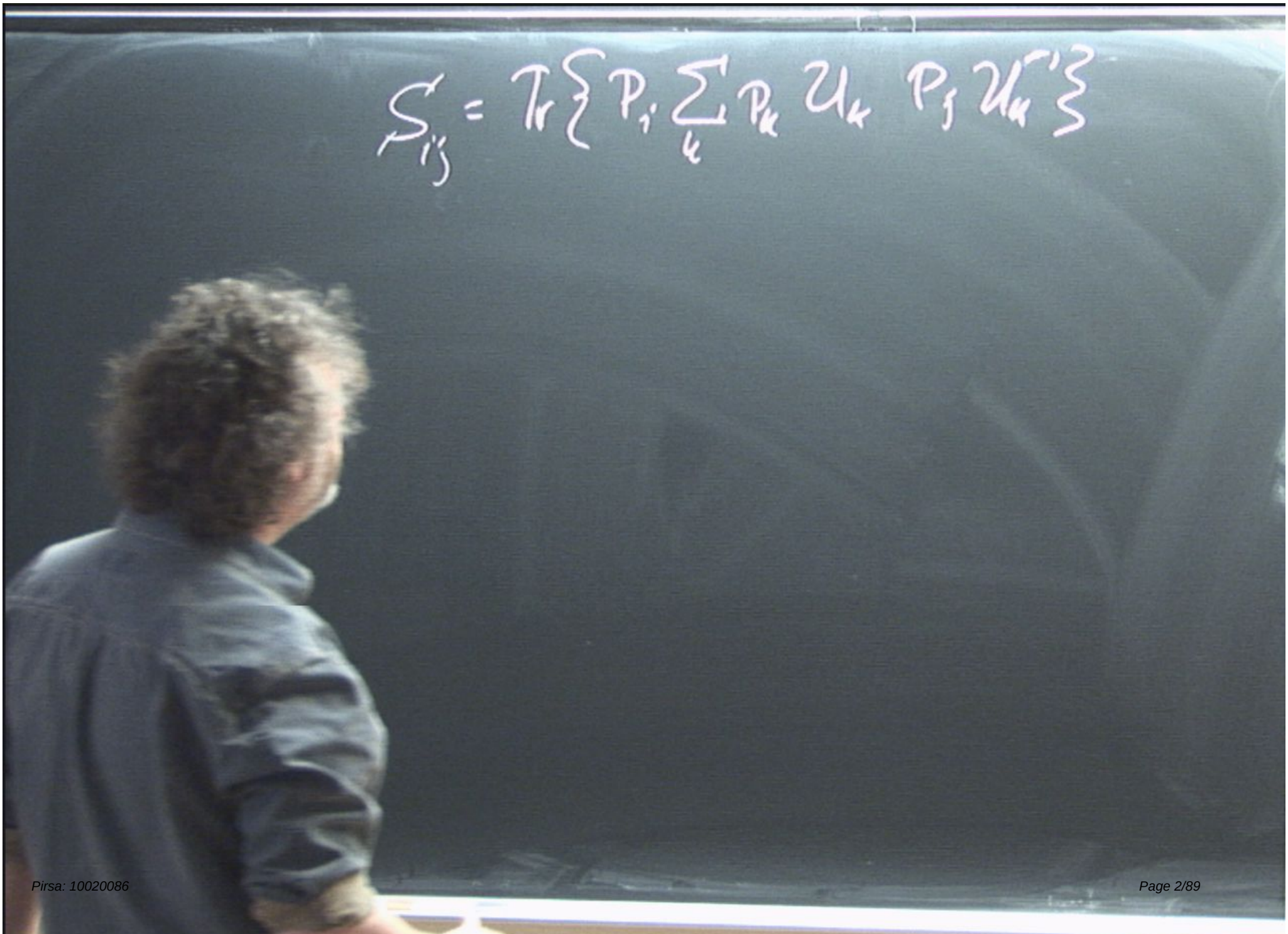
Title: Explorations in Quantum Info. (PHYS 641) - Lecture 3

Date: Feb 18, 2010 09:00 AM

URL: http://pirsa.org/10020086

Abstract:


$S_{ij} = \mathrm{Tr}\{ P_i \sum_k P_k U_k P_j U_k^{-1} \}$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

classical probability

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

p_k ← classical probability

P_i = Pauli

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

p_k ← classical probab. 1.4.10

P_i = Pauli

$$U = e^{i\theta \vec{n}\vec{\sigma}}$$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

classical probability

P_i = Pauli

$$U = e^{i\theta \vec{n}\vec{\sigma}}$$

$$\Omega_i \in \{ |0\rangle\langle 0|, |0\rangle\langle 1|, |1\rangle\langle 0|, |$$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

p_k ← classical probab. dist.

