Title: Explorations in Quantum Info. (PHYS 641) - Lecture 6

Date: Feb 23, 2010 09:00 AM

URL: http://pirsa.org/10020089

Abstract:

$$|\psi\rangle = \alpha|000\rangle + \beta|101\rangle + \gamma|110\rangle + \delta|111\rangle$$

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$C = |\alpha\delta - \beta\gamma|$$

$$|\psi\rangle = \alpha|000\rangle + \beta|011\rangle + \gamma|101\rangle + \delta|111\rangle$$

$$C = 2|\alpha\delta - \beta\gamma|$$

$$0 \leq C \leq 1$$

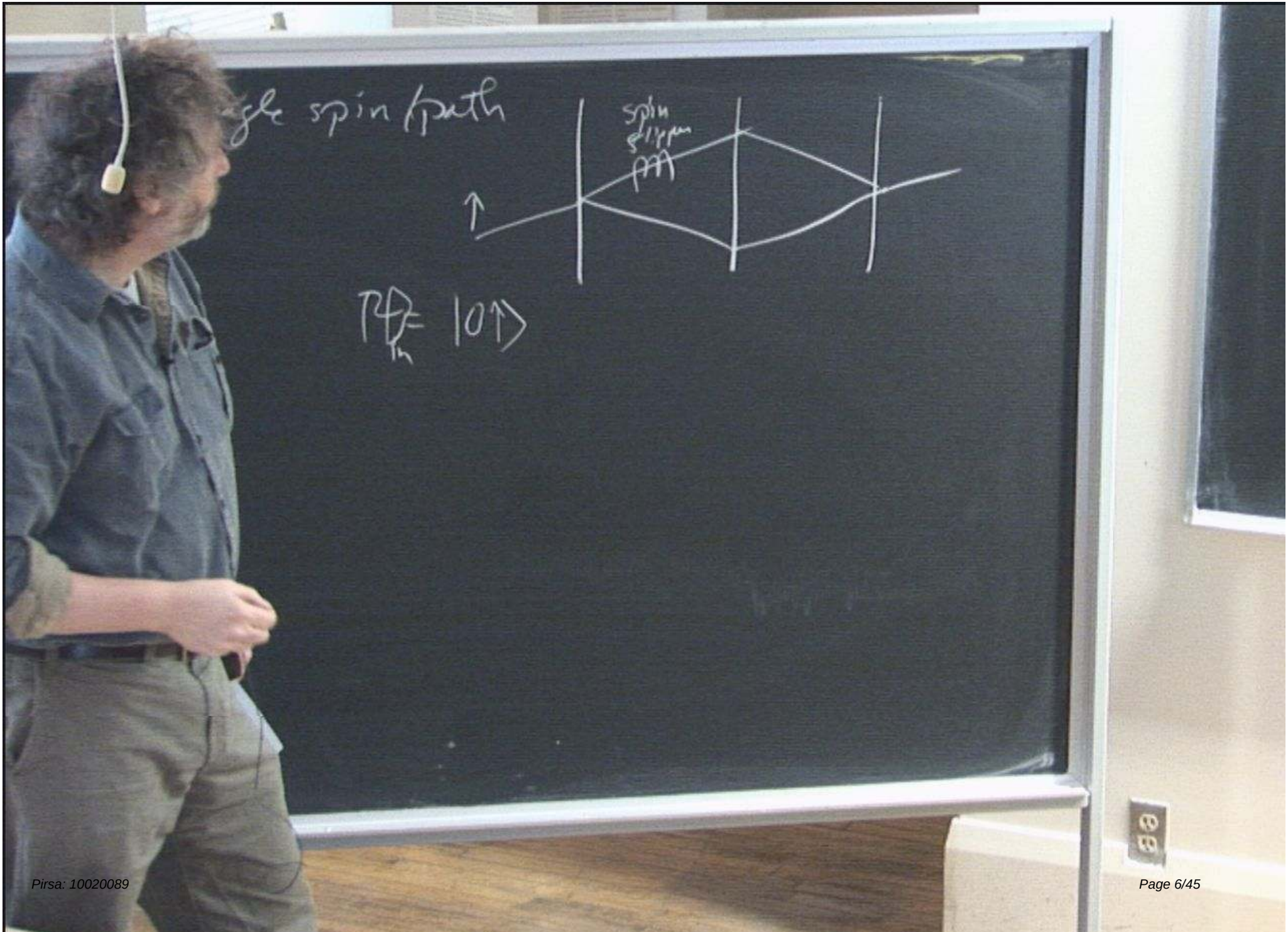
↑ separable

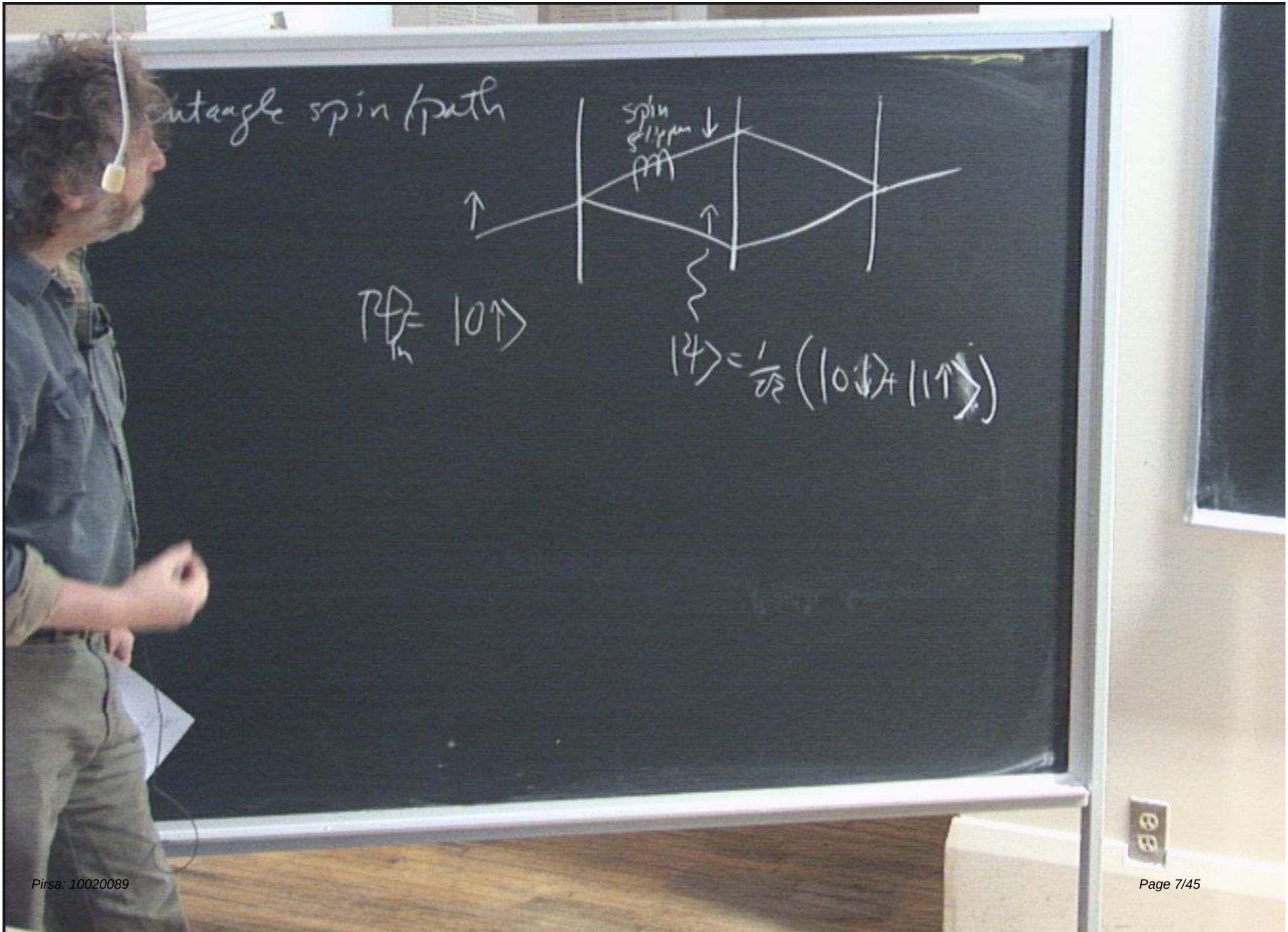
$$|\Psi\rangle = \alpha|000\rangle + \beta|011\rangle + \gamma|101\rangle + \delta|110\rangle$$

