Title: Foundations and Interpretation of Quantum Theory - Lecture 12

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Abstract:

Bohmian Mechanics

Roderich Tumulka

RUTGERS
UNIVERSITY

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Richard Feynman (1959)

Does this mean that my observations become real only when I observe an observer observing something as it happens? This is a horrible viewpoint. Do you seriously entertain the thought that without observer there is no reality? Which observer? Any observer? Is a fly an observer? Is a star an observer? Was there no reality before 10^9 B.C. before life began? Or are you the observer? Then there is no reality to the world after you are dead? I know a number of otherwise respectable physicists who have bought life insurance. By what philosophy will the universe without man be understood?

[R.P. Feynman, F.B. Morinigo, W.G. Wagner: Feynman Lectures on Gravitation (Addison-Wesley Publishing Company, 1959). Edited by Brian Hatfield]

Names of Bohmian mechanics

pilot wave theory (de Broglie),
ontological interpretation of quantum mechanics (Bohm),
de Broglie-Bohm theory (Bell),
...

Definition of Bohmian mechanics

(version suitable for N spinless particles in non-relativistic space-time)

Electrons and other elementary particles have precise positions at every t

$\mathbf{Q}_k(t) \in \mathbb{R}^3$ position of particle k, $Q(t) = (\mathbf{Q}_1(t), \ldots, \mathbf{Q}_N(t))$ config.,

$$\frac{d\mathbf{Q}_k(t)}{dt} = \frac{\hbar}{m_k} \mathrm{Im} \frac{\nabla_k \psi_t}{\psi_t}(Q(t)) \qquad (k = 1, \ldots, N)$$

The wave function $\psi_t : (\mathbb{R}^3)^N \to \mathbb{C}$ evolves according to

$$i\hbar \frac{\partial \psi}{\partial t} = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla_k^2 \psi + V\psi$$

Probability Distribution

$$\rho(Q(t) = q) = |\psi_t(q)|^2$$

[Slater 1923, de Broglie 1926, Bohm 1952, Bell 1966]

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Bohm's Equation of Motion

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$v^{\psi} = (\mathbf{v}_1^{\psi}, \ldots, \mathbf{v}_N^{\psi})$ is a vector field on configuration space $\mathbb{R}^{3N}$

$$\frac{dQ(t)}{dt} = v^{\psi_t}(Q(t))$$

first-order ordinary differential equation (ODE)

Determinism: If, for any "initial time" t_0, initial data ψ_{t_0} and $Q(t_0)$ are given, then the Schrödinger eq determines ψ_t for every t, and Bohm's eq of motion determines $Q(t)$ for every t.

(state at time t) = $(Q(t), \psi_t)$

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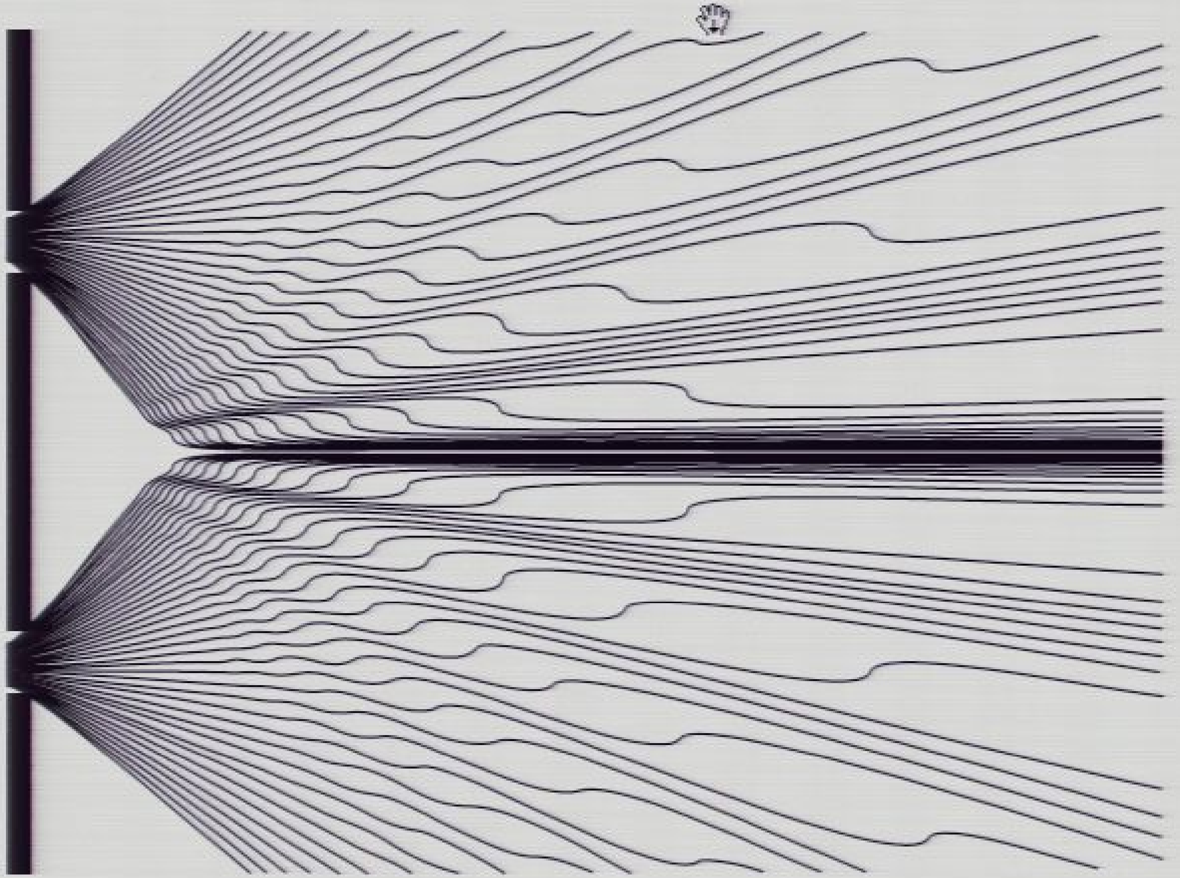
Example of Bohmian trajectories $\mathbf{Q}(t)$: 2-slit experiment



Picture: Gernot Bauer (after Chris Dewdney)

wave-particle duality (in the literal sense)

Probability Axiom

determinism $\Rightarrow$ if initial config. $Q(t_0)$ is random with density $\rho_{t_0}(q)$ then the probability distribution of $Q(t)$ is determined by

$$\frac{\partial \rho_t}{\partial t} = -\sum_{k=1}^{N} \nabla_k \cdot (\rho_t \mathbf{v}_k^{\psi_t})$$

Equivariance theorem (expresses compatibility between the 3 axioms)

If $\rho_{t_0} = |\psi_{t_0}|^2$ then, for every t, $\rho_t = |\psi_t|^2$.

