Title: The Phase Diagram of QCD: Results & Challenges

Date: Mar 05, 2010 02:30 PM

URL: http://pirsa.org/10030028

Abstract: I will review the progress made in our understanding of the QCD phase diagram within an RG approach to QCD and effective QCD models. In particular this includes a discussion of the confinement-deconfinement phase transition/cross-over, the chiral phase transition/cross-over, as well as their interrelation.

The phase diagram of QCD

Results & Challenges

Jan M. Pawlowski
Universität Heidelberg & ExtreMe Matter Institute

Perimeter Institute, March 5th 2010

Outline

- QCD phase diagram
- Quark confinement & chiral symmetry breaking
- Chiral phase structure at finite density
- Summary and outlook

QCD phase diagram

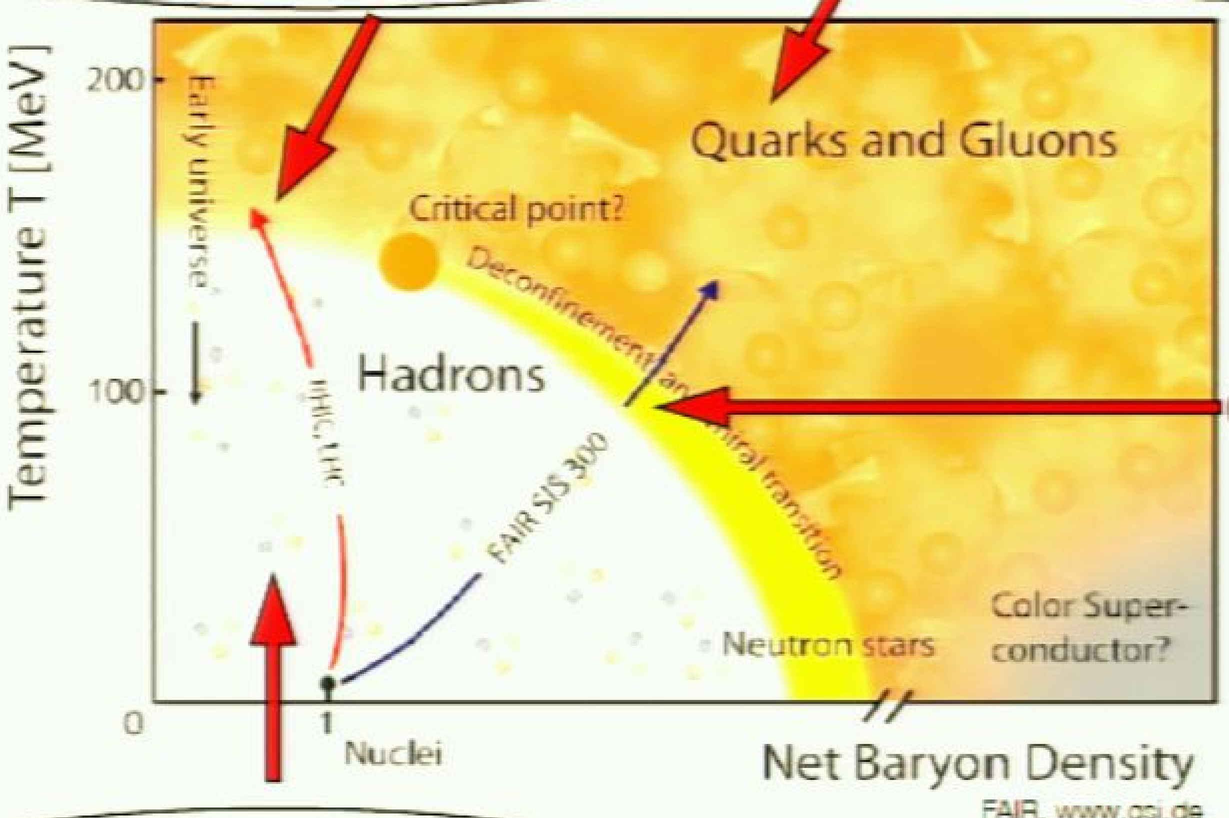
Phase diagram of QCD

Strongly correlated quark-gluon-plasma

'RHIC serves the perfect fluid'

massless quarks (chiral symmetry)

deconfinement



quarkyonic:

confinement & chiral symmetry?

hadronic phase

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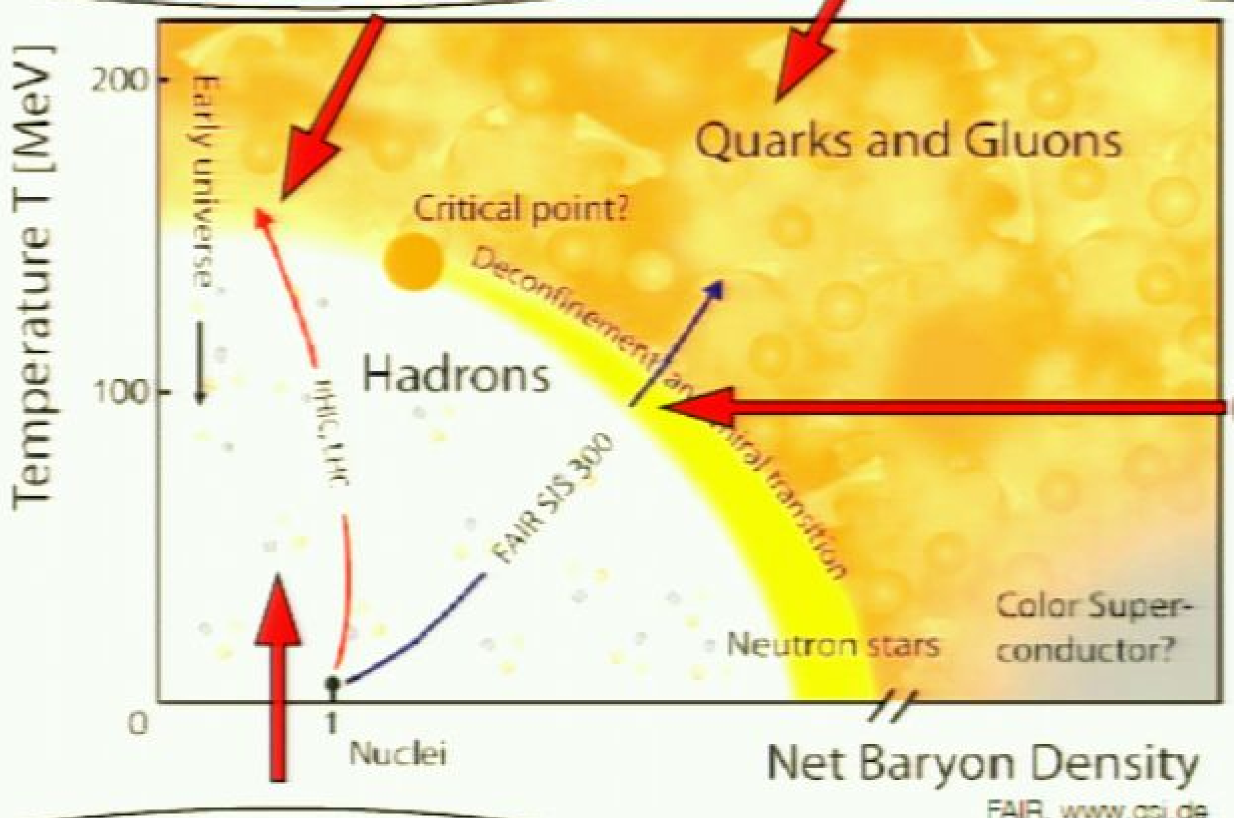

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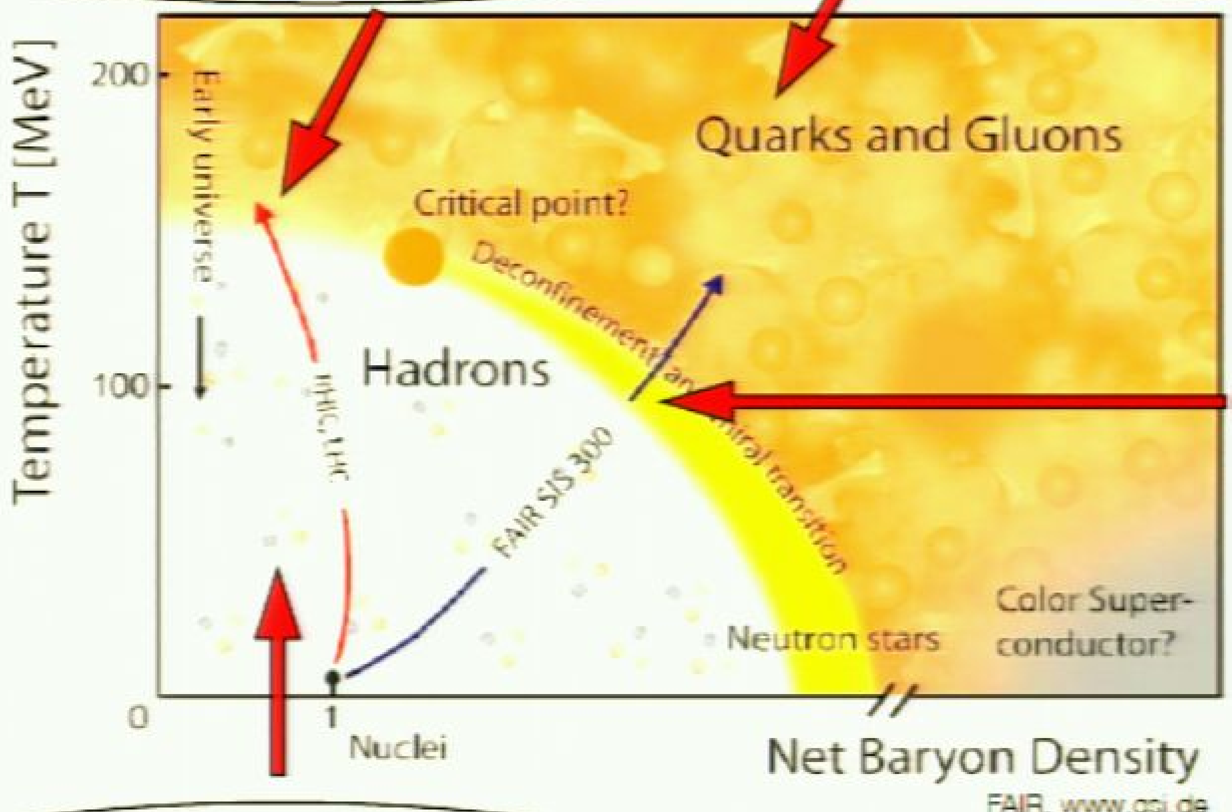

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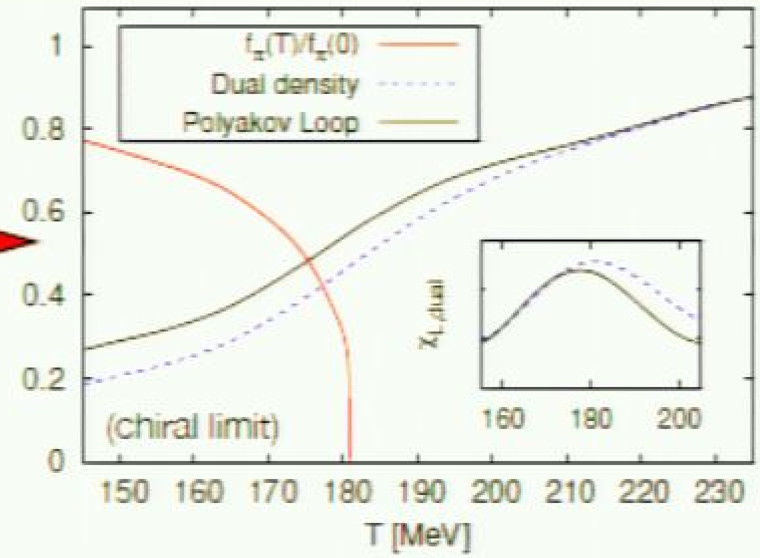
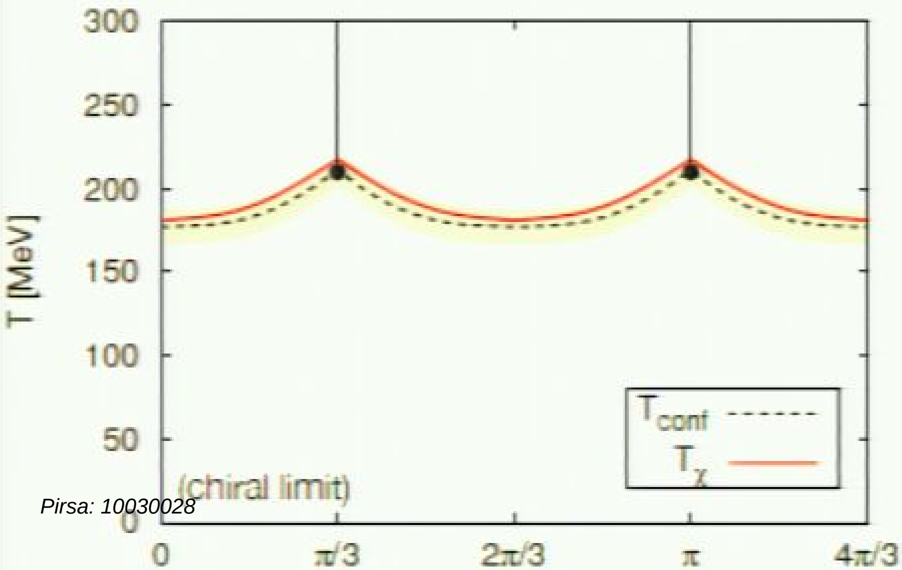
Phase diagram of two flavour QCD

Continuum methods

RG-flows in QCD

Braun, Haas, Marhauser, JMP '09





PNJL & PQM model

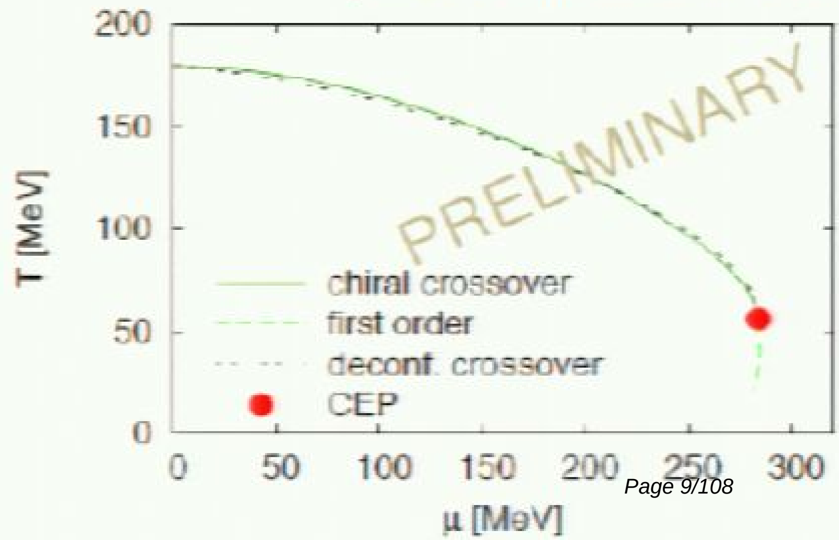
Fukushima '03

Ratti, Thaler, Weise '06

Back-coupling of matter fluctuations to YM-sector

Schaefer, JMP, Wambach '07

Herbst, JMP, Schaefer, in prep



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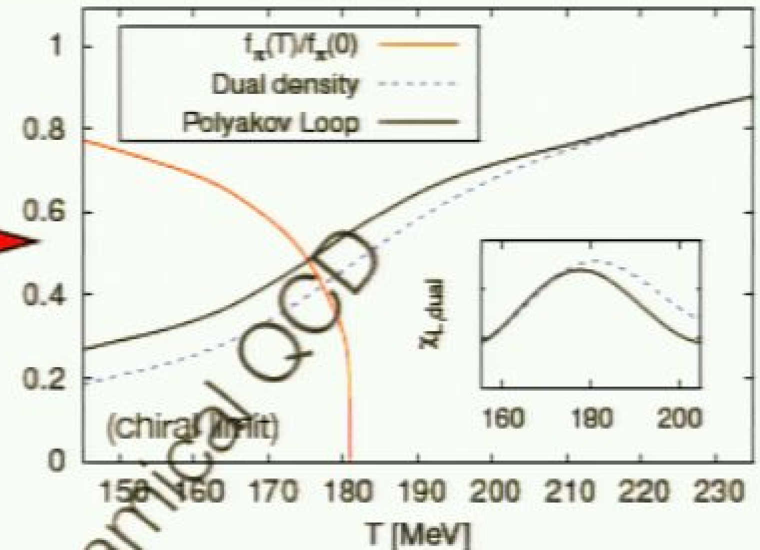

Full dynamical QCD

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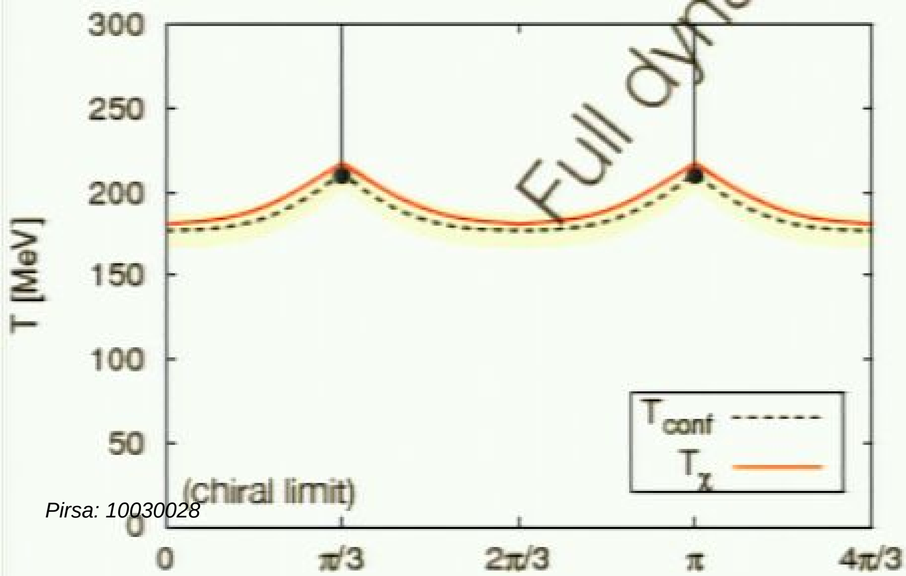

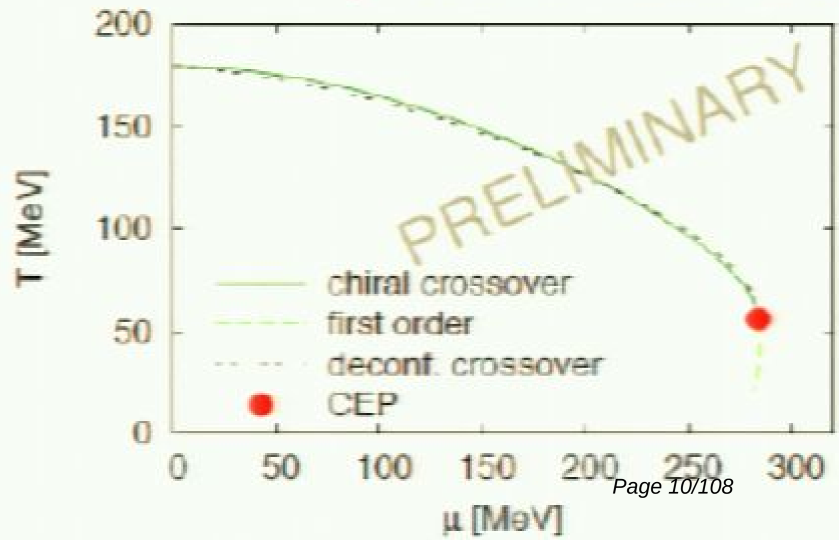

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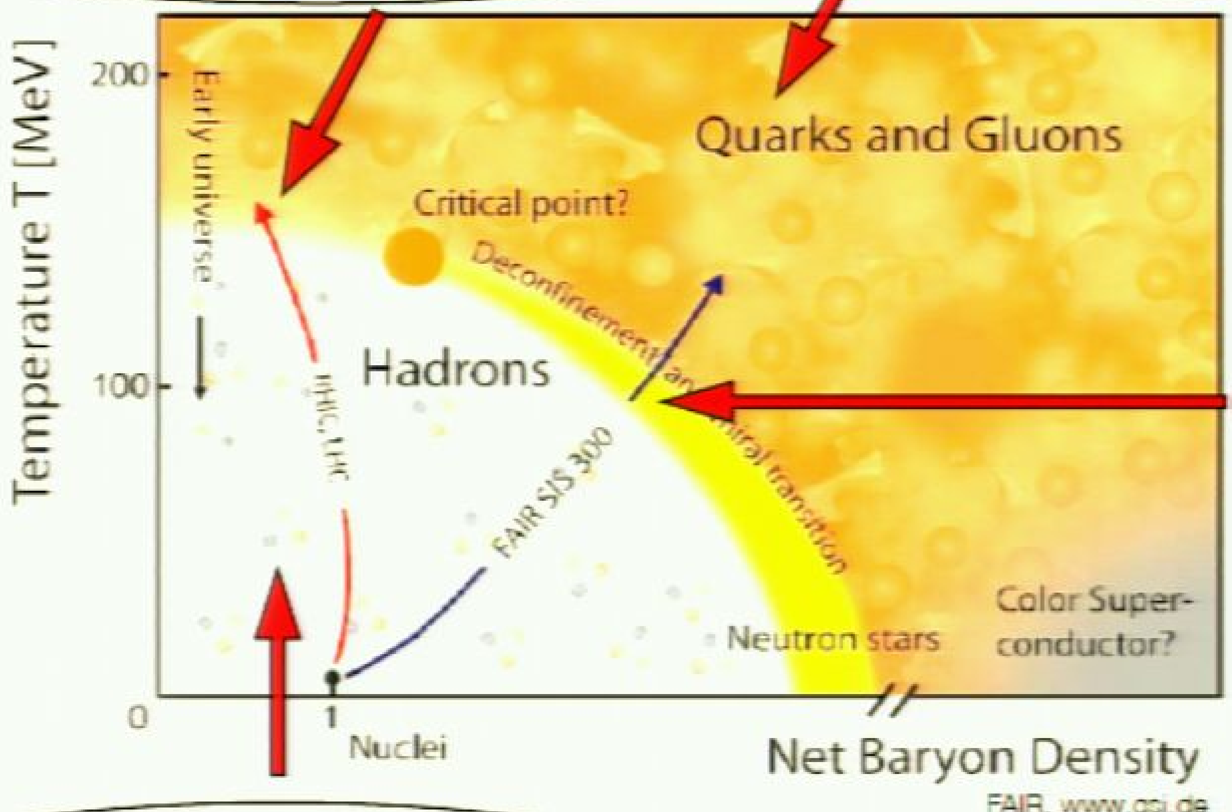

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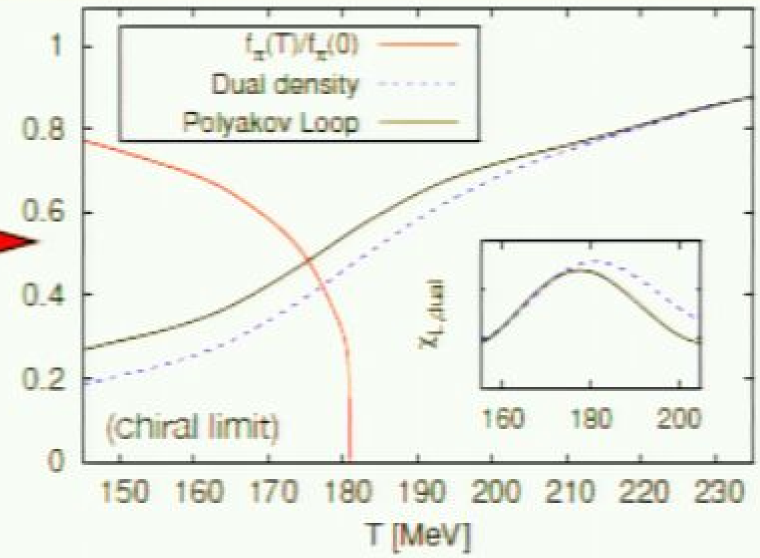

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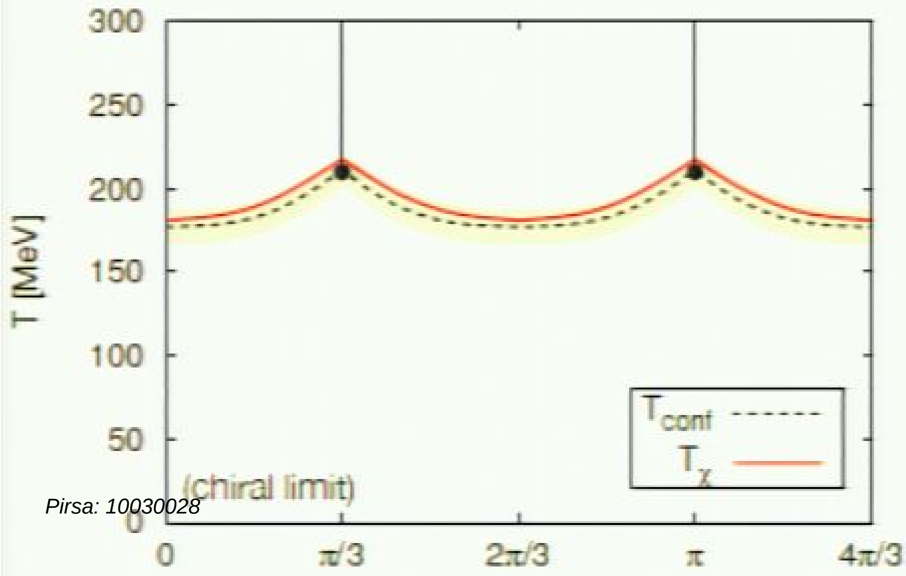

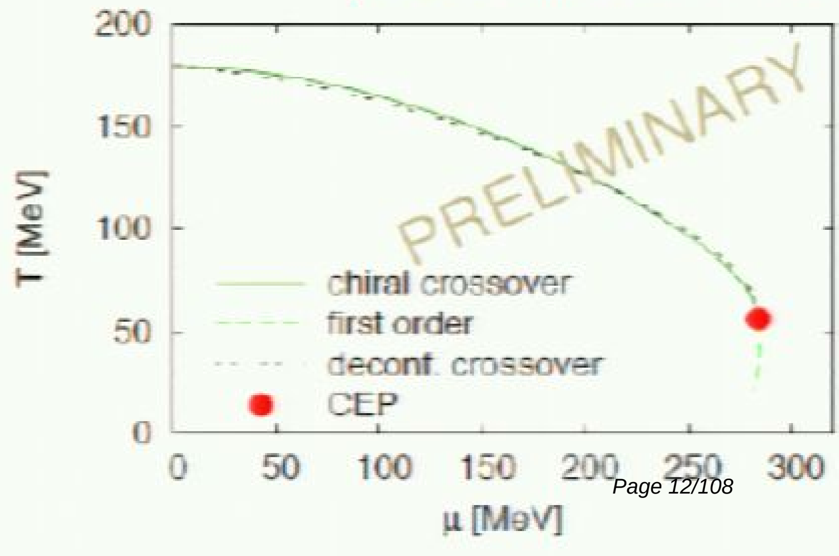

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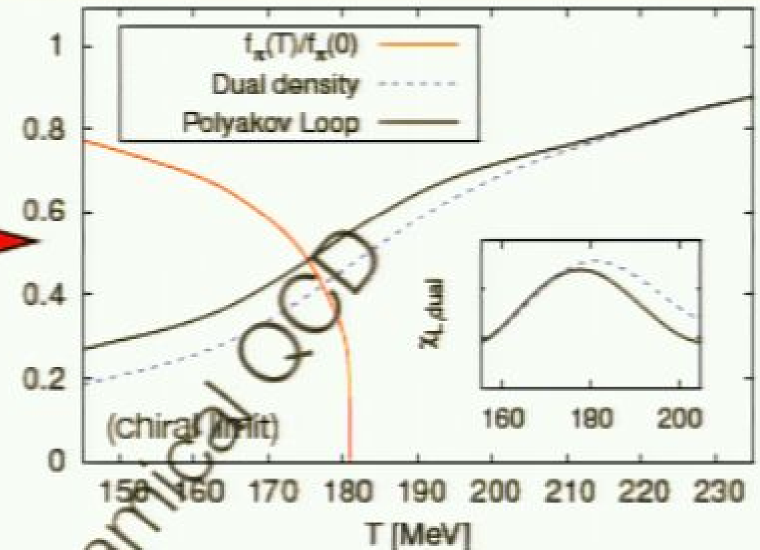

