Title: Explorations in String Theory (PHYS 647) - Lecture 5

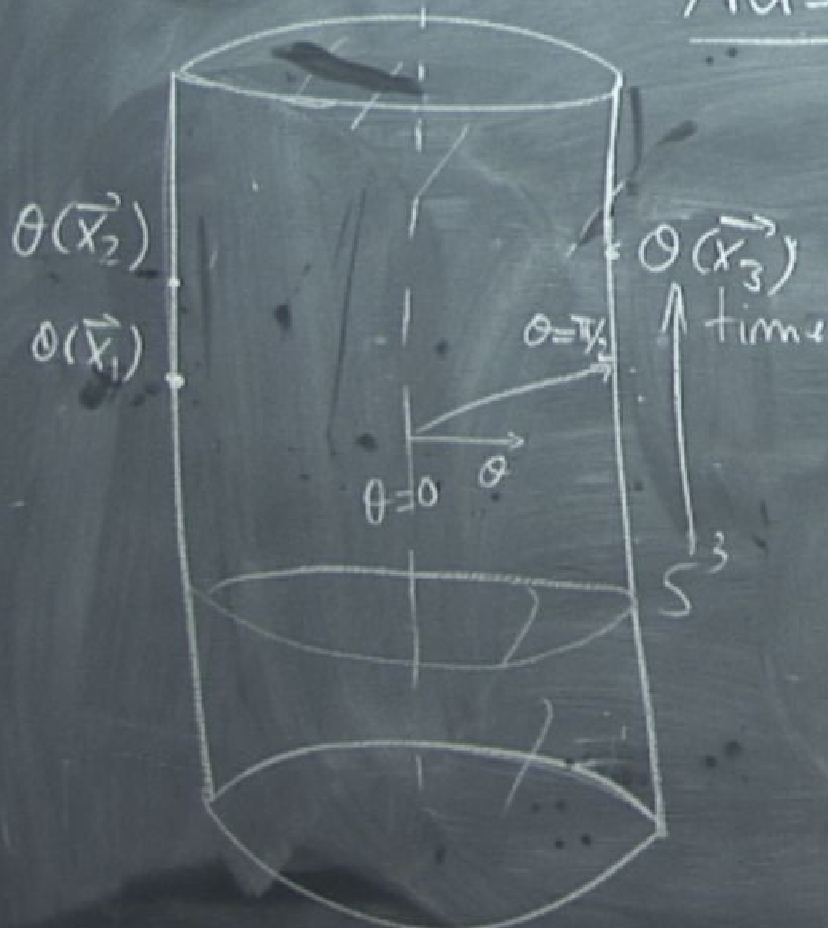
Date: Feb 19, 2010 01:45 PM

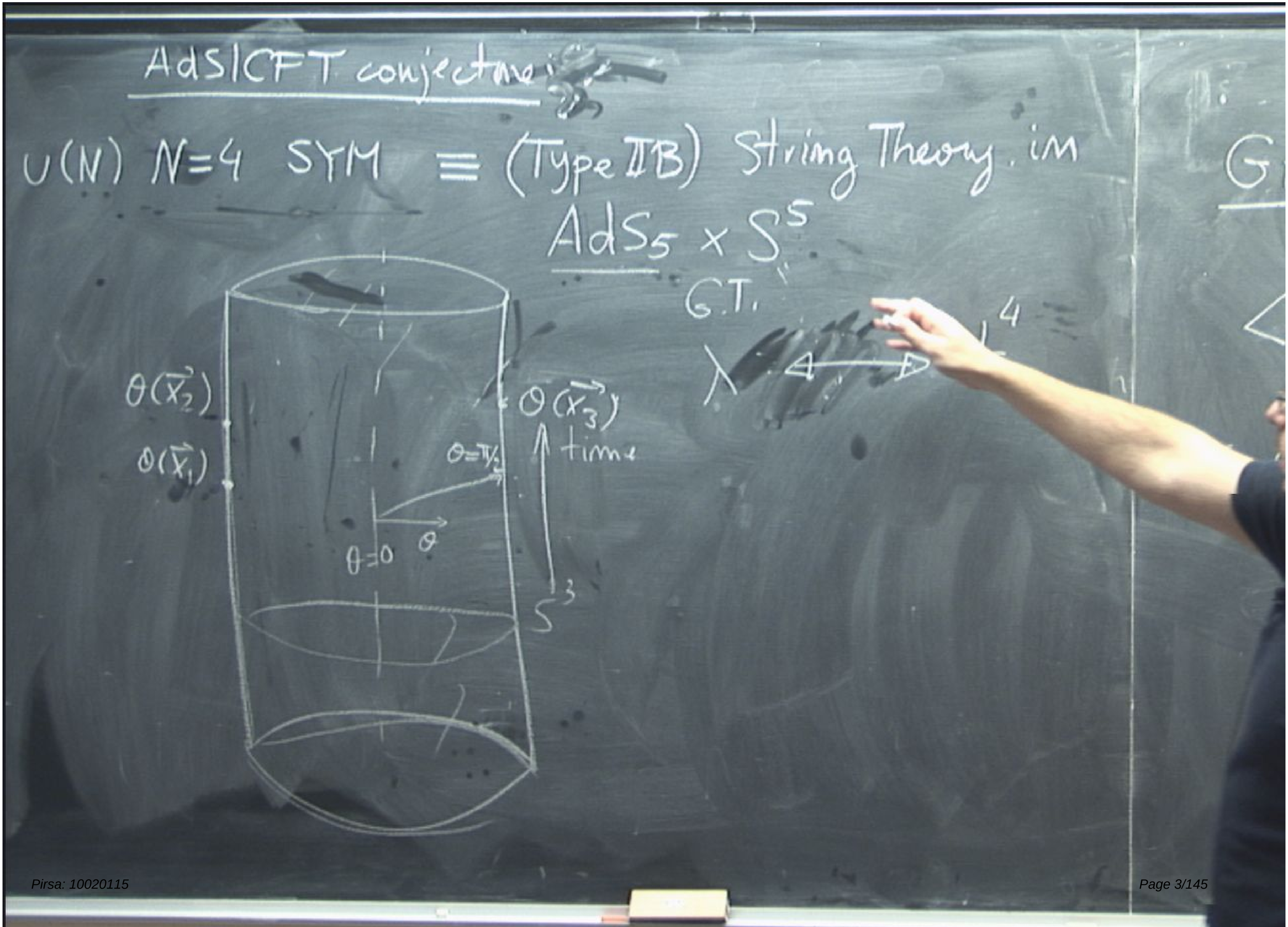
URL: http://pirsa.org/10020115

Abstract:

AdS/CFT conjecture:

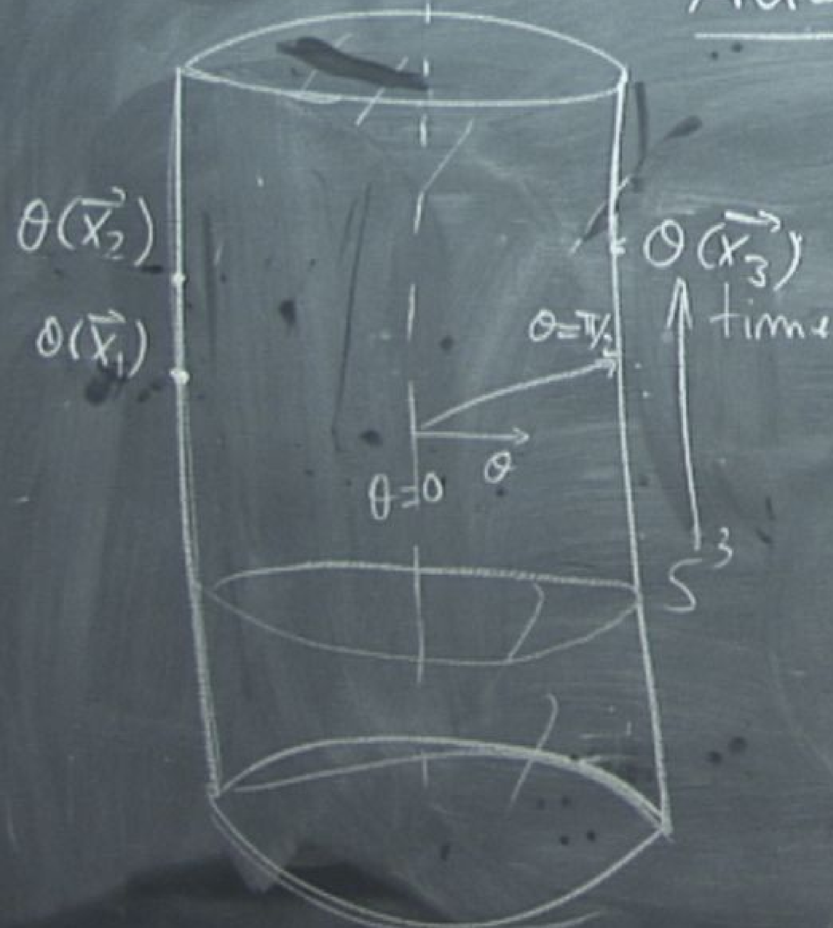
$U(N)$ $\mathcal{N}=4$ SYM $\equiv$ (Type IIB) String Theory in $AdS_5 \times S^5$




AdS/CFT conjecture:
U(N) N=4 SYM ≡ (Type IIB) String Theory in
AdS5 × S5
G.T.
θ(x2)
θ(x1)
θ(x3)
θ=π/2
time
θ=0
θ
S3

AdS/CFT conjecture:

$U(N)$ $\mathcal{N}=4$ SYM $\equiv$ (Type IIB) String Theory in $AdS_5 \times S^5$



G.T.

$\lambda \longleftrightarrow \frac{L^4}{l_s^2}$

$N \longleftrightarrow$

AdS/CFT conjecture:

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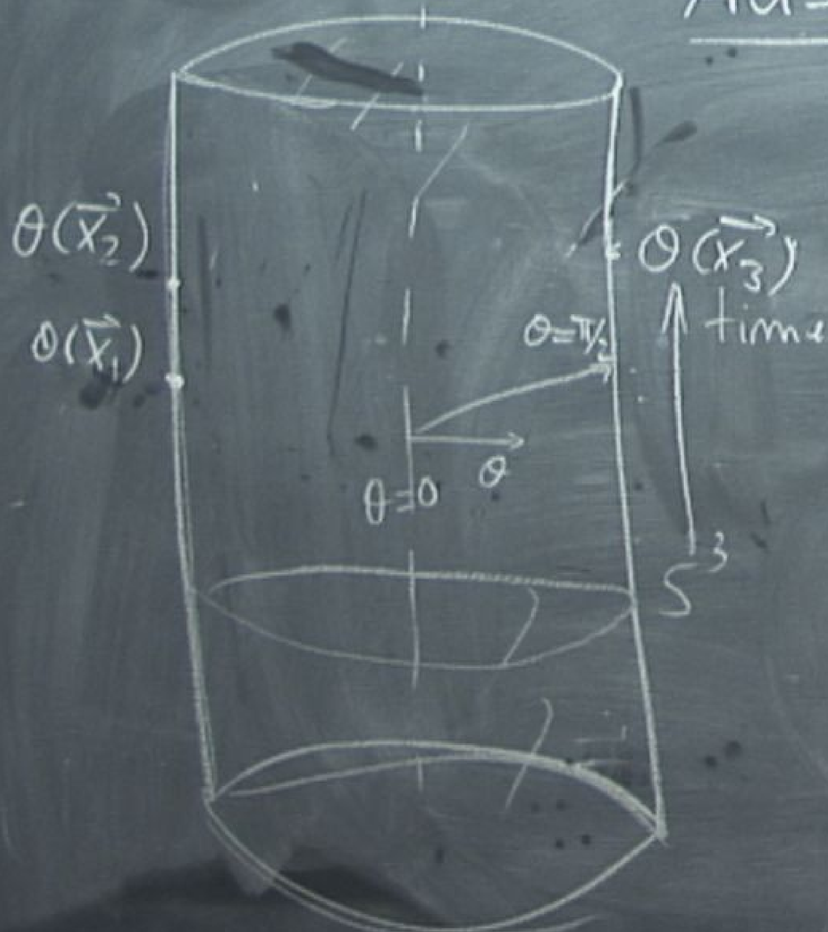

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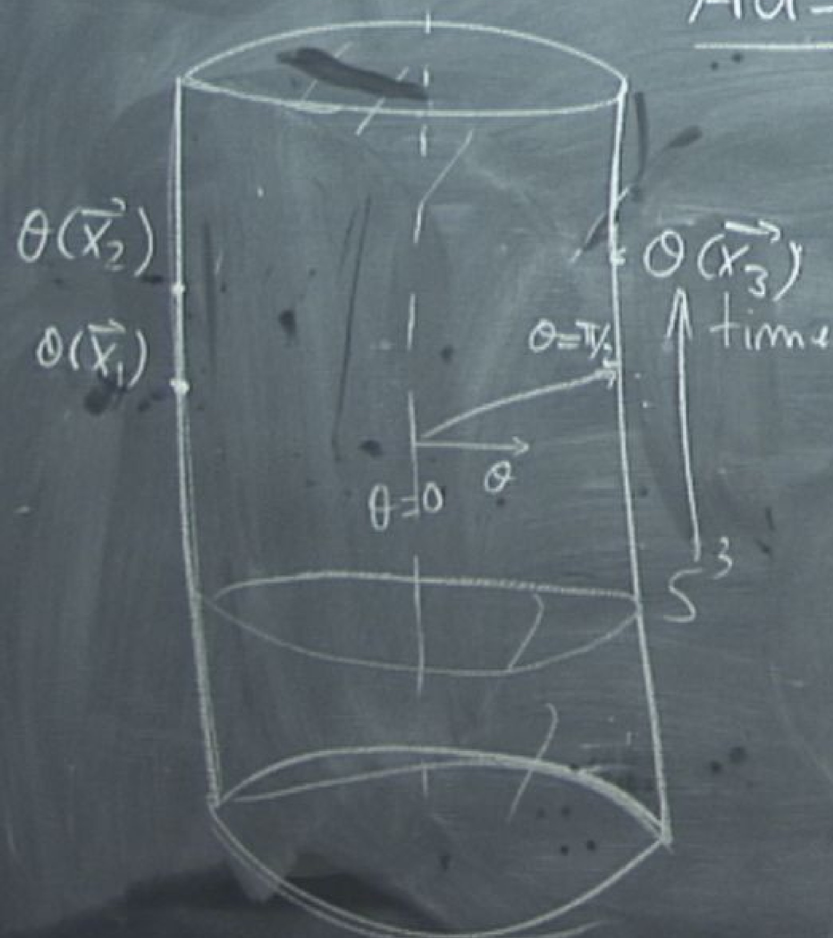

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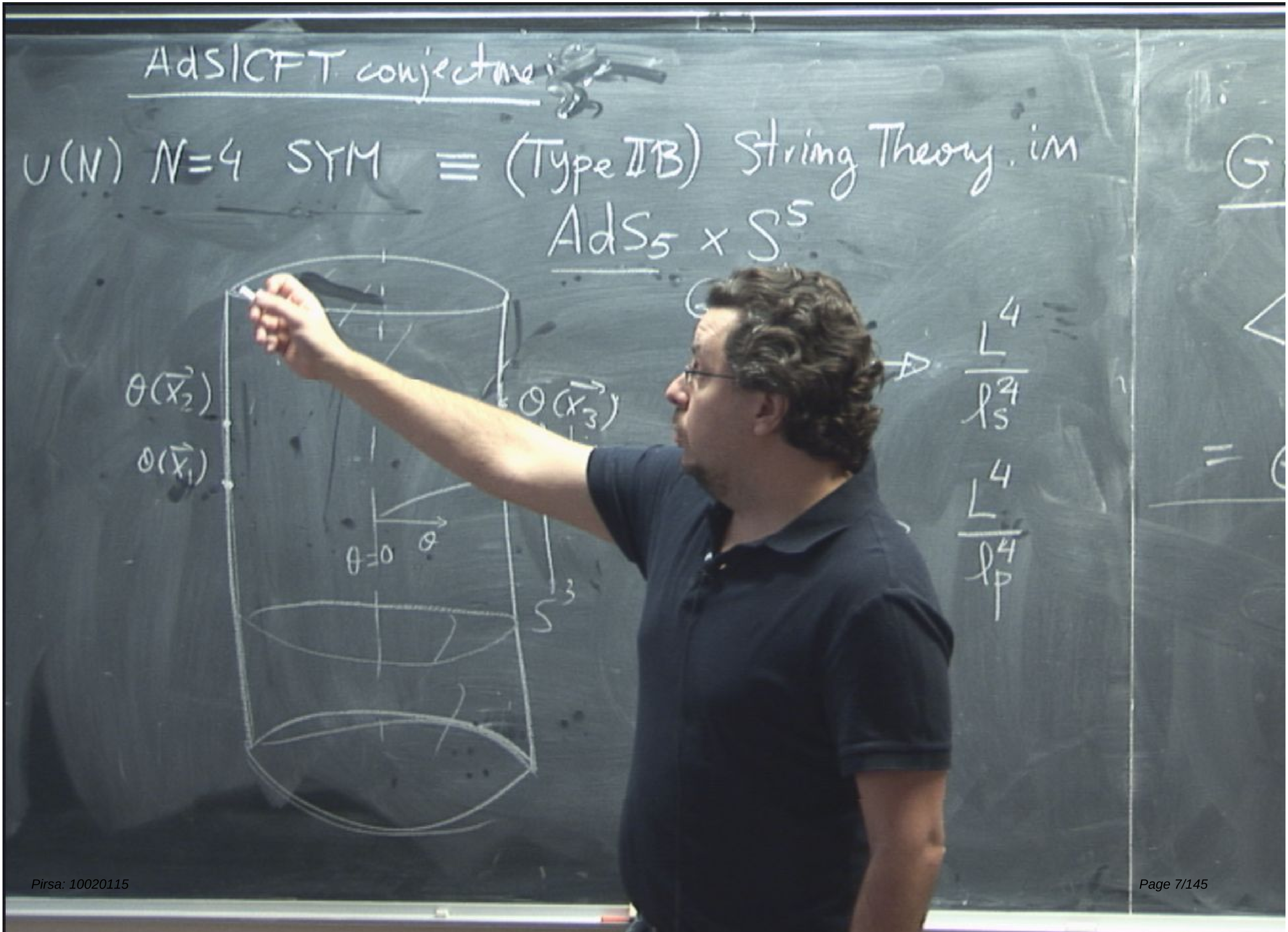
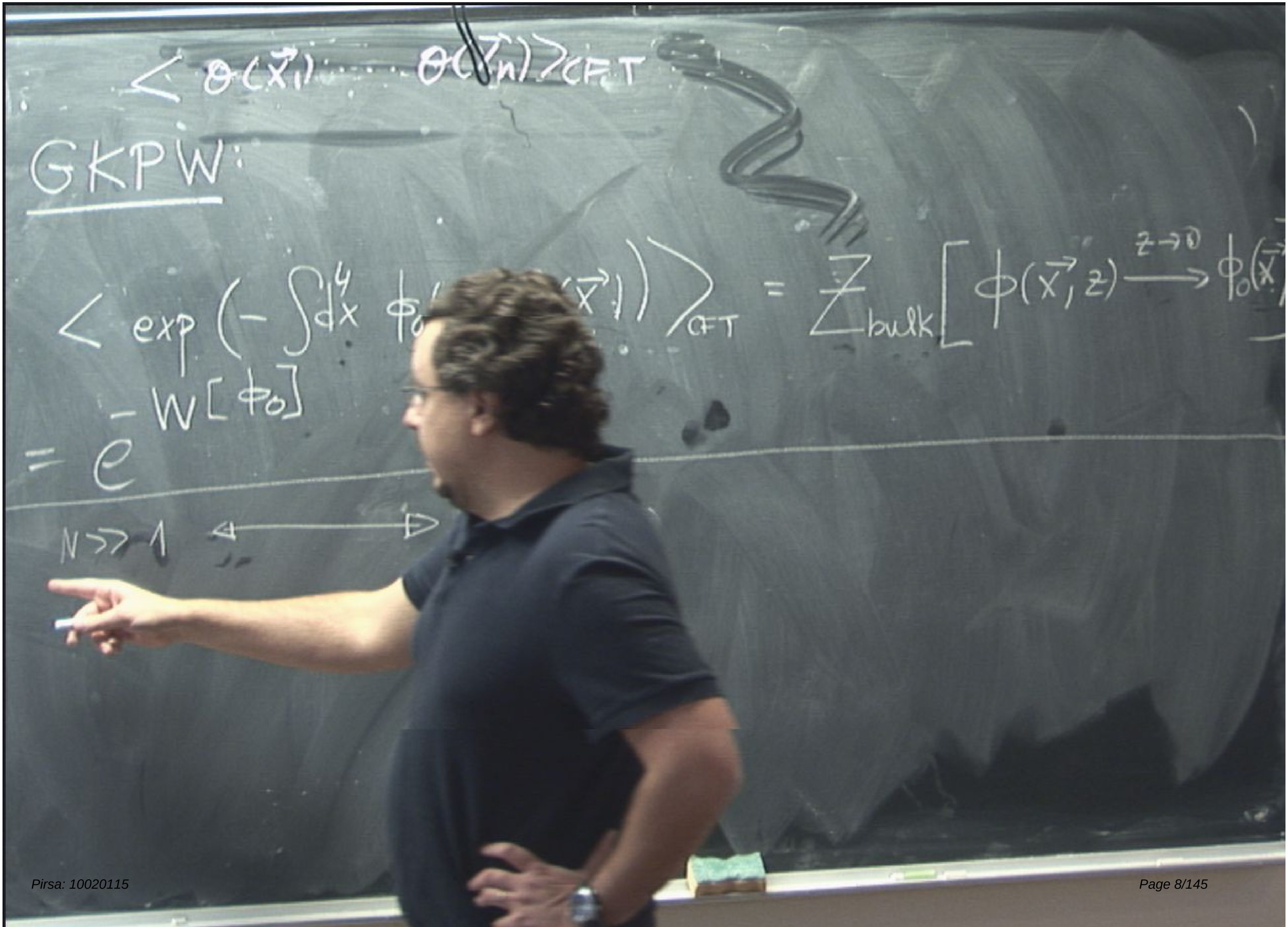
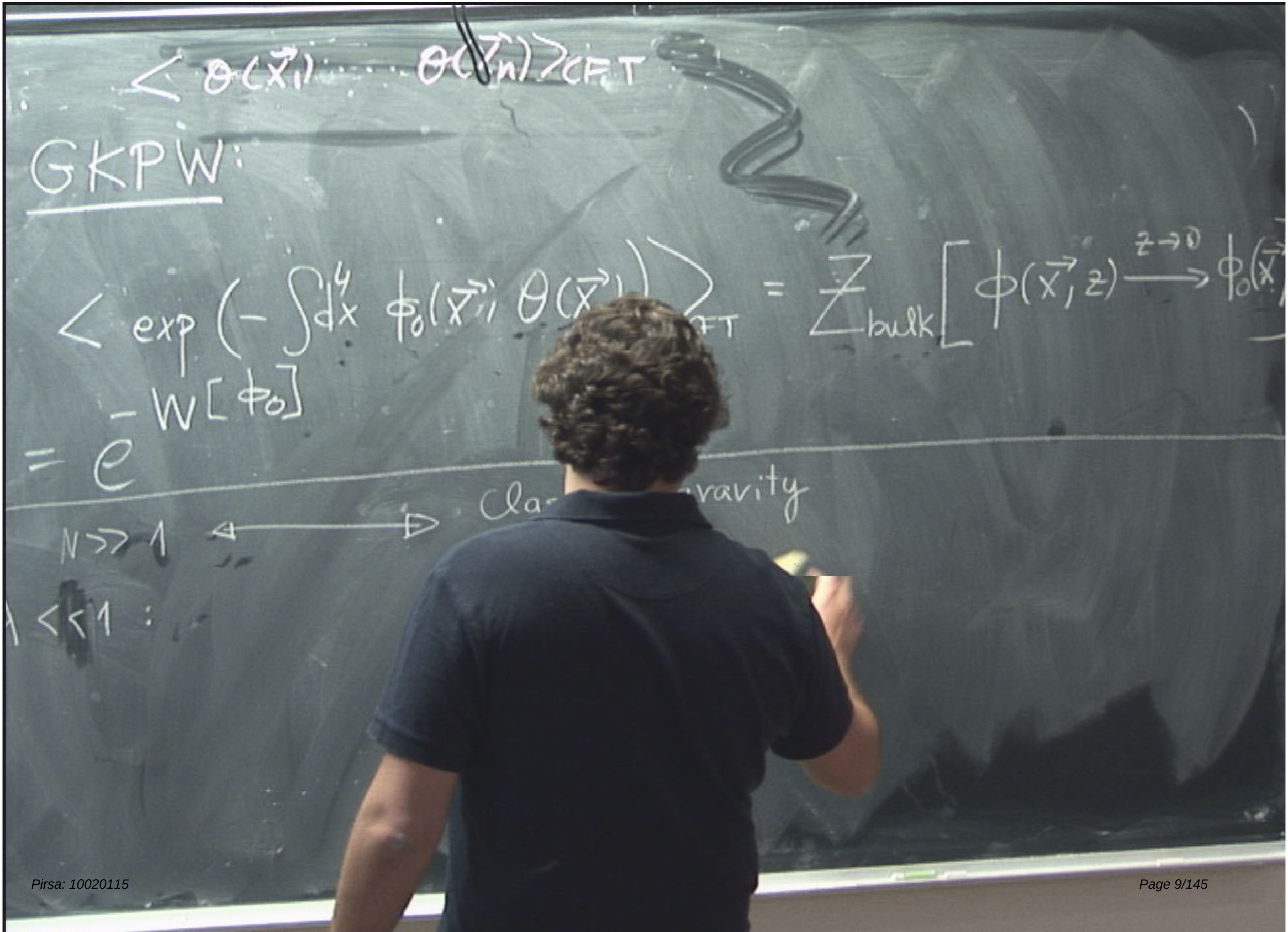

AdS/CFT conjecture:
U(N) N=4 SYM ≡ (Type IIB) String Theory in
AdS5 × S5
θ(x2)
θ(x1)
θ(x3)
θ=0
θ
S3
L4/ls4
L4/lp4


