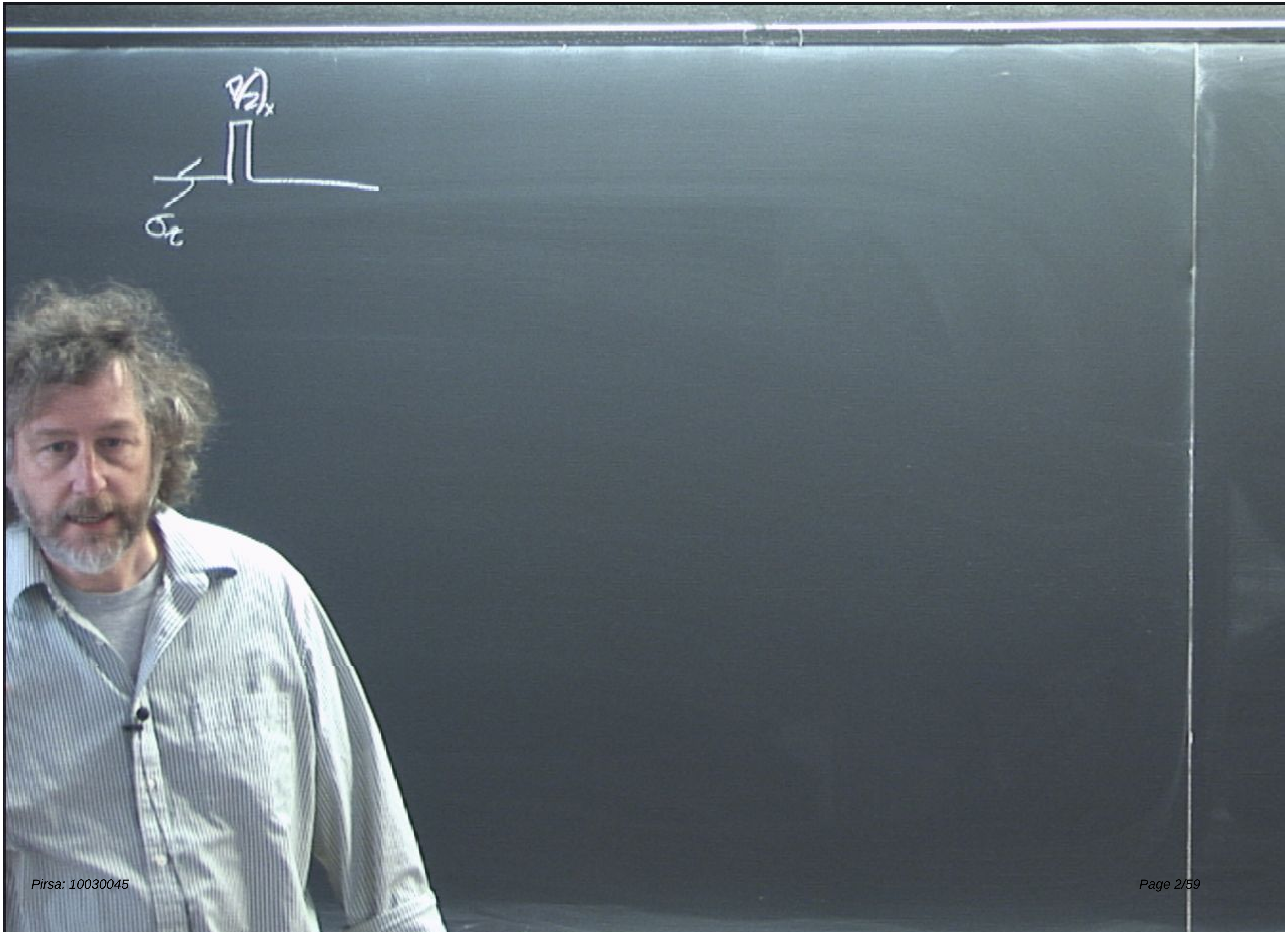
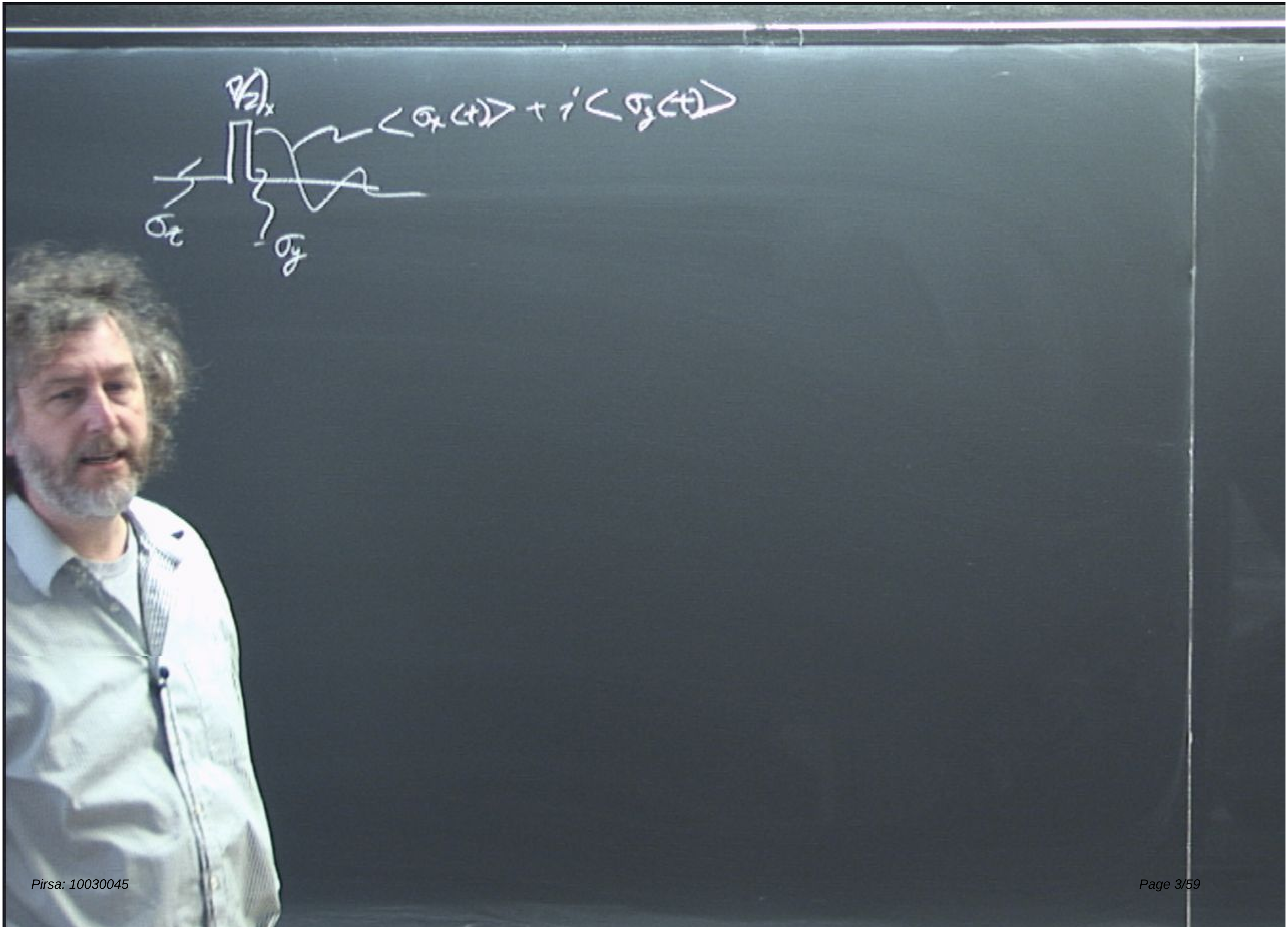
Title: Explorations in Quantum Info. (PHYS 641) - Lecture 12

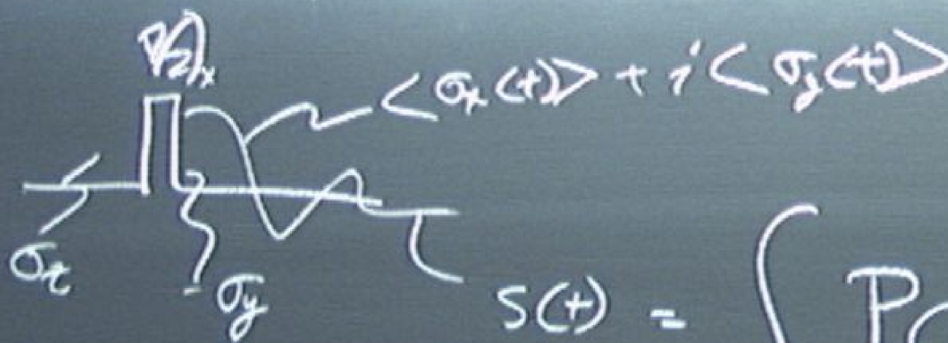
Date: Mar 03, 2010 09:00 AM

URL: http://pirsa.org/10030045

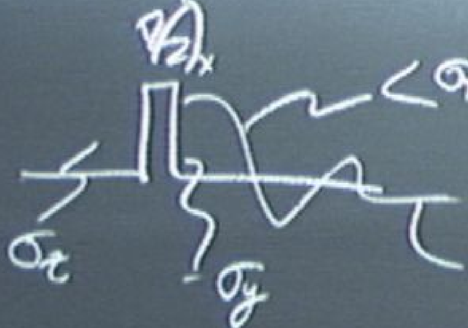
Abstract:




$(\pi/2)_x$
$\langle \sigma_x(t) \rangle + i \langle \sigma_y(t) \rangle$
σ_x
σ_y


$(\pi/2)_x$
$\langle \sigma_x(t) \rangle + i \langle \sigma_y(t) \rangle$
σ_x
σ_y
τ

$$S(t) = \int P(\vec{r})$$



$\langle \sigma_x(t) \rangle + i \langle \sigma_y(t) \rangle$

$$S(t) = \int \underset{\uparrow}{P}(\vec{r})\, e^{-i\gamma \int_0^t \underset{\uparrow}{B}(\vec{r},t)\, dt}\, e^{-t/T_2}\, d\vec{r}$$

distribution

spatially, temporally varying field

$\pi/2_x$

σ_x σ_y

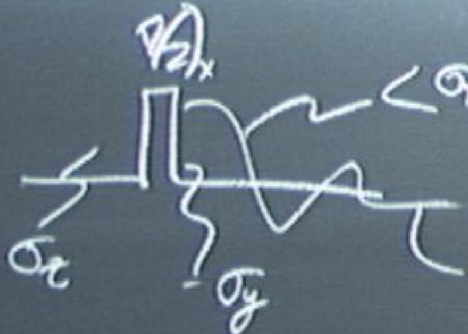
$\langle\sigma_x(t)\rangle + i\langle\sigma_y(t)\rangle$

$$S(t) = \int P(\vec{r})\, e^{-i\gamma\int_0^t B(\vec{r},t)\,dt}\, e^{-t/T_2}\, d\vec{r}$$

distribution

spatially, temporally varying field

$$S(t) = e^{i\omega_0 t}\, e^{-t/T_2^*}$$



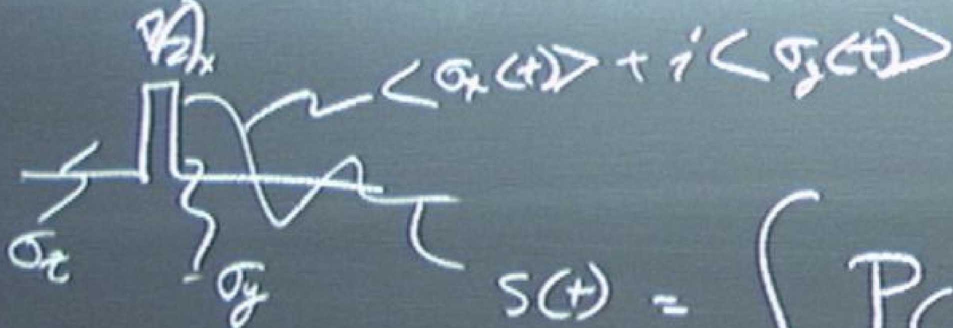
$$S(t) = \int P(\vec{r})\, e^{-i\gamma \int_0^t B(\vec{r},t)\,dt}\, e^{-t/T_2}\, d\vec{r}$$

distribution

spatially, temporally varying field

$$S(t) = e^{i\omega_0 t}\, e^{-t/T_2^*}$$

$$B(r,t) = \frac{\partial B_z}{\partial z} z$$



$$S(t) = \int P(\vec{r})\, e^{-i\gamma \int_0^t B(\vec{r},t)\,dt}\, e^{-t/T_2}\, d\vec{r}$$

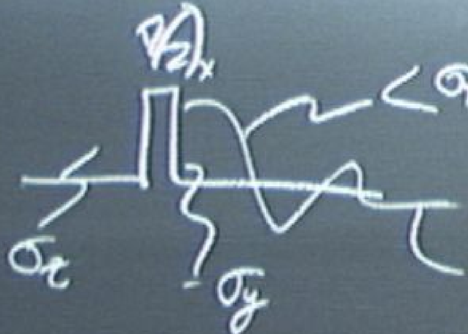
distribution

spatially, temporally varying field

$$S(t) = e^{i\omega_0 t}\, e^{-t/T_2^*}$$

$$B(r,t) = \frac{\partial B_z}{\partial z} z \quad ; \quad k(t) = \gamma \frac{\partial B_z}{\partial z} t$$

$$S(t) = \int P(r)\, e^{-ikz}\, e^{-t/T_2}\, dr$$



$\langle \sigma_x(t) \rangle + i \langle \sigma_y(t) \rangle$

static

$$S(t) = \int P(\vec{r})\, e^{-i\gamma\int_0^t B(\vec{r},t)\,dt}\, e^{-t/T_2}\, d\vec{r}$$

