Title: Implications of background independence for quantum gravity

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Abstract: We will give a short overview of non-perturbative quantum gravity models and discuss some key common problems for these models. In particular we will analyze what background independence requires from a theory of quantum gravity. That I may detect the inmost force Which binds the world, and guides its course; Its germs, productive powers explore, And rummage in empty words no more!

[Goethe, Faust I]

We still have to do with words and a few pictures ...

Overview

- A. Quantum gravity as a state sum model: different approaches
- B. Regulator dependence, diffeomorphism symmetry and constraints in canonical framework
- C. Regulator independence by renormalization flow?
- D. Connecting the problems
- E. Observables in background independent theories
- F. Conclusions

Quantum gravity as a state sum model



$$Z = e^{\frac{i}{\hbar}S(\text{conf }1)} + e^{\frac{i}{\hbar}S(\text{conf }2)} + \cdots$$

Many approaches - including canonical ones - result in a state sum model.

Most of these models can be interpreted as summing over (microscopic) geometries.

Quantum gravity as a state sum model



The sum is just a formal object, to define it more precisely specify:

measure

 (discrete counting measure, continuous measure, ...)
 including what to sum over (triangulations, labels, matter labels, degenerate geometries, twocomplexes, graphs, topologies ...)

(construction principle for) amplitudes

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motivated by

•regularization of path integral

- insight from canonical quantization
- counting of configurations, practicality: sum over less
- make maximal number of features dynamical: some over more

Aims

- •obtain 4d manifolds at large scales
- •that satisfy Einstein equations
- •plus corrections
- ...

To have 4d spacetimes (and more) emerging as fully dynamical objects.

Some models







spin foams: sum over labels (and two-complexes)

large spin limit gives discretized Einstein-Hilbert (Regge) action [Conrady, Freidel '08, Pirsa: 10030026Barret et al '09] (causal) dynamical triangulations: sum over triangulations (not) including topologies

Monte Carlo simulations, phase diagrams, ... [Bialas et al '96, Ambjorn, Jurkiewicz, Loll '98+] sum over labels on graphs, or sum over graphs

quantum graphity, emergent gravity [Konopka, Markopoulou, Smolin '06+;Volovik, Wen, Page 8/72

Causal Dynamical Triangulations





 $d = \infty$

d=2

[Bialas et al '96]



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Lessons

 proof of principle: non-perturbative gravity can be made to work!

 choice of measure can be very important, here restrictions motivated by microscopic implementation of causality

 conformal sickness can be cured: entropy winning over energy

 spacetime dimension is a dynamical concept, not predefined by dimension of building blocks



change of spectral dimension with probing time [Ambjorn, Loll, Jurkiewicz 05]



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Theory with intrinsic ultra violet cut off (dimensional reduction)!

relation to

•asymptotic safety scenario [Weinberg '79, Reuter et al '96+, Lauscher, Reuter '05]: there is a non-Gaussian fix point for renormalization flow

 Horava-Lifshitz gravity [Horava '09]: quantum gravity is Lorentz/diffeomorphsim violating in the ultra violet; spacetime is anisotopic

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- Generalization of causal structure to triangulations with product structure [BD, Loll '05]: universality class, anisotropic scaling in more dimensions?
- Does diffeomorphism symmetry emerge?
- Renormalization (connection to asymptotic safety?)
 -introduces labels (effective degrees of freedom)
 - -can we reabsorb sum over triangulations
- -have an alternative mechanism that results in causality restriction? Page 25/72

 Diffeomorphism symmetry deeply entangled with dynamics of general relativity. [Hojman, Kuchar, Teitelboim '76, Wald '86]
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- Diffeomorphism symmetry deeply entangled with dynamics of general relativity. [Hojman, Kuchar, Teitelboim '76, Wald '86]
 How can we implement diffeomorphism symmetry in discrete models?
- Relation of diffeomorphism symmetry to renormalization?
 Relation of diffeomorphism symmetry to background independence?

