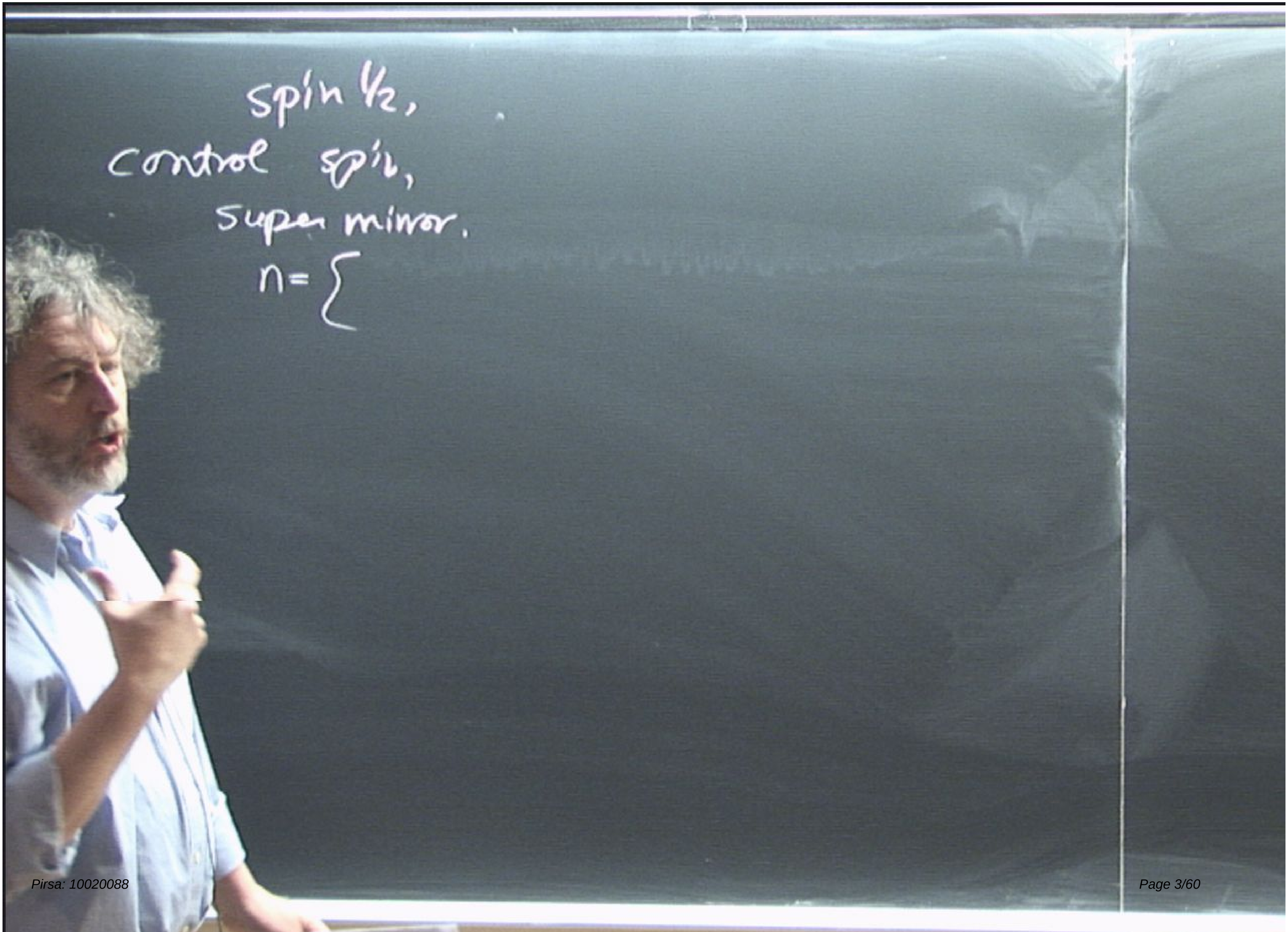
Title: Explorations in Quantum Info. (PHYS 641) - Lecture 5

Date: Feb 22, 2010 09:00 AM

URL: http://pirsa.org/10020088

Abstract:

spin $1/2$,


spin 1/2,
control spin,
super mirror.
n= {

spin 1/2,

control spin,

super mirror.

$$n = \left\{ 1 - \frac{V(r)}{E} \right\}^{1/2}$$

spin ½,

control spin,

super mirror.

$$n = \left\{ 1 - \frac{V(r)}{E} \right\}^{1/2}$$

$$V_{mag} = \mu \cdot B$$

spin 1/2,

control spin,

super mirror.

$$n = \left\{ 1 - \frac{V(r)}{E} \right\}^{1/2} \sim 1 - 10^{-5}$$

$$V_{mag} = \mu \cdot B$$

$$n_+ \neq n_-$$

spin 1/2,

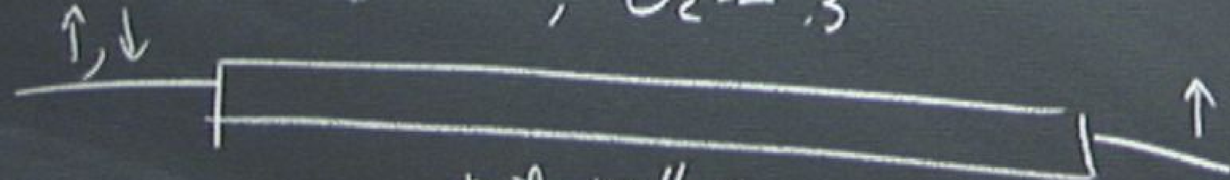
control spin,

super mirror.

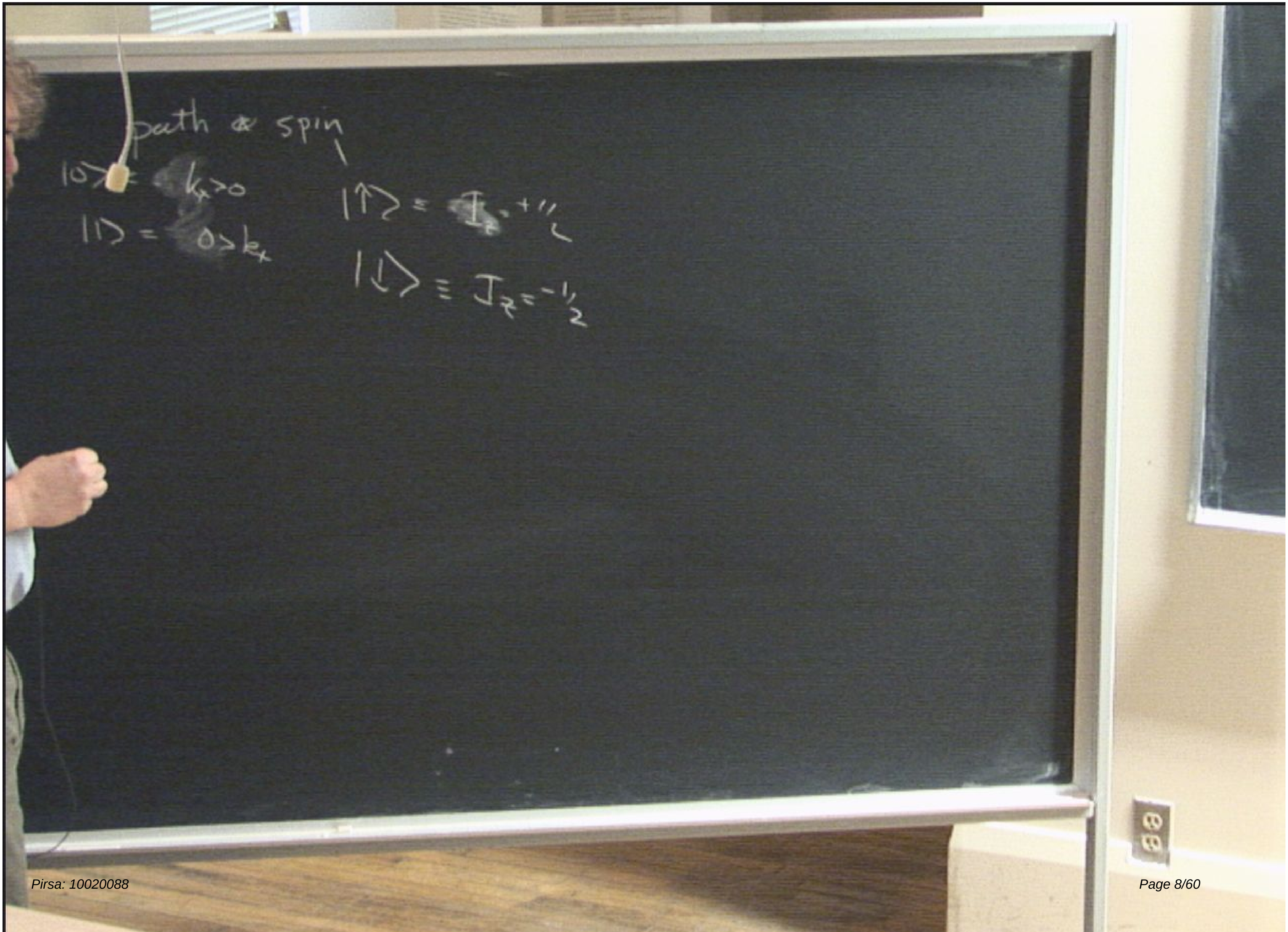
$$n = \left\{ 1 - \frac{V(r)}{E} \right\}^{1/2} \sim 1 - 10^{-5}$$

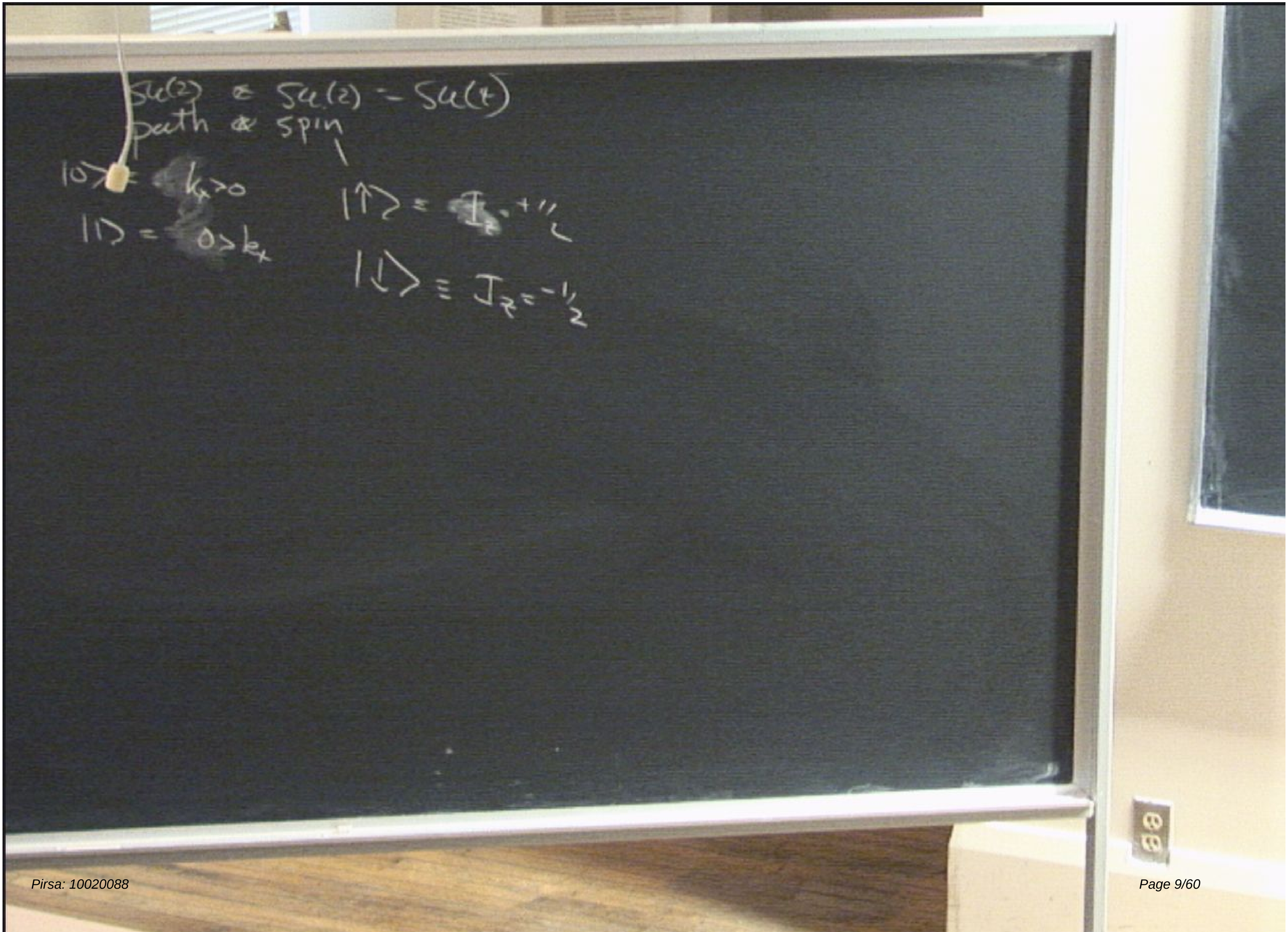
$$V_{mag} = \mu \cdot B$$

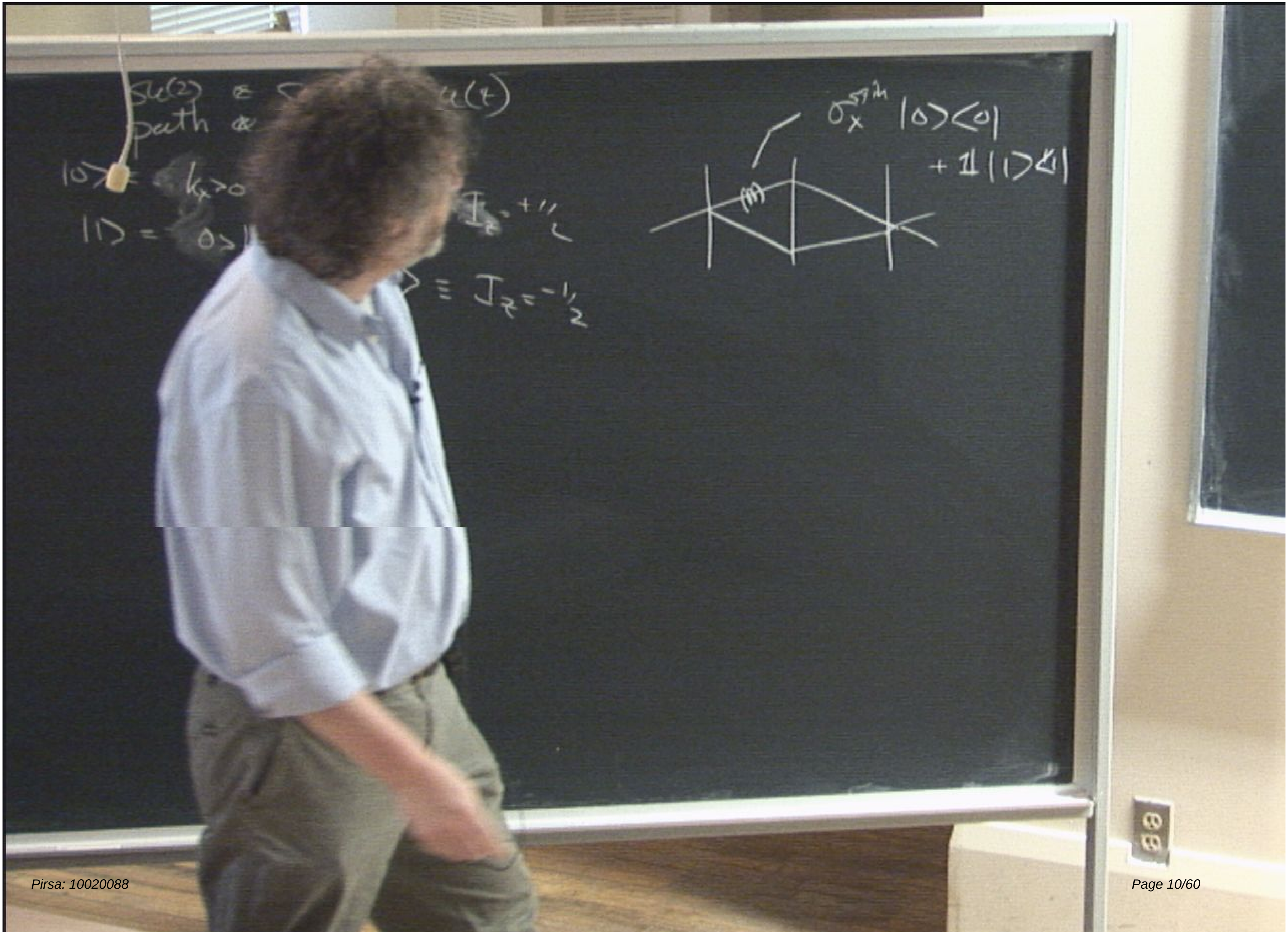
$$n_+ = n_- \; ; \; \theta_c \sim .3^\circ$$



not unitary, a projector.