Proof: This follows from

$$\frac{\partial |\psi_t|^2}{\partial t} = -\sum_{k=1}^{N} \nabla_k \cdot (|\psi_t|^2 \mathbf{v}_k^{\psi_t})$$

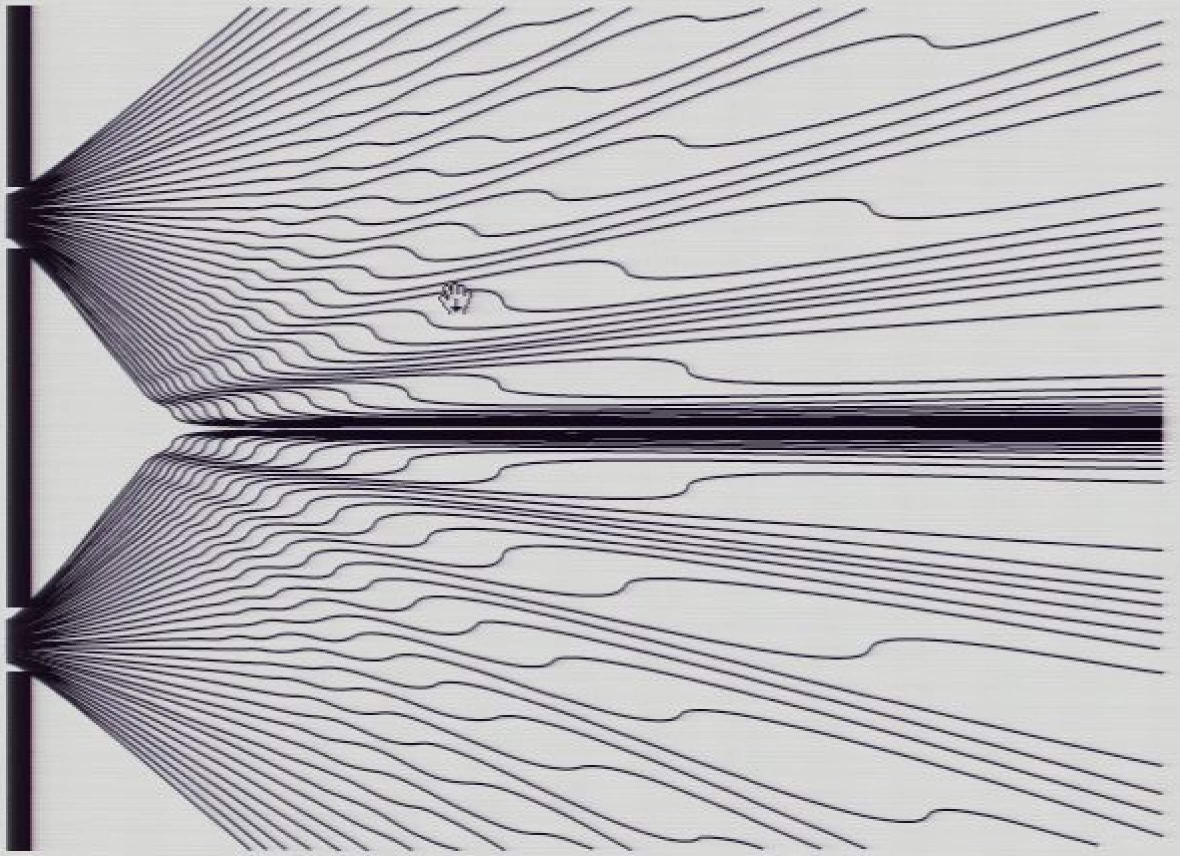
which in turn follows from the Schrödinger equation: Observe that

$$|\psi_t|^2 \mathbf{v}_k = \frac{\hbar}{m_k} \mathrm{Im}\, \psi_t^* \nabla_k \psi_t = \mathbf{j}_k$$

(known as the quantum probability current),

$$\nabla_k \cdot \mathbf{j}_k = \frac{\hbar}{m_k} \mathrm{Im}\, \psi_t^* \nabla_k^2 \psi_t, \quad \text{then do some algebra.}$$

Equivariance



John S. Bell (1986):

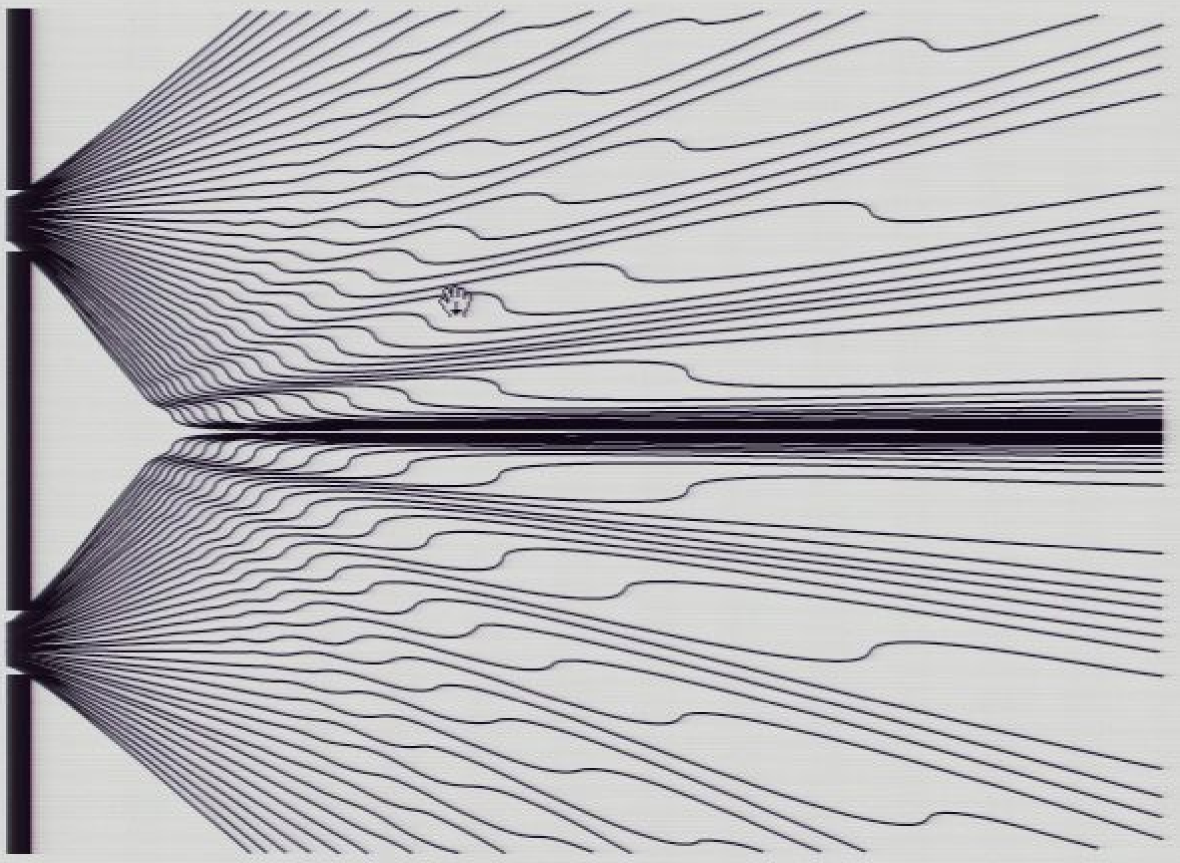
De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. [Speakable and unspeakable in quantum mechanics, page 191]

