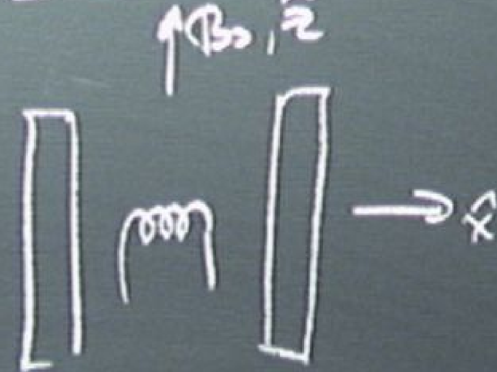
Title: Explorations in Quantum Info. (PHYS 641) - Lecture 13

Date: Mar 04, 2010 09:00 AM

URL: http://pirsa.org/10030046

Abstract:

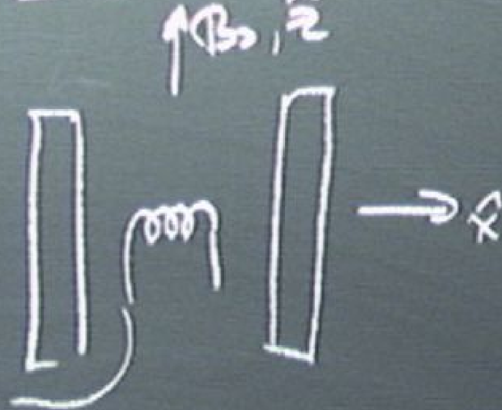
Lab frame



Rotating Frame

Rotating Frame

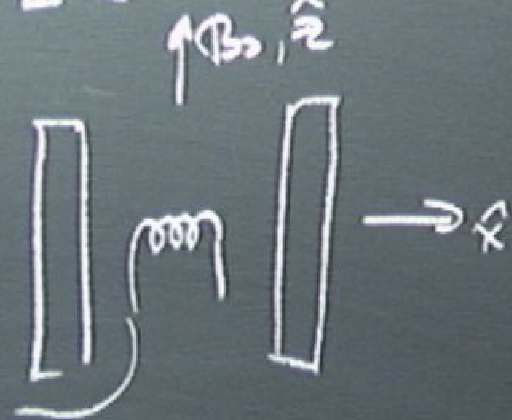
Lab frame



$$B_1 = |B_1| \cos(\omega_T t + \phi)$$

Rotating Frame

Lab frame



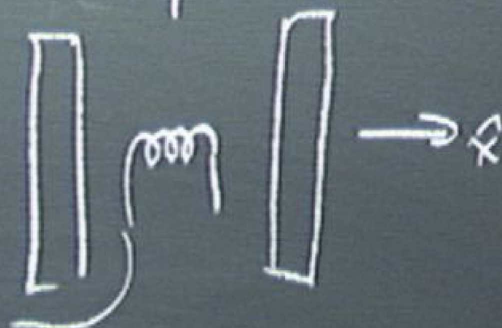
$$B_1 = |B_1| \cos(\omega_T t + \phi) \quad ; \quad 0 < t < t_p$$

Lab frame

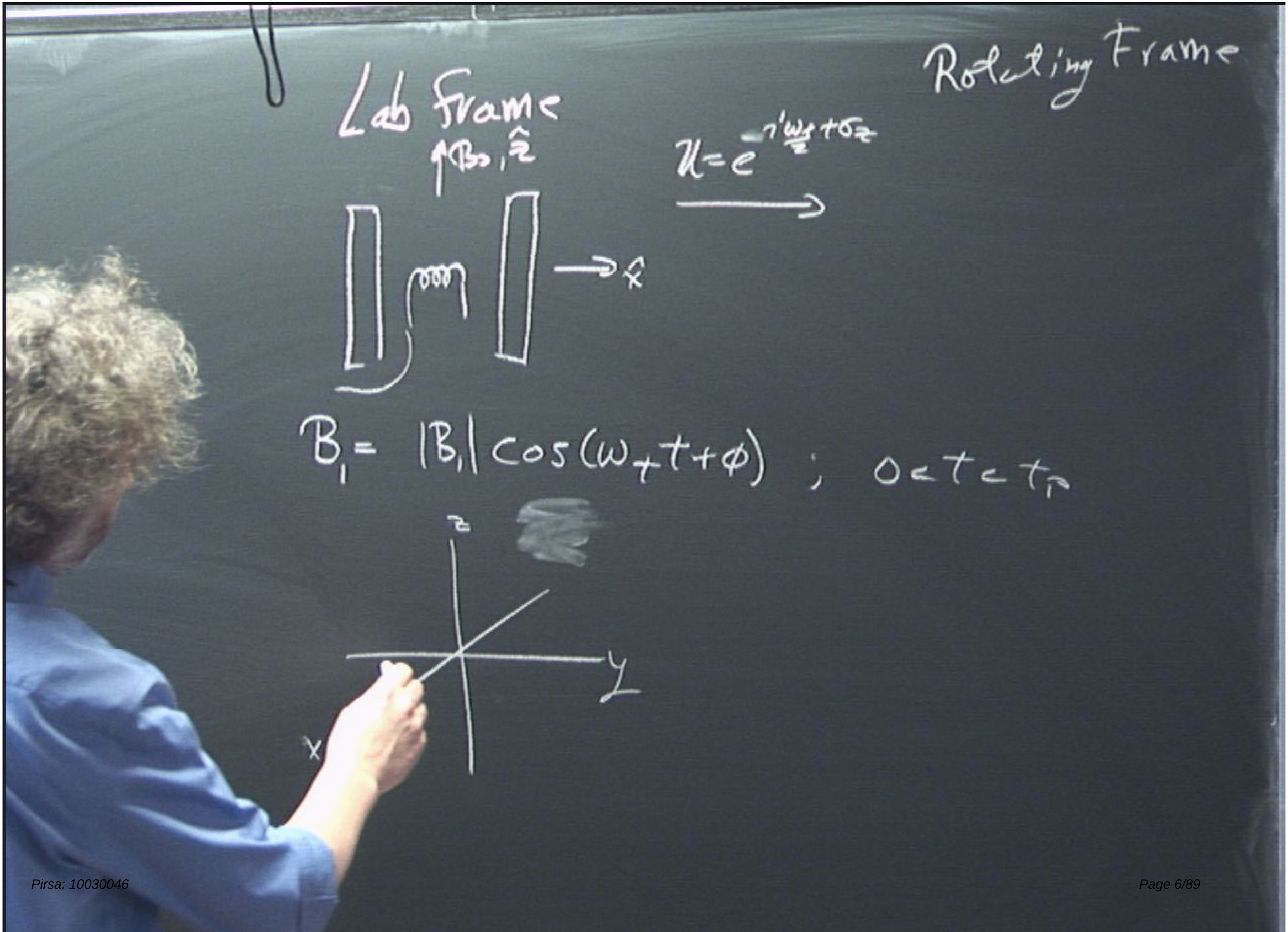
Rotating Frame

$\uparrow B_0, \hat{z}$

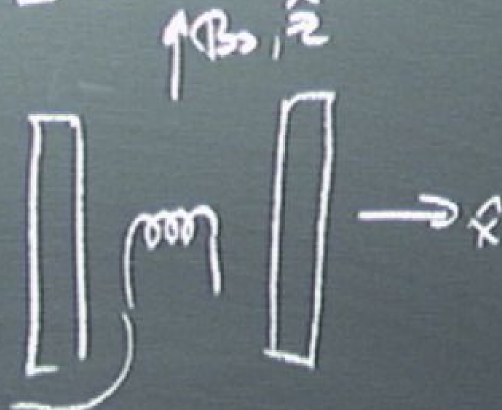
$\mathcal{U} = e^{-i\frac{\omega_T}{2} t \sigma_z}$ $\longrightarrow$



$B_1 = |B_1| \cos(\omega_T t + \phi) \; ; \quad 0 < t < t_p$


Rotating Frame
Lab frame
$B_0, \hat{z}$
$\mathcal{U} = e^{-i\frac{\omega_T}{2} t\sigma_z}$
$\hat{x}$
$B_1 = |B_1| \cos(\omega_T t + \phi)$; $0 < t < t_p$
z
y
x

Lab frame

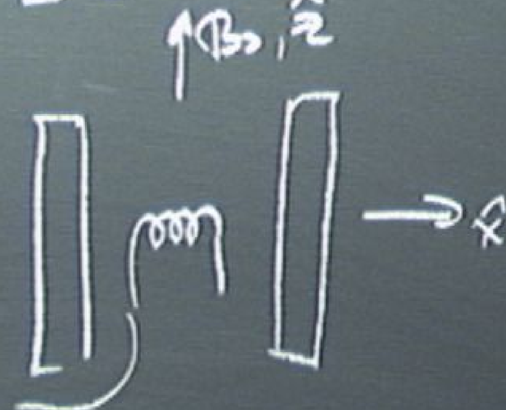


Rotating Frame

$$\mathcal{U} = e^{-i\frac{\omega_T t}{2}\sigma_z} \longrightarrow$$

$$B_1 = |B_1| \cos(\omega_T t + \phi) \; ; \quad 0 < t < t_p$$

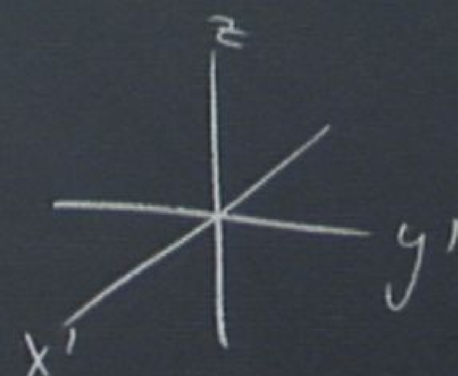
Lab frame

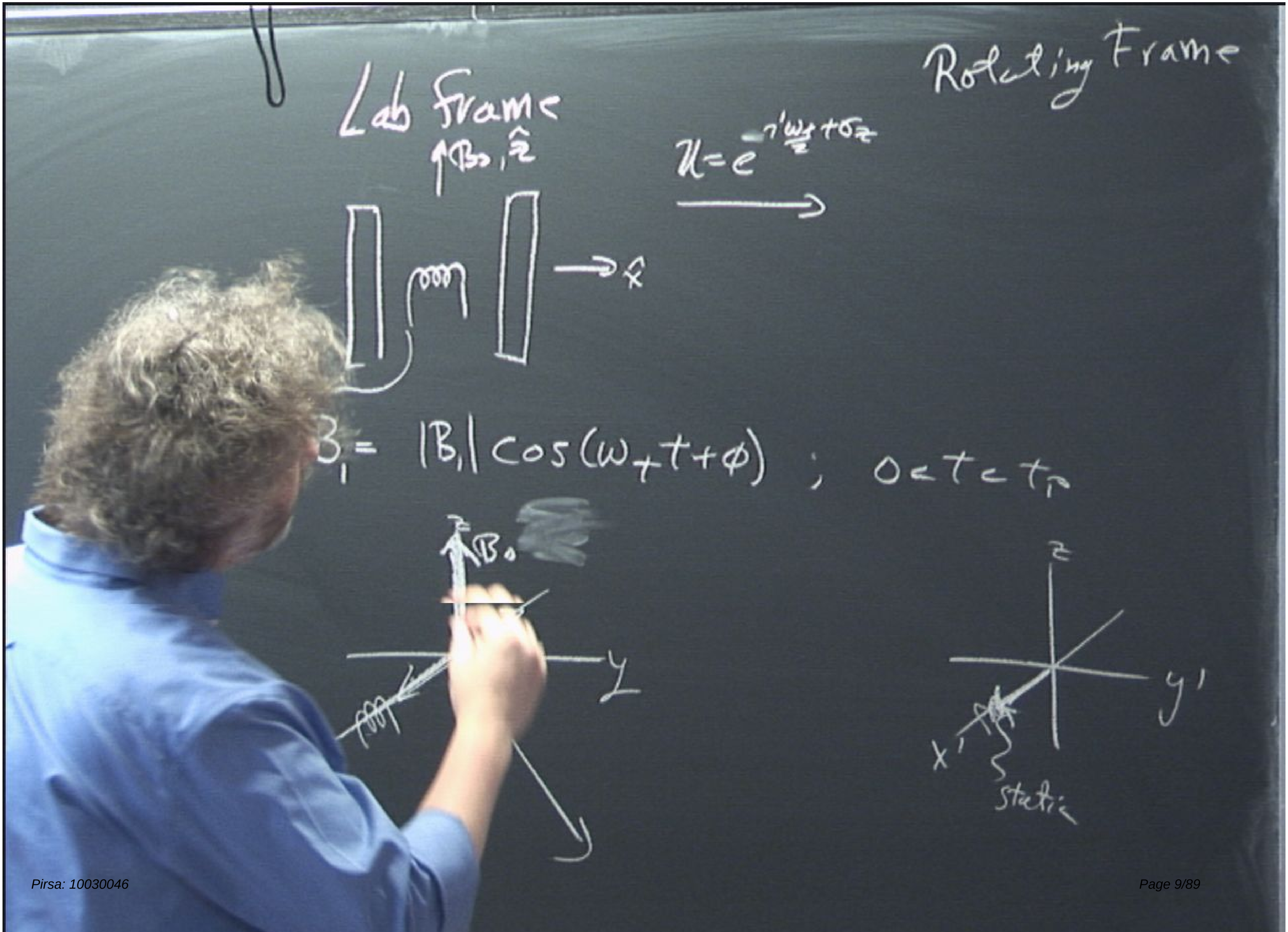


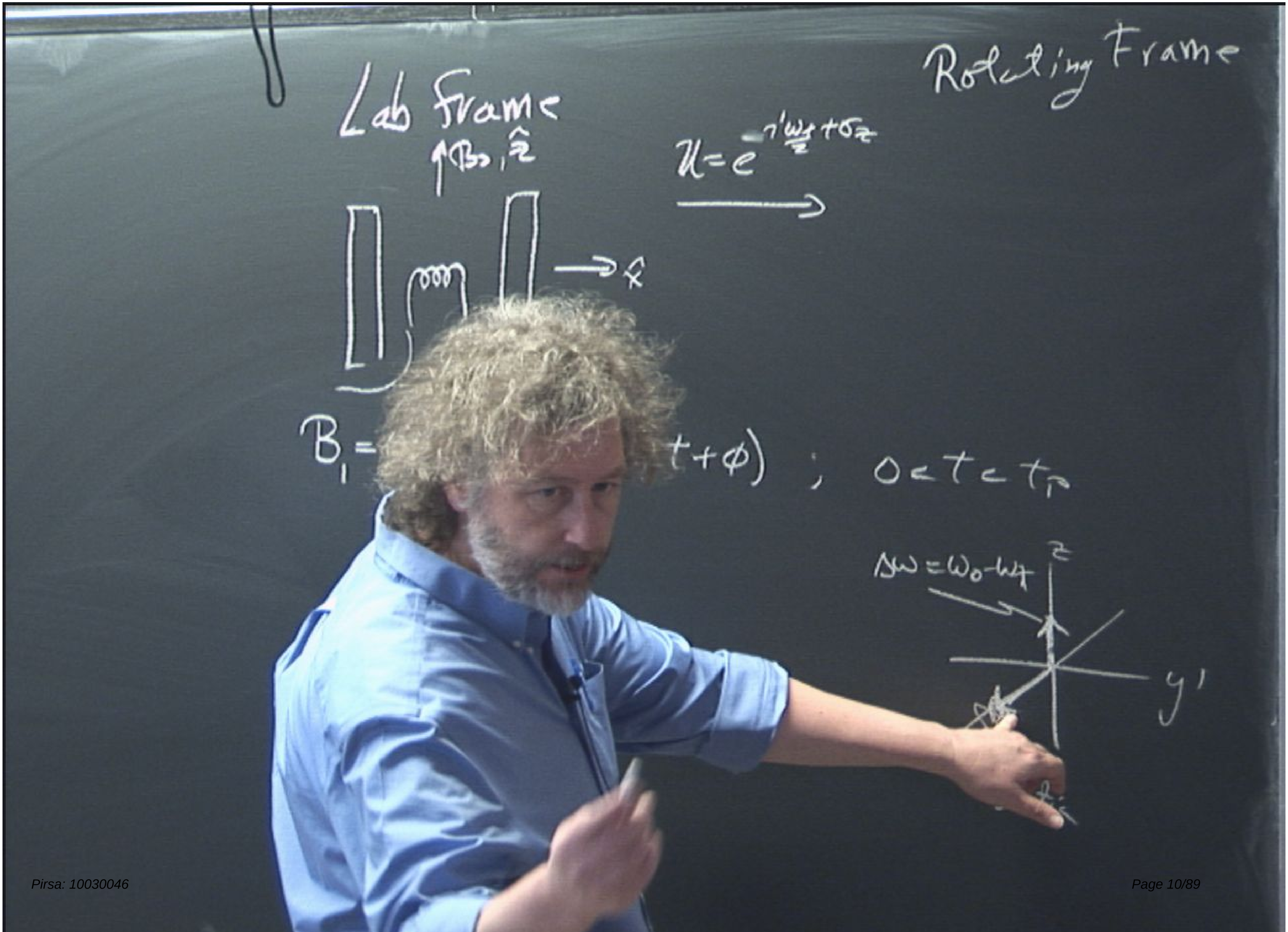
Rotating Frame

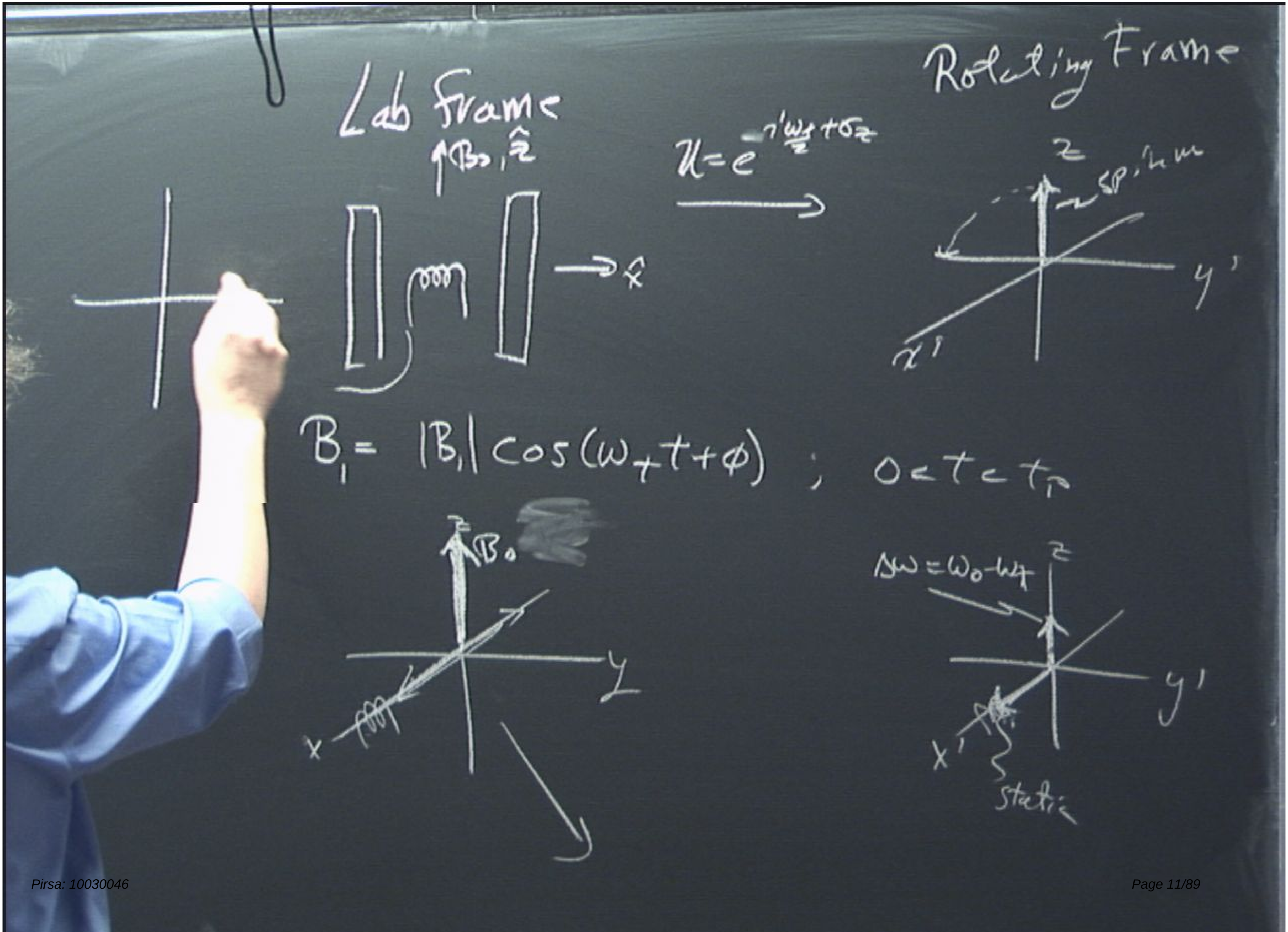
$$\mathcal{U} = e^{-i\frac{\omega_T}{2}t\sigma_z} \longrightarrow$$

$$B_1 = |B_1| \cos(\omega_T t + \phi) \; ; \quad 0 < t < t_p$$

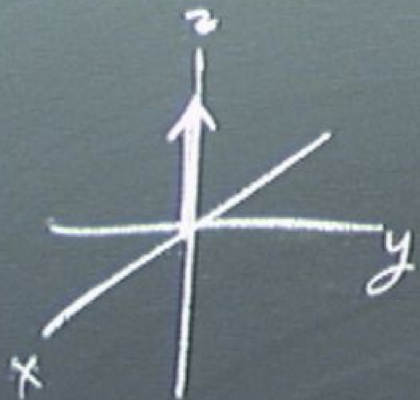


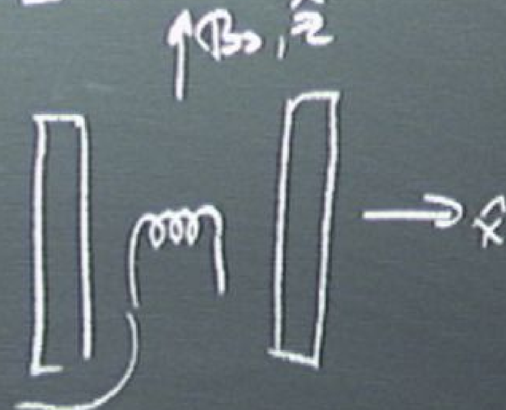

Lab frame
Rotating Frame
$\mathcal{U} = e^{-i\frac{\omega_T}{2} t \sigma_z}$
$B_0, \hat{z}$
$\hat{x}$
$B_1 = |B_1| \cos(\omega_T t + \phi) \; ; \; 0 < t < t_p$
z
B_0
y
z
y'
x'
static


Rotating Frame
Lab frame
B₀, ẑ
x̂
0 < t < t_p
z
y'


Lab frame
$B_0, \hat{z}$
$\hat{x}$
Rotating Frame
$U = e^{-i\frac{\omega_T t}{2}\sigma_z}$
z
spin
y'
x'
$B_1 = |B_1| \cos(\omega_T t + \phi) \;;\; 0 < t < t_p$
z
B_0
y
x
$\Delta\omega = \omega_0 - \omega_T$
z
y'
x'
static

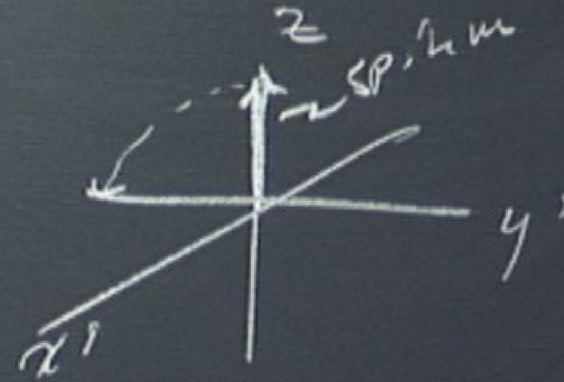
Lab frame



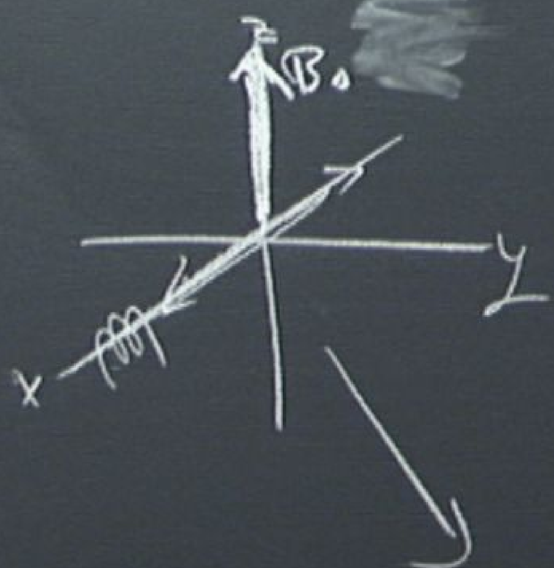


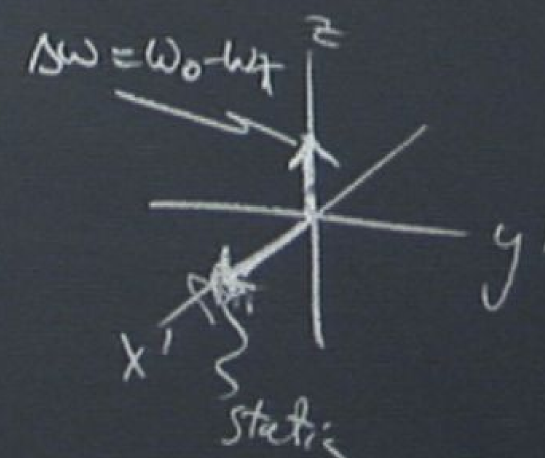
$$\mathcal{U} = e^{-i\frac{\omega_T}{2} t \sigma_z} \Longrightarrow$$

