Title: Quantum Bayesianism---Something Old, Something New

Date: Mar 10, 2010 02:00 PM

URL: http://www.pirsa.org/10030036

Abstract: Quantum Bayesianism is a point of view on quantum foundations that says that there is no such thing as a “measurement problem” because there is no such THING as a quantum state: Quantum states are not things---instead information. But the view doesn’t stop there; it starts there! Taking the idea seriously over the last 15 years has been the direct motivation for a number of theorems and objects in quantum information theory: from the no-broadcasting theorem, to the quantum de Finetti theorem, and even some quantum cryptographic alphabets. I will review some of this, and then move on to the holy grail of present efforts: Finding an efficient representation of quantum states in terms of a singular probability function. Doing so leads to the hard technical problem of demonstrating the existence of a certain very symmetric sets of quantum states, and holds out the hope of understanding the amount of “quantum stuff” in a physical system in terms of a single parameter. (I.e., there is the THING that the quantum state is not).

Quantum Bayesianism,

Something Old
Something New

Christopher Fuchs

P̂I

[redacted]

Quantum Bayesianism,

Something Old Something New

Christopher Fuchs

$\hat{\text{PI}}$

"QBism" – the quantum Bayesian program of

C. M. Caves

R. Schack

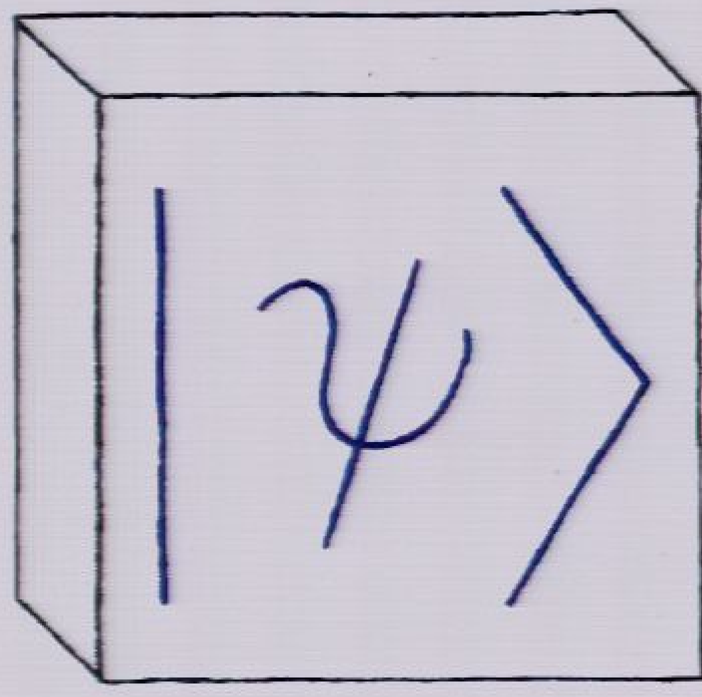
D. M. Appleby

myself

See arXiv.org.

See also:

C. G. Timpson,
"Quantum Bayesianism: A Study"
and
pirsa.org/09080010
09090029


$|\psi\rangle$

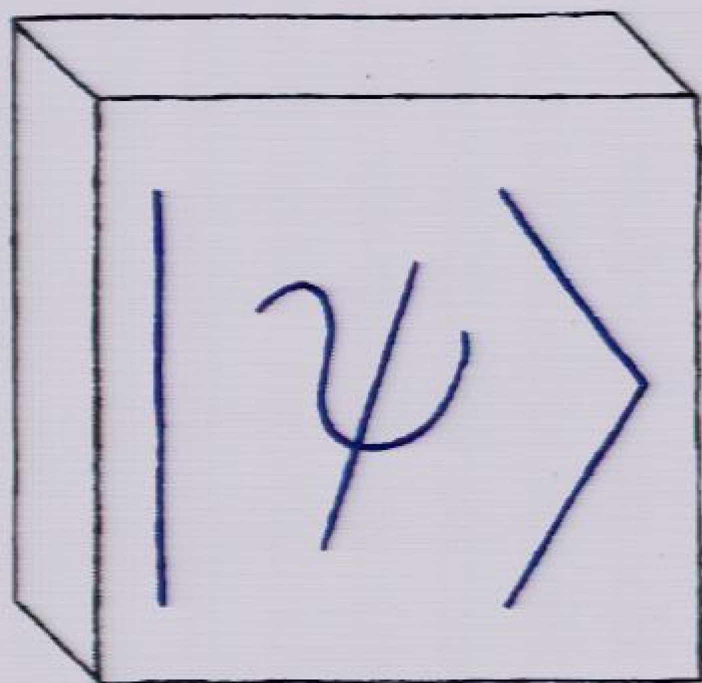
- What makes collapse?

- Action at a distance?

The spook disappears only if one relinquishes the orthodox standpoint, according to which the ψ-function is accepted as a complete description of the single system.

It may appear as if all such considerations were just superfluous learned hairsplitting, which have nothing to do with physics proper. However, it depends precisely upon such considerations in which direction one believes one must look for the future conceptual basis of physics.

— Albert Einstein, 1949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- What makes collapse?

INFORMATION

- Action at a distance?

You know how men have always hankered after unlawful magic, and you know what a great part in magic *words* have always played. If you have his name, … you can control the spirit, genie, afrite, or whatever the power may be. Solomon knew the names of all the spirits, and having their names, he held them subject to his will. So the universe has always appeared … as a kind of enigma, of which the key must be sought in the shape of some illuminating or power-bringing word or name. …

But if you follow the pragmatic method, you cannot look on any such word as closing your quest. You must bring out of each word its practical cash-value, set it at work within the stream of your experience. It appears less as a solution, then, than as a program for more work …

— William James

Density Operators

linear operators

$$\rho \in \mathcal{L}(\mathcal{H}_d)$$

complex vector space

catalog of uncertainties

---

1) $\rho^\dagger = \rho$
2) $\mathrm{tr}\,\rho = 1$
3) $\lambda_i(\rho) \geqslant 0$

eigenvalues

convex hull of the set $\{ |\psi\rangle\langle\psi| : |\psi\rangle \in \mathcal{H}_d \}$

POVMs

Positive Operator Valued Measures

— an immensely useful tool

Let $\mathcal{P} = \{ E : 0 \le \langle\psi|E|\psi\rangle \le 1 \ \forall |\psi\rangle \}$.

Any set of operators

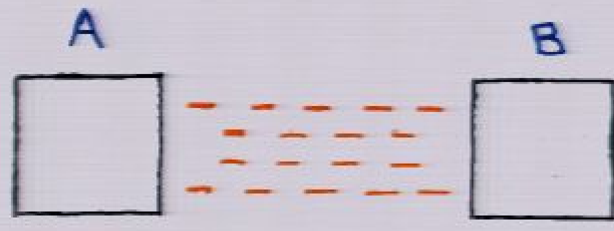
$$\{ E_b : E_b \in \mathcal{P}, \ \sum_b E_b = I \}$$

corresponds to a potential mmt.

Probability of outcome b,

$$p_b = \mathrm{tr}\, \rho E_b .$$

Standard Measurements	Generalized Measurements
$\{\Pi_i\}$	$\{E_b\}$
$\langle\psi\vert\Pi_i\vert\psi\rangle \geq 0, \forall\vert\psi\rangle$	$\langle\psi\vert E_b\vert\psi\rangle \geq 0, \forall\vert\psi\rangle$
$\sum_i \Pi_i = I$	$\sum_b E_b = I$
$p(i) = \mathrm{tr}\,\rho\Pi_i$	$p(b) = \mathrm{tr}\,\rho E_b$
$\Pi_i\Pi_j = \delta_{ij}\Pi_i$	—



$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$$

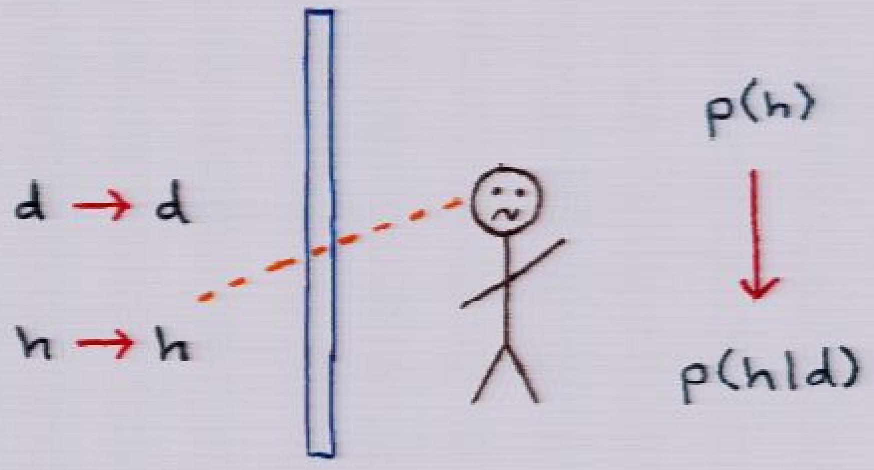
Let Alice measure $|\uparrow\rangle, |\downarrow\rangle$ basis. Bob's system will be in state $|\uparrow\rangle$ or $|\downarrow\rangle$ afterward.

Let Alice measure $|\rightarrow\rangle, |\leftarrow\rangle$ basis. Bob's system will be in state $|\leftarrow\rangle$ or $|\rightarrow\rangle$ afterward.

Conclusion

$|\psi\rangle$ is information.

The Weatherman

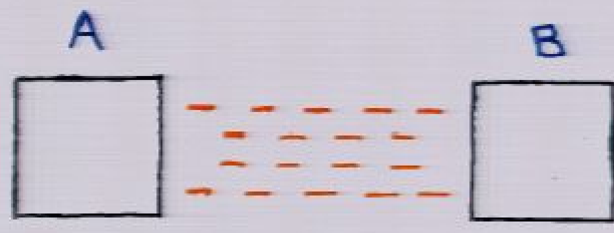


Bayesian Updating

$$p(h) = \sum_d p(h,d)$$

$$= \sum_d p(d)\,p(h|d)$$

$$p(h) \xrightarrow{d} p(h|d)$$



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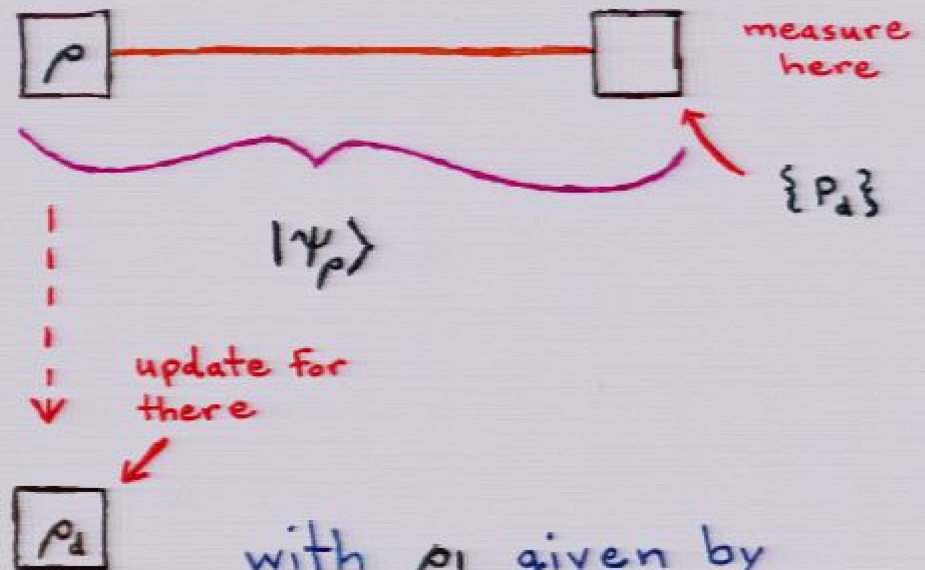
$|\psi\rangle$ **is** information.

Particularly Important Case



with ρ_d given by pure conditionalization!

