Title: Foundations and Interpretation of Quantum Theory - Lecture 13

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Abstract:

Bohmian mechanics as an explanation of QM

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Primitive Ontology

ontology = what (according to a theory) exists = "beables" [Bell]
ontology of Bohmian mechanics = point particles, wave fct
primitive ontology = part of the ontology representing matter in 3-space
primitive ontology of Bohmian mechanics = point particles

Bohmian mechanics provides an explanation of quantum mechanics in terms of a coherent story about a clear and objective (observer-independent) primitive ontology.

Positivist way of thinking: only statements that can be *experimentally tested* are scientific statements.

Realist way of thinking: only statements that fit together as a coherent story about a clear ontology form an acceptable theory.

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Other realist theories explaining QM

- Variants of Bohmian mechanics:
  - Other laws of motion, e.g., "stochastic mechanics" [Nelson 1968]
  - replace point particles by strings or fields
- Theories of spontaneous wave fct collapse, with suitable primitive ontology
  - Ghirardi-Rimini-Weber (GRW) theory [1986] with flash ontology or matter density ontology [also: Pearle]
- Maybe many-worlds
  - usual many-worlds [Everett 1957]: no primitive ontology, just ψ
  - many-worlds theories with primitive ontology: Bell 1986; Schrödinger 1927: matter density $m(\mathbf{x}, t)$

$$m(\mathbf{x}, t) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} d\mathbf{q}_1 \cdots d\mathbf{q}_N \, \delta(\mathbf{q}_i - \mathbf{x}) \, |\psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)|^2$$

[see also Tumulka et al., arXiv:0903.2211]

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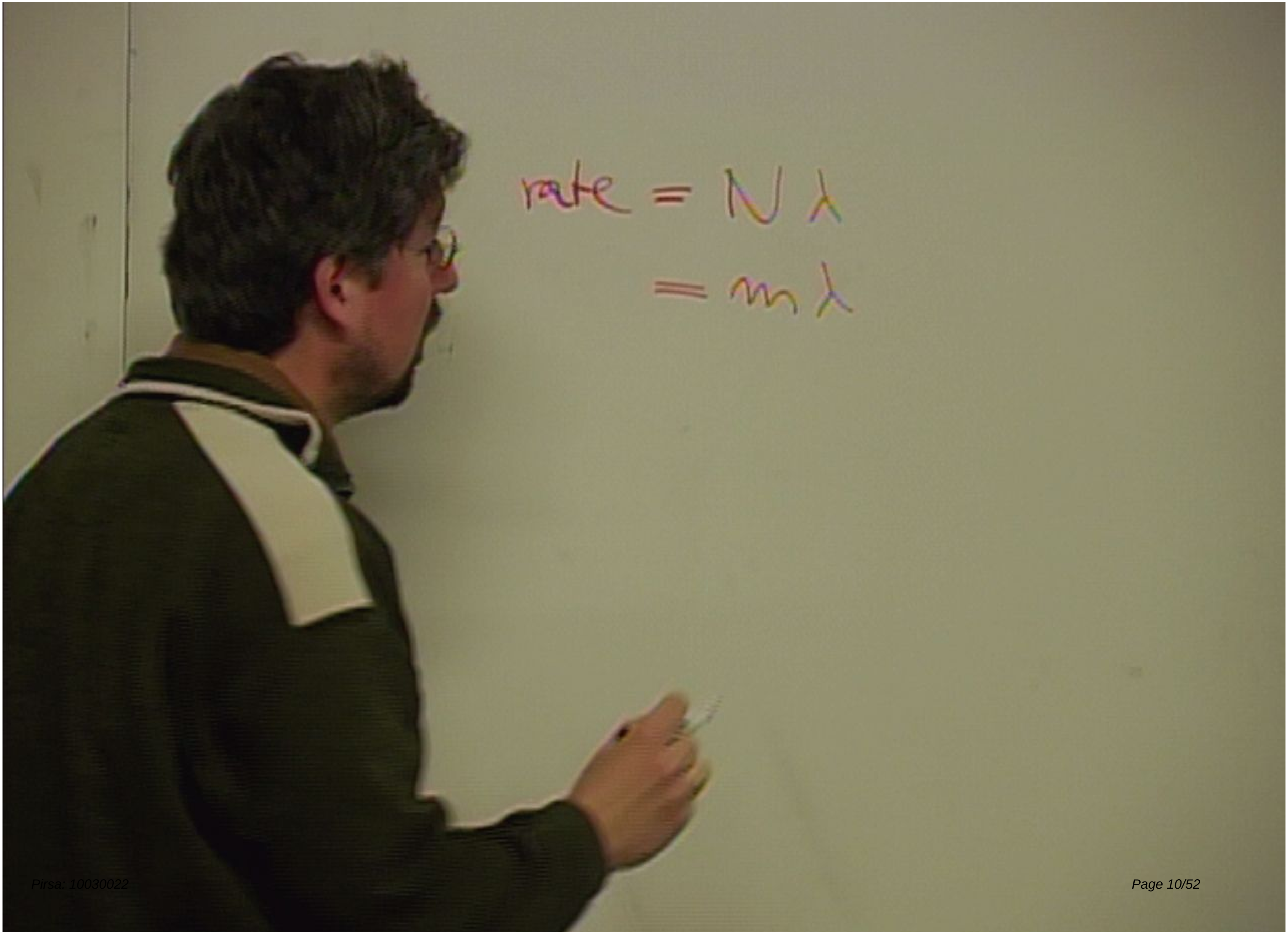
rate = N λ
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$$\text{rate} = N\lambda$$
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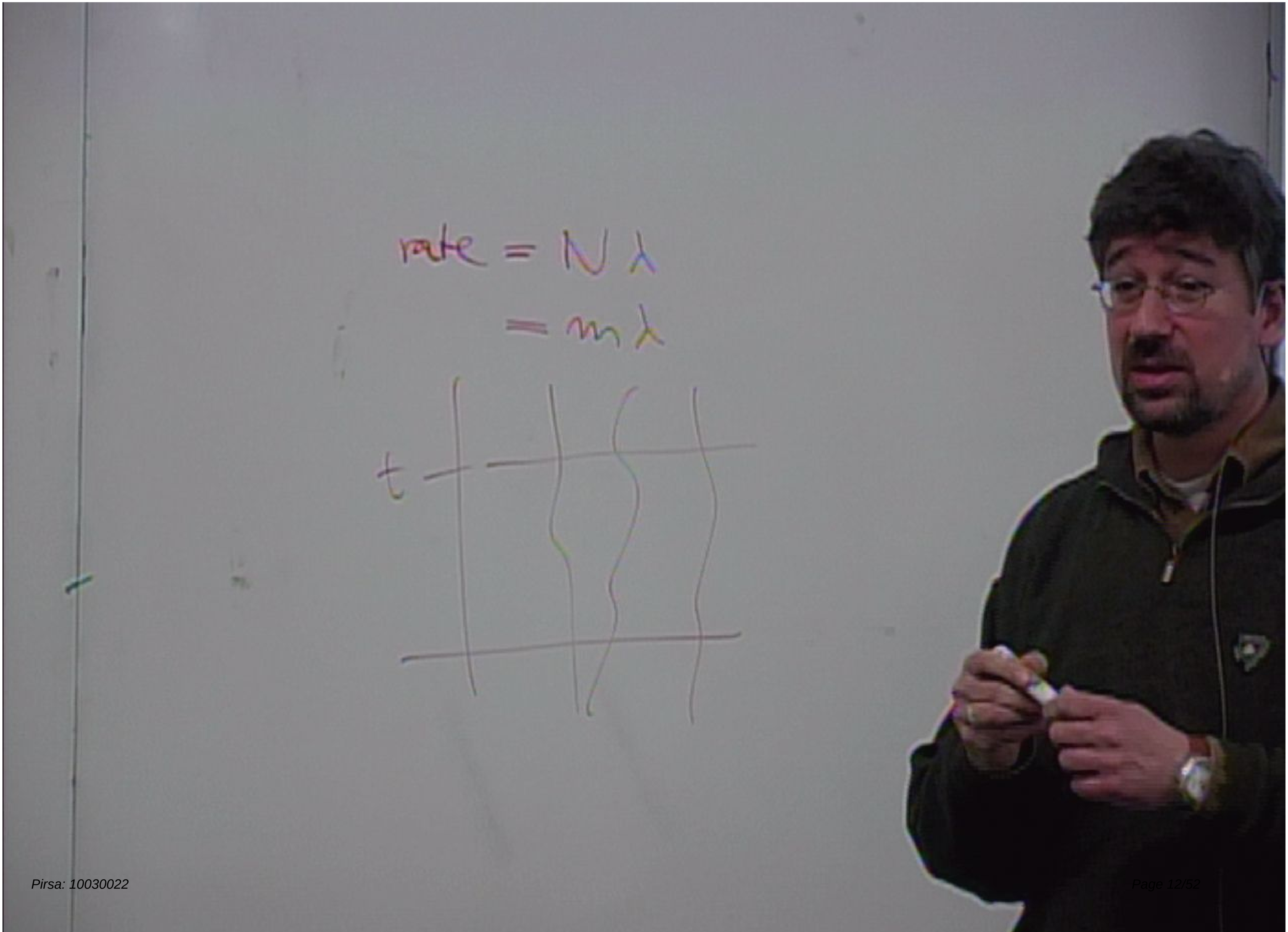
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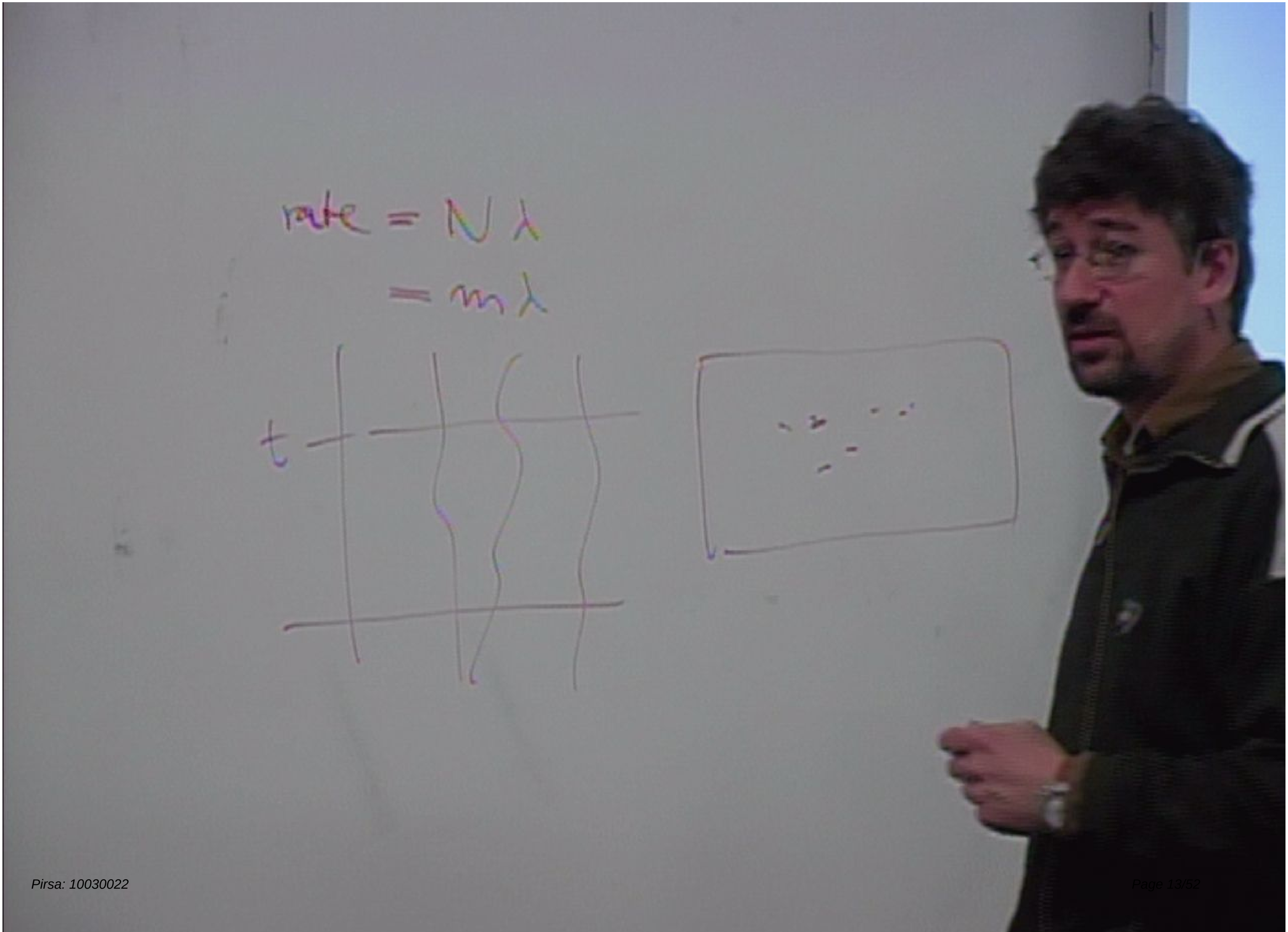
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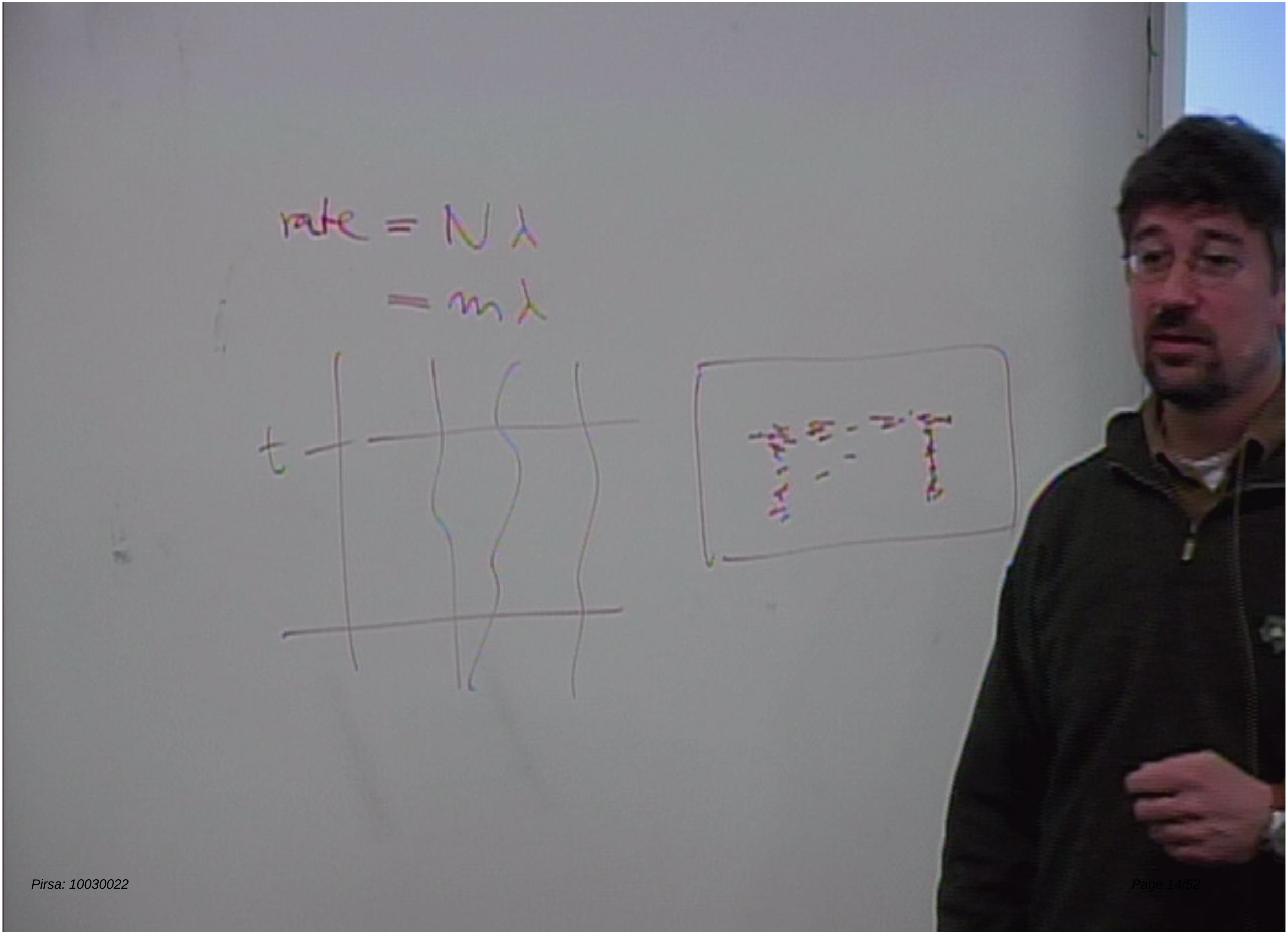
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Bohmian mechanics developed further