< O(x1) ...... O(xn) >CFT
GKPW:
< exp(- ∫d4x φ0 ... (x)) >CFT = Zbulk [φ(x, z) z→0 φ0(x
= e -W[φ0]
N >> 1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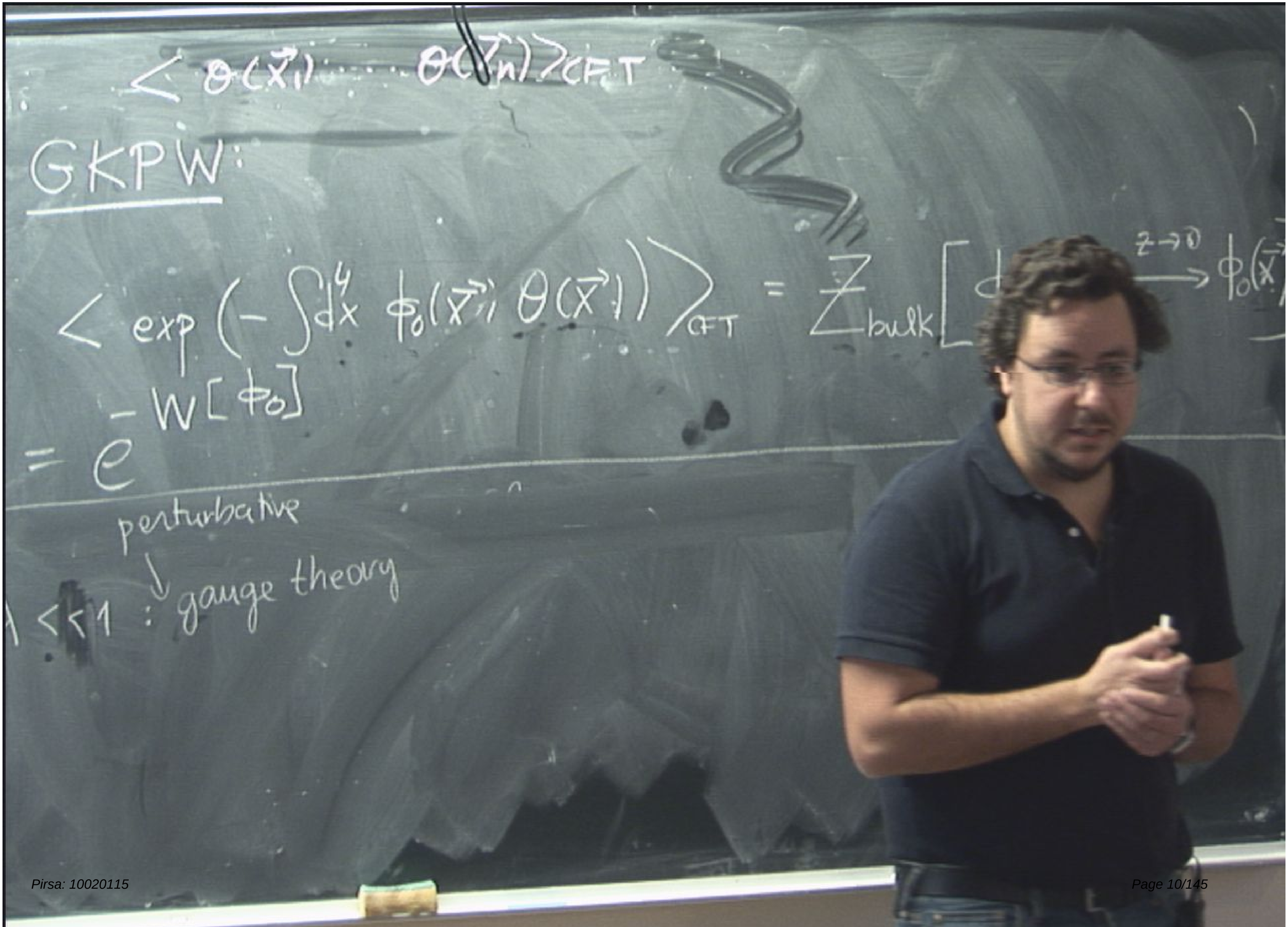
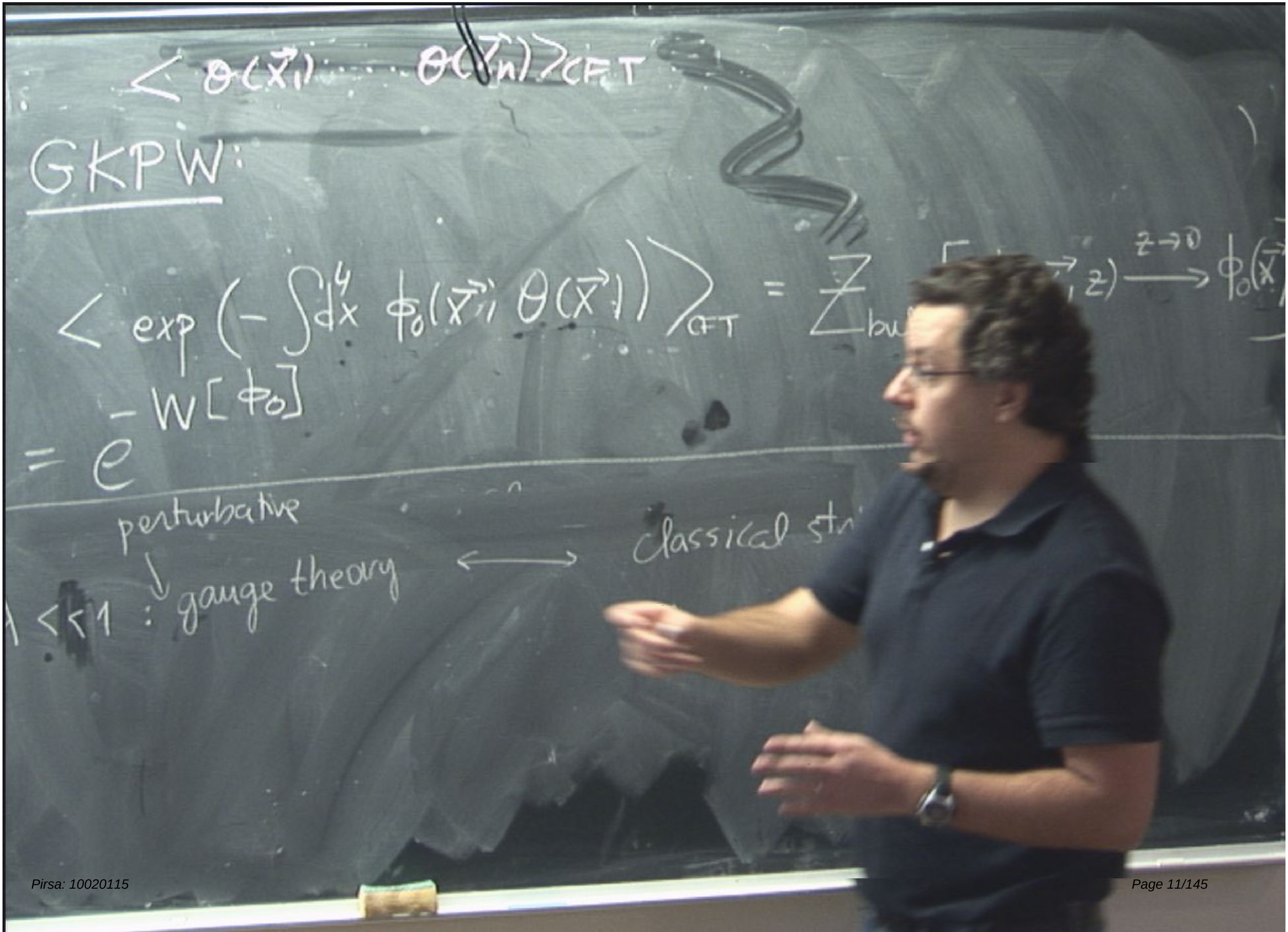
λ << 1 :


$\langle \mathcal{O}(\vec{x}_1) \cdots \mathcal{O}(\vec{x}_n) \rangle_{CFT}$
GKPW:
$\langle \exp\left(-\int d^4x\, \phi_0(\vec{x})\, \mathcal{O}(\vec{x})\right) \rangle_{CFT} = Z_{bulk}\left[\phi \xrightarrow{z \to 0} \phi_0(\vec{x})\right]$
$= e^{-W[\phi_0]}$
perturbative
$\lambda \ll 1$: gauge theory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$z \to 0$
$z) \longrightarrow \phi_0(\vec{x}$
$= e^{-W[\phi_0]}$
perturbative
gauge theory
classical st

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GKPW:

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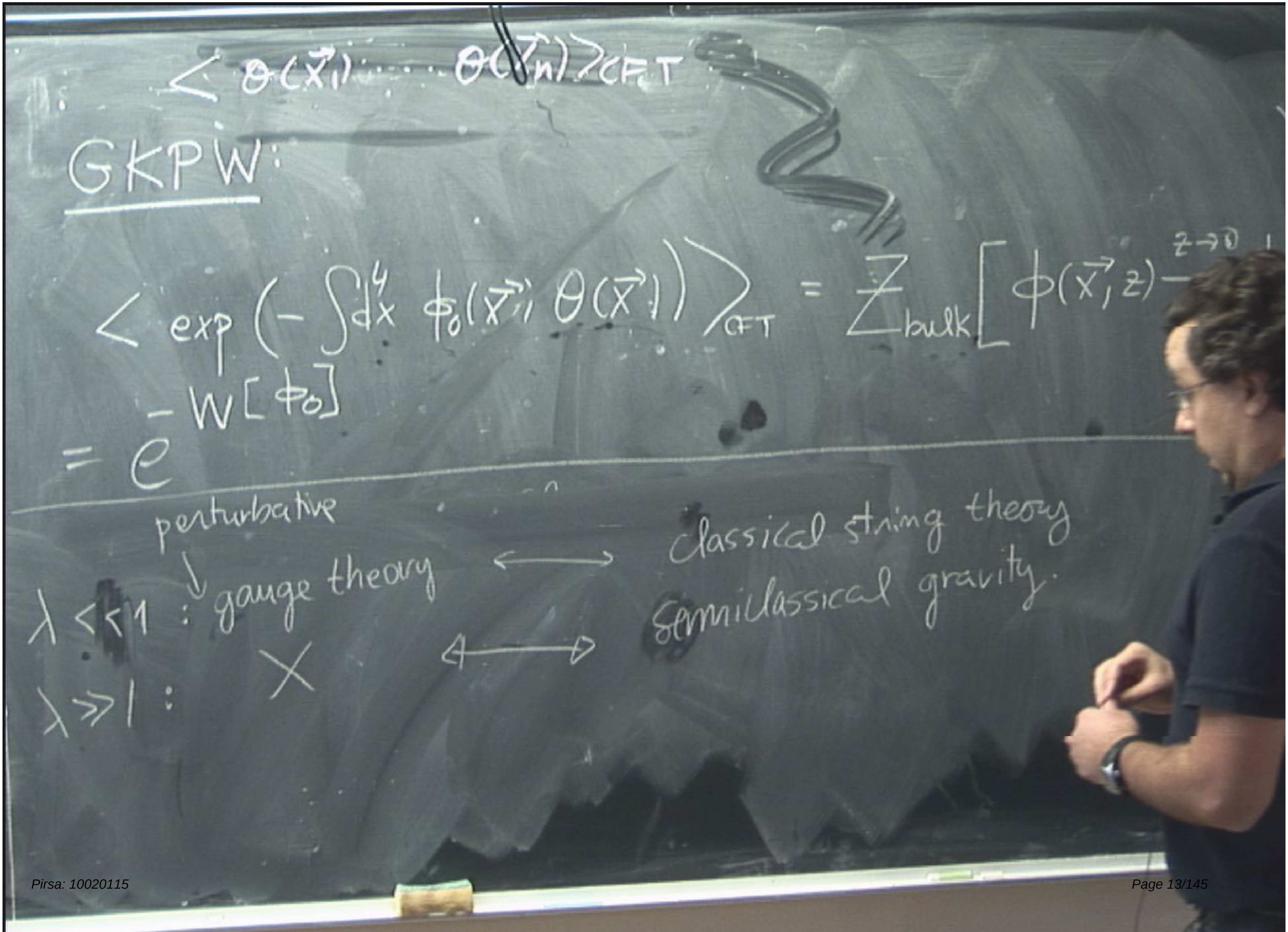
perturbative

$\lambda \ll 1$: gauge theory $\longleftrightarrow$ classical string theory

$\lambda \gg 1$: $\times$



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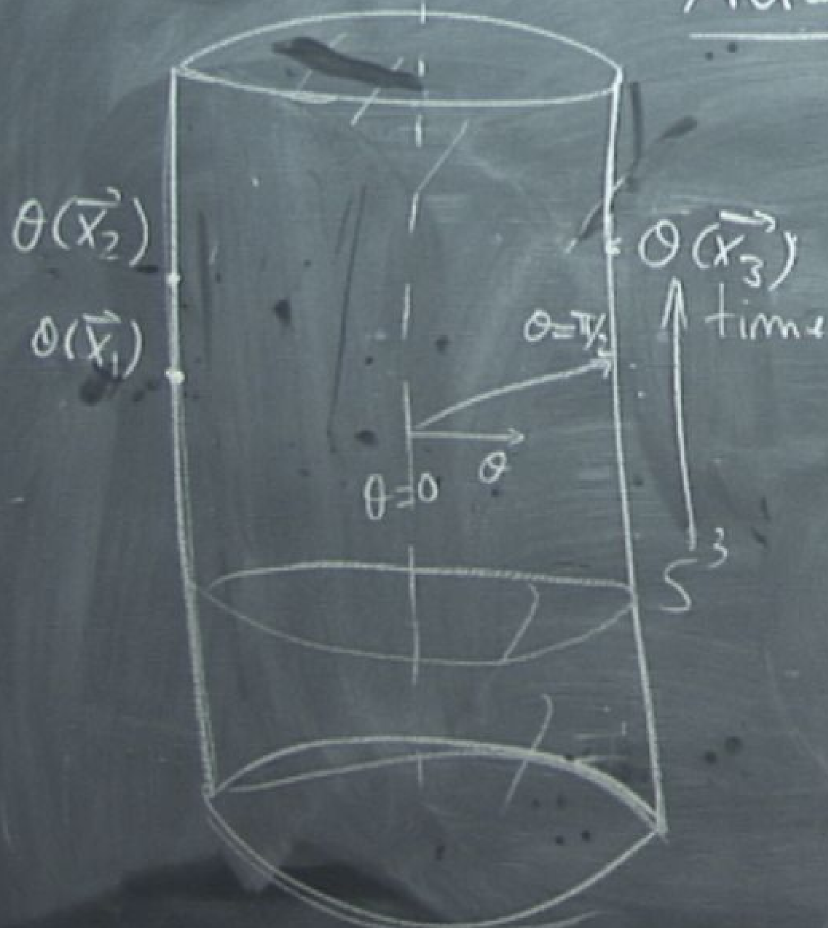
$\lambda \ll 1$: gauge theory ⟷ classical string theory
$\lambda \gg 1$: × ⟷ semiclassical gravity.

AdS/CFT conjecture:

$U(N)$ $\mathcal{N}=4$ SYM $\equiv$ (Type IIB) String Theory in $\underline{AdS_5} \times S^5$

"G.T."



$$\lambda \longleftrightarrow \frac{L^4}{l_s^4}$$

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G

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$\lambda \gg$

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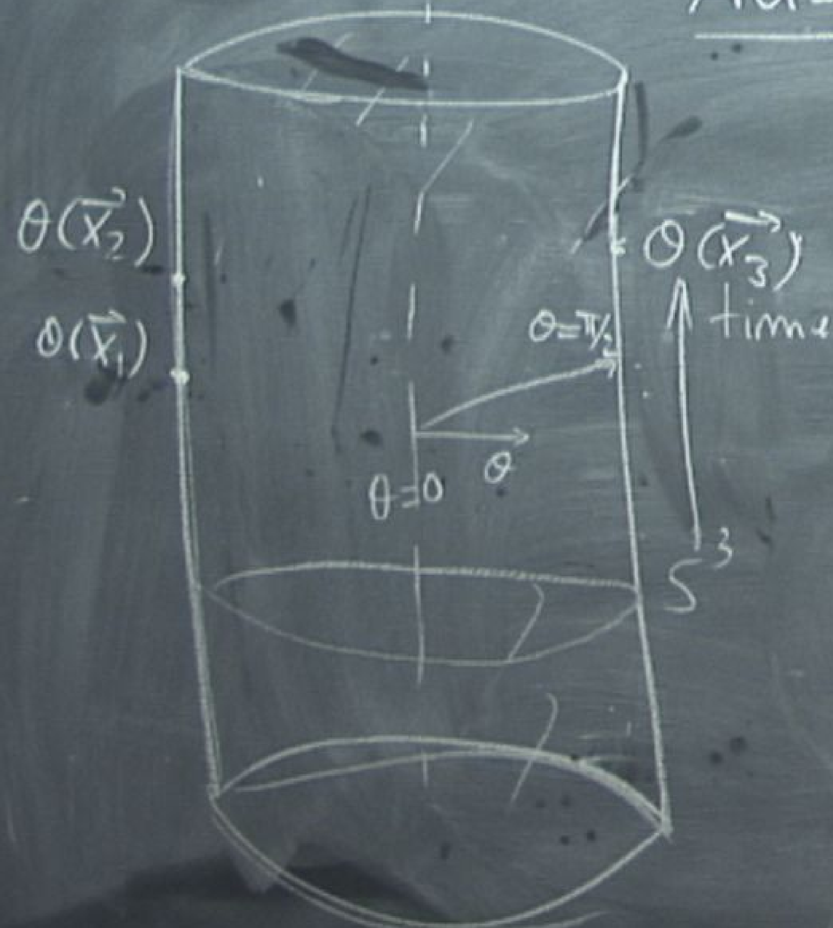

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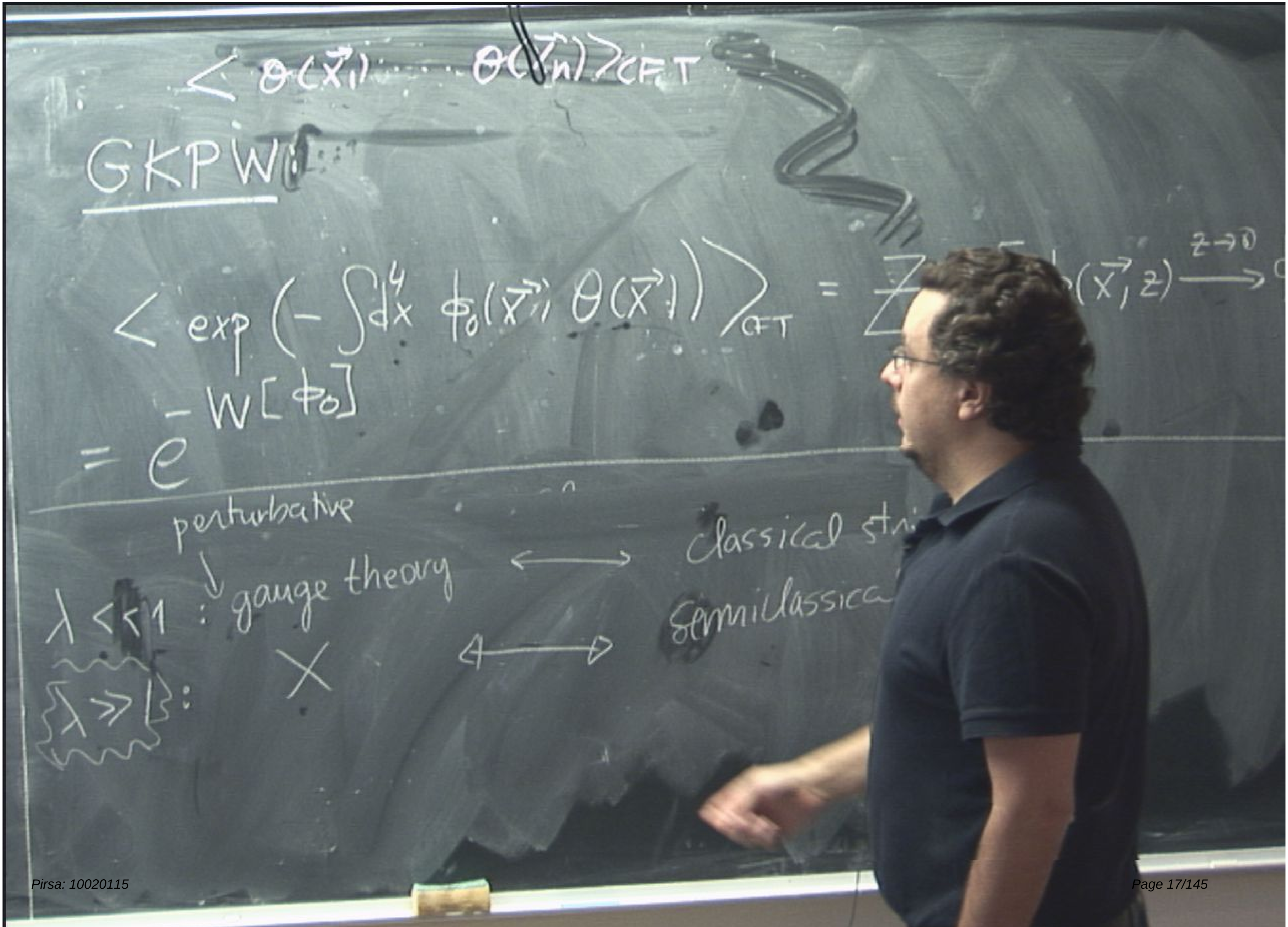
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classical
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$\lambda \ll 1$: perturbative gauge theory $\longleftrightarrow$ classical string theory

$\lambda \gg 1$: $\times$ $\longleftrightarrow$ semiclassical gravity.

$$\langle \theta(\vec{x}_1) \cdots \theta(\vec{x}_n) \rangle_{CFT} \qquad \frac{\delta^2 W}{\delta\phi_0(\vec{x}_1)\,\delta\phi_0(\vec{x}_2)}\Big|_{\phi_0=0} = \langle \theta(\vec{x}_1)\theta(\vec{x}_2) \rangle$$

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perturbative gauge theory $\longleftrightarrow$ classical string theory

$\ll 1$: gauge theory $\longleftrightarrow$ semiclassical gravity.

$\gg 1$: $\times$ $\longleftrightarrow$

$$\langle \mathcal{O}(\vec{x}_1) \cdots \mathcal{O}(\vec{x}_n) \rangle_{CFT} \qquad \frac{\delta^2 W}{\delta \phi_0(\vec{x}_1)\, \delta \phi_0(\vec{x}_2)}\Big|_{\phi_0 = 0} = \langle \mathcal{O}(\vec{x}_1)\, \mathcal{O}(\vec{x}_2) \rangle$$

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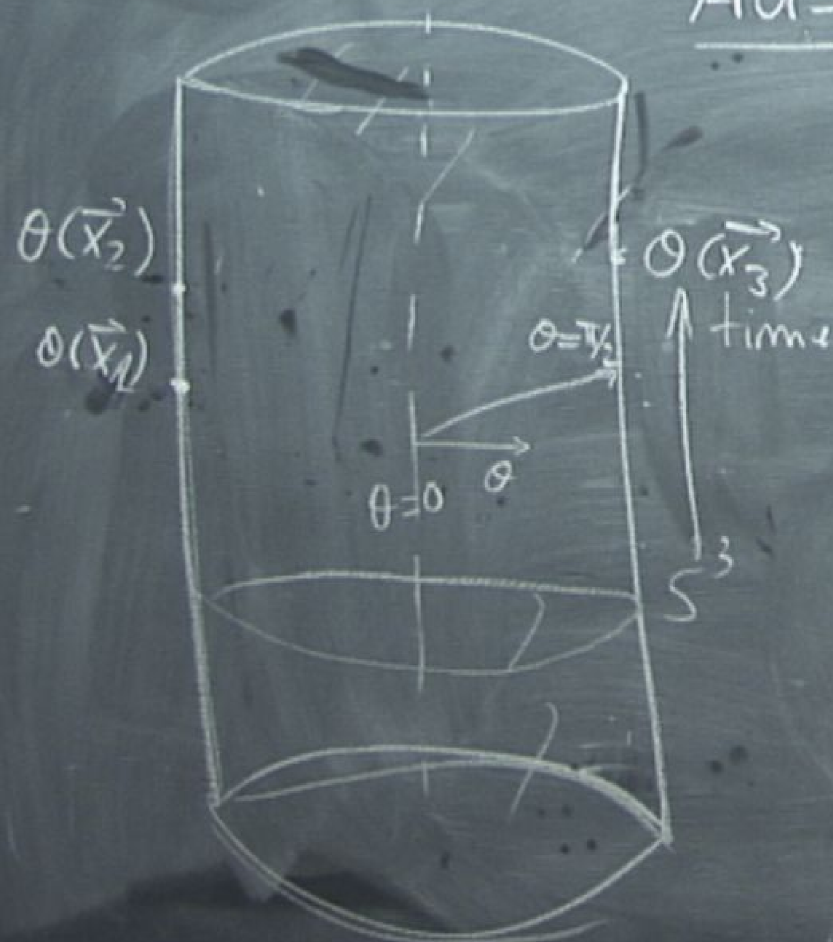

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AdS/CFT conjecture:

$U(N)\ \mathcal{N}=4$ SYM $\equiv$ (Type IIB) String Theory in

$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\rangle_{CFT}$

G

$=$

$\lambda \ll$

$\lambda \gg$

AdS/CFT conjecture:

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$$\phi(\vec{x}, z)$$

G

=

$\lambda \ll 1$

$\lambda \gg 1$

AdS/CFT conjecture:

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$$\langle \mathcal{O}(\vec{x}_1)\, \mathcal{O}(\vec{x}_2) \rangle_{CFT}$$

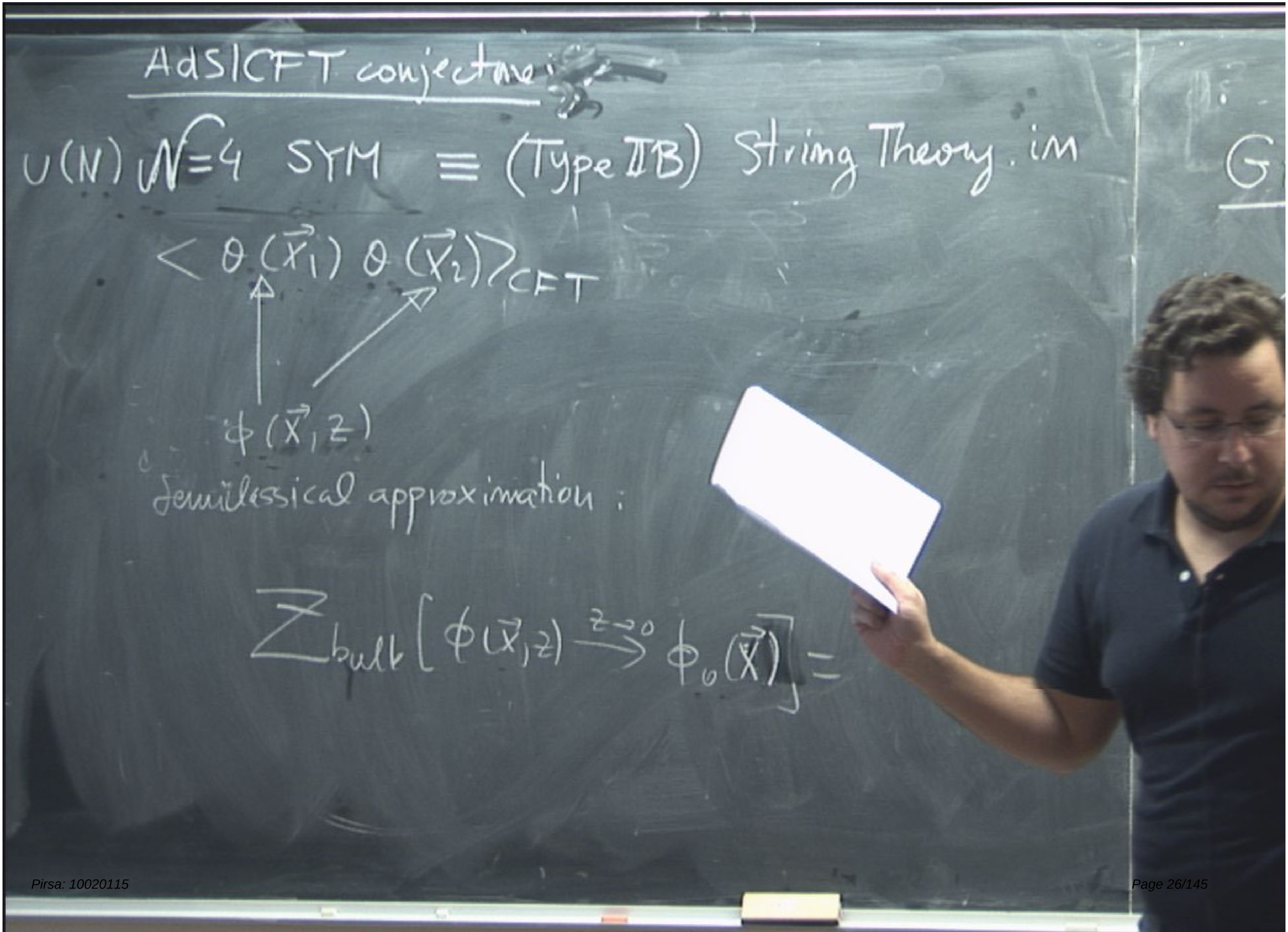
$$\phi(\vec{x}, z)$$

Semiclassical approximation:

$$Z_{bulk}\left[\phi(\vec{x},z) \xrightarrow{z\to 0} \phi_0(\vec{x})\right] =$$

$\lambda \ll 1$

$\lambda \gg 1$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AdS/CFT conjecture:

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$$\phi(\vec{x}, z)$$

Semiclassical approximation:

$$Z_{bulk}\left[\phi(\vec{x},z) \xrightarrow{z\to 0} \phi_0(\vec{x})\right] = e^{-S_{classical}[\Phi(\Phi_0)]}$$

