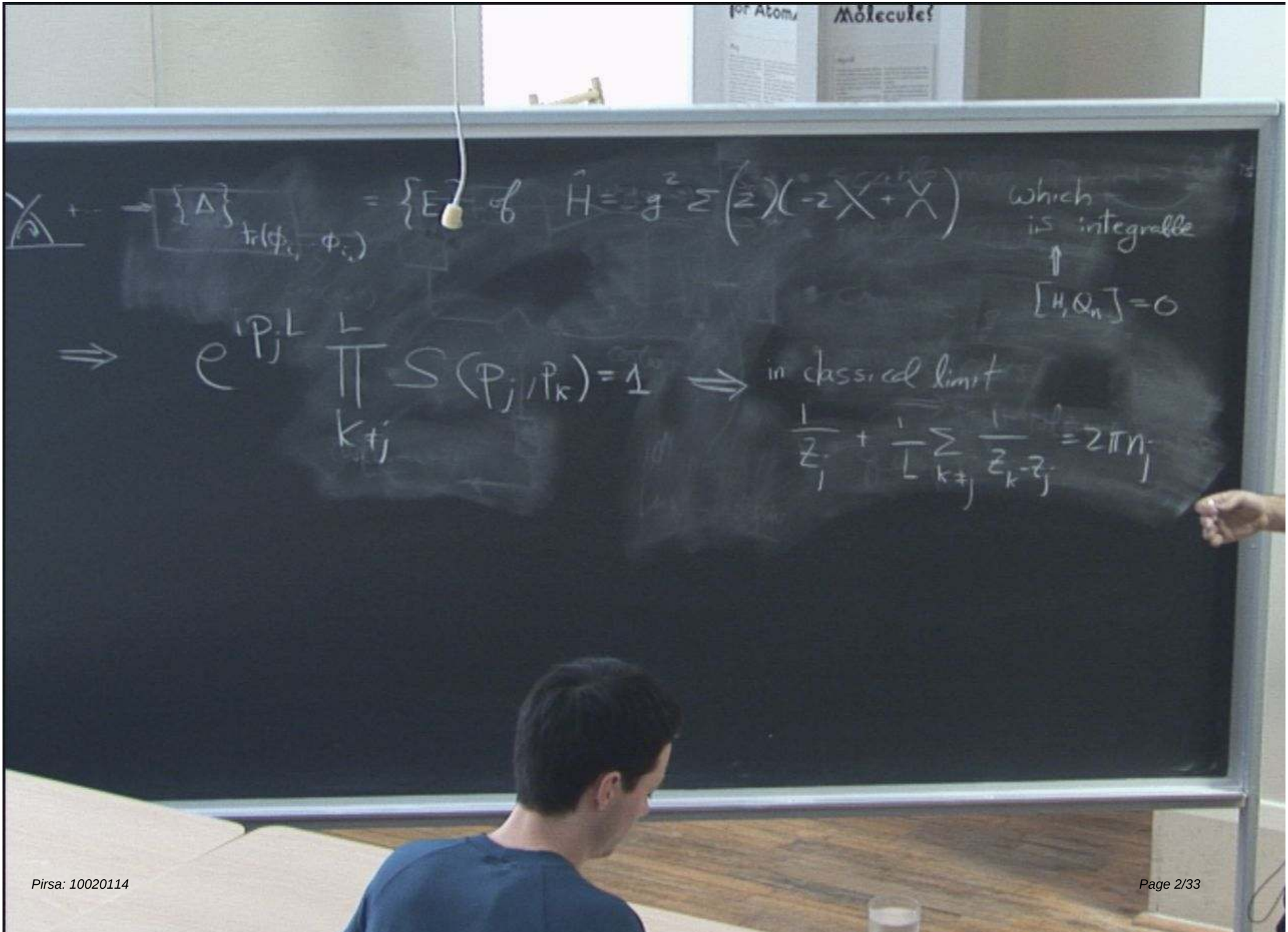
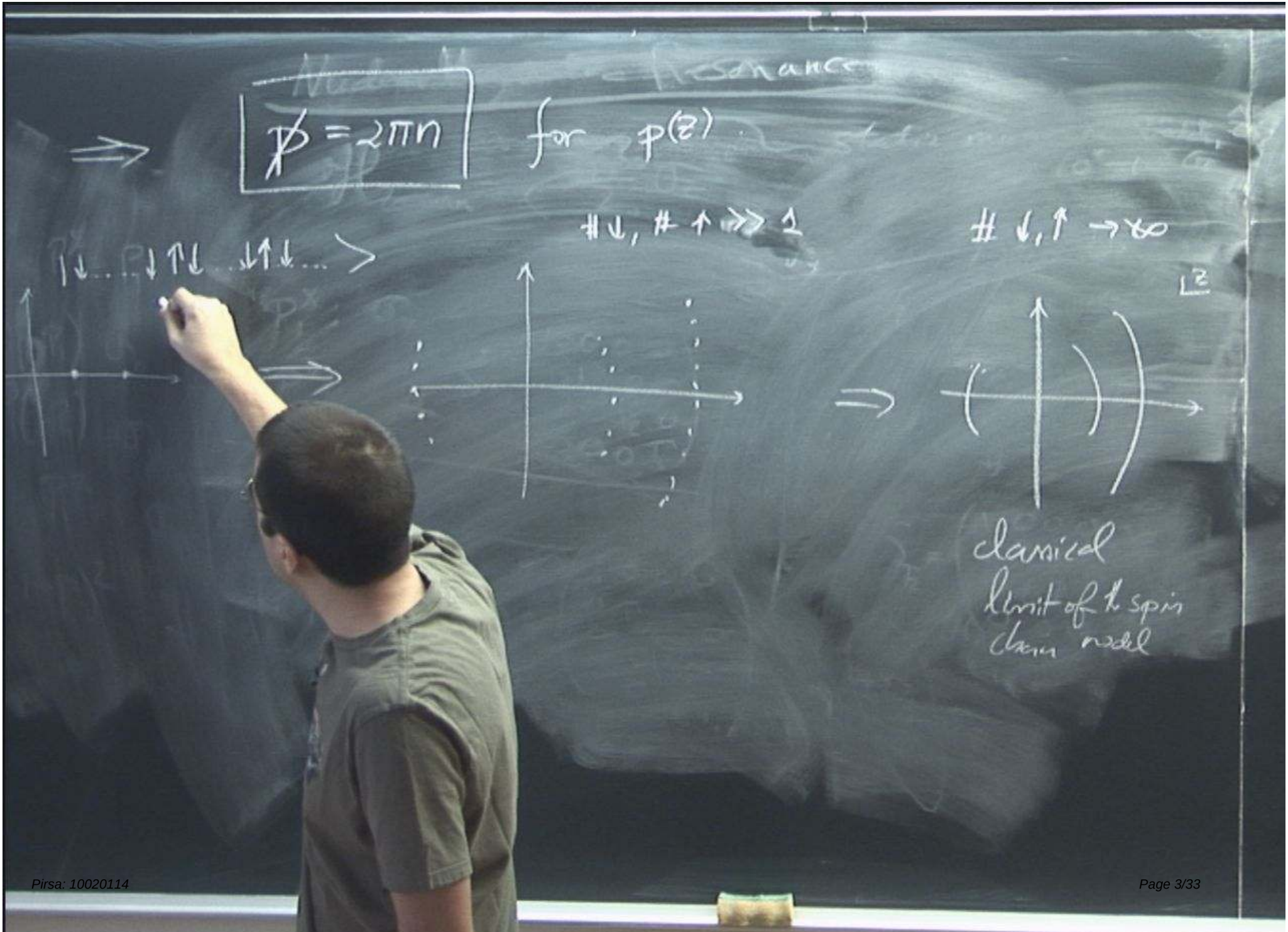
Title: Explorations in String Theory (PHYS 647) - Lecture 10