The symmetrization postulate

For N identical particles, we assume in Bohmian mechanics the same symmetrization postulate as in standard QM: $\psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)$ is either a *symmetric* or an *anti-symmetric* function.

If we take the particle ontology seriously then

the appropriate configuration space of N *identical* particles is
not the set $\mathbb{R}^{3N}$ of *ordered* configurations $(\mathbf{Q}_1, \ldots, \mathbf{Q}_N)$
but the set of *unordered* configurations $\{\mathbf{Q}_1, \ldots, \mathbf{Q}_N\}$,

$$^N\mathbb{R}^3 = \{Q \subset \mathbb{R}^3 : \#Q = N\} = (\mathbb{R}^{3N} \setminus \{\text{collisions}\})/\{\text{permutations}\}.$$

And indeed: If ψ is symmetric or anti-symmetric then v^ψ is permutation-covariant and thus projects consistently to a vector field on $^N\mathbb{R}^3$. For general (asymmetric) ψ, this is not the case.

Spin

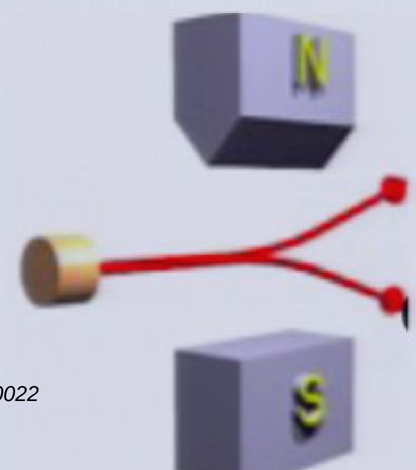
Bohmian mechanics with spin

$\psi_t : \mathbb{R}^{3N} \to (\mathbb{C}^2)^{\otimes N}$. Equation of motion:

$$\frac{d\mathbf{Q}_k(t)}{dt} = \frac{\hbar}{m_k} \mathrm{Im} \frac{\psi_t^* \nabla_k \psi_t}{\psi_t^* \psi_t}(Q(t))$$

where $\phi^* \psi = \sum_{s=1}^{2^N} \phi_s^* \psi_s$ inner product in spin-space

No "actual spin vector" (unlike actual position) needed, no rotational motion needed.



Stern–Gerlach experiment

Wave packet $\psi = \binom{\psi_\uparrow}{\psi_\downarrow}$ splits into two packets, one purely $\uparrow$, the other purely $\downarrow$. Then detect the position of the particle: If it is in the spatial support of the $\uparrow$ packet, say that the outcome is "up."

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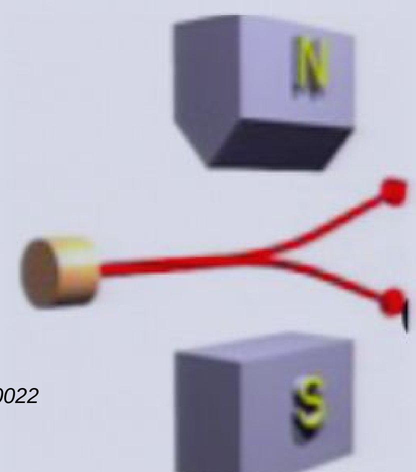

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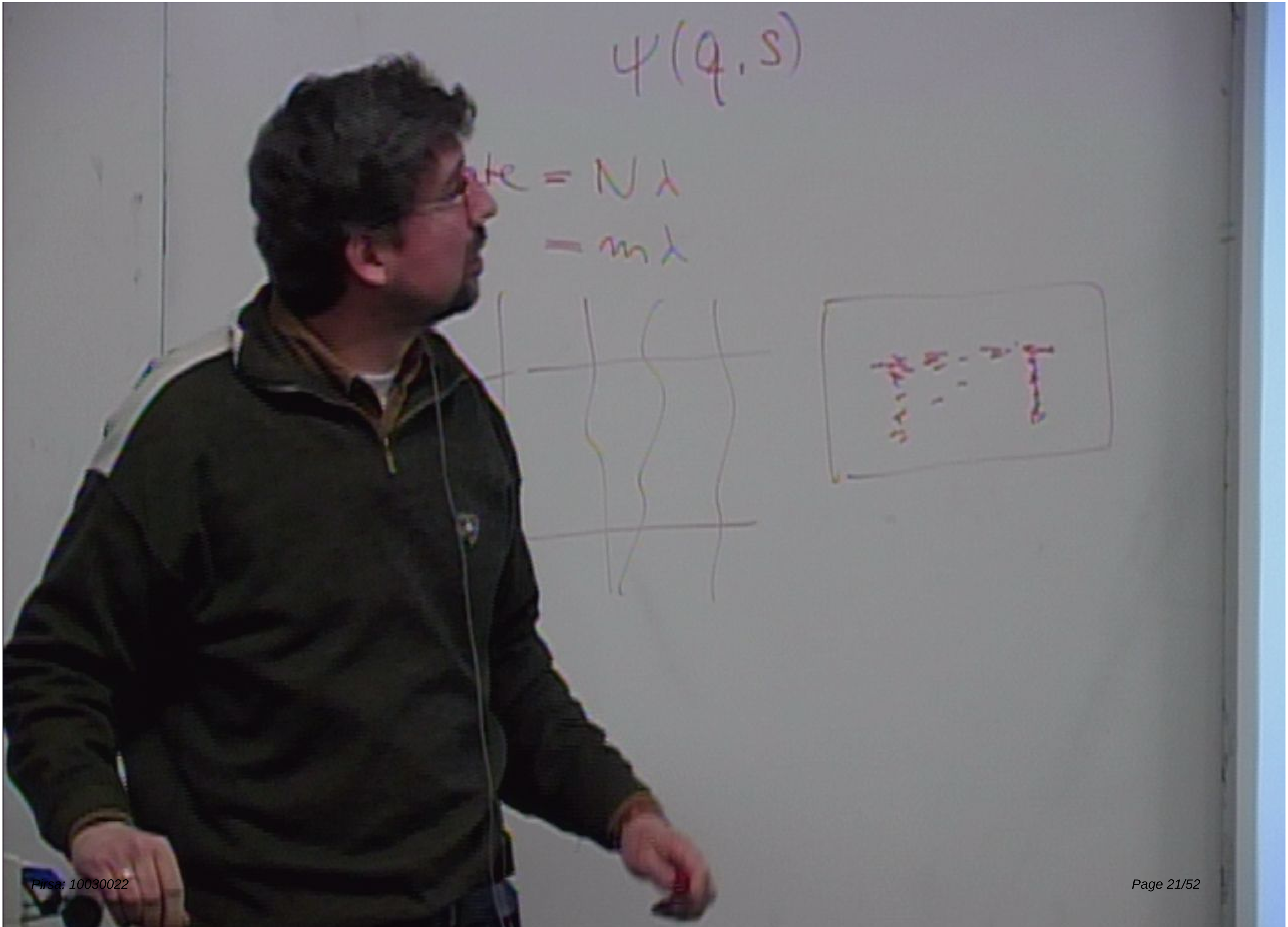
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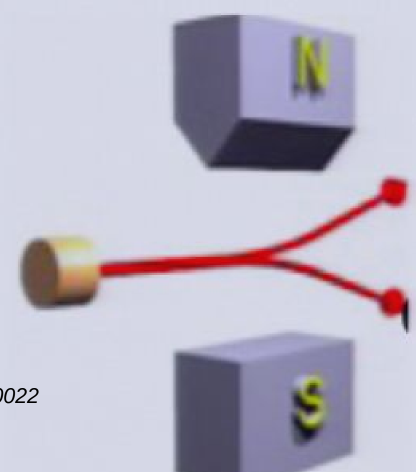

