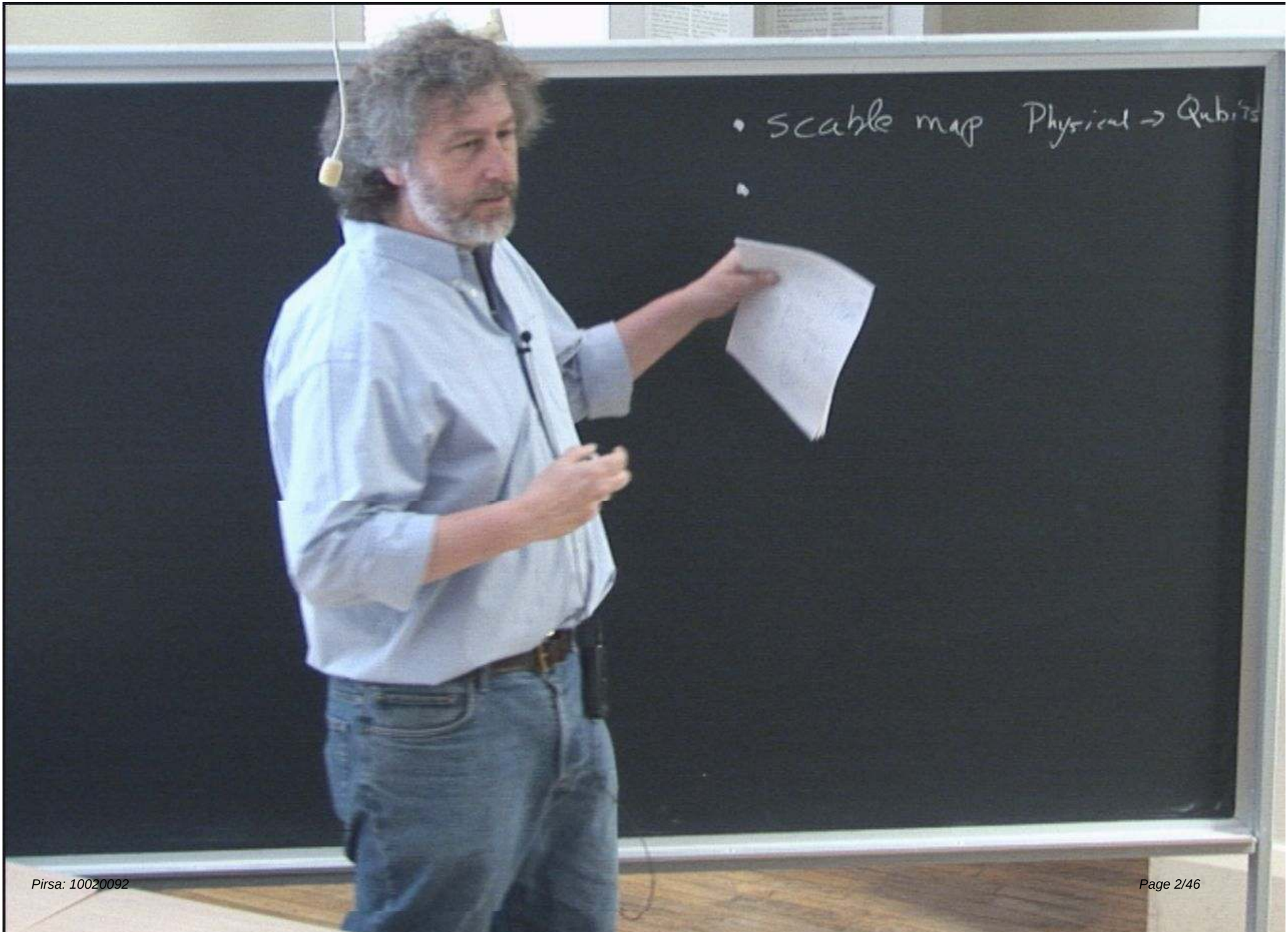
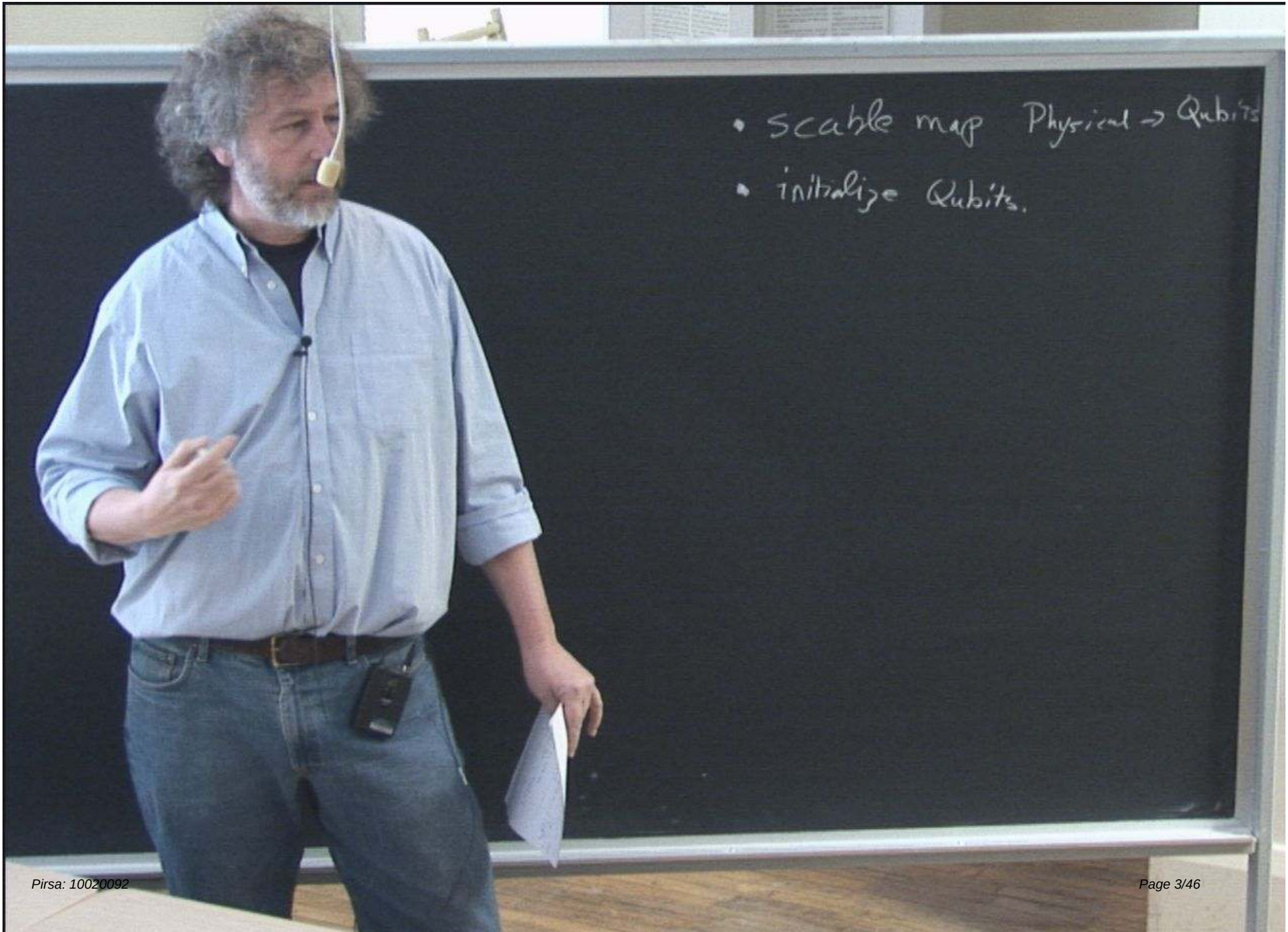
Title: Explorations in Quantum Info. (PHYS 641) - Lecture 9