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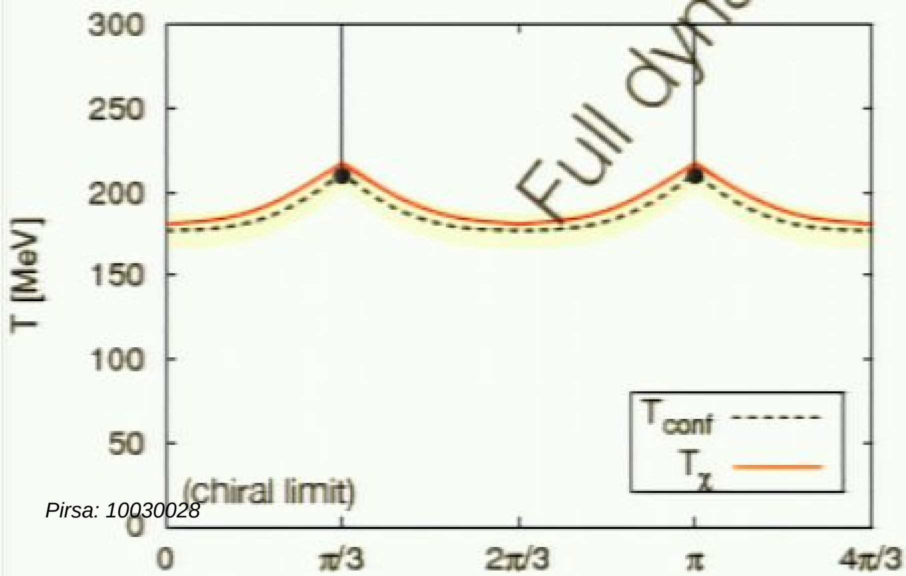

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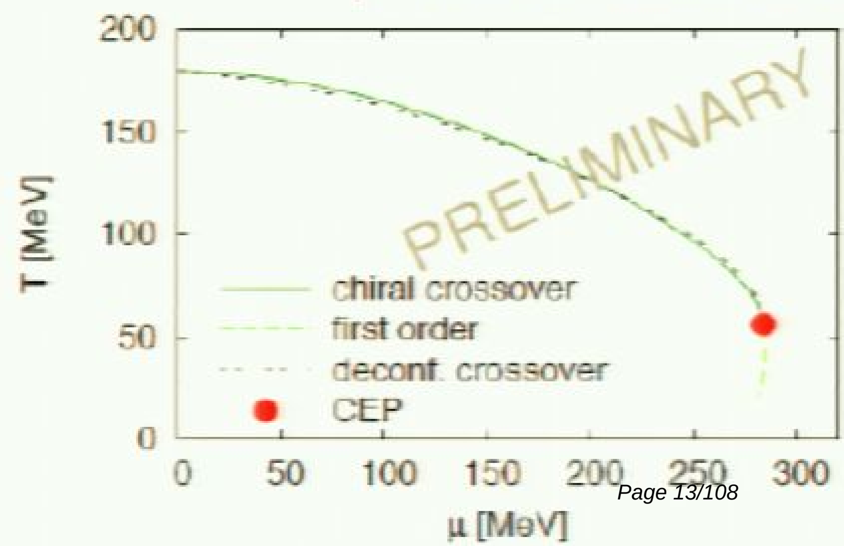
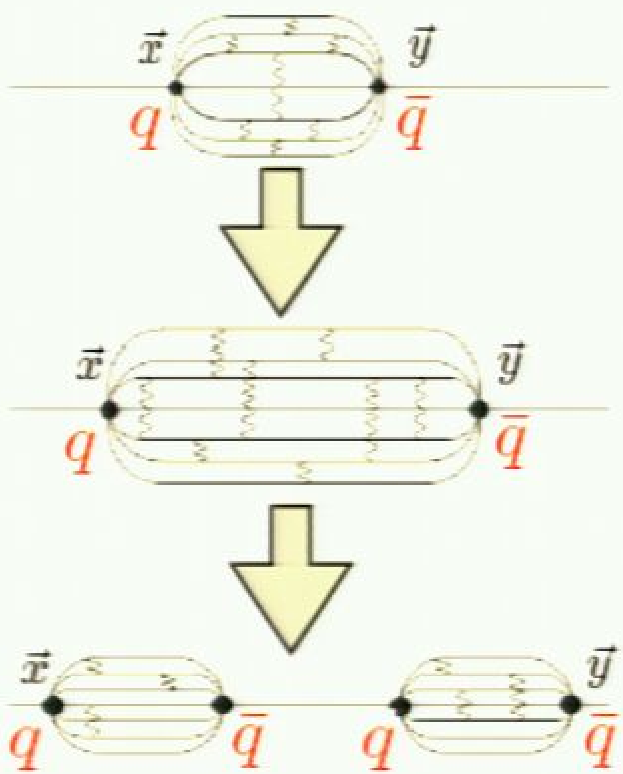


Quark confinement & chiral symmetry breaking

Confinement

$$r = |\vec{x} - \vec{y}|$$



Order parameter $\sim '\langle q \rangle'$

$$\Phi = e^{-\frac{1}{2}\beta F_{q\bar{q}}(\infty)}$$

- Confinement: $\Phi = 0$
- Deconfinement: $\Phi \neq 0$

Φ Polyakov loop

$$\Phi = \frac{1}{3}\langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\}\rangle$$

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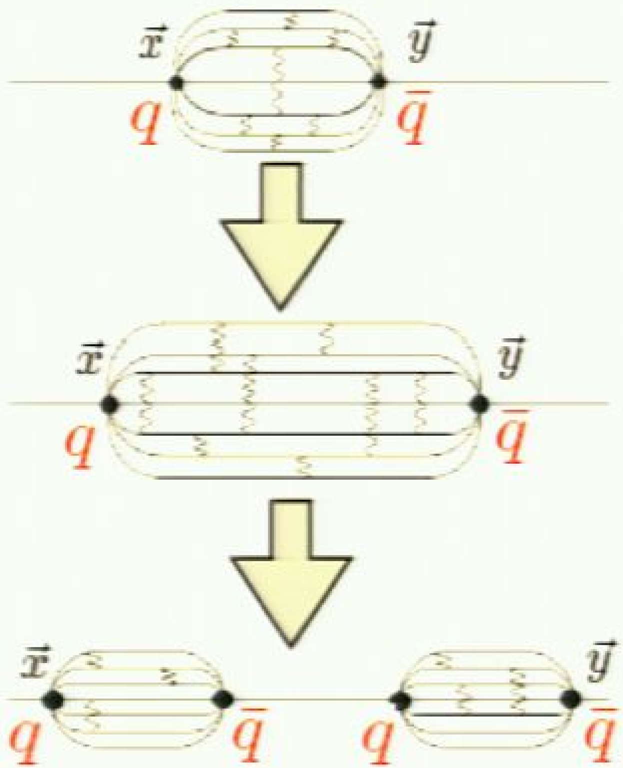

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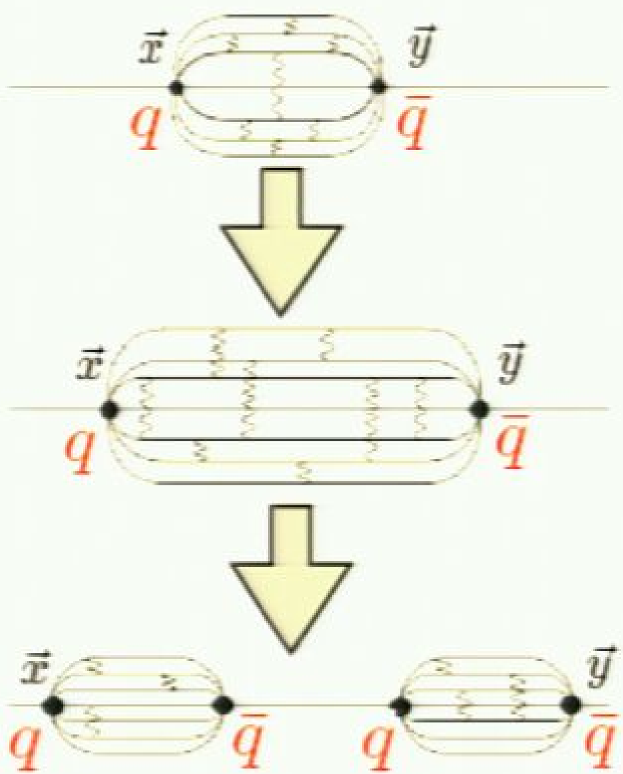

string breaking at $r \approx 1.1 fm$

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Symmetry

- Z_3 - symmetry: $q \rightarrow zq$
- broken by dynamical quarks

Chiral symmetry breaking

chiral symmetry

Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	170×10^3	
Quark	u	c	t	$\frac{2}{3}$
Quark	d	s	b	$-\frac{1}{3}$
Mass [MeV]	4-8	80-130	$(4.1\text{-}4.4) \times 10^3$	

chiral symmetry breaking: $\Delta m \approx 400 MeV$

2 light flavours, one heavy flavour $2 + 1$

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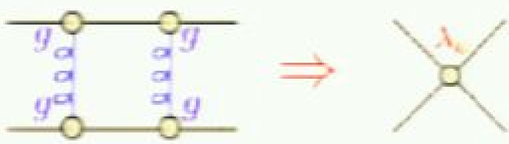

$\Rightarrow$ $\propto \alpha_s^2$

$$\int d^4x\, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2\right]$$

$\langle \bar{q}q \rangle \neq 0$

mass term: $\langle \bar{q}q \rangle\, \bar{q}q$

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Order parameter

$$\sigma = \langle \bar{q}q \rangle$$

chiral condensate

- chiral symmetry: $\sigma = 0$
- symmetry breaking: $\sigma \neq 0$

Symmetry

- $SU_L(N_f) \times SU_R(N_f)$
- broken to $SU(N_f)$

Functional RG

Functional RG

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Wetterich '93

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p)}\; k\partial_k R_k(p)$$

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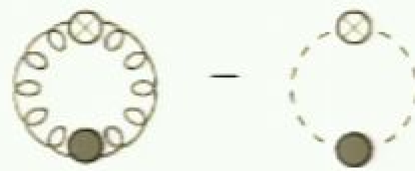

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  - no sign problem numerics as in scalar theories
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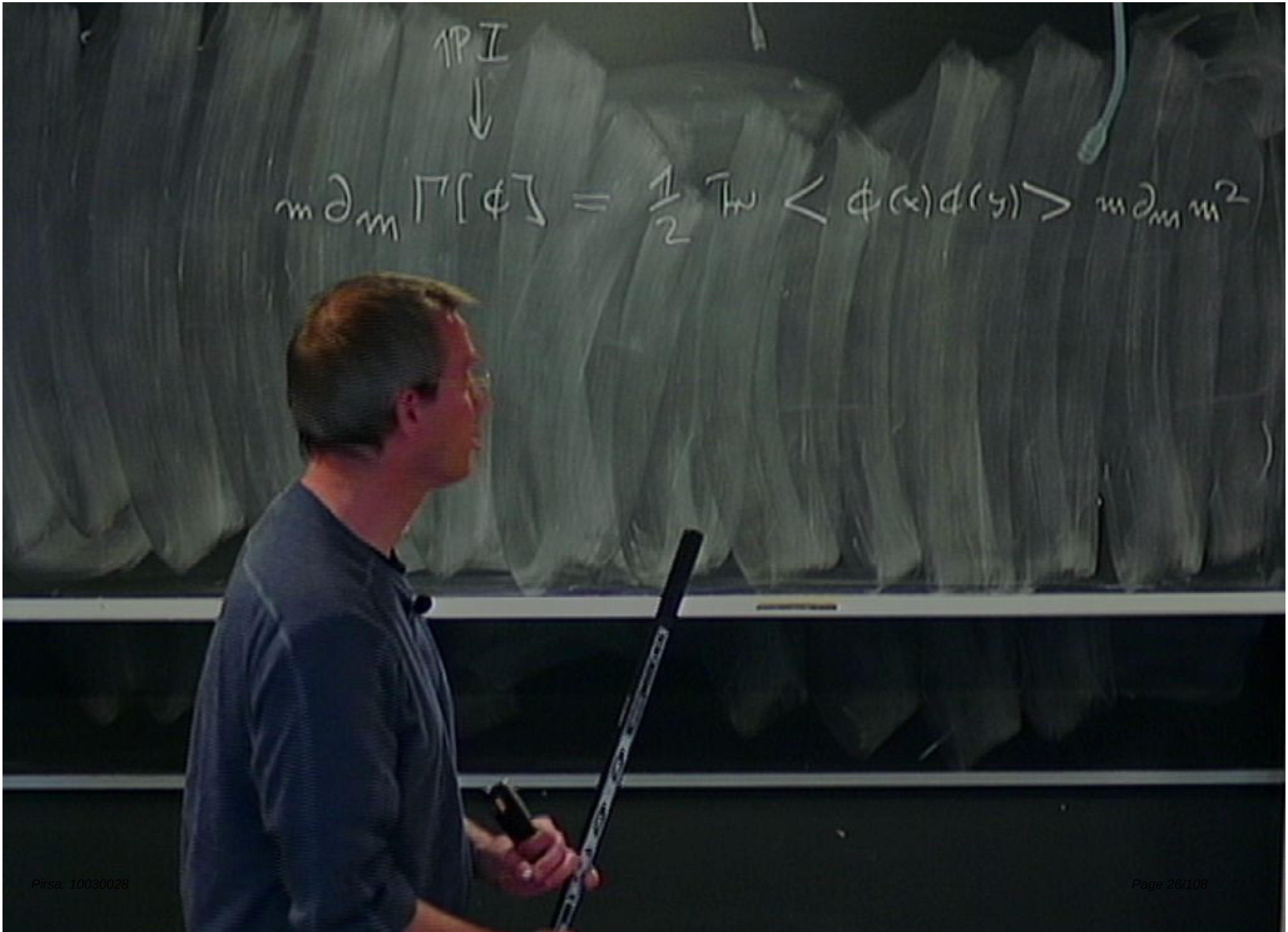
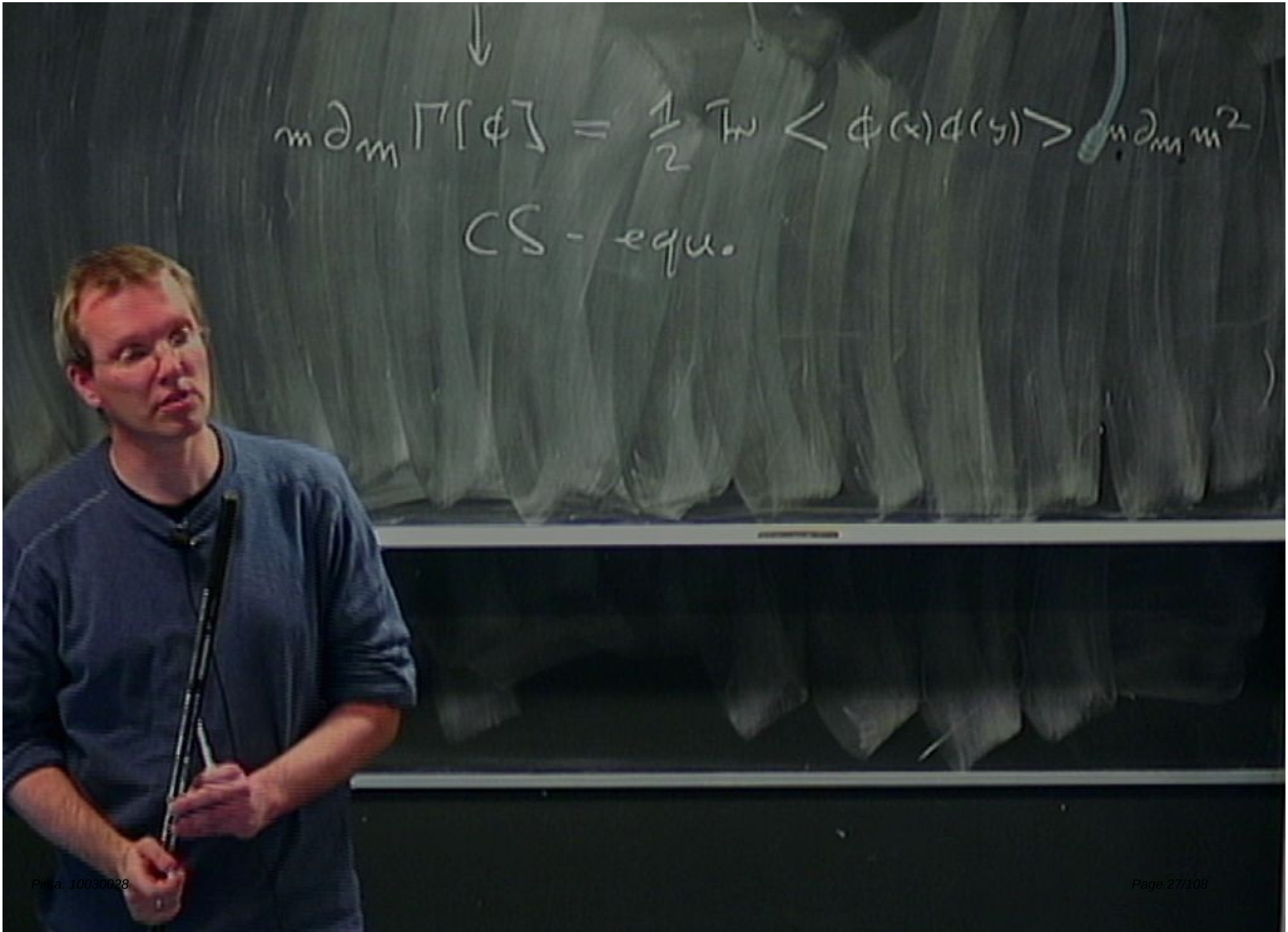

1PI
$m\partial_m \Gamma[\phi] = \frac{1}{2} \mathrm{Tr} \langle \phi(x)\phi(y) \rangle \, m\partial_m m^2$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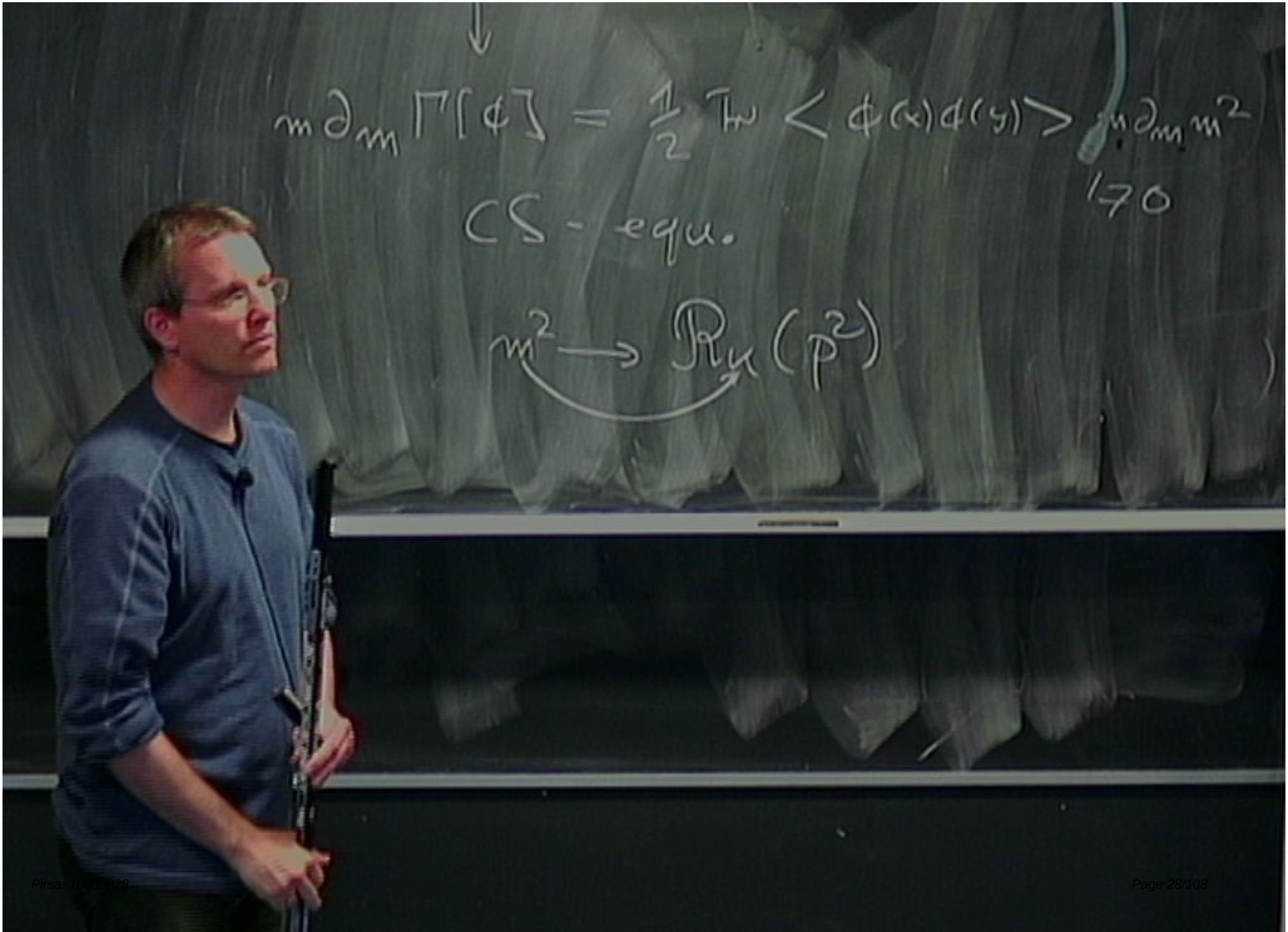
CS - equ.


m∂_m Γ[φ] = 1/2 Tr < φ(x)φ(y)> m∂_m m^2
170
CS - equ.
m^2 → R_k(p^2)

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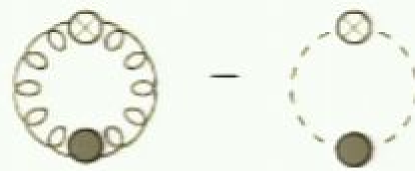
$\partial_t \Gamma_k[\phi] \;=\; \frac{1}{2}$ 

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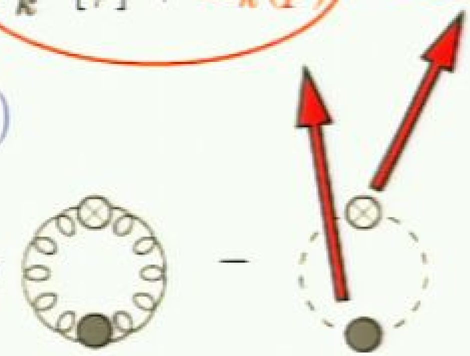

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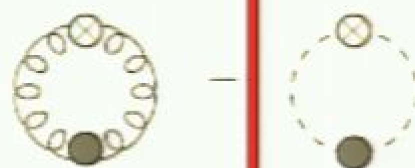
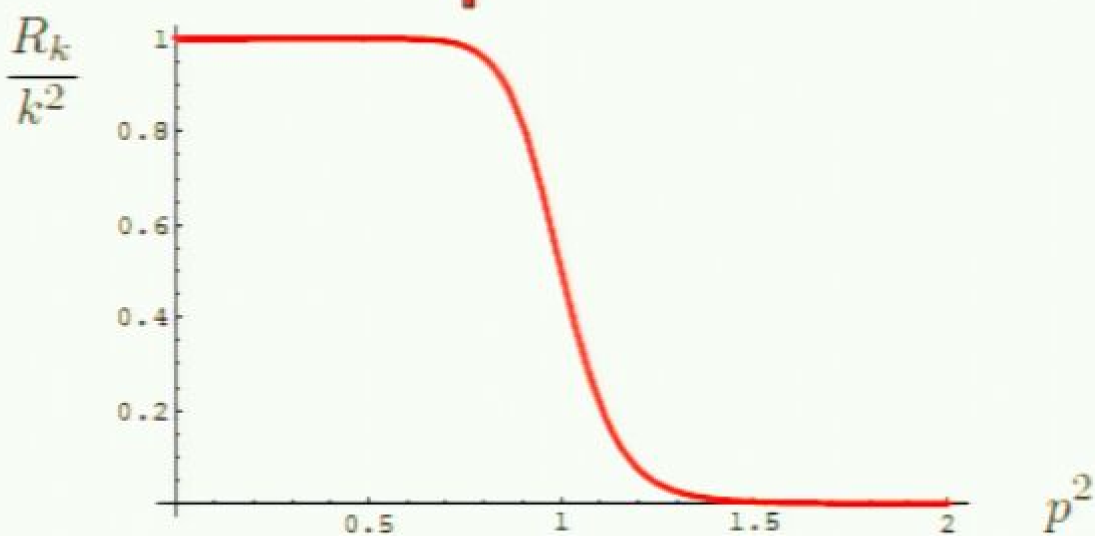
$$\partial_t \Gamma_k[\phi] \;=\; \tfrac{1}{2}$$ 

- Flow infrared finite



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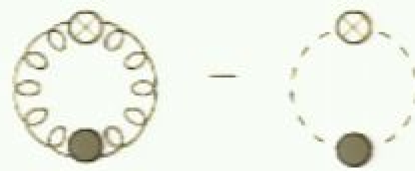
$$\partial_t \Gamma_k[\phi] \;=\; \tfrac{1}{2}$$ 

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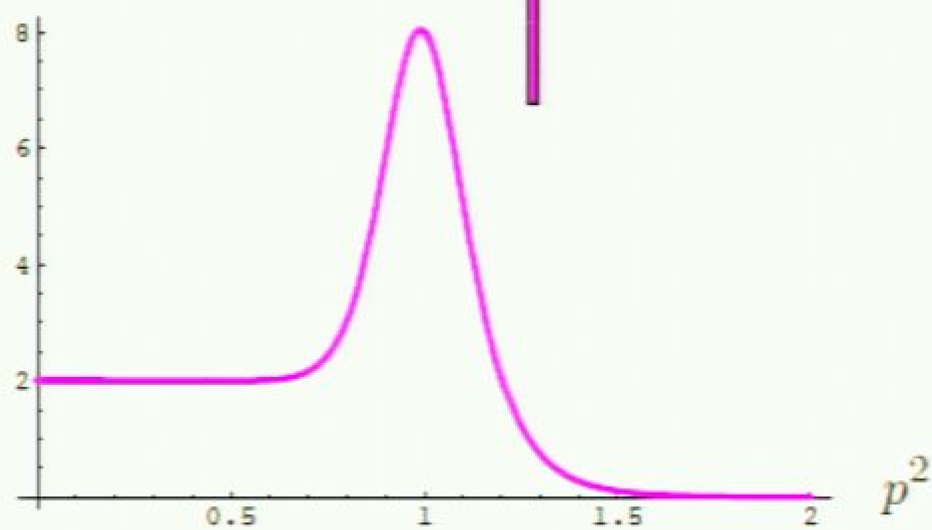

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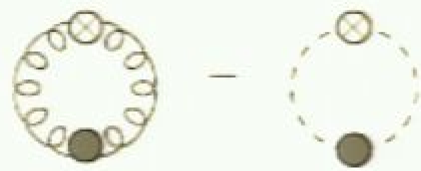
$\partial_t \Gamma_k[\phi] \;=\; \frac{1}{2}$ 

- Perturbation theory

$$\partial_t \Gamma_k[\phi] = \partial_t\, \frac{1}{2}\mathrm{Tr}\log\left(S_{\mathrm{cl}}^{(2)}[\phi] + R_k(p)\right)$$

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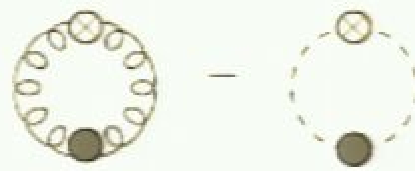
$\partial_t \Gamma_k[\phi] \;=\; \frac{1}{2}$ 

