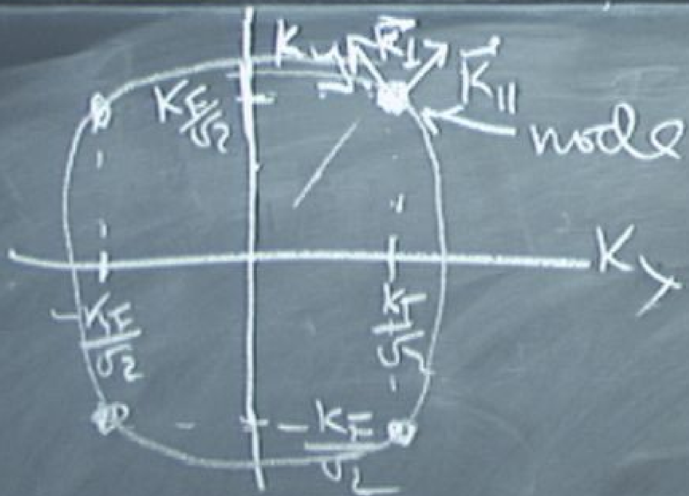
$$\Delta(\vec{K}) = 2\Delta\left[\cos\left(\frac{K_F + K_1 - K_2}{\sqrt{2}}\right) - \cos\left(\frac{K_F + K_1 + K_2}{\sqrt{2}}\right)\right]$$

$$2\Delta\left[\cos\frac{K_F}{\sqrt{2}}\cos\left(\frac{K_1 - K_2}{\sqrt{2}}\right) - \sin\frac{K_F}{\sqrt{2}}\sin\left(\frac{K_1 - K_2}{\sqrt{2}}\right) - \cos\frac{K_F}{\sqrt{2}}\cos\left(\frac{K_1 + K_2}{\sqrt{2}}\right) + \sin\frac{K_F}{\sqrt{2}}\sin\left(\frac{K_1 + K_2}{\sqrt{2}}\right)\right]$$



Near nodes

$$\vec{k} = \vec{k}_F + \vec{k}_{\parallel} + \vec{k}_{\perp}$$

$$\vec{k}_F = k_F\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$

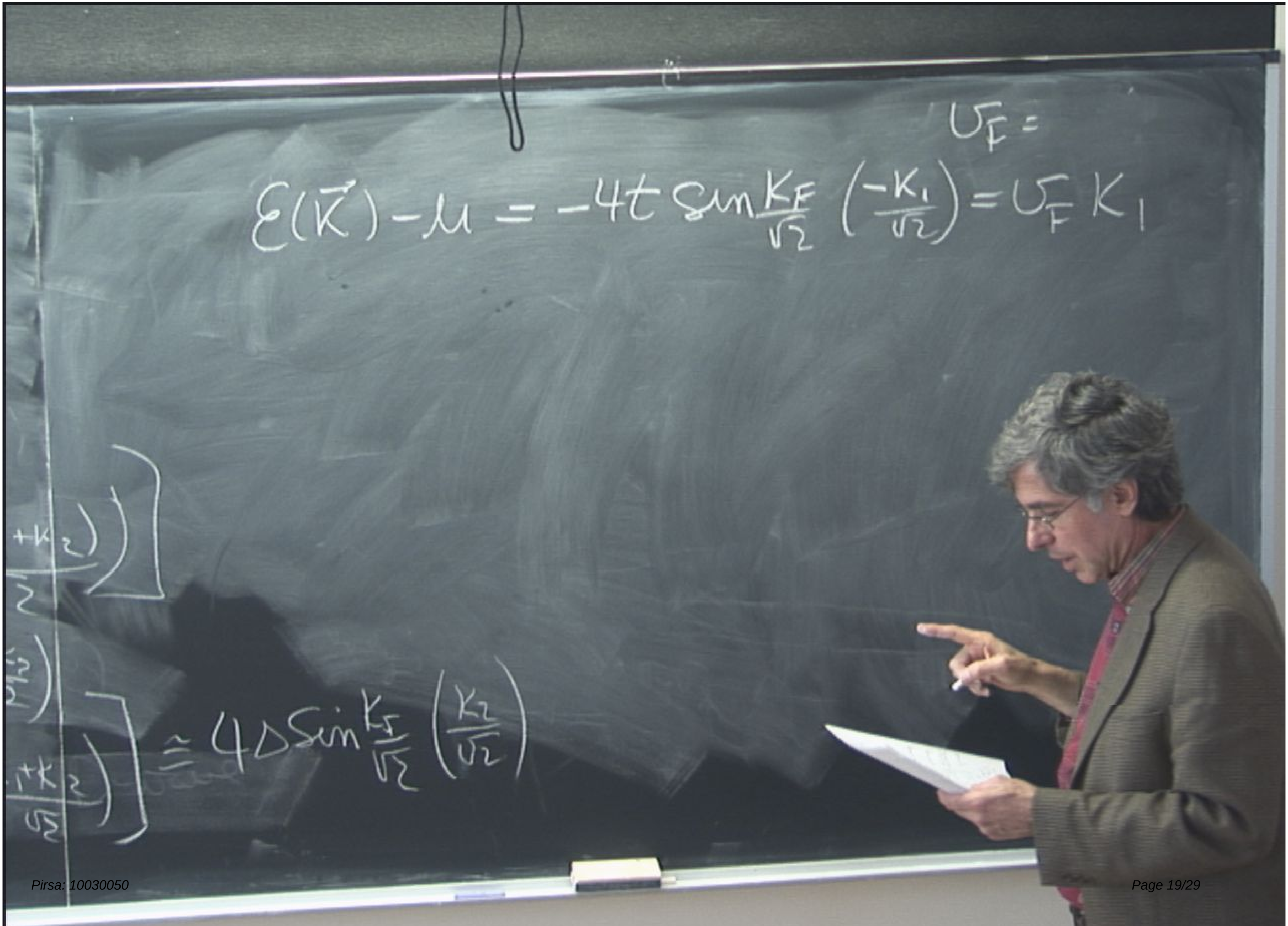
$$\vec{k}_{\parallel} = k_1\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$

