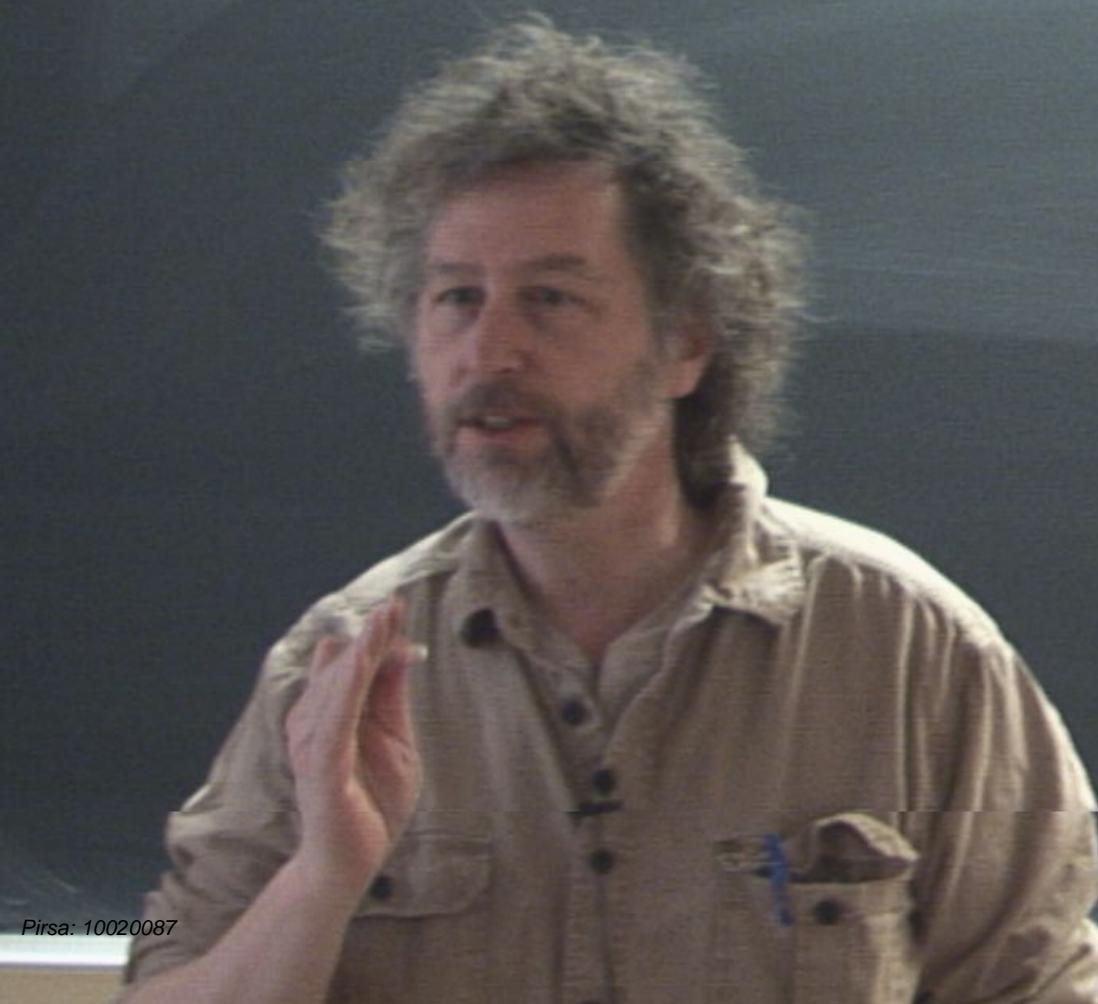
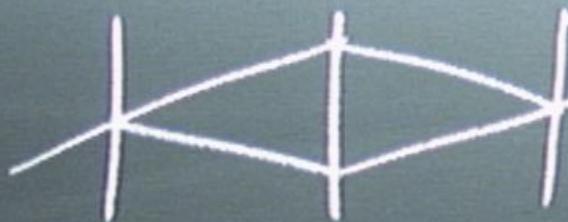


Title: Explorations in Quantum Info. (PHYS 641) - Lecture 4

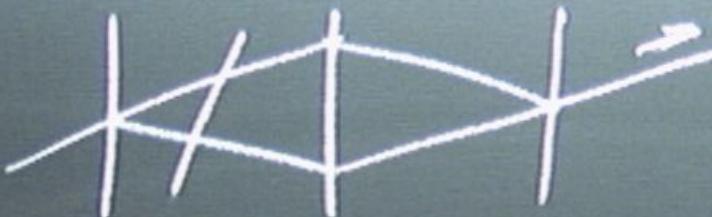
Date: Feb 19, 2010 09:00 AM

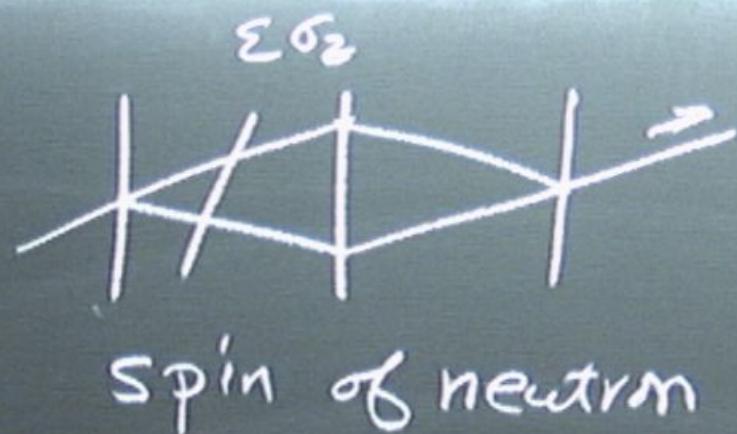
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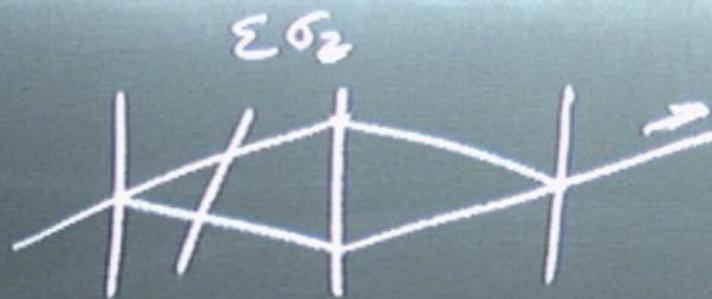
Abstract:

$\Sigma \sigma_2$ 

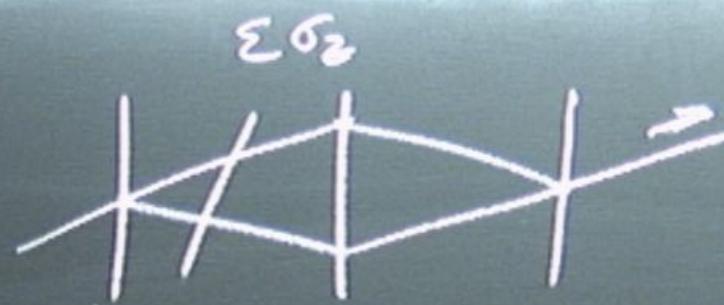
$\Sigma \sigma_2$





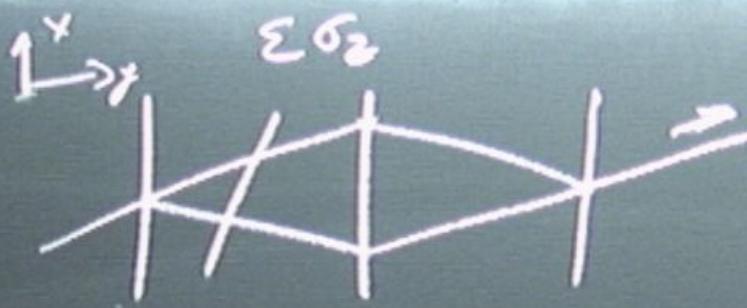


Spin of neutron  
momentum of neutron



spin of neutron

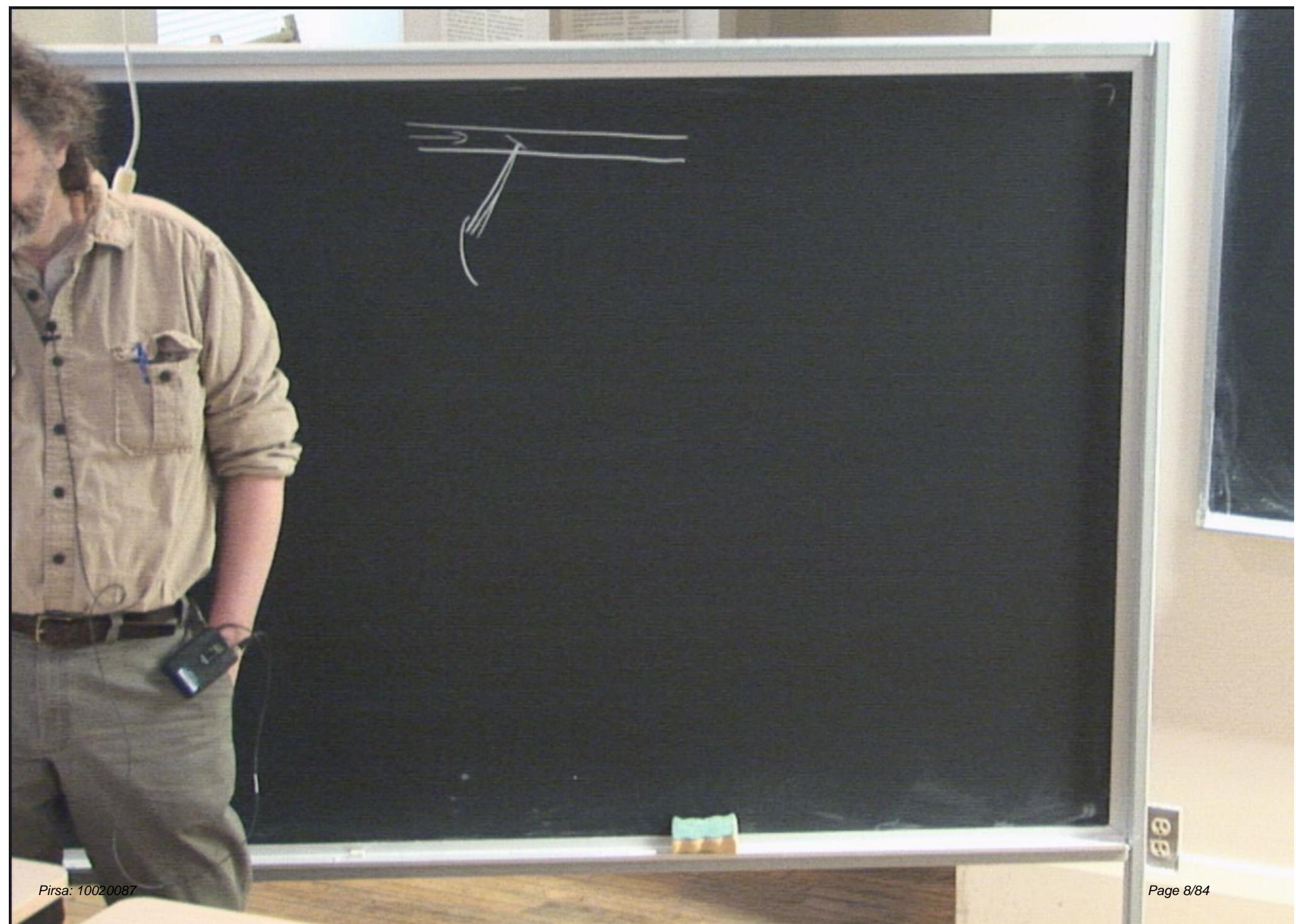
momentum of neutron

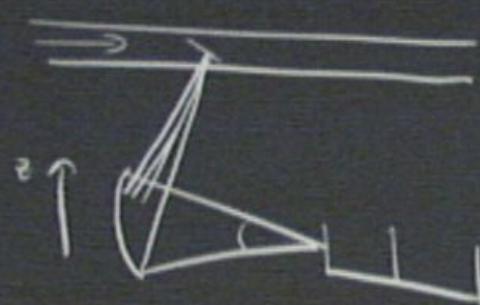


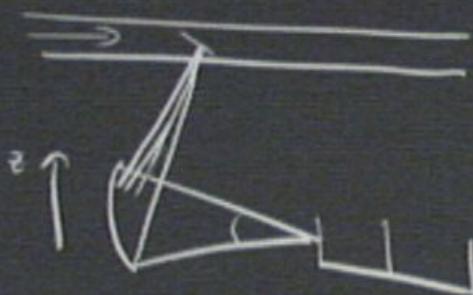
Spin of neutron

Momentum of neutron

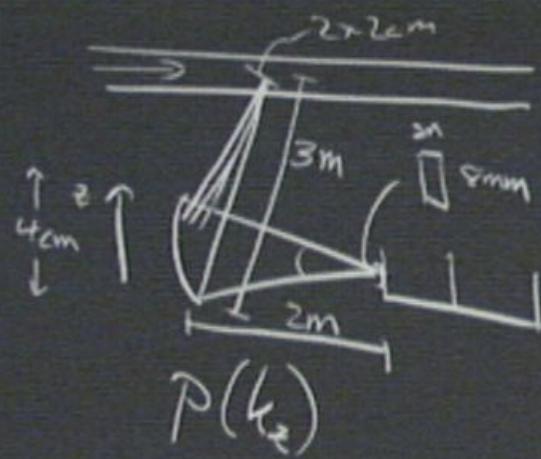
$-z$

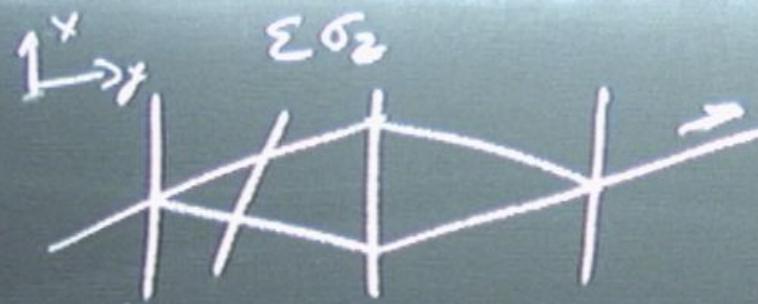






$P(k_z)$



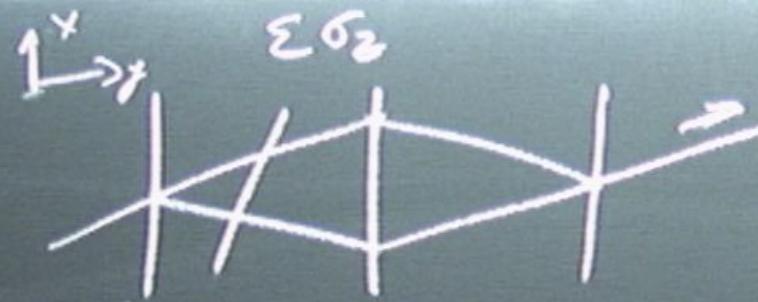


spin of neutrino

momentum of neutrino

-z

$K^2 =$



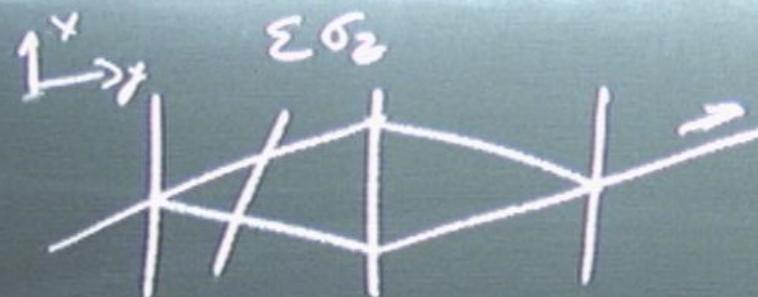
spin of neutron

momentum of neutron

-z

$$K_{(i)}^2 = \frac{2m}{\hbar^2} [E - V(r)]$$

$$\vec{\nabla}^2 \psi(r) + K_{(i)}^2 \psi(r) = 0$$



spin of neutron

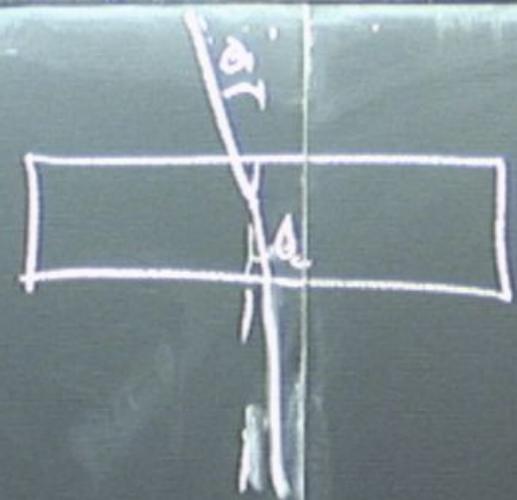
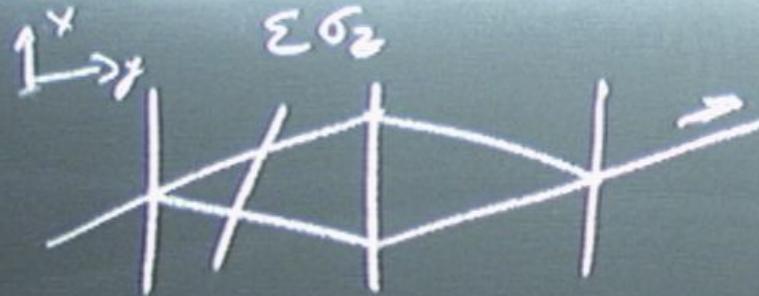
momentum of neutron

$$\underline{-z}$$

$$K_{(r)}^2 = \frac{2m}{\hbar^2} [E - V(r)]$$

$$\vec{\nabla}^2 \psi(r) + K_{(r)}^2 \psi(r) = 0$$

$$n(r) = \frac{K(r)}{k}$$

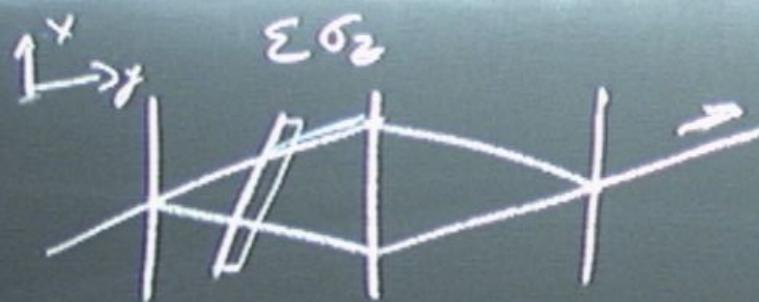


spin of neutron  
momentum of neutron  
-z

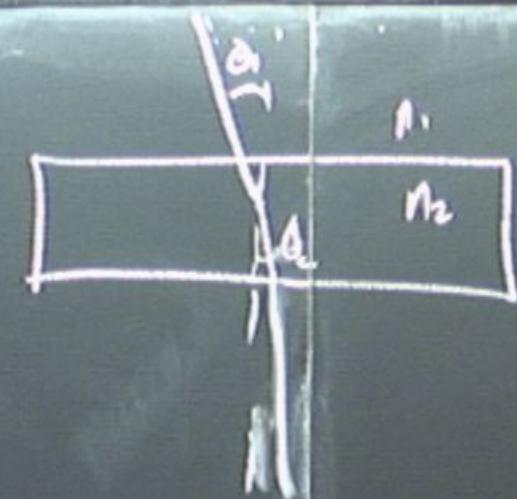
$$K_0^2 = \frac{2m}{\hbar^2} [E - V(r)]$$

$$\nabla^2 \psi(r) + K_0^2 \psi(r) = 0$$

$$n(r) = \frac{K_0 r}{k} = \left[ 1 - \frac{V(r)}{E} \right]^{1/2}$$



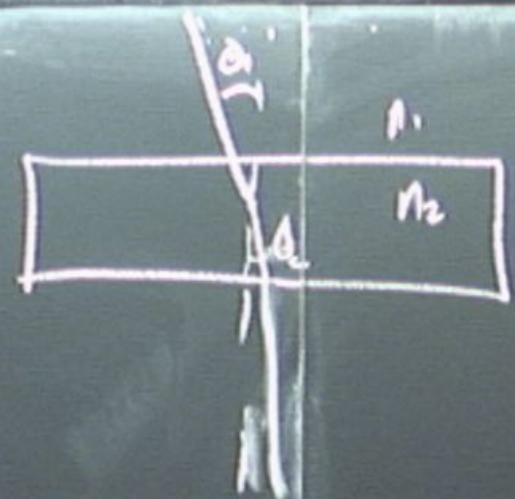
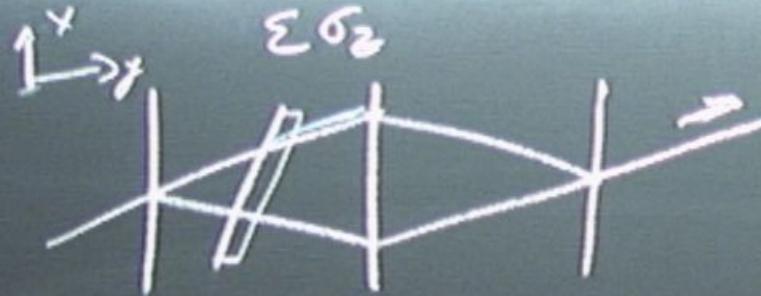
spin of neutron  
momentum of neutron  
 $-\vec{z}$



$$K_0^2 = \frac{2m}{\hbar^2} [E - V(r)]$$

$$\nabla^2 \psi(r) + K_0^2 \psi(r) = 0$$

$$n(r) = \frac{K_0 r}{k} = \left[ 1 - \frac{V(r)}{E} \right]^{1/2}$$

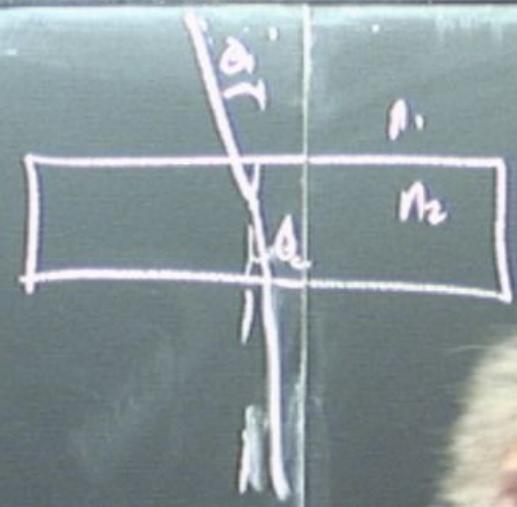
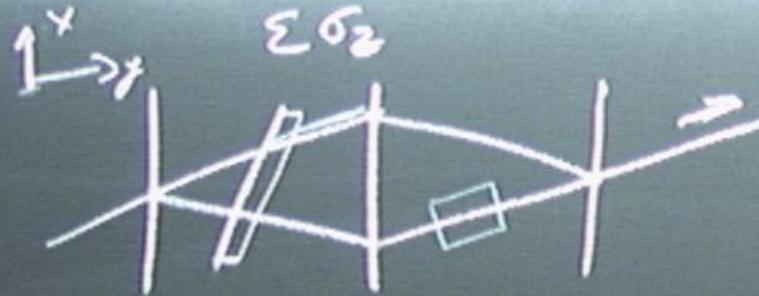


spin of neutron  
momentum of neutron  
-z

$$K_0^2 = \frac{2m}{\hbar^2} [E - V(r)]$$

$$\nabla^2 \psi(r) + K_0^2 \psi(r) = 0$$

$$n(r) = \frac{K_0 r}{k} = \left[ 1 - \frac{V(r)}{E} \right]^{1/2}$$

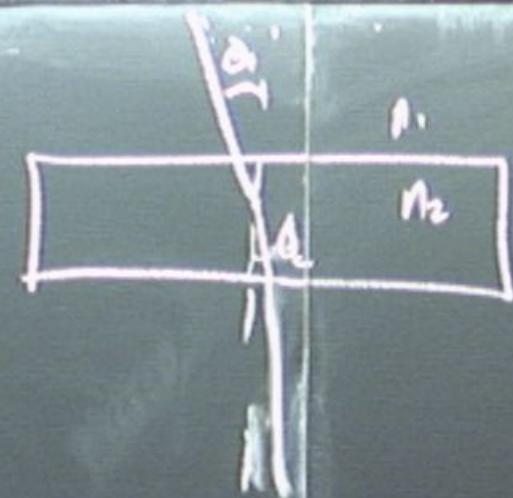
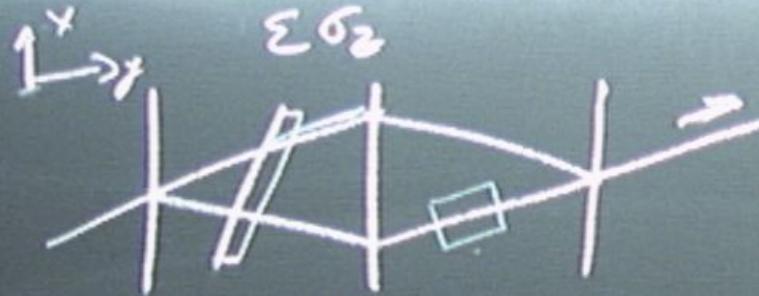