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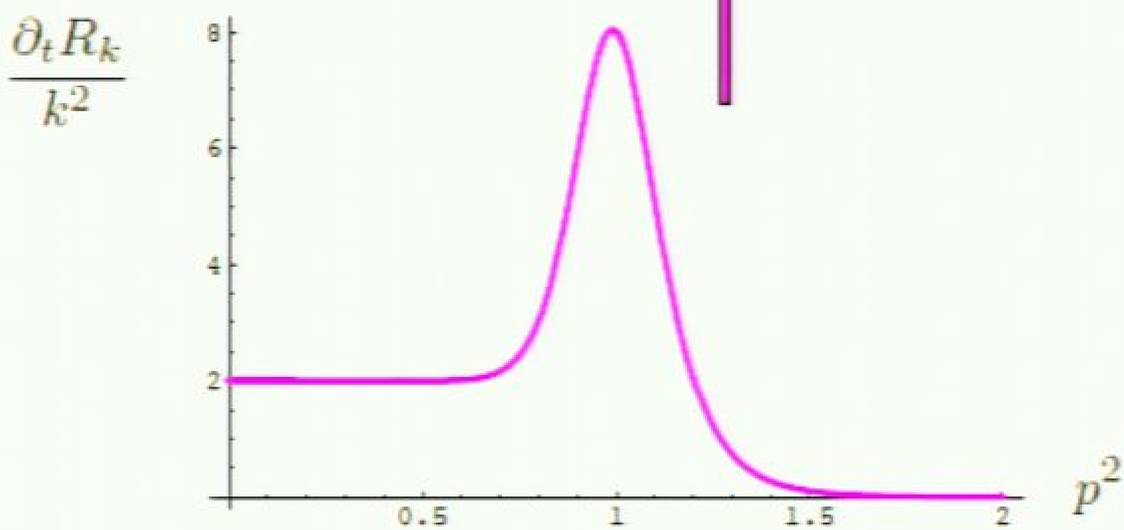

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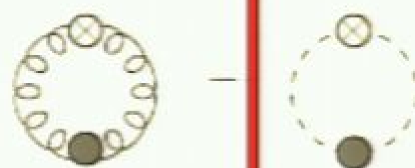

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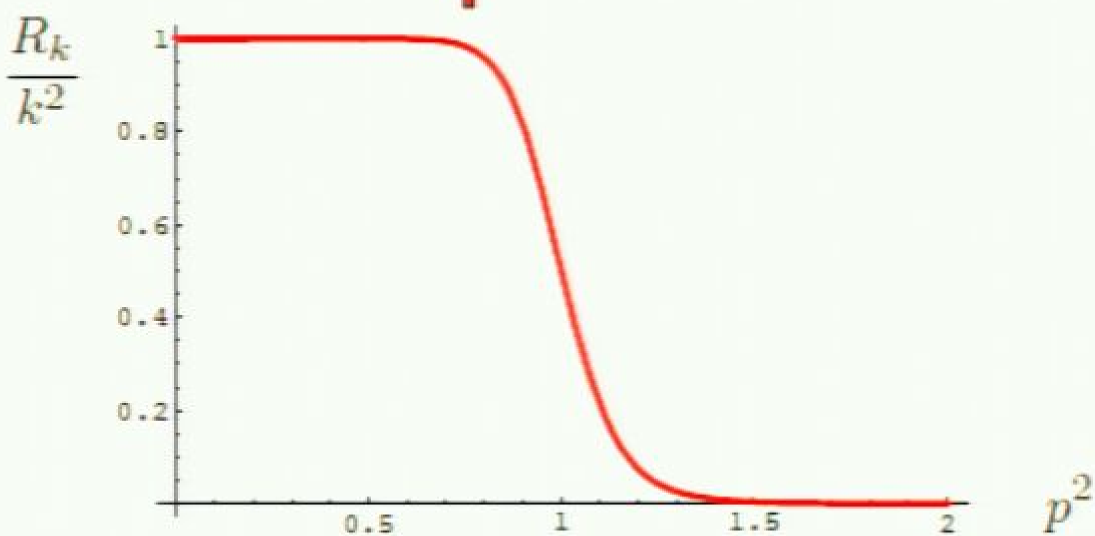

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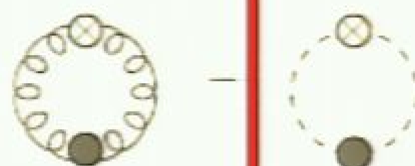

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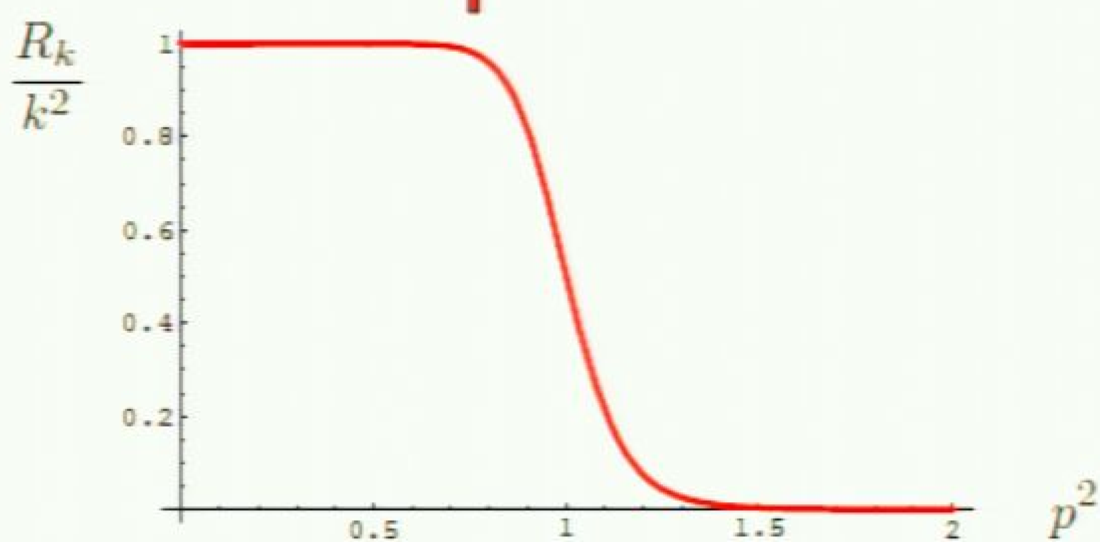

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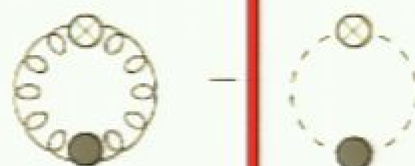

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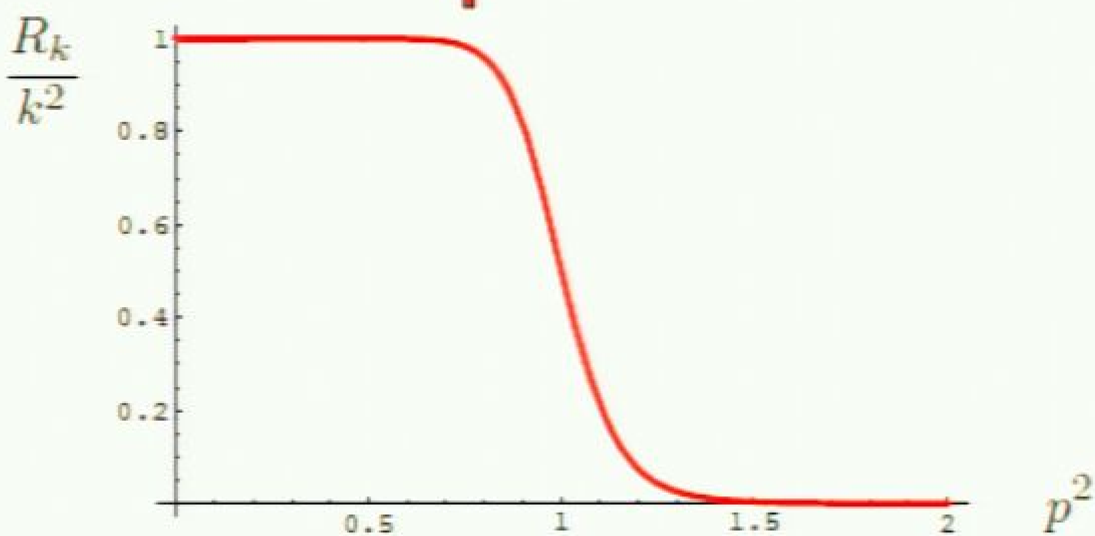
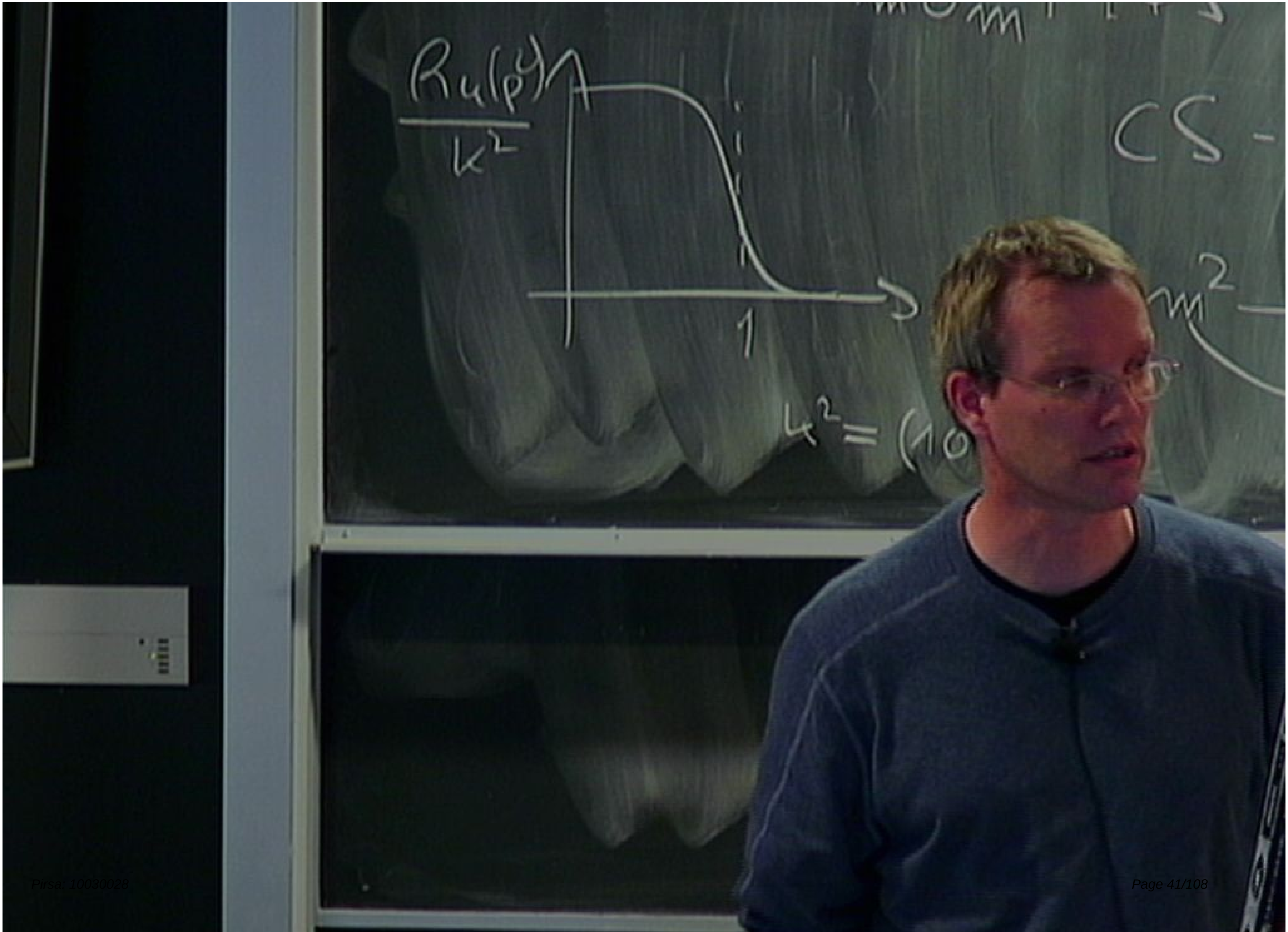


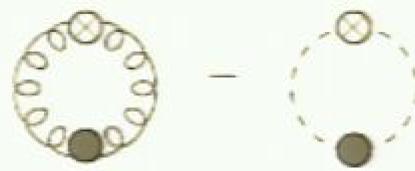


Functional RG

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p)}\; k\partial_k R_k(p)$$

- Yang Mills Theory: $\phi = (A, C, \bar{C})$ — RG-scale k: $t = \ln k$

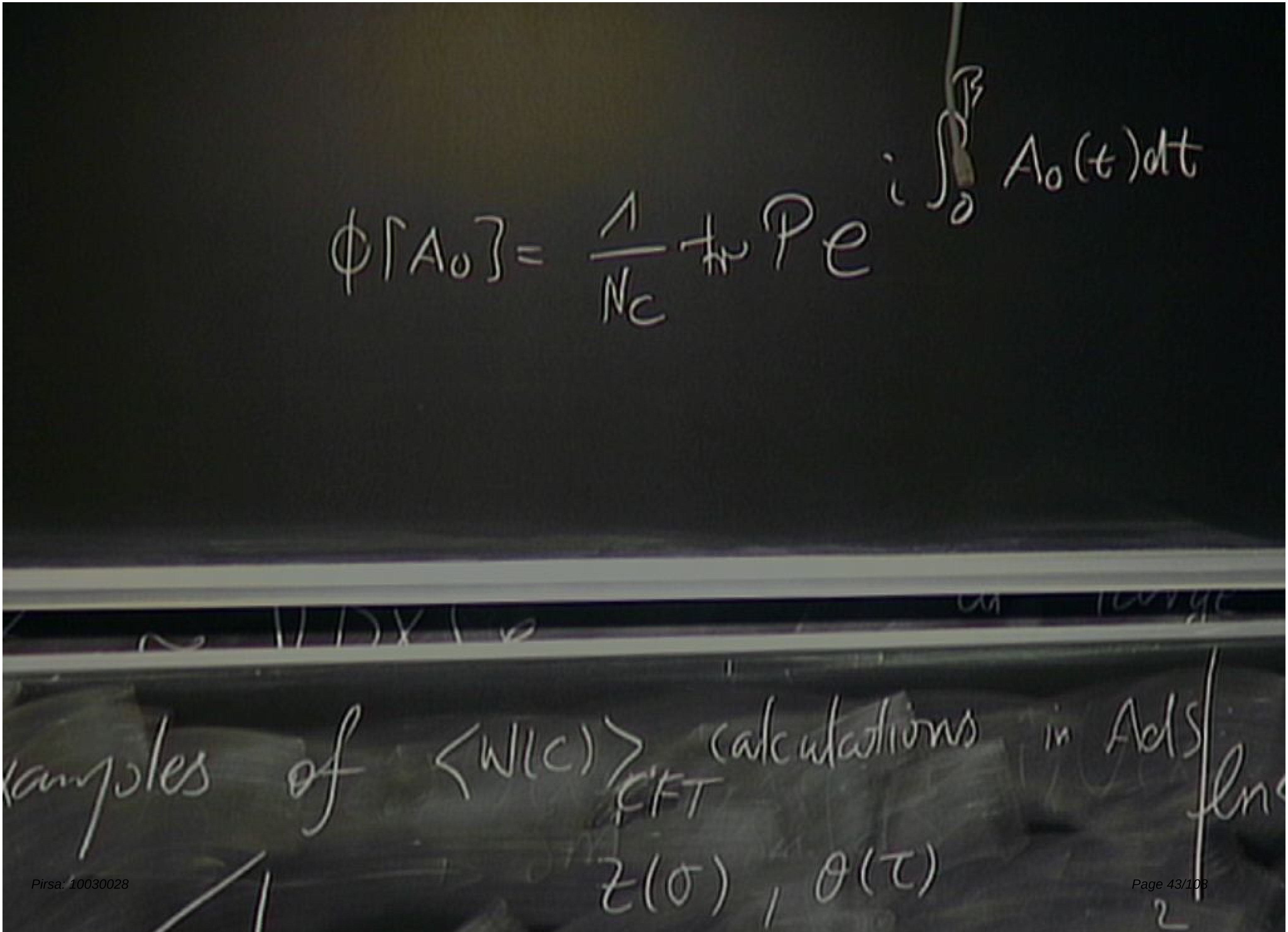
$$\partial_t \Gamma_k[\phi] \;=\; \tfrac{1}{2}$$



- Full flow

$$\partial_t \Gamma_k[\phi] = \partial_t \frac{1}{2}\mathrm{Tr}\log\Big(\Gamma_k^{(2)}[\phi] + R_k(p)\Big) \;+\; \partial_t \Gamma_k^{(2)}[\phi] - \text{terms}$$

RG-improvement

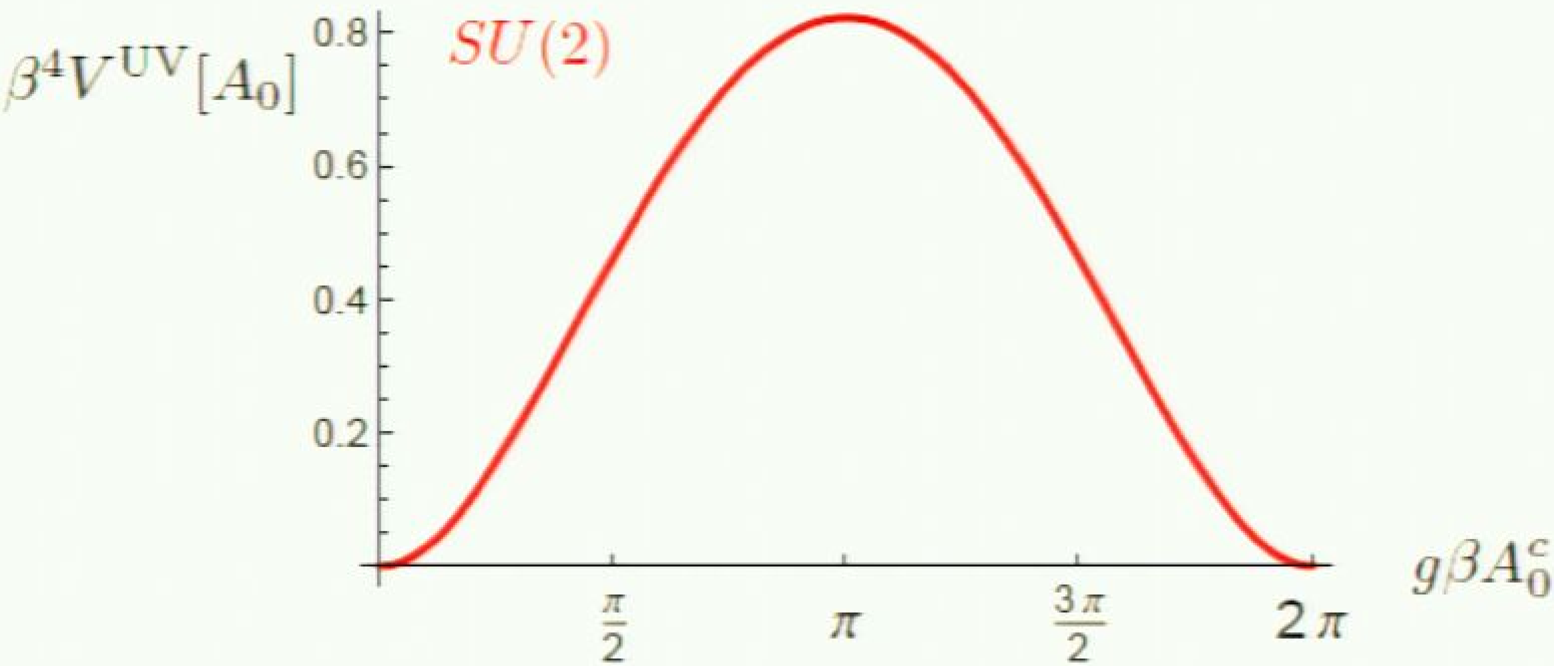


$\phi[A_0] = \frac{1}{N_c} \operatorname{tr} P e^{i\int_0^\beta A_0(t)dt}$
xamples of $\langle W(C)\rangle_{CFT}$ calculations in AdS
$z(\sigma)$, $\theta(\tau)$

Confinement

Perturbation theory

$$V^{\rm UV}[A_0] = \frac{1}{2\Omega}\,{\rm Tr}\log S^{(2)}_{AA}[A_0] - \frac{1}{\Omega}{\rm Tr}\log S^{(2)}_{C\bar{C}}[A_0]$$



$$SU(2): \Phi[A_0] = \cos\frac{1}{2}\beta g A_0^c \qquad \text{with} \qquad A_0 = A_0^c\,\frac{\sigma_3}{2}$$

Confinement

Perturbation theory

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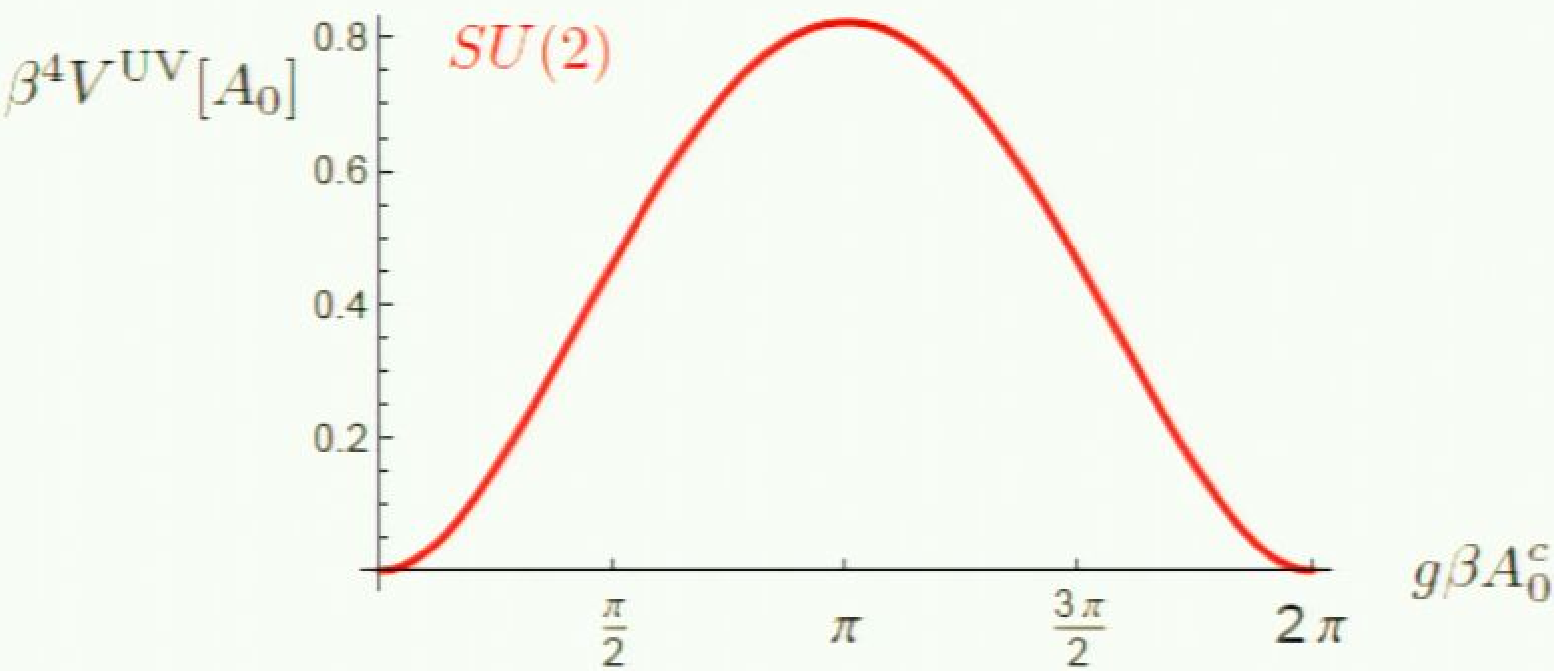

$SU(2): \Phi[A_0] = \cos\frac{1}{2}\beta g A_0^c$ with $A_0 = A_0^c\,\frac{\sigma_3}{2}$

Confinement

Continuum methods ⟸ (Functional RG-flows)

Braun, Gies, JMP '07

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) - \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle) + O(V''[A_0])$$

- Yang-Mills propagators in Landau gauge ('96 - today)
  - DSE, FRG, Lattice, Stochastic Quantisation — von Smekal, Hauck, Alkofer '96

    Alkofer, Aguilar, Binosi, Bicudo, Boucaud, Bogolubsky, Bowman, Braun, Cucchieri, De Soto, Dudal, Fischer, Gies, Gracey, Ilgenfritz, Langfeld, Leinweber, Leroy, Litim, Llanes-Estrada, Natale, Mendes, Micheli, Müller-Preußker, Oliveira, Papavassilio, JMP, Quandt, Reinhardt, Rodriguez-Quintero, Schwenzer, Skullerud, Sorella, Sternbeck, Verschelde, von Smekal, Williams, Zwanziger, ...

