

Title: Resonant non-Gaussianity

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Abstract: Two of the most exciting observables in the cosmic microwave background (CMB) radiation, which could deeply impact our picture of the early universe, are non-Gaussianity and tensor modes. A potential detection of tensor modes can be explained in terms of a model of large field inflation. Theoretical considerations suggest that a symmetry should be invoked in order to protect the flatness of the inflaton potential and hence an axion enjoying a shift symmetry is a natural candidate. As main example, I will present a model of inflation in string theory based on axion monodromy. Non-perturbative effects typically correct the axion potential leading to small sinusoidal modulations on top of an otherwise flat slow roll potential. It can be shown analytically that a resonance between the oscillations of the background and the oscillations of the curvature fluctuations is responsible for the production of an observably large non-Gaussian signal. An explicit expression for the shape of this resonant non-Gaussianity will be presented. There is essentially no overlap between this shape and the local, equilateral, and orthogonal shapes, and in fact resonant non-Gaussianity is not captured by the simplest version of the effective field theory of inflation. Hopefully the analytic expression for resonant non-Gaussianity will be useful to further observationally constrain this class of models.

Resonant non-Gaussianity

based on

Flauger & E.P.

arXiv:1002.0833 (hep-th)

and

Flauger, McAllister, E.P., Westphal & Xu

arXiv:0907.2916 (hep-th)

Enrico Pajer

Cornell University, Ithaca

Perimeter

Mar 2010

Outline

- 1 Motivations
- 2 The model: inflation from axion monodromy
- 3 Non-Gaussianity in the bispectrum
- 4 Summary and conclusions

Cosmological data

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We are living in the **golden age of observational cosmology**: COBE goes to Stockholm, WMAP has measured the CMB with percent accuracy...



and now Planck: the satellite, launched on May 2009, finished the first full sky map (95%) on Feb 14th!



The picture emerging from the data

- Inflation does not solve the horizon and flatness problem but can arguably alleviate it. It provides a mechanism that shifts in the past the initial condition problem.



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- Inflation does not solve the horizon and flatness problem but can arguably alleviate it. It provides a mechanism that shifts in the past the initial condition problem.
- Nevertheless it is a spectacular model to generate cosmological perturbations.
- So far the simplest models of inflation is compatible with the data [see e.g. WMAP7] , i.e. small, scale-invariant but slightly red tilted, Gaussian, adiabatic primordial curvature perturbations.

Exciting signatures in the sky

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- **non-Gaussianity:**
 - Detectable e.g. in the three-point function of T perturbations. Current bounds are of the order a percent (shape dependent).
 - A detection would rule out the simplest class of models (a slowly rolling single canonically normalized field).

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- In a fundamental theory a flat potential over a superplanckian distance is hard to control, e.g. η -problem.
- This is the main motivation to consider axion monodromy inflation

Schematically

Tensor modes \Rightarrow High scale \Rightarrow Large field \Rightarrow more UV-sensitive

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- Within string theory and supergravity many models suffer from an η -problem.
- We need to invoke a symmetry, e.g. shift symmetry.

Axion monodromy

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- Invoke a shift symmetry on a “angular” field



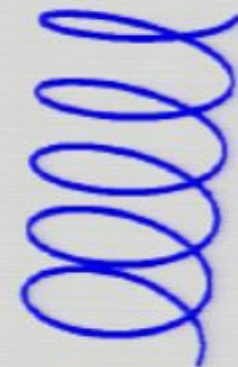
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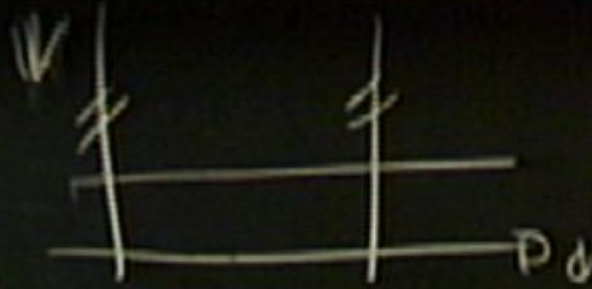
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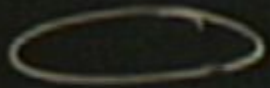
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- The symmetry is broken in a controlled way inducing a monodromy.

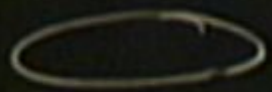




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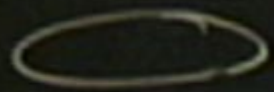


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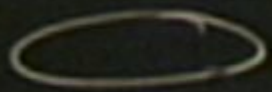


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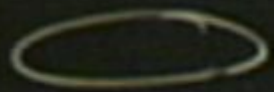


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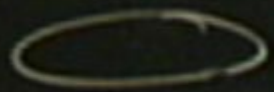
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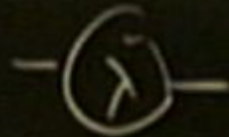


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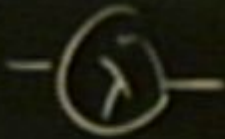


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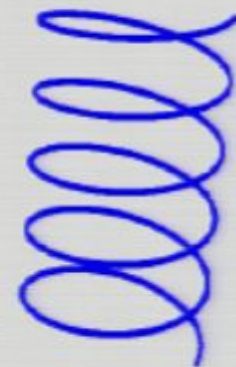
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- Invoke a shift symmetry on a “angular” field
- The symmetry is broken in a controlled way inducing a monodromy.
- This enlarges the field space and provides the potential for inflation.



Generically non-perturbative effects generate modulations.

Axions in field theory

- Axions are scalar fields with only derivative couplings and might arise e.g. from the breaking of a $U(1)$ symmetry [Peccei & Quinn 77]
- They enjoy a continuous shift symmetry at all orders in perturbation theory

$$\phi(x) \rightarrow \phi(x) + \text{constant}$$

Axion in string theory

String theory seen from a low energy 4D observer has in general many axions:

- Model independent axions, e.g. dualizing $B_{\mu\nu}$ or $C_{\mu\nu}$. Model dependent axions from integrating a p-form over a p-cycle of the compact manifold

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- The shift symmetry is broken non-perturbatively, e.g by world-sheet or brane instantons. **Periodic modulations.**

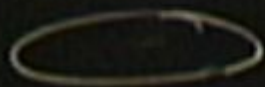
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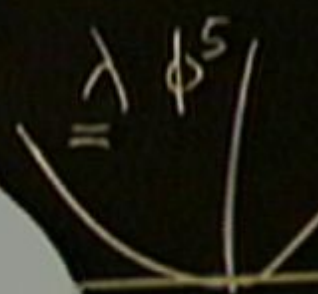
Modulation are generic

Flat potential \Rightarrow Shift symmetry \Rightarrow Axions \Rightarrow non-perturbative modulations



$$V = \text{const.} + m^3 \phi$$

$$= m^2 \phi^2$$



$$Q \sim \frac{\hbar^2}{m} \sim e^{-\frac{1}{g^2}}$$

$$\Delta V \sim \frac{\hbar^4}{m} \cos \frac{\phi}{g}$$



A cartoon of the model

We consider Type IIB (orientifolds) because moduli stabilization is more developed.

- In the $N = 1$, 4D effective theory there is an axion $c(x)$ coming from 10D C_2 integrated over a two-cycle Σ_2
- Wrapping a **5-brane over Σ_2** induces a potential for $c(x)$ (world-sheets with boundary).

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- If the **5-brane is a warped region**, the potential leads to viable inflation (COBE normalization)
- The moduli stabilization á la KKLT does not spoil the shift symmetry.

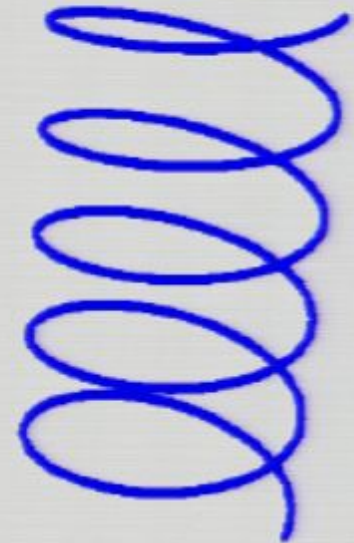
Linear potential for the inflaton

The shift symmetry can be broken in the presence of boundaries.

Consider a D5-brane wrapped on a two-cycle Σ .

The DBI action

$$-T_5 \int d^6 x e^{-\Phi} \sqrt{\det (G^{ind} + B^{ind})}$$



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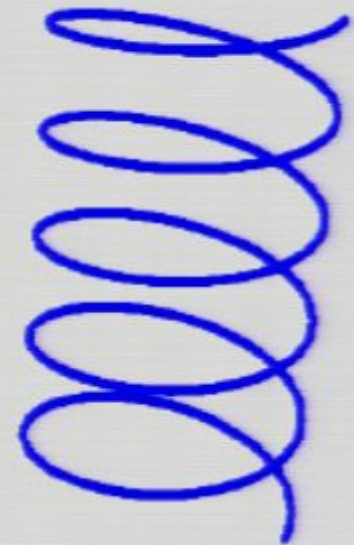
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The shift $a(x) \rightarrow a(x) + \text{const}$ of $a(x) = \int_{\Sigma} B_2$ stores some potential energy.

$$V(b) = T_5 \sqrt{L^4 + a^2} \sim T_5 a \quad \text{for large } a$$

This generates the **linear inflaton potential** (and break SUSY). COBE normalization and control require to red-shift T_5





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$m \rightarrow 0$

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$$\cos \frac{\phi}{f}$$



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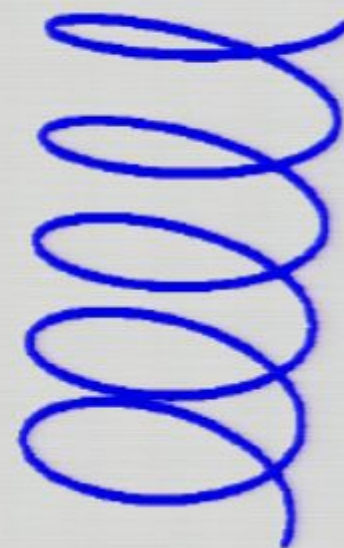
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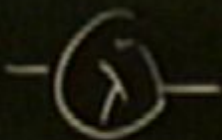


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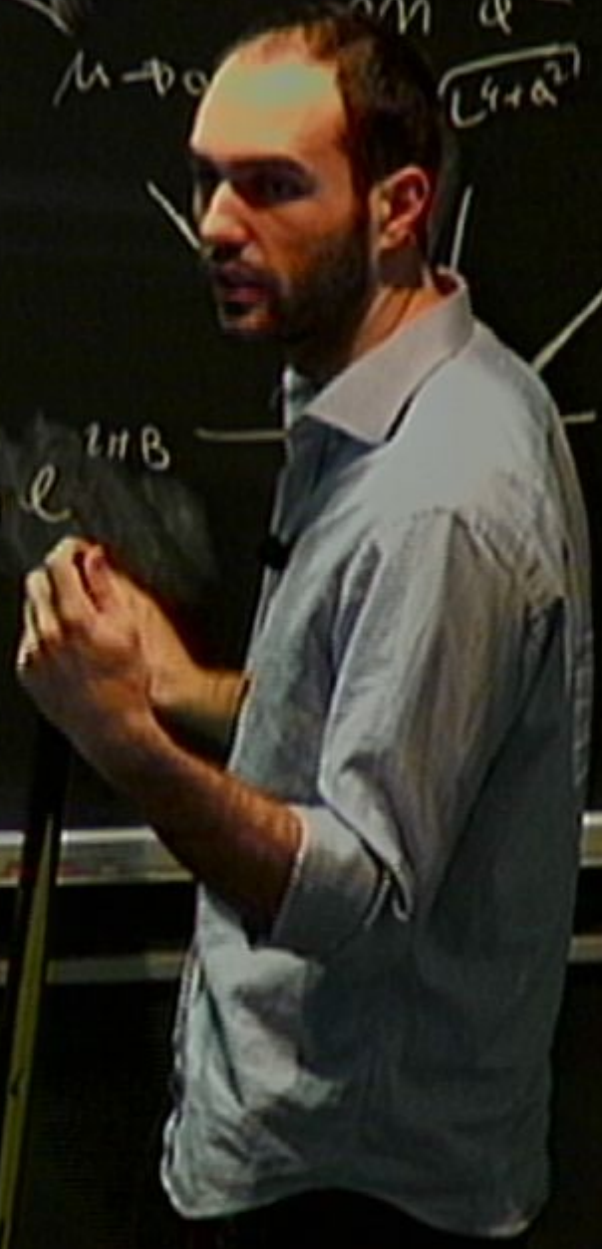
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05

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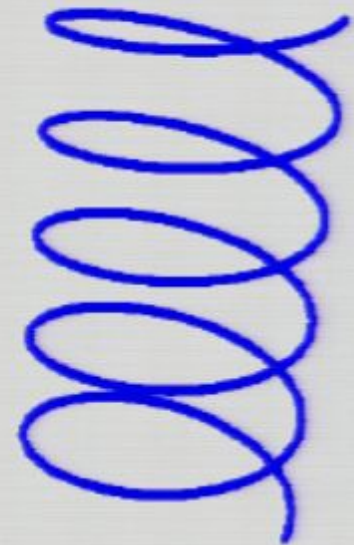
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The effective potential

Inflation is driven by a real scalar field with potential

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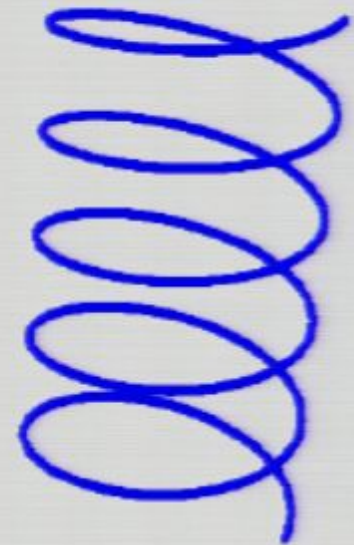
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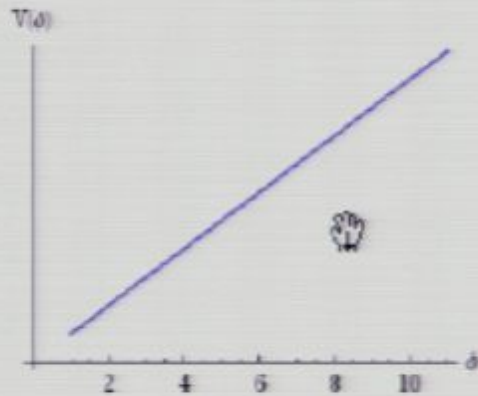


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- $f \ll M_{pl}$ many short ripples. Different from the superplanckian case that seems to be hard to achieve in string theory.

