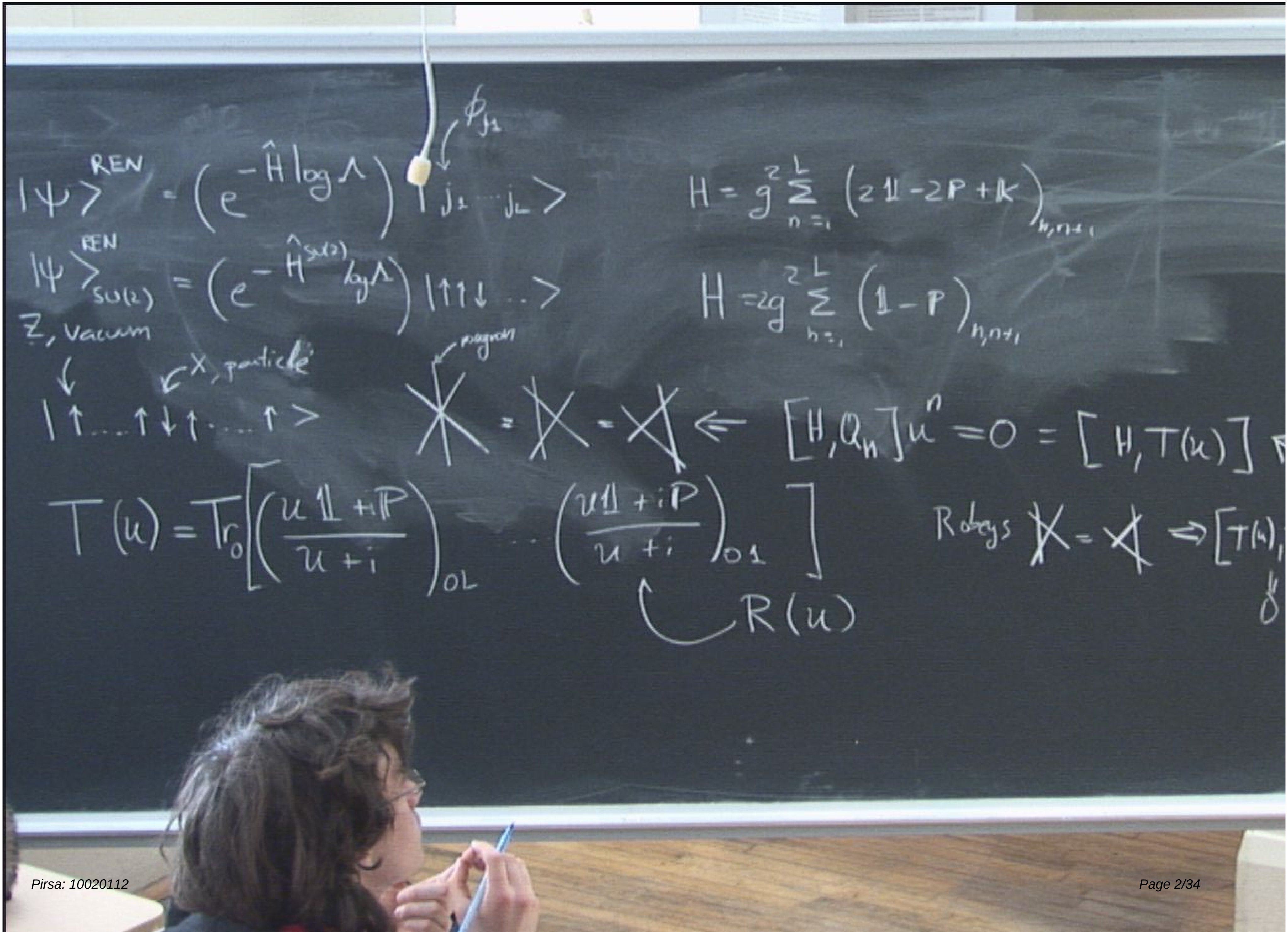
Title: Explorations in String Theory (PHYS 647) - Lecture 8

Date: Feb 24, 2010 11:20 AM

URL: http://pirsa.org/10020112

Abstract:



ϕ_{j_1}

$|\psi\rangle^{REN} = \left(e^{-\hat{H}\log\Lambda}\right)|j_1 \cdots j_L\rangle$

$H = g^2 \sum_{n=1}^{L} \left(2\mathbb{1} - 2P + K\right)_{n,n+1}$

$|\psi\rangle^{REN}_{SU(2)} = \left(e^{-\hat{H}^{SU(2)}\log\Lambda}\right)|\uparrow\uparrow\downarrow \ldots\rangle$

$H = 2g^2 \sum_{n=1}^{L} \left(\mathbb{1} - P\right)_{n,n+1}$

Z, vacuum

X, particle

magnon

$|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\rangle$

$\Leftarrow \quad [H, Q_n]u^n = 0 = [H, T(u)]$

$T(u) = \mathrm{Tr}_0\left[\left(\frac{u\mathbb{1} + iP}{u+i}\right)_{0L} \cdots \left(\frac{u\mathbb{1} + iP}{u+i}\right)_{01}\right]$

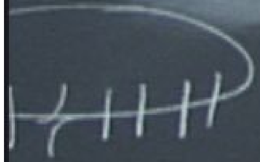
$R(u)$

R obeys $\Rightarrow [T(u),$

$$H^{SU(2)} = T^{-1}(u)\,\frac{d}{du}\,T(u)\Big|_{u=0}$$

magnon factorization

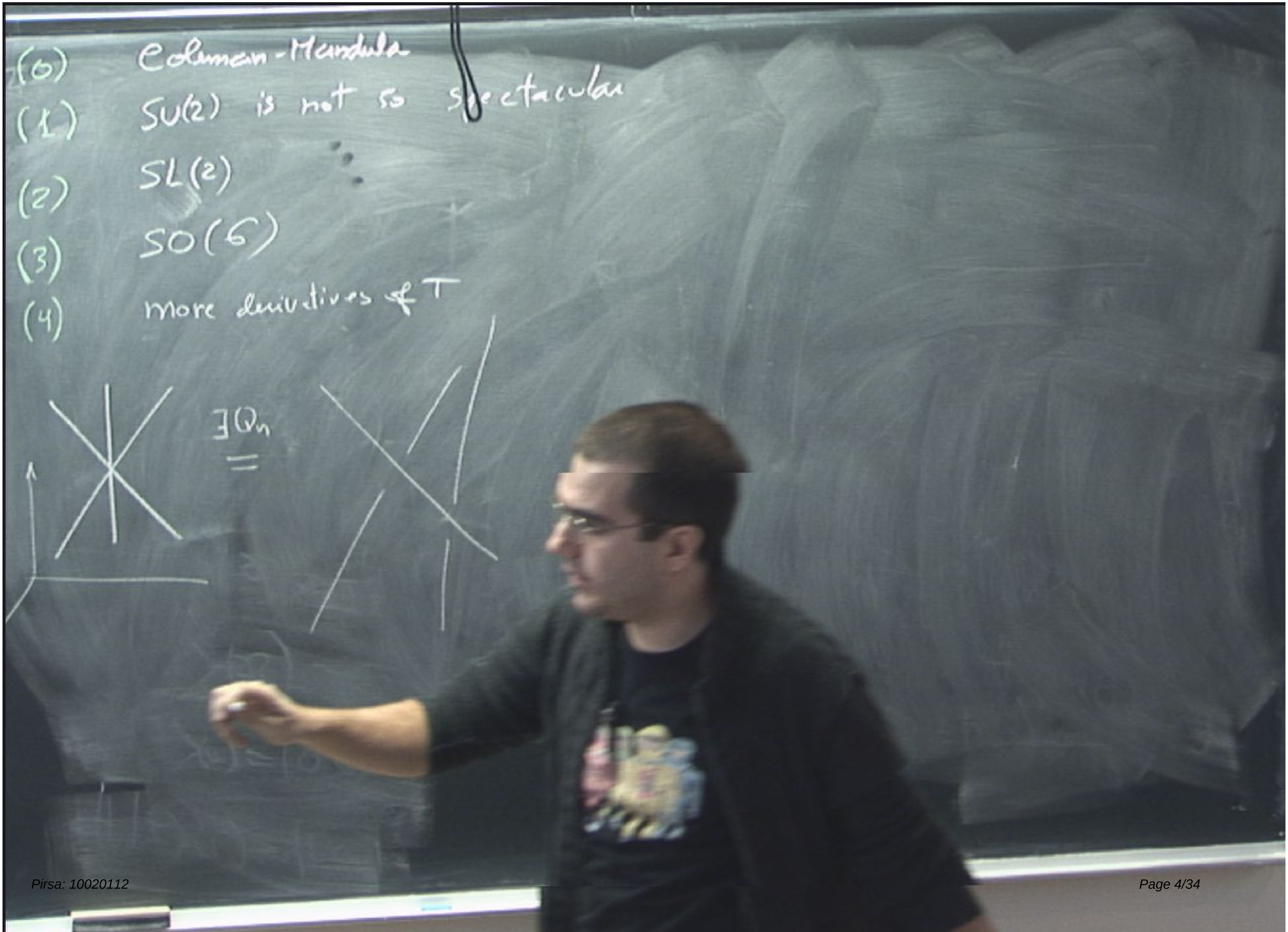
$\Downarrow$



$$e^{ip_j L}\prod_{k\neq j}^{M} S(p_j, p_k) = 1 \qquad E = \sum_{j=1}^{M} \epsilon(p_j)$$

$\Updownarrow$

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k\neq j}^{M} \frac{u_j - u_k + i}{u_j - u_k - i} \ , \quad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}$$



