Title: Probing primordial non-Gaussianity with today's universe

Date: Mar 02, 2010 02:00 PM

URL: http://pirsa.org/10030001

Abstract: Primordial non-Gaussianity has been traditionaly constrained using three-point function of the cosmic microwave background. Two years ago, however, Dalal et al have shown that non-Gaussianity of the local type induces a scale dependent bias for biased tracers of the underlying dark matter structure. This allows constraining of the primordial non-Gaussianity from measurements of large-scale structure provided by redshift surveys. I will discuss the technique, its theoretical aspects,

it surprising resilience towards systematics and current results from the real data. I will also show some preliminary new results: extension to the two field inflationary models and the analogue of the Dalal effect in the Lyman alpha forest.

Primordial non-Gaussianity

with today's Universe

PI, March 2010

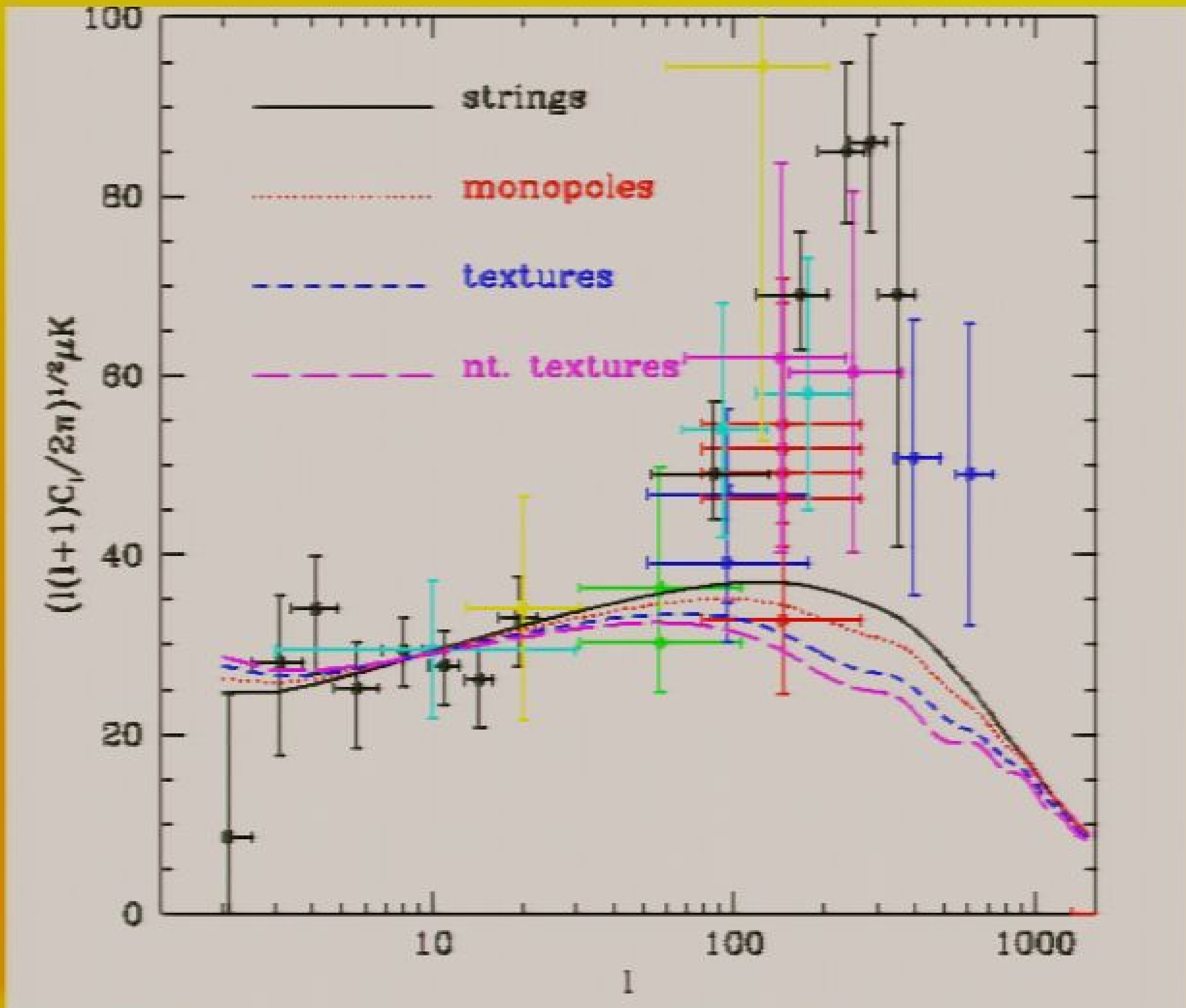
Anže Slosar,

BNL

Historical note:

- In 90s, the source of primordial fluctuations was unknown. There were two options:
  - Inflation – generic predictions of the nearly scale invariant primordial power spectrum
  - Topological defects: model dependent and very different power spectrum

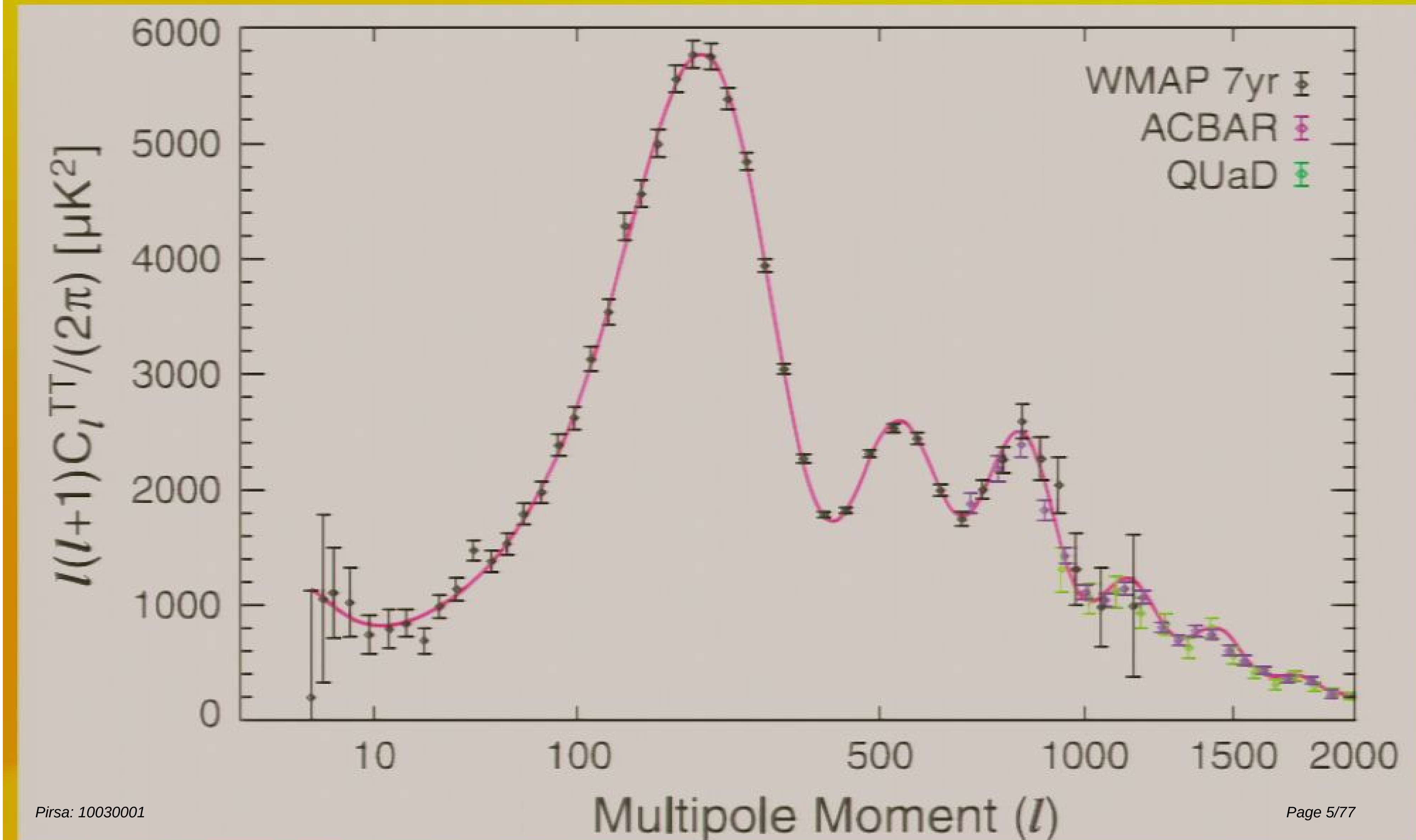
Inflation vs defects:



- Data in the *older days* (late 90s)
- HAHAHA!!!
- Inflation clearly won
- The strength of inflation is its curse:
  - predictions are very model independent

(note the lack of the WMAP plot)

2010:



$l(l+1)C_l^{TT}/(2\pi)$ [μK^2]
6000
5000
4000
3000
2000
1000
0
WMAP 7yr
ACBAR
QUaD
10
100
500
1000
1500
2000
Multipole Moment (l)

Inflation:

How to constrain inflation? Not too many options:

- measure departures from a power-law spectrum
- measure B-modes of CMB polarization
- Measure iso-curvature modes
- measure non-Gaussianity

Linde & Guth

Gaussianity:

In standard inflationary models, primordial curvature perturbations and in turn other observables are *Gaussian:*

Every Fourier mode has a random phase and amplitude randomly taken from a Gaussian distribution with variance P(k)

All odd point statistics vanish, all even points statistics are completely determined by P(k)

Gaussianity can be falsified, non-Gaussianity cannot

Non-Gaussianity:

In minimal inflationary models, curvature perturbation:

$$\mathcal{R} = \frac{H(\phi)}{\dot{\phi}}\delta\phi$$

- if φ is normally distributed, so is R.
- One can break non-Gaussianity:
  - scalar fields non-free
  - non-linear corrections to above equation
  - initial state non Bunch-Davis vacuum

Non-Gaussianity:

A generic way is to postulate that curvature perturbation in the matter domination obeys

$$\Phi = \Phi_{\rm L} + f_{NL}^{\rm local}\Phi_{\rm L}^2.$$

- this is the so-called "local" non-Gaussianity:
  - local in real space and in potential:
- By far not the only possible non-Gaussianty
- If detected, we can check further, most non-Gaussinity will have a projected component
- Note the typical scales – these are very stringent tests

Non-Gaussianity:

What are the typical value of fnl?

2nd order perturbation theory of GR gives fnl~1

Simplest scalar field gives fnl~0.01

More complicated models up to fnl~10

fnl>10 implies very non-concordant physics: New Ekpyriotic scenario, violent reheating, etc.

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How to measure f_{NL}?

In principle, one can use 1-point distribution function

In practice, higher-order correlators give much better signal-to-noise: best to use bispectrum

CMB the cleanest probe (as always)

Difficult to measure, but special estimators have been constructed by Komatsu and company

fnl from CMB

Komatsu et al in WMAP3: $-54 < f_{nl} < 114$

Creminelli et al: $-36 < f_{nl} < 100$

Komatsu et al in WMAP5: $-9 < f_{nl} < 111$

Yadav & Wandelt: $f_{nl} > 0$ at 2.9 sigma

Smith et al $-4 < f_{nl} < 80$

Komatsu et al in WMAP7: $-10 < f_{nl} < 74$

Hikage et al: $-71 < f_{nl} < 91$

Curto et al (wavelets): $-8 < f_{nl} < 111$

Note on sigmas:

Linda Evangelista: "I don't go out of bed for less than $10000."

Andy Albrecht: "I don't go out of bed for less than 4 sigma."

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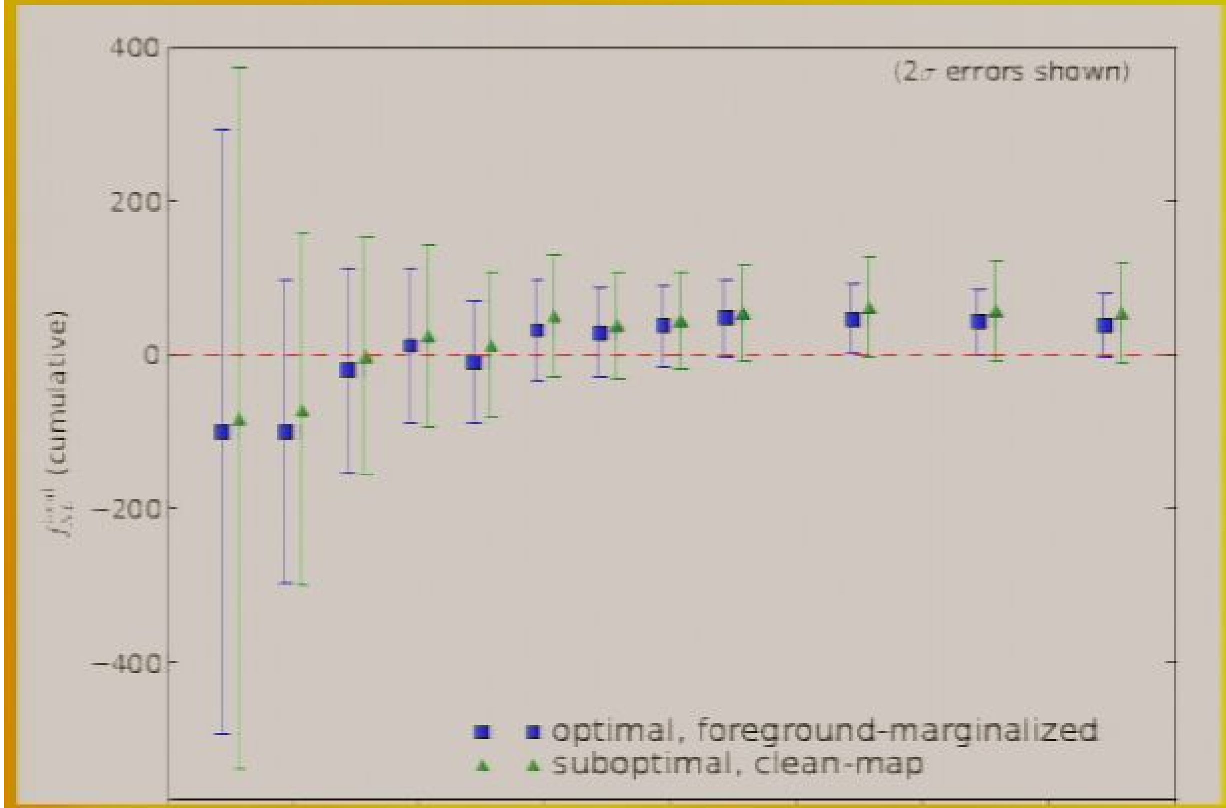
WMAP 5:



- Smith et al
- Most optimal estimator
- The optimality is a compsci rather than physics problem
- Don't see anything exciting, some discrepancies remain

Fnl from LSS?

If you have ability to probe the primordial density fields – see it direct (maybe 21cm at high z?)

It is 3–dimensional so in principle much better

In practice this is never the case: non–linearities induce their own non–Gaussianity

Initial idea:

- Look at the mass function in the exponential tail
- rare peak probability affected a lot in even weakly NG models

Galaxy power spectrum

- Assume that process of formation of galaxies is local on some scale.