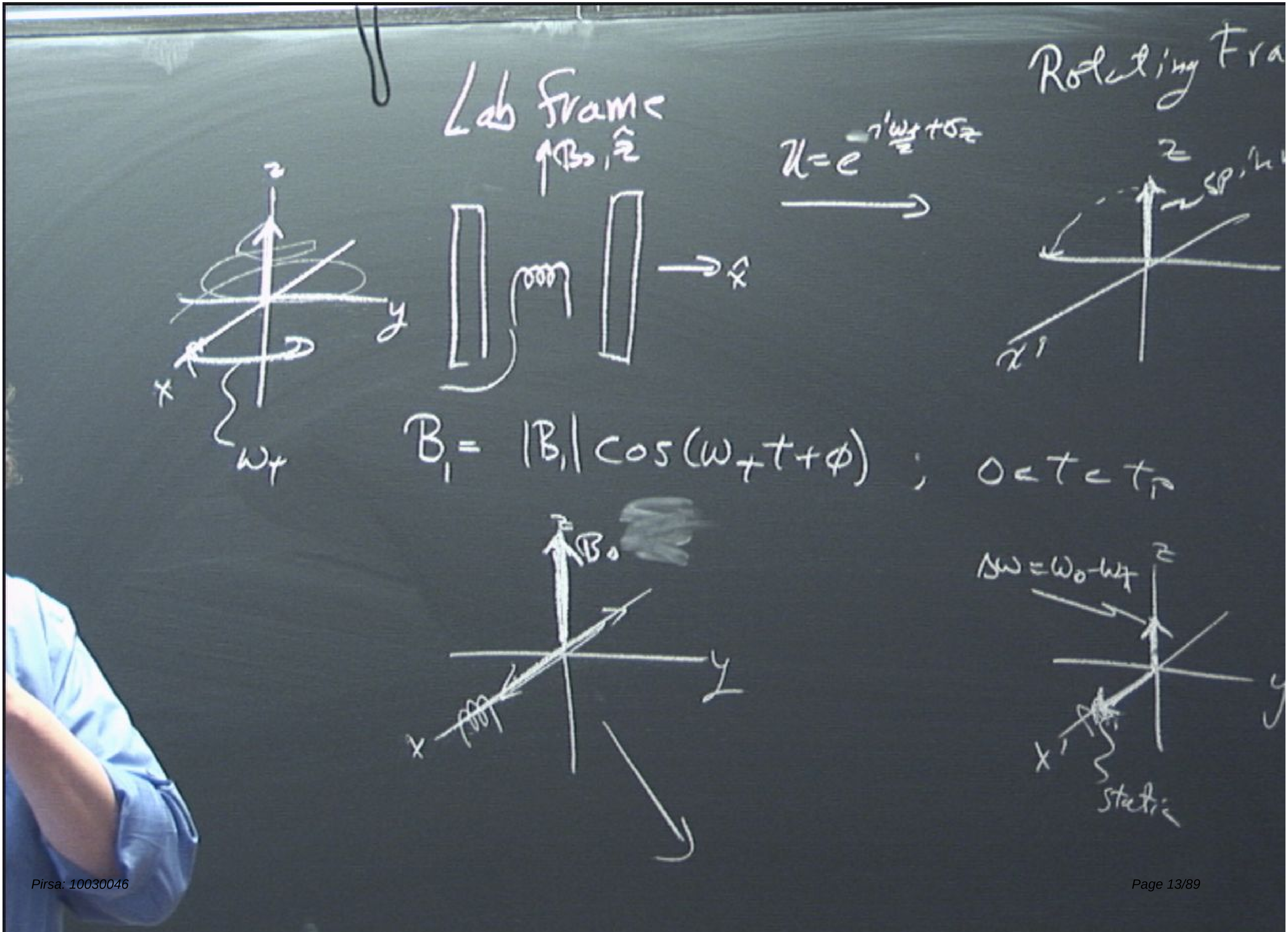
Rotating Frame

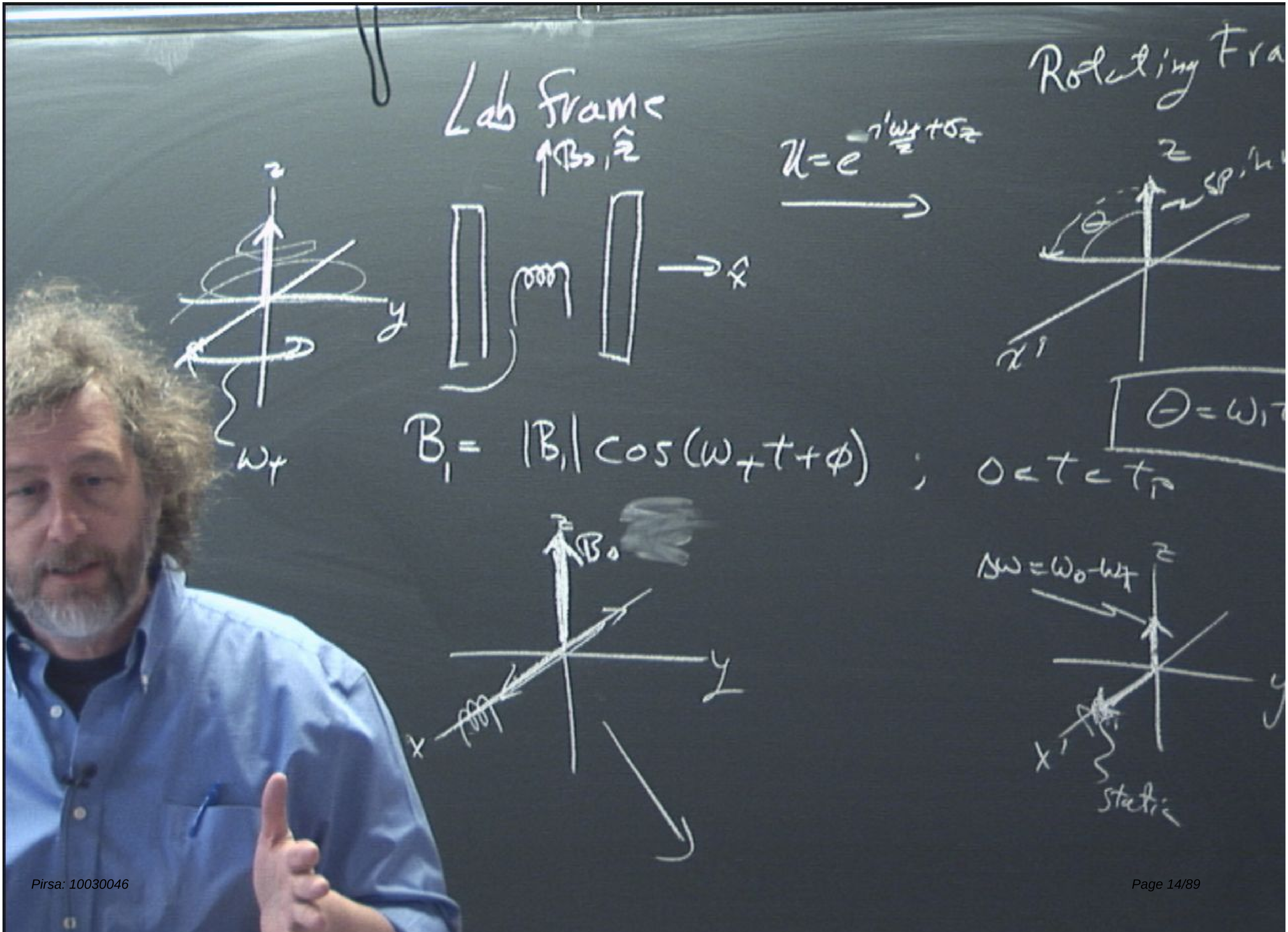


$$B_1 = |B_1| \cos(\omega_T t + \phi) \; ; \quad 0 < t < t_p$$

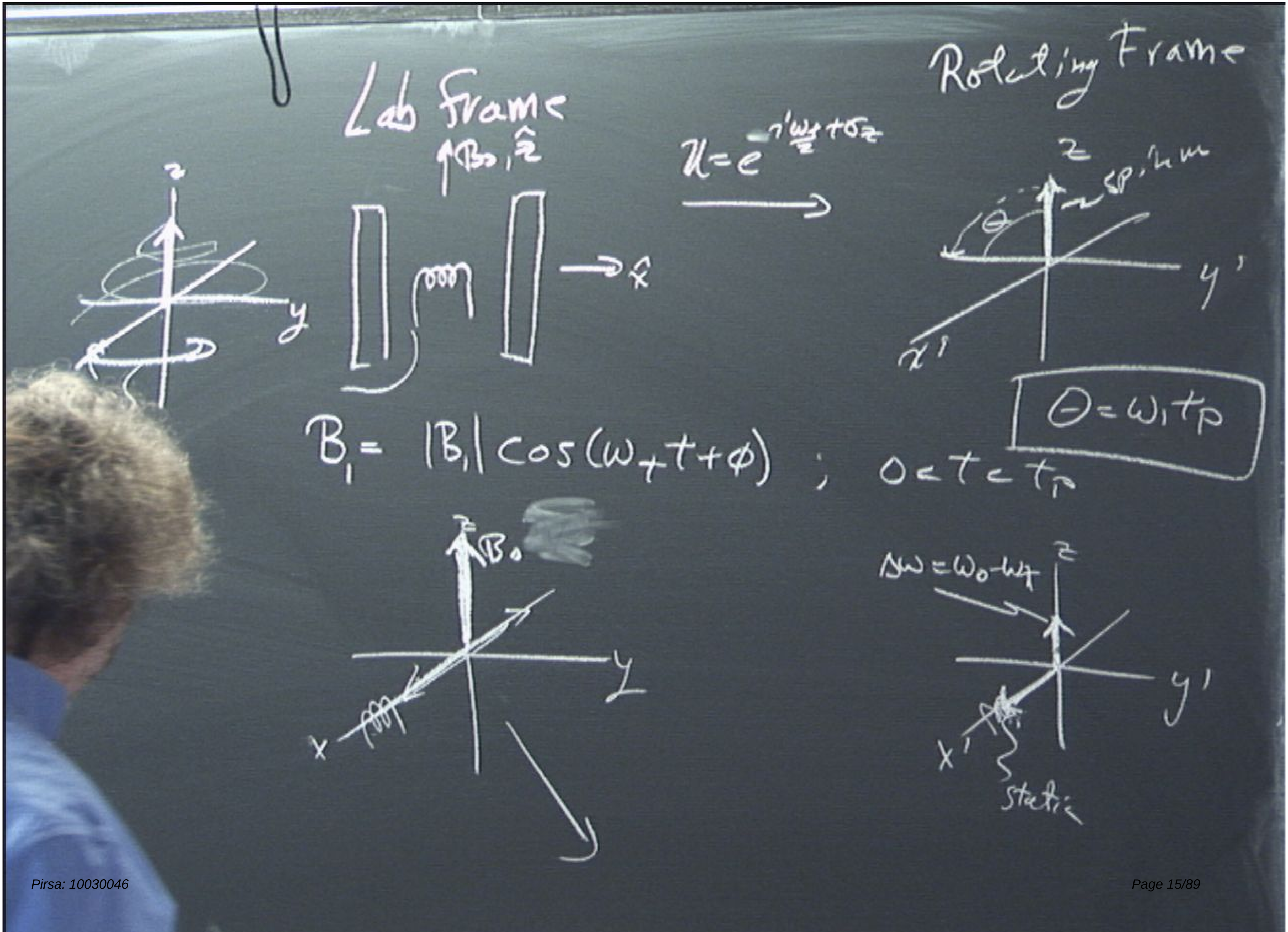


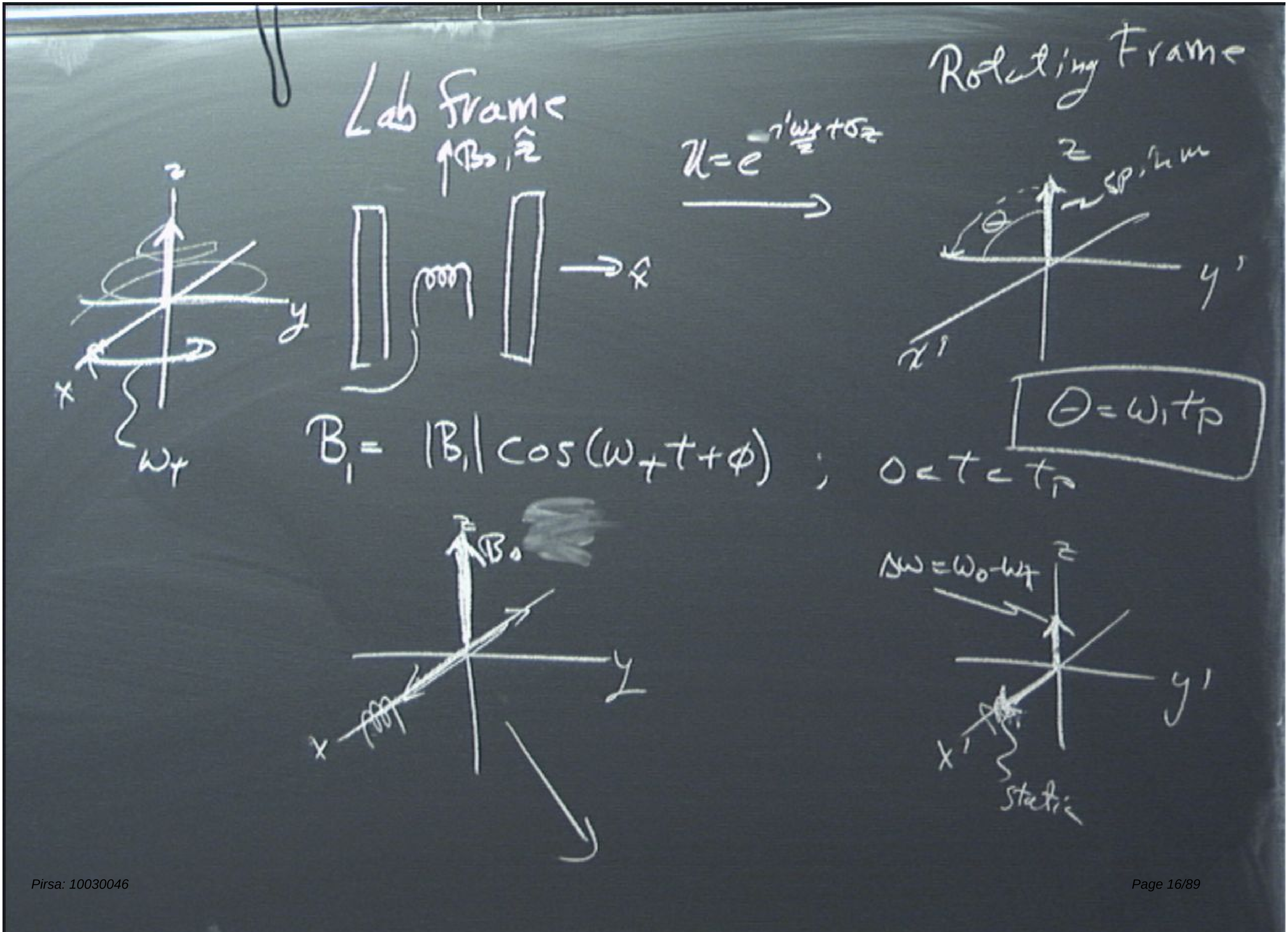



Lab frame
Rotating Fra
$B_1 = |B_1| \cos(\omega_T t + \phi)$; $0 < t < t_p$
$\Delta\omega = \omega_0 - \omega_T$
static

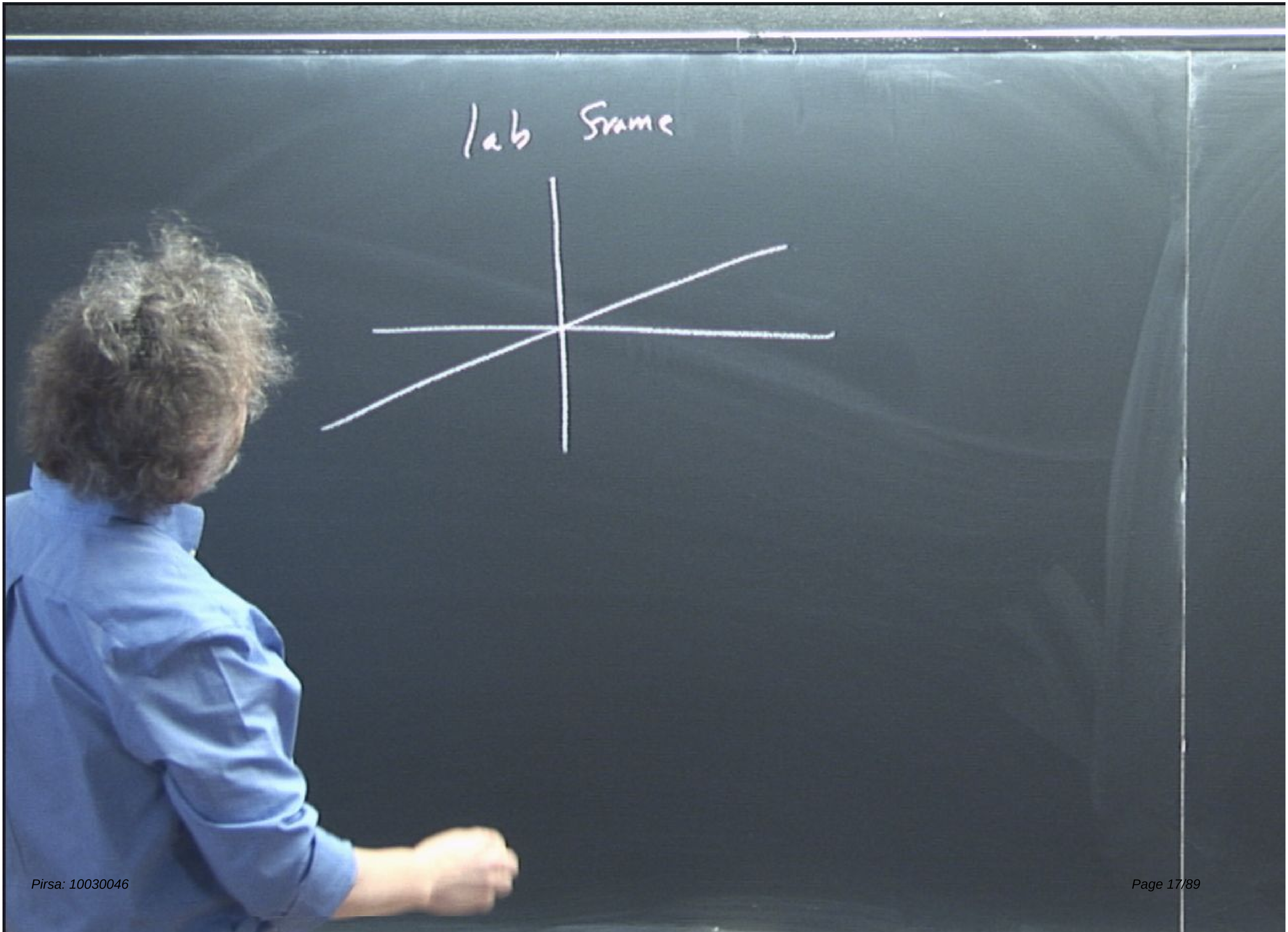


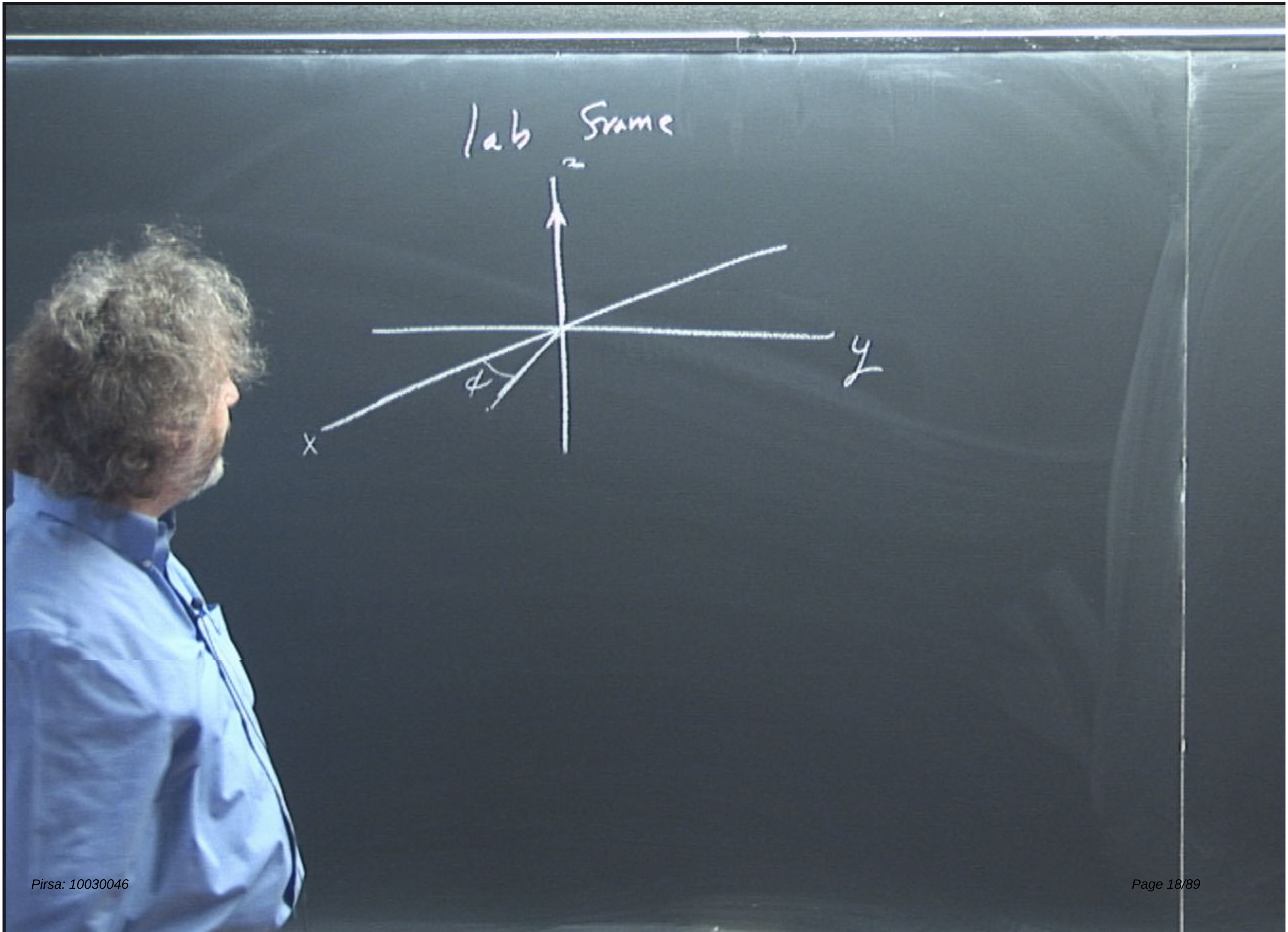
Lab frame
Rotating Fra
B₀, ẑ
x̂
z
y
x
x'
B₁ = |B₁| cos(ω_r t+φ) ; 0<t<t_p
Δω = ω₀ − ω_r
static

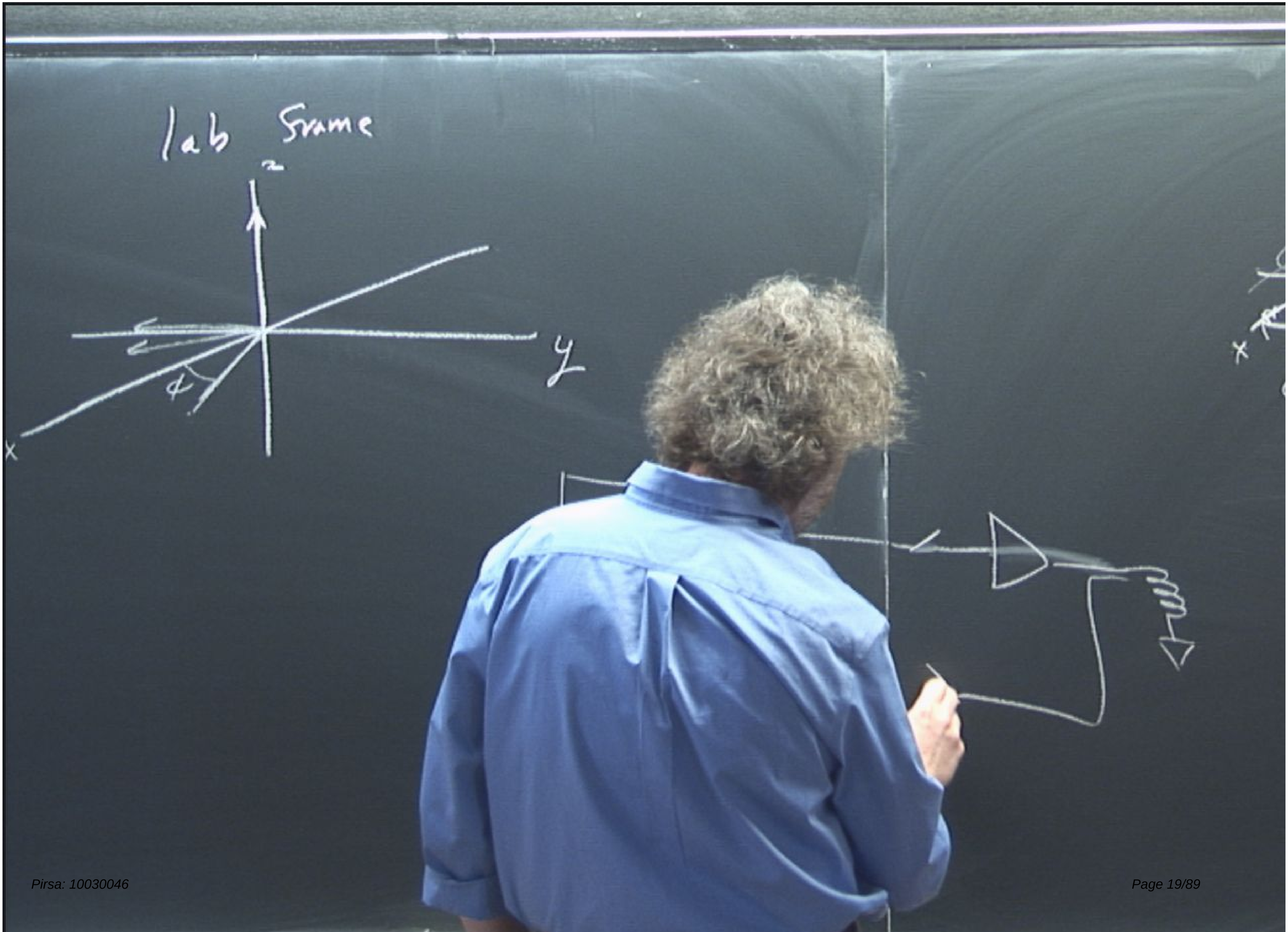

Lab frame
Rotating Frame
$B_0, \hat{z}$
$\hat{x}$
$\mathcal{U} = e^{-i\frac{\omega_T}{2}t\sigma_z}$
spin
θ
y'
x'
$\theta = \omega_1 t_p$
$B_1 = |B_1| \cos(\omega_T t + \phi)$; $0 < t < t_p$
B_0
$\Delta\omega = \omega_0 - \omega_T$
static

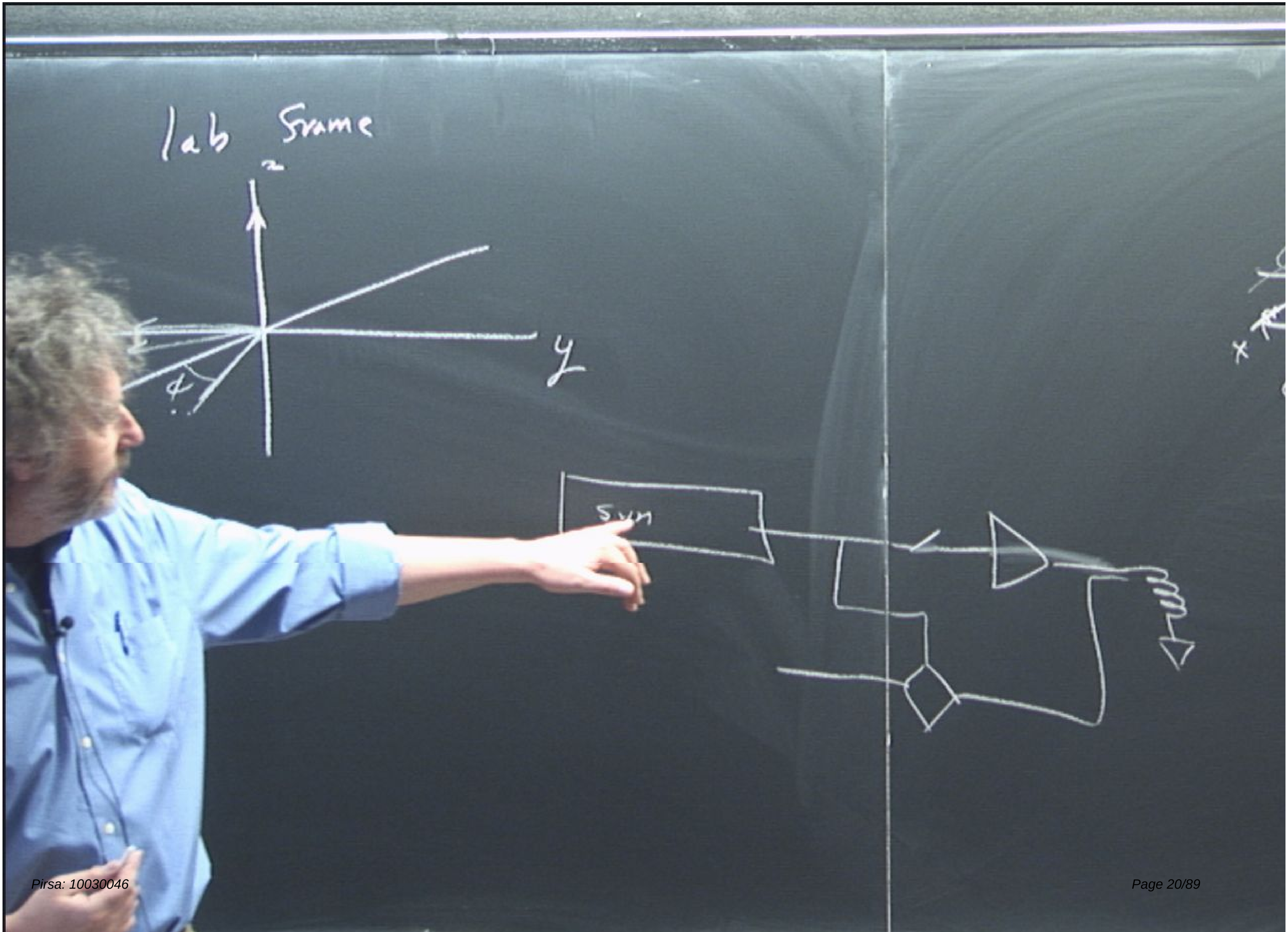


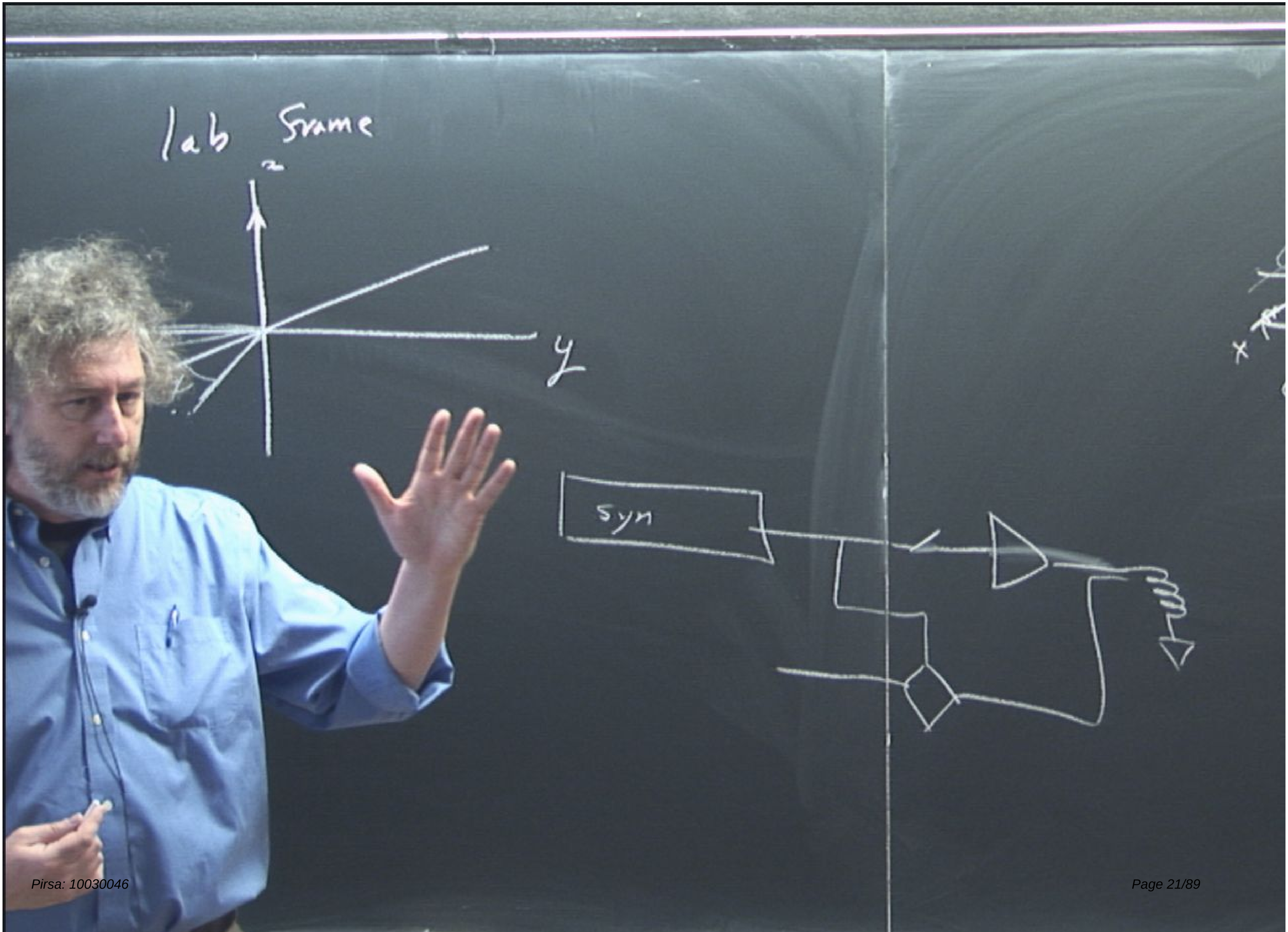
Lab frame
B0, ẑ
U = e^{-i ωT t σz/2}
Rotating Frame
z
spin
θ
y'
x'
θ = ω1 tp
x
y
ωT
B1 = |B1| cos(ωT t + φ) ; 0 < t < tp
B0
Δω = ω0 − ωT
static

