Title: Quantum Field Theory for Cosmology - Lecture 20

Date: Mar 25, 2010 05:00 PM

URL: http://pirsa.org/10030017

Abstract:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

QFT for Cosmology, Achim Kempf, Winter 10, Lecture 20

Note Title 3/22/2006

Recall:

- In Minkowski space, the fluctuation spectrum reads:

$$\delta\phi_\lambda = \frac{1}{\lambda} \qquad (\text{for } m=0)$$

$\rightsquigarrow$ Fluctuations of large spatial extent λ are suppressed.

- We considered a period, $[\eta_i, \eta_f]$, of exponential expansion:

$$a(t) = e^{Ht}$$

i.e.: $a(\eta) = -\frac{1}{H\eta}$

1 / 22

10qft20.jnt - Windows Journal

QFT for Cosmology, Achim Kempf, Winter 10, Lecture 20

Recall:

- In Minkowski space, the fluctuation spectrum reads:

$$\delta\phi_\lambda = \frac{1}{\lambda} \qquad (\text{for } m=0)$$

$\rightsquigarrow$ Fluctuations of large spatial extent λ are suppressed.

- We considered a period, $[\eta_i, \eta_f]$, of exponential expansion:

$$a(t) = e^{Ht}$$

i.e.: $$a(\eta) = -\frac{1}{H\eta}$$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

□ In Minkowski space, the fluctuation spectrum reads:

$$\delta\phi_\lambda = \frac{1}{\lambda} \qquad (\text{for } m=0)$$

⤳ Fluctuations of large spatial extent λ are suppressed.

□ We considered a period, $[\eta_i, \eta_f]$, of exponential expansion:

$$a(t) = e^{Ht}$$

i.e.: $a(\eta) = -\frac{1}{H\eta}$

□ We said that a comoving mode k crosses the Hubble

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

□ In Minkowski space, the fluctuation spectrum reads:

$$\delta\phi_\lambda = \frac{1}{\lambda} \qquad (\text{for } m=0)$$

⇝ Fluctuations of large spatial extent λ are suppressed.

□ We considered a period, $[\eta_i, \eta_f]$, of exponential expansion:

$$a(t) = e^{Ht}$$

i.e.: $$a(\eta) = -\frac{1}{H\eta}$$

□ We said that a comoving mode k crosses the Hubble

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

$$a(t) = e^{Ht}$$

i.e.: $$a(\eta) = -\frac{1}{H\eta}$$

We said that a comoving mode k crosses the Hubble horizon when in its mode equation the sign of $\omega_k^2(\eta)$ changes:

$$v_k''(\eta) + \left(k^2 - \frac{2}{\eta^2}\right) v_k(\eta) = 0$$ (neglecting the mass: $m \ll H$)

This happens when $\eta = \eta_{hor}(k) = -\frac{\sqrt{2}}{k}$, if this time is in $[\eta_i, \eta_f]$.

The case of very small modes: $\eta_{hor}(k) \gg \eta_f$

* In K.G. eqn. k^2 here dominates over $\frac{2}{\ldots}$

10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width

$$a(t) = e^{Ht}$$

i.e.: $$a(\eta) = -\frac{1}{H\eta}$$

- We said that a comoving mode k crosses the Hubble horizon when in its mode equation the sign of $\omega_k^2(\eta)$ changes:

$$v_k''(\eta) + \left(k^2 - \frac{2}{\eta^2}\right) v_k(\eta) = 0$$

(neglecting the mass: $m \ll H$)

This happens when $\eta = \eta_{hor}(k) = -\frac{\sqrt{2}}{k}$, if this time is in $[\eta_i, \eta_f]$.

- <u>The case of very small modes:</u> $\eta_{hor}(k) \gg \eta_f$

* In K.G. eqn.: k^2 keeps dominating over $\frac{2}{}$

1 / 22

10qft20.jnt - Windows Journal

$$a(t) = e^{Ht}$$

i.e.: $$a(\eta) = -\frac{1}{H\eta}$$

▫ We said that a comoving mode k crosses the Hubble horizon when in its mode equation the sign of $\omega_k^2(\eta)$ changes:

$$v_k''(\eta) + \left(k^2 - \frac{2}{\eta^2}\right) v_k(\eta) = 0 \qquad \text{(neglecting the mass: } m \ll H\text{)}$$

This happens when $\eta = \eta_{hor}(k) = -\frac{\sqrt{2}}{k}$, if this time is in $[\eta_i, \eta_f]$.

▫ The case of very small modes: $\eta_{hor}(k) \gg \eta_f$

* In K.G. eqn., k^2 keeps dominating over $-\frac{2}{\eta^2}$.

* Mode functions behave as in Minkowski space

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

- We said that a comoving mode k crosses the Hubble horizon when in its mode equation the sign of $\omega_k^2(\eta)$ changes:

$$v_k''(\eta) + \left(k^2 - \frac{2}{\eta^2}\right) v_k(\eta) = 0$$

(neglecting the mass: $m \ll H$)

This happens when $\eta = \eta_{hor}(k) = -\frac{\sqrt{2}}{k}$, if this time is in $[\eta_i, \eta_f]$.

- The case of very small modes: $\eta_{hor}(k) \gg \eta_f$

  * In K.G. eqn., k^2 keeps dominating over $-\frac{2}{\eta^2}$.
  * Mode functions behave as in Minkowski space.
  * The fluctuation spectrum was found to stay:

$\delta\phi_k = \frac{1}{\ldots}$ ... proper wave length ...

10qft20.jnt - Windows Journal

This happens when $\eta = \eta_{hor}(k) = -\frac{\sqrt{2}}{k}$, if this time is in $[\eta_i, \eta_f]$.

The case of very small modes: $\eta_{hor}(k) \gg \eta_f$

* In K.G. eqn., k^2 keeps dominating over $-\frac{2}{\eta^2}$.
* Mode functions behave as in Minkowski space.
* The fluctuation spectrum was found to stay:

$$\delta\phi_\lambda = \frac{1}{\lambda}$$ ← proper wave length.

Note: $\delta\phi_L$ for fixed L (comoving wave length) decreases over time, because $\lambda = a L$.

The case of medium size modes: $\eta_{hor}(k) \in [\eta_i, \eta_f]$

* In K.G. eqn., from horizon crossing onwards, $-\frac{2}{\eta^2}$ dominates over k^2. This soon means:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

The case of very small modes: $\eta_{hor}(k) \gg \eta_f$

* In K.G. eqn., k^2 keeps dominating over $-\frac{2}{\eta^2}$.
* Mode functions behave as in Minkowski space.
* The fluctuation spectrum was found to stay:

$$\delta\phi_\lambda = \frac{1}{\lambda} \leftarrow \text{proper wave length.}$$

Note: $\delta\phi_L$ for fixed L (comoving wave length) decreases over time, because $\lambda = a\,L$.

The case of medium size modes: $\eta_{hor}(k) \in [\eta_i, \eta_f]$

* In K.G. eqn., from horizon crossing onwards, $-\frac{2}{\eta^2}$ dominates over k^2. This soon means:

$\Rightarrow$ Mode functions then no longer depend on k!

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

Note: $\delta\varphi_L$ for fixed L decreases over time, because $\lambda = a L$.

□ <u>The case of medium size modes:</u> $\eta_{hor}(k) \in [\eta_i, \eta_f]$

* In K.G. eqn., from horizon crossing onwards, $-\frac{2}{\eta^2}$ dominates over k^2. This soon means:

$\Rightarrow$ Mode functions then no longer depend on k!

$\Rightarrow$ The fluctuation spectrum was found after Hubble horizon crossing to become constant:

$$\boxed{\delta\phi_k(\eta_f) \approx H \cdot 2^{3/2}\Gamma(3/2)/\pi}$$

<u>Conclusion:</u>

□ Normally, the size of a mode's fluctuations $\delta\phi_\lambda(\eta)$ decreases as its proper wavelength $\lambda(\eta)$ increases due to the expansion.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

The case of medium size modes: $\eta_{hor}(k) \in [\eta_i, \eta_f]$

* In K.G. eqn., from horizon crossing onwards, $-\frac{2}{\eta^2}$ dominates over k^2. This soon means:

$\Rightarrow$ Mode functions then no longer depend on k!

$\Rightarrow$ The fluctuation spectrum was found after Hubble horizon crossing to become constant:

$$\boxed{\delta\phi_k(\eta_f) \approx H \cdot 2^{3/2}\Gamma(3/2)/\pi}$$

Conclusion:

- Normally, the size of a mode's fluctuations $\delta\phi_\lambda(\eta)$ decreases as its proper wavelength $\lambda(\eta)$ increases due to the expansion.
- But, after modes cross the horizon, $\frac{1}{H}$, they keep their fluctuation amplitudes even as their proper wavelength $\lambda(t)$ increa...

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$\boxed{\delta\phi_\lambda(\eta_f) \approx H \cdot 2^{3/2}\Gamma(3/2)/\pi}$$

Conclusion:

- Normally, the size of a mode's fluctuations $\delta\phi_\lambda(\eta)$ decreases as its proper wavelength $\lambda(\eta)$ increases due to the expansion.
- But, after modes cross the horizon, $\frac{1}{H}$, they keep their fluctuation amplitudes even as their proper wavelength $\lambda(t)$ increases!

Conclusion:

- Assume that the expansion $a(t) = e^{Ht}$ is very fast.
- Thus, the Hubble horizon $\frac{1}{H}$ is very small.
- Thus, as modes cross the Hubble horizon, their proper wavelength is very small $\lambda \sim \frac{1}{H}$.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Conclusion:

- Normally, the size of a mode's fluctuations $\delta\phi_\lambda(\eta)$ decreases as its proper wavelength $\lambda(\eta)$ increases due to the expansion.
- But, after modes cross the horizon, $\frac{1}{H}$, they keep their fluctuation amplitudes even as their proper wavelength $\lambda(t)$ increases!

Conclusion:

- Assume that the expansion $a(t) = e^{Ht}$ is very fast.
- Thus, the Hubble horizon $\frac{1}{H}$ is very small.
- Thus, as modes cross the Hubble horizon, their proper wavelength is very small $\lambda \sim \frac{1}{H}$.
- Thus, modes have large fluctuations $\delta\phi_\lambda(\eta) \sim H$ as they cross the horizon and they keep these large

10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width

Conclusion:

- Assume that the expansion $a(t) = e^{Ht}$ is very fast.
- Thus, the Hubble horizon $\frac{1}{H}$ is very small.
- Thus, as modes cross the Hubble horizon, their proper wavelength is very small $\lambda \sim \frac{1}{H}$.
- Thus, modes have large fluctuations $\delta\phi_\lambda(\eta) \sim H$ as they cross the horizon and they keep these large fluctuations as their proper wavelength further increases.
- If $[\eta_i, \eta_f]$ is long enough, modes with large fluctuations can reach even cosmological proper wavelengths — and can therefore cause the cosmic structure!

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

- Thus, the Hubble horizon $\frac{1}{H}$ is very small.

- Thus, as modes cross the Hubble horizon, their proper wavelength is very small $\lambda \sim \frac{1}{H}$.

- Thus, modes have large fluctuations $\delta\phi_\lambda(\eta) \sim H$ as they cross the horizon and they keep these large fluctuations as their proper wavelength further increases.

- If $[\eta_i, \eta_f]$ is long enough, modes with large fluctuations can reach even cosmological proper wavelengths — and can therefore cause the cosmic structure!

Preliminary estimates:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

proper wavelength is very small $\lambda \sim \frac{1}{H}$.

Thus, modes have large fluctuations $\delta\phi_\lambda(\eta) \sim H$ as they cross the horizon and they keep these large fluctuations as their proper wavelength further increases.

If $[\eta_i, \eta_f]$ is long enough, modes with large fluctuations can reach even cosmological proper wavelengths — and can therefore cause the cosmic structure!

Preliminary estimates:

* If this is the seeding mechanism for cosmic structure formation, then:

fluctuations as their proper wavelength further increases.

If $[\eta_i, \eta_f]$ is long enough, modes with large fluctuations can reach even cosmological proper wavelengths – and can therefore cause the cosmic structure!

Preliminary estimates:

* If this is the seeding mechanism for cosmic structure formation, then:

* H determines the amplitude of the later-observed fluctuations and must be of the right size to conform with observations. Measurements of the CMB indicate:

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

If $[\eta_i, \eta_f]$ is long enough, modes with large fluctuations can reach even cosmological proper wavelengths — and can therefore cause the cosmic structure!

Preliminary estimates:

* If this is the seeding mechanism for cosmic structure formation, then:

* H determines the amplitude of the later-observed fluctuations and must be of the right size to conform with observations. Measurements of the CMB indicate:

$$H^{-1} \approx 10^5 \, l_{planck} \approx 10^{-29} m$$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

□ If $[\eta_i, \eta_f]$ is long enough, modes with large fluctuations can reach even cosmological proper wavelengths – and can therefore cause the cosmic structure!

Preliminary estimates:

* If this is the seeding mechanism for cosmic structure formation, then:

* H determines the amplitude of the late-observed fluctuations and must be of the right size to conform with observations. Measurements of the CMB indicate:

$$H^{-1} \approx 10^5 \, l_{planck} \approx 10^{-29} \, m$$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

wavelengths — and can therefore cause the cosmic structure!