Another way to write Bohm's eq of motion

polar representation of complex number: $\psi(q) = R(q)\, e^{iS(q)/\hbar}$
with R, S real-valued fcts, $R \geq 0$. Then

$$\mathbf{v}_k^{\psi} = \frac{\hbar}{m_k} \mathrm{Im} \frac{\nabla_k \psi}{\psi} = \frac{1}{m_k} \nabla_k S.$$

Equivariance



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Many ways to arrive at Bohm's eq of motion

- Suppose we know Schrödinger's eq and regard $j = (\mathbf{j}_1, \ldots, \mathbf{j}_N)$ as the "prob current." Suppose we want the prob density ρ_t of $Q(t)$ to be $= |\psi_t|^2$ and the prob current $\rho_t v$ to be $= j$. Then we must have

$$\frac{d\mathbf{Q}_k}{dt} = \frac{\mathbf{j}_k^{\psi_t}(Q(t))}{|\psi_t(Q(t))|^2}.$$

- Suppose we want to link a wave and a particle via de Broglie's relation $m\mathbf{v} = \hbar\mathbf{k}$. Let's try generalize this from plane waves, $\psi(\mathbf{q}) = e^{i\mathbf{k}\cdot\mathbf{q}}$, to arbitrary $\psi(\mathbf{q})$ by replacing $\mathbf{k}$ with the "local wave number" $\nabla S/\hbar$. This leads to

$$\mathbf{v} = \tfrac{1}{m}\nabla S.$$

  A similar reasoning (using also Planck's relation $E = \hbar\omega$) leads from $E = \mathbf{p}^2/2m + V$ to the Schrödinger eq.

- Bohm 1952: analogy with Hamilton–Jacobi formulation of classical mechanics (involves fct $S(\mathbf{q}_1, \ldots, \mathbf{q}_N)$, eq of motion $d\mathbf{Q}_k/dt = (1/m_k)\nabla_k S$)

- ...

Wave function of a subsystem

composite system, $\Psi = \Psi(x, y)$, $Q(t) = (X(t), Y(t))$

conditional wave function

$$\psi(x) = \mathcal{N}\,\Psi(x, Y)$$

$\mathcal{N} =$ normalization factor $= (\int dx |\Psi(x, Y)|^2)^{-1/2}$. Time-dependence:

$$\psi_t(x) = \mathcal{N}_t\,\Psi_t(x, Y(t))$$

Does not, in general evolve according to a Schrödinger eq.

Note: conditional probability $\rho(X = x|Y) = |\psi(x)|^2$

Absolute uncertainty

Inhabitants of a Bohmian universe cannot know a particle's position more precisely than the $|\psi|^2$ distribution allows, with ψ the conditional wave function.