$$\delta_g(\vec{x}) = f(\delta_m(\vec{x}), \nabla\delta_m(\vec{x}), \ldots)$$

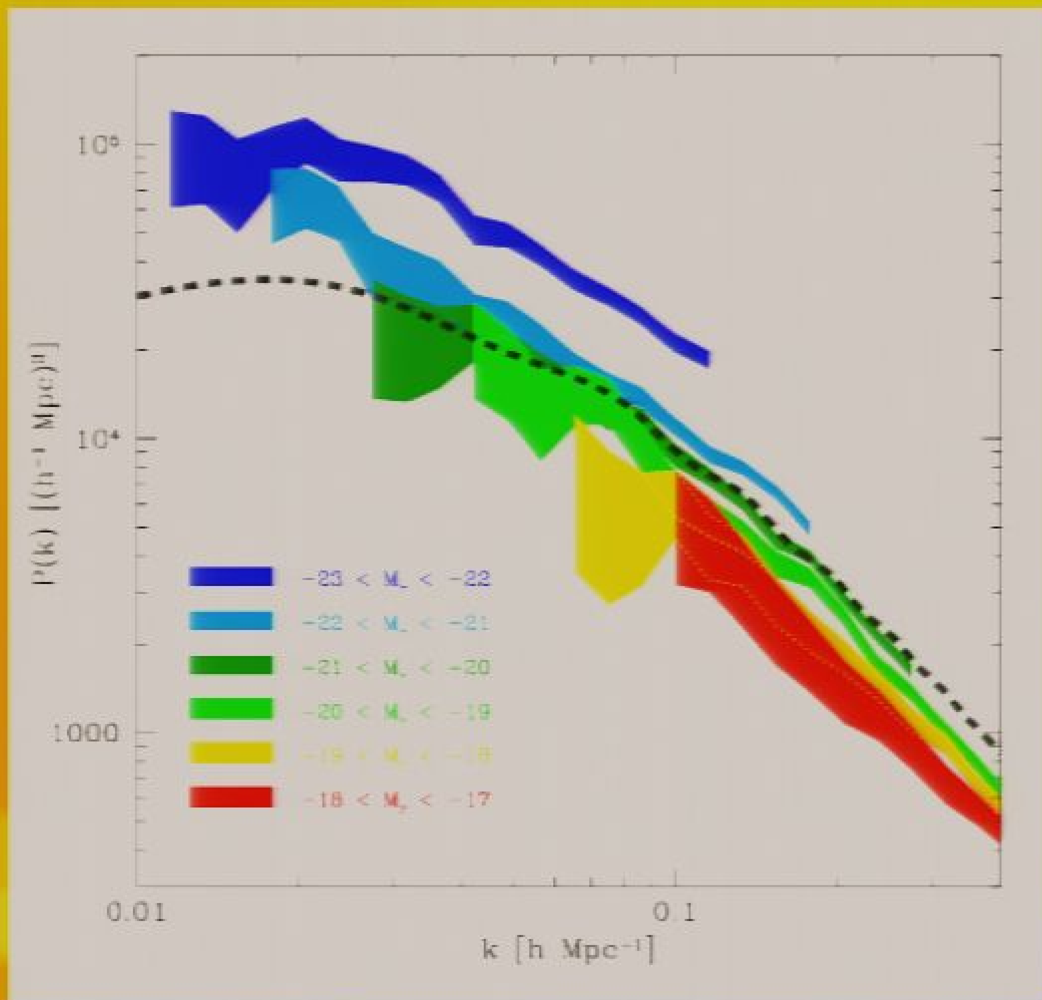
- On large scales, it follows that

$$\delta_g(\vec{k}) = b\,\delta_m(\vec{k}) + s(\vec{k})$$

- And hence

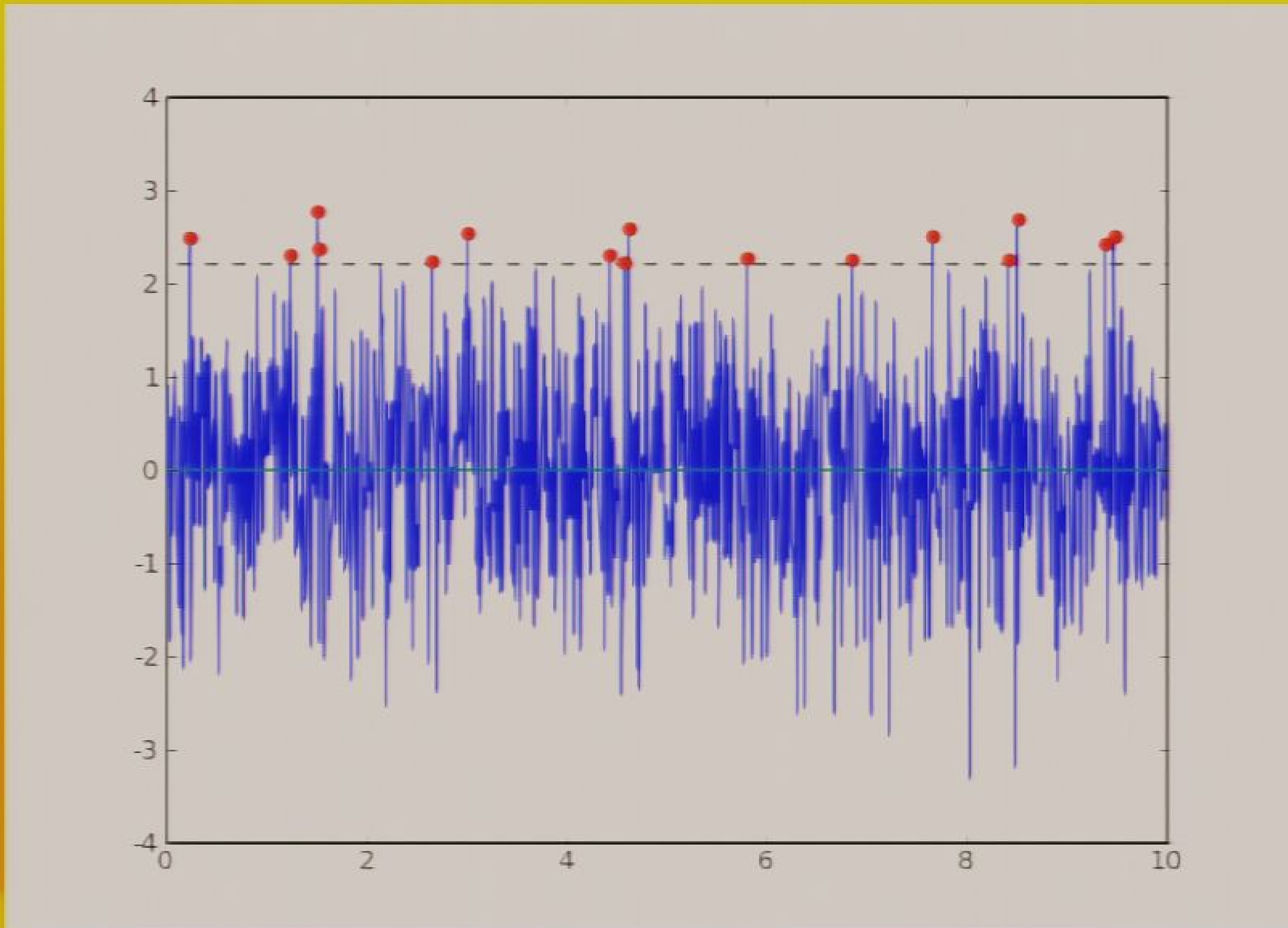
$$P_g(k) = b^2 P_m(k) + const.$$

Power spectrum of galaxies



From Tegmark et al

Detour: rare peaks



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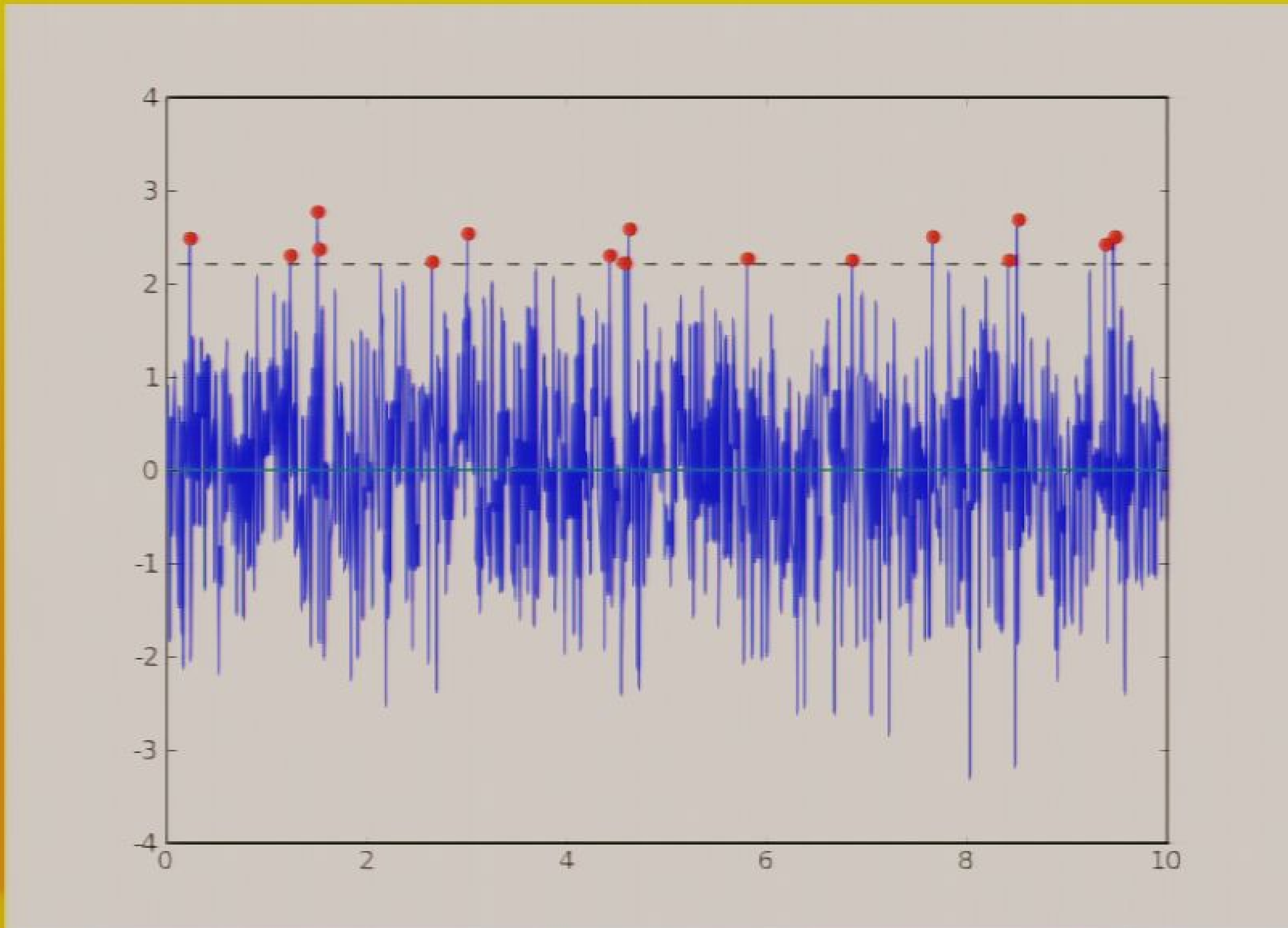

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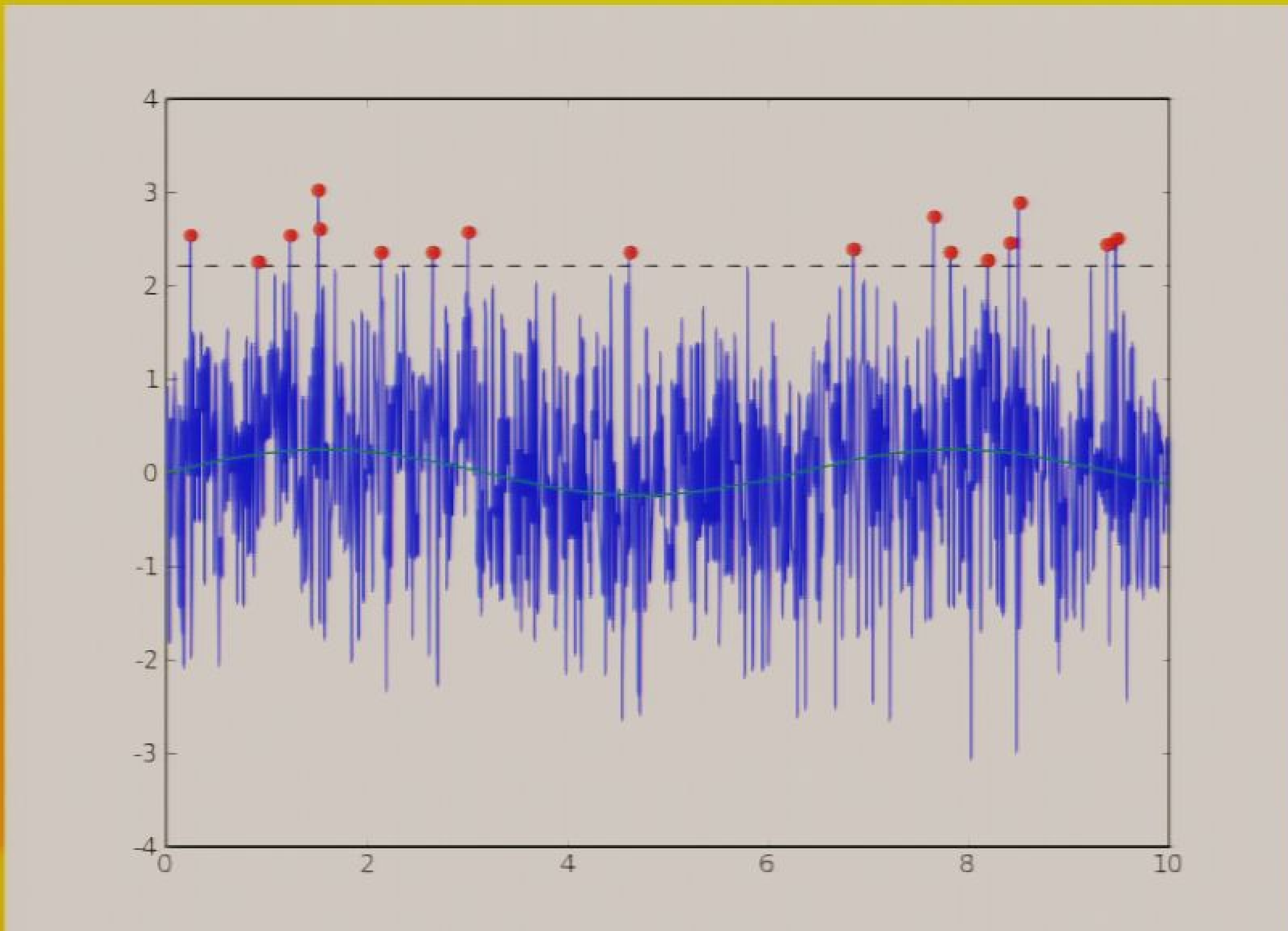

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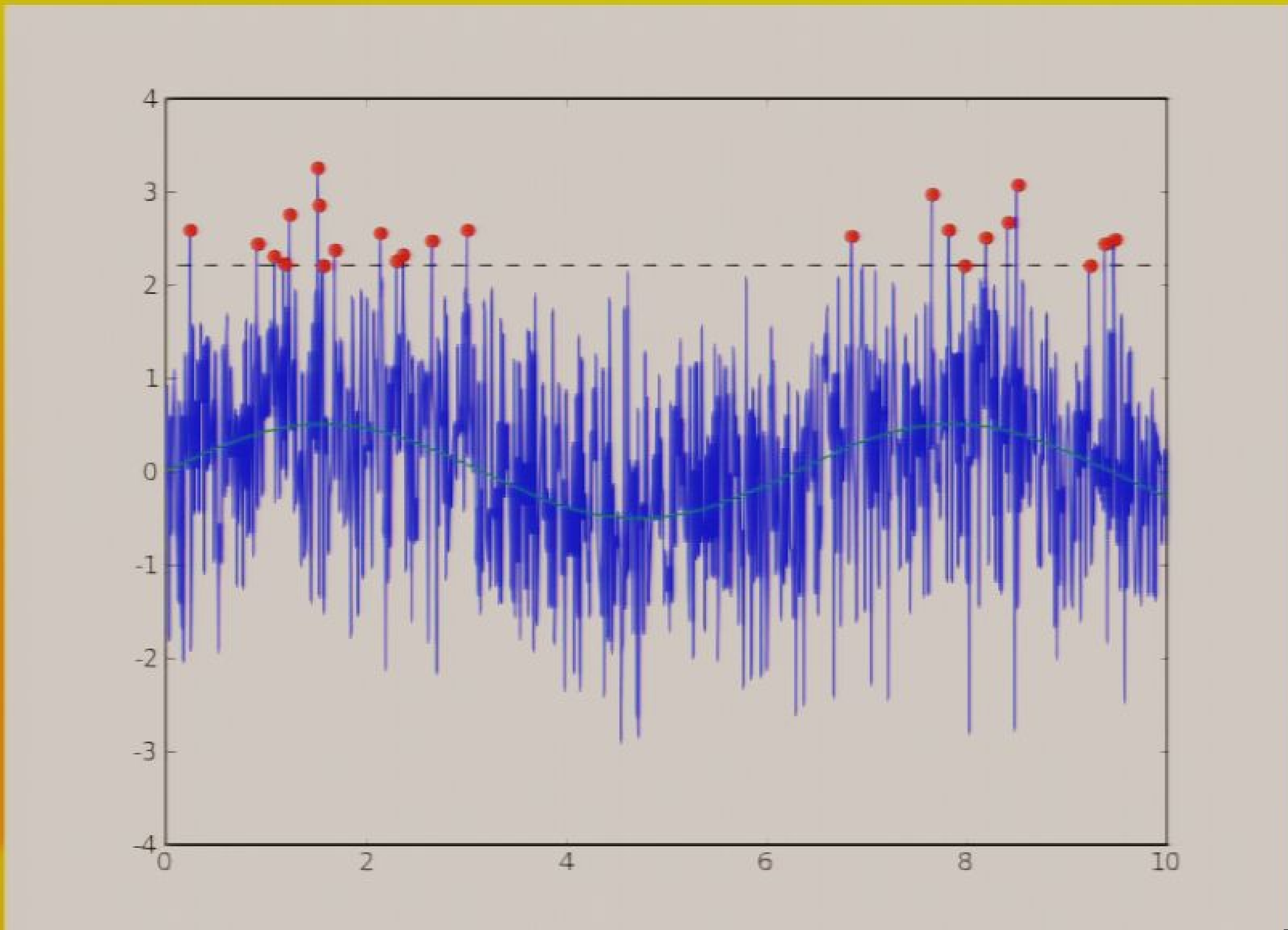

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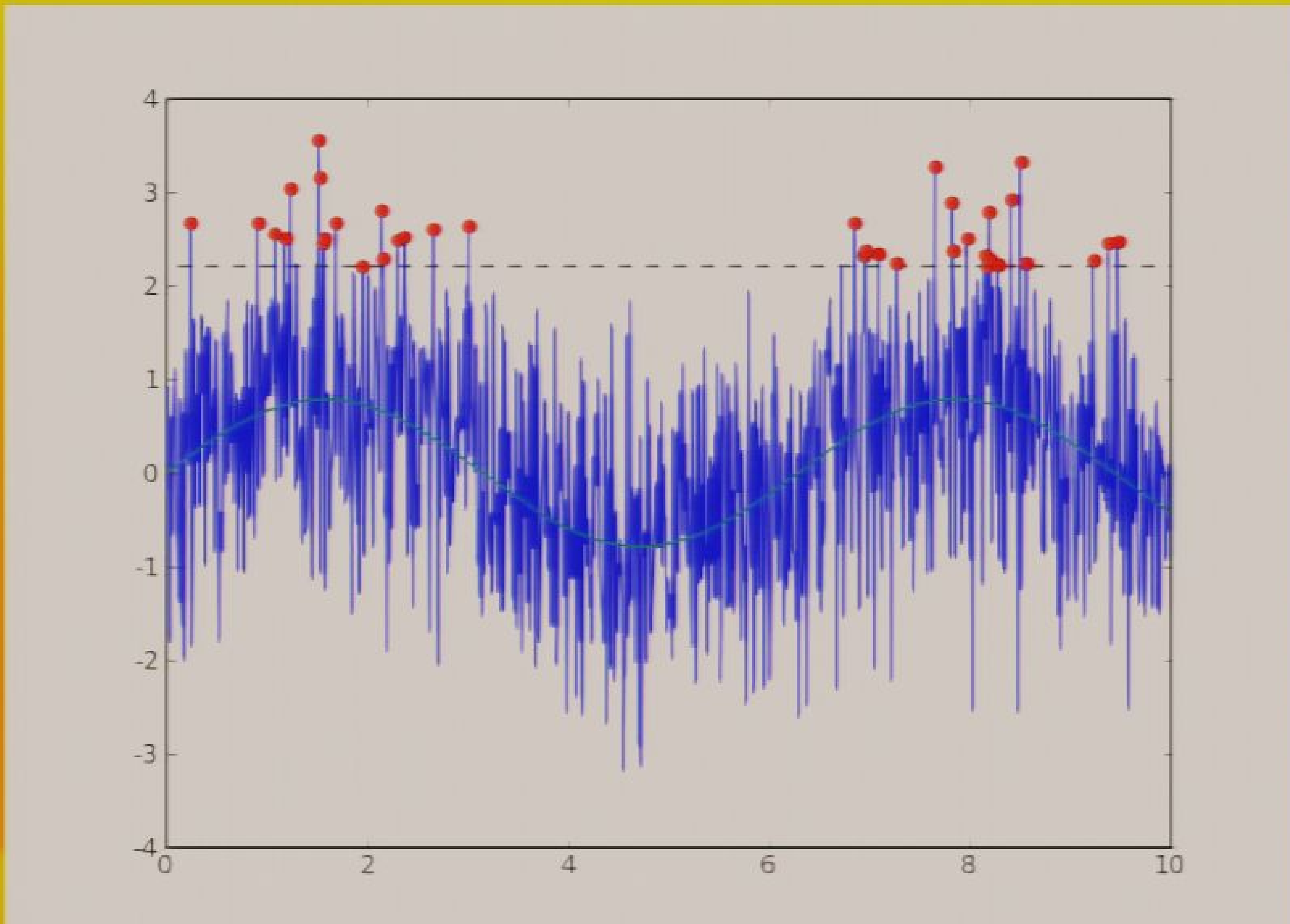

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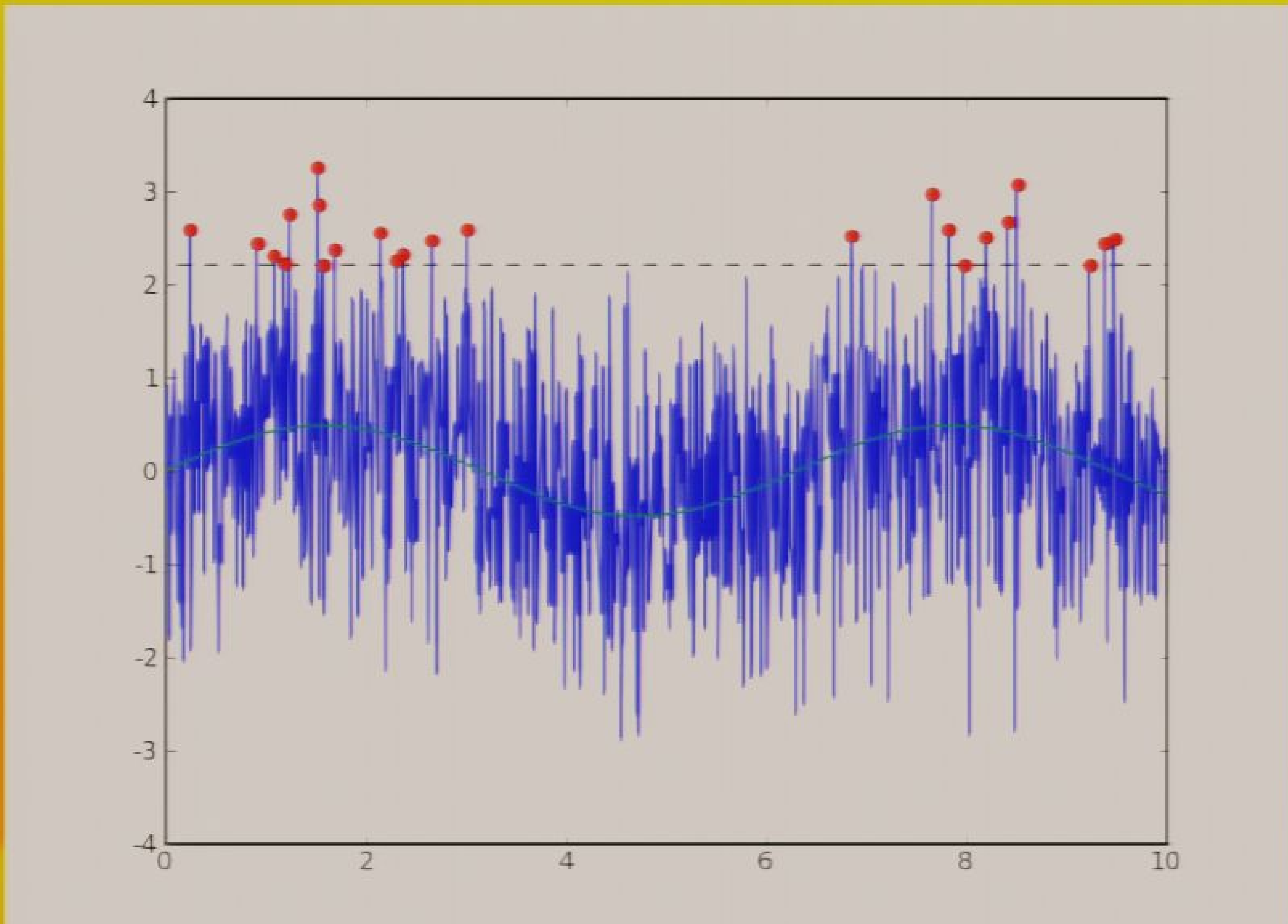