$$\vec{k}_{\perp} = k_2\left(-\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$

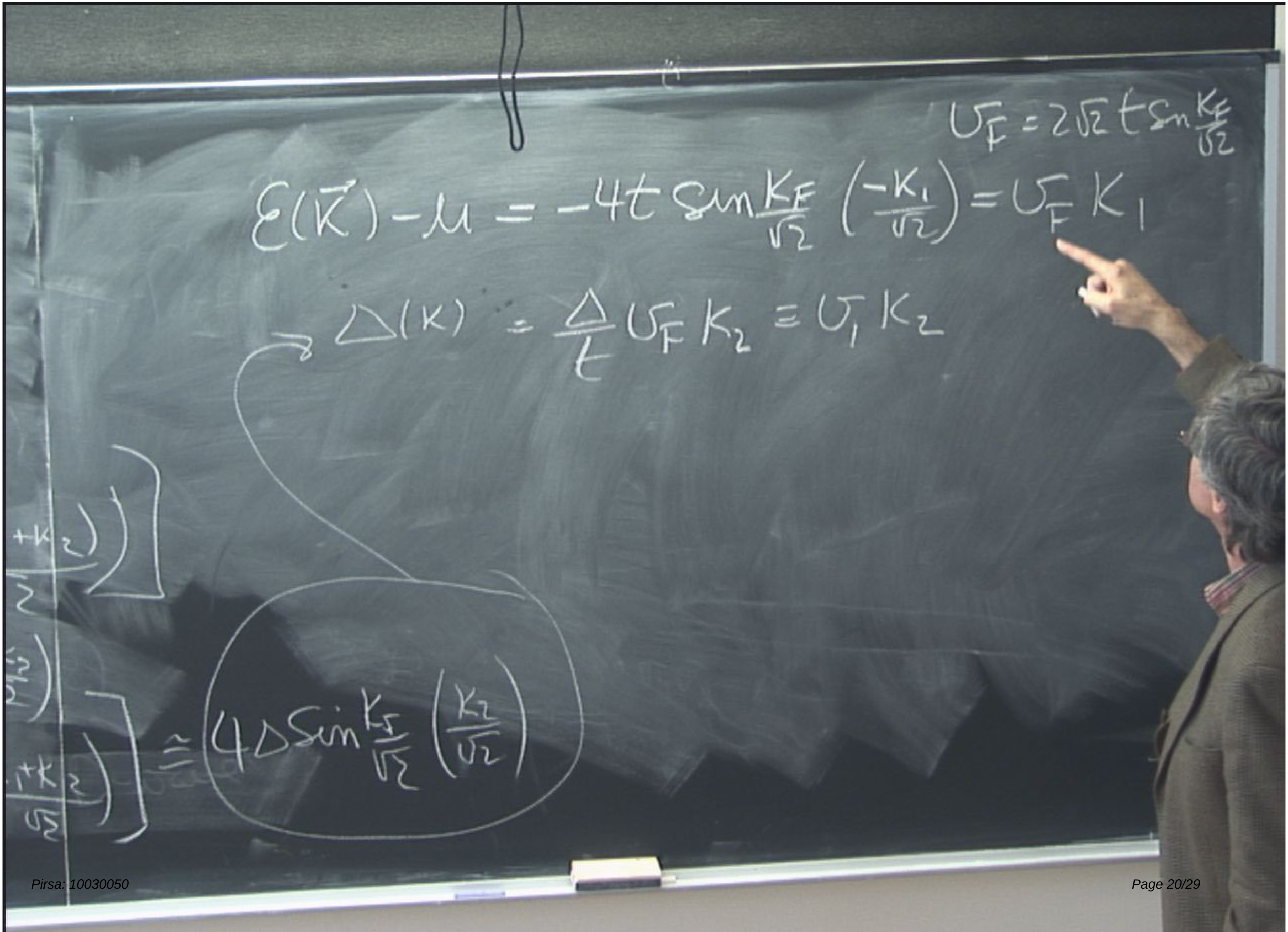
$$\Delta(\vec{k}) = 2\Delta\left[\cos\left(\frac{k_F + k_1 - k_2}{\sqrt{2}}\right) - \cos\left(\frac{k_F + k_1 + k_2}{\sqrt{2}}\right)\right]$$