    - Numerical solutions
    - Analytic IR-asymptotics IR-scaling & Gribov ambiguity

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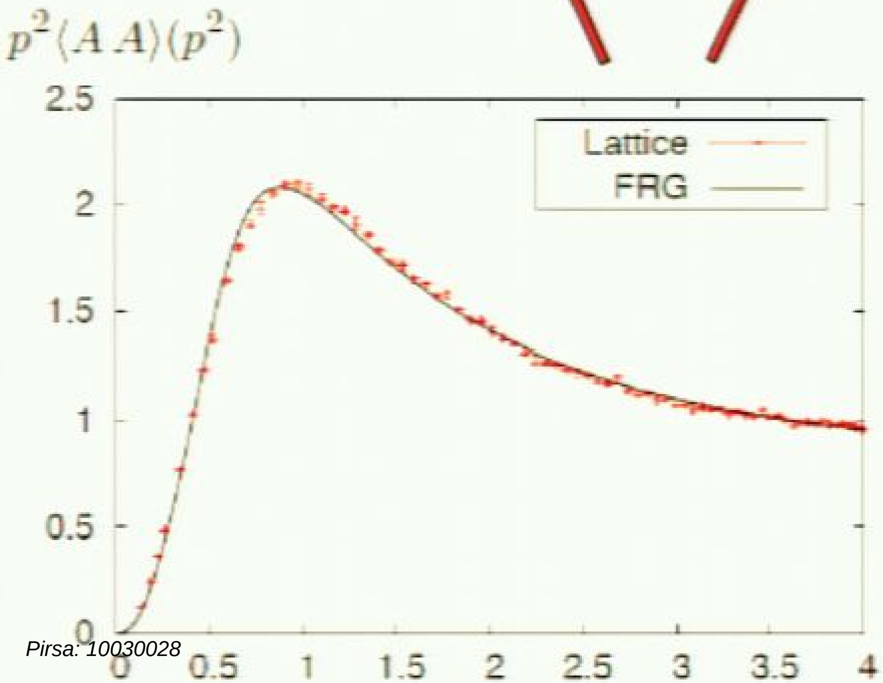
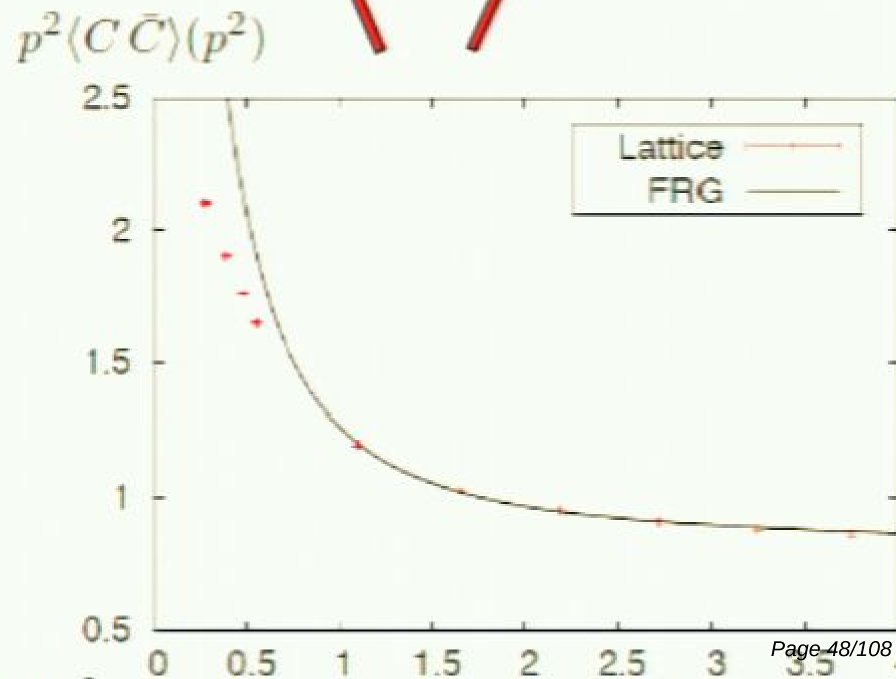
$$p_0 \to 2\pi T n - g A_0$$





p [GeV]

Fischer, Maas, JMP '08

Confinement

Continuum methods ⟸ (Functional RG-flows)

Braun, Gies, JMP '07

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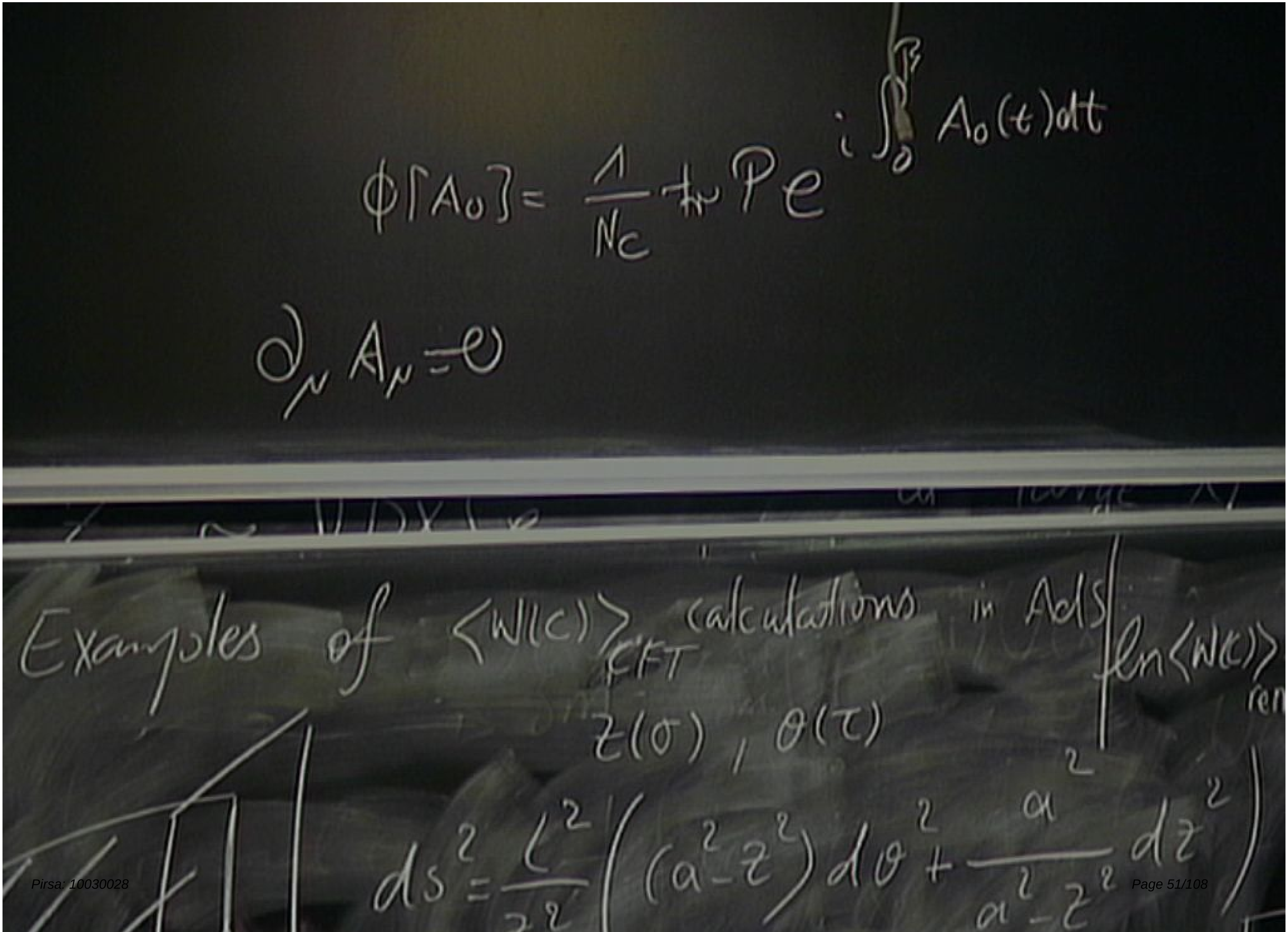
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$\ln\langle W(C)\rangle_{ren}$
$z(\sigma)\,,\ \theta(\tau)$
$ds^2 = \frac{l^2}{z^2}\left((a^2-z^2)\,d\theta^2 + \frac{a^2}{a^2-z^2}\,dz^2\right)$

Confinement

Continuum methods ⟸ (Functional RG-flows)

Braun, Gies, JMP '07

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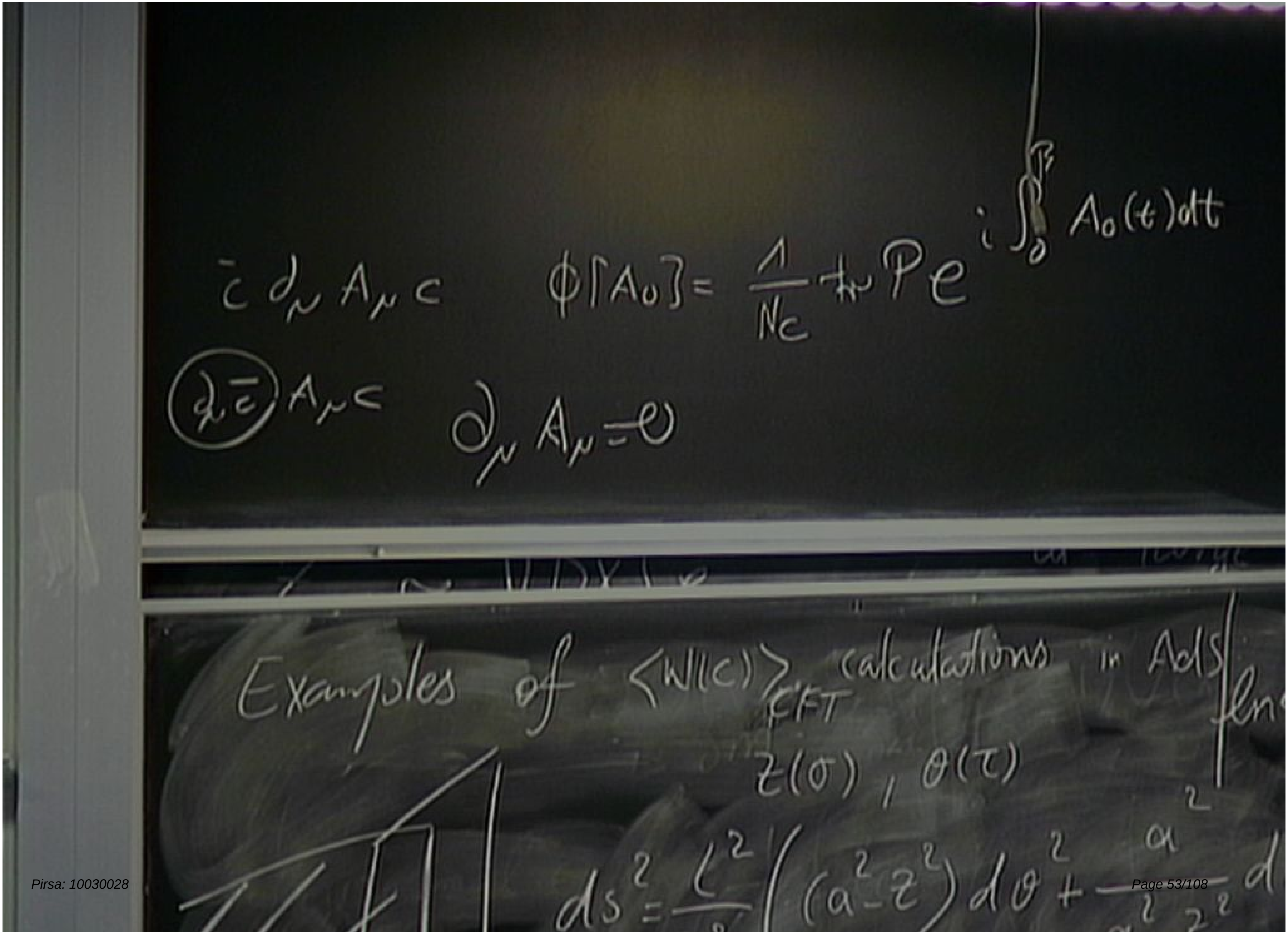
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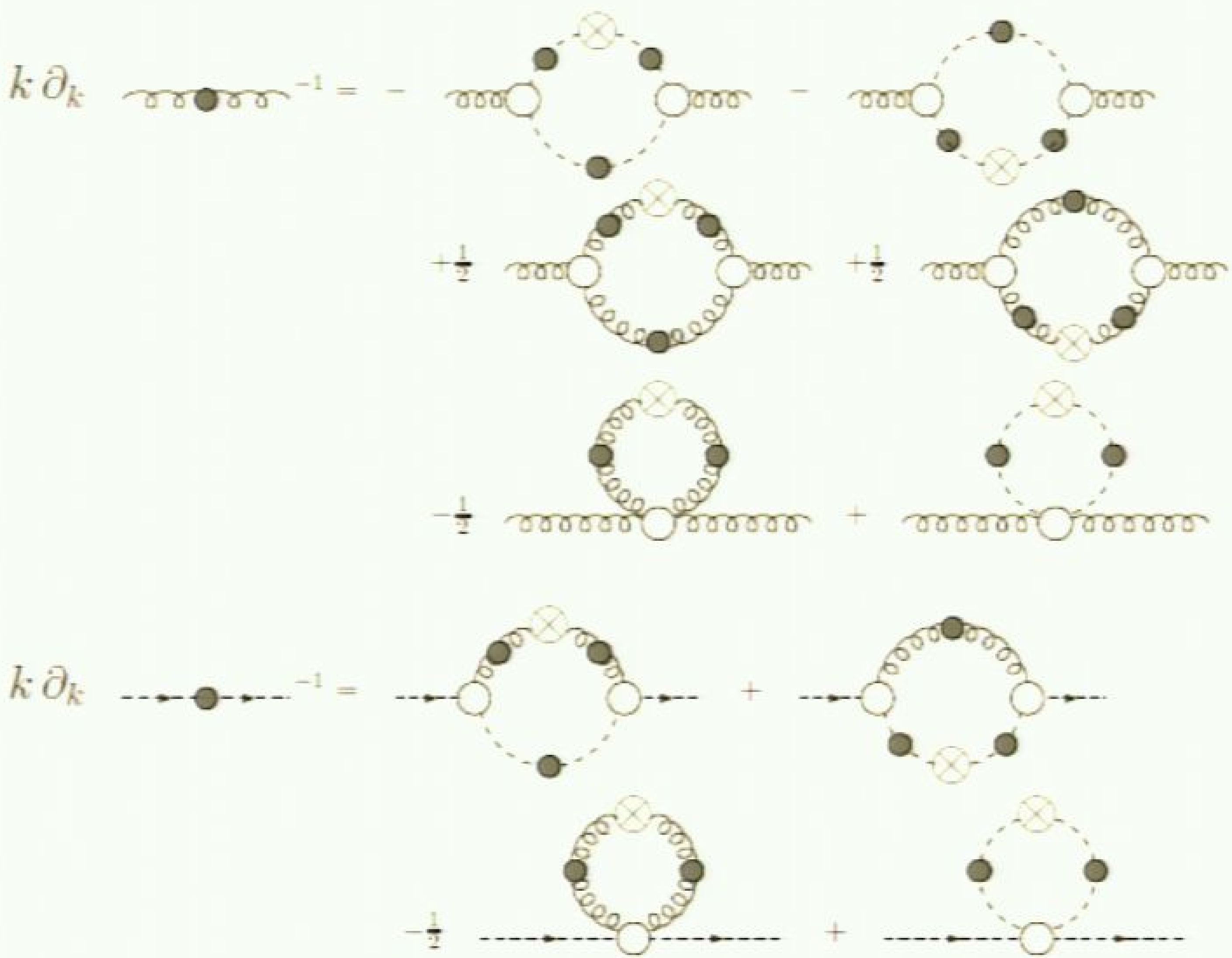
$z(\sigma)$, $\theta(\tau)$

Confinement

Computation of propagators



Confinement

Computation of propagators

- full momentum dependence of propagators
- vertices momentum-dependent RG-dressing
- functional optimisation JMP'05
- functional relations between diagrams: Flow=Flow(DSE)

$$\Rightarrow k\partial_k \langle A(p)\, A(-p)\rangle \;=\; \mathrm{Flow}_A[\langle A\,A\rangle\,.\,\langle C\bar{C}\rangle]$$

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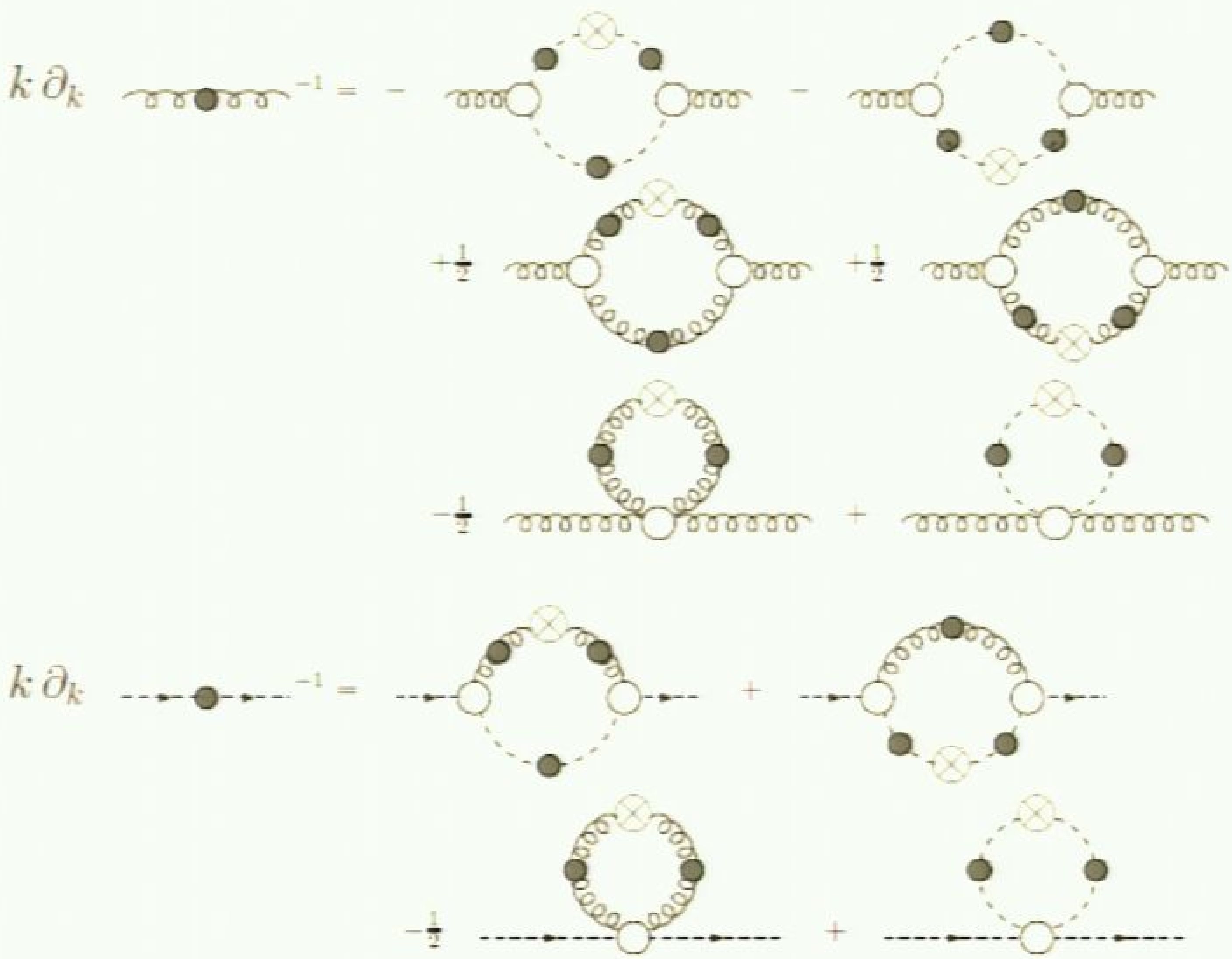
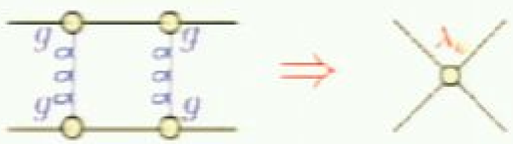


Chiral symmetry breaking



$\propto \alpha_s^2$

$$\int d^4x\, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2\right]$$

$\langle \bar{q}q \rangle \neq 0$

mass term: $\langle \bar{q}q \rangle\, \bar{q}q$

explicitly broken by massive quarks

Order parameter

$$\sigma = \langle \bar{q}q \rangle$$

chiral condensate

- chiral symmetry: $\sigma = 0$
- symmetry breaking: $\sigma \neq 0$

Symmetry

- $SU_L(N_f) \times SU_R(N_f)$
- broken to $SU(N_f)$

Functional RG

- Introduction to Functional RG flows & some results in QCD (talks & lit)
  - Integrals from differential equations: The FRG-idea in 0+0-dimensions
  - Confinement & chiral symmetry breaking from Functional Methods
  - Aspects of the Functional RG

Confinement

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Braun, Gies, JMP '07

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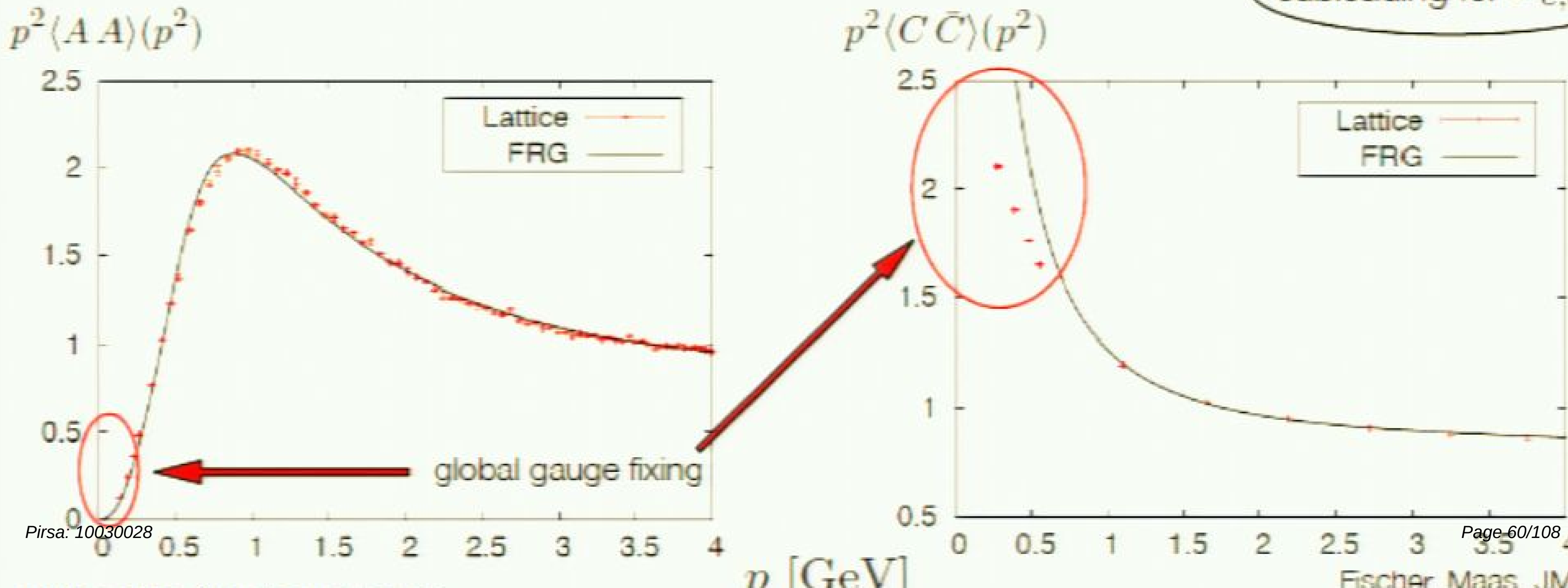
'Polyakov loop potential'

subleading for $T_{c,\mathrm{conf}}$



Fischer, Maas, JMP '08

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Confinement

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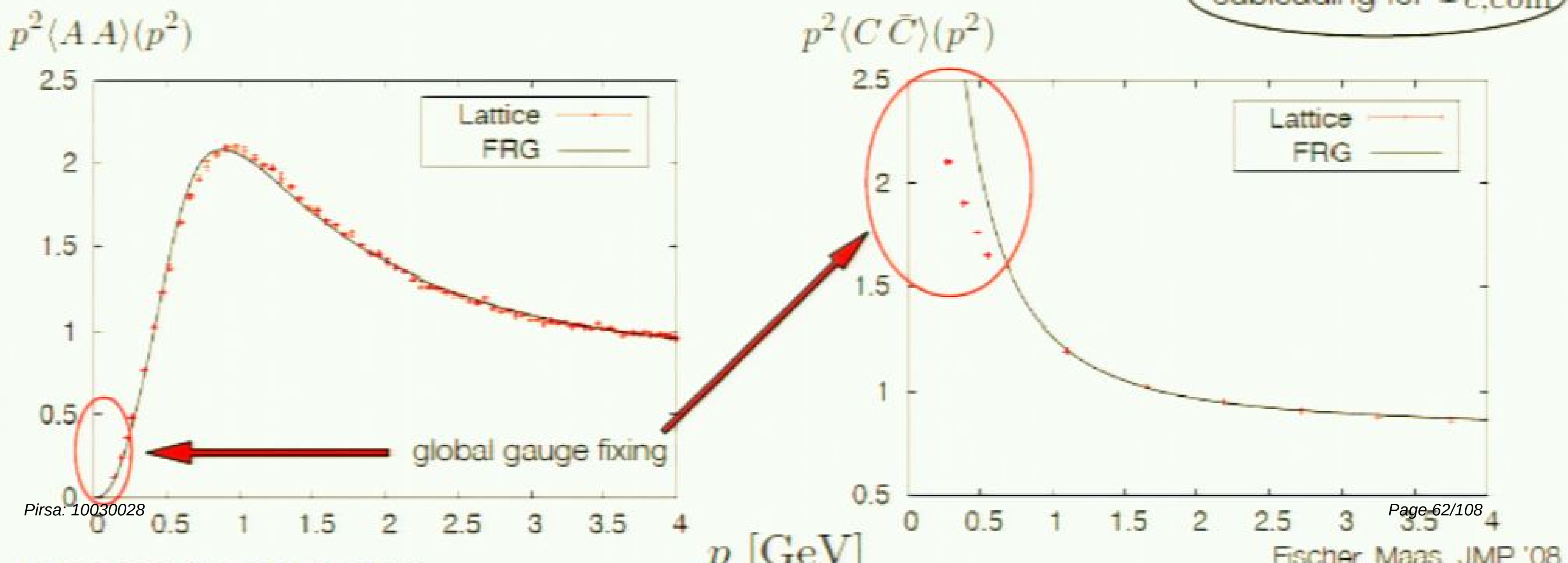

Fischer, Maas, JMP '08

Confinement

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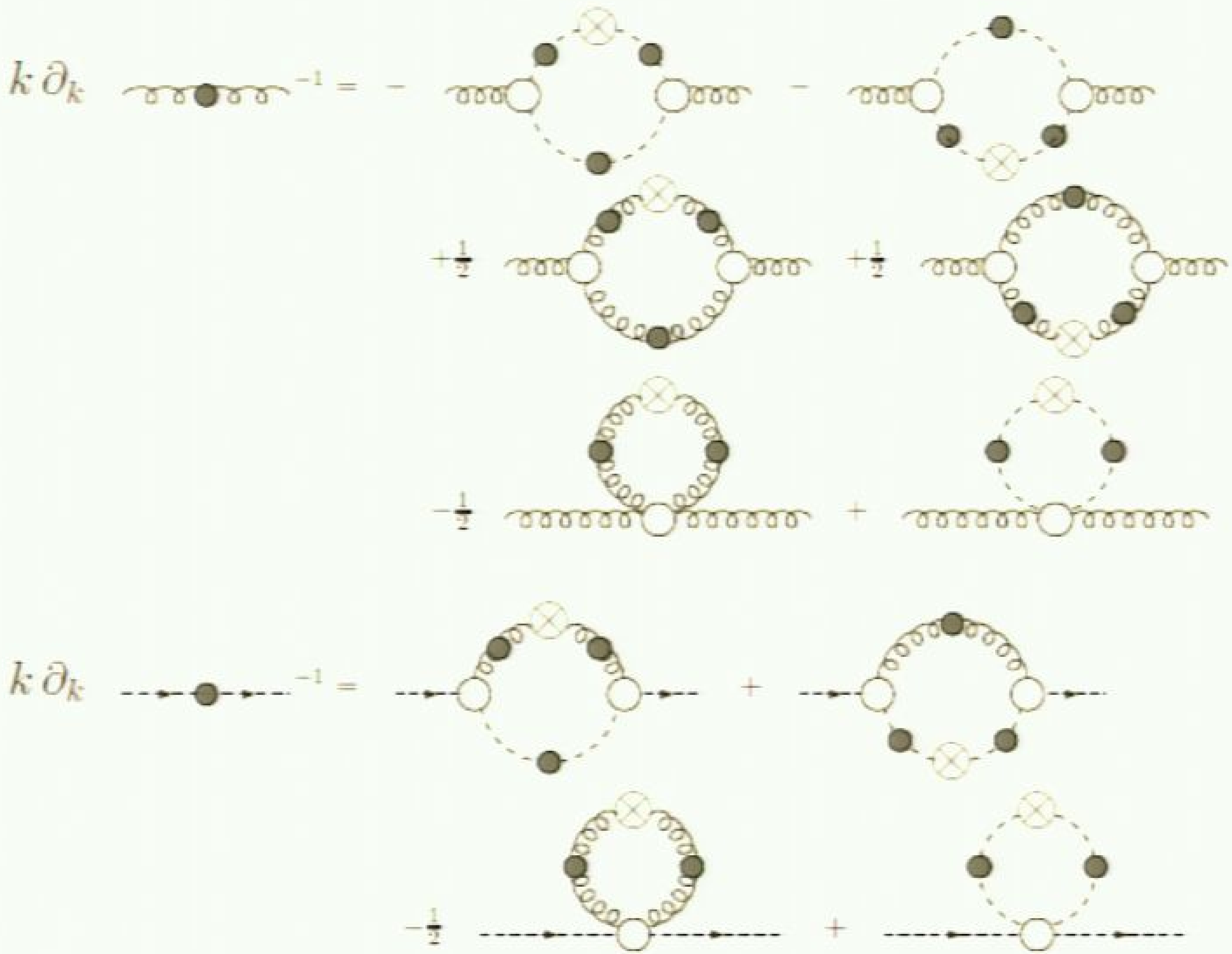
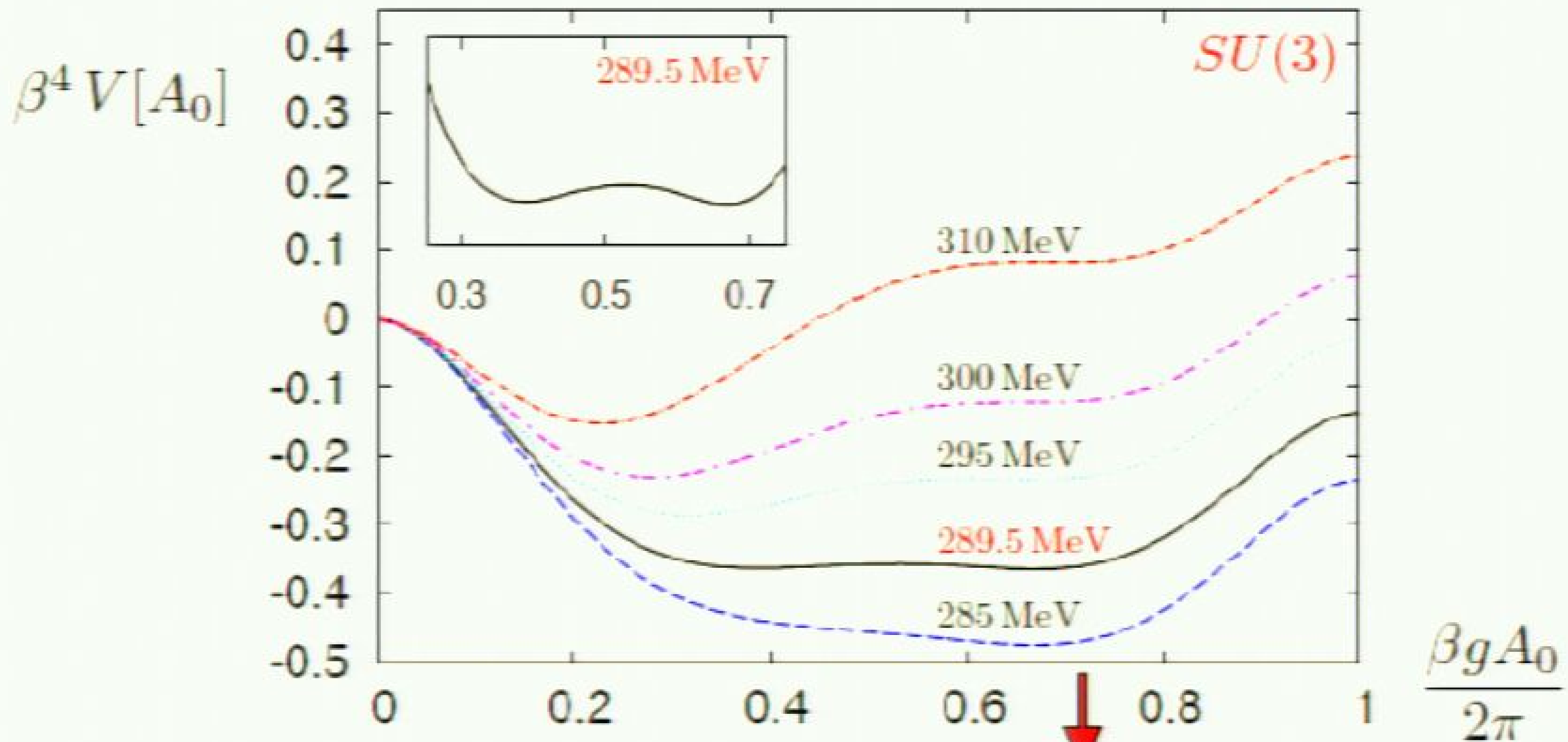