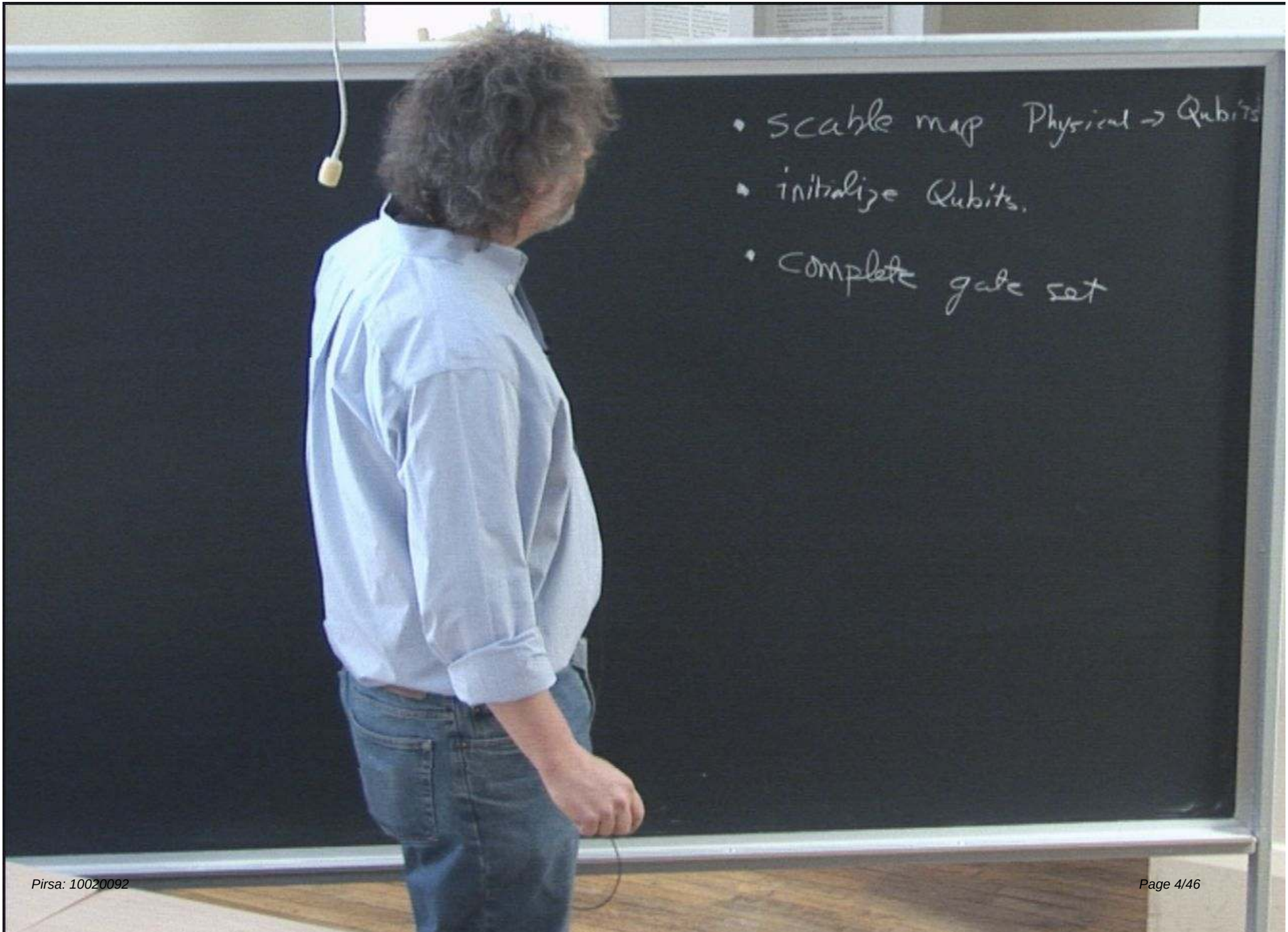
Date: Feb 26, 2010 09:00 AM

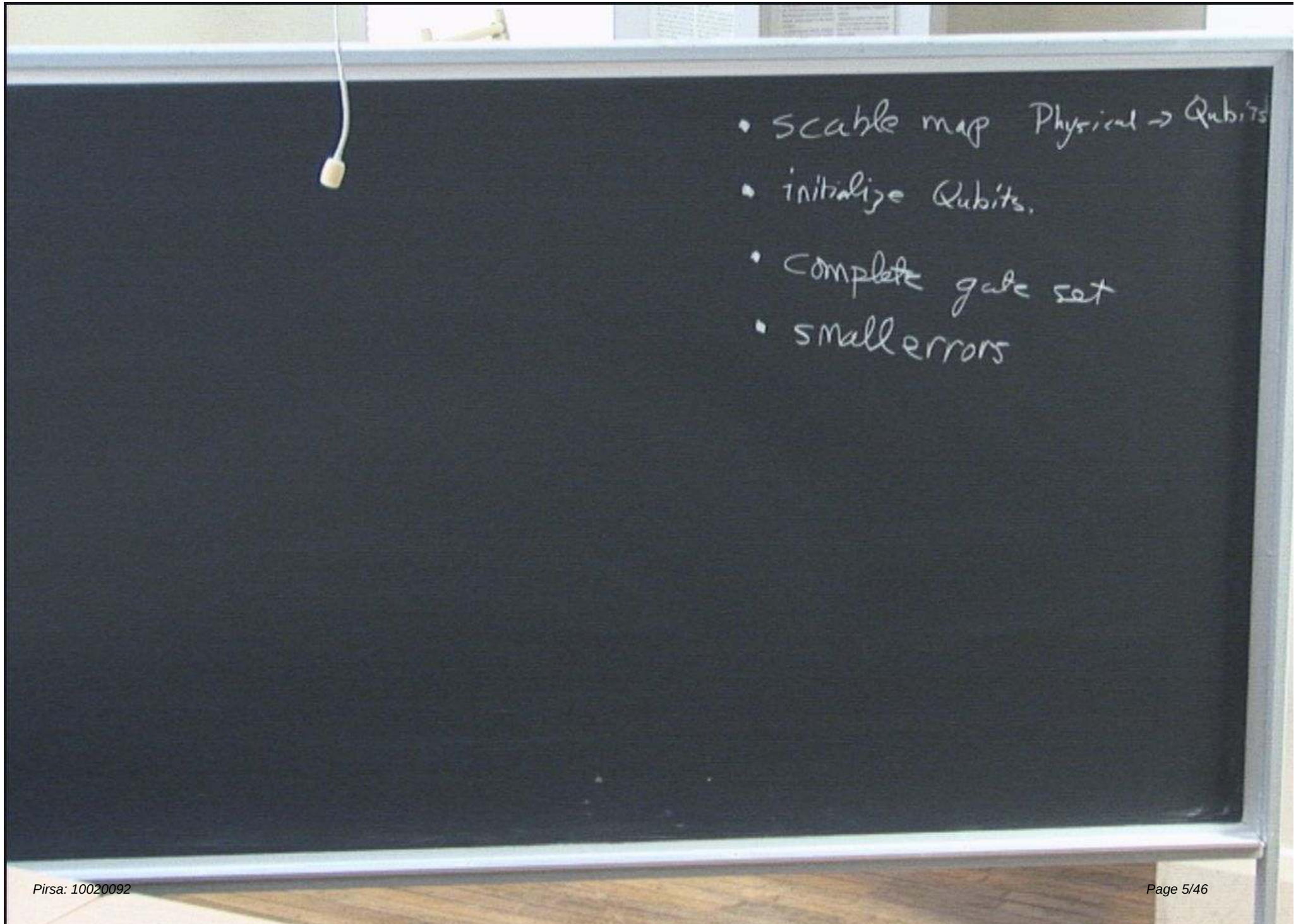
URL: http://pirsa.org/10020092

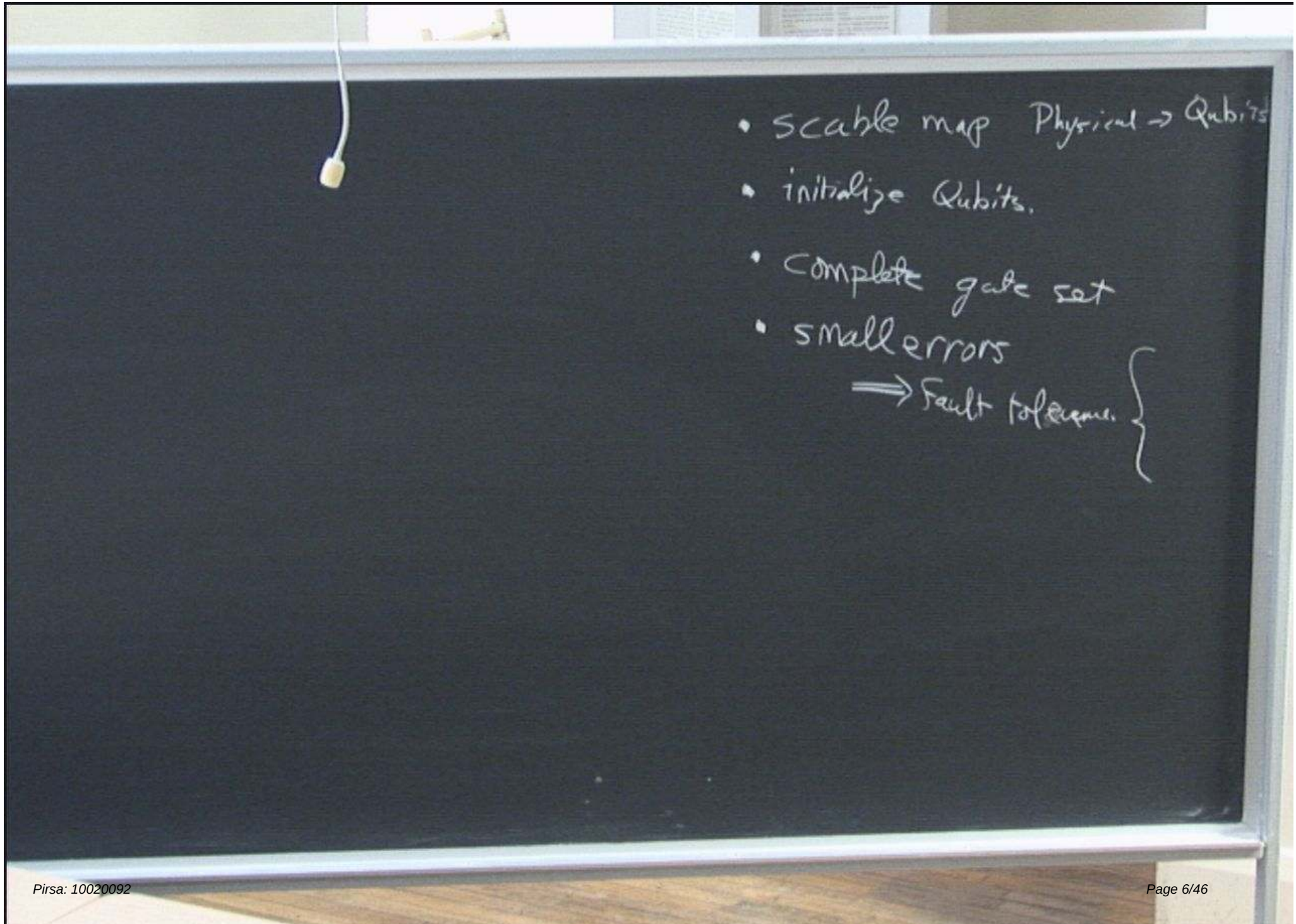
Abstract:


• scable map Physical → Qubits


• scable map Physical → Qubits
• initialize Qubits.


• scable map Physical → Qubits
• initialize Qubits.
• complete gate set


• scable map Physical → Qubits
• initialize Qubits.
• complete gate set
• small errors


• scable map Physical → Qubits
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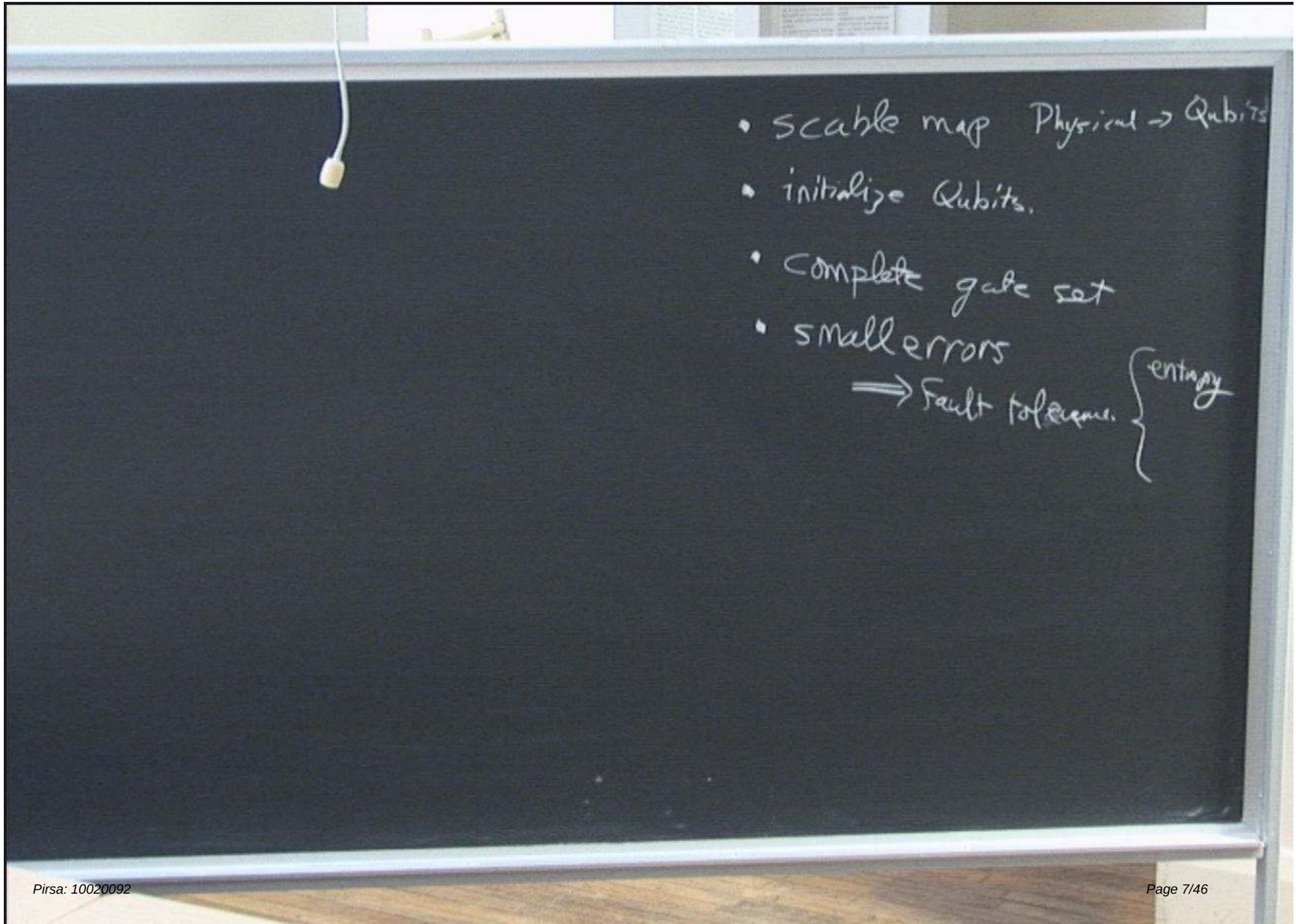
⟹ Fault tolerance.


• scable map Physical → Qubits
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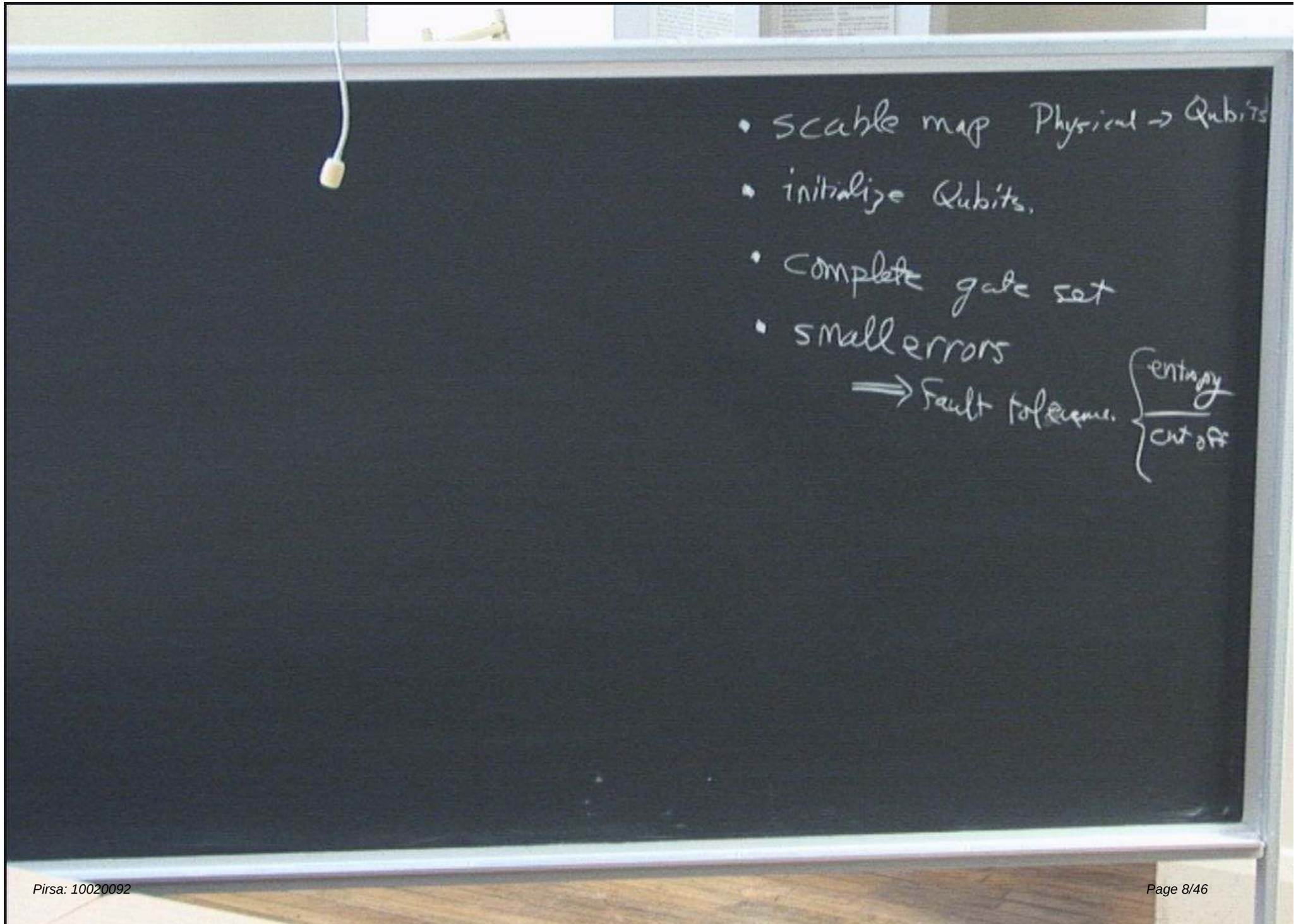
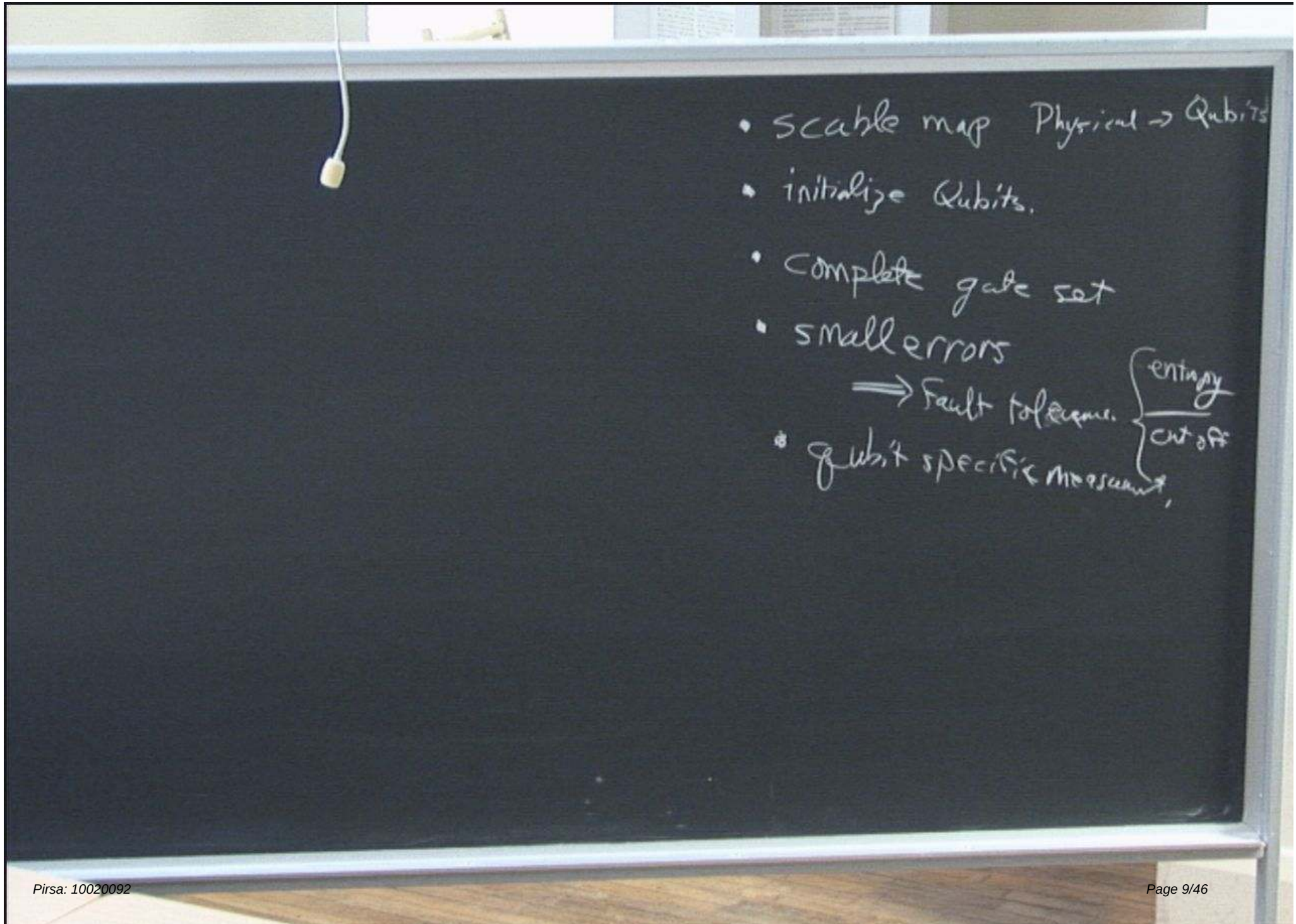
entropy