$$= 2\Delta\left[\cos\frac{k_F}{\sqrt{2}}\cos\left(\frac{k_1 - k_2}{\sqrt{2}}\right) - \sin\frac{k_F}{\sqrt{2}}\sin\left(\frac{k_1 - k_2}{\sqrt{2}}\right)\right.$$

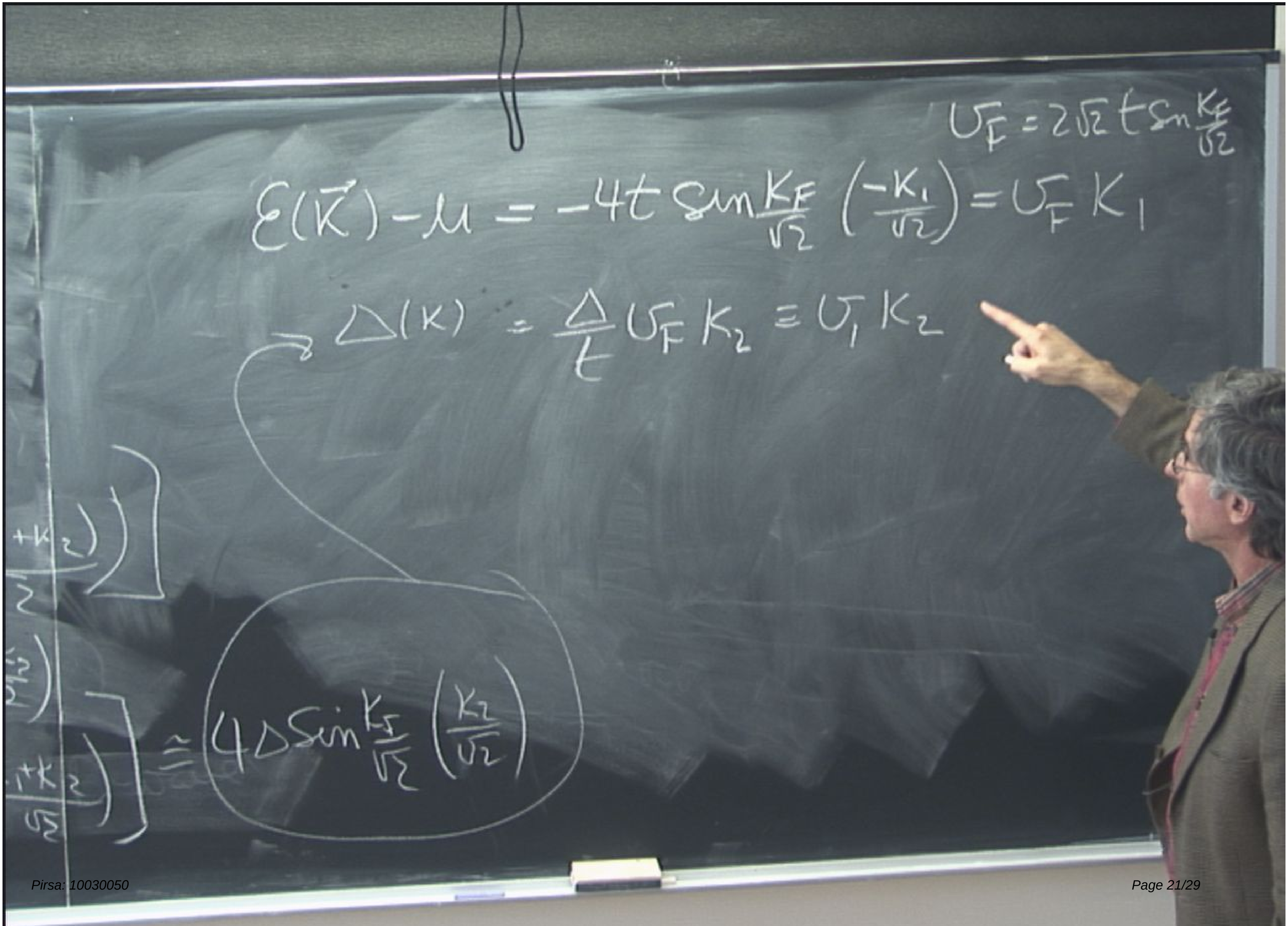
$$\left. - \cos\frac{k_F}{\sqrt{2}}\cos\left(\frac{k_1 + k_2}{\sqrt{2}}\right) + \sin\frac{k_F}{\sqrt{2}}\sin\left(\frac{k_1 + k_2}{\sqrt{2}}\right)\right] \simeq 4\Delta$$



$v_F =$
$\varepsilon(\vec{k}) - \mu = -4t \sin\frac{k_F}{\sqrt{2}}\left(\frac{-k_1}{\sqrt{2}}\right) = v_F k_1$
$\cong 4\Delta \sin\frac{k_F}{\sqrt{2}}\left(\frac{k_2}{\sqrt{2}}\right)$



$v_F = 2\sqrt{2}\, t \sin\frac{k_F}{\sqrt{2}}$
$\varepsilon(\vec{k}) - \mu = -4t \sin\frac{k_F}{\sqrt{2}} \left(\frac{-k_1}{\sqrt{2}}\right) = v_F k_1$
$\Delta(k) = \frac{\Delta}{t} v_F k_2 = v_1 k_2$
$\simeq 4\Delta \sin\frac{k_F}{\sqrt{2}} \left(\frac{k_2}{\sqrt{2}}\right)$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$$v_F = 2\sqrt{2}\, t \sin\frac{k_F}{\sqrt{2}}$$

$$\varepsilon(\vec{k}) - \mu = -4t \sin\frac{k_F}{\sqrt{2}} \left(\frac{-k_1}{\sqrt{2}}\right) = v_F\, k_1$$

$$\Delta(k) = \frac{\Delta}{t} v_F\, k_2 \equiv v_1 k_2$$

$$\simeq 4\Delta \sin\frac{k_F}{\sqrt{2}} \left(\frac{k_2}{\sqrt{2}}\right)$$

Then $\xi(\vec{k}) = \sqrt{(v_F k_1)^2 + (v_1 k_2)^2}$

$$v_F = 2\sqrt{2}\, t \sin\frac{k_F}{\sqrt{2}}$$

$$\varepsilon(\vec{k}) - \mu = -4t \sin\frac{k_F}{\sqrt{2}}\left(\frac{-k_1}{\sqrt{2}}\right) = v_F k_1$$