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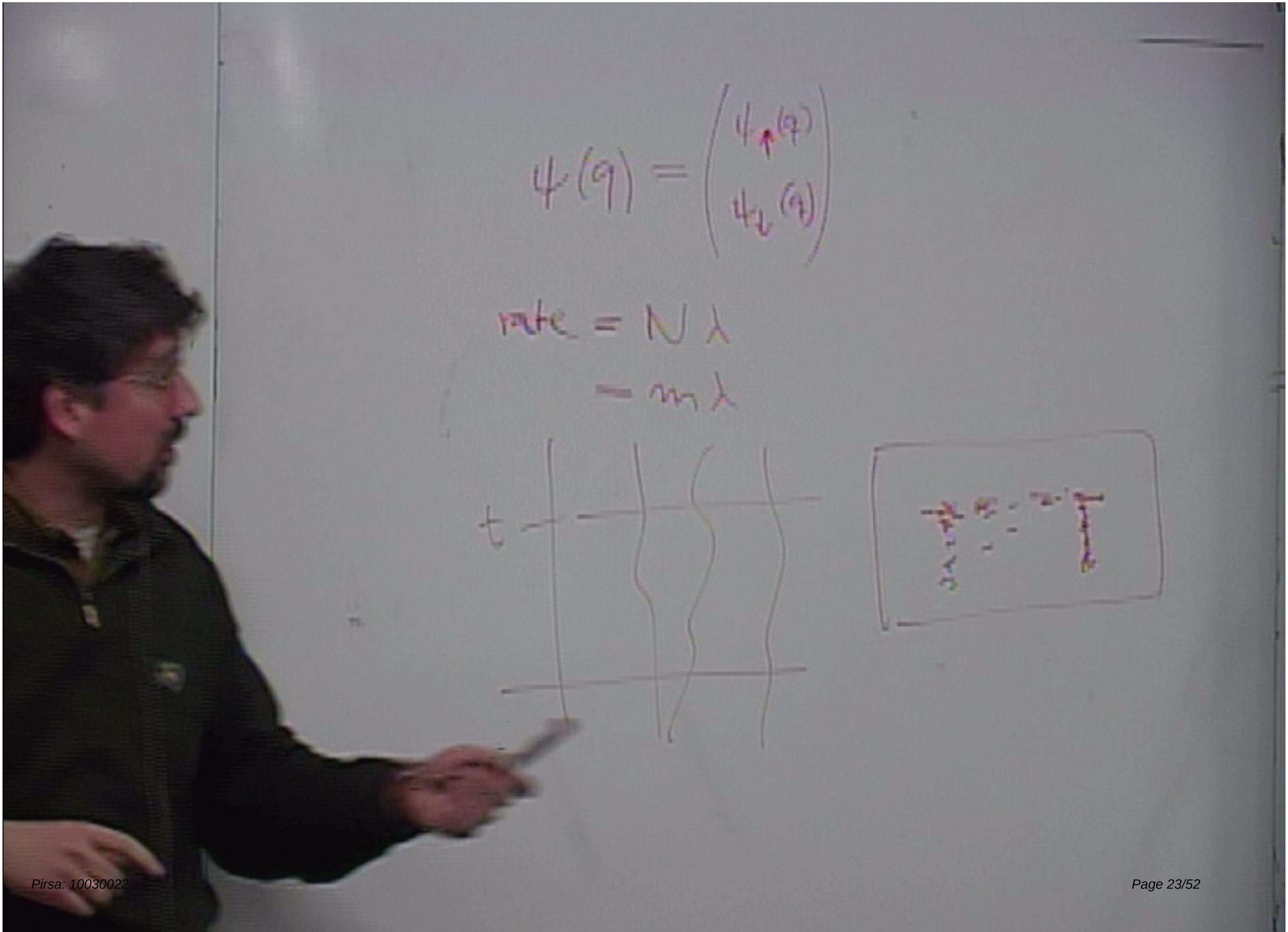
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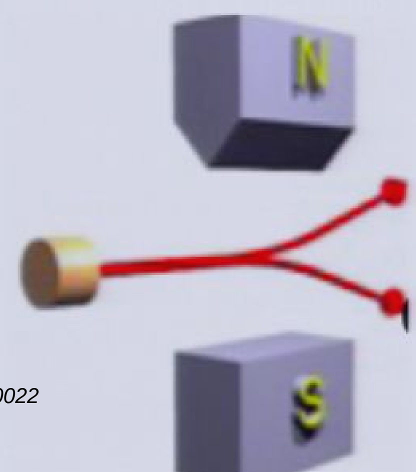

