Title: Quantum Field Theory for Cosmology - Lecture 15

Date: Mar 09, 2010 04:00 PM

URL: http://pirsa.org/10030010

Abstract:

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QFT for Cosmology, Achim Kempf, Winter 10, Lecture 15

Solving the quantized K.G. eqn. on FRW spacetimes

Recall:

1.) We obtain the solution $\hat{\phi}(x,t)$ through the ansatz

$$\hat{\phi}(x,t) = \sum_k u_k(x,t)\, a_k + u_k^*(x,t)\, a_k^+$$

* if we use operators a_k obeying $[a_k, a_{k'}^+] = \delta^3(k-k')$ and

* if we find classical solutions $\{u_k(x,t)\}$ of the K.G. eqn., called mode functions, which obey:

$$\sqrt{|g|}\, g^{0\nu} \sum_k \left(u_k(x,t) \frac{\partial}{\partial x^\nu} u_k^*(x',t) - u_k^*(x,t) \frac{\partial}{\partial x^\nu} u_k(x',t) \right) = i\, \delta^3(\vec{x} - \vec{x}') \quad (C)$$

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2.) Then, we can use the $\{a_k\}$ to build a convenient basis in the Hilbert space:

- Namely: $|0\rangle$ is the vector obeying $a_k|0\rangle = 0$
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3.) Choosing a different set of classical solutions $\{\tilde{u}_k(x,t)\}$ which obey (C) yields the same $\hat{\phi}(x,t)$, namely

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Application to FRW spacetime

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- For convenience (namely, to avoid a "friction"-type term) we aim to solve not for $\hat{\phi}(x,t)$ directly, but instead for:

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- **Define:** $\hat{\chi}_k(\eta) := \int \frac{1}{\ldots}\, \hat{\chi}(\eta,x)\, e^{-ikx}\, d^3x$

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i.e.: $$\hat{x}(\eta,x) = \int \frac{1}{(2\pi)^{3/2}} \hat{x}_k(\eta) e^{ikx} d^3k$$

Analogously:

$$\hat{\pi}^{(x)}_k(\eta) := \int \frac{1}{(2\pi)^{3/2}} \hat{\pi}^{(x)}(\eta,x) e^{-ikx} d^3x$$

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Thus, in terms of $\hat{x}_k(\eta)$, the K.G. eqn. reads:

$$\boxed{\hat{x}_k''(\eta) + \left(k^2 + m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)}\right) \hat{x}_k(\eta) = 0} \quad \text{(EoM)}$$

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Notice:

The frequency $\omega_k(\eta)$ may become imaginary, namely if $a''(\eta)$ is large enough, i.e., if the expansion is rapid enough. Note that the discriminant also depends on k, i.e., some modes may have imaginary frequencies while others don't.

Exercise:

* Show that $\hat{x}_k^+(\eta) = \hat{x}_{-k}(\eta)$, $\hat{\pi}_k^{(x)+}(\eta) = \hat{\pi}_{-k}^{(x)}(\eta)$ (HC)

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- In order to solve EoM, HC, CCR for $\hat{x}_k(\eta)$, we
  we make this ansatz:

convenient later

$$\hat{x}_k(\eta) := \frac{1}{\sqrt{2}}\left(v_k^*(\eta)\, a_k + v_k(\eta)\, a_{-k}^+\right) \qquad (A)$$

- Exercise: What are the mode functions $u_k(\eta,x)$ in terms of the functions $v_k(\eta)$?

- Proposition: The ansatz (A) solves

  1.) the hermiticity condition (HC) by construction.