G

$=$

$\lambda \ll 1$

$\lambda \gg 1$

$$\mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$$

$$S_{bulk} = \int d^5x \sqrt{g}\ \frac{1}{2}$$

$$\langle \mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$$

$$S_{bulk} = \int d^5x \sqrt{g} \left(\frac{1}{2} (\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 \right)$$

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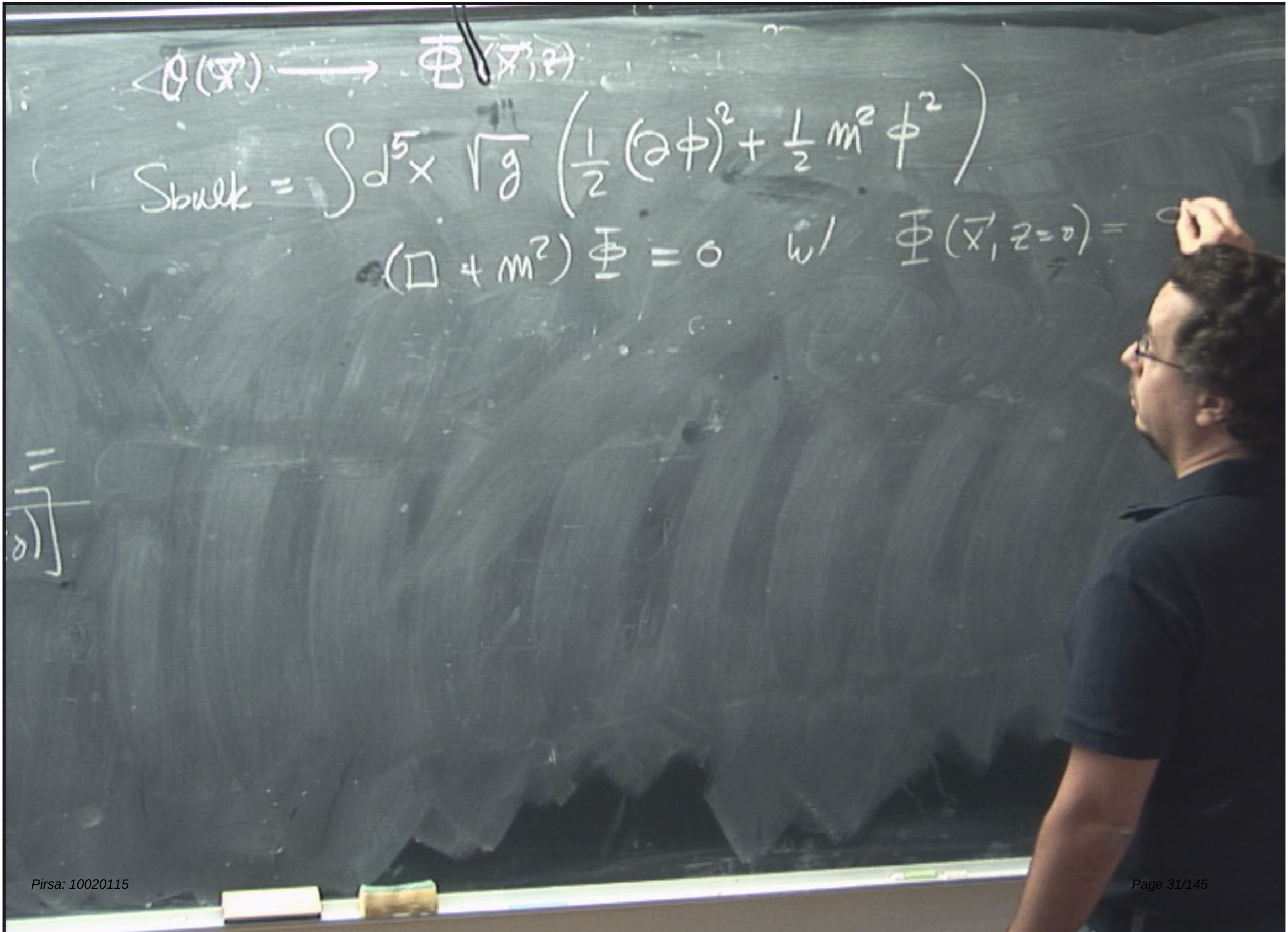
$$(\Box + m^2)\, \Phi = 0$$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$(\Box + m^2) \Phi = 0$ w/ $\Phi(\vec{x}, z=0) =$

$$\mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$$

$$S_{bulk} = \int d^5x \sqrt{g} \left(\frac{1}{2} (\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 \right)$$

$$(\Box + m^2)\Phi = 0 \quad w/ \quad \Phi(\vec{x}, z=0) = \phi_0(\vec{x})$$

$$\Phi(\vec{x}, z) = \int d^4\vec{x}' \; K(z, \vec{x}'; \vec{x}) \; \phi_0(\vec{x}').$$

$$\mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$$

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$$\Phi(\vec{x}, z) = \int d^4\vec{x}' \, K(z, \vec{x}'; \vec{x}) \, \phi_0(\vec{x}')$$

bulk to boundary propaga

$\langle \mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$

$$S_{bulk} = \int d^5x \sqrt{g} \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 \right)$$

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$$\Phi(\vec{x}, z) = \int d^4\vec{x}\,' \; K(z, \vec{x}\,'; \vec{x}) \; \phi_0(\vec{x}\,').$$

bulk to boundary propagator

$\mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$

$$S_{bulk} = \int d^5x \sqrt{g} \left(\frac{1}{2} (\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 \right)$$

$(-\Box + m^2)\Phi = 0$ w/ $\Phi(\vec{x}, z=0) = \phi_0(\vec{x})$

$$\Phi(\vec{x}, z) = \int d^4\vec{x}' \, K(z, \vec{x}'; \vec{x}) \, \phi_0(\vec{x}').$$

bulk to boundary propagator

$$(-\Box + m^2) K = 0.$$

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bulk to boundary propagator

$$(-\Box + m^2) K = 0$$

$$K(z, \vec{x}', \vec{x}) \xrightarrow{z \to 0}$$

$\mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$

$$S_{bulk} = \int d^5x \sqrt{g} \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 \right)$$

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bulk to boundary propagator

$$(-\Box + m^2) K = 0.$$

$$K(z, \vec{x}'; \vec{x}) \xrightarrow{z \to 0} \delta(\vec{x} - \vec{x}')$$

y. im

$=$

$S_{classical}[\Phi(\Phi_0)]$

$\mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$

$S_{bulk} = \int d^5x \sqrt{g} \left(\frac{1}{2} (\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 \right)$

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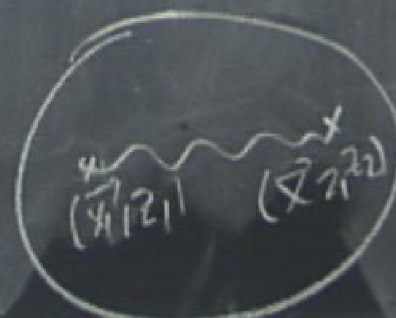
bulk to boundary prop

$(-\Box + m^2) K = 0$

$K(z, \vec{x}'; \vec{x}) \xrightarrow{z \to 0} \delta($



$(-\Delta + m^2) G = \frac{1}{\sqrt{}}$

y. im

$\langle O(\vec{x})\rangle \longrightarrow \Phi(\vec{x}, z)$

$$S_{bulk} = \int d^5x \sqrt{g}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 \phi^2\right)$$

$$(-\Box + m^2)\,\Phi = 0 \quad w/ \quad \Phi(\vec{x}, z$$

$$\Phi(\vec{x}, z) = \int d^4\vec{x}'\; K(z, \vec{x}'; \vec{x})\; \phi_0(\vec{x}$$

bulk to boundary prop

$$=$$

$$S_{classical}[\Phi(\Phi_0)]$$

$$(-\Box + m^2)\,K = 0.$$

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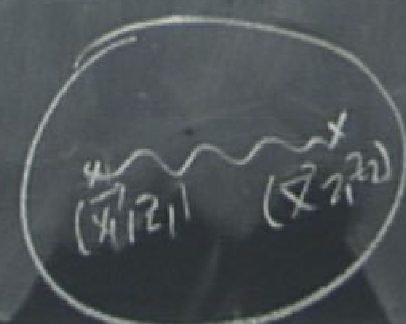

$$(-\Delta + m^2)\,G = \frac{1}{r}$$

$\langle O(\vec{x})\rangle \longrightarrow \Phi(\vec{x}, z)$

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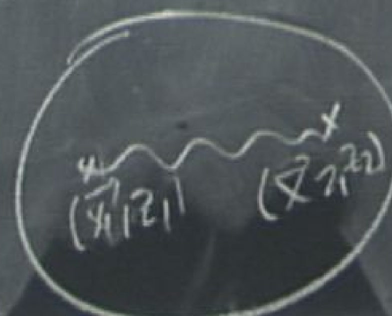

$$(-\Delta + m^2) G = \frac{1}{\sqrt{g}} \delta(\vec{x}_1 - \vec{x}_2)\, \delta(z_1 - z_2)$$

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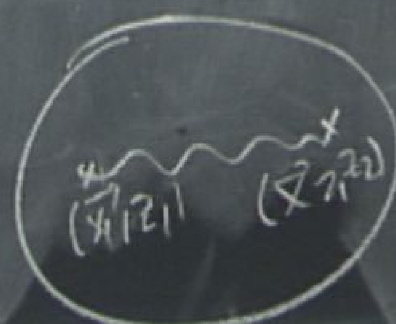
$$K(z, \vec{x}', \vec{x}) \xrightarrow{z \to 0} \delta(\vec{x}' - \vec{x})$$



$$(-\Delta + m^2) G = \frac{1}{\sqrt{g}} \delta(\vec{x}_1 - \vec{x}_2)\, \delta(z_1 - z_2)$$

$\mathcal{O}(\vec{x}) \longrightarrow \Phi(\vec{x}, z)$

$$S_{bulk} = \int d^5x \sqrt{g} \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 + \cdots \right)$$

$$(-\Box + m^2)\Phi = 0 \quad w/ \quad \Phi(\vec{x}, z=0) = \phi_0(\vec{x})$$

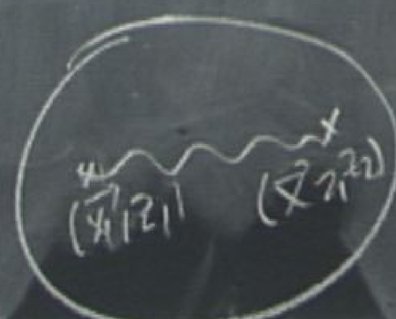
$$\Phi(\vec{x}, z) = \int d^4\vec{x}' \, K(z, \vec{x}'; \vec{x}) \, \phi_0(\vec{x}')$$

bulk to boundary propagator

$$(-\Box + m^2) K = 0$$

$$K(z, \vec{x}', \vec{x}) \xrightarrow{z \to 0} \delta(\vec{x} - \vec{x}')$$

$$(-\Delta + m^2) G = \frac{1}{\sqrt{g}} \delta(\vec{x}_1 - \vec{x}_2) \, \delta(z_1 - z_2)$$

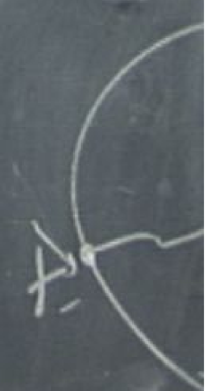


AdS/CFT conjecture:

δS_{bulk}

AdS/CFT conjecture:

$$\frac{\delta S^{\text{on-shell}}_{\text{bulk}}}{\delta \phi_0(\vec{x}_1)\, \delta \phi_0(\vec{x}_2)} = \langle \mathcal{O}(\vec{x}_1)\, \mathcal{O}(\vec{x}_2) \rangle$$



$$\langle \mathcal{O}(\vec{x}) \rangle \longrightarrow \Phi(\vec{x}, z)$$

$$K = \left(\frac{z}{z^2 + \Delta\vec{x}^{\,2}} \right)$$

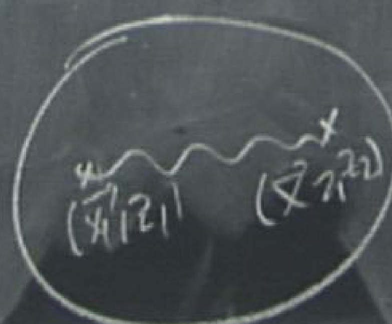
$$S_{bulk} = \int d^5x \sqrt{g} \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 + \cdots \right)$$

$$(\Box + m^2)\,\Phi = 0 \quad \text{w/} \quad \Phi(\vec{x}, z=0) = \phi_0(\vec{x})$$

$$\Phi(\vec{x}, z) = \int d^4\vec{x}^{\,\prime} \; \underbrace{K(z, \vec{x}^{\,\prime}; \vec{x})}_{\text{bulk to boundary propagator}} \; \phi_0(\vec{x}^{\,\prime}).$$

$$(-\Box + m^2)\,K = 0.$$

$$K(z, \vec{x}^{\,\prime}; \vec{x}) \xrightarrow{z \to 0} \delta(\vec{x} - \vec{x}^{\,\prime})$$

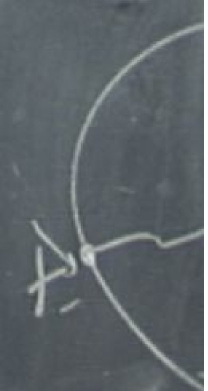


$$(-\Delta + m^2)\,G = \frac{1}{\sqrt{g}}\,\delta(\vec{x}_1 - \vec{x}_2)\,\delta(z_1 - z_2)$$

AdS/CFT conjecture:

$$\frac{\delta S_{bulk}^{on\text{-}shell}}{\delta \phi_0(\vec{x}_1)\, \delta \phi_0(\vec{x}_2)} = \langle \mathcal{O}(\vec{x}_1)\, \mathcal{O}(\vec{x}_2) \rangle$$

$$m^2 = \Delta(\Delta - 4)$$



AdS/CFT conjecture:

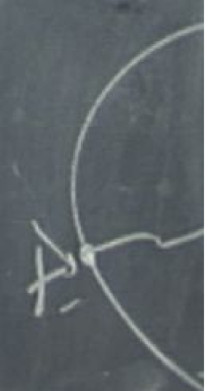
$$\frac{\delta S_{bulk}^{on\text{-}shell}}{\delta \phi_0(\vec{x}_1)\, \delta \phi_0(\vec{x}_2)} = \langle O(\vec{x}_1)\, O(\vec{x}_2) \rangle = \frac{1}{|x_1 - x_2|^{2\Delta}}$$

$$m^2 = \Delta(\Delta - 4)$$

AdS/CFT conjecture:

$$\frac{\delta S_{bulk}^{on\text{-}shell}}{\delta\phi_0(\vec{x}_1)\,\delta\phi_0(\vec{x}_2)} = \langle \mathcal{O}(\vec{x}_1)\,\mathcal{O}(\vec{x}_2)\rangle = \frac{1}{|x_1-x_2|^{2\Delta}}$$

$$m^2 = \Delta(\Delta-4)$$

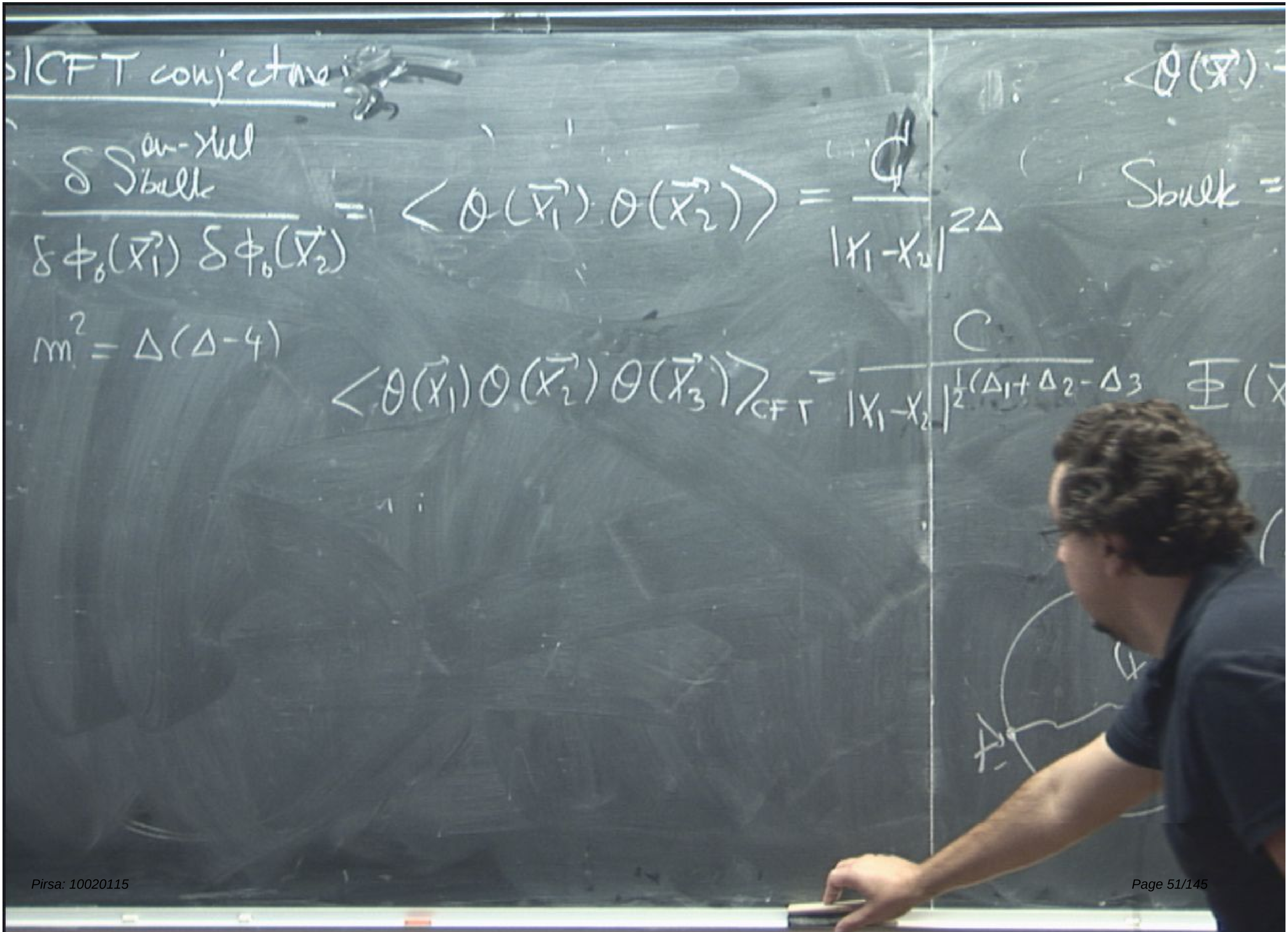


AdS|CFT conjecture:

$$\frac{\delta S_{bulk}^{on\text{-}shell}}{\delta \phi_0(\vec{x}_1)\, \delta \phi_0(\vec{x}_2)} = \langle \mathcal{O}(\vec{x}_1)\, \mathcal{O}(\vec{x}_2) \rangle = \frac{C}{|x_1 - x_2|^{2\Delta}}$$

$$m^2 = \Delta(\Delta - 4)$$

$$\langle \mathcal{O}(\vec{x}_1)\, \mathcal{O}(\vec{x}_2)\, \mathcal{O}(\vec{x}_3) \rangle_{CFT}$$


CFT conjecture:
$\frac{\delta S_{bulk}^{on\text{-}shell}}{\delta\phi_0(\vec{x}_1)\,\delta\phi_0(\vec{x}_2)} = \langle O(\vec{x}_1)\,O(\vec{x}_2)\rangle = \frac{C}{|x_1-x_2|^{2\Delta}}$
$m^2 = \Delta(\Delta-4)$
$\langle O(\vec{x}_1)\,O(\vec{x}_2)\,O(\vec{x}_3)\rangle_{CFT} = \frac{C}{|x_1-x_2|^{\frac{1}{2}(\Delta_1+\Delta_2-\Delta_3}}$
$S_{bulk} =$

S/CFT conjecture:

$$\frac{\delta S_{bulk}^{on\text{-}shell}}{\delta\phi_0(\vec{x}_1)\,\delta\phi_0(\vec{x}_2)} = \langle O(\vec{x}_1)\,O(\vec{x}_2)\rangle = \frac{C}{|x_1-x_2|^{2\Delta}}$$

$$m^2 = \Delta(\Delta-4)$$

$$\langle O(\vec{x}_1)\,O(\vec{x}_2)\,O(\vec{x}_3)\rangle_{CFT} = \frac{C}{|x_1-x_2|^{\frac{1}{2}(\Delta_1+\Delta_2-\Delta_3}} \quad \Phi(\vec{x}$$

$\langle O(\vec{x})$

$S_{bulk} =$

$$\frac{\tilde{z}}{(\Delta_1+\Delta_2-\Delta_3) \quad |x_2-x_3|}$$

$$\tilde{c}$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$\Delta_1 + \Delta_2 - \Delta_3) \quad |x_2 - x_3|^{\frac{1}{2}(\Delta_2 + \Delta_3 - \Delta_1)} \quad |x_1 - x_3|^{\frac{1}{2}(\Delta_1 + \Delta_3 - \Delta_2)}$$

conjecture

...-xtal
...ulk

$$= \langle \mathcal{O}(\vec{x}_1)\,\mathcal{O}(\vec{x}_2)\rangle = \frac{C}{|x_1 - x_2|^{2\Delta}}$$

$$...)\,\delta\phi_0(\vec{x}_2)$$

$$...(\Delta - 4)$$

$$\langle \mathcal{O}(\vec{x}_1)\,\mathcal{O}(\vec{x}_2)\,\mathcal{O}(\vec{x}_3)\rangle_{CFT} = \frac{C}{|x_1 - x_2|^{\frac{1}{2}(\Delta_1 + \Delta_2 - \Delta_3)}\;|x_2 - x_3|^{\frac{1}{2}(\Delta ...}}$$

$$\updownarrow$$

$$S = \int d^5x \sqrt{g}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\right.$$

$$\langle \mathcal{O}(\vec{x}_1)\, \mathcal{O}(\vec{x}_2) \rangle = \frac{c}{|x_1 - x_2|^{2\Delta}}$$

$$\langle \mathcal{O}(\vec{x}_1)\, \mathcal{O}(\vec{x}_2)\, \mathcal{O}(\vec{x}_3) \rangle_{CFT} = \frac{}{|x_1 - x_2|^{\frac{1}{2}(\Delta_1 + \Delta_2 - \Delta_3)}\ |x_2 - x_3|^{\frac{1}{2}(\Delta_2 + \Delta_3 - \Delta_1)}\ |}$$

$$S = \int d^5x \sqrt{g} \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{3} \phi^3 \right)$$

$$\langle \mathcal{O}(\vec{x}_1)\,\mathcal{O}(\vec{x}_2)\rangle = \frac{C}{|x_1-x_2|^{2\Delta}}$$

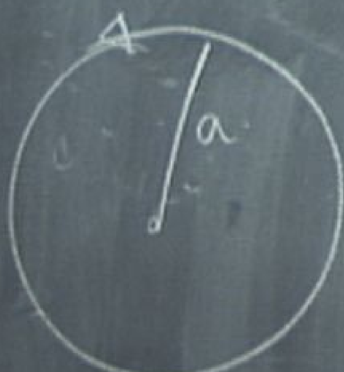
$$\langle \mathcal{O}(\vec{x}_1)\,\mathcal{O}(\vec{x}_2)\,\mathcal{O}(\vec{x}_3)\rangle_{CFT} = \frac{}{|x_1-x_2|^{\frac{1}{2}(\Delta_1+\Delta_2-\Delta_3)}\;|x_2-x_3|^{\frac{1}{2}(\Delta_2+\Delta_3-\Delta_1)}\;|}$$

$$S = \int d^5x\,\sqrt{g}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{3}\phi^3\right)$$

Gauge Theory

A_μ:

$x^\mu(\tau)$



$x^\mu(\tau) = a(\cos\tau, \sin\tau, 0, 0)$

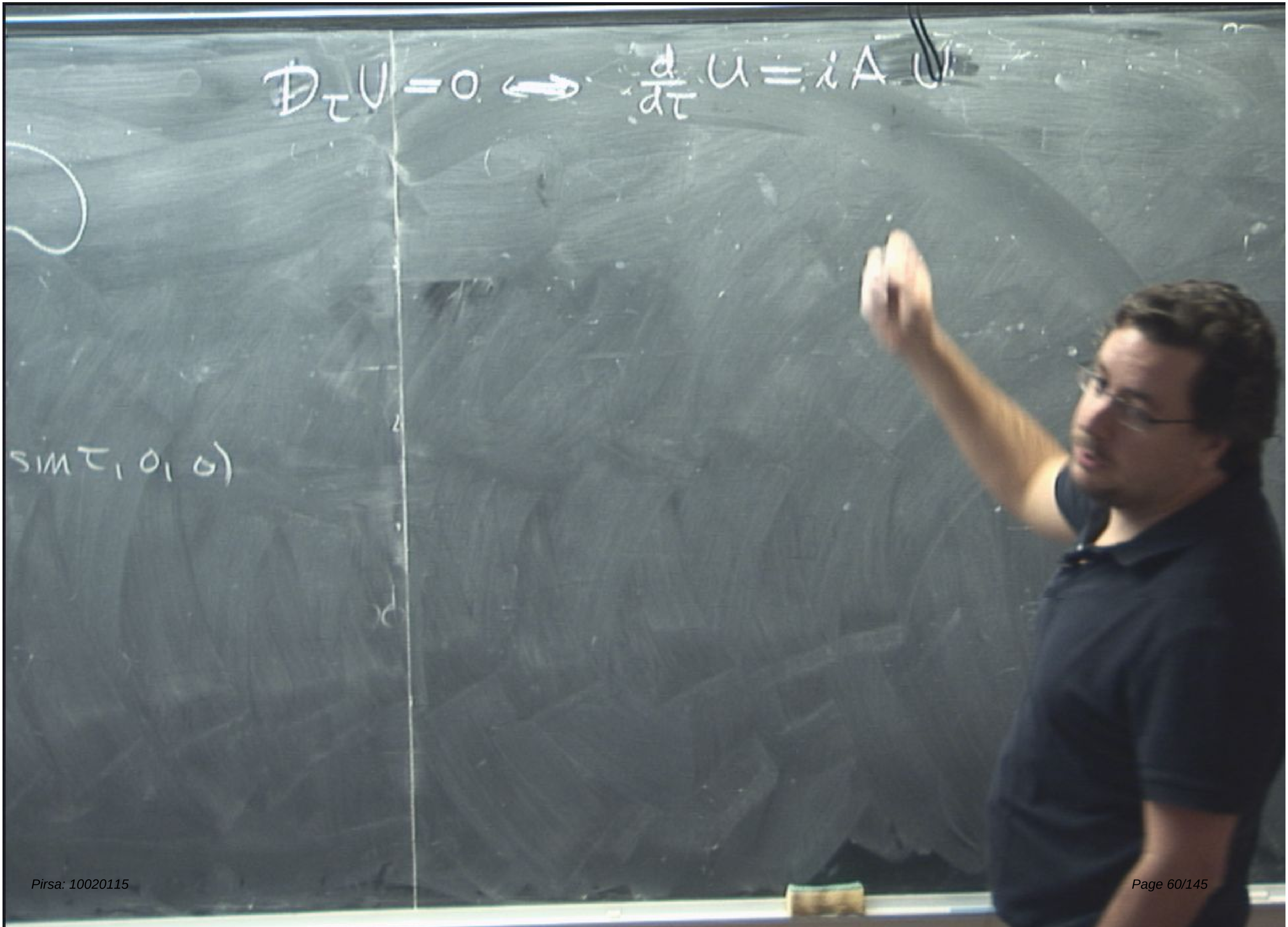
Gauge Theory

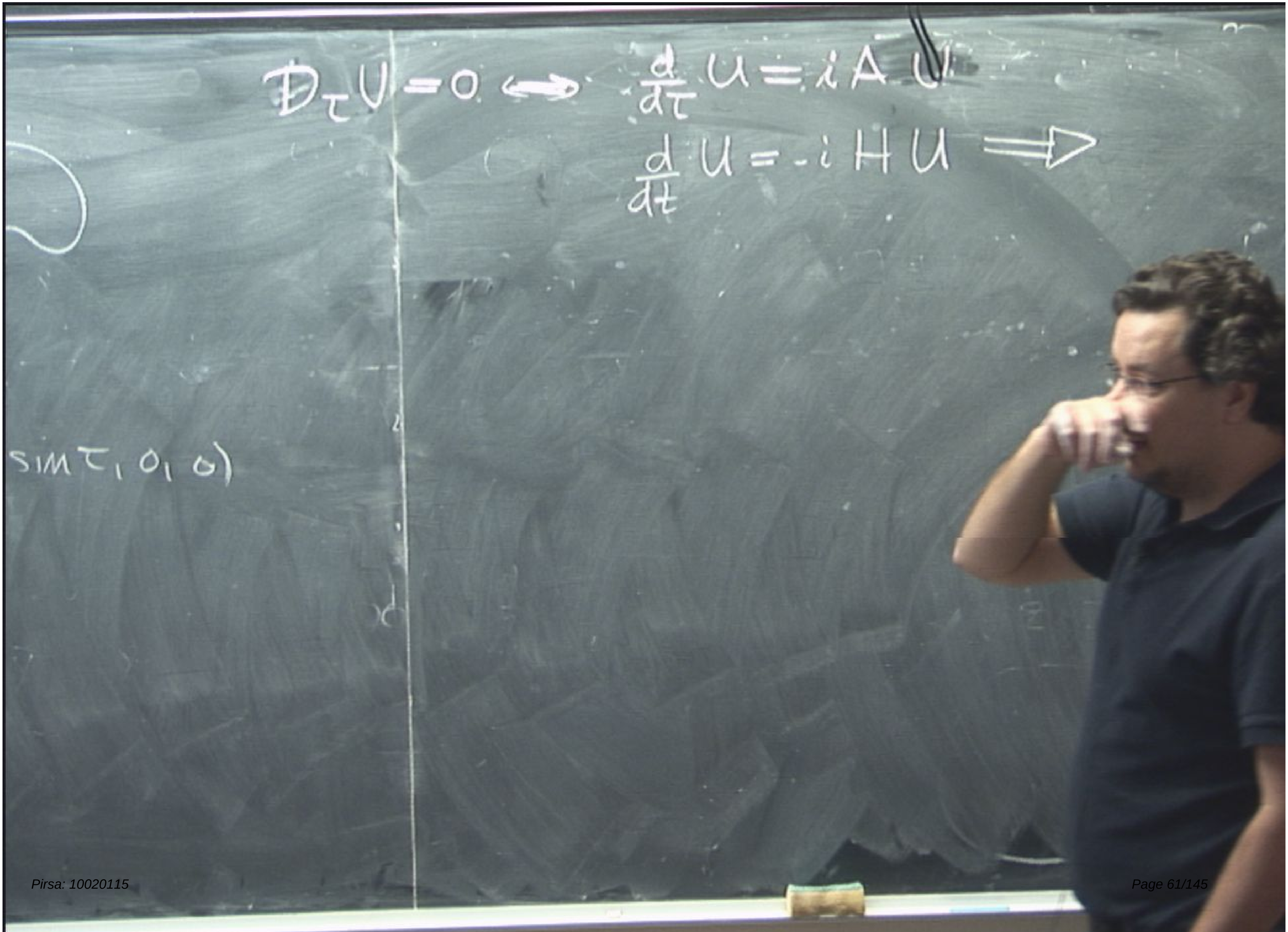
A_μ :