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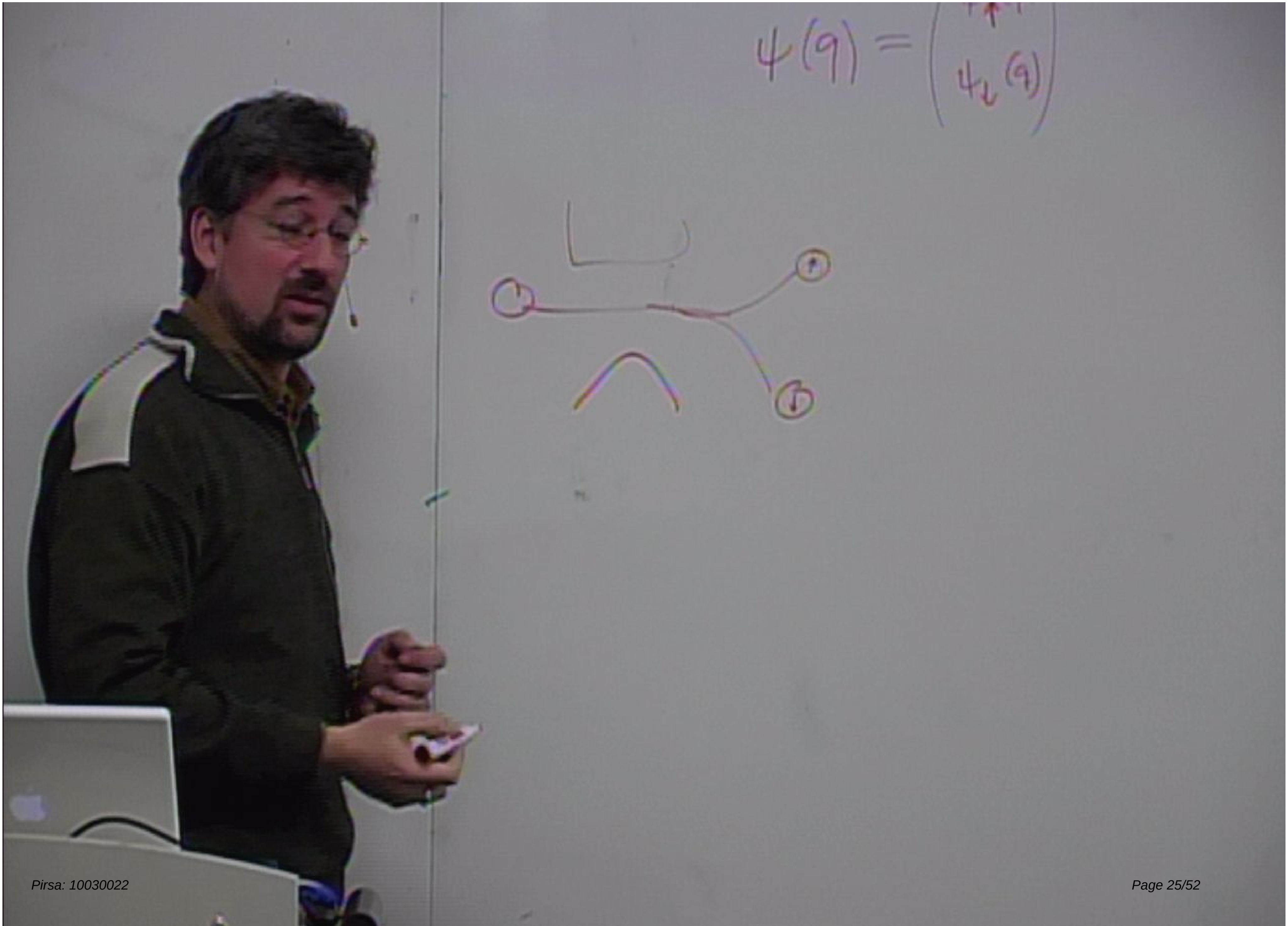
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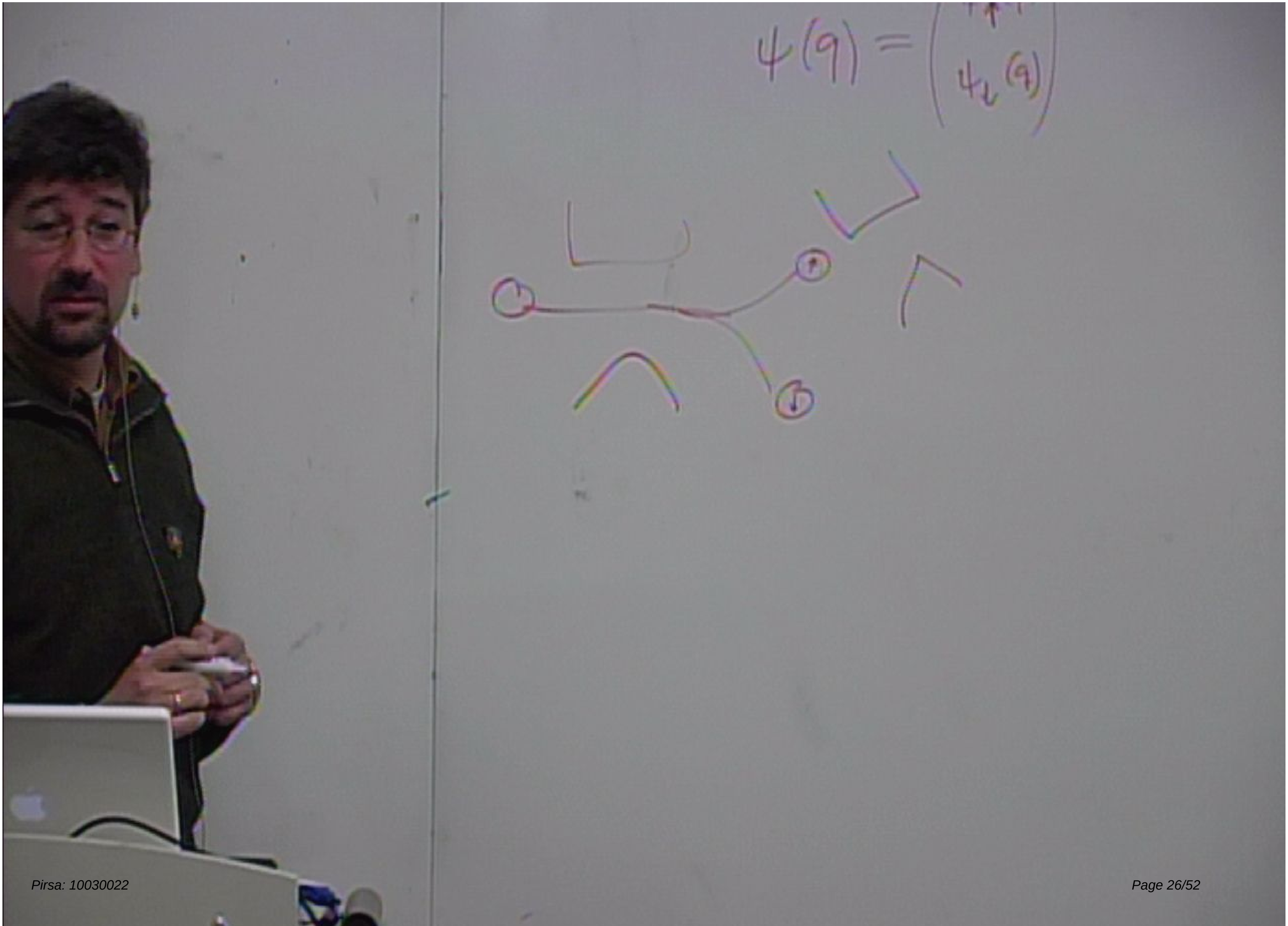
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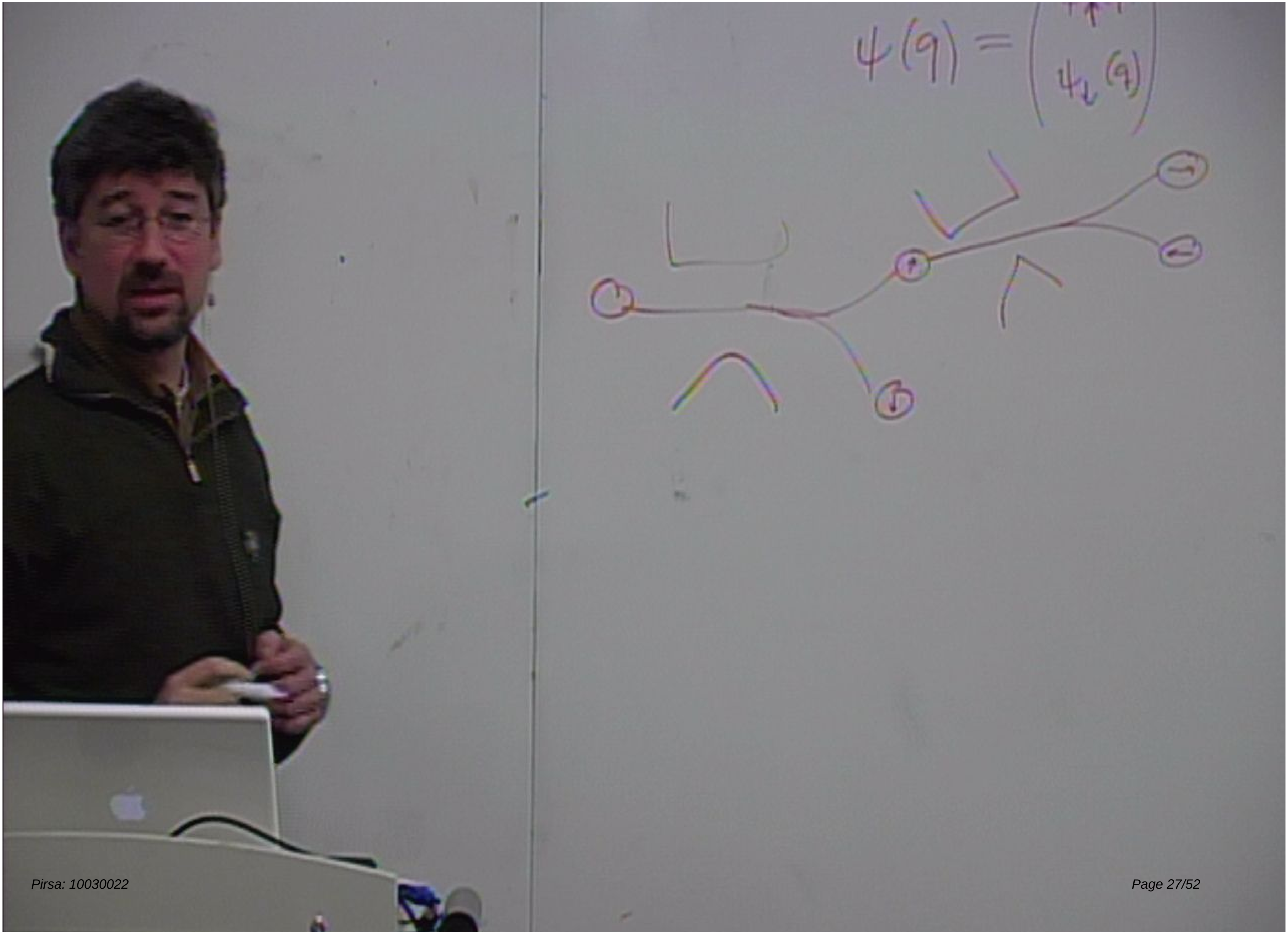
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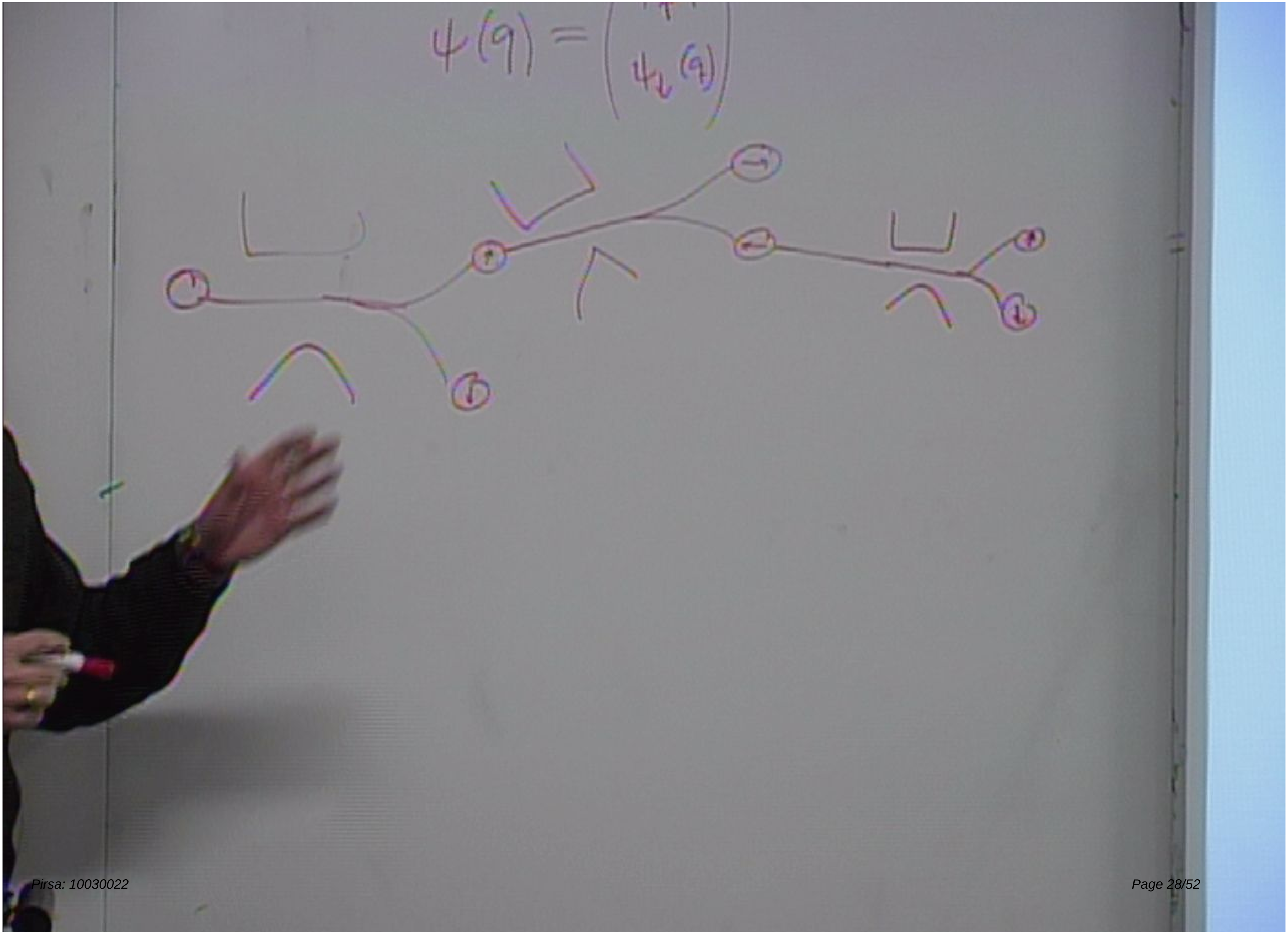
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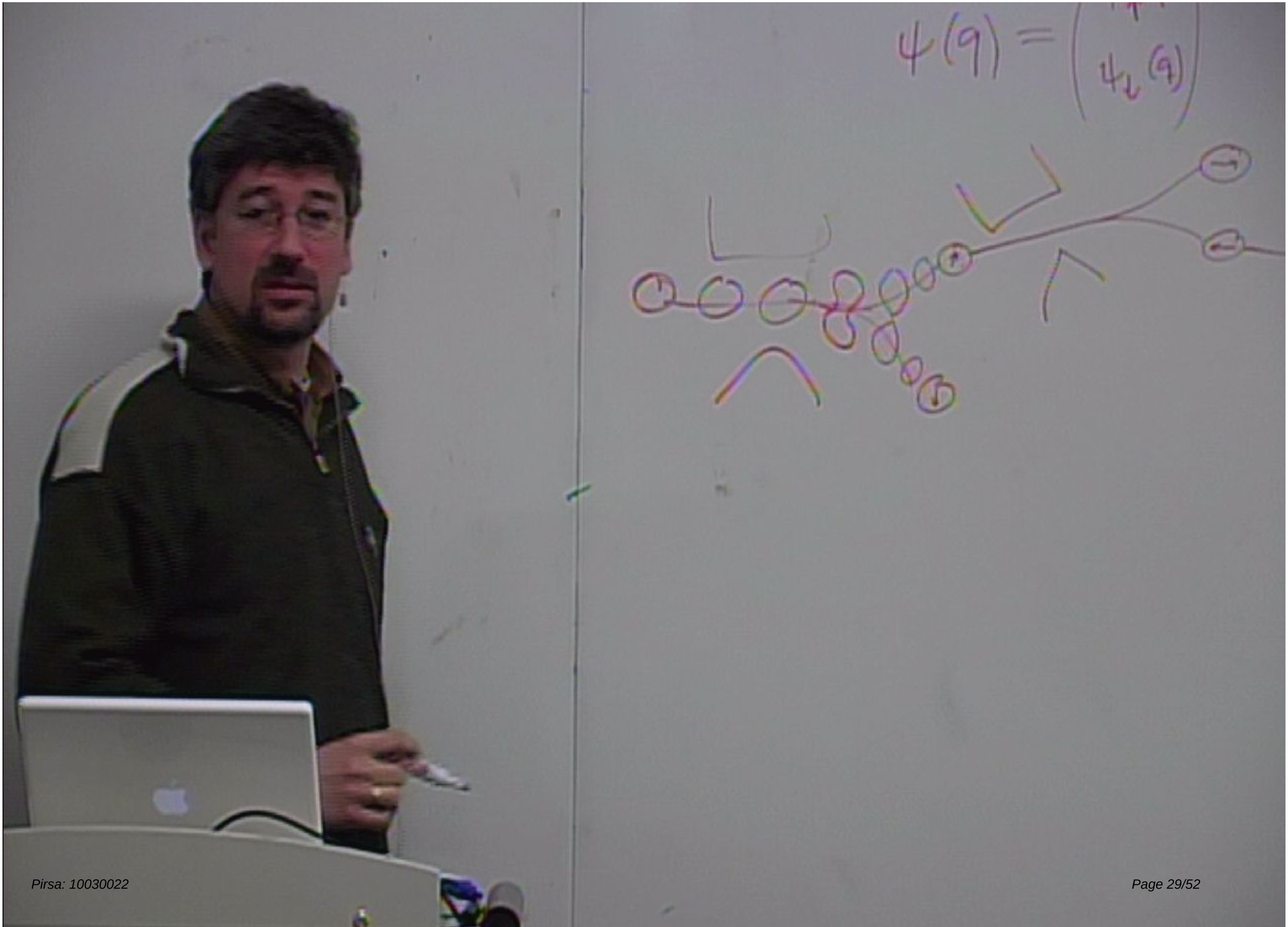
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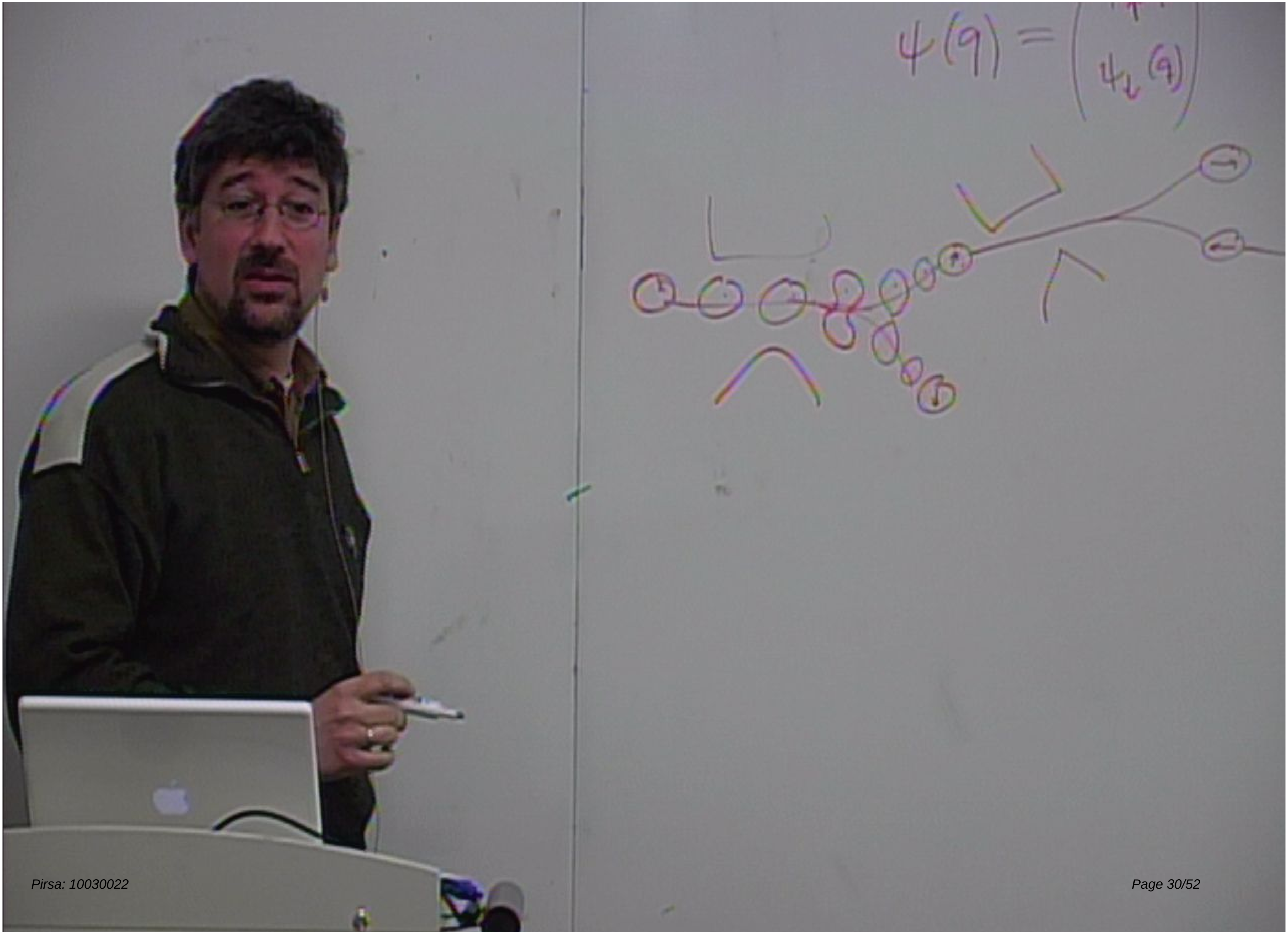
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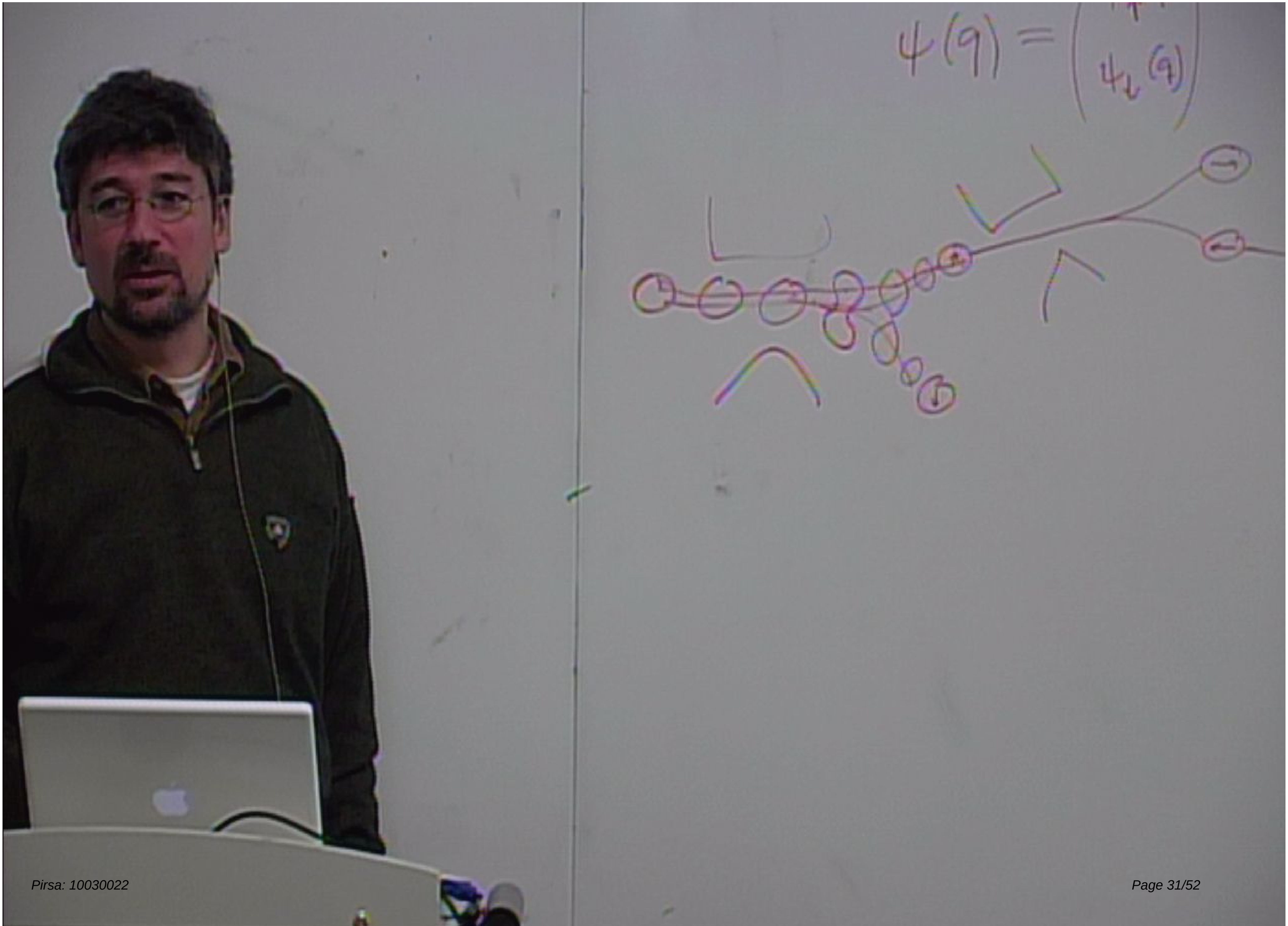
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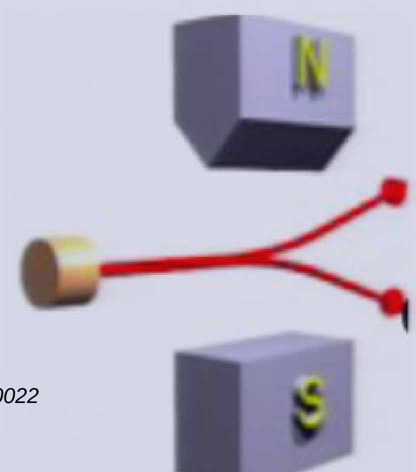