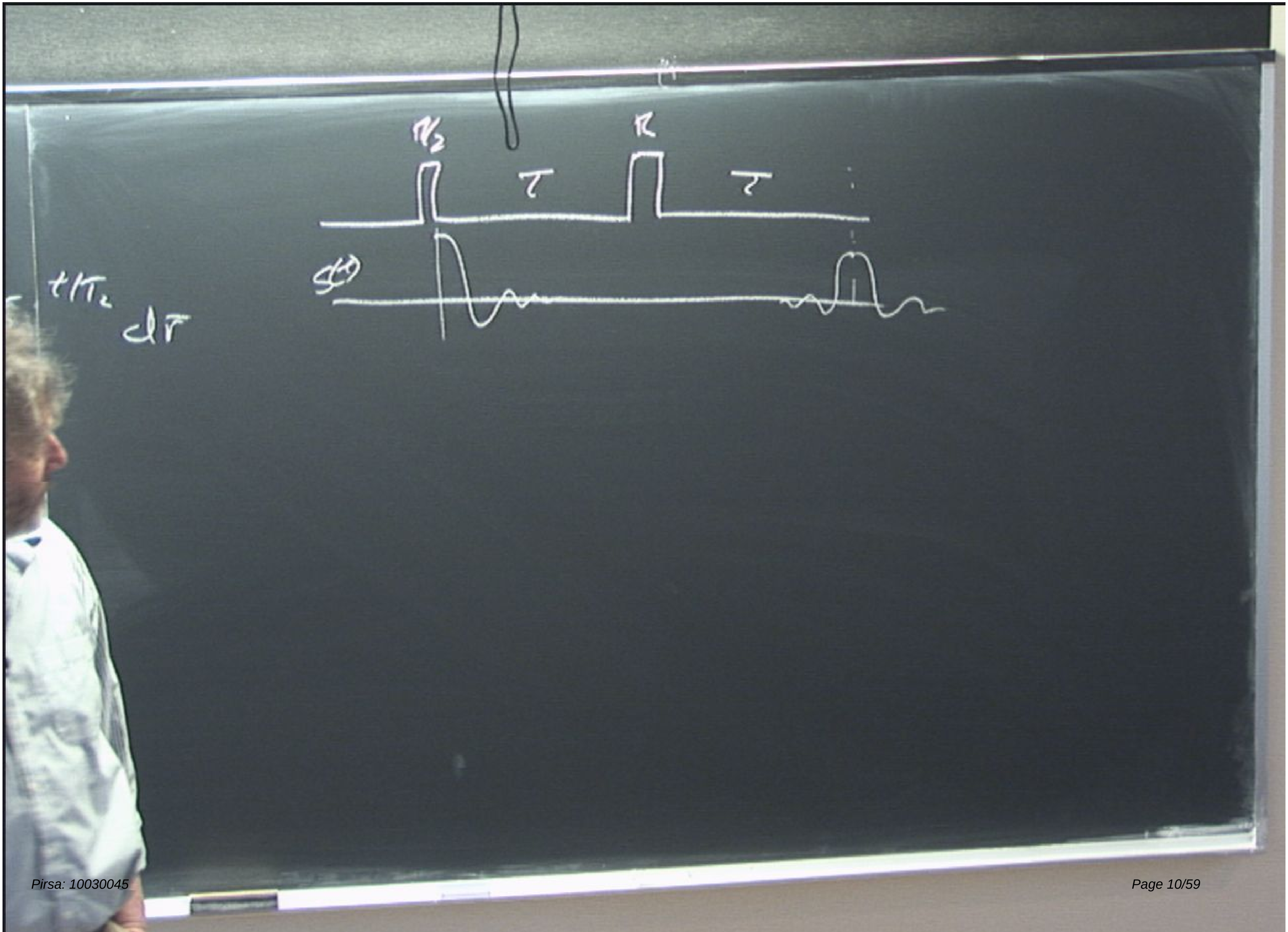
distribution

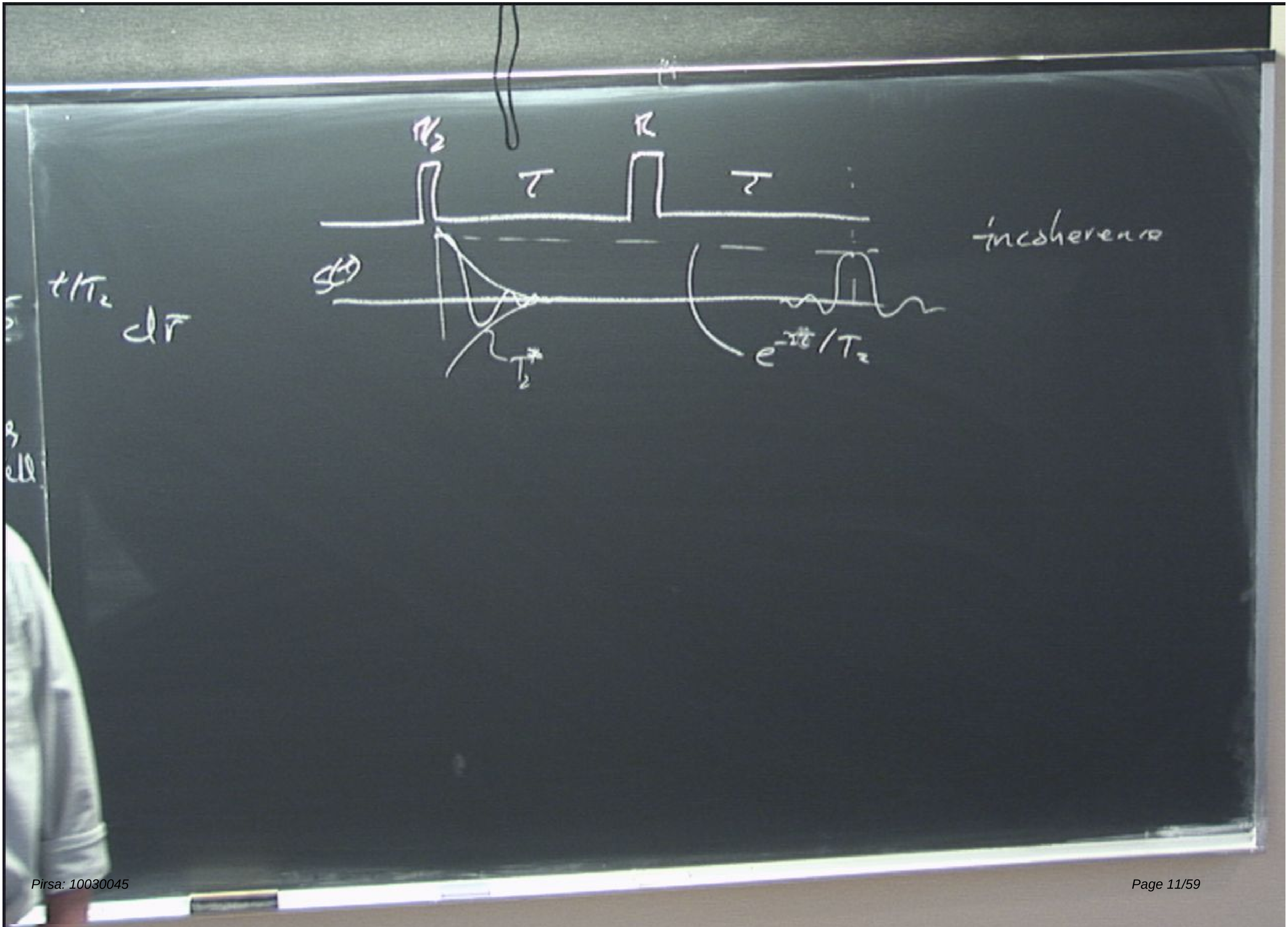
spatially, temporally varying field

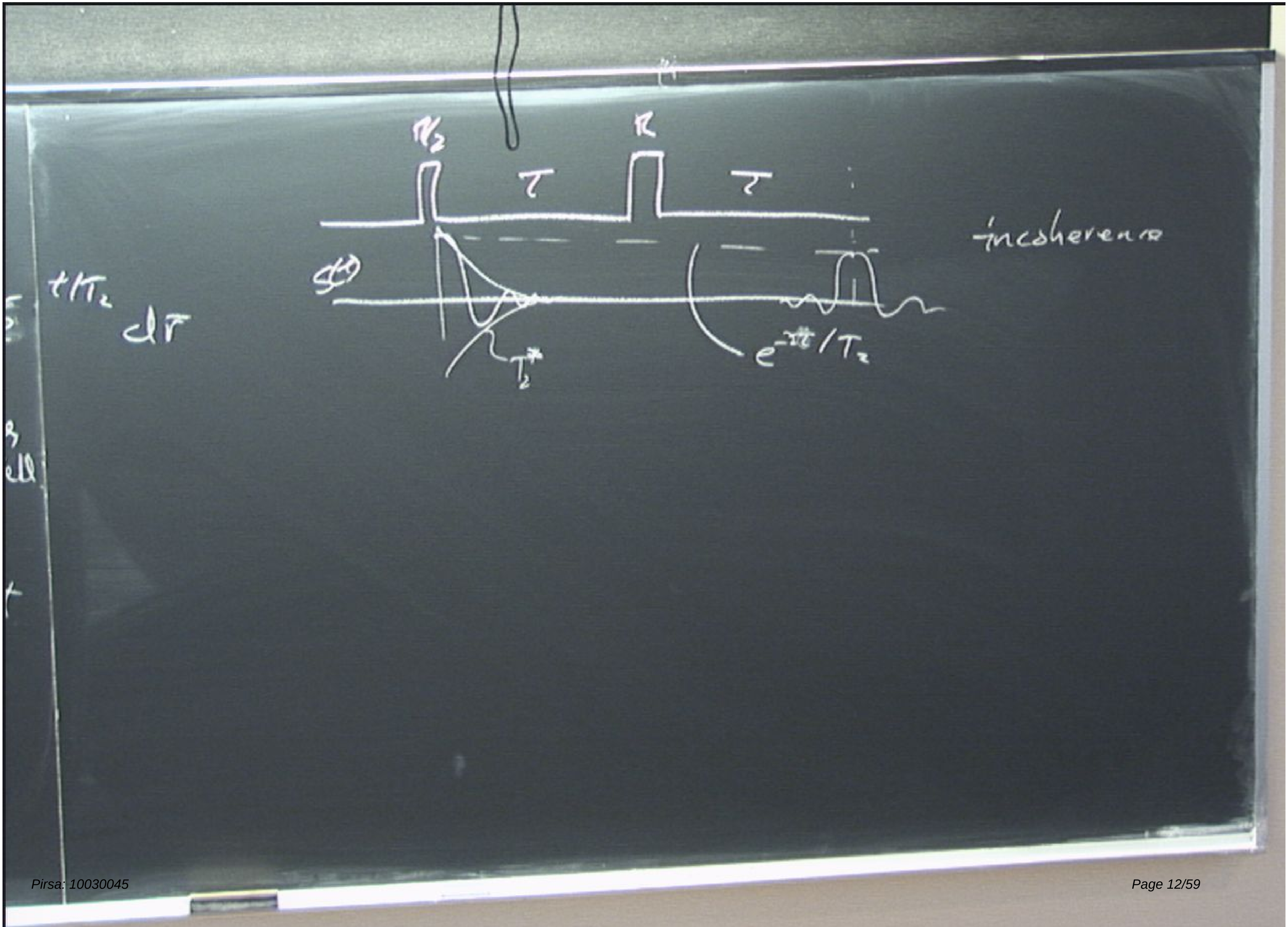
$$\boxed{S(t) = e^{i\omega_0 t}\, e^{-t/T_2^*}}$$

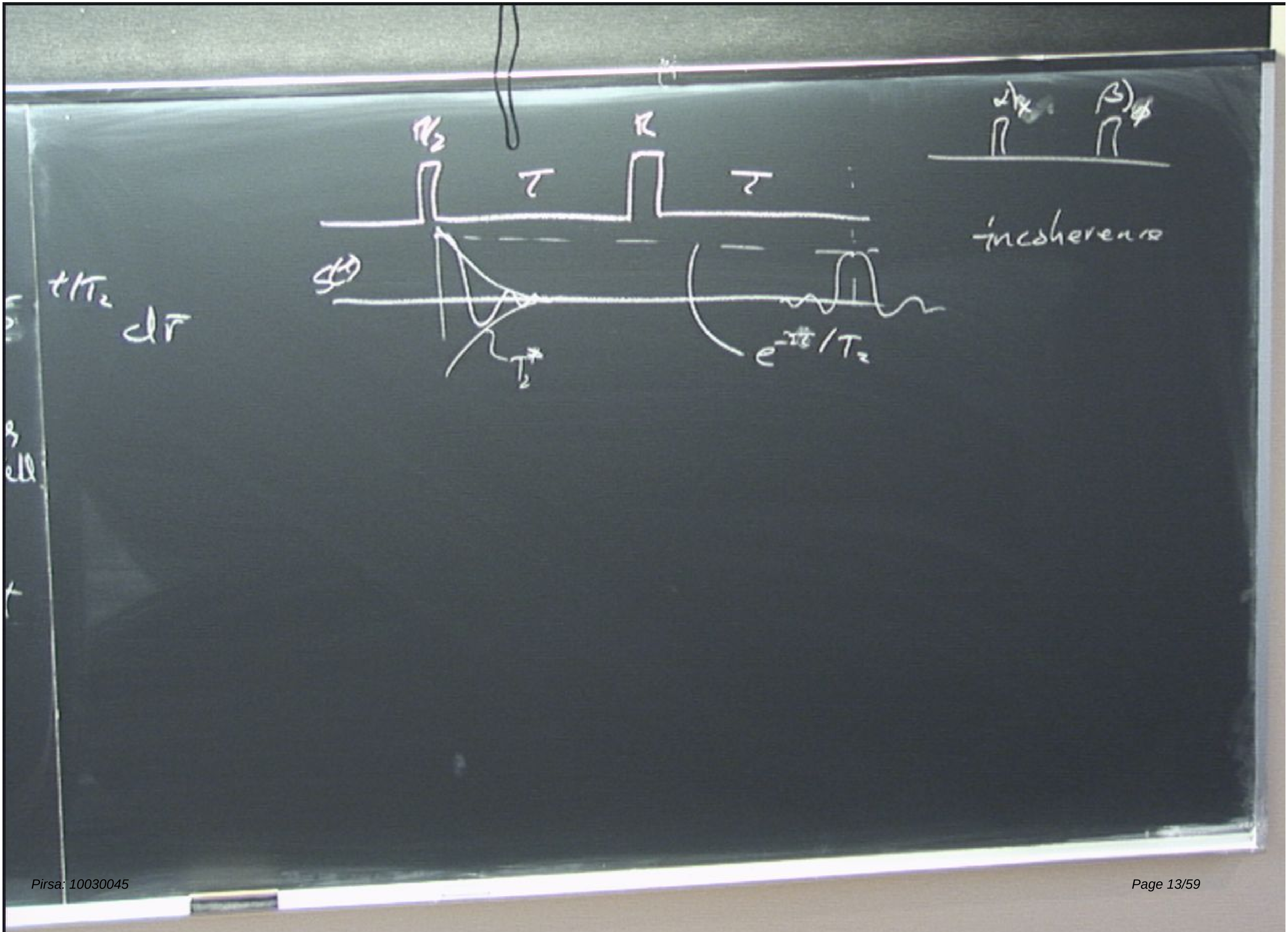
$$B(r,t) = \frac{\partial B_z}{\partial z} z \;;\; k(t) = \gamma \frac{\partial B_z}{\partial z} t$$

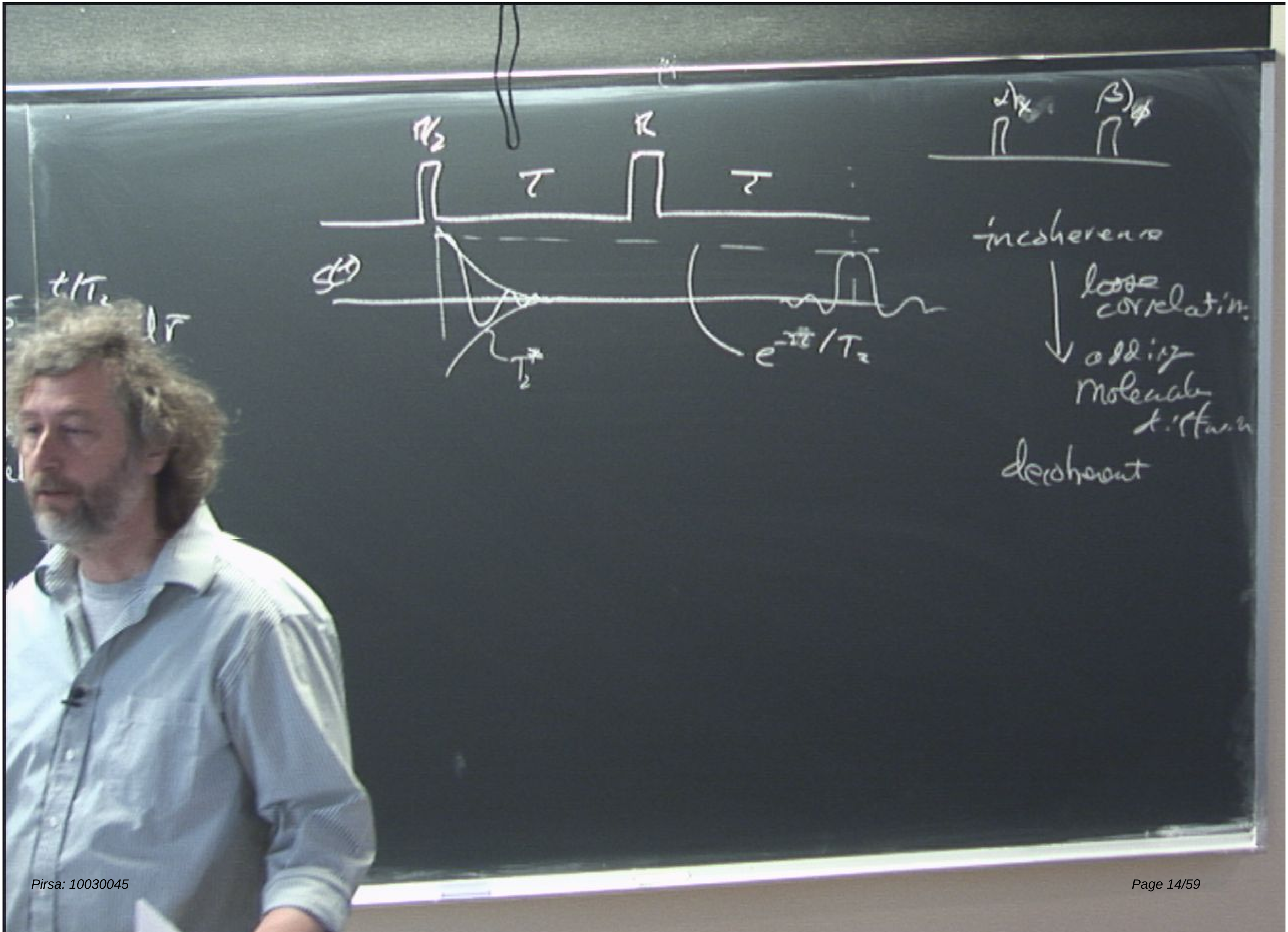
$$S(t) = \int P(r)\, e^{-ikz}\, e^{-t/T_2}\, dr$$

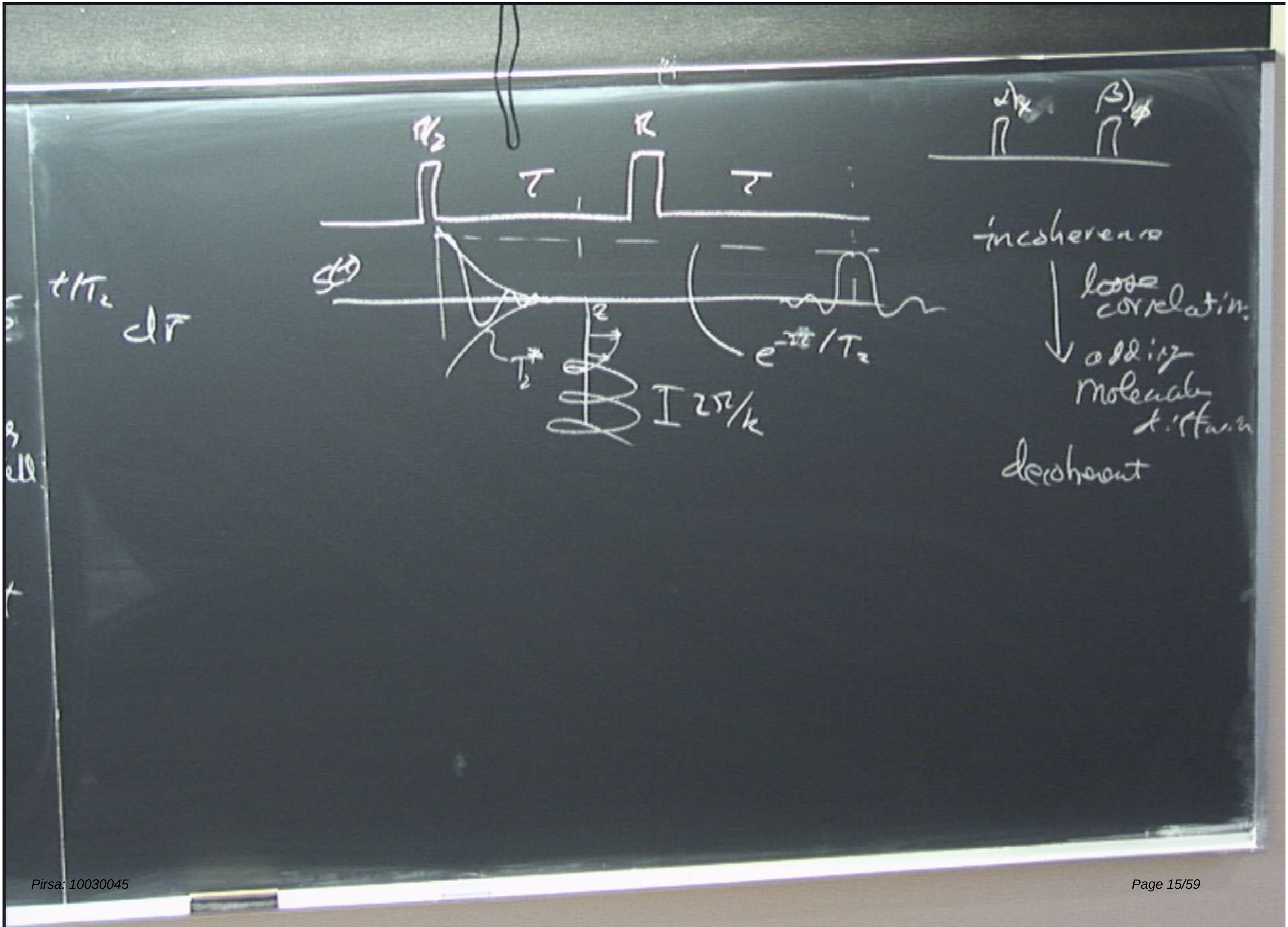

π/2
τ
π
τ
s(t)

