Title: Condensed Matter II - Lecture 7

Date: Feb 24, 2010 10:10 AM

URL: http://pirsa.org/10020103

Abstract:

$\vec{v} = \frac{\vec{p}_i}{m}$

Non-int Bose Gas
condensed in
state where

$$\vec{p}_i = \frac{\vec{P}}{N} \quad i = 1, \ldots N$$

$\vec{v} = \frac{\vec{p}_i}{m}$

Non-int Bose Gas

condensed in state where

$$\vec{p}_i = \frac{\vec{P}}{N} \quad i = 1, \ldots N$$

$$E = \sum_i \frac{p_i^2}{2m} = \frac{P^2}{2mN}$$

$\vec{v} = \frac{\vec{p}_i}{m}$

Non-int Bose Gas

Condensed in state where

$$\vec{p}_i = \frac{\vec{P}}{N} \qquad i = 1, \dots N$$

$$E = \sum_i \frac{p_i^2}{2m} = \frac{P^2}{2mN}$$

Consider an excitation of one particle $\vec{p}_i \to \vec{p}_i + \Delta\vec{p}_i$

$\vec{v} = \frac{\vec{p}_i}{m}$

Non-int Bose Gas

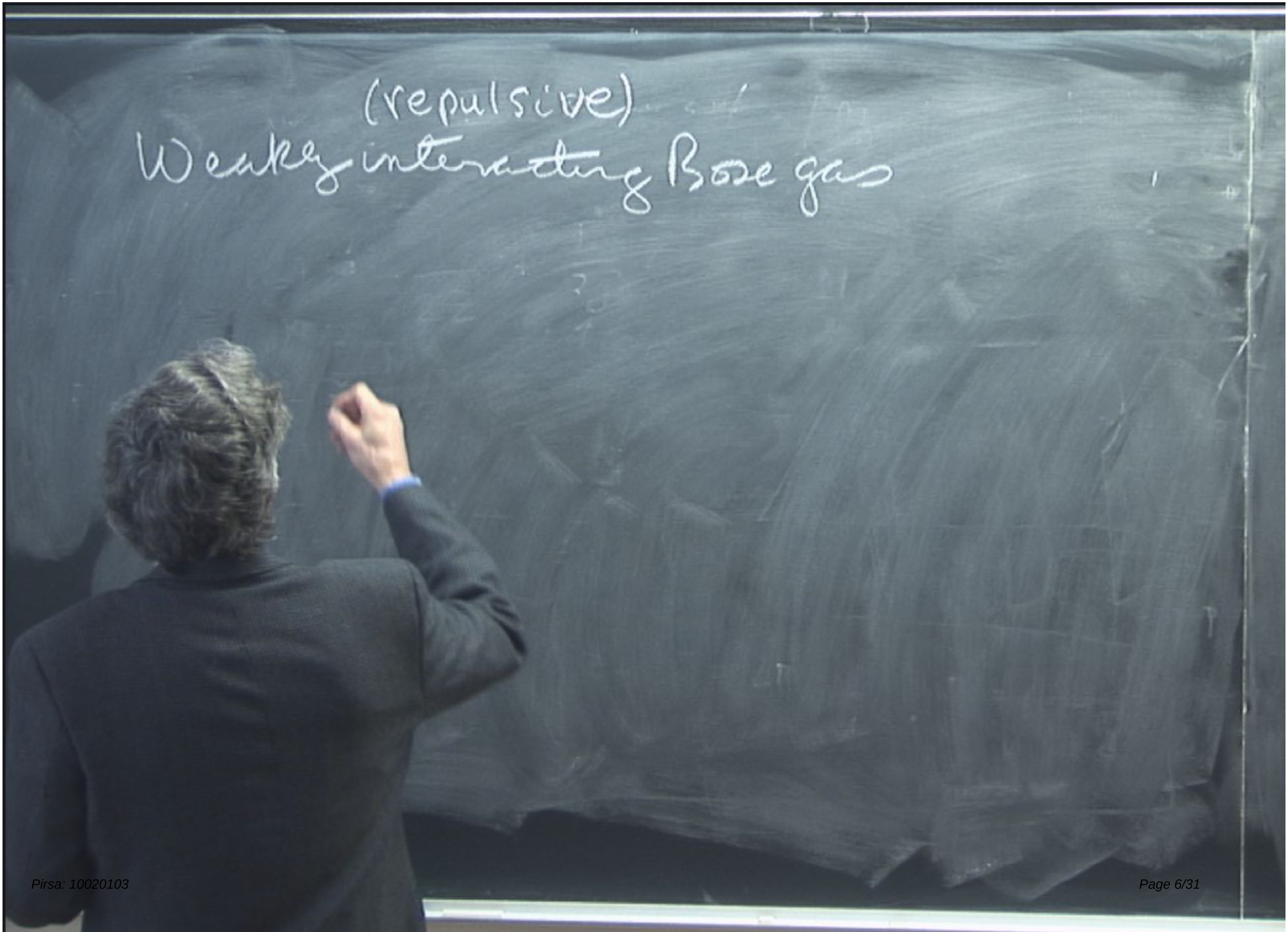
condensed in state where

$$\vec{p}_i = \frac{\vec{P}}{N} \quad i = 1, \ldots N$$

$$E = \sum_i \frac{p_i^2}{2m} = \frac{P^2}{2mN}$$

Consider an excitation of one particle $\vec{p}_i \to \vec{p}_i + \Delta\vec{p}_i$

$$\therefore \Delta E = \frac{(\vec{p}_i + \Delta\vec{p}_i)^2}{2m} - \frac{p_i^2}{2m} = \frac{\vec{p}_i \cdot \Delta\vec{p}_i}{m} + \frac{(\Delta\vec{p}_i)^2}{2m}$$


(repulsive)
Weakly interacting Bose gas

(repulsive)

Weakly interacting Bose gas

Then $\mathcal{E}(\bar{K}) = \hbar c K$ $\left[c = \frac{\hbar}{m}\sqrt{4\pi a n} \right.$

s-wave scattering length

den... of ...