Preliminary estimates:

* If this is the seeding mechanism for cosmic structure formation, then:

* H determines the amplitude of the late-observed fluctuations and must be of the right size to conform with observations. Measurements of the CMB indicate:

$$H^{-1} \approx 10^{5}\, l_{planck} \approx 10^{-29}\, m$$

* The interval $[\eta_i, \eta_f]$ must be long enough so that such small modes have time to expand to

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Preliminary estimates:

* If this is the seeding mechanism for cosmic structure formation, then:

* H determines the amplitude of the later-observed fluctuations and must be of the right size to conform with observations. Measurements of the CMB indicate:

$$H^{-1} \approx 10^5 \, l_{planck} \approx 10^{-29} m$$

* The interval $[\eta_i, \eta_f]$ must be long enough so that such small modes have time to expand to cosmological size. For example this time period would do:

$$[10^{-34} s, 10^{-32} s]$$

10qft20.jnt - Windows Journal

* H determines the amplitude of the later-observed fluctuations and must be of the right size to conform with observations. Measurements of the CMB indicate:

$$H^{-1} \approx 10^5 \, l_{planck} \approx 10^{-29} \, m$$

* The interval $[\eta_i, \eta_f]$ must be long enough so that such small modes have time to expand to cosmological size. For example this time period would do:

$$[10^{-34} s, \, 10^{-32} s]$$

Realistic cosmic inflation

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

* H determines the amplitude of the later-observed fluctuations and must be of the right size to conform with observations. Measurements of the CMB indicate:

$$H^{-1} \approx 10^{5}\, l_{planck} \approx 10^{-29}\, m$$

* The interval $[\eta_i, \eta_f]$ must be long enough so that such small modes have time to expand to cosmological size. For example this time period would do:

$$[10^{-34}s,\ 10^{-32}s]$$

Realistic cosmic inflation

1. How can a period of near exponential expansion be ...

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

* The interval $[\eta_i, \eta_f]$ must be long enough so that such small modes have time to expand to cosmological size. For example this time period would do:

$$[10^{-34}s,\ 10^{-32}s]$$

Realistic cosmic inflation

1. How can a period of near-exponential expansion be caused?

- **Recall the full action:**

$$S = -\frac{1}{16\pi G}\int\Big[2\underset{\uparrow\ \text{cosm. constant}}{\Lambda} + R(x) + \overbrace{\cancel{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \ldots}}^{\text{We neglect such terms by Occam's razor: there is no evidence for their existence as yet.}}\Big]\sqrt{|g|}\, d^4x$$

$$+ \int\Big[\tfrac{1}{2} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\Big]\sqrt{|g|}\, d^4x$$

Note: ϕ is now called the

10qft20.jnt - Windows Journal

observations. Measurements of the CMB indicate:

$$H^{-1} \approx 10^{5}\, l_{planck} \approx 10^{-29}\, m$$

* The interval $[\eta_i, \eta_f]$ must be long enough so that such small modes have time to expand to cosmological size. For example this time period would do:

$$[10^{-34}s,\ 10^{-32}s]$$

Realistic cosmic inflation

1. How can a period of near-exponential expansion be caused?

- Recall the full action:

We neglect such terms by Occam's razor: there is no evidence

10qft20.jnt - Windows Journal

cosmological size. For example this time period would do:

$$[10^{-34}s, 10^{-32}s]$$

Realistic cosmic inflation

1. How can a period of near-exponential expansion be caused?

Recall the full action:

We neglect such terms by Occam's razor: there is no evidence for their existence as yet.

$$S = -\frac{1}{16\pi G}\int\Big[\underbrace{2\Lambda}_{\text{cosm. constant}} + R(x) + \overbrace{\cancel{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \dots}}\Big]\sqrt{|g|}\,d^4x$$

Note: ϕ is now called the "Inflaton" field.

$$+\int\Big[\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\Big]\sqrt{|g|}\,d^4x$$

$$+\cancel{S_{\text{other fields}}}$$ ← We neglect this term because the contribution

10qft20.jnt - Windows Journal

Realistic cosmic inflation

1. How can a period of near-exponential expansion be caused?

Recall the full action:

We neglect such terms by Occam's razor: there is no evidence for their existence as yet.

$$S = -\frac{1}{16\pi G}\int\Big[2\Lambda + R(x) + \underbrace{\cancel{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \dots}}\Big]\sqrt{|g|}\, d^4x$$

$\uparrow$ cosm. constant

$$+\int\left[\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\right]\sqrt{|g|}\, d^4x$$

Note: ϕ is now called the "Inflaton" field.

$$+\cancel{S_{\text{other fields}}}$$

$\leftarrow$ We neglect this term because the contribution of the inflaton field ϕ and of $g_{\mu\nu}$ are assumed to have been dominant in the very early universe.

Example choice of V: $V(\phi) = m\phi^2 + \lambda\phi^4$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Realistic cosmic inflation

1. How can a period of near-exponential expansion be caused?

Recall the full action:

We neglect such terms by Occam's razor: there is no evidence for their existence as yet.

$$S = -\frac{1}{16\pi G}\int \Big[\underbrace{2\Lambda}_{\text{cosm. constant}} + R(x) + \overbrace{\cancel{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \ldots}}\Big]\sqrt{|g|}\, d^4x$$

Note: ϕ is now called the "Inflaton" field.

$$+ \int \left[\frac{1}{2} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\right]\sqrt{|g|}\, d^4x$$

$$+ \cancel{S_{\text{other fields}}}$$

$\leftarrow$ We neglect this term because the contribution of the inflaton field ϕ and of $g_{\mu\nu}$ are assumed to have been dominant in the very early universe.

Example choice of V: $\quad V(\phi) = m\,\phi^2 + \lambda\,\phi^4$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Recall the full action:

We neglect such terms by Occam's razor: there is no evidence for their existence as yet.

$$S = -\frac{1}{16\pi G}\int\Big[2\Lambda + R(x) + \underbrace{\cancel{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \ldots}}\Big]\sqrt{|g|}\, d^4x$$

Λ: cosm. constant

Note: ϕ is now called the "Inflaton" field.

$$+ \int\Big[\frac{1}{2} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\Big]\sqrt{|g|}\, d^4x$$

$$+ \cancel{S_{\text{other fields}}}$$

← We neglect this term because the contribution of the inflaton field ϕ and of $g_{\mu\nu}$ are assumed to have been dominant in the very early universe.

Example choice of V: $\quad V(\phi) = m\,\phi^2 + \lambda\,\phi^4$

Equations of motion:

$\frac{\delta S}{\ldots} = 0$ yields the K.G. eqn.:

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

Recall the full action:

We neglect such terms by Occam's razor: there is no evidence for their existence as yet.

$$S = -\frac{1}{16\pi G}\int\Big[2\Lambda + R(x) + \cancel{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \ldots}\Big]\sqrt{|g|}\,d^4x$$

Λ: cosm. constant

Note: ϕ is now called the "Inflaton" field.

$$+\int\Big[\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\Big]\sqrt{|g|}\,d^4x$$

$$+\cancel{S_{\text{other fields}}}$$

← We neglect this term because the contribution of the inflaton field ϕ and of $g_{\mu\nu}$ are assumed to have been dominant in the very early universe.

Example choice of V: $\quad V(\phi) = m\,\phi^2 + \lambda\,\phi^4$

Equations of motion:

$*\quad \frac{\delta S}{\delta\phi} = 0$ yields the K.G. eqn.:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Occam's razor: there is no evidence for their existence as yet.

$$S = -\frac{1}{16\pi G}\int \Big[2\Lambda + R(x) + \overbrace{\cancel{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \ldots}}\Big]\sqrt{|g|}\, d^4x$$

Λ: cosm. constant

Note: ϕ is now called the "Inflaton" field.

$$+ \int \Big[\frac{1}{2} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\Big]\sqrt{|g|}\, d^4x$$

$$+ \cancel{S_{\text{other fields}}}$$

← We neglect this term because the contribution of the inflaton field ϕ and of $g_{\mu\nu}$ are assumed to have been dominant in the very early universe.

Example choice of V: $V(\phi) = m\,\phi^2 + \lambda\,\phi^4$

Equations of motion:

$*$ $\dfrac{\delta S}{\delta \phi(x)} = 0$ yields the K.G. eqn.:

10qft20.jnt - Windows Journal

$$S = -\frac{1}{16\pi G}\int\Big[2\Lambda + R(x) + \cancel{\overbrace{\mathcal{O}(R\phi) + \mathcal{O}(R^2) + \dots}}\Big]\sqrt{|g|}\,d^4x$$

$\uparrow$ cosm. constant

Note: ϕ is now called the "Inflaton" field.

$$+ \int\Big[\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\Big]\sqrt{|g|}\,d^4x$$

$$+ \cancel{S_{\text{other fields}}}$$

$\leftarrow$ We neglect this term because the contribution of the inflaton field ϕ and of $g_{\mu\nu}$ are assumed to have been dominant in the very early universe.

Example choice of V: $V(\phi) = m\,\phi^2 + \lambda\,\phi^4$

Equations of motion:

$*$ $\frac{\delta S}{\delta\phi(x)} = 0$ yields the K.G. eqn.:

10qft20.jnt - Windows Journal

Example choice of V: $V(\phi) = m\,\phi^2 + \lambda\,\phi^4$

Equations of motion:

* $\frac{\delta S}{\delta \phi(x)} = 0$ yields the K.G. eqn.:

$$\frac{\partial}{\partial x^\mu}\left(g^{\mu\nu}(x)\,\phi_{,\nu}(x)\sqrt{|g(x)|}\right) + \frac{\partial V}{\partial \phi}(x)\sqrt{|g(x)|} = 0 \qquad (KG)$$

* $\frac{\delta S}{\delta g_{\mu\nu}(x)} = 0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x) - g_{\mu\nu}(x)\,R(x) + \Lambda\, g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

10qft20.jnt - Windows Journal

Equations of motion:

* $\frac{\delta S}{\delta \phi(x)} = 0$ yields the K.G. eqn.:

$$\frac{\partial}{\partial x^\mu}\left(g^{\mu\nu}(x)\,\phi_{,\nu}(x)\sqrt{|g(x)|}\right) + \frac{\partial V}{\partial \phi}(x)\sqrt{|g(x)|} = 0 \qquad (KG)$$

* $\frac{\delta S}{\delta g_{\mu\nu}(x)} = 0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x) - g_{\mu\nu}(x) R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu} = \phi_{,\mu}\,\phi_{,\nu} - g_{\mu\nu}\left(g^{\alpha\beta}\phi_{,\alpha}\,\phi_{,\beta} - V(\phi)\right)$$

10qft20.jnt - Windows Journal

Equations of motion:

* $\dfrac{\delta S}{\delta \phi(x)} = 0$ yields the K.G. eqn.:

$$\frac{\partial}{\partial x^\mu}\left(g^{\mu\nu}(x)\,\phi_{,\nu}(x)\sqrt{|g(x)|}\right) + \frac{\partial V}{\partial \phi}(x)\sqrt{|g(x)|} = 0 \qquad (KG)$$

* $\dfrac{\delta S}{\delta g_{\mu\nu}(x)} = 0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x) - g_{\mu\nu}(x) R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi)\right)$$

10qft20.jnt - Windows Journal

Equations of motion:

* $\dfrac{\delta S}{\delta \phi(x)} = 0$ yields the K.G. eqn.:

$$\frac{\partial}{\partial x^\mu}\left(g^{\mu\nu}(x)\,\phi_{,\nu}(x)\sqrt{|g(x)|}\right) + \frac{\partial V}{\partial \phi}(x)\sqrt{|g(x)|} = 0 \qquad (KG)$$

* $\dfrac{\delta S}{\delta g_{\mu\nu}(x)} = 0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x) - g_{\mu\nu}(x)R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu} = \phi_{,\mu}\,\phi_{,\nu} - g_{\mu\nu}\left(g^{\alpha\beta}\phi_{,\alpha}\,\phi_{,\beta} - V(\phi)\right) + T_{\mu\nu}^{(other)}$$

10qft20.jnt - Windows Journal

$\delta g_{\mu\nu}(x)$

$$R_{\mu\nu}(x) - g_{\mu\nu}(x) R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G \, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu} = \phi_{,\mu} \phi_{,\nu} - g_{\mu\nu} \left(g^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} - V(\phi) \right) + T_{\mu\nu}^{(\text{other})}$$

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear partial differential equations which are even classically very hard.

$\rightsquigarrow$ As a lowest order approximation we assume perfect homogeneity & isotropy:

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear partial differential equations which are even classically very hard.

⇝ As a lowest order approximation we assume perfect homogeneity & isotropy:

$$\phi(x,t) = \phi(t)$$

$$g_{\mu\nu}(x,t) = g_{\mu\nu}(t)$$

Note:

This may also be viewed as considering only the $k=0$ modes, neglecting all other modes.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear partial differential equations which are even classically very hard.

→ As a lowest order approximation we assume perfect homogeneity & isotropy:

$$\phi(x,t) = \phi(t)$$

$$g_{\mu\nu}(x,t) = g_{\mu\nu}(t)$$

Note:

This may also be viewed as considering only the $k=0$ modes, neglecting all other modes.

