Title: Quantum Field Theory for Cosmology - Lecture 17

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Abstract:

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QFT for Cosmology, Achim Kempf, Winter 2010, Lecture 17

From the particle picture to the wave picture

Plan:

1. Summary: How spacetime expansion can create particles.

2. Conclusion: Expansion-induced particle production probably not significant in cosmology.

3. Study: how spacetime expansion amplifies quantum fluctuations of fields

Cosmic microwave background (CMB)

4. Conclusion: this effect appears to explain the properties of the CMB

QFT for Cosmology, Achim Kempf, Winter 2010, Lecture 17

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QFT I for Cosmology, Achim Kempf, Winter 2010, Lecture 17

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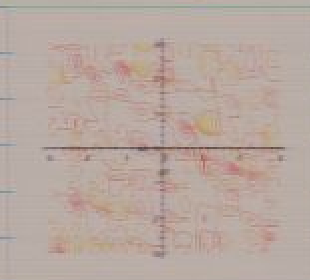

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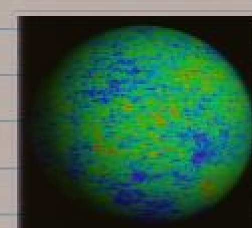

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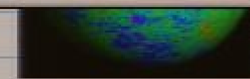

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4. Conclusion: this effect appears to explain the properties of the CMB and the origin of all structure in the universe.

Summary: How spacetime expansion can create particles.

- The quantum field $\hat{\chi}_k(\eta)$ has to obey HC, CCRs and KG eqn.

- The solution can be written in the form

$$\hat{\chi}_k(\eta) = \frac{1}{\sqrt{2}}\left(v_k^*(\eta)\,a_k + v_k(\eta)\,a_{-k}^+\right)$$

where v_k is any mode function, i.e., any complex-valued solution of:

1. $v_k''(\eta) + \overbrace{\omega_k^2(\eta)}^{= k^2 + m^2 a(\eta)^2 - \frac{a''(\eta)}{a(\eta)}}\, v_k(\eta) = 0$ (K.G. eqn)

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and

$$\mathbf{2.}\quad v_k'(\eta)\, v_k^*(\eta) - v_k(\eta)\, v_k^{*\prime}(\eta) = 2i \qquad \text{(Wronskian condition)}$$

which ensures CCRs

- With UV & IR cutoffs, Stone and v. Neumann's theorem guarantees uniqueness of the solution.

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Thus: When using different choices of mode functions, $v_k(\eta)$, $\tilde{v}_k(\eta)$, ... we always represent the same $\hat{x}_k(\eta)$:

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Definition: We define the vectors $|0\rangle$ or $|\tilde{0}\rangle$, ... as those vectors which obey $a_k|0\rangle = 0$ or $\tilde{a}_k|\tilde{0}\rangle = 0$ respectively.

ON Bases: Corresponding Hilbert bases can be built on them:

$|0\rangle$, $a_k^+|0\rangle$, $\frac{1}{\sqrt{n!}}(a_k^+)^n|0\rangle$ etc form ON basis

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* Since the solution space of the KG equation is 2-dimensional, any pair v_k, v_k^* spans it.

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* In general, the vacuum state can be a different vector at different times, which means that particles can be created & destroyed.

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* In general, the vacuum state can be a different vector at different times, which means that particles can be created & destroyed.

* If, at a time η, there exists a no-particle state, $|\text{vacuum at } \eta\rangle$, is it the $|0\rangle$ arising with a suitable mode function v_k? It depends:

* Yes, at least if the evolution is adiabatic around η. Then, it is the mode function v_k which is specified (up to a phase $e^{i\theta}$) by:

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* Why identify the vacuum state using AV?

Thus, we should predict no particle production in the interval $[\eta_1, \eta_2]$ →

o Assume the evolution is adiabatic (at least for the mode k in question) in an interval $[\eta_1, \eta_2]$.

o Then, Eqs. AV are obeyed (and only equations AV) by a single mode function v_k

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Then we should predict)

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Thus, we should predict no particle production in the interval $[\eta_1, \eta_2]$

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and only equations AV

o Then, Eqs. AV are obeyed by a single mode function v_k for all times in the interval of adiabaticity $[\eta_1, \eta_2]$, namely by:

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⇒ When adopting vacuum identification through AV one (correctly) predicts no particle production during adiabatic evolu...

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* Ground state of harm. osc. at η depends only on its momentary frequency $\omega_k(\eta)$.
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$$\cdot\; v_k'(\eta) = \omega_k(\eta) + \frac{i\,\omega_k'(\eta)}{\ldots}$$

← slight frequency shift

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From the particle picture to the wave picture