(repulsive)

Weakly interacting Bose gas

s-wave scattering length

Then $\mathcal{E}(\vec{k}) = \hbar c k$ $\left[c = \frac{\hbar}{m}\sqrt{4\pi a n} \right]$

density of particles

(repulsive)

Weakly interacting Bose gas

s-wave scattering length

density of particles

Then $\mathcal{E}(\bar{K}) = \hbar c K$ $\quad \left[c = \frac{\hbar}{m}\sqrt{4\pi a n} \right]$

(repulsive)

Weakly interacting Bose gas

s-wave scattering length

density of particles

Then $\mathcal{E}(\bar{K}) = \hbar c K$ $\quad \left[c = \frac{\hbar}{m}\sqrt{4\pi a n} \right]$

(repulsive)

Weakly interacting Bose gas

s-wave scattering length

density of particles

Then $\mathcal{E}(\vec{k}) = \hbar c k$ $\quad \left[c = \frac{\hbar}{m}\sqrt{4\pi a n} \right]$

For a uniformly flowing gas

$$\mathcal{E}(\vec{k}) = \hbar c k + \hbar \vec{k} \cdot \vec{v}_s$$

(repulsive)

Weakly interacting Bose gas

s-wave scattering length

Then $\mathcal{E}(\vec{k}) = \hbar c k$ $\quad \left[c = \frac{\hbar}{m}\sqrt{4\pi a n} \right]$ density of particles

For a uniformly flowing gas

$$\mathcal{E}(\vec{k}) = \hbar c k + \hbar \vec{k}\cdot\vec{v}_s$$

Critical velocity

$$|\vec{v}_s| = c$$

(repulsive)

Weakly interacting Bose gas

Then $\mathcal{E}(\bar{K}) = \hbar c K$ $\quad \left[c = \frac{\hbar}{m} \sqrt{4\pi a n} \right.$

s-wave scattering length

density of particles

For a uniformly flowing gas

$$\mathcal{E}(\vec{K}) = \hbar c K + \hbar \vec{K} \cdot \vec{v}_s$$

Critical velocity

$$|\vec{v}_s| = c$$

(repulsive)

Weakly interacting Bose gas

Then $\mathcal{E}(\bar{k}) = \hbar c k$ $\quad \left[c = \frac{\hbar}{m}\sqrt{4\pi a n} \right]$

s-wave scattering length

density of particles

For a uniformly flowing gas

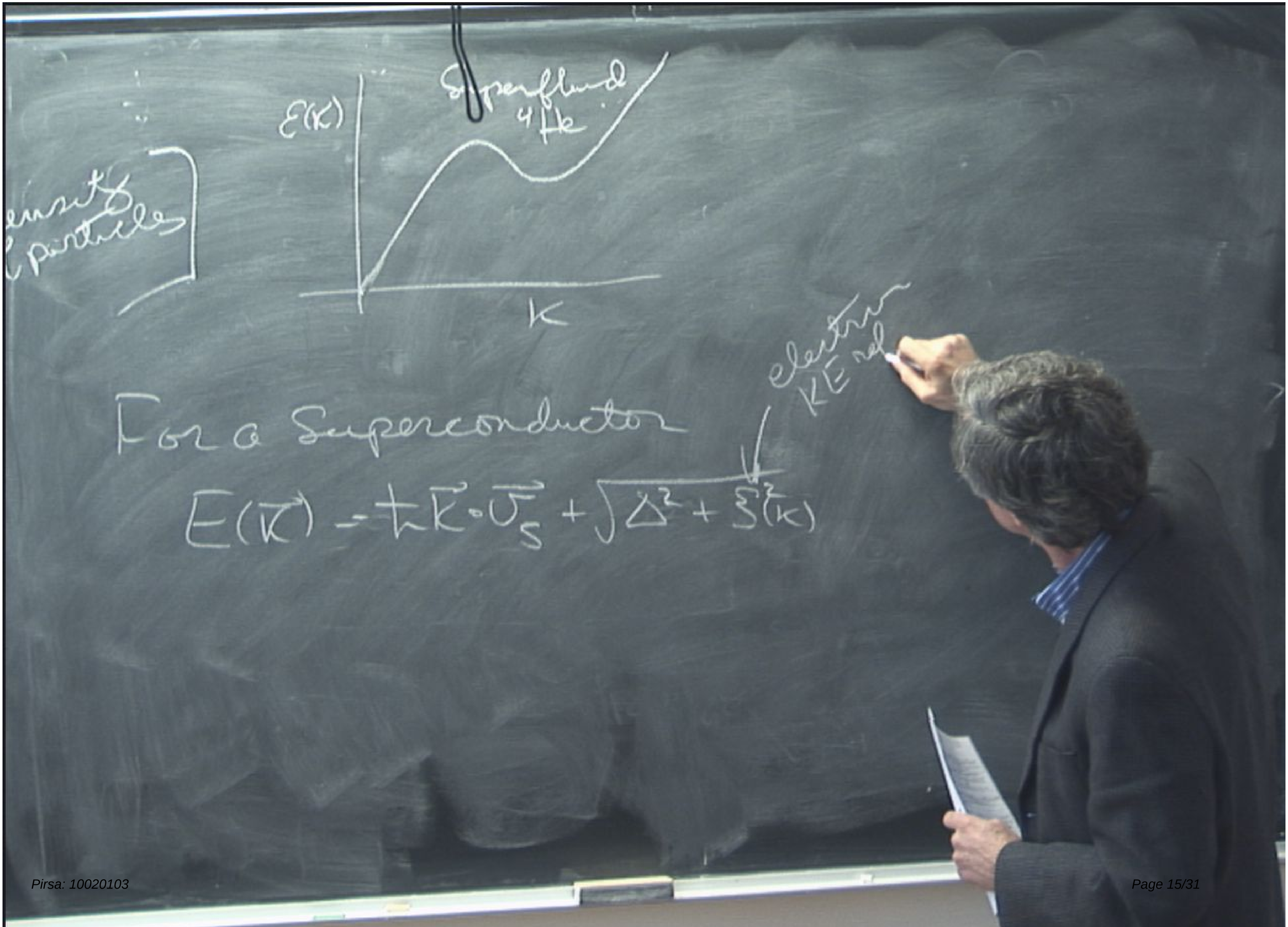
$$\mathcal{E}(\vec{k}) = \hbar c k + \hbar \vec{k}\cdot\vec{v}_s$$