• scable map Physical → Qubits
• initialize Qubits.
• complete gate set
• small errors
⟹ Fault tolerance. { entropy / cut off


• scable map Physical → Qubits
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entropy
cut off
• Qubit specific measurement

Nuclear Magnetic Resonance

Nuclear Magnetic Resonance

$\mathcal{H}_{Zeeman} = \frac{\gamma B_0}{2} \sigma_z$ — static magnetic field $10^4 - 10^5$ G

gyromagnetic ratio, 4,250 $\frac{Hz}{G}$: ^1H

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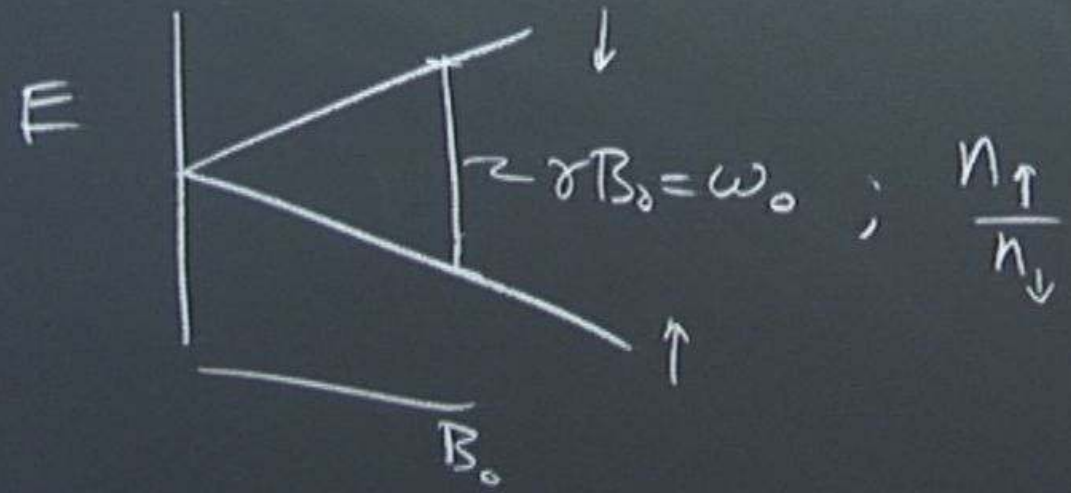

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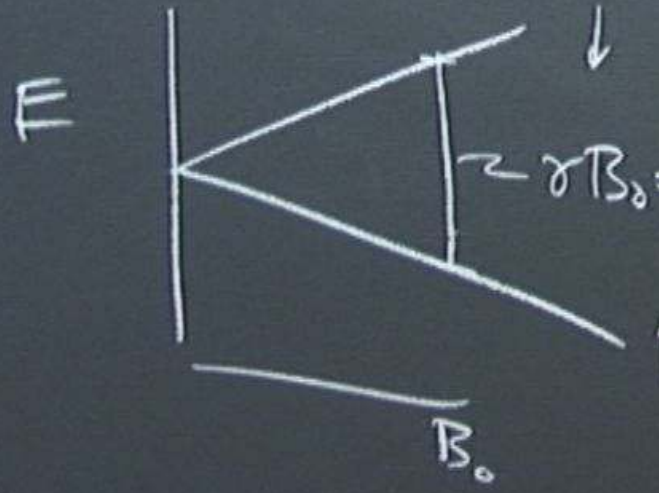

$\frac{n_\uparrow}{n_\downarrow} = e^{-E/kT}$

$\frac{\hbar\omega_0}{kT} = 10^{-5}$

Nuclear Magnetic Resonance

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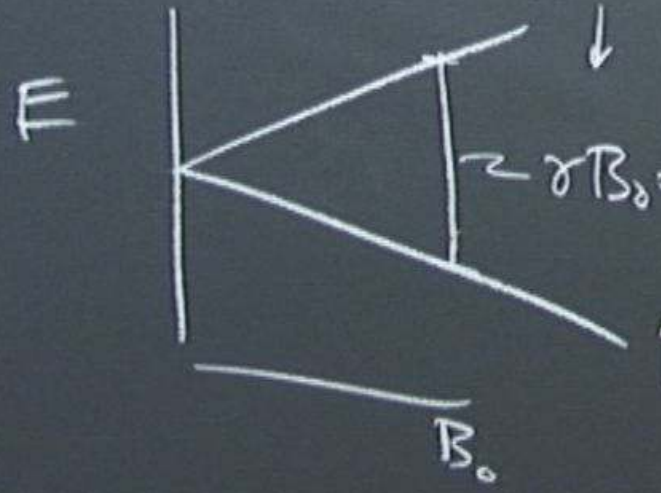

$\frac{n_\uparrow}{n_\downarrow} = e^{-E/kT}$

$\frac{\hbar\omega_0}{kT} = 10^{-5}$

$\rho_z = \mathbb{1} - \frac{\hbar\omega_0}{kT}\sigma_z$

Nuclear Magnetic Resonance

$\mathcal{H}_{Zeeman} = \frac{\gamma B_0}{2}\sigma_z$ — static magnetic field, $\hat{z}$, $10^4 - 10^5$ G

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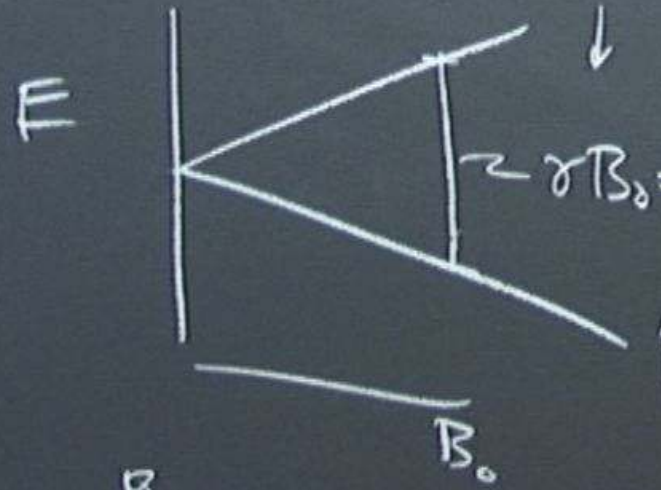
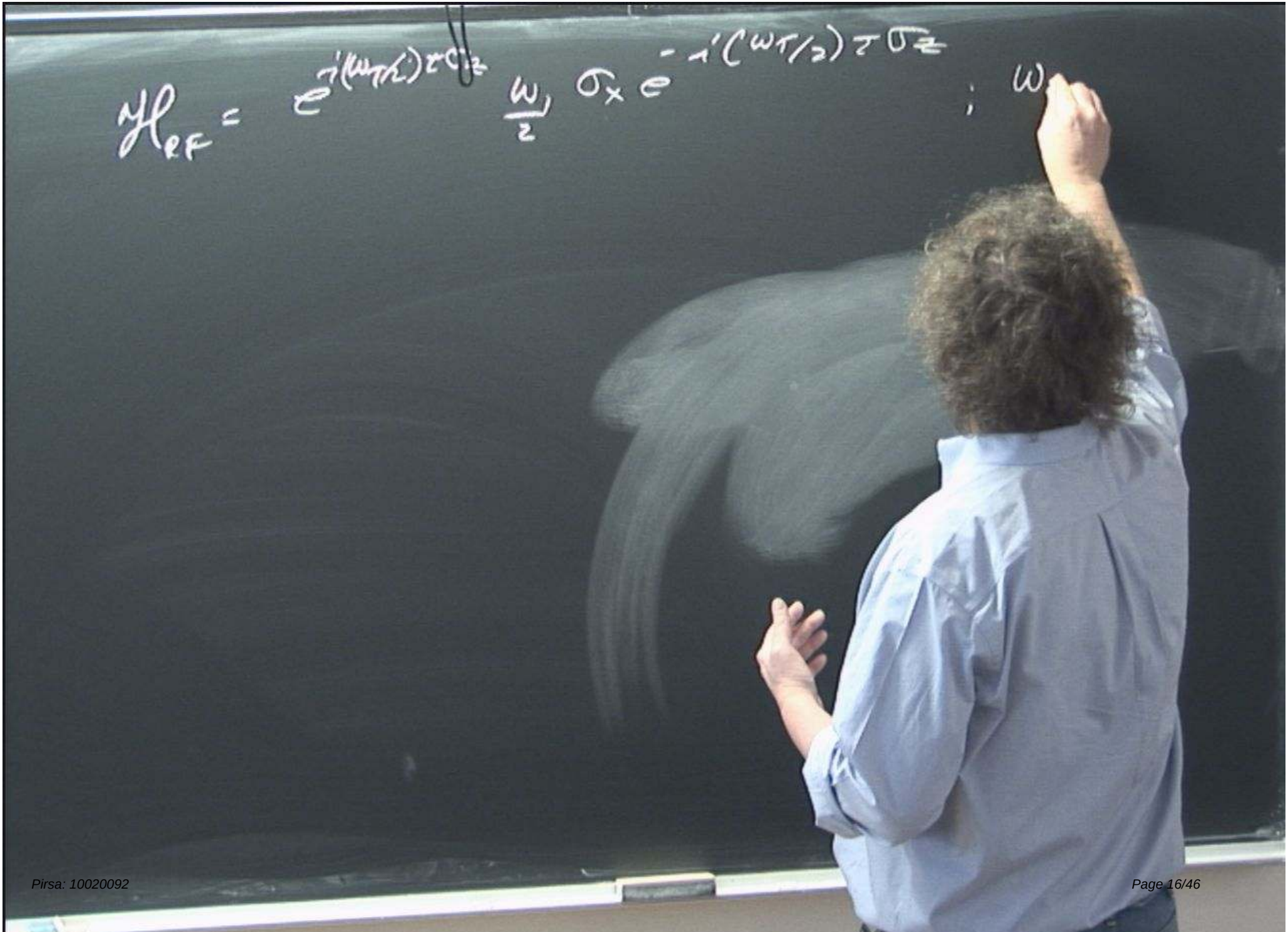