path ⊗ spin
|0⟩ ≡ k_x > 0
|1⟩ ≡ 0 > k_x
|↑⟩ ≡ J_z = +1/2
|↓⟩ ≡ J_z = -1/2


SU(2) ⊗ SU(2) = SU(4)
path ⊗ spin
|0> ≡ k_x > 0
|1> ≡ 0 > k_x
|↑> ≡ J_z = +1/2
|↓> ≡ J_z = -1/2


path
$\sigma_x^{spin} |0\rangle\langle 0|$
$+ \mathbb{1} |1\rangle\langle 1|$
$\equiv J_z = -1/2$

$SU(2) \otimes SU(2) = SU(4)$

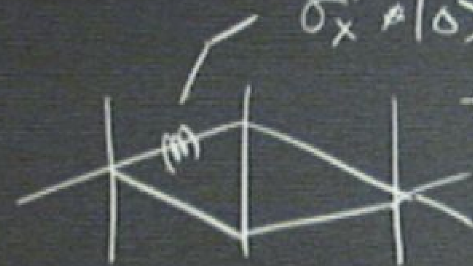
path $\otimes$ spin

$|0\rangle \equiv k_x > 0$

$|1\rangle = 0 > k_x$

$|\uparrow\rangle \equiv J_z = +1/2$

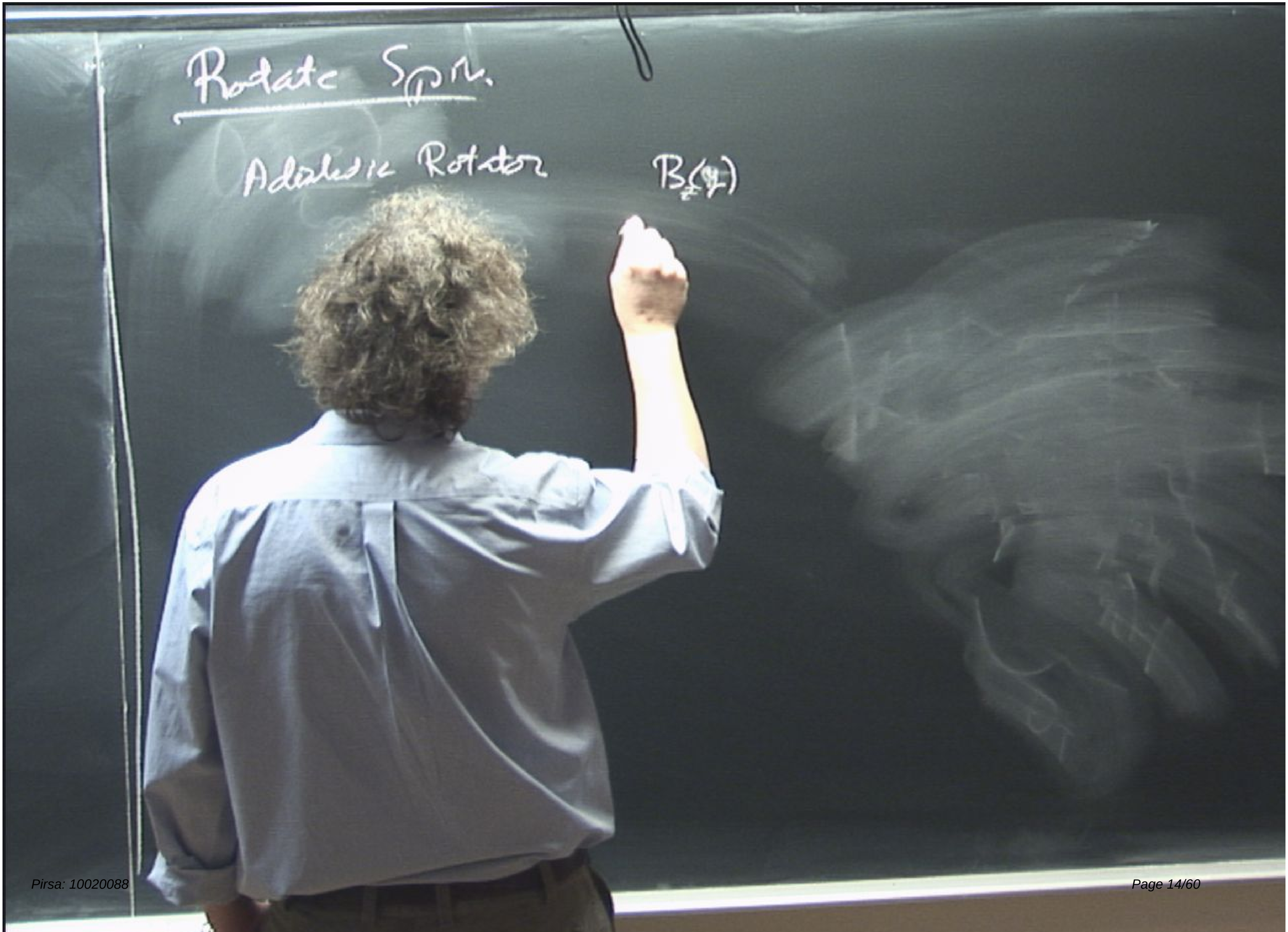
$|\downarrow\rangle \equiv J_z = -1/2$



$\sigma_x^{spin} \otimes |0\rangle\langle 0| + \mathbb{1} \otimes |1\rangle\langle 1|$

He-3


He-3
spin filter


Rotate Spin.
Adiabatic Rotator

Rotate Spin.

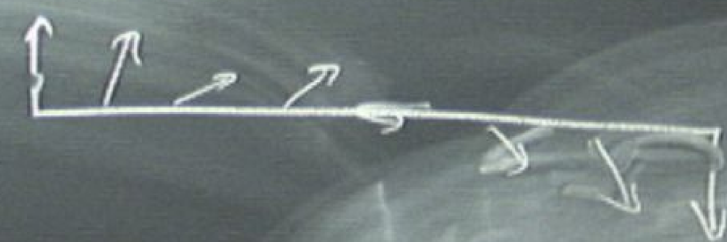
Adiabatic Rotation

$B_z(t)$

Rotate Spin.

Adiabatic Rotator

$B_z(\varphi)$



input state is
an eigenstate of
initial Field

Output state is
an eigenstate of
final Field.

Rotate Spin.

Adiabatic Rotator

input state is an eigenstate of initial Field

Output state is an eigenstate of final Field.

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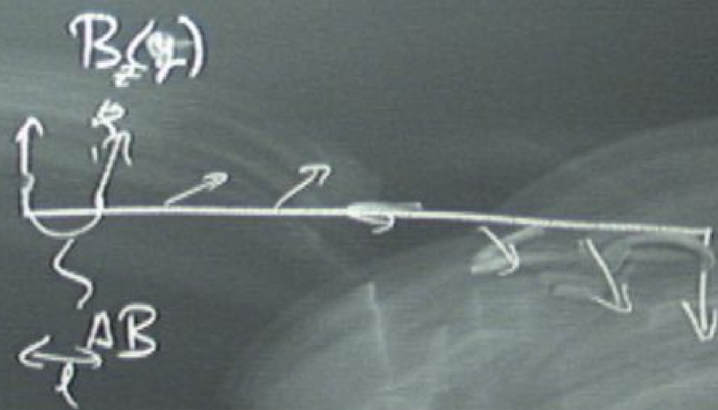
state follows the field



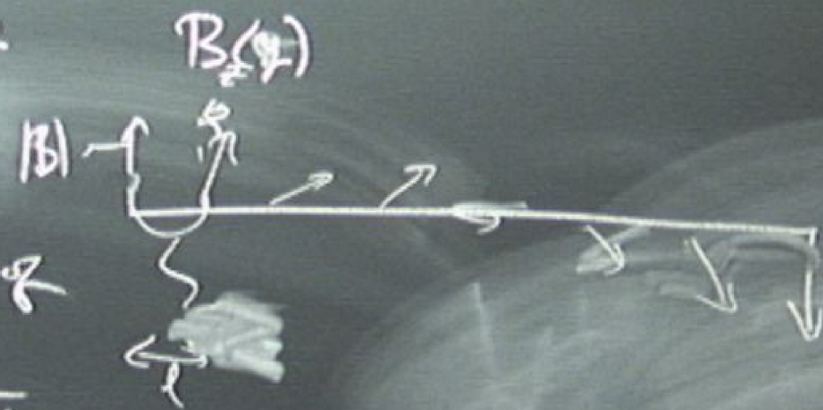
Rotate Spin.

Adiabatic Rotator

input state is an eigenstate of initial field

Output state is an eigenstate of final field.

state follows the field

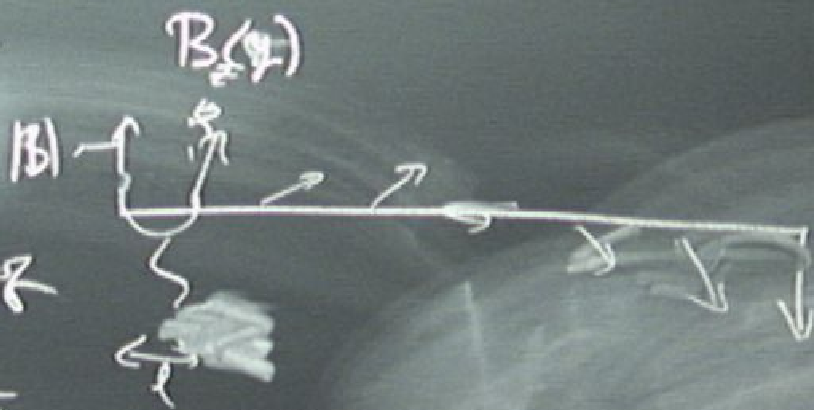


$$\frac{\gamma |B| \sin\theta}{\ell / v_n} < \gamma |B|$$

$|B|$ big enough to dominate stray fields.

Rotate Spin.

Adiabatic Rotator



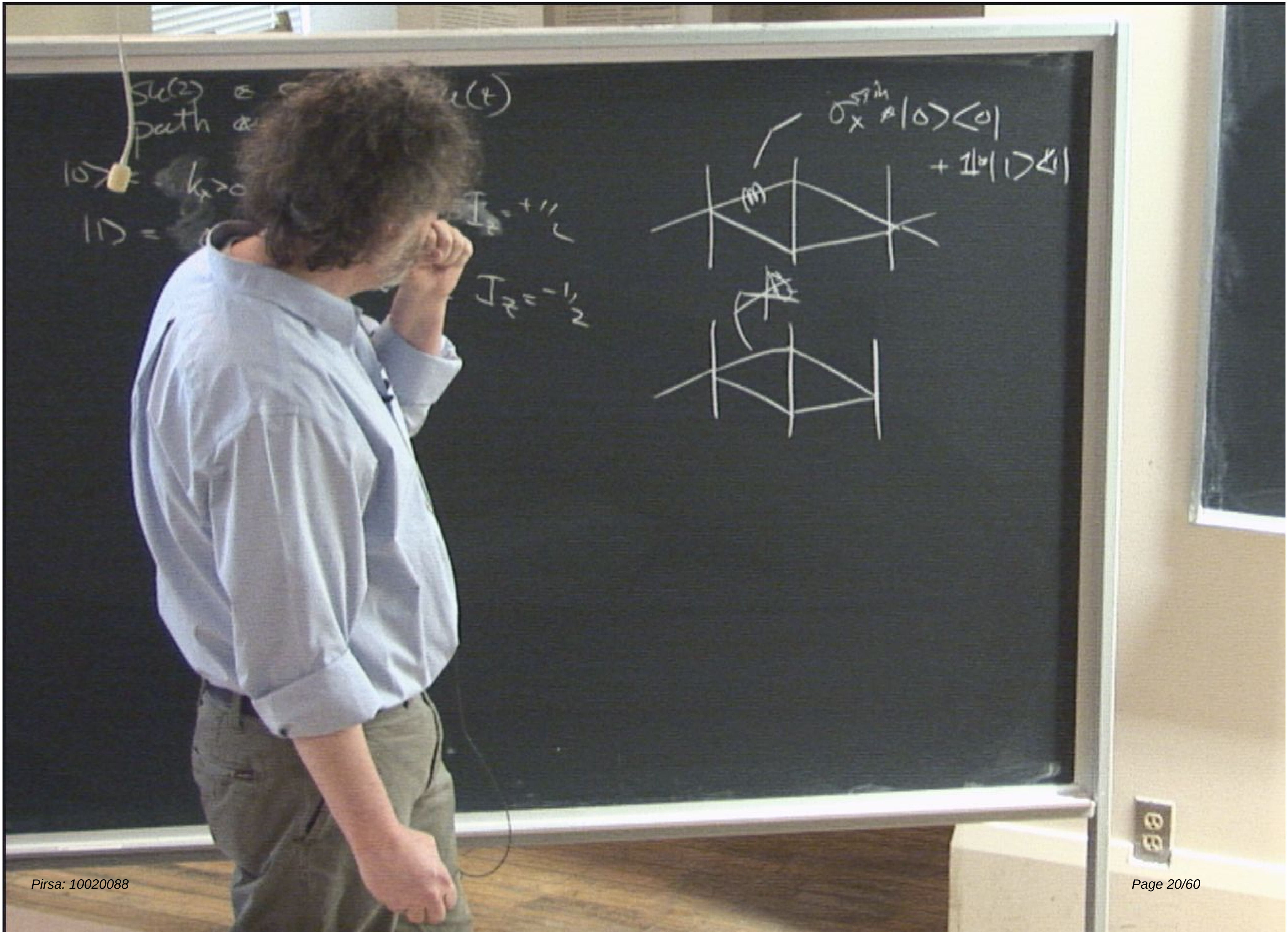
input state is
an eigenstate of
initial Field

Output state is
an eigenstate of
final Field.

