Title: Explorations in String Theory (PHYS 647) - Lecture 1

Date: Feb 16, 2010 11:20 AM

URL: http://pirsa.org/10020106

Abstract:

Jaume Gomis, 456

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Topic: Introduction to the AdS/CFT correspondence

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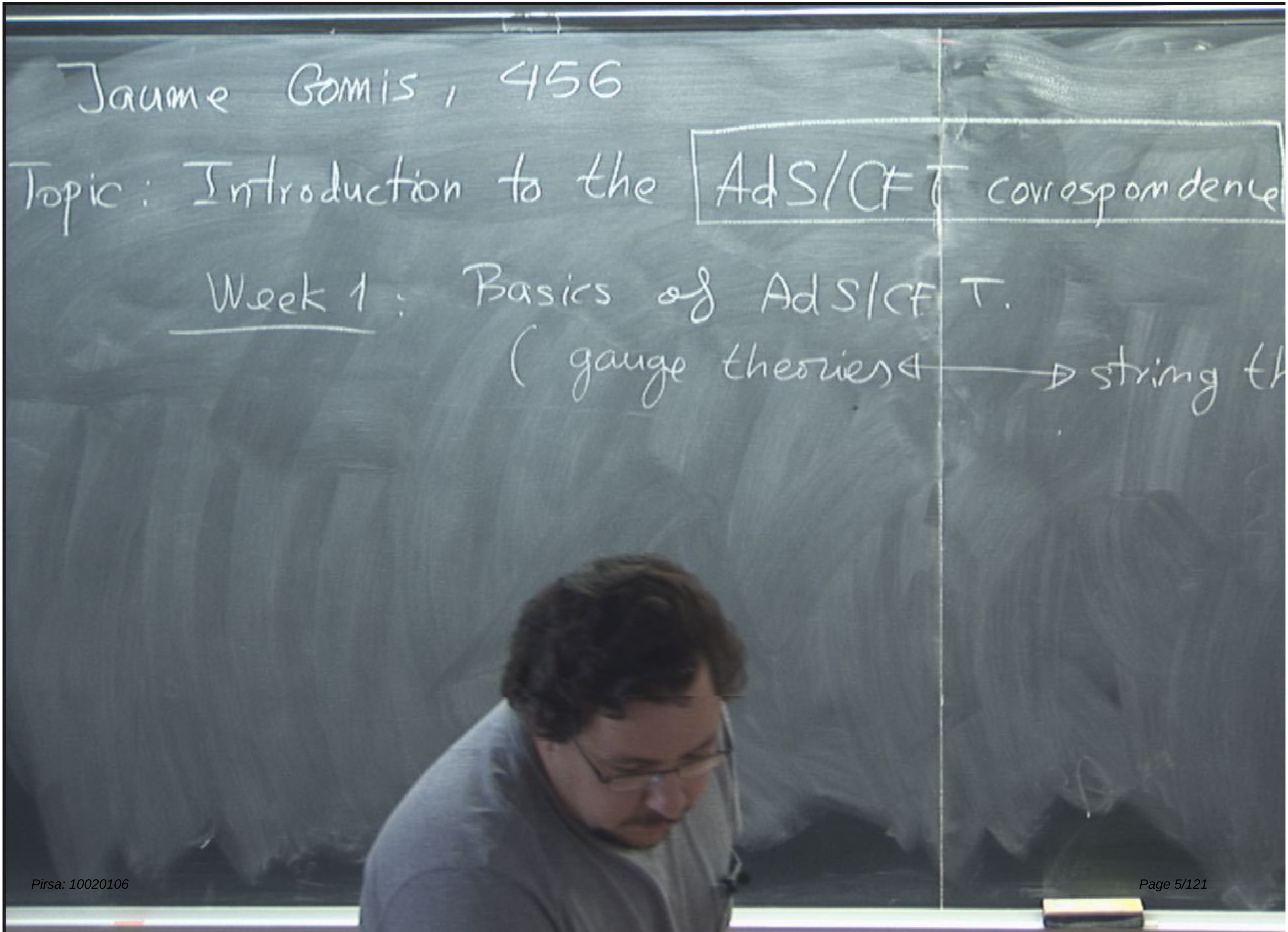
Topic: Introduction to the $\boxed{\text{AdS/CFT correspondence}}$

Week 1: Basics of AdS/CFT.

(gauge theories


Jaume Gomis, 456
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(gauge theories $\longleftrightarrow$ string theory

$\uparrow$

gravity

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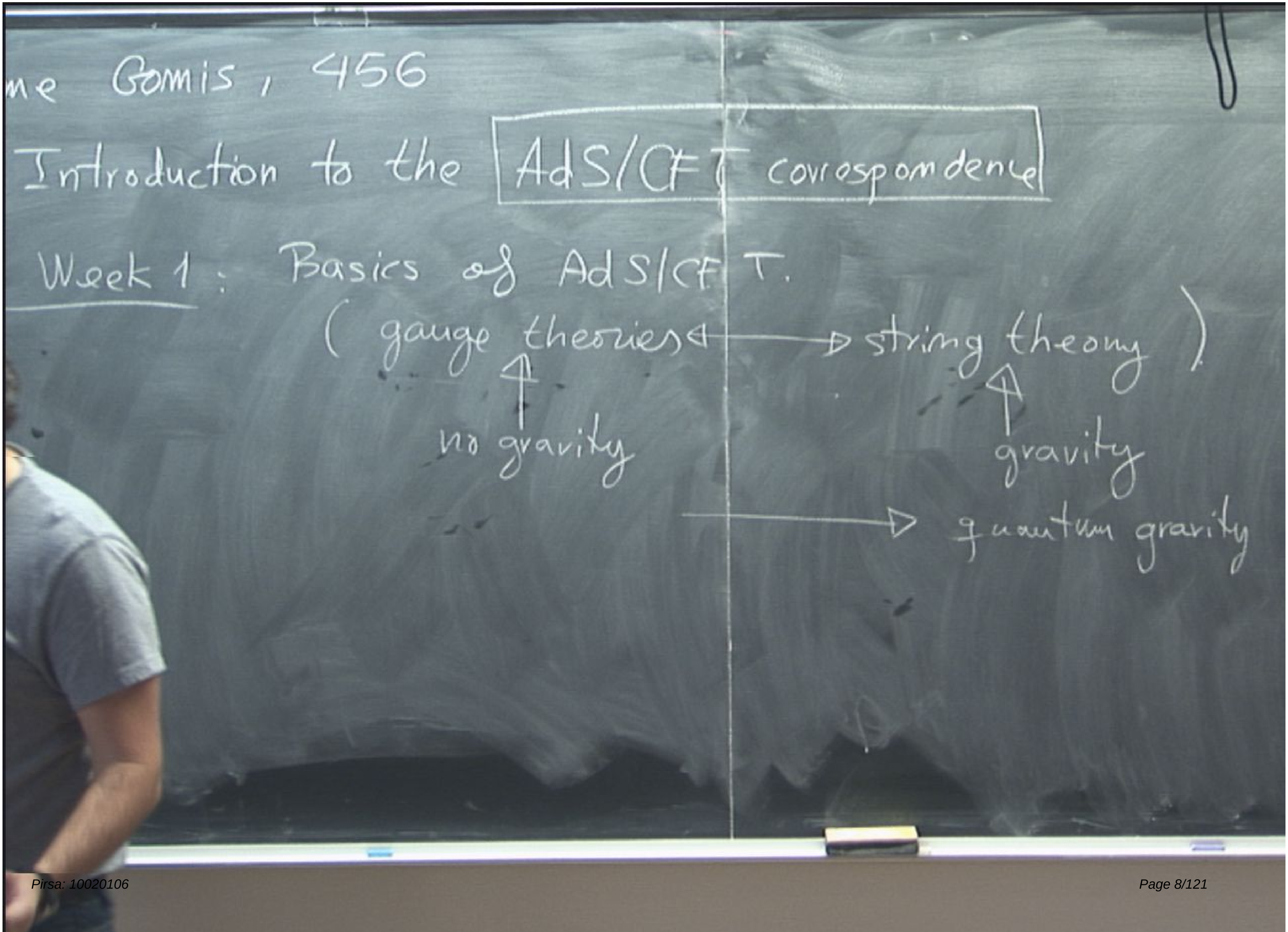
no gravity

gravity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Introduction to the AdS/CFT correspondence
Week 1: Basics of AdS/CFT.
(gauge theories ⟷ string theory)
no gravity
gravity
quantum gravity

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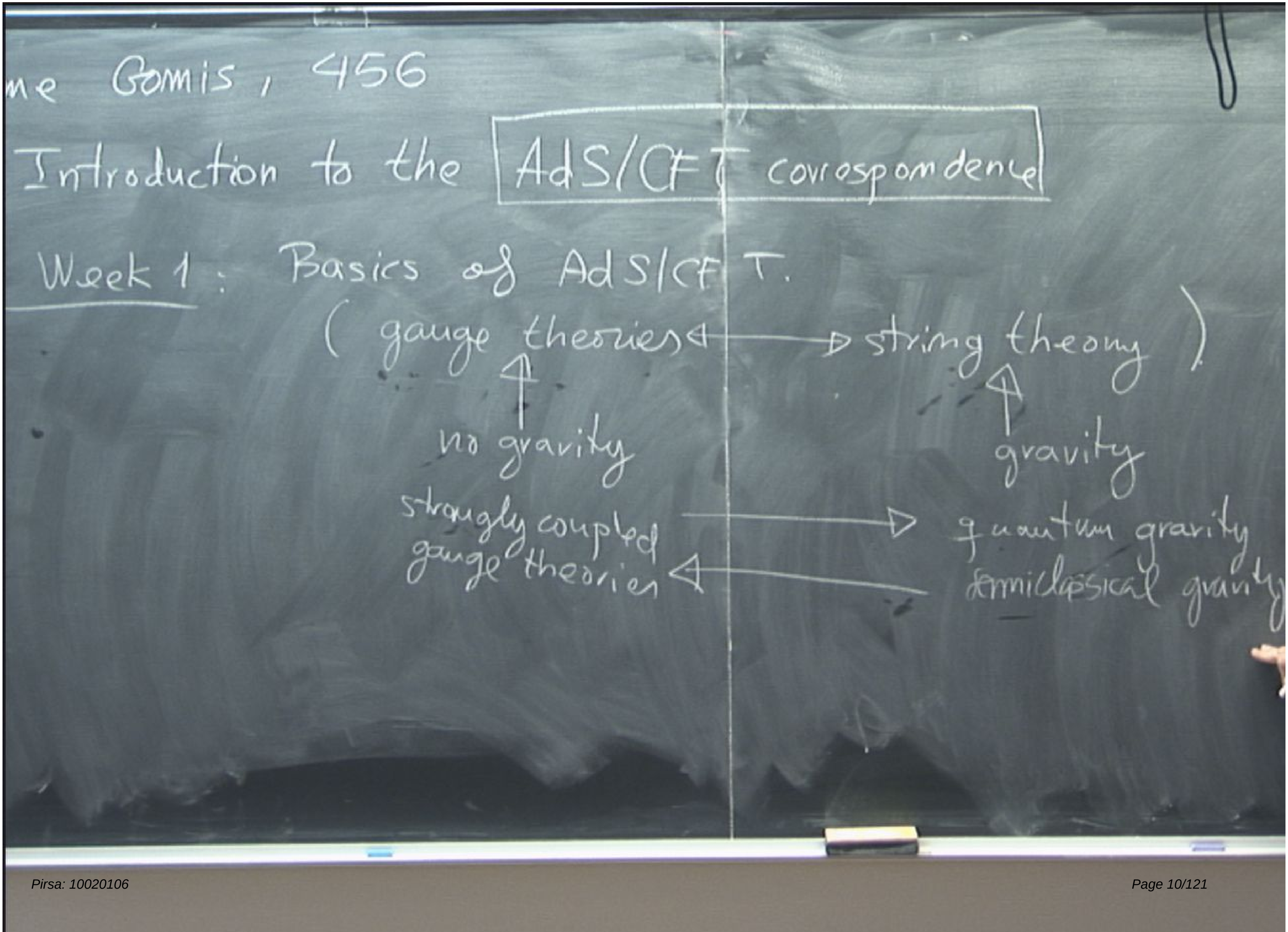
$\longrightarrow$ quantum gravity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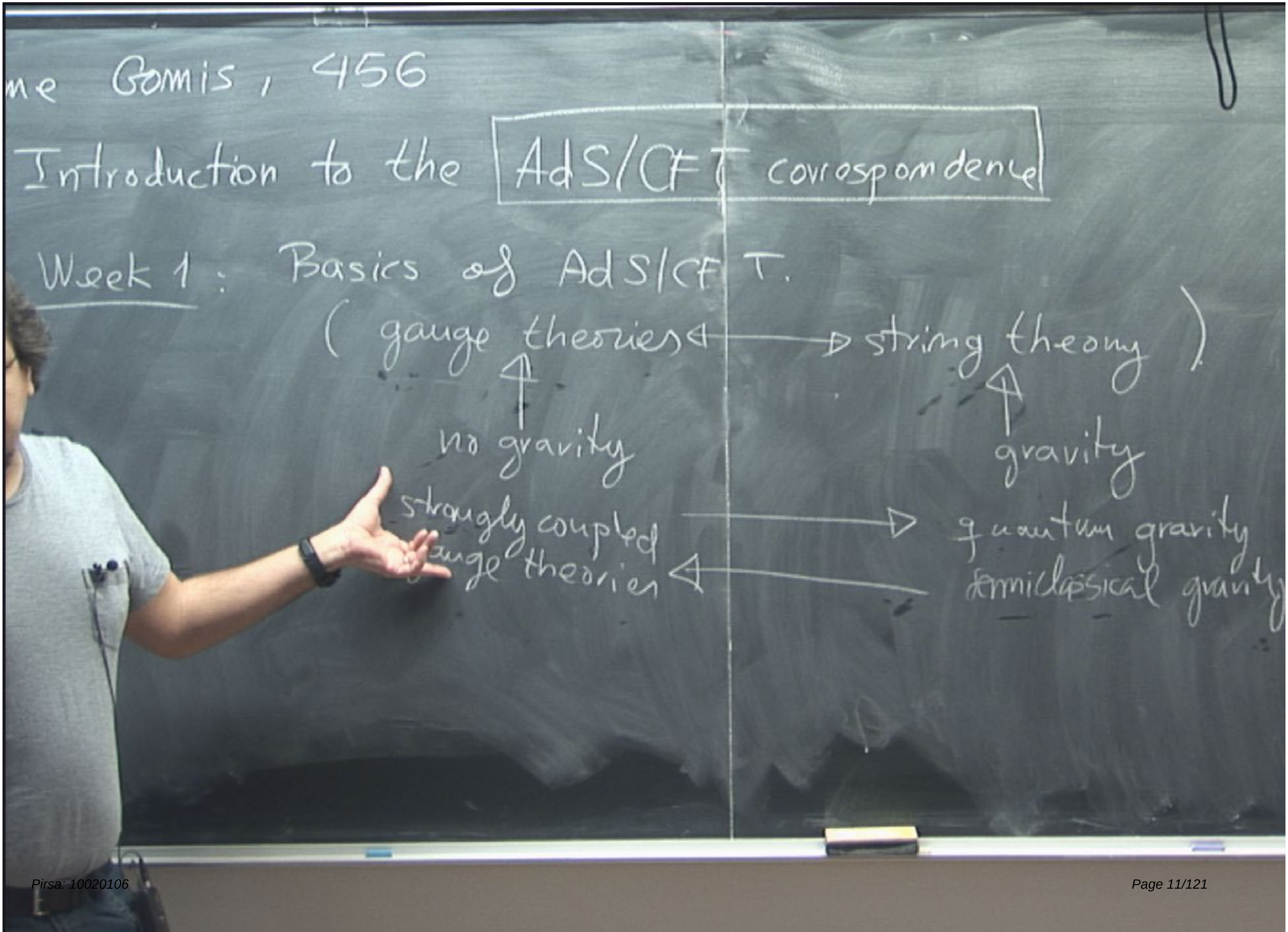
Introduction to the AdS/CFT corrospondence
Week 1: Basics of AdS/CFT.
(gauge theories ⟷ string theory)
no gravity
gravity
strongly coupled gauge theories
quantum gravity
semiclassical gravity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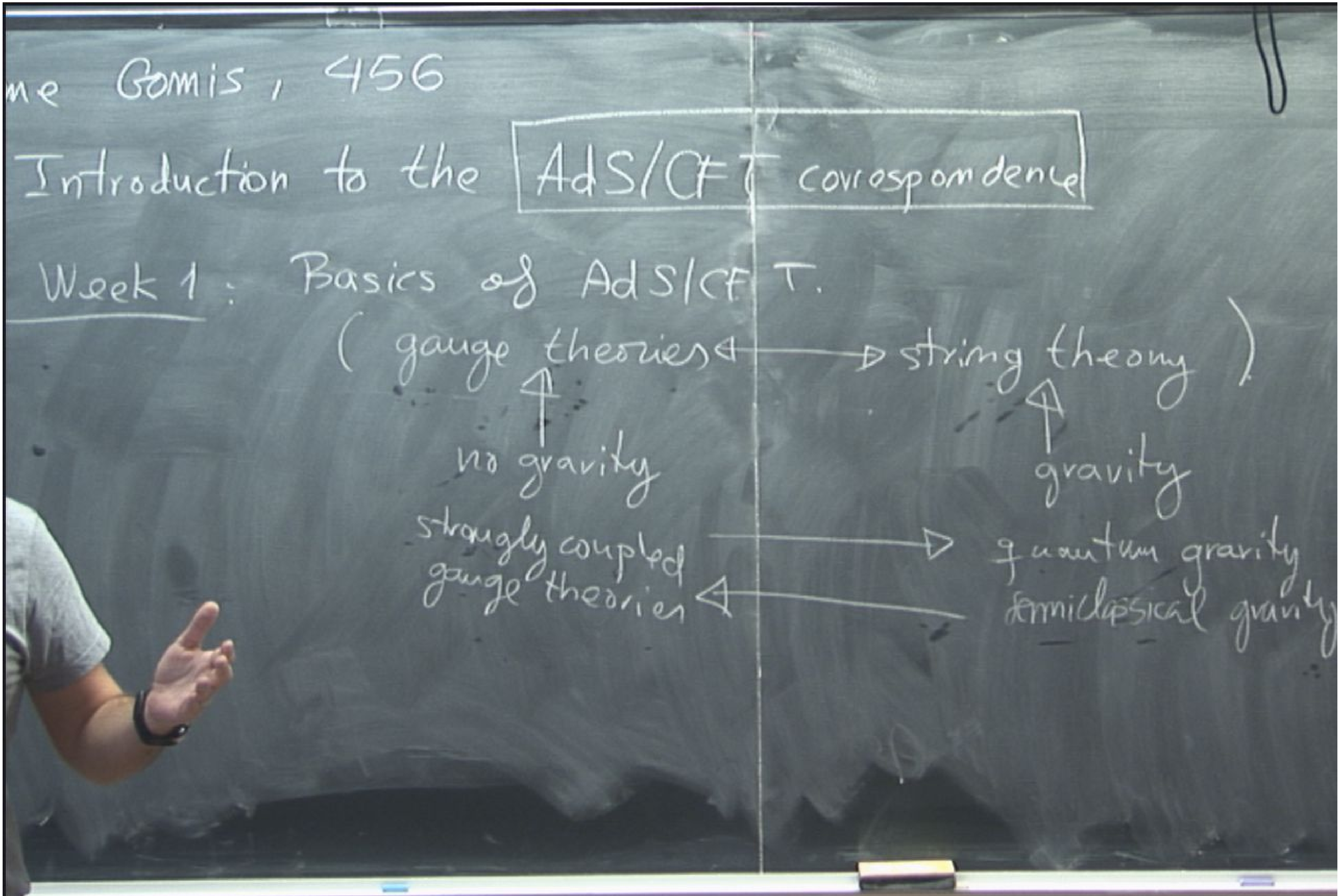
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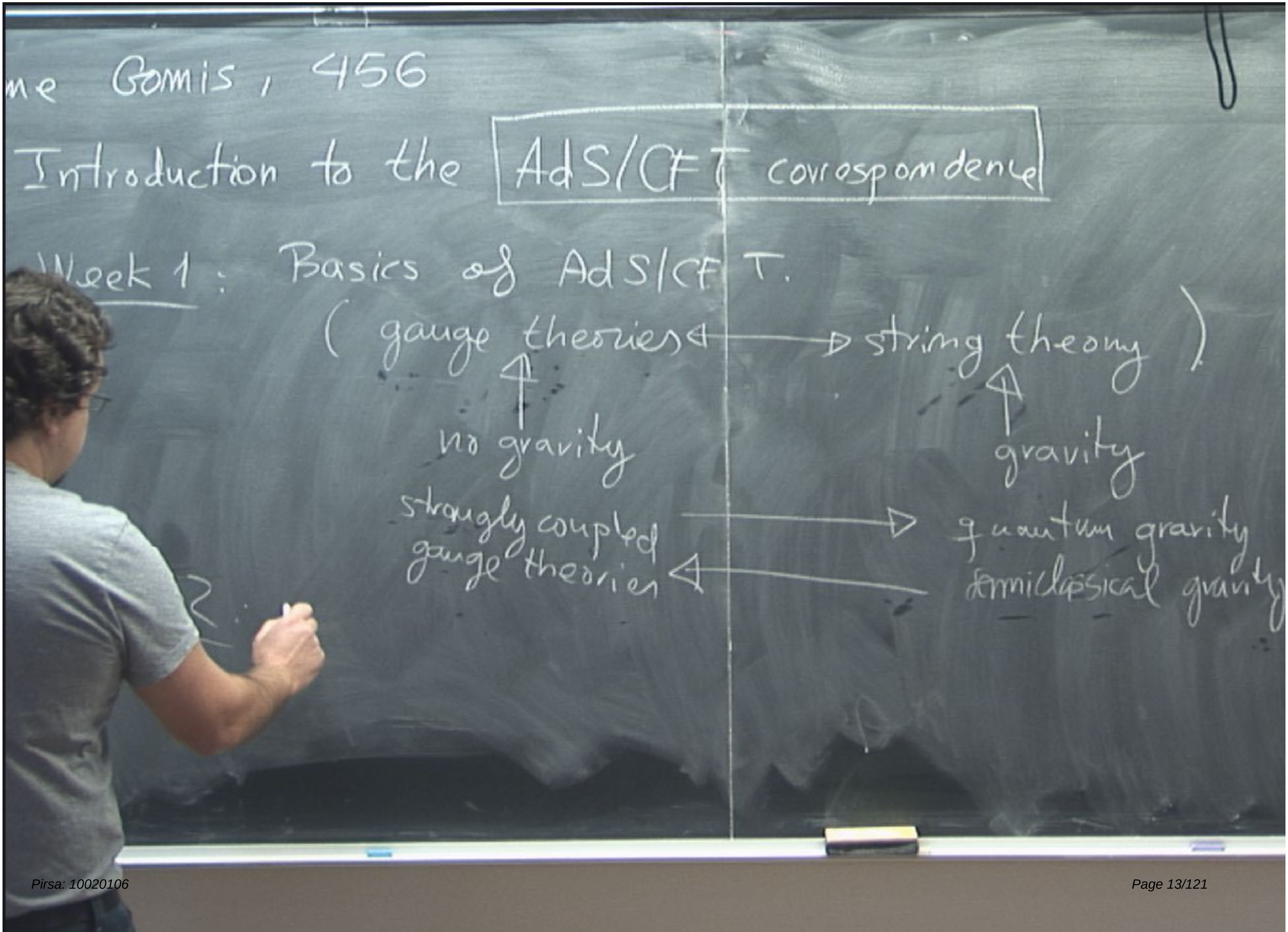
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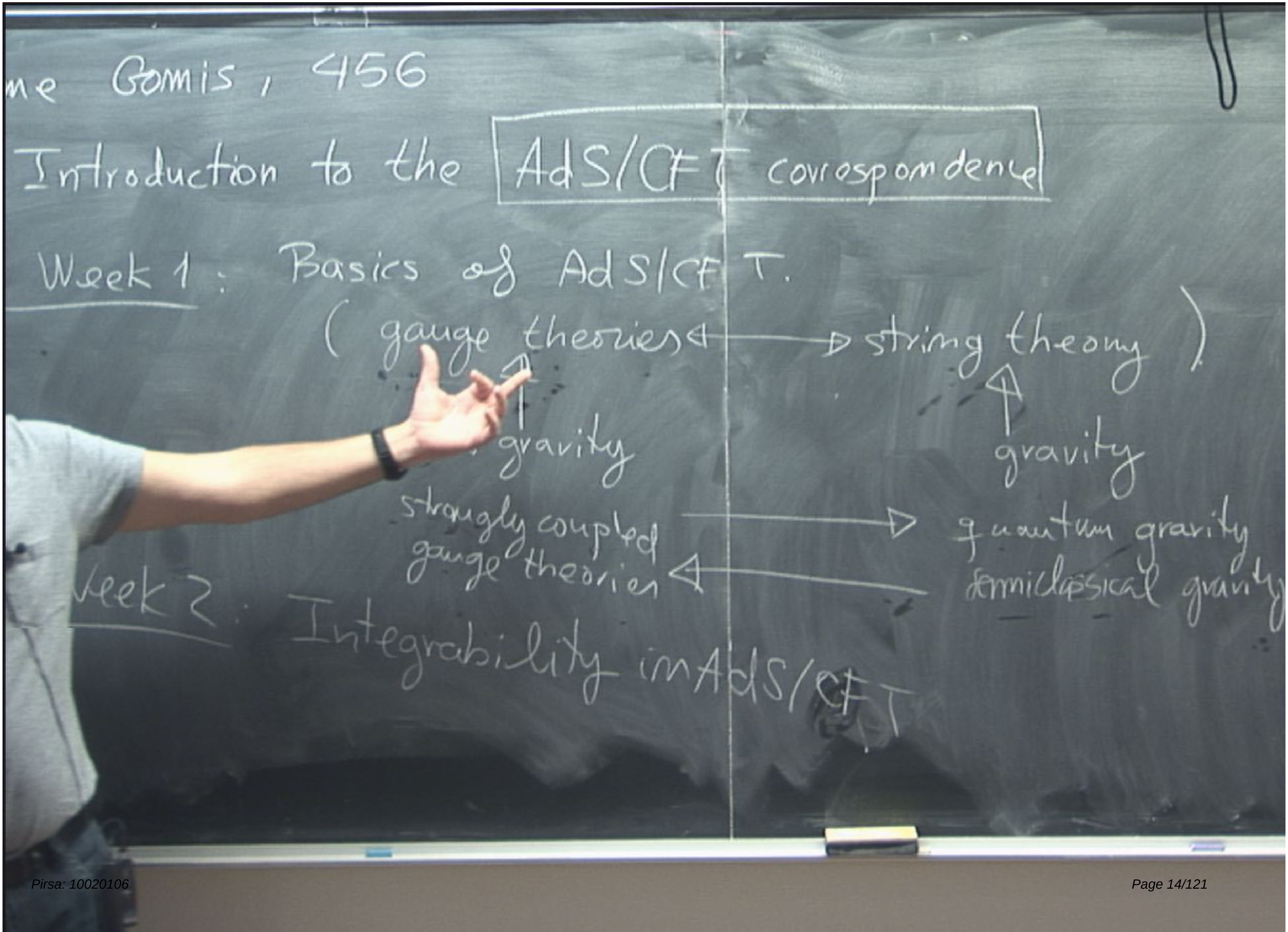
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Introduction to the AdS/CFT correspondence
Week 1: Basics of AdS/CFT.
(gauge theories ⟷ string theory)
gravity
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Week 2: Integrability in AdS/CFT

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Week 1: Basics of AdS/CFT.