[Banks et al. 03]

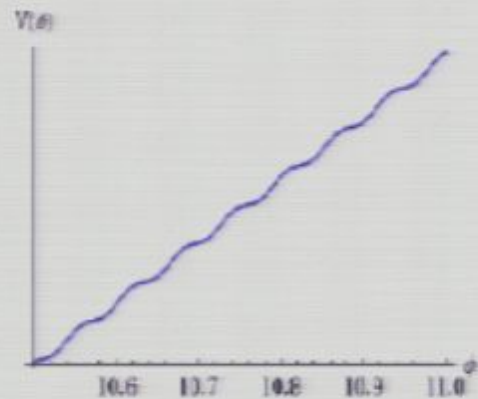
Background evolution

We solve the e.o.m. perturbatively in b :



zeroth order

$$\phi_0 = \left(\phi_{\text{in}}^{3/2} - \frac{\sqrt{3}}{2} \mu^{3/2} t \right)^{2/3}$$



first order

$$\phi_1 \simeq -3bf^2 \phi_0 \sin\left(\frac{\phi_0}{f}\right)$$

Background oscillations

The (Hubble) slow-roll parameters oscillate

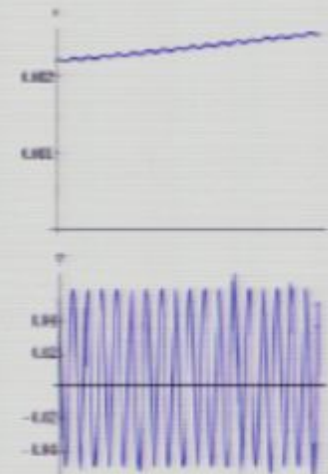
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$$\eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \simeq \eta_0 + \eta_{osci} \sin\left(\frac{\phi_0}{f}\right)$$

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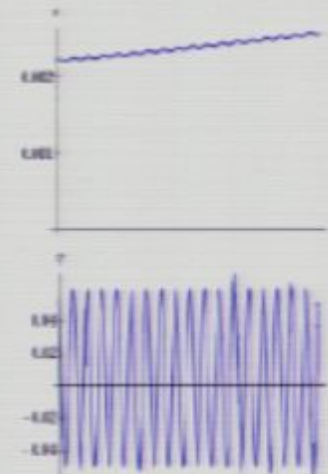
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Notice that $\dot{\eta} \gg \epsilon$ so **one can not use slow-roll formulae to compute the perturbations.**



Spectrum of scalar perturbations

The oscillations in the potential induce **oscillations in the spectrum**:

$$P_s(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1} \left[1 + \delta n_s \cos \left(\frac{\phi_k}{f} \right) \right]$$

$$\Rightarrow A_s \left(\frac{k}{k_*} \right)^{n_s-1 + \frac{\delta n_s}{\ln(k/k_*)} \cos \left(\frac{\phi_k}{f} \right)}$$

$$\cos\left(\frac{\psi}{\gamma}\right) =$$

$$AV \sim \cos \frac{\psi}{\gamma}$$

$2\pi B$
 e
 $B 2\pi$

$$\cos\left(\frac{\phi_k}{T_s}\right) = \cos\left(\frac{\cos \phi_k}{f_s}\right)$$

$$AV \sim \sum_n \cos\left(\frac{\phi_n}{T_s}\right) e^{-\gamma n}$$

05

$$\sum_n \int C_n e^{i n B}$$

$$\int C_n \Lambda B 2\pi$$

Cosmological perturbation theory

- During single-field inflation, there is a single scalar degree of freedom after fixing the gauge (diffeomorphisms).
- We choose to have in the metric, i.e. the uniform density gauge $\delta\phi = 0$ and we are left with $\zeta(x)$ in

$$g_{ij} = a^2 \delta_{ij} [1 + 2\zeta(x)]$$

- The background inflationary solution is quasi de Sitter, i.e.

$$a(t) = a_0 e^{tH}$$

with slowly changing Hubble parameter.

- The action expanded to quadratic order around this classical background leads to the Mukhanov-Sasaki equation for **the curvature perturbation $\zeta(x)$**

Cosmological perturbation theory

- During single-field inflation, there is a single scalar degree of freedom after fixing the gauge (diffeomorphisms).
- We choose to have in the metric, i.e. the uniform density gauge $\delta\phi = 0$ and we are left with $\zeta(x)$ in

$$g_{ij} = a^2 \delta_{ij} [1 + 2\zeta(x)]$$

- The background inflationary solution is quasi de Sitter, i.e.

$$a(t) = a_0 e^{tH}$$

with slowly changing Hubble parameter.

- The action expanded to quadratic order around this classical background leads to the Mukhanov-Sasaki equation for **the curvature perturbation $\zeta(x)$**

The Mukhanov-Sasaki equation

Mukhanov-Sasaki equation leads to the power spectrum

- Slow roll is not enough because ϵ and δ are **not** approximatively constant, in fact oscillate fast.

$$\frac{d^2 \zeta_k}{dx^2} - \frac{2(1 + 2\epsilon + \delta)}{x} \frac{d\zeta_k}{dx} + \zeta_k = 0,$$

where $x \equiv -k\tau$ and τ is the conformal time $a d\tau \equiv dt$

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$$\zeta_k(x) \simeq i \sqrt{\frac{\pi}{2}} x^{3/2} H_{3/2}^{(1)}(x) - c_k^- i \sqrt{\frac{\pi}{2}} x^{3/2} H_{3/2}^{(2)}(x),$$

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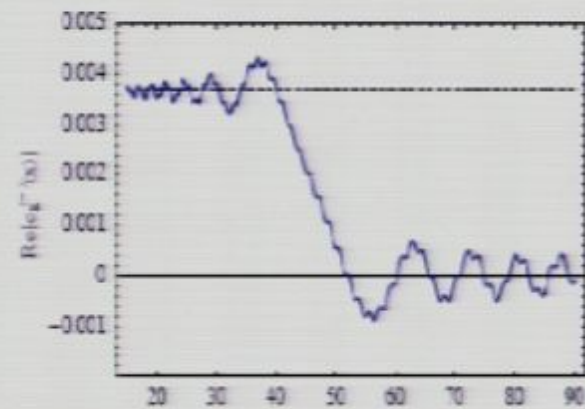
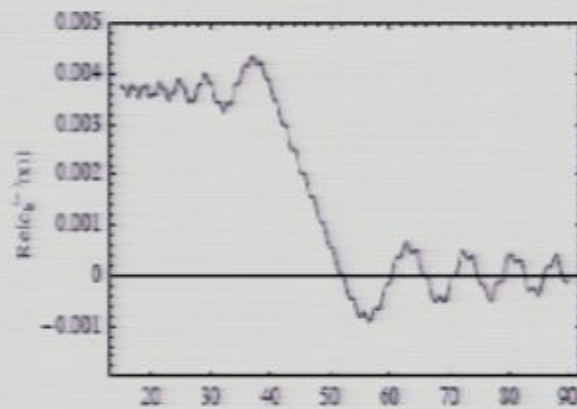
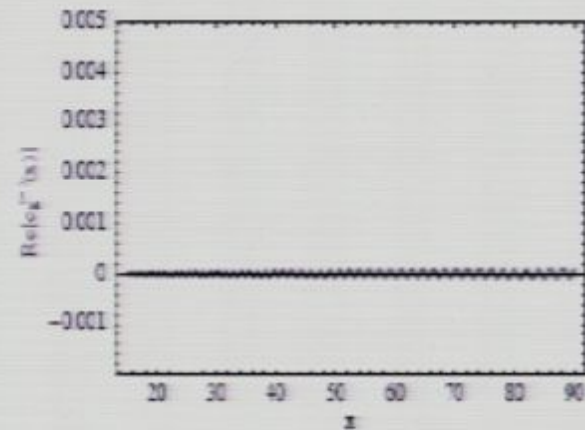
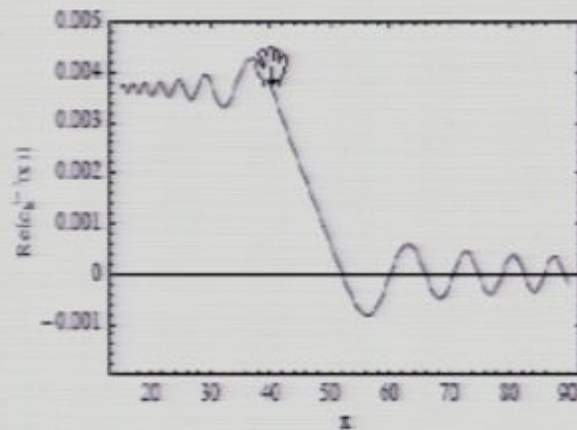
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The effect of the resonance on the modes

- The equation for $c_k^-(x)$ can be solved exactly

$$c_k^-(x) \sim bf\phi_* e^{-i\frac{\log k}{f}} \Gamma\left[1 - \frac{i}{f\phi_*}, -2ix\right]$$

- the resonance is seen numerically and analytically



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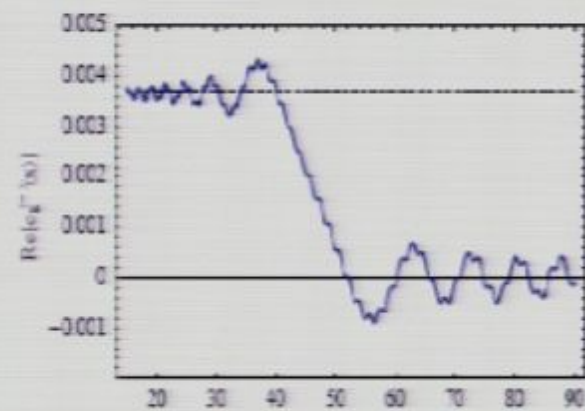
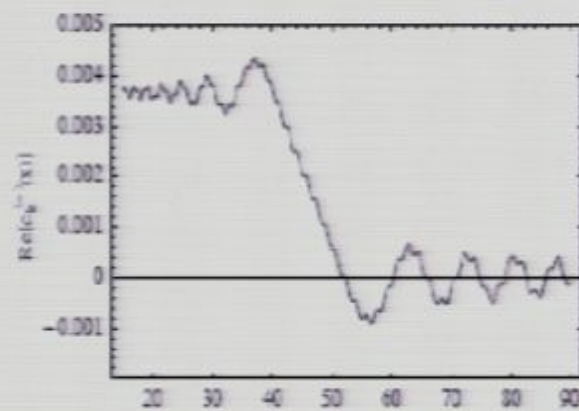
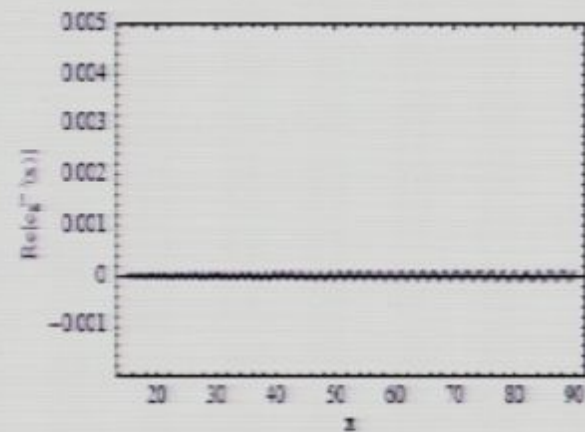
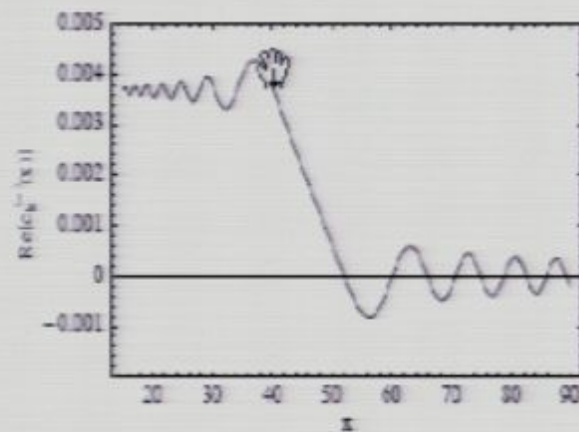


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$$\cos\left(\frac{\phi k}{g}\right) = \cos\left(\frac{\log k}{g}\right)$$

$$e^{ix}$$

$$\begin{aligned} \phi &\gg M_p \\ g &\ll M_p \end{aligned}$$

$$\Delta V \sim \uparrow \cos \frac{\phi}{g}$$

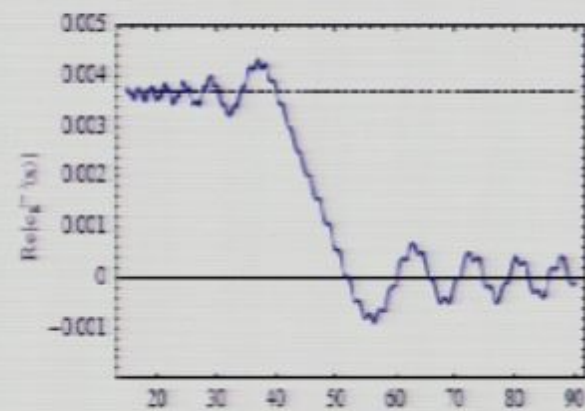
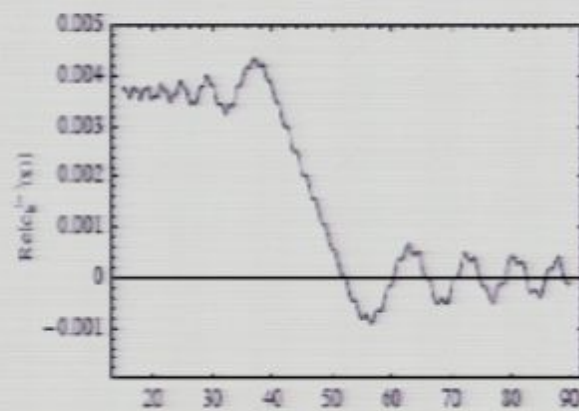
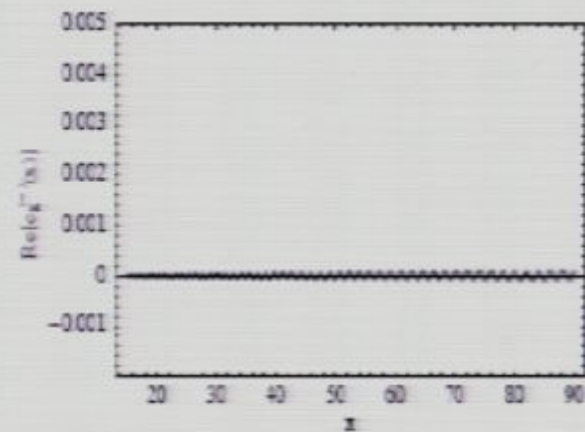
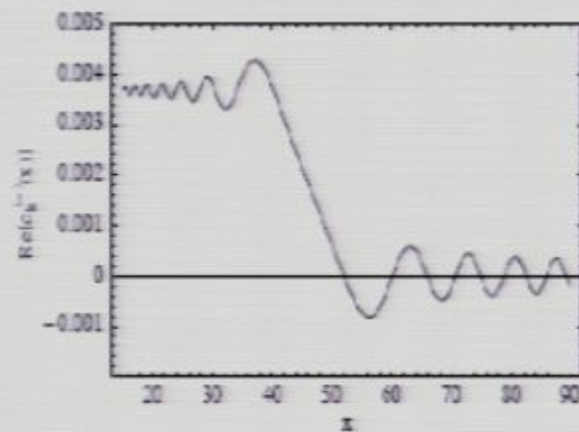


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$$c_k^-(x) \sim bf\phi_* e^{-i\frac{\log k}{f}} \Gamma \left[1 - \frac{i}{f\phi_*}, -2ix \right]$$

