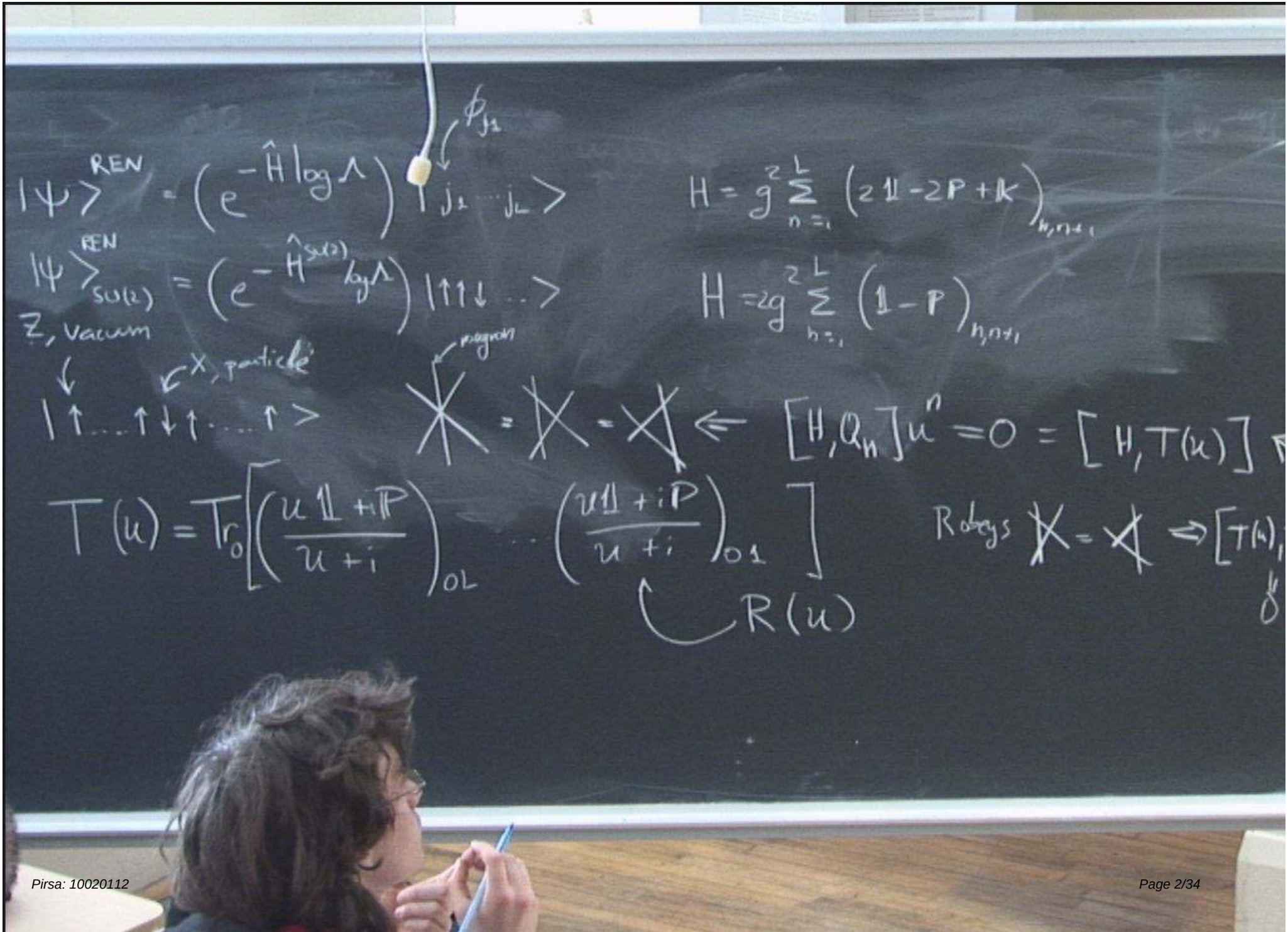
Title: Explorations in String Theory (PHYS 647) - Lecture 8

Date: Feb 24, 2010 11:20 AM

URL: http://pirsa.org/10020112

Abstract:

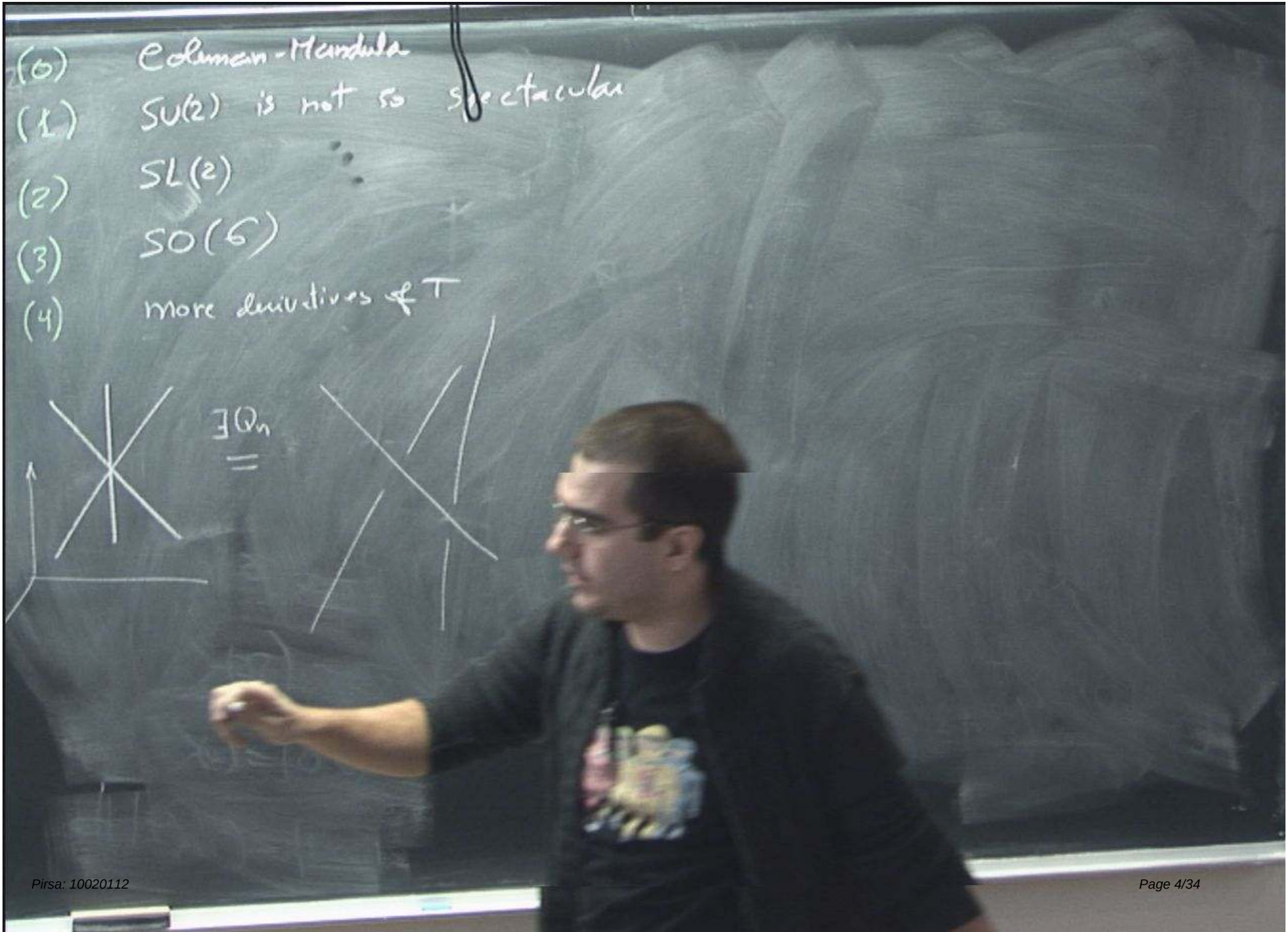

$|\psi\rangle^{REN} = \left(e^{-\hat{H}\log\Lambda}\right)|j_1 \dots j_L\rangle$
$H = g^2 \sum_{n=1}^{L} \left(2\mathbb{1} - 2P + K\right)_{n,n+1}$
$|\psi\rangle^{REN}_{SU(2)} = \left(e^{-\hat{H}^{SU(2)}\log\Lambda}\right)|\uparrow\uparrow\downarrow \dots\rangle$
$H = 2g^2 \sum_{n=1}^{L} \left(\mathbb{1} - P\right)_{n,n+1}$
Z, Vacuum
X, particle
$|\uparrow \dots \uparrow\downarrow\uparrow \dots \uparrow\rangle$
$[H, Q_n]u^n = 0 = [H, T(u)]$
$T(u) = Tr_0\left[\left(\frac{u\mathbb{1} + iP}{u+i}\right)_{0L} \dots \left(\frac{u\mathbb{1} + iP}{u+i}\right)_{01}\right]$
$R(u)$

$$H^{SU(2)} = T^{-1}(u)\,\frac{d}{du}\,T(u)\Big|_{u=0}$$

magnon factorization

$$e^{ip_j L}\prod_{k\neq j}^{M} S(p_j,p_k) = 1 \qquad E = \sum_{j=1}^{M}\epsilon(p_j)$$