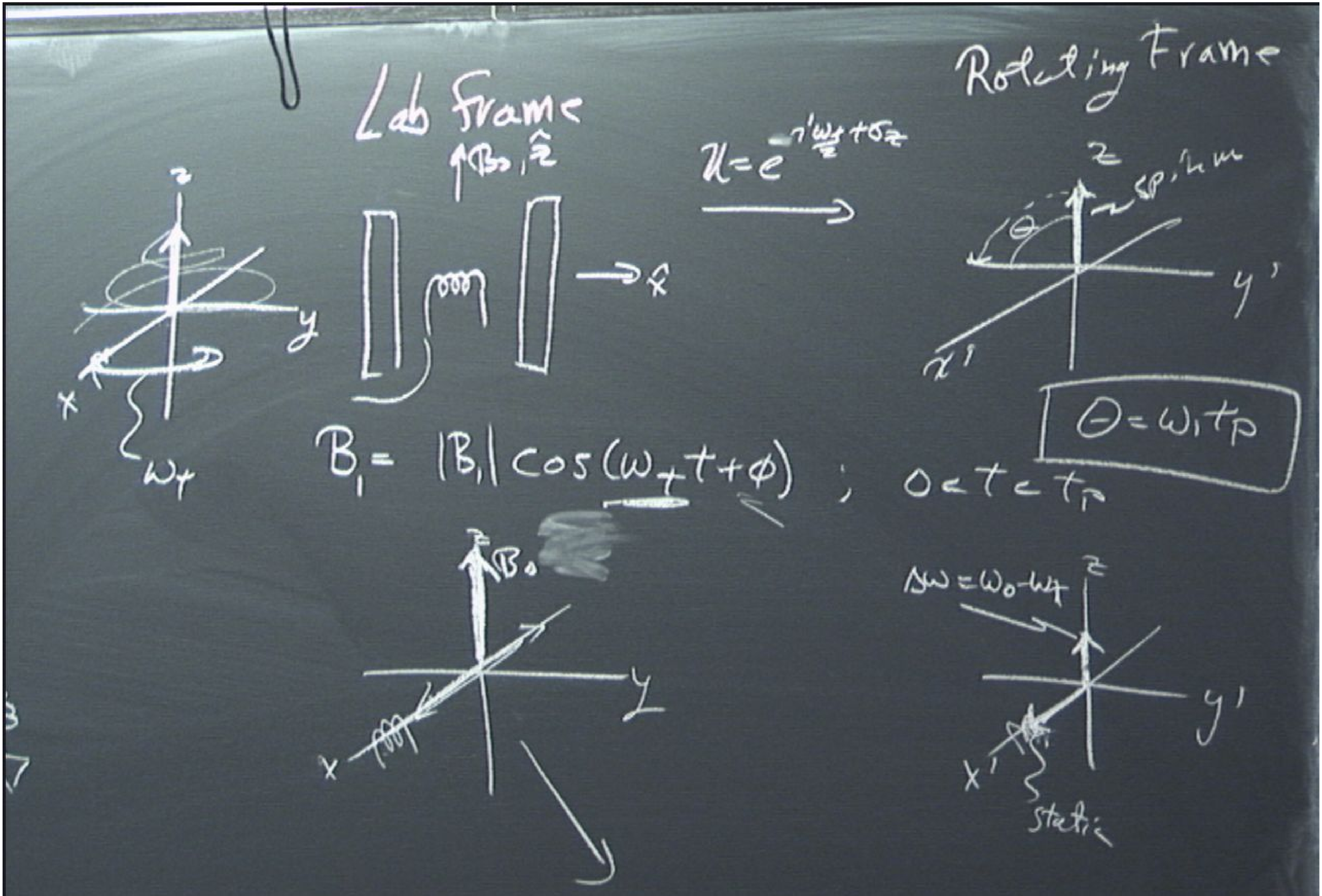

lab frame

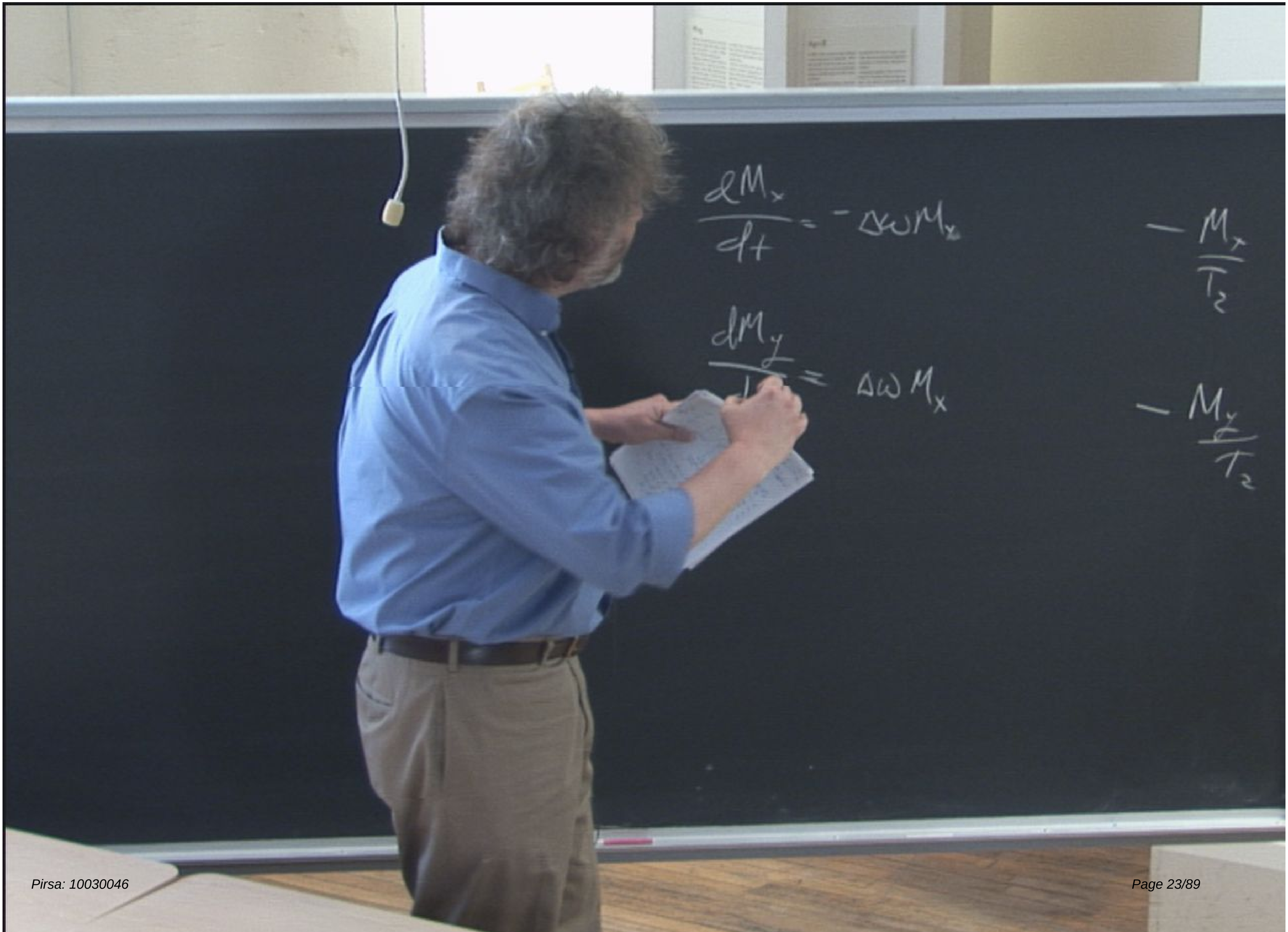

lab frame
y
x

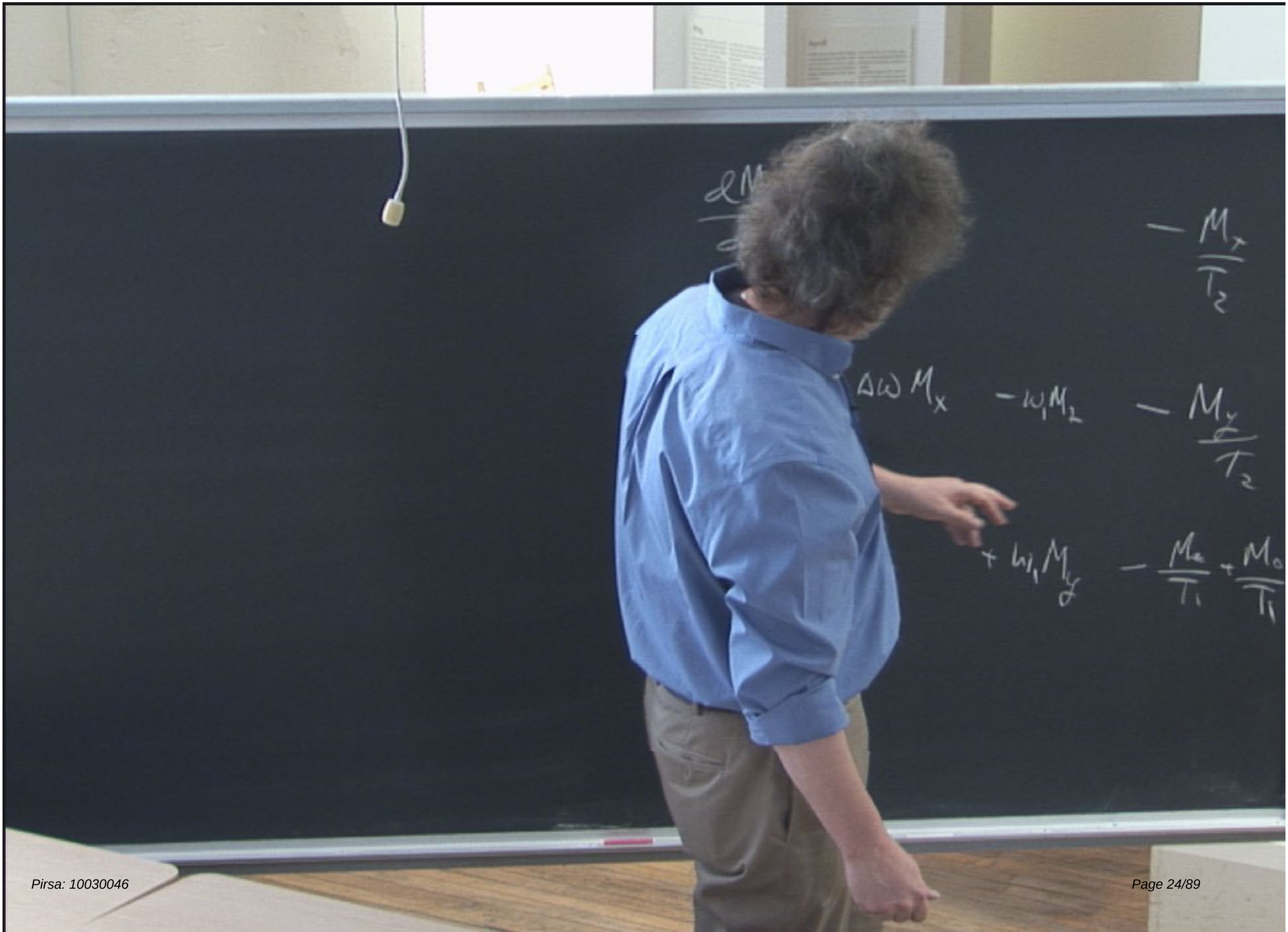

lab frame
z
y
x

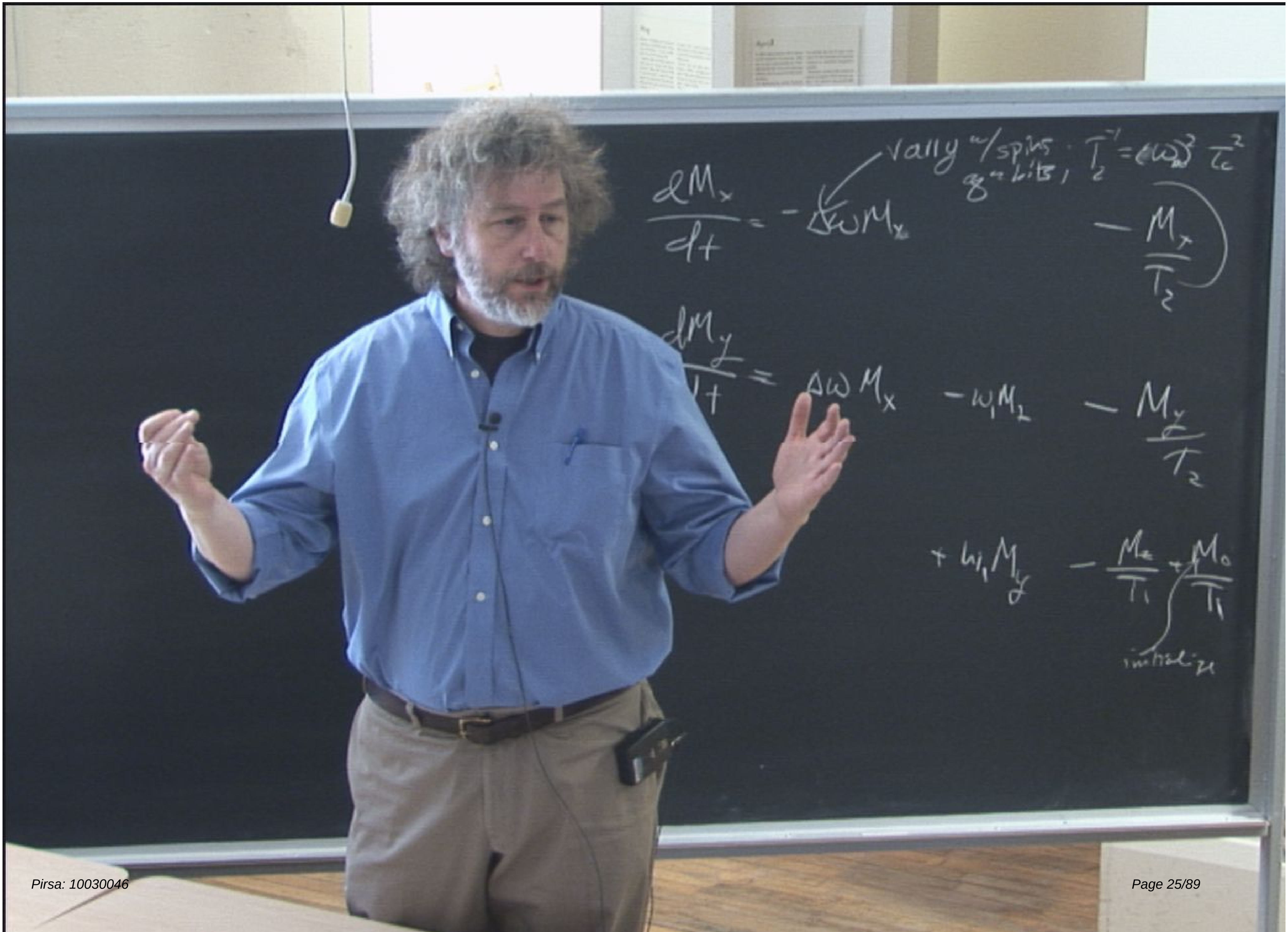

lab frame
z
y
φ
Sum

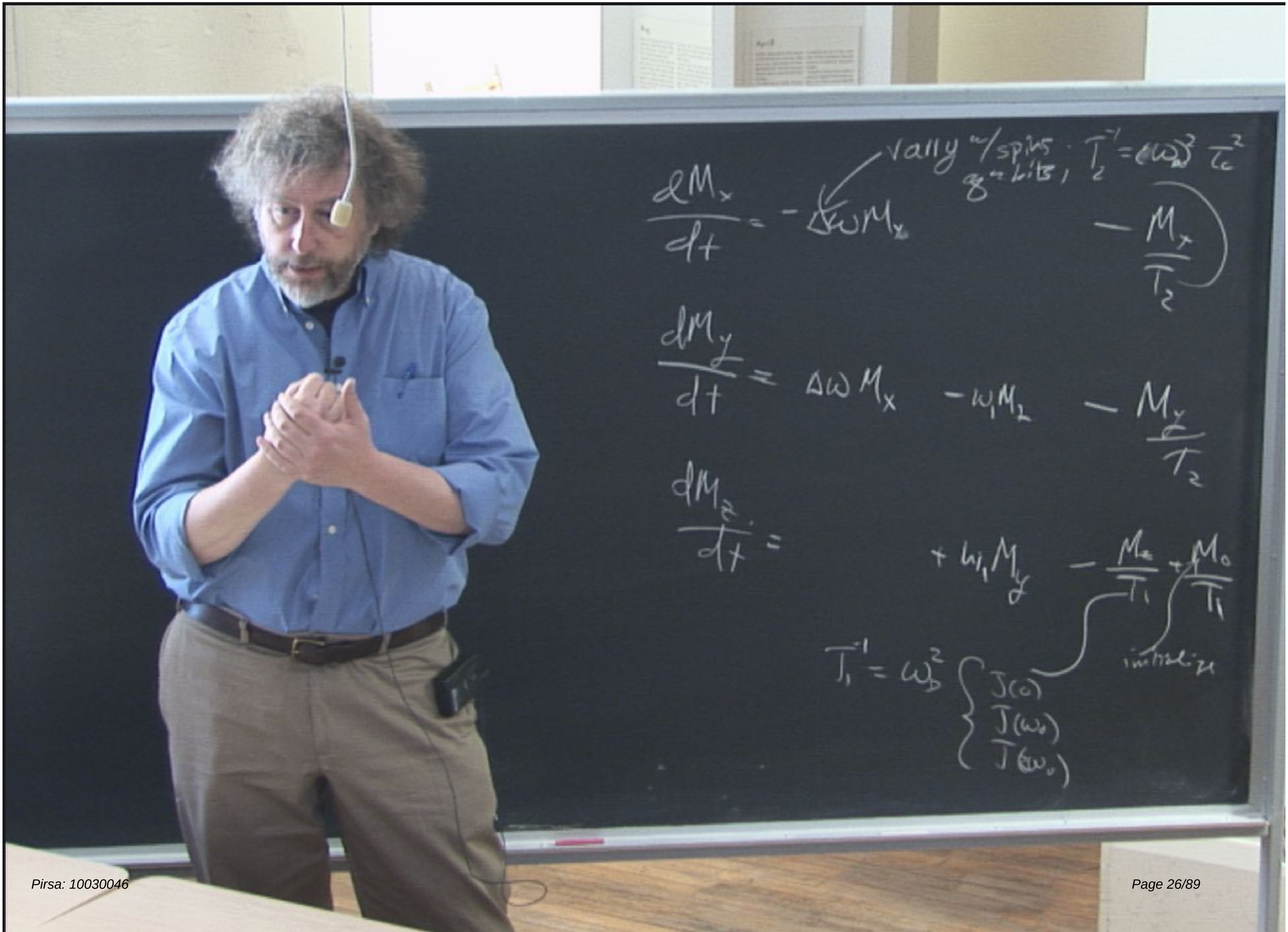

lab frame
z
y
syn
x

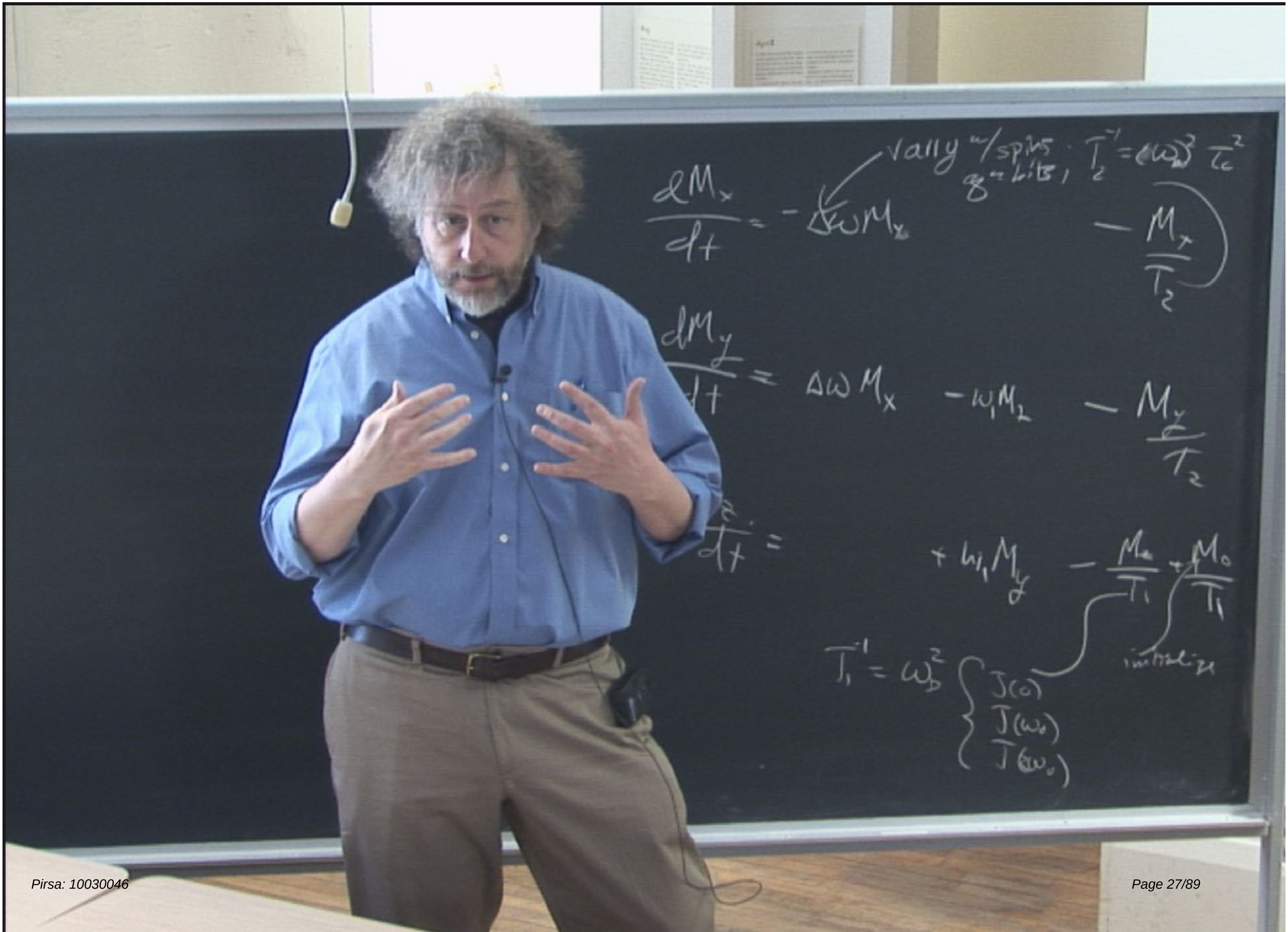

Lab frame
Rotating Frame
$B_0, \hat{z}$
$U = e^{-i\frac{\omega_t}{2} t \sigma_z}$
spin
$\hat{x}$
ω_t
$\theta = \omega_1 t_p$
$B_1 = |B_1| \cos(\omega_t t + \phi) \; ; \; 0 < t < t_p$
B_0
$\Delta\omega = \omega_0 - \omega_t$
static
x
y
z
x'
y'

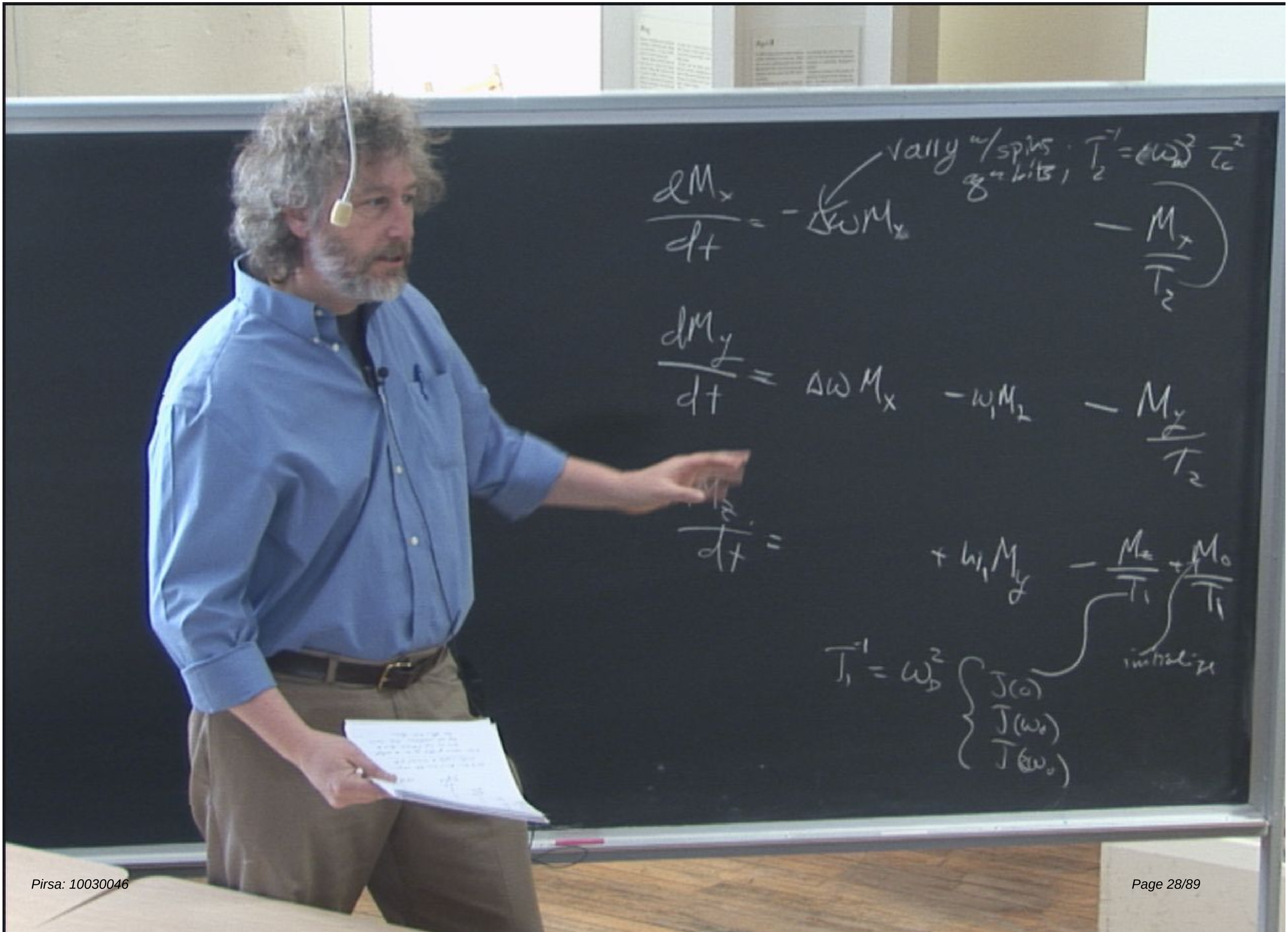

$\frac{dM_x}{dt} = -\Delta\omega M_x \qquad - \frac{M_x}{T_2}$
$\frac{dM_y}{dt} = \Delta\omega M_x \qquad - \frac{M_y}{T_2}$


$- \frac{M_x}{T_2}$
$\Delta\omega M_x \quad - \omega_1 M_z \quad - \frac{M_y}{T_2}$
$+ \omega_1 M_y \quad - \frac{M_z}{T_1} + \frac{M_0}{T_1}$


vally w/spins
g-bits
$T_2^{-1} = (\Delta\omega)^2 \tau_c^2$
$\frac{dM_x}{dt} = -\Delta\omega M_x - \frac{M_x}{T_2}$
$\frac{dM_y}{dt} = \Delta\omega M_x - \omega_1 M_z - \frac{M_y}{T_2}$
$+ \omega_1 M_y - \frac{M_z}{T_1} + \frac{M_0}{T_1}$


vally w/spins
g-bits; $T_2^{-1} = \Delta\omega^2 \tau_c^2$
$\frac{dM_x}{dt} = -\Delta\omega M_x - \frac{M_x}{T_2}$
$\frac{dM_y}{dt} = \Delta\omega M_x - \omega_1 M_z - \frac{M_y}{T_2}$
$\frac{dM_z}{dt} = + \omega_1 M_y - \frac{M_z}{T_1} + \frac{M_0}{T_1}$
initialize
$T_1^{-1} = \omega_D^2 \{ J(0), J(\omega_0), J(2\omega_0)$


dMx/dt = − ΔωMx − Mx/T2
dMy/dt = ΔωMx − ω1Mz − My/T2
dMz/dt = + ω1My − Mz/T1 + M0/T1
T1^−1 = ω_D^2 { J(0) J(ω0) J(2ω0)


dMx/dt = -ΔωMx — Mx/T2
dMy/dt = ΔωMx - ω1Mz — My/T2
dMz/dt = + ω1My — Mz/T1 + M0/T1
T1^-1 = ωD^2 J(0) J(ω0) J(2ω0)

Errors

T_2

vally w/spins, g** bits; $T_2^{-1} = (\Delta\omega)^2 \tau_c^2$

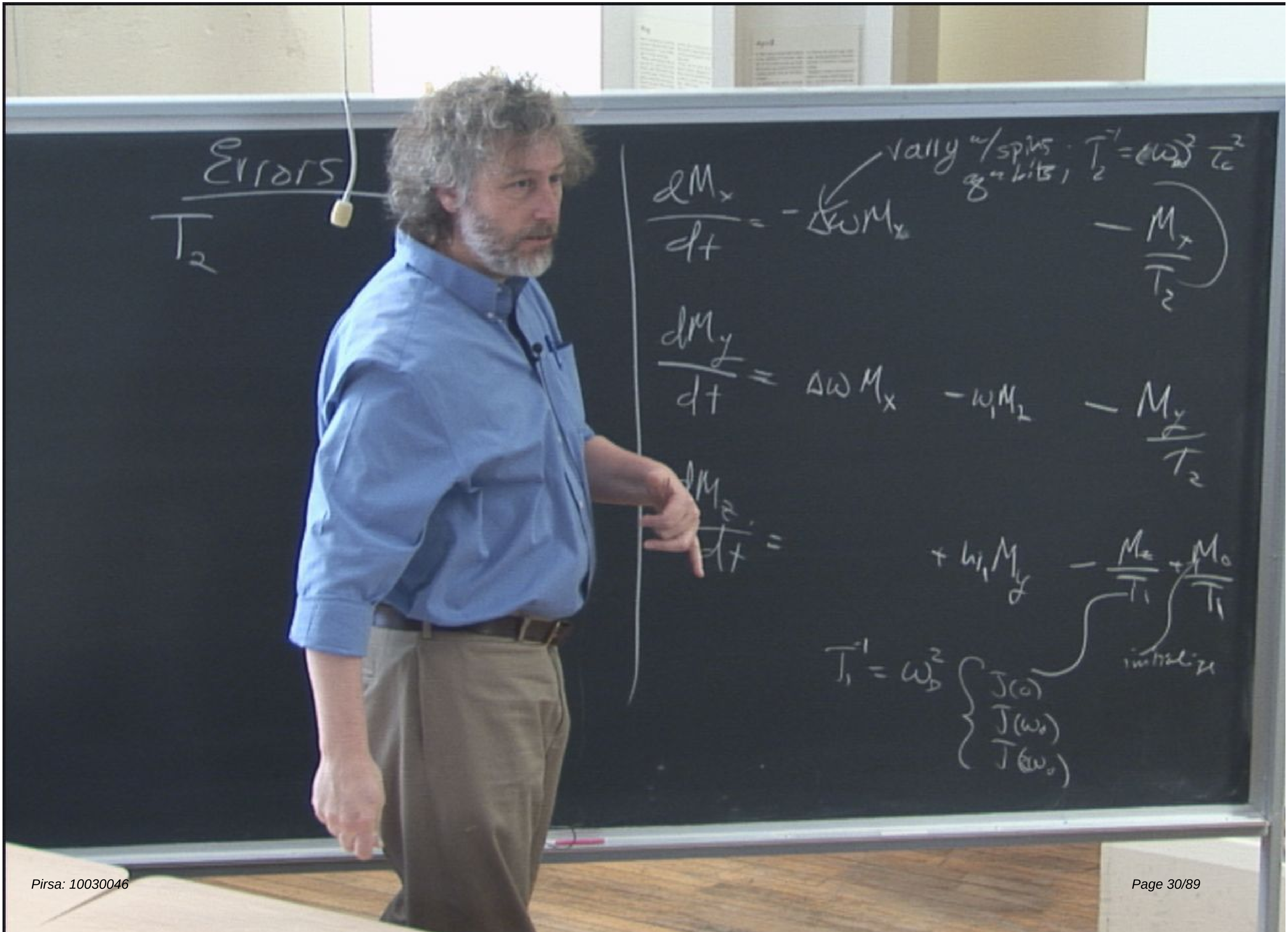
$$\frac{dM_x}{dt} = -\Delta\omega M_x - \frac{M_x}{T_2}$$

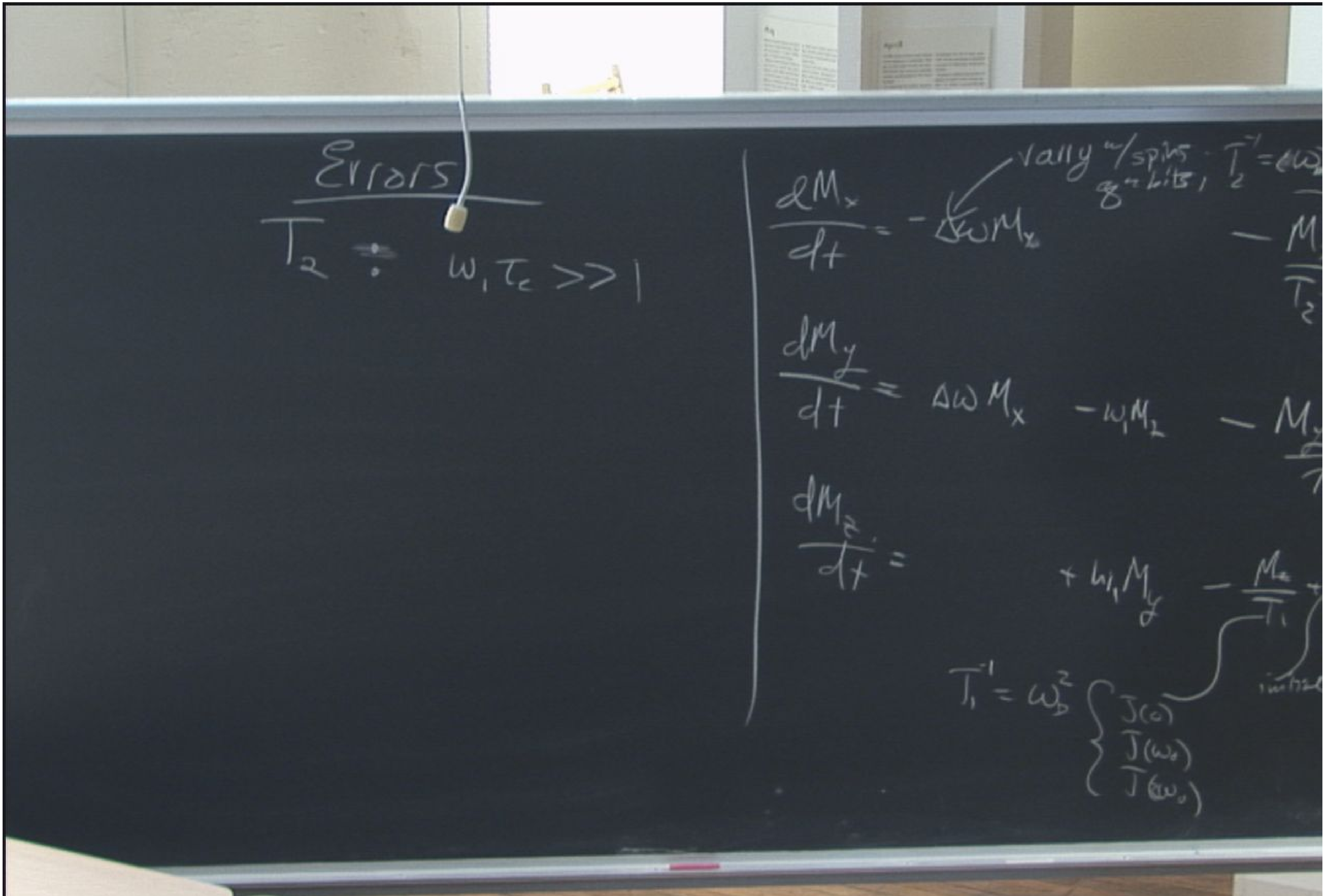
$$\frac{dM_y}{dt} = \Delta\omega M_x - \omega_1 M_z - \frac{M_y}{T_2}$$

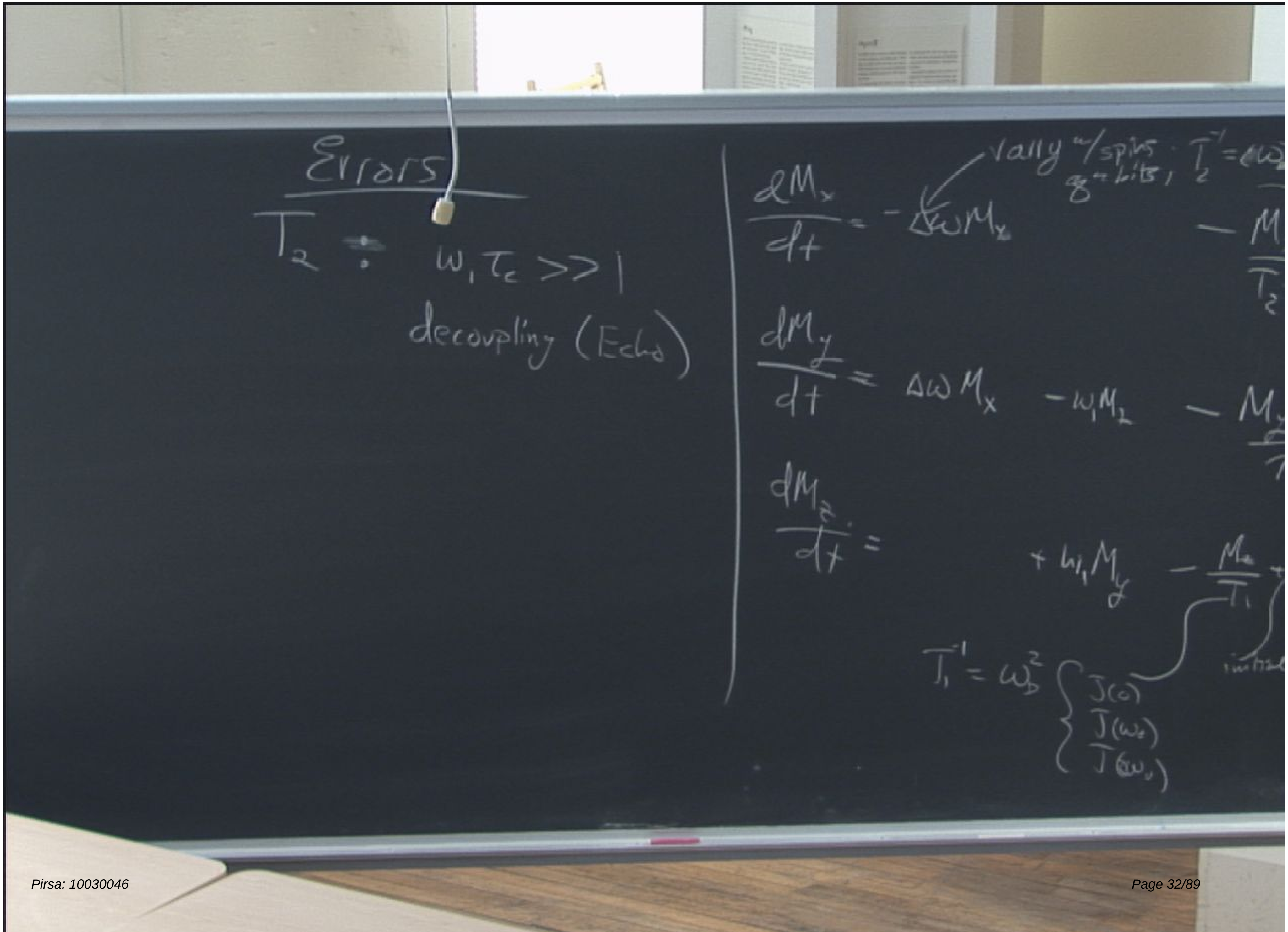
$$\frac{dM_z}{dt} = + \omega_1 M_y - \frac{M_z}{T_1} + \frac{M_0}{T_1}$$

