

Title: Explorations in Quantum Info. (PHYS 641) - Lecture 7

Date: Feb 24, 2010 09:00 AM

URL: <http://pirsa.org/10020090>

Abstract:

# Spinor Dynamics.

initial state  $|0\rangle$  ;

$$H = \omega \sigma_z$$

$$U(t) = e^{-i \frac{\theta}{2} \sigma_x}$$

$$\rho(t) = \cos \frac{\theta}{2} |0\rangle \langle 0| - i \sin \frac{\theta}{2} |1\rangle \langle 0|$$

# Spinor Dynamics.

initial state  $|0\rangle$  ;

$$\mathcal{H} = \omega \sigma_z$$

$$U(t) = e^{-i \frac{\theta}{2} \sigma_x}$$

$$|4\rangle(t) = \cos \frac{\theta}{2} |0\rangle - i \sin \frac{\theta}{2} |1\rangle$$

$$|2\rangle(t) = |0\rangle$$

$$|\pi\rangle = -i |1\rangle$$

$$|2\pi\rangle = -|0\rangle$$

# Spinor Dynamics.

initial state  $|0\rangle$  ;

$$H = \omega \sigma_x$$

$$U(t) = e^{-i \frac{\theta}{2} \sigma_x}$$

$$|4\rangle(t) = \cos \frac{\theta}{2} |0\rangle - i \sin \frac{\theta}{2} |1\rangle$$

$$|4\rangle(0) = |0\rangle$$

$$(\pi) = -i |1\rangle$$

$$(2\pi) = |0\rangle$$

global phase

$$|4\rangle = |100\rangle$$

$$|7\rangle = |110\rangle$$

$$|14\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |110\rangle)$$

$$|4\rangle = |00\rangle$$

$$|110\rangle$$

$$|4\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$U = |0\rangle\langle 0| + |1\rangle\langle 1| + \dots$$

$$|2\rangle = |00\rangle$$

$$|1\rangle = |10\rangle$$

$$|2\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$$U = |0\rangle\langle 0| + |1\rangle\langle 1| e^{i\theta/2\sigma_z}$$

$$|2\rangle(t) = \frac{1}{\sqrt{2}} \left[ |00\rangle + \cos\frac{\theta}{2} |10\rangle - i \sin\frac{\theta}{2} |11\rangle \right]$$

$$|2\rangle(0) = \frac{1}{\sqrt{2}} (|00\rangle + |10\rangle)$$

$$|2\rangle(\pi) = \frac{1}{\sqrt{2}} (|00\rangle - |10\rangle)$$

$$| \Psi \rangle = | 00 \rangle$$

$$| \Psi \rangle = | 00 \rangle$$

$$| \Psi \rangle = \frac{1}{\sqrt{2}} (| 00 \rangle + | 10 \rangle)$$

$$U = | 0 \rangle \langle 0 | \otimes \mathbb{1} + | 1 \rangle \langle 1 | \otimes e^{i\theta/2 \sigma_x}$$

$$| \Psi \rangle(t) = \frac{1}{\sqrt{2}} \left[ | 00 \rangle + \cos \frac{\theta}{2} | 10 \rangle - i \sin \frac{\theta}{2} | 11 \rangle \right]$$

$$| \Psi \rangle(0) = \frac{1}{\sqrt{2}} (| 00 \rangle + | 10 \rangle) = \frac{1}{\sqrt{2}} (| 0 \rangle + | 1 \rangle) \otimes | 0 \rangle$$

$$| \Psi \rangle(\pi) = \frac{1}{\sqrt{2}} (| 00 \rangle - | 10 \rangle) = \frac{1}{\sqrt{2}} (| 0 \rangle - | 1 \rangle) \otimes | 0 \rangle$$



$$|\psi\rangle = |00\rangle$$

$$H|10\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$$U = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes C$$

$$|\psi\rangle(t) = \frac{1}{\sqrt{2}} \left[ |00\rangle + \cos\frac{\theta}{2} |10\rangle - i \sin\frac{\theta}{2} |11\rangle \right]$$

$$|\psi\rangle(0) = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

$$|\psi\rangle(\pi) = \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |0\rangle$$

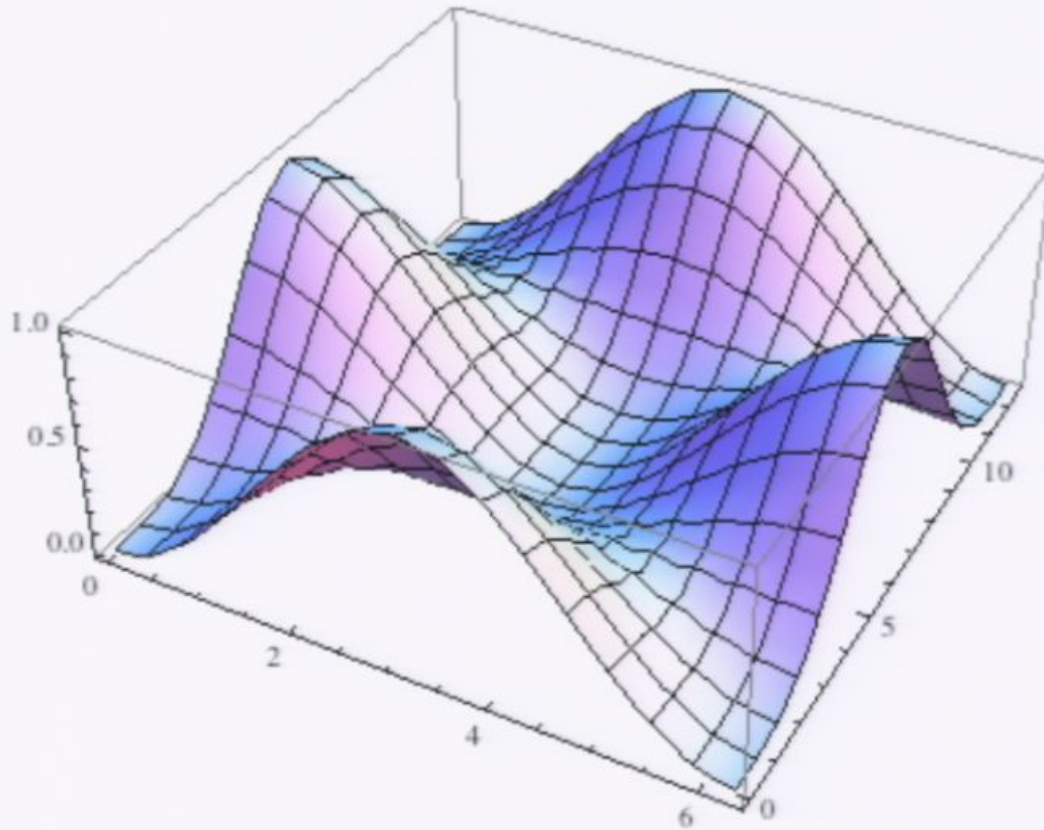
$$H|\psi\rangle(0) = |00\rangle$$

$$H|\psi\rangle(\pi) = |10\rangle$$

$$|00\rangle \xrightarrow{H \otimes I} |00\rangle + |11\rangle \xrightarrow{\frac{1}{\sqrt{2}} \sigma_x} |00\rangle \xrightarrow{H \otimes I}$$

$$\frac{1}{8} \left( 3 - 2 \cos \left[ a - \frac{t}{2} \right] - 2 \cos \left[ a + \frac{t}{2} \right] + \cos [t] \right)$$