Critical velocity

$$|\vec{v}_s| = c$$

For

$\mathcal{E}$


Superfluid
4He
E(K)
K
For a Superconductor
$E(\vec{K}) = \hbar\vec{K}\cdot\vec{v}_s + \sqrt{\Delta^2 + \xi^2(k)}$
electron
KE rel

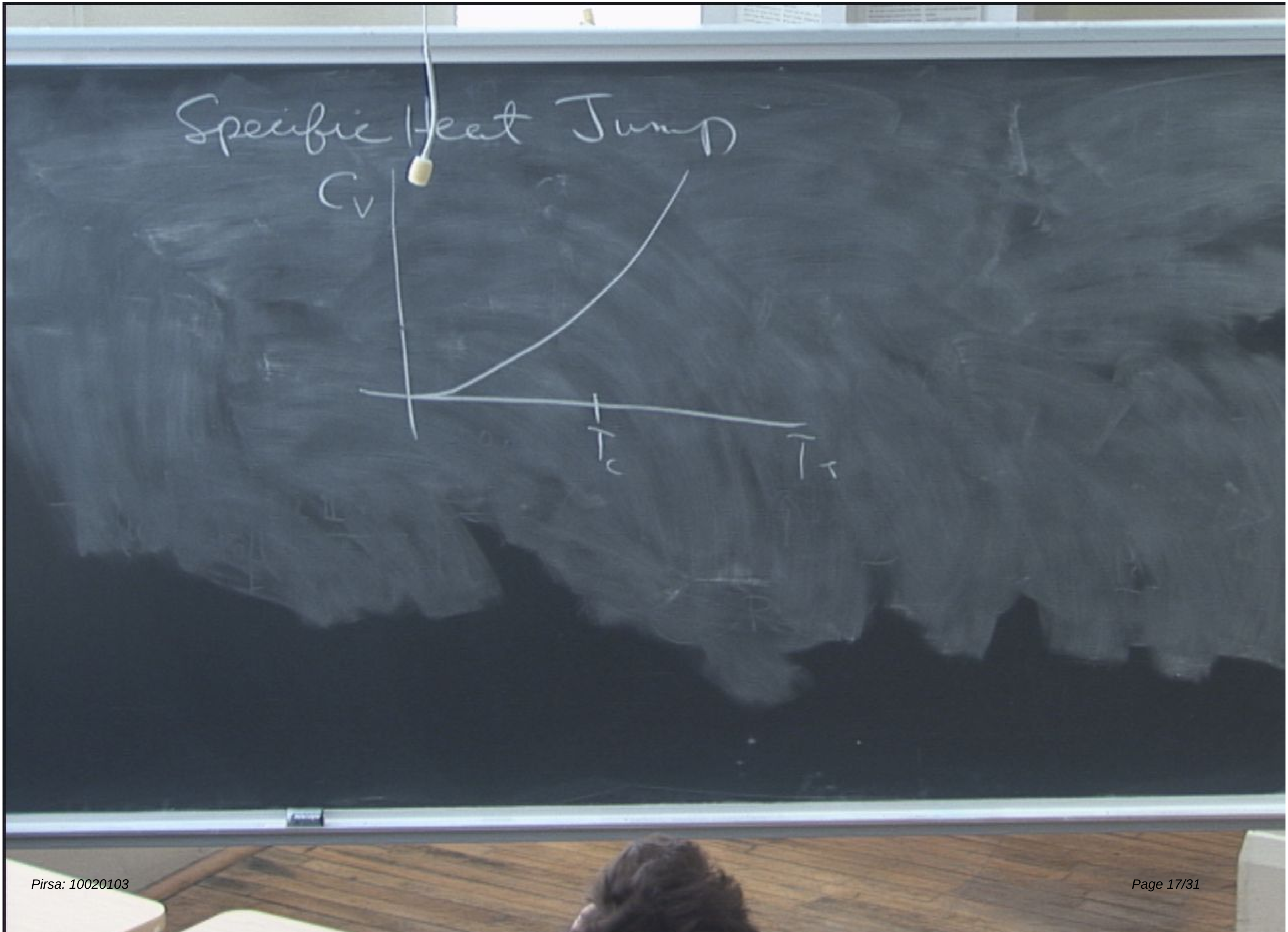