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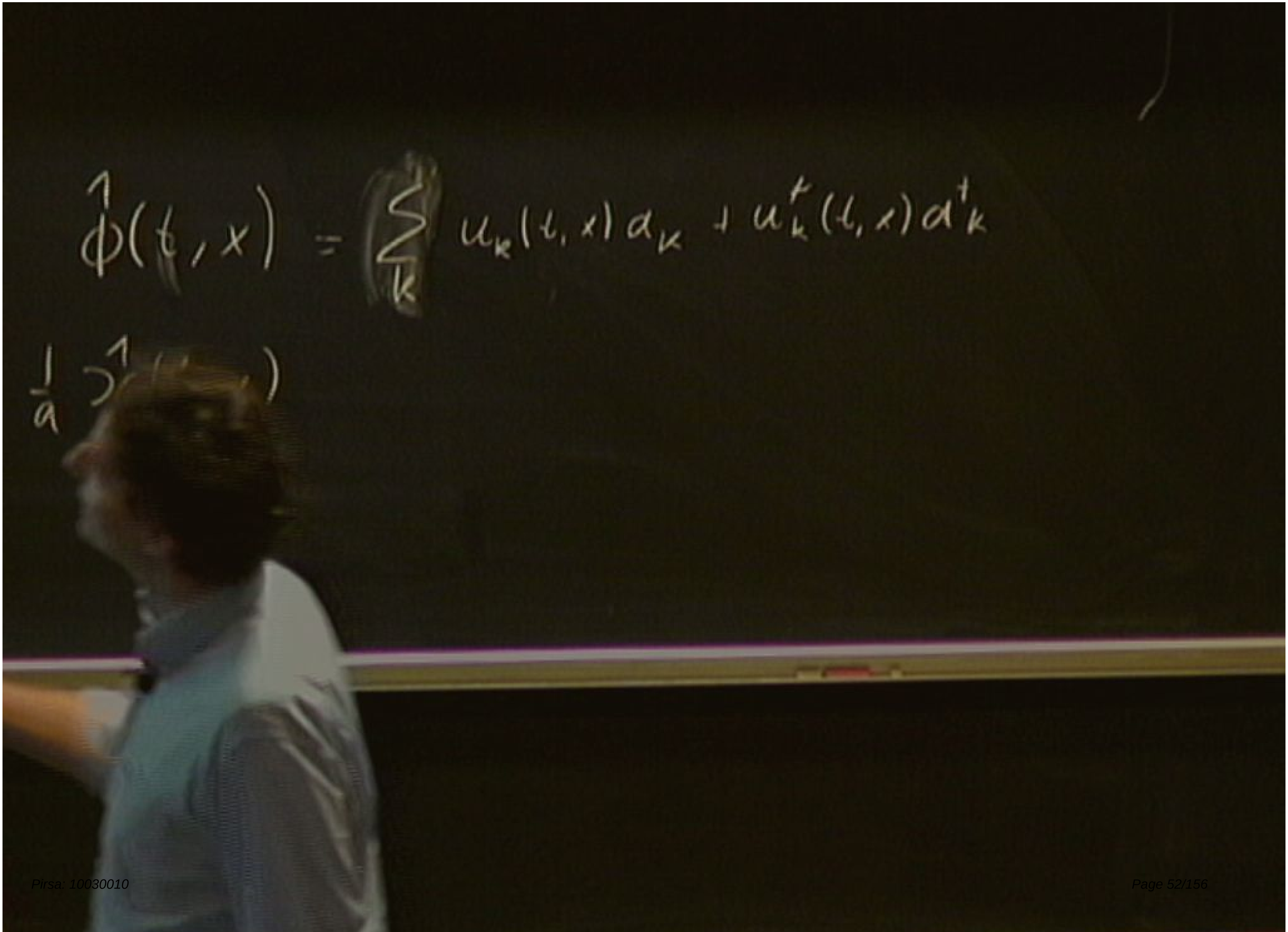
$$\hat{\phi}(t,x) = \sum_k u_k(t,x)\, a_k + u_k^{+}(t,x)\, a_k^{+}$$


$\hat{\phi}(t,x) = \sum_k u_k(t,x) a_k + u_k^*(t,x) a_k^\dagger$

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$$\boxed{v_k'(\eta)\, v_k^{*}(\eta) - v_k(\eta)\, v_k^{*}(\eta)' = 2i \qquad (W)}$$

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Show that if (W) holds at one time then it holds at all times.

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Conclusion:

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B) Make sure $v_k(\eta)$ obeys **(W)**, if need be by multiplying with a constant. (Recall exercise 6))

C) Build a basis in the Hilbert space:

$a_k |0\rangle = 0, \quad a_k^+ |0\rangle, \quad a_k^+ a_{k'}^+ |0\rangle$, etc ...