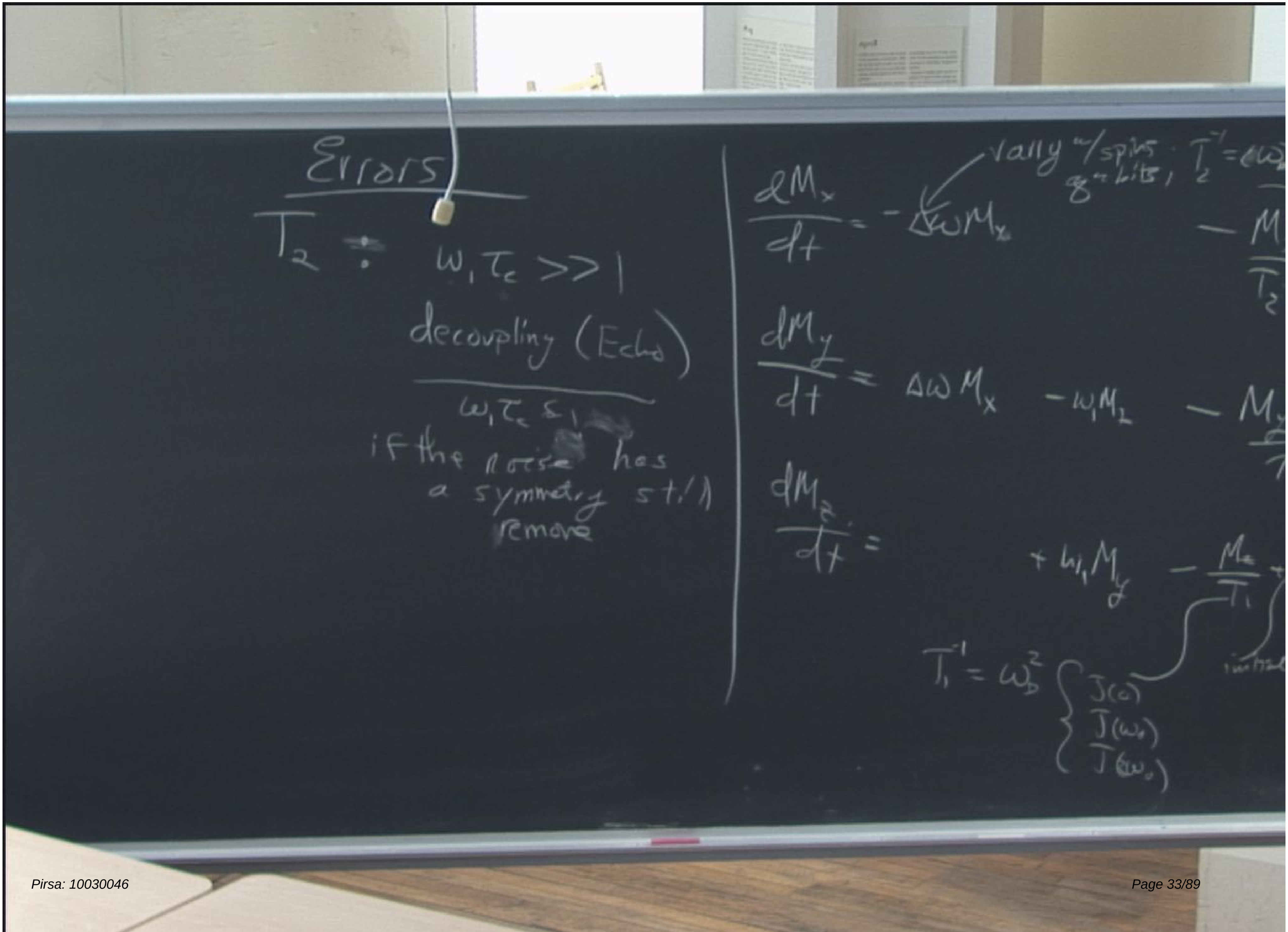
initialize

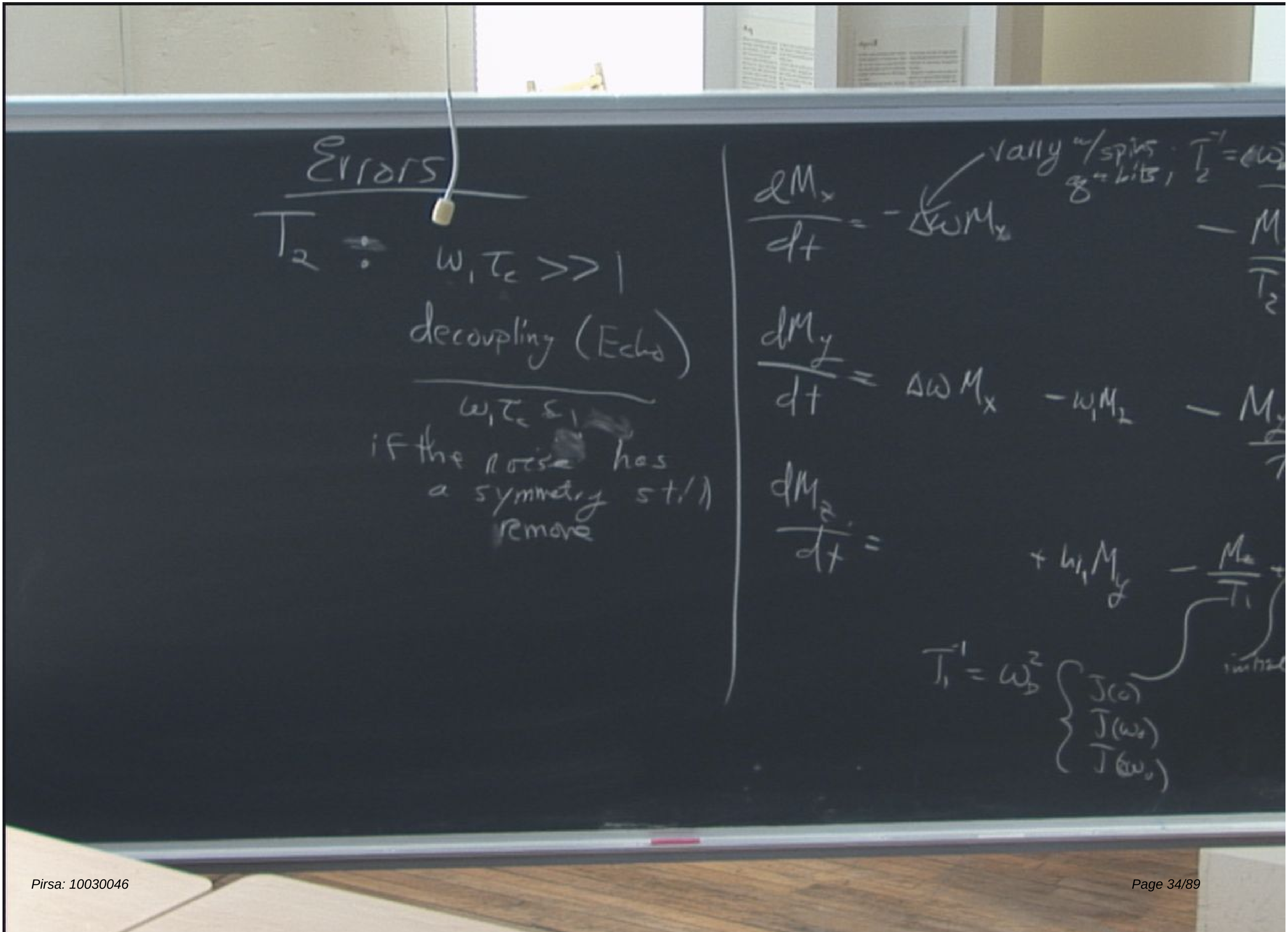
$$T_1^{-1} = \omega_D^2 \begin{cases} J(0) \\ J(\omega_0) \\ J(2\omega_0) \end{cases}$$

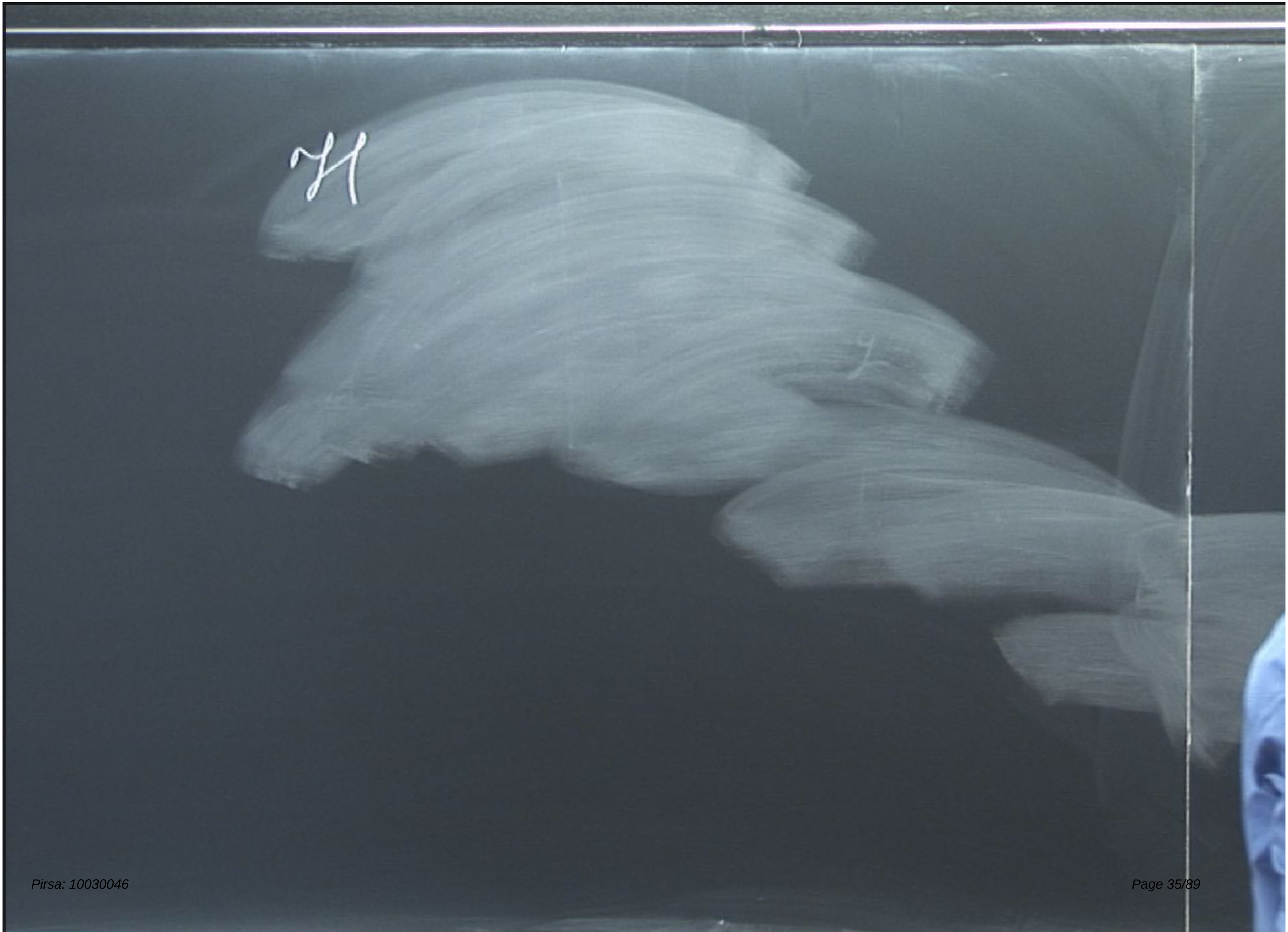

Errors
T2
really w/ spins qubits; T2^-1 = Δω^2 τc^2
dMx/dt = -ΔωMx - Mx/T2
dMy/dt = ΔωMx - ω1Mz - My/T2
dMz/dt = + ω1My - Mz/T1 + M0/T1
T1^-1 = ω^2 { J(0), J(ω0), J(2ω0)
initialize

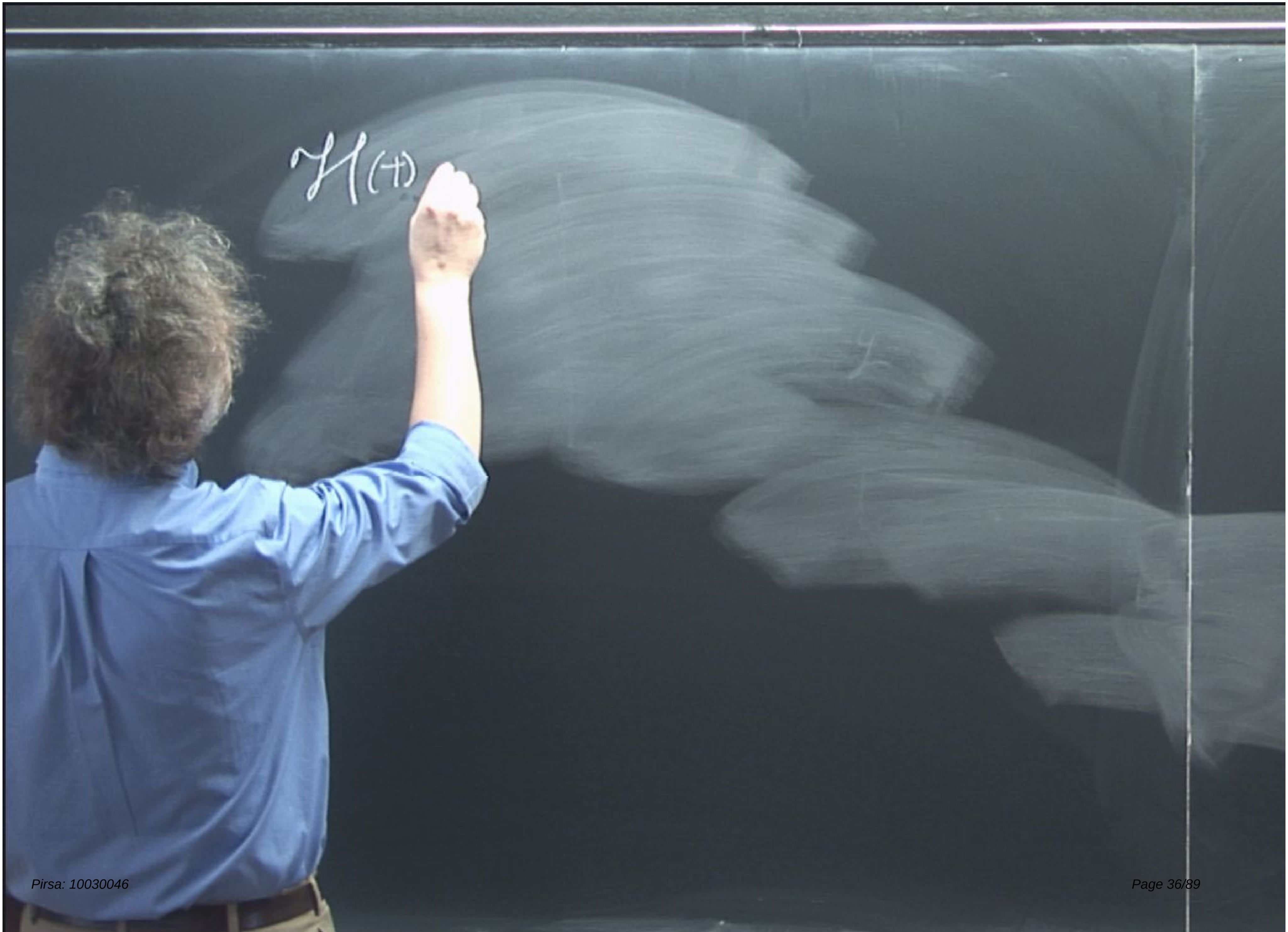

Errors
T_2 : $\omega_1 \tau_c >> 1$
$\frac{dM_x}{dt} = -\Delta\omega M_x$
$\frac{dM_y}{dt} = \Delta\omega M_x - \omega_1 M_z$
$\frac{dM_z}{dt} = +\omega_1 M_y - \frac{M_z}{T_1}$
$T_1^{-1} = \omega_D^2 \{ J(0), J(\omega_0), J(2\omega_0)$

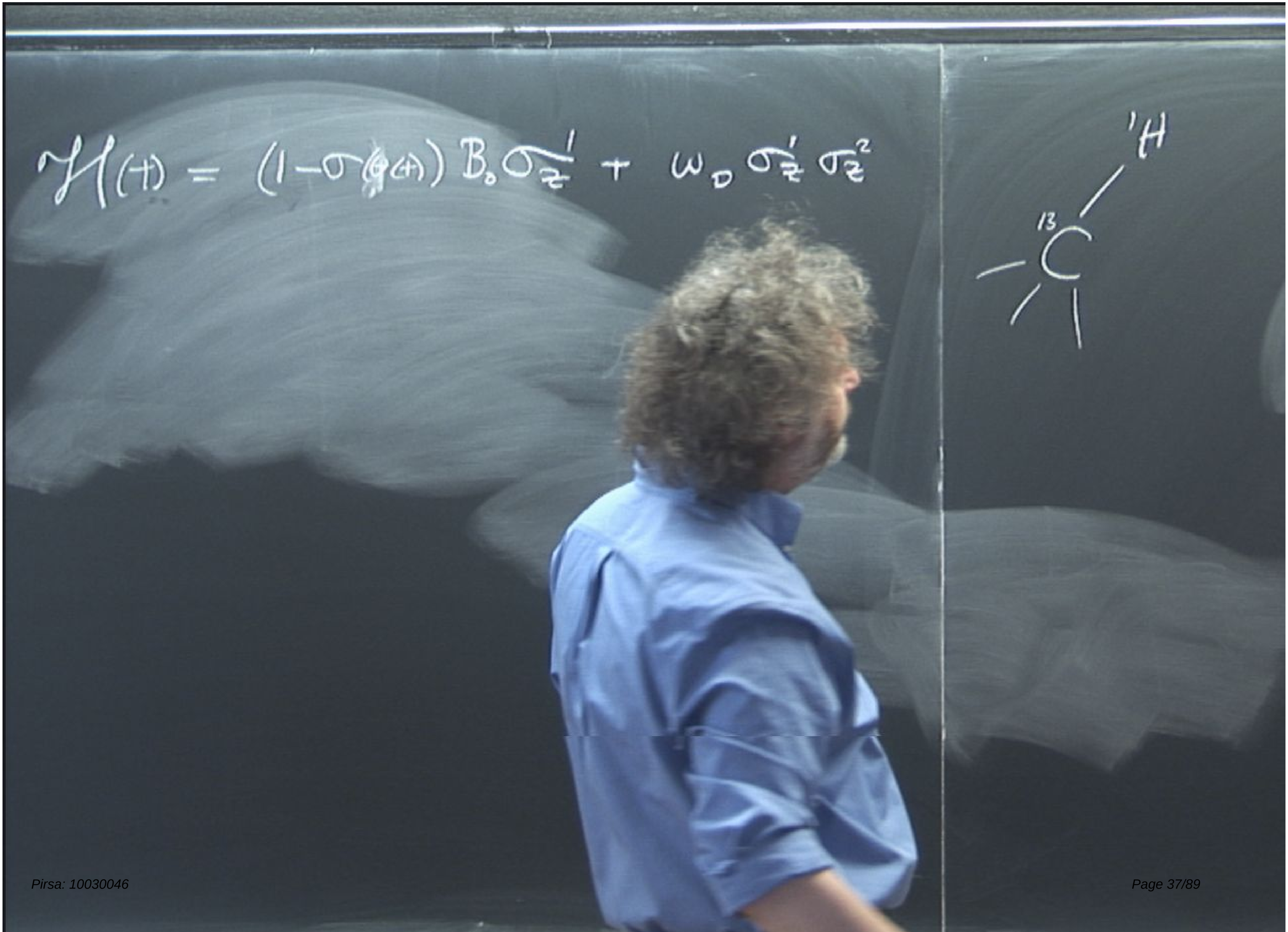

Errors
T2 : ω1 τc >> 1
decoupling (Echo)
dMx/dt = −ΔωMx
dMy/dt = ΔωMx − ω1Mz
dMz/dt = + ω1My − Mz/T1
vary w/ spins, qubits
T1^-1 = ωD^2
J(0)
J(ωz)
J(ω0)


Errors
T2 : ω1 τc >> 1
decoupling (Echo)
ω1 τc ≲ 1
if the noise has
a symmetry still
remove
dMx/dt = − ΔωMx − Mx/T2
dMy/dt = ΔωMx − ω1Mz − My
dMz/dt = + ω1My − Mz/T1
vary w/spins
g+ bits
T2^−1 = ω
T1^−1 = ωD^2
J(0)
J(ω1)
J(ω0)

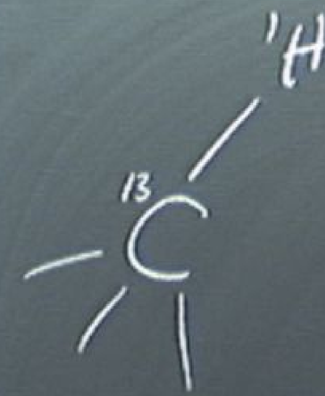

Errors
T2 : ω1 τc >> 1
decoupling (Echo)
ω1 τc ≤ 1
if the noise has
a symmetry st/λ
remove
dMx/dt = − Δω Mx
dMy/dt = Δω Mx − ω1 Mz
dMz/dt = + ω1 My − Mz/T1
T1^−1 = ωD^2 { J(0) J(ωz) J(ω0)
vary w/ spins


H

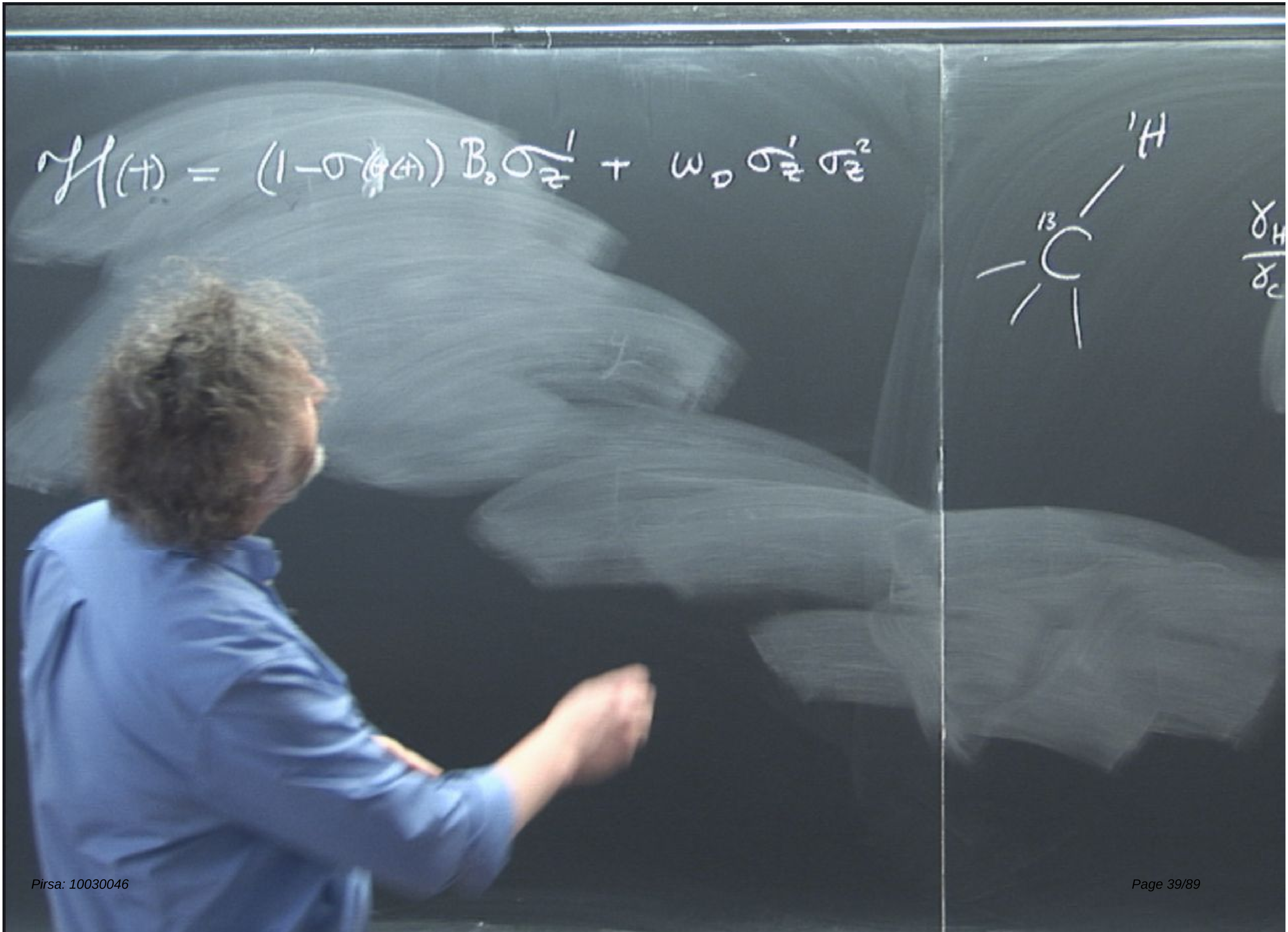


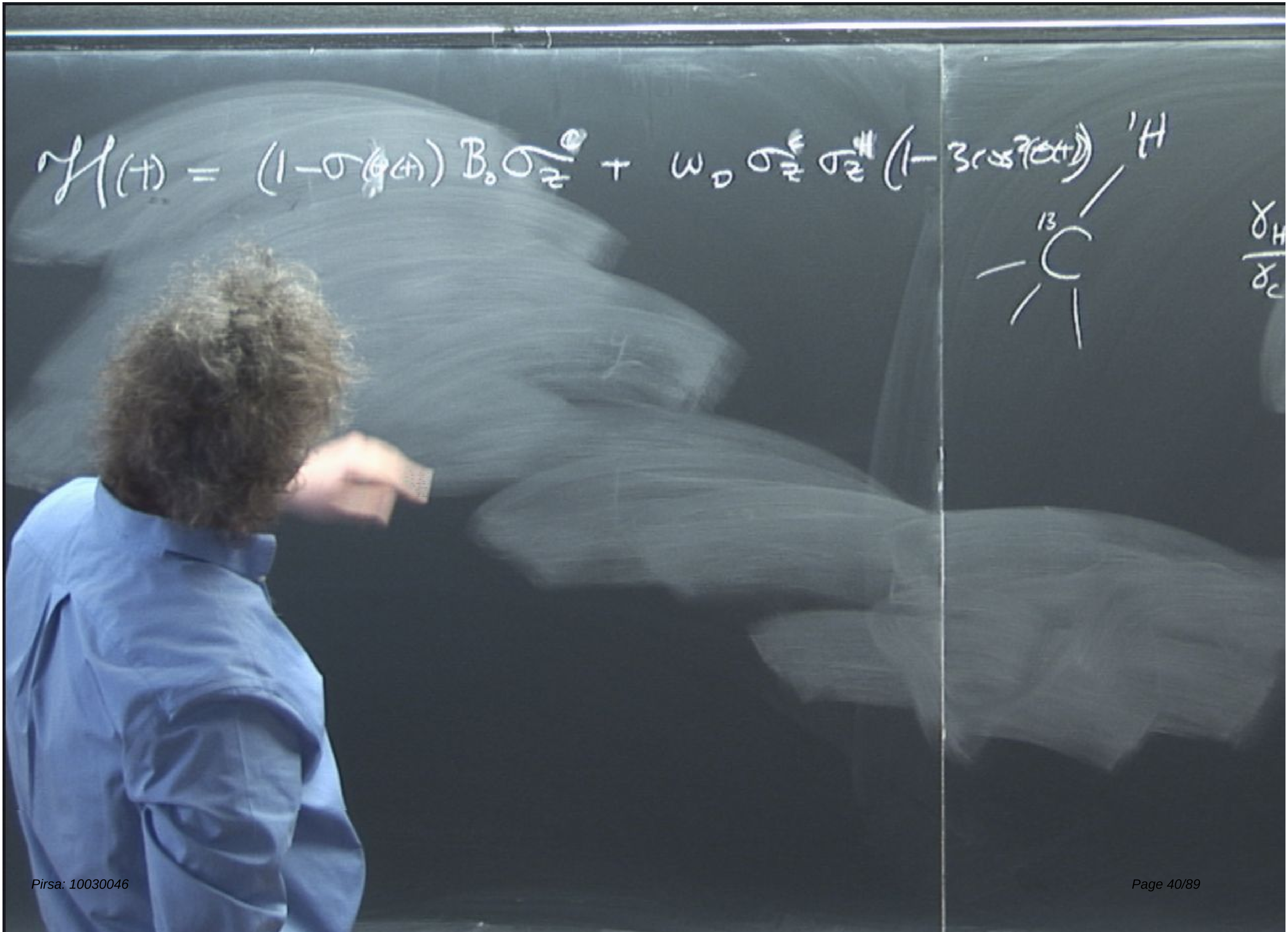

$\mathcal{H}(t) = (1-\sigma(\theta(t)))\, B_0 \sigma_z^1 + \omega_D \sigma_z^1 \sigma_z^2$
^{1}H
^{13}C

$$\mathcal{H}(t) = (1-\sigma(\theta(t)))\, B_0\, \sigma_z^1 + \omega_D\, \sigma_z^1 \sigma_z^2$$

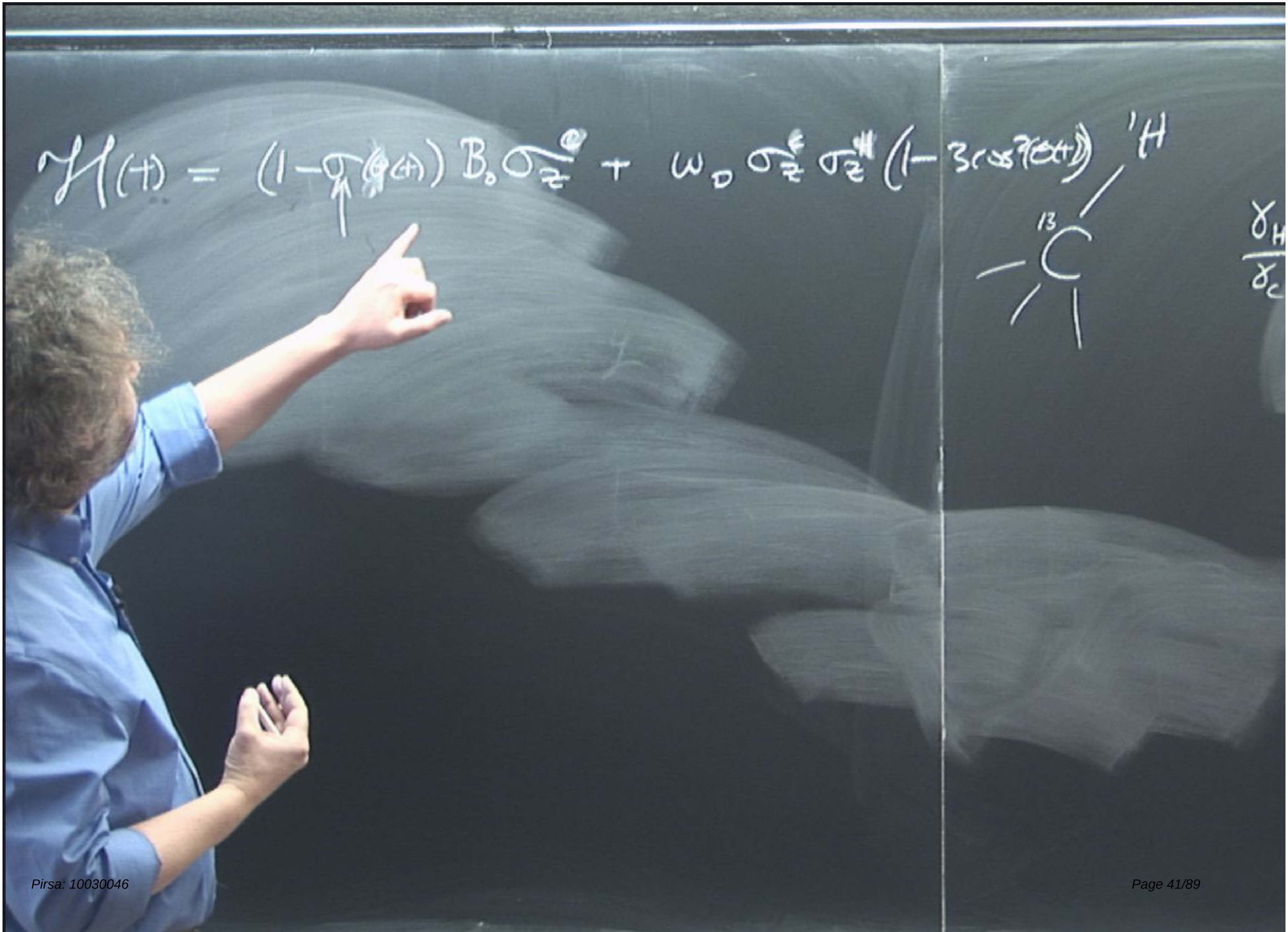


$$\frac{\gamma_H}{\gamma_C}$$

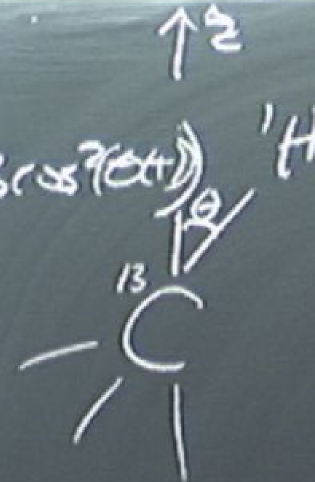

$\mathcal{H}(t) = (1-\sigma(\theta(t)))B_0\sigma_z^1 + \omega_D\sigma_z^1\sigma_z^2$
^{13}C
^{1}H