CAF, quant-ph/0205039

State Change at a Distance

$$|\psi_\rho\rangle = \sum_i \sqrt{\lambda_i}\, |a_i\rangle |b_i\rangle$$

Measurement causes update:

$$|\psi_\rho\rangle\langle\psi_\rho| \longrightarrow (P_d \otimes I)\, |\psi_\rho\rangle\langle\psi_\rho|\, (P_d \otimes I)$$

Partial trace:

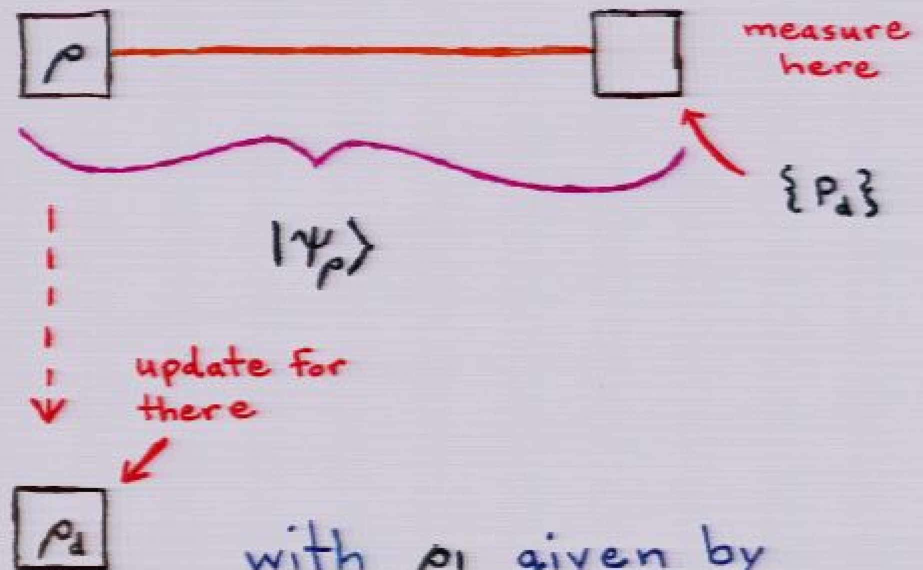
$$\begin{aligned}
tr_A(\cdot) &= \sum_{ijk} \sqrt{\lambda_j}\sqrt{\lambda_k}\, \langle a_i| P_d \otimes I |a_j\rangle |b_j\rangle \langle a_k| \langle b_k| P_d \otimes I |a_i\rangle \\
&= \sum_{ijk} \sqrt{\lambda_j}\sqrt{\lambda_k}\, \langle a_k| P_d |a_i\rangle\langle a_i| P_d |a_j\rangle\, |b_j\rangle\langle b_k| \\
&= \sum_{jk} \sqrt{\lambda_j}\sqrt{\lambda_k}\, \langle a_j| P_d^T |a_k\rangle\, |b_j\rangle\langle b_k| \\
&= \Big(\sum_j \sqrt{\lambda_j}\, |a_j\rangle\langle a_j|\Big)\, P_d^T \Big(\sum_k \sqrt{\lambda_k}\, |a_k\rangle\langle a_k|\Big) \\
&= \rho^{1/2} P_d\, \rho^{1/2} \\
&\equiv p(d)\, \rho_d
\end{aligned}$$

By making $p(d) = tr\,\rho P_d$ and $\rho_d = \frac{1}{p(d)} \rho^{1/2} P_d\, \rho^{1/2}$.

Note!

$$\boxed{\rho = \sum_d p(d)\, \rho_d}$$

Particularly Important Case



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Emphasis

Classical

$$p(H) = \sum_D p(D)\, p(H|D)$$

$$p(H) \xrightarrow{D} p(H|D)$$

Quantum

$$\rho = \sum_b p(b)\, \rho_b$$

$$\rho \xrightarrow{b} \rho_b$$

Quantum No Cloning

Want a device that

$$|\psi_i\rangle|s\rangle \longrightarrow |\psi_i\rangle|\psi_i\rangle$$

regardless of $i \in \{0,1\}$.

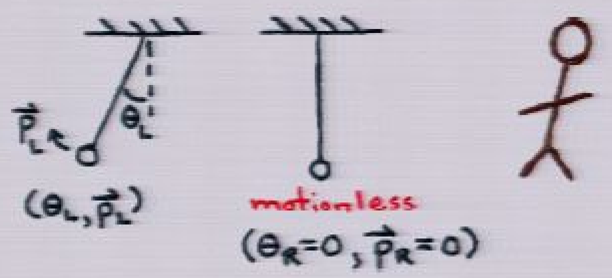
Cannot build if $0 < |\langle\psi_0|\psi_1\rangle| < 1$.

Because on total Hilbert space

initial inner product $= \langle\psi_0|\psi_1\rangle \underbrace{\langle s|s\rangle}_{=1}$

final inner product $= \langle\psi_0|\psi_1\rangle\langle\psi_0|\psi_1\rangle$

Belief Cloning?



$P_i(\theta_L, \vec{p}_L)$

$i \in \{0,1\}$

Is there any device I can build that will cause the first observer to describe the pendula according to

$$P_i(\theta_L, \vec{p}_L) \times P_i(\theta_R, \vec{p}_R) \ ?$$

Not if $0 < \int \sqrt{P_0(\theta_L, \vec{p}_L) P_1(\theta_L, \vec{p}_L)}\, d\theta_L d\vec{p}_L < 1$.

Because Liousville mechanics is phase-space volume preserving.

CAF, quant-ph/9601020

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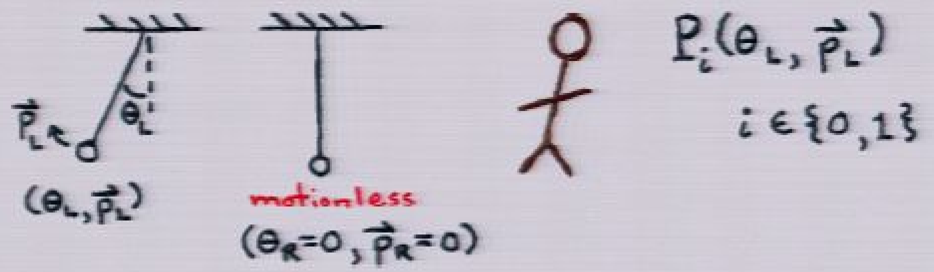

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CAF, quant-ph/9601020

$$P(h)$$

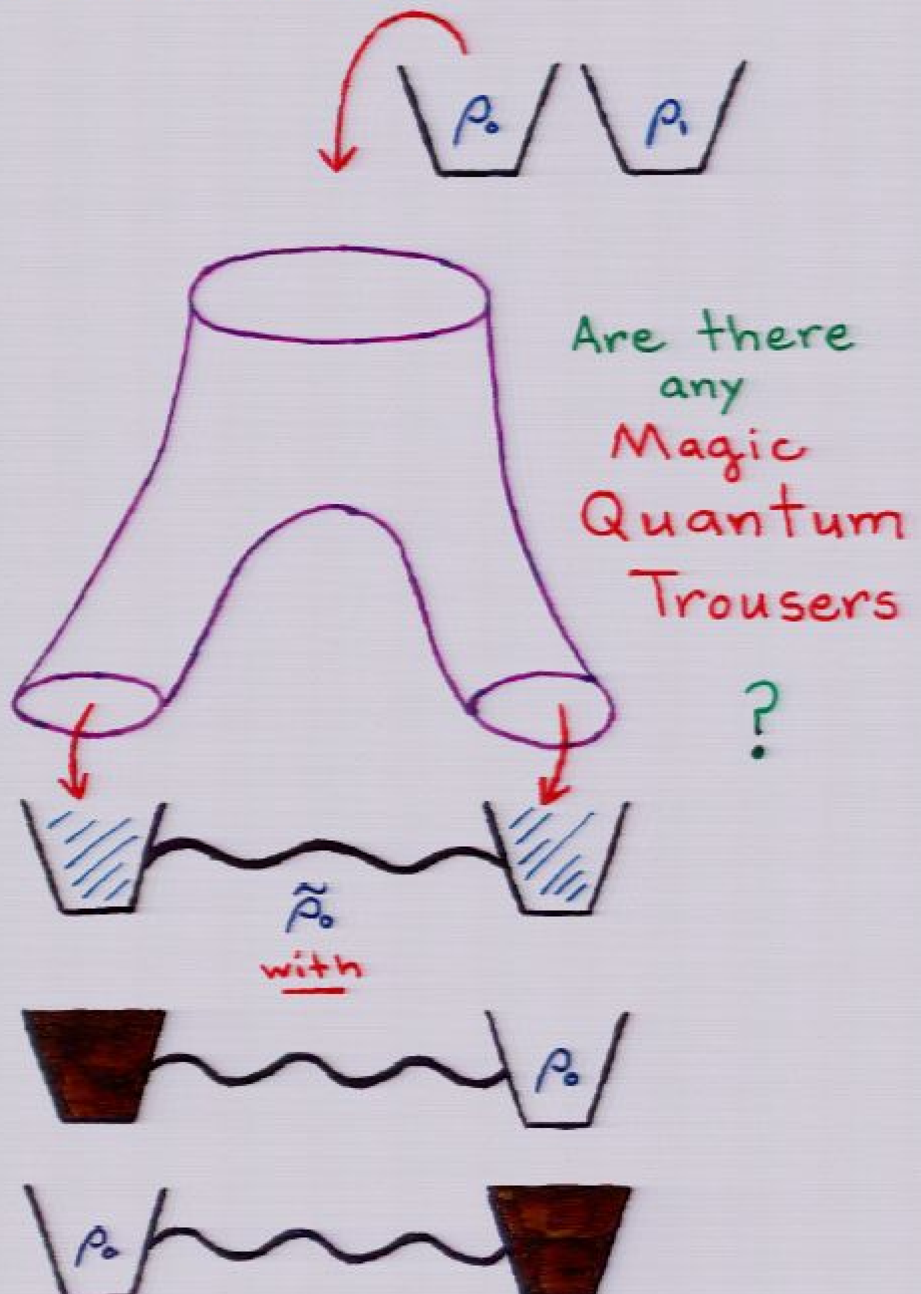
~~states of pre-existent reality~~

consequences of "measurement" interactions

CAF & Peres, Phys. Today <u>53</u> (3), 70 (2000)

Barnum, Caves, CAF, Jozsa, Schumacher

PRL 76 (1996) 2818



Broadcasting Commuting States

$$\rho_0 = \sum_b \lambda_b^0 |b\rangle\langle b|$$

$$\rho_1 = \sum_b \lambda_b^1 |b\rangle\langle b|$$

Make U such that

$$\tilde{\rho}_s = U(\rho_s \otimes \Sigma)U^+$$

$$= \sum_b \lambda_b^s |b\rangle|b\rangle\langle b|\langle b|$$

What are the
necessary and sufficient
conditions on

ρ_0 and ρ_1

?

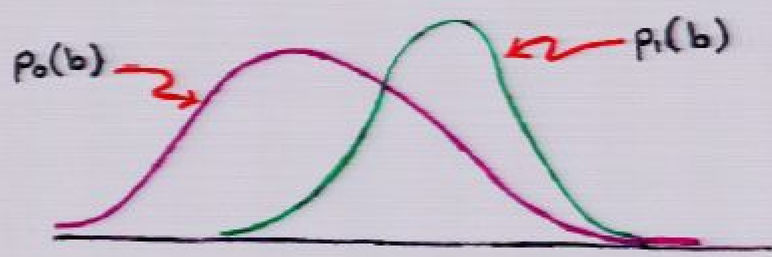
Example: Overlap Criterion

Suppose $\pi_0 = \pi_1 = 1/2$.

A measurement $\{E_b\}$ generates two probability distributions:

$$p_0(b) = \mathrm{tr}\,\rho_0 E_b$$
$$p_1(b) = \mathrm{tr}\,\rho_1 E_b .$$

Maybe a good measurement is the one that minimizes their overlap?



$$F(p_0, p_1) = \sum_b \sqrt{p_0(b)}\sqrt{p_1(b)}$$

CAF & Caves, Open Sys. Info. Dyn. 3 (1995) 345.

Using the Schwarz

$$F = \sum_b \sqrt{\mathrm{tr}\,\rho_0 E_b}\sqrt{\mathrm{tr}\,\rho_1 E_b}$$

$$= \sum_b \sqrt{\mathrm{tr}\, \underbrace{E_b^{1/2}\rho_0^{1/2}}_{A^\dagger}\underbrace{\rho_0^{1/2}E_b^{1/2}}_{A}}\;\sqrt{\mathrm{tr}\, \underbrace{E_b^{1/2}\rho_1^{1/2}}_{B^\dagger}\underbrace{\rho_1^{1/2}E_b^{1/2}}_{B}}$$

Insert also
$I = U^\dagger U$
Why not? Can't hurt.

$$\geq \sum_b \left|\mathrm{tr}\, E_b^{1/2}\rho_0^{1/2}U\rho_1^{1/2}E_b^{1/2}\right| \qquad \text{by Schwarz}$$

$$\geq \left|\sum_b \mathrm{tr}\,\rho_0^{1/2}U\rho_1^{1/2}E_b\right|$$

$$= \left|\mathrm{tr}\,U\rho_1^{1/2}\rho_0^{1/2}\right| \qquad \text{because } \sum_b E_b = I$$

Now maximize over U to tighten.

Get $F \geq \mathrm{tr}\sqrt{\rho_0^{1/2}\rho_1\rho_0^{1/2}}$.

Equality achieved by suitable $\{E_b\}$.

Fidelity

$$F(\rho_0, \rho_1) = \mathrm{tr}\sqrt{\rho_0^{1/2}\rho_1\rho_0^{1/2}}$$

YUK.!