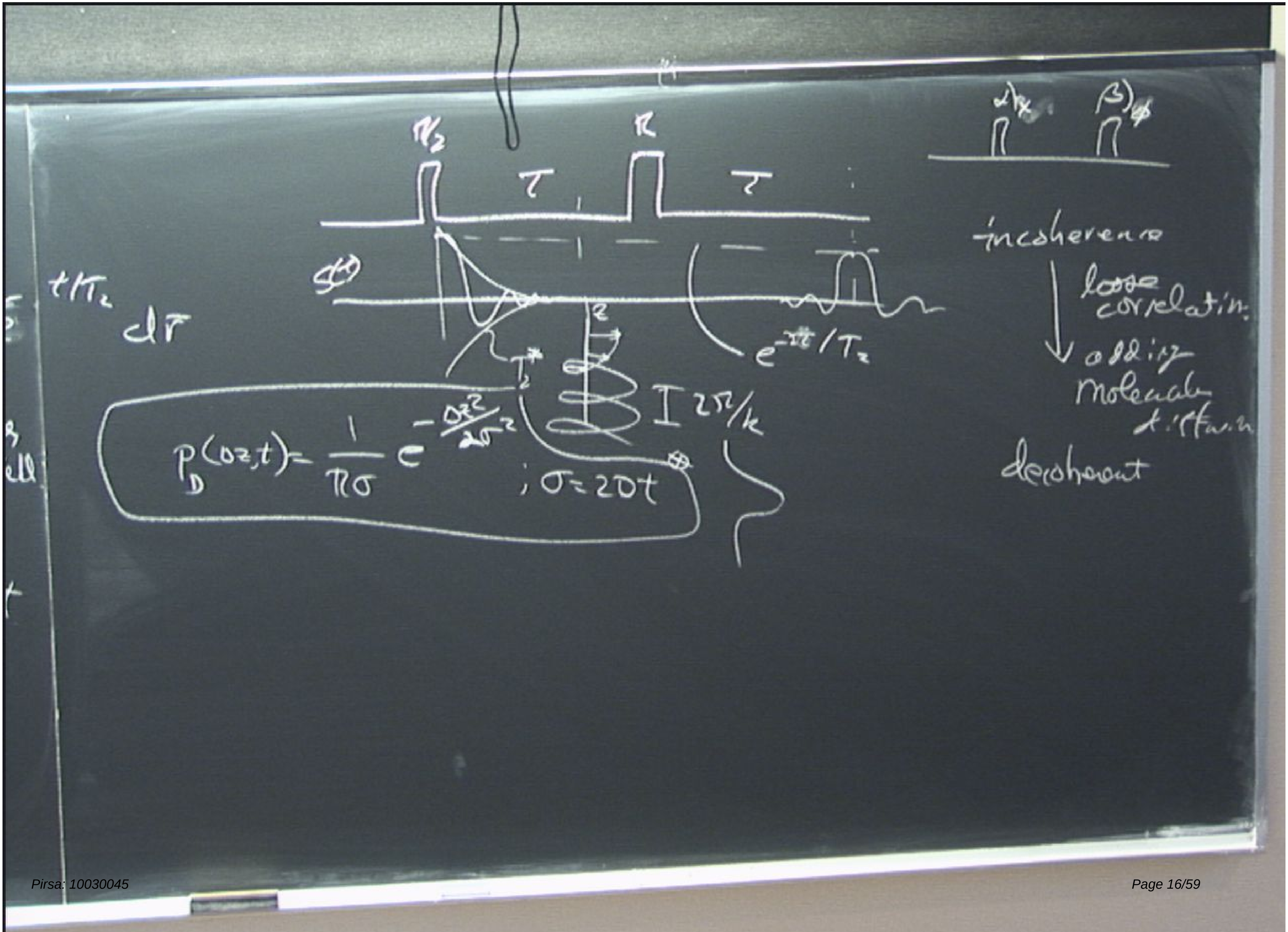

π/2
π
τ
τ
incoherence
S(t)
t/T₂
dF
T₂*
e^{-2τ}/T₂


π/2
τ
π
τ
incoherence
S(t)
T2*
e^-2τ/T2

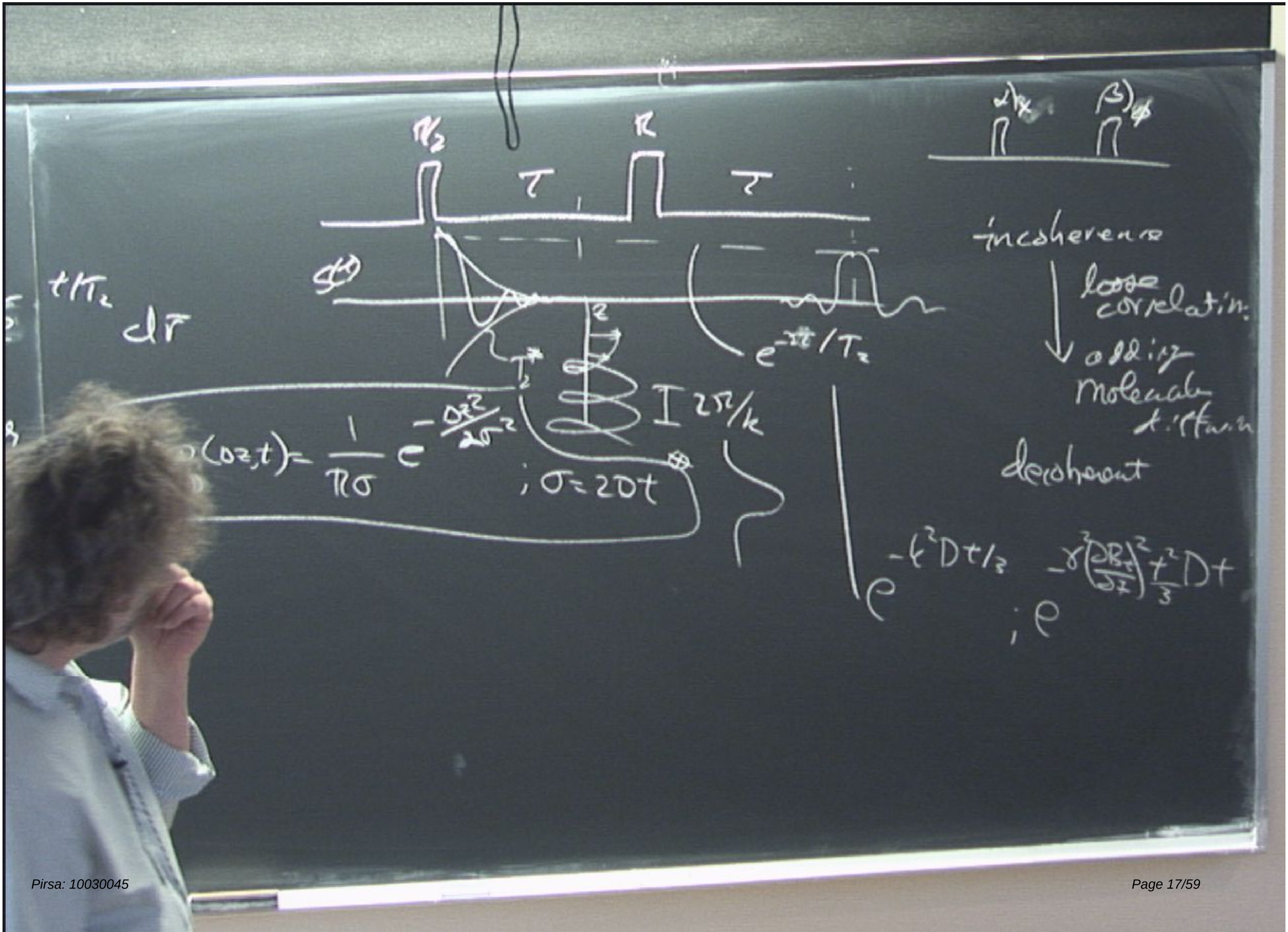

π/2
τ
π
τ
incoherence
S(t)
t/T₂
T₂*
e^{-2τ}/T₂

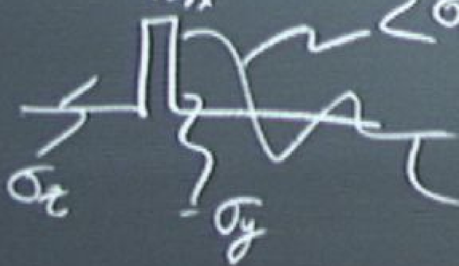

π/2
π
τ
τ
S(t)
T_2^*
$e^{-2\tau}/T_2$
α)x
β)φ
incoherent
loose correlation
adding molecular diffusion
decoherent


π/2
π
τ
τ
α)x
β)φ
S(t)
t/πT2
dr
T2*
e^{-2τ/T2}
I 2π/k
incoherence
loose correlation
adding molecular diffusion
decoherent



π/2
π
τ
τ
α)x
β)φ
incoherence
loose correlation
adding molecular diffusion
decoherent
e^(-2τ/T2)
T2*
2π/k
P_D(Δz,t) = 1/(πσ) e^(-Δz²/2σ²) ; σ = 2Dt


incoherence
correlation
decoherent



$\langle \sigma_x(t) \rangle + i \langle \sigma_y(t) \rangle$

static

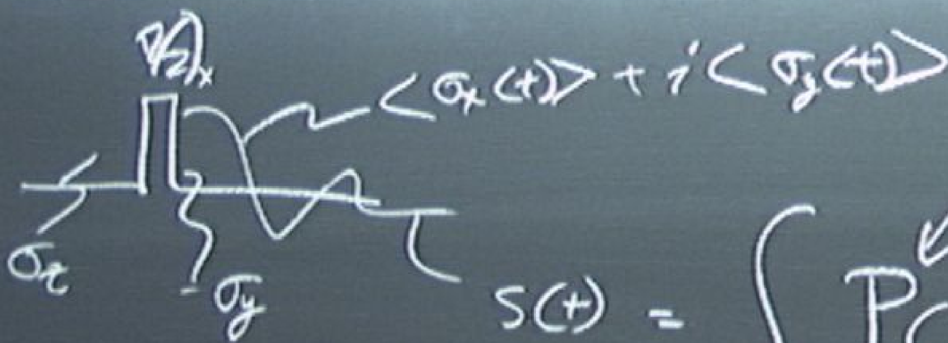
$$S(t) = \int P(\vec{r})\, e^{-i\gamma \int_0^t B(\vec{r}, t)\, dt} \quad e^{-t/T_2}$$

distribution

spatially, temporally varying field

$$\boxed{S(t) = e^{i\omega_0 t}\, e^{-t/T_2^*}}$$

$$T^{-1}_{2,\mathrm{Diff}} = \gamma^2 \left(\frac{\partial B_z}{\partial z}\right)^2 \frac{D}{3}\, t^2$$



static

$$S(t) = \int P(\vec{r})\, e^{-i\gamma\int_0^t B(\vec{r},t)\,dt} \qquad e^{-t/T_2}$$

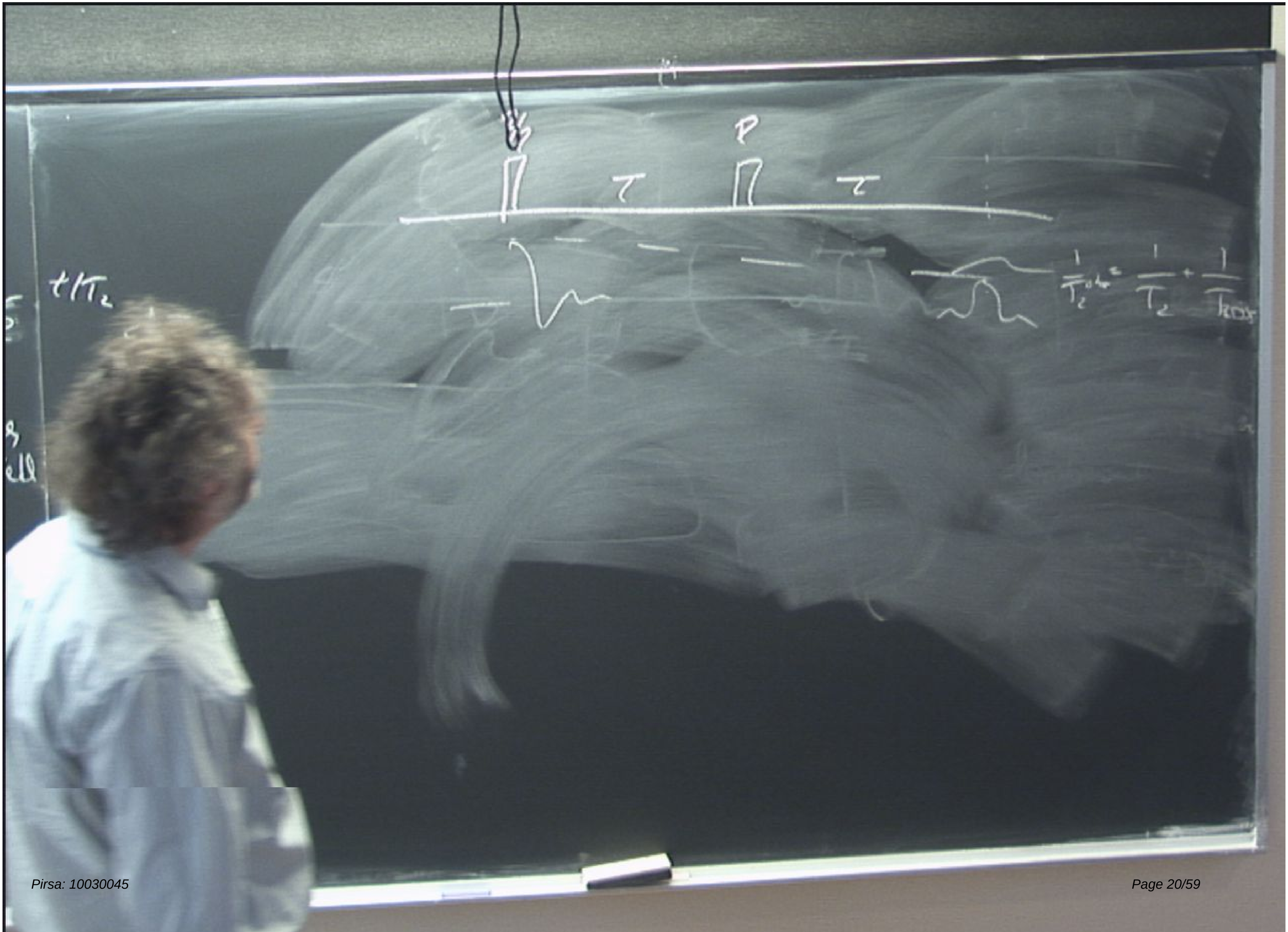
distribution

spatially, temporally varying field

$$\boxed{S(t) = e^{i\omega t}\, e^{-t/T_2^*}}$$

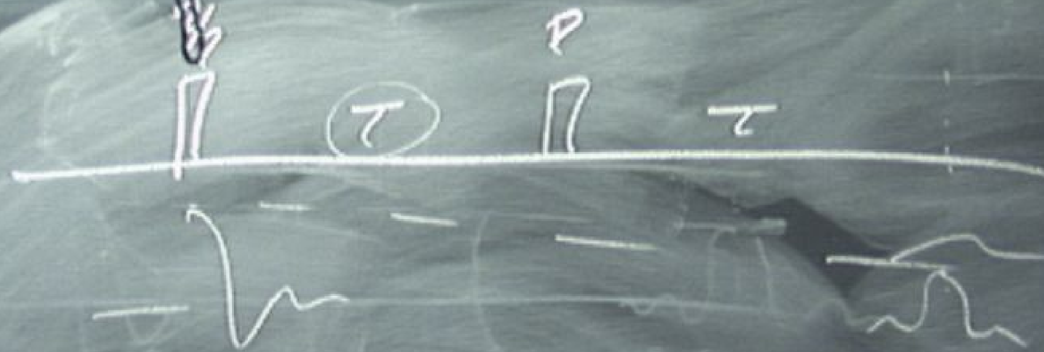
$$T^{-1}_{2,\mathrm{Diff}} = \underbrace{\gamma^2 \left(\frac{\partial B_z}{\partial z}\right)^2 \frac{D}{3}}_{\text{fluctuating field}}\ t^2$$