$$\Delta(k) = \frac{\Delta}{t} v_F k_2 \equiv v_1 k_2$$

Then $\xi(\vec{k}) = \sqrt{(v_F k_1)^2 + (v_1 k_2)^2}$

For $\vec{k}$ near $\vec{k}_F = \left(\frac{\pm k_F}{\sqrt{2}}, \frac{\pm k_F}{\sqrt{2}}\right)$

$$\simeq 4\Delta \sin\frac{k_F}{\sqrt{2}}\left(\frac{k_2}{\sqrt{2}}\right)$$

$$v_F = 2\sqrt{2}\, t \sin\frac{k_F}{\sqrt{2}}$$

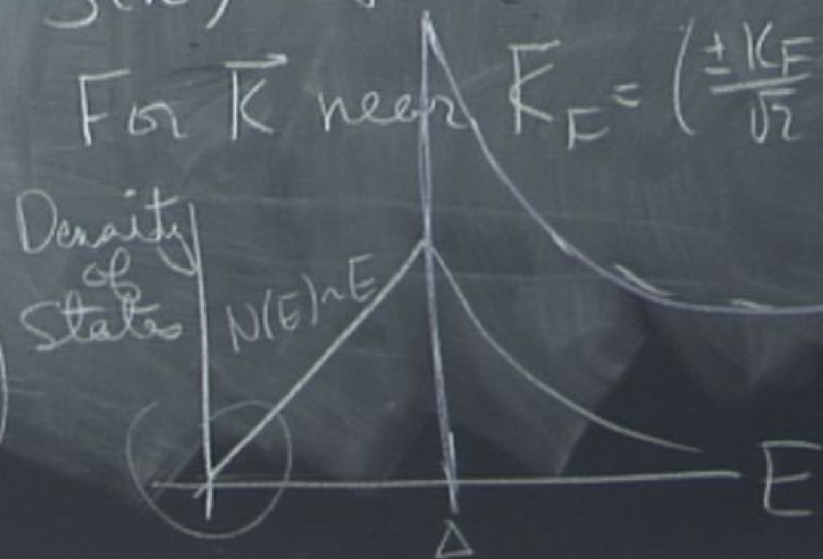
$$\mathcal{E}(\vec{k}) - \mu = -4t \sin\frac{k_F}{\sqrt{2}}\left(\frac{-k_1}{\sqrt{2}}\right) = v_F k_1$$