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no gravity

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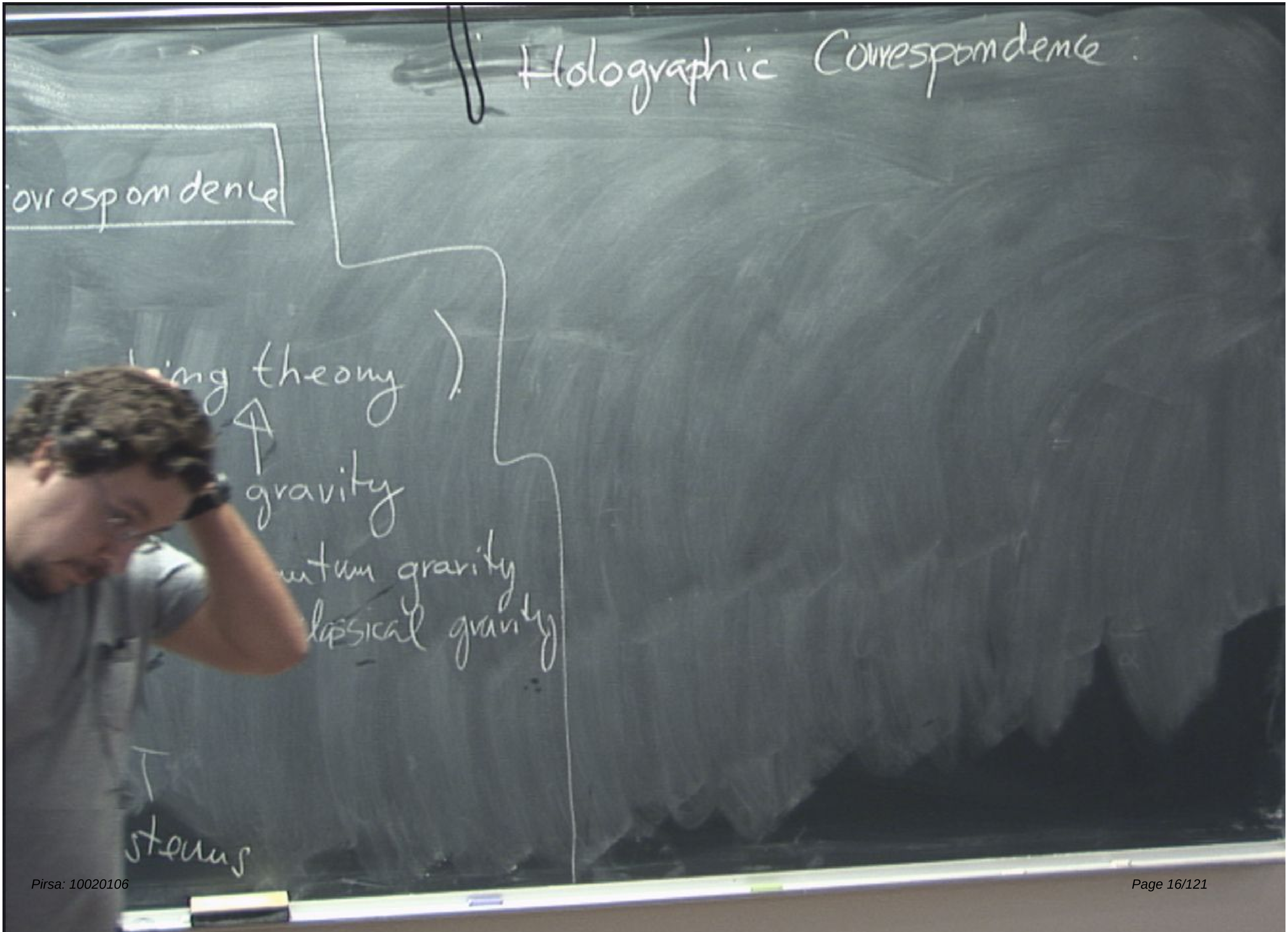
strongly coupled gauge theories ⟶ quantum gravity

strongly coupled gauge theories ⟵ semiclassical gravity

Week 2: Integrability in AdS/CFT

Week 3: Applications to physical systems


Holographic Correspondence :
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gravity
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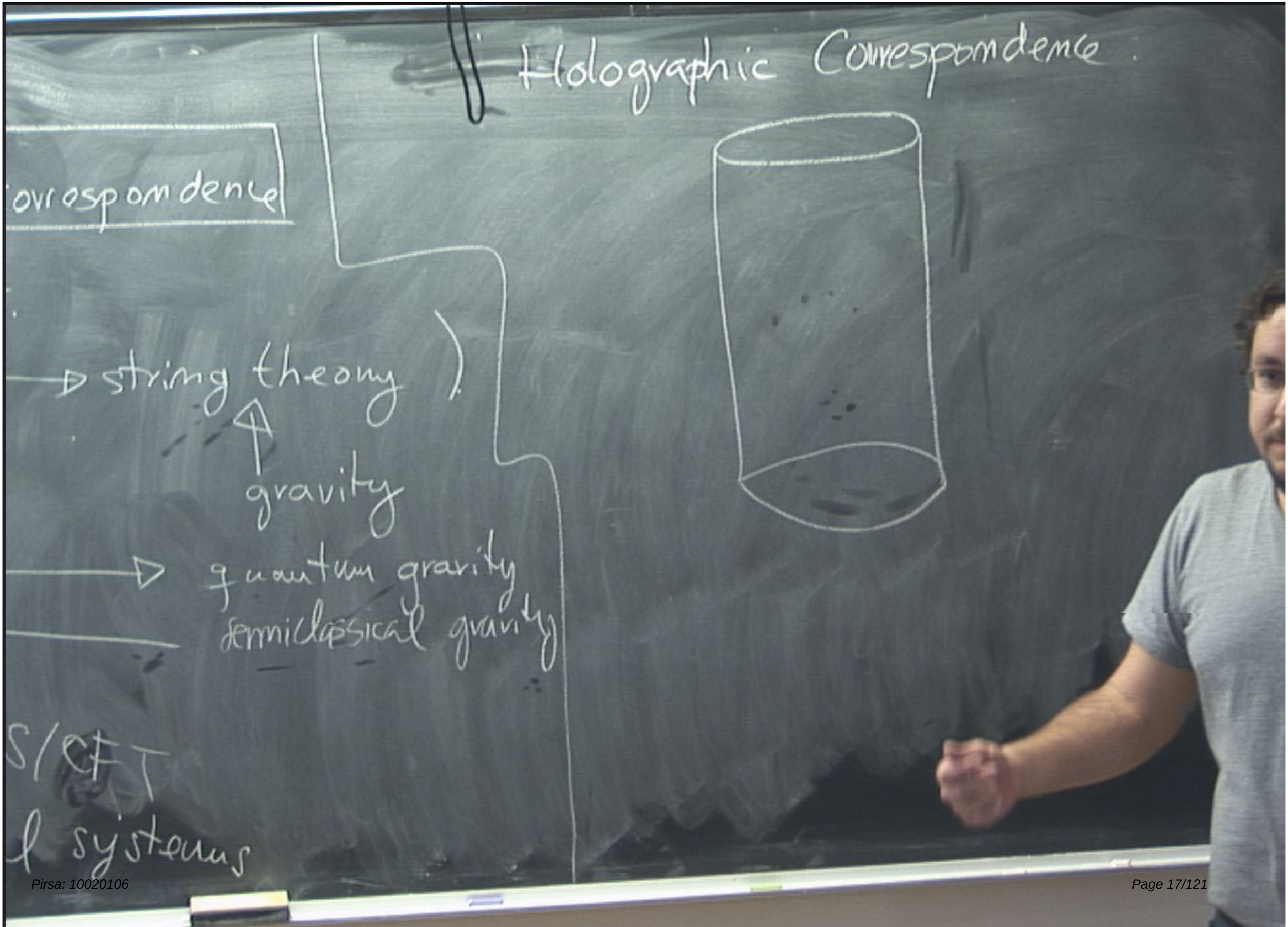
Holographic Correspondence:
string theory
gravity
quantum gravity
semiclassical gravity
systems

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→ string theory)

gravity

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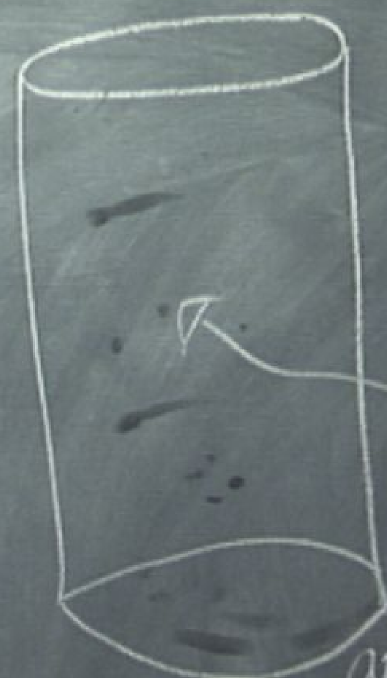
S/CFT

l systems

Holographic Correspondence :



quantum gravity

In semiclassical approximation.

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→ string theory)

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gravity

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S/CFT

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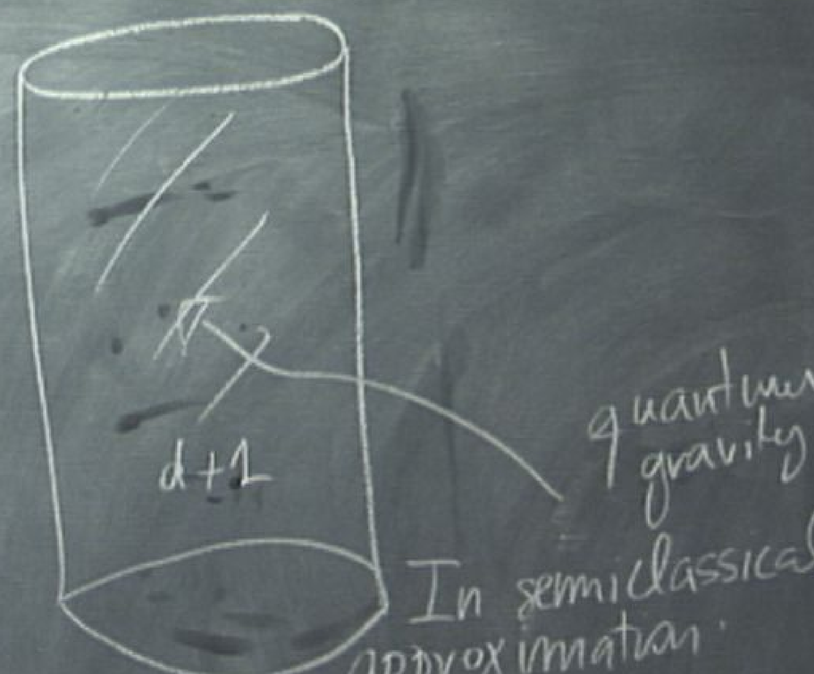
Holographic Correspondence:



In semiclassical approximation:

$$S = \frac{1}{16\pi G_5} \int d^{d+1}x \sqrt{g}\, (R - 2\Lambda$$

orrespondence

→ string theory)

gravity

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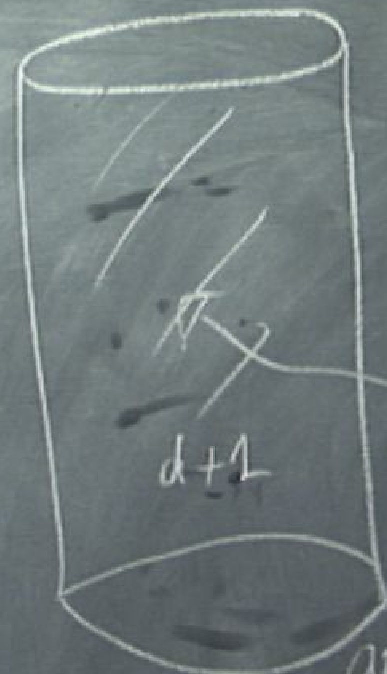
S/CFT

l systems

Holographic Correspondence:



quantum gravity

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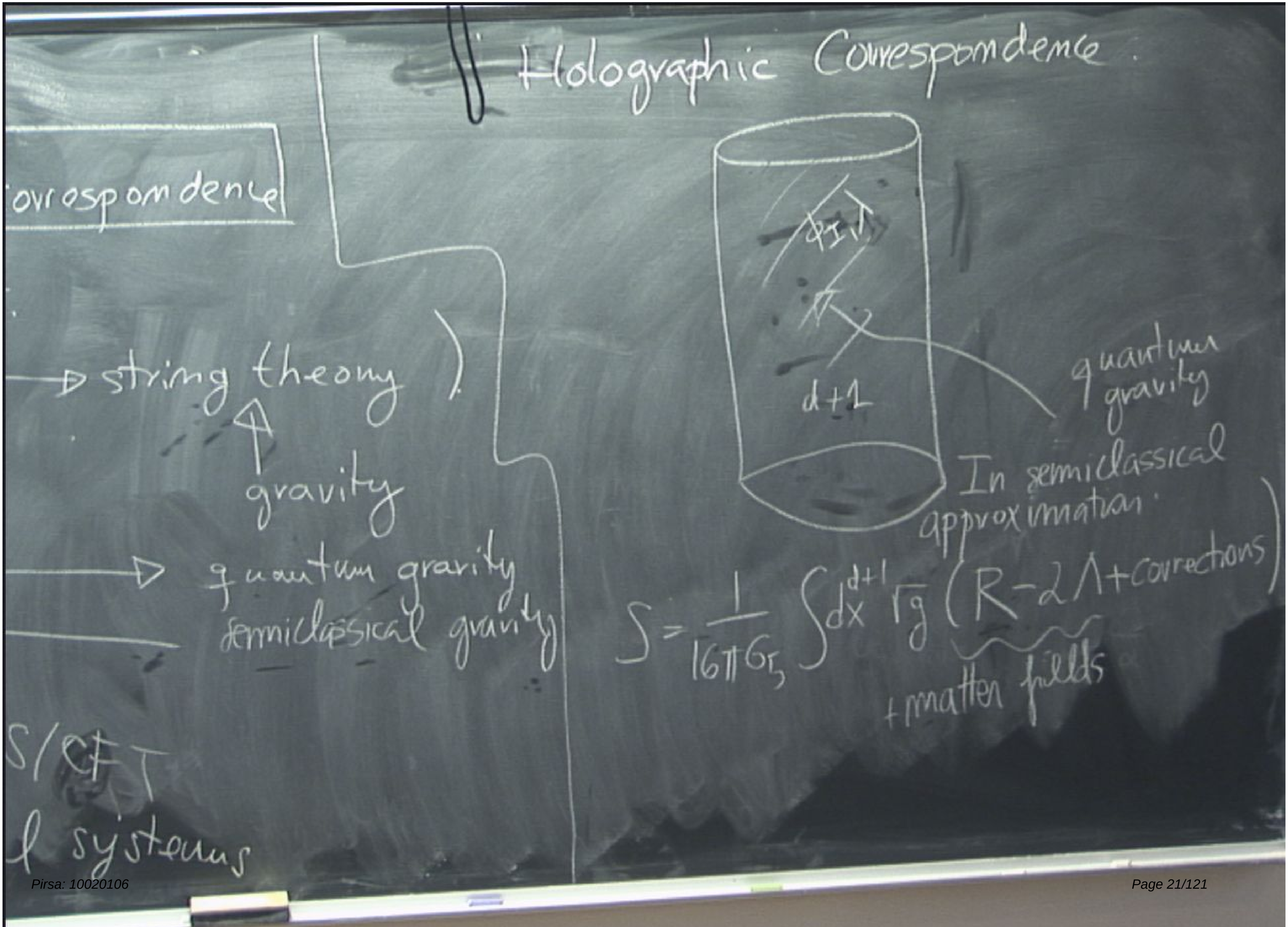
$$S = \frac{1}{16\pi G_5} \int d^{d+1}x \sqrt{g} \Big(R - 2\Lambda + \underbrace{\text{corrections}}_{+\text{matter fields}} \Big)$$



orrespondence
→ string theory)
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S/CFT
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Holographic Correspondence:
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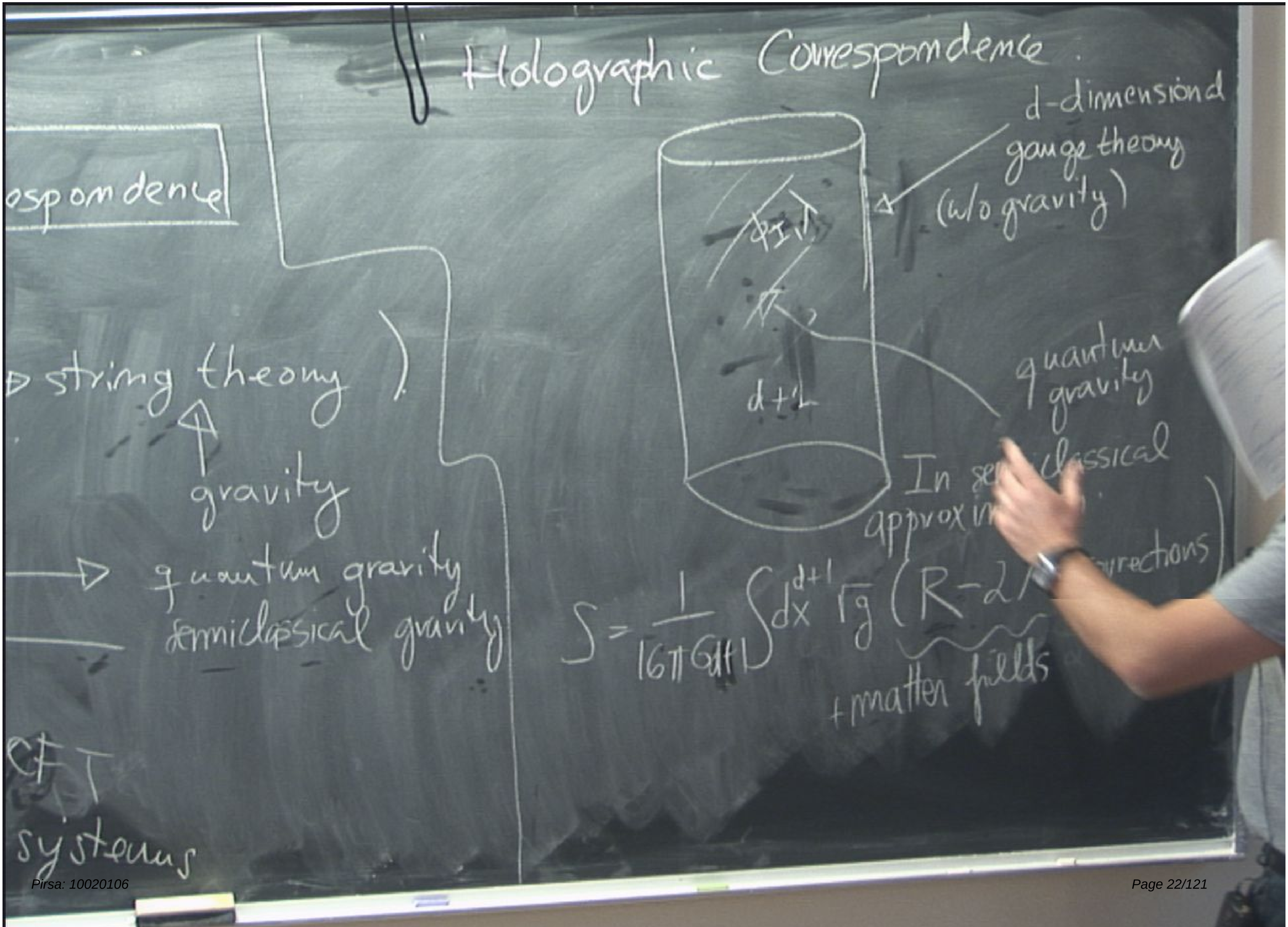
Holographic Correspondence
d-dimensional
gauge theory
(w/o gravity)
d+1
quantum
gravity
In semiclassical
approx
corrections
+ matter fields
string theory
gravity
quantum gravity
semiclassical gravity
systems

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→ string theory)

↑

gravity

→ quantum gravity

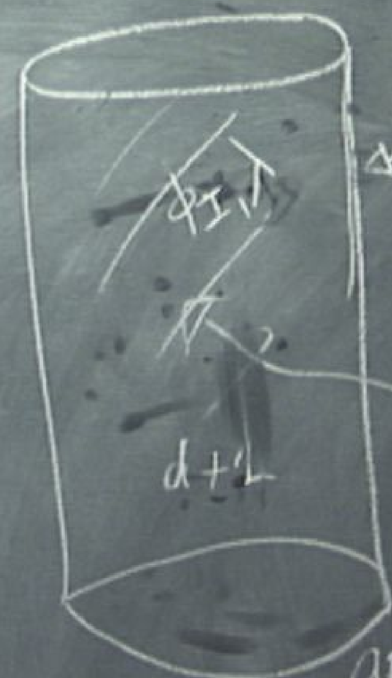
semiclassical gravity

CFT

systems

Holographic Correspondence.

d-dimensional gauge theory (w/o gravity)



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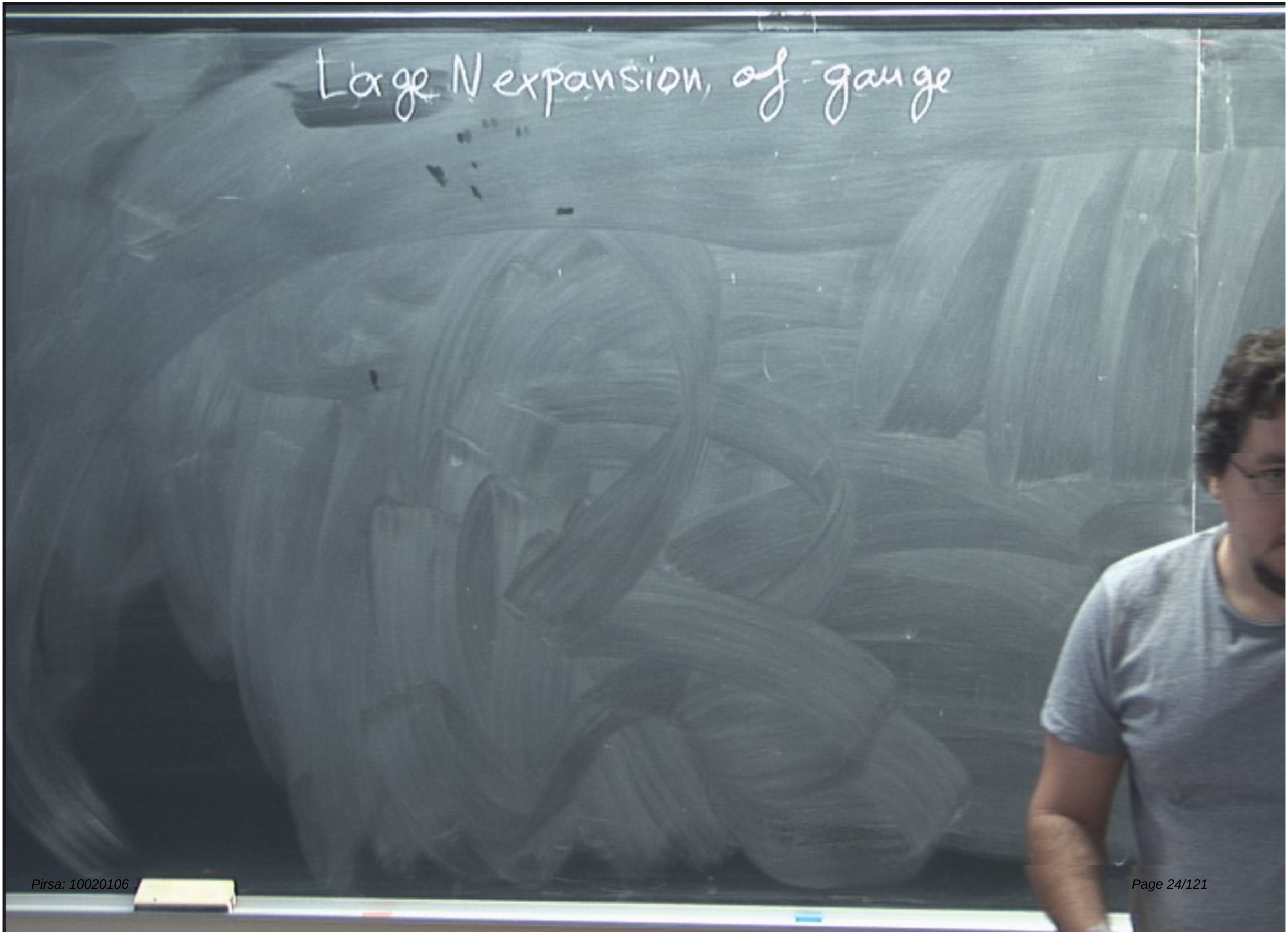
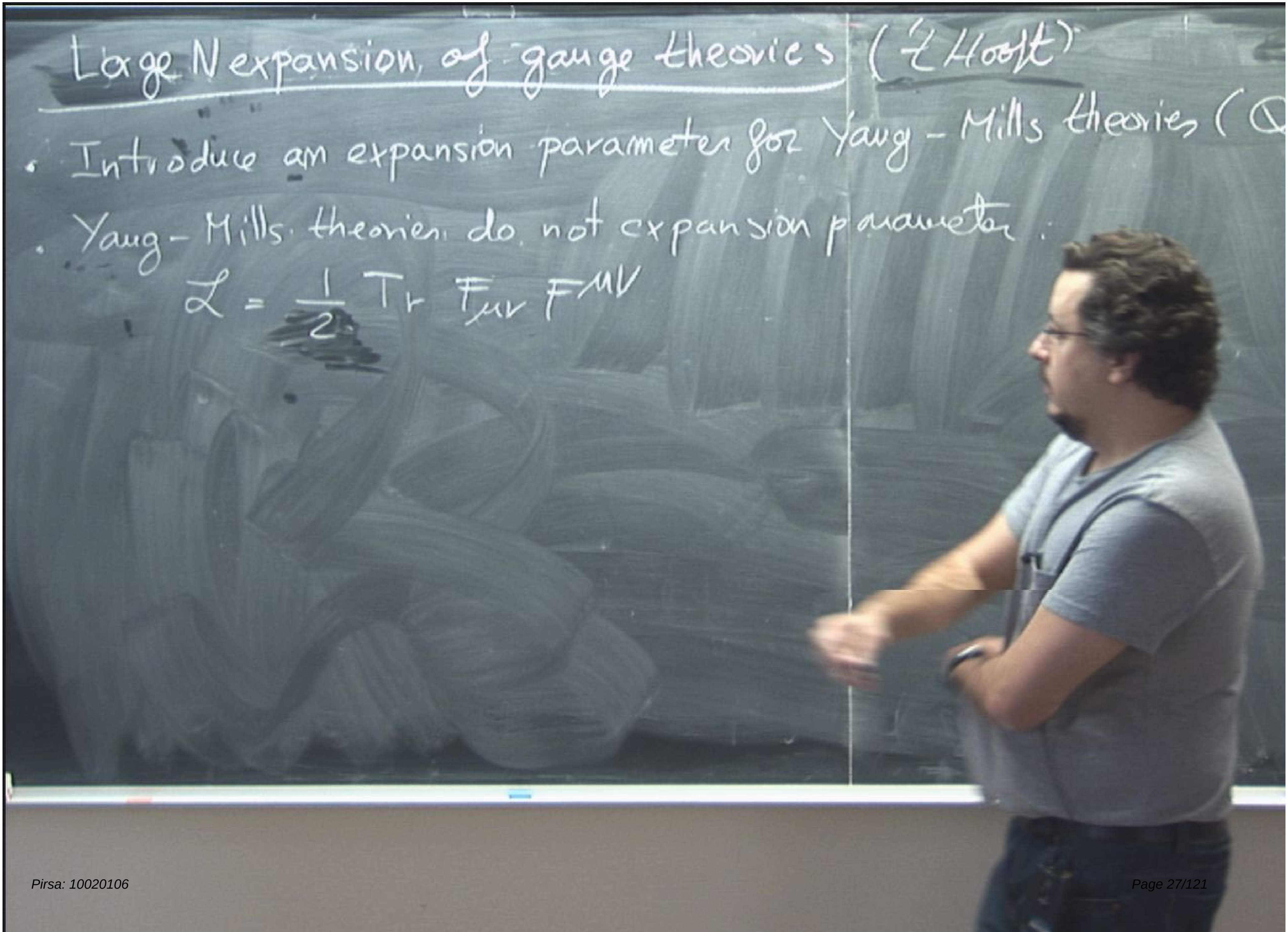