Confinement

$T_c = 289.5 \pm 10\,\mathrm{MeV}$ $\quad$ $T_c/\sqrt{\sigma} = 0.658 \pm 0.023$ $\quad$ lattice : $T_c/\sqrt{\sigma} = 0.646$



$$\Phi[A_0] = \frac{1}{3}(1 + 2\cos\frac{1}{2}\beta g A_0) \longrightarrow \Phi[\frac{4}{3}\pi\frac{1}{\beta g}] = 0$$

Braun, Gies, JMP '07

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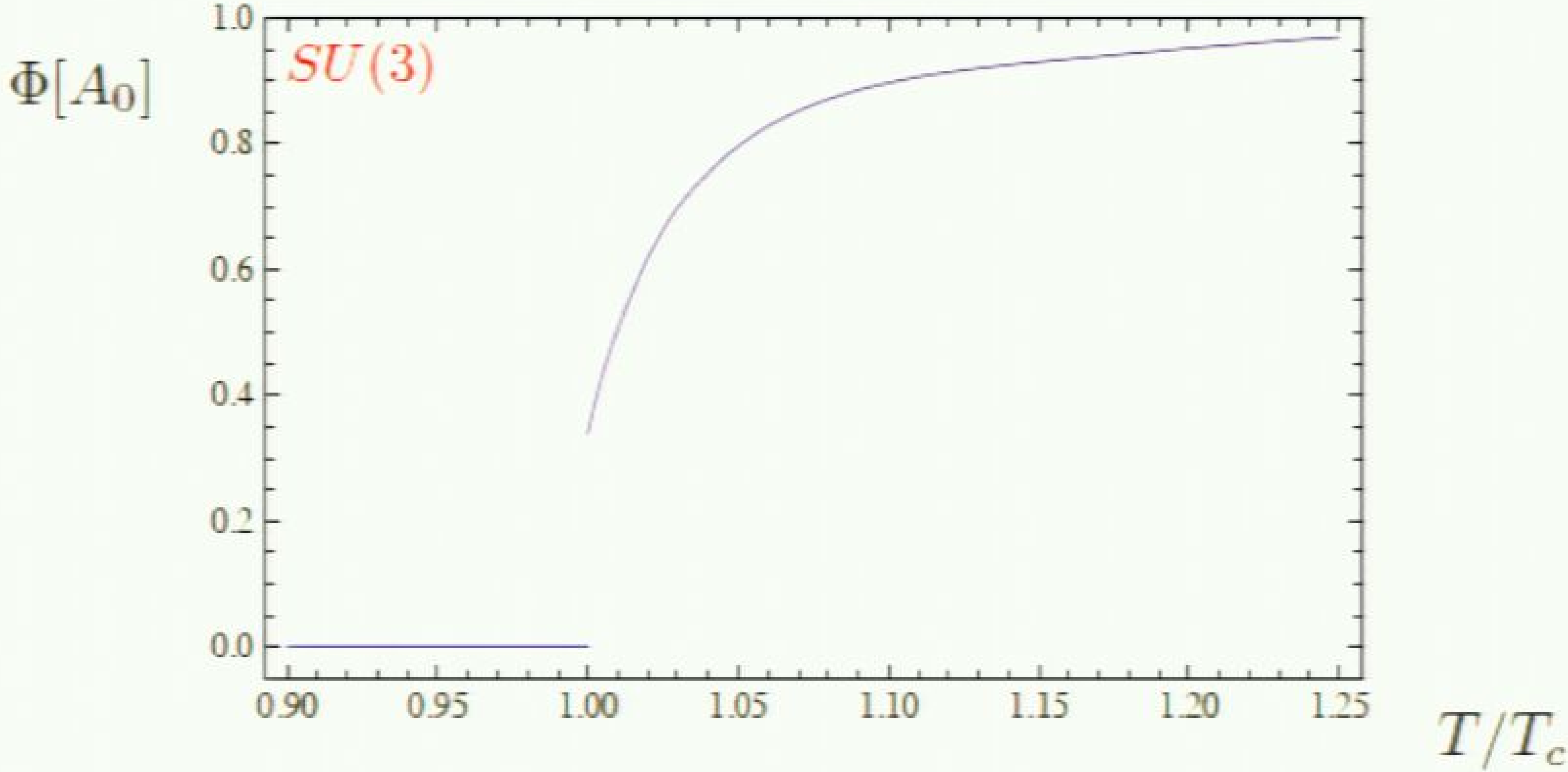
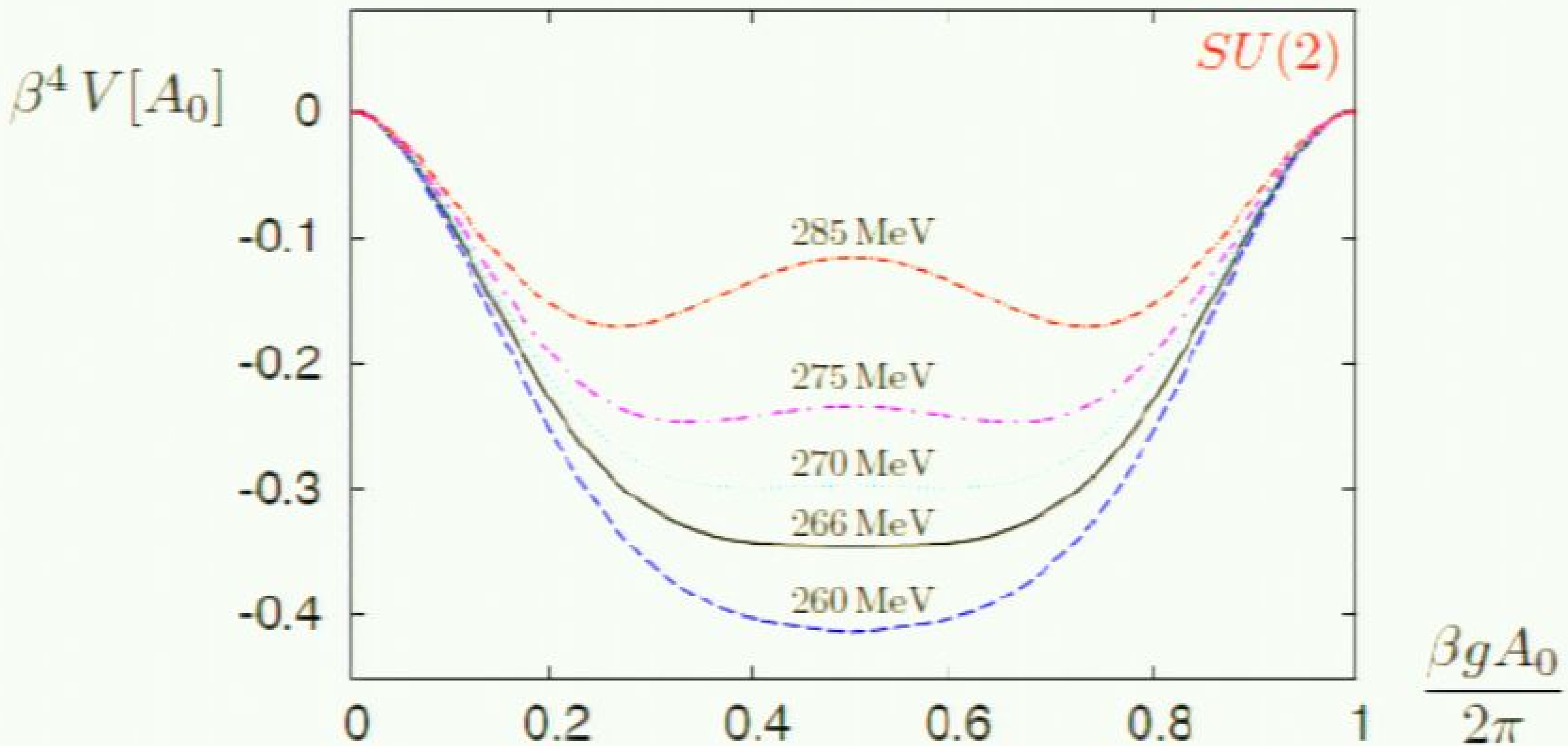


SU(N), G(2), Sp(2): Braun, Eichhorn, Gies, JMP, in preparation

Braun, Gies, JMP '07

Confinement

$T_c = 266 \pm 10\,\mathrm{MeV}$ $\qquad T_c/\sqrt{\sigma} = 0.605 \pm 0.023$ $\qquad$ lattice : $T_c/\sqrt{\sigma} = 0.709$



$\Phi[A_0] = \cos \frac{1}{2}\beta g A_0 \longrightarrow \Phi[\pi/(\beta g)] = 0$

Braun, Gies, JMP '07

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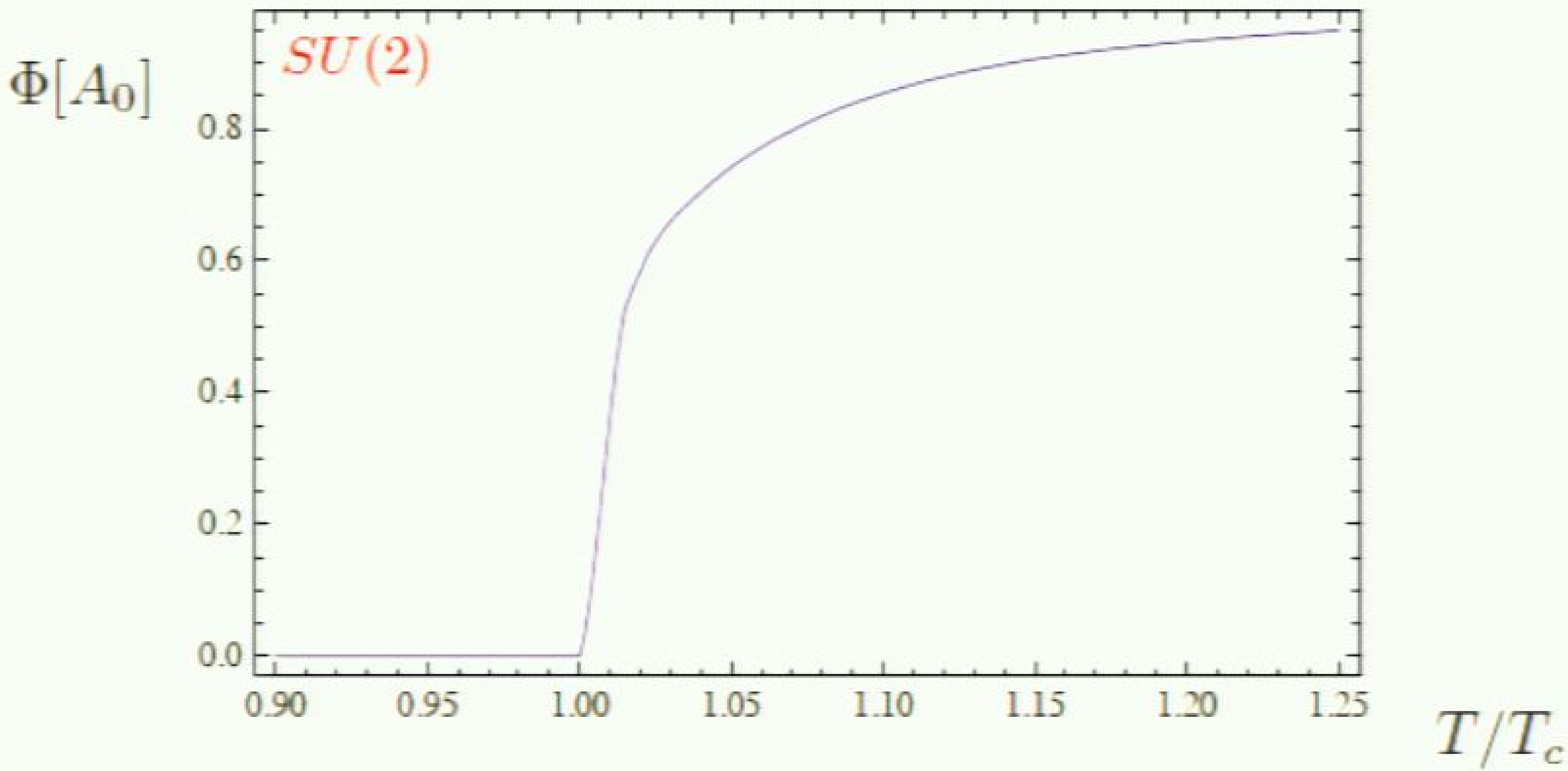
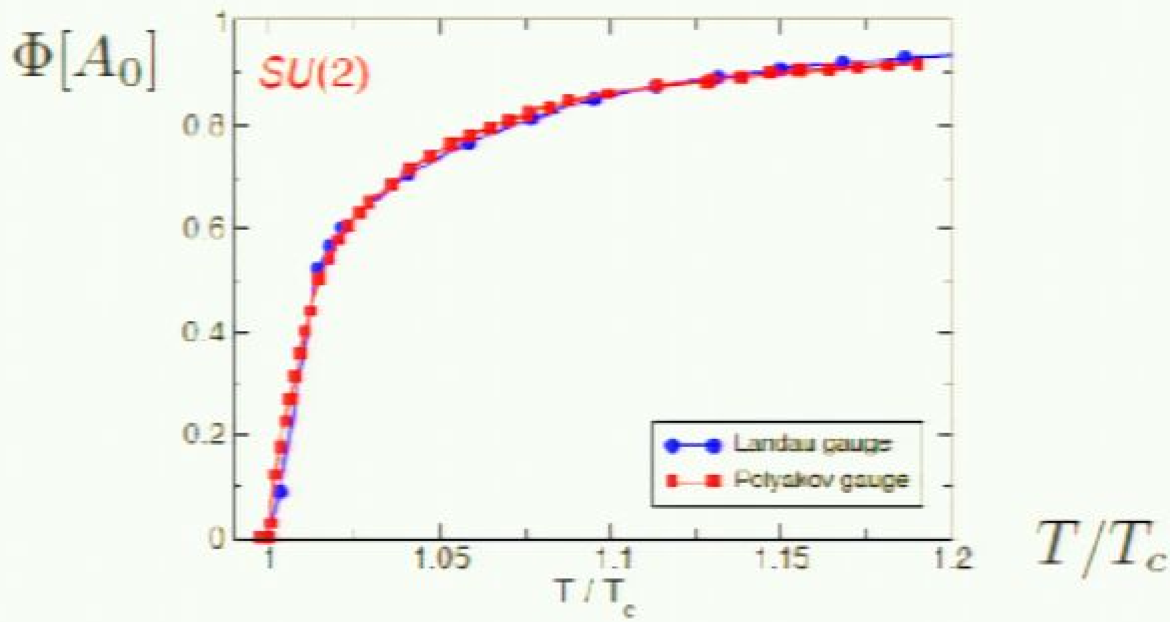


Braun, Gies, JMP '07

Universal properties & gauge independence

Polyakov gauge: $A_0 = A_0^c(\vec{x})\sigma_3$

$$\text{RG-flow}: \quad V[A_0] = -\int dt\, \text{flow}[V''[A_0], \alpha_s]$$



- $--$: Polyakov gauge: crit. exp. $\nu = 0.65$ $\qquad \nu_{\text{Ising}} = 0.63$
- $--$: Landau gauge propagators

JMP, Marhauser '08

Phase structure at vanishing density

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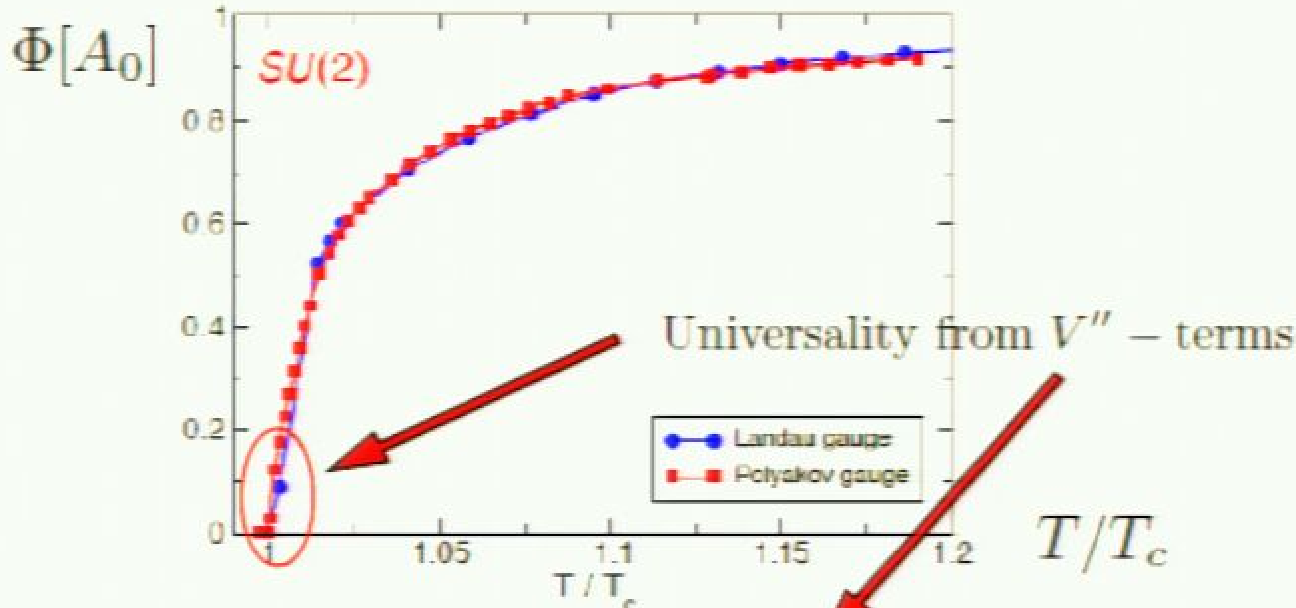

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JMP, Marhauser '08

Phase structure at vanishing density

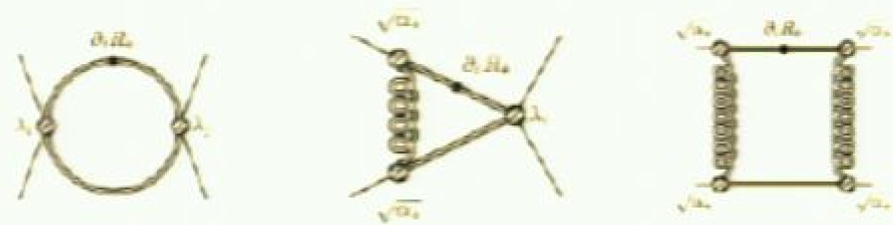
Chiral symmetry breaking

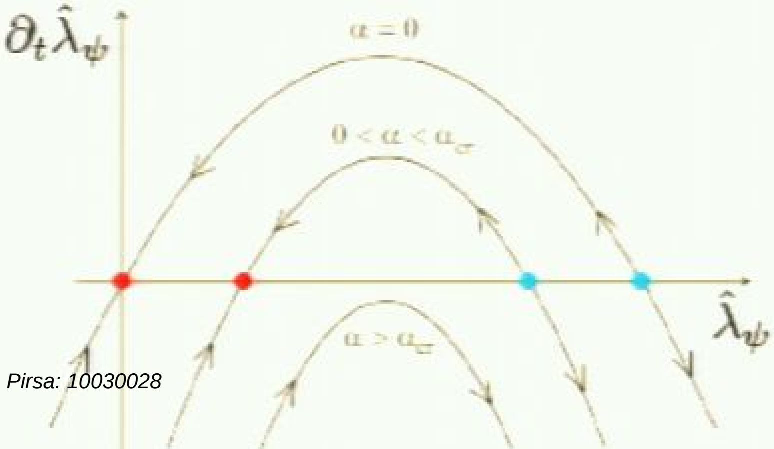
A glimpse at chiral symmetry breaking

Flow of four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi - A(\tfrac{T}{k})\,\hat{\lambda}_\psi^2 - B(\tfrac{T}{k})\,\hat{\lambda}_\psi\,\alpha_s - C(\tfrac{T}{k})\,\alpha_s^2$$

$\Uparrow \quad \Uparrow \quad \Uparrow$





$m^2 \propto \frac{1}{\lambda}$ $\Longrightarrow$ $\alpha_s > \alpha_{s,crit}$: chiral symmetry breaking

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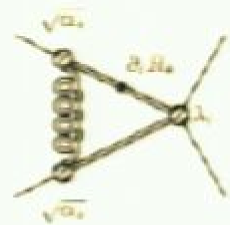
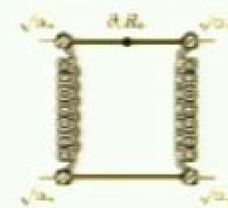
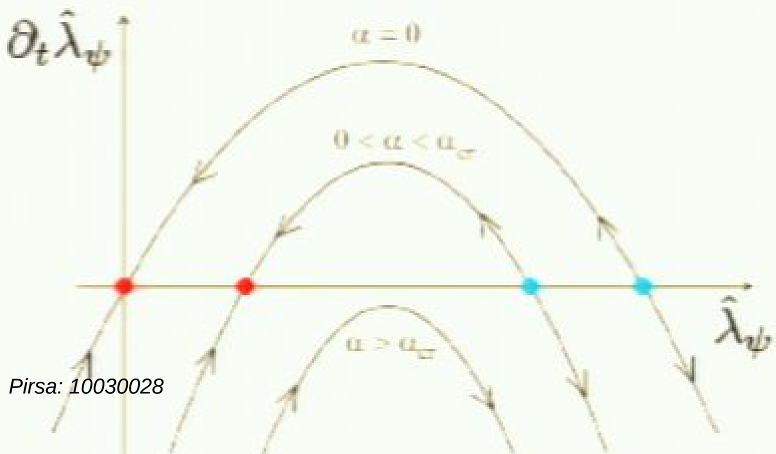
$\Uparrow$ $\Uparrow$ $\Uparrow$







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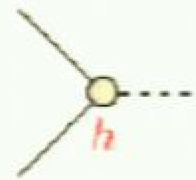
Chiral symmetry breaking

$g\,\bar{\psi}\not{A}\,\psi \quad \xrightarrow{\int_p}$

Hubbard-Stratonovitch

$\lambda_\psi\,(\bar{\psi}\psi)^2 = h\,\bar{\psi}\psi\,\sigma - \frac{1}{2}m^2\sigma^2$

with $m^2 = -\frac{h^2}{2\lambda_\psi}$ and EoM(σ) $\Longrightarrow$



+Baryons and Glueballs

+Baryonisation

Dynamical degrees of freedom

Quarks, Gluons ψ, A $\Longrightarrow$ ψ, A + Mesons, Baryons $\phi \sim \bar{\psi}\psi$, $b \sim \psi^3$

Chiral symmetry breaking

Dynamical hadronisation ⟸ (Functional RG-flows)

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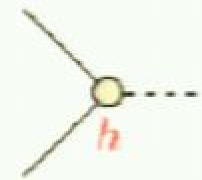

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Gies, Wetterich '01

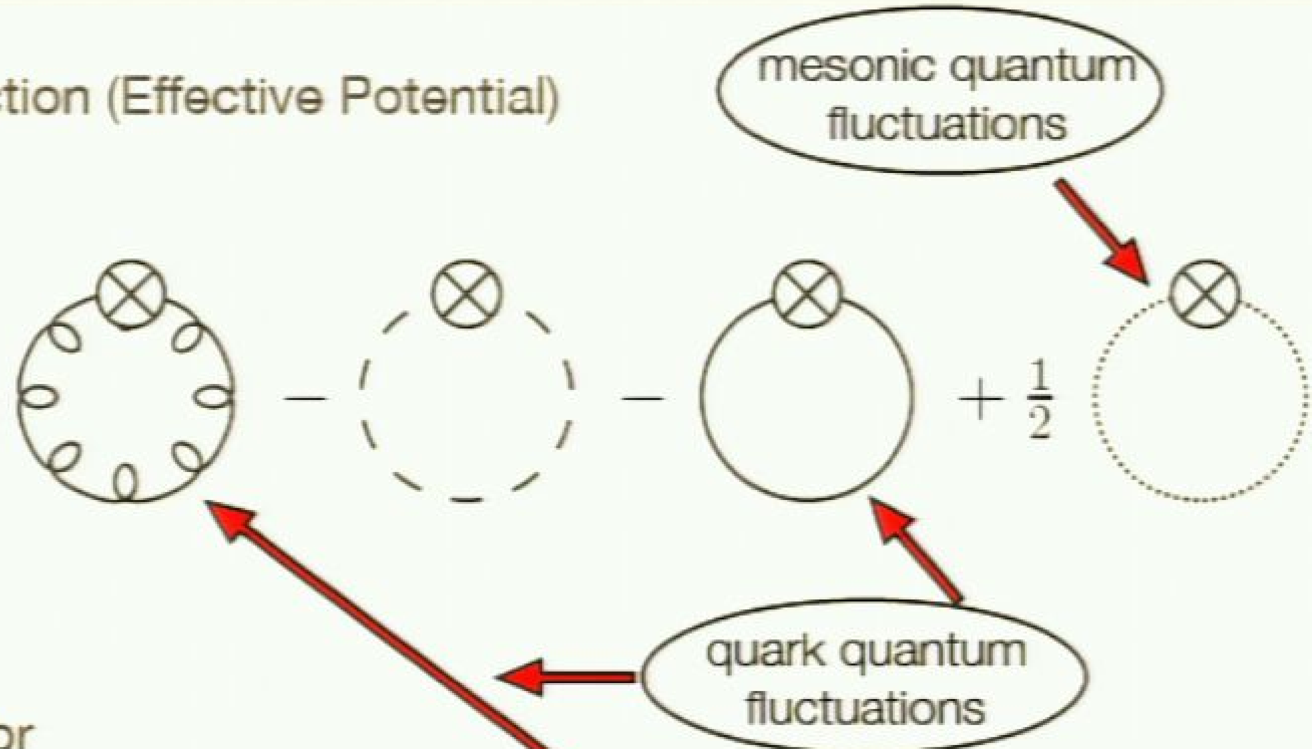
JMP '05

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods ⟸ (Functional RG-flows)