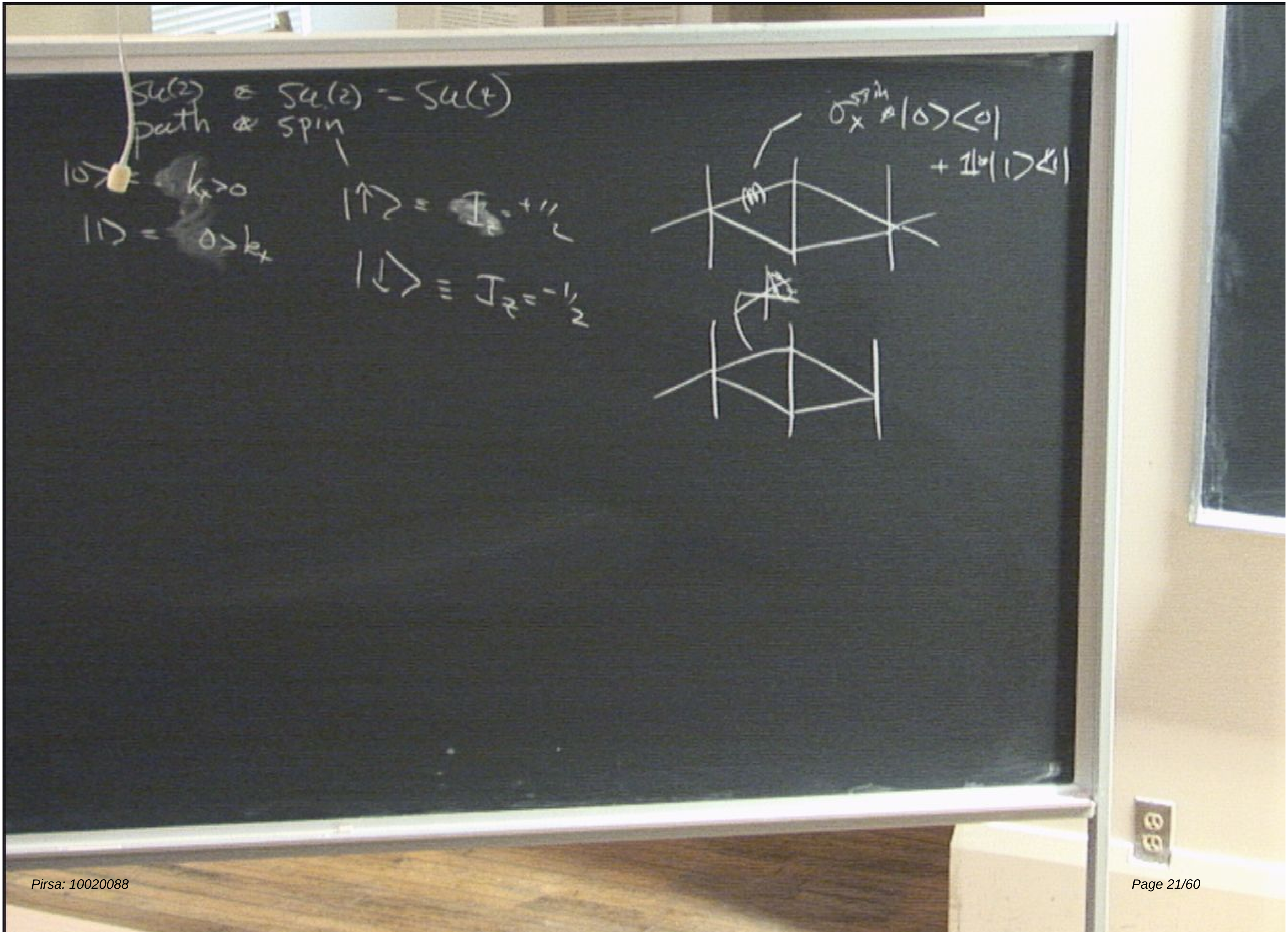
state follows
the field

$$\frac{\gamma |B| \sin\theta}{\ell / v_n} < \gamma |B|$$

$|B|$ big enough to
dominate stray fields.

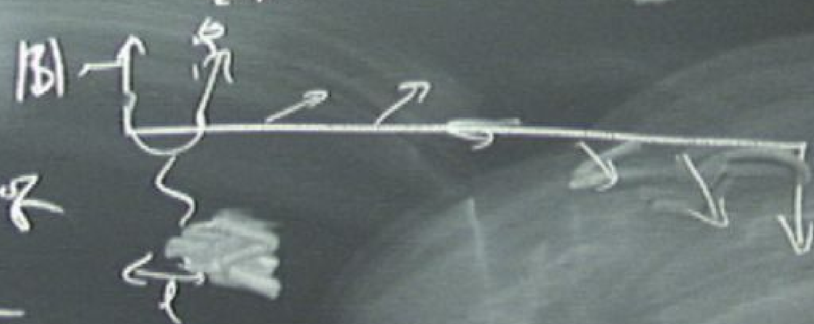


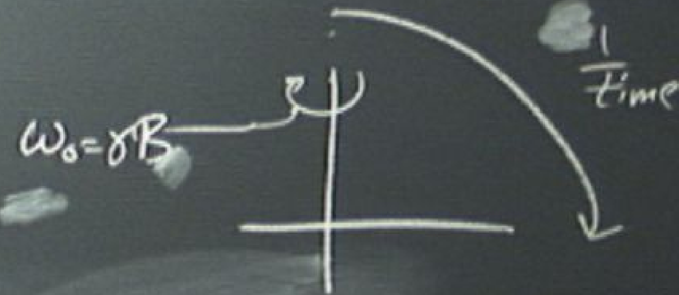
path
$\sigma_x^{spin} \otimes |0\rangle\langle 0|$
$J_z = +1/2$
$J_z = -1/2$


SU(2) ⊗ SU(2) = SU(4)
path ⊗ spin
|0> = kx > 0
|1> = 0 > kx
|↑> ≡ Jz = +1/2
|↓> ≡ Jz = −1/2
σx spin ⊗ |0><0|
+ 1 ⊗ |1><1|

Rotate Spin.

Adiabatic Rotator





input state is an eigenstate of initial field

Output state is an eigenstate of final field.

state follows the field

$$\frac{\gamma |B| \sin\theta}{\ell / v_n} < \gamma |B|$$

$|B|$ big enough to dominate stray fields.

Rotate Spin.

Adiabatic Rotator

$\omega_0 = \gamma B$

$\frac{1}{time}$

$B_z(y)$

$|B|$

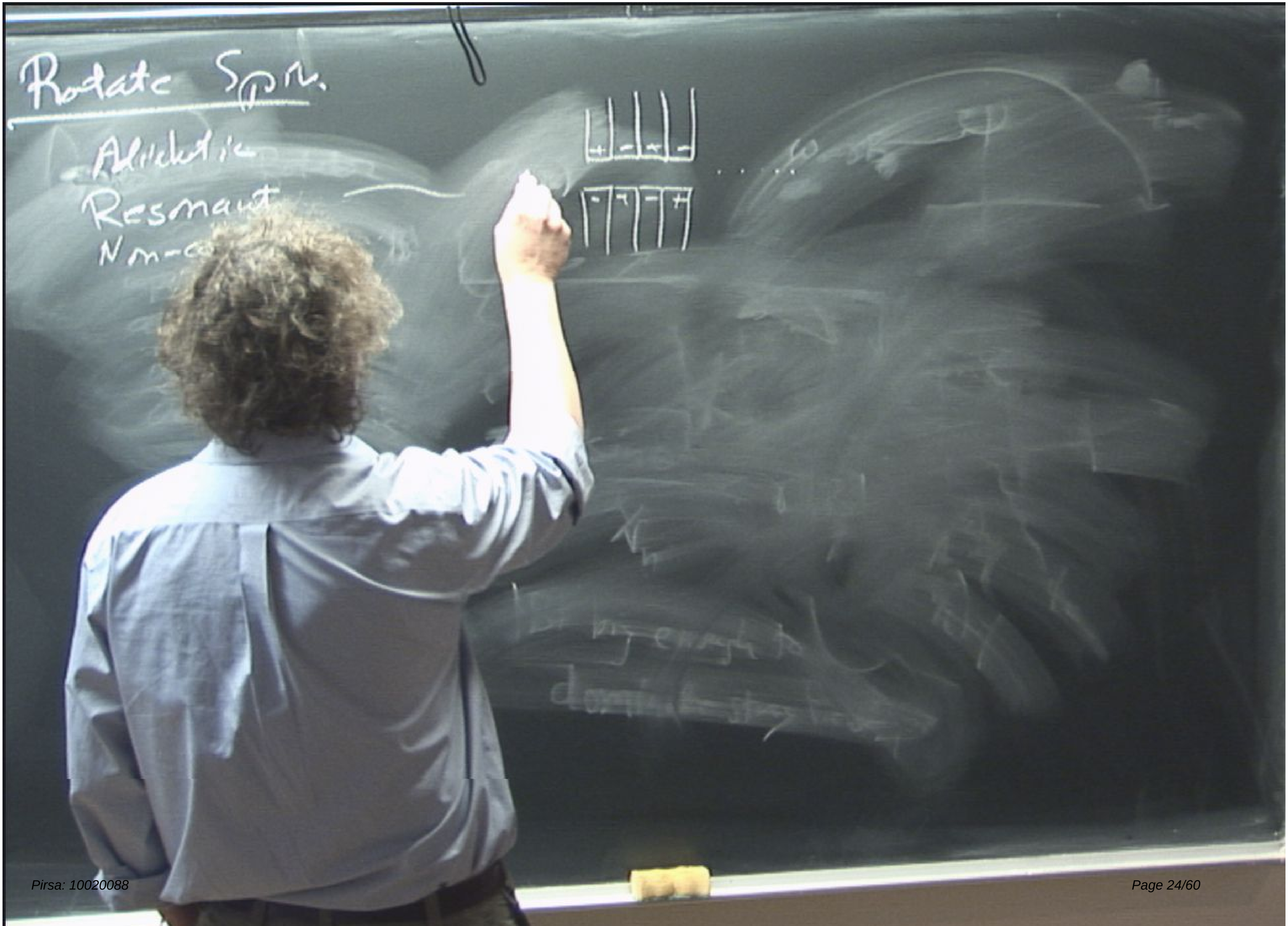
input state is an eigenstate of initial Field

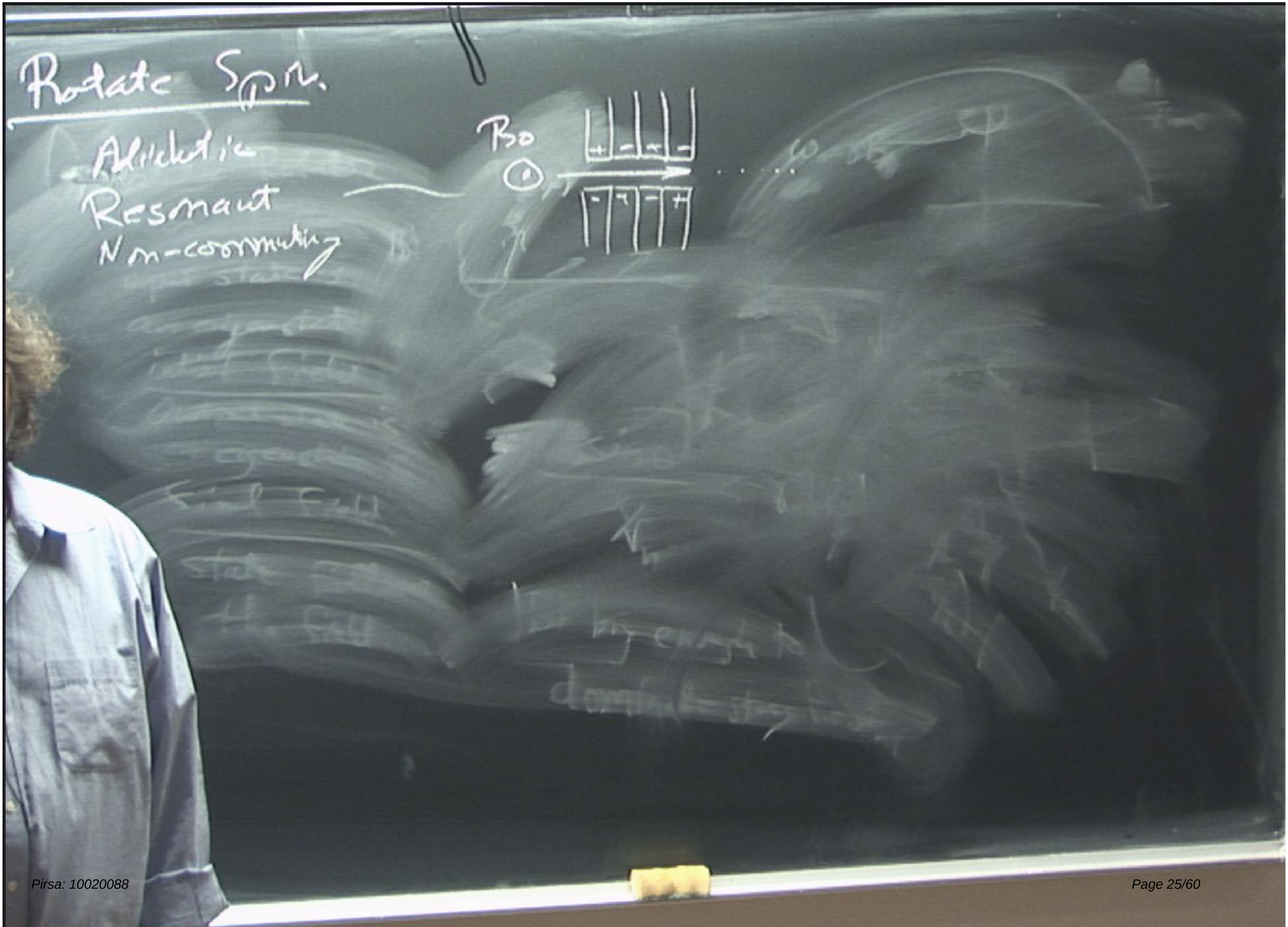
Output state is an eigenstate of final Field.

state follows the field

$$\frac{\sin\theta}{\ell / v_n} < \gamma |B|$$

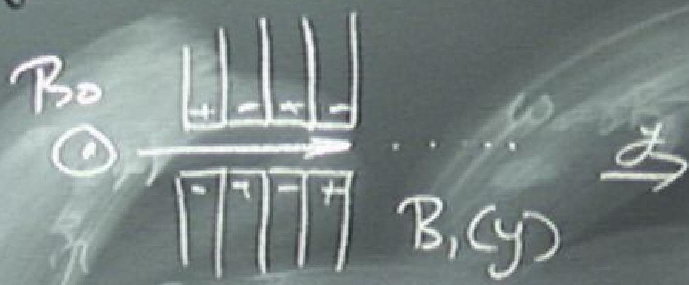
$|B|$ big enough to dominate stray fields.