Properties

1) $0 \leq F(\rho_0, \rho_1) \leq 1$

$=$ iff $\rho_0\rho_1 = 0$ $\quad\quad$ $=$ iff $\rho_0 = \rho_1$

2) symmetric $0 \leftrightarrow 1$

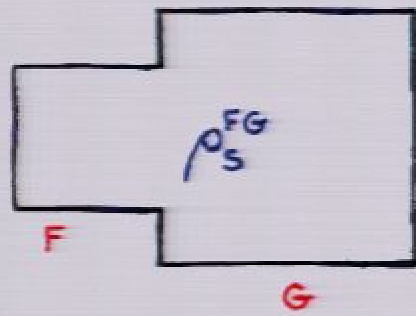
3) invariant $\rho_0 \rightarrow U\rho_0 U^\dagger$

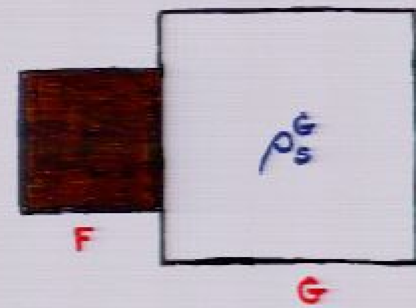
$\rho_1 \rightarrow U\rho_1 U^\dagger$

5) $F(\rho_0 \otimes \sigma_0, \rho_1 \otimes \sigma_1) = F(\rho_0, \rho_1) F(\sigma_0, \sigma_1)$

and

Partial Trace Property



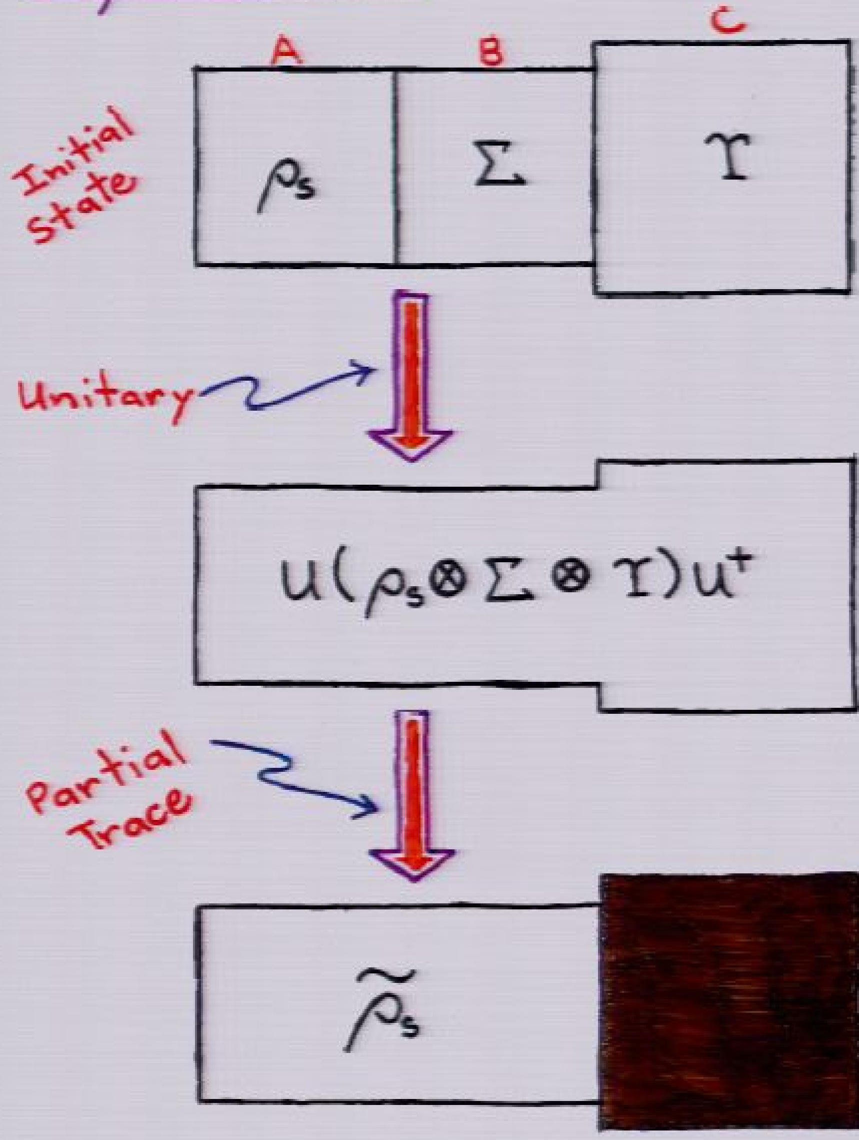


$$\rho_s^G = \mathrm{tr}_F(\rho_s^{FG})$$

Then

$$F(\rho_0^{FG}, \rho_1^{FG}) \leq F(\rho_0^G, \rho_1^G)$$

Dynamics



$$\tilde{\rho}_s = \mathrm{tr}_C\left[U(\rho_s \otimes \Sigma \otimes \Upsilon)U^\dagger\right]$$

Broadcasting Requires:

$$F_A(\rho_0,\rho_1) = F(\tilde{\rho}_0,\tilde{\rho}_1) = F_B(\rho_0,\rho_1)$$

$\Rightarrow$ relation on optimal measurements

another story !

Hard Part

Implies

$$[\rho_0,\rho_1] = 0 .$$

✓

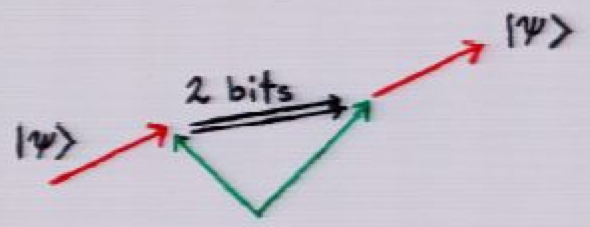
End of story.

Unknown States

We <u>try</u> to clone them:

$$|\psi\rangle \not\longrightarrow |\psi\rangle|\psi\rangle$$

We teleport them:

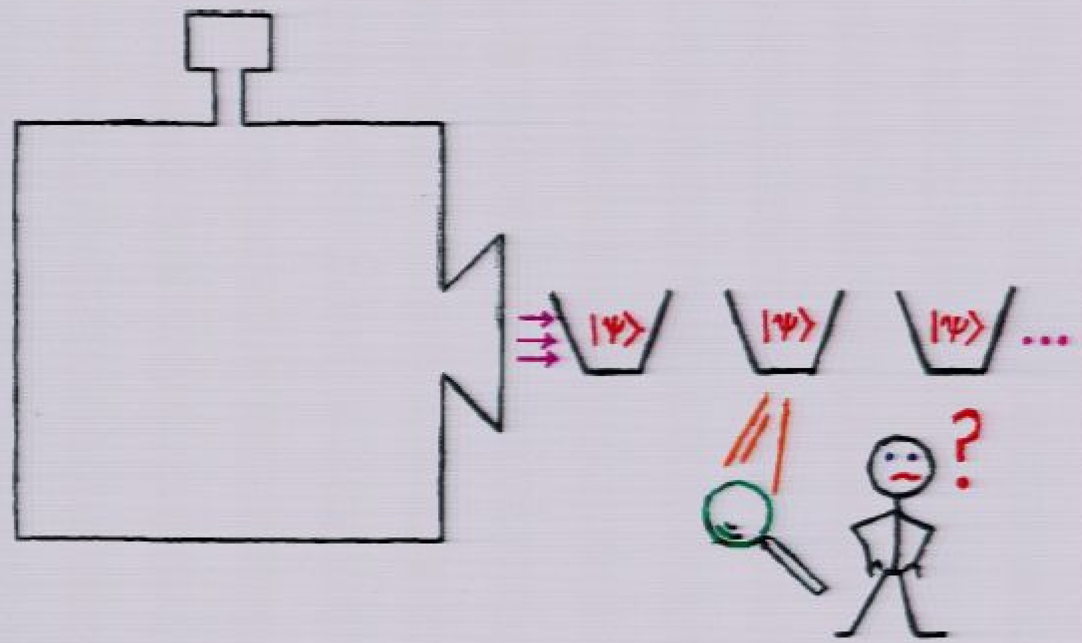


We protect them:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\longrightarrow \alpha\left(|000\rangle + |111\rangle\right)^{\otimes 3} + \beta\left(|000\rangle - |111\rangle\right)^{\otimes 3}$$

Quantum State Tomography



Tomography on a Qubit

Operator space is a linear vector space in its own right.

$(\hat{A},\hat{B}) = \mathrm{tr}\,\hat{A}^\dagger\hat{B}$ — inner product

If state is $\hat{\pi} = |\psi\rangle\langle\psi|$,

"projections"

$$1 = |\langle\psi|\psi\rangle|^2 = \mathrm{tr}\,\hat{\pi}\hat{I}$$

$$\overline{\sigma_x} = \langle\psi|\hat{\sigma}_x|\psi\rangle = \mathrm{tr}\,\hat{\pi}\hat{\sigma}_x$$

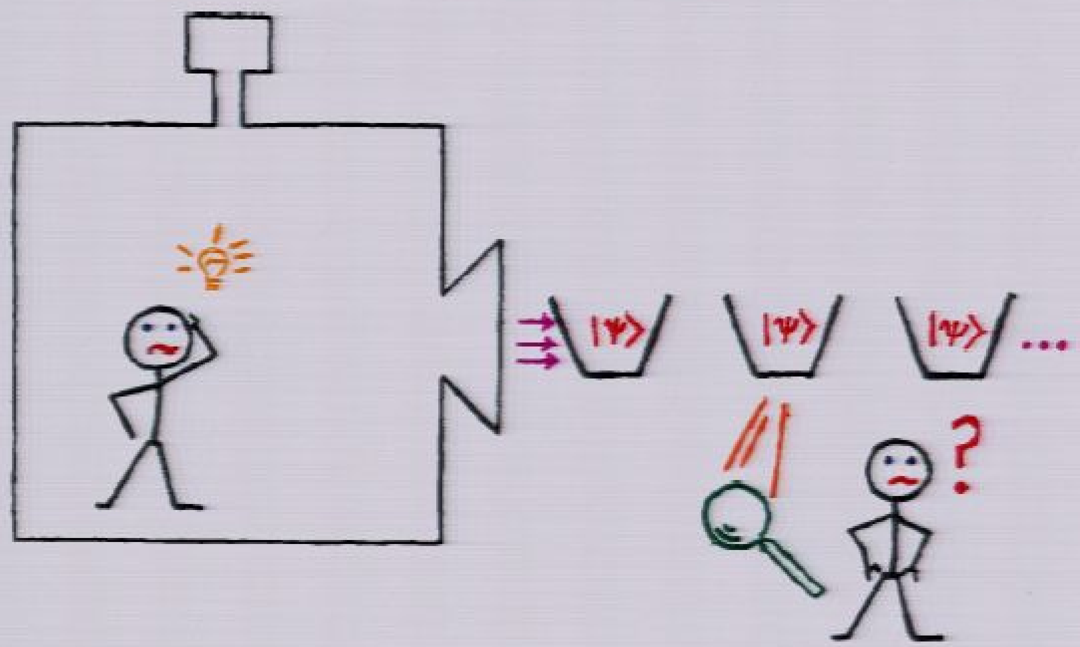
$$\overline{\sigma_y} = \langle\psi|\hat{\sigma}_y|\psi\rangle = \mathrm{tr}\,\hat{\pi}\hat{\sigma}_y$$

$$\overline{\sigma_z} = \langle\psi|\hat{\sigma}_z|\psi\rangle = \mathrm{tr}\,\hat{\pi}\hat{\sigma}_z$$

fix the state uniquely.

$\hat{I}, \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ — linearly indep.

Quantum State Tomography



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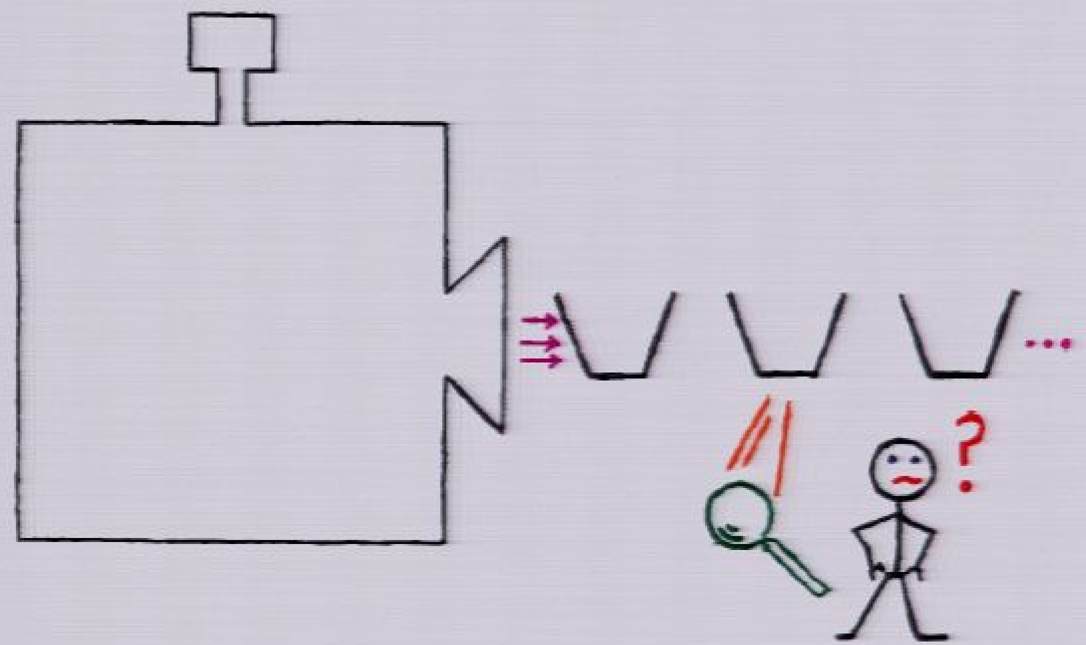
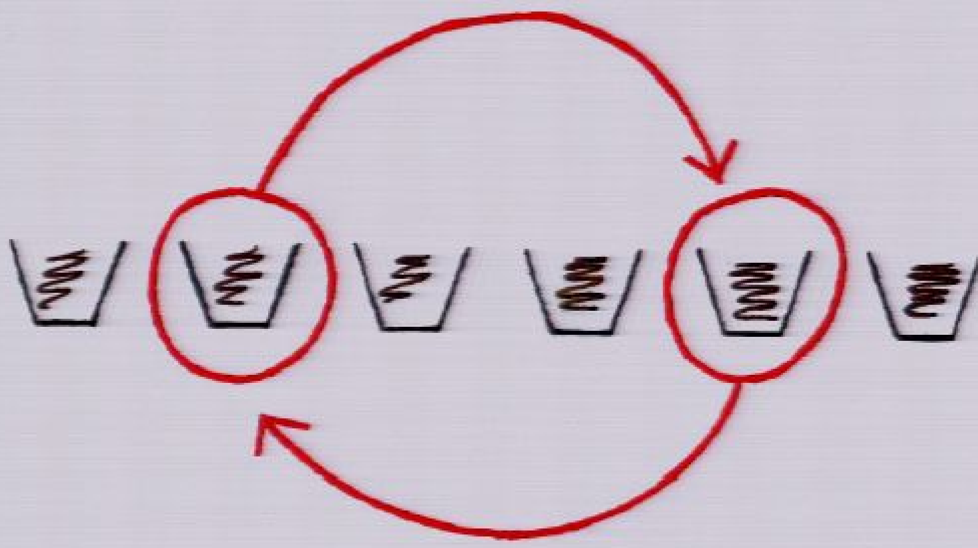