(0) Coleman-Mandula

(1) SU(2) is not so spectacular

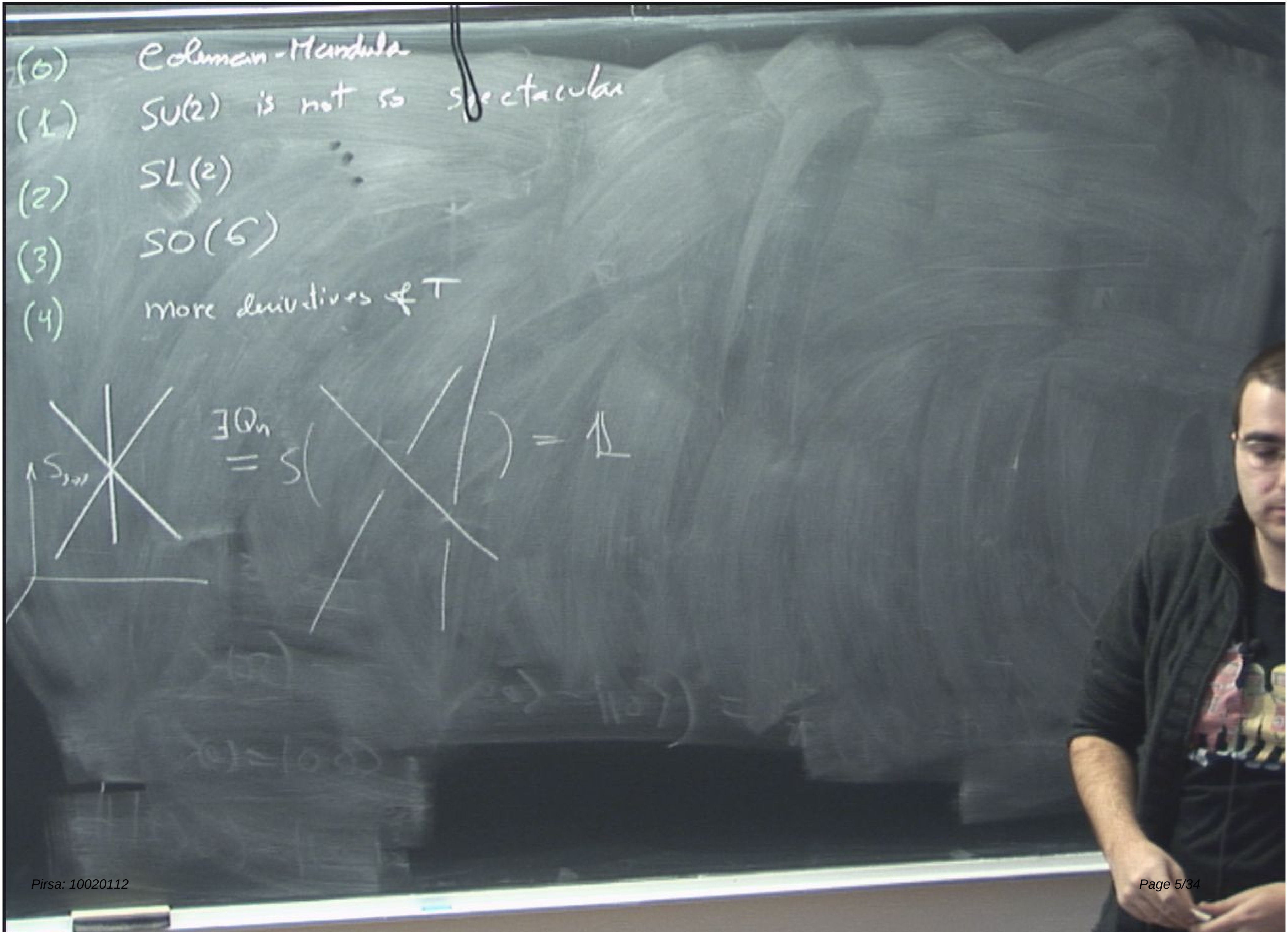
(2) SL(2)

(3) SO(6)

(4) more derivatives of T

$\exists Q_n$

$=$



(0) Coleman-Mandula

(1) SU(2) is not so spectacular

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$\exists Q_n$

$= S(\qquad) = \mathbb{1}$

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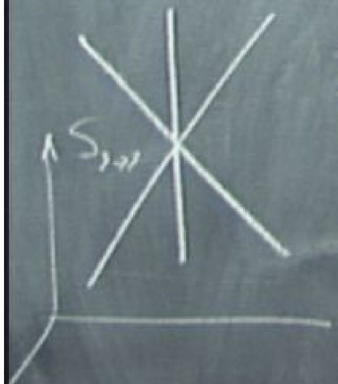
(4) more derivatives of T



$\exists Q_n \Rightarrow S($  $) = 1$

(1) We only have ℙ

(2) SL(2) sector

$|Z \ldots Z D Z Z \ldots \overset{\text{particle}}{ZDZ} \ldots ZDZ \ldots\rangle$

light cone cov. derivative

(0) Coleman-Mandula

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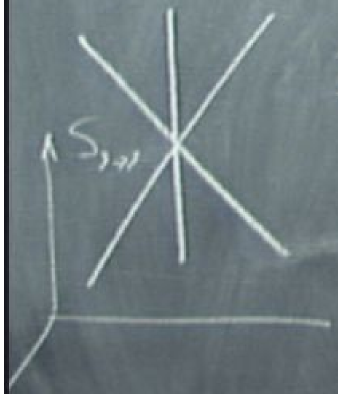

$\exists Q_n$

$= S($

$) = 1$

(1) We only have $\mathbb{P}$

(2) SL(2) sector

$|Z..ZDZZ....ZDZ..ZD^2Z...\rangle$

particle

light cone cov. derivative

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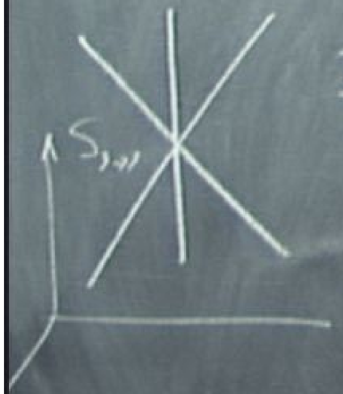


$\exists Q_n$

$= S($  $) = \mathbb{1}$

(1) We only have $\mathbb{P}$

(2) <u>SL(2) Sector</u>

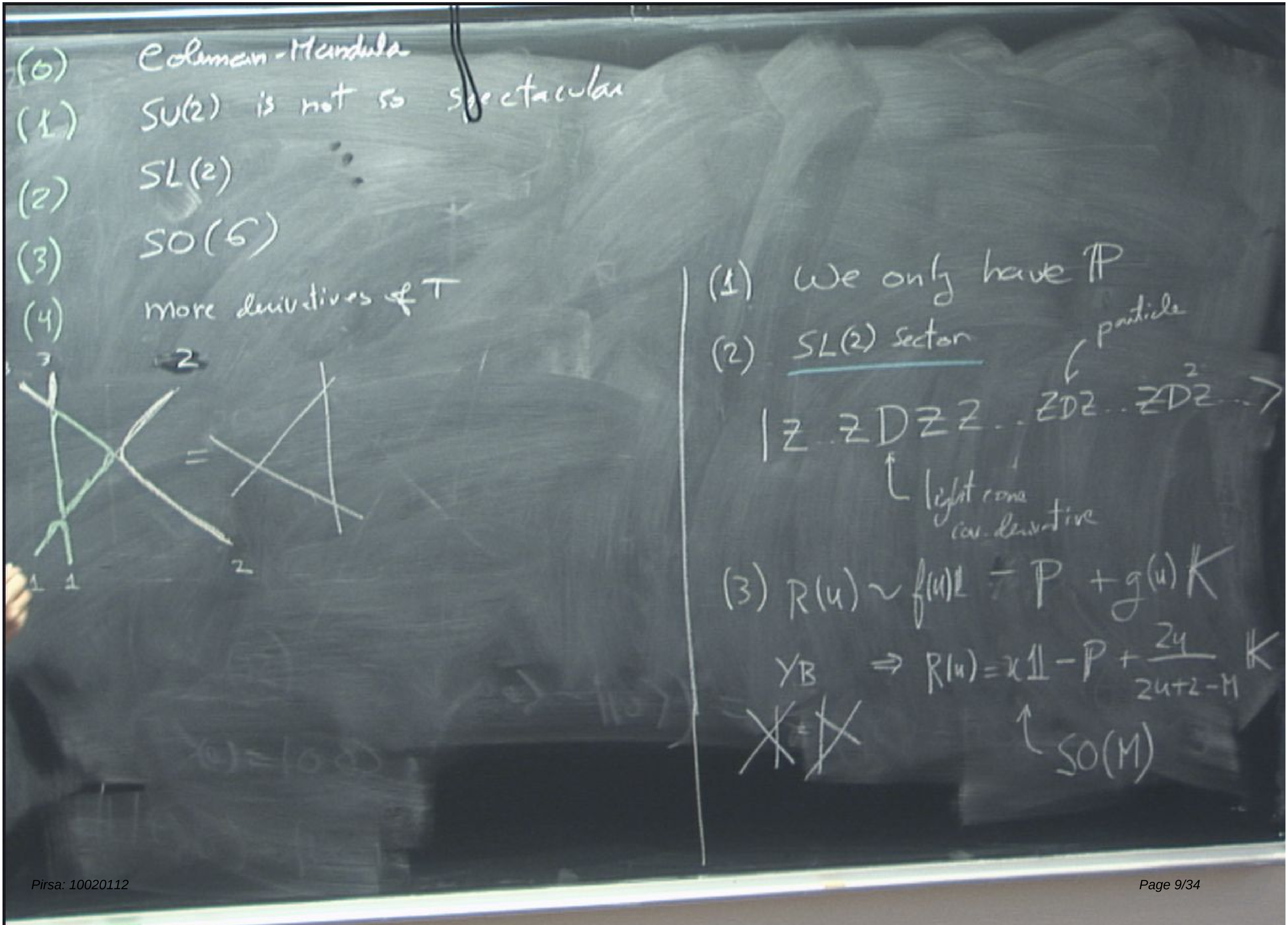
particle

$|Z \ldots Z D Z Z \ldots ZDZ \ldots ZD^2Z \ldots \rangle$

light cone cov. derivative

(3) $R(u) \sim f(u)\mathbb{1} \pm P + g(u) K$

YB $\Rightarrow R(u) = x\mathbb{1} - P + \frac{2y}{2u+2-M} K$



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$$|Z \ldots Z D Z Z \ldots \overset{\text{particle}}{ZDZ} \ldots Z D^2 Z \ldots \rangle$$