Large N expansion, of gauge

Large N expansion of gauge theories

Large N expansion of gauge theories ('t Hooft)

Introduce an expansion parameter for Yang-Mills theories


Large N expansion of gauge theories ('t Hooft)
• Introduce an expansion parameter for Yang-Mills theories (Q
• Yang-Mills theories do not expansion parameter :
$\mathcal{L} = \frac{1}{2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}$

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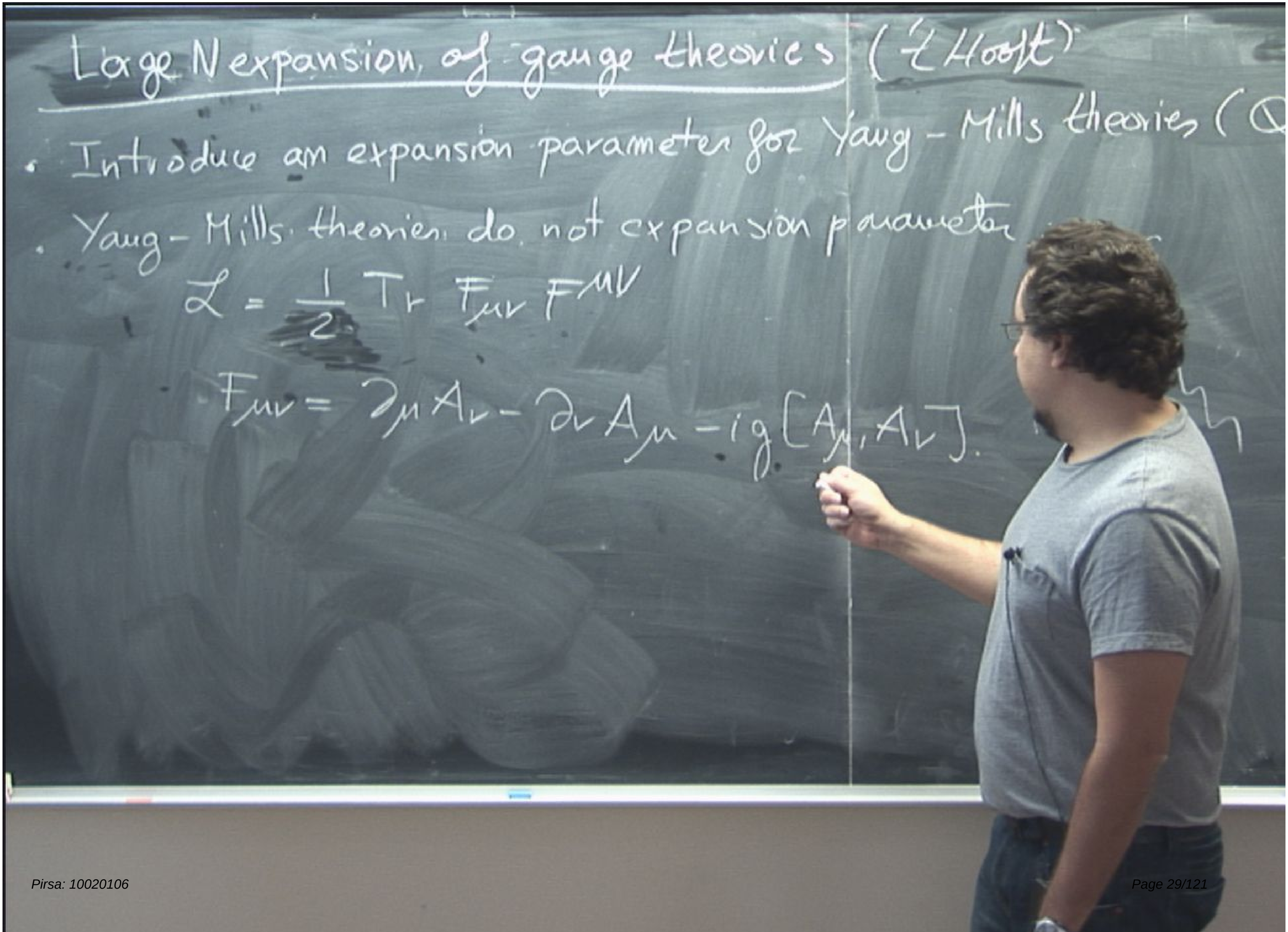
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu].$$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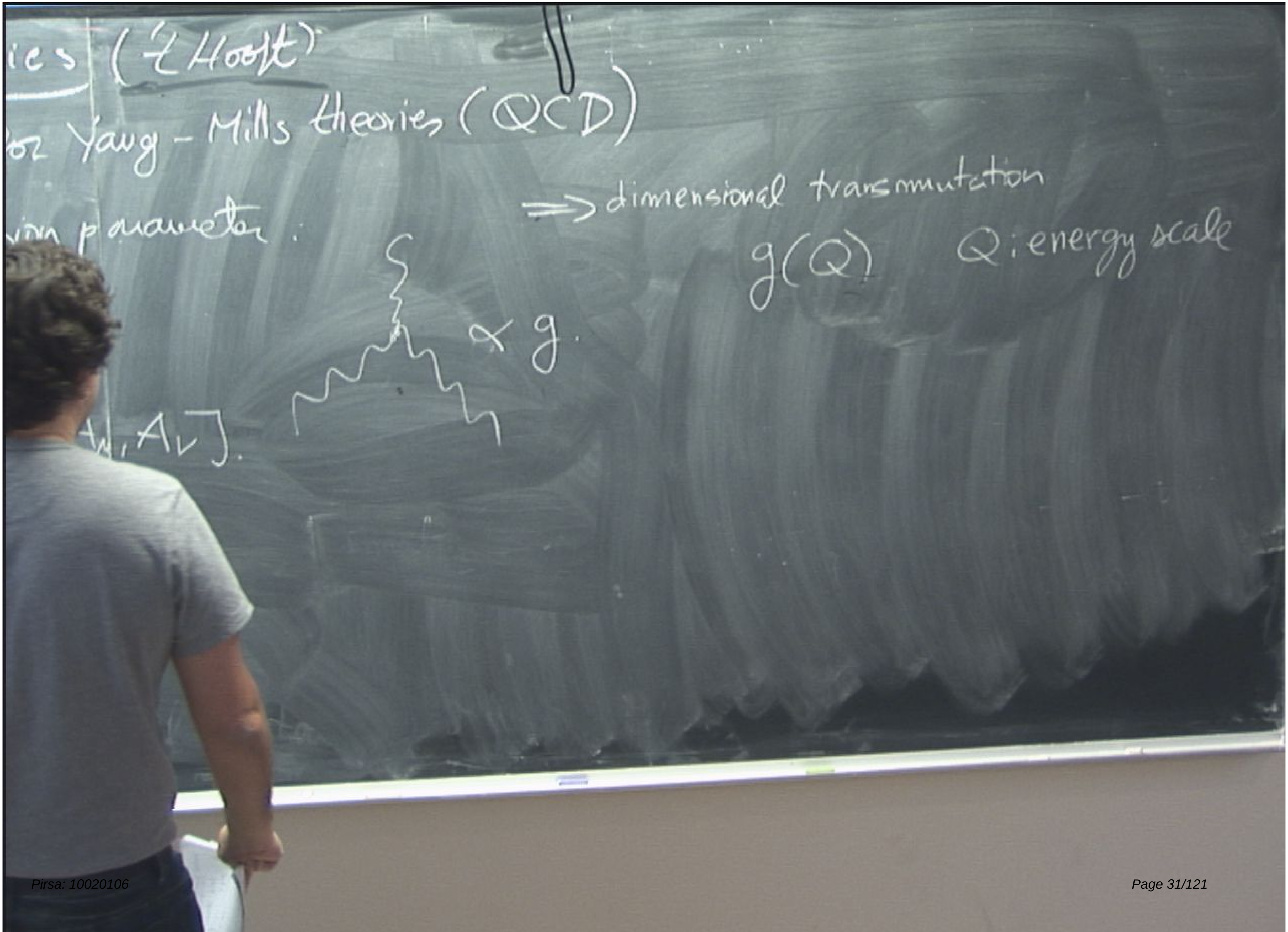
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for Yang - Mills theories (QCD)
⟹ dimensional transmutation
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Q: energy scale
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$\propto g.$

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$g[A_\mu, A_\nu]$.

- consequence of RG equation:

$$\frac{d}{d\log Q}\, g^2 = \beta(g) = \frac{-g^4}{32\pi^2}\, b_0$$

$$S = \frac{1}{g^2} \mathrm{Tr}\, F_{\mu\nu}^2$$

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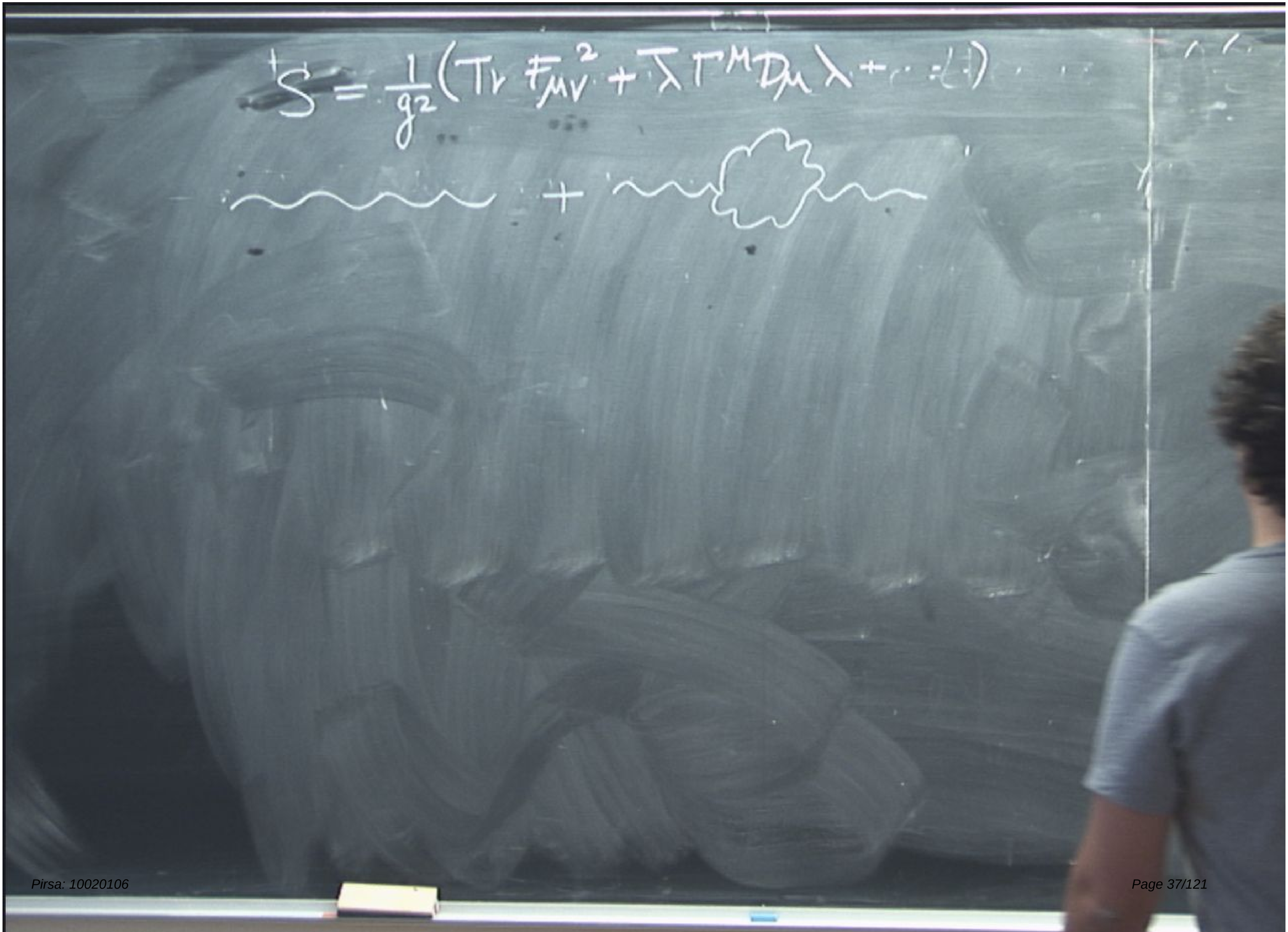
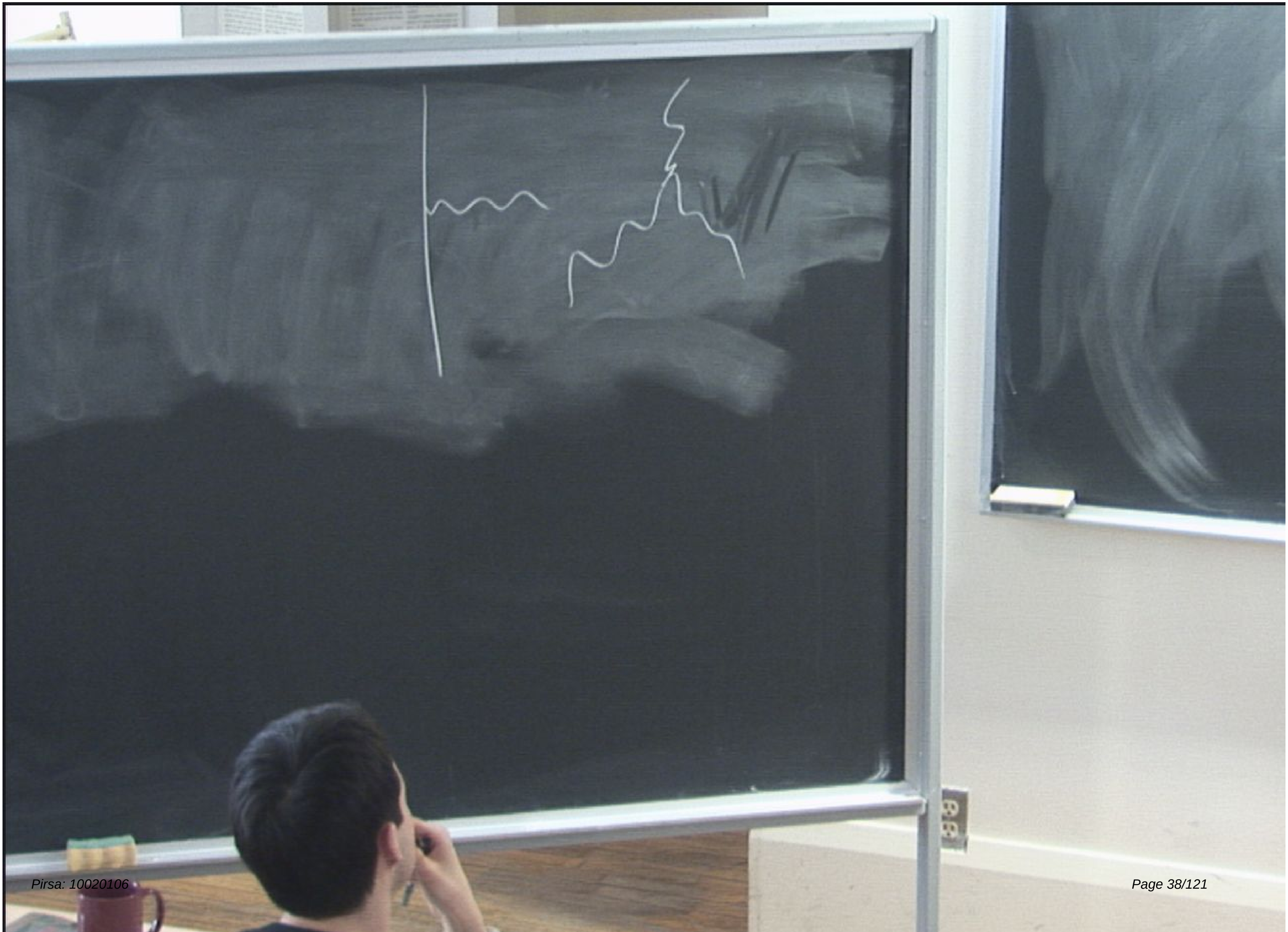

$S = \frac{1}{g^2}\left(\mathrm{Tr}\, F_{\mu\nu}^2 + \bar{\lambda}\,\Gamma^\mu D_\mu \lambda + \cdots\right)$
+



$$S = \frac{1}{g^2}\left(\mathrm{Tr}\, F_{\mu\nu}^2 + \bar{q}\,\Gamma^\mu D_\mu q + \cdots\right)$$

$$D_\mu q = \partial_\mu q - i A_\mu q$$

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$$D_\mu q = \partial_\mu q - i A_\mu q$$

$$A_\mu : \quad G = U(1)$$

$$A_{\mu\, i}{}^{j}(x) \qquad {}_i\!\left(\quad \right)^{j}$$

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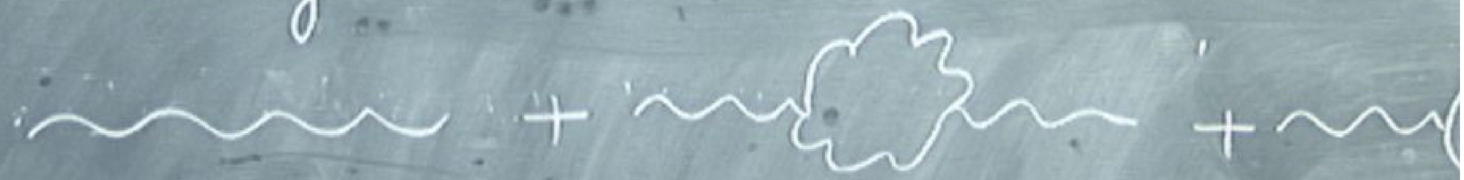

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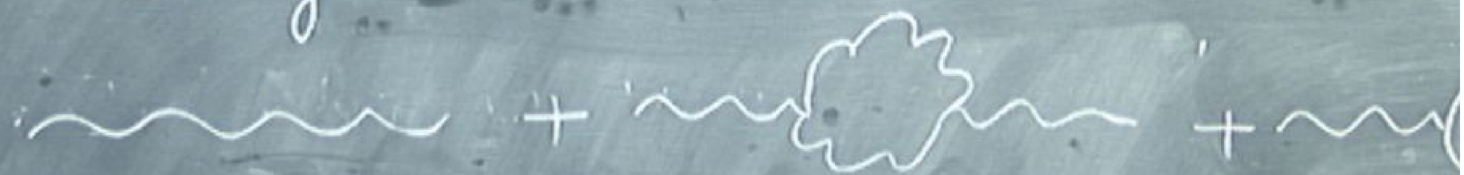

$$G = SU(N)$$

$$A_{\mu\,i}^{\;j}$$

$$S = \frac{1}{g^2}\left(\mathrm{Tr}\, F_{\mu\nu}^2 + \bar{q}\,\Gamma^\mu D_\mu q + \cdots\right)$$

$G = SU(N)$

$A_\mu{}^i{}_j$: $N \times N$ Hermitean + traceless.

$i, j = 1 \ldots N$

q^a_i : $a = 1, \ldots, N_F$

$b_0 =$

$$S = \frac{1}{g^2}\left(\mathrm{Tr}\, F_{\mu\nu}^2 + \bar{q}\, \Gamma^\mu D_\mu q + \cdots\right)$$

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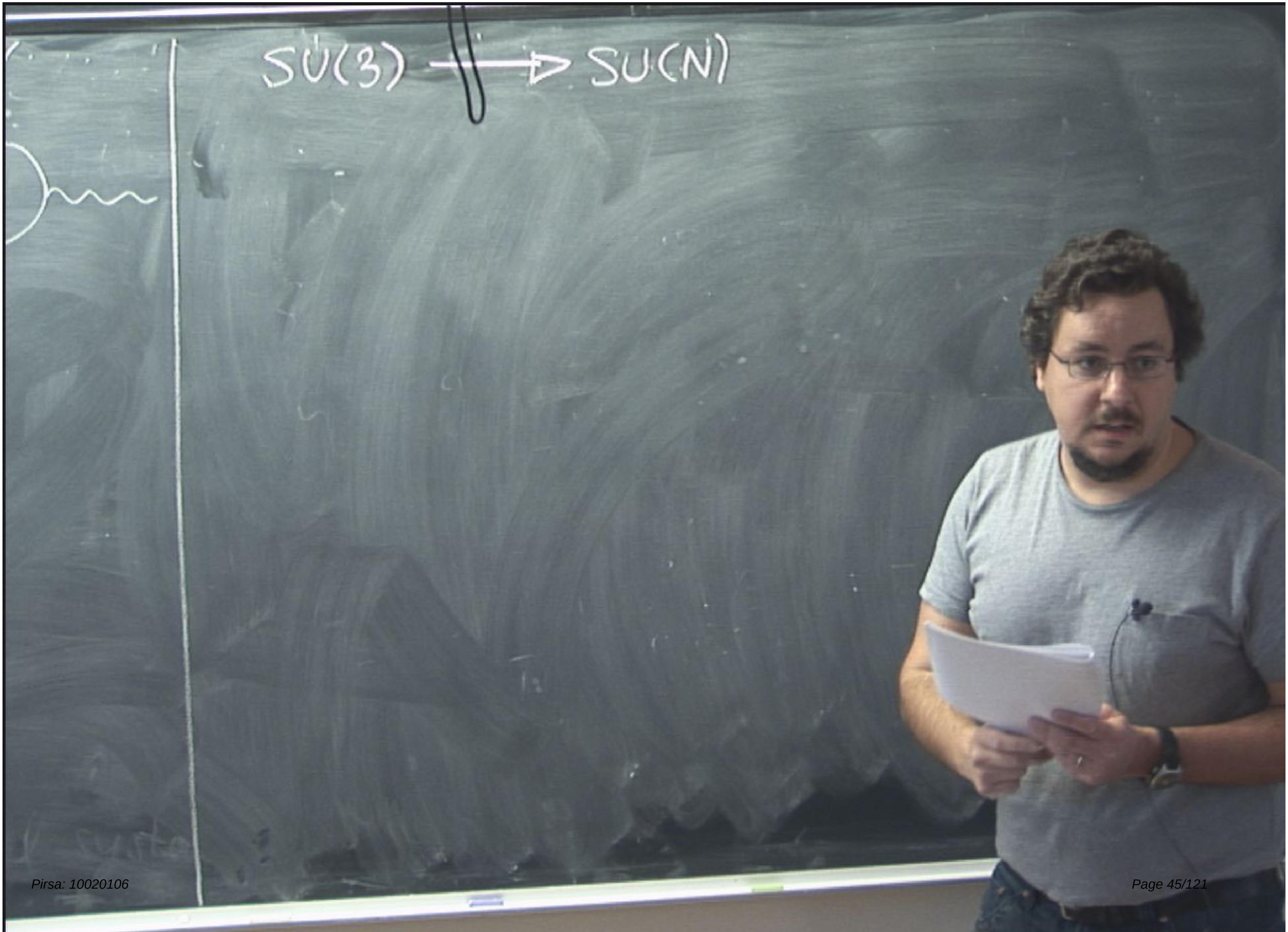
q^a_i : $a = 1, \dots, N_F$

$$b_0 = \frac{11}{3} N - \frac{2}{3} N_F$$


SU(3) → SU(N)

$SU(3) \longrightarrow SU(N)$

$\exists$ limit ('t Hooft limit) where theory simplifies.

. Theory acquires an exp parameter :

$1/N$

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. Theory acquires an exp. parameter:

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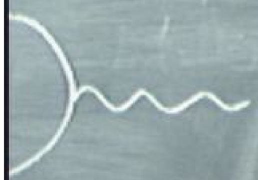
. Expansion resembles perturbative string theory



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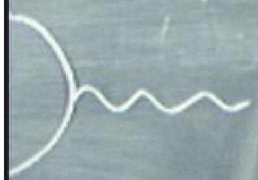
$$g_s = 1/N$$



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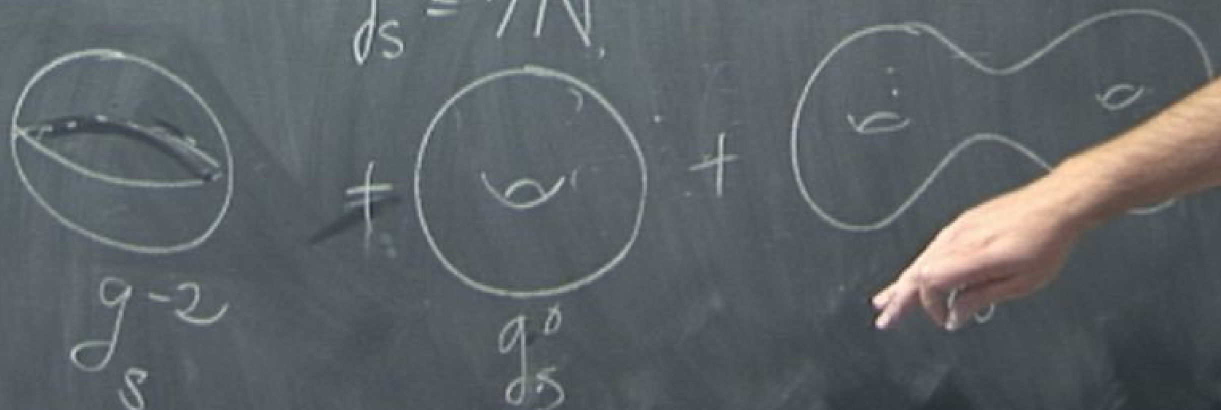