Essence is that $\hat{\rho}_o$ evolve toward $\hat{\rho} \otimes \hat{\rho} \otimes \hat{\rho} \otimes \cdots$ with mmt.

nothing left to be "learned"

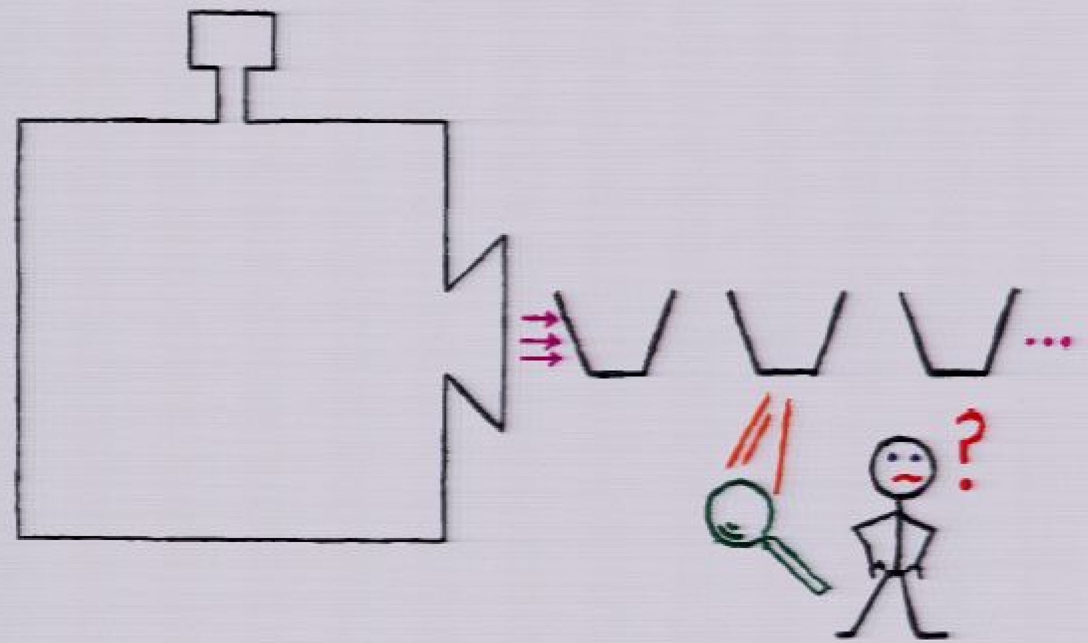
Condition for Tomography



$$\hat{\rho}^{(n)} \longrightarrow \hat{\rho}^{(n)}$$

[redacted]

Quantum State Tomography



Essence is that $\hat{\rho}_0$ evolve toward $\hat{\rho} \otimes \hat{\rho} \otimes \hat{\rho} \otimes \cdots$ with mmt.

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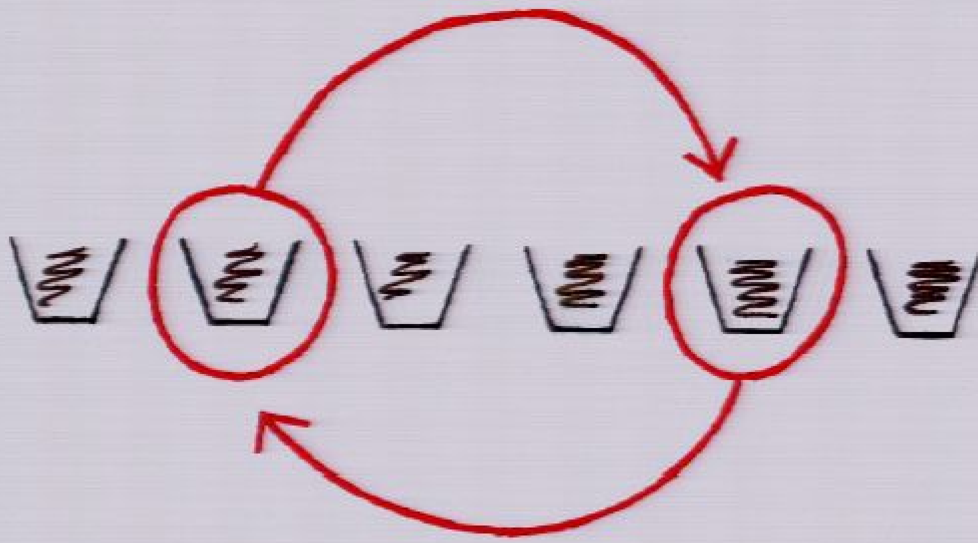


$$\hat{\rho}^{(n)} \longrightarrow \hat{\rho}^{(n)}$$

Quantum de Finetti Theorem

- Technical way to exorcise box boy.

Def: Density operator $\rho^{(n)} \in \mathcal{L}(\mathcal{H}^{\otimes n})$ is n-exchangeable if permutation invariant.

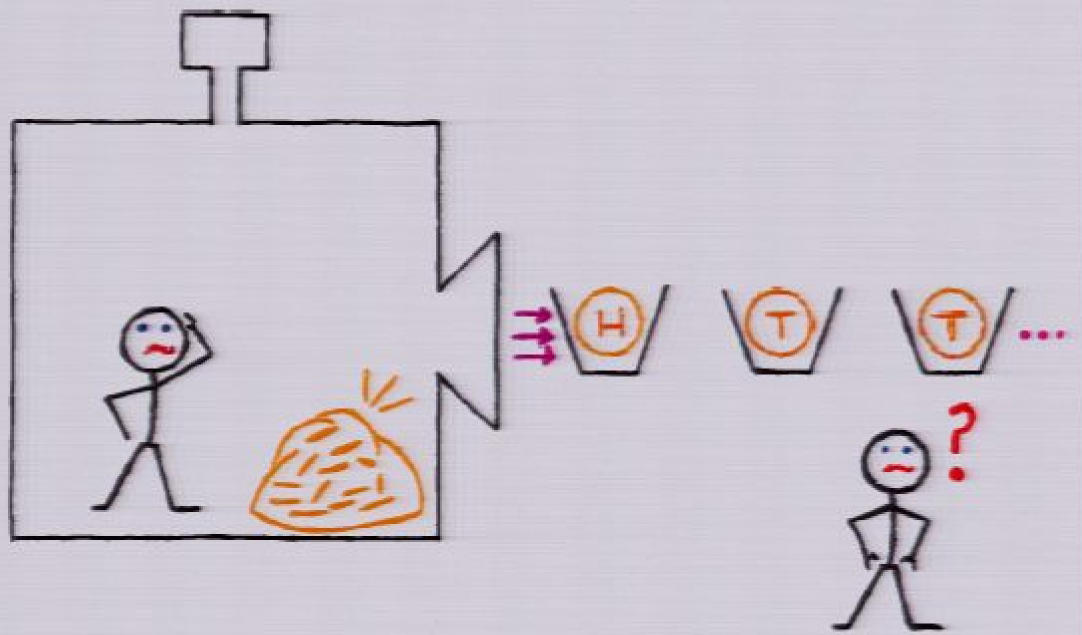
Def: $\{\rho^{(n)}\}_{n=1}^{\infty}$ is an exchangeable sequence if 1) each $\rho^{(n)}$ is n-exchangeable, and
2) $\rho^{(n)} = \mathrm{tr}_{n+1}\, \rho^{(n+1)}$ for all n.

Theorem:

$\{\rho^{(n)}\}_{n=1}^{\infty}$ forms an exchangeable sequence iff there exists a probability density $P(\rho)$ such that

$$\rho^{(n)} = \int P(\rho)\, \rho^{\otimes n}\, d\rho\,.$$

Caves, CAF, Schack, J. Math. Phys. **43** (2002) 4537.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The Born Rule

Given ρ and $\{E_i\}$,

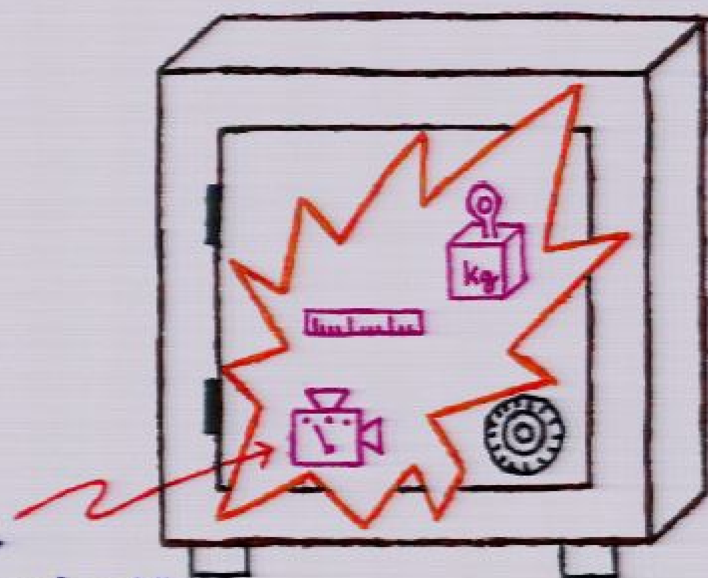
quantum state

POVM measurement

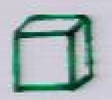
$$\boxed{p(i) = \mathrm{tr}\,\rho E_i}$$

"The Born Rule"

Bureau of Standards



the
"standard"
quantum measurement



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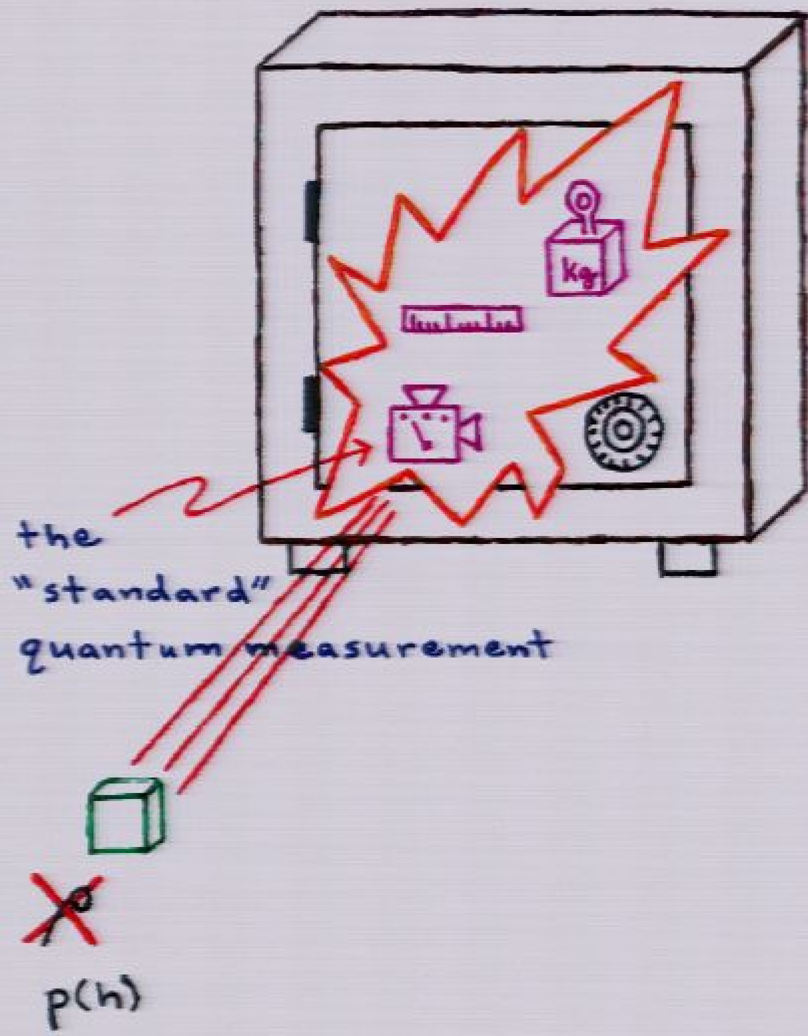
Bureau of Standards



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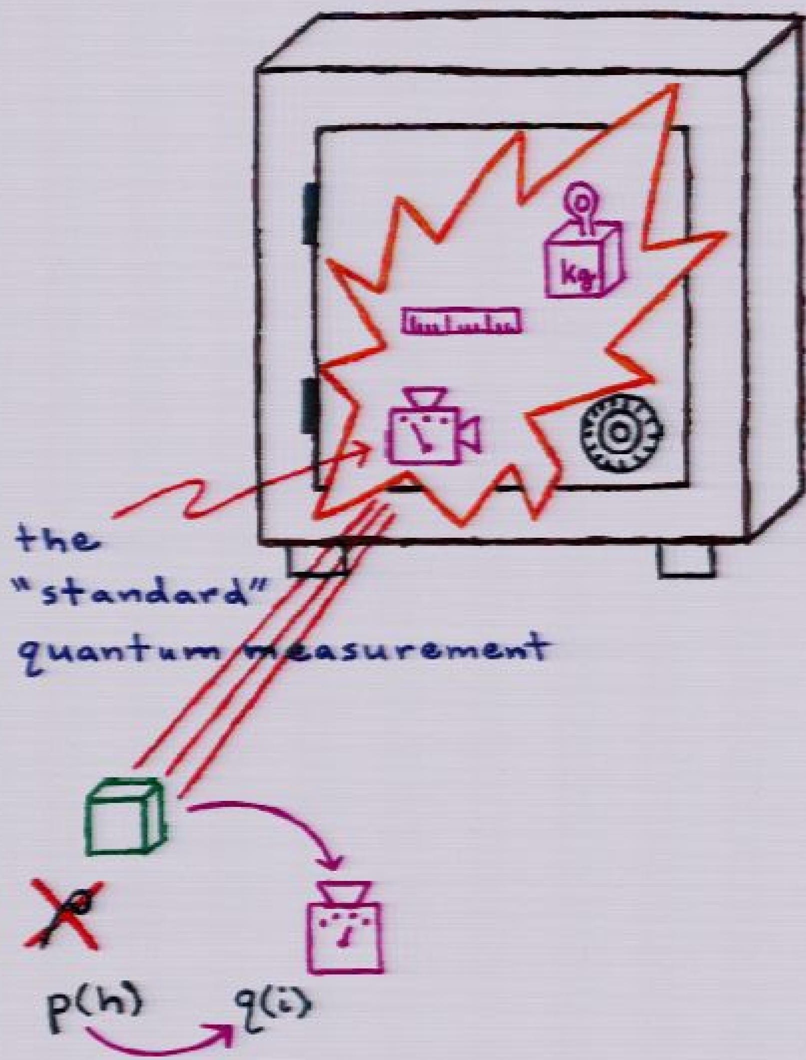


Measure of Orthonormality

Appleby, Dang, CAF, 0707.2071

Suppose A_i, $i = 1, \ldots, d^2$

positive semi-definite.