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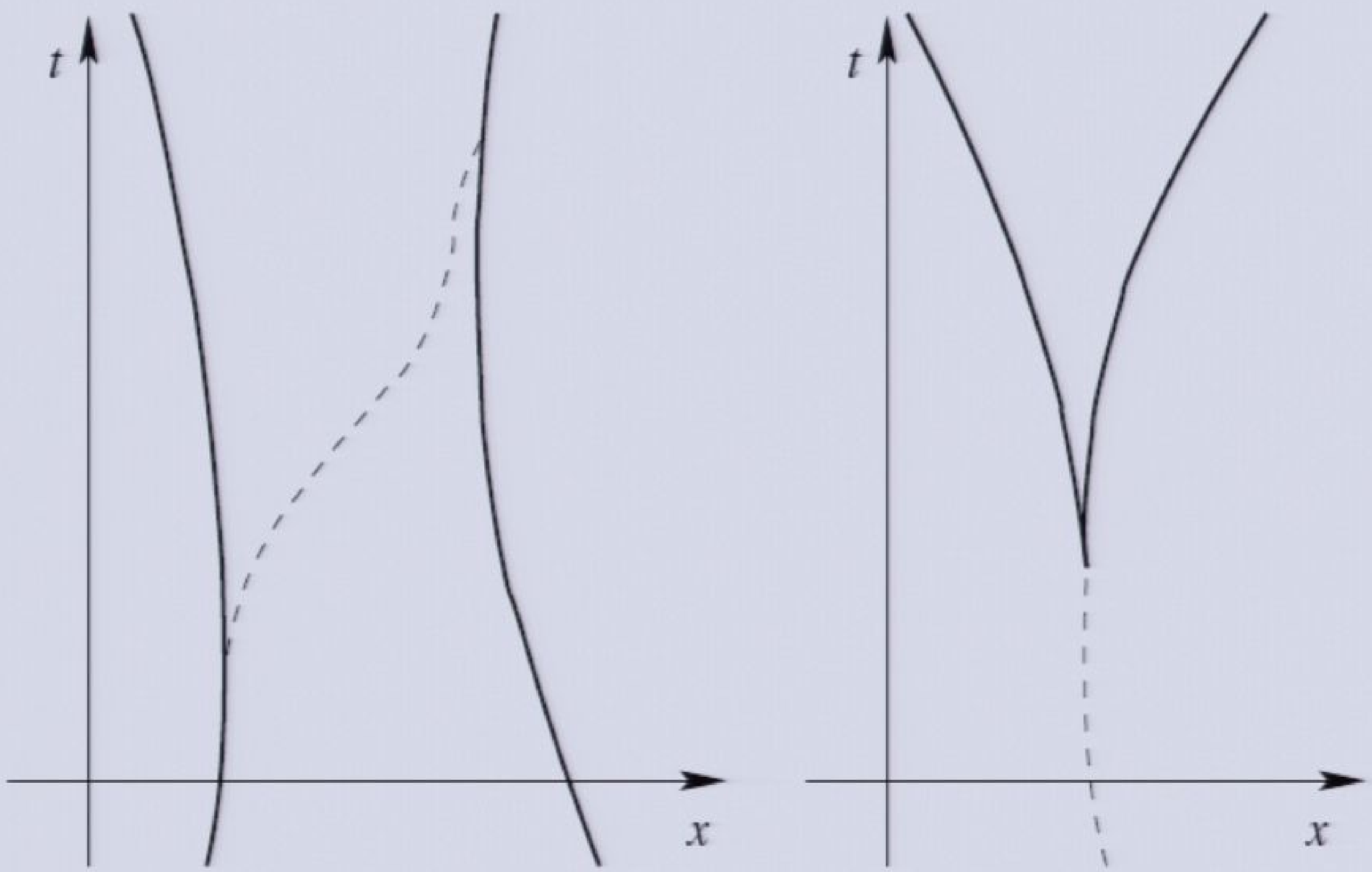
Bohmian mechanics and quantum field theory

Particle creation and annihilation:



(a) (b)