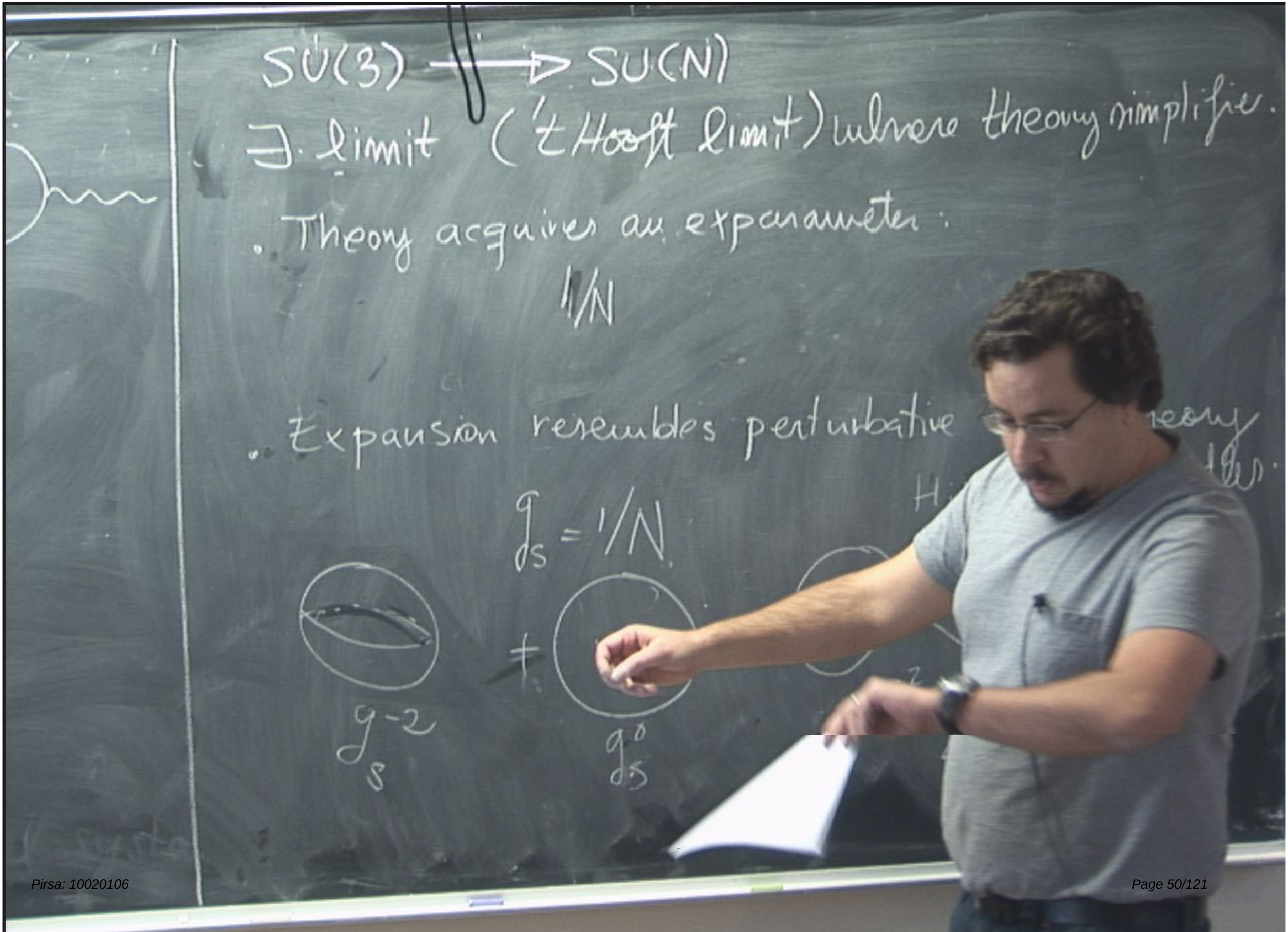

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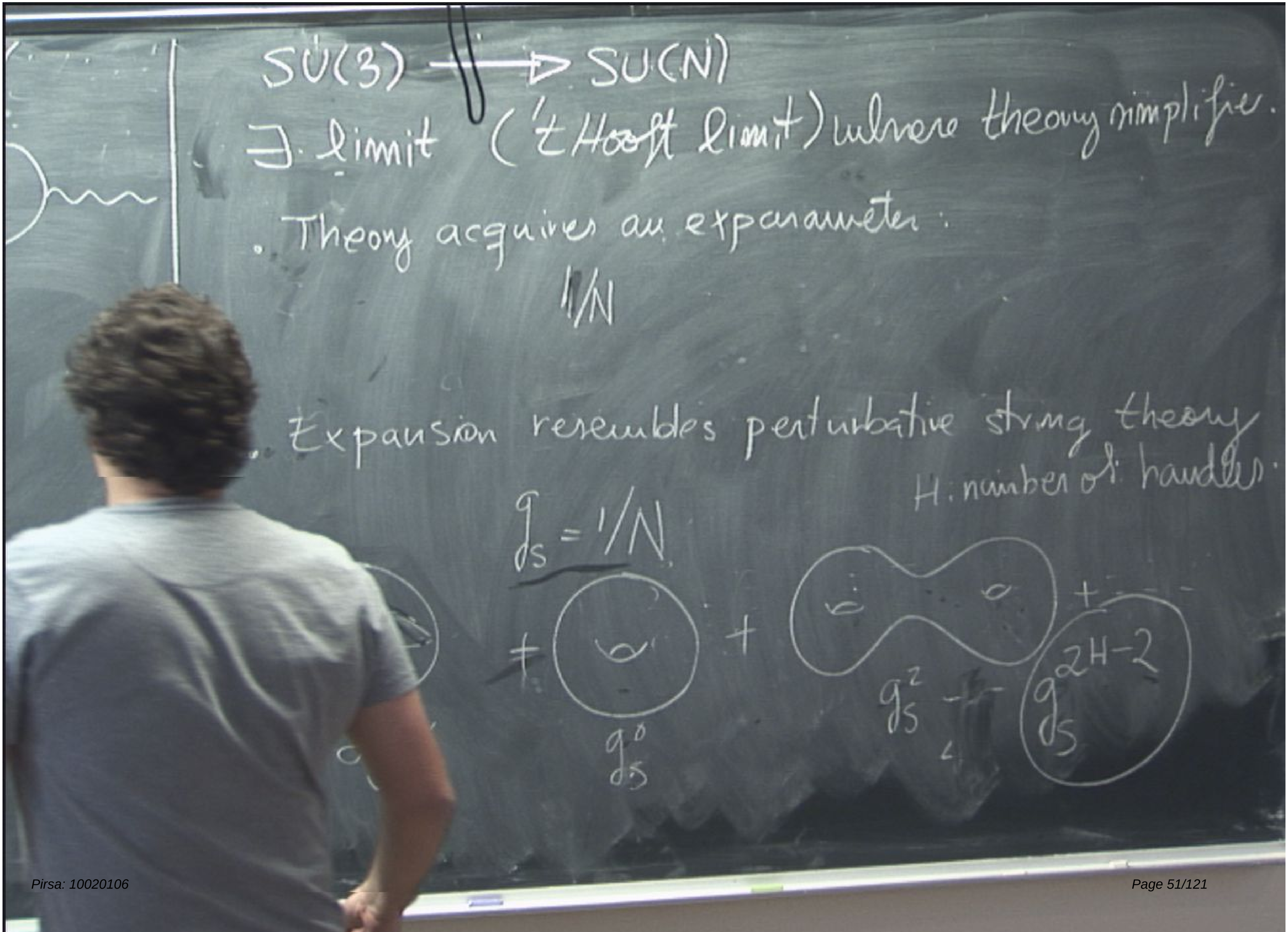
Expansion resembles perturbative
$g_s = 1/N$
g_s^{-2}
g_s^0


SU(3) → SU(N)
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H: number of handles.
$g_s = 1/N$
+ + + ...
g_s^0
g_s^2
g_s^{2H-2}

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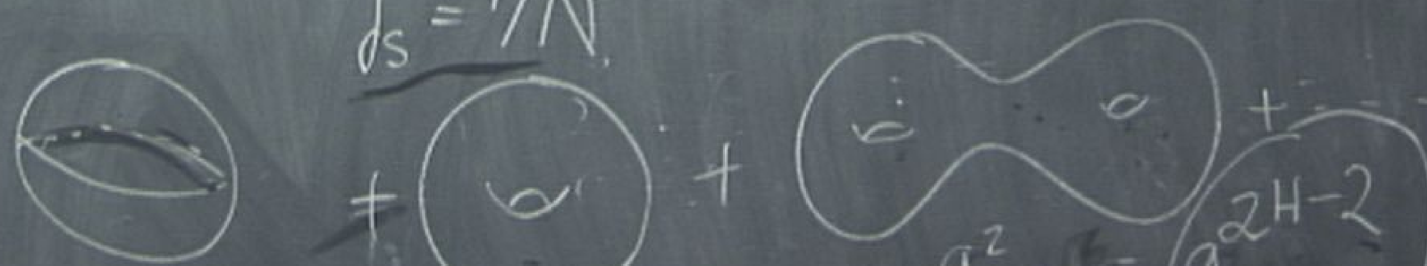
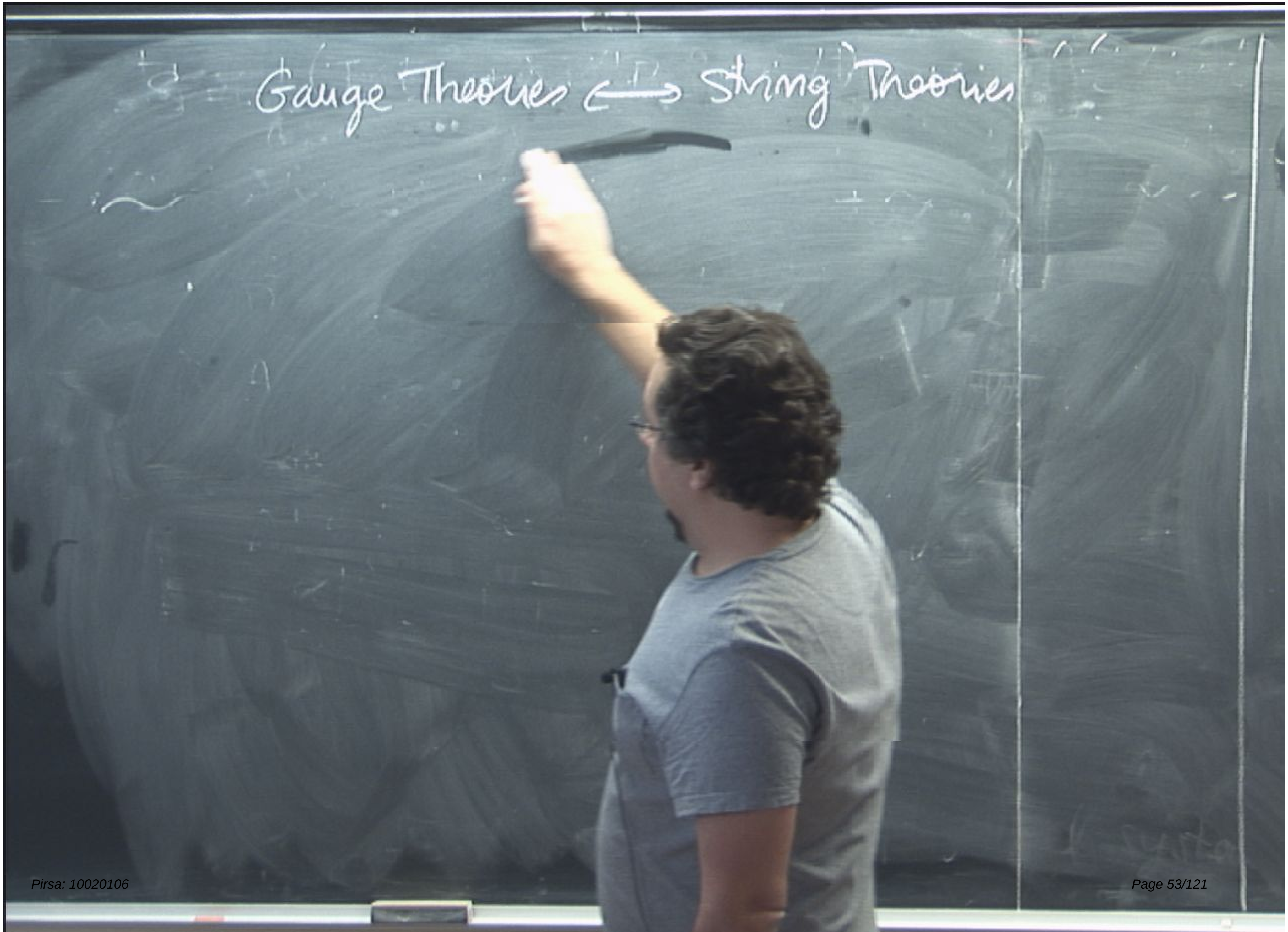
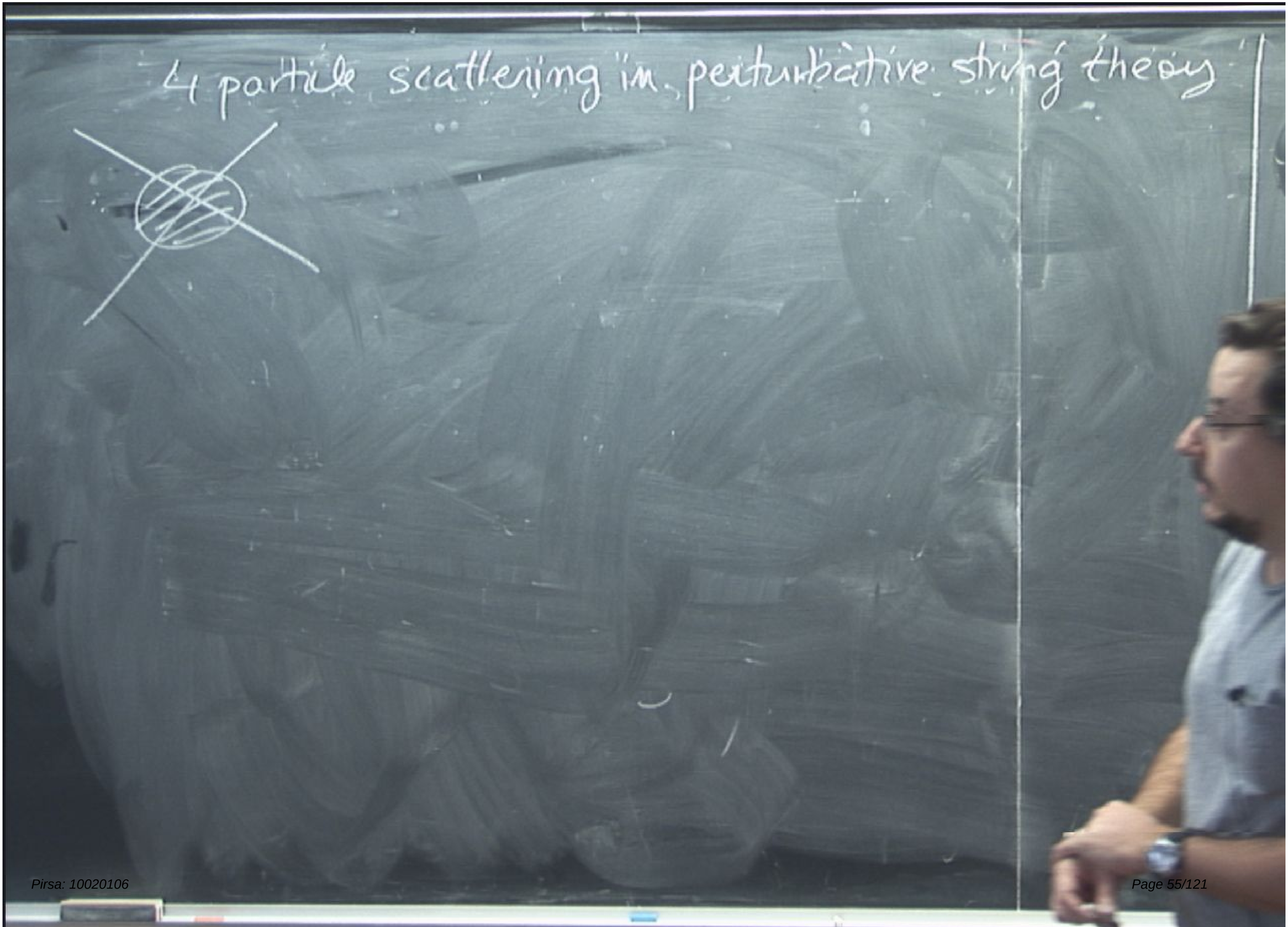


$$g_s^{-2} + g_s^{0} + g_s^{2} + \dots + g_s^{2H-2}$$


Gauge Theories ⟷ String Theories

4 particle scattering in perturbative string theory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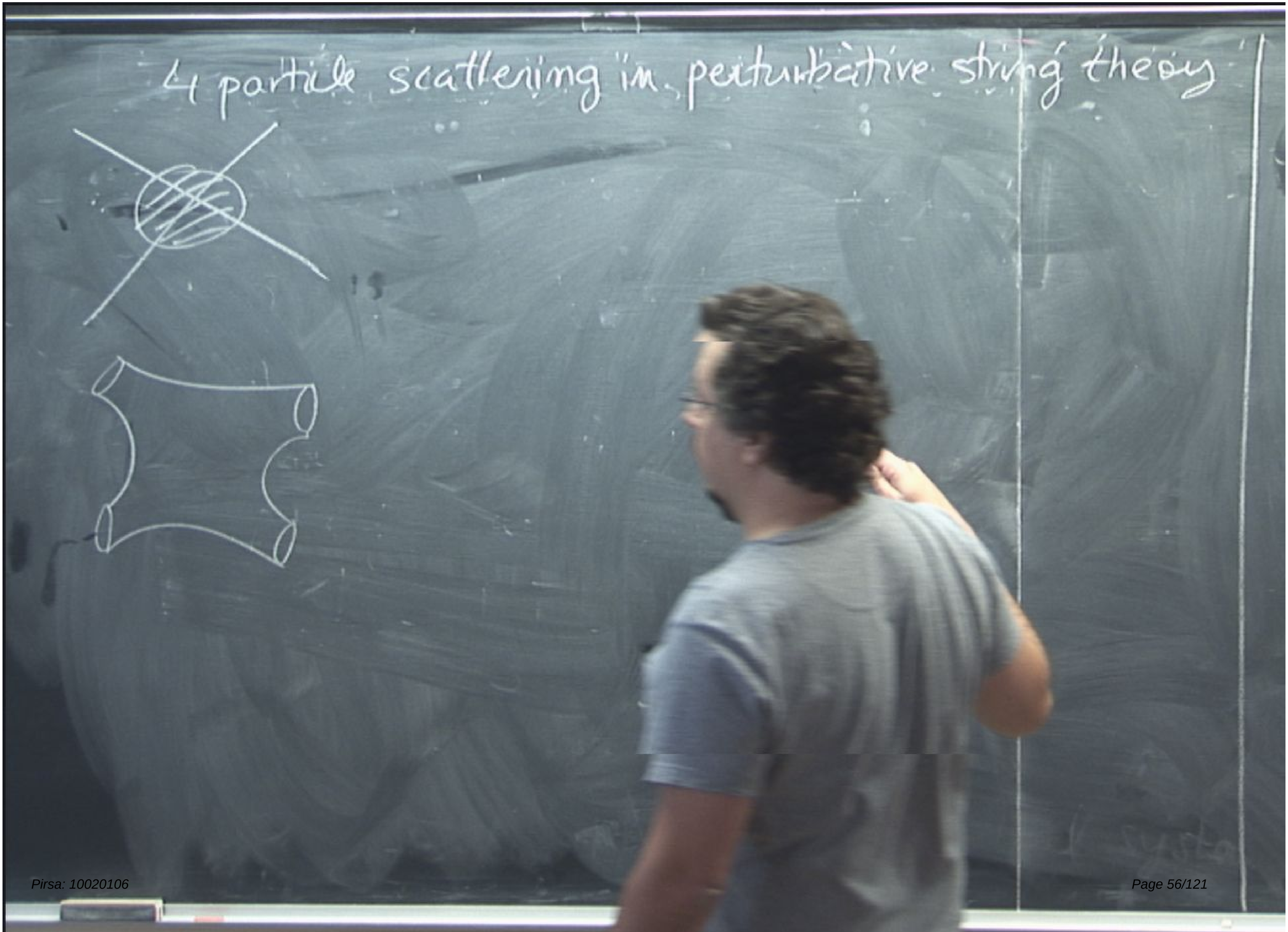
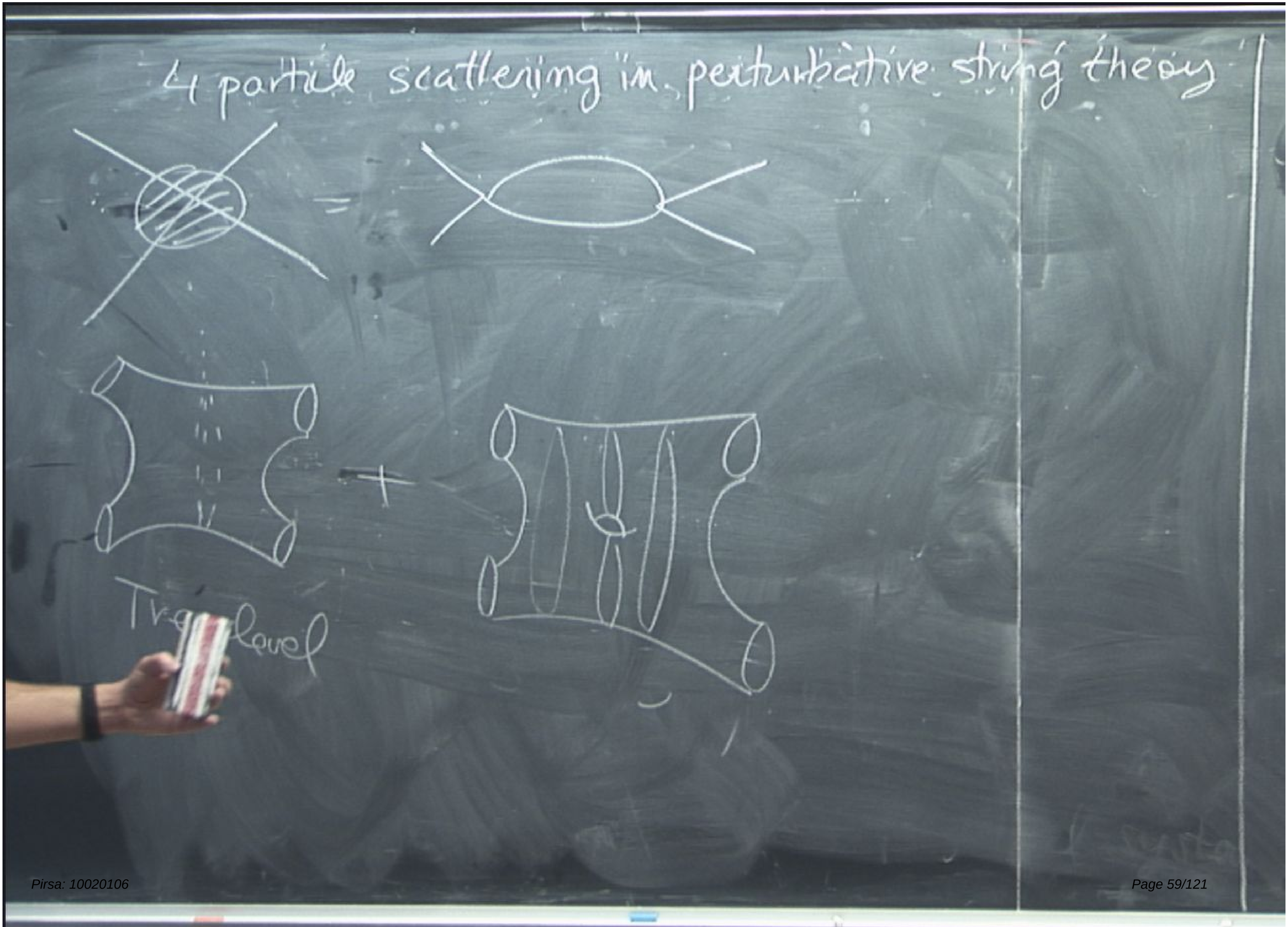

4 particle scattering in perturbative string theory

4 particle scattering in perturbative string theory

Tree level

4 particle scattering in perturbative string theory

Tree level


4 particle scattering in perturbative string theory
=
+
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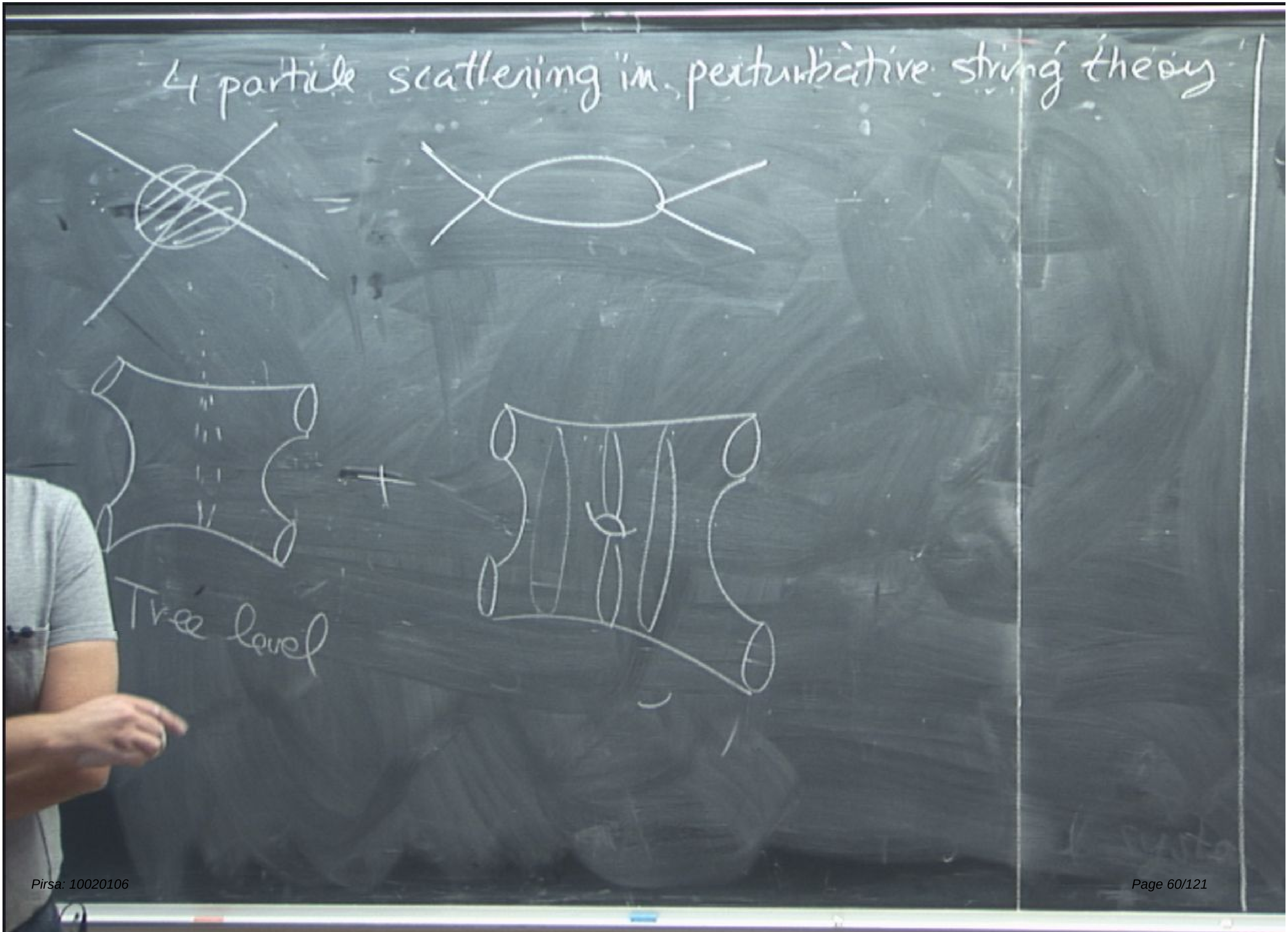
4 particle scattering in perturbative string theory
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4 particle scattering in perturbative string theory

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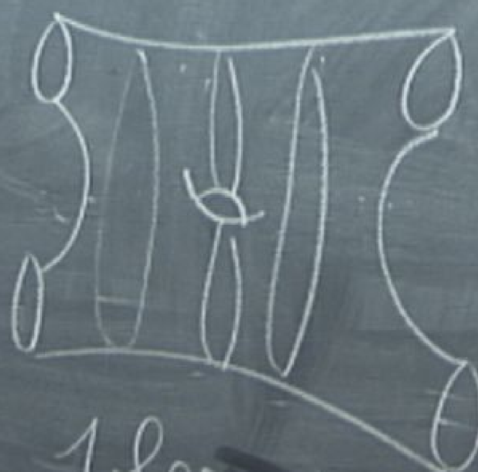


