

Title: Surprising phenomena in a rich new class of inflationary models

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Abstract: We report on a new class of fast-roll inflationary models. In a part of its parameter space, inflationary perturbations exhibit quite unusual phenomena such as scalar and tensor modes freezing out at widely different times, as well as scalar modes reentering the horizon during inflation. One specific point in parameter space is characterized by extraordinary behavior of the scalar perturbations. Freeze-out of scalar perturbations as well as particle production at horizon crossing are absent. Also the behavior of the perturbations around this quasi-de Sitter background is dual to a quantum field theory in flat space-time. Finally, the form of the primordial power spectrum is determined by the interaction between different modes of scalar perturbations.

# Surprising phenomena in inflation

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arXiv:0911.3397 [astro-ph.CO]



# Outline

Brief review of Inflation

Some fast roll models

Surprising features

The curious model  $p = -2$

Conclusions

# Inflation in a Nutshell

Scalar Field in FRW:

- ▶ FRW Metric

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2, H = \frac{\dot{a}}{a}$$

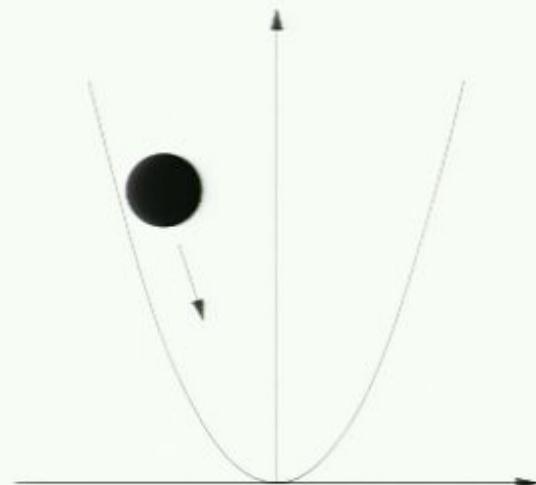
- ▶ Einstein Equations

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2$$

$$H^2 = \frac{1}{3} \left( \frac{1}{2}\dot{\phi}^2 + V(\phi) \right)$$

- ▶ Scalar Field

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = 0$$

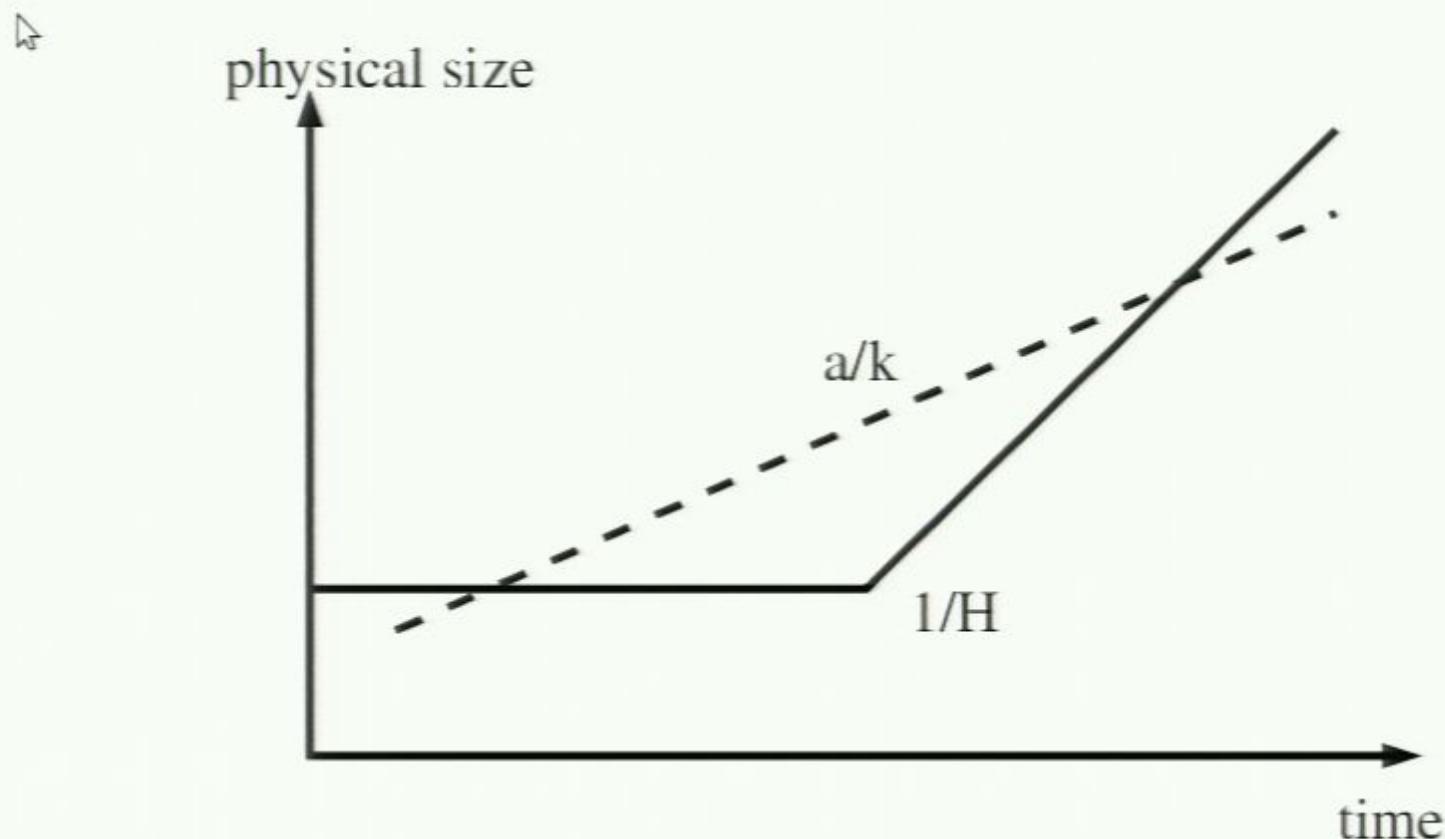


Slowly rolling field:

$$\begin{aligned}\dot{\phi} &\approx 0 \\ \Rightarrow H &= \text{const} \\ \Rightarrow a &= a_0 e^{Ht}\end{aligned}$$

Exponential Expansion

## Successes of Inflation



- ▶ Horizon Problem
- ▶ Flatness Problem
- ▶ Monopole Problem
- ▶ CMB temperature fluctuations

# Perturbations during inflation 1

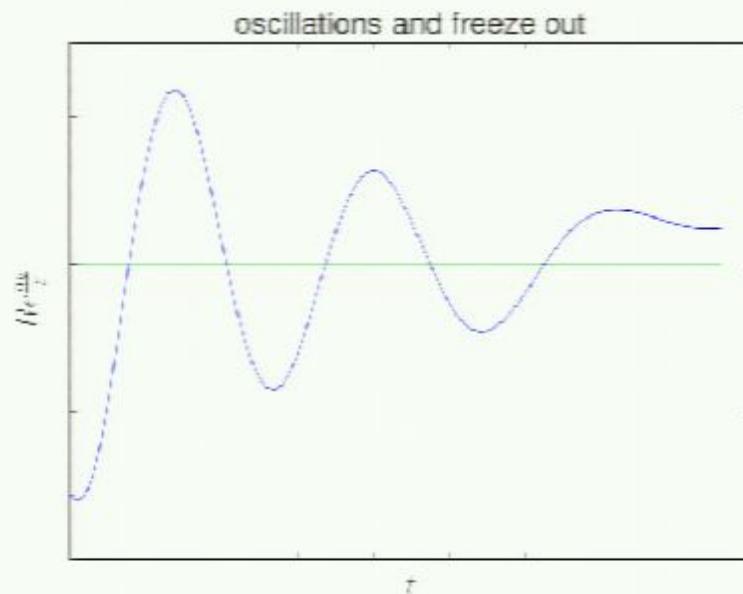
$$u_k''(\tau) + \left( k^2 - \frac{z''}{z} \right) u_k(\tau) = 0$$

- ▶  $u_k = -\frac{\dot{a}\phi}{H} \left( \Psi - \frac{H}{\dot{\phi}} \delta\phi \right)$
- ▶  $\delta\phi$ : perturbation of the inflaton field  $\phi$
- ▶  $\Psi$ : scalar perturbation of the spatial part of the metric
- ▶  $\tau$ : conformal time
- ▶  $z = \sqrt{2\epsilon}a$
- ▶  $0 < \epsilon = -\frac{\dot{H}}{H^2} < 1$ : accelerated expansion

power spectrum  $\mathcal{P}_S = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|_{k=aH} \equiv A_S k^{n_s - 1}$   
scalar spectral index  $n_s - 1 = 2\eta - 4\epsilon$  in slow roll

## Perturbations during inflation 2

$$u_k'' + \left( k^2 - \frac{z''}{z} \right) u_k = 0$$



two regimes

- ▶ harmonic oscillator  $k^2 \gg \frac{z''}{z}$ :  $u_k \propto e^{ik\tau}$ , vacuum

- ▶ freeze out:  $k^2 \ll \frac{z''}{z}$ :  $u_k \propto z$

$$P(k) \propto \left| \frac{u}{z} \right|^2 = \text{const}$$

modes leave the horizon

# Superhorizon Perturbations

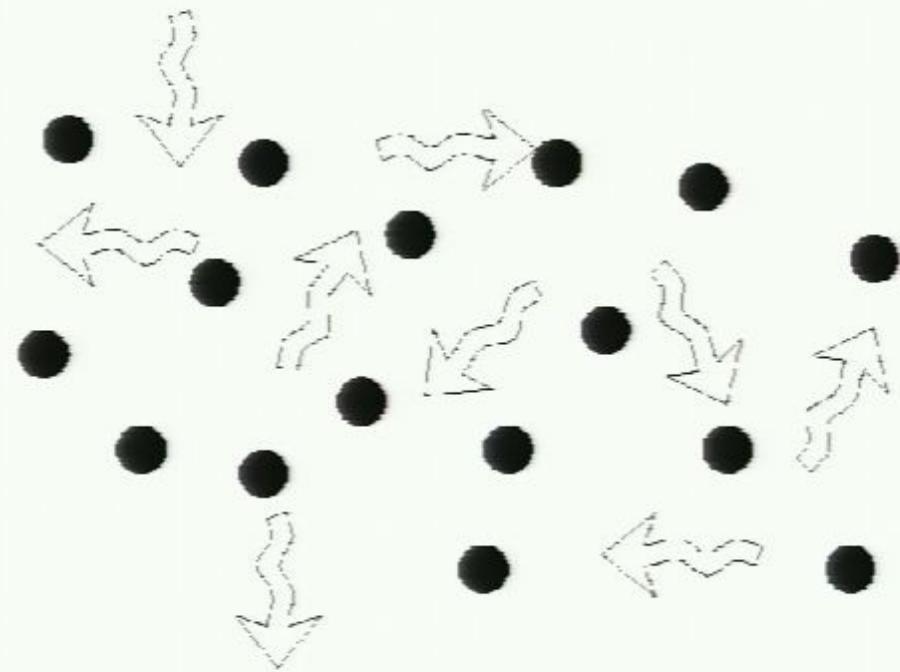


- ▶ inflation ends
- ▶ reheating dumps inflaton energy into SM matter
- ▶ gravity wells from inflationary perturbations
- ▶ photons
- ▶ baryons

# Before Last Scattering

at  $z > 1100$ :

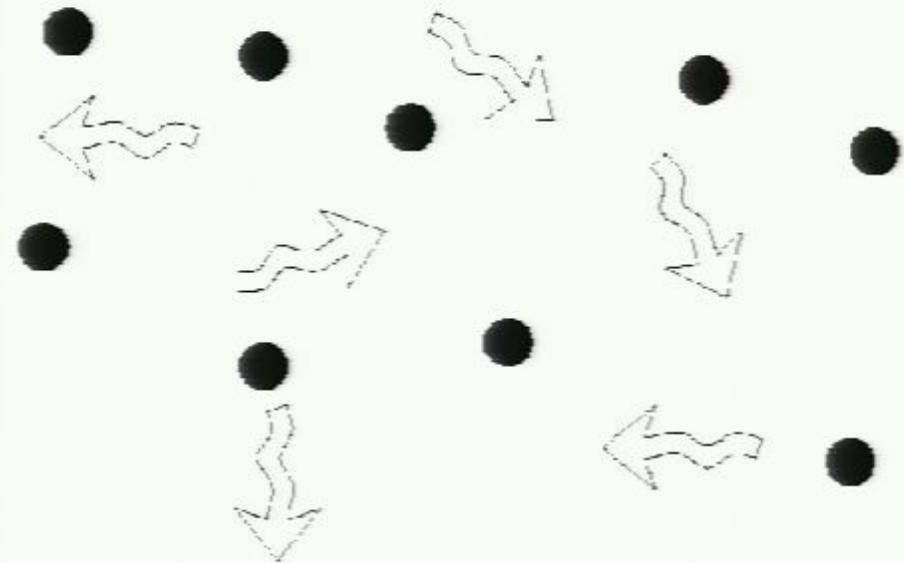
- ▶ photon-baryon plasma
- ▶ curvature perturbations influence homogeneous fluid
- ▶ baryons want to collapse into gravity wells
- ▶ photon pressure resists
- ▶ oscillations in the photon-baryon fluid



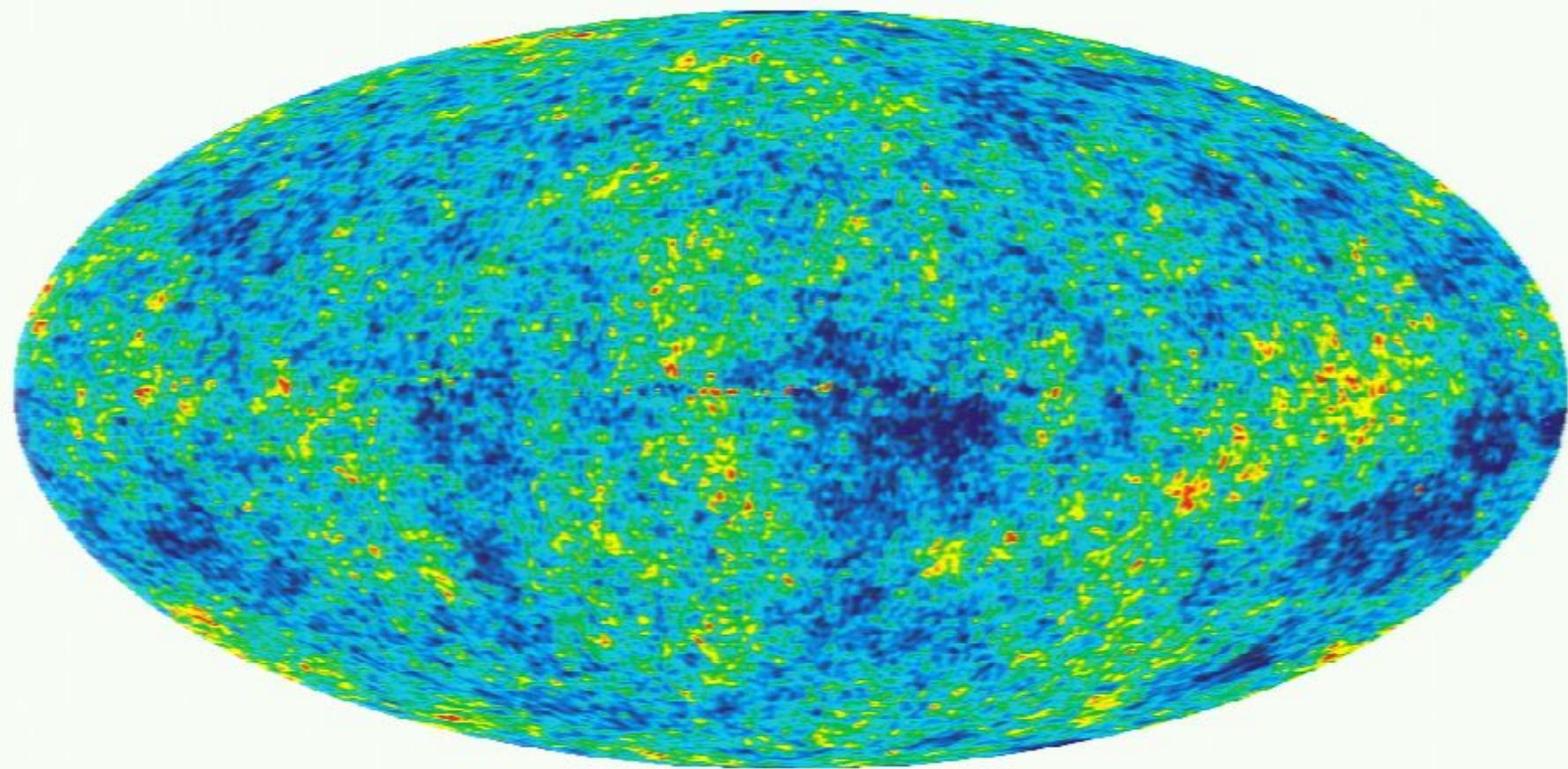
# Last Scattering Surface

at  $z \approx 1100$ :

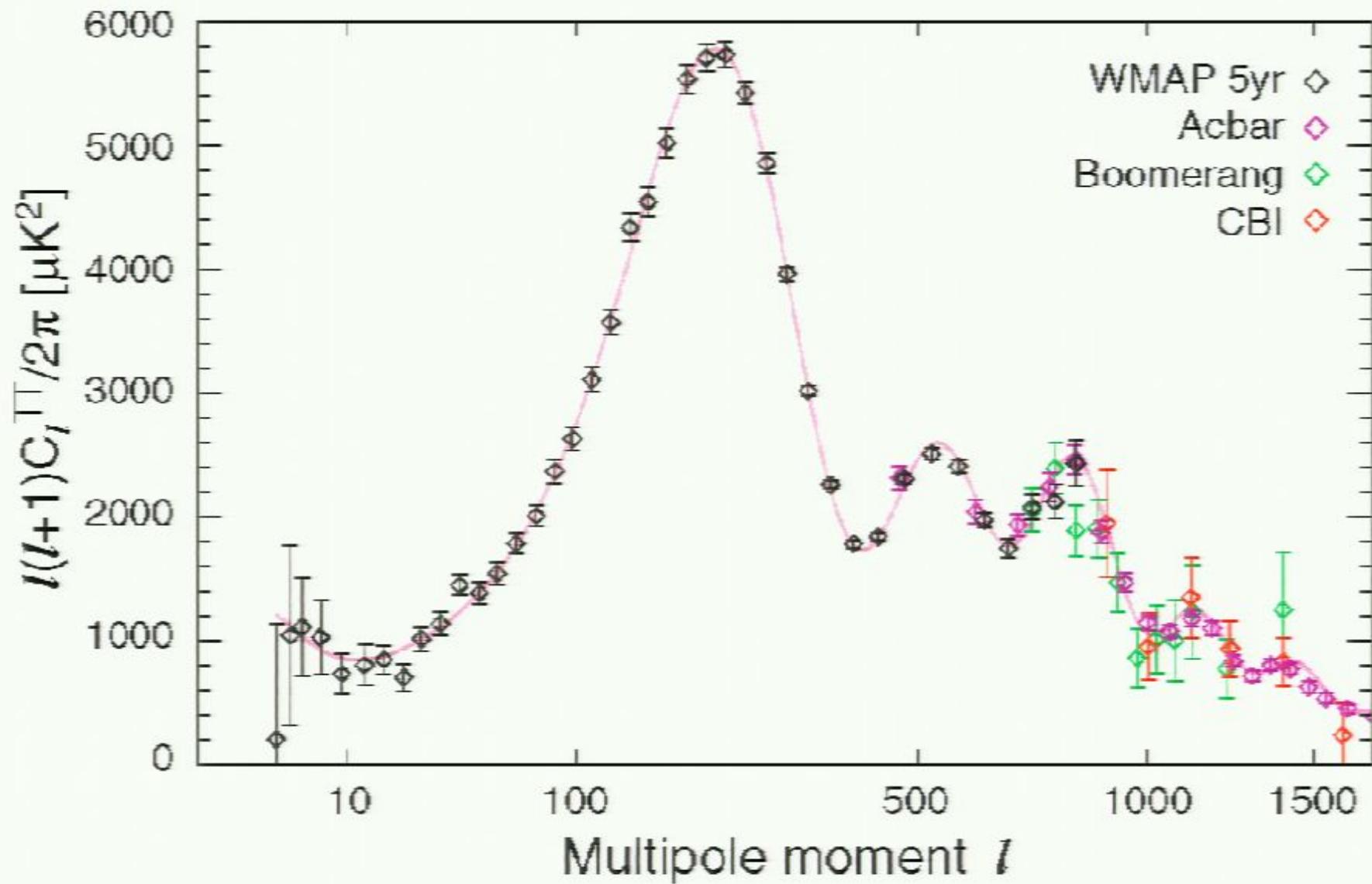
- ▶ (universe cools)
- ▶ photons and baryons fall out of equilibrium
- ▶ imprint of oscillations in the photon temperature
- ▶ from then on: free streaming photons



