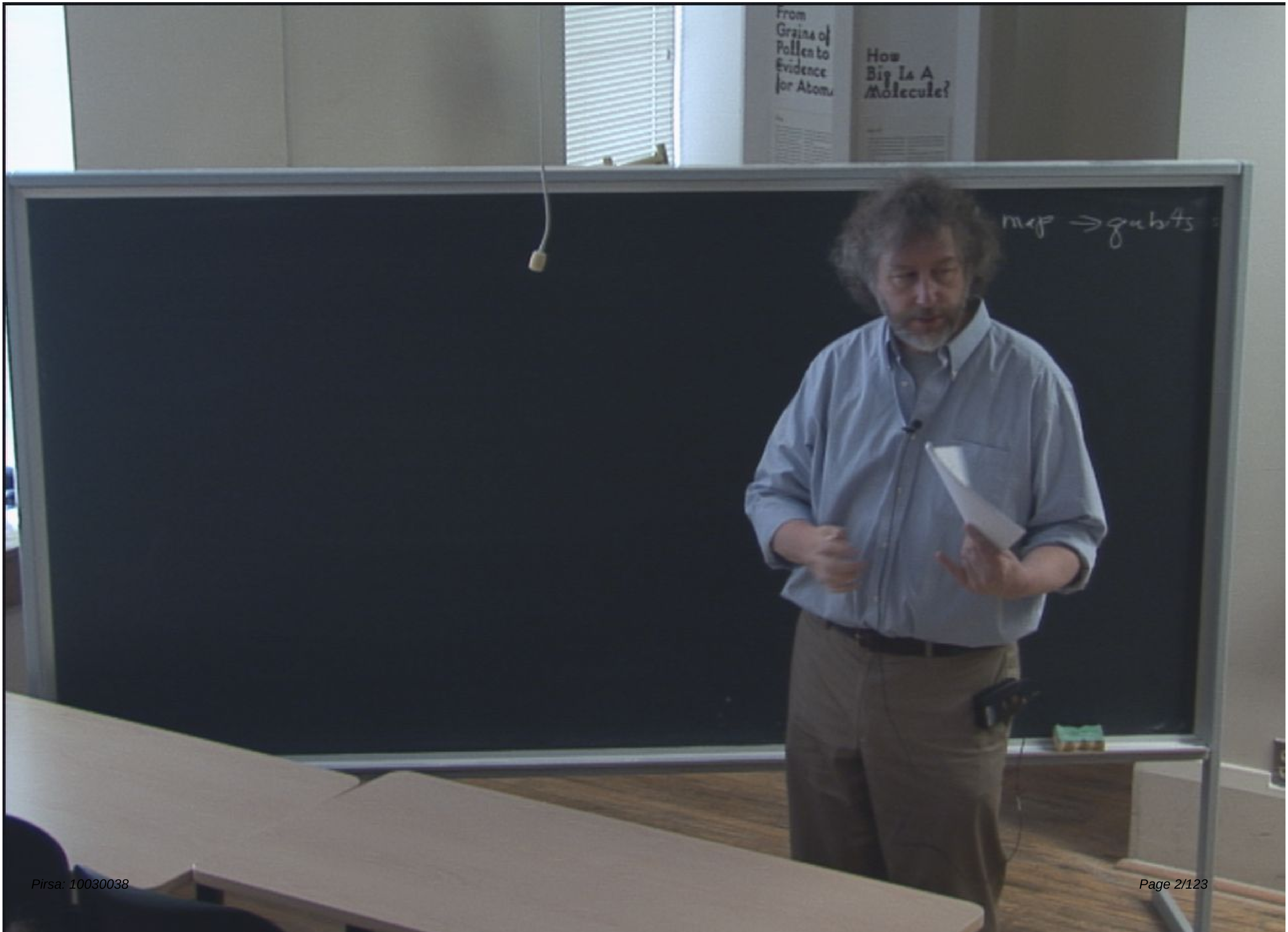
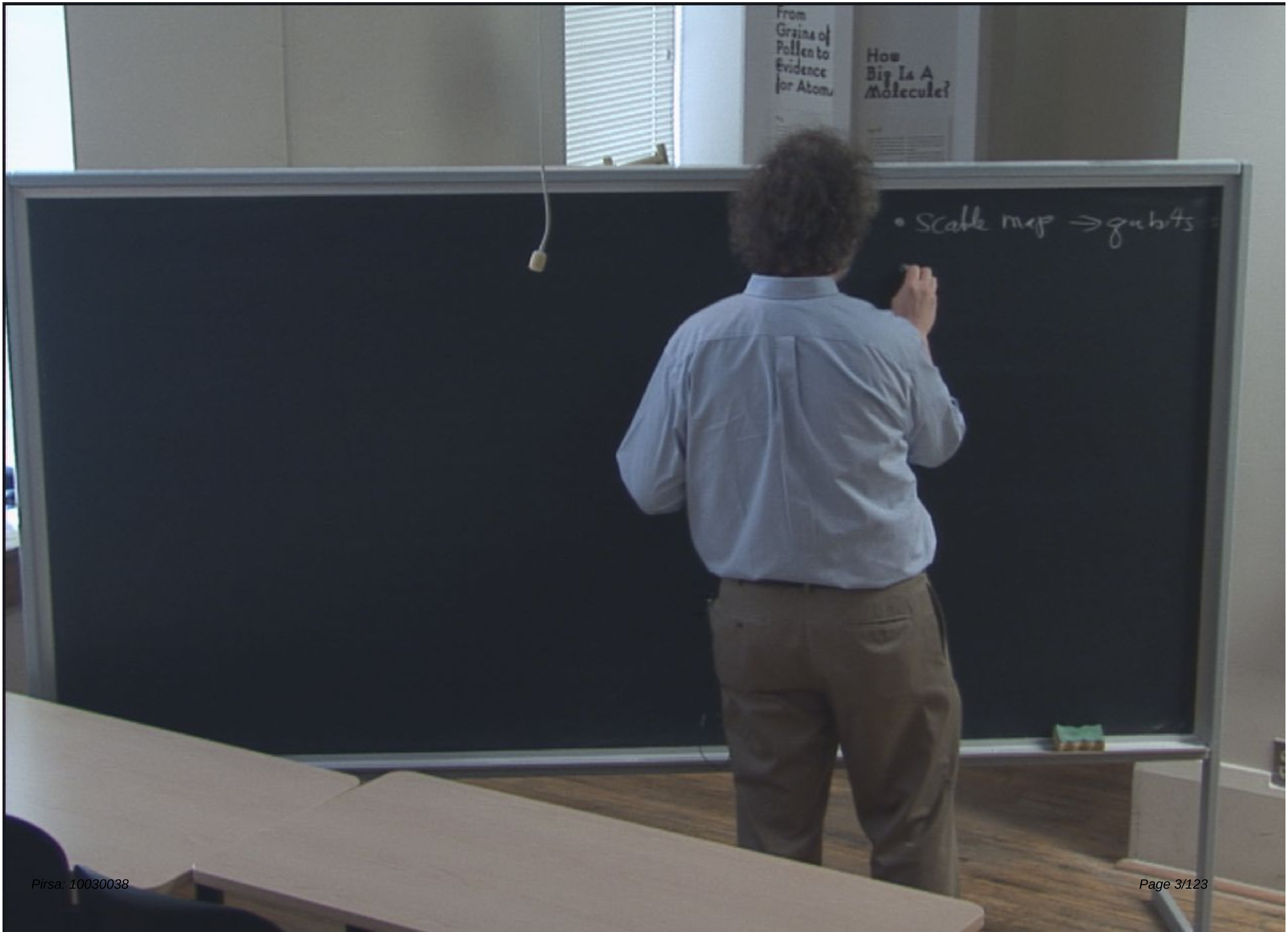
Title: Explorations in Quantum Info. (PHYS 641) - Lecture 10

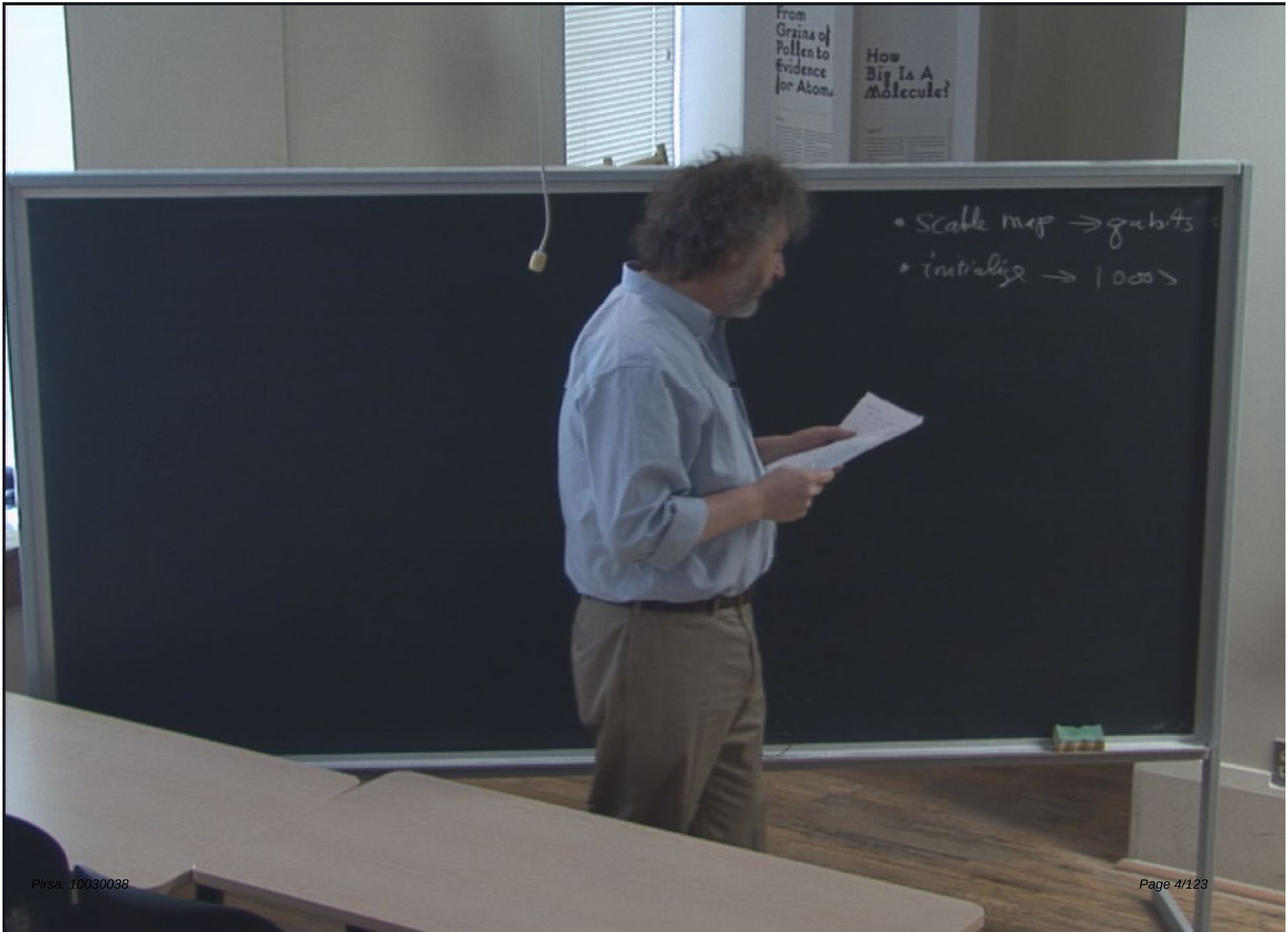
Date: Mar 01, 2010 09:00 AM

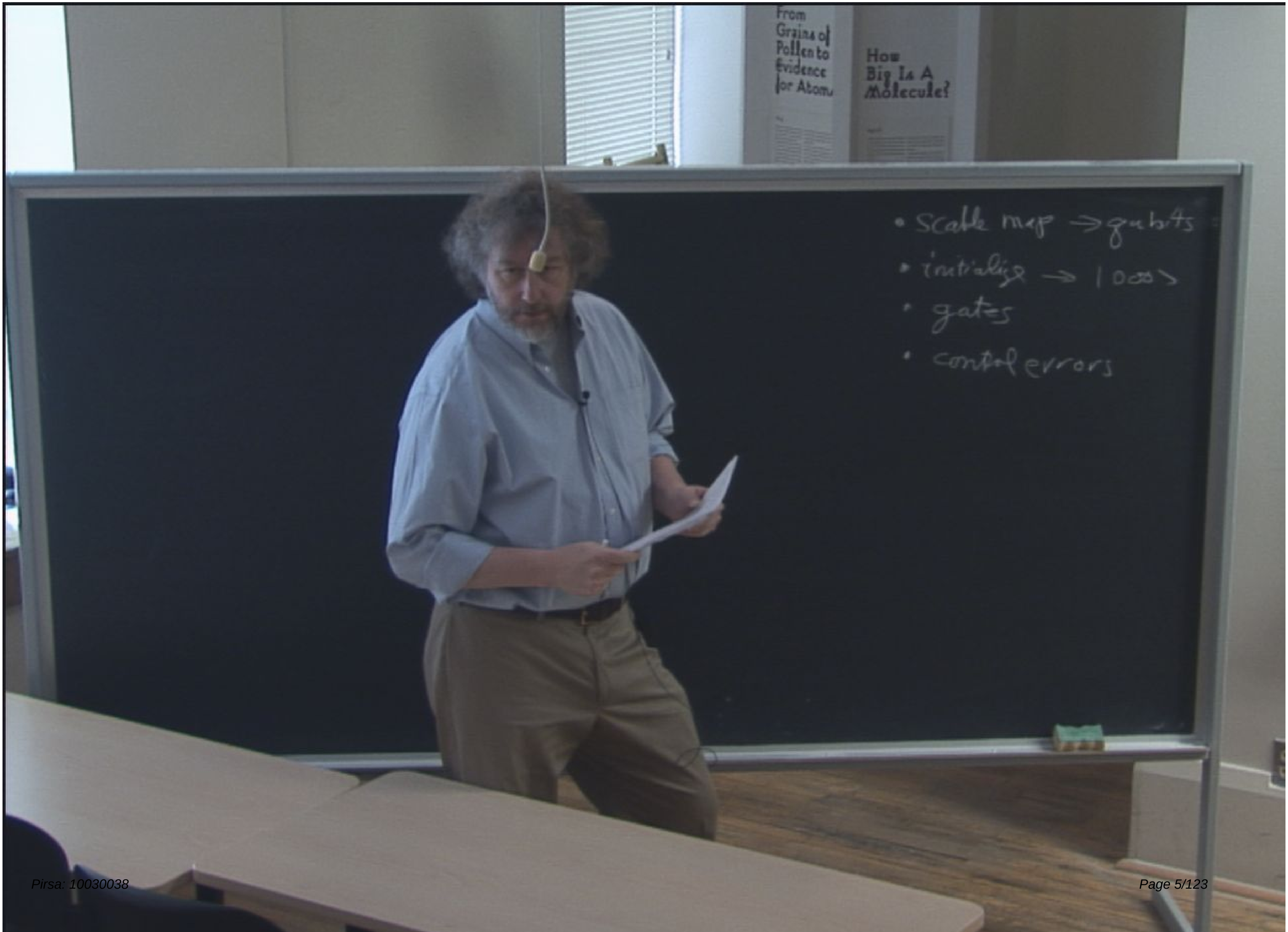
URL: http://pirsa.org/10030038

Abstract:

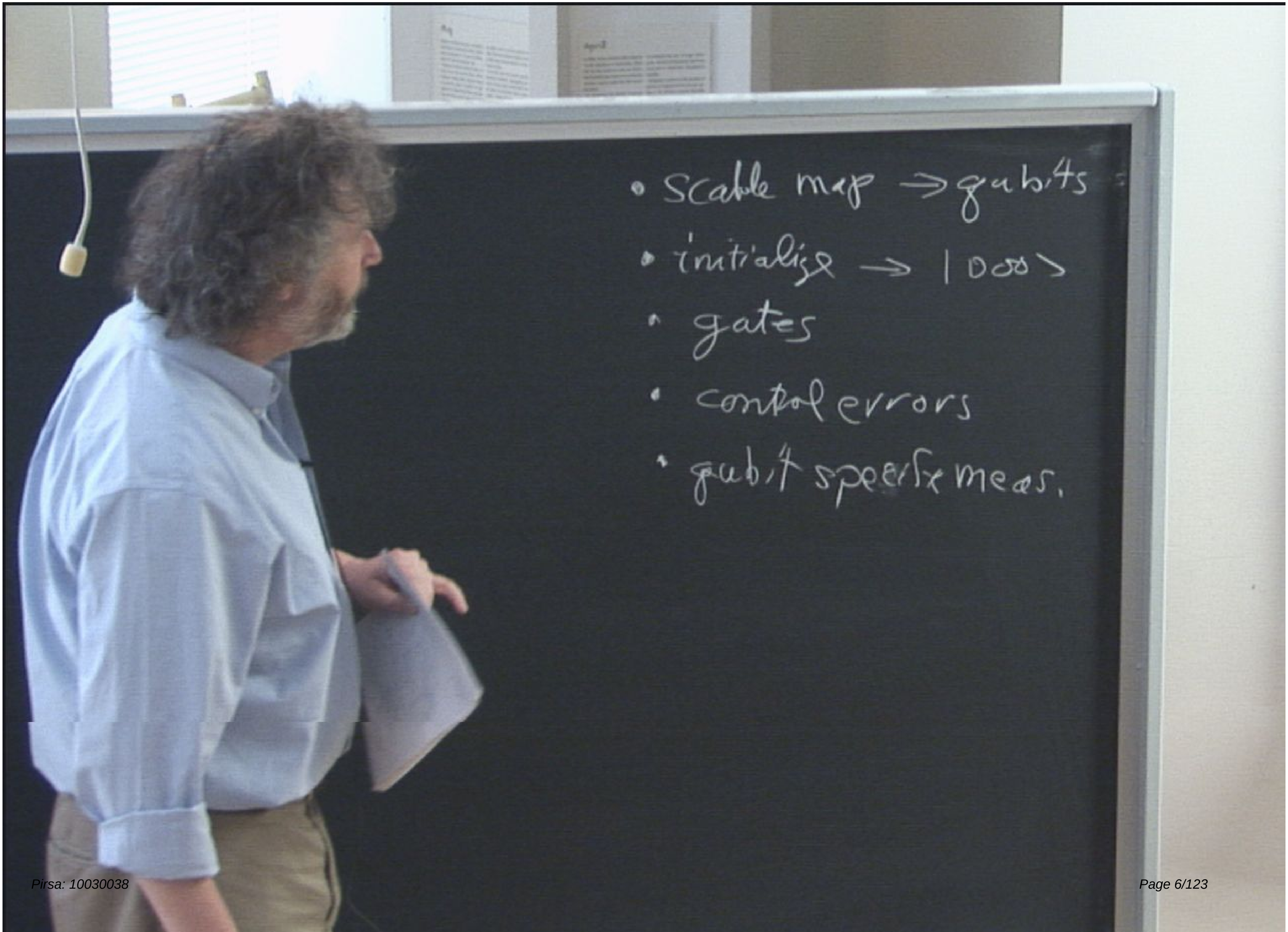

From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
map → qubits


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
• Scable map ⇒ qubits

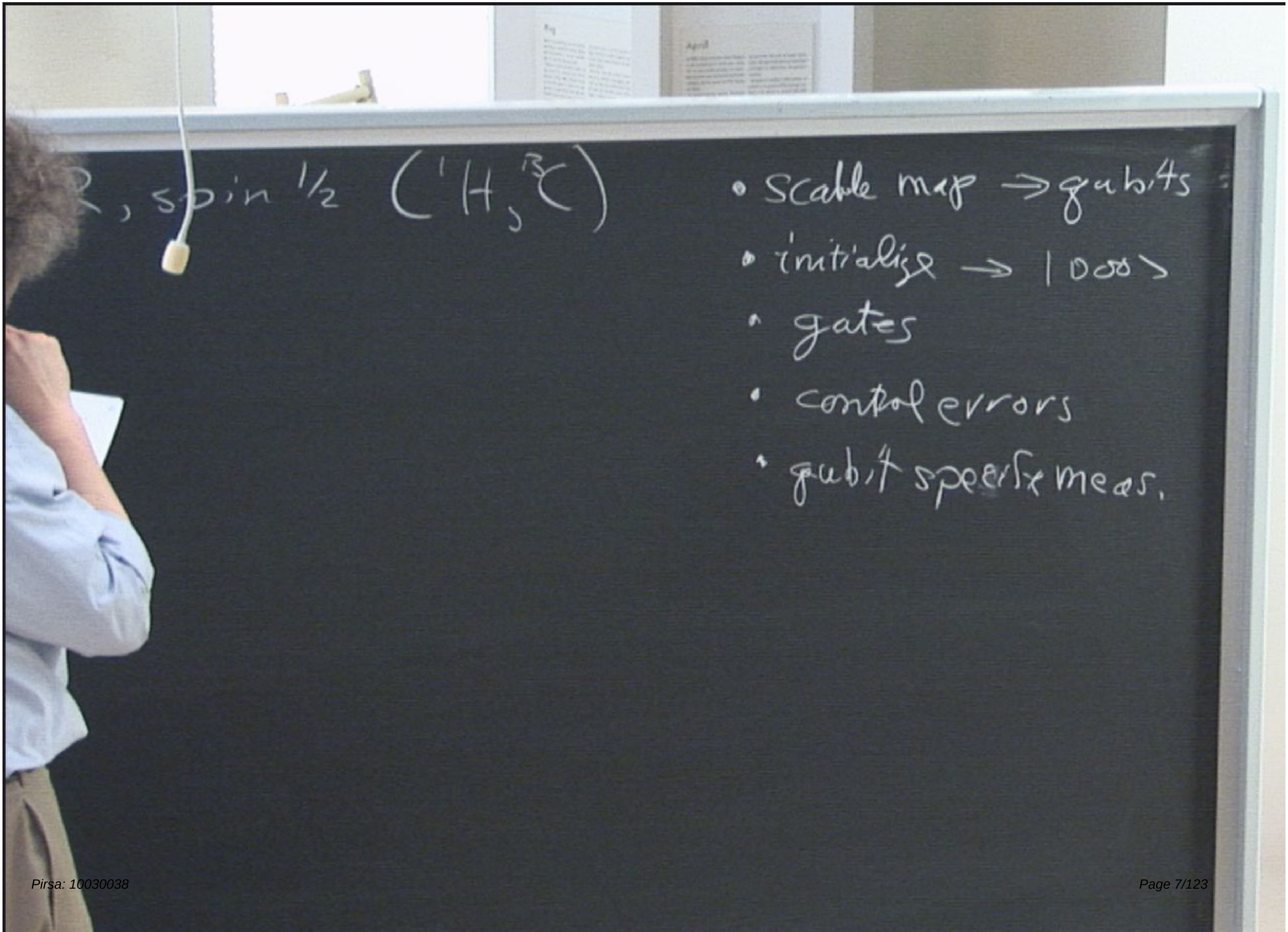

From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
• Scalable map ⇒ qubits
• initialize → $|000\rangle$



- Scable map → qubits
- initialize → $|000\rangle$
- gates
- control errors



- Scalable map → qubits
- initialize → $|000\rangle$
- gates
- control errors
- qubit specific meas.



2, spin 1/2 (^1H, ^{13}C)

- Scalable map ⇒ qubits
- initialize → $|000\rangle$
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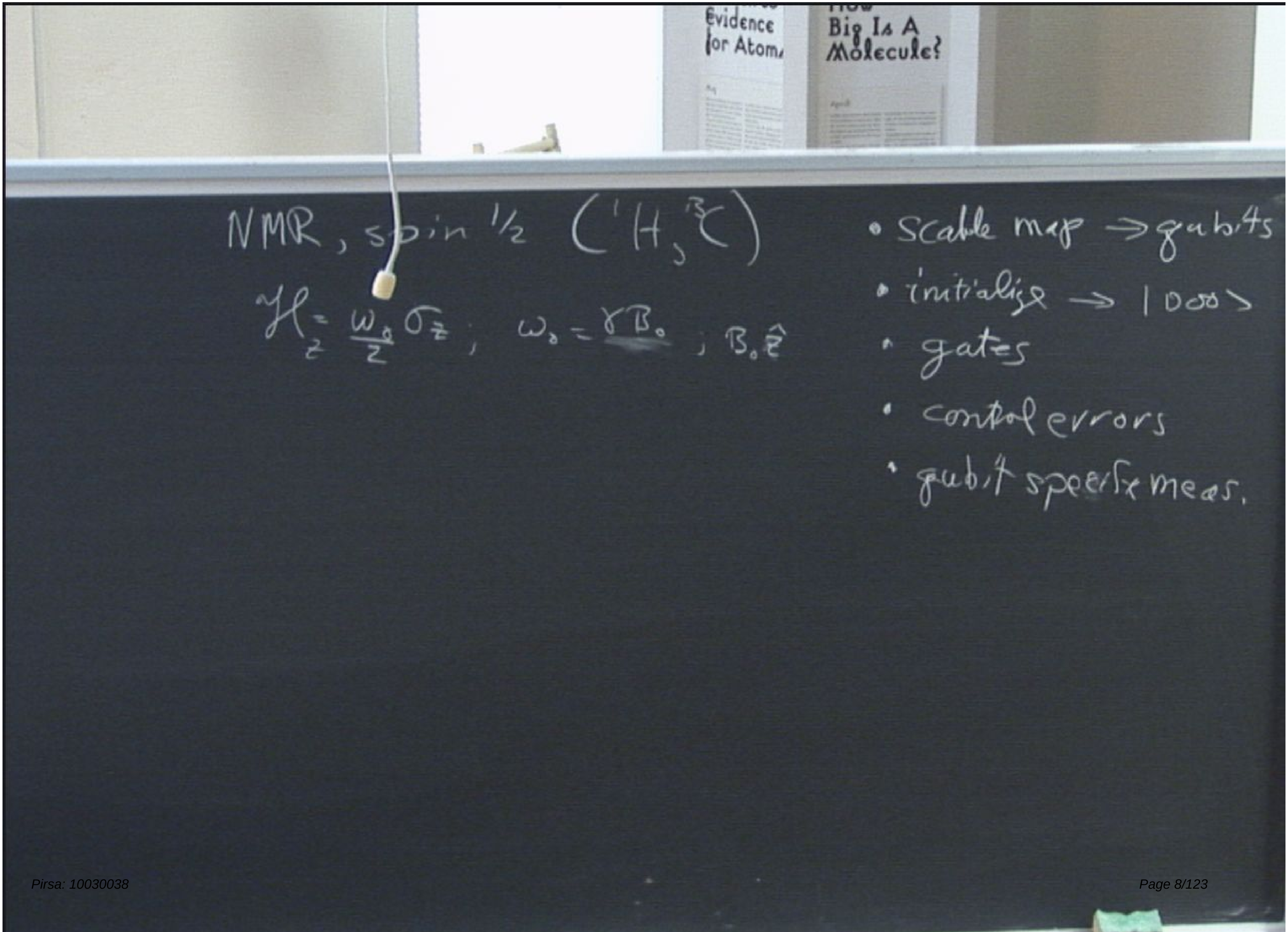

NMR, spin 1/2 (^{1}H, ^{13}C)

$\mathcal{H}_z = \frac{\omega_0}{2}\sigma_z$; $\omega_0 = \gamma B_0$; $B_0 \hat{z}$

- scalable map → qubits
- initialize → $|000\rangle$
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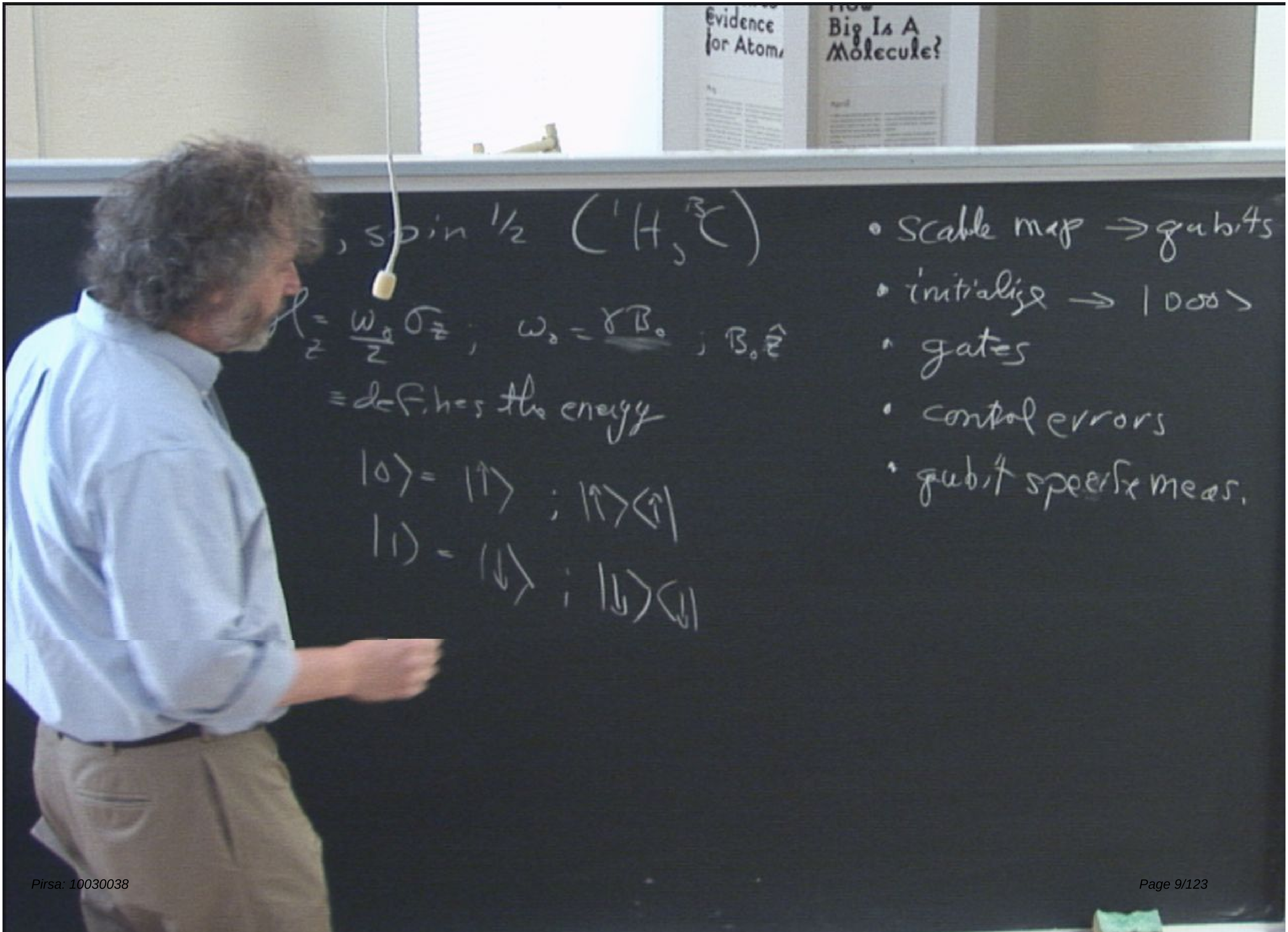
- qubit specific meas.



, spin 1/2 $({}^{1}H_3{}^{13}C)$

$\mathcal{H}_z = \frac{\omega_0}{2}\sigma_z$; $\omega_0 = \gamma B_0$; $B_0\hat{z}$

= defines the energy

$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow|$

$|1\rangle = |\downarrow\rangle$; $|\downarrow\rangle\langle\downarrow|$

- Scable map → qubits
- initialize → $|000\rangle$
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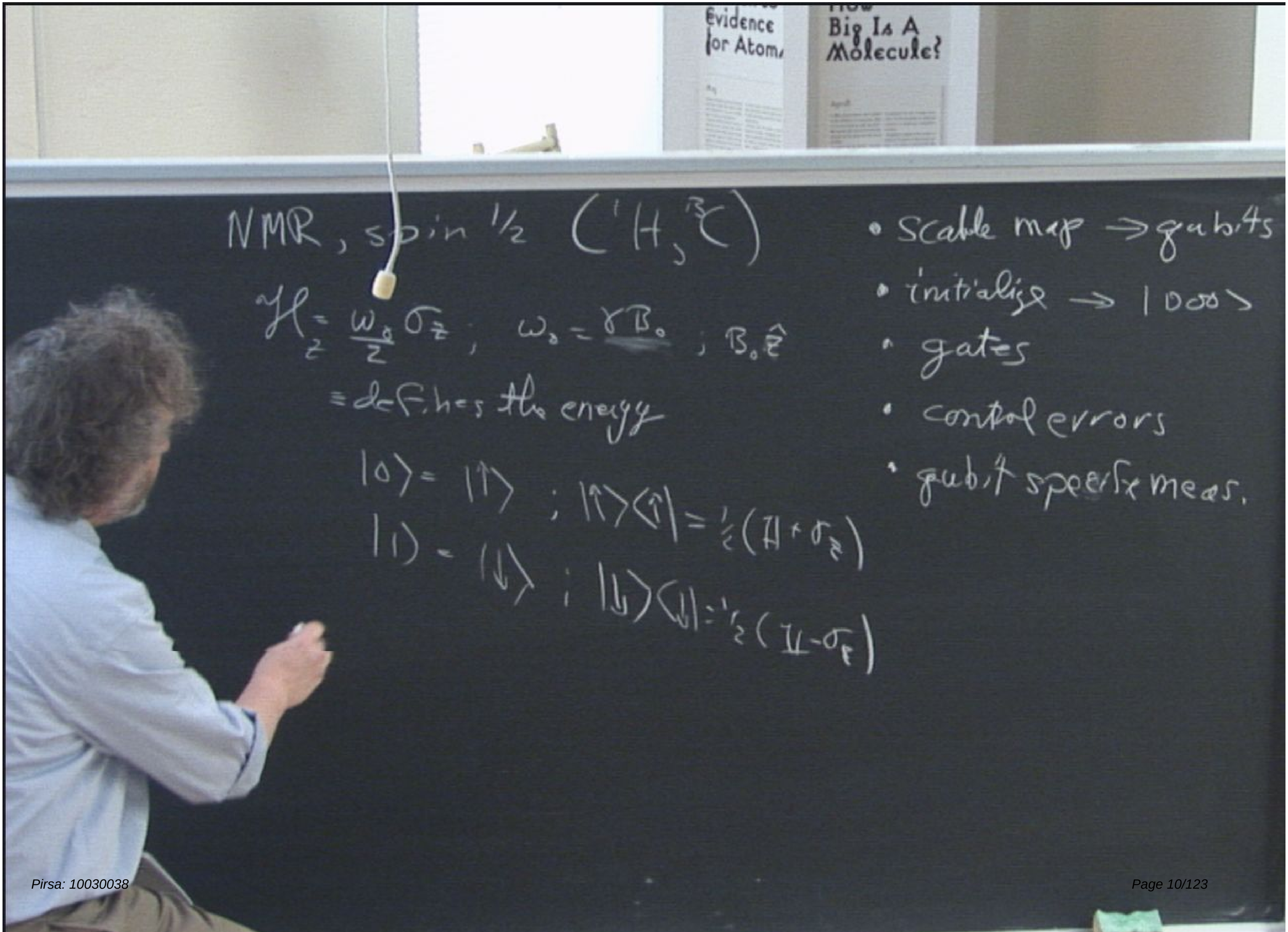

NMR, spin 1/2 (^1H, ^{13}C)

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- scalable map → qubits
- initialize → $|000\rangle$
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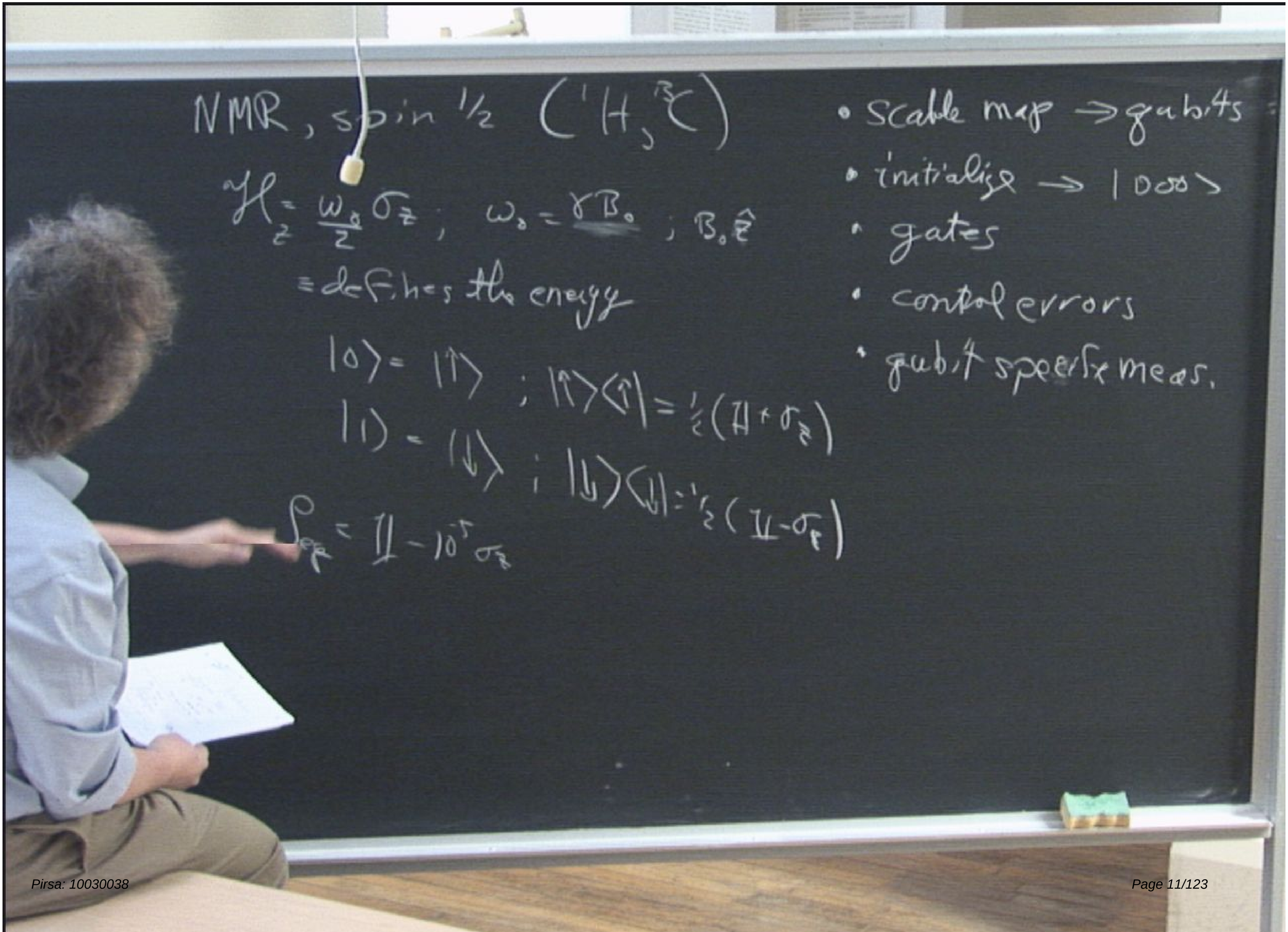

NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H} = \frac{\omega_0}{2}\sigma_z$; $\omega_0 = \gamma B_0$; $B_0 \hat{z}$

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$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{1} + \sigma_z)$

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$\rho_{eq} = \mathbb{1} - 10^{-5}\sigma_z$

- Scalable map → qubits
- initialize → $|000\rangle$
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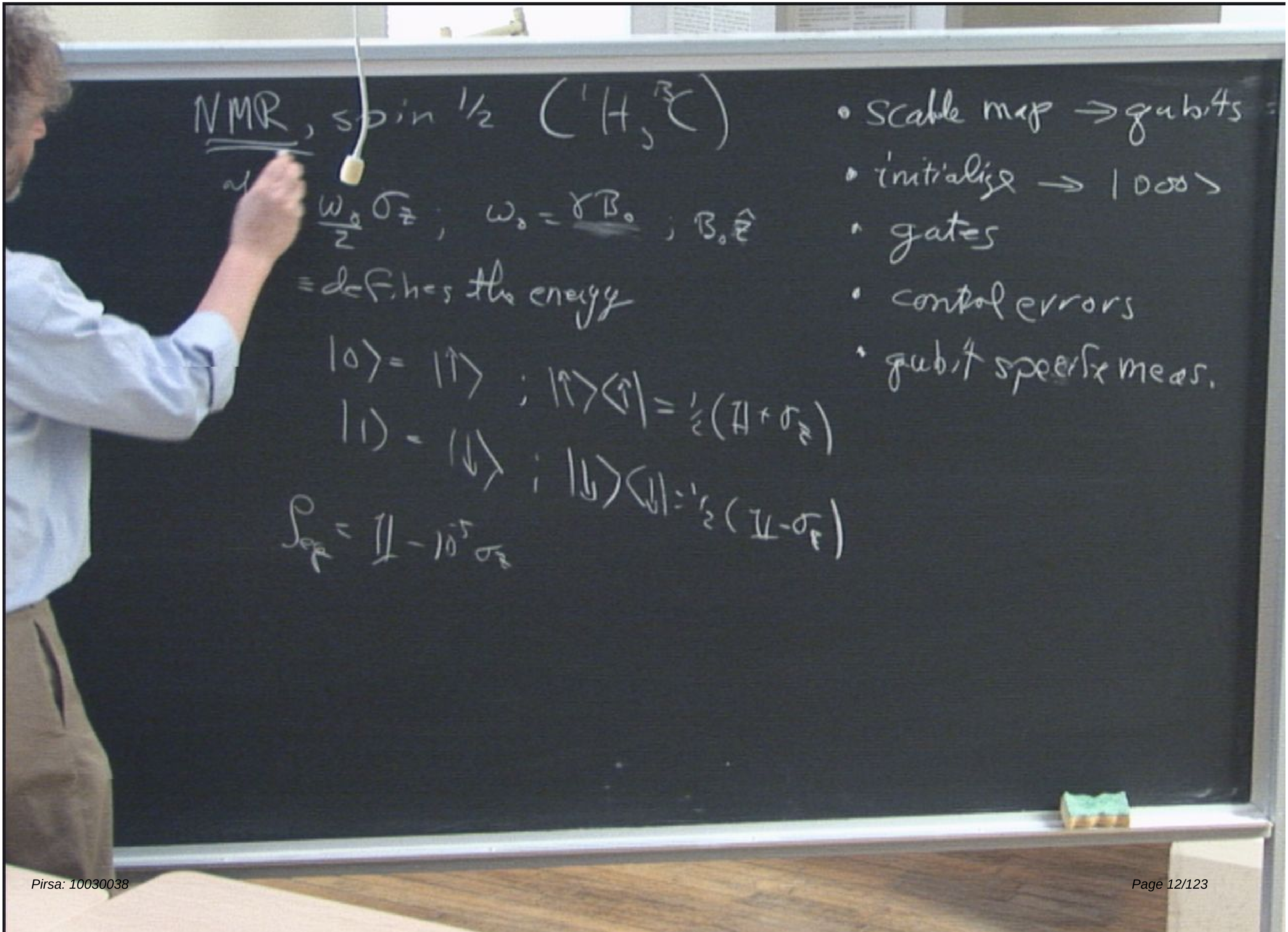

NMR, spin 1/2 $({}^{1}H, {}^{13}C)$

$\frac{\omega_0}{2}\sigma_z$; $\omega_0 = \gamma B_0$; $B_0\hat{z}$

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- scalable map → qubits
- initialize → $|000\rangle$
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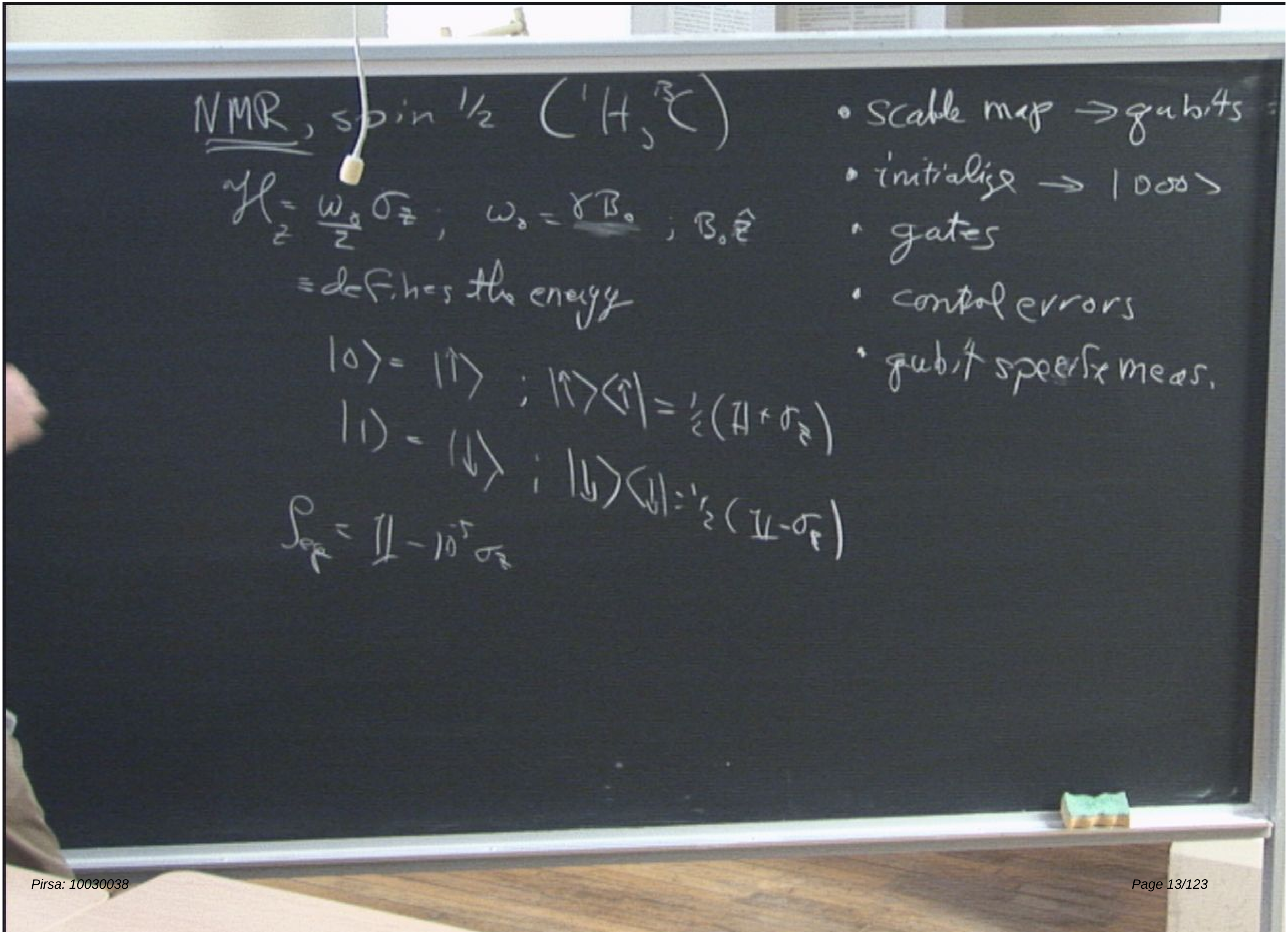


NMR, spin 1/2 (^1H, ^{13}C)

$$\mathcal{H}_Z = \frac{\omega_0}{2}\sigma_z \; ; \quad \omega_0 = \gamma B_0 \; ; \; B_0\hat{z}$$

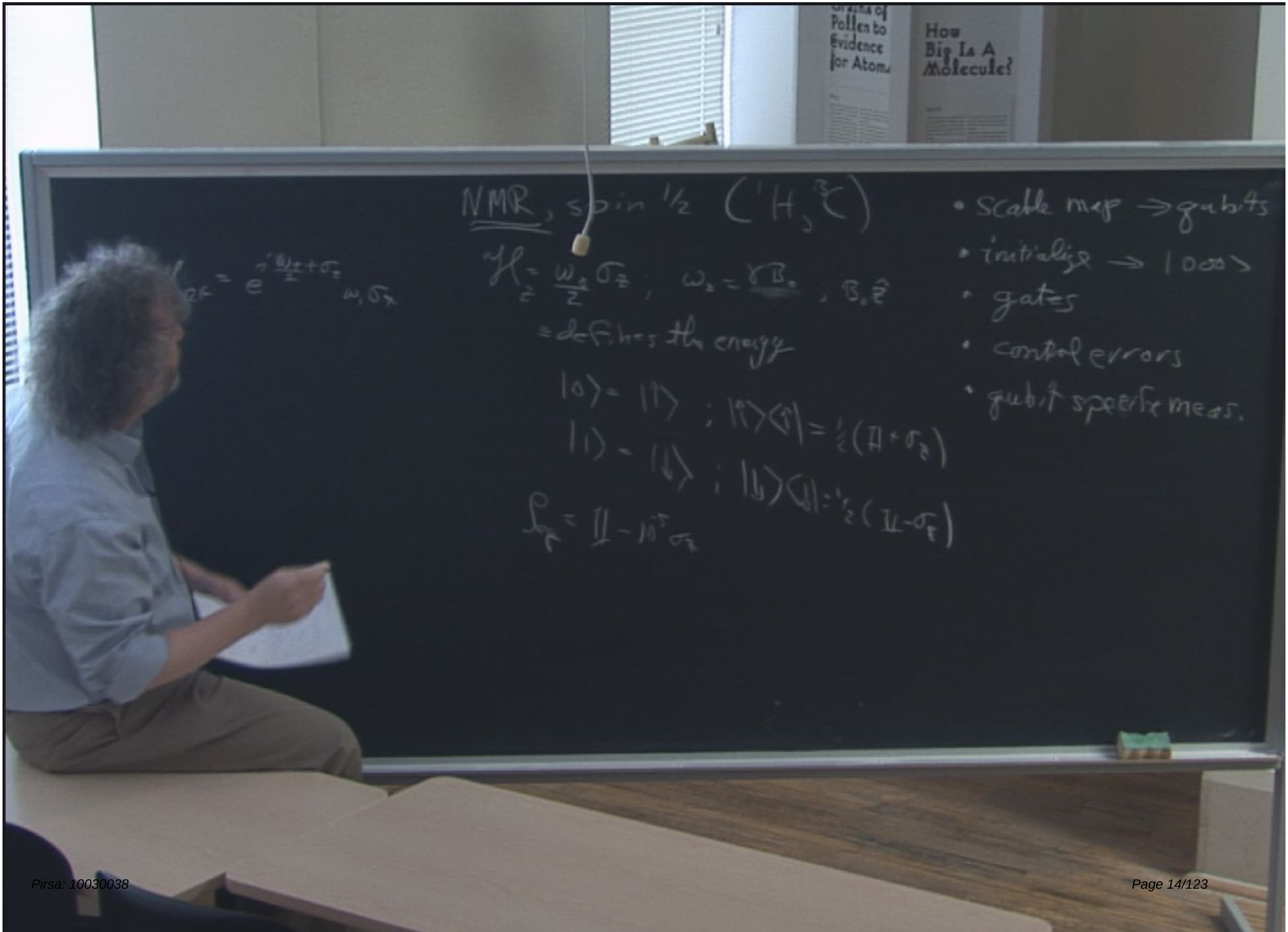
≡ defines the energy

$$|0\rangle = |\uparrow\rangle \; ; \; |\uparrow\rangle\langle\uparrow| = \tfrac{1}{2}(\mathbb{1}+\sigma_z)$$

$$|1\rangle = |\downarrow\rangle \; ; \; |\downarrow\rangle\langle\downarrow| = \tfrac{1}{2}(\mathbb{1}-\sigma_z)$$

$$\rho_{eq} = \mathbb{1} - 10^{-5}\sigma_z$$

- Scable map ⇒ qubits
- initialise → $|000\rangle$
- gates
- control errors
- qubit specific meas.