correlation time

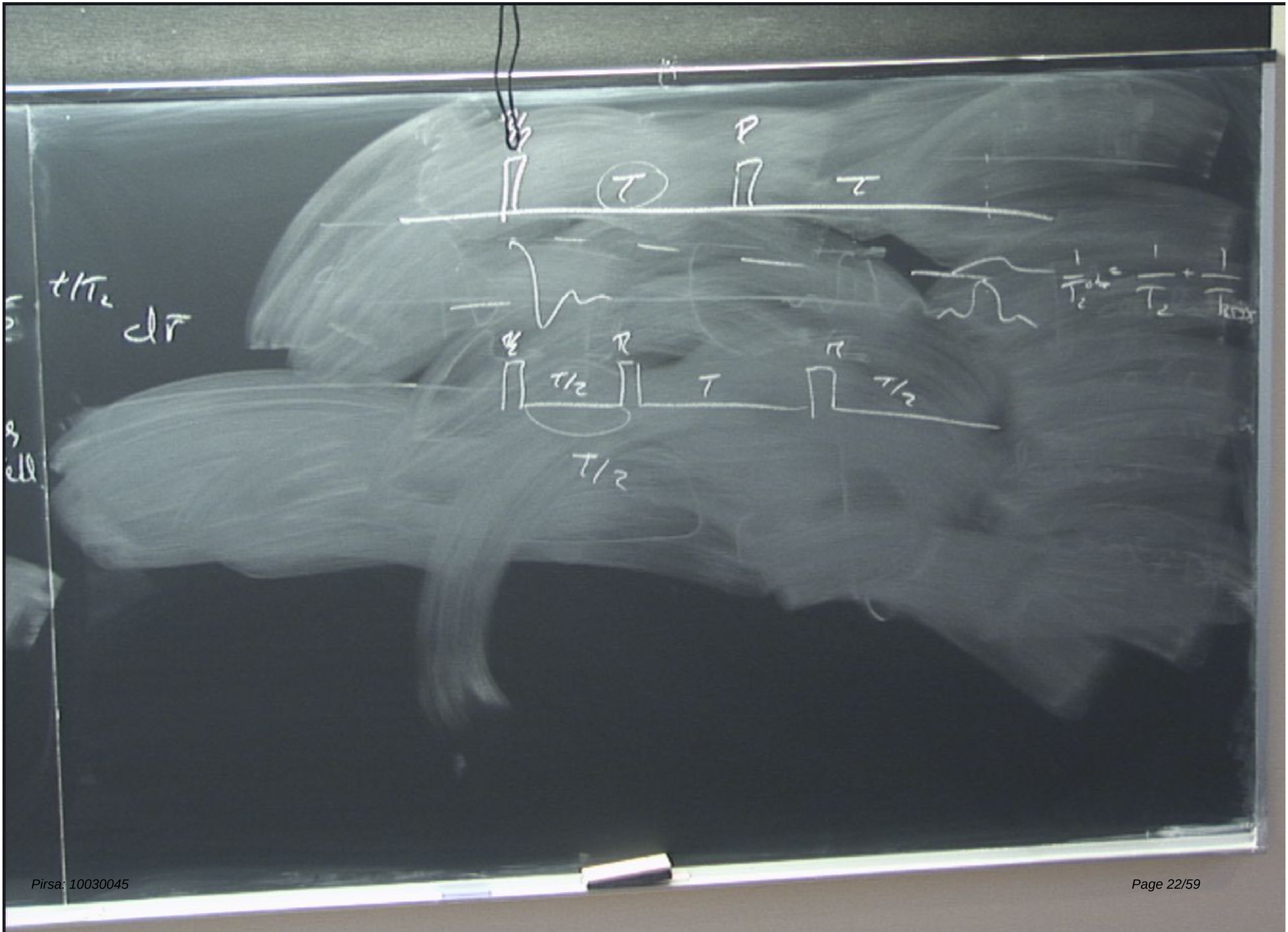

P
τ
τ

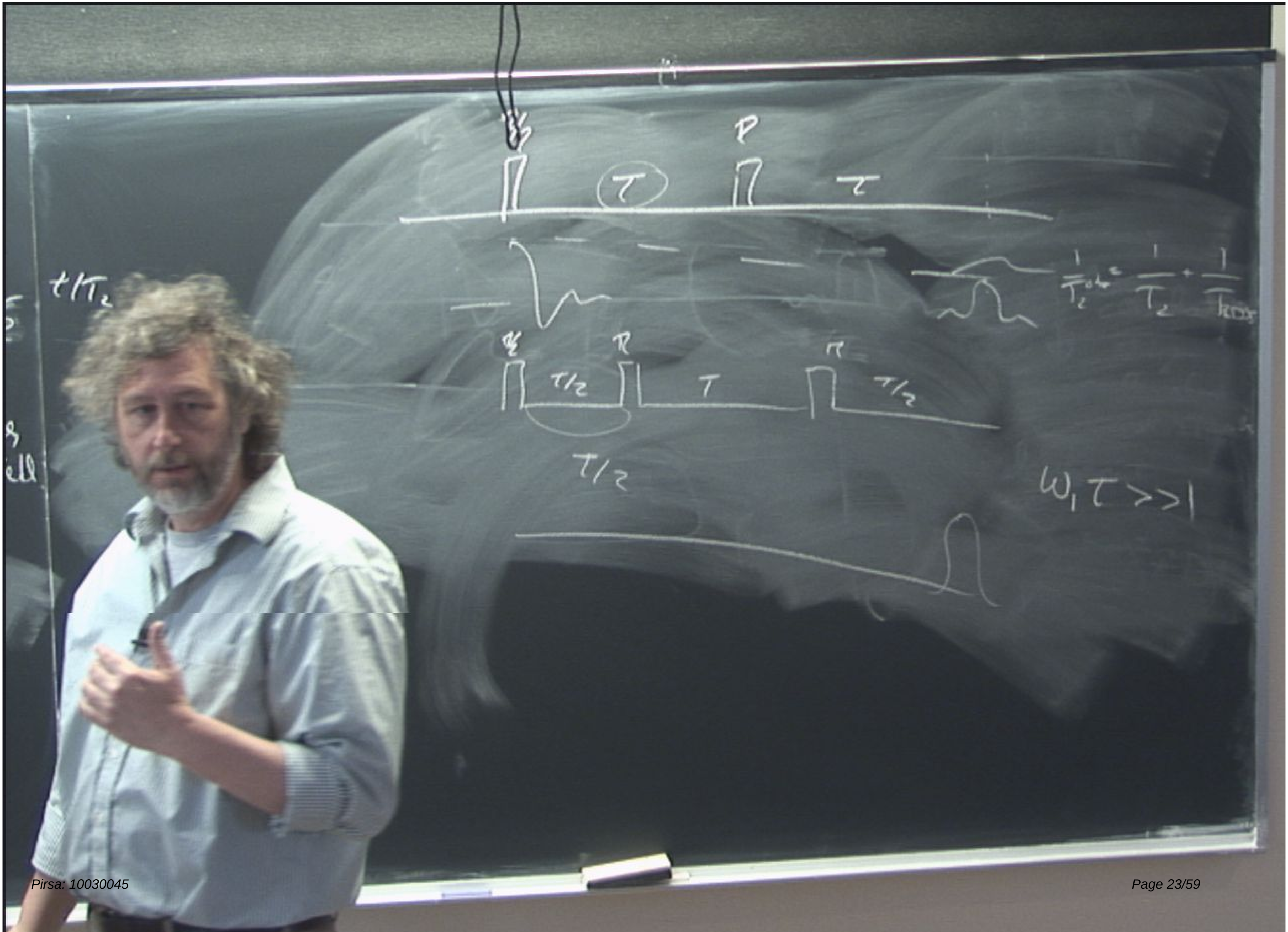
t/T_2

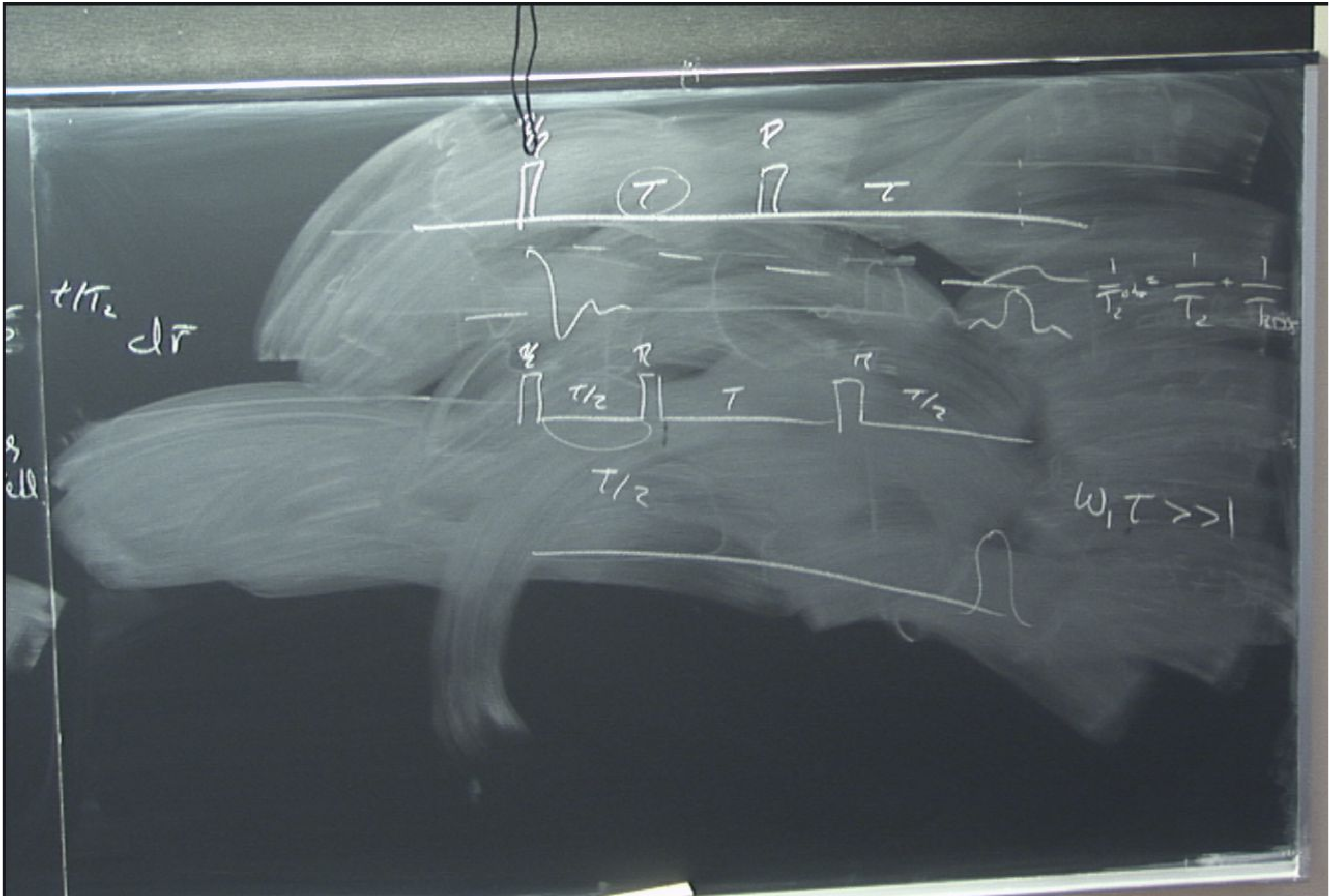
$d\vec{r}$



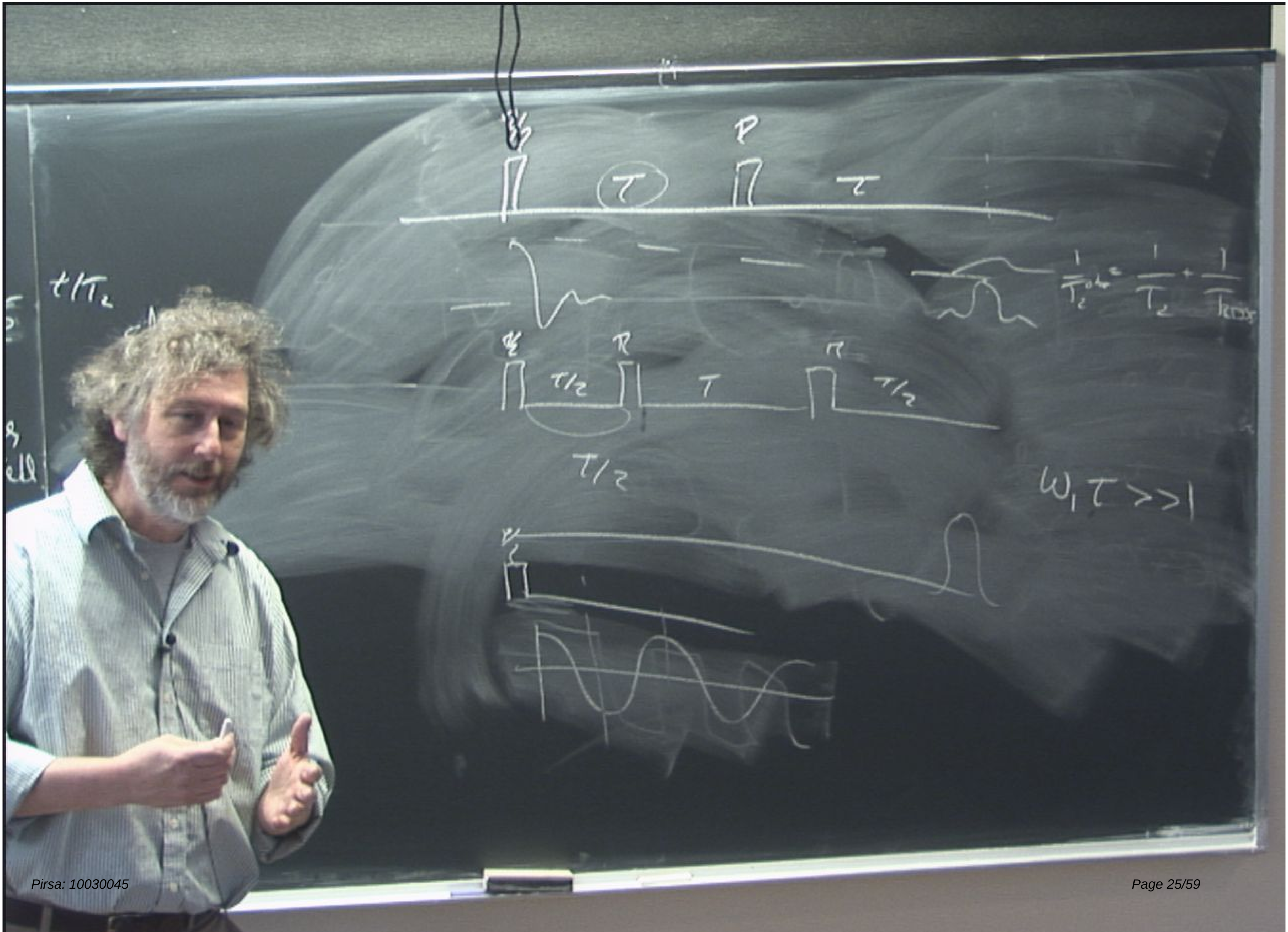
$$\frac{1}{T_2^{obs}} = \frac{1}{T_2} + \frac{1}{T_{loss}}$$