P_i = Pauli

$$i\Omega_i \in \left\{ |0\rangle\langle 0|,\ |0\rangle\langle 1|,\ |1\rangle\langle 0|,\ |1\rangle\langle 1| \right\}$$

$$\frac{(\mathbb{1}+\sigma_z)}{2},\quad \sigma_+,\quad \sigma_-,\quad \left(\frac{\mathbb{1}-\sigma_z}{2}\right)$$

$e^{i\theta \vec{n}\cdot\vec{\sigma}}$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

p_k classical probab. 1.4.10

P_i = Pauli

$$U = e^{i\theta \vec{n}\vec{\sigma}}$$

Transition $\Omega_i \in \{ |0\rangle\langle 0|, |0\rangle\langle 1|, |1\rangle\langle 0|, |1\rangle\langle 1| \}$

$$\frac{(\mathbb{1} + \sigma_z)}{2}, \quad \sigma_+, \quad \sigma_-, \quad \left(\frac{\mathbb{1} - \sigma_z}{2}\right)$$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

p_k classical probability

P_i = Pauli

$$U = e^{i\theta \vec{n}\cdot\vec{\sigma}}$$

Transition $\Omega_i \in \{ |0\rangle\langle 0|, |0\rangle\langle 1|, |1\rangle\langle 0|, |1\rangle\langle 1| \}$

$$\left(\frac{\mathbb{1}+\sigma_z}{2}\right), \quad \sigma_+, \quad \sigma_-, \quad \left(\frac{\mathbb{1}-\sigma_z}{2}\right)$$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