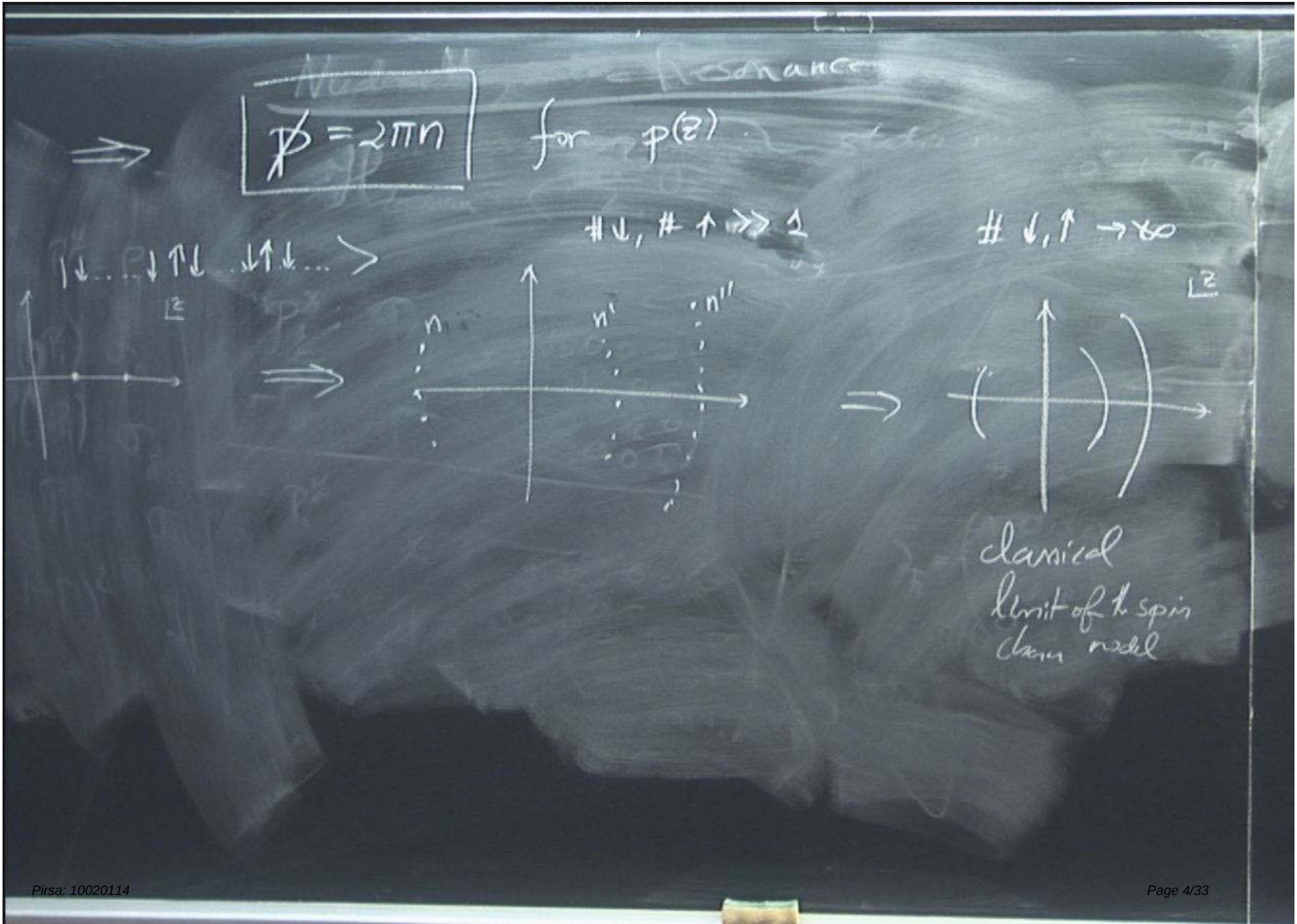
Date: Feb 26, 2010 11:20 AM

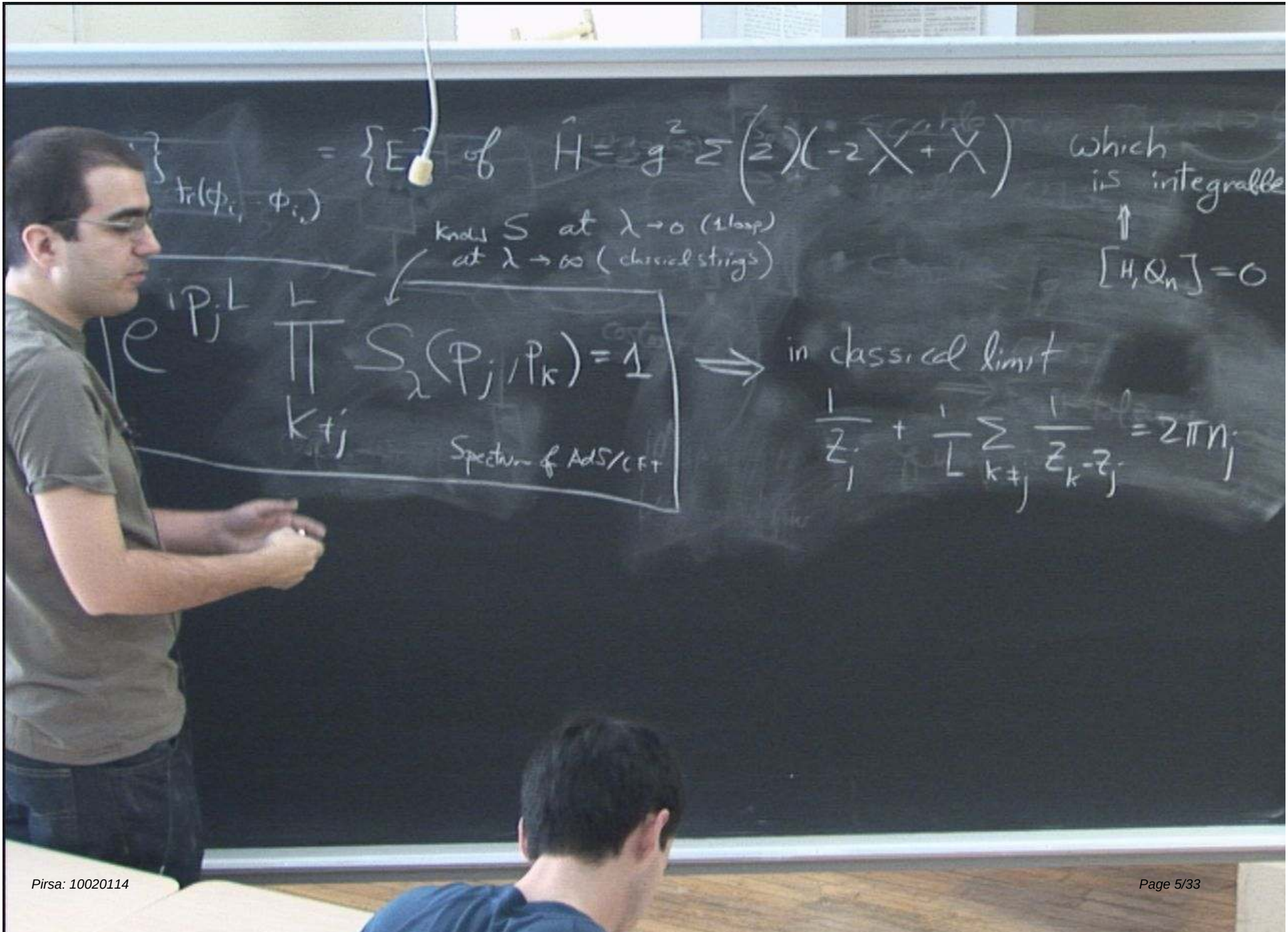
