

Title: Condensed Matter II - Lecture 2

Date: Feb 17, 2010 10:10 AM

URL: <http://pirsa.org/10020098>

Abstract:

Motion in a Periodic Potential



Books on Condensed Matter Physics

Standard classic texts are:

Charles Kittel: *Introduction to Solid State Physics*.

Ashcroft and Mermin: *Solid State Physics*.

Charles Kittel: *Quantum Theory of Solids*.

W. A. Harrison *Solid State Theory*.

J.M Ziman *Principles of the Theory of Solids*.

J.M Ziman *Electrons and Phonons in Metals*.

A. Abrikosov, *Fundamentals of the Theory of Metals*, (Half of the book is on superconductivity.)

G. Mahan, *Many-Particle Physics*.

Some of the more modern references include:

X.-G. Wen, *Quantum Field Theory of Many-Body Systems*.

E. Fradkin, *Field Theory of Condensed Matter Systems*.

H. Bruus and K. Flensberg, *Many-Body Quantum Theory in Condensed Matter Physics*.

J.M Ziman *Principles of the Theory of Solids*.

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E. Fradkin, *Field Theory of Condensed Matter Systems*.

H. Bruus and K. Flensberg, *Many-Body Quantum Theory in Condensed Matter Physics*.

For Landau's Fermi Liquid Theory, the original article by Landau will be posted on the wiki. The books by Wen and by Bruus and Flensberg are useful as is the book by

A.J. Leggett, *Quantum Liquids: Bose Condensation and Cooper Pairing in Condensed Matter Physics*. (See also A.J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).)

For Superconductivity, I will mainly refer to:

P.G. deGennes, *Superconductivity of Metals and Alloys*.

Motion in a Periodic Potential
Translational Symmetry

Motion in a Periodic Potential

Translational Symmetry

Discrete Symmetry

$$G_T = \left\{ T_{R_{e,m,n}} \mid m, n = 0, \dots, L-1 \right\}$$

Motion in a Periodic Potential

Translational Symmetry

Discrete Symmetry

$$G_T = \left\{ T_{\vec{R}_{l,m,n}} \mid l, m, n = 0, \dots, L-1 \right\}$$

$$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$$

Motion in a Periodic Potential

Translational Symmetry

Discrete Symmetry

$$G_T = \left\{ T_{\vec{R}_{l,m,n}} \mid l, m, n = 0, \dots, L-1 \right\}$$

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Motion in a Periodic Potential

Translational Symmetry

Discrete Symmetry

$$G_T = \left\{ T_{\vec{R}_{l,m,n}} \mid l, m, n = 0, \dots, L-1 \right\}$$

$$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$$

$$T_{\vec{R}_{L,0,0}} = E$$

identity



Motion in a Periodic Potential

Translational Symmetry

Discrete Symmetry

$$G_T = \left\{ T_{\vec{R}_{l,m,n}} \mid l, m, n = 0, \dots, L-1 \right\} \quad T_{\vec{R}_{L,0,0}} = E$$

$$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$$

$\vec{a}, \vec{b}, \vec{c}$ = primitive translation vectors

Motion in a Periodic Potential

Translational Symmetry

Discrete Symmetry

$$\left(\sum_{l,m,n} T_{\vec{R}_{l,m,n}} \mid l, m, n = 0, \dots L-1 \right) T_{\vec{R}_{L,0,0}} = E$$

$$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$$

$\vec{a}, \vec{b}, \vec{c}$ = primitive translation vectors

$$|\vec{a} \cdot (\vec{b} \times \vec{c})| \neq 0$$

= (Volume of unit cell)

identity
↓

Motion in a Periodic Potential

Translational Symmetry

Discrete Symmetry

$$G_T = \left\{ T_{\vec{R}_{l,m,n}} \mid l, m, n = 0, \dots, L-1 \right\} \quad T_{\vec{R}_{L,0,0}} = E$$

$$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$$

$\vec{a}, \vec{b}, \vec{c}$ = primitive translation vectors

$$|\vec{a} \cdot (\vec{b} \times \vec{c})| \neq 0$$

= \rightarrow volume of unit cell

identical



Functions with the Symmetry of G_T

Identify

$$\downarrow \\ = E$$

σ_D

tions

Functions with the Symmetry of G_T

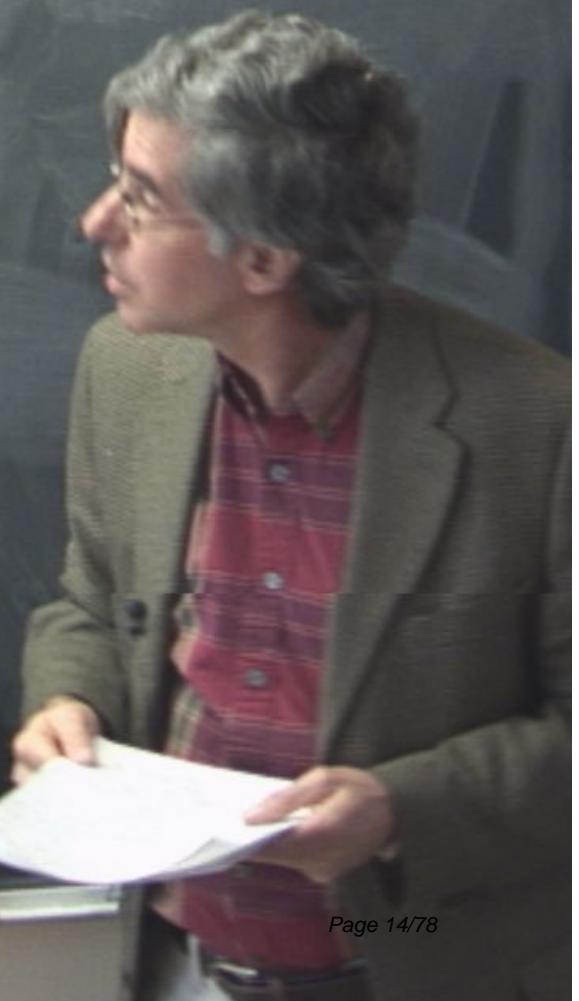
$$T_{\vec{R}} \varphi(\vec{x}) = \varphi(\vec{r})$$

Newton's

$$\downarrow \\ = E$$

σ_D

tions



Functions with the Symmetry of G_T

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

Newton's

$$\downarrow \\ = E$$

σ_D

tions

Functions with the Symmetry of G_T .

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

Then $\varphi(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q}) e^{i \vec{Q} \cdot \vec{r}}$

Newton
↓
 $= E$

ω_0

tions

Newton
↓
= E

Functions with the Symmetry of G_T

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

Then $\varphi(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q}) e^{i \vec{Q} \cdot \vec{r}}$

where $\vec{Q} \cdot \vec{R} = 2\pi n$

Newton
↓
 $= E$

Functions with the Symmetry of G_T .

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

Then $\varphi(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q}) e^{i \vec{Q} \cdot \vec{r}}$

where $\vec{Q} \cdot \vec{R} = 2\pi n$

Then $\vec{Q} = L \vec{A} + M \vec{B} + N \vec{C}$ L, M, N
integers

Functions with the Symmetry of G_I

$$T_{\vec{R}} \mathcal{Q}(\vec{r}) = \mathcal{Q}(\vec{r} + \vec{R}) = \mathcal{Q}(\vec{r})$$

where $\vec{Q} \cdot \vec{R} = 2\pi n$

$$\vec{A} = 2\pi \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}}$$

Motion in a Periodic Potential

Translational Symmetry

Discrete Symmetry

$$G_T = \left\{ T_{\vec{R}_{l,m,n}} \mid l, m, n = 0, \dots, L-1 \right\}$$

$$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$$

$\vec{a}, \vec{b}, \vec{c}$ = primitive translation

$$|\vec{a} \cdot \vec{b} \times \vec{c}| \neq 0$$

=> volume of
unit cell

↓
= E

Functions with the Symmetry of G_T

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

Then $\varphi(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q}) e^{i \vec{Q} \cdot \vec{r}}$

where $\vec{Q} \cdot \vec{R} = 2\pi n$

Then $\vec{Q} = L \vec{A} + M \vec{B} + N \vec{C}$ L, M, N
integers

$$\vec{A} = 2\pi \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}} + \text{etc}$$

Functions with the Symmetry of G_T

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

Then $\varphi(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q}) e^{i \vec{Q} \cdot \vec{r}}$

where $\vec{Q} \cdot \vec{R} = 2\pi n$

Then $\vec{Q} = L \vec{A} + M \vec{B} + N \vec{C}$ L, M, N
integers

$$\vec{A} = 2\pi \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}} + \text{etc}$$

$\{\vec{Q}\}$ are called Reciprocal Lattice Vectors

Newton's
law

$$\downarrow = E$$

ω_0

tors

Functions with the Symmetry of G_F

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

$$\text{Then } \varphi(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q}) e^{i \vec{Q} \cdot \vec{r}}$$

$$\text{where } \vec{Q} \cdot \vec{R} = 2\pi n$$

$$\text{Then } \vec{Q} = L \vec{A} + M \vec{B} + N \vec{C} \quad \begin{matrix} L, M, N \\ \text{integers} \end{matrix}$$

$$\vec{A} = 2\pi \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}} \quad \text{etc}$$

$\{\vec{Q}\}$ are called Reciprocal Lattice Vectors

Eigenfs of Translation operators



Eigenfs of Translation operators

$$T_{\vec{R}} \Psi_{\vec{k}}(\vec{r}) = \Psi_{\vec{k}}(\vec{r} + \vec{R}) = e^{i \vec{k} \cdot \vec{R}} \Psi_{\vec{k}}(\vec{r})$$



Eigenfs of Translation operators

$$T_R \Psi_{\vec{K}}(\vec{r}) = \Psi_{\vec{K}}(\vec{r} + \vec{R}) = e^{i \vec{K} \cdot \vec{R}} \Psi_{\vec{K}}(\vec{r})$$

Eigenfns of Translation operators

$$T_R \Psi_{\vec{K}}(\vec{r}) = \Psi_{\vec{K}}(\vec{r} + \vec{R}) = e^{i\vec{K}\cdot\vec{R}} \Psi_{\vec{K}}(\vec{r})$$

For p.b.c.'s

$$\vec{R} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$

Functions with the Symmetry of G_F

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

Then $\varphi(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q}) e^{i \vec{Q} \cdot \vec{r}}$

where $\vec{Q} \cdot \vec{R} = 2\pi n$

Then $\vec{Q} = L \vec{A} + M \vec{B} + N \vec{C}$ L, M, N
integers

$$\vec{A} = 2\pi \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}} + \text{etc}$$