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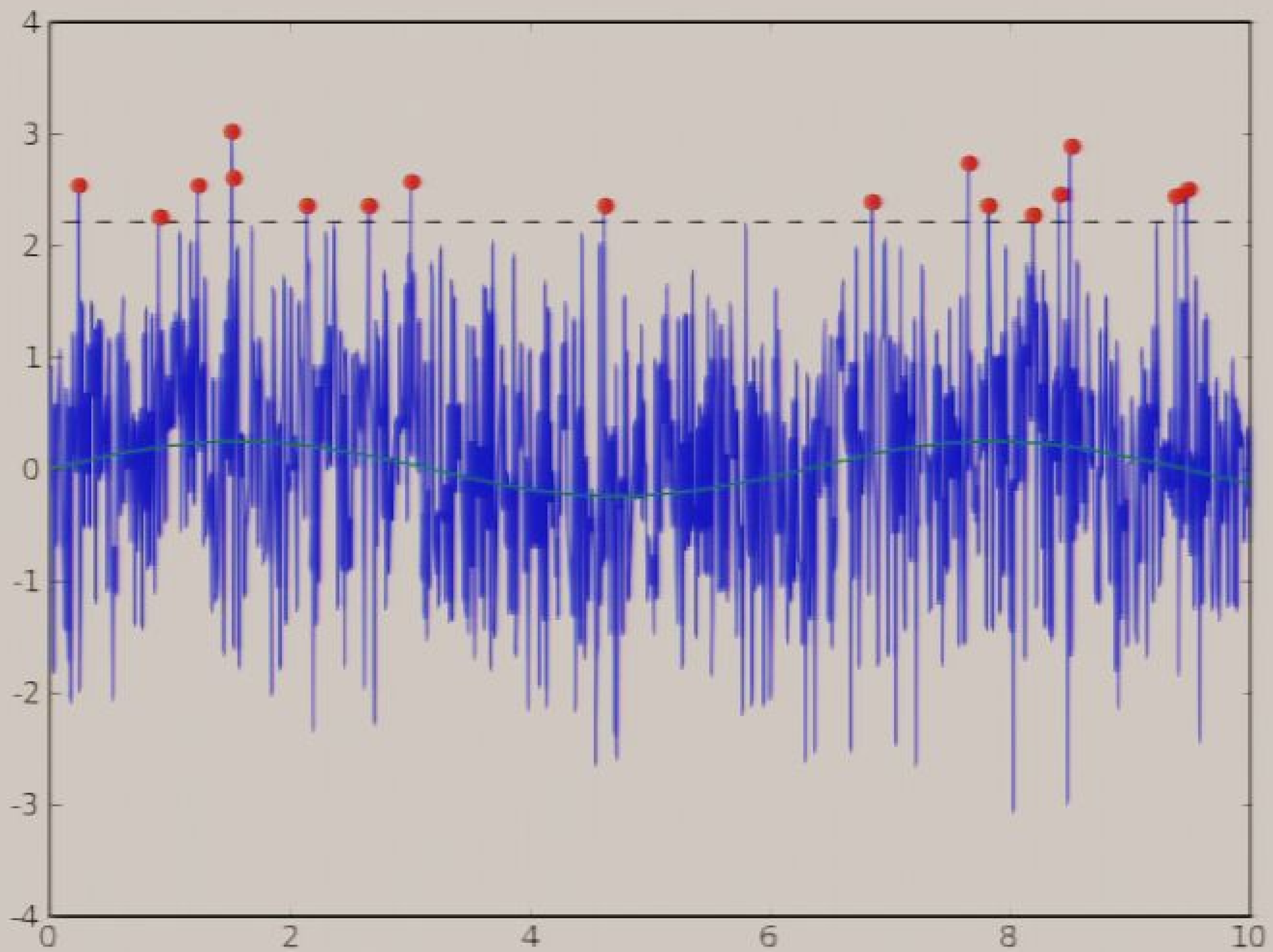

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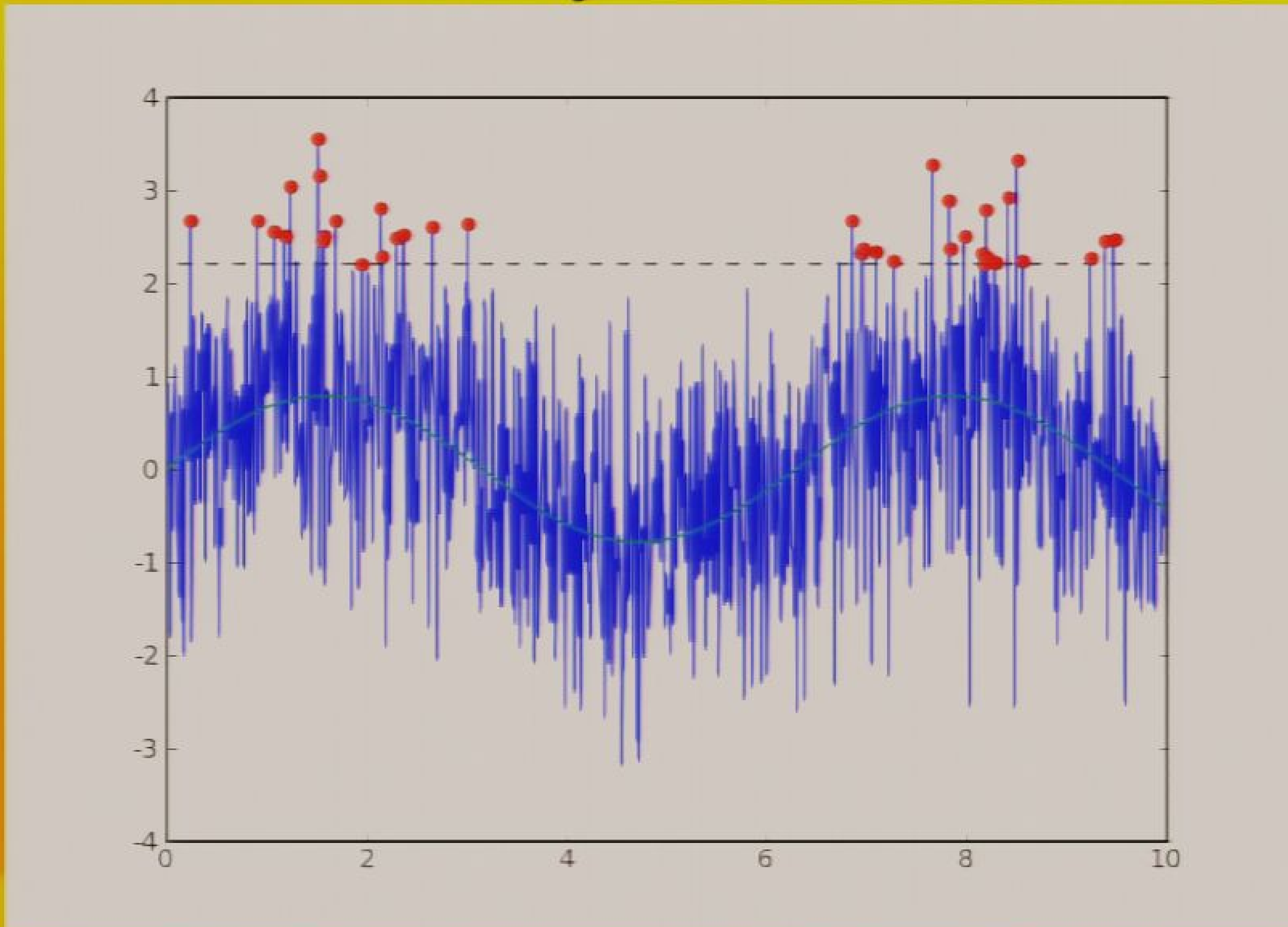


Scale dependent bias:

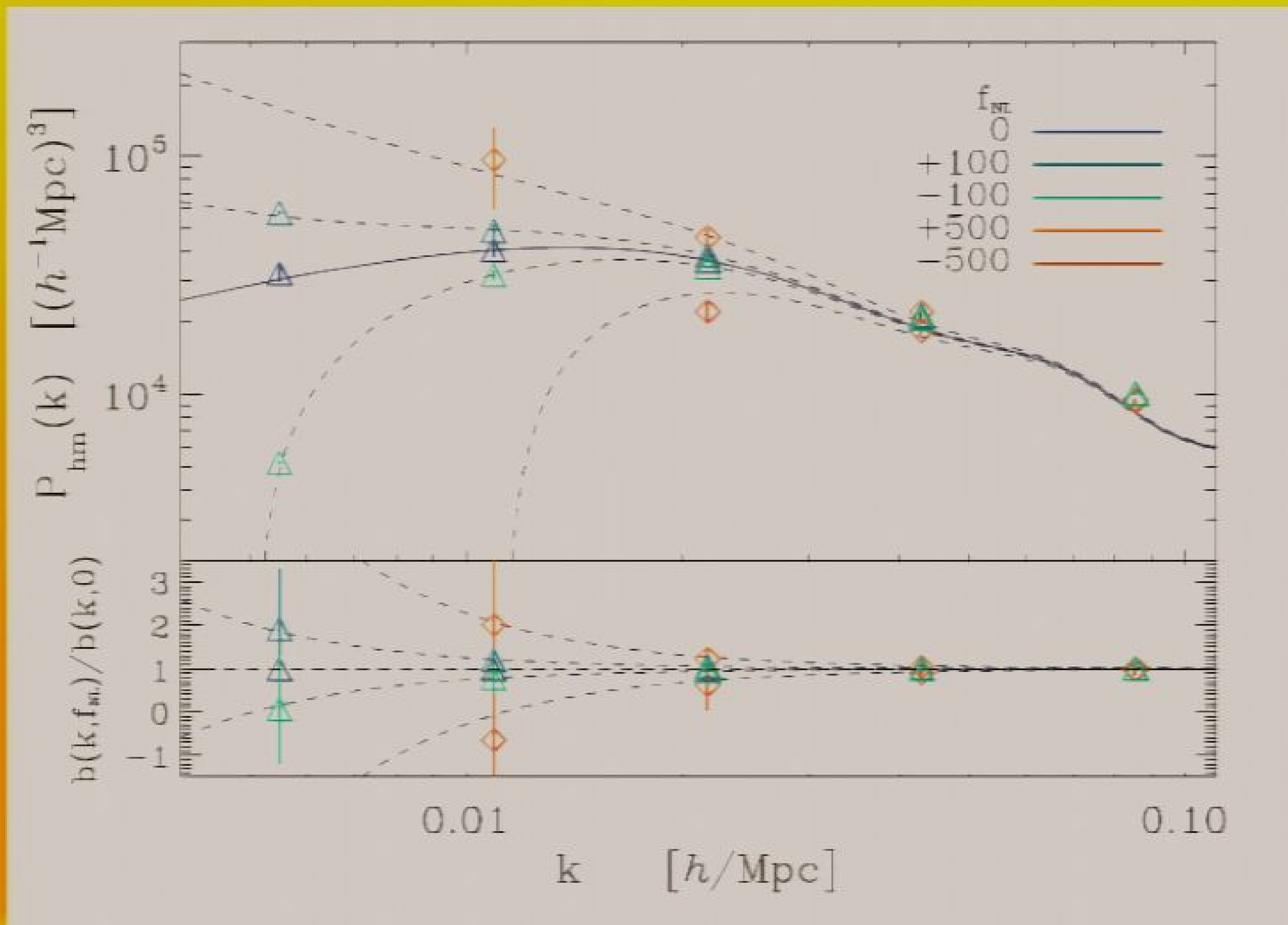
Dalal et al note a very different and surprising effect

- scale dependent bias in clustering of rare peaks in fr
  cosmologies.
- Get scale dependent bias in clustering of highly biased
  objects. Formula is:

$$\Delta b(M, k) = 3 f_{\mathrm{NL}} (b-1) \delta_c \frac{\Omega_m}{k^2 T(k) D(z)} \left(\frac{H_0}{c} \right)^2$$

- Matarrese & Verde, Afshordi and Tolley, McDonald, Slosar et, Giannantonio & Porciani, etc derive an equivalent formula using a multitiude of approaches.

- Note k and b dependence.



f_{NL}
0
+100
−100
+500
−500
10^5
10^4
$P_{hm}(k)$ $[(h^{-1}\mathrm{Mpc})^3]$
$b(k,f_{NL})/b(k,0)$
3
2
1
0
−1
0.01
0.10
k [h/Mpc]

Hirata's magic:

$$\Delta b(M, k) = \frac{3\Omega_m H_0^2}{c^2 k^2 T(k) D(z)} f_{\rm NL} \frac{\partial \ln n}{\partial \ln \sigma_8}.$$

- It can be shown that all PS–like theories in which mass function is function of $\nu = \delta_c^2/\sigma^2(M)$ only reduce to Dalal et al formula.
- But they need not be. For objects tracing recent mergers one gets:

$$\frac{\partial \ln n_{\rm merger}}{\partial \ln \sigma_8} = \delta_c(b - 1 - \delta_c^{-1}).$$

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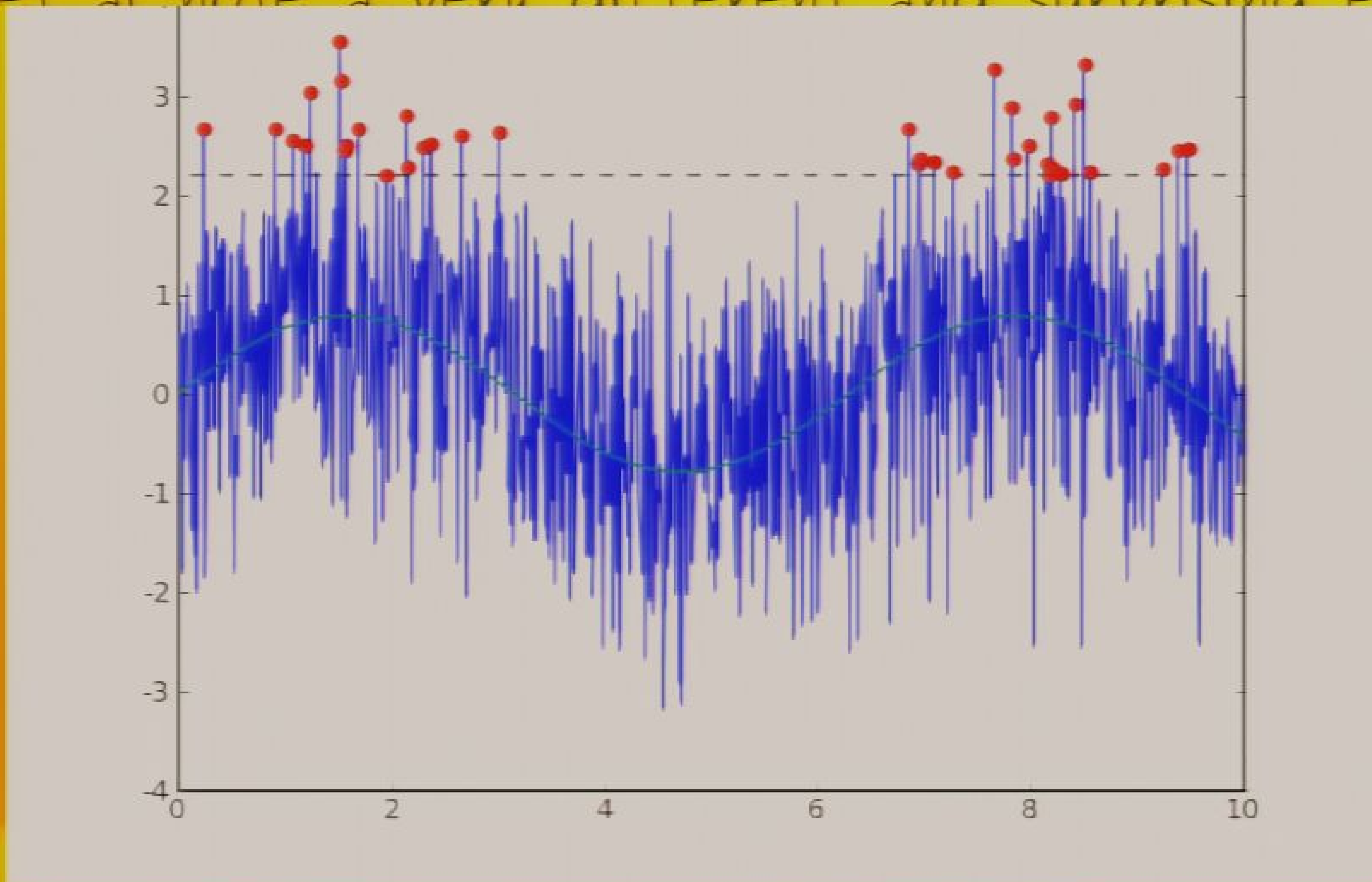