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In order to obtain the solution $\hat{\phi}(\eta, x)$, we do:

A) Find for each mode $k \in \mathbb{R}^3$ a solution $v_k(\eta)$ to (M), i.e., a solution to the classical harmonic oscillator with time-dependent frequency.

B) Make sure $v_k(\eta)$ obeys (W), if need be by multiplying with a constant. (Recall exercise 6))

C) Build a basis in the Hilbert space:

$a_k |0\rangle = 0, \quad a_k^+ |0\rangle, \quad a_k^+ a_{k'}^+ |0\rangle$, etc ...

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3.) the commutation relations (CCR) if the v_k are chosen such that they also obey:

$$\boxed{v_k'(\eta)\, v_k^*(\eta) - v_k(\eta)\, v_k^*(\eta)' = 2i \qquad (W)}$$

Exercise:

a) Prove the proposition.

b.) Assume that $v_k(\eta)$ is any solution of (EoM). Show that if (W) holds at one time then it holds at all time.

(Note: The LHS of (W) is the "Wronskian" of the ODE)

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Choice of mode solutions $\{v_k(\eta)\}$

- For each choice, say $\{v_k(\eta)\}_{k \in \mathbb{R}^3}$ or $\{\tilde{v}_k(\eta)\}_{k \in \mathbb{R}^3}$ we obtain the same $\hat{\phi}(x,t)$ but the bases

  $|0\rangle, \; a_k^+|0\rangle, \; a_k^+ a_{k'}^+ |0\rangle, \dots$

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- Why?

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Recall: In the Heisenberg picture, the system's state vector is always the same Hilbert space vector. Therefore, since things do change, of course, the meaning of each Hilbert space vector changes over time (technically, because observables do).

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How many possible choices of

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- We can consider each mode, k, separately:

- The solution space of (M), for fixed k,

$$v_k''(\eta) + \left(k^2 + m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)}\right) v_k(\eta) = 0$$

has of course 2 complex dimensions.

Note: Every solution obeying (M) must be complex-valued. Why?

- If v_k is a complex-valued solution, then

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- The actual dimensionality is 3!

The solution space thus has 4 real dimensions, but one real dimension is lost because the solutions $v_k, \tilde{v}_k$, etc must also obey (W), i.e.:

$$v_k'(\eta)\, v_k^*(\eta) - v_k(\eta)\, v_k^*(\eta)' = 2i \qquad (W)$$

(i.e. $\mathrm{Im}(v' v^*) = 1$, which is only one real equation)

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Assume v_k obeys (W). Then, $\tilde{v}_k$ defined through

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Proof: Exercise

$\Rightarrow$ we easily obtain:

$$\left.\begin{aligned} \hat{\chi}_k(\eta) &= \frac{1}{\sqrt{2}}\left(v_k^*(\eta)\, a_k + v_k(\eta)\, a_{-k}^+\right) \\ &= \frac{1}{\sqrt{2}}\left(\tilde{v}_k^*(\eta)\, \tilde{a}_k + \tilde{v}_k(\eta)\, \tilde{a}_{-k}^+\right) \end{aligned}\right\} (P)$$

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Terminology: Such transformations from one choice $\{v_k\}$, a_k and corresponding basis

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$$\left.\begin{aligned} \hat{x}_k(\eta) &= \frac{1}{\sqrt{2}}\left(v_k^*(\eta)\, a_k + v_k(\eta)\, a_{-k}^+\right) \\ &= \frac{1}{\sqrt{2}}\left(\tilde{v}_k^*(\eta)\, \tilde{a}_k + \tilde{v}_k(\eta)\, \tilde{a}_{-k}^+\right) \end{aligned}\right\} (P)$$

$$= \dots$$

- Terminology: Such transformations from one choice $\{v_k\}, a_k$ and corresponding basis

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$$\tilde{v}_k(\eta) = \alpha_k\, v_k(\eta) + \beta_k\, v_k^*(\eta) \qquad \text{(B)}$$

also obeys (W), iff the coefficients α_k, β_k obey:

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

- Proof: Exercise

⇒ we easily obtain:

$$\left.\begin{aligned} \hat{\chi}_k(\eta) &= \frac{1}{\sqrt{2}}\left(v_k^*(\eta)\, a_k + v_k(\eta)\, a_{-k}^+\right) \\ &= \frac{1}{\sqrt{2}}\left(\tilde{v}_k^*(\eta)\, \tilde{a}_k + \tilde{v}_k(\eta)\, \tilde{a}_{-k}^+\right) \end{aligned}\right\} \text{(P)}$$

$$= \dots$$

- **Terminology:** Such transformations from one choice $\{v_k\}, a_k$ and corresponding basis

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- <u>Terminology</u>: Such transformations from one choice $\{v_k\}$, a_k and corresponding basis

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$\Rightarrow$ we easily obtain: $\hat{x}_k(\eta) = \frac{1}{\sqrt{2}}\left(v_k^*(\eta)\, a_k + v_k(\eta)\, a_{-k}^+\right)$

$$= \frac{1}{\sqrt{2}}\left(\tilde{v}_k^*(\eta)\, \tilde{a}_k + \tilde{v}_k(\eta)\, \tilde{a}_{-k}^+\right) \Bigg\} (P)$$

$$= \dots$$

Terminology: Such transformations from one choice $\{v_k\}$, a_k and corresponding basis

$|0\rangle, a^+|0\rangle, a^+ a^+ |0\rangle$ ...

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$$v_k'(\eta)\, v_k^{*}(\eta) - v_k(\eta)\, v_k^{*}(\eta)' = 2i \qquad (W)$$

(i.e. $\mathrm{Im}\,(v' v^*) = 1$, which is only one real equation)

Proposition:

Assume v_k obeys (W). Then, $\tilde{v}_k$ defined through

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□ Terminology: Such transformations from one choice

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$\rightarrow$ we easily obtain: $\chi_k(\eta) = \frac{1}{\sqrt{2}}\left(v_k^*(\eta)\, a_k + v_k(\eta)\, a_{-k}^+\right)$

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- **Terminology:** Such transformations from one choice $\{v_k\}$, a_k and corresponding basis

$$|0\rangle,\ a_k^+|0\rangle,\ a_k^+ a_{k'}^+|0\rangle, \ldots$$

to some $\{\tilde{v}_k\}$, $|\tilde{0}\rangle$, $\tilde{a}_k$ and their basis

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* Make Bogolubov Hilbert basis transforms explic...

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Bogolubov transformations of Hilbert bases

- How can we express the basis vectors

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Terminology: Such transformations from one choice $\{v_k\}, a_k$ and corresponding basis

$|0\rangle,\ a_k^+|0\rangle,\ a_k^+ a_{k'}^+|0\rangle,$

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Bogolubov transformations of Hilbert bases

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- Try to solve for $|0\rangle$ using ansatz: $|0\rangle := \left(\prod_k f_k(\tilde{a}_k^+, \tilde{a}_{-k}^+)\right)|\tilde{0}\rangle$

- P...

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Proposition:

$$|0\rangle = \left[\prod \ \frac{1}{\ } \ \ e^{-\frac{\beta_k}{2\alpha_k^*}\tilde{a}_k^+ \tilde{a}_{-k}^+}\right]|\tilde{0}\rangle$$

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- Proof: Exercise.

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- **Proof:** Exercise.

Hint: Use $a e^{\lambda a^+} = e^{\lambda a^+} a + \lambda e^{\lambda a^+} \dots$

Interpretation of (T):

- Assume, e.g., that $|0\rangle$ and $|\tilde{0}\rangle$ are those Hilbert

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Interpretation of (T):

- Assume, e.g., that $|0\rangle$ and $|\tilde{0}\rangle$ are those Hilbert space vectors which happen to be the vacuum