$\{\vec{Q}\}$ are called Reciprocal Lattice Vectors

Eigenfs of Translation operators

$$T_{\vec{R}} \Psi_{\vec{k}}(\vec{r}) = \Psi_{\vec{k}}(\vec{r} + \vec{R}) = e^{i\vec{k} \cdot \vec{R}} \Psi_{\vec{k}}(\vec{r})$$

For p.b.c.'s

$$\vec{R} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$



Eigenfs of Translation operators

$$T_{\vec{R}} \Psi_{\vec{k}}(\vec{r}) = \Psi_{\vec{k}}(\vec{r} + \vec{R}) = e^{i \vec{k} \cdot \vec{R}} \Psi_{\vec{k}}(\vec{r})$$

For p.b.c.'s

$$\vec{k} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$

$$l = 0, \dots L-1$$

Functions with the Symmetry of G_T

$$T_{\vec{R}} \varphi(\vec{r}) = \varphi(\vec{r} + \vec{R}) = \varphi(\vec{r})$$

Then $\varphi(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q}) e^{i \vec{Q} \cdot \vec{r}}$

where $\vec{Q} \cdot \vec{R} = 2\pi n$

Then $\vec{Q} = L \vec{A} + M \vec{B} + N \vec{C}$ L, M, N
integers

$$\vec{A} = 2\pi \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot \vec{b} \times \vec{c}} \quad \text{etc}$$

$\{\vec{Q}\}$ are called Reciprocal Lattice Vectors

Eigenfns of Translation operators

$$T_{\vec{R}} \Psi_{\vec{k}}(\vec{r}) = \Psi_{\vec{k}}(\vec{r} + \vec{R}) = e^{i\vec{k} \cdot \vec{R}} \Psi_{\vec{k}}(\vec{r})$$

For P.b.c.'s

$$\vec{R} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$

$$l=0, \dots L-1 \quad \text{or} \quad \frac{l}{2}-1 \dots 0, \dots \frac{L}{2}$$

Even

Eigenfunctions of Translation operators

$$T_{\vec{R}} \Psi_{\vec{k}}(\vec{r}) = \Psi_{\vec{k}}(\vec{r} + \vec{R}) = e^{i\vec{k} \cdot \vec{R}} \Psi_{\vec{k}}(\vec{r})$$

For p.b.c.'s

$$\vec{R} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$

$$l=0, \dots L-1 \quad \text{or} \quad -\frac{L+1}{2}, \dots 0, \frac{L}{2}$$

Even

Block's Theorem

$$\Psi_{\vec{K}}(\vec{r}) = e^{i\vec{K} \cdot \vec{r}} u_{\vec{K}}(\vec{r})$$

where $u_{\vec{K}}(\vec{r} + \vec{R}) = u_{\vec{K}}(\vec{r})$

Eigenfns of Translation operators

$$T_R \Psi_{\vec{K}}(\vec{r}) = \Psi_{\vec{K}}(\vec{r} + \vec{R}) = e^{i\vec{K} \cdot \vec{R}} \Psi_{\vec{K}}(\vec{r})$$

For p.b.c.'s

$$\vec{R} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$

$$l = 0, \dots, L-1 \quad m = -\frac{L+1}{2}, \dots, 0, \dots, \frac{L-1}{2} \quad \text{even}$$

Bloch's Theorem

$$\Psi_{\vec{K}}(\vec{r}) = e^{i\vec{K} \cdot \vec{r}} u_{\vec{K}}(\vec{r})$$

$$\text{where } u_{\vec{K}}(\vec{r} + \vec{R}) = u_{\vec{K}}(\vec{r})$$

Eigenfunctions of Translation operators

$$T_R \Psi_{\vec{K}}(\vec{r}) = \Psi_{\vec{K}}(\vec{r} + \vec{R}) = e^{i\vec{K} \cdot \vec{R}} \Psi_{\vec{K}}(\vec{r})$$

For p.b.c.'s

$$\vec{R} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$

$$l=0, \dots L-1 \quad m=-\frac{L+1}{2}, \dots 0, \quad \begin{cases} n \\ \text{even} \end{cases}$$

Bloch's Theorem

$$\Psi_{\vec{K}}(\vec{r}) = e^{i\vec{K} \cdot \vec{r}} u_{\vec{K}}(\vec{r})$$

$$\text{where } u_{\vec{K}}(\vec{r} + \vec{R}) = u_{\vec{K}}(\vec{r})$$

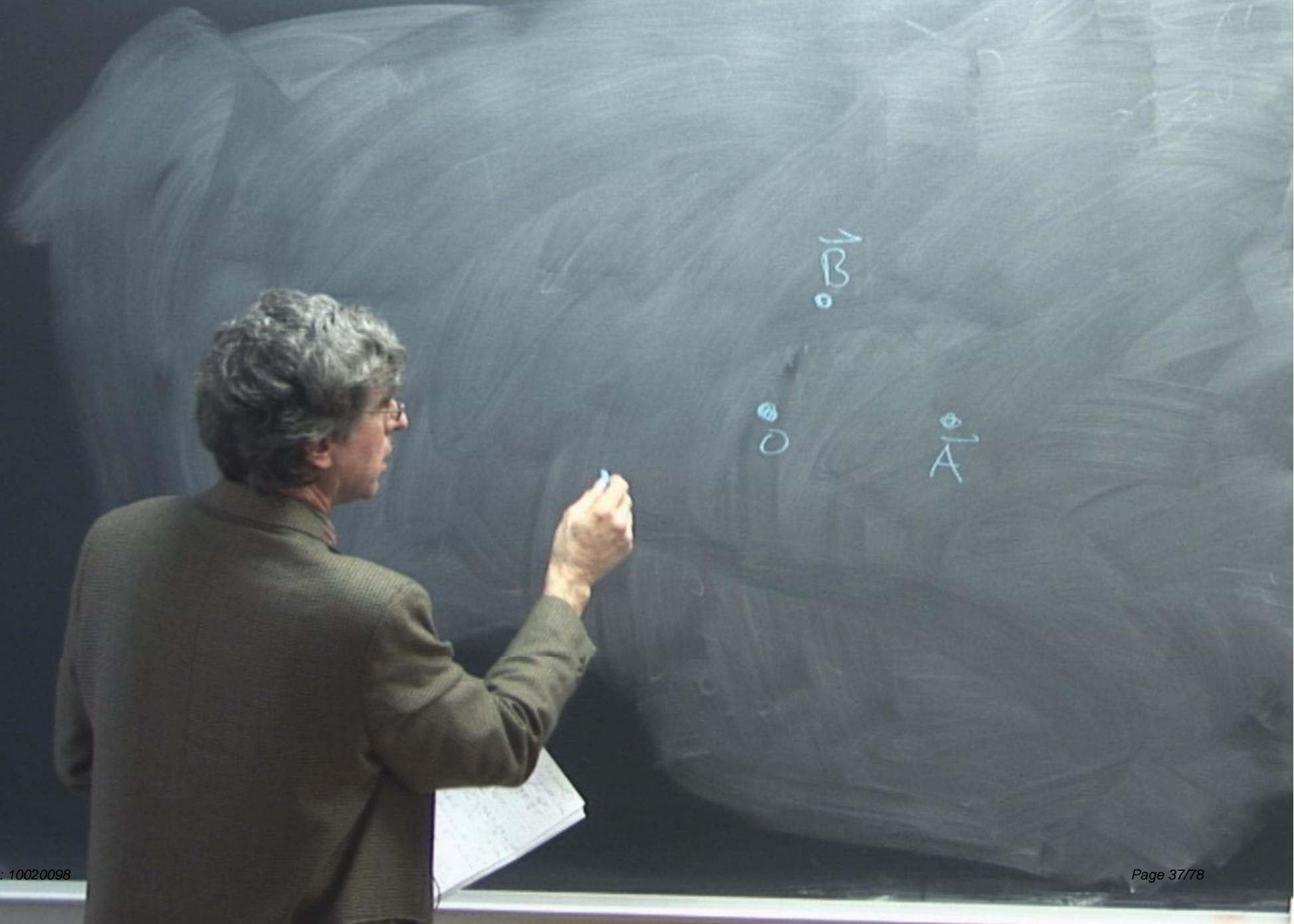
Eigenfunctions of Translation operators

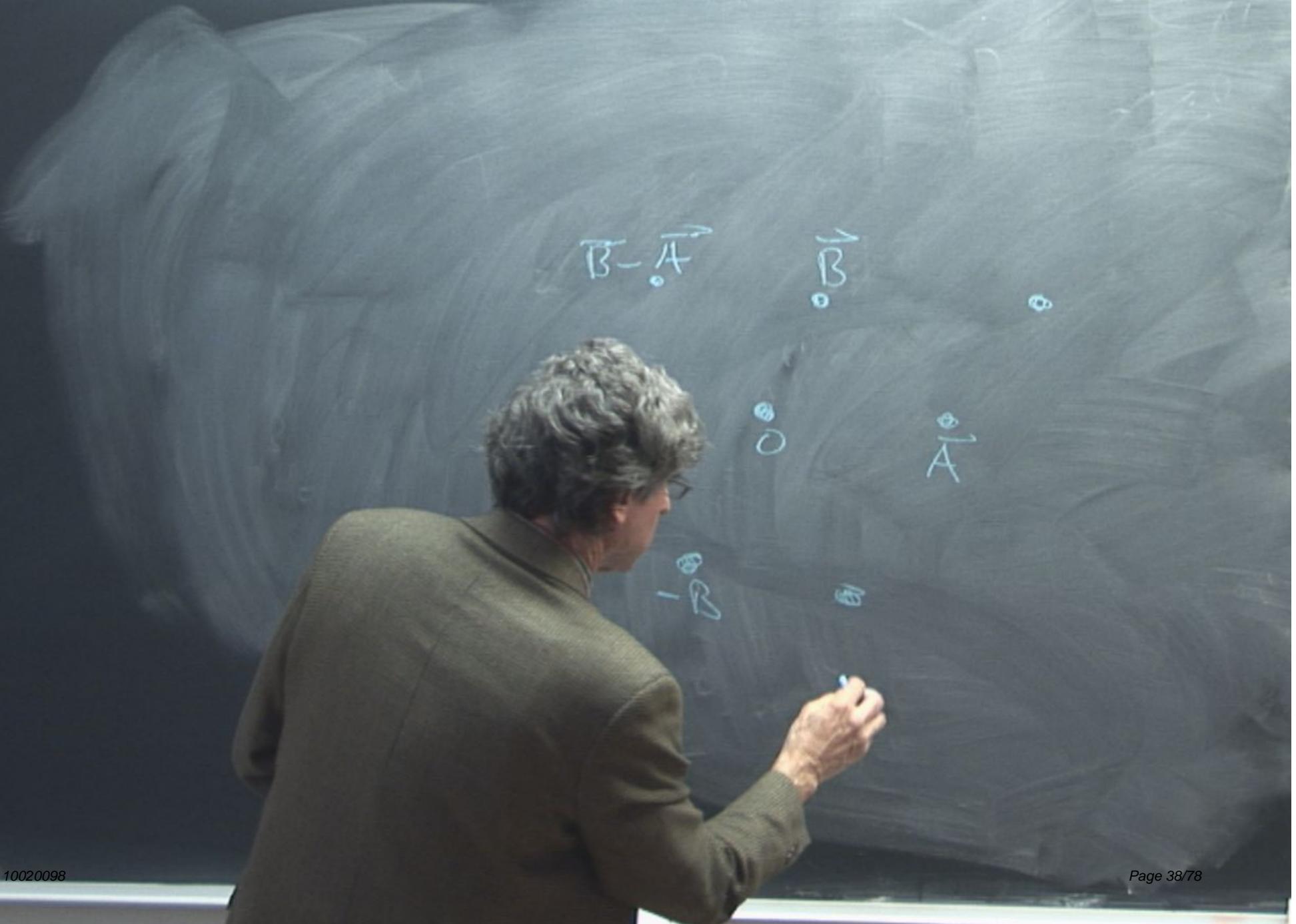
$$T_R \Psi_{\vec{K}}(\vec{r}) = \Psi_{\vec{K}}(\vec{r} + \vec{R}) = e^{i\vec{K} \cdot \vec{R}} \Psi_{\vec{K}}(\vec{r})$$