So far: Studied the effect of spacetime expansion on the

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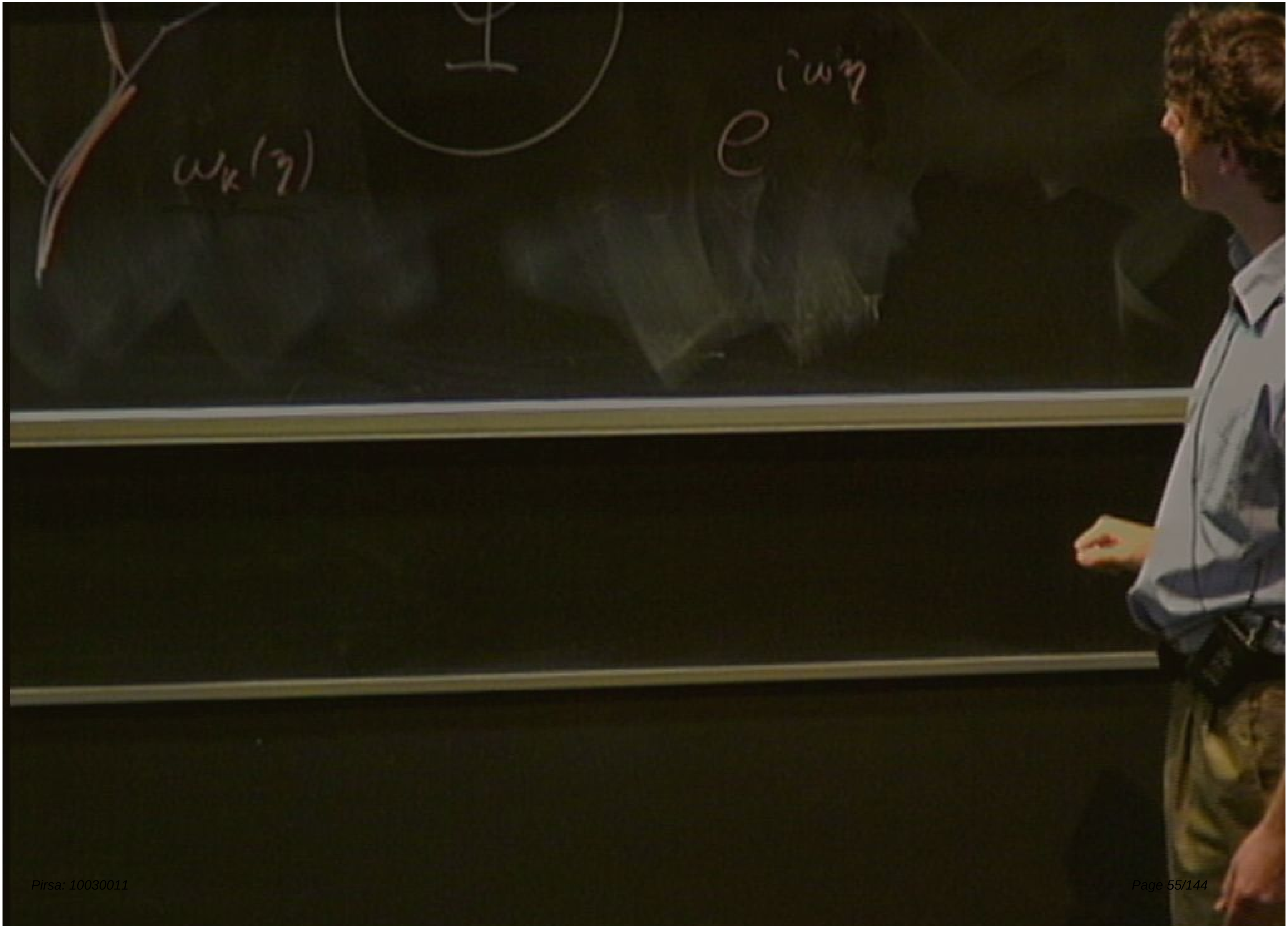
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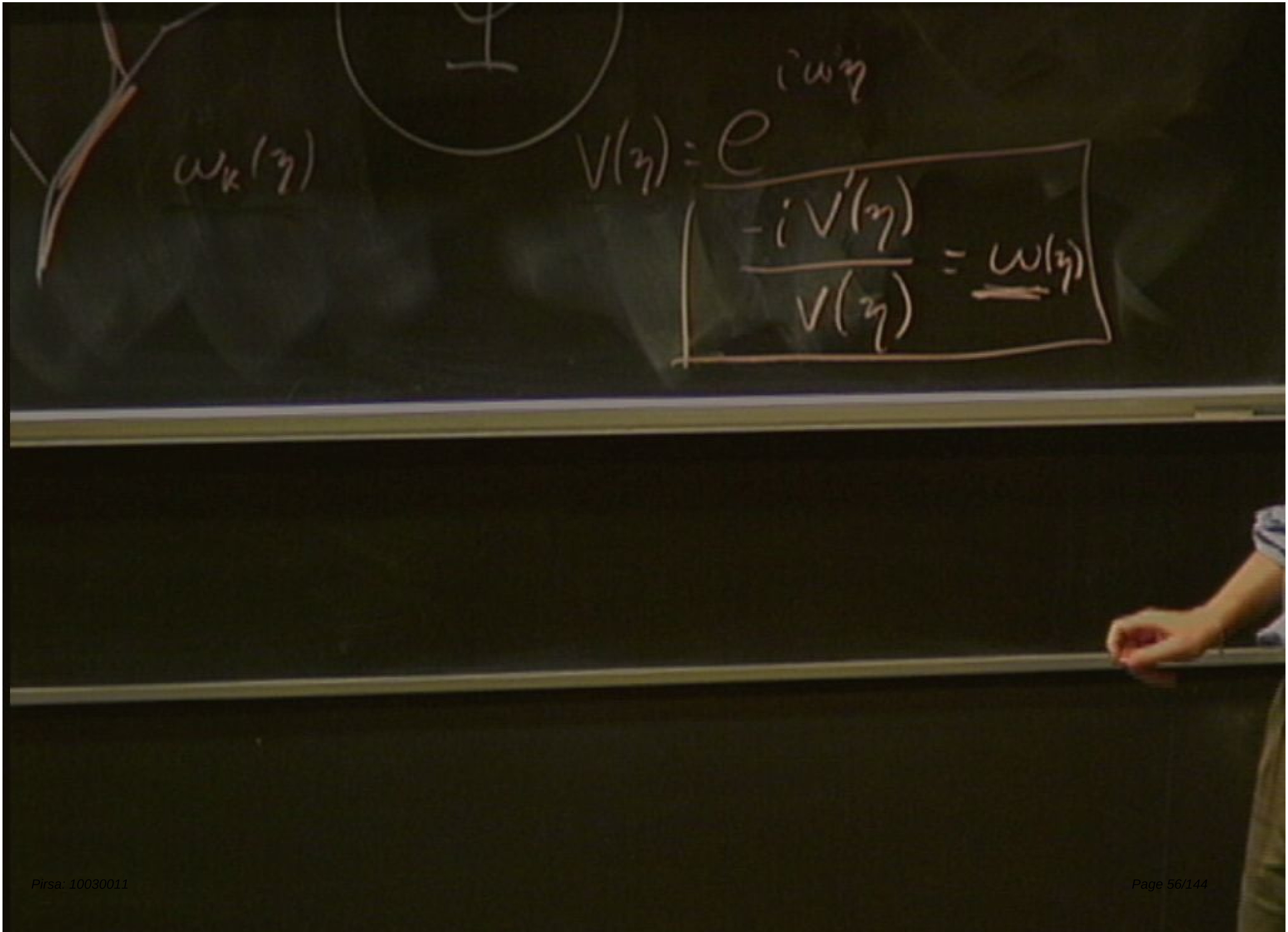
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* Else, e.g., if evolution non-adiabatic or even $\omega_k^2(\eta) < 0$, then we don't know how to identify the vacuum state, and in the latter case it probably does not exist.