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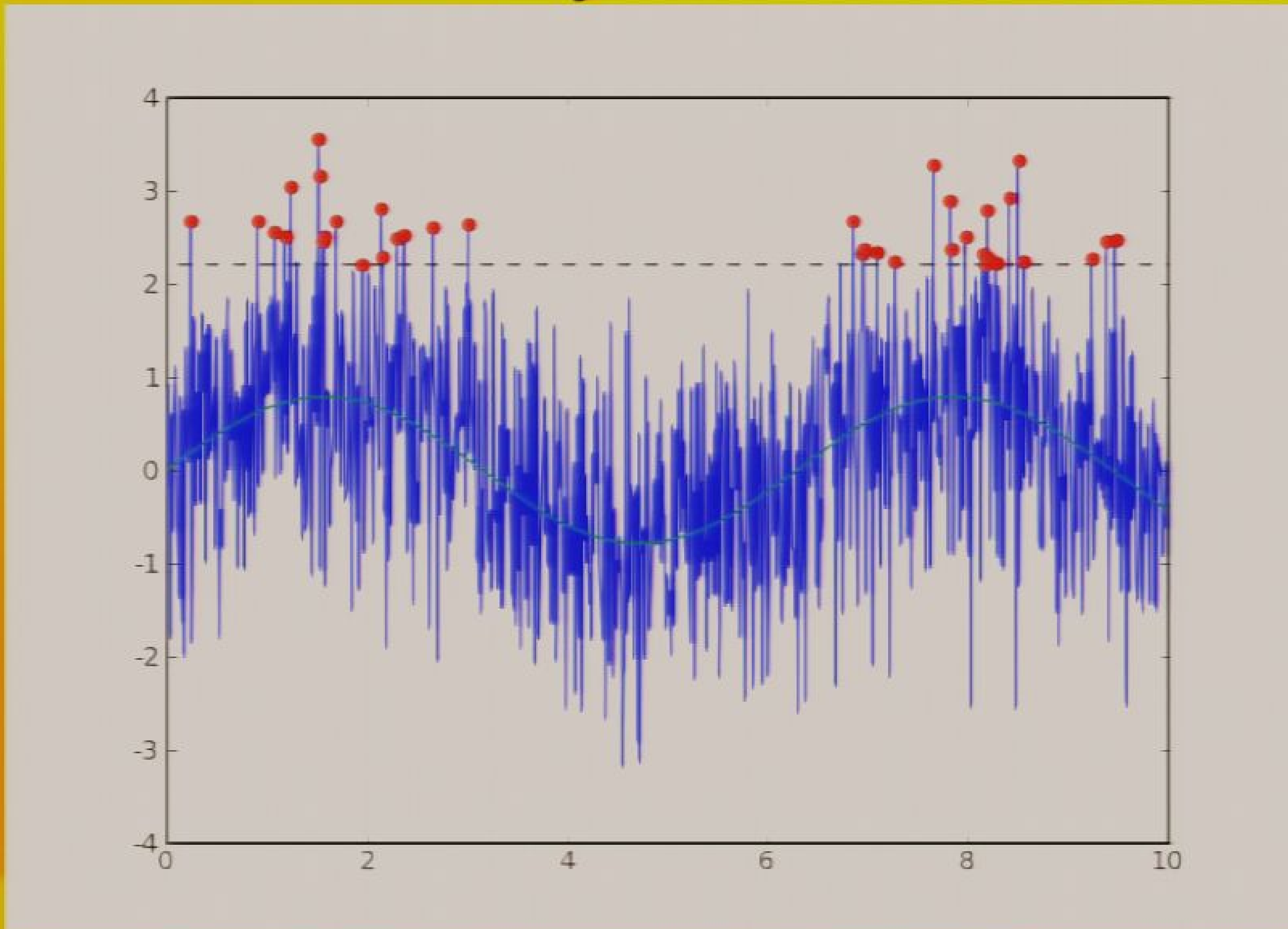
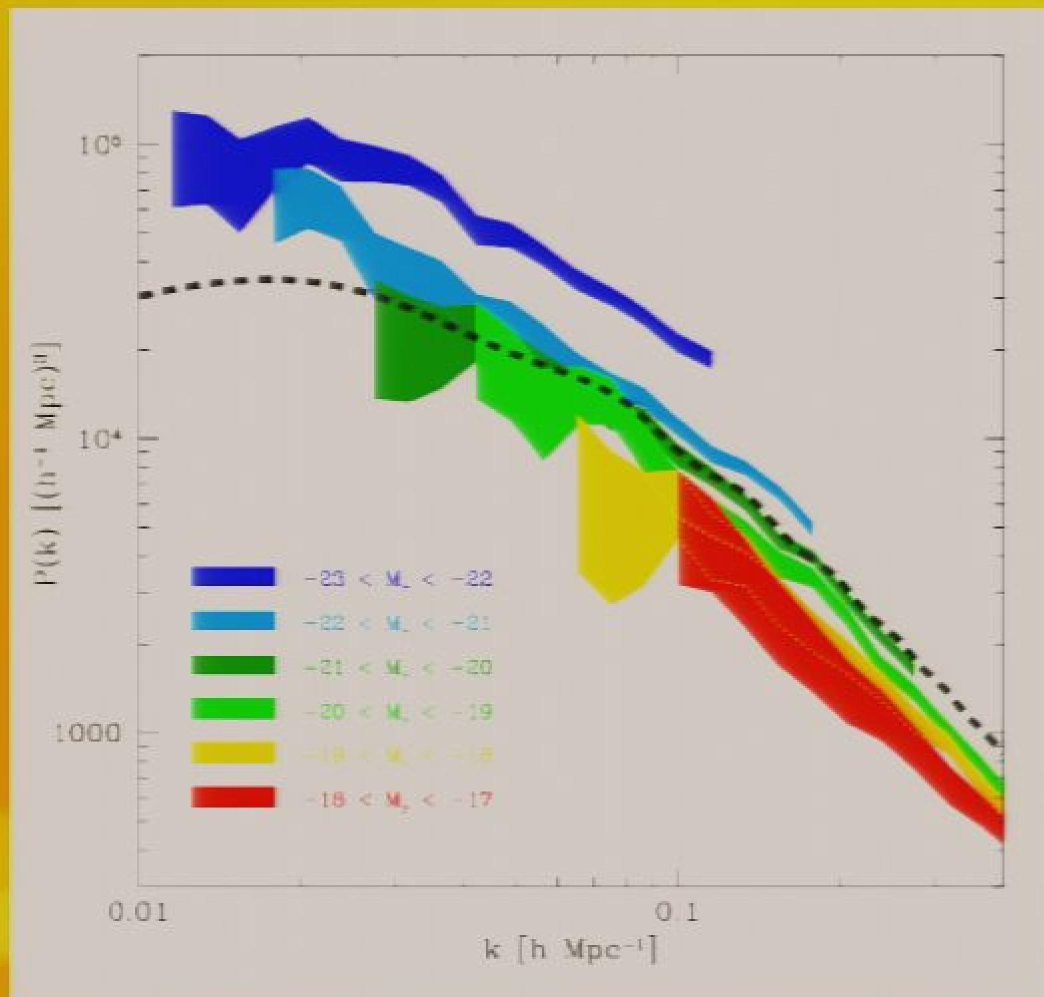


Power spectrum of galaxies



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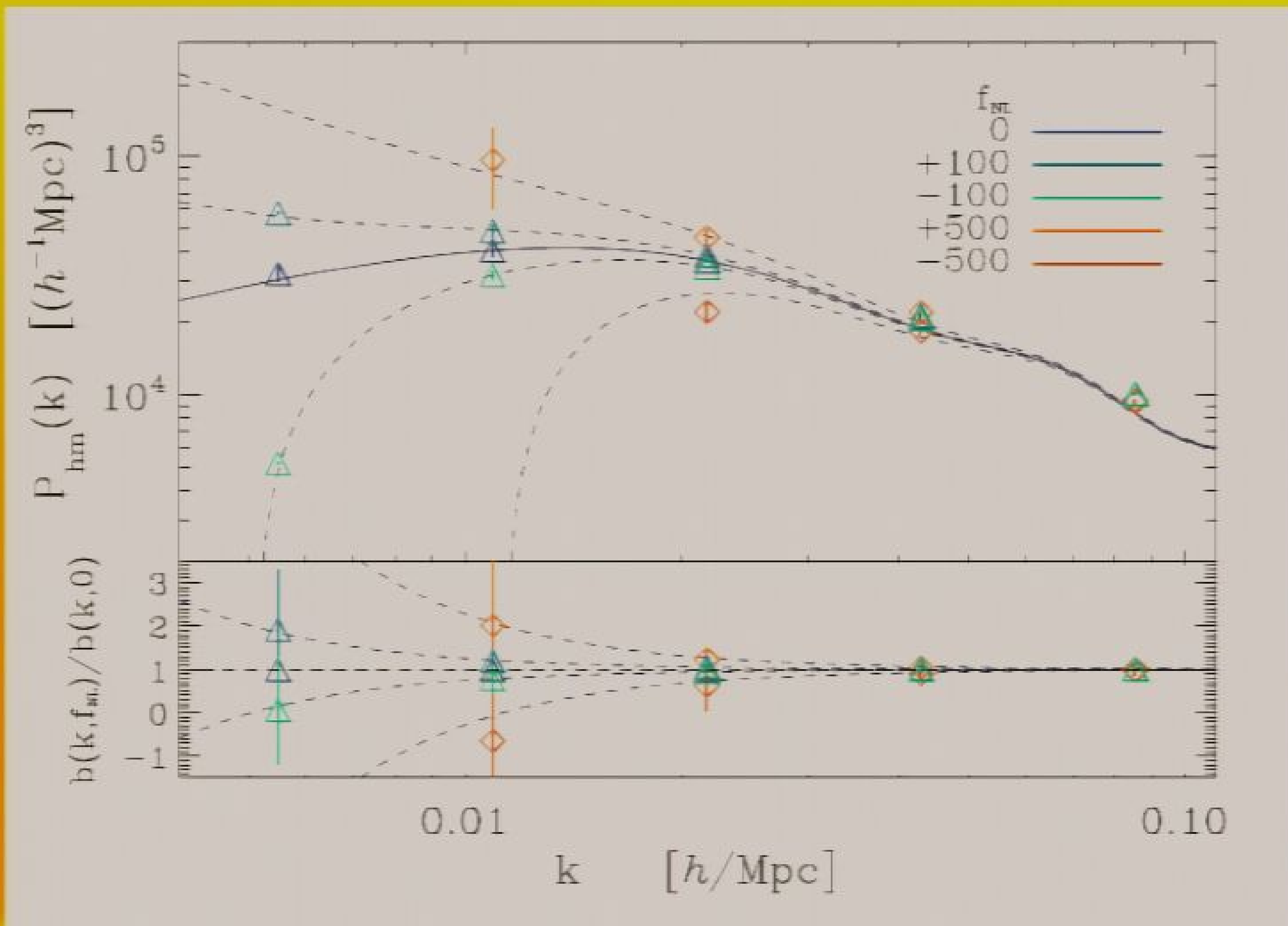

$P_{hm}(k)$ $[(h^{-1}\text{Mpc})^3]$
10^5
10^4
f_{NL}
0
+100
−100
+500
−500
$b(k,f_{NL})/b(k,0)$
3
2
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k [h/Mpc]

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Verde & Matarrese's magic:

- Two point function of peaks can be expressed as

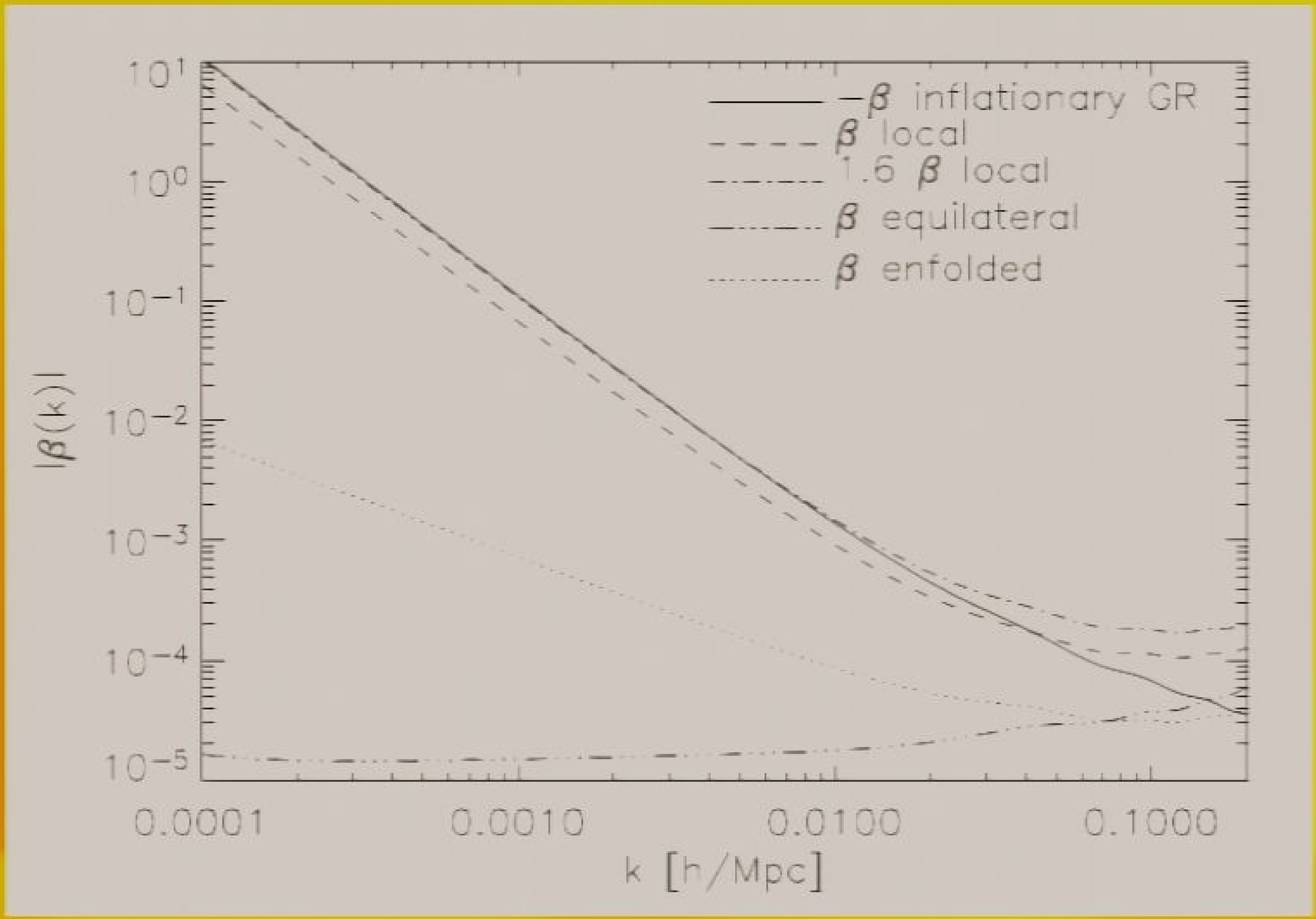
$$\xi_{h,M}(|\mathbf{x}_1-\mathbf{x}_2|) = -1 + \exp\left\{\sum_{N=2}^{\infty}\sum_{j=1}^{N-1}\frac{\nu^N\sigma_R^{-N}}{j!(N-1)!}\xi^{(N)}\left[\underbrace{\mathbf{x}_1,\ldots,\mathbf{x}_1}_{j\,times},\underbrace{\mathbf{x}_2,\ldots\ldots,\mathbf{x}_2}_{(N-j)\,times}\right]\right\}. \quad (4)$$

- They get:

$$P_{\rm h}(k,z) = \frac{\delta_c^2(z)P_{\delta\delta}(k,z)}{\sigma_R^4 D^2(z)}\left[1+4f_{\rm NL}\delta_c(z)\frac{P_{\phi\delta}(k)\mathcal{F}_R(k)}{P_{\delta\delta}(k)}\right]$$

- Can write two-point function of peaks for **any** non-Gaussianity as an integral over higher order correlators

Verde & Matarrese's magic:



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$$P_{\mathrm{h}}(k, z) = \frac{\delta_c^2(z) P_{\delta\delta}(k, z)}{\sigma_R^4 D^2(z)} \left[1 + 4 f_{\mathrm{NL}} \delta_c(z) \frac{P_{\phi\delta}(k) \mathcal{F}_R(k)}{P_{\delta\delta}(k)} \right]$$

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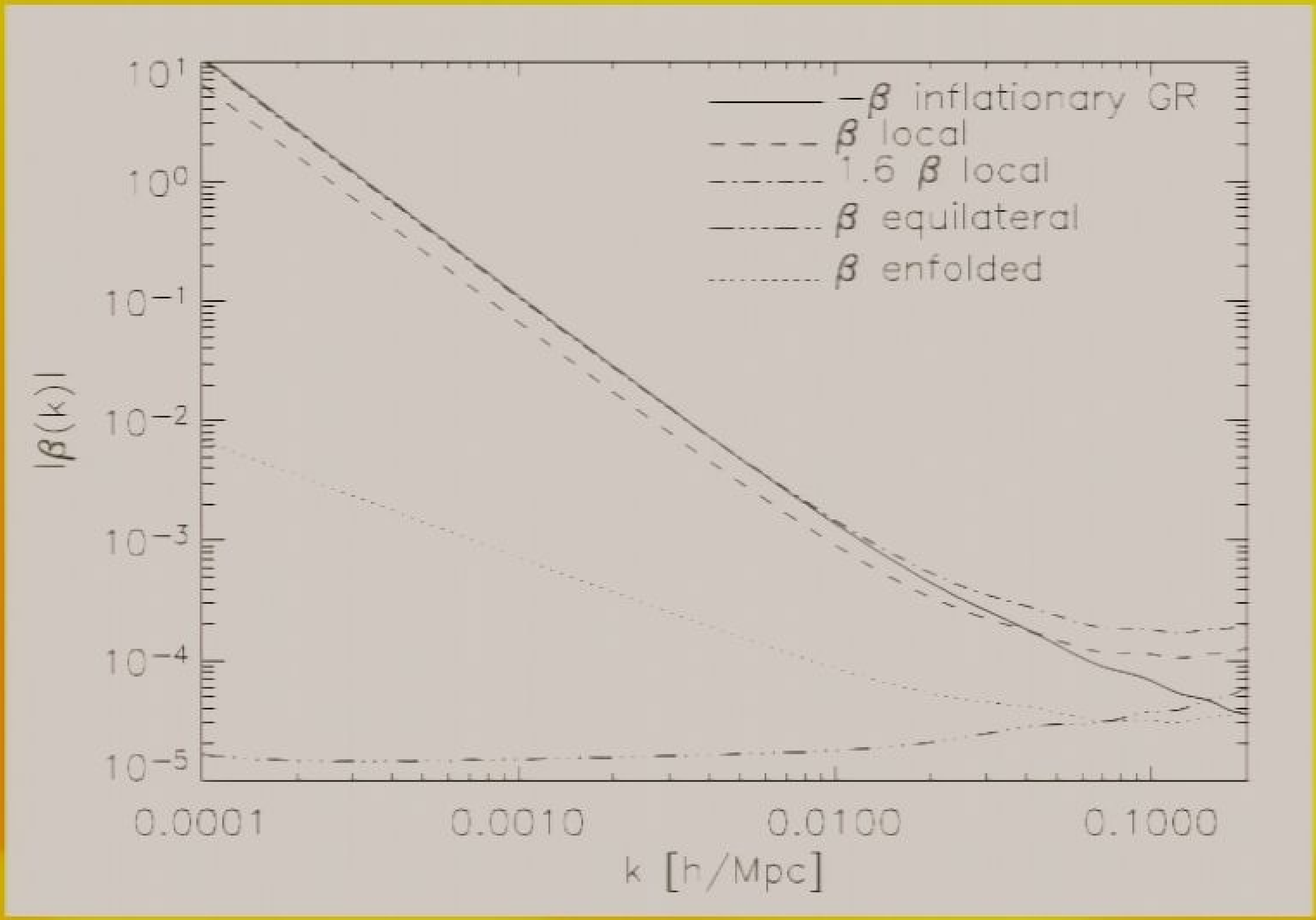
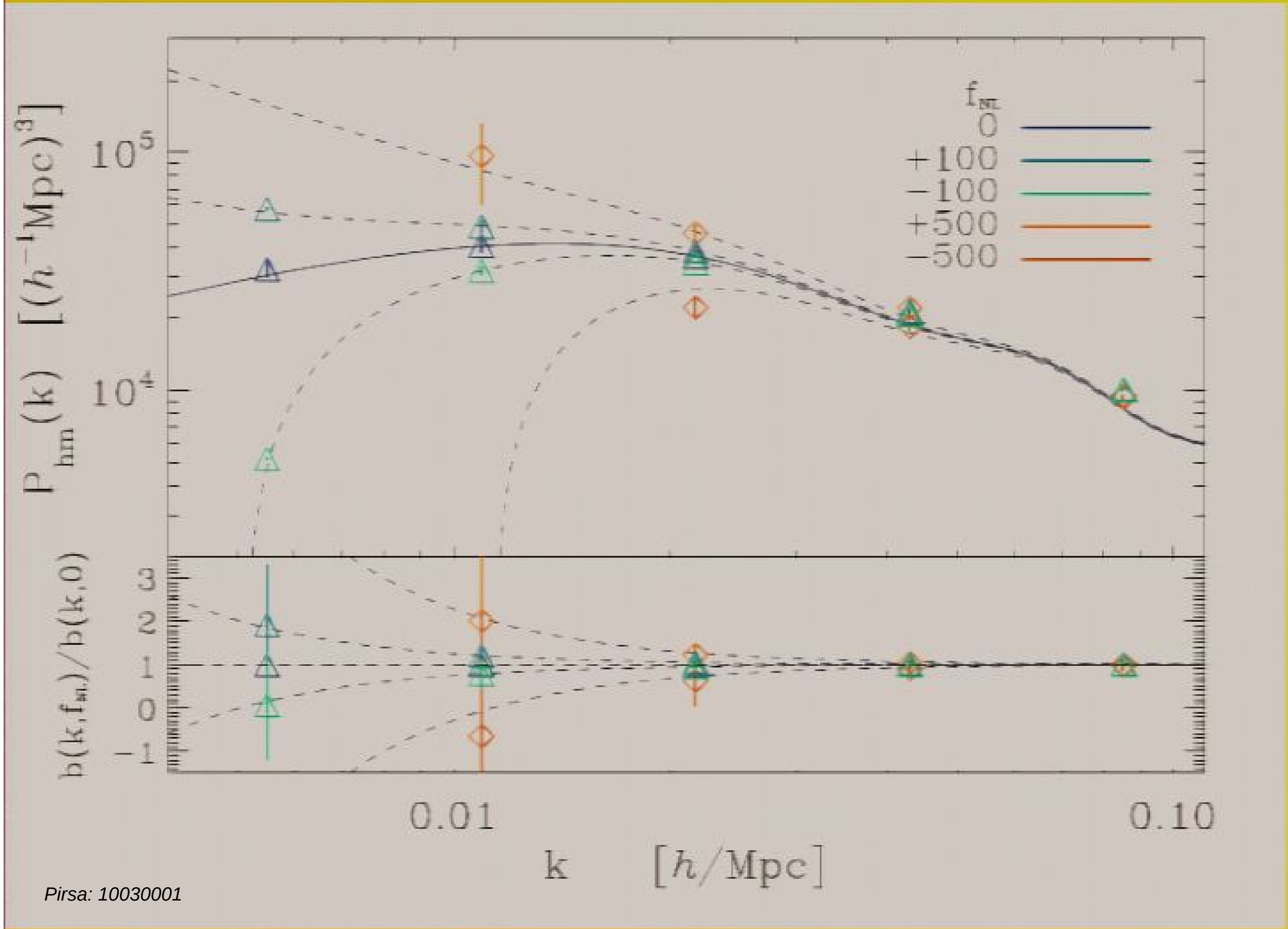


Simulations:



- Formula tested in cross-correlation!!
- seems to work like magic!
- (from Dalal et al)
- Note that one is safe from NL physics.