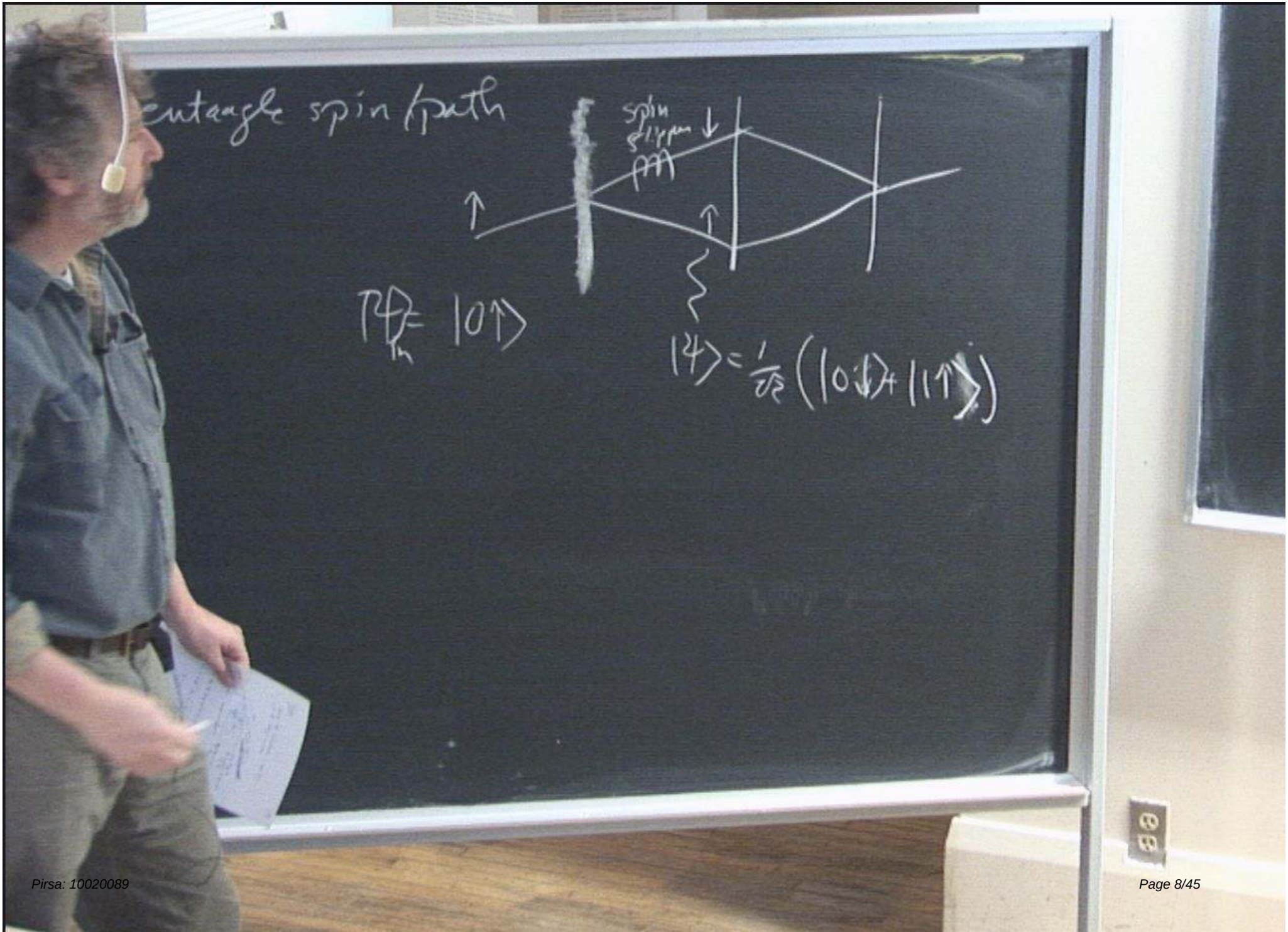
$$C = 2|\alpha\delta - \beta\gamma|$$

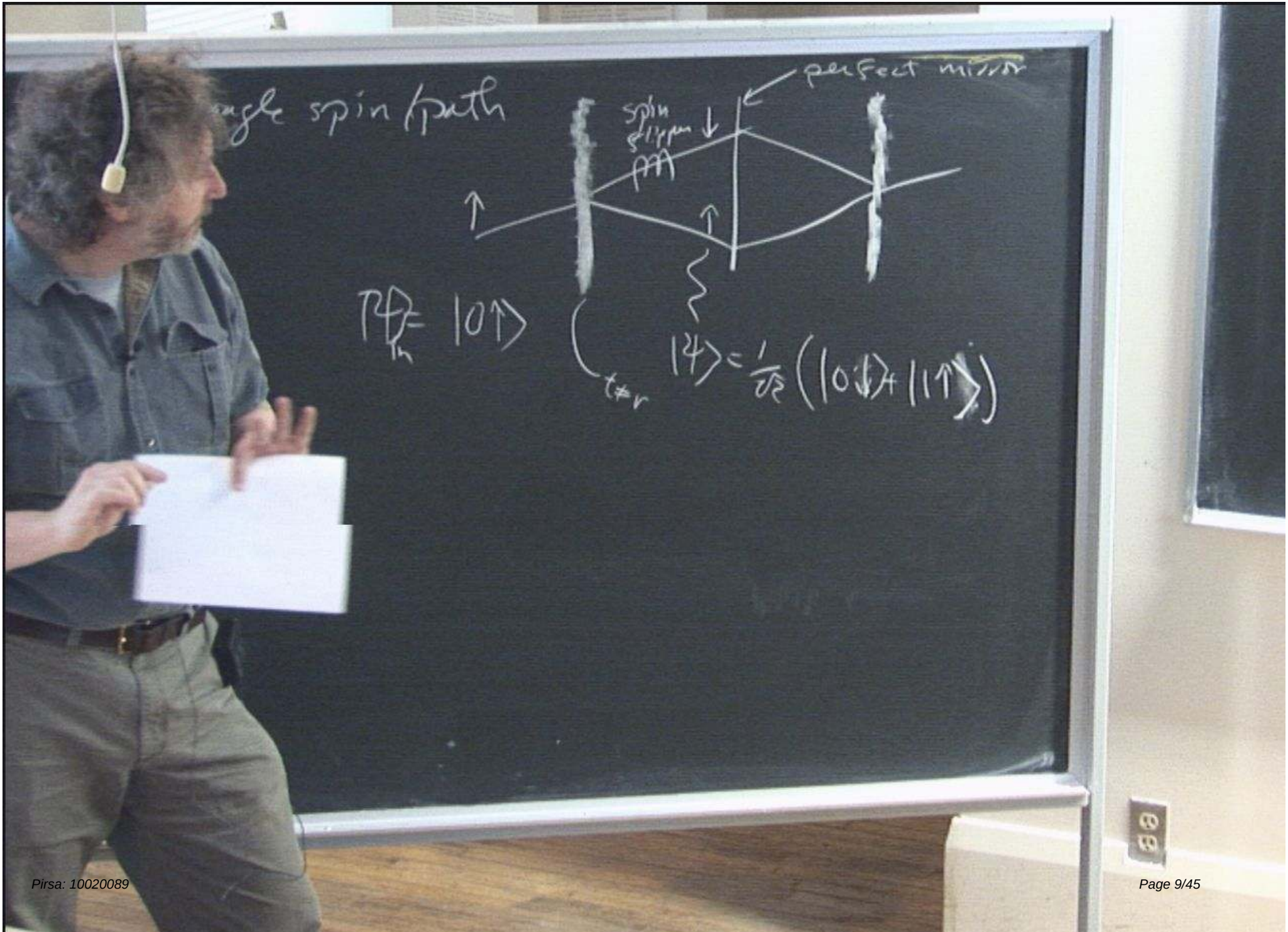
$$0 \leq C \leq 1$$

↑ Separable ↑ Bell

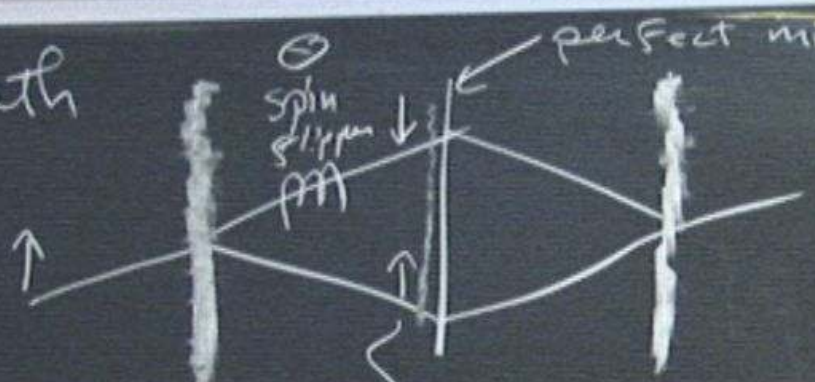

spin flip


spin
$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\downarrow\rangle + |1\uparrow\rangle\right)$


entangle spin/path
spin flipper
$|\psi_{in}\rangle = |0\uparrow\rangle$
$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\downarrow\rangle + |1\uparrow\rangle)$


perfect mirror
spin flipper
$|\psi\rangle = |0\uparrow\rangle$
$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\downarrow\rangle + |1\uparrow\rangle)$
$t \neq r$