And $\operatorname{tr} A_i^2 = 1$.

"Orthonormality"

$$K = \sum_{i \neq j} (\operatorname{tr} A_i A_j)^2$$

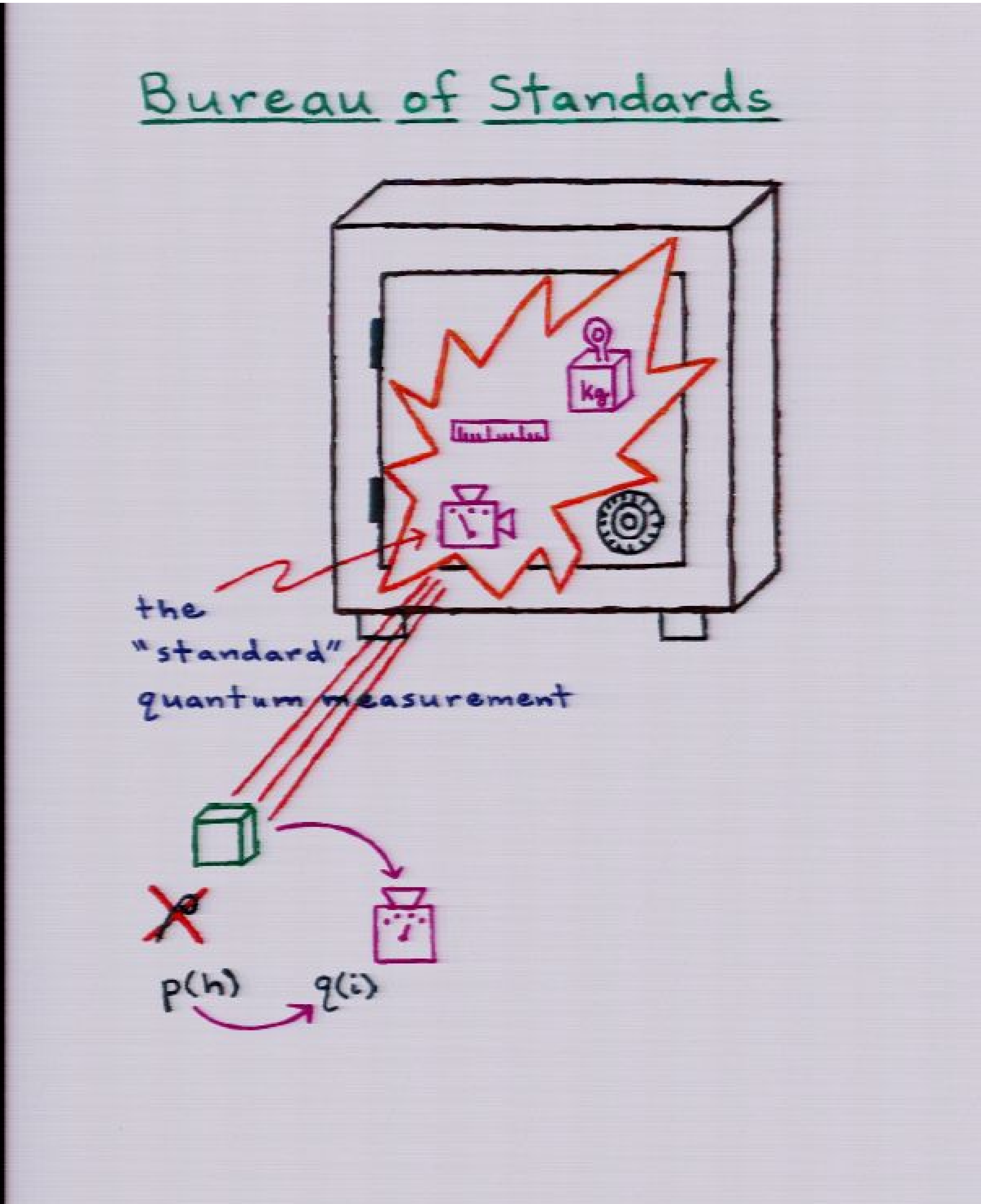
smaller the better

Can prove

$$K \geq \frac{d^2(d-1)}{d+1}$$ with = iff

$$\operatorname{tr} A_i A_j = \frac{1}{d+1} \quad \forall i \neq j$$

A_i — rank-1



Bureau of Standards
kg
the
"standard"
quantum measurement
p(h)
q(i)

Measure of Orthonormality

Appleby, Dang, CAF, 0707.2071

Suppose A_i , $i = 1, \ldots, d^2$

positive semi-definite.

And $\operatorname{tr} A_i^2 = 1$.

"Orthonormality"

$$K = \sum_{i \neq j} (\operatorname{tr} A_i A_j)^2$$

smaller the better

Can prove

$$K \geq \frac{d^2(d-1)}{d+1} \quad \text{with } = \text{ iff}$$

$$\operatorname{tr} A_i A_j = \frac{1}{d+1} \quad \forall i \neq j$$

$$A_i - \text{rank-1}$$

A Very Fundamental Mmt ?

Suppose d^2 projectors $\Pi_i = |\psi_i\rangle\langle\psi_i|$
satisfying

$$\mathrm{tr}\, \Pi_i \Pi_j = \frac{1}{d+1} \quad , \quad i \neq j$$

exist. ← Called SIC.

Can prove:

1) the Π_i linearly independent

2) $\sum_i \frac{1}{d} \Pi_i = I$

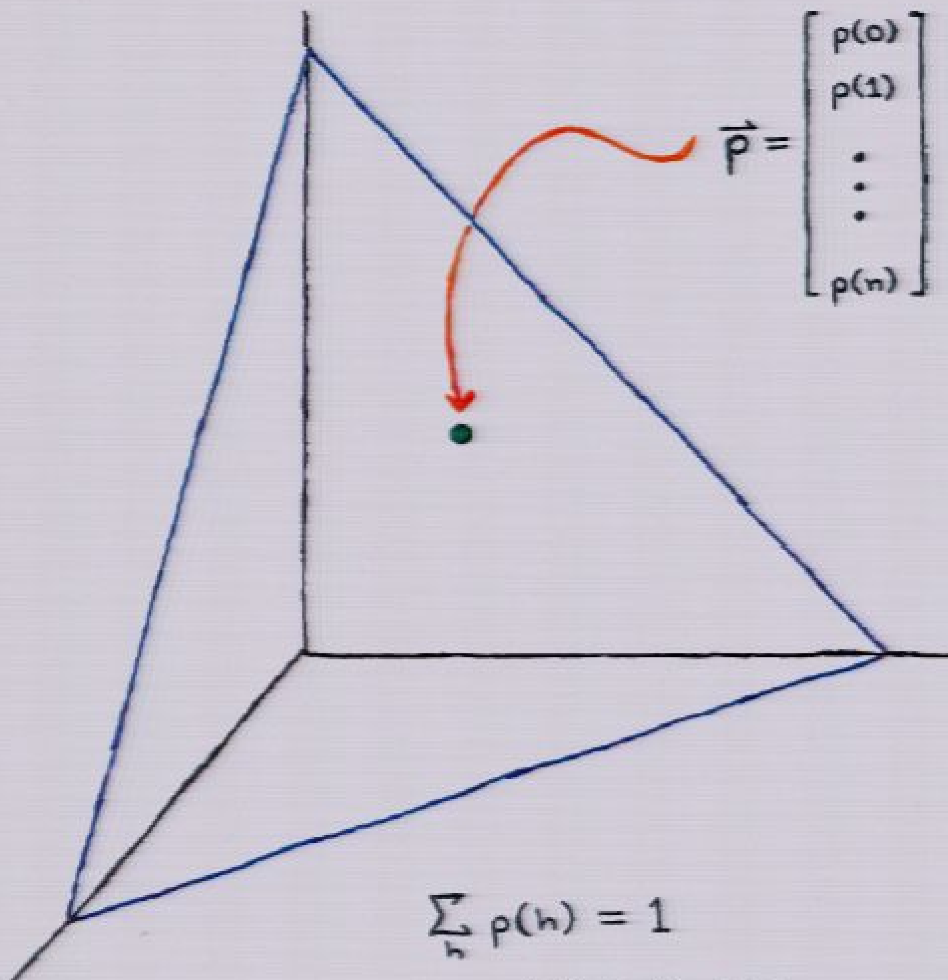
So good for Bureau of Standards.

Also
$$p(i) = \frac{1}{d} \mathrm{tr}\, \rho \Pi_i$$

$$\boxed{\rho = \sum_i \left[(d+1) p(i) - \frac{1}{d} \right] \Pi_i}$$

CAF, QIC 4, 467 (2004)

Probability Simplex



$$\sum_h p(h) = 1$$

$$p(h) \geq 0 \quad \forall h$$

If They Exist ...

1) the $|\psi_i\rangle$ form a set of states maximally sensitive to eavesdropping in quantum crypto settings

CAF, quant-ph/0404122

2) are optimal for some natural cases of quantum tomography

A. J. Scott, quant-ph/0604049

3) in prime d form "minimum uncertainty" states, in analogy to coherent states, for complete sets of mutually unbiased bases

Appleby, Dang, CAF, 0707.2071

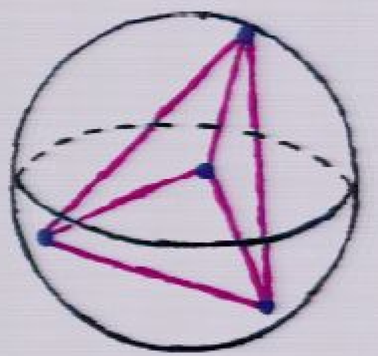
But do they

EXIST

?