$\mathbf{Q}_k(t)$ often called "hidden variable"—better: uncontrollable variable

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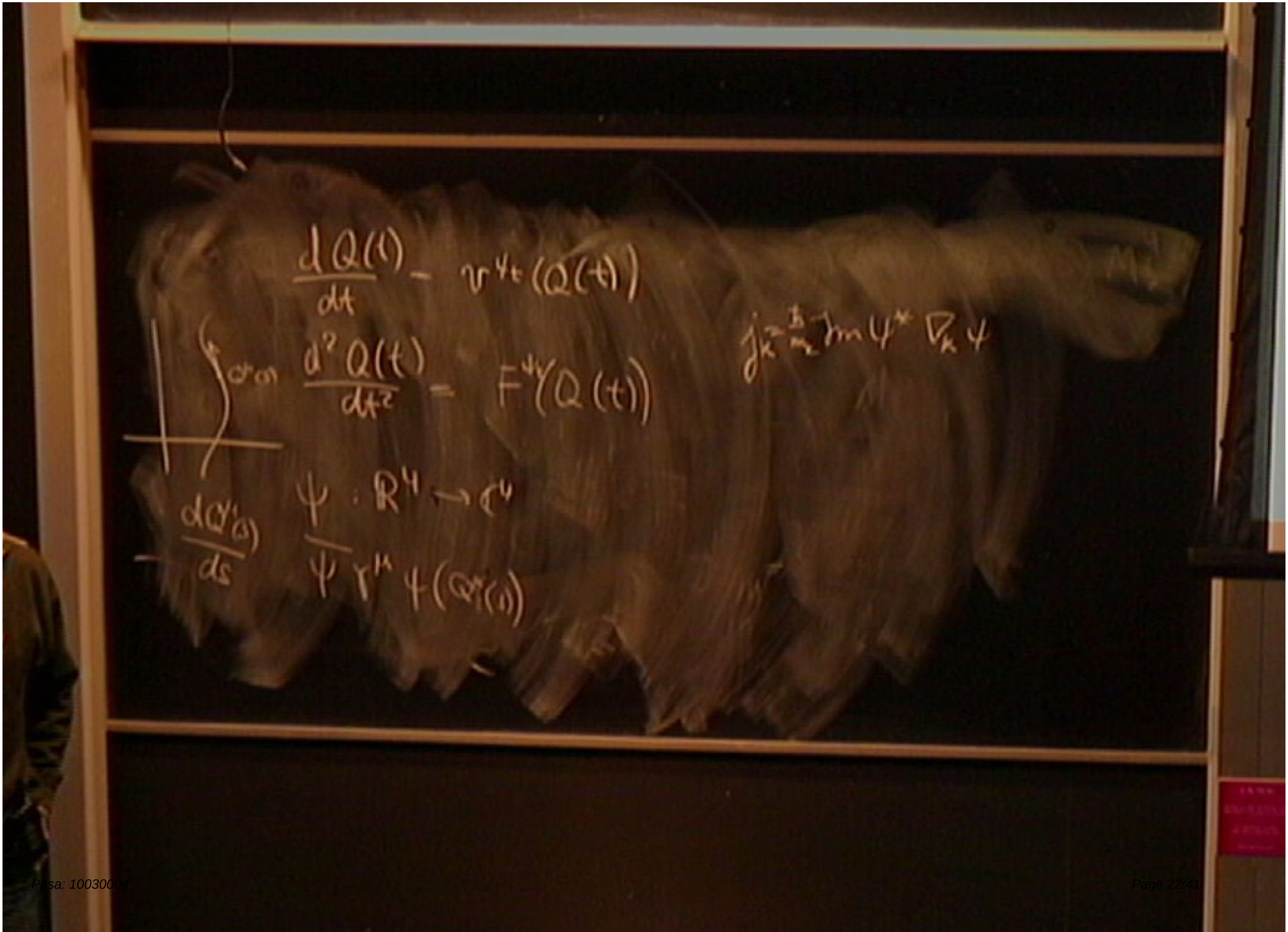
$\frac{dQ(t)}{dt} = v^{\psi_t}(Q(t))$
$m \frac{d^2 Q(t)}{dt^2} = F^{\psi}(Q(t))$
$\psi : \mathbb{R}^4 \to \mathbb{C}^4$
$\frac{dQ^\mu(s)}{ds} = \frac{\bar{\psi}\gamma^\mu \psi(Q(s))}{\bar{\psi}\psi}$
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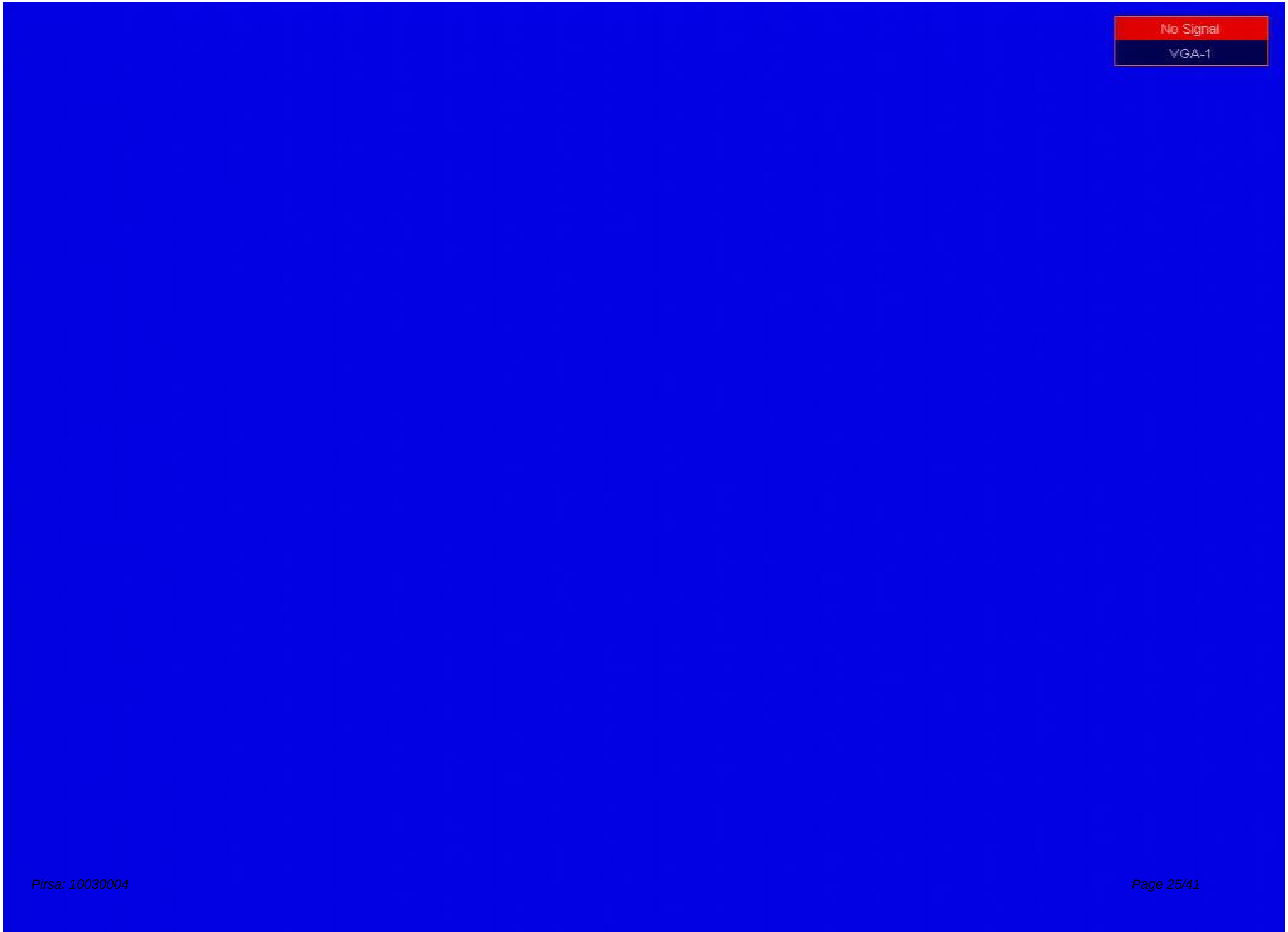
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VGA-1