$\mathcal{U}_{ot} = e^{i\frac{\omega_z}{2}t\sigma_z}$ ω, σ_x

NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H}_z = \frac{\omega_z}{2}\sigma_z$; $\omega_z = \gamma B_0$; $B_0\hat{z}$

= defines the energy

$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{1}+\sigma_z)$

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$\rho_{eq} = \mathbb{1} - 10^{-5}\sigma_z$

- scalable map → qubits
- initialize → $|000\rangle$
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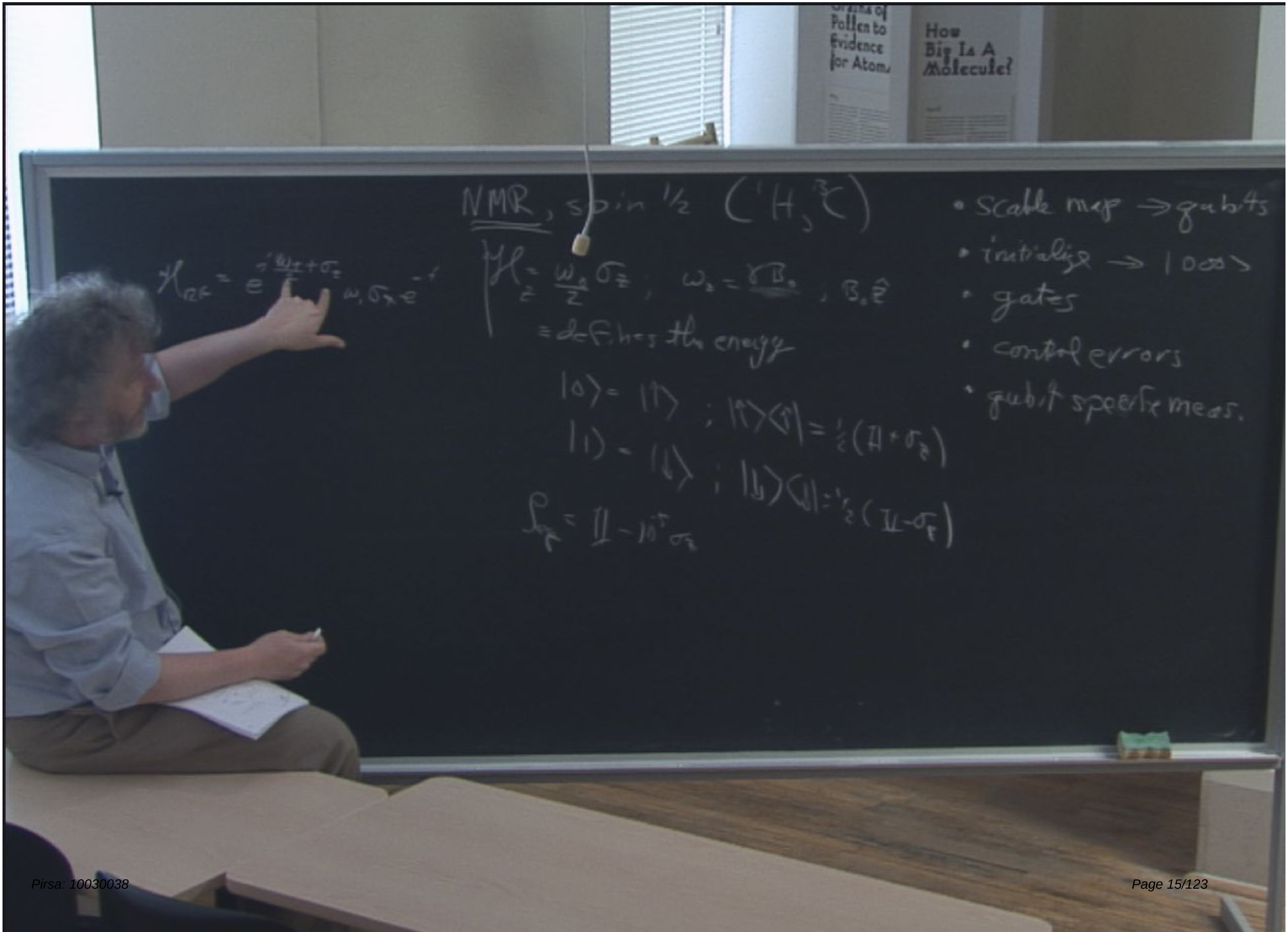
- qubit specific meas.



NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H}_{RF} = e^{i\frac{\omega_z t}{2}\sigma_z}\,\omega_1\sigma_x e^{-i}$

$\mathcal{H}_z = \frac{\omega_z}{2}\sigma_z$; $\omega_z = \gamma B_0$; $B_0\hat{z}$

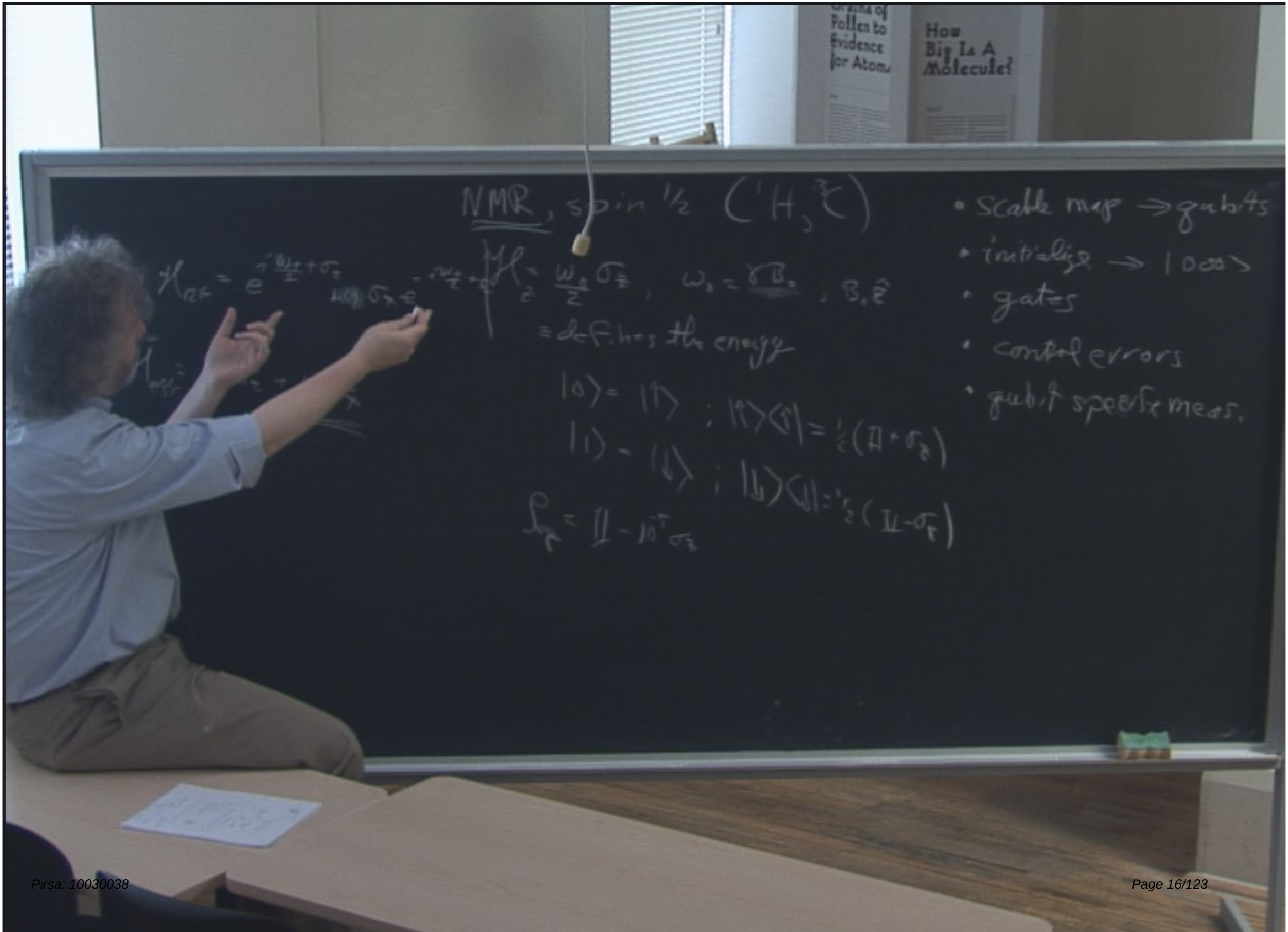
= defines the energy

$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{I}+\sigma_z)$

$|1\rangle = |\downarrow\rangle$; $|\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\mathbb{I}-\sigma_z)$

$\rho_{eq} = \mathbb{I} - 10^{5}\sigma_z$

- Scalable map → qubits
- initialize → $|000\rangle$
- gates
- control errors
- qubit specific meas.



NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H}_{RF} = e^{i\frac{\omega_0 t}{2}\sigma_z}\,\omega_1\sigma_x\,e^{-i\frac{\omega_0 t}{2}\sigma_z}$

$\mathcal{H} = \frac{\omega_0}{2}\sigma_z$; $\omega_0 = \gamma B_0$; $B_0\hat{z}$

= defines the energy

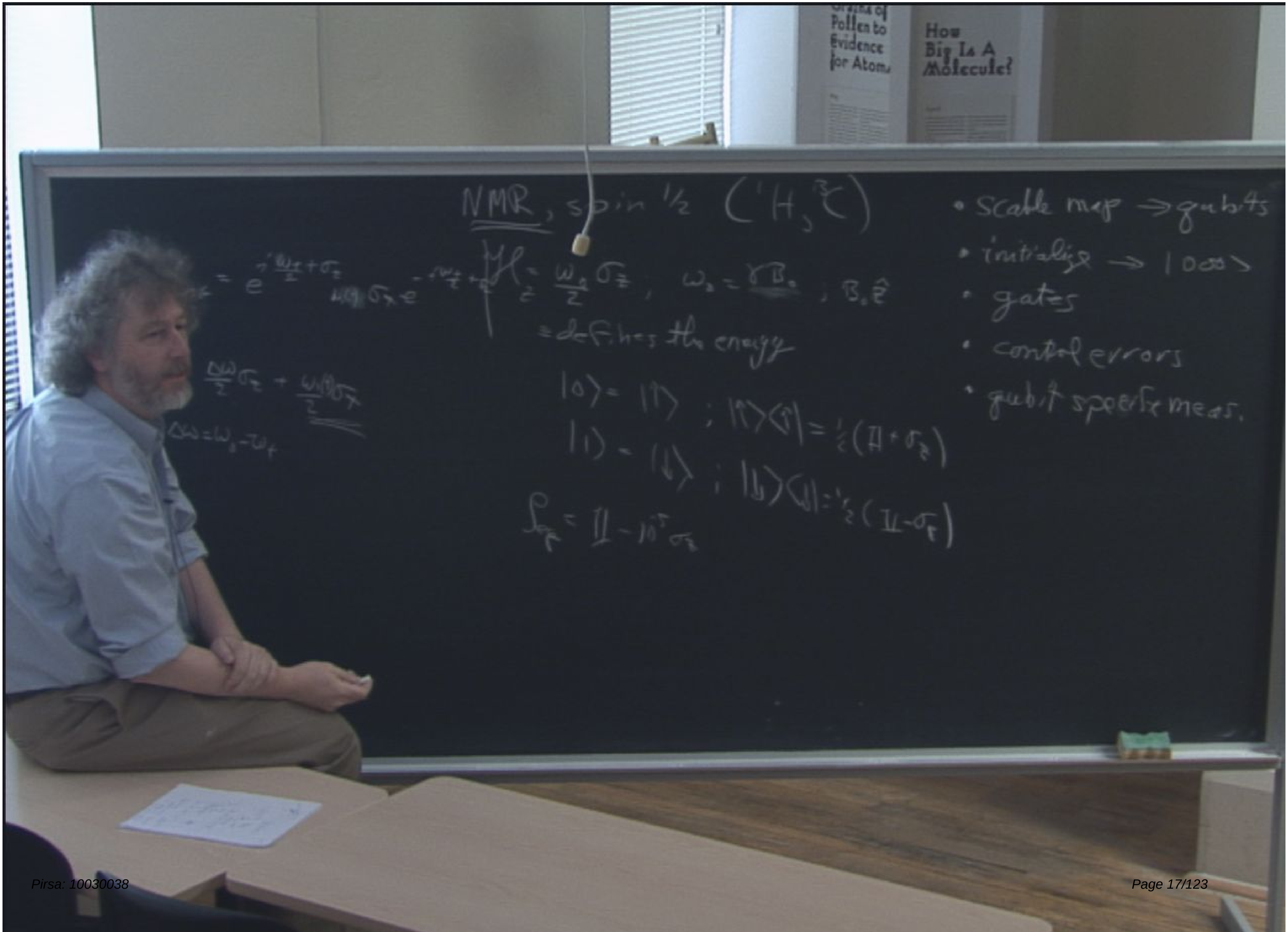
$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{1} + \sigma_z)$

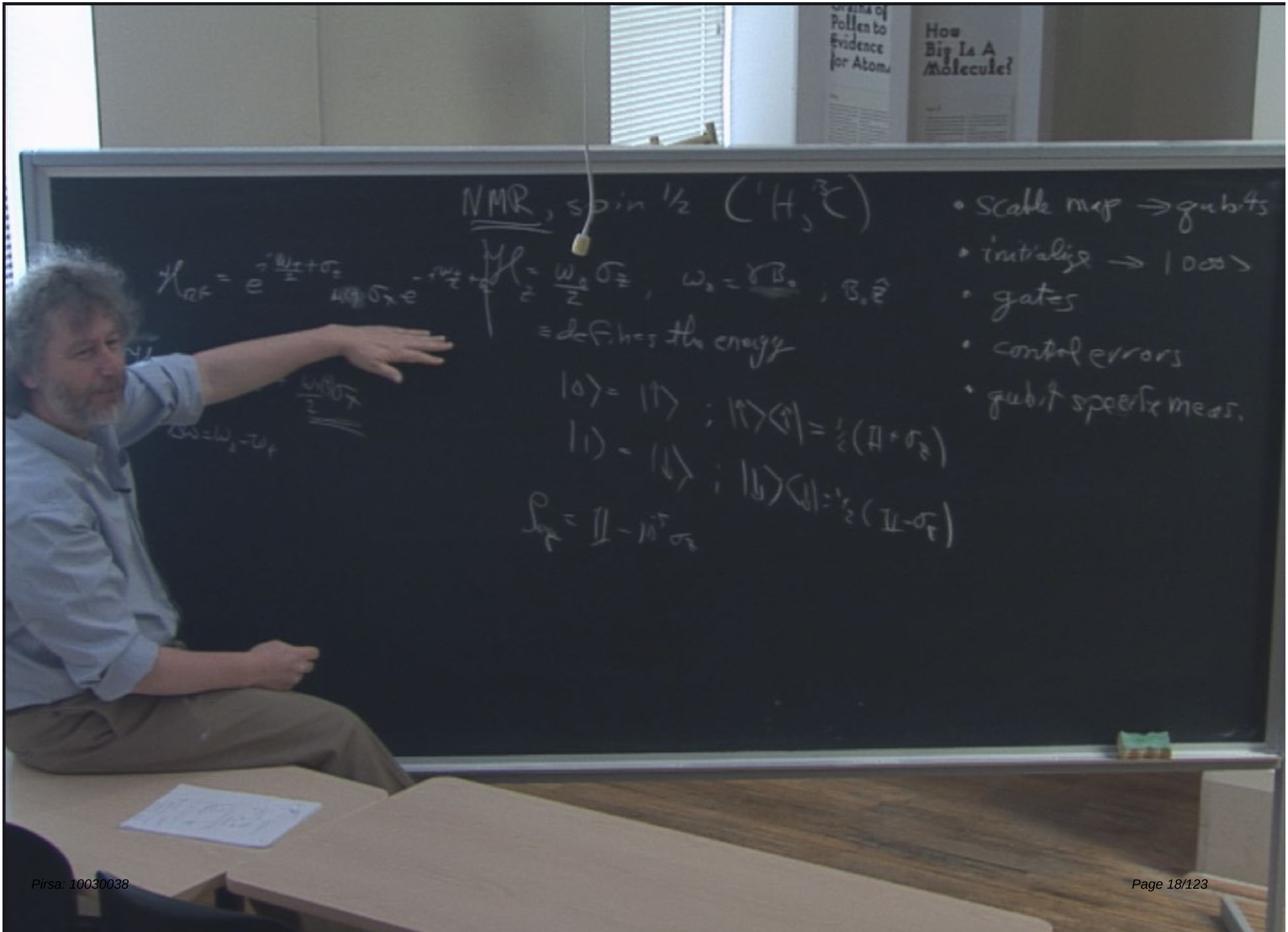
$|1\rangle = |\downarrow\rangle$; $|\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\mathbb{1} - \sigma_z)$

$\rho_{eq} = \mathbb{1} - 10^{-5}\sigma_z$

$\mathcal{H}_{eff} =$ [illegible]

- scalable map ⇒ qubits
- initialize → $|0\cdots0\rangle$
- gates
- control errors
- qubit specific meas.


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
NMR, spin ½ (¹H, ¹³C)
H = ω₀/2 σz ; ω₀ = γB₀ ; B₀ẑ
= defines the energy
|0⟩ = |↑⟩ ; |↑⟩⟨↑| = ½(𝟙 + σz)
|1⟩ = |↓⟩ ; |↓⟩⟨↓| = ½(𝟙 − σz)
ρeq = 𝟙 − 10⁻⁵ σz
= e^{i ωr t/2 σz} ω₁(t) σx e^{−i ωr t/2 σz}
Δω/2 σz + ω₁(t)σx/2
Δω = ω₀ − ωr
• scalable map → qubits
• initialize → |000⟩
• gates
• control errors
• qubit specific meas.



NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H}_{RF} = e^{-i\frac{\omega_r t}{2}\sigma_z} \omega_1 \sigma_x e^{-i\frac{\omega_r t}{2}\sigma_z}$

$\frac{\omega_1 \sigma_x}{2}$

$\Delta\omega = \omega_0 - \omega_r$

$\mathcal{H} = \frac{\omega_0}{2}\sigma_z$, $\omega_0 = \gamma B_0$; $B_0 \hat{z}$

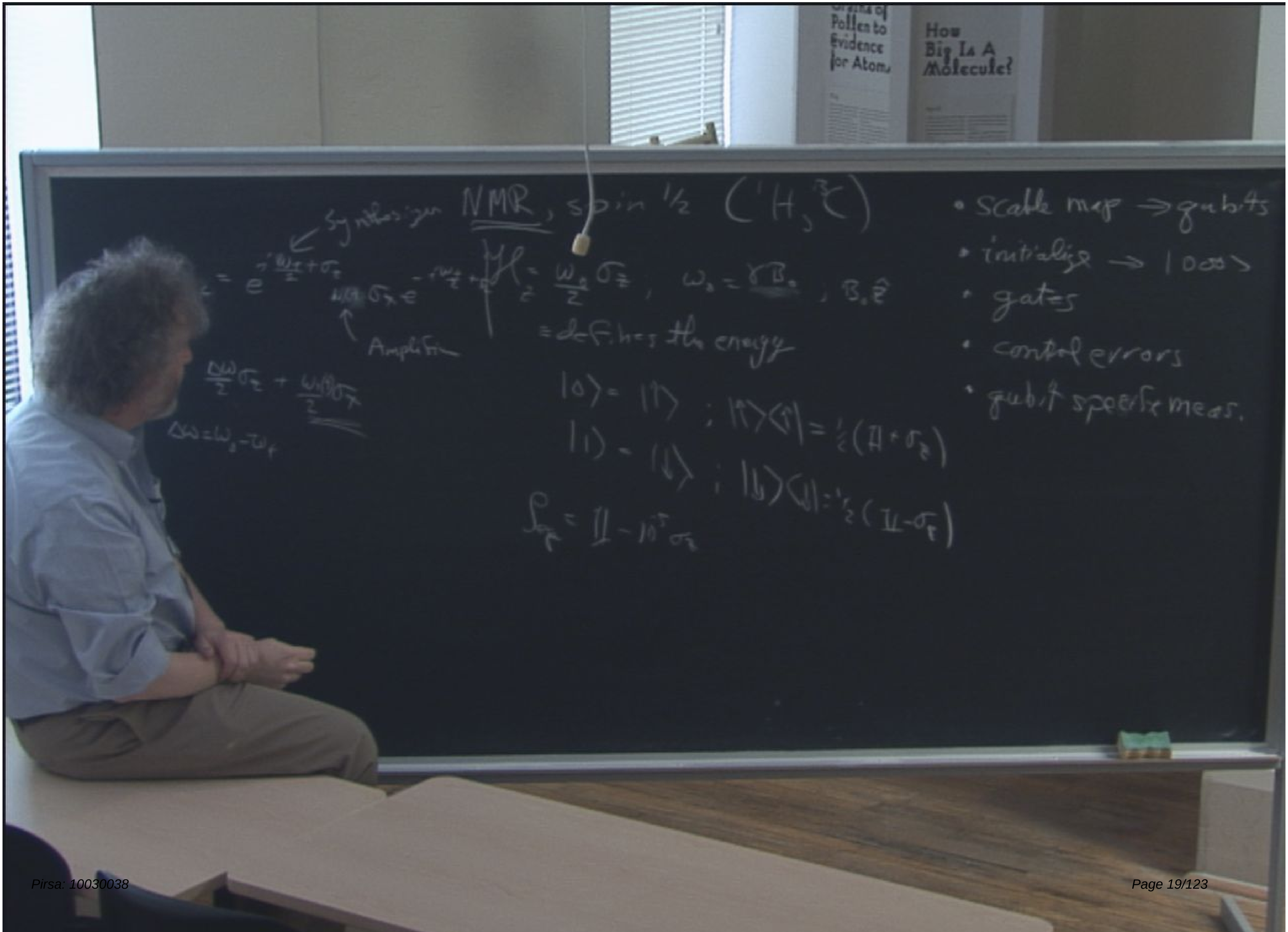
= defines the energy

$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{1}+\sigma_z)$

$|1\rangle = |\downarrow\rangle$; $|\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\mathbb{1}-\sigma_z)$

$\rho_T = \mathbb{1} - 10^{-5}\sigma_z$

- scalable map → qubits
- initialize → $|000\rangle$
- gates
- control errors
- qubit specific meas.



Synthesizer

NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H} = \frac{\omega_0}{2}\sigma_z$, $\omega_0 = \gamma B_0$; $B_0\hat{z}$

= defines the energy

$= e^{-i\frac{\omega_r t}{2}\sigma_z}\,\omega_1(t)\,\sigma_x\, e^{-i\frac{\omega_r t}{2}\sigma_z}$

Amplifier

$\frac{\Delta\omega}{2}\sigma_z + \frac{\omega_1(t)\sigma_x}{2}$

$\Delta\omega = \omega_0 - \omega_r$

$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{I}+\sigma_z)$

$|1\rangle = |\downarrow\rangle$; $|\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\mathbb{I}-\sigma_z)$

$\rho_{eq} = \mathbb{I} - 10^{5}\sigma_z$

- scalable map → qubits
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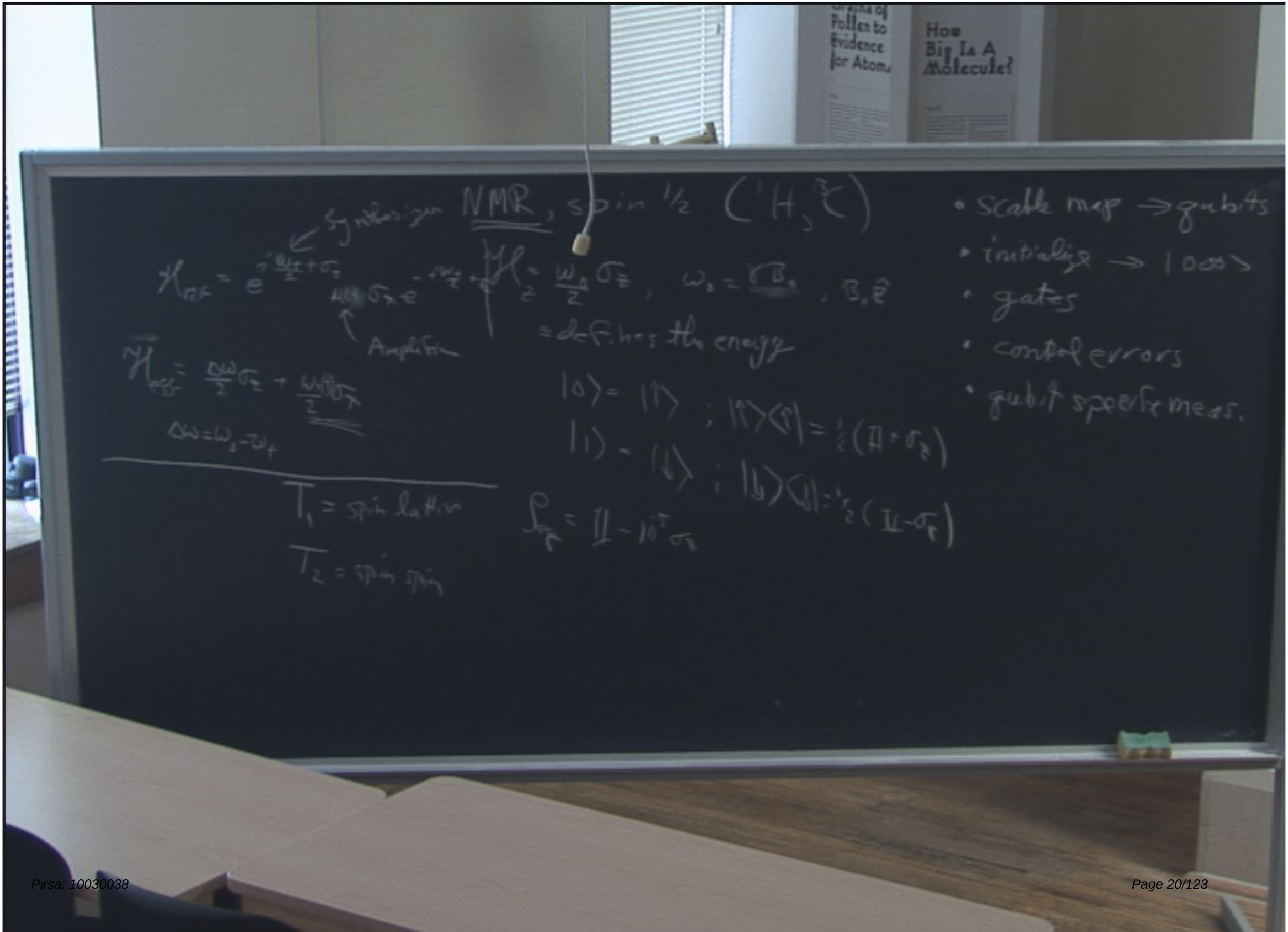
- control errors
- qubit specific meas.



NMR, spin 1/2 (^1H, ^{13}C)

Synthesizer

$$\mathcal{H}_{RF} = e^{-i\frac{\omega_t}{2}t\sigma_z}\,\omega_1(t)\,\sigma_x\, e^{-i\frac{\omega_t}{2}t\sigma_z}$$

Amplitude

$$\mathcal{H}_z = \frac{\omega_0}{2}\sigma_z, \quad \omega_0 = \gamma B_0 \;;\; B_0\hat{z}$$

= defines the energy

$$\tilde{\mathcal{H}}_{eff} = \frac{\Delta\omega}{2}\sigma_z + \frac{\omega_1(t)\sigma_x}{2}$$

$$\Delta\omega = \omega_0 - \omega_t$$

$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{I} + \sigma_z)$

$|1\rangle = |\downarrow\rangle$; $|\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\mathbb{I} - \sigma_z)$

$\rho_{eq} = \mathbb{I} - 10^{5}\sigma_z$

T_1 = spin lattice

T_2 = spin spin

- Scalable map → qubits
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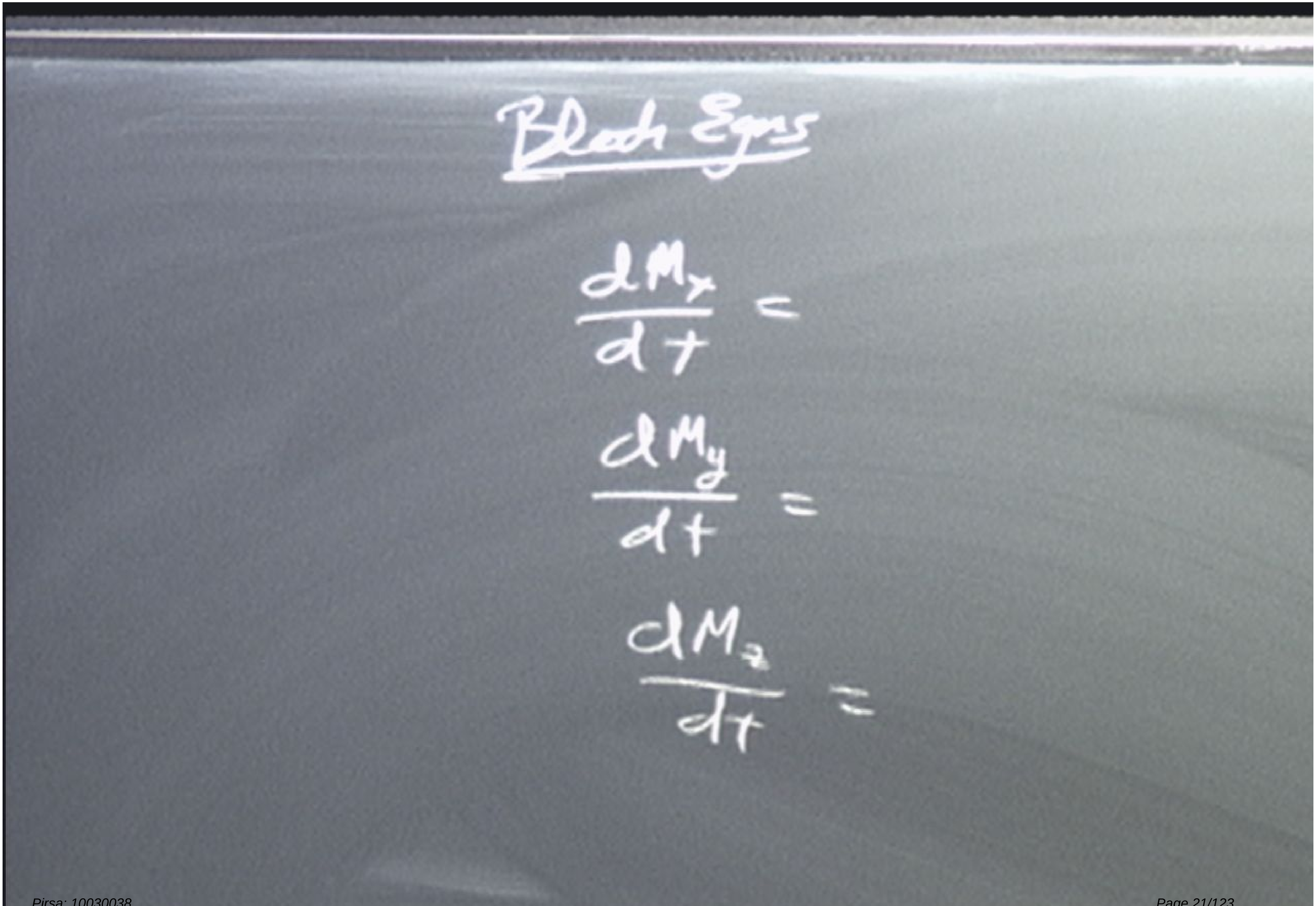
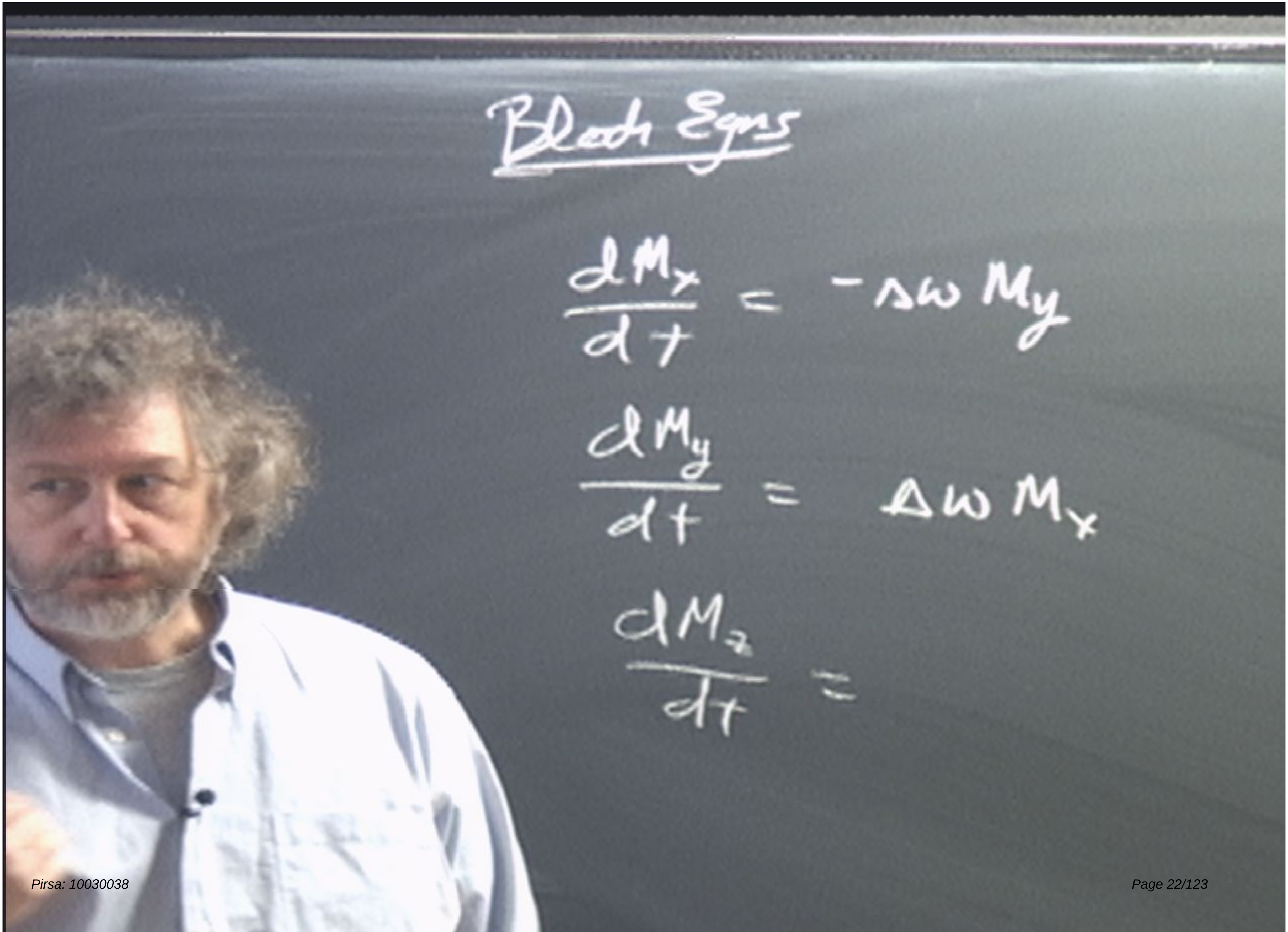


Bloch Eqns

$$\frac{dM_x}{dt} =$$

$$\frac{dM_y}{dt} =$$

$$\frac{dM_z}{dt} =$$

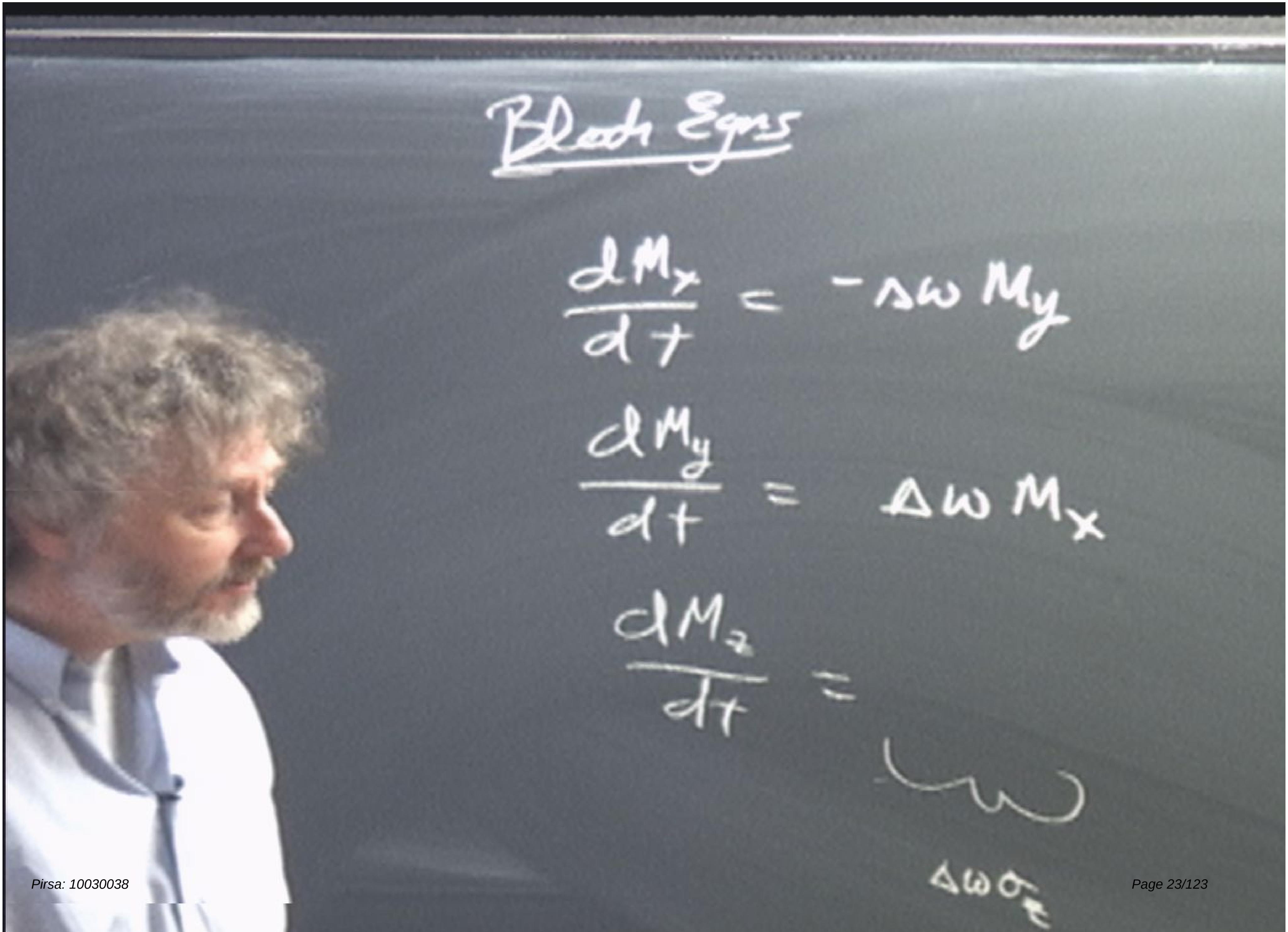


Bloch Eqns

$$\frac{dM_x}{dt} = -\Delta\omega M_y$$

$$\frac{dM_y}{dt} = \Delta\omega M_x$$

$$\frac{dM_z}{dt} =$$



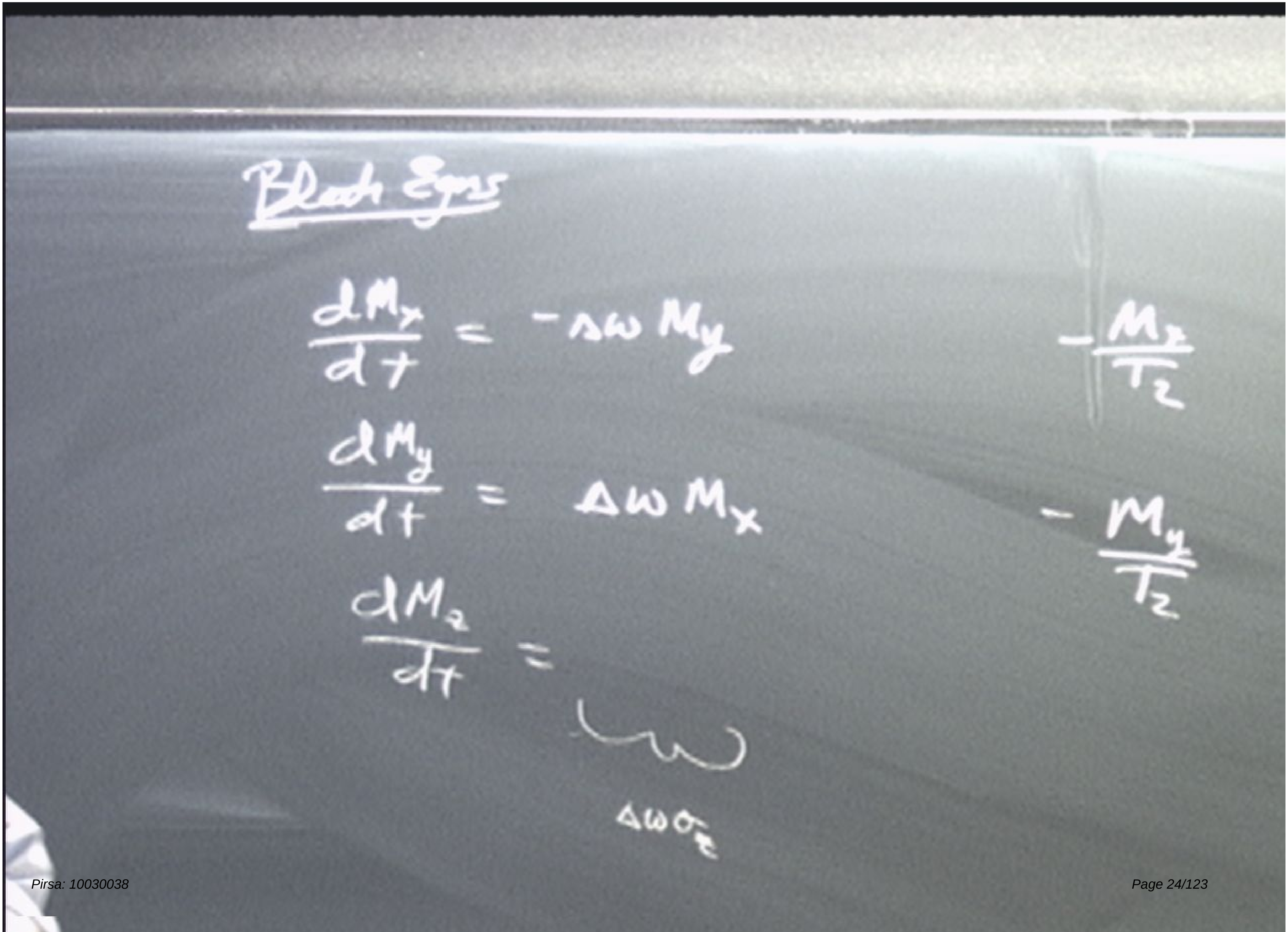
Bloch Eqns

$$\frac{dM_x}{dt} = -\Delta\omega M_y$$

$$\frac{dM_y}{dt} = \Delta\omega M_x$$

$$\frac{dM_z}{dt} =$$

$\Delta\omega\sigma_z$



Bloch Eqns

$$\frac{dM_x}{dt} = -\Delta\omega M_y \qquad -\frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \Delta\omega M_x \qquad -\frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \underbrace{\qquad\qquad}_{\Delta\omega\sigma_z}$$

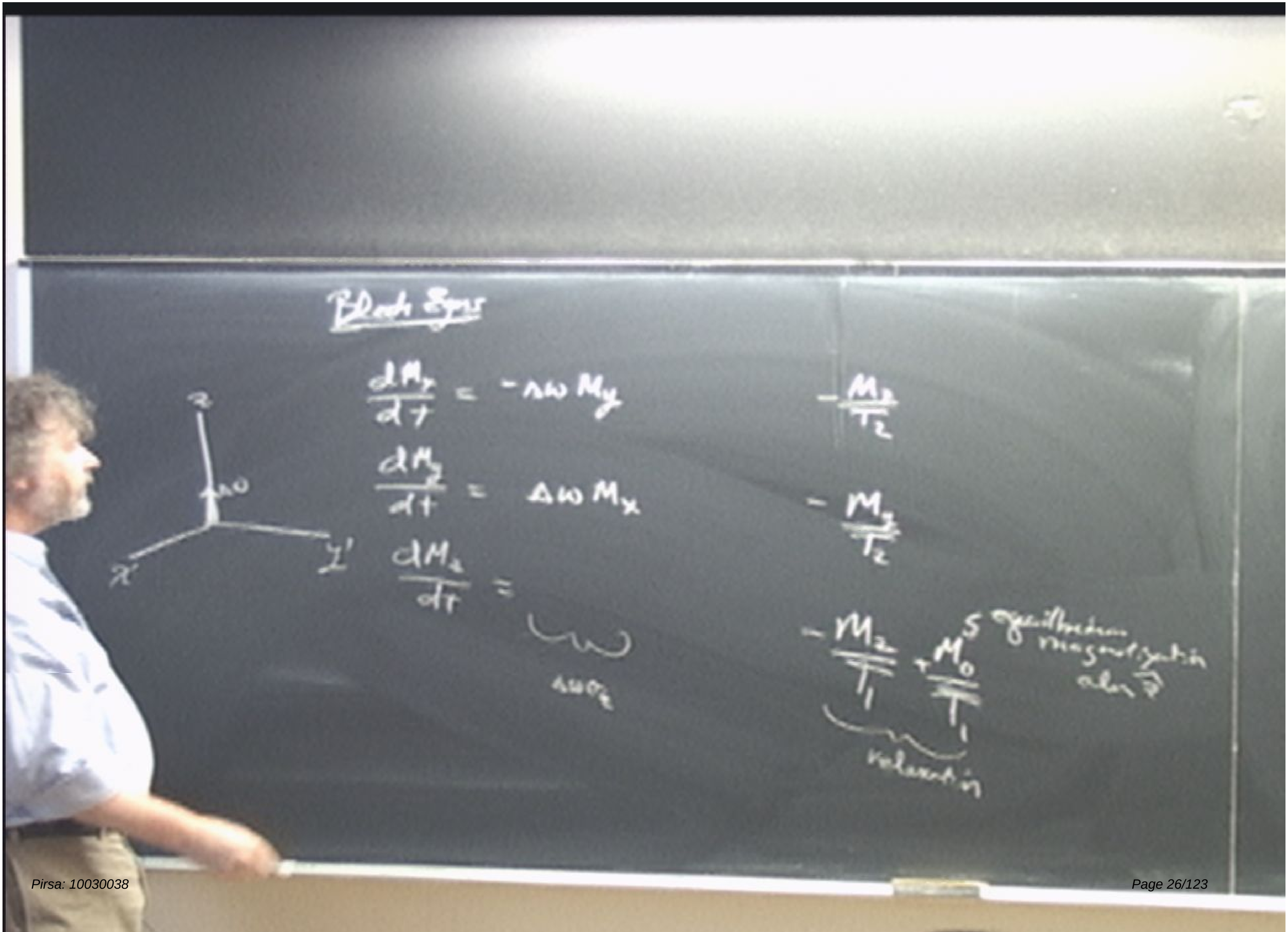
Bloch Eqns

$$\frac{dM_x}{dt} = -\Delta\omega M_y - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \Delta\omega M_x - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \underbrace{\qquad\qquad}_{\Delta\omega\sigma_z} - \frac{M_z}{T_1} + \frac{M_0}{T_1}$$