`Plot3D[MlsOp[t, a], {a, 0, 2 π}, {t, 0, 4 π}]`



`MlsHp[t_, a_] := Tr[Uzpdwn . resls[t, a]]`

`MlsHp[t, a]`

$$\frac{1}{8} \left( 3 + 2 \cos \left[ a - \frac{t}{2} \right] + 2 \cos \left[ a + \frac{t}{2} \right] + \cos [t] \right)$$

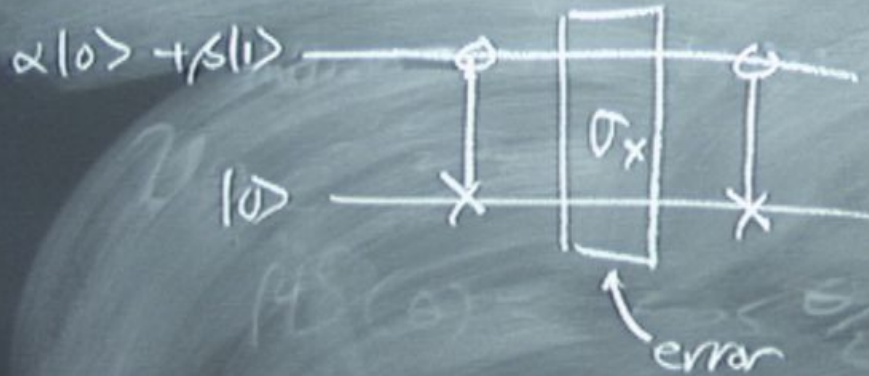
`Plot3D[MlsHp[t, a], {a, 0, 2 π}, {t, 0, 4 π}]`

# Error Detection

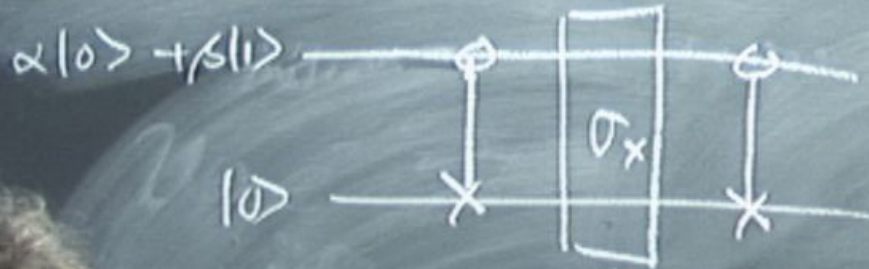
$$\alpha|0\rangle + \beta|1\rangle$$

$$|0\rangle$$

# Error Detection

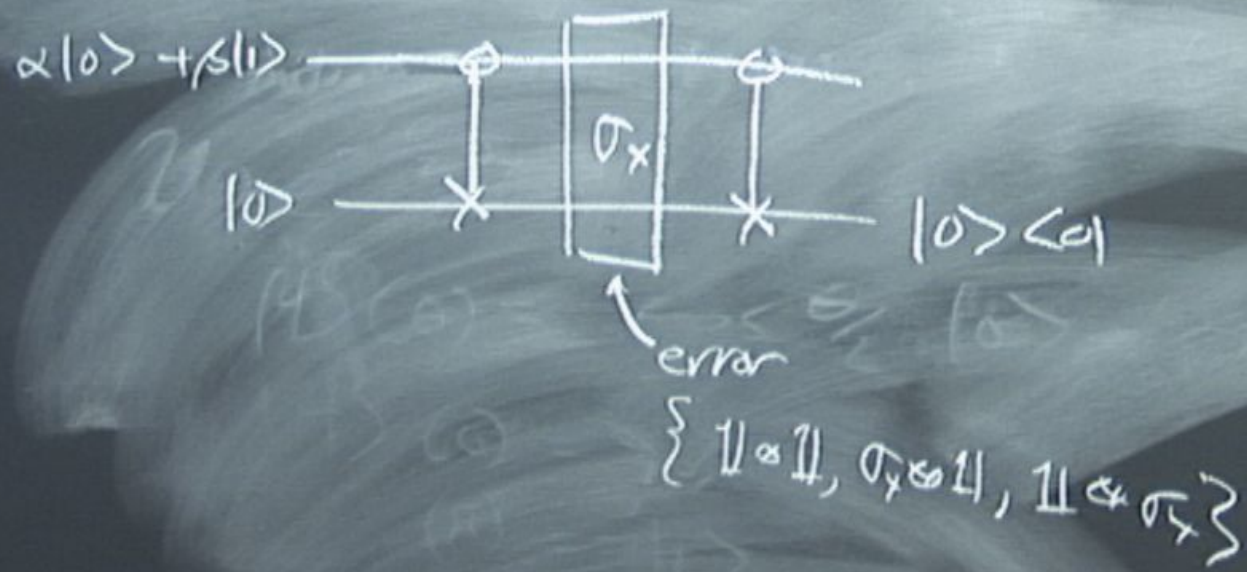


# Error Detection



$$\{ I \otimes I, \sigma_x \otimes I, I \otimes \sigma_x \}$$

# Error Detection



$$(\alpha|b\rangle + \beta|10\rangle) \xrightarrow{CNOT} \alpha|00\rangle + \beta|11\rangle$$





$$(\alpha|b\rangle + \beta|11\rangle) \otimes |10\rangle \xrightarrow{CNOT} \alpha|00\rangle + \beta|11\rangle$$

$$\alpha|00\rangle + \beta|11\rangle \xrightarrow{P_{11}} \alpha|00\rangle + \beta|11\rangle \xrightarrow{CNOT} (\alpha|b\rangle + \beta|11\rangle) |0\rangle$$

$$P_{11} \rightarrow \alpha|10\rangle + \beta|01\rangle \rightarrow \alpha|11\rangle + \beta|01\rangle$$

$$P_{10} \rightarrow \alpha|01\rangle + \beta|10\rangle \rightarrow (\alpha|1\rangle + \beta|0\rangle) |1\rangle$$

$$P_{01} \rightarrow \alpha|01\rangle + \beta|10\rangle$$

$$(\alpha|b\rangle + \beta|11\rangle) \otimes |10\rangle \xrightarrow{CNOT} \alpha|00\rangle + \beta|11\rangle$$