ensity
l particles

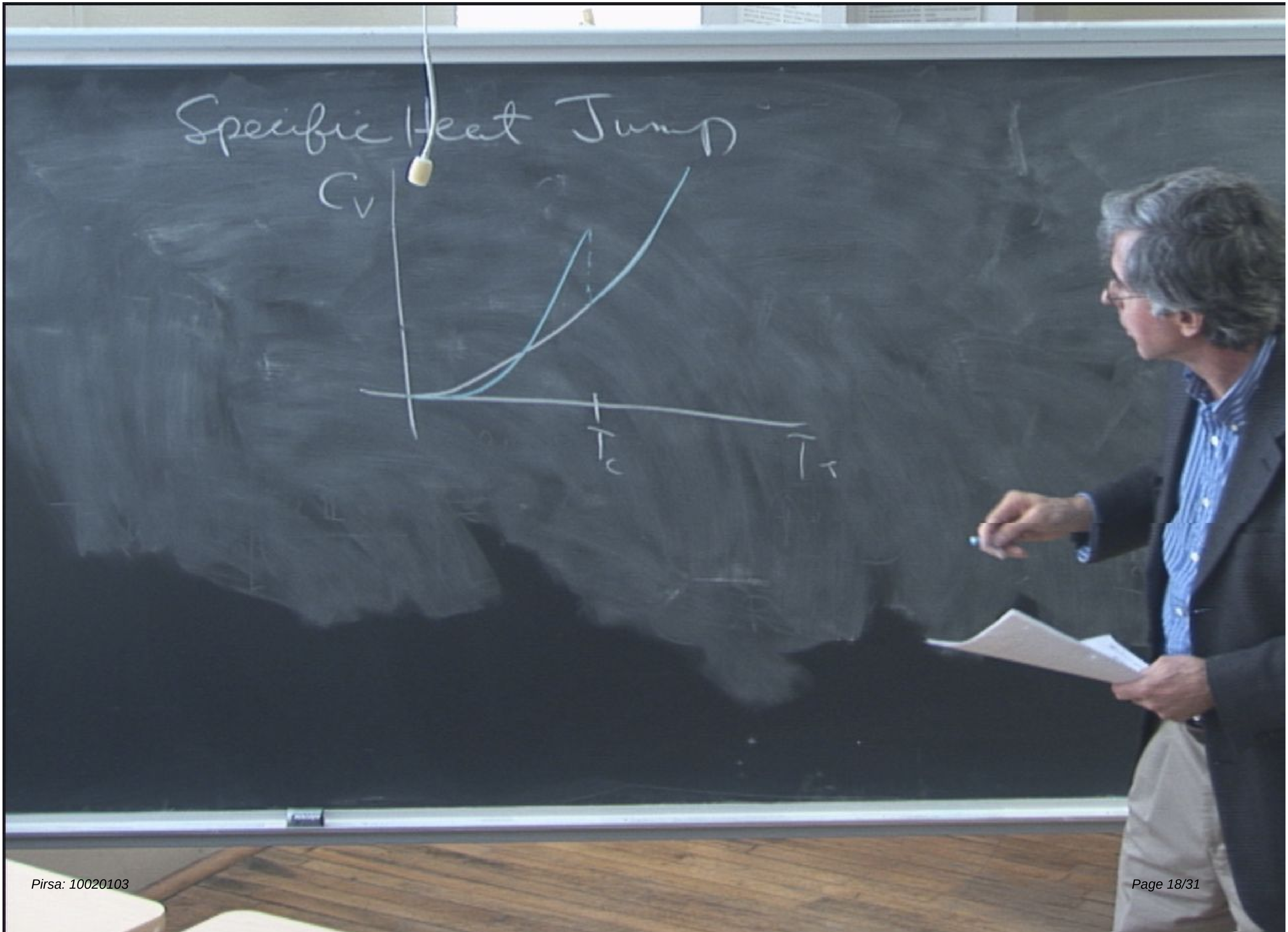
For a Superconductor

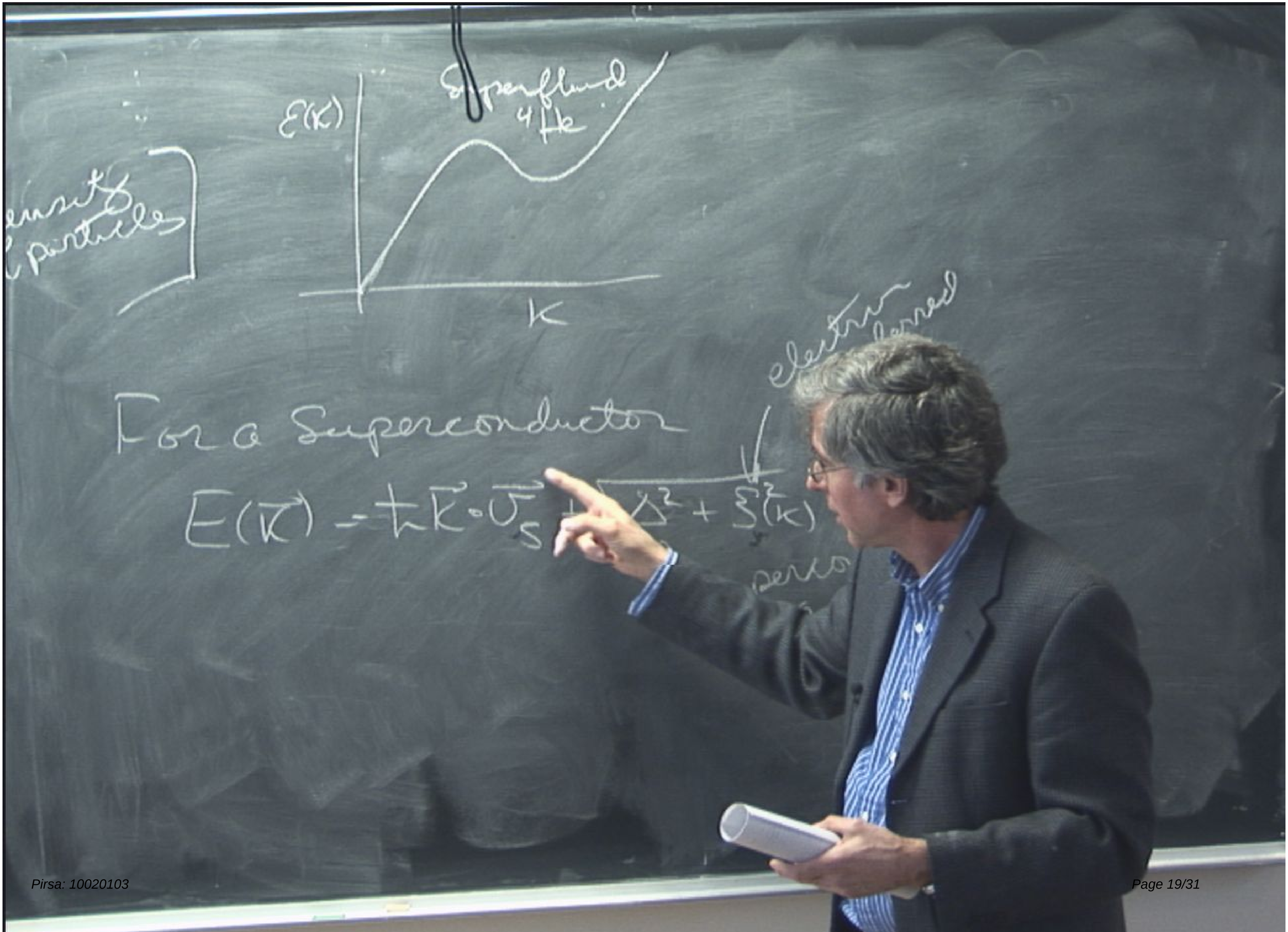
$$E(\vec{k}) = \hbar\vec{k}\cdot\vec{v}_s + \sqrt{\Delta^2 + \xi^2(k)}$$

