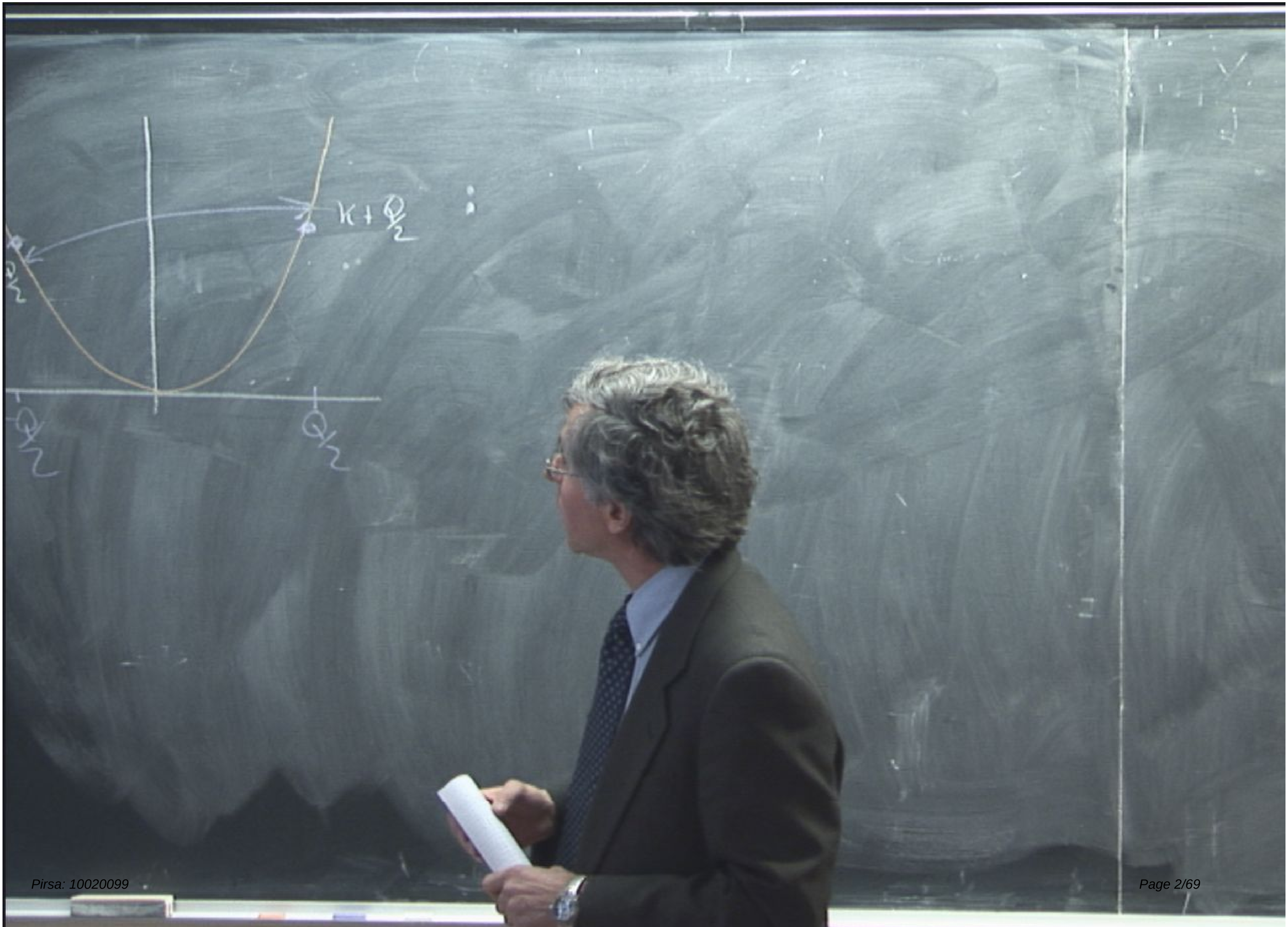
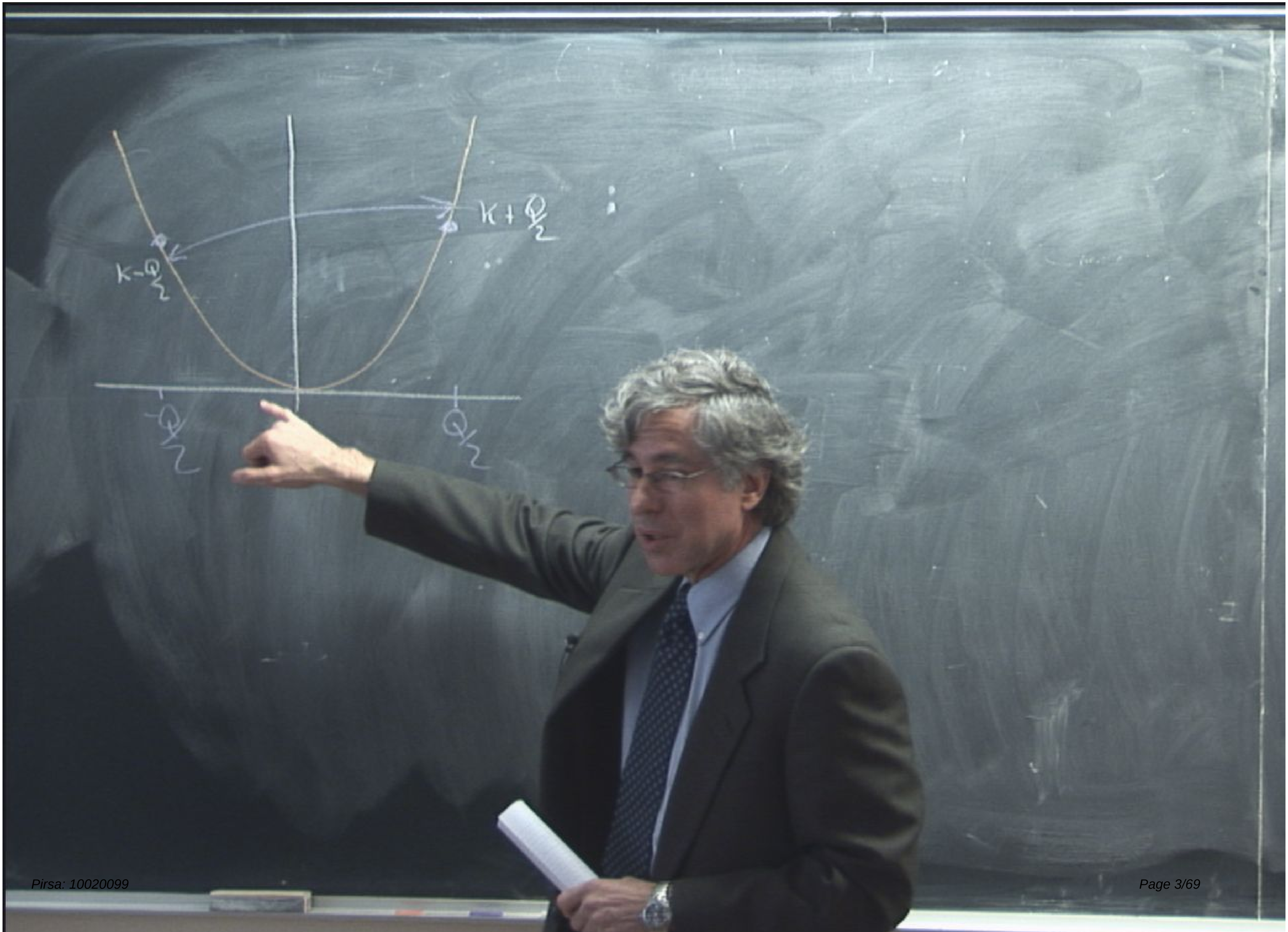
Title: Condensed Matter II - Lecture 3

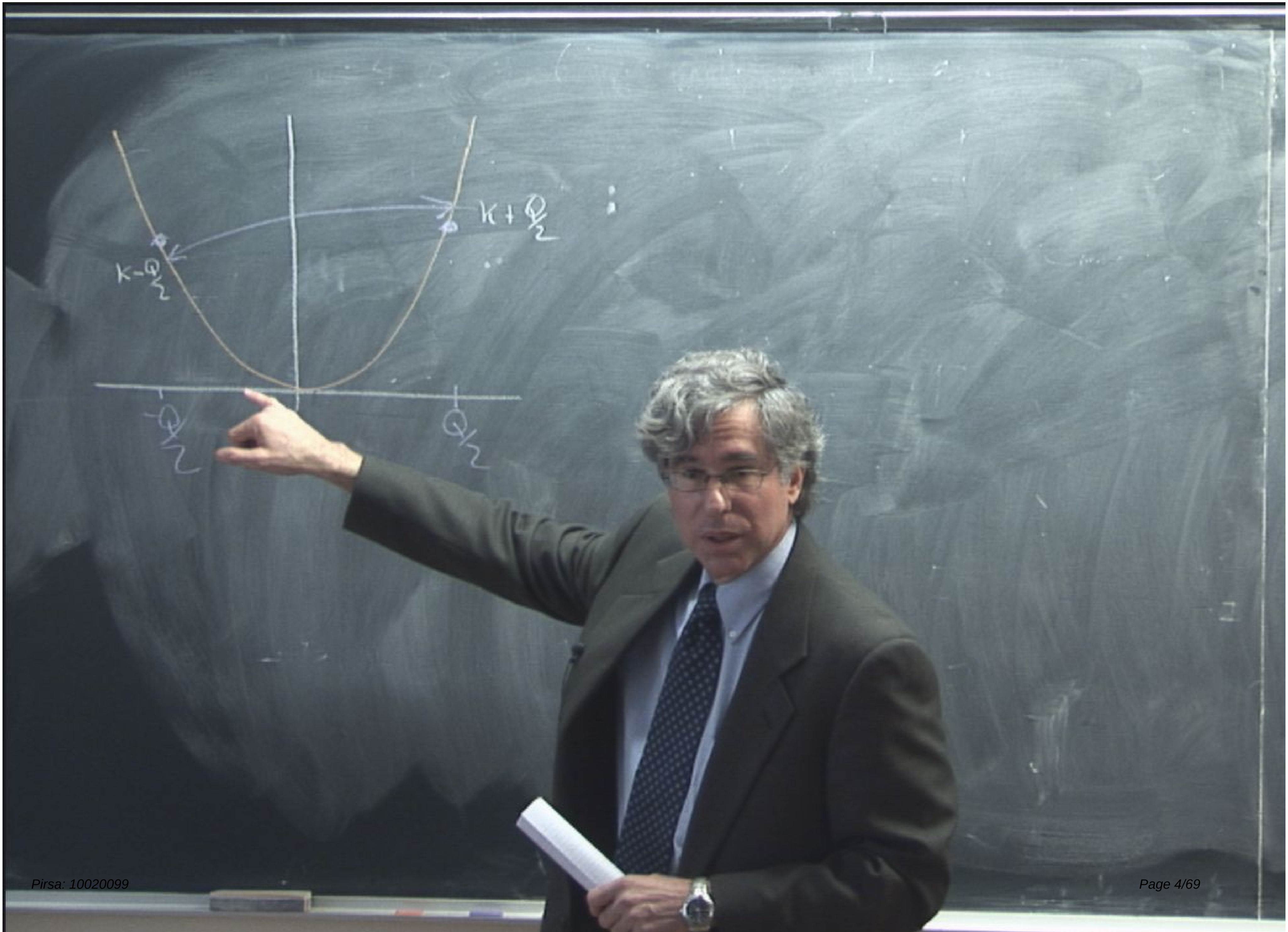
Date: Feb 18, 2010 10:10 AM

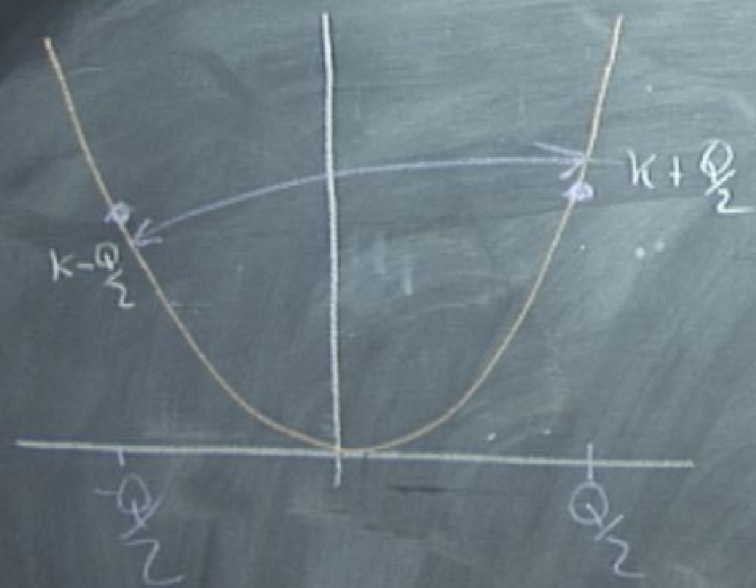
URL: http://pirsa.org/10020099

Abstract:

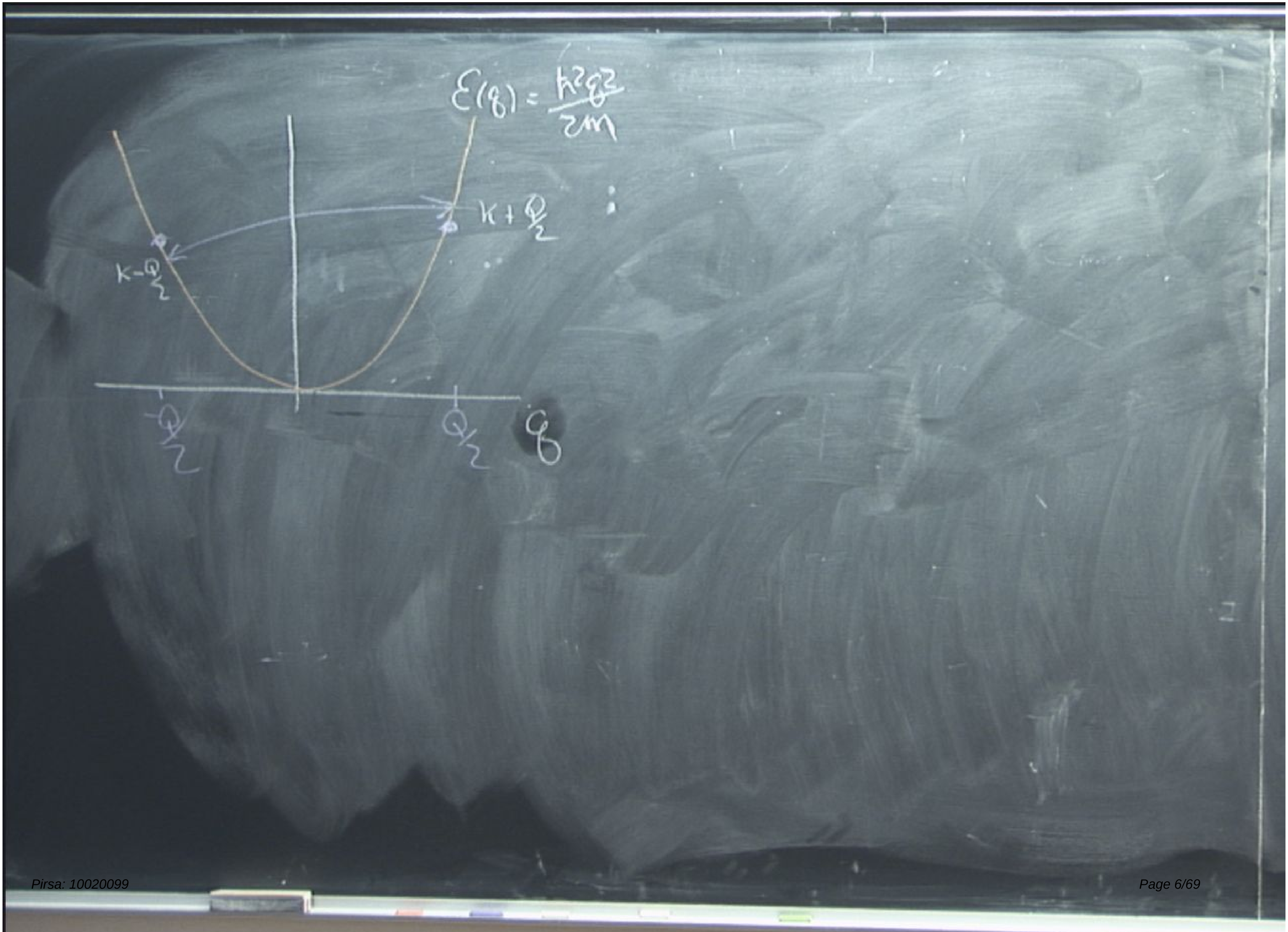

k + Q/2
Q/2

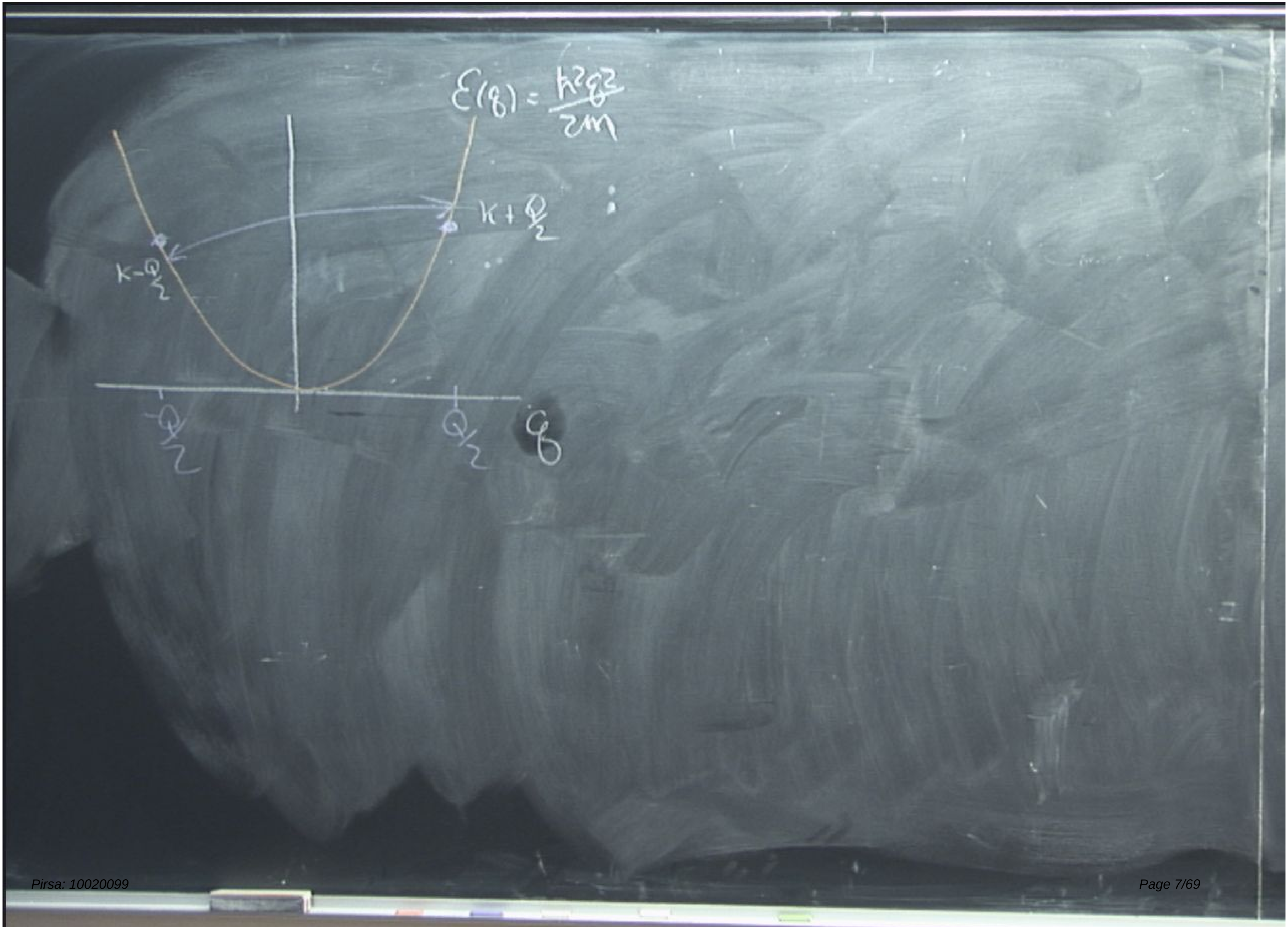

K + Q/2
K - Q/2
-Q/2
Q/2


K + Q/2
K - Q/2
-Q/2
Q/2



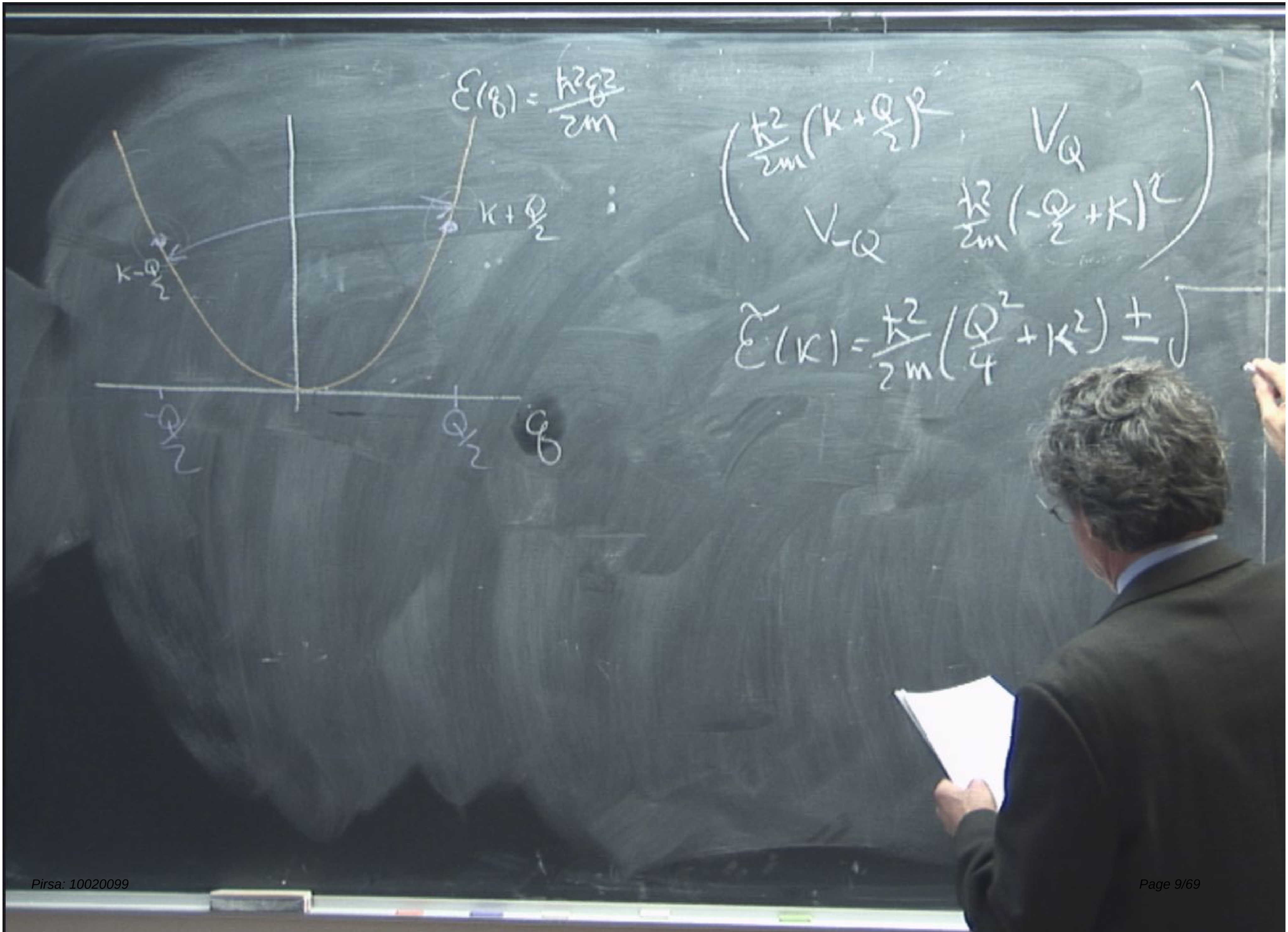
k + Q/2
k − Q/2
−Q/2
Q/2


$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$
$k + \frac{Q}{2}$
$k - \frac{Q}{2}$
$-\frac{Q}{2}$
$Q/2$
q


$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$
$k + \frac{Q}{2}$
$k - \frac{Q}{2}$
$-\frac{Q}{2}$
$Q/2$
q

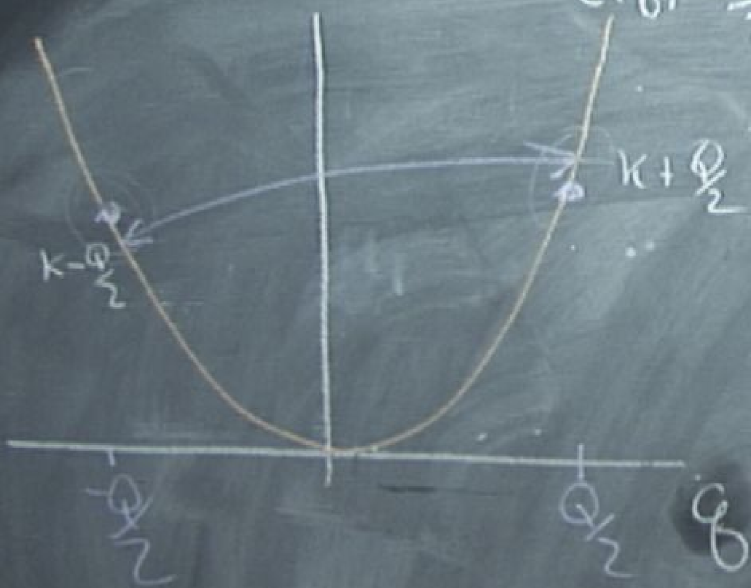
$$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$$

$$\begin{pmatrix} \frac{\hbar^2}{2m}\left(k + \frac{Q}{2}\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2} + k\right)^2 \end{pmatrix}$$



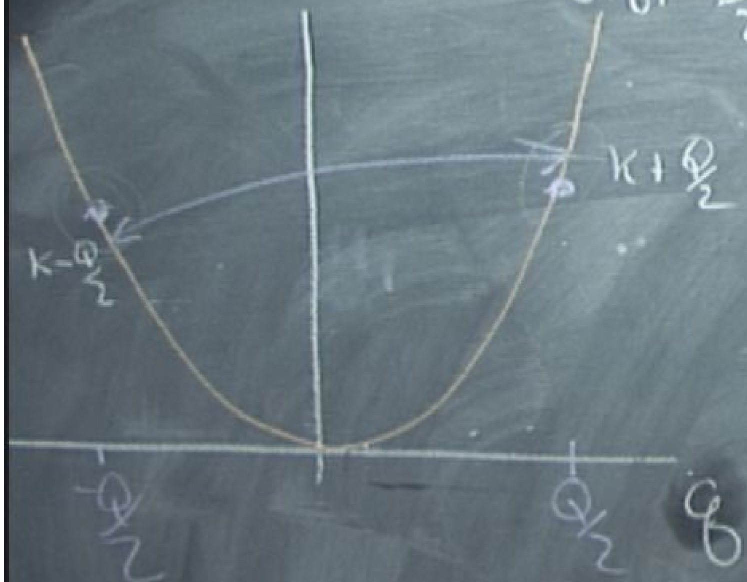
$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$
$k + \frac{Q}{2}$
$k - \frac{Q}{2}$
$-\frac{Q}{2}$
$\frac{Q}{2}$
q
$\begin{pmatrix} \frac{\hbar^2}{2m}\left(k+\frac{Q}{2}\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+k\right)^2 \end{pmatrix}$
$\tilde{\mathcal{E}}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+k^2\right) \pm \sqrt{}$

$$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$$



$$\begin{pmatrix} \frac{\hbar^2}{2m}\left(k+\frac{Q}{2}\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+k\right)^2 \end{pmatrix}$$

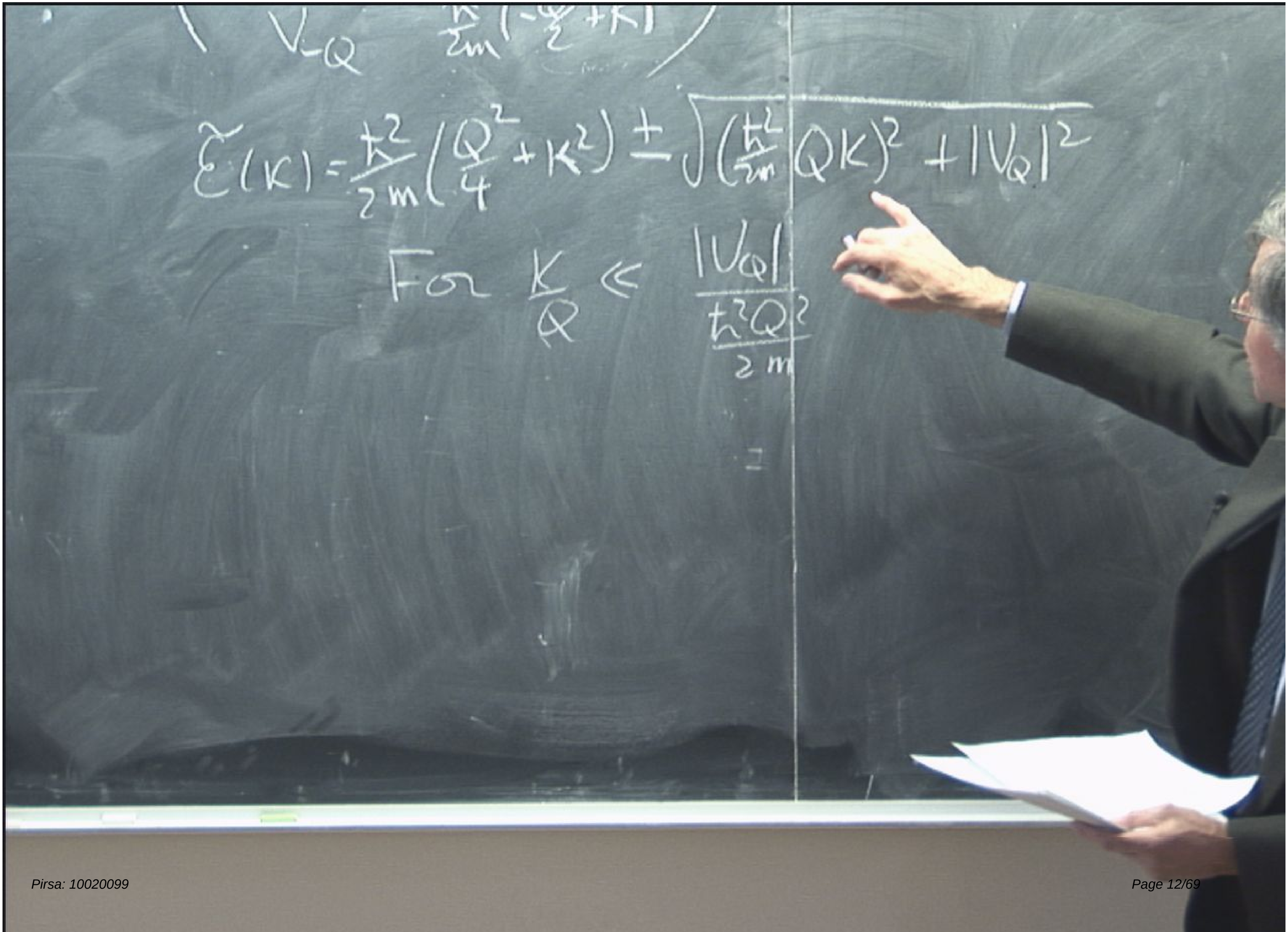
$$\tilde{\mathcal{E}}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+k^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}\right.}$$

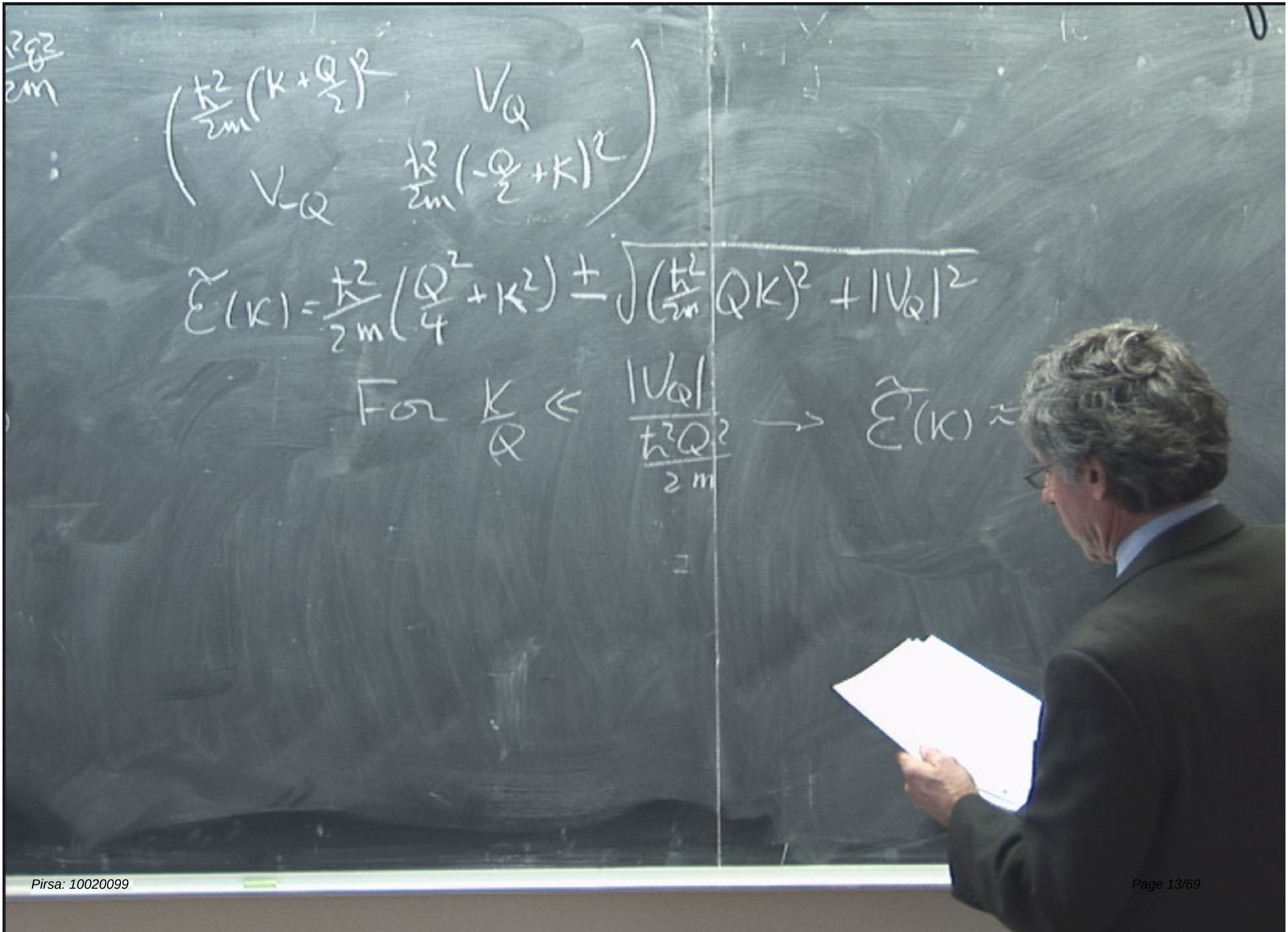


$$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$$

$$\begin{pmatrix} \frac{\hbar^2}{2m}\left(K+\frac{Q}{2}\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+K\right)^2 \end{pmatrix}$$

$$\tilde{\mathcal{E}}(K) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+K^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}QK\right)^2}$$