$$\alpha|00\rangle + \beta|11\rangle \xrightarrow{P_{11}} \alpha|00\rangle + \beta|11\rangle \xrightarrow{CNOT} (\alpha|10\rangle + \beta|11\rangle)$$

$$P_{x1} \rightarrow \alpha|10\rangle + \beta|01\rangle \rightarrow \alpha|11\rangle + \beta|10\rangle$$

$$P_{x2} \rightarrow \alpha|01\rangle + \beta|10\rangle \rightarrow (\alpha|11\rangle + \beta|10\rangle)$$

$$(\alpha|b\rangle + \beta|10\rangle) \otimes |10\rangle \xrightarrow{CNOT} \alpha|00\rangle + \beta|11\rangle$$

$$\alpha|00\rangle + \beta|11\rangle \xrightarrow{P_{11}} \alpha|00\rangle + \beta|11\rangle \xrightarrow{CNOT} (\alpha|b\rangle + \beta|10\rangle) |10\rangle$$

$$P_{10} \rightarrow \alpha|10\rangle + \beta|01\rangle \rightarrow \alpha|11\rangle + \beta|01\rangle$$

$$( \alpha|1\rangle + \beta|0\rangle ) |1\rangle$$

$$P_{10} \rightarrow \alpha|01\rangle + \beta|10\rangle \rightarrow \alpha|01\rangle + \beta|11\rangle$$

$$( \alpha|0\rangle + \beta|1\rangle ) |1\rangle$$

$$(\alpha|b\rangle + \beta|11\rangle) \otimes |10\rangle \xrightarrow{CNOT} \alpha|00\rangle + \beta|11\rangle$$

$$\alpha|00\rangle + \beta|11\rangle \xrightarrow{P_{11}} \alpha|00\rangle + \beta|11\rangle \xrightarrow{CNOT} \boxed{(\alpha|b\rangle + \beta|11\rangle) |10\rangle}$$

$$P_{x1} \rightarrow \alpha|10\rangle + \beta|01\rangle \rightarrow \alpha|11\rangle + \beta|01\rangle$$

$$(\alpha|1\rangle + \beta|0\rangle) |1\rangle$$

$$P_{x2} \rightarrow \alpha|01\rangle + \beta|10\rangle \rightarrow \alpha|01\rangle + \beta|11\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle) |1\rangle$$

$$(\alpha|b\rangle + \beta|11\rangle)|0\rangle \xrightarrow{CNOT} \alpha|00\rangle + \beta|11\rangle$$

$$\alpha|00\rangle + \beta|11\rangle \xrightarrow{P_{11}} \alpha|00\rangle + \beta|11\rangle \xrightarrow{CNOT} \boxed{(\alpha|b\rangle + \beta|11\rangle)|0\rangle}$$

 $P_{10}$ 

$$\alpha|10\rangle + \beta|01\rangle \longrightarrow \alpha|11\rangle + \beta|01\rangle$$

$$(\alpha|1\rangle + \beta|0\rangle)|1\rangle$$

 $P_{10}$ 

$$\alpha|01\rangle + \beta|10\rangle \longrightarrow \alpha|01\rangle + \beta|11\rangle$$

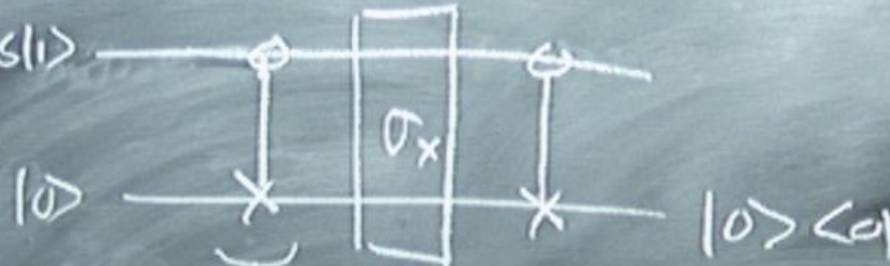
$$(\alpha|0\rangle + \beta|1\rangle)|1\rangle$$

# Error Detection

(spin)

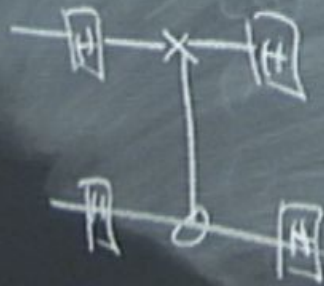
$$\propto |0\rangle + |1\rangle$$

(path)