CMB



# Angular Powerspectrum



## From model to observables

- ▶ write down potential  $V(\phi)$
- ▶ verify that inflation lasts sufficiently long
- ▶ compute perturbation spectrum
- ▶ build detector + fly satellite
- ▶ take data
- ▶ analyse it
- ▶ obtain values for  $A_s$ ,  $n_s$  from Markov Chain Monte Carlo algorithms

$$P_S(k) = A_s \left( \frac{k}{k_*} \right)^{n_s - 1}$$

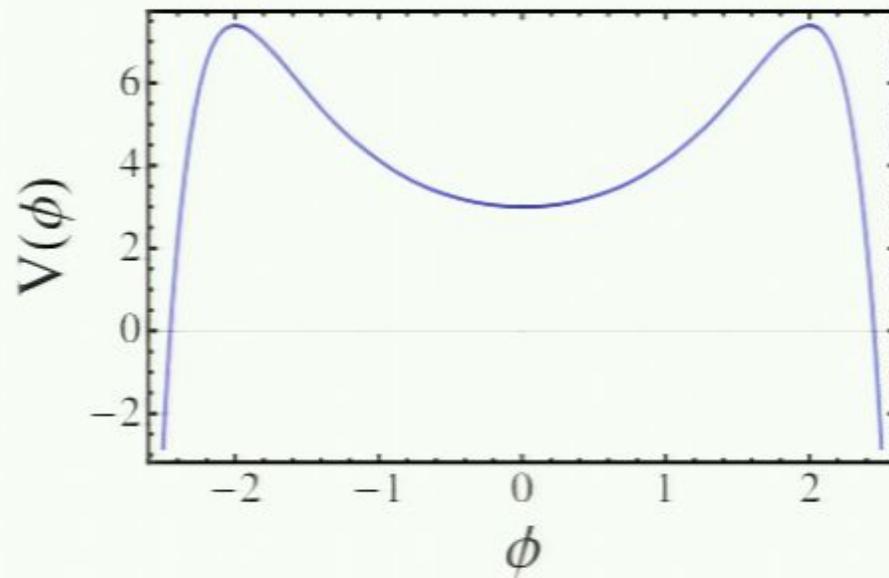


C O M P A R E

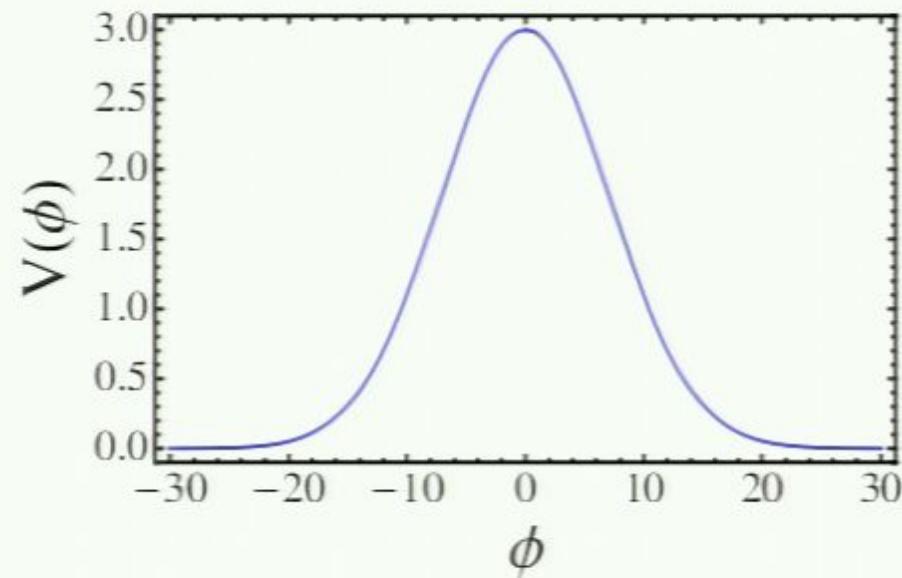
# CMB parameters (WMAP5)

WMAP Cosmological Parameters			
Model: $\Lambda$ cdm+sz+lens			
Data: wmap5+bao+snall			
$10^2 \Omega_b h^2$	$2.265 \pm 0.059$	$1 - n_s$	$0.040^{+0.013}_{-0.014}$
$1 - n_s$	$0.014 < 1 - n_s < 0.067$ (95% CL)	$A_{\text{BAO}}(z = 0.35)$	$0.473 \pm 0.010$
$C_{220}$	$5748 \pm 41$	$d_A(z_{\text{eq}})$	$14172^{+141}_{-139}$ Mpc
$d_A(z_*)$	$14006^{+142}_{-141}$ Mpc	$\Delta_{\mathcal{R}}^2$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
$h$	$0.701 \pm 0.013$	$H_0$	$70.1 \pm 1.3$ km/s/Mpc
$k_{\text{eq}}$	$0.01000 \pm 0.00027$	$\ell_{\text{eq}}$	$140.0 \pm 2.5$
$\ell_*$	$302.11^{+0.84}_{-0.82}$	$n_s$	$0.960^{+0.014}_{-0.013}$
$\Omega_b$	$0.0462 \pm 0.0015$	$\Omega_b h^2$	$0.02265 \pm 0.00059$
$\Omega_c$	$0.233 \pm 0.013$	$\Omega_c h^2$	$0.1143 \pm 0.0034$
$\Omega_\Lambda$	$0.721 \pm 0.015$	$\Omega_m$	$0.279 \pm 0.015$
$\Omega_m h^2$	$0.1369 \pm 0.0037$	$r_{\text{hor}}(z_{\text{dec}})$	$283.6 \pm 1.9$ Mpc
$r_s(z_d)$	$152.1 \pm 1.4$ Mpc	$r_s(z_d)/D_v(z = 0.2)$	$0.1889 \pm 0.0035$
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$R$	$1.7283^{+0.0093}_{-0.0092}$	$\sigma_8$	$0.817 \pm 0.026$
$A_{\text{SZ}}$	$0.99^{+1.00}_{-0.99}$	$t_0$	$13.73 \pm 0.12$ Gyr
$\tau$	$0.084 \pm 0.016$	$\theta_*$	$0.010399^{+0.000028}_{-0.000029}$
$\theta_*$	$0.5958^{+0.0016}_{-0.0017}$ $\circ$	$t_*$	$375938^{+3148}_{-3115}$ yr
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# One parameter family of fast roll models - background



$p = -2$



$p = 0.04$

$$V(\phi) = H_0^2 e^{-\frac{p}{4}\phi^2} \left( 3 - \frac{p^2}{8}\phi^2 \right)$$

$$a = e^N, H = H_0 e^{-\frac{\epsilon_0}{p} e^{pN}}, \epsilon = \epsilon_0 e^{pN}, \phi = \frac{2\sqrt{2\epsilon_0}}{p} e^{\frac{pN}{2}}, dN = +Hdt$$

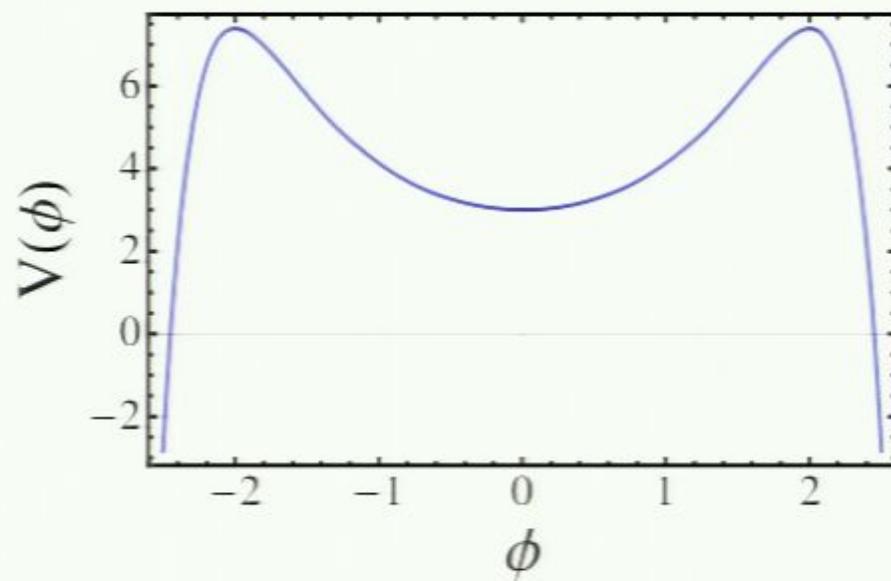
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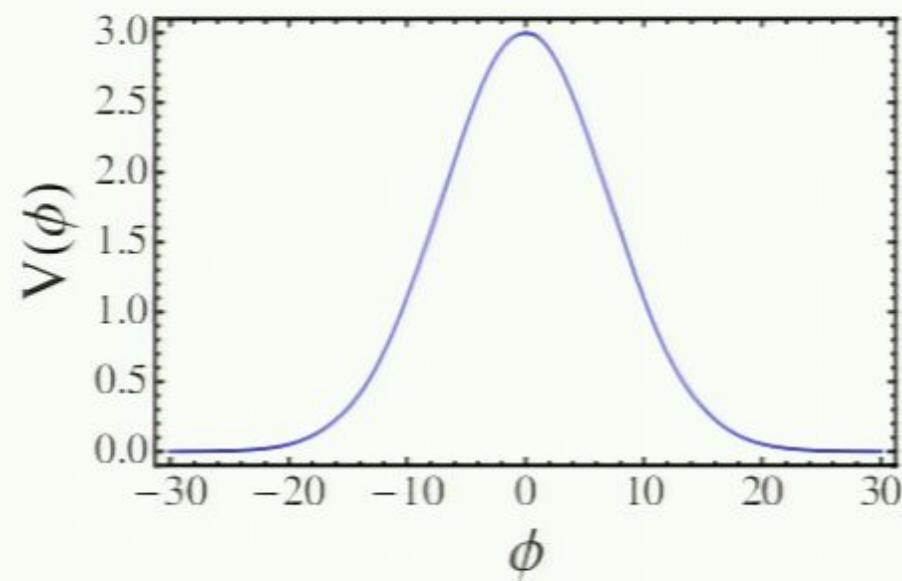
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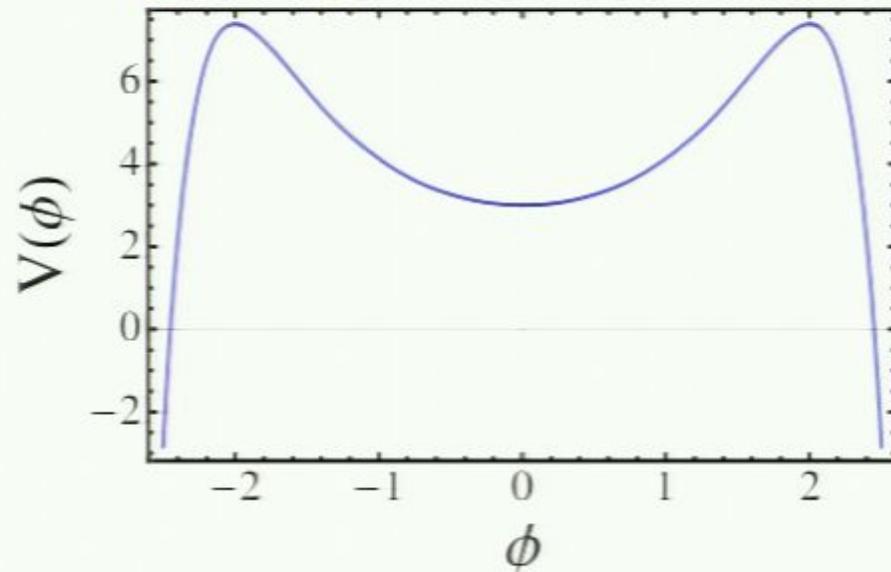
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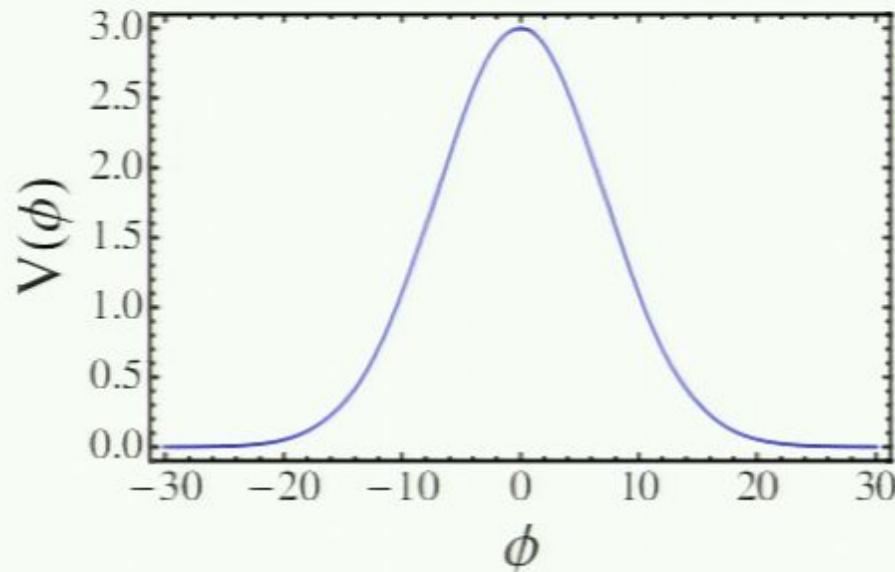
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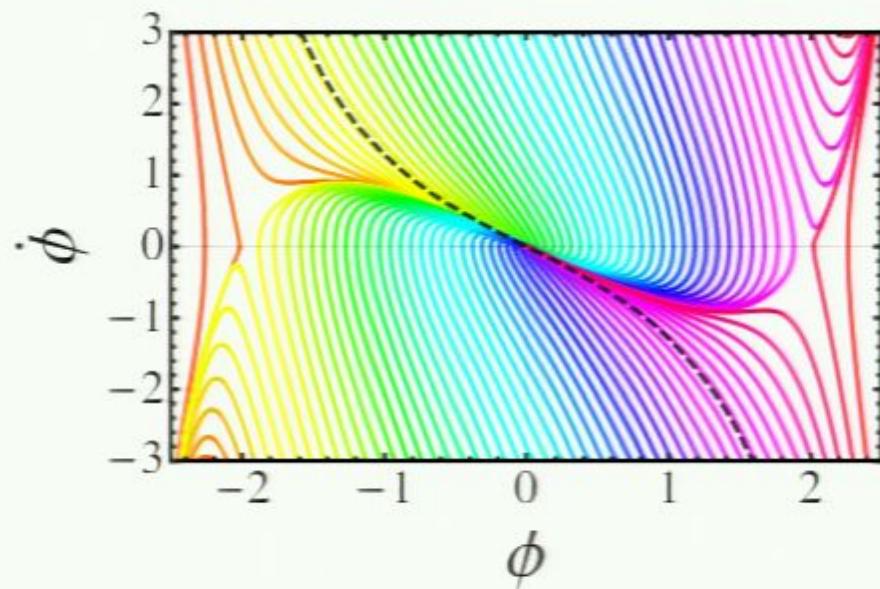
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$\phi_*$ : arbitrary position between the zero crossings  $|\phi_*| < \frac{\sqrt{24}}{|p|}$

- ▶ avoids problems with negative values of  $V$  (collapse)
- ▶ not really necessary: not really realistic anyways

## Background evolution for negative p ( $p=-2$ )

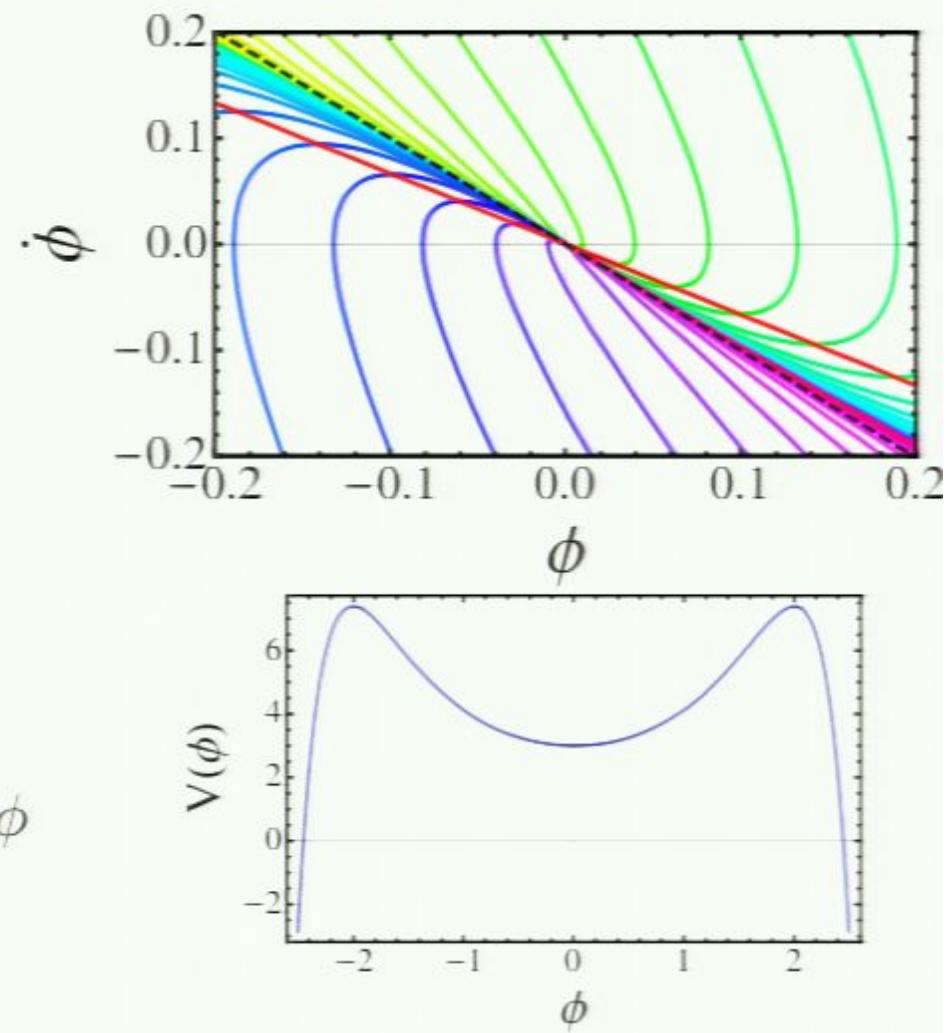


slow roll “attractor”(solid red)