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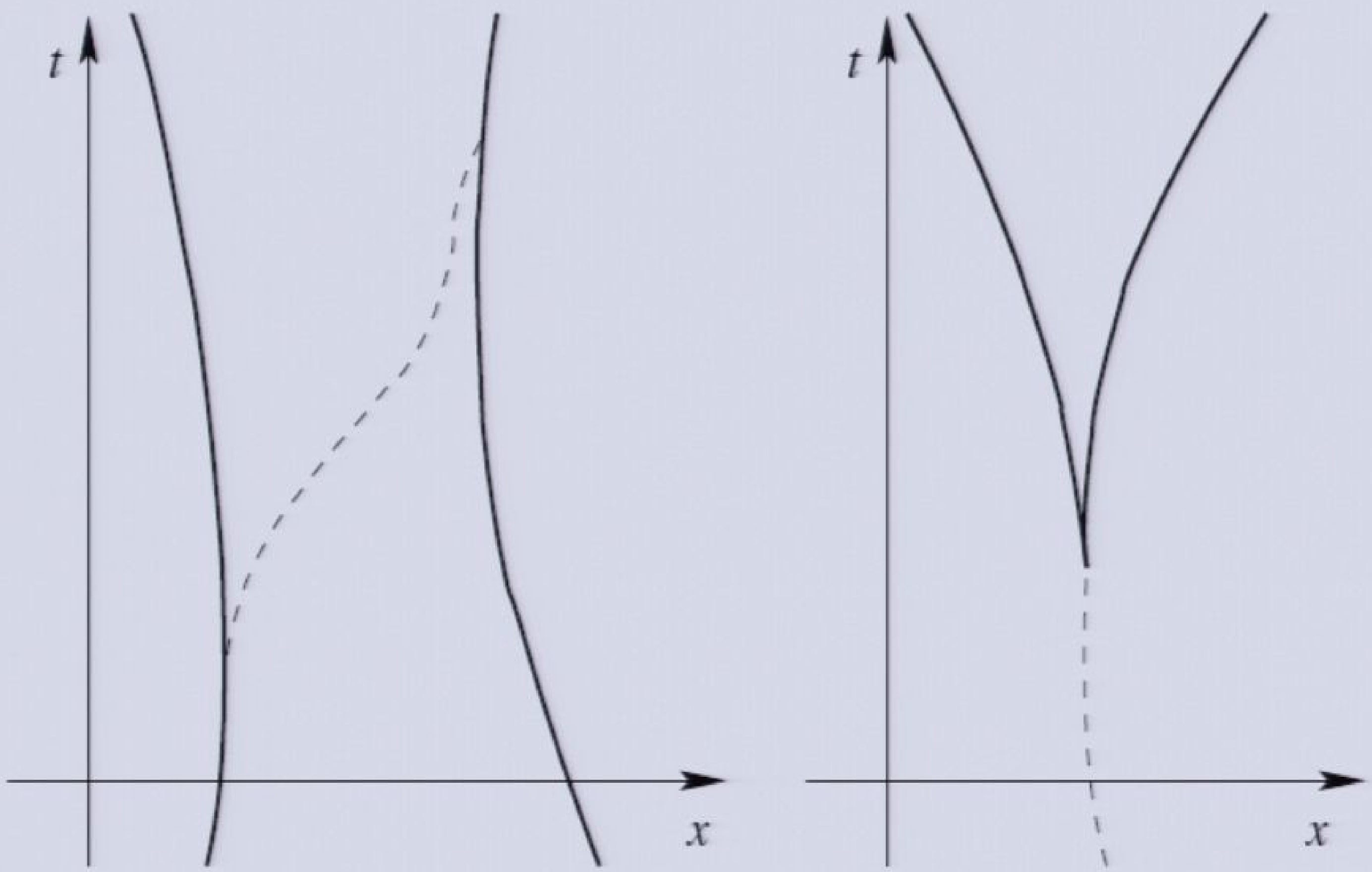

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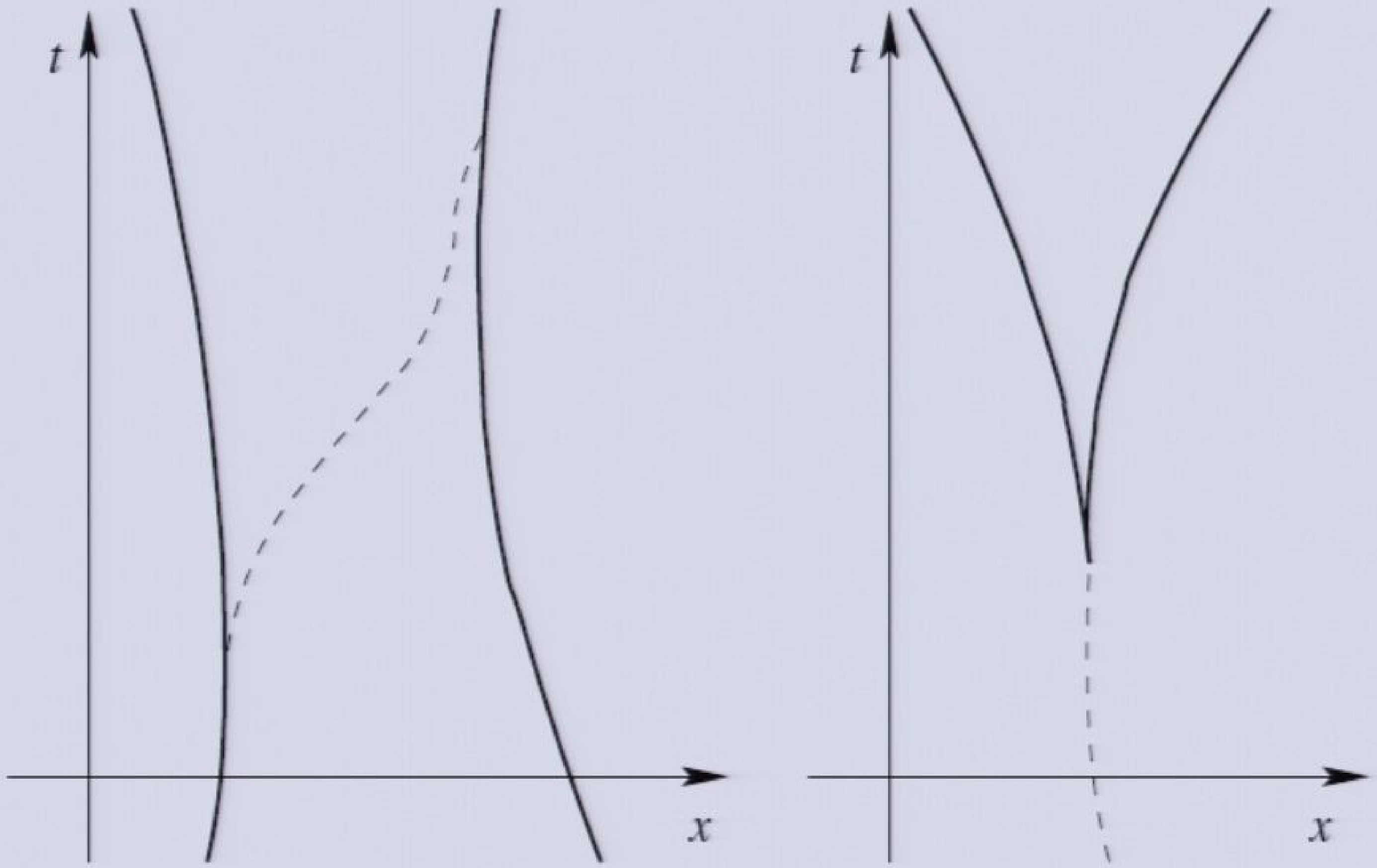
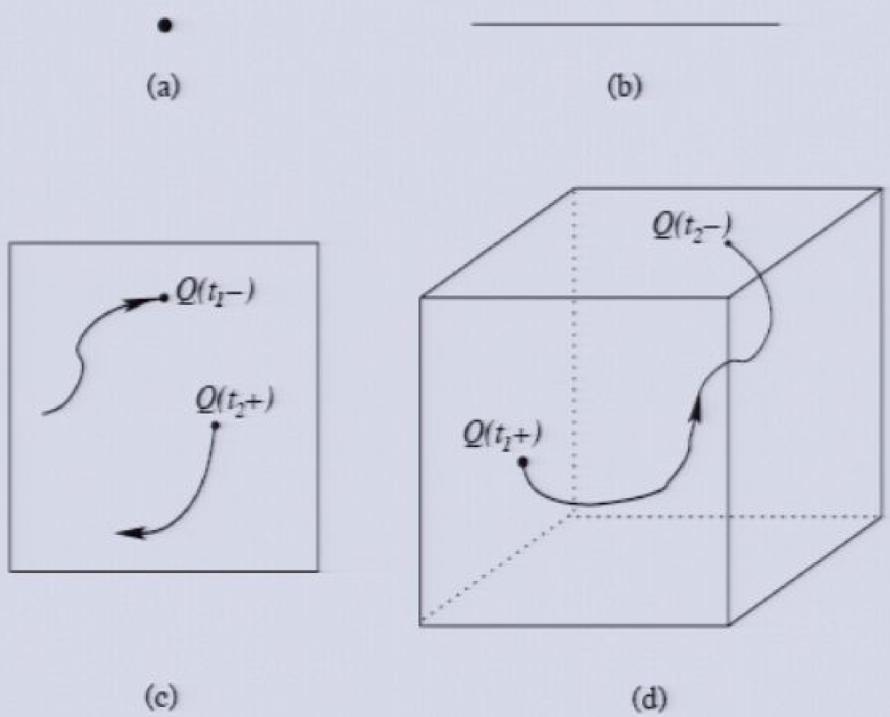


(a) (b)

Bohmian mechanics and quantum field theory (2)

$\Psi \in$ Fock space $= \bigoplus_{N=0}^{\infty} \mathscr{H}_N$,

configuration space of a variable number of (identical) particles $=$ $\{$finite subsets of $\mathbb{R}^3\} =$ $\bigcup_{N=0}^{\infty} (\mathbb{R}^{3N} \setminus \{\text{coll.}\})/\{\text{perm.}\}$



[Dürr, Goldstein, Tumulka, Zanghì 2003]

Bohmian trajectories interrupted by stochastic jumps $q' \to q$ with rate

$$\sigma^{\Psi_t}(q|q') = \frac{\left[\frac{2}{\hbar}\,\mathrm{Im}\,\Psi_t(q)\langle q|H_I|q'\rangle\Psi_t(q')\right]^+}{|\Psi_t(q')|^2}$$

$H_I =$ interaction Hamiltonian, $x^+ = \max(x,0)$

Bohmian mechanics and quantum field theory (3)

Other proposals:

- For bosonic fields: actual field configuration $\phi(\mathbf{x})$, with $\Psi = \Psi(\phi)$ [Bohm 1952]; see [Struyve 2007] for a review
- Take the Dirac sea literally: Positrons are not real particles, but there are many, many electrons of negative energy that we normally don't notice [Colin et al. 2005].
- ...

Nonlocality in Bohmian mechanics

$\frac{d\mathbf{Q}_1}{dt}$ depends on $\mathbf{Q}_2(t)$, no matter the distance $|\mathbf{Q}_1(t) - \mathbf{Q}_2(t)|$.

Nonlocality

Bell's nonlocality theorem (1964)

Certain statistics of outcomes (predicted by QM) are possible only if spacelike separated events sometimes influence each other. (No matter which interpretation of QM is right.)

These statistics were confirmed in experiment [Aspect 1982 etc.].

Bell's lemma (1964)

Non-contextual hidden variables are impossible in the sense that they cannot reproduce the statistics predicted by QM for certain experiments.

Upshot of Einstein-Podolsky-Rosen's argument (1935)

Assume that influences between spacelike separated events are impossible. Then there must be non-contextual hidden variables for all local observables.

Note: EPR + Bell's lemma $\Rightarrow$ Bell's theorem

Bohmian mechanics in relativistic space-time

Requires a preferred space-like foliation $\mathcal{F} = \{\Sigma\}$ (against the common understanding of relativity).