spin of neutron  
momentum of neutron  
-z

$$K_0^2 = \frac{2m}{\hbar^2} [E - V(r)]$$

$$\nabla^2 \psi(r) + K_0^2 \psi(r) = 0$$

$$n(r) = \frac{K_0}{k} = \left[ 1 - \frac{V(r)}{E} \right]^{1/k}$$



spin of neutron

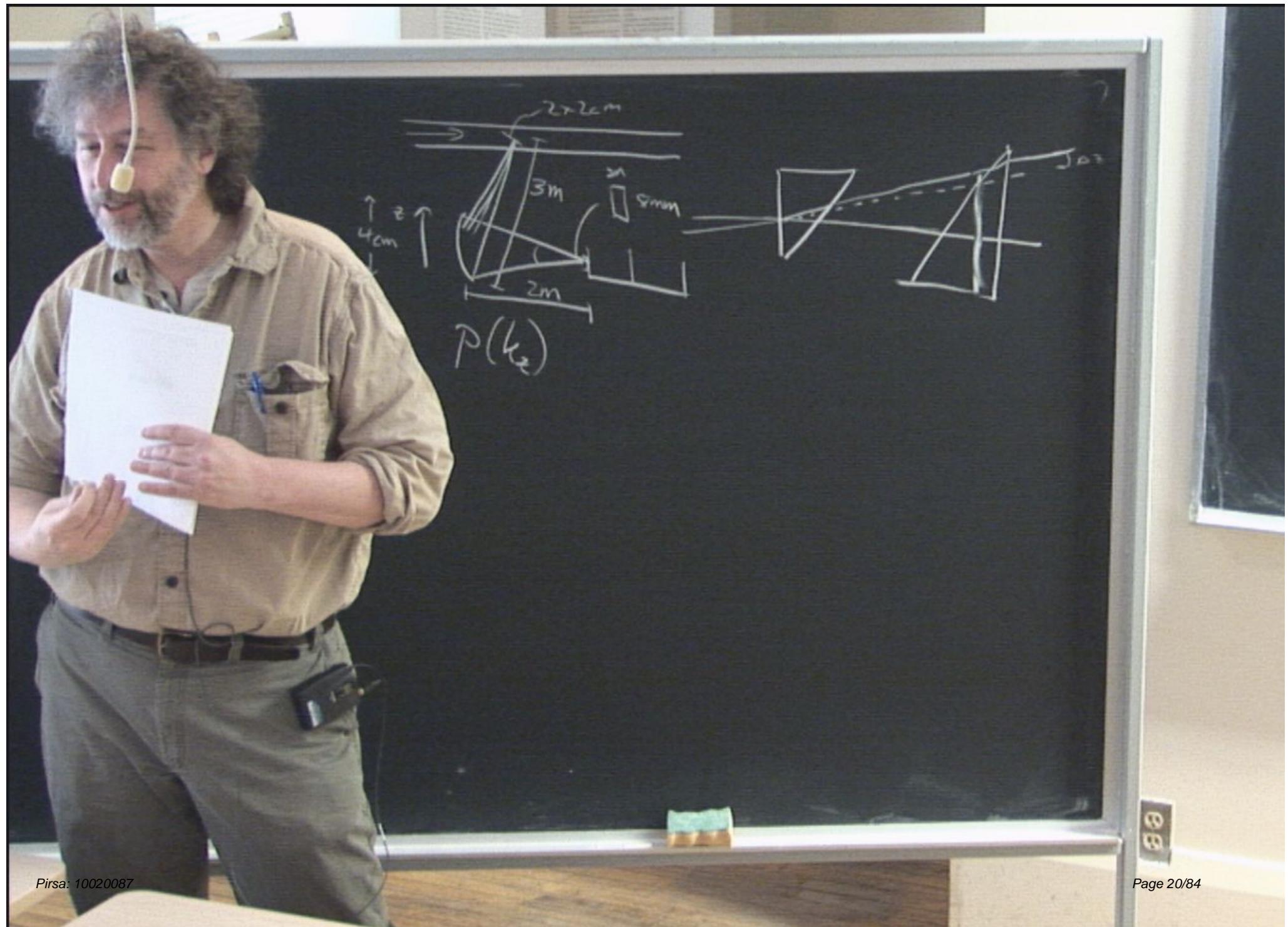
momentum of neutron

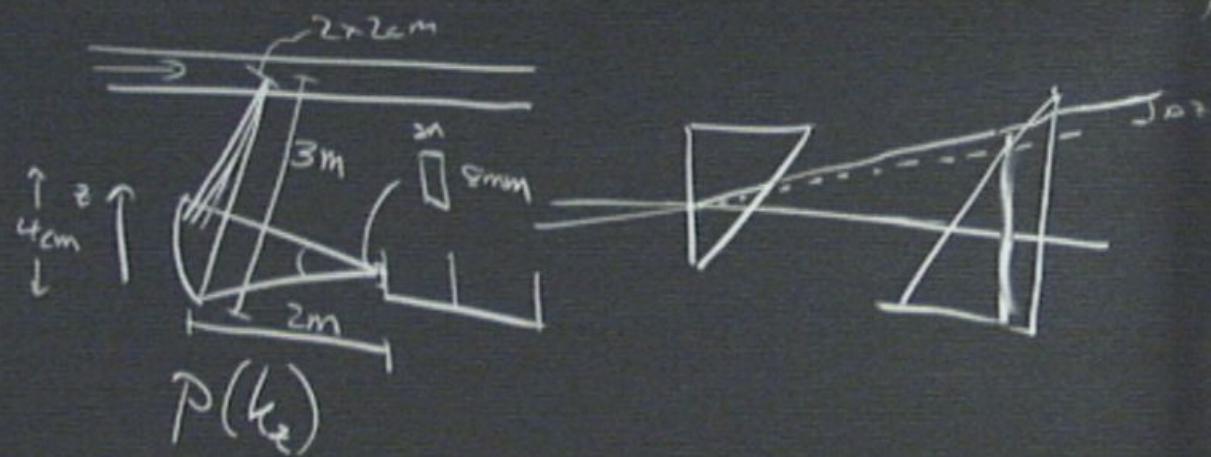
$$\underline{-z}$$

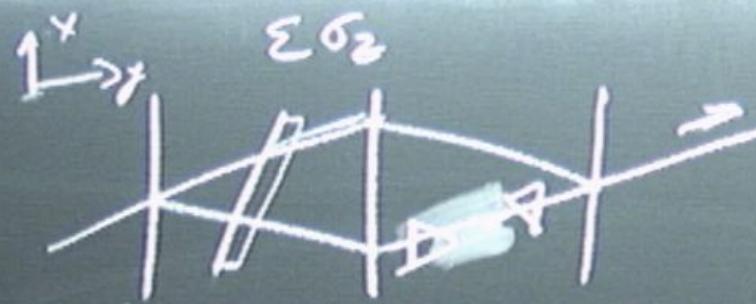
$$K_0^2 = \frac{2m}{\hbar^2} [E - V(r)]$$

$$\nabla^2 \psi(r) + K_0^2 \psi(r) = 0$$

$$n(r) = \frac{K_0 r}{k} = \left[ 1 - \frac{V(r)}{E} \right]^{1/k}$$







spin of neutron

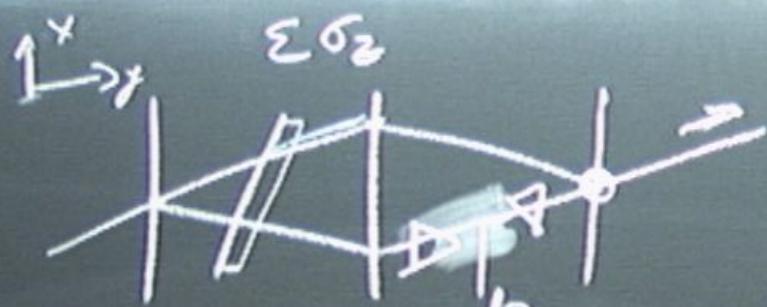
momentum of neutron

$$\underline{-z}$$

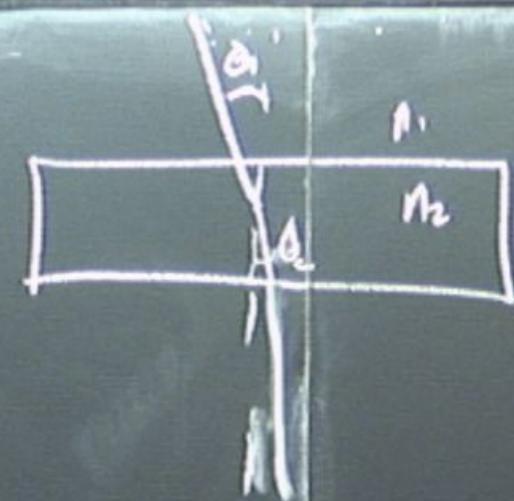
$$K^2 = \frac{2m}{r^2} [E - V(r)]$$

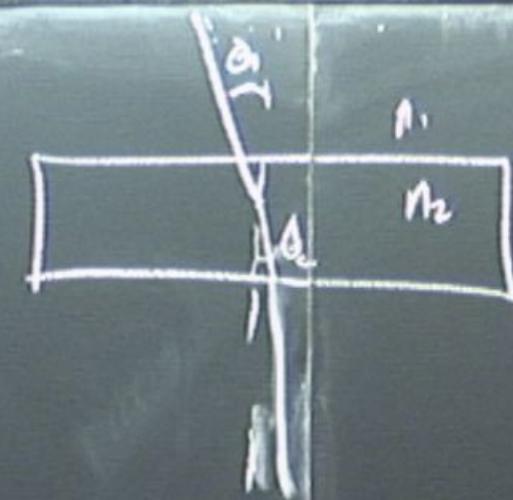
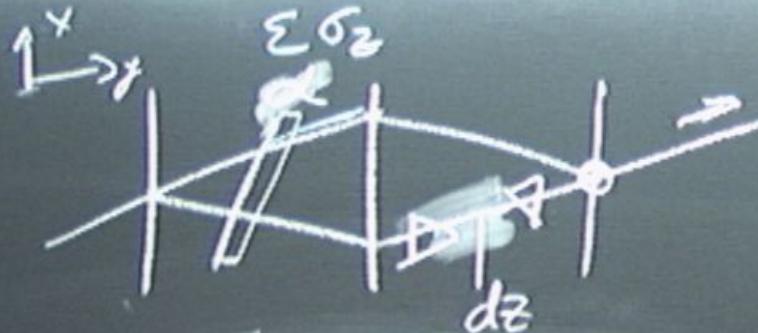
$$\nabla^2 \psi(r) + K^2(r) \psi(r) = 0$$

$$n(r) = \frac{K(r)}{k} = \left[ 1 - \frac{V(r)}{E} \right]^{1/k}$$



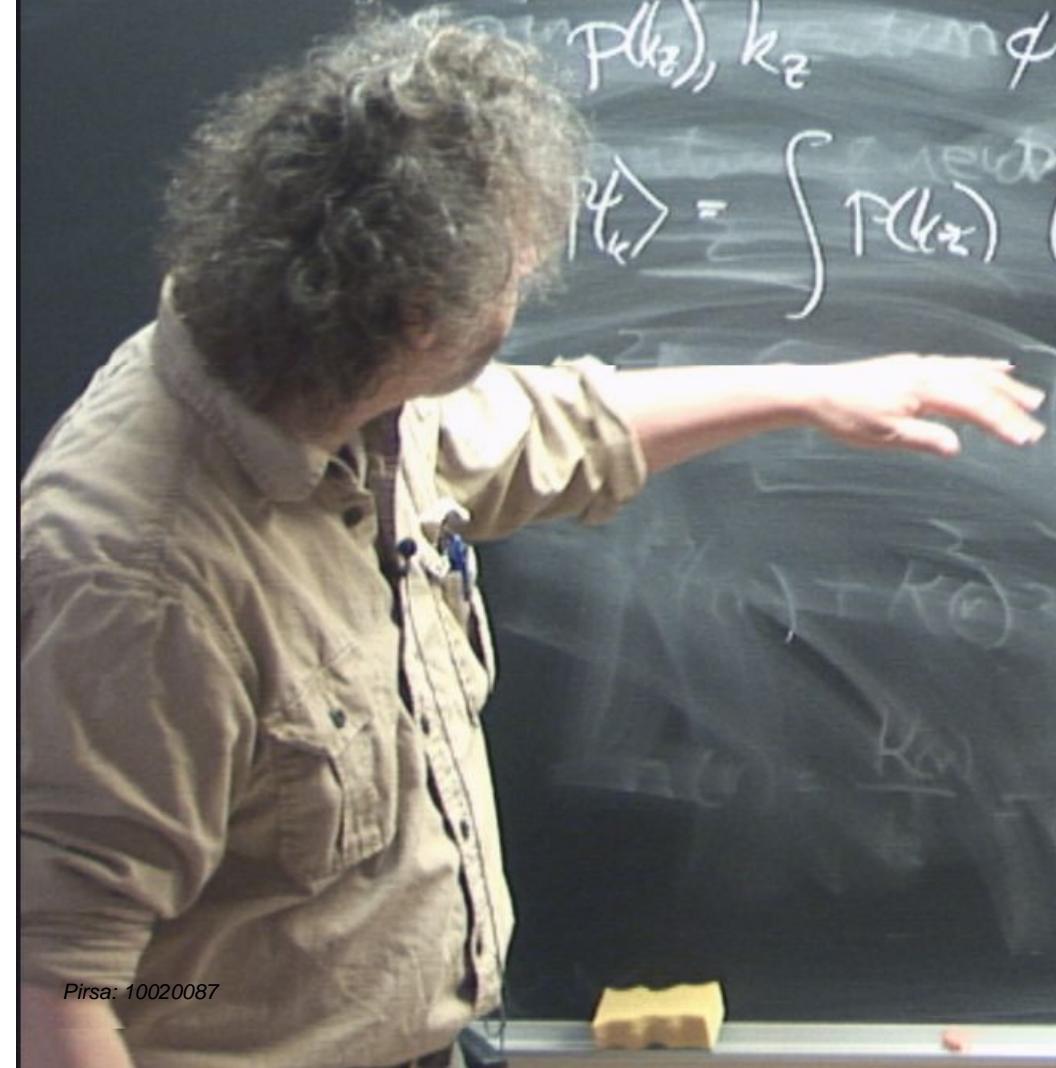
$\text{Spin } P(k_z), k_z \text{ trans}$   
momentum of neutron

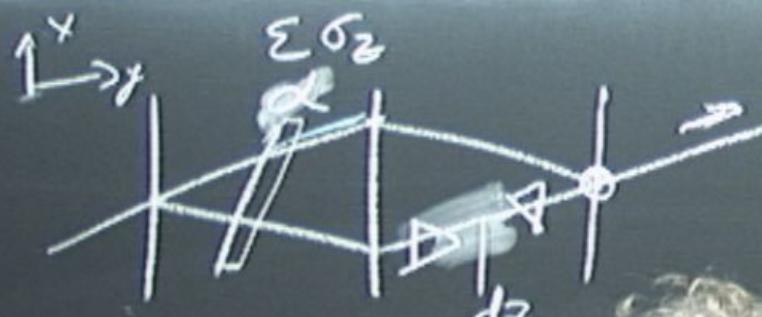




$$P(k_z), k_z \quad \text{and} \quad d\sigma = dz k_z$$

$$|k\rangle = \int P(k_z) (e^{ik_z z} |0\rangle + e^{-ik_z(z-dz)} |1\rangle)$$





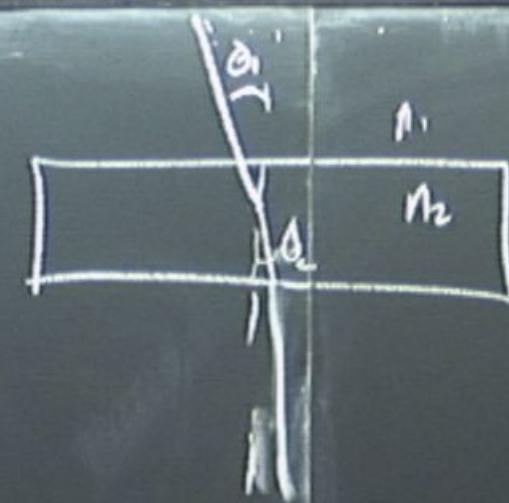
Spin  $P(k_z), k_z$

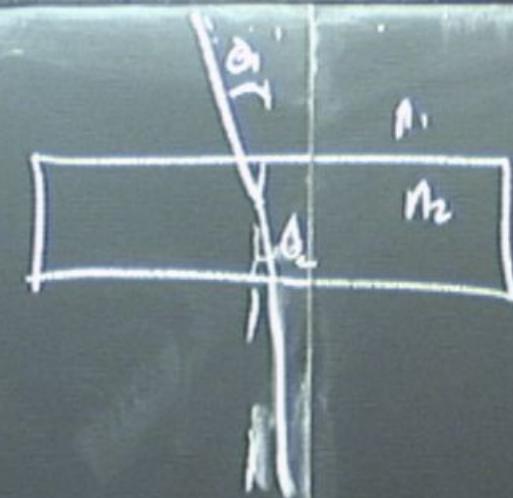
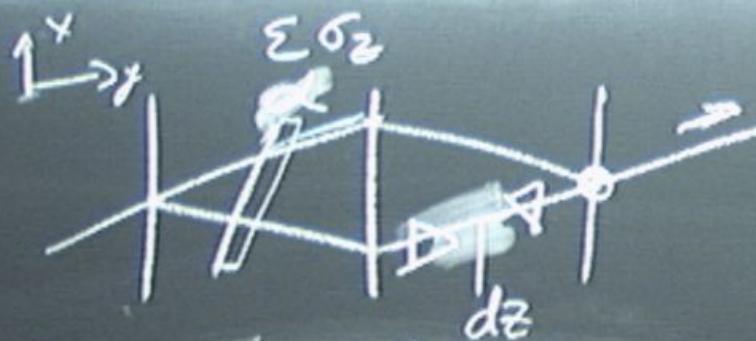
max

$$|P_k\rangle =$$

$dz k_z$

$$= e^{-ik_z z} |0\rangle + e^{-ik_z(z-dz)} |1\rangle$$





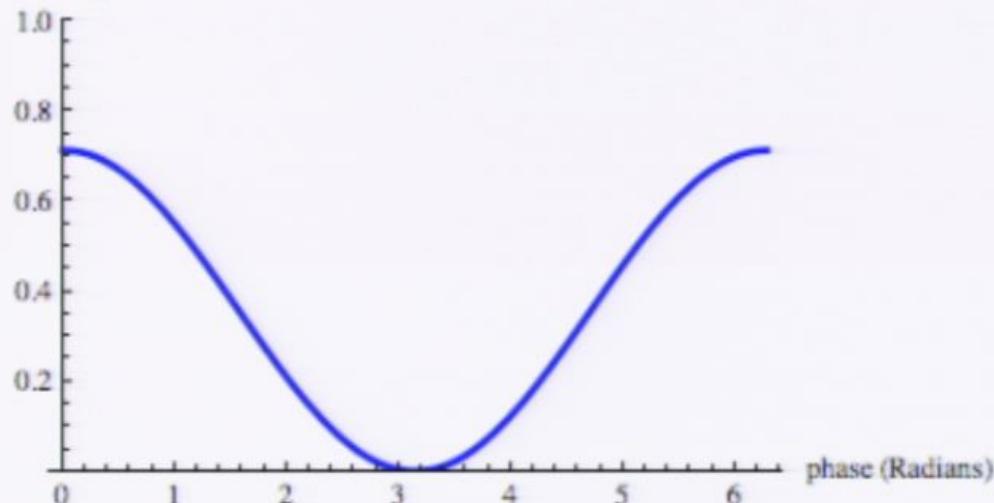
$$\text{Spin } P(k_z), k_z \text{ } t \sin \phi = dz k_z$$

$$P_k = \int P(k_z) (e^{ik_z z} |0\rangle + e^{-ik_z z} |1\rangle)$$

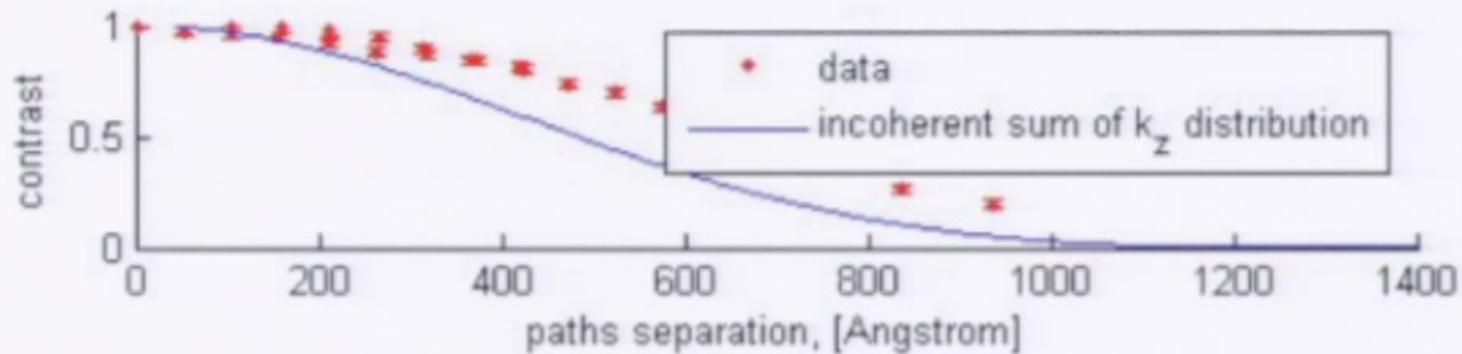
\$Aborted

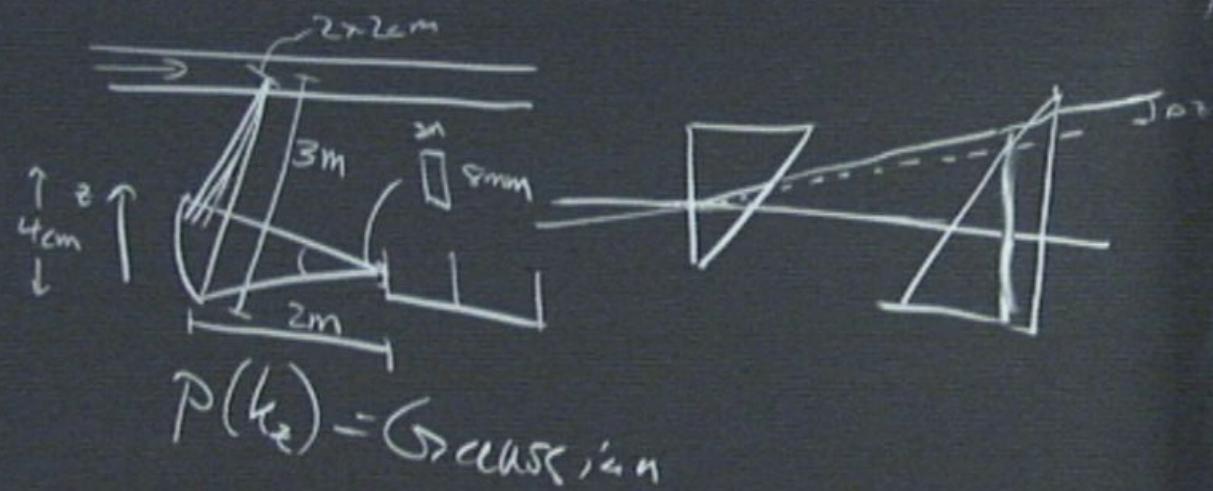
```
Plot[M140[a, .02], {a, 0, 2 π}, {AxesLabel -> {"phase (Radians)", "intensity"}, PlotStyle -> {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange -> {0, 1}}]
```

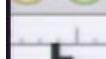
intensity



xpected the contrast is a function of momentum spread.