S equilibrium magnetization along $\vec{z}$



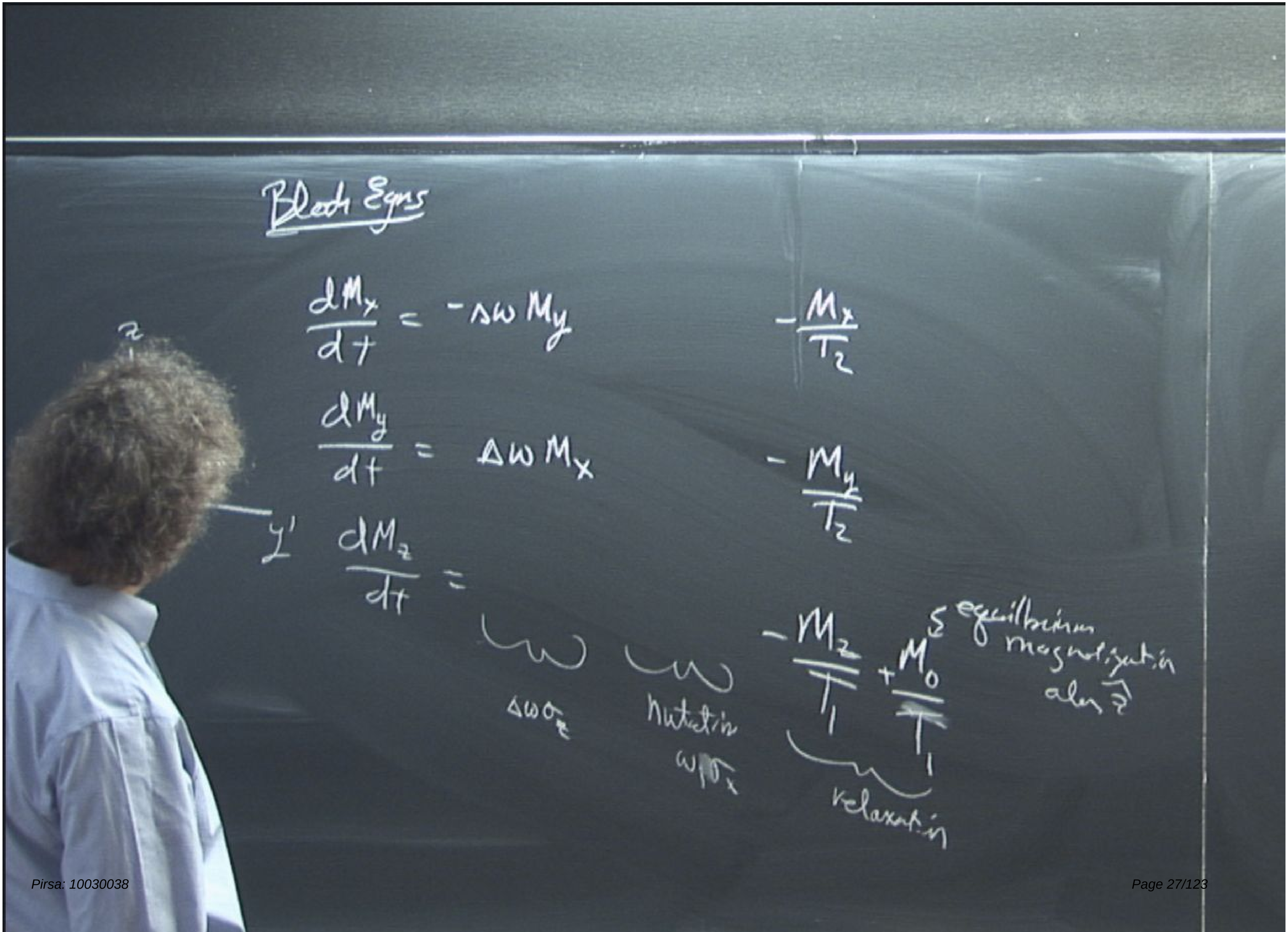
Bloch Eqns

$$\frac{dM_x}{dt} = -\Delta\omega M_y \qquad - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \Delta\omega M_x \qquad - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \underbrace{\qquad}_{\Delta\omega\sigma_z} \qquad \underbrace{- \frac{M_z}{T_1} + \frac{M_0}{T_1}}_{\text{relaxation}}$$

M_0 equilibrium magnetization along $\vec{z}$



Bloch Eqns

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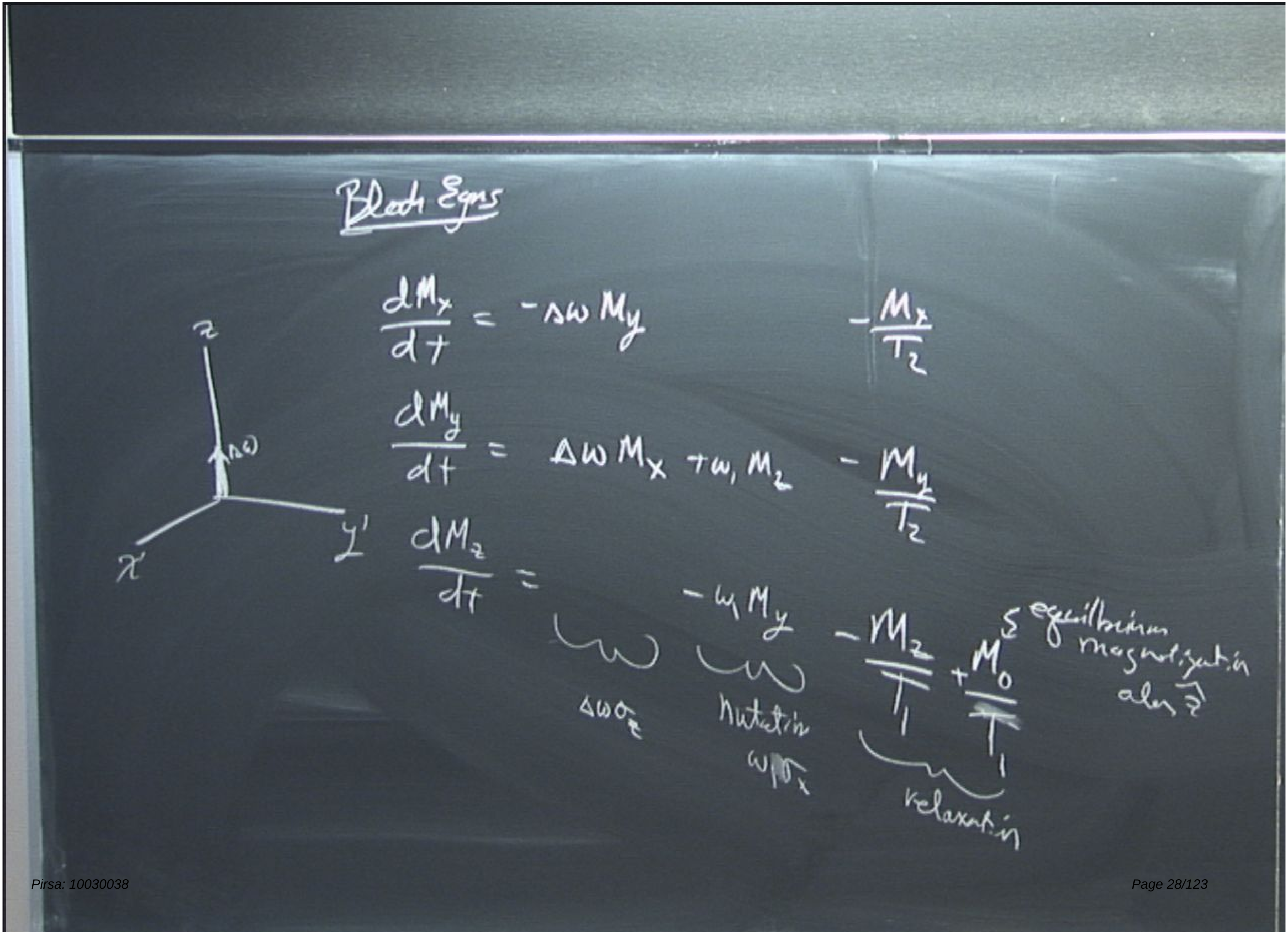
$$\frac{dM_y}{dt} = \Delta\omega M_x \qquad - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \underbrace{\qquad}_{\Delta\omega\sigma_z} \quad \underbrace{\qquad}_{\text{nutation } \omega\sigma_x} \quad \underbrace{-\frac{M_z}{T_1} + \frac{M_0}{T_1}}_{\text{relaxation}}$$

equilibrium magnetization along $\vec{z}$



Bloch Eqns

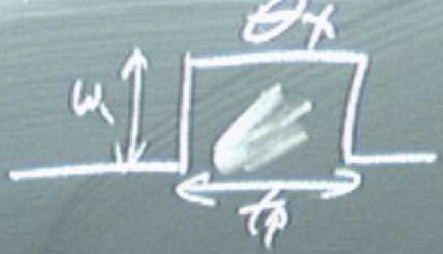
$$\frac{dM_x}{dt} = -\Delta\omega M_y \qquad - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \Delta\omega M_x + \omega_1 M_z - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \underbrace{\qquad}_{\Delta\omega\sigma_z} \; \underbrace{-\omega_1 M_y}_{\substack{\text{nutation} \\ \omega_1\sigma_x}} \; \underbrace{- \frac{M_z}{T_1} + \frac{M_0}{T_1}}_{\text{relaxation}}$$

M_0 { equilibrium magnetization along $\vec{z}$

Rabi

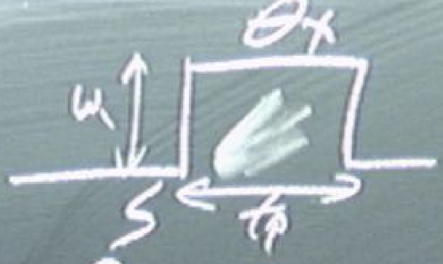


equilibrium magnetization along $\vec{z}$

$+\frac{M_0}{T_1}$

elaxation

Rabi



$\rho_{eq} \propto \sigma_z$

$+\frac{M_0}{T_1}$ equilibrium magnetization along $\vec{z}$

laxation

$+\frac{M_0}{T_1}$ equilibrium magnetization along $\vec{z}$

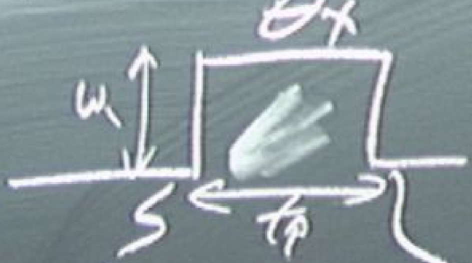
laxation

Rabi

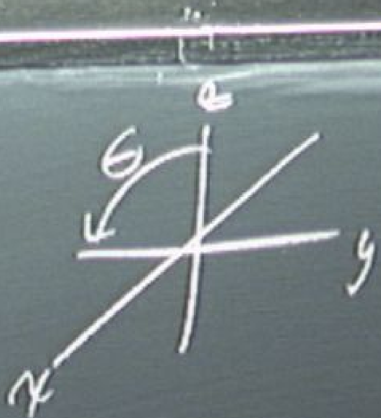
$\Delta\omega = 0$

$\rho_{eq} \propto \sigma_z$

Rabi



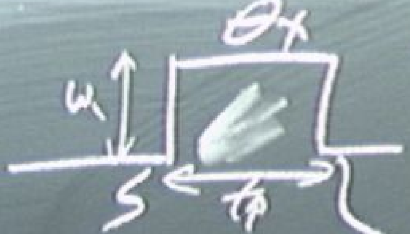
$\Delta\omega = 0$



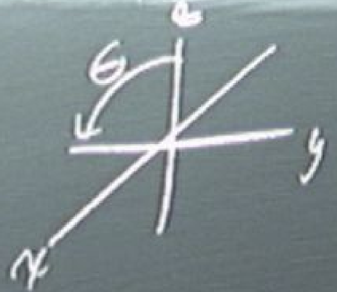
$\rho_{eg} \propto \sigma_z$

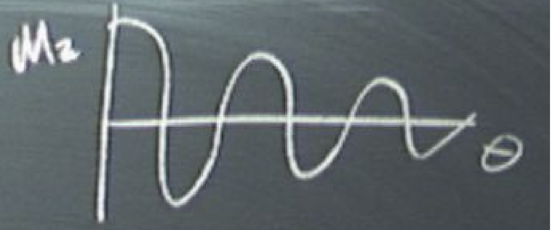
$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

Rabi



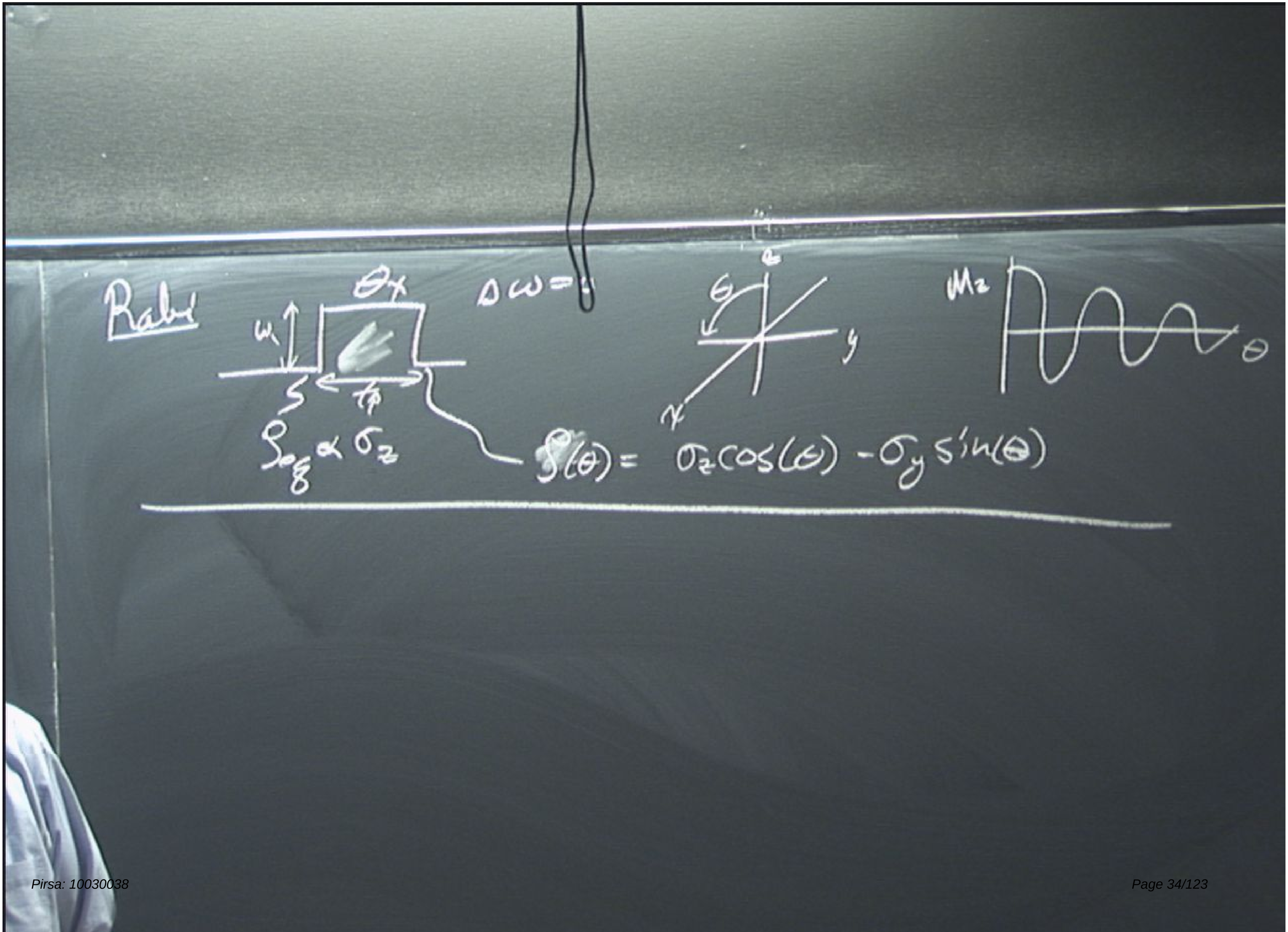
$\Delta\omega = 0$





$\rho_{eg} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

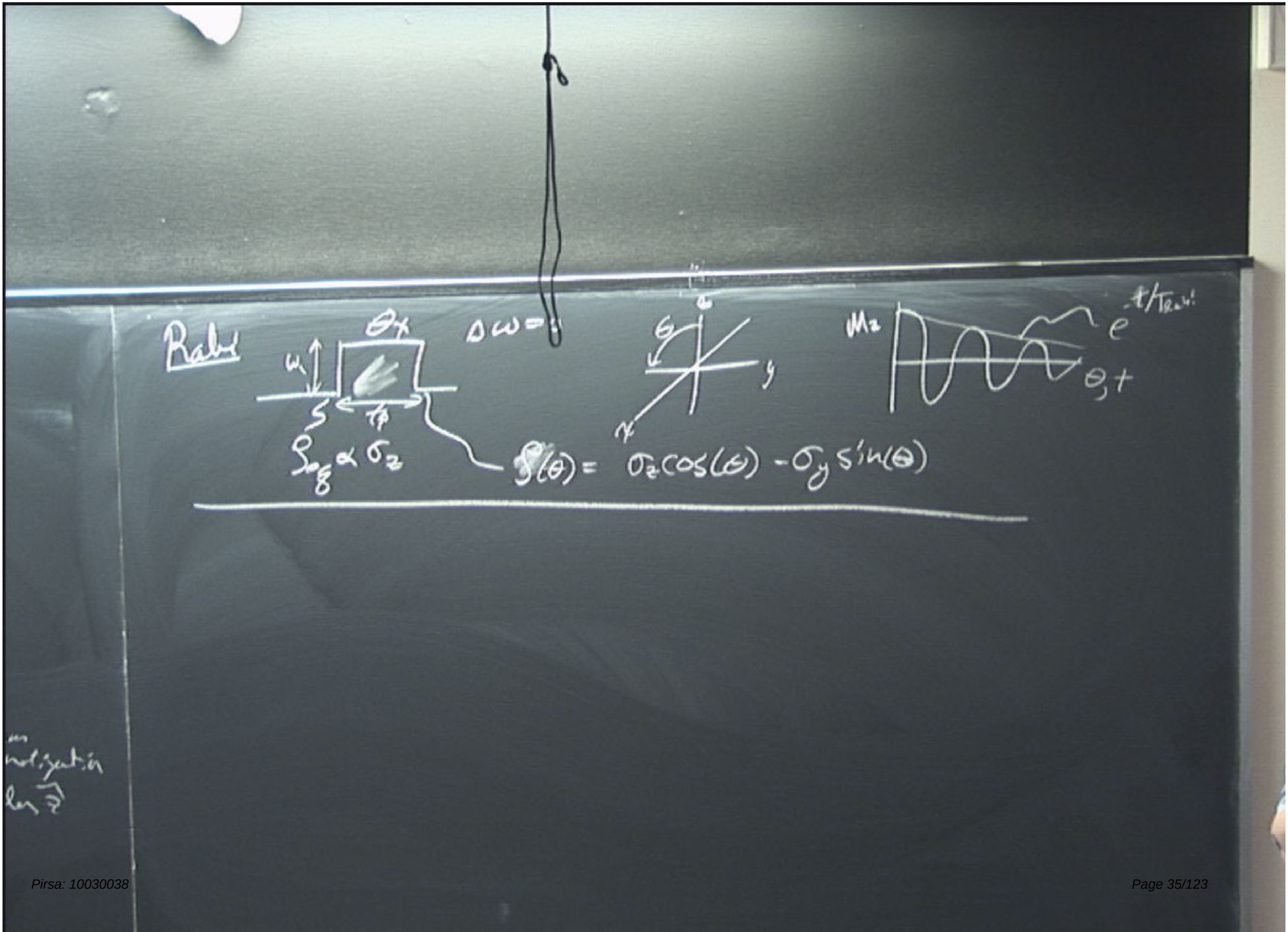


Rabi

$\Delta\omega =$

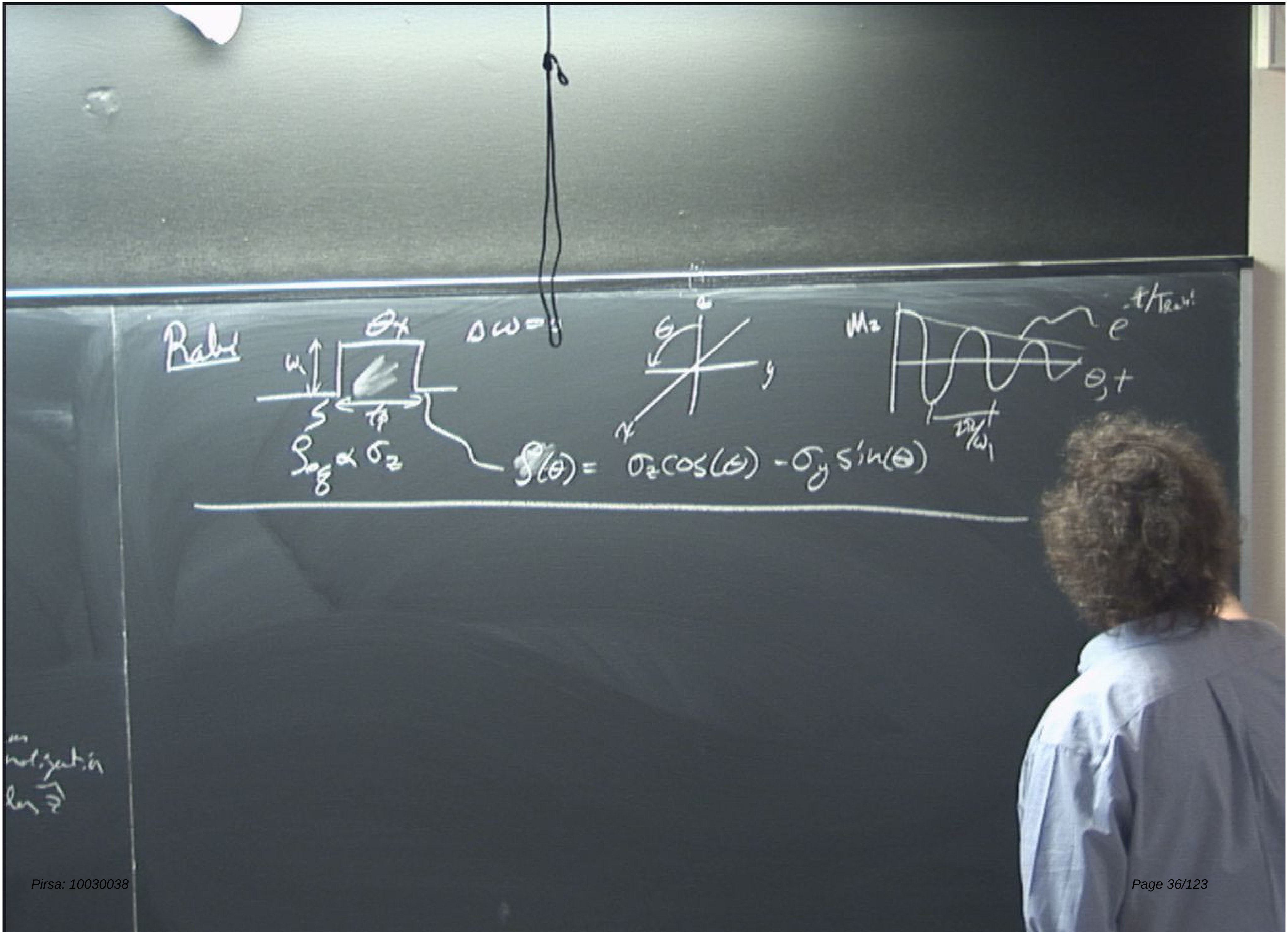
$\rho_{eq} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$



$\rho_{eg} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$



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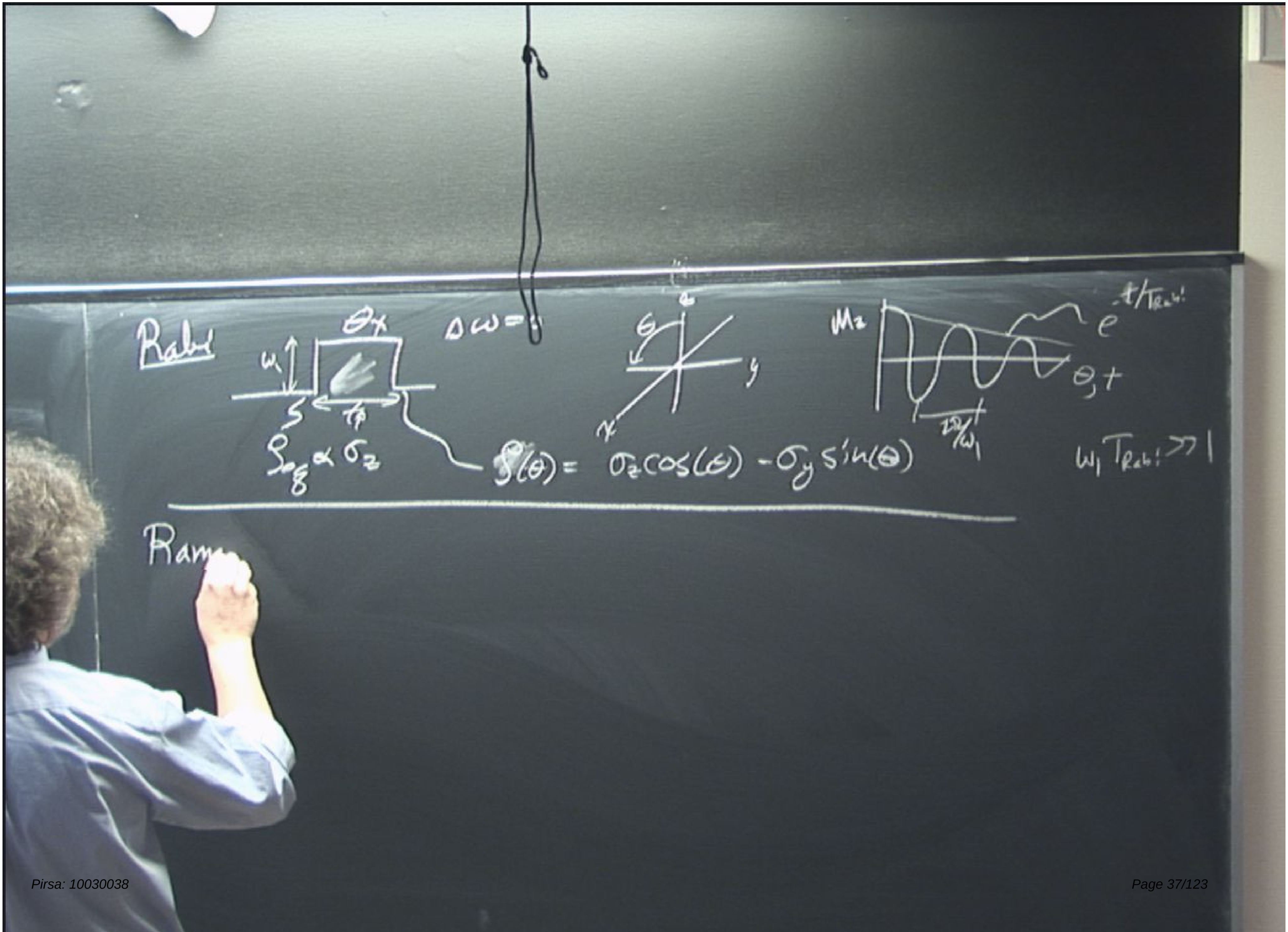
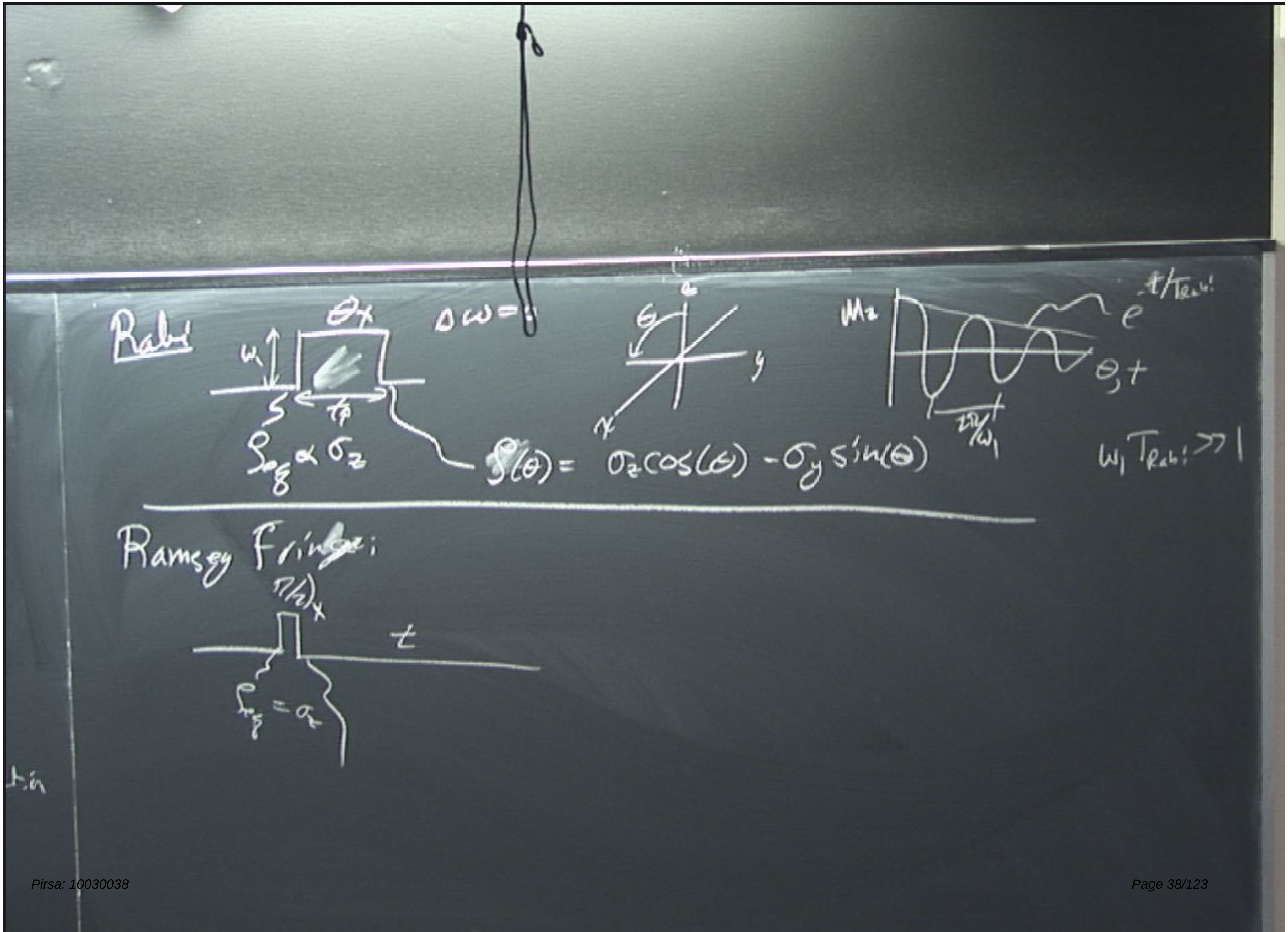


$\rho_{eg} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

$\omega_1 T_{Rabi} >> 1$

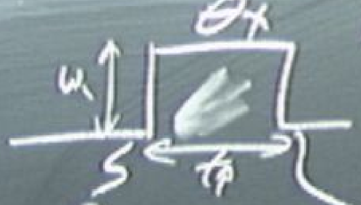


$$\rho_{eg} \propto \sigma_z$$

$$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$$

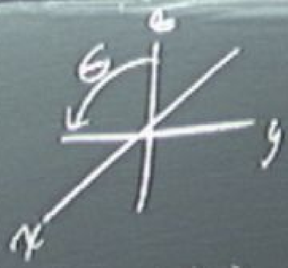
$$\omega_1 T_{Rabi} \gg 1$$

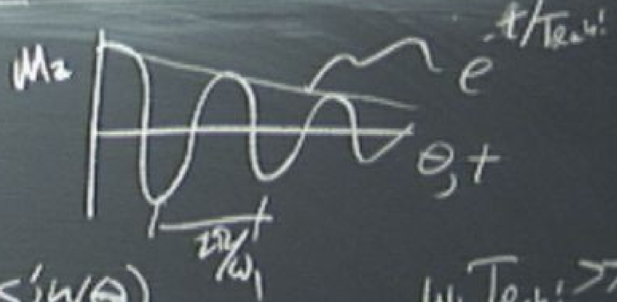
Rabi



$\rho_{eq} \propto \sigma_z$

$\Delta\omega = 0$

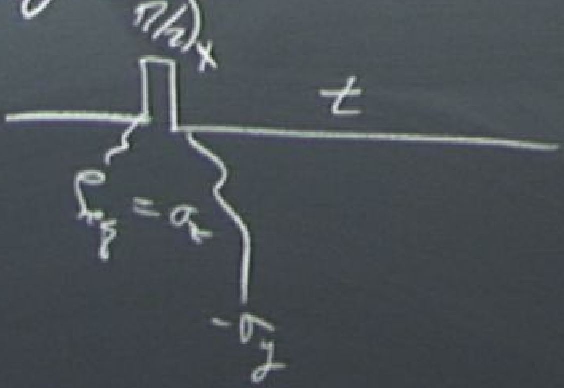




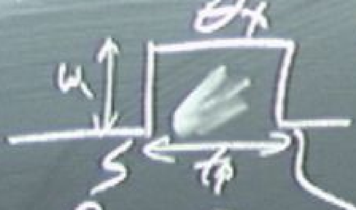
$$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$$

$\omega_1 T_{Rabi} >> 1$

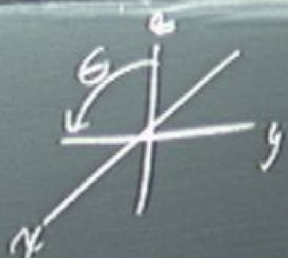
Ramsey Fringes:

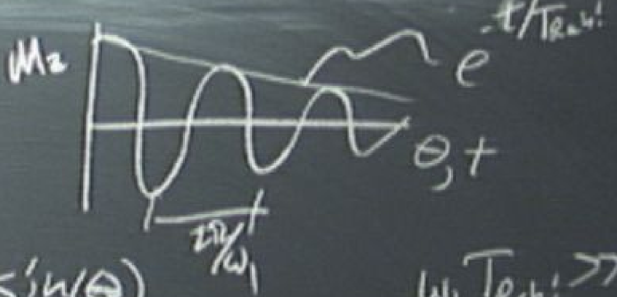


Rabi



$\Delta\omega = 0$





$\rho_{eg} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

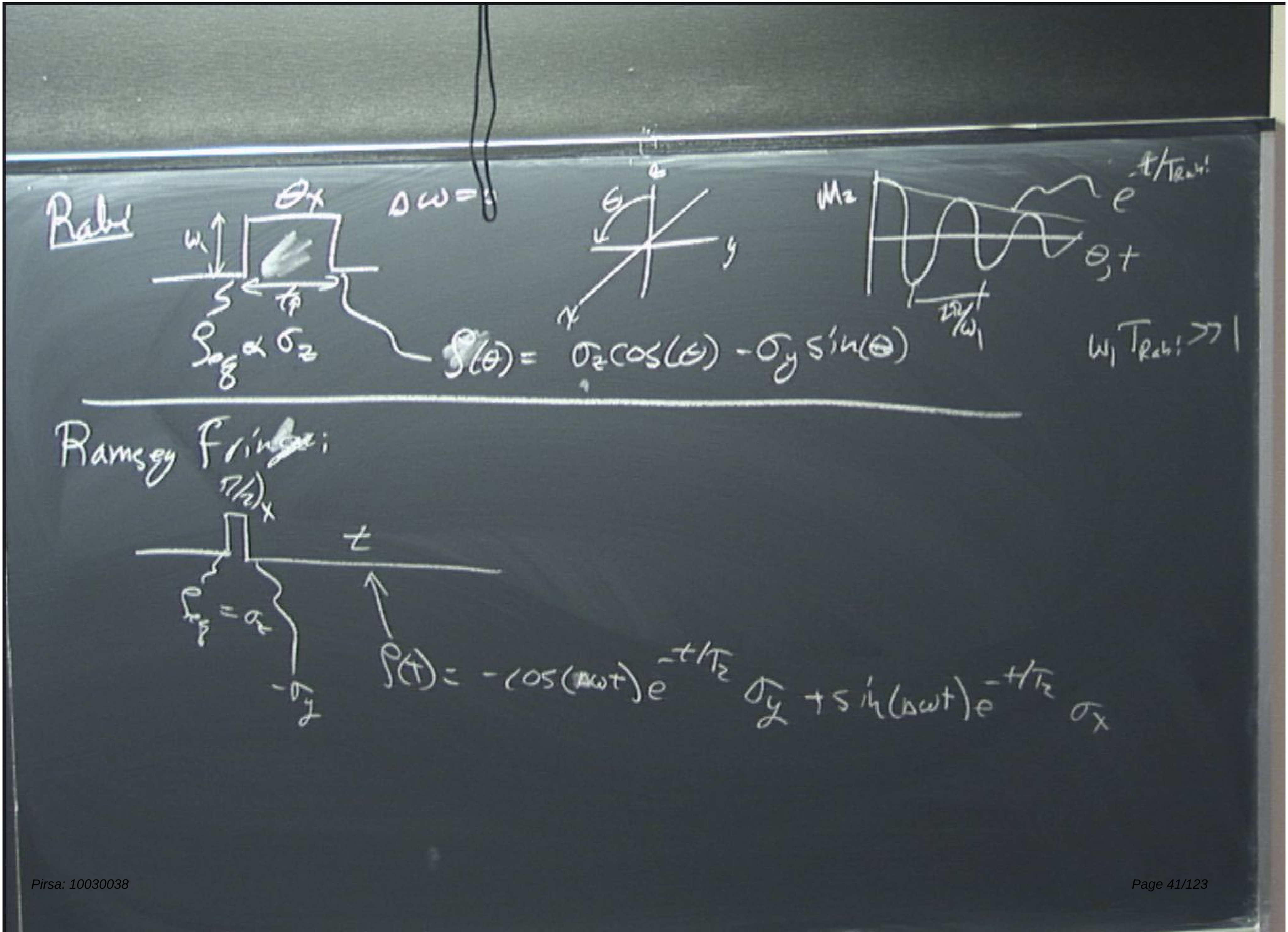
$\omega_1 T_{Rabi} \gg 1$

Ramsey Fringes:

$\rho_{eg} = \sigma_z$

$-\sigma_y$

$\rho(t) = -\cos(\Delta\omega t) e^{-t/T_2} \sigma_y$



Rabi

θ_x

ω_1

t_p

$\Delta\omega = 0$

$\rho_{eg} \propto \sigma_z$

$$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$$

$e^{-t/T_{Rabi}}$

$\omega_1 T_{Rabi} \gg 1$

Ramsey Fringe:

$(\pi/2)_x$

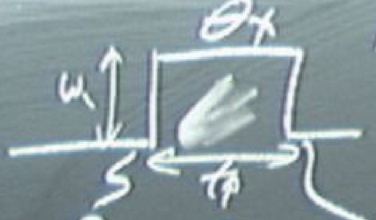
t

$\rho_{eg} = \sigma_z$

$-\sigma_y$

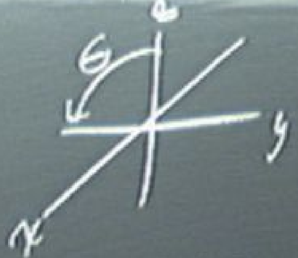
$$\rho(t) = -\cos(\Delta\omega t)\, e^{-t/T_2}\, \sigma_y + \sin(\Delta\omega t)\, e^{-t/T_2}\, \sigma_x$$

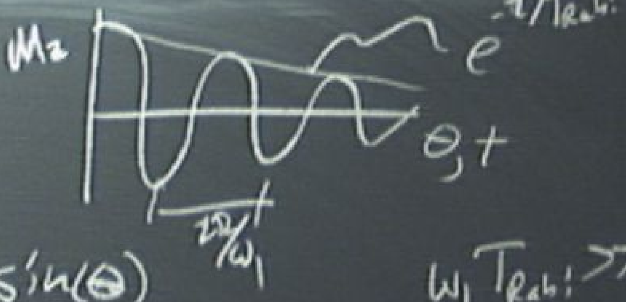
Rabi



$\rho_{eg} \propto \sigma_z$

$\Delta\omega =$





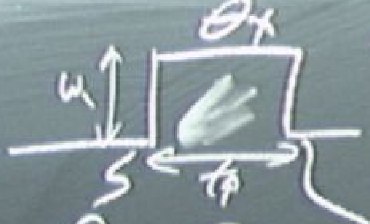
$$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$$

$\omega_1 T_{Rabi} \gg 1$

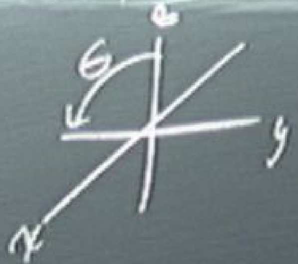
Ramsey Fringe:

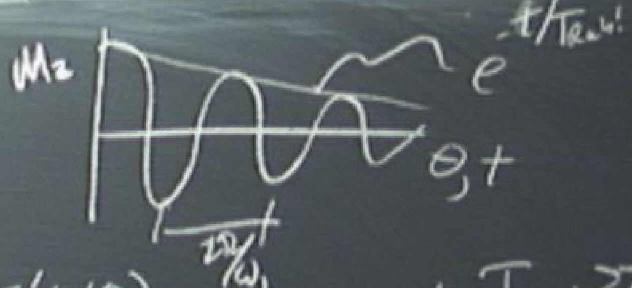
$$\rho(t) = -\cos(\Delta\omega t) e^{-t/T_2} \sigma_y + \sin(\Delta\omega t) e^{-t/T_2} \sigma_x + \left(1 - e^{-t/T_1}\right)\sigma_z$$

Rabi



$\Delta\omega = 0$



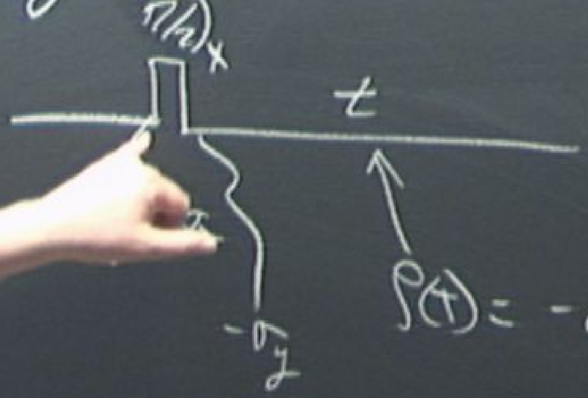


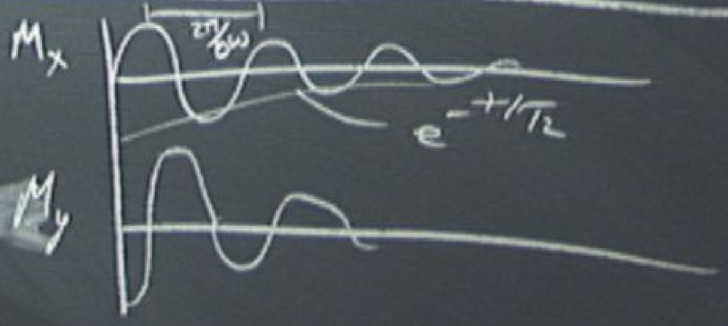
$\rho_{eq} \propto \sigma_z$

$$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$$

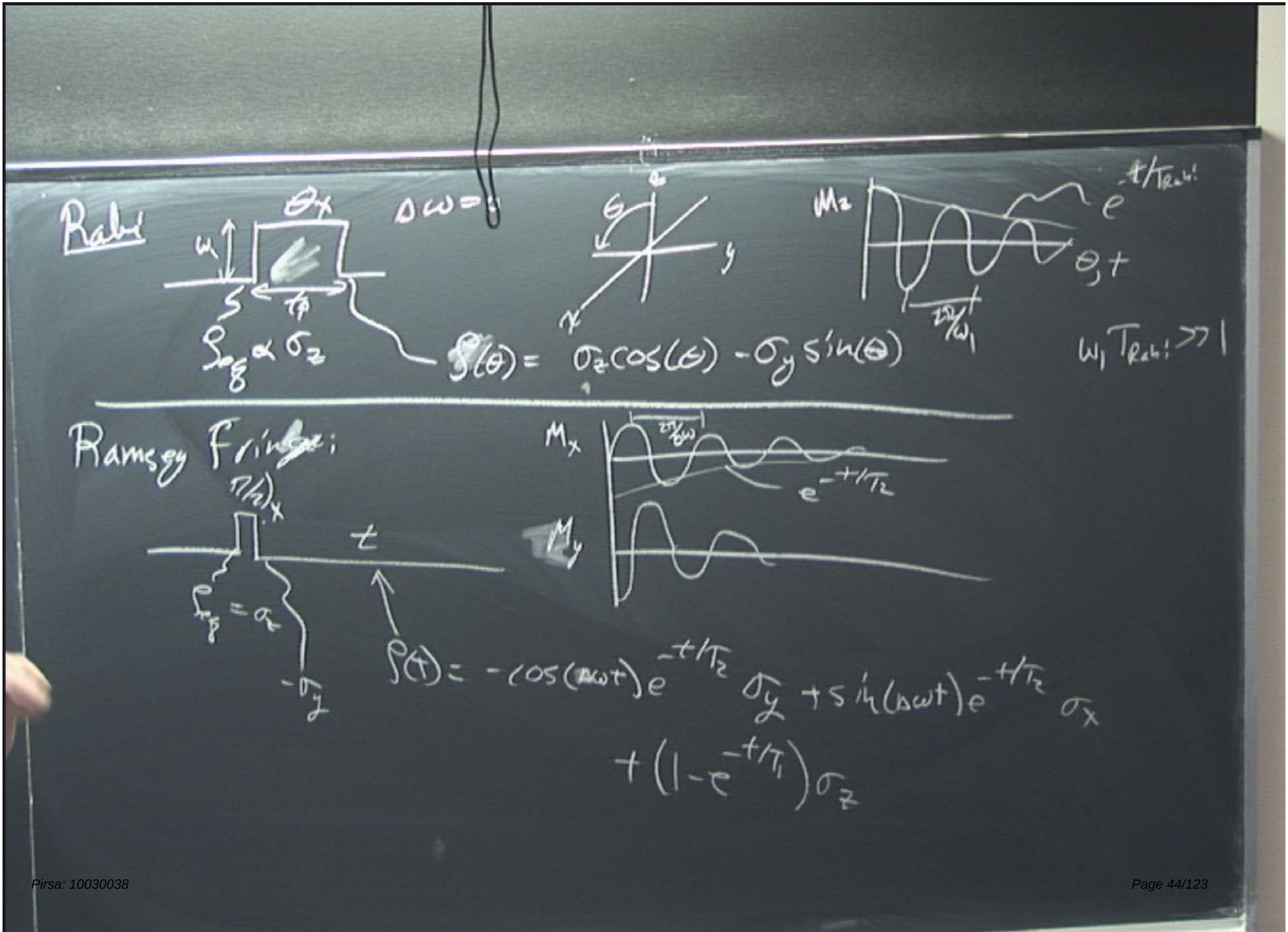
$\omega_1 T_{Rabi} \gg 1$

Ramsey Fringe:





$$\rho(t) = -\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x + (1-e^{-t/T_1})\sigma_z$$