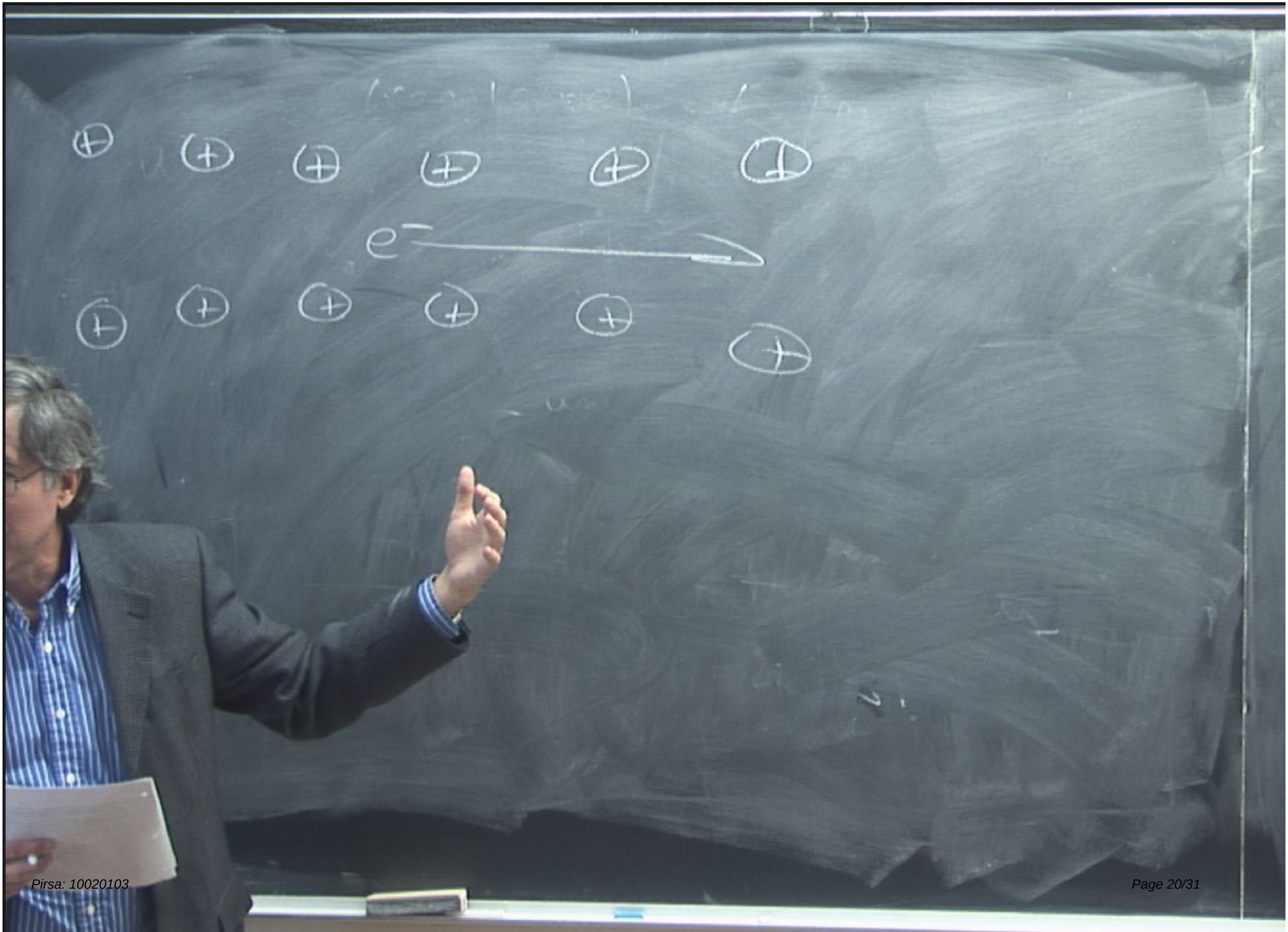
electron KE referred to E_F

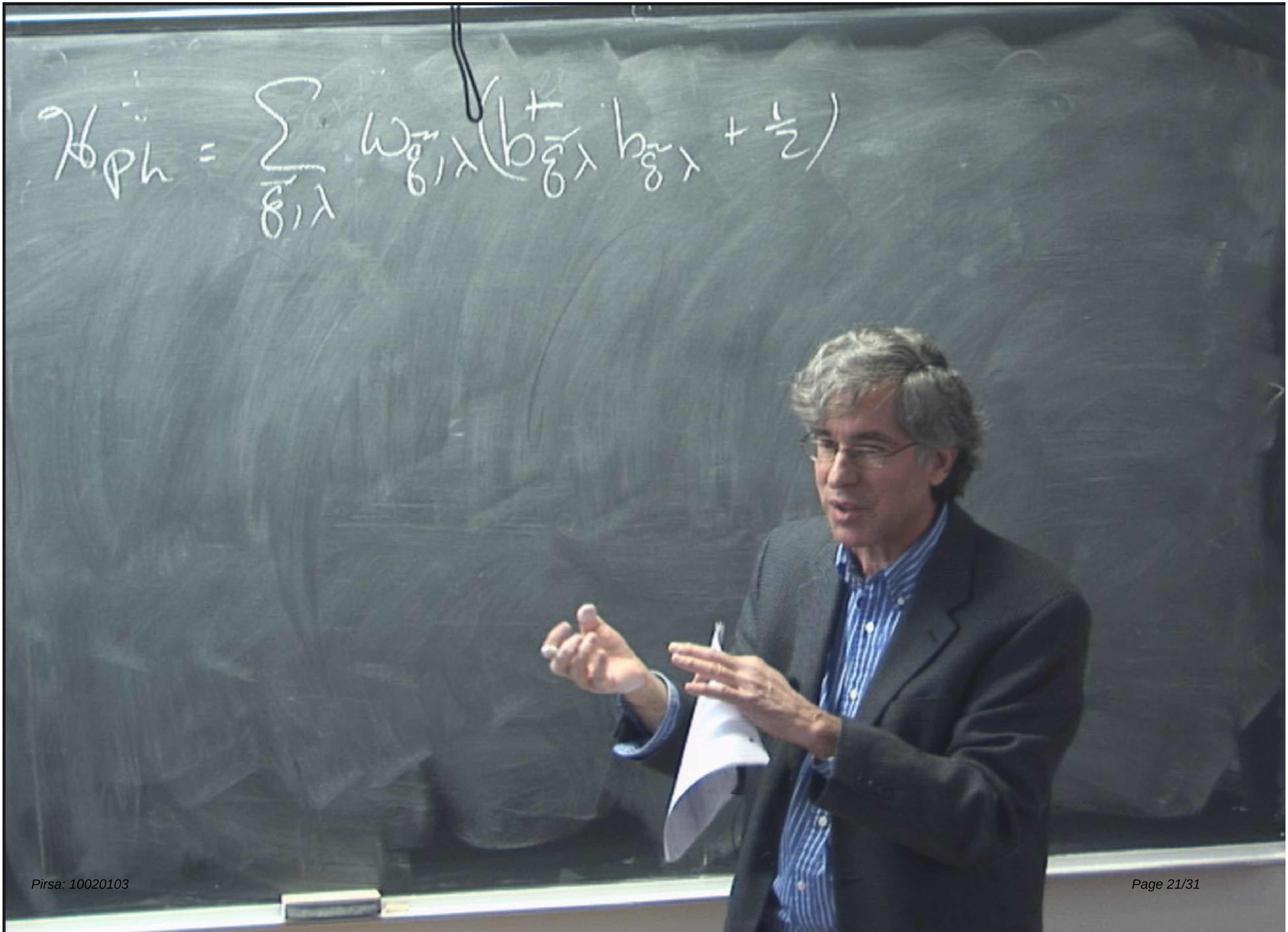
Superconducting gap $|\Delta|$


Specific Heat Jump
C_V
T_c


Specific Heat Jump
C_V
T_c
T


Superfluid
4He
E(K)
K
electron
For a Superconductor


e^-


$\mathcal{H}_{ph} = \sum_{\vec{q},\lambda} \omega_{\vec{q},\lambda}\left(b^{+}_{\vec{q}\lambda} b_{\vec{q}\lambda} + \frac{1}{2}\right)$

$$\mathcal{H}_{ph} = \sum_{\vec{q},\lambda} \omega_{\vec{q},\lambda} \left(b^{\dagger}_{\vec{q}\lambda} b_{\vec{q}\lambda} + \frac{1}{2} \right)$$

$$\mathcal{H}_{el-ph} = \frac{1}{N} \sum_{\vec{k},\sigma} \sum_{\vec{q},\lambda} g_{\vec{q},\lambda} C^{\dagger}_{\vec{k}+\vec{q},\sigma} C_{\vec{k},\sigma} \left(b^{\dagger}_{\vec{q}\lambda} + b_{-\vec{q},\lambda} \right)$$

$$\mathcal{H}_{ph} = \sum_{\vec{q},\lambda} \omega_{\vec{q},\lambda} \left(b^{+}_{\vec{q},\lambda} b_{\vec{q},\lambda} + \frac{1}{2} \right)$$

$$\mathcal{H}_{el-ph} = \frac{1}{N} \sum_{\vec{k},\sigma} \sum_{\vec{q},\lambda} g_{\vec{q},\lambda} C^{+}_{\vec{k}-\vec{q},\sigma} C_{\vec{k},\sigma} \left(b^{+}_{\vec{q},\lambda} + b_{-\vec{q},\lambda} \right)$$