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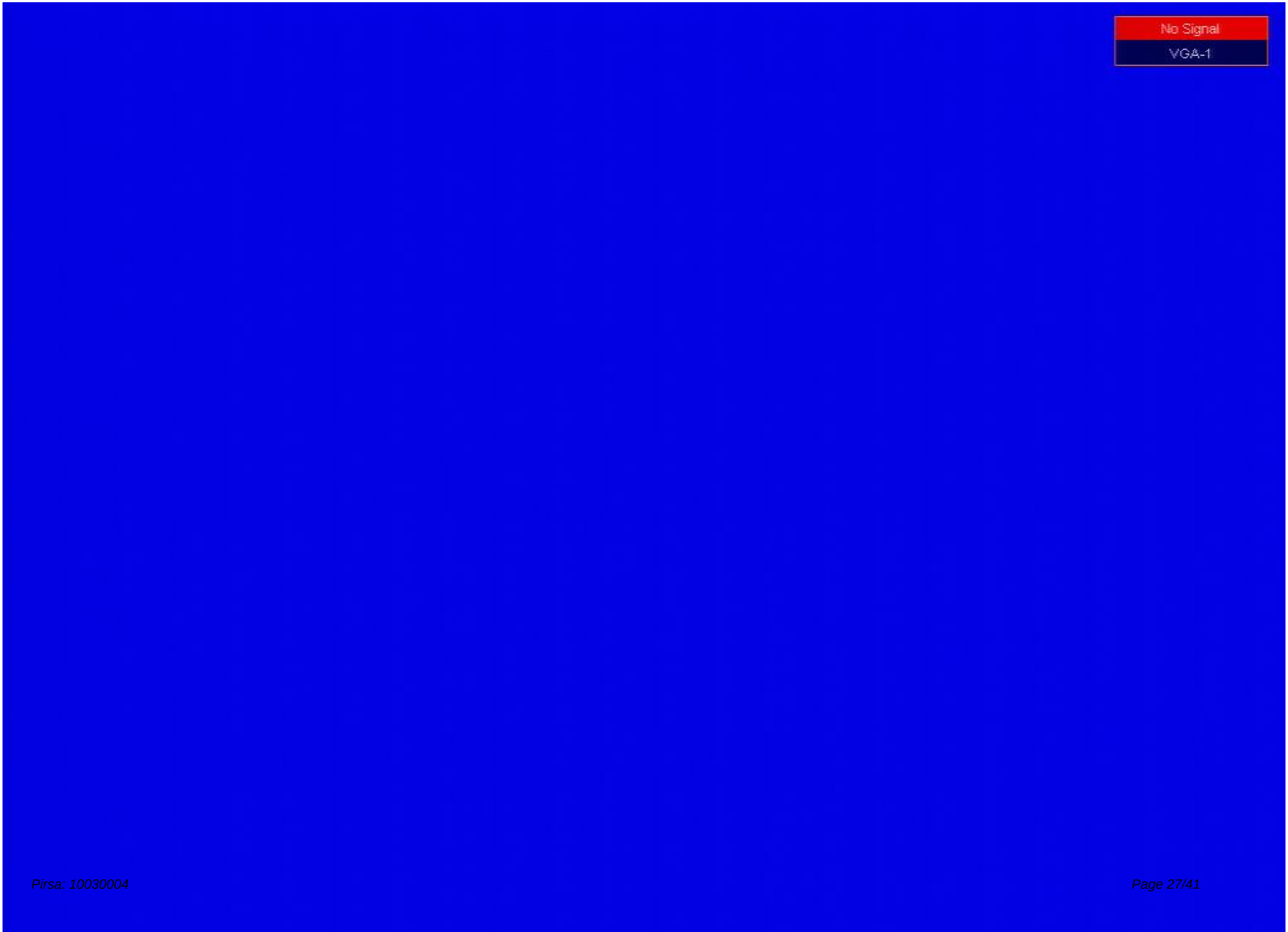
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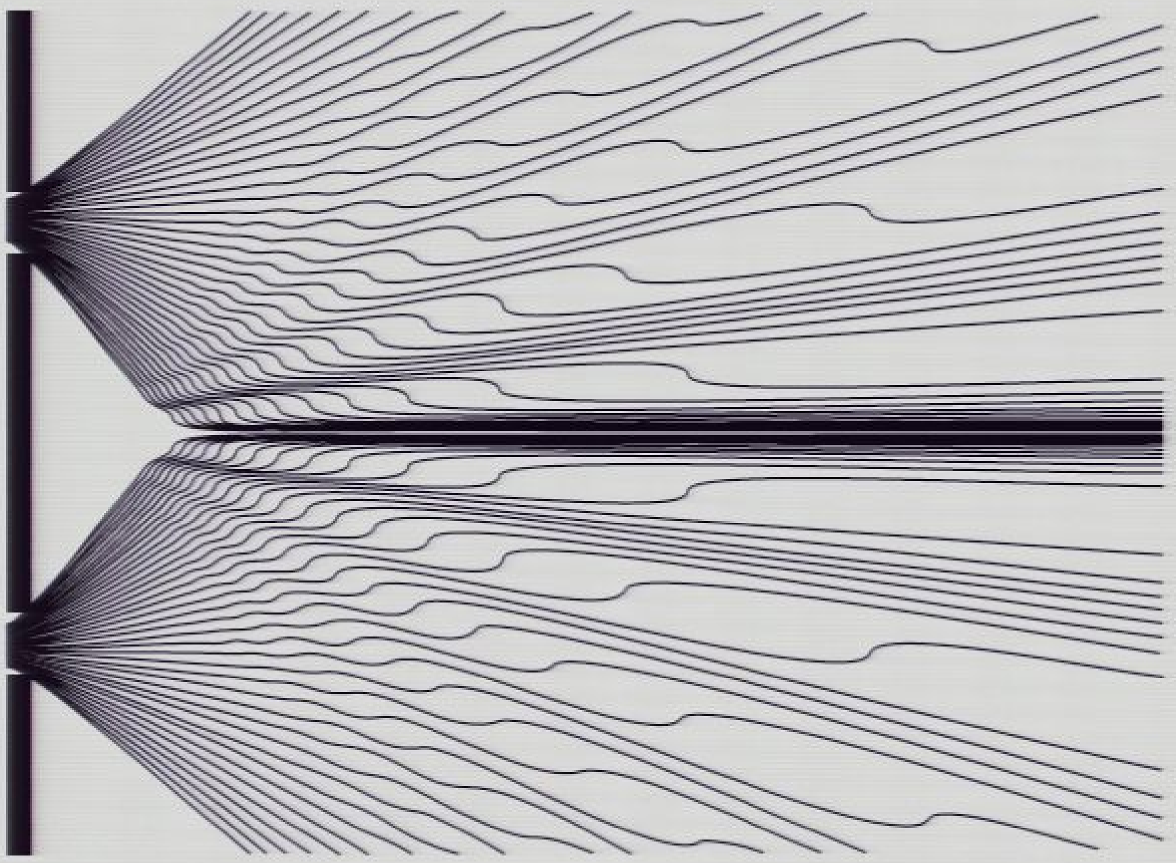
Heisenberg uncertainty in Bohmian mechanics