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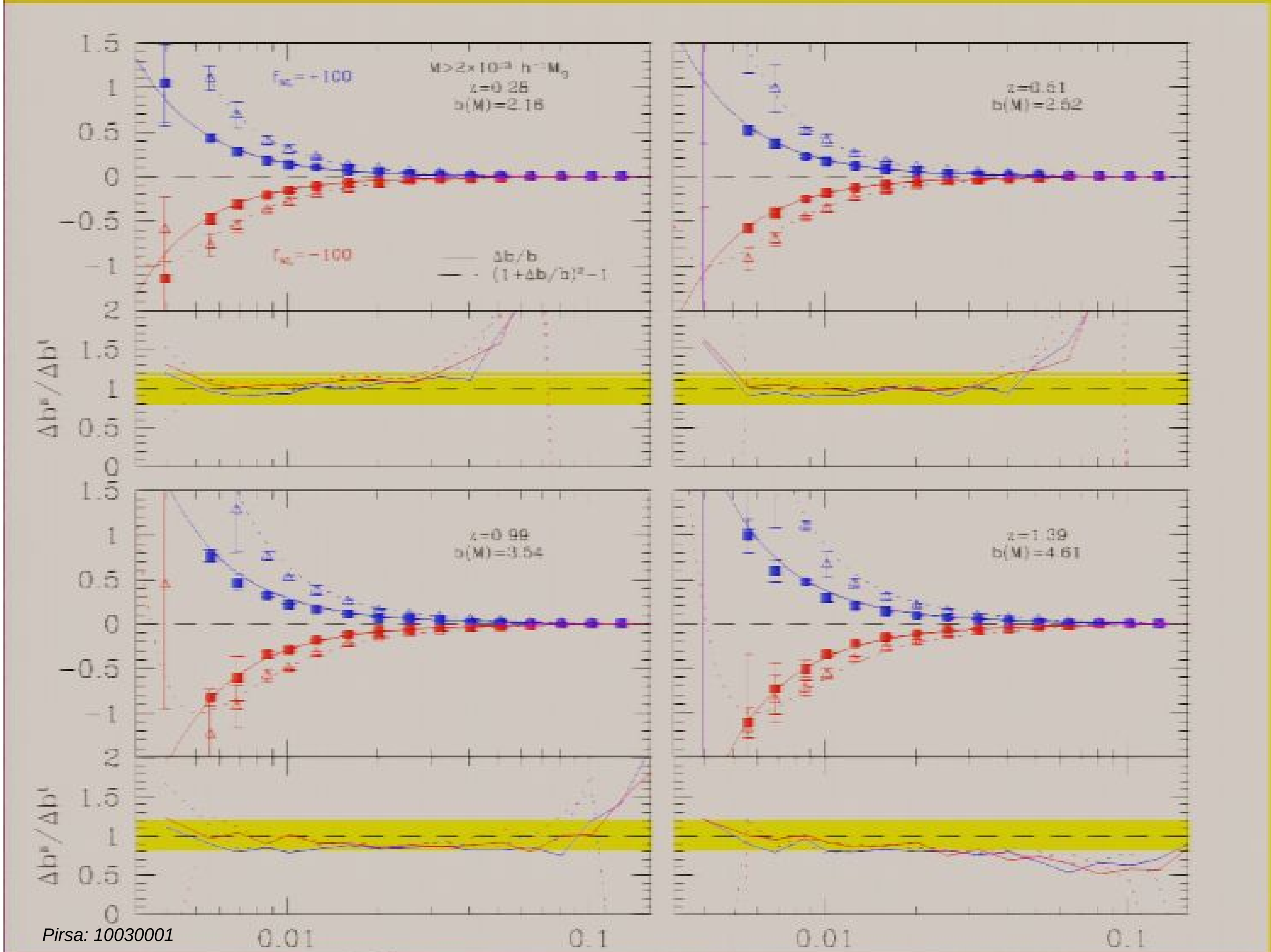
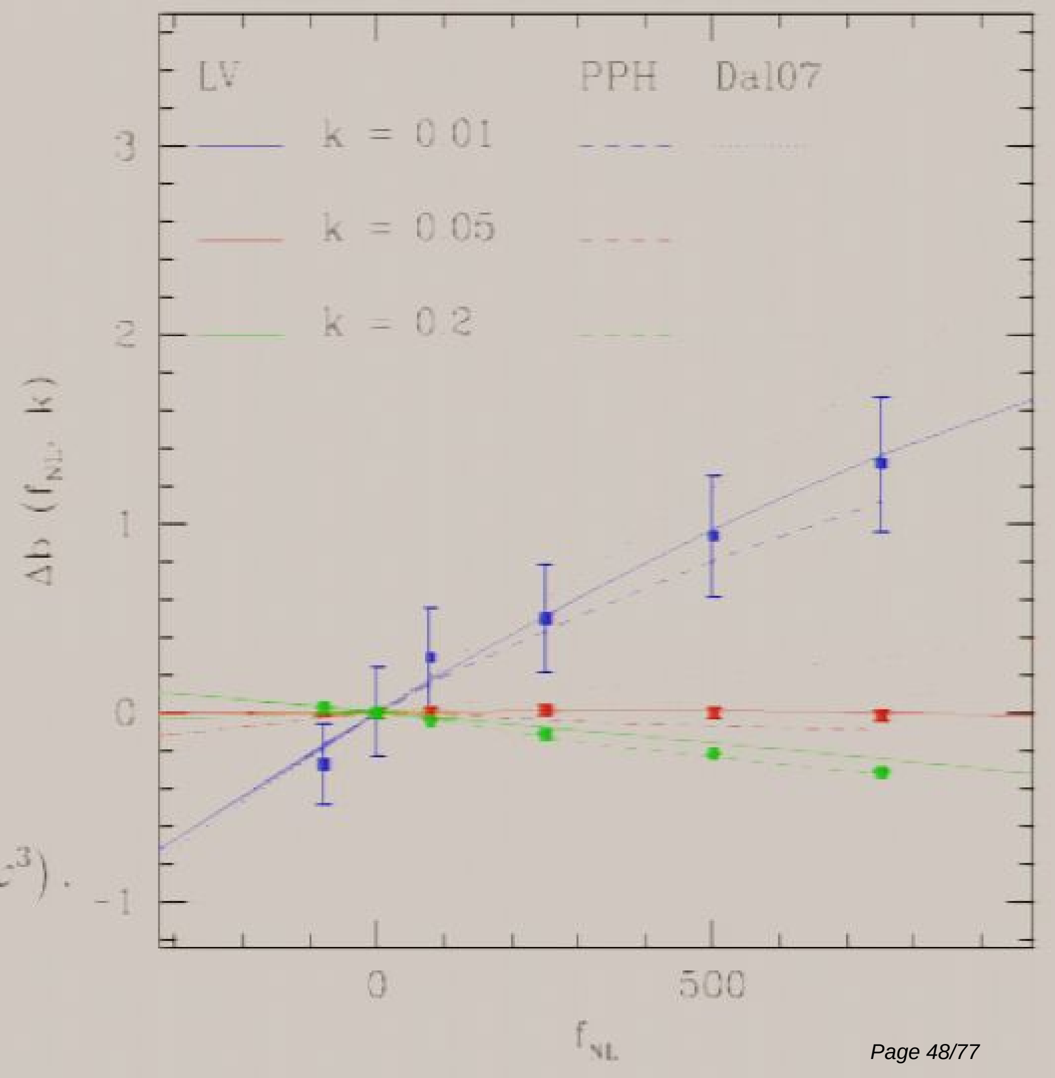


- seems to work like magic!
- from Desjacques e al
- they also see Taruya et all effect and scale indep. Bias.
- Licia Verde and collabs disagree?

Giannantonio & Porciani's magic



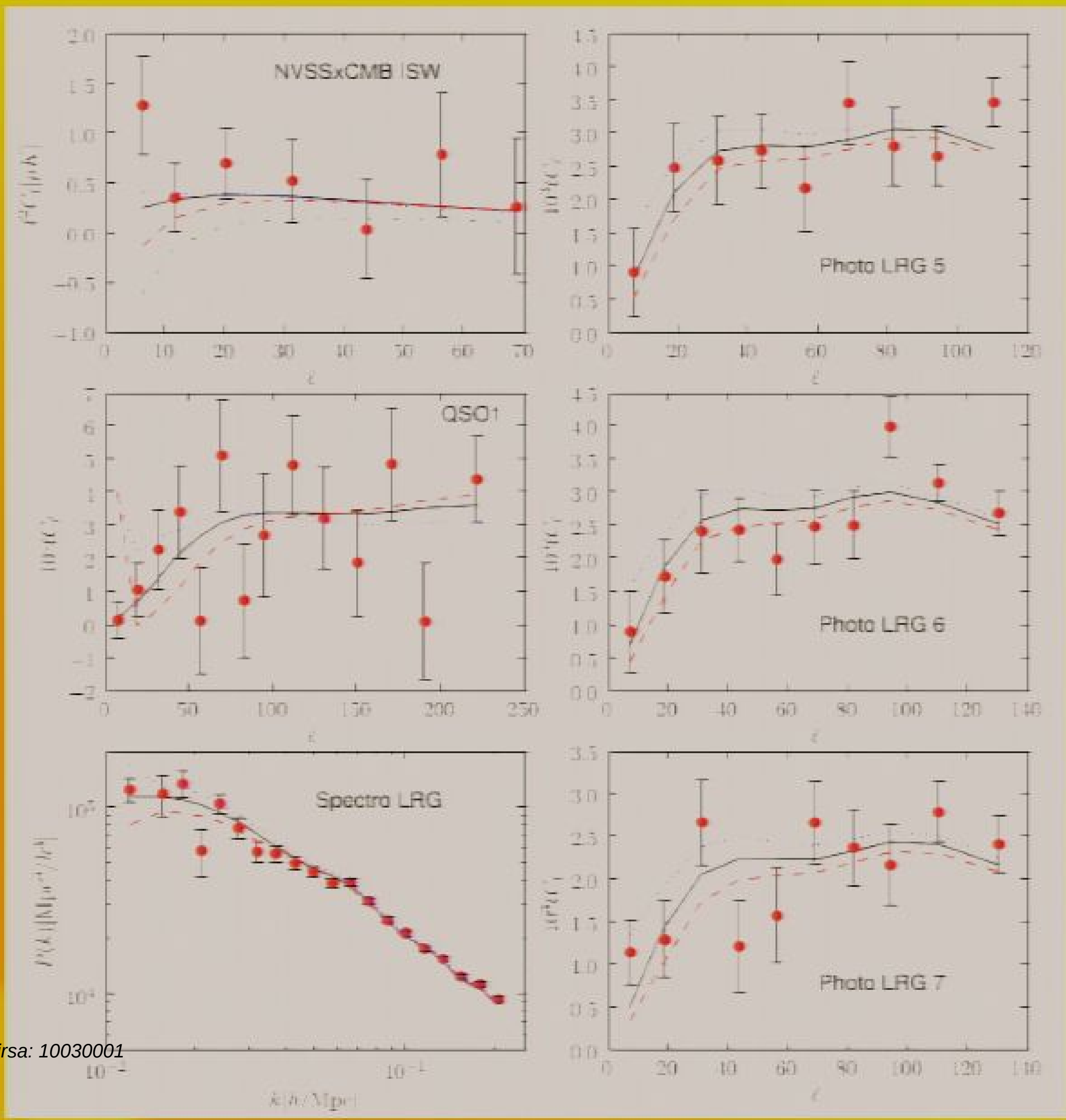
$$
\begin{aligned}
\delta_h(\mathbf{x}) &= b_{10}\,\delta + b_{01}\,\varphi + \\
&+ \frac{1}{2!}\left(b_{20}\,\delta^2 + 2\,b_{11}\,\delta\varphi + b_{02}\,\varphi^2\right) + \\
&+ \frac{1}{3!}\left(b_{30}\,\delta^3 + 3\,b_{21}\,\delta^2\varphi + 3\,b_{12}\,\delta\varphi^2 + b_{03}\,\varphi^3\right).
\end{aligned}
$$

How to measure it?

$$\Delta b(M, k) = 3 f_{\rm NL}(b-1)\delta_c \frac{\Omega_m}{k^2 T(k) D(z)} \left(\frac{H_0}{c}\right)^2$$

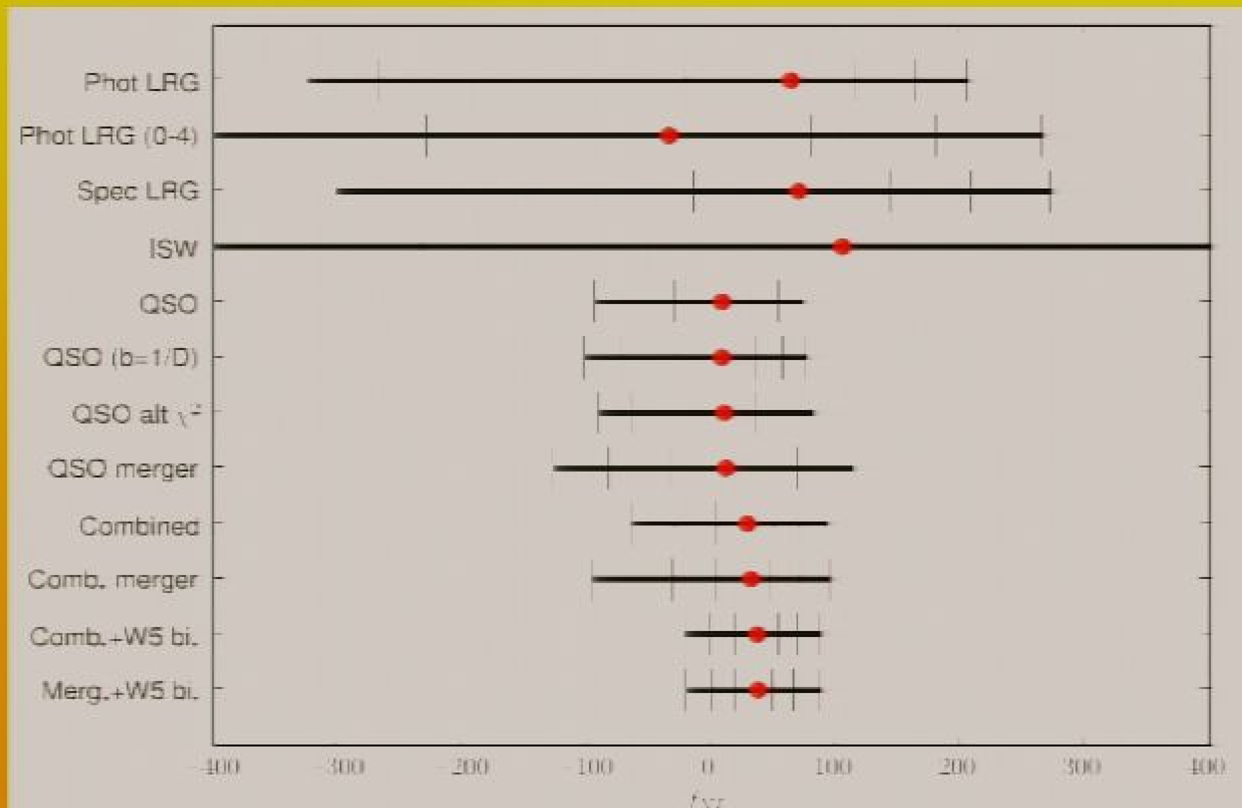
- Need large volume surveys (small k) of highly biased tracers of structure.
- We use:
  - LRGs (photo, spectro)
  - QSO
  - ISW

Datasets & Effects



- ISW and QSO data same as in Hirata et al. & Ho et al.
- Spectroscopic Tegmark's LRGs
- Photometric Padmanabhan's LRGs
- We don't find great sensitivity in ISW as opposed to Afshordi

Results (0805.3580):



- Limits:

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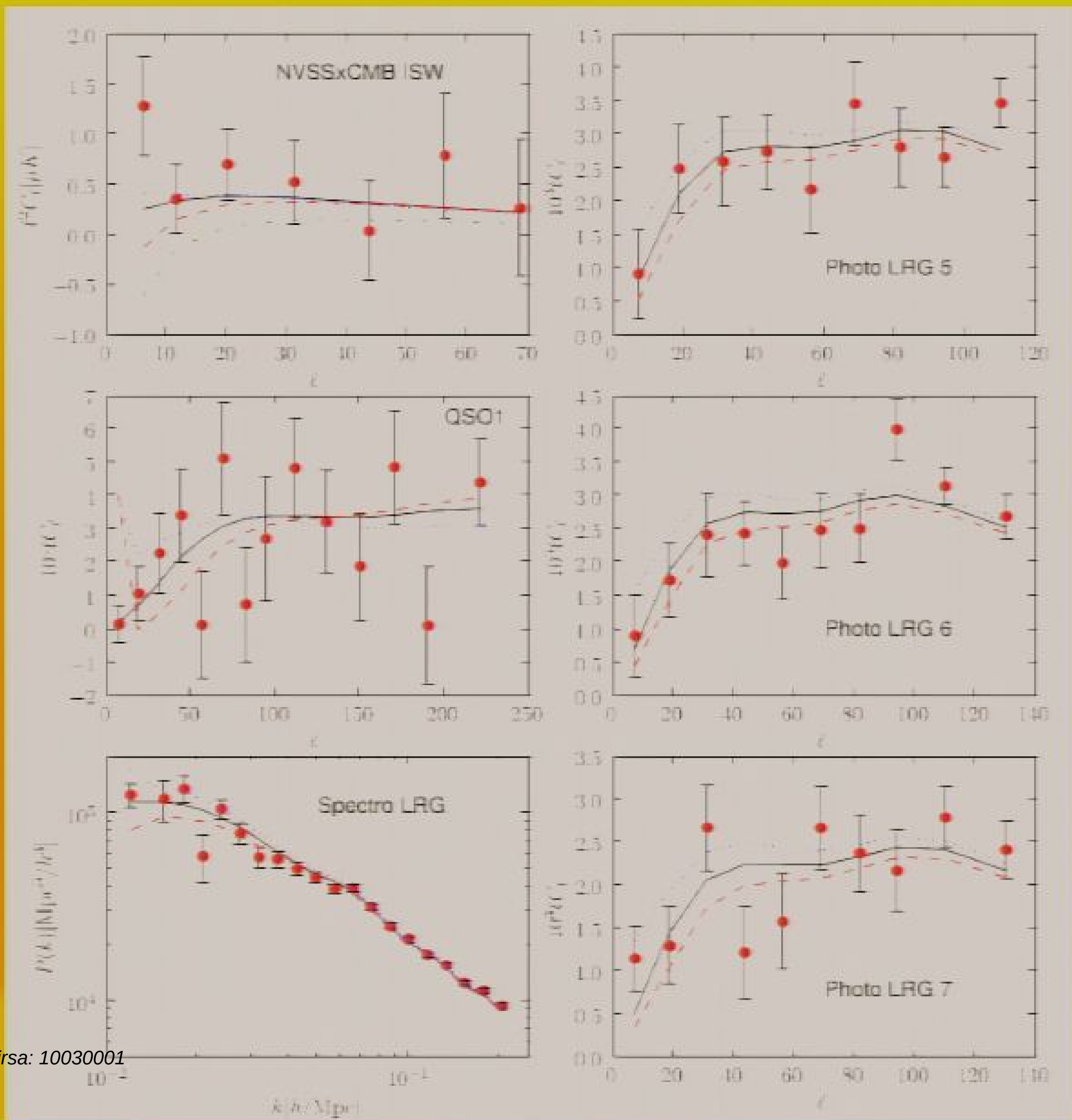

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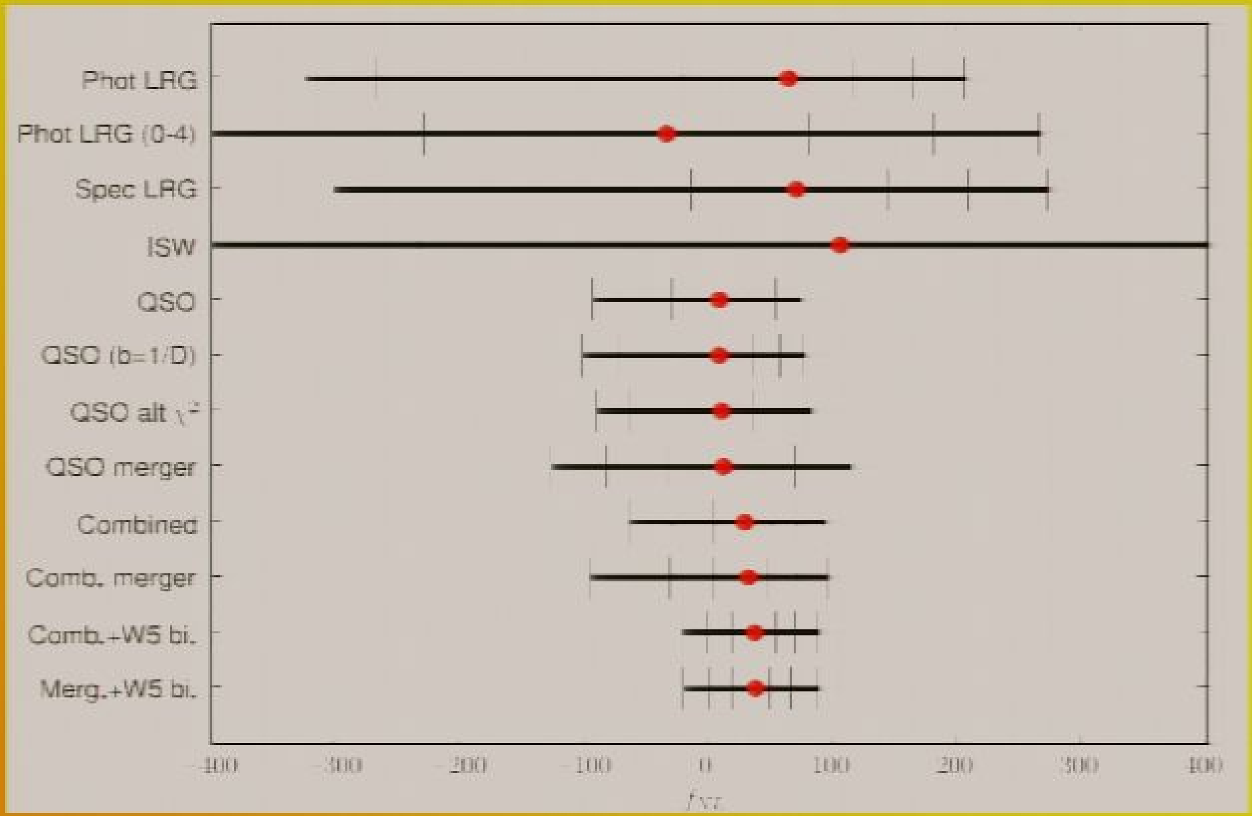

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Gen.: Bispectrum of a tracer

- Jeong + Komatsu et al, 0904.0497:

$$B_h(k_1,k_2,k_3,z) = b_1^3(z)D^4(z)\left[B_m^G(k_1,k_2,k_3) + \frac{b_2(z)}{b_1(z)}\{P_R(k_1)P_R(k_2) + \text{(cyclic)}\} + f_{\rm NL}\frac{B^{nG}_{f_{\rm NL}}(k_1,k_2,k_3)}{D(z)} + f_{\rm NL}^2\frac{B^{nG}_{f^2_{\rm NL}}(k_1,k_2,k_3)}{D^2(z)} + g_{\rm NL}\frac{B^{nG}_{g_{\rm NL}}(k_1,k_2,k_3)}{D^2(z)}\right].$$

- Different "triangle" dependence
- At higher redshift NG goes up, NL goes down

Infrared divergence?

- For small k, P(k) goes as k^-4 and correlation function diverges. Is this a problem?

$$\xi(r) = \frac{1}{(2\pi)^3} \int d^3k P(k) e^{i\mathbf{k}\cdot\mathbf{r}}.$$

- Measured correlation function always Ok:

$$\left\langle \tilde{\xi}(r) \right\rangle = \xi(r) - \sigma_W^2.$$

Infrared divergence?

- Any other GR corrections on large scales?
- Two issues:
  - What do you really measure on super-horizon scales?
    - Yoo et al working on it, sounds promising
  - What is the correct form of the Poisson equation to use when dealing with collapsing objects.
    - Somewhat trivially no effects within spherical collapse (Wands & Slosar), the implicit gauge is Newtonian

Two Fields

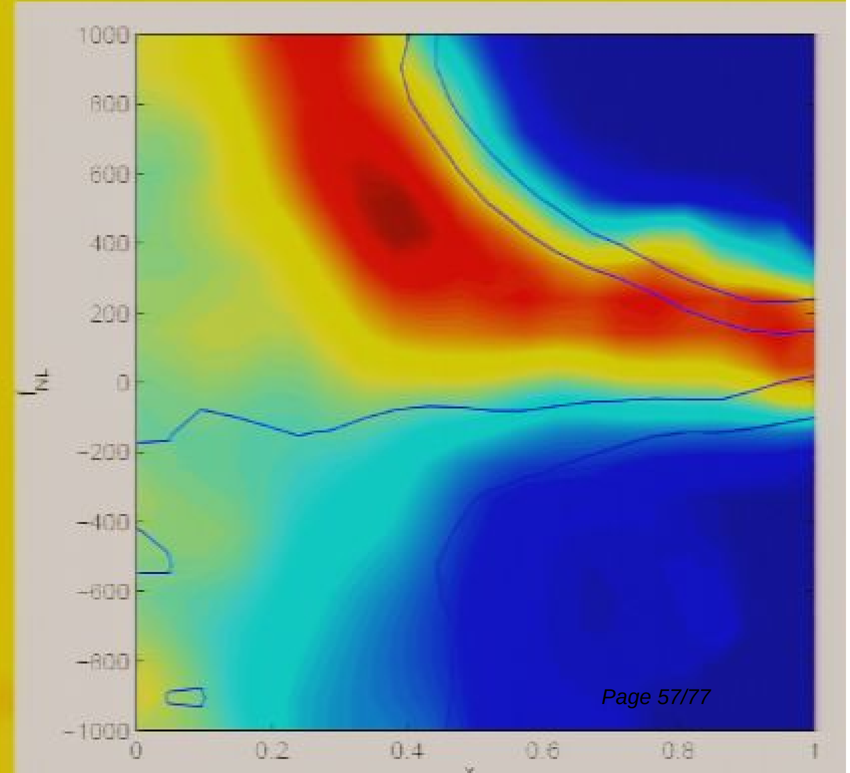
- With Tseliakovich & Hirata

$$\Phi = \phi_1 + \phi_2 + \tilde{f}_{\rm NL}\phi_2^2.$$

$$P_L^h = b_g^2 \frac{P^{lin}}{1+\xi^2}\left(\xi^2 + \left(1 + \frac{2\tilde{f}_{\rm NL}\alpha^{-1}\delta_c}{1+\xi^2}\right)^2\right)$$

$$\xi = \sigma_1/\sigma_2$$

$$x_2 = 1/(1+\xi^2).$$

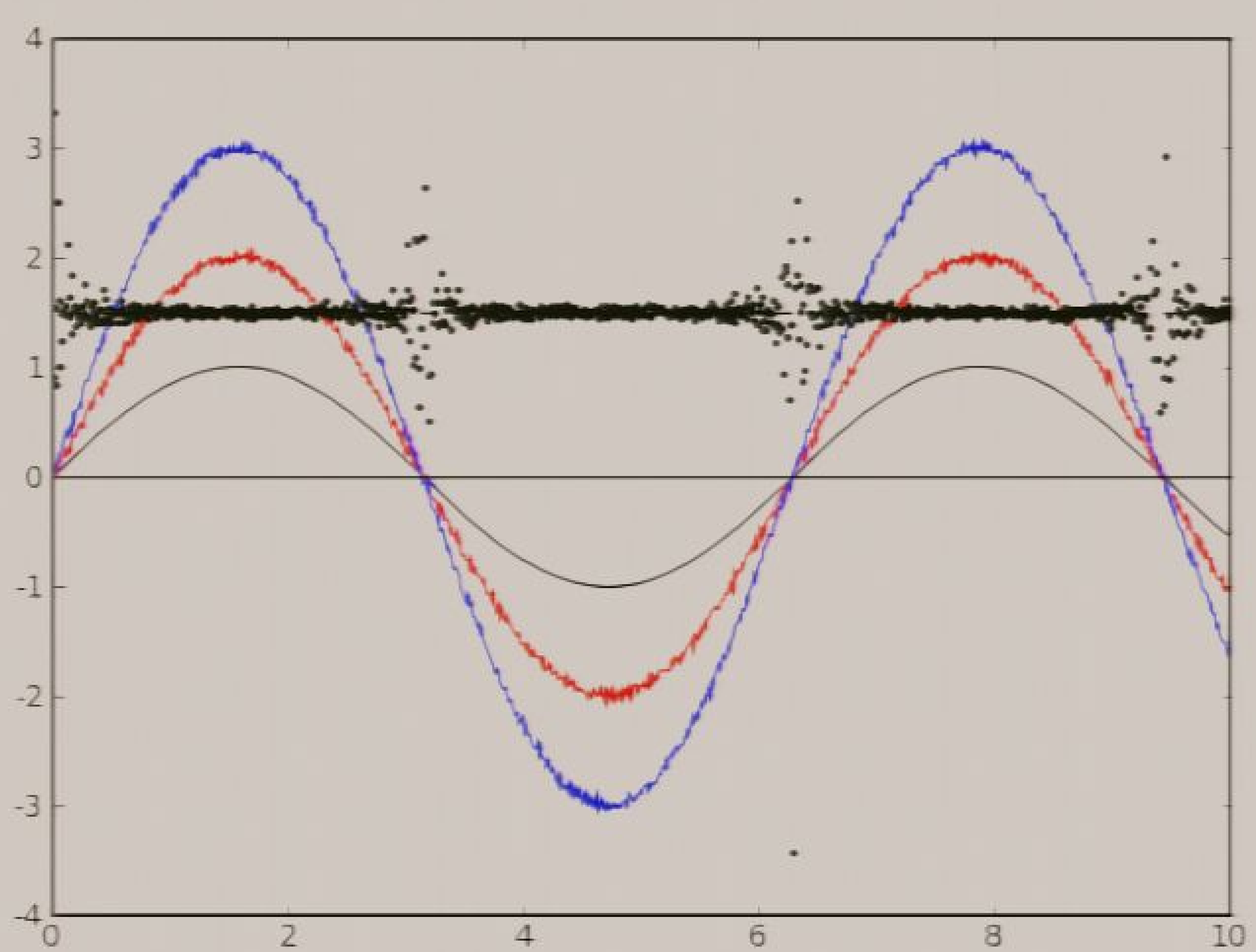


Avoiding sample variance.

- The above method relied on the scale dependent bias detected through power spectrum.
- Large scales → fewer modes → large sample variance
- Seljak (arxiv:0807.1770) proposed way around it
- Measure the same mode using two tracers with different intrinsic bias
- In the ratio, the mode amplitude cancels and one remains with bias ratio

- (useless at the moment as we're still Poisson limited)

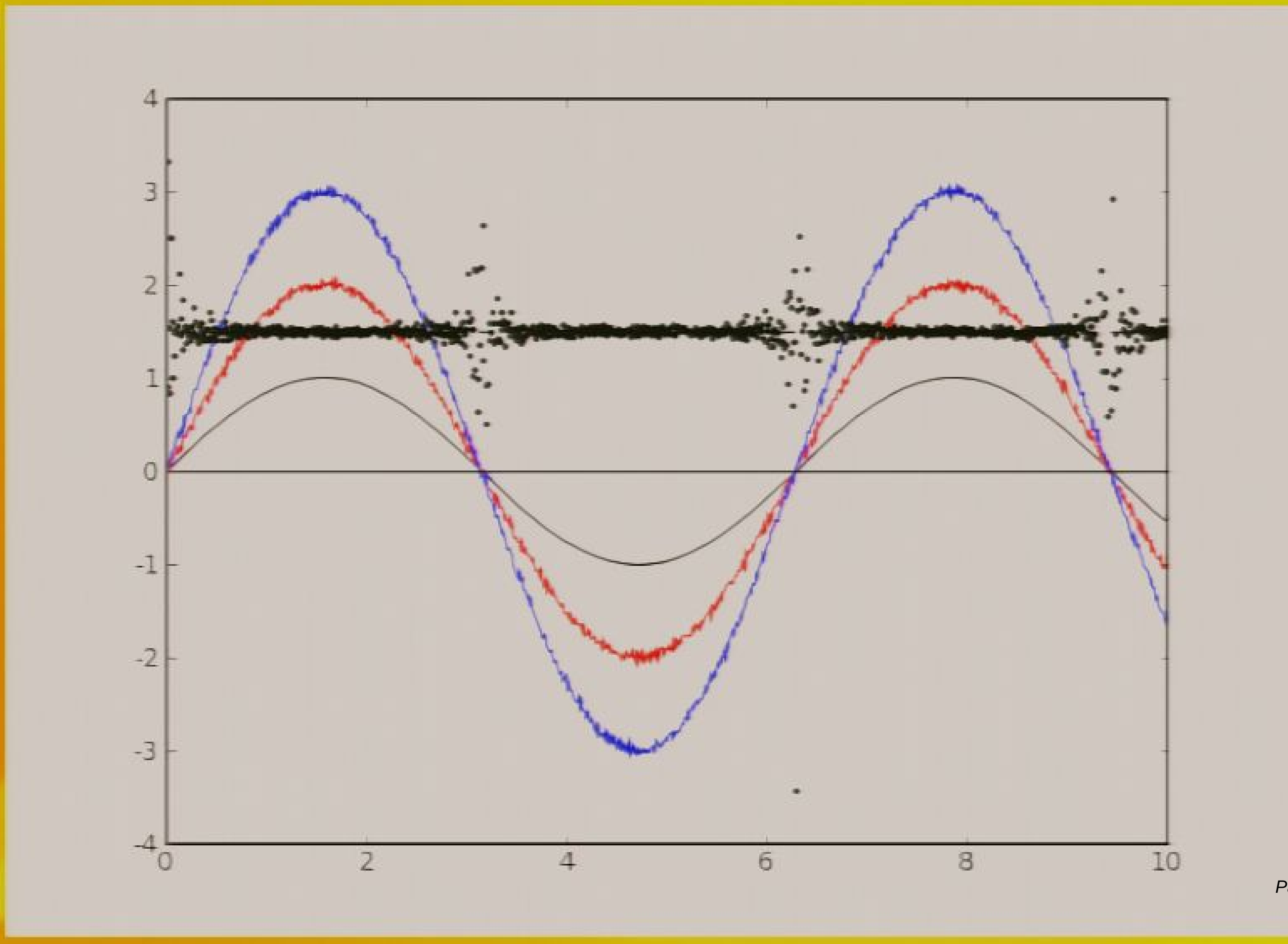
Avoiding sample variance.



FNL with Lyman-alpha forest

- Dalal's effect works, because the mass function is exponentially sensitive to PS normalisation.
- In Lyman-alpha forest, flux is also exponentially sensitive to delta under (over?) simplifying assumptions

Avoiding sample variance.