light cone cov. derivative

(3) $R(u) \sim f(u)\mathbb{1} + \mathbb{P} + g(u) \mathbb{K}$

YB $\Rightarrow R(u) = x\mathbb{1} - \mathbb{P} + \frac{2u}{2u+2-M}\mathbb{K}$

SO(M)

$$H' = \frac{d^2}{du^2}\log T\Big|_{u=0} = \sum_{n=1}^{L}$$  $+$ 

$$H = \frac{d}{du}\log T\Big|_{u=0} = \sum$$ 

$$\Downarrow$$

$$\boxed{\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k\neq j}^{M}\frac{u_j-u_k\pm i}{u_j-u_k\mp i}\,,\quad E=\sum_{j=1}^{M}\frac{2g^2}{u_j^2+1/4}}$$

SL(2)

$$H' = \frac{d^2}{du^2} \log T \Big|_{u=0} = \sum_{n=1}^{L} \underline{X} + \underline{X}$$

$$H = \frac{d}{du} \log T \Big|_{u=0} = \sum \underline{X}$$

$$\boxed{\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k \pm i}{u_j - u_k \mp i} \quad , \quad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}}$$

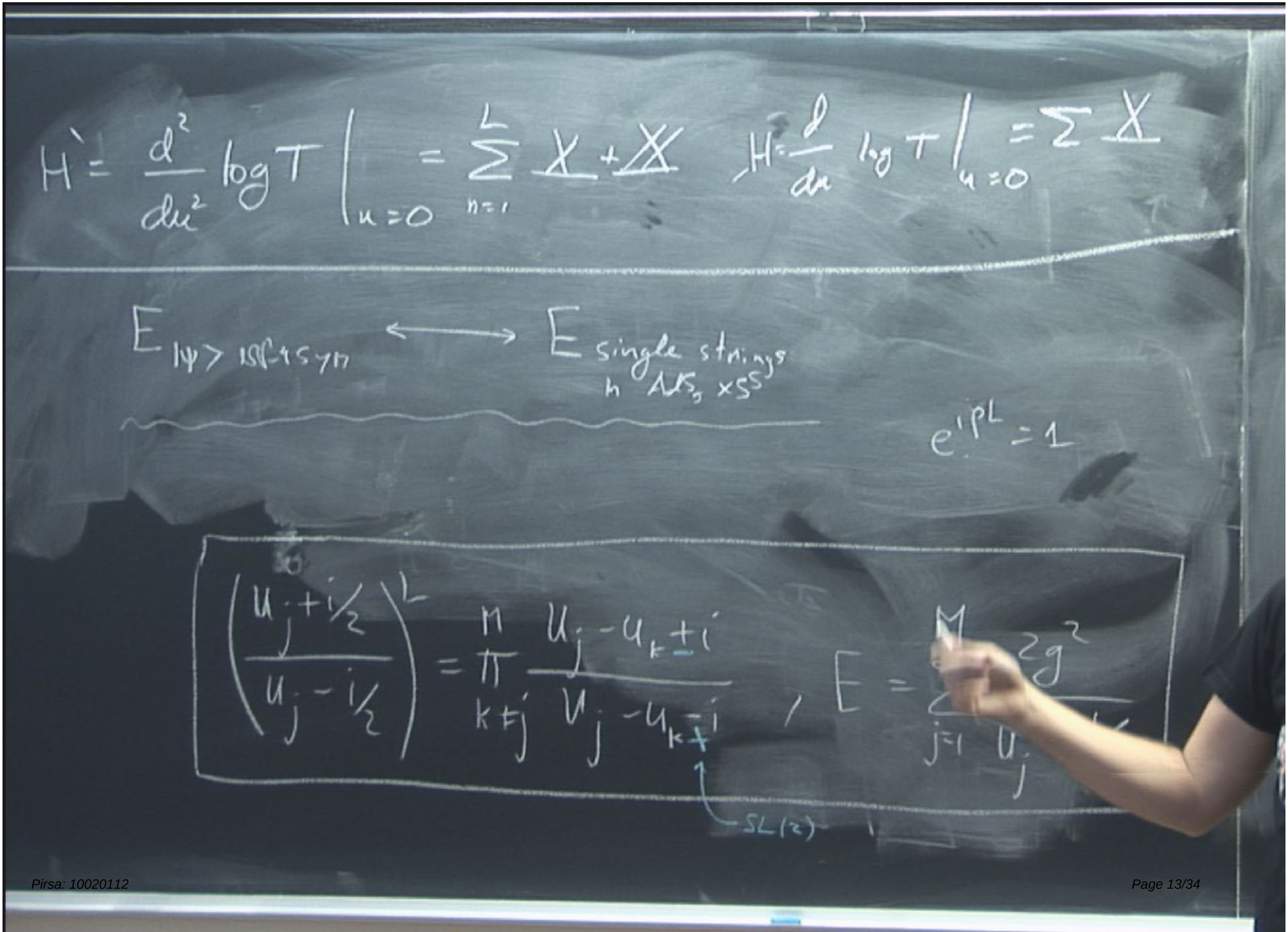
SL(2)

$$H' = \frac{d^2}{du^2} \log T \Big|_{u=0} = \sum_{n=1}^{L} \underline{X} + \underline{X} \qquad H = \frac{d}{du} \log T \Big|_{u=0} = \sum \underline{X}$$

$$E_{|\psi\rangle\, N=4\, SYM} \longleftrightarrow E_{\text{single strings in } AdS_5 \times S^5}$$

$$\boxed{\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k \pm i}{u_j - u_k \mp i} \,, \quad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}}$$

SL(2)



$$H' = \frac{d^2}{du^2}\log T\Big|_{u=0} = \sum_{n=1}^{L} \underline{X} + \underline{\cancel{X}} \qquad H = \frac{d}{du}\log T\Big|_{u=0} = \sum \underline{X}$$

$$E_{|\psi\rangle\ \text{ISF+sym}} \longleftrightarrow E_{\text{single strings in } AdS_5 \times S^5}$$

$$e^{ipL} = 1$$

$$\boxed{\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k \underline{+} i}{u_j - u_k \underset{+}{-} i}, \quad E = \sum_{j=1}^{M} \frac{2g^2}{u_j}}$$

SL(2)

$$H' = \frac{d^2}{du^2}\log T\Big|_{u=0} = \sum_{n=1}^{L} \underline{X} + \underline{X} \qquad H = \frac{d}{du}\log T\Big|_{u=0} = \sum \underline{X}$$

$$E_{|\psi\rangle\ \mathcal{N}=4\ \mathrm{SYM}} \longleftrightarrow E_{\substack{\text{single strings}\\ \text{in } AdS_5\times S^5}}$$

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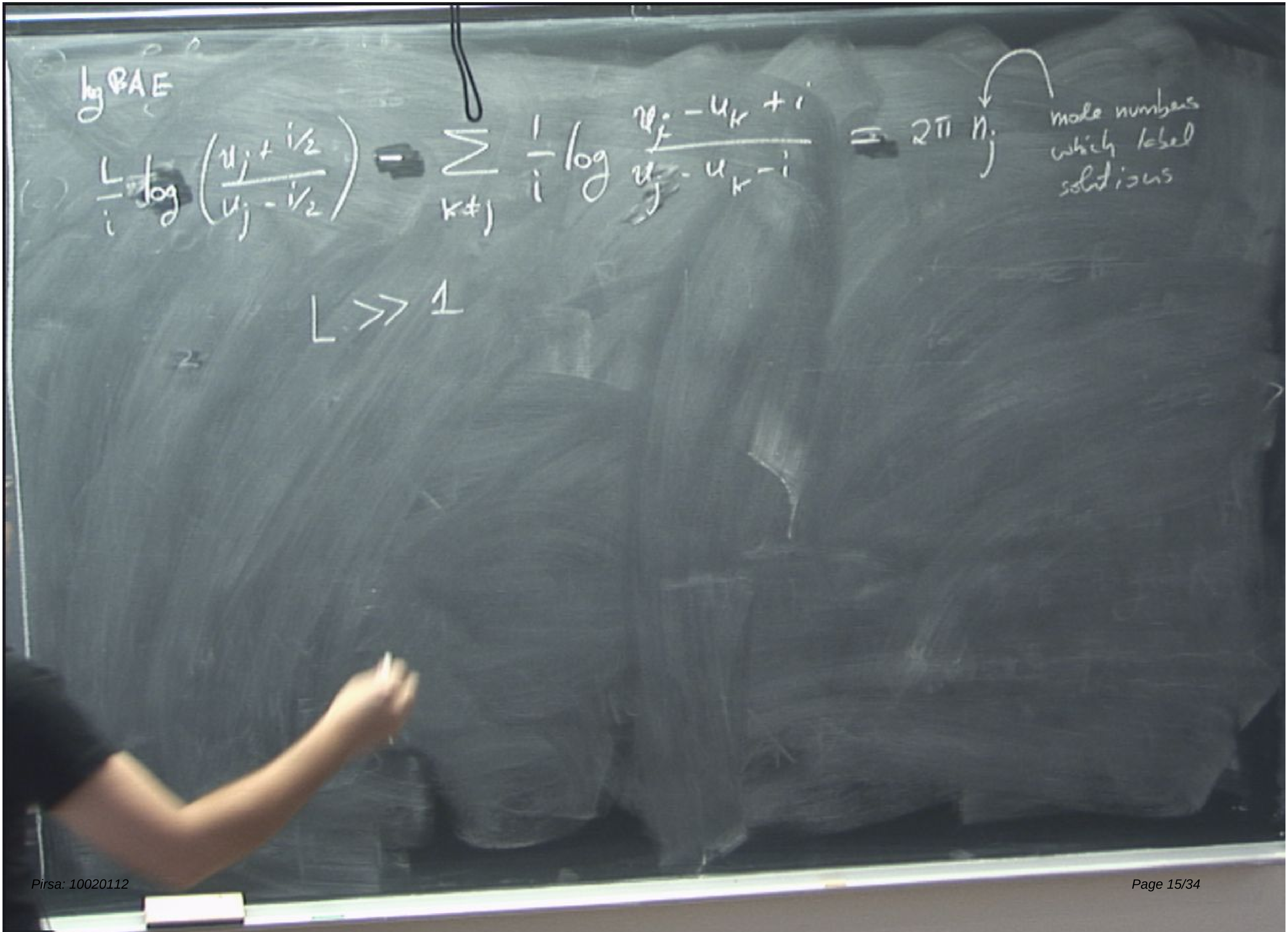
$$\boxed{\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k\neq j}^{M}\frac{u_j-u_k\pm i}{u_j-u_k\underset{\underset{SL(2)}{\uparrow}}{\mp} i}\,,\qquad E=\sum_{j=1}^{M}\frac{2g^2}{u_j^2+1/4}}$$