entangle spin/path



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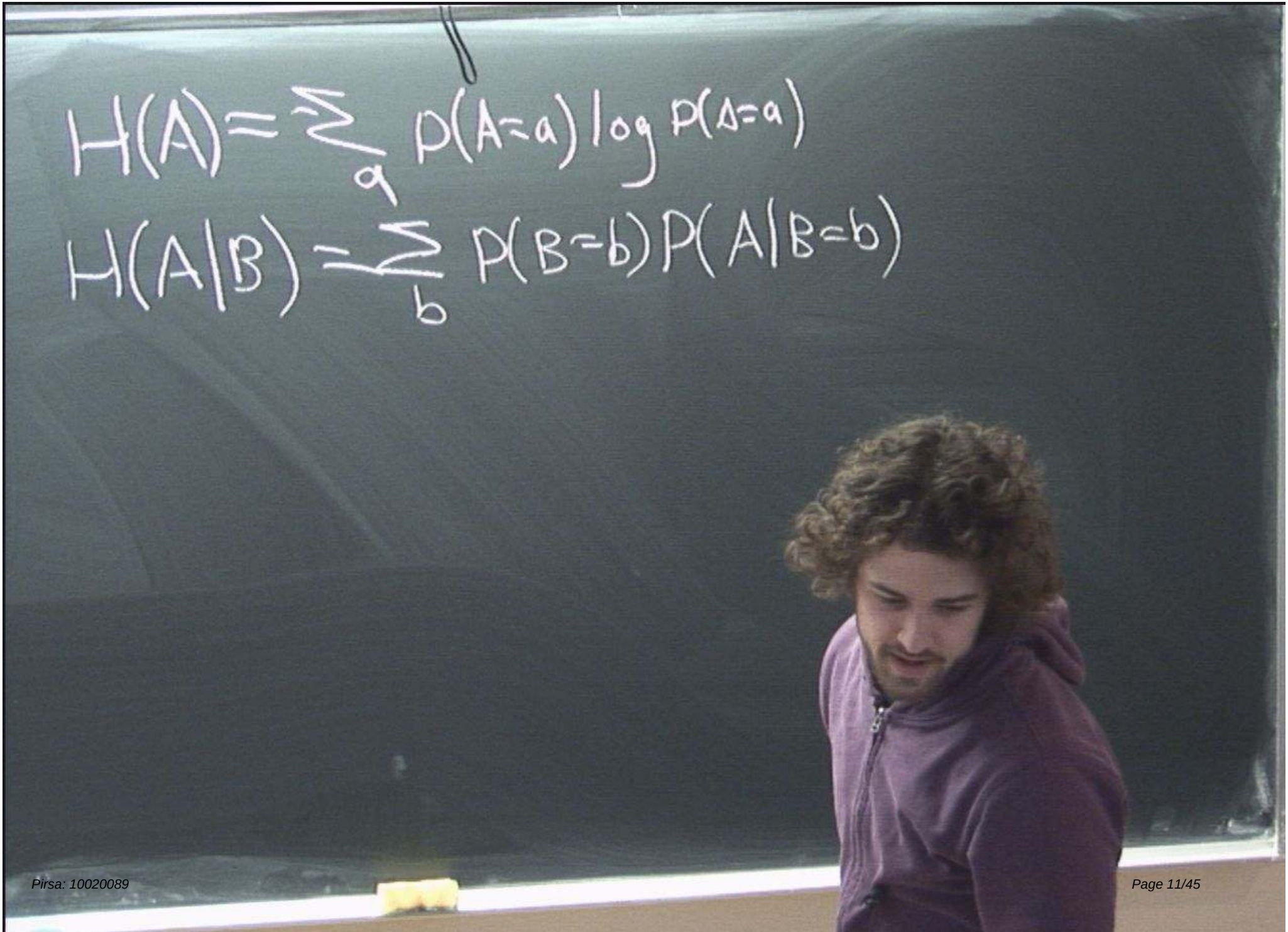
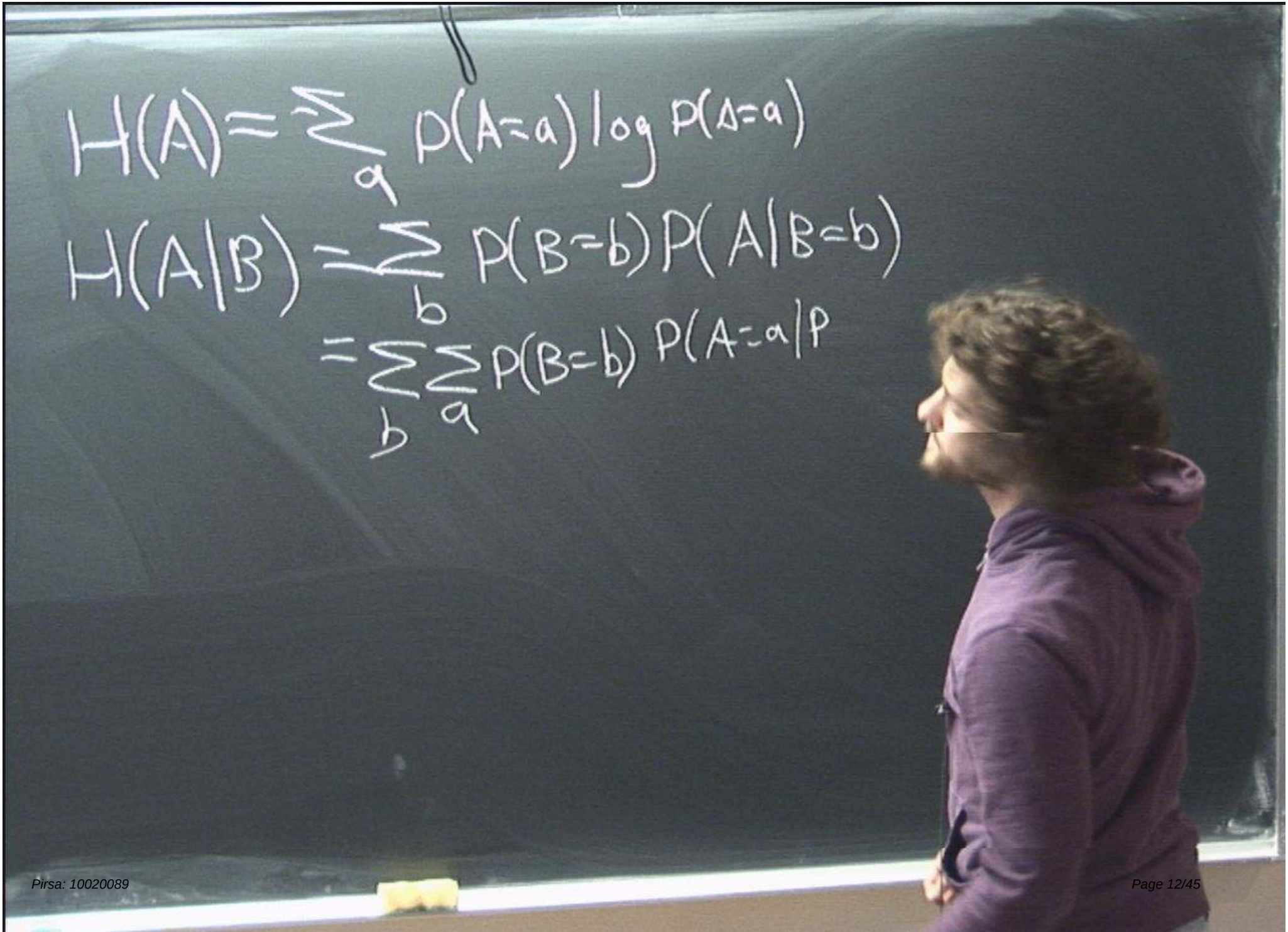
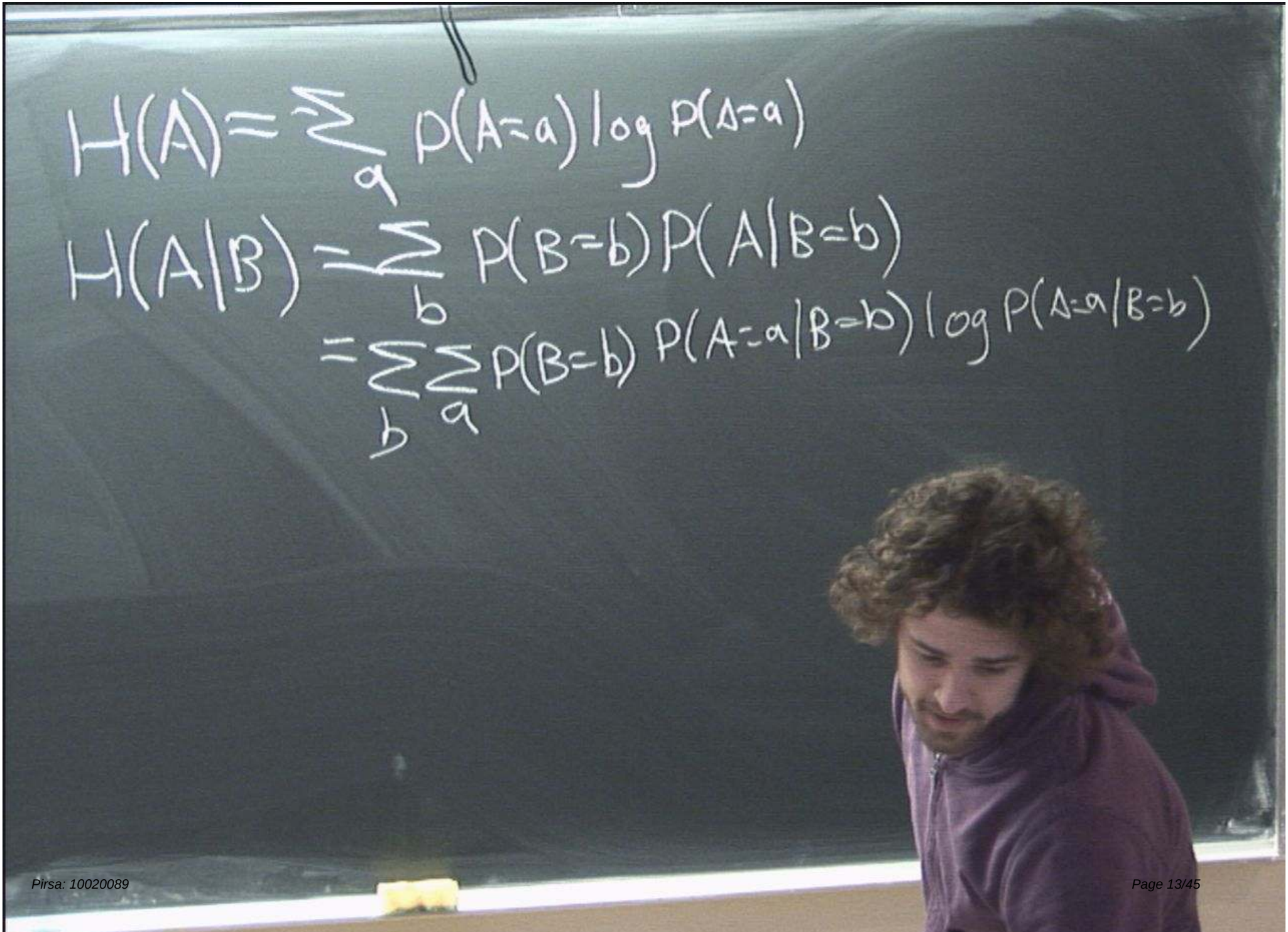
$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\downarrow\rangle + |1\uparrow\rangle)$