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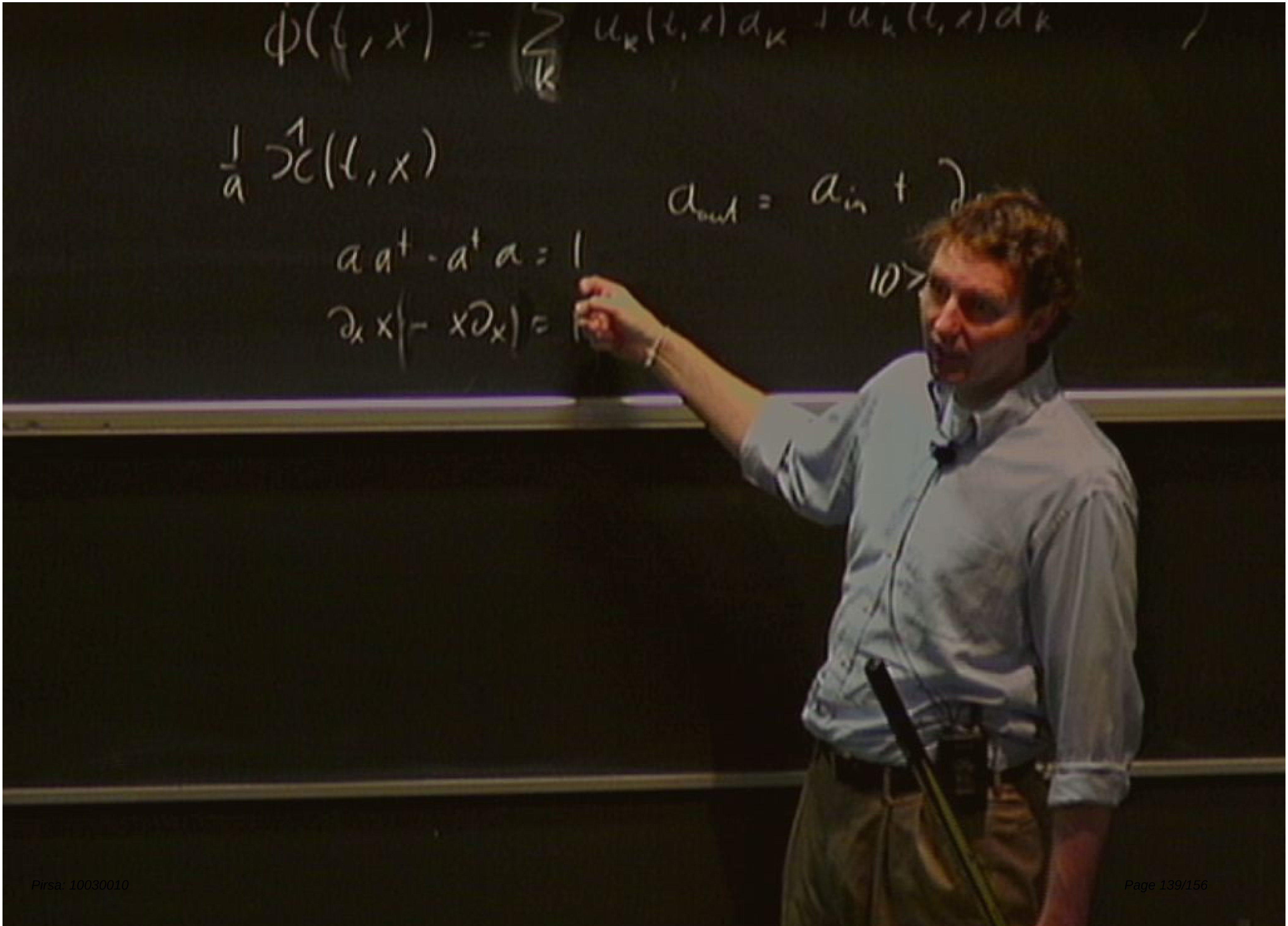
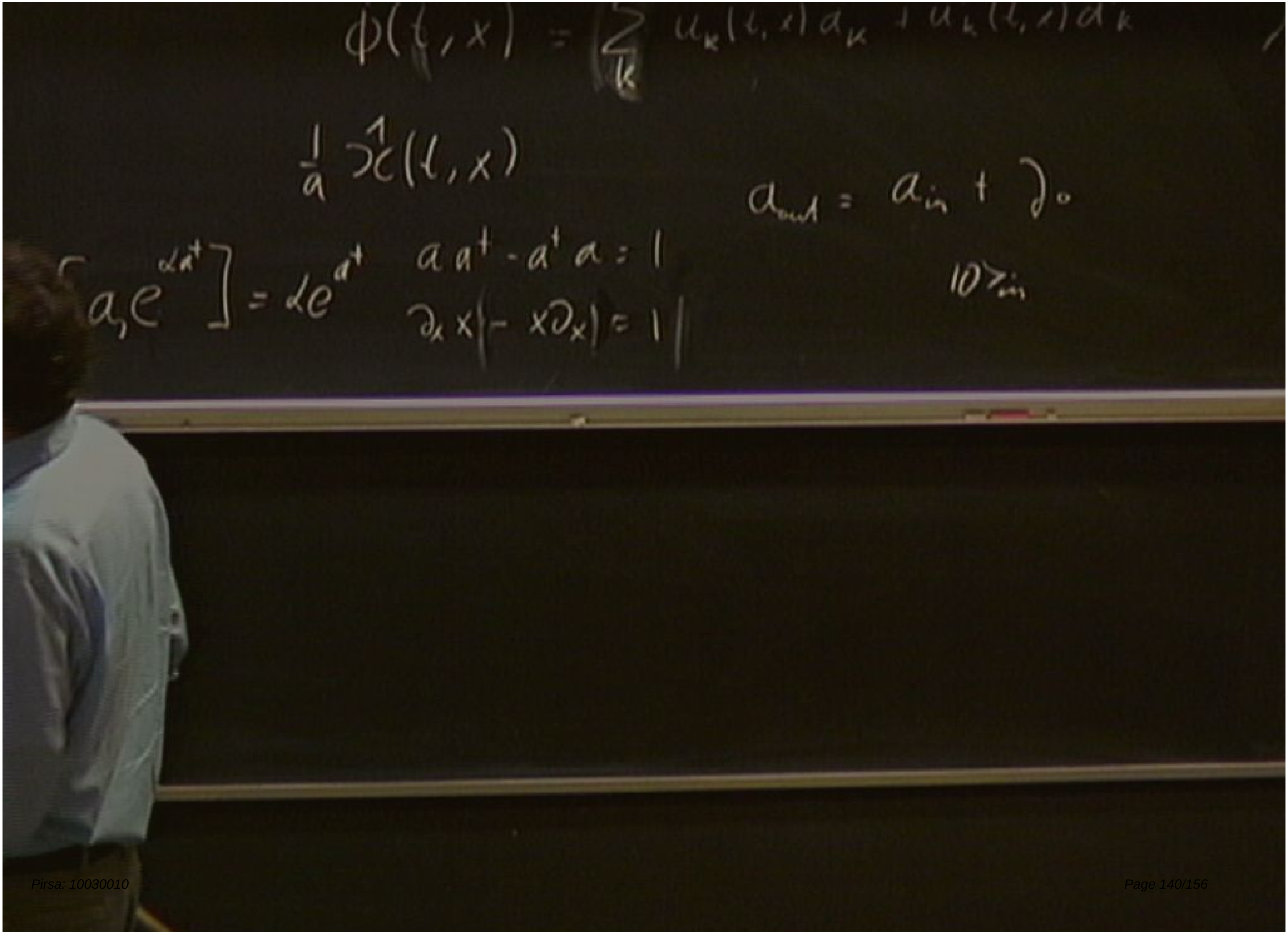