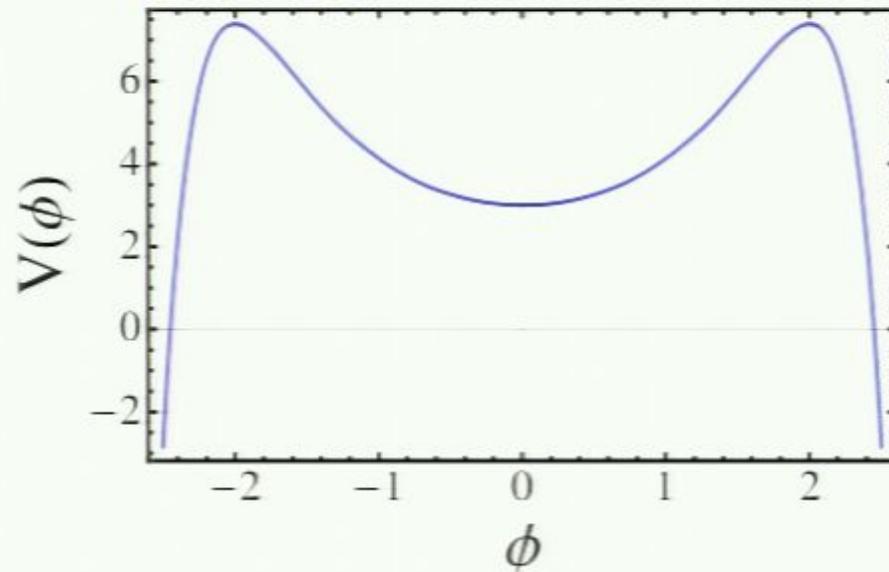
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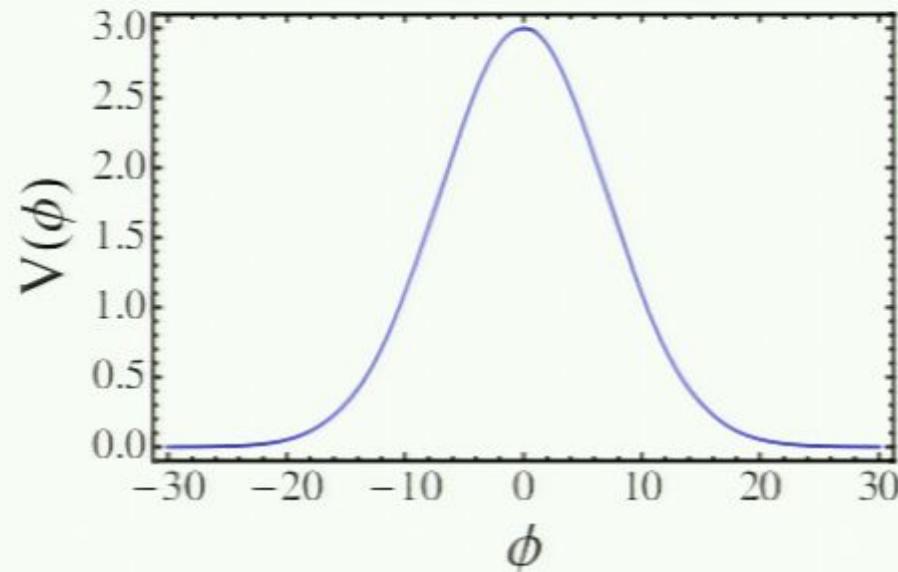
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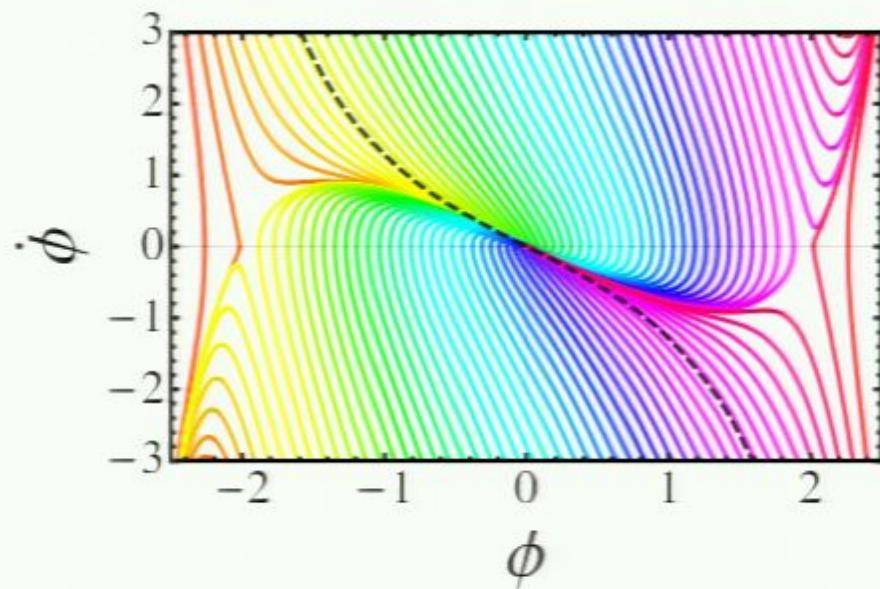
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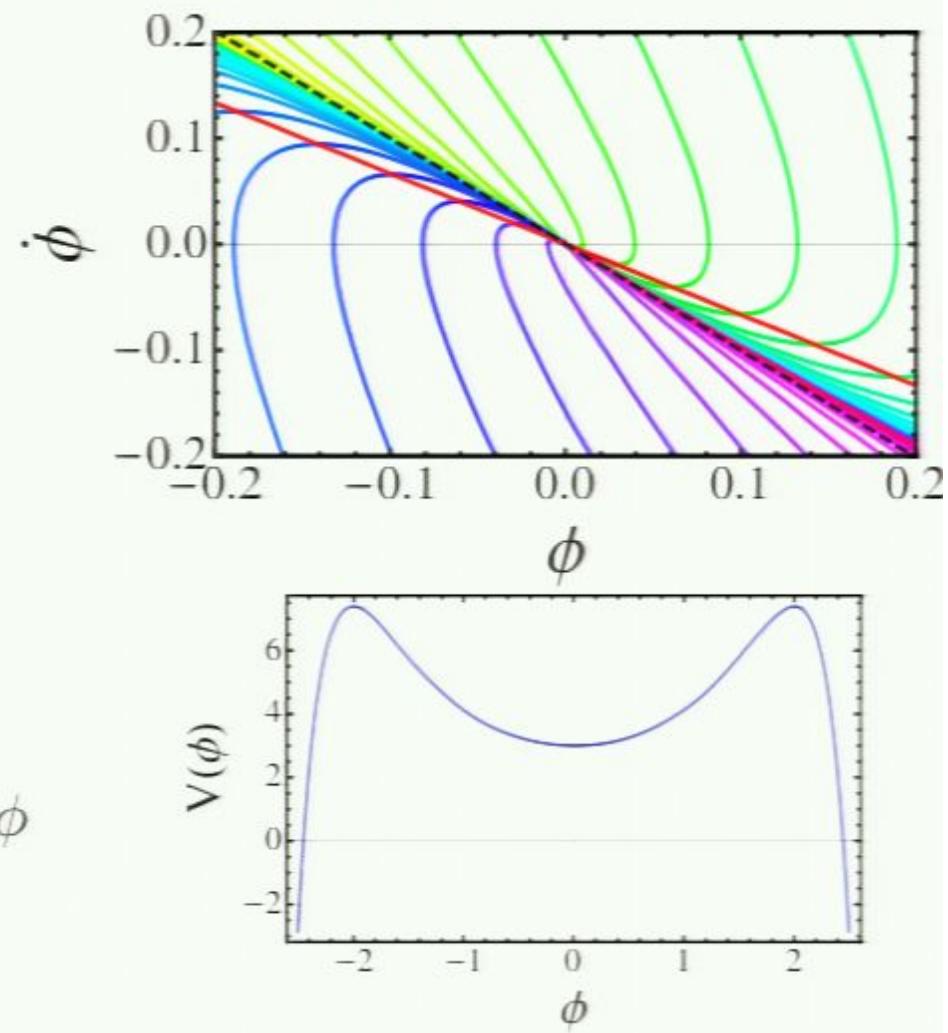
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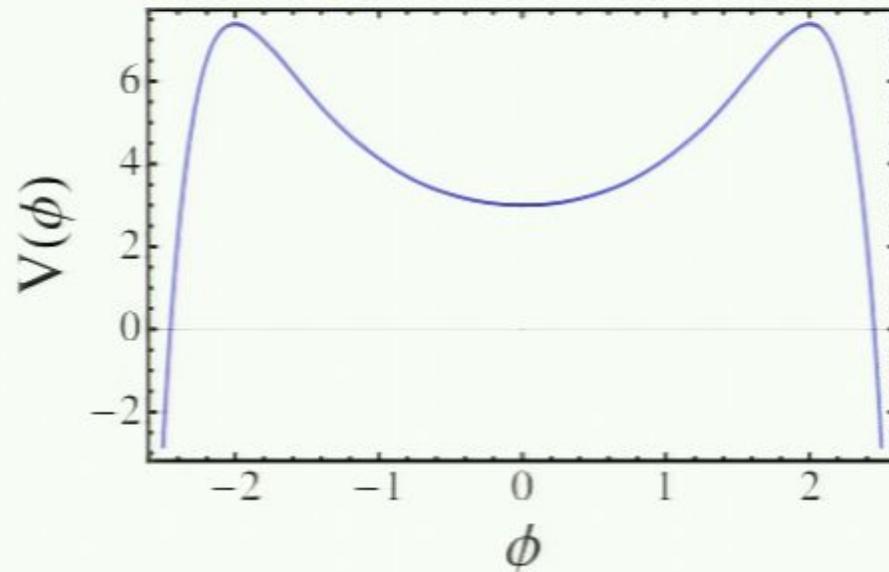
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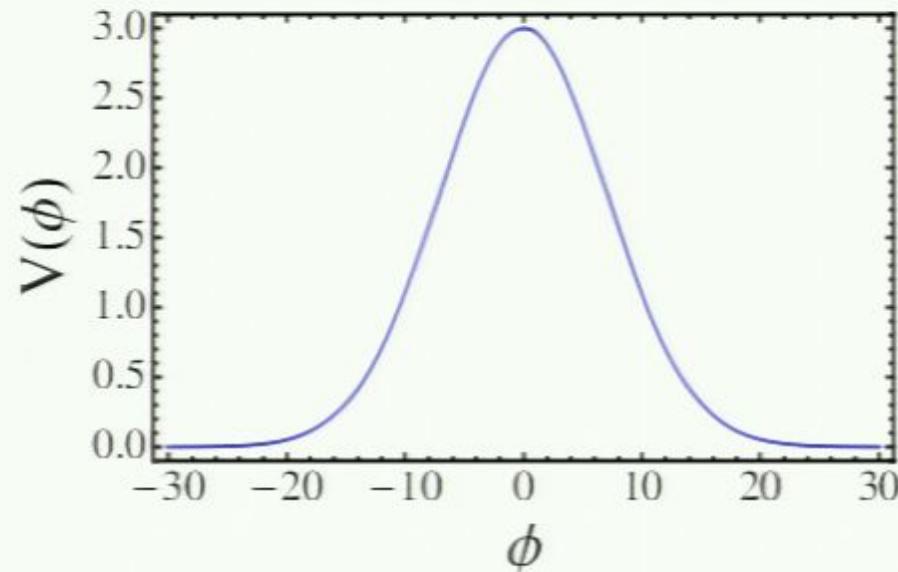
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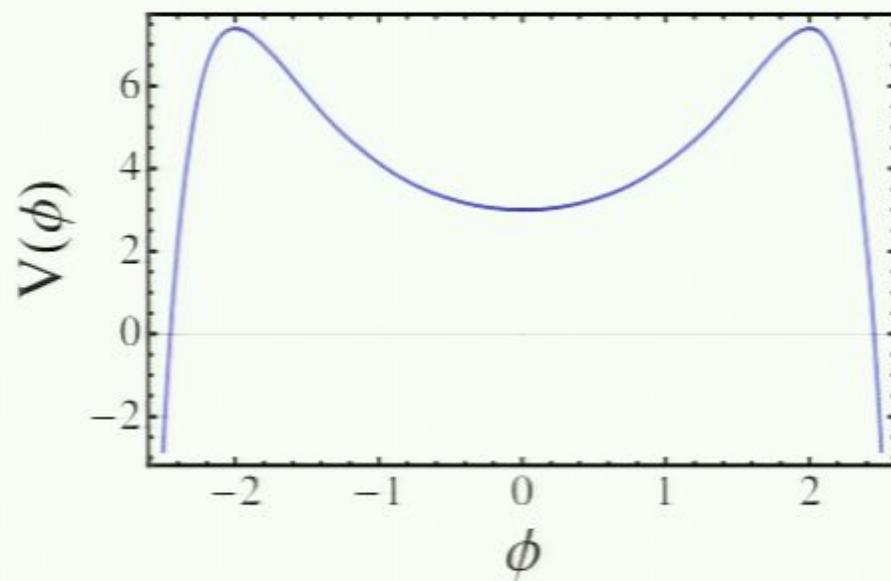
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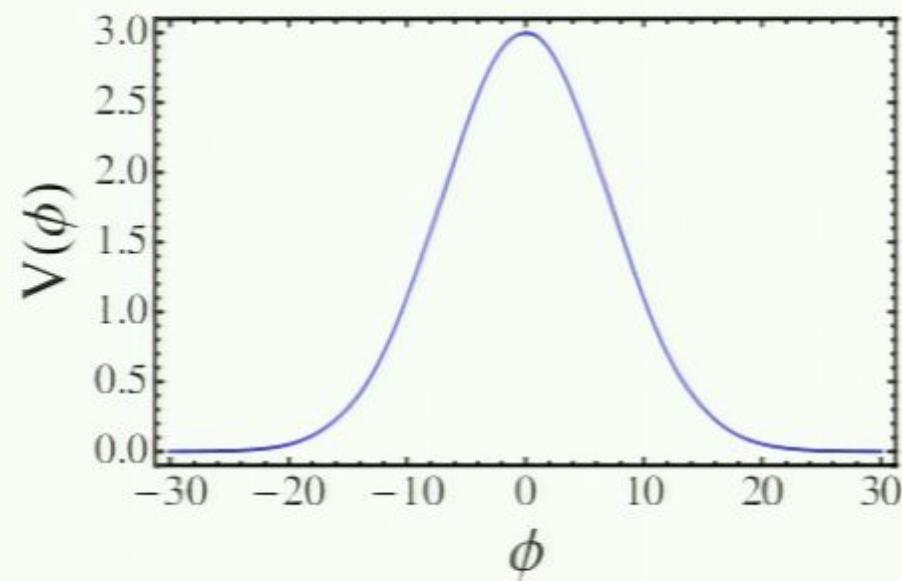
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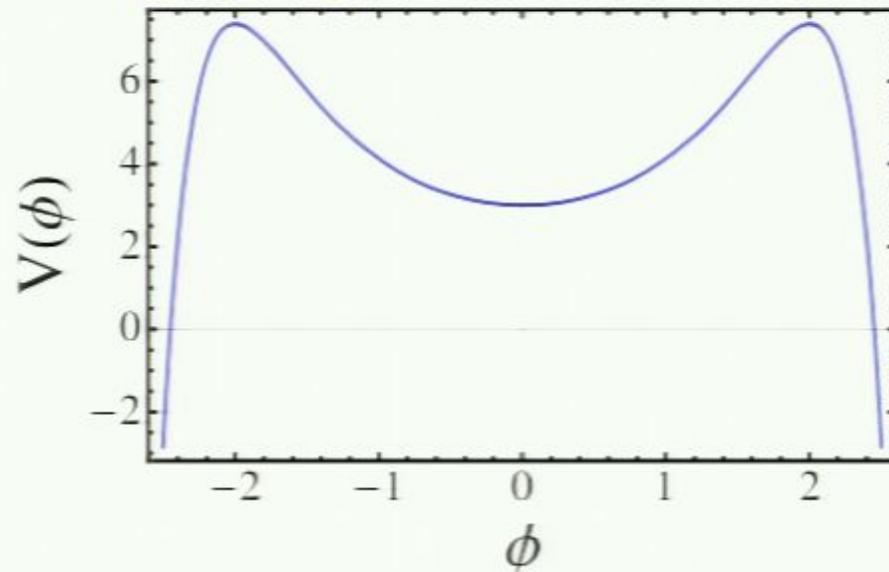
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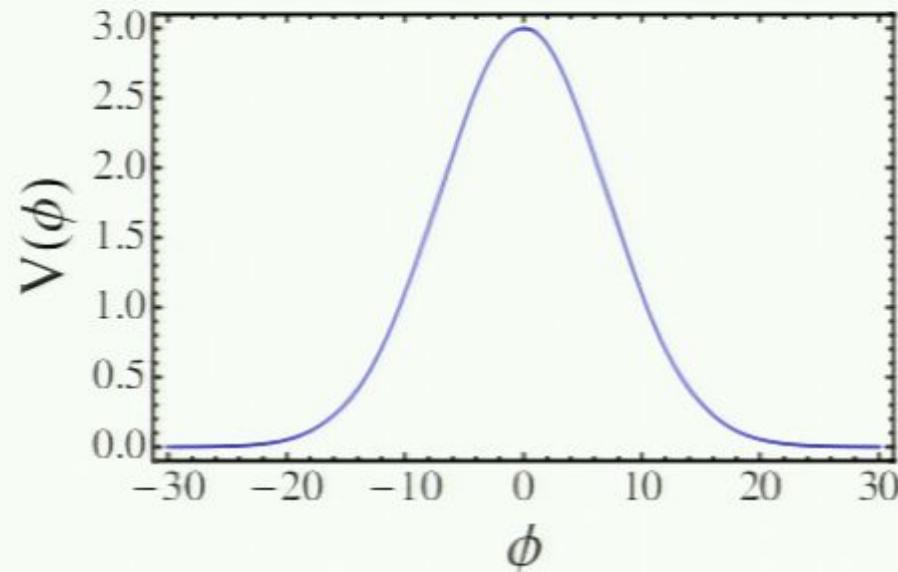
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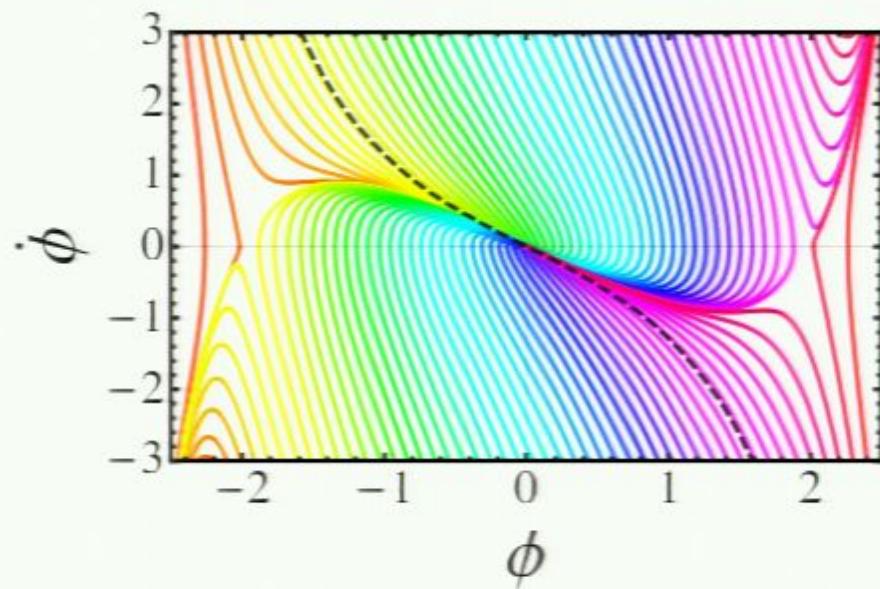
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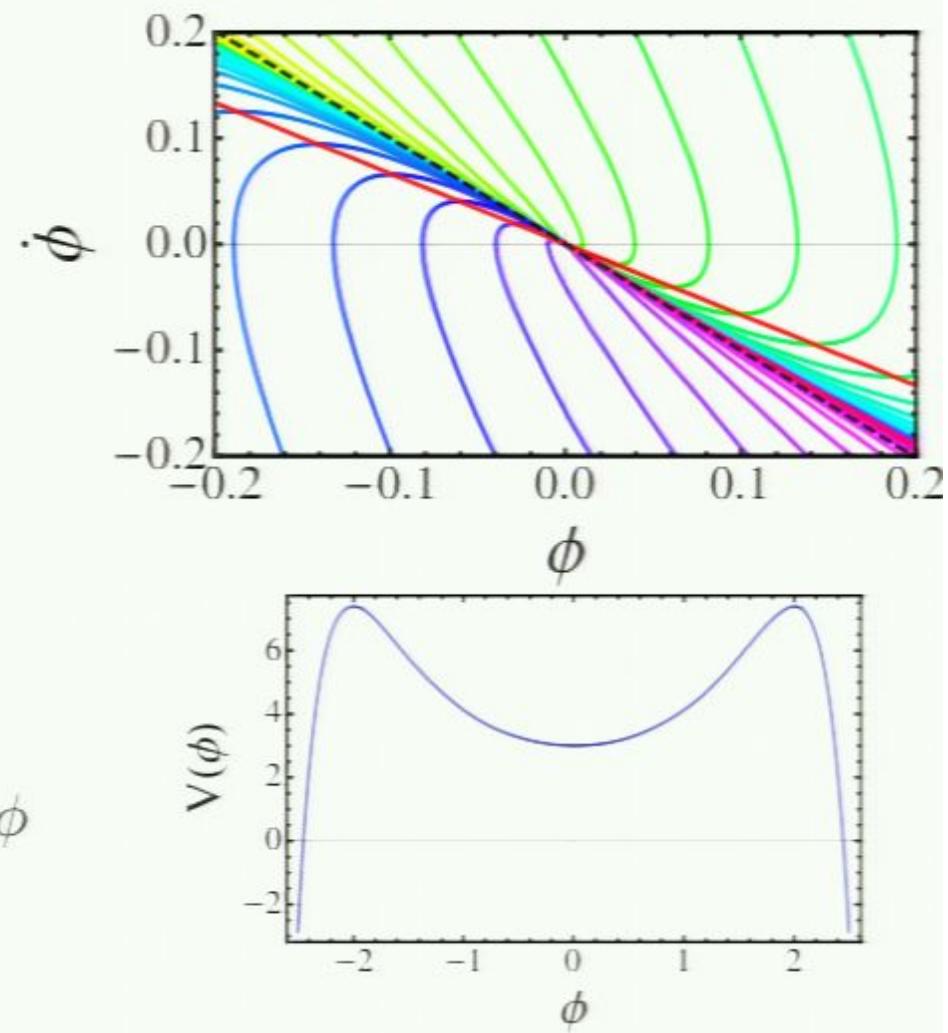