- the resonance is seen numerically and analytically



$$e^{ix} \quad \cos \frac{\log x}{g}$$

$$\cos\left(\frac{\phi x}{g}\right) = \cos\left(\frac{\log k}{g}\right)$$

$$e^{ix}$$

$$\begin{aligned} \phi > \pi_p \\ g < \pi_p \end{aligned}$$

$$\sqrt{g}$$

2π

$$e^{ix} = \cos\left(\frac{\log x}{\sqrt{8}}\right)$$

$$\cos\left(\frac{\phi \kappa}{\sqrt{8}}\right) = \cos\left(\frac{\log \kappa}{\sqrt{8}}\right) \quad M_p = 1$$

$$e^{ix}$$

$$\phi > 1$$

$$\sqrt{8\phi}$$

$$\int C_4 \wedge B \text{ zu}$$

$$e^{ix + \frac{\log x}{89}}$$

$$\int dx e^{ix} \cos\left(\frac{\log x}{84}\right)$$

$$\cos\left(\frac{\phi x}{8}\right) = \cos\left(\frac{\log x}{8}\right) \quad M_p = 1$$

$$e^{ix}$$

$$\begin{aligned} \phi &> \pi_p \\ 8 &< \pi_p \end{aligned}$$

$$e^{nix + \frac{\log x}{\delta \eta}}$$

$$\int dx e^{nix} \cos\left(\frac{\log x}{\delta \eta}\right)$$

$$X_{\text{res}} = \frac{1}{2\delta\eta}$$

$$\cos\left(\frac{\phi k}{\delta}\right) = \cos\left(\frac{\log k}{\delta}\right) \quad M_p = 1$$

$$\begin{aligned} \phi &> \pi_p \\ \delta &< \pi_p \end{aligned}$$

$$\sqrt{\delta \phi}$$

$$\int \mathcal{L}_4 \wedge B_{2\pi}$$

$$e^{nix + \frac{\log x}{\gamma}}$$

$$\int dx e^{nix} \cos\left(\frac{\log x}{\gamma}\right)$$

$$X_{\text{res}} = \frac{1}{2\gamma\phi}$$

$$\cos\left(\frac{\phi k}{\gamma}\right) = \cos\left(\frac{\log k}{\gamma}\right) \quad M_p = 1$$

$$\begin{aligned} \phi &> \pi_p \\ \gamma &< \pi_p \end{aligned}$$

$$\sqrt{\gamma\phi}$$

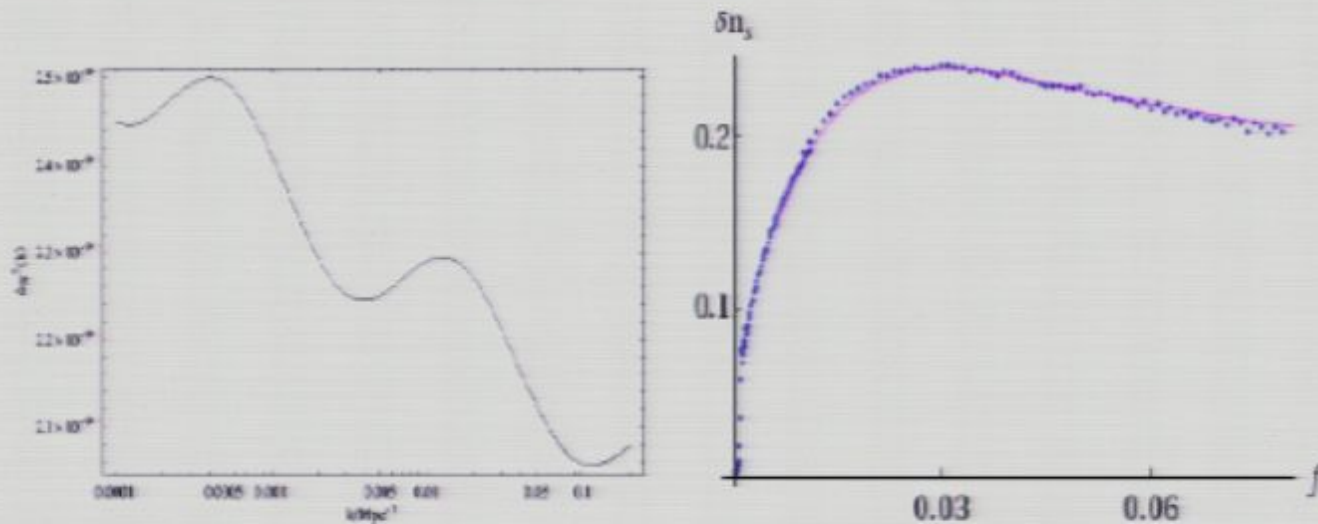
$$\int C_4 \Lambda B 2\pi$$

Solution of the Mukhanov-Sasaki equation

The amplitude of oscillations in the spectrum is then

$$\delta n_s = -12b \sqrt{\frac{\frac{\pi}{8} \coth\left(\frac{\pi}{2f\phi}\right) f\phi}{(1 + (3f\phi)^2)}} \sim 3b\sqrt{2\pi f\phi},$$

valid for $\phi \gg M_{pl}$, $f \ll M_{pl}$ and $b \ll 1$. Excellent agreement with the numerics.

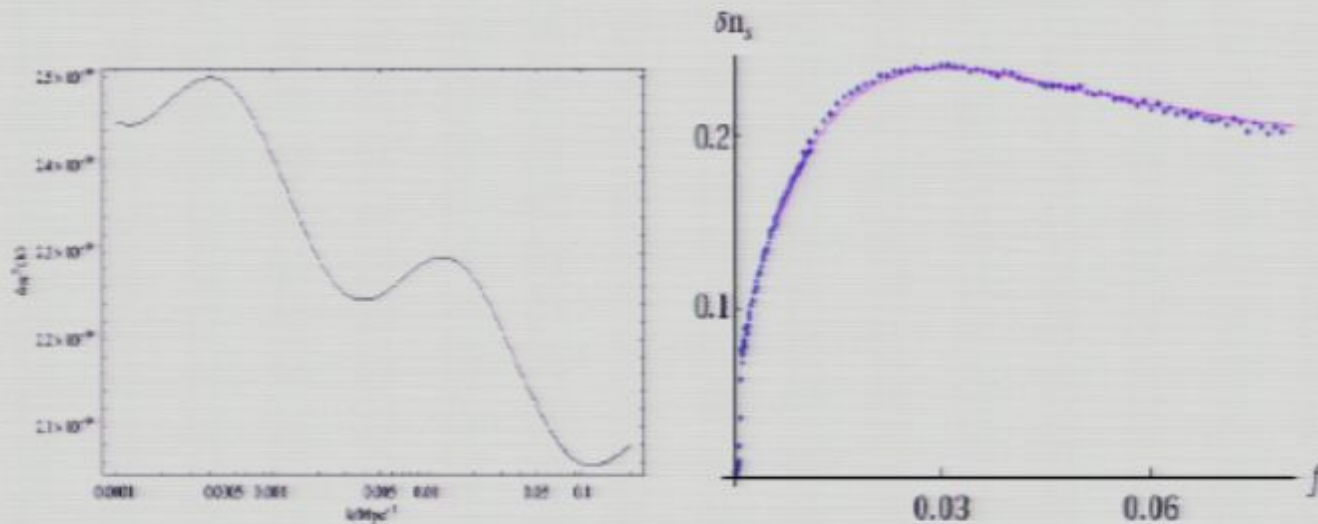


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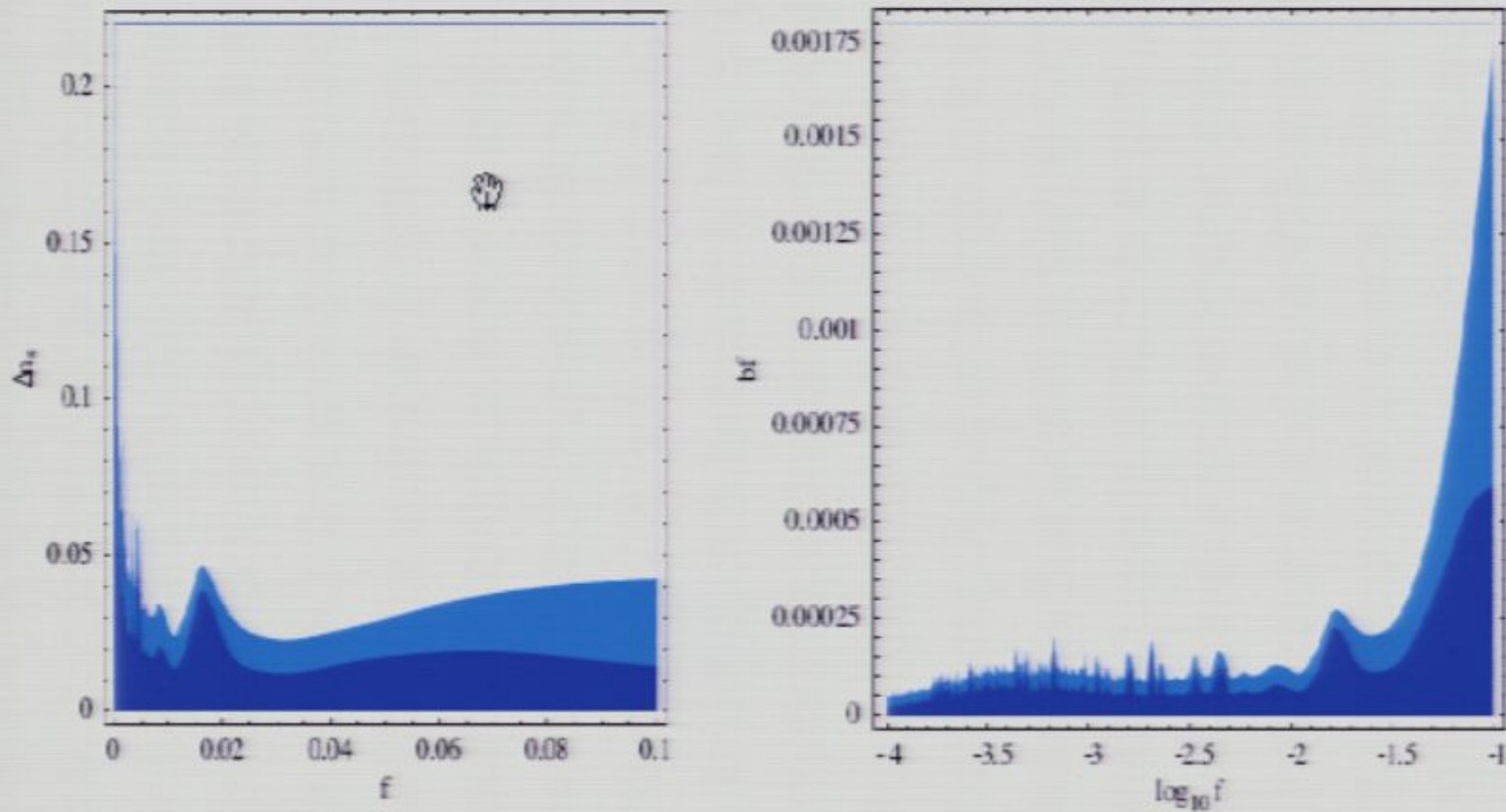
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Observational constraints on the spectrum

We have evolved this signal with CAMB and compared it with WMAP5. The one- and two-sigma exclusion contours are

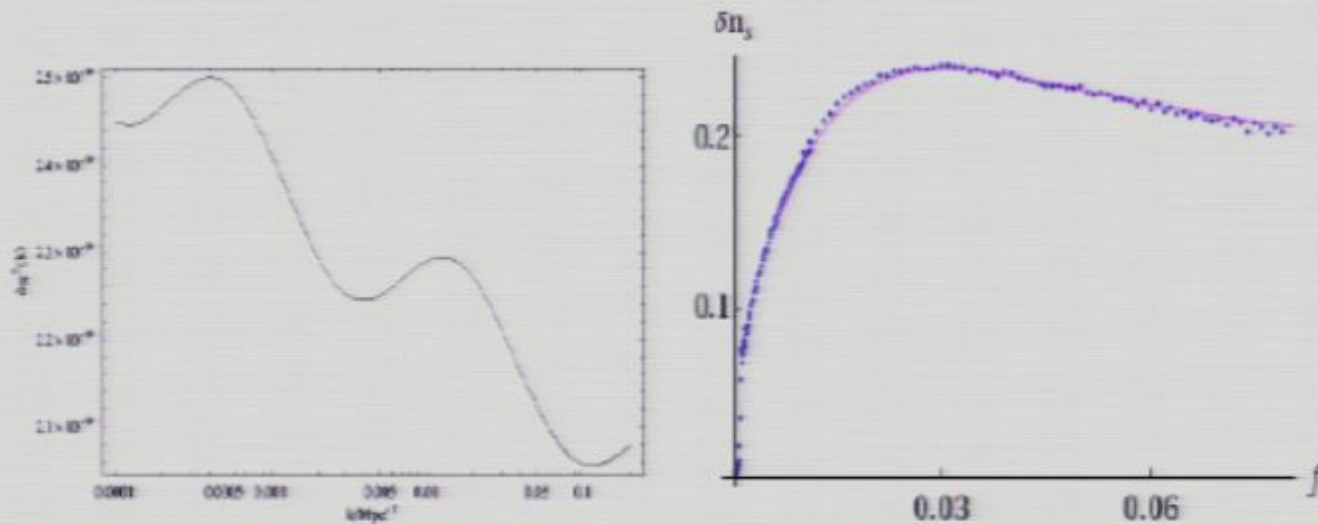


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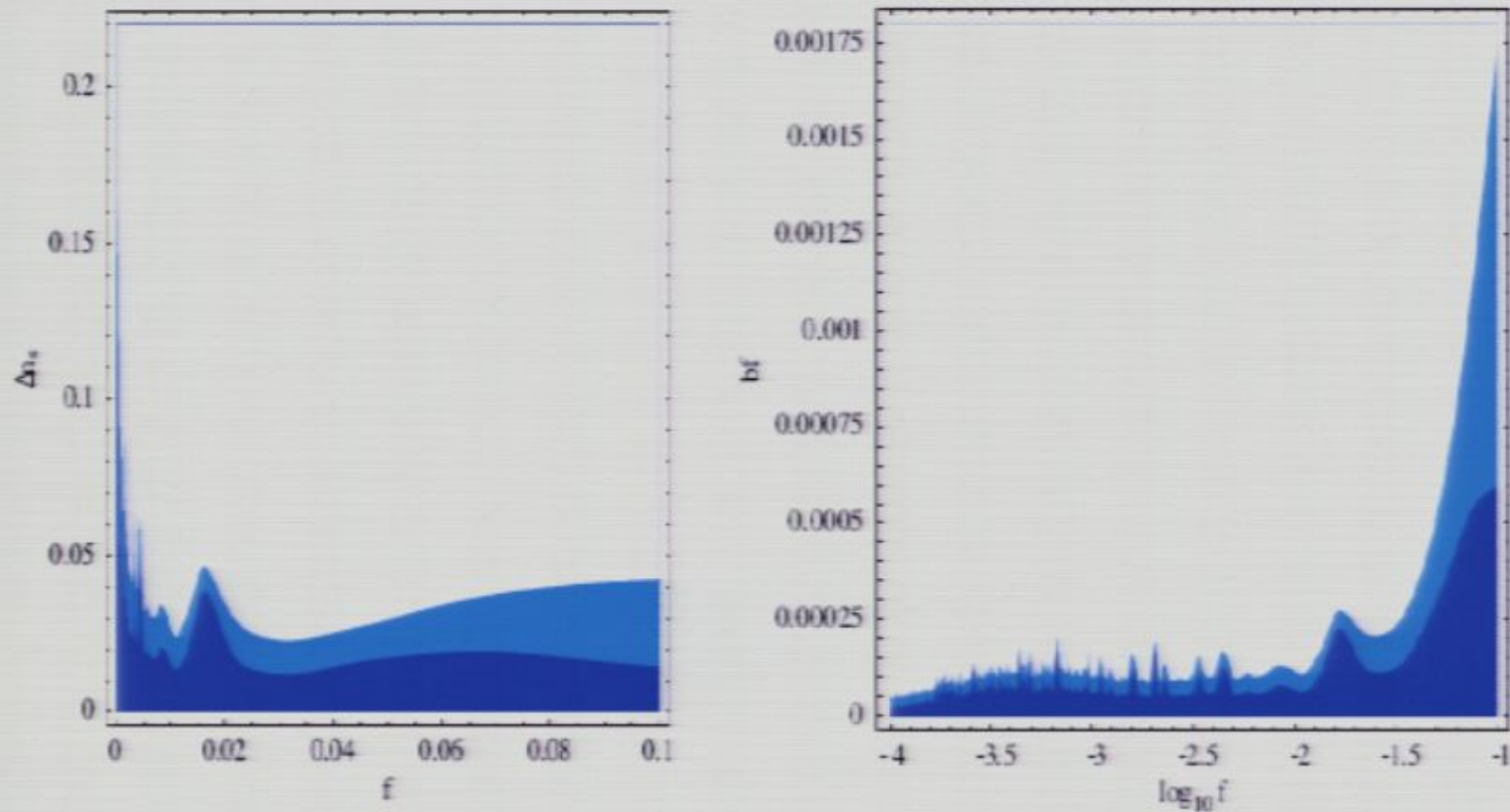
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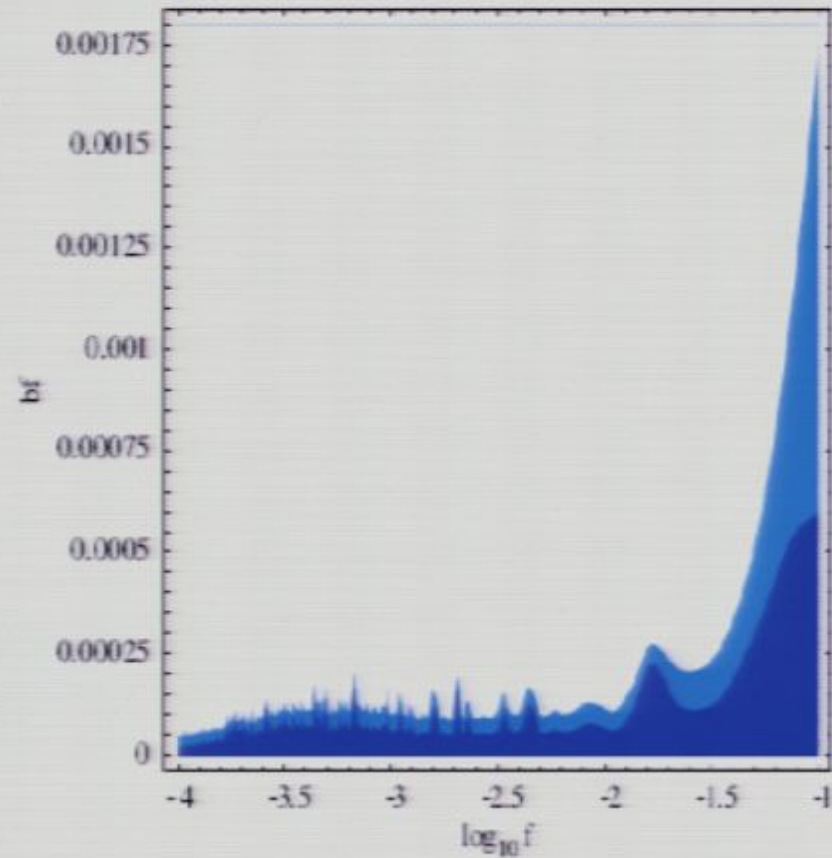
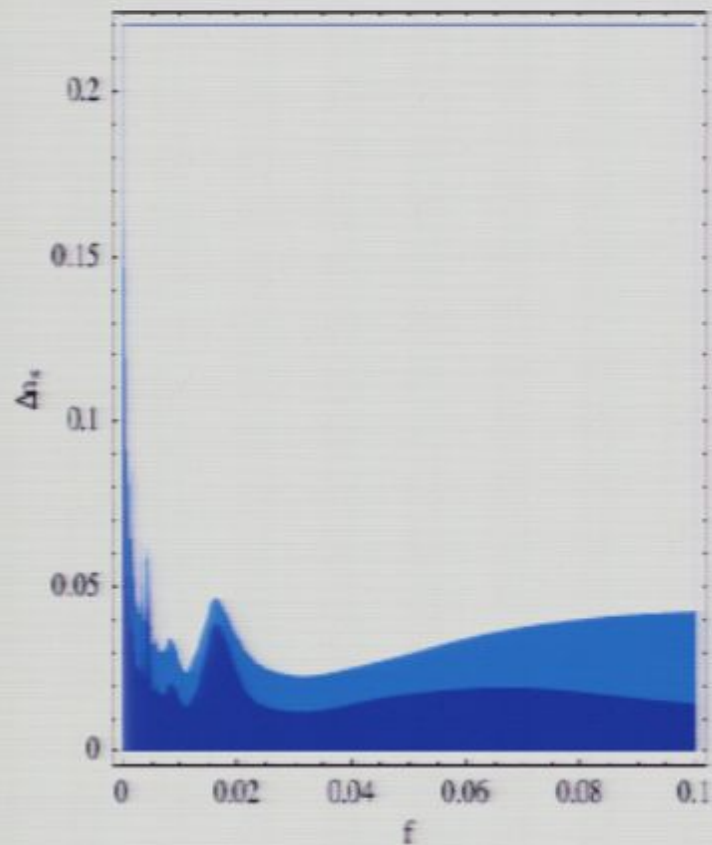
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δ, b