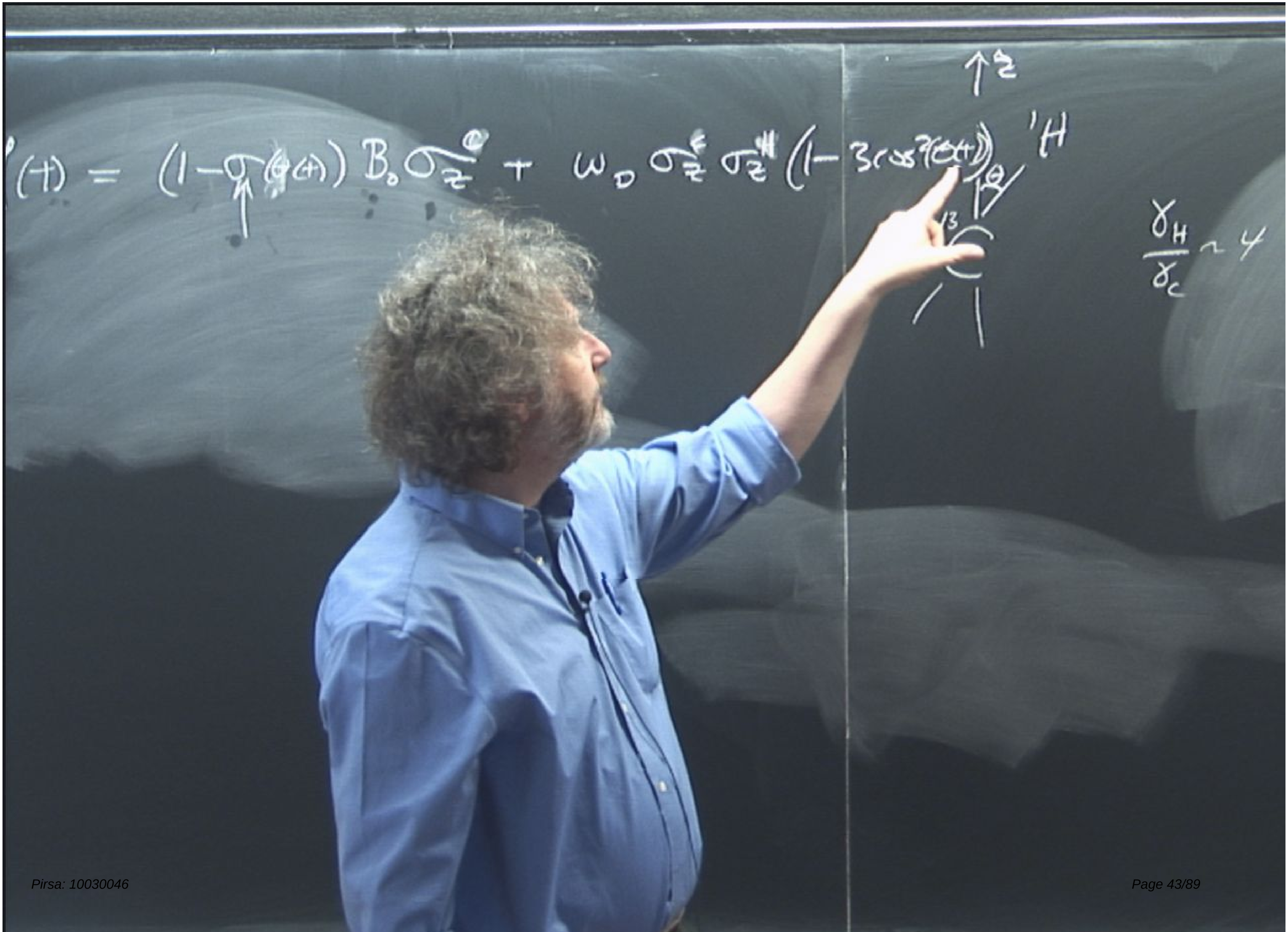
$\mathcal{H}(t) = (1-\sigma(\theta(t)))B_0\sigma_z^C + \omega_D\sigma_z^C\sigma_z^H(1-3\cos^2(\theta(t)))$
^1H
^{13}C
$\frac{\gamma_H}{\gamma_C}$


$\mathcal{H}(t) = (1-\sigma(\theta(t)))B_0\sigma_z^C + \omega_D\sigma_z^C\sigma_z^H(1-3\cos^2\theta(t))$
^1H
^{13}C

$$\mathcal{H}(t) = (1-\sigma(\theta(t)))\, B_0 \sigma_z^{C} + \omega_D \sigma_z^{C} \sigma_z^{H} (1 - 3\cos^2\theta(t))$$

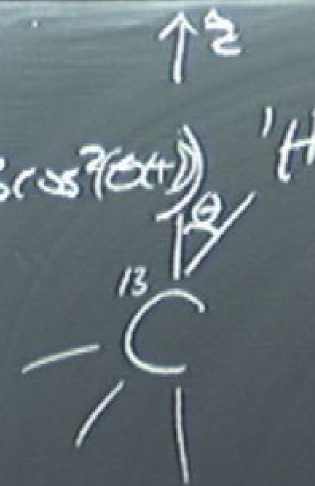


$\frac{\gamma_H}{\gamma_C}$



'H
13
$\frac{\gamma_H}{\gamma_C} \sim 4$

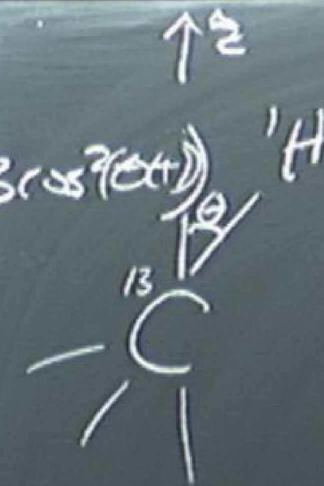
$$(t) = (1-\sigma(\theta(t)))B_0\sigma_z^c + \omega_D\sigma_z^c\sigma_z^H(1-3\cos^2(\theta(t)))$$



$$\frac{\gamma_H}{\gamma_C}\sim 4$$

$$= |\sigma|(1-3\cos^2\theta(t))B_0\sigma_z^c + \omega_D(1-3\cos^2(\theta(t)))\sigma_z^c\sigma_z^H$$

$$(t) = (1-\sigma(\theta(t)))\, B_0\, \sigma_z^c + \omega_D\, \sigma_z^c\, \sigma_z^H \left(1 - 3\cos^2(\theta(t))\right)$$



$$\frac{\gamma_H}{\gamma_C} \sim 4$$

$$= |\sigma| \left(1-3\cos^2\theta(t)\right) B_0\, \sigma_z^c\, \mathbb{1}^H + \omega_D \left(1-3\cos^2(\theta(t))\right) \sigma_z^c\, \sigma_z^H$$

$E_+^H + E_-^H$

$(E_+^H - E_-$

$$\mathcal{H}(t) = \left(\left(1 - \sigma(\theta(t))\right) B_0\right) \sigma_z^c + \omega_D \sigma_z^c \sigma_z^H \left(1 - 3\cos^2(\theta(t))\right)$$

ω_0

$\uparrow z$

${}^1H$

θ

${}^{13}C$

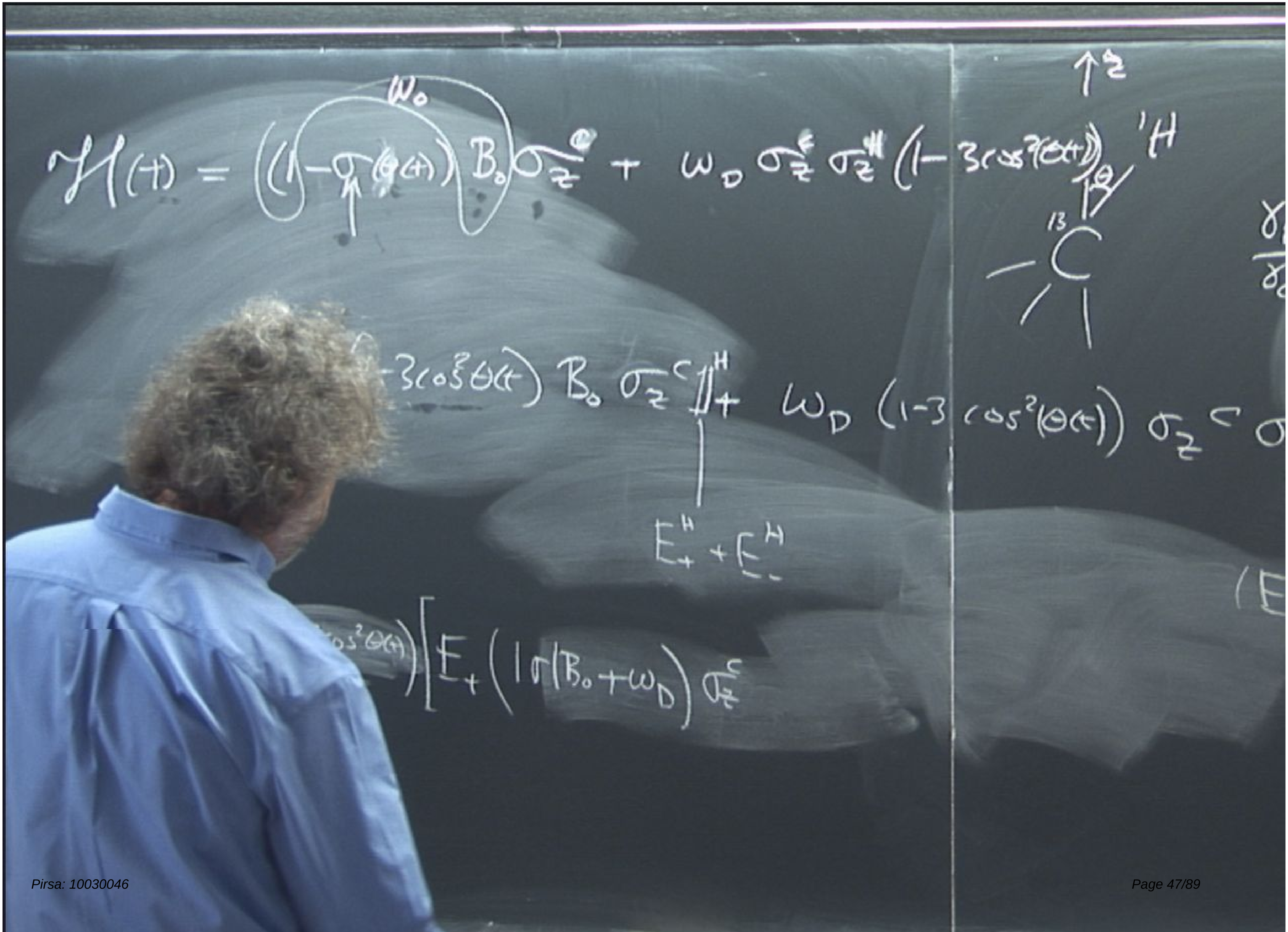
$$= |\sigma| \left(1 - 3\cos^2\theta(t)\right) B_0 \sigma_z^c \mathbb{1}^H + \omega_D \left(1-3\cos^2(\theta(t))\right) \sigma_z^c \sigma$$

$E_+^H + E_-^H$

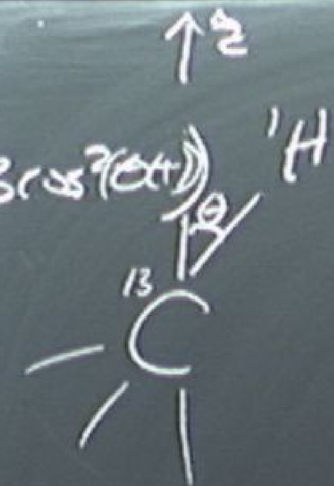
γ

γ

$(E$


$\mathcal{H}(t) = ((1 - \sigma(\theta(t))) B_0 \sigma_z^c + \omega_D \sigma_z^c \sigma_z^H (1 - 3\cos^2(\theta(t)))$
ω_0
^1H
^{13}C
θ
z
$3\cos^2\theta(t)) B_0 \sigma_z^c \mathbb{1}^H + \omega_D (1-3\cos^2(\theta(t))) \sigma_z^c$
$E_+^H + E_-^H$
$\cos^2\theta(t))[E_+ (1\sigma(B_0 + \omega_D) \sigma_z^c$

$$\mathcal{H}(t) = \overbrace{\left((1-\sigma(\theta(t))\right) B_0}^{\omega_0} \sigma_z^{c} + \omega_D \sigma_z^{c} \sigma_z^{H} \left(1 - 3\cos^2(\theta(t))\right)$$



$$= |\sigma| \left(1 - 3\cos^2\theta(t)\right) B_0 \sigma_z^{c} \, \mathbb{1}^{H} + \omega_D \left(1-3\cos^2(\theta(t))\right) \sigma_z^{c} \sigma$$

$$\mathbb{1}^{H} = E_+^{H} + E_-^{H}$$

$$\mathcal{H}(t) = \left(1-3\cos^2\theta(t)\right)\left[E_+\left(|\sigma| B_0 + \omega_D\right)\sigma_z^{c} + E_-\left(|\sigma| B_0 - \omega_D\right)\sigma_z^{c}\right]$$

$\frac{\gamma}{\gamma}$ $(E$

$\uparrow z$

$$\mathcal{H}(t) = \left((1-\sigma(\theta(t))\right) B_0 \sigma_z^{c} + \omega_D \sigma_z^{c} \sigma_z^{H} \left(1 - 3\cos^2(\theta(t))\right)$$

^1H

θ

^{13}C

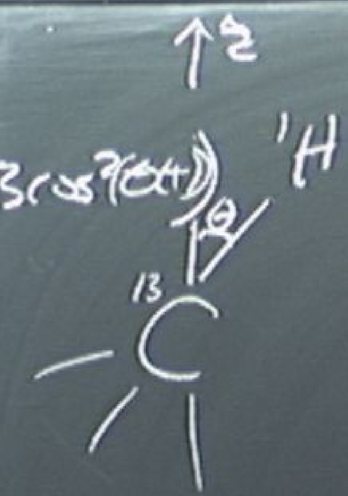
$$= |\sigma| \left(1-3\cos^2\theta(t)\right) B_0 \sigma_z^{c} \mathbb{1}^{H} + \omega_D \left(1-3\cos^2(\theta(t))\right) \sigma_z^{c} \sigma$$

$E_+^H + E_-^H$

$$\mathcal{H}(t) = \left(1-3\cos^2\theta(t)\right)\left[E_+\left(|\sigma| B_0 + \omega_D\right)\sigma_z^{c} + E_-\left(|\sigma| B_0 - \omega_D\right)\sigma_z^{c}\right]$$

θ^0

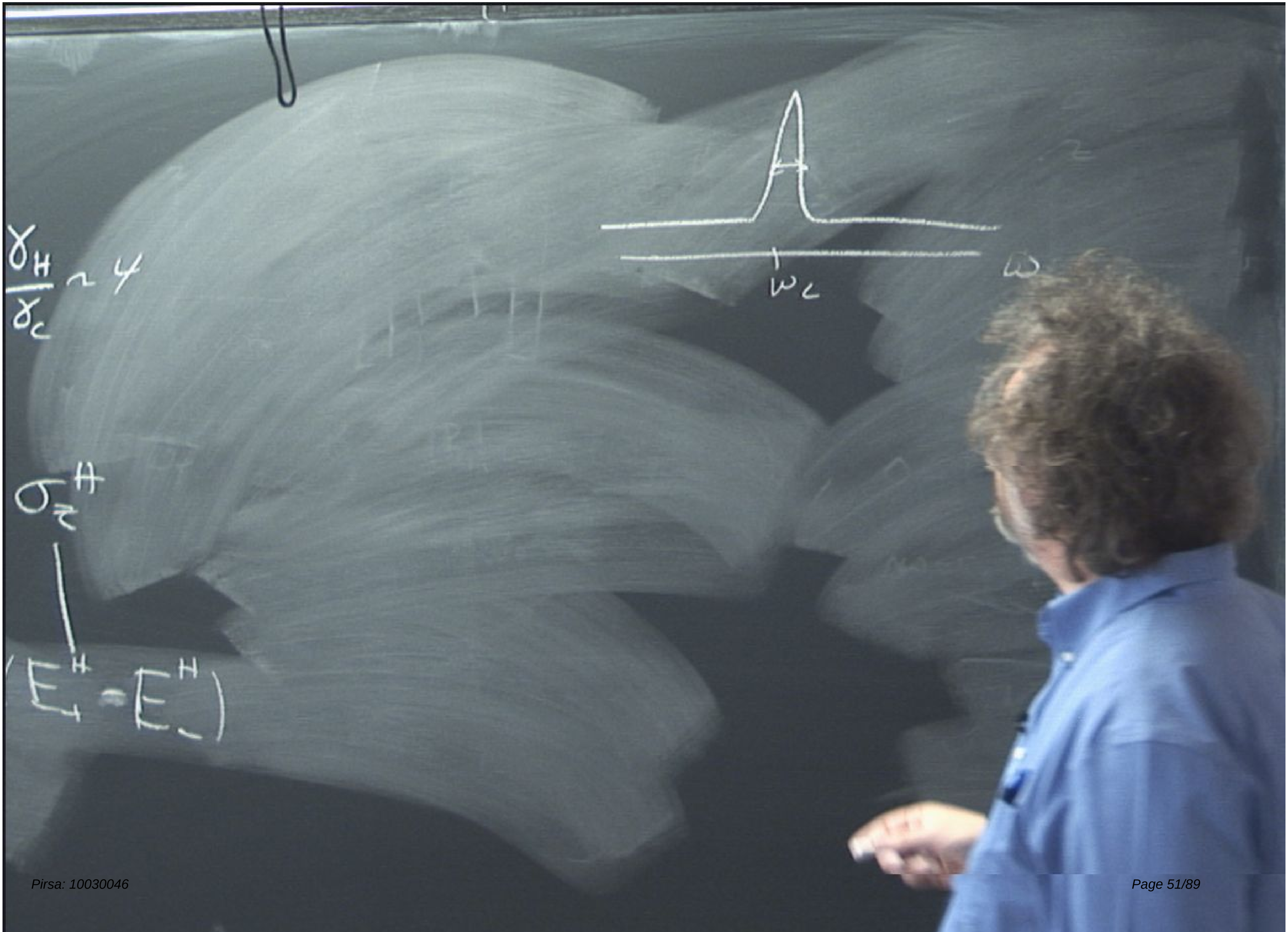
$$\mathcal{H}(t) = \Big(\overbrace{\big(1-\sigma(\theta(t))\big) B_0}^{\omega_0}\Big)\sigma_z^C + \omega_D \sigma_z^C \sigma_z^H \big(1-3\cos^2(\theta(t))\big)$$

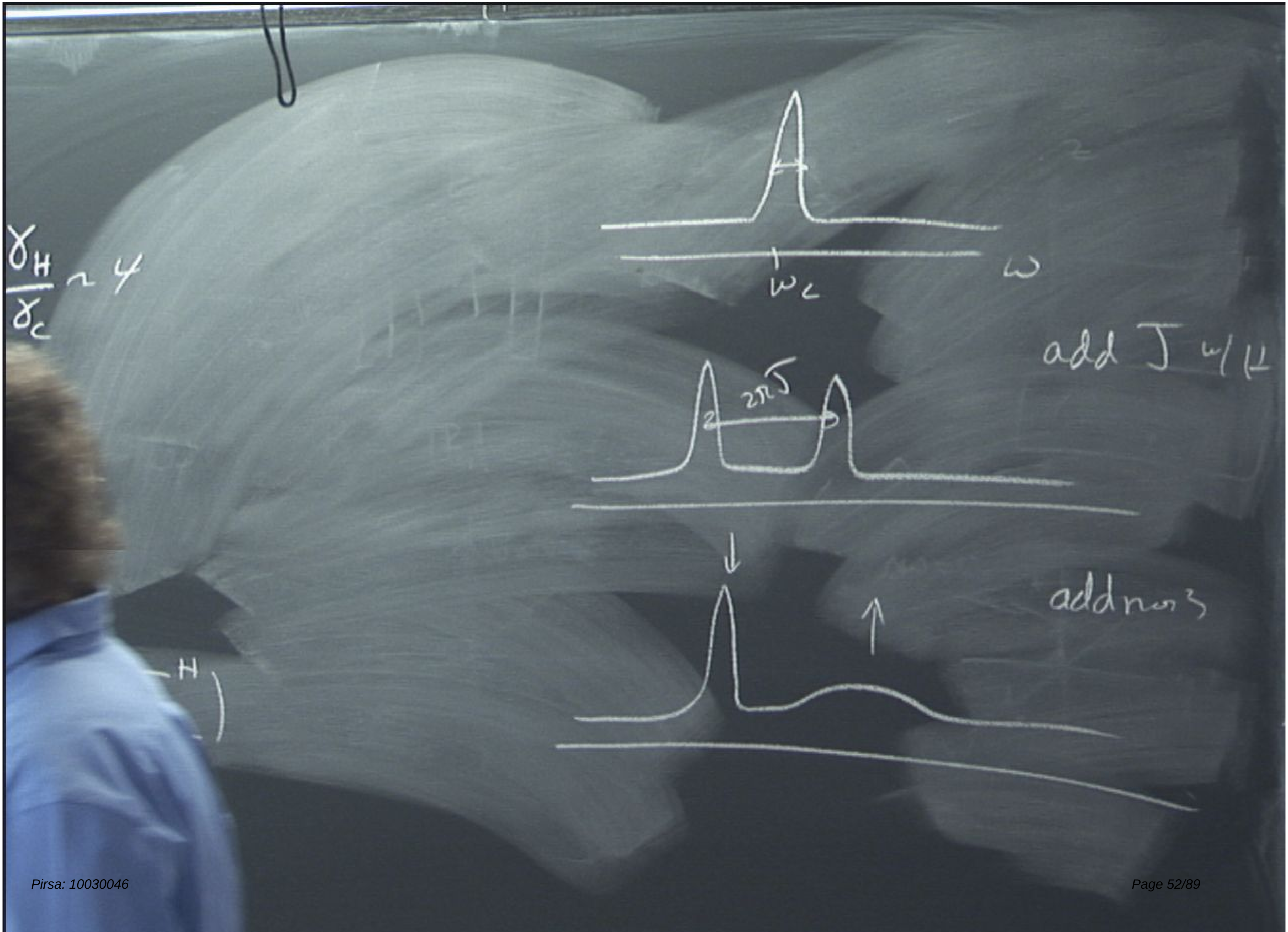


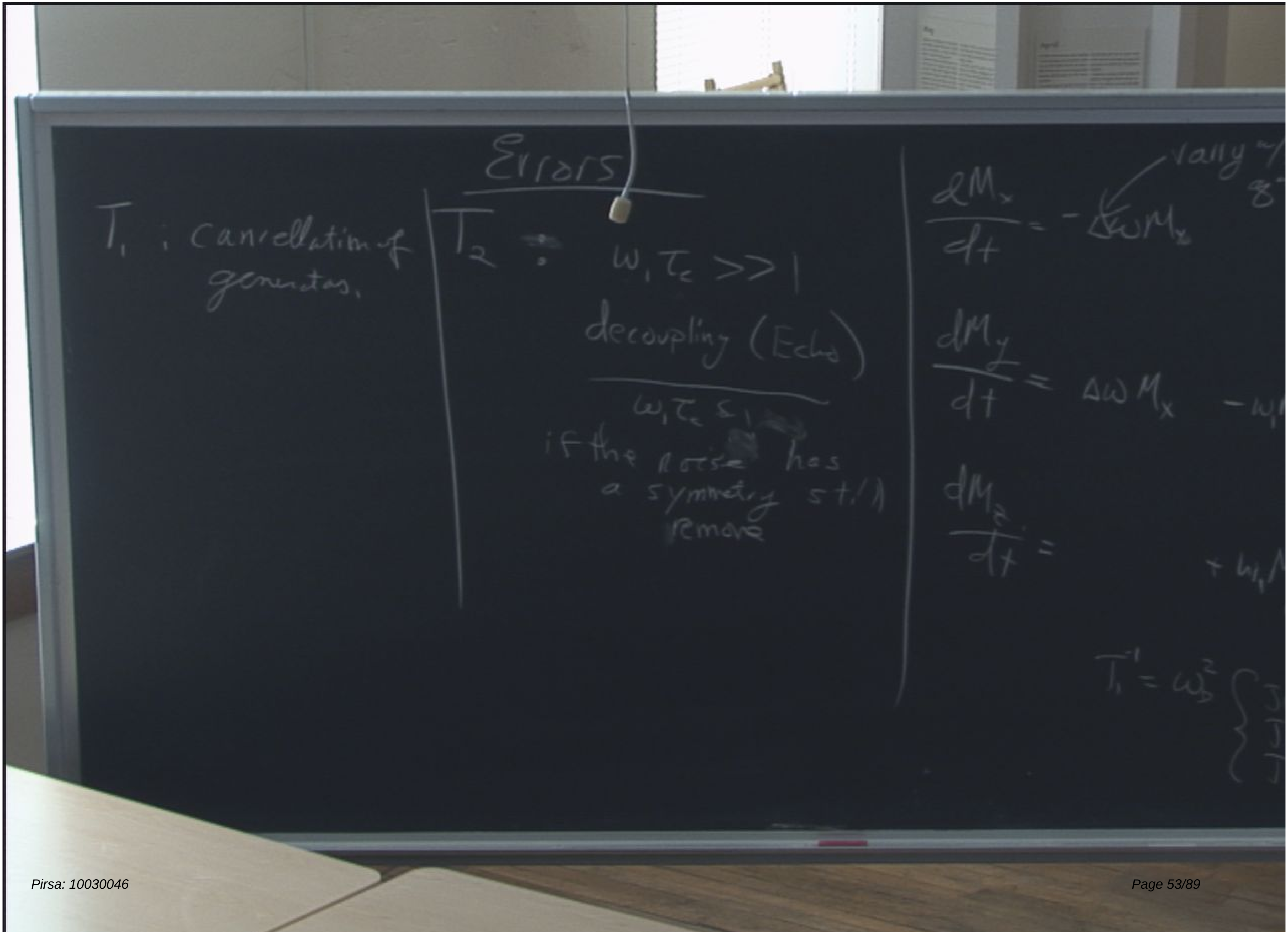
$$= |\sigma|\big(1-3\cos^2\theta(t)\big) B_0 \sigma_z^C \mathbb{1}^H + \omega_D \big(1-3\cos^2(\theta(t))\big) \sigma_z^C \sigma$$

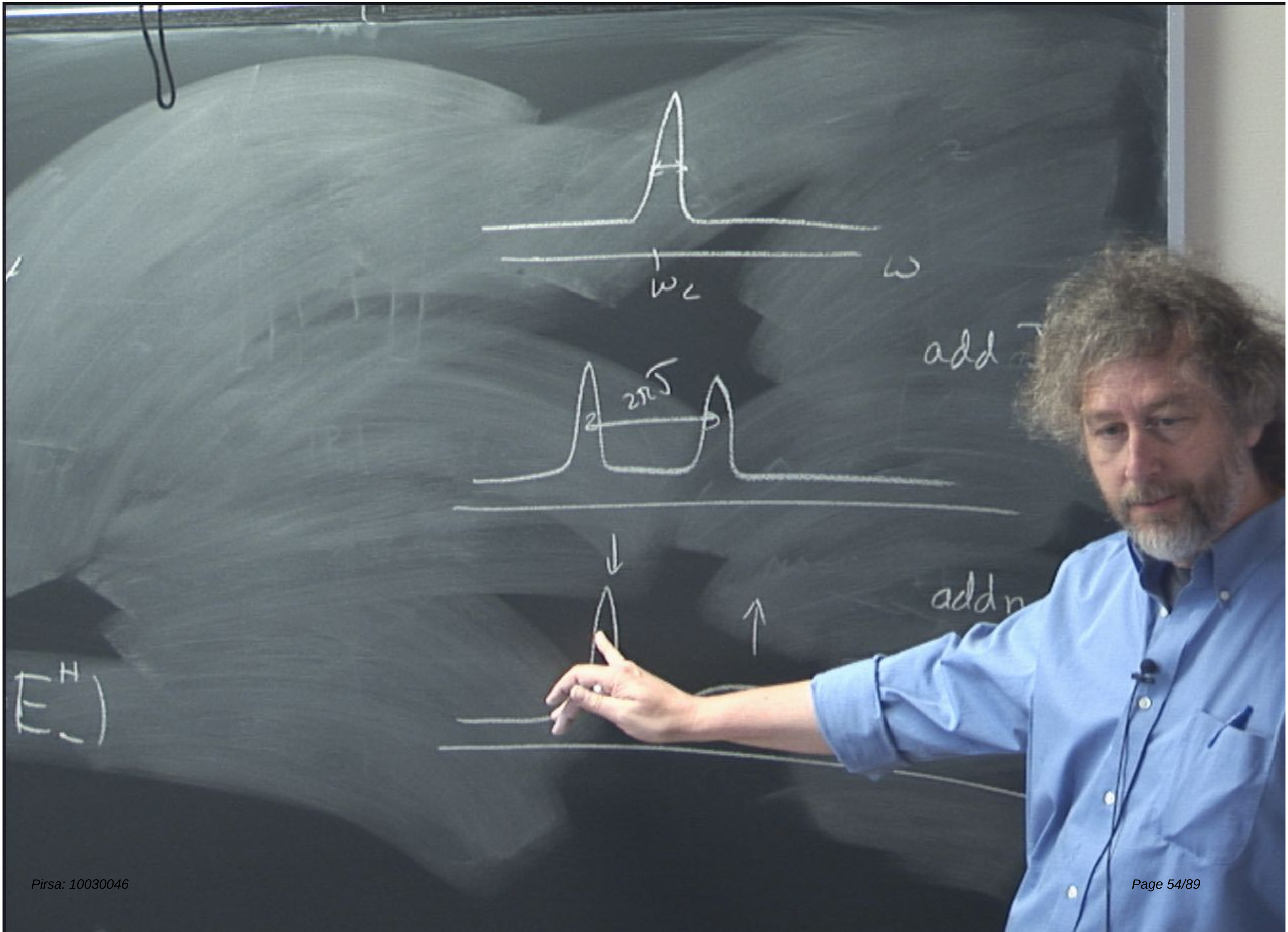
$$\mathbb{1}^H = E_+^H + E_-^H$$

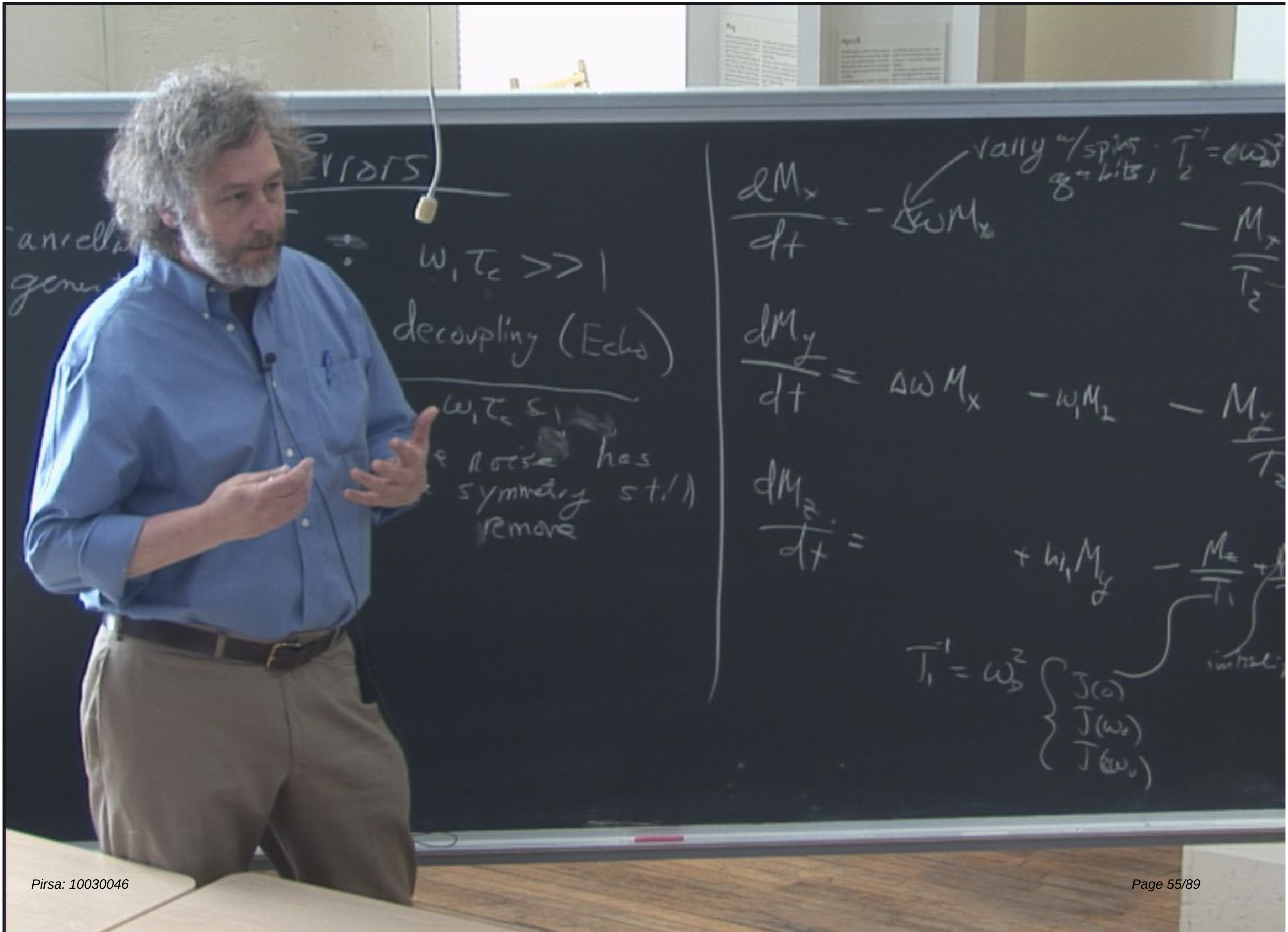
$$\mathcal{H}(t) = \big(1-3\cos^2\theta(t)\big)\Big[E_+\big(|\sigma|B_0+\omega_D\big)\sigma_z^C + E_-\big(|\sigma|B_0-\omega_D\big)\sigma_z^C\Big]$$


$\gamma_H / \gamma_C \sim 4$
σ_z^H
$(E_+^H - E_-^H)$
ω_c
ω


γH/γC ~ 4
ωc
ω
add J w/ H
2πJ
add noise


Errors
T1 : cancellation of generators.
T2 : ω1 τc >> 1
decoupling (Echo)
ω1 τc ≲ 1
if the noise has a symmetry st/ remove
dMx/dt = -ΔωMx
dMy/dt = ΔωMx
dMz/dt =


ω_L
ω
add
$2\pi J$
add n
$E^H_-)$


Errors
$\omega_1 \tau_c \gg 1$
decoupling (Echo)
$\omega_1 \tau_c \lesssim 1$
noise has
symmetry still
remove
$\frac{dM_x}{dt} = -\Delta\omega M_x$
valley w/spins
T_2^{-1}
$\frac{dM_y}{dt} = \Delta\omega M_x - \omega_1 M_z$
$\frac{dM_z}{dt} = + \omega_1 M_y - \frac{M_z}{T_1}$
$T_1^{-1} = \omega_b^2$
$J(0)$
$J(\omega_0)$

cancellation of generators.