$$\mathcal{H}_{ph} = \sum_{\vec{q},\lambda} \omega_{\vec{q},\lambda}\left(b^{\dagger}_{\vec{q}\lambda} b_{\vec{q}\lambda} + \frac{1}{2}\right)$$

$$\mathcal{H}_{el-ph} = \frac{1}{N}\sum_{\vec{k},\sigma}\sum_{\vec{q},\lambda} g_{\vec{q},\lambda}\, c^{\dagger}_{\vec{k}-\vec{q},\sigma}\, c_{\vec{k},\sigma}\left(b^{\dagger}_{\vec{q}\lambda} + b_{-\vec{q},\lambda}\right)$$

$$\mathcal{H}_{ph} = \sum_{\vec{q},\lambda} \omega_{\vec{q},\lambda}\left(b^{+}_{\vec{q}\lambda} b_{\vec{q}\lambda} + \frac{1}{2}\right)$$

$$\mathcal{H}_{el-ph} = \frac{1}{N} \sum_{\vec{k},\sigma} \sum_{\vec{q},\lambda} g_{\vec{q},\lambda}\, C^{+}_{\vec{k}-\vec{q},\sigma}\, C_{\vec{k},\sigma} \left(b^{+}_{\vec{q}\lambda} + b_{-\vec{q},\lambda}\right)$$

Effective Int

$$V_{eff}(\vec{q}) = |g(q)|^2 \left\{ \frac{1}{\mathcal{E}(k+q) - \mathcal{E}(k) - \omega_q} - \frac{1}{\mathcal{E}(k+q) - \mathcal{E}(k) + \omega(q)} \right\}$$

$$= |g(q)|^2 \frac{2\omega_q}{\omega^2 - \omega_q^2}, \quad \omega \equiv \mathcal{E}(k+q) - \mathcal{E}$$