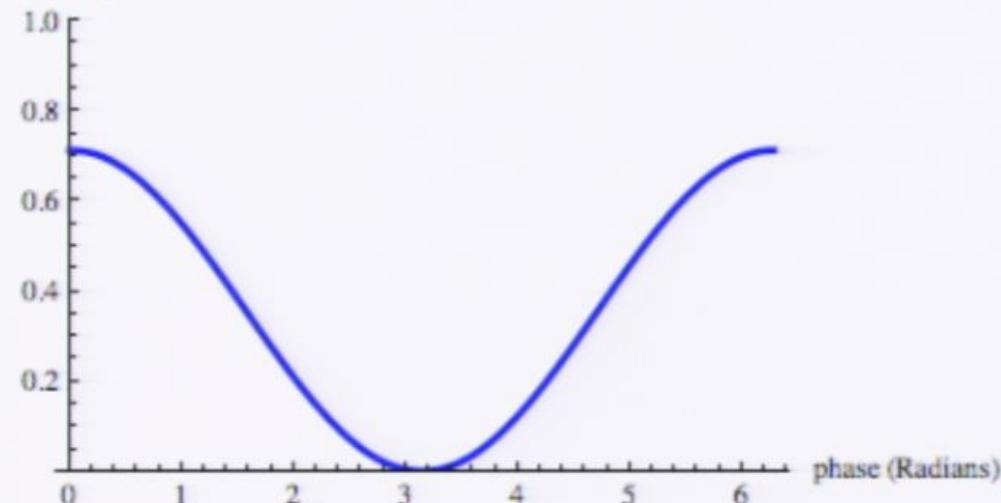




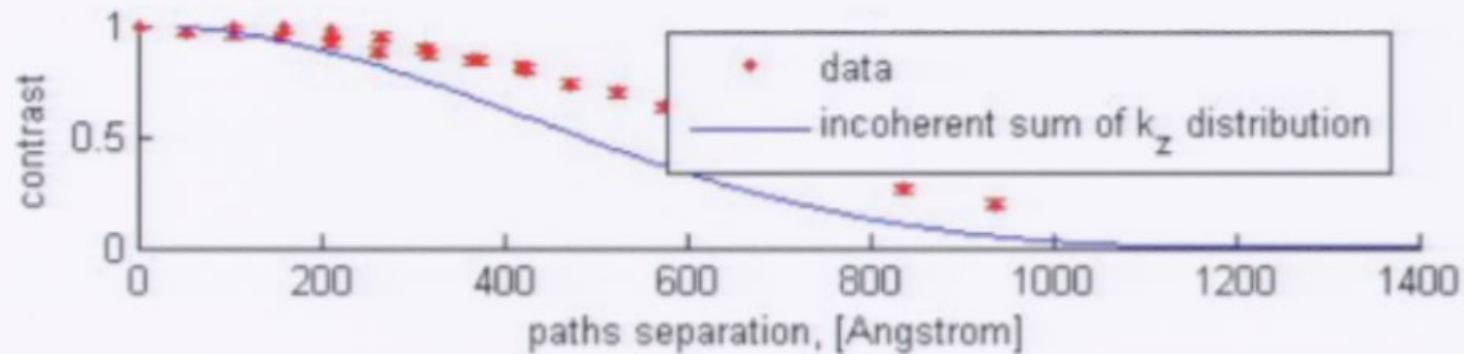
\$Aborted

```
Plot[M140[a, .02], {a, 0, 2 π}, {AxesLabel -> {"phase (Radians)", "intensity"}, PlotStyle -> {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange -> {0, 1}}]
```

intensity



Expected the contrast is a function of momentum spread.



a Gaussian distribution of momenta along z.

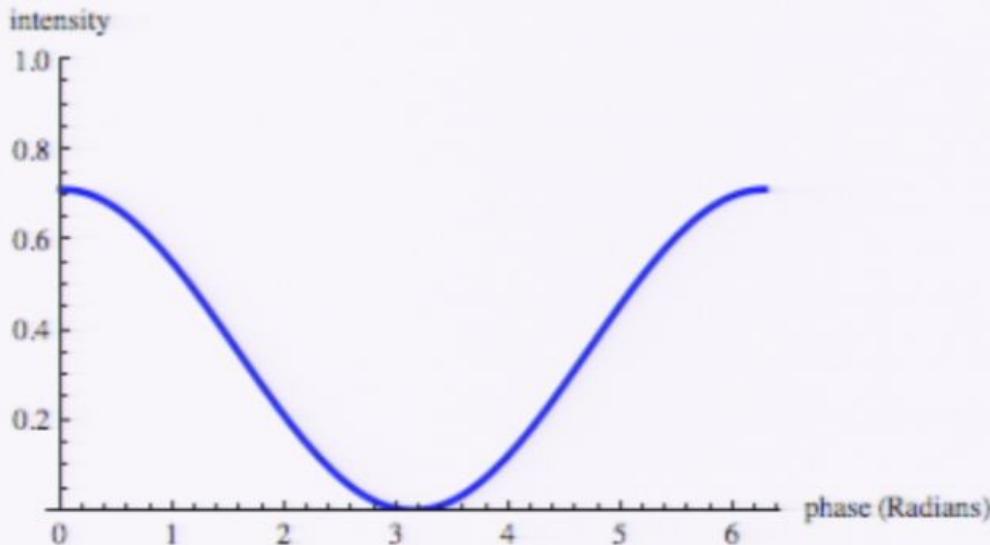
```
nd[x_, sd_] := Exp[-x^2 / (2 sd^2)] / (sd Sqrt[2 \[Pi]]);

M14O[a_, sd_] :=
  Sum[Sum[nd[kz, sd] Tr[Ezp . res14[kz, z, a]] / 2, {kz, -0.1, 0.1, 0.001}],
    {z, -.5, .5, .025}] / 40 / 500

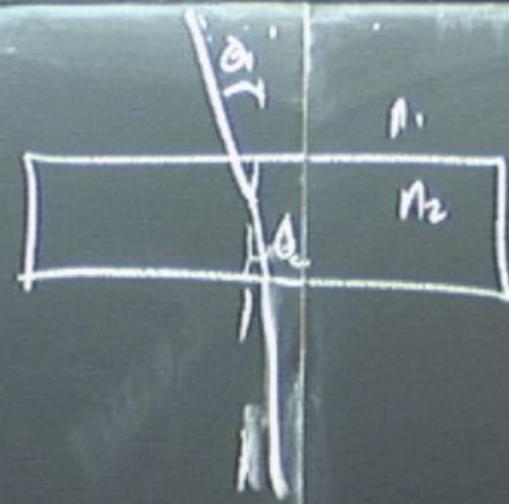
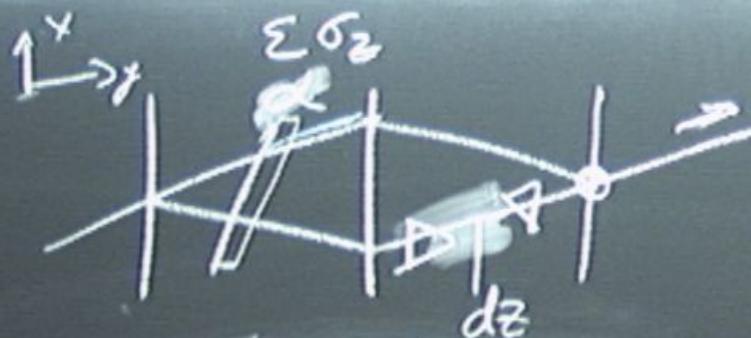
Plot[M14O[a, .01], {a, 0, 2 \[Pi]}, {AxesLabel \[Rule] {"phase (Radians)", "intensity"}, PlotStyle \[Rule] {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange \[Rule] {0, 1}}]

$Aborted

Plot[M14O[a, .02], {a, 0, 2 \[Pi]}, {AxesLabel \[Rule] {"phase (Radians)", "intensity"}, PlotStyle \[Rule] {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange \[Rule] {0, 1}}]
```



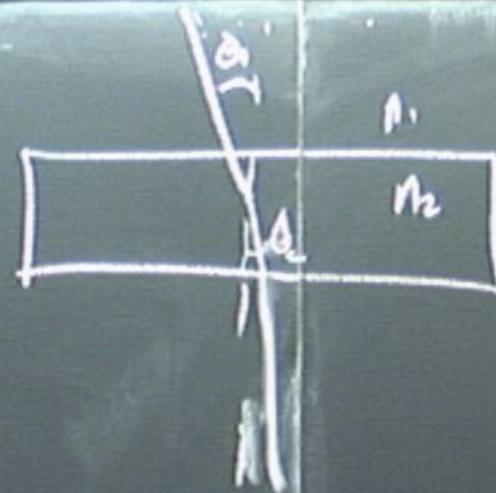
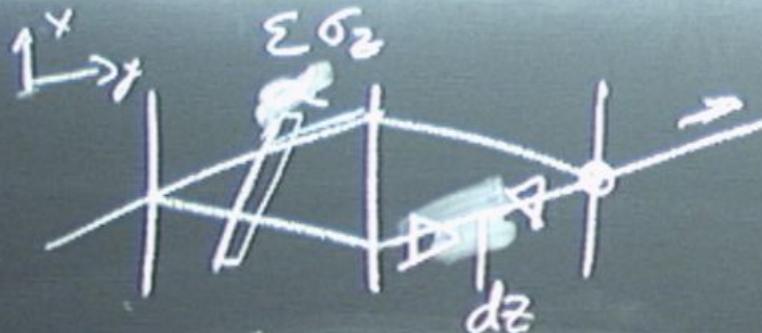
xpected the contrast is a function of momentum spread.



spin  $P(k_z), k_z \tan \phi = dz k_z$

$$|k\rangle = \int P(k_z) (e^{ik_z z} |0\rangle + e^{-ik_z z} |1\rangle)$$

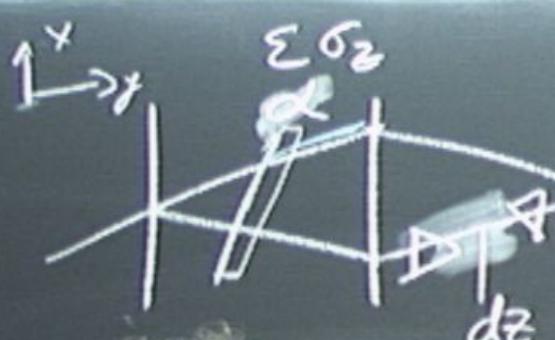




spin  $P(k_z), k_z \tan \phi = dz \mu_z$

$$|\Psi_k\rangle = \int P(k_z) \left( e^{i k_z z} |0\rangle + e^{-i k_z (z - a)} |1\rangle \right) dk_z$$

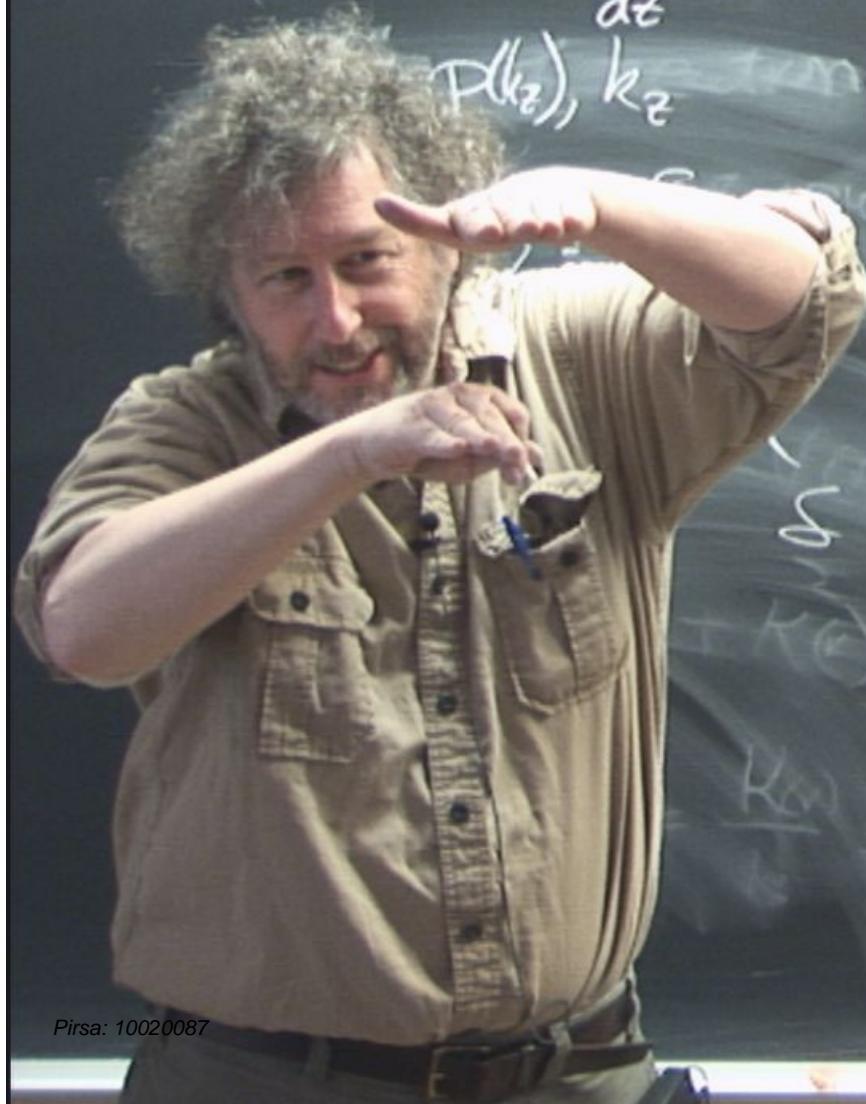




$$\frac{1}{\sqrt{2}} (|0>|1\rangle_{d\rightarrow d} + |0>|1\rangle_{d\leftarrow d})$$

$$P(k_z), k_z$$

$$(e^{i k_z x} e^{-i k_z z} |0\rangle + e^{-i k_z (z-x)} |1\rangle) dk_z$$



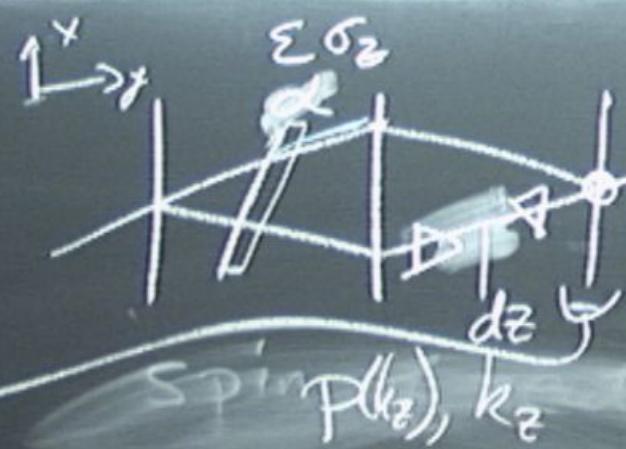
$$\frac{1}{\delta z} (|0\rangle \langle 1| |dz=0\rangle + |0\rangle \langle 0| |dz=dz\rangle)$$

$$(|0\rangle \langle 0| |dz=0\rangle |dz=0\rangle + |0\rangle \langle 0| |dz=dz\rangle |dz=dz\rangle)$$

$$\phi = dz \mu_z$$

$$\int F(k_z) (e^{ik_z z} e^{-ik_z z} |0\rangle + e^{ik_z(z-dz)} |1\rangle) dk_z$$

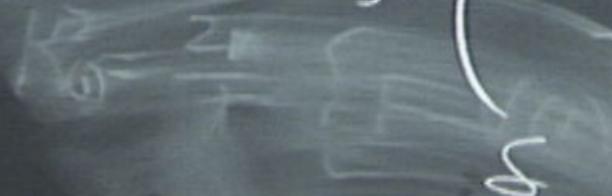
unshifted  
shifted

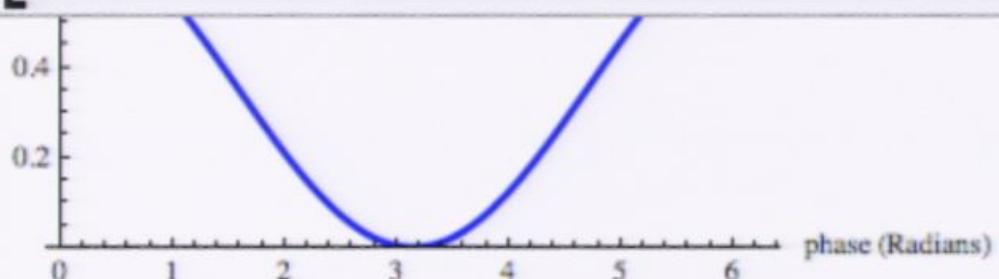


$$\frac{1}{V} \left( |0>_{k1} |dz=0> + |0>_{k1} |dz=0> \right) \\ (|0>_{k1} |dz=0>) (dz=0) +$$

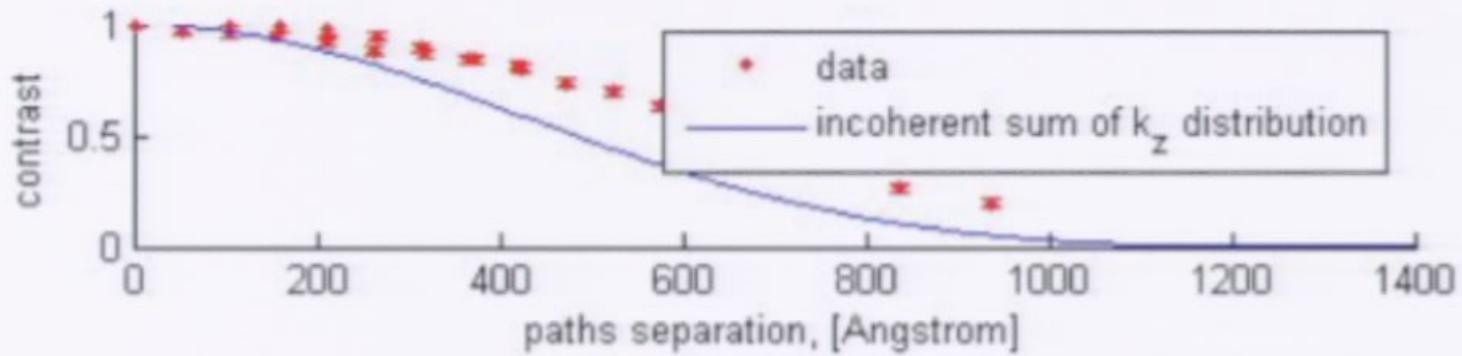
$$\phi = dz k_z$$

$$|k> = \int P(k_z) \left( e^{-ik_z z} |0> + e^{ik_z z} |1> \right) dk_z$$





expected the contrast is a function of momentum spread.

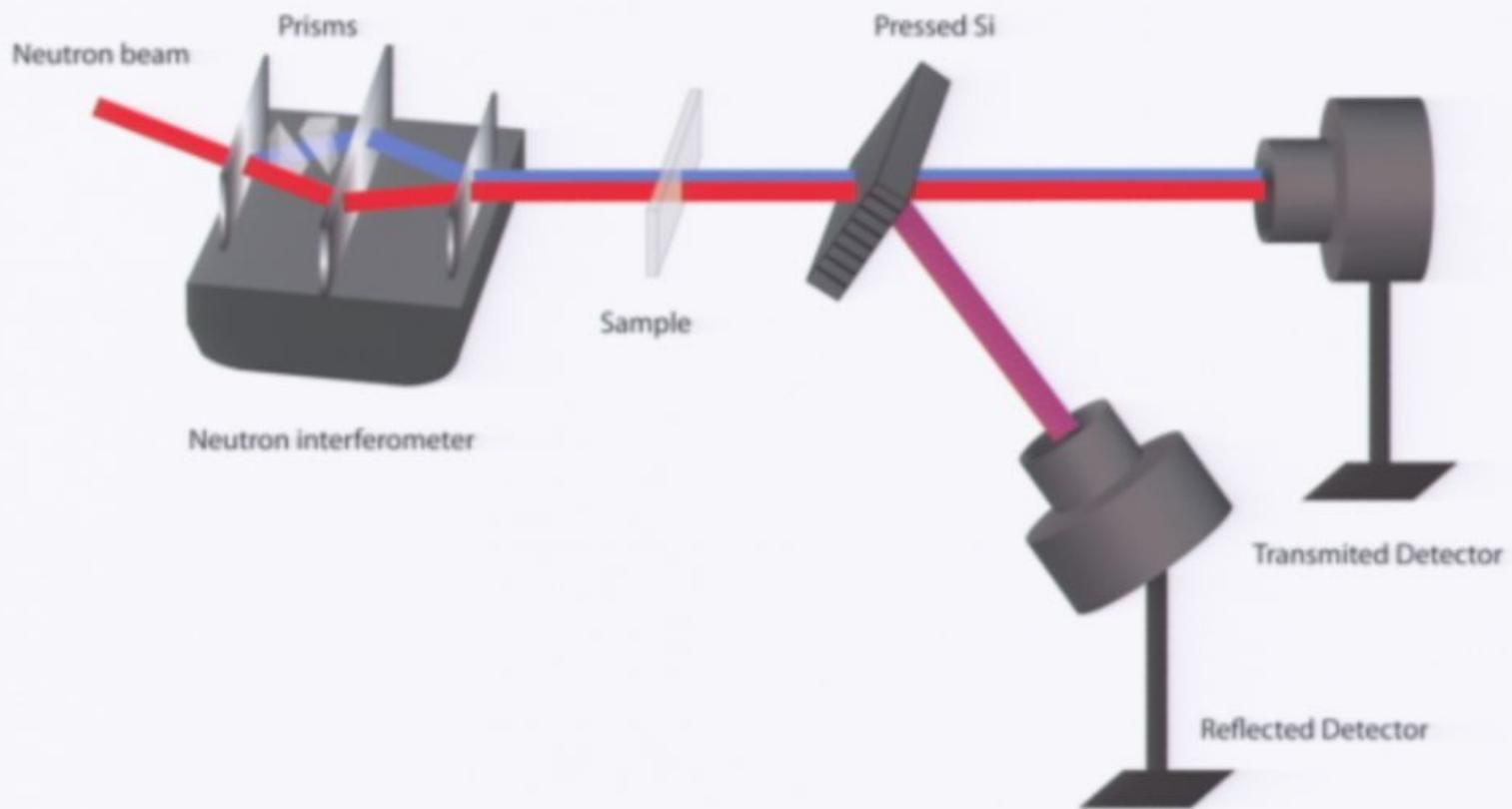


Experimentally we see the same thing. Note that the contrast curve is not a Gaussian.

### Experiment 15: outgoing wave

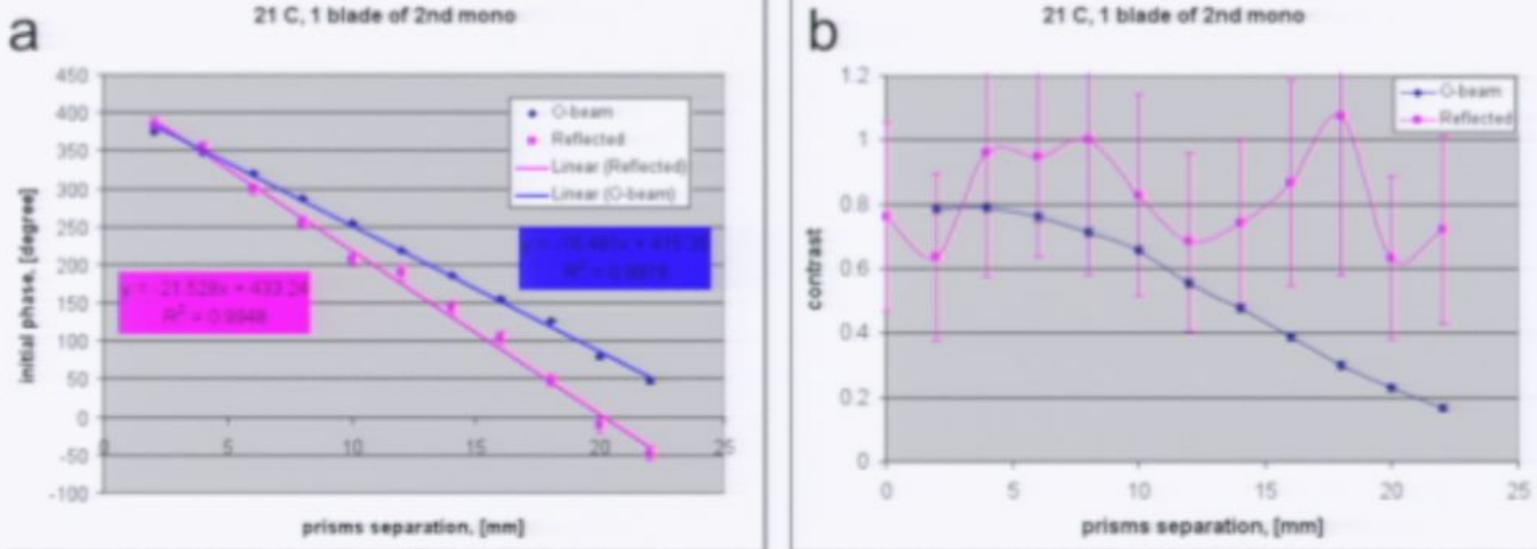


now that contrast remains we can recombine the blades with a "thick crystal" momentum filter.



thick crystal is conveniently thought of as combining the displaced beams.

data is noisy but shows that the scattered (recombined) beam retains contrast, while the direct beam loses contrast with increased separation of the beams.