[BD, Freidel, Speziale '07; BD '08; BD, Ryan '08; Bahr, BD '09 a.b.c; BD, Hoehn '09; Bahr, BD, He wip] Consider state sum models based on triangulations or other discrete structures. For the moment viewed as regulator.

Different proposals:

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defines fundamental theory

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take refinement limit sum over triangulations (and more)

is background independent if partition function is already invariant under triangulation

how to find such models?

argument: sum over labels incorporates sum over (coarser) triangulations

hope: regulator independence in limit how to do sum? measure(s)?

group field theories [Boulatov '92, ..., Gurau '09]

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effective action via coarse graining

effective measure? (fixed topology)

How to find such regulator independent models?

Is there a relation to diffeomorphism symmetry?

Issue of diffeomorphism symmetry is involved in discrete models. Often argument [CDT, Rovelli,], that discrete configurations represent already diffeomorphism invariant geometric objects.

> However, derivation of path integral from canonical framework suggest: symmetries are related to constraints.

Path integral from canonical quantization

canonical quantization in 5 sec:

Hilbert space, states

 $\psi \in \mathcal{H}$

central: constraint operators

 \hat{C}_I

encode dynamics: generate (diffeomorphism) symmetries

physical (gauge invariant states):

 $\hat{C}_I \psi_{phys} = 0$

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Pirsa: 10030026 [for instance BD, Thiemann 04: Testing the Master Constraint Programme: Lto VI
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Pirsa: 10030026 [for instance BD, Thiemann 04: Testing the Master Constraint Programme: Lto VI How to obtain gauge invariant states? 'group' average: [Ashtekar et al 95, ...]

 $\psi_{phys} \sim \int_{\text{Gauge orbits}} e^{i\hat{C}} \psi \ d\mu$

physical inner product path integral w/ boundary data [Reisenberger, Rovelli '97,...] $<\psi_1|\psi_2>_{phys}\sim\int_{v_1}^{v_2}e^{iS}d\nu$

Constraints imply symmetries in path integral.

Symmetries of path integral imply constraints. [Halliwell, Hartle '91]

What are these gauge symmetries?

We will apply a dynamical definition: non-uniqueness of solutions. Implies equations of motions are not independent.

Covariant symmetries and canonical constraints

Firstly:

Need canonical formalism with discrete time allowing for local evolution and reproducing exactly the covariant dynamics. Problem: hypersurfaces in 4d triangulation? [Gambini,Pullin '03] & [Sorkin '75] ⇒ [BD '08; Bahr, BD '09; BD, Hoehn 09]

Covariant symmetries and canonical constraints

Theorem [Bahr. BD '09	for s	ystems with discrete time evolutions]:	
covariant			canonical
symmetries exact	⇒	eom not independent	\Rightarrow constraints (first class)
broken	⇒	eom (weakly) not independent	⇒pseudo-constraints





Analysis of 1d models, 3d gravity with and without cosmological constant, 4d linearized gravity, 4d flat sector [only canonical: Waelbroeck, Zapata '93+], [BD '08], [Bahr, BD '09], [BD, Hoehn '09]:

covariant		canonical
symmetry under vertex translations	\Rightarrow	Hamiltonian and diffeomorphism constraints
exact symmetries \Rightarrow		closed constraint algebra for discrete geometries!

Vertex translations



For fixed boundary length there is a three parameter set of solutions for inner length variables corresponding to position of vertex inside the tetrahedron.

Corresponds to the action of Hamiltonian and Diffeomorphism constraints.

Actions with symmetries?

Do we have 4d actions with such vertex translation symmetries (for curved solutions)?