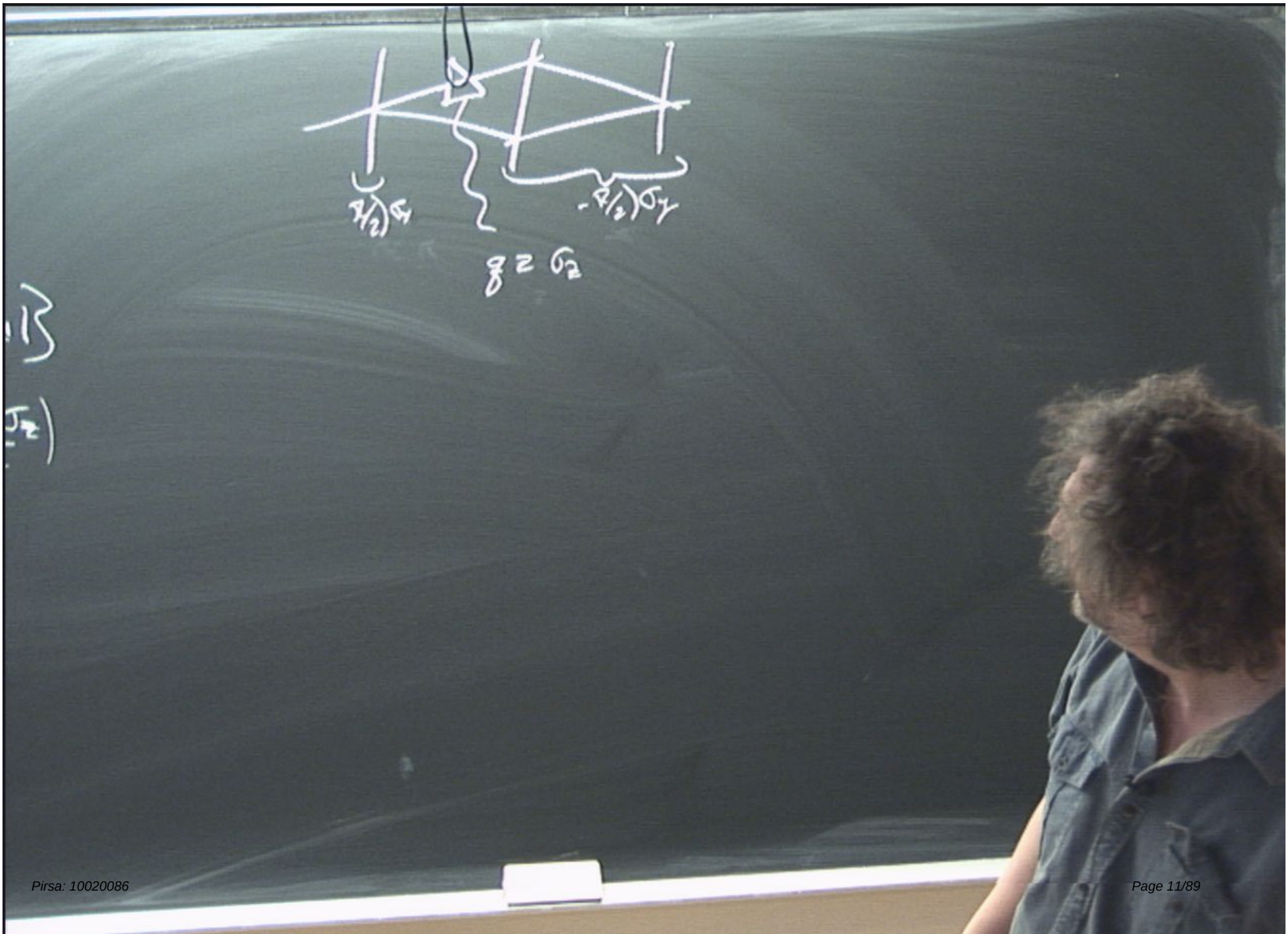
p_k ← classical probability

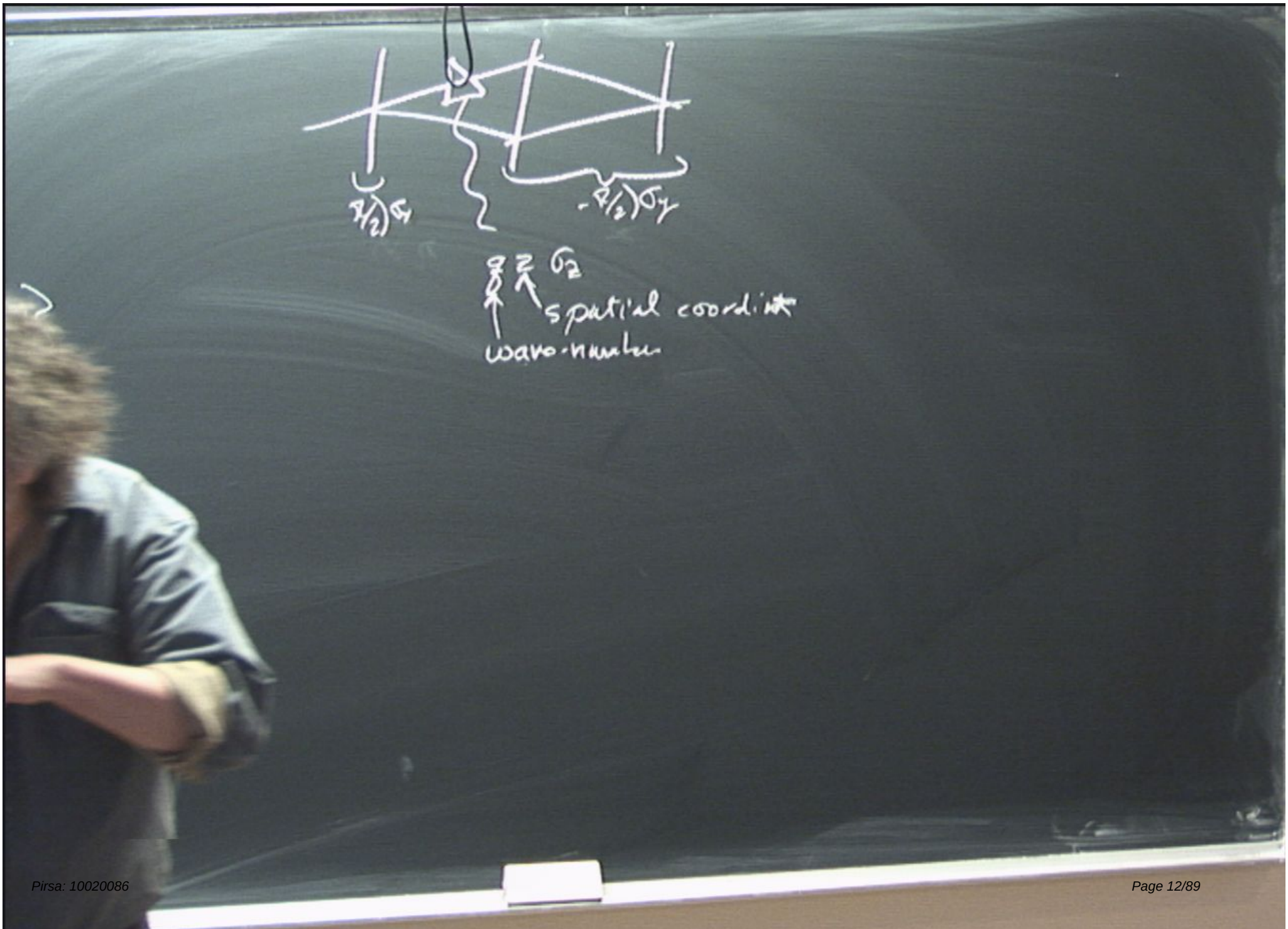
P_i = Pauli

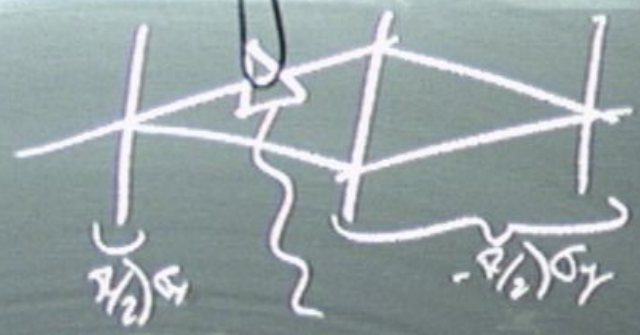
$$U = e^{i\theta \vec{n}\vec{\sigma}}$$

Transition $\Omega_i \in \{ |0\rangle\langle 0|, |0\rangle\langle 1|, |1\rangle\langle 0|, |1\rangle\langle 1| \}$

$$\frac{(\mathbb{1} + \sigma_z)}{2}, \quad \sigma_+, \quad \sigma_-, \quad \left(\frac{\mathbb{1} - \sigma_z}{2}\right)$$