$\frac{n_\uparrow}{n_\downarrow} = e^{-E/kT}$

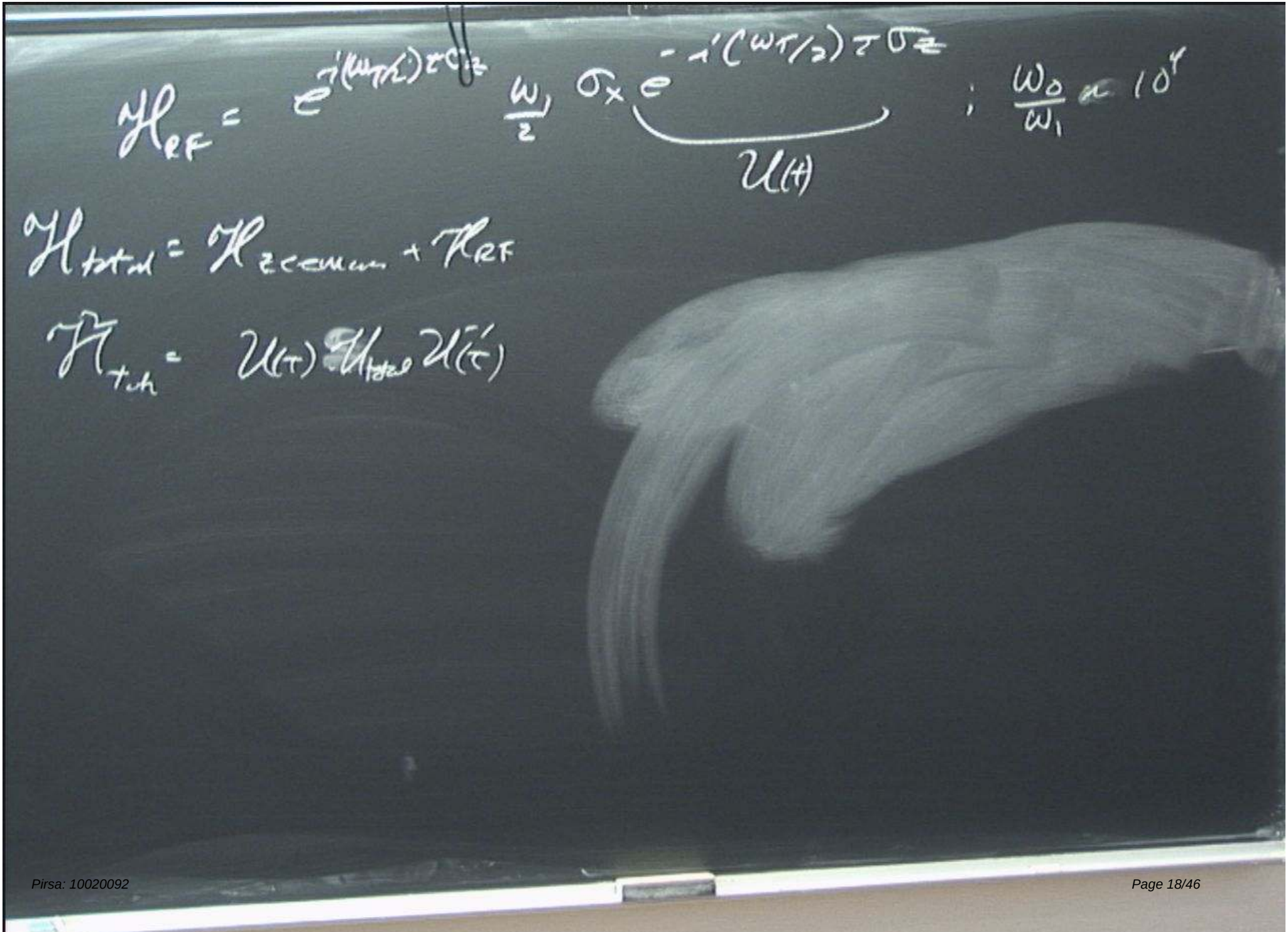
$\frac{\hbar\omega_0}{kT} = 10^{-5}$

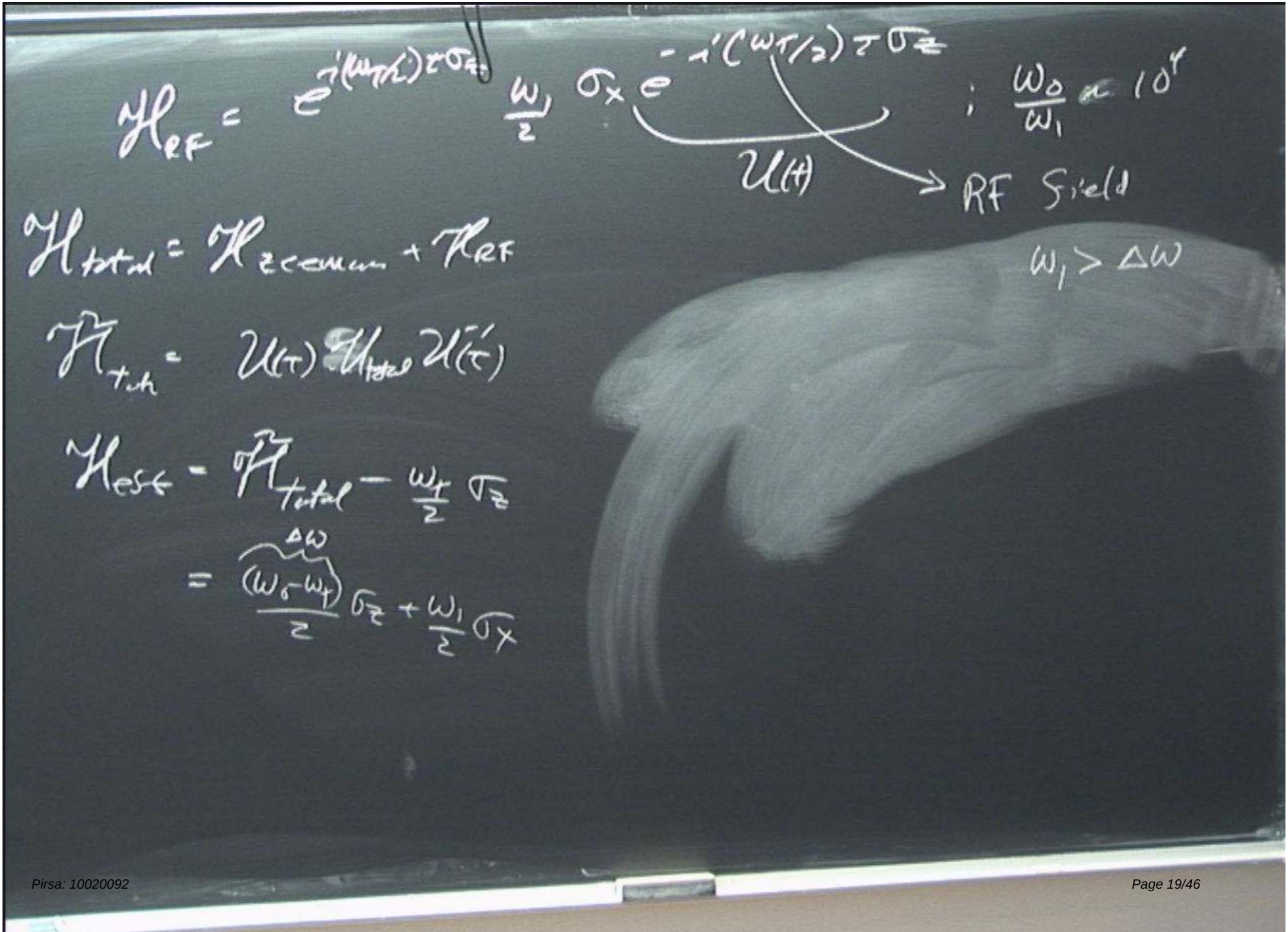
$\frac{R}{R} \sim \sigma_z$

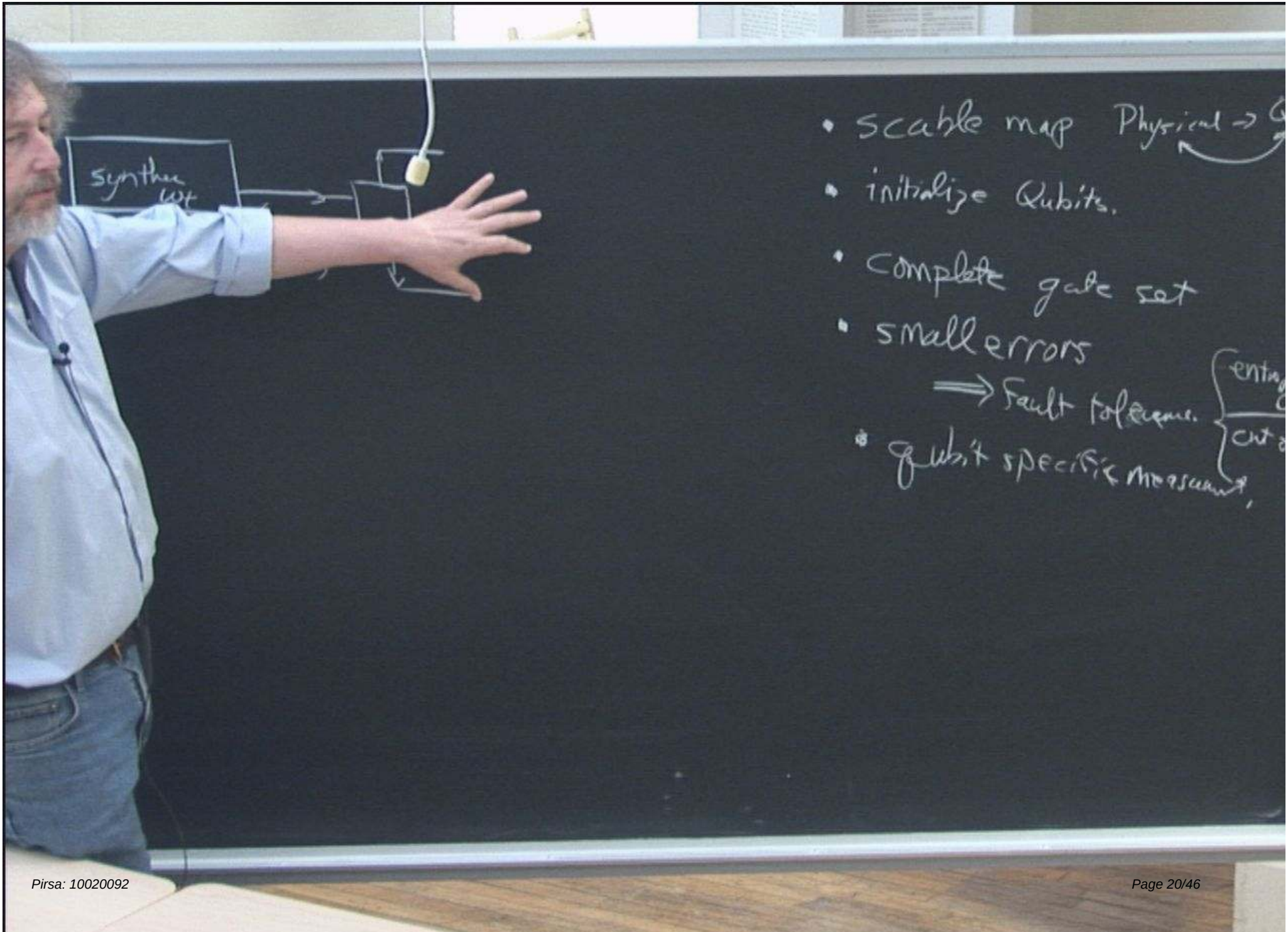
$\rho_{eq} = \mathbb{1} - \frac{\hbar\omega_0}{kT}\sigma_z$

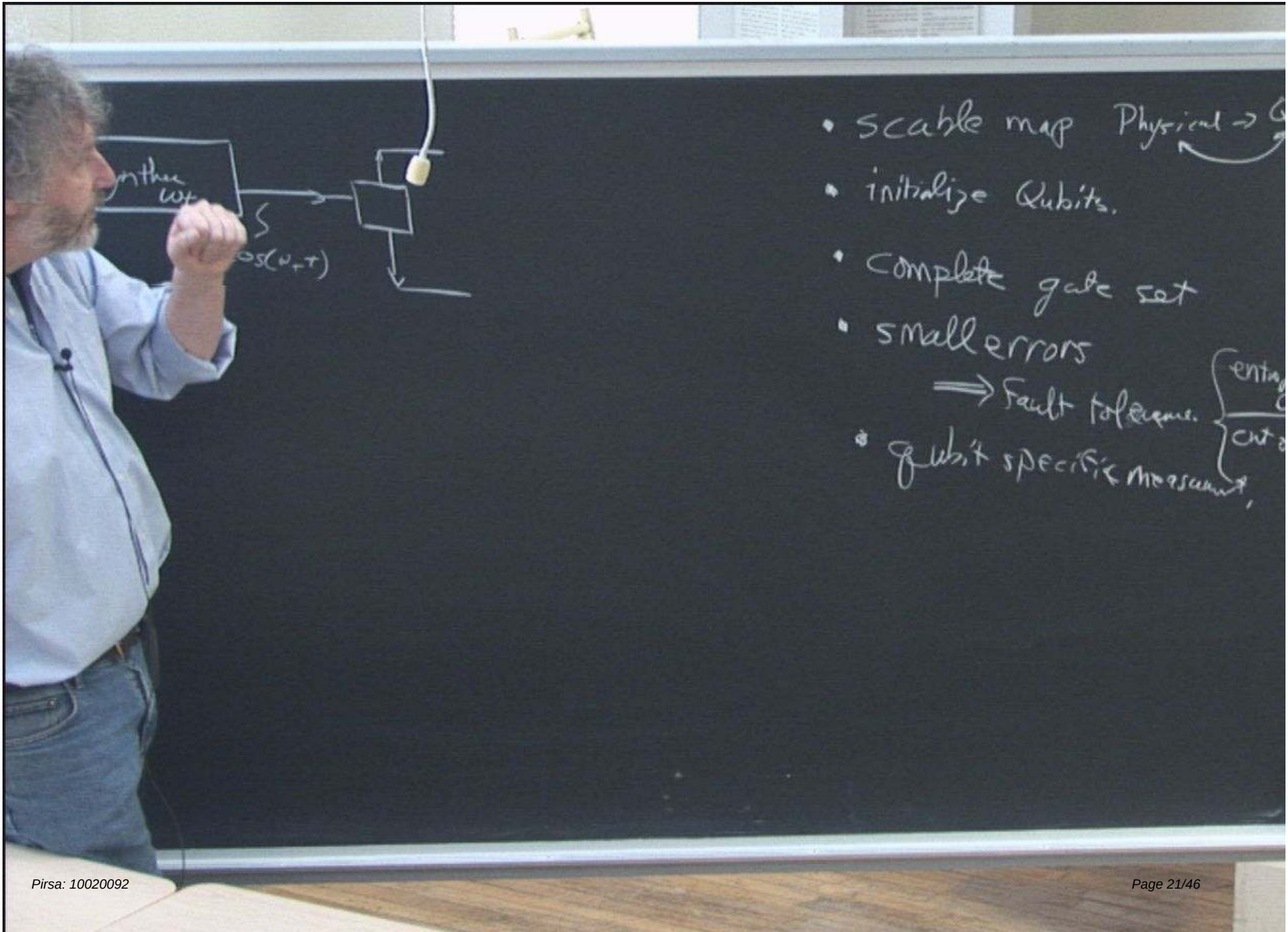

$\mathcal{H}_{RF} = e^{-i(\omega_T/2)\tau\sigma_z} \frac{\omega_1}{2}\sigma_x e^{-i(\omega_T/2)\tau\sigma_z}$; ω

$$\mathcal{H}_{RF} = e^{-i(\omega_{RF}t)\frac{\sigma_z}{2}} \frac{\omega_1}{2}\sigma_x e^{-i(\omega_{RF}t/2)\frac{\sigma_z}{2}} \quad ; \quad \frac{\omega_0}{\omega_1} \approx 10^4$$


$\mathcal{H}_{RF} = e^{-i(\omega_T/2)\tau\sigma_z} \frac{\omega_1}{2}\sigma_x \underbrace{e^{-i(\omega_T/2)\tau\sigma_z}}_{U(t)}$; $\frac{\omega_0}{\omega_1} \approx 10^4$
$\mathcal{H}_{total} = \mathcal{H}_{zeeman} + \mathcal{H}_{RF}$
$\mathcal{H}_{int} = U(\tau)^\dagger \mathcal{H}_{hyp} U(\tau)$


$\mathcal{H}_{RF} = e^{-i(\omega_T/2)\tau\sigma_z} \frac{\omega_1}{2}\sigma_x e^{-i(\omega_T/2)\tau\sigma_z}$
$\frac{\omega_0}{\omega_1} \approx 10^4$
$U(t)$
RF Field
$\mathcal{H}_{total} = \mathcal{H}_{zeeman} + \mathcal{H}_{RF}$
$\omega_1 > \Delta\omega$
$\mathcal{H}_{int} = U(\tau)\mathcal{H}_{total}U(\tau)$
$\mathcal{H}_{est} = \mathcal{H}_{total} - \frac{\omega_T}{2}\sigma_z$
$= \frac{(\omega_0 - \omega_T)}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x$
$\Delta\omega$