error

$$\{ I \otimes I, \sigma_x \otimes I, I \otimes \sigma_x \}$$

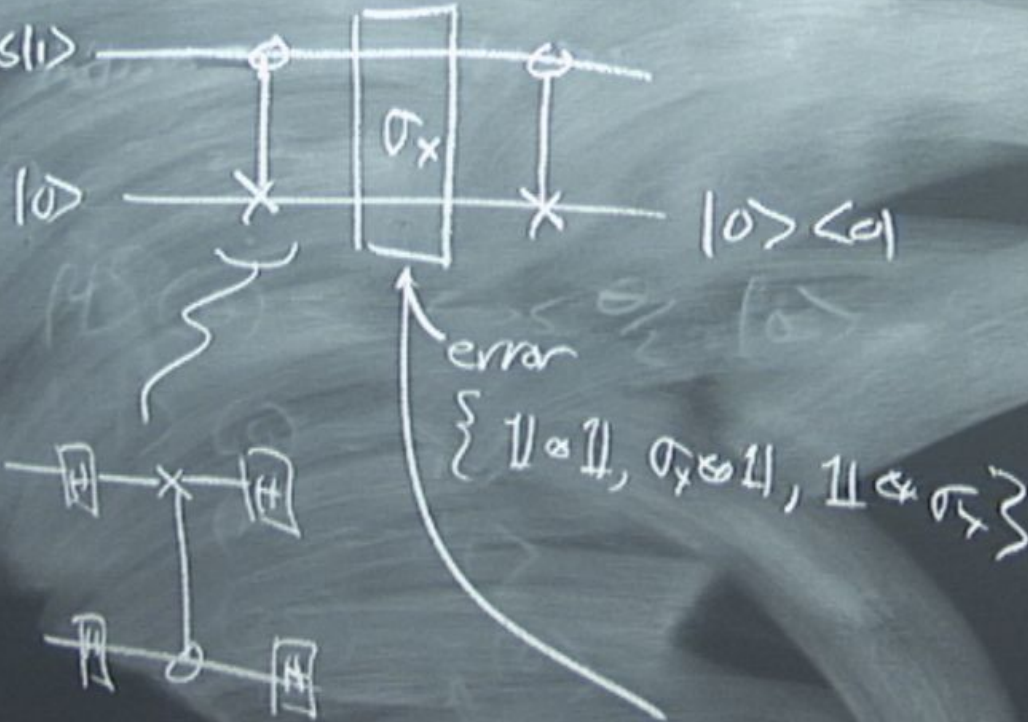


# Error Detection

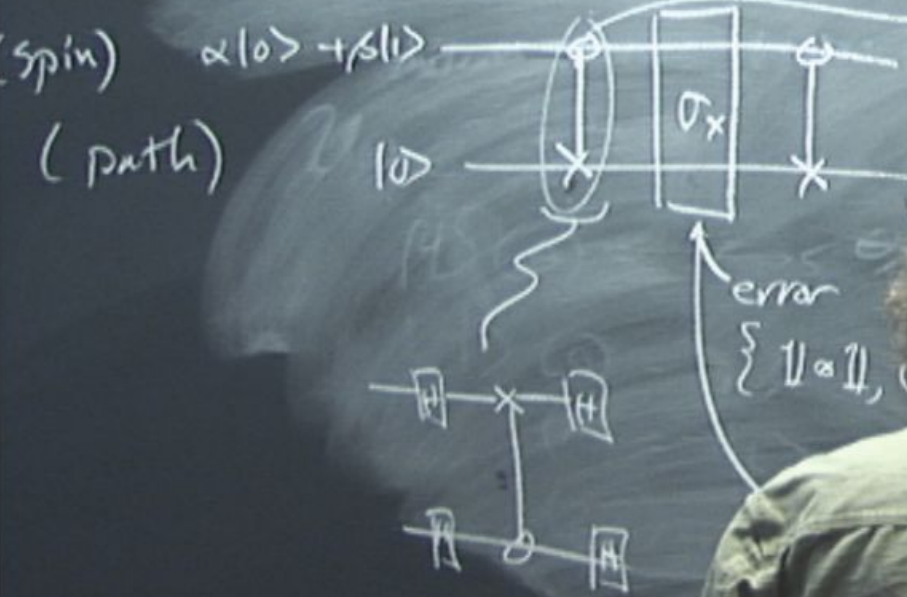
(spin)

$$\propto |0\rangle + |1\rangle$$

(path)

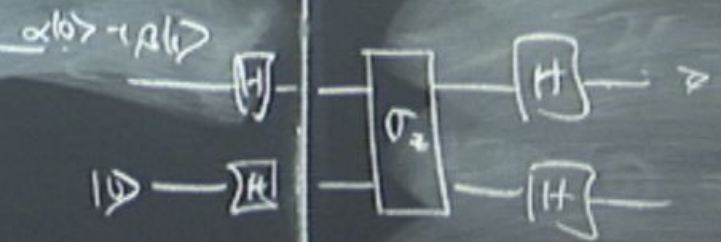


# Error Detection



swap paths  
 if the  
 spin is  $\uparrow$

$(\alpha|b\rangle + \beta|1\rangle) \otimes |0\rangle$   
 $\alpha|00\rangle + \beta|11\rangle$





# Error Detection

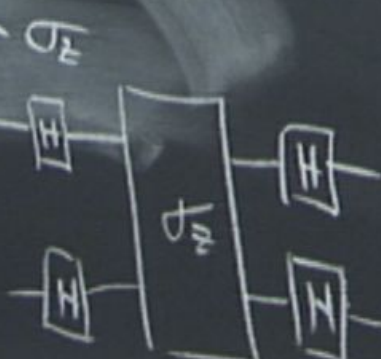
Spin  
(path)