π/2)σ_x
-π/2)σ_y
g = σ_z

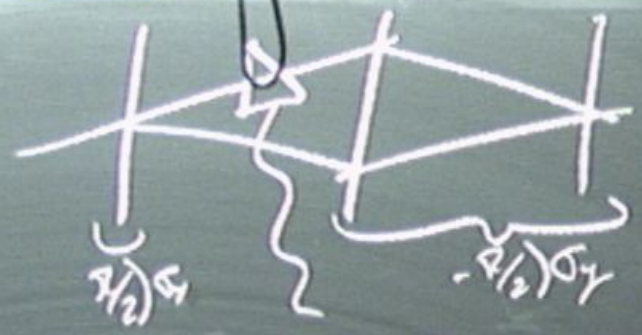

$(d/2)\sigma_1$
$-(d/2)\sigma_y$
σ_z
spatial coordinate
wave-number



σ_z

spatial coordinate

wave-number

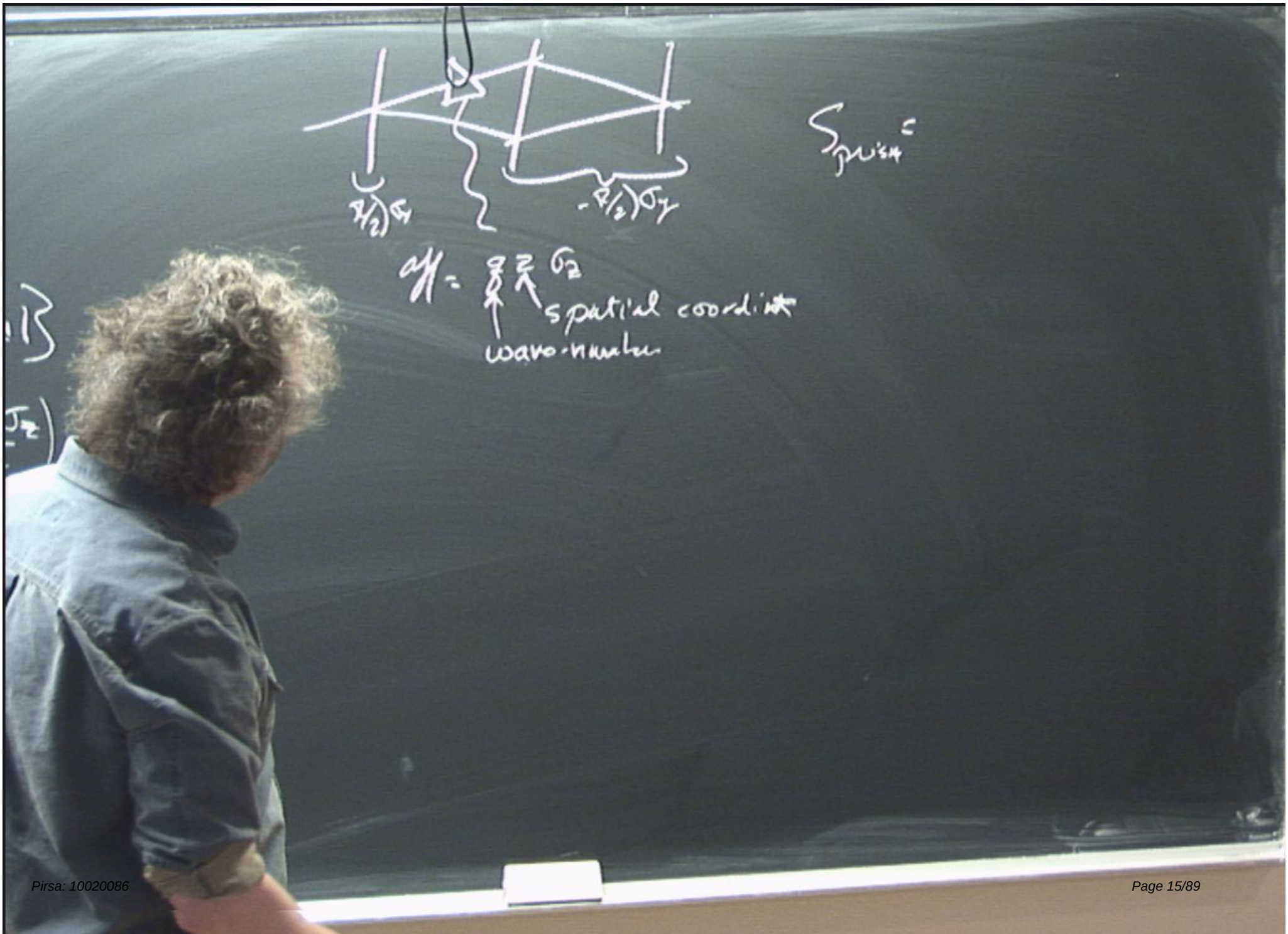


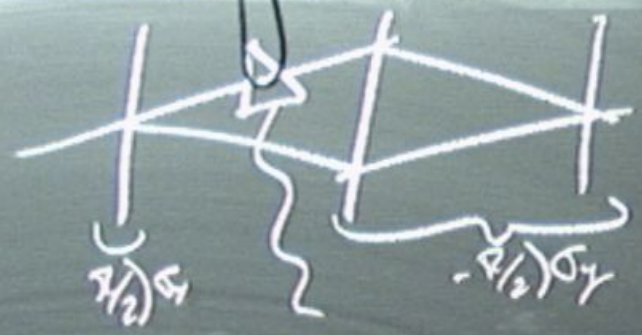
$S_{prism} =$

σ_z

spatial coordinate

wave-number


spatial coordinate
wave-number



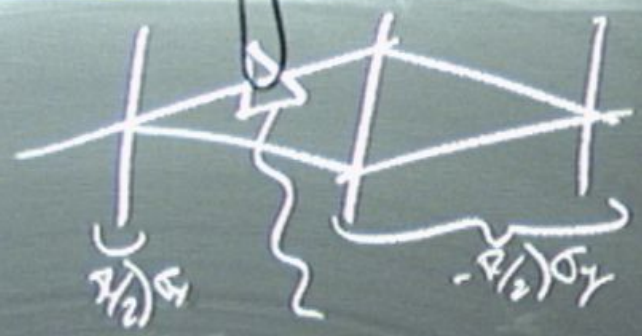
S_{prism} =

σ_z

spatial coordinate

wave-number

$$U(z) = e^{i\frac{qz}{2}\sigma_z}$$



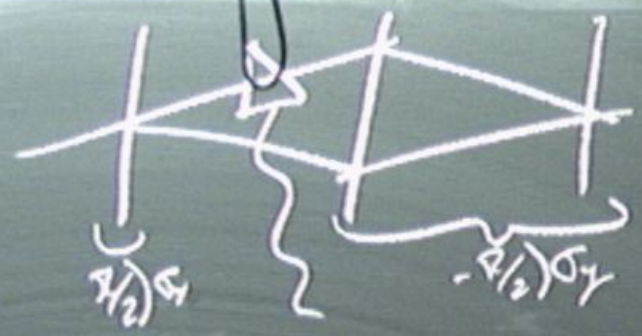
$S_{prism} =$

$q \quad z \quad \sigma_z$

spatial coordinate

wave-number

$$U_p(z) = e^{i\frac{qz}{2}\sigma_z} \quad ; \quad -0.5 < z < 0.5$$



$S_{prism} =$

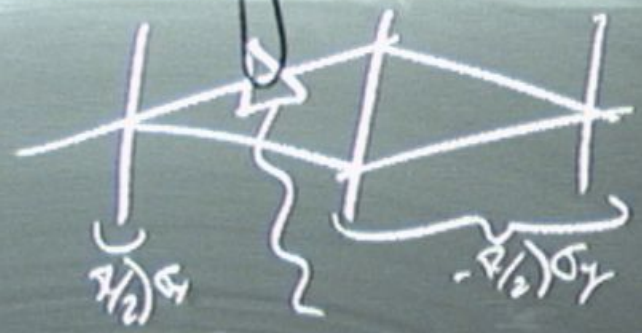
$z\,\sigma_z$

spatial coordinate

wave-number

$$U_p(z) = e^{i\frac{qz}{2}\sigma_z} \quad ; \quad -0.5 < z < 0.5$$

$P(z)$



$$S_{\text{pulse}} = \int_{-0.5}^{0.5} dz\, \mathrm{Tr}\left\{ P_f\, U_p(z)\, P_f\, U_p^{\dagger}(z) \right\}$$

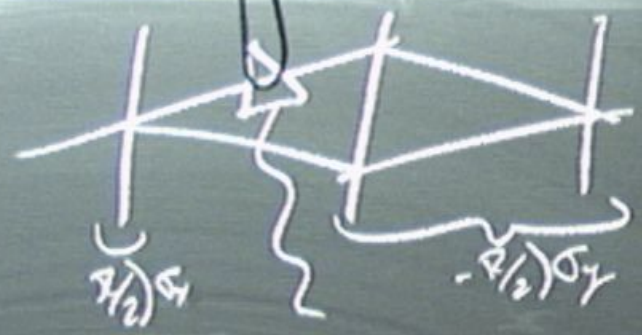
q z σ_z

spatial coordinate

wave-number