Another point to remember here is that the contrast in the transmitted beam is lost due to a distribution of momenta. Since the neutron momenta in free space do not change, this can easily be recovered. Also note that we can compose arbitrary states of momenta through a series of such filters.

### Experiment 16: coherence length with a focusing monochromator

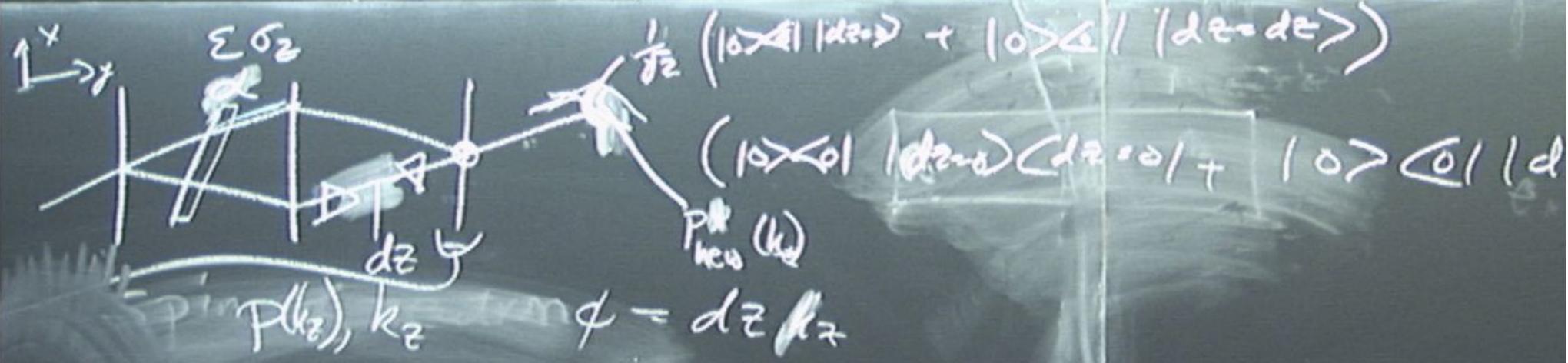
distribution in  $k_z$  has two contributions: a set of focusing monochromator blades; and the thermal width of each

unshifted  
 shifted

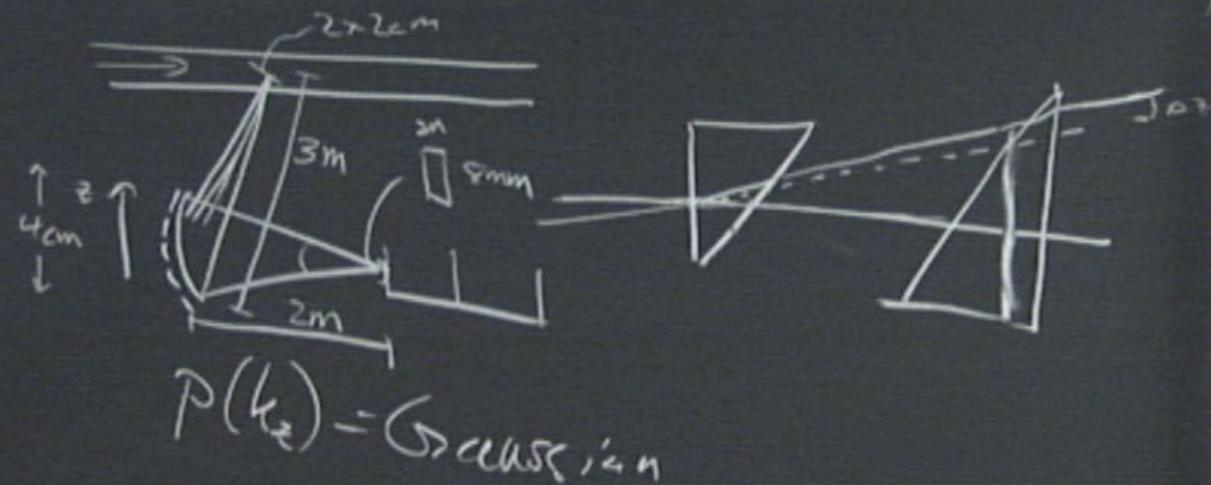
$$\Sigma \delta z$$

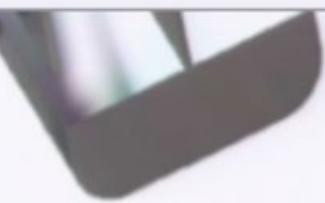
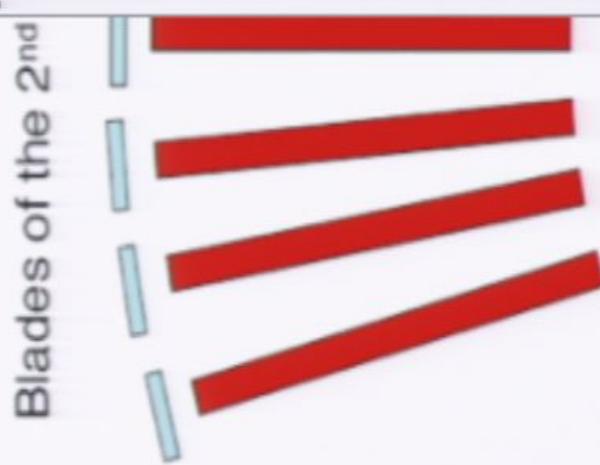
$$R(k_z) = \int_S R(k_z) (e^{ik_z x} e^{-ik_z z} |D| + e^{-ik_z x} e^{ik_z z} |D|)$$

$$\phi = dz k_z$$



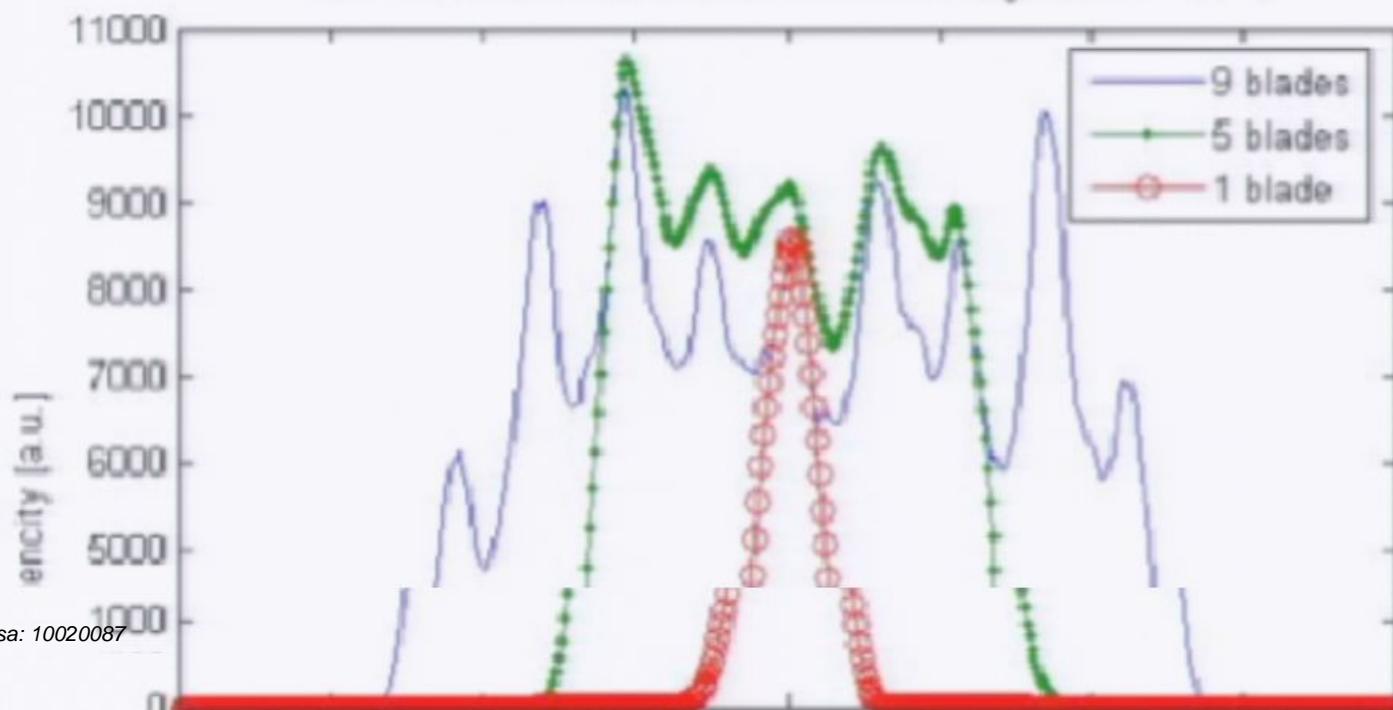
$$|\psi_k\rangle = \int P(k_z) \left( e^{ik_z z} e^{-i k_z z} |0\rangle + e^{i k_z z} e^{-i k_z z} |1\rangle \right) dk_z$$

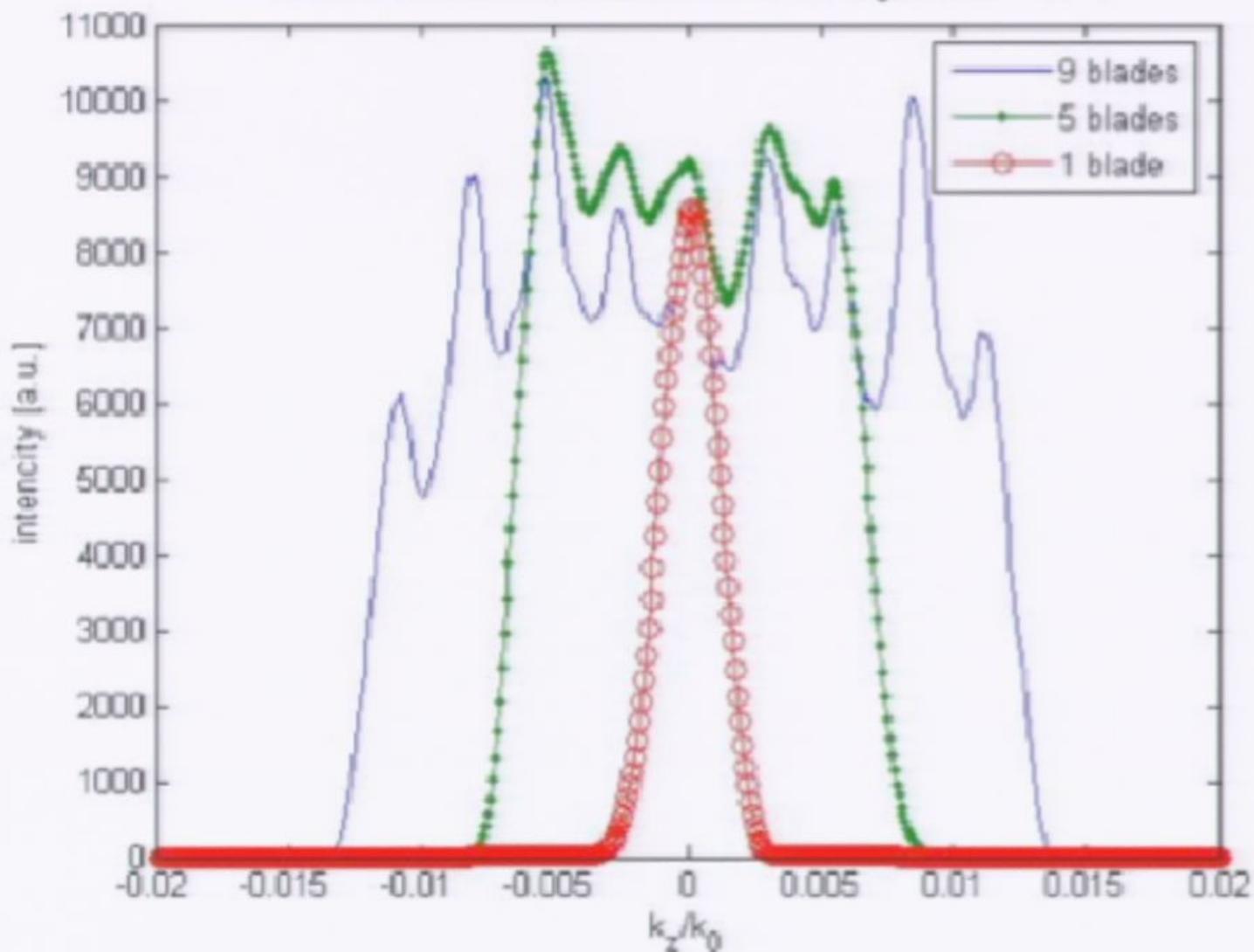




Interferometer

Vertical Neutron Momentum Distribution ( $k_0 = 2.3 \times 10^{10} \text{ m}^{-1}$ )

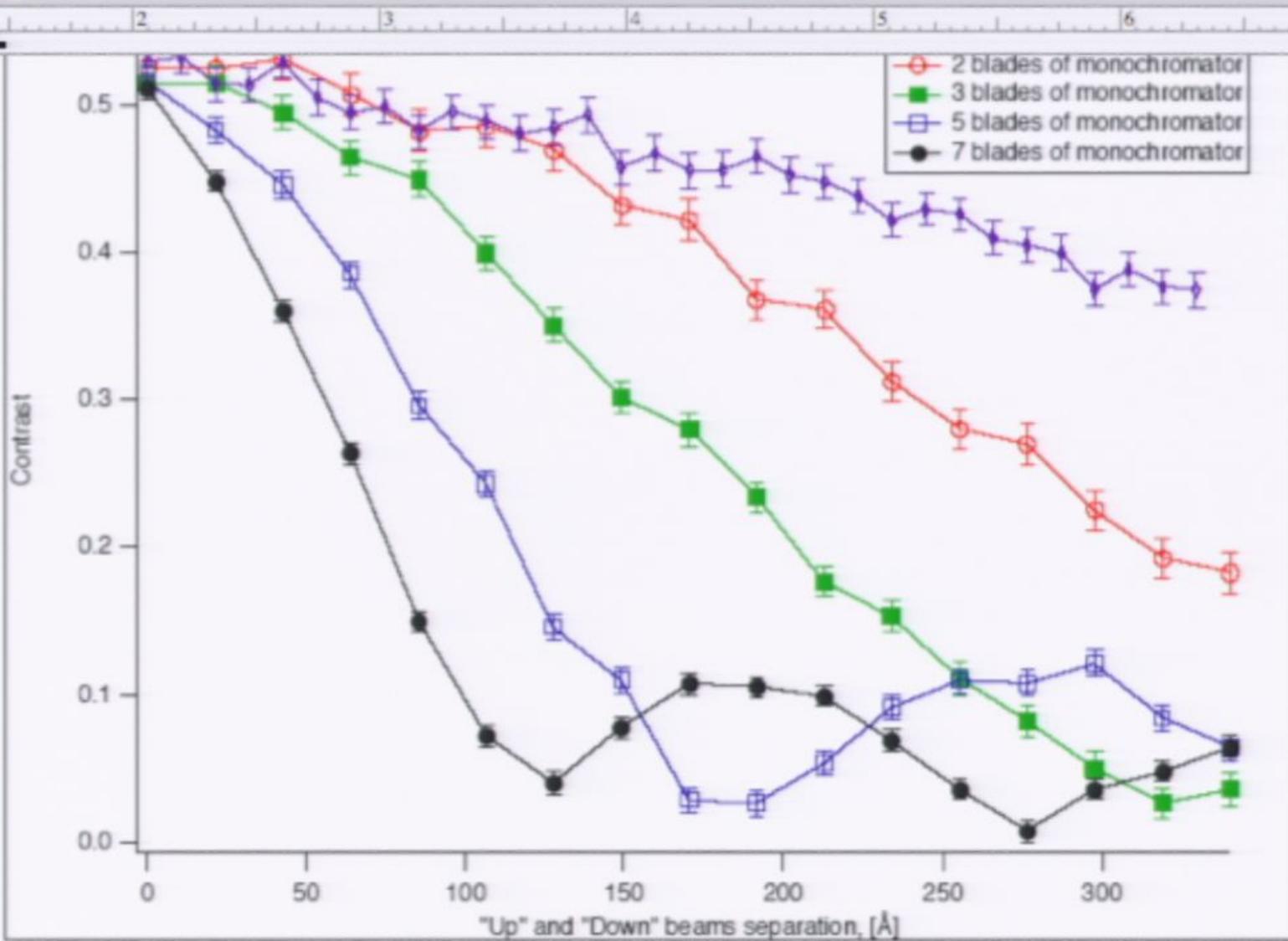


Vertical Neutron Momentum Distribution ( $k_0 = 2.5 \times 10^{-4} \text{ m}^{-1}$ )

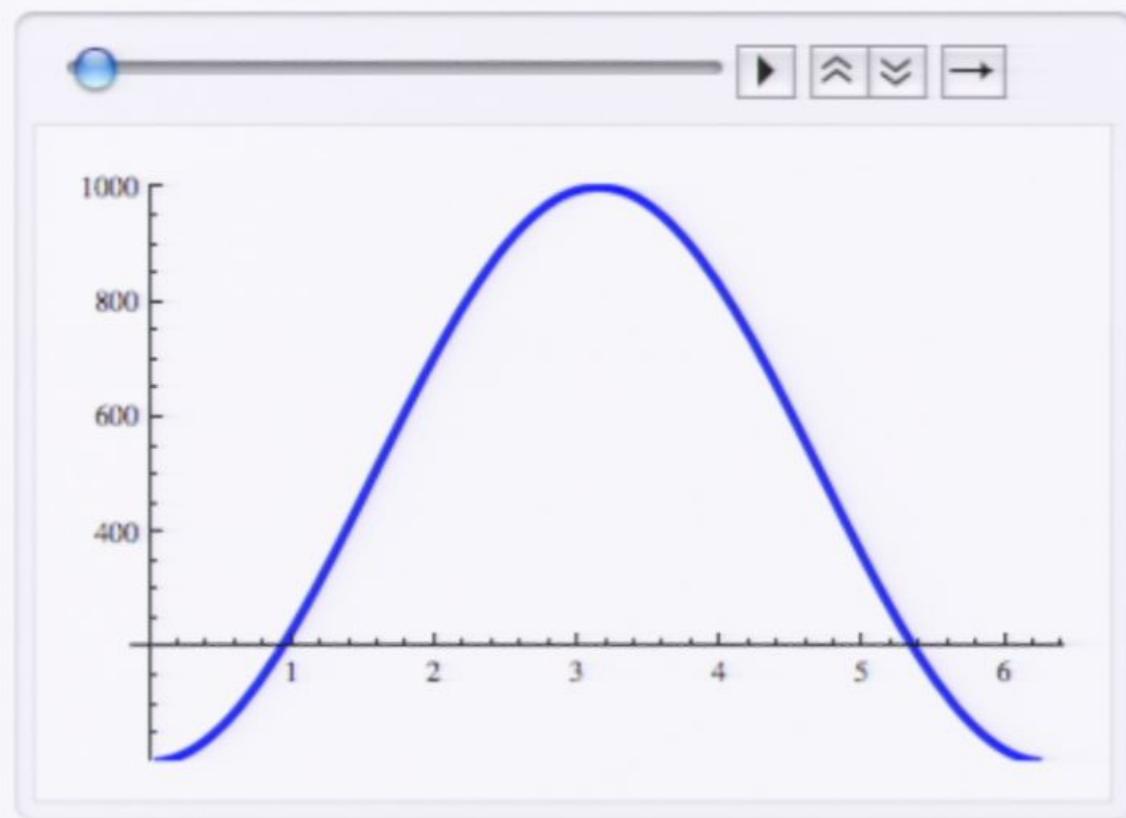
```
Pkz[kz_] := Sum[nd[kz - d, 0.001], {d, 0.012, -0.012, -0.003}] / 9
```

Pirsa: 10020087

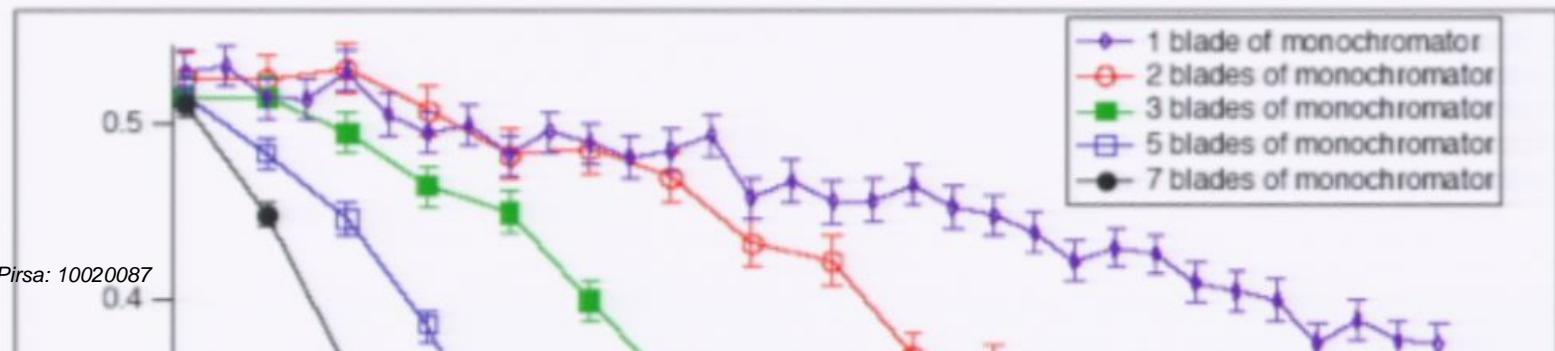
```
Plot[Pkz[kz], {kz, -0.02, 0.02}, PlotStyle -> {RGBColor[0, 0, 1], Thickness[0.01]}]
```

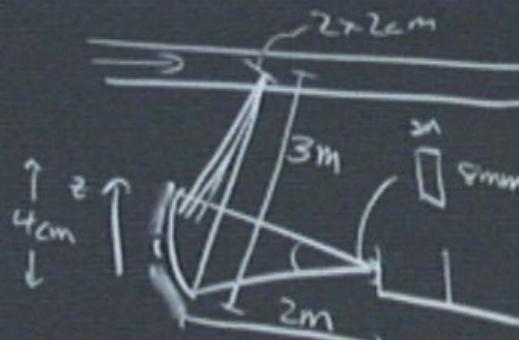
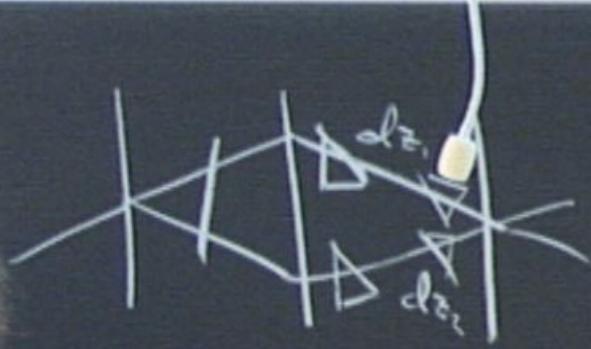


that the beats in the above data are predicted since the momentum spread has features. The Coheence curve is the Fourier transform of the momemtum distribution.