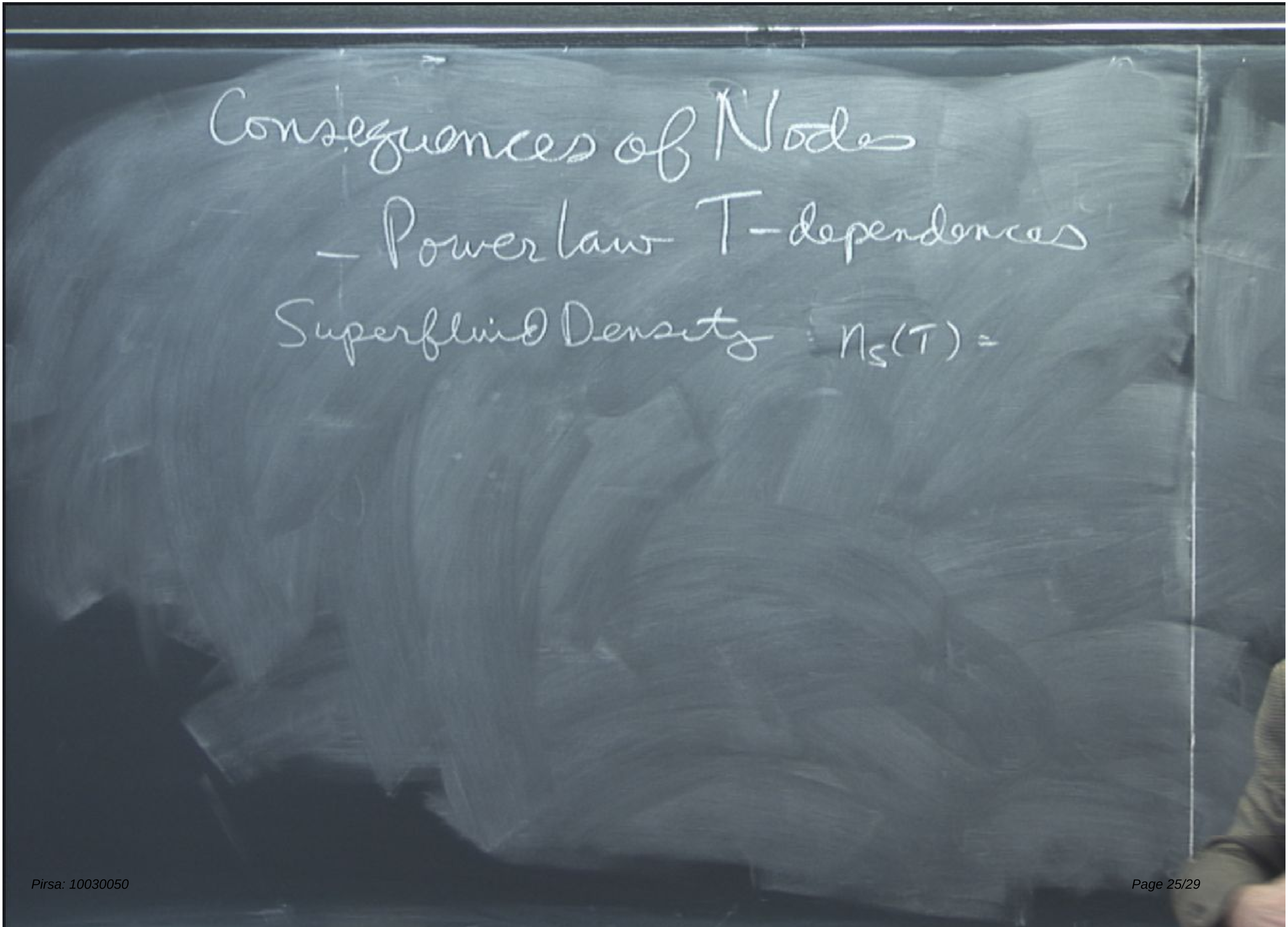
$$\Delta(k) = \frac{\Delta}{t} v_F k_2 \equiv v_1 k_2$$