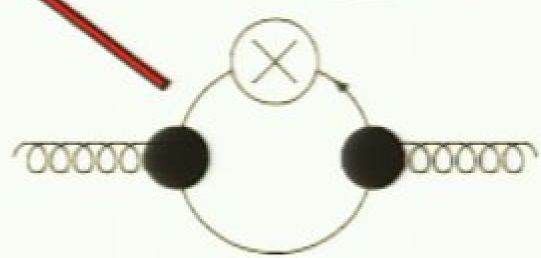
- RG-flow of Effective Action (Effective Potential)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



- flow of gluon propagator

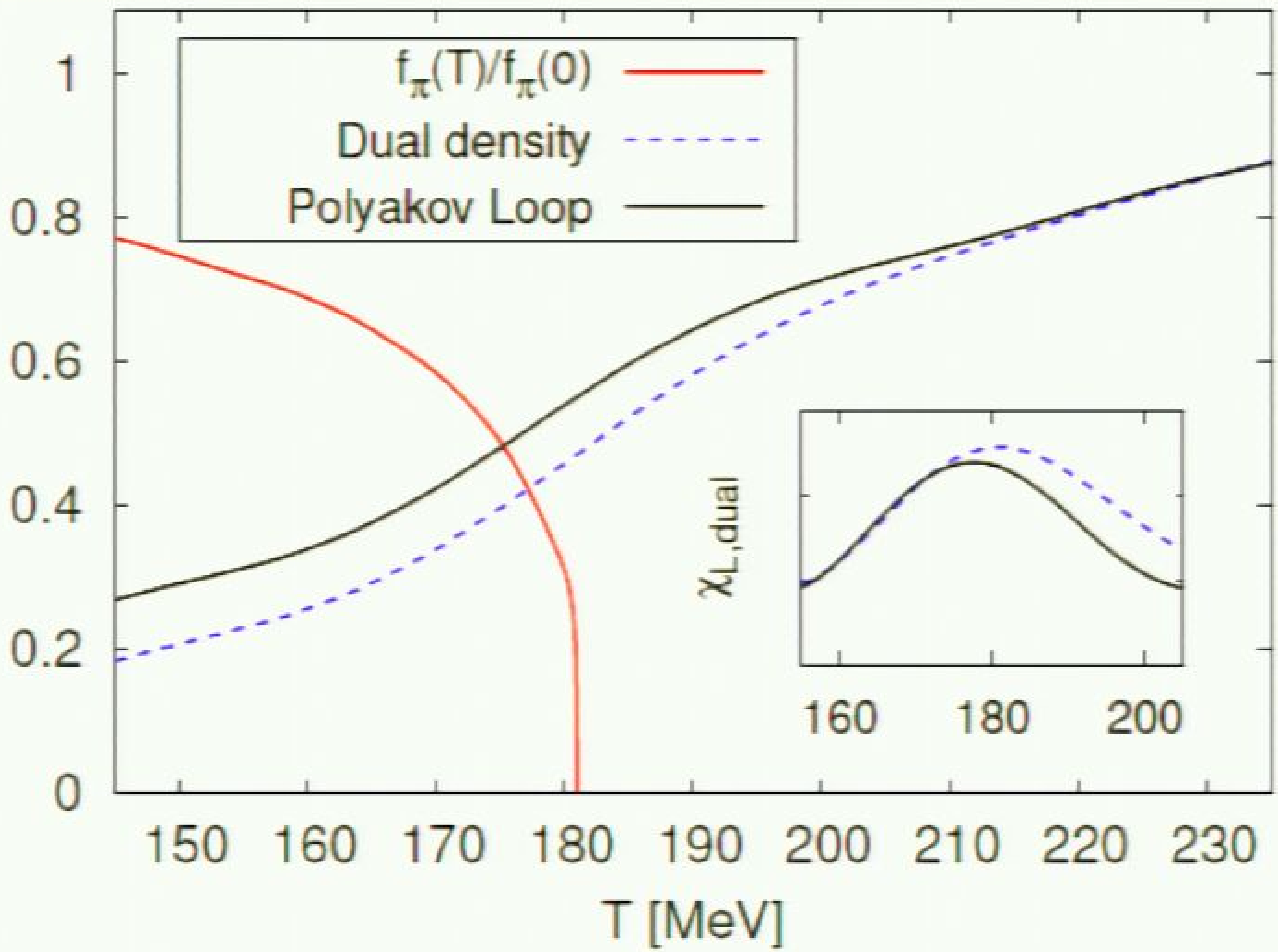
pure gauge theory flow +



+ ⋯

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods



$$T_\chi \simeq T_{conf} \simeq 180\text{MeV}$$

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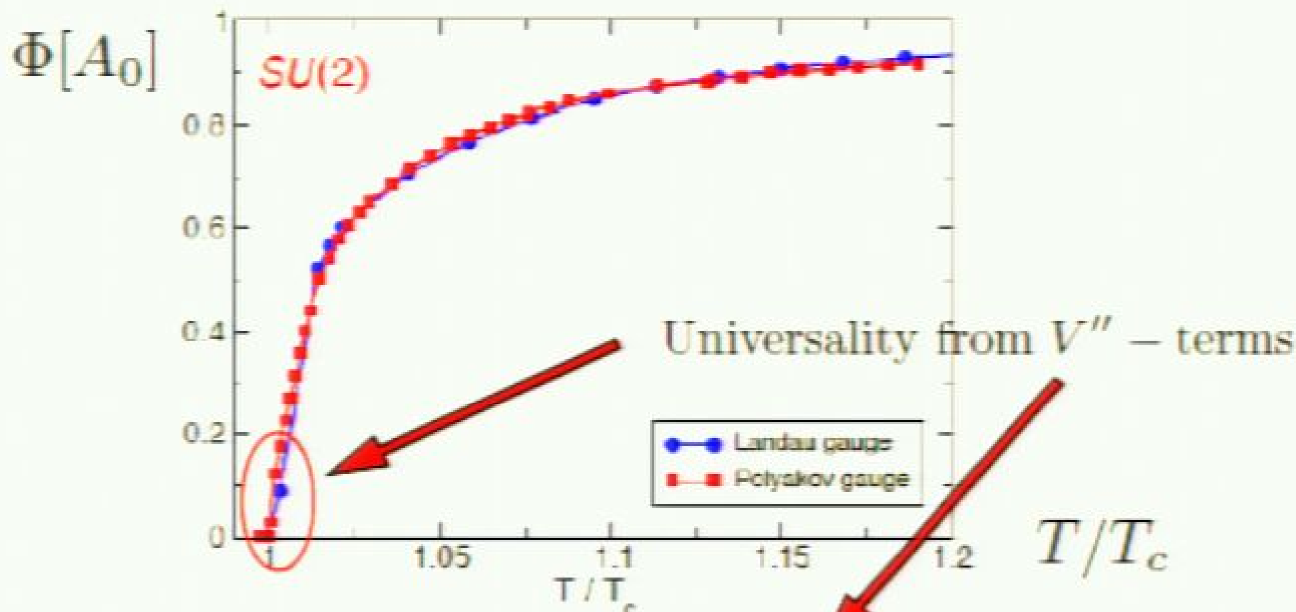
Polyakov gauge: $A_0 = A_0^c(\vec{x})\sigma_3$

RG-flow : $V[A_0] = -\int dt\, \mathrm{flow}[V''[A_0], \alpha_s]$



- – –: Polyakov gauge: crit. exp. $\nu = 0.65$ $\nu_{\mathrm{Ising}} = 0.63$
- – –: Landau gauge propagators

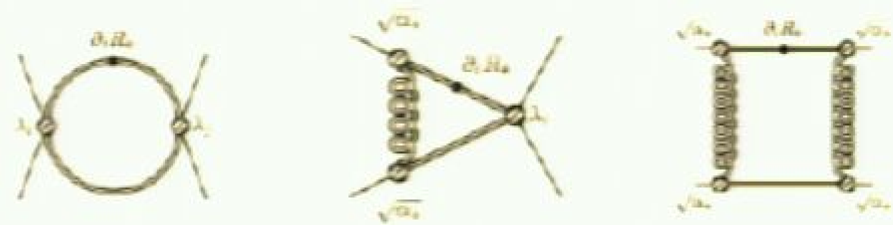
JMP, Marhauser '08

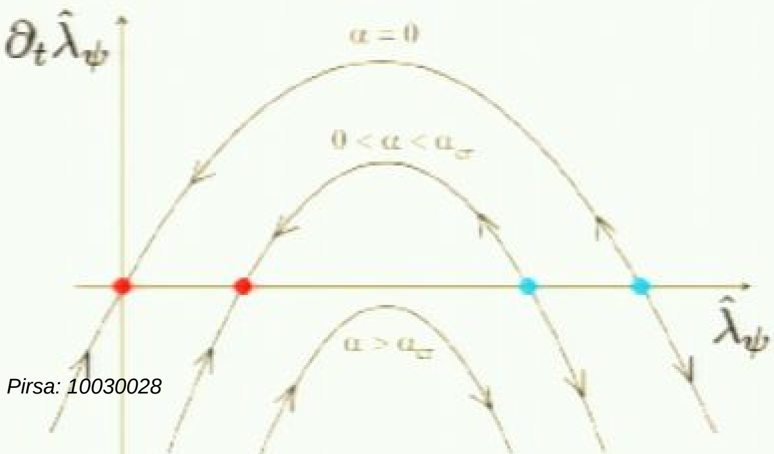
Chiral symmetry breaking

A glimpse at chiral symmetry breaking

Flow of four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi \quad - \quad A(\tfrac{T}{k})\,\hat{\lambda}_\psi^2 \quad - \quad B(\tfrac{T}{k})\,\hat{\lambda}_\psi\,\alpha_s \quad - \quad C(\tfrac{T}{k})\,\alpha_s^2$$





$m^2 \propto \frac{1}{\lambda}$ $\Longrightarrow$ $\alpha_s > \alpha_{s,crit}$: chiral symmetry breaking

Chiral symmetry breaking

A glimpse at chiral symmetry breaking

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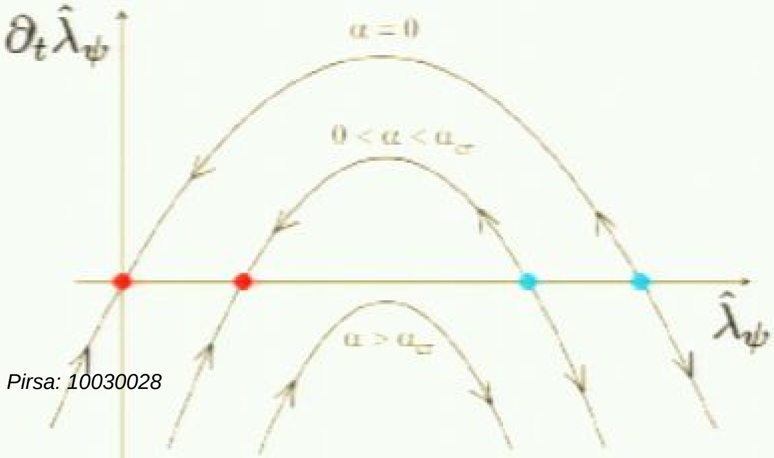

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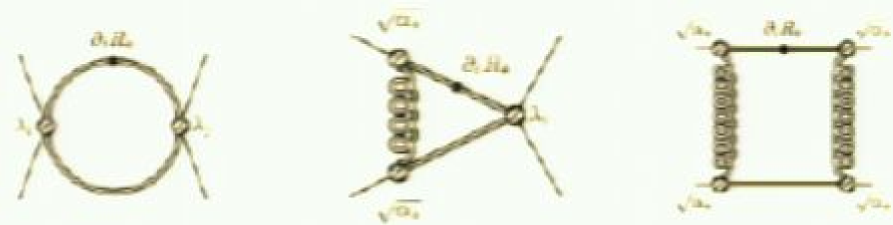

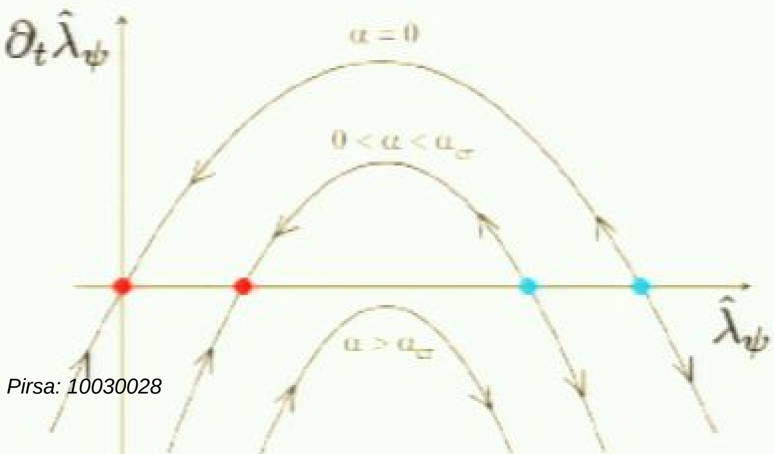

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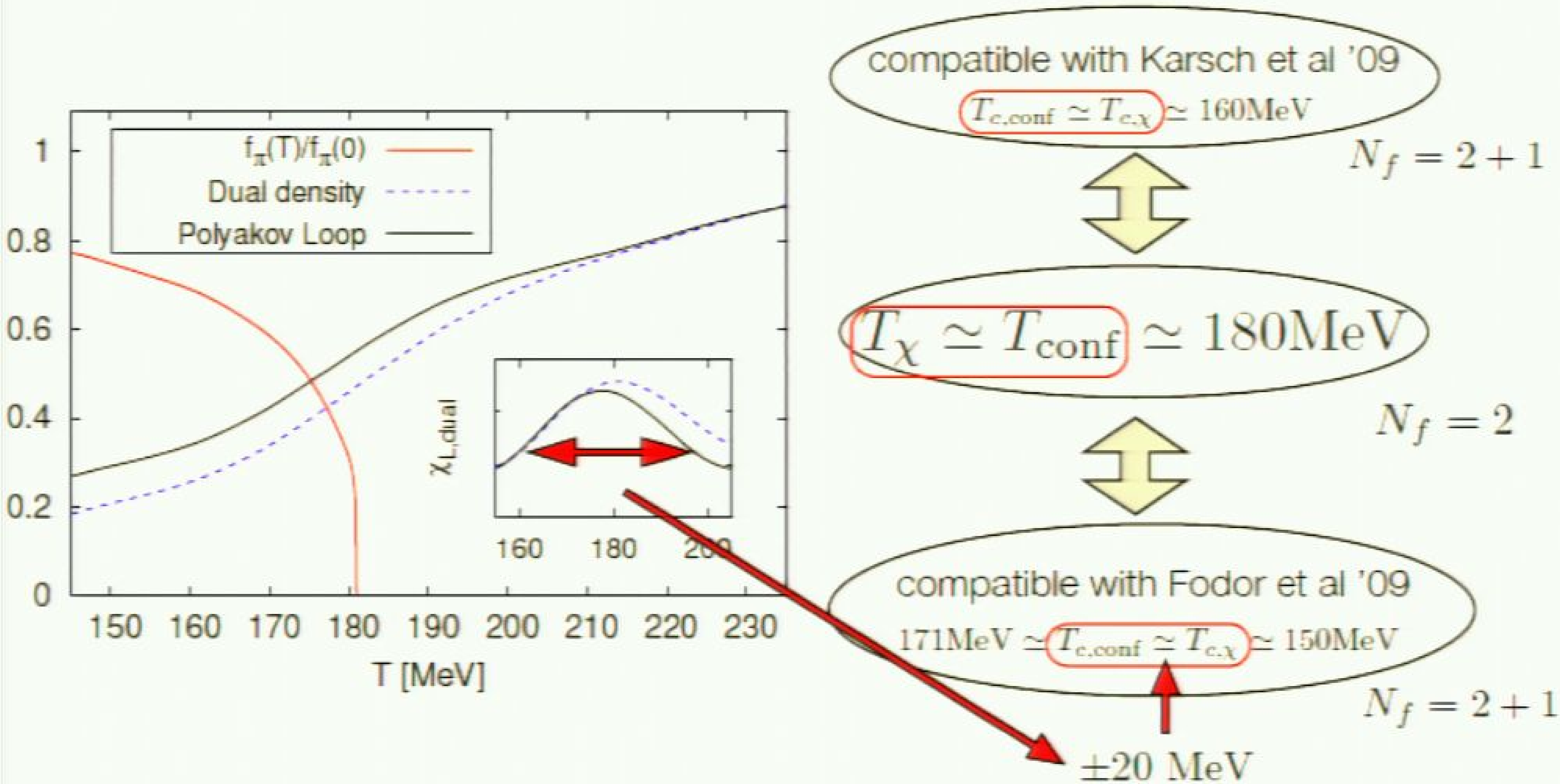
Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods & lattice

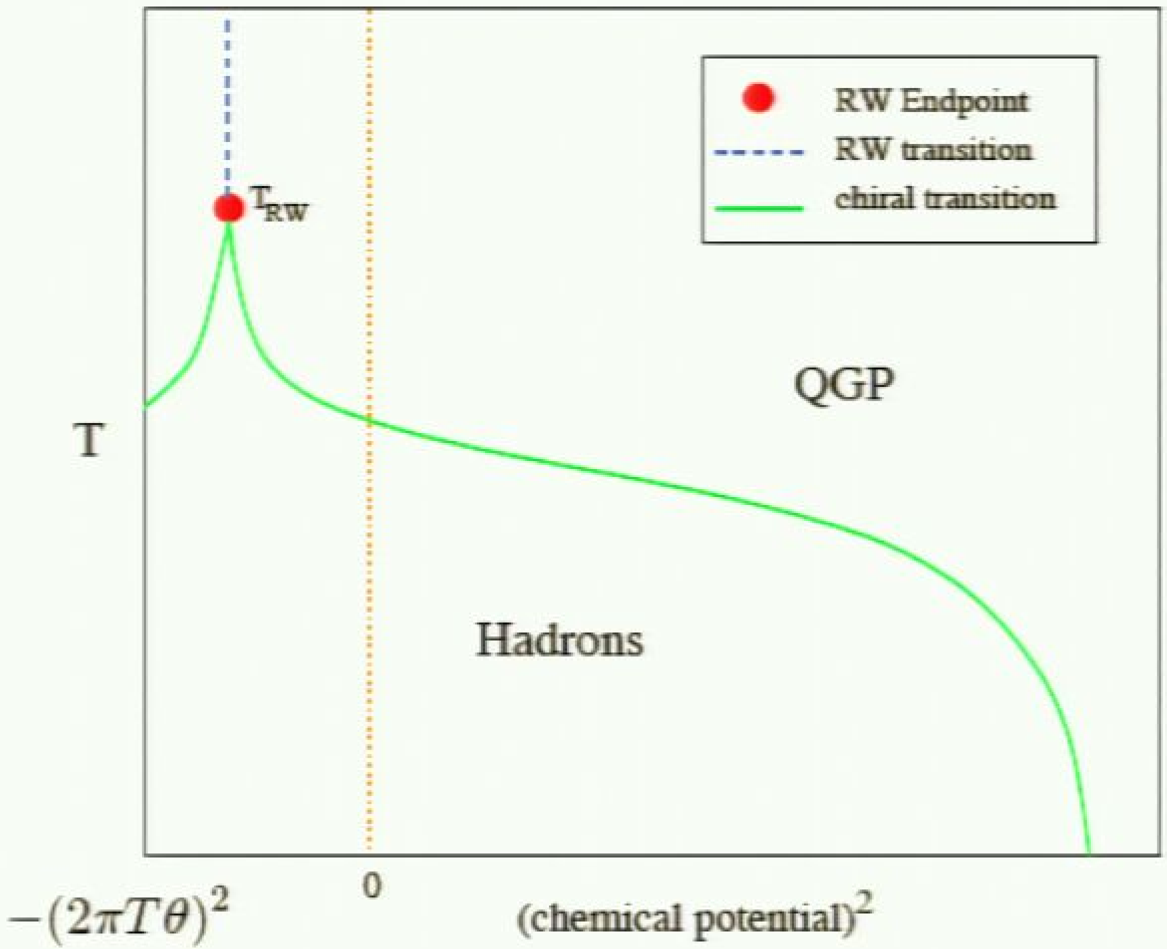


Phase structure at finite density

Imaginary chemical potential

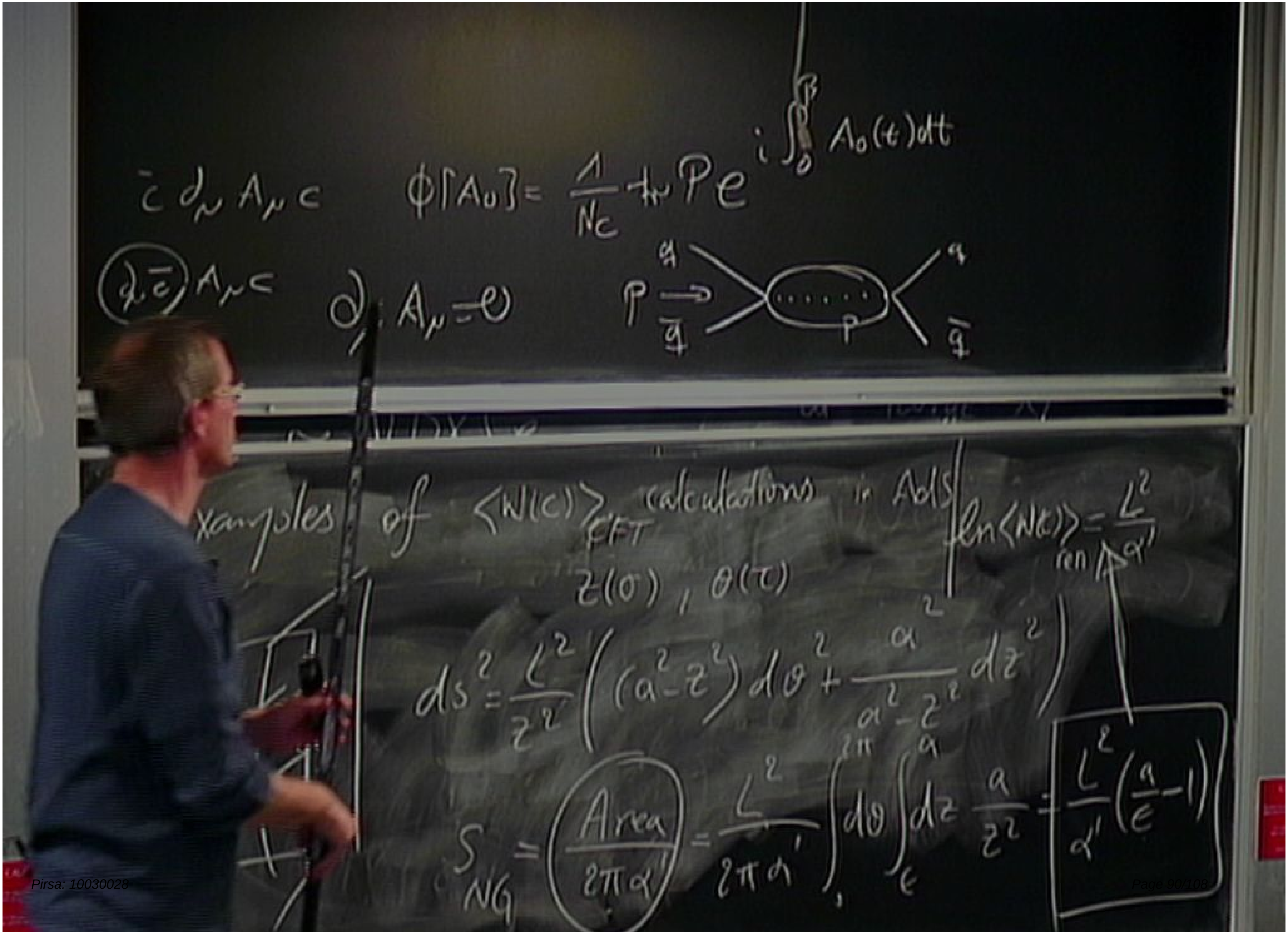
Lattice & Continuum QCD

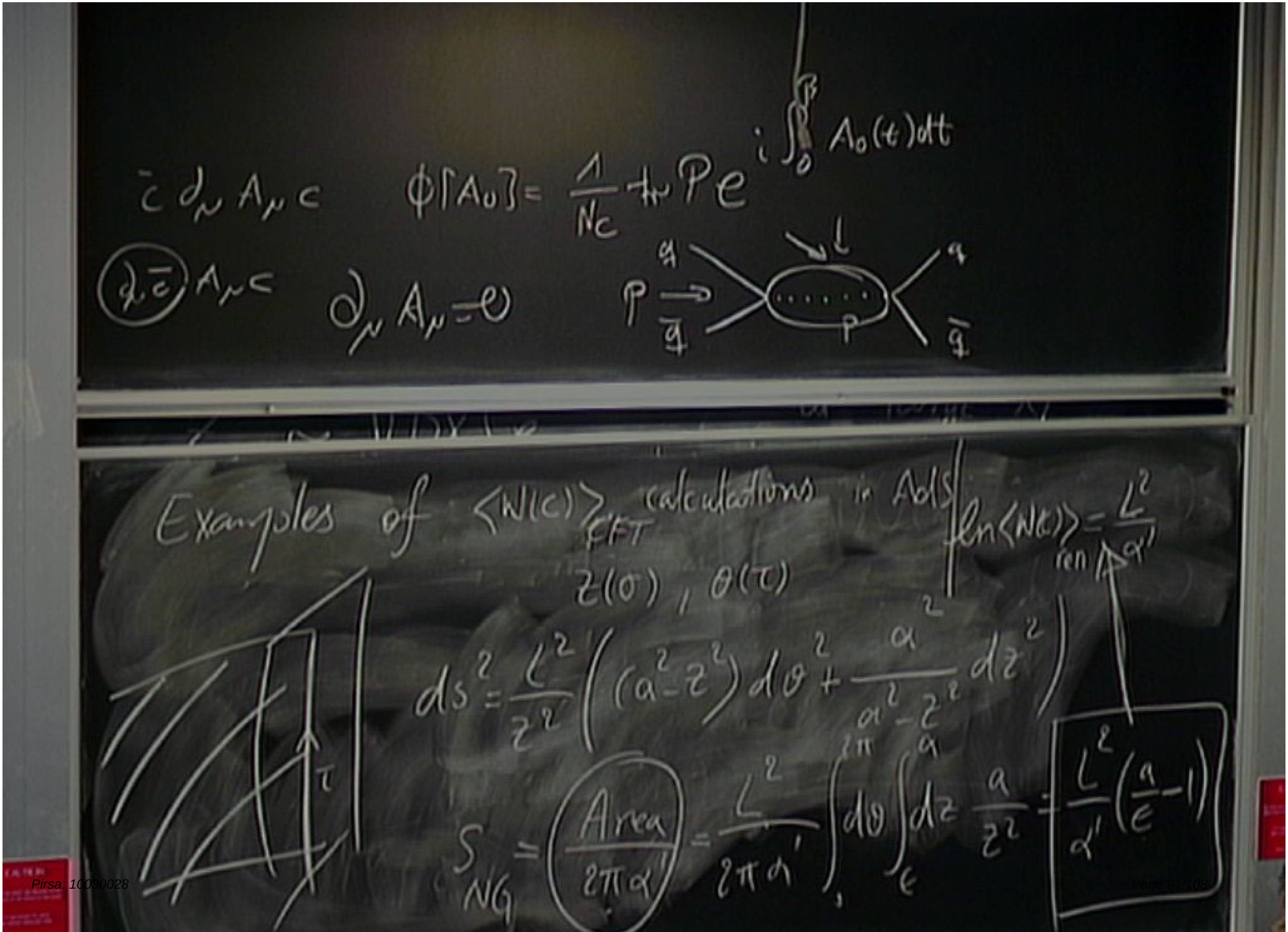
$$\psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x) \qquad \text{with} \qquad \mu_I = 2\pi T\theta$$