Rabi

$\Delta\omega = 0$

$\rho_{eg} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

$\omega_1 T_{Rabi} \gg 1$

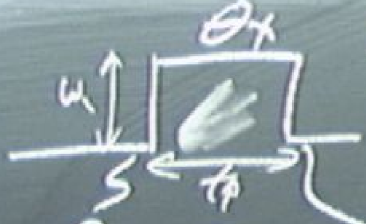
Ramsey Fringes:

$\rho_{eg} = \sigma_z$

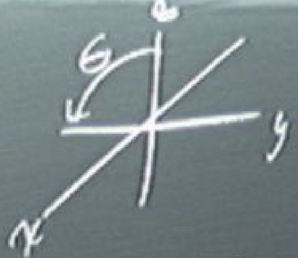
$-\sigma_y$

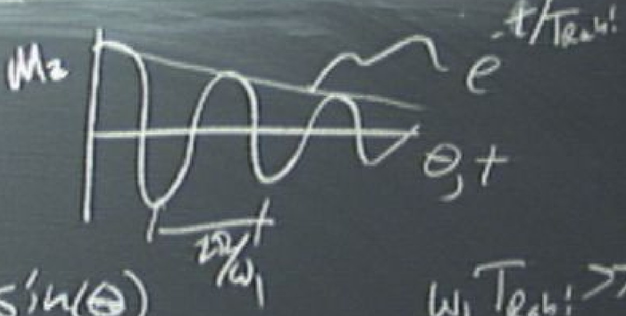
$$\rho(t) = -\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x + (1-e^{-t/T_1})\sigma_z$$

Rabi



$\Delta\omega = 0$



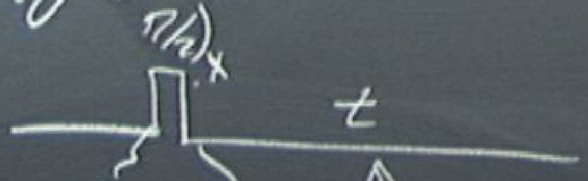


$\rho_{eg} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

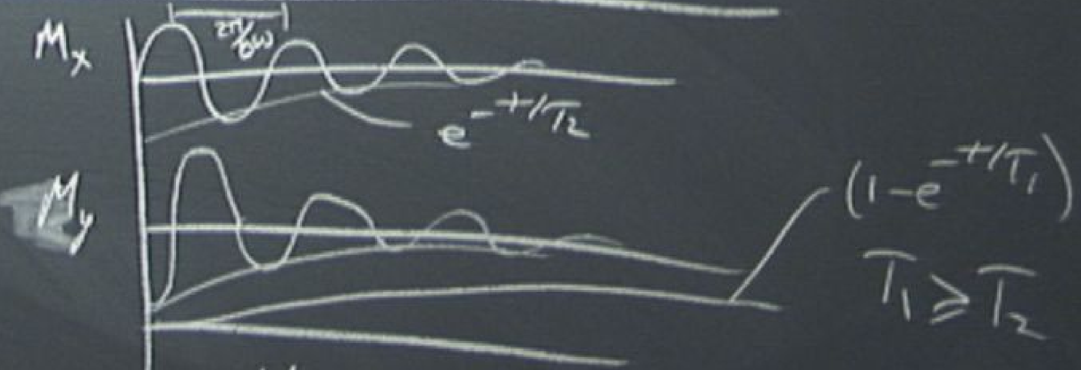
$\omega_1 T_{Rabi} \gg 1$

Ramsey Fringe:

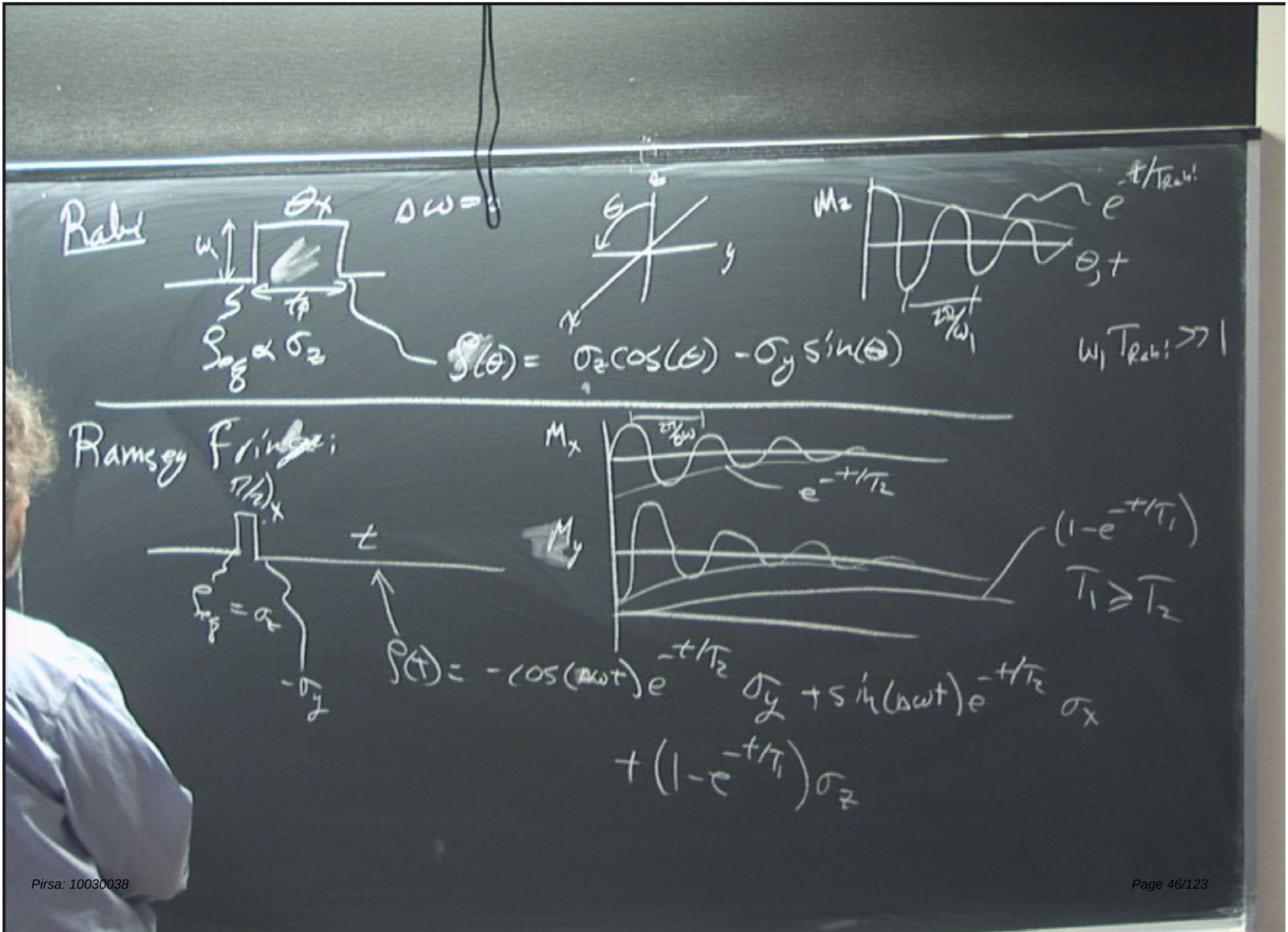


$\rho_{eg} = \sigma_z$

$-\sigma_y$



$$\rho(t) = -\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x + (1-e^{-t/T_1})\sigma_z$$



Rabi

θ_x

$\Delta\omega = 0$

$\rho_{eq} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

$\omega_1 T_{Rabi} \gg 1$

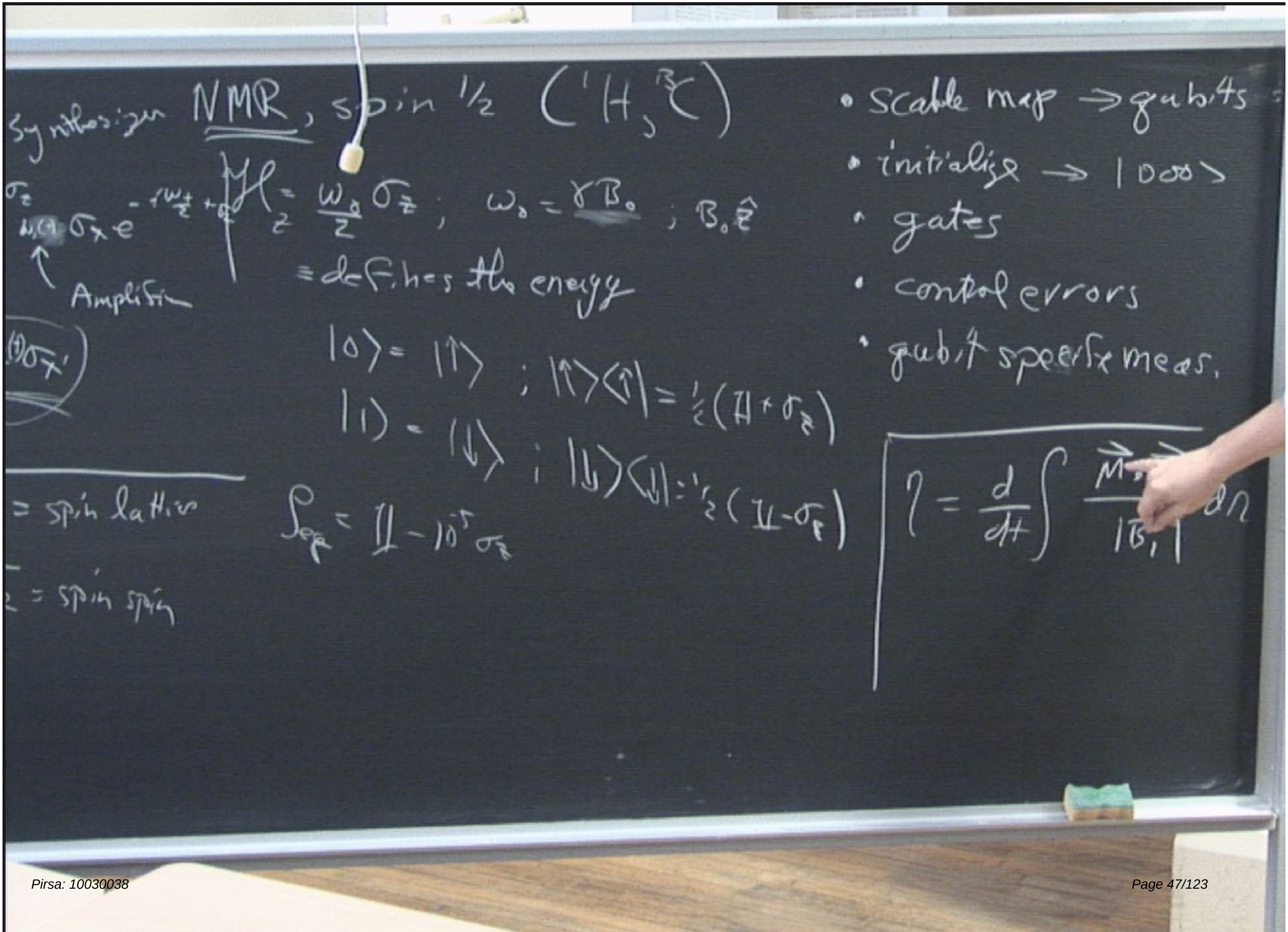
Ramsey Fringe:

$\rho_{eq} = \sigma_z$

$-\sigma_y$

$T_1 \geq T_2$

$$\rho(t) = -\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x + (1-e^{-t/T_1})\sigma_z$$



Synthesizer NMR, spin 1/2 (^1H, ^{13}C)

σ_z

$\omega_1(t)\,\sigma_x e^{-i\frac{\omega_t}{2}t}$

Amplifier

$(t)\sigma_x'$

= spin lattice

= spin spin

$\mathcal{H}_Z = \frac{\omega_0}{2}\sigma_z$; $\omega_0 = \gamma B_0$; $B_0\hat{z}$

= defines the energy

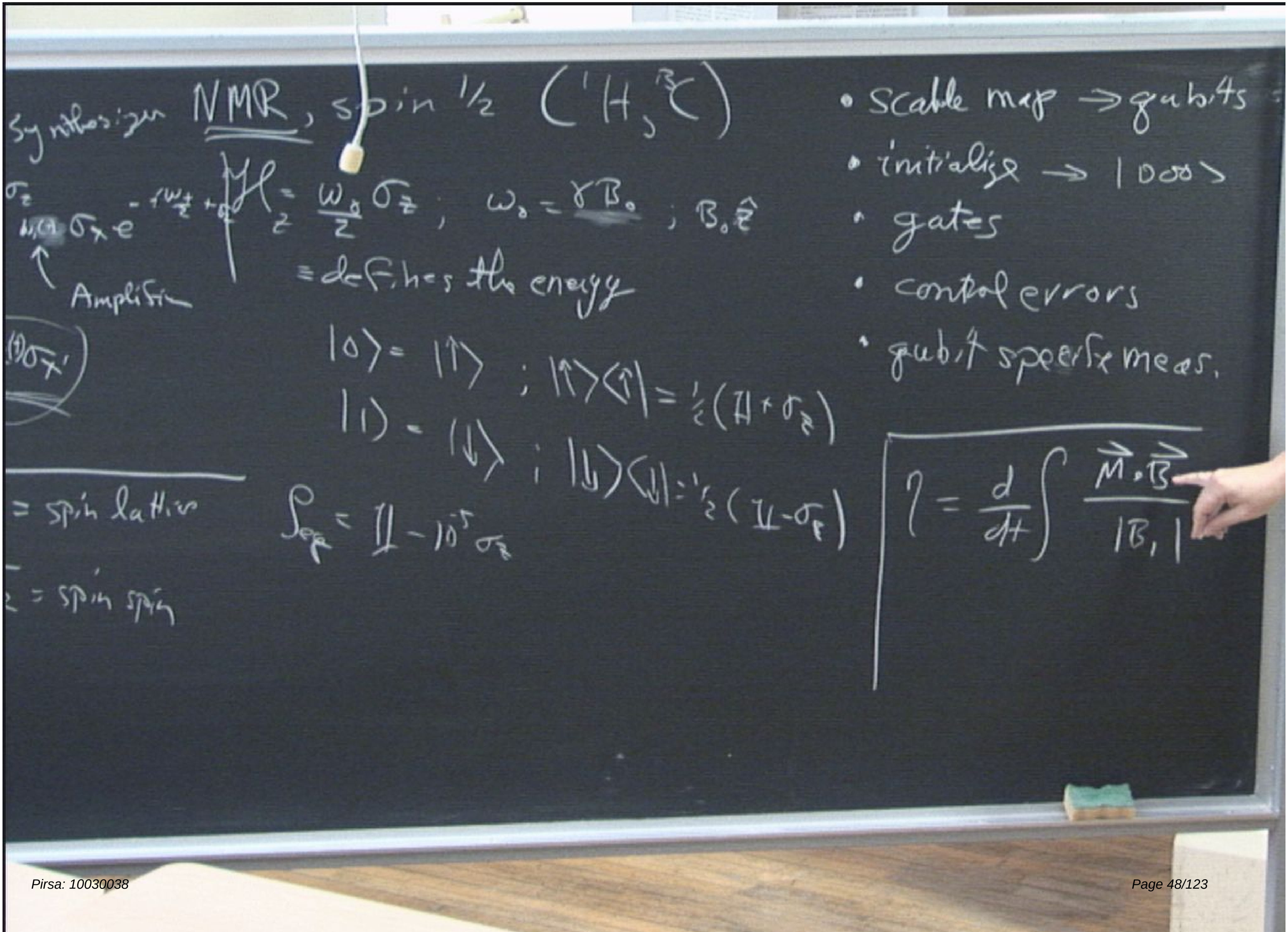
$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{1}+\sigma_z)$

$|1\rangle = |\downarrow\rangle$; $|\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\mathbb{1}-\sigma_z)$

$\rho_{eq} = \mathbb{1} - 10^{-5}\sigma_z$

- scalable map ⇒ qubits
- initialize → $|000\rangle$
- gates
- control errors
- qubit specific meas.

$$\eta = \frac{d}{dt}\int \frac{\vec{M}\,[\text{illegible}]}{|B_1|}\,d\Omega$$



Synthesizer

σ_z

$\omega_1(t)\,\sigma_x e^{-i\omega_z t + \phi}$

↑ Amplifier

$(t)\sigma_x'$

= spin lattice

= spin spin

NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H}_z = \frac{\omega_0}{2}\sigma_z$; $\omega_0 = \gamma B_0$; $B_0\hat{z}$

≡ defines the energy

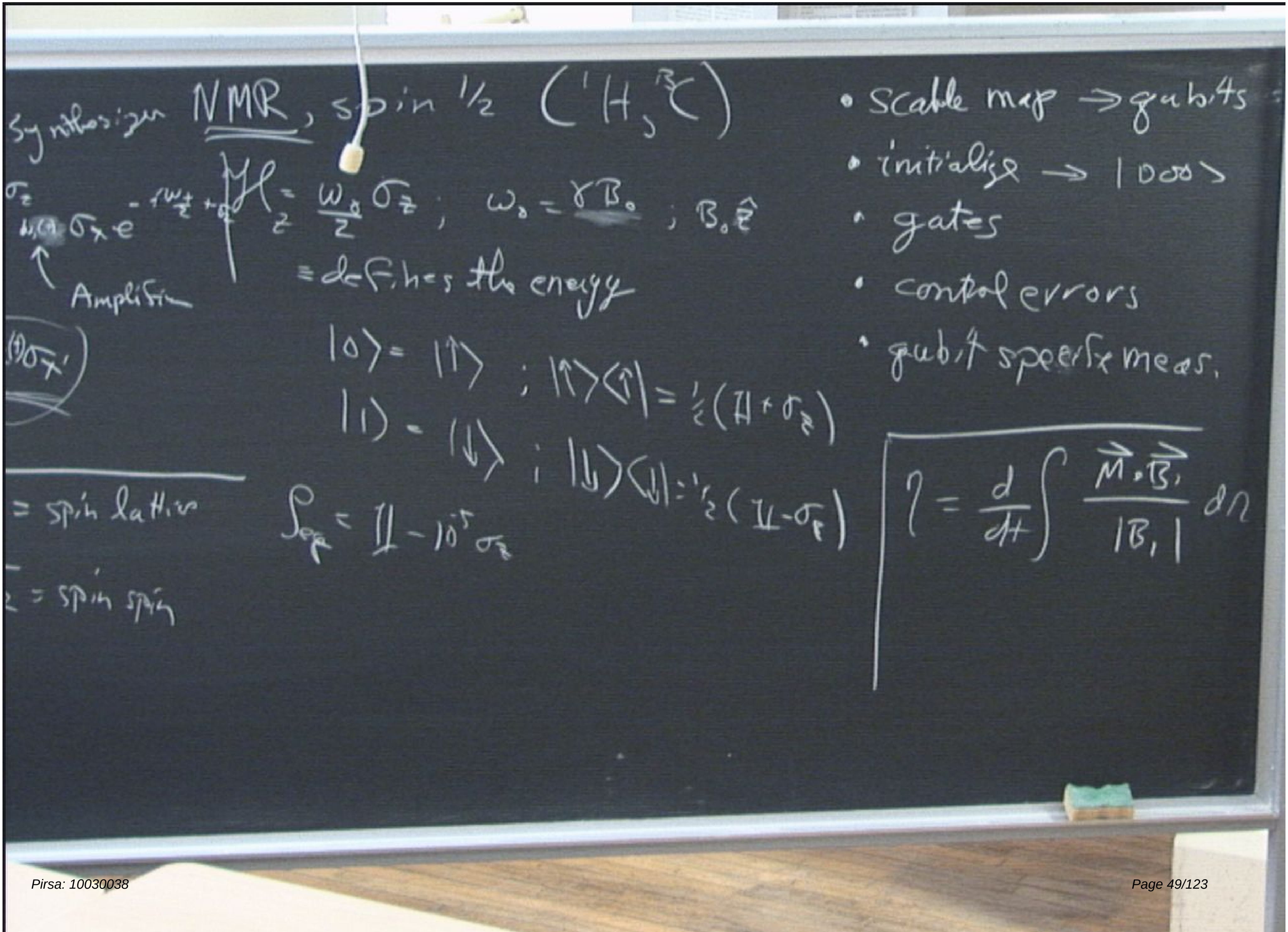
$|0\rangle = |\uparrow\rangle$; $|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\mathbb{1} + \sigma_z)$

$|1\rangle = |\downarrow\rangle$; $|\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\mathbb{1} - \sigma_z)$

$\rho_{eq} = \mathbb{1} - 10^{-5}\sigma_z$

- Scalable map → qubits
- initialize → $|000\rangle$
- gates
- control errors
- qubit specific meas.

$$V = \frac{d}{dt}\int \frac{\vec{M}\cdot\vec{B}}{|B_1|}$$



Synthesizer

σ_z

$\omega_1(t)\sigma_x e^{-i\frac{\omega_t}{2}t}$

Amplifier

$(t)\sigma_x'$

= spin lattice

= spin spin

NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H}_z = \frac{\omega_0}{2}\sigma_z$; $\omega_0 = \gamma B_0$; $B_0\hat{z}$

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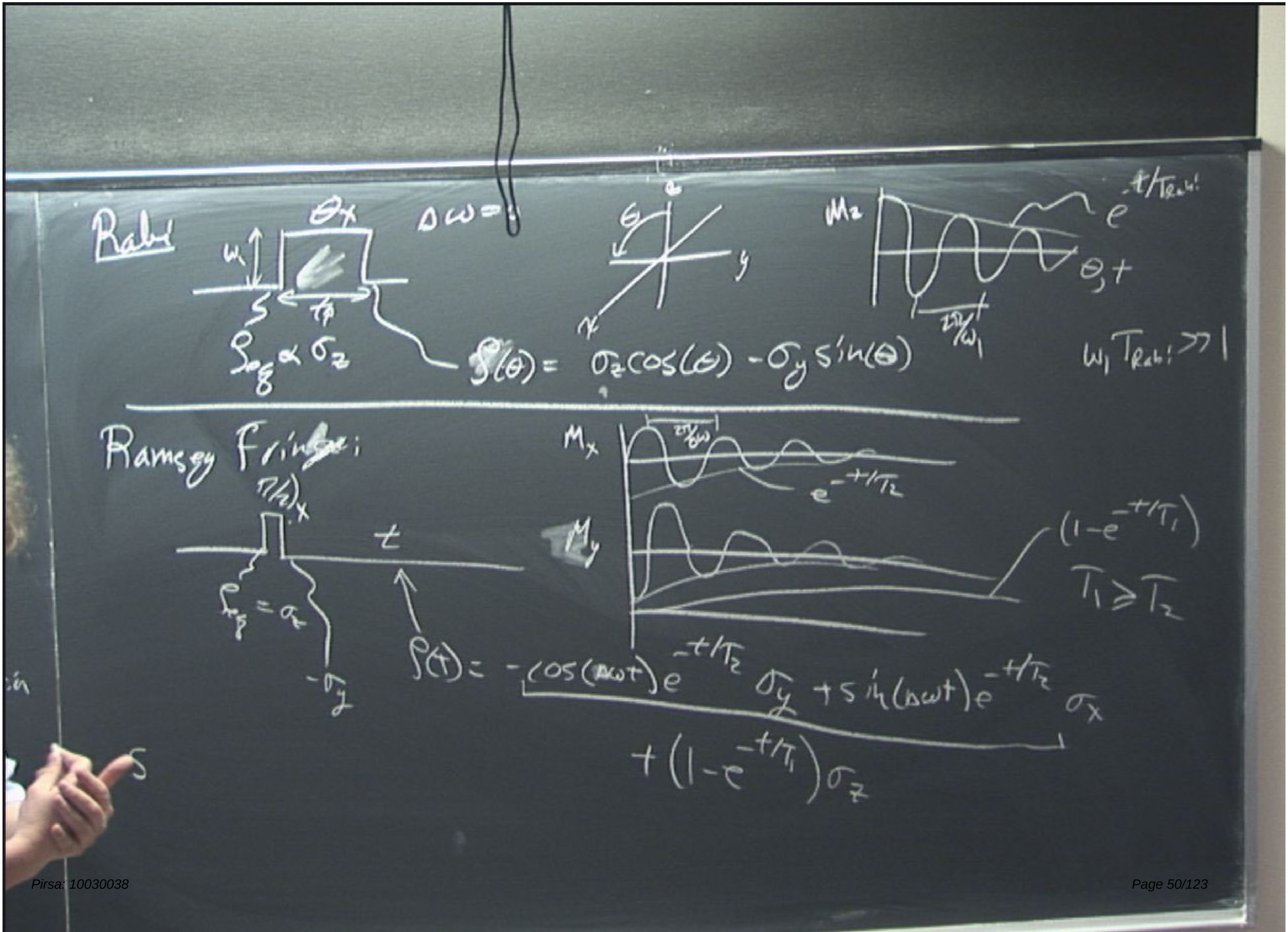
$\rho_{eq} = \mathbb{1} - 10^{-5}\sigma_z$

- Scalable map ⇒ qubits
- initialize → $|000\rangle$
- gates
- control errors
- qubit specific meas.

$$? = \frac{d}{dt}\int \frac{\vec{M}\cdot\vec{B}_1}{|B_1|}\,d\Omega$$



Rabi

$\Delta\omega =$

$\rho_{eq} \propto \sigma_z$

$$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$$

$\omega_1 T_{Rabi} >> 1$

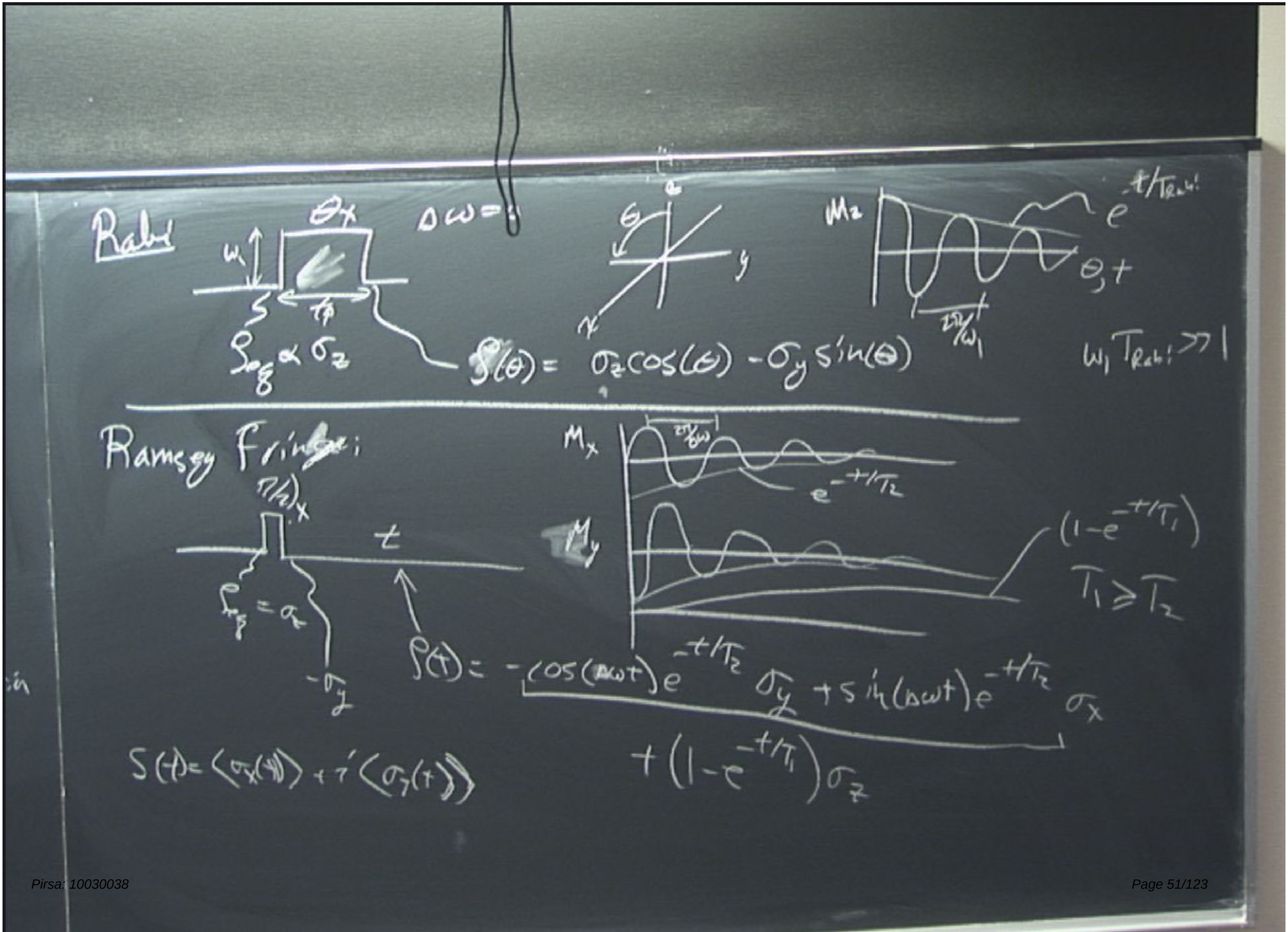
Ramsey Fringes:

$\rho_{eq} = \sigma_z$

$-\sigma_y$

$T_1 \geq T_2$

$$\rho(t) = \underbrace{-\cos(\Delta\omega t) e^{-t/T_2} \sigma_y + \sin(\Delta\omega t) e^{-t/T_2}}\sigma_x$$
$$+ (1 - e^{-t/T_1}) \sigma_z$$



Rabi

θ_x $\Delta\omega = 0$

$\rho_{eq} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

$\omega_1 T_{Rabi} \gg 1$

Ramsey Fringe:

$(\pi/2)_x$ t

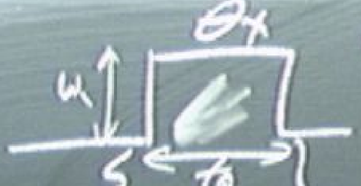
$\rho_{eq} = \sigma_z$ $-\sigma_y$

$T_1 \geq T_2$

$$\rho(t) = \underbrace{-\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x} + (1-e^{-t/T_1})\sigma_z$$

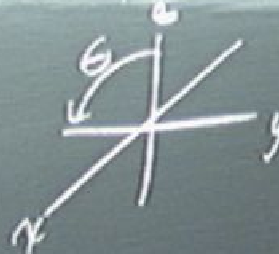
$$S(t) = \langle\sigma_x(t)\rangle + i\langle\sigma_y(t)\rangle$$

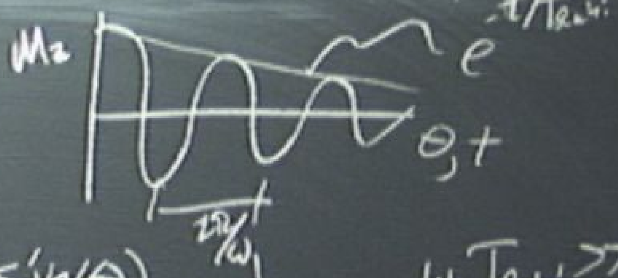
Rabi



$\rho_{eg} \propto \sigma_z$

$\Delta\omega = 0$

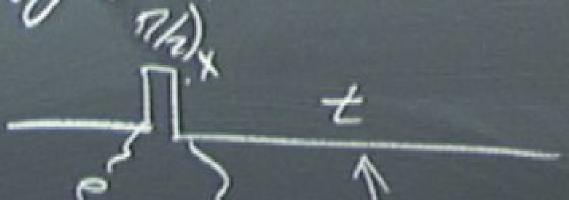




$\omega_1 T_{Rabi} >> 1$

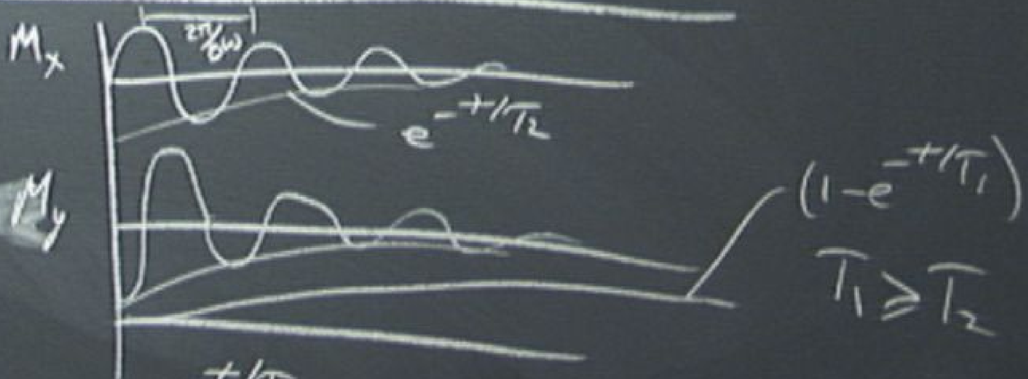
$$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$$

Ramsey Fringes:



$\rho_{eg} = \sigma_z$

$-\sigma_y$



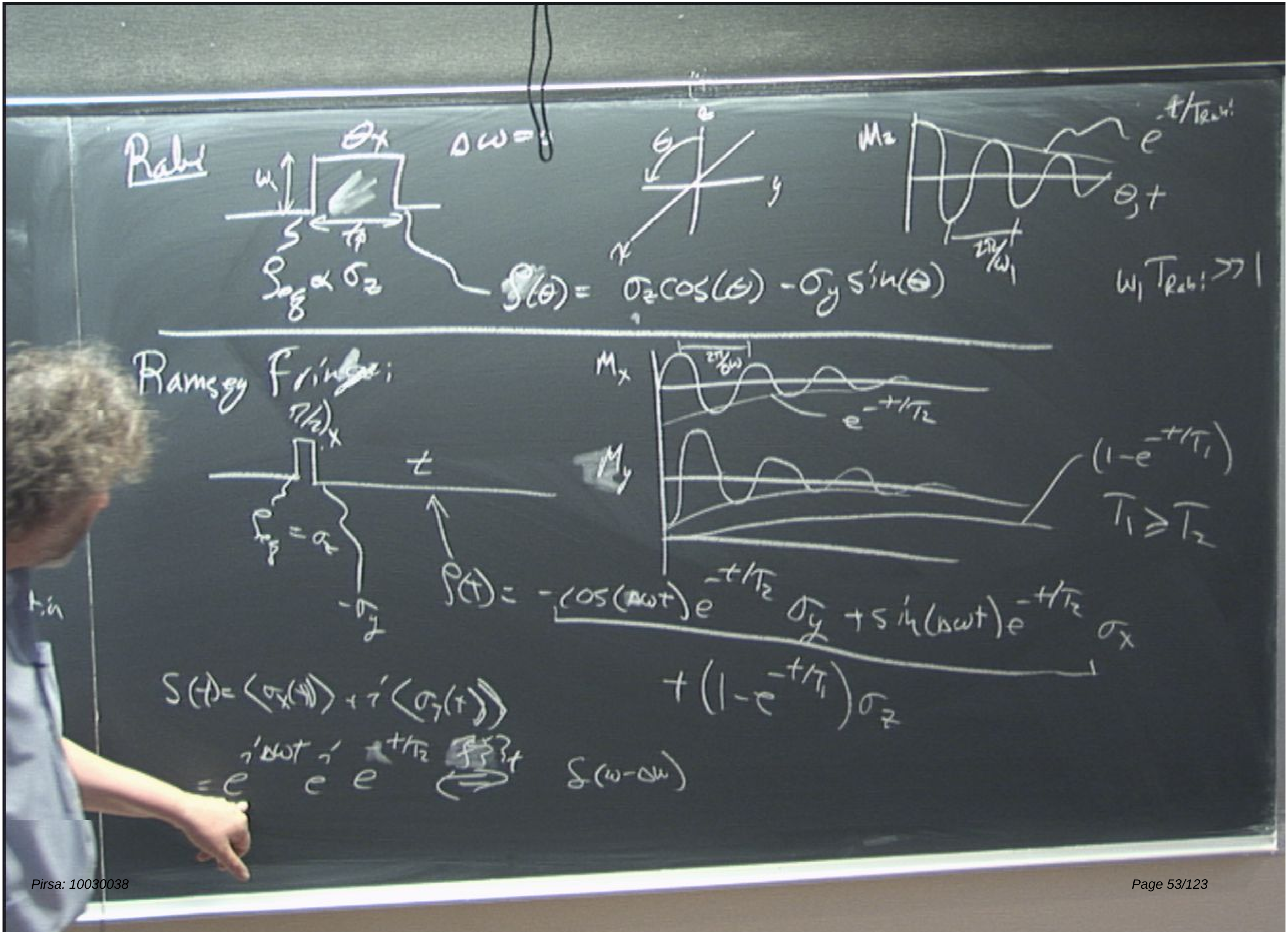
$T_1 \geq T_2$

$$\rho(t) = \underbrace{-\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x}$$

$$+ (1-e^{-t/T_1})\sigma_z$$

$$S(t) = \langle\sigma_x(t)\rangle + i\langle\sigma_y(t)\rangle$$

$$= e^{i\Delta\omega t} e^{i} e^{-t/T_2}$$



$\rho_{eg} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

$\omega_1 T_{Rabi} >> 1$

$\rho_{eg} = \sigma_z$

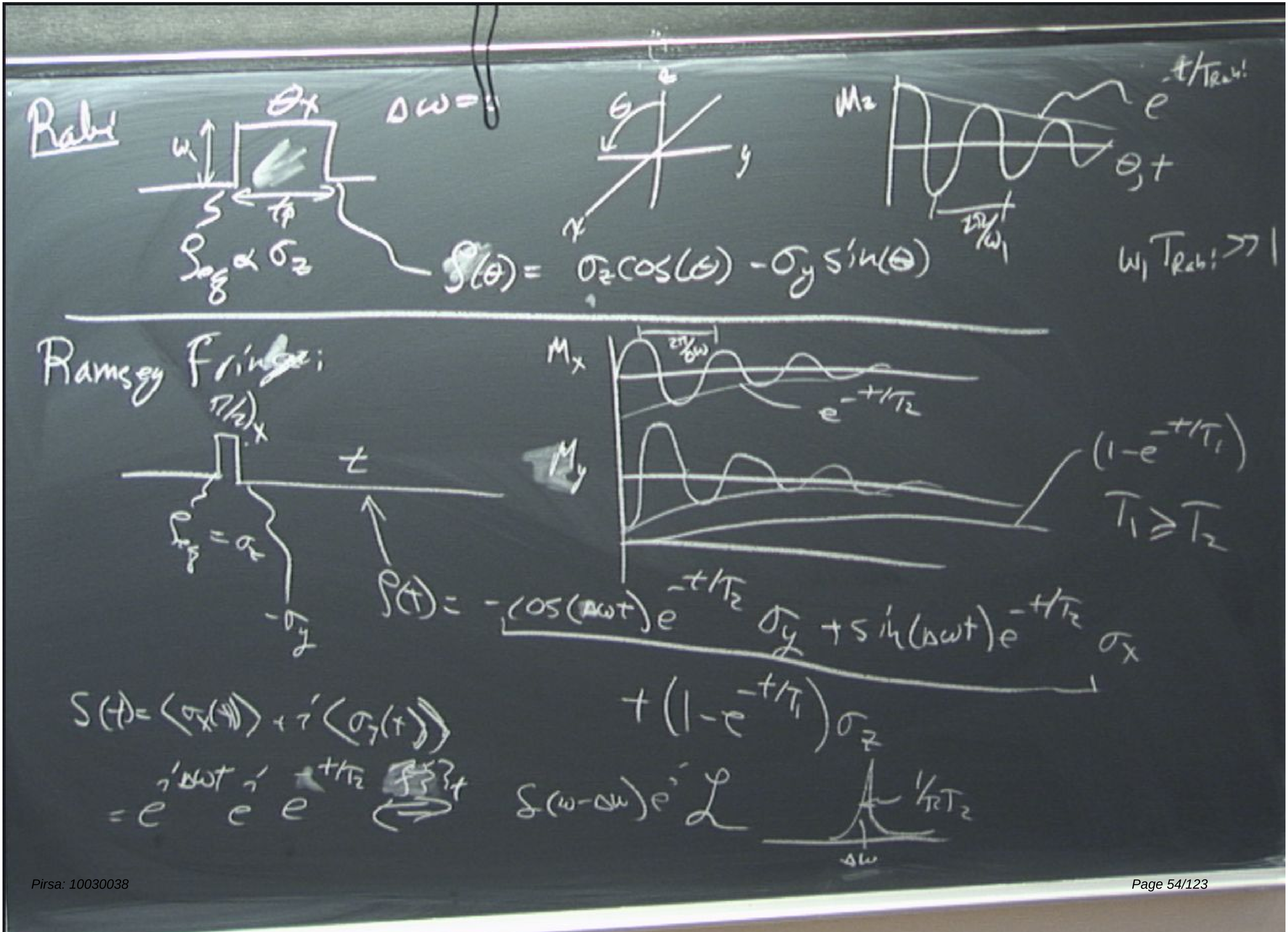
$-\sigma_y$

$T_1 \geq T_2$

$$\rho(t) = -\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x + (1-e^{-t/T_1})\sigma_z$$

$$S(t) = \langle \sigma_x(t) \rangle + i\langle \sigma_y(t) \rangle$$

$$= e^{i\Delta\omega t} e^{i} e^{-t/T_2} \mathcal{F}\{\}_t \qquad \delta(\omega - \Delta\omega)$$



Rabi

θ_x

$\Delta\omega =$

ω_1

t_p

$\rho_{eq} \propto \sigma_z$

$\rho(\theta) = \sigma_z\cos(\theta) - \sigma_y\sin(\theta)$

θ, x, y

M_z

$e^{-t/T_{Rabi}}$

θ, t

$2\pi/\omega_1$

$\omega_1 T_{Rabi} \gg 1$

Ramsey Fringes:

$\pi/2)_x$

t

$\rho_{eq} = \sigma_z$

$-\sigma_y$

M_x

$2\pi/\Delta\omega$

e^{-t/T_2}

M_y

$(1-e^{-t/T_1})$

$T_1 \geq T_2$

$$\rho(t) = \underbrace{-\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x} + (1-e^{-t/T_1})\sigma_z$$

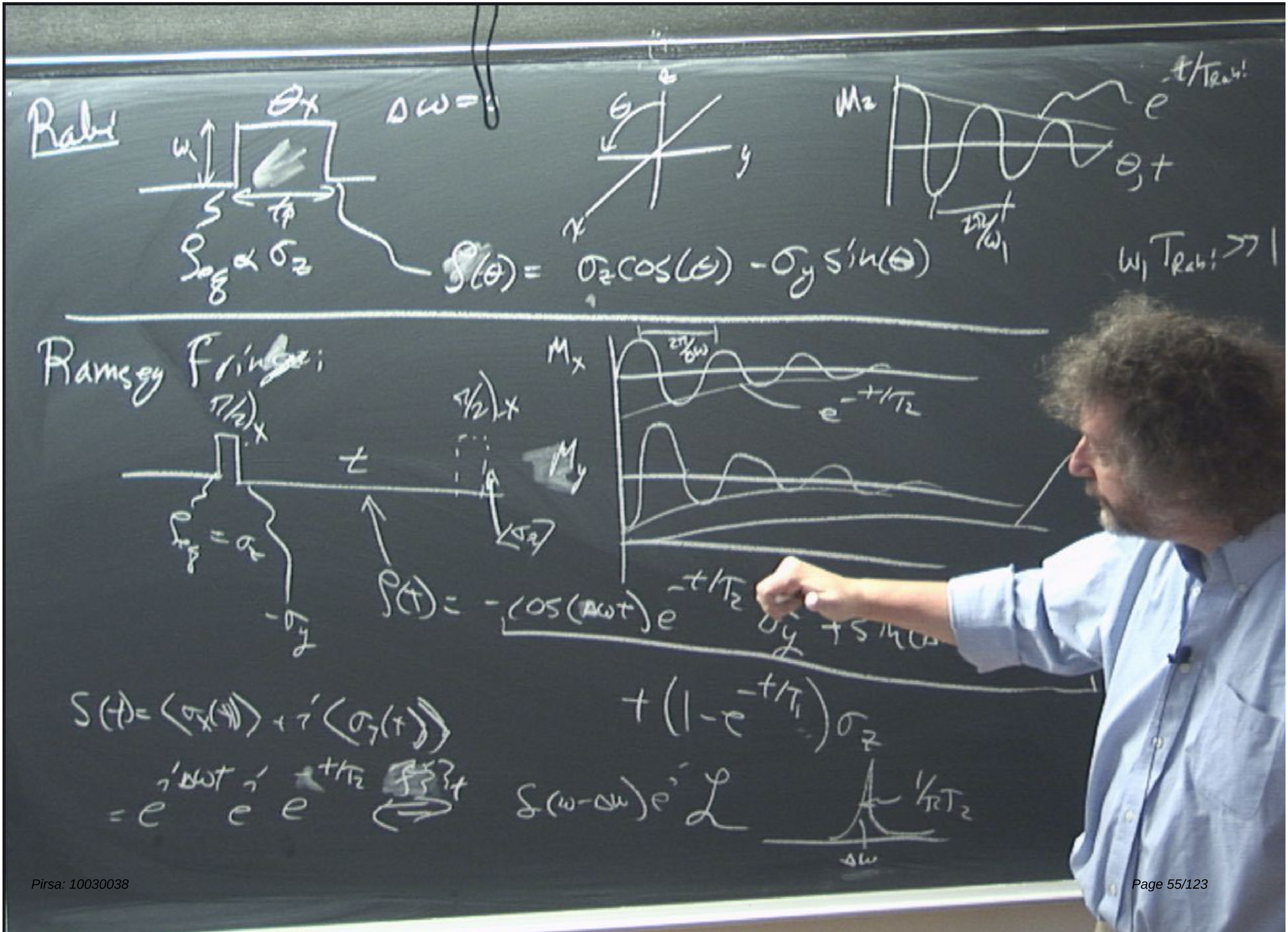
$S(t) = \langle\sigma_x(t)\rangle + i\langle\sigma_y(t)\rangle$