SIC Sets

dimension 2



any regular tetrahedron

dimension 3

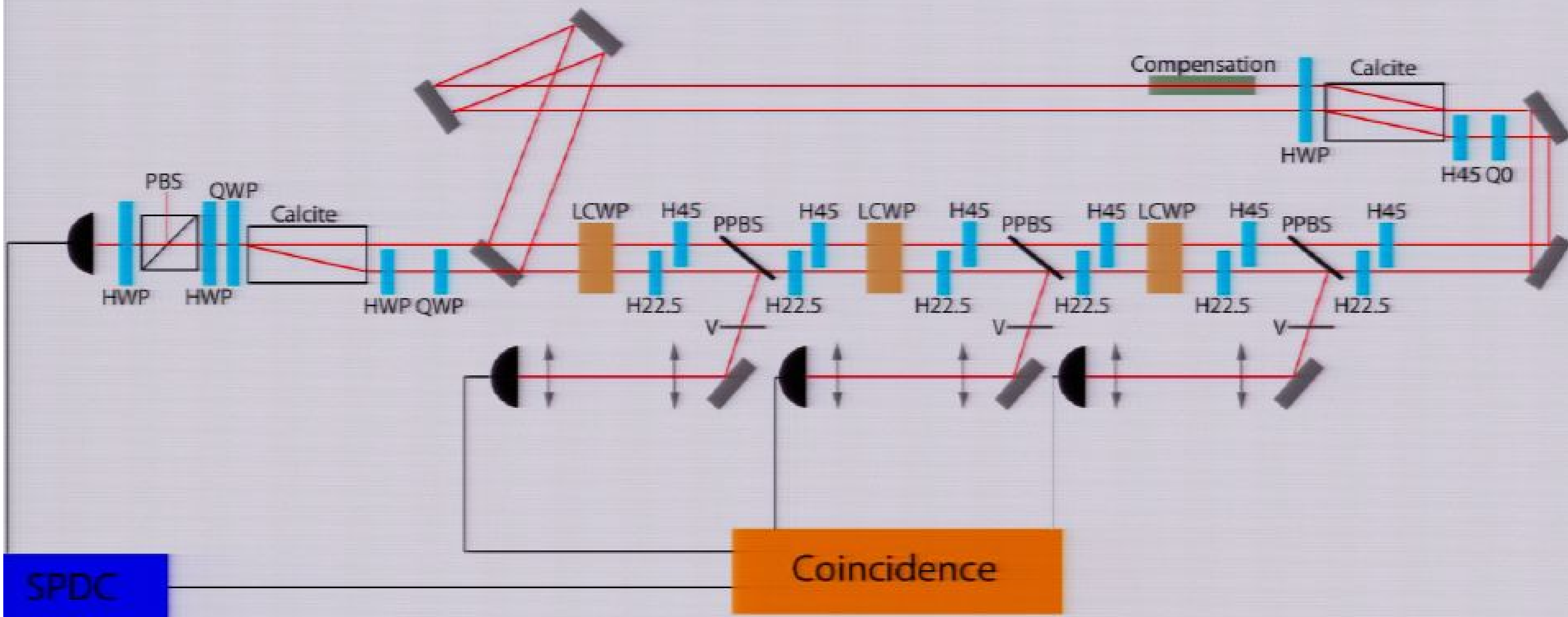
Let $\omega = e^{\frac{2\pi i}{3}}$.

$$\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ \omega \\ -\bar{\omega} \end{bmatrix} \quad \begin{bmatrix} 0 \\ \bar{\omega} \\ -\omega \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 0 \\ \bar{\omega} \end{bmatrix} \quad \begin{bmatrix} -1 \\ 0 \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -\omega \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -\bar{\omega} \\ 0 \end{bmatrix}$$

Medendorp, Torres-Ruiz, Shalm, CAF, Steinberg, at QELS 2010



Dimension 6

$$|\psi\rangle = \frac{\alpha}{3\sqrt{2}}\begin{pmatrix} f_+ \\ \sigma^5 f_- \\ \sigma^8 f_+ \\ \sigma^{-3} f_- \\ \sigma^8 f_+ \\ \sigma^5 f_- \end{pmatrix} + \frac{\beta_- e^{i\theta_+}}{3\sqrt{2}}\begin{pmatrix} \sigma^8 f_- \\ \sigma^{-7} f_+ \\ f_- \\ \sigma^{-7} f_+ \\ \sigma^8 f_- \\ \sigma^9 f_- \end{pmatrix} + \frac{\beta_+ e^{i\theta_-}}{3\sqrt{2}}\begin{pmatrix} \sigma^8 f_- \\ \sigma^9 f_+ \\ \sigma^8 f_- \\ \sigma^{-7} f_+ \\ f_- \\ \sigma^{-7} f_- \end{pmatrix}$$

where

$$\sigma = e^{i\pi/12}$$

$$f_\pm = \sqrt{3 \pm \sqrt{3}}$$

$$g = \sqrt{6\sqrt{21} - 18}$$

$$\alpha = \sqrt{\frac{7 - \sqrt{21}}{14}}$$

$$\beta_\pm = \sqrt{\frac{7 + \sqrt{21} \pm \sqrt{14\sqrt{21} - 42}}{28}}$$

$$e^{i\theta_\pm} = \frac{1}{2}\left(\sqrt{46 - 6\sqrt{21} \mp 6g} \pm i\sqrt{18 + 6\sqrt{21} \pm 6g}\right)^{\frac{1}{3}}$$

Evidence for Existence

Analytical Constructions

$d = 2 - 13, \ 14, \ 15, \ 19, \ 24, \ 35, \ 48$

Numerical ($\epsilon \le \cancel{10^{-[illegible]}}$) 10^{-38} !

$d = 2 - \cancel{47} \ 67$

<u>Conditions for Existence</u>

Want $E_j = \frac{1}{d}|\psi_j\rangle\langle\psi_j|$ with

$$\langle\psi_j|\psi_k\rangle = \frac{1}{\sqrt{d+1}}\, e^{i\theta_{jk}}\,.$$

Then a necessary condition on the θ_{jk} follows from:

For any $j \neq \ell$,

$$E_j E_\ell = E_j I E_\ell = \sum_k E_j E_k E_\ell$$

$\Rightarrow$

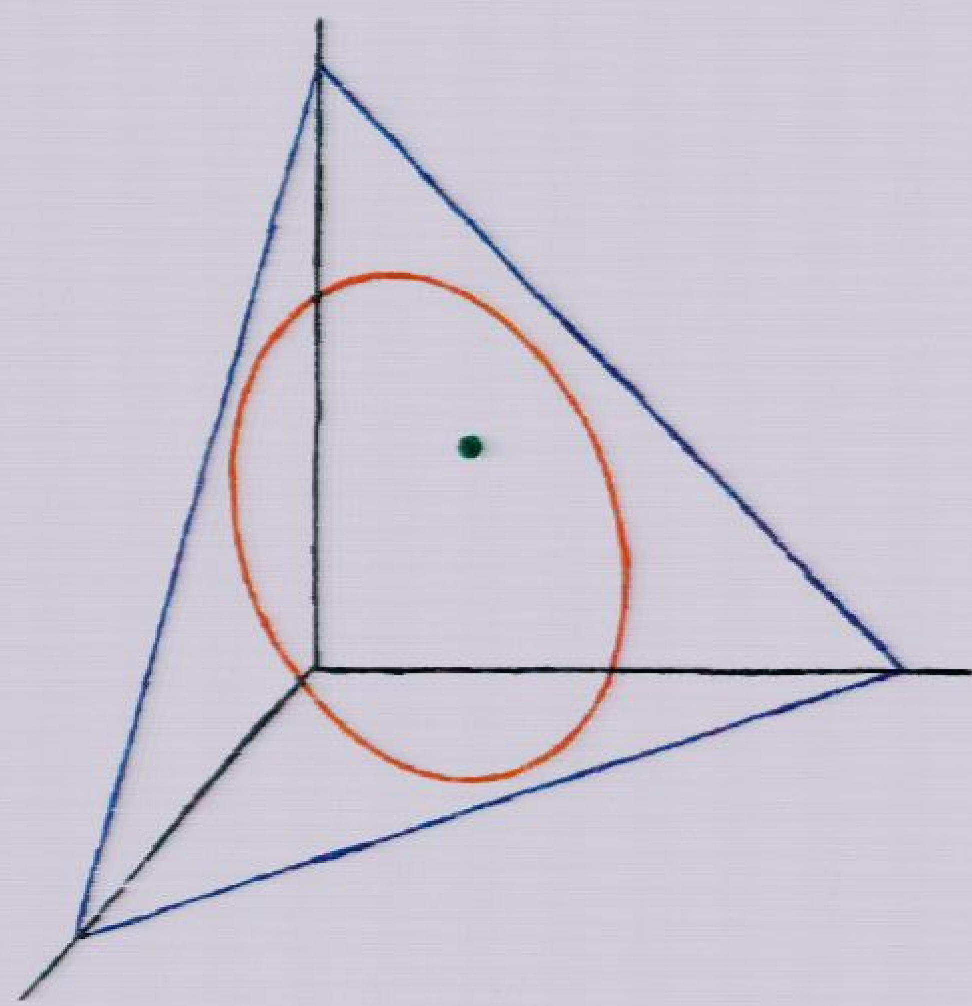
$$\frac{1}{d^2\sqrt{d+1}} = \sum_k \mathrm{tr}\, E_j E_k E_\ell = \frac{1}{d^3} \sum_k \langle\psi_j|\psi_k\rangle\langle\psi_k|\psi_\ell\rangle\langle\psi_\ell|\psi_j\rangle$$

$\Rightarrow$

$$\boxed{\sum_{k\neq j,\ell} e^{i(\theta_{jk}+\theta_{k\ell}-\theta_{j\ell})} = (d-2)\sqrt{d+1} \quad \text{for all } j\neq\ell}$$

Also sufficient!

Appleby, Flammia, CAF, 1001.0004



Group Covariant Case

Let $|\psi\rangle = \sum_{i=0}^{d-1} a_i |i\rangle$.

Generate $|\psi_{jk}\rangle$ in Renes et al way.

Then

$$K = d^3 \sum_{jn} \left| \sum_s a_s^* a_{s+j} a_{s+n} a_{s+n+j}^* \right|^2 - d^2 .$$

By previous theorem $K \geq \frac{d^2(d-1)}{d+1}$.

Get equality iff

$$\boxed{\sum_s a_s^* a_{s+j} a_{s+n} a_{s+n+j}^* = \frac{1}{d+1}(\delta_{n0} + \delta_{j0})}$$

Appleby, Dang, CAF, 0707.2071

Pure States in SIC Language

Conditions

$$\rho^\dagger = \rho \quad , \quad \mathrm{tr}\,\rho^2 = \mathrm{tr}\,\rho^3 = 1$$

translate to

$$\sum_i p(i)^2 = \frac{2}{d(d+1)}$$

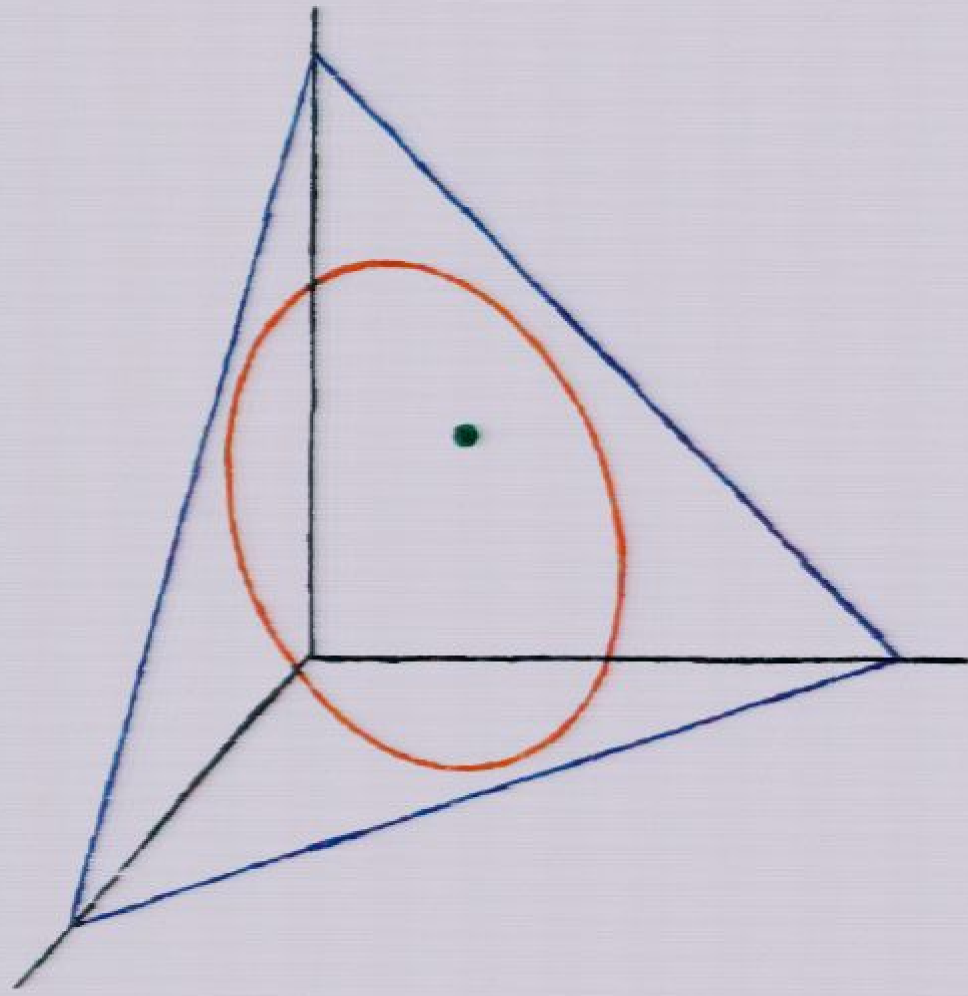
and

$$\sum_{jkl} c_{jkl}\, p(j)\, p(k)\, p(l) = \frac{d+7}{(d+1)^3}$$

where

$$c_{jkl} = \mathrm{Re}\ \mathrm{tr}\, \Pi_j \Pi_k \Pi_l$$

Could these be independently motivatable physical constants?