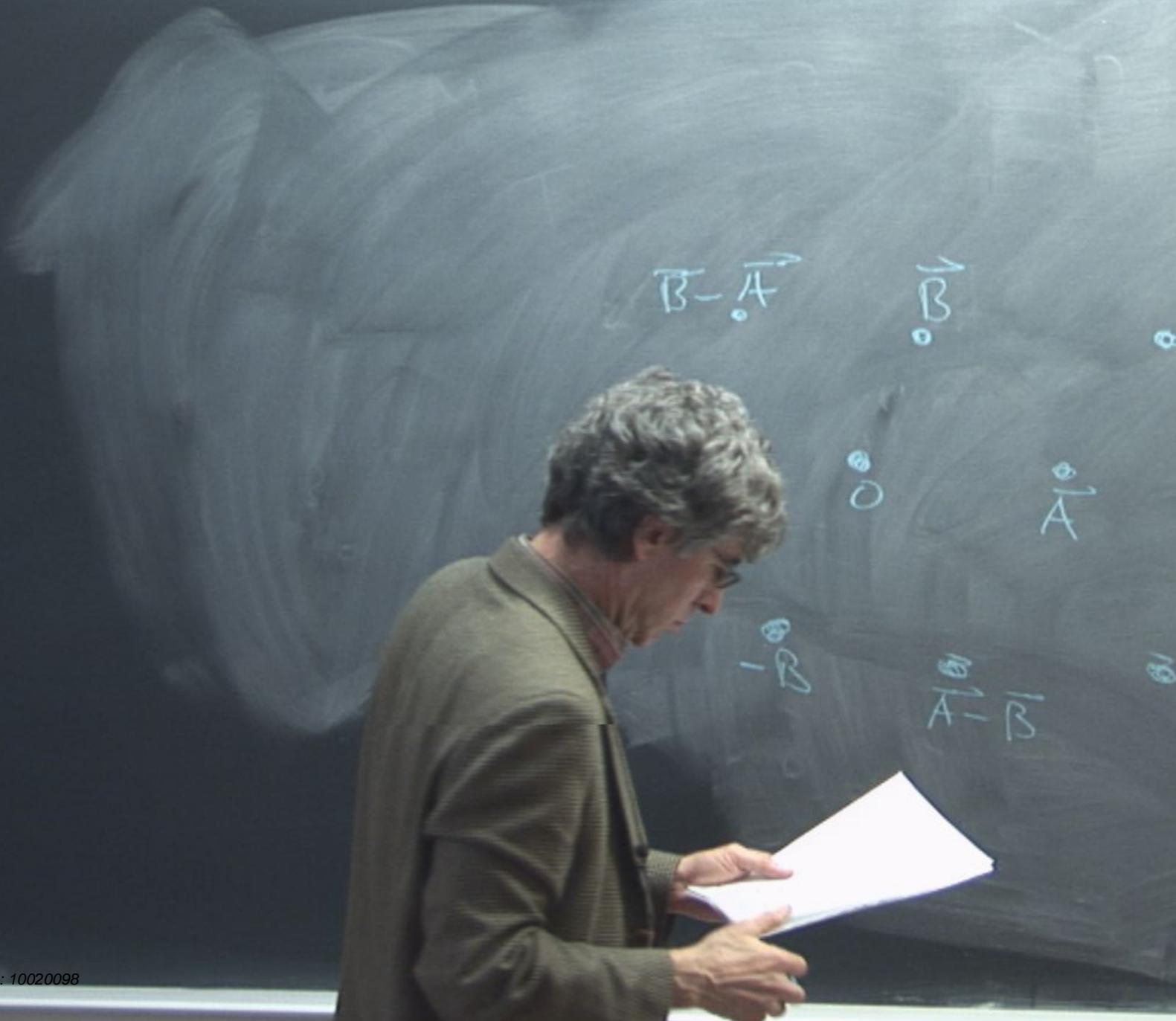
For p.b.c.'s

$$\vec{R} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$

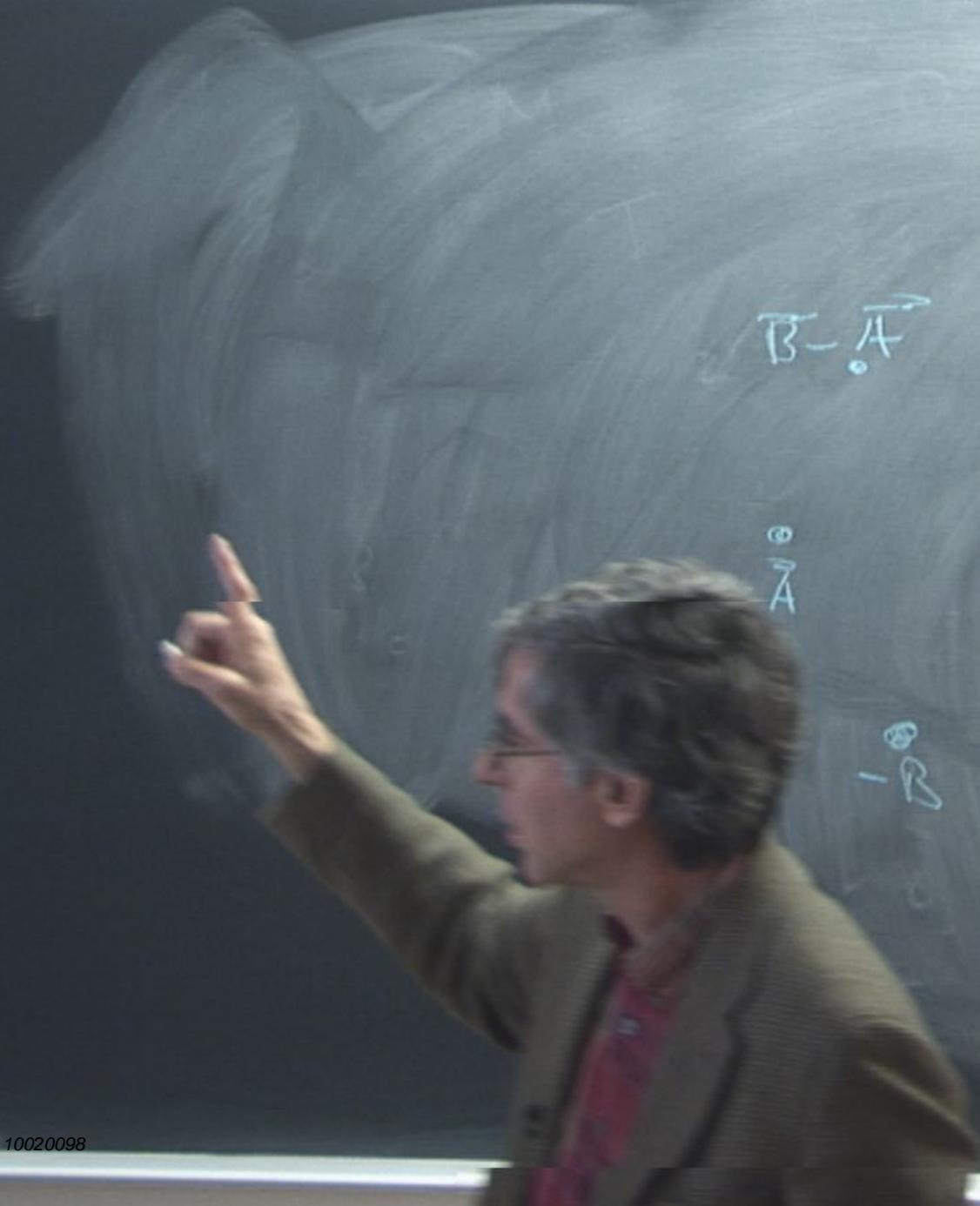
$$l=0, \dots L-1 \quad m=-\frac{L+1}{2}, \dots 0, \quad n \text{ even}$$







$\vec{B} - \vec{A}$ \vec{B} $-\vec{A}$ \vec{O} \vec{A} $-\vec{B}$ $\vec{A} - \vec{B}$



$$\overrightarrow{B-A}$$

$$\overrightarrow{\vec{A}}$$

$$-\overrightarrow{B}$$

$$\overrightarrow{B}$$

$$\overrightarrow{A-B}$$

$$\overrightarrow{\vec{A}}$$

Eigenfns of Translation operators

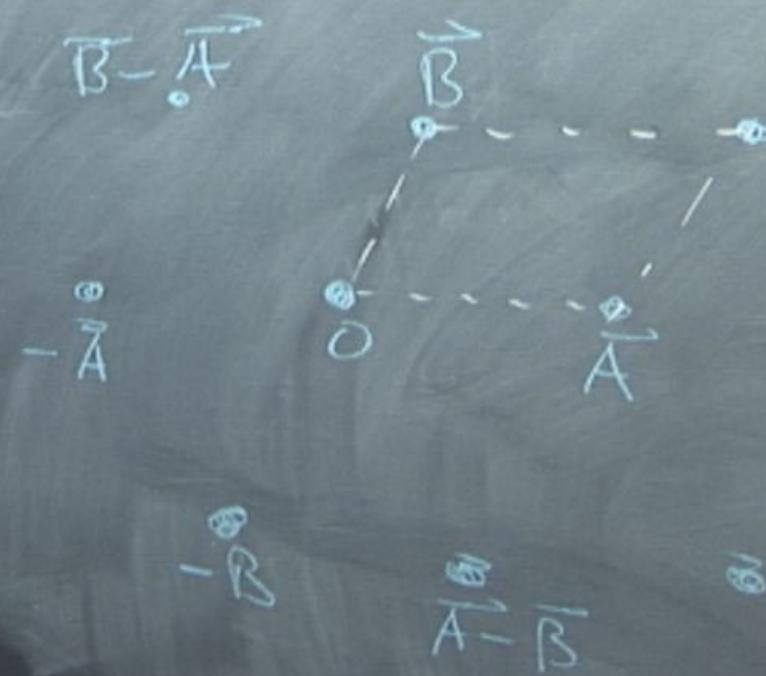
where $\vec{K} \cdot \vec{r} \Psi_{\vec{K}}(\vec{r})$

$$T_{\vec{R}} \Psi_{\vec{K}}(\vec{r}) = \Psi_{\vec{K}}(\vec{r} + \vec{R}) = e^{i\vec{K} \cdot \vec{R}} \Psi_{\vec{K}}(\vec{r})$$