$\tilde{\mathcal{E}}(K) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4} + K^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m} QK\right)^2 + |V_Q|^2}$
For $\frac{K}{Q} \ll \frac{|V_Q|}{\frac{\hbar^2 Q^2}{2m}}$



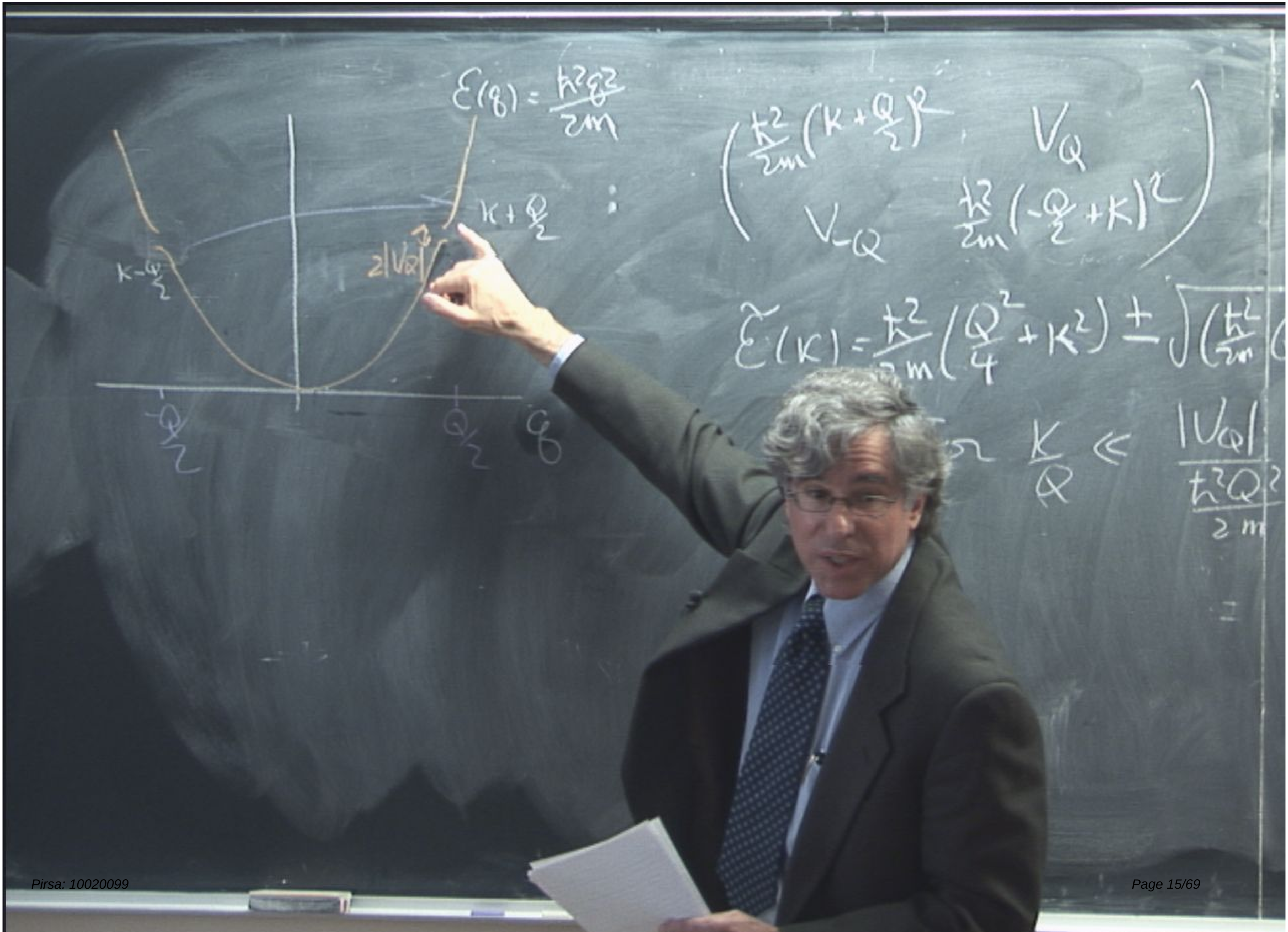
$\begin{pmatrix} \frac{\hbar^2}{2m}(k+\frac{Q}{2})^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}(-\frac{Q}{2}+k)^2 \end{pmatrix}$
$\tilde{\varepsilon}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+k^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}Qk\right)^2 + |V_Q|^2}$
For $\frac{k}{Q} \ll \frac{|V_Q|}{\frac{\hbar^2 Q^2}{2m}} \longrightarrow \tilde{\varepsilon}(k) \approx$

$+\frac{Q}{2})^2 \qquad V_Q$

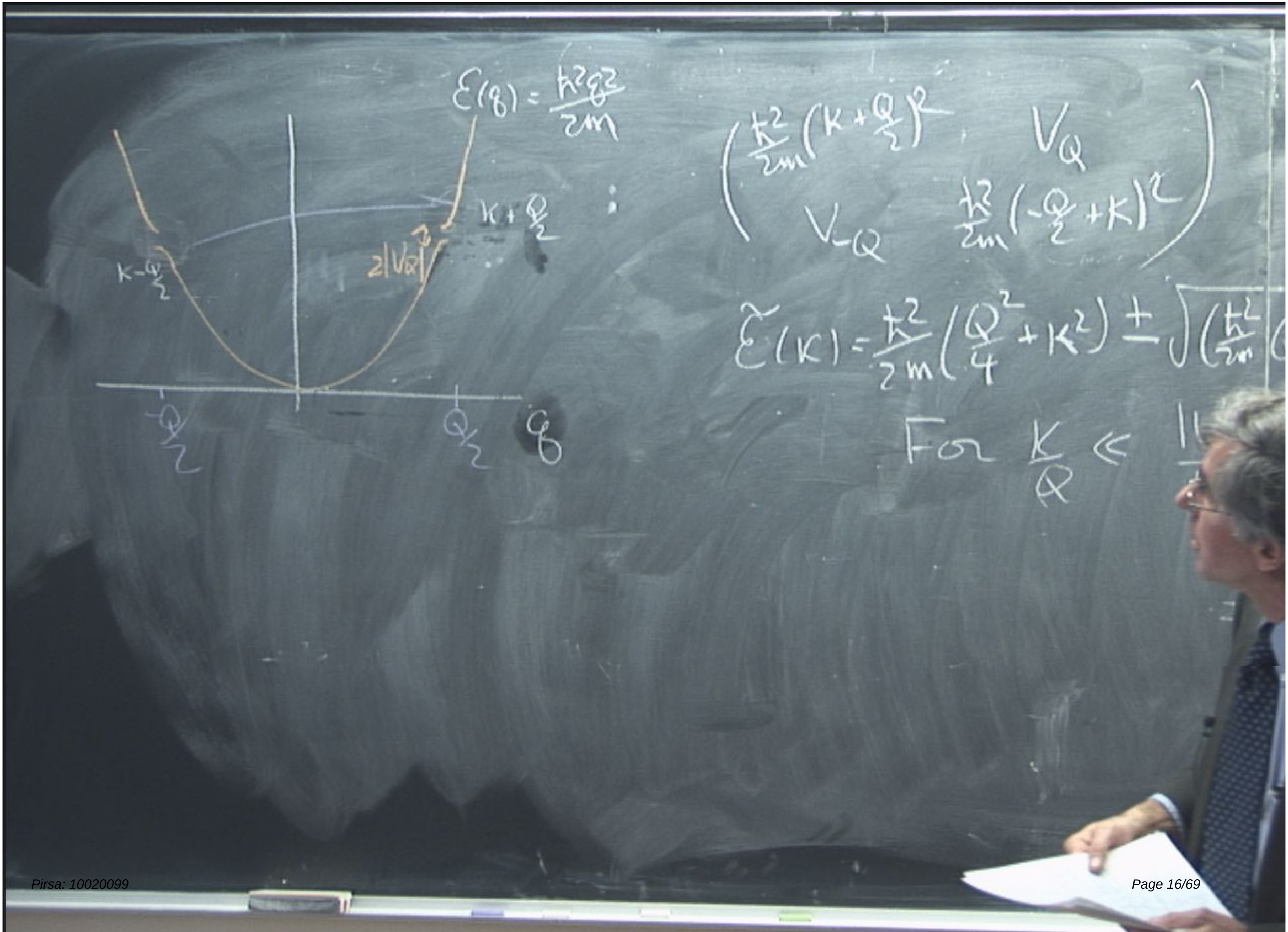
$Q \qquad \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+K\right)^2$

$= \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+K^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}QK\right)^2 + |V_Q|^2}$

For $\frac{K}{Q} \ll \frac{|V_Q|}{\frac{\hbar^2 Q^2}{2m}} \longrightarrow \tilde{\varepsilon}(K) \approx \frac{\hbar^2 Q^2}{8m} \pm |V_Q|\left(1 \pm \frac{1}{2}\frac{\hbar^2 Q^2}{2m|V_Q|}\right.$



$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$
$k + \frac{Q}{2}$
$k - \frac{Q}{2}$
$2|V_Q|$
$-\frac{Q}{2}$
$Q/2$
q
$\begin{pmatrix} \frac{\hbar^2}{2m}(k+\frac{Q}{2})^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}(-\frac{Q}{2}+k)^2 \end{pmatrix}$
$\tilde{\mathcal{E}}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+k^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}\right.}$
$\frac{k}{Q} \ll \frac{|V_Q|}{\frac{\hbar^2 Q^2}{2m}}$



$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$
$k - \frac{Q}{2}$
$k + \frac{Q}{2}$
$2|V_Q|$
$-\frac{Q}{2}$
$\frac{Q}{2}$
q
$\begin{pmatrix} \frac{\hbar^2}{2m}\left(k+\frac{Q}{2}\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+k\right)^2 \end{pmatrix}$
$\tilde{\mathcal{E}}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4} + k^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}\right.}$
For $\frac{k}{Q} \ll$

$$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$$

$$\begin{pmatrix} \frac{\hbar^2}{2m}\left(k+\frac{Q}{2}\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+k\right)^2 \end{pmatrix}$$

$$\tilde{\mathcal{E}}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+k^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}\right.}$$

For $\frac{k}{Q} \ll \frac{|V_Q|}{\frac{\hbar^2 Q^2}{2m}}$

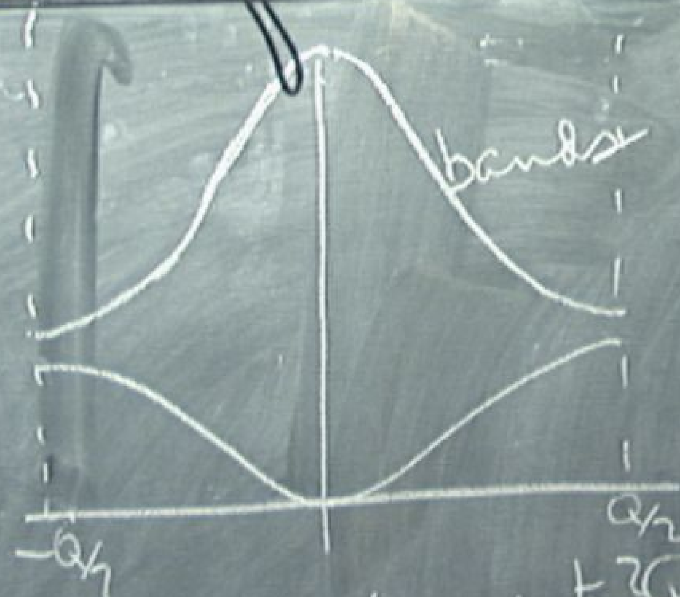
$$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$$

$$\begin{pmatrix} \frac{\hbar^2}{2m}\left(k+\frac{Q}{2}\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+k\right)^2 \end{pmatrix}$$

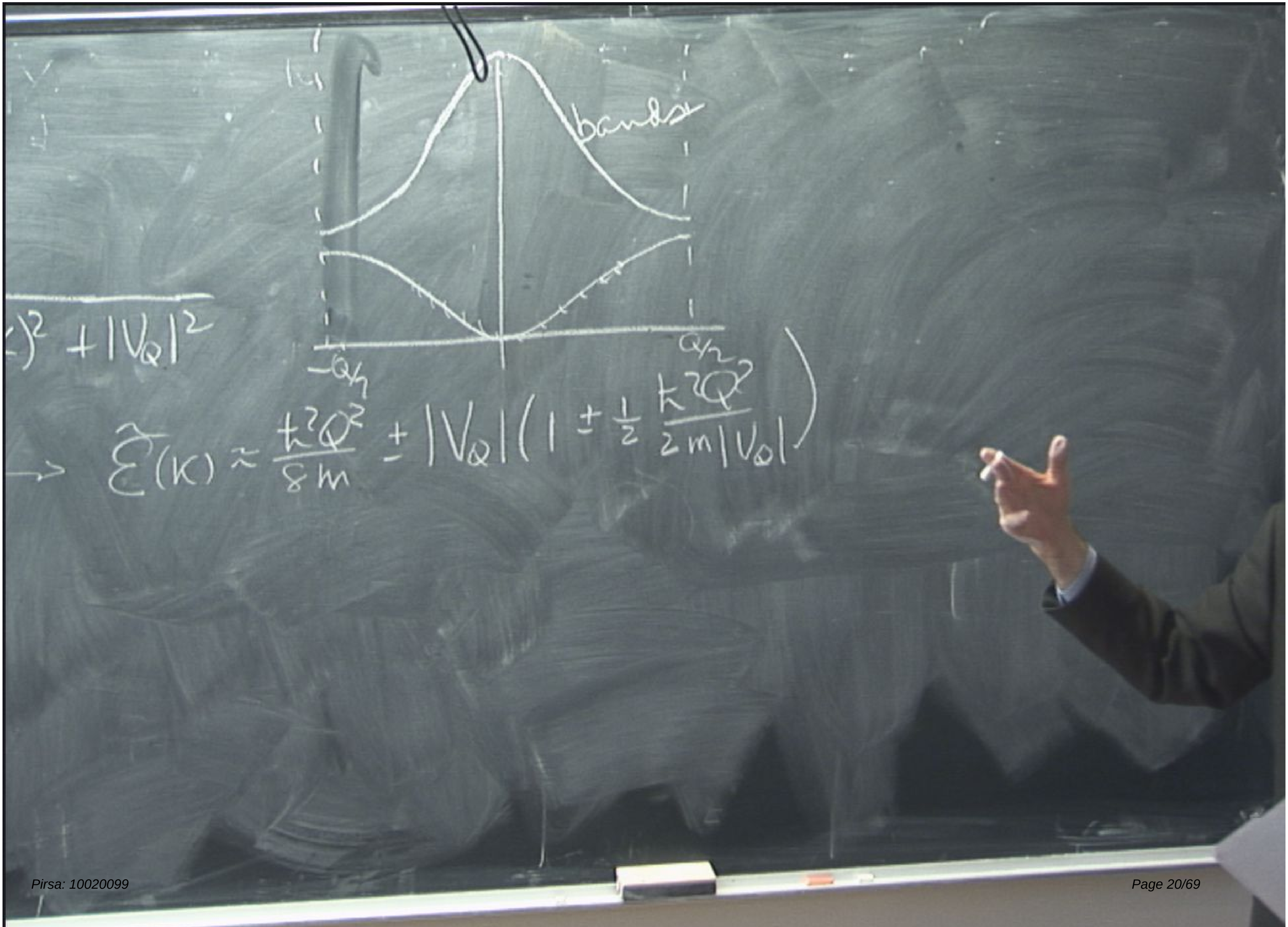
$$\tilde{\mathcal{E}}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+k^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}\right.\cdots}$$

For $\frac{k}{Q} \ll \frac{|V_Q|}{\frac{\hbar^2 Q^2}{2m}}$

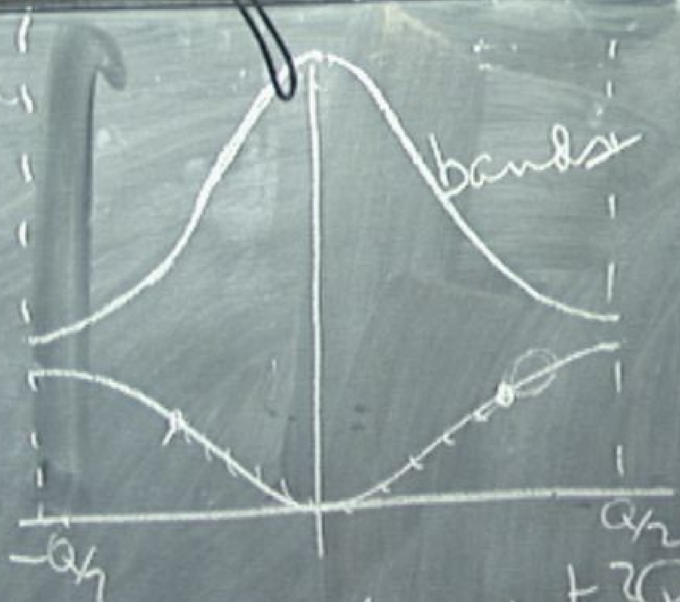
$$\ldots)^2 + |V_Q|^2$$



$$\rightarrow \quad \tilde{\mathcal{E}}(k) \approx \frac{\hbar^2 Q^2}{8m} \pm |V_Q| \left(1 \pm \frac{1}{2} \frac{\hbar^2 Q^2}{2m|V_Q|} \right)$$