synthesis
• scable map Physical →
• initialize Qubits.
• complete gate set
• small errors
⟹ Fault tolerance.
• Qubit specific measurement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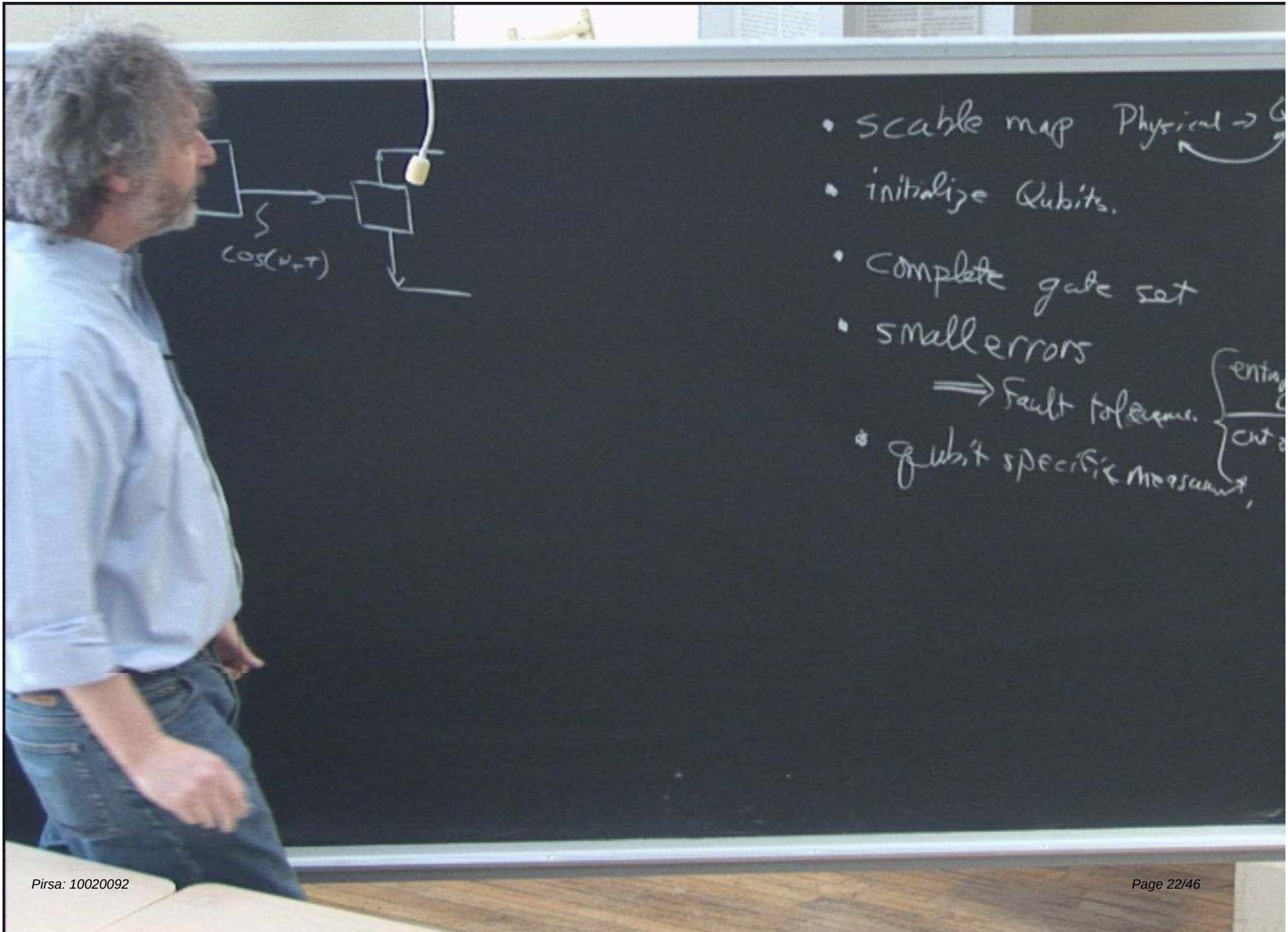
cos(ν+T)
• scable map Physical →
• initialize Qubits.
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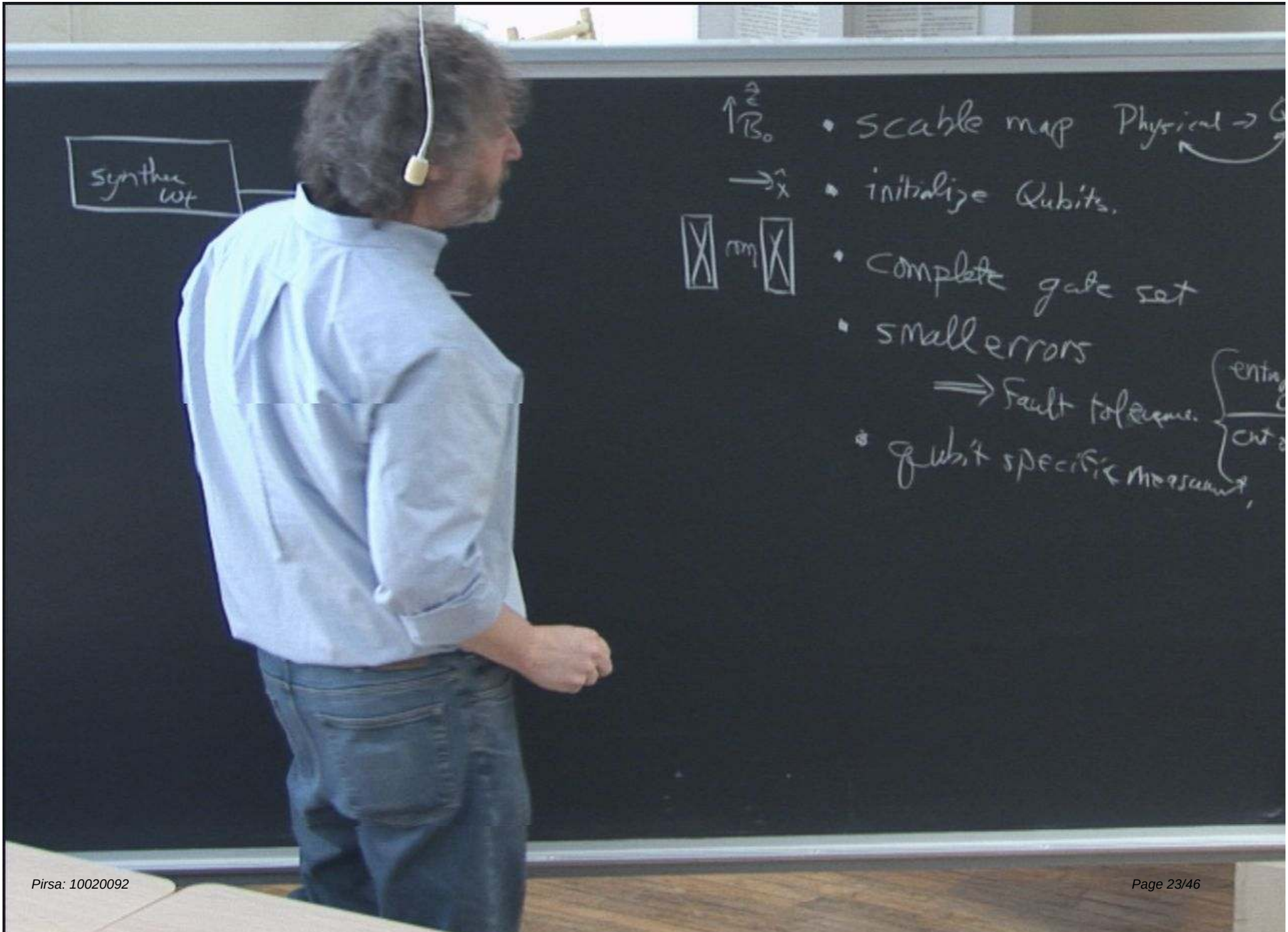
synthesis
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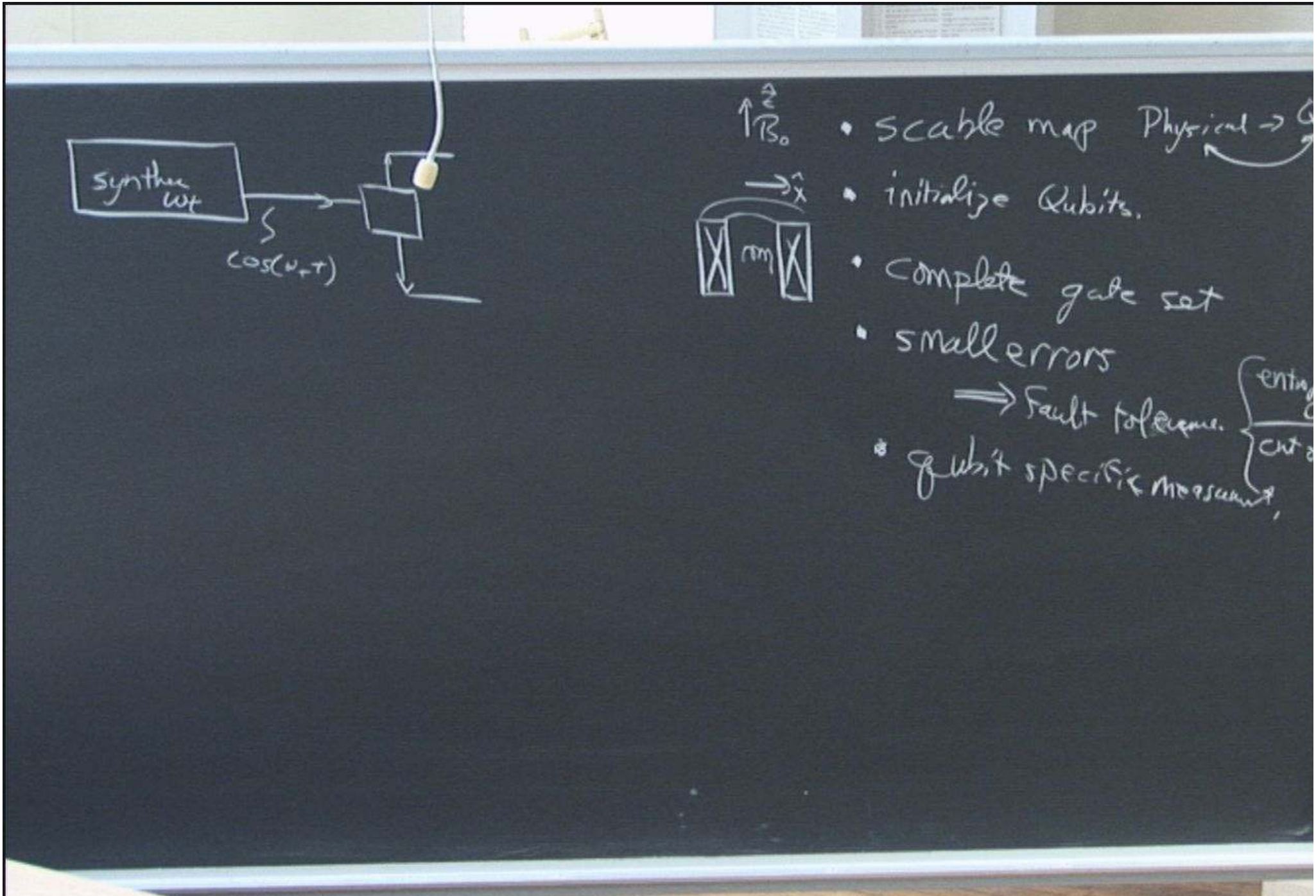
synthesizer
cos(ω₀t)
$\hat{z}$
B_0
$\hat{x}$
rf
• scable map Physical →
• initialize Qubits.
• complete gate set
• small errors
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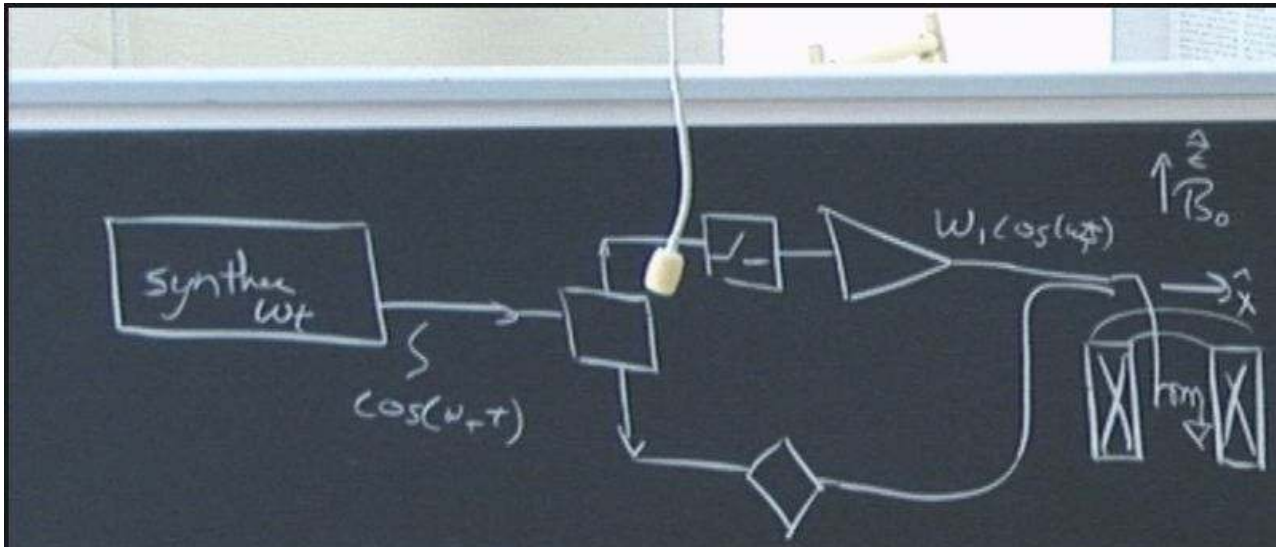

- scable map Physical →
- initialize Qubits.
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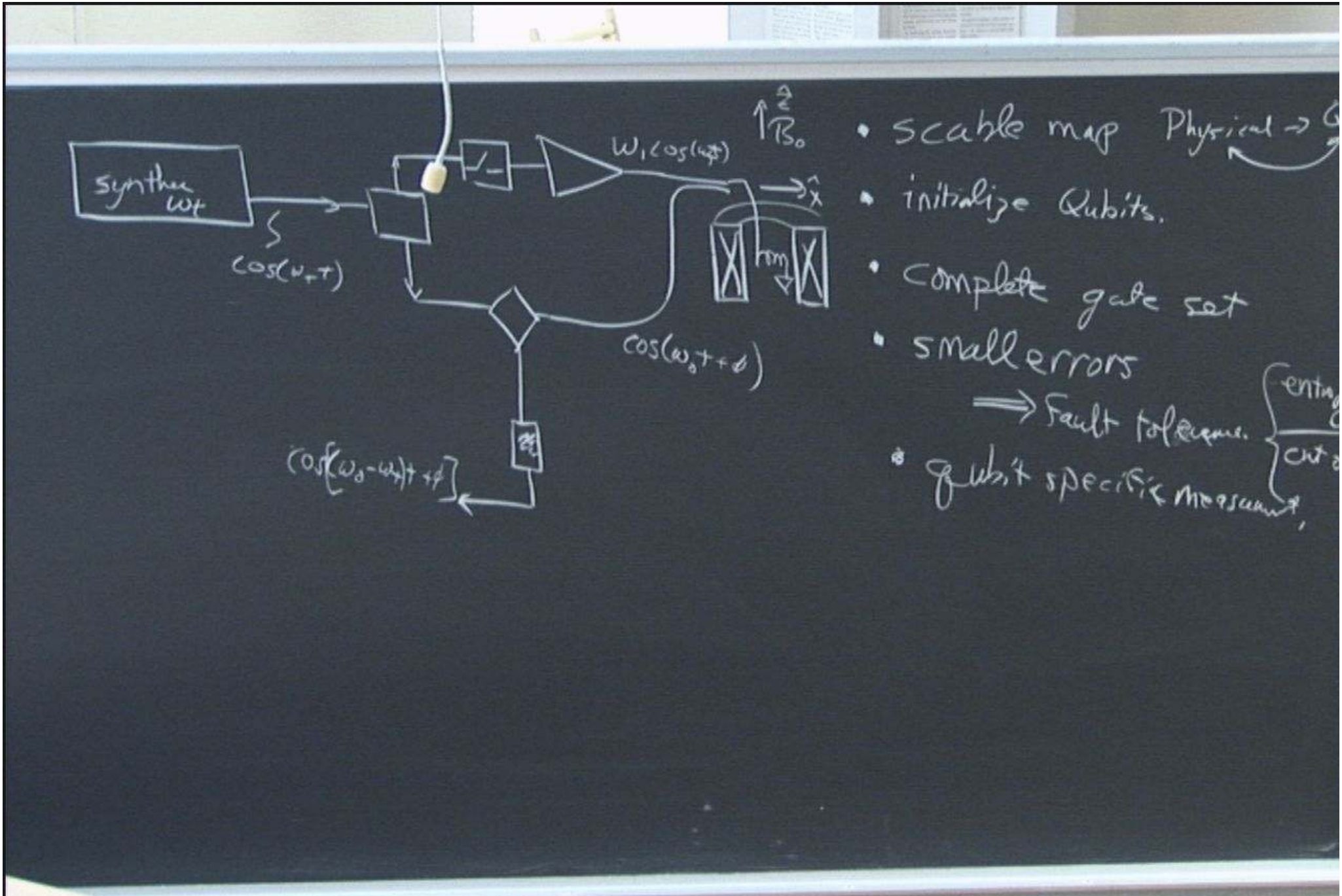
- small errors
  ⟹ Fault tolerance.
- Qubit specific measurement.