URL: http://pirsa.org/10020114

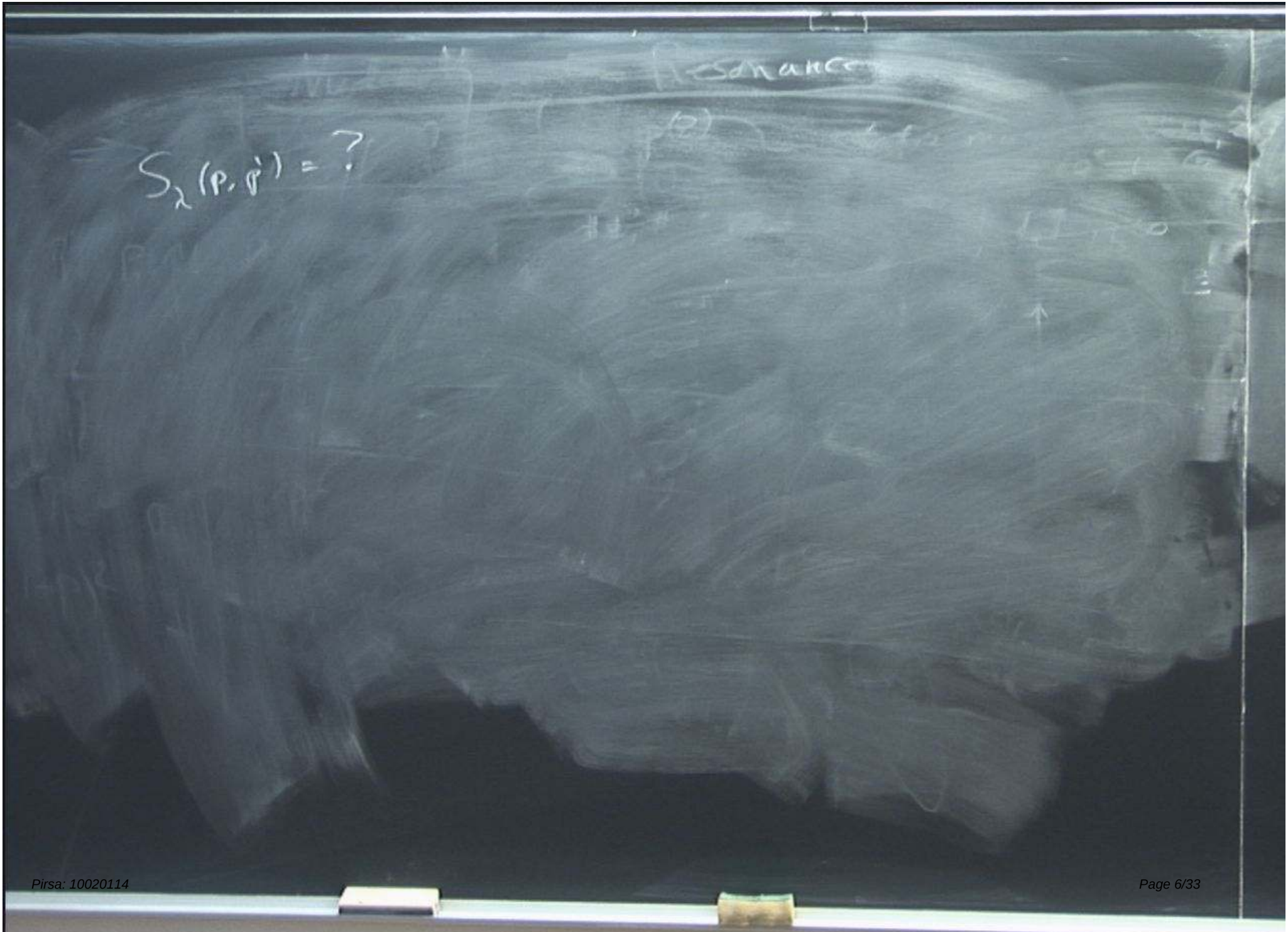
Abstract:

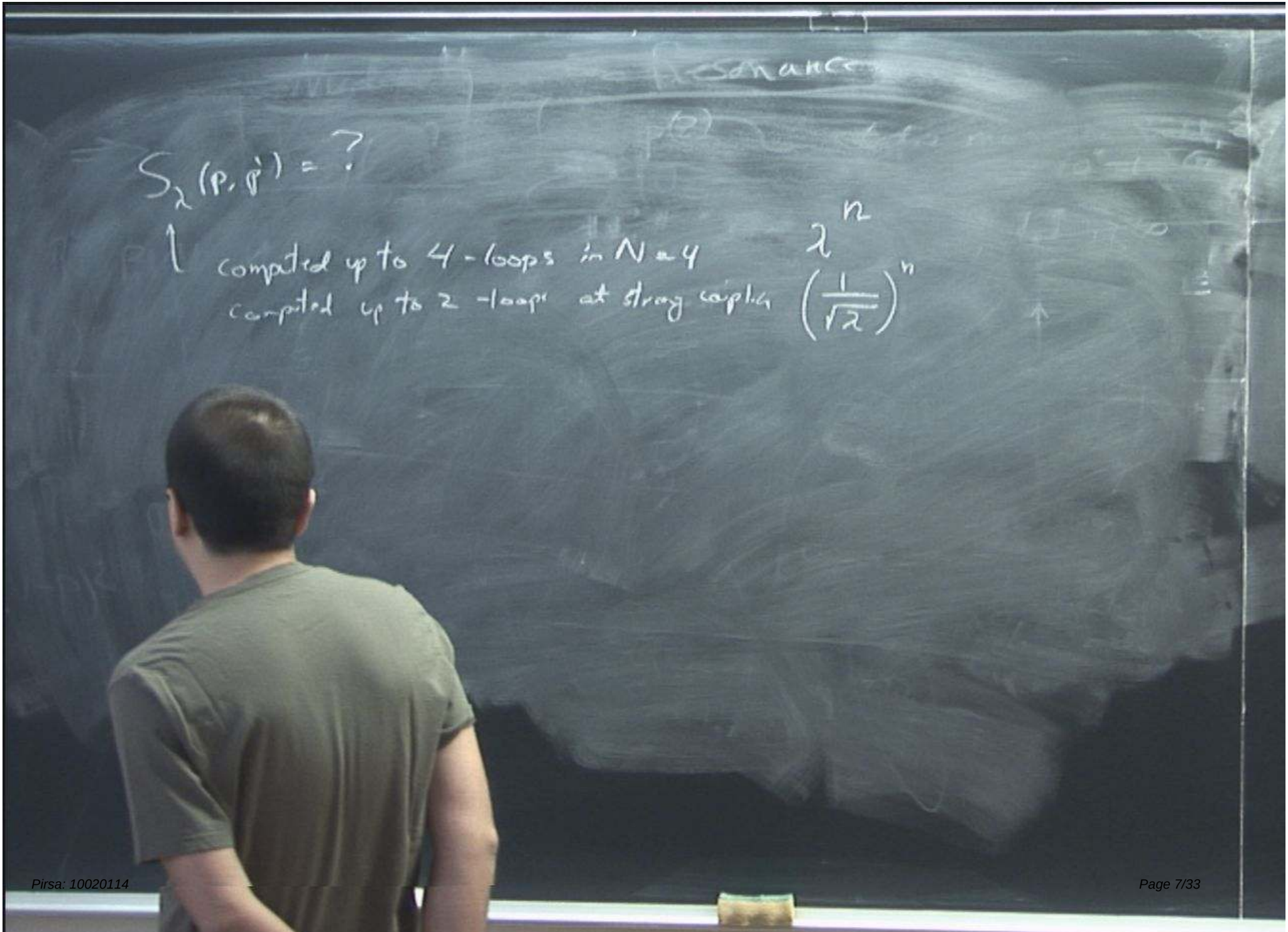

for Atom
Molecule?
$\{\Delta\}_{tr(\Phi_i \Phi_j)} = \{E\}$ of $\hat{H} = g^2 \Sigma(z)(-2X + X)$
which is integrable
$[H, Q_n] = 0$
$\Rightarrow e^{iP_j L} \prod_{k \neq j}^{L} S(P_j, P_k) = 1 \Rightarrow$ in classical limit
$\frac{1}{z_j} + \frac{1}{L} \sum_{k \neq j} \frac{1}{z_k - z_j} = 2\pi n_j$


$\oint p = 2\pi n$ for $p(z)$
$\#\downarrow, \#\uparrow \gg 1$
$\#\downarrow, \uparrow \to \infty$
$|\downarrow \ldots \downarrow\uparrow\downarrow \ldots \downarrow\uparrow\downarrow \ldots\rangle$
$L^2$
classical
limit of the spin
chain model


$\Rightarrow$ $\boxed{\not{p} = 2\pi n}$ for $p(z)$
$\#\downarrow, \#\uparrow \gg 1$
$\#\downarrow, \uparrow \to \infty$
$|\downarrow \ldots \downarrow\uparrow\downarrow \ldots \downarrow\uparrow\downarrow \ldots\rangle$
$L^z$
$n$
$n'$
$n''$
classical
limit of the spin
chain model


$= \{E\}$ of $\hat{H} = g^2 \sum (2)(-2X^{+}\bar{X})$
which is integrable
$[H, Q_n] = 0$
$\mathrm{tr}(\Phi_{i_1} \cdots \Phi_{i_L})$
known S at $\lambda \to 0$ (1 loop)
at $\lambda \to \infty$ (classical string)
$e^{ip_j L} \prod_{k \neq j}^{L} S_\lambda(p_j, p_k) = 1$
Spectrum of AdS/CFT
$\Rightarrow$ in classical limit
$\frac{1}{z_j} + \frac{1}{L} \sum_{k \neq j} \frac{1}{z_k - z_j} = 2\pi n_j$


$S_2(p, p') = ?$


$S_\lambda(p,p') = ?$
computed up to 4-loops in $N=4$
computed up to 2-loops at strong coupling
$\lambda^n$
$\left(\frac{1}{\sqrt{\lambda}}\right)^n$

$S_\lambda(p, p') = ?$

↑ computed up to 4-loops in $N=4$

computed up to 2-loops at strong coupling $\lambda^n \left(\frac{1}{\sqrt{2}}\right)^n$

We can impose crossing sym.

$S_\lambda(p, p') = ?$

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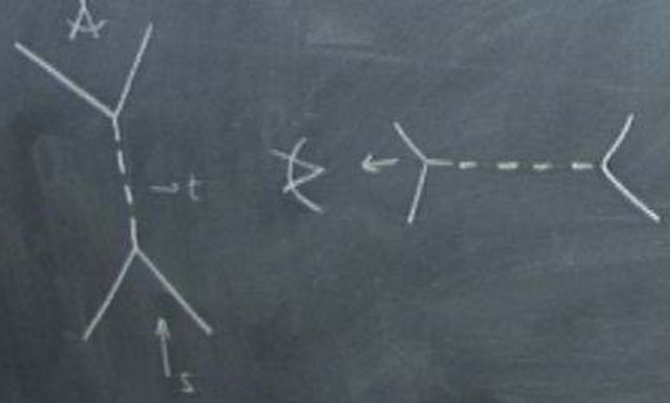
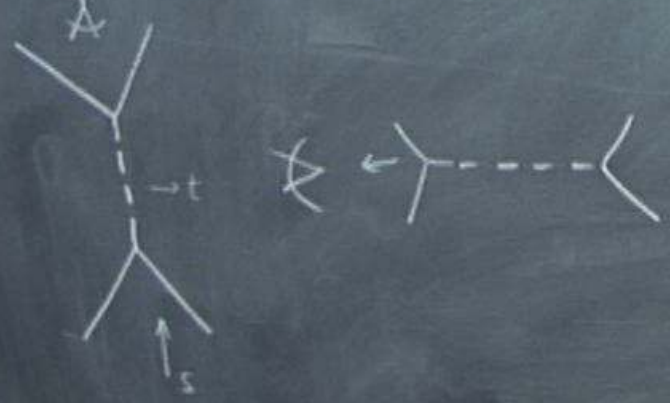



s, t, u mandelstam var

$S_2(p, p') = ?$

↑ computed up to 4-loops in $N=4$ $\lambda^n$
computed up to 2-loops at strong coupling $\left(\frac{1}{\sqrt{2}}\right)^n$

We can impose crossing sym.