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(Within the adiabatic approximation)

o Thus: Eqs. AV yield the same vector, the $|0\rangle$ belonging to v_k, at all times $\eta \in [\eta_1, \eta_2]$.

$\Rightarrow$ When adopting vacuum identification through AV one (correctly) predicts no particle production during adiabatic evolution.

Experimental evidence?

* Astronomical observations:

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* Astronomical observations:
  - The universe is currently evolving adiabatically
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* Recall:
  - We tried the idea vacuum = lowest energy state
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Intuition?

* Ground state of harm. osc. at η depends only on its momentary frequency $\omega_k(\eta)$.
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$$-i\frac{v_k'}{v_k}(\eta) = \omega_k(\eta) + \frac{i}{2}\frac{\omega_k'(\eta)}{\omega_k^2(\eta)}$$

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(Recall fixed oscillator: $v_k = e^{i\omega\eta}$, $v_k' = e^{i\omega\eta} i\omega$ $\Rightarrow -i\frac{v_k'}{v_k} = \omega$)

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The effect of spacetime expansion on field's vacuum fluctuations

Strategy:

a. Consider the typical amplitude of quantum fluctuations as a function of their spatial size.

b. See how this relationship is affected by cosmic expansion.

Definition:

- Consider a real-valued function $f: \mathbb{R}^3 \to \mathbb{R}$ and a time η_0.
- Then, we define the state $|f\rangle$ (normalized) as the joint eigenstate of all operators $\hat{\phi}(x, \eta_0)$ with eigenvalues $f(x)$:

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Quantum fluctuations:

- There are no quantum fluctuations of the quantum field $\hat{\phi}$ if the system is in such a state $|f\rangle$:

Expectation value: $\bar{\phi}(x,\eta_0) = \langle f|\hat{\phi}(x,\eta_0)|f\rangle$

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Variance: $\Delta\phi^2(x,\eta_0) = \langle f|\left(\hat{\phi}(x,\eta_0) - \bar{\phi}(x,\eta_0)\right)^2|f\rangle$

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Exercise: Show this.

Hint: For these states, $\Delta\phi = 0$, and so $\Delta\pi^\phi = \infty$

But $\hat{H}^{(x)}$ contains a term $\hat{\pi}^2 \ldots$

- Crucial consequence:

Exercise:

What is the analogue of this observation in the case of the

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- Assume the system is in some finite energy state $|\Psi\rangle$.

- Assume at time η_0 we could measure $\hat{\phi}(x,\eta_0)$ for all $x \in \mathbb{R}^3$.

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<u>Recall Minkowski case:</u>

If the system is in the vacuum state, then:

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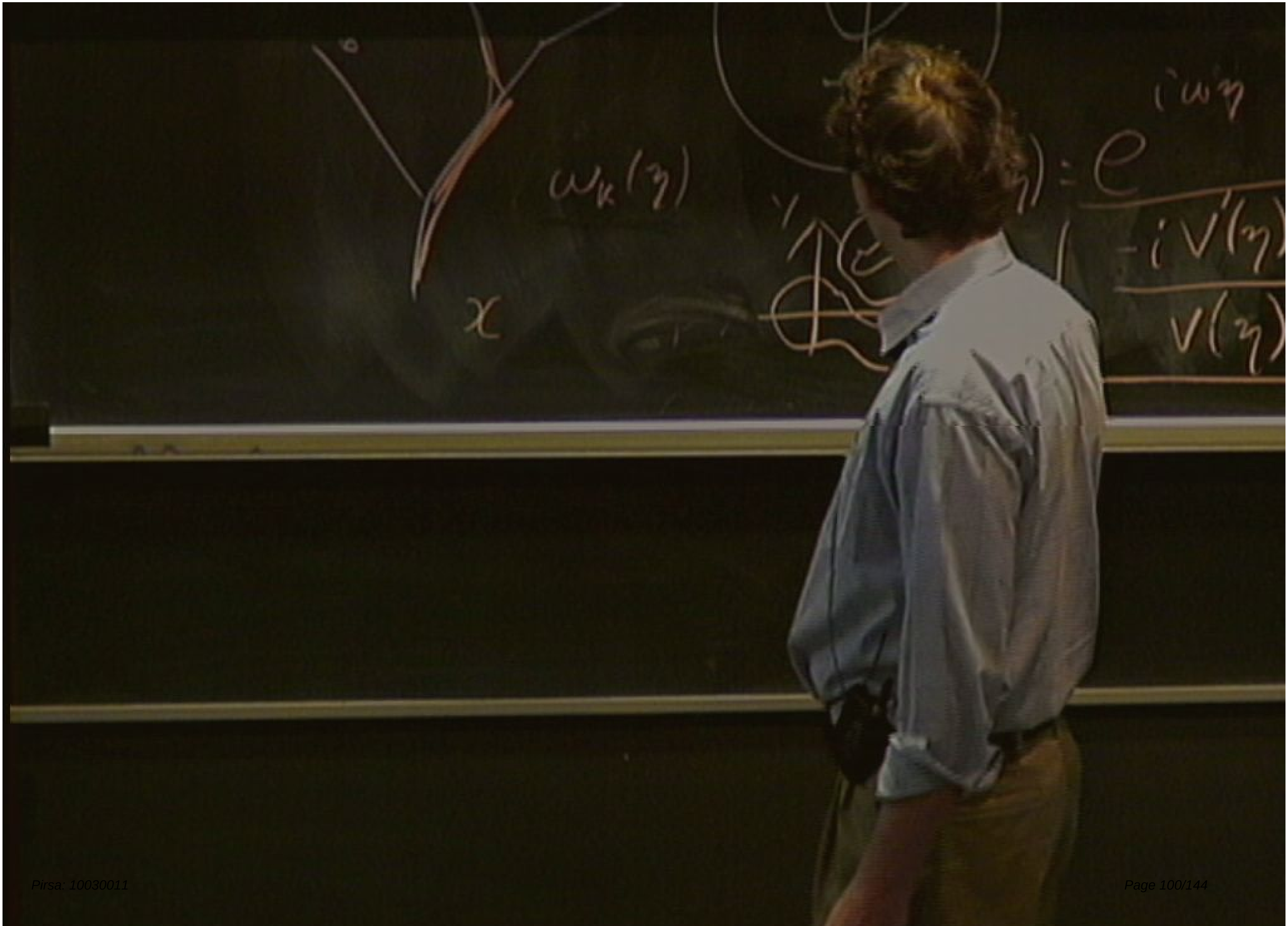
ωk(η)
x
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-iV'(η)
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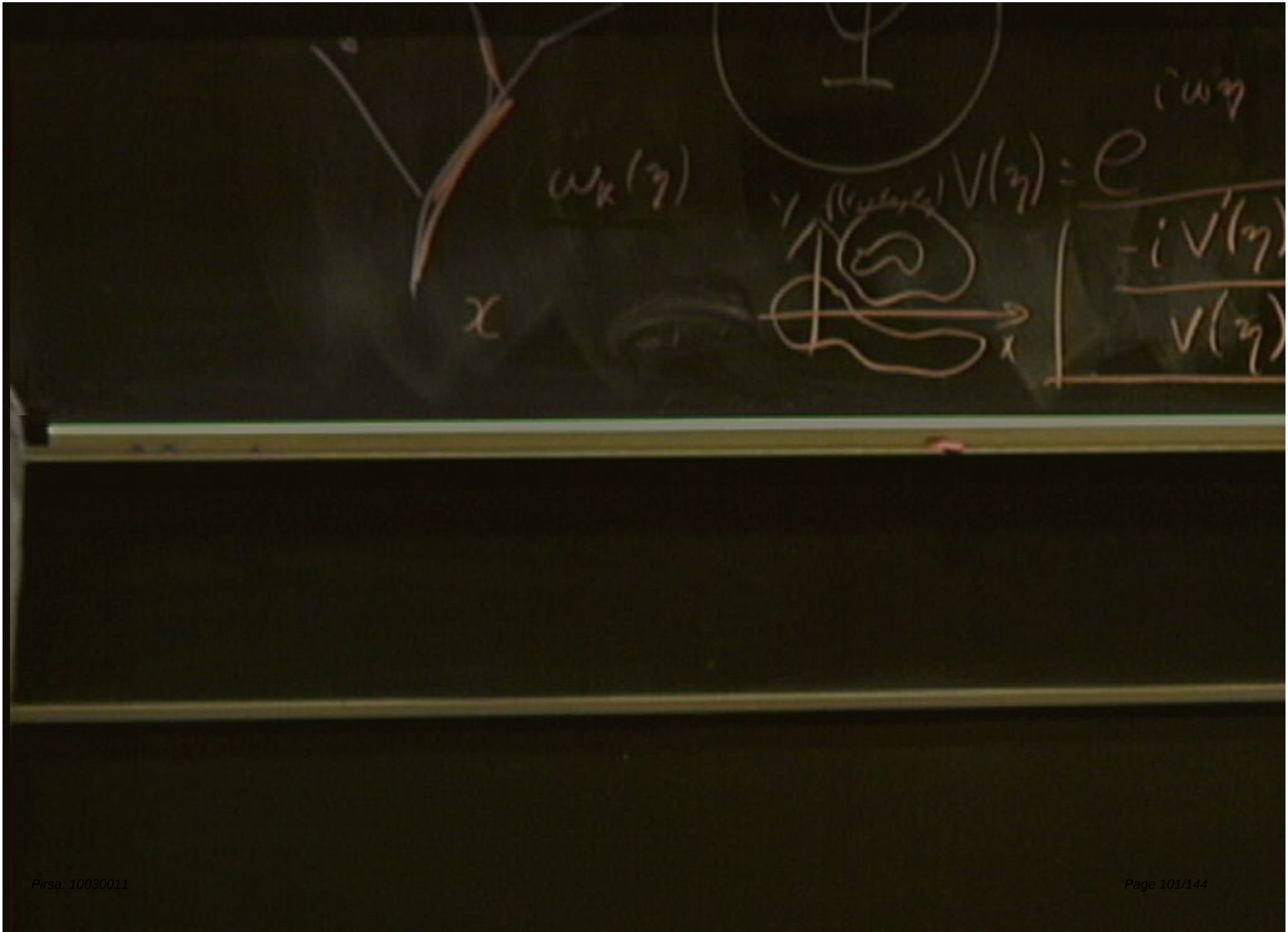

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Realistically, we could measure an average of the values of $\hat{\phi}$ over regions $B \subset \mathbb{R}^3$ of not too small volume V,

$$\hat{\phi}_B(\eta) := \int_{\mathbb{R}^3} \hat{\phi}(x,\eta)\, W(x)\, d^3x ,$$

with some "window function" W which obeys:

$$W(x) = \begin{cases} \approx 0 & \text{for all } x \notin B \\ \approx V^{-1} & \text{for all } x \in B \end{cases}$$

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Note:

In proper coordinates, this is a box of increasing size $a(\eta)L \times a(\eta)L \times a(\eta)L$.

- Assume that at η_0 the system's state, $|\Omega\rangle$, is the vacuum state:

$$|\Omega\rangle = |vac_{\eta_0}\rangle = |0\rangle$$

↑ for suitable choice of mode functions, $\{v_k\}$

- The expectation value of the region-averaged field at a time $\eta \geq \eta_0$:

$$\bar{\phi}_B(\eta) = \langle \Omega | \hat{\phi}_B(\eta) | \Omega \rangle = \langle vac_{\eta_0} | \hat{\phi}_B(\eta) | vac_{\eta_0} \rangle$$

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In proper coordinates, this is a box of increasing size $a(\eta)L \times a(\eta)L \times a(\eta)L$.