1 loop

4 particle scattering in perturbative string theory

Tree level

1 loop

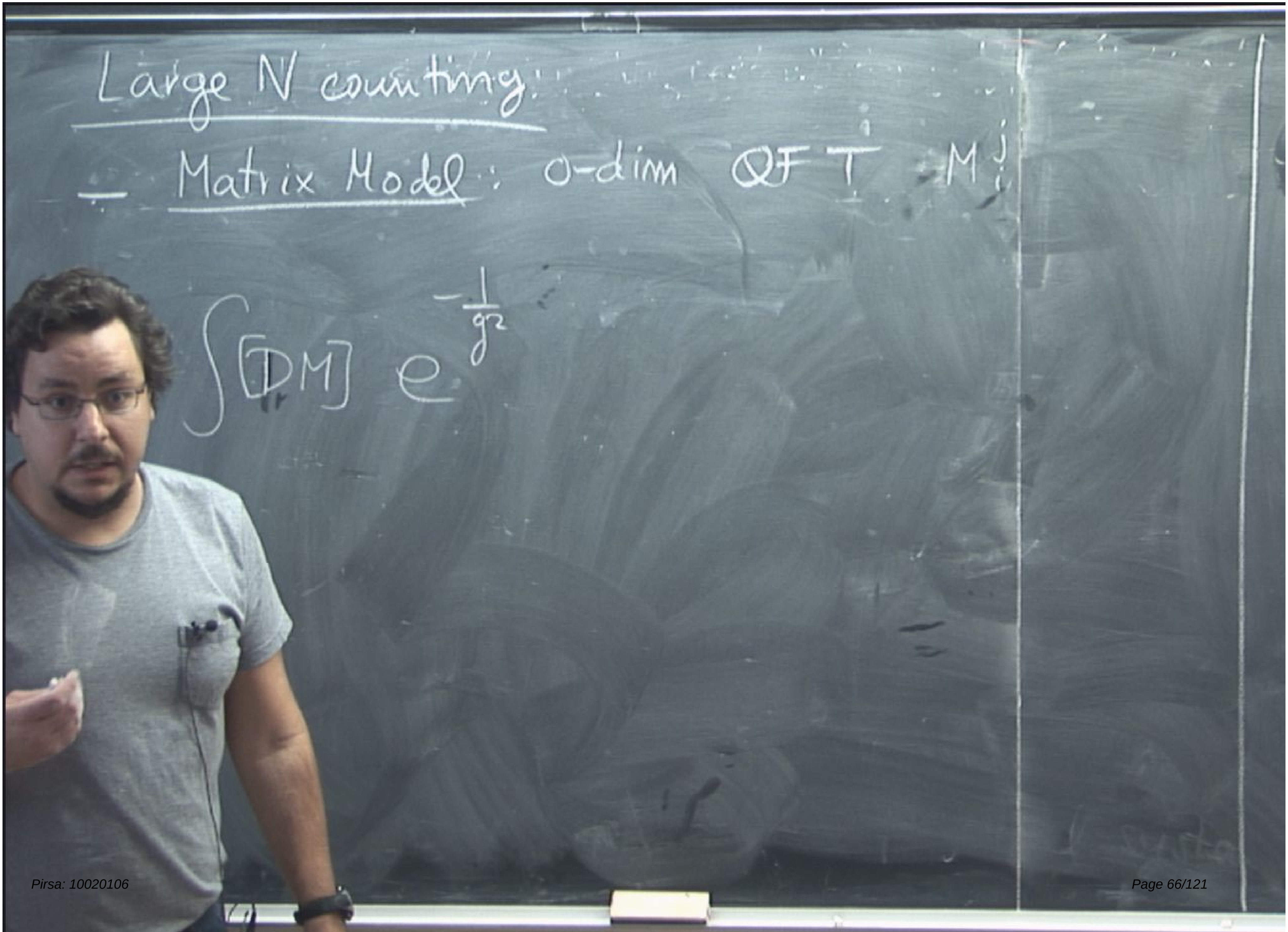
Large N counting:

Large N counting:

– Matrix Model

Large N counting:

– Matrix Model: 0-dim QFT M_i^j


Large N counting:
- Matrix Model: 0-dim QFT M_i^j
$\int [DM]\, e^{-\frac{1}{g^2}}$

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$$Z = \int [DM]\, e^{-\frac{1}{g^2} \mathrm{Tr}\left(M^2 + \lambda_3 M^3 + \lambda_4 M^4 + \dots\right)}$$

M_i^j

Large N counting:

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M_i^j: $N \times N$ Hermitean matrix

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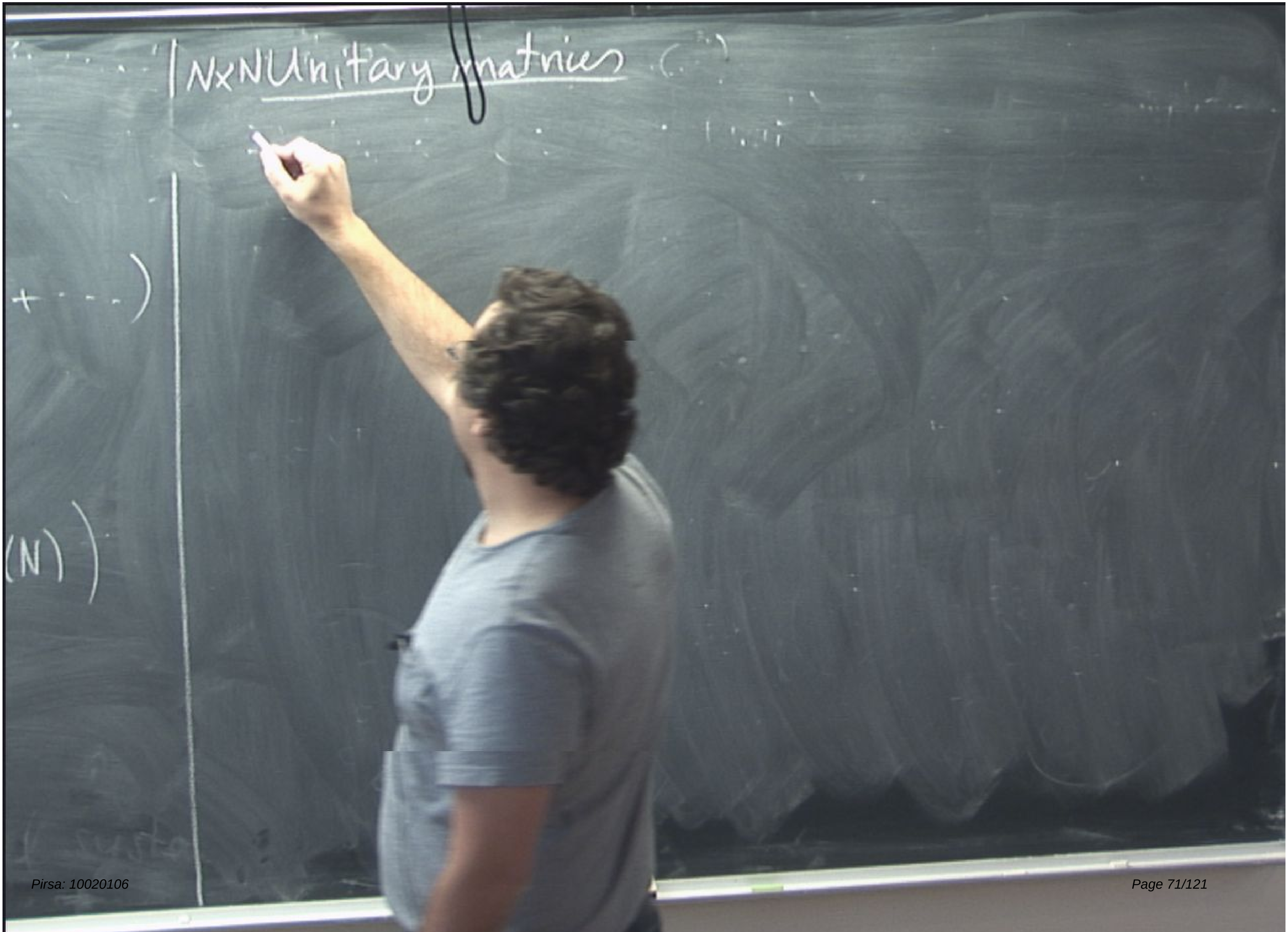
M_i^j: $N \times N$ Hermitean matrix $\left(U(N) \right)$
$\quad\;\; |||$

M is an element of U(N) Lie algebra.


N×N Unitary matrices
+ - - -)
(N))

$+ \cdots)$

$(N))$

N×N Unitary matrices U

- complex
- invertible

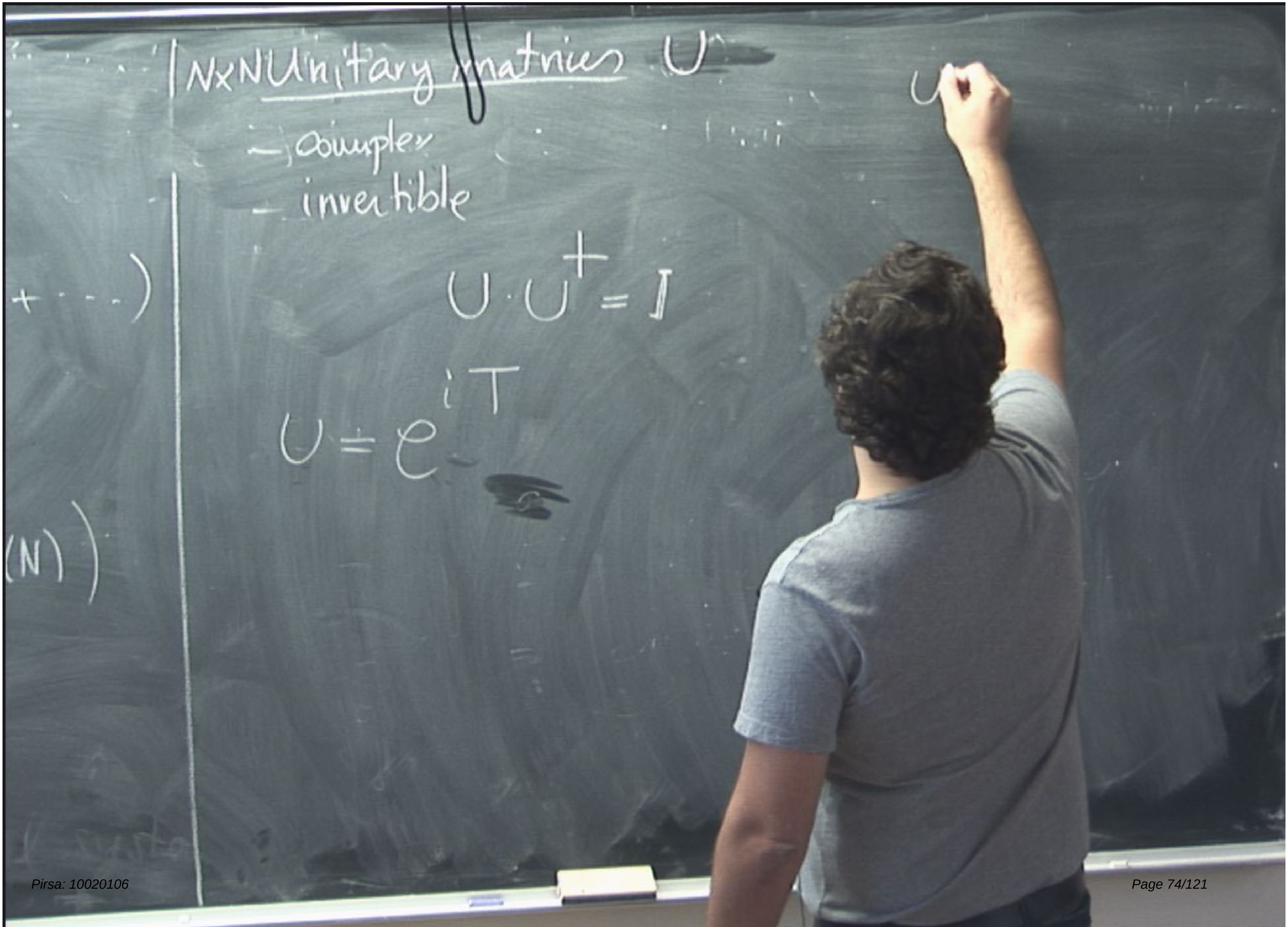
$(+ \cdots)$

$(N))$

N×N Unitary matrices U

- complex
- invertible

$$U \cdot U^{\dagger} = \mathbb{I}$$


N×N Unitary matrices U
- complex
- invertible
U·U† = I
U = e^{iT}

NxN Unitary matrices U

- complex
- invertible

$$U \cdot U^{\dagger} = \mathbb{I}$$

$$U = e^{i\varepsilon T}$$

$$(1+\varepsilon i T)(1-i\varepsilon T^{\dagger}) = 1$$

$$\boxed{T = T^{\dagger}}$$

SU(2)

$$U = e^{i\frac{\vec{\sigma}}{2}\cdot\vec{n}}$$

$SU(2)$ $U = e^{i \frac{\vec{\sigma}}{2} \cdot \vec{n}}$

$+ \dots)$

$(N))$

N×N Unitary matrices U

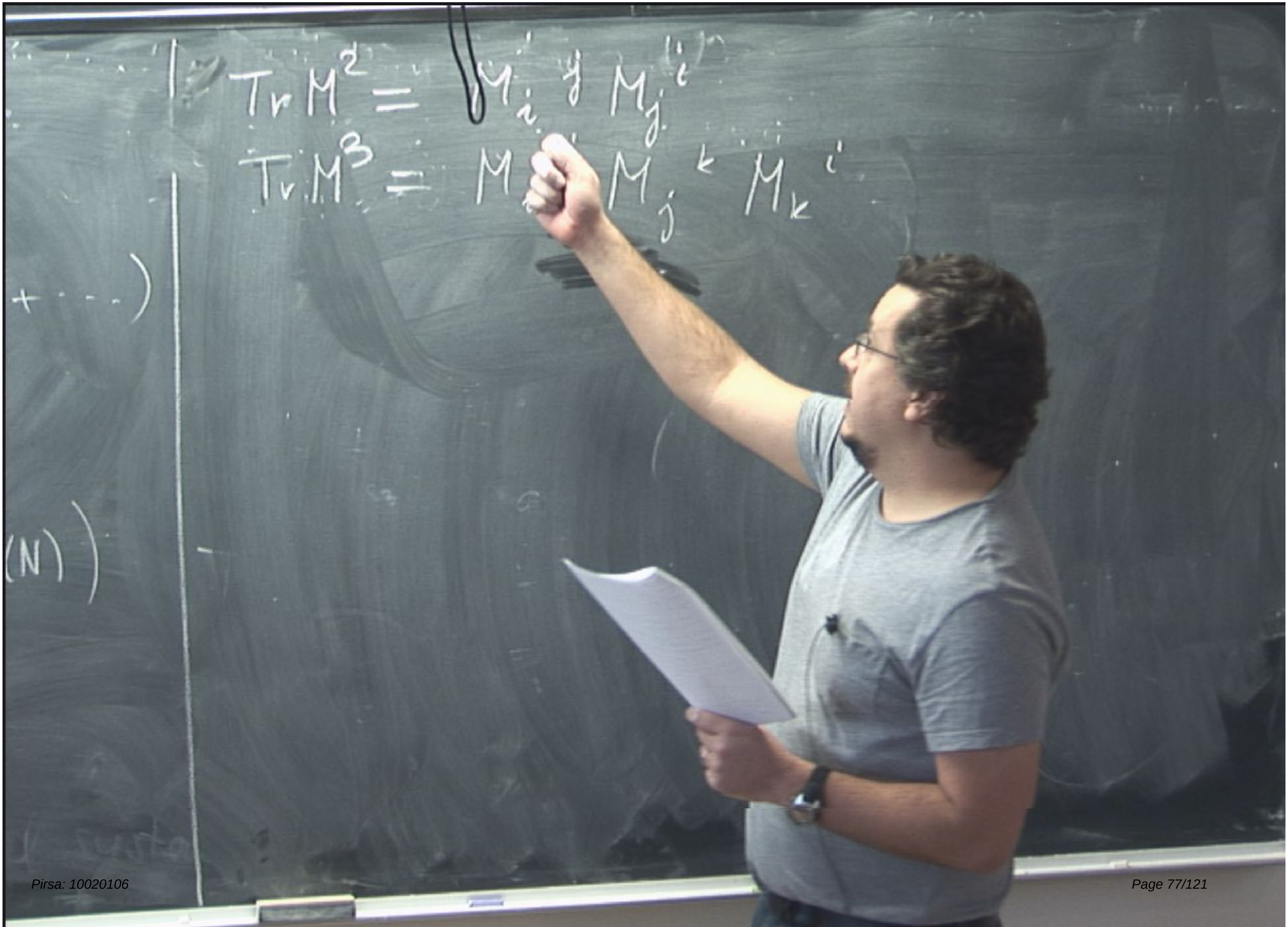
- complex
- invertible

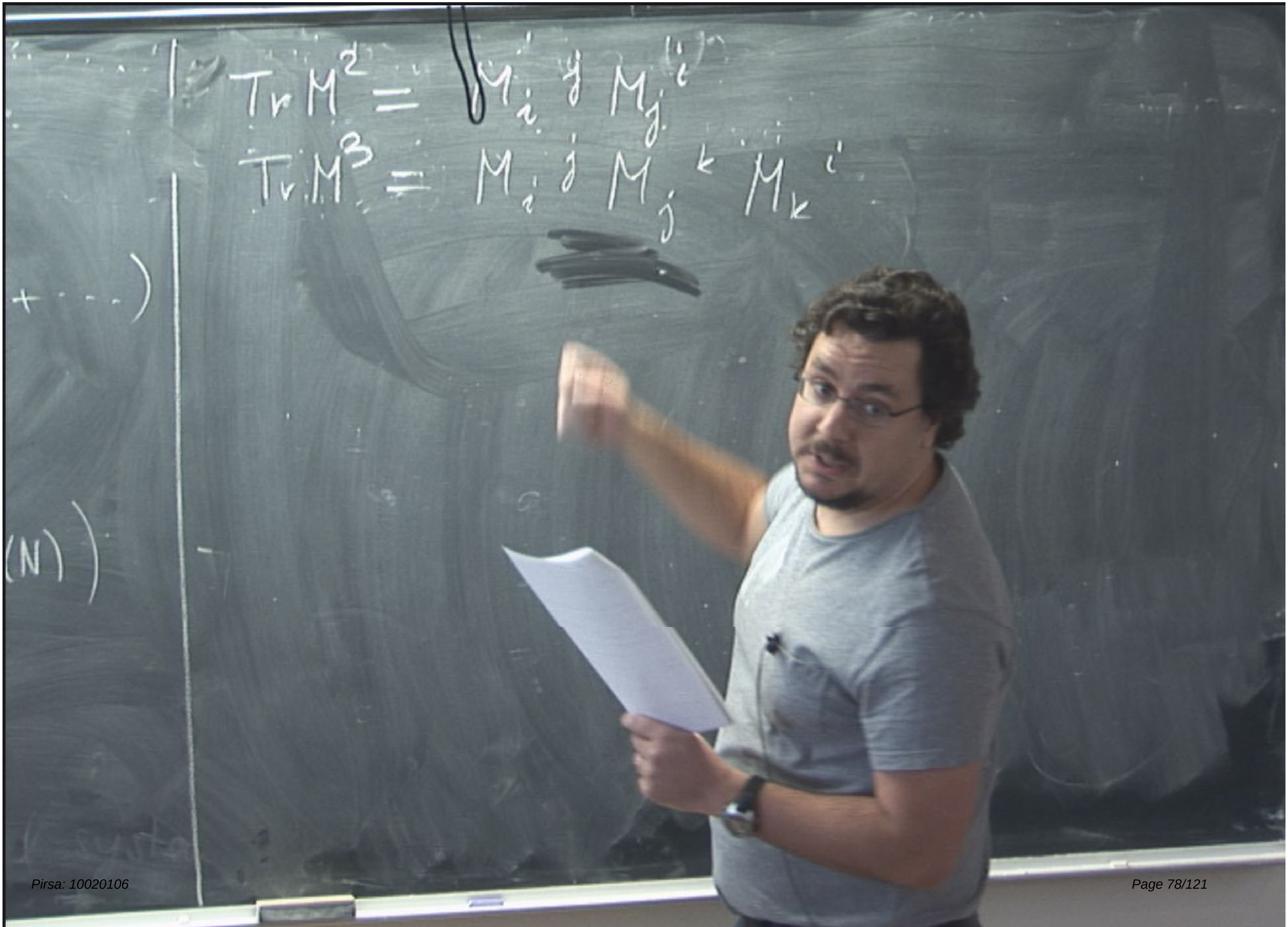
$$U \cdot U^{\dagger} = \mathbb{I}$$

$$U = e^{i\varepsilon T} \qquad (1 + \varepsilon i T)(1 - i \varepsilon T^{\dagger}) = 1$$

$$\boxed{T = T^{\dagger}}$$

T : elements in the Lie algebra


Tr M^2 = M_i^j M_j^i
Tr M^3 = M_i^j M_j^k M_k^i
+ . . .)
(N))


Tr M^2 = M_i^j M_j^i
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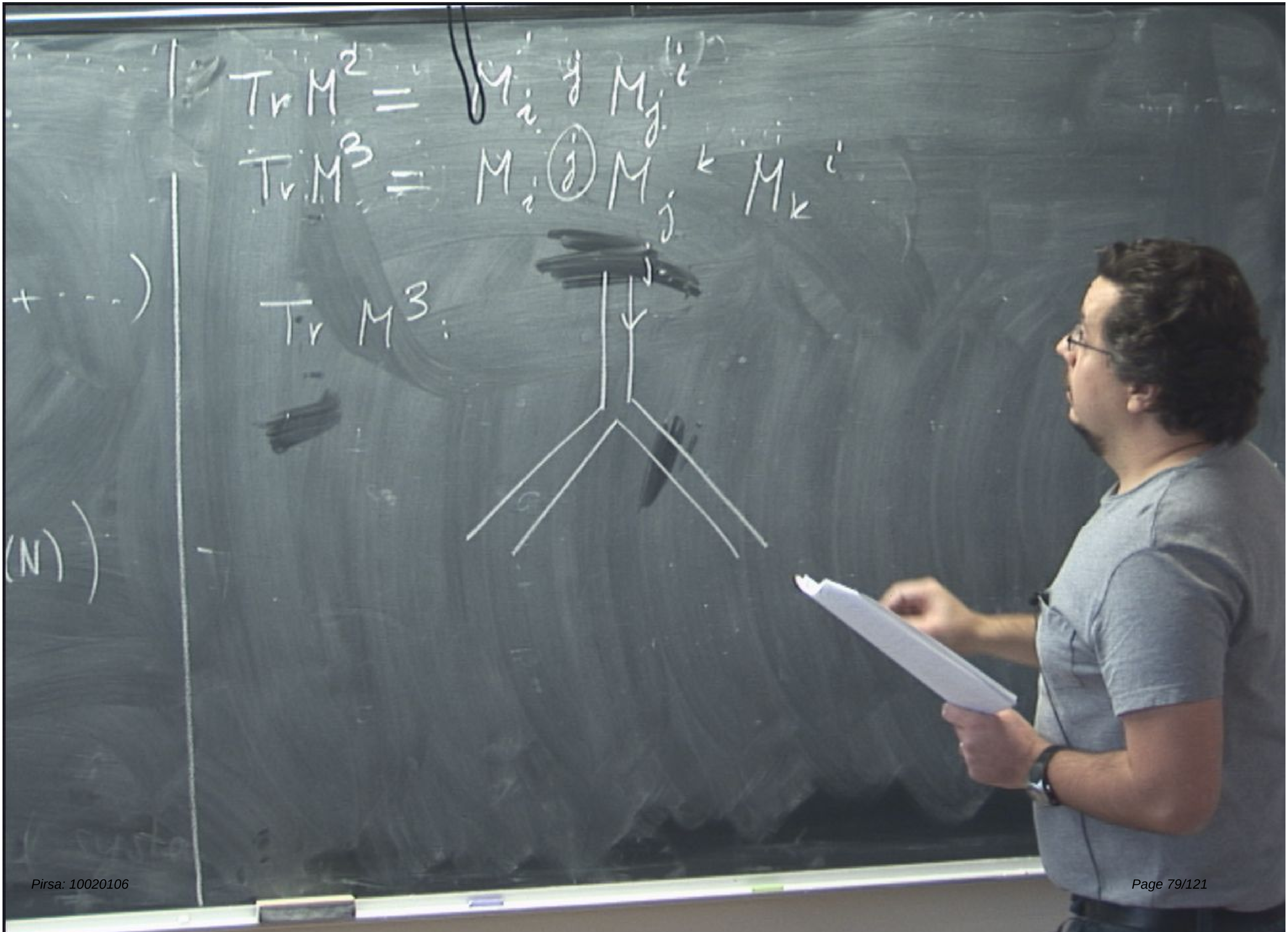
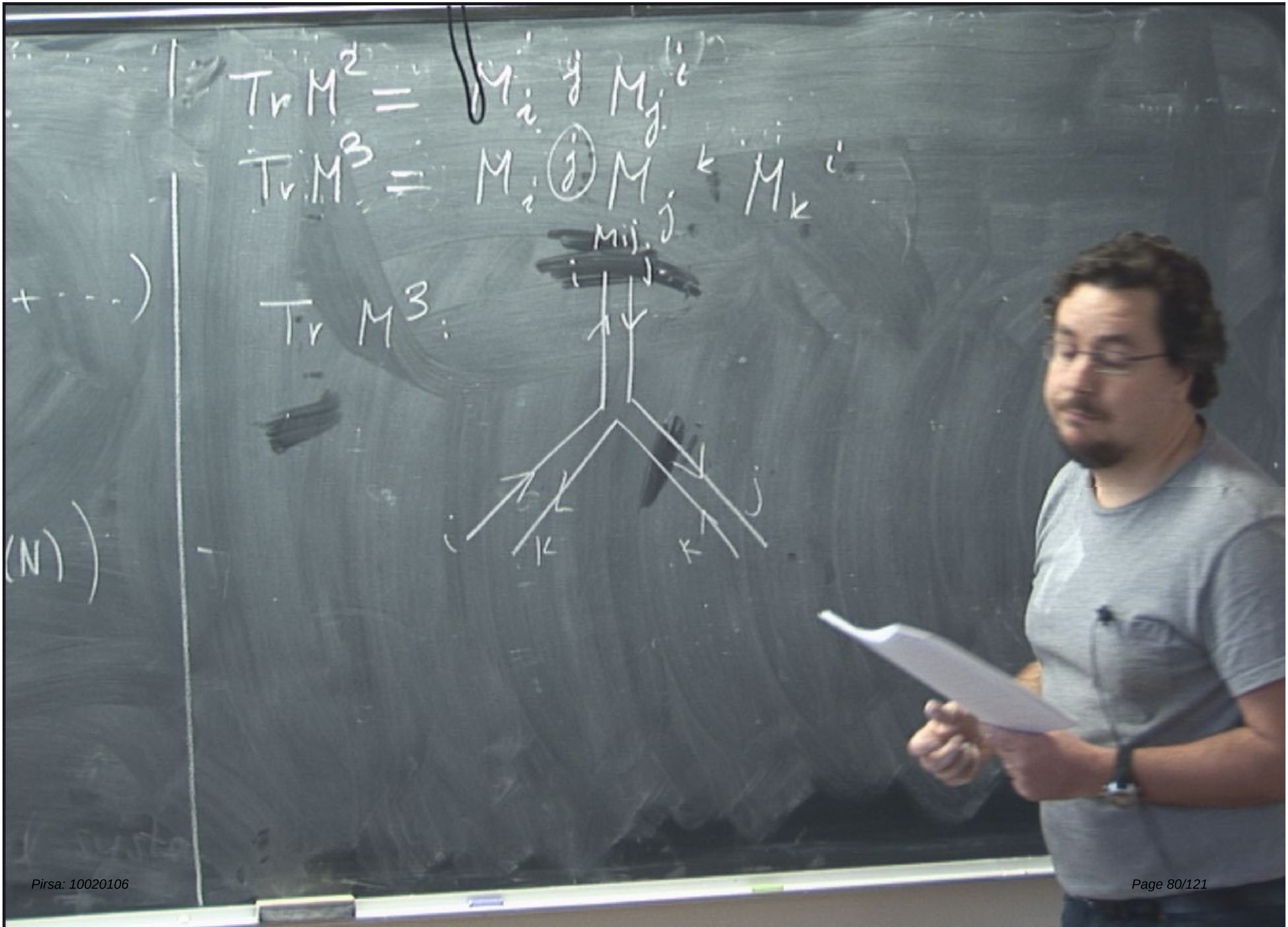

Tr M^2 = M_i^j M_j^i
Tr M^3 = M_i^j M_j^k M_k^i
Tr M^3 :
+ . . .)
(N))


Tr M^2 = M_i^j M_j^i
Tr M^3 = M_i^j M_j^k M_k^i
Tr M^3 :
Mij
i j
i k k j
(N)