C

$x^\mu(\tau)$

$$x^\mu(\tau) = a(\cos\tau, \sin\tau, 0, 0)$$

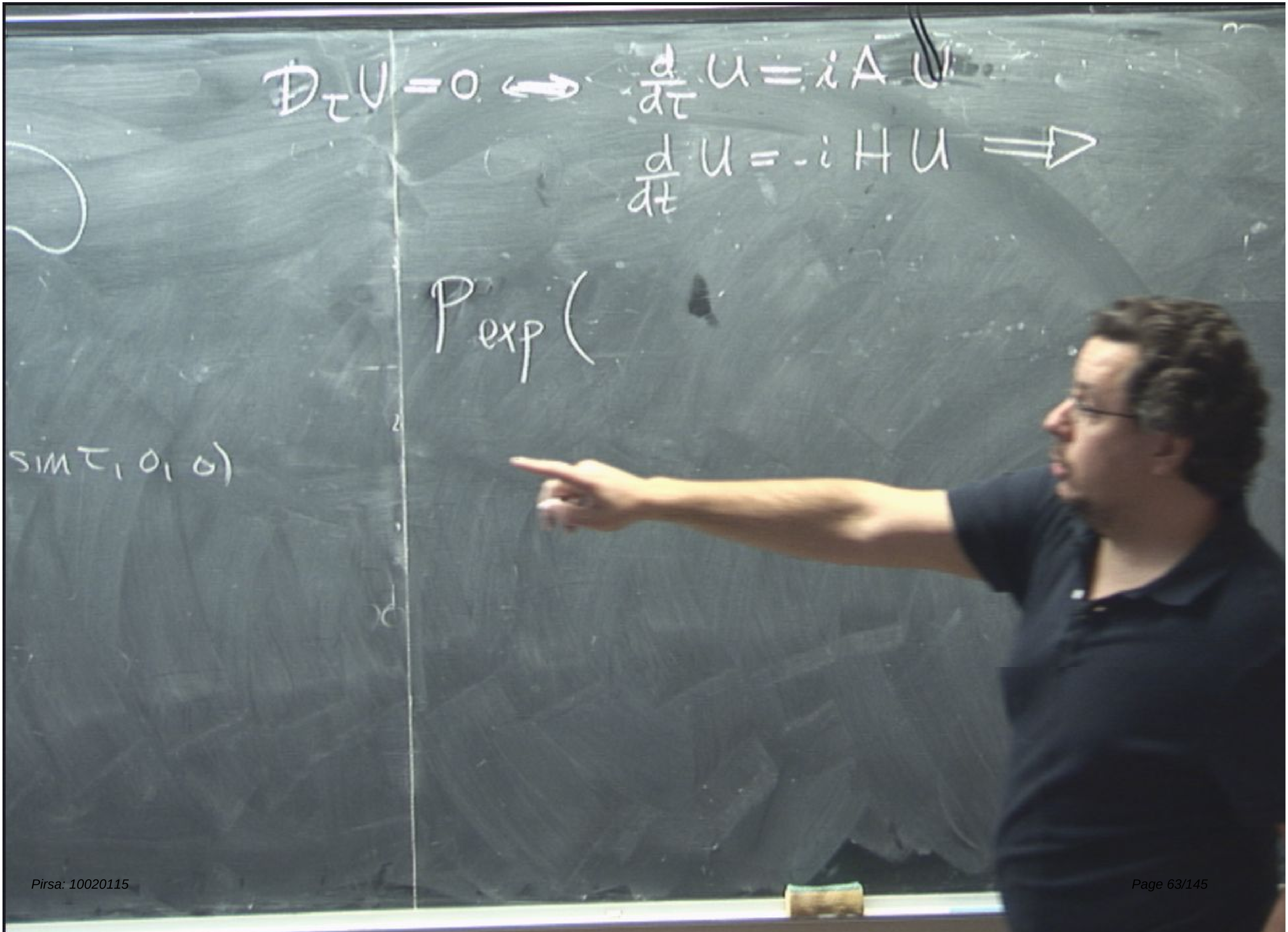

$D_\tau U = 0 \Longleftrightarrow \frac{d}{d\tau} U = i A U$
$\sin\tau, 0, 0)$

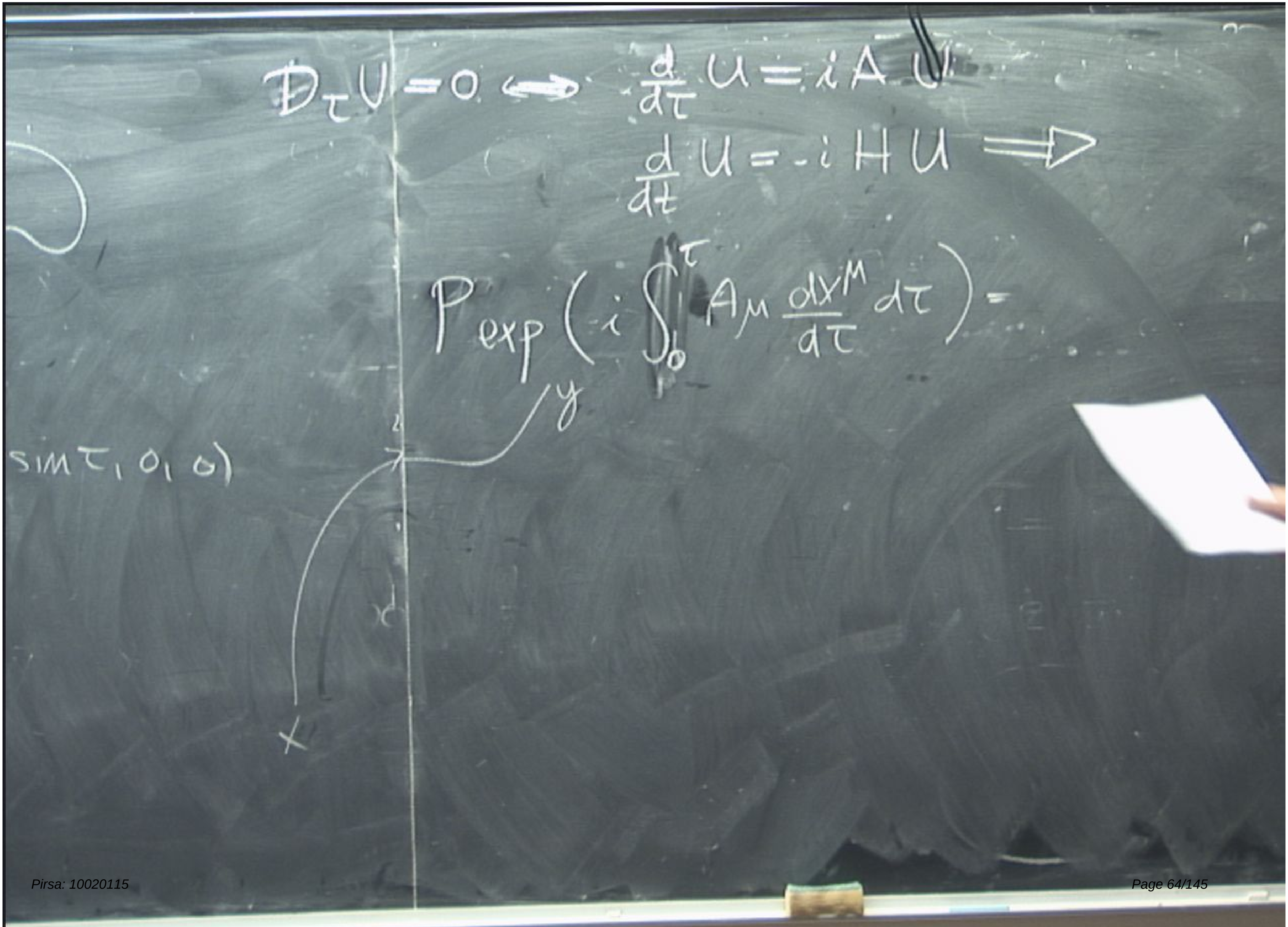

$D_\tau U = 0 \Longleftrightarrow \frac{d}{d\tau} U = iAU$
$\frac{d}{dt} U = -iHU \Longrightarrow$
$\sin\tau, 0, 0)$

$$\mathcal{D}_\tau U = 0 \Longleftrightarrow \frac{d}{d\tau} U = iA\,U$$

$$\frac{d}{dt} U = -iHU \Longrightarrow$$

$\sin\tau, 0, 0)$


D_τ U = 0 ⟺ d/dτ U = i A U
d/dt U = -i H U ⟹
P exp (
sin τ, 0, 0)


$D_\tau U = 0 \iff \frac{d}{d\tau} U = iAU$
$\frac{d}{dt} U = -iHU \Longrightarrow$
$P\exp\left(i\int_0^\tau A_\mu \frac{dx^\mu}{d\tau} d\tau\right) =$
y
x
sin τ, 0, 0)

$$D_\tau U = 0 \iff \frac{d}{d\tau} U = iAU$$

$$\frac{d}{dt} U = -iHU \implies$$

$$P\exp\left(i\int_0^\tau \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{{}''A} d\tau\right) = 1 + i\int$$

$\sin\tau, 0, 0)$

$$D_\tau U = 0 \Longleftrightarrow \frac{d}{d\tau} U = iAU$$

$$\frac{d}{dt} U = -iHU \Longrightarrow$$

$$P\exp\left(i\int_0^\tau \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{``A"} d\tau\right) = 1 + i \int^\tau$$

sin τ, 0, 0)

y

x'

x

$$\ldots U = 0 \Longleftrightarrow \frac{d}{d\tau} U = i A U$$

$$\frac{d}{dt} U = -i H U \Longrightarrow$$

$$P \exp\Big(i \int_0^{\tau} \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{\text{“}A\text{”}} d\tau\Big)_y = 1 + i \int_0^{\tau'} d\tau_1\, A(\tau_1)$$

$$+ i^2 \int_0^{\tau} d\tau_1 \int d\tau_2 \quad A(\tau_1) A(\ldots$$

$$0 \Longleftrightarrow \frac{d}{d\tau} U = i A U$$

$$\frac{d}{dt} U = -i H U \Longrightarrow$$

$$P_y \exp\left(i \int_0^\tau \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{= A} d\tau \right) = 1 + i \int_0^{\tau} d\tau_1 \, A(\tau_1) + i^2 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \quad A(\tau_1) A(\tau_2)$$

$$\Longleftrightarrow \quad \frac{d}{d\tau} U = i A U$$

$$\frac{d}{dt} U = -i H U \Longrightarrow$$

$$\exp\left(i \int_0^\tau \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{\text{"}A} d\tau \right) = 1 + i \int_0^{\tau} d\tau_1 \, A(\tau_1)$$

$$+ i^2 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \quad A(\tau_1) A(\tau_2)$$

$$+ i^3 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 \; A(\tau_1) A(\tau_2) A(\tau_3)$$

y

$\Longleftrightarrow \quad \frac{d}{d\tau} U = iAU$

$\frac{d}{dt} U = -iHU \Longrightarrow$

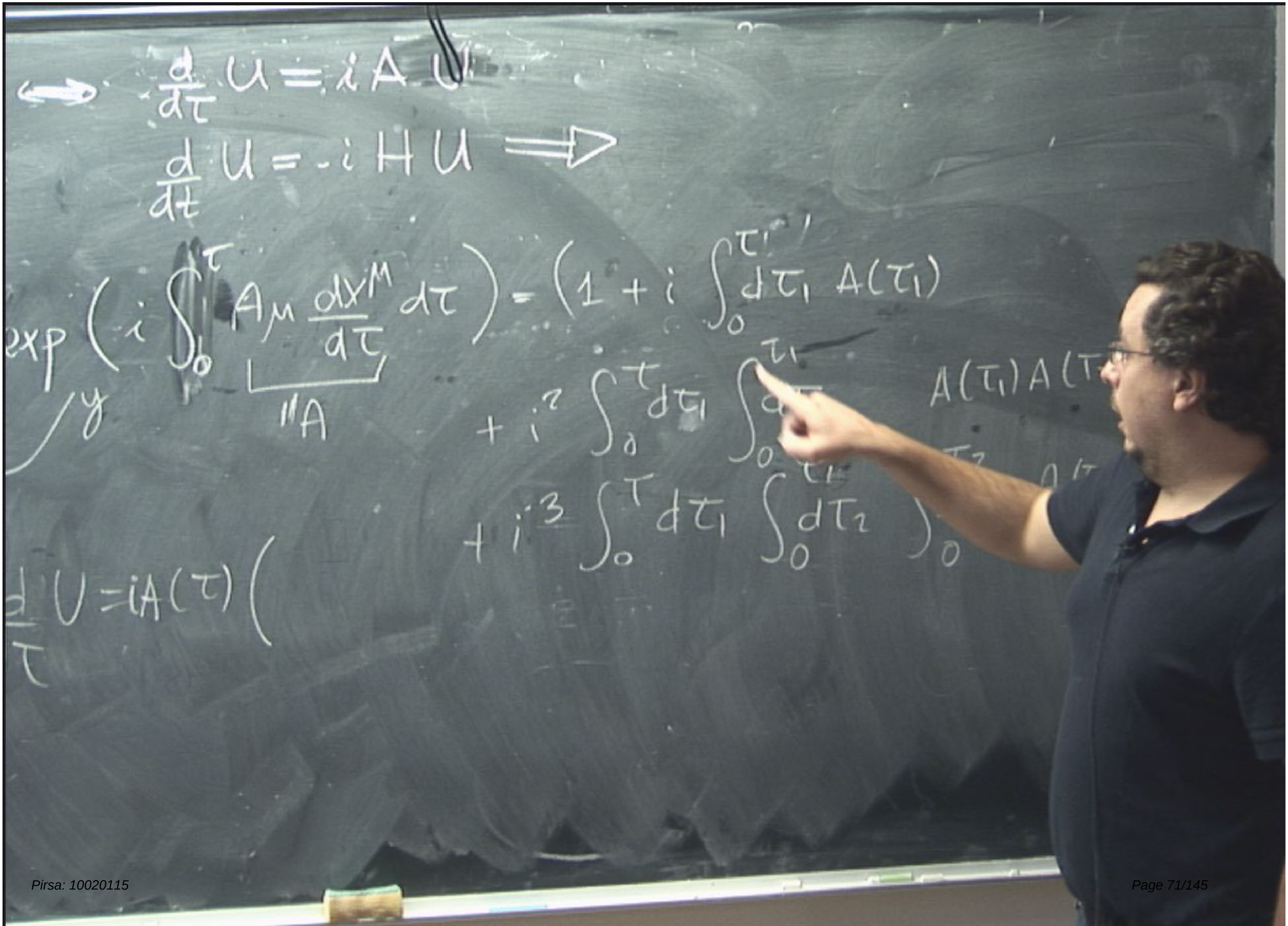
$$\exp\left(i \int_0^\tau \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{\text{"}A} d\tau\right) = \Big(1 + i \int_0^{\tau} d\tau_1\, A(\tau_1)$$

$$+ i^2 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \quad A(\tau_1) A(\tau_2)$$

$$+ i^3 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3\, A(\tau_1)A(\tau_2)A(\tau_3) + \ldots \Big)$$

y

τ



$\Longleftrightarrow \frac{d}{d\tau} U = iA\,U$
$\frac{d}{dt} U = -iHU \Longrightarrow$
$\exp\left(i \int_0^\tau \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{A} d\tau \right) = \left(1 + i \int_0^{\tau'} d\tau_1\, A(\tau_1) \right.$
$+ i^2 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2\, A(\tau_1) A(\tau_2)$
$+ i^3 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2}$
y
$\frac{d}{d\tau} U = iA(\tau)\,($

$$\Longleftrightarrow \quad \frac{d}{d\tau} U = iAU$$

$$\frac{d}{dt} U = -iHU \Longrightarrow$$

$$\exp\left(i\int_0^\tau \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{=A} d\tau\right) = \Big(1 + i\int_0^{\tau} d\tau_1\, A(\tau_1)$$

y

$$+ i^2 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \quad A(\tau_1)A(\tau_2)$$

$$+ i^3 \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3\, A \ldots A(\tau_2)A(\tau_3)\Big)$$

$$+ \ldots$$

$$\frac{d}{d\tau} U = iA(\tau)\left(1 + \int_0^\tau d\tau_2\, A(\tau_2) + \ldots\right.$$

$$\Longleftrightarrow \quad \frac{d}{d\tau} U = i A U$$

$$\frac{d}{dt} U = -i H U \Longrightarrow$$

$$\exp_{\gamma}\Big(i \int_0^{\tau} \underbrace{A_\mu \frac{dx^\mu}{d\tau}}_{``A"} d\tau \Big) = \Big(1 + i \int_0^{\tau} d\tau_1 \, A(\tau_1)$$

$$+ \, i^2 \int_0^{\tau} d\tau_1 \int_0^{\tau_1} d\tau_2 \quad A(\tau_1) A(\tau_2)$$

$$+ \, i^3 \int_0^{\tau} d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 \; A(\tau_1) A(\tau_2) A(\tau_3) + \ldots \Big)$$

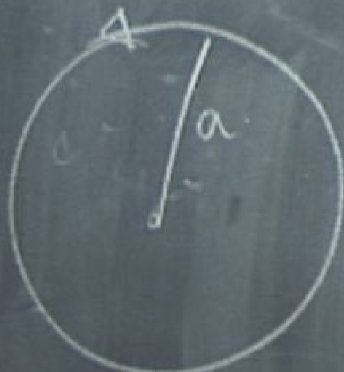
$$\frac{d}{d\tau} U = i A(\tau) \Big(1 + \int_0^{\tau} d\tau_2 \; A(\tau_2) + \ldots$$

Gauge Theory

A_μ :

C

$x^\mu(\tau)$

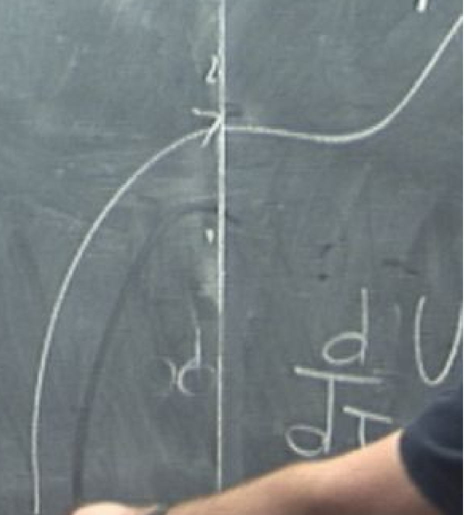


$x^\mu(\tau) = a(\cos\tau, \sin\tau, 0, 0)$

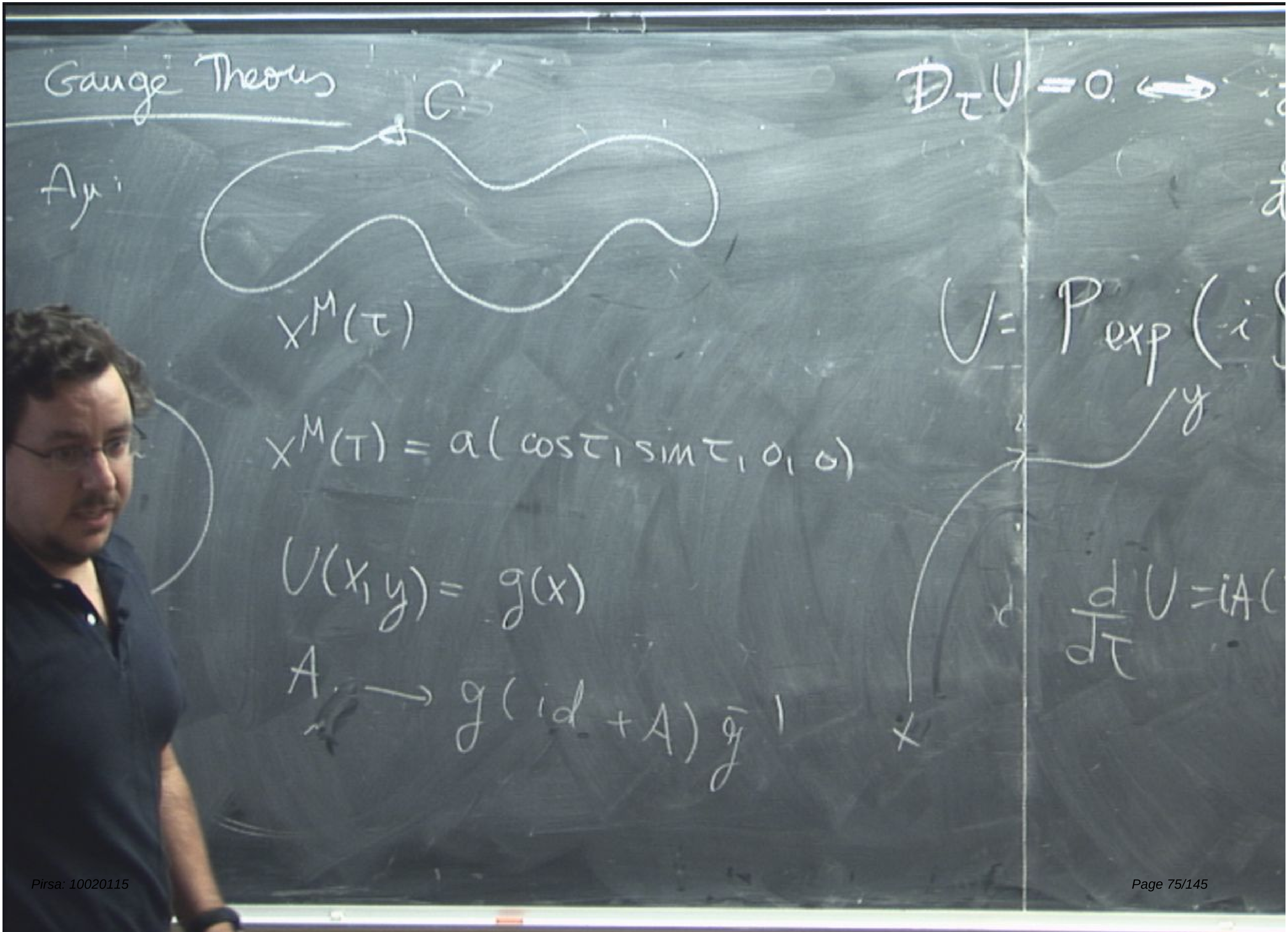
$U(x, y)$

$D_\tau U = 0 \Leftarrow$

$U = P\exp$



$\frac{d}{d\tau} U$


Gauge Theory
C
A_μ :
$x^M(\tau)$
$x^M(\tau) = a(\cos\tau, \sin\tau, 0, 0)$
$U(x,y) = g(x)$
$A \longrightarrow g(id + A)g^{-1}$
$D_\tau U = 0 \Longleftrightarrow$
$U = P\exp(i$
y
x
$\frac{d}{d\tau}U = iA($

Gauge Theory

A_μ:

$x^M(\tau)$

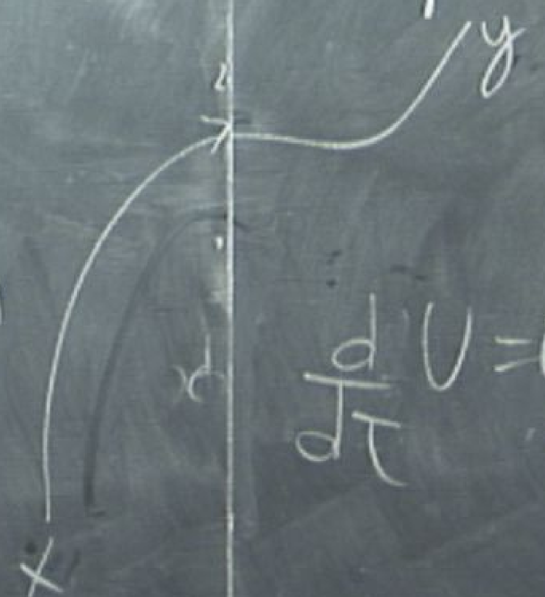
$x^M(\tau) = a(\cos\tau, \sin\tau, 0, 0)$