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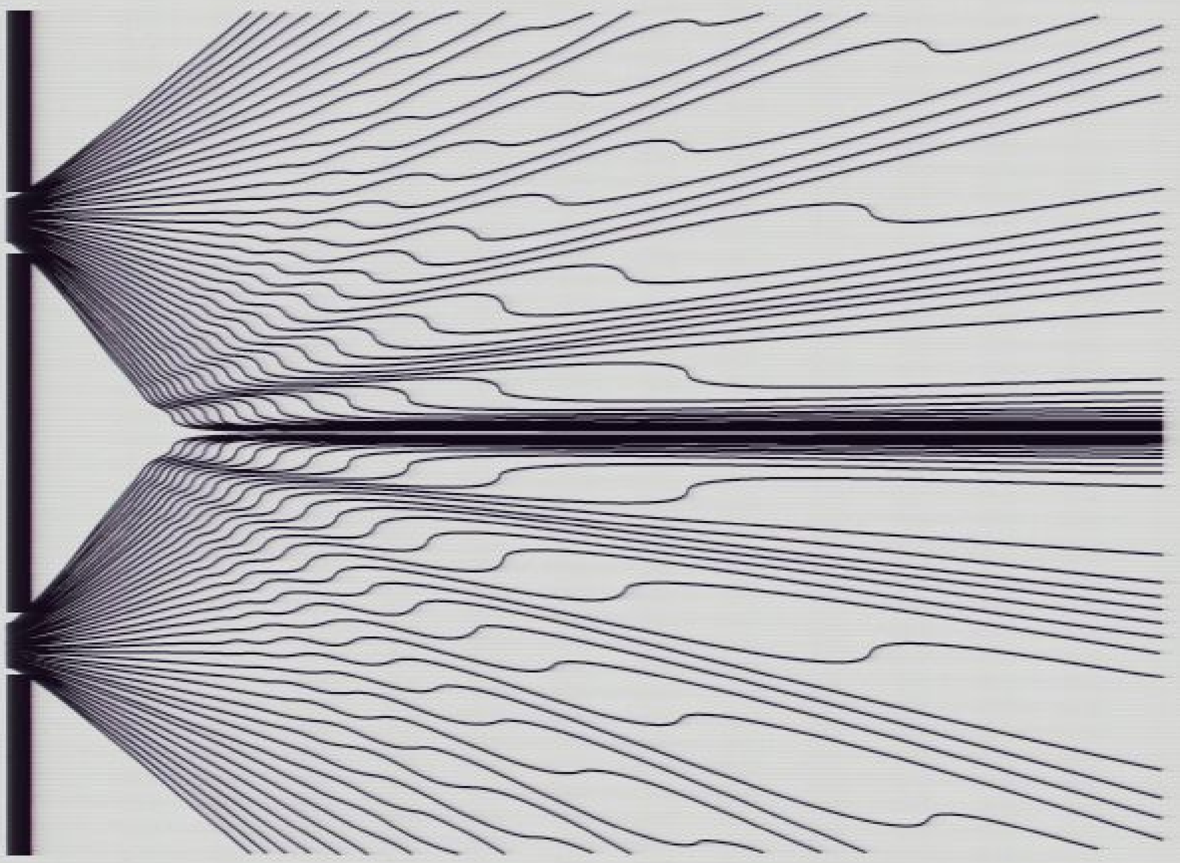


Empirical predictions of Bohmian mechanics

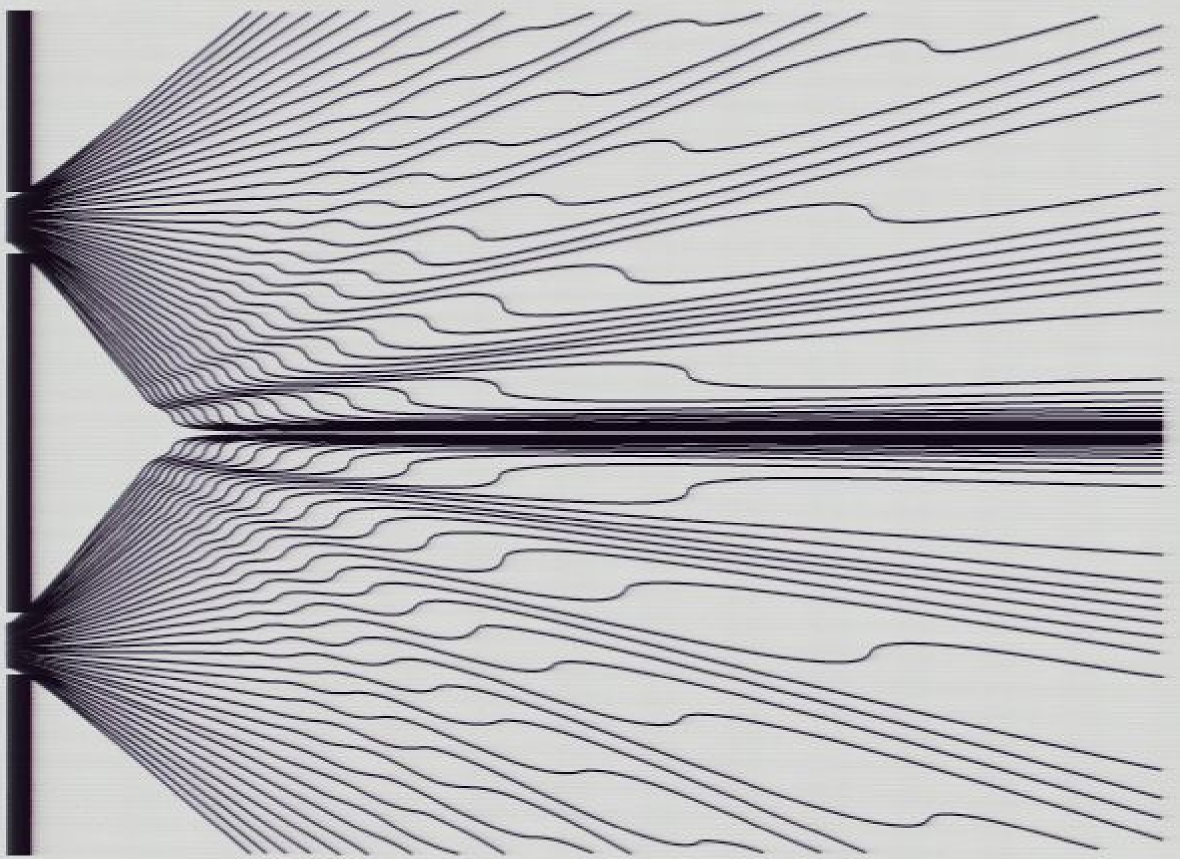
Central fact

Inhabitants of a Bohmian universe would observe outcomes in agreement with the predictions of quantum mechanics.