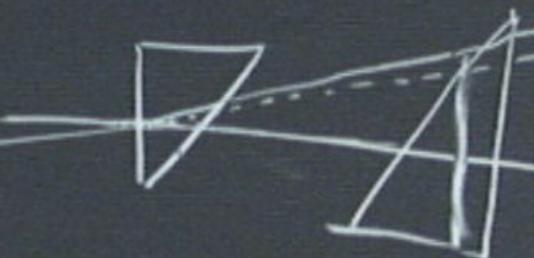


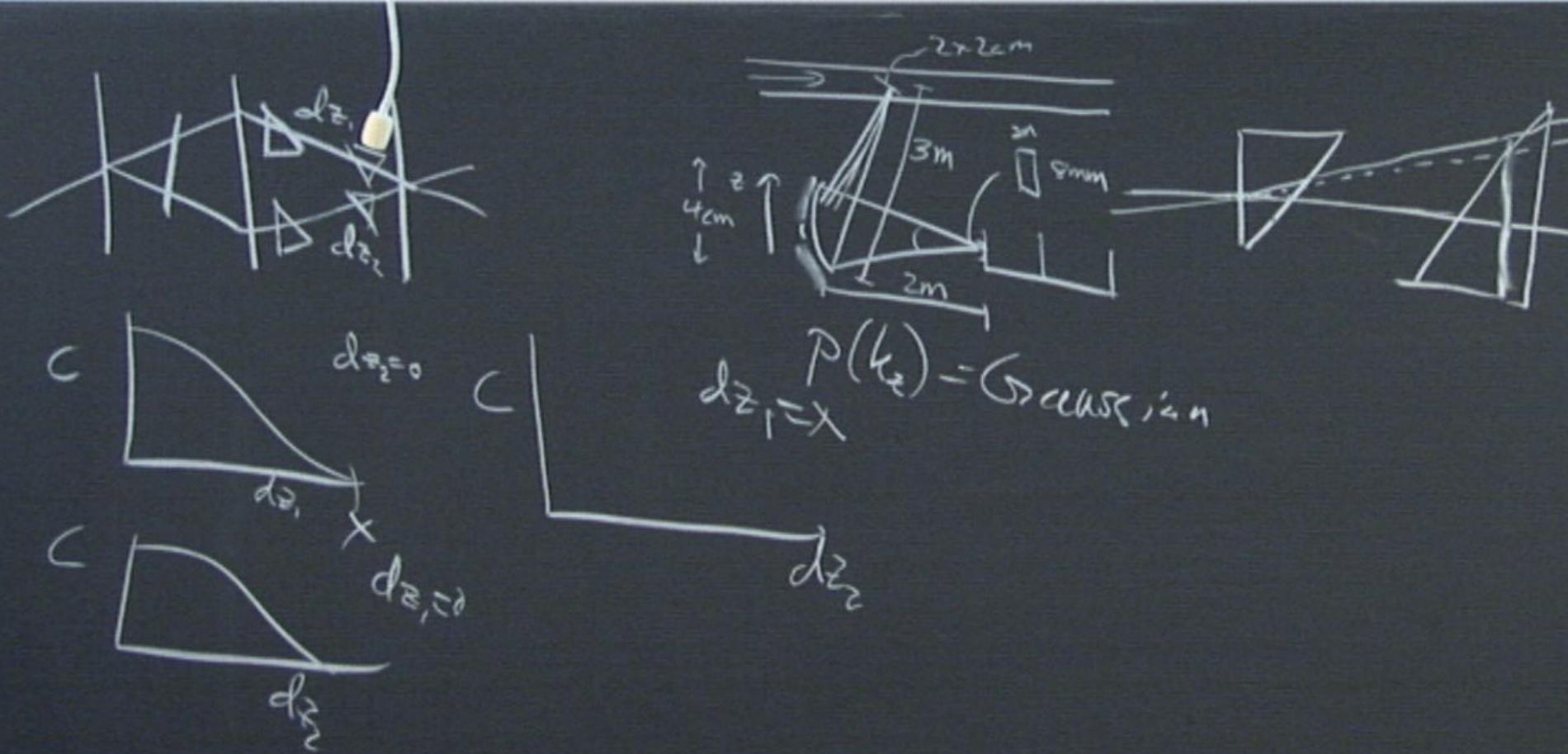
experiments show the expected behavior.

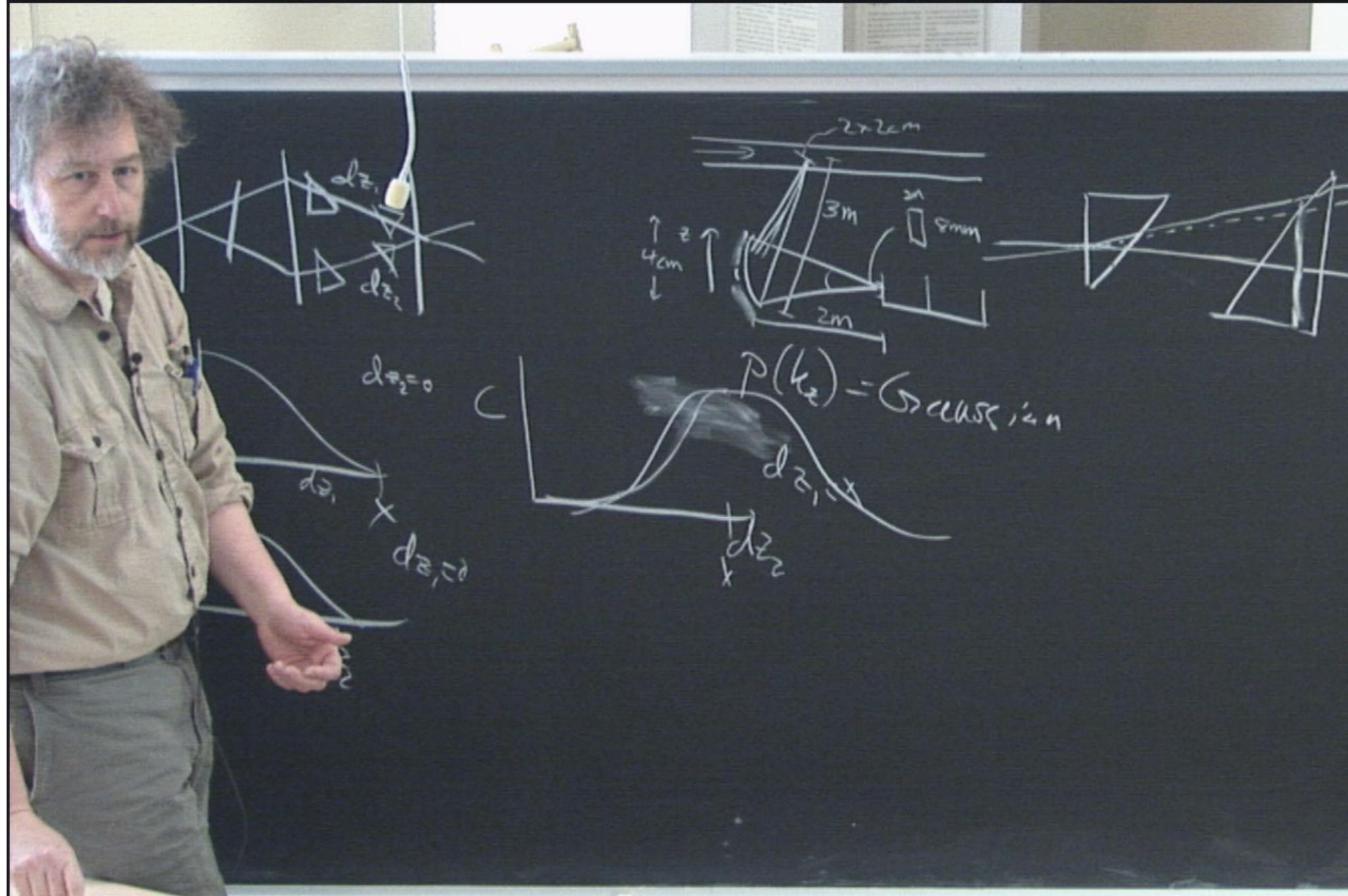


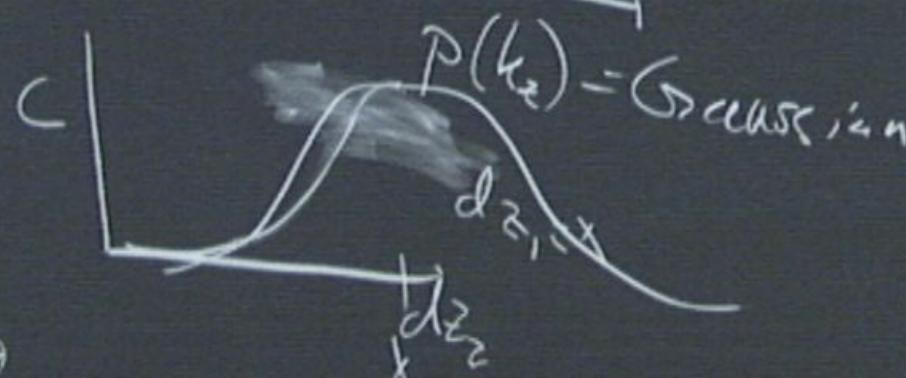
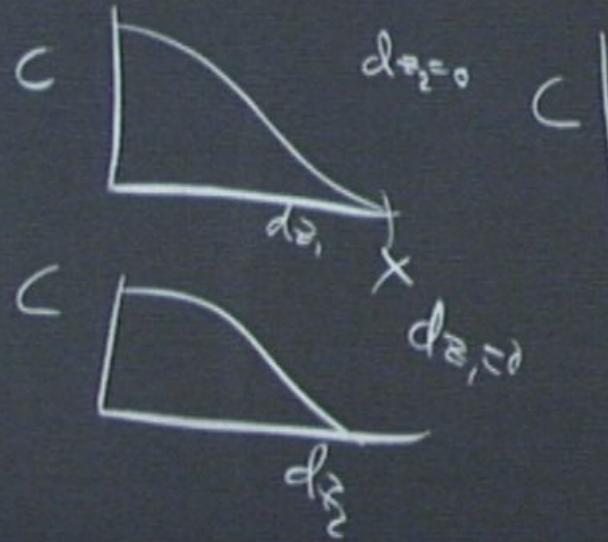
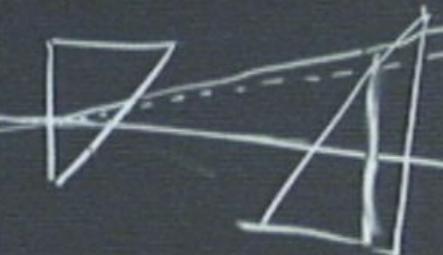
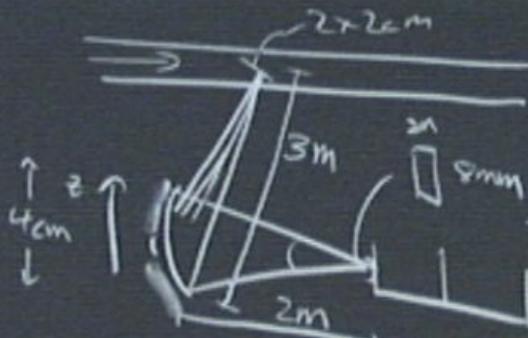
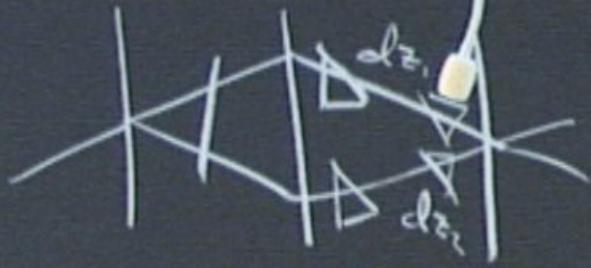


$$P(k_e) = G_{\text{class}, in}$$

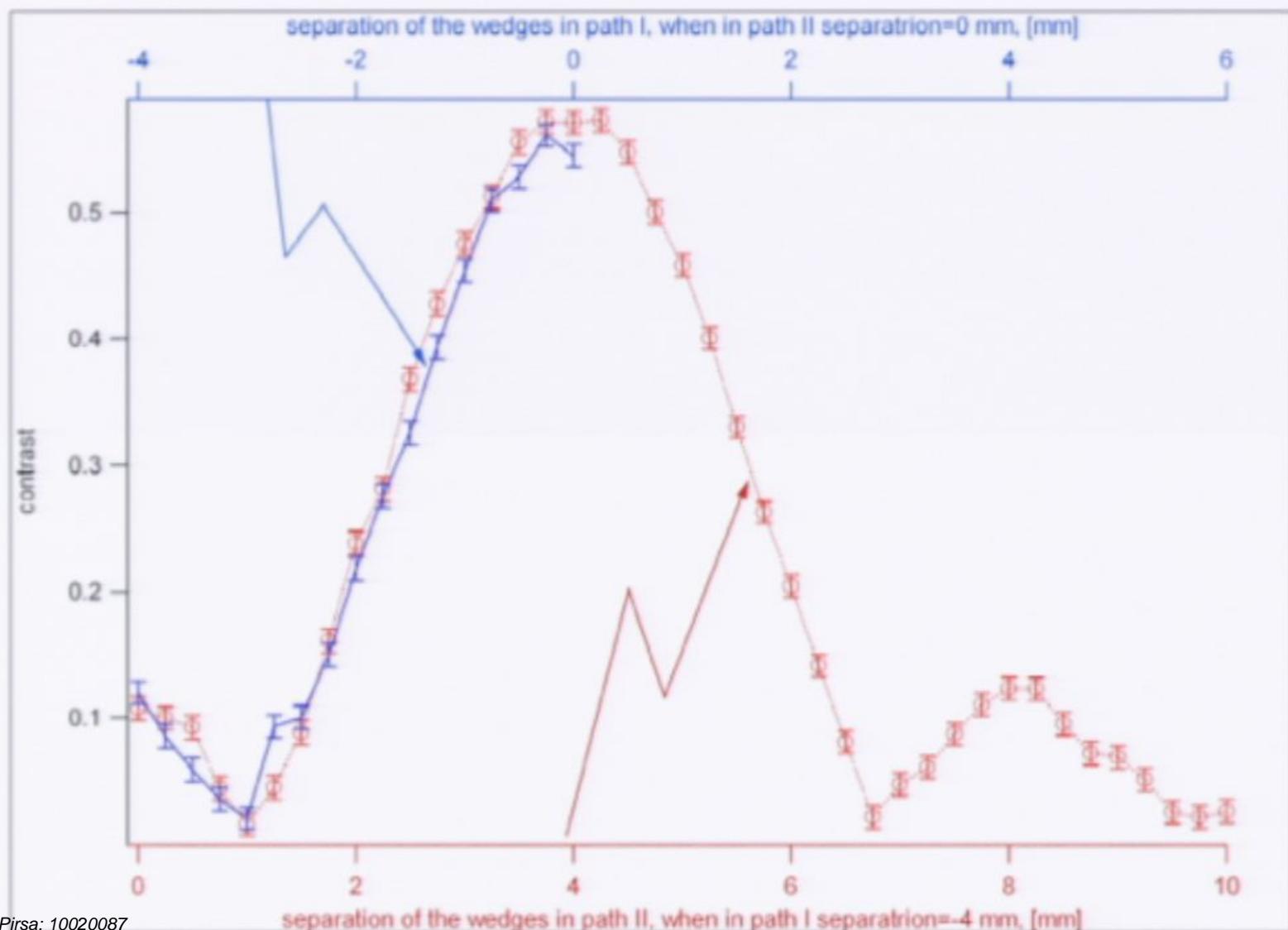






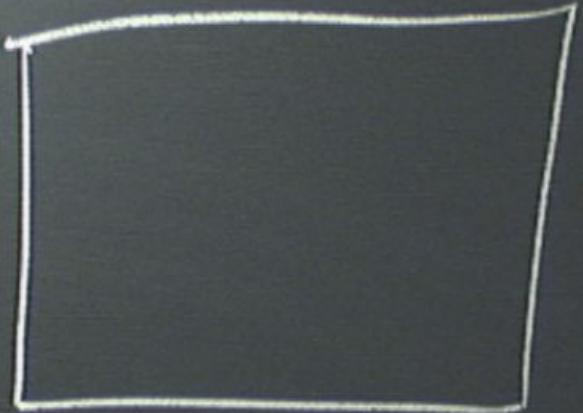


we put a set of prisms in each arm and show that the contrast is fully recovered as we displace the beam in both of the interferometer.



Wigner Function

$k_z$



$\mathcal{Z}$

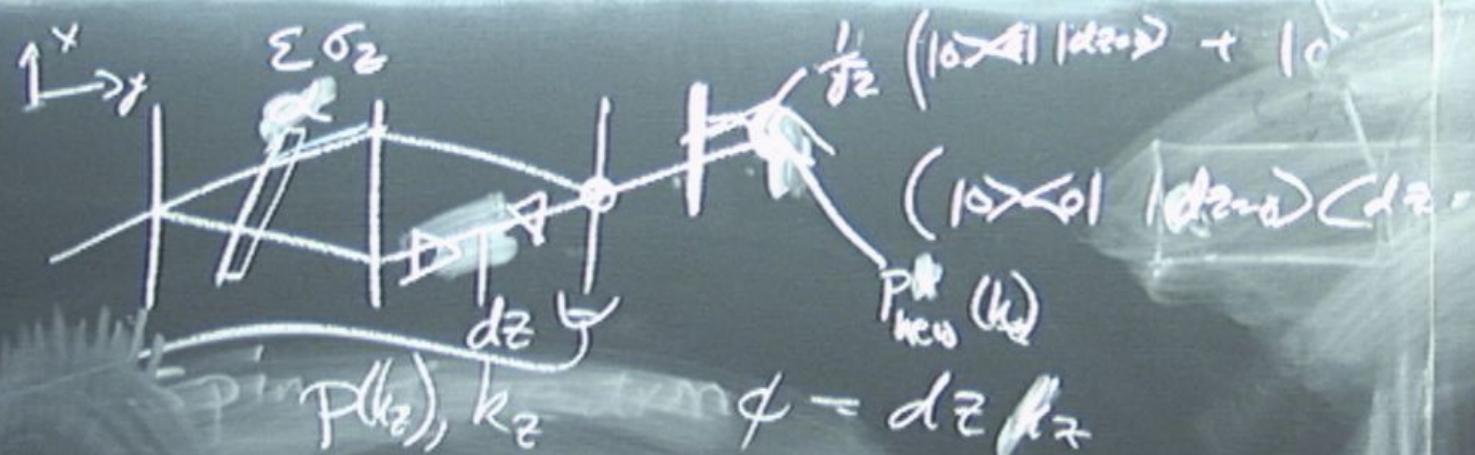
Wigner Function

$W(p, k_z)$

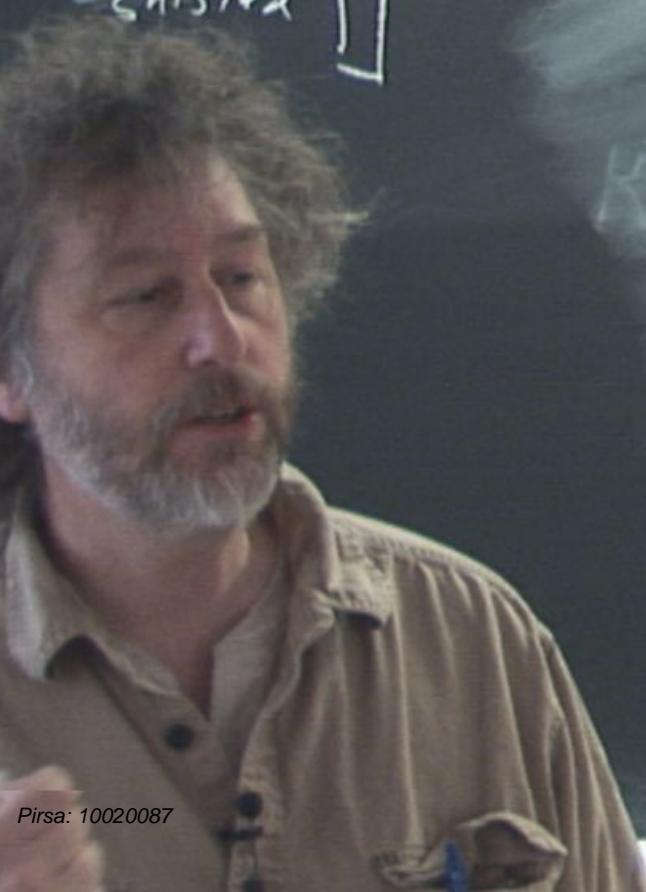
$k_z$



unshifted  
shifted



$$|P_k\rangle = \int P(k_z) (e^{ik_z z} e^{-ik_z z} |0\rangle + e^{ik_z z} e^{-ik_z z} |1\rangle + \dots)$$

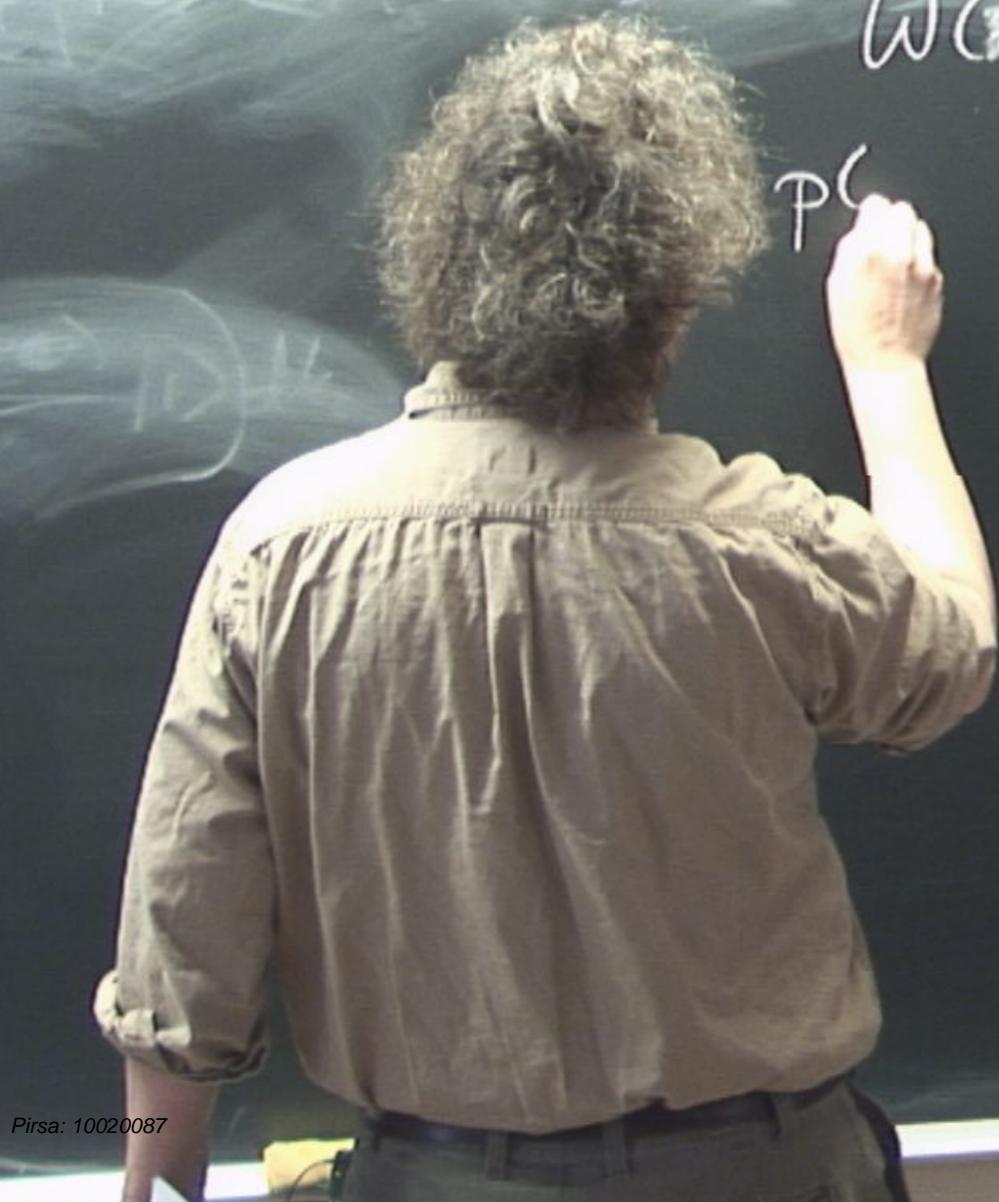
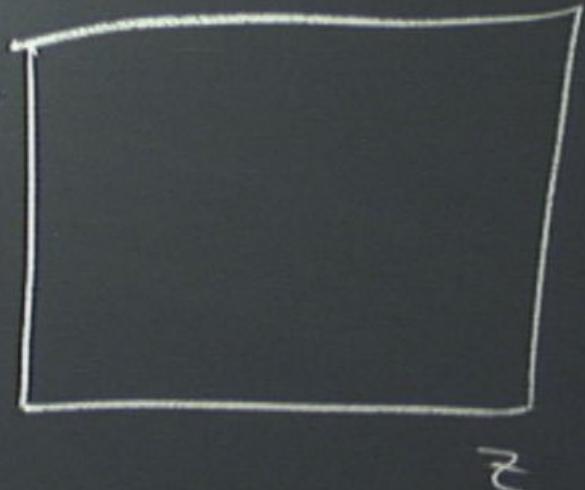