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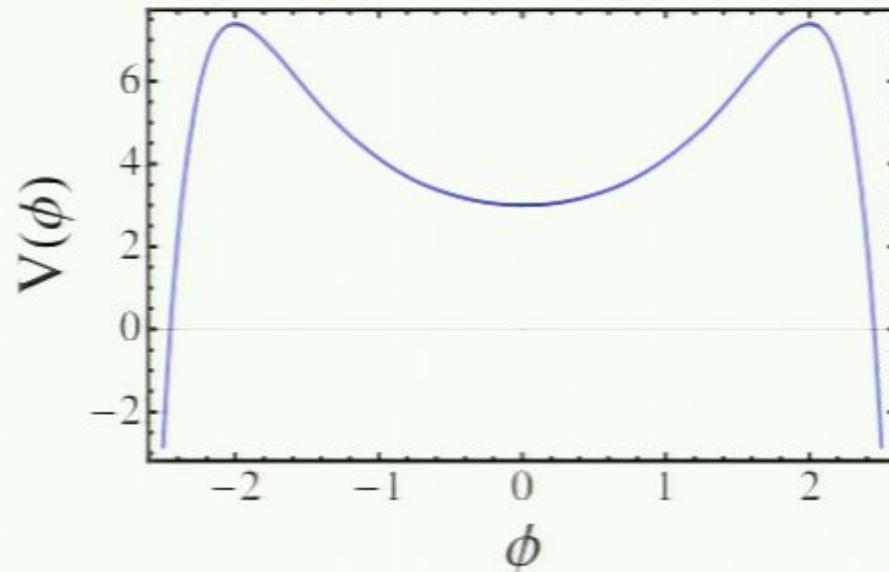
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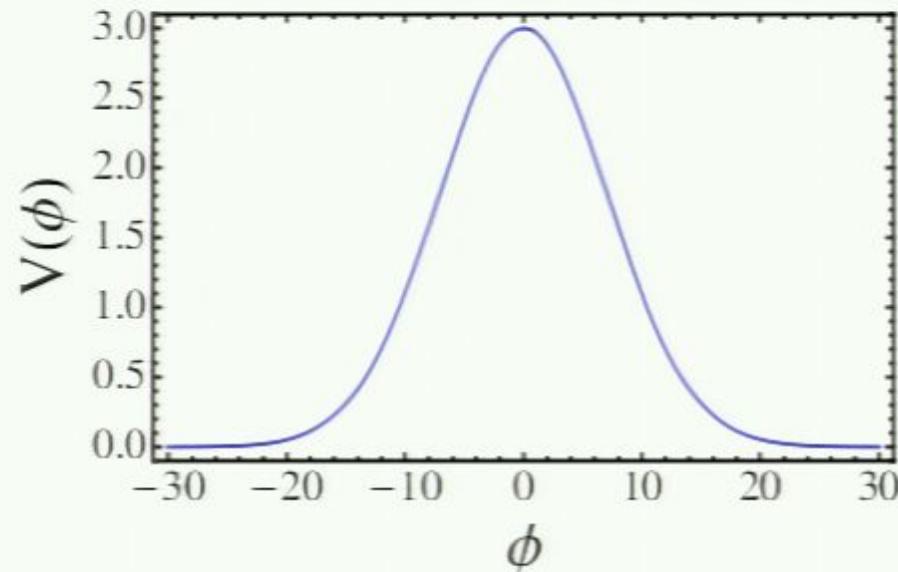
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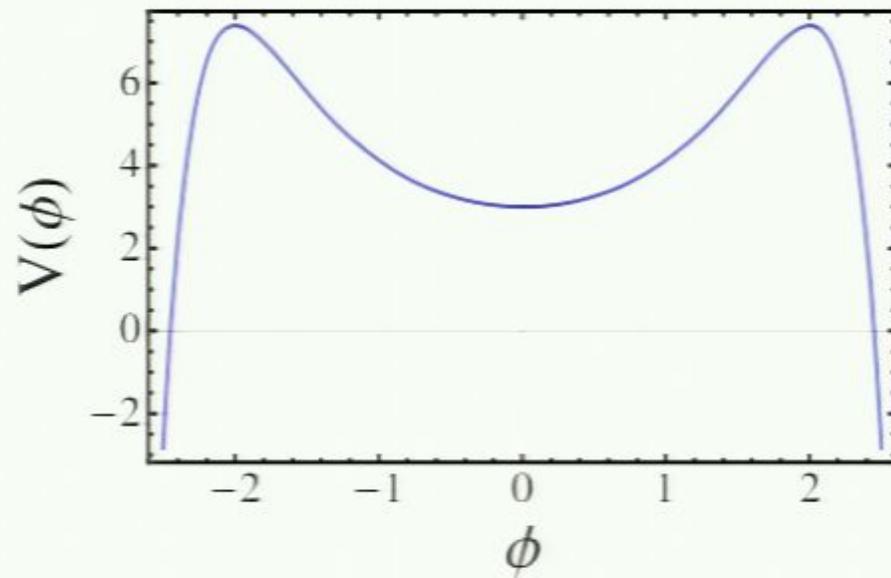
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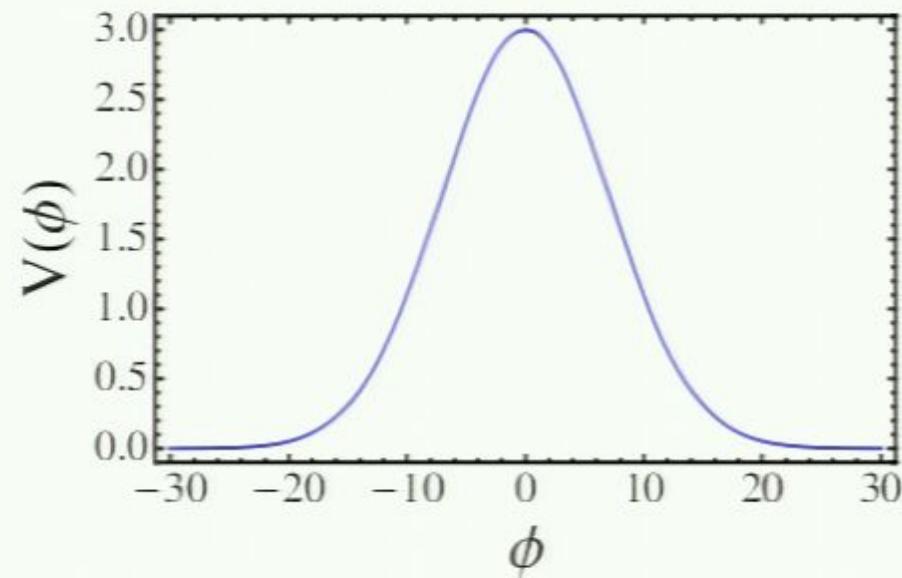
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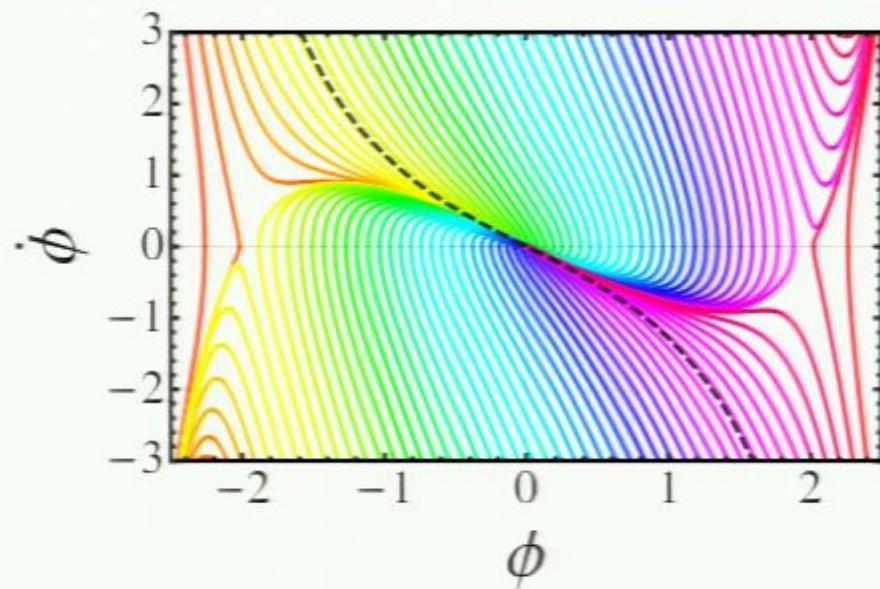
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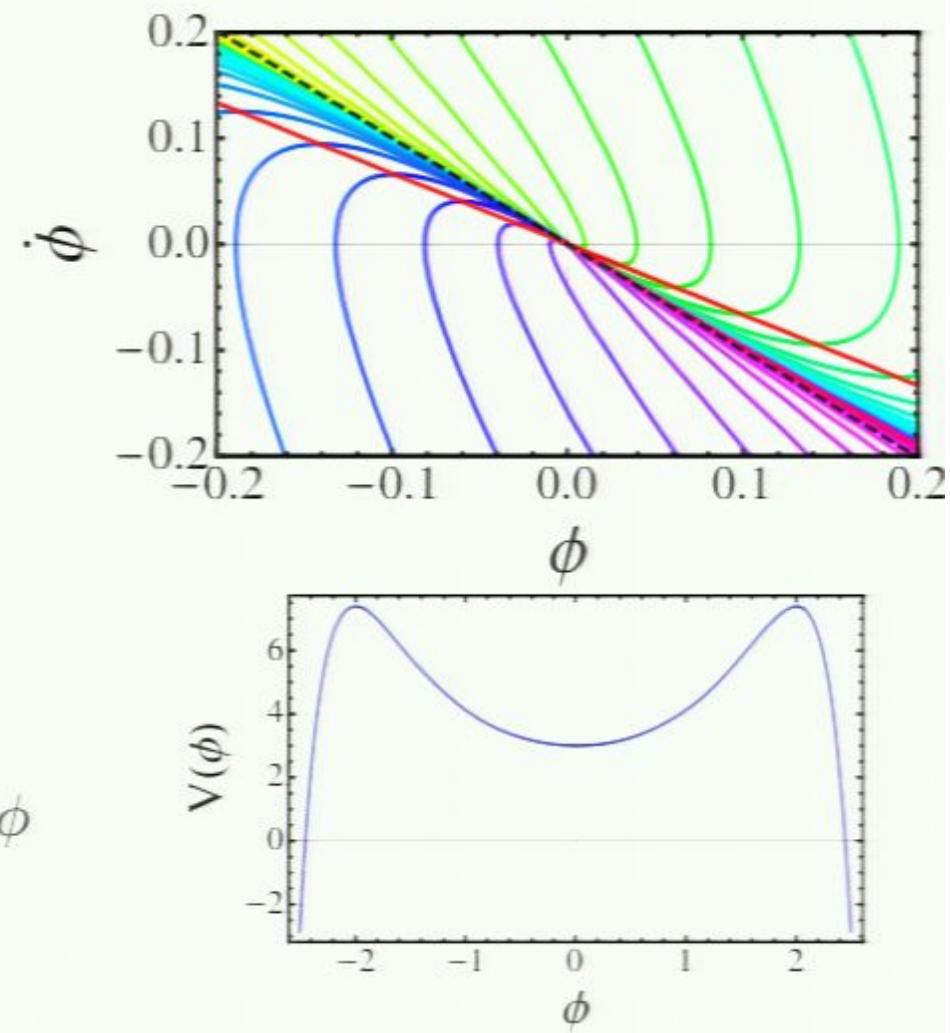


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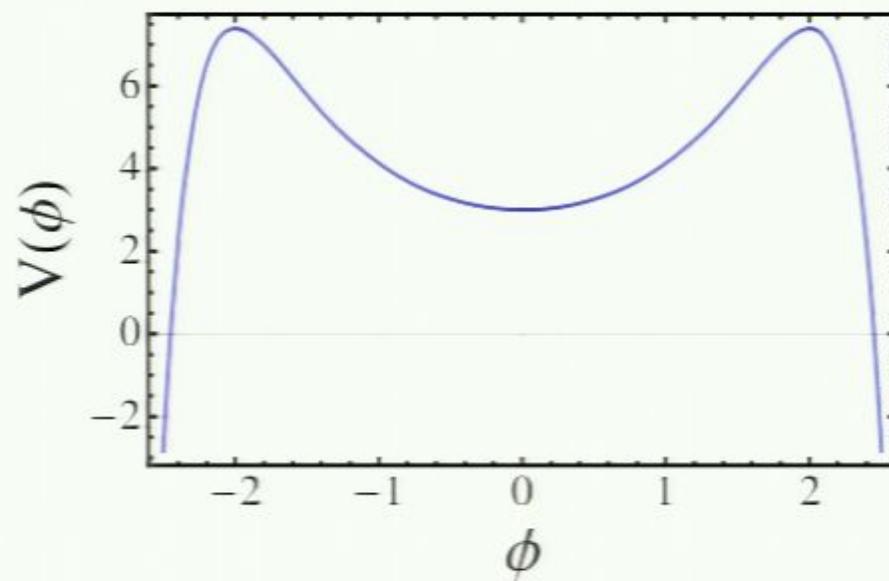
$$\dot{\phi}_{\text{SR}} = -\frac{\partial_\phi V}{3H^2} \approx \frac{H_0}{12} p(6+p)\phi = -\frac{2}{3} H_0 \phi$$

exact solution (dashed black)

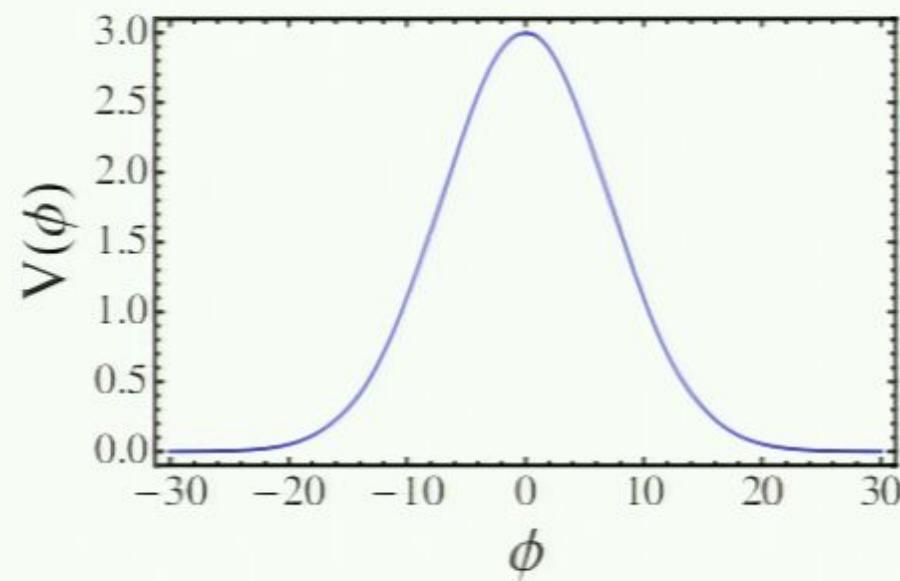
$$\dot{\phi}_{\text{exact}} = \frac{p}{2} H_0 \phi e^{-p \frac{\phi^2}{8}} \approx \frac{p}{2} H_0 \phi = -H_0 \phi$$



# One parameter family of fast roll models - background



$p = -2$



$p = 0.04$

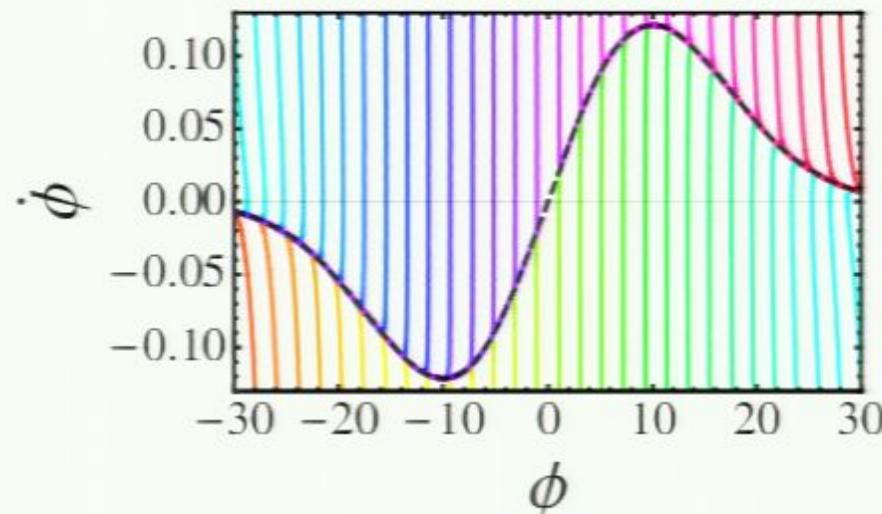
$$V(\phi) = H_0^2 e^{-\frac{p}{4}\phi^2} \left( 3 - \frac{p^2}{8}\phi^2 \right)$$

$$a = e^N, H = H_0 e^{-\frac{\epsilon_0}{p} e^{pN}}, \epsilon = \epsilon_0 e^{pN}, \phi = \frac{2\sqrt{2\epsilon_0}}{p} e^{\frac{pN}{2}}, dN = +Hdt$$

$\epsilon$  exponentially small

$\eta$  potentially large,  $\eta = 2 \frac{\partial^2 H}{H} = -\frac{p}{2} + \epsilon$

## Background evolution for positive p (p=0.04)

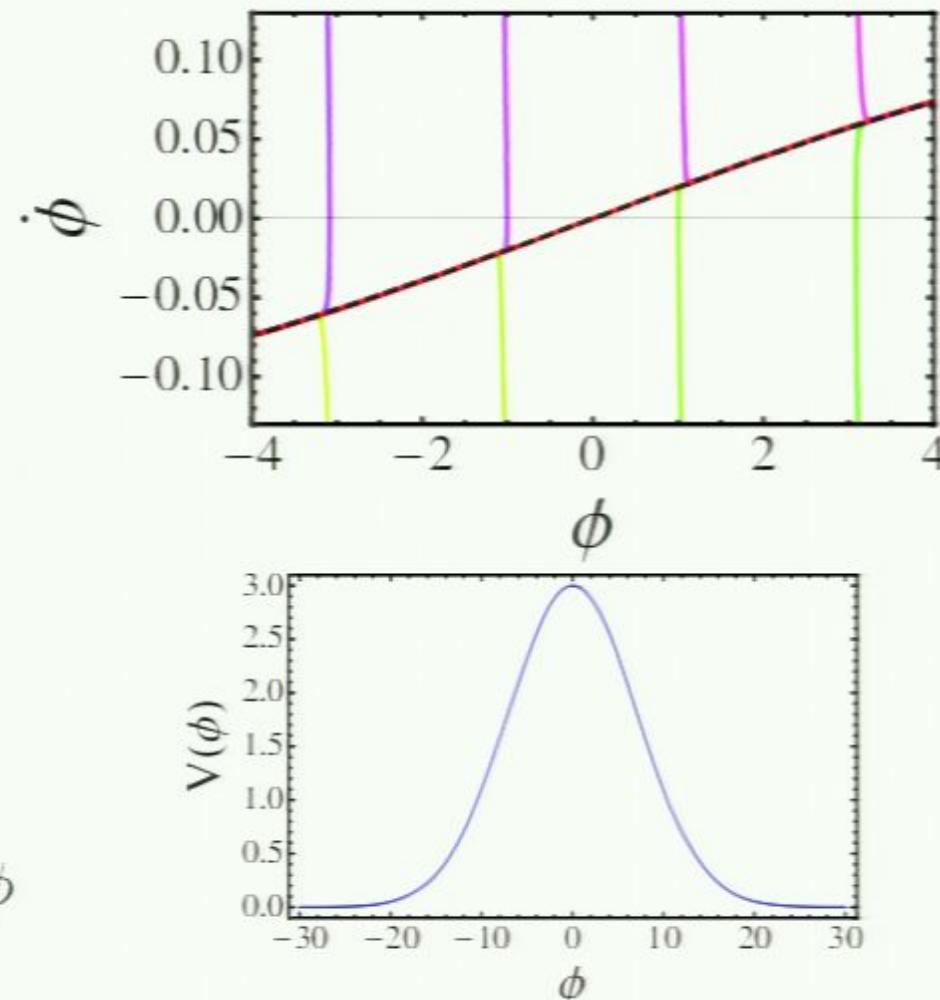


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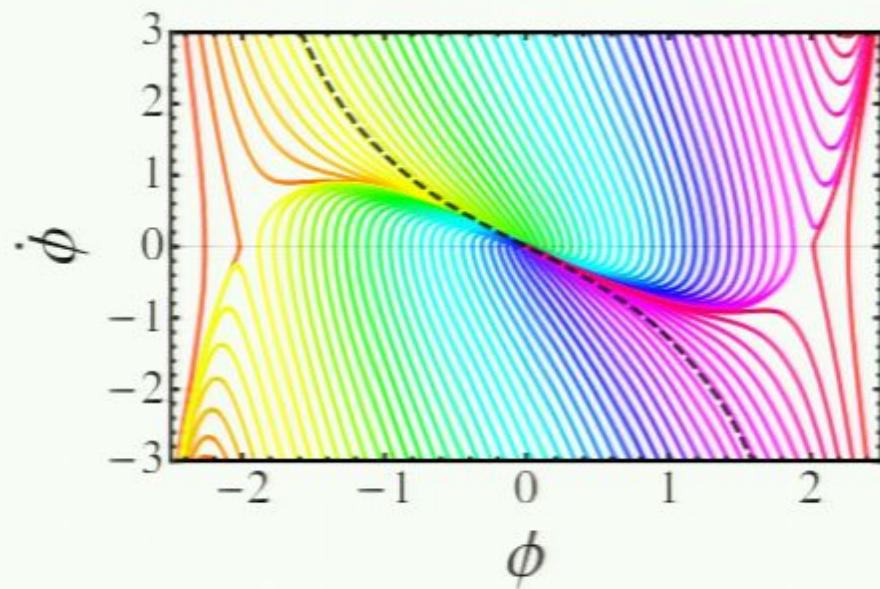
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$$\dot{\phi}_{\text{exact}} = \frac{p}{2} H_0 \phi e^{-p \frac{\phi^2}{8}} \approx \frac{p}{2} H_0 \phi = \frac{2}{10} H_0 \phi$$



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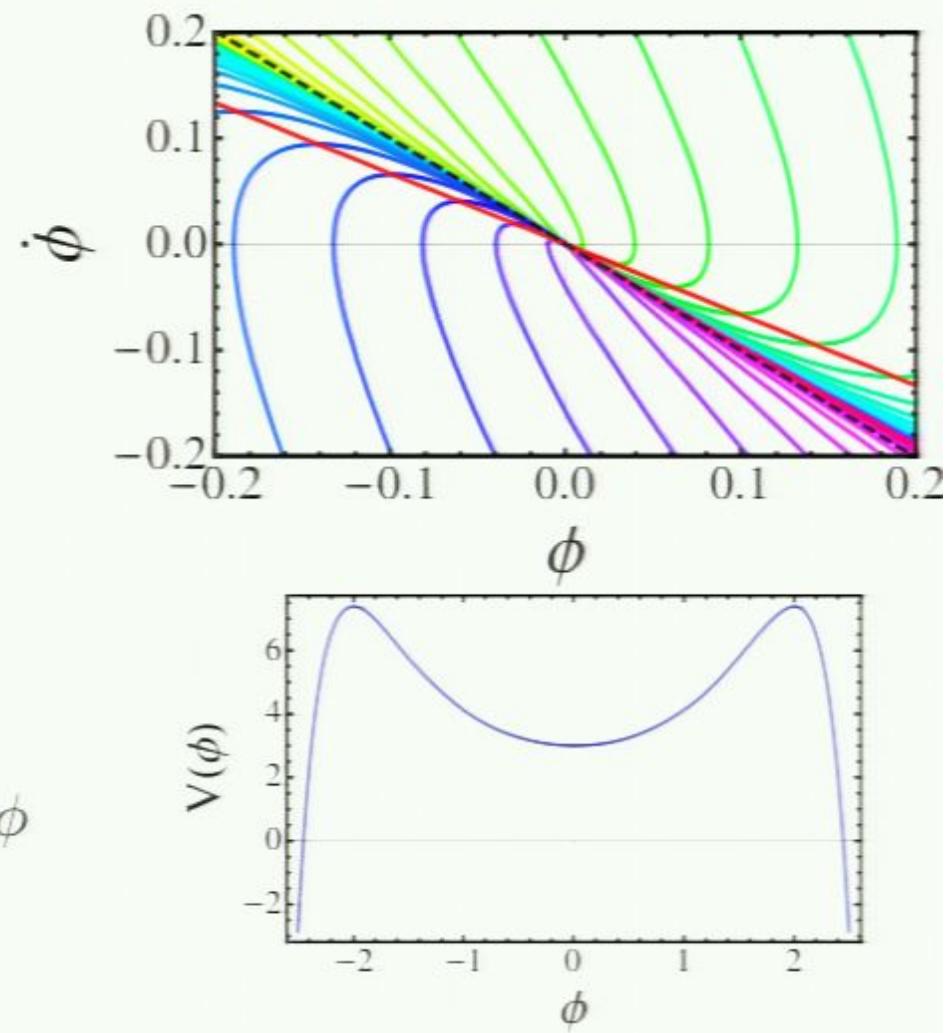