t_{arr}


$H(A) = \sum_a P(A=a) \log P(A=a)$
$H(A|B) = \sum_b P(B=b) P(A|B=b)$


$H(A) = \sum_a P(A=a) \log P(A=a)$
$H(A|B) = \sum_b P(B=b) P(A|B=b)$
$= \sum_b \sum_a P(B=b) P(A=a|B$


$H(A) = -\sum_a P(A=a) \log P(A=a)$
$H(A|B) = \sum_b P(B=b) P(A|B=b)$
$= \sum_b \sum_a P(B=b) P(A=a|B=b) \log P(A=a|B=b)$

$$H(A) = \sum_a P(A=a) \log P(A=a)$$

$$H(A|B) = \sum_b P(B=b) H(A|B=b)$$

$$= \sum_b \sum_a P(B=b)\, P(A=a|B=b) \log P(A=a|B=b)$$

$$J(A:B) = H(A) - H(A|B)$$

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$C = 2|\alpha\delta - \beta\gamma|$$

$$0 \leq C \leq 1$$

0 ↑ Separable

1 ↑ Bell

$$H(A|B) = H(A,B) - H(B)$$

$$H(A) = \sum_a P(A=a) \log P(A=a)$$

$$H(A|B) = \sum_b P(B=b) H(A|B=b)$$

$$= \sum_b \sum_a P(B=b)\, P(A=a|B=b) \log P(A=a|B=b)$$

(B)

$$J(A:B) = H(A) - H(A|B)$$

$$I(A:B) = H(A) - H(B) - H(A,B)$$

$$H(A) = \sum_a P(A=a) \log P(A=a)$$

$$H(A|B) = \sum_b P(B=b) H(A|B=b)$$

$$= \sum_b \sum_a P(B=b)\, P(A=a|B=b) \log P(A=a|B=b)$$

(B)

$$J(A:B) = H(A) - H(A|B)$$

$$I(A:B) = H(A) - H(B) - H(A,B)$$

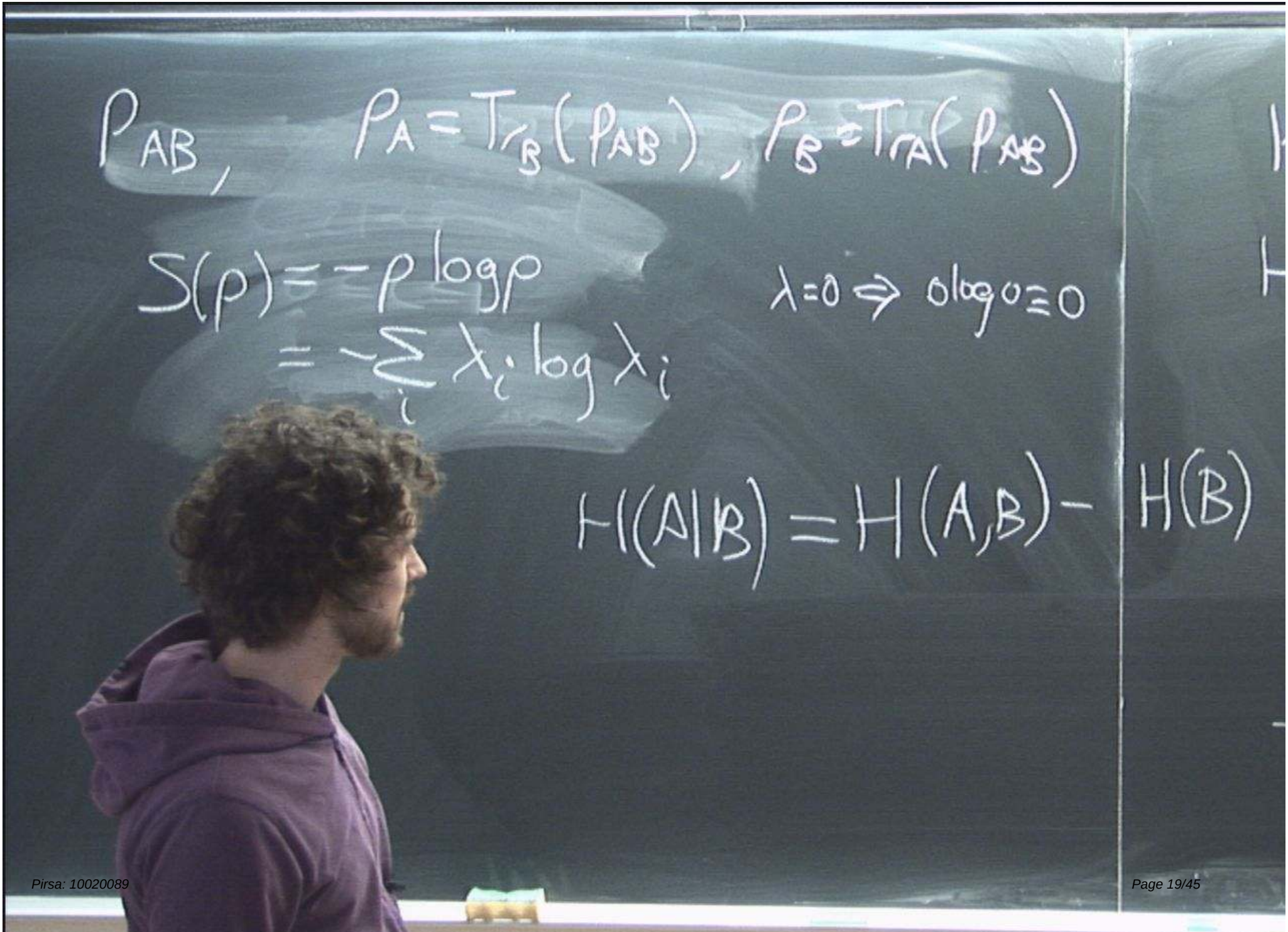
ρ_{AB}, $\rho_A = Tr_B(\rho_{AB})$, $\rho_B = Tr_A(\rho_{AB})$