$= e^{i\Delta\omega t} e^{i} e^{-t/T_2}$

$\delta(\omega-\Delta\omega)e^{i}\mathcal{L}$

$1/\pi T_2$

$\Delta\omega$



$\rho_{eq} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

$e^{-t/T_{Rabi}}$

$\omega_1 T_{Rabi} >> 1$

e^{-t/T_2}

$\rho_{eq} = \sigma_z$

$-\sigma_y$

$\rho(t) = -\cos(\Delta\omega t) e^{-t/T_2} \sigma_y + \sin($

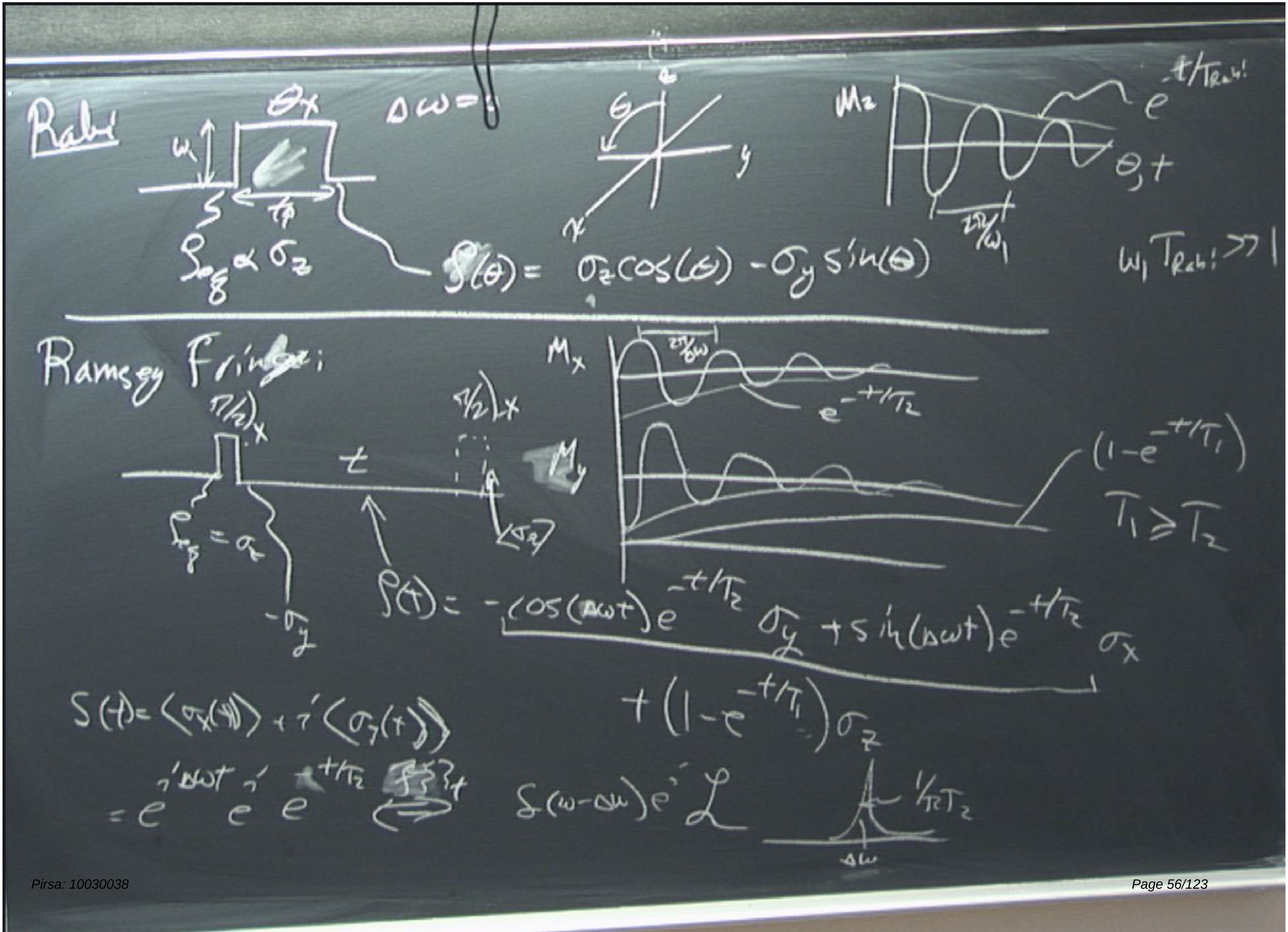
$+ (1 - e^{-t/T_1}) \sigma_z$

$S(t) = \langle \sigma_x(t) \rangle + i \langle \sigma_y(t) \rangle$

$= e^{i\Delta\omega t} e^{i} e^{-t/T_2}$

$\delta(\omega - \Delta\omega) e^{i} \mathcal{L}$

$1/\pi T_2$



Rabi

θ_x $\Delta\omega = $

ω_1 t_p

$\rho_{eq} \propto \sigma_z$

$\rho(\theta) = \sigma_z\cos(\theta) - \sigma_y\sin(\theta)$

M_z $e^{-t/T_{Rabi}}$ θ, t $2\pi/\omega_1$

$\omega_1 T_{Rabi} \gg 1$

Ramsey Fringes:

$\pi/2)_x$ t $\pi/2)_x$ $\langle\sigma_z\rangle$

$\rho_{eq} = \sigma_z$ $-\sigma_y$

M_x $2\pi/\Delta\omega$ e^{-t/T_2}

M_y $(1-e^{-t/T_1})$

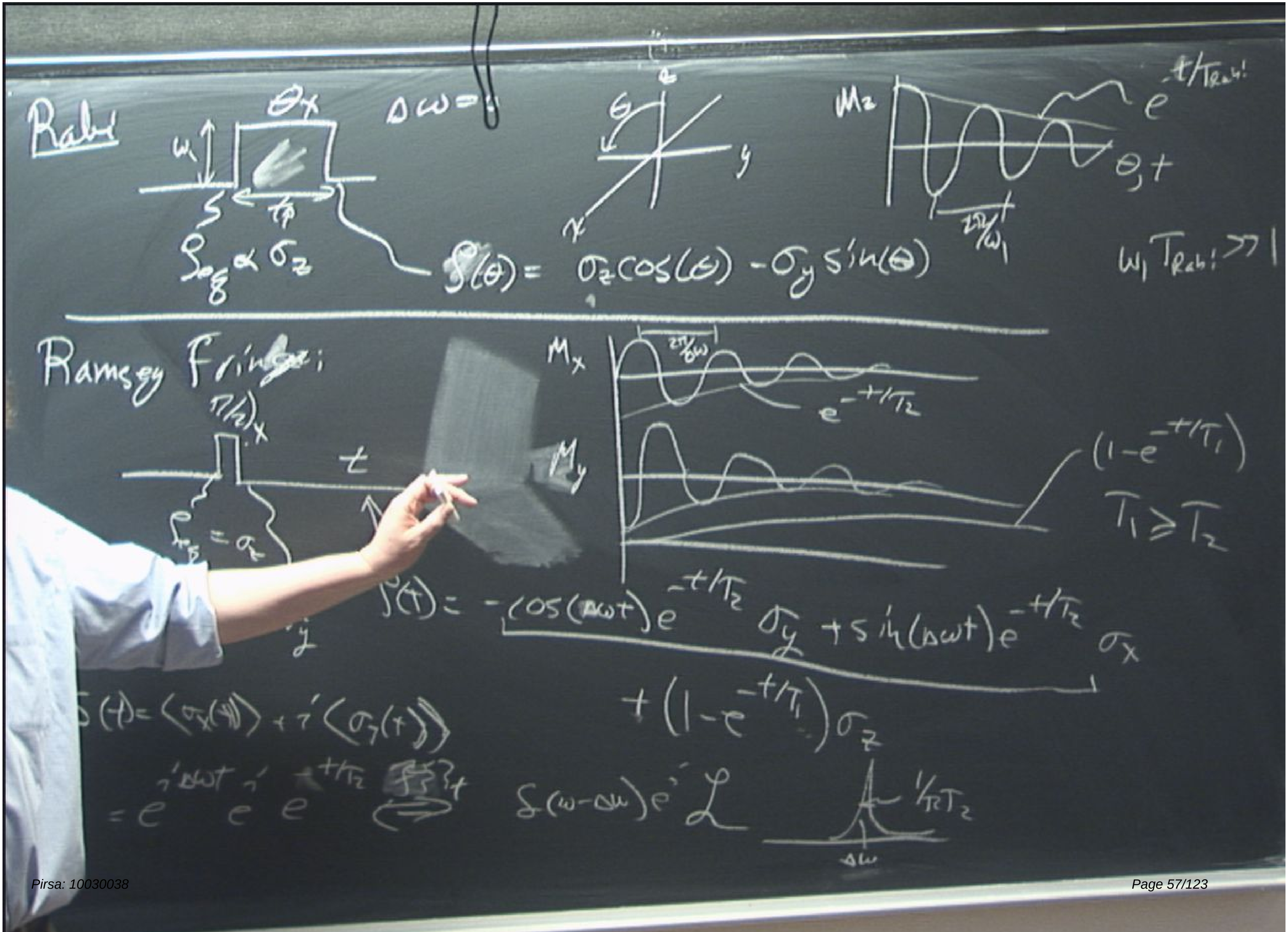
$T_1 \geq T_2$

$$\rho(t) = \underbrace{-\cos(\Delta\omega t)e^{-t/T_2}\sigma_y + \sin(\Delta\omega t)e^{-t/T_2}\sigma_x} + (1-e^{-t/T_1})\sigma_z$$

$$S(t) = \langle\sigma_x(t)\rangle + i\langle\sigma_y(t)\rangle$$

$$= e^{i\Delta\omega t}\, e^{i}\, e^{t/T_2} \quad \text{[illegible]}_t \quad \delta(\omega-\Delta\omega)\, e^{i}\, \mathcal{L}$$

$1/\pi T_2$ $\Delta\omega$



$\rho_{eq} \propto \sigma_z$

$\rho(\theta) = \sigma_z \cos(\theta) - \sigma_y \sin(\theta)$

$\omega_1 T_{Rabi} \gg 1$

$\rho_{eq} = \sigma_z$

$T_1 \geq T_2$

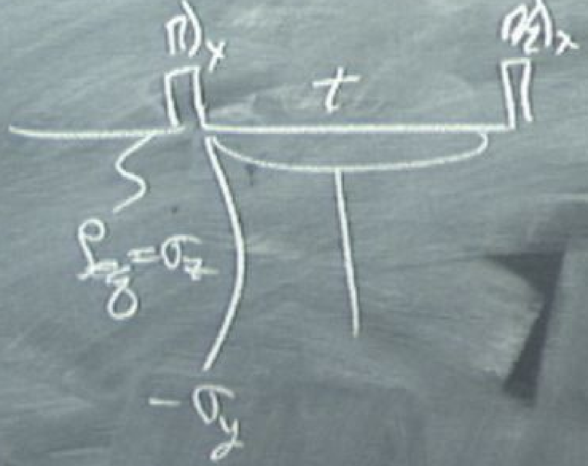
$$\rho(t) = \underbrace{-\cos(\Delta\omega t) e^{-t/T_2} \sigma_y + \sin(\Delta\omega t) e^{-t/T_2} \sigma_x}_{} + (1 - e^{-t/T_1}) \sigma_z$$

$S(t) = \langle \sigma_x(t) \rangle + i \langle \sigma_y(t) \rangle$

$= e^{i\Delta\omega t} e^{i} e^{-t/T_2} \quad \mathcal{F}\{\}_t \quad \delta(\omega - \Delta\omega) e^{i} \mathcal{L}$

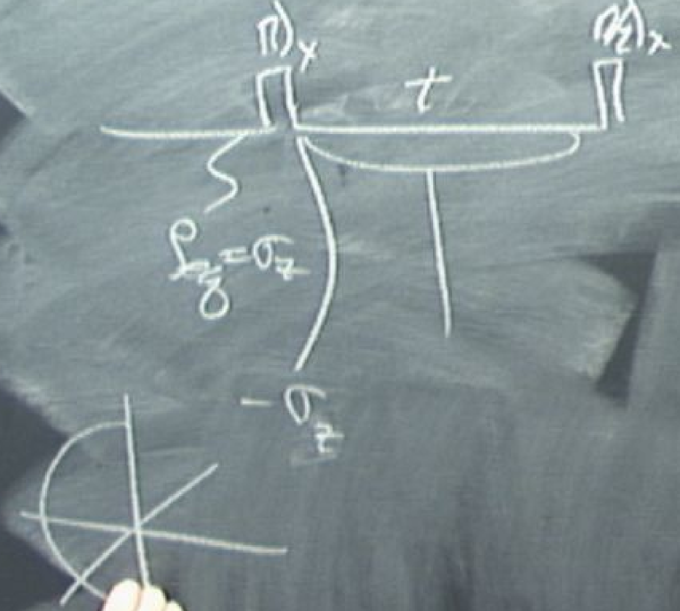
B Inversion Recovery

Inversion Recovery

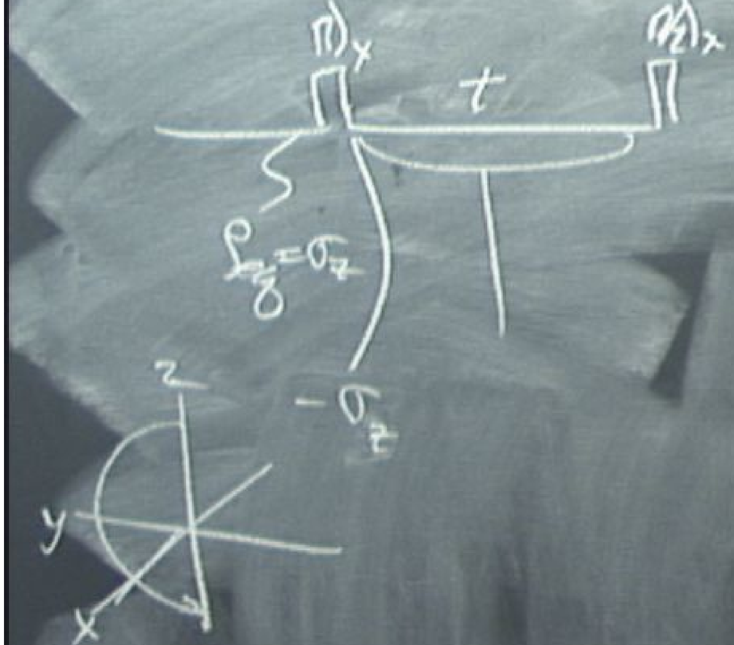


B

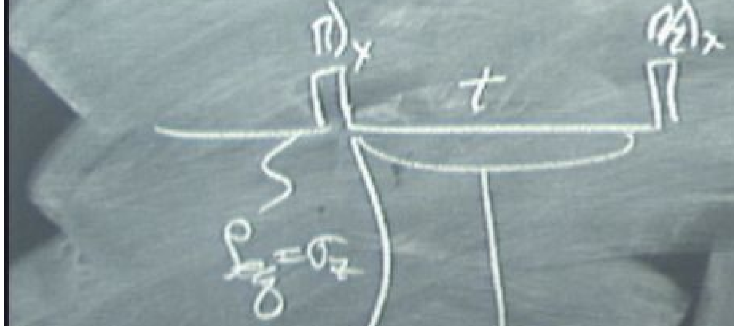
Inversion Recovery

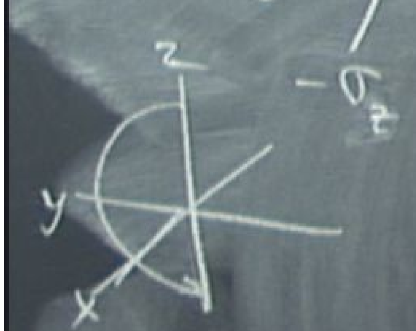


Inversion Recovery



B Inversion Recovery

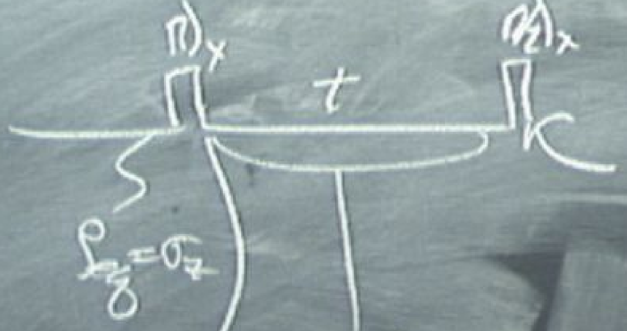




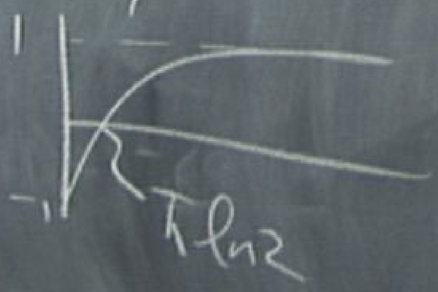
$$\rho(t) = (1 - 2e^{-t/T_1})$$

B

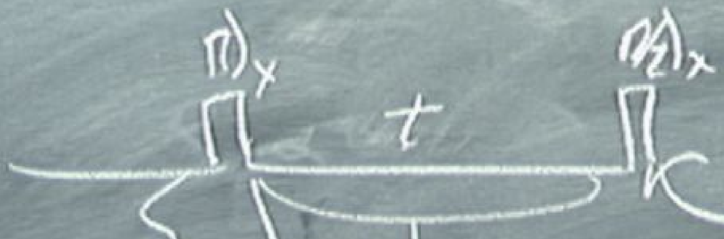
Inversion Recovery



$$\rho(t) = \left(1 - 2e^{-t/T_1}\right)\hat{z}$$



Inversion Recovery



$-(1-2e^{-t/T_1})\,\sigma_y$

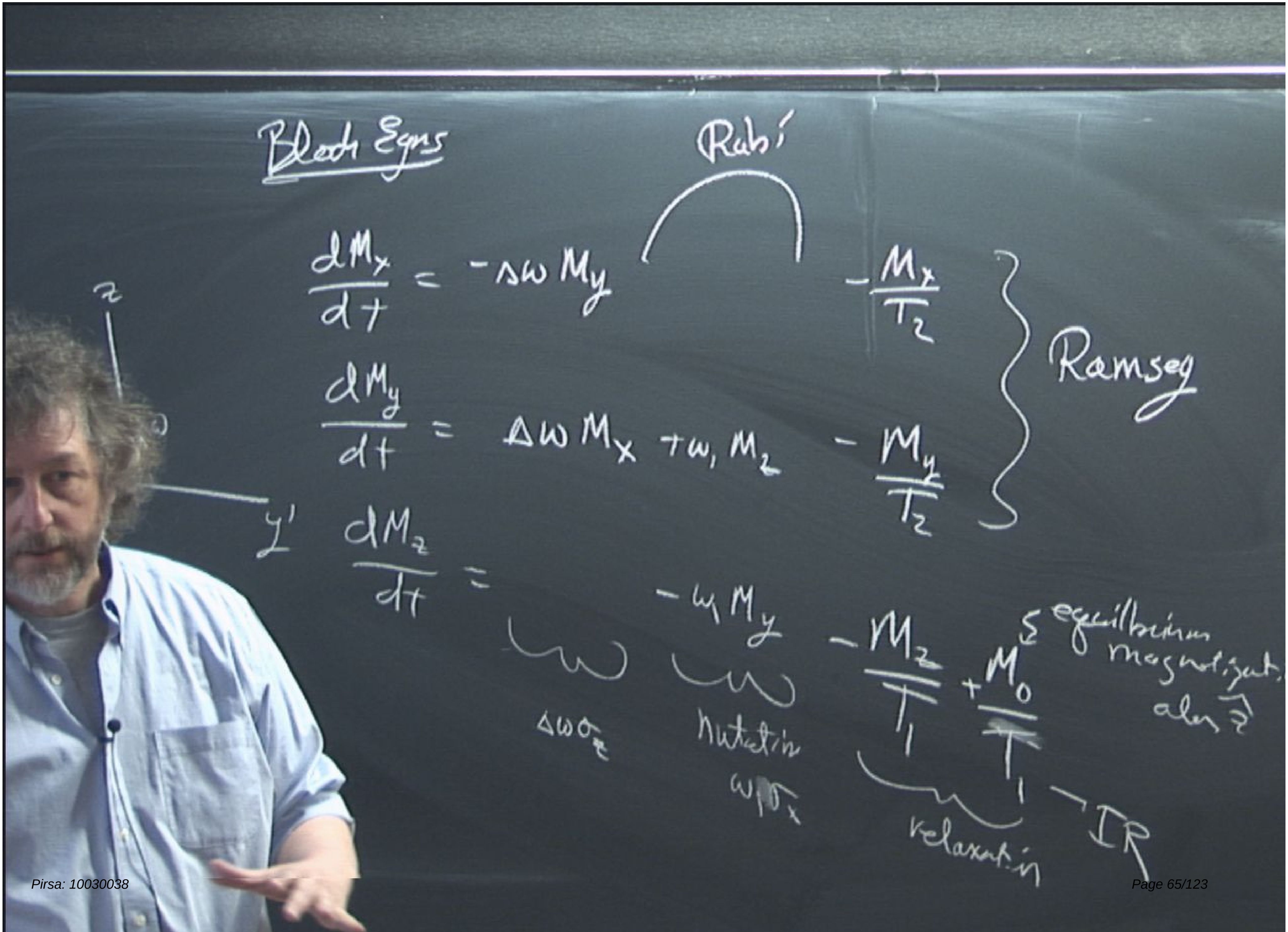
$\rho_z = \sigma_z$

$-\sigma_z$

$\rho(t) = (1-2e^{-t/T_1})\,\sigma_z$

tigation

z



Bloch Eqns

Rabi

$$\frac{dM_x}{dt} = -\Delta\omega M_y \qquad - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \Delta\omega M_x + \omega_1 M_z \qquad - \frac{M_y}{T_2}$$

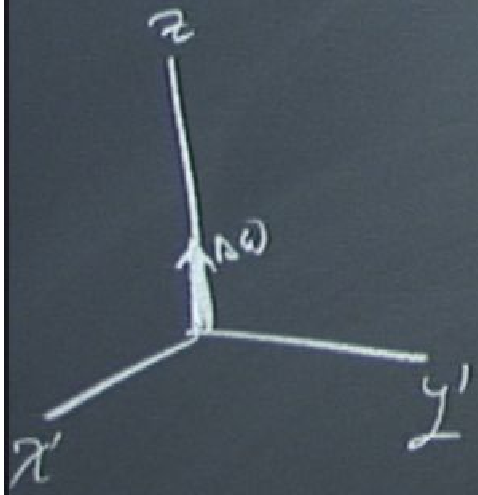
Ramsey

$$\frac{dM_z}{dt} = \underbrace{\qquad}_{\Delta\omega\sigma_z} \underbrace{-\omega_1 M_y}_{\text{nutation } \omega_1\sigma_x} \underbrace{-\frac{M_z}{T_1} + \frac{M_0}{T_1}}_{\text{relaxation}}$$

M_0 equilibrium magnetizat. along $\vec{z}$

$T_1 \to$ IR

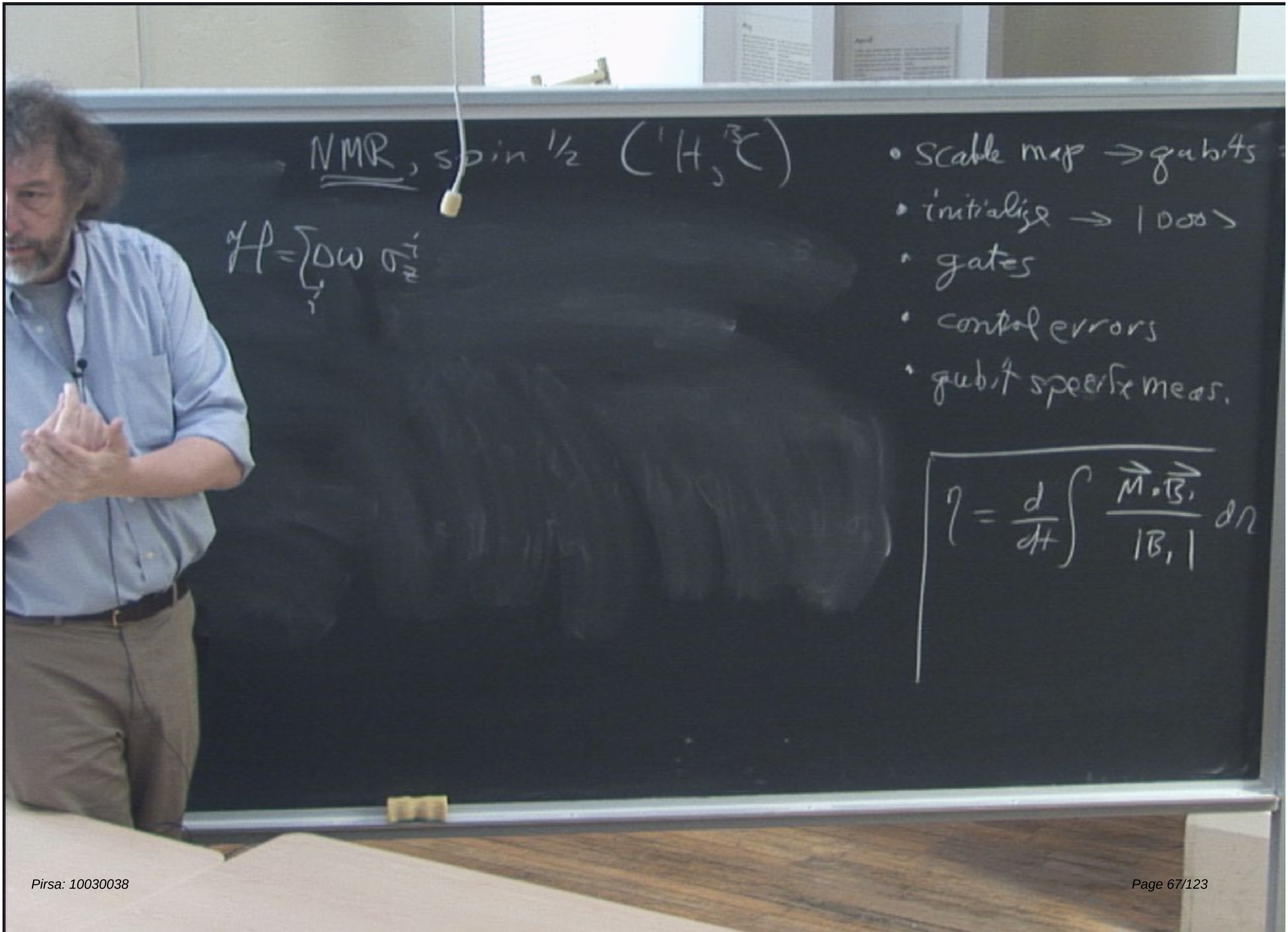
Bloch Eqns



$$\left.\begin{aligned} \frac{dM_x}{dt} &= -\Delta\omega M_y \overbrace{\qquad\qquad}^{\text{Rabi}} - \frac{M_x}{T_2} \\ \frac{dM_y}{dt} &= \Delta\omega M_x + \omega_1 M_z - \frac{M_y}{T_2} \end{aligned}\right\} \text{Ramsey}$$

$$\frac{dM_z}{dt} = \underbrace{\qquad\qquad}_{\Delta\omega\sigma_z} \underbrace{-\omega_1 M_y}_{\text{nutation } \omega_1\sigma_x} \underbrace{-\frac{M_z}{T_1} + \frac{\overset{\text{equilibrium magnetization along } \vec{z}}{M_0}}{T_1}}_{\text{relaxation}}$$

$T_1 \to$ IR

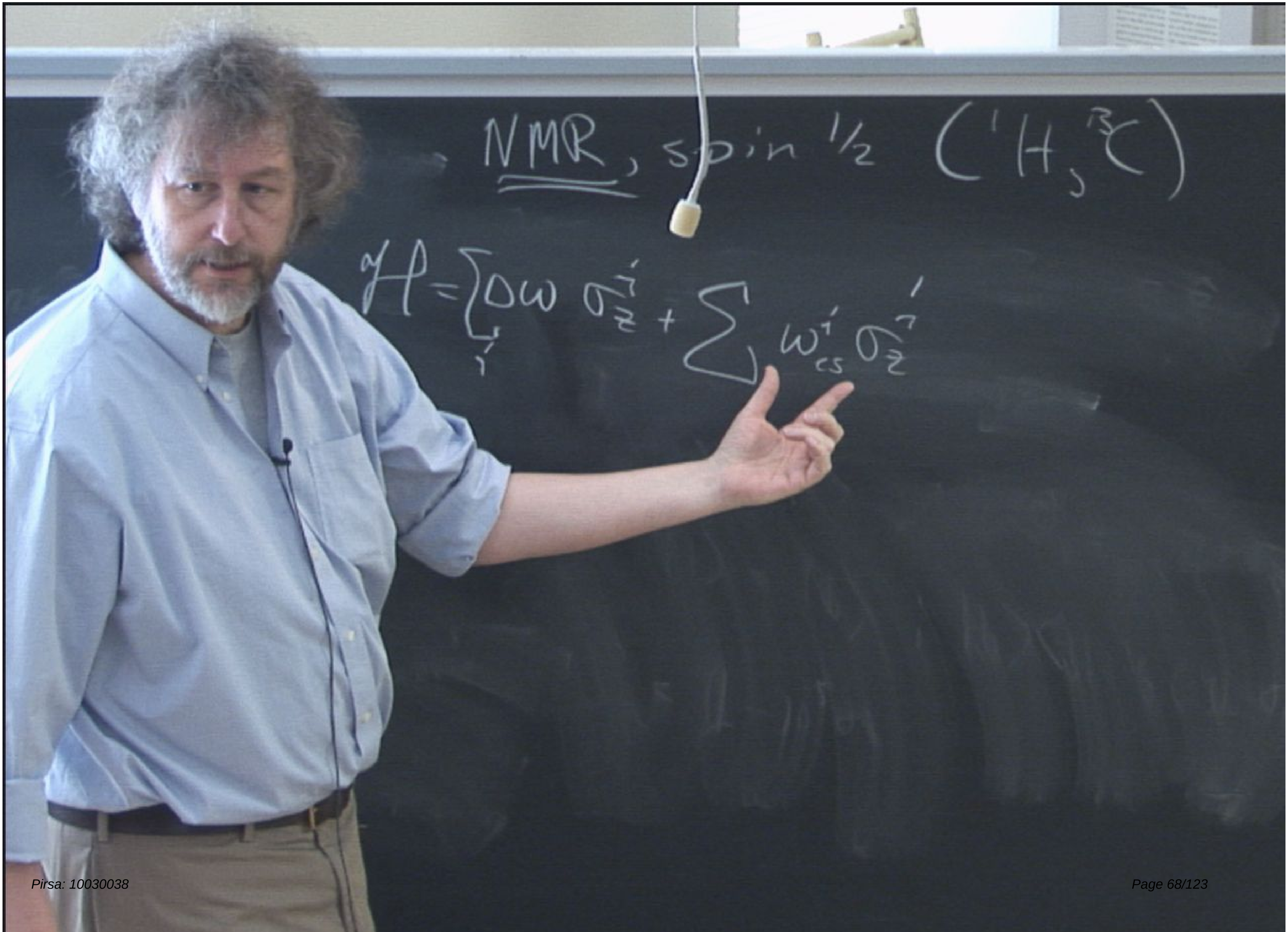


NMR, spin 1/2 (^1H, ^{13}C)

$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i$

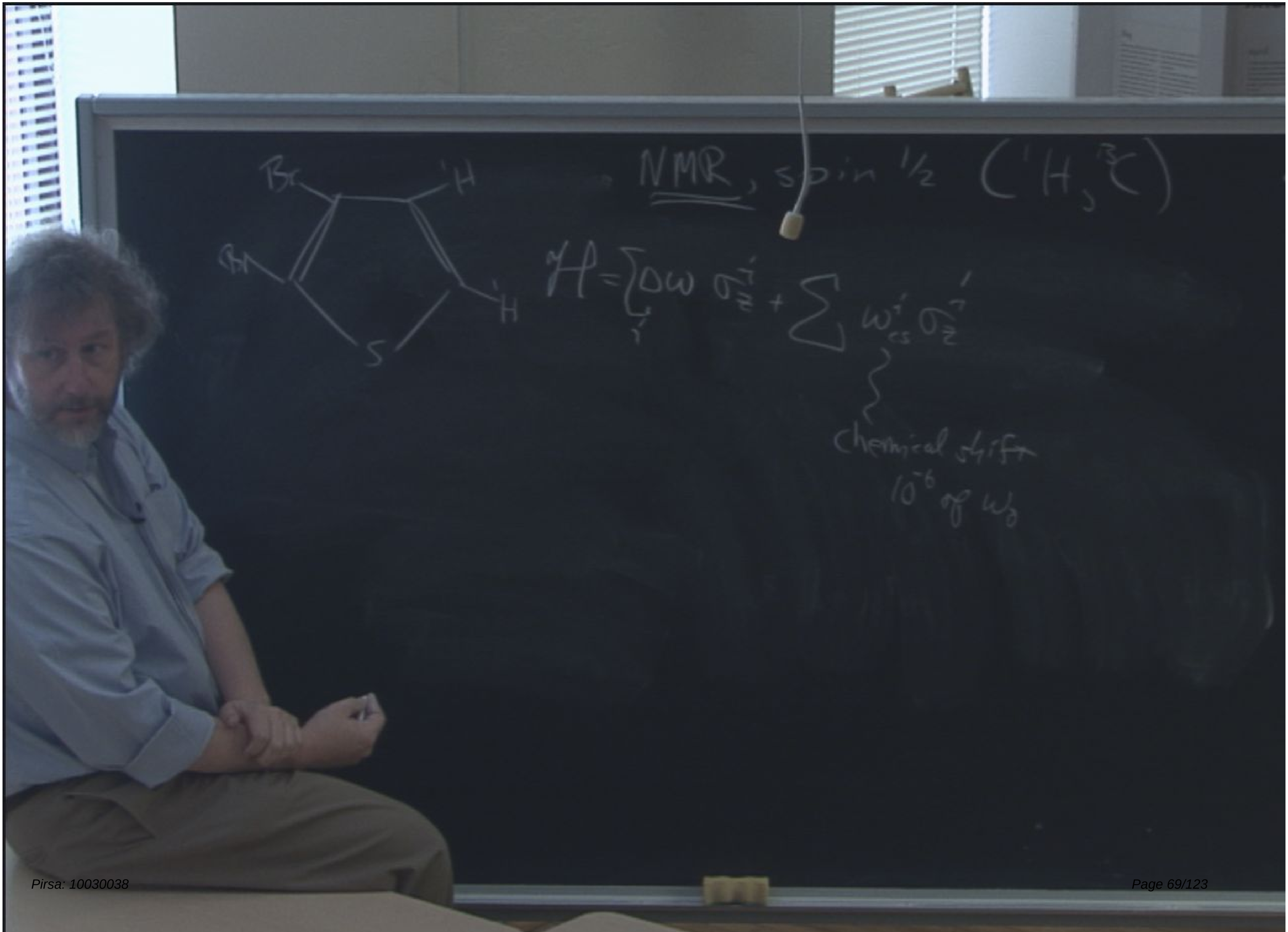
- Scable map ⇒ qubits
- initialize → $|000\rangle$
- gates
- control errors
- qubit specific meas.

$$\boxed{\eta = \frac{d}{dt}\int \frac{\vec{M}\cdot\vec{B}_1}{|B_1|}\, d\Omega}$$



NMR, spin 1/2 (^1H, ^{13}C)

$$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \sum_i \omega_{cs}^i \sigma_z^i$$

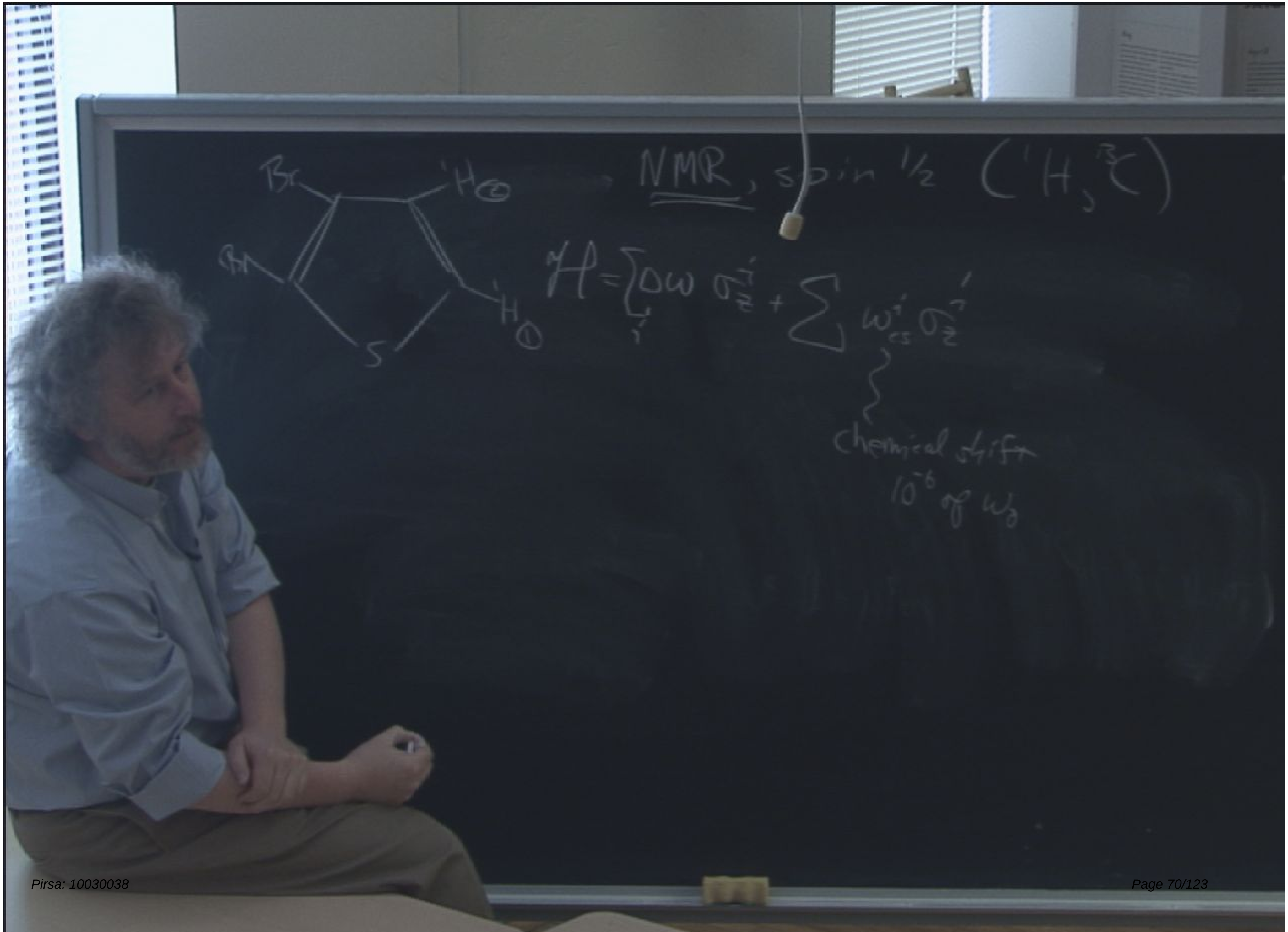


NMR, spin 1/2 $({}^1H, {}^{13}C)$

$$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \sum_i \omega_{cs}^i\, \sigma_z^i$$

chemical shift

10^{-6} of ω_0

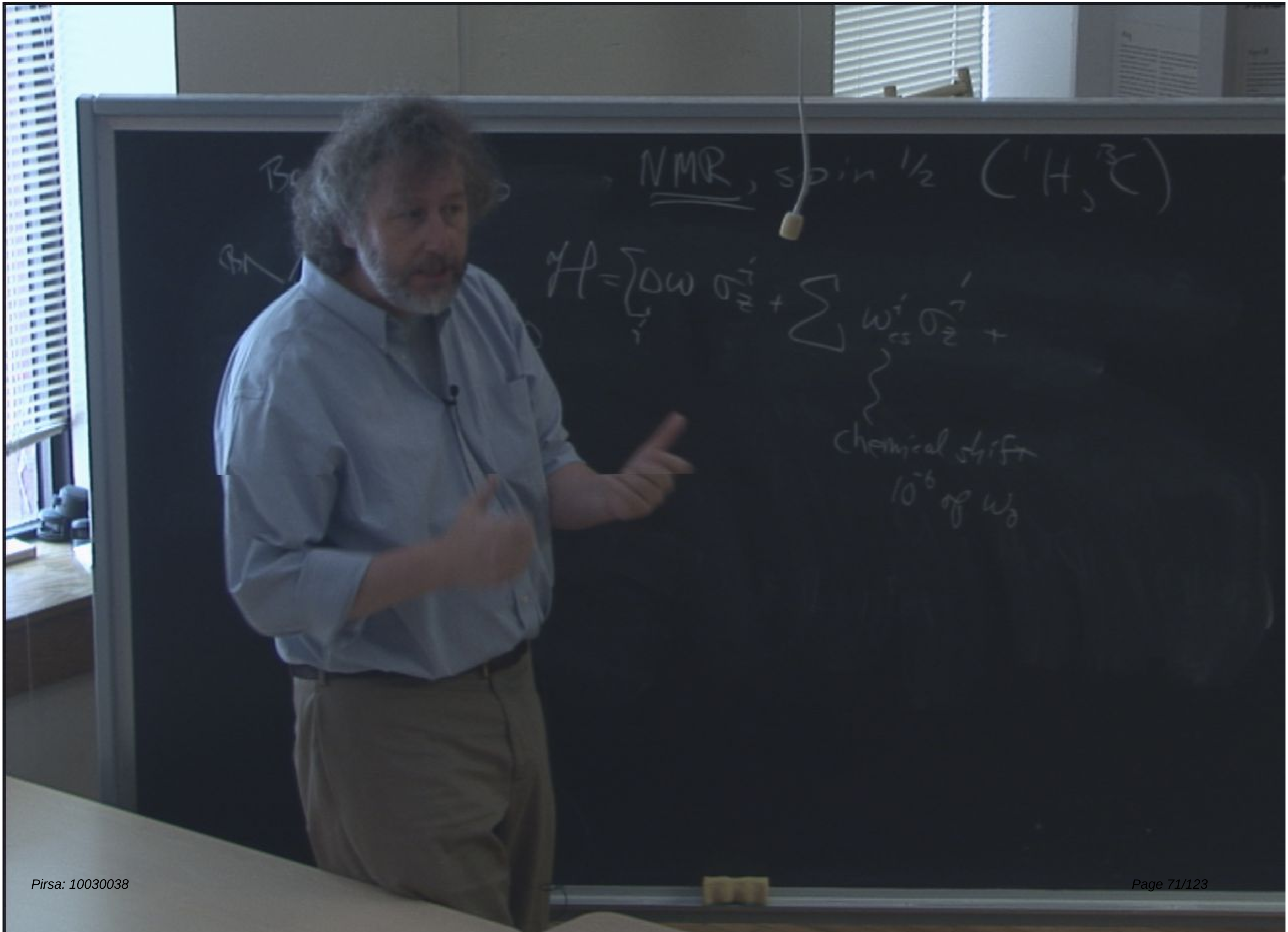


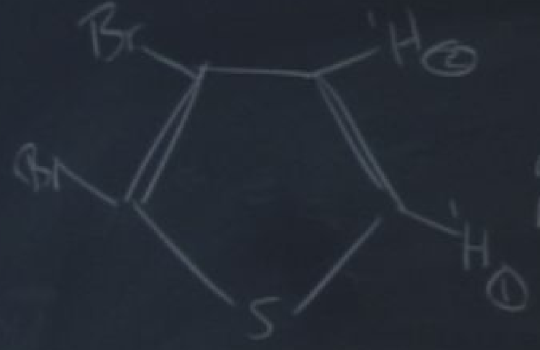
NMR, spin 1/2 (^1H, ^{13}C)

$$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \sum_i \omega_{cs}^i \sigma_z^i$$

chemical shift

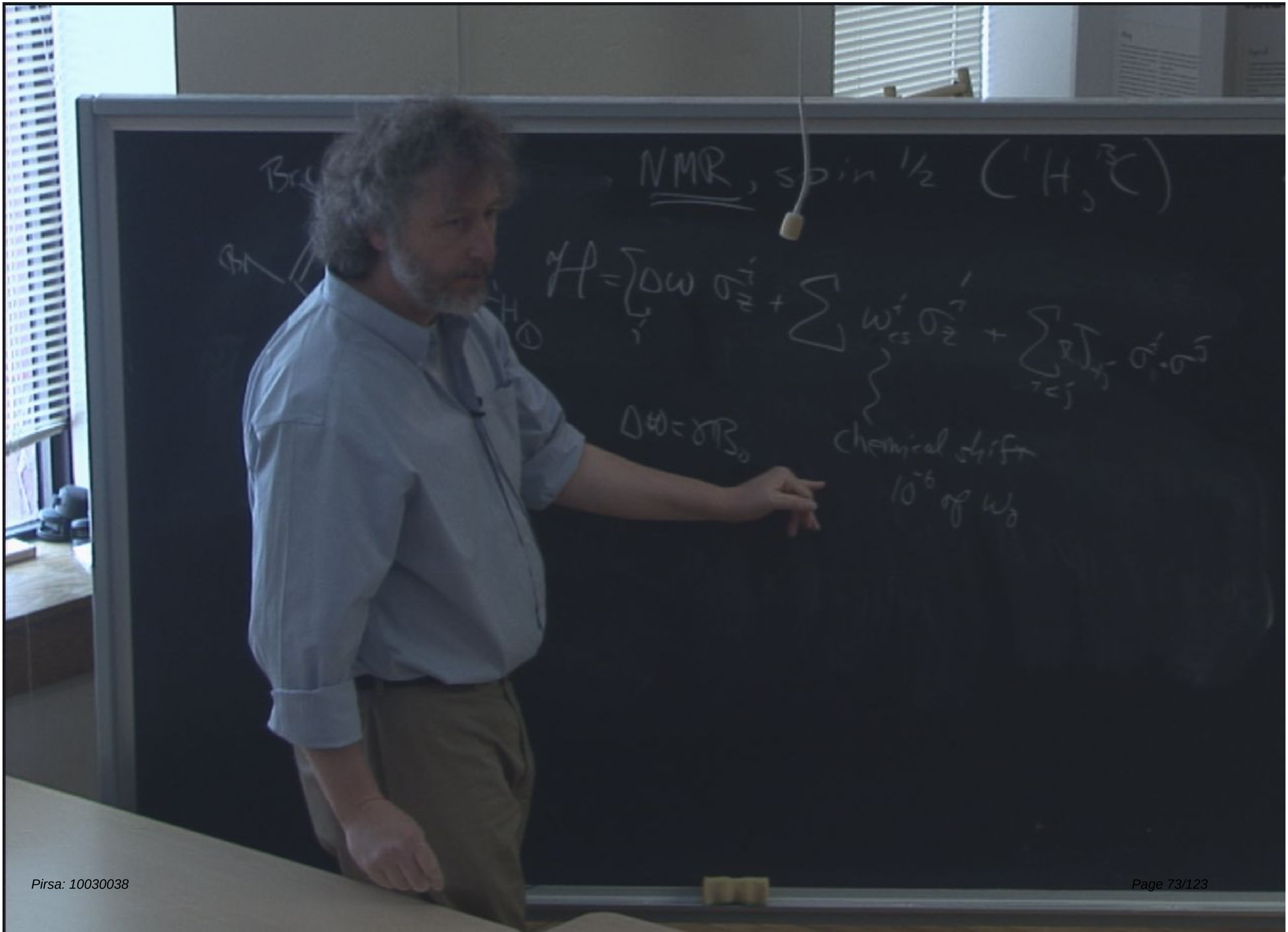
10^{-6} of ω_0

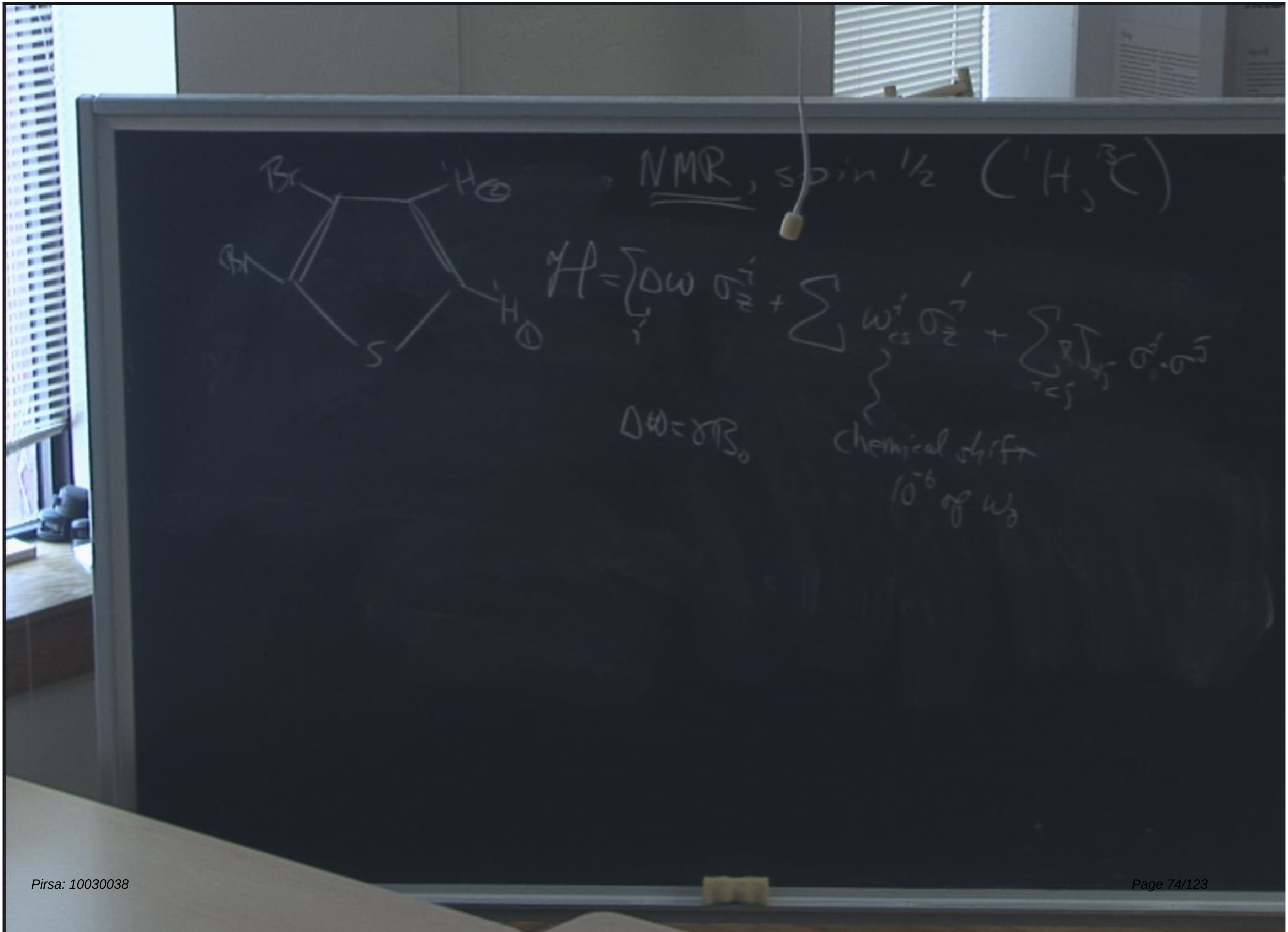

NMR, spin 1/2 (1H, 13C)
$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \sum \omega^i_{cs} \sigma_z^i +$
chemical shift
10^{-6} of ω_0