# Wigner Function

$$W(p, k_z)$$

p

$k_z$



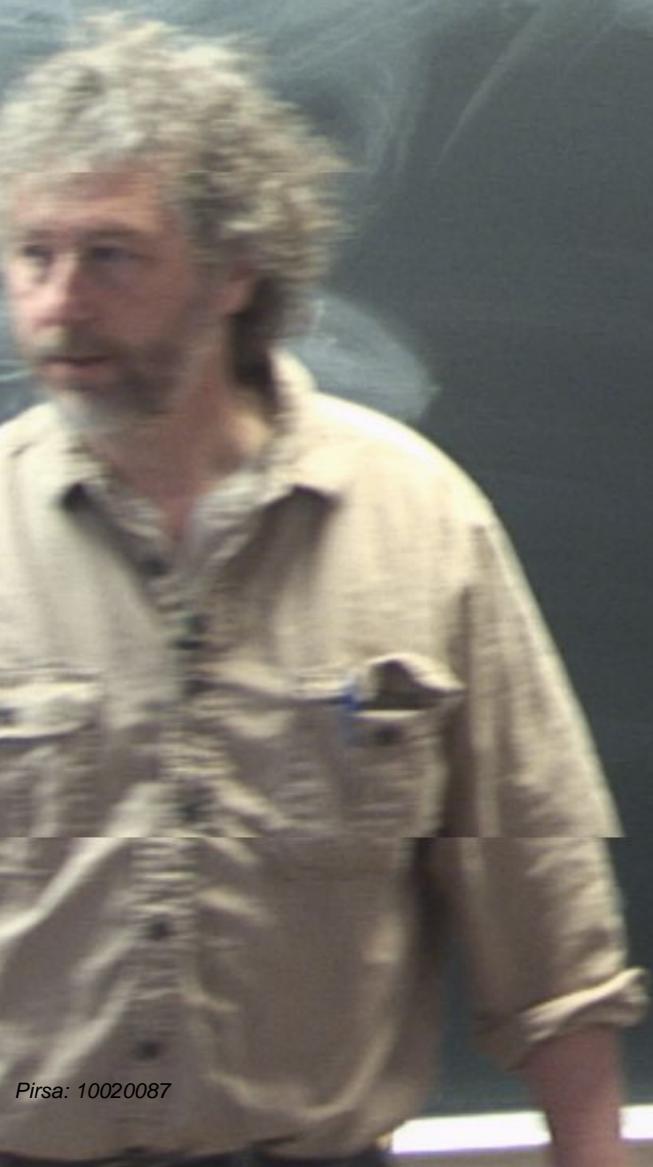
# Wigner Function

$$W(z, k_z)$$

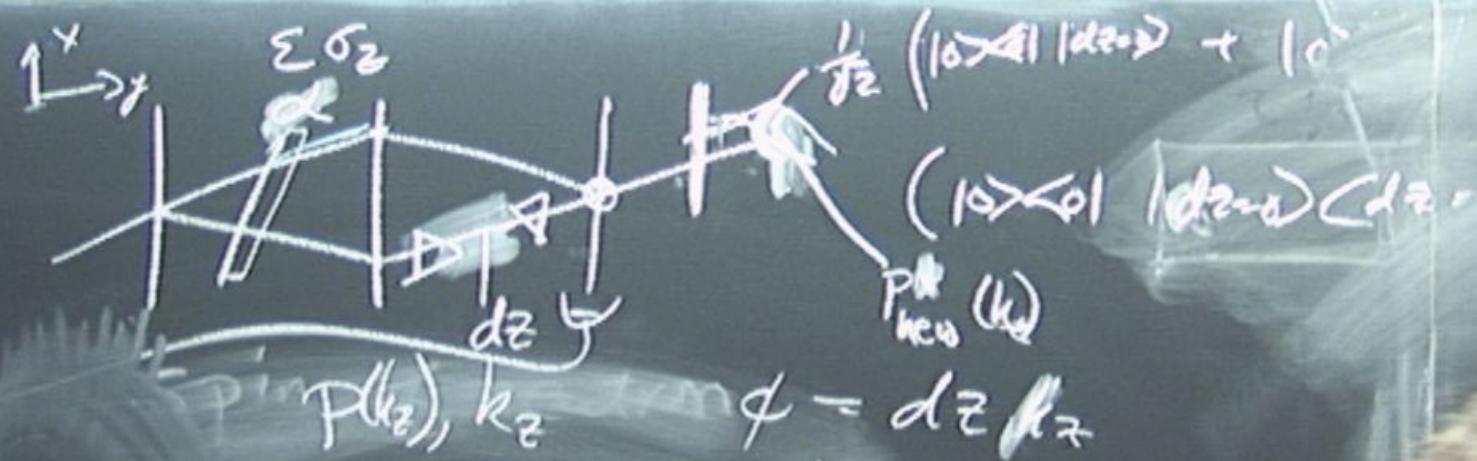
$$P(z) = \int W(z, k_z) dk_z$$

$k_z$

$z$



unshifted  
shifted



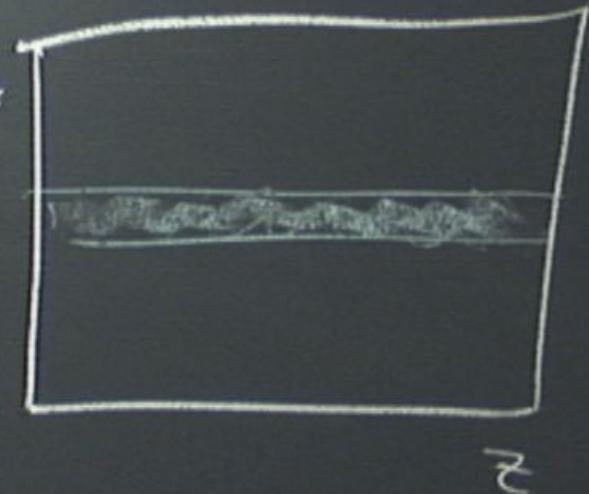
$$P(k_z) = \int P(k_z) (e^{-ik_z z} e^{ik_z z}) |0> +$$

# Wigner Function

$$W(z, k_z)$$

$$P(z) = \int W(z, k_z) dk_z$$

$k_z$

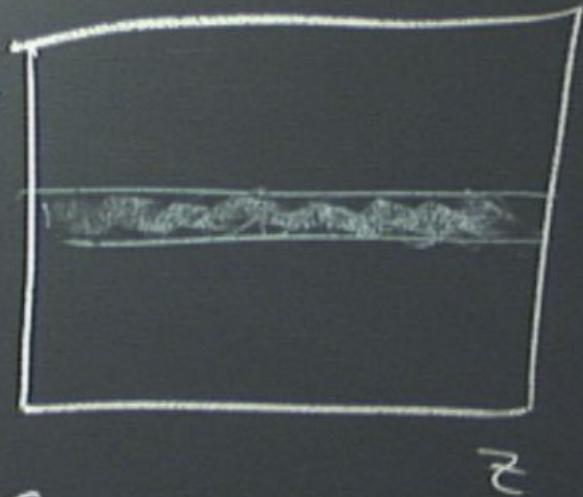


# Wigner Function

$$W(z, k_z)$$

$$P(z) = \int W(z, k_z) dk_z$$

$k_z$

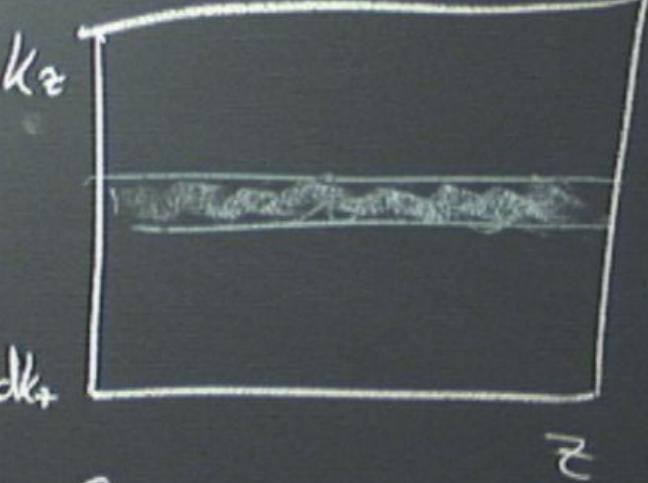


$$W(z, k) = \int e^{-izk} \psi_{(z+z')} \psi_{(z-z')} dz'$$

# Wigner Function

$$W(z, k_z)$$

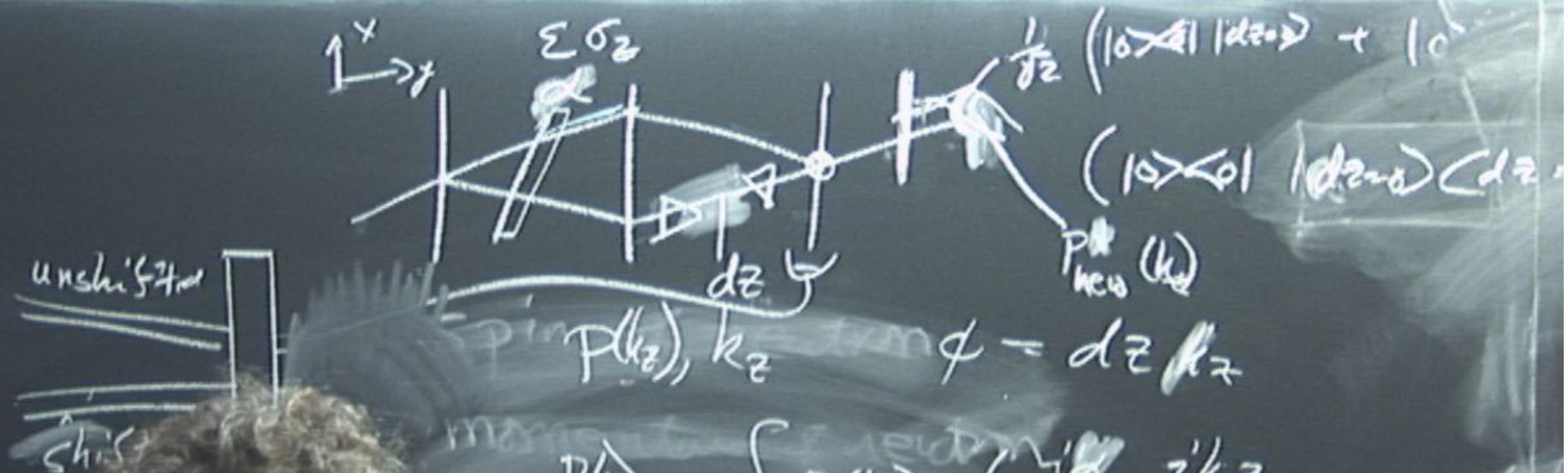
$$P(z) = \int W(z, k_z) dk_z$$



$$W(z, k) = \int e^{-ikz} \psi^*(z + \frac{z'}{2}) \psi(z - \frac{z'}{2}) dz'$$

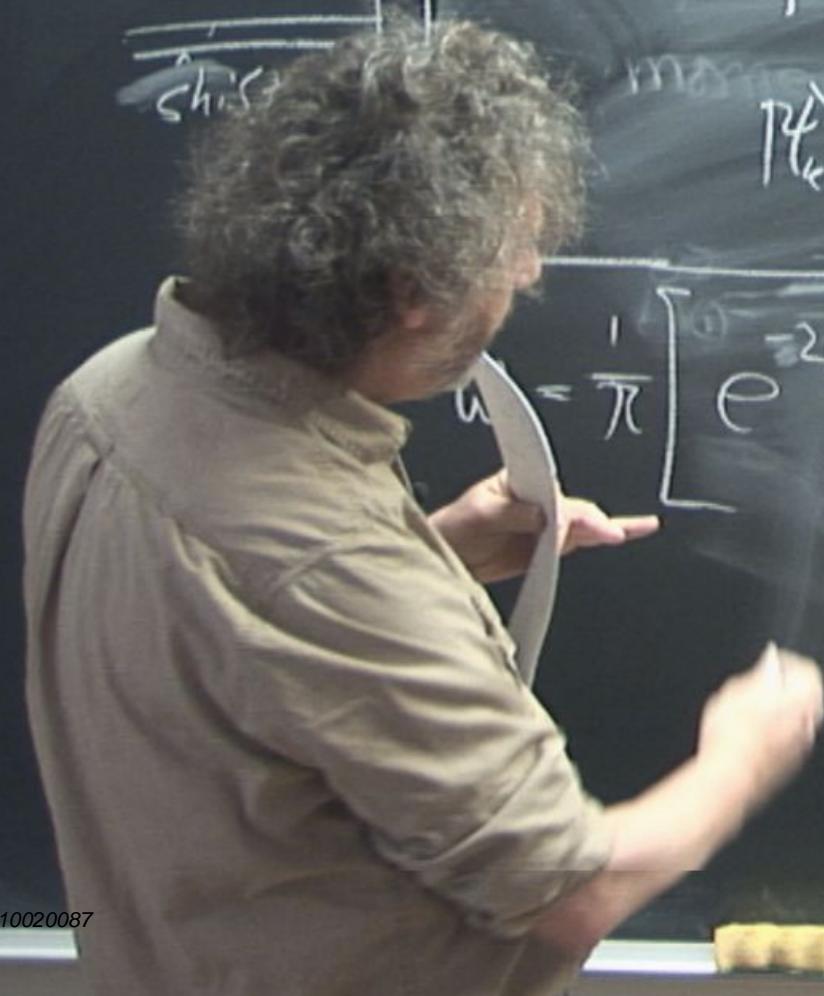
$$P_f = \int P(k_z) (e^{ik_z x} e^{-ik_z z} |0\rangle + \dots)$$

$$\omega = \frac{1}{\pi} \left[ e^{-2(z-\delta)^2} \right]$$



$$P(k_z) = \int P(k_z) (e^{ik_z z} e^{-ik_z(z+dz)} |0\rangle + \dots)$$

$$\omega = \frac{1}{\pi} \left[ e^{-2(z-\delta)^2} + e^{-\delta^2(z+dz)^2} + \dots \right]$$



$$\sum \sigma_z$$

$\frac{1}{\delta z} (|0\rangle\langle 0| |dz\rangle + |0\rangle$

$(|0\rangle\langle 0| |dz\rangle)(dz)$

$P_{\text{new}}^*(k_z)$

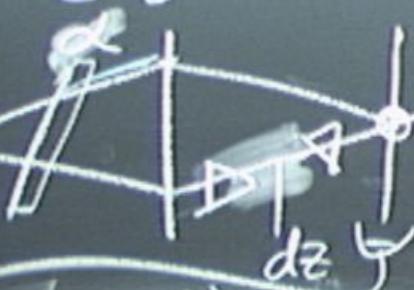
$\phi = dz k_z$

$$P_k = \int P(k_z) (e^{ikz} e^{-ik_z z} |0\rangle +$$

$$e^{-2(\frac{z-\delta}{\delta})^2} - e^{-\delta^2(z+\delta)^2}$$

$$+ e^{+\delta^2(z+\delta)^2} + 2\cos(d_z k + \alpha) e^{-\frac{\delta^2(z+\delta)^2}{2}}$$

$$e^{-\frac{1}{2}\left(\frac{k-k_m}{\delta}\right)^2}$$

$\sum \sigma_2$ 

$$\frac{1}{\delta z} (|0> \otimes |1> + |0>$$

$$(|0> \otimes |0>) (|dz> - |0> \otimes |1>)$$

$P_{\text{new}}(k_z)$

$$P(k_z), k_z \quad \phi = dz k_z$$

$$|k_z\rangle = \int P(k_z) (e^{-ik_z z} |0\rangle + e^{ik_z z} |1\rangle)$$

$$\left[ e^{-2(z-\delta)^2} + e^{-\delta^2(z+\delta z)^2} + 2\cos(dz k_z + \alpha) e^{\delta^2(z+\frac{dz}{2})^2} \right] e^{-\frac{1}{2}\left(\frac{z-l_m}{\delta}\right)^2}$$

$P(k_z)$

$A$

$K(x)$

$\frac{V(x)}{E}$

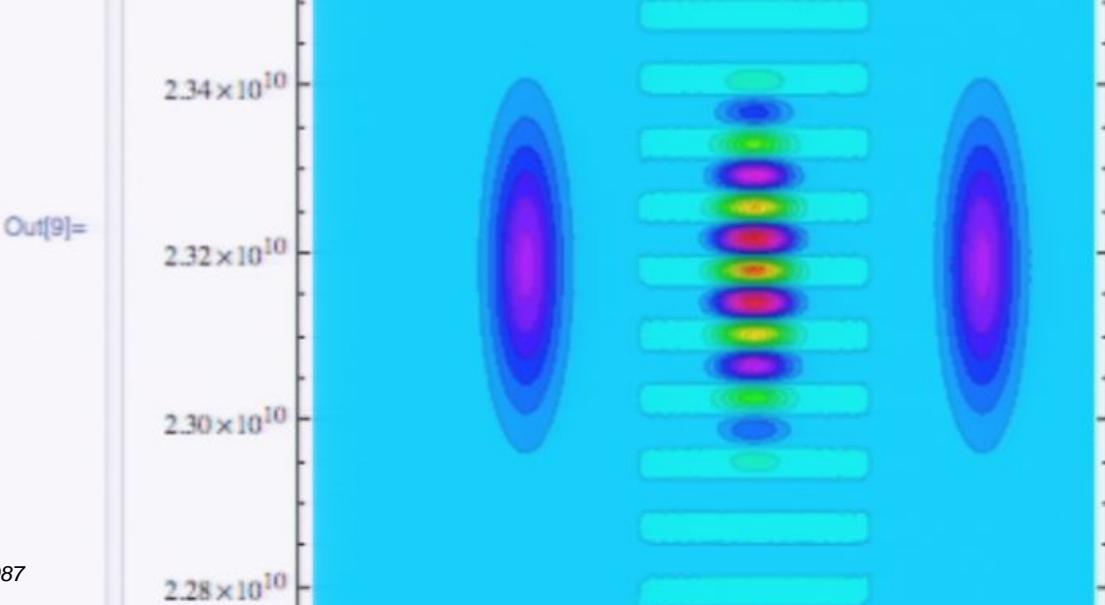
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

wigner\_dima\_v3\_small

- $6 \times 10^{-8}$  - $4 \times 10^{-8}$  - $2 \times 10^{-8}$  0  $2 \times 10^{-8}$

```
In[9]:= Manipulate[ContourPlot[Ws[z, dz, k, 0],  
{z, -1200*10^(-10), 200*10^(-10)},  
{k, 2.2721*10^10, 2.3649*10^10}, PlotRange -> {-1, 1.5},  
Contours -> Function[{min, max}, Range[min, max, 0.05]],  
ColorFunction -> Hue], {dz, 0, 1000*10^(-10), 10^(-9)}]
```

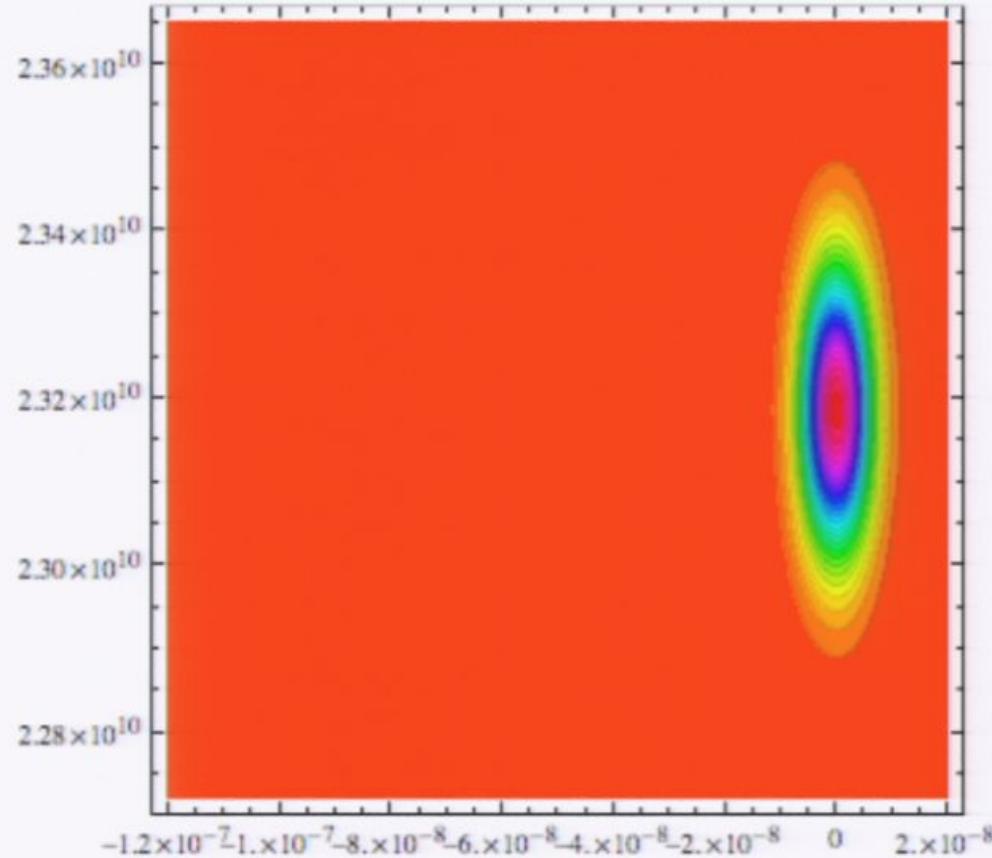
dz



wigner\_dima\_v3\_small

ColorFunction -&gt; Hue], {dz, 0, 1000 \* 10^(-10), 10^(-9)}]]

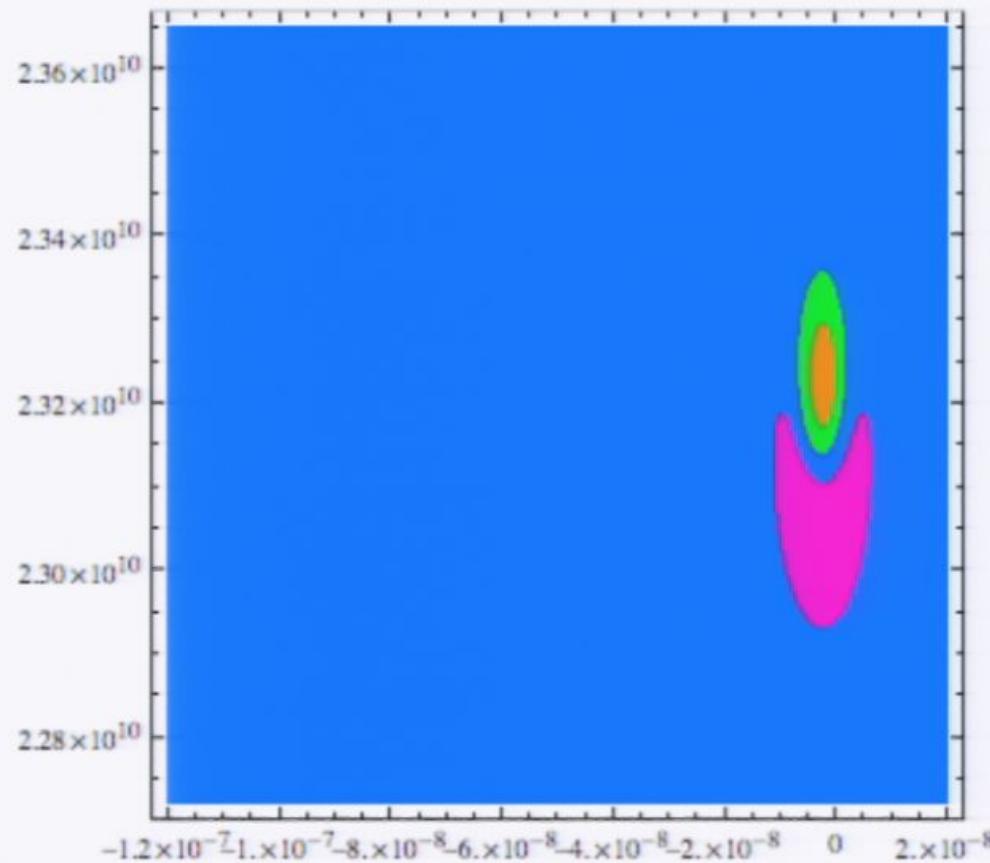
dz



wigner\_dima\_v3\_small

ColorFunction -&gt; Hue], {dz, 0, 1000 \* 10^(-10), 10^(-9)}]]

dz

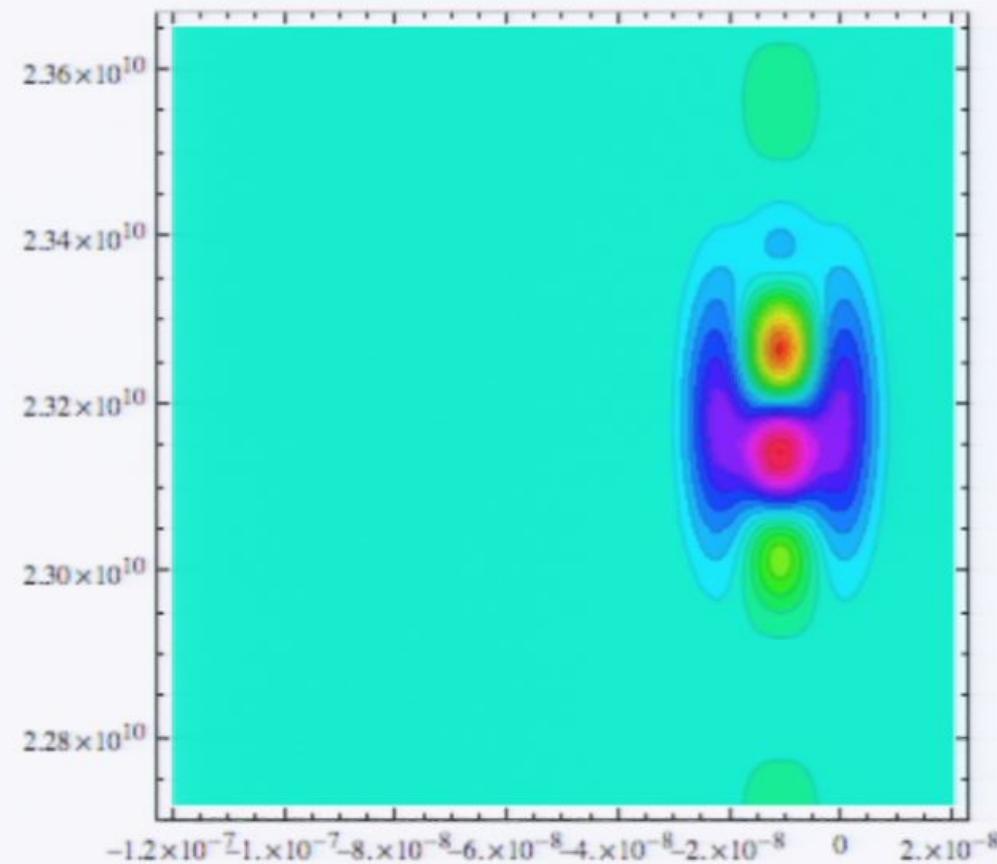


wigner\_dima\_v3\_small

ColorFunction -> Hue], {dz, 0, 1000 \* 10^(-10), 10^(-9)}]]