$S(\rho) = -\rho \log \rho$

$= -\sum_i \lambda_i \log \lambda_i$

$\lambda = 0 \Rightarrow 0 \log 0 \equiv 0$

$H(A|B) = H(A,B) -$ $H(B)$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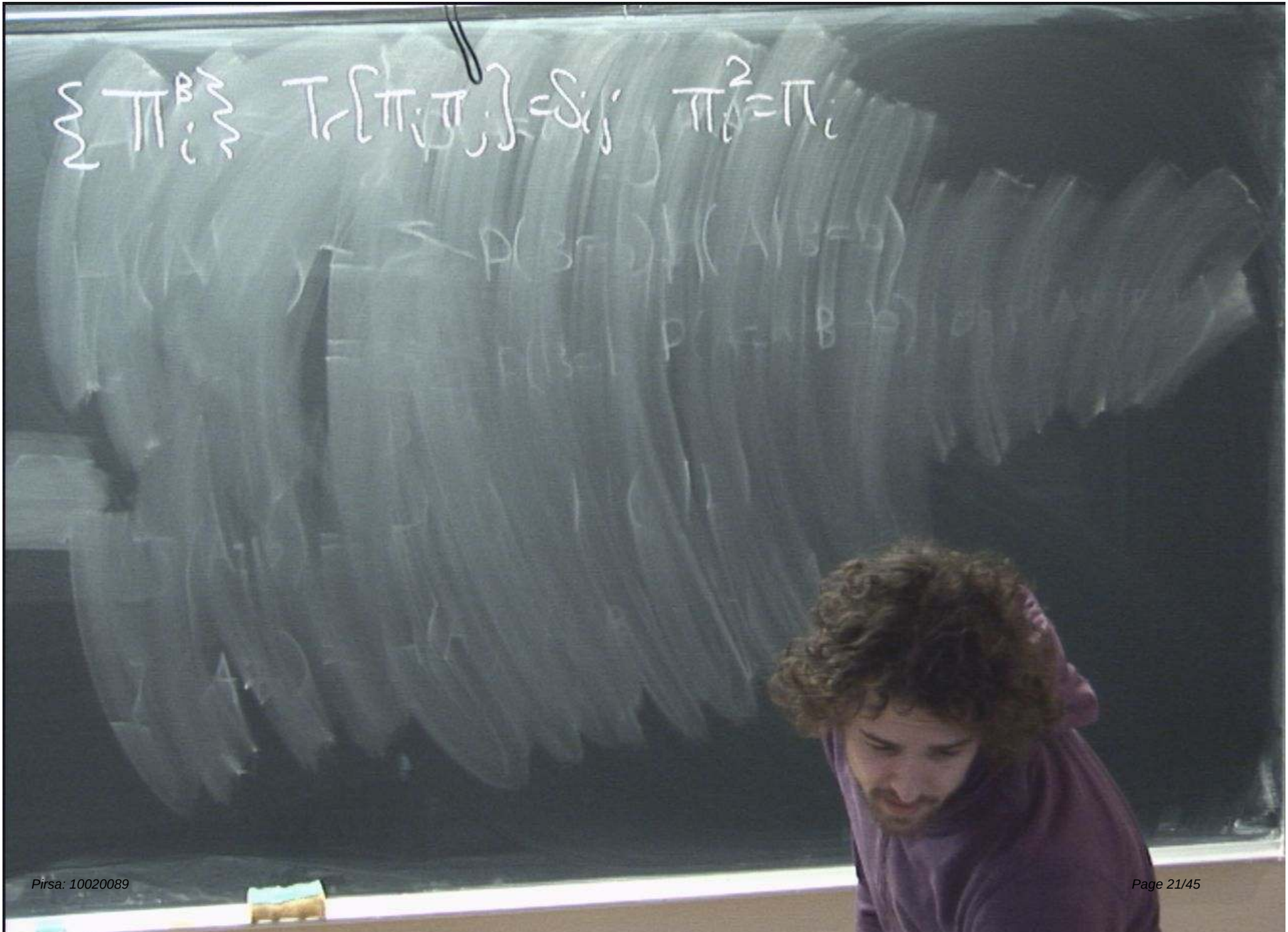
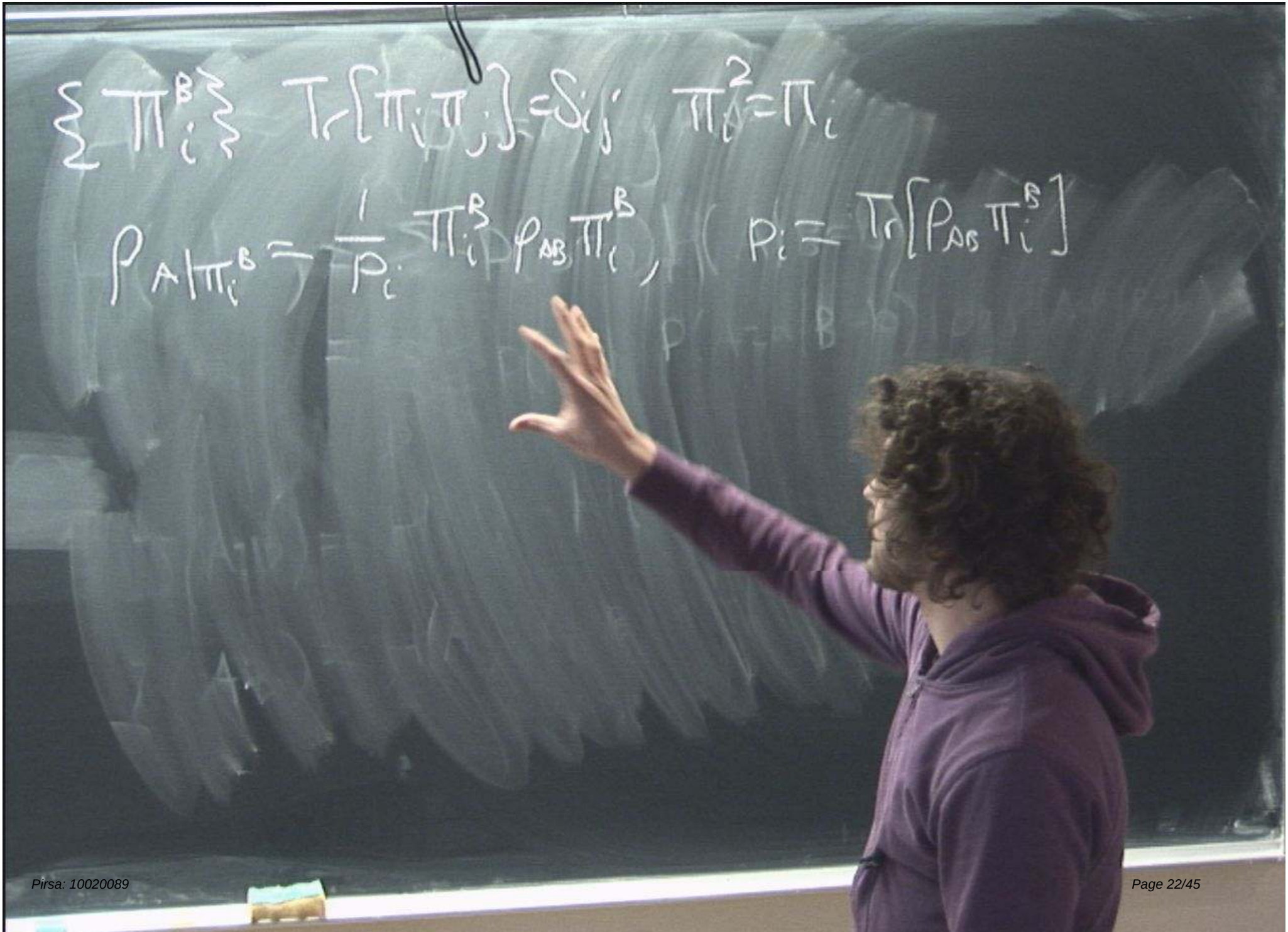
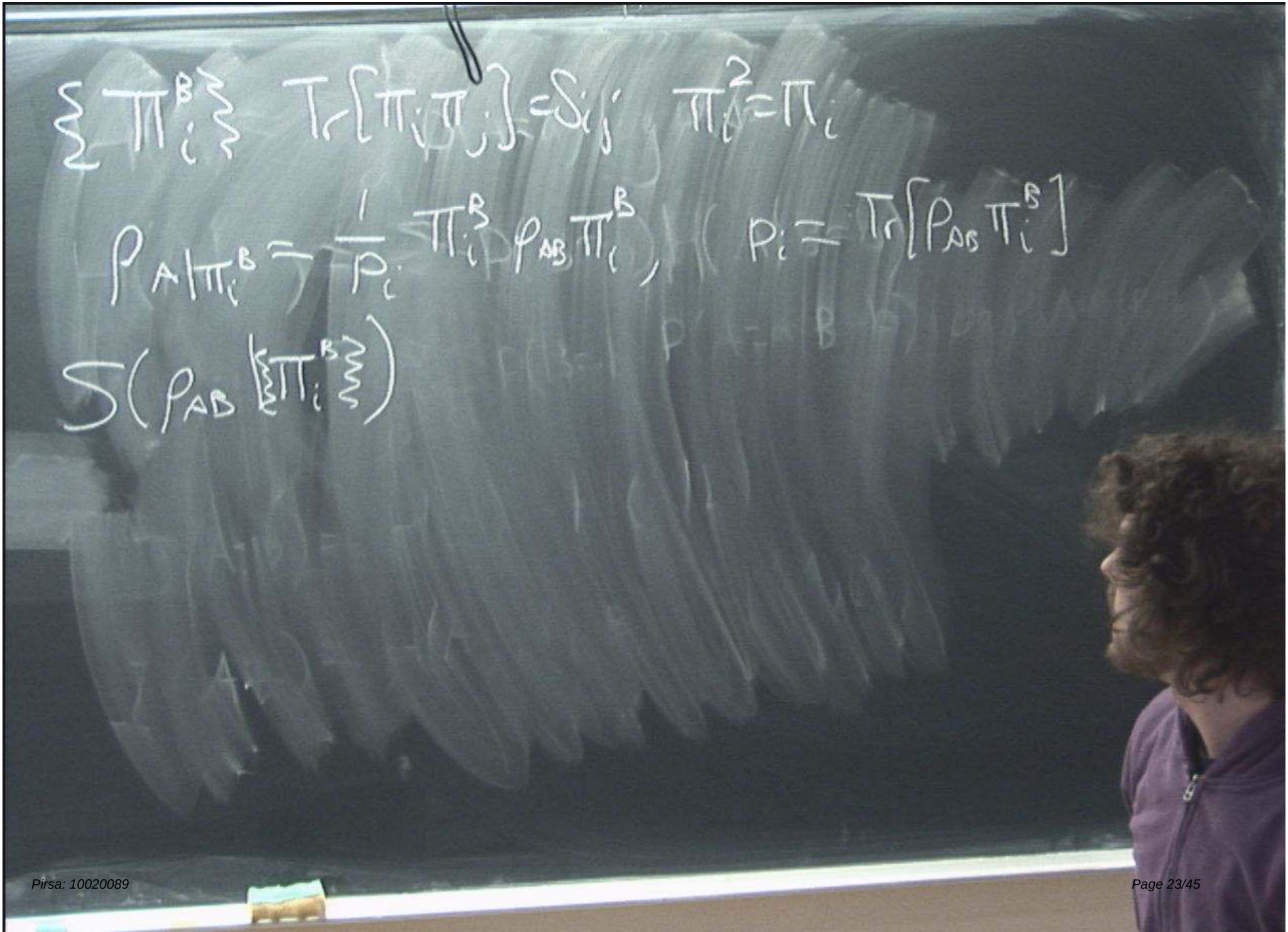
$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$