Example for N Dirac particles

For every $\Sigma \in \mathcal{F}$, $\quad \psi_\Sigma : \Sigma^N \to (\mathbb{C}^4)^{\otimes N}$.

$$Q(\Sigma) = (Q_1 \cap \Sigma, \ldots, Q_N \cap \Sigma) = \text{configuration on } \Sigma,$$

Equation of motion:

$$\frac{dQ_k^\mu(s)}{ds} \propto j_k^\mu(Q(\Sigma)),$$

$$j^{\mu_1 \cdots \mu_N} = \overline{\psi}[\gamma^{\mu_1} \otimes \cdots \otimes \gamma^{\mu_N}]\psi,$$

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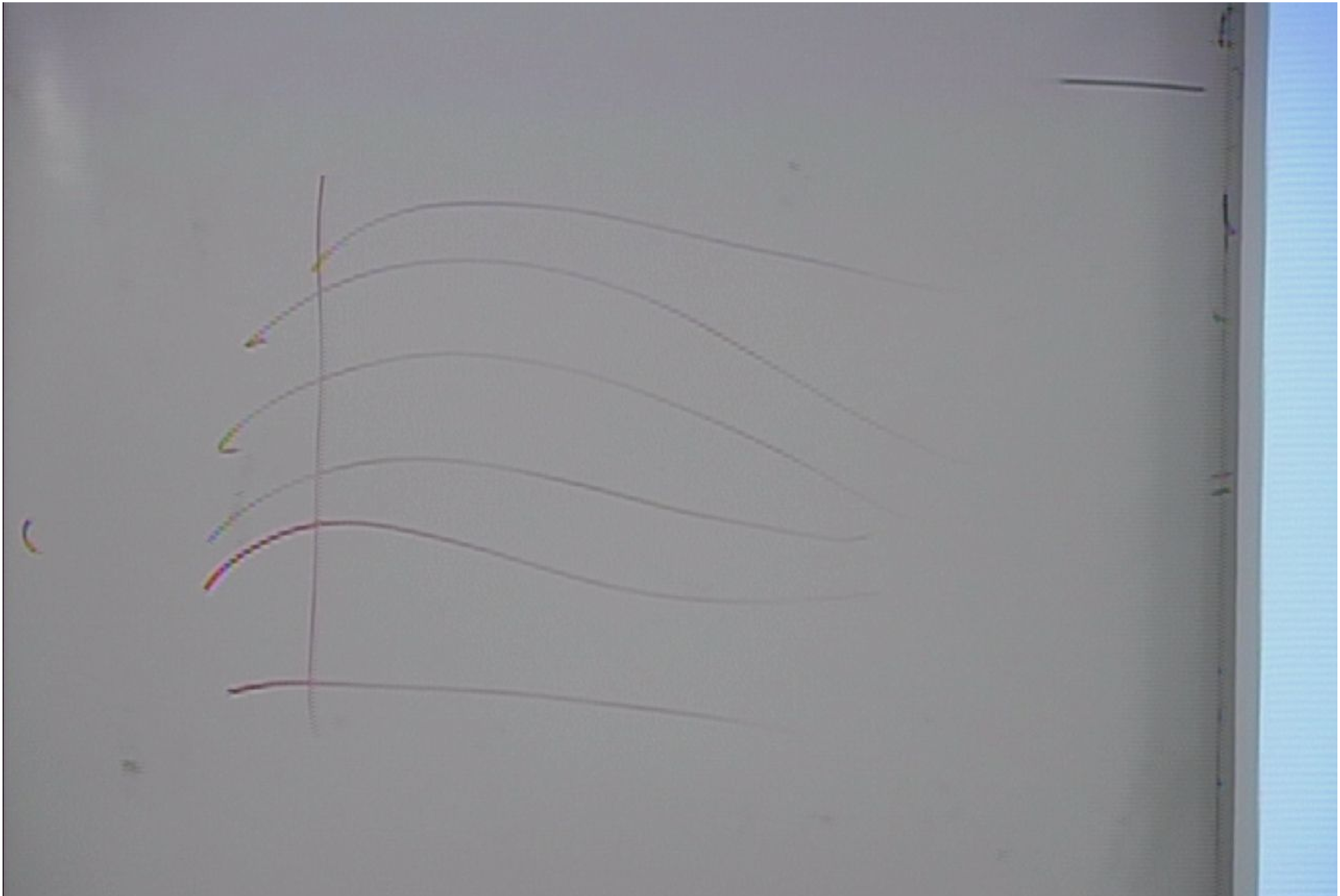
with $n_\mu(x) =$ unit normal vector to Σ at $x \in \Sigma$.



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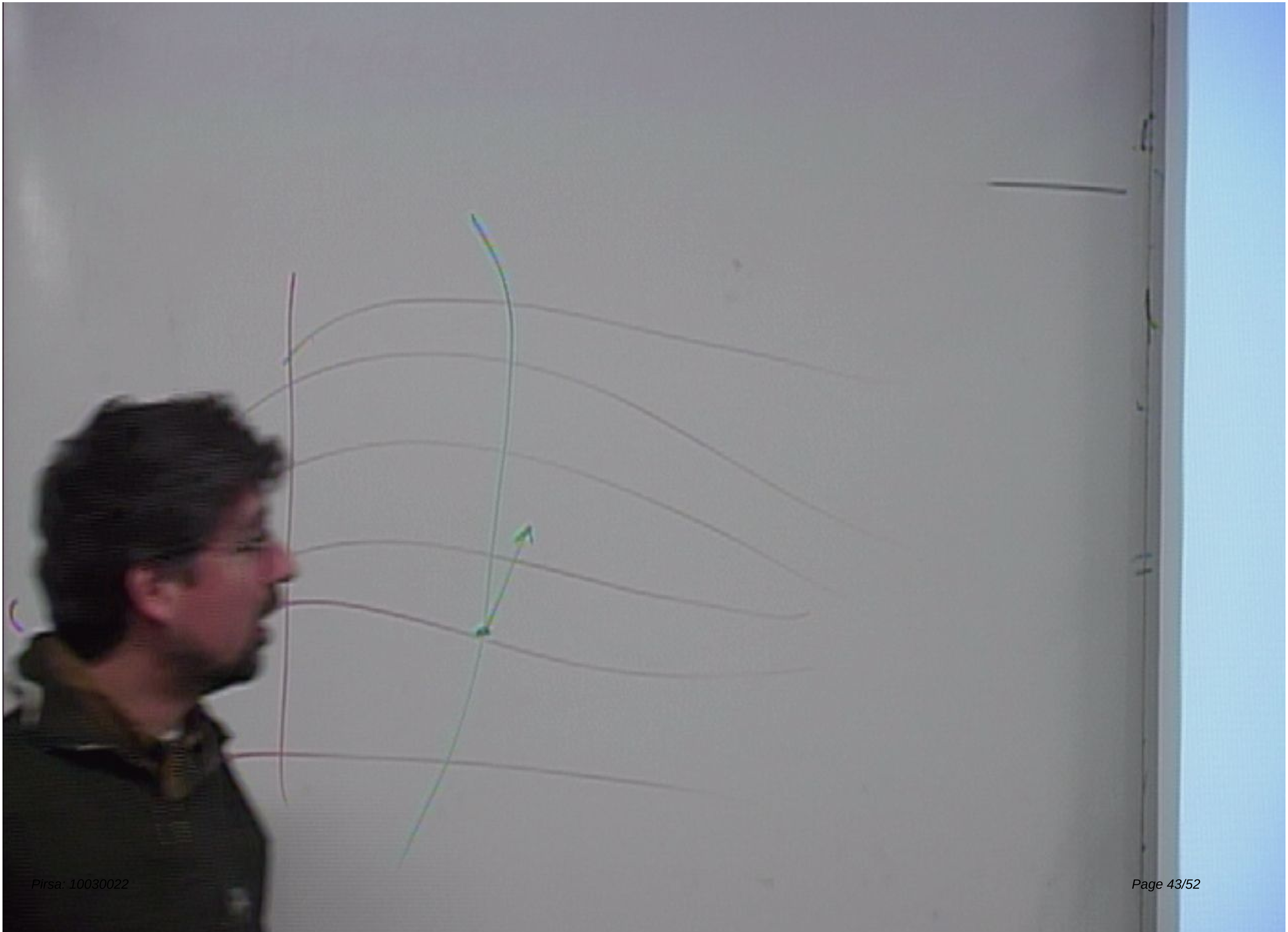

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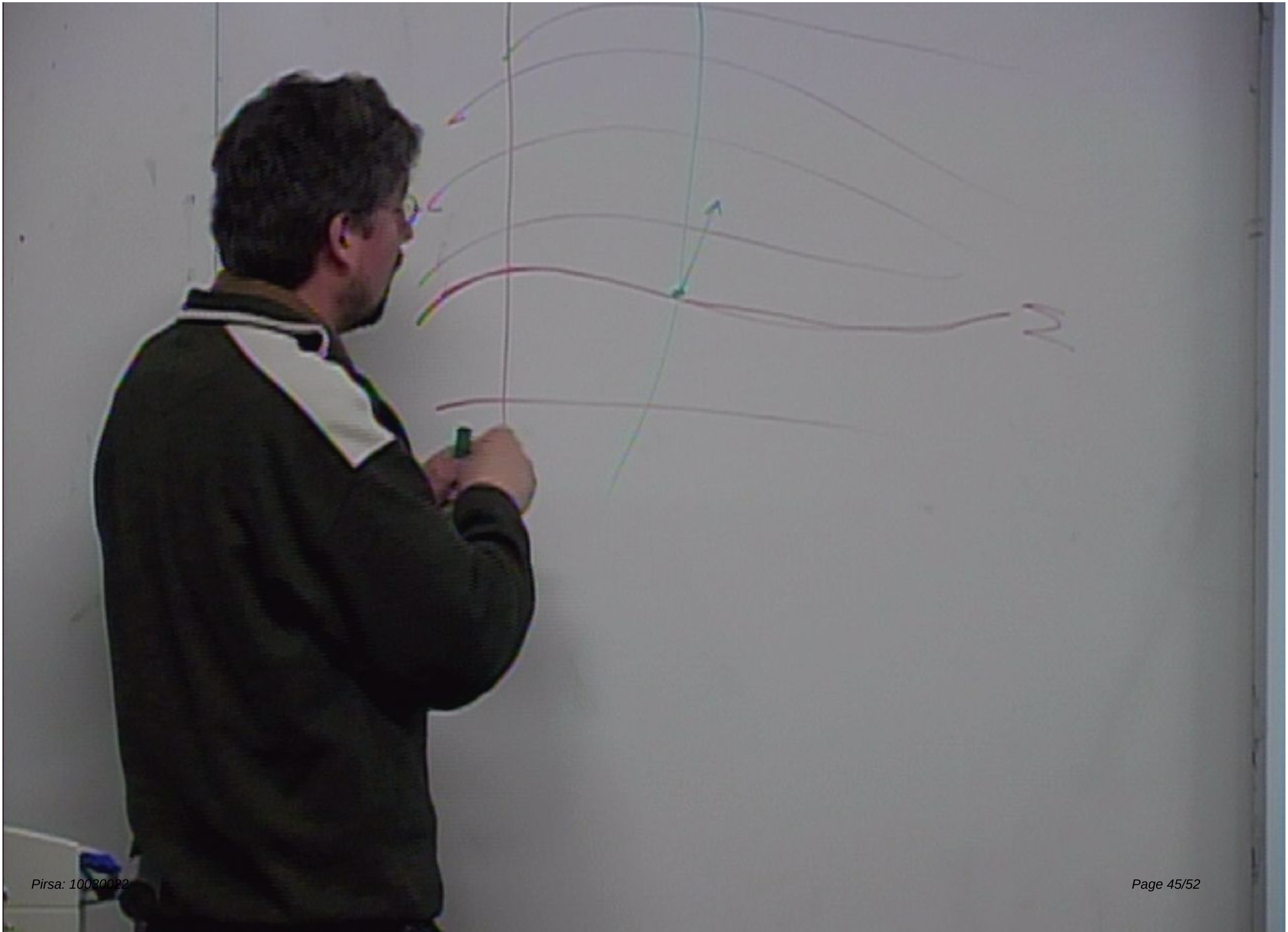