dz

Out[9]=

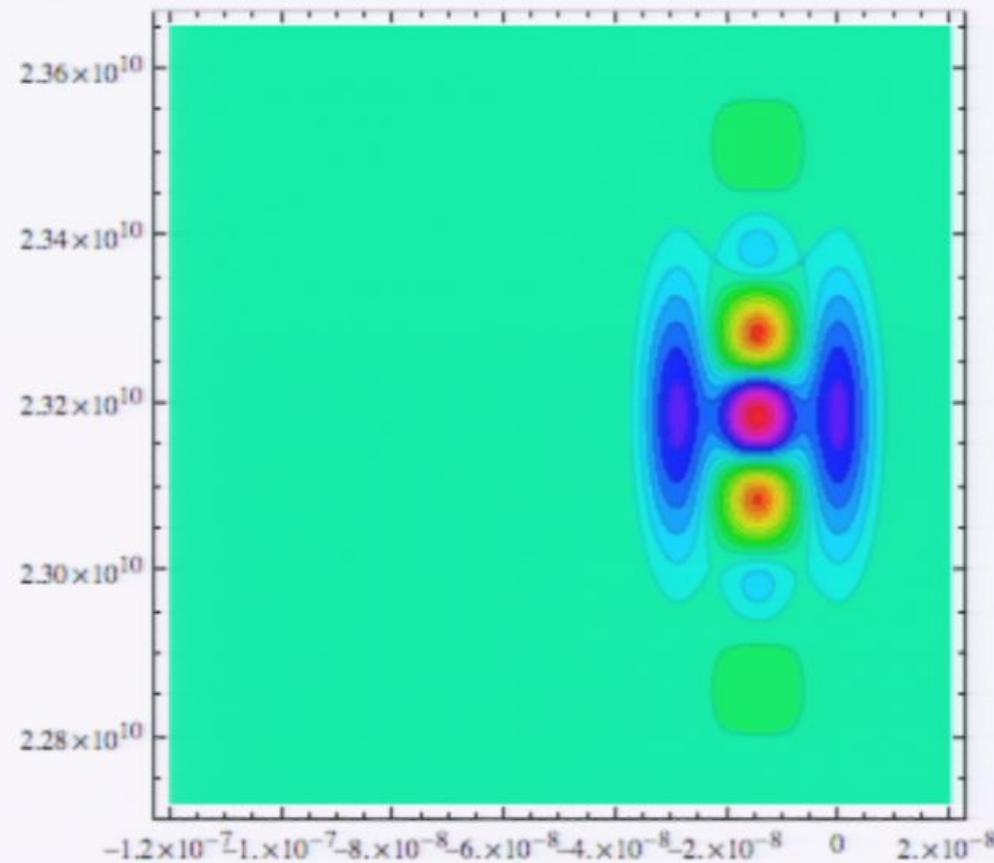


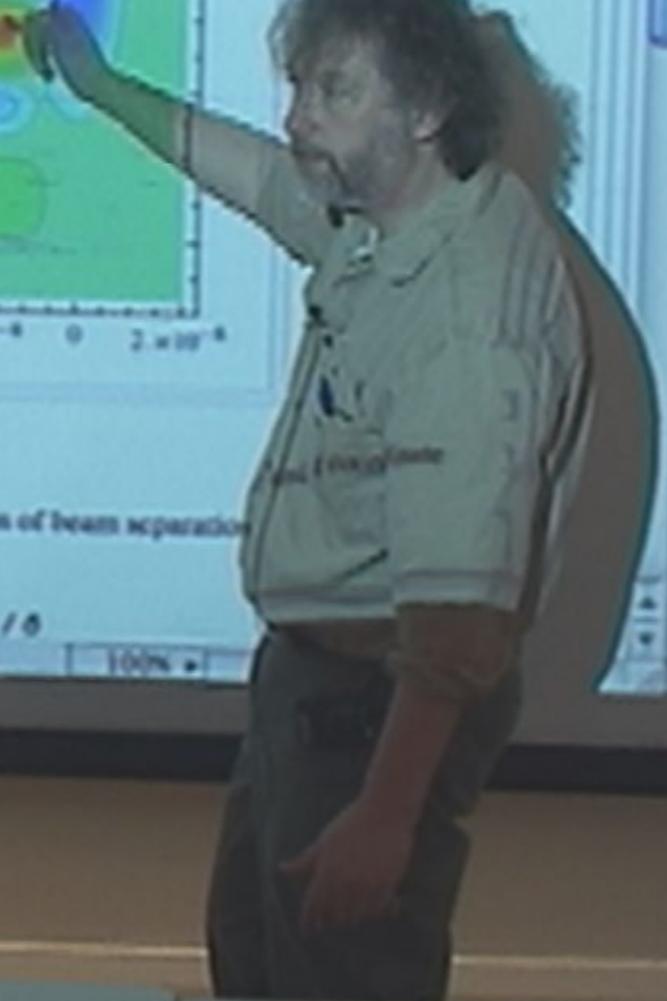
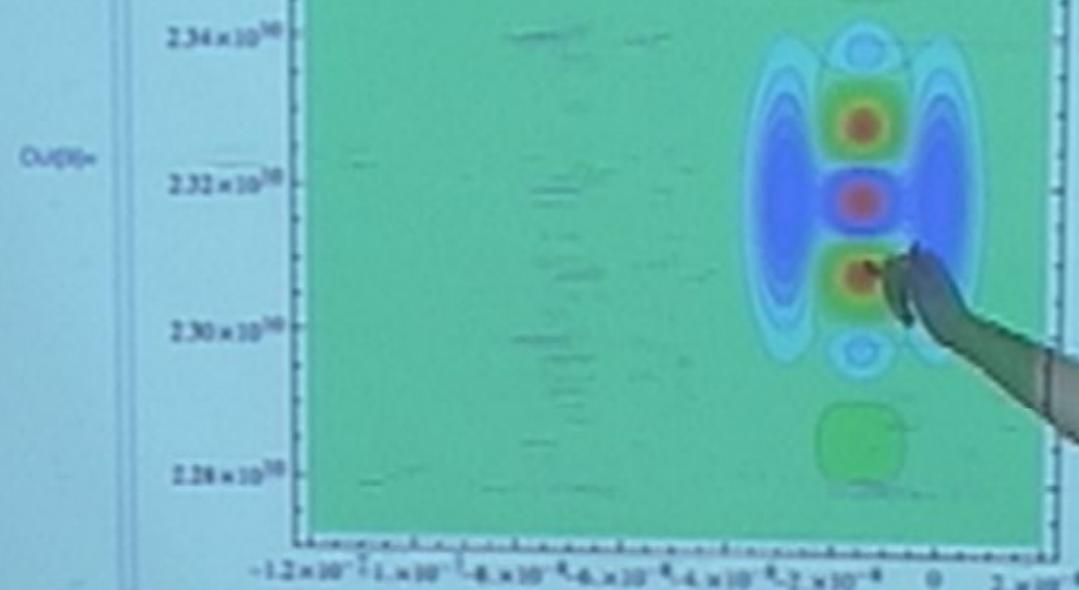
wigner\_dima\_v3\_small

ColorFunction -&gt; Hue], {dz, 0, 1000 \* 10^(-10), 10^(-9)}]]

dz

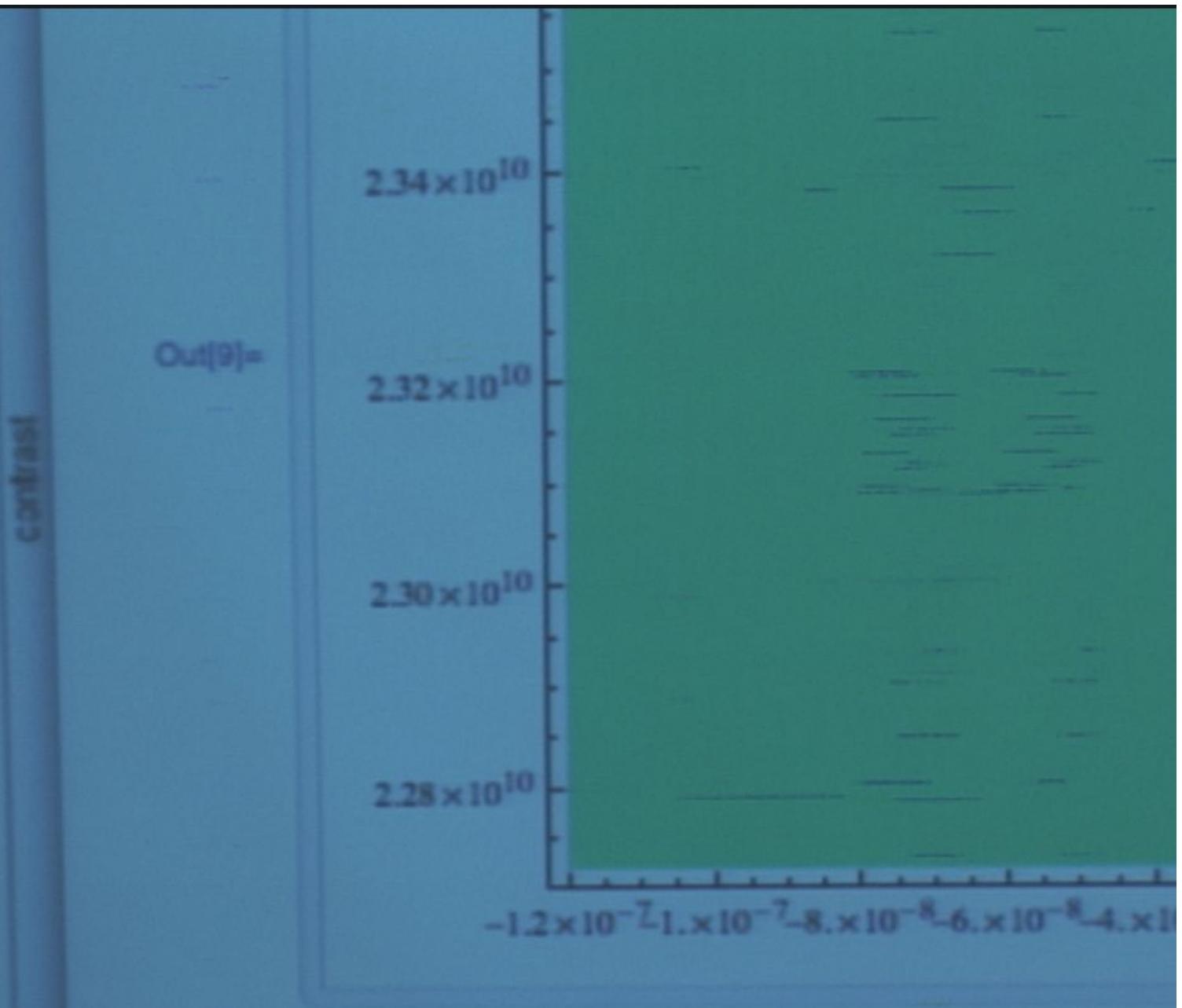
Out[9]=





We can also plot Intensity on the O+ beam detector as function of beam separation

$$\text{Intensity}[x_{\pm}, dx_{\pm}] = \psi_0[x, dx] * \psi_{00}[x, dx] / \phi$$

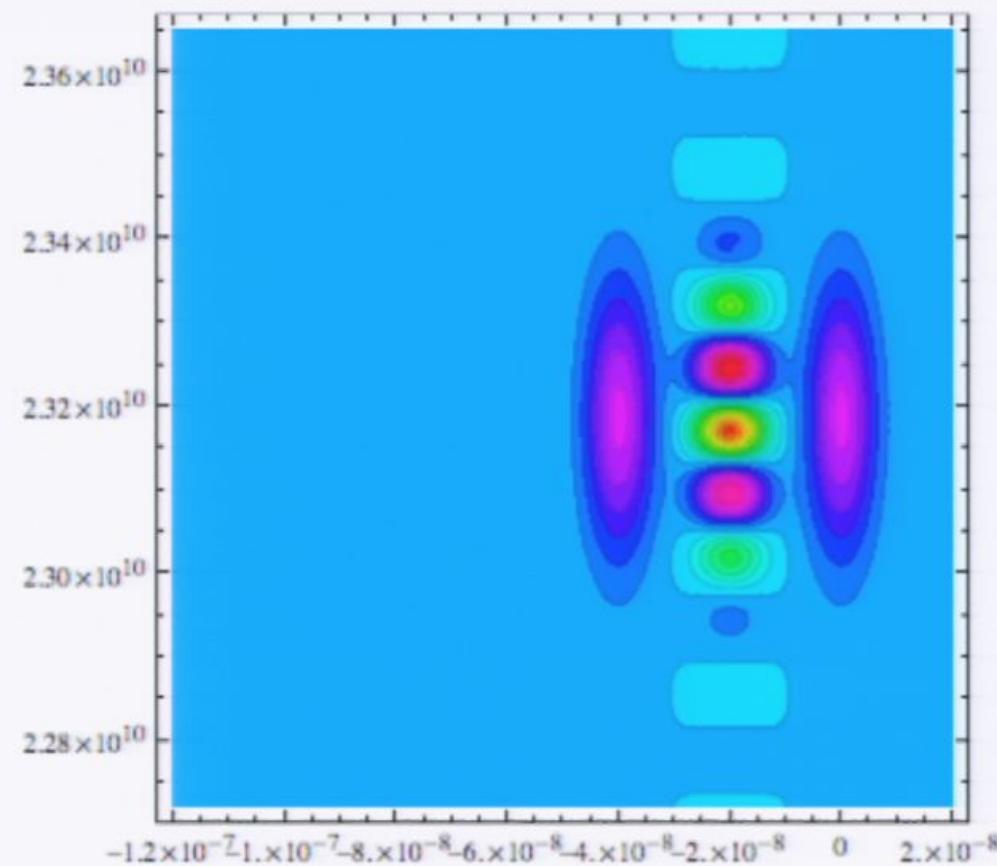


wigner\_dima\_v3\_small

ColorFunction -&gt; Hue], {dz, 0, 1000 \* 10^(-10), 10^(-9)}]]

dz

Out[9]=

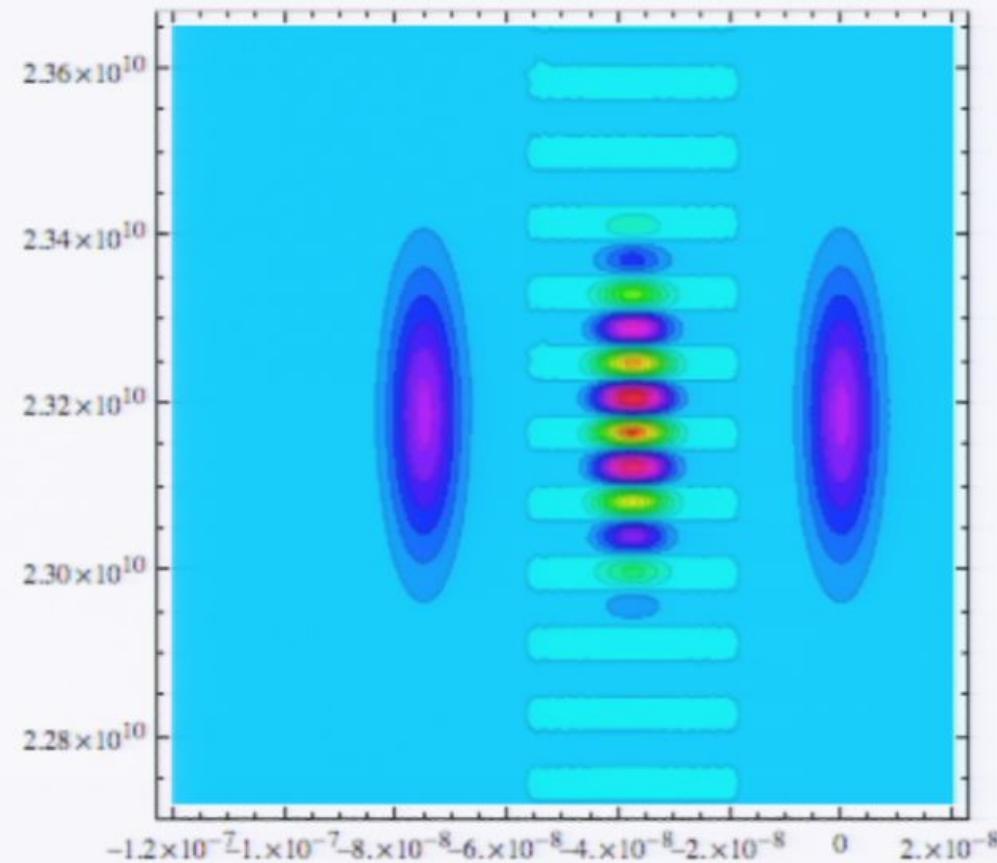


wigner\_dima\_v3\_small

ColorFunction -&gt; Hue], {dz, 0, 1000 \* 10^(-10), 10^(-9)}]]

dz

Out[9]=

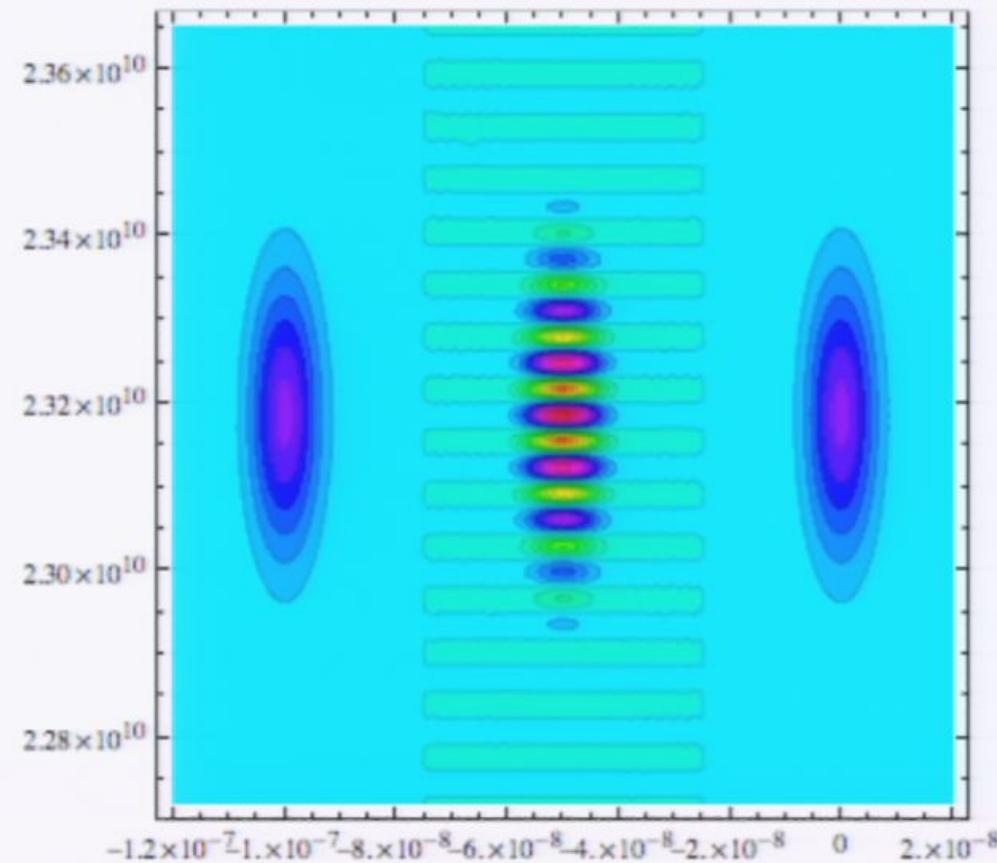


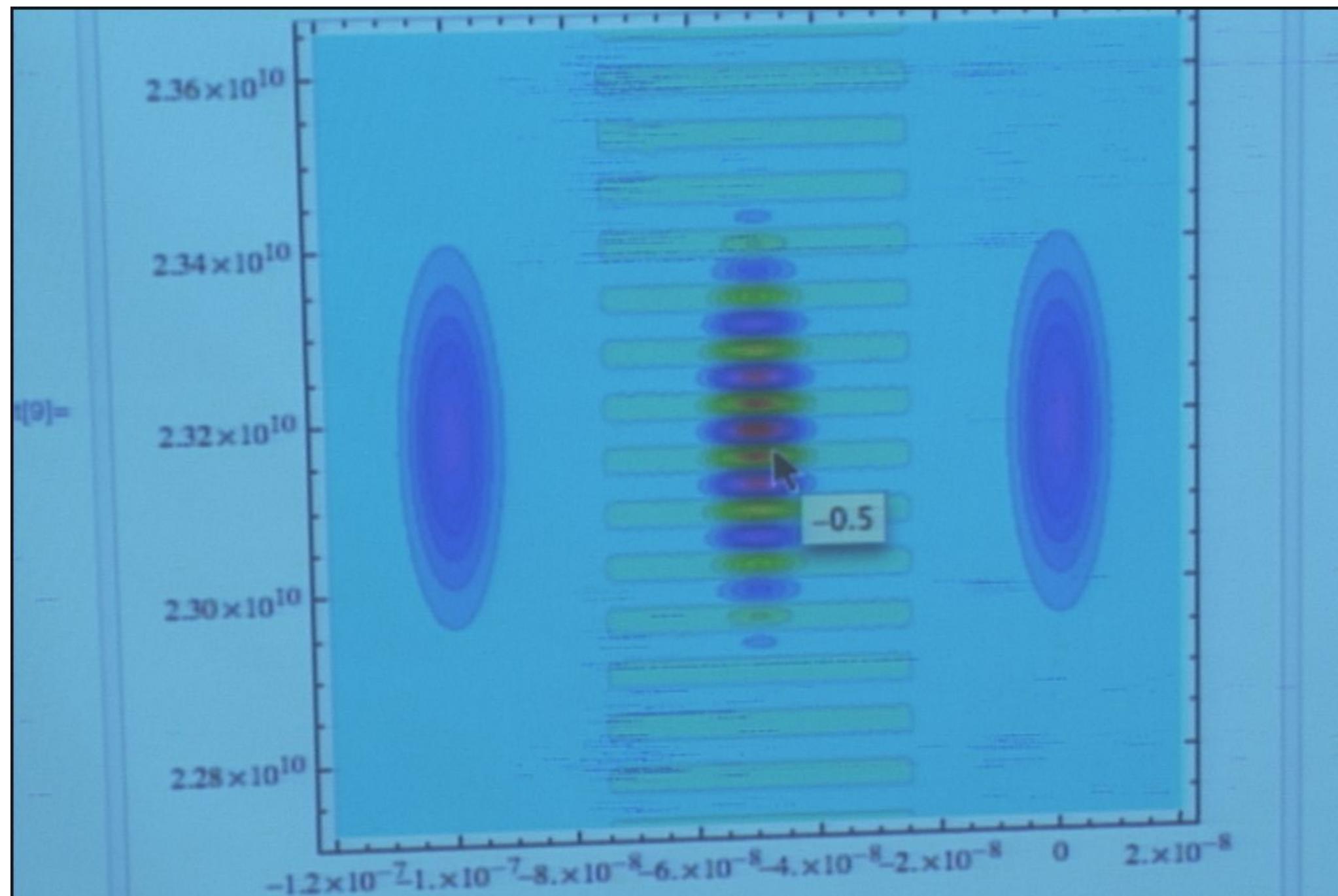
wigner\_dima\_v3\_small

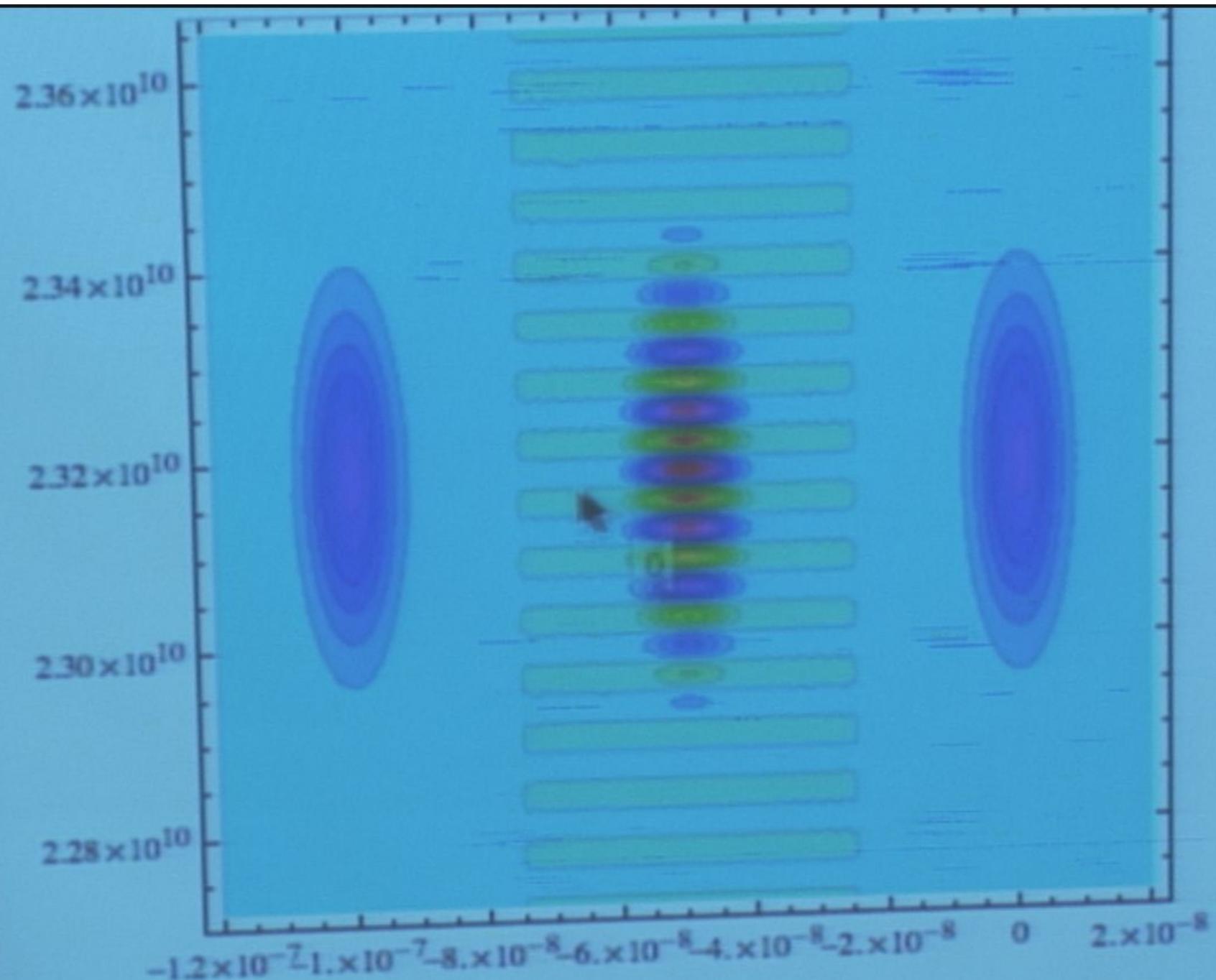
ColorFunction -&gt; Hue], {dz, 0, 1000 \* 10^(-10), 10^(-9)}]]