$\{\Pi_i^B\}$ $\mathrm{Tr}[\Pi_i \Pi_j] = \delta_{ij}$ $\Pi_i^2 = \Pi_i$
$\rho_{A|\Pi_i^B} = \frac{1}{p_i} \Pi_i^B \rho_{AB} \Pi_i^B$ $p_i = \mathrm{Tr}[\rho_{AB} \Pi_i^B]$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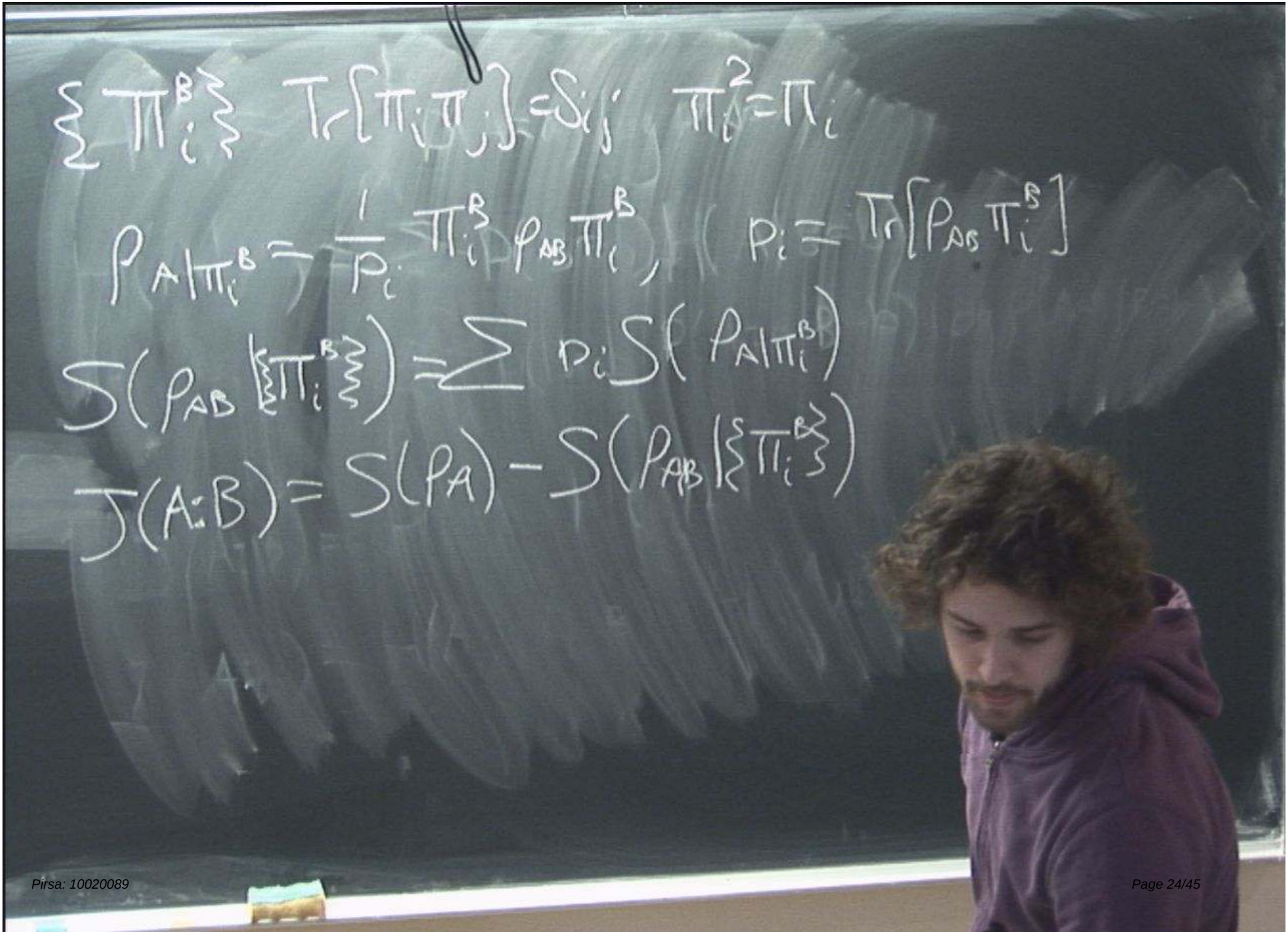
$S(\rho_{AB} \| \{\Pi_i^B\})$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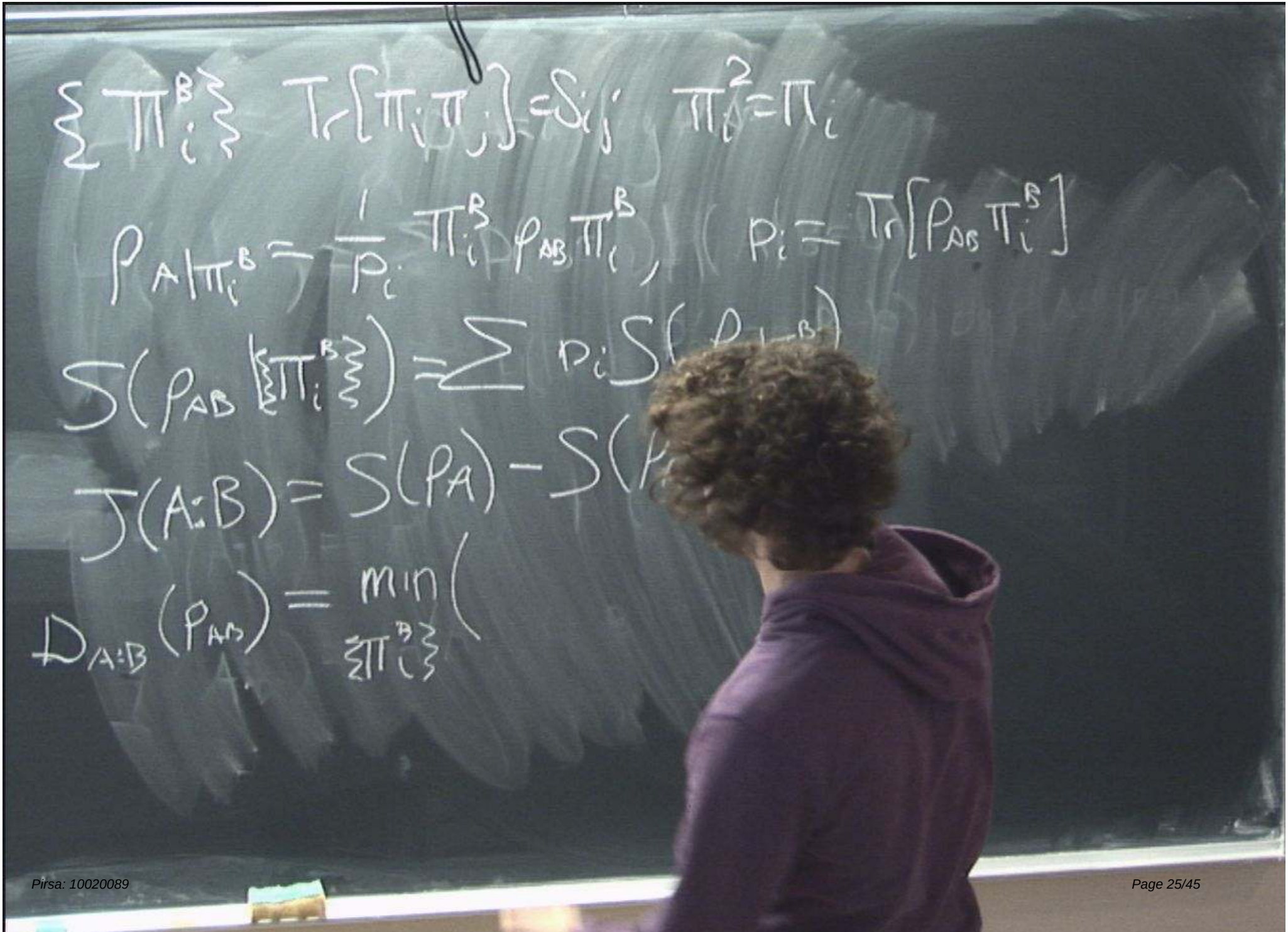
$\rho_{A|\Pi_i^B} = \frac{1}{p_i} \Pi_i^B \rho_{AB} \Pi_i^B$, $p_i = \mathrm{Tr}[\rho_{AB} \Pi_i^B]$
$S(\rho_{AB}|\{\Pi_i^B\}) = \sum p_i S(\rho_{A|\Pi_i^B})$
$J(A:B) = S(\rho_A) - S(\rho_{AB}|\{\Pi_i^B\})$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$S(\rho_{AB} \| \{\Pi_i^B\}) = \sum p_i S($
$J(A:B) = S(\rho_A) - S($
$D_{A:B}(\rho_{AB}) = \min_{\{\Pi_i^B\}} ($