Phase structure at finite density

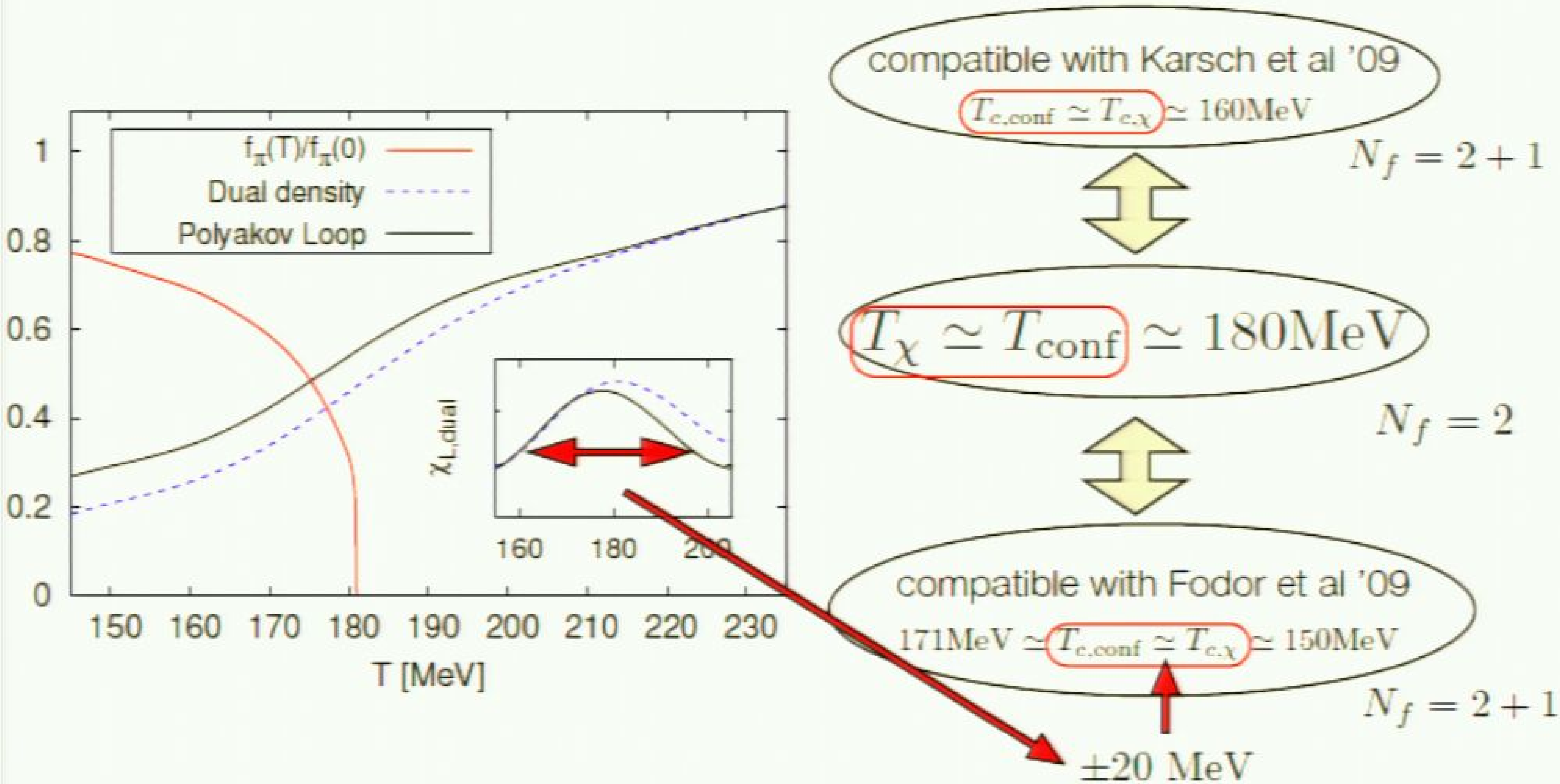
[redacted]


Examples of ⟨W(C)⟩ calculations in AdS
EFT
ln⟨W(C)⟩ = L²/α'
ren
z(σ), θ(τ)
Area


i ∂_μ A_μ c
Φ[A_0] = 1/N_c tr P e^{i ∫_0^T A_0(t) dt}
∂_μ A_μ = 0
P
q
q̄
Examples of ⟨W(C)⟩_CFT calculations in AdS
ln⟨W(C)⟩_ren = L²/α'
z(σ), θ(τ)
ds² = L²/z² ((a² − z²) dθ² + a²/(a² − z²) dz²)
S_NG = Area/(2πα') = L²/(2πα') ∫_0^{2π} dθ ∫_ε^a dz a/z² = L²/α' (a/ε − 1)

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods & lattice



Full dynamical QCD: N_f = 2 & chiral limit

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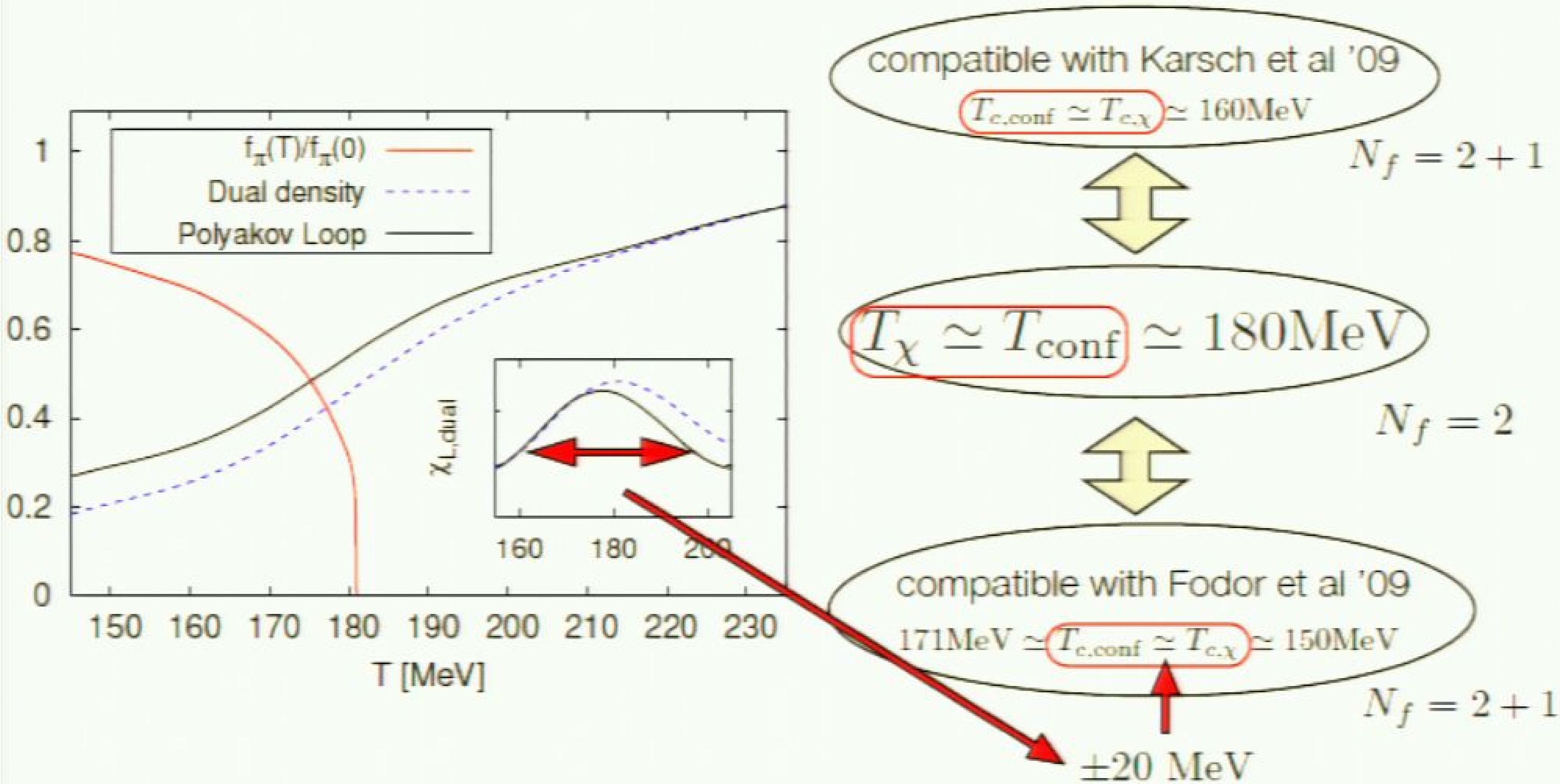

Full dynamical QCD: N_f = 2 & chiral limit

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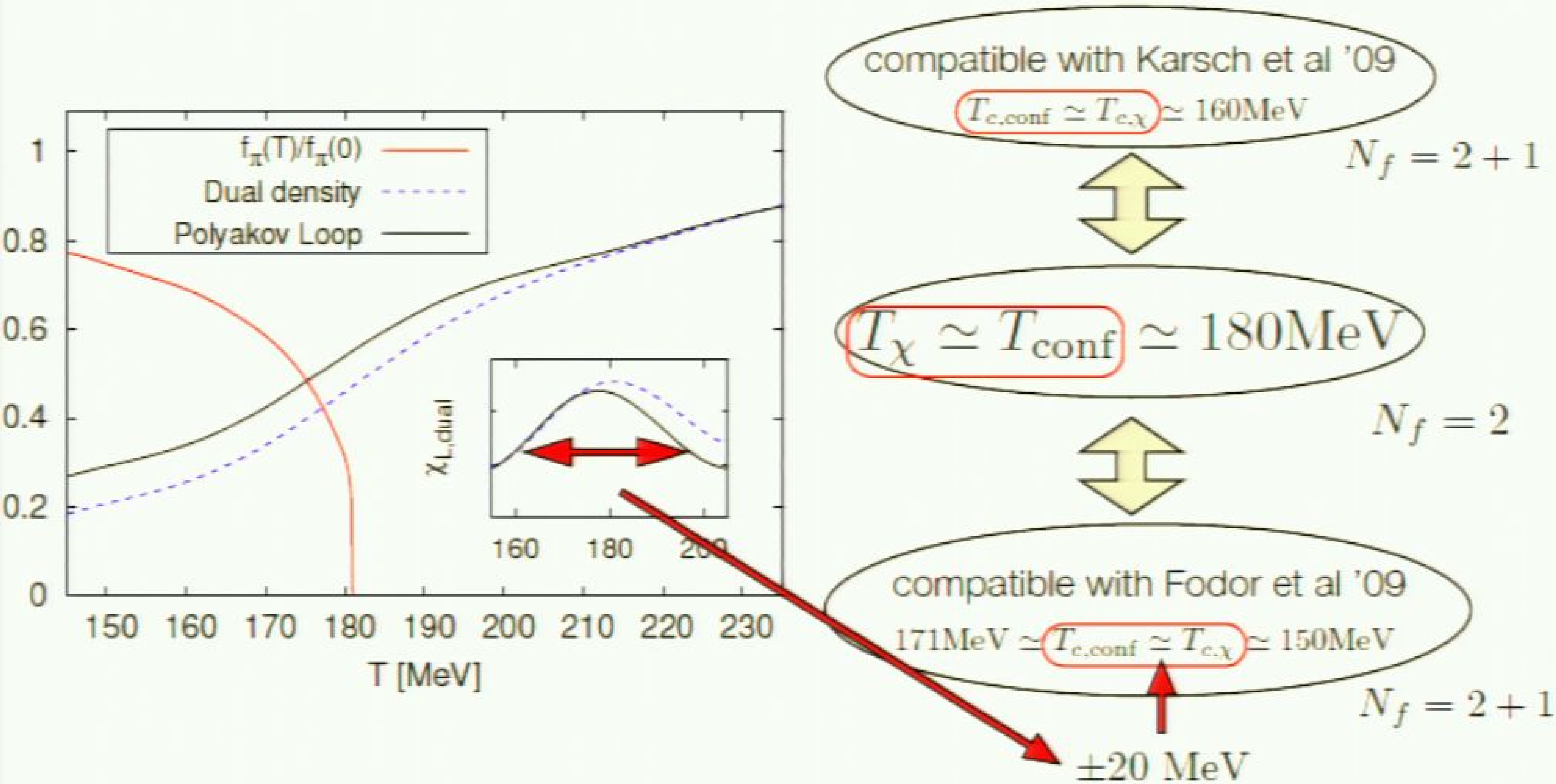

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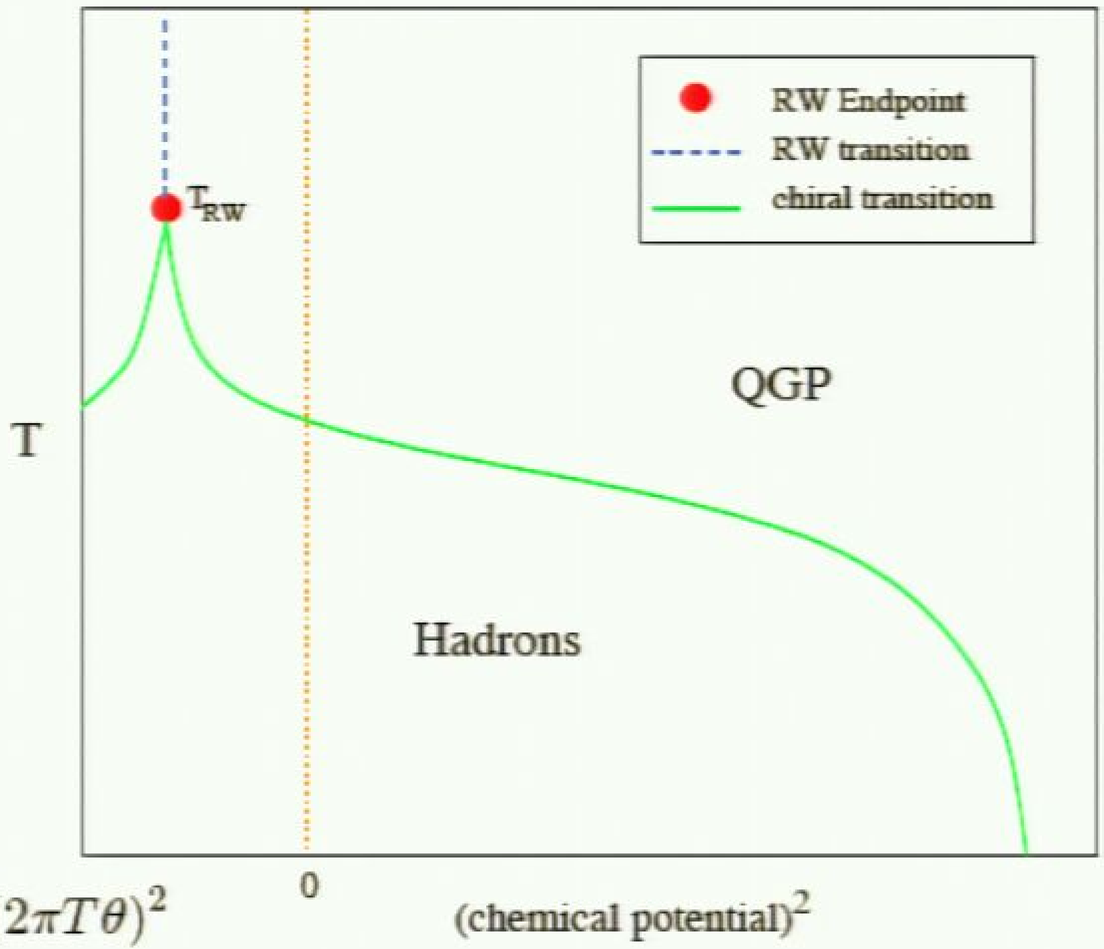

Full dynamical QCD: N_f = 2 & chiral limit

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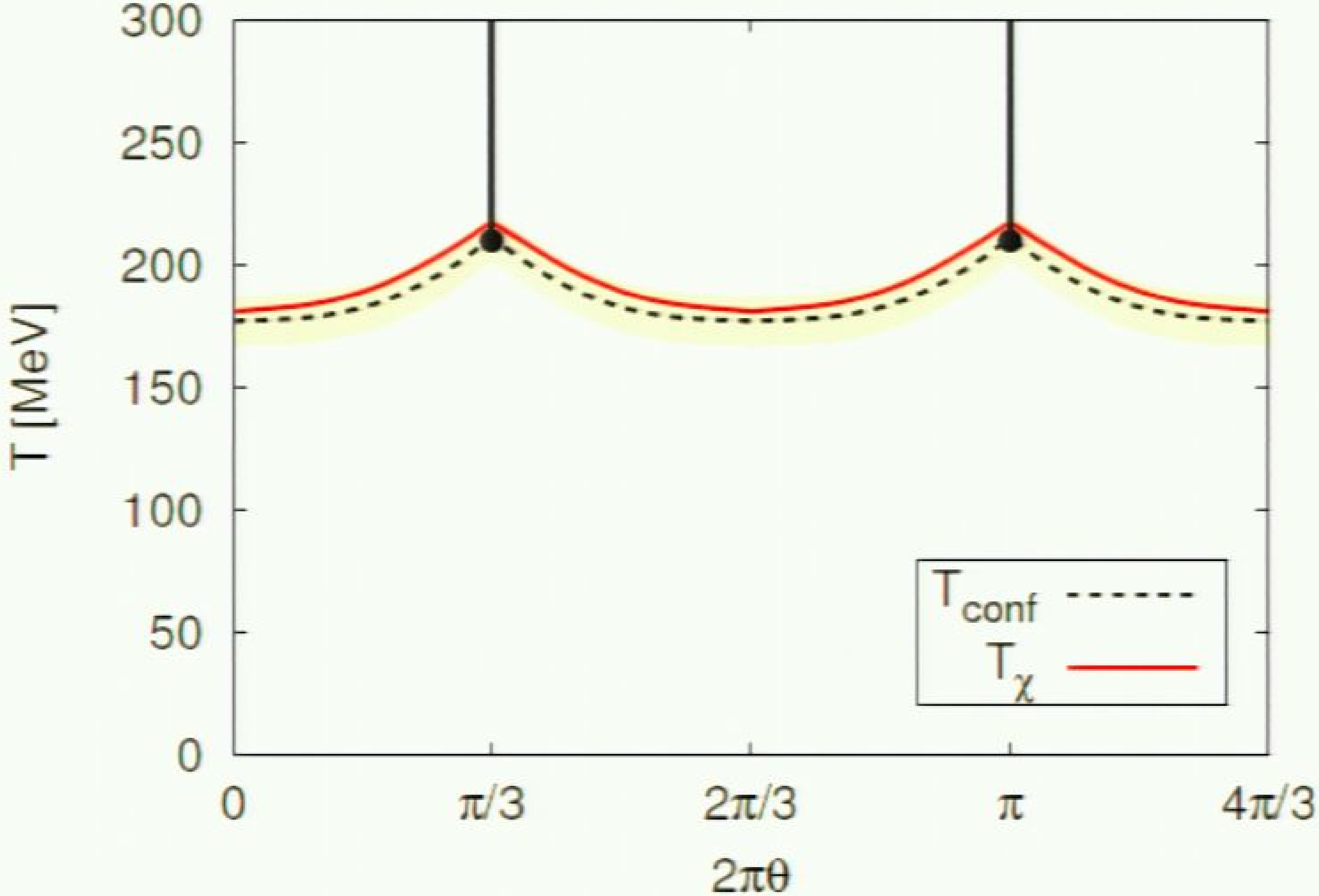
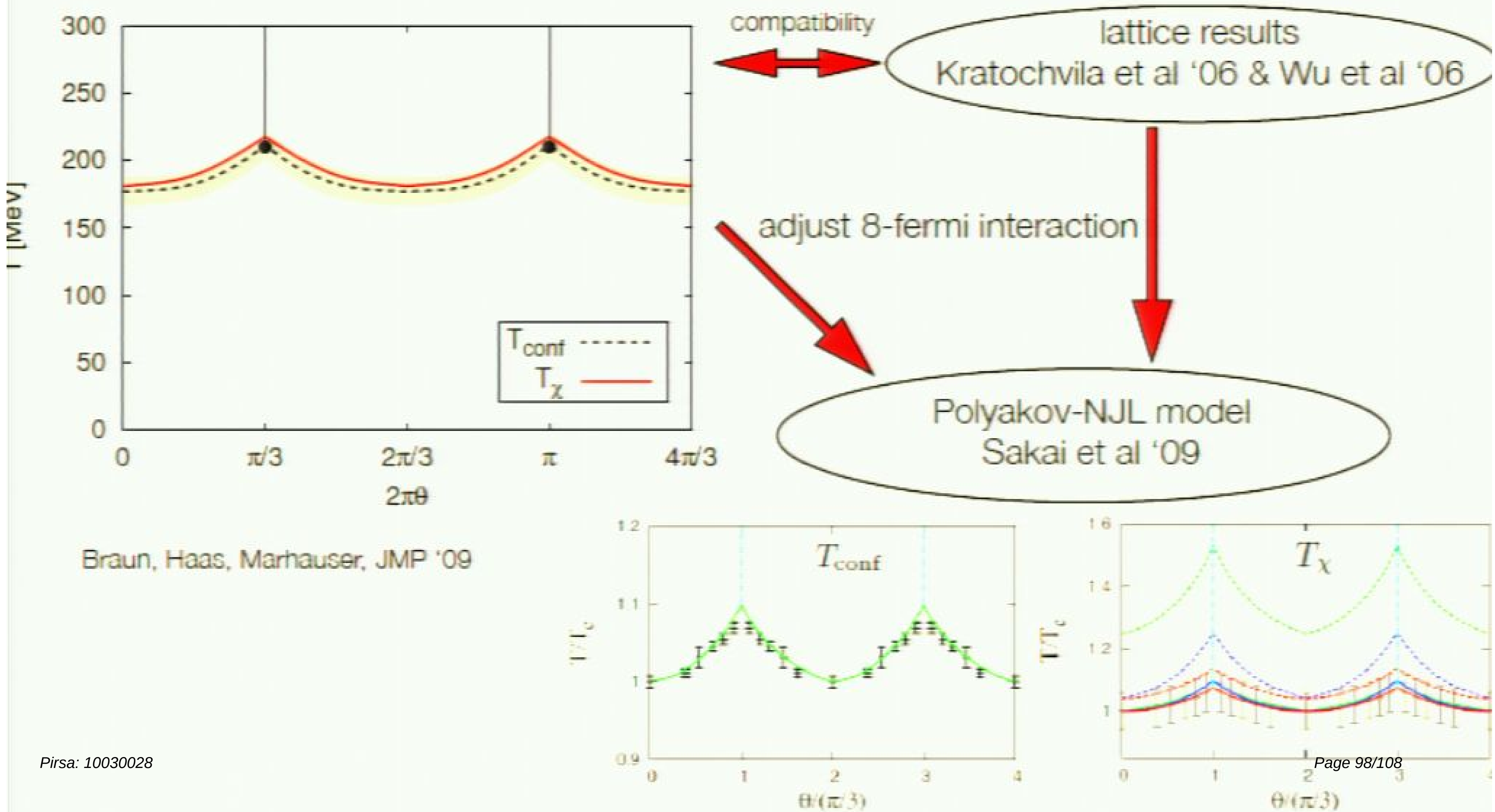