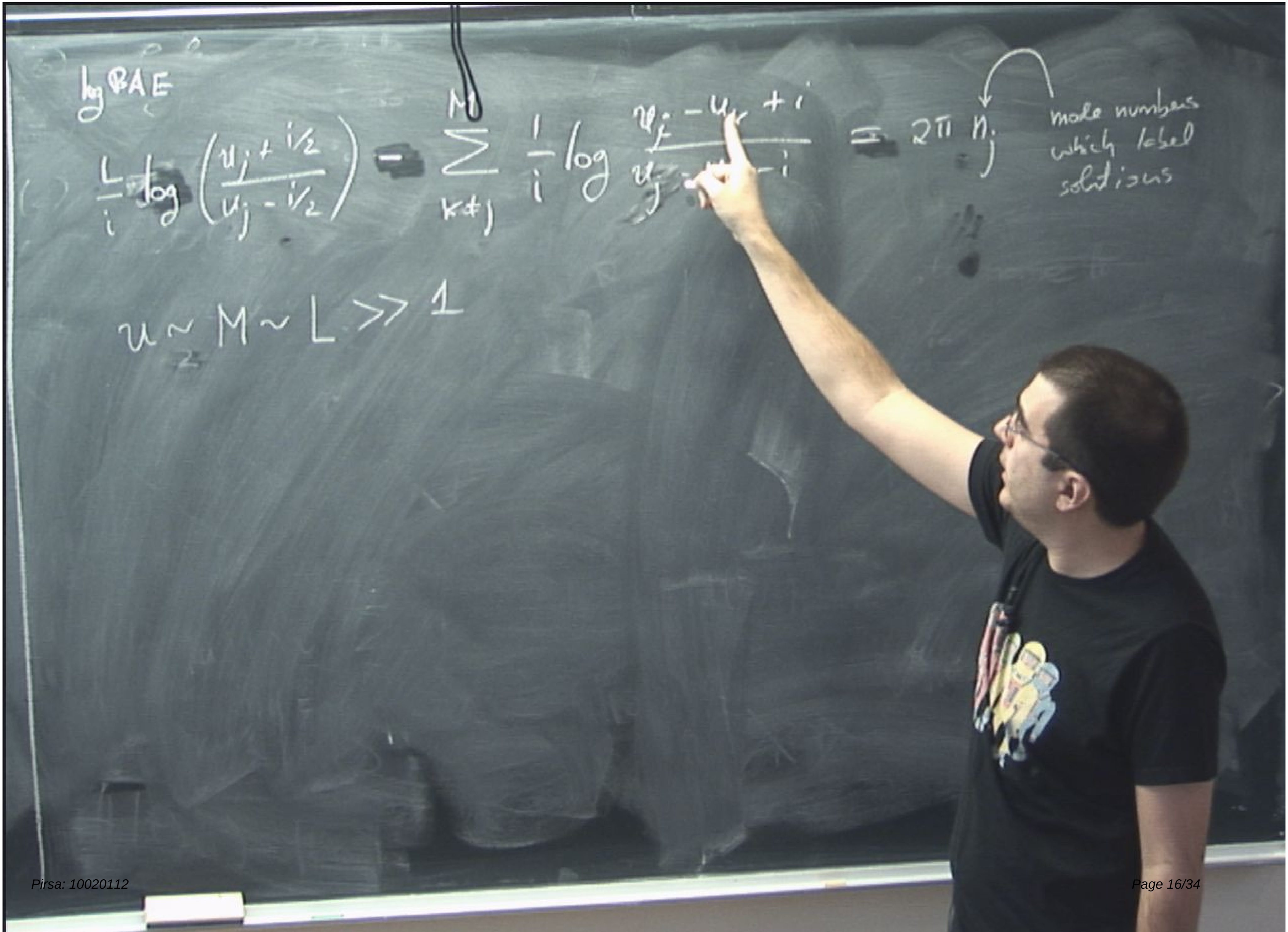


log BAE

$$\frac{L}{i}\log\left(\frac{u_j+i/2}{u_j-i/2}\right) - \sum_{k\neq j}\frac{1}{i}\log\frac{u_j-u_k+i}{u_j-u_k-i} = 2\pi\, n_j$$

mode numbers which label solutions

$L >> 1$

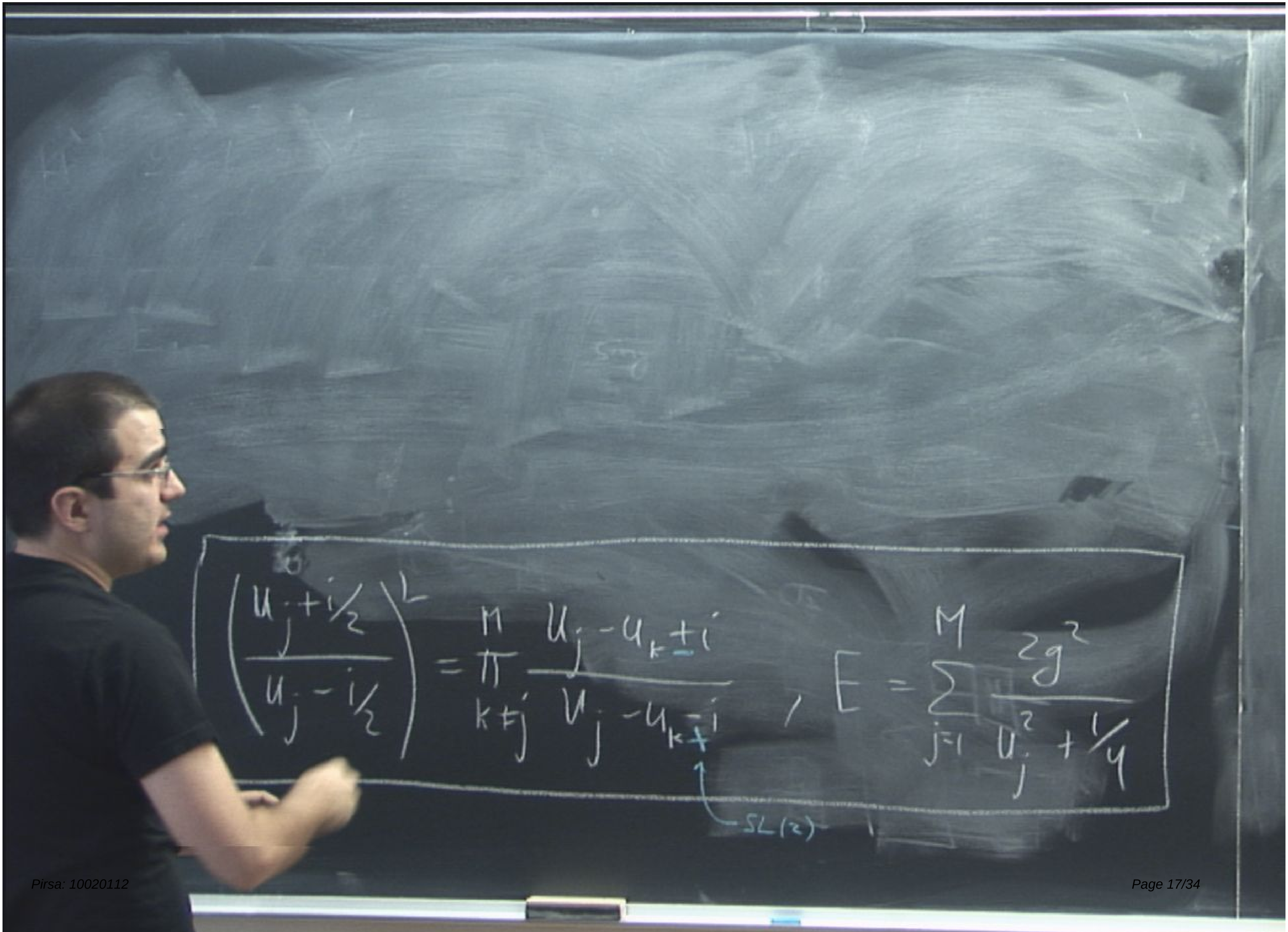


log BAE

$$\frac{L}{i}\log\left(\frac{u_j+i/2}{u_j-i/2}\right) - \sum_{k\neq j}^{M}\frac{1}{i}\log\frac{u_j-u_k+i}{u_j-u_k-i} = 2\pi\, n_j$$