$$\alpha|0\rangle + \beta|1\rangle$$



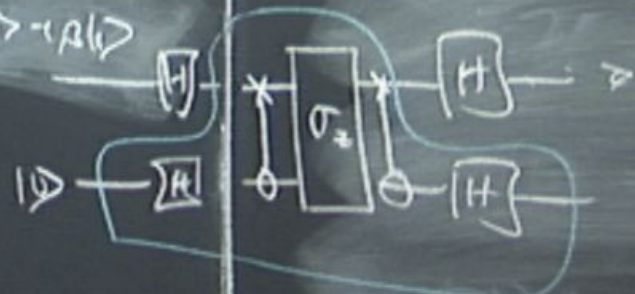
swap paths  
if the  
spin is ↑

error  
 $\{I \otimes I, \sigma_y \otimes I, I \otimes \sigma_y\}$



$$(\alpha|b\rangle + \beta|1\rangle) \otimes |0\rangle$$

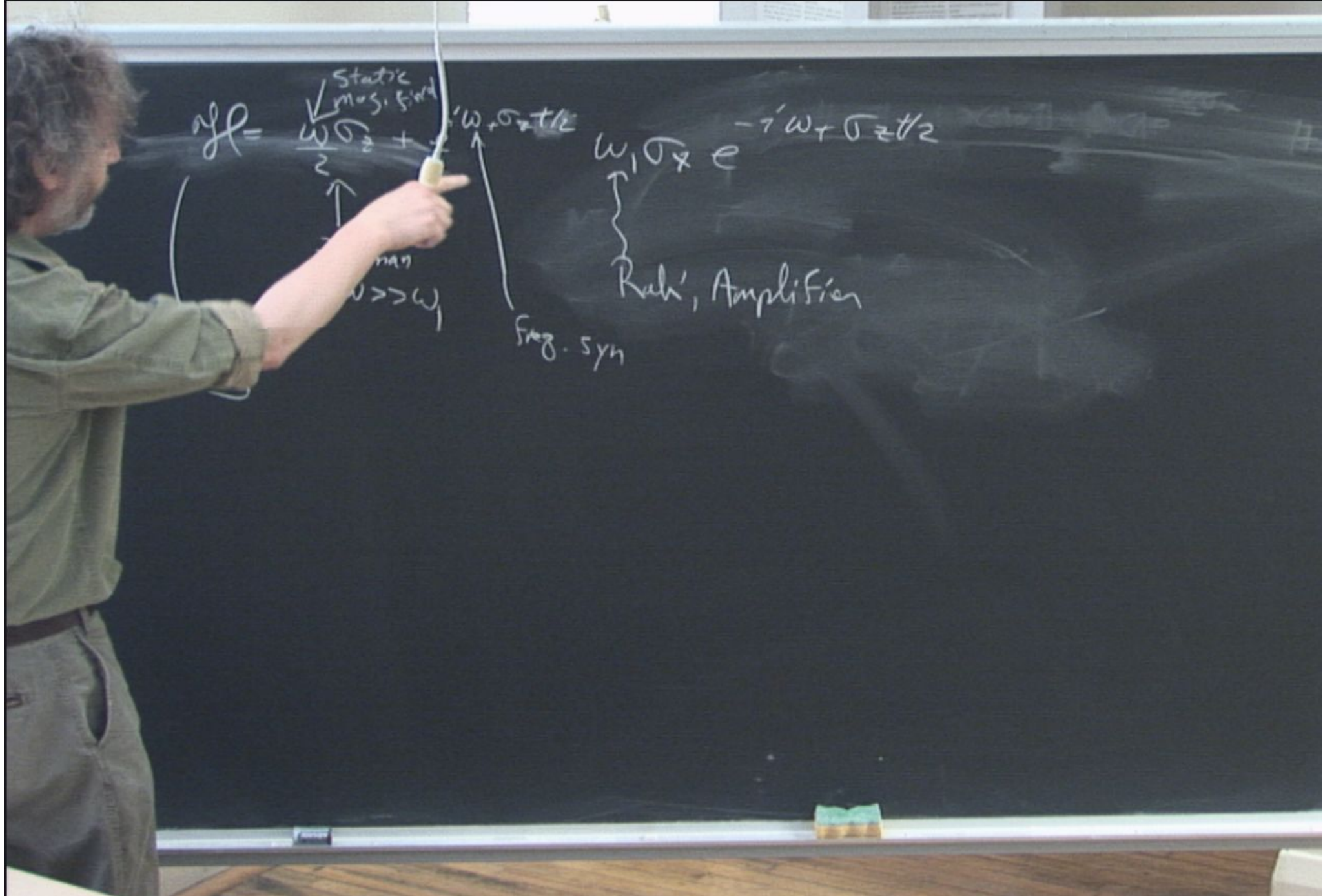
$$\alpha|00\rangle + \beta|11\rangle$$



$$\mathcal{H} = \frac{\omega \sigma_z}{2} + \omega_1 \sigma_x e^{-i(\omega + \sigma_z)t/2}$$

$\uparrow$   
 Zeeman  
 $\omega \gg \omega_1$

$\uparrow$   
 Rabi



$\mathcal{L} = \frac{\omega \sigma_z}{z} + \dots$

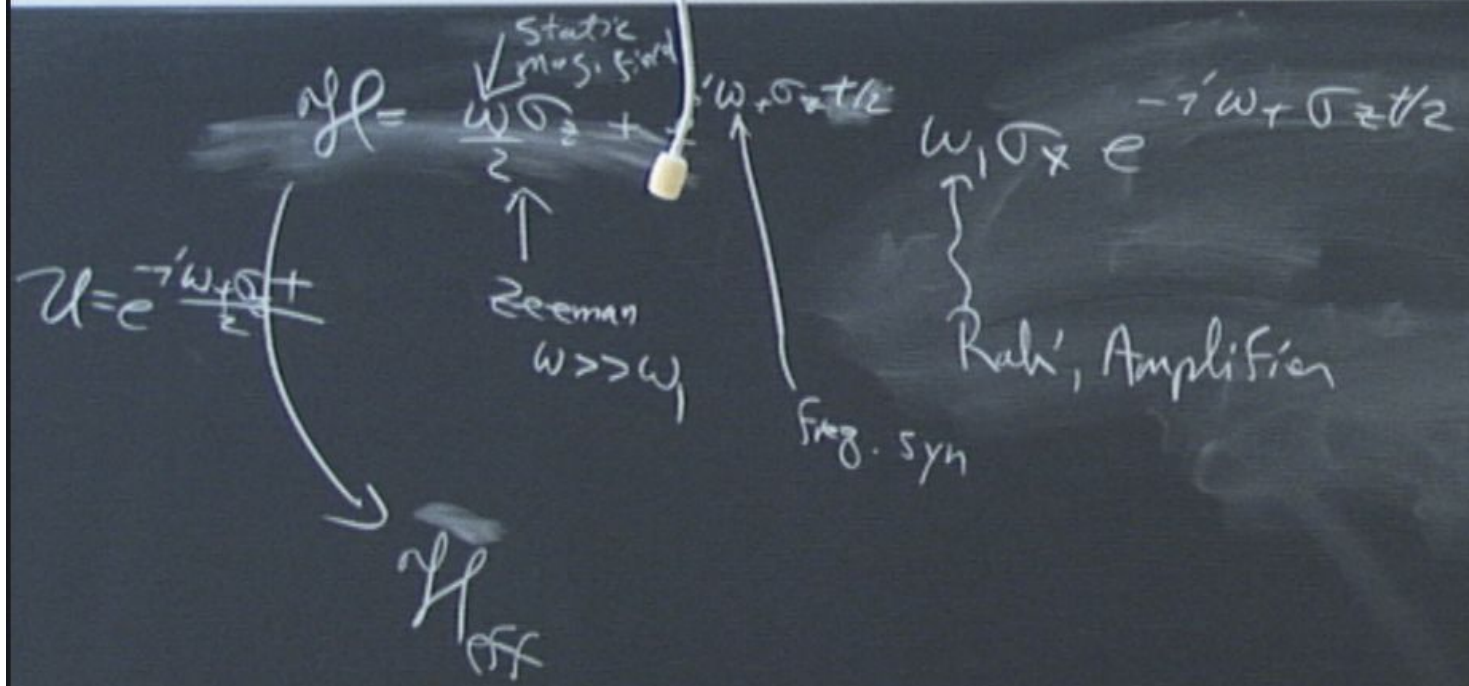
Static mag. field  $\omega + \sigma_z t/2$

$\omega_1 \sigma_x e^{-i\omega + \sigma_z t/2}$

$\omega \gg \omega_1$

Resonance, Amplifier

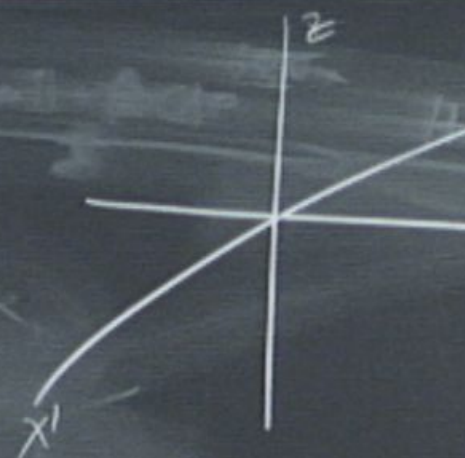
Frag. syn



$$\mathcal{H} = \frac{\omega_0 \sigma_z}{2} + \dots$$

Static mag. field  $\omega_0$   
 Zeeman  $\omega \gg \omega_0$   
 Frag. syn  $\omega_+ \sigma_x + \frac{1}{2}$   
 $\omega_+ \sigma_x e^{-i\omega_+ t/2}$   
 Rot., Amplifier

$$U = e^{-i\omega_+ t/2}$$

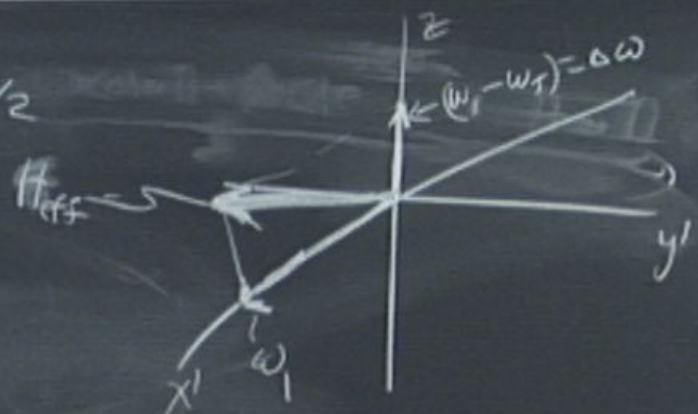


$$\mathcal{H}_{\text{eff}} = \frac{(\omega - \omega_+)}{2} \sigma_z + \omega_+ \sigma_x$$

$$\mathcal{H} = \frac{\omega_0 \sigma_z}{2} + \frac{\omega_+ \sigma_x}{2} e^{-i\omega_+ \sigma_z t/2}$$