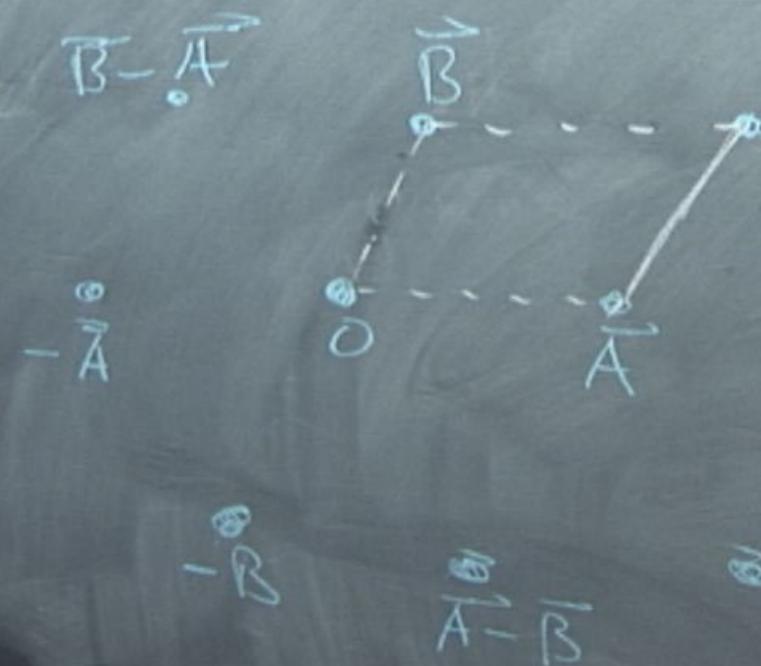
For p.b.c.'s

$$\Psi_{\vec{K}}(\vec{r} + \vec{R}) = \Psi_{\vec{K}}(\vec{r}) \quad \vec{K} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$
$$l = 0, \dots, L-1 \quad \text{or} \quad -\frac{L}{2}+1, \dots, 0, \frac{L}{2} \quad \text{L even}$$