mode numbers which label solutions

$$u \sim M \sim L \gg 1$$



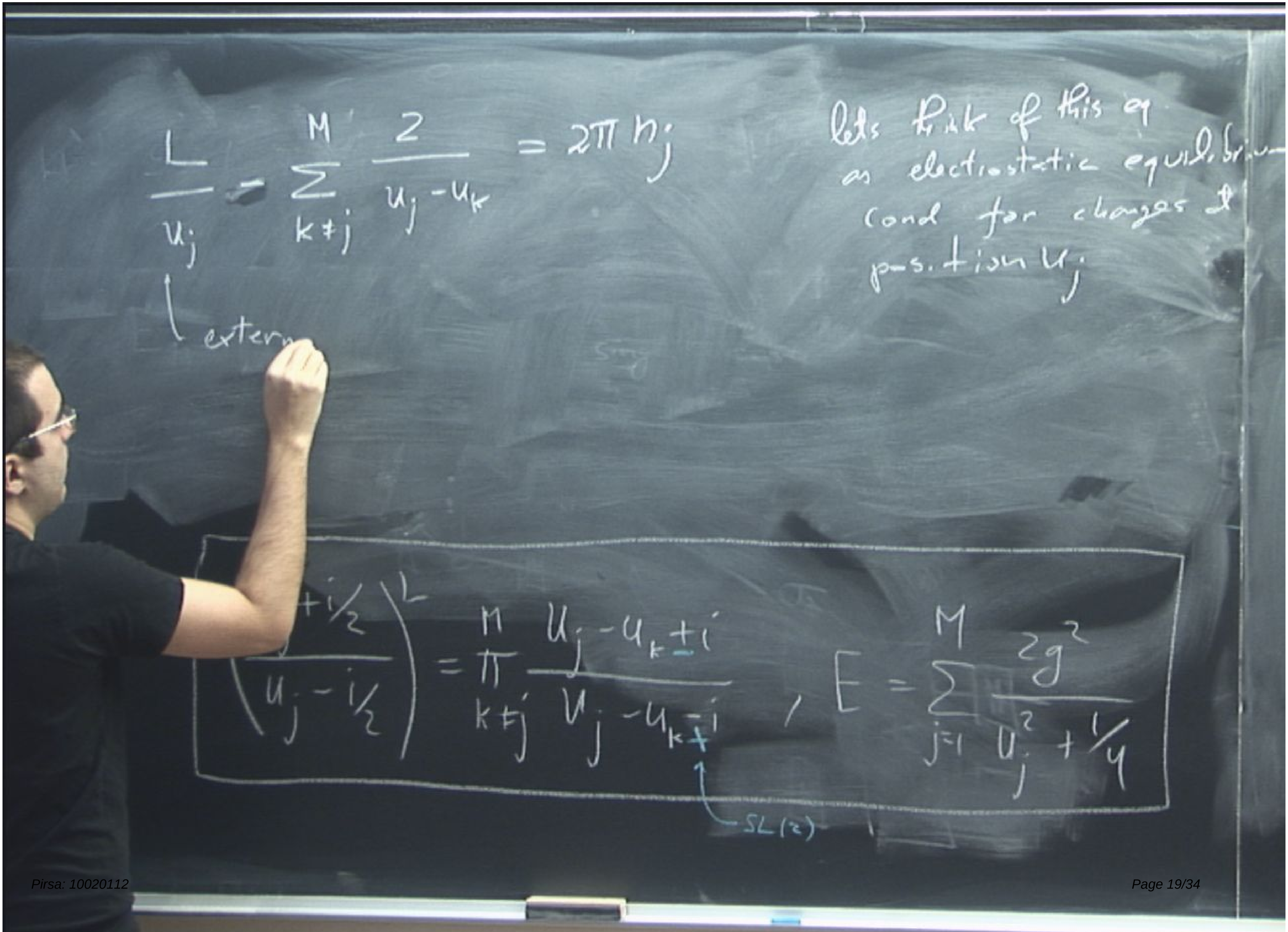
$$\boxed{\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k\neq j}^{M}\frac{u_j-u_k\pm i}{u_j-u_k\mp i}\ ,\quad E=\sum_{j=1}^{M}\frac{2g^2}{u_j^2+1/4}}$$

SL(2)

$$\frac{L}{u_j} - \sum_{k \neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

$$\boxed{\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k \pm i}{u_j - u_k \underset{\uparrow}{\mp} i}\ , \quad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}}$$

SL(2)



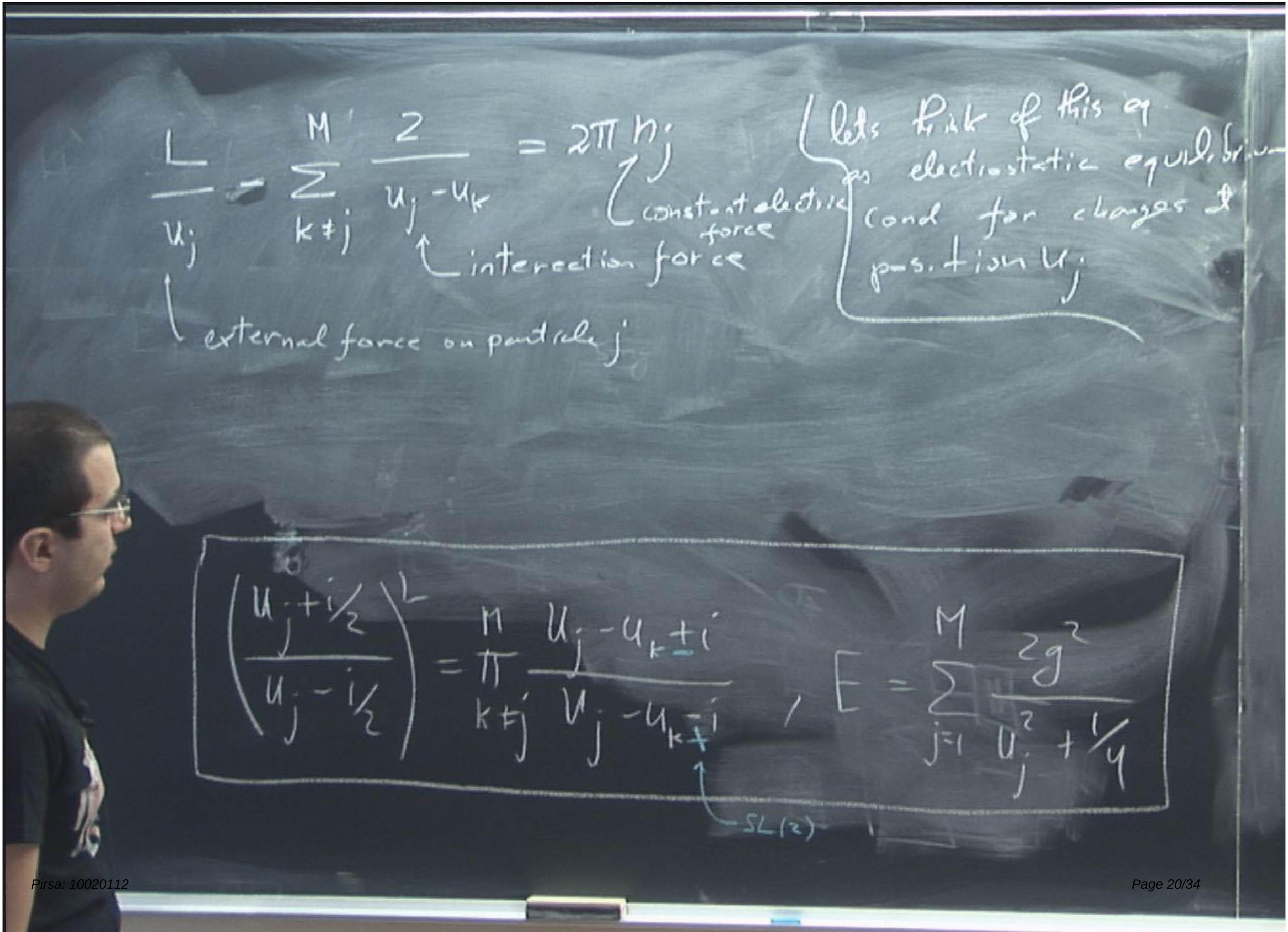
$$\frac{L}{u_j} - \sum_{k\neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

u_j ← extern

lets think of this eq as electrostatic equilibri cond for charges at position u_j

$$\boxed{\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k\neq j}^{M} \frac{u_j - u_k \pm i}{u_j - u_k \mp i}\ , \quad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}}$$

SL(2)



$$\frac{L}{u_j} - \sum_{k\neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

constant electric force

interection force

external force on particle j

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SL(2)