NMR, spin 1/2 (^1H, ^{13}C)

$$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \underbrace{\sum_i \omega_{cs}^i\, \sigma_z^i}_{\substack{\text{chemical shift} \\ 10^{-6} \text{ of } \omega_0}} + \sum_{i<j} 2J_{ij}\, \sigma^i \cdot \sigma^j$$


NMR, spin 1/2 (^1H, ^{13}C)
$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \sum \omega_{cs}^i \sigma_z^i + \sum_{i<j} 2\pi J_{ij}\, \sigma^i \cdot \sigma^j$
H_0 ①
$\Delta\omega = \gamma B_0$
chemical shift
10^{-6} of ω_0

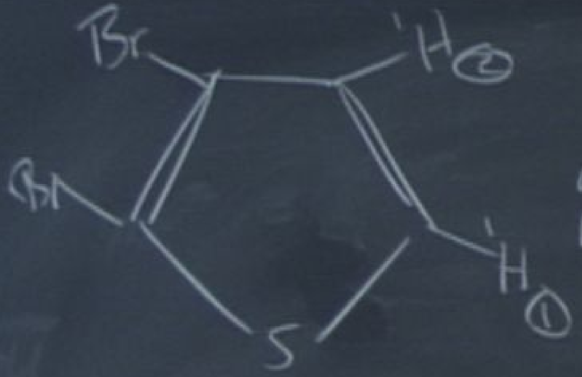


→ NMR, spin ½ (^1H, ^{13}C)

$$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \sum_i \underbrace{\omega_{cs}^i}_{\text{chemical shift}} \sigma_z^i + \sum_{i<j} 2\pi J_{ij}\, \sigma^i \cdot \sigma^j$$

$\Delta\omega = \gamma B_0$

10^{-6} of ω_0



NMR, spin 1/2 (^1H, ^{13}C)

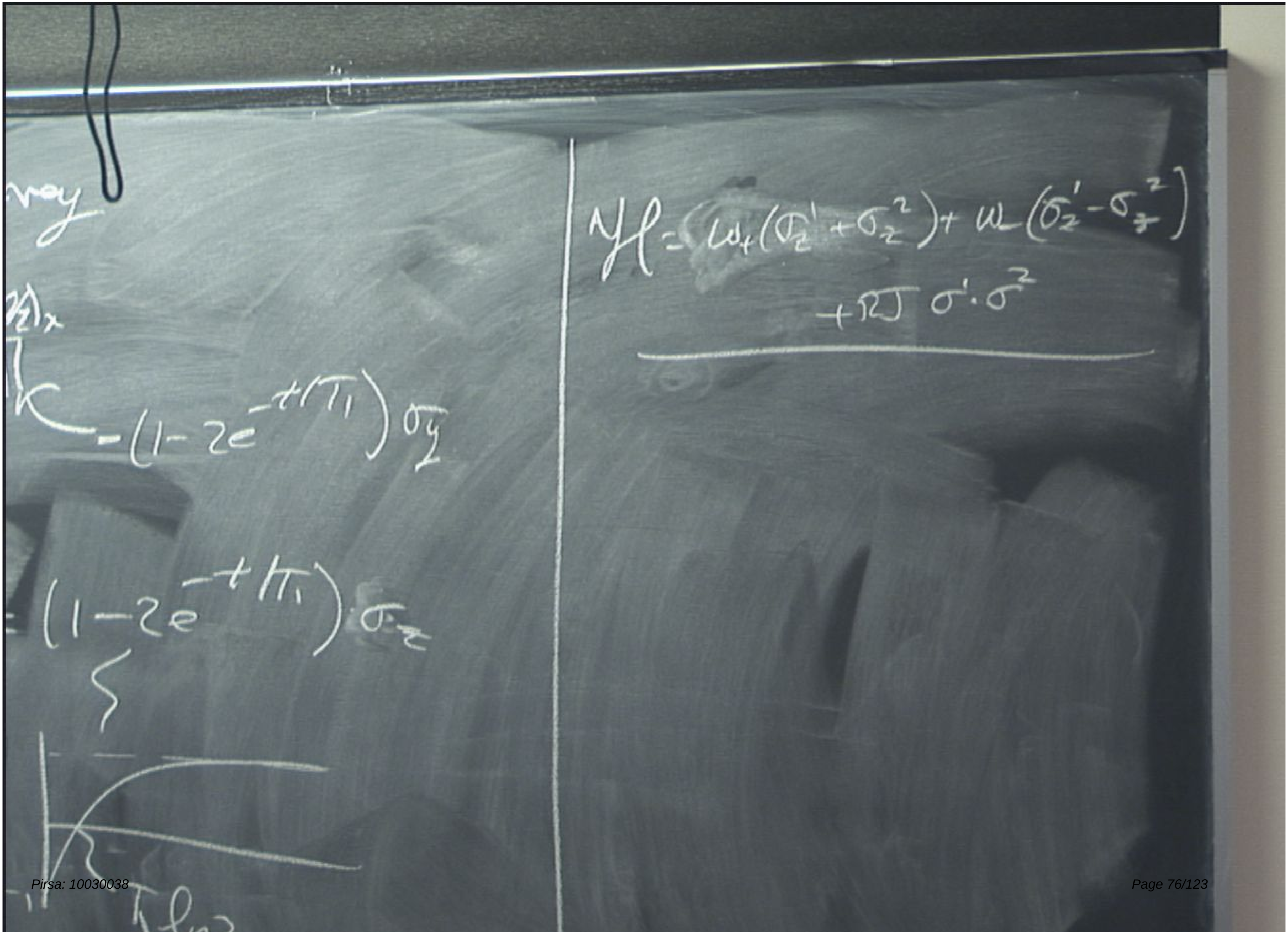
$$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \sum_i \omega_{cs}^i\, \sigma_z^i + \sum_{i<j} 2J_{ij}\, \sigma^i \cdot \sigma^j$$

$\Delta\omega = \gamma B_0$

chemical shift 10^{-6} of ω_0

Scalar Coupling

- sc
- in
- g
- c
- g



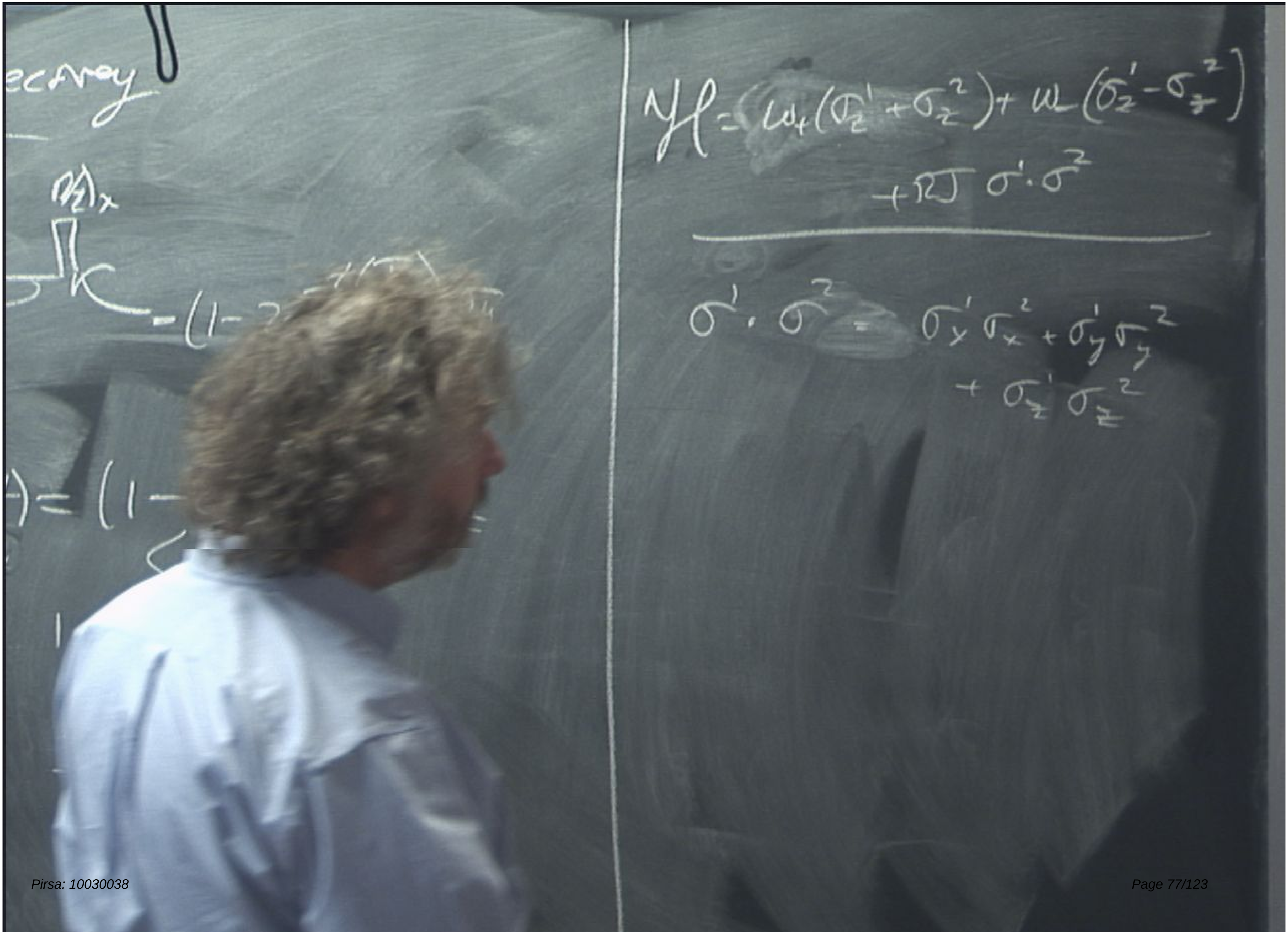
way

$z)_x$

$-(1-2e^{-t/T_1})\sigma_y$

$(1-2e^{-t/T_1})\sigma_z$

$$\mathcal{H} = \omega_+(\sigma_z^1 + \sigma_z^2) + \omega_-(\sigma_z^1 - \sigma_z^2)$$
$$+ \Omega J\, \sigma^1 \cdot \sigma^2$$



ecovery

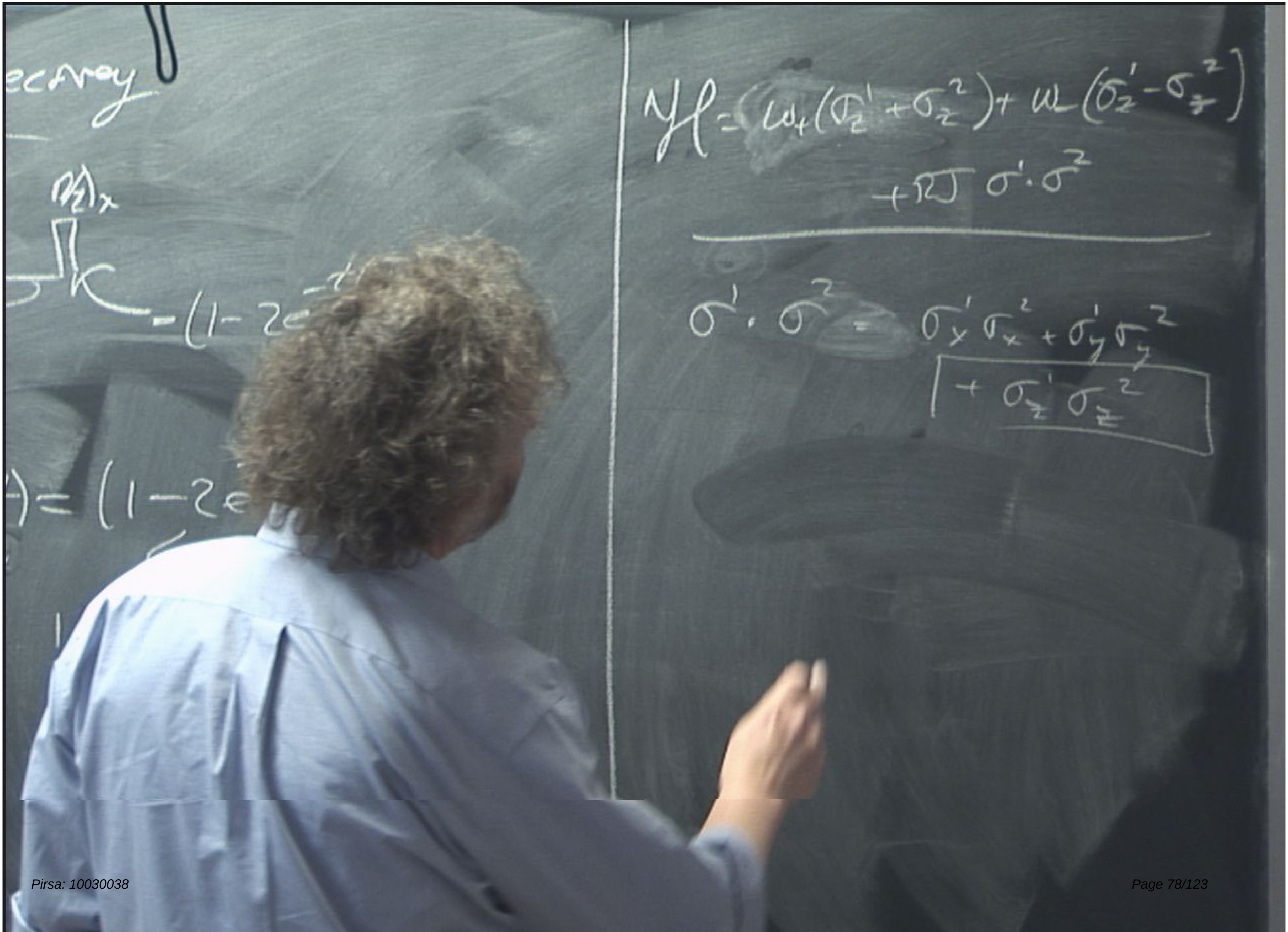
$(\sigma_z)_x$

$-(1-$

$)=(1-$

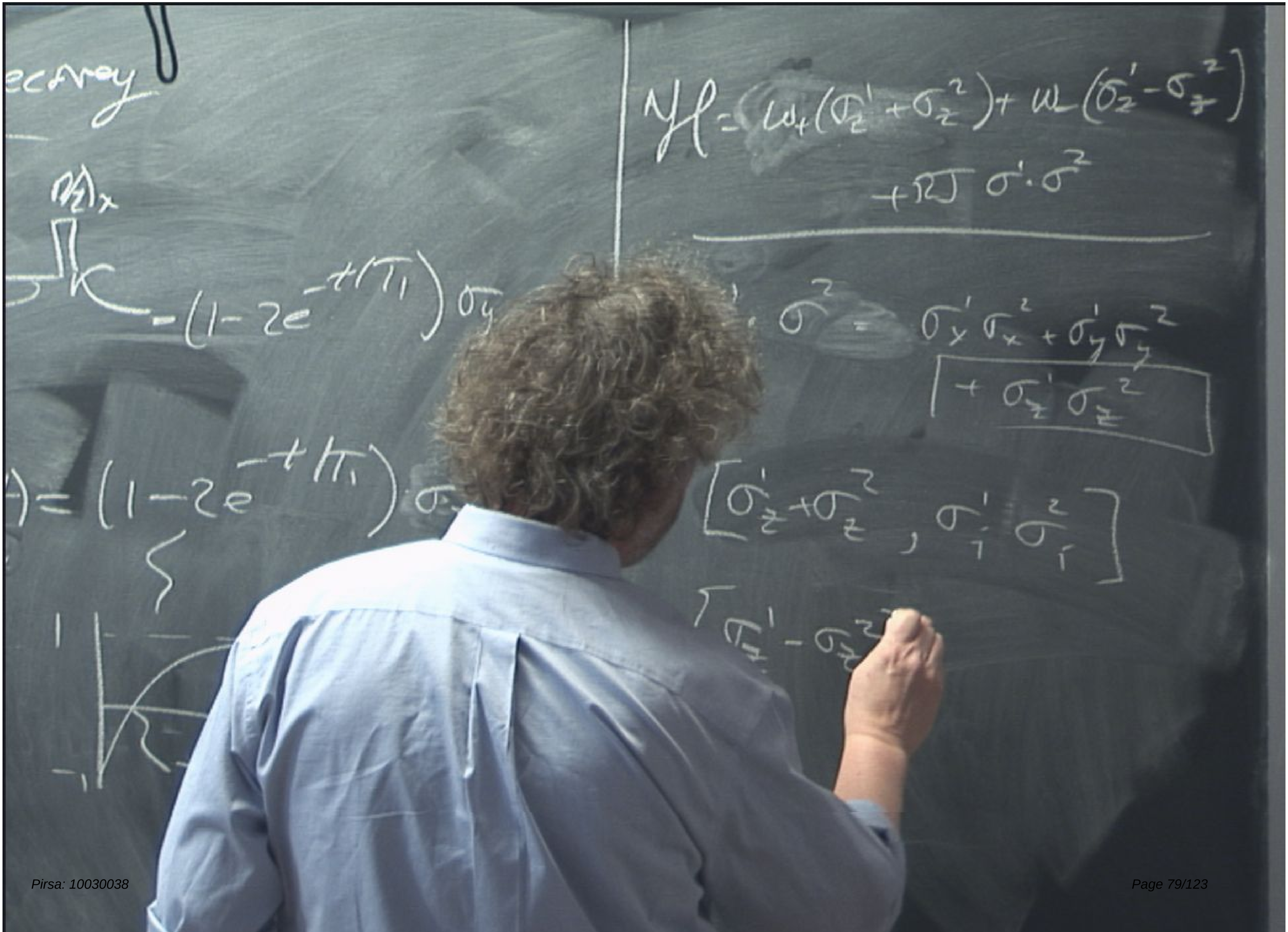
$$\hat{\mathcal{H}} = \omega_+(\sigma_z^1 + \sigma_z^2) + \omega_-(\sigma_z^1 - \sigma_z^2) + \Omega J\, \sigma^1 \cdot \sigma^2$$

$$\sigma^1 \cdot \sigma^2 = \sigma_x^1 \sigma_x^2 + \sigma_y^1 \sigma_y^2 + \sigma_z^1 \sigma_z^2$$



$$\mathcal{H} = \omega_+(\sigma_z^1 + \sigma_z^2) + \omega_-(\sigma_z^1 - \sigma_z^2) + \Omega J\, \sigma^1\cdot\sigma^2$$

$$\sigma^1\cdot\sigma^2 = \sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2 \boxed{+ \sigma_z^1\sigma_z^2}$$



ecovery

$\langle\sigma_z\rangle_x$

$-(1-2e^{-t/T_1})\sigma_y$

$) = (1-2e^{-t/T_1})\sigma_z$

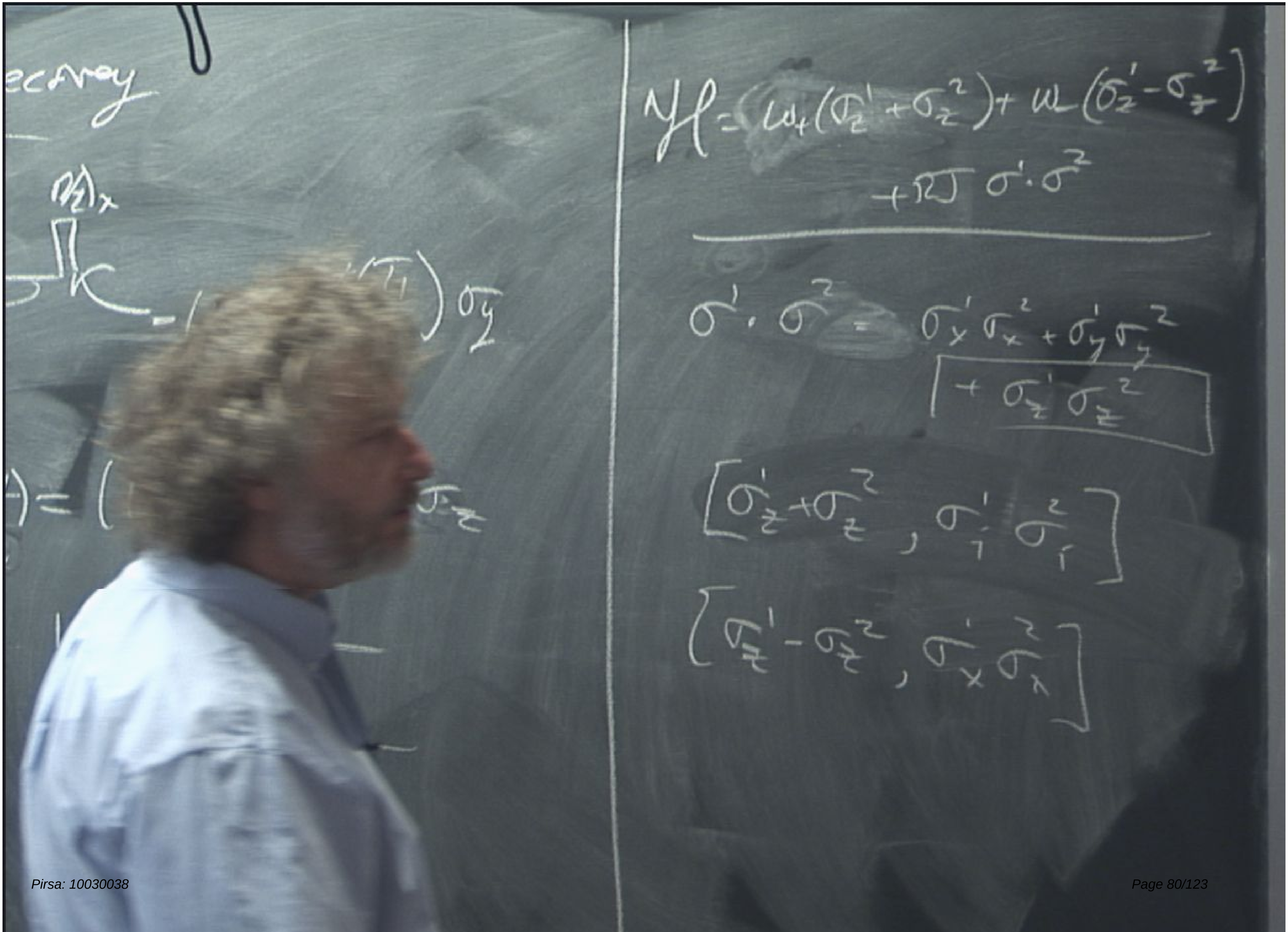
$$\mathcal{H} = \omega_+(\sigma_z^1+\sigma_z^2) + \omega_-(\sigma_z^1-\sigma_z^2)$$

$$+ \Omega J\, \sigma^1\cdot\sigma^2$$

$$\sigma^1\cdot\sigma^2 = \sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2 \boxed{+\sigma_z^1\sigma_z^2}$$

$$[\sigma_z^1+\sigma_z^2 , \sigma_i^1\sigma_i^2]$$

$$[\sigma_z^1-\sigma_z^2$$



$$\mathcal{H} = \omega_+(\sigma_z^1+\sigma_z^2) + \omega_-(\sigma_z^1-\sigma_z^2)$$

$$+\Omega J\;\sigma^1\cdot\sigma^2$$

$$\sigma^1\cdot\sigma^2 = \sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2$$

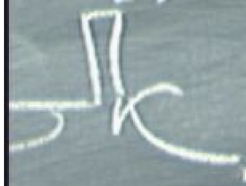
$$\boxed{+\,\sigma_z^1\sigma_z^2}$$

$$[\sigma_z^1+\sigma_z^2\,,\;\sigma_i^1\sigma_i^2]$$

$$[\sigma_z^1-\sigma_z^2\,,\;\sigma_x^1\sigma_x^2]$$

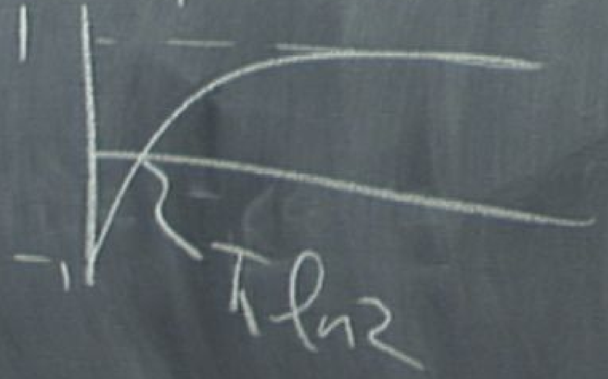
ecovery

$\langle\sigma_z\rangle_x$



$$-(1-2e^{-t/T_1})\sigma_y$$

$$\rangle = (1-2e^{-t/T_1})\sigma_z$$



$$\mathcal{H} = \omega_+(\sigma_z^1+\sigma_z^2)+\omega_-(\sigma_z^1-\sigma_z^2) + \Omega J\,\sigma^1\cdot\sigma^2$$

$$\sigma^1\cdot\sigma^2 = \sigma_x^1\sigma_x^2+\sigma_y^1\sigma_y^2 \boxed{+\sigma_z^1\sigma_z^2}$$

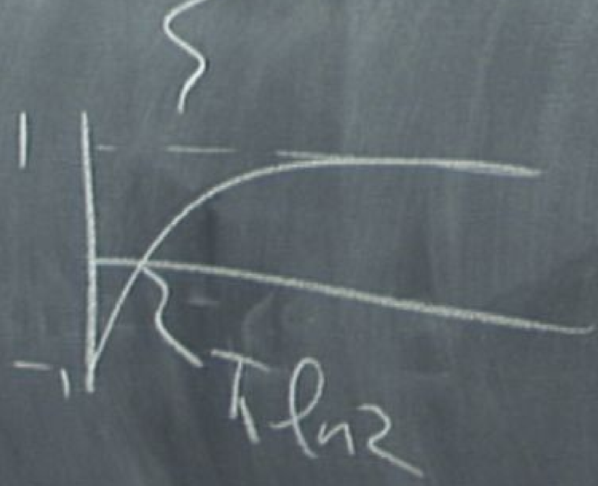
$$[\sigma_z^1+\sigma_z^2,\ \sigma_i^1\sigma_i^2]=0$$

$$[\sigma_z^1-\sigma_z^2,\ \sigma_x^1\sigma_x^2]\neq 0$$

ecovery

$$-(1-2e^{-t/T_1})\sigma_y$$

$$) = (1-2e^{-t/T_1})\sigma_z$$

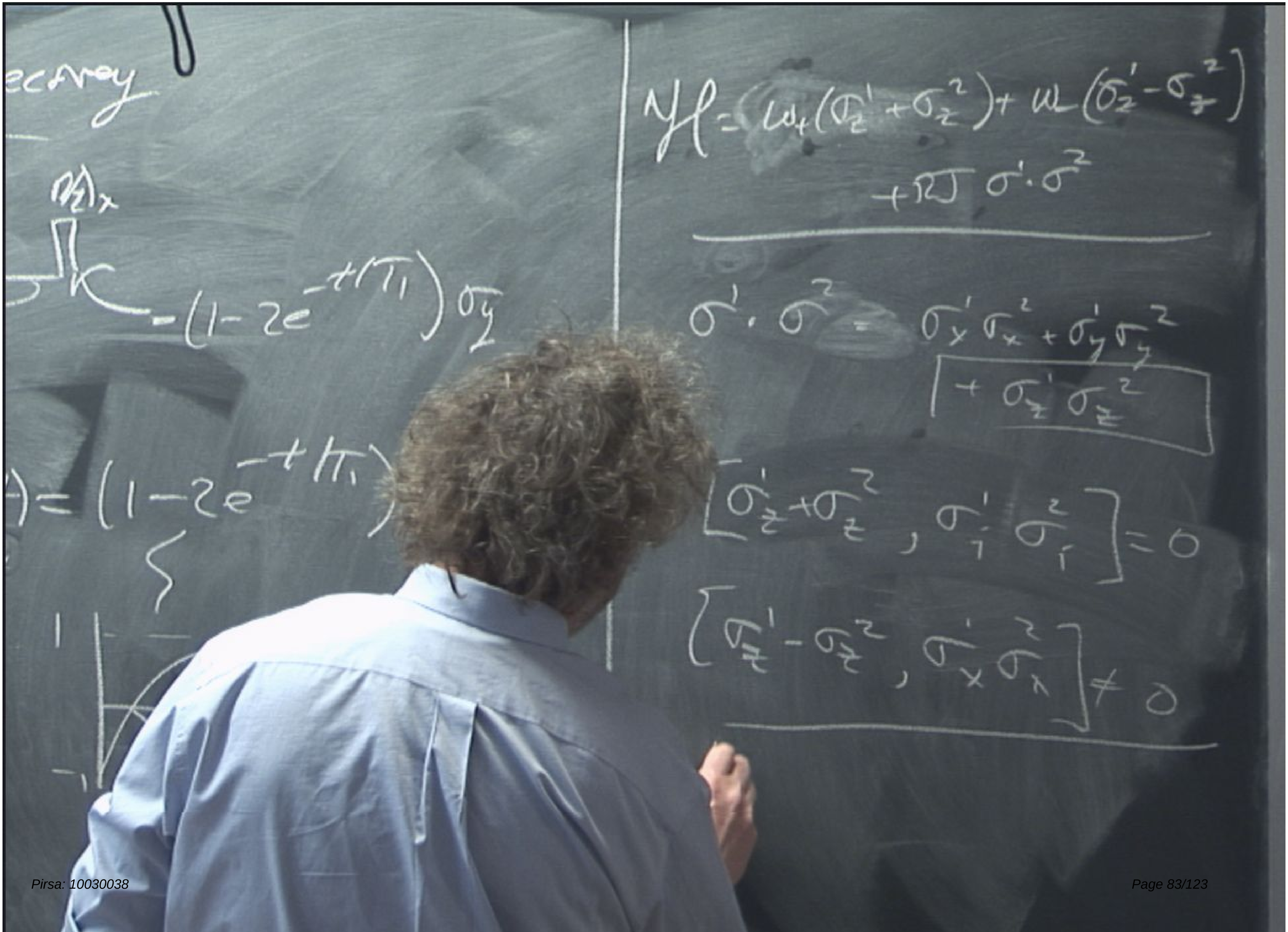


$$\mathcal{H} = \omega_+(\sigma_z^1+\sigma_z^2) + \omega_-(\sigma_z^1-\sigma_z^2) + \Omega J\, \sigma^1\cdot\sigma^2$$

$$\sigma^1\cdot\sigma^2 = \sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2 \boxed{+\sigma_z^1\sigma_z^2}$$

$$[\sigma_z^1+\sigma_z^2,\ \sigma_i^1\sigma_i^2] = 0$$

$$[\sigma_z^1-\sigma_z^2,\ \sigma_x^1\sigma_x^2] \neq 0$$



ecovery

$\langle\sigma_z\rangle_x$

$-(1-2e^{-t/T_1})\sigma_z$

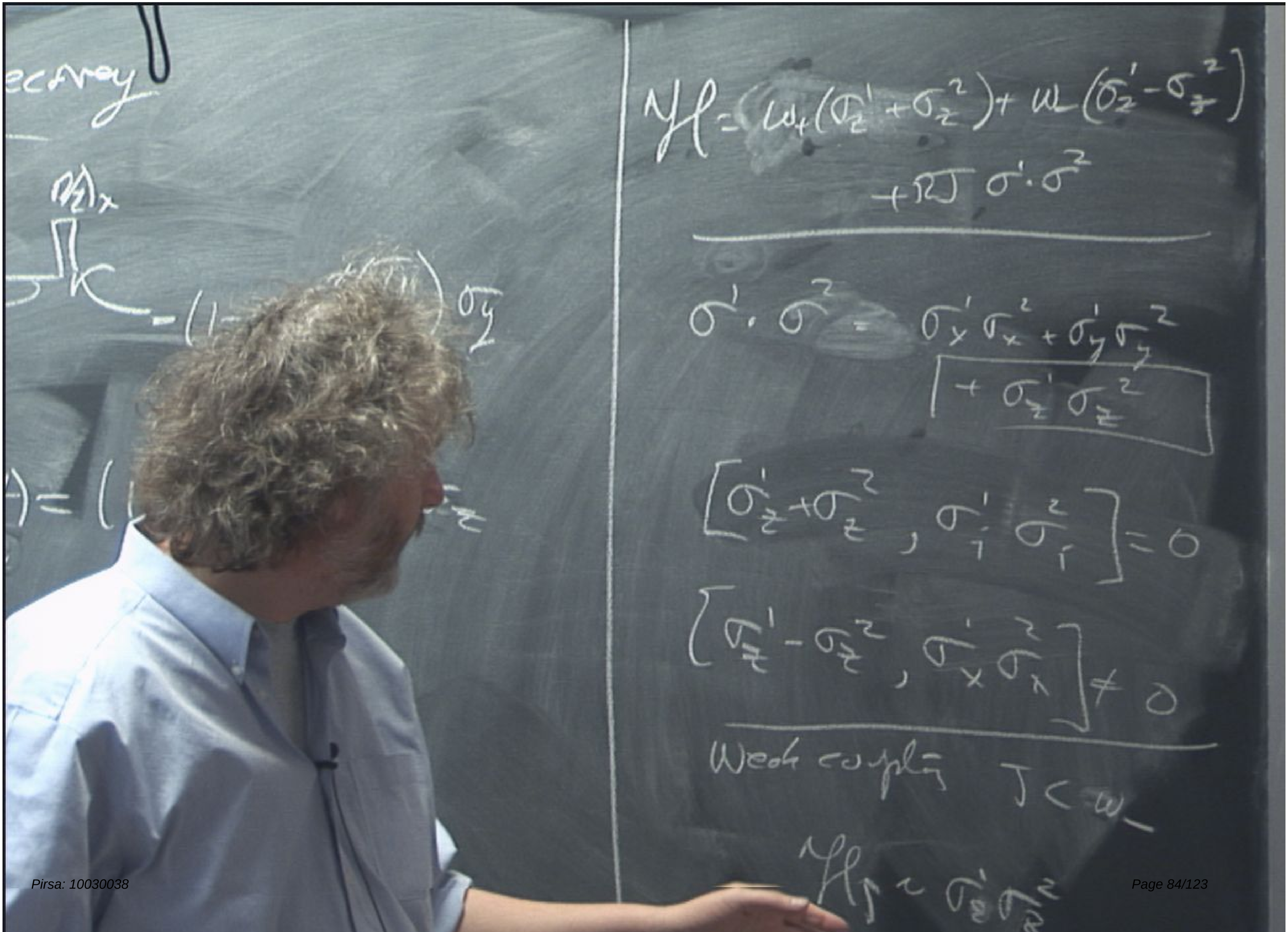
$) = (1-2e^{-t/T_1})$

$$\mathcal{H} = \omega_+(\sigma_z^1 + \sigma_z^2) + \omega_-(\sigma_z^1 - \sigma_z^2)$$
$$+ \Omega J\, \sigma^1\cdot\sigma^2$$

$$\sigma^1\cdot\sigma^2 = \sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2 \boxed{+ \sigma_z^1\sigma_z^2}$$

$$[\sigma_z^1 + \sigma_z^2, \sigma_i^1\sigma_i^2] = 0$$

$$[\sigma_z^1 - \sigma_z^2, \sigma_x^1\sigma_x^2] \neq 0$$



ecovery

$$\mathcal{H} = \omega_+(\sigma_z^1 + \sigma_z^2) + \omega_-(\sigma_z^1 - \sigma_z^2)$$
$$+ \hbar J\, \sigma^1 \cdot \sigma^2$$

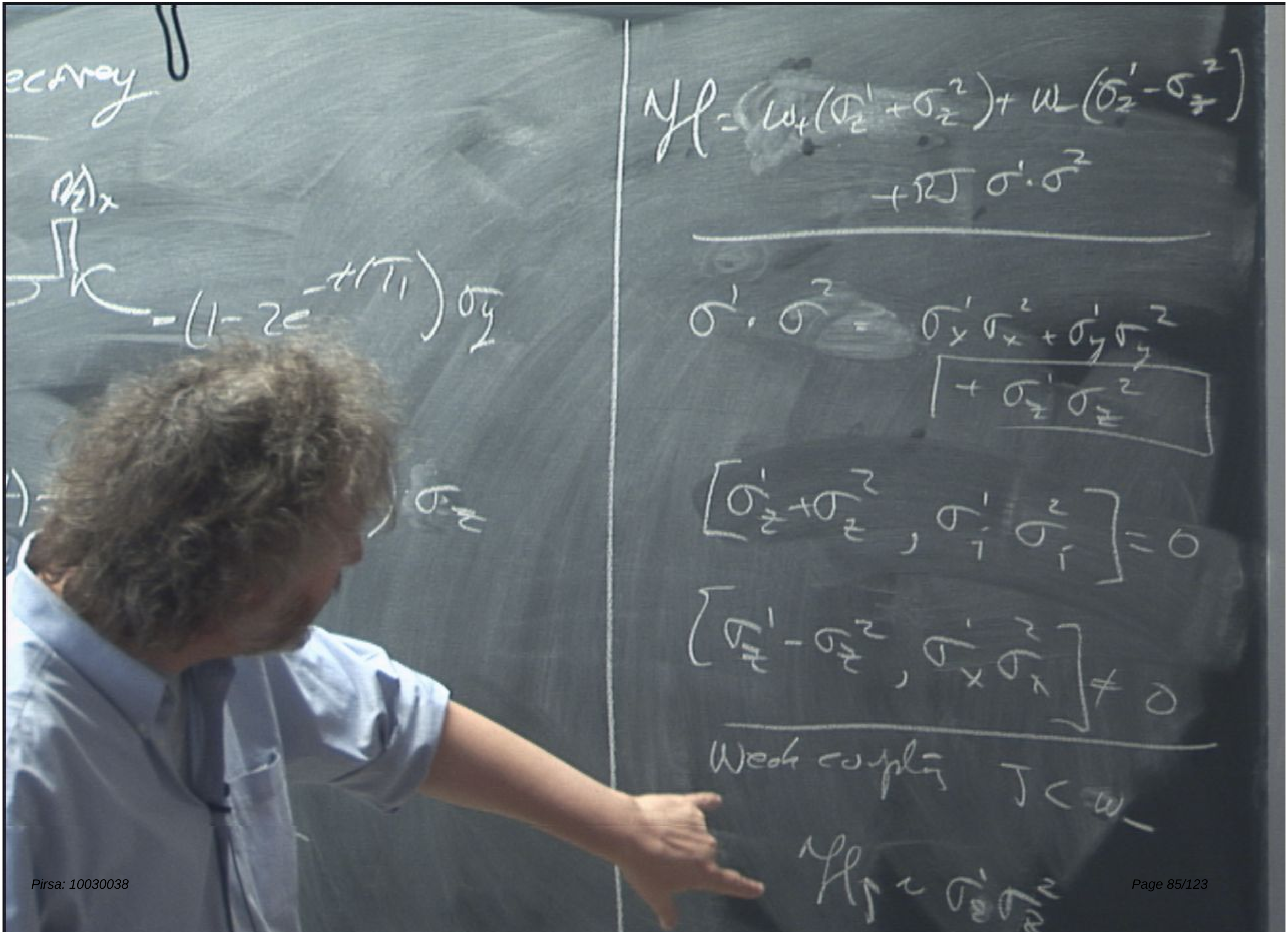
$$\sigma^1 \cdot \sigma^2 = \sigma_x^1 \sigma_x^2 + \sigma_y^1 \sigma_y^2$$
$$\boxed{+ \sigma_z^1 \sigma_z^2}$$

$$[\sigma_z^1 + \sigma_z^2 , \sigma_i^1 \sigma_i^2] = 0$$

$$[\sigma_z^1 - \sigma_z^2 , \sigma_x^1 \sigma_x^2] \neq 0$$

Weak coupling $J < \omega_-$

$$\mathcal{H}_J \sim \sigma_z^1 \sigma_z^2$$



$$\mathcal{H} = \omega_+(\sigma_z^1+\sigma_z^2) + \omega_-(\sigma_z^1-\sigma_z^2)$$
$$+\pi J\,\sigma^1\cdot\sigma^2$$

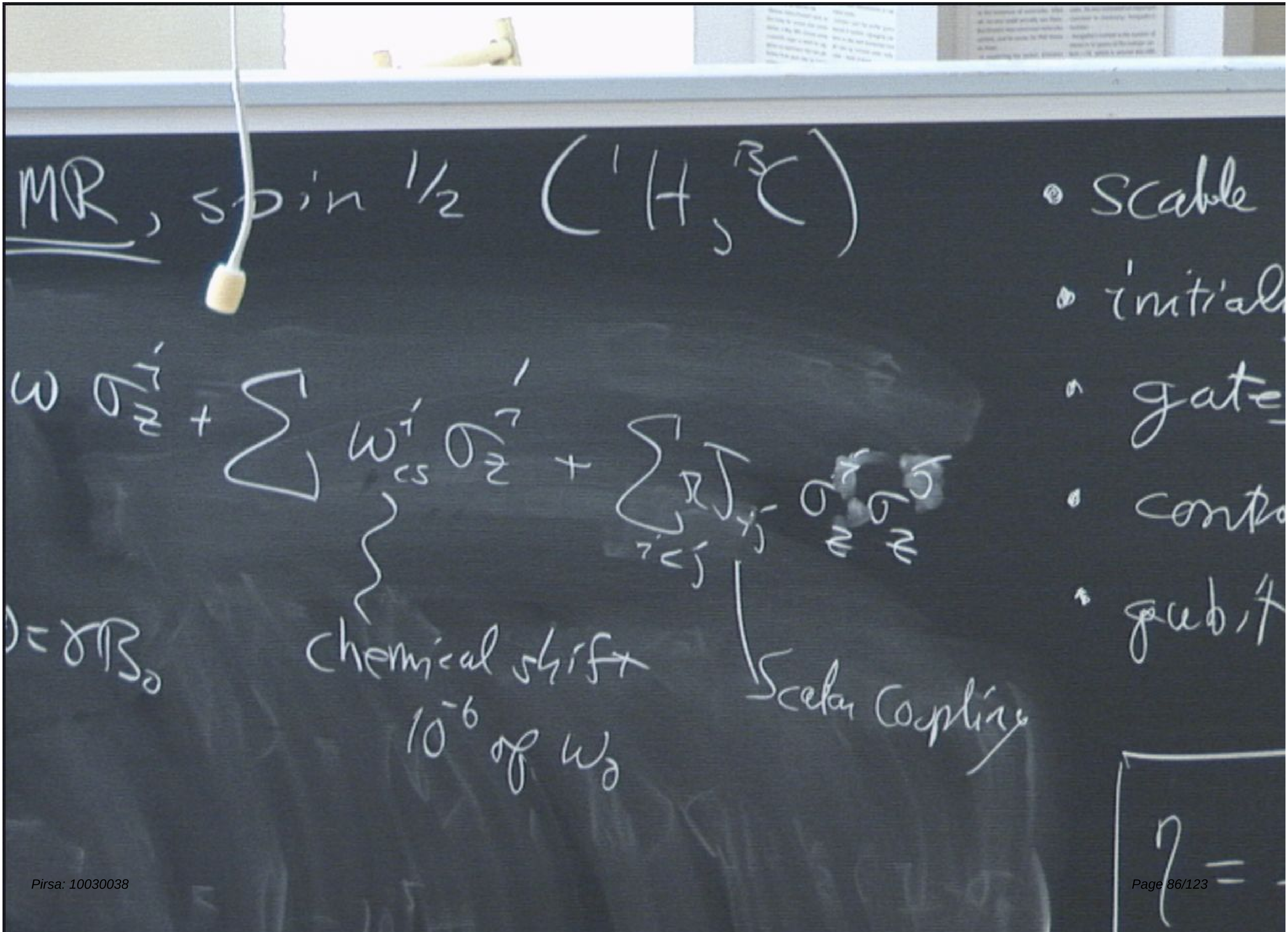
$$\sigma^1\cdot\sigma^2 = \sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2 \boxed{+\sigma_z^1\sigma_z^2}$$

$$[\sigma_z^1+\sigma_z^2,\ \sigma_i^1\sigma_i^2] = 0$$

$$[\sigma_z^1-\sigma_z^2,\ \sigma_x^1\sigma_x^2] \neq 0$$

Weak coupling $J < \omega_-$

$$\mathcal{H}_J \simeq \sigma_z^1\sigma_z^2$$



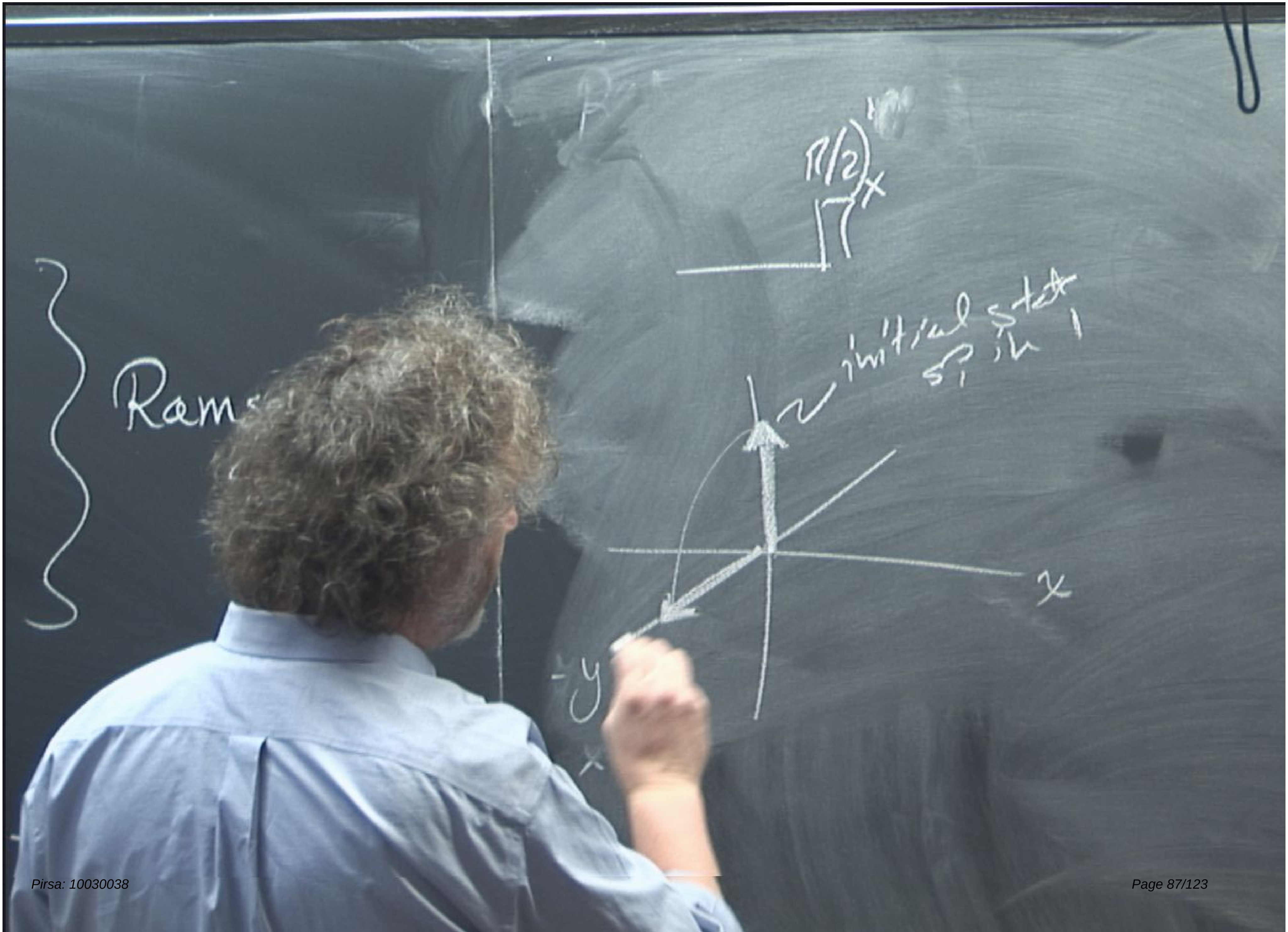
MR, spin 1/2 $({}^1H, {}^{13}C)$

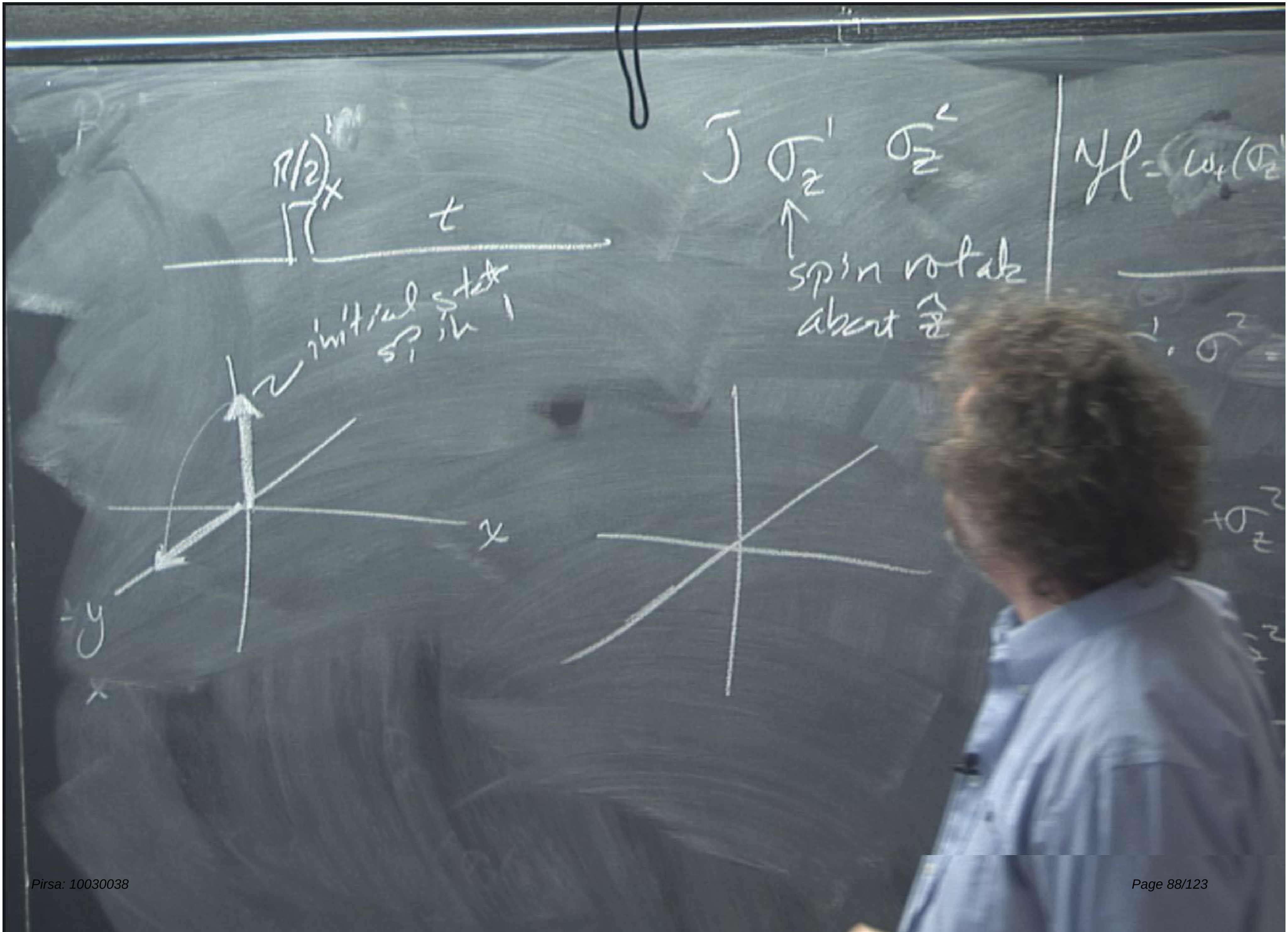
$$\omega \sigma_z^i + \sum \underbrace{\omega_{cs}^i}_{\substack{\text{chemical shift} \\ 10^{-6} \text{ of } \omega_0}} \sigma_z^i + \sum_{i<j} \underbrace{\pi J_{ij}}_{\text{Scalar Coupling}} \sigma_z^i \sigma_z^j$$