200-199

$$b < \frac{10^{-4}}{\delta}$$

$$\int dx e^{ix} \cos\left(\frac{\log x}{\delta b}\right)$$

$X_{res} =$

$$\cos\left(\frac{\log K}{\delta}\right) = \cos\left(\frac{\log K}{\delta}\right)$$

$M_p = 1$

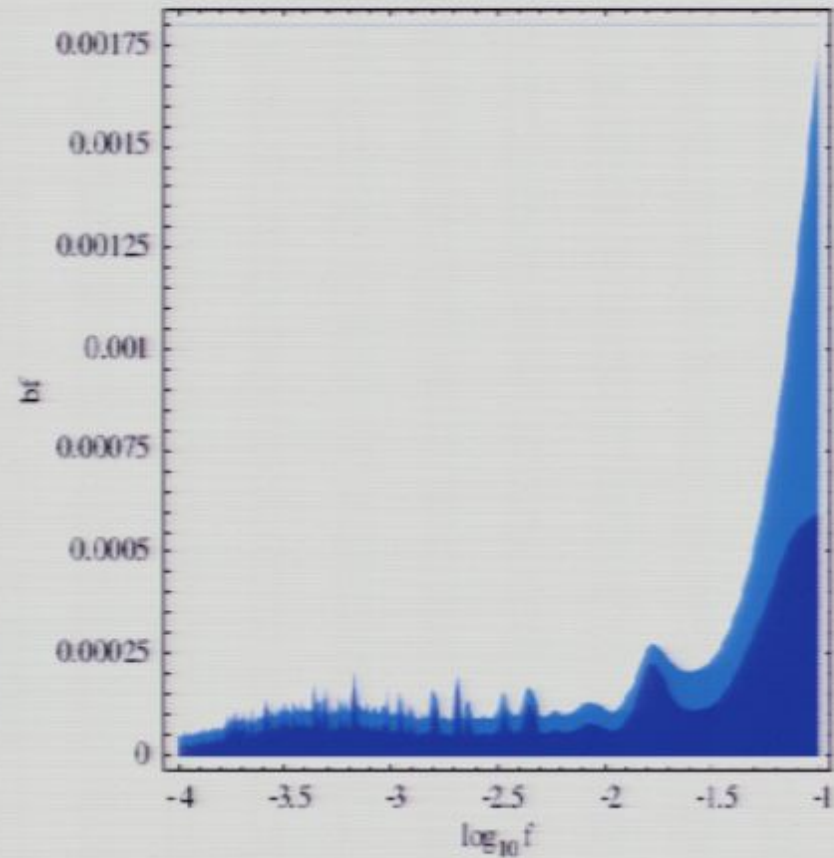
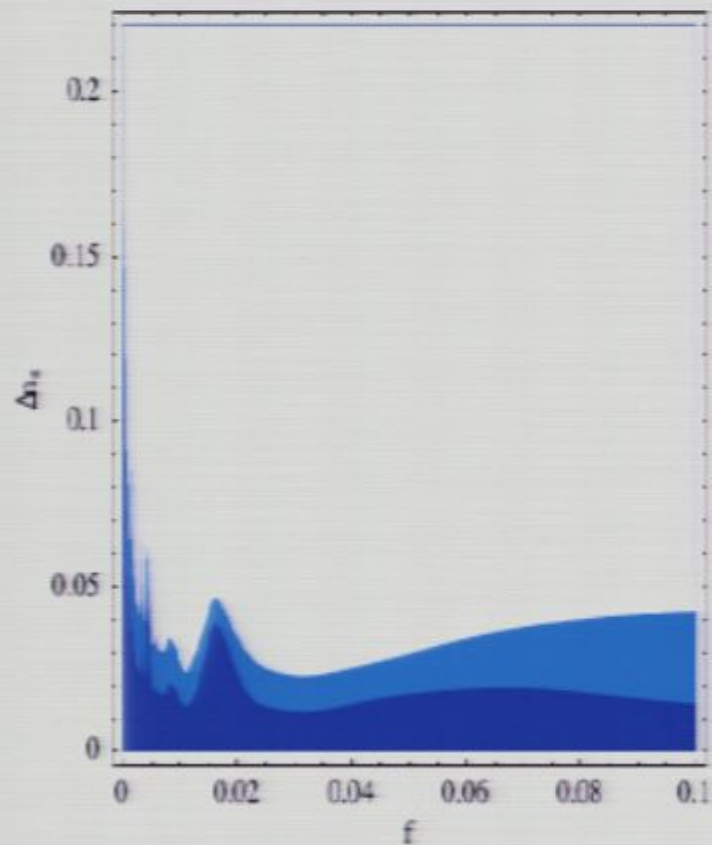
e^{ix}

$\phi > \pi_p$
 $\delta < \pi_p$

$\sqrt{\delta b}$

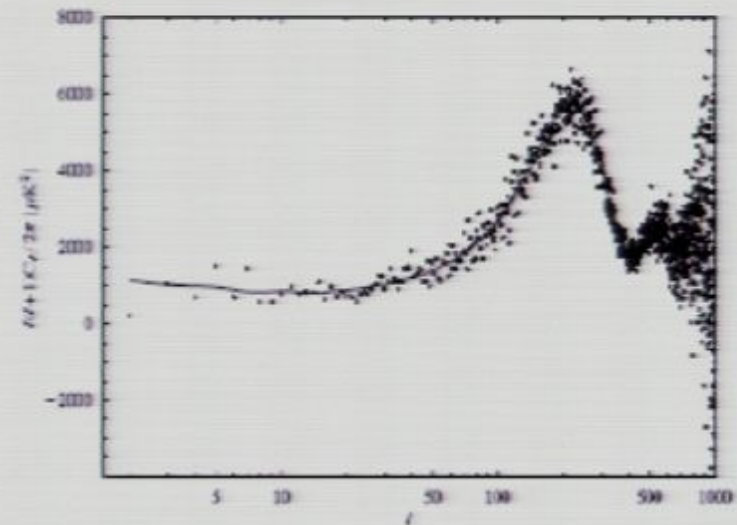
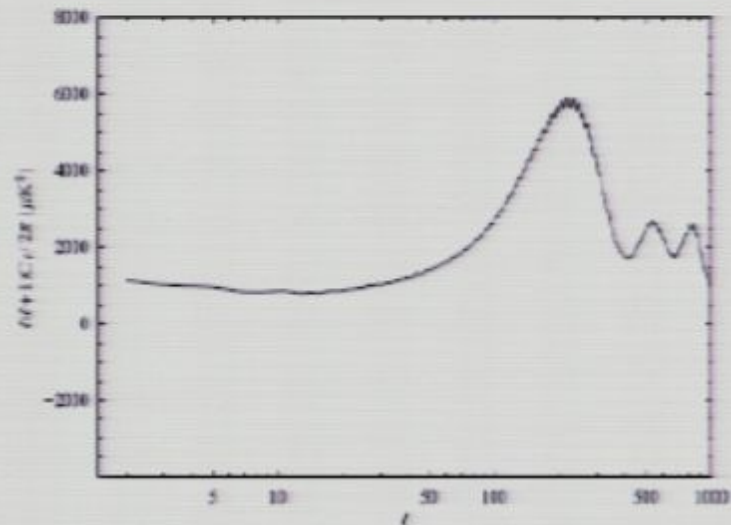
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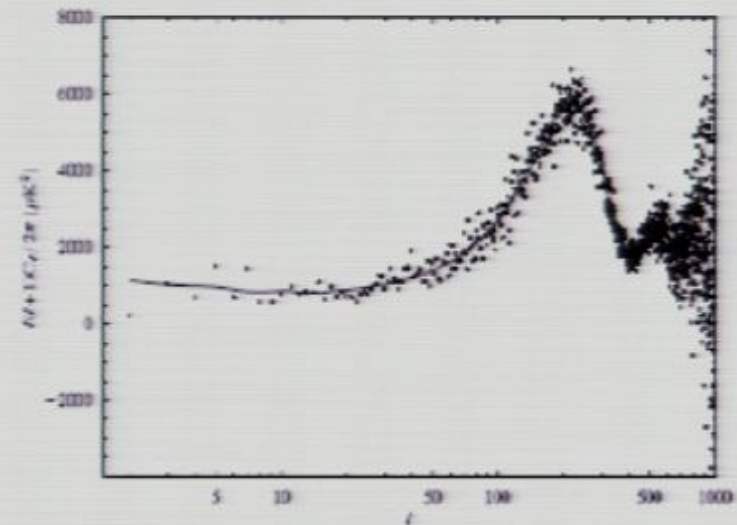
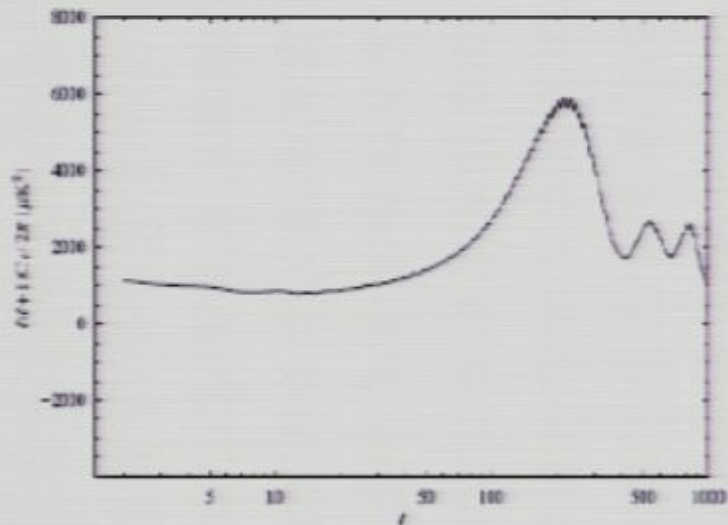
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$$b \propto e^{-\frac{\sqrt{E_2}}{f, b}}$$

$$f, b$$

$$b < \frac{10^{-4}}{f}$$

$$P_{32} = A_s \left(\frac{K}{K_X} \right)^{5.7} \left[1 + \delta m_s \cos \right]$$



$$e^{ix} \cos \left(\frac{\log x}{f, b} \right)$$

$$\cos \left(\frac{\log K}{f, b} \right) \quad M_p = 1$$

$$\sqrt{f, b}$$

e^{-ix}
 M_p
 $K M_p$

$$b \propto e^{-\frac{\sqrt{Kx}}{g}}$$

$$g, b$$

$$b < \frac{10^{-4}}{g}$$

$$P_{32} = A_5 \left(\frac{K}{K_x} \right)^{5/2} \left[1 + \delta m s \cos \omega \right]$$



$$\int dx \cos \left(\frac{\log x}{g} \right)$$

$$\cos \left(\frac{K}{g} \right) \quad M_p = 1$$

$$e^{ix}$$

$$\sqrt{\frac{K}{g}}$$

$$b \propto e^{-\frac{\sqrt{E_2}}{2f}}$$

$$f, b$$

$$b < \frac{10^{-4}}{f}$$

$$P_{32} = A_s \left(\frac{k}{k_x} \right)^5 \left[1 + \delta m_s \cos \omega \right]$$



$$\int dx \cos \left(\frac{\log x}{f^4} \right)$$

$$\cos \left(\frac{\phi}{f} \right)$$

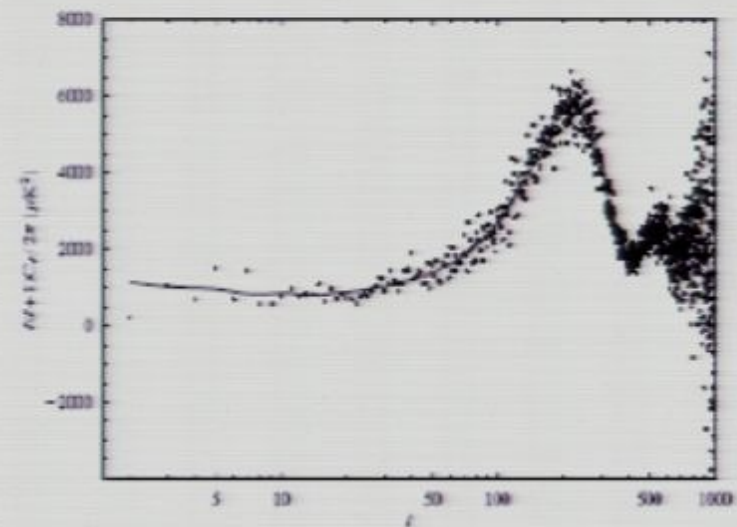
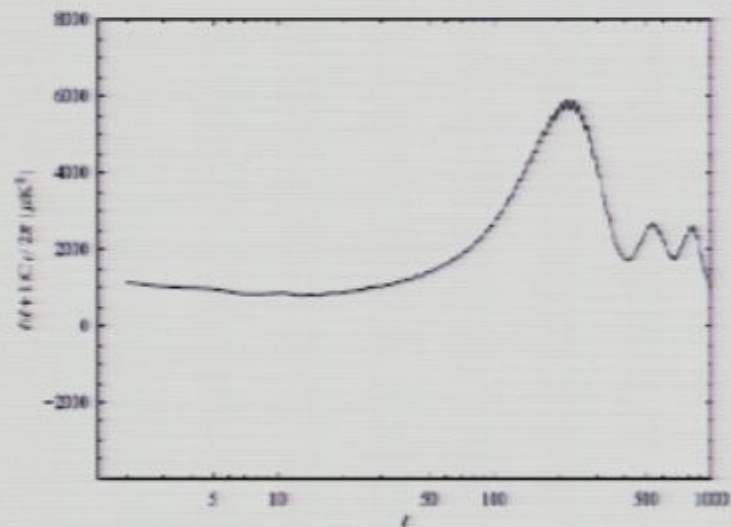
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$$f^4$$