a a† - a† a = 1
∂x x - x∂x = 1
a_out = a_in +
|0>


$\phi(t,x) = \sum_k u_k(t,x) a_k + u_k^*(t,x) a_k^\dagger$
$\frac{1}{a}\hat{\chi}(t,x)$
$a_{out} = a_{in} + \partial_0$
$[a, e^{\alpha a^\dagger}] = \alpha e^{\alpha a^\dagger}$
$a a^\dagger - a^\dagger a = 1$
$\partial_x x| - x\partial_x| = 1$
$|0\rangle_{in}$

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- Try to solve for $|0\rangle$ using ansatz: $|0\rangle := \left(\prod_k f_k(\tilde{a}_k^+, \tilde{a}_{-k}^+)\right)|\tilde{0}\rangle$

- <u>Proposition:</u>

$$|0\rangle = \left[\prod_k \frac{1}{|\alpha_k|^{1/2}} e^{-\frac{\beta_k}{2\alpha_k^*}\tilde{a}_k^+ \tilde{a}_{-k}^+}\right]|\tilde{0}\rangle \quad (T)$$

needed for normalization

- <u>Proof:</u> Exercise.

Hint: Use $a e^{\lambda a^+} = e^{\lambda a^+} a + \lambda e^{\lambda a^+} \ldots$

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- Assume, e.g., that $|0\rangle$ and $|\tilde{0}\rangle$ are those Hilbert space vectors which happen to be the vacuum