Errors

T_2 : $\omega_1 \tau_c >> 1$

decoupling (Echo)

$\omega_1 \tau_c \lesssim 1$

if the noise has a symmetry still remove

vally w/spins g-bits, $T_2^{-1} = \dots$

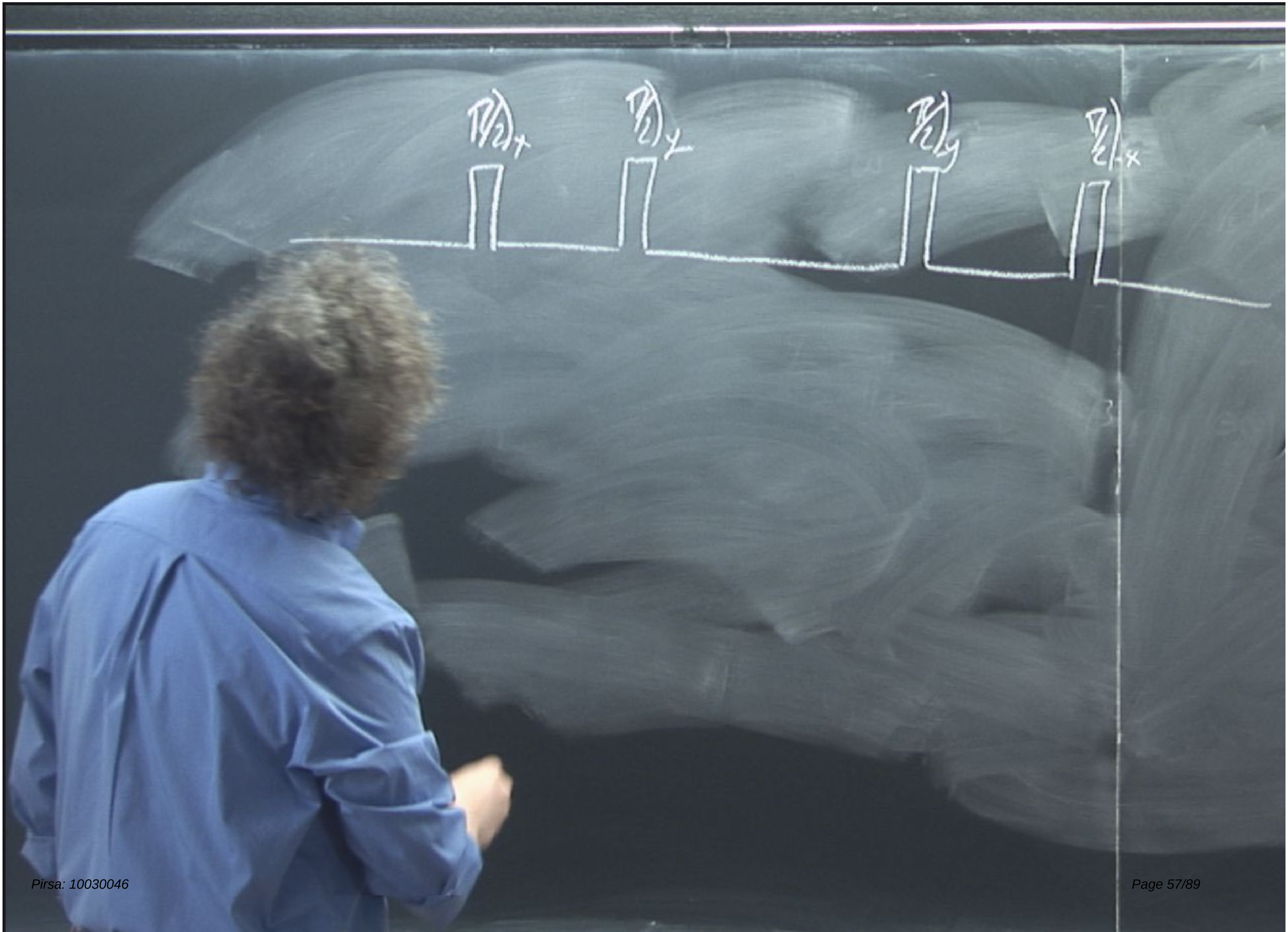
$$\frac{dM_x}{dt} = -\Delta\omega M_x \qquad - \frac{M_x}{T_2}$$

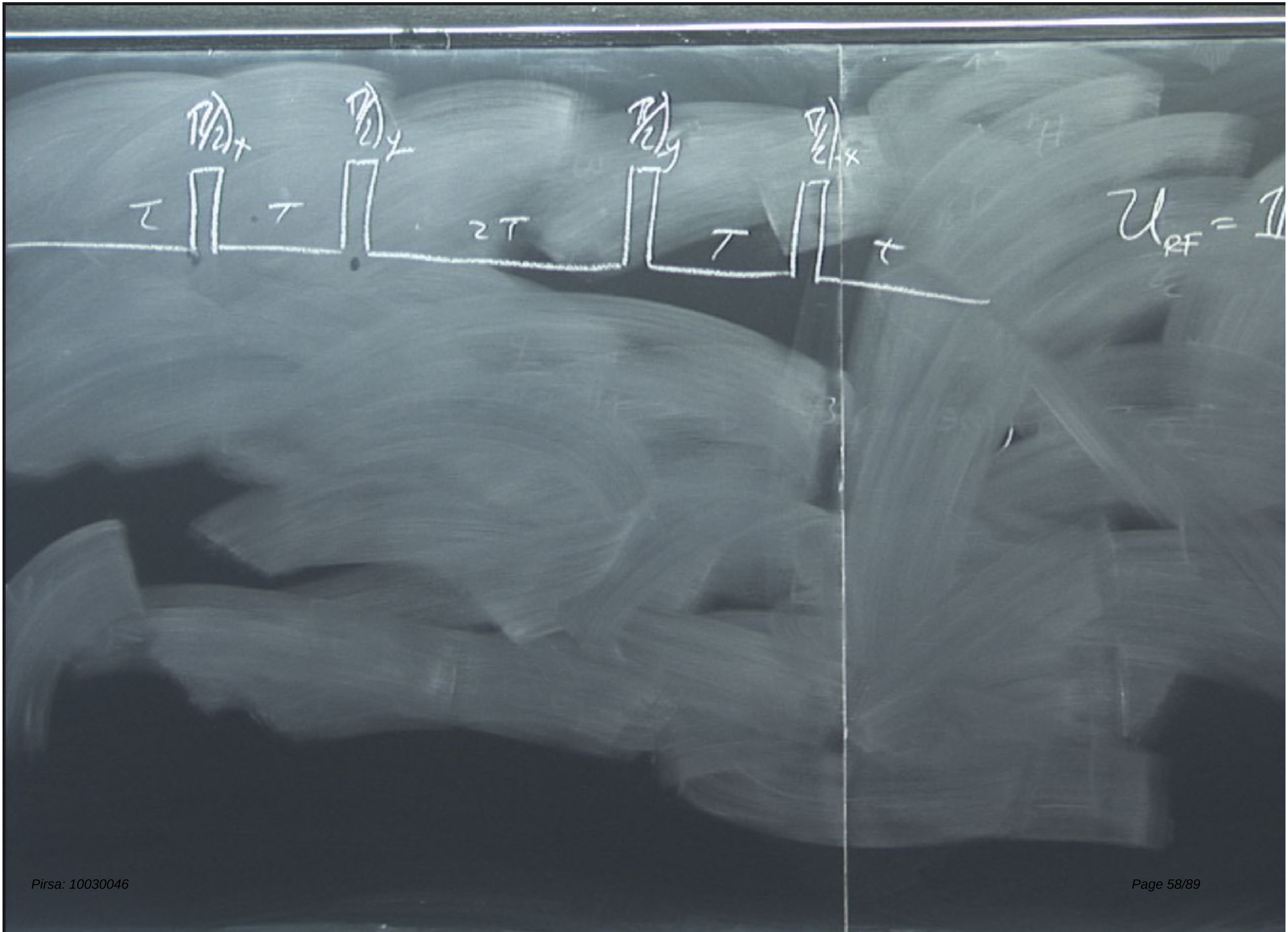
$$\frac{dM_y}{dt} = \Delta\omega M_x \quad - \omega_1 M_z \quad - \frac{M_y}{T_2}$$

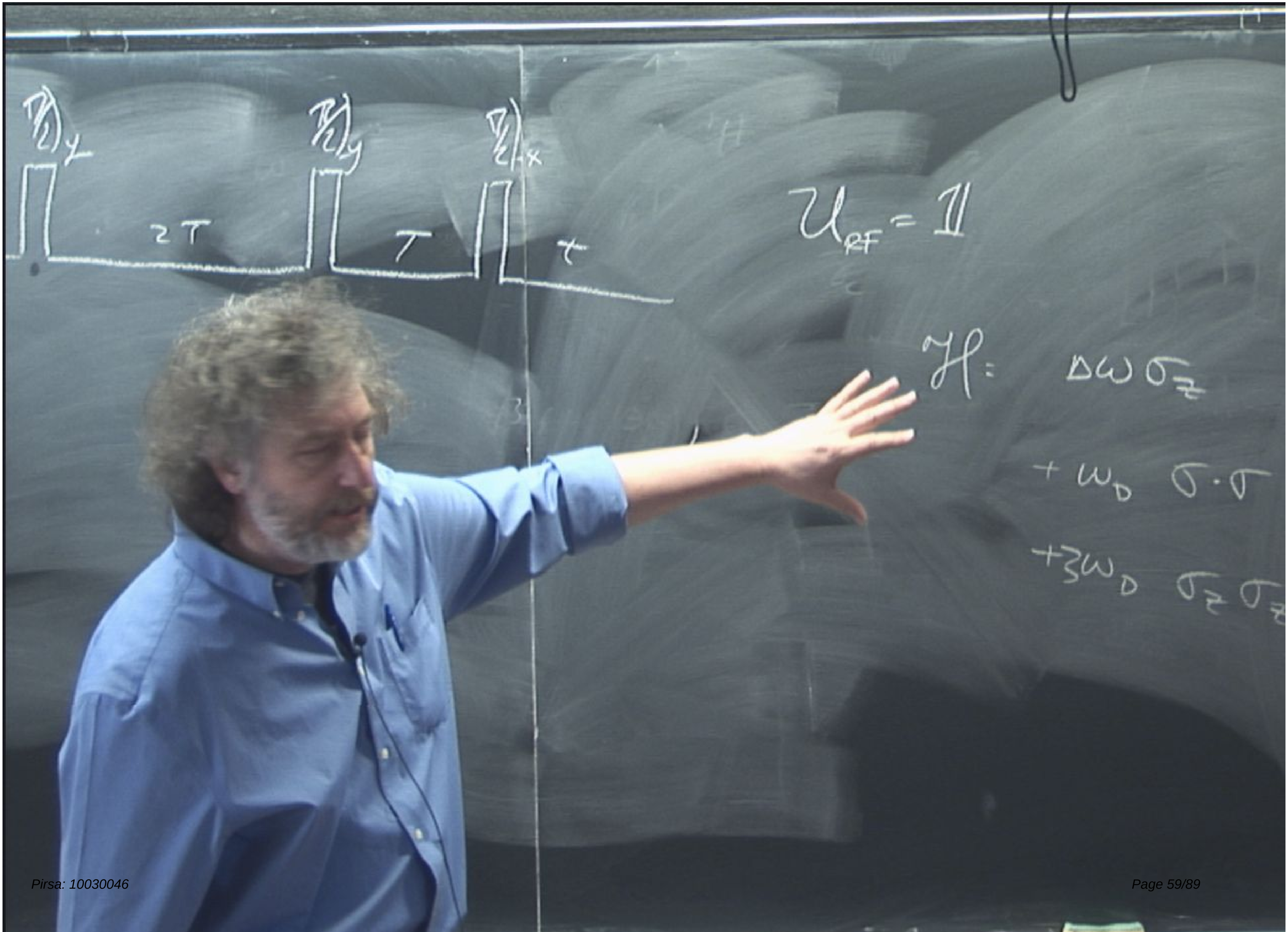
$$\frac{dM_z}{dt} = \qquad + \omega_1 M_y \quad - \frac{M_z}{T_1} + \dots$$

$$T_1^{-1} = \omega_D^2 \left\{ \begin{array}{l} J(0) \\ J(\omega_0) \\ J(2\omega_0) \end{array} \right)$$

initial


π/2)x
π/2)y
π/2)y
π/2)-x


τ
π/2)x
τ
π/2)y
2τ
π/2)y
τ
π/2)-x
τ
U_RF = 1


π/2)y
2T
π/2)y
T
π/2)-x
t
U_RF = 1
H: Δω σz
+ ωD σ·σ
+3ωD σz σz

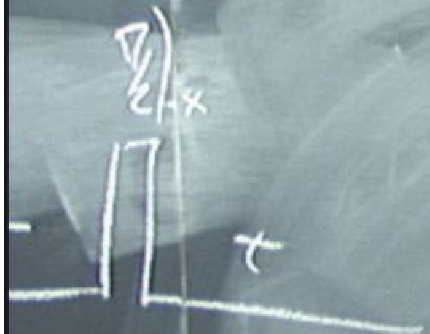
$$\mathcal{U}_{RF} = \mathbb{1}$$

$$\mathcal{H}: \quad \Delta\omega\,\sigma_z \quad \equiv \text{chemical shift}$$

$$+\,\omega_D\,\sigma\cdot\sigma$$

$$+\,3\omega_D\,\sigma_z\sigma_z$$

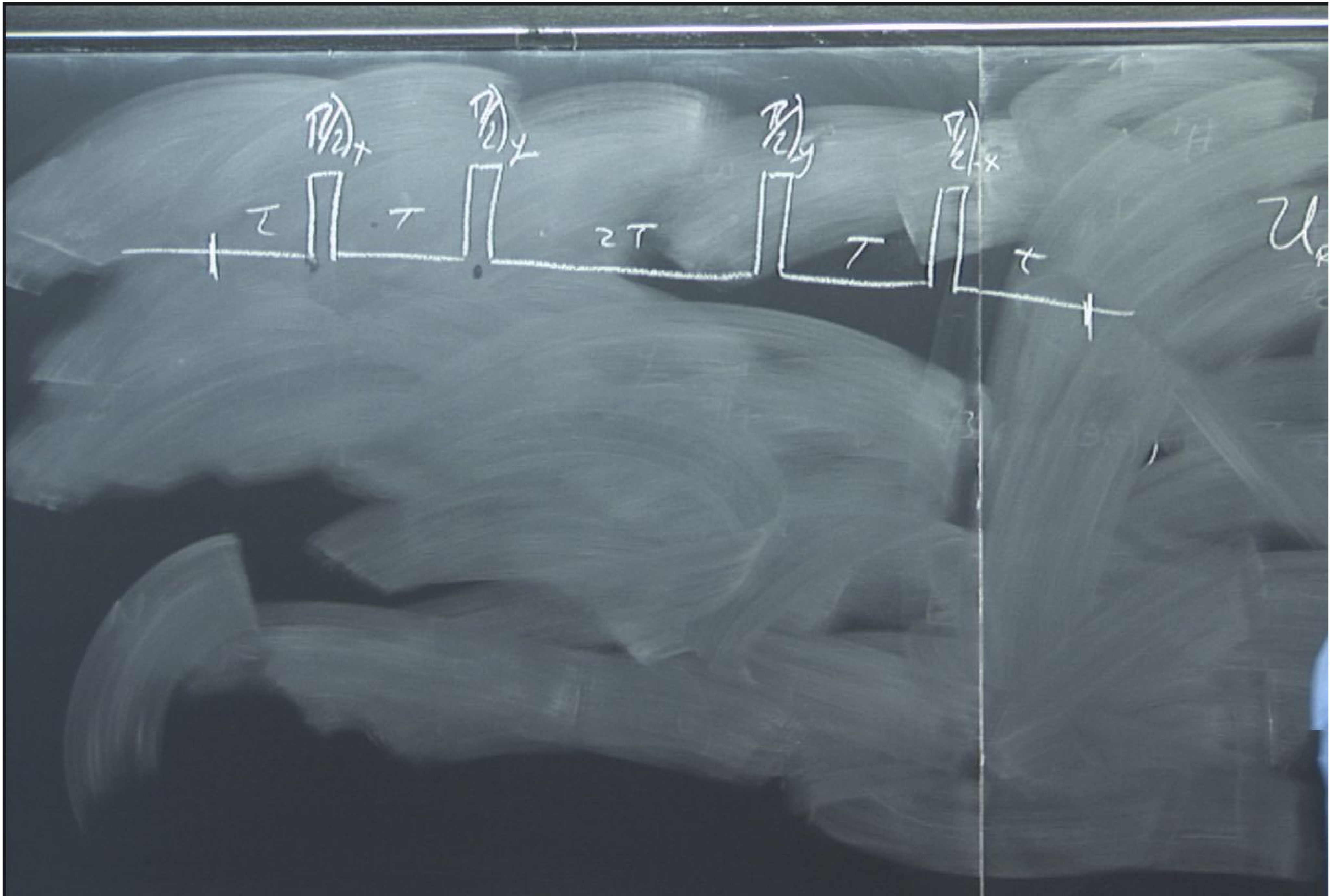
dipolar interaction

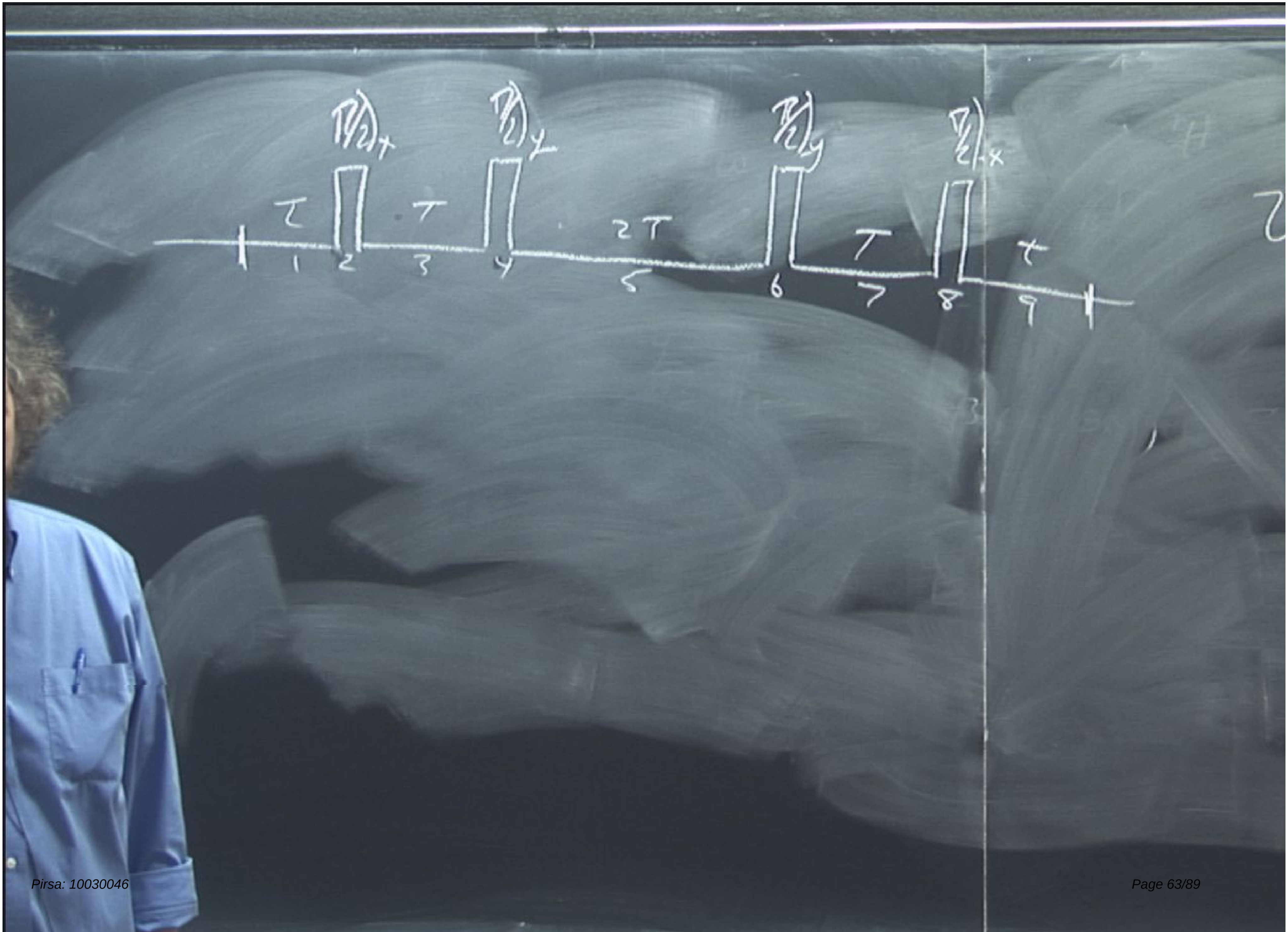


$$\mathcal{U}_{RF} = \mathbb{1}$$

$$\mathcal{H} = \Delta\omega\,\sigma_z \equiv \text{chemical shift}$$

$$\left.\begin{aligned} &+\omega_D\,\sigma\cdot\sigma \\ &+3\omega_D\,\sigma_z\sigma_z \end{aligned}\right\}\ \text{dipolar interact...}$$


π/2)x
π/2)y
π/2)y
π/2)-x
τ
τ
2τ
τ
τ

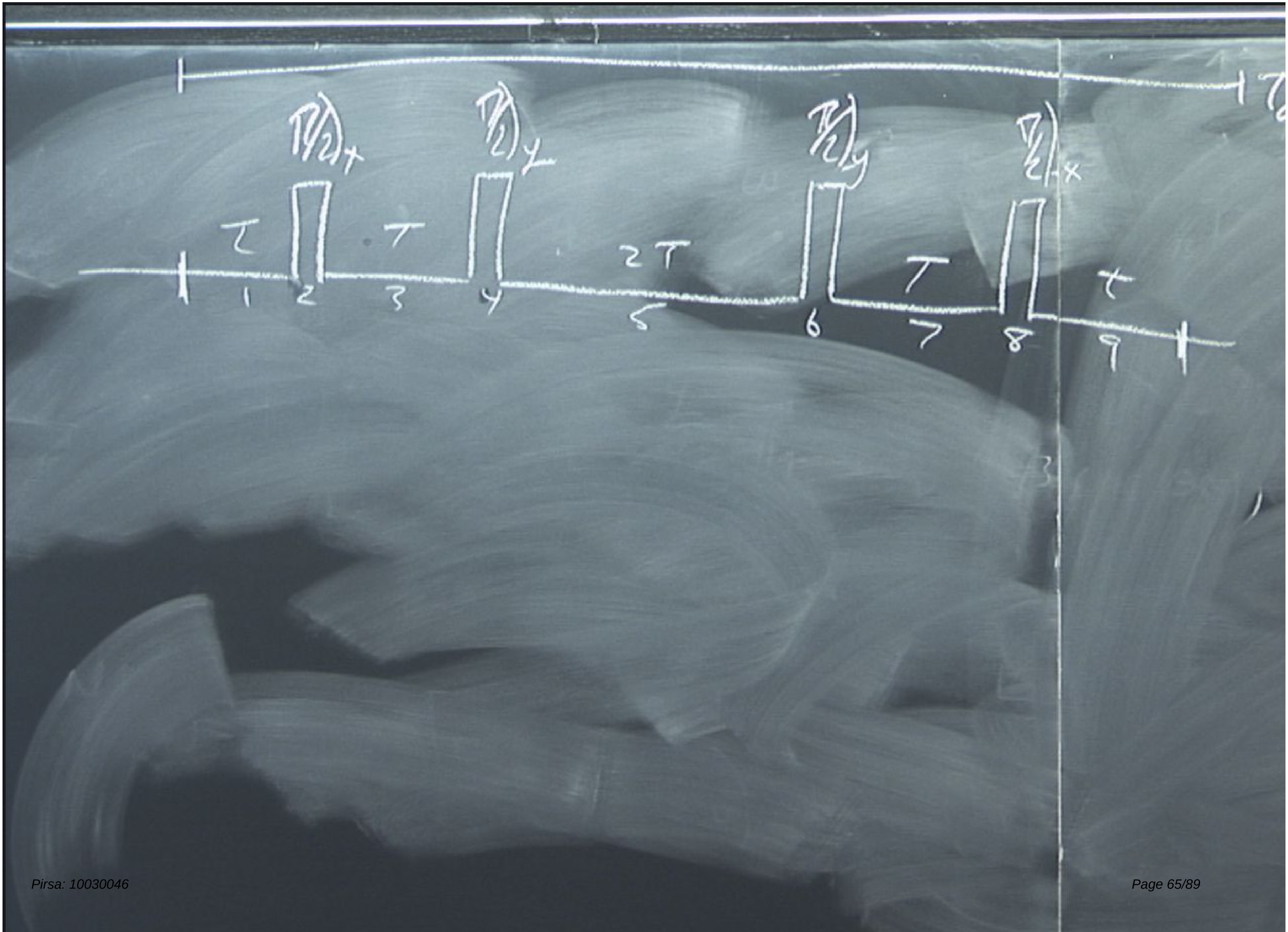

π/2)x
π/2)y
π/2)y
π/2)-x
τ
τ
2τ
τ
τ
1 2 3 4 5 6 7 8 9

$$\mathcal{U}_{cycle} = e^{-i\mathcal{H}_q t_q} \cdots\cdots e^{-i\mathcal{H}_2 t_2} e^{-i\mathcal{H}_1 t_1}$$

$$\mathcal{U}_{RF} = \mathbb{1}$$

$$\mathcal{H} = \Delta\omega\, \sigma_z \equiv \text{chemical shift}$$

$$\left.\begin{aligned} &+ \omega_D\, \sigma \cdot \sigma \\ &+ 3\omega_D\, \sigma_z \sigma_z \end{aligned}\right\} \text{dipolar interaction}$$


π/2)x
π/2)y
π/2)y
π/2)x
τ
τ
2τ
τ
t
1 2 3 4 5 6 7 8 9

$$U_{\mathrm{cycle}} = e^{-i\mathcal{H}_q t_q} \cdots \cdots e^{-i\mathcal{H}_2 t_2}\, e^{-i\mathcal{H}_1 t_1}$$

$$U_{RF} = \mathbb{1}$$

$$= e^{-i\bar{\mathcal{H}}\tau_c} \qquad \bar{\mathcal{H}} = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} + \cdots$$

$$\mathcal{H}^{(0)} = \frac{1}{\tau_c}\sum_i \mathcal{H}_i \tau_i$$

$\mathcal{H}$: $\Delta\omega\, \sigma_z$ $\equiv$ chemical shift

$$\left.\begin{array}{l} +\,\omega_D\ \sigma\cdot\sigma \\ +\,3\omega_D\ \sigma_z\sigma_z \end{array}\right\}\ \text{dipolar interaction}$$

$$\mathcal{U}_{cycle} = e^{-i\mathcal{H}_q t_q} \cdots\cdots e^{-i\mathcal{H}_2 t_2} e^{-i\mathcal{H}_1 t_1}$$

$${}_{2F} = \mathbb{1}$$

$$= e^{-i\bar{\mathcal{H}}\tau_c} \qquad \bar{\mathcal{H}} = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} + \cdots$$

$$\left(\frac{1}{\tau_c}\sum_i \mathcal{H}_i \tau_i\right)$$

$$\frac{-i}{2\tau_c}\sum_{i<j}[\mathcal{H}_j t_j, \mathcal{H}_i t_i]$$

$\mathcal{H}$: $\Delta\omega\,\sigma_z$ $\equiv$ chemical shift

$+\,\omega_D\;\sigma\cdot\sigma$

$+3\omega_D\;\sigma_z\sigma_z$ $\Big\}$ dipolar interaction

$$U_{cycle} = e^{-i\mathcal{H}_q t_q} \cdots\cdots e^{-i\mathcal{H}_2 t_2} e^{-i\mathcal{H}_1 t_1}$$

$$_{\mathrm{F}} = \mathbb{1}$$

$$= e^{-i\bar{\mathcal{H}}\tau_c} \qquad \bar{\mathcal{H}} = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} + \cdots$$

$$\left(\frac{1}{\tau_c} \sum_i \mathcal{H}_i \tau_i \right)$$

$$\mathcal{H}: \quad \Delta\omega\,\sigma_z \equiv \text{chemical shift}$$

$$\frac{-i}{2\tau_c} \sum_{i<j} [\mathcal{H}_j t_j, \mathcal{H}_i t_i]$$

$$\left.\begin{array}{l} +\omega_D\ \sigma\cdot\sigma \\ +3\omega_D\ \sigma_z\sigma_z \end{array}\right\} \text{dipolar interaction}$$

$$\boxed{\omega \gg \omega_D}$$

$$\mathcal{U}_{\text{cycle}} = e^{-i\mathcal{H}_q t_q} \cdots\cdots e^{-i\mathcal{H}_2 t_2}\, e^{-i\mathcal{H}_1 t_1}$$

$_{2F} = \mathbb{1}$

$$= e^{-i\bar{\mathcal{H}}\tau_c} \qquad \bar{\mathcal{H}} = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} + \cdots$$

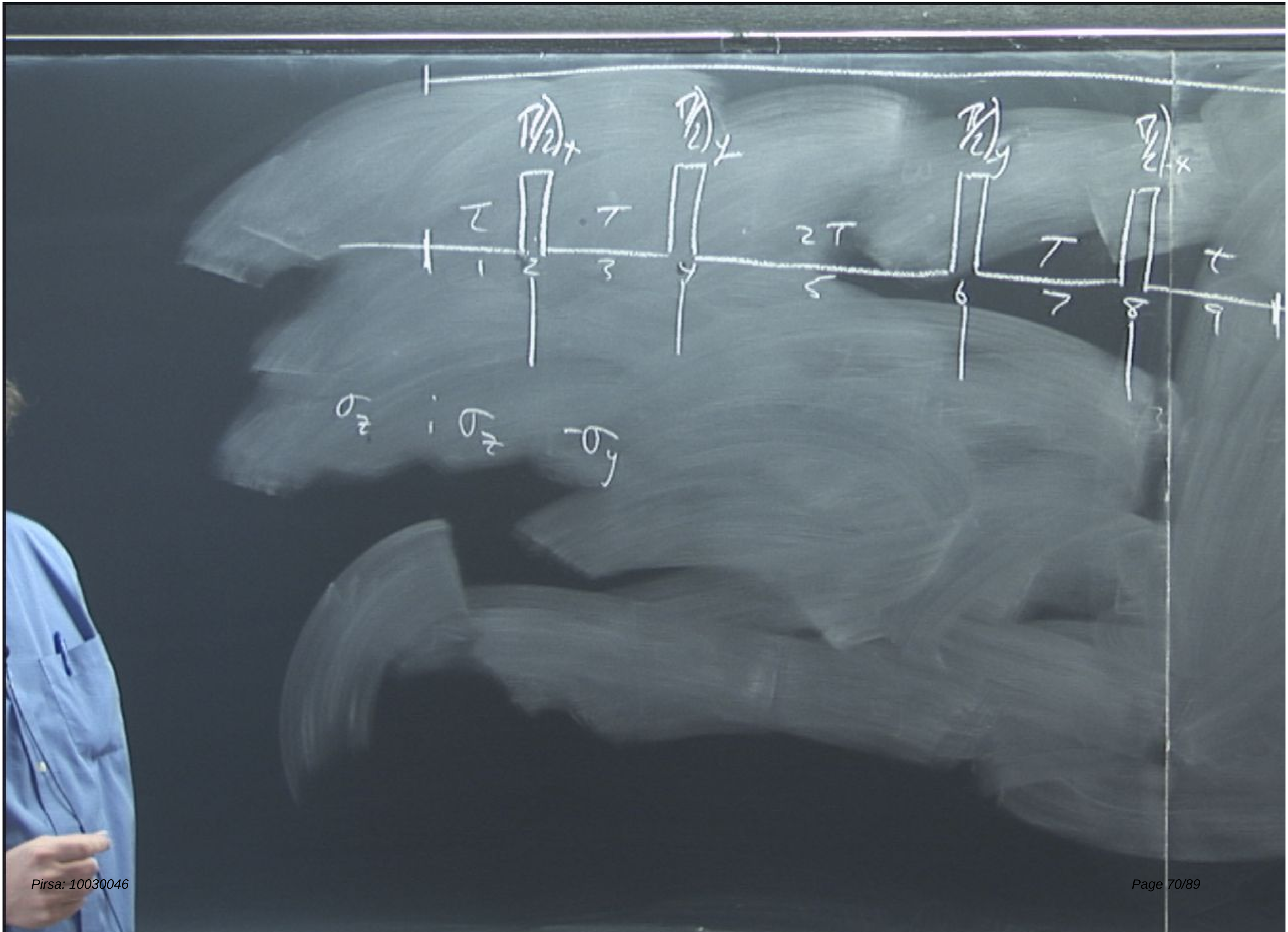
$$\left(\frac{1}{\tau_c}\sum_i \mathcal{H}_i \tau_i\right)$$