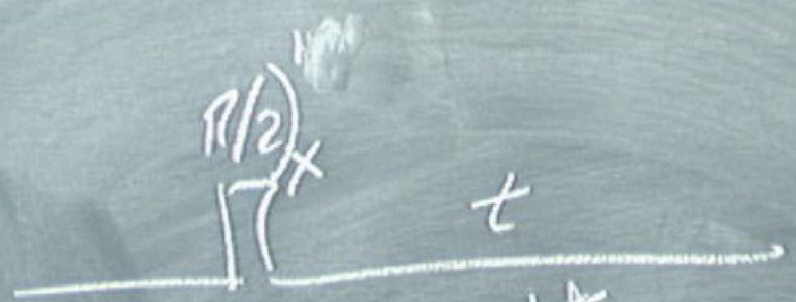
$)=\gamma B_0$

- scable
- initial
- gate
- contro
- qubit

$\eta =$


$\pi/2)_x$
$\pi)_x$
initial state
spin 1
Rams
x
-y
x

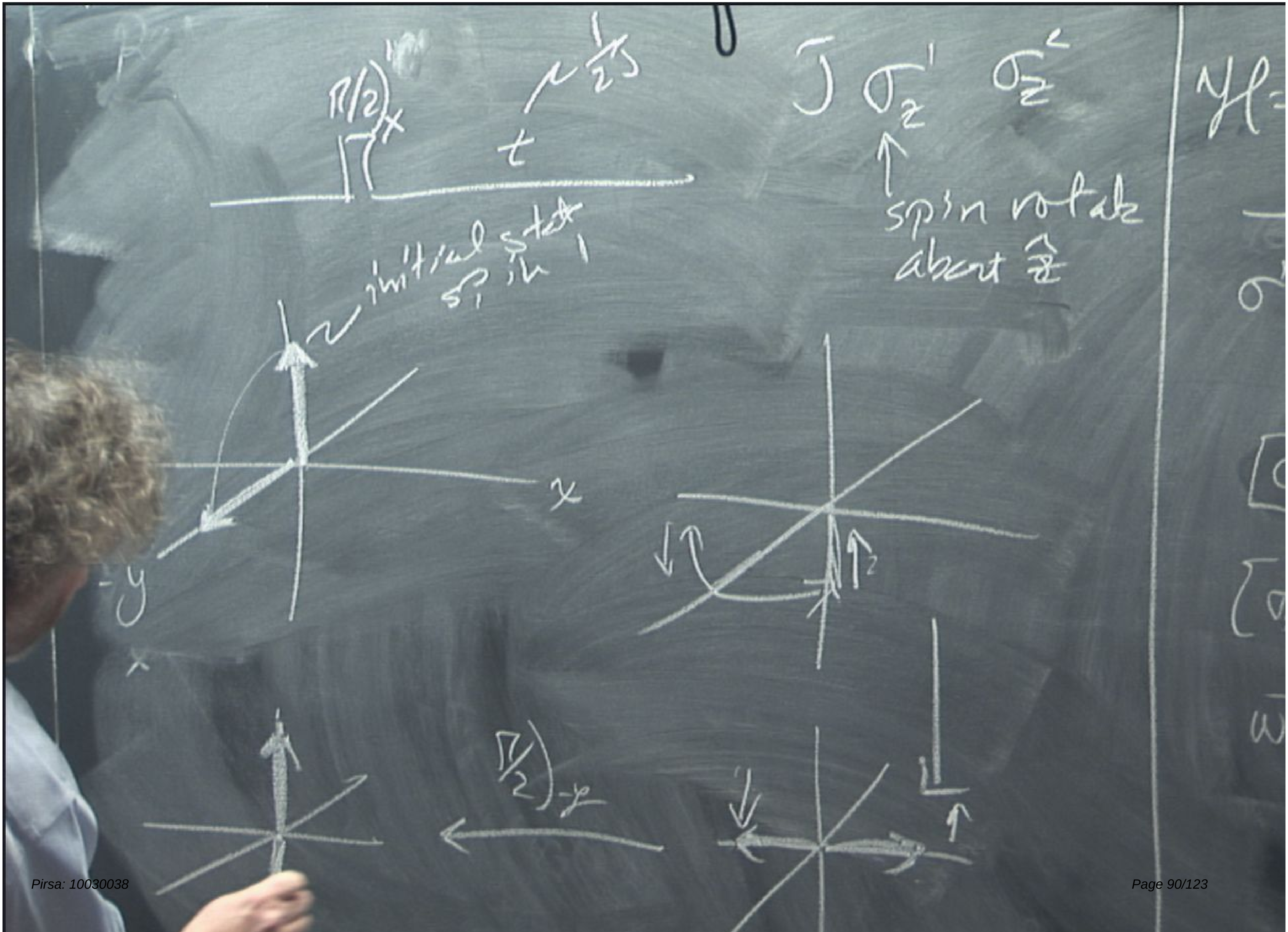

π/2)x
t
initial state
spin 1
x
y
J σz1 σz2
spin rotate
about ẑ
H = ω1(σz

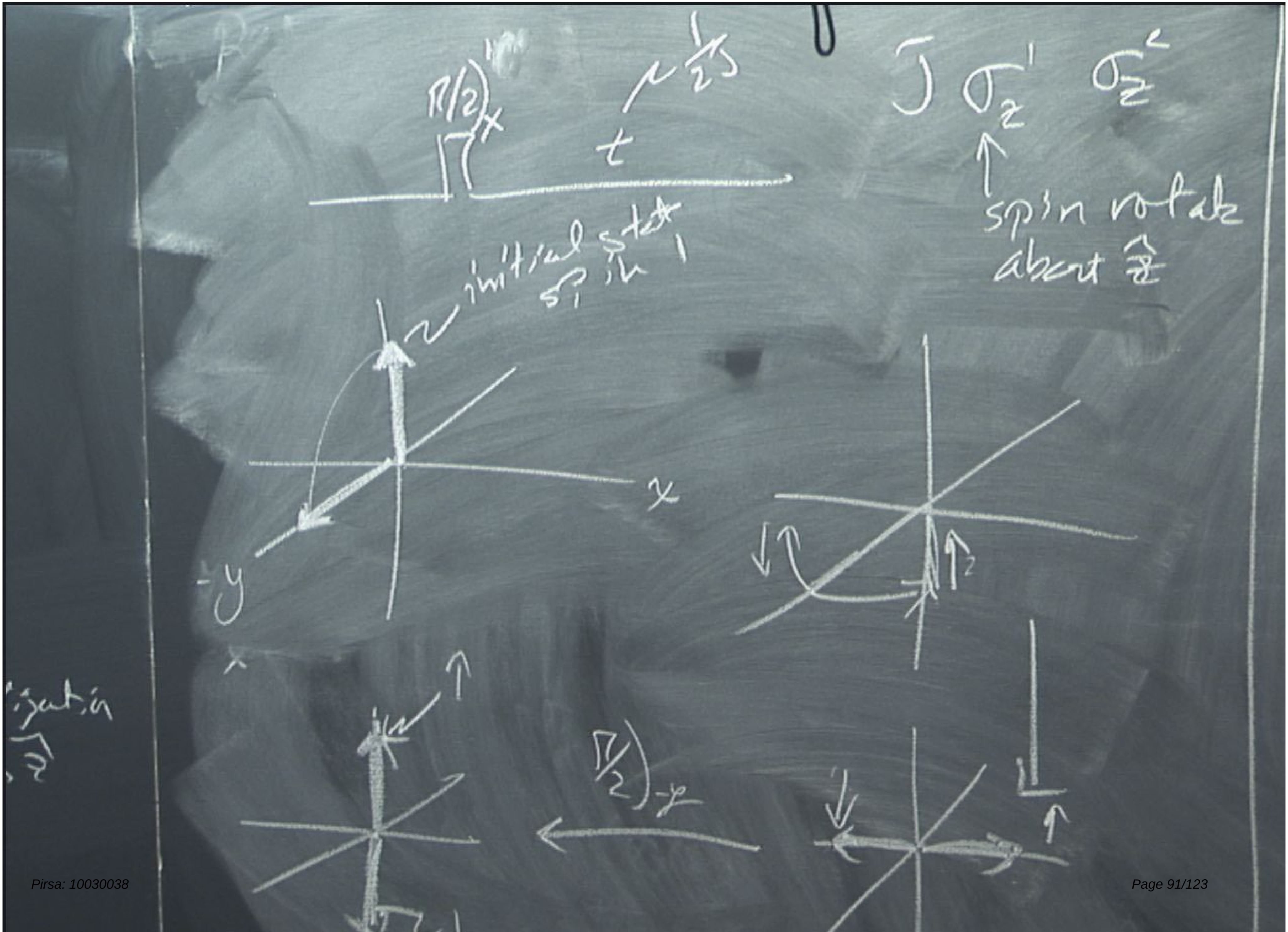


$J\sigma_z^1\sigma_z^2$

spin rotate about $\hat{z}$

$\mathcal{H}=$

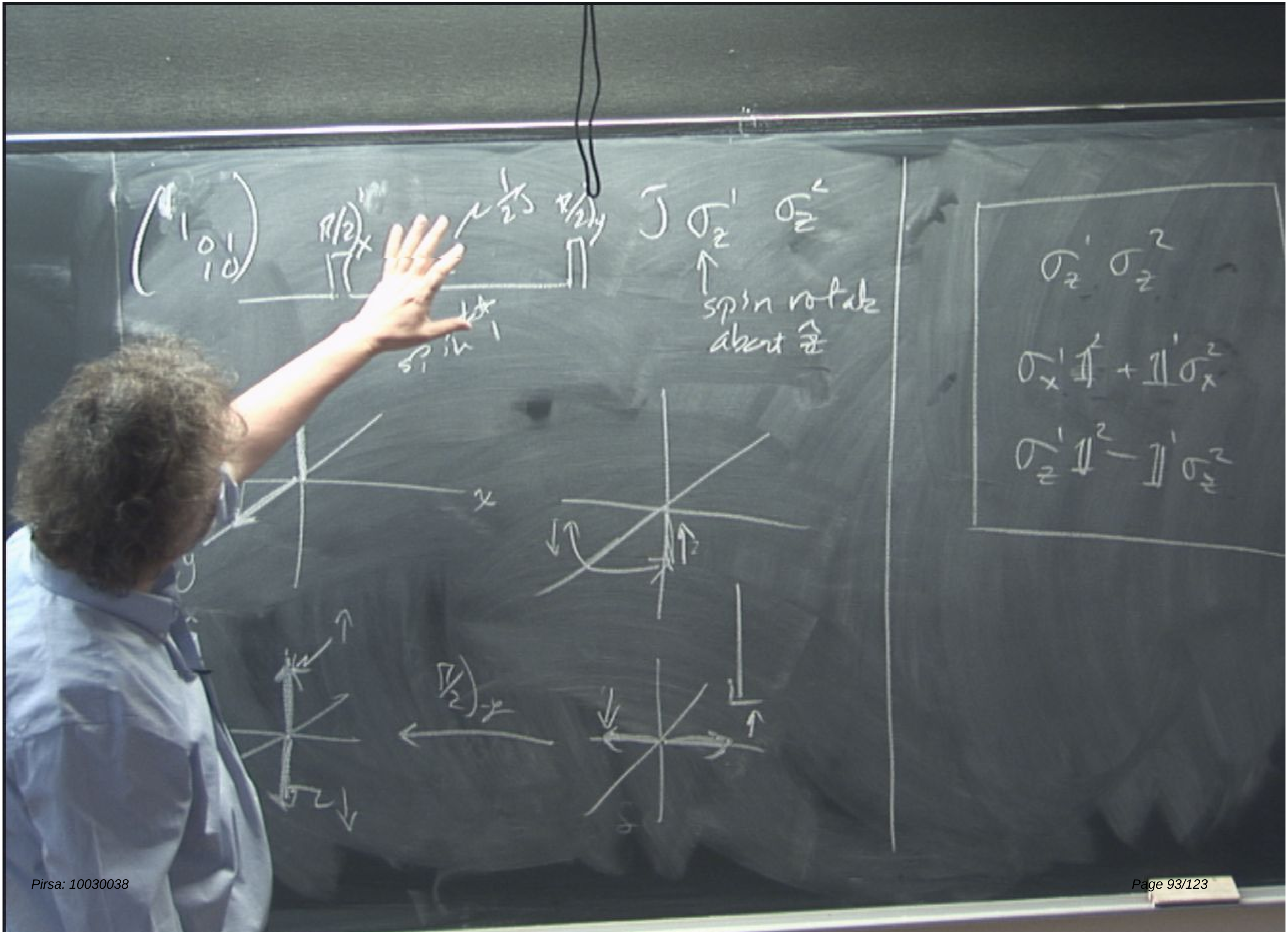

π/2)_x
t
1/2J
J σ_z^1 σ_z^2
spin rotate about ẑ
initial state spin 1
x
-y
π/2)_{-y}
ℋ=

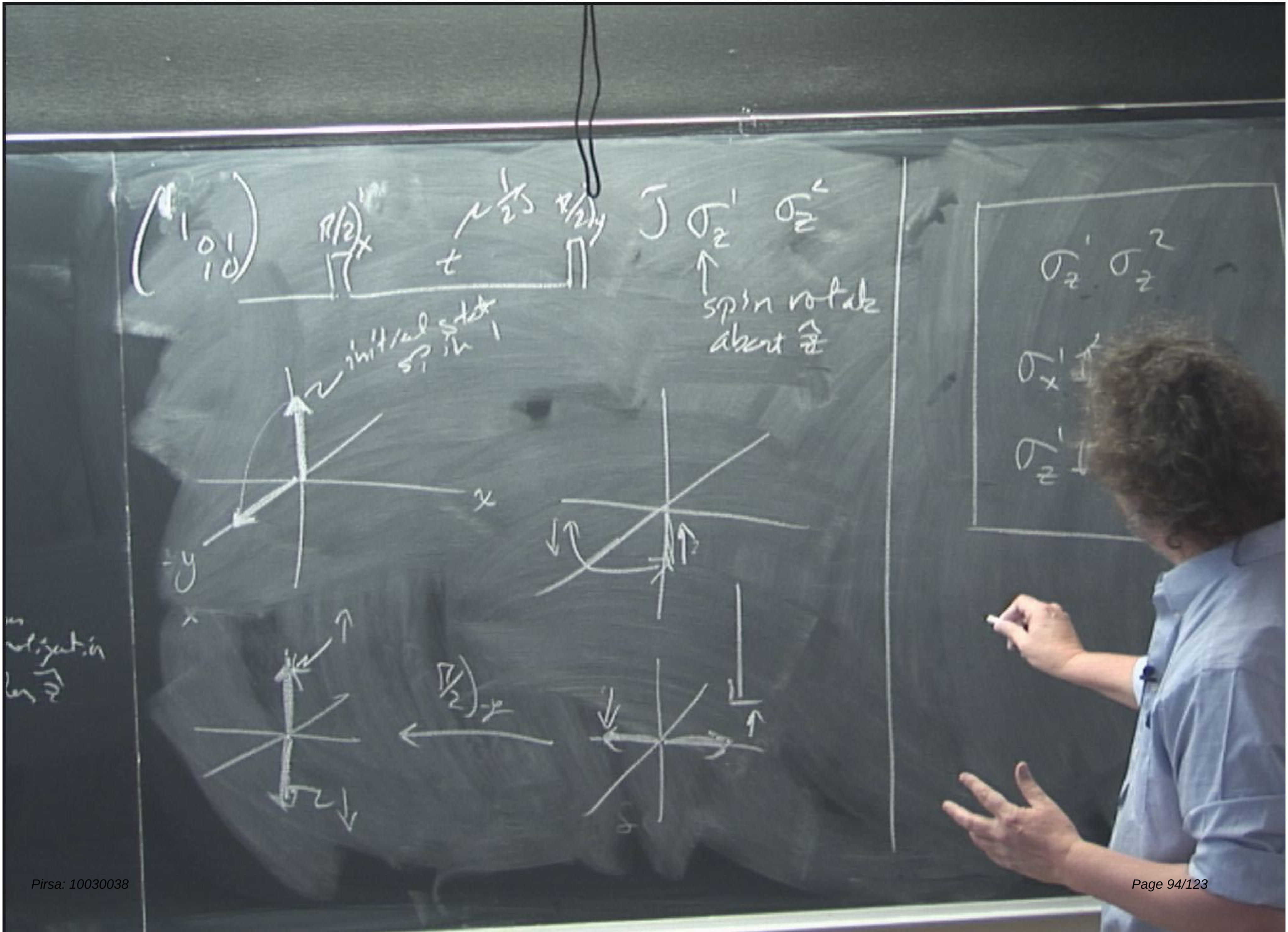

π/2)x
t
J σz¹ σz²
spin rotate about ẑ
initial state spin 1
x
-y
↑2
π/2)-y

$\sigma_z \qquad \sigma_z$

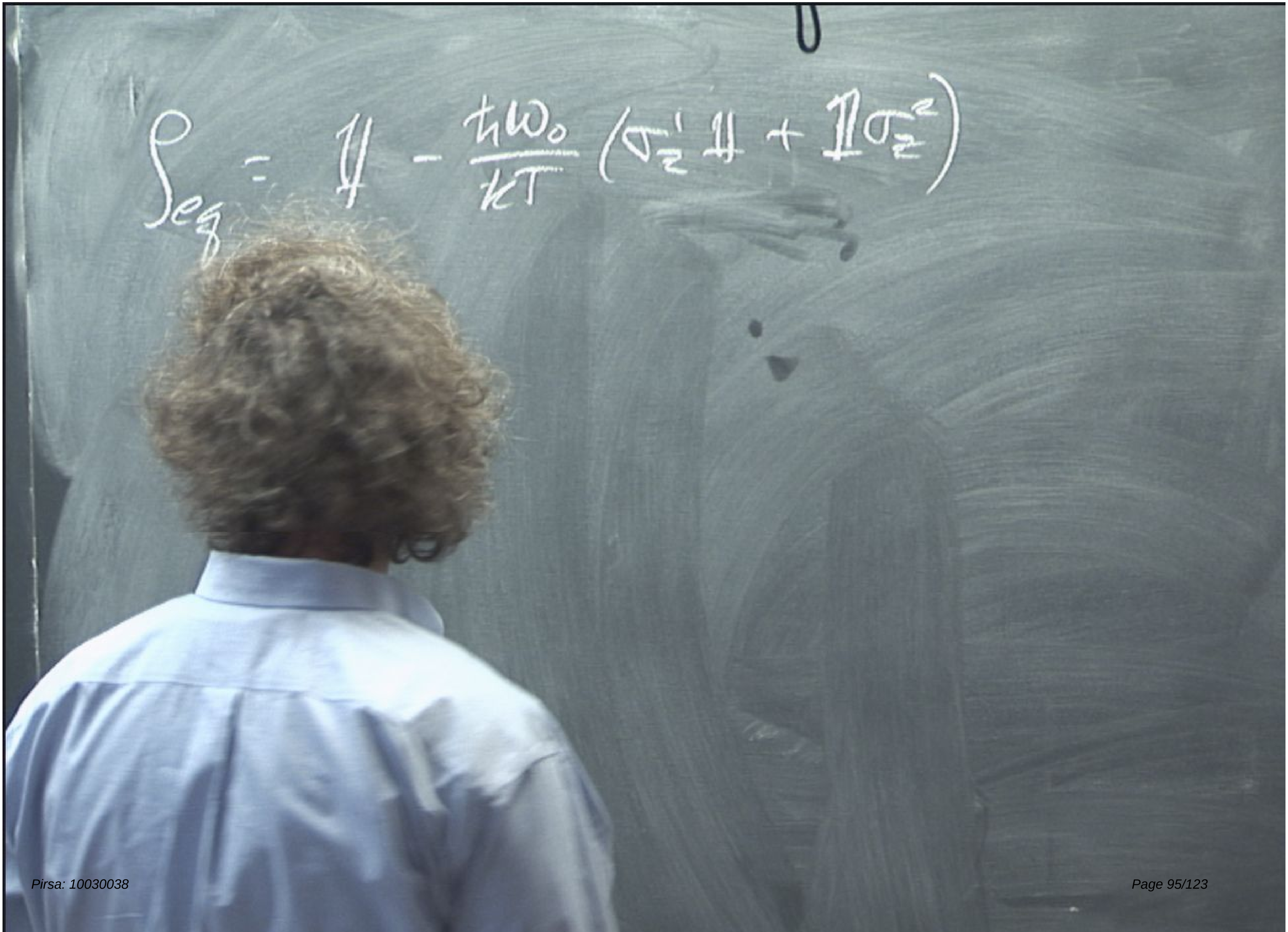
$\uparrow$ spin rotate about $\hat{z}$

$$\boxed{\begin{array}{c} \sigma_z^1\,\sigma_z^2 \\ \\ \sigma_x^1\,\mathbb{1}^2 + \mathbb{1}^1\sigma_x^2 \\ \\ \sigma_z^1\,\mathbb{1}^2 - \mathbb{1}^1\sigma_z^2 \end{array}}$$


π/2)_x
π/2)_y
J σ_z^1 σ_z^2
spin rotate about ẑ
spin 1
x
y
z
π/2)_-y
σ_z^1 σ_z^2
σ_x^1 𝟙^2 + 𝟙^1 σ_x^2
σ_z^1 𝟙^2 − 𝟙^1 σ_z^2



π/2)x
t
½J
π/2)y
J σz¹ σz²
spin rotate about ẑ
initial state spin 1
z
x
-y
σz¹ σz²
σx¹
σz¹
π/2)-y
olarization
along ẑ



$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\left(\sigma_z^1 \mathbb{1} + \mathbb{1}\sigma_z^2\right)$$

$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\left(\sigma_z^1\,\mathbb{1} + \mathbb{1}\,\sigma_z^2\right)$$

$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\left(\sigma_z^1 \mathbb{1} + \mathbb{1}\sigma_z^2\right)$$

$$|0\rangle \doteq \frac{1}{2}\left(\mathbb{1} + \sigma_z\right)$$

$$|1\rangle = \frac{1}{2}\left(\mathbb{1} - \sigma_z\right)$$

$$|0\rangle = \tfrac{1}{2}(\mathbb{1} + \sigma_z)$$

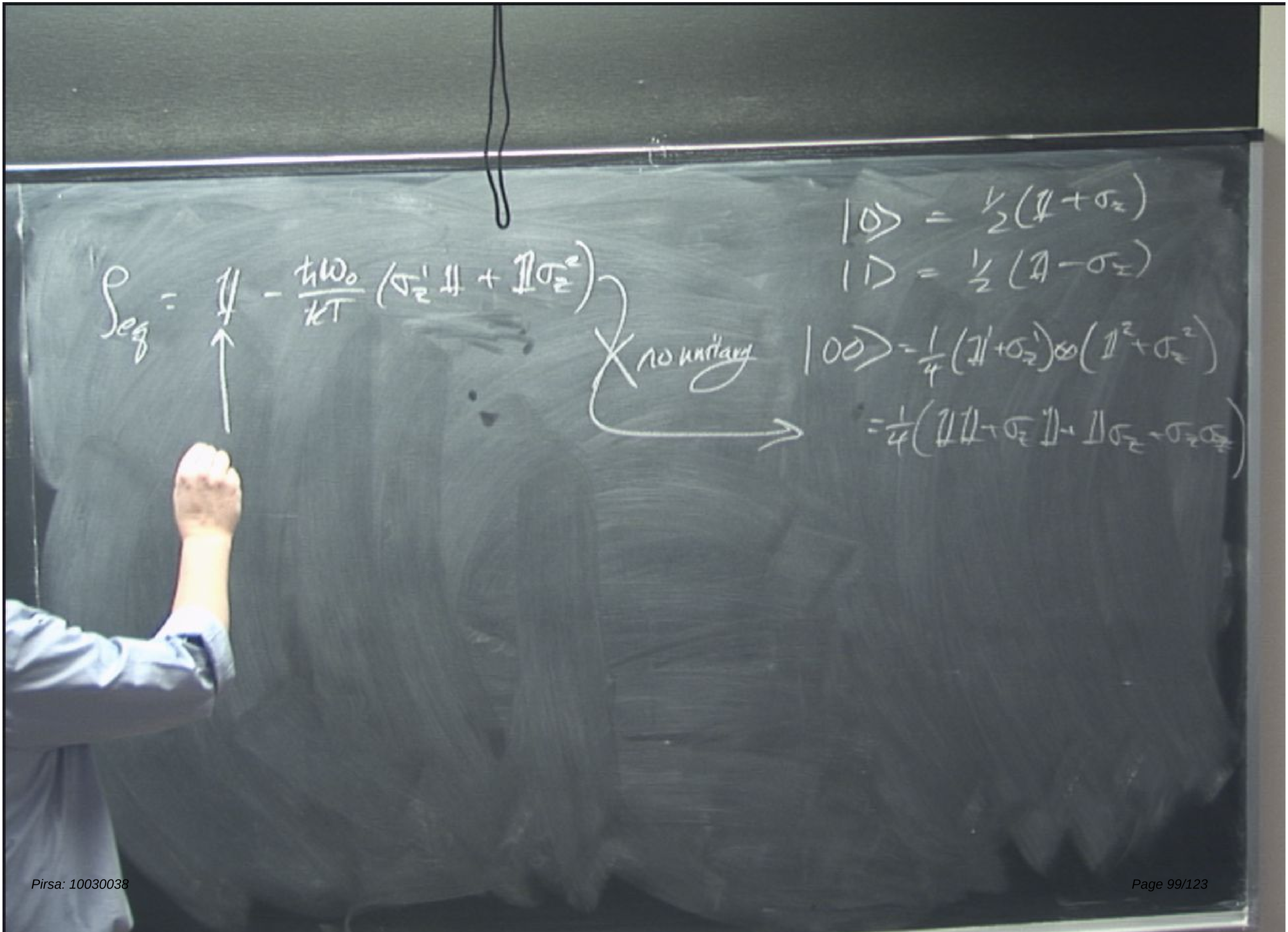
$$|1\rangle = \tfrac{1}{2}(\mathbb{1} - \sigma_z)$$

$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\left(\sigma_z^1\,\mathbb{1} + \mathbb{1}\,\sigma_z^2\right)$$

$\not\longrightarrow$ no unitary

$$|00\rangle = \frac{1}{4}(\mathbb{1}^1 + \sigma_z^1) \otimes (\mathbb{1}^2 + \sigma_z^2)$$

$$= \frac{1}{4}(\mathbb{1}\mathbb{1} + \sigma_z\mathbb{1} + \mathbb{1}\sigma_z + \sigma_z\sigma_z)$$



$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\left(\sigma_z^1 \mathbb{1} + \mathbb{1}\sigma_z^2\right)$$

$$\uparrow$$

$$\cancel{\xrightarrow{\text{no unitary}}}$$

$$|0\rangle = \tfrac{1}{2}(\mathbb{1} + \sigma_z)$$

$$|1\rangle = \tfrac{1}{2}(\mathbb{1} - \sigma_z)$$

$$|00\rangle = \frac{1}{4}(\mathbb{1}^1 + \sigma_z^1) \otimes (\mathbb{1}^2 + \sigma_z^2)$$

$$= \frac{1}{4}(\mathbb{1}\mathbb{1} + \sigma_z\mathbb{1} + \mathbb{1}\sigma_z + \sigma_z\sigma_z)$$

$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\left(\sigma_z^1 \mathbb{1} + \mathbb{1}\sigma_z^2\right)$$

$\uparrow$

remains

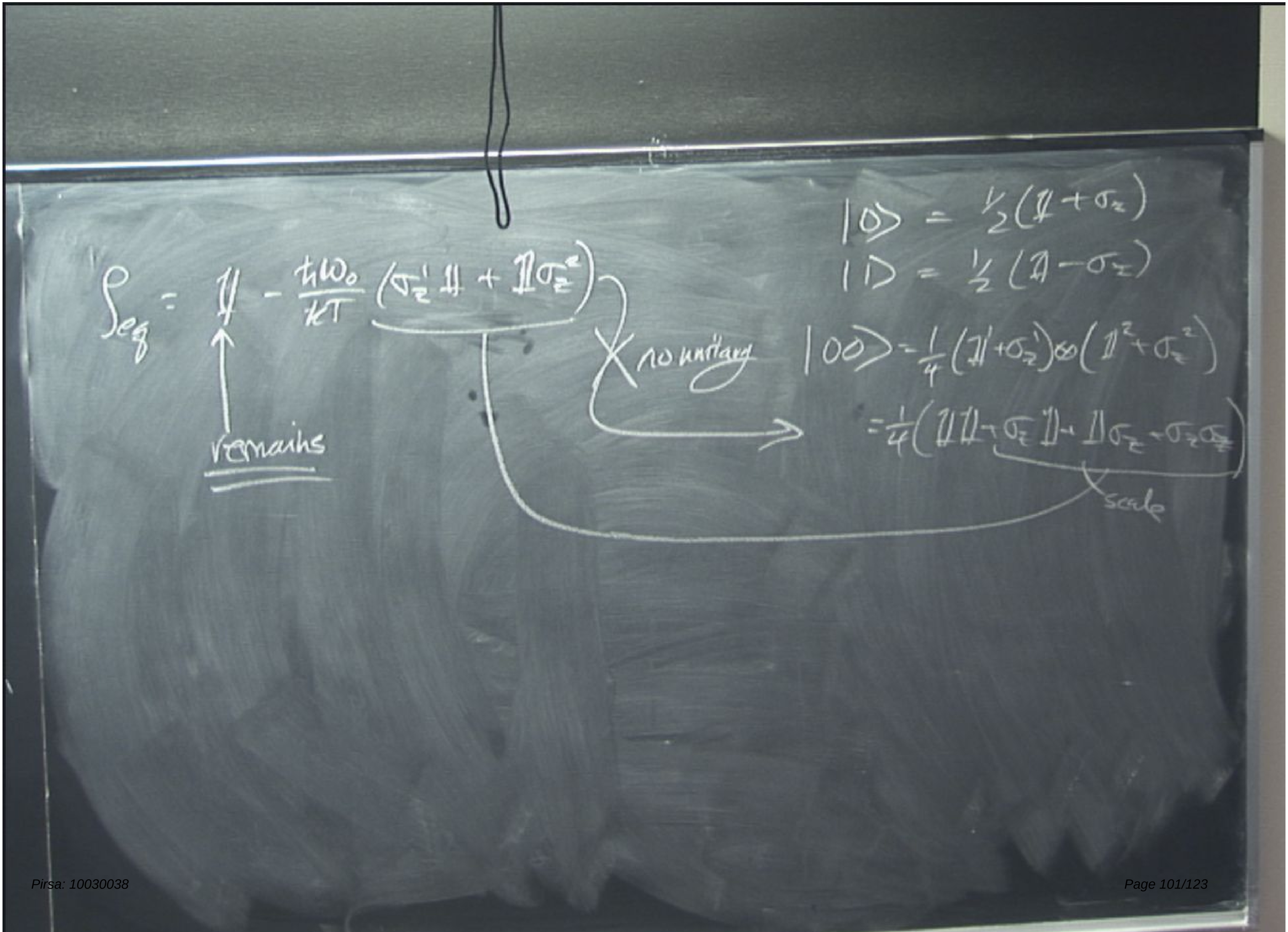
$\times$ no unitary $\longrightarrow$

$$|0\rangle = \tfrac{1}{2}(\mathbb{1} + \sigma_z)$$

$$|1\rangle = \tfrac{1}{2}(\mathbb{1} - \sigma_z)$$

$$|00\rangle = \frac{1}{4}(\mathbb{1}^1 + \sigma_z^1) \otimes (\mathbb{1}^2 + \sigma_z^2)$$

$$= \frac{1}{4}\left(\mathbb{1}\mathbb{1} + \sigma_z \mathbb{1} + \mathbb{1}\sigma_z + \sigma_z\sigma_z\right)$$



$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\underbrace{(\sigma_z^1\mathbb{1} + \mathbb{1}\sigma_z^2)}$$

remains

no unitary

$$|0\rangle = \tfrac{1}{2}(\mathbb{1} + \sigma_z)$$

$$|1\rangle = \tfrac{1}{2}(\mathbb{1} - \sigma_z)$$

$$|00\rangle = \frac{1}{4}(\mathbb{1}^1 + \sigma_z^1) \otimes (\mathbb{1}^2 + \sigma_z^2)$$

$$= \frac{1}{4}(\mathbb{1}\mathbb{1} + \underbrace{\sigma_z\mathbb{1} + \mathbb{1}\sigma_z + \sigma_z\sigma_z}_{\text{scale}})$$

$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\underbrace{\left(\sigma_z^1\,\mathbb{1} + \mathbb{1}\,\sigma_z^2\right)}$$

remains

no unitary

$$|0\rangle = \tfrac{1}{2}(\mathbb{1} + \sigma_z)$$

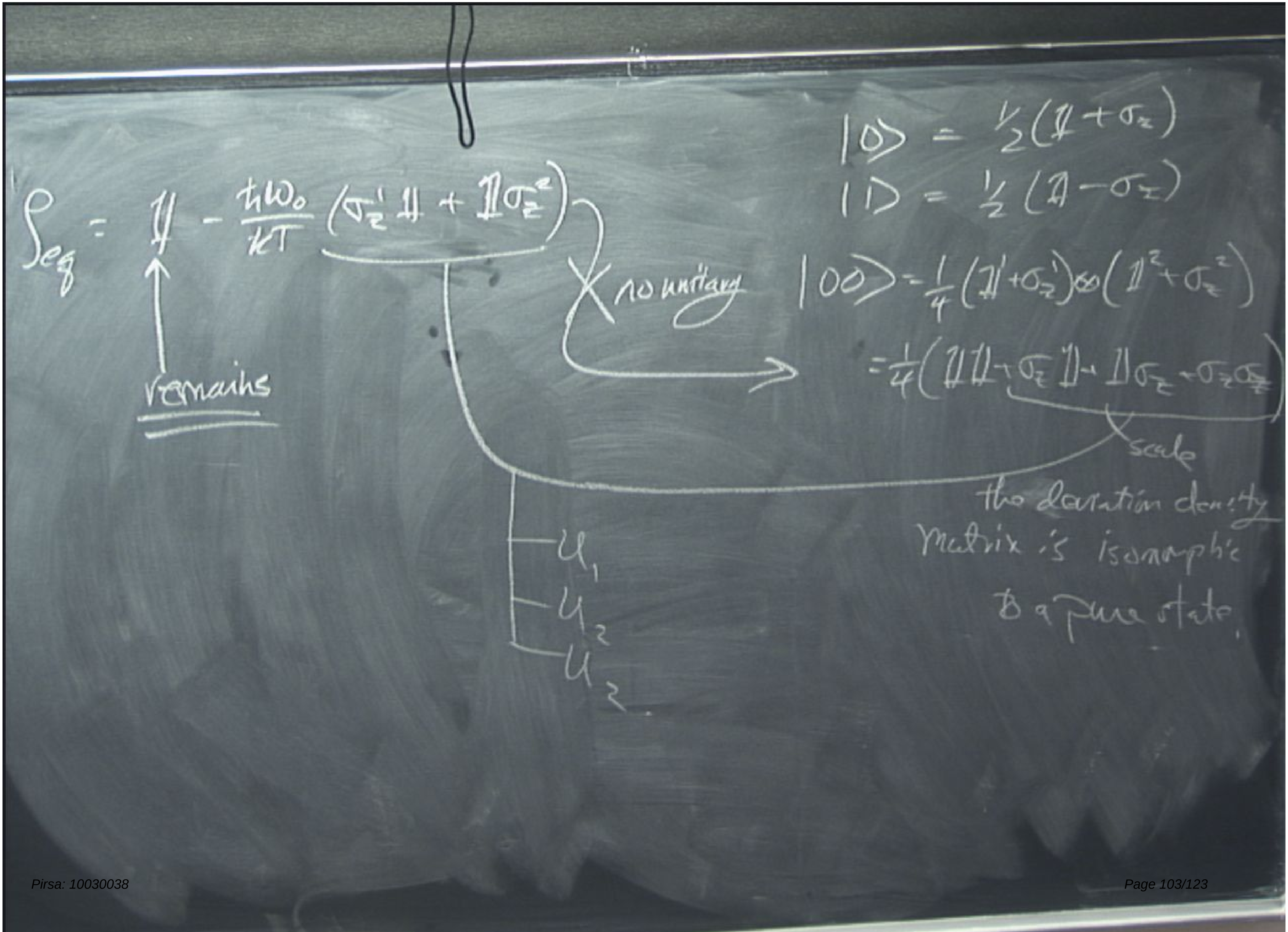
$$|1\rangle = \tfrac{1}{2}(\mathbb{1} - \sigma_z)$$

$$|00\rangle = \frac{1}{4}(\mathbb{1}^1 + \sigma_z^1)\otimes(\mathbb{1}^2 + \sigma_z^2)$$

$$= \frac{1}{4}\left(\mathbb{1}\mathbb{1} + \underbrace{\sigma_z\mathbb{1} + \mathbb{1}\sigma_z + \sigma_z\sigma_z}\right)$$

scale

the deviation density matrix is isomorphic to a pure state.



$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\left(\sigma_z^1\mathbb{1} + \mathbb{1}\sigma_z^2\right)$$

remains

no unitary

$$|0\rangle = \tfrac{1}{2}(\mathbb{1} + \sigma_z)$$

$$|1\rangle = \tfrac{1}{2}(\mathbb{1} - \sigma_z)$$

$$|00\rangle = \tfrac{1}{4}(\mathbb{1}^1 + \sigma_z^1) \otimes (\mathbb{1}^2 + \sigma_z^2)$$

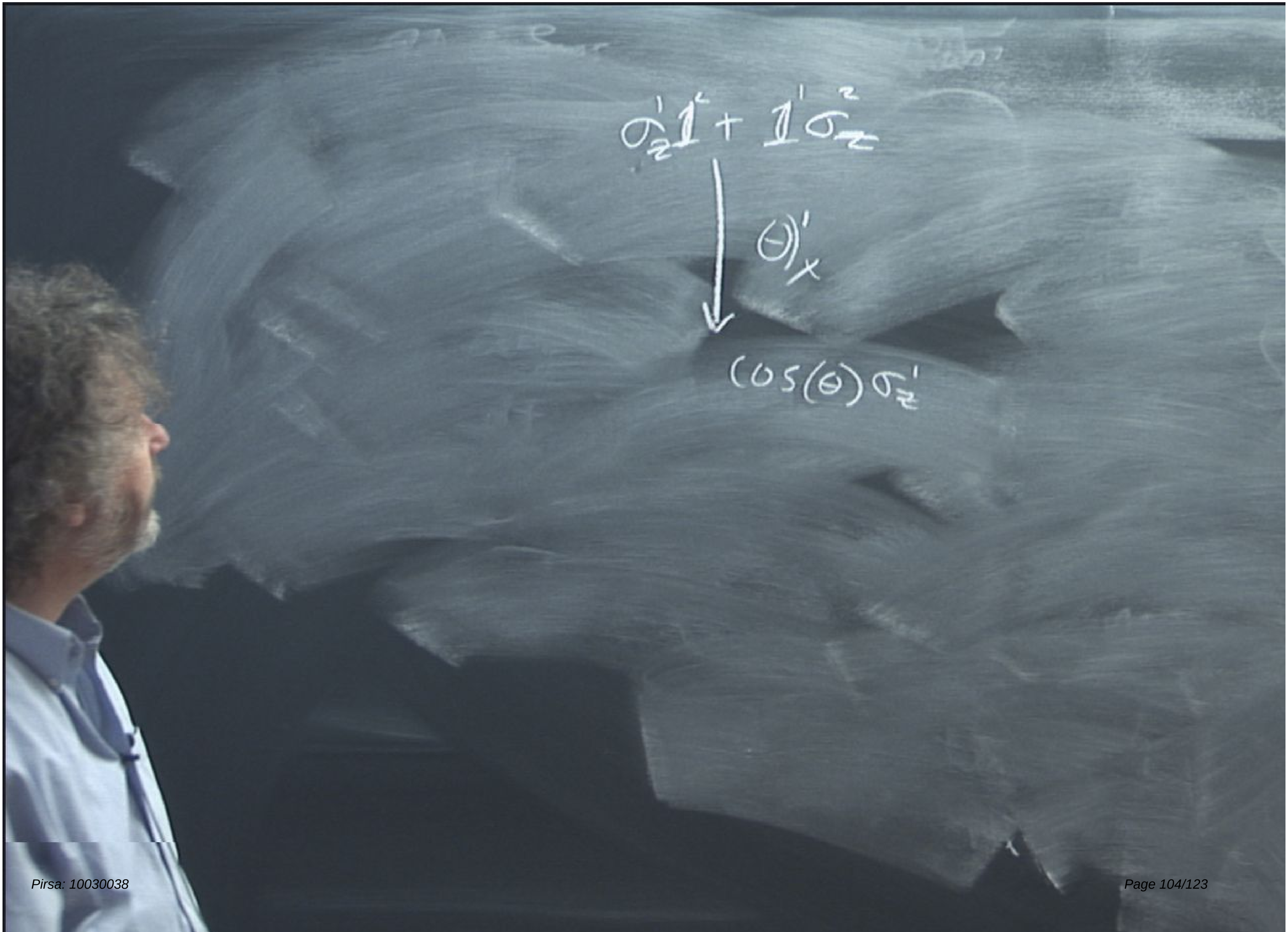
$$= \tfrac{1}{4}(\mathbb{1}\mathbb{1} + \underbrace{\sigma_z\mathbb{1} + \mathbb{1}\sigma_z + \sigma_z\sigma_z}_{\text{scale}})$$

the deviation density matrix is isomorphic to a pure state.