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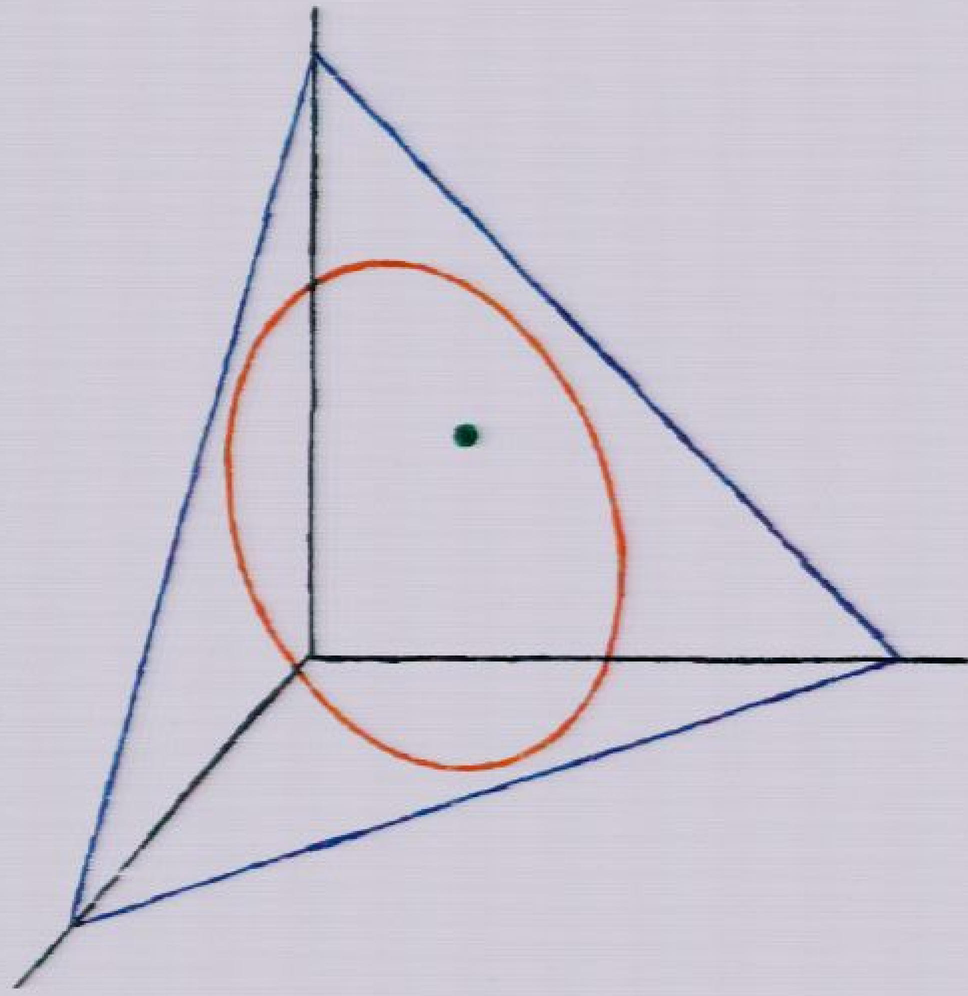
Appleby, Dang, CAF, 0707.2071



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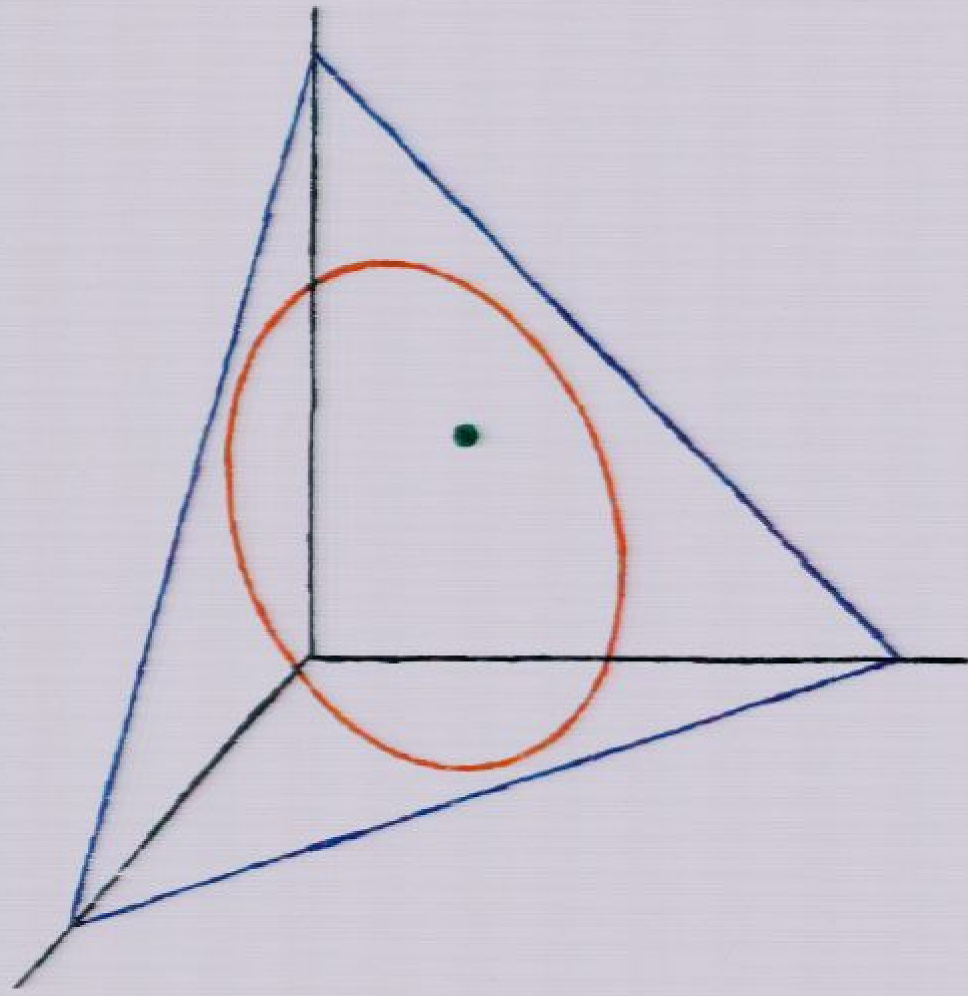

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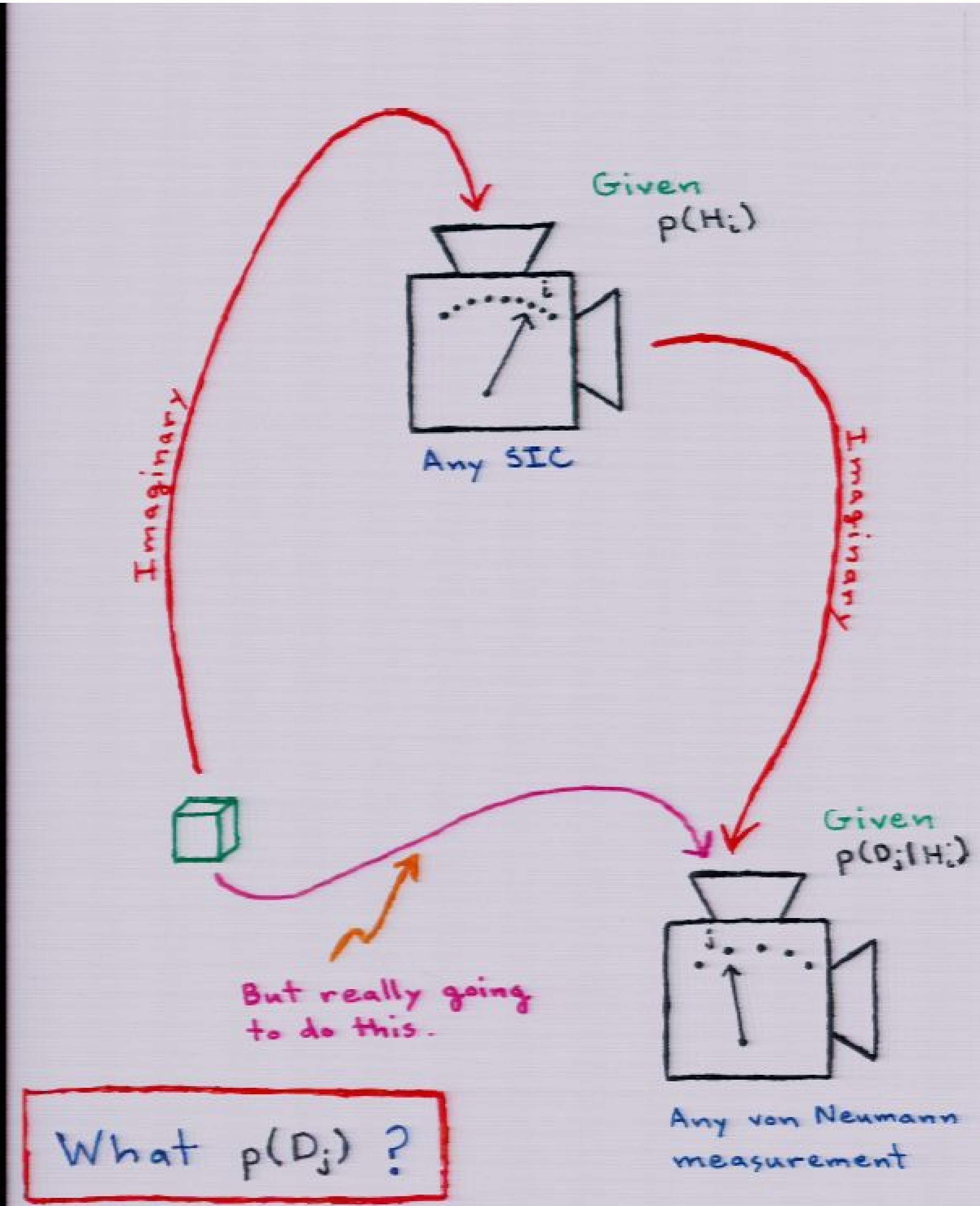
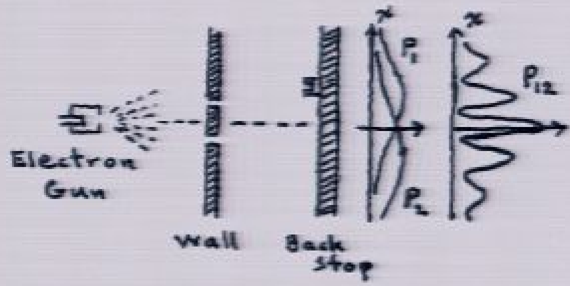

Given
p(H_i)
Any SIC
Imaginary
Imaginary
Given
p(D_j|H_i)
But really going
to do this.
Any von Neumann
measurement
What p(D_j) ?

Feynman 1



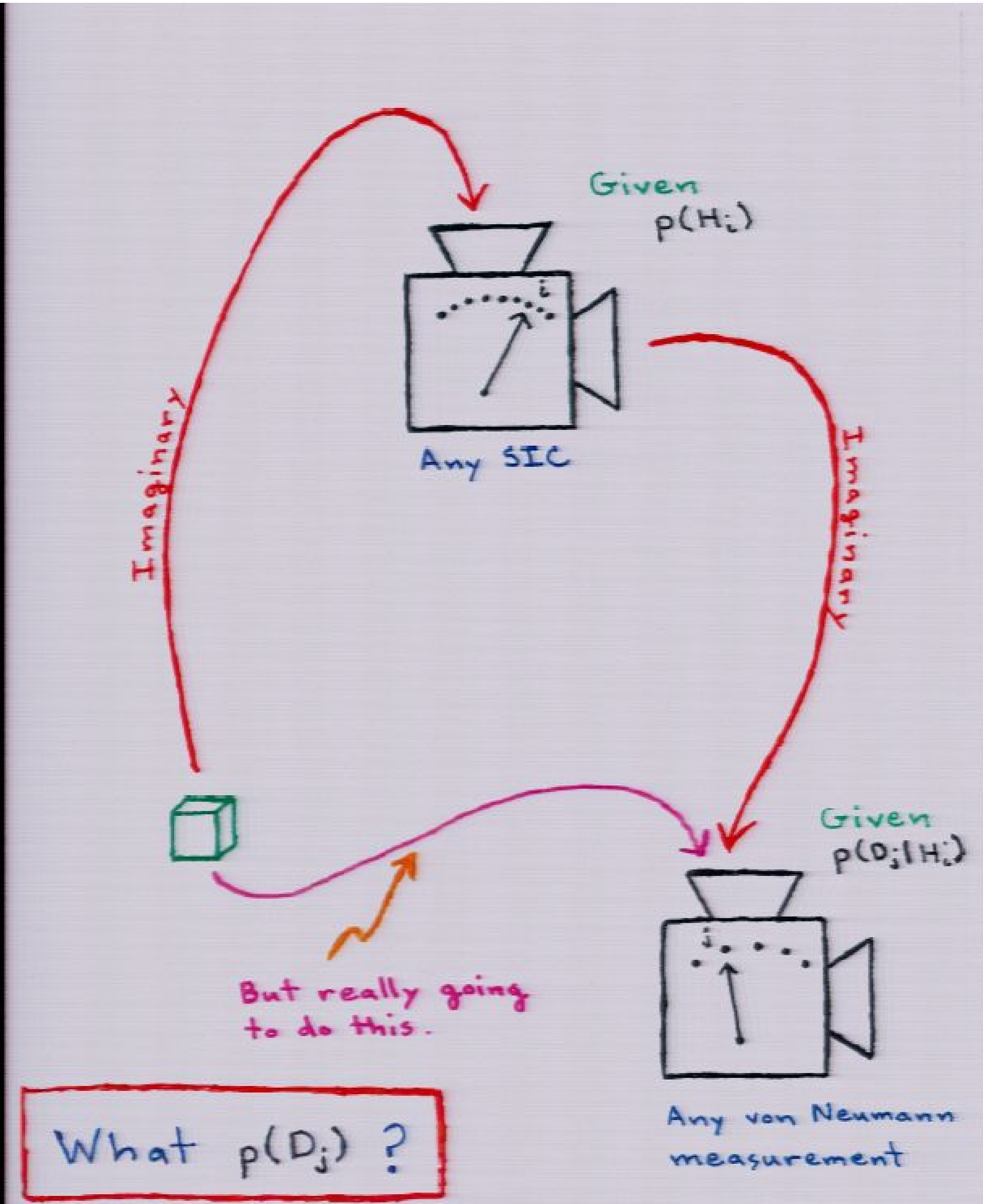
"The result P_{12} obtained with both holes is clearly not the sum of P_1 and P_2, the probabilities for each hole alone. In analogy with our water-wave experiment, we say: 'There is interference.'

For electrons: $P_{12} \neq P_1 + P_2$."