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^{L} = \prod_{k\neq j}^{M}\frac{u_j-u_k+i}{u_j-u_k-i} \qquad , \qquad E = \sum_{j=1}^{M}\frac{2g^2}{u_j^2+1/4}$$


(0) Coleman-Mandula
(1) SU(2) is not so spectacular
(2) SL(2)
(3) SO(6)
(4) more derivatives of T
∃Qn
=

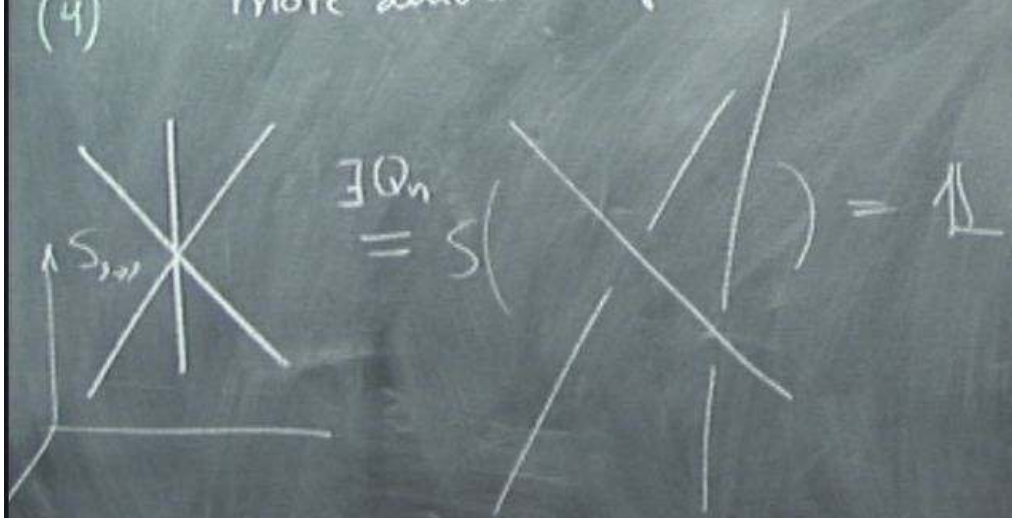
(0) Coleman-Mandula

(1) SU(2) is not so spectacular

(2) SL(2)

(3) SO(6)

(4) more derivatives of T



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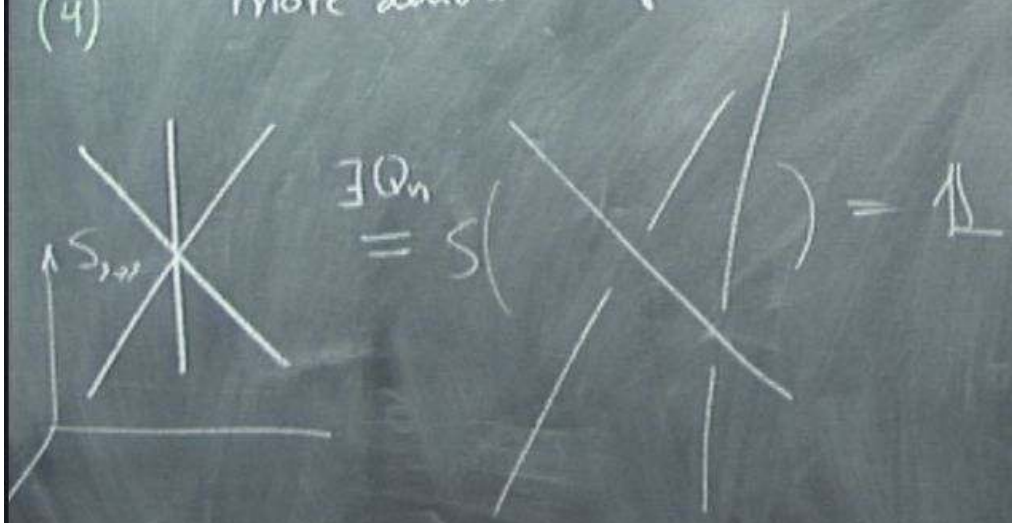

(1) We only have P

(2) SL(2) sector

$|Z\ ZDZZ\ldots ZDZ\ldots ZDZ\ldots\rangle$

(particle

↑ light cone cov. derivative

(0) Coleman-Mandula

(1) SU(2) is not so spectacular

(2) SL(2)

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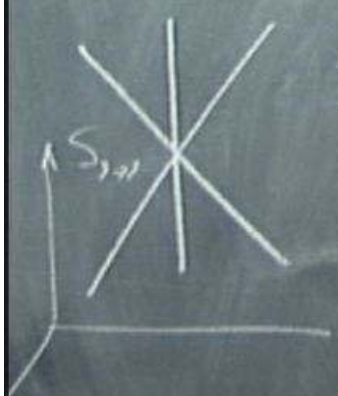

$\exists Q_n = S($ $) = \mathbb{1}$

(1) We only have $\mathbb{P}$

(2) SL(2) sector

particle

$|Z \ldots ZDZZ \ldots ZDZ \ldots ZD^2Z \ldots\rangle$

light cone cov. derivative

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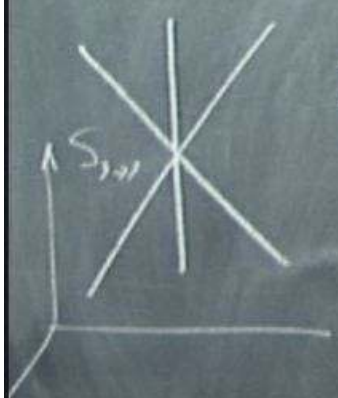
(4) more derivatives of T



$\exists Q_n$

$= S($ $) = \mathbb{1}$

(1) We only have $\mathbb{P}$

(2) SL(2) sector

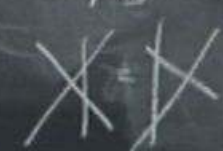
particle

$|Z \ldots ZDZZ \ldots ZDZ \ldots ZD^2Z \ldots\rangle$

light cone cov. derivative

(3) $R(u) \sim f(u)\mathbb{1} - \mathbb{P} + g(u)\mathbb{K}$

YB $\Rightarrow R(u) = u\mathbb{1} - \mathbb{P} + \frac{2u}{2u+2-M}\mathbb{K}$



(0) Coleman-Mandula

(1) SU(2) is not so spectacular

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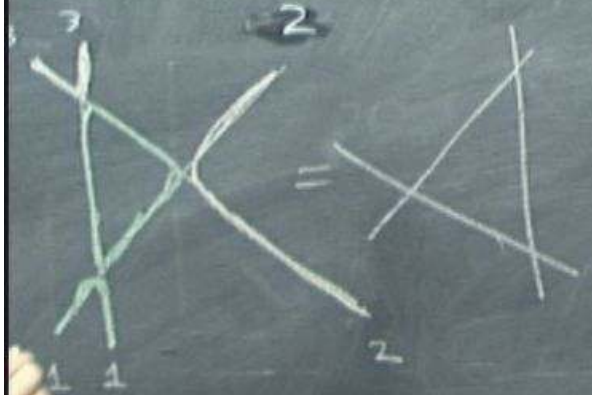
(3) SO(6)

(4) more derivatives of T



(1) We only have $\mathbb{P}$

(2) SL(2) sector

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$$|Z\ldots ZDZZ\ldots ZDZ\ldots ZD^2Z\ldots\rangle$$

light cone cov. derivative

(3) $R(u) \sim f(u)\mathbb{1} - \mathbb{P} + g(u)\mathbb{K}$

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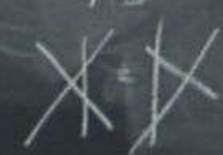