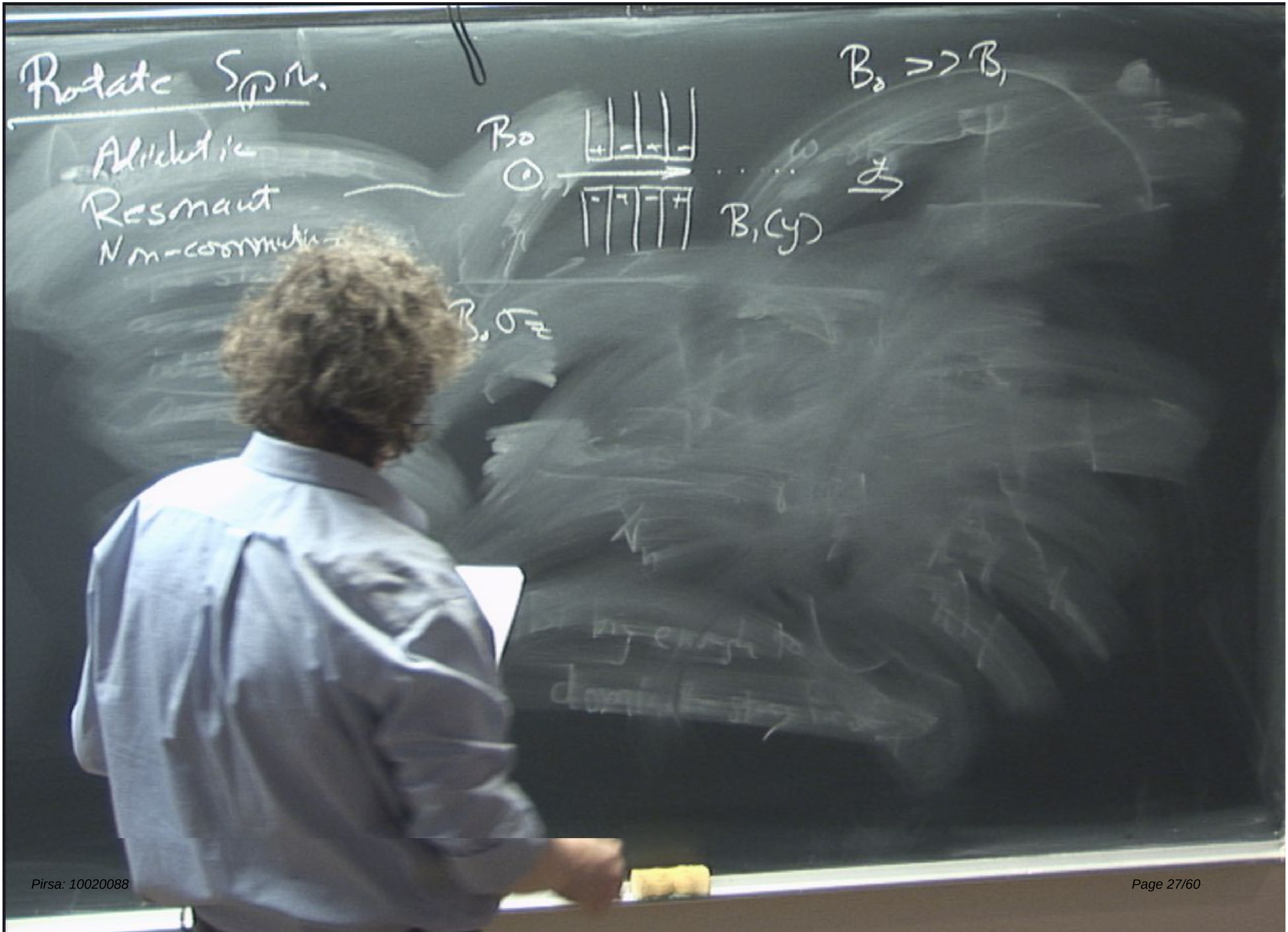

Rotate Spin
Adiabatic
Resonant

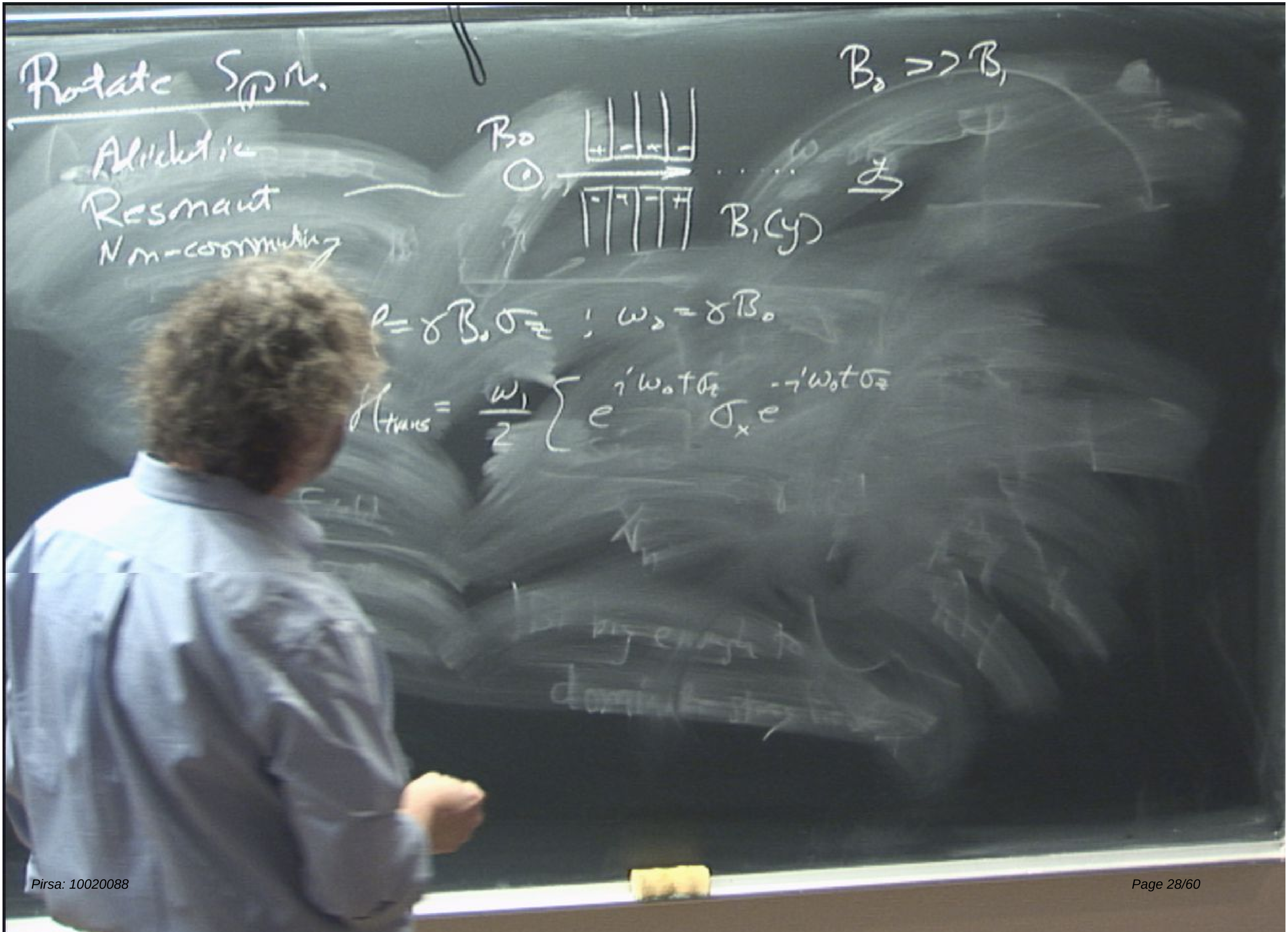

Rotate Spin
Adiabatic
Resonant
Non-commuting
B0

Rotate Spin

Adiabatic
Resonant
Non-commuting




Rotate Spin.
Adiabatic
Resonant
Non-commuting
$B_0 >> B_1$
B_0
$B_1(y)$
y
$B_0 \sigma_z$



Rotate Spin
Adiabatic
Resonant
Non-commuting
$B_0 >> B_1$
B_0
$B_1(y)$
y
$H = \gamma B_0 \sigma_z$; $\omega_0 = \gamma B_0$
$H_{trans} = \frac{\omega_1}{2} \{ e^{i\omega_0 t \sigma_z} \sigma_x e^{-i\omega_0 t \sigma_z}$

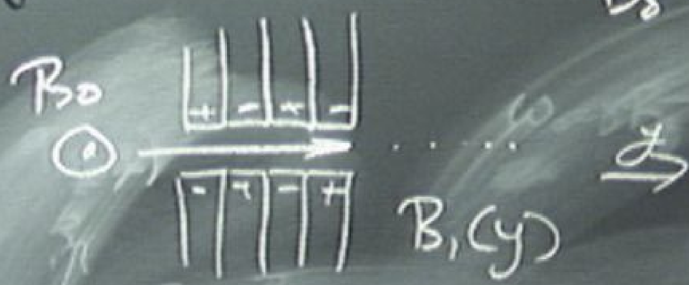
Rotate Spin.

Adiabatic

Resonant

Non-commuting

$B_0 >> B_1$



$$\mathcal{H} = \gamma B_0 \sigma_z \; ; \; \omega_0 = \gamma B_0$$

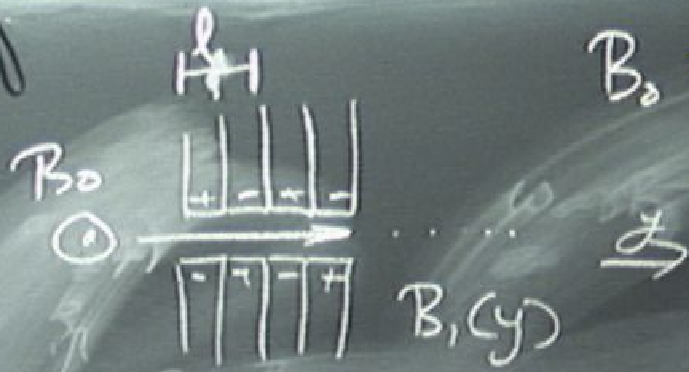
$$\mathcal{H}_{trans} = \frac{\omega_1}{2} \left[e^{i\omega_0 t \sigma_z} \sigma_x e^{-i\omega_0 t \sigma_z} + e^{-i\omega_0 t \sigma_z} \sigma_x e^{i\omega_0 t \sigma_z} \right]$$

Rotate Spin.

Adiabatic

Resonant

Non-commuting



$B_0 >> B_1$

$$\frac{l}{v_n} = \frac{2\pi}{\omega_0}$$

$$H = \gamma B_0 \sigma_z \; ; \; \omega_0 = \gamma B_0$$

$$H_{trans} = \frac{\omega_1}{2} \left[e^{i\omega_0 t \sigma_z} \sigma_x e^{-i\omega_0 t \sigma_z} + e^{-i\omega_0 t \sigma_z} \sigma_x e^{i\omega_0 t \sigma_z} \right]$$

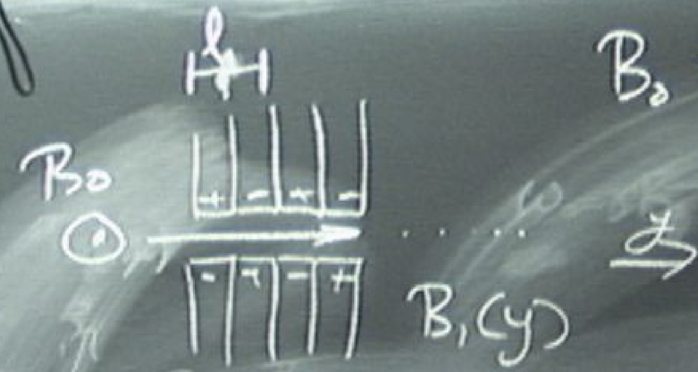
Rotate Spin

Adiabatic

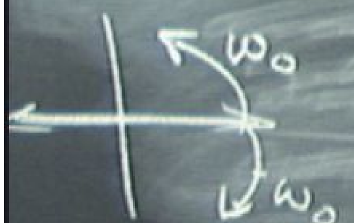
Resonant

Non-commuting

$B_0 >> B_1$

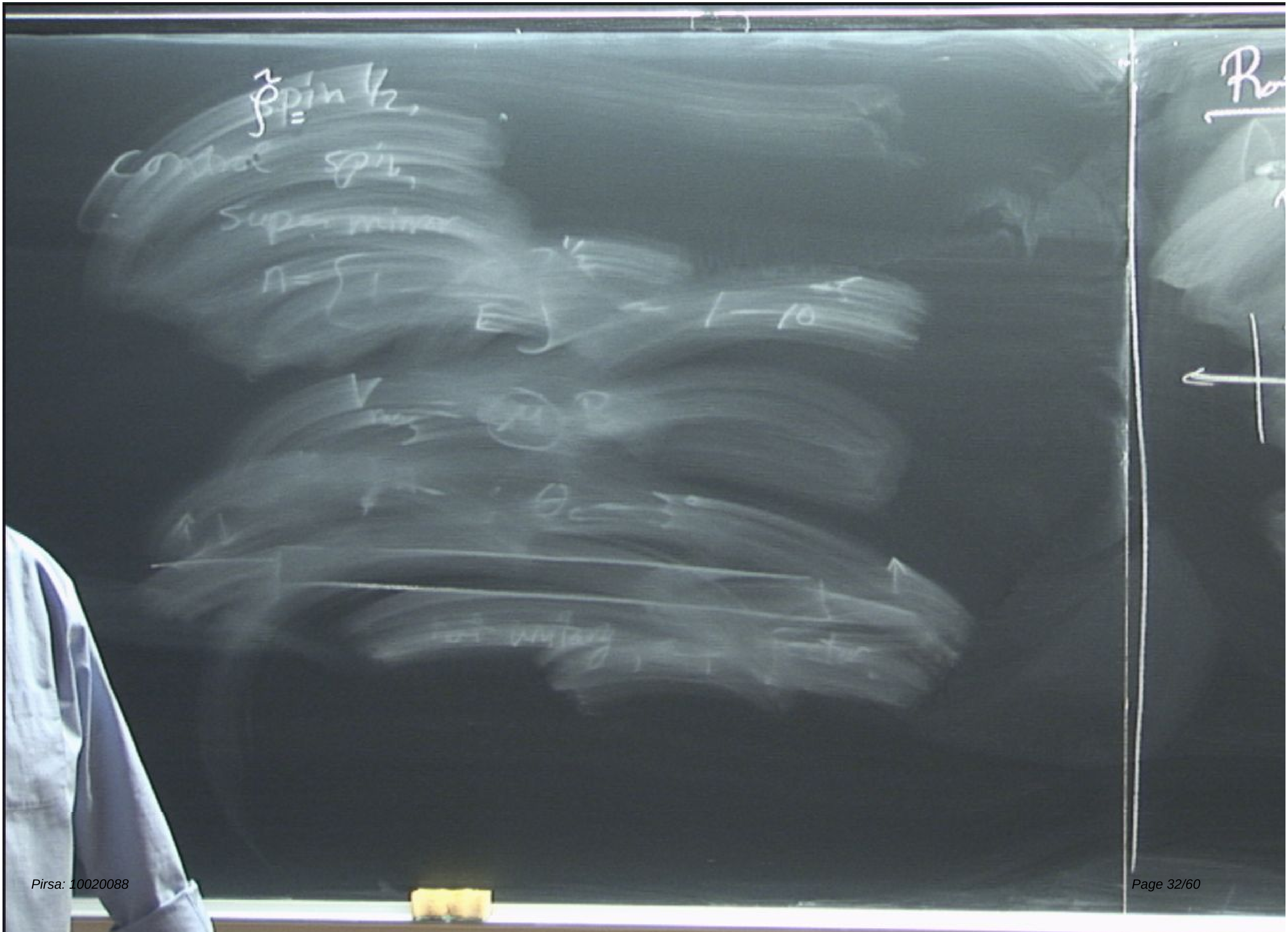


$$\frac{l}{v_n} = \frac{2\pi}{\omega_0}$$



$$\mathcal{H} = \gamma B_0 \sigma_z \;;\; \omega_0 = \gamma B_0$$

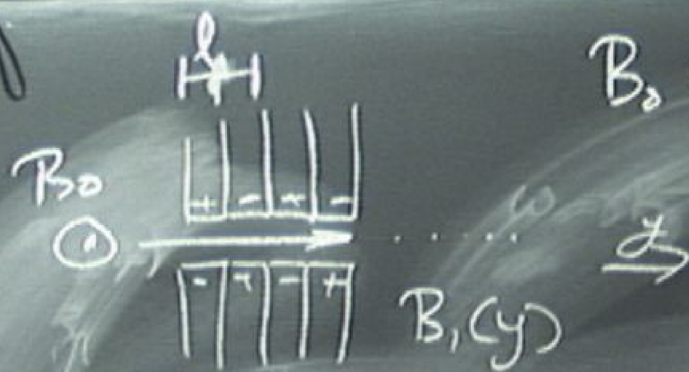
$$\mathcal{H}_{trans} = \frac{\omega_1}{2} \left\{ e^{i\omega_0 t \sigma_z} \sigma_x e^{-i\omega_0 t \sigma_z} + e^{-i\omega_0 t \sigma_z} \sigma_x e^{i\omega_0 t \sigma_z} \right]$$


spin 1/2,

$\tilde{\rho} = \mathcal{U}\rho\,\mathcal{U}^{-1}$; $\mathcal{U} = e^{-iH_{\alpha}t}$

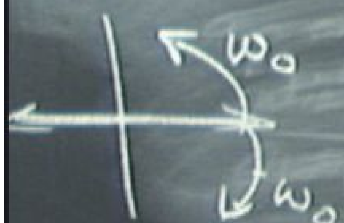
Rotate Spin.