$U(x,y) = g(x)\,U(x,y)\,g(y)^{-1}$

$A \longrightarrow g(id + A)g^{-1}$

$D_\tau U = 0 \Longleftrightarrow$

$U = P\exp(i\int$



$\frac{d}{d\tau} U = iA($

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

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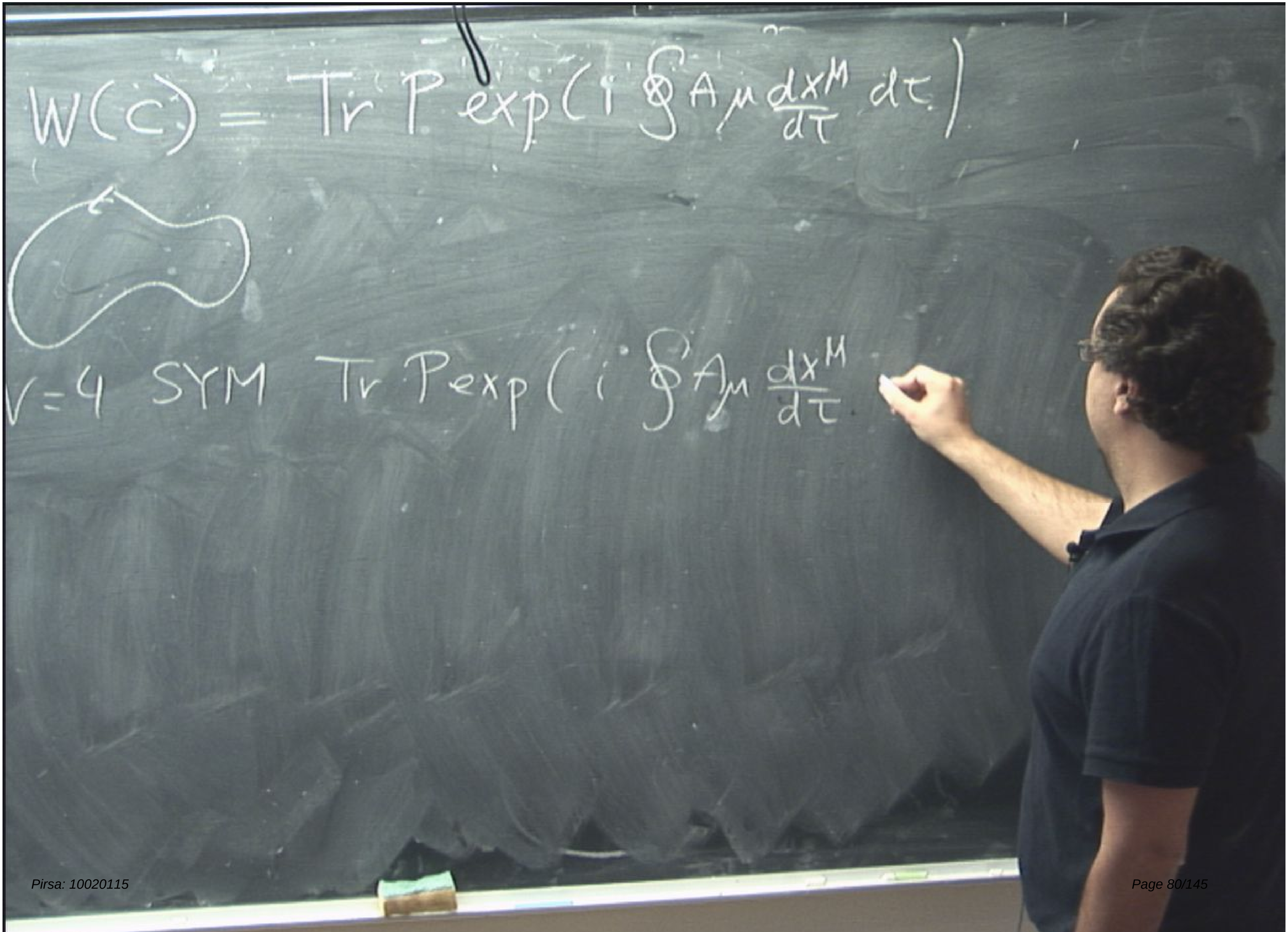
$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau} d\tau\right)$
$\mathcal{N}=4$ SYM $\mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\right.$

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

$\mathcal{N}=4$ SYM $\quad \mathrm{Tr}\, P \exp\left(\oint \left(i A_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2}\right) d\tau\right)$

$$\sum_{I=1}^{6} n^I n^I = 1$$

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

$$\mathcal{N}=4 \text{ SYM } \quad \frac{1}{N}\mathrm{Tr}\, P \exp\left(\oint \left(iA_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2}\right) d\tau\right)$$

$$\sum_{I=1}^{6} n^I n^I =$$

$$\langle W \rangle = 1 +$$

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

$\mathcal{N}=4$ SYM $\quad \frac{1}{N}\mathrm{Tr}\, P \exp\left(\oint \left(iA_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2}\right) d\tau\right)$

$$\langle W \rangle = 1 - \oint d\tau_1\, d\tau_2$$

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

$$\langle \phi(x)\, \phi(y) \rangle$$

$$=$$

$$\mathcal{N}=4 \text{ SYM} \quad \frac{1}{N}\mathrm{Tr}\, P \exp\left(\oint \left(i A_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2}\right) d\tau\right)$$

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$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

$$\langle \phi(x)\, \phi(y) \rangle \propto \frac{1}{(x-y)^2}$$

$\mathcal{N}=4$ SYM $\frac{1}{N}\mathrm{Tr}\, P \exp\left(\oint \left(iA_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2}\right) \right)$

$$\langle W \rangle = 1 - \oint d\tau_1\, d\tau_2$$

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

$$\langle \phi(x)\, \phi(y) \rangle \propto \frac{1}{(x-y)^2}$$

$$\mathcal{N}=4 \ \text{SYM} \quad \frac{1}{N}\mathrm{Tr}\, P\exp\left(\oint \left(iA_\mu \frac{dx^\mu}{d\tau} \ldots {}^I n^I \sqrt{\dot{x}^2}\right) d\tau\right)$$

$$\langle W \rangle = 1 - \oint d\tau_1\, d\tau_2 \, \frac{\dot{x}(\tau_1)\cdot \dot{x}(\tau_2)}{(x(\tau_1)-x(\tau_2))^2}$$

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

$$\langle \phi(x)\, \phi(y) \rangle \propto \frac{1}{(x-y)^2}$$

$$\mathcal{N}=4 \text{ SYM } \frac{1}{N}\mathrm{Tr}\, P\exp\left(\oint \left(iA_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2}\right) d\tau\right)$$

$$\langle W \rangle = 1 - \oint d\tau_1\, d\tau_2 \; \frac{\left(\dot{x}(\tau_1)\cdot \dot{x}(\tau_2) - \right.}{(x(\tau_1) - x(\tau_2))^2}$$

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

$$\langle \phi(x)\, \phi(y) \rangle \propto \frac{1}{(x-y)^2}$$

ϕ^I

$$\mathcal{N}=4 \ \mathrm{SYM} \quad \frac{1}{N}\mathrm{Tr}\, P \exp\left(\oint \left(i A_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2}\right) d\tau \right)$$

$$\langle W \rangle = 1 - \oint d\tau_1\, d\tau_2 \, \frac{\left(\dot{x}(\tau_1)\cdot \dot{x}(\tau_2) - |\dot{x}(\tau_1)|\,|\dot{x}(\tau_2)|\right)}{\left(x(\tau_1) - x(\tau_2)\right)^2}$$

$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

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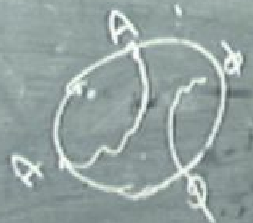
$$\phi^2$$

$$\mathcal{N}=4 \text{ SYM } \frac{1}{N} \mathrm{Tr}\, P \exp\left(\oint \left(i A_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2} \right) d\tau \right)$$

$$\langle W \rangle = 1 - g^2 N \oint d\tau_1\, d\tau_2 \; \frac{\left(\dot{x}(\tau_1) \cdot \dot{x}(\tau_2) - |\dot{x}(\tau_1)|\,|\dot{x}(\tau_2)| \right)}{\left(x(\tau_1) - x(\tau_2) \right)^2} + \cdots$$

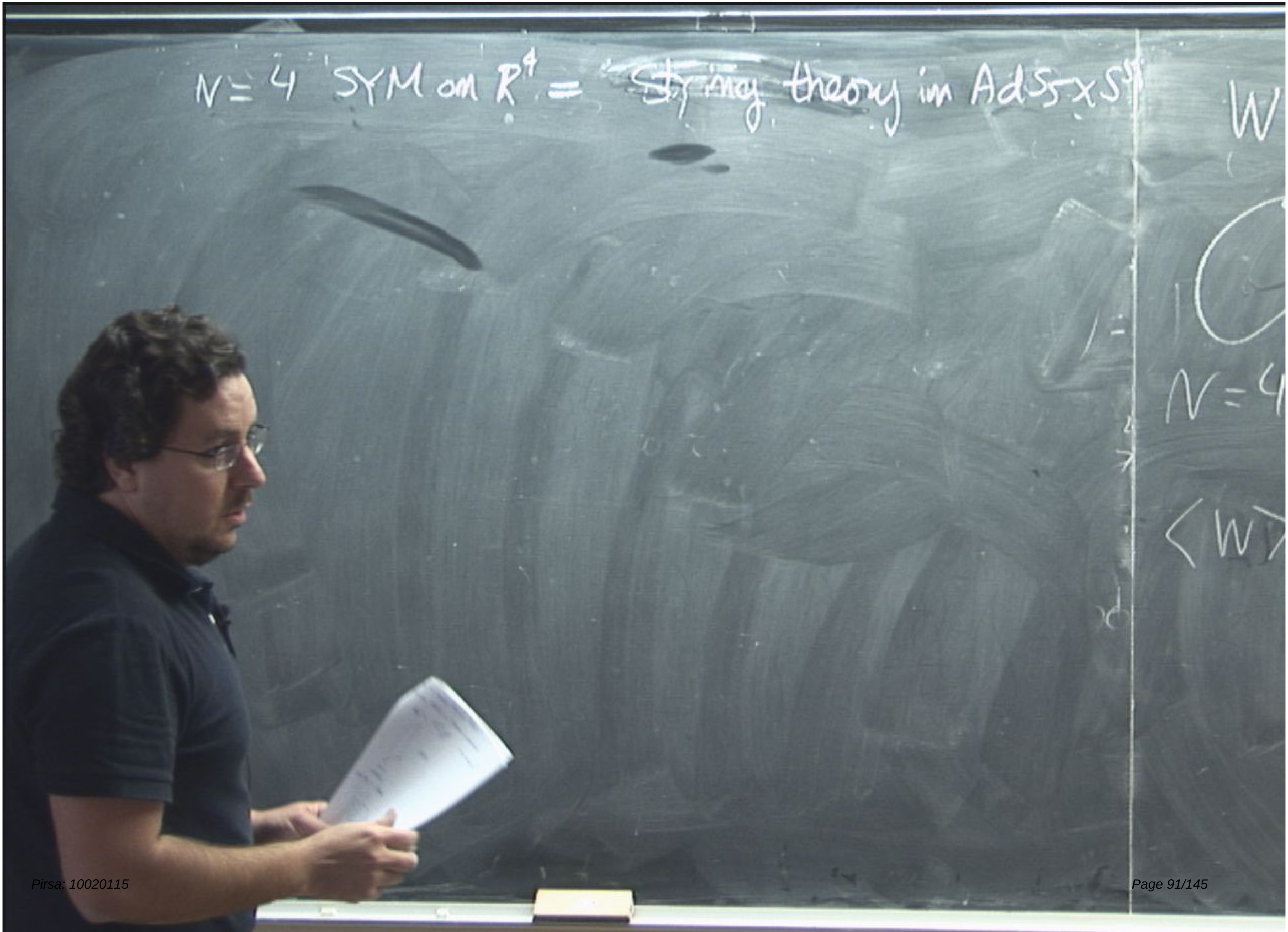
$$W(C) = \mathrm{Tr}\, P \exp\left(i \oint A_\mu \frac{dx^\mu}{d\tau}\, d\tau\right)$$

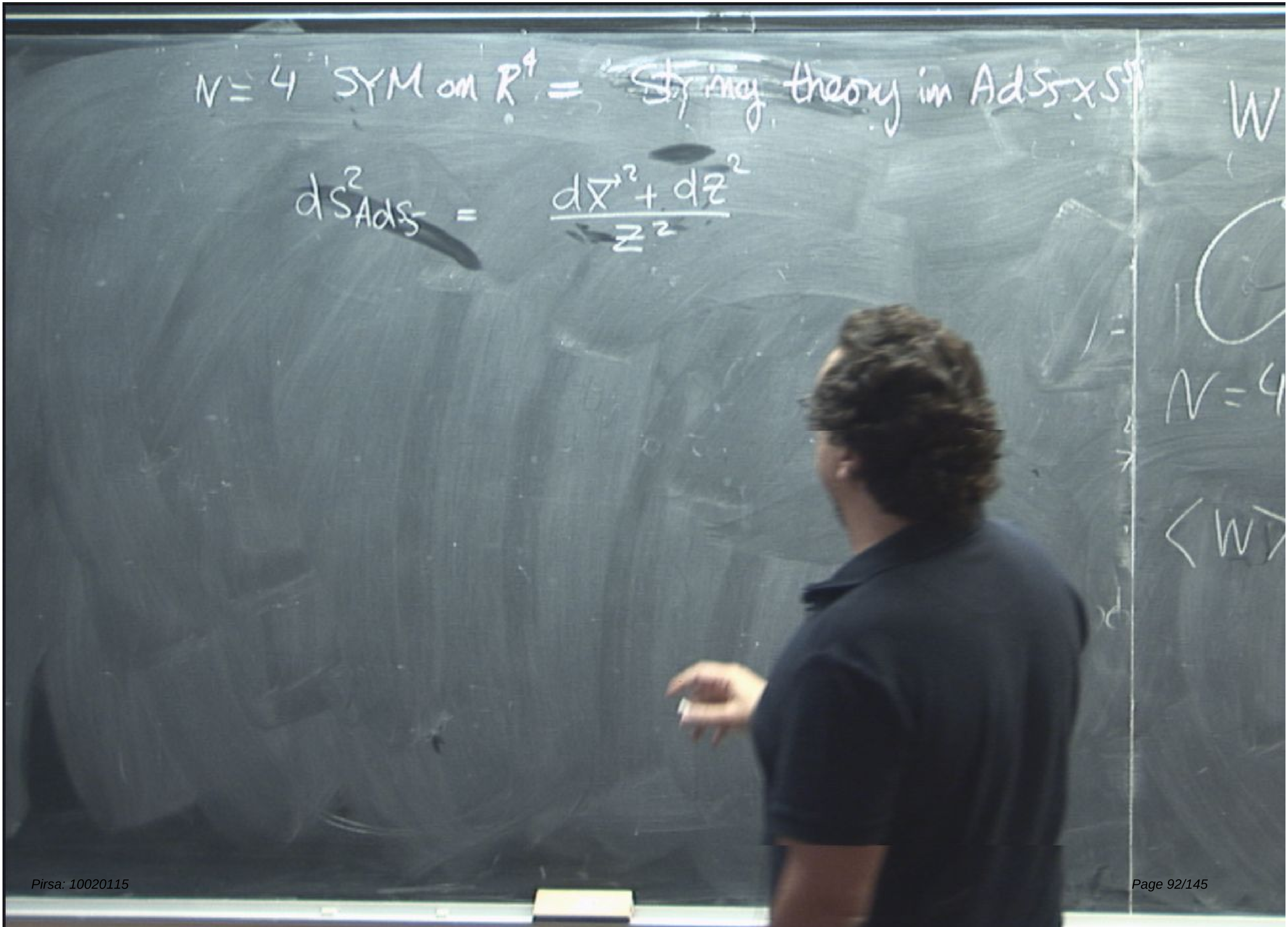


$$\langle \phi(x)\, \phi(y) \rangle \propto \frac{1}{(x-y)^2}$$

$\mathcal{N}=4$ SYM $$\frac{1}{N} \mathrm{Tr}\, P \exp\left(\oint \left(i A_\mu \frac{dx^\mu}{d\tau} + \phi^I n^I \sqrt{\dot{x}^2}\right) d\tau\right)$$

$$\langle W \rangle = 1 - g^2 N \oint d\tau_1\, d\tau_2\, \frac{\left(\dot{x}(\tau_1)\cdot\dot{x}(\tau_2) - |\dot{x}(\tau_1)||\dot{x}(\tau_2)|\right)}{(x(\tau_1) - x(\tau_2))^2} + \cdots$$


N = 4 SYM on R^4 = "String theory in AdS5xS5"


N = 4 SYM on R^4 = String theory in AdS5 x S5
ds²AdS5 = (dx⃗² + dz²) / z²
W
N=4
<W>

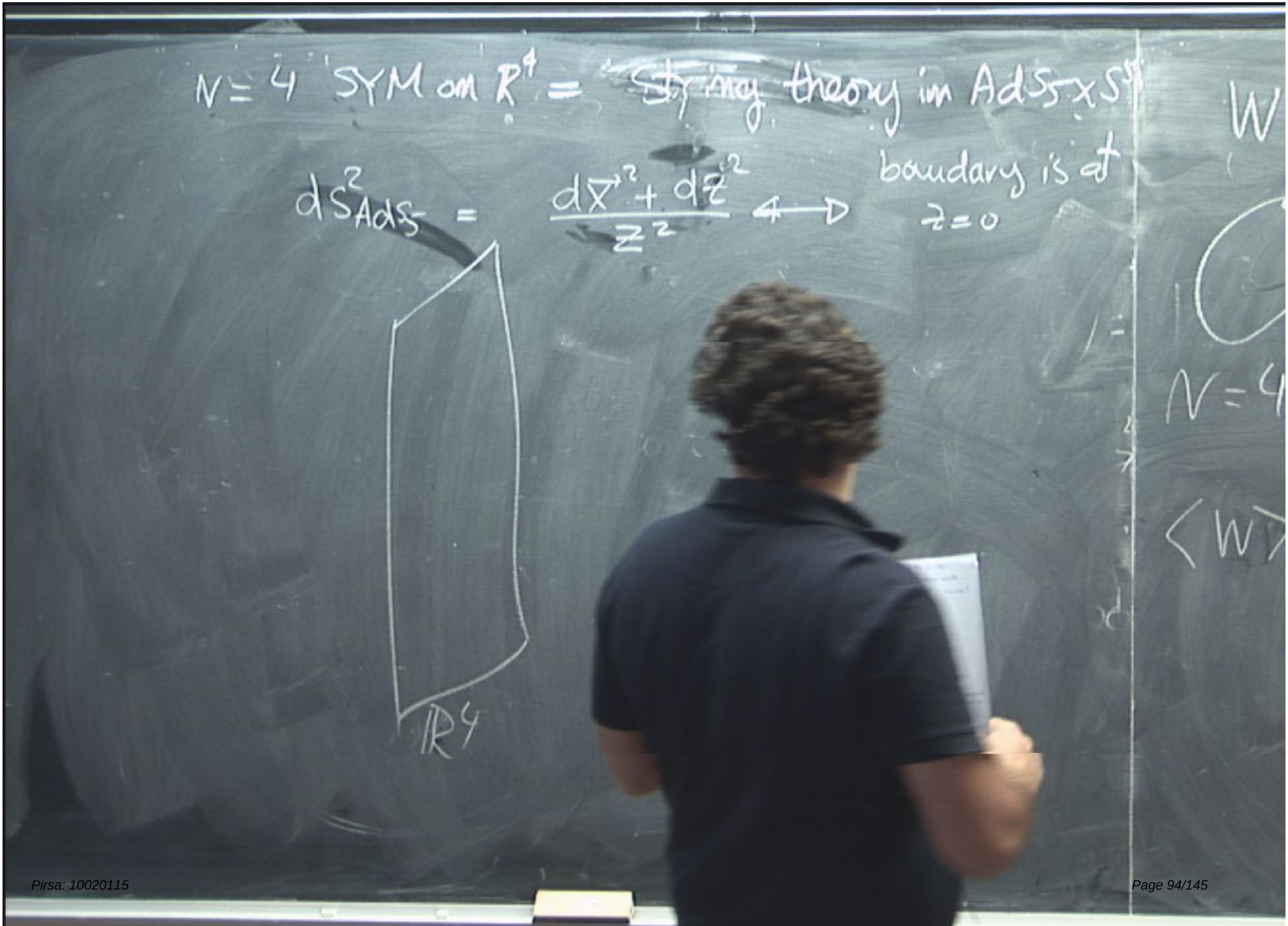
$\mathcal{N}=4$ SYM on $\mathbb{R}^4$ = "String theory in $AdS_5 \times S^5$"

$$ds^2_{AdS_5} = \frac{d\vec{x}^2 + dz^2}{z^2} \quad 4 \longrightarrow$$

W

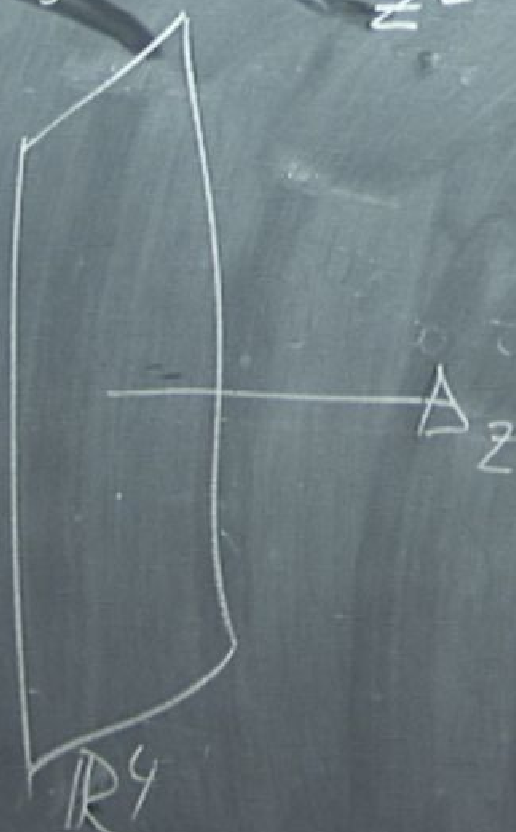
$\mathcal{N}=4$

$\langle W \rangle$


N = 4 SYM on R⁴ = String theory in AdS5 x S5
ds²AdS5 = (dx⃗² + dz²)/z² ⟷ boundary is at z = 0
IR4
W
N = 4
<W>

$\mathcal{N}=4$ SYM on $\mathbb{R}^4$ = "String theory in $AdS_5 \times S^5$"

$$ds^2_{AdS_5} = \frac{d\vec{x}^2 + dz^2}{z^2} \longleftrightarrow \text{boundary is at } z=0$$



W

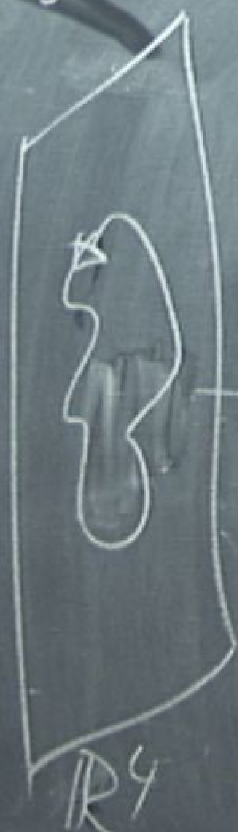
$\mathcal{N}=4$

$\langle W \rangle$

$\mathcal{N}=4$ SYM on R^4 = String theory in $AdS_5 \times S^5$

$$ds^2_{AdS_5} = \frac{d\vec{x}^2 + dz^2}{z^2} \longleftrightarrow$$

boundary is at $z=0$



W

$\mathcal{N}=4$

$\langle W \rangle$

$\mathcal{N}=4$ SYM on $\mathbb{R}^4$ = String theory in $AdS_5 \times S^5$

$$ds^2_{AdS_5} = \frac{d\vec{x}^2 + dz^2}{z^2} \longleftrightarrow$$

boundary is at $z=0$

W

$\mathcal{N}=4$

$\langle W \rangle$

$\mathcal{N}=4$ SYM on $\mathbb{R}^4$ = String theory in $AdS_5 \times S^5$

$$ds^2_{AdS_5} = \frac{d\vec{x}^2 + dz^2}{z^2} \longleftrightarrow$$ boundary is at $z=0$

W

$\mathcal{N}=4$

$\langle W \rangle$

$\mathcal{N}=4$ SYM on $\mathbb{R}^4$ = String theory in $AdS_5 \times S^5$

$$ds^2_{AdS_5} = \frac{d\vec{x}^{\,2} + dz^2}{z^2} \longleftrightarrow$$ boundary is at $z=0$