SO(M)

$$H = \frac{d^2}{du^2} \log T \Big|_{u=0} = \sum_{n=1}^{L} X + \cancel{X} \qquad H = \frac{d}{du} \log T \Big|_{u=0} = \Sigma X$$

$$\left(\frac{u_j + i/2}{u_j - i/2} \right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k + i}{u_j - u_k - i} \qquad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}$$

SL(2)

$$H' = \frac{d^2}{du^2} \log T \Big|_{u=0} = \sum_{n=1}^{L} \chi + \chi \qquad H = \frac{d}{du} \log T \Big|_{u=0} = \sum \chi$$

$$\left(\frac{u_j + i/2}{u_j - i/2} \right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k + i}{u_j - u_k - i} \qquad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}$$

$SL(2)$

$$H' = \frac{d^2}{du^2} \log T \Big|_{u=0} = \sum_{n=1}^{L} \underline{X} + \underline{\mathbb{X}} \qquad H = \frac{d}{du} \log T \Big|_{u=0} = \sum \underline{X}$$

$$E_{|\psi\rangle \text{ BSF-4 sym}} \longleftrightarrow E_{\text{single strings in } AdS_5 \times S^5}$$

$$\boxed{\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k \pm i}{u_j - u_k \mp i}, \quad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}}$$

SL(2)

$$H^{(1)} = \frac{d^2}{du^2} \log T \Big|_{u=0} = \sum_{n=1}^{L} X + \bar{X} \qquad H = \frac{d}{du} \log T \Big|_{u=0} = \sum X$$

$$E_{|\psi\rangle \text{ ISG-system}} \longleftrightarrow E_{\text{single strings in } AdS_5 \times S^5}$$

$$e^{iPL} = 1$$

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k + i}{u_j - u_k - i} \qquad E = \sum_{j=1}^{M} \frac{2g^2}{u_j}$$

$SL(2)$

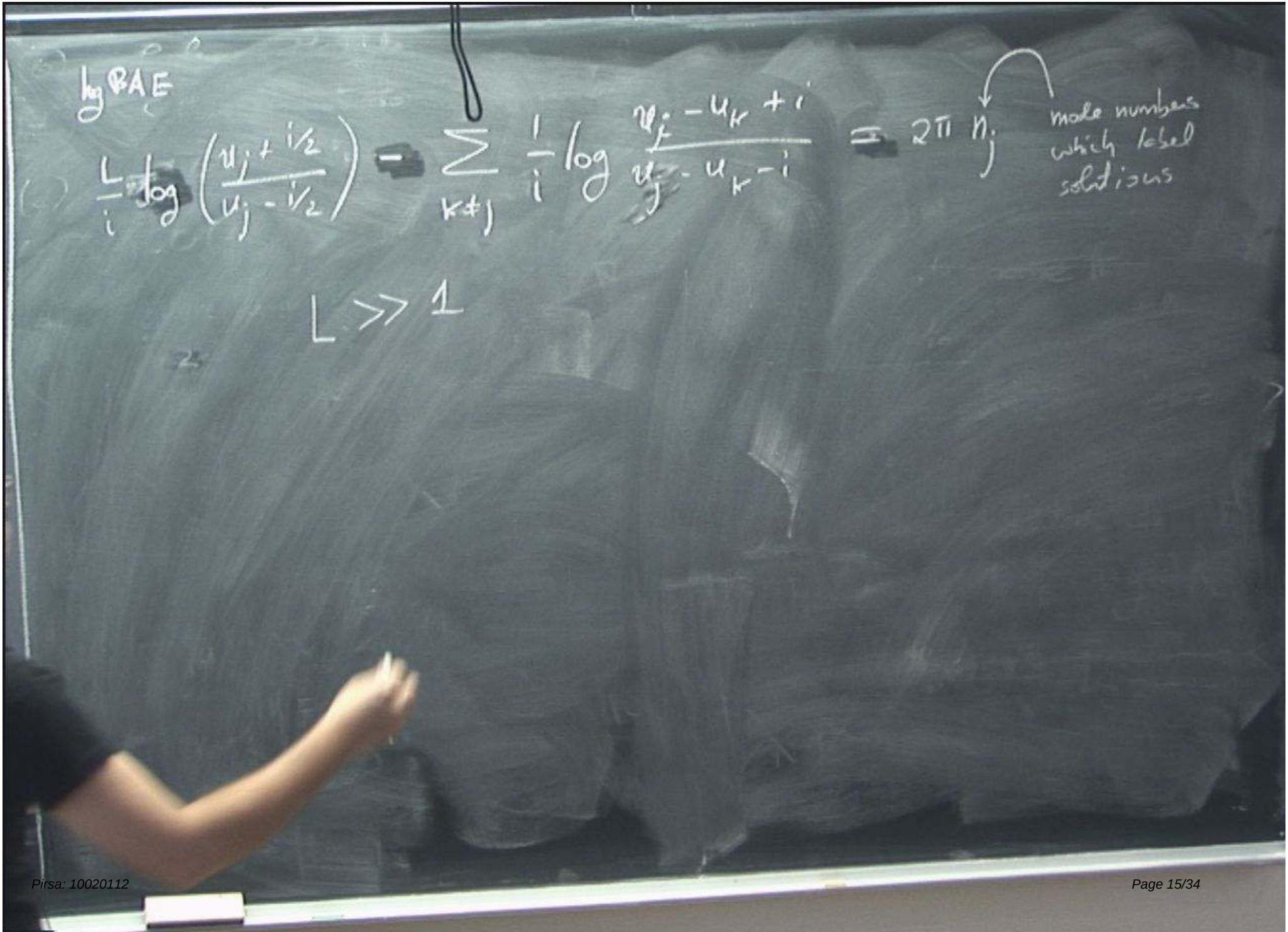
$$H' = \frac{d^2}{du^2} \log T \Big|_{u=0} = \sum_{n=1}^{L} \underline{X} + \underline{\mathbb{X}} \qquad H = \frac{d}{du} \log T \Big|_{u=0} = \sum \underline{X}$$

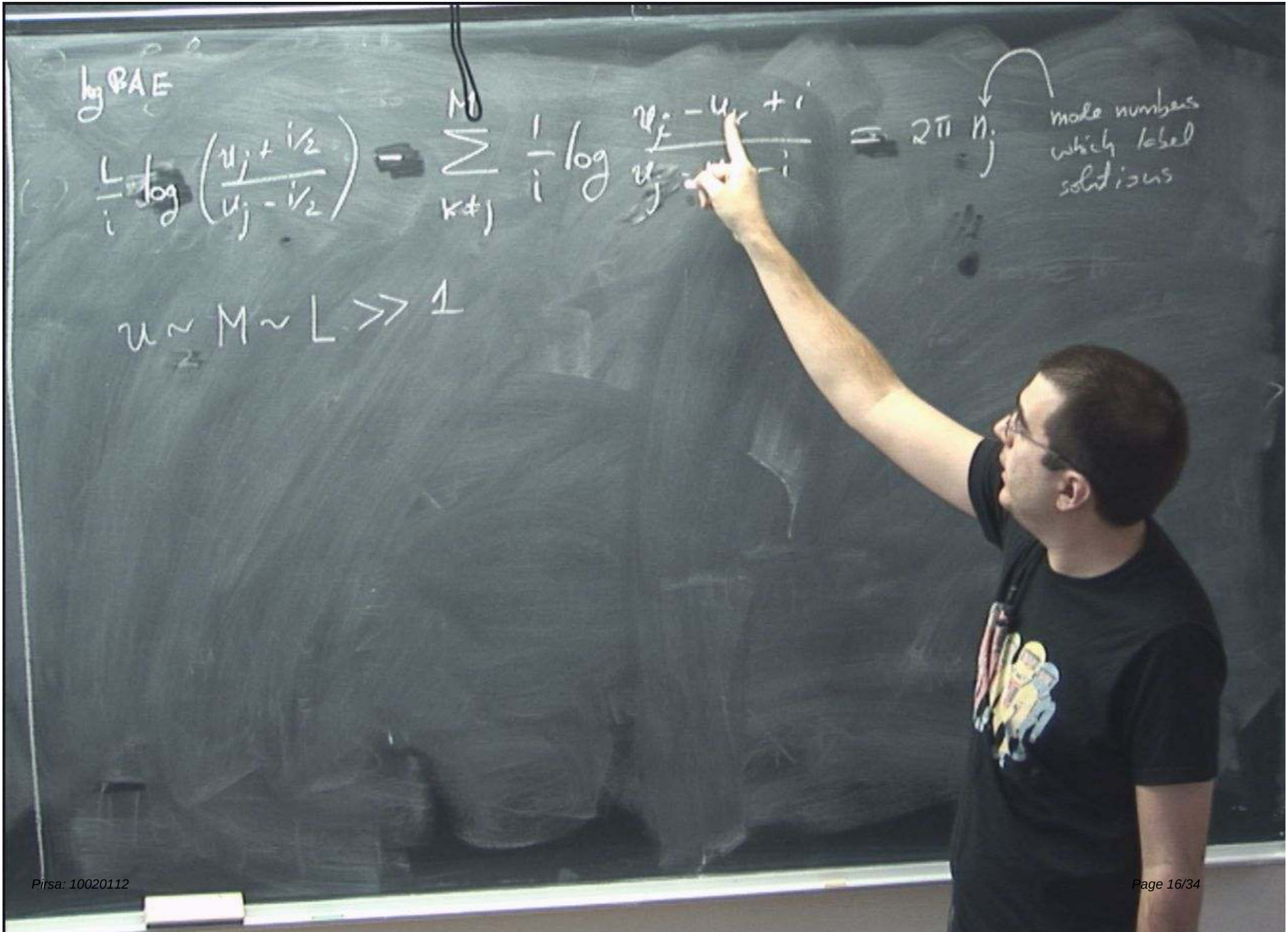
$$E_{|\psi\rangle \text{ BPS-4 sym}} \longleftrightarrow E_{\text{single strings in } AdS_5 \times S^5}$$