Adiabatic
Resonant
Non-commuting



$B_0 >> B_1$

$$\frac{l}{v_n} = \frac{2\pi}{\omega_0}$$



$$\mathcal{H} = \gamma B_0 \sigma_z \; ; \; \omega_0 = \gamma B_0$$

$$\mathcal{H}_{trans} = \frac{\omega_1}{2} \left\{ e^{i\omega_0 t \sigma_z} \sigma_x e^{-i\omega_0 t \sigma_z} + e^{-i\omega_0 t \sigma_z} \sigma_x e^{i\omega_0 t \sigma_z} \right]$$

$$\mathcal{H}_{tot} = \mathcal{H} + \mathcal{H}_{trans}$$

$$\tilde{\rho} = \mathcal{U}\rho\,\mathcal{U}^{-1} \quad ; \quad \mathcal{U} = e^{-i\mathcal{H}_R t}$$

$$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$

$$\frac{d\tilde{\rho}}{dt} = -i\left[\tilde{\mathcal{H}}_{(t)}, \tilde{\rho}\right]$$

$$\tilde{\rho} = \mathcal{U}\rho\,\mathcal{U}^{-1} \quad ; \quad \mathcal{U} = e^{-i\mathcal{H}_e t}$$

$$\frac{d\rho}{dt} = -i[\mathcal{H},\rho] \qquad \frac{d\mathcal{U}}{dt} = -i\mathcal{H}_e\,\mathcal{U}$$

$$\frac{d\tilde{\rho}}{dt} = -i\left[\mathcal{H}_{(t)}, \tilde{\rho}\right] = \frac{d\mathcal{U}}{dt}\rho\,\mathcal{U}^{-1} + \mathcal{U}\frac{d\rho}{dt}\mathcal{U}^{-1} + \mathcal{U}\rho\frac{d\mathcal{U}^{-1}}{dt}$$

Ro

$$\tilde{\rho} = \mathcal{U}\rho\,\mathcal{U}^{-1} \quad ; \quad \mathcal{U} = e^{-i\mathcal{H}_R t}$$

$$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] \qquad \frac{d\mathcal{U}}{dt} = -i\mathcal{H}_R\,\mathcal{U}$$

$$\frac{d\tilde{\rho}}{dt} = -i\left[\mathcal{H}_{eff}, \tilde{\rho}\right] = \frac{d\mathcal{U}}{dt}\rho\,\mathcal{U}^{-1} + \mathcal{U}\frac{d\rho}{dt}\mathcal{U}^{-1} + \mathcal{U}\rho\frac{d\mathcal{U}^{-1}}{dt}$$

$$= -i\left[\tilde{\mathcal{H}}, \tilde{\rho}\right] - i\left[-\mathcal{H}_R, \rho\right]$$

$$\tilde{\mathcal{H}} = \mathcal{U}\,\mathcal{H}_{total}\,\mathcal{U}^{-1}$$

$$\tilde{\rho} = \mathcal{U} \rho \mathcal{U}^{-1} \; ; \quad \mathcal{U} = e^{-i \mathcal{H}_R t}$$

$$\frac{d\rho}{dt} = -i [\mathcal{H}, \rho] \qquad \frac{d\mathcal{U}}{dt} = -i \mathcal{H}_R \mathcal{U}$$

$$\frac{d\tilde{\rho}}{dt} = -i [\mathcal{H}_{eff}, \tilde{\rho}] = \frac{d\mathcal{U}}{dt} \rho \mathcal{U}^{-1} + \mathcal{U} \frac{d\rho}{dt} \mathcal{U}^{-1} + \mathcal{U} \rho \frac{d\mathcal{U}^{-1}}{dt}$$

$$= -i [\tilde{\mathcal{H}}, \tilde{\rho}] - i [-\mathcal{H}_R, \tilde{\rho}]$$

$$\tilde{\mathcal{H}} = \mathcal{U} \mathcal{H} \mathcal{U}^{-1}$$

$$\mathcal{H}_{eff} = \tilde{\mathcal{H}} - \mathcal{H}_R$$

Rotate Spin.

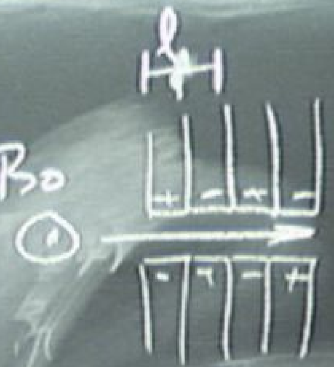
Adiabatic

Resonant

Non-commuting

$B_0 \gg B_1$

B_0



y

$B_1(y)$

$\frac{l}{v_n} = \frac{2\pi}{\omega_0}$

$\mathcal{H} = \gamma B_0 \sigma_z \; ; \; \omega_0 = \gamma B_0$

ω_0

ω_0

$\mathcal{H}_{trans} = \frac{\omega_1}{2}\left[e^{i\omega_0 t \sigma_z} \sigma_x e^{-i\omega_0 t \sigma_z} + e^{-i\omega_0 t \sigma_z} \sigma_x e^{i\omega_0 t \sigma_z} \right]$

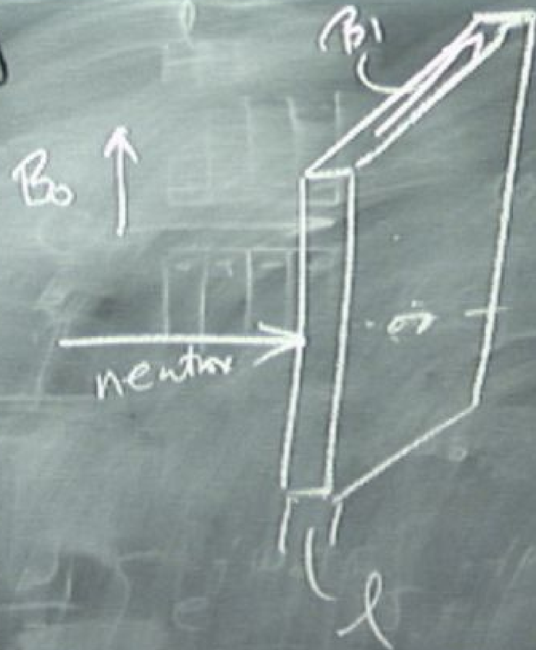
$U = e^{-i\omega_0 t \sigma_z}$ $\quad \mathcal{H} = \omega_0 t \sigma_z$

$\mathcal{H}_{tot} = \mathcal{H} + \mathcal{H}_{trans}$

$\tilde{\mathcal{H}} = \gamma B_0 \sigma_z + \frac{\omega_1}{2} \sigma_x + e^{-i2\omega_0 t \sigma_z} \sigma_x e^{+i2\omega_0 t \sigma_z}$

Rotate Spin

Adiabatic
Resonant
Non-commuting

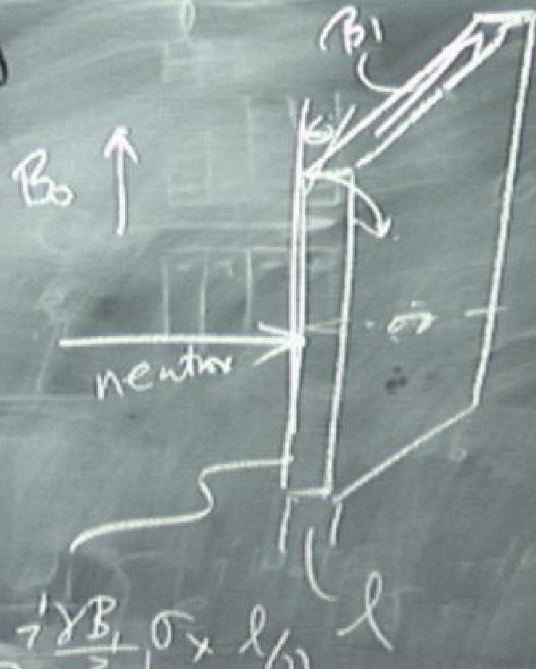


Rotate Spin.

Adiabatic

Resonant

Non-commuting



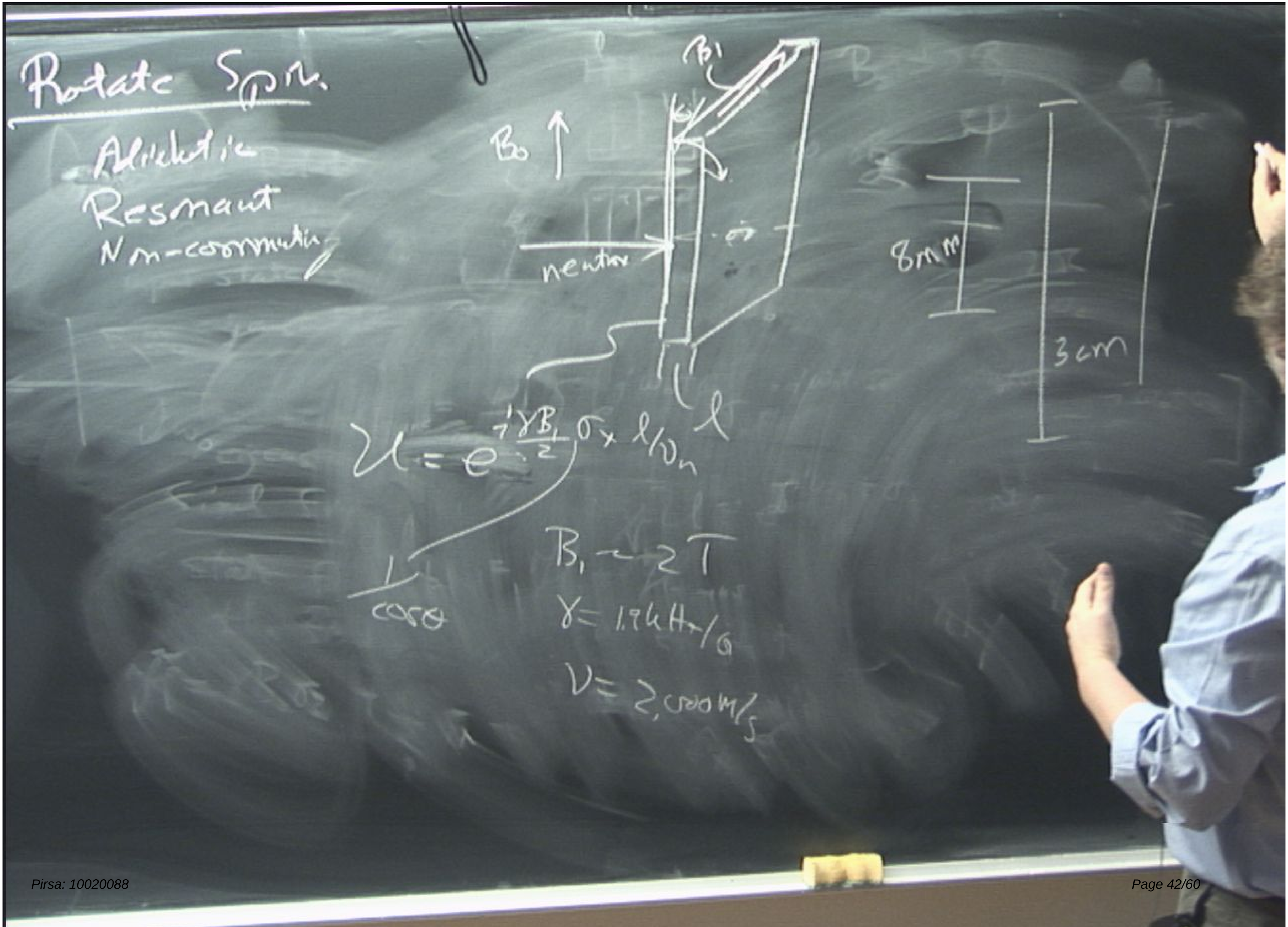
$$U = e^{-i\frac{\gamma B_1}{2}\sigma_x l/v_n}$$

$\frac{1}{\cos\theta}$

$B_1 \sim 2\,T$

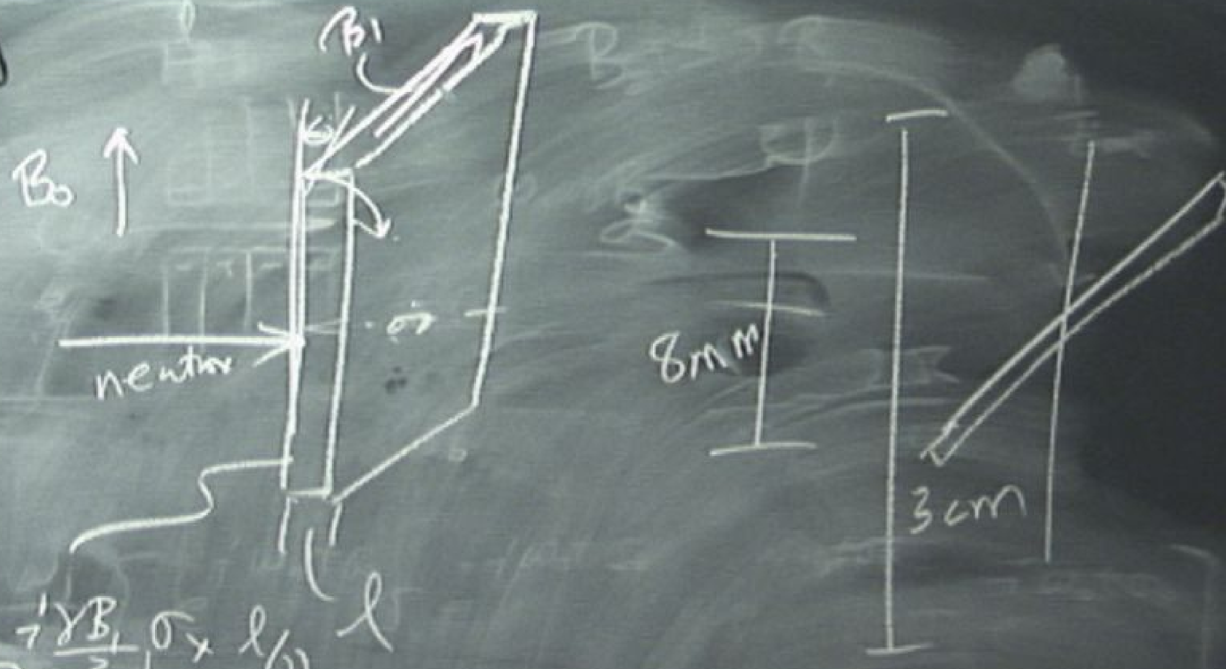
$\gamma = 1.94\,Hz/G$

$v = 2{,}000\,m/s$


Rotate Spin
Adiabatic
Resonant
Non-commuting
B0
B1
neutron
8mm
3cm
U = e^{-i γB1/2 σx l/vn}
1/cosθ
B1 ~ 2T
γ = 1.9 kHz/G
V = 2,000 m/s