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear partial differential equations which are even classically very hard.

⇝ As a lowest order approximation we assume perfect homogeneity & isotropy:

$$\phi(x,t) = \phi(t)$$

$$g_{\mu\nu}(x,t) = g_{\mu\nu}(t)$$

Note:

This may also be viewed as considering only the $k=0$ modes, neglecting all other modes.

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

$$\phi(x,t) = \phi(t)$$

$$g_{\mu\nu}(x,t) = g_{\mu\nu}(t)$$

Note:

This may also be viewed as considering only the $k=0$ modes, neglecting all other modes.

Thus, the eqns of motion simplify:

$\left(\dot{\ } = \frac{\partial}{\partial t}\right)$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Note:

This may also be viewed as considering only the $k=0$ modes, neglecting all other modes.

Thus, the eqns of motion simplify:

$\left(\dot{} = \frac{\partial}{\partial t} \right)$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\begin{matrix}\text{the 0,0 component of} \\ \text{the Einstein equation}\end{matrix}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\begin{matrix}\text{the i,i components of} \\ \text{the Einstein equation}\end{matrix}\right)$$

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

Thus, the eqns of motion simplify:

$$\left(\dot{} = \frac{\partial}{\partial t}\right) \qquad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\begin{array}{l}\text{the i,i components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Thus, the eqns of motion simplify:

$\left(\dot{} = \frac{\partial}{\partial t}\right)$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3}T^0_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3}T^\mu_\mu \qquad \left(\begin{array}{l}\text{the i,i components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Thus, the eqns of motion simplify:

$$\left(\dot{\ } = \frac{\partial}{\partial t}\right) \qquad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T_0^{\ 0} \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T_\mu^{\ \mu} \qquad \left(\begin{array}{l}\text{the i,i components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \;=:\; \rho(t) \quad \text{(the energy density of } \phi)$$

$$T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) \;=:\; p(t) \quad \text{(the pressure of } \phi)$$

□ Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter."

$$T^0_{\ 0} = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t) \quad \text{(the energy density of } \phi)$$

$$T^i_{\ i} = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t) \quad \text{(the pressure of } \phi)$$

- **Definition:** The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter"

- **Solvability:**

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear partial differential equations which are even classically very hard.

⇝ As a lowest order approximation we assume perfect homogeneity & isotropy:

$$\phi(x,t) = \phi(t)$$

$$g_{\mu\nu}(x,t) = g_{\mu\nu}(t)$$

Note:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

* $\frac{\delta S}{\delta g_{\mu\nu}(x)} = 0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x) - g_{\mu\nu}(x) R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi)\right) + T_{\mu\nu}^{(other)}$$

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear partial differential equations which are even classically very hard.

10qft20.jnt - Windows Journal

$$R_{\mu\nu}(x) - g_{\mu\nu}(x) R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu} = \phi_{,\mu}\, \phi_{,\nu} - g_{\mu\nu}\left(g^{\alpha\beta} \phi_{,\alpha}\, \phi_{,\beta} - V(\phi)\right) + T_{\mu\nu}^{(other)}$$

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear partial differential equations which are even classically very hard.

$\rightsquigarrow$ As a lowest order approximation we assume perfect homogeneity & isotropy:

10qft20.jnt - Windows Journal

$$\left(\frac{\dot a}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

(the 0,0 component of the Einstein equation)

$$\frac{\ddot a}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu$$

(the i,i components of the Einstein equation)

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_{\ 0} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \;=:\; \rho(t) \qquad \text{(the energy density of } \phi)$$

$$T^i_{\ i} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \;=:\; p(t) \qquad \text{(the pressure of } \phi)$$

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter."

Solvability:

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

$$T^i_{\ i} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \;=:\; p(t) \quad \text{(the pressure of } \phi\text{)}$$

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter."

Solvability:

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable for $a(t)$ and $\phi(t)$, at least numerically.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

- Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter."

- Solvability:

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable for $a(t)$ and $\phi(t)$, at least numerically.

- 1^{st} possibility for causing exponential expansion

* Observation:

10qft20.jnt - Windows Journal

for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad =: \rho(t) \quad \text{(the energy density of } \phi\text{)}$$

$$T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad =: p(t) \quad \text{(the pressure of } \phi\text{)}$$

□ Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter"

□ Solvability:

10qft20.jnt - Windows Journal

$\left(\dot{\ } = \frac{\partial}{\partial t}\right)$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\text{the } 0,0 \text{ component of the Einstein equation}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\text{the } i,i \text{ components of the Einstein equation}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t) \qquad (\text{the energy density of } \phi)$$

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

$(\dot{\ } = \frac{\partial}{\partial t})$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$ (K.G. eqn.)

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$ (the 0,0 component of the Einstein equation)

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu$$ (the i,i components of the Einstein equation)

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^{\mu}_{\mu}$$ (the i,i components of the Einstein equation)

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t)$$ (the energy density of ϕ)

$$T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t)$$ (the pressure of ϕ)

Definition: The function

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Thus, the eqns of motion simplify:

$$\left(\cdot = \frac{\partial}{\partial t}\right) \qquad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\begin{array}{l}\text{the i,i components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Thus, the eqns of motion simplify:

$$\left(\dot{} = \frac{\partial}{\partial t}\right) \qquad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\begin{array}{l}\text{the i,i components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

$$\left(\frac{\dot a}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$ (the 0,0 component of the Einstein equation)

$$\frac{\ddot a}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu$$ (the i,i components of the Einstein equation)

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot\phi^2 + V(\phi) =: \rho(t)$$ (the energy density of ϕ)

$$T^i_i = \frac{1}{2}\dot\phi^2 - V(\phi) =: p(t)$$ (the pressure of ϕ)

10qft20.jnt - Windows Journal

Thus, the eqns of motion simplify:

$\left(\cdot = \frac{\partial}{\partial t}\right)$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$ (K.G. eqn.)

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$ (the 0,0 component of the Einstein equation)

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu$$ (the i,i components of the Einstein equation)

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^{\mu}_{\mu}$$ (the i,i components of the Einstein equation)

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^{0}_{0} = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t)$$ (the energy density of ϕ)

$$T^{i}_{i} = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t)$$ (the pressure of ϕ)

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t) \quad \text{(the energy density of } \phi\text{)}$$

$$T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t) \quad \text{(the pressure of } \phi\text{)}$$

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter."

10qft20.jnt - Windows Journal

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_{\ 0} = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t) \quad \text{(the energy density of } \phi\text{)}$$

$$T^i_{\ i} = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t) \quad \text{(the pressure of } \phi\text{)}$$

□ Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter."

10qft20.jnt - Windows Journal

$$\left(\frac{\dot a}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_{\ 0}$$ (the 0,0 component of the Einstein equation)

$$\frac{\ddot a}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_{\ \mu}$$ (the i,i components of the Einstein equation)

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_{\ 0} = \frac{1}{2}\dot\phi^2 + V(\phi) =: \rho(t)$$ (the energy density of ϕ)

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$\left(\dot{\ } = \frac{\partial}{\partial t}\right)$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3}T^0_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3}T^\mu_\mu \qquad \left(\begin{array}{l}\text{the i,i components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t) \quad \text{(the energy density of } \phi)$$

$$T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t) \quad \text{(the pressure of } \phi)$$

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter".

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

$$T^0_{\ 0} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad =: \rho(t) \quad \text{(the energy density of } \phi)$$

$$T^i_{\ i} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad =: p(t) \quad \text{(the pressure of } \phi)$$

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter".

Solvability:

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable for $a(t)$ and $\phi(t)$, at least numerically.

10qft20.jnt - Windows Journal

$$T^0_{\ 0} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad =: \rho(t) \quad \text{(the energy density of } \phi\text{)}$$

$$T^i_{\ i} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad =: p(t) \quad \text{(the pressure of } \phi\text{)}$$

□ Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter."

□ Solvability:

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable for $a(t)$ and $\phi(t)$, at least numerically.

10qft20.jnt - Windows Journal

is called the "Equation of state parameter."

Solvability:

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable for $a(t)$ and $\phi(t)$, at least numerically.

1st possibility for causing exponential expansion

* Observation:

The cosmological constant Λ, when brought to the RHS of the Einstein eqn, can be viewed as

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Solvability:

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable for $a(t)$ and $\phi(t)$, at least numerically.

1^{st} possibility for causing exponential expansion

* Observation:

The cosmological constant Λ, when brought to the RHS of the Einstein eqn, can be viewed as a diagonal contribution to the energy-momentum tensor.

* Exercise:

10 / 22

10qft20.jnt - Windows Journal

1^{st} possibility for causing exponential expansion

*** Observation:**

The cosmological constant Λ, when brought to the RHS of the Einstein eqn, can be viewed as a diagonal contribution to the energy-momentum tensor.

*** Exercise:**

Show that in this interpretation, a positive Λ plays the rôle of a positive energy density and of a negative pressure, thus yielding

$$w(t) = -1$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0{}_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu{}_\mu \qquad \left(\begin{array}{l}\text{the } i,i \text{ components of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0{}_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu{}_\mu \qquad \left(\begin{array}{l}\text{the } i,i \text{ components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0{}_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad =: \rho(t) \qquad (\text{the energy density of } \phi)$$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Thus, the eqns of motion simplify:

$\left(\dot{\cdot} = \frac{\partial}{\partial t}\right)$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$ (K.G. eqn.)

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$ (the 0,0 component of the Einstein equation)

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu$$ (the i,i components of the Einstein equation)

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Thus, the eqns of motion simplify:

$(\dot{\ } = \frac{\partial}{\partial t})$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\text{the 0,0 component of the Einstein equation}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\text{the } i,i \text{ components of the Einstein equation}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

* $\frac{\delta S}{\delta g_{\mu\nu}(x)} = 0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x) - g_{\mu\nu}(x) R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi)\right) + T_{\mu\nu}^{(other)}$$

Homogeneity & isotropy

Eqns. (KG) and (E) are a set of coupled nonlinear

10qft20.jnt - Windows Journal

$$\frac{\partial}{\partial x^\mu}\left(g^{\mu\nu}(x)\,\phi_{,\nu}(x)\sqrt{|g(x)|}\right) + \frac{\partial V}{\partial \phi}(x)\sqrt{|g(x)|} = 0 \qquad (KG)$$

* $\frac{\delta S}{\delta g_{\mu\nu}(x)} = 0$ yields the Einstein eqn.:

$$R_{\mu\nu}(x) - g_{\mu\nu}(x) R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G\, T_{\mu\nu}(x) \qquad (E)$$

where the energy-momentum tensor (for ϕ only) reads:

$$T_{\mu\nu} = \phi_{,\mu}\,\phi_{,\nu} - g_{\mu\nu}\left(g^{\alpha\beta}\phi_{,\alpha}\,\phi_{,\beta} - V(\phi)\right) + T_{\mu\nu}^{(other)}$$

Homogeneity & isotropy

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Thus, the eqns of motion simplify:

$$\left(\cdot = \frac{\partial}{\partial t}\right) \qquad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\begin{array}{l}\text{the i,i components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Thus, the eqns of motion simplify:

$$\left(\dot{} = \frac{\partial}{\partial t}\right) \qquad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\begin{array}{l}\text{the 0,0 component of}\\ \text{the Einstein equation}\end{array}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\begin{array}{l}\text{the i,i components of}\\ \text{the Einstein equation}\end{array}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Thus, the eqns of motion simplify:

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad \text{(K.G. eqn.)}$$

$\left(\dot{} = \frac{\partial}{\partial t}\right)$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0 \qquad \left(\text{the 0,0 component of the Einstein equation}\right)$$

$$\frac{\ddot{a}}{a} - \frac{1}{3}\Lambda = -\frac{4\pi G}{3} T^\mu_\mu \qquad \left(\text{the i,i components of the Einstein equation}\right)$$

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t) \quad \text{(the energy density of } \phi\text{)}$$

$$T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t) \quad \text{(the pressure of } \phi\text{)}$$

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter"

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Also, the components of the energy-momentum tensor for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad =: \rho(t) \quad \text{(the energy density of } \phi\text{)}$$

$$T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad =: p(t) \quad \text{(the pressure of } \phi\text{)}$$

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter"

10qft20.jnt - Windows Journal

for the inflaton field ϕ simplify and now take this form:

$$T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t) \quad \text{(the energy density of } \phi)$$

$$T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t) \quad \text{(the pressure of } \phi)$$

Definition: The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter"

Solvability:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$T^0_{\ 0} = \frac{1}{2}\dot{\phi}^2 + V(\phi) =: \rho(t) \quad \text{(the energy density of } \phi\text{)}$$

$$T^i_{\ i} = \frac{1}{2}\dot{\phi}^2 - V(\phi) =: p(t) \quad \text{(the pressure of } \phi\text{)}$$

- **Definition:** The function

$$w(t) := \frac{p(t)}{\rho(t)}$$

is called the "Equation of state parameter"

- **Solvability:**

For any given choice of potential function $V(\phi)$

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

For any given choice of potential function $V(\phi)$ the above set of equations is now solvable for $a(t)$ and $\phi(t)$, at least numerically.