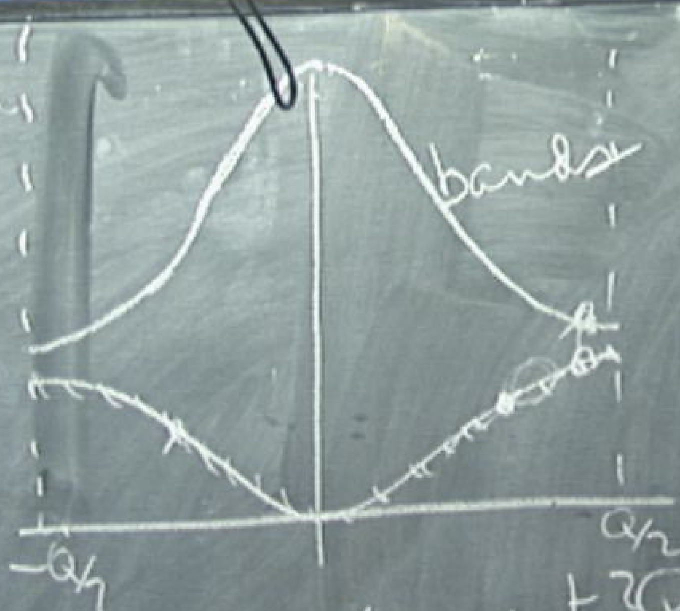

bands
$-Q/2$
$Q/2$
$)^2 + |V_Q|^2$
$\rightarrow \quad \tilde{\mathcal{E}}(k) \approx \frac{\hbar^2 Q^2}{8m} \pm |V_Q| \left(1 \pm \frac{1}{2}\frac{\hbar^2 Q^2}{2m|V_Q|}\right)$

$)^2 + |V_Q|^2$



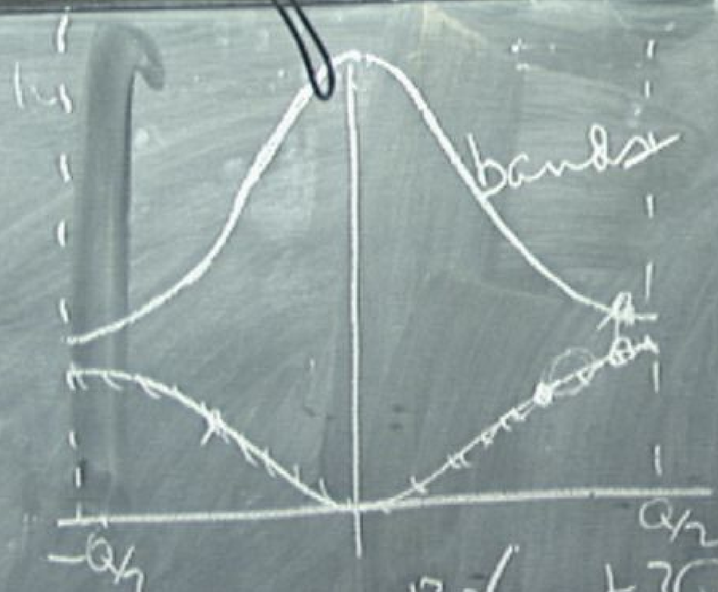
$$\rightarrow \quad \tilde{\mathcal{E}}(k) \approx \frac{\hbar^2 Q^2}{8m} \pm |V_Q| \left(1 \pm \frac{1}{2}\frac{\hbar^2 Q^2}{2m|V_Q|}\right)$$

$$\ldots)^2 + |V_Q|^2$$

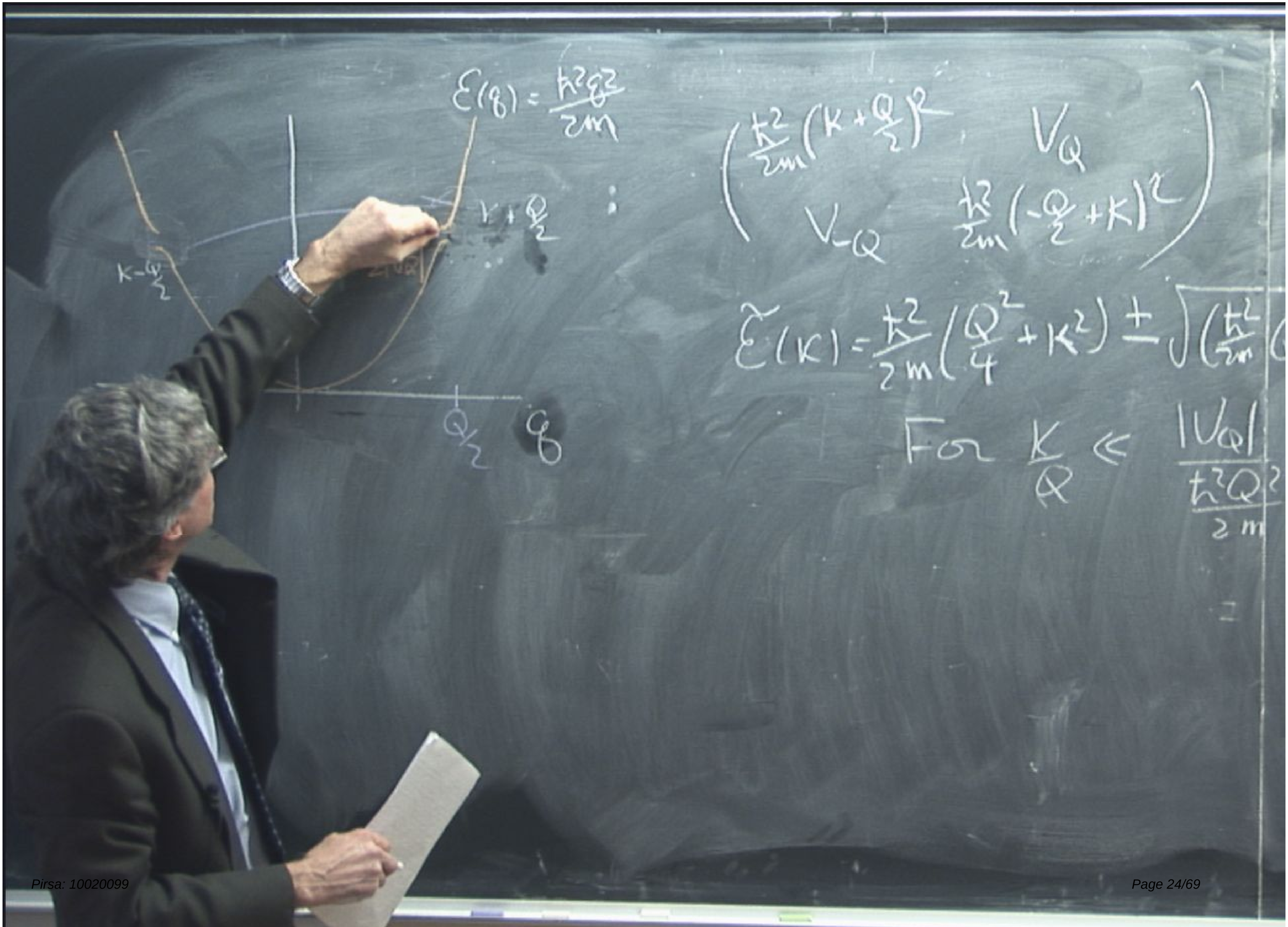


$$\rightarrow \tilde{\varepsilon}(k) \simeq \frac{\hbar^2 Q^2}{8m} \pm |V_Q| \left(1 \pm \frac{1}{2}\frac{\hbar^2 Q^2}{2m|V_Q|}\right)$$

$$)^2 + |V_Q|^2$$



$$\rightarrow \quad \tilde{\varepsilon}(k) \approx \frac{\hbar^2 Q^2}{8m} \pm |V_Q| + \frac{\hbar^2 q^2}{2m}\left(1 \pm \frac{1}{2}\,\frac{\hbar^2 Q^2}{2m|V_Q|}\right)$$


$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$
$k + \frac{Q}{2}$
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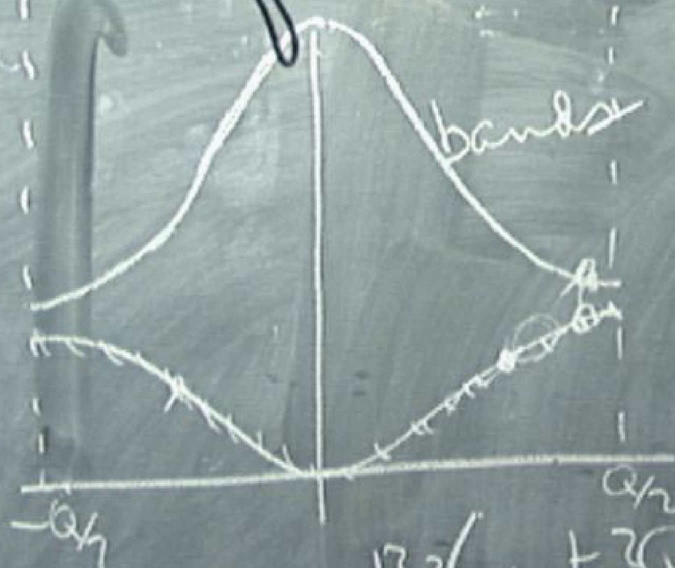
$\tilde{\mathcal{E}}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+k^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}\right.}$
For $\frac{k}{Q} \ll \frac{|V_Q|}{\frac{\hbar^2 Q^2}{2m}}$

$$\mathcal{E}(q) = \frac{\hbar^2 q^2}{2m}$$

$$\begin{pmatrix} \frac{\hbar^2}{2m}\left(k+\frac{Q}{2}\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+k\right)^2 \end{pmatrix}$$

$$\tilde{\mathcal{E}}(k) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+k^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}\right) \cdots}$$

For $\frac{k}{Q} \ll \frac{|V_Q|}{\frac{\hbar^2 Q^2}{2m}}$



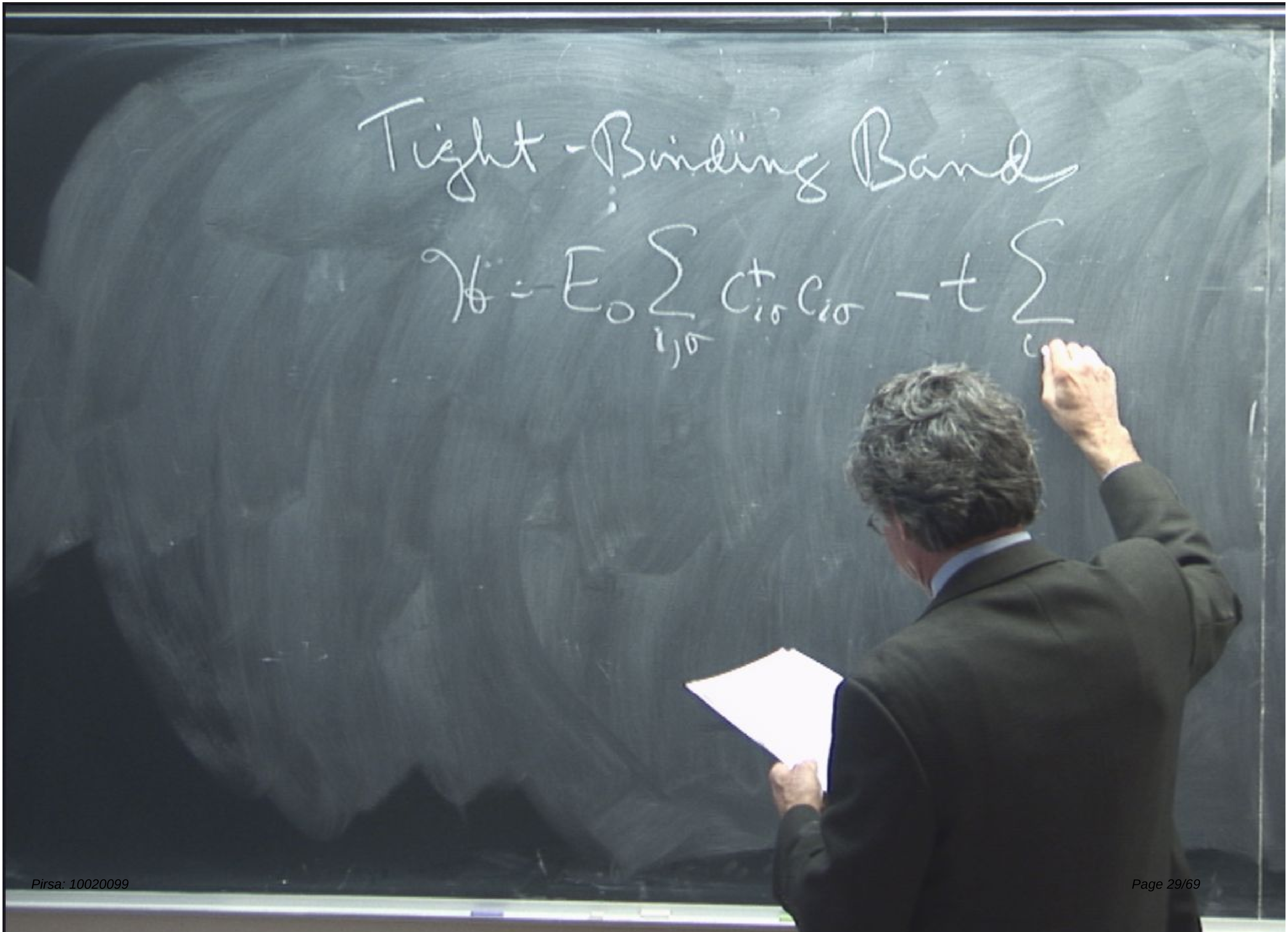
$$\ldots)^2 + |V_Q|^2$$

$$\rightarrow \quad \tilde{\mathcal{E}}(k) \approx \frac{\hbar^2 Q^2}{8m} \pm |V_Q| + \frac{\hbar^2 k^2}{2m}\left(1 \pm \frac{1}{2}\,\frac{\hbar^2 Q^2}{2m|V_Q|}\right)$$

Tight-Binding Band

Tight-Binding Band

$$\mathcal{H} = E_0 \sum_{i,\sigma} c^\dagger_{i\sigma} c_{i\sigma}$$

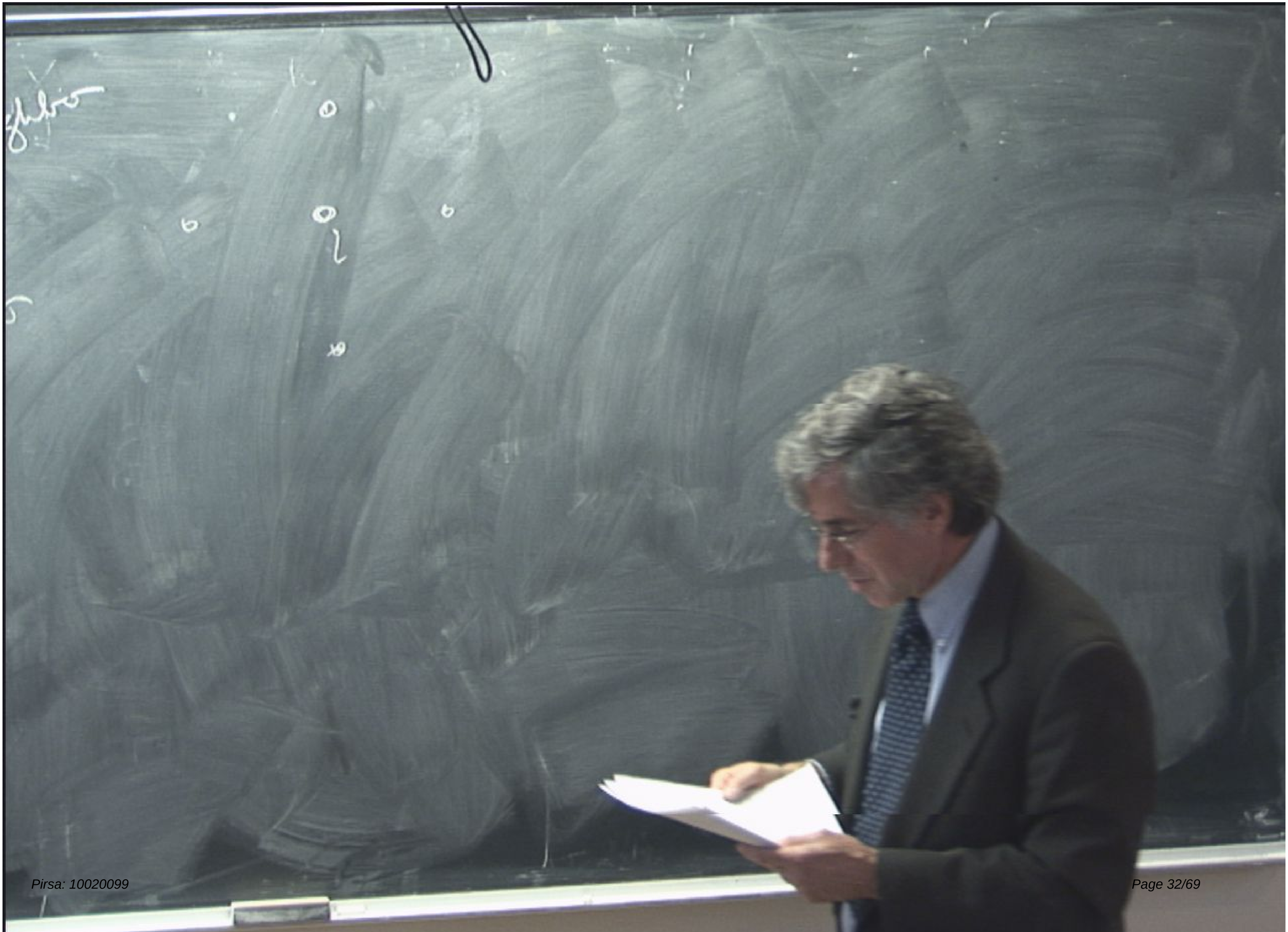

Tight-Binding Band
$\mathcal{H} = E_0 \sum_{i,\sigma} c^{\dagger}_{i\sigma} c_{i\sigma} - t \sum_{i}$