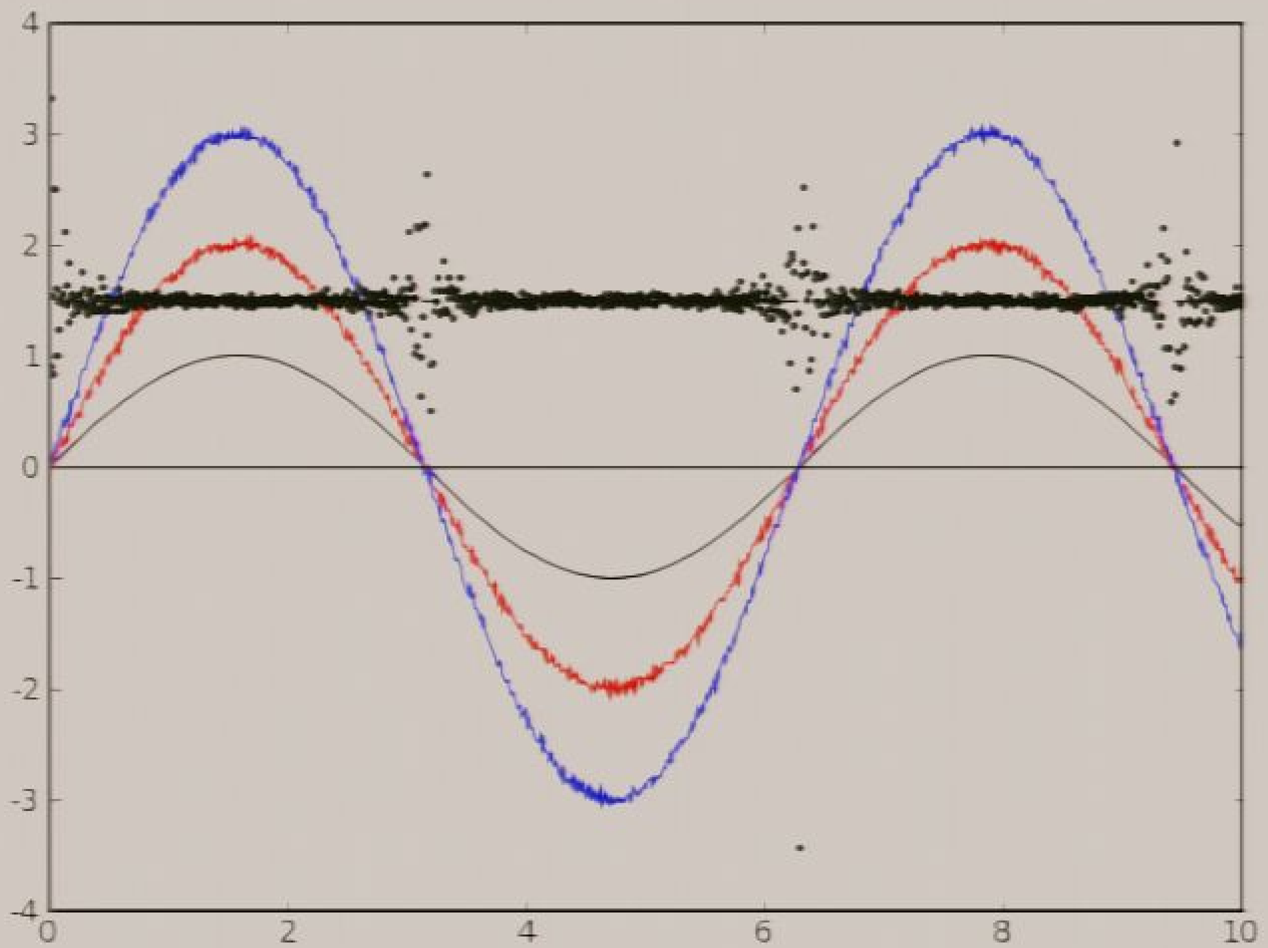


Avoiding sample variance.

- The above method relied on the scale dependent bias detected through power spectrum.
- Large scales → fewer modes → large sample variance
- Seljak (arxiv:0807.1770) proposed way around it
- Measure the same mode using two tracers with different intrinsic bias
- In the ratio, the mode amplitude cancels and one remains with bias ratio

- (useless at the moment as we're still Poisson limited)

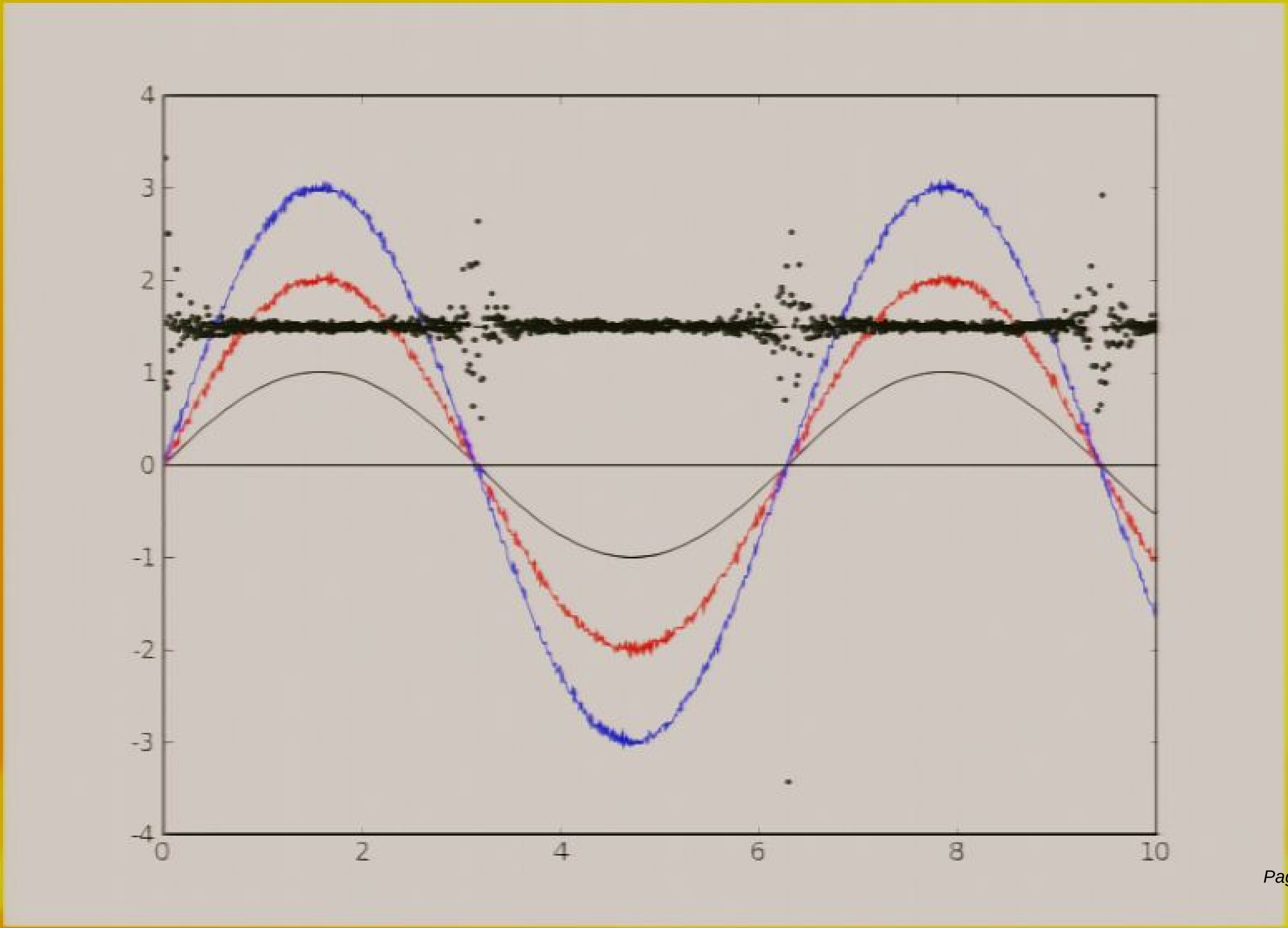
Avoiding sample variance.



Avoiding sample variance.

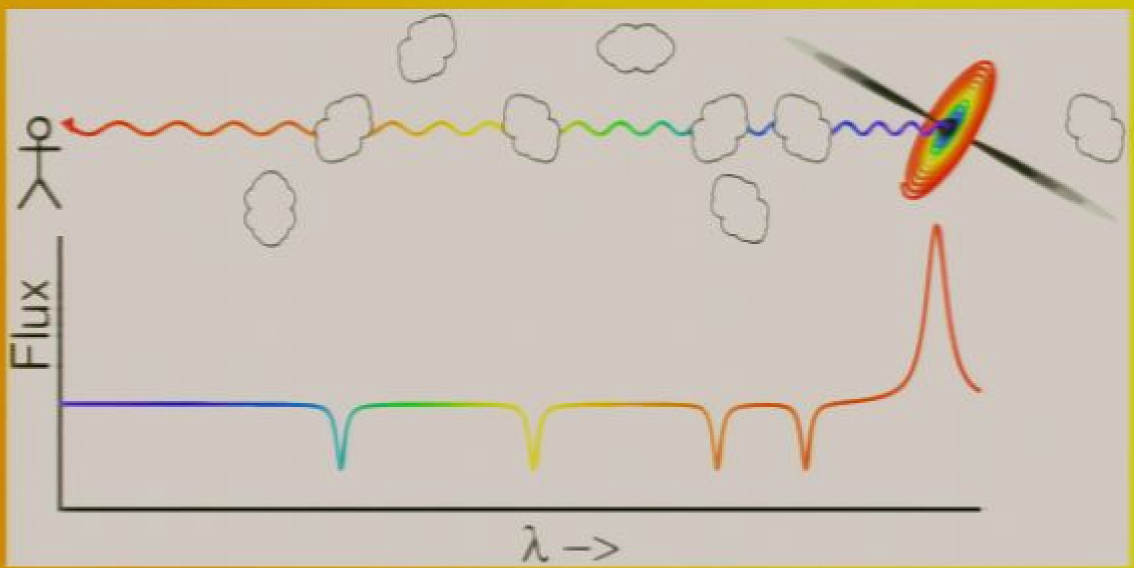
- Final errors now dominated by the Poisson error, that are subdominant on large scales.
- In other words, replace large scale mode counting with small scale mode counting – plenty of small scale modes.
- In principle a single mode is good enough. In practice, measure two power spectra and take ratio
- Degeneracies with other parameters also disappear
- Projections: with SNAP–like survey one gets to fnl ~1

Avoiding sample variance.



Lyman-alpha forest

- clouds of hydrogen absorb light from distant quasars, blueward of Lyman-alpha emission



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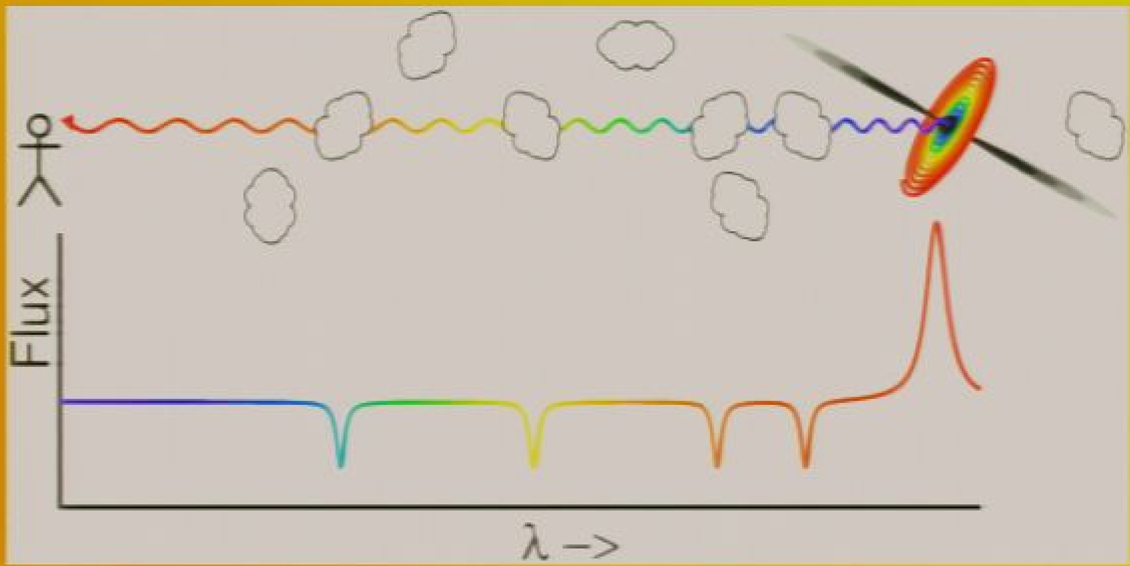

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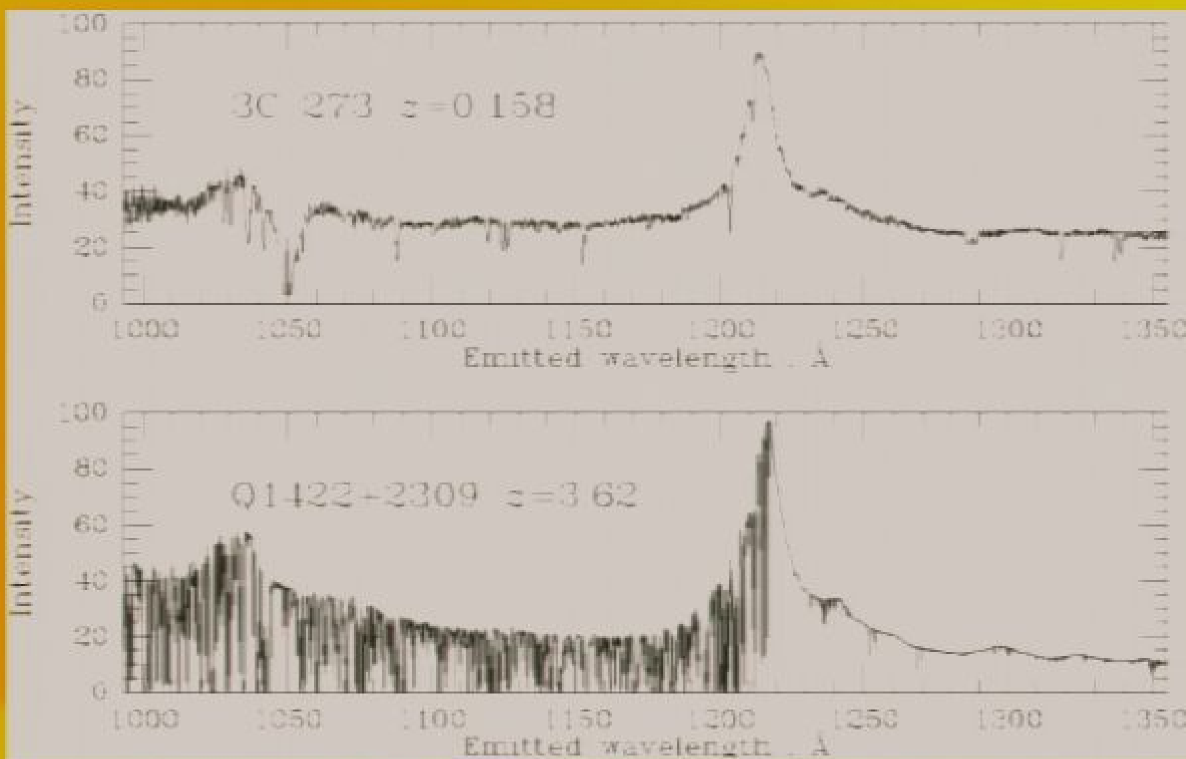
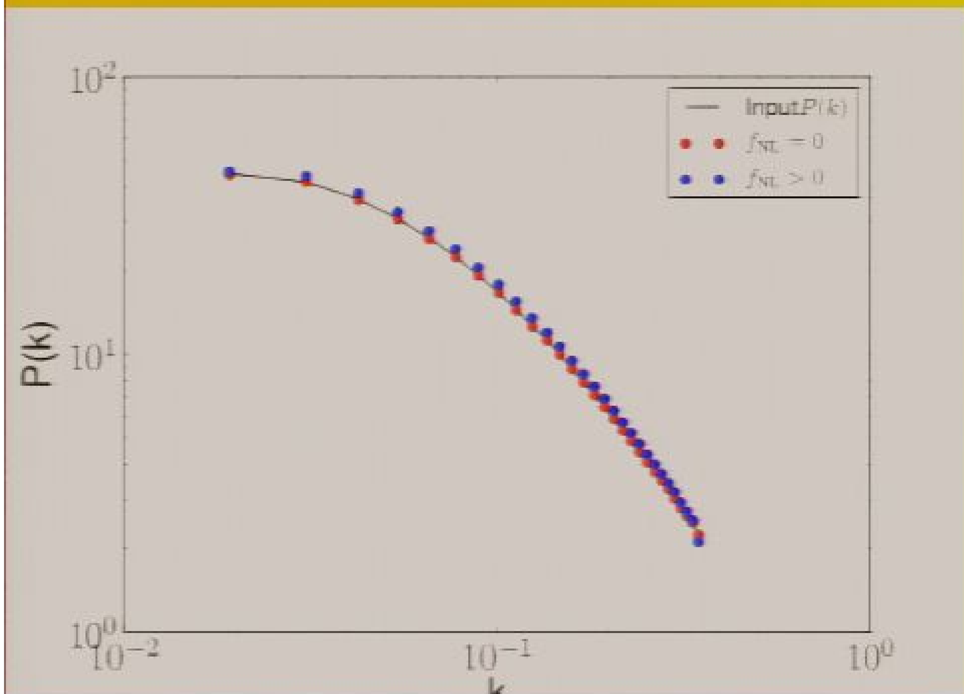


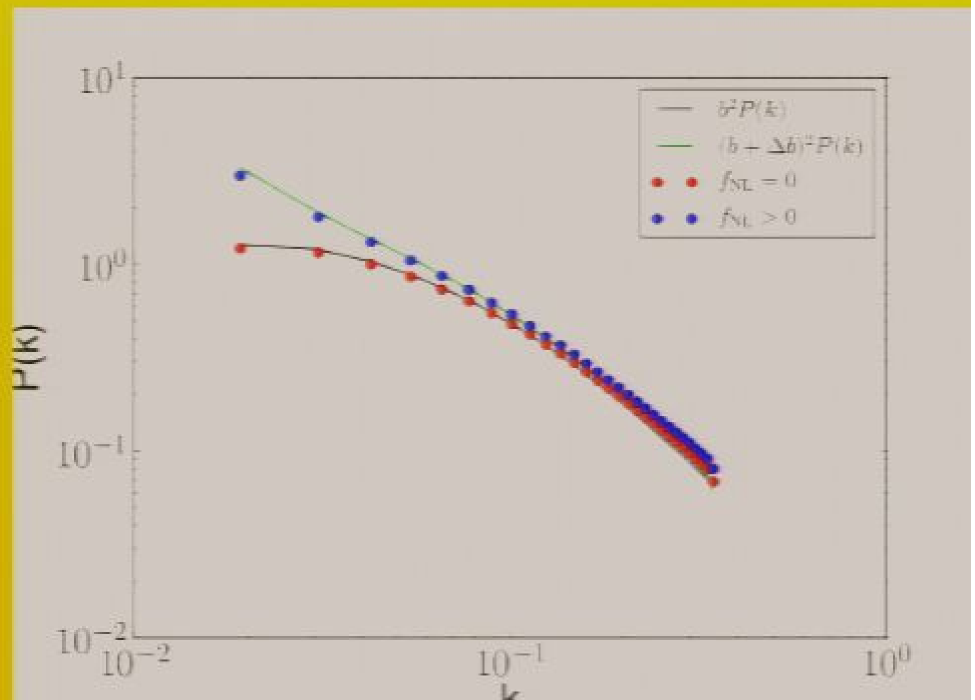
Assuming photo-ionization equilibrium:

$$\tau \propto (1+\delta)^{\beta}$$

$$f = e^{-\tau}$$

Results using Gaussian fields





Gaussan field converted into fluxes using FGPA

Solid lines are simple analytical results using P-bg split

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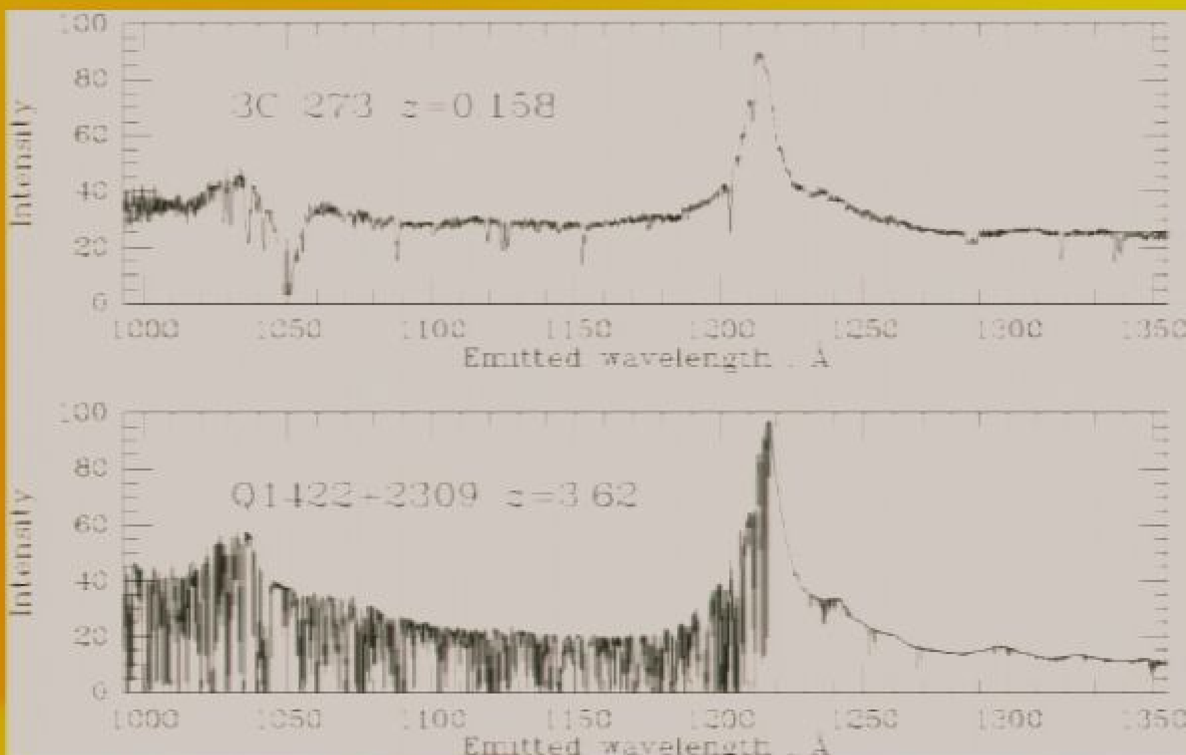