1st possibility for causing exponential expansion

* Observation:

The cosmological constant Λ, when brought to the RHS of the Einstein eqn, can be viewed as a diagonal contribution to the energy-momentum tensor.

* Exercise:

Show that in this interpretation, a positive Λ plays the

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

The cosmological constant Λ, when brought to the RHS of the Einstein eqn, can be viewed as a diagonal contribution to the energy-momentum tensor.

* Exercise:

Show that in this interpretation, a positive Λ plays the rôle of a positive energy density and of a negative pressure, thus yielding

$$w(t) = -1$$

if there is no other $T^{\mu}{}_{\nu}$.

* Assume that Λ dominates over $T^{0}{}_{0}$:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

the RHS of the Einstein eqn, can be viewed as a diagonal contribution to the energy-momentum tensor.

* Exercise:

Show that in this interpretation, a positive Λ plays the rôle of a positive energy density and of a negative pressure, thus yielding

$$w(t) = -1$$

if there is no other $T^\mu{}_\nu$.

* Assume that Λ dominates over T^0_0:

Then, the $0,0$ component of Einstein's equation

11 / 22

10qft20.jnt - Windows Journal

* Exercise:

Show that in this interpretation, a positive Λ plays the rôle of a positive energy density and of a negative pressure, thus yielding

$$w(t) = -1$$

if there is no other $T^{\mu}{}_{\nu}$.

* Assume that Λ dominates over T^{0}_{0}:

Then, the 0,0 component of Einstein's equation,

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^{0}_{0}$$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

* Assume that Λ dominates over T^0_0:

Then, the 0,0 component of Einstein's equation,

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\Lambda$$

whose solution has the desired behavior

$$a(t) = a_0 e^{tH}$$

with: $H = \sqrt{\Lambda/3}$

* Problem: Since Λ is by definition a constant, such an inflation would never end!

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

* **Assume that Λ dominates over T^0_0:**

Then, the 0,0 component of Einstein's equation,

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\Lambda$$

whose solution has the desired behavior

$$a(t) = a_0 e^{tH}$$

with: $H = \sqrt{\Lambda/3}$

* Problem: Since Λ is by definition a constant, such an inflation would never end!

10qft20.jnt - Windows Journal

Realistic possibility

* Instead of a fixed Λ, consider this case:
  - Universe may be in a generic state
  - I.e. we have spatially varying curvature, expansion or contraction, and possibly much matter and radiation.
  - One of the physical fields may be a real scalar field obeying the K.G. eqn with a potential term $V(\phi)$.
  - Then, at some time, it can happen that ϕ fluctuates such that, in some patch of significant size, the $V(\phi)$ is temporarily dominant over all other terms in the T^0_0 of the full energy momentum tensor of all fields.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

* Instead of a fixed Λ, consider this case:

- Universe may be in a generic state

- I.e. we have spatially varying curvature, expansion or contraction, and possibly much matter and radiation.

- One of the physical fields may be a real scalar field obeying the K.G. eqn with a potential term $V(\phi)$.

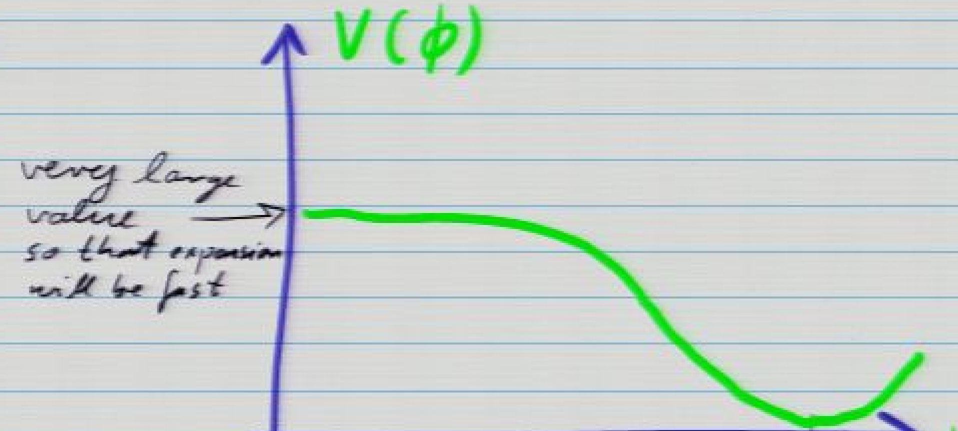
- Then, at some time, it can happen that ϕ fluctuates such that, in some patch of significant size, the $V(\phi)$ is temporarily dominant over all other terms in the T^0_0 of the full energy momentum tensor of all fields.

10qft20.jnt - Windows Journal

Then, at some time, it can happen that ϕ fluctuates such that, in some patch of significant size, the $V(\phi)$ is **temporarily dominant** over all other terms in the $T^{\circ}{}_{\circ}$ of the full energy momentum tensor of all fields.

$\Rightarrow$ This patch can inflate and spawn a new bubble, in effect a new universe!

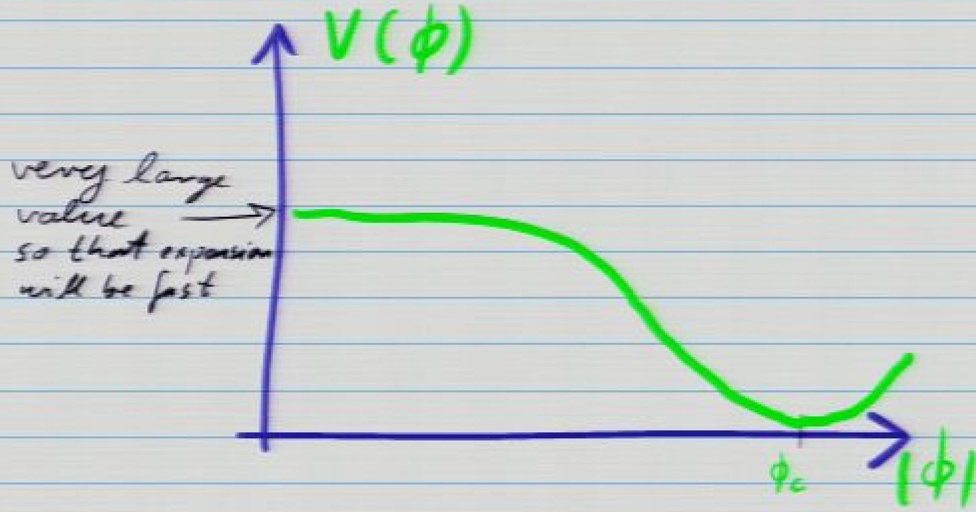
Example potential:



10qft20.jnt - Windows Journal

⇒ This patch can inflate and spawn a new bubble, in effect a new universe!

Example potential:

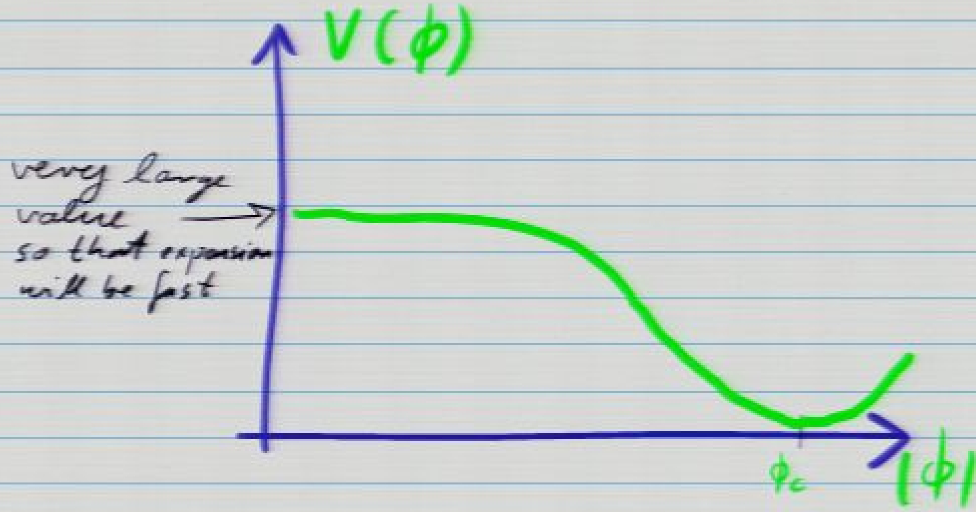


- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi=0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi = \phi_c$ is reached, $V(\phi)=0$, and inflation has ended.

10qft20.jnt - Windows Journal

bubble, in effect a new universe!

Example potential:

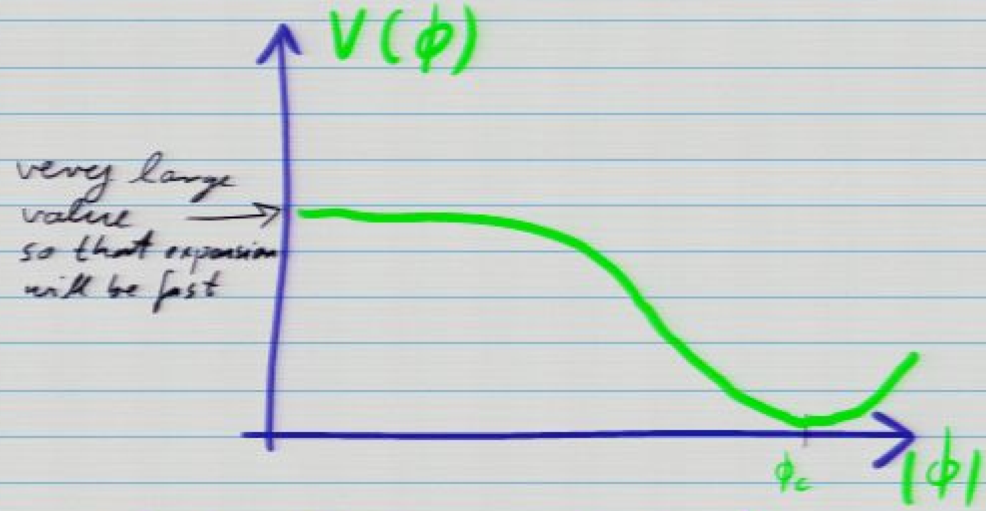


- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi=0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi=\phi_c$ is reached, $V(\phi)=0$, and inflation has ended.

10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width

bubble, in effect a new universe!

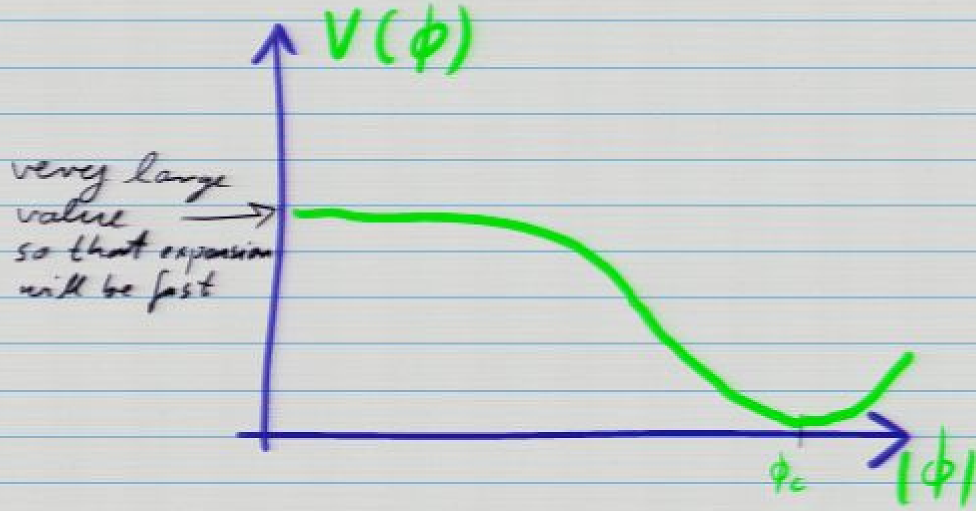
Example potential:



- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi = 0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi = \phi_c$ is reached, $V(\phi) = 0$, and inflation has ended.

bubble, in effect a new universe!

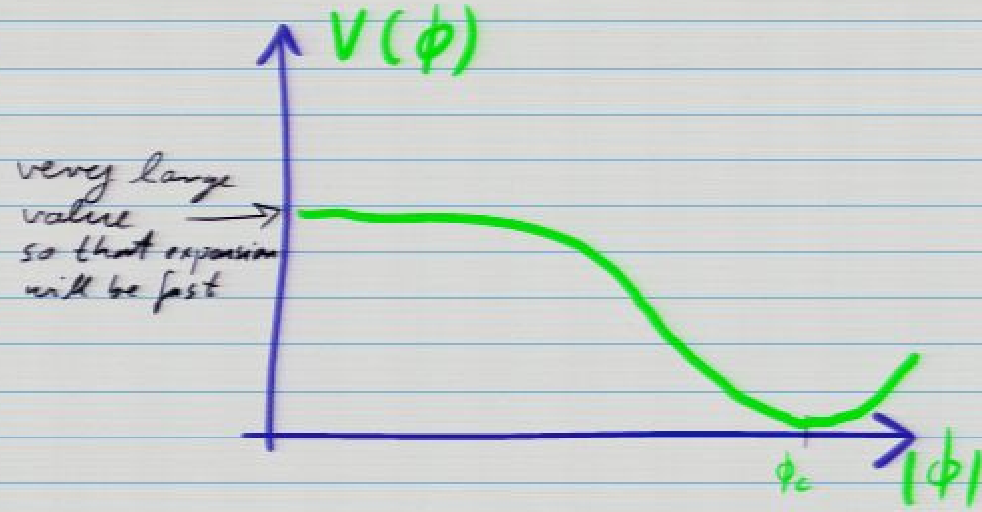
Example potential:



- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi = 0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi = \phi_c$ is reached, $V(\phi) = 0$, and inflation has ended.