- Assume that at η_0 the system's state, $|\Omega\rangle$, is the vacuum state:

$$|\Omega\rangle = |vac_{\eta_0}\rangle = |0\rangle$$

↑ for suitable choice of mode functions, $\{v_k\}$

- The expectation value of the region-averaged field at a time $\eta \geq \eta_0$:

$$\bar{\phi}_B(\eta) = \langle \Omega | \hat{\phi}_B(\eta) | \Omega \rangle = \langle vac_{\eta_0} | \hat{\phi}_B(\eta) | vac_{\eta_0} \rangle$$

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Application to FRW spacetime

- Choose conformal time η and comoving coordinates x.

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$$= 0$$

⇒ The average amplitude of $\hat{\phi}_B$ vanishes in the vacuum st[...] 15 / 19

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$\Rightarrow$ The average amplitude of $\hat{\phi}_B$ vanishes in the vacuum state.

The vacuum fluctuations

While $\overline{\Phi_B}(\eta)$ vanishes, measurement outcomes for $\hat{\phi}_B(\eta)$ are not fully predictable because subject to fluctuations around zero with this standard deviation:

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While $\bar{\phi}_B(\eta)$ vanishes, measurement outcomes for $\hat{\phi}_B(\eta)$ are not fully predictable because subject to fluctuations around zero with this standard deviation:

$$\Delta \phi_B^2(\eta) = \langle \Omega | \left(\hat{\phi}_B(\eta) - \cancel{\bar{\phi}_B(\eta)} \right)^2 | \Omega \rangle = \langle 0 | \hat{\phi}_B(\eta)^2 | 0 \rangle$$

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$= \ldots$ Exercise: fill in the steps

$$= \frac{1}{2a(\eta)^2}\frac{1}{(2\pi)^3}\int_{\mathbb{R}^3}|v_k(\eta)|^2\,\tilde{W}(k)\,d^3k$$

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The vacuum fluctuations

While $\bar{\phi}_8(\eta)$ vanishes, measurement outcomes for $\hat{\phi}_8(\eta)$ are not fully predictable because subject to fluctuations around zero with this standard deviation:

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$$= \frac{1}{a(\eta)^2} \langle 0 | \left(\int_{\mathbb{R}^3} \hat{x}(x,\eta)\, W(x)\, d^3x \right)^2 | 0 \rangle$$

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Fourier transform of

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Fourier transform of the window function $W(x)$.

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Fourier transform of the window function $W(x)$.

Assume for simplicity that B is spherical with radius L. Then use spherical coordinates:

$$= \frac{1}{2a(\eta)^2} \frac{1}{(2\pi)^3} \int_0^\infty k^2\, 4\pi\, |v_k(\eta)|^2\, \widetilde{W}(k)\, dk$$

$k = \sqrt{k_1^2 + k_2^2 + k_3^2}$

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Approximation: Consider that: $w(x)$

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$$a(\eta)^2 \quad \int_{\mathbb{R}^3}$$

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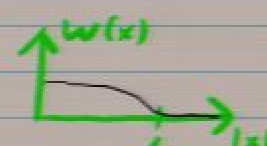

typical scale is L

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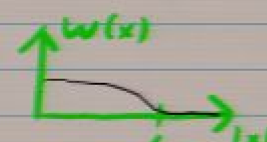

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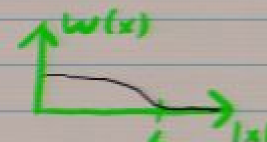

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(using Fourier) $\Rightarrow$ We can assume that, roughly

$$\tilde{W}(k) \approx 0 \text{ for } |k| > \frac{2\pi}{L}$$

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Example: If $W(x) =$ … then $\tilde{W}(k) =$ …

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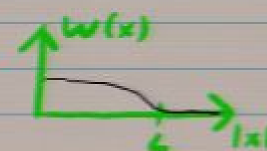

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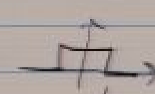
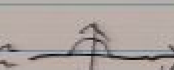
Example: If $W(x) =$  then $\tilde{W}(k) =$ 

and we approximate that $\tilde{W}(k) \approx$ 

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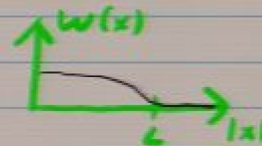

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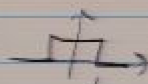
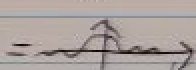
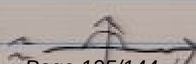
Example: If $W(x) =$  then $\tilde{W}(k) =$ 

and we approximate that $\tilde{W}(k) \approx$ 

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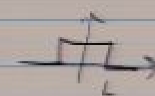
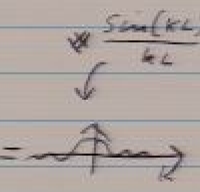
Example: If $W(x) =$  then $\tilde{W}(k) =$ 

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$$\Longrightarrow \quad \Delta\phi_\delta^2(\eta) \approx \text{const.} \frac{1}{a(\eta)^2} \int_0^{2\pi/L} k^2 \, |v_k(\eta)|^2 \, dk$$

In the integral, the values of $|v_k(\eta)|^2$ for small k are suppressed by k^2.

$\Rightarrow$ Can approximately replace $|v_k(\eta)|$ by its value at $k = 2\pi/L$:

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$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in

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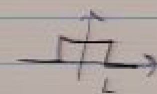
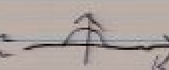
Example: If $W(x) =$  then $\tilde{W}(k) =$ 

and we approximate that $\tilde{W}(k) \approx$ 

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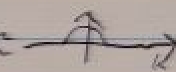
and we approximate that $\tilde{w}(k) \approx$ 

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In the integral, the values of $|v_k(\eta)|^2$ for small k are suppressed by k^2.