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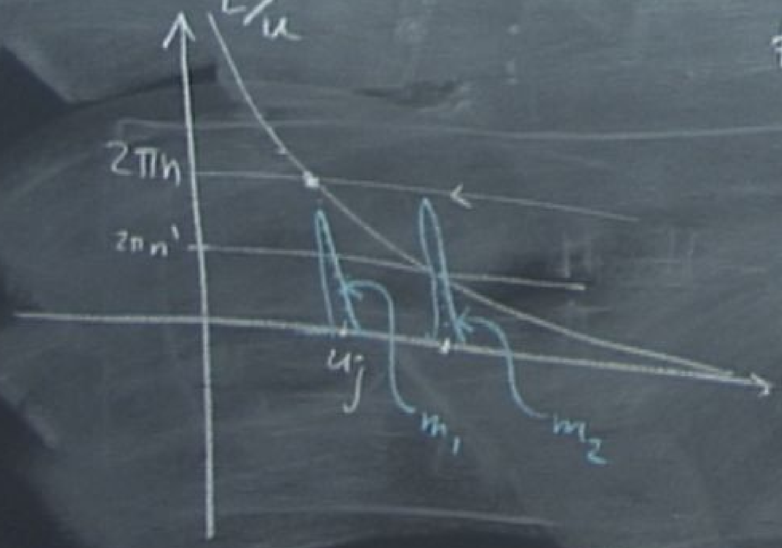
lets think of this eq as electrostatic equilibrium cond for charges at position u_j

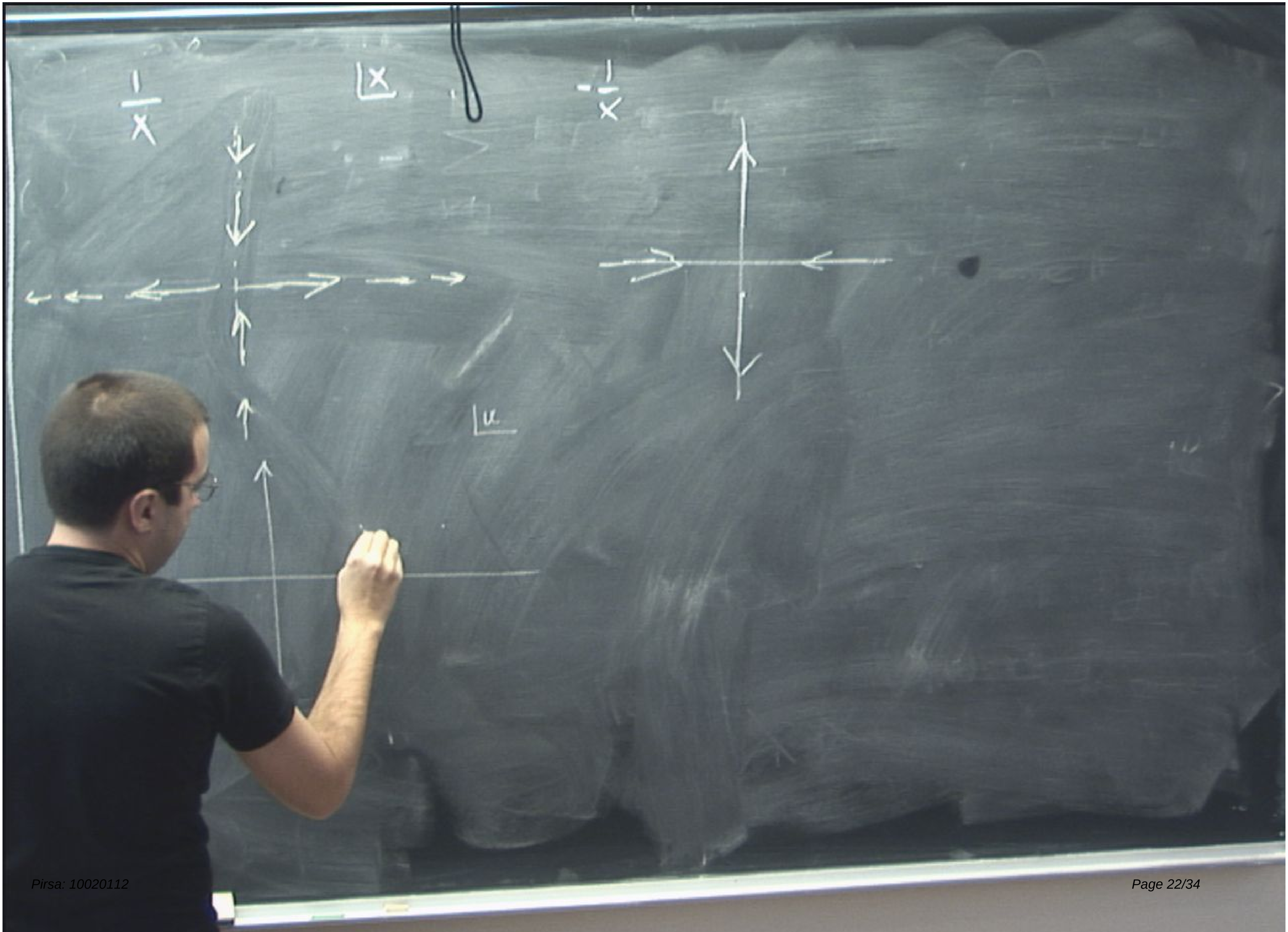
1) turnoff interactions

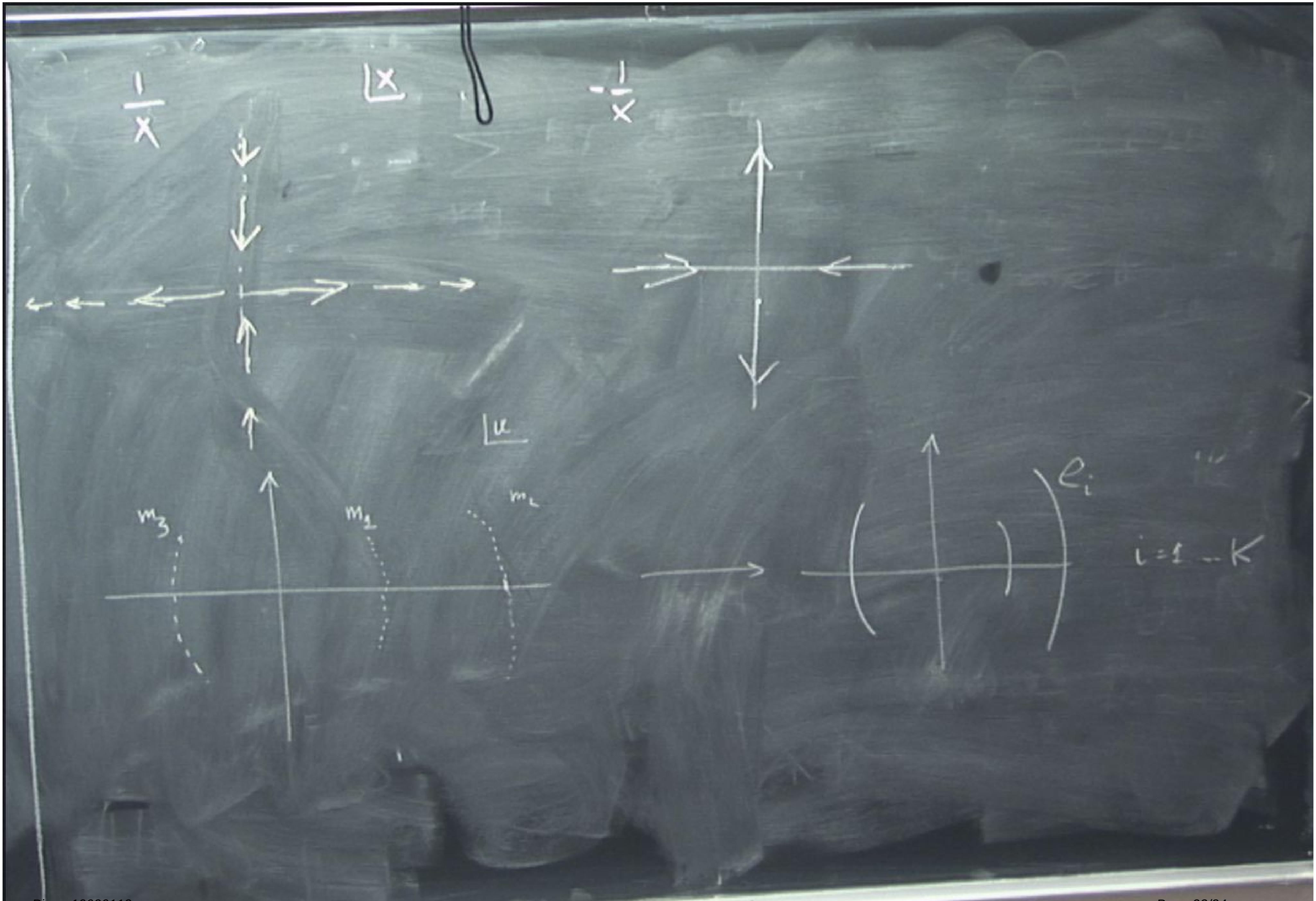
$$\{n_j\} = \{\underbrace{m_1, m_1 \cdots m_1}_{M_1}, \underbrace{m_2, \cdots, m_2}_{M_2}, \cdots\}$$