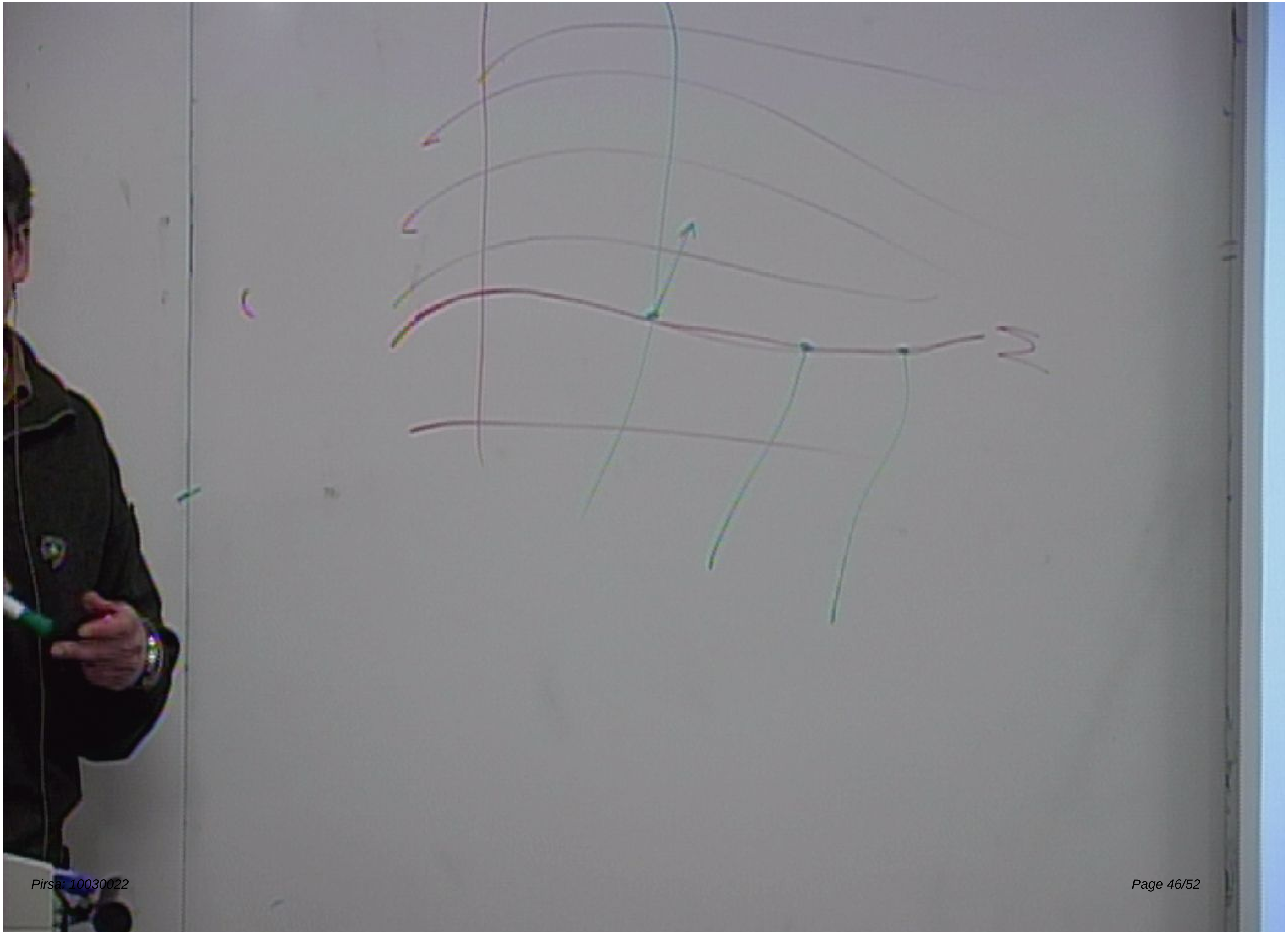

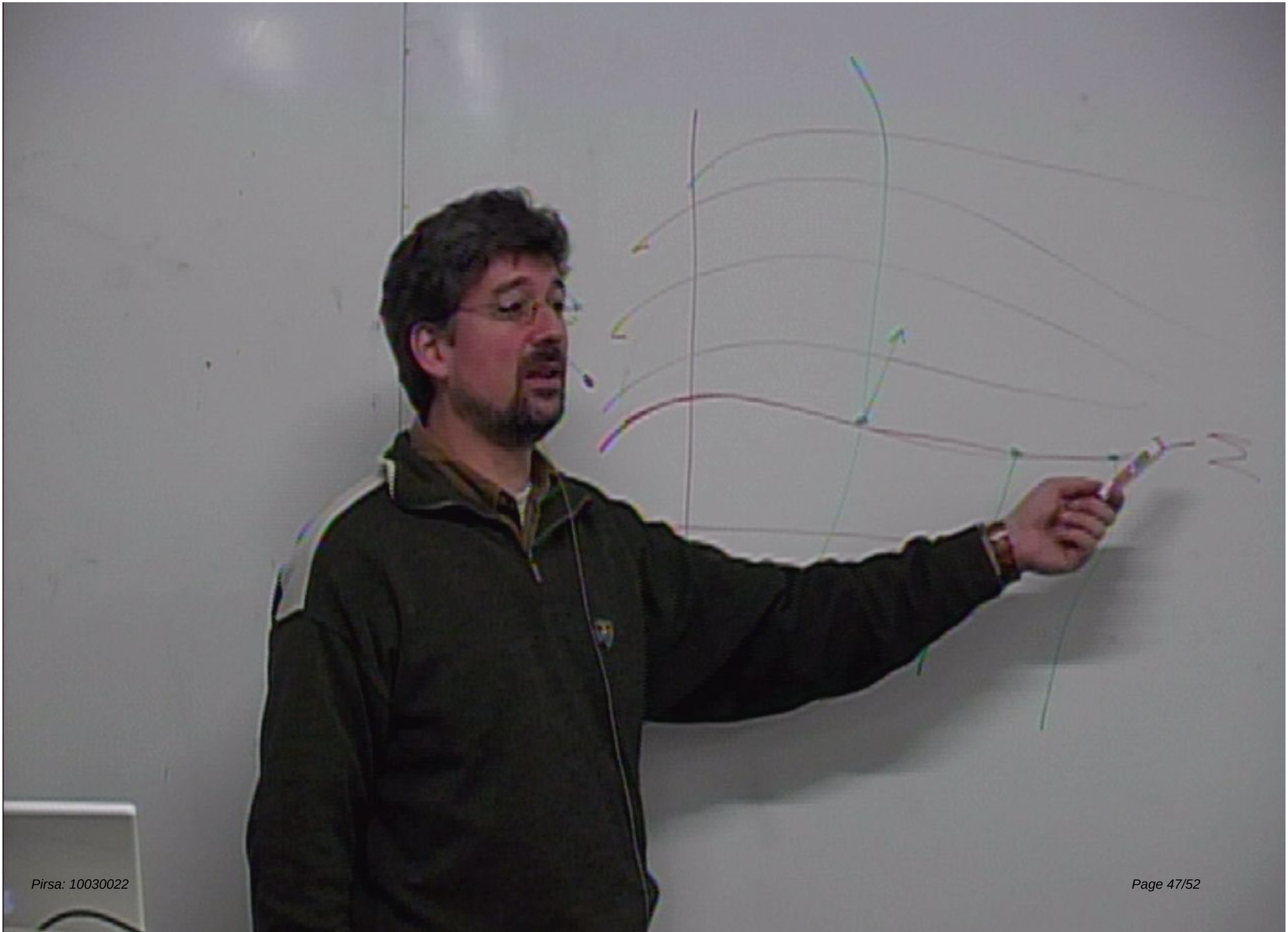

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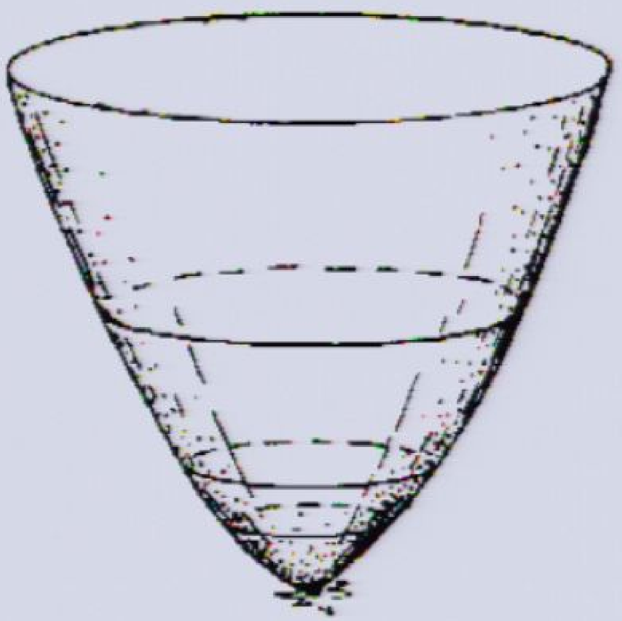
Spacelike foliation

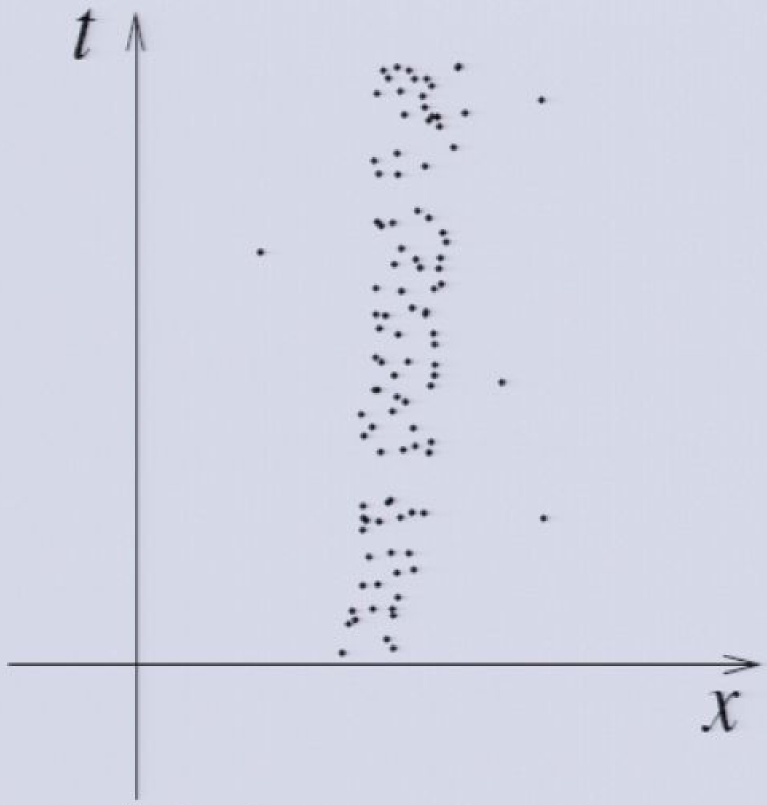


For example, let $\mathcal{F}$ be the level sets of the function T : (space-time) $\to \mathbb{R}$, $T(x) =$ timelike-distance$(x,$ big bang$)$.

Drawing: R. Penrose

Alternatively, $\mathcal{F}$ might be given by some (covariant) law involving the wave fct Ψ.

Relativistic GRW collapse theory



1986 Ghirardi-Rimini-Weber: non-relativistic theory of spontaneous wave fct collapse

1987 Bell: flash ontology (instead of world lines, discrete random world points)

2006 Tumulka: relativistic version for N non-interacting particles

$\approx 10^9$ flashes per second in a cubic centimeter of water,
$\approx 10^6$ flashes per second in a cubic centimeter of air, zero in vacuum.

The theory specifies the joint probability distribution of the flashes by a covariant law involving the (initial) wave fct and N initial flashes. No preferred foliation (or similar structure) involved.

The empirical predictions of GRW theories are very close to those of

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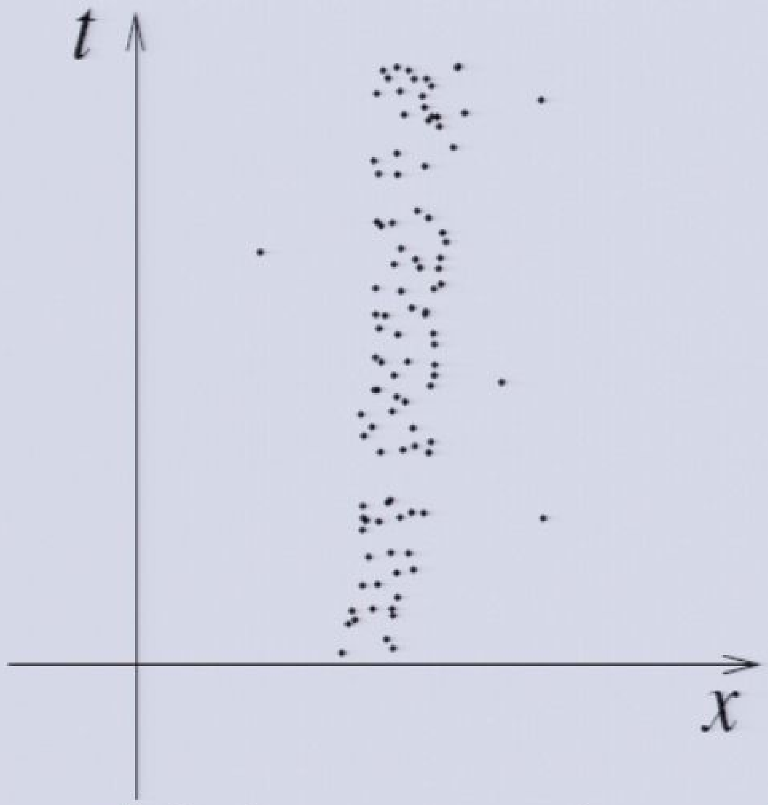

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Thank you for your attention