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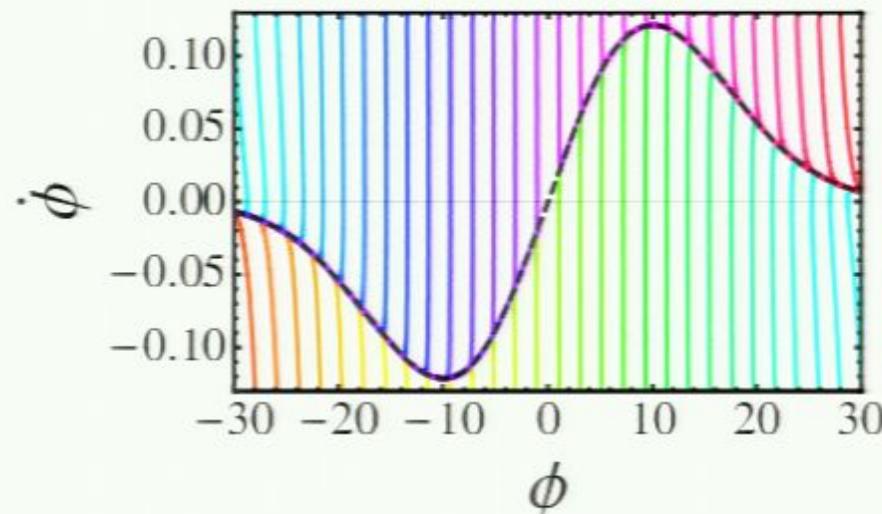
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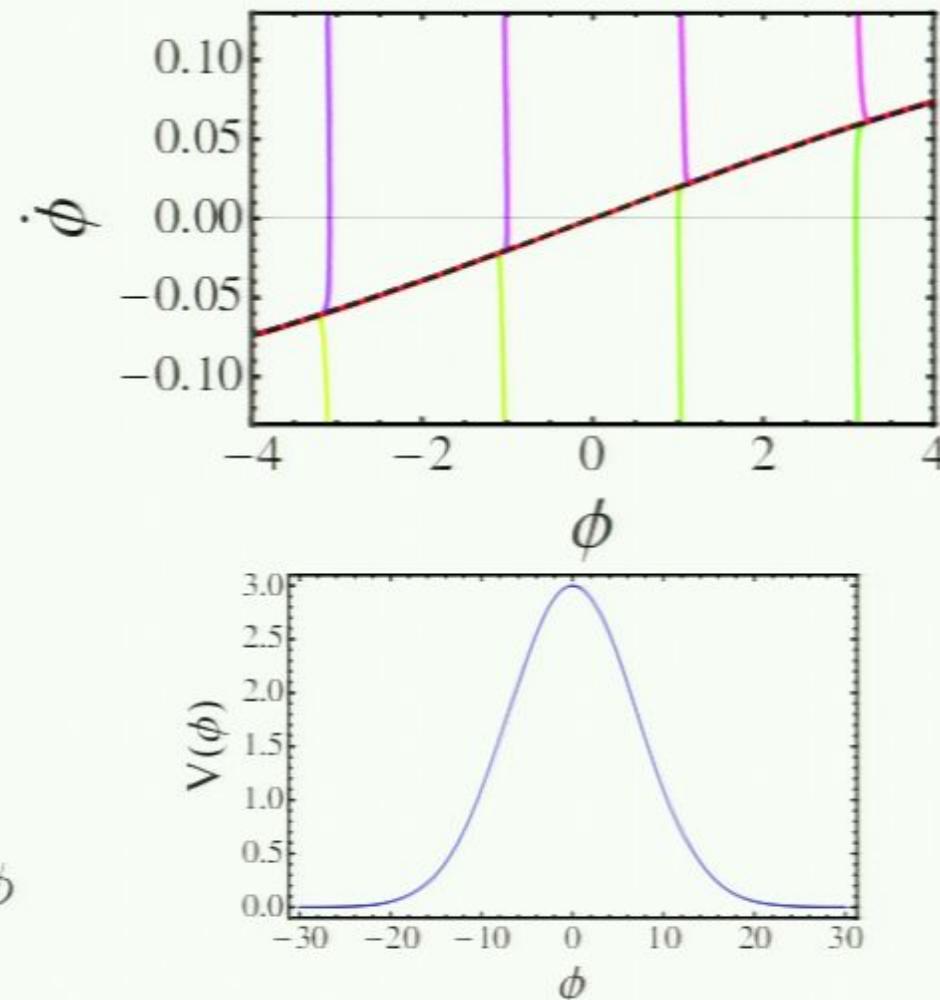


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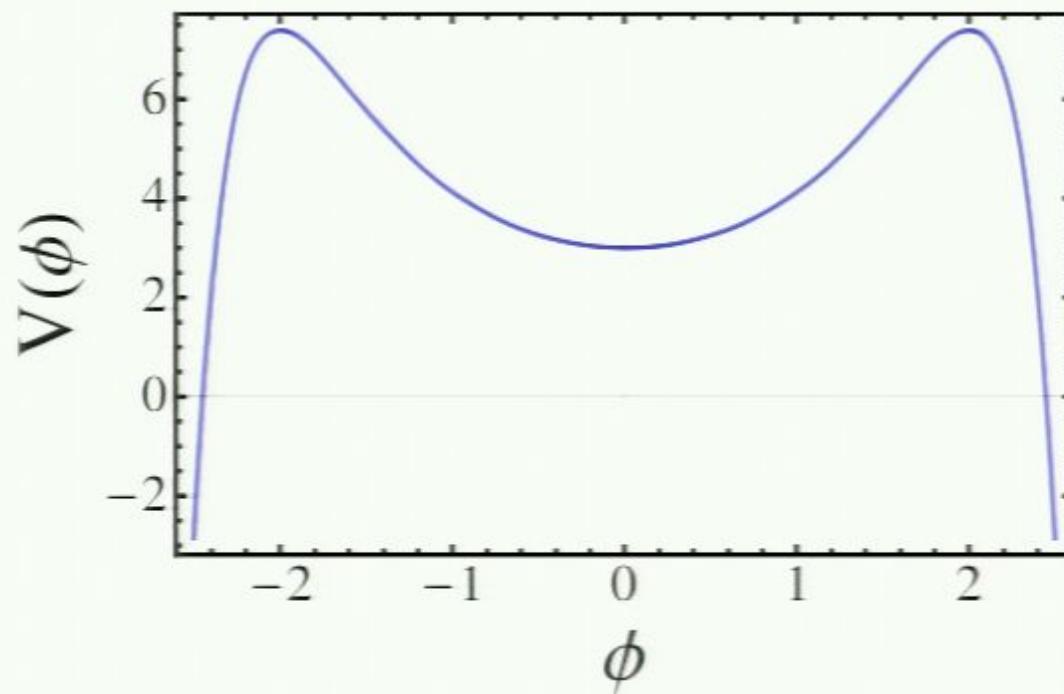
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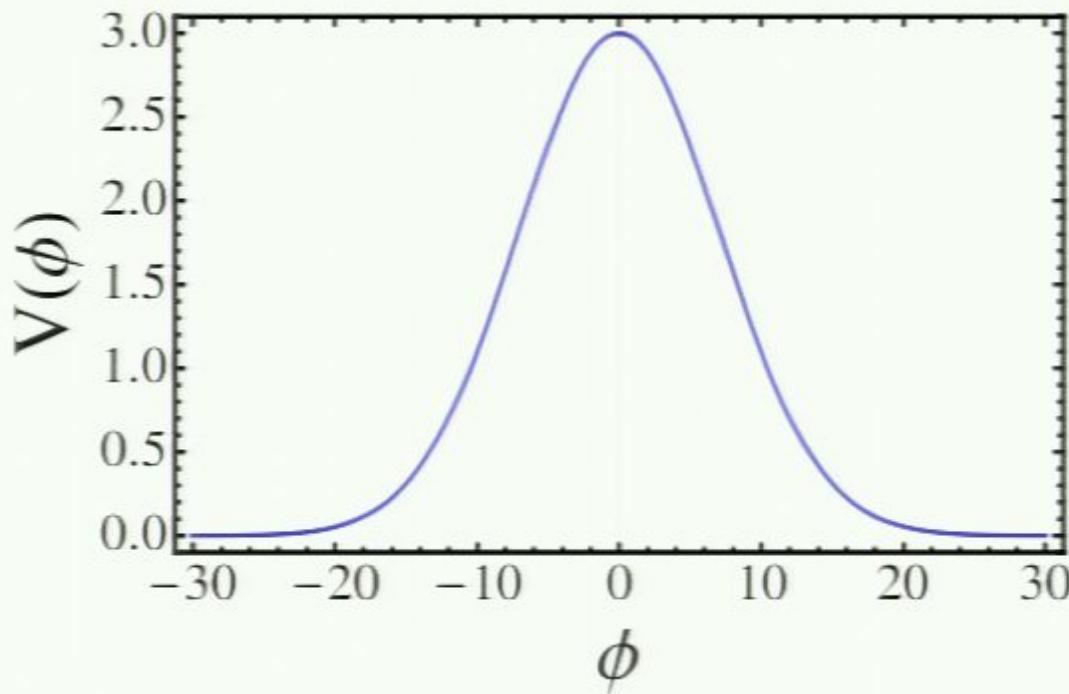
## End of Inflation for negative p – Tunneling



Hawking Moss tunneling time ( $H_0 \ll M_P$ ,  $\kappa \sim O(1)$ )

$$t_{\text{inf}} \sim H_0^{-1} \exp \left( \kappa \frac{M_P^2}{H_0^2} \right)$$

## End of inflation for positive $p - \epsilon = 1$

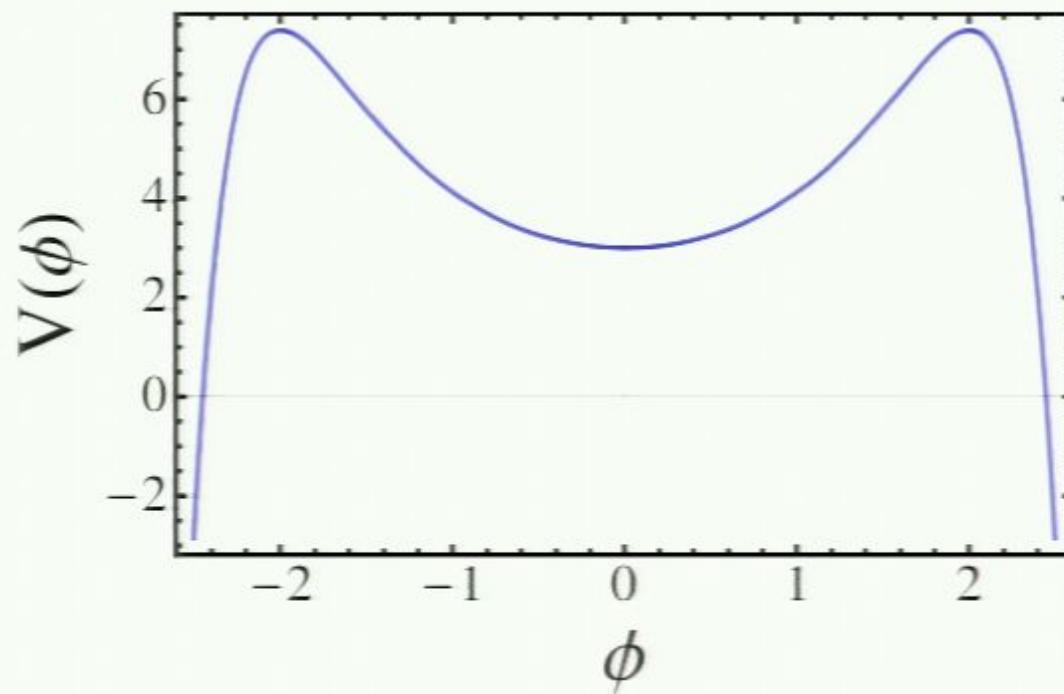


Inflation ends at  $\epsilon = 1$ :

$$\ddot{a} = aH^2(1 - \epsilon) \stackrel{!}{\leq} 0$$

about  $\Delta N \approx \frac{1}{p} \ln 3$  efolds before  $V < 0$ .

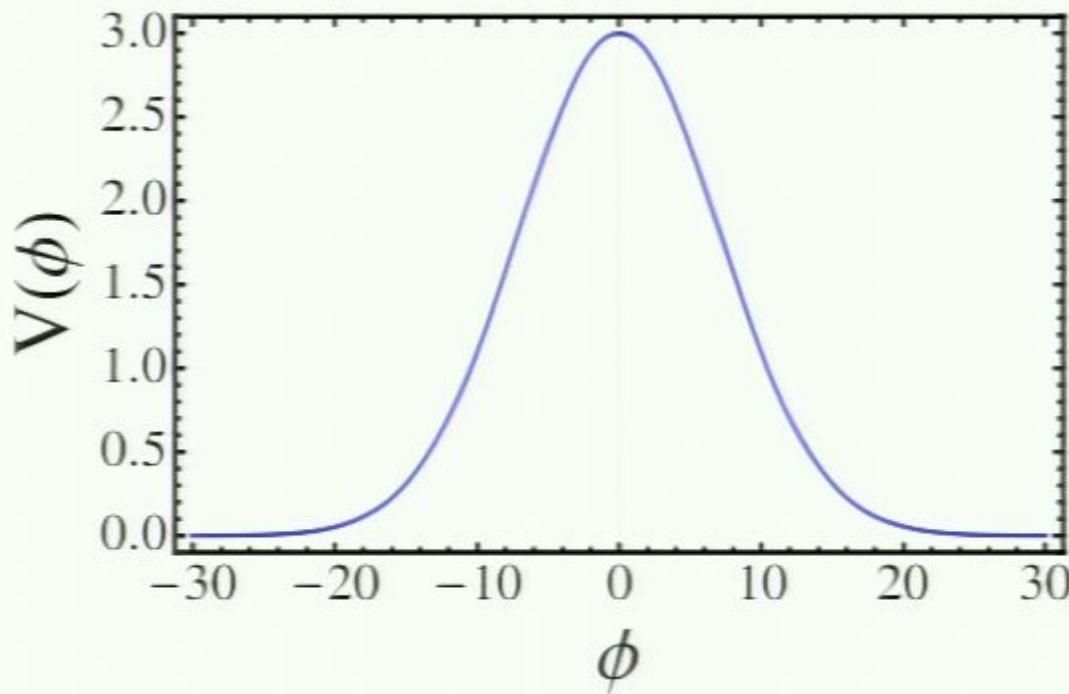
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## Status:

### Background:

- ▶ models with arbitrary  $\eta$
- ▶  $p > 0$  vs  $p < 0$
- ▶ arbitrarily long inflation (by choosing initial conditions)
- ▶ inflation ends by tunneling or  $\epsilon = 1$
- ▶ late time attractor given by exact solution

How does it compare with observations?  $\Rightarrow$  Perturbations

## Linear scalar perturbations

$$u_k'' + \left[ k^2 - \frac{z''}{z} \right] u_k = 0, \quad (z = \sqrt{2\epsilon}a),$$
$$u_k'' + \left[ k^2 - a^2 H^2 \left( \frac{p}{2} + 1 \right) \left( \frac{p}{2} + 2 - \epsilon \right) \right] u_k = 0$$

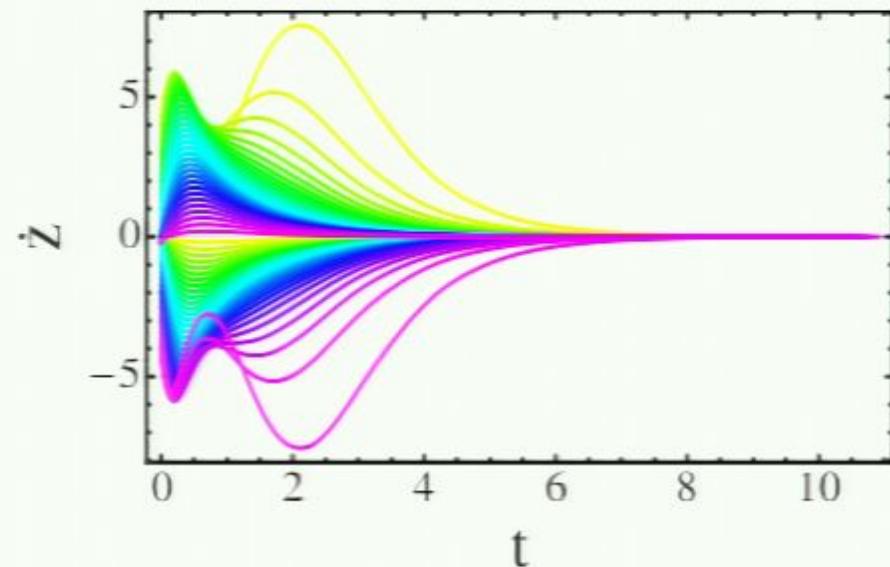
solvable for  $\epsilon = \epsilon_0 e^{pN} \ll 1$

$$u_k(\tau) = e^{i(\frac{p}{4}+1)\pi} \frac{\sqrt{\pi}}{2} \sqrt{-\tau} H_{\frac{3+p}{2}}^{(1)}(-k\tau)$$
$$\mathcal{P}_s = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|^2$$
$$= \frac{2^{p-1}}{\pi^3} \frac{3.81 \times 10^{56p}}{\epsilon_0} \left[ \Gamma \left( \frac{p+3}{2} \right) \right]^2 \left( \frac{H_0}{M_p} \right)^{p+2} \left( \frac{k}{\text{Mpc}^{-1}} \right)^{-p}$$

scalar spectral index  $n_s - 1 = -p$

## How good is the attractor?

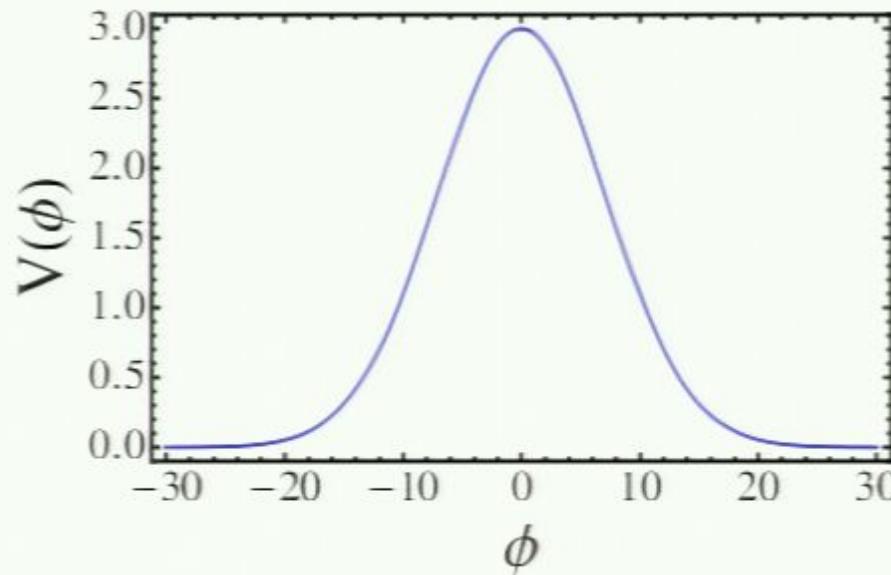
$$u_k'' + \left( k^2 - \frac{z''}{z} \right) u_k = 0, \quad (z = \sqrt{2\epsilon}a)$$



- ▶ numerical computation of  $z$  for different initial conditions
- ▶  $z$  becomes constant after a short time
- ⇒ substitute exact solutions into perturbation equation

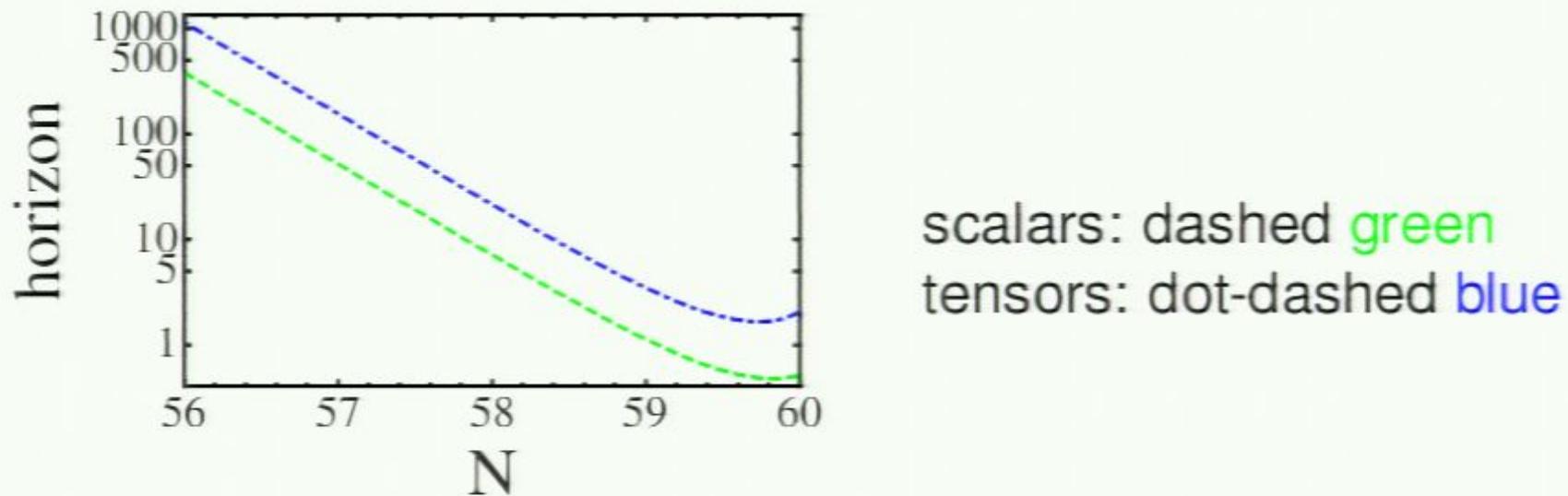
## Realistic model

WMAP 5 yr:  $n_s = 0.91 \dots 0.99$ (95%CL)



- ▶  $\mathcal{P}_S = A_s k^{n_s - 1} = A_s k^{-p}$ ,  $\Rightarrow p = 0.01 \dots 0.09$
- ▶ inflation lasts for arbitrary number of efolds (determined by initial position)
- ▶ after inflation,  $V > 0$  for  $\Delta N = 110 \dots 12$  efolds w/o uplift

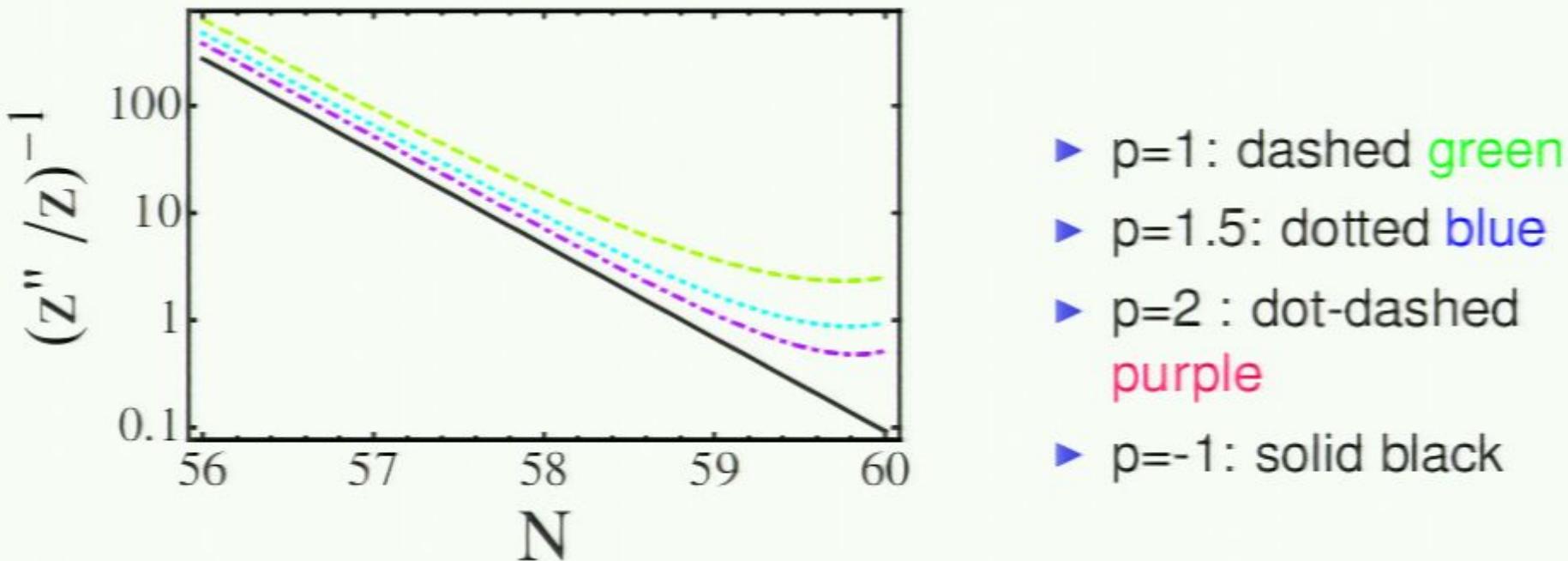
## Freeze-out at different times for scalars and tensors