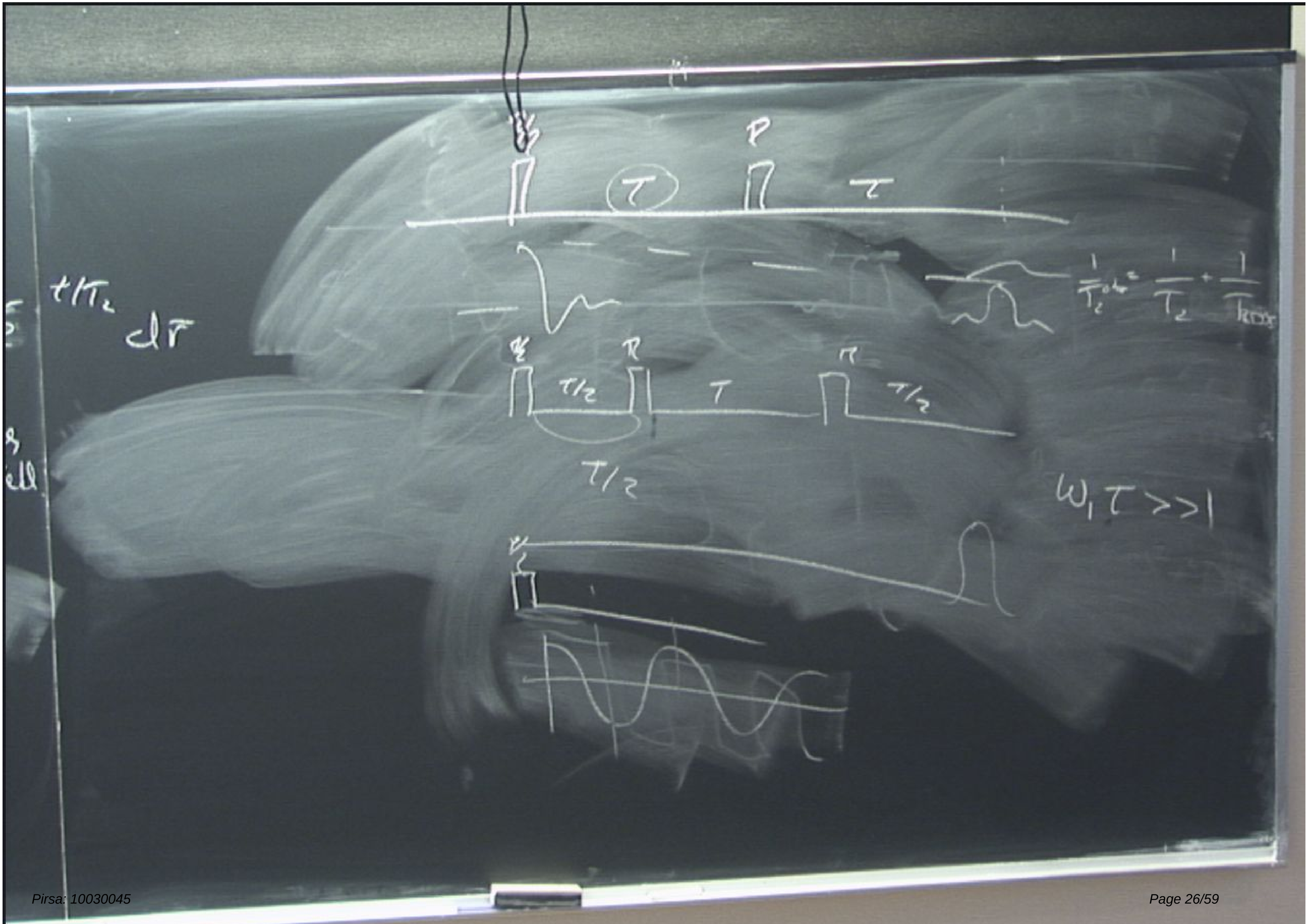

P
τ
t/T₂
dF
R
R
T/2
T
T/2
T/2


τ
τ
t/T₂
T/2
T
T/2
T/2
ω₁τ >> 1

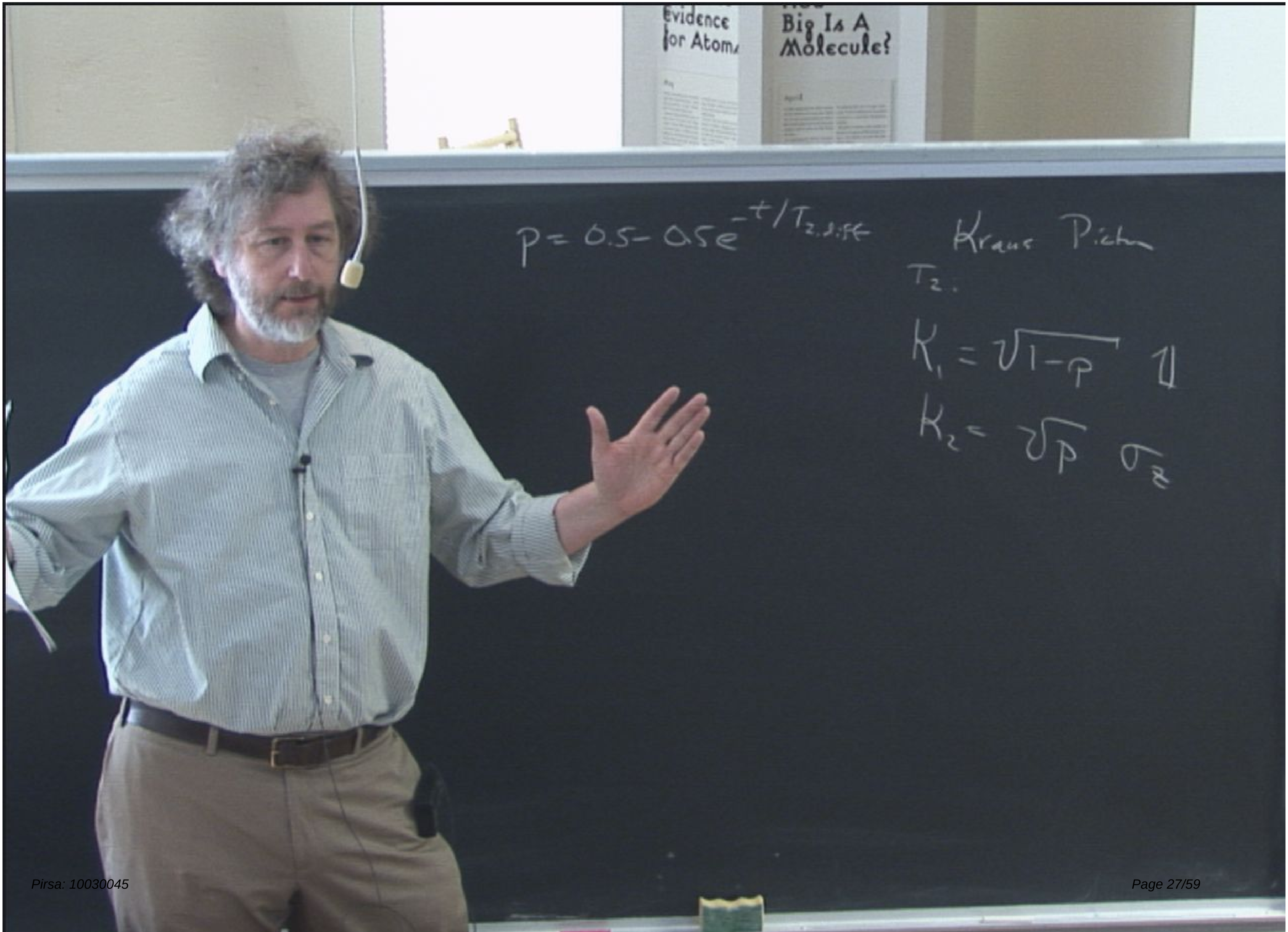


π/2
π
τ
τ
t/T2
dr
1/T2obs = 1/T2 + 1/Tloss
π/2
π
π
τ/2
τ
τ/2
τ/2
ω1 τ >> 1


t/T_2
π/2
π
τ
π/2
π
π
τ/2
τ
τ/2
T/2
ω_1 τ >> 1
1/T_2^obs = 1/T_2 + 1/T_diff



t/T₂
dF
P
τ
1/T₂obs = 1/T₂ + 1/T_loss
π/2
π
τ/2
τ
τ/2
ω₁τ >> 1


Evidence
for Atom
Big Is A
Molecule?
$p = 0.5 - 0.5 e^{-t/T_{2,diff}}$
Kraus Pict
T_2:
$K_1 = \sqrt{1-p}\ \mathbb{1}$
$K_2 = \sqrt{p}\ \sigma_z$

$$p = 0.5 - 0.5\, e^{-t/T_{2,\text{diff}}}$$

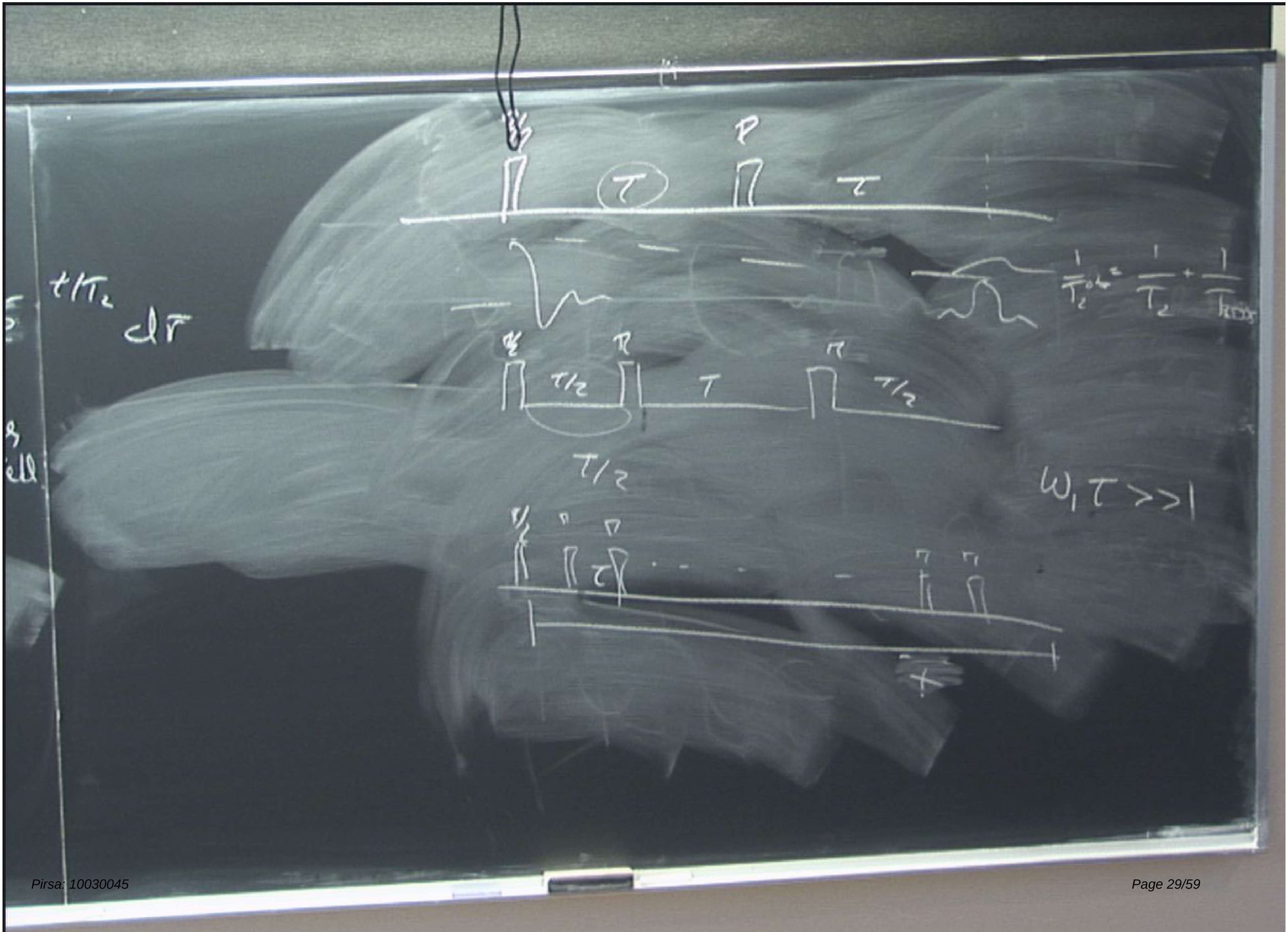
$$p = t/_{t,\text{diff}}$$

Kraus Picture

T_2:

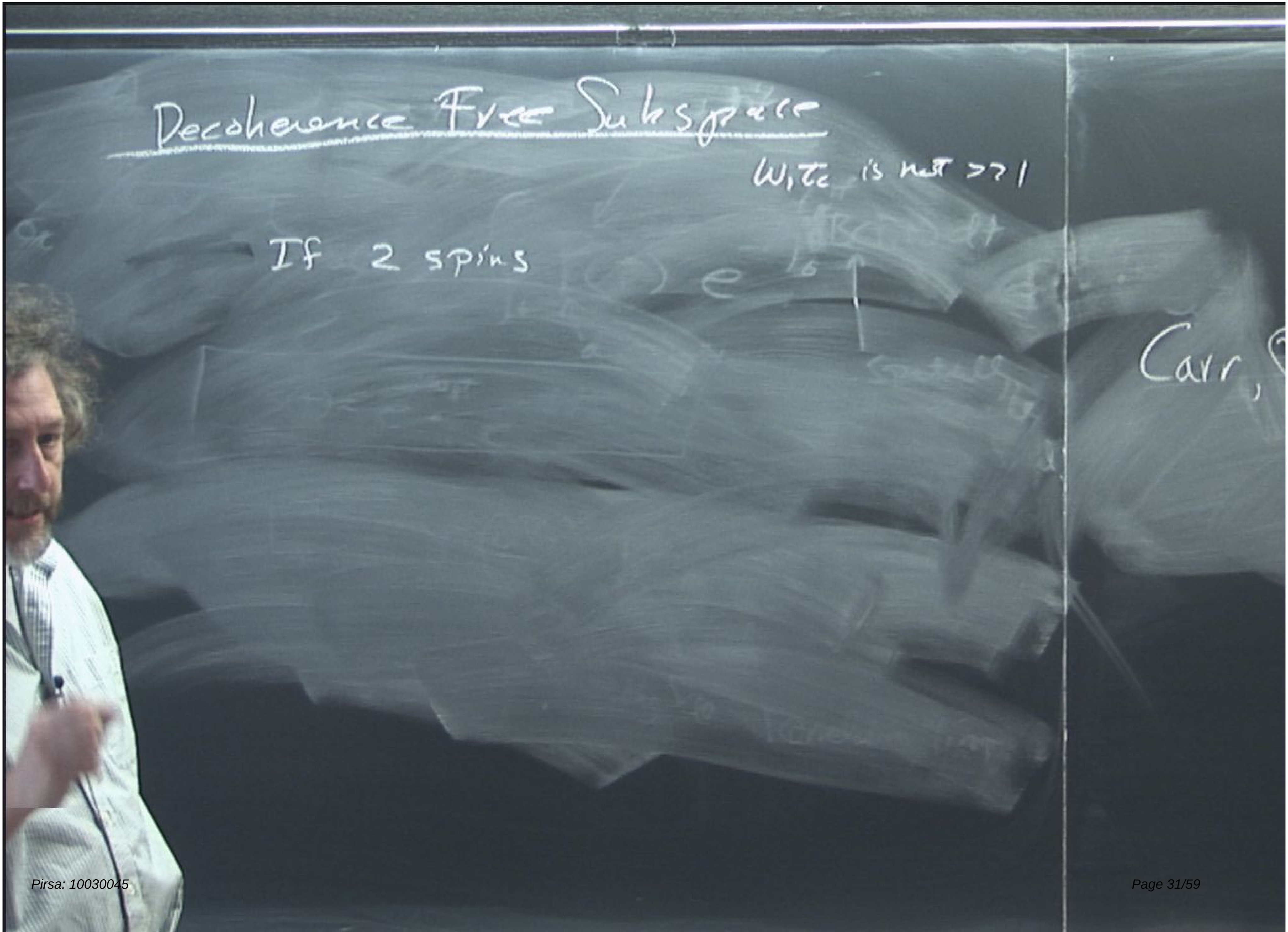
$$K_1 = \sqrt{1-p}\ \mathbb{1}$$

$$K_2 = \sqrt{p}\ \sigma_z$$


t/T2
dr
π/2
π
τ
τ/2
τ/2
τ/2
τ
ω1 τ >> 1
1/T2obs = 1/T2 + 1/Tloss

Decoherence Free Subspace

$\omega_1 \tau_c$ is not $\gg 1$

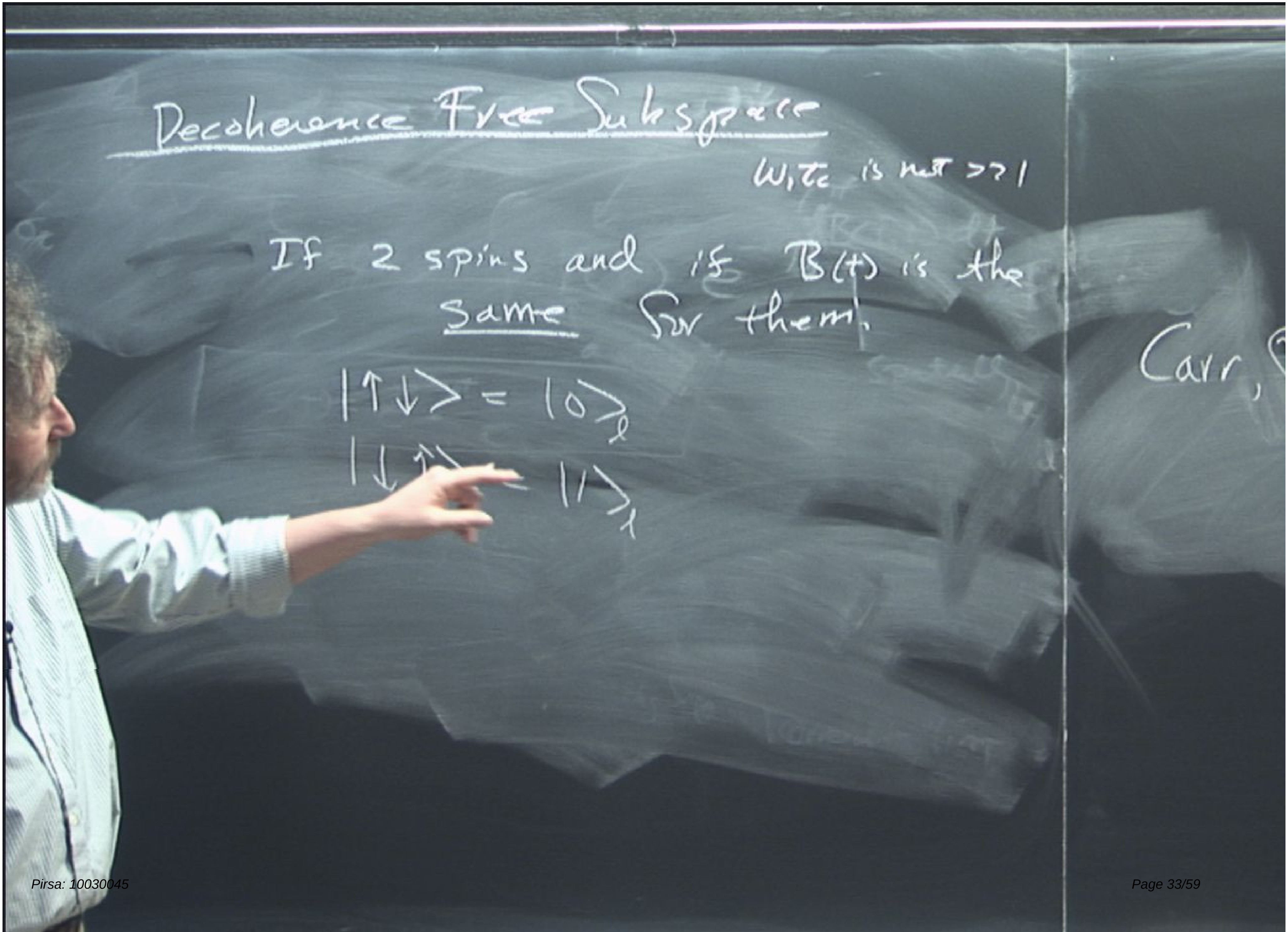

Decoherence Free Subspace
$\omega_1 \tau_c$ is not >> 1
If 2 spins
Carr,

Decoherence Free Subspace

$\omega_1 \tau_c$ is not $\gg 1$

If 2 spins and if $B(t)$ is the <u>same</u> for them.

Carr, ...


Decoherence Free Subspace
ω₁τc is not >> 1
If 2 spins and if B(t) is the same for them.
|↑↓> = |0>l
|↓↑> = |1>l
Carr,

Decoherence Free Subspace

$\omega_1 \tau_c$ is not $\gg 1$

If 2 spins and if $B(t)$ is the <u>same</u> for them.

$$|\uparrow\downarrow\rangle = |0\rangle_L$$
$$|\downarrow\uparrow\rangle = |1\rangle_L$$

Carr,

Decoherence Free Subspace

$\omega_1 \tau_c$ is not $\gg 1$

If 2 spins and if $B(t)$ is the same for them.

$$|\uparrow\downarrow\rangle = |0\rangle_l$$
$$|\downarrow\uparrow\rangle = |1\rangle_l$$

$$\begin{array}{c} |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{array} \left(\quad \boxed{} \quad \right)$$

operators $\mathbb{1}^l = \sigma_z^1 \sigma_z^2$

$$\sigma_z^l = \sigma_z^1 \mathbb{1}^2 - \mathbb{1}^1 \sigma_z^2$$
$$\sigma_x^l = \sigma_x^1 \sigma_x^2 + \sigma_y^1 \sigma_y^2$$
$$\sigma_y^l = \sigma_y^1 \sigma_x^2 - \sigma_x^1 \sigma_y^2$$

Carr,

Decoherence Free Subspace

$\omega_1 \tau_c$ is not $\gg 1$

If 2 spins and if $B(t)$ is the same for them.

$$|\uparrow\downarrow\rangle = |0\rangle_l$$
$$|\downarrow\uparrow\rangle = |1\rangle_l$$

$$\begin{array}{l} |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{array} \left(\quad \boxed{} \quad \right)$$

operators $\mathbb{1}^l = \sigma_z^1 \sigma_z^2$

$$\sigma_z^l = \sigma_z^1 \mathbb{1}^2 - \mathbb{1}^1 \sigma_z^2$$
$$\sigma_x^l = \sigma_x^1 \sigma_x^2 + \sigma_y^1 \sigma_y^2$$
$$\sigma_y^l = \sigma_y^1 \sigma_x^2 - \sigma_x^1 \sigma_y^2$$

Carr,

Decoherence Free Subspace

$\omega_1 \tau_c$ is not $\gg 1$

If 2 spins and if $B(t)$ is the <u>same</u> for them.