$$\cong 4\Delta \sin\frac{k_F}{\sqrt{2}}\left(\frac{k_2}{\sqrt{2}}\right)$$

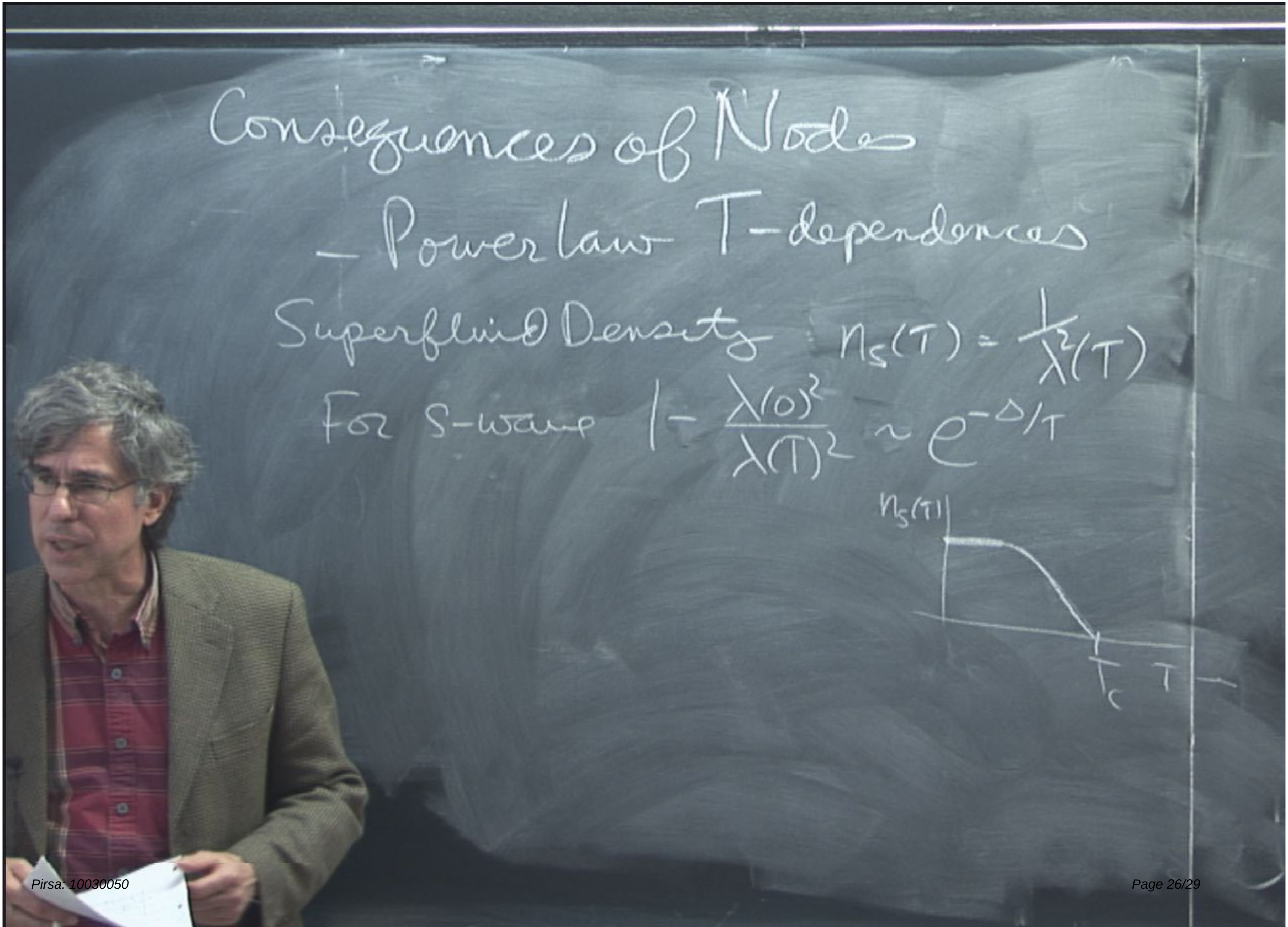
Then $\xi(\vec{k}) = \sqrt{(v_F k_1)^2 + (v_1 k_2)^2}$

For $\vec{k}$ near $\vec{k}_F = \left(\frac{\pm k_F}{\sqrt{2}}, \frac{\pm k_F}{\sqrt{2}}\right)$





Consequences of Nodes
– Powerlaw T-dependences
Superfluid Density $n_s(T) =$


Consequences of Nodes
– Powerlaw T-dependences
Superfluid Density $n_s(T) = \frac{1}{\lambda^2(T)}$
For s-wave $1 - \frac{\lambda(0)^2}{\lambda(T)^2} \sim e^{-\Delta/T}$
$n_s(T)$
T_c
T