Let us proceed slowly towards understanding the reasons behind this statement.

Niels Bohr: impossibility of explanation of quantum mechanics in terms of objective reality

Heisenberg uncertainty in Bohmian mechanics



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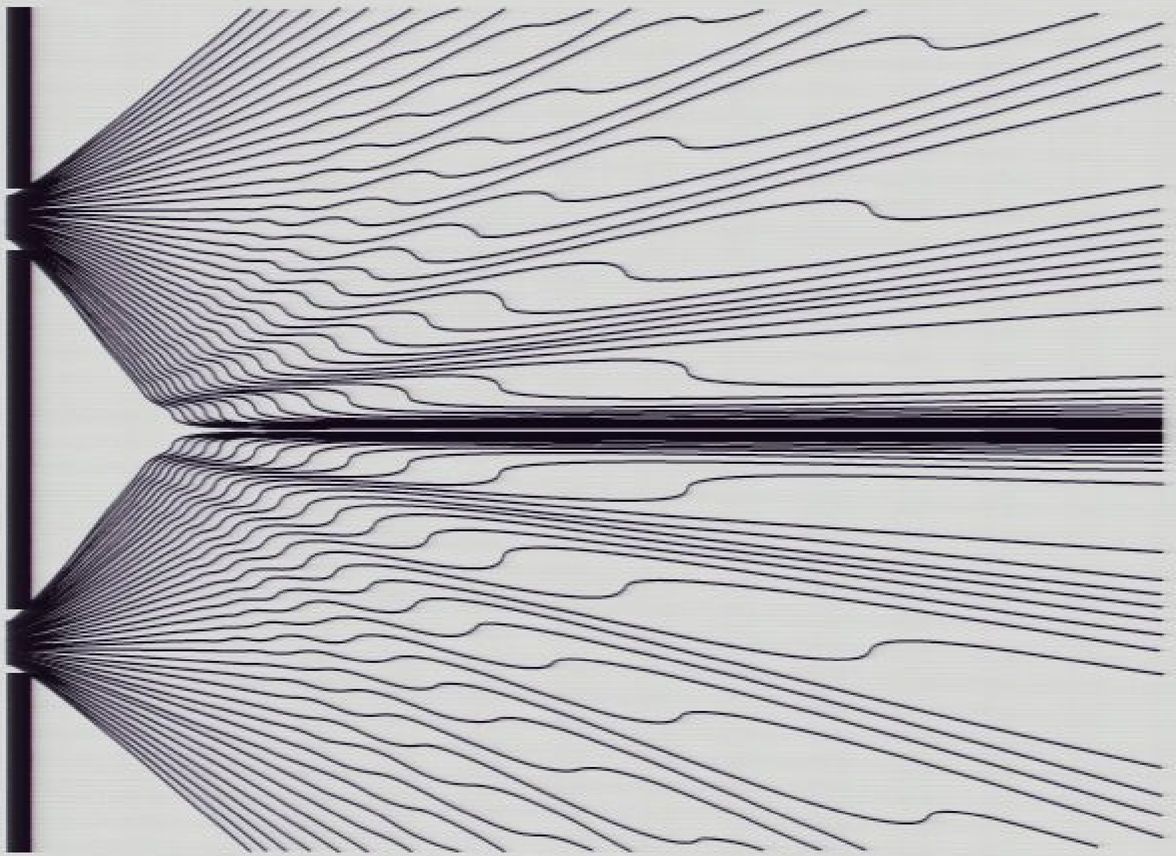
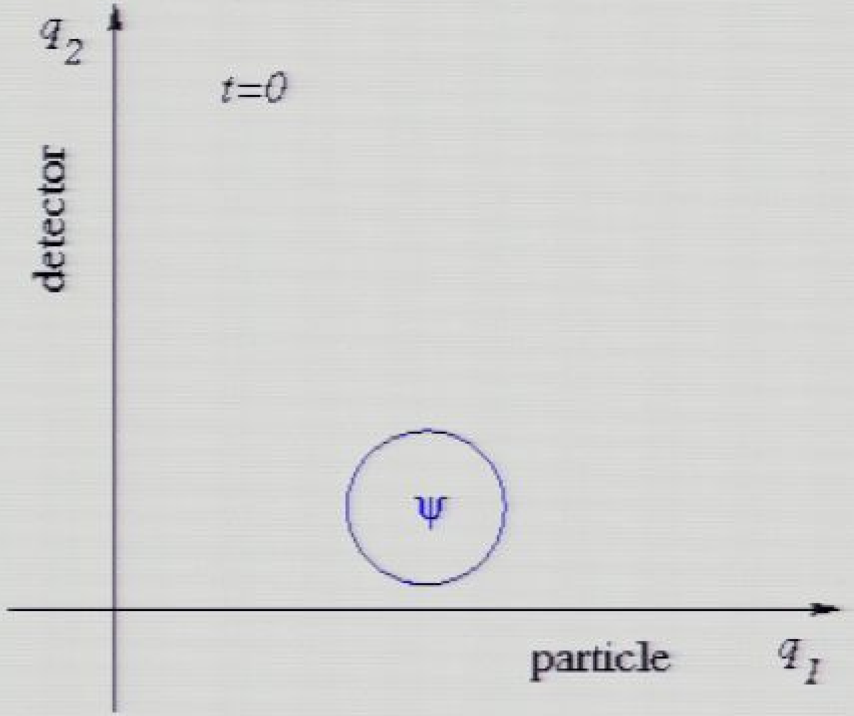
The 2-slit experiment



Particles do arrive on the screen at random positions with $\rho = |\psi|^2$.
If one hole is closed, the interference pattern will be different.
It is easy to know through which slit the particle passed.
Why, in Bohmian mechanics, does detection at a slit destroy the interference pattern?