$$\{\Pi_i^B\} \quad \mathrm{Tr}[\Pi_i \Pi_j] = \delta_{ij} \quad \Pi_i^2 = \Pi_i$$

$$\rho_{A|\Pi_i^B} = \frac{1}{p_i} \Pi_i^B \rho_{AB} \Pi_i^B, \quad p_i = \mathrm{Tr}[\rho_{AB} \Pi_i^B]$$

$$S(\rho_{AB} | \{\Pi_i^B\}) = \sum p_i S(\rho_{A|\Pi_i^B})$$

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{AB} | \{\Pi_i^B\})$$

$$D_{A:B}(\rho_{AB}) = \min_{\{\Pi_i^B\}} \left(I_{A:B}(\rho_{AB}) - J_{A:B}(\rho_{AB}) \right)$$

$$\{\Pi_i^B\} \quad \mathrm{Tr}[\Pi_i \Pi_j] = \delta_{ij} \quad \Pi_i^2 = \Pi_i$$

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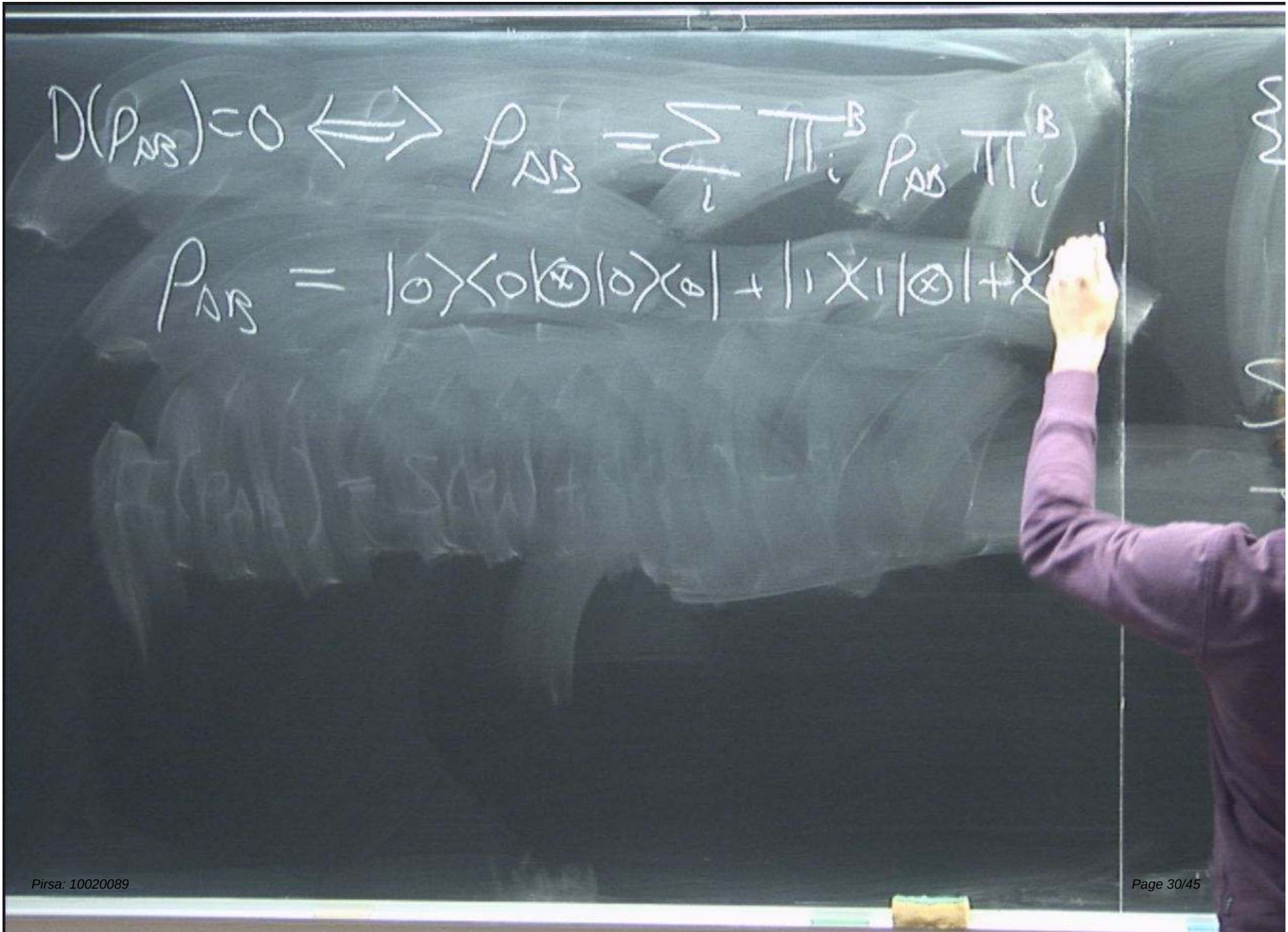
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$\rho_{AB} = |0\rangle\langle 0| \otimes |0\rangle\langle 0| + |1\rangle\langle 1| \otimes |1\rangle\langle 1| + \ldots$

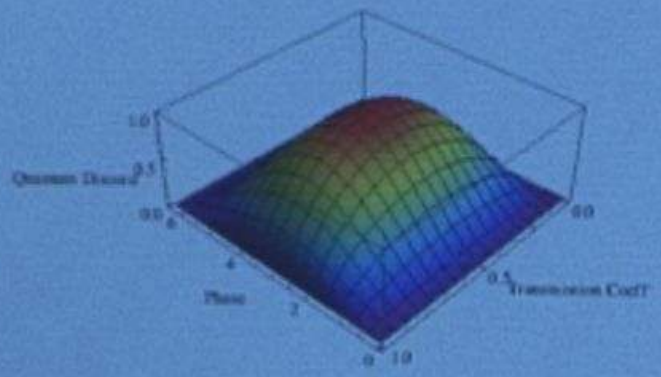
$$D(\rho_{AB}) = 0 \iff \rho_{AB} = \sum_i \Pi_i^B \rho_{AB} \Pi_i^B$$

$$\rho_{AB} = \frac{|0\rangle\langle 0| \otimes |0\rangle\langle 0|}{2} + \frac{|1\rangle\langle 1| \otimes |+\rangle\langle +|}{2}$$

3 blade with spin

- Quantum discord functions
  - Partial Trace
  - Entropy and Conditional Entropy
  - Functions for calculating quantum discord
- Unitaries for Ni elements
- Quantum discord of output state
  - Output State
  - Quantum discord of output state
  - Discord as function of spin phase and blade transmission coefficient

```
Plot3D[
 DiscordAB[StateOut[ϕ, x], 10], {ϕ, 0.0, 2.0 Pi}, {x, 0.0, 1.0},
 AxesLabel -> {"Phase", "Transmission Coeff", "Quantum Discord"},
 ColorFunction → "Rainbow"]
```

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Plot3D[
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```