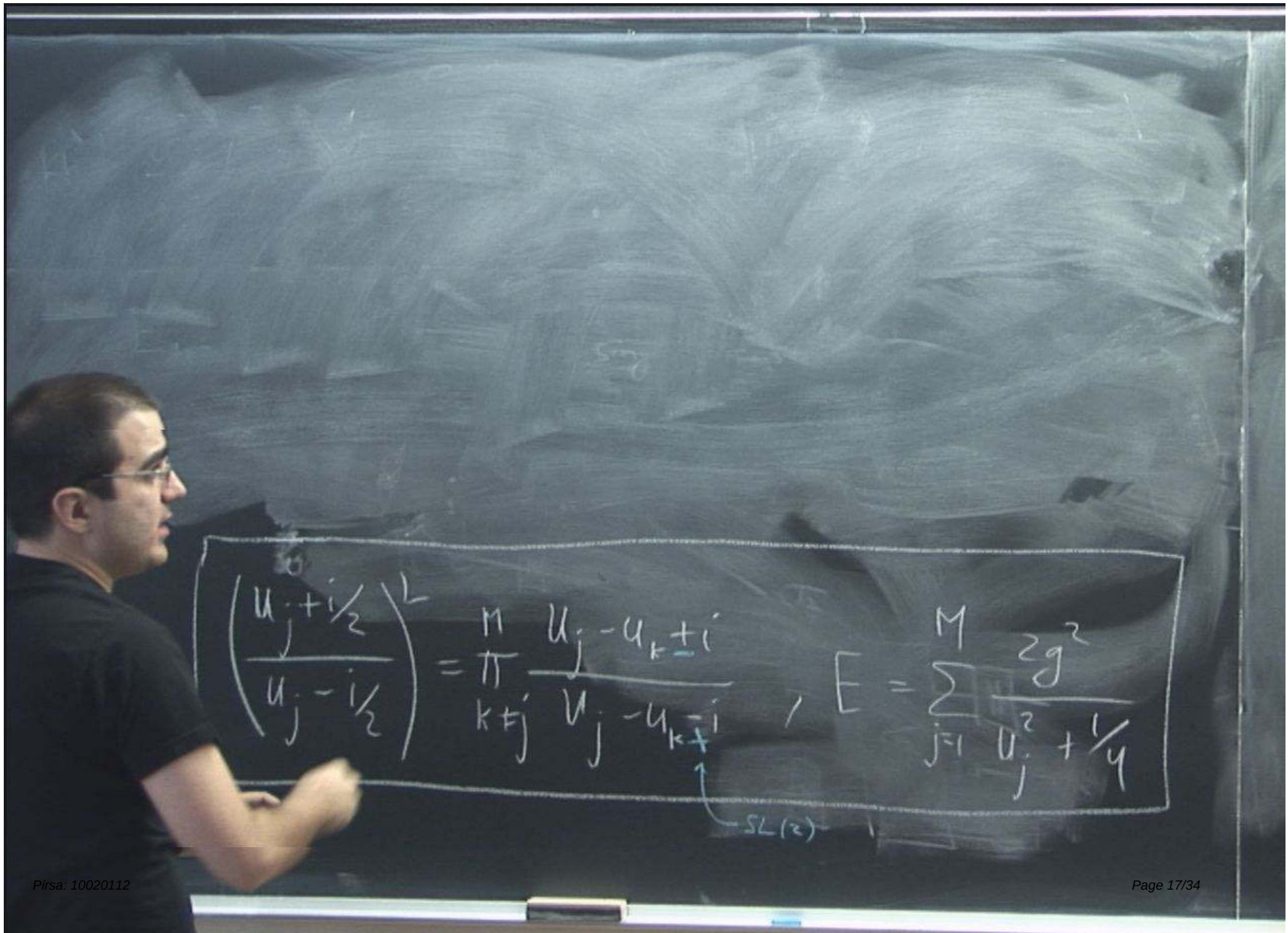
$$e^{ipL} = 1$$

$$\boxed{\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k \pm i}{u_j - u_k \mp i}, \quad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}}$$

SL(2)


log BAE
$\frac{L}{i}\log\left(\frac{u_j + i/2}{u_j - i/2}\right) - \sum_{k\neq j} \frac{1}{i}\log\frac{u_j - u_k + i}{u_j - u_k - i} = 2\pi n_j$
mode numbers which label solutions
$L \gg 1$