[Hamber, Williams 97] (conjectured) yes for Regge calculus [Bahr, BD 09] no, there are examples with broken symmetries

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Related: Do we have a consistent constraint algebra for discrete geometries?

main problem for canonical lattice models (also in numerical relativity)

•consistent constraints for 'flat or homogeneously curved dynamics' [BD, Ryan '08, Bahr, BD '09]

- •first formulation of consistent constraints in linearized discrete gravity for arbitrary triangluations [BD, Hoehn '09]
- based on vertices (as opposed to dual vertices), corresponding to vertex translations
 no consistent algebra for non-linear order yet

•LQG (as continuum theory): anomaly free [Thiemann '96] because graph is changed without Pirsa: 10030026 Interaction between dual vertices. However dynamical interpretation not very clear? Obtaining anomaly free constraints is equivalent to constructing an action with exact symmetries.

How can we construct actions with exact symmetries?

Can we construct actions with exact symmetries?

- (broken) symmetries are properties of action
- •idea: construct actions that capture better continuum dynamics [Improved and Perfect actions: ..., Symanzik, Wilson, Hasenfratz et al in QCD: avoid Lorentz symmetry breaking!]

•by renormalization group transformation!

- •fine grain and integrate out fine grained degrees of freedom
- obtain effective action on coarse grained lattice, capturing dynamics of fine grained lattice
 take infinite refinement limit to find fixed point action

Question: Do we regain local gauge symmetries from continuum?

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Yes, in 1d examples and in 3d with cosmological constant. [Bahr, BD 09] Quantum 3d with cosmological constant? [to do]

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Non-topological theories [Bahr, BD, He wip] : non-local actions.

4d Regge calculus, perturbative expansion

[BD, Hoehn 09] [Bahr, BD, He wip] 3d Regge calculus with matter

[Banisch, BD wip]

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Set up: Regge calculus

(classical theory corresponding to spin foam models, lattice loop quantum gravity)

approximate space time by piecewise flat triangulation
length variables on edges fix geometry
discrete action defines dynamics





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3d Regge calculus

without cosmological constant

- any triangulation of flat space is a solution
- at every vertex 3dim translation symmetry
- triangulation independent
- zero physical degrees of freedom

with cosmological constant

- unique solutions to equations of motion
- there is no translation symmetry acting
- not triangulation independent
- all degrees of freedom physical

exact diffeo symmetry



approximate diffeo symmetry







3d Regge with cosmological constant

$$S_{\mathcal{T}} = \sum_{e} l_e \epsilon_e - \Lambda \sum_{\sigma} V_{\sigma}$$

action for flat simplices

approximate symmetries, riterritering. dependent

3d Regge with curved simplices

$$S_{\mathcal{T}}^{\kappa} = \sum_{E} L_{E} \epsilon_{E}^{\kappa} + 2\kappa \sum_{\sigma} V_{\Sigma}^{\kappa}$$

action for simplices with curvature

 $\kappa = \Lambda$

exact symmetries, triang. independ@हा/१

Conjecture: Actions with exact diffeomorphism invariance are triangulation independent.

supported by:

- •examples
- canonical framework: should support both continuous (lapse, shift for every vertex) and discrete (which vertex is evolved) evolution parameters
- •considerations based on category theory [Pfeiffer '04] : smooth 4d manifolds/diffeos ⇔ piecewise linear/ Pachner moves

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Causal Dynamical Triangulations: sum only over 'foliation preserving' triangulations.

Summary

a)Actions for 4d with exact diffeomorphism invariance might be constructed by renormalization group methods.

b) Such actions would allow construction of anomaly free canonical lattice models.

c) Such actions might be triangulation independent.

d) Is there a (universality) class of associated statistical models?

Connections between proposals.

defines fundamental theory

take refinement limit sum over triangulations (and more)

is background independent if partition function is already invariant under triangulation

how to find such models?

argument: sum over labels incorporates sum over (coarser) triangulations

> hope: background independence in limit

how to do sum? measure(s)?

group field theories [Boulatov '92, ..., Gurau '09]

(causal) dynamical triangulations

effective action via coarse graining

effective measure? (fixed topology)



Some implications for the quantum gravity practitioner

take diffeomorphism symmetry in discrete models seriously!
understand interplay of renormalization and diffeomorphism symmetry, order parameters, observables, ...

be not afraid of divergencies in spin foam models [Perini, Rovelli, Speziale '08]
rather understand the divergencies in spin foams induced by symmetry
constraints corresponding to vertex translations are based on vertices of triangulation, not on dual vertices

restricting ambiguities by requiring (emergence of) diffeomorphism symmetry?
understand measure on space of geometries induced by different models
can we relate sum over triangulations to sum over labels of a fine triangulation?