$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0, \quad v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$$\sqrt{\Delta k^2} = \sqrt{\left| \frac{z''}{z} - \frac{a''}{a} \right|} = aH\sqrt{\frac{p}{2} \left( \frac{p}{2} + 3 - \epsilon \right)}$$

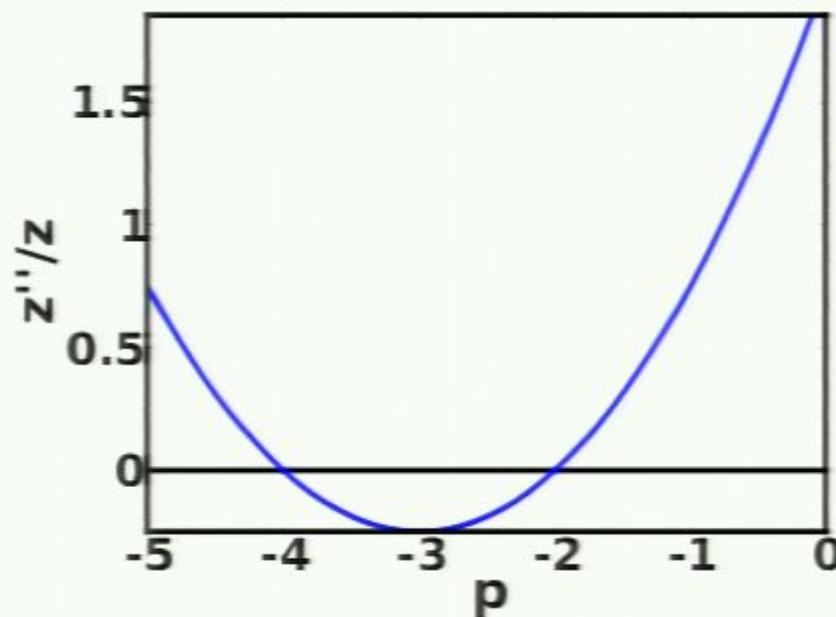
## Modes reenter the horizon



$z''/z = a^2 H^2 \left(\frac{\rho}{2} + 1\right) \left(\frac{\rho}{2} + 2 - \epsilon\right)$  has extremum (see Leach et al. 2000, 2001)

⇒ comoving wavenumbers leave and reenter the horizon during inflation

## No freeze out for scalars



$$u_k'' + \left[ k^2 - a^2 H^2 \left( \frac{p}{2} + 1 \right) \left( \frac{p}{2} + 2 - \epsilon \right) \right] u_k = 0$$

for  $-2 > p > -4$ ,  $z''/z$  changes sign  
⇒ positive mass square and no freeze-out

$$u'' + (\lambda^2 - \frac{c''}{x}) u_x = 0$$

$$u''_x + \frac{c''}{x} u_x = 0$$

$$u_{1x} \sim \frac{c}{x}$$



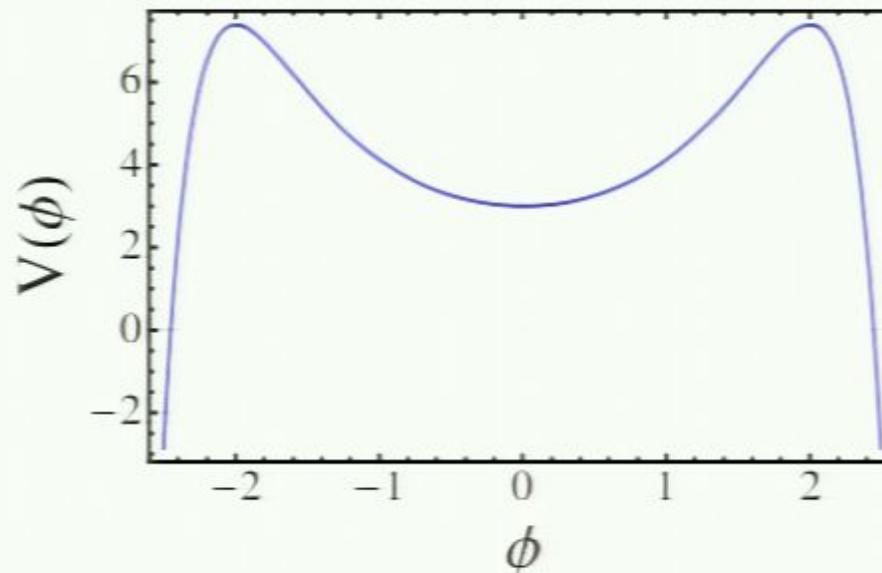
$$u'' + \left(\hbar^2 - \frac{e''}{\epsilon}\right)u_\epsilon = 0$$

$$u_\epsilon'' + \left|\frac{e''}{\epsilon}\right| u_\epsilon = 0$$

$$u_\epsilon \sim \epsilon$$



## The curious model $p = -2$



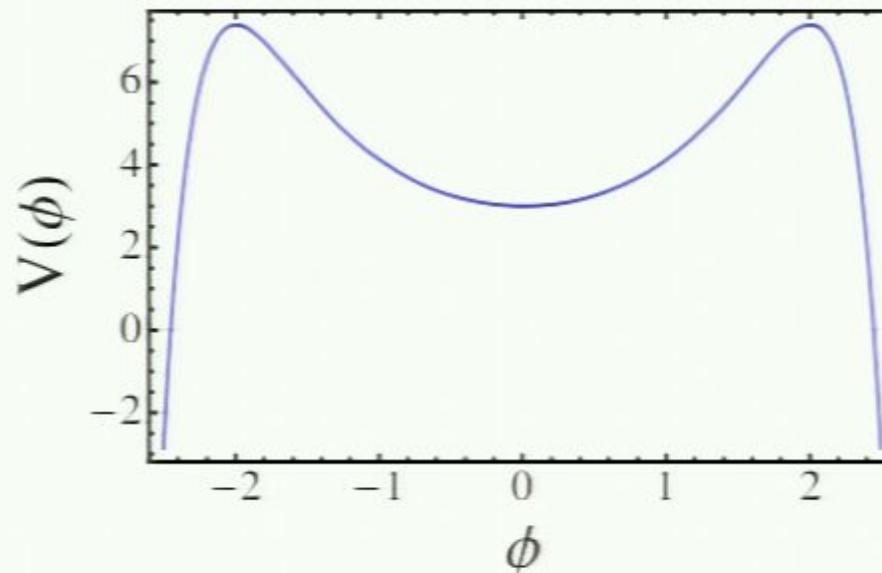
background

$$\begin{aligned}V &= H_0^2 e^{-\frac{1}{2}\phi^2} \left( 3 - \frac{1}{2}\phi^2 \right) \\ \epsilon &= e^{-2N}, a = e^N\end{aligned}$$

$\epsilon$  decreases with time  
perturbations

$$u_k'' + k^2 u_k = 0, \quad z = \sqrt{2\epsilon}a = \text{const!}$$

## The curious model $p = -2$



background

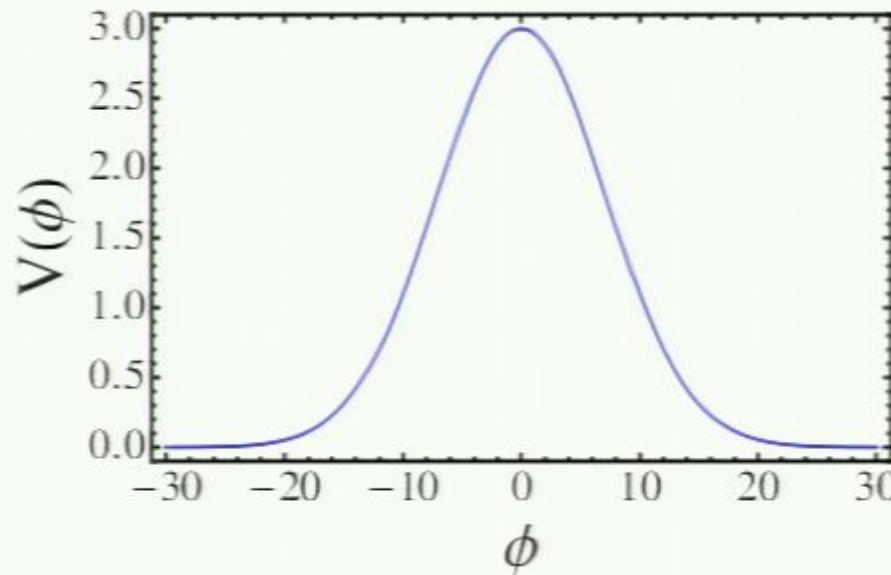
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## Realistic model

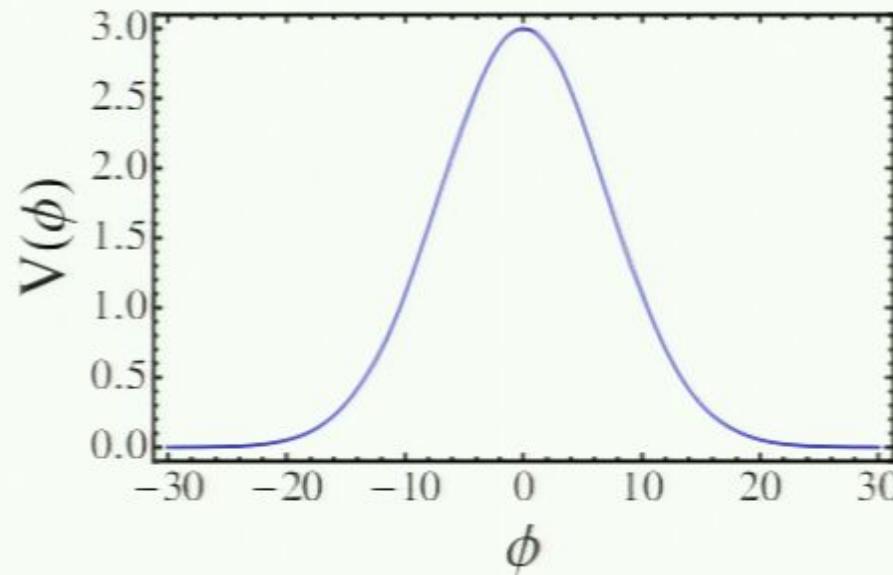
WMAP 5 yr:  $n_s = 0.91 \dots 0.99$ (95%CL)



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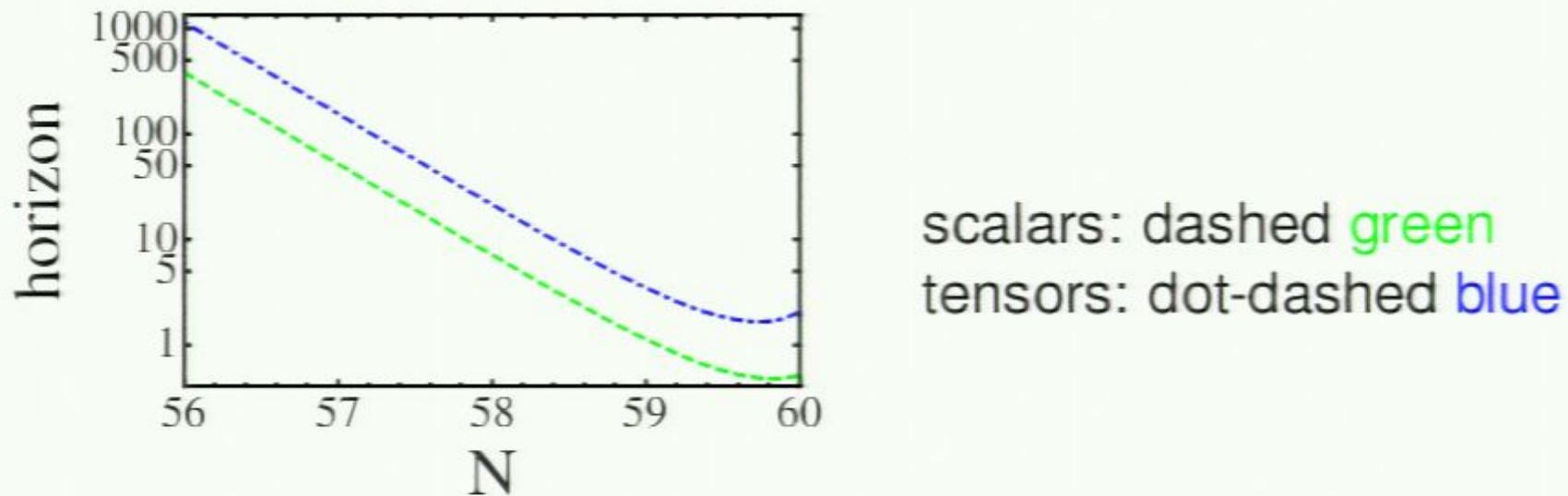
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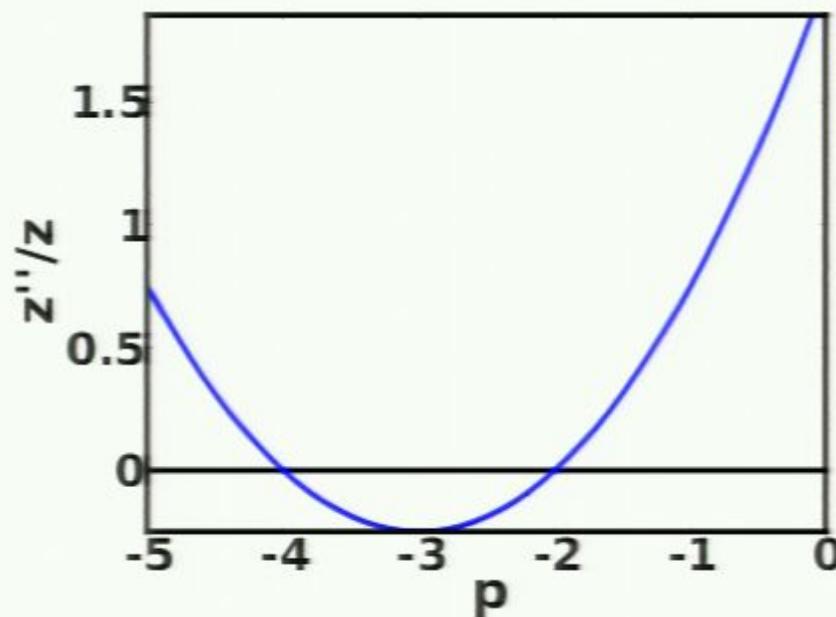
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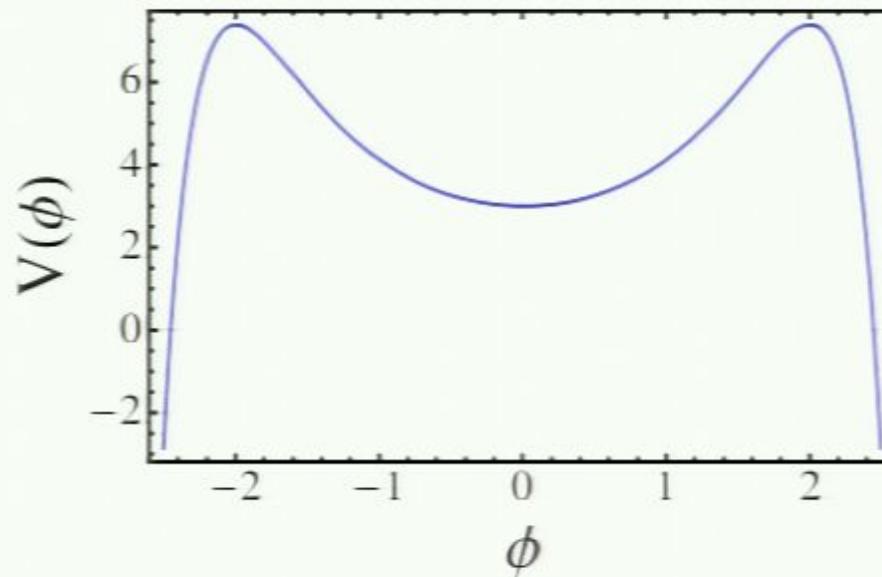
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## The curious model $p = -2$



background

$$\begin{aligned}V &= H_0^2 e^{-\frac{1}{2}\phi^2} \left( 3 - \frac{1}{2}\phi^2 \right) \\ \epsilon &= e^{-2N}, a = e^N\end{aligned}$$

$\epsilon$  decreases with time  
perturbations

$$u_k'' + k^2 u_k = 0, \quad z = \sqrt{2\epsilon}a = \text{const!}$$

## Importance of initial state - vacuum

$$u_k'' + \left[ k^2 - a^2 H^2 \left( \frac{p}{2} + 1 \right) \left( \frac{p}{2} + 2 - \epsilon \right) \right] u_k = 0$$
$$p = -2 \Rightarrow u_k'' + k^2 u_k = 0$$

initial state: Bunch Davis vacuum  $u_k = \frac{1}{\sqrt{2k}} e^{-ik\tau}$ :

- ▶ modes never freeze out
- ▶ no gravitational particle production
- ▶ no stochastic kicks
- ▶ no eternal inflation
- ▶ power spectrum  $\mathcal{P}_S \propto |\frac{u_k}{z}| = \frac{1}{8\pi^2} k^2$  (see Easter 1996)
- ▶ modes never become classical! (see Starobinski 2005)

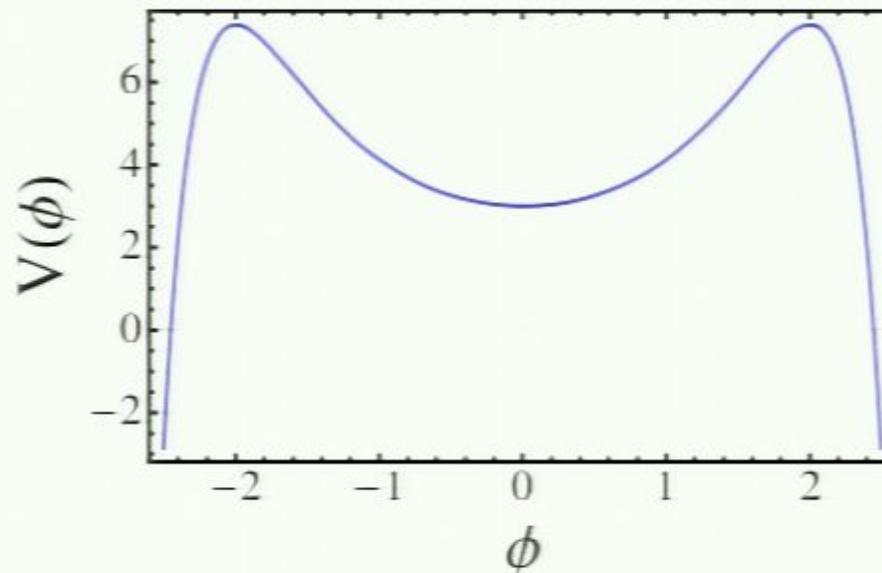
## Intuition

Action for curvature perturbations  $\zeta$

$$S = \int d^4x \sqrt{-g} \epsilon g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$$

- ▶ normally,  $S \propto a^{\text{number}}$  grows exponentially  
⇒ path integral  $\propto \int e^{iS/\hbar}$  becomes classical (equivalent to  $\hbar \rightarrow 0$ )
- ▶ in conformal time  $\sqrt{-g} = a^4$ ,  $\epsilon \propto a^{-2}$ ,  $g^{\mu\nu} = a^{-2} \eta^{\mu\nu}$   
⇒  $S$  becomes  $a$ -independent  
⇒ path integral does not become classical

## The curious model $p = -2$



background

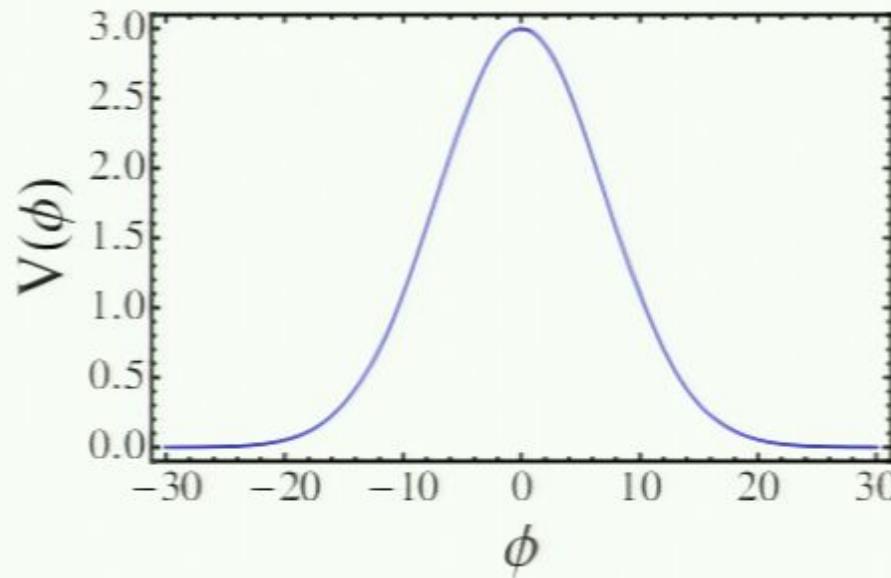
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$$u_k'' + k^2 u_k = 0, \quad z = \sqrt{2\epsilon}a = \text{const!}$$