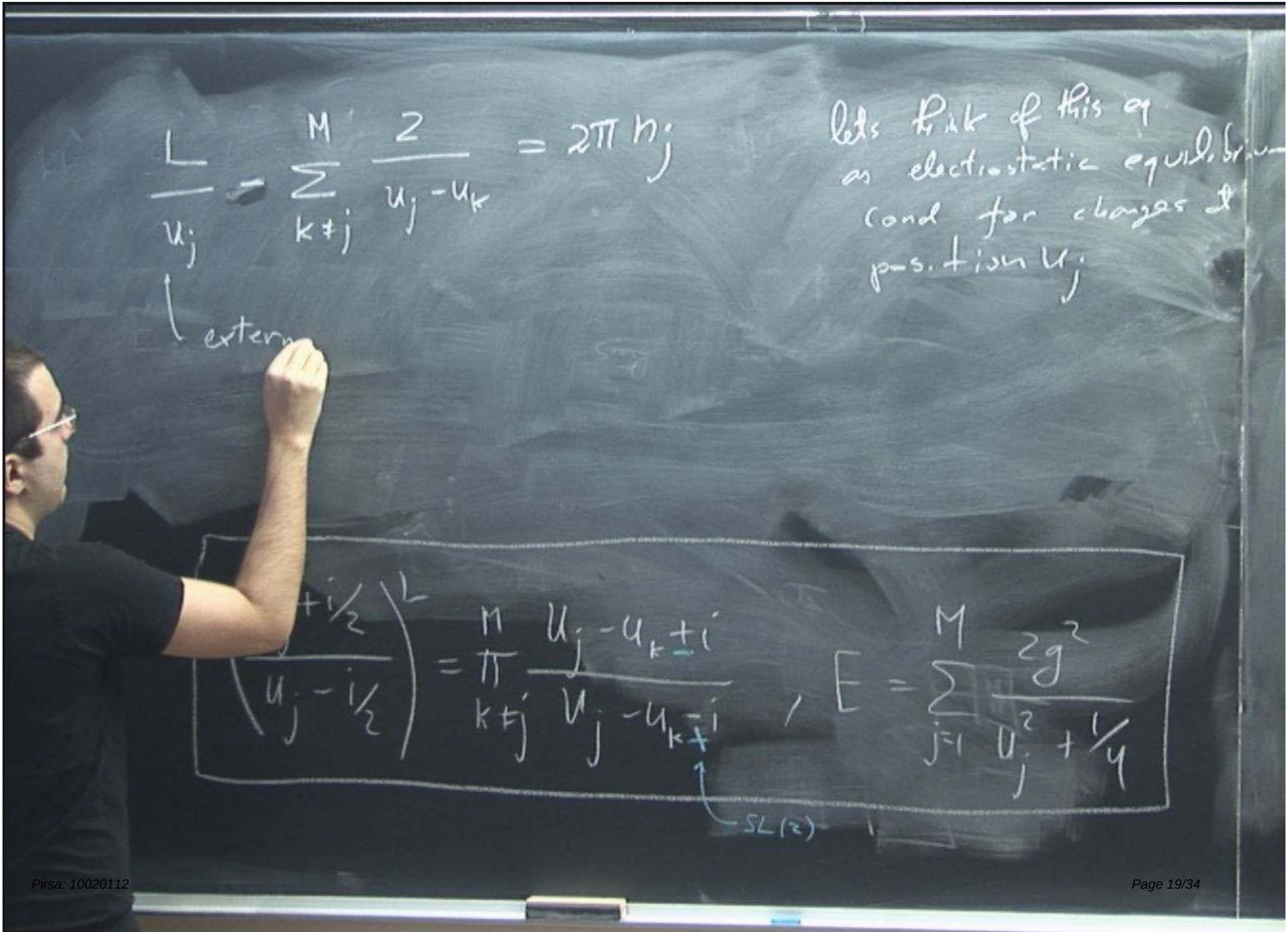

log BAE
$\frac{L}{i}\log\left(\frac{u_j+i/2}{u_j-i/2}\right) - \sum_{k\neq j}^{M}\frac{1}{i}\log\frac{u_j-u_k+i}{u_j-u_k-i} = 2\pi n_j$
mode numbers which label solutions
$u \sim M \sim L >> 1$


$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k + i}{u_j - u_k - i}$
$E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}$
SL(2)

$$\frac{L}{u_j} - \sum_{k\neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k\neq j}^{M} \frac{u_j - u_k + i}{u_j - u_k - i} \qquad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}$$

SL(2)


$\frac{L}{u_j} = \sum_{k \neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$
lets think of this eq as electrostatic equil. bw cond for charges at position u_j
extern
$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k + i}{u_j - u_k - i}$
$E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}$
SL(2)

$$\frac{L}{u_j} - \sum_{k \neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

u_j: external force on particle j

$\sum_{k \neq j}$: interaction force

$2\pi n_j$: constant electric force

Let's think of this eq as electrostatic equilibrium cond for charges at position u_j

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{M} \frac{u_j - u_k + i}{u_j - u_k - i} \qquad E = \sum_{j=1}^{M} \frac{2g^2}{u_j^2 + 1/4}$$

$SL(2)$

$$\frac{L}{u_j} - \sum_{k \neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

n_j: constant electric force

u_j: external force on particle j

$u_j - u_k$: interaction force

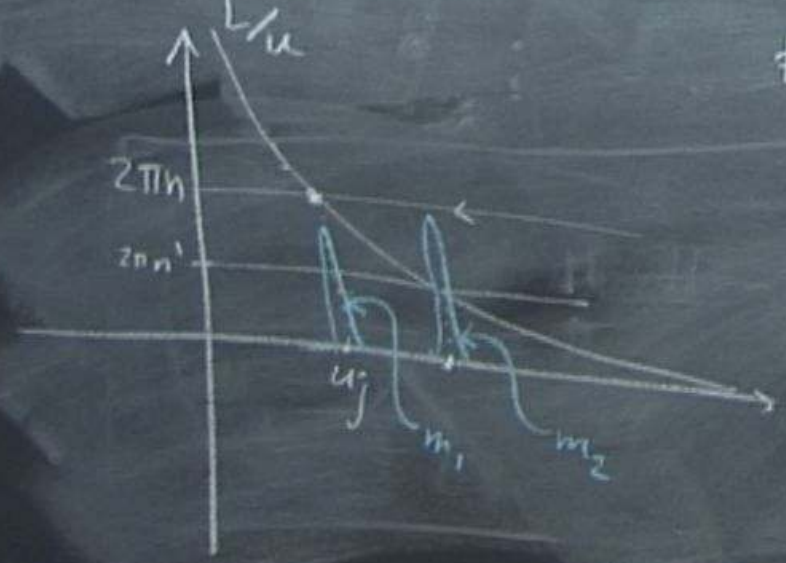
lets think of this eq as electrostatic equilibrium cond for charges at position u_j