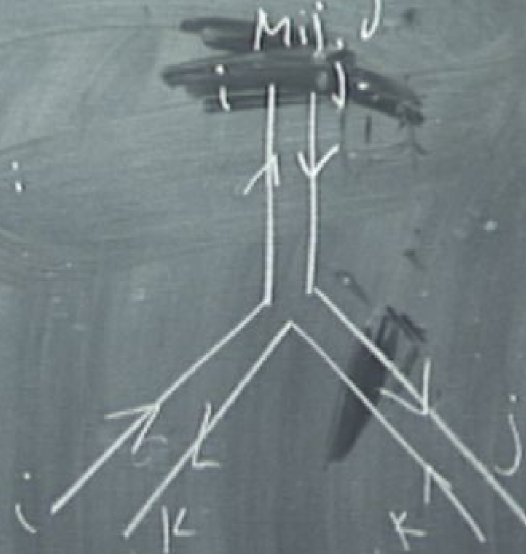
$+ \cdots)$

$(N))$

$$\mathrm{Tr}\, M^2 = M_i{}^j M_j{}^i$$

$$\mathrm{Tr}\, M^3 = M_i{}^j M_j{}^k M_k{}^i$$

$\mathrm{Tr}\, M^3$:



Large N counting:

- Matrix Model: 0-dim QFT M^j_i

$$Z = \int [DM]\, e^{-\frac{1}{g^2} \mathrm{Tr}\left(M^2 + \lambda_3 M^3 + \lambda_4 M^4 + \dots\right)}$$

- Propagator:

Large N counting:

- Matrix Model: 0-dim QFT $M_i^{\,j}$

$$Z = \int [DM]\, e^{-\frac{1}{g^2}\mathrm{Tr}\left(M^2 + \lambda_3 M^3 + \lambda_4 M^4 + \cdots\right)}$$

- Propagator: $\langle M_i^{\,j} \; M_k^{\,l} \rangle$

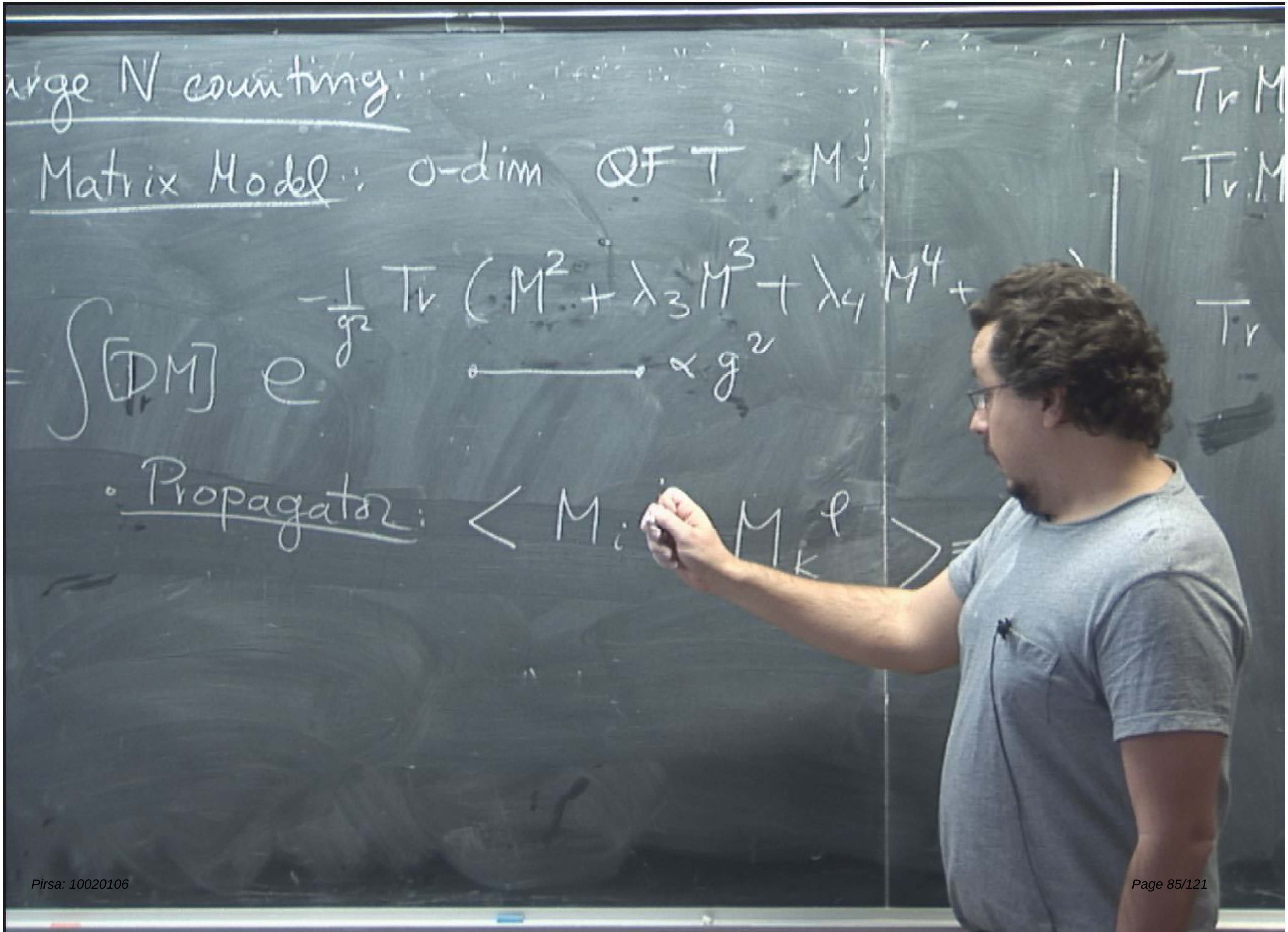
Large N counting:

- Matrix Model: 0-dim QFT $M_i^{\ j}$

$$Z = \int [DM]\, e^{-\frac{1}{g^2} \mathrm{Tr}\left(M^2 + \lambda_3 M^3 + \lambda_4 M^4 + \dots\right)}$$

$\propto g^2$

- Propagator: $\langle M_i^{\ j} \; M_k^{\ \ell} \rangle$


arge N counting:
Matrix Model: 0-dim QFT M_i^j
$= \int [DM] \, e^{-\frac{1}{g^2} \mathrm{Tr} \left(M^2 + \lambda_3 M^3 + \lambda_4 M^4 + \ldots \right)}$
$\propto g^2$
Propagator: $< M_i \ \ M_k^{\ell} >$
Tr M
Tr M
Tr

ing:

el: 0-dim QFT $M_i^{\,j}$

$-\frac{1}{g^2}\,\mathrm{Tr}\,(M^2+\lambda_3 M^3+\lambda_4 M^4+\dots)$

$\propto g^2$

gator: $\langle M_i^{\,j}\, M_k^{\,l}\rangle = g^2\,\delta_i^{\,l}\,\delta_k^{\,j}$

$j \longrightarrow$

$\mathrm{Tr}\,M^2 = M_i^{\,j}\, M$

$\mathrm{Tr}\,M^3 = M_i^{\,j}\, M$

$\mathrm{Tr}\,M^3$:

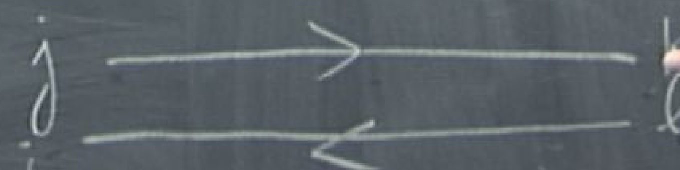
...ing:

...l: 0-dim QFT $M_i^{\ j}$

$$-\frac{1}{g^2}\,\mathrm{Tr}\,(M^2 + \lambda_3 M^3 + \lambda_4 M^4 + \dots)$$

o————o $\propto g^2$

...gator: $\langle M_i^{\ j}\, M_k^{\ \ell} \rangle = g^2 \delta_i^{\ \ell}\, \delta_k^{\ j}$



$\mathrm{Tr}\, M^2 = M_i^{\ j} M$...

$\mathrm{Tr}\, M^3 = M_i^{\ j} M$...

Mi...

$\mathrm{Tr}\, M^3$:

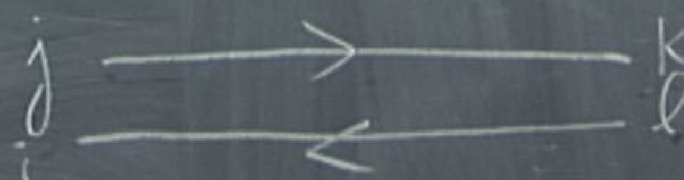
...ing:

...el: 0-dim QFT $M_i^{\,j}$

$-\frac{1}{g^2}\,\mathrm{Tr}\,(M^2+\lambda_3 M^3+\lambda_4 M^4+\dots)$

$\propto g^2$

...gator: $\langle M_i{}^j \; M_k{}^\ell \rangle = g^2\,\delta_i^{\,\ell}\,\delta_k^{\,j}$

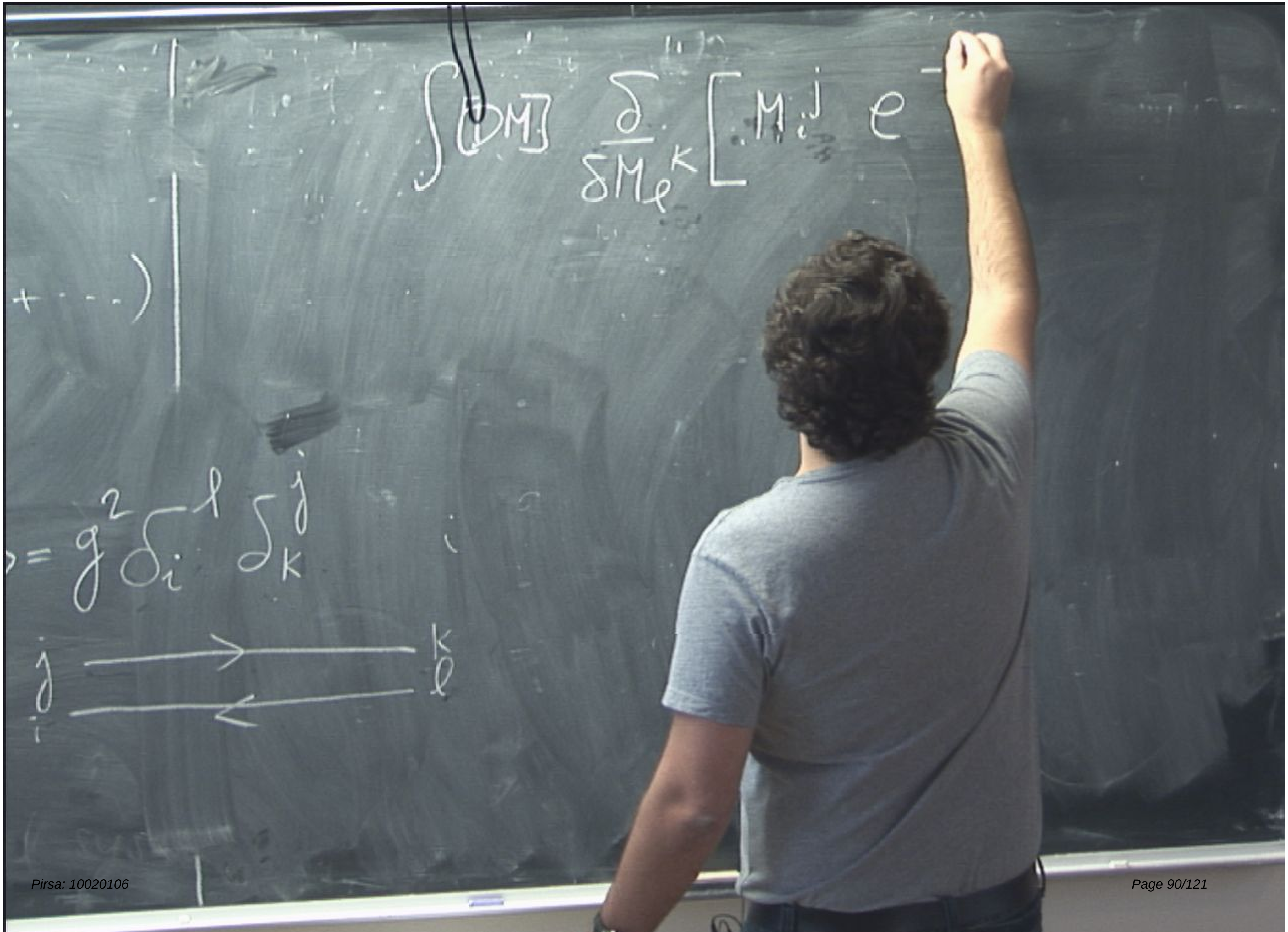


$\mathrm{Tr}\,M^2 = M_i{}^j\, M$...

$\mathrm{Tr}\,M^3 = M_i{}^j\, M$...

$\mathrm{Tr}\,M^3$:


∫[DM] δ
+ . . .)
= g² δ_i^l δ_k^j

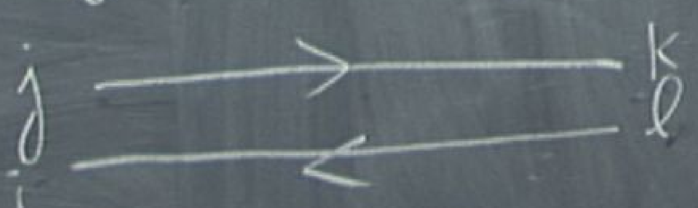


$\int [DM] \frac{\delta}{\delta M_\ell{}^k} [M_i{}^j e$
$+ \cdots)$
$= g^2 \delta_i{}^\ell \delta_k{}^j$

$$\int [DM] \frac{\delta}{\delta M_\ell{}^k}\left[M_i{}^j \, e^{-\frac{1}{2g^2} M_m{}^n M_n{}^m}\right]$$

$$\left(+ \cdots \right)$$

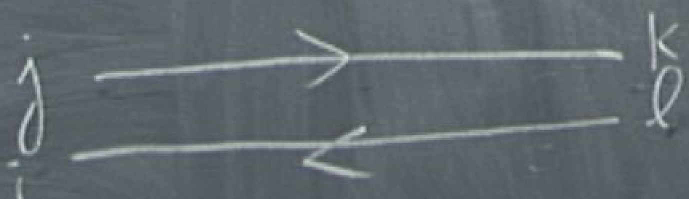
$$= g^2 \delta_i{}^\ell \, \delta_k{}^j$$

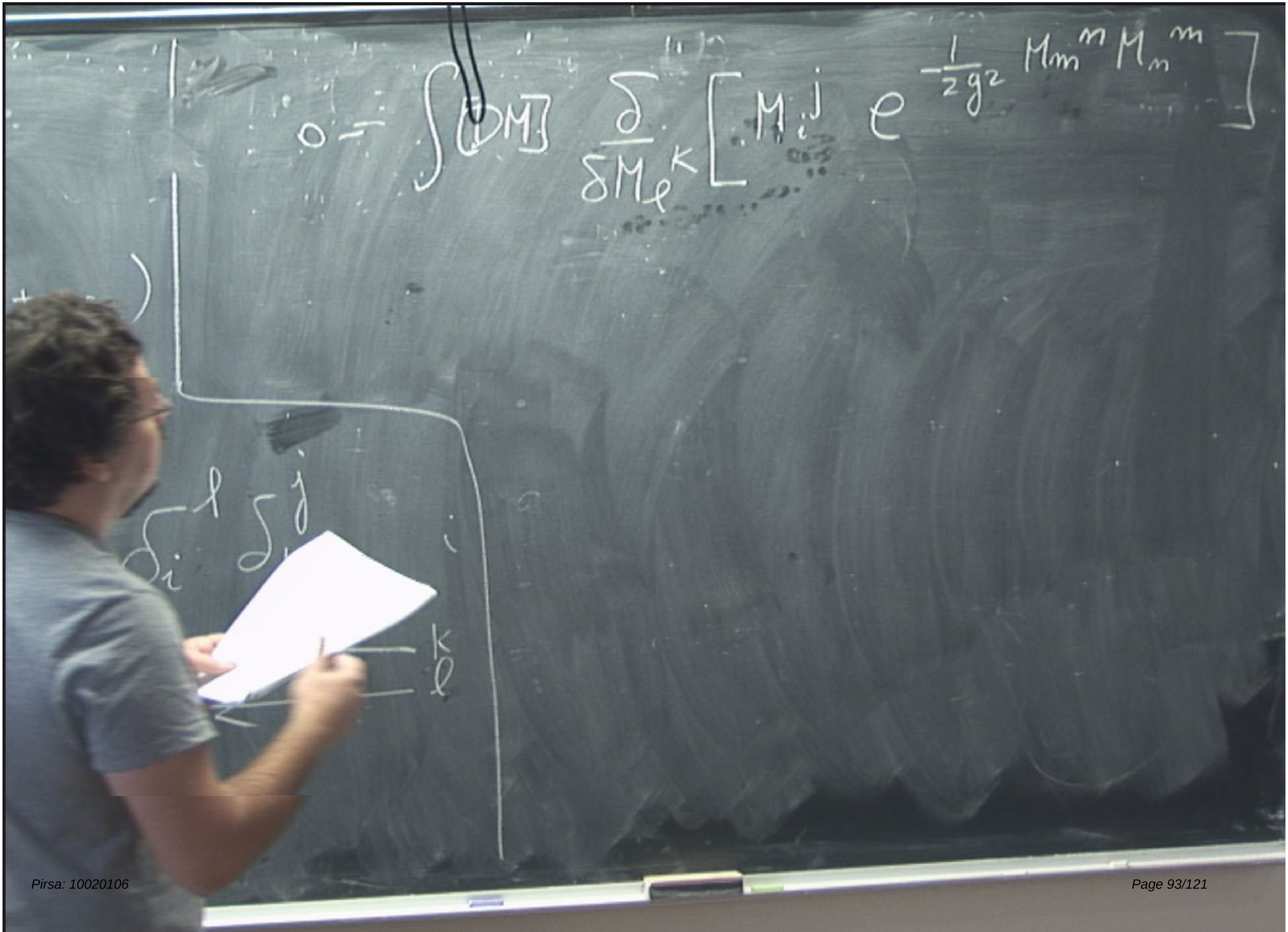


$$0 = \int [DM] \, \frac{\delta}{\delta M_\ell{}^k}\left[M_i{}^j \, e^{-\frac{1}{2g^2} M_m{}^n M_n{}^m} \right]$$

$+ \cdots)$

$$= g^2 \delta_i{}^\ell \, \delta_k{}^j$$




$0 = \int [DM] \frac{\delta}{\delta M_\ell{}^k}\left[M_i{}^j \, e^{-\frac{1}{2g^2} M_m{}^n M_n{}^m} \right]$
$\delta_i{}^\ell \, \delta$
k
ℓ

$$0 = \int [DM] \frac{\delta}{\delta M_\ell{}^k}\left[M_i{}^j \, e^{-\frac{1}{2g^2} M_m{}^n M_n{}^m} \right]$$

$$+ \dots)$$

$$\frac{\delta M_i{}^j}{\delta M_\ell{}^k} = \delta_i^\ell \, \delta_k^j$$

$$\rangle = g^2 \delta_i^\ell \, \delta_k^j$$

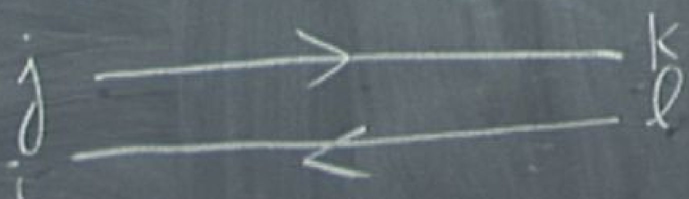
$$0 = \int [DM] \frac{\delta}{\delta M_\ell{}^k}\left[M_i{}^j \, e^{-\frac{1}{2g^2} M_m{}^n M_n{}^m} \right]$$

$+ \cdots)$

$$\frac{\delta M_i{}^j}{\delta M_\ell{}^k} = \delta_i^\ell \, \delta_k^j$$

$$= g^2 \delta_i^\ell \, \delta_k^j$$

$$\delta_i^\ell \, \delta_k^j -$$



$$0 = \int [DM] \frac{\delta}{\delta M_\ell{}^k} \left[M_i{}^j \, e^{-\frac{1}{2g^2} M_m{}^n M_n{}^m} \right]$$

$$+ \cdots)$$

$$\frac{\delta M_i{}^j}{\delta M_\ell{}^k} = \delta_i^\ell \delta_k^j$$

$$\rangle = g^2 \delta_i{}^\ell \, \delta_k^j$$

$$\delta_i^\ell \delta_k^j - \frac{1}{g^2} \langle M_i{}^j M_k{}^\ell \rangle = 0$$

$$j \longrightarrow k$$

$$i \longleftarrow \ell$$

$+ \cdots)$

$$0 = \int [DM] \frac{\delta}{\delta M_\ell{}^k}\left[M_i{}^j \, e^{-\frac{1}{2g^2} M_m{}^n M_n{}^m} \right]$$

$$\frac{\delta M_i{}^j}{\delta M_\ell{}^k} = \delta_i^\ell \, \delta_k^j$$

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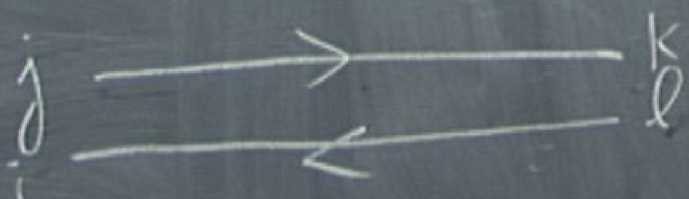
$$= g^2 \delta_i^\ell \, \delta_k^j$$



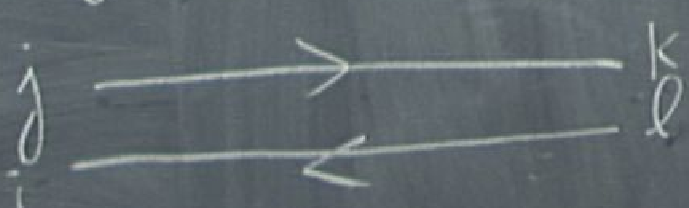
$$0 = \int [DM]\, \frac{\delta}{\delta M_\ell{}^k}\left[M_i{}^j \, e^{-\frac{1}{2g^2} M_m{}^n M_n{}^m} \right]$$