- $|\uparrow\downarrow\rangle = |0\rangle_\ell$
- $|\downarrow\uparrow\rangle = |1\rangle_\ell$

$$\begin{array}{c} |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{array} \begin{pmatrix} & & & \\ & \boxed{\begin{array}{cc} \cdot & \cdot \\ \cdot & \end{array}} & \\ & & & \end{pmatrix}$$

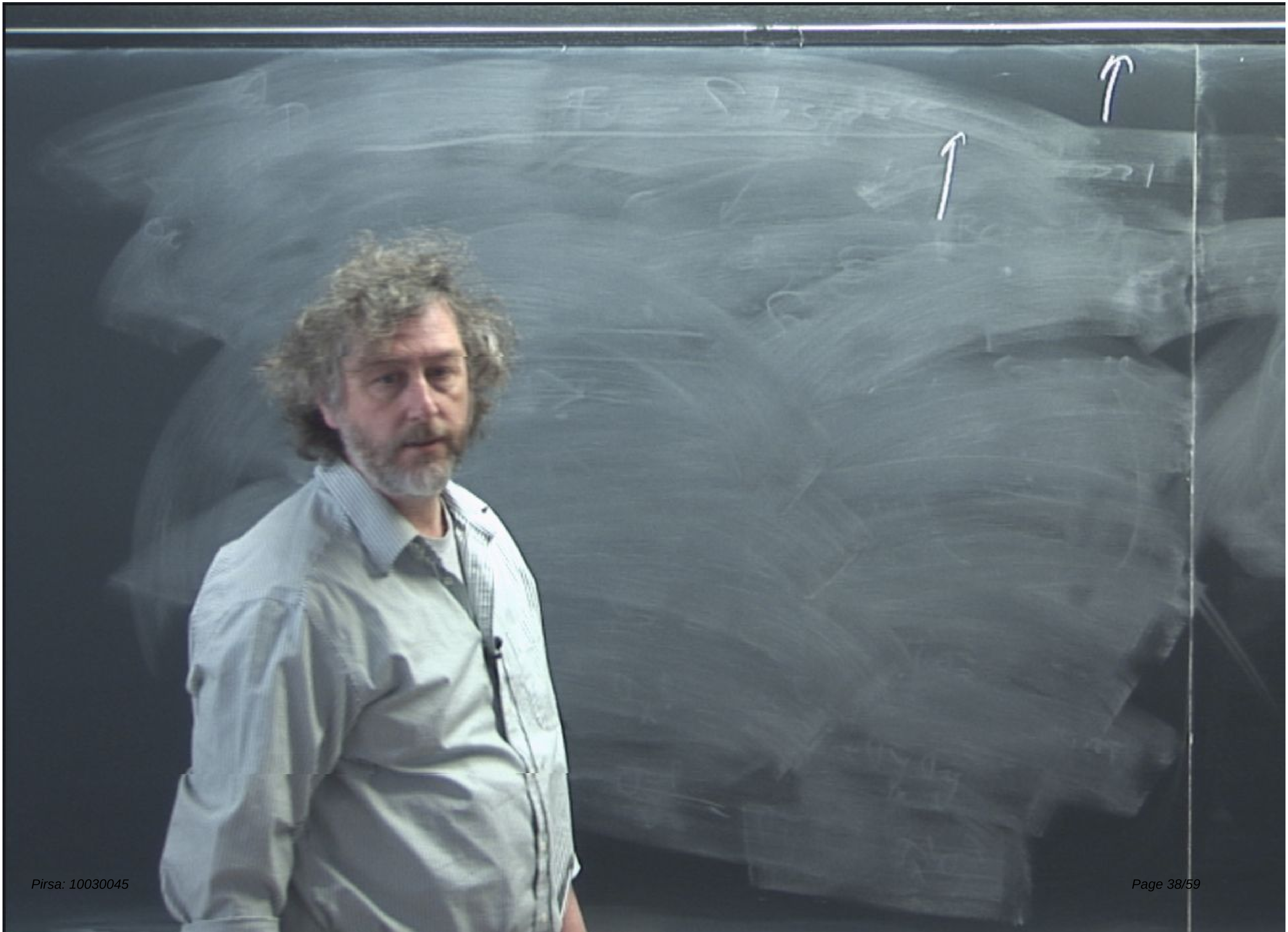
operators $\mathbb{1}^\ell = \sigma_z^1 \sigma_z^2$

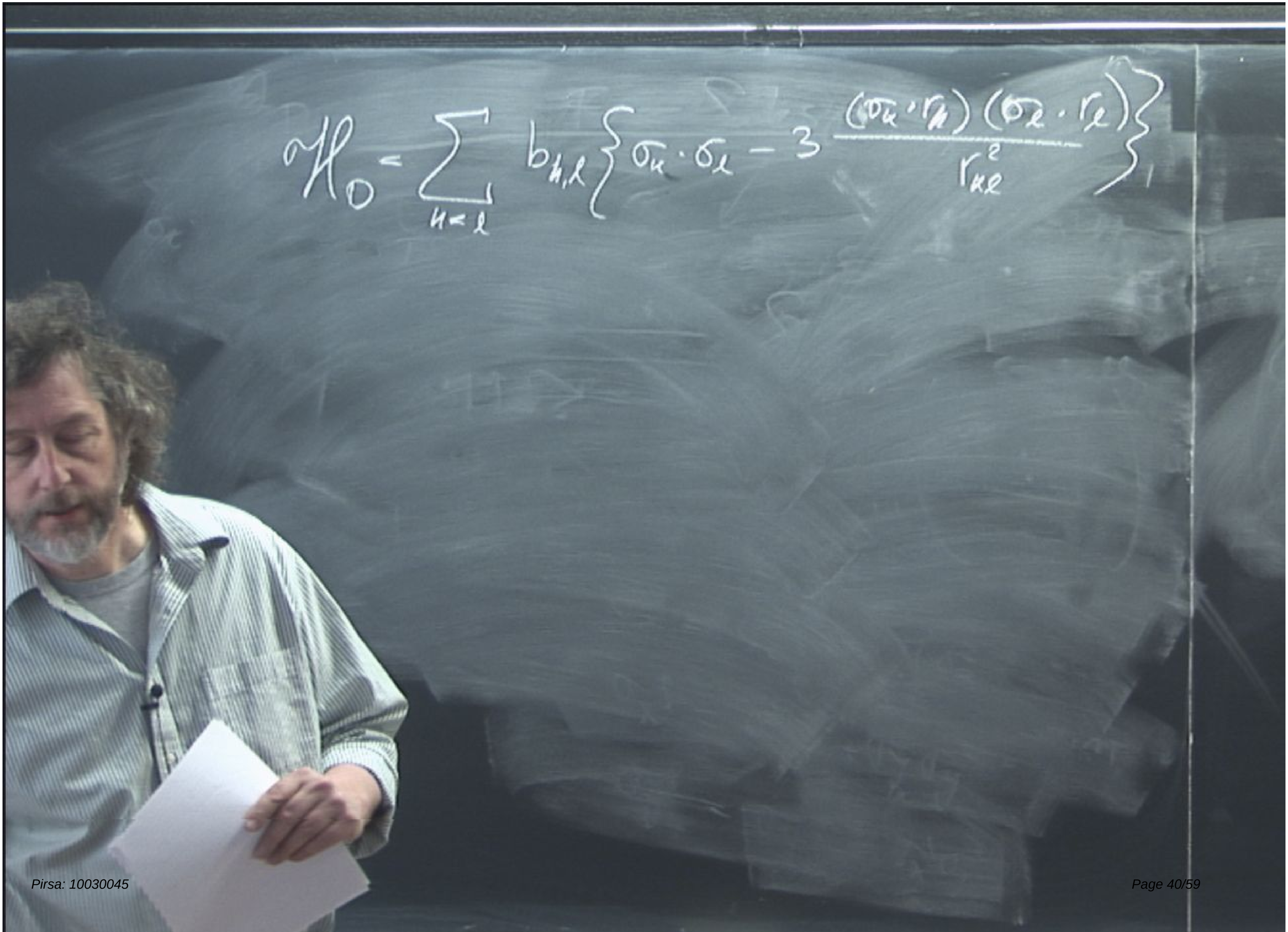
$$\sigma_z^\ell = \sigma_z^1 \mathbb{1}^2 - \mathbb{1}^1 \sigma_z^2$$

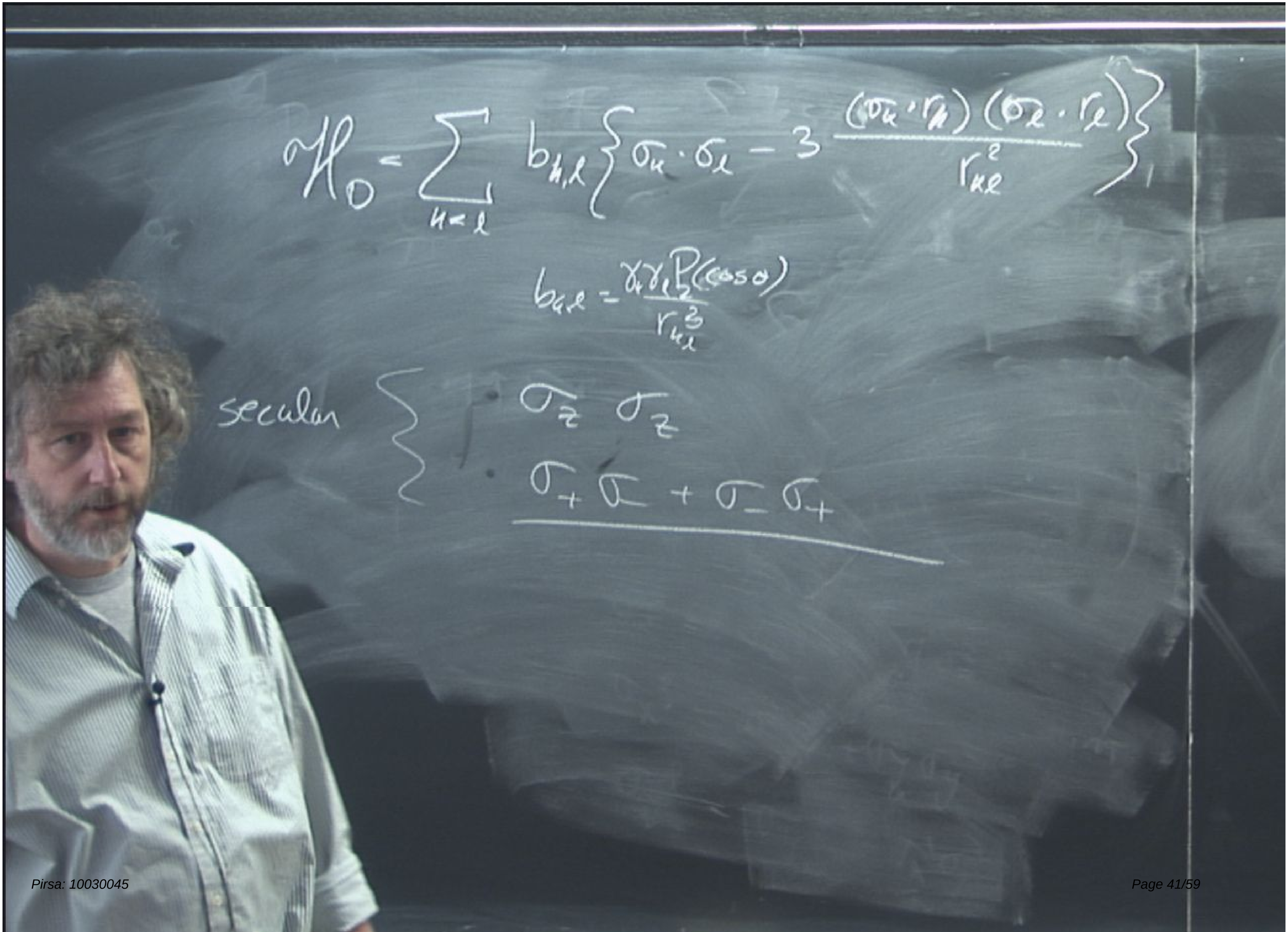
$$\sigma_x^\ell = \sigma_x^1 \sigma_x^2 + \sigma_y^1 \sigma_y^2$$

$$\sigma_y^\ell = \sigma_y^1 \sigma_x^2 - \sigma_x^1 \sigma_y^2$$

Carr, ...




$\mathcal{H}_D = \sum_{k<l} b_{k,l}\left\{\sigma_k\cdot\sigma_l - 3\frac{(\sigma_k\cdot r_k)(\sigma_l\cdot r_l)}{r_{kl}^2}\right\}$


$\mathcal{H}_D = \sum_{k<l} b_{k,l} \left\{ \sigma_k \cdot \sigma_l - 3 \frac{(\sigma_k \cdot r_{kl})(\sigma_l \cdot r_{kl})}{r_{kl}^2} \right\}$
$b_{kl} = \frac{\gamma_k \gamma_l P_2(\cos\theta)}{r_{kl}^3}$
secular
$\sigma_z \sigma_z$
$\sigma_+ \sigma_- + \sigma_- \sigma_+$

$$\mathcal{H}_D = \sum_{k<l} b_{k,l} \left\{ \sigma_k \cdot \sigma_l - 3 \frac{(\sigma_k \cdot r_k)(\sigma_l \cdot r_l)}{r_{kl}^2} \right\}$$

$$b_{kl} = \frac{\gamma_k \gamma_l P_2(\cos\theta)}{r_{kl}^3}$$

secular $\left\{ \begin{array}{l} \cdot\ \sigma_z\ \sigma_z \\ \cdot\ \sigma_+ \sigma_- + \sigma_- \sigma_+ \end{array} \right.$ $\left(1 - 3\cos^2\theta\right)$

$$\mathcal{H}_D = \sum_{k<l} b_{k,l}\left\{ \sigma_k \cdot \sigma_l - 3\,\frac{(\sigma_k \cdot r_k)(\sigma_l \cdot r_l)}{r_{kl}^2} \right\},$$

$$b_{kl} = \frac{\gamma_k \gamma_l P_2(\cos\theta)}{r_{kl}^3}$$

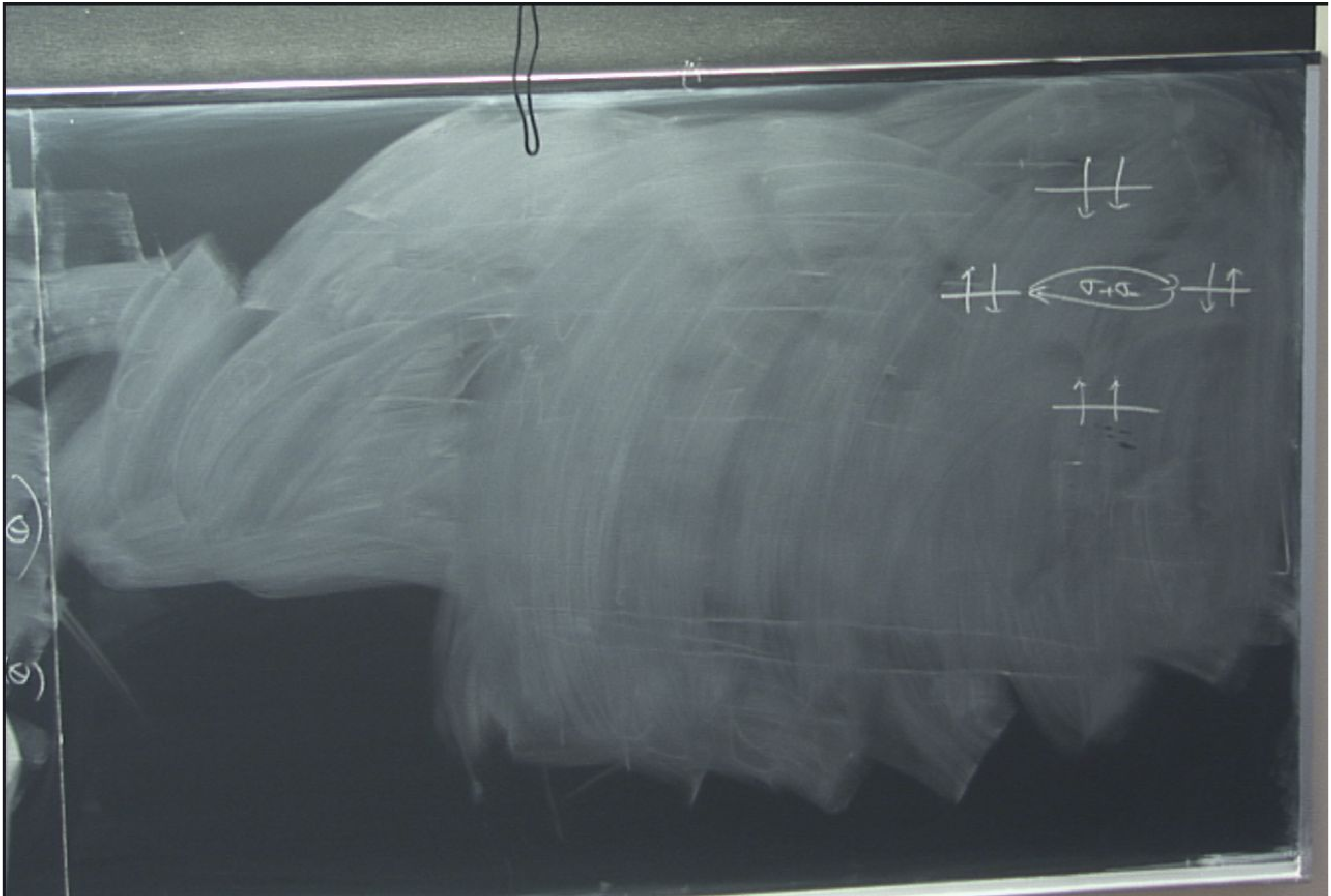
secular $\left\{ \begin{array}{l} \sigma_z \sigma_z \\ \sigma_+ \sigma_- + \sigma_- \sigma_+ \end{array} \right.$ $\quad (1 - 3\cos^2\theta)$