Effective Int

$$V_{eff}(\vec{q}) = |g(q)|^2 \left\{ \frac{1}{\mathcal{E}(k+q) - \mathcal{E}(k) - \omega_q} - \frac{1}{\mathcal{E}(k+q) - \mathcal{E}(k) + \omega(q)} \right\}$$

$$= |g(q)|^2 \frac{2\omega_q}{\omega^2 - \omega_{\vec{q}}^2}, \quad \omega \equiv \mathcal{E}(k+q) - \mathcal{E}(\vec{k})$$

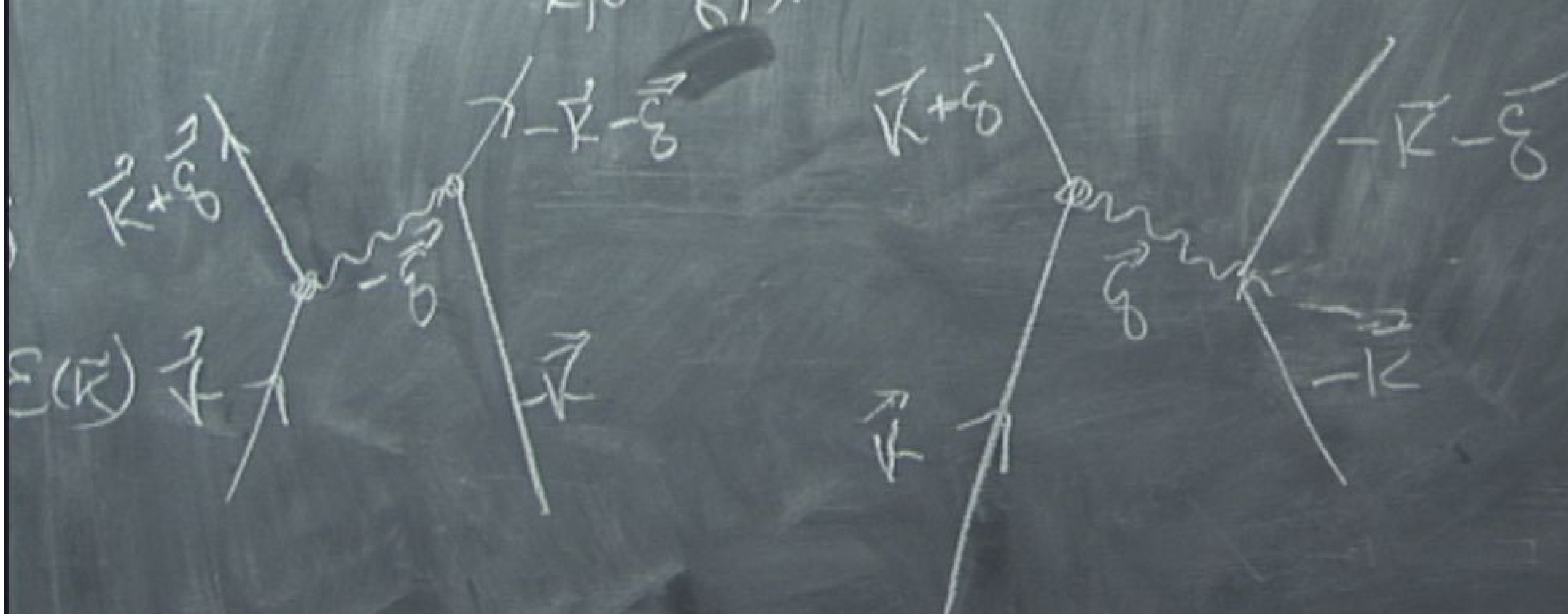
$\mathcal{H}_{ph}$

$\mathcal{H}_{el-}$

$\vec{k}+\vec{q}$

$$\mathcal{H}_{ph} = \sum_{\vec{q},\lambda} \omega_{\vec{q},\lambda}\left(b^{\dagger}_{\vec{q}\lambda} b_{\vec{q}\lambda} + \frac{1}{2}\right)$$

$$\mathcal{H}_{el-ph} = \frac{1}{N}\sum_{\vec{k},\sigma}\sum_{\vec{q},\lambda} g_{\vec{q},\lambda}\, C^{\dagger}_{\vec{k}-\vec{q},\sigma} C_{\vec{k},\sigma}\left(b^{\dagger}_{\vec{q}\lambda} + b_{-\vec{q},\lambda}\right)$$



Effective Int

$$V_{eff}(\vec{q}) = |g(q)|^2 \left\{ \frac{1}{\mathcal{E}(k+q) - \mathcal{E}(k) - \omega_q} - \frac{1}{\mathcal{E}(k+q) - \mathcal{E}(k) + \omega(q)} \right\}$$

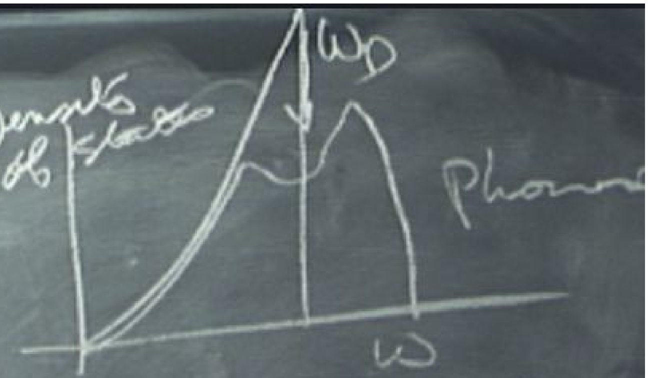
$$= |g(\vec{q})|^2 \frac{2\omega_q}{\omega^2 - \omega_q^2}, \quad \omega \equiv \mathcal{E}(k+q) -$$