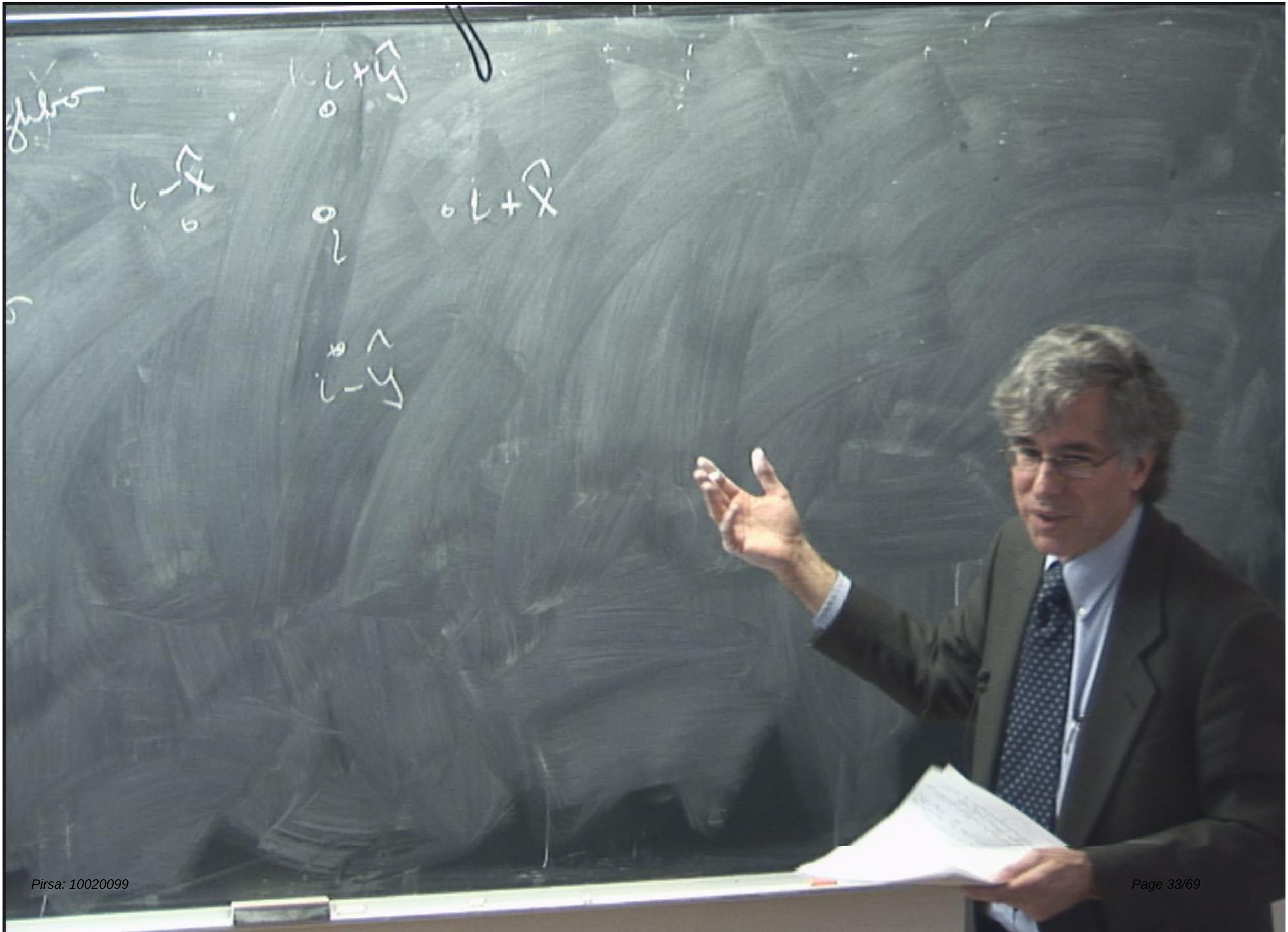
Tight-Binding Band

$$\mathcal{H} = -E_0 \sum_{i,\sigma} c^{\dagger}_{i\sigma} c_{i\sigma} - t \sum_{\substack{i,\delta \\ \sigma}} c^{\dagger}_{i+\delta,\sigma} c_{i\sigma}$$

Tight-Binding Band

$$\mathcal{H} = -E_0 \sum_{i,\sigma} c^\dagger_{i\sigma} c_{i\sigma} - t \sum_{\substack{i,\delta \\ \sigma}} c^\dagger_{i+\delta,\sigma} c_{i\sigma}$$




$i+\hat{y}$
$i-\hat{x}$
i
$i+\hat{x}$
$i-\hat{y}$

ight - Binding Band

nearest neighbor

$$\mathcal{H} = E_0 \sum_{i,\sigma} c^{+}_{i\sigma} c_{i\sigma} - t \sum_{\substack{i,\delta \\ \sigma}} c^{+}_{i+\delta,\sigma} c_{i\sigma}$$

$i - \hat{x}$

i

i

$i -$

Tight-Binding Band

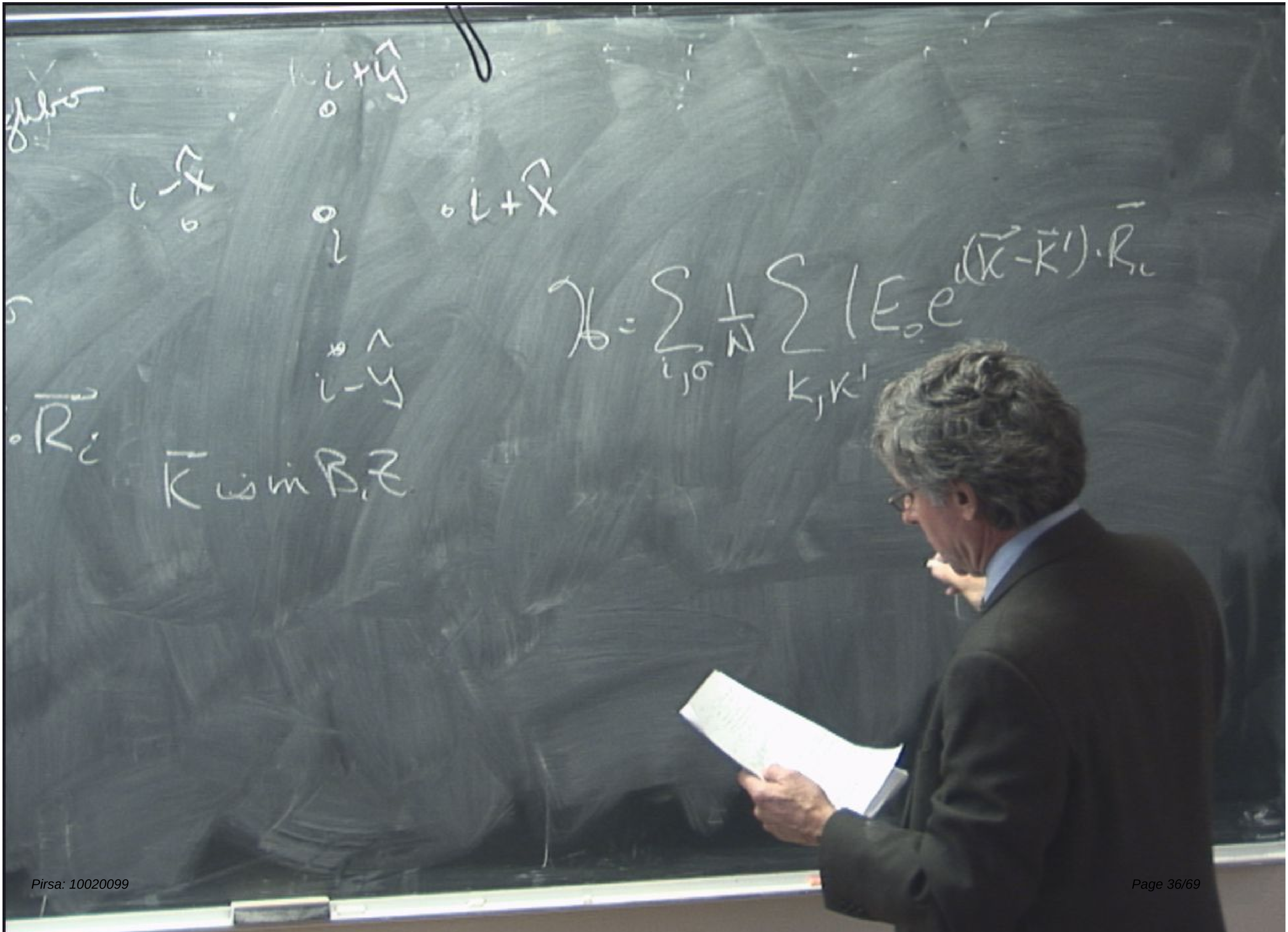
$$\mathcal{H} = E_0 \sum_{i,\sigma} c^+_{i\sigma} c_{i\sigma} - t \sum_{\substack{i,\delta \\ \sigma}} c^+_{i+\delta,\sigma} c_{i\sigma}$$

nearest neighbor

$i - \hat{x}$

Let $c^+_{i\sigma} = \frac{1}{\sqrt{N}} \sum_{\vec{k}} c^+_{k,\sigma} e^{i\vec{k}\cdot\vec{R}_i}$

$\vec{k}$ is in


i+ŷ
i−x̂
i
i+x̂
i−ŷ
K is in B.Z
H = Σ_{i,σ} 1/N Σ_{k,k'} (E₀ e^{i(k−k')·R_i}

$\circ\, i+\hat{y}$

$i-\hat{x}$ $\circ$ $\circ\, i$ $\circ\, i+\hat{x}$

$\circ\, i-\hat{y}$

$\cdot \vec{R}_i$

$\vec{K}$ is in B.Z

$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{k,k'} \Big(E_0 e^{i(\vec{K}-\vec{K}')\cdot \vec{R}_i}$$

$$- t \sum_{\delta} e^{i\vec{K}(\vec{R}_i+\vec{\delta})} e^{-i\vec{K}'\cdot \vec{R}_i} \Big)$$

$$\times C^{\dagger}_{\vec{K},\sigma} C_{\vec{K}',\sigma}$$

Tight-Binding Band

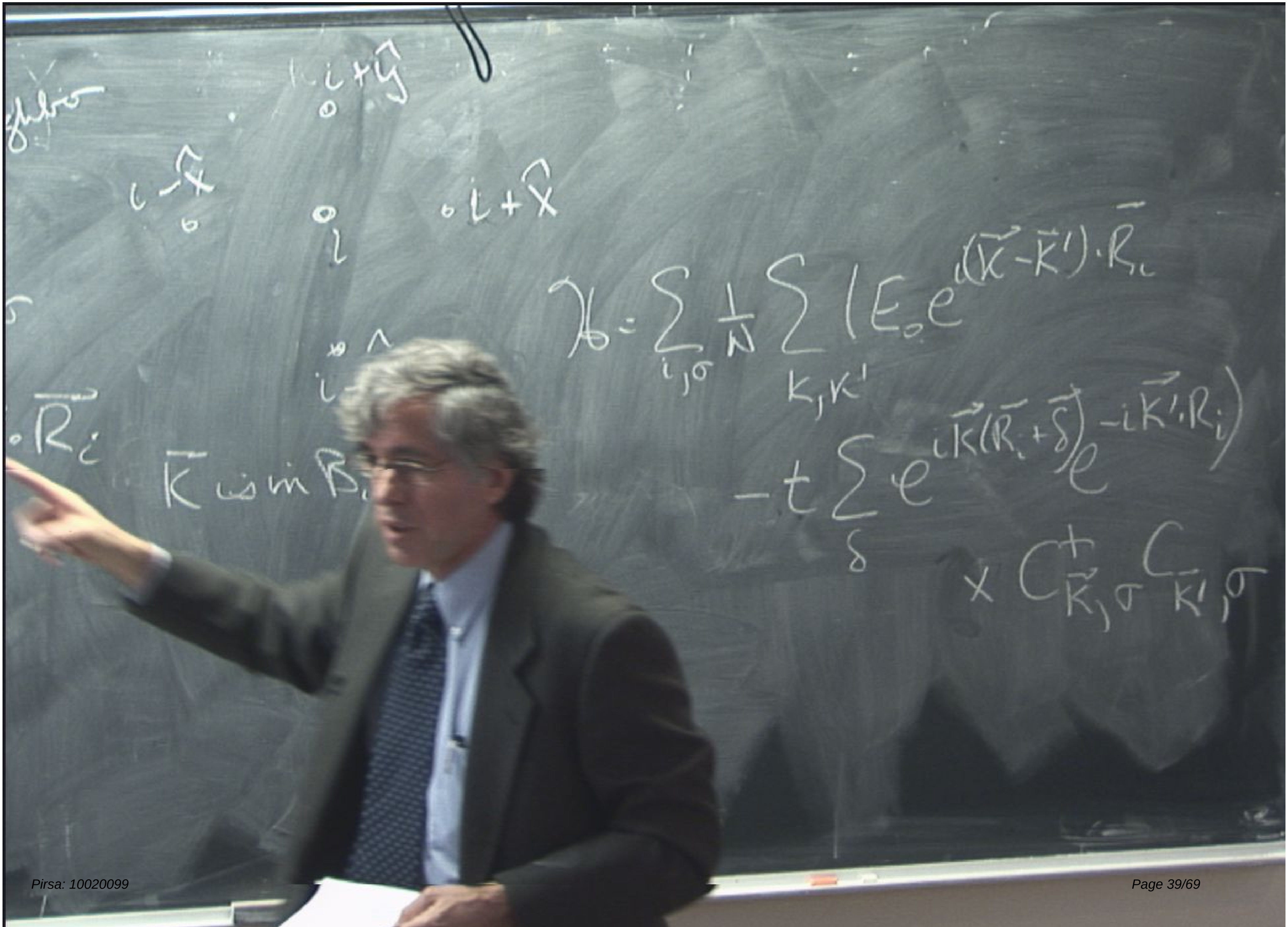
nearest neighbor

$$\mathcal{H} = E_0 \sum_{i,\sigma} c^\dagger_{i\sigma} c_{i\sigma} - t \sum_{\substack{i,\delta \\ \sigma}} c^\dagger_{i+\delta,\sigma} c_{i\sigma}$$

$i \to \hat{x}$

Let $c^\dagger_{i\sigma} = \frac{1}{\sqrt{N}} \sum_{\bar{k}} c^\dagger_{k,\sigma} e^{i \bar{k} \cdot \bar{R}_i}$

$\bar{k}$


$i+\hat{y}$
$i-\hat{x}$
i
$i+\hat{x}$
$\cdot \vec{R}_i$
$\vec{K}$ is in B
$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{k,k'} \Big(E_0 e^{i(\vec{k}-\vec{k}')\cdot \vec{R}_i}$
$- t \sum_{\delta} e^{i\vec{k}(\vec{R}_i+\vec{\delta})} e^{-i\vec{k}'\cdot R_i} \Big)$
$\times \, C^{\dagger}_{\vec{k},\sigma} C_{\vec{k}',\sigma}$

Tight-Binding Band

nearest neighbor

$$\mathcal{H} = -E_0 \sum_{i,\sigma} c^{\dagger}_{i\sigma} c_{i\sigma} - t \sum_{i,\delta,\sigma} c^{\dagger}_{i+\delta,\sigma} c_{i\sigma}$$

Let $c^{\dagger}_{i\sigma} = \frac{1}{\sqrt{N}} \sum_{\vec{k}} c^{\dagger}_{k,\sigma} e^{i\vec{k}\cdot\vec{R}_i}$ $\quad \vec{k}$ is in B.

$$\sum_i e^{i(\vec{k}-\vec{k}')\cdot R_i} = N\delta_{\vec{k},\vec{k}'}$$

$i-\hat{x}$

i

i

$i-$

$i+\hat{y}$

$i-\hat{x}$ $\quad i \quad$ $i+\hat{x}$

$i-\hat{y}$

$\bar{K}$ is in B.Z.

What is $-t\sum_i e^{i\vec{K}\cdot\vec{\delta}}$

$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{K,K'} \Big(E_0 e^{i(\vec{K}-\vec{K}')\cdot\vec{R}_i} - t\sum_{\delta} e^{i\vec{K}(\vec{R}_i+\vec{\delta})} e^{-i\vec{K}'\cdot R_i} \Big) \times C^{\dagger}_{\vec{K},\sigma} C_{\vec{K}',\sigma}$$

$\delta_{\vec{K},\vec{K}'}$

$i+\hat{y}$

$i-\hat{x}$ $\quad i \quad$ $i+\hat{x}$

$i-\hat{y}$

$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{k,k'} \Big(E_0 e^{i(\vec{k}-\vec{k}')\cdot \vec{R}_i} - t \sum_{\delta} e^{i\vec{k}(\vec{R}_i+\vec{\delta})} e^{-i\vec{k}'\cdot R_i} \Big) \times C^{\dagger}_{\vec{k},\sigma} C_{\vec{k}',\sigma}$$

$\cdot \vec{R}_i$

$\bar{k}$ is in B.Z.