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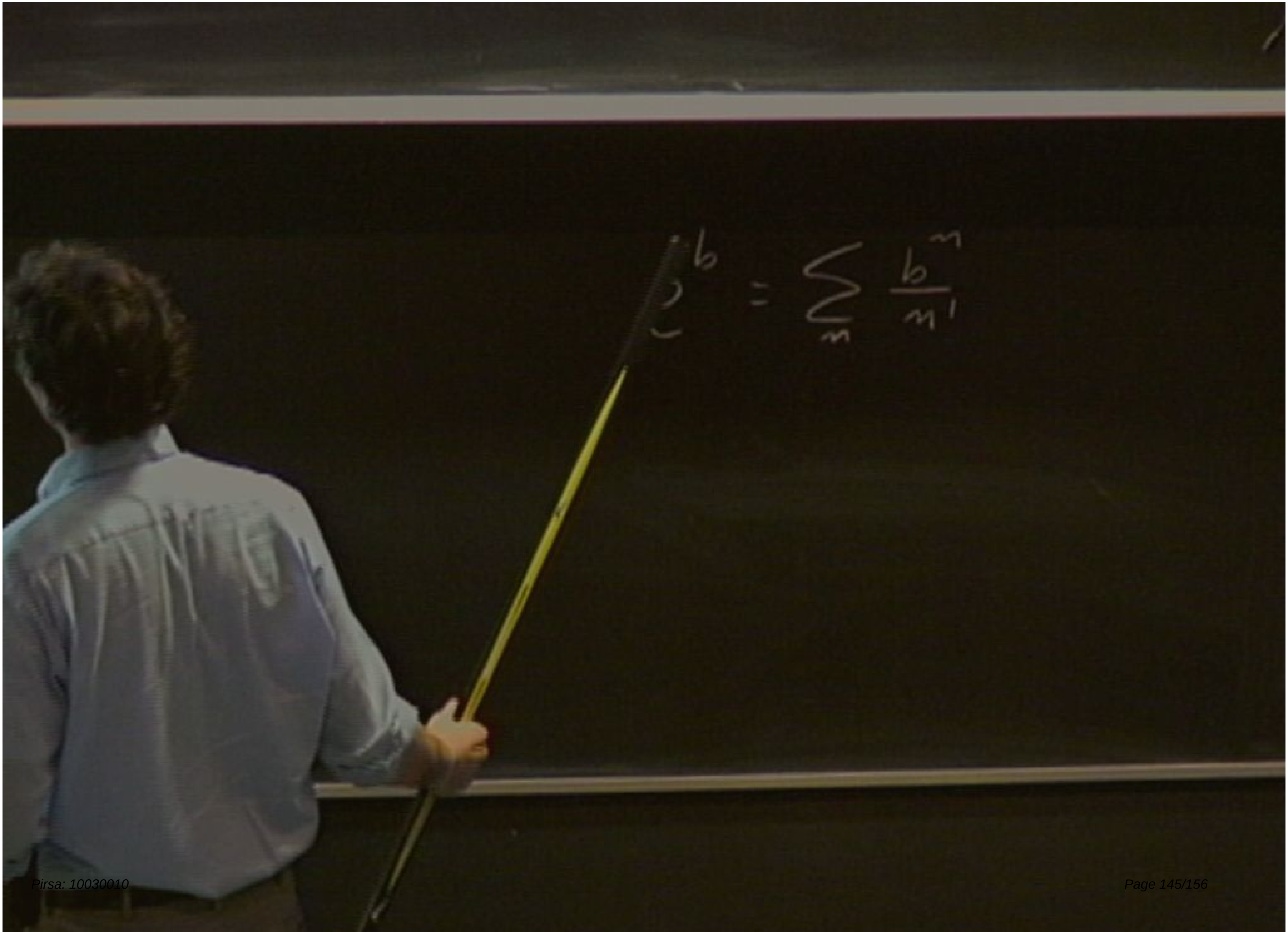