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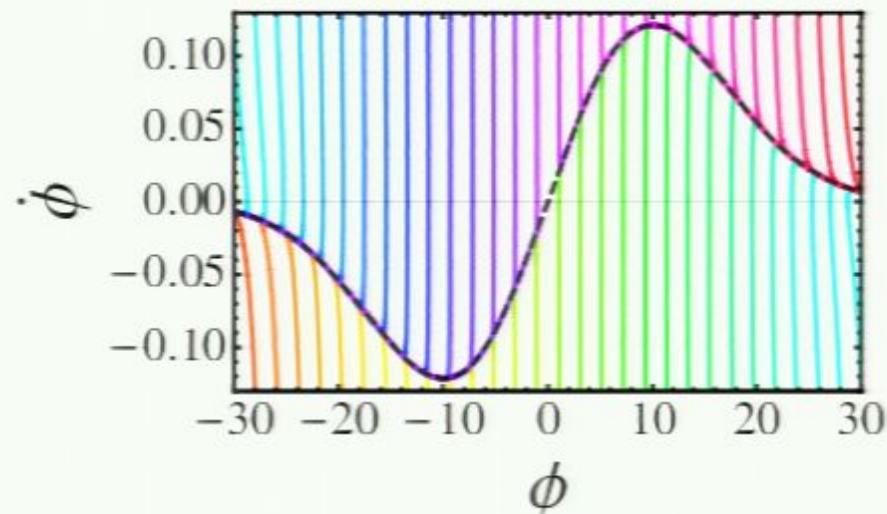
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## Background evolution for positive p (p=0.04)

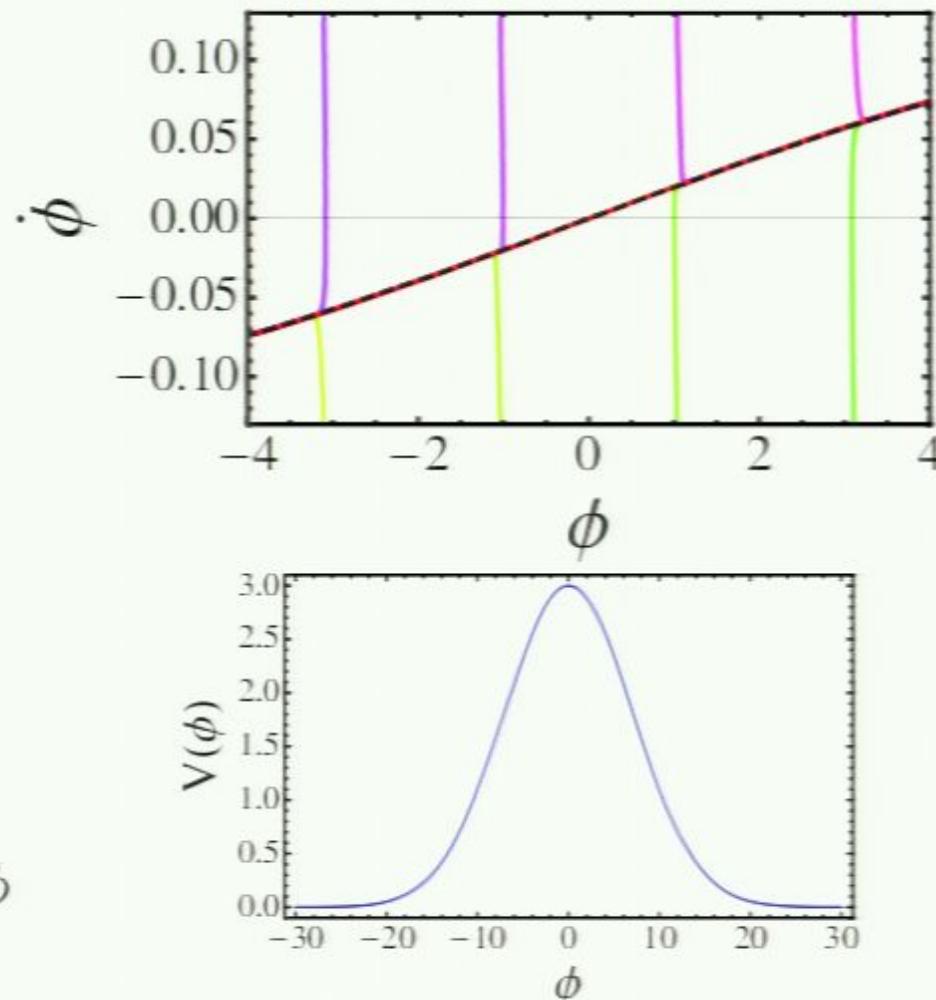


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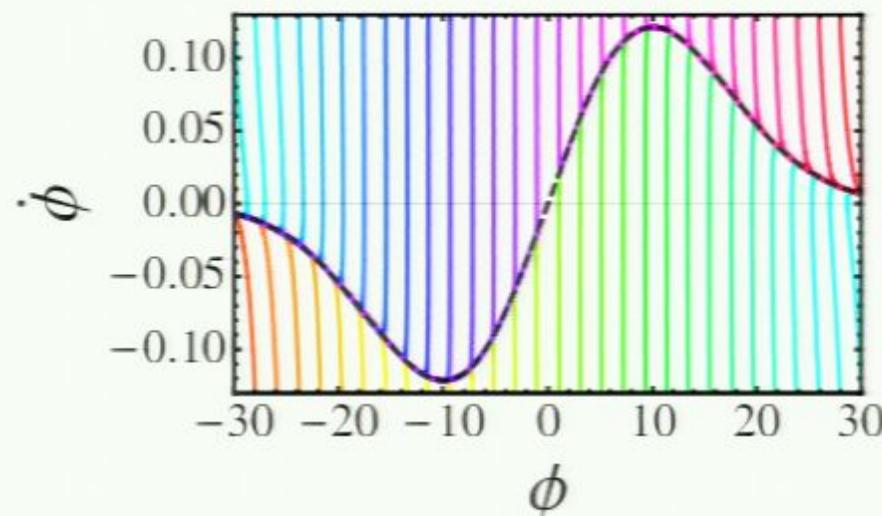
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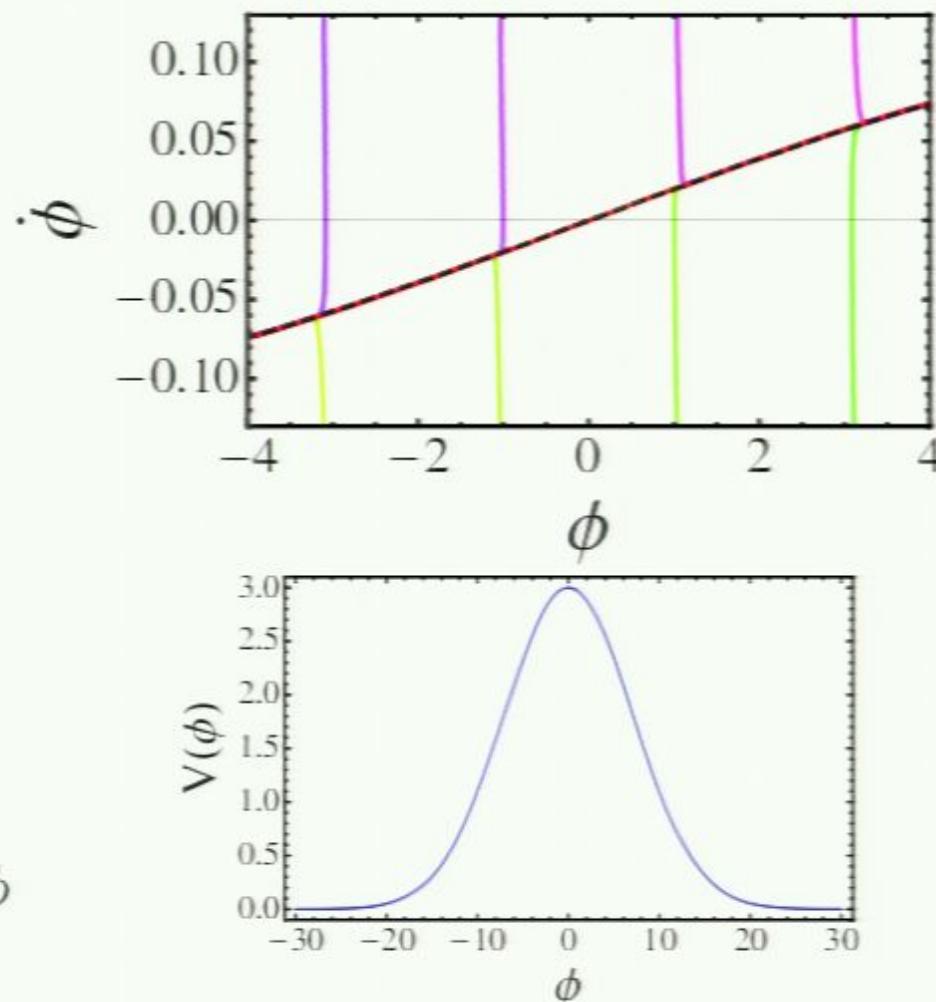


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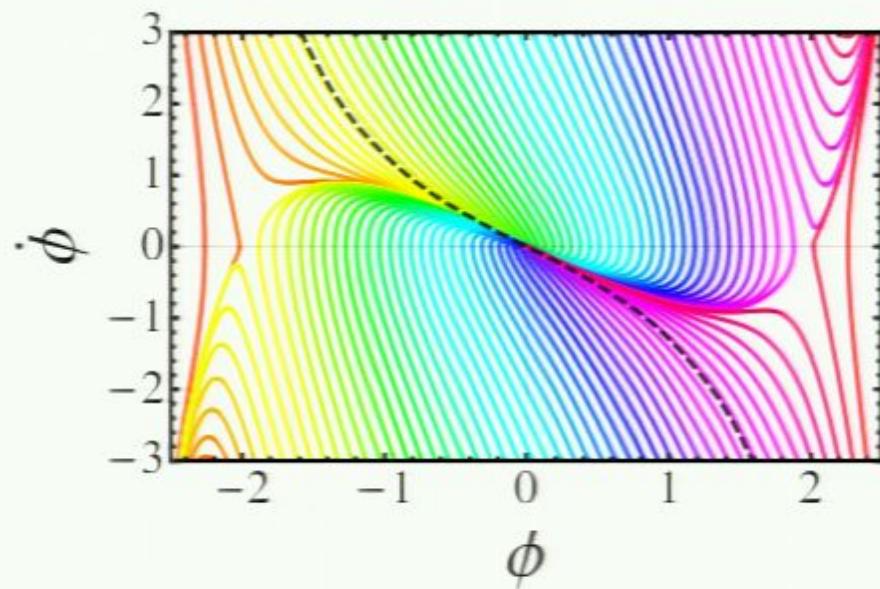
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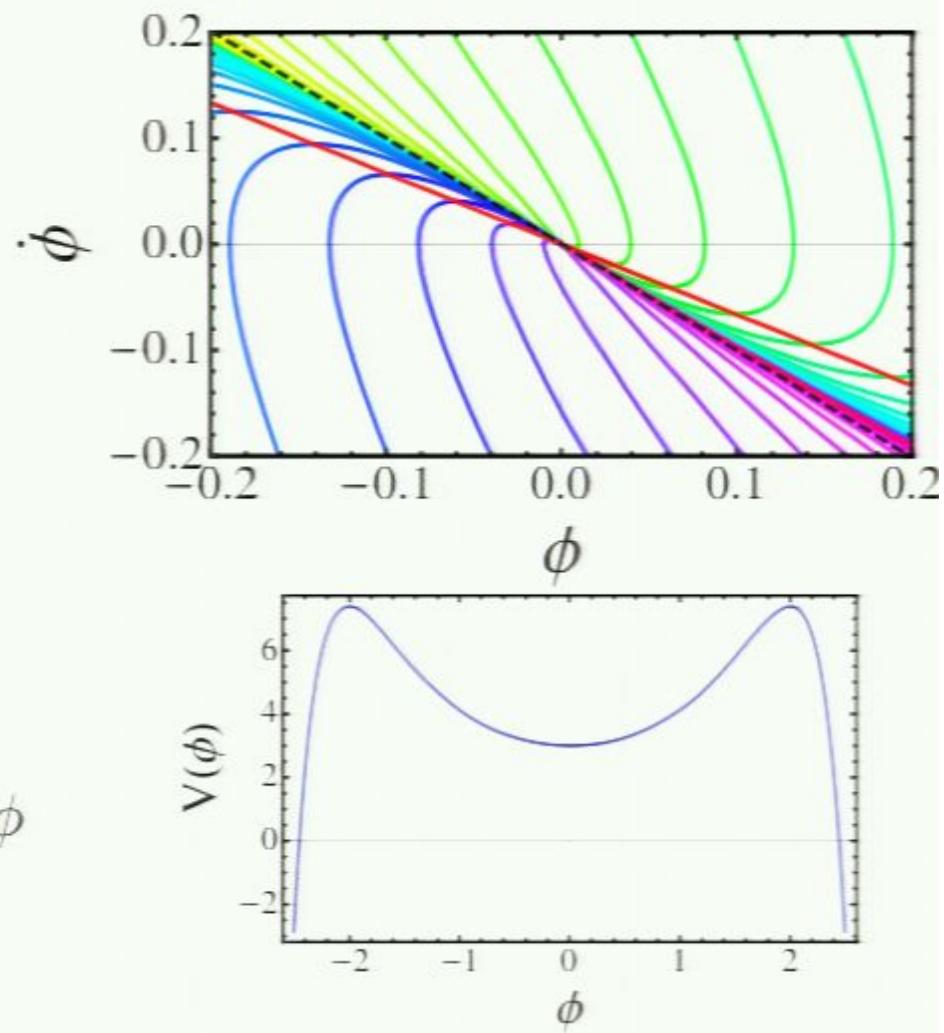


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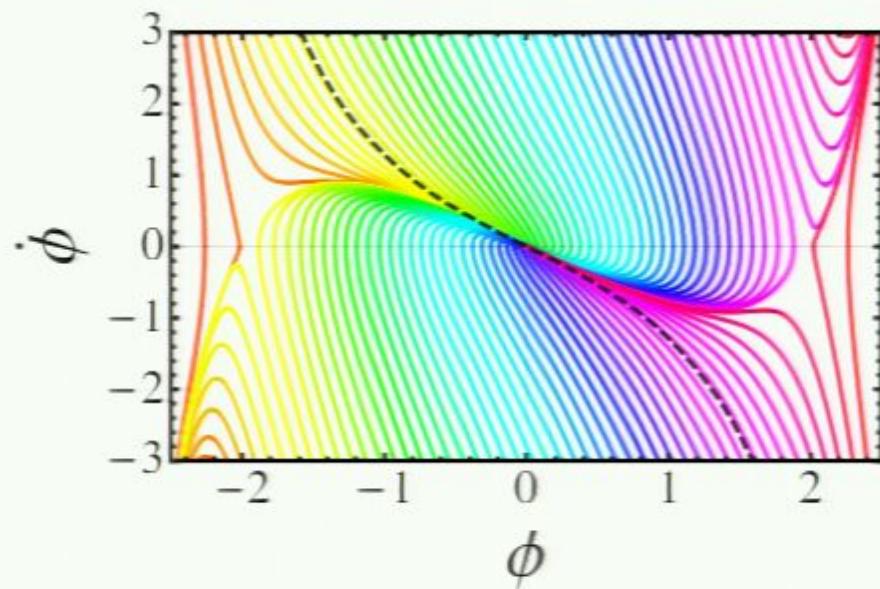
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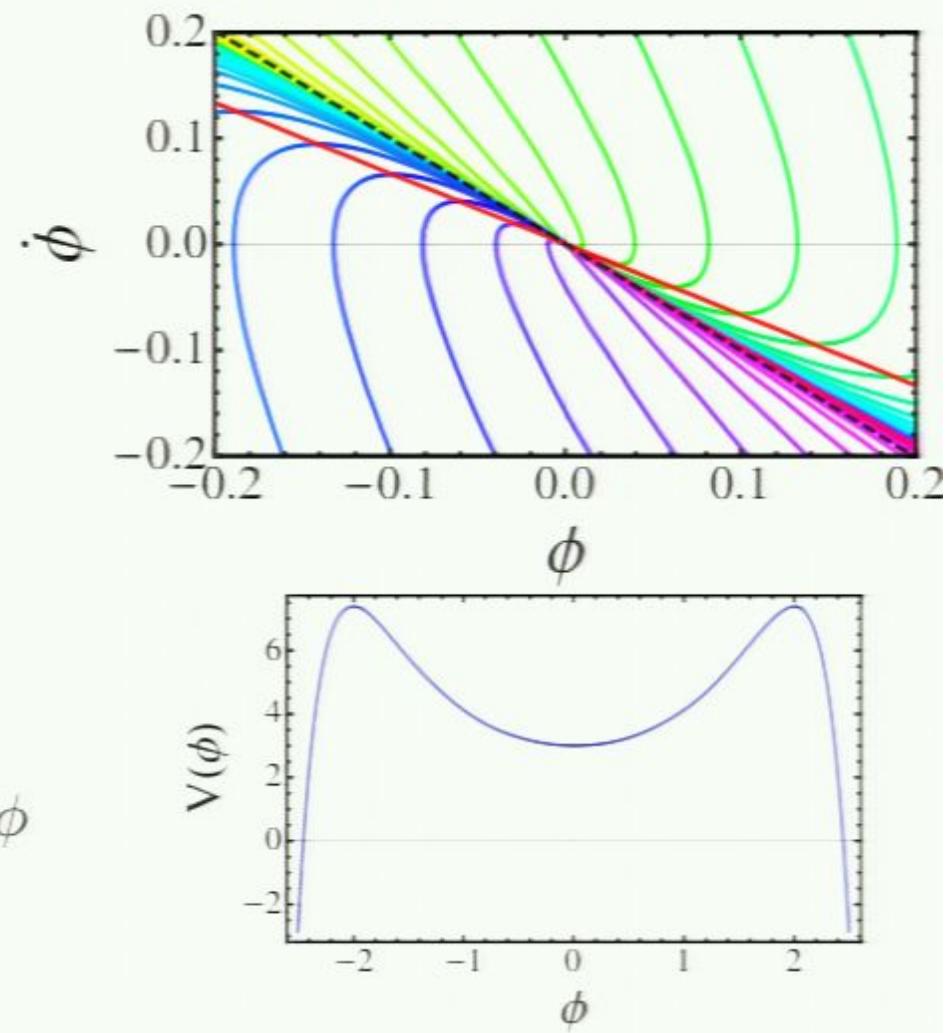


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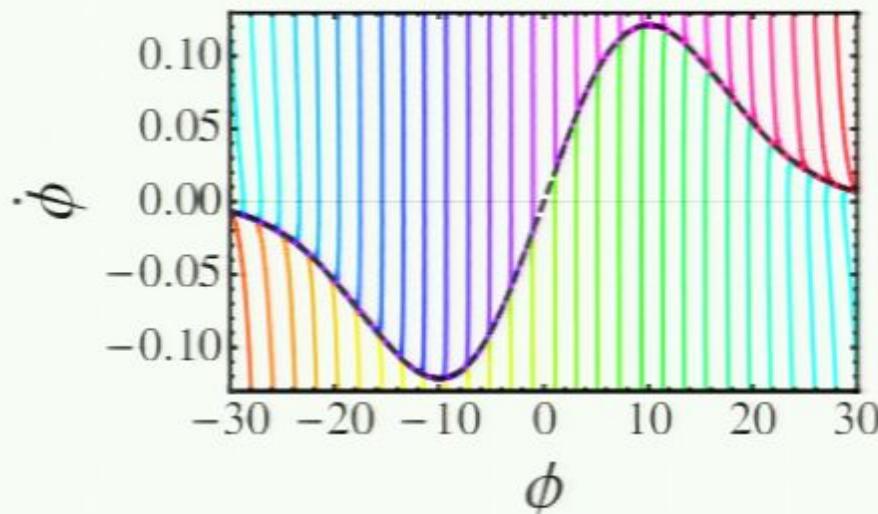
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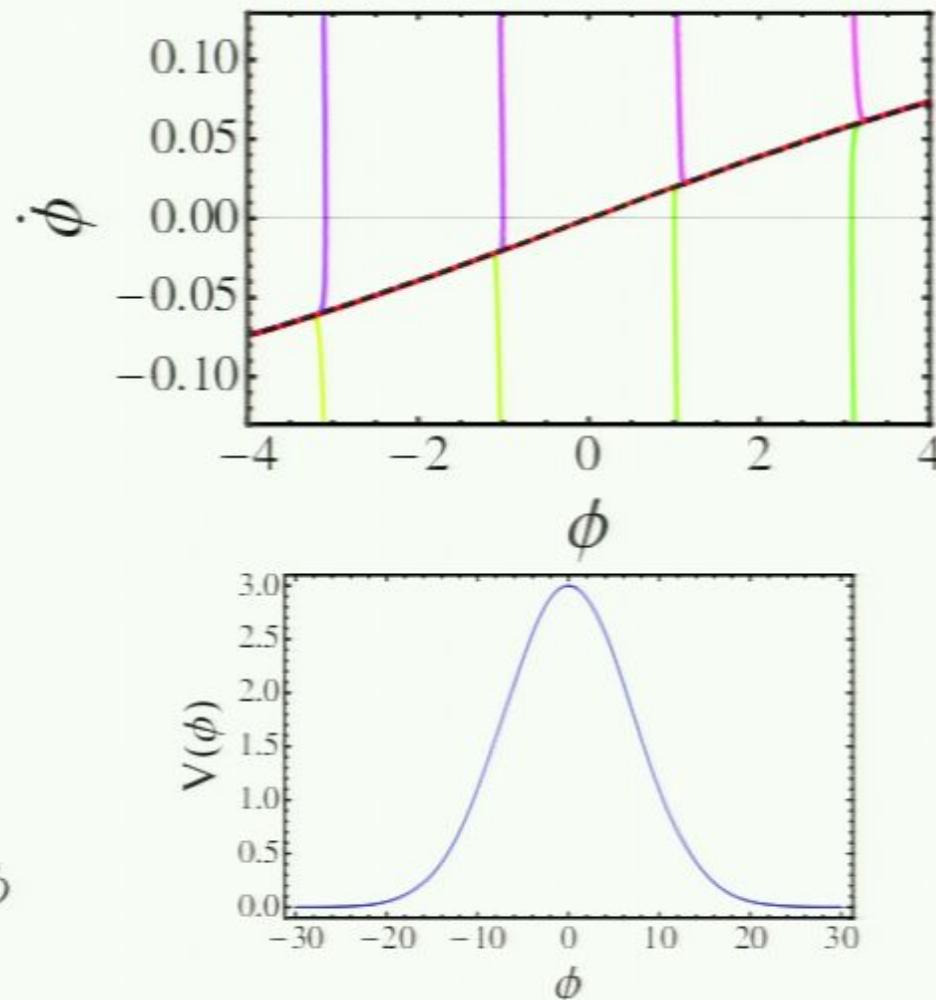