synthesis ω_t
$\cos(\omega_t t)$
$\omega_1 \cos(\omega_t t)$
$\hat{z}$
B_0
$\hat{x}$
$\cos(\omega_0 t + \phi)$
$\cos[(\omega_0 - \omega_t)t + \phi]$
• scable map Physical →
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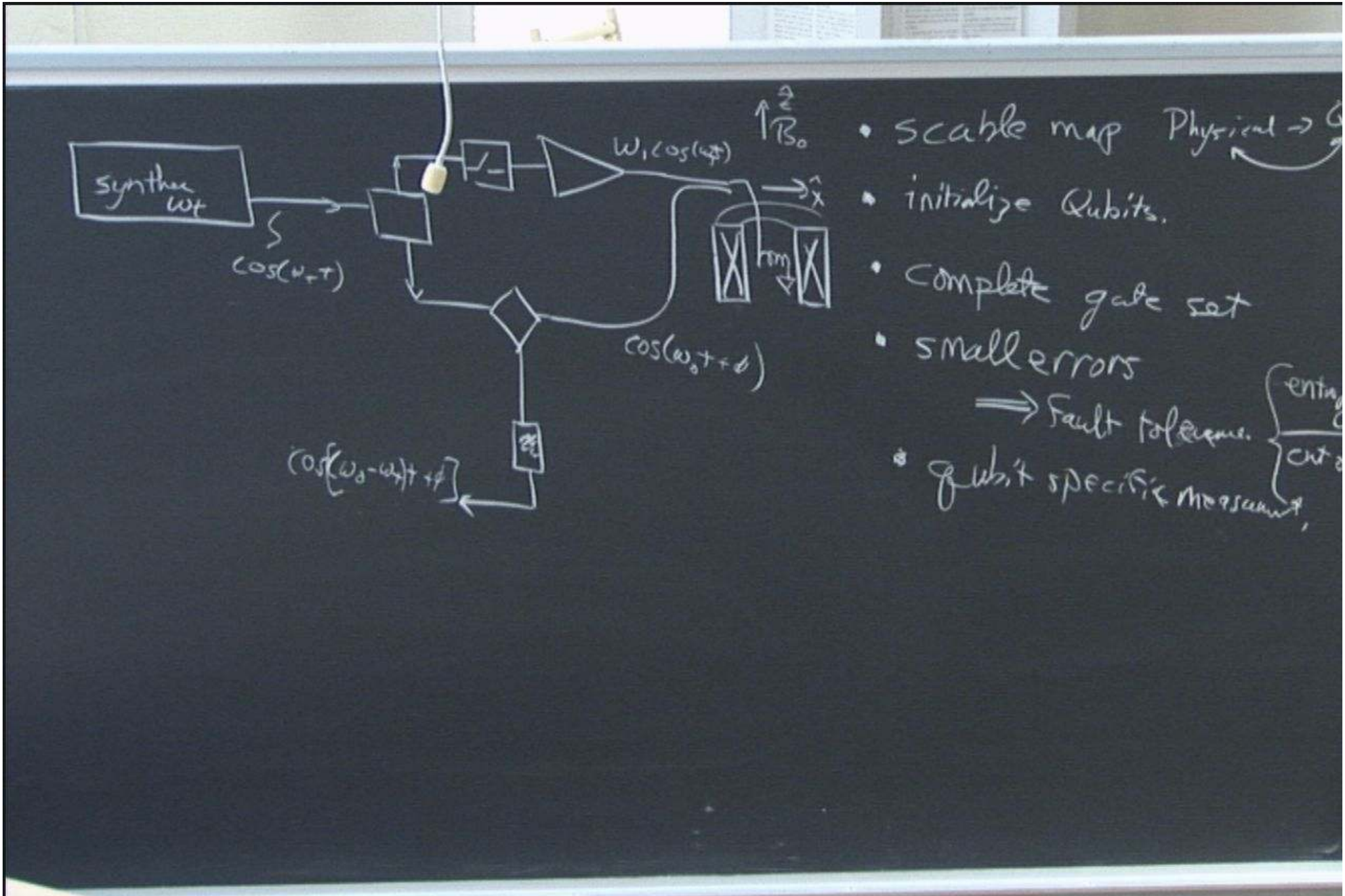
• complete gate set
• small errors
⟹ Fault tolerance.
• qubit specific measurement.


synthesis ω_t
$\cos(\omega_t t)$
$\omega_1 \cos(\omega_t t)$
$\hat{z}$
B_0
$\hat{x}$
rom
$\cos(\omega_0 t + \phi)$
$\cos[(\omega_0 - \omega_t)t + \phi]$
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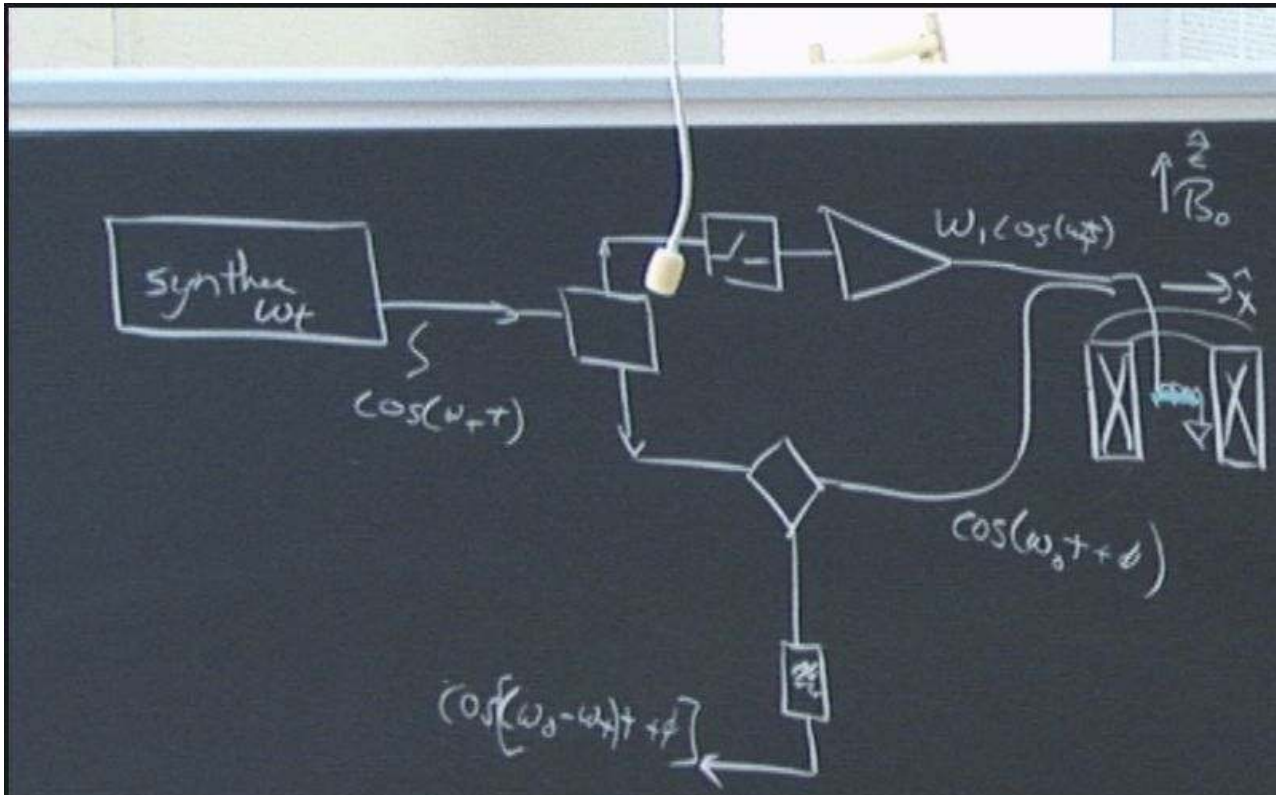

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$H_{eff} = \frac{\Delta\omega}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x$

$\rho(\omega) = \sigma_z$;

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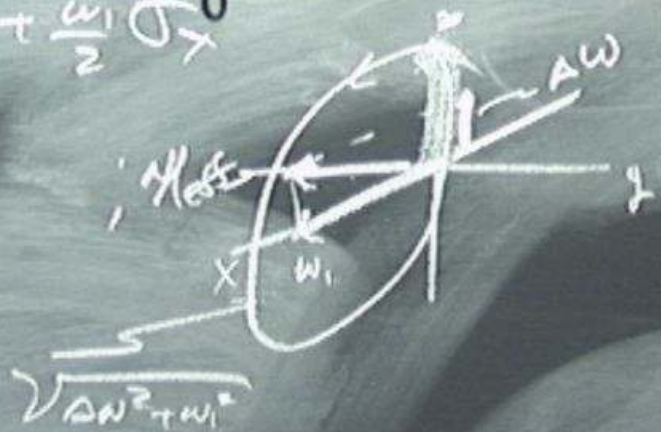
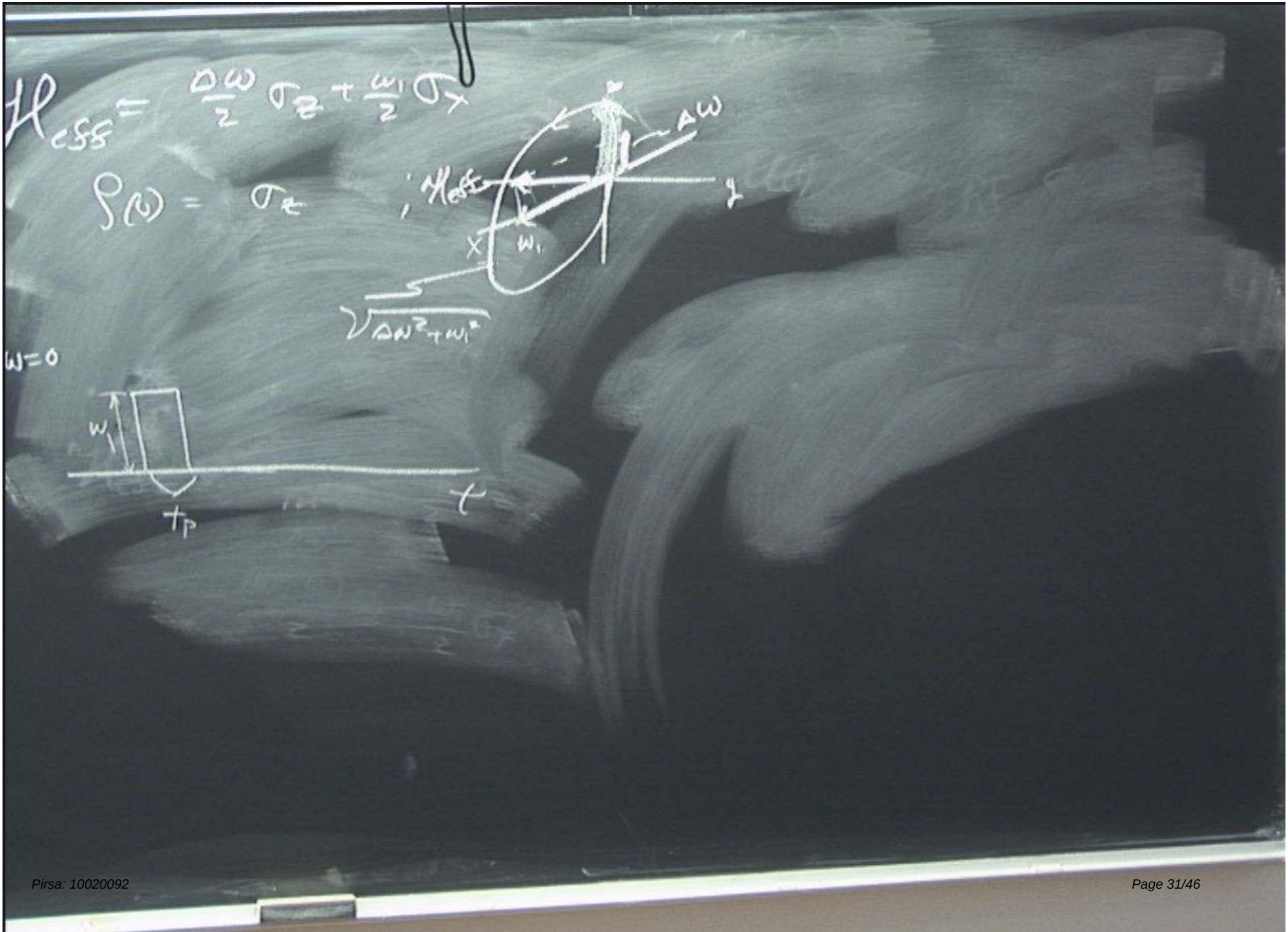
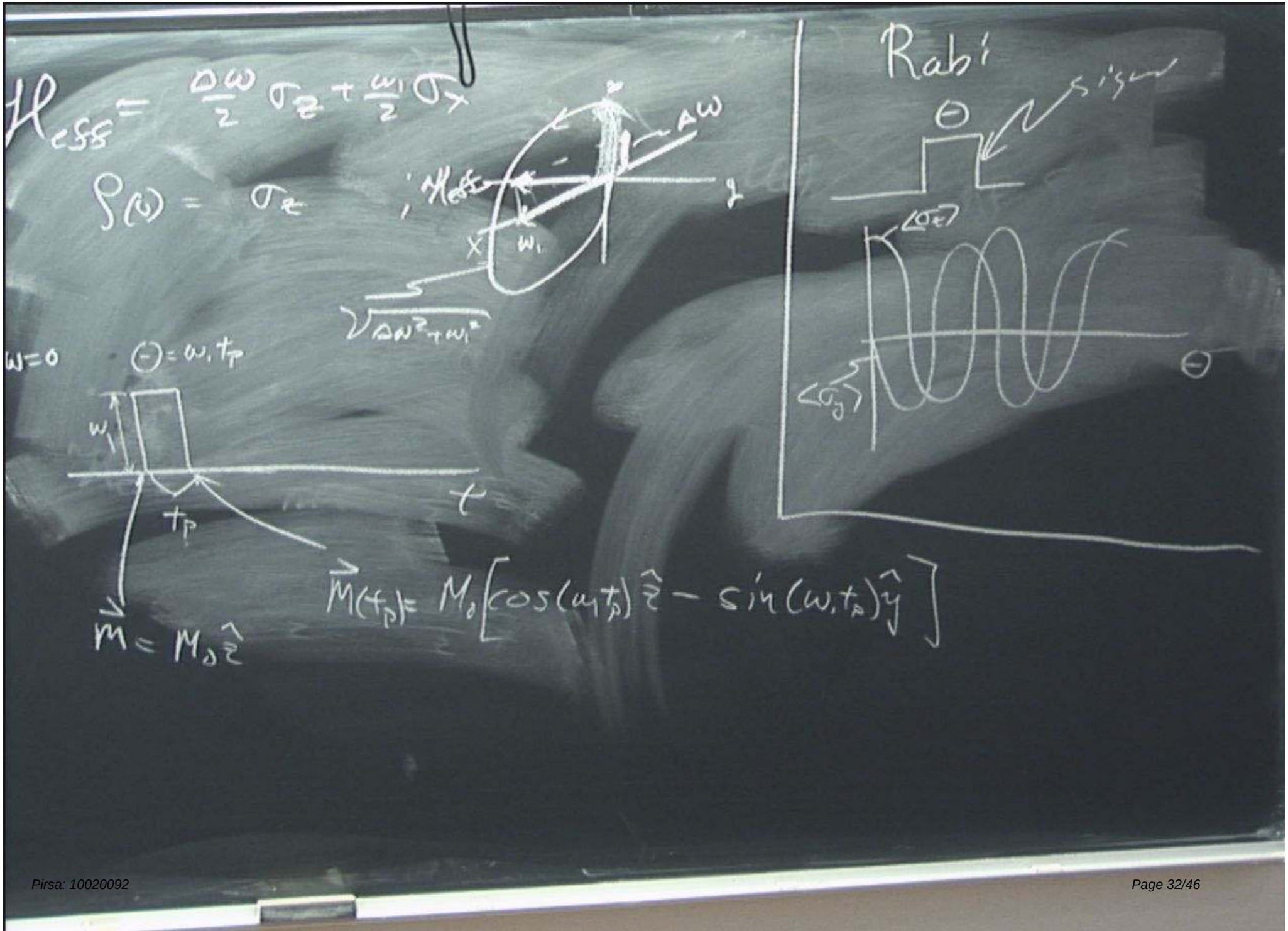
$\rho(\omega) = \sigma_z$; H_{eff}