$S(s,t,u)^{cd}_{ab}$

$= S(u,t,s)^{cb}_{ad}$

Janik wrote ↑ down and BES, BHL solved it

s,t,u mandelstam var

the problem is not
solved yet
we need L larger

$L \gg 1$

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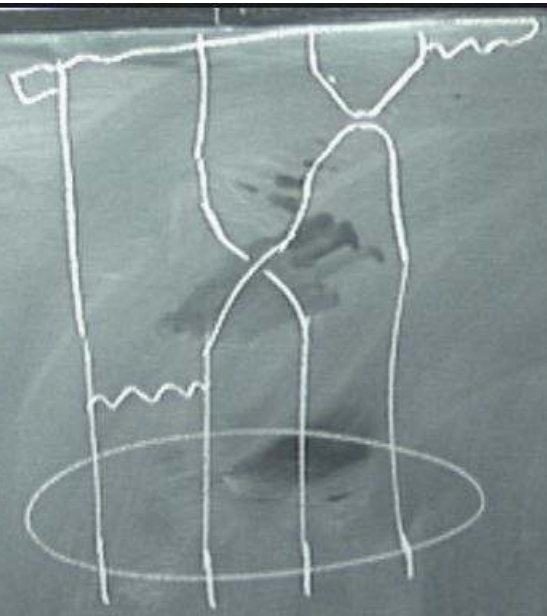
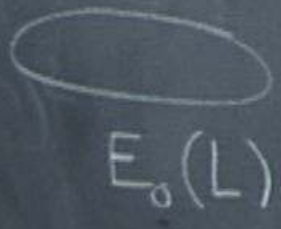


the problem is not solved yet we need L large

$L \gg 1$

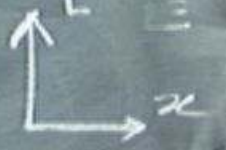
B-eth Ansatz fails at $O(\lambda^L)$

Thermodynamic Bethe Ansatz

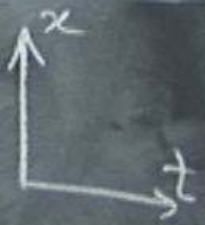


# Thermodynamic Bethe Ansatz

$$Z = \int_{\substack{R\to\infty \\ L \text{ fixed}}} \mathcal{D}\phi \; e^{-S[\phi]} = e^{-E_0(L)\,R}$$




$$Z = \sum_n e^{-RE_n}$$



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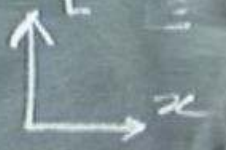
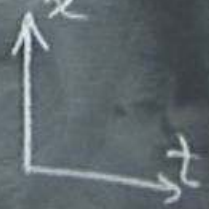
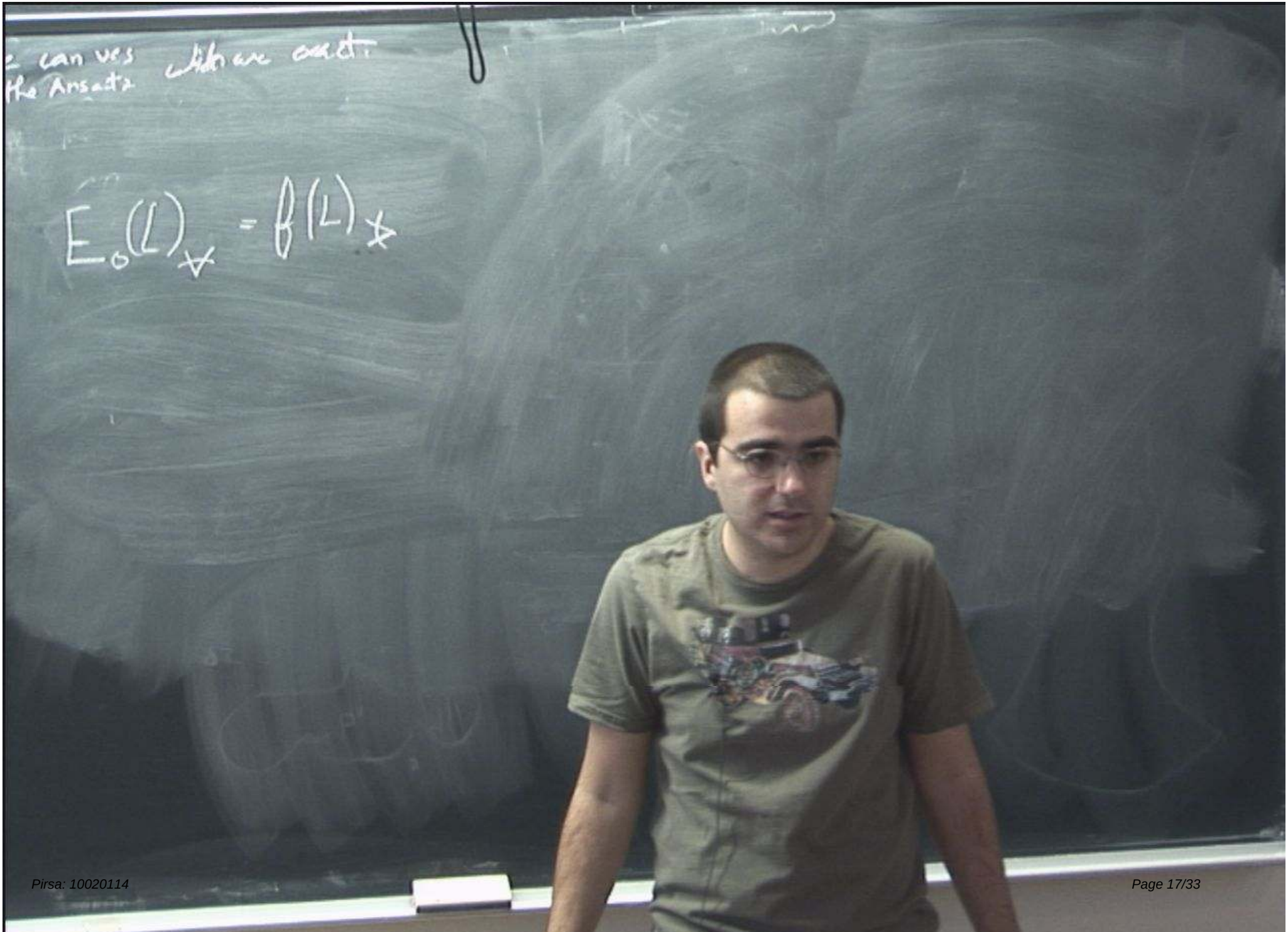



$$Z = \sum_n e^{-RE_n} = e^{-F(L,R)} = e^{-R f(L)}$$




free energy of my model for space R and temperature $1/L$


can ves
the Ansatz
$E_0(l)_{\forall} = f(l)_{\star}$

e can use this one as the Ansatz

if I don't have Lorentz:

$$E_0(L)_{\forall} = f(L)_{\star}$$

$$\frac{L}{i}\log\frac{u_j+\frac{i}{2}}{u_j-\frac{i}{2}} + \sum_{k\neq j}\frac{1}{i}\log\frac{u_j-u_k+i}{u_j-u_k-i} = 2\pi n_j$$