What is $-t\sum_i e^{i\vec{k}\cdot\vec{\delta}}$?

$N\,\delta_{\vec{k},\vec{k}'}$

$i+\hat{y}$

$i-\hat{x}$ $\quad i \quad$ $i+\hat{x}$

$i-\hat{y}$

$\cdot \vec{R}_i$

$\bar{K}$ is in B.Z.

$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{k,k'} \Big(E_0 e^{i(\vec{k}-\vec{k}')\cdot \vec{R}_i} - t \sum_{\delta} e^{i\vec{k}(\vec{R}_i+\vec{\delta})} e^{-i\vec{k}'\cdot R_i} \Big) \times C^{\dagger}_{\vec{k},\sigma} C_{k',\sigma}$$

What is $-t \sum_i e^{i\vec{k}\cdot\vec{\delta}}$?

$$= -2t\boxed{\cos k_x + \cos k_y}$$

$N \delta_{\vec{k},\vec{k}'}$

$i+\hat{y}$

$i-\hat{x}$ $\quad i \quad$ $i+\hat{x}$

$i-\hat{y}$

$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{k,k'} \Big(E_0 e^{i(\vec{k}-\vec{k}')\cdot \bar{R}_i} - t \sum_{\delta} e^{i\vec{k}(\bar{R}_i+\vec{\delta})} e^{-i\vec{k}'\cdot R_i} \Big) \times c^{\dagger}_{\vec{k},\sigma} c_{\vec{k}',\sigma}$$

$\bar{k}$ is in B.Z

What is $-t \sum_i e^{i\vec{k}\cdot\vec{\delta}}$?

$$= \boxed{-2t[\cos k_x + \cos k_y]}$$

Tight-Binding Band

$$\mathcal{H} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k})\, C^{\dagger}_{\vec{k},\sigma} C_{\vec{k},\sigma}$$

$$\varepsilon(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

neighbo

neighbo

Tight-Binding Band

$$\mathcal{H} = \sum_{\vec{k},\sigma} \mathcal{E}(\vec{k}) C^{\dagger}_{\vec{k},\sigma} C_{\vec{k},\sigma}$$

$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

If $k_x, k_y \ll 1$

$$\mathcal{E}(\vec{k}) = E_0 - 4t(k_x^2 + k_y^2)$$

Tight-Binding Band

$$\mathcal{H} = \sum_{\vec{k},\sigma} \mathcal{E}(\vec{k})\, C^{+}_{\vec{k},\sigma} C_{\vec{k},\sigma}$$

$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

If $k_x, k_y \ll 1$

$$\mathcal{E}(\vec{k}) = E_0 - 4t(k_x^2 + k_y^2)$$

like $\frac{\hbar^2 k^2}{2m^*}$

neighbo

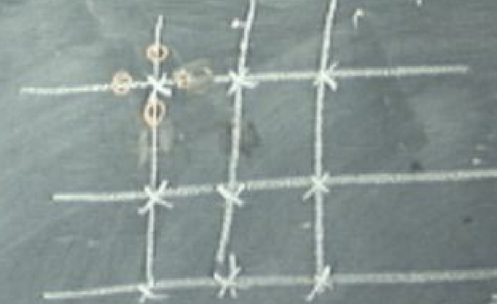
$\vec{k}$ is in

Wha

$i+\hat{y}$

$i-\hat{x}$ $\quad$ i $\quad$ $i+\hat{x}$

$i-\hat{y}$



$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{k,k'} \Big(E_0\, e^{i(\vec{k}-\vec{k}')\cdot \vec{R}_i} - t \sum_{\delta} e^{i\vec{k}(\vec{R}_i+\vec{\delta}) - i\vec{k}'\cdot \vec{R}_i} \Big) \times C \; C_{k',\sigma}$$

$\vec{k}$ is in B.Z.

What is $-t\sum_{i} e^{i\vec{k}\cdot\vec{\delta}}$?

$$= -2t\,\boxed{\cos k_x + \cos k_y}$$

$i+\hat{y}$

$i-\hat{x}$ $\quad i \quad i+\hat{x}$

$i-\hat{y}$

$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{k,k'} \Big(E_0 e^{i(\vec{k}-\vec{k}')\cdot R_i} - t \sum_{\delta} e^{i\vec{k}(R_i+\delta)} e^{-i\vec{k}'\cdot R_i} \Big) \times C^{\dagger}_{\vec{k},\sigma} C_{k',\sigma}$$

$\bar{k}$ is in B.Z

What is $-t\sum_i e^{i\vec{k}\cdot\vec{\delta}}$?

$$= \boxed{-2t[\cos k_x + \cos k_y]}$$

$y)$

$k_y^2)$

Tight-Binding Band

$$\mathcal{H} = \sum_{\vec{k},\sigma} \mathcal{E}(\vec{k}) C^{\dagger}_{\vec{k},\sigma} C_{\vec{k},\sigma}$$

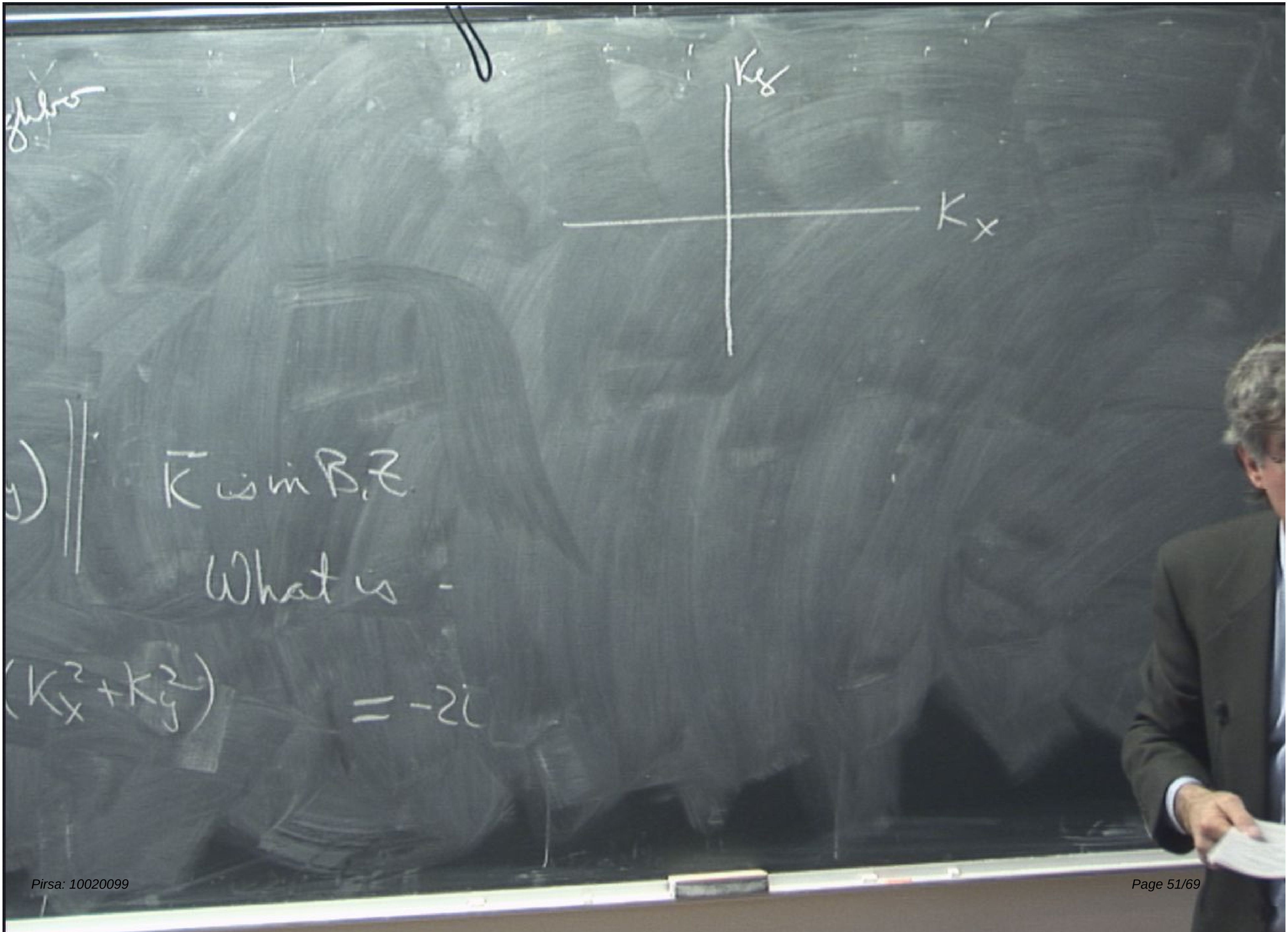
$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

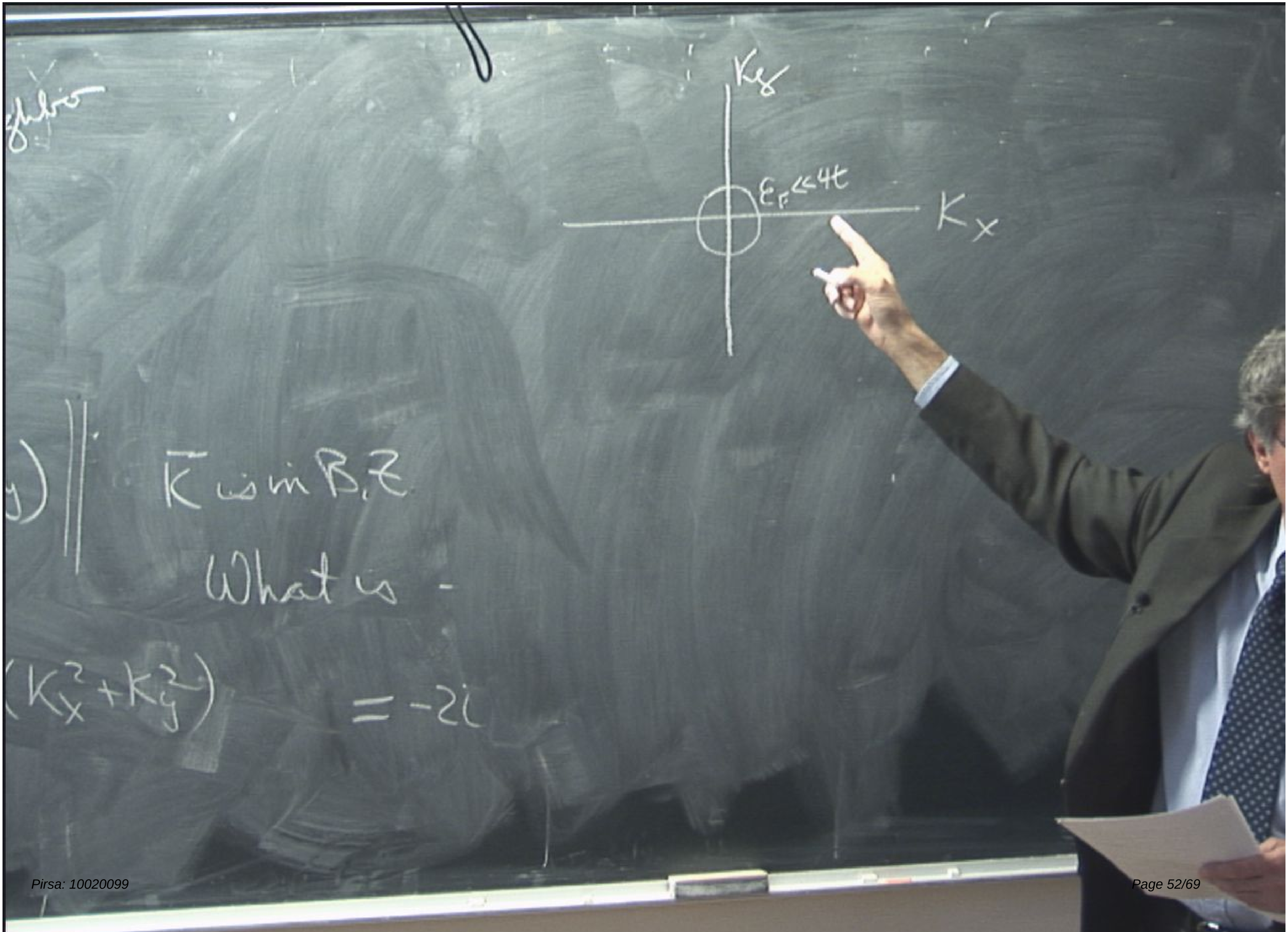
If $k_x, k_y \ll 1$

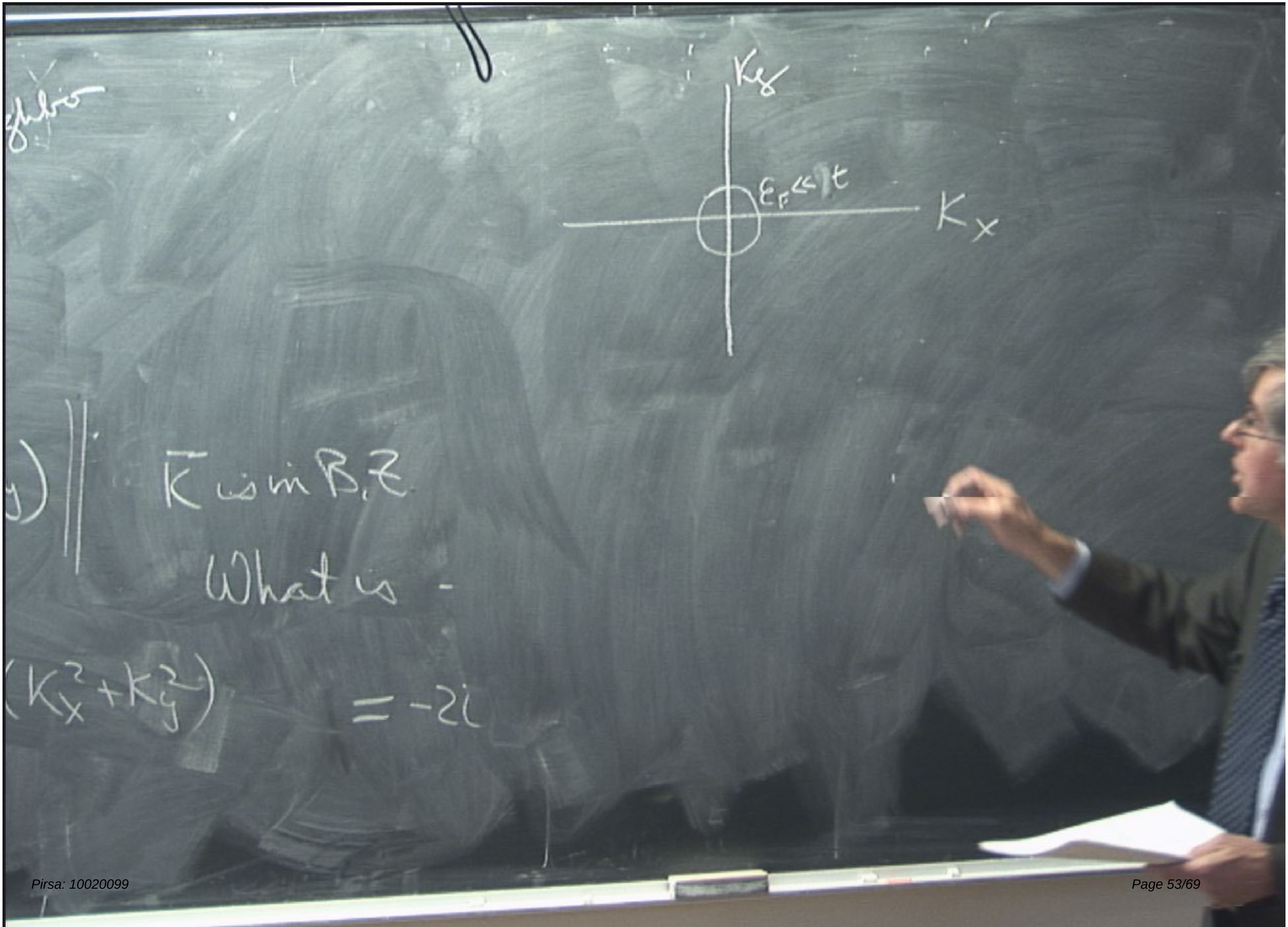
$$\mathcal{E}(\vec{k}) = E_0 - 4t + t(k_x^2 + k_y^2$$