10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width

Example potential:

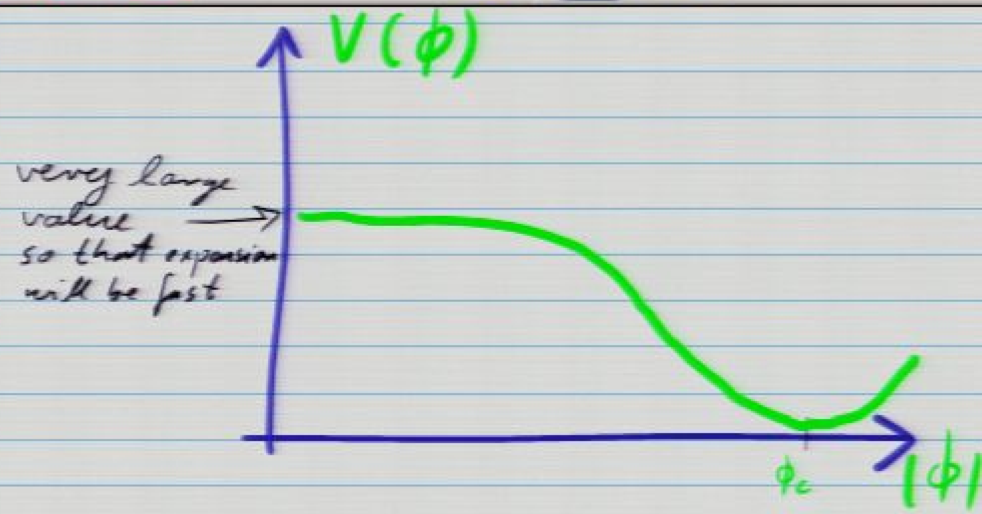


- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi=0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi=\phi_c$ is reached, $V(\phi)=0$, and inflation has ended.

* Concretely:

10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width

Example potential:



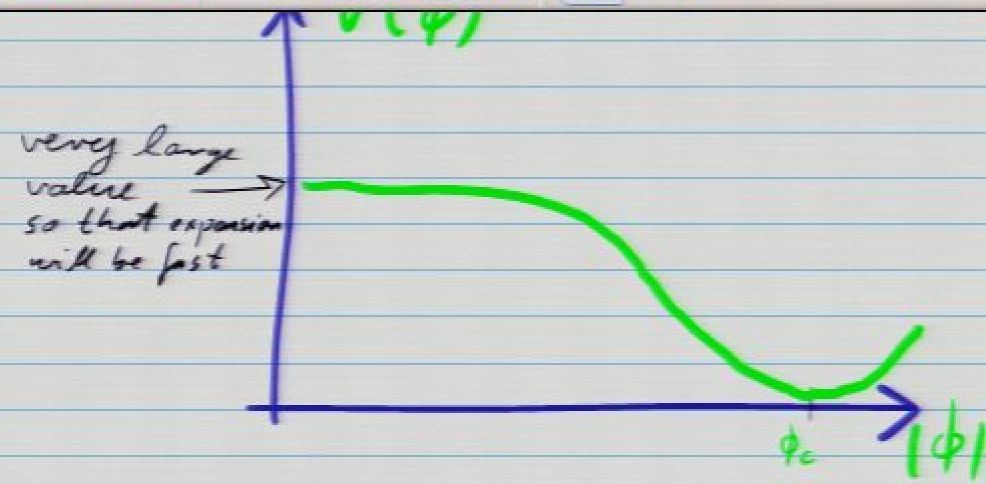
- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi = 0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi = \phi_c$ is reached, $V(\phi) = 0$, and inflation has ended.

* **Concretely:**

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Example potential:



- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi = 0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi = \phi_c$ is reached, $V(\phi) = 0$, and inflation has ended.

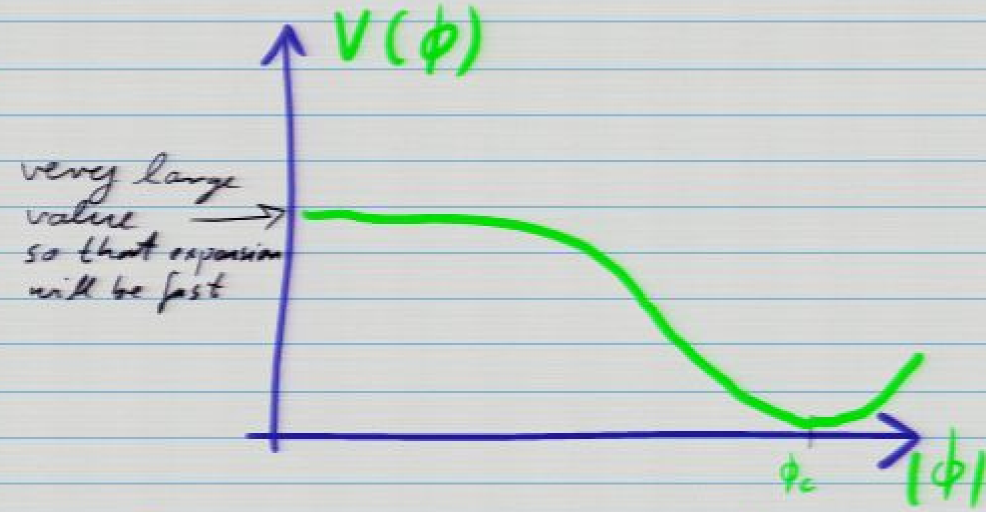
* **Concretely:**

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

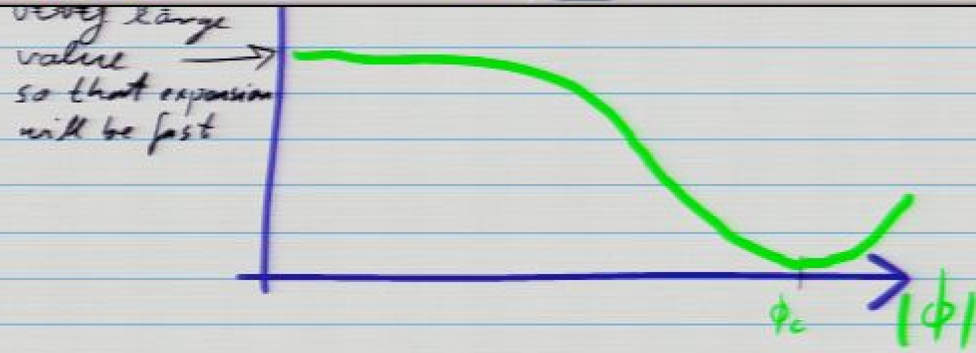
Page Width

Example potential:



- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi = 0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi = \phi_c$ is reached, $V(\phi) = 0$, and inflation has ended.

* **Concretely:**



- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)
- Then, after ϕ starts out at $\phi=0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.
- Once $\phi = \phi_c$ is reached, $V(\phi)=0$, and inflation has ended.

---

* **Concretely:**

The Klein Gordon equation reads:

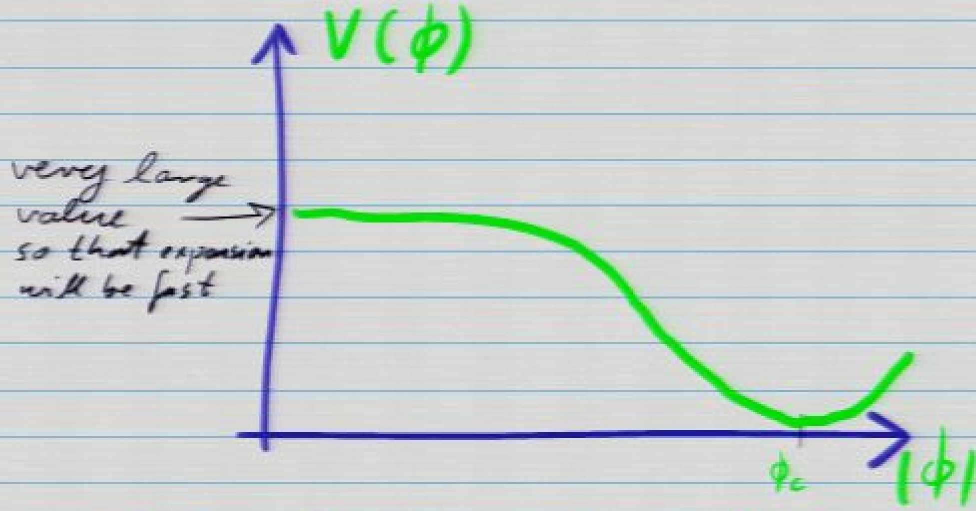
friction term

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

bubble, in effect a new universe!

Example potential:



- Then, inflation starts, if in a patch, ϕ is very small, even though it is energetically expensive (a rare quantum fluctuation)

- Then, after ϕ starts out at $\phi = 0$ and large $V(\phi)$, it will slowly evolve towards ϕ_c while the universe inflates, thus flattens, and the matter dilutes.

- Once $\phi = \phi_c$ is reached, $V(\phi) = 0$, and inflation has ended.

10qft20.jnt - Windows Journal

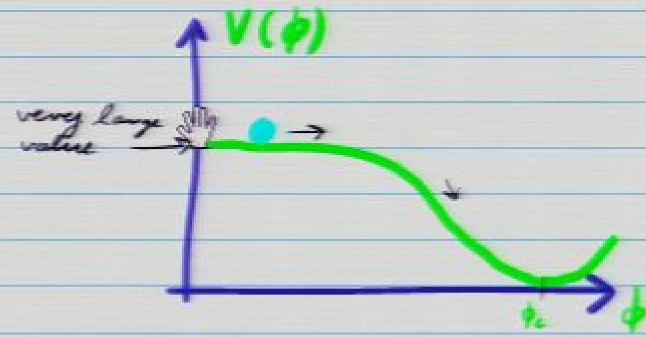
* Concretely:

The Klein Gordon equation reads:

friction term

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} - \frac{dV}{d\phi}$$

This is like the equation of motion of a ball rolling down a hill, with friction:



$\left(-\frac{dV}{d\phi}\right)$ acts to pull ϕ down the potential hill.

$\left(-3\frac{\dot{a}}{a}\dot{\phi}\right)$ acts as a "friction" term.

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

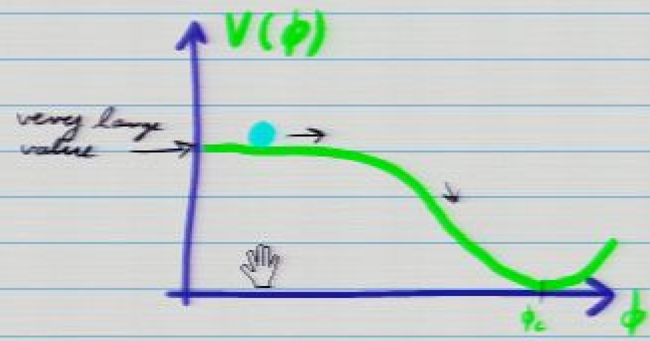
* Concretely:

The Klein Gordon equation reads:

friction term

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} - \frac{dV}{d\phi}$$

This is like the equation of motion of a ball rolling down a hill, with friction:



$\left(-\frac{dV}{d\phi}\right)$ acts to pull ϕ down the potential hill.

$\left(-3\frac{\dot{a}}{a}\dot{\phi}\right)$ acts as a "friction" term.

10qft20.jnt - Windows Journal

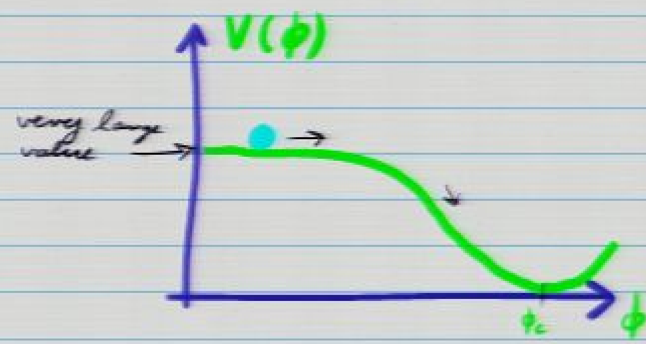
Edit View Insert Actions Tools Help

Page Width

The Klein Gordon equation reads:

$$\ddot{\phi} = -3\,\frac{\dot{a}}{a}\,\underbrace{\dot{\phi}}_{\text{friction term}} - \frac{dV}{d\phi}$$

This is like the equation of motion of a ball rolling down a hill, with friction:



$\left(-\frac{dV}{d\phi}\right)$ acts to pull ϕ down the potential hill.