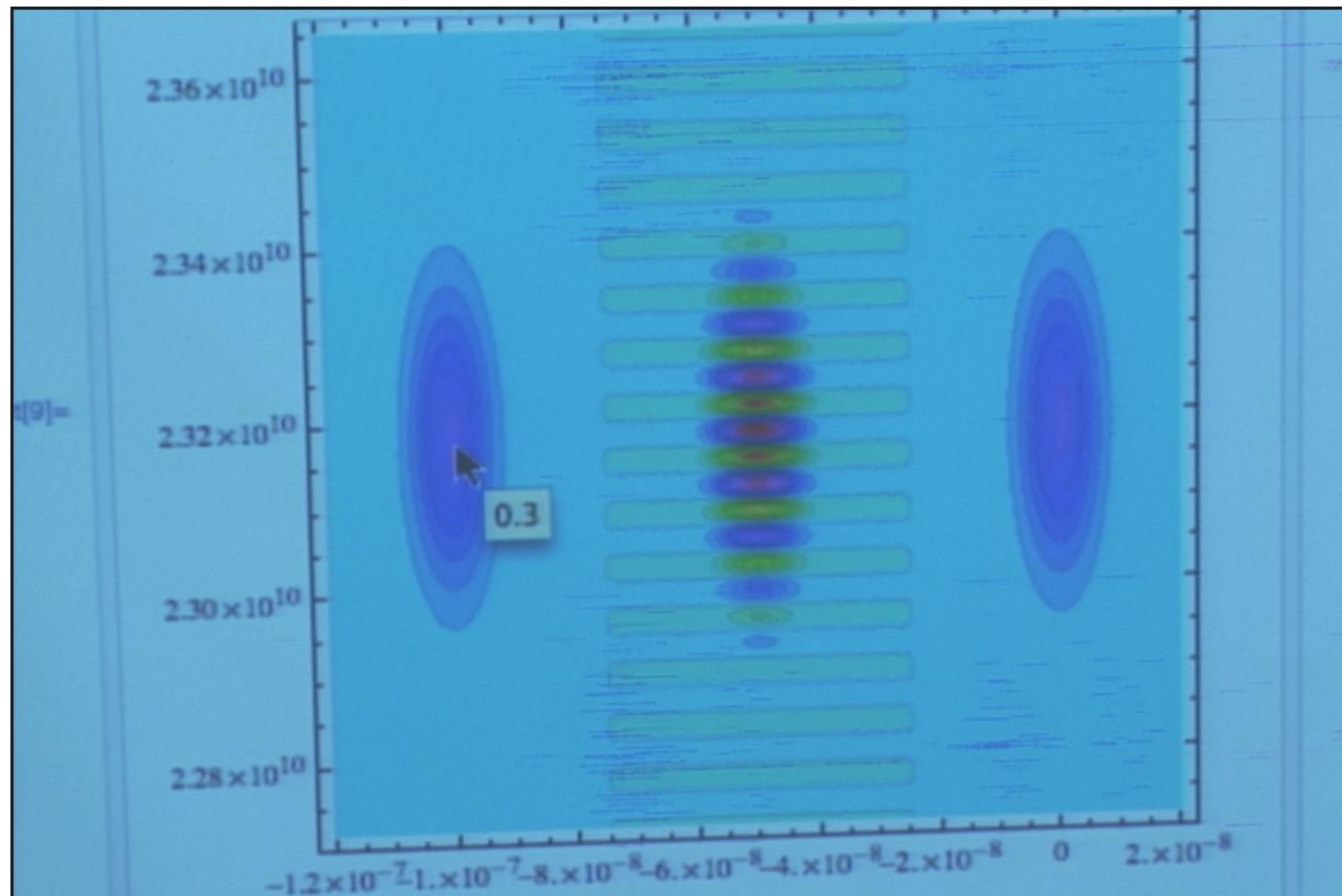
dz

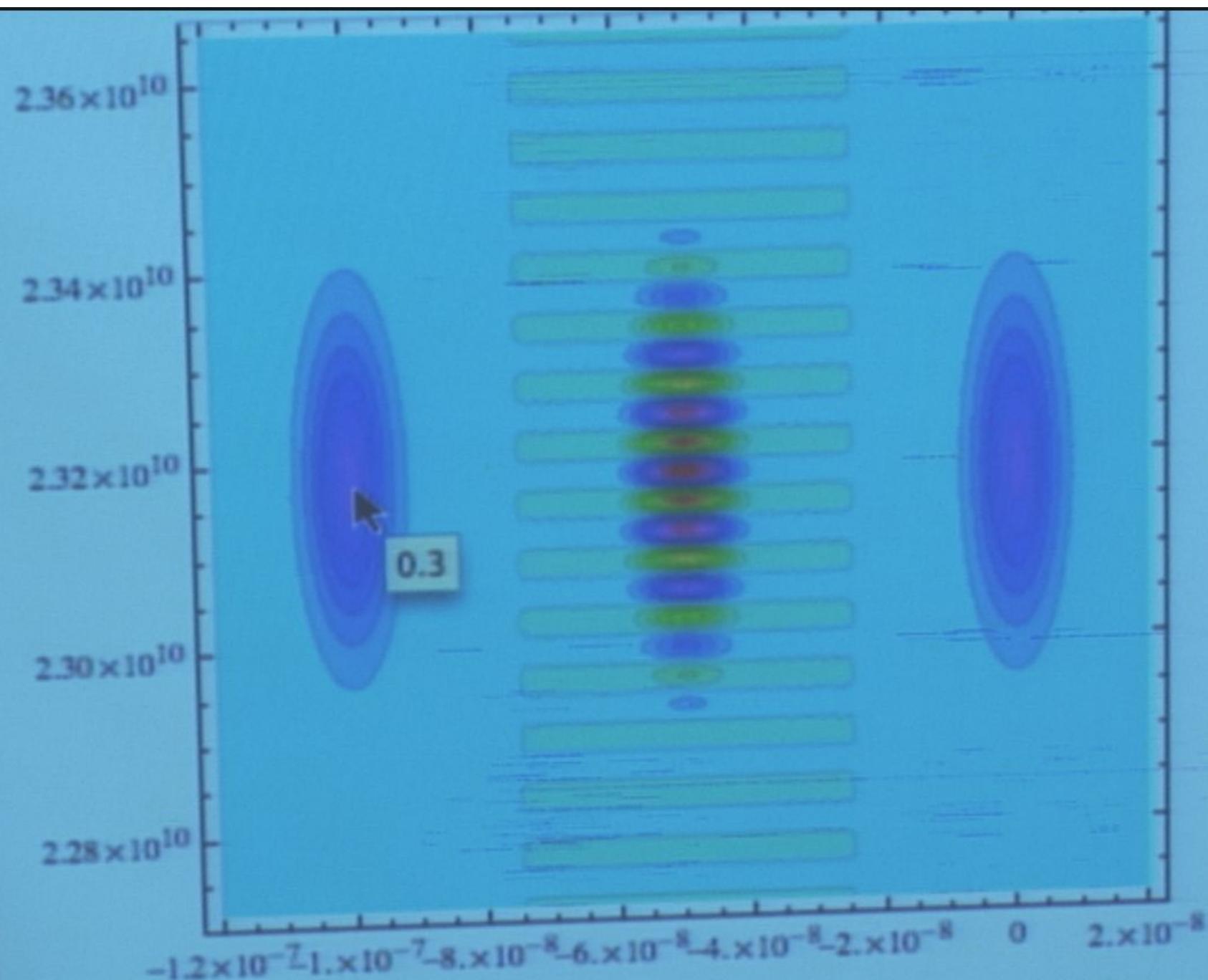
Out[9]=

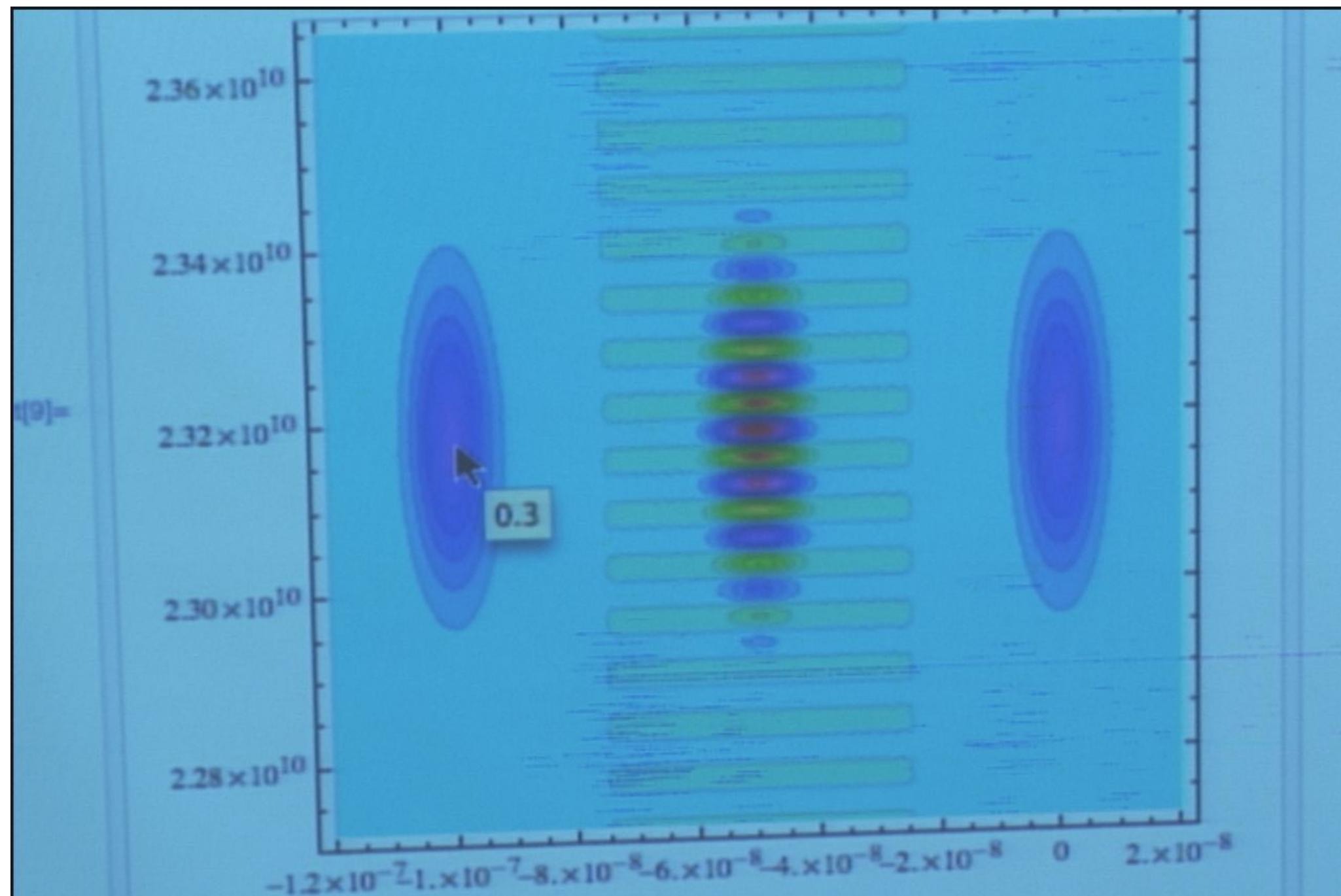




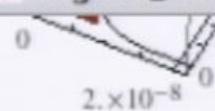




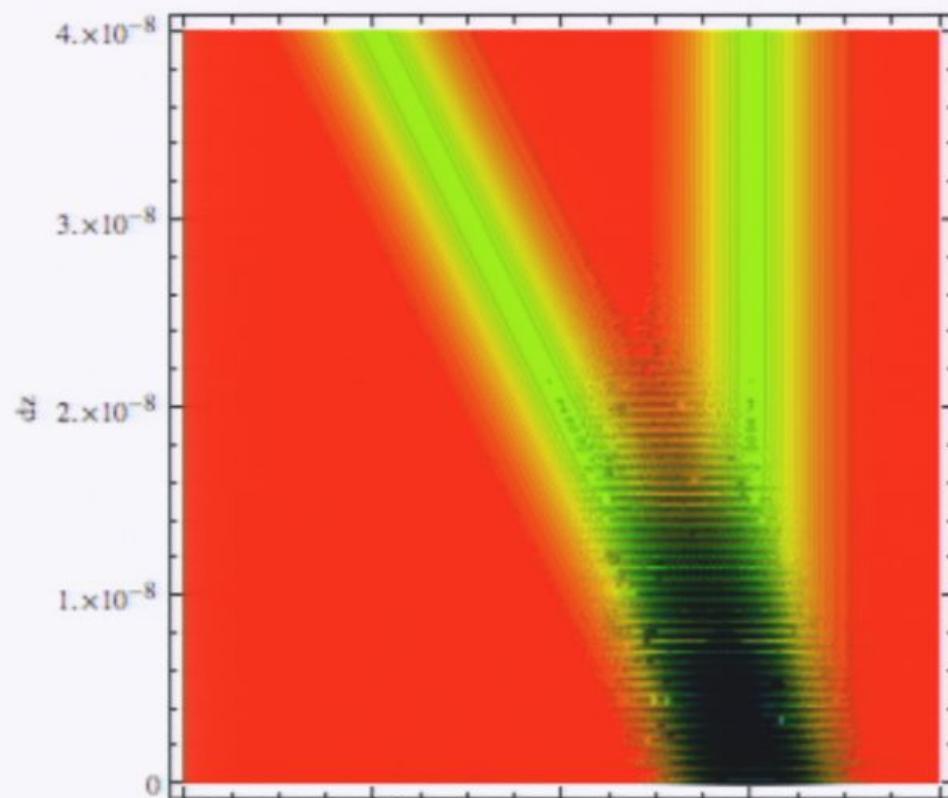




wigner\_dima\_v3\_small



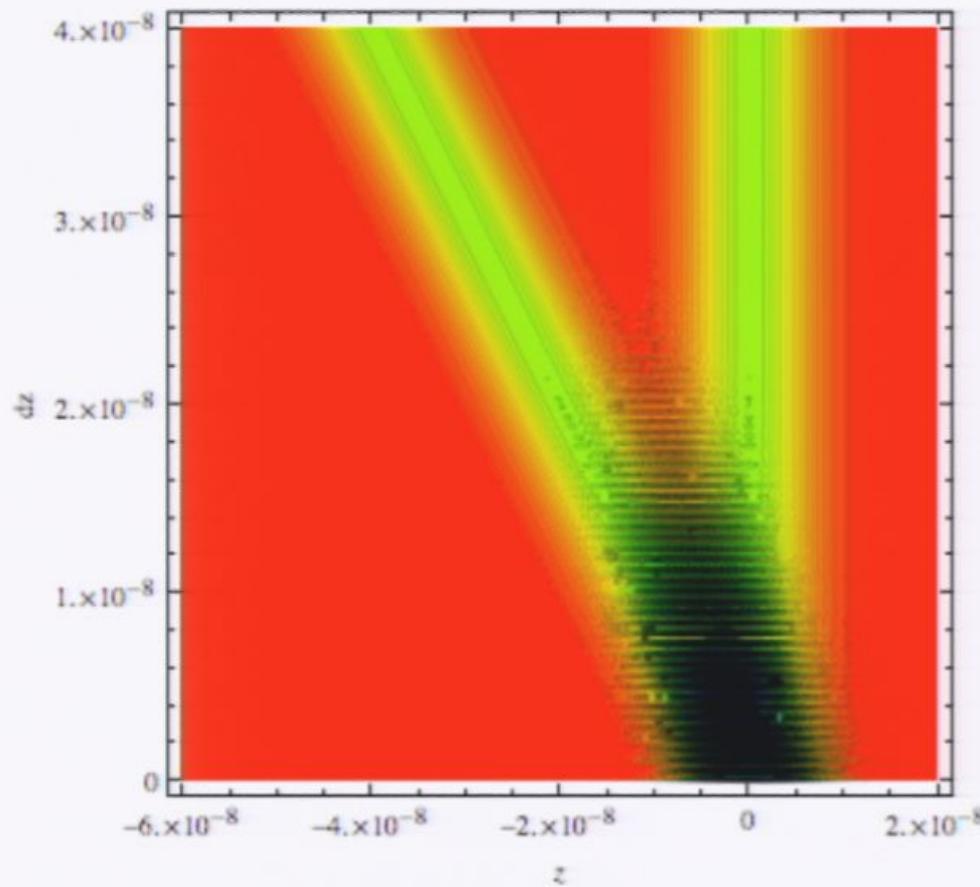
```
ContourPlot[Intensity[z, dz], {z, -600*10^(-10), 200*10^(-10)},  
{dz, 0, 400*10^(-10)}, PlotRange -> {0, 3.5},  
Contours -> Function[{min, max}, Range[min, max, 0.05]],  
ColorFunction -> Hue, FrameLabel -> {z, dz}]
```



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wigner\_dima\_v3\_small

```
{dz, 0, 400 * 10^(-10)}, PlotRange -> {0, 3.5},  
Contours -> Function[{min, max}, Range[min, max, 0.05]],  
ColorFunction -> Hue, FrameLabel -> {z, dz}]
```

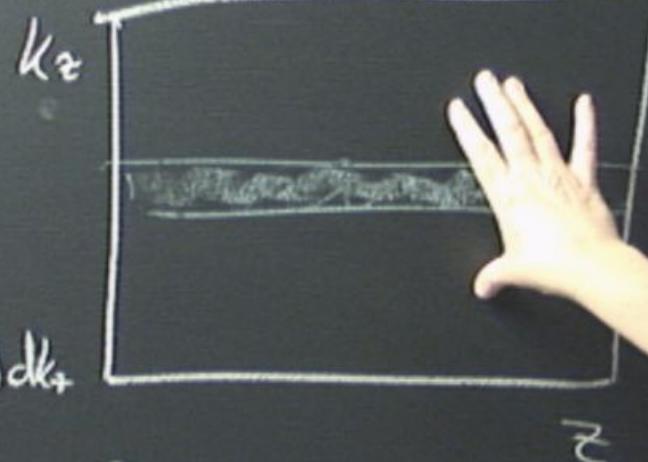


```
Plot3D[Intensity[z, dz], {z, -600 * 10^(-10), 200 * 10^(-10)},  
{dz, 0, 400 * 10^(-10)}, PlotRange -> {0, 3.5},  
ColorFunction -> Function[{x, y, z}, Hue[z]], PlotLabel -> "Intensity",  
AxesLabel -> Automatic]
```

# Wigner Function

$$W(z, k_z)$$

$$P(z) = \int W(z, k_z) dk_z$$



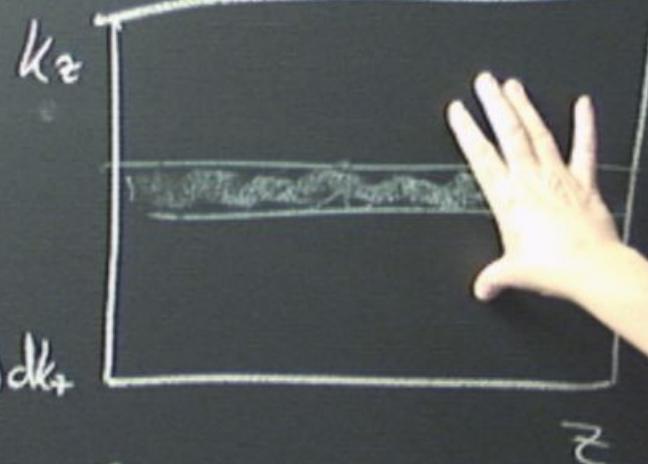
$$W(z, k) = \int e^{-izk} \psi_{(z+z')}^\dagger \psi_{(z-z')} dz'$$

$$\left[ \left( z + \frac{dz}{2} \right)^2 \right] e^{-\frac{1}{2} \left( \frac{k - k_0}{\sigma} \right)^2}$$

# Wigner Function

$$W(z, k_z)$$

$$P(z) = \int W(z, k_z) dk_z$$



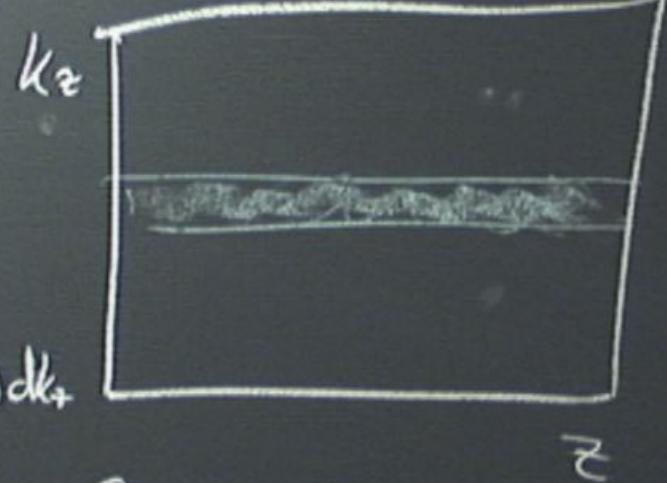
$$W(z, k) = \int e^{-ikz} \psi_{(z+z')}^\dagger \psi_{(z-z')} dz'$$

$$\left[ \left( z + \frac{dz}{2} \right)^2 \right] e^{-\frac{1}{2} \left( \frac{z - k_z}{\sigma} \right)^2}$$

# Wigner Function

$$W(z, k_z)$$

$$P(z) = \int W(z, k_z) dk_z$$



$$W(z, k) = \int e^{-ikz} \psi_{(z+z')}^\dagger \psi_{(z-z')} dz'$$

$$\left[ \left( z + \frac{dz}{2} \right)^2 \right] e^{-\frac{1}{2} \left( \frac{k - k_0}{\Delta k} \right)^2}$$