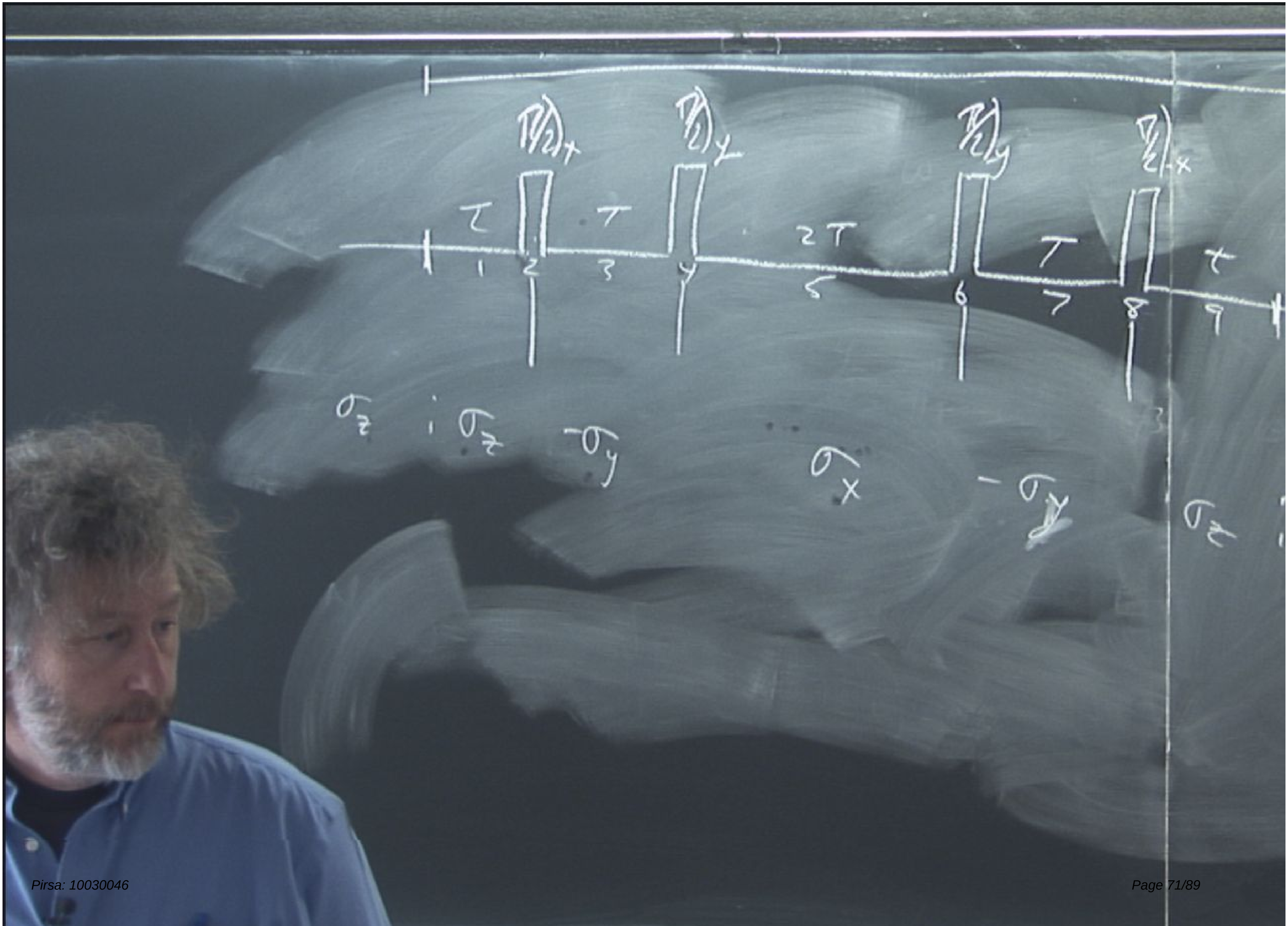
$$\frac{-i}{2\tau_c}\sum_{i<j}\left[\mathcal{H}_j t_j, \mathcal{H}_i t_i\right]$$

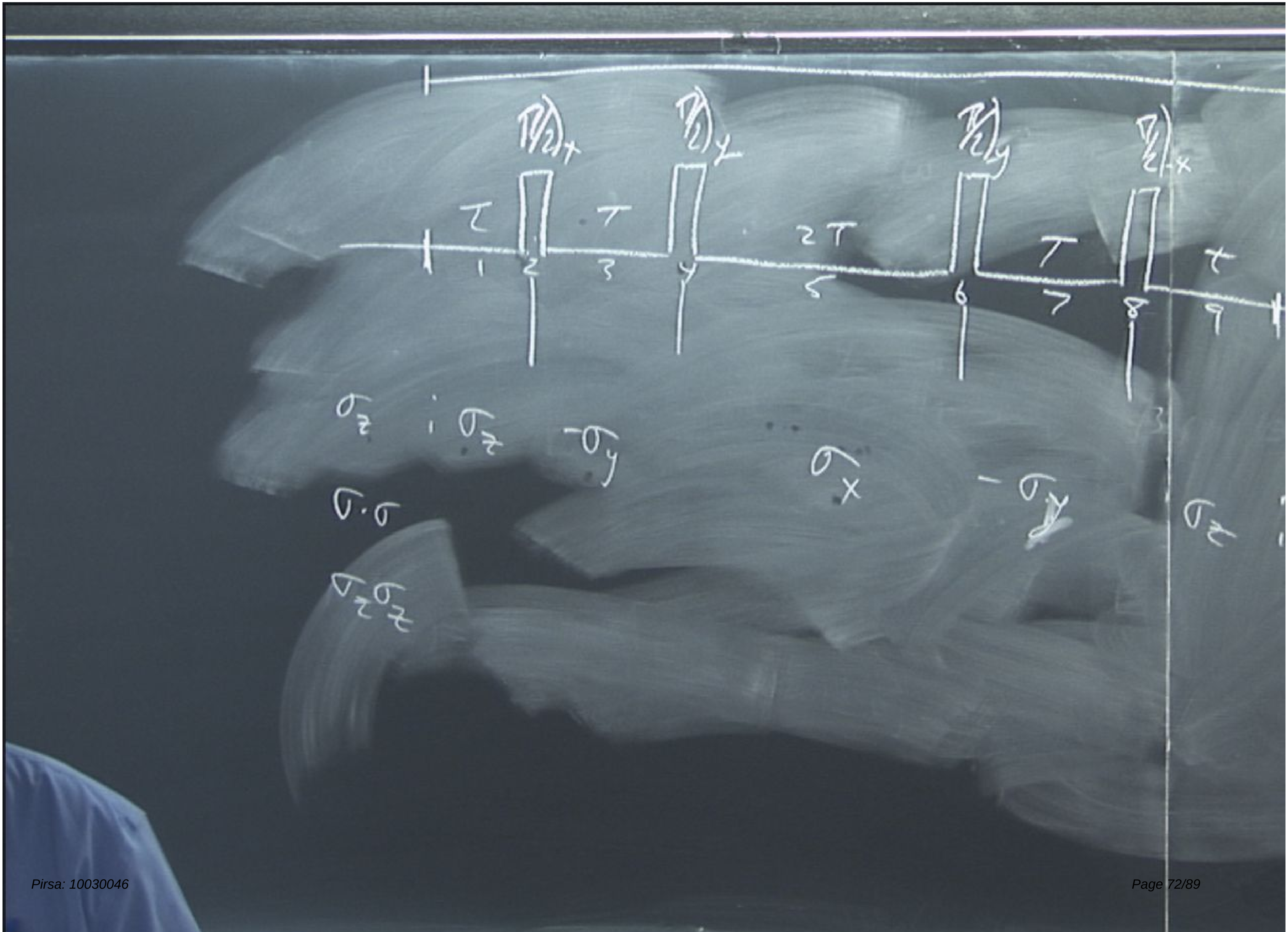
$\mathcal{H}$: $\Delta\omega\, \sigma_z$ $\equiv$ chemical shift

$+\,\omega_D\, \sigma\cdot\sigma$

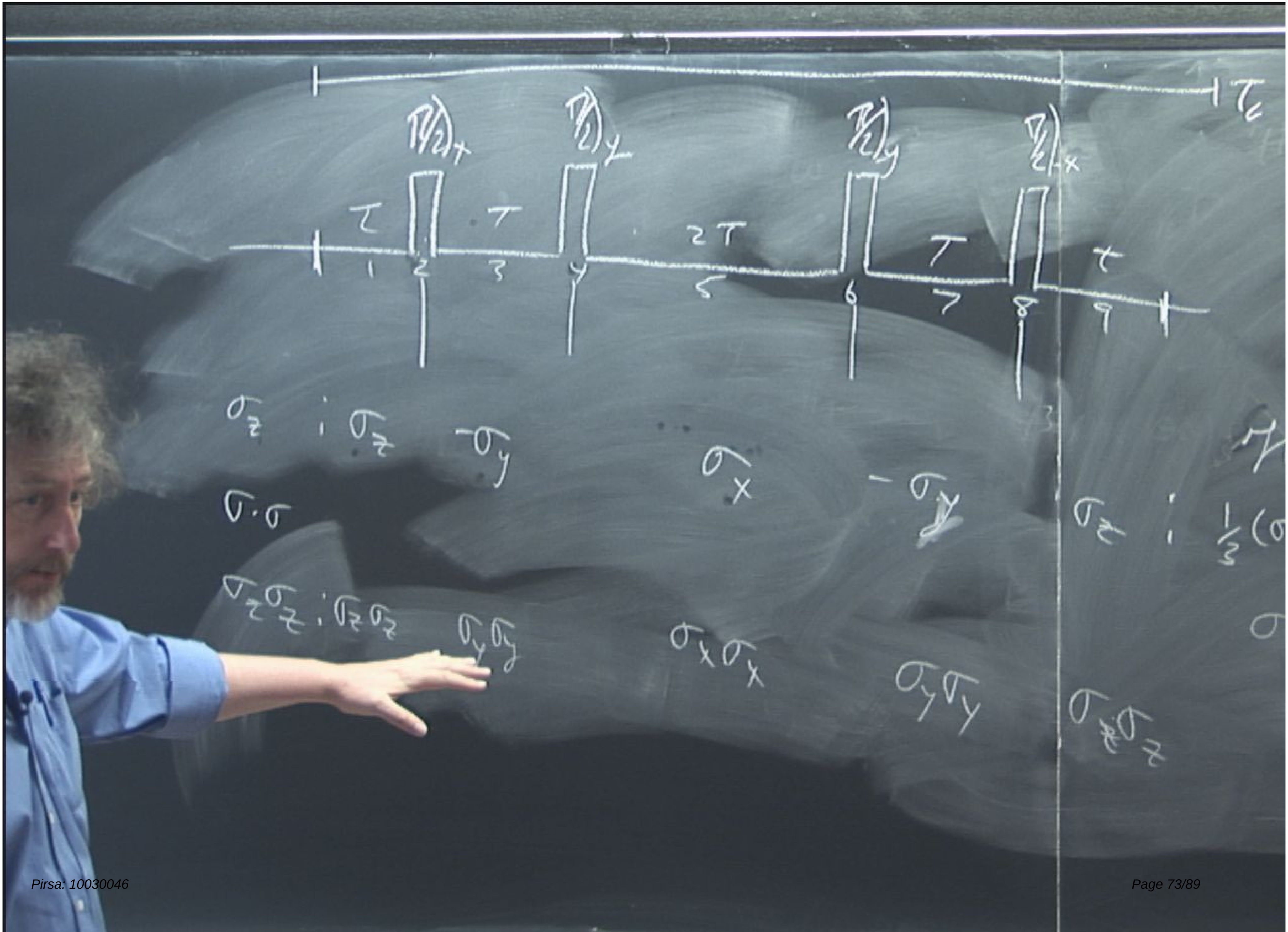
$+\,3\omega_D\, \sigma_z \sigma_z$ } dipolar interaction

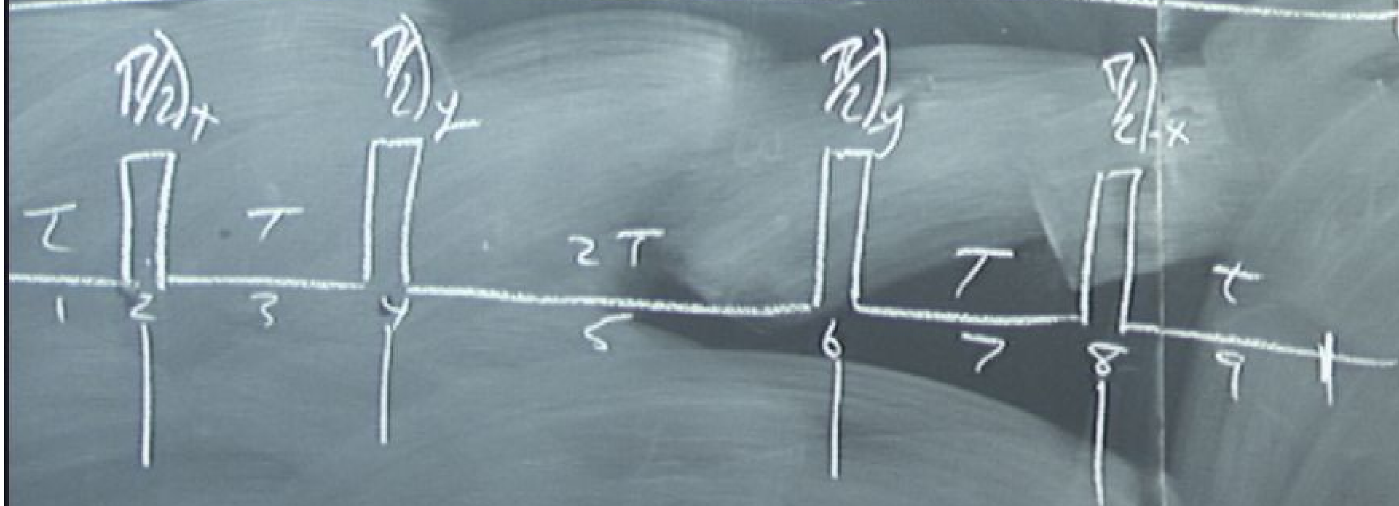

π/2)x
π/2)y
π/2)y
π/2)-x
τ
τ
2τ
τ
τ
1
2
3
4
5
6
7
8
9
σz
σz
-σy


π/2)x
π/2)y
π/2)y
π/2)x
τ
τ
2τ
τ
τ
1
2
3
4
5
6
7
8
9
σz
σz
-σy
σx
-σy
σz



$\pi/2)_x$
$\pi/2)_y$
$\pi/2)_y$
$\pi/2)_{-x}$
τ τ 2τ τ τ
1 2 3 4 5 6 7 8 9
σ_z σ_z $-\sigma_y$ σ_x $-\sigma_y$ σ_z
$\sigma\cdot\sigma$
$\sigma_z\sigma_z$


τ_c
π/2)_x
π/2)_y
π/2)_y
π/2)_x
τ
τ
2τ
τ
t
1 2 3 4 5 6 7 8 9
σ_z ; σ_z
-σ_y
σ_x
-σ_y
σ·σ
σ_z ; 1/3(σ
σ_zσ_z ; σ_zσ_z
σ_yσ_y
σ_xσ_x
σ_yσ_y
σ_zσ_z



$U_{RF} = \mathbb{1}$

$\mathcal{H}:$

$\mathcal{H}^{(0)}$

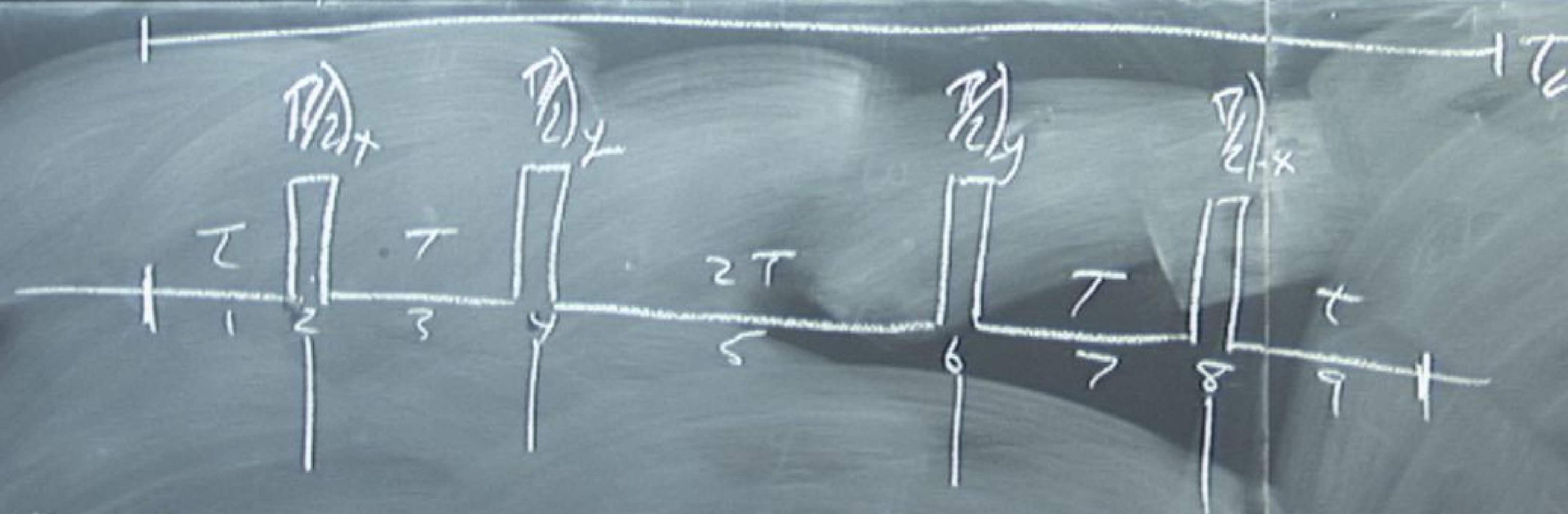
$\sigma_z \quad -\sigma_y \quad \sigma_x \quad -\sigma_y$

$\sigma_z : \frac{1}{3}(\sigma_x - \sigma_y + \sigma_z)$

$\sigma_z\sigma_z \quad \sigma_y\sigma_y \quad \sigma_x\sigma_x \quad \sigma_y\sigma_y$

$: \sigma\cdot\sigma \cdot \omega_D$

$\sigma_z\sigma_z : \frac{1}{3}(\sigma\cdot\sigma) \cdot 3\omega_D$



$\mathcal{U}$

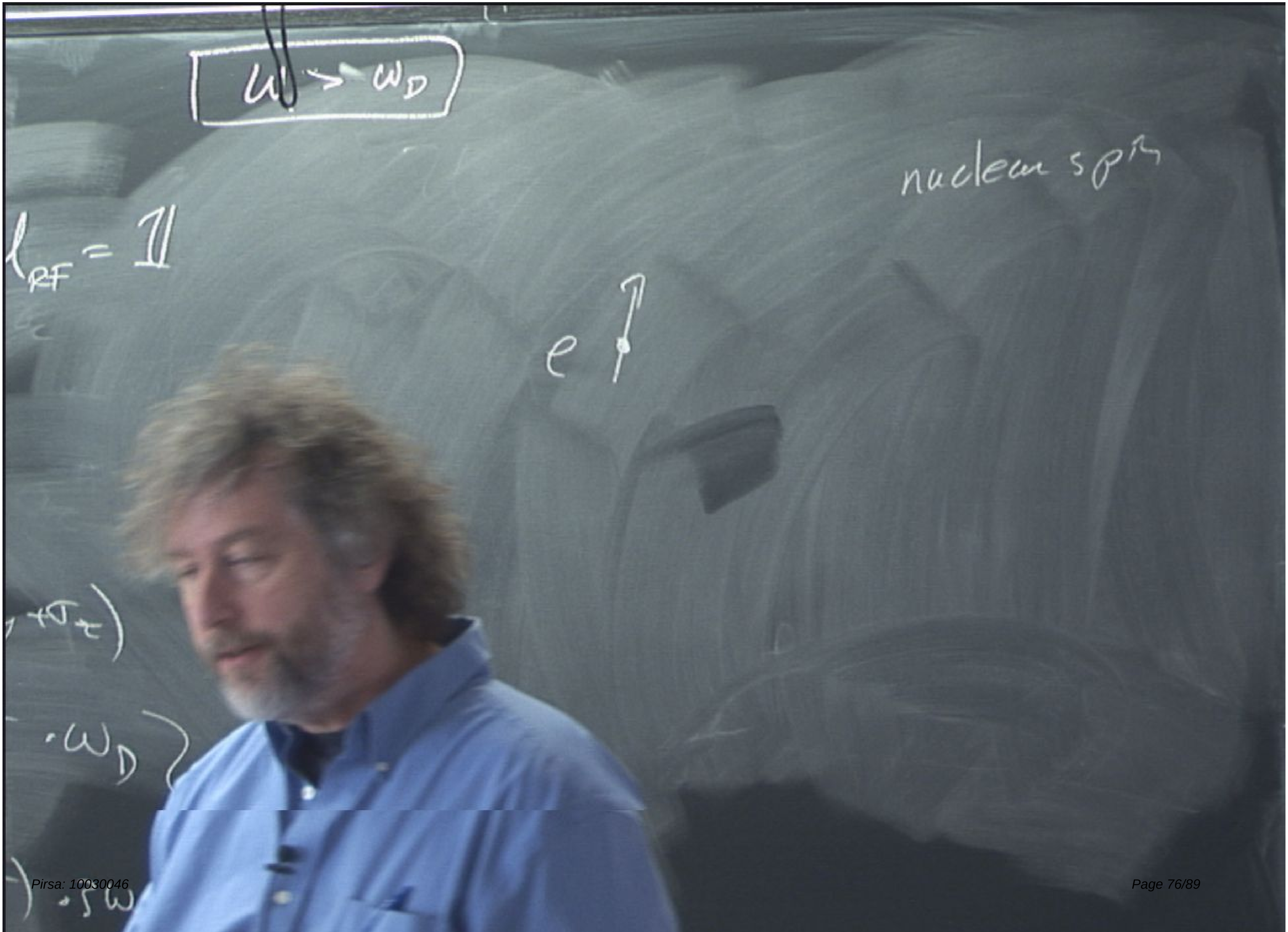
$\mathcal{H}^{(0)}$

$\sigma_z \; : \; \sigma_z \quad -\sigma_y \quad \sigma_x \quad -\sigma_y$ $\qquad \sigma_z \; : \; \frac{1}{3}(\sigma_x + \sigma_y + \ldots$

$\sigma\cdot\sigma$ $\qquad : \; \sigma\cdot\sigma$

$\sigma_z\sigma_z \; : \; \sigma_z\sigma_z \quad \sigma_y\sigma_y \quad \sigma_x\sigma_x \quad \sigma_y\sigma_y$