chemical potential : $\mu = 2\pi i T \theta$

Full dynamical QCD: N_f = 2 & chiral limit

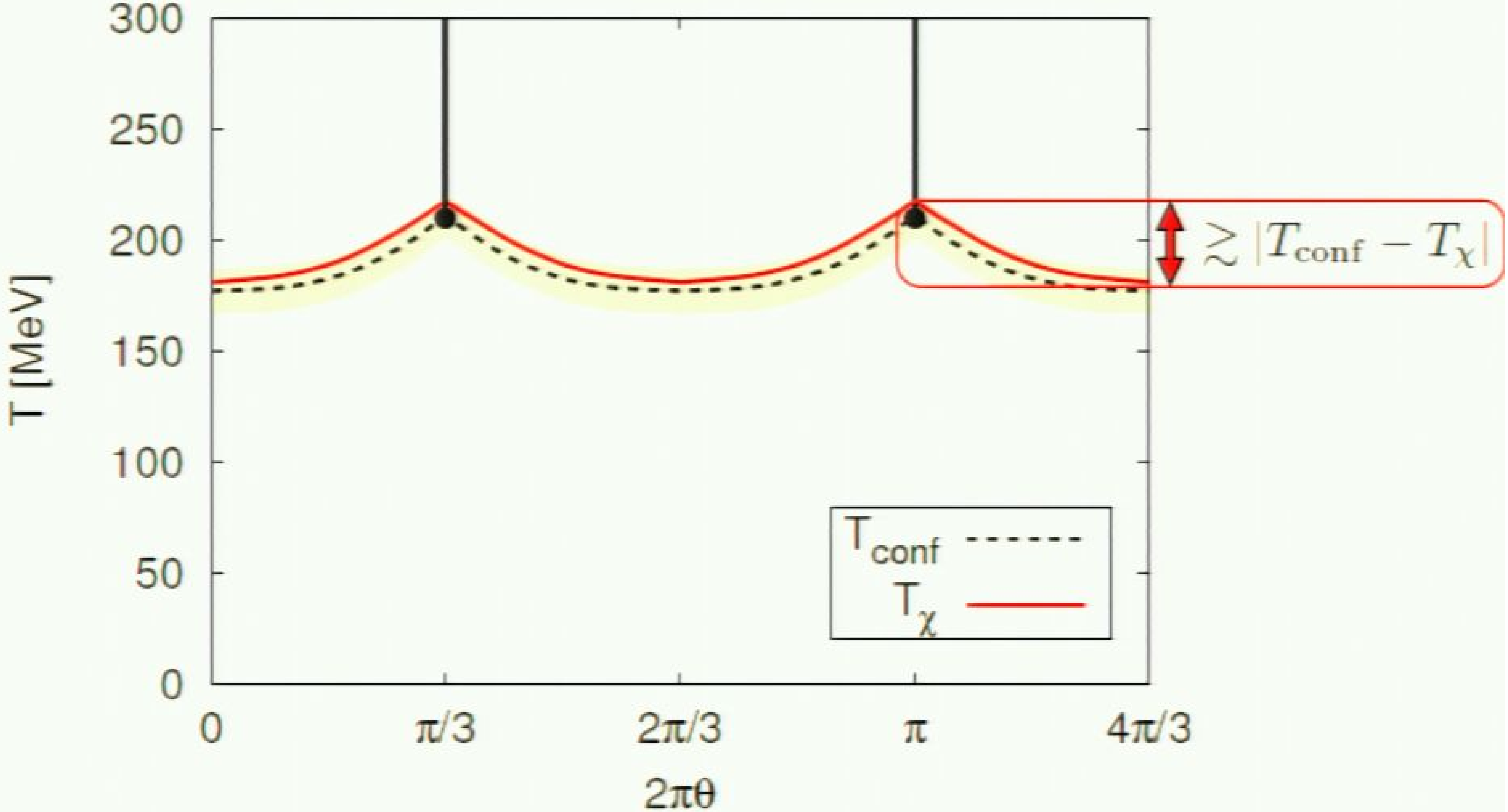
Continuum methods & lattice



Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f = 2 & chiral limit

Remark on dual order parameters for confinement



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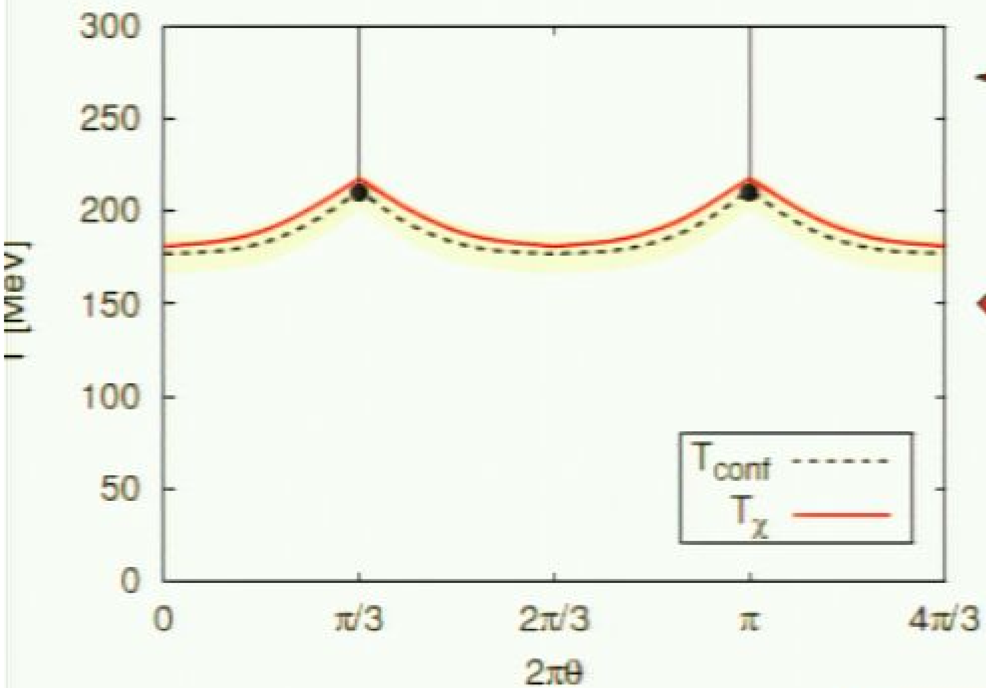
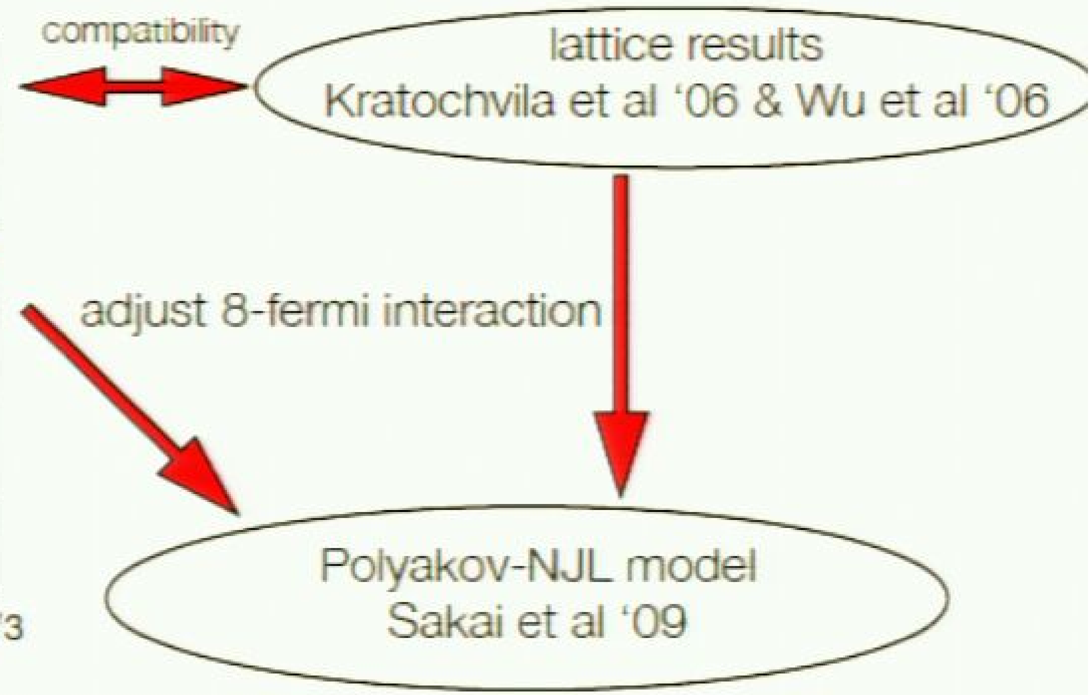
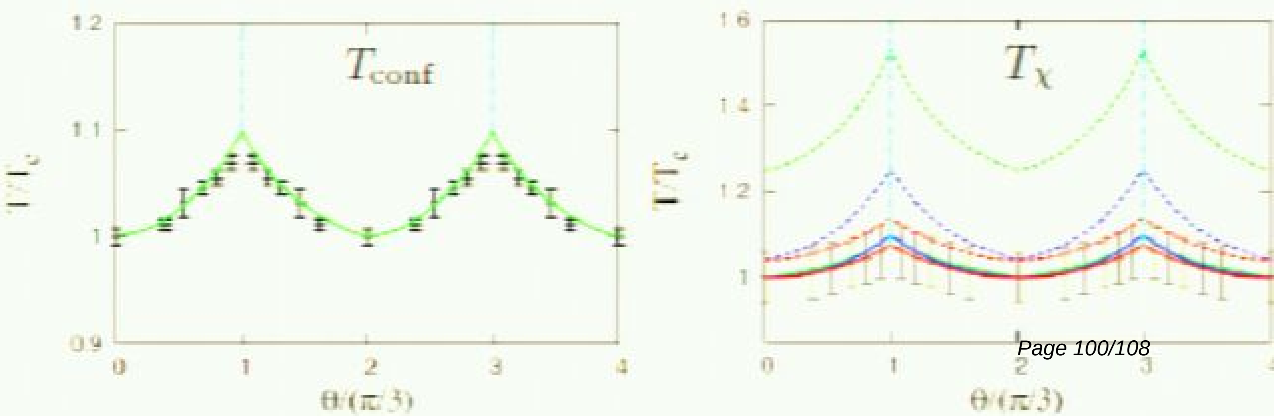


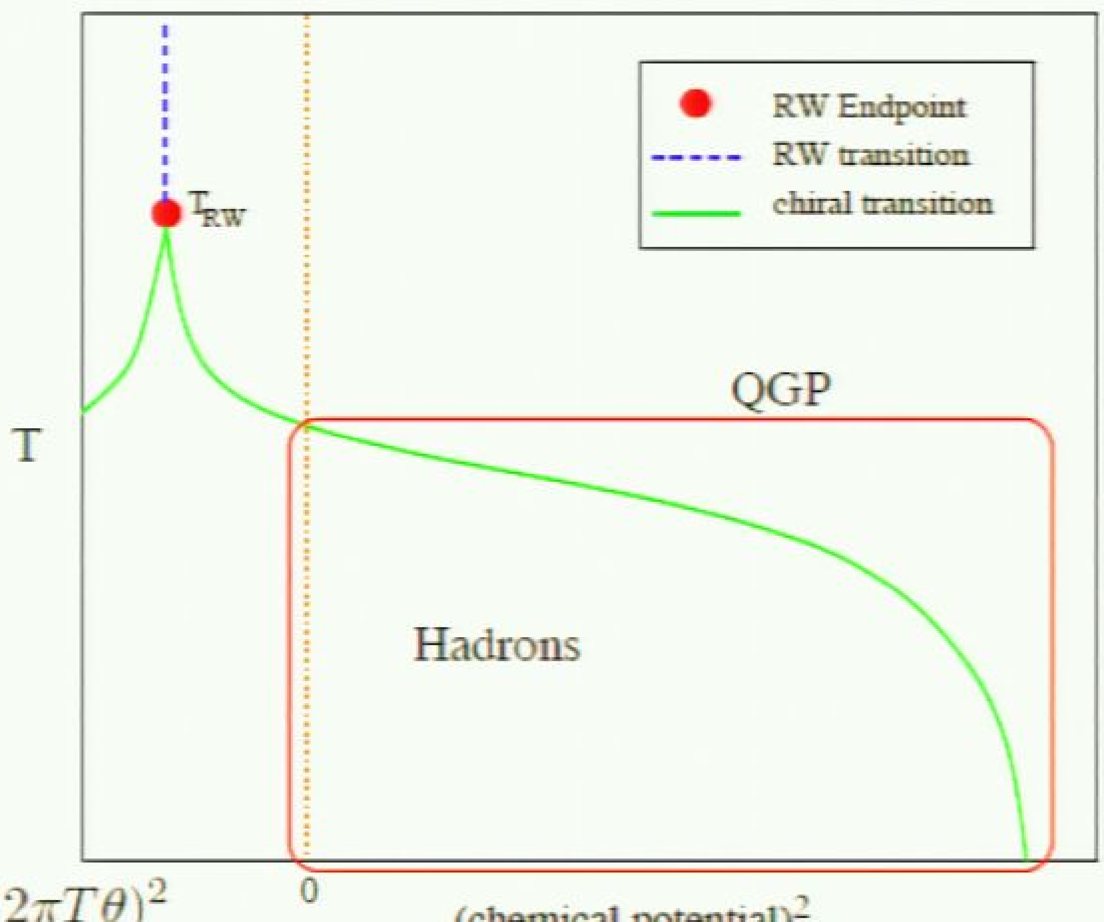
Braun, Haas, Marhauser, JMP '09





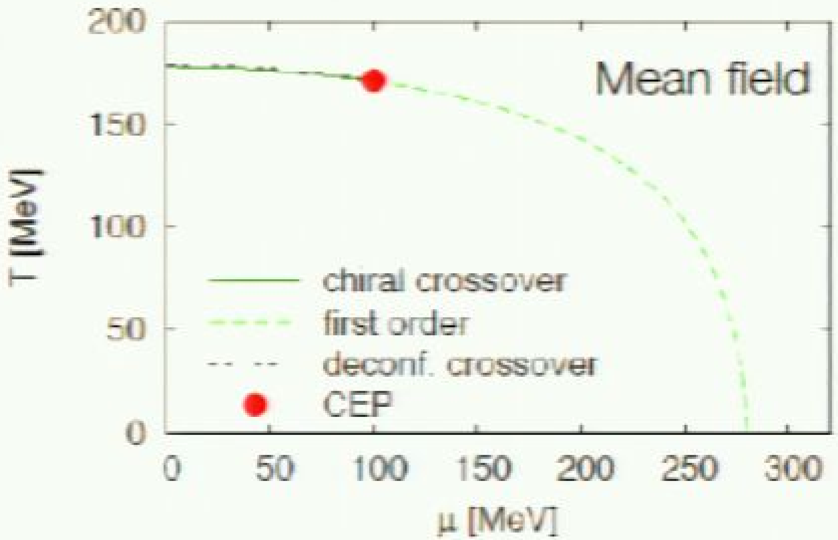
Real chemical potential

$$\psi_\theta(t+\beta,\vec{x}) = -\psi(t,x)$$



A glimpse at real chemical potential

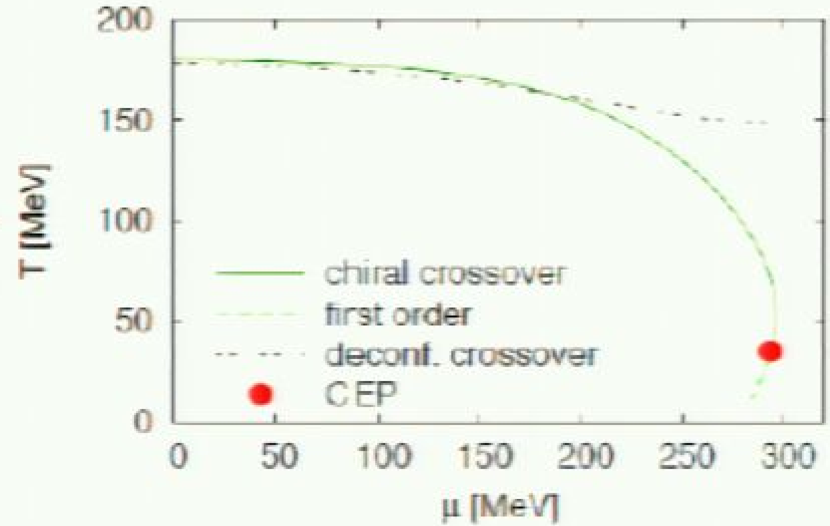
Polyakov - Quark-Meson model



RG

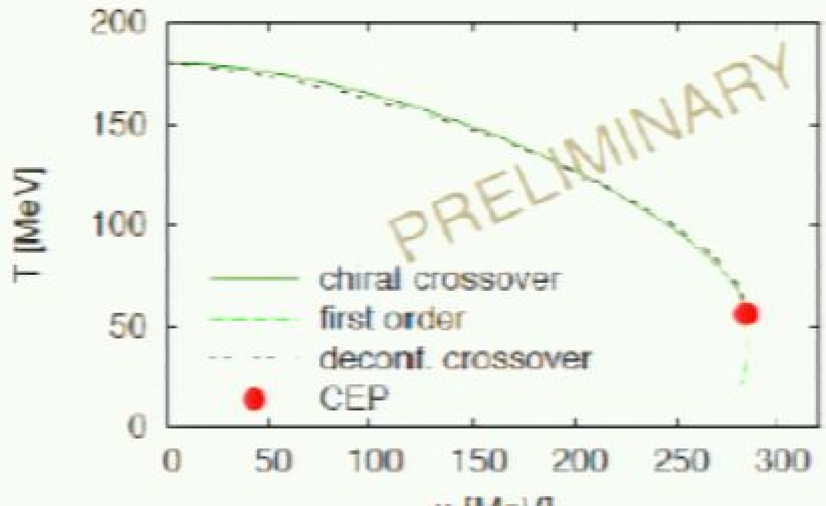
quark-meson

fluctuations



$N_f = 2$

HTL/HDL



quark fluctuations

in YM sector

Schaefer, JMP, Wambach '07

Summary & Outlook

A glimpse at real chemical potential

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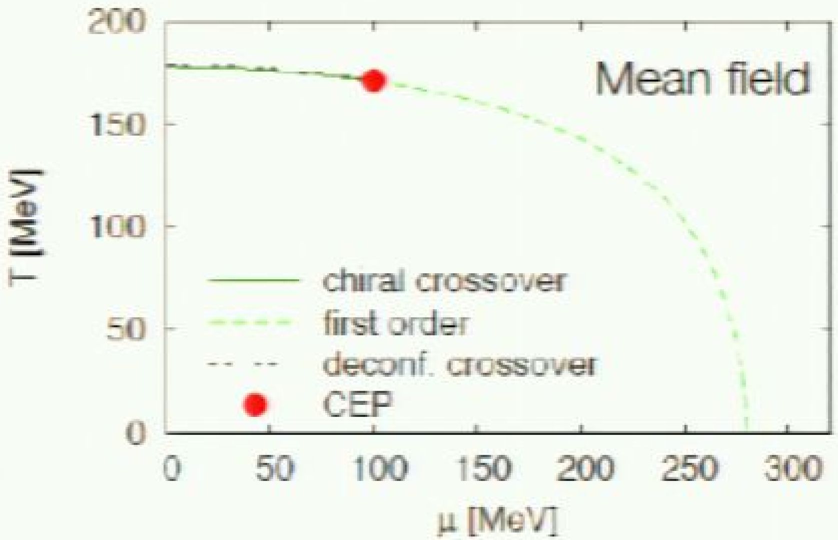

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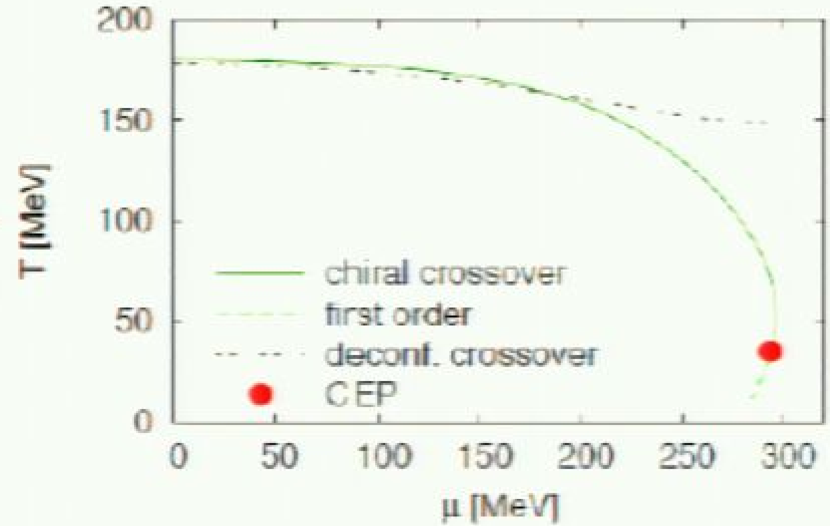

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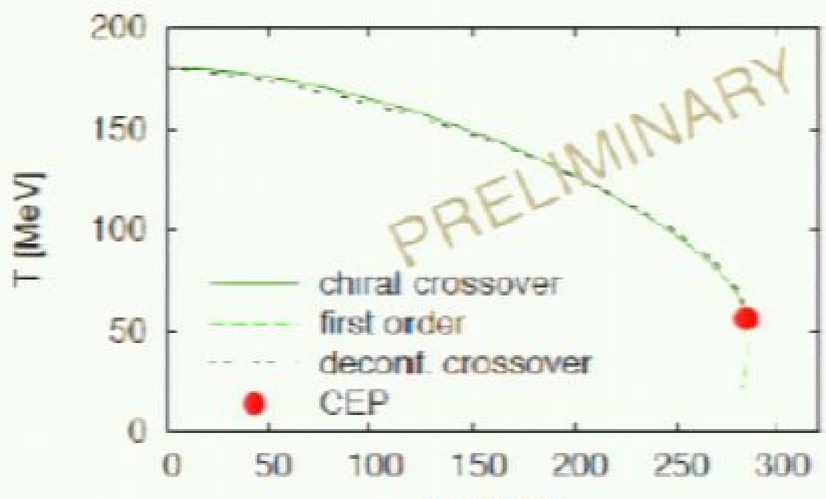
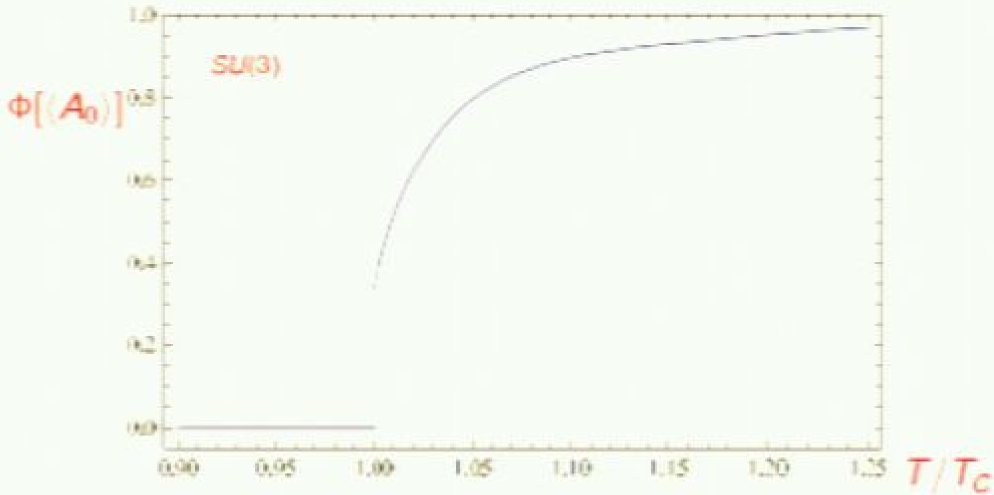
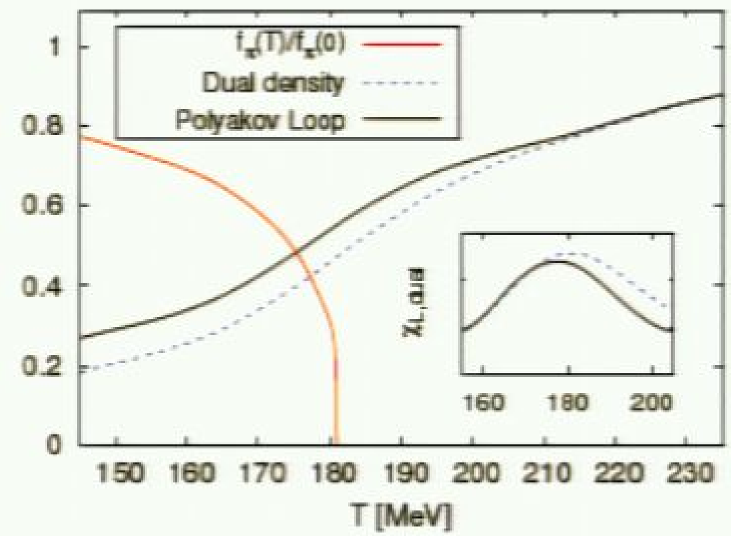


Summary & outlook

- Phase diagram of QCD
  - Confinement & chiral symmetry breaking at finite temperature





Summary & outlook

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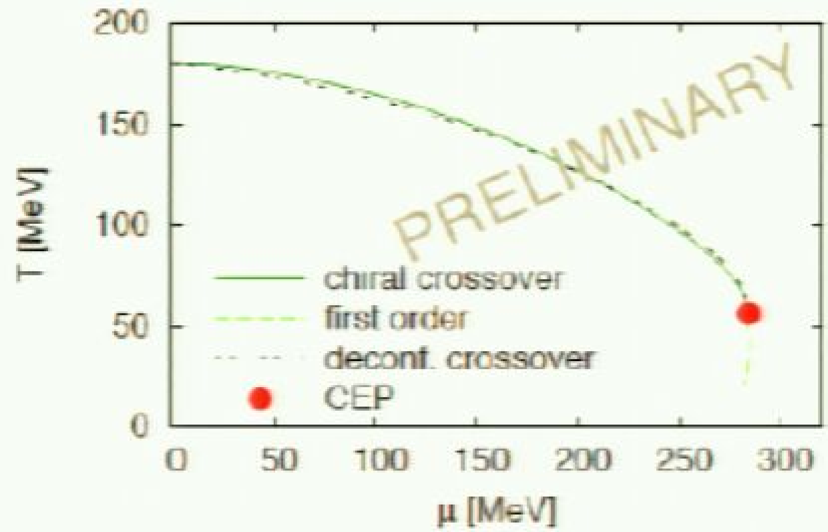
        - Dynamical hadronisation
    - critical point and phase lines in effective theories



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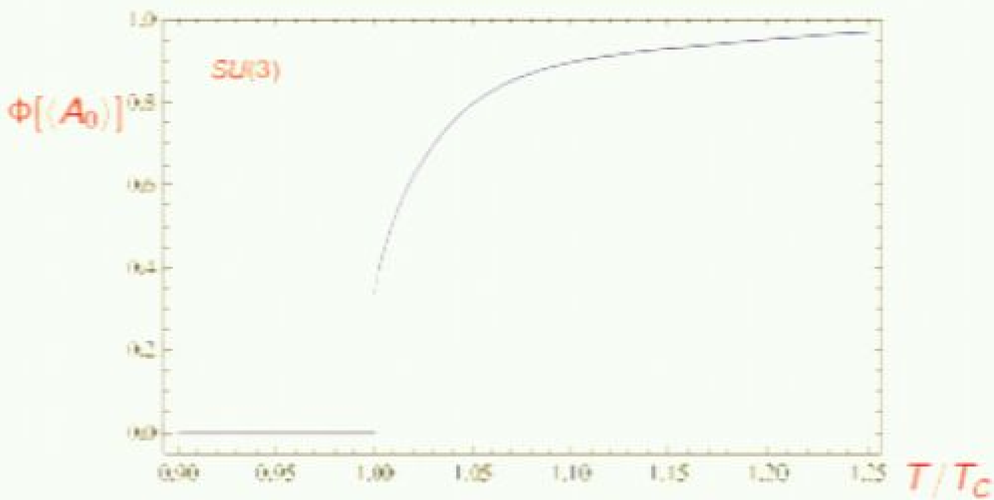

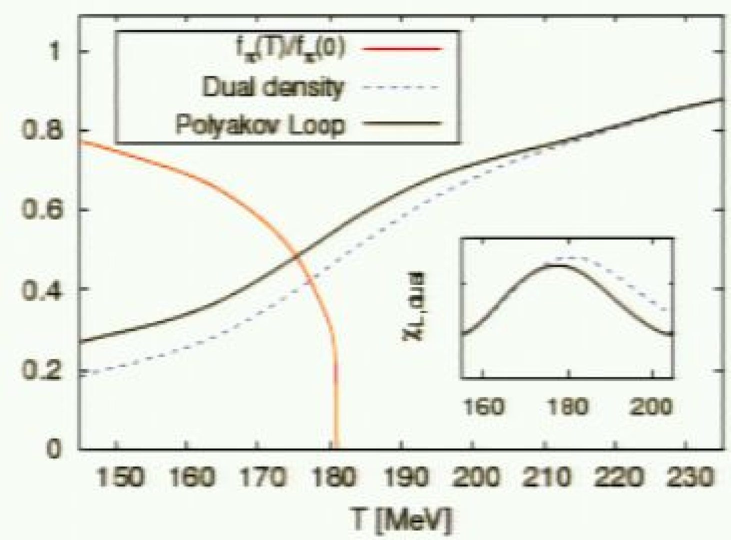

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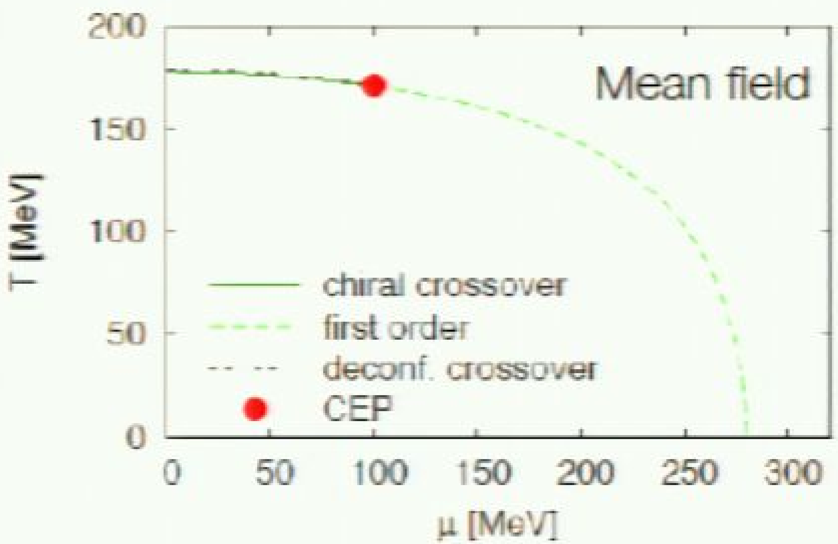

RG

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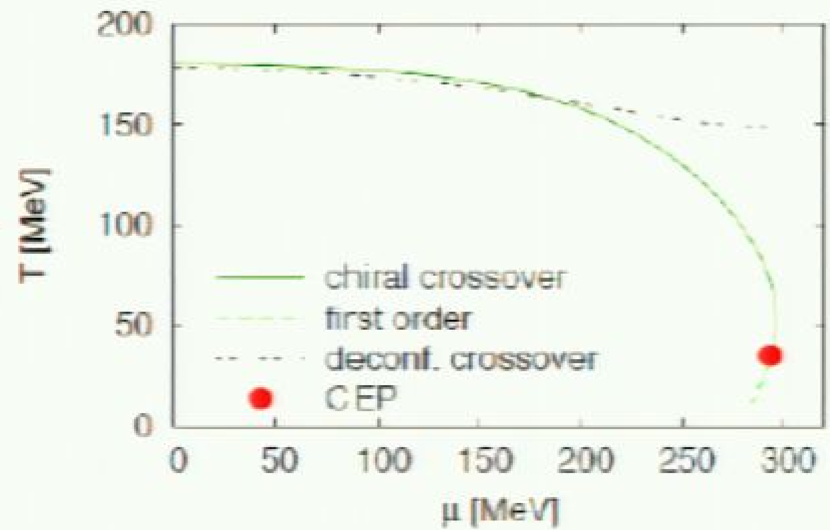

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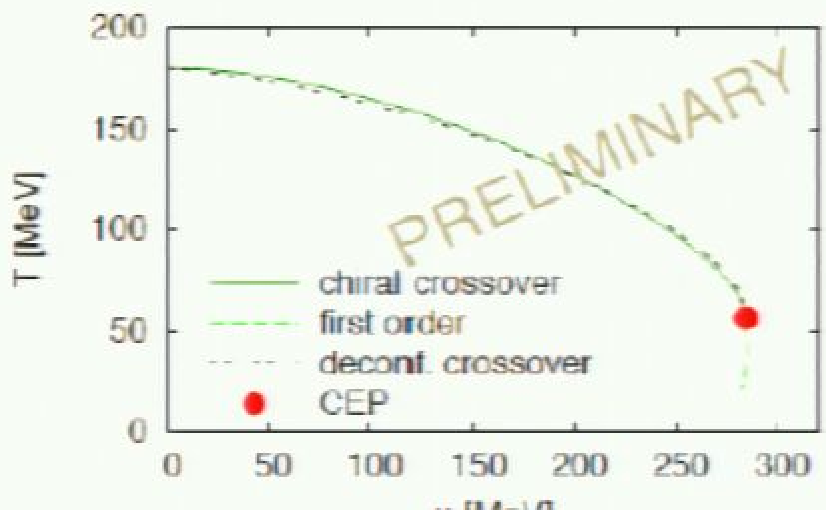

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Schaefer, JMP, Wambach '07