Rotate Spin

Adiabatic
Resonant
Non-commuting



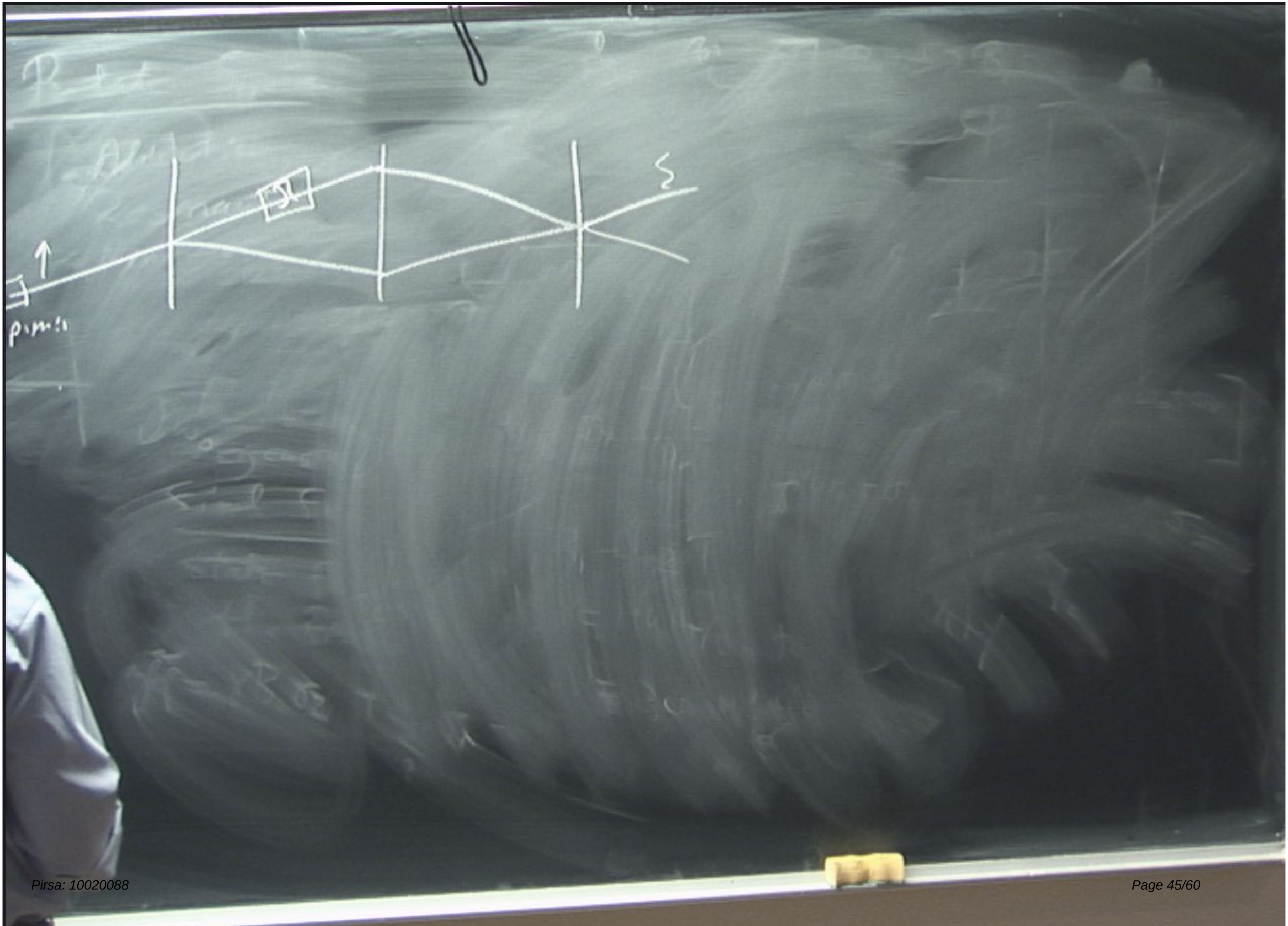
$$U = e^{-i\frac{\gamma B_1}{2}\sigma_x\, l/v_n}$$

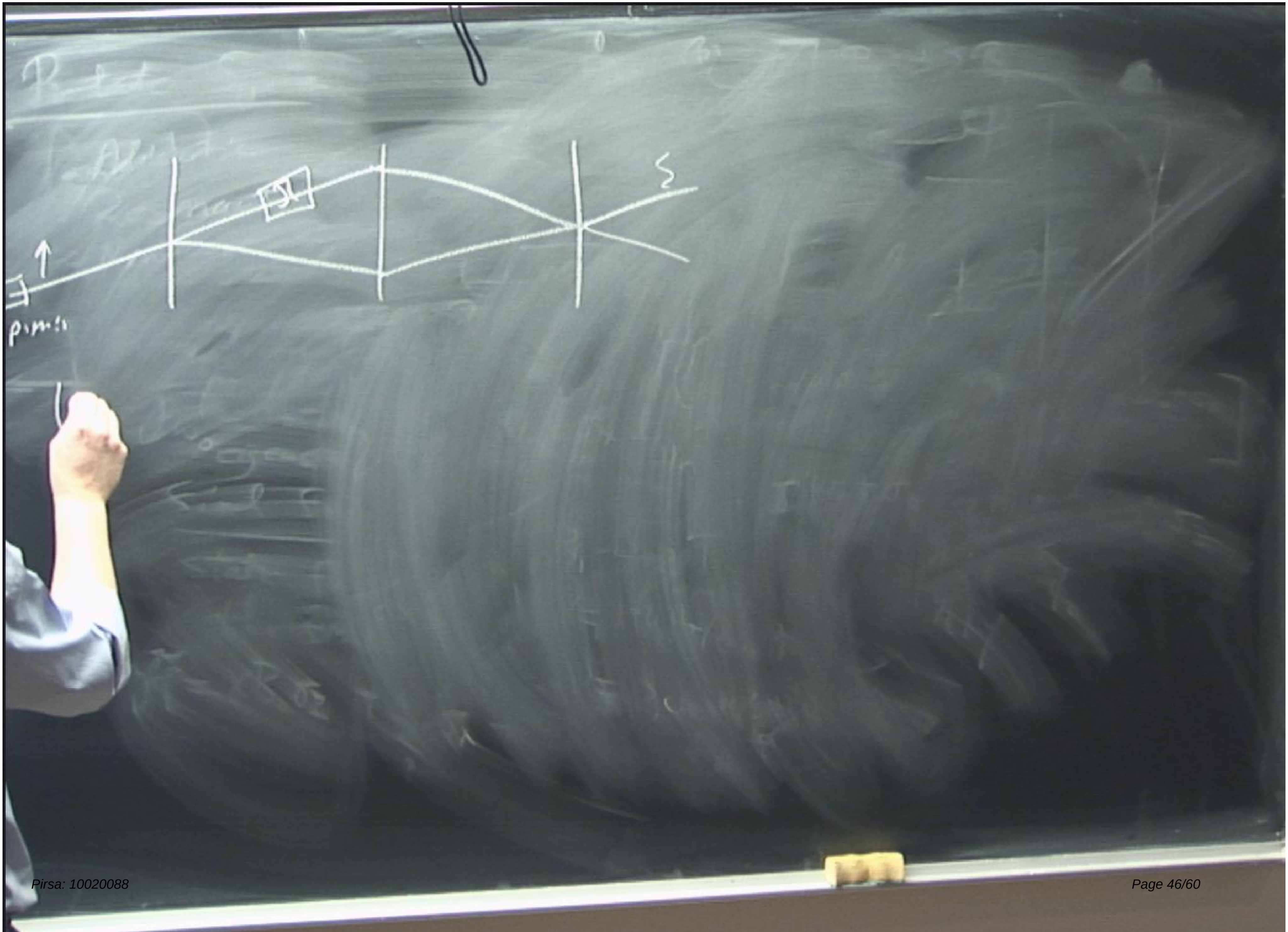
$$\frac{1}{\cos\theta}$$

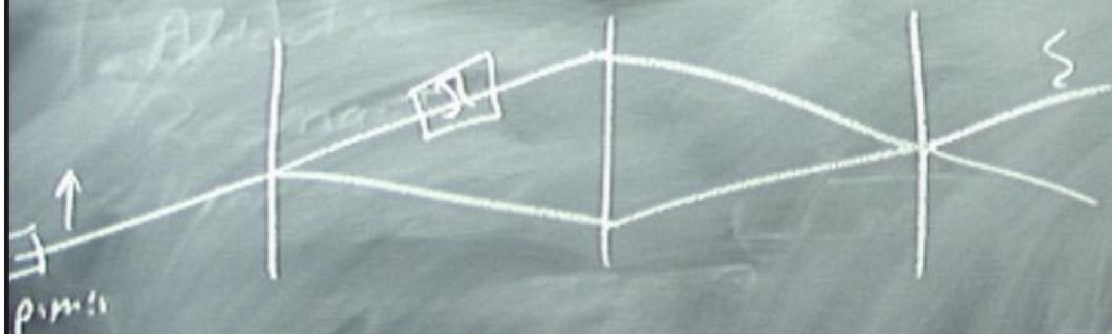
$B_1 \sim 2T$

$\gamma = 1.94\ \mathrm{kHz}/G$

$v = 2{,}000\ \mathrm{m/s}$







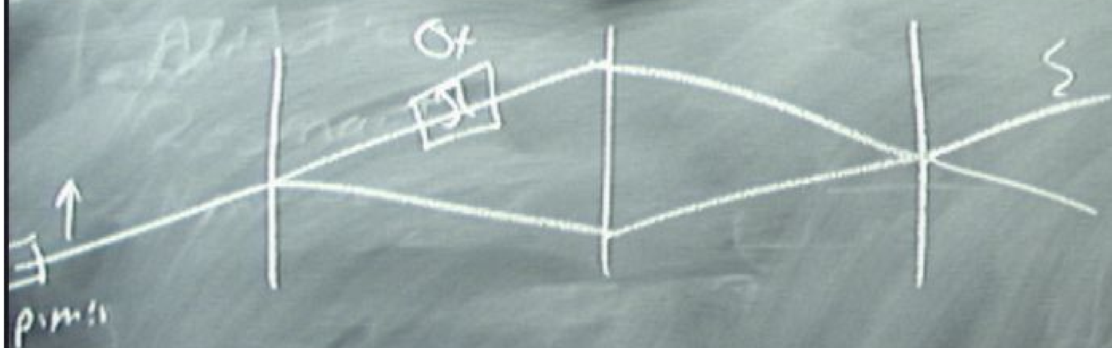
$|0\uparrow\rangle$

$|0\uparrow\rangle$

$|1\uparrow\rangle$

$|0\downarrow\rangle$

$|1\downarrow\rangle$



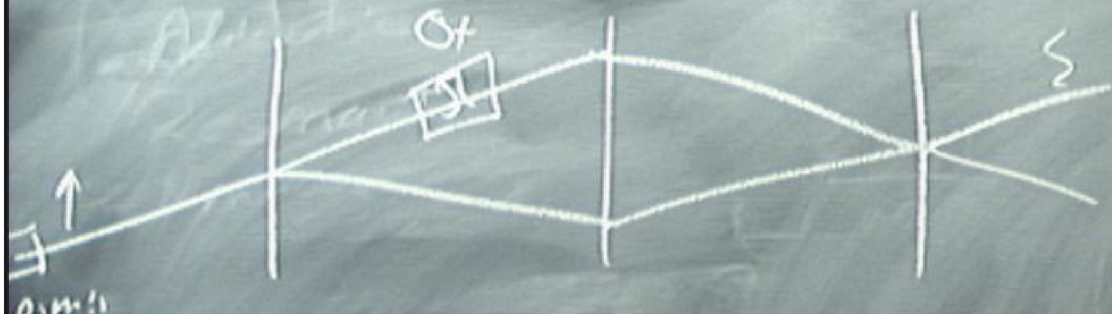
$$|0\uparrow\rangle \xrightarrow[H\otimes \mathbb{1}]{} $$

$|0\uparrow\rangle$

$|1\uparrow\rangle$

$|0\downarrow\rangle$

$|1\downarrow\rangle$



$$|0\uparrow\rangle \xrightarrow[H\otimes \mathbb{1}]{\text{[illegible]}} \frac{1}{\sqrt{2}}\left(|0\uparrow\rangle + |1\uparrow\rangle\right)$$

$$H|\uparrow\rangle\langle\uparrow| + H|\downarrow\rangle\langle\downarrow|$$

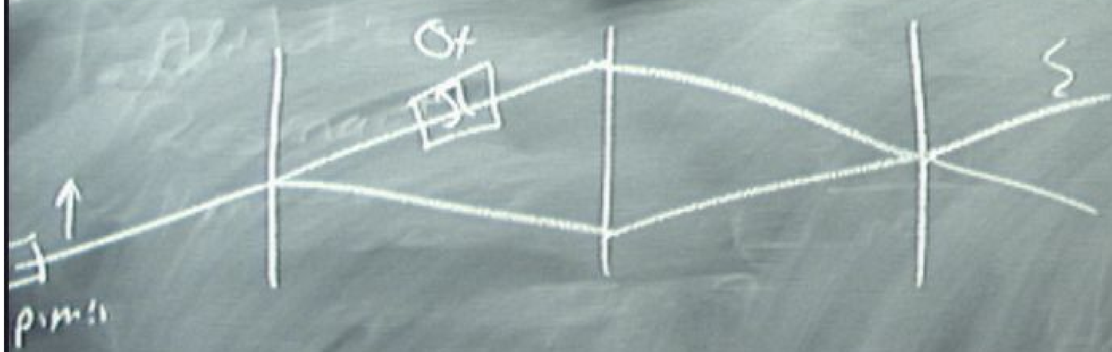
$|0\uparrow\rangle$

$|1\uparrow\rangle$

$|0\downarrow\rangle$

$|1\downarrow\rangle$

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$



$$|0\uparrow\rangle \xrightarrow[H\otimes \mathbb{1}]{\text{Splitter}} \frac{1}{\sqrt{2}}\left(|0\uparrow\rangle + |1\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\otimes|\uparrow\rangle$$

$$H|\uparrow\rangle\langle\uparrow| + H|\downarrow\rangle\langle\downarrow|$$

$\xrightarrow{\text{spin flipper}}$

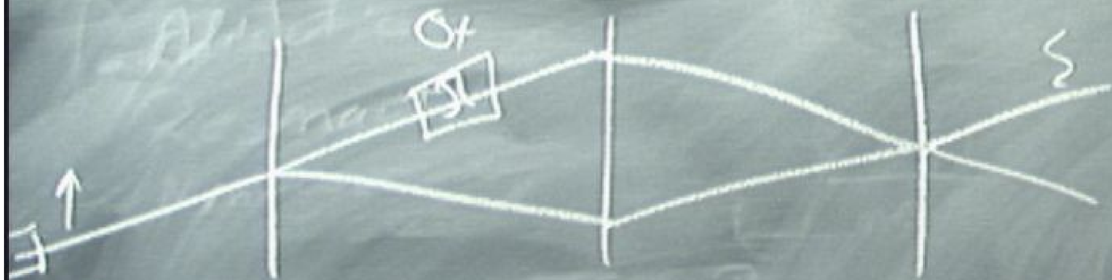
$|0\uparrow\rangle$

$|1\uparrow\rangle$

$|0\downarrow\rangle$

$|1\downarrow\rangle$

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$



$$|0\uparrow\rangle \xrightarrow[H\otimes \mathbb{1}]{\text{Silver plate}} \frac{1}{\sqrt{2}}\left(|0\uparrow\rangle + |1\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\otimes|\uparrow\rangle$$