Evolution of ψ in configuration space of particle + detector:



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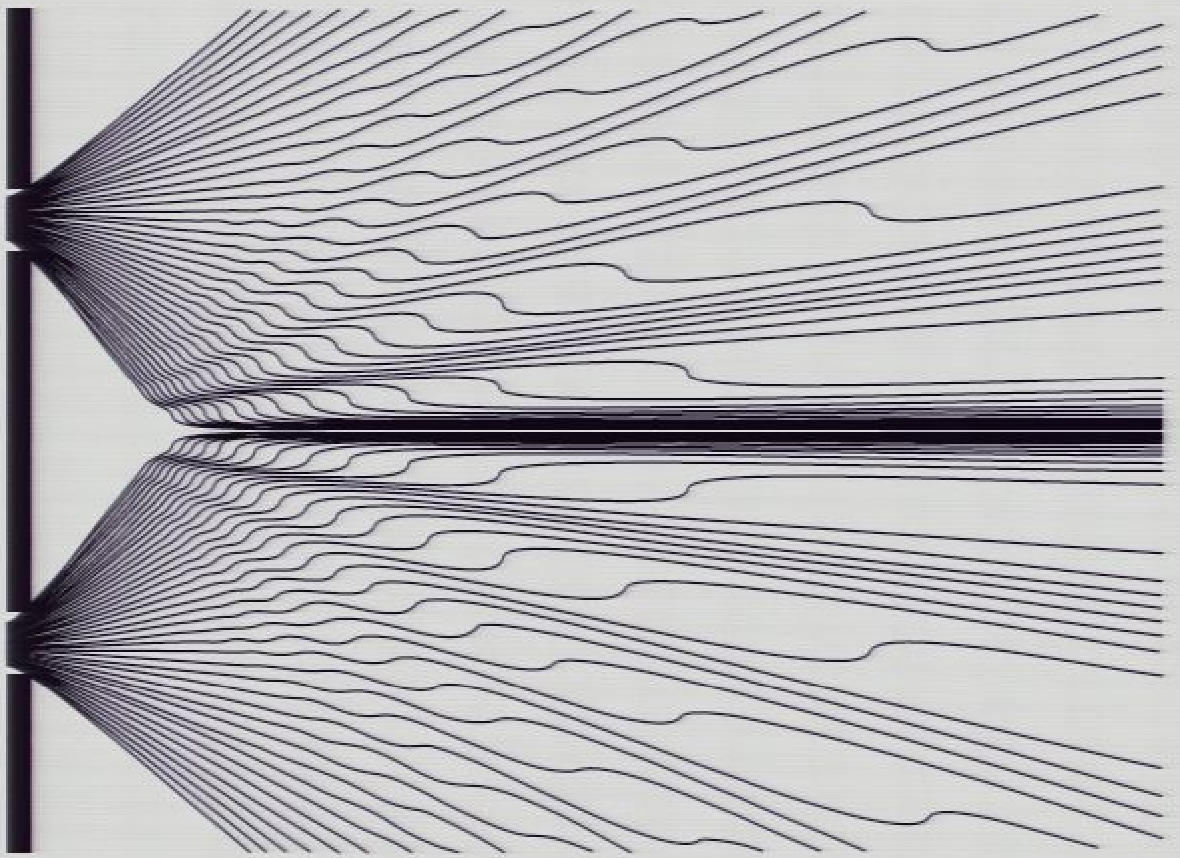

Richard Feynman (1965): “absolutely impossible to explain”

John Bell (1986): De Broglie's explanation “seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.”

Let us now proceed to more general “quantum measurements.”

Conventional Axioms For Quantum Mechanics

Schrödinger Equation

$\psi_t : (\mathbb{R}^3)^N \to \mathbb{C}^m$ wave function of a system. While the system is closed,

$$i\hbar \frac{\partial \psi}{\partial t} = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla_k^2 \psi + V\psi$$

Measurement Postulate

If an observer measures the observable with operator $A = \sum_\alpha \alpha P_\alpha$ at time t, then

$$\mathrm{Prob}(\mathrm{outcome} = \alpha) = \langle \psi_t | P_\alpha \psi_t \rangle \,,$$

and if outcome $= \alpha$ then (collapse)

$$\psi_{t+0} = \frac{P_\alpha \psi_t}{\| P_\alpha \psi_t \|} \,.$$

Analysis of quantum measurement

Consider system + apparatus, experiment during time interval $[t_1, t_2]$.

$$\Psi_{t_1}(x, y) = \psi_{t_1}(x)\phi(y) = \psi_{t_1} \otimes \phi$$

with ϕ = ready state of apparatus.
self-adjoint operator A, orthonormal set of eigenfunctions $A\psi_\alpha = \alpha\psi_\alpha$.
"$\rightarrow$" = unitary time evolution (Schrödinger eq) from t_1 to t_2. Suppose

$$\psi_\alpha \otimes \phi \rightarrow \psi_\alpha \otimes \phi_\alpha$$

with ϕ_α a state of the apparatus displaying the outcome α. Then, by the linearity of the Schrödinger eq, for $\psi = \sum_\alpha c_\alpha \psi_\alpha$,

$$\psi \otimes \phi \rightarrow \sum_\alpha c_\alpha \psi_\alpha \otimes \phi_\alpha .$$

Analysis of quantum measurement (2)

Suppose $\psi_\alpha \otimes \phi \to \psi_\alpha \otimes \phi_\alpha$.

Then, for $\psi = \sum_\alpha c_\alpha \psi_\alpha$, $\quad \psi \otimes \phi \to \sum_\alpha c_\alpha \psi_\alpha \otimes \phi_\alpha$.

In Bohmian mechanics: The pointer of the apparatus consists of Bohmian particles, too, and thus points in some direction. $Q(t_2) = (X(t_2), Y(t_2))$ = the configuration of system + apparatus, has distribution

$$|\Psi_{t_2}(x, y)|^2 = \sum_\alpha |c_\alpha|^2 \, |\psi_\alpha(x)|^2 \, |\phi_\alpha(y)|^2,$$

using that the ϕ_α usually have (approx.) disjoint supports in configuration space. Thus, the probability that the pointer points in the direction corresponding to α_0 is

$$\int dx \int_{\mathrm{support}(\phi_{\alpha_0})} dy \, |\Psi_{t_2}(x, y)|^2 = |c_{\alpha_0}|^2,$$

in agreement with the measurement rule of QM.

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Thank you for your attention