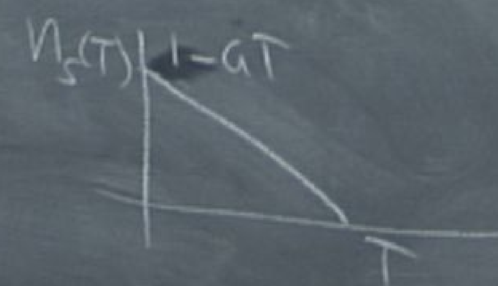
Consequences of Nodes

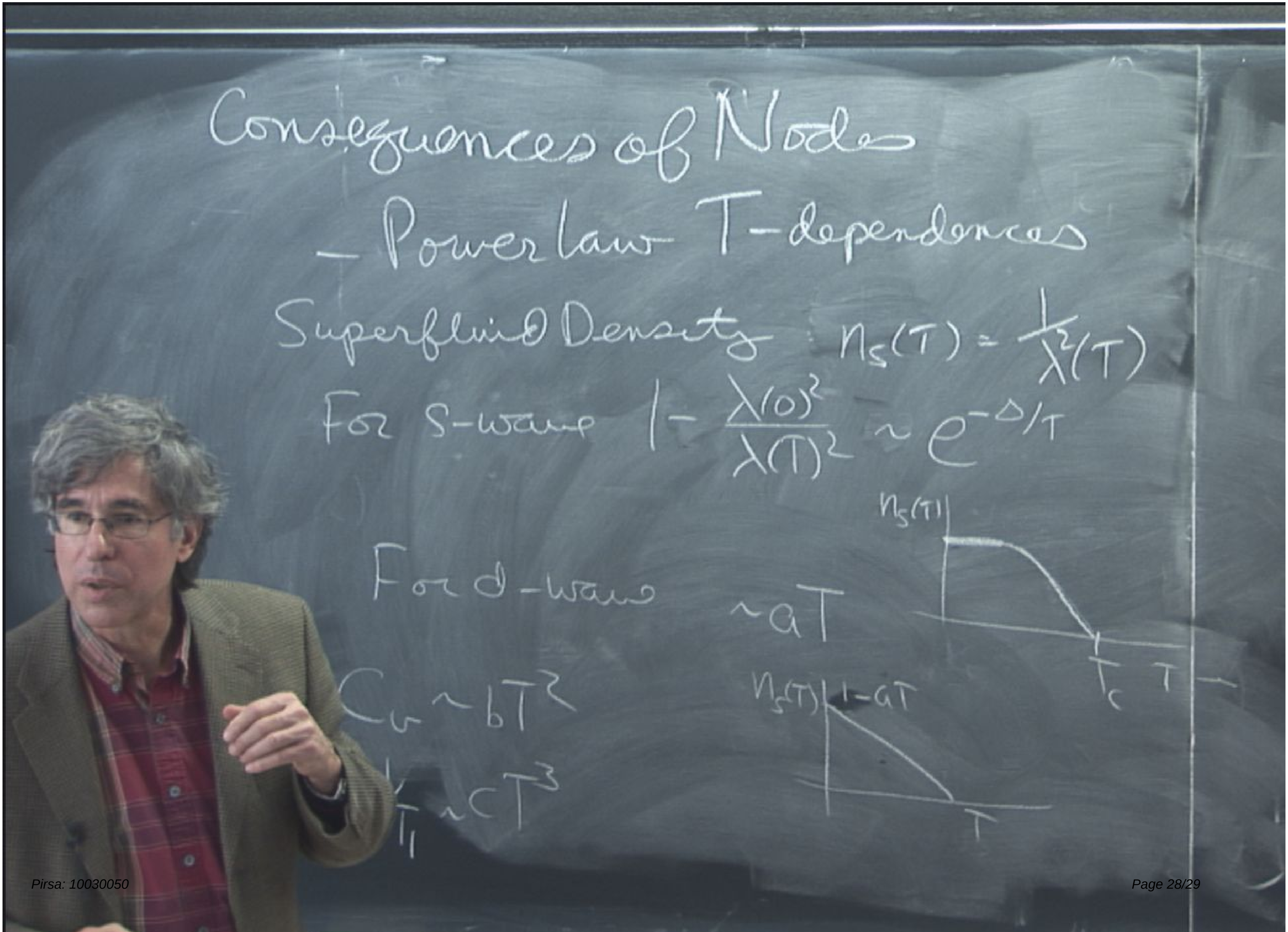
– Powerlaw T-dependences

Superfluid Density : $n_s(T) = \frac{1}{\lambda^2(T)}$

For s-wave $1 - \frac{\lambda(0)^2}{\lambda(T)^2} \sim e^{-\Delta/T}$

For d-wave $\sim aT$




Consequences of Nodes
– Powerlaw T–dependences
Superfluid Density $n_s(T) = \frac{1}{\lambda^2(T)}$
For s-wave $1 - \frac{\lambda(0)^2}{\lambda(T)^2} \sim e^{-\Delta/T}$
For d-wave $\sim aT$
$C_v \sim bT^2$
$\frac{1}{T_1} \sim cT^3$
$n_s(T)$
T_c
T
$n_s(T)$
$1-aT$
T

Consequences of Nodes

– Powerlaw T–dependences

Superfluid Density $n_s(T) = \frac{1}{\lambda^2(T)}$

For s-wave $1 - \frac{\lambda(0)^2}{\lambda(T)^2} \sim e^{-\Delta/T}$

For d-wave $\sim aT$

$C_v \sim bT^2$

$1/T_1 \sim cT^3$