Observational constraints on the spectrum

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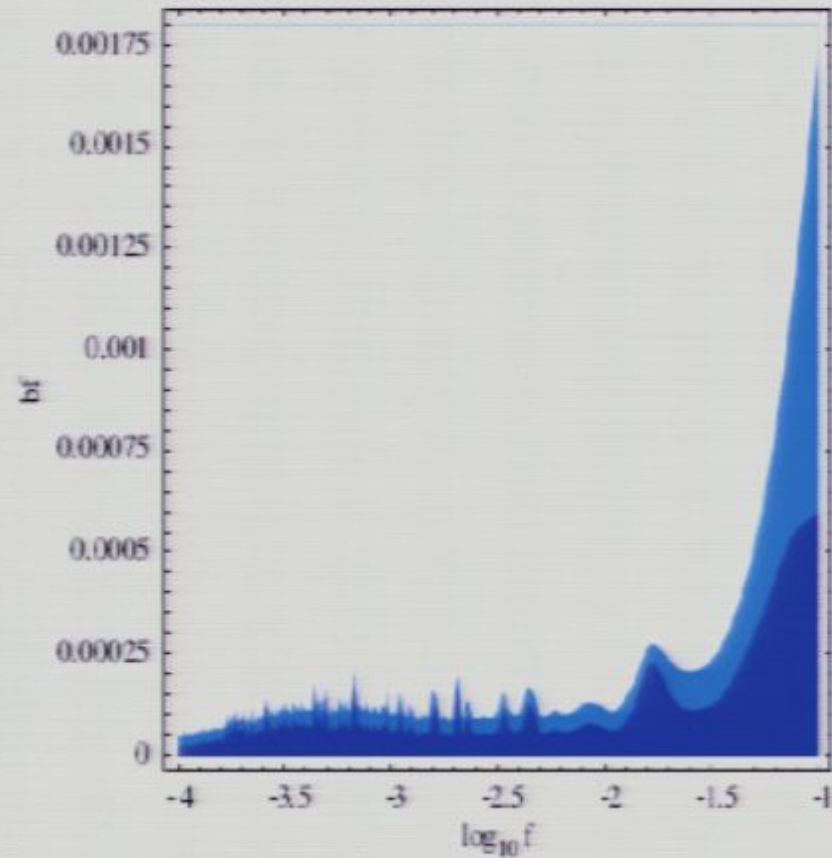
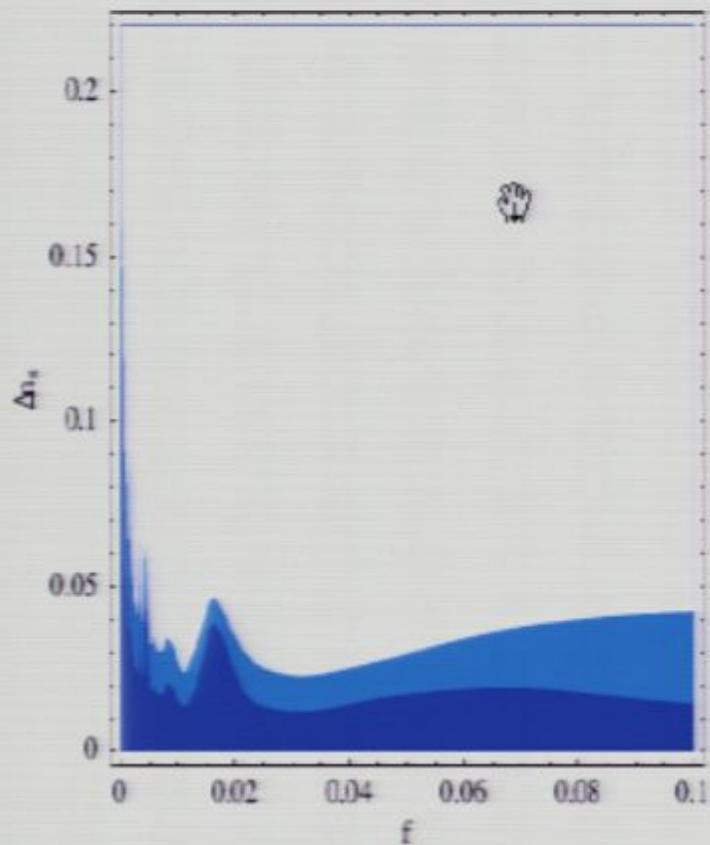
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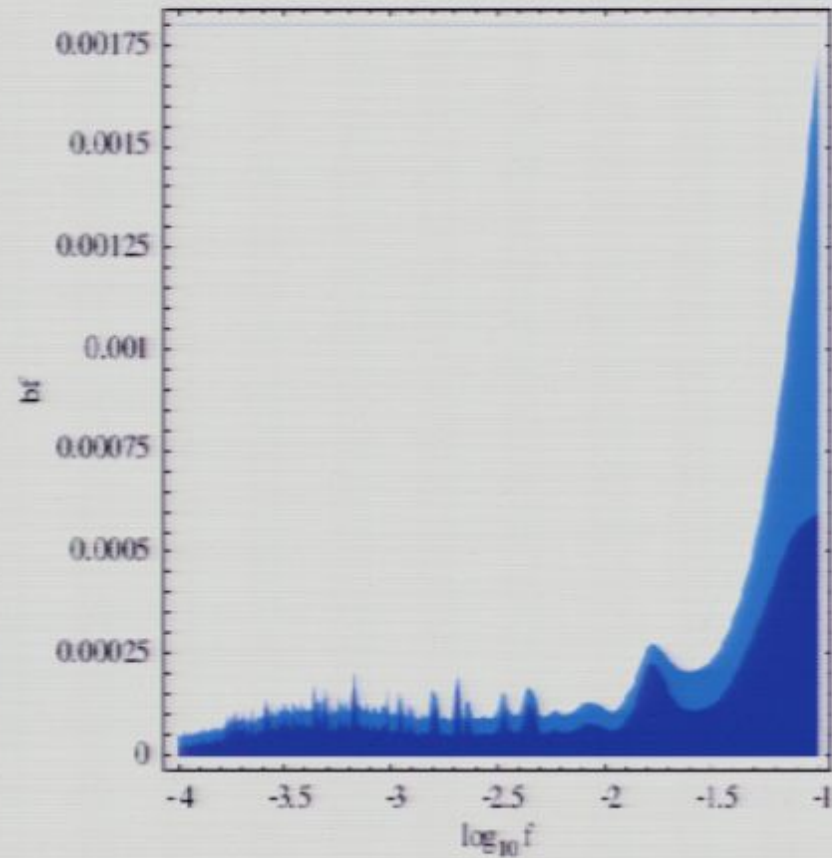
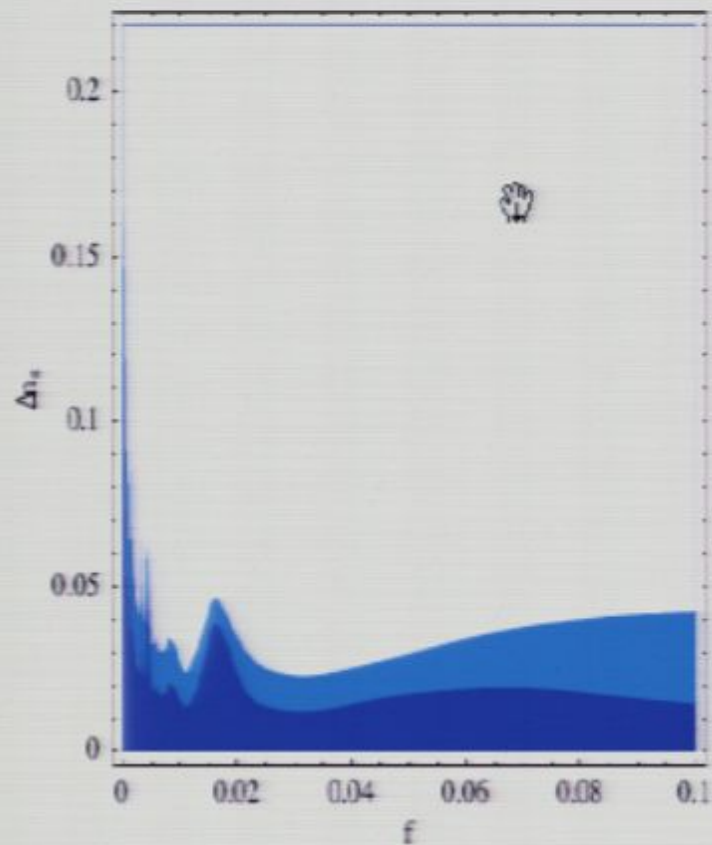
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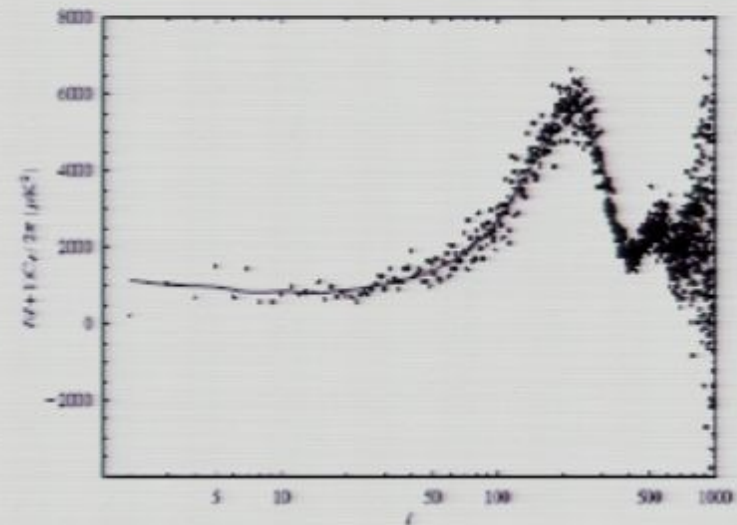
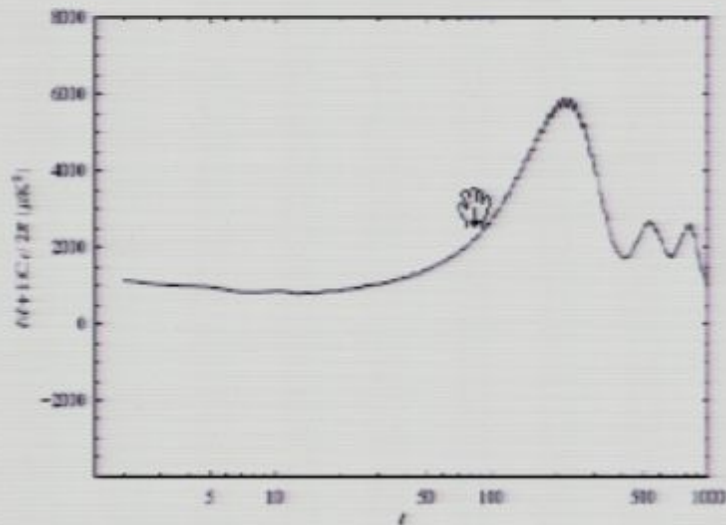
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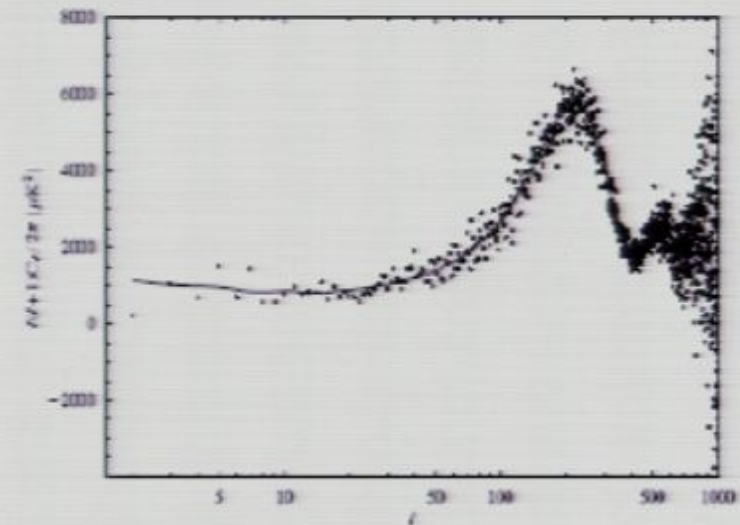
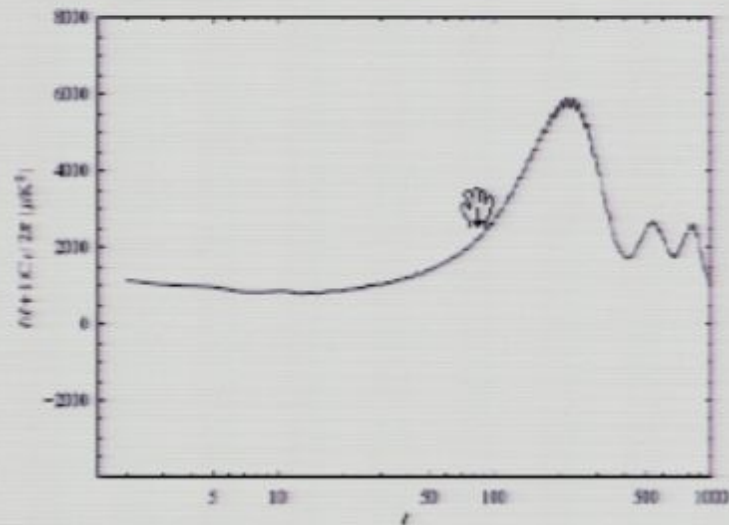
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Tensor modes

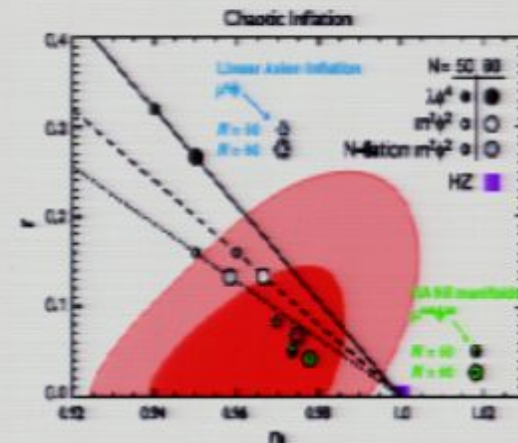
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For $b = 0$ and using slow roll

$$r \simeq 0.07$$



This is within Planck sensitivity!