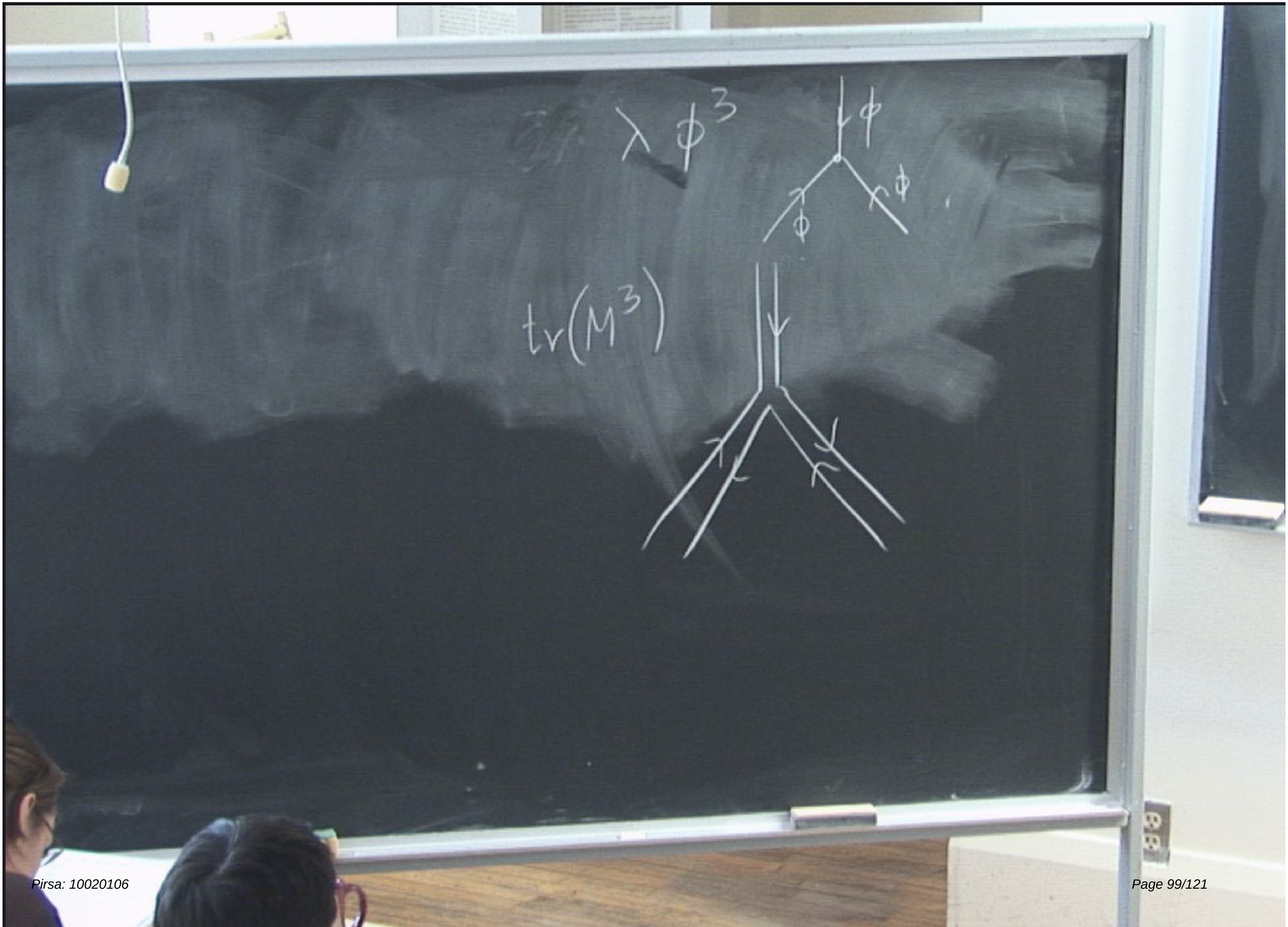
$+ \cdots)$

$$\frac{\delta M_i{}^j}{\delta M_\ell{}^k} = \delta_i^\ell \, \delta_k^j$$

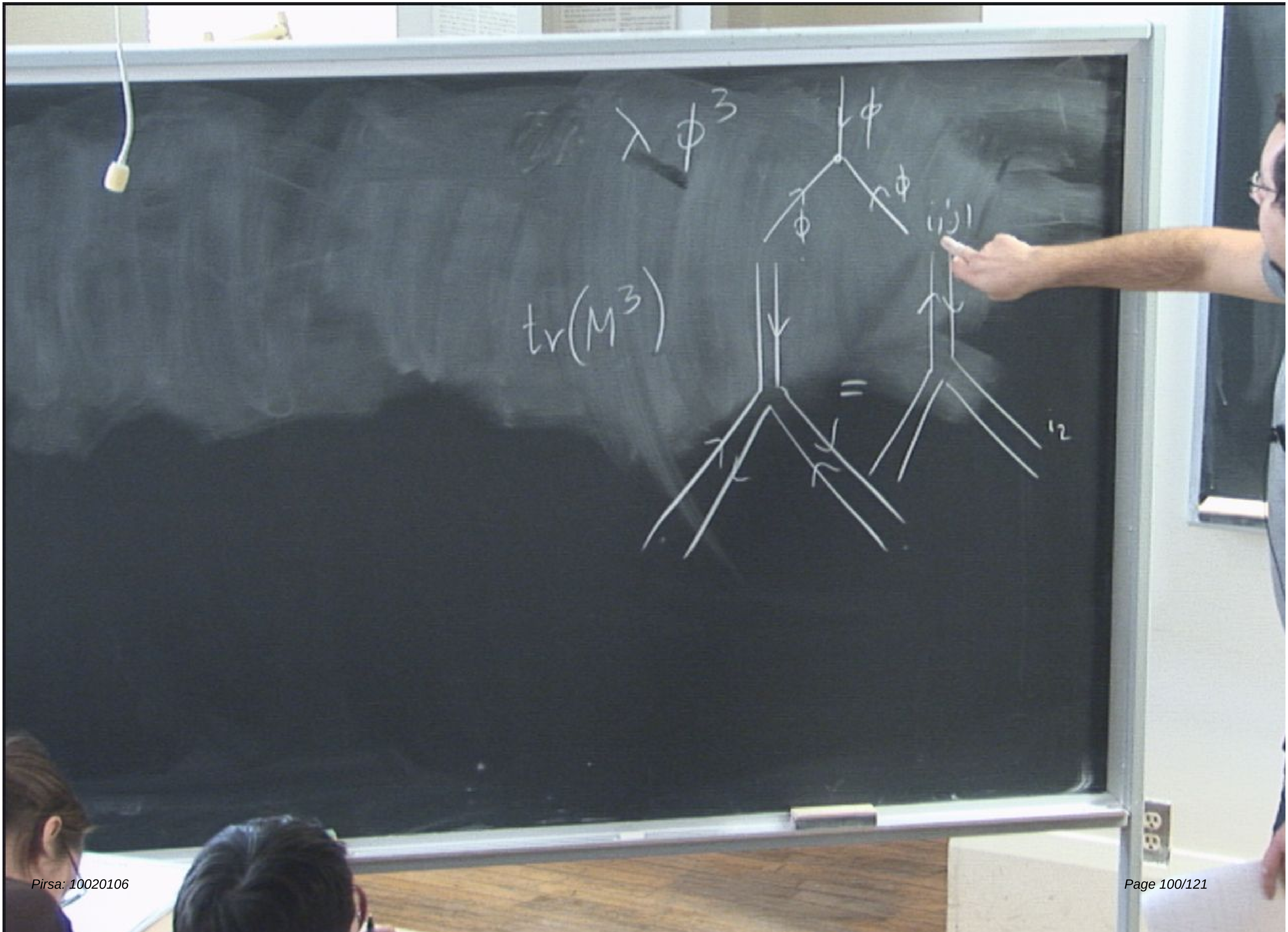
$$= g^2 \delta_i^\ell \, \delta_k^j$$

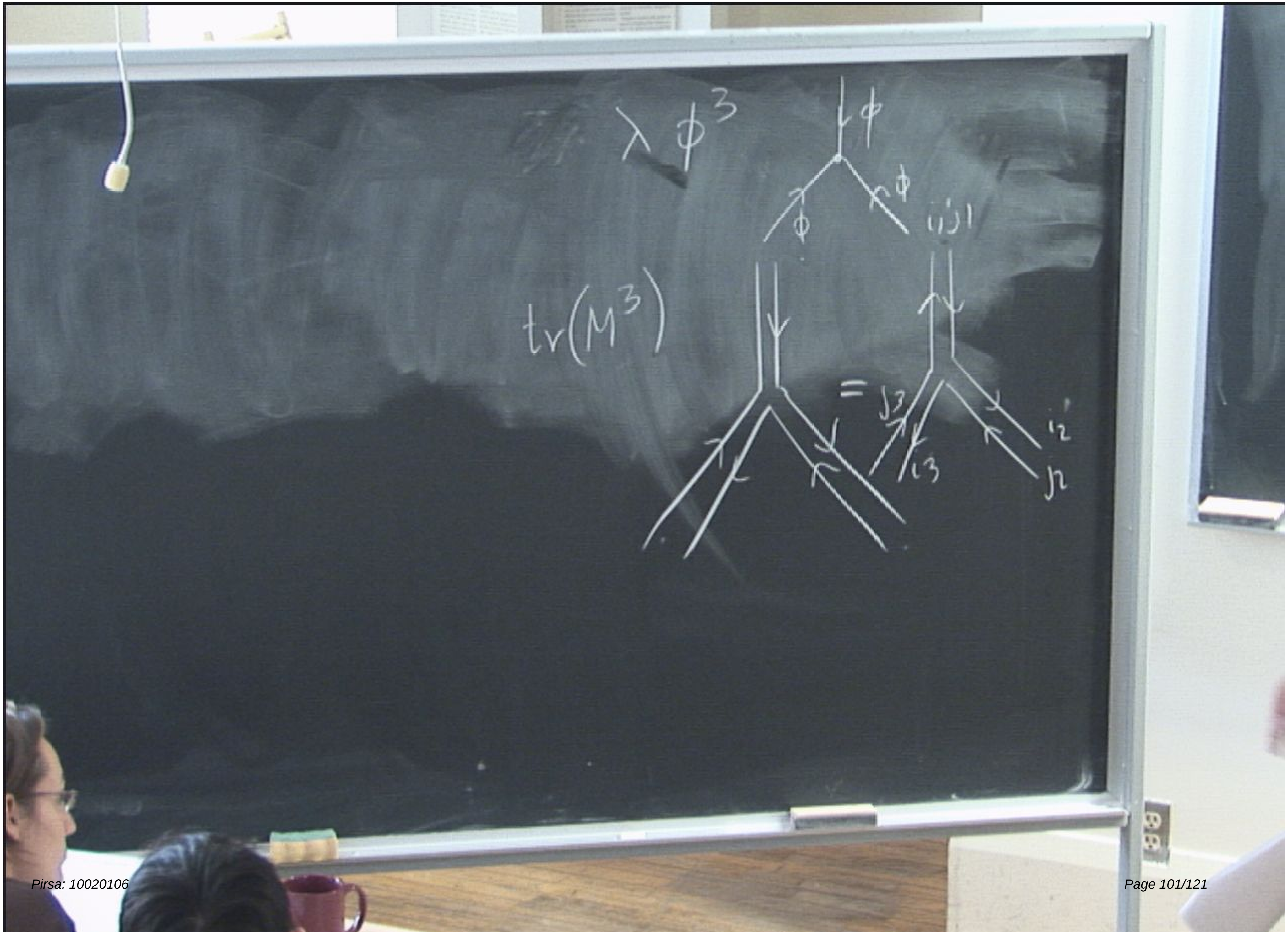
$$\delta_i^\ell \, \delta_k^j - \frac{1}{g^2} \langle M_i{}^j M_k{}^\ell \rangle = 0$$



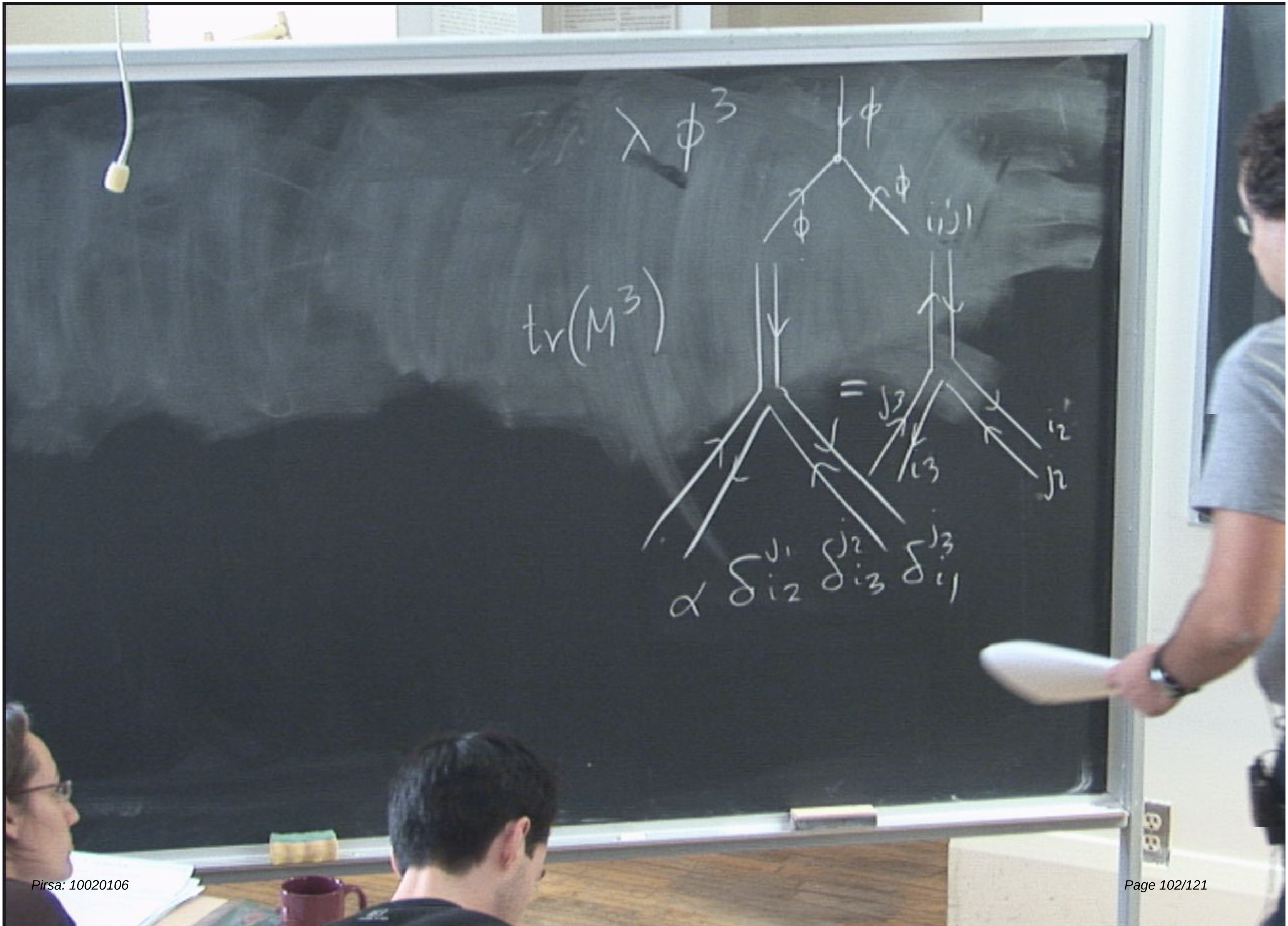


$\lambda\phi^3$
ϕ
ϕ
ϕ
$tr(M^3)$

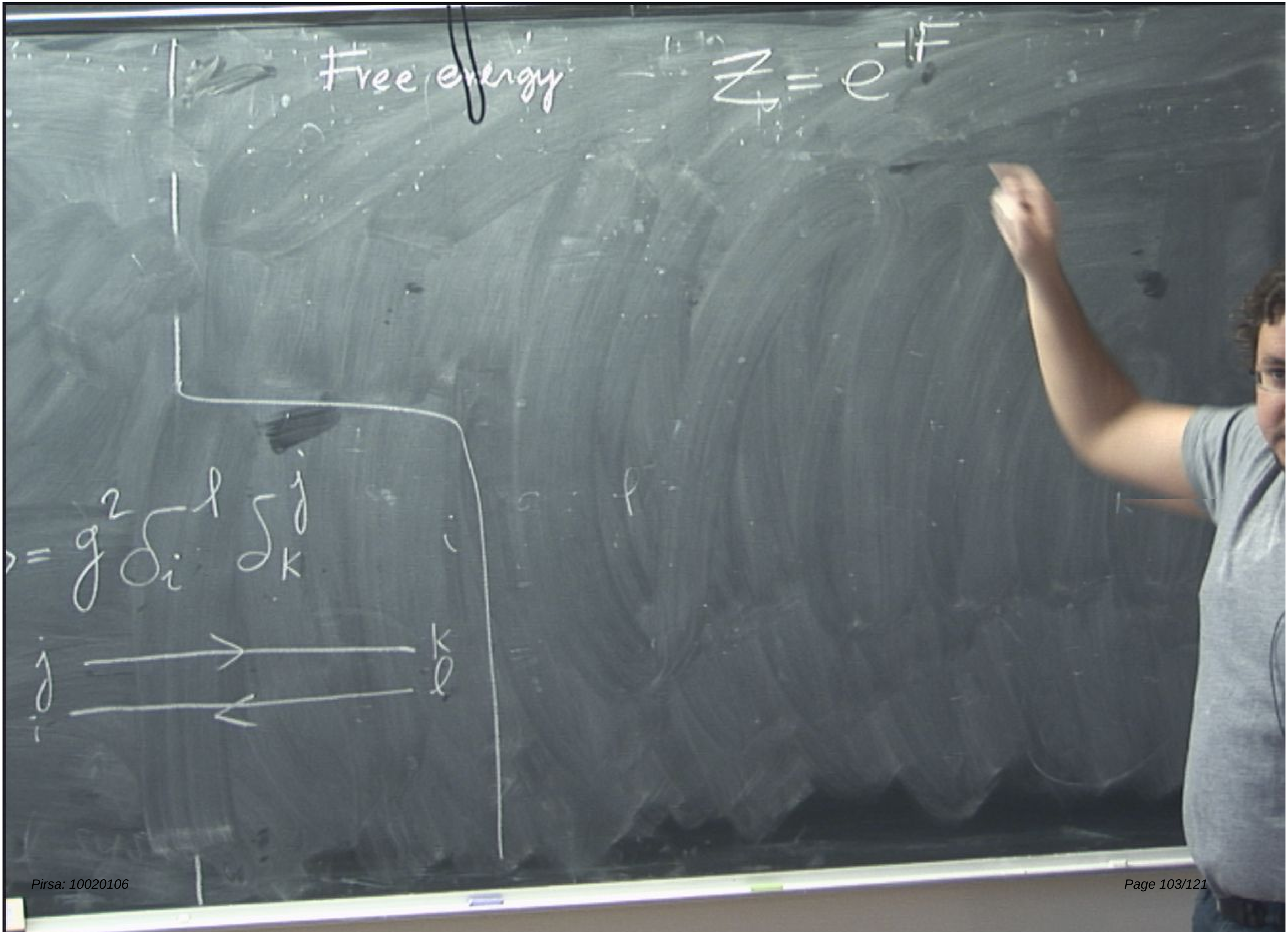

$\lambda\phi^3$
ϕ
ϕ
ϕ
$tr(M^3)$
=

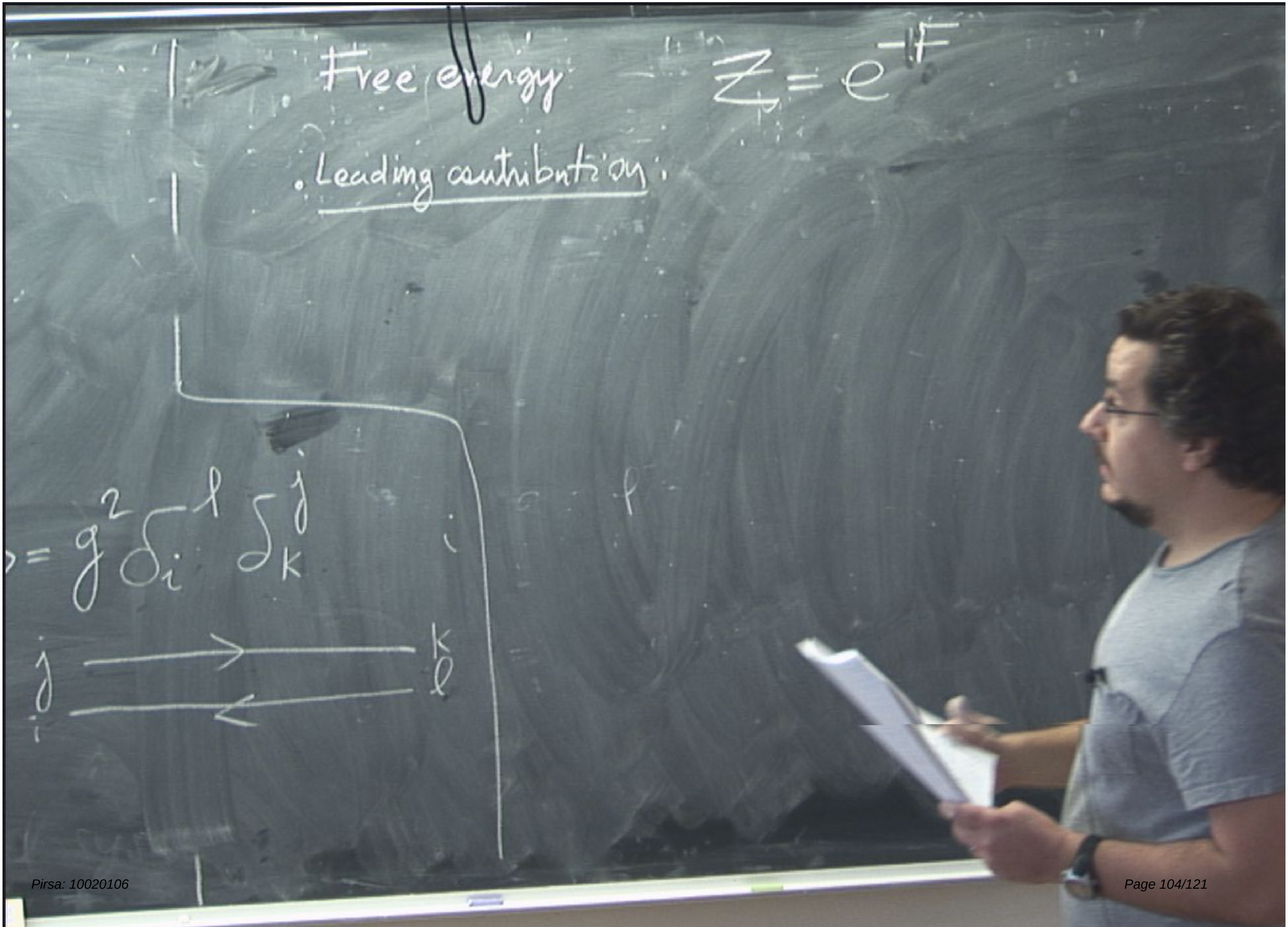


λ φ³
tr(M³)
φ



$\lambda \phi^3$
ϕ
ϕ
ϕ
$tr(M^3)$
$=$
$\propto \delta^{j_1}_{i_2} \delta^{j_2}_{i_3} \delta^{j_3}_{i_1}$


Free energy
Z = e^{-F}
= g^2 δ_i^l δ_k^j

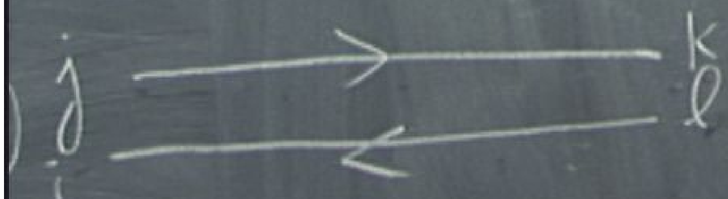

Free energy
Z = e^-F
Leading contribution:
= g^2 δ_i^l δ_k^j
j
i
k
l

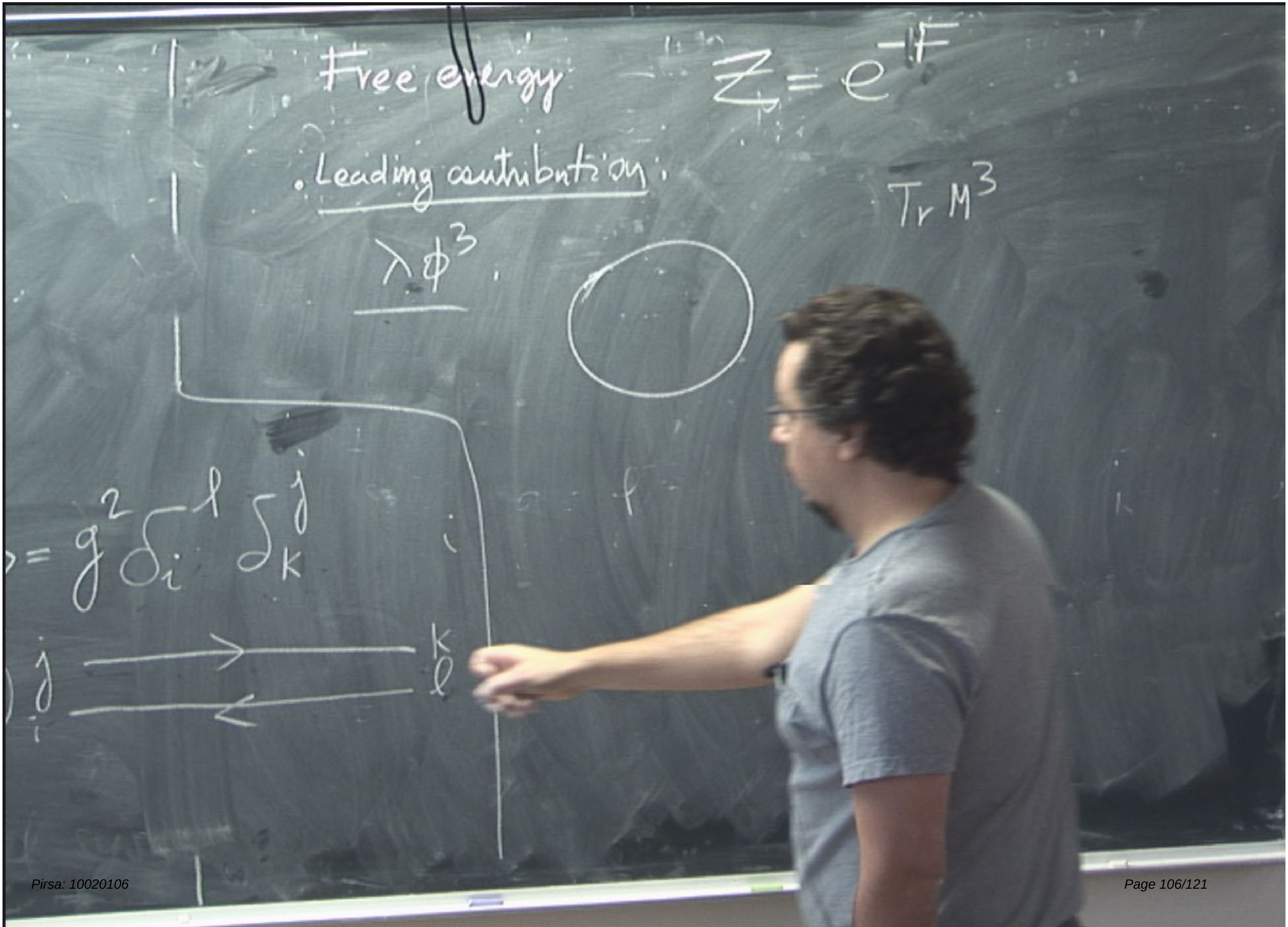
Free energy $\quad Z = e^{-F}$

. Leading contribution:

$\lambda \phi^3$

$= g^2 \delta_i^{\ l} \, \delta_k^{\ j}$





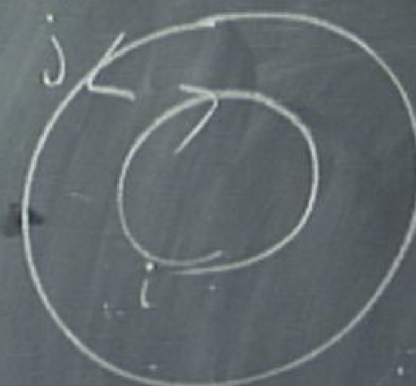
Free energy
$Z = e^{-F}$
Leading contribution:
$\lambda\phi^3$
$\mathrm{Tr}\, M^3$
$= g^2 \delta_i^{\,l} \delta_k^{\,j}$

Free energy $\quad Z = e^{-F}$

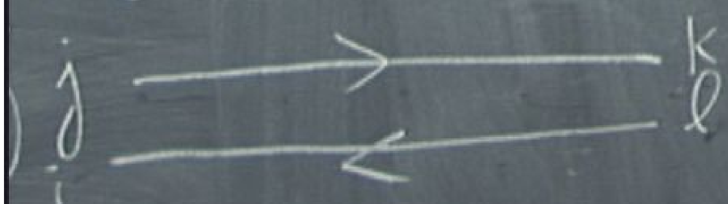
. Leading contribution:

$\lambda \phi^3$:

$Tr\, M^3$



$$= g^2 \delta_i^{\ l} \delta_k^{\ j}$$

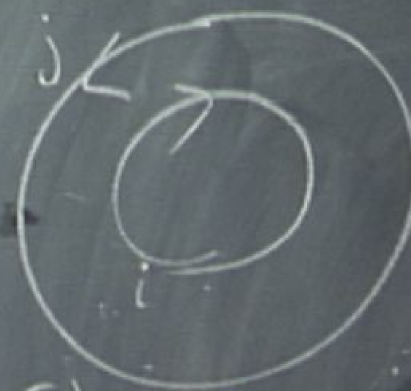


Free energy $\quad Z = e^{-F}$

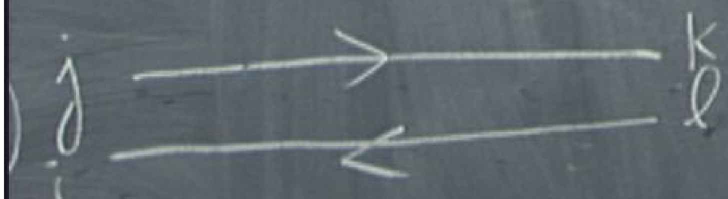
. Leading contribution:

$\lambda \phi^3$.