can use this one
the Ansatz

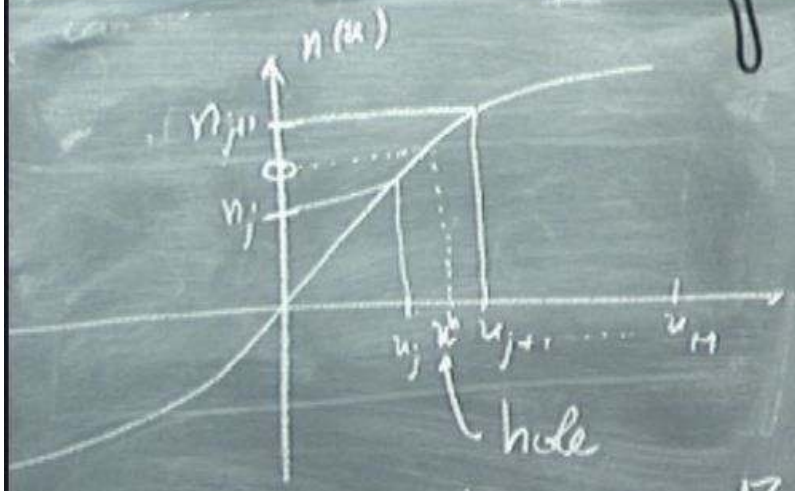
If I don't have Lorentz

$$\{n_k\} \longrightarrow \{u_k\} \longrightarrow n(u)$$

$$E_0(L)_{\forall} = f(L)_{\star}$$

$$\frac{L}{i}\log\frac{u_j+i/2}{u_j-i/2} - \sum_{k\neq j}^{M}\frac{1}{i}\log\frac{u_j-u_k+i}{u_j-u_k-i} = 2\pi n_j$$

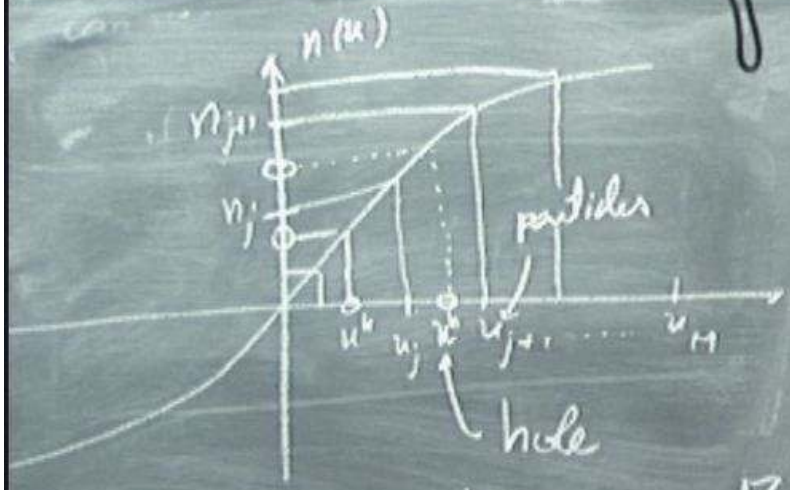
$$\frac{L}{i}\log\frac{u+i/2}{u-i/2} - \sum_{k=1}^{M}\frac{1}{i}\log\frac{u-u_k+i}{u-u_k-i} = 2\pi n(u)$$




$$\{n_k\} \longrightarrow \{u_k\} \longrightarrow n(u)$$

$$\frac{L}{i}\log\frac{u_j+i/2}{u_j-i/2} - \sum_{k\neq j}^{M}\frac{1}{i}\log\frac{u_j-u_k+i}{u_j-u_k-i} = 2\pi n_j$$

$$+\frac{L}{i}\log\frac{u+i/2}{u-i/2} - \sum_{k=1}^{M}\frac{1}{i}\log\frac{u-u_k+i}{u-u_k-i} \equiv 2\pi n(u)$$

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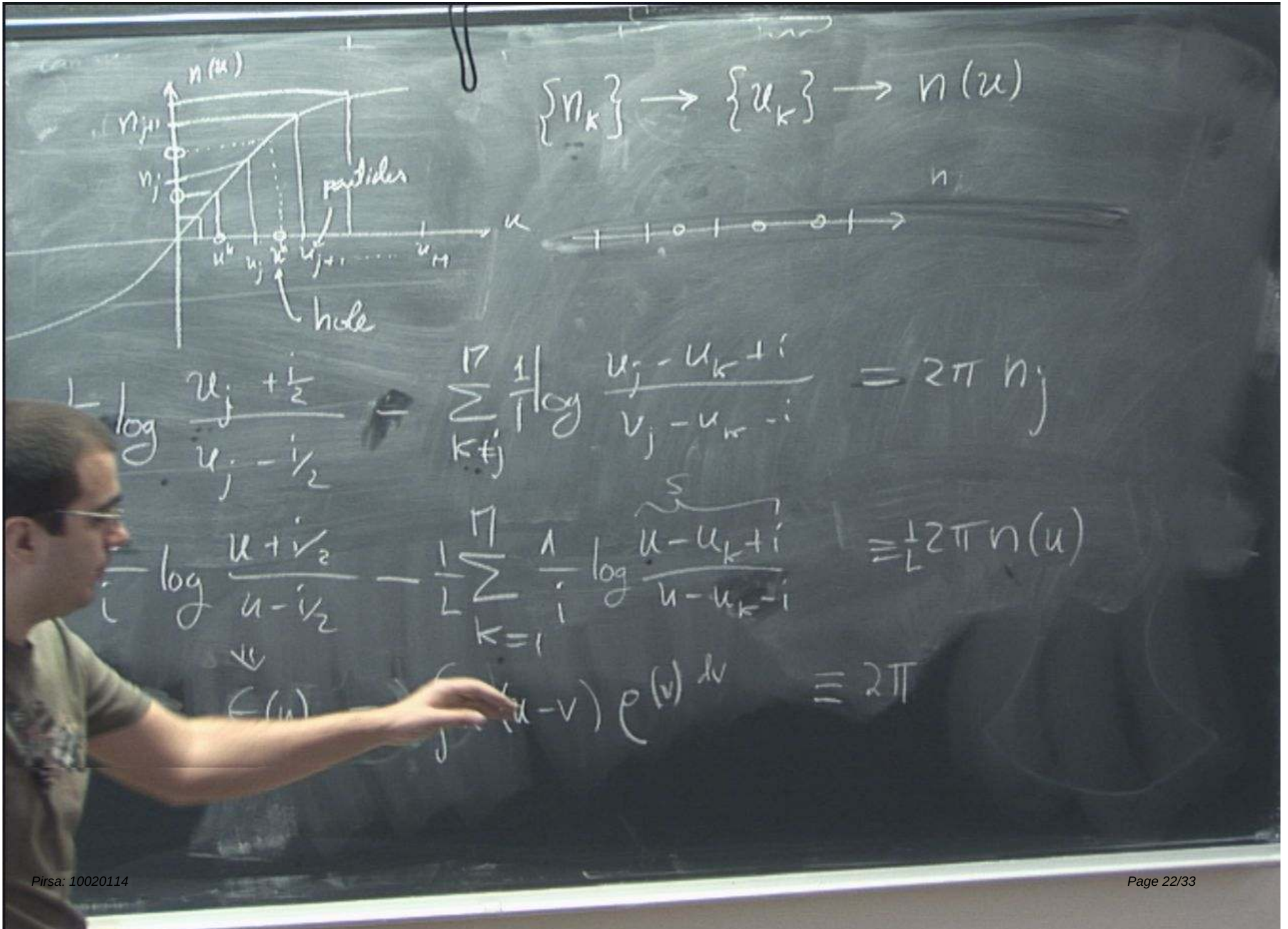
n(u)
$n_{j+1}$
$n_j$
particles
hole
$\{n_k\} \to \{u_k\} \to n(u)$
n
u
$\log \frac{u_j + i/2}{u_j - i/2} + \sum_{k \neq j}^{M} \frac{1}{i} \log \frac{u_j - u_k + i}{u_j - u_k - i} = 2\pi n_j$
$\frac{1}{i} \log \frac{u + i/2}{u - i/2} - \frac{1}{L} \sum_{k=1}^{M} \frac{1}{i} \log \frac{u - u_k + i}{u - u_k - i} \equiv \frac{1}{L} 2\pi n(u)$
$\epsilon(u)$
$(u - v)\, \rho(v)\, dv \equiv 2\pi$