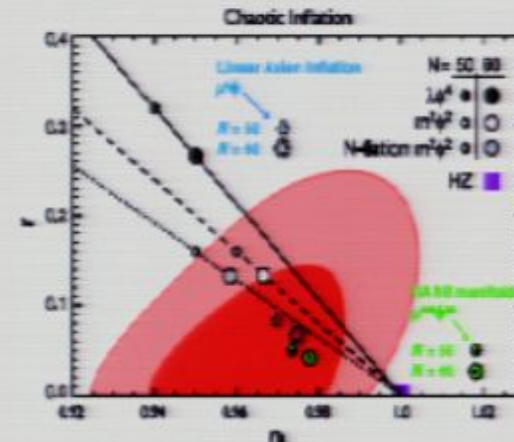
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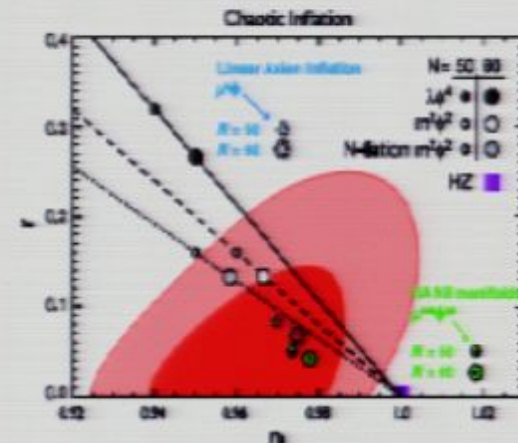
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Oscillations

Oscillations in the tensor spectrum are suppressed w.r.t. those in the scalar spectrum due to a hierarchy in the slow-roll parameters

$$\epsilon \sim \epsilon_0 + \epsilon_{osci} \cos(\phi/f)$$

$$\eta \sim \eta_0 + \eta_{osci} \sin(\phi/f)$$

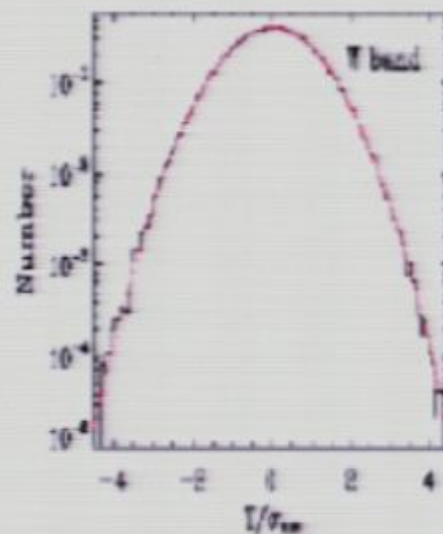
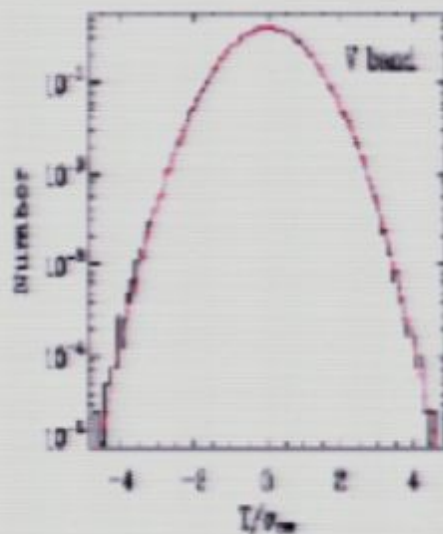
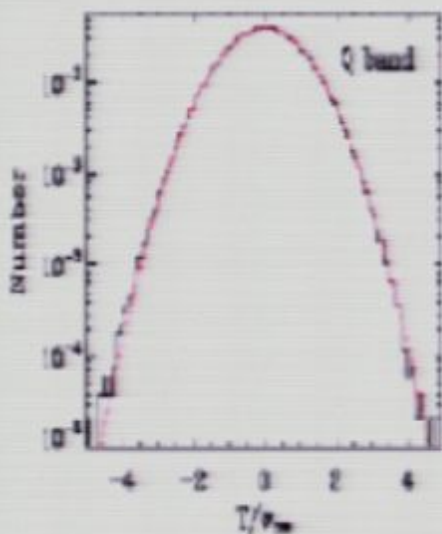
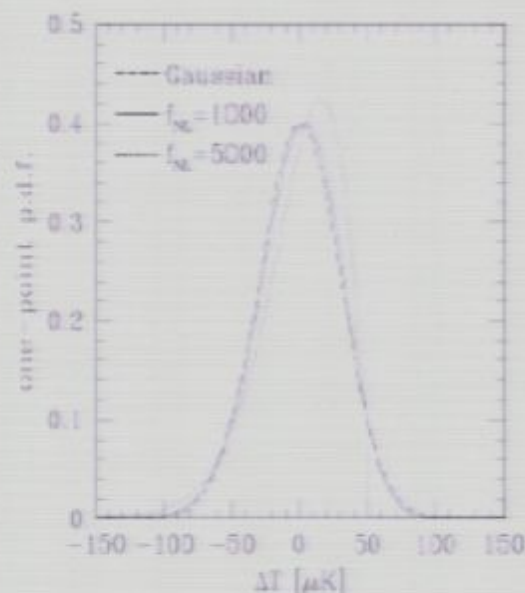
$$\eta_{osci} \sim \epsilon_{osci} \phi/f$$

Outline

- 1 Motivations
- 2 The model: inflation from axion monodromy
- 3 Non-Gaussianity in the bispectrum**
- 4 Summary and conclusions

Can you tell Gaussianity from non-Gaussianity?

Looking at a distribution is hard to detect small deviations from Gaussianity



n-point functions

- For a Gaussian distributed variable ζ

$$\begin{aligned}\langle \zeta^{2n+1} \rangle &= 0, \\ \langle \zeta^{2n} \rangle &\propto \langle \zeta^2 \rangle^n\end{aligned}$$

The three-point ^{hand}function $\langle \zeta^3 \rangle$, or **bispectrum** is the best observable to detect non-Gaussianity.

Computing the bispectrum

Primordial perturbations from inflation are computed using the in-in formalism [Maldacena 03] :

$$\langle \zeta_{k_1}(t) \zeta_{k_2}(t) \zeta_{k_3}(t) \rangle = -i \int_{t_0}^t dt' \langle [\zeta_{k_1}(t) \zeta_{k_2}(t) \zeta_{k_3}(t), H_I(t')] \rangle$$

where the interaction Hamiltonian at order ζ^3 is obtained expanding around an inflationary solution

$$H_I = \int a\epsilon^2 \zeta \zeta'^2 + a\epsilon^2 \zeta (\partial\zeta)^2 - 2\epsilon\zeta' (\partial\zeta) (\partial\chi) \\ + \frac{a}{2} \epsilon \dot{\eta} \zeta^2 \zeta' + \frac{\epsilon}{2a} (\partial\zeta) (\partial\chi) (\partial^2\chi) + \frac{\epsilon}{4a} (\partial^2\zeta) (\partial\chi)^2, \\ \chi \equiv a^2 \epsilon \partial^{-2} \dot{\zeta}$$

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Symmetries, sizes and shapes

A priori, $\langle \zeta^3 \rangle$ depends on three momenta $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$, 9 real numbers.

- Rotational invariance fixes 3 numbers (6 scalar products $\mathbf{k}_i \cdot \mathbf{k}_j$).



Symmetries, sizes and shapes

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- Rotational invariance fixes 3 numbers (6 scalar products $\mathbf{k}_i \cdot \mathbf{k}_j$).
- Conservation of momentum fixes 3 numbers

$$\langle \zeta^3 \rangle \equiv \left(\frac{2\pi}{L}\right)^3 f_{NL} F(k_1, k_2, k_3) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

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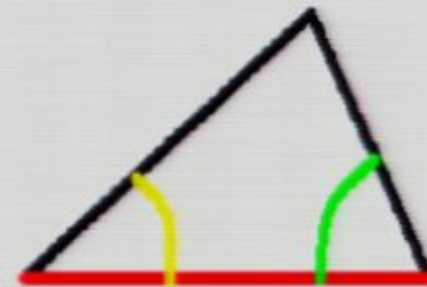
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$\langle \zeta^3 \rangle$ and $F(k_1, k_2, k_3)$ depend on 3 numbers: two angles and one side.



From primordial perturbations to the CMB

- The 2D temperature fluctuations decomposed in spherical harmonics

$$\frac{\Delta T}{T}(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}) .$$



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$$\begin{aligned} \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle &\propto \int \frac{d^3 \vec{k}_1}{(2\pi)^3} \frac{d^3 \vec{k}_2}{(2\pi)^3} \frac{d^3 \vec{k}_3}{(2\pi)^3} Y_{l_1 m_1}^*(\hat{k}_1) Y_{l_2 m_2}^*(\hat{k}_2) Y_{l_3 m_3}^*(\hat{k}_3) \\ &\times \delta^3 \left(\sum_i \mathbf{k}_i \right) F(k_1, k_2, k_3) \Delta_{l_1}^T(k_1) \Delta_{l_2}^T(k_2) \Delta_{l_3}^T(k_3) . \end{aligned}$$

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- $Y_{l,m}$ and the radiation transfer function Δ_l^T oscillate fast.
- Number of operations: $l_{max}^3 \sim 10^9$ integrals with 10^{10} operations each.
- 10^{19} operations, **numerically very challenging!**


The cosine of shapes

How can we look for **any** non-Gaussian signal when even a single one requires computational superpowers? This is an open problem...



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- The cosine is close to ± 1 , the two shapes are similar.
- The observational constraints on one apply to the other as well [\[Babich et al. 04\]](#) .

The scalar product

- Following [Fergusson & Shellard], we chose the scalar product

$$F \cdot F' \equiv \int \frac{dk_1 dk_2 dk_3}{k_1 + k_2 + k_3} F(k_1, k_2, k_3) F'(k_1, k_2, k_3)$$

which has the same scaling as the optimal CMB estimator.

The state of the art

Ruling out the "simplest" model of inflation?

Single-field slow-roll inflation with canonical kinetic term, i.e. vanilla inflation, leads to undetectable non-Gaussianity.



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Ruling out the "simplest" model of inflation?

Single-field slow-roll inflation with canonical kinetic term, i.e. vanilla inflation, leads to undetectable non-Gaussianity.

On the other hand, models with large non-Gaussianity (various shapes):

- non-canonical kinetic terms (e.g. DBI)
- extra light spectator fields (e.g. curvaton)
- multifields
- **violations of slow roll.**

Only a few shapes have been constrained by observations

[WMAP7, Smith et al. 09] . They are **all scale invariant.**

Observational constraints

Observational constraints [WMAP7] on three different shapes:

Local

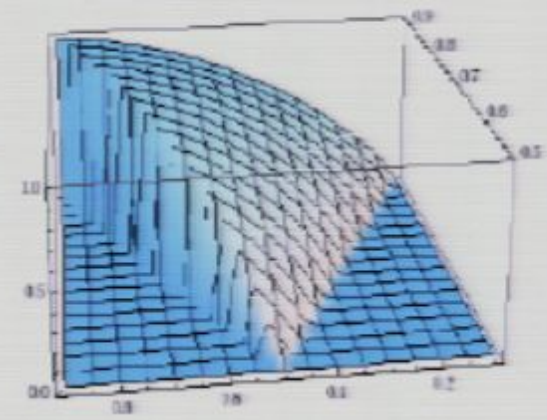
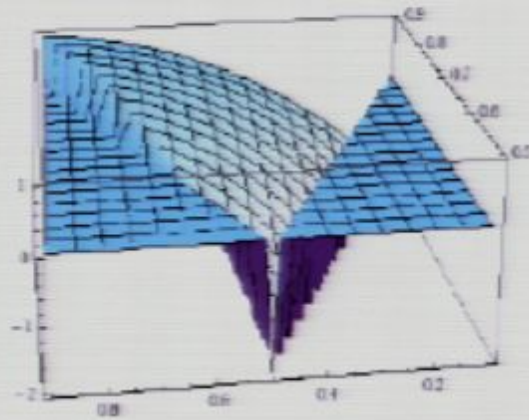
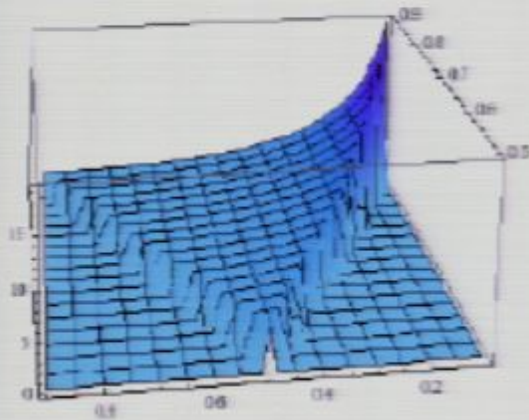
$$-10 < f^{loc} < 74$$

Orthogonal

$$-410 < f^{ort} < 6$$

Equilateral

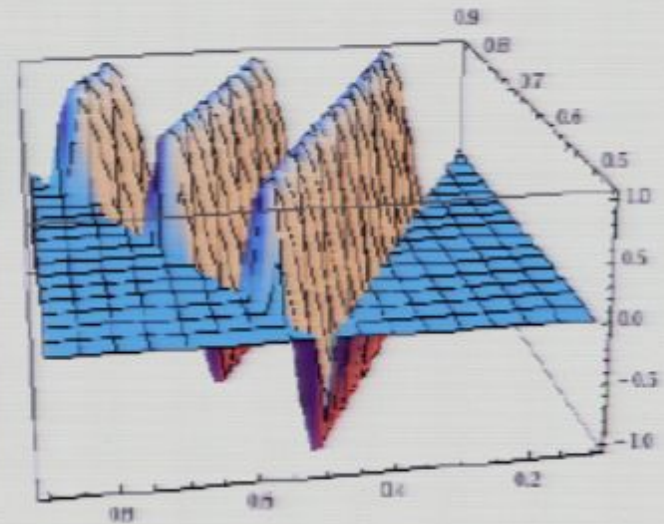
$$-214 < f^{equi} < 266$$



Resonant non-Gaussianity

Large non-Gaussianity from modulations

Modulations on the potential **violate slow roll** and can induce large non-Gaussianity.



- We now present **resonant non-Gaussianity**.

Calculation of resonant of non-Gaussianity

Schematically [Chen et al 08, Flauger & E.P. 10]

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \simeq (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \times \\ \int dt' 2a^3 \epsilon \dot{\delta} \left[\zeta_{k_1}^* \zeta_{k_2}^* \zeta_{k_3}^* + 2 \text{ perm.} \right] + c.c.,$$

$$\zeta_k \underset{\text{hand}}{\sim} x^{3/2} H^{(1)}(x) \sim e^{ix}$$

$$\dot{\delta} \sim \sin\left(\frac{\log x}{f\phi}\right)$$

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The frequency of ζ_k is stretched by the expansion from M_{pl} to H when it exits the horizon.

Necessary condition

$$H < \omega < M_{pl} \quad \Rightarrow \quad f\phi \ll M_{pl}^2 \text{ and } f \ll M_{pl}$$

where ω is the frequency of background oscillations.