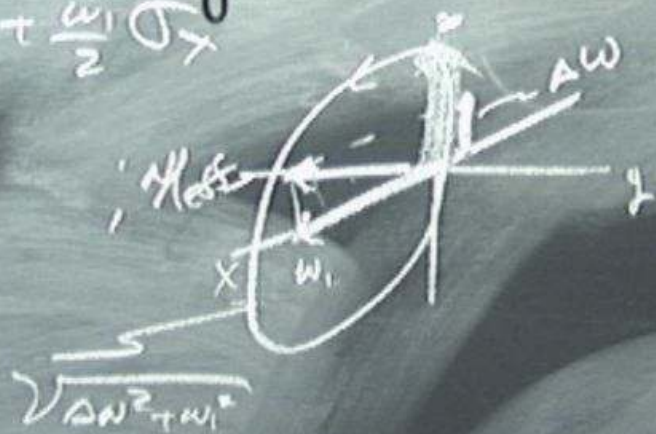

$H_{eff} = \frac{\Delta\omega}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x$
$\rho(\omega) = \sigma_z$
$\Delta\omega$
ω_1
$\sqrt{\Delta\omega^2 + \omega_1^2}$
$\omega = 0$
ω_1
t_p
t



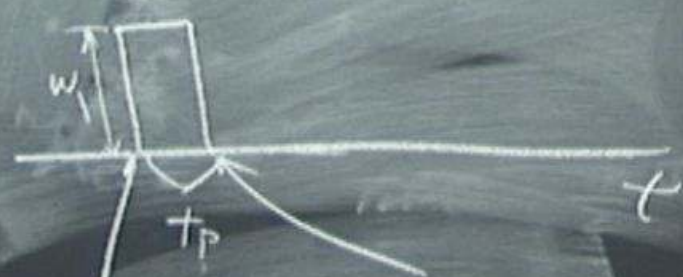
$\mathcal{H}_{eff} = \frac{\Delta\omega}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x$
$\rho(0) = \sigma_z$
$\Delta\omega$
ω_1
$\sqrt{\Delta\omega^2+\omega_1^2}$
$\omega=0$
$\Theta = \omega_1 t_p$
ω_1
t_p
t
$\vec{M} = M_0\hat{z}$
$\vec{M}(t_p) = M_0\left[\cos(\omega_1 t_p)\hat{z} - \sin(\omega_1 t_p)\hat{y}\right]$
Rabi
Θ
$\langle\sigma_z\rangle$
$\langle\sigma_y\rangle$
Θ

$$\mathcal{H}_{eff} = \frac{\Delta\omega}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x$$

$$\rho(0) = \sigma_z$$



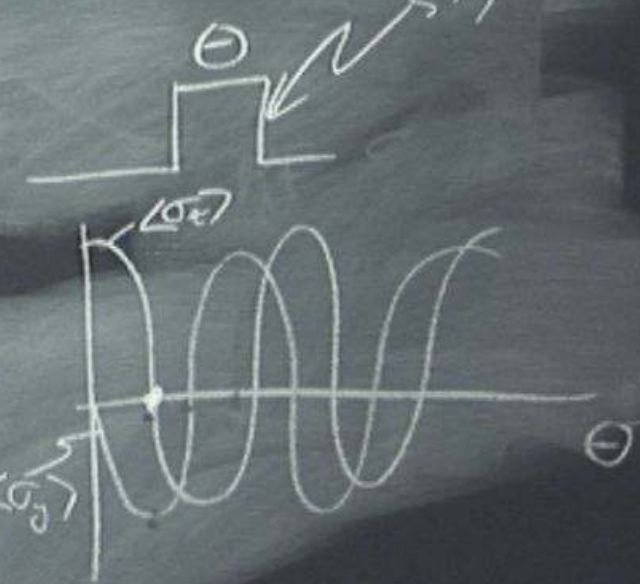
$\omega = 0$ $\quad \Theta = \omega_1 t_p$



$$\vec{M}(t_p) = M_0\left[\cos(\omega_1 t_p)\,\hat{z} - \sin(\omega_1 t_p)\,\hat{y}\right]$$

$$\vec{M} = M_0\,\hat{z}$$

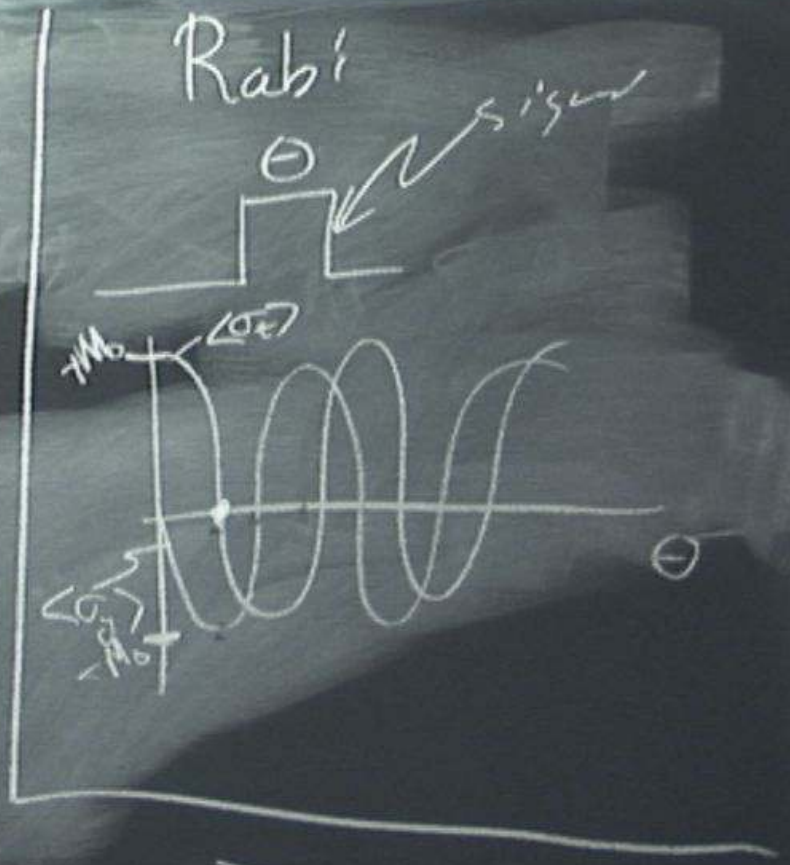
Rabi



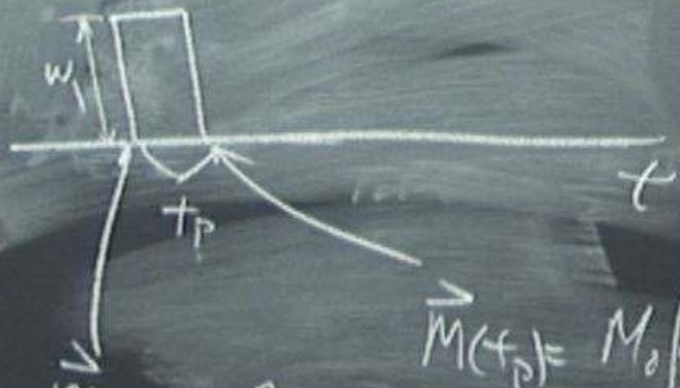
$H_{eff} = \frac{\Delta\omega}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x$

$\rho(\omega) = \sigma_z$

Rabi



$\omega = 0$ $\quad \theta = \omega_1 t_p$



$\vec{M} = M_0 \hat{z}$

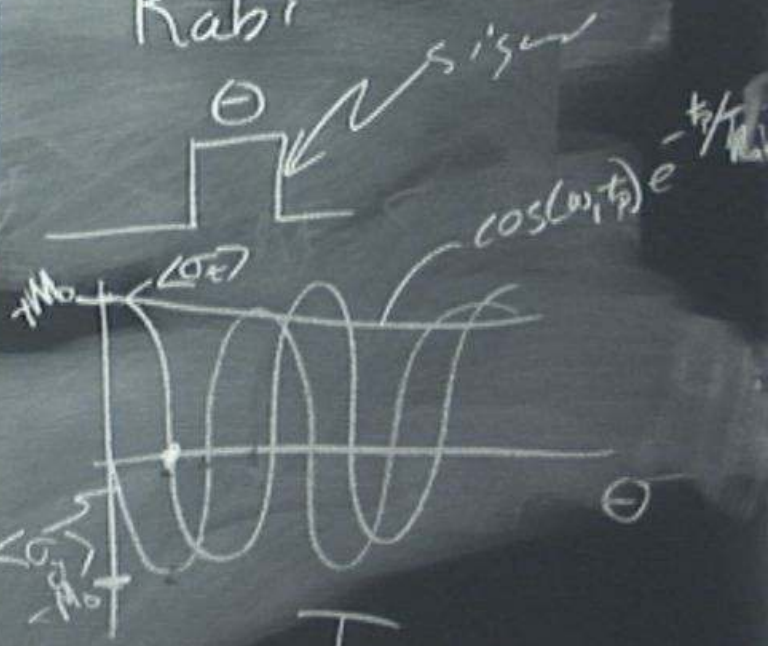
$\vec{M}(t_p) = M_0\left[\cos(\omega_1 t_p)\hat{z} - \sin(\omega_1 t_p)\hat{y}\right]$

"Relaxation"

T_1 = spin lattice relaxation, populations → eg. energy exchange,

T_2 = spin-spin relaxation coherence,

Rabi

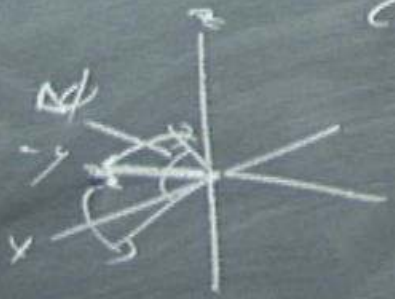


$T_{Rabi}\ \omega_1 >> 1$

Relaxation

T_1 = spin lattice relaxation, populations → eg. energy exchange,

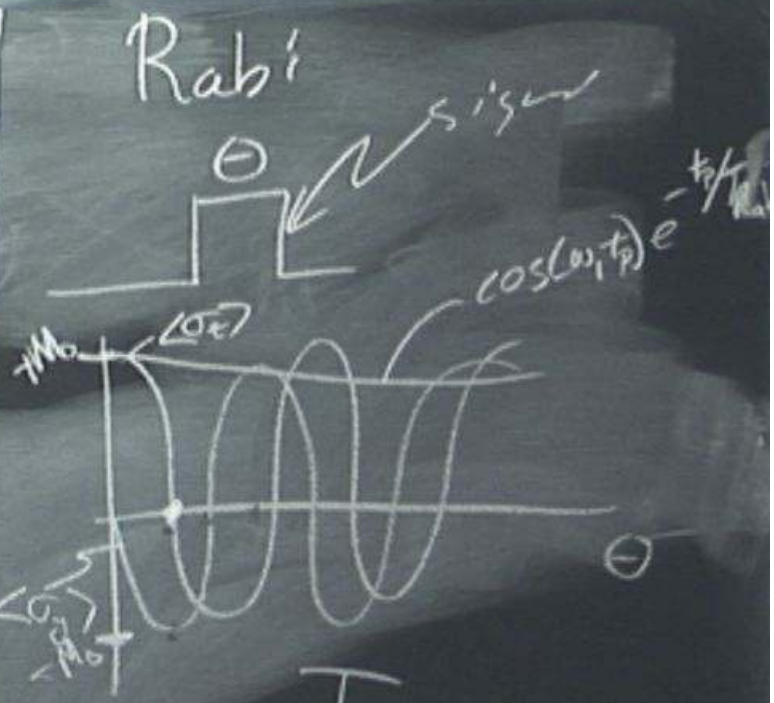
T_2 = spin-spin relaxation coherence,



$\Delta\varphi = \Delta B\, \tau$

$M(y,t) = \cos(\Delta\varphi(t))$

Rabi



$T_{Rabi}\, \omega_1 >> 1$

...nce

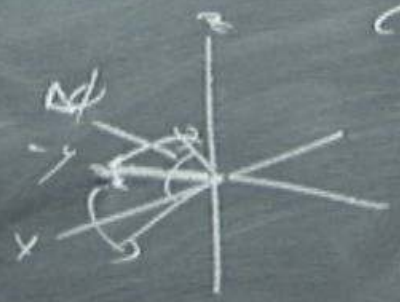
- static magnetic field, $\hat{z}$
  $10^4 - 10^5$ G