$$U_p(z) = e^{i \frac{q z}{2} \sigma_z} \quad ; \quad -0.5 < z < 0.5$$

$$P(z) = 1$$



$$S'_{\substack{\text{prism}\\ \gamma\gamma'}} = \int_{-0.5}^{0.5} dz\, \mathrm{Tr}\left\{ P_{\gamma'}\, U_p(z)\, P_\gamma\, U_p^{-1}(z) \right\}$$

q z σ_z

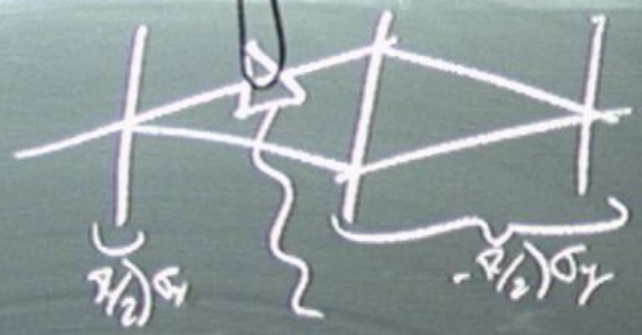
spatial coordinate

wave-number

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sin q/2}{q} & 0 & 0 \\ 0 & 0 & \frac{\sin q/2}{q} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U_p(z) = e^{i\frac{qz}{2}\sigma_z} \quad ; \quad \underline{-0.5 < z < 0.5}$$

$$P(z) = 1$$



$$S'_{\text{prism}} = \int_{-0.5}^{0.5} dz\, \mathrm{Tr}\left\{ P_f\, U_p(z)\, P_f\, U_p^{-1}(z) \right\}$$

q z σ_z

spatial coordinate

wave-number

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sin q/2}{q} & 0 & 0 \\ 0 & 0 & \frac{\sin q/2}{q} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U_p(z) = e^{i\frac{qz}{2}\sigma_z} \quad ; \quad \underline{-0.5 < z < 0.5}$$

$$P(z) = 1$$

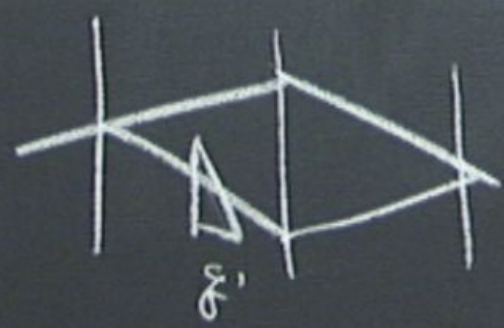
$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