like $\frac{\hbar^2 k^2}{2m^*}$

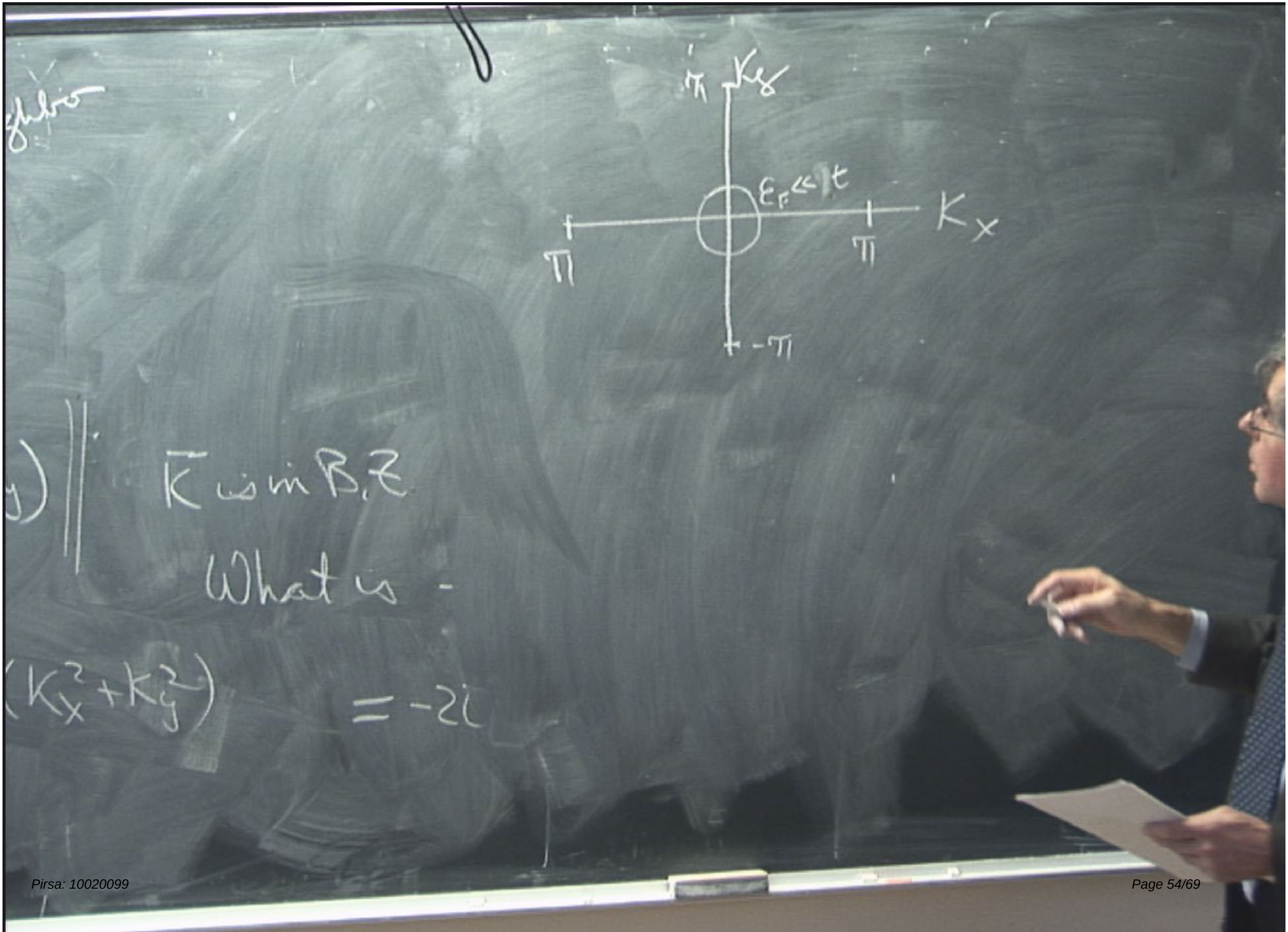
neighbo


K_y
K_x
$\bar{K}$ is in B.Z.
What is -
$(K_x^2+K_y^2)$
$= -2i$

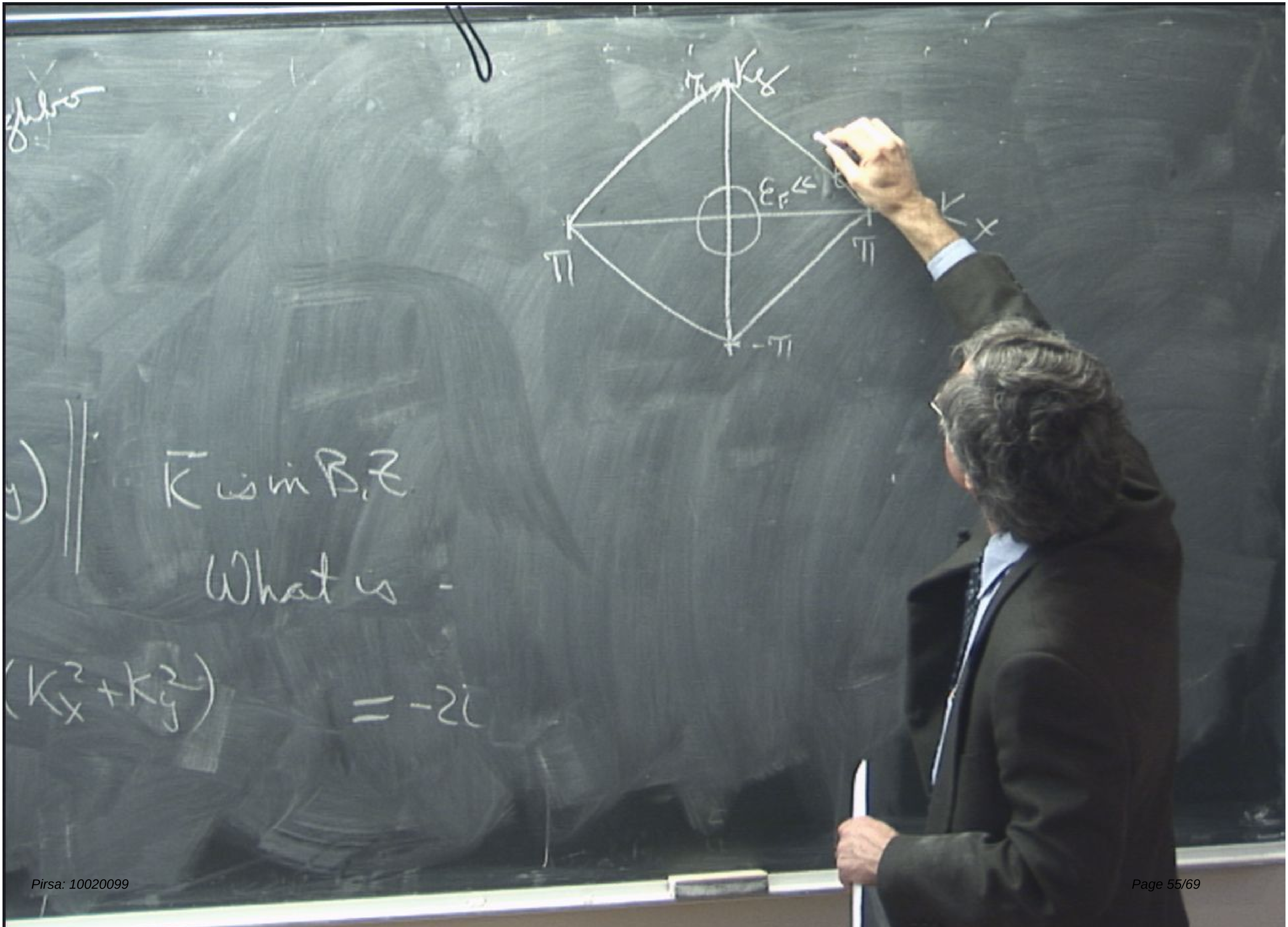

K_y
$\varepsilon_F \ll 4t$
K_x
$\bar{K}$ is in B.Z
What is
$(K_x^2+K_y^2)$
$= -2i$

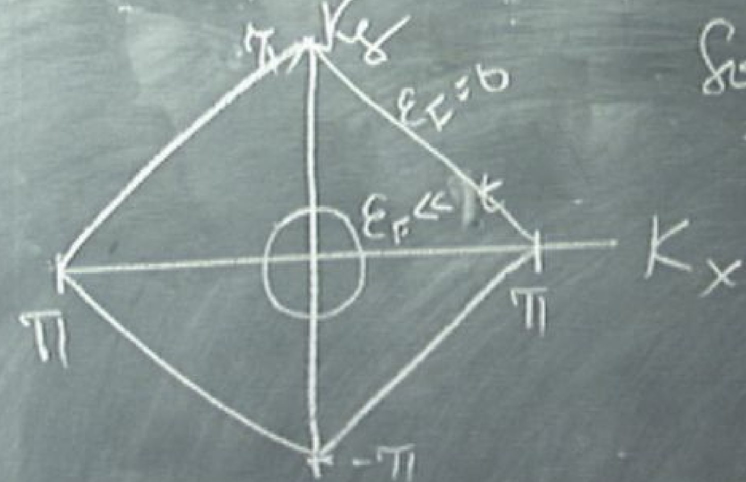


K_y
$\varepsilon_F \ll t$
K_x
$\bar{K}$ is in B.Z
What is
$(K_x^2+K_y^2)$
$= -2i$



π
k_y
$\varepsilon_F \ll t$
π
π
k_x
−π
$\bar{k}$ is in B.Z.
What is
$(k_x^2+k_y^2)$
$= -2i$


ky
kx
π
π
-π
εF ≪
K is in B.Z
What is -
(Kx²+Ky²)
= -2i



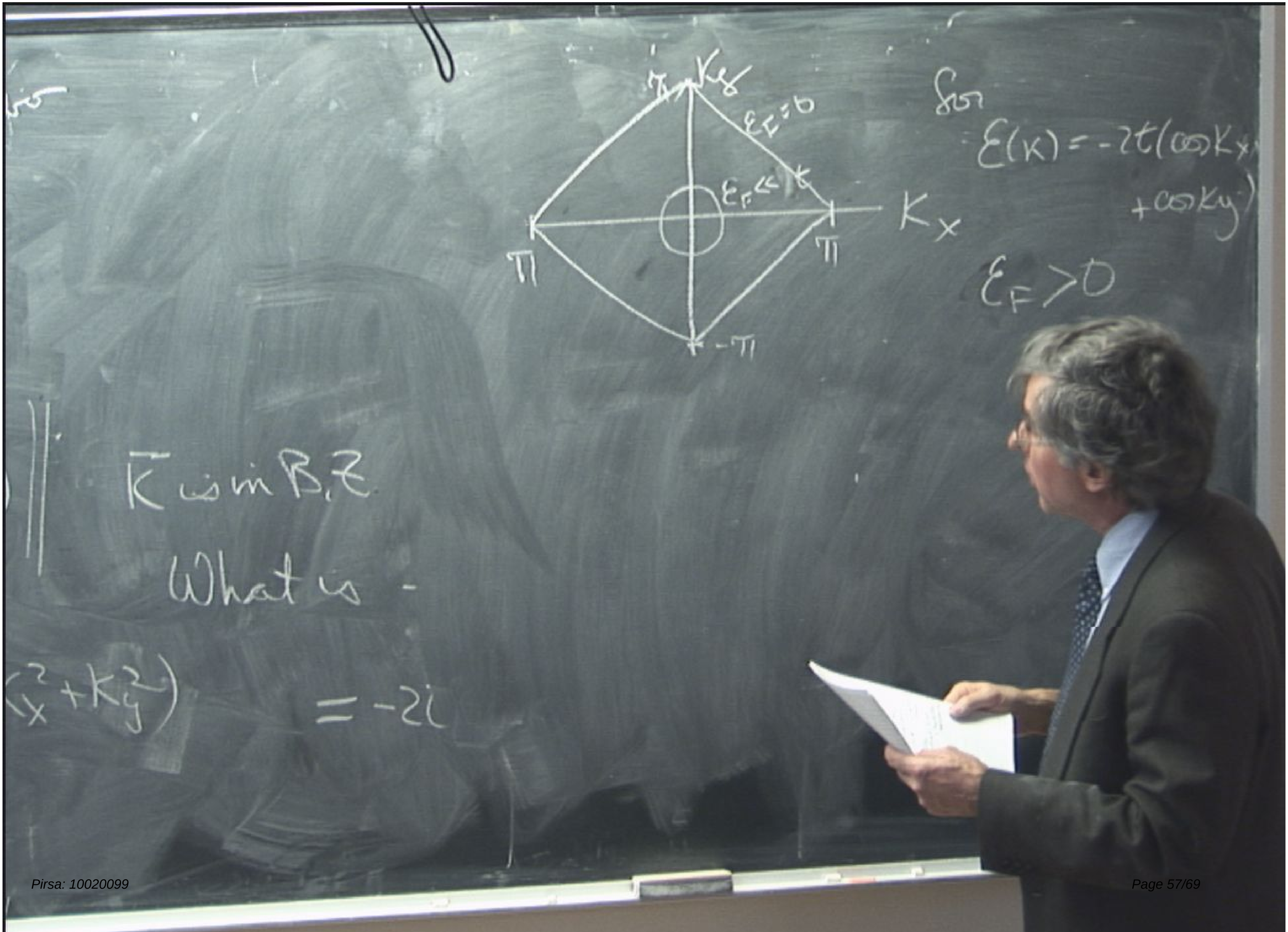
for

$\mathcal{E}(k) = -2t(\cos k_x + \cos k_y)$

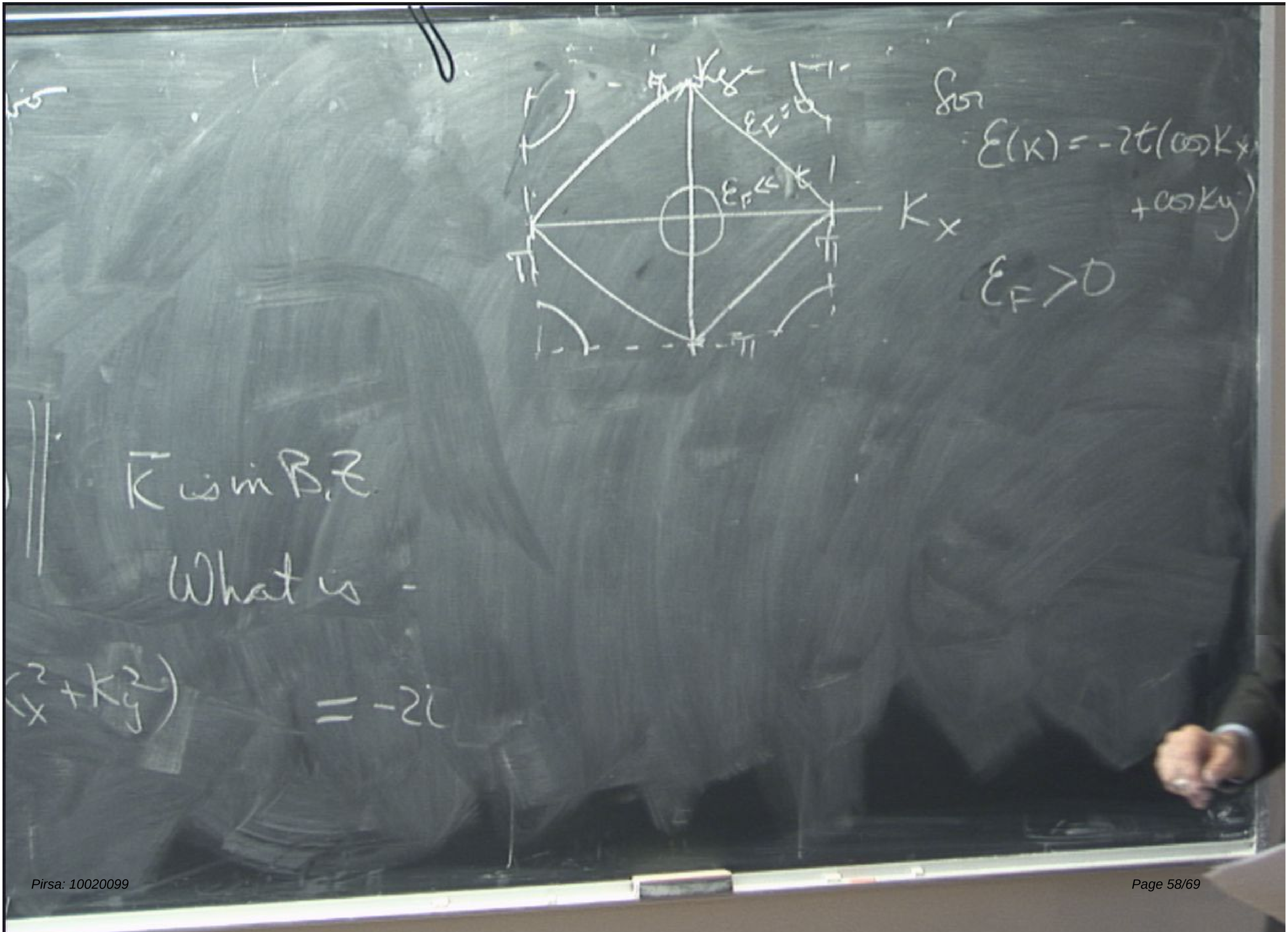
$\bar{k}$ is in B.Z

What is

$(k_x^2 + k_y^2)$ $= -2t$



Ky
εF = 0
εF << t
Kx
π
π
-π
for
ε(K) = -2t(cos Kx
+ cos Ky)
εF > 0
K is in B.Z
What is
= -2t



k_y
$\varepsilon_F = 0$
$\varepsilon_F \ll t$
k_x
π
for
$\varepsilon(k) = -2t(\cos k_x + \cos k_y)$
$\varepsilon_F > 0$
$\bar{k}$ is in B.Z
What is
$= -2t$

for $\epsilon(k) = -2t(\cos k_x + \cos k_y)$

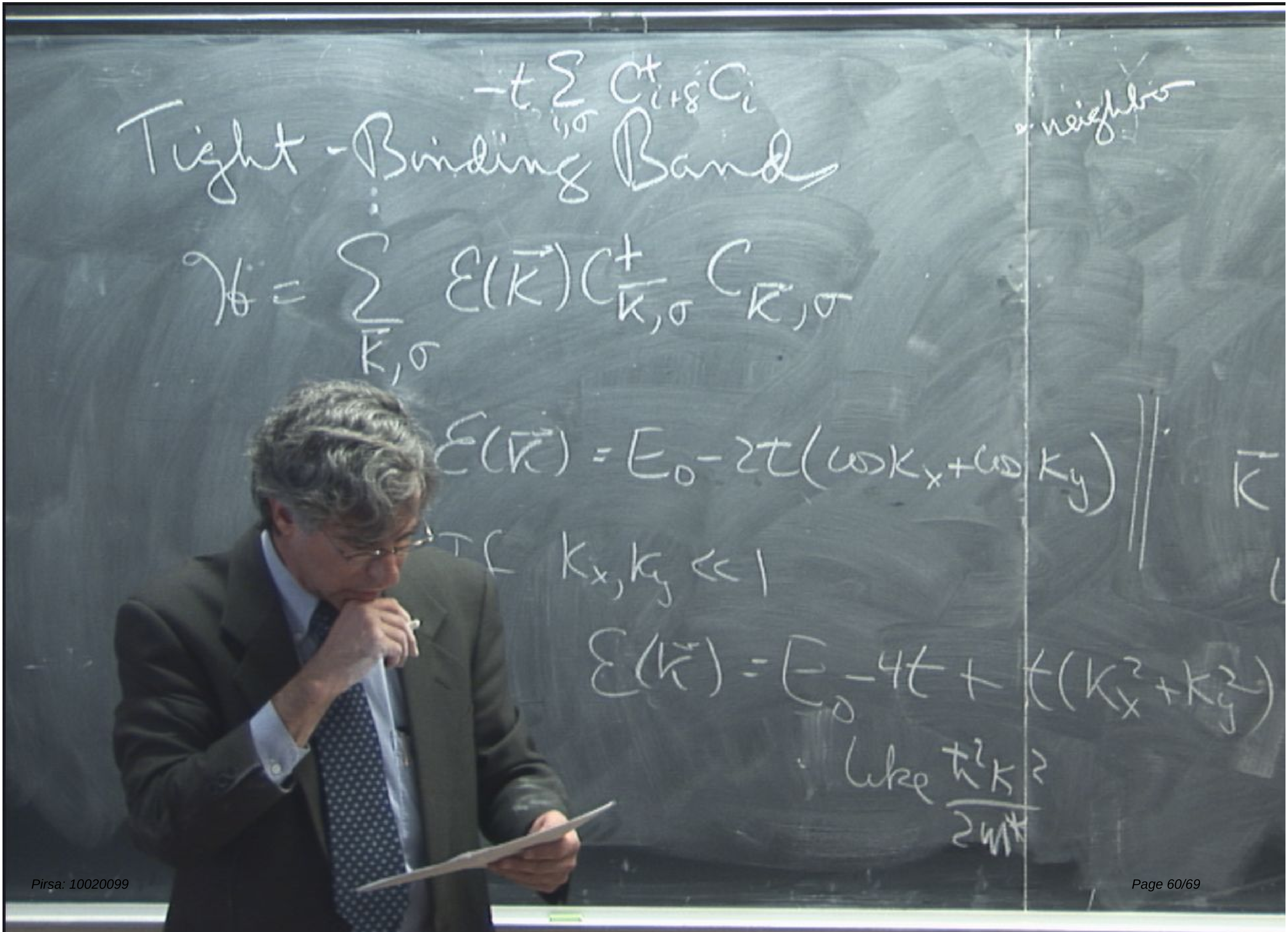
$\epsilon_F > 0$

$\bar{k}$ is in B.Z.