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$$\Delta\phi_B^2(\eta) \approx \text{const.}\, \frac{1}{a(\eta)^2} \int_0^{2\pi/L} k^2 \, |v_{2\pi/L}(\eta)|^2 \, dk$$

$\Rightarrow$

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and we approximate that $\omega(k) \approx \dots$

$$\Longrightarrow \quad \Delta\phi_\delta^2(\eta) \approx \text{const.} \frac{1}{a(\eta)^2} \int_0^{2\pi/L} k^2 \, |v_k(\eta)|^2 \, dk$$

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$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in

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$$\Longrightarrow \quad \Delta\phi_B^2(\eta) \approx \text{const.}\ \frac{1}{a(\eta)^2}\int_0^{2\pi/L} k^2\,|v_k(\eta)|^2\,dk$$

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$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in

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$$\Longrightarrow \quad \Delta\phi_{\mathcal{B}}^2(\eta) \approx \text{const.}\, \frac{1}{a(\eta)^2} \int_0^{2\pi/L} k^2 \, |v_k(\eta)|^2 \, dk$$

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$\Rightarrow$ Can approximately replace $|v_k(\eta)|$ by its value at $k = 2\pi/L$:

$$\Delta\phi_{\mathcal{B}}^2(\eta) \approx \text{const.}\, \frac{1}{a(\eta)^2} \int_0^{2\pi/L} k^2 \, |v_{1/L}(\eta)|^2 \, dk$$

$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in vacuum state at time η:

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$$\Longrightarrow \quad \Delta\phi_B^2(\eta) \approx \text{const.} \frac{1}{a(\eta)^2} \int_0^{2\pi/L} k^2 \, |v_k(\eta)|^2 \, dk$$

In the integral, the values of $|v_k(\eta)|^2$ for small k are suppressed by k^2.

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$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in vacuum state at time η:

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$$\Longrightarrow \quad \Delta\phi_B^2(\eta) \approx \text{const.}\, \frac{1}{a(\eta)^2} \int_0 k^2\, |v_k(\eta)|^2\, dk$$

In the integral, the values of $|v_k(\eta)|^2$ for small k are suppressed by k^2.

$\Rightarrow$ Can approximately replace $|v_k(\eta)|$ by its value at $k = 2\pi/L$:

$$\Delta\phi_B^2(\eta) \approx \text{const.}\, \frac{1}{a(\eta)^2} \int_0^{2\pi/L} k^2\, |v_{1/L}(\eta)|^2\, dk$$

$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in vacuum state at time η :

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$\Rightarrow$ Can approximately replace $|v_k(\eta)|$ by its value at $k = 2\pi/L$:

$$\Delta\phi_L^2(\eta) \approx \text{const.} \frac{1}{a(\eta)^2} \int_0^{2\pi/L} k^2 |v_k(\eta)|^2 dk$$

$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in vacuum state at time η :

$$\boxed{\Delta\phi_L^2(\eta) \approx \text{const.} \frac{1}{a(\eta)^2} k^3 |v_k(\eta)|^2 \Big|_{\text{at } k=2\pi/L}}$$

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$$\Delta\phi_L^2(\eta) \approx \text{const.}\,\frac{1}{a(\eta)^2}\int_0^{2\pi/L} k^2\,|v_{1/L}(\eta)|^2\,dk$$

$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in vacuum state at time η:

$$\boxed{\begin{gathered}\Delta\phi_L^2(\eta) \approx \text{const.}\,\frac{1}{a(\eta)^2}\,k^3\,|v_k(\eta)|^2\Big|_{\text{at } k=2\pi/L}\\[1em] \Delta\phi_L^2(\eta) \approx \text{const}\,\frac{1}{a(\eta)^2}\,\frac{(2\pi)^3}{L^3}\,|v_{\frac{2\pi}{L}}(\eta)|^2\end{gathered}}$$

Note: The expansion of the universe enters through $a(\eta)$ and $v_k($ ...

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$$\Delta\phi_{\mathcal{B}}^2(\eta) \approx \text{const.}\ \frac{1}{a(\eta)^2}\int_0^{2\pi/L} k^2\,|v_{1/k}(\eta)|^2\,dk$$

$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in vacuum state at time η:

$$\boxed{\begin{aligned}\Delta\phi_{\mathcal{B}}^2(\eta) &\approx \text{const.}\ \frac{1}{a(\eta)^2}\, k^3\, |v_k(\eta)|^2 \Big|_{\text{at } k=2\pi/L}\\ \Delta\phi_{\mathcal{B}}^2(\eta) &\approx \text{const}\ \frac{1}{a(\eta)^2}\,\frac{(2\pi)^3}{L^3}\,\left|v_{\frac{2\pi}{L}}(\eta)\right|^2\end{aligned}}$$

Note: The expansion of the universe enters through $a(\eta)$ and $v_k(\eta)$.