$$\epsilon - K * e = e + \tilde{e}$$

$$E_0(L) = \beta\left(\frac{L}{1}\right)^{-\frac{1}{2}}$$

inverse temp

$F =$

$$= e^{-E_0(L)\,R}$$

free energy per unit R

$$= e^{-F(L,R)} \quad -R f(L)$$

$$= e \Big/ = e$$

free energy of my model for space R and temperature $1/L$

$$\epsilon - K * \rho = \rho + \bar{\rho}$$

$$E_0(L) = \beta\left(\frac{L}{2}\right)^{\frac{1}{1}}$$

inverse temp

entropy

$$F = L \sum \epsilon(j) - S$$

$$= L \int \underbrace{\rho(u)\epsilon(u)\,du}_{E} - \int \log \frac{(\rho+\bar{\rho})\,du!}{\rho\,du!\;\bar{\rho}\,du!}$$

$$= \int L\rho(u)\epsilon(u) - \bar{\rho}\log\left(1+\frac{\rho}{\bar{\rho}}\right) - \rho\log\left(1+\frac{\bar{\rho}}{\rho}\right)$$

$$\epsilon(u) = \frac{d}{du} \frac{1}{2\pi i} p(u)$$

$$k(u) = \frac{d}{du} \frac{1}{2\pi i} \log S(u)$$

$$F = F|_{\delta F = 0}$$

$$\delta F = 0 = \int \epsilon(u)\, \delta\rho(u) - \delta\rho(u) \log\left(1 + \frac{\rho}{\bar{\rho}}\right)$$

$$- \delta\bar{\rho} \log\left(1 + \frac{\bar{\rho}}{\rho}\right) + \text{zero}$$

$$\epsilon(u) = \frac{d}{du}\,\frac{1}{2\pi i}\,p(u)$$

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BAE $\Rightarrow$ $\delta\tilde{\epsilon} = -k * \delta\epsilon - \delta\epsilon$

$$F = F\big|_{\delta F = 0}$$

$$\delta F = 0 = \int \epsilon(u)\,\delta\rho(u) - \delta\rho(u)\log\left(1+\frac{\rho}{\rho}\right) - \delta\tilde{\rho}\log\left(1+\frac{\rho}{\rho}\right) + \text{zero}$$

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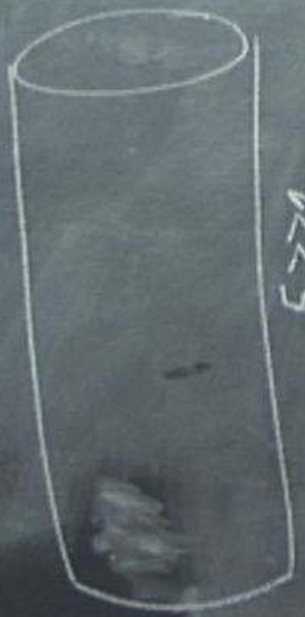
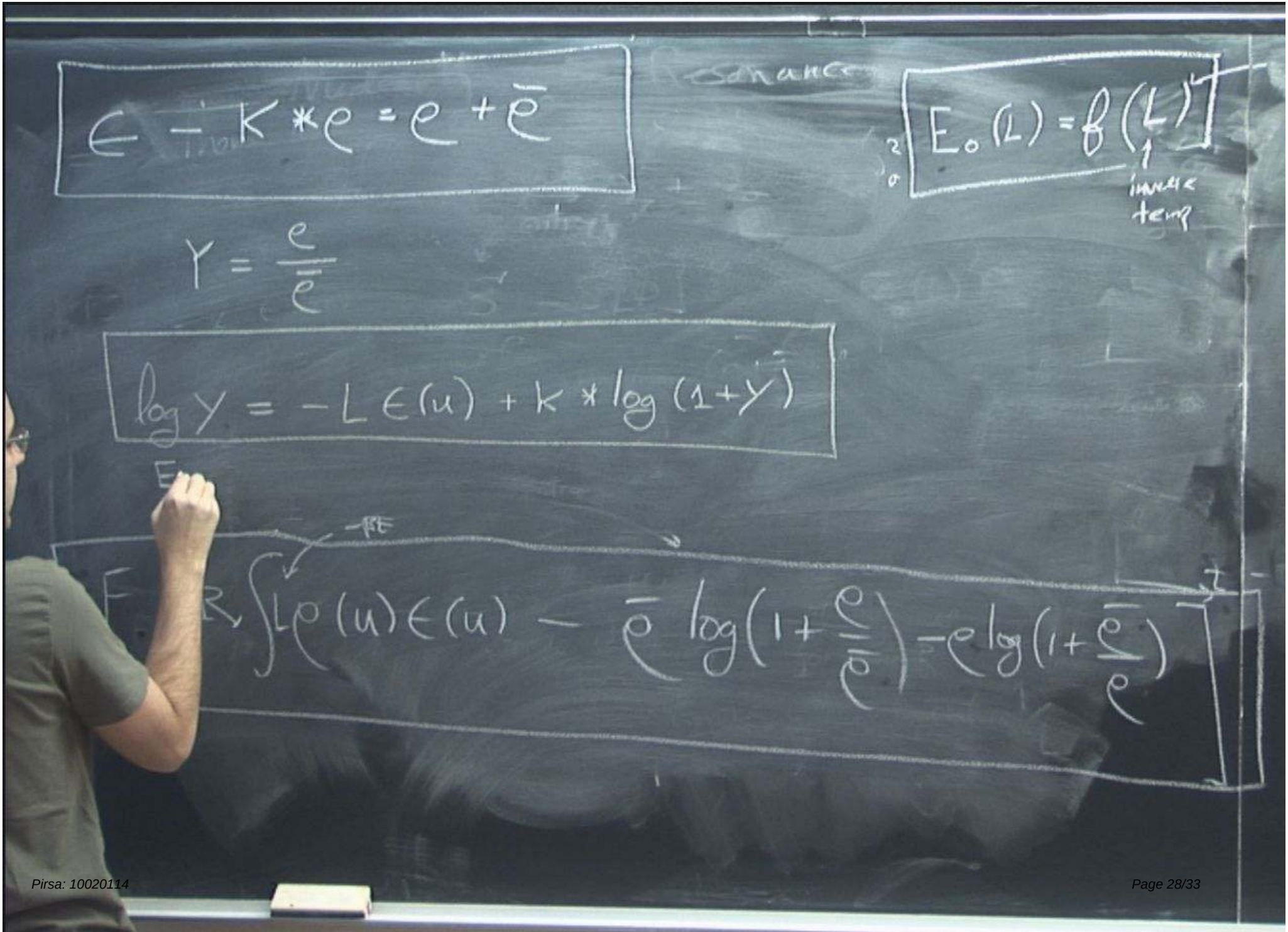