Static mag. field  
 Zeeman  $\omega \gg \omega_+$   
 frag. sym  
 Rabi, Amplifier  
 $\omega_+ > (\omega - \omega_+)$

$$U = e^{-i\omega_+ \sigma_x t/2}$$



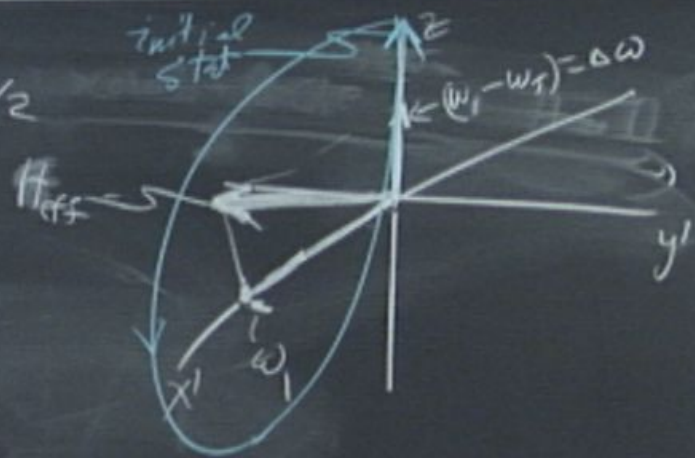
$$\mathcal{H}_{eff} = \frac{(\omega - \omega_+)}{2} \sigma_z + \omega_+ \sigma_x$$

$$\mathcal{H} = \frac{\omega \sigma_z}{2} + \omega_1 \sigma_x e^{-i\omega_1 t} e^{-i\omega_+ \sigma_z t/2}$$

Static mag. field  $\omega$   
 Zeeman  $\omega \gg \omega_1$   
 frag. sym  $\omega_1 \sigma_x$   
 Rabi, Amplifier  $\omega_1 > (\omega - \omega_+)$

$$U = e^{-i\omega_+ \sigma_z t/2}$$

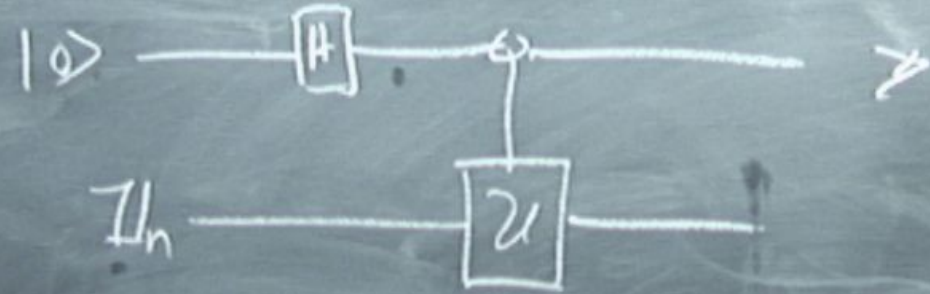
$$\mathcal{H}_{\text{eff}} = \frac{(\omega - \omega_+)}{2} \sigma_z + \omega_1 \sigma_x$$

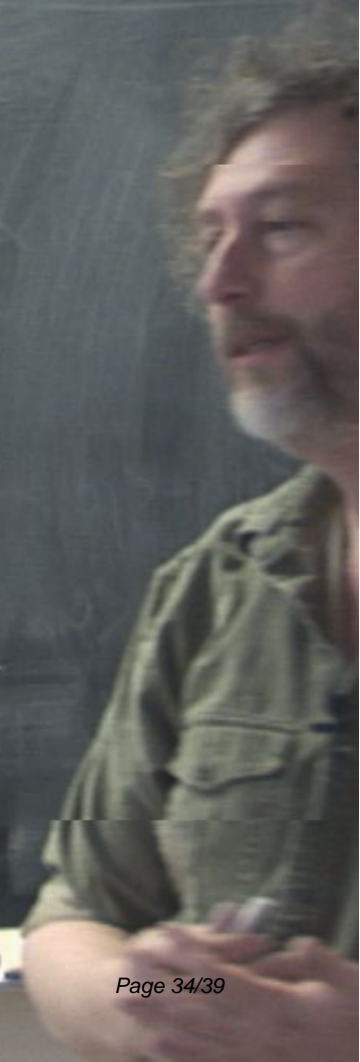
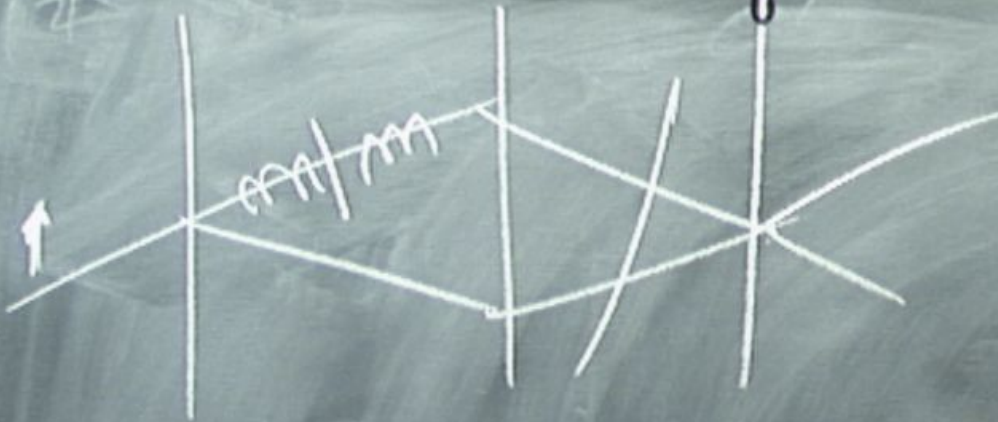