$\mathrm{Tr}\, M^3$



$\ldots = g^2 \delta_i^{\,l} \, \delta_k^{\,j}$

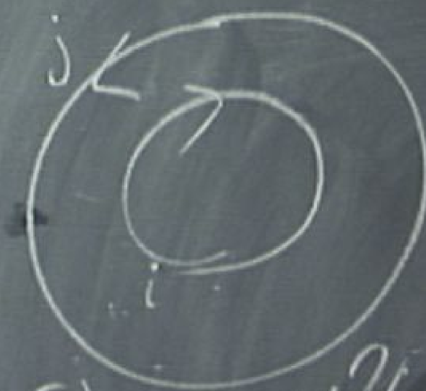


$$\sum_{i,j=1}^{N} \delta_i^{\,i} \delta_j^{\,j}$$

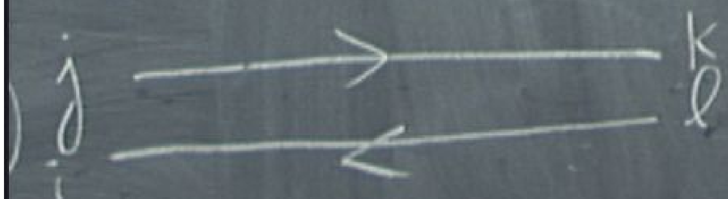
Free energy $\qquad Z = e^{-F}$

Leading contribution:

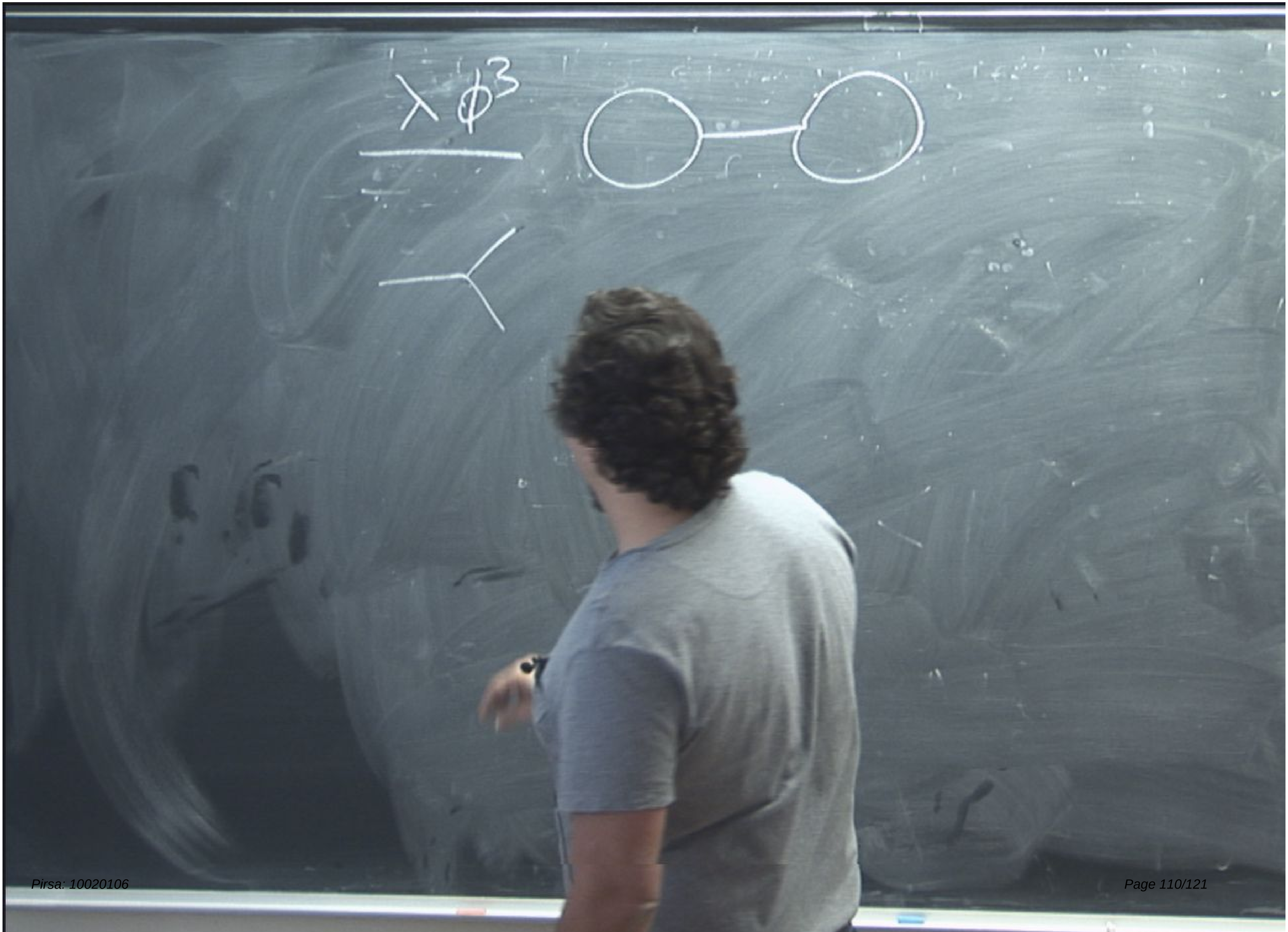
$\lambda\phi^3$ $\qquad$ $\mathrm{Tr}\, M^3$



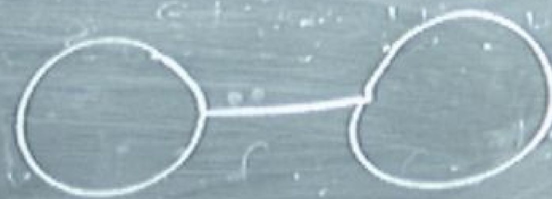
$$\ldots = g^2 \delta_i^{\ l} \delta_k^{\ j}$$



$$\sum_{i,j=1}^{N} \delta_i^{\ i} \delta_j^{\ j} = N^2$$

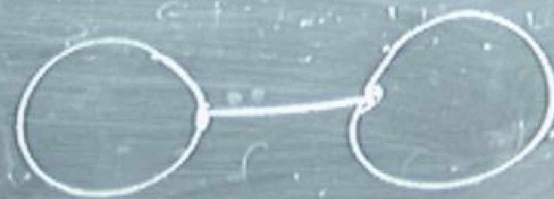

λφ³

$\lambda\phi^3$



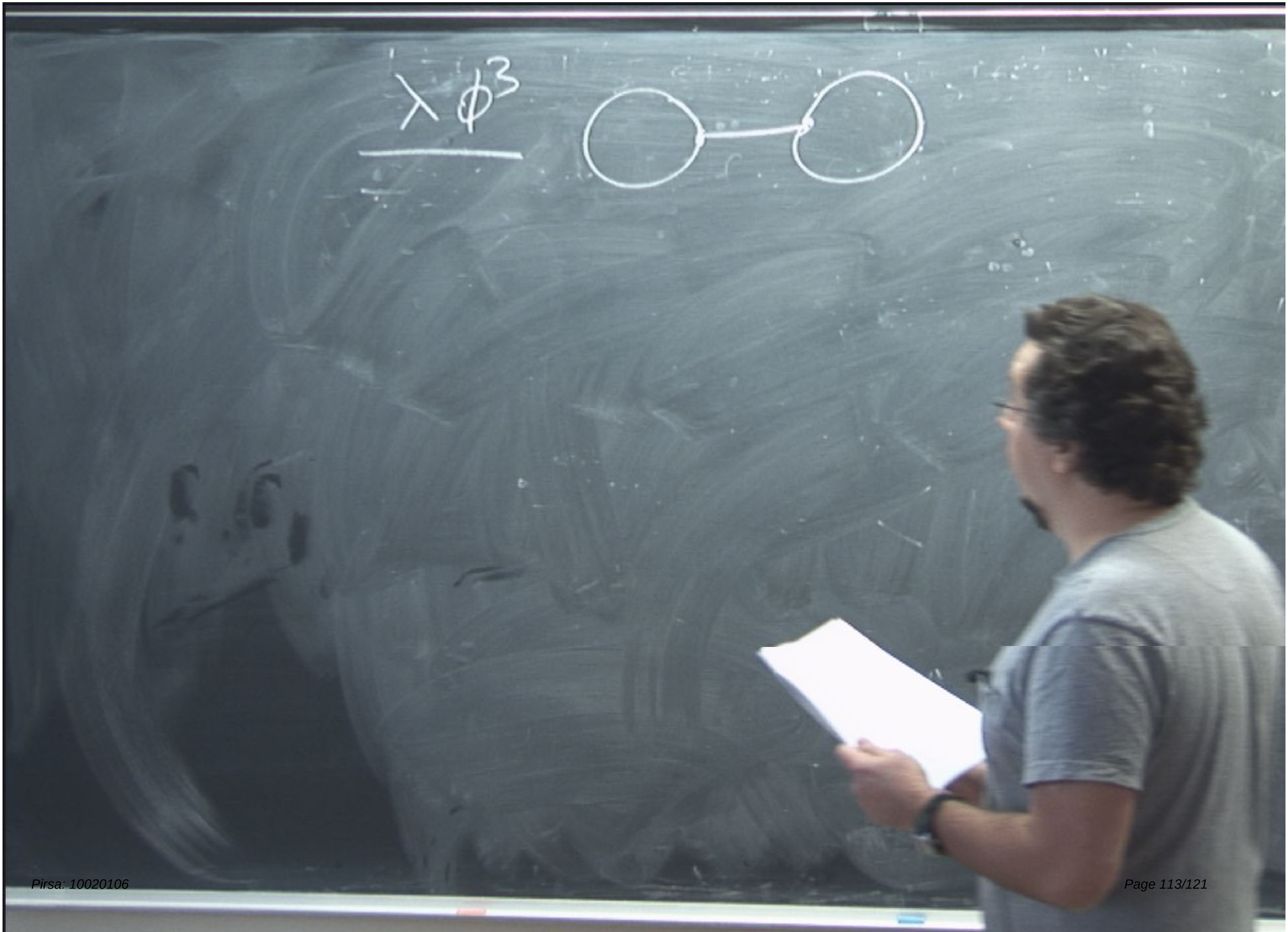
$$\int dx \; x^3 \, e^{-\frac{1}{2}x^2} = 0$$

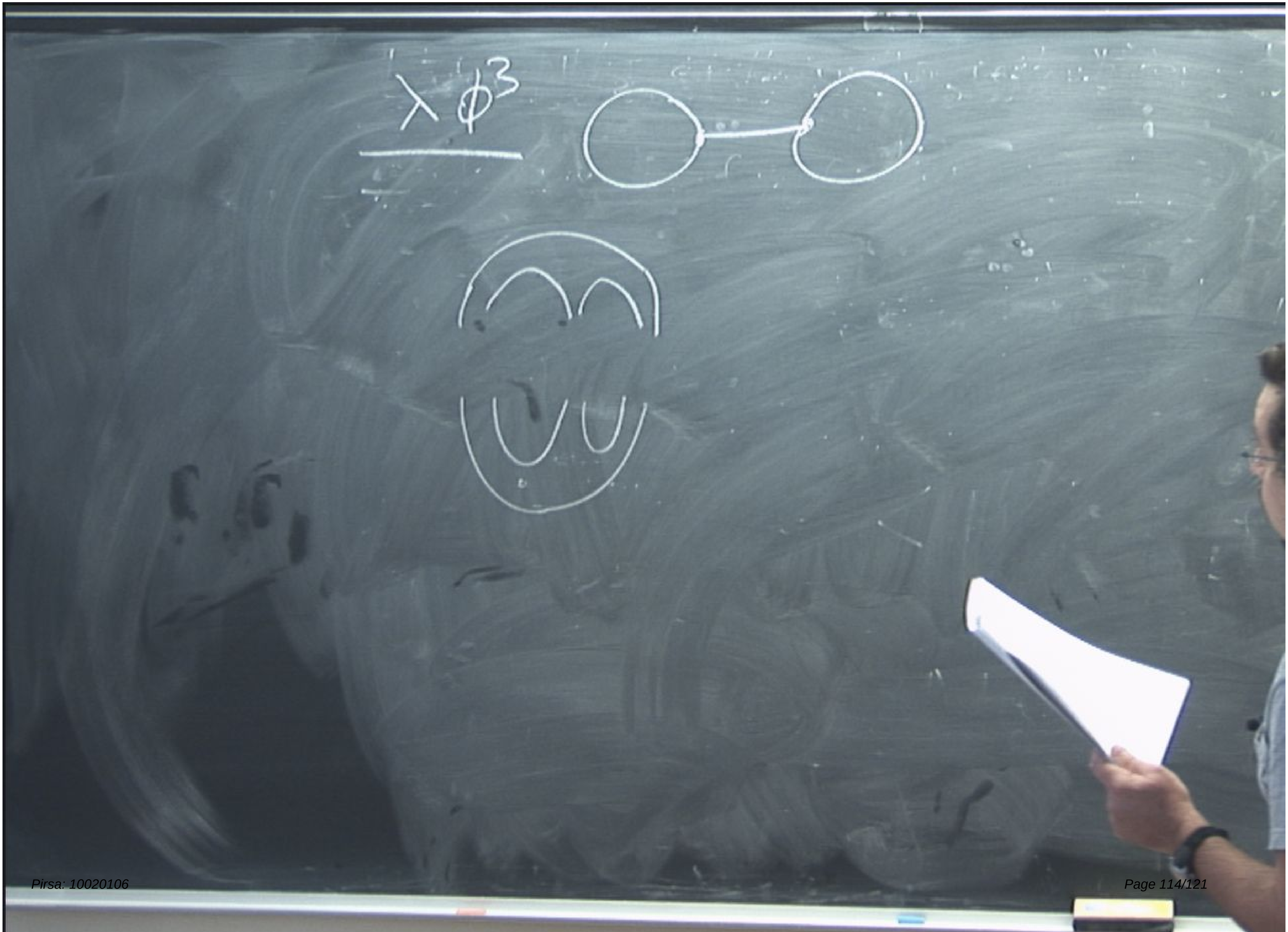
$\lambda \phi^3$

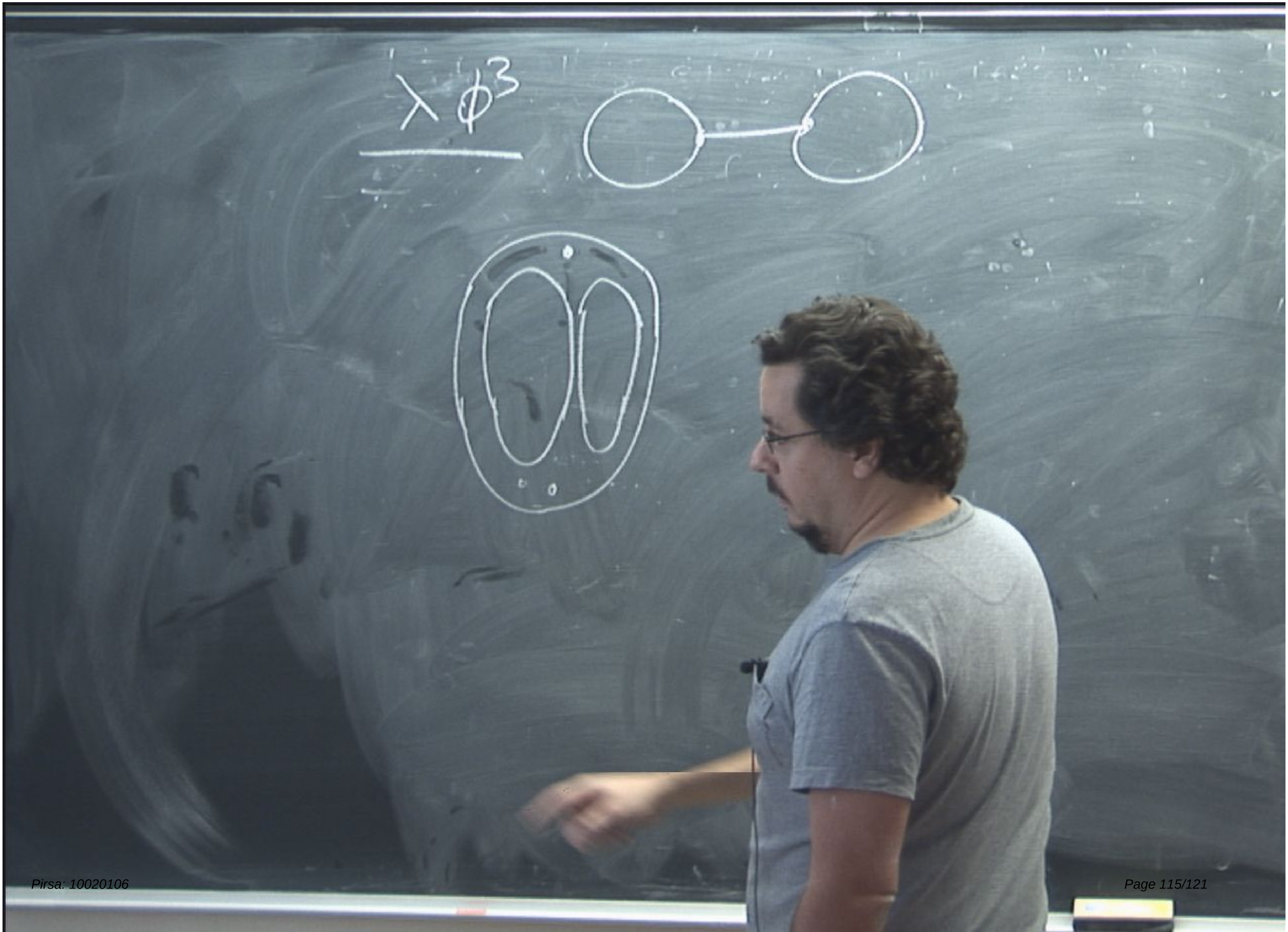


$$\int dx \quad x^3 \qquad -\frac{1}{2}x^2$$

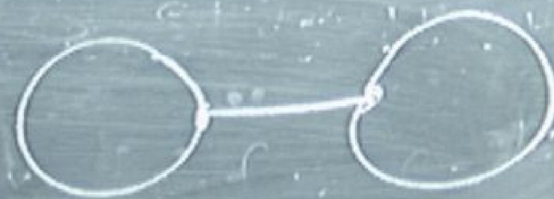
$$= 0$$

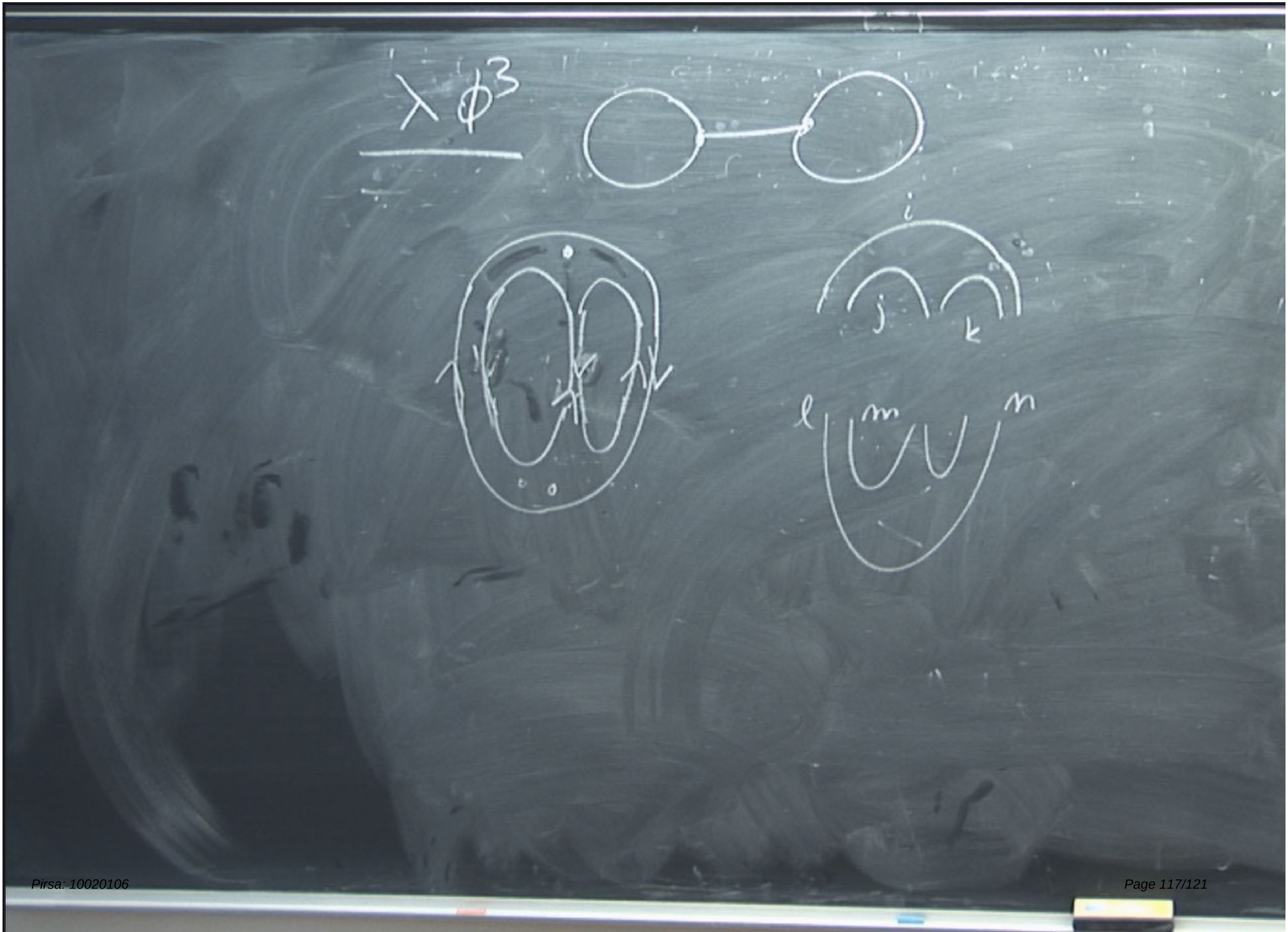

$\lambda \phi^3$


$\lambda \phi^3$

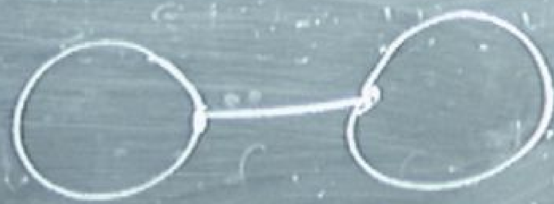

λφ³

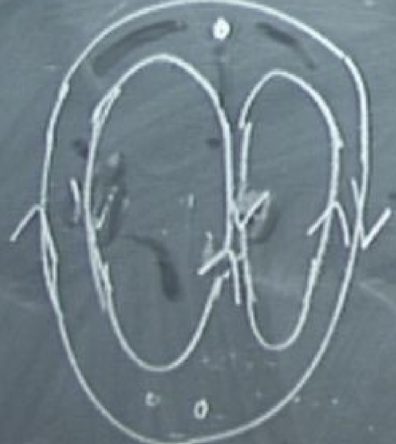
$\lambda \phi^3$

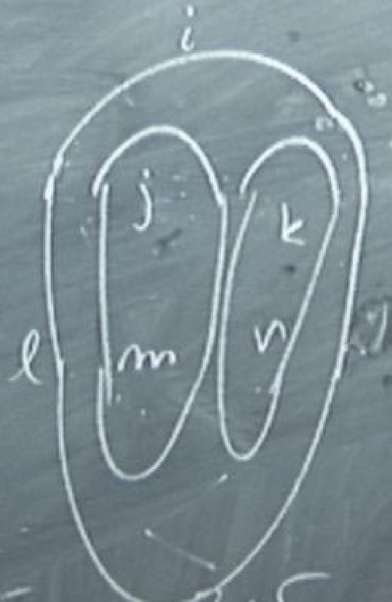



λφ³
i
j
k
l
m
n

$\lambda \phi^3$







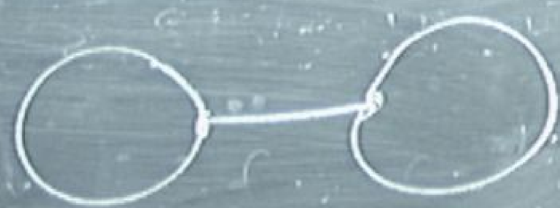
$\propto N^3 g^2$

$$\sum_{i,j,k,l,m,n} (\delta_{il})^2 (\delta_{jm})^2 (\delta_{kn})^2 = \delta_{ii}\delta_{jj}\delta_{kk} = N^3$$

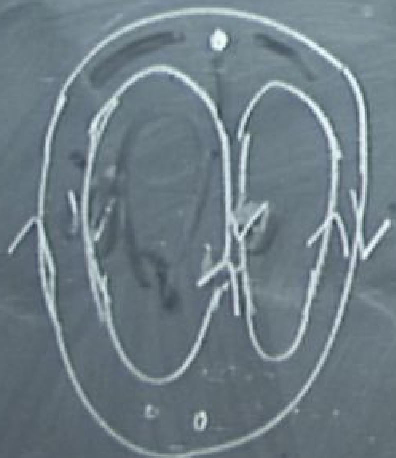
$= g^2 \delta_i$

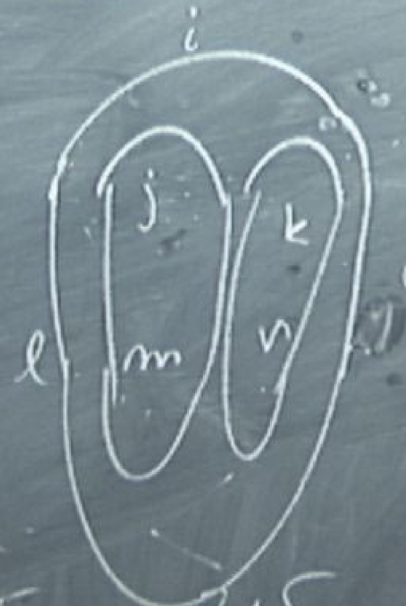
$(1)\ j_i$

$\lambda \phi^3$



$= \delta_{ii} = N$

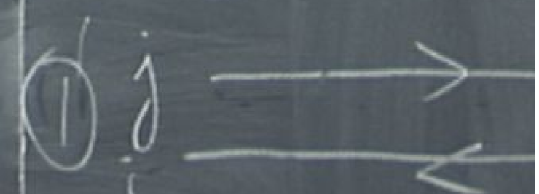


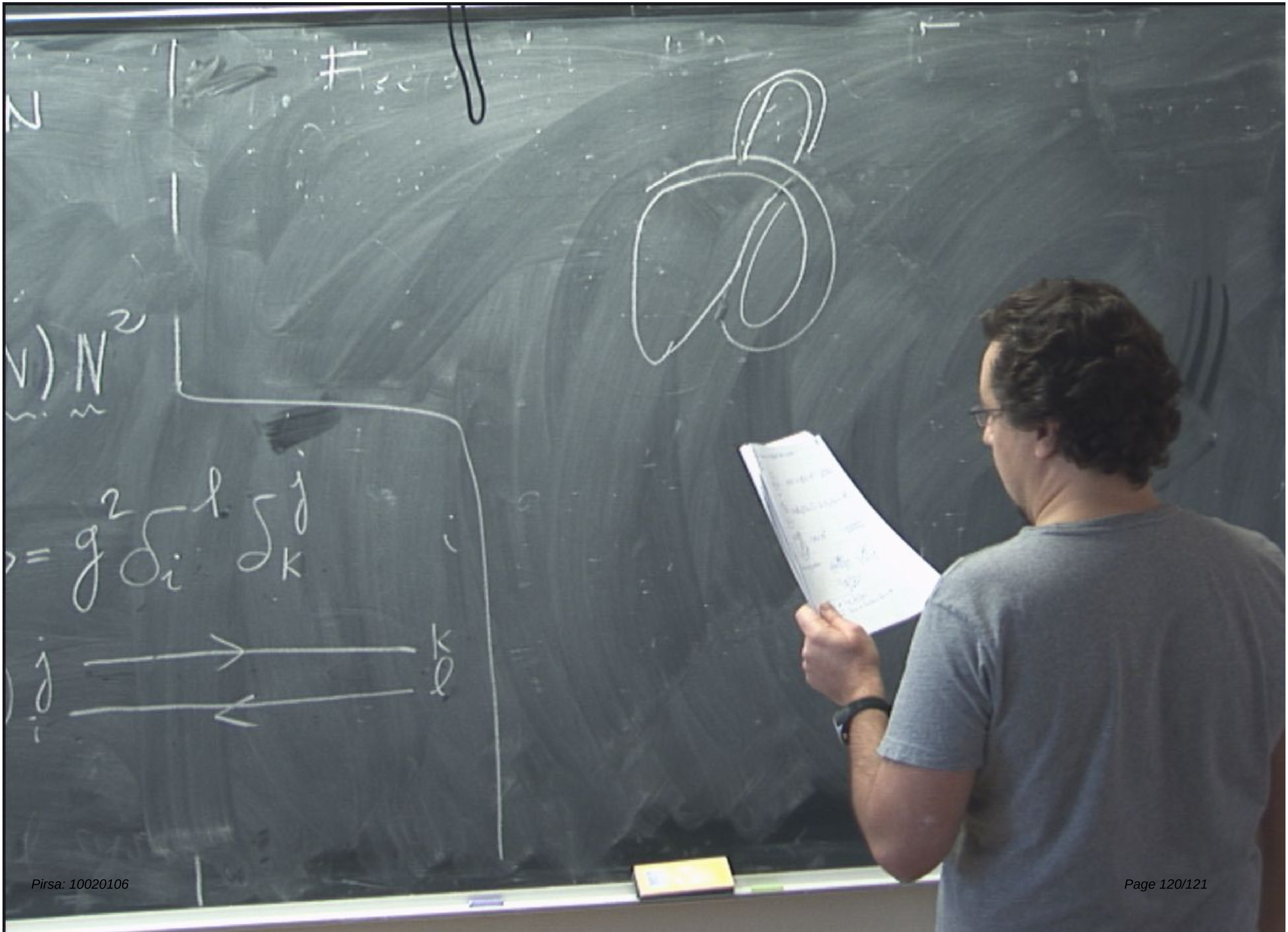


$\propto N^3 g^2 = (g^2 N) N^2$

$$\sum_{i,j,k,\ell,m,n} (\delta_{i\ell})^2 (\delta_{jm})^2 (\delta_{kn})^2 = \delta_{ii}\delta_{jj}\delta_{kk} = g^2 \delta_i^{\ \ell} \delta$$

$$= N^3$$




$= g^2 \delta_i^l \delta_k^j$