$\left(-3\,\frac{\dot{a}}{a}\,\dot{\phi}\right)$ acts as a "friction" term.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

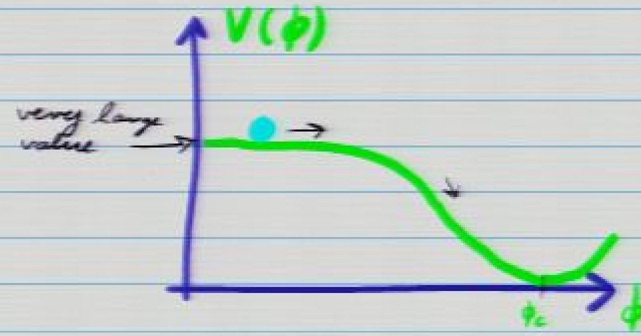
Page Width

The Klein Gordon equation reads:

friction term

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} - \frac{dV}{d\phi}$$

This is like the equation of motion of a ball rolling down a hill, with friction:

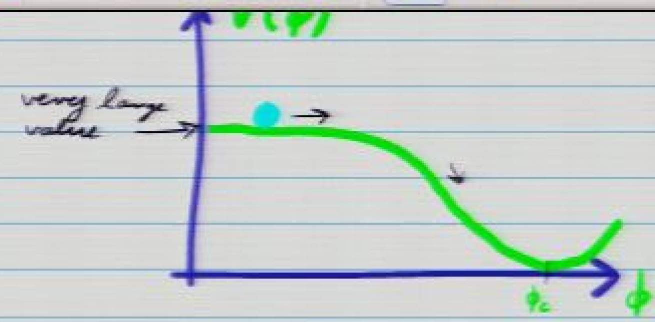


$\left(-\frac{dV}{d\phi}\right)$ acts to pull ϕ down the potential hill.

$\left(-3\frac{\dot{a}}{a}\dot{\phi}\right)$ acts as a "friction" term.

* Definition:

10qft20.jnt - Windows Journal



$\left(-\frac{dV}{d\phi}\right)$ acts to pull ϕ down the potential hill.

$\left(-3\frac{\dot{a}}{a}\dot{\phi}\right)$ acts as a "friction" term.

* **Definition:**

If the initial value of $V(\phi)$ is very large and if the initial slope is very flat, i.e., if the ball for a period rolls slowly, with approximately constant $V(\phi)$, we call this

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

* Definition:

If the initial value of $V(\phi)$ is very large and if the initial slope is very flat, i.e., if the ball for a period rolls slowly, with approximately constant $V(\phi)$, we call this a period of "Slow Roll Inflation".

* Observation:

During the slow roll period, we have, in particular, that $V(\phi)$ dominates over $\frac{1}{2}\dot{\phi}^2$.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

* Definition:

If the initial value of $V(\phi)$ is very large and if the initial slope is very flat, i.e., if the ball for a period rolls slowly, with approximately constant $V(\phi)$, we call this a period of "Slow Roll Inflation".

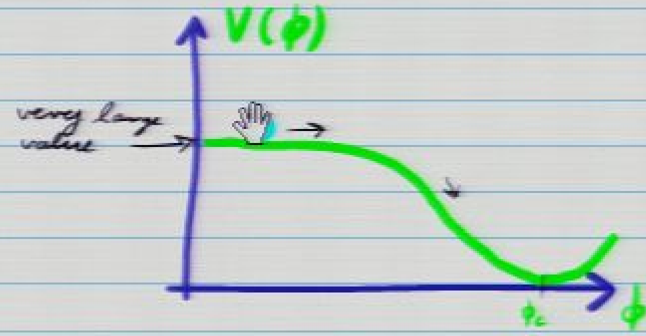
* Observation:

During the slow roll period, we have, in particular, that $V(\phi)$ dominates over $\frac{1}{2}\dot{\phi}^2$.

$\Rightarrow \quad w(t) = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\ldots} \simeq -1 \quad$ (temporarily

10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width

This is like the equation of motion of a ball rolling down a hill, with friction:



$\left(-\dfrac{dV}{d\phi}\right)$ acts to pull ϕ down the potential hill.

$\left(-3\dfrac{\dot{a}}{a}\dot{\phi}\right)$ acts as a "friction" term.

---

* Definition:

If the initial value of $V(\phi)$ is very large and if the initial slope is very flat, i.e., if

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

* Definition:

If the initial value of $V(\phi)$ is very large and if the initial slope is very flat, i.e., if the ball for a period rolls slowly, with approximately constant $V(\phi)$, we call this a period of "Slow Roll Inflation".

* Observation:

During the slow roll period, we have, in particular, that $V(\phi)$ dominates over $\frac{1}{2}\dot{\phi}^2$.

$$\Rightarrow \quad w(t) = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \simeq -1 \quad (\text{temporarily})$$

approximately constant $V(\phi)$, we call this a period of "Slow Roll Inflation".

* Observation:

During the slow roll period, we have, in particular, that $V(\phi)$ dominates over $\frac{1}{2}\dot{\phi}^2$.

$$\Rightarrow \quad w(t) = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \simeq -1 \quad (\text{temporarily})$$

* But, do we also get temporary exponential inflation?

Indeed, the 0,0 component of the Einstein equation

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

* Observation:

During the slow roll period, we have, in particular, that $V(\phi)$ dominates over $\frac{1}{2}\dot{\phi}^2$.

$$\Rightarrow \qquad w(t) = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \approx -1 \quad (\text{temporarily})$$

* But, do we also get temporary exponential inflation?

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

is during the slow roll period:

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

During the slow roll period, we have, in particular, that $V(\phi)$ dominates over $\frac{1}{2}\dot{\phi}^2$.

$$\Rightarrow \qquad w(t) = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \approx -1 \quad (\text{temporarily})$$

* But, do we also get temporary exponential inflation?

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T_0^0$$

is during the slow roll period:

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

$$\Rightarrow \qquad w(t) = \frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)} \approx -1 \quad (\text{temporarily})$$

* But, do we also get temporary exponential inflation?

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot a}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

is during the slow roll period:

$$\left(\frac{\dot a}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$\Rightarrow \qquad w(t) = \frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)} \simeq -1 \quad (\text{temporarily})$$

* <u>But, do we also get temporary exponential inflation?</u>

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot a}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

is during the slow roll period:

$$\left(\frac{\dot a}{a}\right)^2 \simeq \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \simeq a_0\, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$\Rightarrow \qquad w(t) = \frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)} \approx -1 \quad (\text{temporarily})$$

* <u>But, do we also get temporary exponential inflation?</u>

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot a}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T_0^0$$

is during the slow roll period:

$$\left(\frac{\dot a}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0\, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$\Rightarrow \qquad w(t) = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \simeq -1 \quad (\text{temporarily})$$

* But, do we also get temporary exponential inflation?

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

is during the slow roll period:

$$\left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \simeq a_0\, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

* But, do we also get temporary exponential inflation?

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T_0^0$$

is during the slow roll period:

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

* But, do we also get temporary exponential inflation?

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T_0^0$$

is during the slow roll period:

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

10qft20.jnt - Windows Journal

* **But, do we also get temporary exponential inflation?**

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T_0^0$$

is during the slow roll period:

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

Page Width

* But, do we also get temporary exponential inflation?

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T_0^0$$

is during the slow roll period:

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Indeed, the 0,0 component of the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3} T^0_0$$

is during the slow roll period:

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0 e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$ (ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

is during the slow roll period:

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0\, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

10qft20.jnt - Windows Journal

is during the slow roll period:

$$\left(\frac{\dot a}{a}\right)^2 \approx \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \approx a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

Definition:

The function $H(t) := \frac{\dot a(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$

10qft20.jnt - Windows Journal

whose solution during slow roll is

$$a(t) \simeq a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

whose solution during slow roll is

$$a(t) \simeq a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$ (ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

$a(t) \simeq a_0\, e^{t\sqrt{\frac{\quad}{3}}\, V(\phi)}$ (over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

$$H(t) = \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

Remark:

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

$$H(t) = \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

Remark:

* Since as $V(t)$ decreases also $H(t)$ decreases, inflation predicts

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

Page Width

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

$$H(t) \simeq \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

Remark:

* Since as $V(t)$ decreases also $H(t)$ decreases, inflation predicts that $\delta\phi_L$ decreases for late horizon crossing modes, i.e., for smaller and smaller wavelength modes.

* The WMAP satellite's CMB data for the first time show some small evidence for this.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

$$H(t) = \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

Remark:

* Since as $V(t)$ decreases also $H(t)$ decreases, inflation predicts that $\delta\phi_k$ decreases for later horizon crossing modes, i.e., for smaller and smaller wavelength modes.

* The WMAP satellite's CMB data for the first time show some small evidence for this.

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

$$H(t) = \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

Remark:

* Since as $V(t)$ decreases also $H(t)$ decreases, inflation predicts that $\delta\phi_L$ decreases for late horizon crossing modes, i.e., for smaller and smaller wavelength modes.

* The WMAP satellite's CMB data for the first time show some small evidence for this.

The end of inflation:

(Note that this is a misnomer: We do not necessarily assume ...

$V(\phi)$

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$\left(\frac{\dot a}{a}\right)^2 \simeq \frac{8\pi G}{3} V(\phi)$$

whose solution during slow roll is

$$a(t) \simeq a_0 \, e^{t\sqrt{\frac{8\pi G}{3} V(\phi)}}$$

(ϕ and $V(\phi)$ change slowly over time in slow roll.)

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

Definition:

The function $H(t) := \frac{\dot a(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

* Thus, during slow roll, we have effectively a slowly varying Hubble parameter!

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

$$H(t) = \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

Remark:

* Since as $V(t)$ decreases also $H(t)$ decreases, inflation predicts that $\delta\phi$ decreases for late horizon crossing modes, i.e., fo

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

The function $H(t) := \frac{\dot{a}(t)}{a(t)}$ is called the Hubble parameter function.

* In the case $a(t) = e^{Ht}$ we recover $H = H(t)$.

* In the case of slow roll inflation, we have

$$H(t) = \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

Remark:

* Since as $V(t)$ decreases also $H(t)$ decreases, inflation predicts that $\delta\phi_k$ decreases for late horizon crossing modes, i.e., for smaller and smaller wavelength modes.

* The WMAP satellite's CMB data for the first time show some small evidence for this

18 / 22

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$H(t) = \sqrt{\frac{8\pi G}{3} V(\phi(t))}$$

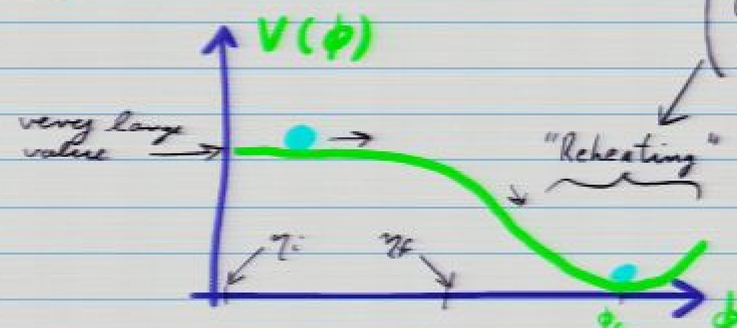
Remark:

- Since as $V(t)$ decreases also $H(t)$ decreases, inflation predicts that $\delta\phi_L$ decreases for late horizon crossing modes, i.e., for smaller and smaller wavelength modes.

- The WMAP satellite's CMB data for the first time show some small evidence for this.

The end of inflation:

(Note that this is a misnomer: We do not necessarily assume the existence of a prior hot phase)



10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width

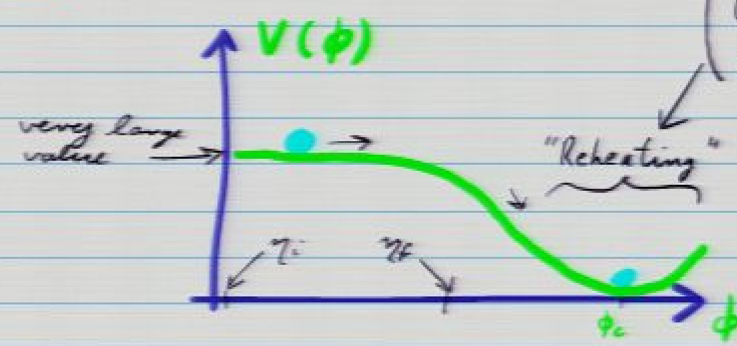
Remark:

* Since as $V(t)$ decreases also $H(t)$ decreases, inflation predicts that $\delta\phi_L$ decreases for late horizon crossing modes, i.e., for smaller and smaller wavelength modes.

* The WMAP satellite's CMB data for the first time show some small evidence for this.