$$H|\uparrow\rangle\langle\uparrow| + H|\downarrow\rangle\langle\downarrow|$$

$|0\uparrow\rangle$

$|1\uparrow\rangle$

$|0\downarrow\rangle$

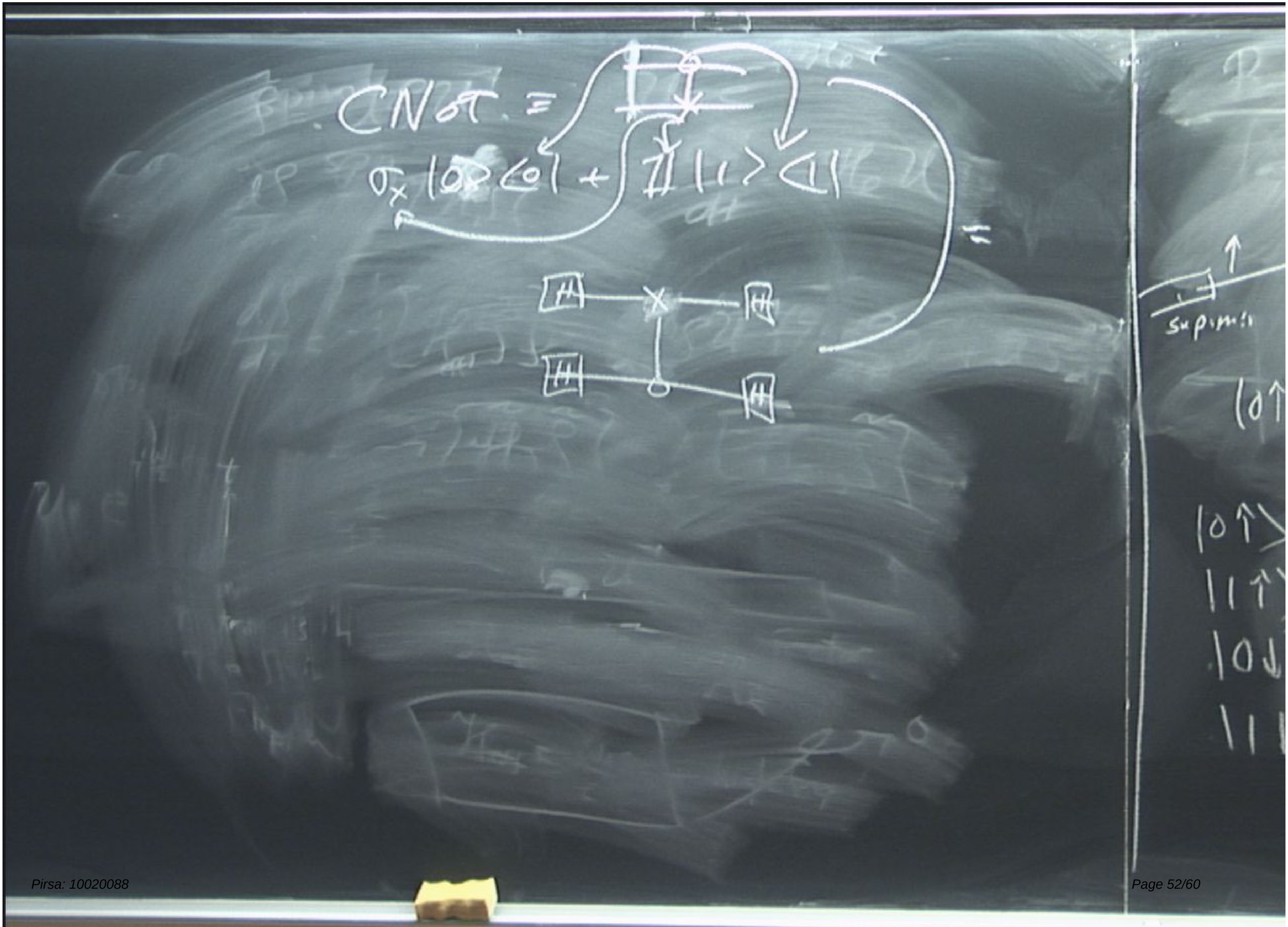
$|1\downarrow\rangle$

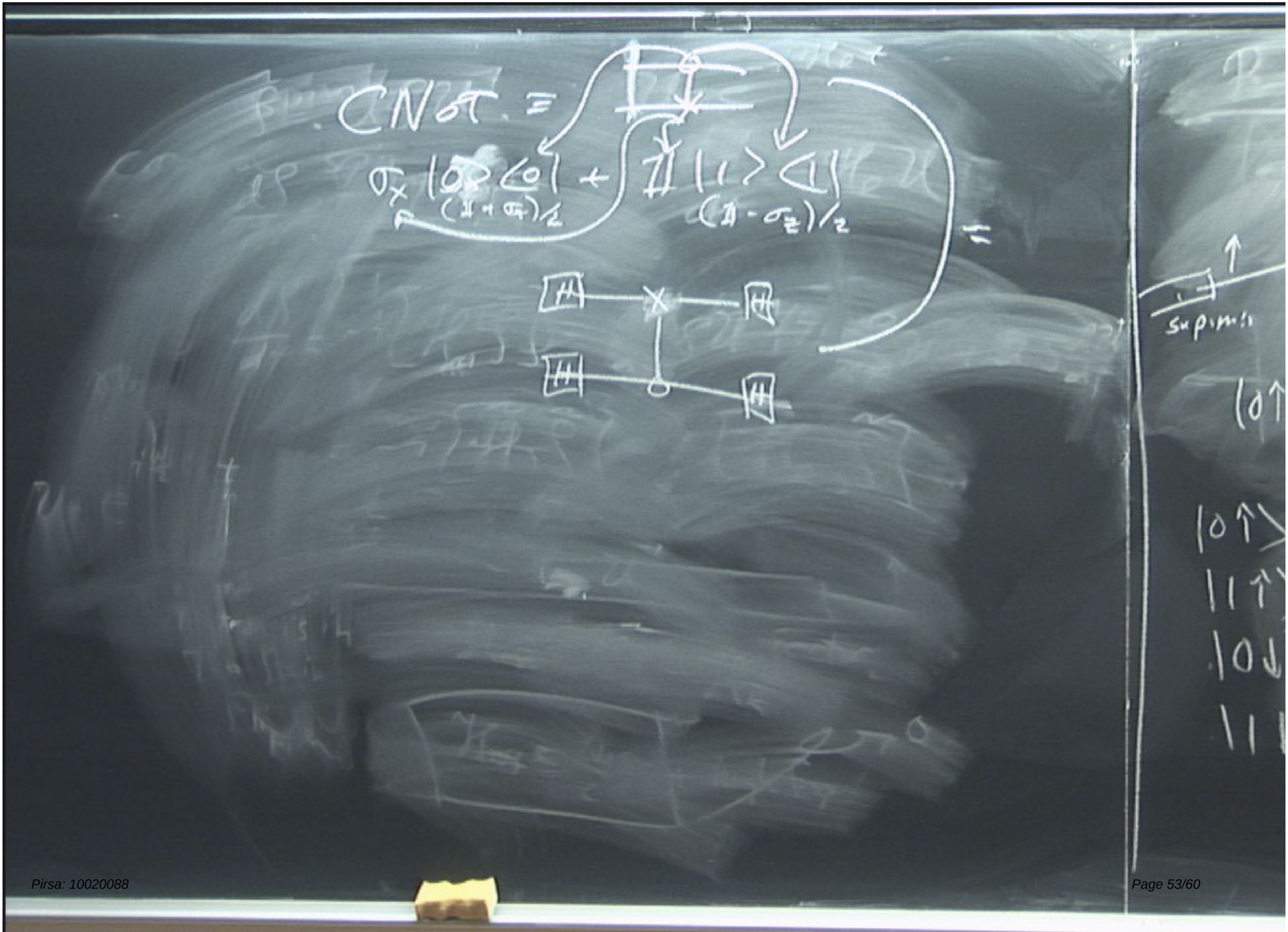
$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

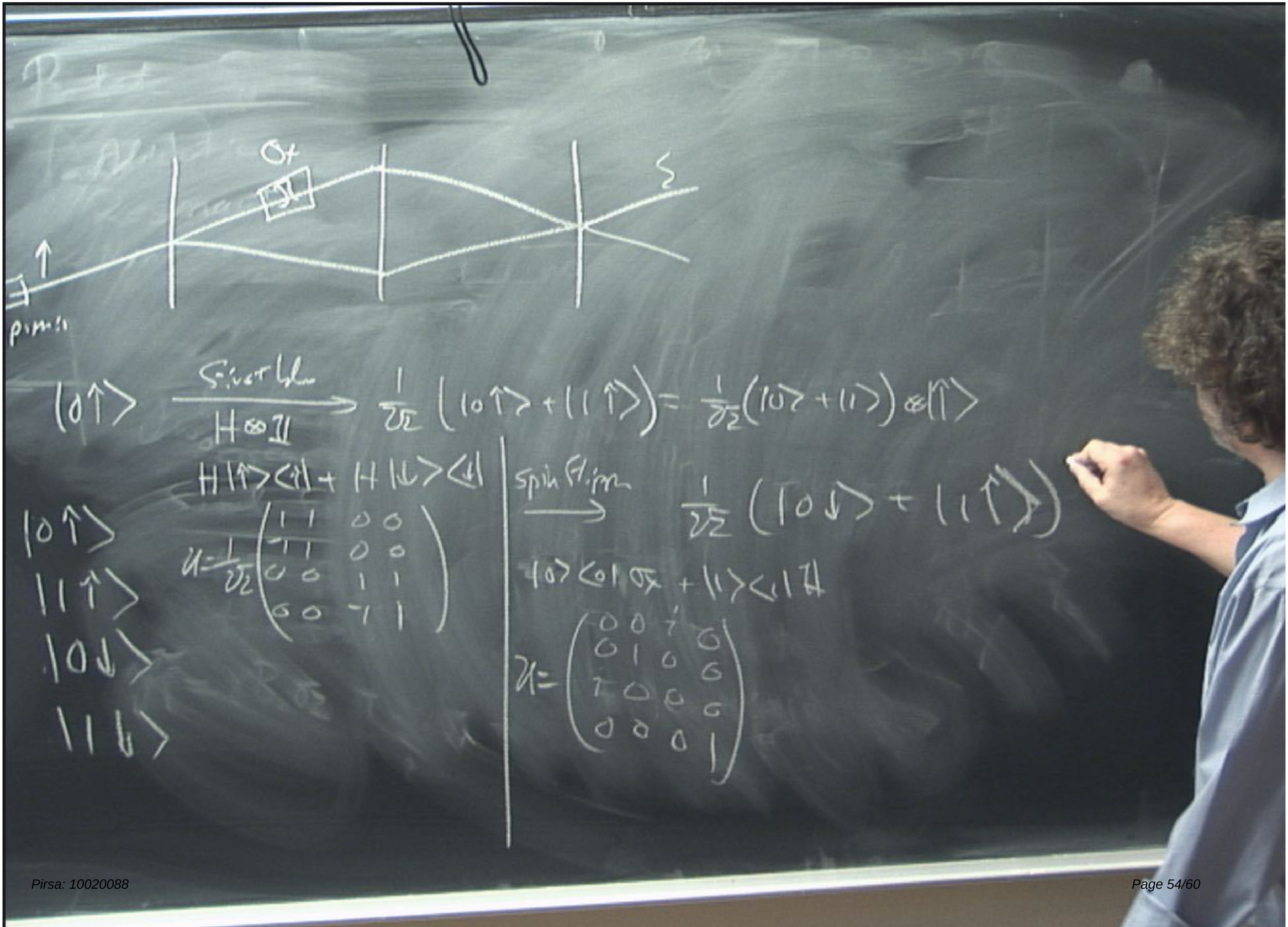
$$\xrightarrow{\text{spin flipper}}$$

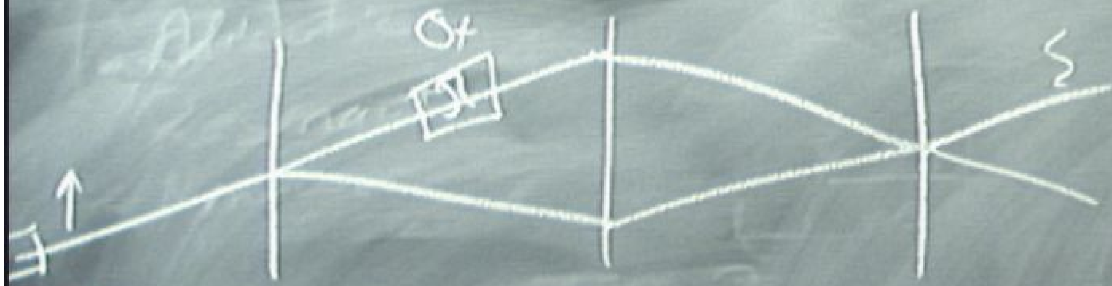
$$|0\rangle\langle 0|\,\sigma_x + |1\rangle\langle 1|\,\mathbb{1}$$

$$U = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$


CNot =
$\sigma_x |0\rangle\langle 0| + \mathbb{1} |1\rangle\langle 1|$
=
H
H
H
H


CNOT ≡
σx |0⟩⟨0| + 𝟙 |1⟩⟨1|
(𝟙 + σz)/2
(𝟙 − σz)/2
=


σx
|0↑⟩
H⊗𝟙
1/√2 (|0↑⟩ + |1↑⟩) = 1/√2 (|0⟩ + |1⟩) ⊗ |↑⟩
H |↑⟩⟨↑| + H |↓⟩⟨↓|
spin flipper
1/√2 (|0↓⟩ + |1↑⟩)
|0⟩⟨0| σx + |1⟩⟨1| 𝟙
|0↑⟩
|1↑⟩
|0↓⟩
|1↓⟩



$$|0\uparrow\rangle \xrightarrow[H\otimes\mathbb{1}]{} \frac{1}{\sqrt{2}}\left(|0\uparrow\rangle + |1\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\otimes|\uparrow\rangle$$