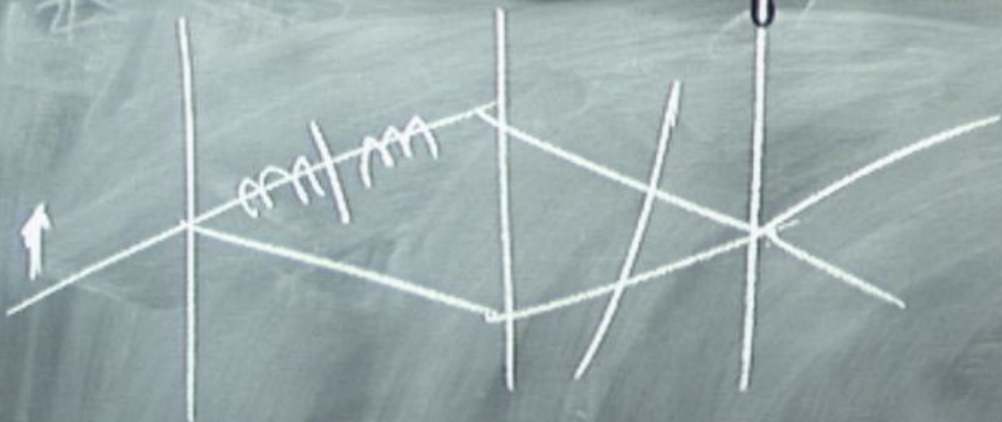
DOCI

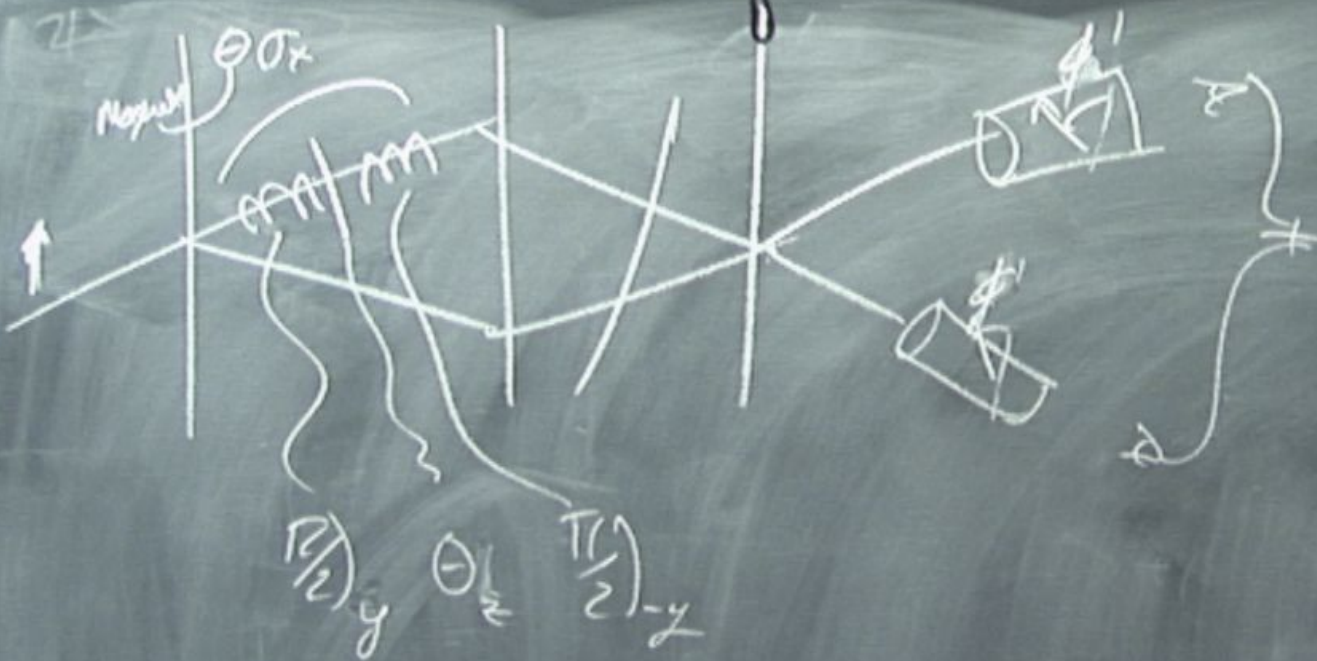


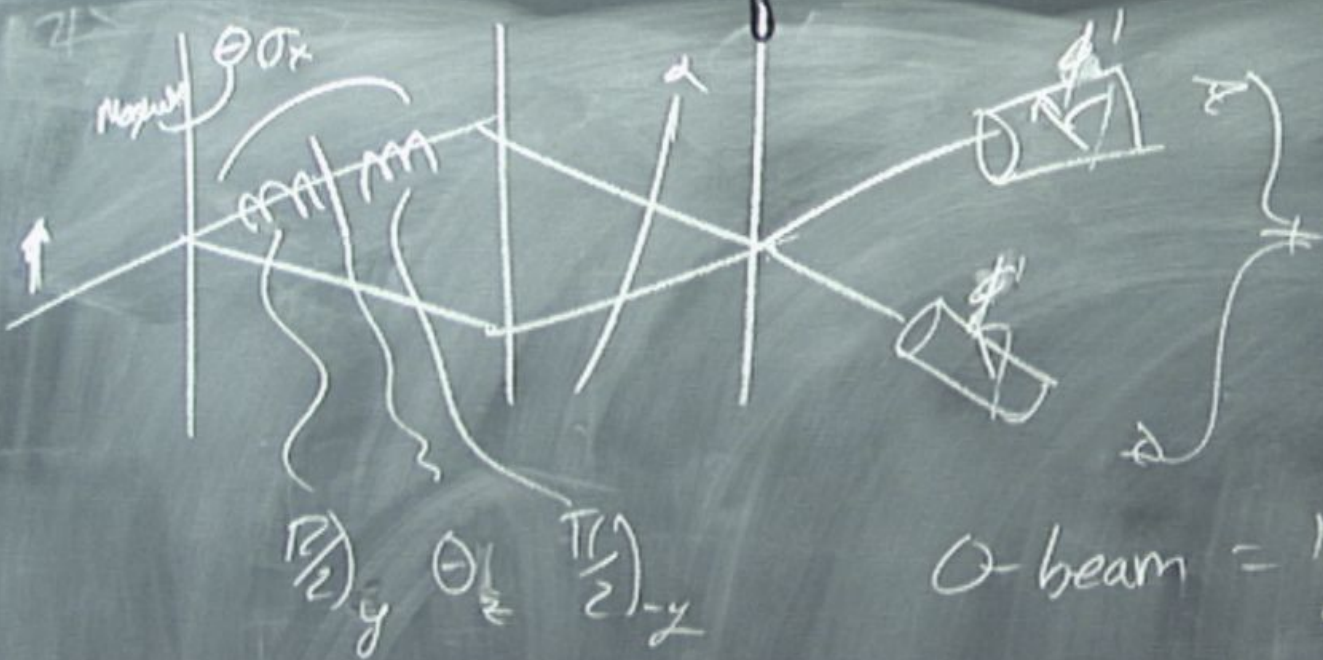
DQC1





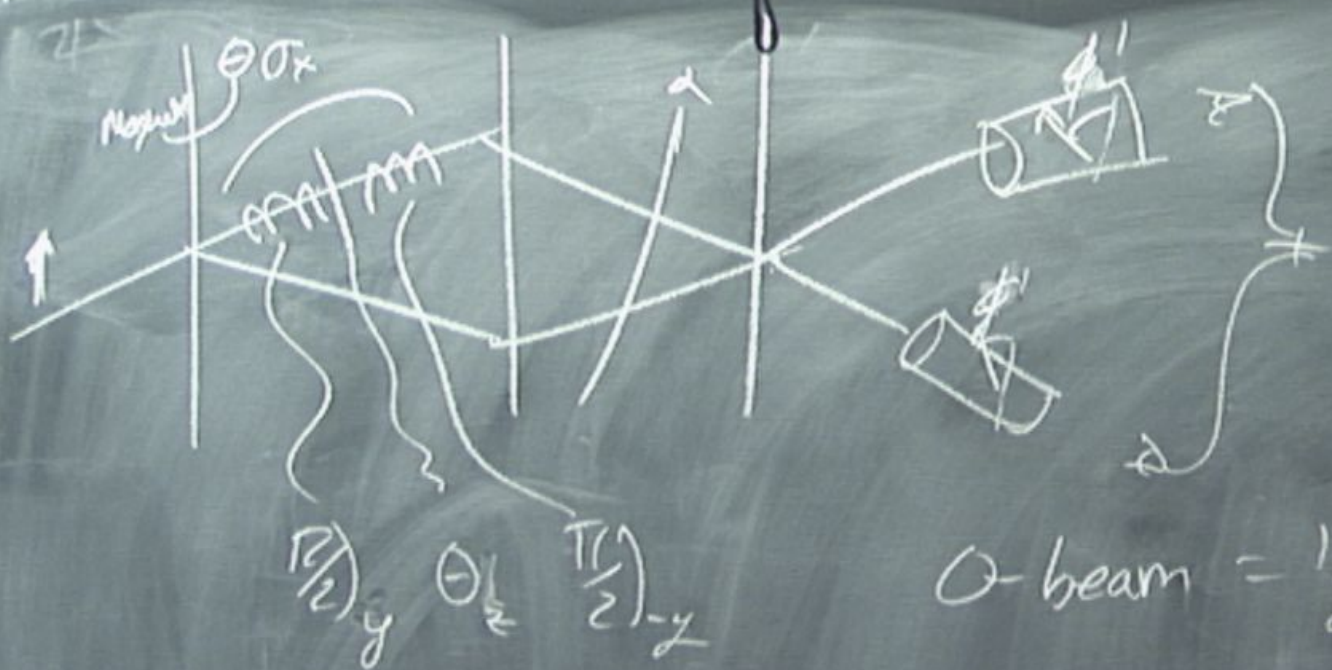






$$O\text{-beam} = \frac{1}{2} |0\rangle \left[ e^{i\phi} \right]$$

H-beam



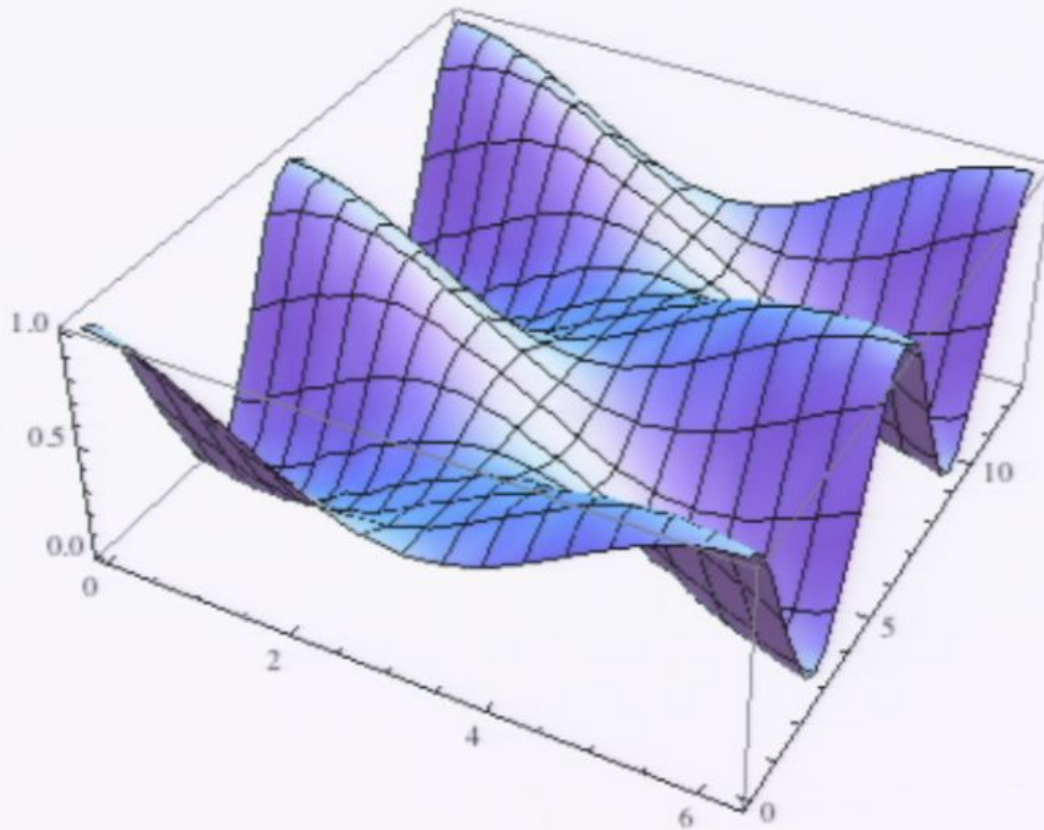
$$O\text{-beam} = \frac{1}{2} |0\rangle \left[ e^{i\theta} |\uparrow\rangle - (\cos\theta + i\sin\theta) |\downarrow\rangle \right]$$

$$H\text{-beam} = \frac{1}{2} |1\rangle \left[ e^{i\theta} |\uparrow\rangle + (\cos\theta + i\sin\theta) |\downarrow\rangle \right]$$

```

Mpq[a_, b_, c_, d_] := Tr[(Ukpsp + Ukmsp) . Uspinx[d, d] . pq[a, b, c] . Uspinxinv[d, d]]
IntMpq[b_, d_] := Integrate[
  Tr[(Ukpsp + Ukmsp) . Uspinx[d, d] . pq[a, b, 0] . Uspinxinv[d, d]], {a, 0, 2π}]/(2π)
Plot3D[Mpq[0, b, 0, d], {b, 0, 2π}, {d, 0, 4π}]

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```

Plot3D[IntMpq[b, d], {b, 0, 2π}, {d, 0, 4π}]

```