What is -

$(k_x^2 + k_y^2)$

$= -2t$


Tight-Binding Band
$-t \sum_{i,\sigma} C^{+}_{i+\delta} C_i$
neighbor
$\mathcal{H} = \sum_{\vec{k},\sigma} \mathcal{E}(\vec{k}) C^{+}_{\vec{k},\sigma} C_{\vec{k},\sigma}$
$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$
If $k_x, k_y \ll 1$
$\mathcal{E}(\vec{k}) = E_0 - 4t + t(k_x^2 + k_y^2)$
like $\frac{\hbar^2 k^2}{2m^*}$

$-t\sum_{i,\sigma} C^{+}_{i+\delta} C_i$

neighbo

Tight-Binding Band

$$\mathcal{H} = \sum_{\vec{k},\sigma} \mathcal{E}(\vec{k}) C^{+}_{\vec{k},\sigma} C_{\vec{k},\sigma}$$

$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

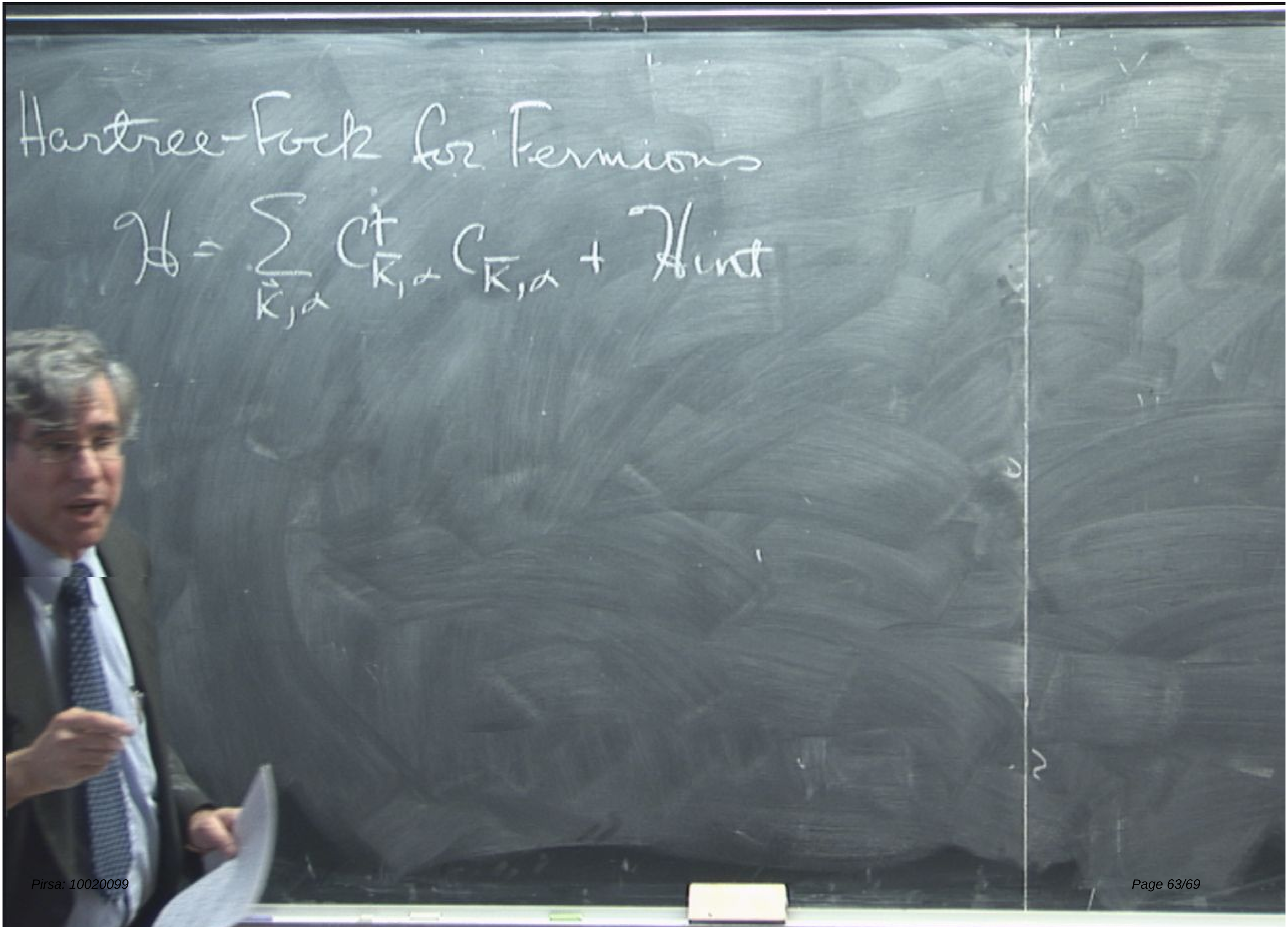
If $k_x, k_y \ll 1$

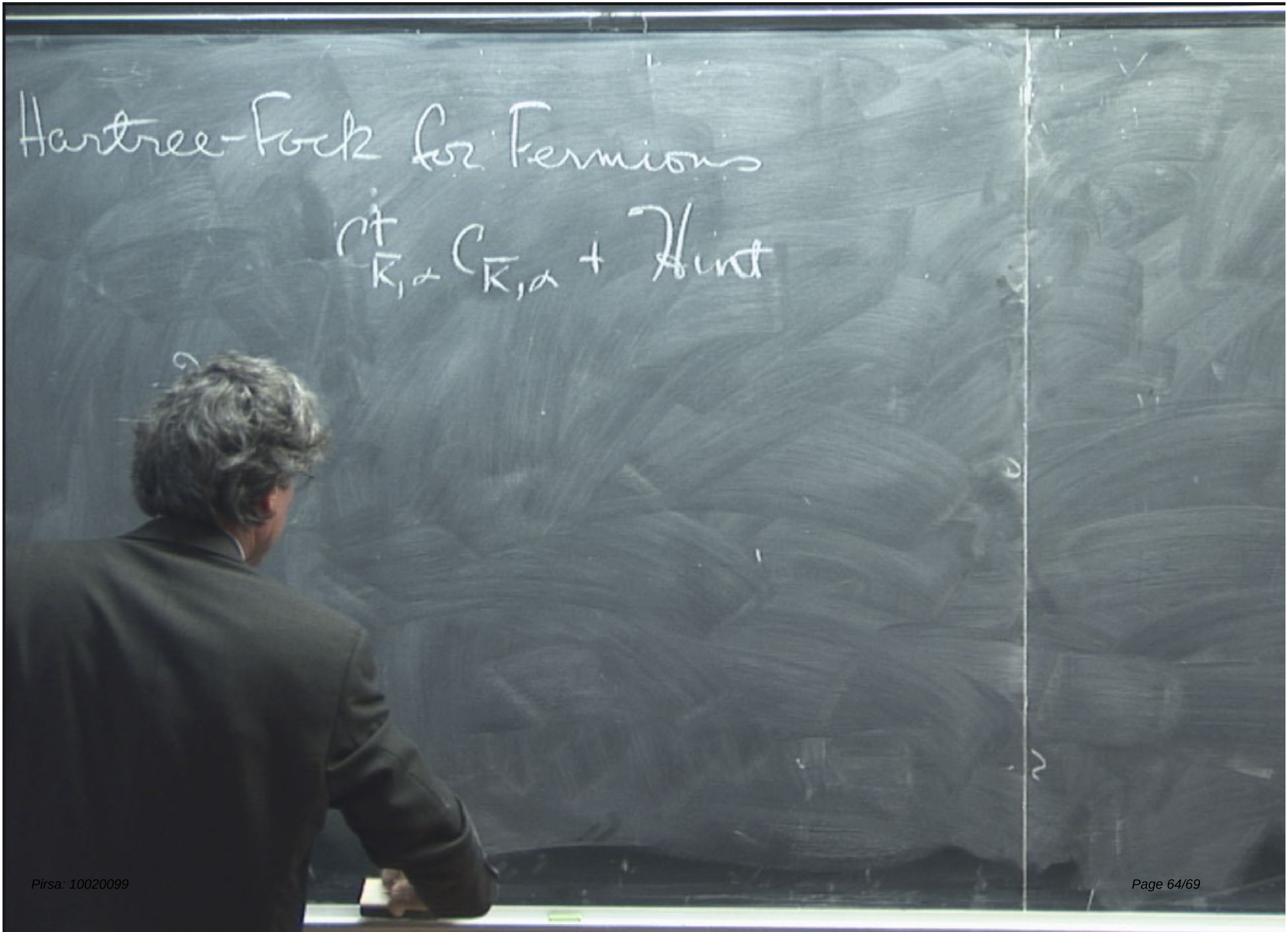
$$\mathcal{E}(\vec{k}) = E_0 - 4t + t(k_x^2 + k_y^2)$$

like $\frac{\hbar^2 k^2}{2m^*}$

$\vec{k}$


Hartree-Fock for Fermions

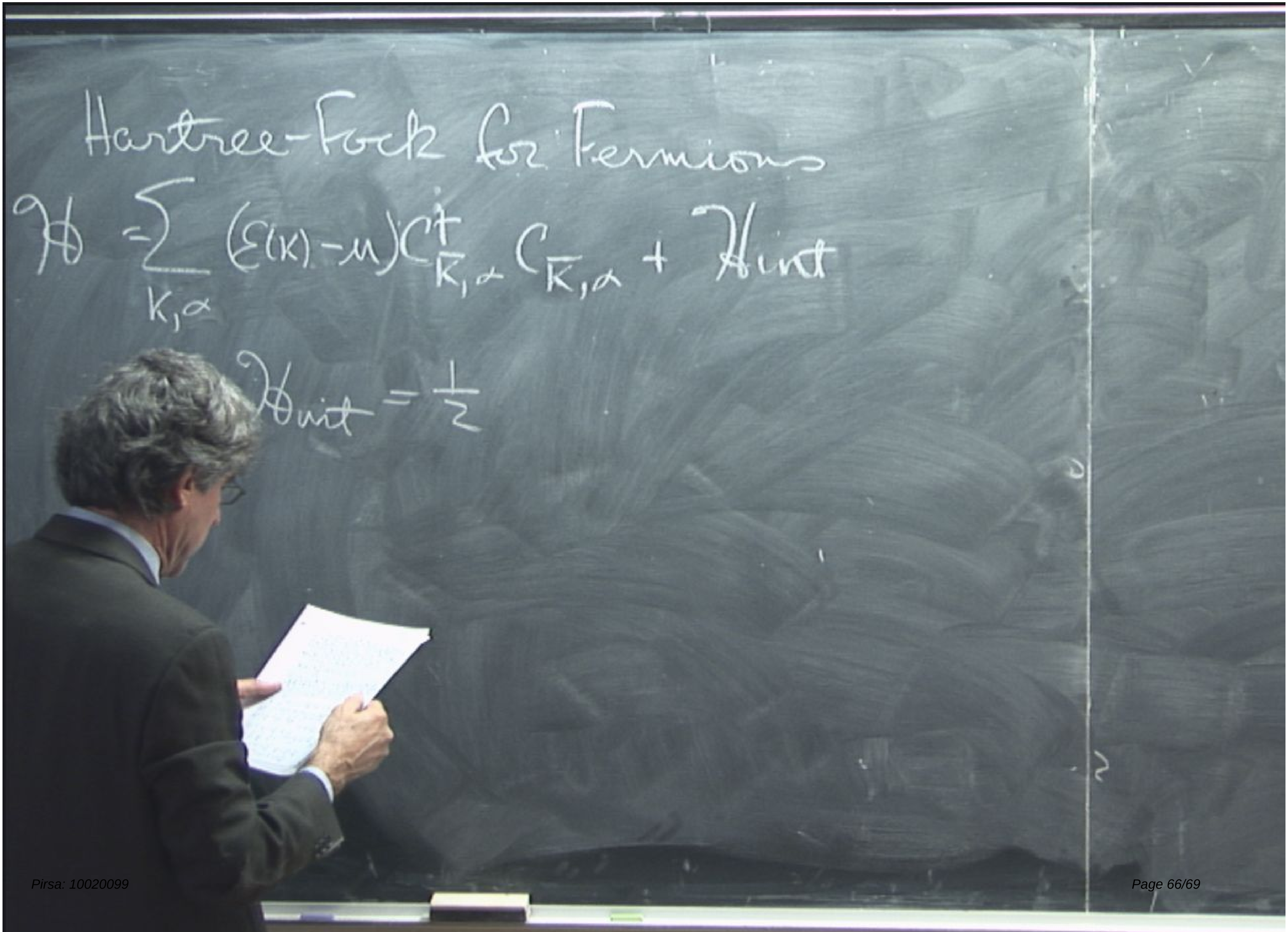

Hartree-Fock for Fermions
$\mathcal{H} = \sum_{\vec{k},\alpha} c^\dagger_{\vec{k},\alpha} c_{\vec{k},\alpha} + \mathcal{H}_{int}$


Hartree-Fock for Fermions
$c^{\dagger}_{\vec{k},\alpha} c_{\vec{k},\alpha} + \mathcal{H}_{int}$

Hartree-Fock for Fermions

$$= \sum_{\vec{k},\alpha} (\varepsilon(\vec{k}) - \mu) C^{\dagger}_{\vec{k},\alpha} C_{\vec{k},\alpha} + \mathcal{H}_{int}$$

$$\mathcal{H}_{int}$$


Hartree-Fock for Fermions
$\mathcal{H} = \sum_{k,\alpha} (\varepsilon(k) - \mu) C^{\dagger}_{\bar{k},\alpha} C_{\bar{k},\alpha} + \mathcal{H}_{int}$
$\mathcal{H}_{int} = \frac{1}{2}$

Hartree-Fock for Fermions

$$\mathcal{H} = \sum_{k,\alpha} (\varepsilon(k) - \mu) C^\dagger_{\bar{k},\alpha} C_{\bar{k},\alpha} + \mathcal{H}_{int}$$

$$\mathcal{H}_{int} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} V(\bar{R}_i - \bar{R}_j) C^\dagger_{i\alpha} C^\dagger_{j\beta} C_{j\beta} C_{i\alpha}$$

tree-Fock for Fermions C.f. Wen 5.2

$$\sum_{\bar{k},\alpha} (\varepsilon(k)-\mu) C^{\dagger}_{\bar{k},\alpha} C_{\bar{k},\alpha} + \mathcal{H}_{int}$$

$$\mathcal{H}_{int} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} V(\bar{R}_i - \bar{R}_j) C^{\dagger}_{i\alpha} C^{\dagger}_{j\beta} C_{j\beta} C_{i\alpha}$$

tree-Fock for Fermions C.f. Wen 5.2

$$(\varepsilon(k)-\mu)C^{\dagger}_{\bar{k},\alpha}C_{\bar{k},\alpha} + \mathcal{H}_{int}$$

$$C^{\dagger}_{i\alpha}C_{i\alpha}V(R_i-R_j)C^{\dagger}_{j\beta}C_{j\beta}$$

$$\mathcal{H}_{int} = \frac{1}{2}\sum_{\substack{i,j\\ \alpha,\beta}} V(\bar{R}_i-\bar{R}_j)C^{\dagger}_{i\alpha}C^{\dagger}_{j\beta}C_{j\beta}C_{i\alpha}$$