Calculation of resonant of non-Gaussianity

The integral can be computed exactly. The final result is [\[Flauger & E.P.\]](#)

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^7 \Delta_\zeta^4 \frac{\delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)}{k_1^2 k_2^2 k_3^2} f^{res} \times$$

$$\left[\sin\left(\frac{\log K/k_*}{f\phi_*}\right) + f\phi_* \sum_{i,j} \frac{k_i}{k_j} \cos\left(\frac{\log K/k_*}{f\phi_*}\right) \right],$$

$$f^{res} \equiv \frac{3\sqrt{2\pi}b}{8(f\phi_*)^{3/2}}.$$

Resonant enhancement of non-Gaussianity

- The size of the resonant non-Gaussianity is

$$f_{res} \simeq \frac{3\sqrt{2\pi}}{8} \frac{b}{(f\phi)^{3/2}}$$



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The non Gaussian signal is

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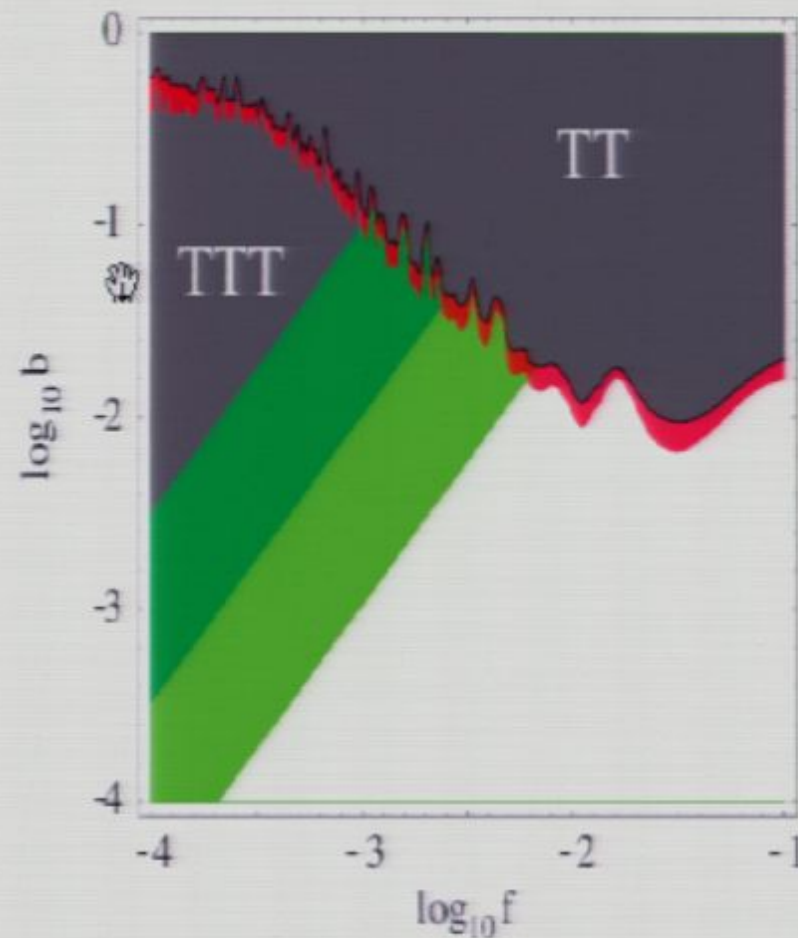
Large resonant non-Gaussianity

The non Gaussian signal is

- Linear in b as for the spectrum
- Fixing b , one finds $f_{res} \propto f^{-3/2}$. Remember that $\delta n_s \propto f^{1/2}$. Both spectrum and bispectrum are potential observables.
- Non-scale-invariant due to the sinusoidal oscillation. It is scale invariant if averaged over the period

Oscillations in the spectrum and bispectrum

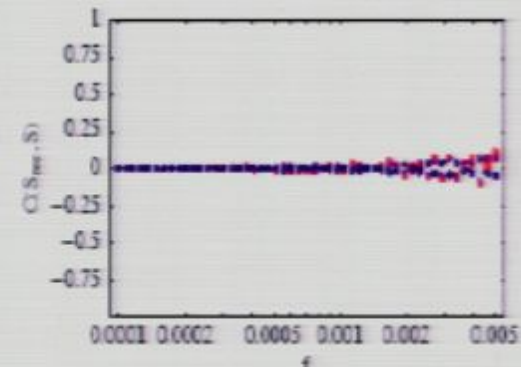
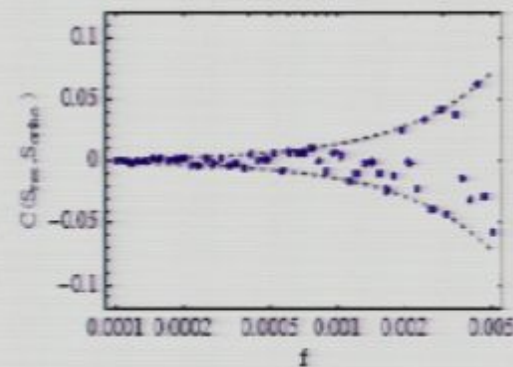
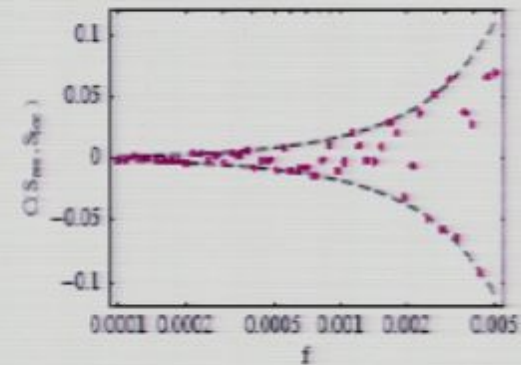
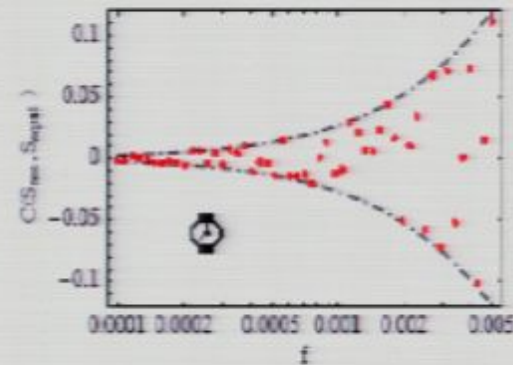
These are the observational constraints from the spectrum together with a contour plot of f^{res} as functions of f and b .



- For small f the bispectrum is the most relevant observable.

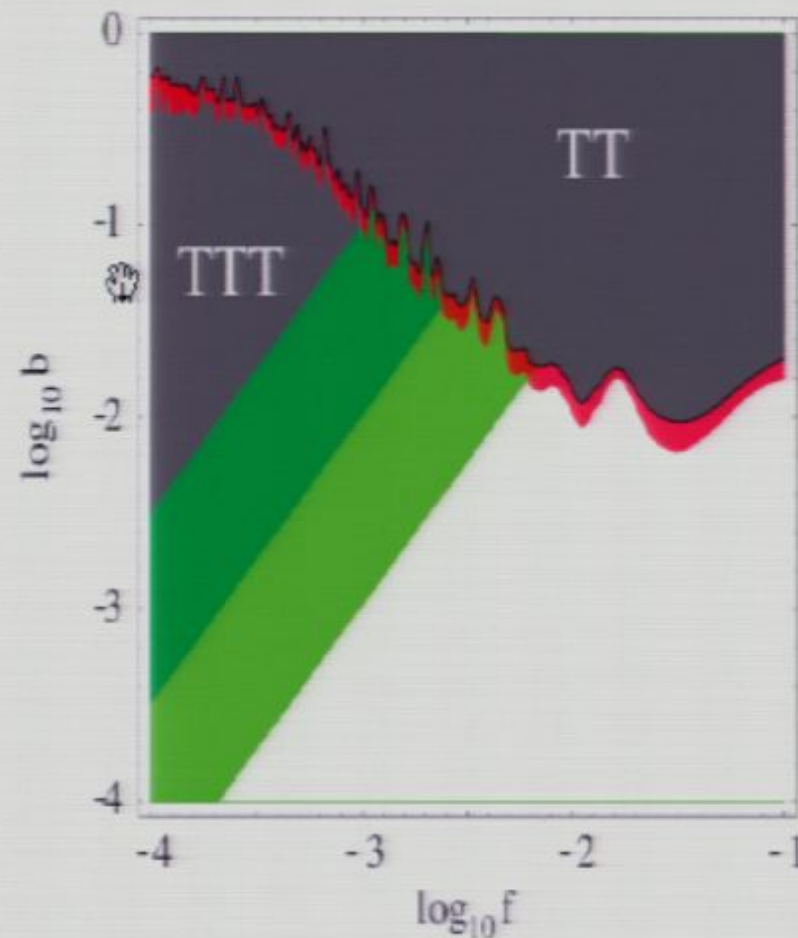
Correlation with other shapes: numerics

- Numerical computation of the cosine between resonant non-Gaussianity and local, equilateral and orthogonal shapes gives

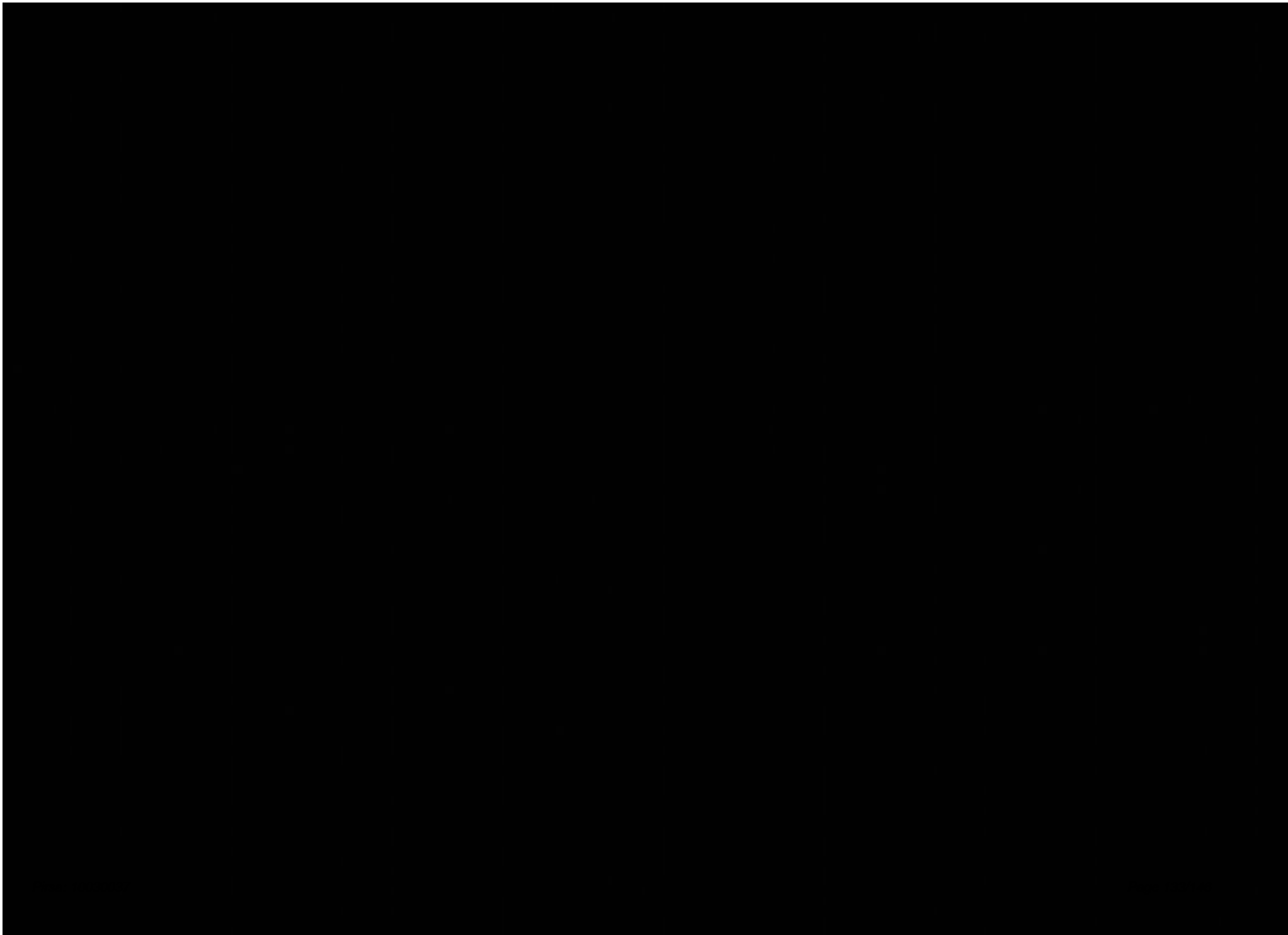


Oscillations in the spectrum and bispectrum

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$$\sin\left(\frac{\sin \kappa}{\gamma \phi}\right)$$

$$P_{3^2} = A_s \left(\frac{\kappa}{\kappa_x}\right)^{m_s-1} \left[1 + \delta m_s \cos(\phi)\right]$$



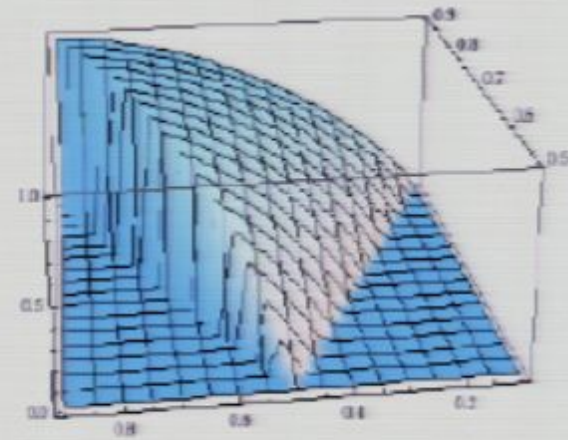
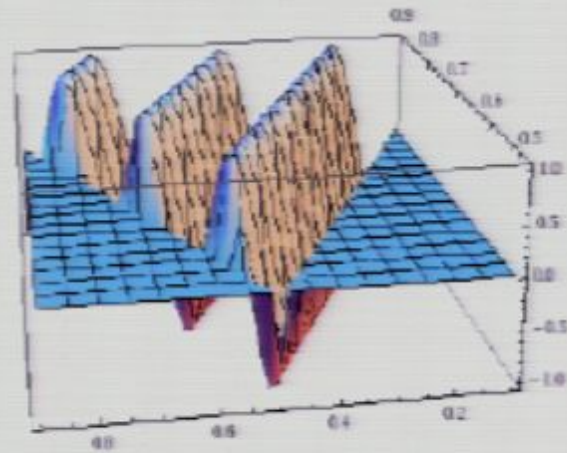
$$\alpha e^{-\frac{1}{2} \frac{m_s}{\kappa_x}}$$