The end of inflation:

(Note that this is a misnomer. We do not necessarily assume the existence of a prior hot phase)



* In the period called "Reheating", the energy of ϕ

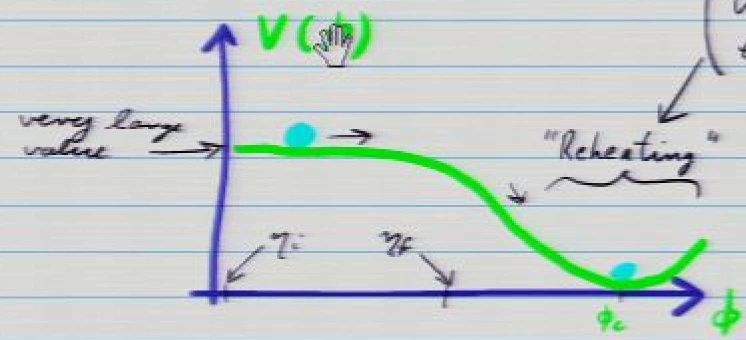
10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width

smaller and smaller wavelength modes.

* The WMAP satellite's CMB data for the first time show some small evidence for this.

The end of inflation:

(Note that this is a misnomer. We do not necessarily assume the existence of a prior hot phase)



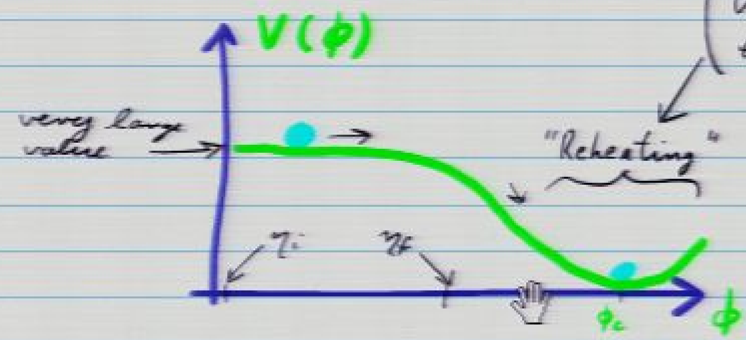
* In the period called "Re-heating", the energy of ϕ is transferred into the mode oscillators of the usual (low mass) particles, i.e., the inflaton particles decay and thereby create a high energetic, i.e. hot

10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help

* The WMAP satellite's CMB data for the first time show some small evidence for this.

The end of inflation:

(Note that this is a misnomer. We do not necessarily assume the existence of a prior hot phase)



* In the period called "Re-heating", the energy of ϕ is transferred into the mode oscillators of the usual (low mass) particles, i.e., the inflaton particles decay and thereby create a high energetic, i.e. hot, plasma of literally all sorts of particles.

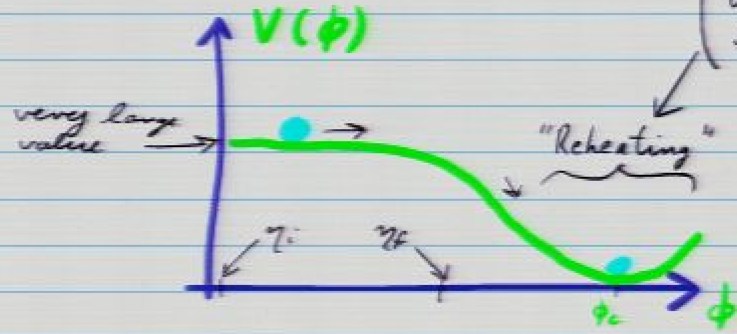
10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

The end of inflation:

(Note that this is a misnomer: We do not necessarily assume the existence of a prior hot phase)



* In the period called "Re-heating", the energy of ϕ is transferred into the mode oscillators of the usual (low mass) particles, i.e., the inflaton particles decay and thereby create a high energetic, i.e. hot, plasma of literally all sorts of particles.

* From thereon, the usual big bang cosmology is assumed to have followed.

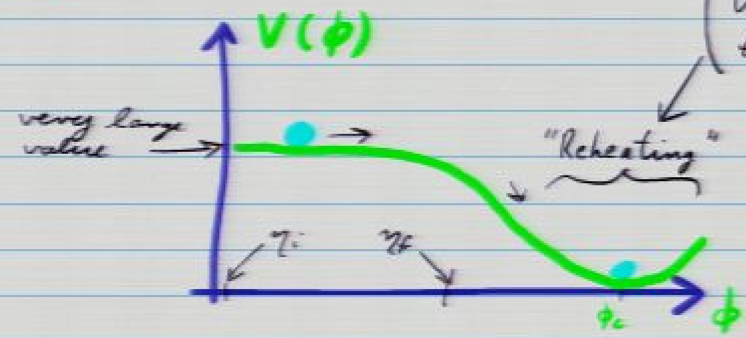
10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

The end of inflation:

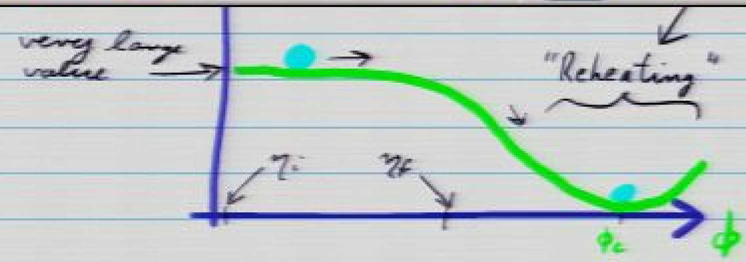
(Note that this is a misnomer: We do not necessarily assume the existence of a prior hot phase)



* In the period called "Re-heating", the energy of ϕ is transferred into the mode oscillators of the usual (low mass) particles, i.e., the inflaton particles decay and thereby create a high energetic, i.e. hot, plasma of literally all sorts of particles.

* From thereon, the usual big bang cosmology is assumed to have followed.

10qft20.jnt - Windows Journal

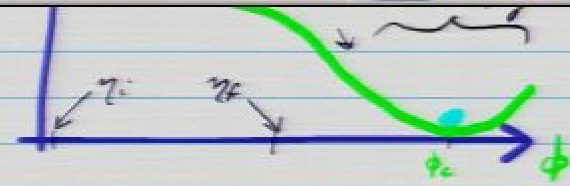


* In the period called "Re-heating", the energy of ϕ is transferred into the mode oscillators of the usual (low mass) particles, i.e., the inflaton particles decay and thereby create a high energetic, i.e. hot, plasma of literally all sorts of particles.

* From thereon, the usual big bang cosmology is assumed to have followed.

Quantum fluctuations

10qft20.jnt - Windows Journal
Edit View Insert Actions Tools Help
Page Width



* In the period called "Re-heating", the energy of ϕ is transferred into the mode oscillators of the usual (low mass) particles, i.e., the inflaton particles decay and thereby create a high energetic, i.e. hot, plasma of literally all sorts of particles.

* From thereon, the usual big bang cosmology is assumed to have followed.

Quantum fluctuations

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Quantum fluctuations

Strategy:

- We assume some $V(\phi)$ and suitable initial conditions which yield classical solutions $\phi_0(t)$ and $a_0(t)$ which exhibit slow roll inflation for a suitable finite time interval.

- We allow small inhomogeneities in the inflaton field:

$$\phi(x,\eta) = \phi_0(\eta) + \varphi(x,\eta) \quad \text{with} \quad |\varphi(x,\eta)| \ll |\phi_0(\eta)|$$

10qft20.jnt - Windows Journal

Quantum fluctuations

Strategy:

- We assume some $V(\phi)$ and suitable initial conditions which yield classical solutions $\phi_0(t)$ and $a_0(t)$ which exhibit slow roll inflation for a suitable finite time interval.

- We allow small inhomogeneities in the inflaton field:

$$\phi(x,\eta) = \phi_0(\eta) + \varphi(x,\eta) \quad \text{with} \quad |\varphi(x,\eta)| \ll |\phi_0(\eta)|$$

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

slow roll inflation for a suitable finite time interval.

- We allow small inhomogeneities in the inflaton field:

$$\phi(x,\eta) = \phi_0(\eta) + \varphi(x,\eta) \quad \text{with} \quad |\varphi(x,\eta)| \ll |\phi_0(\eta)|$$

- We allow also small inhomogeneities in the metric: $g_{\mu\nu}(x,\eta) = a(\eta)\,\eta_{\mu\nu} + \gamma_{\mu\nu}(x,\eta)$

$|\gamma_{\mu\nu}(x,\eta)| \ll 1$

- The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

10qft20.jnt - Windows Journal

- We allow small inhomogeneities in the inflaton field:

$$\phi(x,\eta) = \phi_0(\eta) + \varphi(x,\eta) \quad \text{with} \quad |\varphi(x,\eta)| \ll |\phi_0(\eta)|$$

- We allow also small inhomogeneities in the metric: $g_{\mu\nu}(x,\eta) = a(\eta)\,\eta_{\mu\nu} + \gamma_{\mu\nu}(x,\eta)$

$|\gamma_{\mu\nu}(x,\eta)| \ll 1$

- The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

a) The part of $\gamma_{\mu\nu}$ which can be derived from

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

- We allow also small inhomogeneities in the metric: $g_{\mu\nu}(x,\eta) = a(\eta)\,\eta_{\mu\nu} + \gamma_{\mu\nu}(x,\eta)$

  $|\gamma_{\mu\nu}(x,\eta)| \ll 1$

- The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

  a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

  b) The part of $\gamma_{\mu\nu}$ which can be derived from vector fields

  c) The part of $\gamma_{\mu\nu}$ which is purely tensor.

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

$$\phi(x,\eta) = \phi_0(\eta) + \varphi(x,\eta) \quad \text{with} \quad |\varphi(x,\eta)| \ll |\phi_0(\eta)|$$

- We allow also small inhomogeneities in the metric: $g_{\mu\nu}(x,\eta) = a(\eta)\,\eta_{\mu\nu} + \gamma_{\mu\nu}(x,\eta)$

  $|\gamma_{\mu\nu}(x,\eta)| \ll 1$

- The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

  a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

  b) The part of $\gamma_{\mu\nu}$ which can be derived from

10qft20.jnt - Windows Journal

metric : $g_{\mu\nu}(x,\eta) = a(\eta)\,\eta_{\mu\nu} + \gamma_{\mu\nu}(x,\eta)$

$|\gamma_{\mu\nu}(x,\eta)| \ll 1$

□ The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

b) The part of $\gamma_{\mu\nu}$ which can be derived from vector fields

c) The part of $\gamma_{\mu\nu}$ which is purely tensor.

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

b) The part of $\gamma_{\mu\nu}$ which can be derived from vector fields

c) The part of $\gamma_{\mu\nu}$ which is purely tensor.

This decomposition is convenient because the three types obey independent equations of motion.

Remark:

The scalar part of $\gamma_{\mu\nu}$ and the inflaton

10qft20.jnt - Windows Journal

a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

b) The part of $\gamma_{\mu\nu}$ which can be derived from vector fields

c) The part of $\gamma_{\mu\nu}$ which is purely tensor.

This decomposition is convenient because the three types obey independent equations of motion.

Remark:

The scalar part of $\gamma_{\mu\nu}$ and the inflaton

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

b) The part of $\gamma_{\mu\nu}$ which can be derived from vector fields

c) The part of $\gamma_{\mu\nu}$ which is purely tensor.

This decomposition is convenient because the three types obey independent equations of motion.

Remark:

The scalar part of $\gamma_{\mu\nu}$ and the inflaton

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

$$\left|\gamma_{\mu\nu}(x,\eta)\right| \ll 1$$

☐ The perturbations $\gamma_{\mu\nu}$ of the metric tensor can be decomposed into three types:

a) The part of $\gamma_{\mu\nu}$ which can be derived from scalar functions

b) The part of $\gamma_{\mu\nu}$ which can be derived from vector fields

c) The part of $\gamma_{\mu\nu}$ which is purely tensor.

☐ This decomposition is convenient because the

21 / 22

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

- This decomposition is convenient because the three types obey independent equations of motion.

Remark:

The scalar part of $\chi_{\mu\nu}$ and the inflaton perturbation φ will yield one combined scalar equation of motion (which is similar but not identical to the K.G. eqn.

- We quantize these perturbation fields $\hat{\chi}_{\mu\nu}(x,\eta)$ and $\hat{\varphi}(x,\eta)$ but we keep the $g_{0\mu\nu} = a(\eta)\,\eta_{\mu\nu}$ and $\phi_0(x,\eta)$ as fixed classical background fields.

- We calculate the 3 quantum fluctuation spectra

21 / 22

10qft20.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

- This decomposition is convenient because the three types obey independent equations of motion.

Remark:

The scalar part of $\gamma_{\mu\nu}$ and the inflaton perturbation φ will yield one combined scalar equation of motion (which is similar but not identical to the K.G. eqn.

- We quantize these perturbation fields $\hat{\gamma}_{\mu\nu}(x,\eta)$ and $\hat{\varphi}(x,\eta)$ but we keep the $g_{0\mu\nu} = a(\eta)\,\eta_{\mu\nu}$ and $\phi_0(x,\eta)$ as fixed classical background fields.

- We calculate the 3 quantum fluctuation spectra.

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

- This decomposition is convenient because the three types obey independent equations of motion.

Remark:

The scalar part of $\gamma_{\mu\nu}$ and the inflaton perturbation φ will yield one combined scalar equation of motion (which is similar but not identical to the K.G. eqn.