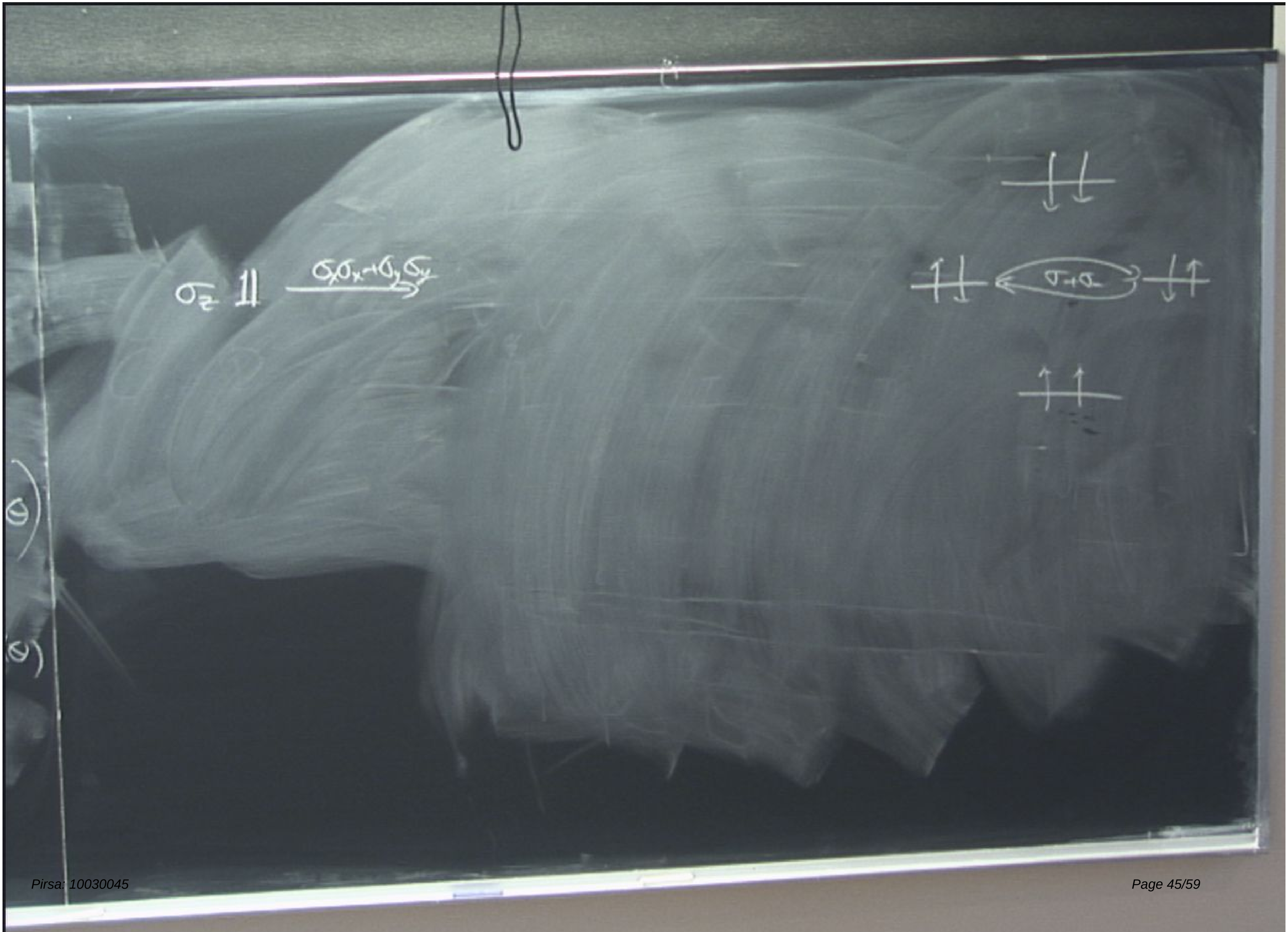
$\sigma_z \sigma_+$

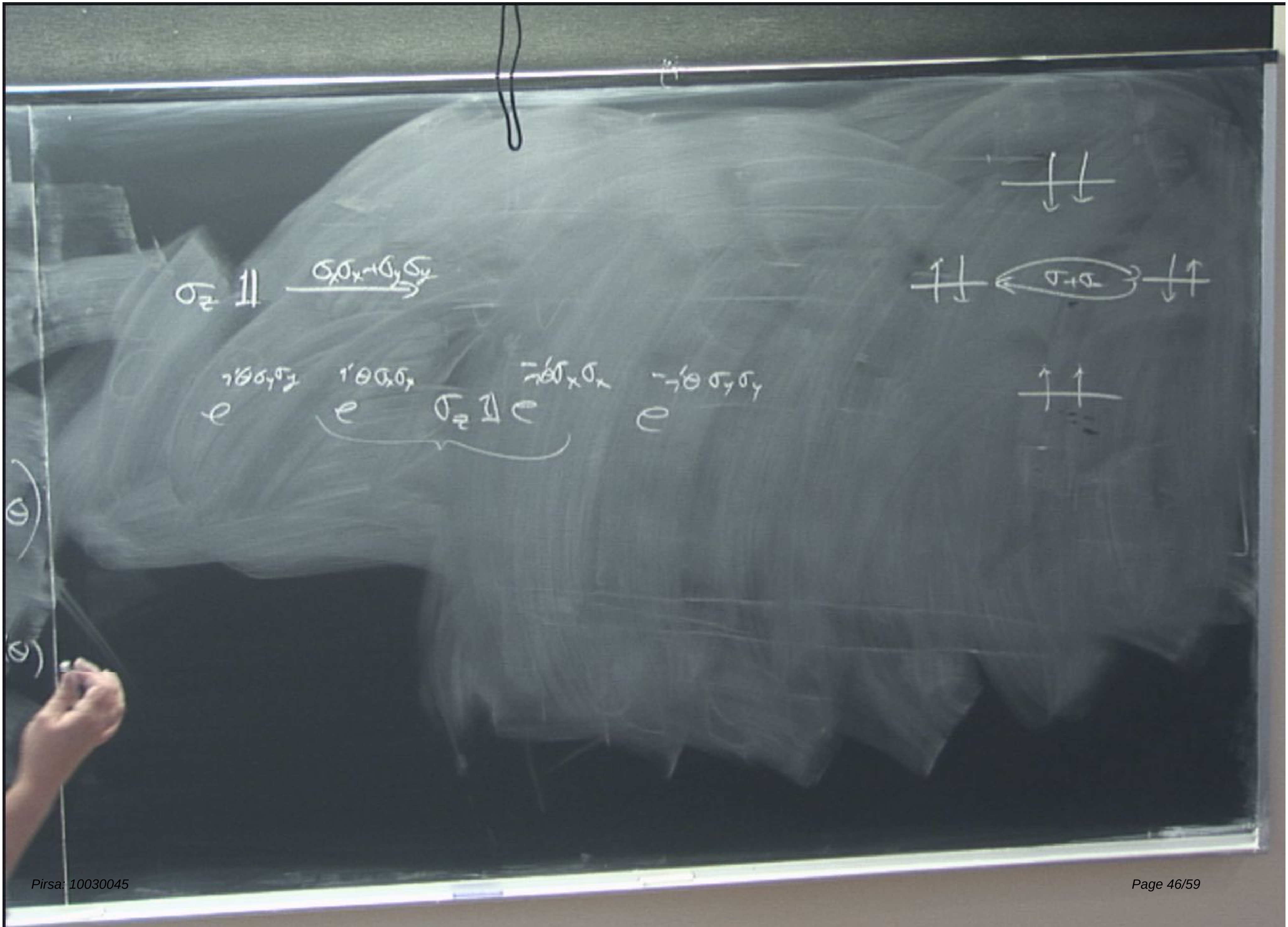
$\sigma_z \sigma_-$ $\quad \sin(\theta)\cos(\theta)$

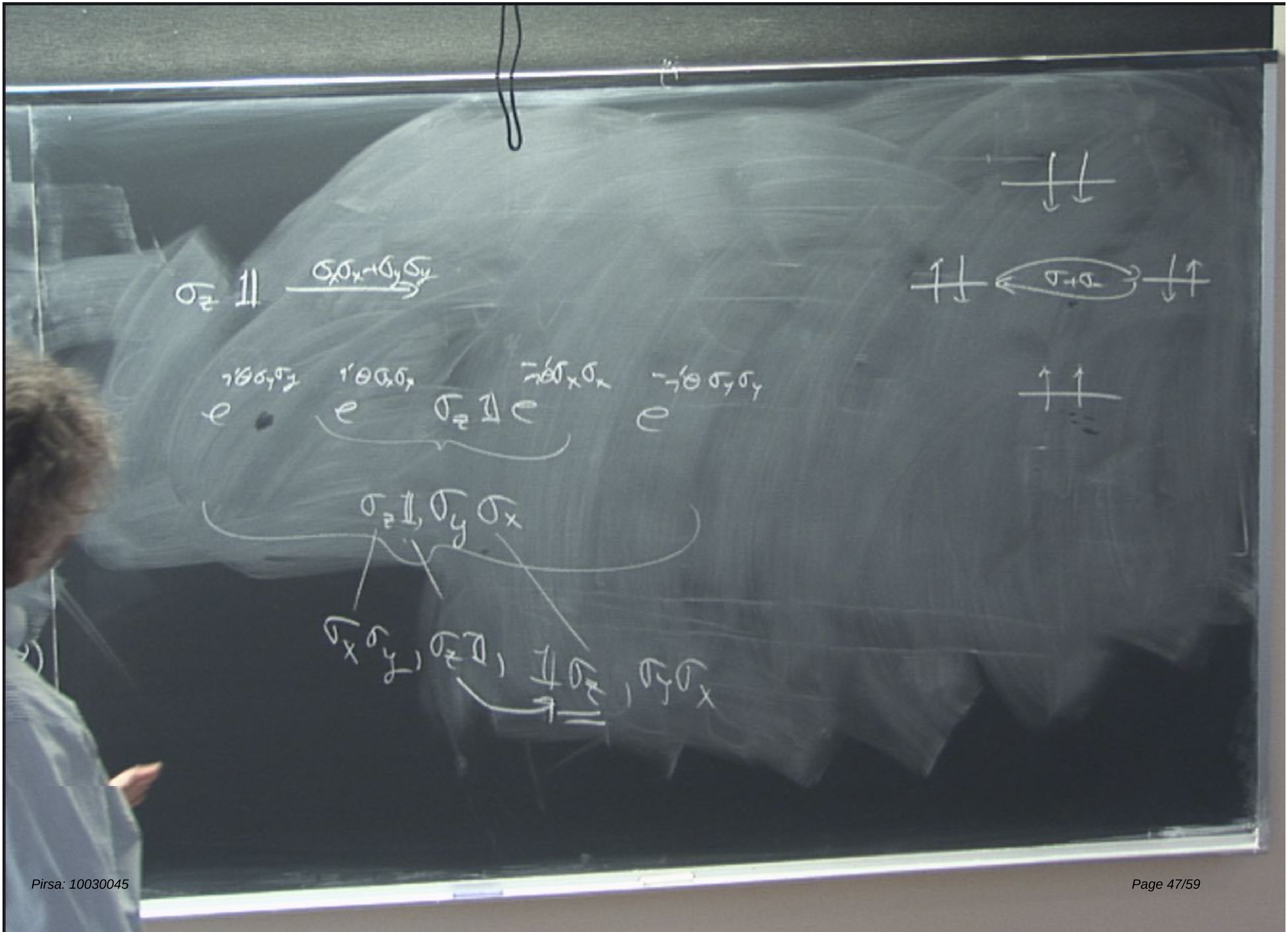
$\sigma_+ \sigma_+$

$\sigma_- \sigma_-$ $\quad \sin^2\theta$


$\sigma_+\sigma_-$


σ_z ↑↓ $\xrightarrow{\sigma_x\sigma_x+\sigma_y\sigma_y}$
$\sigma_+\sigma_-$


σz 𝟙 σxσx+σyσy
e iθσyσy e iθσxσx σz 𝟙 e −iθσxσx e −iθσyσy
σ+σ−

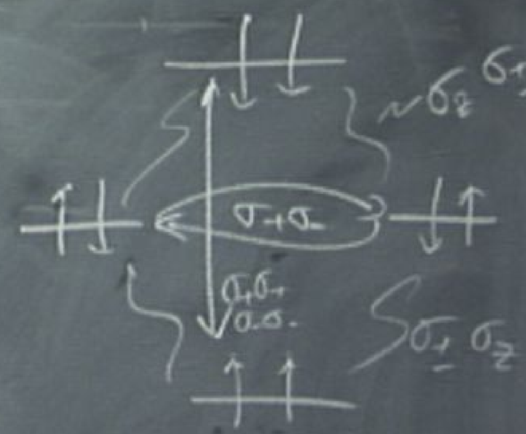

$\sigma_z \mathbb{1} \xrightarrow{\sigma_x\sigma_x + \sigma_y\sigma_y}$
$\sigma_+\sigma_-$
$e^{i\theta\sigma_y\sigma_y}\, e^{i\theta\sigma_x\sigma_x}\, \sigma_z \mathbb{1}\, e^{-i\theta\sigma_x\sigma_x}\, e^{-i\theta\sigma_y\sigma_y}$
$\sigma_z \mathbb{1}, \sigma_y \sigma_x$
$\sigma_x\sigma_y, \sigma_z\mathbb{1}, \mathbb{1}\sigma_z, \sigma_y\sigma_x$

$\sigma_z \mathbb{1} \xrightarrow{\sigma_x\sigma_x+\sigma_y\sigma_y}$

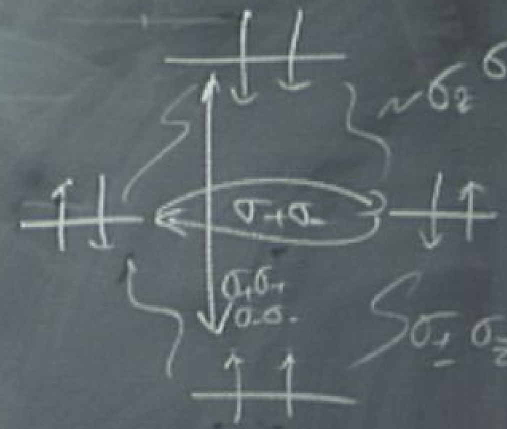
$e^{i\theta\sigma_y\sigma_y}\, e^{i\theta\sigma_x\sigma_x}\, \sigma_z\mathbb{1}\, e^{-i\theta\sigma_x\sigma_x}\, e^{-i\theta\sigma_y\sigma_y}$

$\sigma_z\mathbb{1},\ \sigma_y\sigma_x$

$\sigma_x\sigma_y,\ \sigma_z\mathbb{1},\ \mathbb{1}\sigma_z,\ \sigma_y\sigma_x$

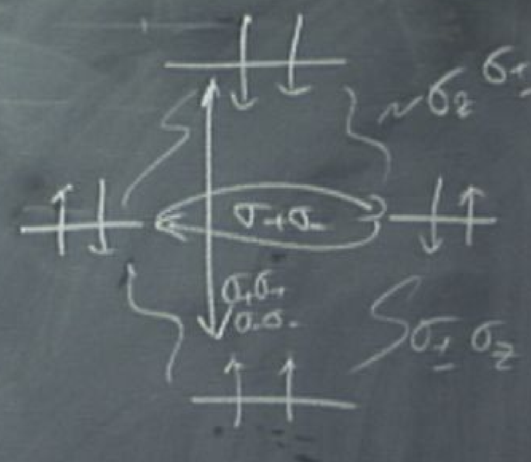


$$\boxed{\frac{d\rho}{dt} = -\int_0^t d\tau \left[\tilde{\mathcal{H}}_{eff}(t), \left[\tilde{\mathcal{H}}_{eff}(t-\tau), \rho(t) - \rho_0\right]\right]}$$



$$\boxed{\frac{d\rho}{dt} = -\int_0^t d\tau \left[\overline{\mathcal{H}_{eff}(t), \left[\mathcal{H}_{eff}(t-\tau), \rho(t)\right.} - \rho_0\right]\Big]}$$