Simplifying assumption of BCS

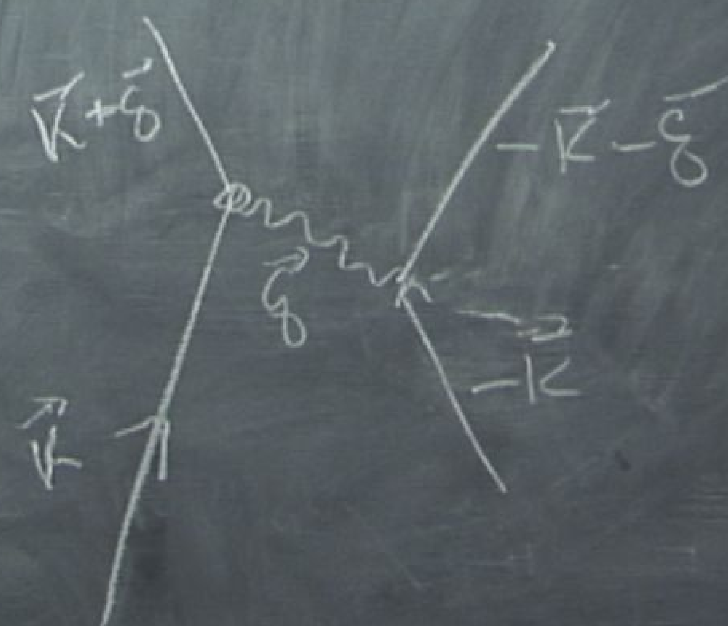
$V_{eff} = -V/N$ if $\mathcal{E}(k)$ and $\mathcal{E}(k+q)$ are within ω_D of E_F

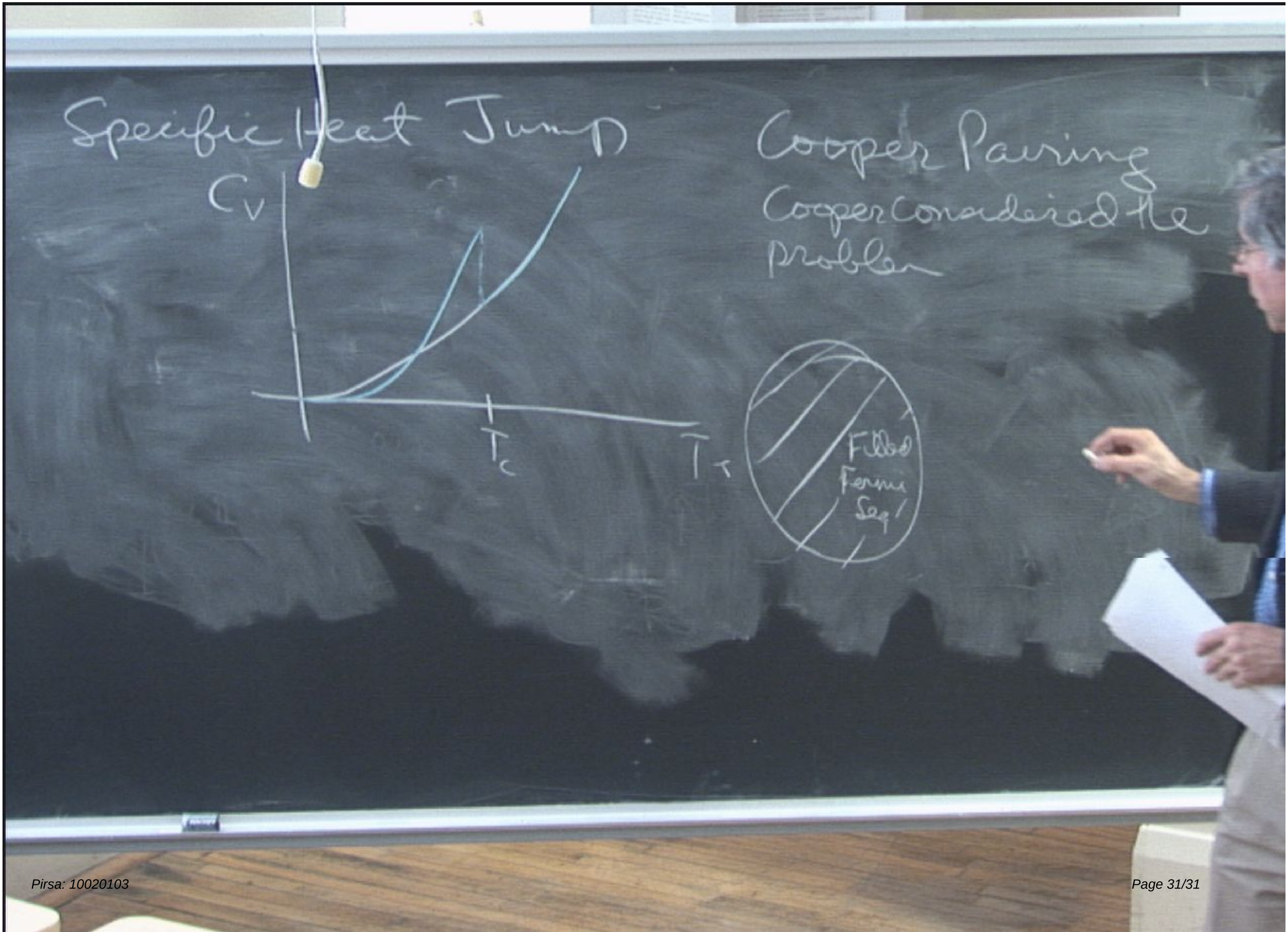
$= 0$ otherwise

$$\mathcal{H}_{ph} = \sum_{\vec{q},\lambda} \omega_{\vec{q},\lambda} \left(b^{\dagger}_{\vec{q}\lambda} b_{\vec{q}\lambda} + \frac{1}{2} \right)$$



$$\mathcal{H}_{el-ph} = \frac{1}{N} \sum_{\vec{k},\sigma} \sum_{\vec{q},\lambda} g_{\vec{q},\lambda} C^{\dagger}_{\vec{k}-\vec{q},\sigma} C_{\vec{k},\sigma} \left(b^{\dagger}_{\vec{q}\lambda} + b_{-\vec{q},\lambda} \right)$$




Specific Heat Jump
C_V
T_c
T
Cooper Pairing
Cooper considered the problem
Filled Fermi Sea!