p_k ← classical probability

P_i = Pauli

$$U = e^{i\theta \vec{n}\vec{\sigma}}$$

Transitions: $\Omega_i \in \left\{ |0\rangle\langle 0|, \; |0\rangle\langle 1|, \; |1\rangle\langle 0|, \; |1\rangle\langle 1|, \; \frac{(\mathbb{1}+\sigma_z)}{2}, \; \sigma_+, \; \sigma_-, \right.$



$$U_{pol}(z) = e^{-i\xi' z \sigma_z/2}$$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

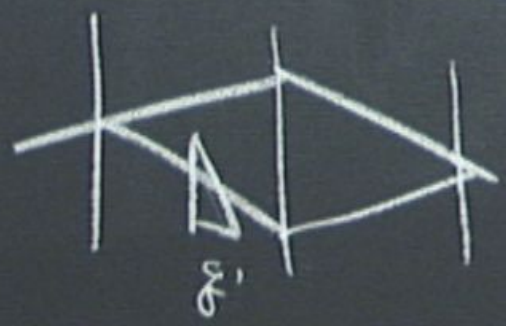
p_k ← classical probability

P_i = Pauli

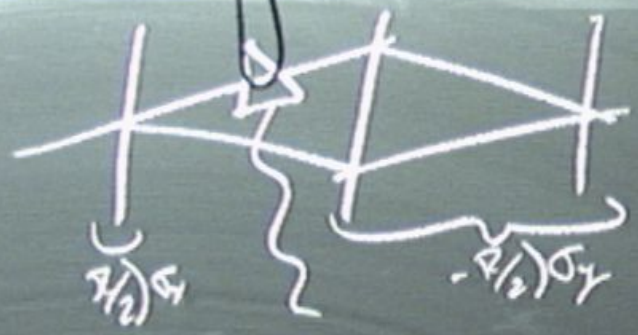
$$U = e^{i\theta \vec{n}\vec{\sigma}}$$

Transitions: $\Omega_i \in \left\{ |0\rangle\langle 0|, \; |0\rangle\langle 1|, \; |1\rangle\langle 1|, \; |1\rangle \right.$

$\frac{(\mathbb{1}+\sigma_z)}{2}$, σ_+, σ_-, $(\mathbb{1}$



$$U_{pd}(z) = e^{-i\xi' z \sigma_z/2}$$



$$S'_{\text{prism}} = \int_{-0.5}^{0.5} dz\, \mathrm{Tr}\left\{ P_f\, U_p(z)\, P_f\, U_p^{\dagger}(z) \right\}$$

q z σ_z

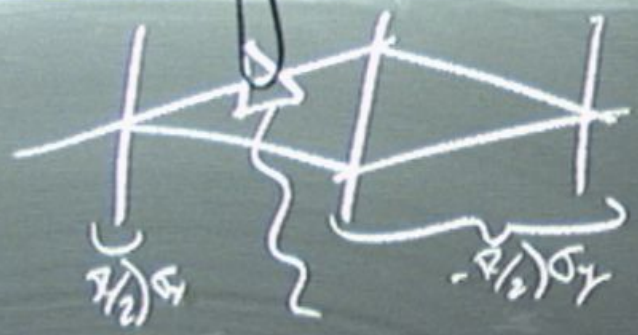
spatial coordinate

wave-number

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sin q/2}{q} & 0 & 0 \\ 0 & 0 & \frac{\sin q/2}{q} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U_p(z) = e^{i\frac{qz}{2}\sigma_z} \quad ; \quad -0.5 < z < 0.5$$

$$P(z) = 1$$



$$S'_{\substack{\text{prism}\\ \uparrow\downarrow}} = \int_{-0.5}^{0.5} dz\, \mathrm{Tr}\left\{ P_{\uparrow}\, U_p(z)\, P_{\uparrow}\, U_p^{\dagger}(z) \right\}$$