$$f, b$$

$$b < \frac{10^{-4}}{f}$$

$$\int dx e^{nix} \cos\left(\frac{\log x}{\gamma \phi}\right)$$

Correlation with other shapes: heuristic



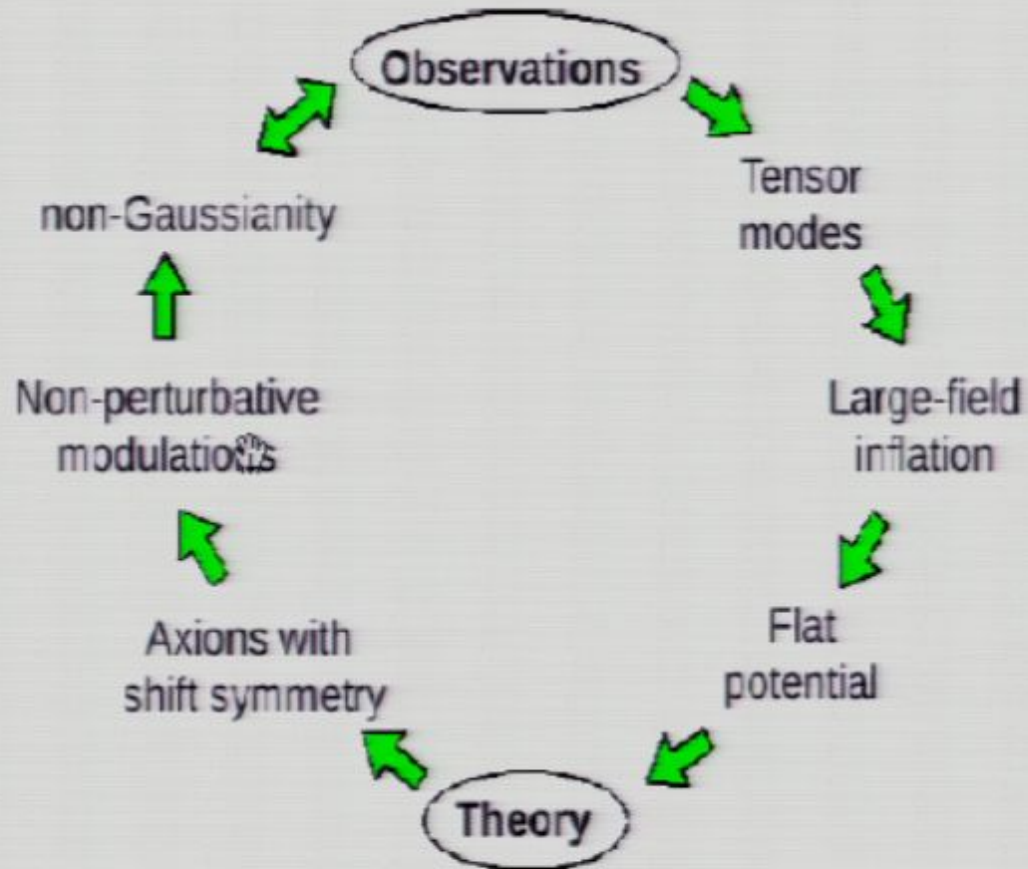
- The resonant shape oscillates as

$$F^{res}(k_1, k_2, k_3) \propto \cos\left(\frac{\log K}{f\phi}\right)$$

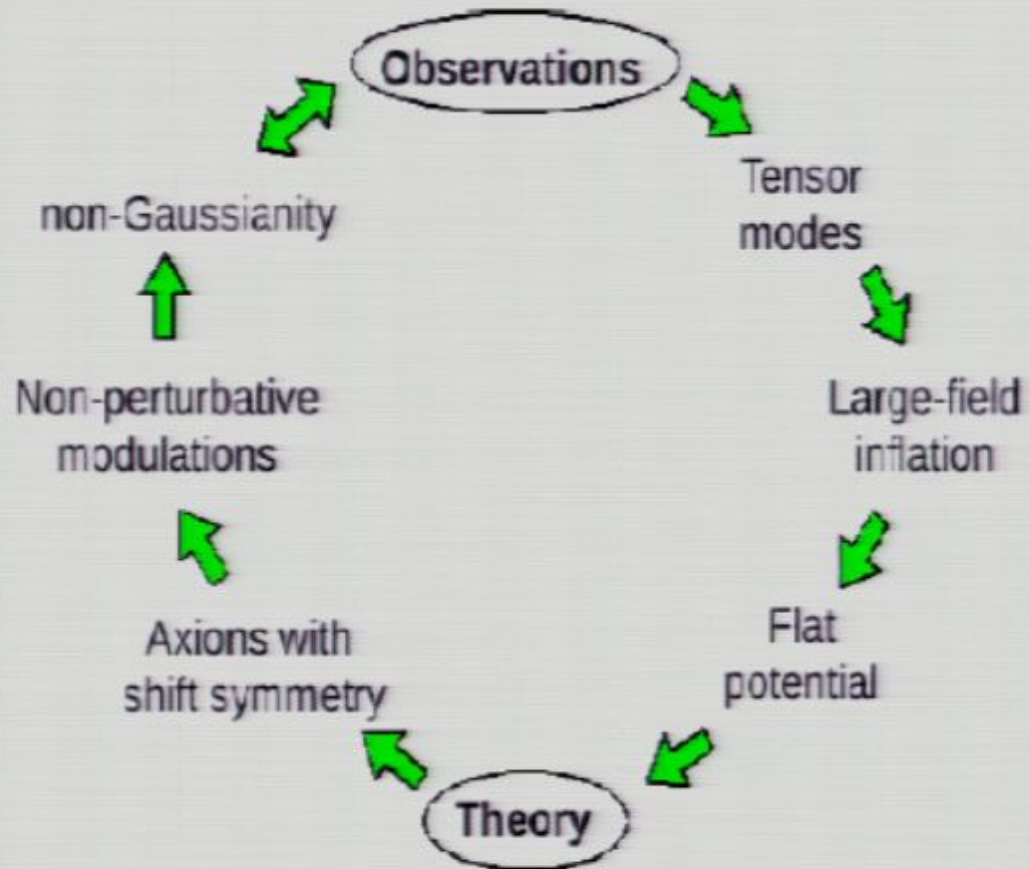
Outline

- 1 Motivations
- 2 The model: inflation from axion monodromy
- 3 Non-Gaussianity in the bispectrum
- 4 Summary and conclusions

Summary

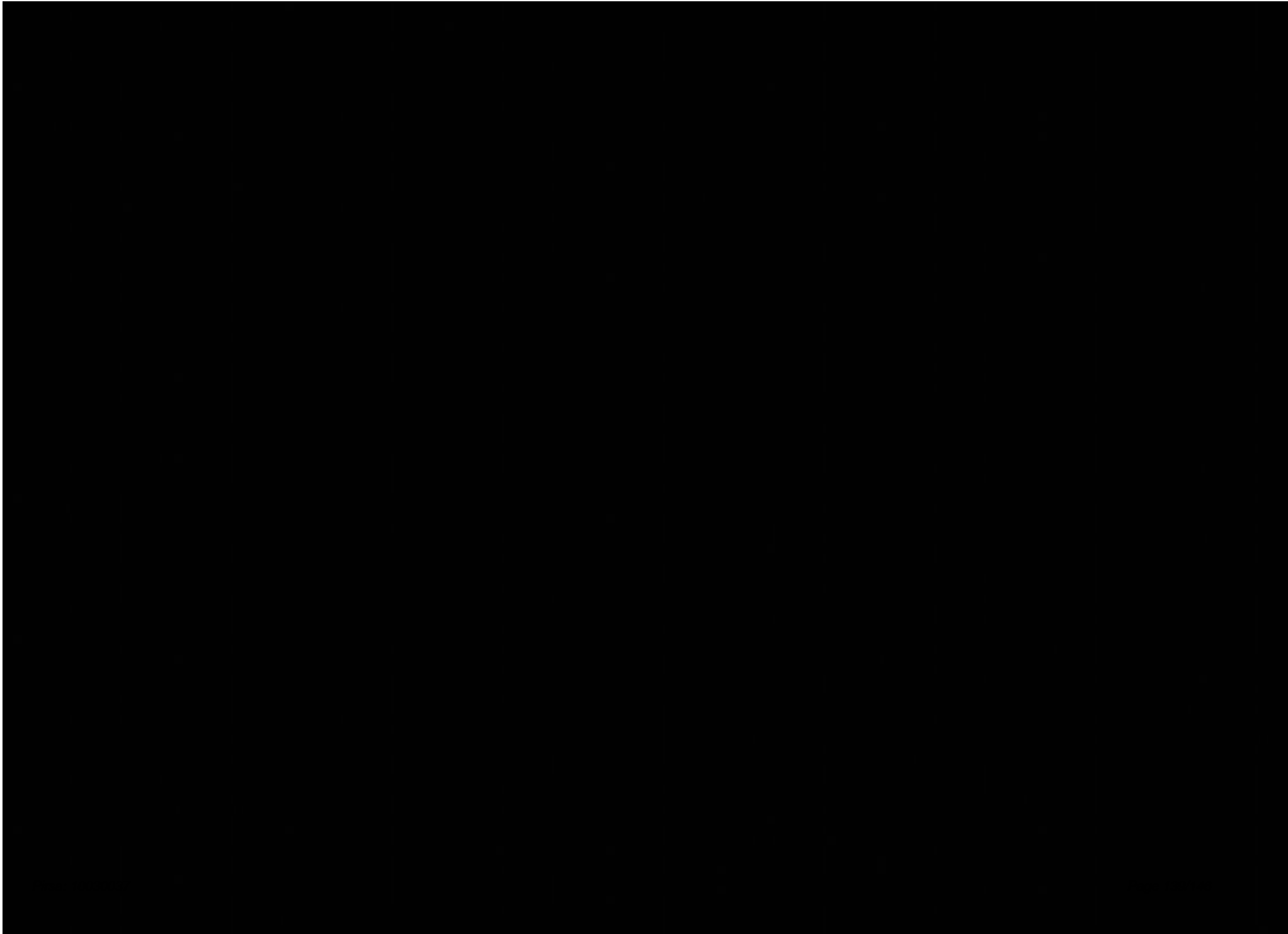


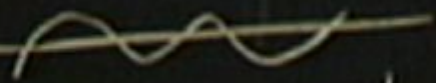
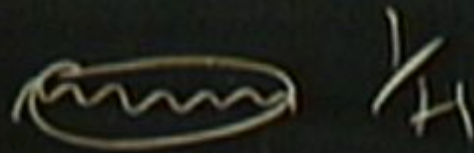
Summary



Thanks!





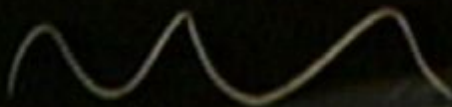


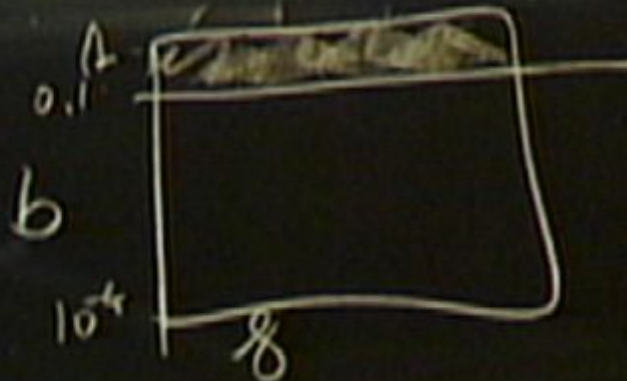
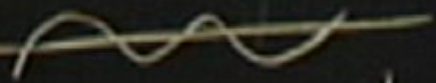
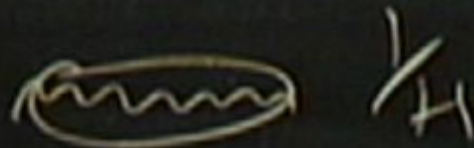
$$b \sim \frac{10^{-4}}{8}$$



$$\mu m \left(\frac{0.2 \text{ m}}{8} \right)$$

$$P_{32} \propto A_s \left(\frac{K}{K_{xx}} \right)^{m_3-1} [1 + \delta m_3]$$





$$\mu m \left(\frac{\phi \kappa}{\gamma \phi} \right)$$

$$\left(\frac{m}{3} - 1 \right) \left[1 + \delta m s \cos \left(\frac{\phi \kappa}{\gamma \phi} \right) \right]$$

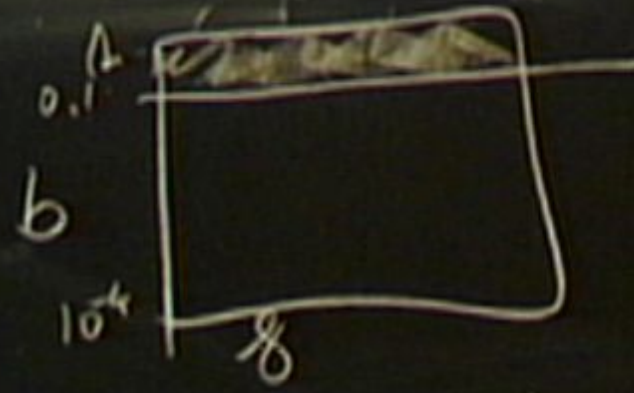


$$\frac{1}{H}$$



$$\frac{1}{H}$$

$$b \sim \frac{10^{-4}}{g}$$



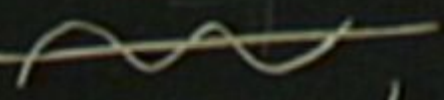
$$\text{mm} \left(\frac{\partial m \kappa}{g \phi} \right)$$

$$b \sim \Lambda \sim e^{-\frac{1}{2} \frac{m}{M_{\text{Pl}}}}$$

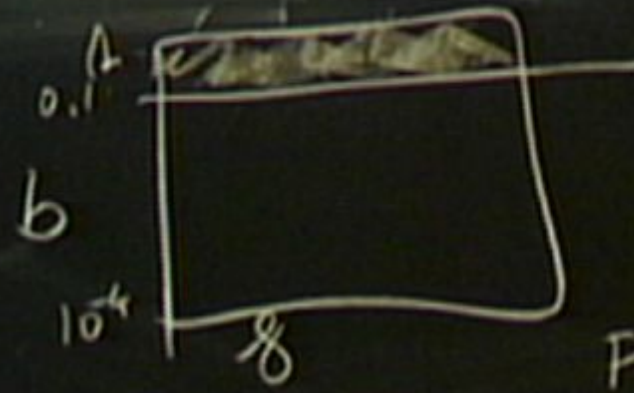
$$P_{3^2} = A_5 \left(\frac{\kappa}{\kappa_{\text{X}}} \right)^{m/3-1} \left[1 + \delta m s \cos \left(\frac{\phi \kappa}{g} \right) \right]$$



200 - 199



$$b \sim \frac{10^{-4}}{\delta}$$

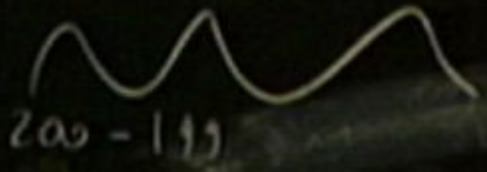


$$\text{mm} \left(\frac{\partial m \kappa}{\gamma \phi} \right)$$

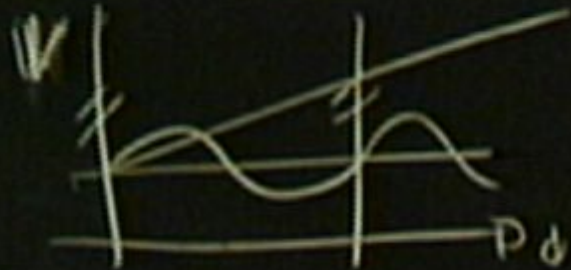
$$H \quad b \sim \Lambda \sim e^{-\frac{1}{2} \frac{\gamma^2}{\partial m \kappa}}$$

$$ST \quad e^{-\sqrt{s_2}}$$

$$P_{3^2} \sim A_5 \left(\frac{\kappa}{\kappa_X} \right)^{3-1} \left[1 + \delta m s \omega \left(\frac{\phi \kappa}{\delta} \right) \right]$$



$$\sqrt{L^2 + a^2}$$



$$V = \text{const} + \mu^3 \phi$$

$\mu \rightarrow 0$

$$\frac{\Delta m^2 \phi^2}{8 T_5 \sqrt{L^2 + a^2}}$$



$$\mu \sim e^{-\frac{1}{2} \mu^2}$$

OS

$$\mu \sim \frac{\phi}{L}$$

$\sim \left(\frac{\phi}{L} \right)^{2+1B}$

$$\left(\frac{k}{k_H} \right)^{m_3 - 1} \left[1 + \delta m_3 \cos \left(\frac{\phi k}{L} \right) \right]$$



-133





$V = \text{constant} + m \phi$

$u \rightarrow 0$

$$E = \frac{m^2 \phi^2}{2 \sqrt{L^2 + a^2}}$$

$$A \sim e^{-\frac{1}{2} \phi^2}$$

OS

$$\phi \sim \frac{\phi}{L}$$

11B



$$s \left(\frac{k}{k_x} \right)^{m-1} \left[1 + \delta m s \cos \left(\frac{\phi k}{\lambda} \right) \right]$$



-199

$$e^{i k x} \cos \left(\frac{\text{const} \cdot x}{\lambda_0} \right)$$

$$X_{\text{res}} = \frac{1}{2 \rho_1}$$



$V = \text{constant} = m \dot{\phi}$

$n \rightarrow \infty$

$\frac{m^2 \dot{\phi}^2}{2} = \frac{m \dot{\phi}^2}{2}$

$E = \frac{1}{2} m \dot{\phi}^2 \sqrt{L^2 + a^2}$

$\lambda \sim e^{-\frac{1}{2} \gamma^2}$

OS

$\frac{\phi}{L}$

$\left(\frac{1}{L} \right) \text{11B}$



$\left(\frac{K}{K_0} \right)^{\frac{m-1}{2}} \left[1 + \delta m s \cos \left(\frac{\phi K}{L} \right) \right]$



$e^{iKx} \cos \left(\frac{L \cos x}{L_0} \right)$

$X_{res} = \frac{1}{2L_0}$