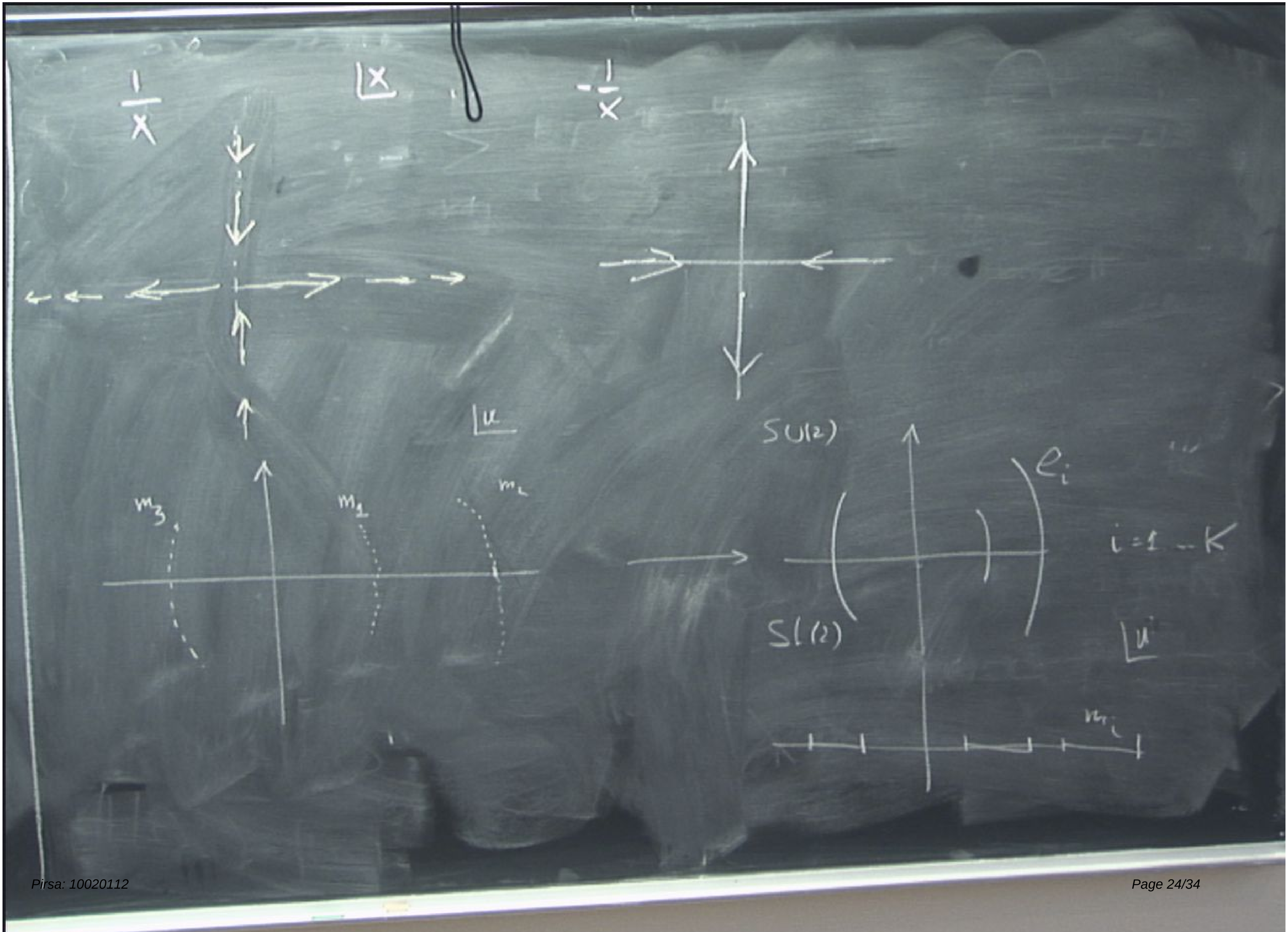
#groups → K

$$\sum_{j=1} M_j = M$$




$\frac{1}{X}$
$\lfloor X$
$-\frac{1}{X}$
$\lfloor u$


$\frac{1}{x}$
x
$-\frac{1}{x}$
u
m_3
m_1
m_2
e_i
$i=1 \dots K$


$\frac{1}{X}$
$\lfloor X$
$-\frac{1}{X}$
$\lfloor u$
m_3
m_1
m_2
$SU(2)$
e_i
$i=1 \ldots K$
$SL(2)$
$\lfloor u$
m_i

$$\frac{L}{u_j} - \sum_{k \neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

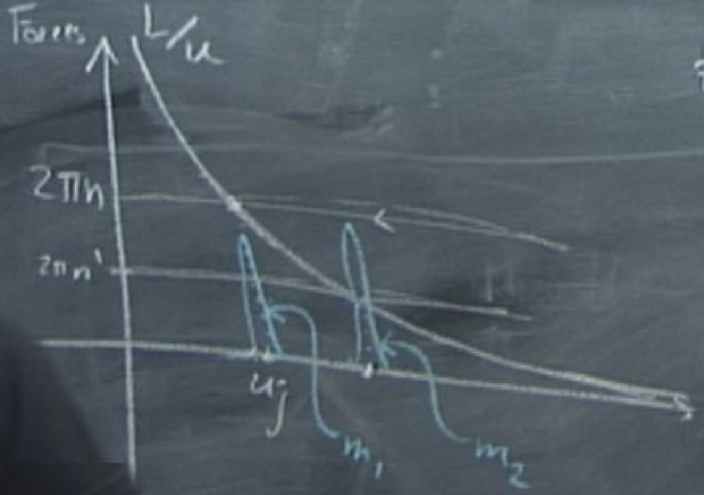
constant electric force

interaction force

external force on particle j

lets think of this eq as electrostatic equilibri
cond for charges at
position u_j

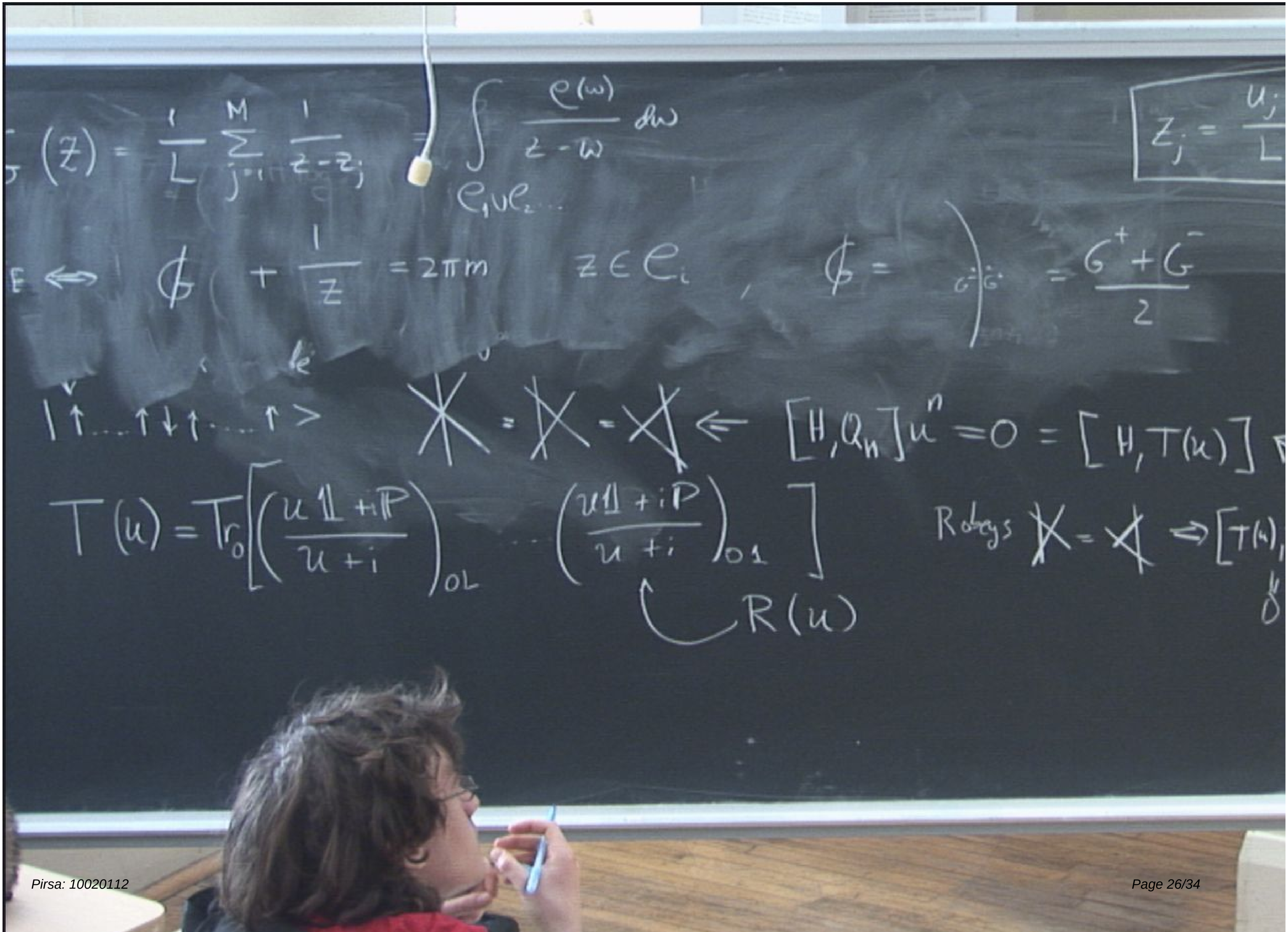
1) turn off interactions



$$\{n_j\} = \{\underbrace{m_1, m_1, \dots m_1}_{M_1}, \underbrace{m_2, \dots, m_2}_{M_2}, \dots\}$$

$\#\text{groups} \longrightarrow K$

$$\sum_{j=1} M_j = M$$



$$G(z) = \frac{1}{L}\sum_{j=1}^{M}\frac{1}{z-z_j} = \int_{\mathcal{C}_1\cup\mathcal{C}_2\dots}\frac{\rho(\omega)}{z-\omega}\,d\omega \qquad \boxed{z_j = \frac{u_j}{L}}$$

$$E \Longleftrightarrow \not{G} + \frac{1}{z} = 2\pi m \qquad z\in\mathcal{C}_i \,, \qquad \not{G} = \quad G^-\big|G^+ = \frac{G^+ + G^-}{2}$$

$$|\uparrow\dots\uparrow\downarrow\uparrow\dots\uparrow\rangle$$

$$= \quad = \quad \Leftarrow \quad [H, Q_n]\,u^n = 0 = [H, T(u)]$$

$$T(u) = \mathrm{Tr}_0\left[\left(\frac{u\mathbb{1} + i\mathbb{P}}{u+i}\right)_{0L}\cdots\left(\frac{u\mathbb{1} + i\mathbb{P}}{u+i}\right)_{01}\right]$$