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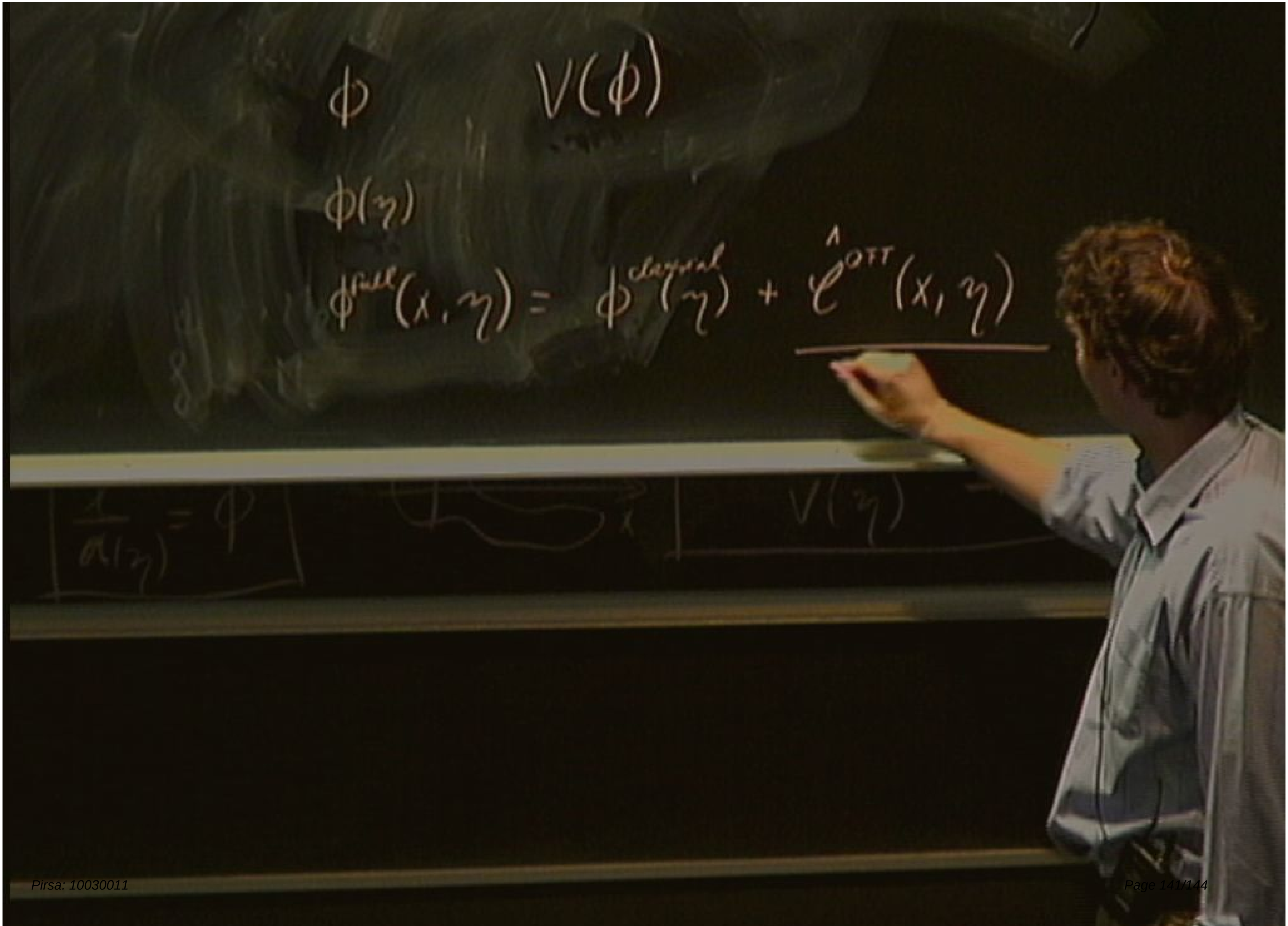
$$\Delta\phi_\delta^2(\eta) \approx \text{const.}\,\frac{1}{a(\eta)^2}\int_0^{2\pi/L} k^2\,|v_{1/L}(\eta)|^2\,dk$$