∂AdS_5

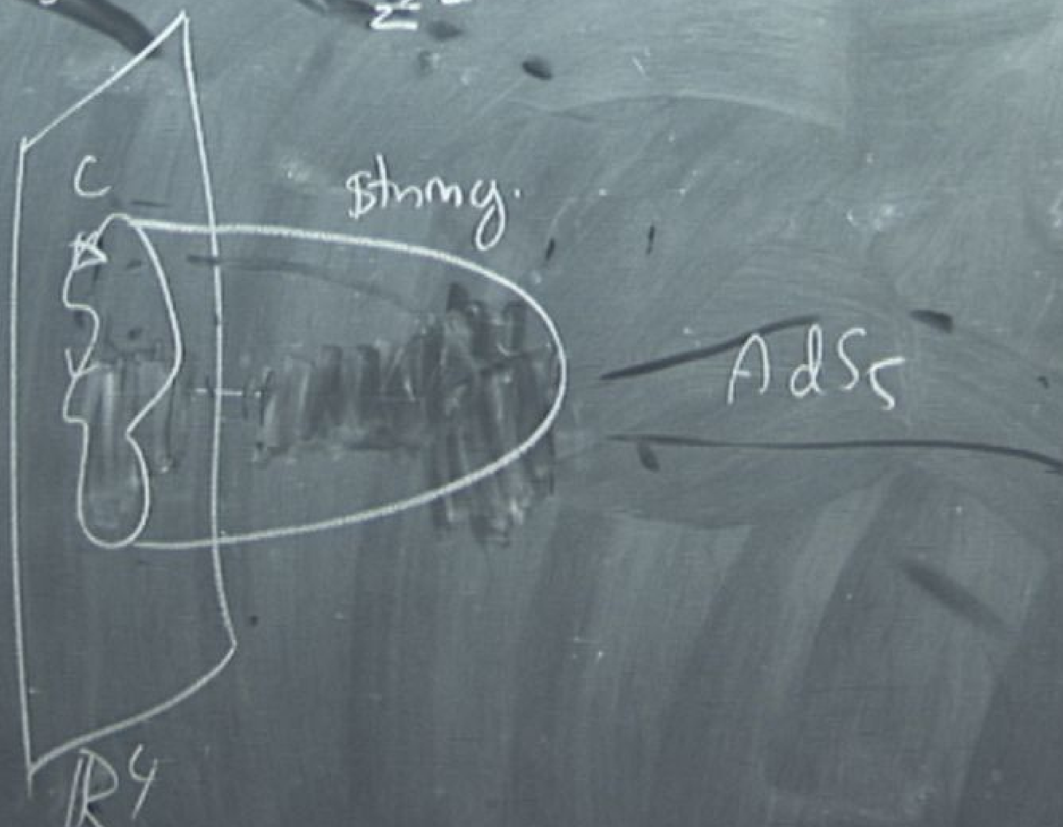
W

$\mathcal{N}=4$

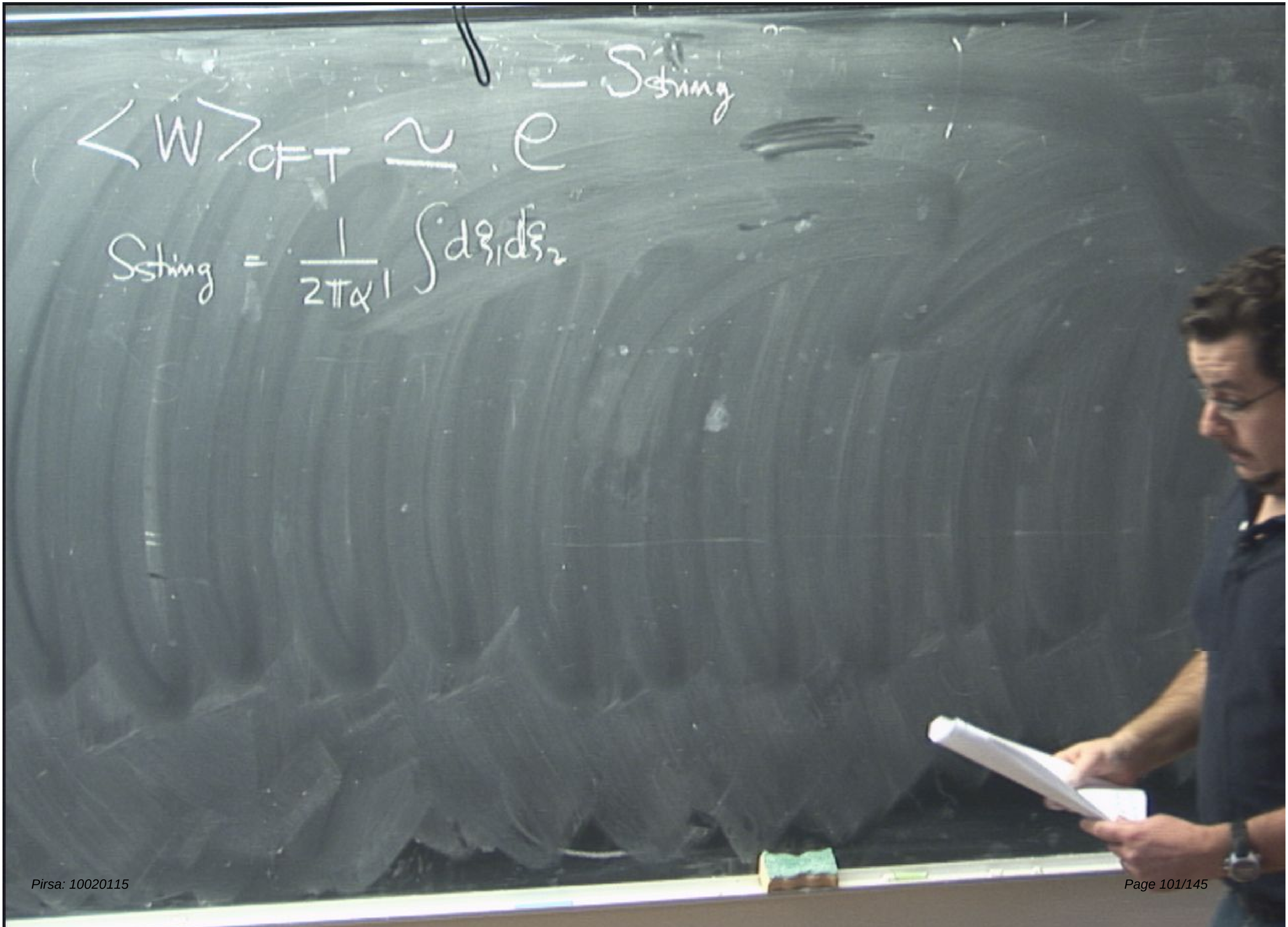
$\langle W \rangle$

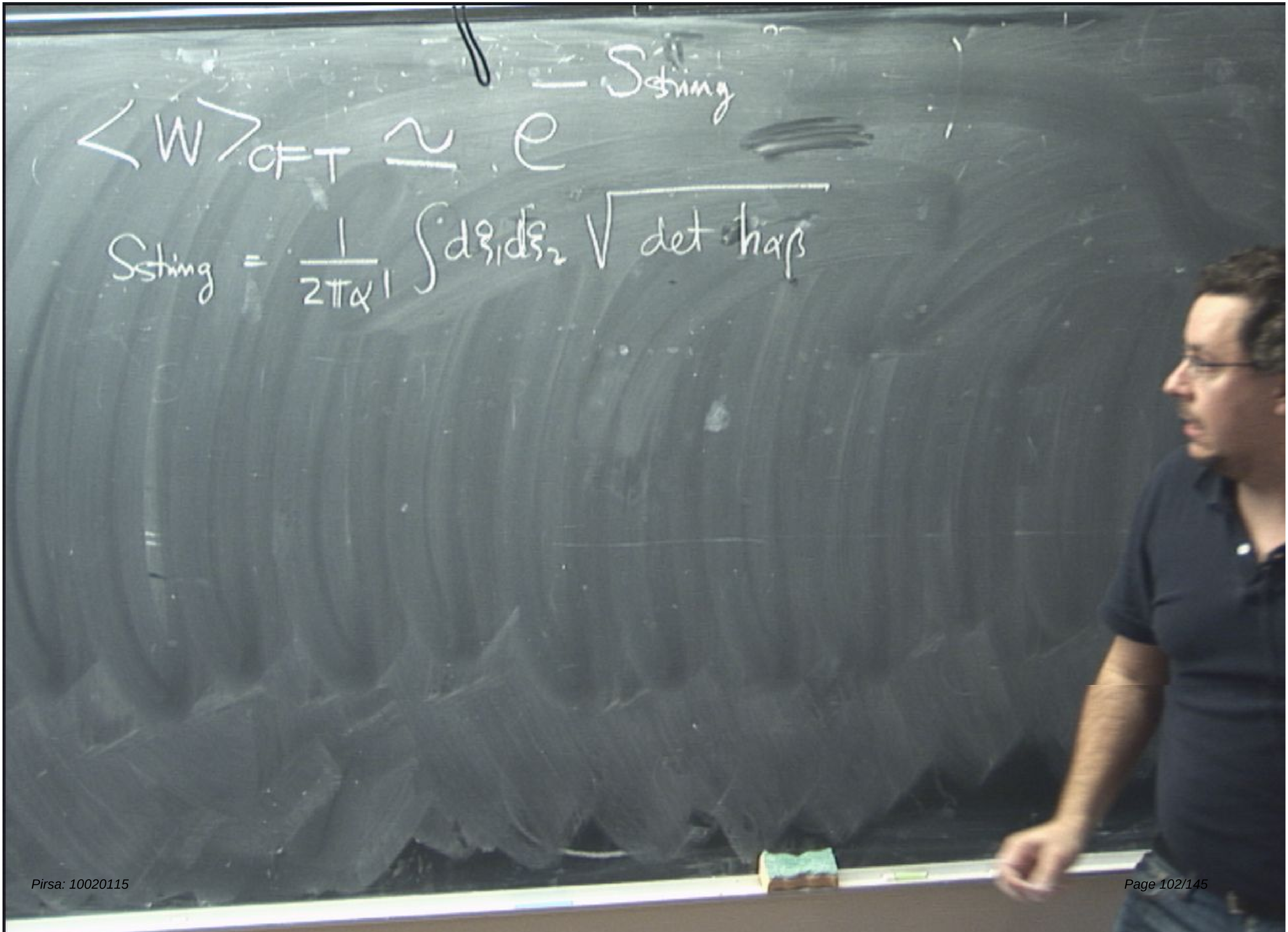
$\mathcal{N}=4$ SYM on $\mathbb{R}^4$ = String theory in $AdS_5 \times S^5$

$$ds^2_{AdS_5} = \frac{d\vec{x}^{\,2} + dz^2}{z^2} \longleftrightarrow$$ boundary is at $z=0$



∂AdS_5


$\langle W \rangle_{CFT} \simeq e^{-S_{string}}$
$S_{string} = \frac{1}{2\pi\alpha'} \int d\xi_1 d\xi_2$


$\langle W \rangle_{CFT} \simeq e^{-S_{string}}$
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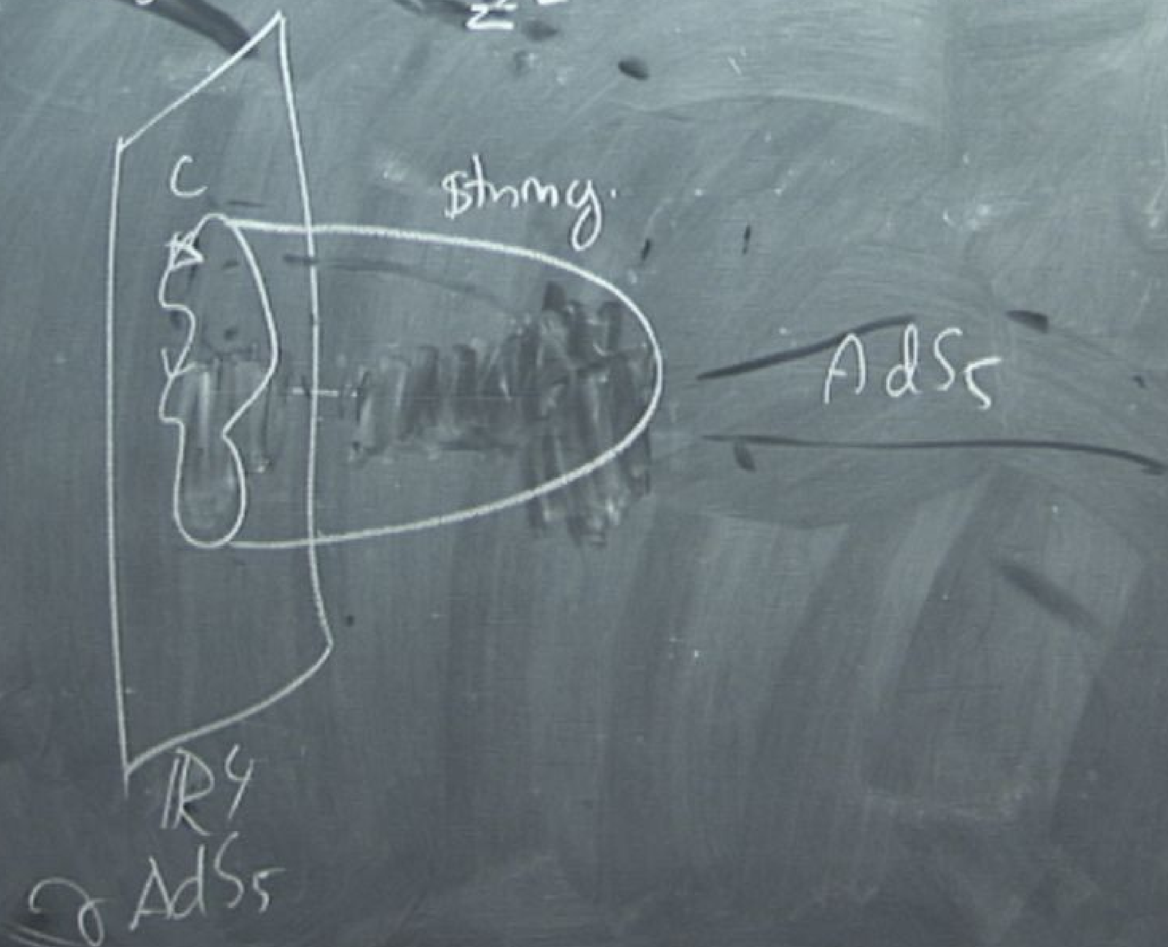
$$h_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N \, g_{MN}$$

$\mathcal{N}=4$ SYM on $\mathbb{R}^4$ = String theory in $AdS_5 \times S^5$

$$ds^2_{AdS_5} = \frac{d\vec{x}^2 + dz^2}{z^2} \longleftrightarrow \text{boundary is at } z=0$$



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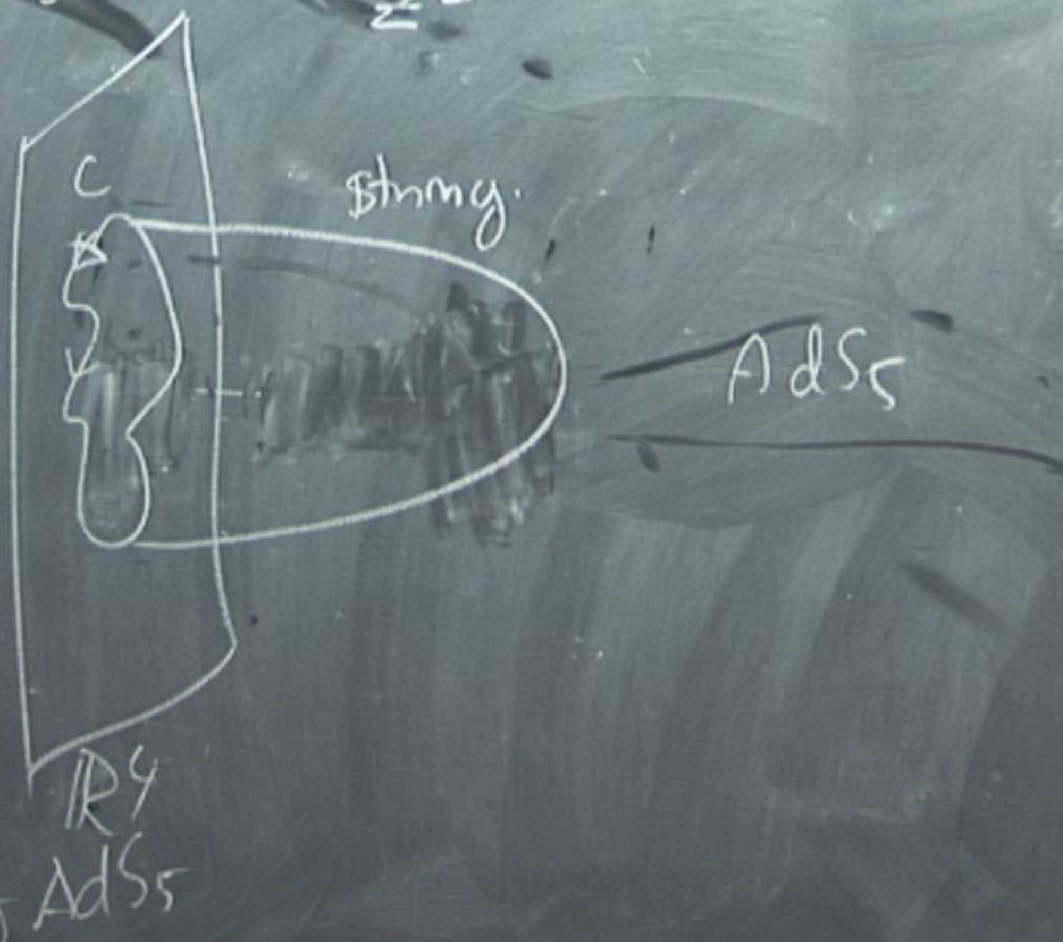

$$\langle W \rangle_{CFT} \simeq e^{-S_{string}}$$

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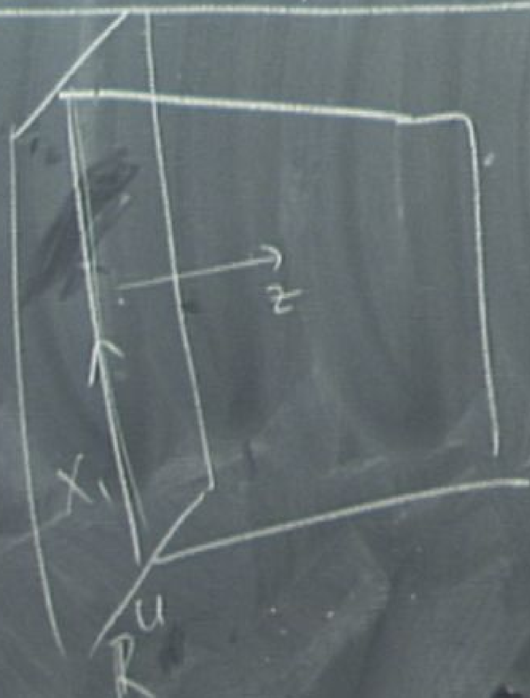

$$\begin{cases} X_1 = \xi_1 \\ z = \xi_2 \end{cases}$$

$$\langle W \rangle_{CFT} \simeq e^{-S_{string}}$$

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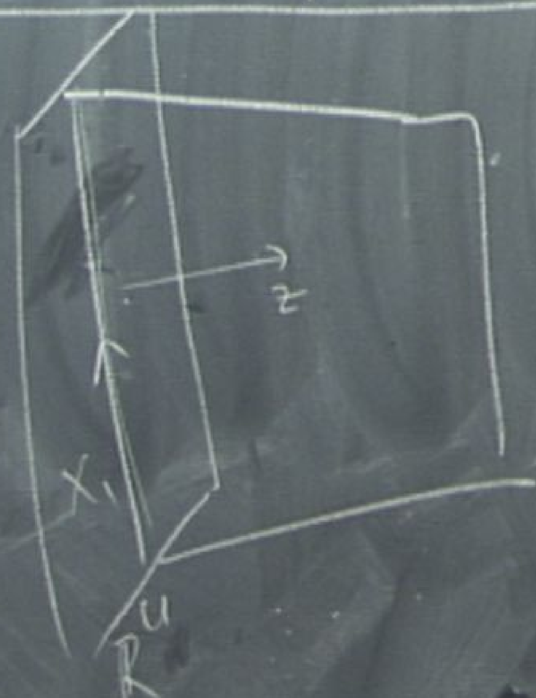

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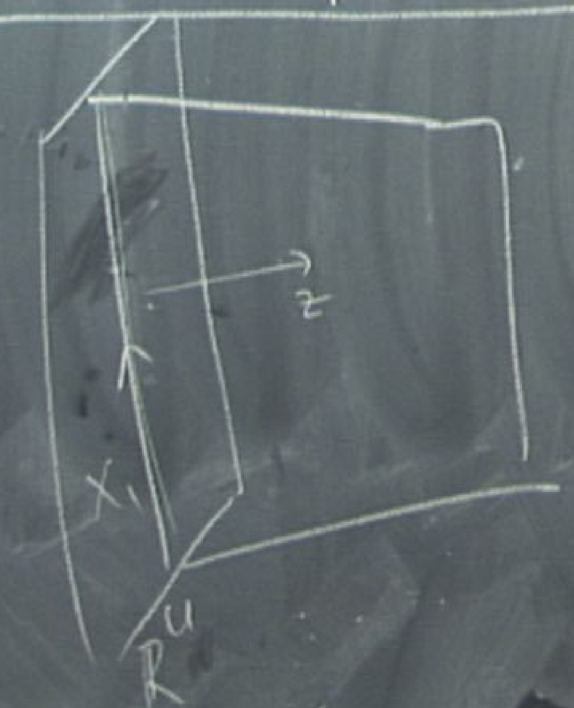

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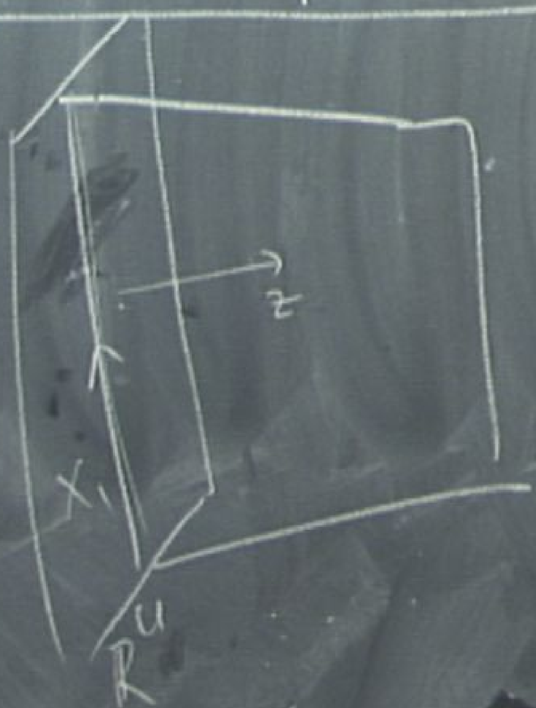
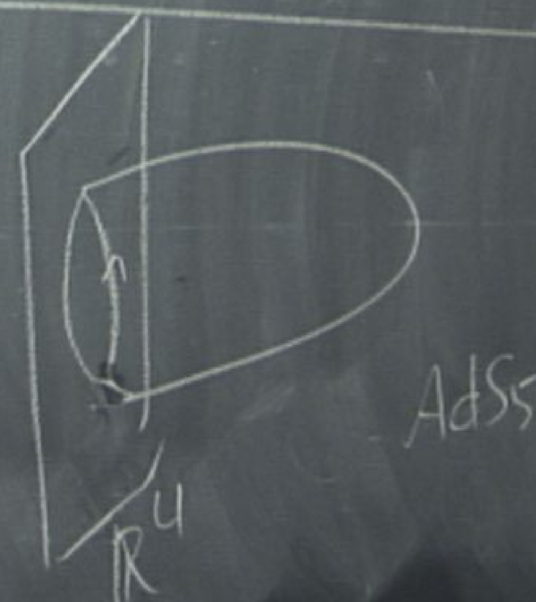
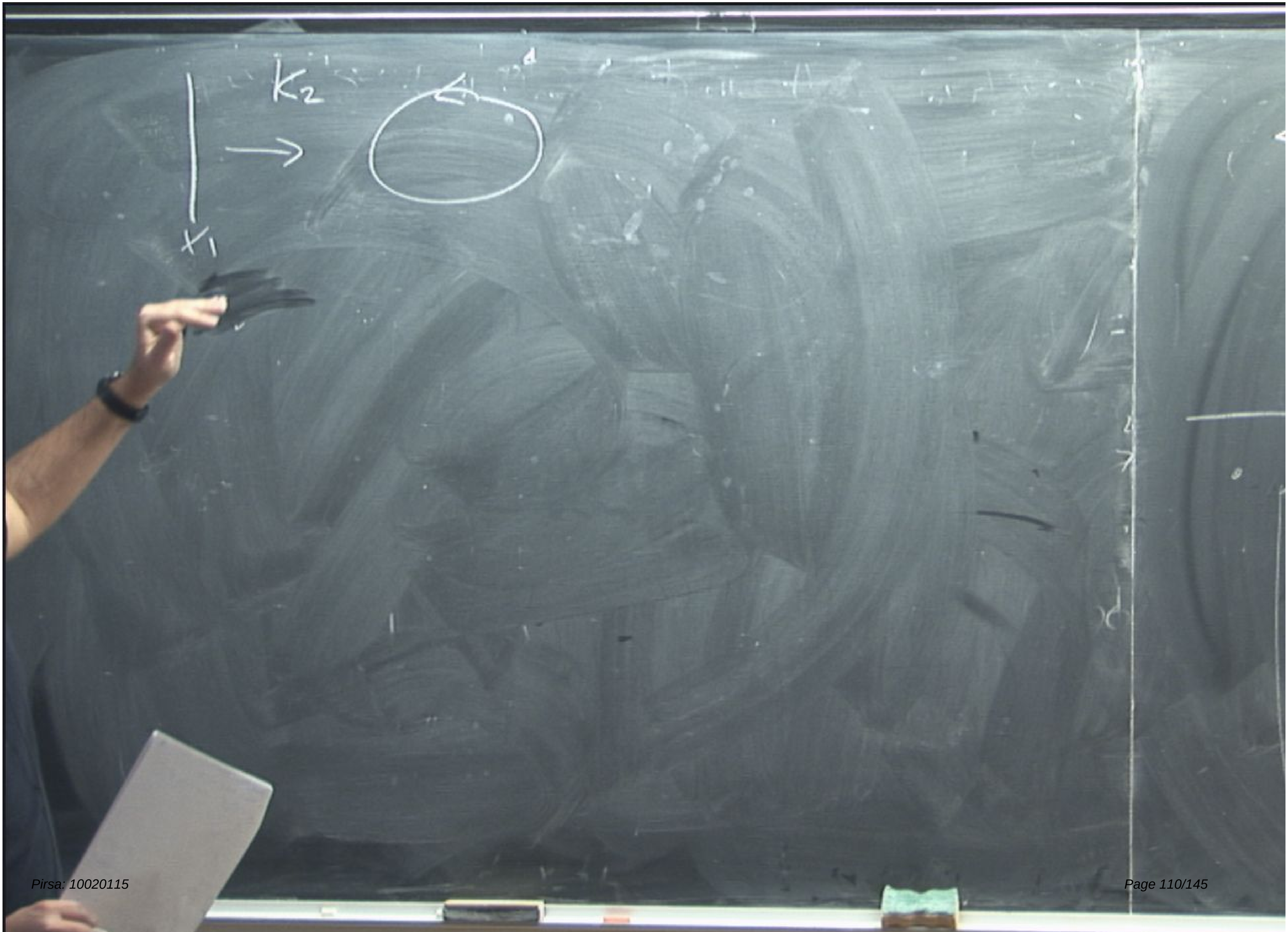
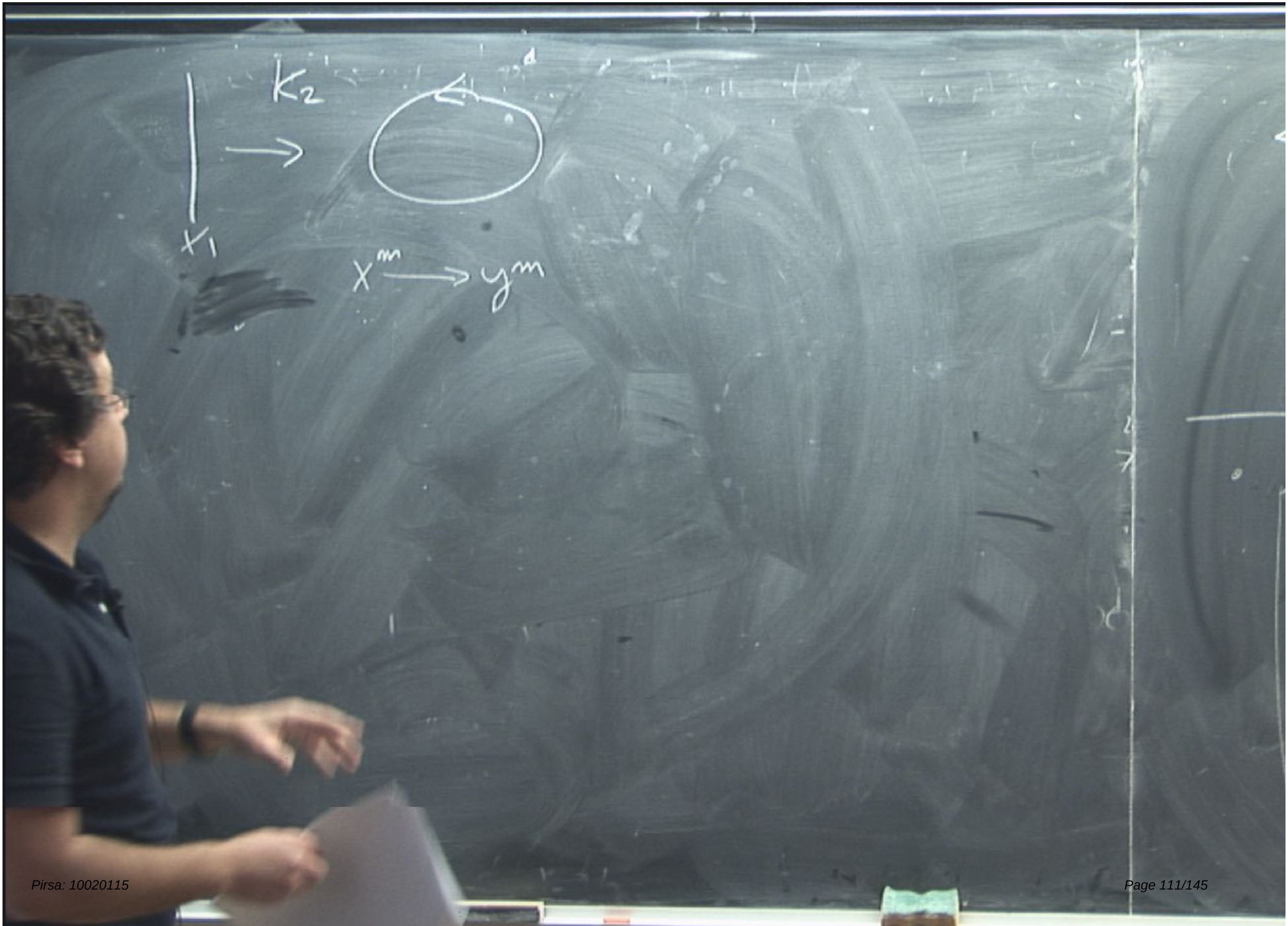