$R(u)$

R obeys $=$ $\Rightarrow [T(u),$

$$\frac{L}{u_j} - \sum_{k\neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

external force on particle j

interection force

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lets think of this eq as electrostatic equilibri cond for charges & position u_j

$$\{n_j\} = \{\underbrace{m_1, m_1, \ldots m_1}_{M_1}, \underbrace{m_2, \ldots, m_2}_{M_2}, \ldots\}$$

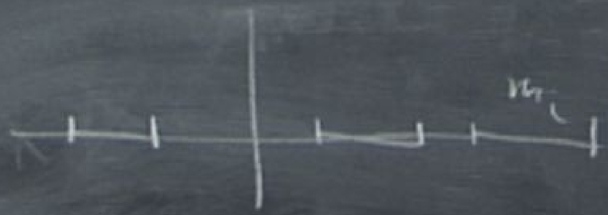
#groups → K

$$\sum_{j=1} M_j = M$$

$$\frac{1}{x \pm i0} = P\frac{1}{x} \mp i\pi\delta(x)$$

$$\frac{1}{x+i0}$$

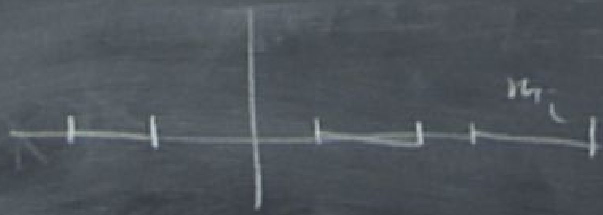
$$\frac{G^{+}+G^{-}}{z}=\frac{1}{2}\int\frac{\rho(\omega)\,d\omega}{z-\omega+i0}+\frac{\rho(\omega)\,d\omega}{z-\omega-i0}=P\int\frac{\rho(\omega)\,d\omega}{z-\omega}$$



$$G^+ + G^- = \int \frac{\rho(\omega)\,d\omega}{z-\omega+i0} + \frac{\rho(\omega)\,d\omega}{z-\omega-i0} = 2P\int \frac{\rho(\omega)\,d\omega}{z-\omega}$$

III

$$\boxed{\phi = \frac{1}{z} - 2\pi n \qquad z \in \mathcal{C}}$$



$$G^+ + G^- = \int \frac{\rho(\omega)\,d\omega}{z-\omega+i0} + \frac{\rho(\omega)\,d\omega}{z-\omega-i0} = 2P\int \frac{\rho(\omega)\,d\omega}{z-\omega}$$

III

$$\boxed{\phi = \frac{1}{z} - 2\pi n \qquad z \in \mathbb{C}}$$

$$2g^2 G'(0) = E$$

$$\boxed{\frac{\Delta G}{2\pi i} = \frac{G^+ - G^-}{2\pi i} = \rho(z)}$$

$$G^+ + G^- = \int \frac{\rho(\omega)\,d\omega}{z-\omega+i0} + \frac{\rho(\omega)\,d\omega}{z-\omega-i0} = 2P\int \frac{\rho(\omega)\,d\omega}{z-\omega}$$

|||

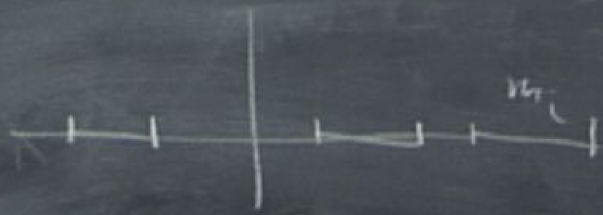
$$\boxed{\phi = \frac{1}{z} - 2\pi n \qquad z \in \mathcal{C}}$$

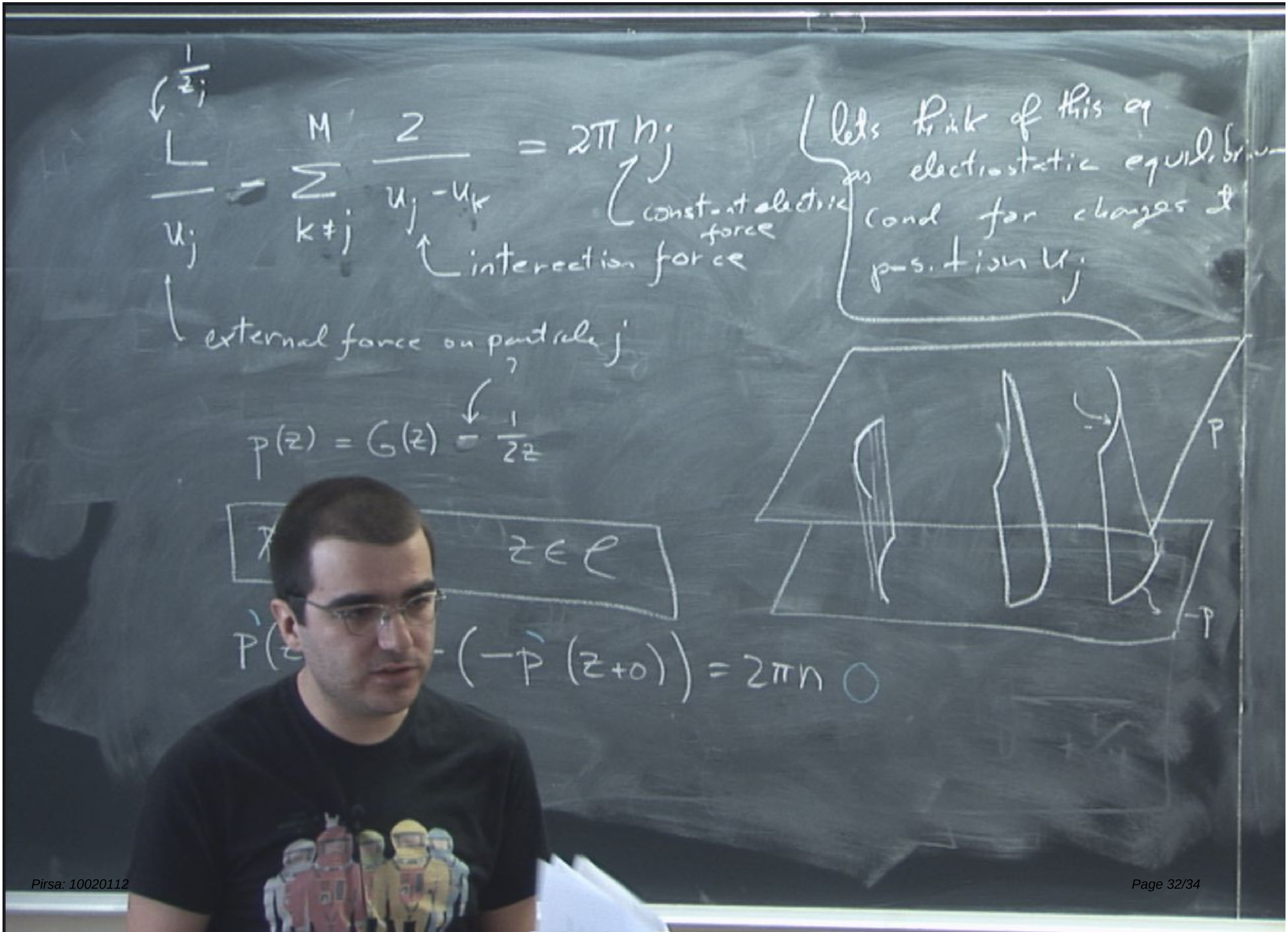
$$\boxed{-2g^2 G'(0) = E}$$

||

$$\frac{1}{L}\sum_{j=1}^{M}\frac{2g^2}{z_j^2} = E \checkmark$$

$$\boxed{\frac{\Delta G}{2\pi i} = \frac{G^+ - G^-}{2\pi i} = \rho(z)}$$





$$\overset{\frac{1}{z_j}}{\frac{L}{u_j}} - \sum_{k\neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

external force on particle j

interaction force

constant electric force

lets think of this eq as electrostatic equilibrium cond for charges at position u_j

$$p(z) = G(z) - \overset{?}{\frac{1}{2z}}$$

$\boxed{X \quad z \in \mathcal{C}}$

$$p'(z \ldots - \left(-p'(z+0)\right) = 2\pi n$$ ○

P lives in 2-sheeted Riemann surf

Such that

$z \to 0 \quad p(z) \simeq -\frac{1}{2z} + \underset{\uparrow\, G'(0)}{\frac{E}{2g^2}}\, z$

$z \to \infty \quad P$

lets think of this eq
as electrostatic equilibrium
i.e. cond for charges at
position u_j

$$\boxed{\not{p} = 2\pi m \qquad z \in \mathcal{C}}$$

$$\grave{p}(z-i0) - \left(-\grave{p}(z+0)\right) = 2\pi n \;○$$

$$P = G - \frac{1}{2z}$$

$$\swarrow$$

$$\frac{M}{Lz}$$

) P lives in 2-sheeted Riemann surf

Such that

2) $z \to 0$ $\quad p(z) \simeq -\frac{1}{2z} + \frac{E}{2g^2} z$

$\uparrow G'(0)$

3) $z \to \infty$ $\quad p(z) \simeq \frac{M}{Lz} + \cdots$

4) $\oint_{e_i} p(z) = K_i$

$$\boxed{\not{p} = 2\pi m \qquad z \in \mathcal{C}}$$

$$p'(z - i0) - \left(-p'(z+0)\right) = 2\pi n$$

Lets think of this eq as electrostatic equilibrium cond for charges at positions u_j

$$P = G - \frac{1}{2z}$$

$$\overset{\swarrow}{\frac{M}{Lz}}$$