$\sigma_z\sigma_z \; : \; \frac{1}{3}(\sigma\cdot\sigma)$


ω > ω_D
nuclear spin
e

$\boxed{\omega > \omega_D}$

nuclear spin

$l_{RF} = \mathbb{1}$

$e \uparrow$

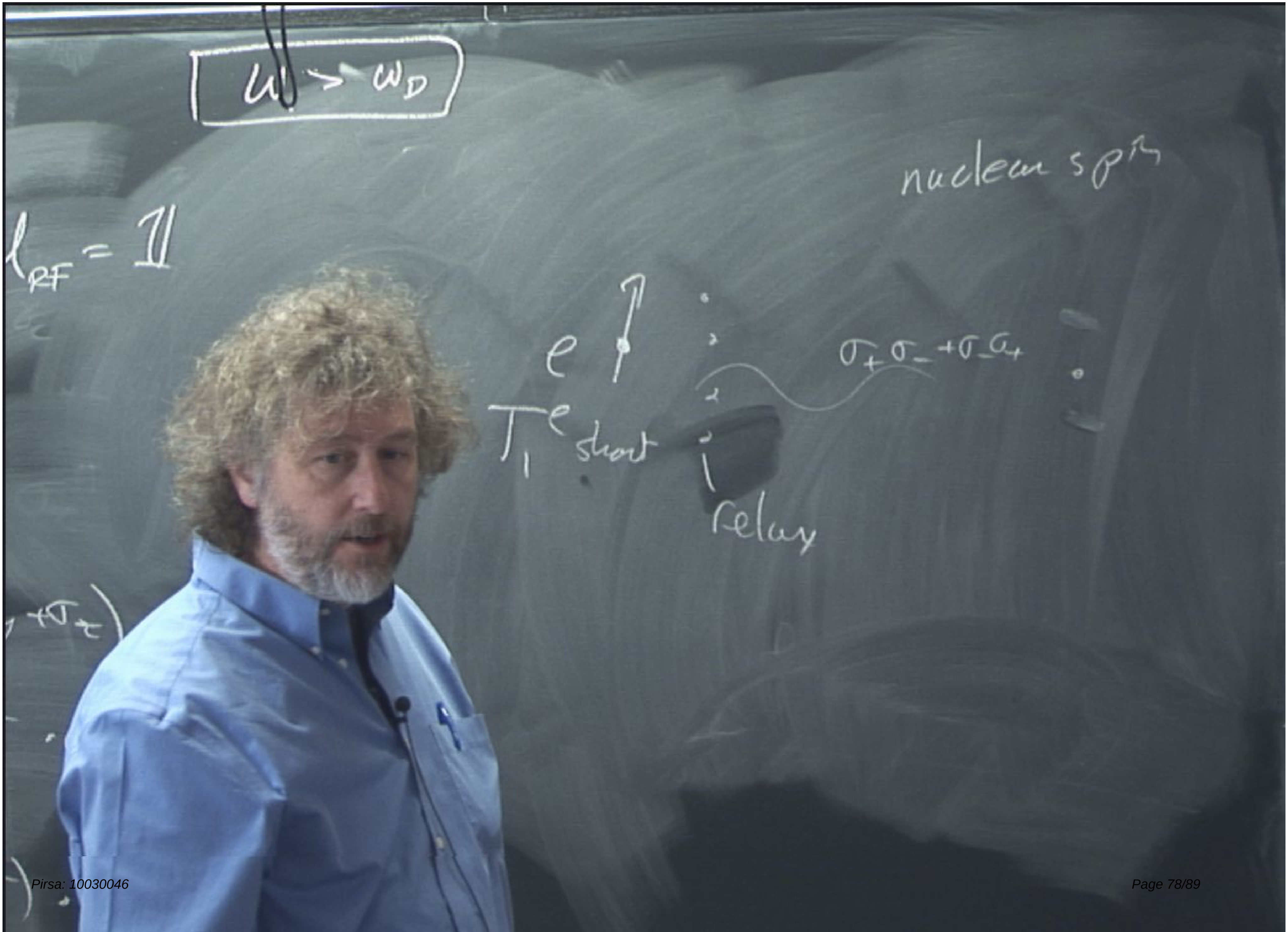
T_1^e short

$+\sigma_z)$

$\cdot\omega_D$

$\rightarrow 0$

$) \cdot 5\omega_D$


ω > ω_D
nuclear spin
e
σ+σ- +σ-σ+
T1e short
relax

$\boxed{\omega \gg \omega_D}$

nuclear spin

$\rho_{RF} = \mathbb{1}$

$e \uparrow$

$\sigma_+\sigma_- + \sigma_-\sigma_+$

T_1^e short

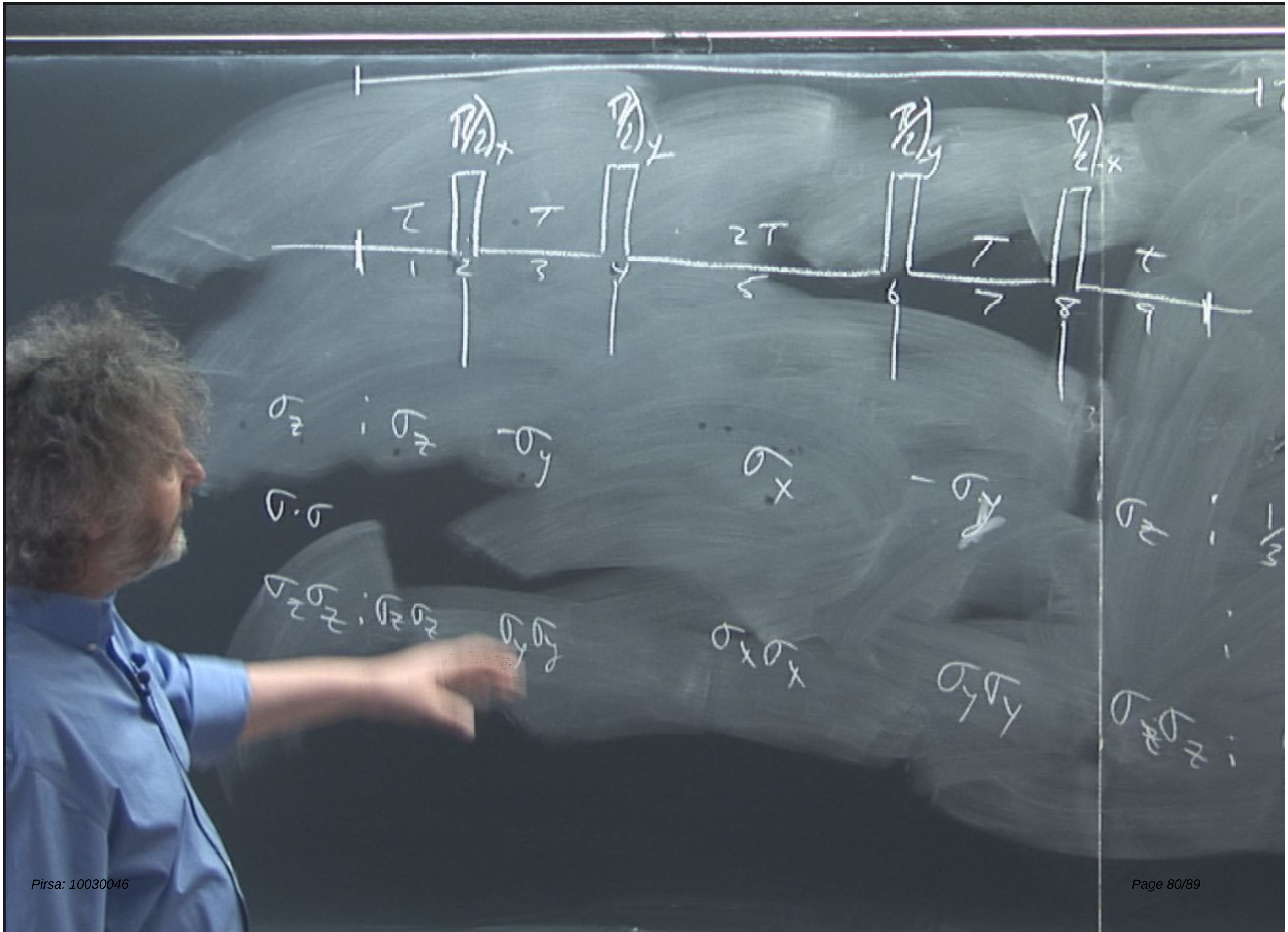
relax

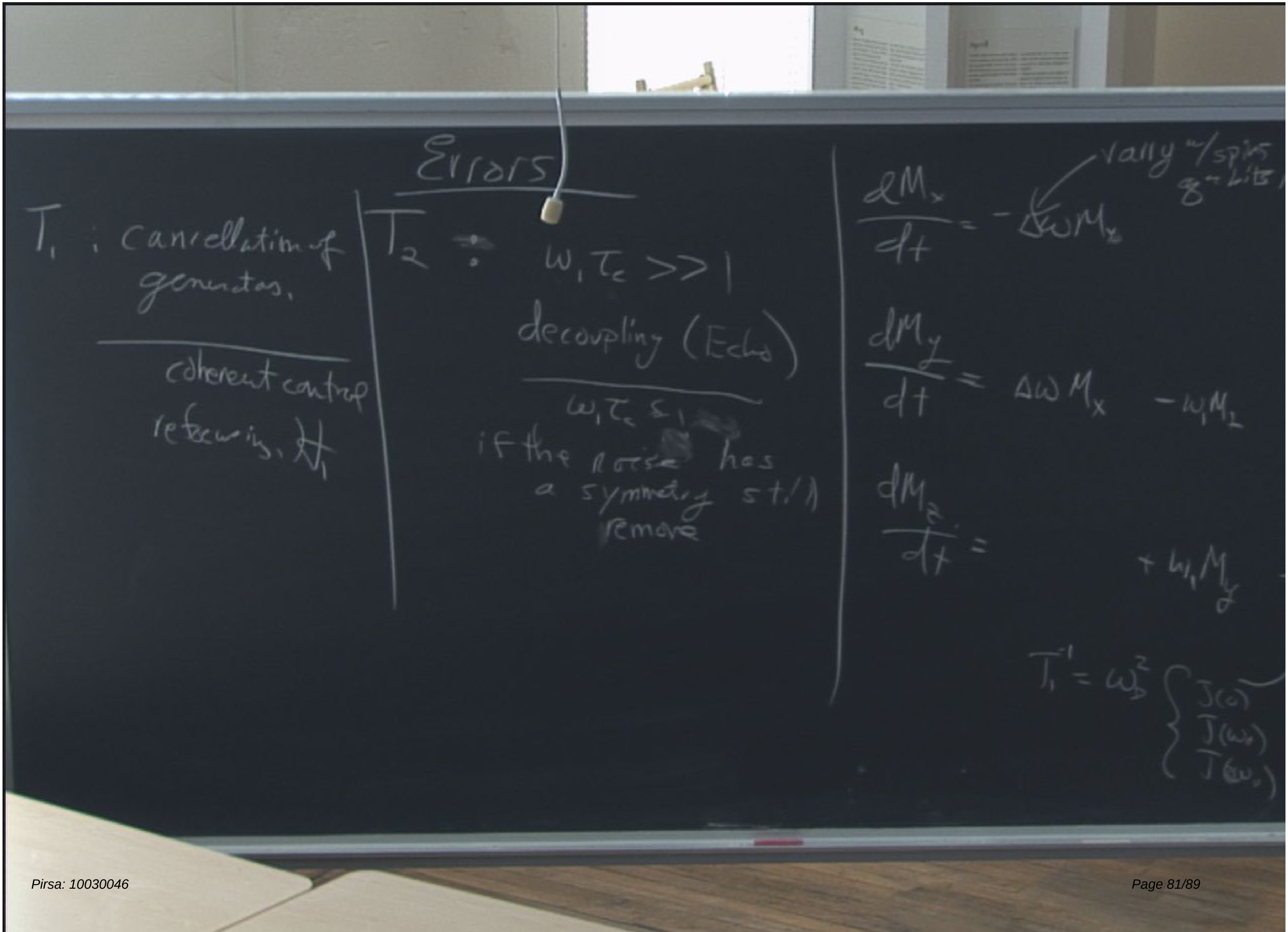
$+\sigma_z)$

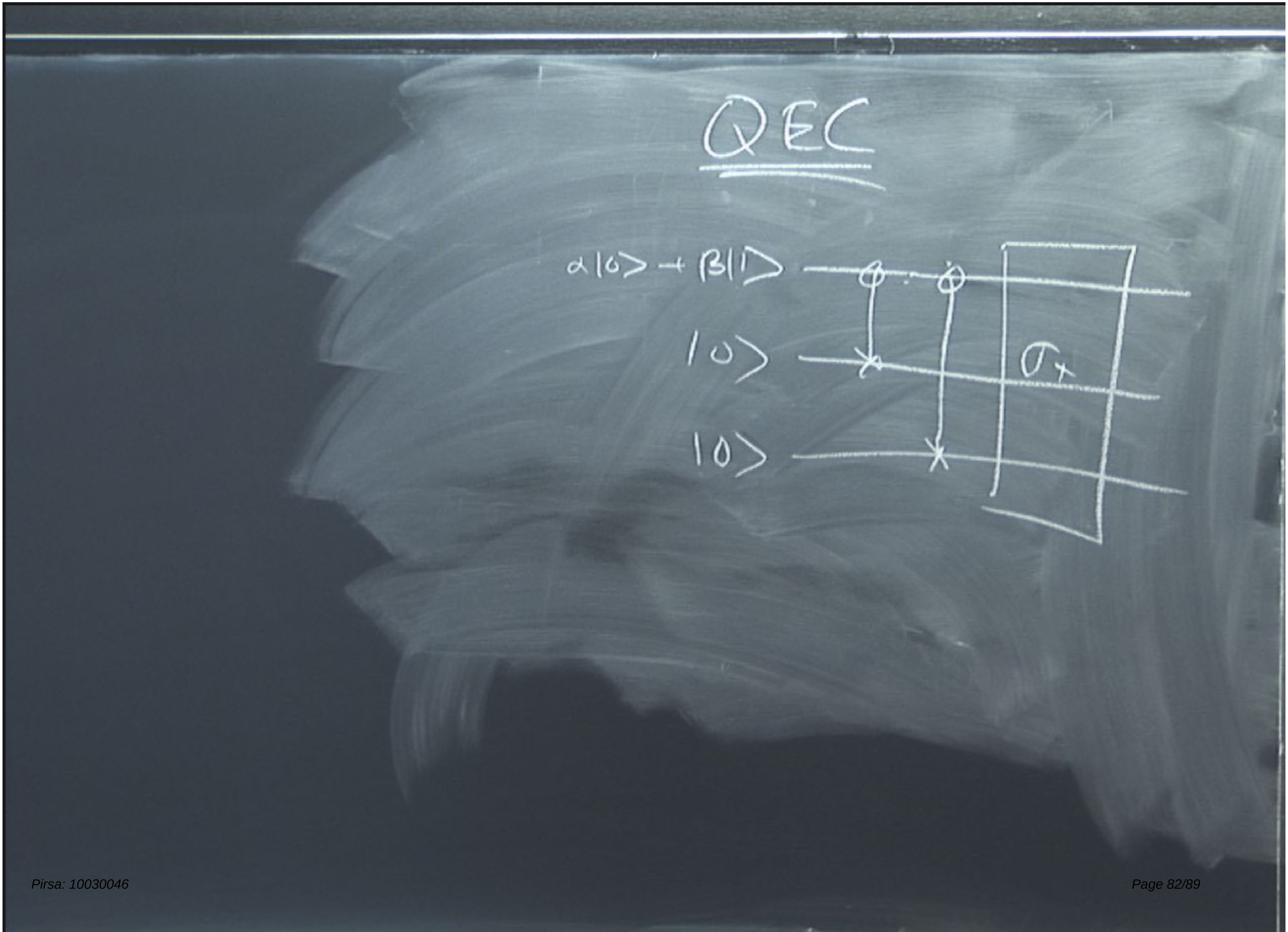
$\cdot\,\omega_D$

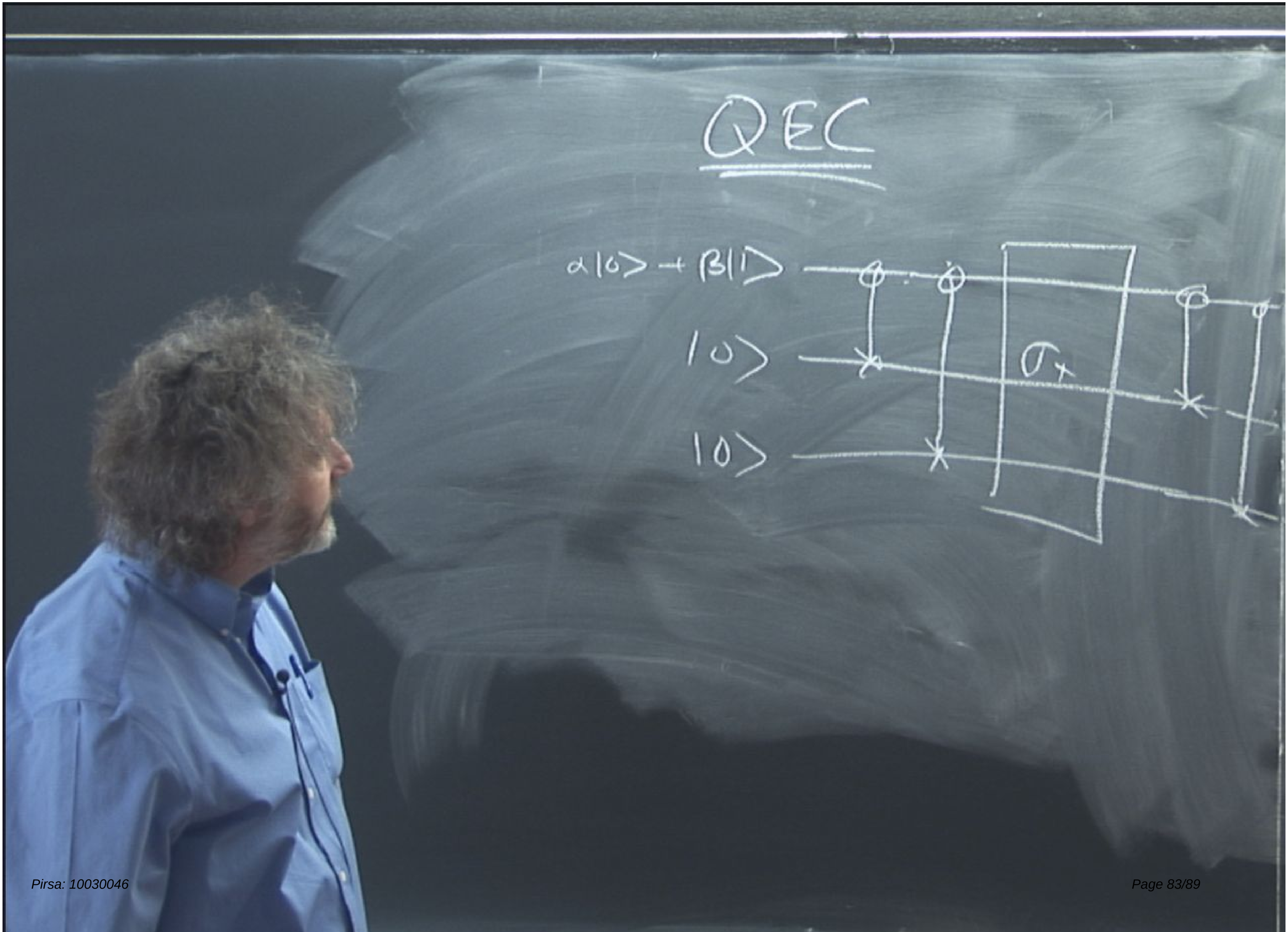
$\longrightarrow 0$

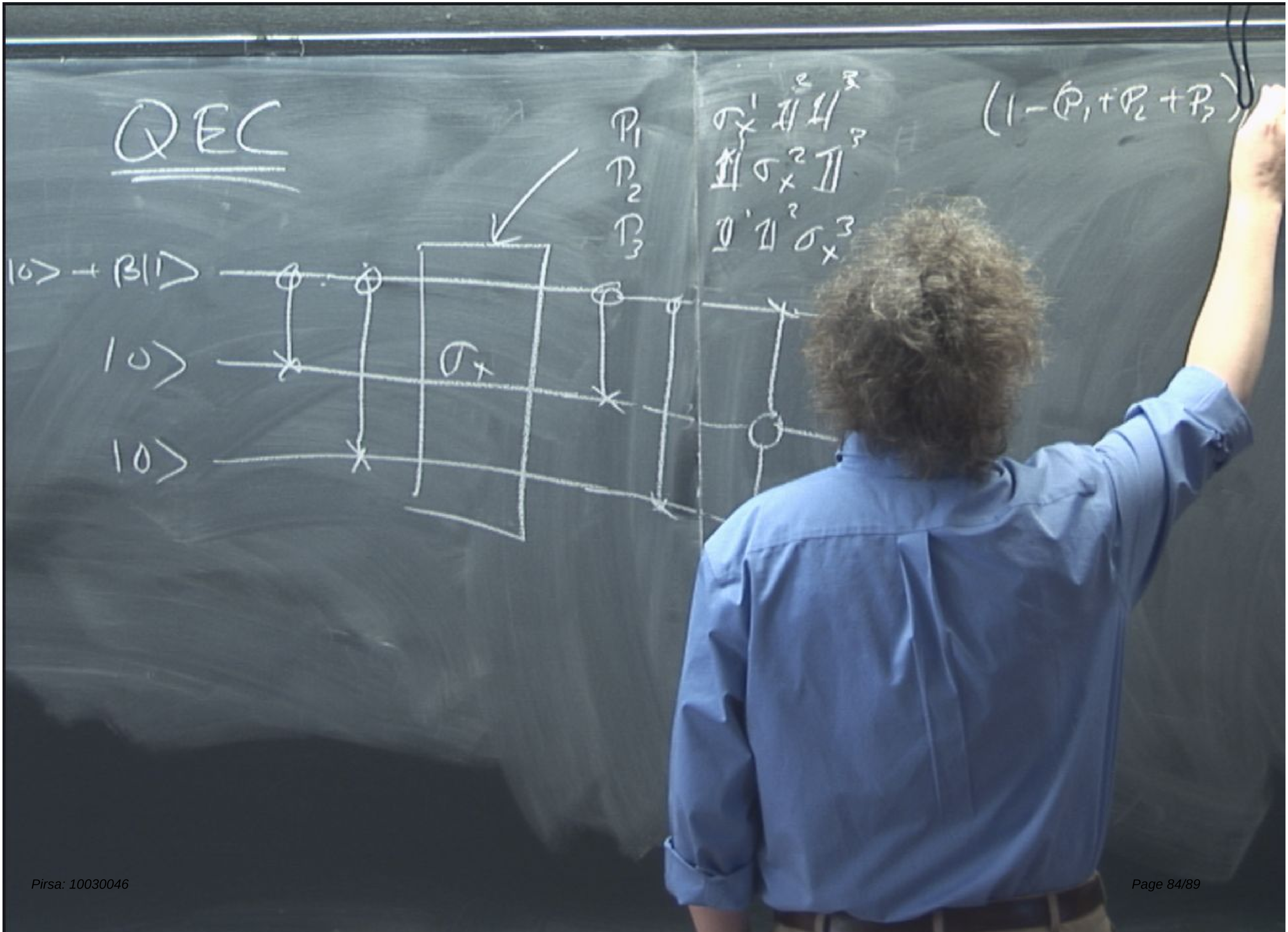
$\cdot\,5\omega_D$


π/2)x
π/2)y
π/2)y
π/2)-x
τ
τ
2τ
τ
τ
1 2 3 4 5 6 7 8 9
σz ; σz
-σy
σx
-σy
σz ; 1/3
σ·σ
σzσz ; σzσz
σyσy
σxσx
σyσy
σzσz ;


Errors
T1 : cancellation of generators.
coherent control
refocusing, H1
T2 : ω1 τc >> 1
decoupling (Echo)
ω1 τc ≤ 1
if the noise has a symmetry st/
remove
vary w/ spins
dMx/dt = - ΔωMx
dMy/dt = ΔωMx - ω1Mz
dMz/dt = + ω1My
T1^-1 = ωD^2 J(ω) J(ω0)


QEC
α|0> + β|1>
|0>
|0>
σx

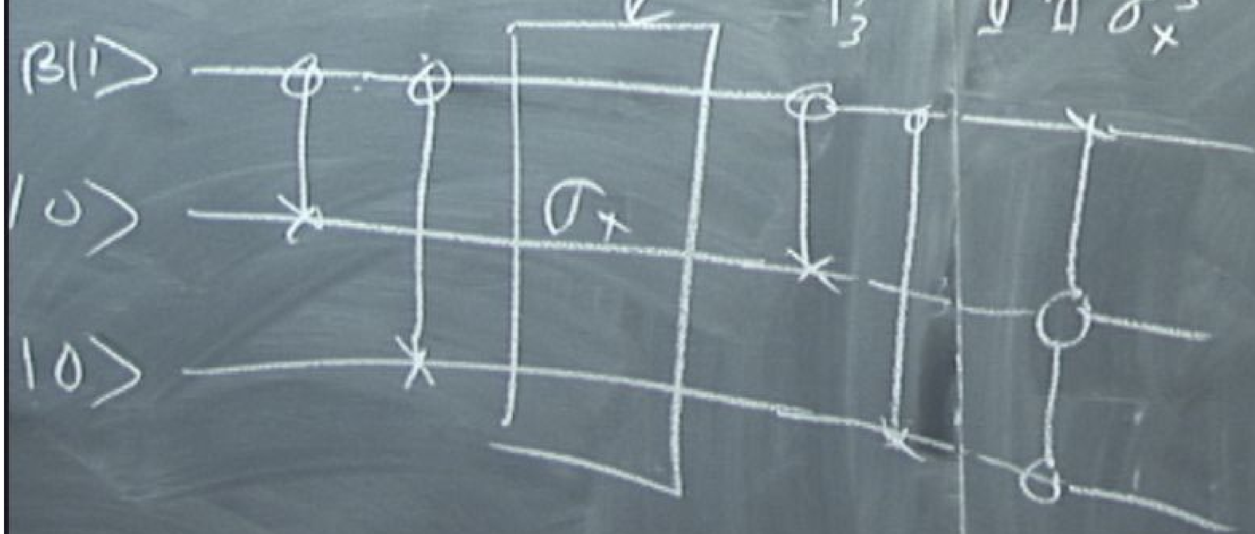

QEC
α|0> + β|1>
|0>
|0>
σx

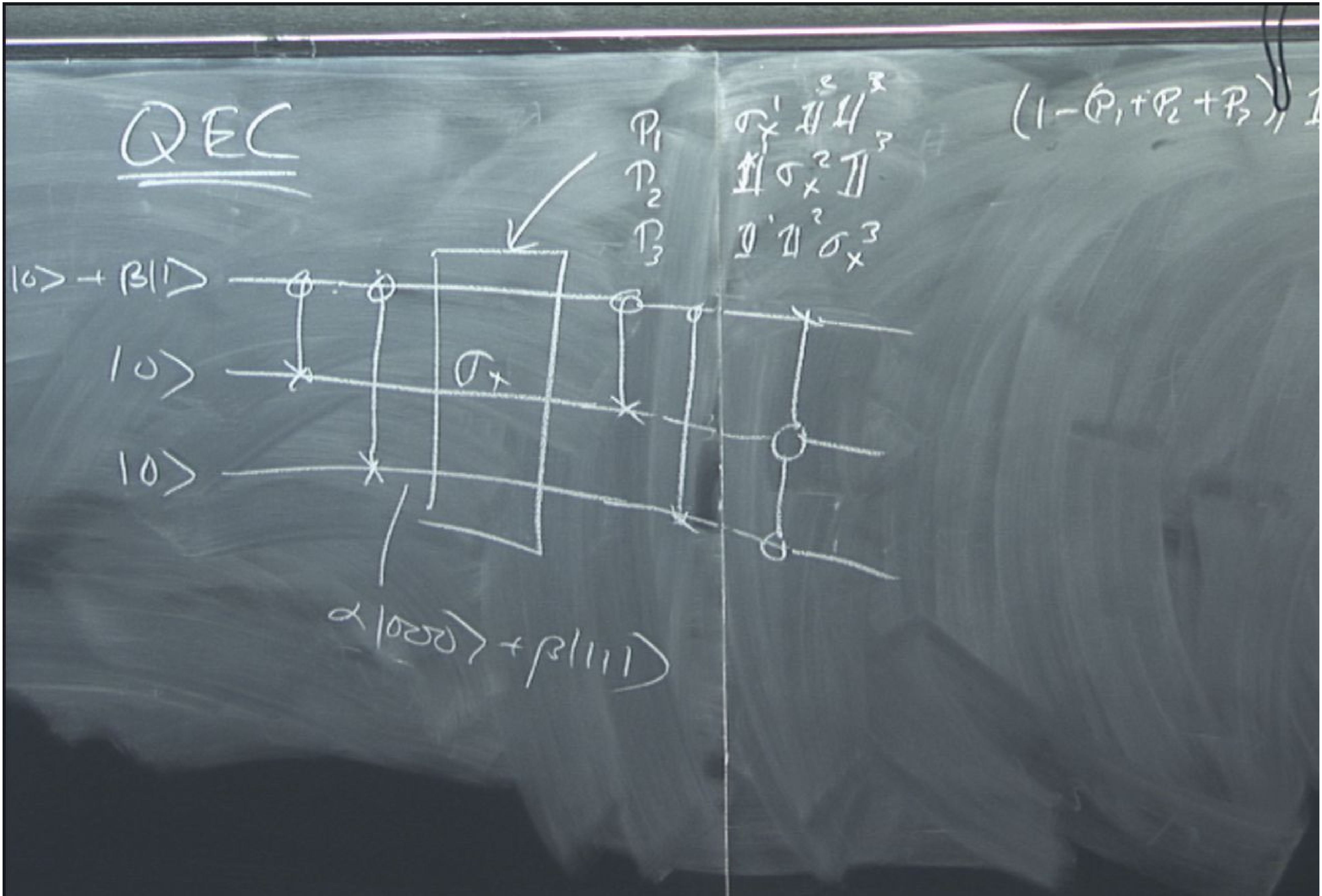

QEC
P_1 $\sigma_x^1 \mathbb{1}^2 \mathbb{1}^3$
P_2 $\mathbb{1}^1 \sigma_x^2 \mathbb{1}^3$
P_3 $\mathbb{1}^1 \mathbb{1}^2 \sigma_x^3$
$(1-(P_1+P_2+P_3)$
$|0\rangle + \beta|1\rangle$
$|0\rangle$
$|0\rangle$
σ_x

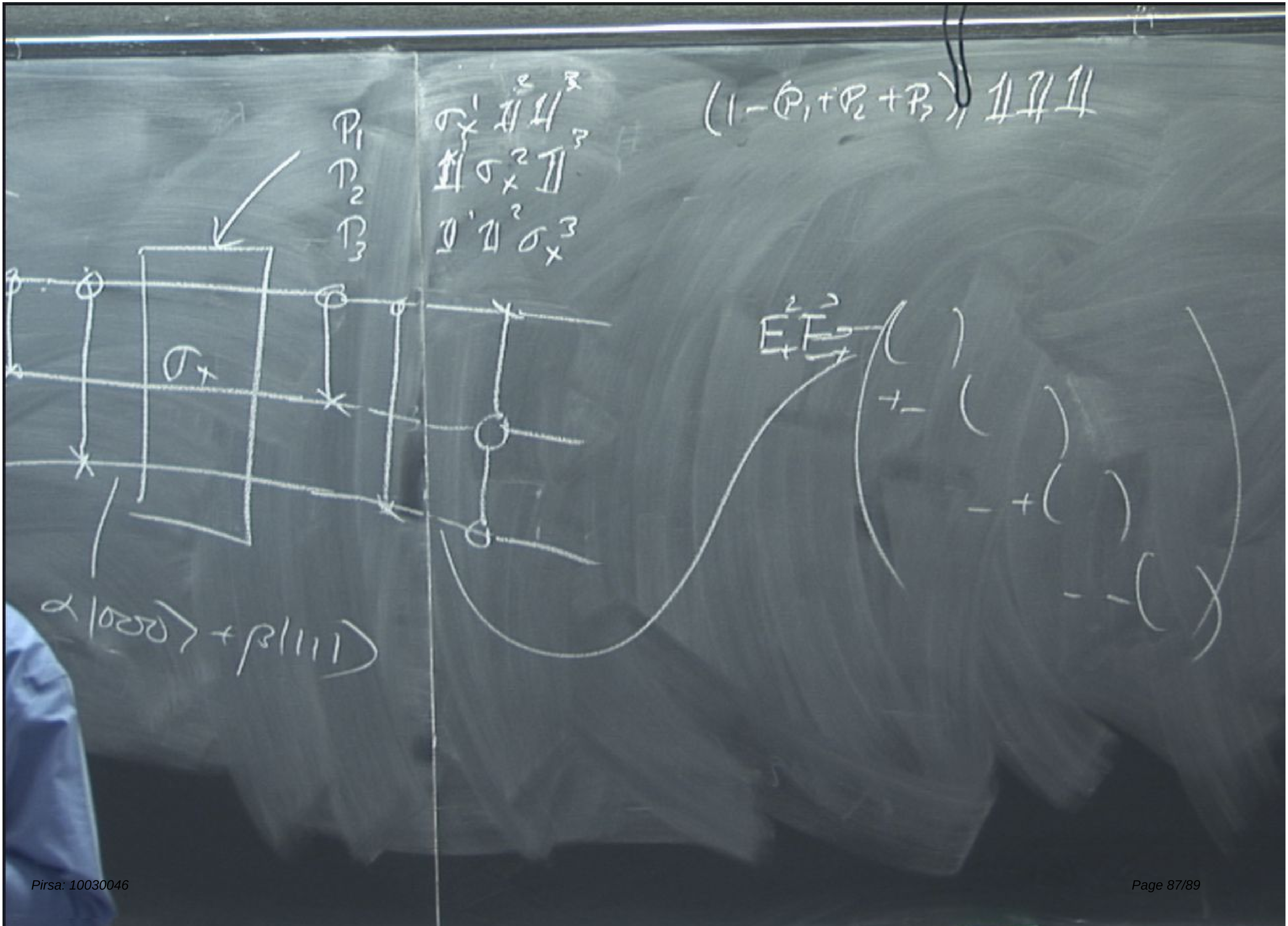
QEC

$$P_1 \quad \sigma_x^1 \, \mathbb{1}^2 \, \mathbb{1}^3$$
$$P_2 \quad \mathbb{1}^1 \, \sigma_x^2 \, \mathbb{1}^3$$
$$P_3 \quad \mathbb{1}^1 \, \mathbb{1}^2 \, \sigma_x^3$$

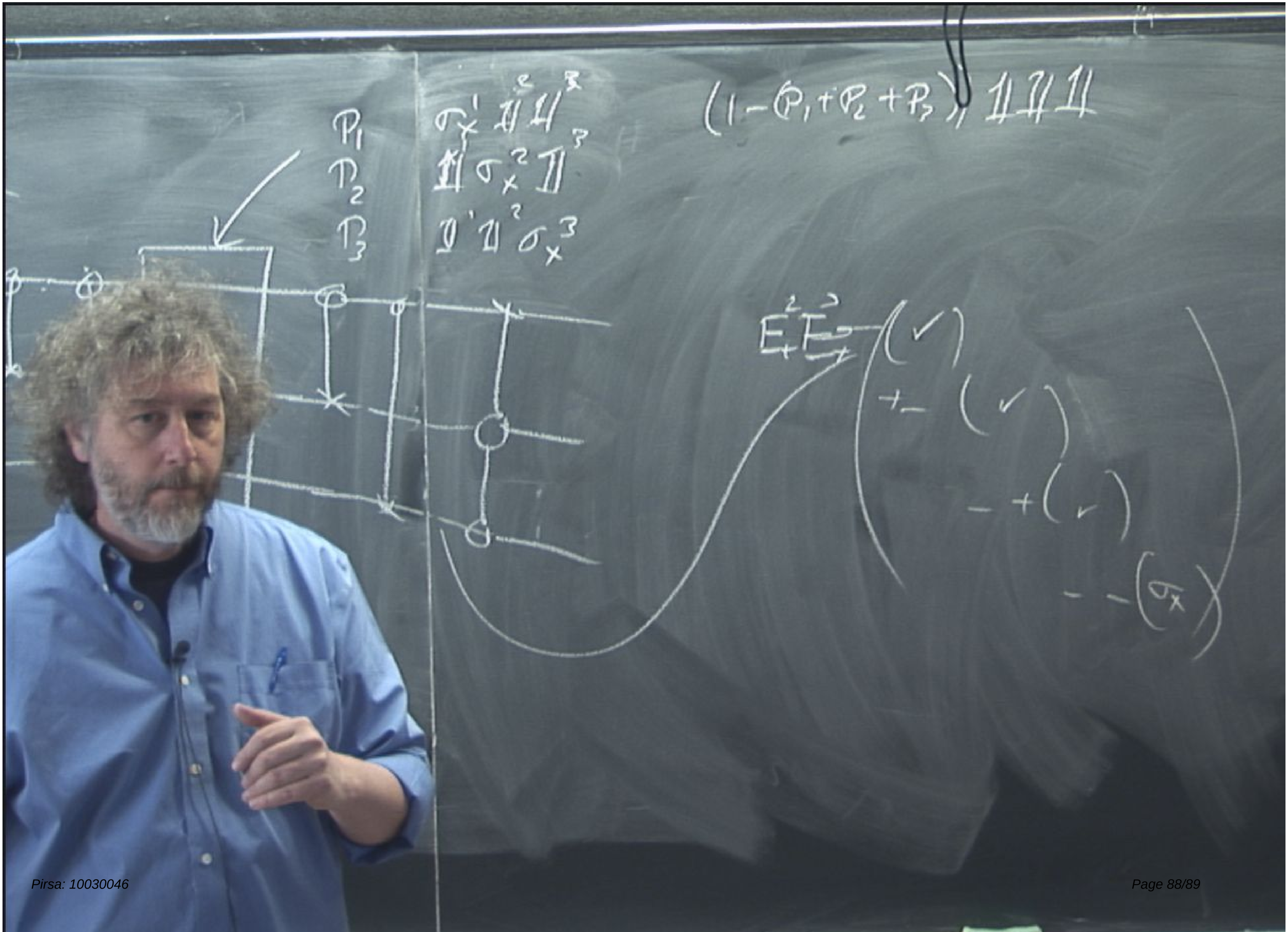
$$(1-(P_1+P_2+P_3)) \, \mathbb{1}\,\mathbb{1}\,\mathbb{1}$$

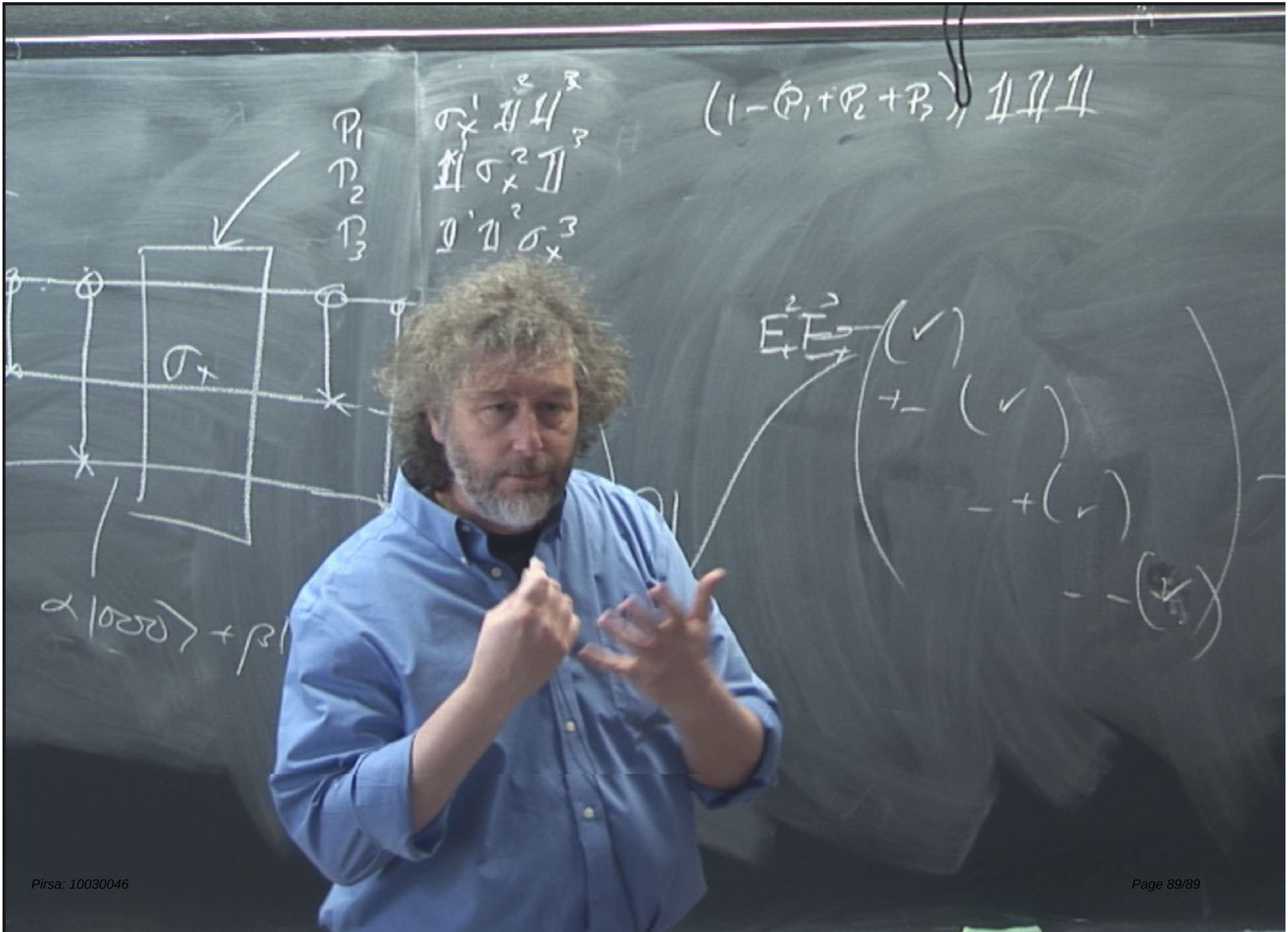



QEC
$P_1 \quad \sigma_x^1 \mathbb{1}^2 \mathbb{1}^3$
$P_2 \quad \mathbb{1}^1 \sigma_x^2 \mathbb{1}^3$
$P_3 \quad \mathbb{1}^1 \mathbb{1}^2 \sigma_x^3$
$(1-(P_1+P_2+P_3))$
$|0\rangle + \beta|1\rangle$
$|0\rangle$
$|0\rangle$
σ_x
$\alpha|000\rangle + \beta|111\rangle$



P_1 $\sigma_x^1 \, 1\!\!1^2 \, 1\!\!1^3$
P_2 $1\!\!1^1 \, \sigma_x^2 \, 1\!\!1^3$
P_3 $1\!\!1^1 \, 1\!\!1^2 \, \sigma_x^3$
$(1-(P_1+P_2+P_3)) \, 1\!\!1 \, 1\!\!1 \, 1\!\!1$
σ_x
$\alpha|000\rangle + \beta|111\rangle$
$E_+^2 E_-^3$




P1
P2
P3
σx
α|000⟩ + β|