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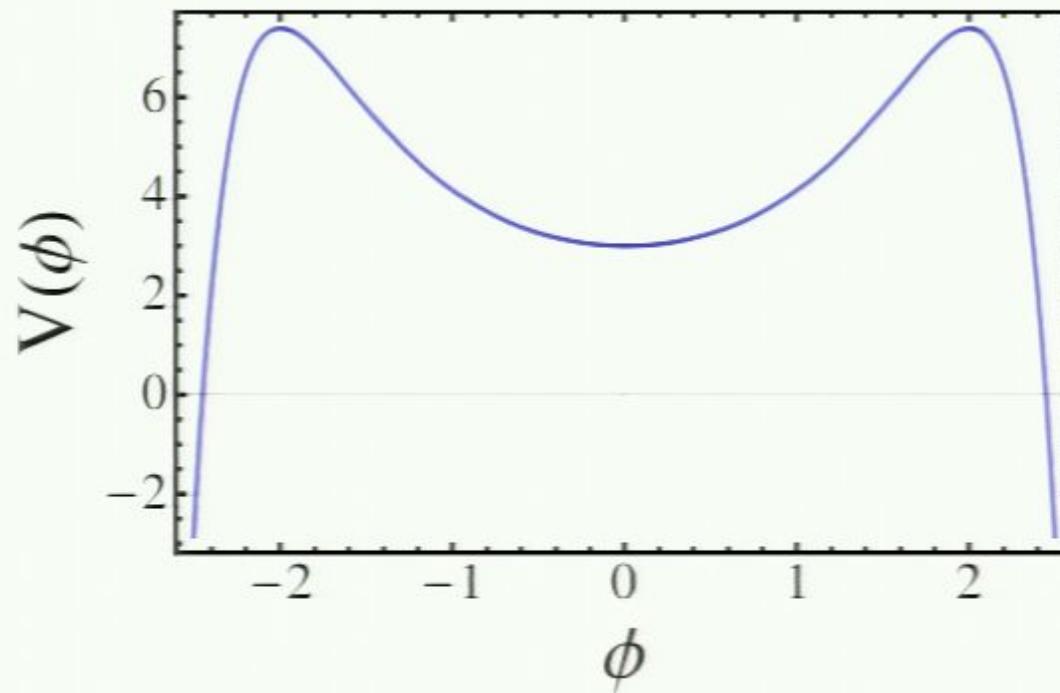
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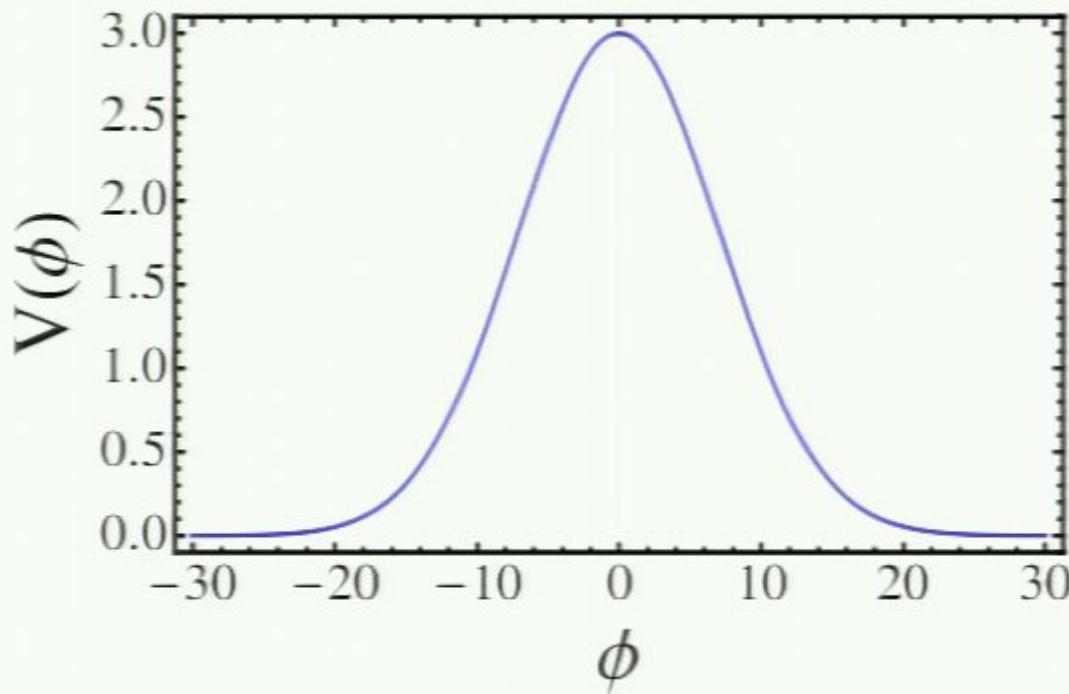
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$$t_{\text{inf}} \sim H_0^{-1} \exp \left( \kappa \frac{M_P^2}{H_0^2} \right)$$

## End of inflation for positive $p - \epsilon = 1$



Inflation ends at  $\epsilon = 1$ :

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## Linear scalar perturbations

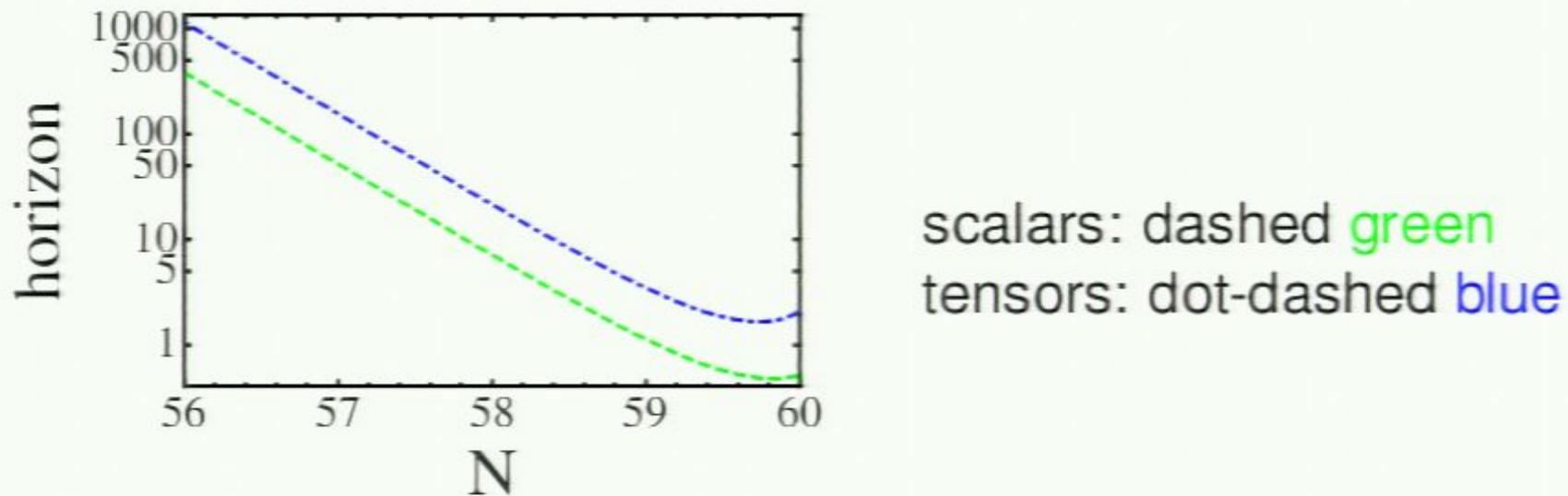
$$u_k'' + \left[ k^2 - \frac{z''}{z} \right] u_k = 0, \quad (z = \sqrt{2\epsilon}a),$$
$$u_k'' + \left[ k^2 - a^2 H^2 \left( \frac{p}{2} + 1 \right) \left( \frac{p}{2} + 2 - \epsilon \right) \right] u_k = 0$$

solvable for  $\epsilon = \epsilon_0 e^{pN} \ll 1$

$$u_k(\tau) = e^{i(\frac{p}{4}+1)\pi} \frac{\sqrt{\pi}}{2} \sqrt{-\tau} H_{\frac{3+p}{2}}^{(1)}(-k\tau)$$
$$\mathcal{P}_s = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|^2$$
$$= \frac{2^{p-1}}{\pi^3} \frac{3.81 \times 10^{56p}}{\epsilon_0} \left[ \Gamma \left( \frac{p+3}{2} \right) \right]^2 \left( \frac{H_0}{M_p} \right)^{p+2} \left( \frac{k}{\text{Mpc}^{-1}} \right)^{-p}$$

scalar spectral index  $n_s - 1 = -p$

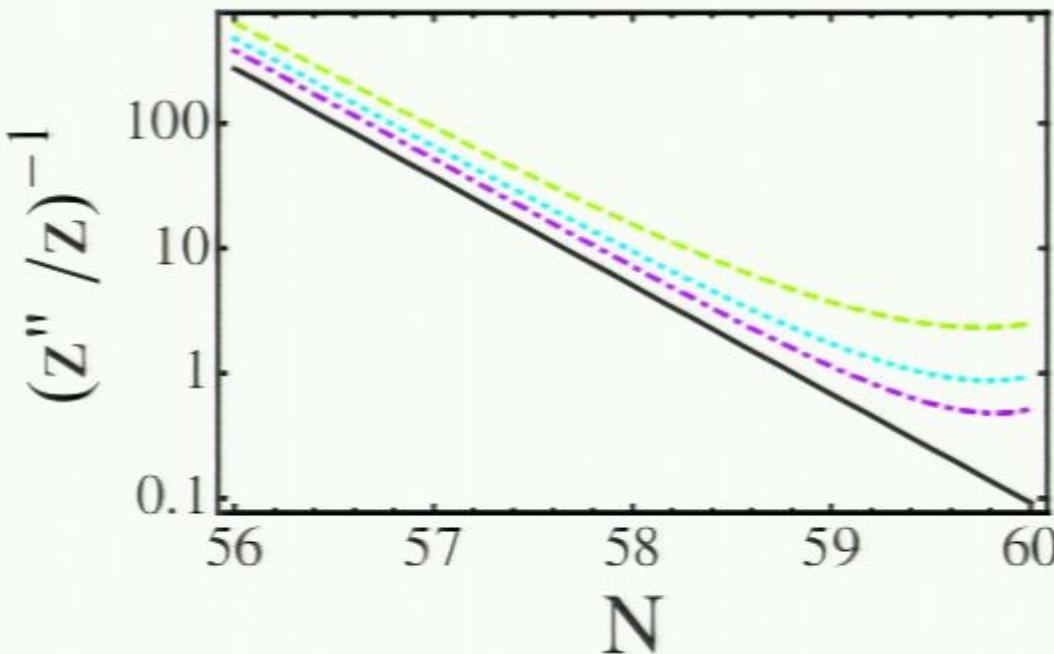
## Freeze-out at different times for scalars and tensors



$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0, \quad v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$$\sqrt{\Delta k^2} = \sqrt{\left| \frac{z''}{z} - \frac{a''}{a} \right|} = aH\sqrt{\frac{p}{2} \left( \frac{p}{2} + 3 - \epsilon \right)}$$

## Modes reenter the horizon

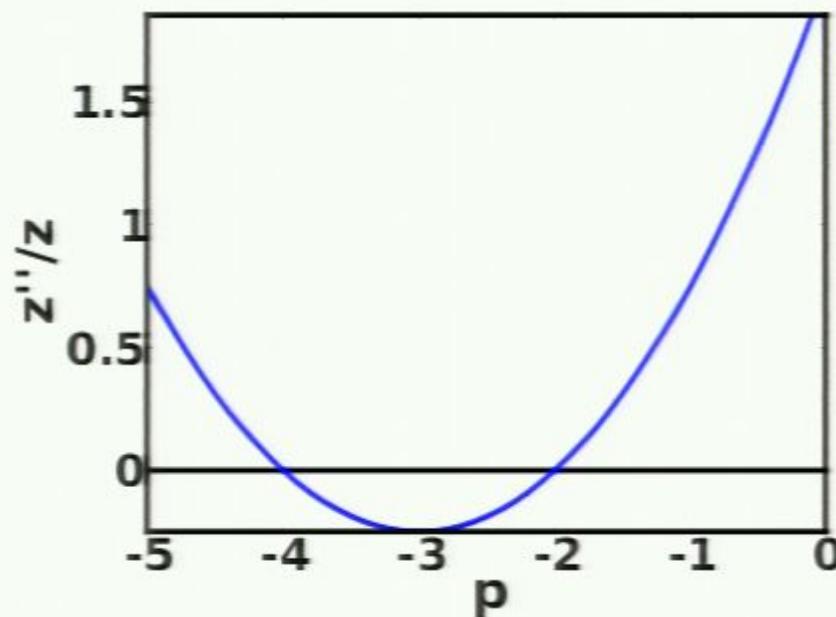


- ▶  $p=1$ : dashed green
- ▶  $p=1.5$ : dotted blue
- ▶  $p=2$  : dot-dashed purple
- ▶  $p=-1$ : solid black

$z''/z = a^2 H^2 \left(\frac{p}{2} + 1\right) \left(\frac{p}{2} + 2 - \epsilon\right)$  has extremum (see Leach et al. 2000, 2001)

⇒ comoving wavenumbers leave and reenter the horizon during inflation

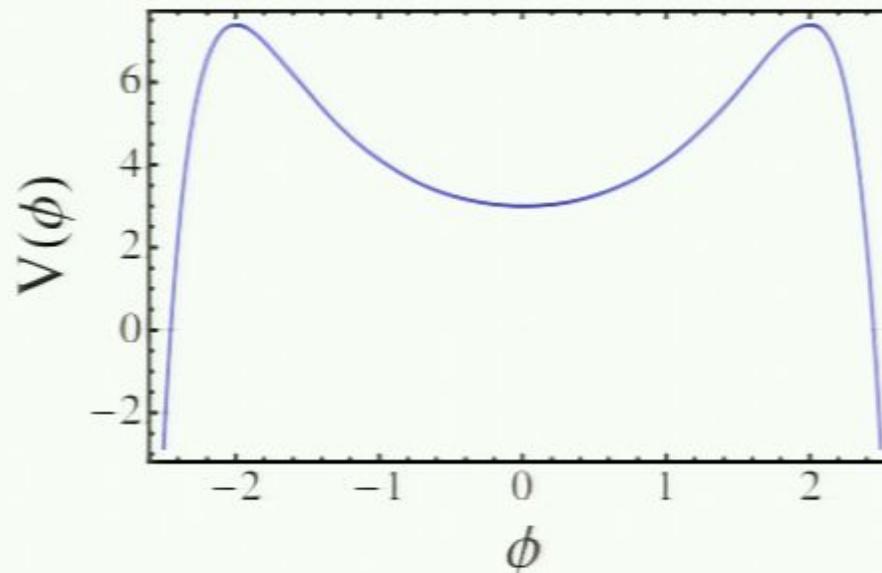
## No freeze out for scalars



$$u_k'' + \left[ k^2 - a^2 H^2 \left( \frac{p}{2} + 1 \right) \left( \frac{p}{2} + 2 - \epsilon \right) \right] u_k = 0$$

for  $-2 > p > -4$ ,  $z''/z$  changes sign  
⇒ positive mass square and no freeze-out

## The curious model $p = -2$



background

$$\begin{aligned}V &= H_0^2 e^{-\frac{1}{2}\phi^2} \left( 3 - \frac{1}{2}\phi^2 \right) \\ \epsilon &= e^{-2N}, a = e^N\end{aligned}$$

$\epsilon$  decreases with time  
perturbations

$$u_k'' + k^2 u_k = 0, \quad z = \sqrt{2\epsilon}a = \text{const!}$$

## Importance of initial state - vacuum

$$u_k'' + \left[ k^2 - a^2 H^2 \left( \frac{p}{2} + 1 \right) \left( \frac{p}{2} + 2 - \epsilon \right) \right] u_k = 0$$
$$p = -2 \Rightarrow u_k'' + k^2 u_k = 0$$

initial state: Bunch Davis vacuum  $u_k = \frac{1}{\sqrt{2k}} e^{-ik\tau}$ :

- ▶ modes never freeze out
- ▶ no gravitational particle production
- ▶ no stochastic kicks
- ▶ no eternal inflation
- ▶ power spectrum  $\mathcal{P}_S \propto |\frac{u_k}{z}| = \frac{1}{8\pi^2} k^2$  (see Easter 1996)
- ▶ modes never become classical! (see Starobinski 2005)

## Intuition

Action for curvature perturbations  $\zeta$

$$S = \int d^4x \sqrt{-g} \epsilon g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$$

- ▶ normally,  $S \propto a^{\text{number}}$  grows exponentially  
⇒ path integral  $\propto \int e^{iS/\hbar}$  becomes classical (equivalent to  $\hbar \rightarrow 0$ )
- ▶ in conformal time  $\sqrt{-g} = a^4$ ,  $\epsilon \propto a^{-2}$ ,  $g^{\mu\nu} = a^{-2} \eta^{\mu\nu}$   
⇒  $S$  becomes  $a$ -independent  
⇒ path integral does not become classical

$$u'' + \lambda - \frac{\varepsilon}{2} |u|^2 u = 0$$

$$u'' + \left| \frac{z''}{\varepsilon} \right| u = 0$$

$$\varepsilon = -\frac{H}{4}$$



## Importance of initial state - many-particle state

What if the initial state is not the vacuum but  $n_k > 1$  for some  $k$ ?

Need to take into account non-linearities

$\Rightarrow$  effective action (in the late time limit  $a \rightarrow \infty$ )

$$S = \int d\tau d^3x \left( \frac{1}{2}(u')^2 - \frac{1}{2}(\partial_x u)^2 - W(u) \right),$$

$$W = h_u u^3, h_u = \frac{H_0^2}{4M_P^2} U,$$

where  $U = a\phi$  is the background solution and  $u_k = a\delta\phi$  the perturbation

*In the limit  $1/a \rightarrow 0$ , the physics of fluctuations around a quasi-de Sitter space is described by a quantum field theory living in flat spacetime*

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# Thermalization

- ▶ thermalization temperature

$$T \sim \left( \frac{\int d^3x \left( \frac{1}{2}(u')^2 + \frac{1}{2}(\nabla u)^2 + W(u) \right)}{1/H_0^3} \right)^{1/4}$$

- ▶ long time scale  $\tau_{\text{therm}} \sim H_0^{-1} (M_P/H_0)^{5/2}$
  - ▶ inflation ends during thermalization  $n_k \sim k^{-3/2} \Rightarrow n_s = \frac{3}{2}$
  - ▶ inflation ends after thermalization  $n_k \sim k^{-1} \Rightarrow n_s = 2$  (in infra-red)
  - ▶ tunneling time scale ( $p < 0$ )  
$$\tau_{\text{tun}} = \sim -\frac{1}{H_0} \exp \left( \exp \left( \text{Const.} \frac{M_P^2}{H_0^2} \right) \right) \gg \tau_{\text{therm}}$$
- ⇒ scalar spectral index  $n_s = 2$

## SUGRA?

$$V_{\text{SUGRA}} = e^K \left( K^{ij} D_i W D_j \bar{W} - 3|W|^2 \right),$$

$$V = H_0^2 M_P^2 e^{-\frac{p}{4} \frac{\phi^2}{M_P^2}} \left( 3 - \frac{p^2}{8} \frac{\phi^2}{M_P^2} \right)$$

Assume a superpotential  $W = H_0$ , Kähler potential  $K = \frac{1}{2}\phi\bar{\phi}$

$$K^{ij} = (\partial_{\phi^i} \partial_{\bar{\phi}^j} K)^{-1}$$

$$D_i W = \partial_{\phi^i} W + W \partial_{\phi^i} K$$

$$\Rightarrow V = -V_{\text{SUGRA}}$$

## More models with this behaviour?

$$u_k'' + \left[ k^2 - \frac{z''}{z} \right] u_k = 0$$

- ▶  $z' = c = \text{const}$
- $\Rightarrow H'' = -\frac{c^2}{2} \left[ 2\frac{\tau}{a} H^2 - \tau^2 H^3 + 2\frac{\tau^2}{a} HH' \right]$
- ▶ background  $H = \frac{2}{c}\tau^{-2}$ ,  $\epsilon = 2$

## Conclusions

$$V(\phi) = H_0^2 e^{\frac{p}{4}\phi^2} \left( 3 - \frac{p^2}{8}\phi^2 \right)$$

- ▶ general family of models
  - ▶ inflation can last arbitrarily long (despite  $\eta = O(1)$ )
  - ▶ compatible with observations
  - ▶ SUGRA?
- ▶ curious behavior for different values of  $p$ 
  - ▶ initial state = vacuum
    - ▶ NO FREEZE OUT
    - ▶ no gravitational particle production
  - ▶ initial state = many particle state
    - ▶ self-interaction, thermalization instead of gravitational particle production
    - ▶ tunneling to end inflation
    - ▶ power spectra with  $n_s = \frac{3}{2}$  or  $n_s = 2$  (fully thermalized)
  - ▶ duality between field theory in Minkowski space and inflationary model

# Model Overview

value of p	model features
$-\infty < p < -6$	widely separated freeze-out $\infty$ long inflation for one initial condition $ \eta  > 1$
$-6 < p < -4$	widely separated freeze-out $ \eta  > 1$ $\infty$ long classical inflation
$-4 < p < -2$	no freeze out $ \eta  > 1$ $\infty$ long classical inflation
$p = -2$	no freeze out no gravitational particle production no stochastic eternal inflation non-suppressed mode interactions (pre)thermalization dual to a QFT in flat space $ \eta  \approx 1$ $\infty$ long classical inflation
$-2 < p < -2 + \sqrt{3}$	modes reenter the horizon $\infty$ long classical inflation
$-2 + \sqrt{3} < p < 0$	$\infty$ long classical inflation
$p = 0$	exact de Sitter space-time
$0 < p < 0.01$	finite duration of inflation
$0.01 < p < 0.07$	compatible with observations finite duration of inflation
$0.07 < p < \infty$	$ \eta $ large finite duration of inflation

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