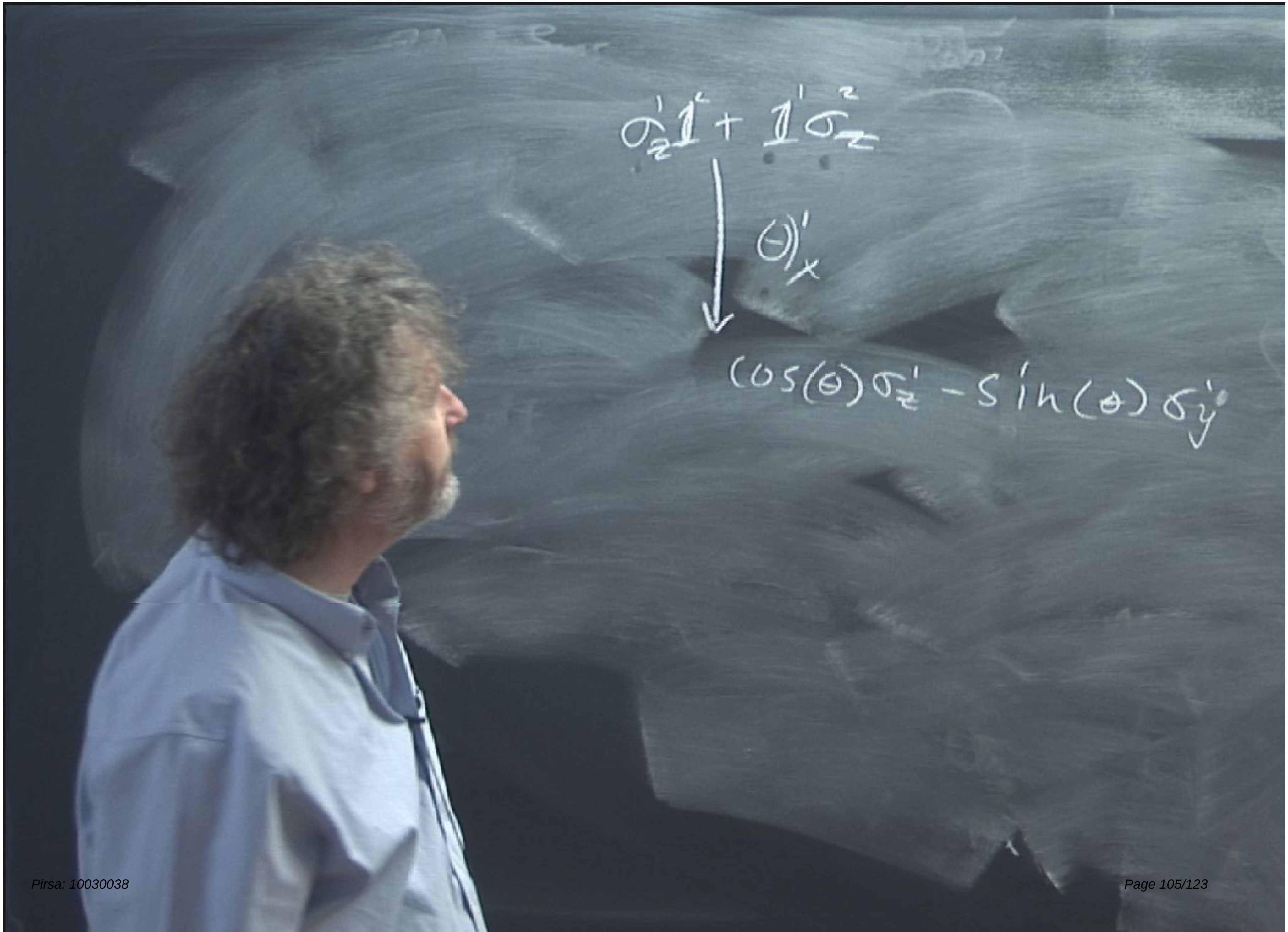
U_1

U_2

U_2



$$\begin{tikzcd}
\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2 \arrow[d, "\Theta_x^1"] \\
\cos(\theta)\sigma_z^1
\end{tikzcd}$$

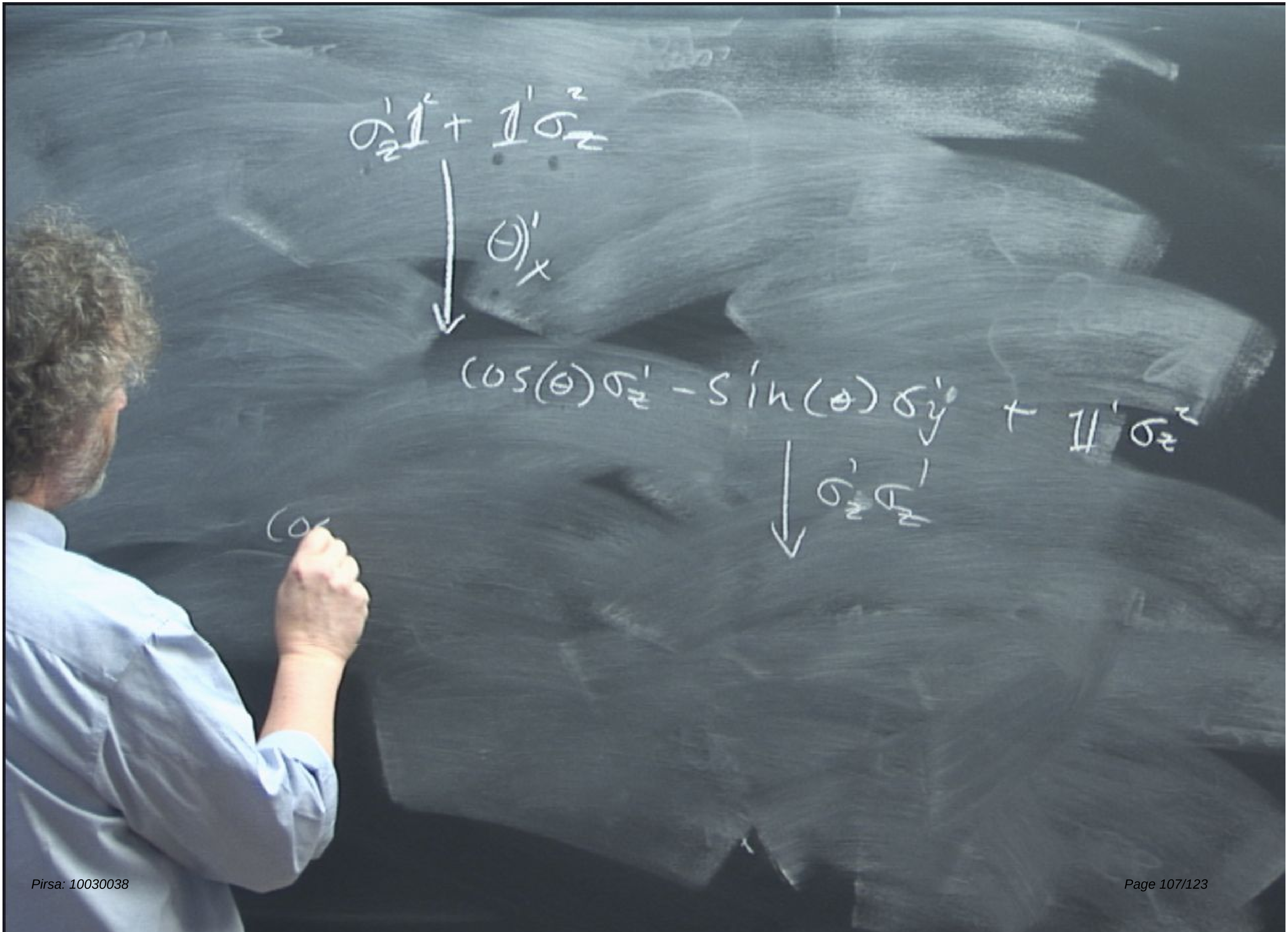


$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \ \Theta_x^1$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1$$

$$\begin{tikzcd} \sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2 \arrow[d, "\Theta_x^1"] \\ \cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1 \sigma_z^2 \end{tikzcd}$$



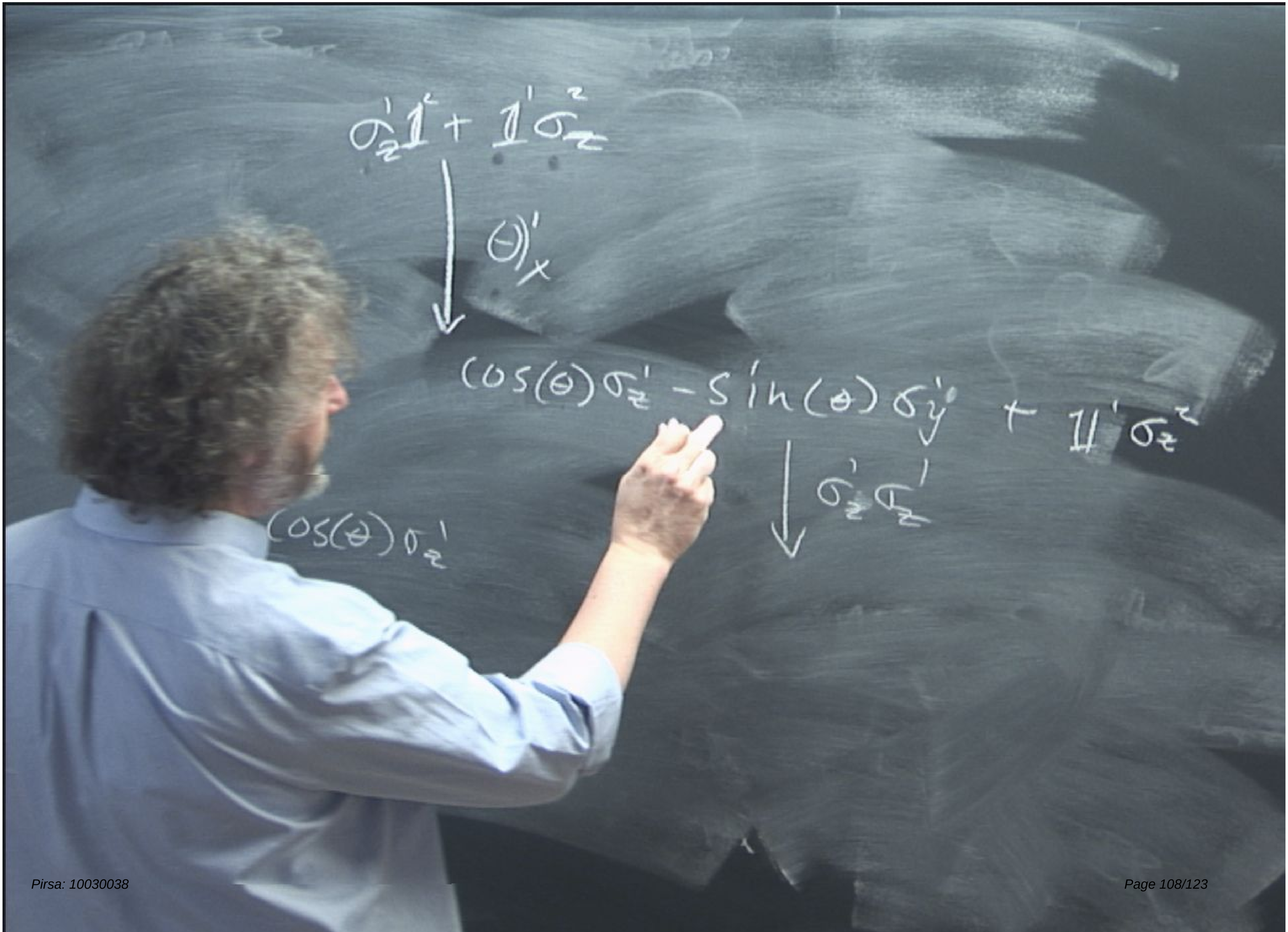
$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \; \Theta_x^1$$

$$\cos(\theta)\,\sigma_z^1 - \sin(\theta)\,\sigma_y^1 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \; \sigma_z^1 \sigma_z^1$$

co



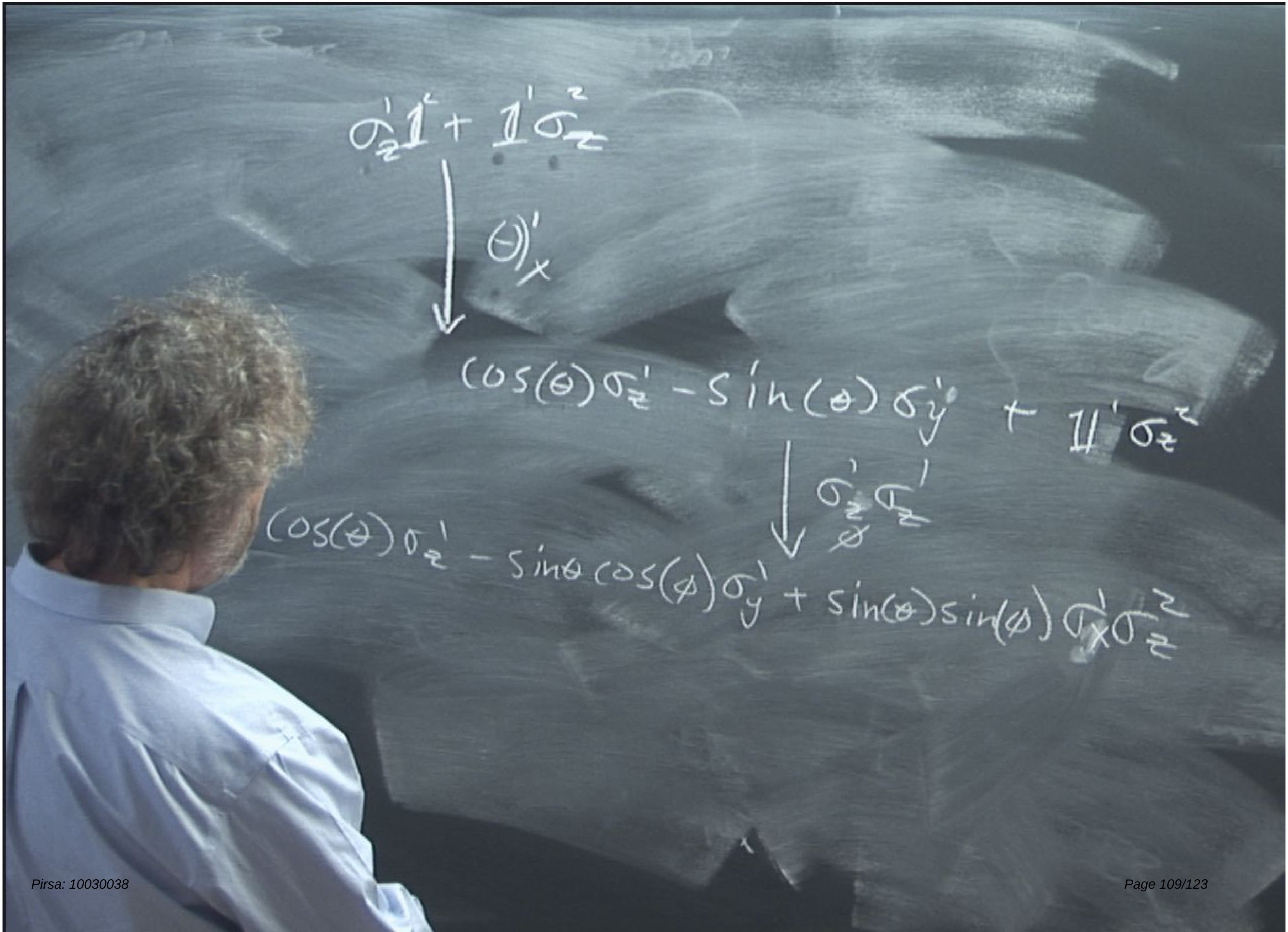
$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \; \Theta_x^1$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \; \sigma_z^1 \sigma_z^1$$

$$\cos(\theta)\sigma_z^1$$



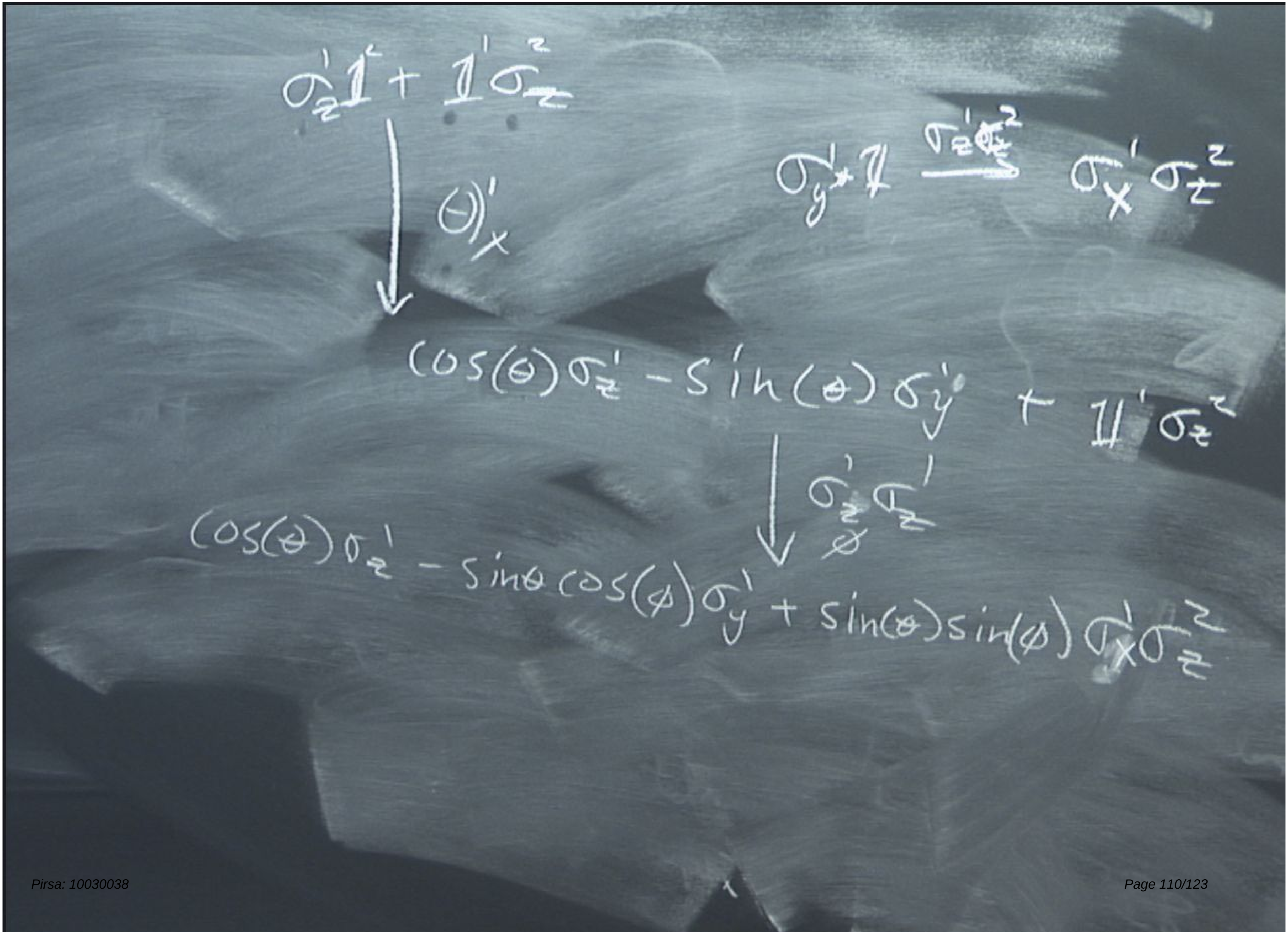
$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \; \Theta_x^1$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1\sigma_z^2$$

$$\Big\downarrow \; \overset{\sigma_z^1\sigma_z^1}{\phi}$$

$$\cos(\theta)\sigma_z^1 - \sin\theta\cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2$$



$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \; \Theta_x^1$$

$$\sigma_y^1 \mathbb{1} \qquad \underline{\sigma_z^1 \sigma_z^2} \qquad \sigma_x^1 \sigma_z^2$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \; \sigma_z^1 \sigma_z^1 \; \phi$$

$$\cos(\theta)\sigma_z^1 - \sin\theta\cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2$$

$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \, \Theta_x^1$$

$$\sigma_y^1 \mathbb{1} \xrightarrow{\sigma_z^1 \sigma_z^2} \sigma_x^1 \sigma_z^2$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \, \overset{\sigma_z^1 \sigma_z^1}{\phi}$$

$$\cos(\theta)\sigma_z^1 - \sin\theta\cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2 + \mathbb{1}^1\sigma_z^2$$

$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\sigma_y^1 \mathbb{1} \quad \underline{\sigma_z^1 \sigma_z^2} \quad \sigma_x^1 \sigma_z^2$$

$$\Big\downarrow \; \Theta_x^1$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \; \overset{\sigma_z^1 \sigma_z^1}{\phi}$$

$$\cos(\theta)\sigma_z^1 - \sin\theta\cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2 + \mathbb{1}^1\sigma_z^2$$

$$\Big\downarrow \; \alpha_y^1$$

$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\sigma_y^1 \mathbb{1} \quad \underline{\sigma_z^1 \sigma_z^2} \quad \sigma_x^1 \sigma_z^2$$

$$\Big\downarrow \; \Theta)_x^1$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1 \sigma_z^2$$

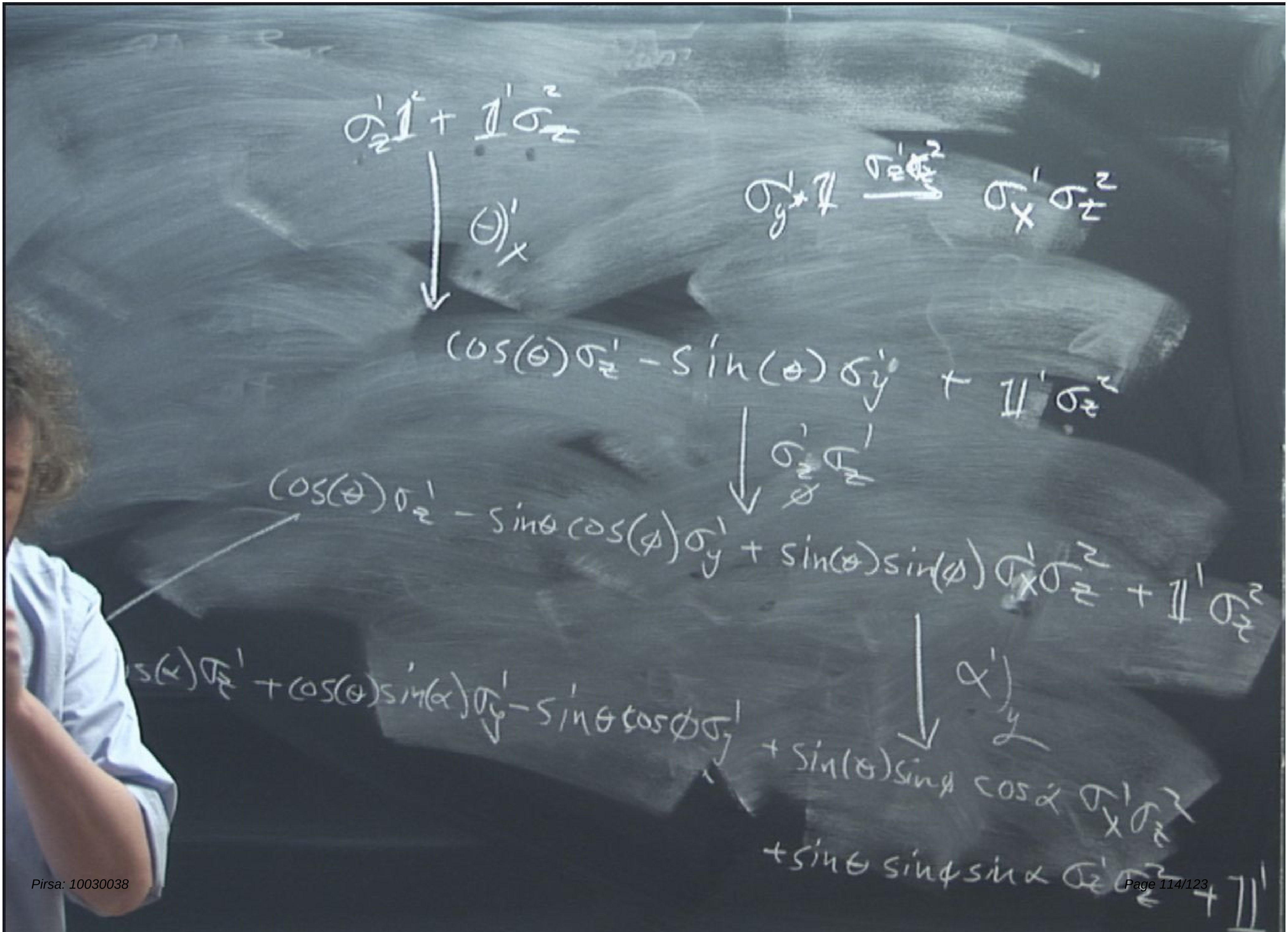
$$\Big\downarrow \; \underset{\phi}{\sigma_z^1 \sigma_z^1}$$

$$\cos(\theta)\sigma_z^1 - \sin\theta\cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2 + \mathbb{1}^1\sigma_z^2$$

$$\Big\downarrow \; \alpha^1)_y$$

$$\cos(\theta)\cos(\alpha)\sigma_z^1 + \cos(\theta)\sin(\alpha)\sigma_y^1 - \sin\theta\cos\phi\,\sigma_y^1 + \sin(\theta)\sin\phi\cos\alpha\,\sigma_x^1\sigma_z^2$$

$$+\sin\theta\sin\phi\sin$$



$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \ \Theta_x^1$$

$$\sigma_y^1 \mathbb{1} \qquad \underline{\sigma_z^1 \sigma_z^2} \qquad \sigma_x^1 \sigma_z^2$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow \ \underset{\phi}{\sigma_z^1} \sigma_z^1$$

$$\cos(\theta)\sigma_z^1 - \sin\theta \cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2 + \mathbb{1}^1\sigma_z^2$$

$$\Big\downarrow \ \alpha_y^1$$

$$s(\alpha)\sigma_z^1 + \cos(\theta)\sin(\alpha)\sigma_y^1 - \sin\theta\cos\phi\,\sigma_y^1 + \sin(\theta)\sin\phi\cos\alpha\,\sigma_x^1\sigma_z^2 + \sin\theta\sin\phi\sin\alpha\,\sigma_z^1\sigma_z^2 + \mathbb{1}^1$$

$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow\ \Theta_x^1$$

$$\sigma_y^1 \mathbb{1} \quad \underline{\sigma_z^1 \sigma_z^2} \quad \sigma_x^1 \sigma_z^2$$

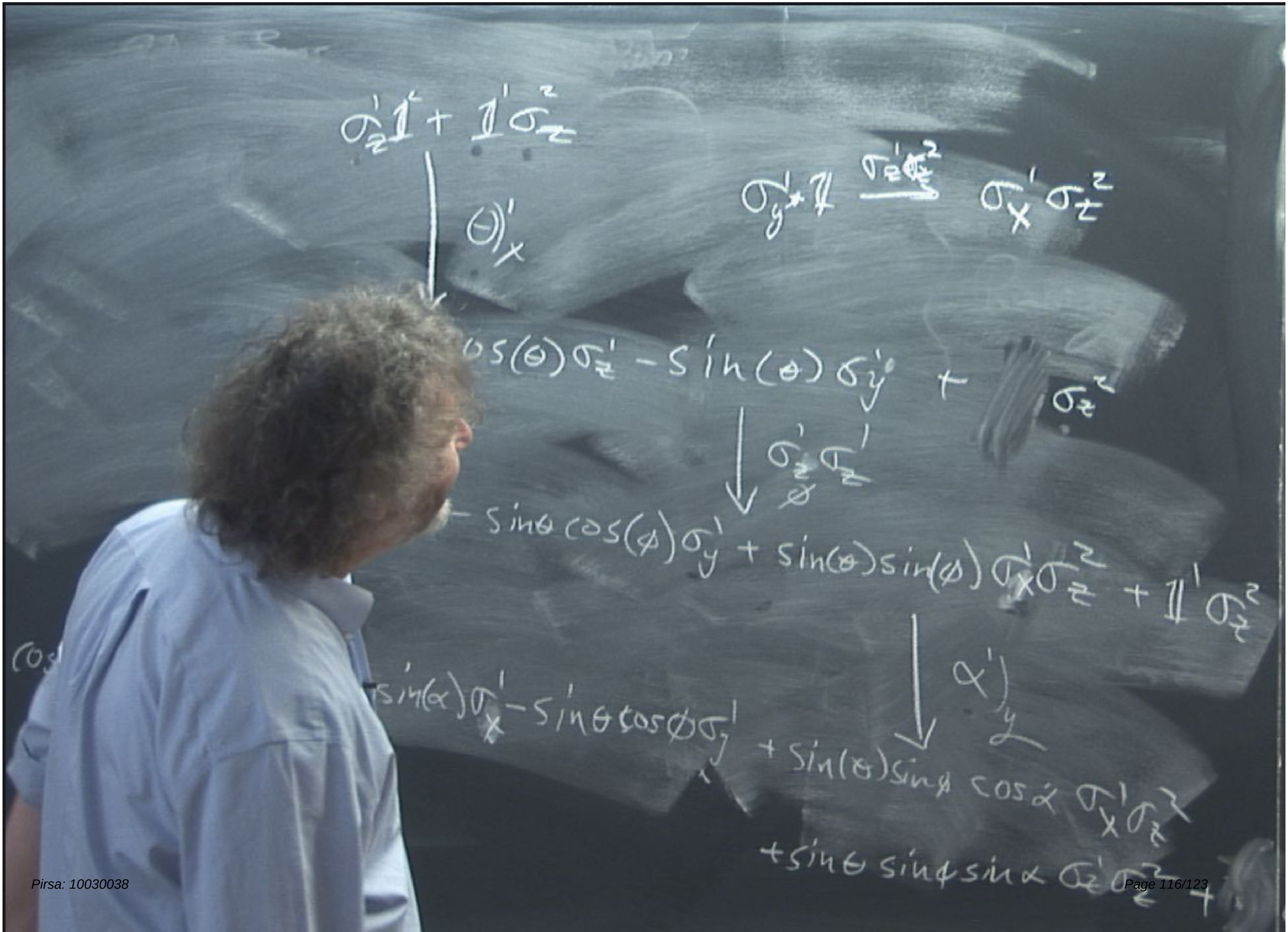
$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \mathbb{1}^1 \sigma_z^2$$

$$\Big\downarrow\ \sigma_z^1 \sigma_z^1 \ \phi$$

$$\cos(\theta)\sigma_z^1 - \sin\theta \cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2 + \mathbb{1}^1\sigma_z^2$$

$$\Big\downarrow\ \alpha_y^1$$

$$\cos(\theta)\cos(\alpha)\sigma_z^1 + \cos(\theta)\sin(\alpha)\sigma_y^1 - \sin\theta\cos\phi\,\sigma_y^1 + \sin(\theta)\sin\phi\cos\alpha\,\sigma_x^1\sigma_z^2 + \sin\theta\sin\phi\sin\alpha\,\sigma_z^1\sigma_z^2 + \mathbb{1}^1$$



$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$

$\Big\downarrow \; \Theta)^1_x$

$\sigma_y^1 \mathbb{1}$ $\quad \underline{\sigma_z^1 \sigma_z^2}$ $\quad \sigma_x^1 \sigma_z^2$

$os(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 \; + \; \sigma_z^2$

$\Big\downarrow \; \sigma_z^1 \sigma_z^1$

ϕ

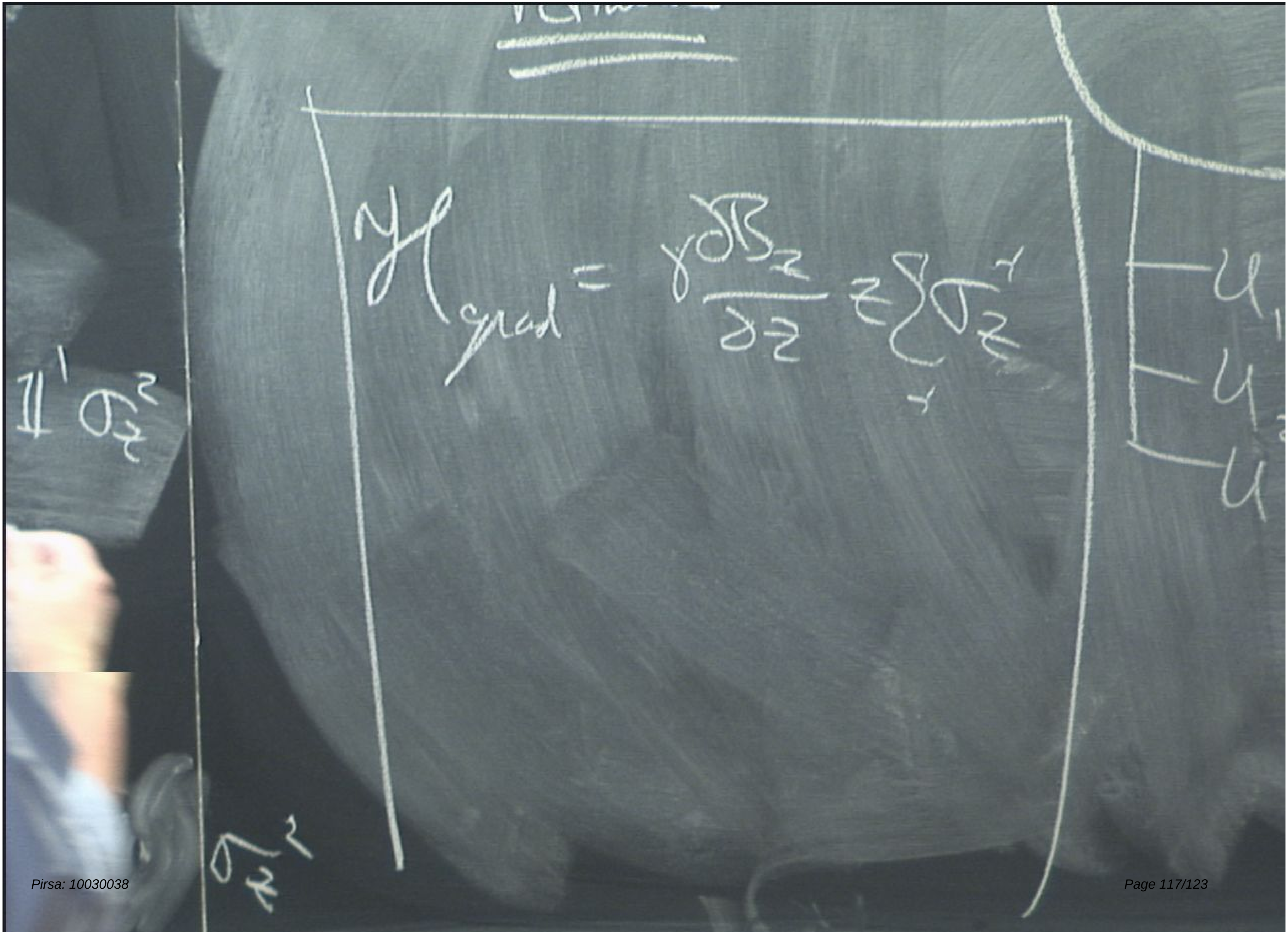
$- \sin\theta \cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\,\sigma_x^1 \sigma_z^2 + \mathbb{1}^1 \sigma_z^2$

$\Big\downarrow \; \alpha)^1_y$

$\cos$

$\sin(\alpha)\sigma_x^1 - \sin\theta\cos\phi\,\sigma_y^1 + \sin(\theta)\sin\phi\cos\alpha\,\sigma_x^1\sigma_z^2$

$+ \sin\theta\sin\phi\sin\alpha\,\sigma_z^1\sigma_z^2 +$



$$\mathcal{H}_{grad} = \gamma \frac{\partial B_z}{\partial z} z \sum_i \sigma_z^i$$

$\mathbb{1}^1 \sigma_z^2$

σ_z^1

U_1

U_2

U

$\mathbb{1}^1 \sigma_z^2$

$$\mathcal{H}_{grad} = \gamma \frac{\partial B_z}{\underset{\uparrow}{\partial z}} z \sum_r \sigma_z^r$$

u_1

u_2

u

σ_z^2

$$\cos(\theta)\sigma_z' - \sin(\theta)\sigma$$

$$\sigma_z' \sigma_z$$

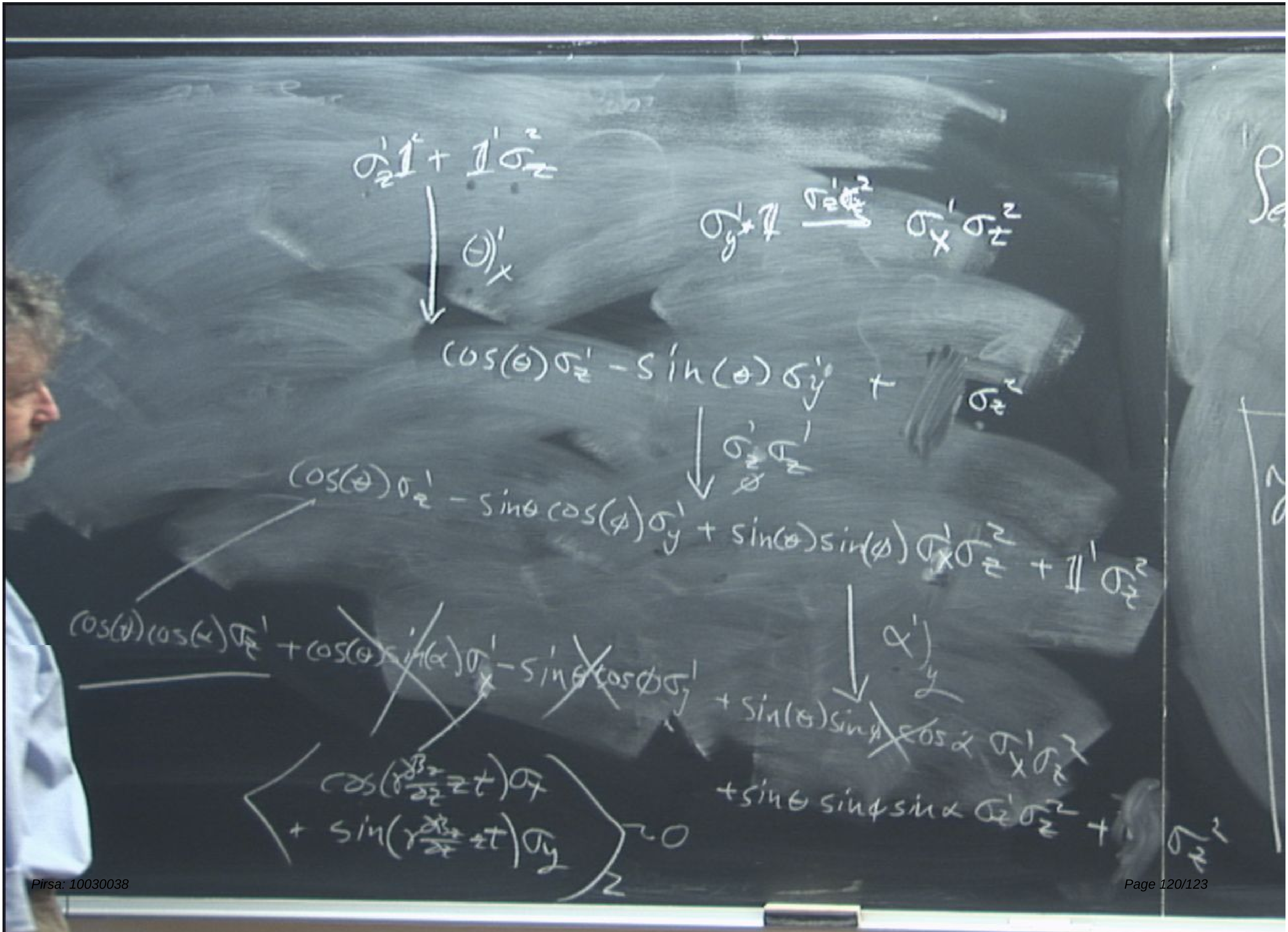
$$\phi$$

$$\cos(\theta)\sigma_z' - \sin\theta\cos(\phi)\sigma_y' + \sin(\theta$$

$$\cos(\theta)\cos(\alpha)\sigma_z' \cancel{+ \cos(\theta)\sin(\alpha)\sigma_x'} - \sin\theta\cos\phi\sigma_y' + \sin($$

$$+\sin\theta$$

$$\left\langle \cos\left(\gamma\frac{\partial B_z}{\partial z} z t\right)\sigma_x + \sin\left(\gamma\frac{\partial B_z}{\partial z} z t\right)\sigma_y \right\rangle_z \sim 0$$



$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\sigma_y^1 \mathbb{1} \quad \underline{\sigma_z^1 \sigma_z^2} \quad \sigma_x^1 \sigma_z^2$$

$$\Big\downarrow (\Theta)_x^1$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \sigma_z^2$$

$$\Big\downarrow \sigma_z^1 \sigma_z^1 \; \phi$$

$$\cos(\theta)\sigma_z^1 - \sin\theta\cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2 + \mathbb{1}^1\sigma_z^2$$

$$\Big\downarrow \alpha_y^1$$

$$\underline{\cos(\theta)\cos(\alpha)\sigma_z^1} + \cancel{\cos(\theta)\sin(\alpha)\sigma_x^1} - \sin\cancel{\theta\cos}\phi\sigma_y^1 + \sin(\theta)\sin\cancel{\phi\cos}\alpha\,\sigma_x^1\sigma_z^2 + \sin\theta\sin\phi\sin\alpha\,\sigma_z^1\sigma_z^2 + \sigma_z^2$$

$$\left\langle \cos\left(\gamma\frac{\partial B_z}{\partial z} z t\right)\sigma_x + \sin\left(\gamma\frac{\partial B_z}{\partial z} z t\right)\sigma_y \right\rangle_z \sim 0$$

$$\sigma_z^1 \mathbb{1}^2 + \mathbb{1}^1 \sigma_z^2$$

$$\sigma_y^1 \mathbb{1} \quad \underline{\sigma_z^1 \sigma_z^2} \quad \sigma_x^1 \sigma_z^2$$

$$\Big\downarrow \; \Theta_x^1$$

$$\cos(\theta)\sigma_z^1 - \sin(\theta)\sigma_y^1 + \sigma_z^2$$

$$\Big\downarrow \; \underset{\phi}{\sigma_z^1 \sigma_z^1}$$

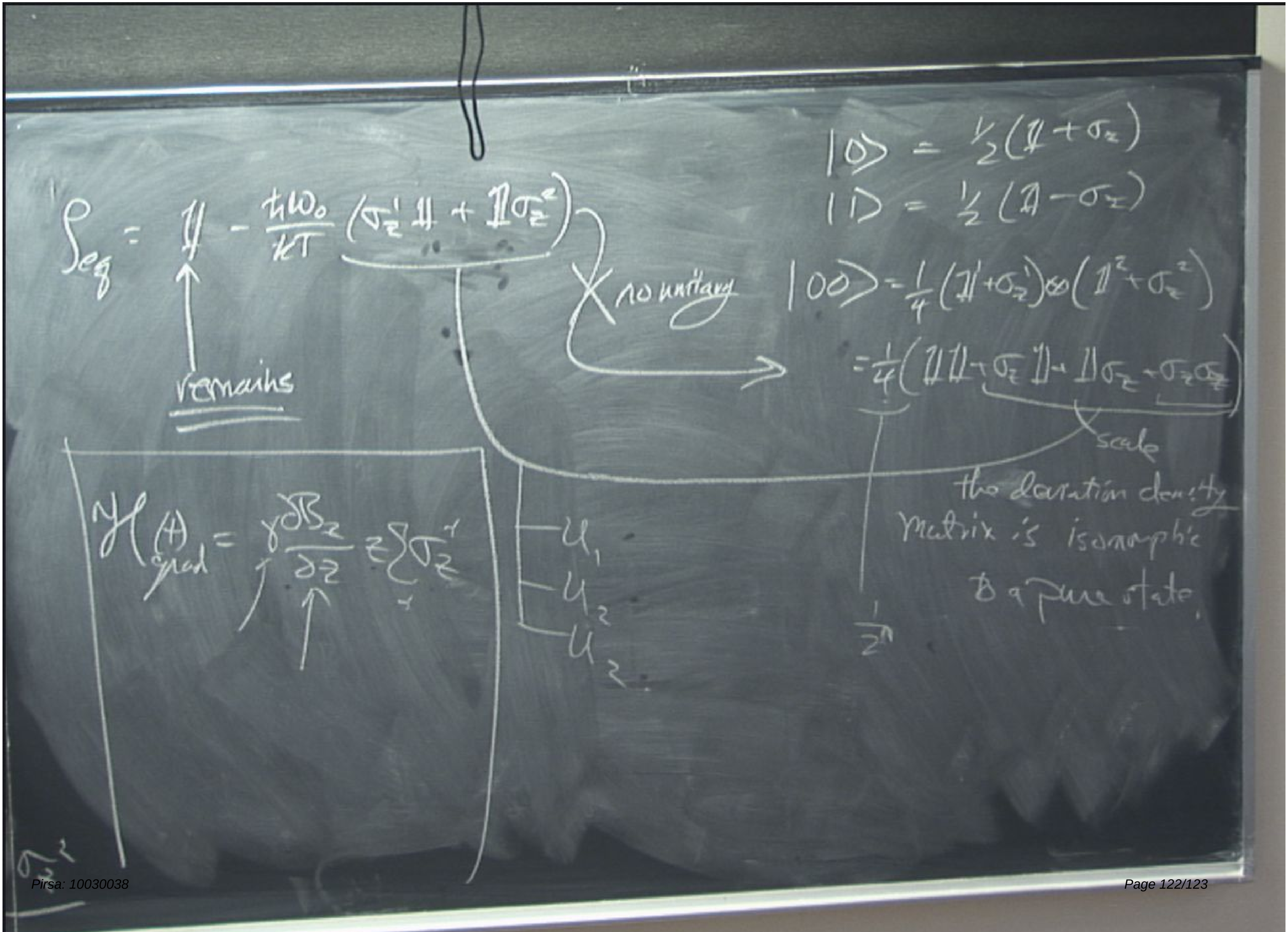
$$\cos(\theta)\sigma_z^1 - \sin\theta\cos(\phi)\sigma_y^1 + \sin(\theta)\sin(\phi)\sigma_x^1\sigma_z^2 + \mathbb{1}^1\sigma_z^2$$

$$\Big\downarrow \; \alpha_y^1$$

$$\underline{\cos(\theta)\cos(\alpha)\sigma_z^1} + \cancel{\cos(\theta)\sin(\alpha)\sigma_x^1} - \cancel{\sin\theta\cos\phi\,\sigma_y^1} + \sin(\theta)\sin\phi\,\cancel{\cos\alpha}\,\sigma_x^1\sigma_z^2$$

$$\underline{+\sin\theta\sin\phi\sin\alpha\,\sigma_z^1\sigma_z^2 + \sigma_z^2}$$

$$\left\langle \cos\left(\gamma\frac{\partial B_z}{\partial z} z t\right)\sigma_x + \sin\left(\gamma\frac{\partial B_z}{\partial z} z t\right)\sigma_y \right\rangle_z \sim 0$$



$$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\left(\sigma_z^1 \mathbb{1} + \mathbb{1}\sigma_z^2\right)$$

remains

no unitary

$$|0\rangle = \tfrac{1}{2}(\mathbb{1} + \sigma_z)$$

$$|1\rangle = \tfrac{1}{2}(\mathbb{1} - \sigma_z)$$

$$|00\rangle = \frac{1}{4}(\mathbb{1}^1 + \sigma_z^1) \otimes (\mathbb{1}^2 + \sigma_z^2)$$

$$= \frac{1}{4}\left(\mathbb{1}\mathbb{1} + \sigma_z \mathbb{1} + \mathbb{1}\sigma_z + \sigma_z\sigma_z\right)$$

scale

the deviation density matrix is isomorphic to a pure state.

$$\frac{1}{2^n}$$

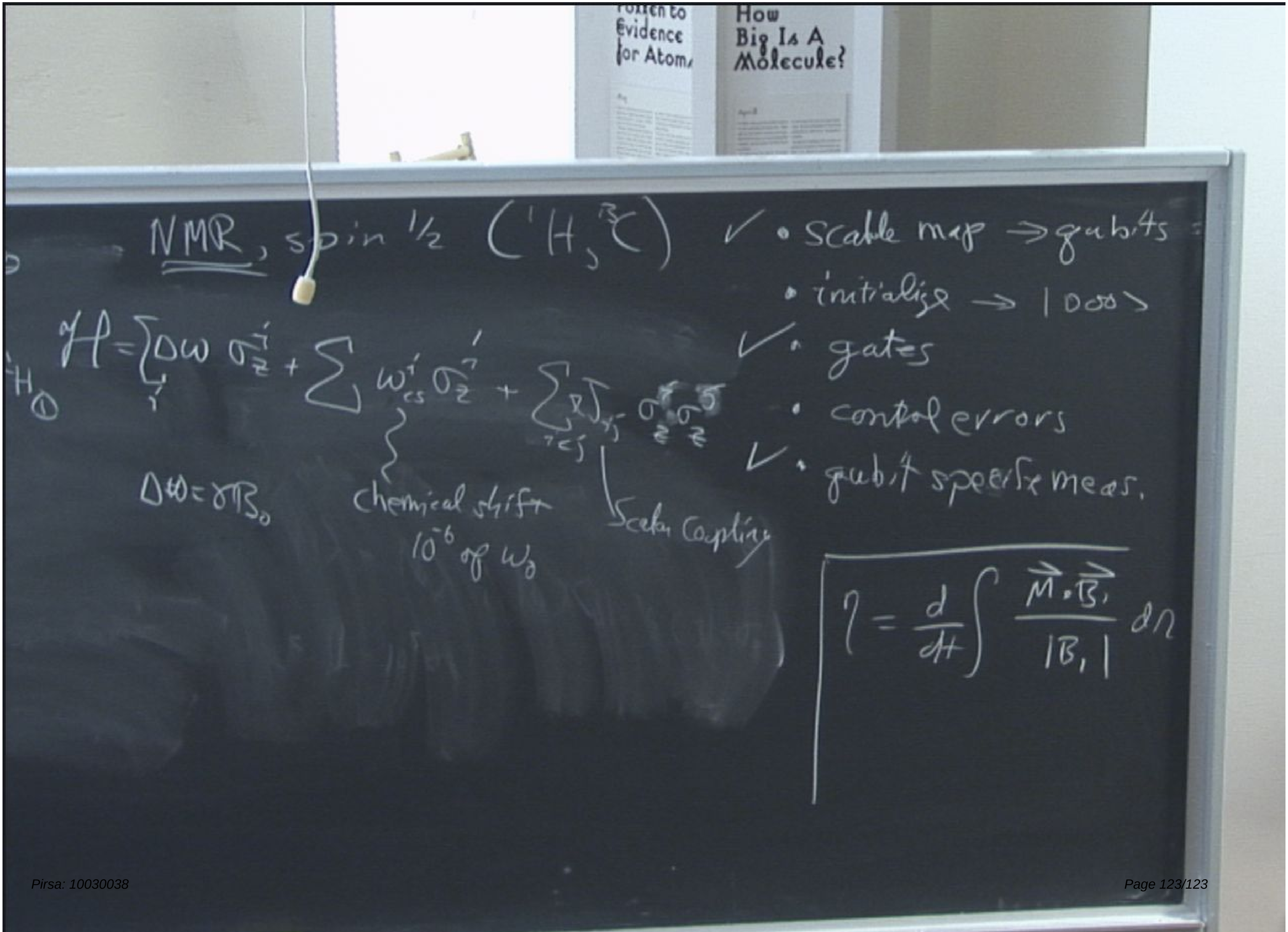
$$\mathcal{H}^{(A)}_{grad} = \gamma \frac{\partial B_z}{\partial z} z \sum_i \sigma_z^i$$

U_1

U_2

U_2

σ_z^2



NMR, spin 1/2 (^1H, ^{13}C)

✓ • scalable map ⇒ qubits

• initialize → $|000\rangle$

✓ • gates

• control errors

✓ • qubit specific meas.

H ①

$$\mathcal{H} = \sum_i \Delta\omega\, \sigma_z^i + \underbrace{\sum \omega_{cs}^i \sigma_z^i}_{\text{chemical shift } 10^{-6} \text{ of } \omega_0} + \underbrace{\sum_{i<j} 2J_{ij}\, \sigma_z^i \sigma_z^j}_{\text{Scalar Coupling}}$$

$\Delta\omega = \gamma B_0$

$$\boxed{\eta = \frac{d}{dt}\int \frac{\vec{M}\cdot\vec{B}_1}{|B_1|}\, d\Omega}$$