$$H|\uparrow\rangle\langle\uparrow| + H|\downarrow\rangle\langle\downarrow|$$

$|0\uparrow\rangle$

$|1\uparrow\rangle$

$|0\downarrow\rangle$

$|1\downarrow\rangle$

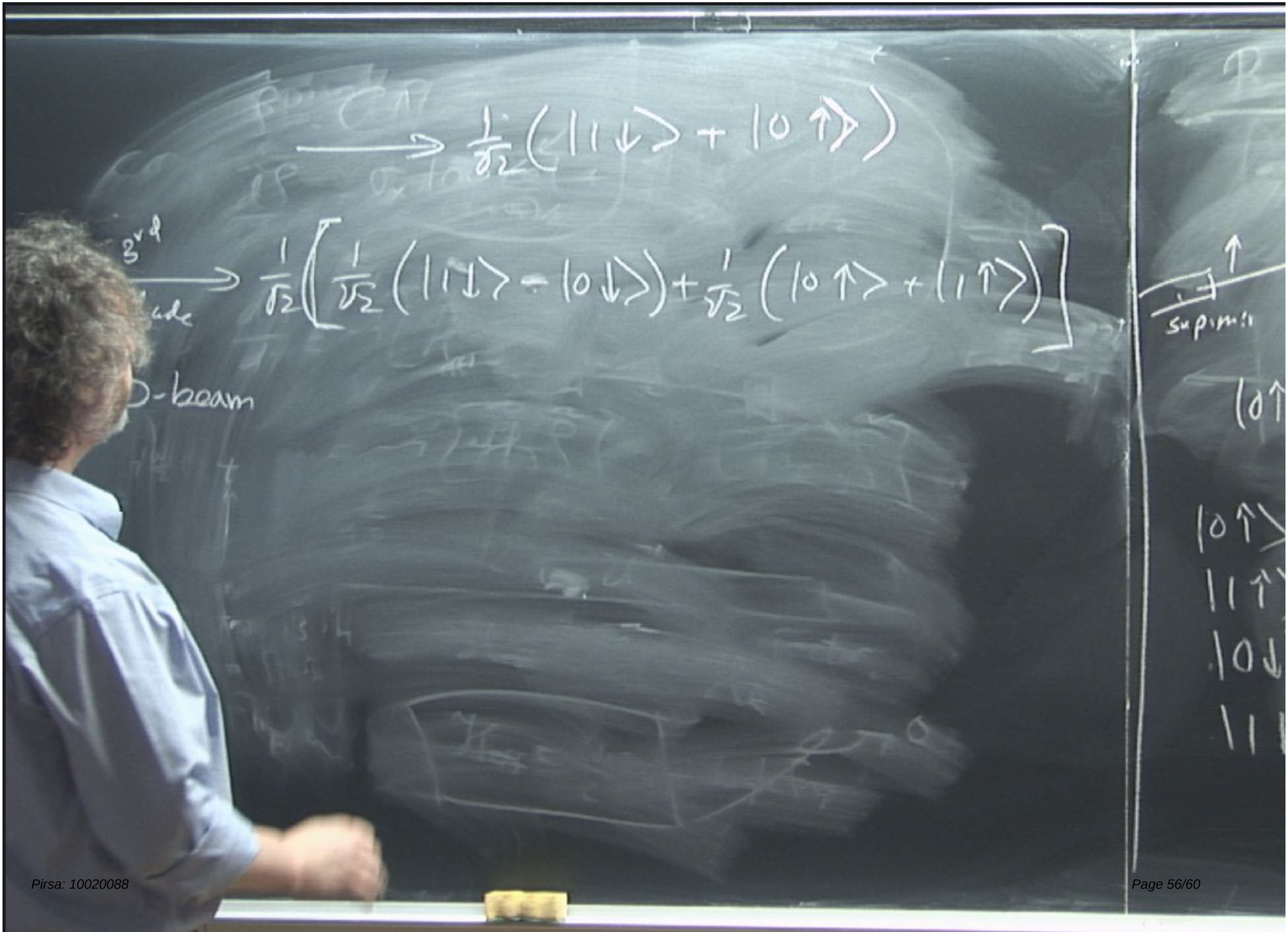
$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{spin flip}} \frac{1}{\sqrt{2}}\left(|0\downarrow\rangle + |1\uparrow\rangle\right)$$

entangled state

$$|0\rangle\langle 0|\,\sigma_x + |1\rangle\langle 1|\,\mathbb{1}$$

$$U = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$


$\longrightarrow \frac{1}{\sqrt{2}}\left(|1\downarrow\rangle + |0\uparrow\rangle\right)$
3rd
$\longrightarrow \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|1\downarrow\rangle - |0\downarrow\rangle\right) + \frac{1}{\sqrt{2}}\left(|0\uparrow\rangle + |1\uparrow\rangle\right)\right]$
-beam

$$\longrightarrow \frac{1}{\sqrt{2}}\left(|1\downarrow\rangle + |0\uparrow\rangle\right)$$

$$\xrightarrow[\text{blade}]{3^{rd}} \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|1\downarrow\rangle - |0\downarrow\rangle\right) + \frac{1}{\sqrt{2}}\left(|0\uparrow\rangle + |1\uparrow\rangle\right)\right]$$

$$\text{0-beam} = \frac{1}{2}\left[|0\rangle\underbrace{\left(|\uparrow\rangle - |\downarrow\rangle\right)}_{\text{coherence}}\right]$$

supim

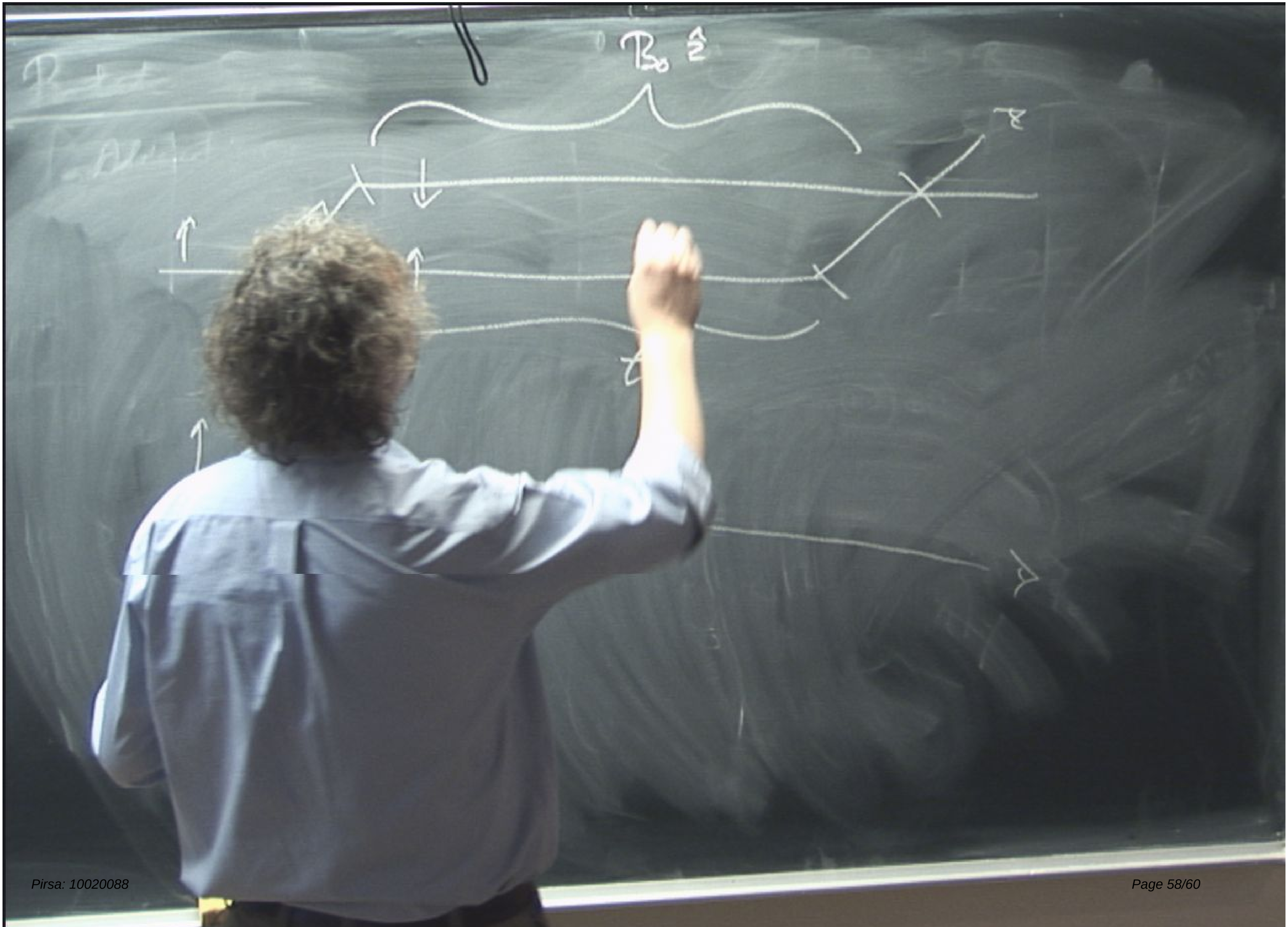
$|0\uparrow$

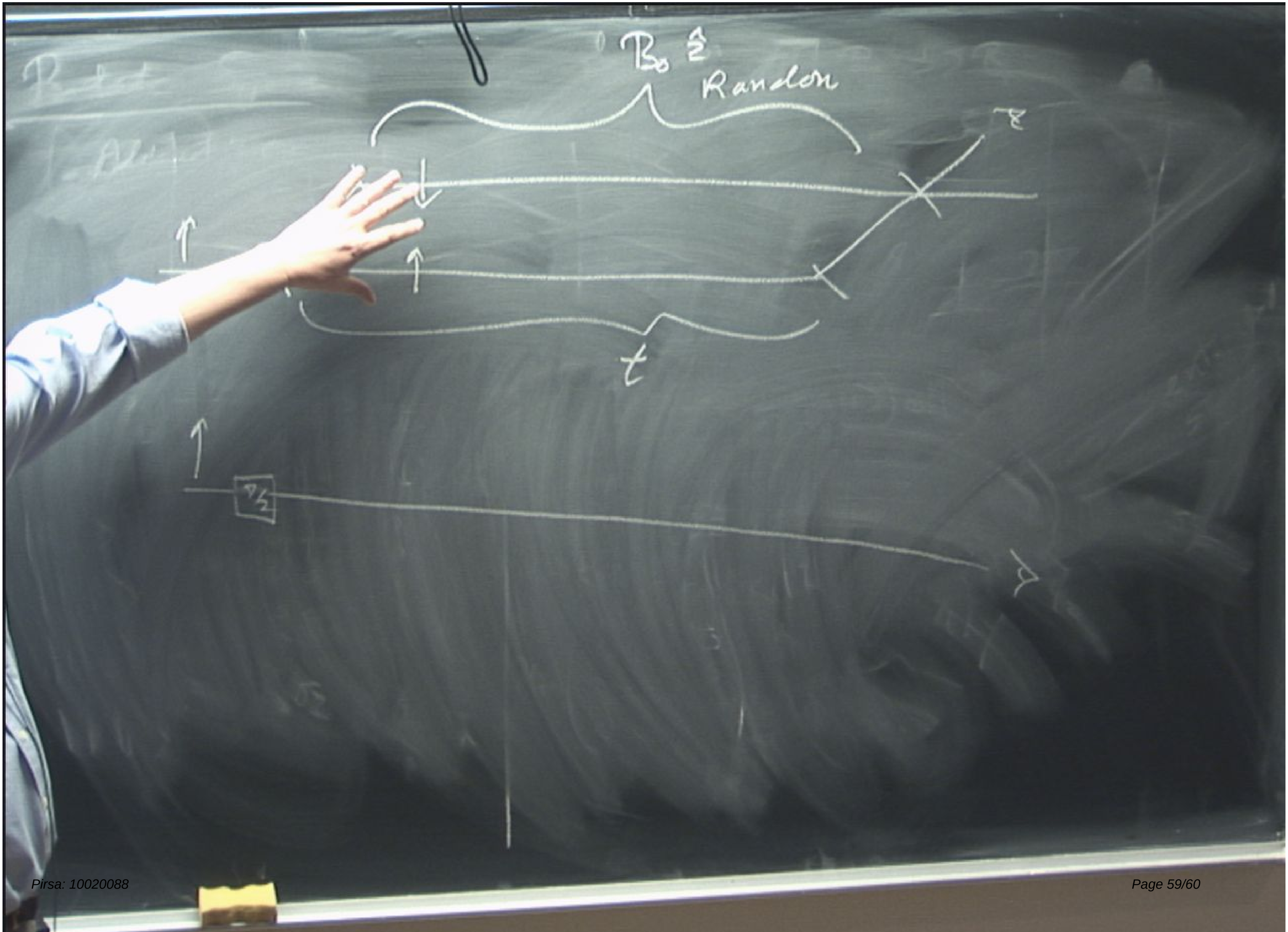
$|0\uparrow\rangle$

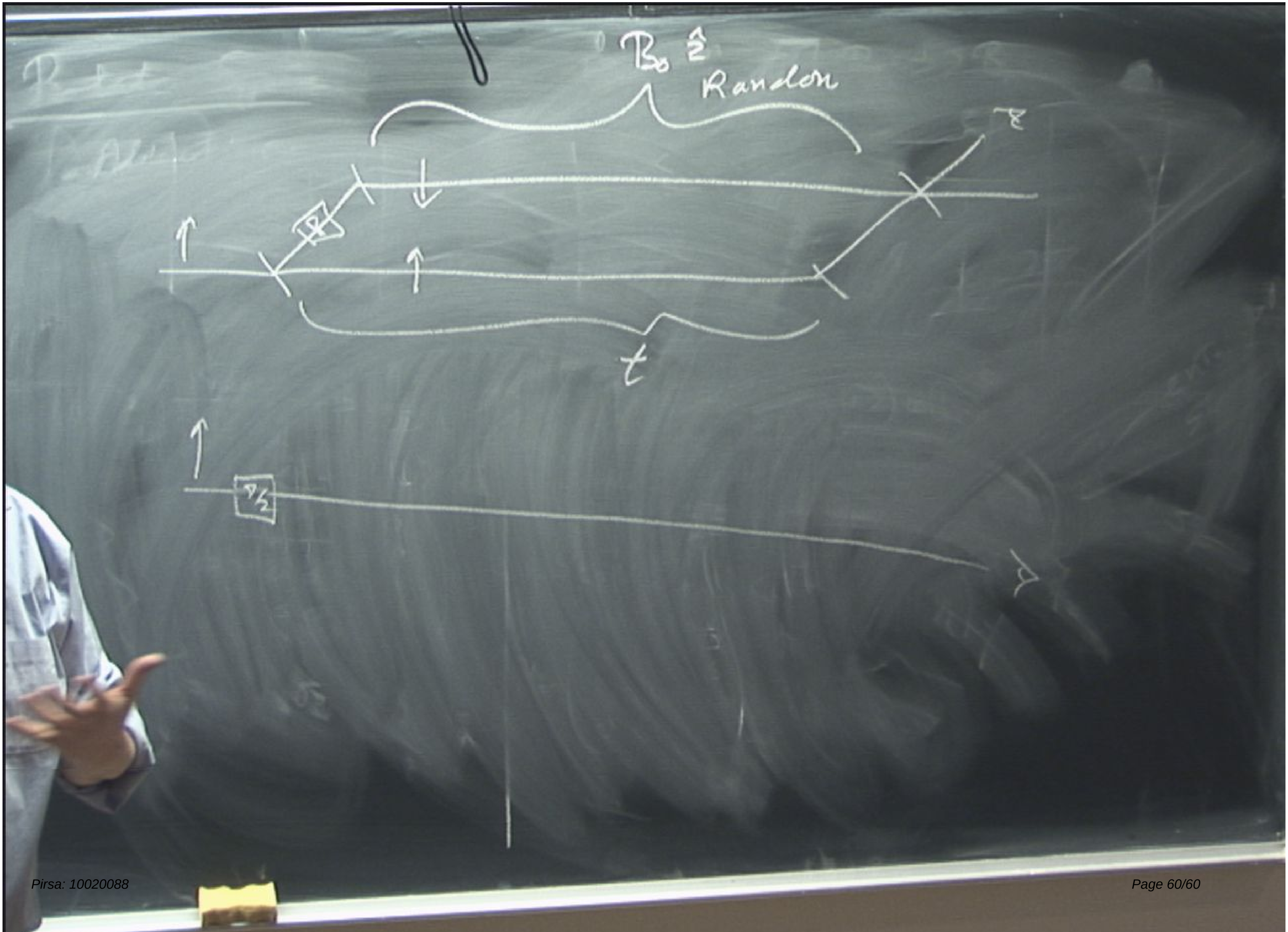
$|1\uparrow$

$|0\downarrow$

$|1$


$B_0 \hat{z}$


$B_0 \hat{z}$
Random
t
π/2



$B_0\hat{z}$
Random
t