$$\begin{cases} X_1 = \xi_1 \\ z = \xi_2 \end{cases}$$

$\longrightarrow$

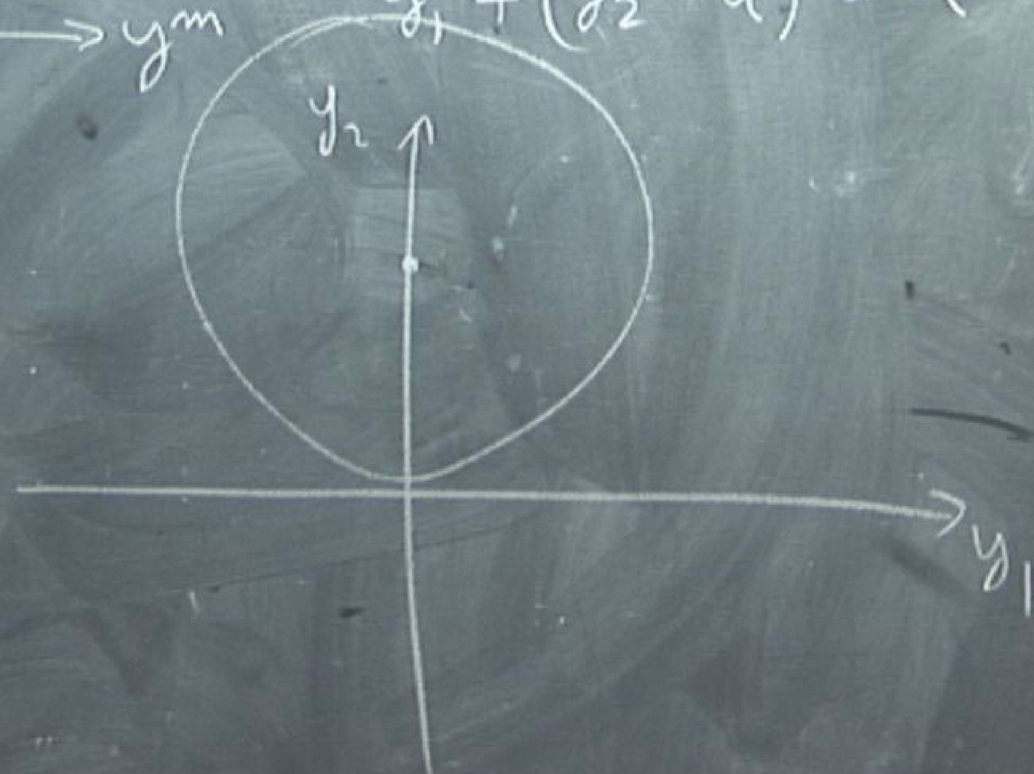



K2
x1


K_2
$X^m \longrightarrow y^m$

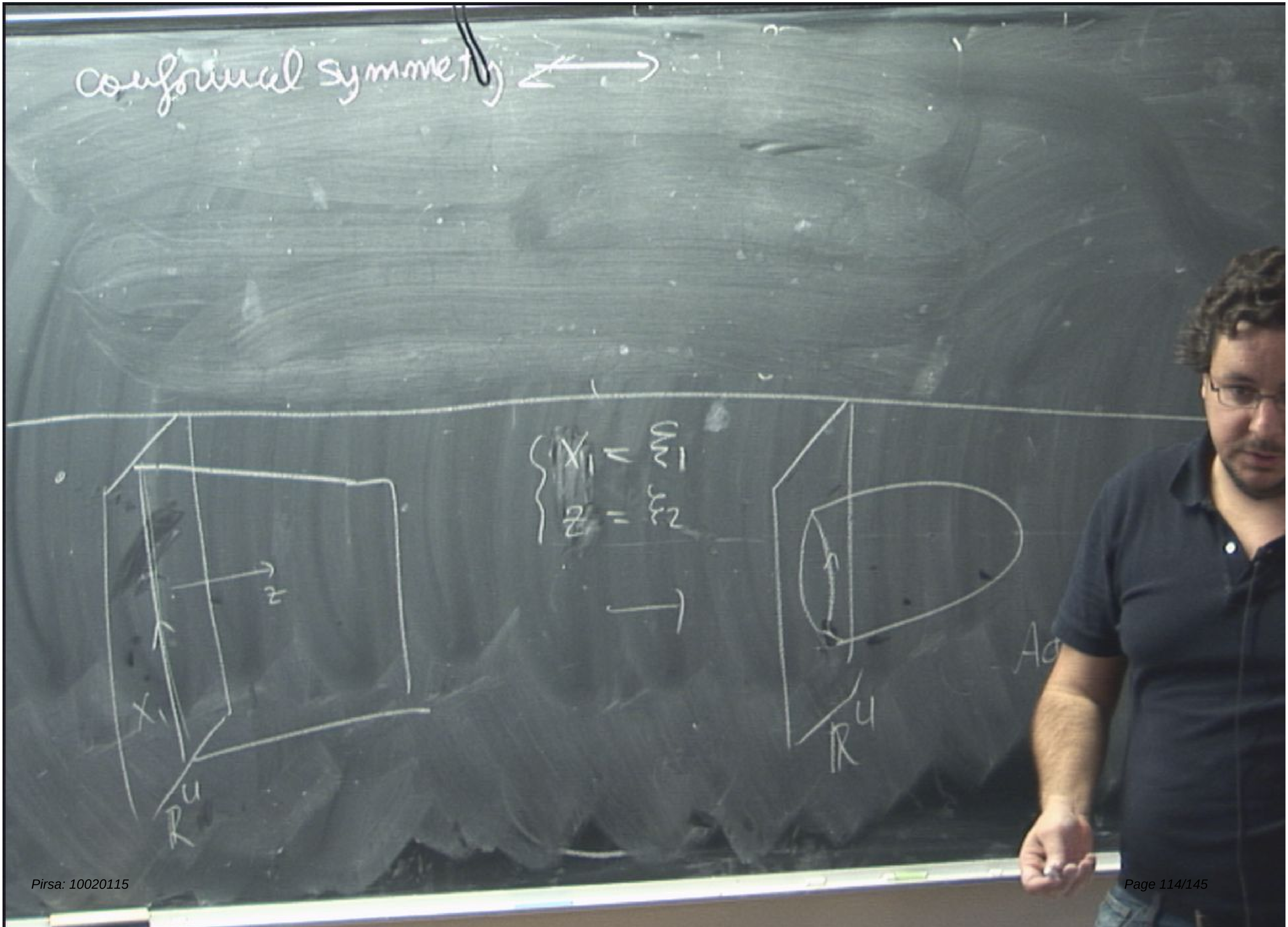
$$x^m \longrightarrow y^m$$

$$y_1^2 + (y_2 - a)^2 = a^2$$



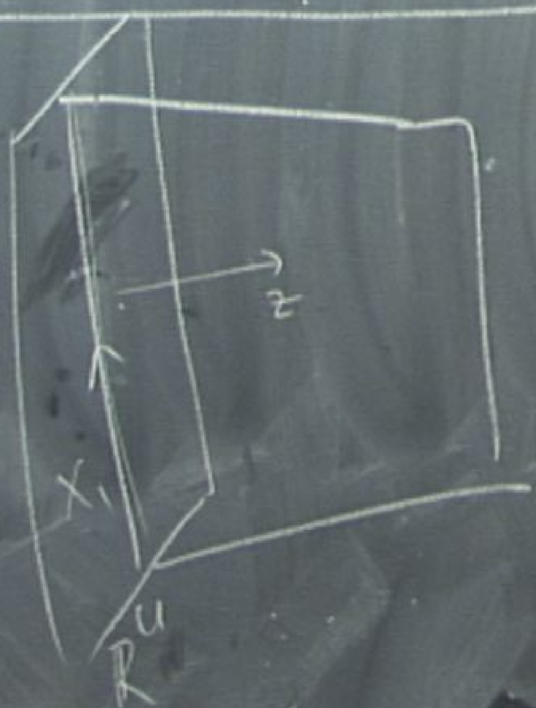
$x^m \longrightarrow y^m$

$y_1^2 + (y_2 - a)^2 = a^2$

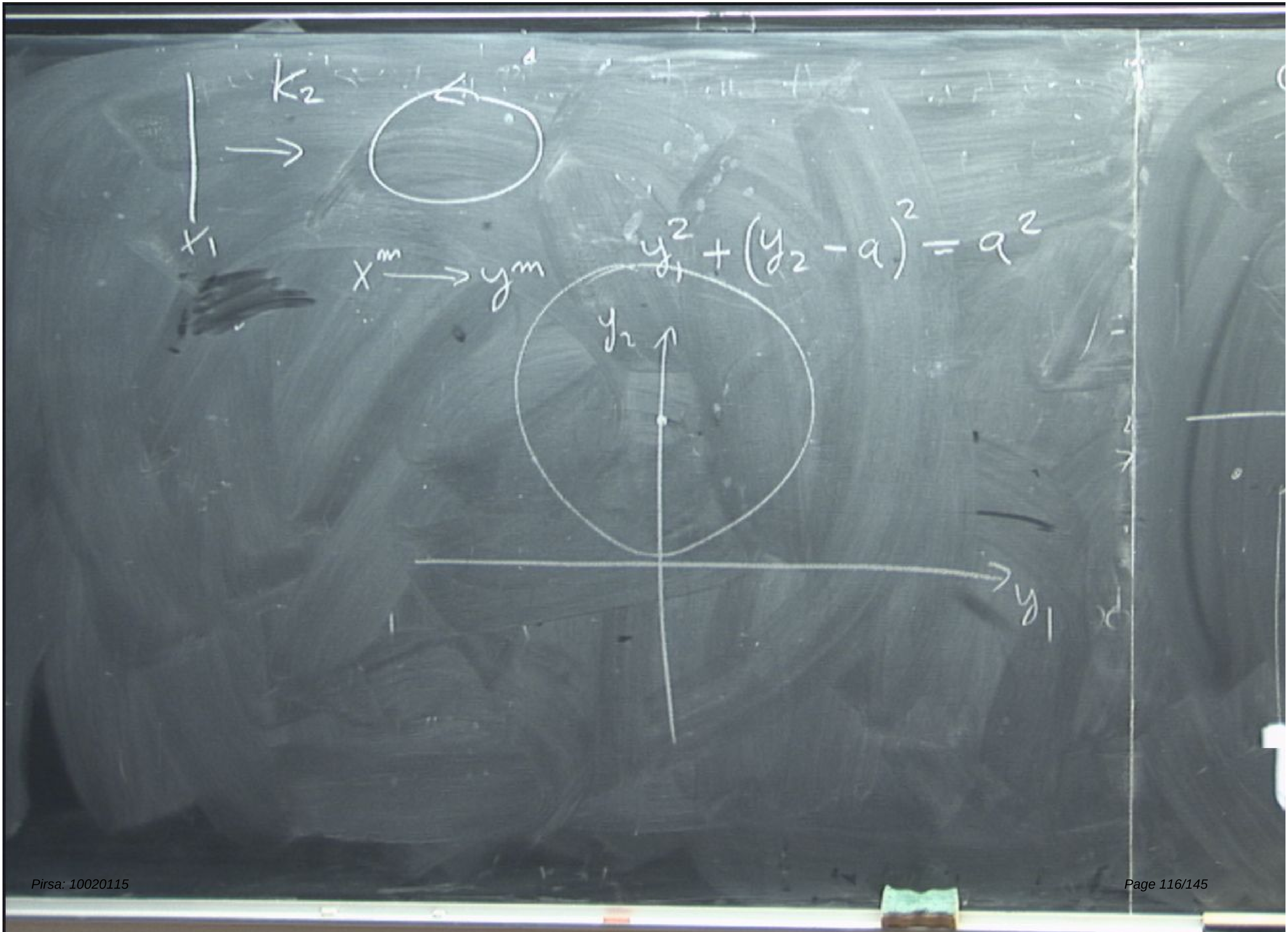

conformal symmetry
X1 = ξ1
z = ξ2
z
X1
R4
R4
Ad

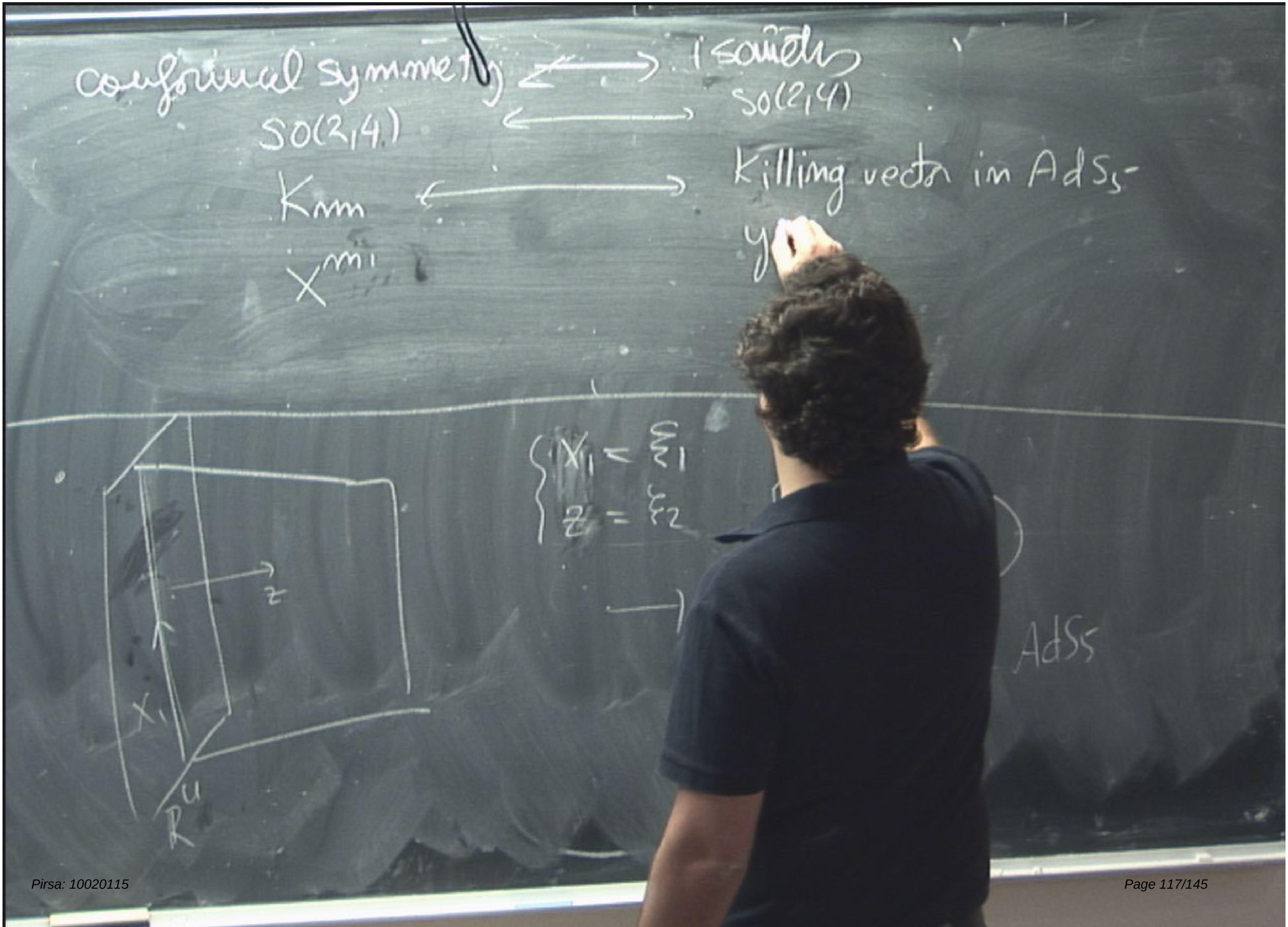
conformal symmetry $\longleftrightarrow$ isometry

SO(2,4) $\longleftrightarrow$ SO(2,4)



$$\begin{cases} X_1 = \xi_1 \\ z = \xi_2 \end{cases}$$


K_2
x_1
$x^m \longrightarrow y^m$
$y_1^2 + (y_2 - a)^2 = a^2$
y_2
y_1


conformal symmetry ⟷ isometry
SO(2,4) ⟷ SO(2,4)
Kmn ⟷ Killing vector in AdS5
Xm
y
X1 = ξ1
z = ξ2
z
X1
R4
AdS5

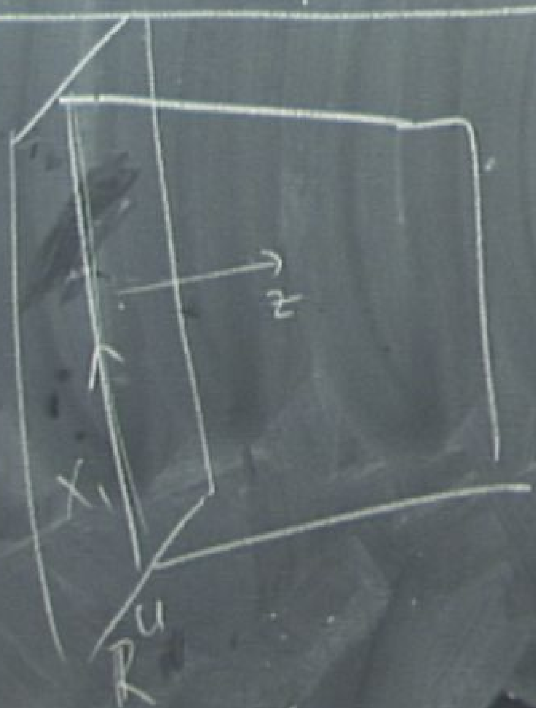
conformal symmetry $\longleftrightarrow$ isometry

$SO(2,4) \longleftrightarrow SO(2,4)$

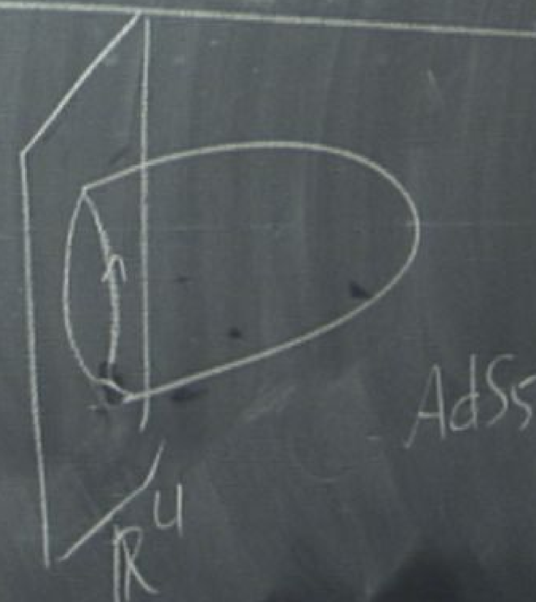
$K_{mn} \longleftrightarrow$ Killing vector in AdS_5

$x^{m'}$

$$y^m = \frac{x^m + b^m x^2}{1 + 2b\cdot x + b^2 x^2}$$



$$\begin{cases} x_1 = \xi_1 \\ z = \xi_2 \end{cases}$$



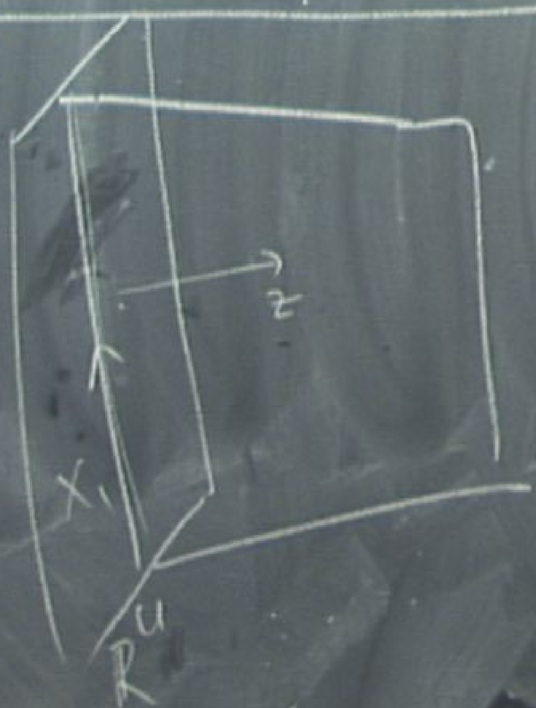
conformal symmetry $\longleftrightarrow$ isometry

$SO(2,4) \longleftrightarrow SO(2,4)$

$K_{mn} \longleftrightarrow$ Killing vector in AdS_5

$x^{m'}$

$$y^m = \frac{x^m + b^m(\vec{x}^2 + z^2)}{1 + 2b\cdot x + b^2 \vec{x}^2}$$



$$\begin{cases} x_1 = \xi_1 \\ z = \xi_2 \end{cases}$$

conformal symmetry $\longleftrightarrow$ isometry

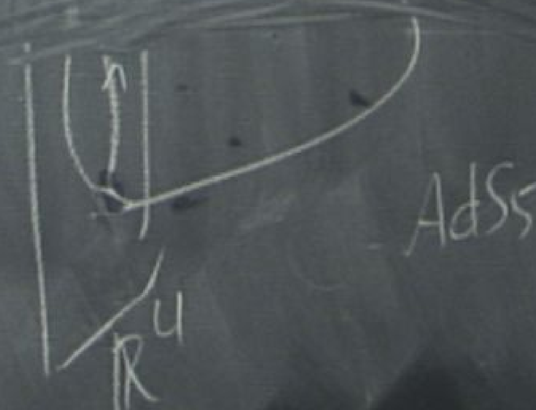
$SO(2,4) \longleftrightarrow SO(2,4)$

$K_m \longleftrightarrow$ Killing vector in AdS_5

$x^m \quad\quad y^m = \dfrac{x^m + b^m(\vec{x}^2+z^2)}{1+2b\cdot x + b^2(\vec{x}^2+z^2)}$

$$w = \frac{z}{1+2b\cdot x + b^2(\vec{x}^2+z^2)}$$

$$\begin{cases} x_1 = \xi_1 \\ z = \xi_2 \end{cases}$$



conformal symmetry $\longleftrightarrow$ isometry

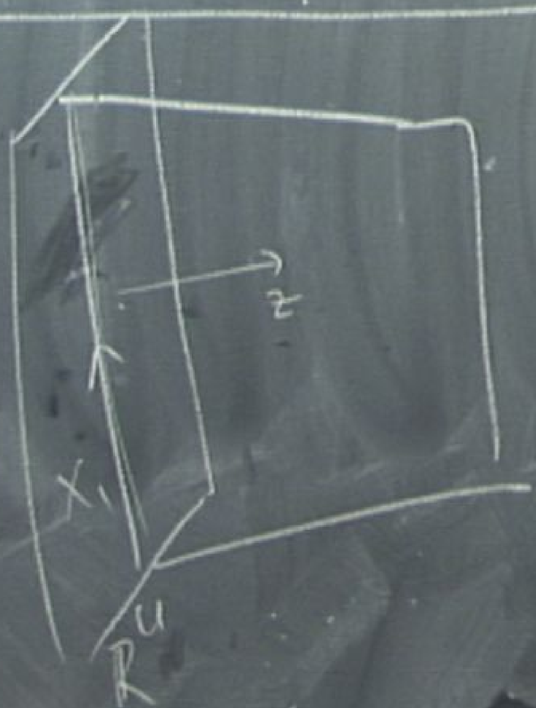
$SO(2,4) \longleftrightarrow SO(2,4)$

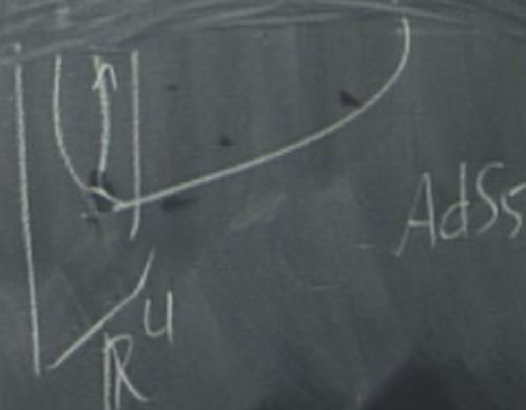
$K_{mn} \longleftrightarrow$ Killing vector in AdS_5

$X^{m'}$

$$\begin{cases} y^m = \dfrac{x^m + b^m(\vec{x}^2 + z^2)}{1 + 2b\cdot x + b^2(\vec{x}^2 + z^2)} \\[2ex] w = \dfrac{z}{1 + 2b\cdot x + b^2(\vec{x}^2 + z^2)} \end{cases}$$

$$\begin{cases} x_1 = \xi_1 \\ z = \xi_2 \end{cases}$$





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$$\begin{cases} y^m = \dfrac{x^m + b^m(\vec{x}^2+z^2)}{1+2b\cdot x + b^2(\vec{x}^2+z^2)} \\[2ex] w = \dfrac{z}{1+2b\cdot x + b^2(\vec{x}^2+z^2)} \end{cases}$$

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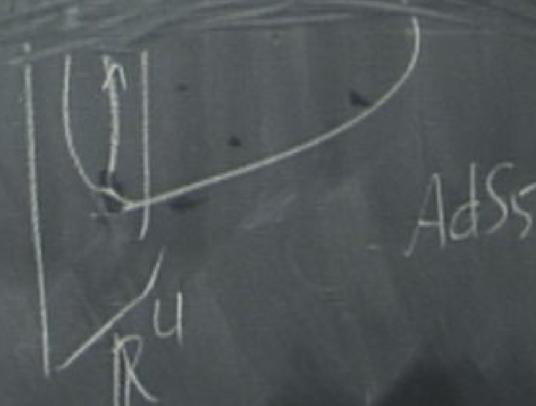
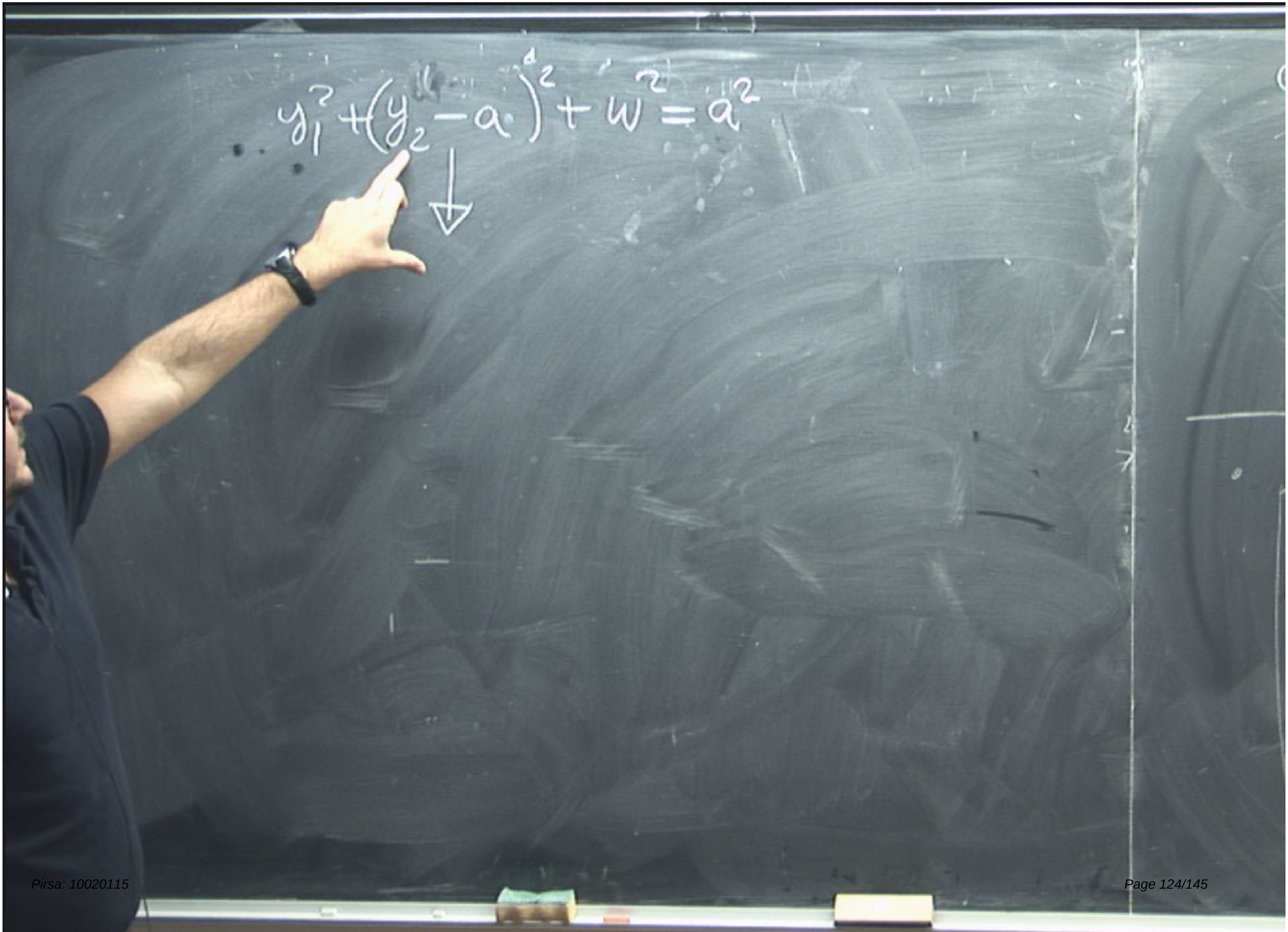