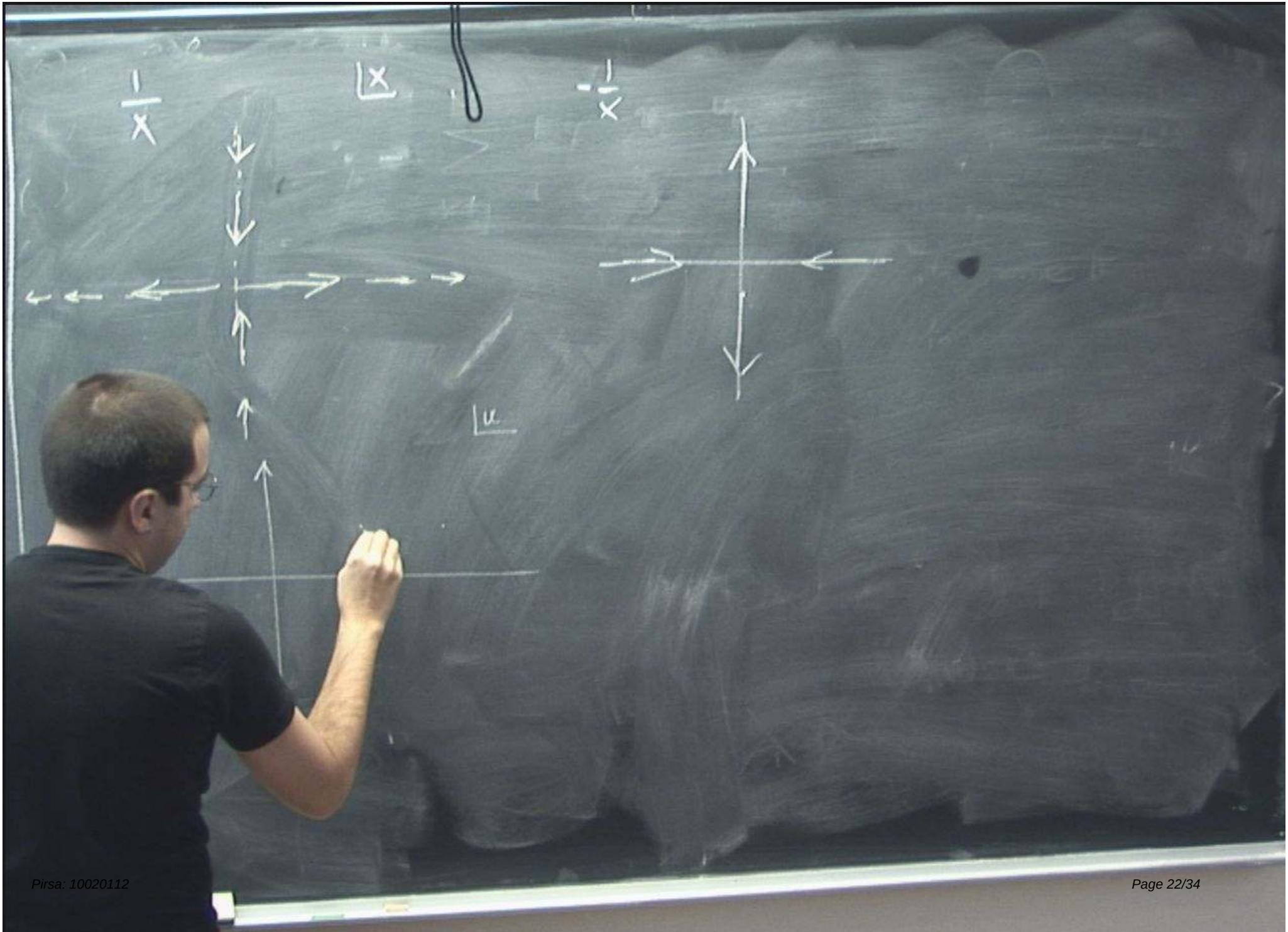
1) turnoff interactions

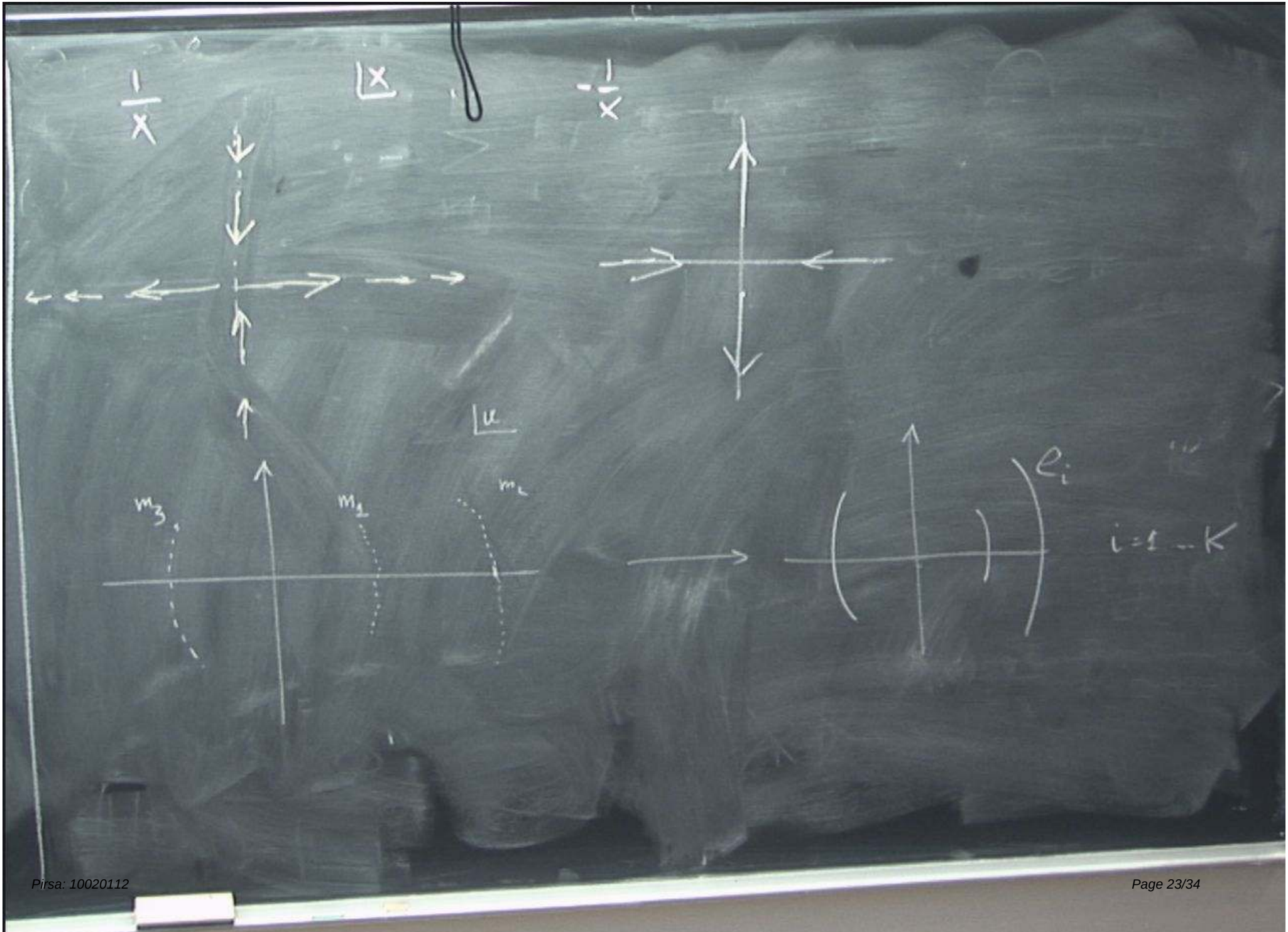
$$\{n_j\} = \{\underbrace{m_1, m_1 \dots m_1}_{M_1}, \underbrace{m_2, \dots, m_2}_{M_2}, \dots\}$$

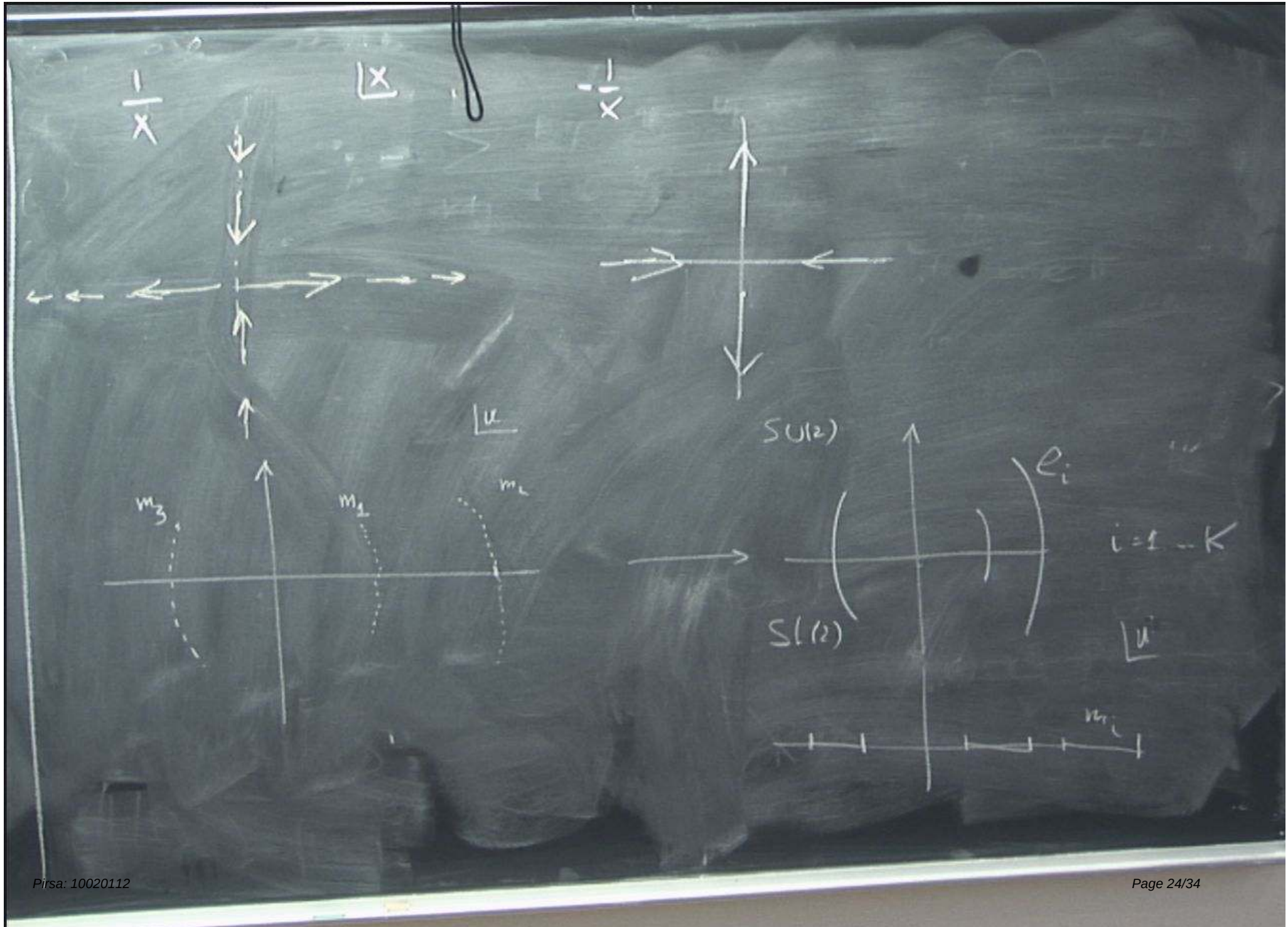
$$\#\text{groups} \longrightarrow K$$

$$\sum_{j=1}^{K} M_j = M$$




1/x
|x
-1/x
|u


1/x
|x
-1/x
|u
m_3
m_1
m_2
e_i
i = 1 .. K



1/x
x
-1/x
u
m3
m1
m2
SU(2)
SL(2)
e_i
i = 1 .. K
u
m_i

$$\frac{L}{u_j} - \sum_{k \neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

u_j: external force on particle j

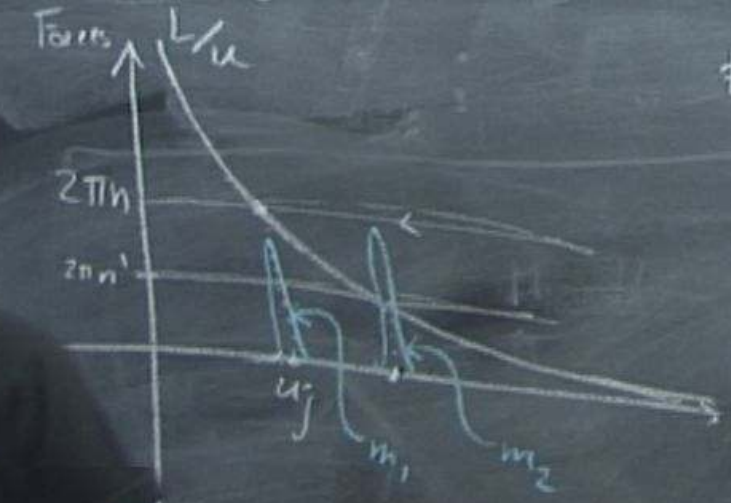
$\frac{2}{u_j - u_k}$: interaction force