Brillouin Zones



Brillouin Zones



Brillouin Zones

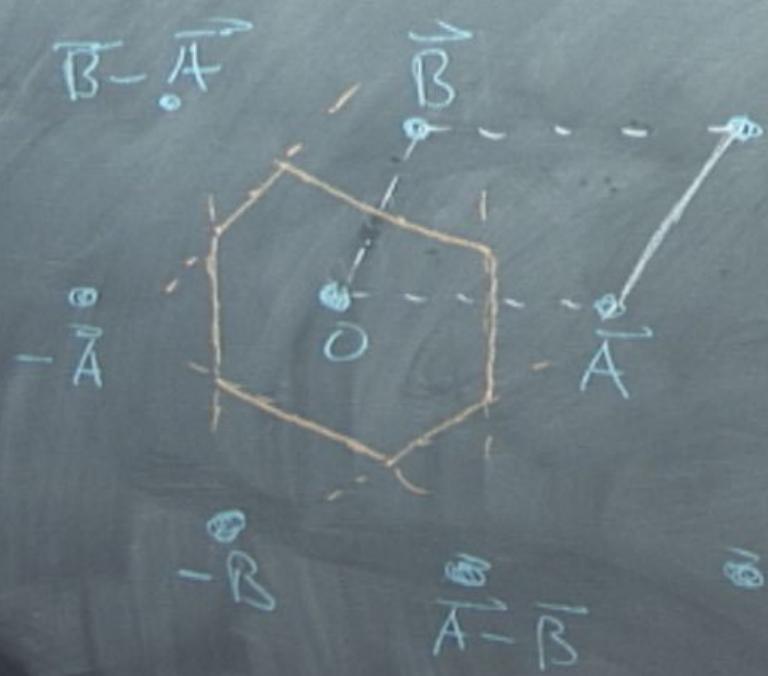
$\vec{B} - \vec{A}$



Wigner-Seitz
Cell

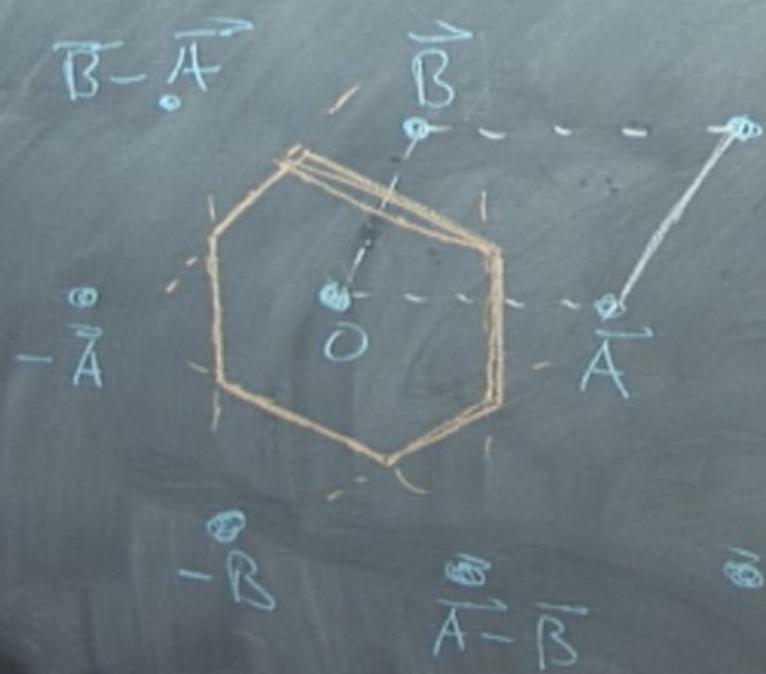
$\vec{A} - \vec{B}$

Brillouin Zones



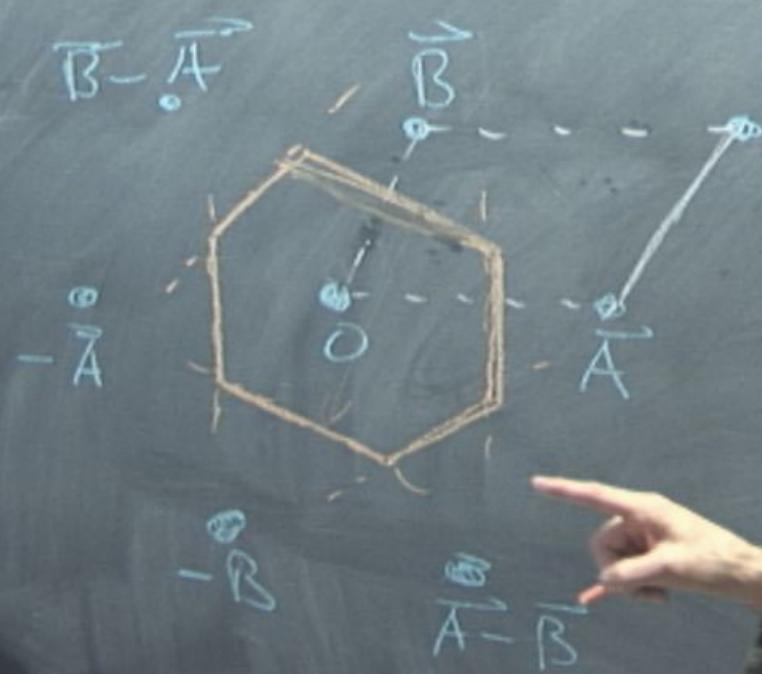
Wigner-Seitz
Cell

Brillouin Zones

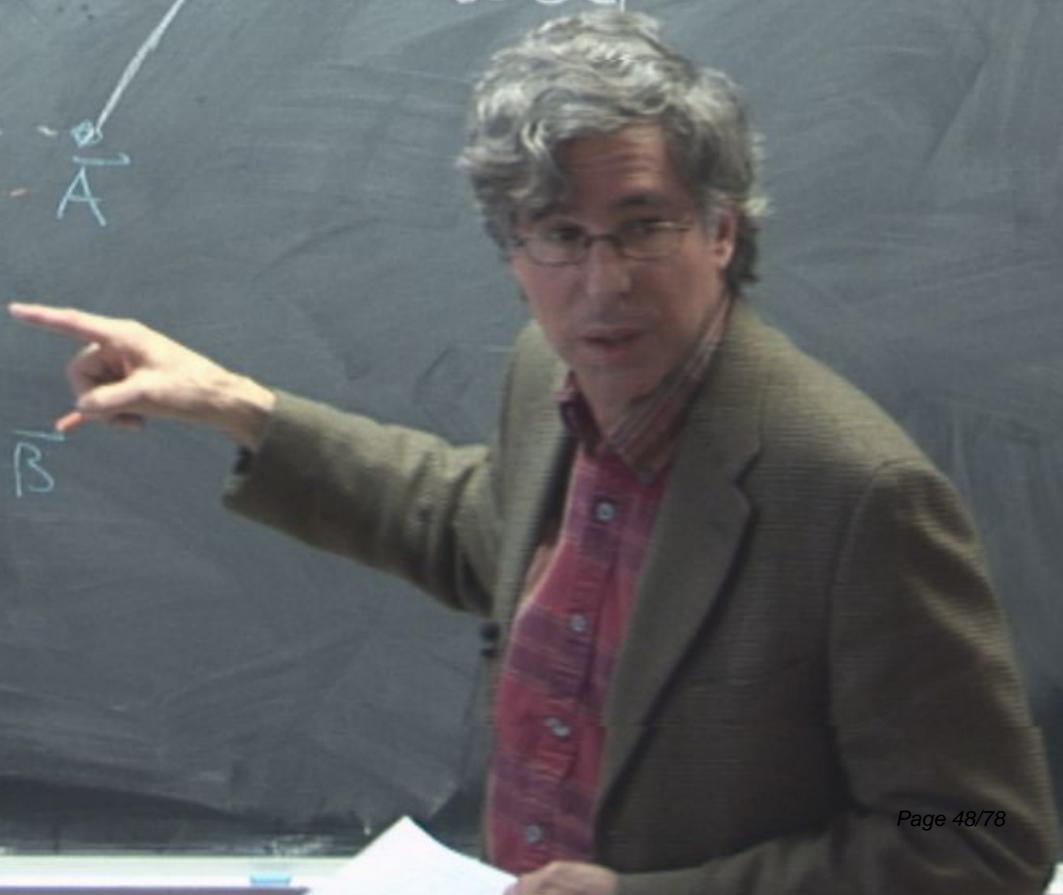


Wigner-Seitz
Cell

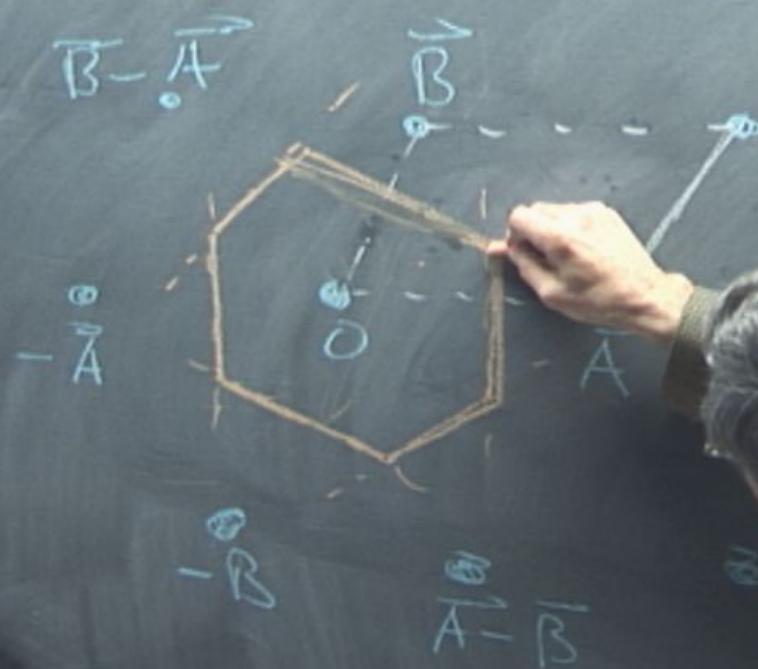
Brillouin Zones



Wigner-Seitz
Cell

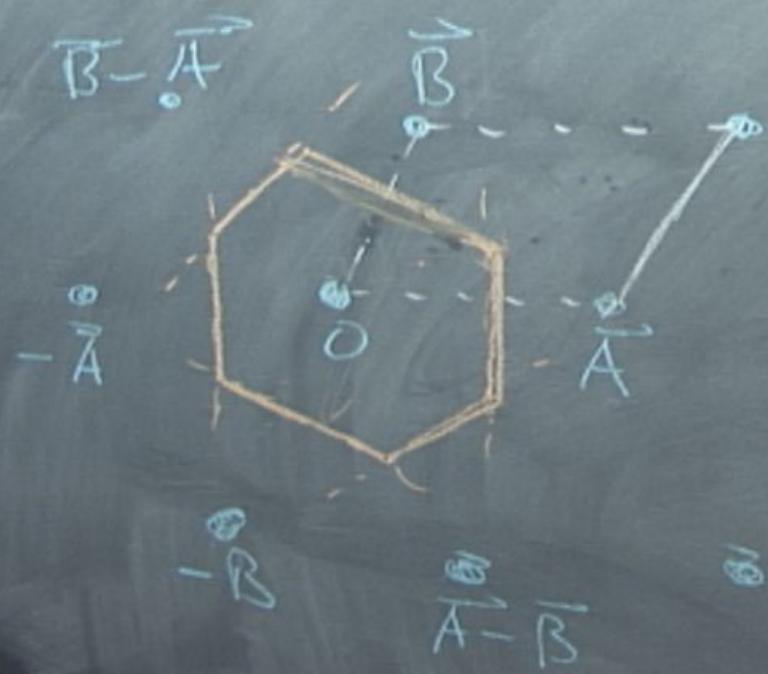


Brillouin Zones



Wigner-Seitz
Cell

Brillouin Zones



Wigner-Seitz
Cell

Group Theory
Tells us that if
 $[x_0, T_R]$ for $\{T_R\}$

Group Theory
Tells us that if
 $[H, T_R] \text{ for } \{T_R\}$
Then H and $\{T_R\}$ can
be simultaneously
diagonalized

Group Theory
Tells us that if

$$[\mathcal{H}, \{T_R\}] \text{ for } \{T_R\}$$

Then \mathcal{H} and $\{T_R\}$ can
be simultaneously
diagonalized

Eigenfns of Translation operators

$$\text{where } \vec{k} \cdot \vec{r} \quad \underbrace{\psi_{\vec{k}}(\vec{r})}_{\psi_{\vec{k}+\vec{R}}(\vec{r})} \quad T_{\vec{R}} \Psi_{\vec{k}}(\vec{r}) = \Psi_{\vec{k}}(\vec{r} + \vec{R}) = e^{i\vec{k} \cdot \vec{R}} \psi_{\vec{k}}(\vec{r})$$

$$U_{\vec{k}}(\vec{r} + \vec{R}) = U_{\vec{k}}(\vec{r}) \quad \vec{k} = \frac{l}{L} \vec{A} + \frac{m}{L} \vec{B} + \frac{n}{L} \vec{C}$$

$$l=0, \dots L-1 \quad \text{or} \quad -\frac{L}{2}+1, \dots 0, \frac{L}{2}$$

Group Theory
Tells us that if

$$[\mathcal{H}, \{T_R\}] \text{ for } \{T_R\}$$

Then \mathcal{H} and $\{T_R\}$ can
be simultaneously
diagonalized

Energy Bands



Energy Bands
No lattice potential →

Energy Bands
No lattice potential $\rightarrow E(k)$

Energy Bands

No lattice potential $\rightarrow E(k) =$

$$\frac{\hbar^2 k^2}{2m}$$

Energy Bands

$$\text{No lattice potential} \rightarrow E(K) = \frac{\hbar^2 K^2}{2m}$$

Effect of Periodic Potential

$$V(\vec{r}) = \sum_Q V_Q e^{i\vec{Q} \cdot \vec{r}}$$

Q summed over RLV's

Energy Bands

$$\text{No lattice potential} \rightarrow E(k) = \frac{\hbar^2 k^2}{2m}$$

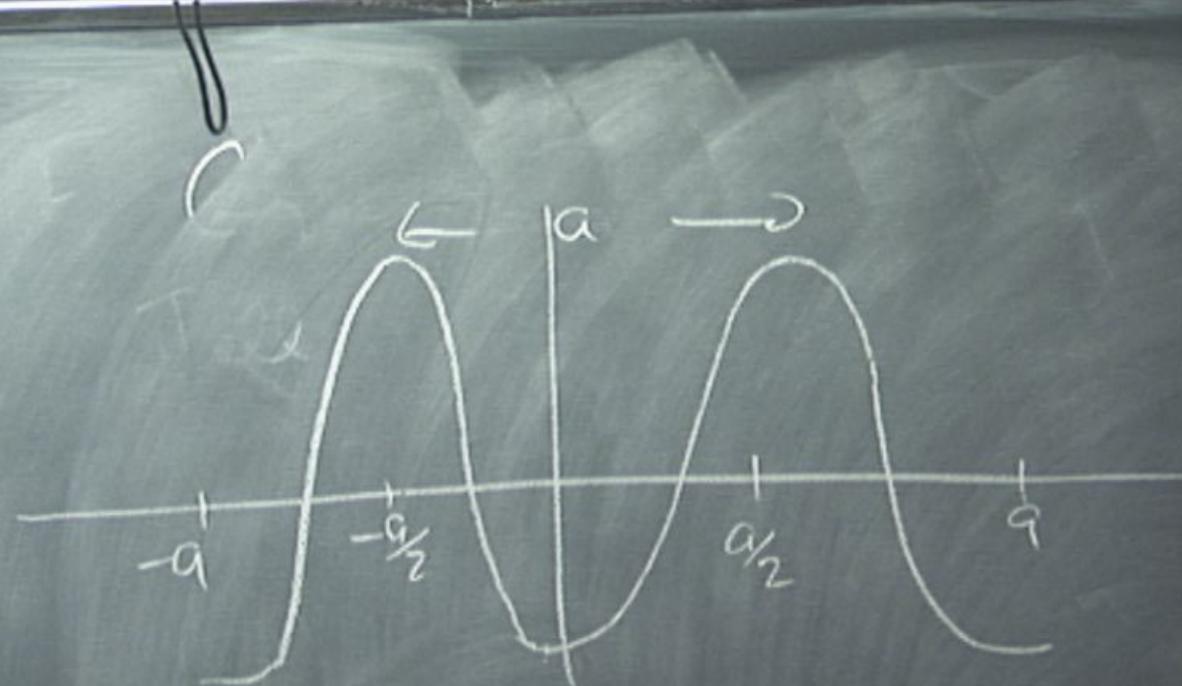
Effect of Periodic Potential

$$V(\vec{r}) = \sum_Q V_Q e^{i\vec{Q} \cdot \vec{r}}$$

Q summed over RLUs

Consider one $\vec{Q}, -\vec{Q}$ pair

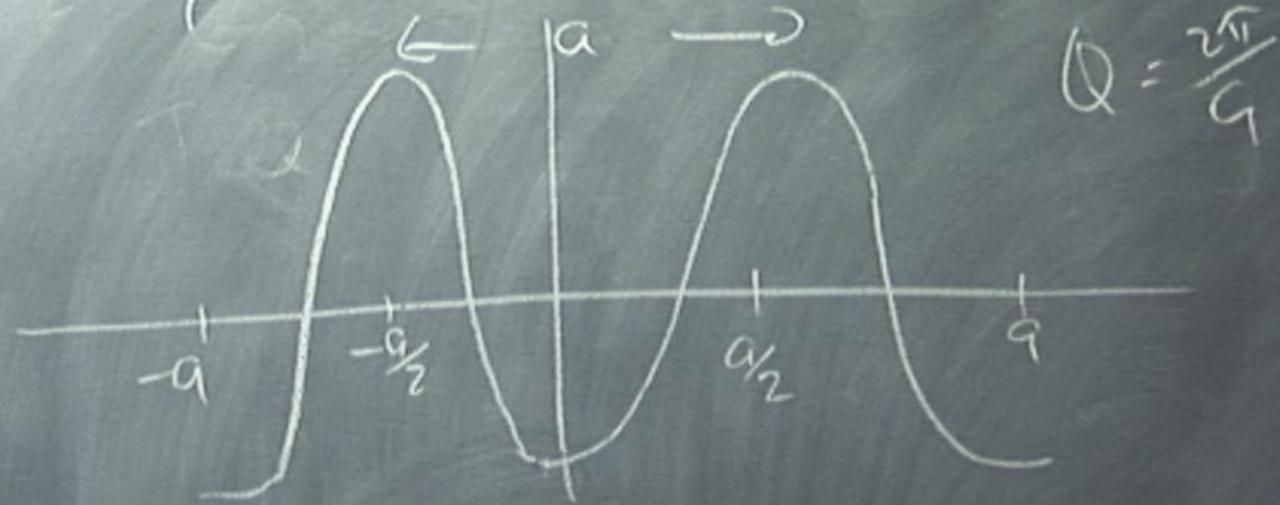
$$V(\vec{r}) = V_Q e^{iQx} + V_{-\vec{Q}} e^{-iQx} = 2|V_Q| \cos(Qx + Q)$$