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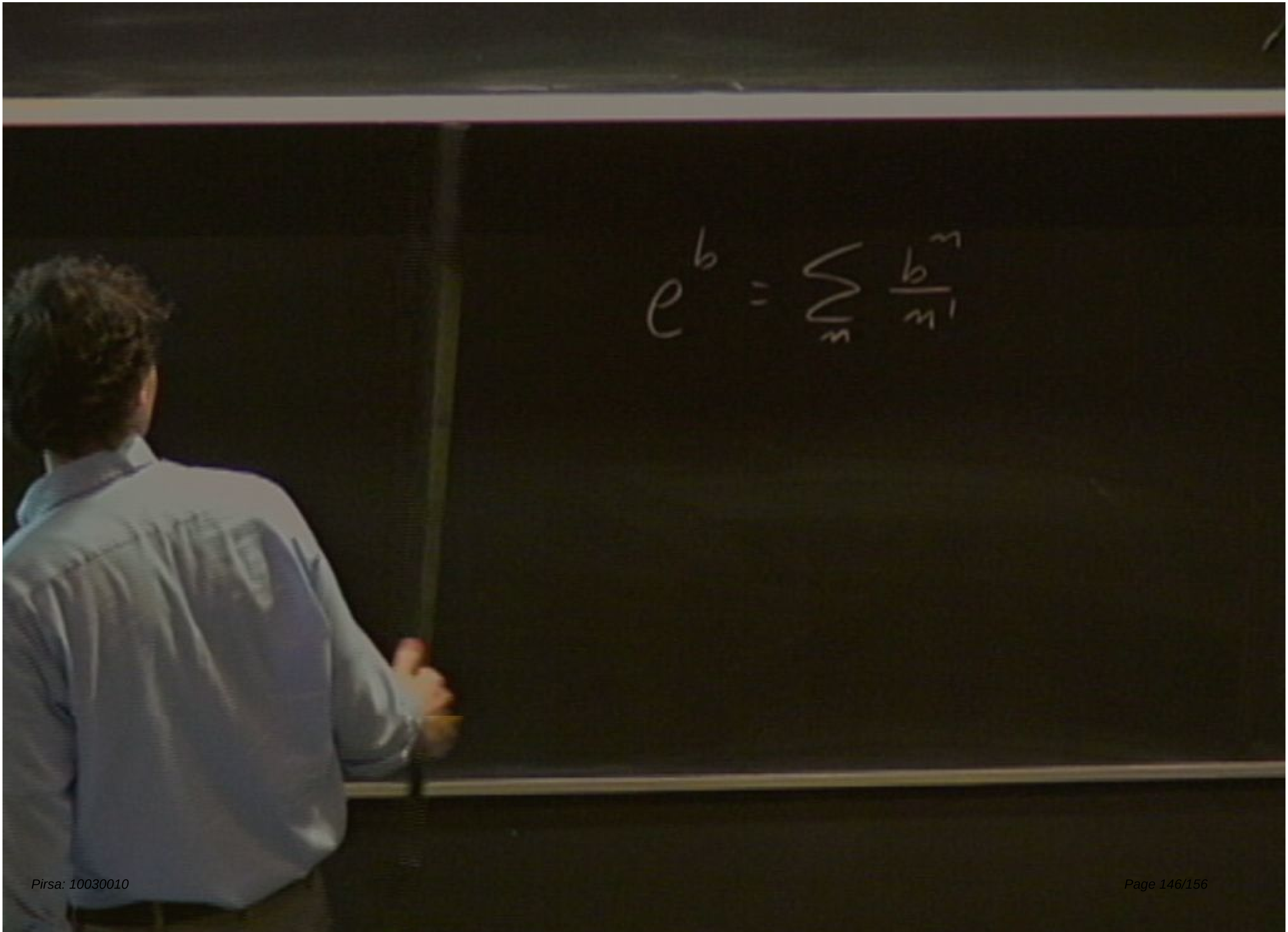
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