$2\pi n_j$: constant electric force

lets think of this eq as electrostatic equilibrium cond for charges at position u_j

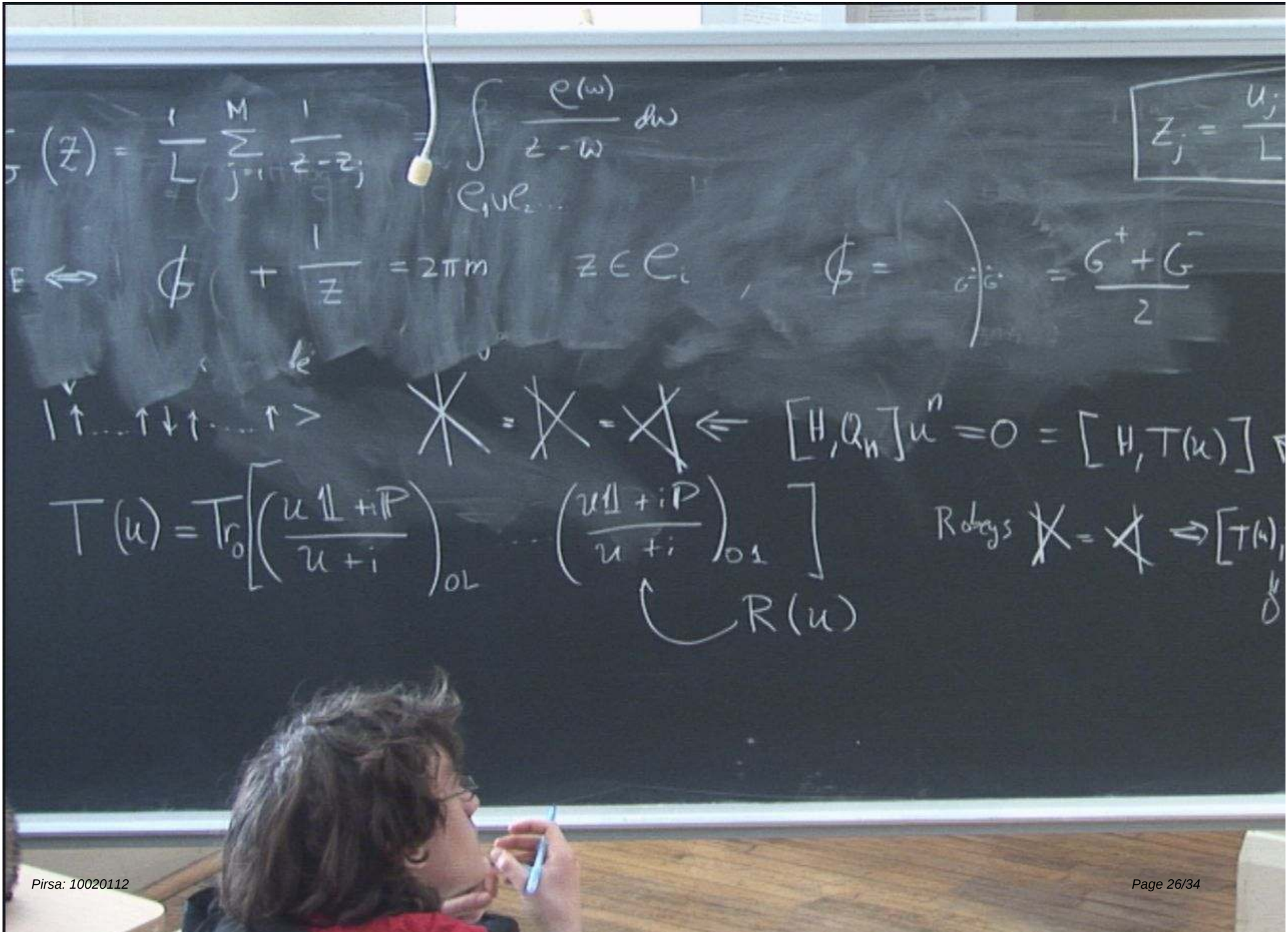
1) Thomson interactions

$$\{n_j\} = \{\underbrace{m_1, m_1, \dots m_1}_{M_1}, \underbrace{m_2, \dots, m_2}_{M_2}, \dots\}$$



groups $\longrightarrow k$

$$\sum_{j=1}^{k} M_j = M$$


$\frac{1}{L}\sum_{j=1}^{M}\frac{1}{z-z_j}$
$\int \frac{\rho(\omega)}{z-\omega}\,d\omega$
$z_j = \frac{u_j}{L}$
$\phi + \frac{1}{z} = 2\pi m \quad z \in C_i$
$\phi = \frac{G^+ + G^-}{2}$
$|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\rangle$
$[H, Q_n]u^n = 0 = [H, T(u)]$
$T(u) = Tr_0\left[\left(\frac{u\mathbb{1} + iP}{u+i}\right)_{0L} \cdots \left(\frac{u\mathbb{1} + iP}{u+i}\right)_{01}\right]$
$R(u)$

$$\frac{L}{u_j} - \sum_{k\neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$$

↑ external force on particle j

↑ interaction force

(constant electric force

(let's think of this eq as electrostatic equilib. cond for charges at position u_j

$$\{n_j\} = \{\underbrace{m_1, m_1, \dots m_1}_{M_1}, \underbrace{m_2, \dots, m_2}_{M_2}, \dots\}$$

$$\frac{1}{x \pm i0} = P\frac{1}{x} \mp i\pi\delta(x)$$

groups $\longrightarrow K$

$$\sum_{j=1}^{K} M_j = M$$

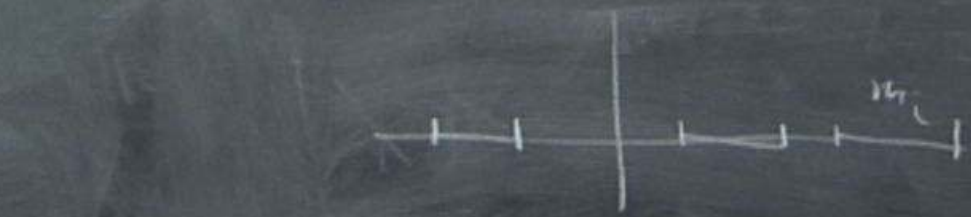
$\frac{1}{x+i0}$

$$\frac{G^{+}+G^{-}}{2} = \frac{1}{2}\int \frac{\rho(\omega)\,d\omega}{z-\omega+i0} + \frac{\rho(\omega)\,d\omega}{z-\omega-i0} = P\int \frac{\rho(\omega)\,d\omega}{z-\omega}$$

$$G^+ + G^- = \int \frac{\rho(\omega)\,d\omega}{z-\omega+i0} + \frac{\rho(\omega)\,d\omega}{z-\omega-i0} = 2P\int \frac{\rho(\omega)\,d\omega}{z-\omega}$$

III

$$\oint = \frac{1}{z} - 2\pi n \qquad z \in \mathcal{C}$$



$$G^+ + G^- = \int \frac{\rho(\omega)\,d\omega}{z-\omega+i0} + \frac{\rho(\omega)d\omega}{z-\omega-i0} = 2P\int \frac{\rho(\omega)\,d\omega}{z-\omega}$$