mixed
RLV's

$$d(Qx + \ell)$$

$\zeta t = \tau_1$



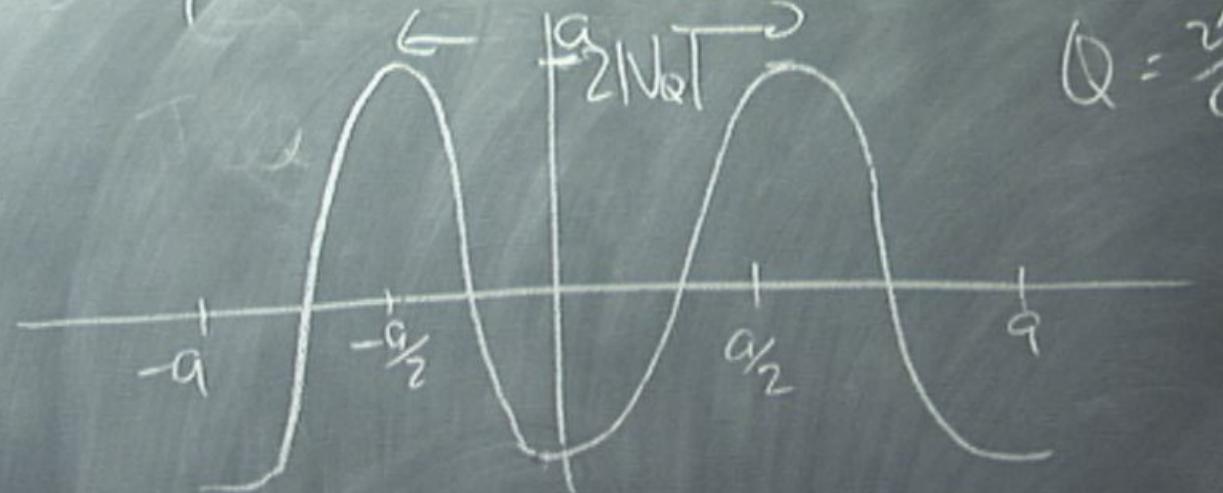
$$Q = \frac{2\pi}{a}$$

mixed
RLV's

$$\text{let } t = \pi$$

$$d(Qx + Q)$$

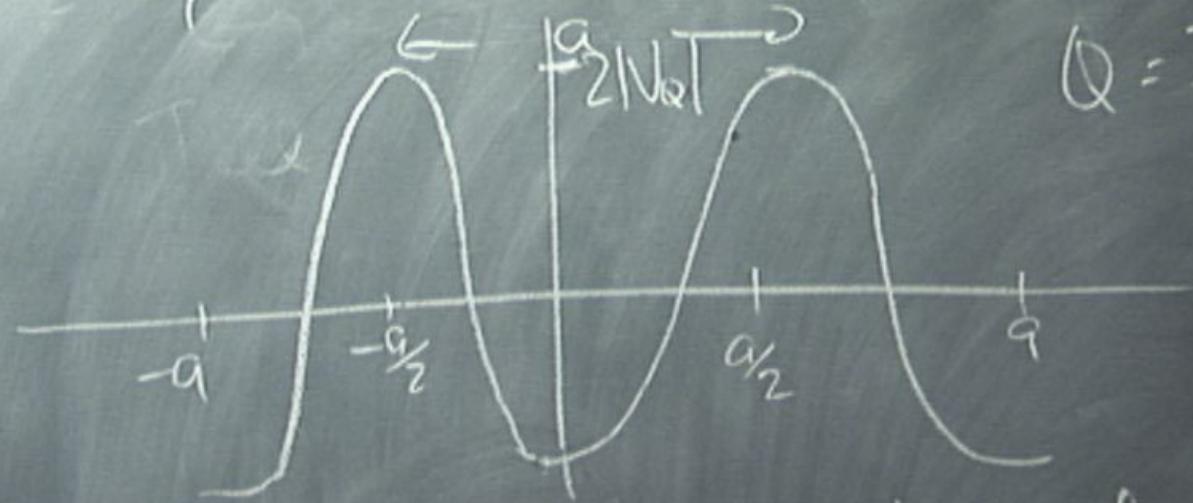
$$Q = \frac{2\pi}{G}$$



mixed
RLV's

$$\Delta(Qx + Q) \quad \text{let } t = \pi$$

$$Q = \frac{2\pi}{\omega}$$



Consider effect of V_{un} pert. theory

mixed
RLV's

$$\sin(Qx + \phi) \quad \text{let } \phi = \pi$$

Energy Bands

$$\text{No lattice potential} \rightarrow E(k) = \frac{\hbar^2 k^2}{2m}$$

Effect of Periodic Potential

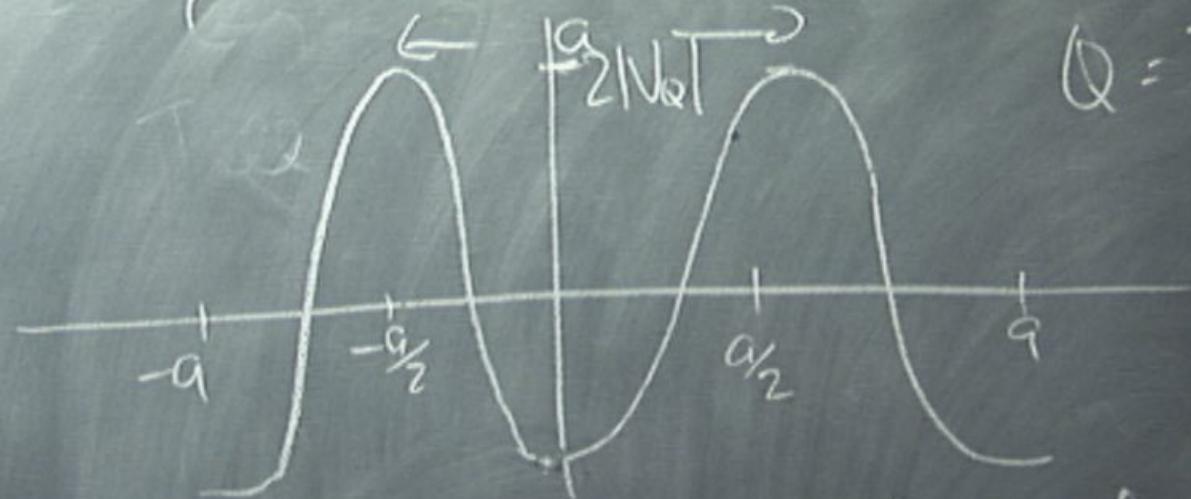
$$V(\vec{r}) = \sum_{\vec{Q}} V_{\vec{Q}} e^{i \vec{Q} \cdot \vec{r}}$$

\vec{Q} summed
over RLV's

Consider only $\vec{Q}, -\vec{Q}$ pairs

$$V(\vec{r}) = V_Q e^{i Q x} + V_{-\vec{Q}} e^{-i Q x} = 2 |V_Q| \cos(Qx + \phi)$$