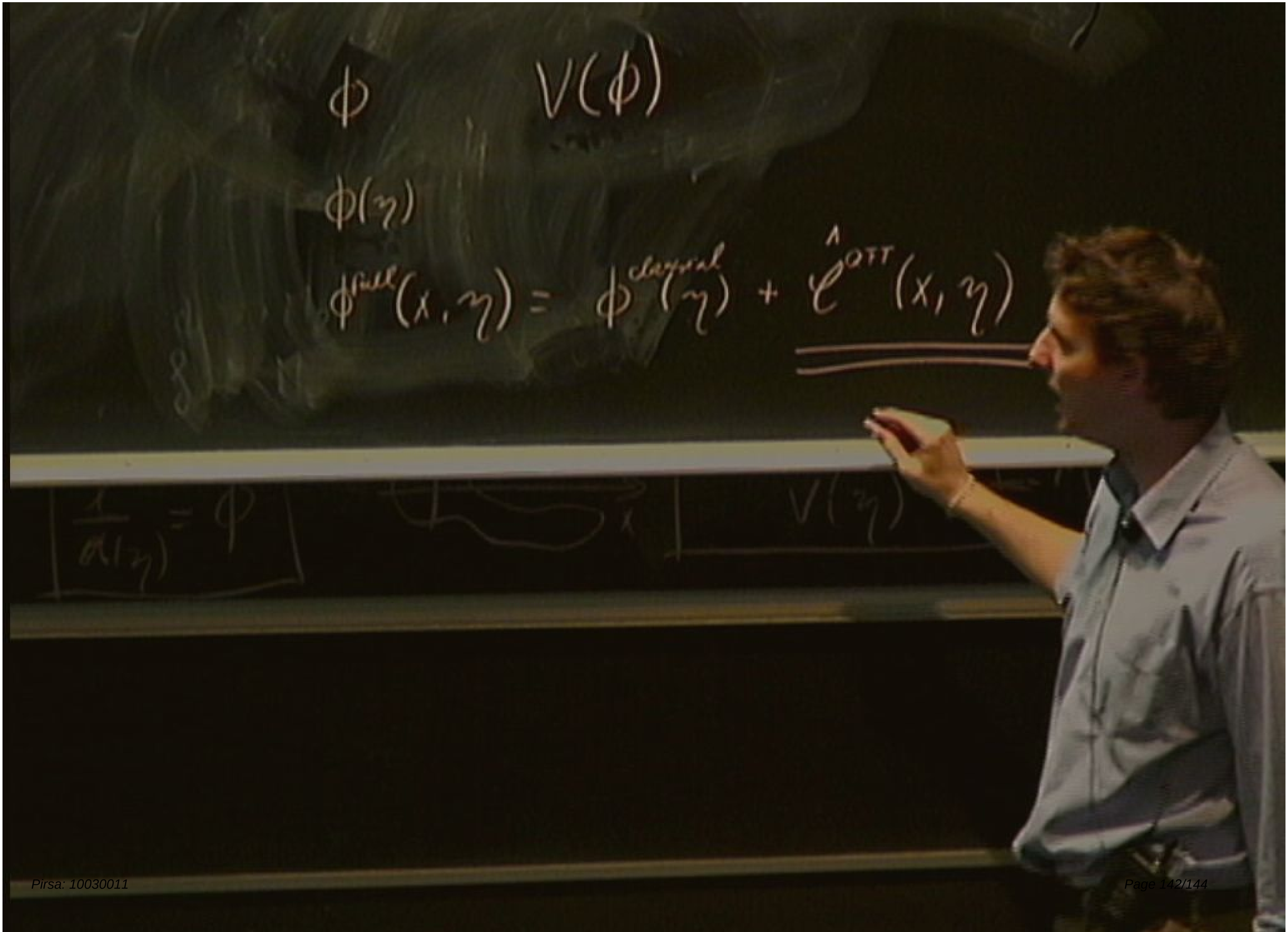
$\Rightarrow$

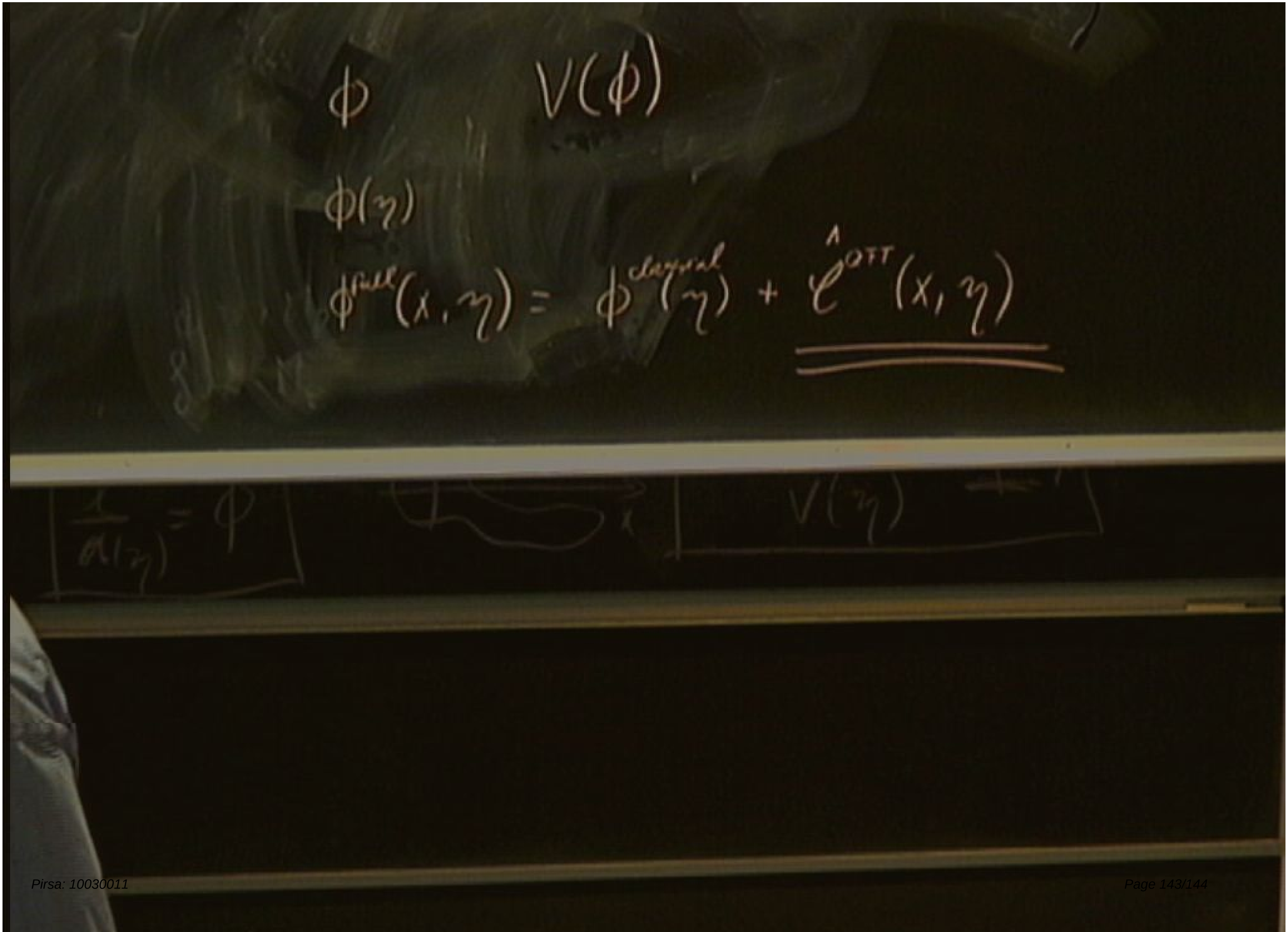
Typical amplitude of fluctuations of size L at time η, if system is in vacuum state at time η:

$$\boxed{\begin{aligned} \Delta\phi_\delta^2(\eta) &\approx \text{const.}\,\frac{1}{a(\eta)^2}\,k^3\,|v_k(\eta)|^2\Big|_{\text{at } k=2\pi/L} \\ \Delta\phi_\delta^2(\eta) &\approx \text{const}\,\frac{1}{a(\eta)^2}\,\frac{(2\pi)^3}{L^3}\,|v_{\frac{2\pi}{L}}(\eta)|^2 \end{aligned}}$$

Note: The expansion of the universe enters through $a(\eta)$ and $v_k(\eta)$...


ϕ
$V(\phi)$
$\phi(\eta)$
$\phi^{full}(x, \eta) = \phi^{classical}(\eta) + \hat{\varphi}^{QFT}(x, \eta)$
$\frac{1}{a(\eta)} = \phi$
$V(\eta)$


φ
V(φ)
φ(η)
φ^full(x, η) = φ^classical(η) + ψ̂^QFT(x, η)
1/a(η) = φ
x
V(η)


ϕ
$V(\phi)$
$\phi(\eta)$
$\phi^{full}(x,\eta) = \phi^{classical}(\eta) + \hat{\phi}^{QFT}(x,\eta)$
$\frac{1}{a(\eta)} = \phi$
$V(\eta)$

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$$\Delta\phi_B^2(\eta) \approx \text{const.}\,\frac{1}{a(\eta)^2}\int_0^{2\pi/L} k^2\,|v_{1/L}(\eta)|^2\,dk$$

$\Rightarrow$

Typical amplitude of fluctuations of size L at time η, if system is in vacuum state at time η:

$$\boxed{\begin{aligned}&\Delta\phi_B^2(\eta) \approx \text{const.}\,\frac{1}{a(\eta)^2}\,k^3\,|v_k(\eta)|^2\Big|_{\text{at } k=2\pi/L}\\ &\Delta\phi_B^2(\eta) \approx \text{const}\,\frac{1}{a(\eta)^2}\,\frac{(2\pi)^3}{L^3}\,|v_{\frac{2\pi}{L}}(\eta)|^2\end{aligned}}$$

Note: The expansion of the universe enters through $a(\eta)$ and $v_k(\eta)$...

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