III

$$\oint = \frac{1}{z} - 2\pi n \qquad z \in \ell$$

$$2g^2 G'(0) = E$$

$$\frac{\Delta G}{2\pi i} = \frac{G^+ - G^-}{2\pi i} = \rho(E)$$

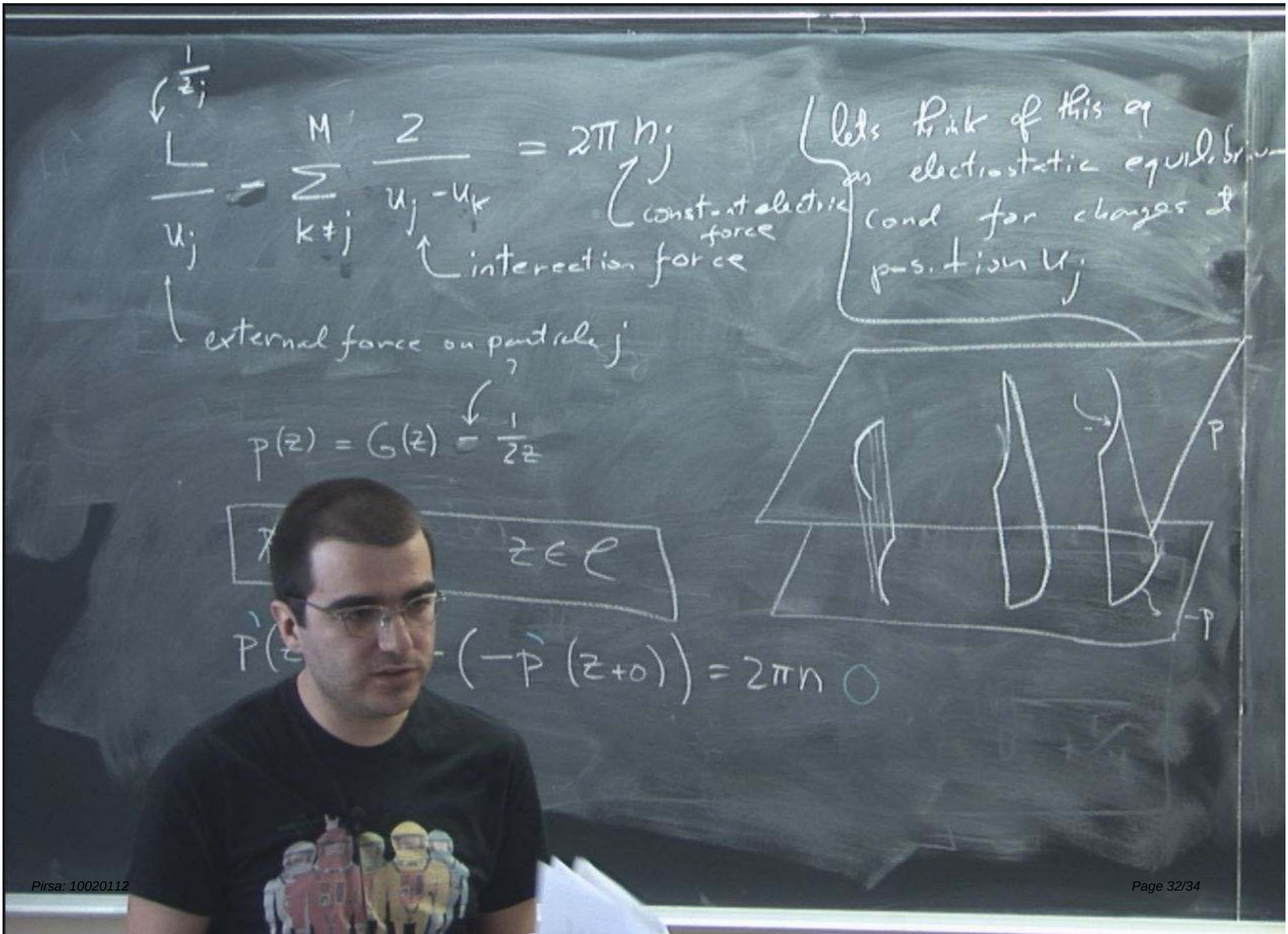
$$G^+ + G^- = \int \frac{\rho(\omega)\,d\omega}{z-\omega+i0} + \frac{\rho(\omega)\,d\omega}{z-\omega-i0} = 2P\int \frac{\rho(\omega)\,d\omega}{z-\omega}$$

$$\phi = \frac{1}{z} - 2\pi n \qquad z \in \rho$$

$$-2g^2 G'(0) = E$$

$$\frac{1}{L}\sum_{j=1}^{n} \frac{2g^2}{z_j^2} = E \checkmark$$

$$\frac{\Delta G}{2\pi i} = \frac{G^+ - G^-}{2\pi i} = \rho(E)$$



$\frac{1}{z_j}$
$\frac{L}{u_j} - \sum_{k \neq j}^{M} \frac{2}{u_j - u_k} = 2\pi n_j$
constant electric force
interaction force
external force on particle j
lets think of this eq as electrostatic equilibrium cond for charges at position u_j
$p(z) = G(z) - \frac{1}{2z}$
$z \in \mathcal{C}$
$p'(z$
$(-p'(z+0)) = 2\pi n$
p
-p

P lives in 2-sheeted Riemann surf

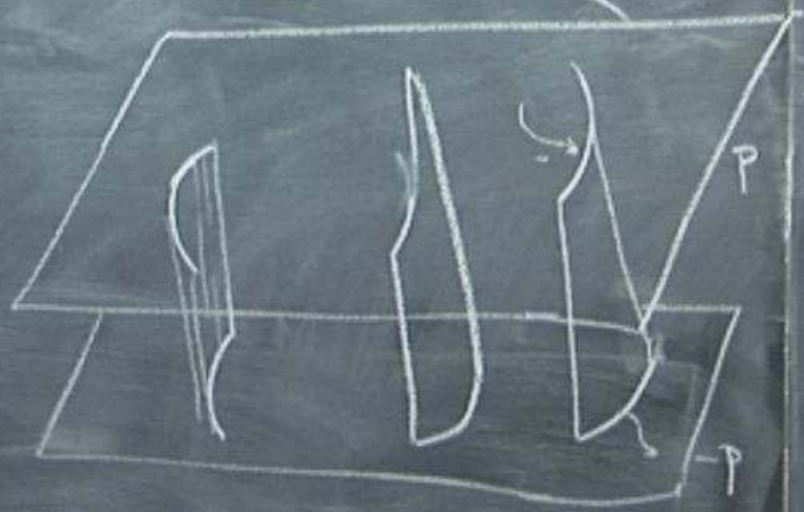
Such that

$z \to 0 \quad P(z) \simeq -\frac{1}{2z} + \frac{E}{2g^2} z$

$\uparrow G'(0)$

$z \to \infty \quad P$

lets think of this eq as electrostatic equilibrium cond for charges at position u_j



$$\boxed{\not{p} = 2\pi m \quad z \in \mathcal{C}}$$

$$P'(z-0) - (-P'(z+0)) = 2\pi n$$

$$P = G - \frac{1}{2z}$$

$\swarrow \frac{M}{Lz}$

p lives in 2-sheeted Riemann surf

such that

2) $z \to 0$ $\quad p(z) \simeq -\frac{1}{2z} + \frac{E}{2g^2} z$

$\uparrow G'(0)$

3) $z \to \infty$ $\quad p(z) \simeq \frac{M}{Lz} + \cdots$

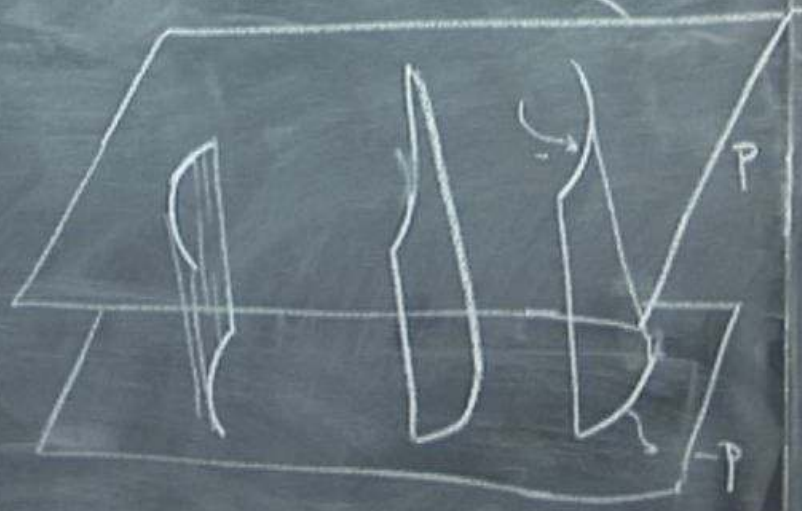
4) $\oint_{e_i} p(z) = K_i$

$$\boxed{\not{p} = 2\pi m \qquad z \in \mathcal{C}}$$

$$p'(z-0) - \left(-p'(z+0)\right) = 2\pi n$$

lets think of this eq as electrostatic equilibrium

ie cond for charges at position u_j



$$p = G - \frac{1}{2z}$$

$\swarrow \frac{M}{Lz}$