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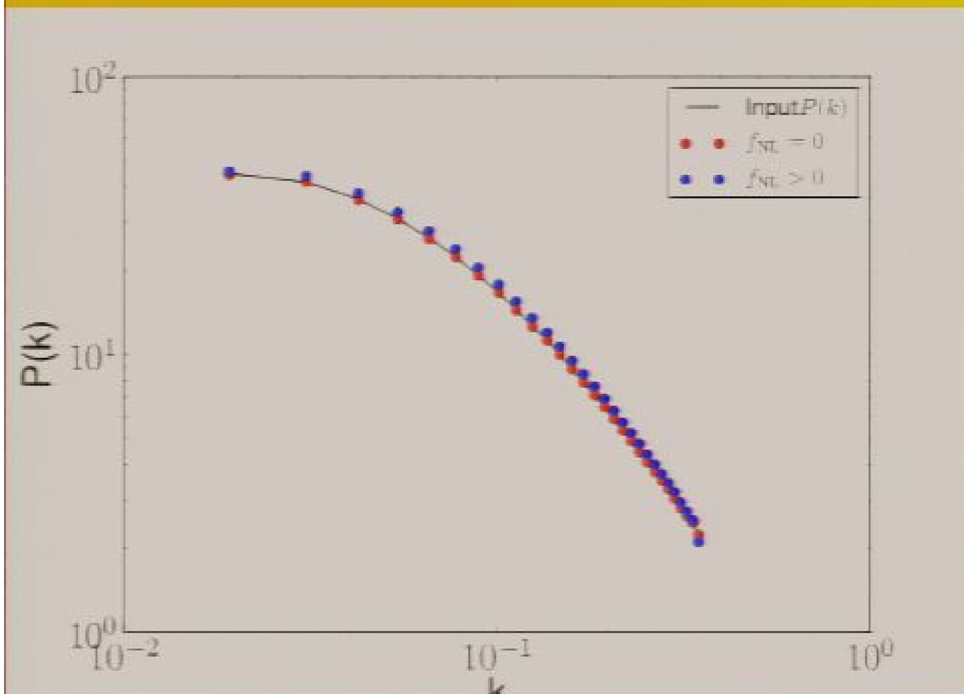

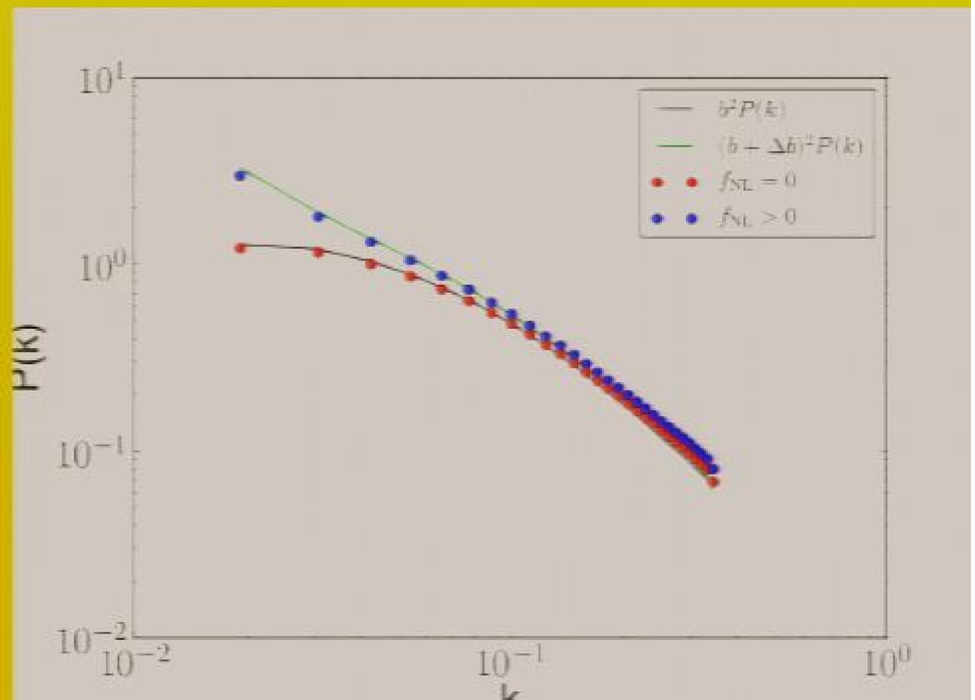

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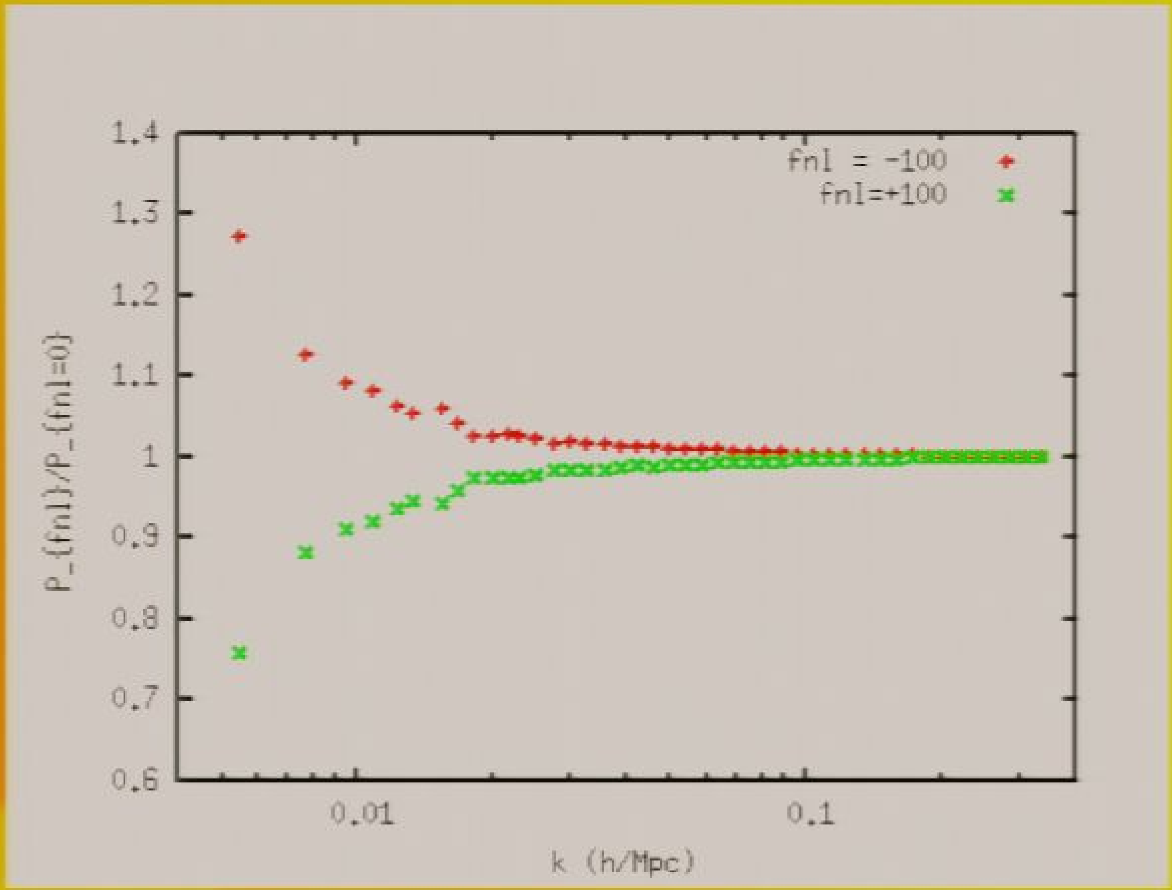
N-body sims (still FGPA)



Conclusions.

- Still many checks to be done:
  - Currently even power spectrum is suboptimal with the present data: can do better.
  - simulations: seems to work very well
  - Future data such as LSST, BigBoss can push down a lot to fnl (a few)
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- Should remain very active field
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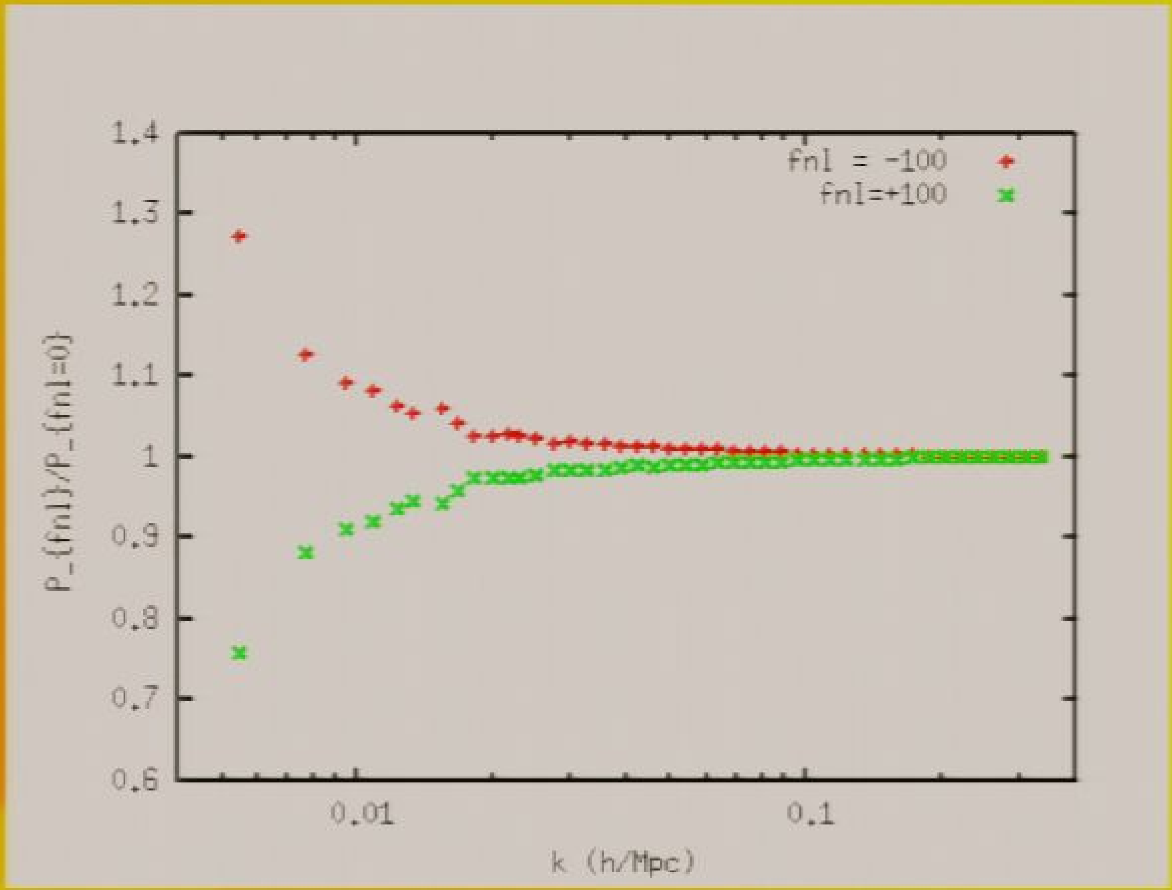

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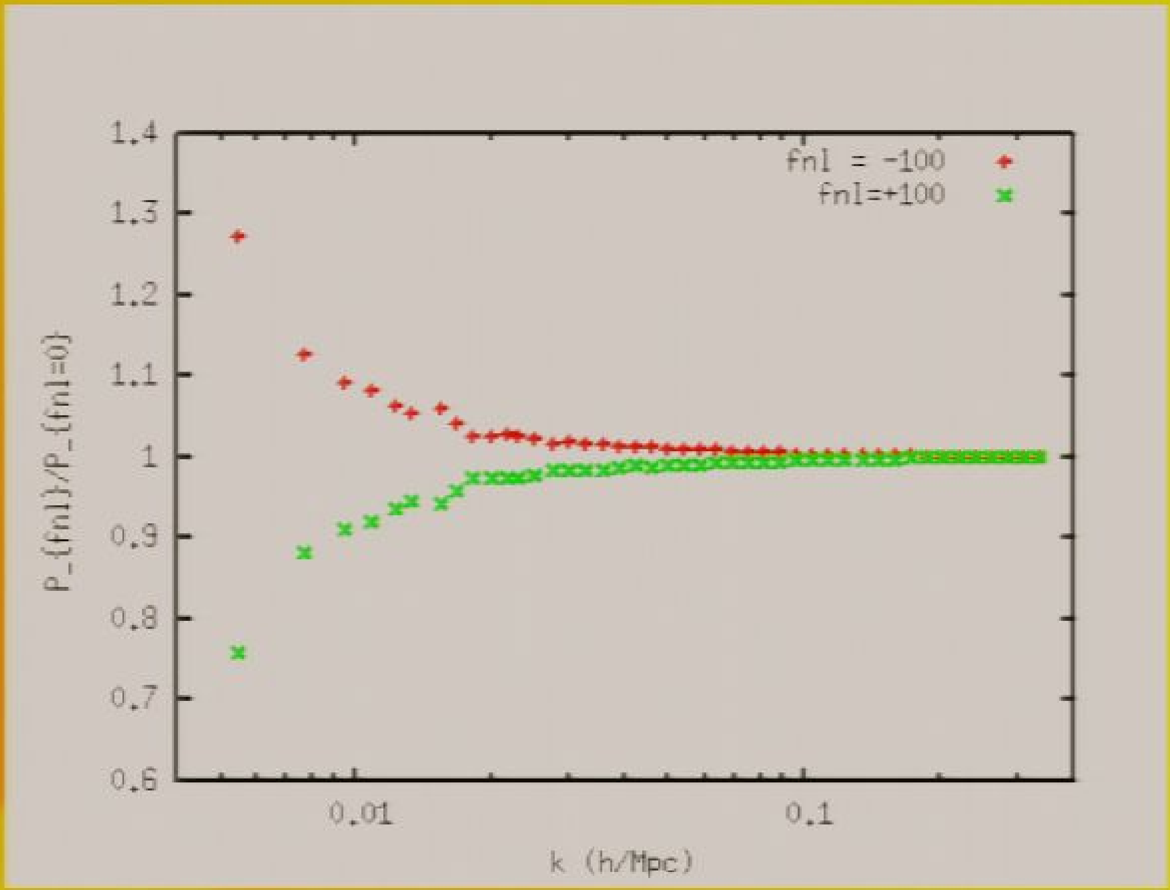

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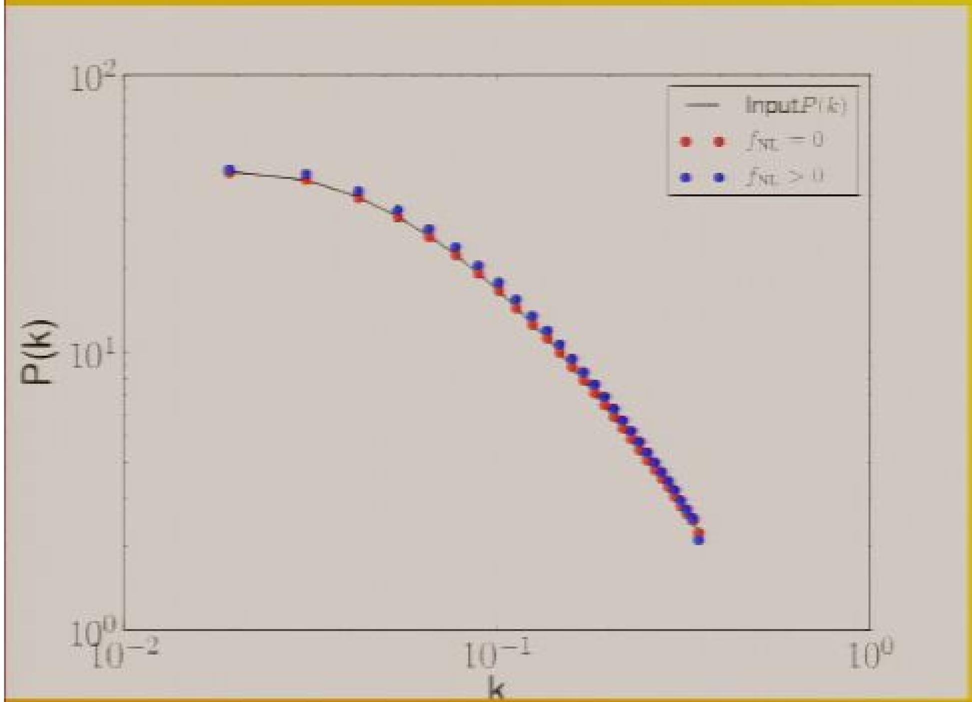

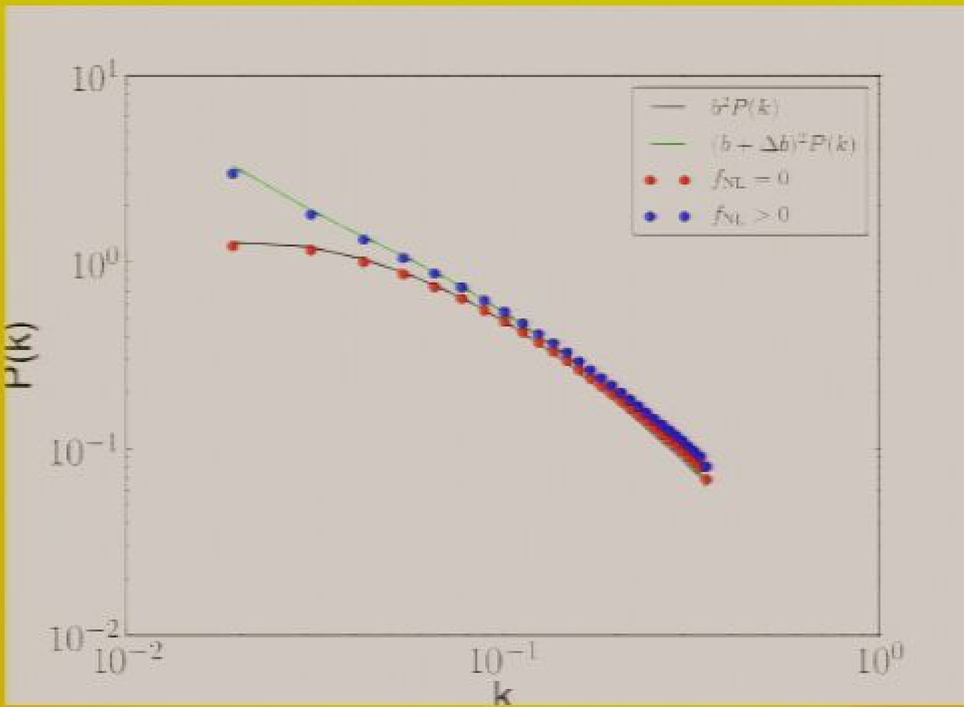

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