Instead $P_1 = |\varphi_1|^2$, $P_2 = |\varphi_2|^2$, $P_{12} = |\varphi_1 + \varphi_2|^2$.

"We shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery."

— R. P. Feynman, 1964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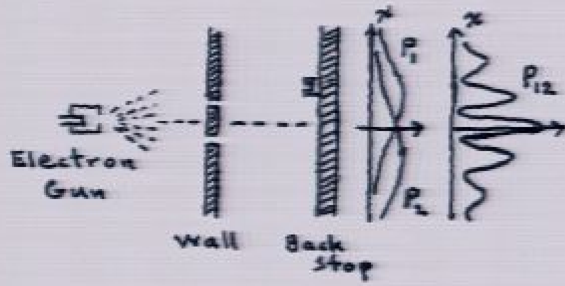

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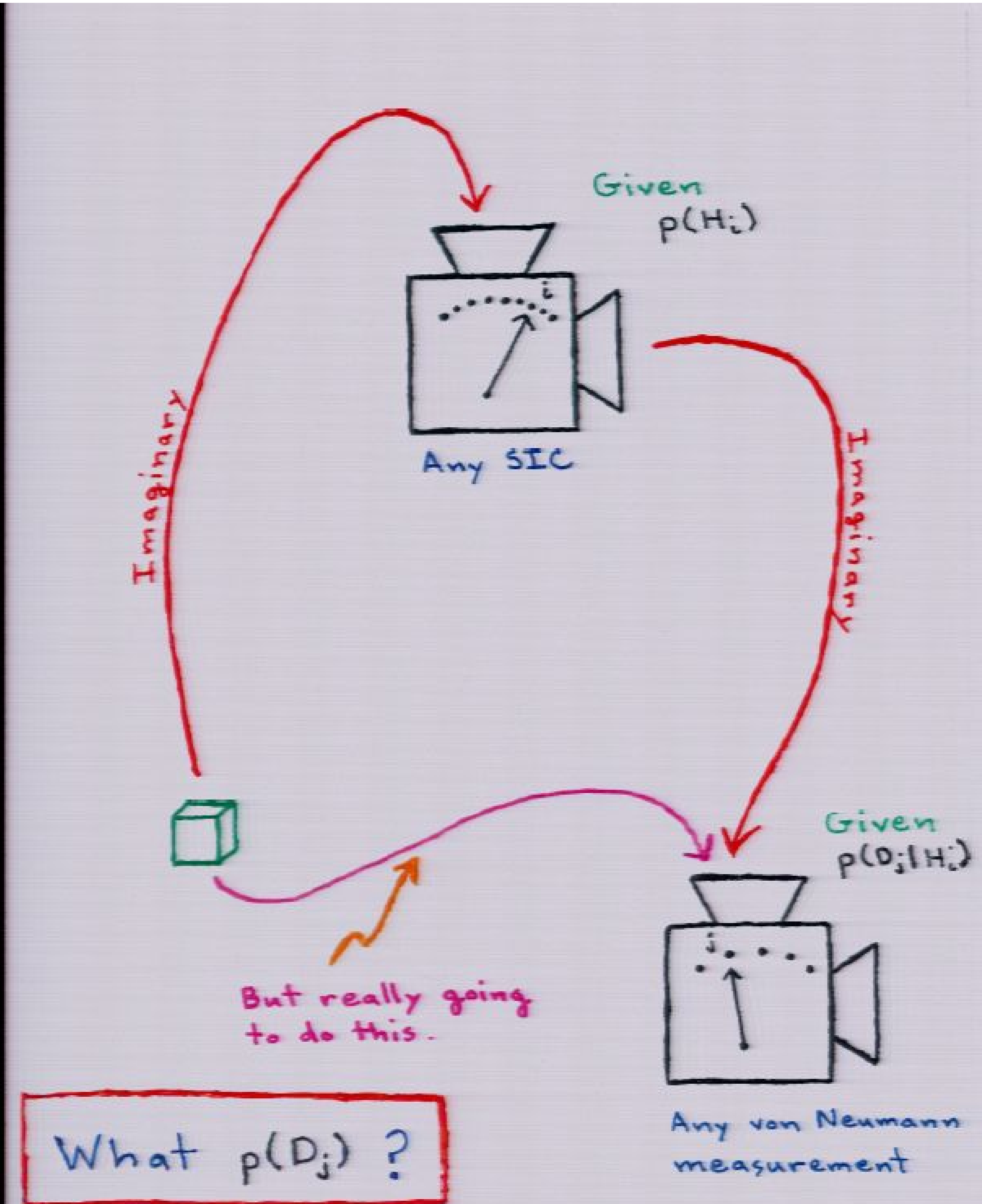
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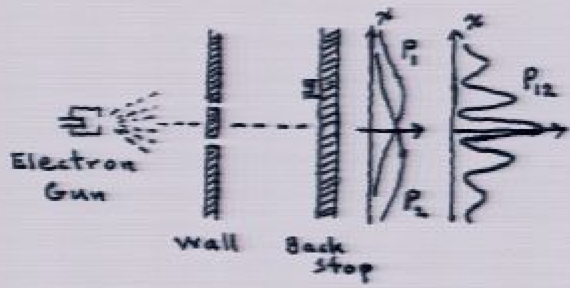

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Magic!

CAF & Schack, 0906.2187 & Rev. Mod. Phys.

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dimensionality of the system

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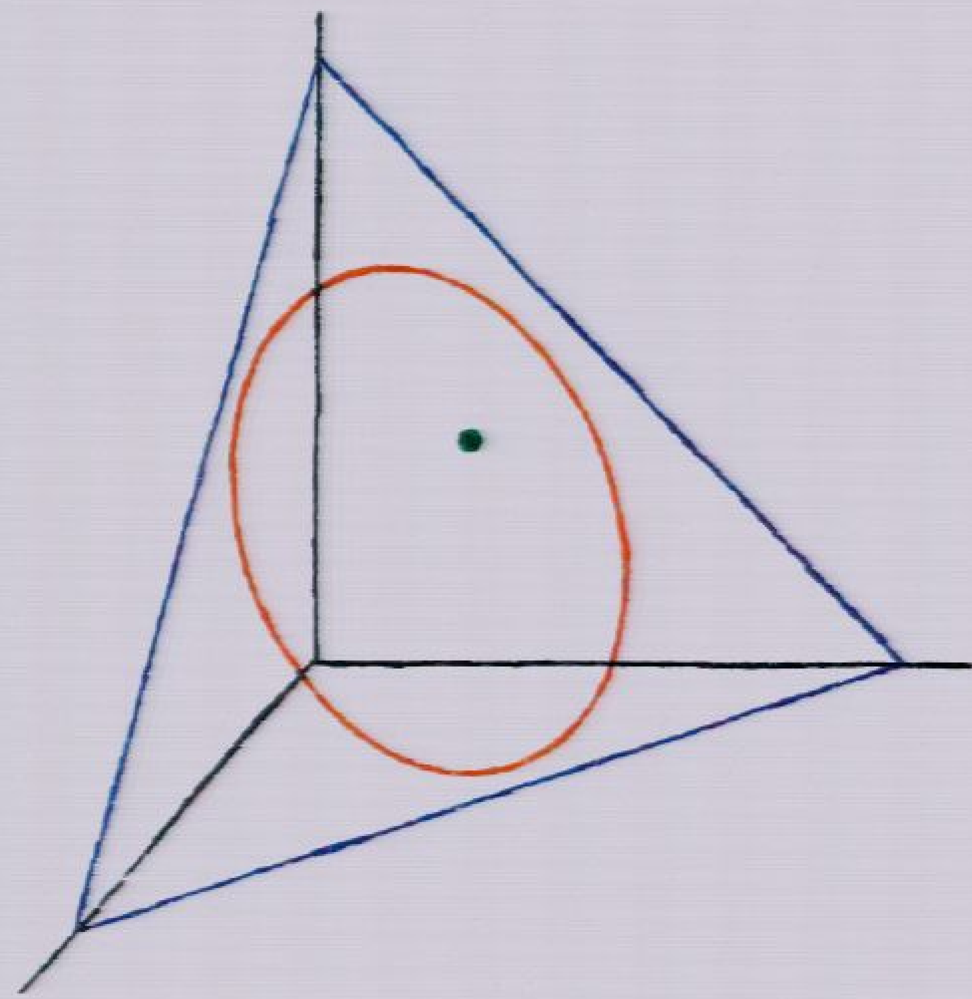

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Homework

containing the $\vec{e}_k$

Call a set $\mathcal{S} \subseteq \Delta_{d^2}$ within the probability simplex

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$$\frac{1}{d(d+1)} \leq \vec{p} \cdot \vec{q} \leq \frac{2}{d(d+1)} ,$$

b) maximal if adding any further $\vec{p} \in \Delta_{d^2}$ makes it inconsistent

Example: If $\mathcal{S}$ is set of quantum states, it is consistent & maximal.

Problem: Characterize all such $\mathcal{S}$; compare to quantum.

Appleby, Ericsson, CAF, 0910.2750

The Future

- Get these damned SICs under control
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arXiv: 1003.???? v1 [quant-ph]

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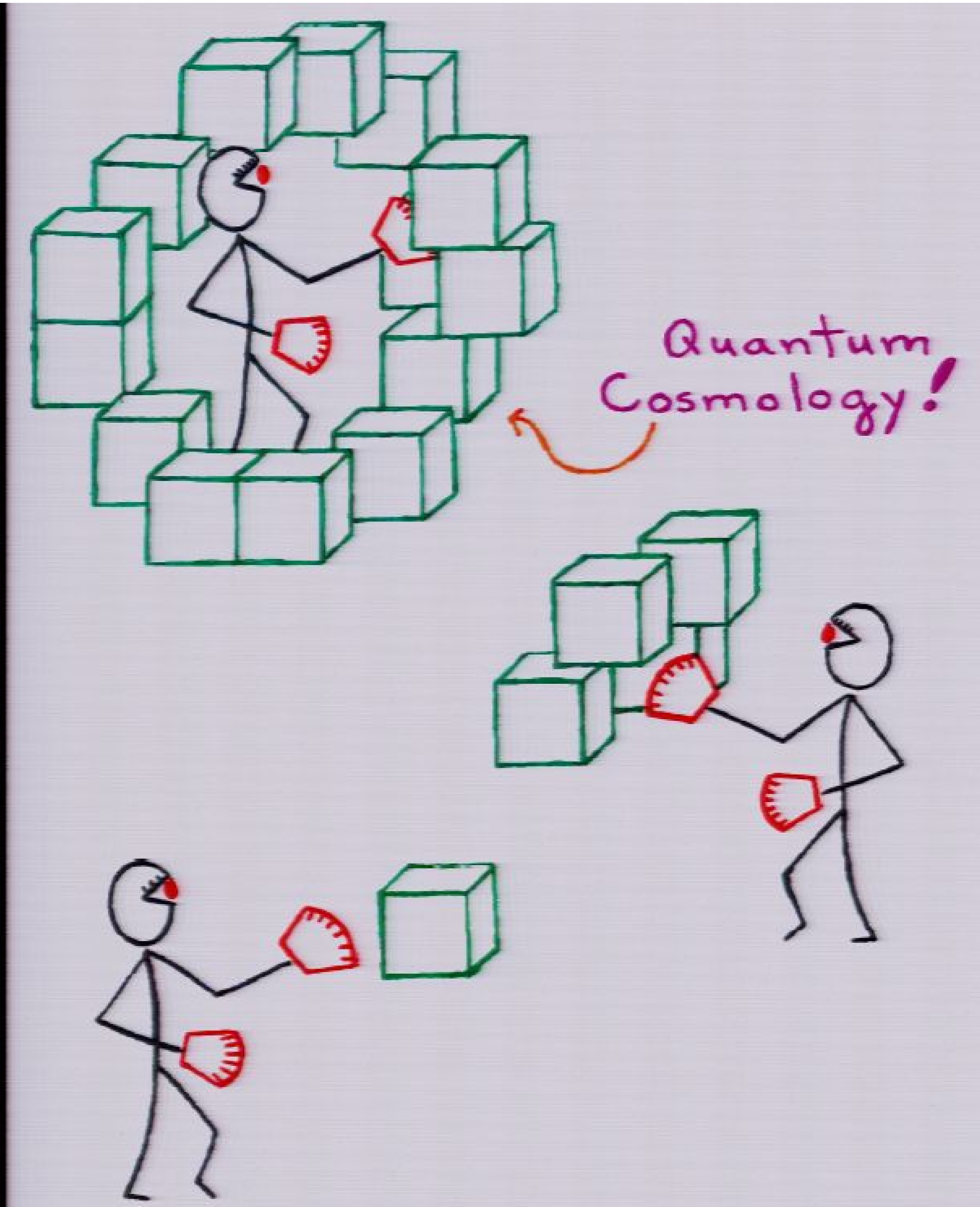
Quantum Cosmology!

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[redacted]

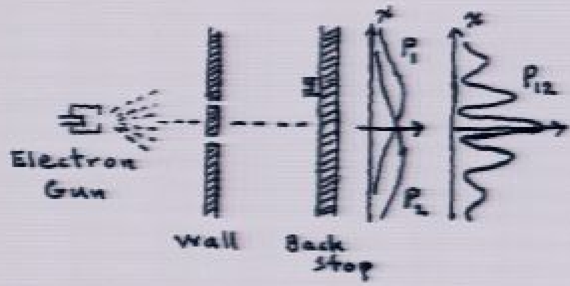
$P(h)$

~~states of pre-existent reality~~

consequences of "measurement" interactions

CAF & Peres, Phys. Today 53 (3), 70 (2000)

Feynman 1



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