- We quantize these perturbation fields $\hat{\gamma}_{\mu\nu}(x,\eta)$ and $\hat{\varphi}(x,\eta)$ but we keep the $g_{0\mu\nu} = a(\eta)\eta_{\mu\nu}$ and $\phi_0(x,\eta)$ as fixed classical background fields.

- We calculate the 3 quantum fluctuation spectra.

10qft20.jnt - Windows Journal

File Edit View Insert Actions Tools Help

Note Title 3/22/2006

QFT for Cosmology, Achim Kempf, Winter 10, Lecture 20

Recall:

- In Minkowski space, the fluctuation spectrum reads:

$$\delta\phi_\lambda = \frac{1}{\lambda} \qquad \text{(for } m=0\text{)}$$

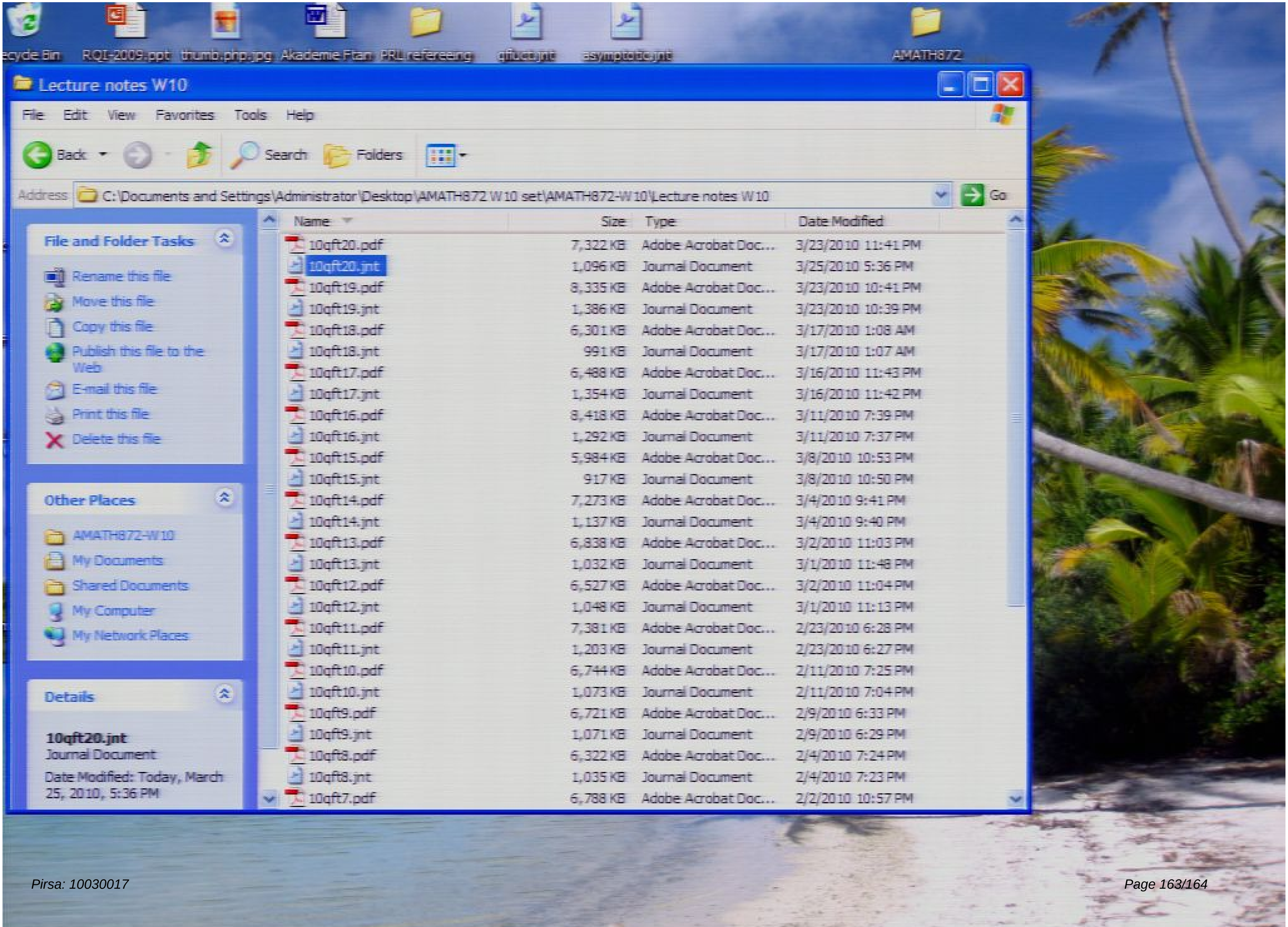
$\rightsquigarrow$ Fluctuations of large spatial extent λ are suppressed.

- We considered a period, $[\eta_i, \eta_f]$, of exponential expansion:

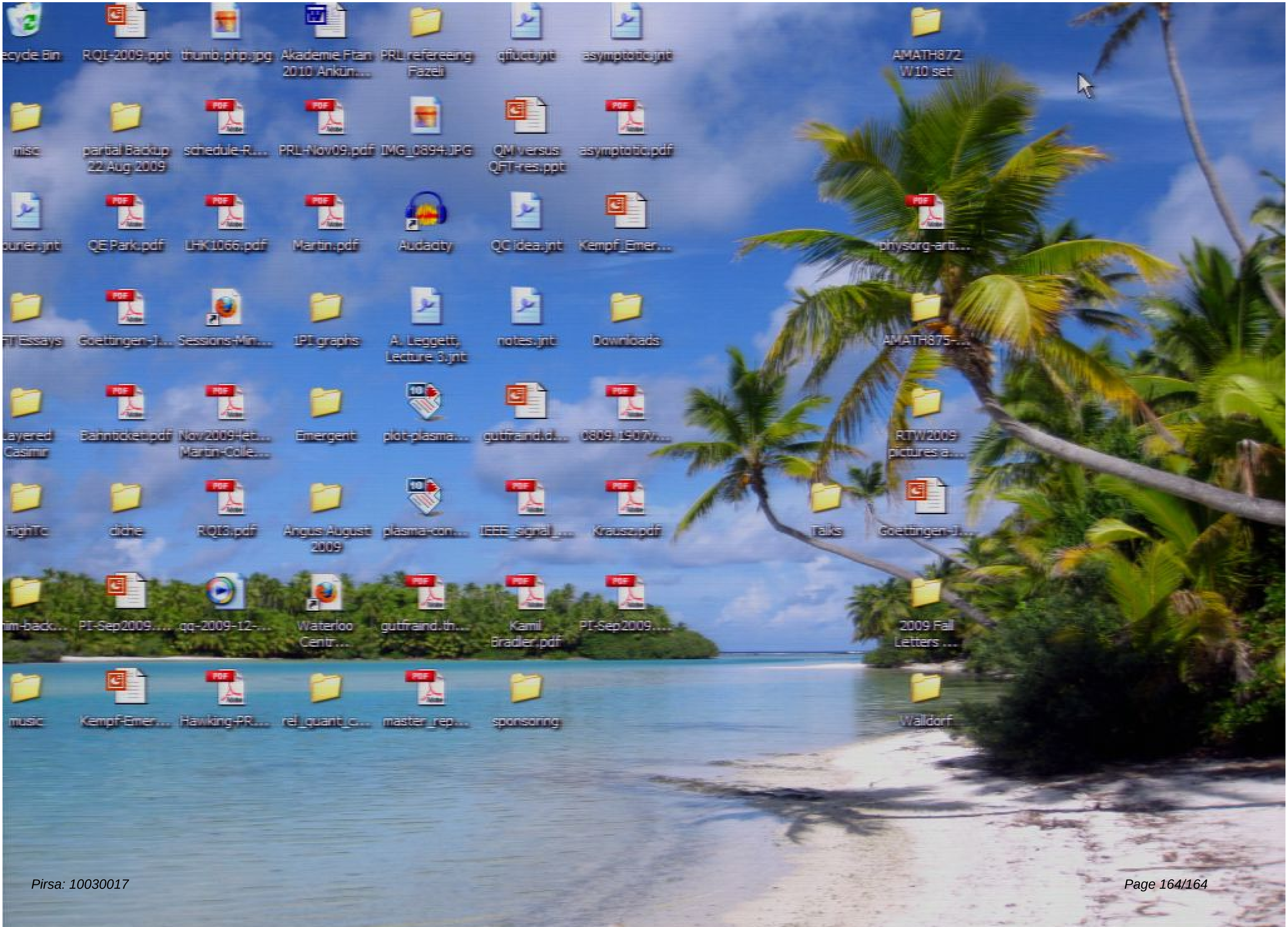
$$a(t) = e^{Ht}$$

i.e.: $a(\eta) = -\frac{1}{H\eta}$

1 / 22



RQI-2009.ppt
thumb.php.jpg
Akademe Plan
PRL refereeing
asymptotic.jnt
AMATH872
Lecture notes W10
File Edit View Favorites Tools Help
Back
Search
Folders
Address C:\Documents and Settings\Administrator\Desktop\AMATH872 W10 set\AMATH872-W10\Lecture notes W10
Go
File and Folder Tasks
Rename this file
Move this file
Copy this file
Publish this file to the Web
E-mail this file
Print this file
Delete this file
Other Places
AMATH872-W10
My Documents
Shared Documents
My Computer
My Network Places
Details
10qft20.jnt
Journal Document
Date Modified: Today, March 25, 2010, 5:36 PM
Name Size Type Date Modified
10qft20.pdf 7,322 KB Adobe Acrobat Doc... 3/23/2010 11:41 PM
10qft20.jnt 1,096 KB Journal Document 3/25/2010 5:36 PM
10qft19.pdf 8,335 KB Adobe Acrobat Doc... 3/23/2010 10:41 PM
10qft19.jnt 1,386 KB Journal Document 3/23/2010 10:39 PM
10qft18.pdf 6,301 KB Adobe Acrobat Doc... 3/17/2010 1:08 AM
10qft18.jnt 991 KB Journal Document 3/17/2010 1:07 AM
10qft17.pdf 6,488 KB Adobe Acrobat Doc... 3/16/2010 11:43 PM
10qft17.jnt 1,354 KB Journal Document 3/16/2010 11:42 PM
10qft16.pdf 8,418 KB Adobe Acrobat Doc... 3/11/2010 7:39 PM
10qft16.jnt 1,292 KB Journal Document 3/11/2010 7:37 PM
10qft15.pdf 5,984 KB Adobe Acrobat Doc... 3/8/2010 10:53 PM
10qft15.jnt 917 KB Journal Document 3/8/2010 10:50 PM
10qft14.pdf 7,273 KB Adobe Acrobat Doc... 3/4/2010 9:41 PM
10qft14.jnt 1,137 KB Journal Document 3/4/2010 9:40 PM
10qft13.pdf 6,838 KB Adobe Acrobat Doc... 3/2/2010 11:03 PM
10qft13.jnt 1,032 KB Journal Document 3/1/2010 11:48 PM
10qft12.pdf 6,527 KB Adobe Acrobat Doc... 3/2/2010 11:04 PM
10qft12.jnt 1,048 KB Journal Document 3/1/2010 11:13 PM
10qft11.pdf 7,381 KB Adobe Acrobat Doc... 2/23/2010 6:28 PM
10qft11.jnt 1,203 KB Journal Document 2/23/2010 6:27 PM
10qft10.pdf 6,744 KB Adobe Acrobat Doc... 2/11/2010 7:25 PM
10qft10.jnt 1,073 KB Journal Document 2/11/2010 7:04 PM
10qft9.pdf 6,721 KB Adobe Acrobat Doc... 2/9/2010 6:33 PM
10qft9.jnt 1,071 KB Journal Document 2/9/2010 6:29 PM
10qft8.pdf 6,322 KB Adobe Acrobat Doc... 2/4/2010 7:24 PM
10qft8.jnt 1,035 KB Journal Document 2/4/2010 7:23 PM
10qft7.pdf 6,788 KB Adobe Acrobat Doc... 2/2/2010 10:57 PM


RQI-2009.ppt
thumb.php.jpg
Akademie Fian 2010 Ankün...
PRL refereeing Fazeli
qfluct.jnt
asymptotic.jnt
AMATH872 W10 set
misc
partial Backup 22 Aug 2009
schedule-R...
PRL-Nov09.pdf
IMG_0894.JPG
QM versus QFT-res.ppt
asymptotic.pdf
QE Park.pdf
LHK1066.pdf
Martin.pdf
Audacity
QC idea.jnt
Kempf_Emer...
physorg-arti...
Goettingen-I...
Sessions-Min...
1PI graphs
A. Leggett, Lecture 3.jnt
notes.jnt
Downloads
AMATH875-...
Layered Casimir
Bahnticket.pdf
Nov2009-let... Martin-Cole...
Emergent
plot-plasma...
gutfraind.d...
0809.1507v...
RTW2009 pictures a...
HighTc
cliche
RQI8.pdf
Angus August 2009
plasma-con...
IEEE_signal_...
Krausz.pdf
Talks
Goettingen-I...
PI-Sep2009....
qq-2009-12-...
Waterloo Centr...
gutfraind.th...
Kamil Bradler.pdf
PI-Sep2009....
2009 Fall Letters ...
music
Kempf-Emer...
Hawking-PR...
rel_quant_c...
master_rep...
sponsoring
Walldorf