$$Q = \frac{2\pi}{G}$$

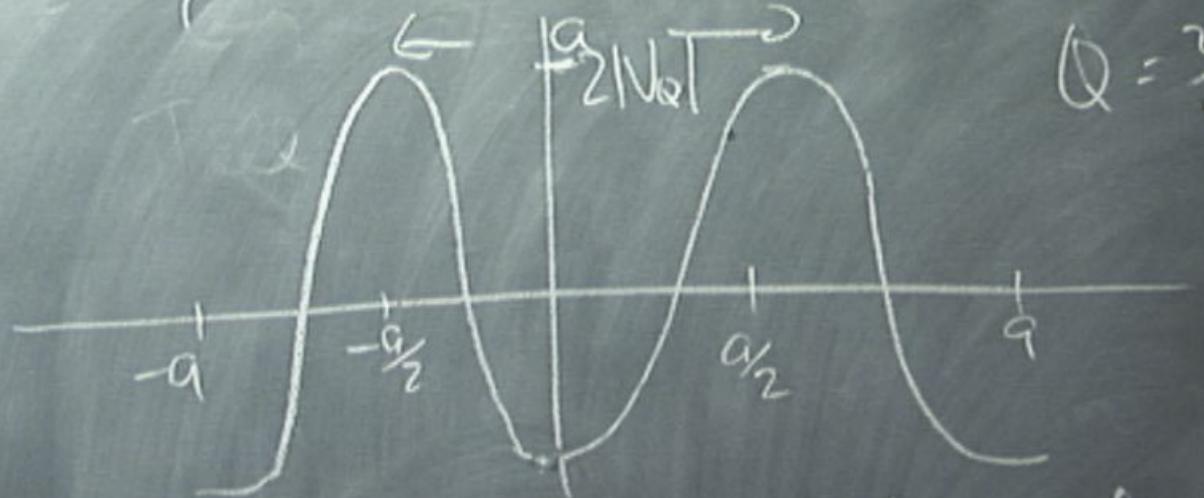


Consider effect of V_{ini} pert. then

mixed
RLV's

$$\Delta(QX + Q) \quad \text{Let } t = \pi$$

$$Q = \frac{2\pi}{G}$$



Consider effect of V in pert. theory

$$\frac{|V_Q|}{|\mathcal{E}(K \pm Q) - \mathcal{E}(K)|}$$

mixed
RLV's

$$\langle \sin(Qx + Q) \rangle$$

$$\frac{k^2}{m}$$

mixed
RLV's

$$\langle Qx + Q \rangle$$

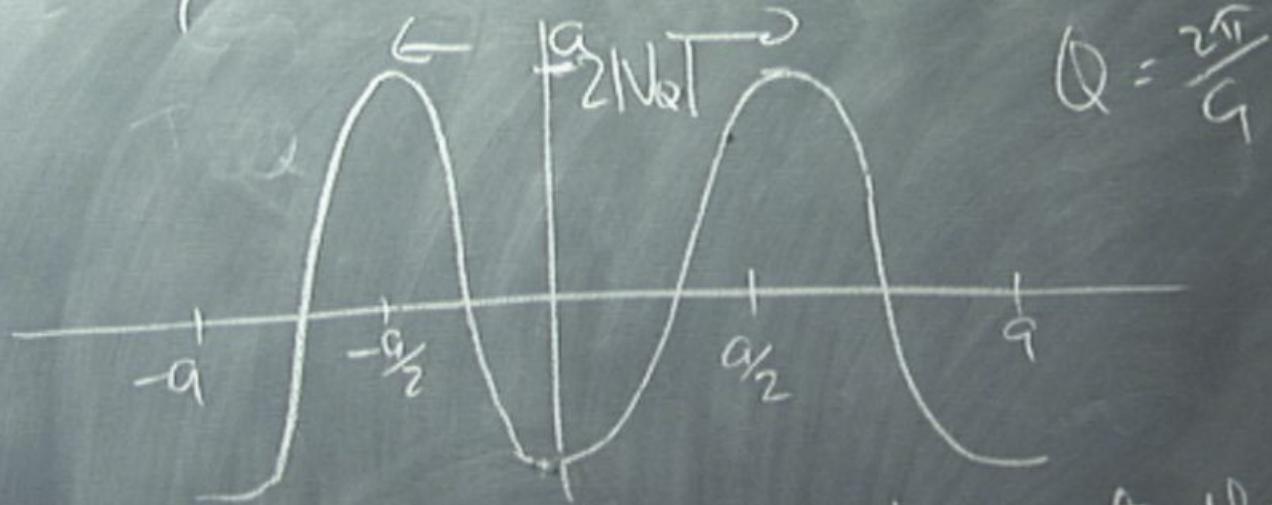
$\omega = \pi$



$$Q = \frac{\sqrt{n}}{a}$$

Consider effect of V in pert. theory

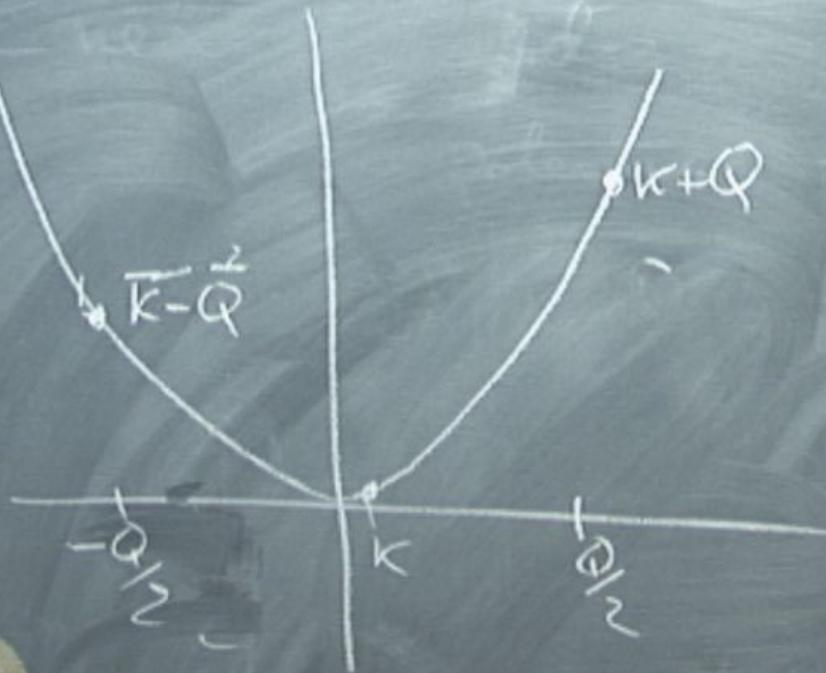
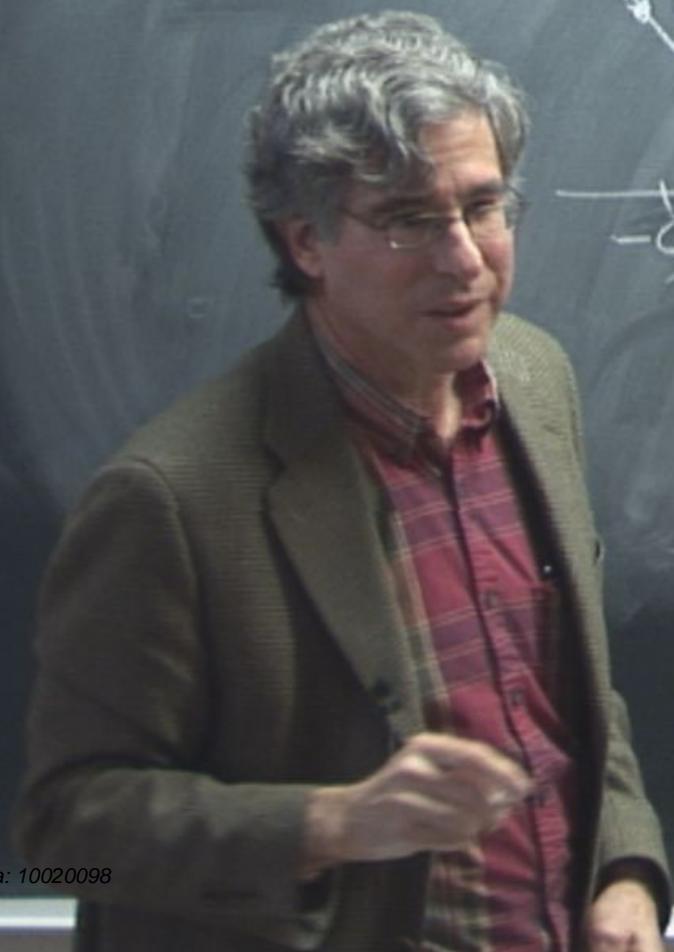
$$\frac{|V_Q|}{|\mathcal{E}(K \pm Q) - \mathcal{E}(K)|} \quad \text{small for } |K| \ll Q$$

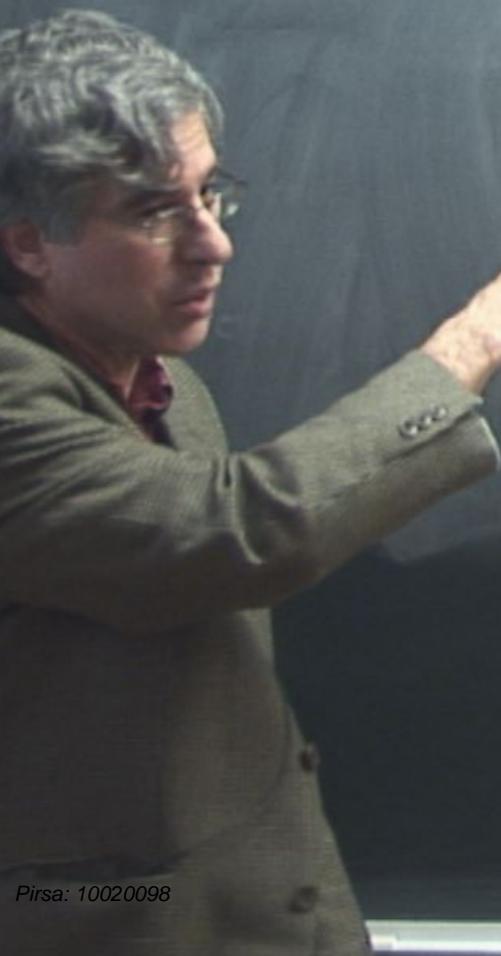
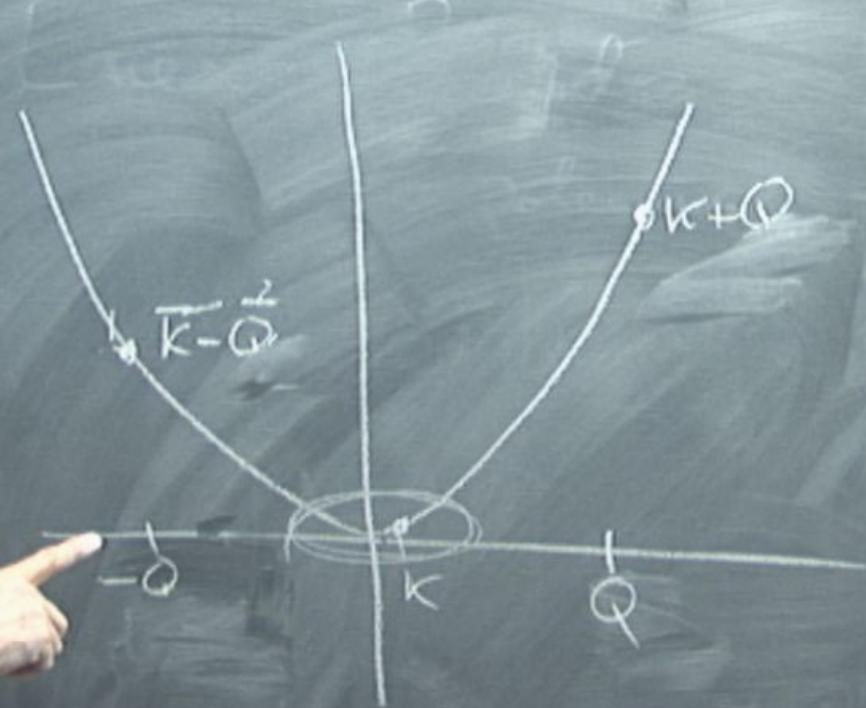