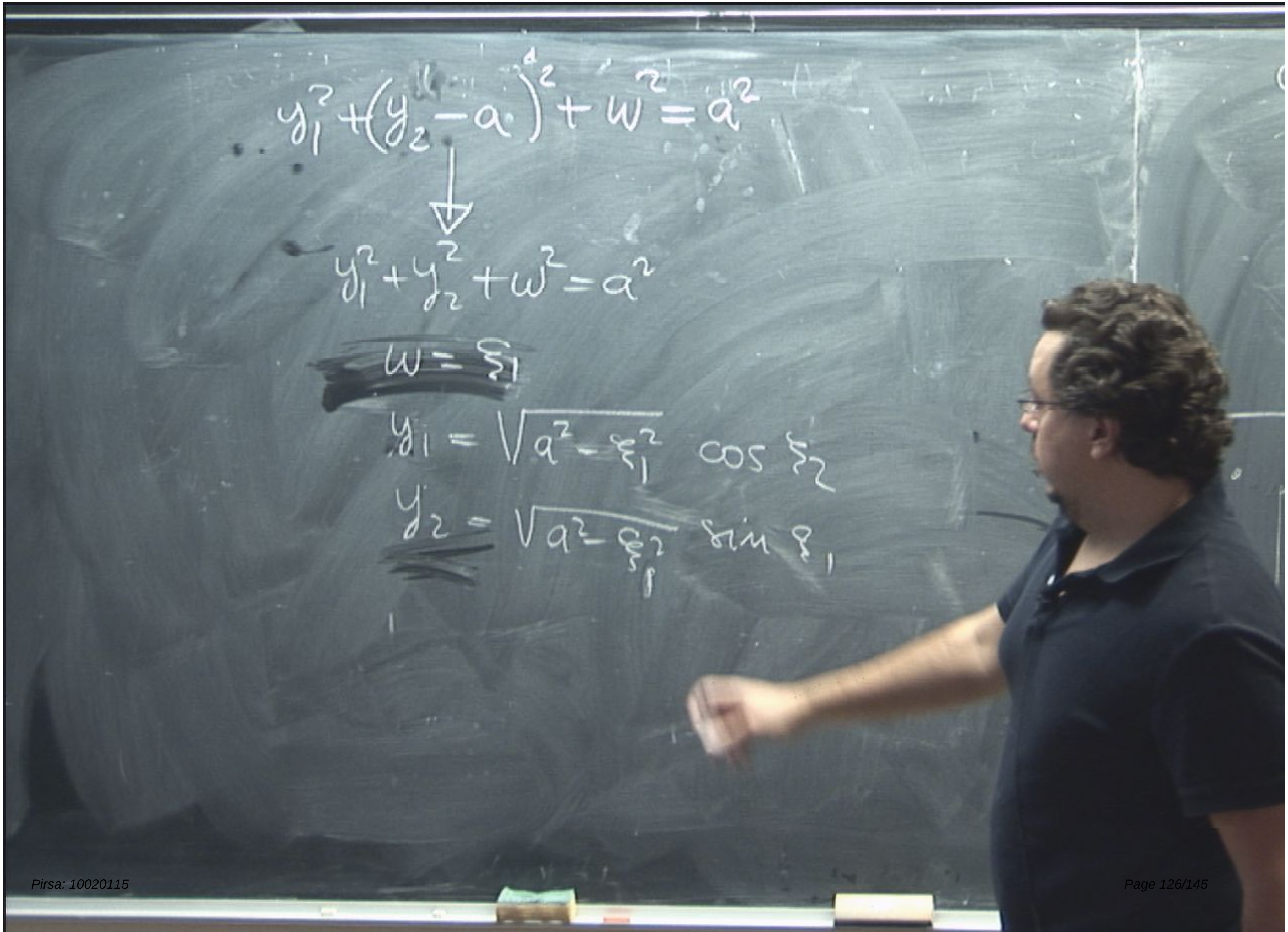
$$y_1^2 + (y_2 - a)^2 + w^2 = a^2$$


$y_1^2 + (y_2 - a)^2 + w^2 = a^2$

$$y_1^2 + (y_2 - a)^2 + w^2 = a^2$$

$$\Downarrow$$

$$y_1^2 + y_2^2 + w^2 = a^2$$


$y_1^2 + (y_2 - a)^2 + w^2 = a^2$
$y_1^2 + y_2^2 + w^2 = a^2$
$w = \xi_1$
$y_1 = \sqrt{a^2 - \xi_1^2}\ \cos \xi_2$
$y_2 = \sqrt{a^2 - \xi_1^2}\ \sin \xi_1$

$$y_1^2 + (y_2 - a)^2 + w^2 = a^2$$

$$\Downarrow$$

$$y_1^2 + y_2^2 + w^2 = a^2$$

$$w = \xi_1$$

$$y_1 = \sqrt{a^2 - \xi_1^2}\,\cos \xi_2$$

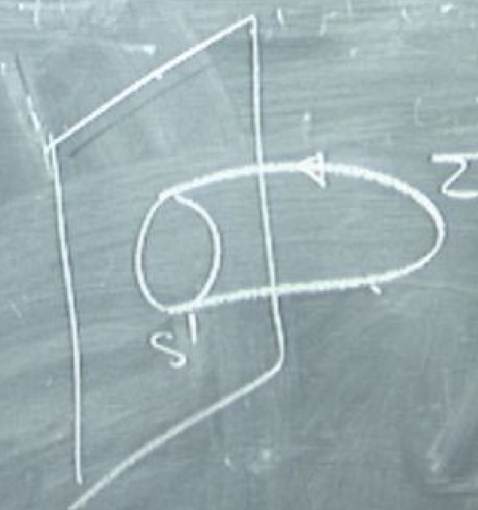
$$y_2 = \sqrt{a^2 - \xi_1^2}\,\sin \xi_1$$

$$y_1^2 + (y_2 - a)^2 + w^2 = a^2$$

$$\Downarrow$$

$$y_1^2 + y_2^2 + w^2 = a^2$$

$$\begin{cases} w = \xi_1 \\ y_1 = \sqrt{a^2 - \xi_1^2}\, \cos \xi_2 \\ y_2 = \sqrt{a^2 - \xi_1^2}\, \sin \xi_1 \end{cases}$$

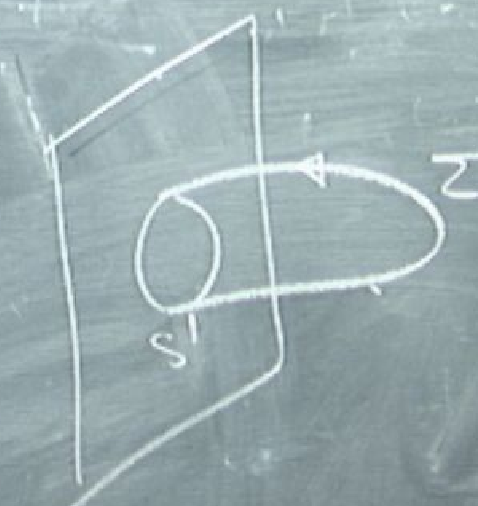


$$ds^2_{ind} =$$

$$y_1^2 + (y_2 - a)^2 + w^2 = a^2$$

$$\Downarrow$$

$$y_1^2 + y_2^2 + w^2 = a^2$$



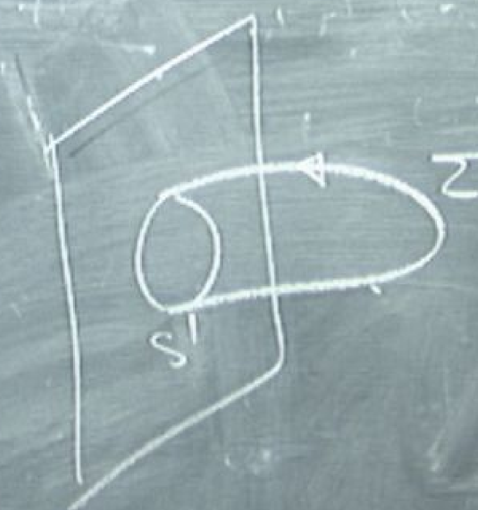
$$\begin{cases} w = \xi_1 \\ y_1 = \sqrt{a^2 - \xi_1^2}\,\cos\xi_2 \\ y_2 = \sqrt{a^2 - \xi_1^2}\,\sin\xi_2 \end{cases}$$

$$ds^2_{ind} = \frac{L^2}{\xi_1^2}\left((a^2 - \xi_1^2)\, d\xi_2^2 + \frac{a^2}{a^2 - \xi_1^2}\, d\xi_1^2 \right)$$

$$y_1^2 + (y_2 - a)^2 + w^2 = a^2$$

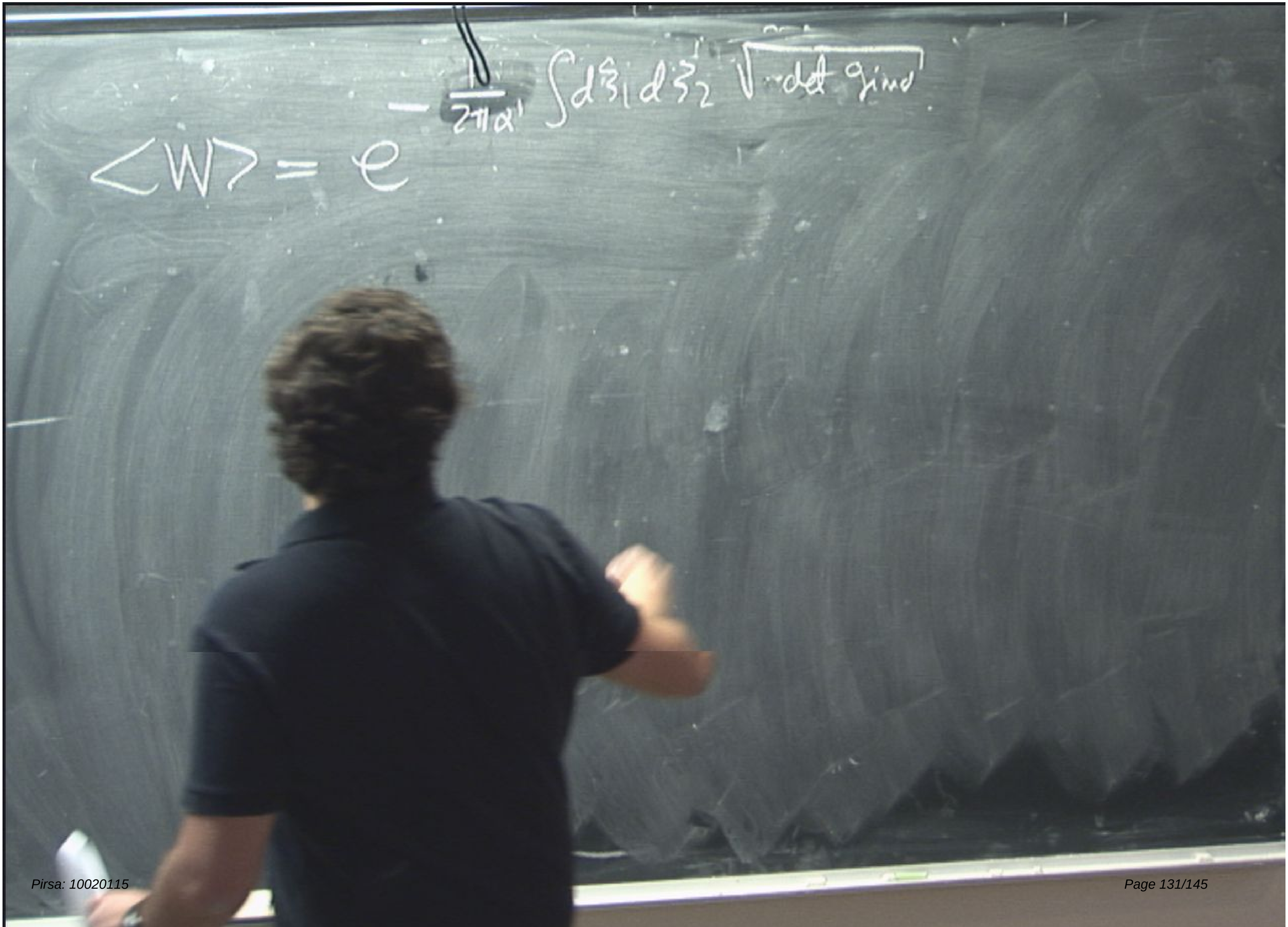
$$\Downarrow$$

$$y_1^2 + y_2^2 + w^2 = a^2$$

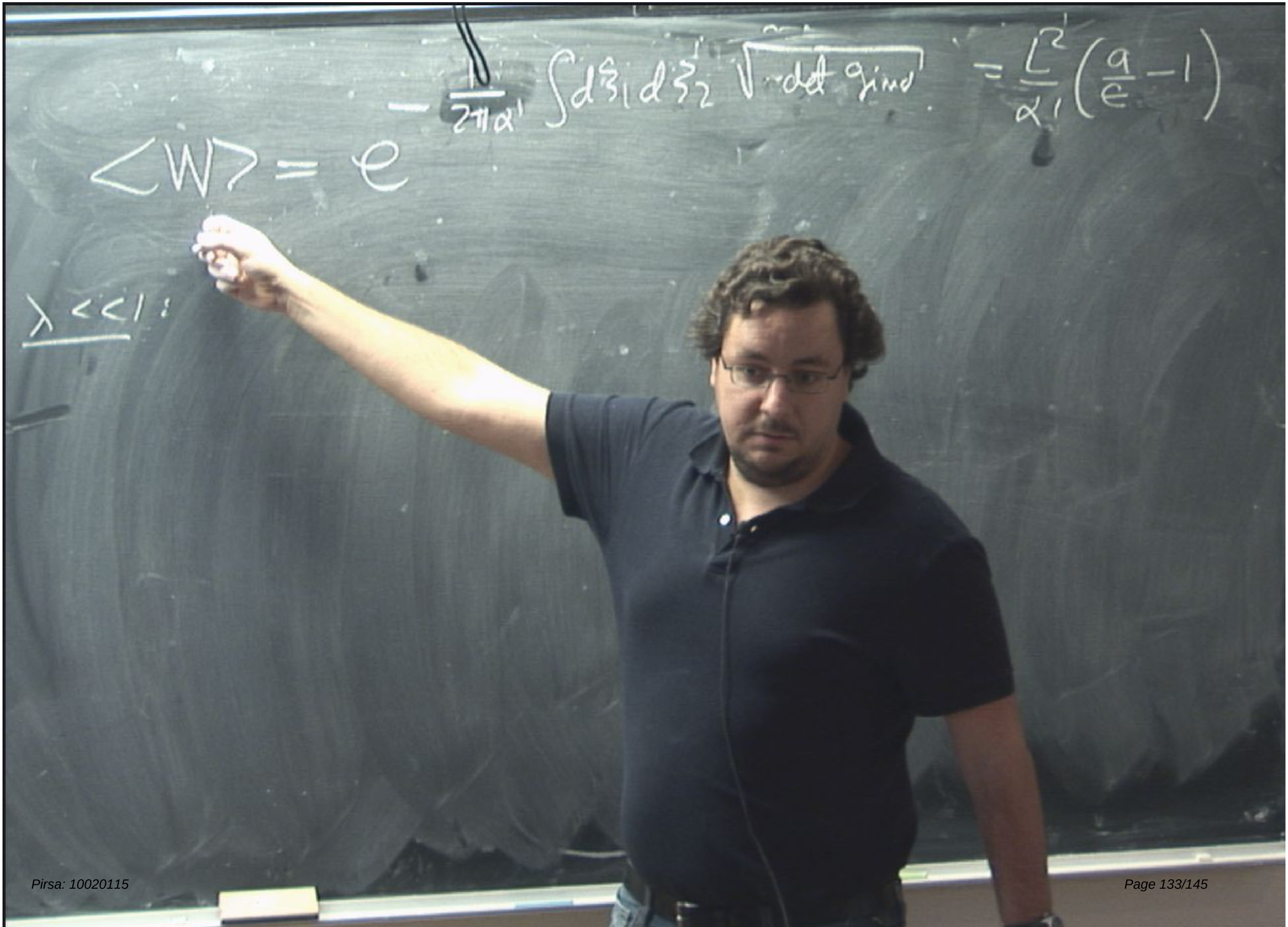


$$\begin{cases} w = \xi_1 \\ y_1 = \sqrt{a^2 - \xi_1^2}\,\cos \xi_2 \\ y_2 = \sqrt{a^2 - \xi_1^2}\,\sin \xi_2 \end{cases}$$

$$ds^2_{wd} = \frac{L^2}{\xi_1^2}\left((a^2 - \xi_1^2)\, d\xi_2^2 + \frac{a^2}{a^2 - \xi_1^2}\, d\xi_1^2 \right)$$


$\langle W \rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1 d\xi_2 \sqrt{-\det g_{ind}}}$

$$\langle W \rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1 d\xi_2 \sqrt{-\det g_{ind}}} = \frac{L^2}{\alpha'}\left(\frac{a}{\epsilon} - 1\right)$$



<W> = e^(− 1/(2πα') ∫dξ1 dξ2 √(−det g_ind)) = L²/α' (a/e − 1)
λ << 1:

$$\langle W \rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1 d\xi_2 \sqrt{-\det g_{ind}}} = \frac{L^2}{\alpha'}\left(\frac{a}{\epsilon}-1\right)$$

$\lambda \ll 1$: $1 + c\lambda + \cdots$

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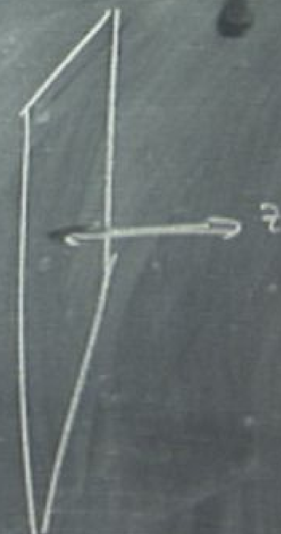
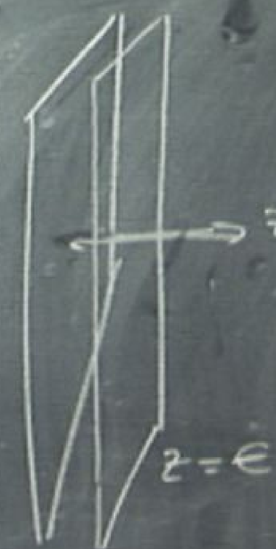


$$\langle W \rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1 d\xi_2 \sqrt{-\det g_{ind}}} \qquad = \frac{L^2}{\alpha'}\left(\frac{a}{\epsilon}-1\right)$$



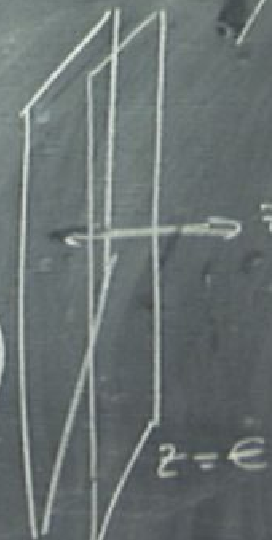
$\underline{\lambda \ll 1}$: $1 + c\lambda + \ldots$

$\underline{\lambda \gg 1}$ $\qquad \dfrac{L^4}{l_s^4} = \lambda$

$$\langle W \rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1\, d\xi_2 \sqrt{-\det g_{ind}}} = \frac{L^2}{\alpha'}\left(\frac{a}{\epsilon}-1\right)$$

$\underline{\lambda \ll 1:}$ $\quad 1 + c\lambda + \ldots\ldots = \langle W \rangle$

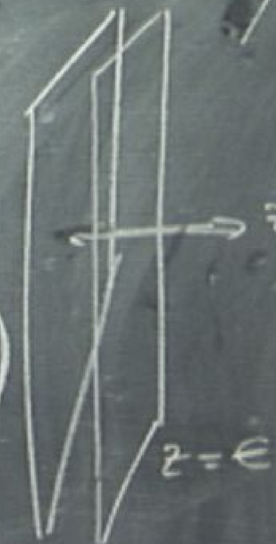
$\underline{\lambda \gg 1}$ $\quad \frac{L^4}{l_s^4} = \lambda \qquad \langle W \rangle = \exp\left(\sqrt{\lambda}\right)$



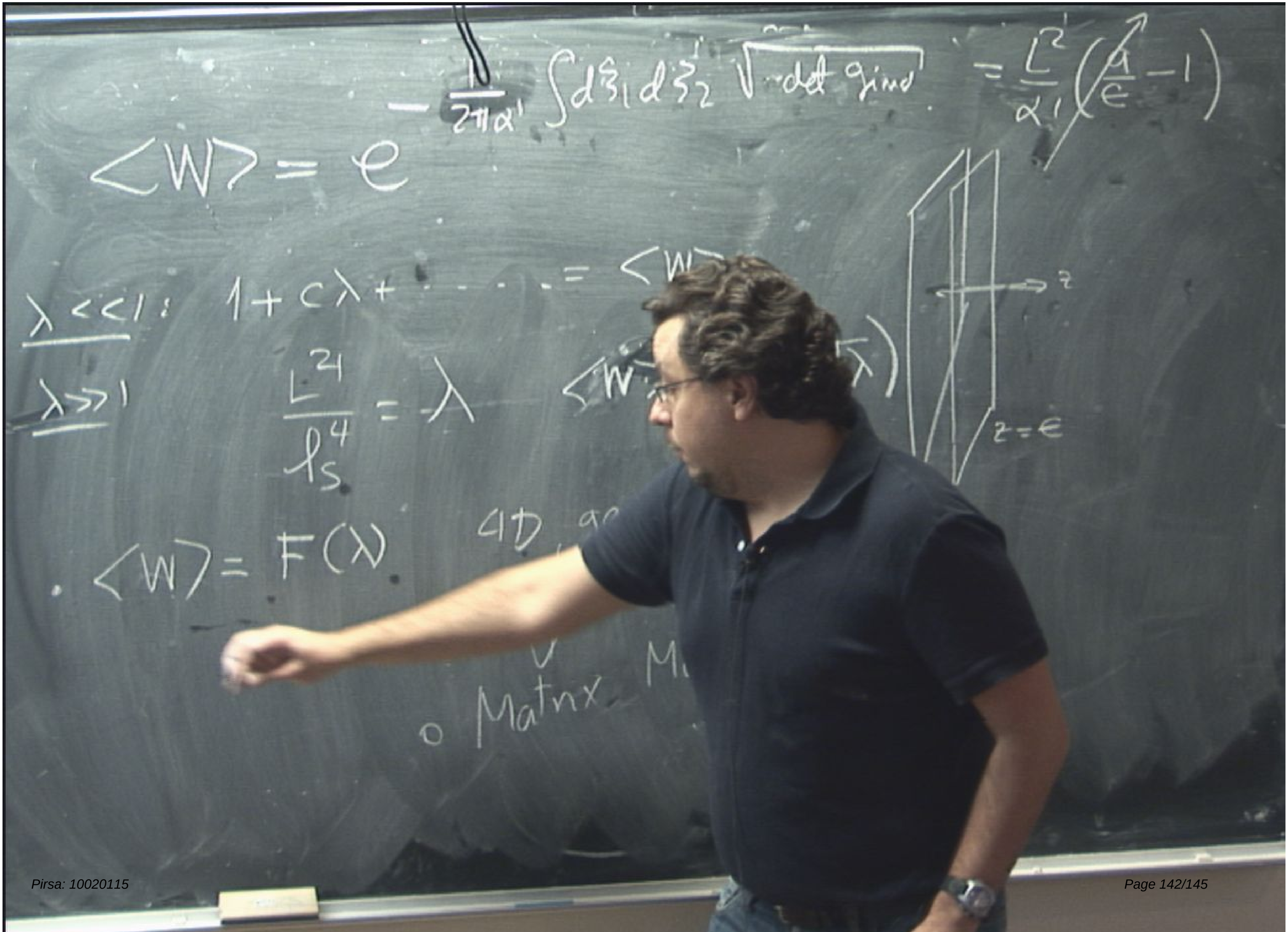
$$\langle W \rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1 d\xi_2 \sqrt{\det g_{ind}}} = \frac{L^2}{\alpha'}\left(\cancel{\frac{A}{\epsilon}} - 1\right)$$

$\underline{\lambda \ll 1}$: $\quad 1 + c\lambda + \ldots = \langle W \rangle$

$\underline{\lambda \gg 1}$ $\quad \frac{L^4}{l_s^4} = \lambda \qquad \langle W \rangle = \exp(\sqrt{\lambda})$



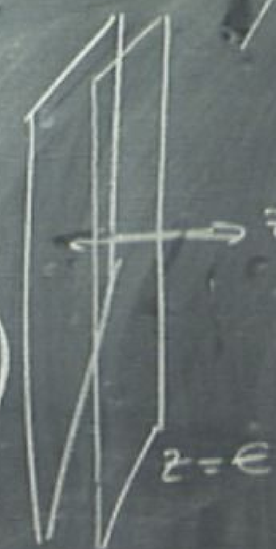
$$\langle W \rangle = F(\lambda)$$


$\langle W \rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1 d\xi_2 \sqrt{\det g_{ind}}}$
$= \frac{L^2}{\alpha'}\left(\frac{\Lambda}{\epsilon} - 1\right)$
$\lambda \ll 1: \quad 1 + c\lambda + \ldots = \langle W \rangle$
$\lambda \gg 1$
$\frac{L^4}{l_s^4} = \lambda$
$z = \epsilon$
$\langle W \rangle = F(\lambda)$
4D
Matrix

$$\langle W\rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1 d\xi_2 \sqrt{\det g_{ind}}} \qquad = \frac{L^2}{\alpha'}\left(\frac{a}{\epsilon}-1\right)$$

$\underline{\lambda \ll 1}$: $\quad 1 + c\lambda + \dots = \langle W\rangle$

$\underline{\lambda \gg 1}$ $\quad \frac{L^4}{l_s^4} = \lambda \qquad \langle W\rangle = \exp(\sqrt{\lambda})$



• $\langle W\rangle = F(\lambda)$ $\quad$ 4D gauge theory

$\downarrow$

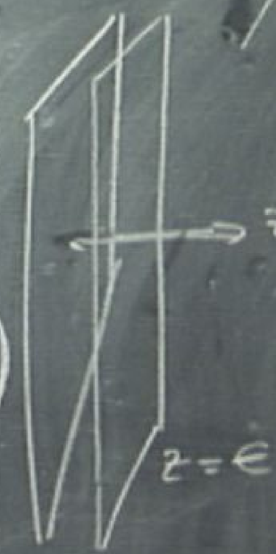
∘ Matrix Model

$F(\lambda \to \infty) = \exp(\sqrt{\lambda})$

$$\langle W\rangle = e^{-\frac{1}{2\pi\alpha'}\int d\xi_1\, d\xi_2 \sqrt{-\det g_{ind}}} \qquad = \frac{L^2}{\alpha'}\left(\frac{a}{\epsilon}-1\right)$$

$\underline{\lambda \ll 1}$: $1 + c\lambda + \ldots = \langle W\rangle$

$\underline{\lambda \gg 1}$ $\frac{L^4}{l_s^4} = \lambda$ $\langle W\rangle = \exp(\sqrt{\lambda})$



• $\langle W\rangle = F(\lambda)$

4D gauge theory

$\downarrow$

∘ Matrix Model

$F(\lambda \to \infty) = \exp(\sqrt{\lambda})$

$$y_1^2 + y_2^2 + w^2 = a^2$$

$$\begin{cases} w = \xi_1 \\ y_1 = \sqrt{a^2 - \xi_1^2} \\ y_2 = \sqrt{a^2 - \xi_1^2} \end{cases}$$

$$ds^2_{ind} = \frac{L^2}{\xi_1^2}\left((a^2 - \xi_1^2 \right.$$