P_z σ_z

Relaxation

$T_1 \equiv$ spin lattice relaxation, populations $\Rightarrow$ eg. energy exchange,

$T_2 \equiv$ spin-spin relaxation coherence,



$\Delta\phi = \Delta B\, \tau$

$M(y,t) = \cos(\Delta\phi(t))$

...nce

- static magnetic field, $\hat{z}$
  $10^4 - 10^5$ G

Relaxation

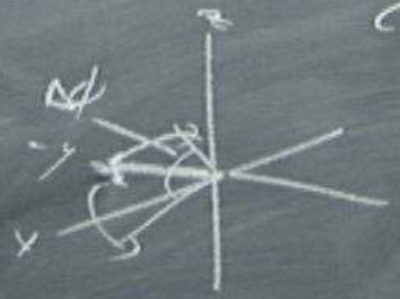
$T_1 \equiv$ spin lattice relaxation, populations $\Rightarrow$ e.g. energy exchange,

$T_2 \equiv$ spin-spin relaxation coherence,

$P_2 : \sigma_z$

$1 - P_2 : \mathbb{1}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cdot & 0 & 0 \\ 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$\Delta\phi = \Delta B \tau$

$M_{(y)}(t) = \cos(\Delta\phi(t))$

...nce

- static magnetic field, $\hat{z}$
  10^{4}–10^{5} G

Relaxation

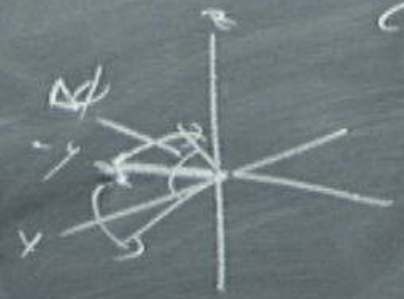
T_1 = spin lattice relaxation, populations → eg. energy exchange,

T_2 = spin-spin relaxation coherence,

$P_z : \sigma_z$

$1 - P_z : \mathbb{1}$

$$\sum_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & T & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$\Delta\phi = \Delta B \tau$

$M(y,t) = \cos(\Delta\phi(t))$

ance

- static magnetic field, $\hat{z}$
  10^{4}–10^{5} G

Relaxation

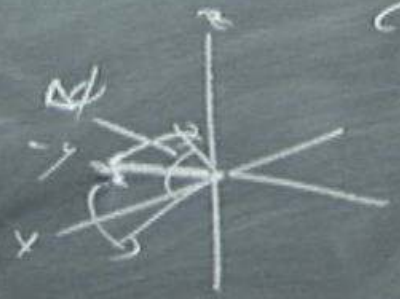
T_1 = spin lattice relaxation, populations → eg. energy exchange,

T_2 = spin-spin relaxation coherence,

$P_2 : \sigma_z$

$1 - P_2 : \mathbb{1}$

$$\Sigma_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \uparrow & 0 & 0 \\ 0 & 0 & \downarrow & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$\Delta\phi = \Delta B \tau$

$M(y,t) = \cos(\Delta\phi(t))$

...nce

– static magnetic field, $\hat{z}$
$10^4 - 10^5$ G

σ_y

$p_2 : \sigma_z$

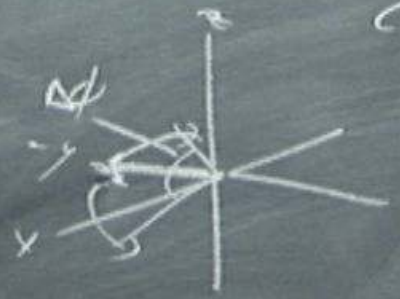
$1 - p_2 : \mathbb{1}$

$$S_{T_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & T & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Relaxation

$T_1 \equiv$ spin lattice relaxation, populations $\Rightarrow$ eq. energy exchange.

$T_2 \equiv$ spin-spin relaxation coherence.



$\Delta\phi = \Delta B \tau$

$M(y,t) = \cos(\Delta\phi(t))$

Nuclear Magnetic Resonance

$$\mathcal{H}_{Zeeman} = \frac{\omega_0}{2}\sigma_z$$

static magnetic field, $\hat{z}$
$10^4 - 10^5$ G

σ_x or σ_y

P_1^x : σ_x

$1 - P_1^x$: $\mathbb{1}$

P_2 : σ_z

$1 - P_2$: $\mathbb{1}$

$$\rho_{t_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & T & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Nuclear Magnetic Resonance

$\mathcal{H}_{Zeeman} = \frac{\gamma B_0}{2} \sigma_z$ — static magnetic field, $\hat{z}$, $10^4 - 10^5$ G

σ_x or σ_y

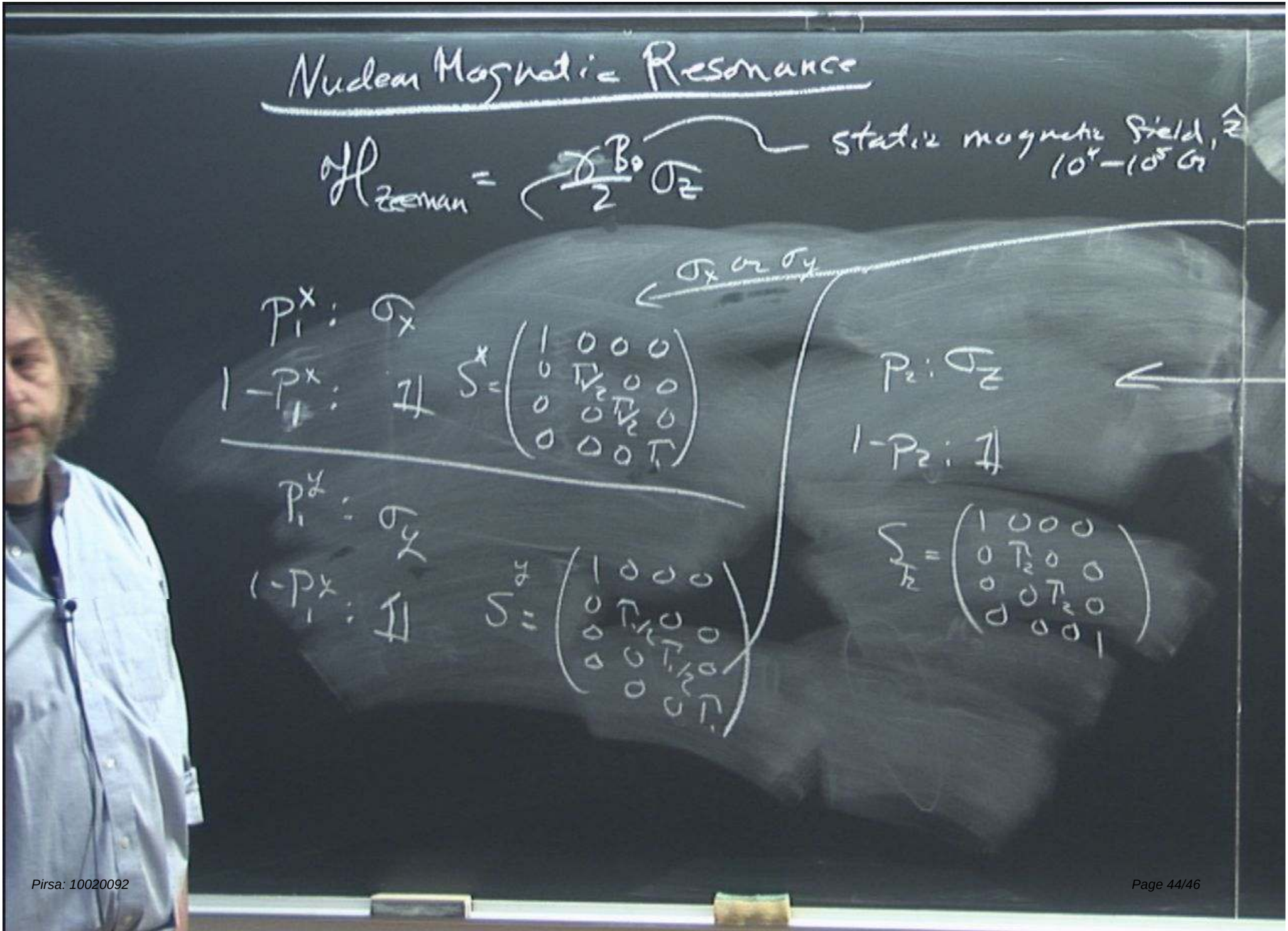
P_1^x : σ_x

$1 - P_1^x$: $\mathbb{1}$

P_2 : σ_z

$1 - P_2$: $\mathbb{1}$

$$\mathcal{S}_{P_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & T & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$


Nuclear Magnetic Resonance
$\mathcal{H}_{Zeeman} = \frac{\omega_0}{2}\sigma_z$
static magnetic field, $\hat{z}$
$10^4 - 10^5$ G
σ_x or σ_y
$P_1^x : \sigma_x$
$1 - P_1^x : \mathbb{1}$
$S^x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_1 \end{pmatrix}$
$P_1^y : \sigma_y$
$1 - P_1^y : \mathbb{1}$
$S^y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_1 \end{pmatrix}$
$P_2 : \sigma_z$
$1 - P_2 : \mathbb{1}$
$S_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Nuclear Magnetic Resonance

$\mathcal{H}_{Zeeman} = \frac{\omega_0}{2}\sigma_z$ — static magnetic field $10^4 - 10^5$ G

$P_1^x = P_1^y = P_1$

$P_1(1-P_1)$ σ_x

$P_1(1-P_1)$ σ_y

P_1^2 σ_z

$(1-P)^2$ $\mathbb{1}$

$P_1^x : \sigma_x$

$1-P_1^x : \mathbb{1}$

$S^x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \Gamma_2 & 0 & 0 \\ 0 & 0 & \Gamma_2 & 0 \\ 0 & 0 & 0 & \Gamma_1 \end{pmatrix}$

$P_1^y : \sigma_y$

$1-P_1^y : \mathbb{1}$

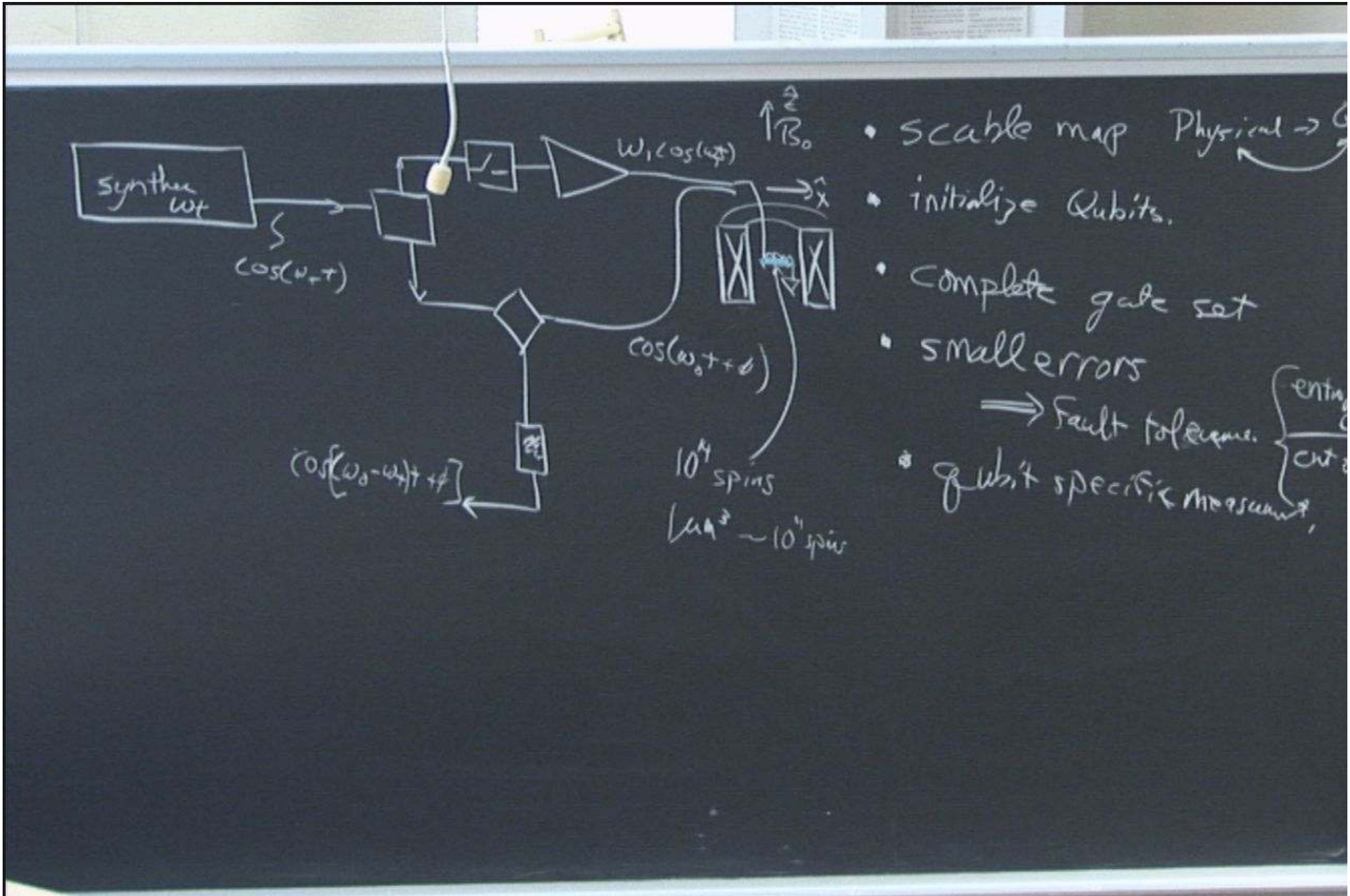
$S^y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \Gamma_2 & 0 & 0 \\ 0 & 0 & \Gamma_2 & 0 \\ 0 & 0 & 0 & \Gamma_1 \end{pmatrix}$

σ_x or σ_y

$P_2 : \sigma_z$

$1-P_2 : \mathbb{1}$

$S_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \Gamma_2 & 0 & 0 \\ 0 & 0 & \Gamma_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$


synthesizer
ω_r
$\cos(\omega_r t)$
$\omega_1 \cos(\omega_r t)$
$\hat{z}$
B_0
$\hat{x}$
$\cos(\omega_0 t + \phi)$
$\cos[(\omega_0 - \omega_r)t + \phi]$
10^4 spins
$1\mu m^3 \sim 10^{11}$ spins
• scable map Physical →
• initialize Qubits.
• complete gate set
• small errors
⟹ Fault tolerance.
• Qubit specific measurement.