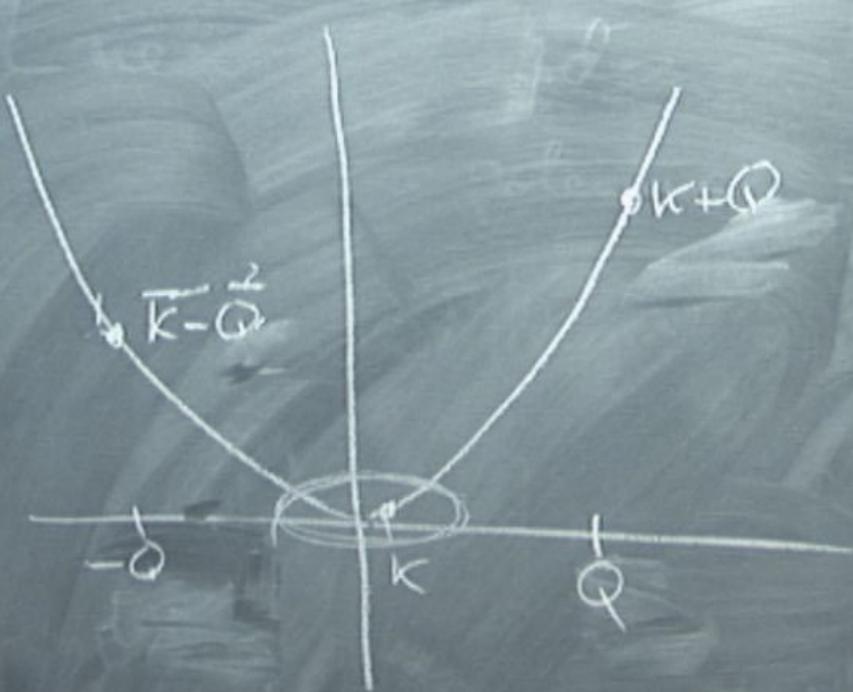
Consider effect of V in pert. theory

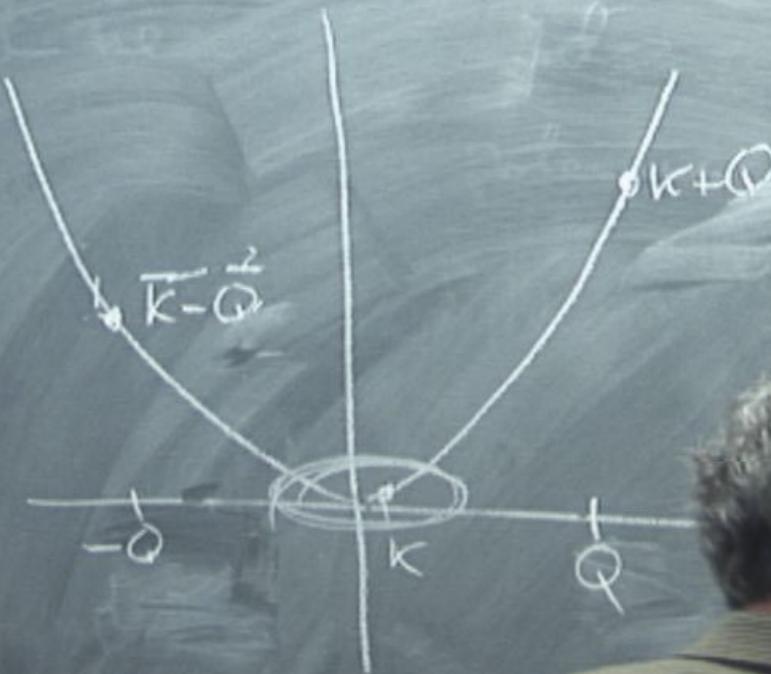
$$\frac{|V_Q|}{|\mathcal{E}(k \pm Q) - \mathcal{E}(k)|} \quad \text{small for } |k| \ll Q$$

if $\frac{|V_Q|}{k^2 Q^2} \ll 1$
 $\frac{1}{2m}$

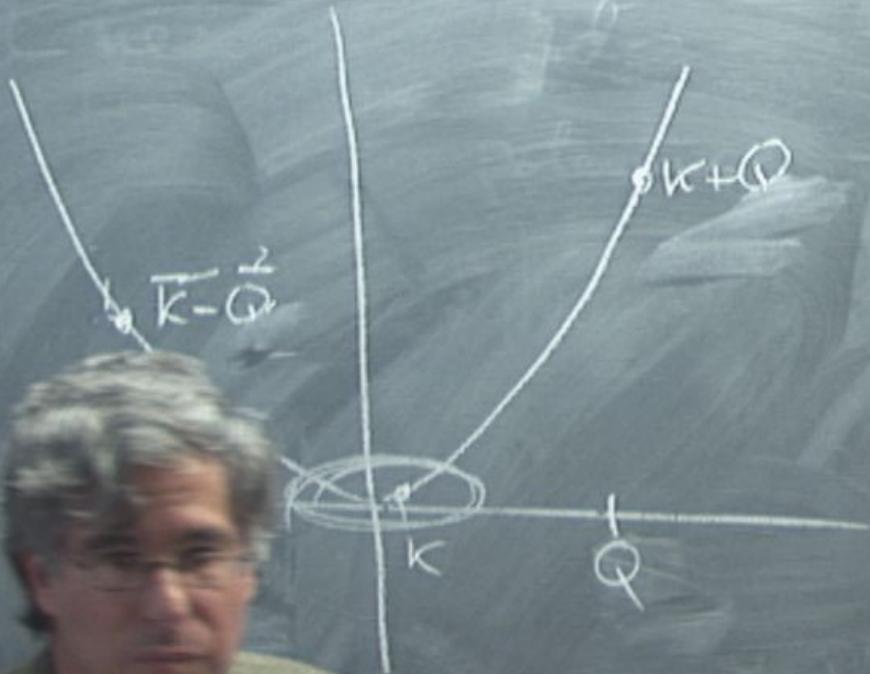




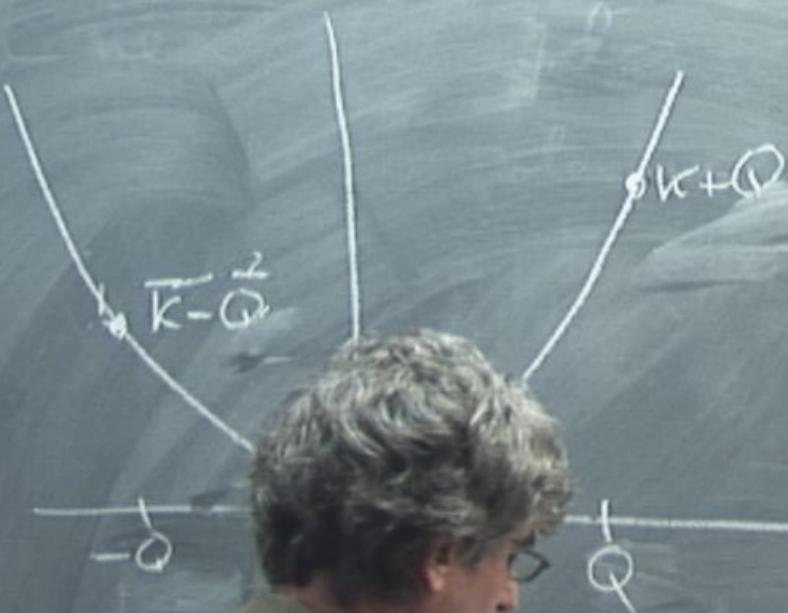




$$\Delta E(k) = -|V_Q|^2$$



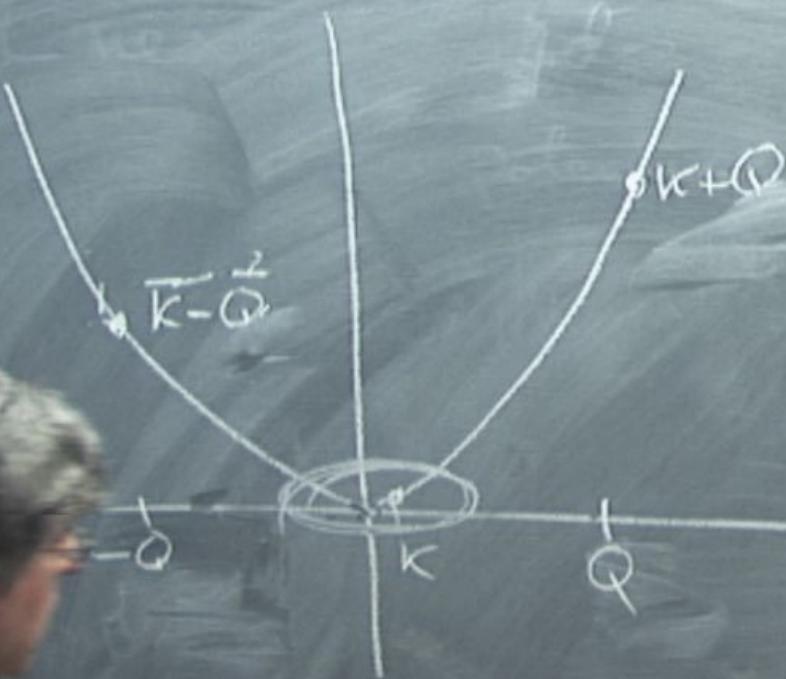
$$\Delta E(k) = -|V_Q|^2 \left\{ \frac{1}{E(k+Q) - E(k)} + \frac{1}{E(k-Q) - E(k)} \right\}$$



$$\Delta E(k) = -|V_Q|^2 \left\{ \frac{1}{E(k+Q) - E(k)} + \frac{1}{E(k-Q) - E(k)} \right\}$$

2 Effects

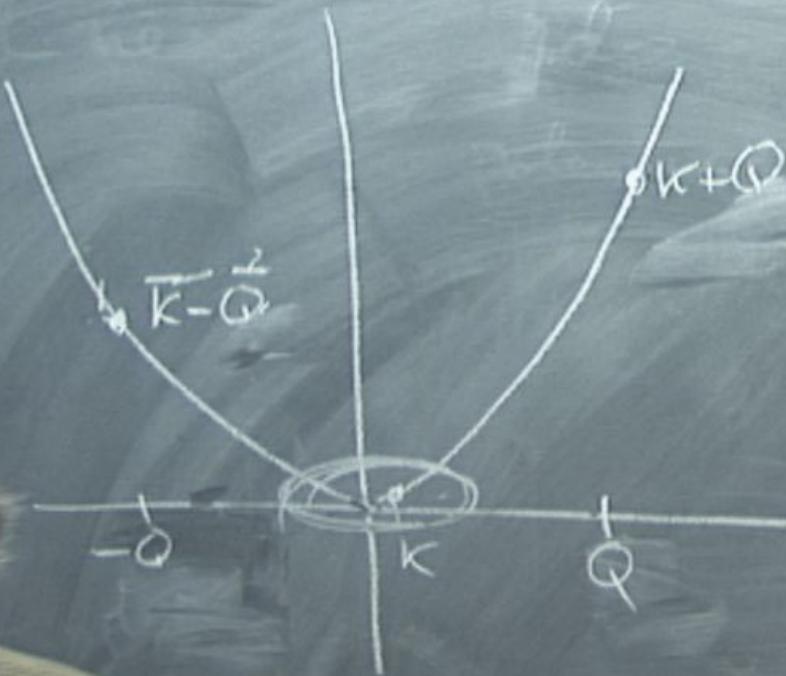
$$\tilde{E}(k) =$$



$$\Delta E(k) = -|V_Q|^2 \left\{ \frac{1}{E(k+Q) - E(k)} + \frac{1}{E(k-Q) - E(k)} \right\}$$

2 Effects

$$\tilde{E}(k) = \Delta E(0) + \frac{\hbar^2 k^2}{2m^*}$$



$$\Delta E(k) = -|V_Q|^2 \left\{ \frac{1}{E(k+Q) - E(k)} + \frac{1}{E(k-Q) - E(k)} \right\}$$

2 Effects

$$E(k) = \Delta E(0) + \frac{\hbar^2 k^2}{2m^*}$$

Shift ↑

increased $m^* > m$