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poison in o

2010 GRT Transit Map

Pang.pdf

Lugo. Dif geometry

Mathem Differenti

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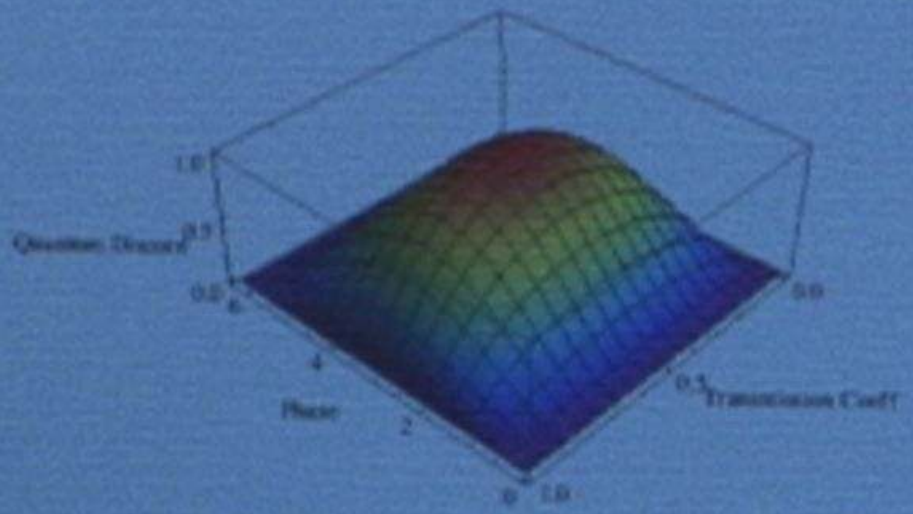


```
Plot3D[
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```

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- Partial Trace
- Entropy and Conditional Entropy
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Unitaries for NI elements

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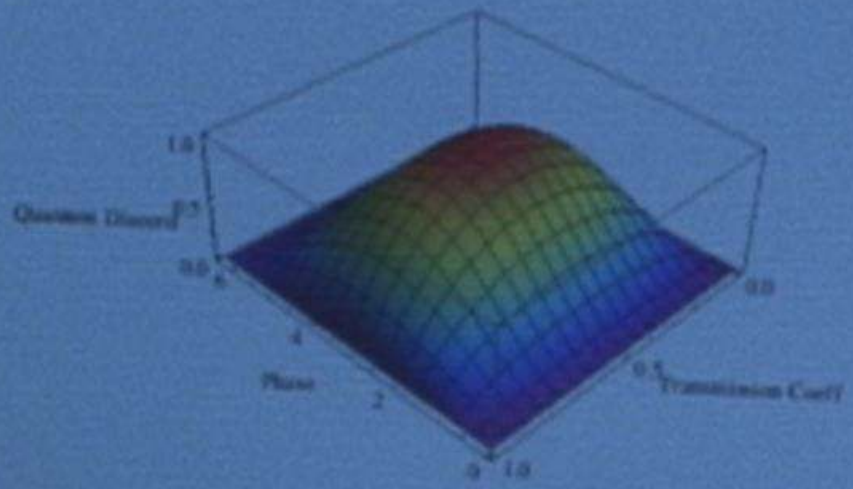


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$$D(\rho_{AB}) = 0 \iff \rho_{AB} = \sum_i \Pi_i^B \rho_{AB} \Pi_i^B$$

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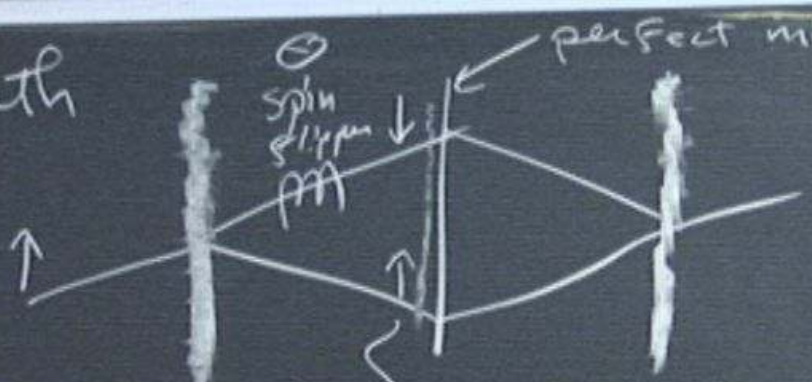
$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$\left(|100\rangle + |010\rangle + |001\rangle\right)$$

entangle spin/path



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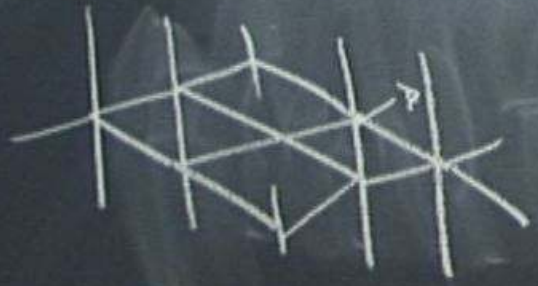

$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

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D_A

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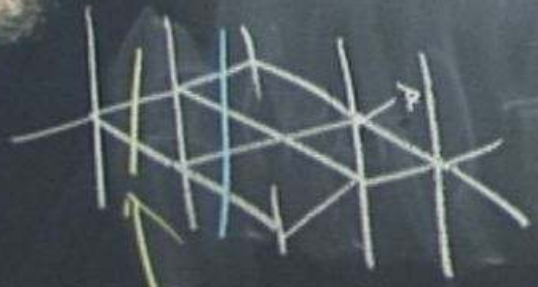

$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$\left(|100\rangle + |010\rangle + |001\rangle\right)$$

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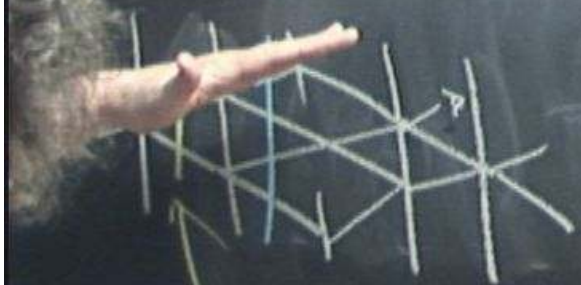

$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$\left(|100\rangle + |010\rangle + |001\rangle\right)$$

control the prob. that neutrons remain in NZ

D_A

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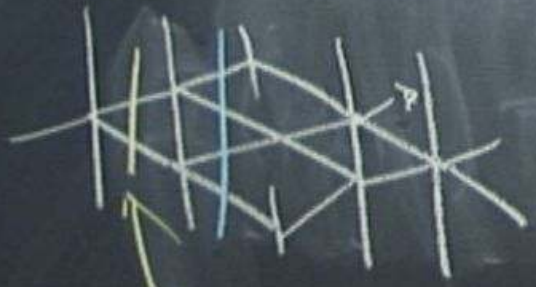

control the probs that neutrons remain in NZ

$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$\left(|100\rangle + |010\rangle + |001\rangle\right)$$

$\sum$

D_A

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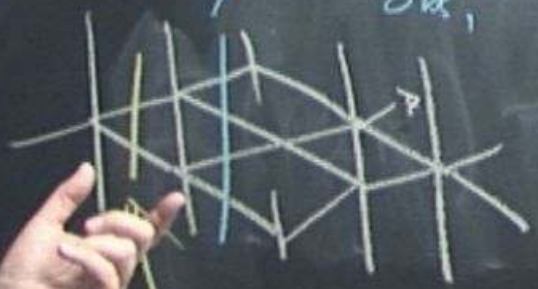

$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$\left(|100\rangle + |010\rangle + |001\rangle\right)$$

control the Tesla that neutrons remain in NZ

D_A

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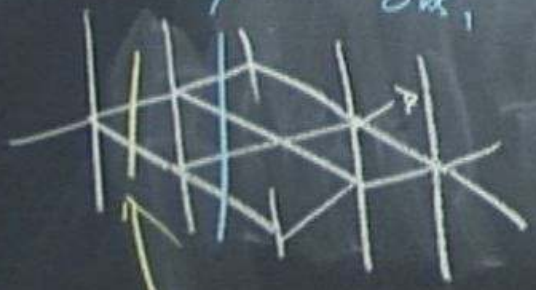

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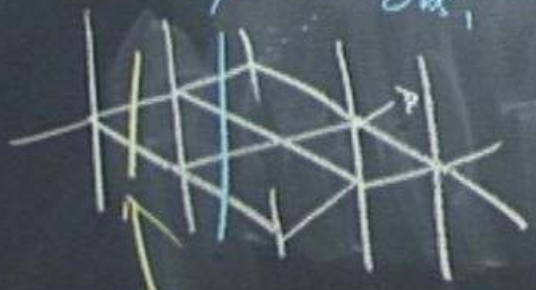

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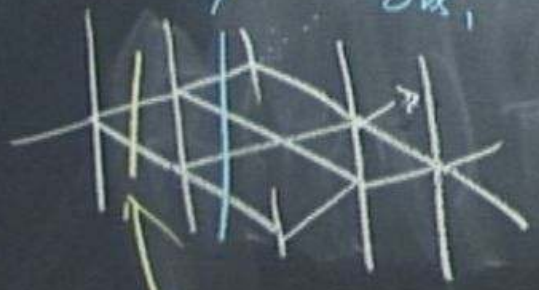

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