$$\delta F = 0 = \int \epsilon(u)\,\delta\rho(u) - \delta\rho(u)\log\left(1+\frac{\tilde{\rho}}{\rho}\right)$$

$$- \delta\tilde{\rho}\,\log\left(1+\frac{\rho}{\tilde{\rho}}\right) \quad + \text{zero}$$

$$= \int \delta\rho\left[\epsilon - \log\left(1+\frac{\tilde{\rho}}{\rho}\right) + \log\left(1+\frac{\rho}{\tilde{\rho}}\right) + k * \log\left(1+\frac{\rho}{\tilde{\rho}}\right)\right]$$

$$= 0$$


$\epsilon = K * \rho = e + \bar{e}$
$E_0(L) = \beta \left(\frac{L}{L_1}\right)^{\gamma}$
$Y = \frac{e}{\bar{e}}$
$\log Y = -L\epsilon(u) + k * \log(1+Y)$
$F = R \int \rho(u)\epsilon(u) - \bar{e}\log\left(1+\frac{e}{\bar{e}}\right) - e\log\left(1+\frac{\bar{e}}{e}\right)$

$$\epsilon - K * e = e + \bar{e}$$

$$E_0(L) = \beta \left(\frac{L}{1}\right)^{\gamma}$$

inverse temp

$$Y = \frac{e}{\bar{e}}$$

$$\log Y = -L\epsilon(u) + k * \log(1+Y)$$

$E_0(L) =$

$$F = R \int \left[ e(u)\epsilon(u) - \bar{e} \log\left(1 + \frac{e}{\bar{e}}\right) - e \log\left(1 + \frac{\bar{e}}{e}\right) \right]$$

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inverse temp

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$$E_0(L) = \int \epsilon(u) \log(1+y)$$

$$F = R \int \left[ \rho(u)\epsilon(u) - \bar{e} \log\left(1+\frac{e}{\bar{e}}\right) - e \log\left(1+\frac{\bar{e}}{e}\right) \right]$$

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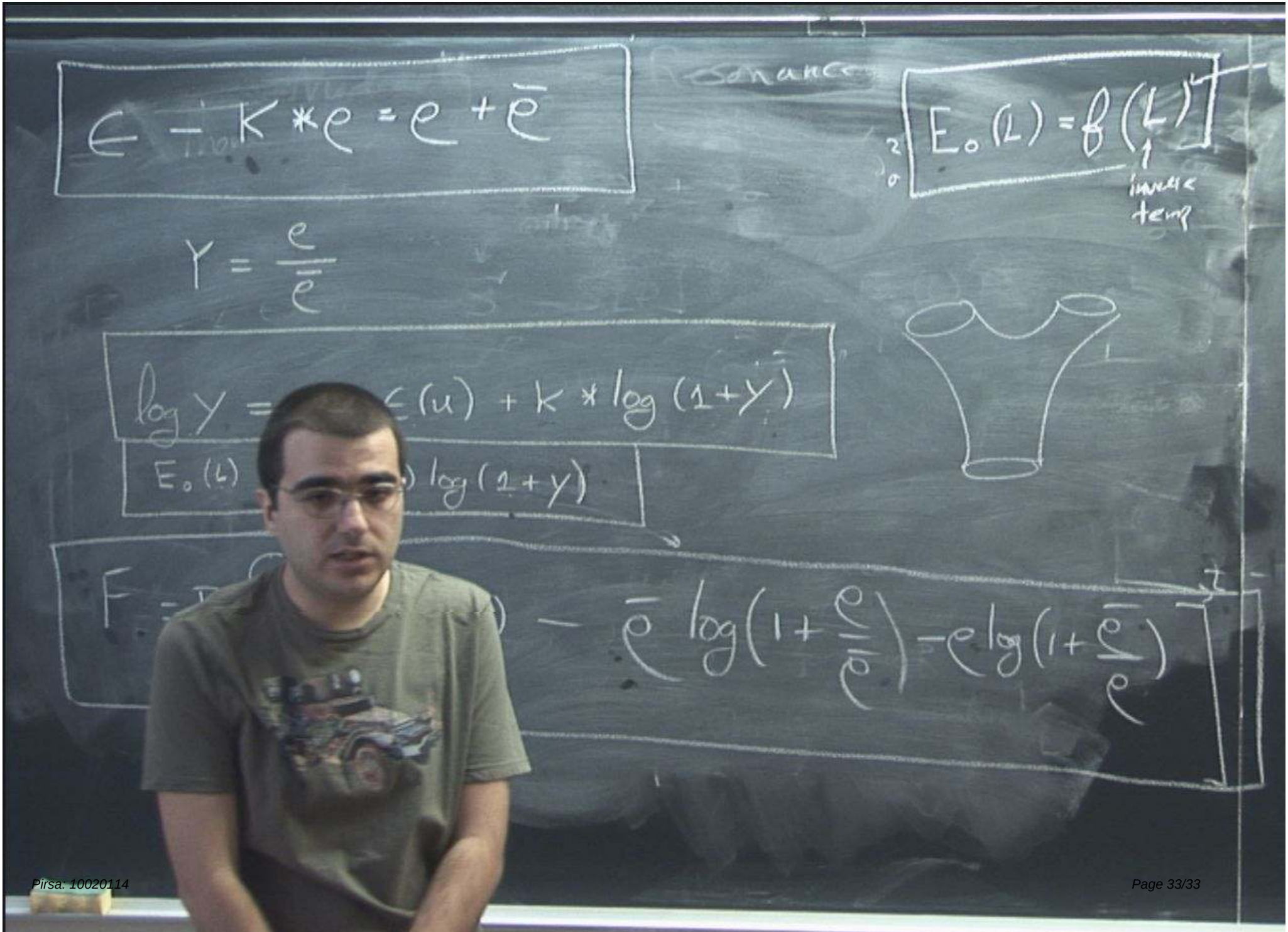
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inverse temp
$Y = \frac{e}{\bar{e}}$
$\log Y = \epsilon(u) + k * \log(1+Y)$
$E_0(L) \quad \log(1+y)$
$- \bar{e} \log\left(1+\frac{e}{\bar{e}}\right) - e \log\left(1+\frac{\bar{e}}{e}\right)$