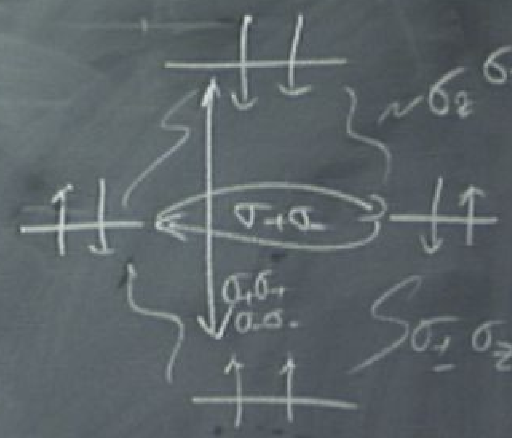
$$\overline{\mathcal{H}_{eff}} = 0$$



ensemble average

$$\boxed{\frac{d\rho}{dt} = -\int_0^t d\tau \, \overline{\left[\mathcal{H}_{eff}(t), \left[\mathcal{H}_{eff}(t-\tau), \rho(t) - \rho_0\right]\right]}}$$

$$\overline{\mathcal{H}_{eff}} = 0$$

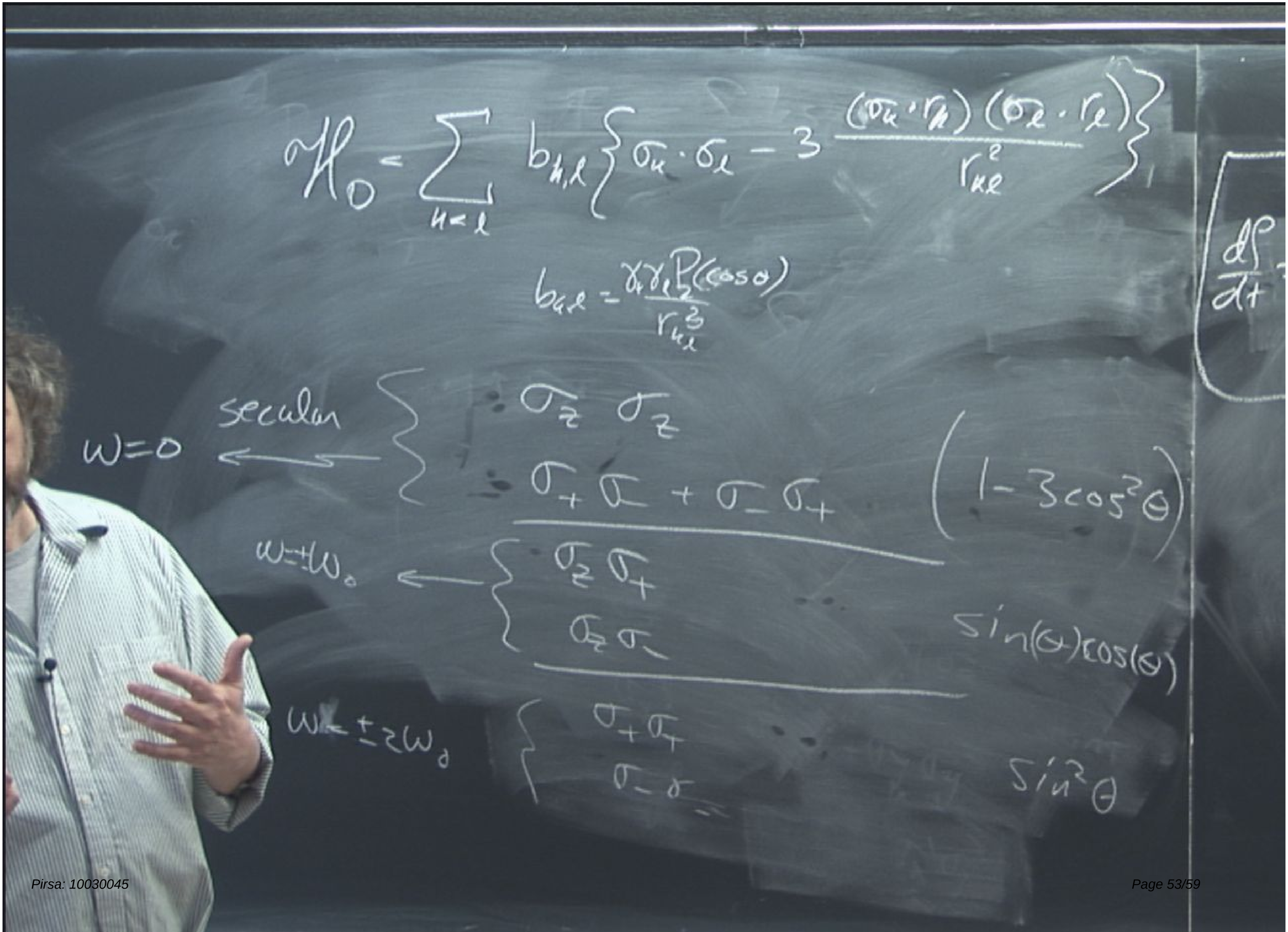


ensemble average

$$\boxed{\frac{d\rho}{dt} = -\int_0^t d\tau \left[\overline{\mathcal{H}_{eff}(t), \left[\mathcal{H}_{eff}(t-\tau), \rho(t) - \rho_0\right]}\right]}$$

equilibrium.

$$\overline{\mathcal{H}_{eff}} = 0$$

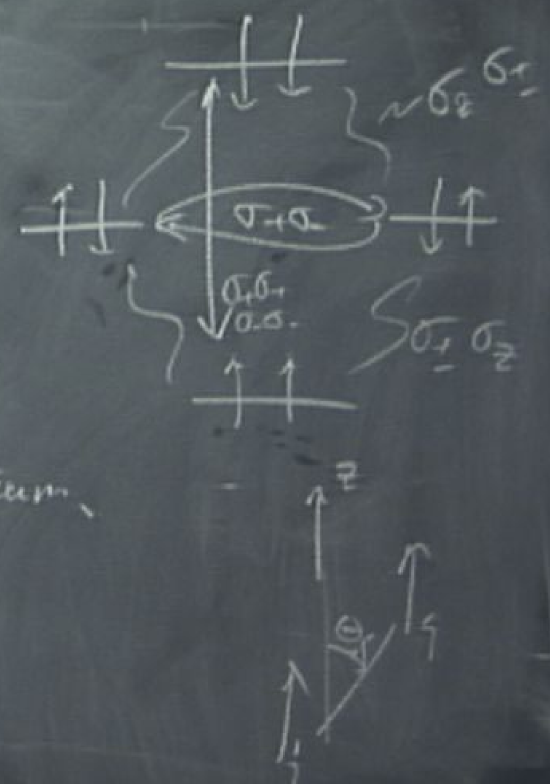

$\mathcal{H}_D = \sum_{k<l} b_{k,l} \left\{ \sigma_k \cdot \sigma_l - 3 \frac{(\sigma_k \cdot r_k)(\sigma_l \cdot r_l)}{r_{kl}^2} \right\}$
$b_{kl} = \frac{\gamma_k \gamma_l P_2(\cos\theta)}{r_{kl}^3}$
secular
$\omega = 0 \longleftarrow$
$\sigma_z \; \sigma_z$
$\sigma_+ \sigma_- + \sigma_- \sigma_+$
$(1 - 3\cos^2\theta)$
$\omega = \pm\omega_0 \longleftarrow$
$\sigma_z \sigma_+$
$\sigma_z \sigma_-$
$\sin(\theta)\cos(\theta)$
$\omega = \pm 2\omega_0$
$\sigma_+ \sigma_+$
$\sigma_- \sigma_-$
$\sin^2\theta$
$\frac{d\rho}{dt}$

ensemble average

$$\boxed{\frac{d\rho}{dt} = -\int_0^t d\tau \, \overline{\left[\mathcal{H}_{eff}(t), \left[\mathcal{H}_{eff}(t-\tau), \rho(t) - \rho_0\right]\right]}}$$

equilibrium.

$$\overline{\mathcal{H}_{eff}} = 0$$



$$\mathcal{H}_D = \sum_{k<l} b_{k,l}\left\{\sigma_k\cdot\sigma_l - 3\,\frac{(\sigma_k\cdot r_{kl})(\sigma_l\cdot r_{kl})}{r_{kl}^2}\right\},$$

$$b_{kl} = \frac{\gamma_k\gamma_l P_2(\cos\theta)}{r_{kl}^3}$$

freq of the

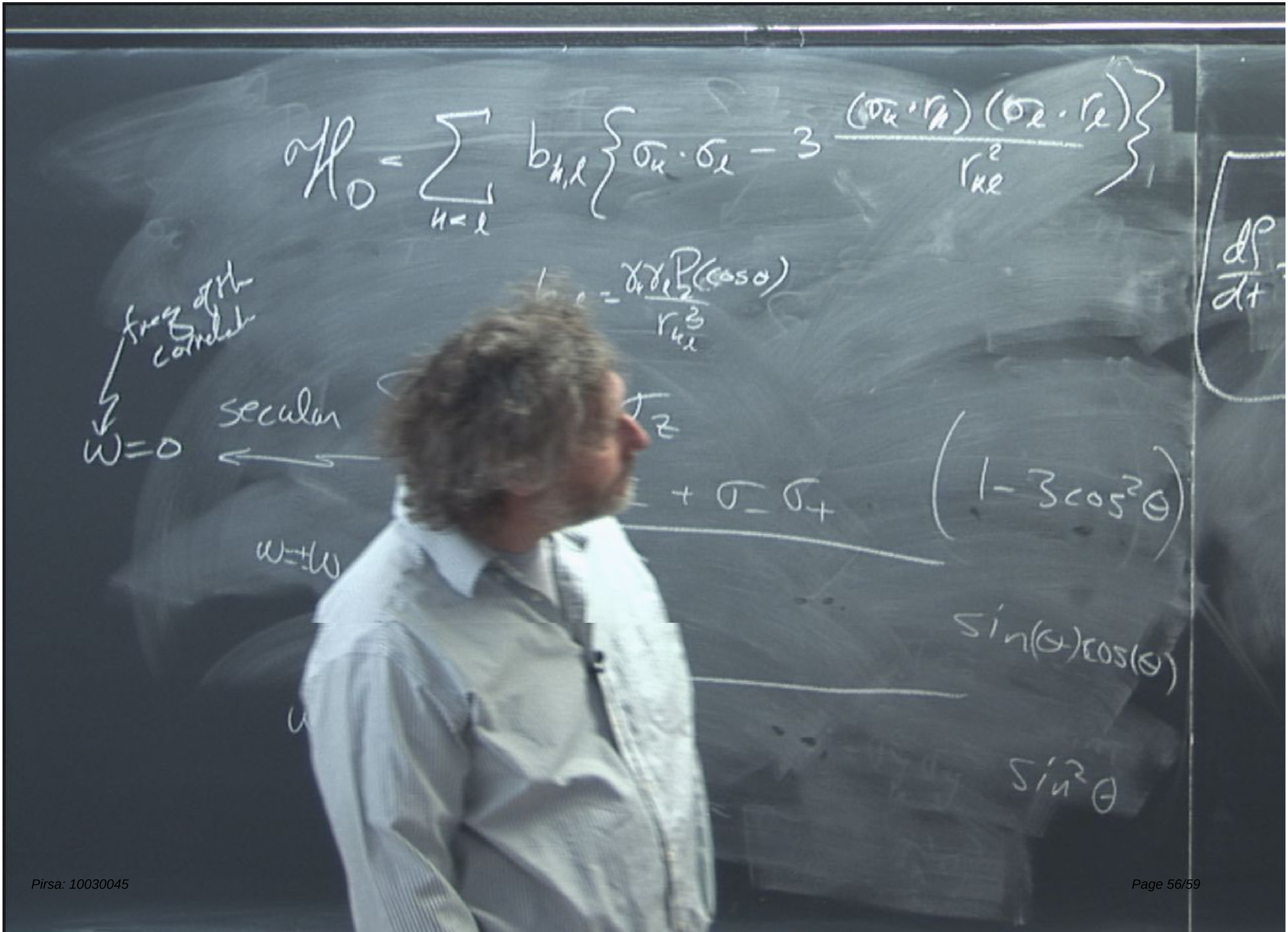
secular

$\omega = 0$ $\left\{ \begin{array}{l} \sigma_z\,\sigma_z \\ \sigma_+\sigma_- + \sigma_-\sigma_+ \end{array} \right.$ $\quad (1 - 3\cos^2\theta)$

$\omega = \pm\omega_0 \leftarrow \left\{ \begin{array}{l} \sigma_z\sigma_+ \\ \sigma_z\sigma_- \end{array} \right.$ $\quad \sin(\theta)\cos(\theta)$

$\omega = \pm 2\omega_0$ $\left\{ \begin{array}{l} \sigma_+\sigma_+ \\ \sigma_-\sigma_- \end{array} \right.$ $\quad \sin^2\theta$

$\frac{d\rho}{dt} =$



$\mathcal{H}_D = \sum_{k<l} b_{k,l}\left\{\sigma_k\cdot\sigma_l - 3\frac{(\sigma_k\cdot r_k)(\sigma_l\cdot r_l)}{r_{kl}^2}\right\}$
secular
$\omega=0$
$\omega=\pm\omega$
$+\sigma_-\sigma_+$
$(1-3\cos^2\theta)$
$\sin(\theta)\cos(\theta)$
$\sin^2\theta$
$\frac{d\rho}{dt}$

$$\mathcal{H}_D = \sum_{k<l} b_{k,l}\left\{\sigma_k \cdot \sigma_l - 3\,\frac{(\sigma_k \cdot r_{kl})(\sigma_l \cdot r_{kl})}{r_{kl}^2}\right\}$$

$$b_{kl} = \frac{\gamma_k \gamma_l P_2(\cos\theta)}{r_{kl}^3}$$

freq of th correlat

$\omega = 0$ $\longleftarrow$ secular $\left\{ \begin{array}{l} \sigma_z\,\sigma_z \\ \sigma_+\sigma_- + \sigma_-\sigma_+ \end{array} \right.$ $\quad (1 - 3\cos^2\theta)$

$\omega = \pm\omega_0$ $\longleftarrow$ $\left\{ \begin{array}{l} \sigma_z\,\sigma_+ \\ \sigma_z\,\sigma_- \end{array} \right.$ $\quad \sin(\theta)\cos($

$\omega = \pm 2\omega_0$ $\left\{ \begin{array}{l} \sigma_+\sigma_+ \\ \sigma_-\sigma_- \end{array} \right.$ $\quad \sin^2($

$\frac{d\rho}{dt}$

$$\mathcal{H}_D = \sum_{k<l} b_{k,l} \left\{ \sigma_k \cdot \sigma_l - 3 \frac{(\sigma_k \cdot r_k)(\sigma_l \cdot r_l)}{r_{kl}^2} \right\}$$

$$b_{kl} = \frac{\gamma_k \gamma_l P_2(\cos\theta)}{r_{kl}^3} \qquad E_\pm = \frac{1}{2}(\mathbb{1} \pm \sigma_z)$$

freq of the correlation

$\omega = 0$ ⟵ secular

$$\left\{ \begin{array}{l} \sigma_z \sigma_z \\ \sigma_+ \sigma_- + \sigma_- \sigma_+ \end{array} \right. \qquad (1 - 3\cos^2\theta)$$

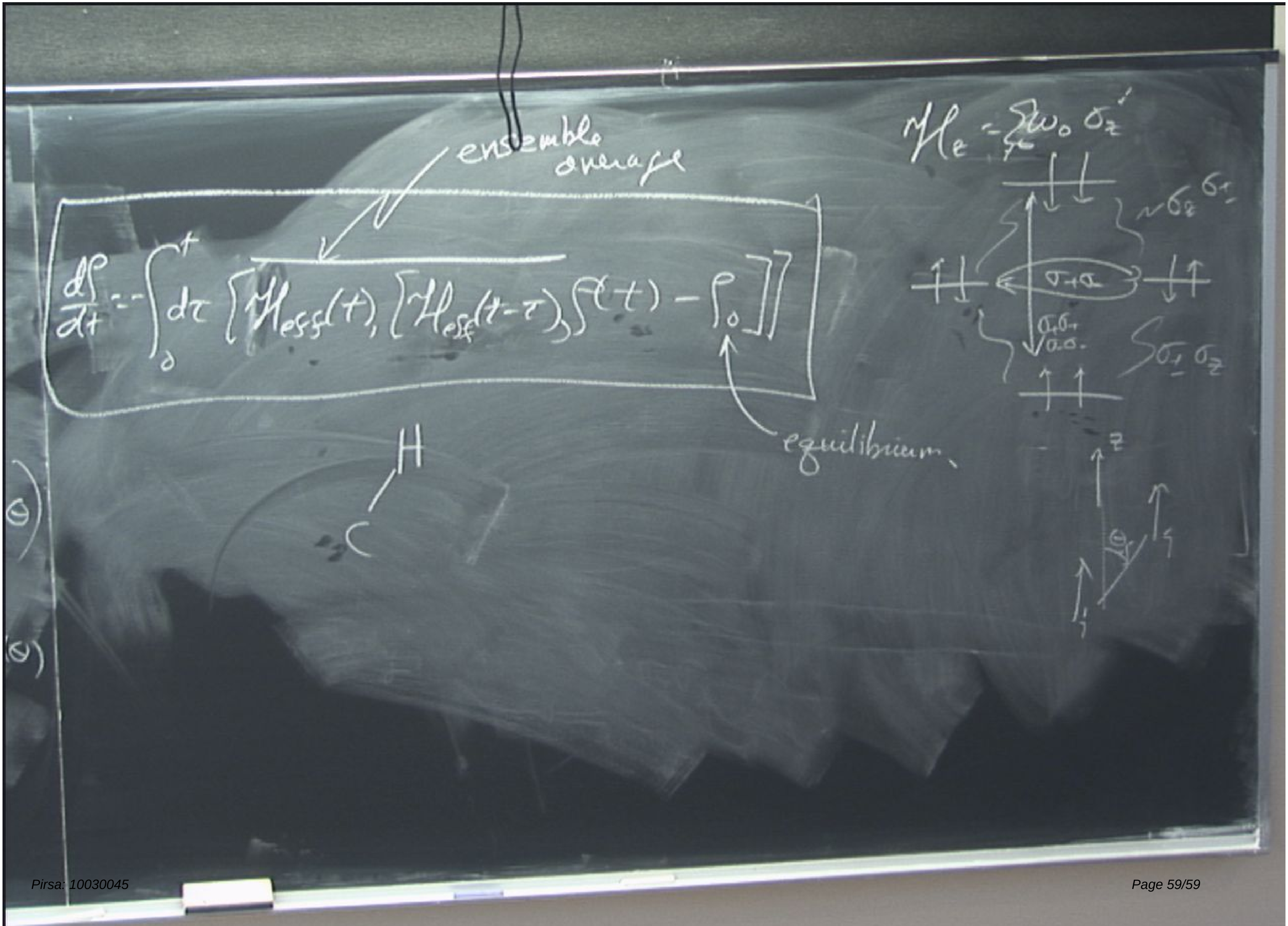
$\omega = \pm\omega_0$ ⟵

$$\left\{ \begin{array}{ll} \sigma_z \sigma_+ & E_+ \sigma_{x,y} + \sigma_{x,y} E_+ \\ \sigma_z \sigma_- & E_- \sigma_{x,y} + \sigma_{x,y} E_+ \end{array} \right. \qquad \sin(\theta)\cos(\theta)$$

$\omega = \pm 2\omega_0$

$$\left\{ \begin{array}{l} \sigma_+ \sigma_+ \\ \sigma_- \sigma_- \end{array} \right. \qquad \sin^2\theta$$

$$\frac{d\rho}{dt} =$$


ensemble average
$\frac{d\rho}{dt} = -\int_0^t d\tau \, [\overline{\mathcal{H}_{eff}(t), [\mathcal{H}_{eff}(t-\tau), \rho(t) - \rho_0}]]$
equilibrium
$\mathcal{H}_z = -\sum_i \omega_0 \sigma_z^i$
$\sigma_+ \sigma_-$
$\sim \sigma_z \sigma_+$
$\sigma_+\sigma_+$
$\sigma_-\sigma_-$
$\sigma_\pm \sigma_z$
H
C
z