•relation to asymptotic safety scenario?

•phenomological implications: Lorentz symmetry breaking, deformation or not?

 be careful with perturbative expansions (graviton scattering) in theories with broken symmetries [tomorrow]
 •relating covariant and canonical models [tomorrow and thursday]

Observables in background independent theories

Whether you can observe a thing or not depends on the theory which you use. It is the theory which decides what can be observed.

[Einstein | 926]

What can we learn about a theory with background independent observables?

Observables in background independent models

Diffeomorphism invariant observables characterize solutions of the theory.

Time tested method to parametrize solutions (and quantize): phase space.



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Choosing a hypersurface in a space time induces phase space data.

But with gauge symmetries: parametrization is not one to one, rather there are constraint hypersurfaces and gauge orbits.



Data induced by different hypersurfaces in a solution are gauge equivalent.

Pirsa: 10030026 Diffeomorphism invariant observables are constants on gauge orbits.

Relational observables

Idea [Einstein '20... Kuchar 90s... Rovelli '91]: Use some variables to define a reference system \Rightarrow clocks. Make observations relative to this reference system.



To find observables requires to partially solve the theory! Computational framework?

Results

Computational framework:

•general computational framework that allows discussions of structural properties (locality and causality properties), relation to gauge fixing and reduced phase space [BD 04]

•application to general relativity, clocks that simplify calculations, equivalence of canonical and covariant observables, Abelian diffeomorphism invariant Hamiltonian constraints [BD 05]

•perturbative framework around flat spacetime: field observables in lowest order, non-local corrections, expansion in Feynman (tree) diagrams [BD, Tambornino 06]

•first canonical framework for gauge invariant cosmological perturbations to arbitrary high order with fully dynamical background variables (average over full phase space) [BD, Tambornino 07]

Can be used to discuss structural properties of observable algebra!

The problem of time (in general relativity)

One aspect: there are no perfect clocks (with standard matter).

Change this by adding non-standard matter [Aether: Jacobson et al, Brown-Kuchar-Dust, TeVeS, ...]

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or embrace it!

Derive limitations on observables as structural properties of quantum gravity. [Brunnetti, Fredenhagen '01, Giddings, Hartle, Marolf 05, BD, Tambornino 06,]

What can we observe?

Basic idea: we cannot consider the resolution of space time points in background manifold. Instead we have to consider regions specified by values of matter or curvature fields.

Commutator of field operators will obtain correction due to using a matter reference system: [BD, Tambornino 06]

$$\{\mathbf{f}[\tau], \mathbf{f}[\tau + \epsilon]\} = \mathbf{G}(\tau, \tau + \epsilon) \left(1 + \frac{\mathsf{Energy}(\mathbf{f})}{\mathsf{Energy}(\mathsf{clocks})}\right)$$

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Because of black hole formation cannot make energy of clocks arbitrary large

⇒ (super holographic) locality bound [Giddings, Hartle, Marolf '05]

Quantum mechanical toy examples:

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Background independence requires relational observables. These only allow for a finite resolution of space time. Hint for quantum gravity theories?

For approaches using lattice/triangulation as regulator: Observables should be independent from choice of lattice. Excludes site-based observables.

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Conclusions

- encouraging results in different approaches
- convergence of approaches
 - canonical and covariant (LQG and Spin foams)
 - asymptotic safety scenario and non-perturbative models

 interplay of diffeomorphism symmetry and renormalization group theory should lead to an even better understanding of differences and similarities: group background independent models into universality classes