q z σ_z

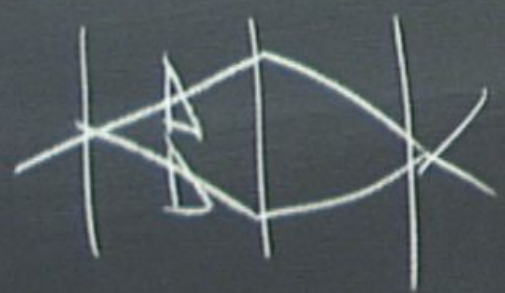
spatial coordinate

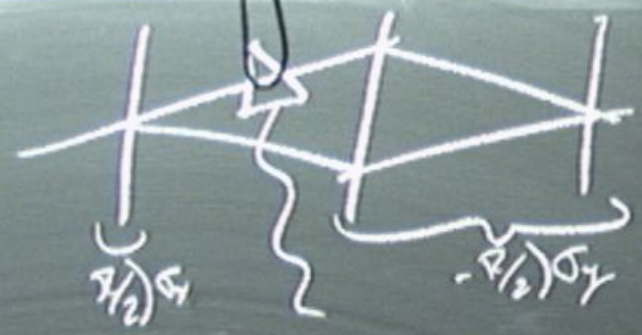
wave-number

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sin q/2}{q} & 0 & 0 \\ 0 & 0 & \frac{\sin q/2}{q} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U_p(z) = e^{i\frac{qz}{2}\sigma_z} \quad ; \quad \underline{-0.5 < z < 0.5}$$

$$P(z) = 1$$





$$S'_{\text{prism}} = \int_{-0.5}^{0.5} dz\, \mathrm{Tr}\left\{ P_f\, U_p(z)\, P_f\, U_p^{\dagger}(z) \right\}$$

q z σ_z

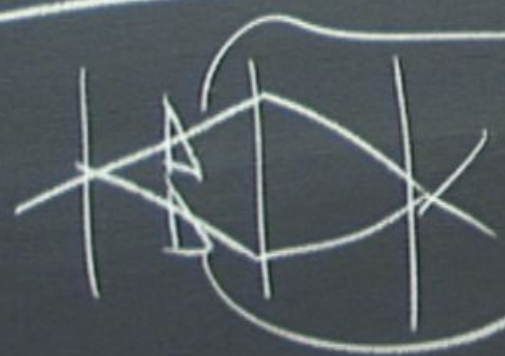
spatial coordinate

wave-number

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sin q/2}{q} & 0 & 0 \\ 0 & 0 & \frac{\sin q/2}{q} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U_p(z) = e^{i\frac{qz}{2}\sigma_z} \quad ; \quad -0.5 < z < 0.5$$

$$P(z) = 1$$



$$\begin{pmatrix} e^{-iqz} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{iqz} \end{pmatrix}$$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

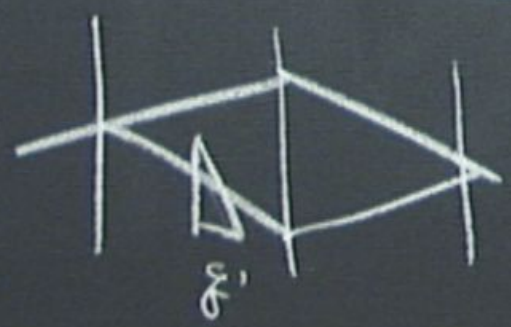
p_k ← classical probab. 1.4.ie.

P_i = Pauli

$$U = e^{i\theta \vec{n}\vec{\sigma}}$$

Transition $\Omega_i \in$ $\left\{ |0\rangle\langle 0|,\ |0\rangle\langle 1|,\ |1\rangle\langle 0|,\ |1\rangle\langle 1| \right\}$

$$\frac{(\mathbb{1}+\sigma_z)}{2},\quad \sigma_+,\quad \sigma_-,\quad \frac{(\mathbb{1}-\sigma_z)}{2}$$



$$S_{Pd} = S_P$$

$$U_{Pd}(z) = e^{-i\xi' z \sigma_z/2}$$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \sum_k p_k \, U_k \, P_j \, U_k^{-1} \right\}$$

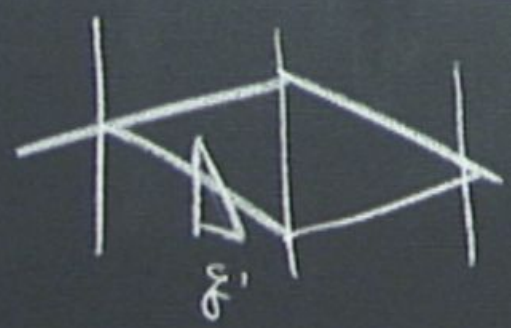
p_k ← classical probabilities

P_i = Pauli

$$U = e^{i\theta \vec{n}\vec{\sigma}}$$

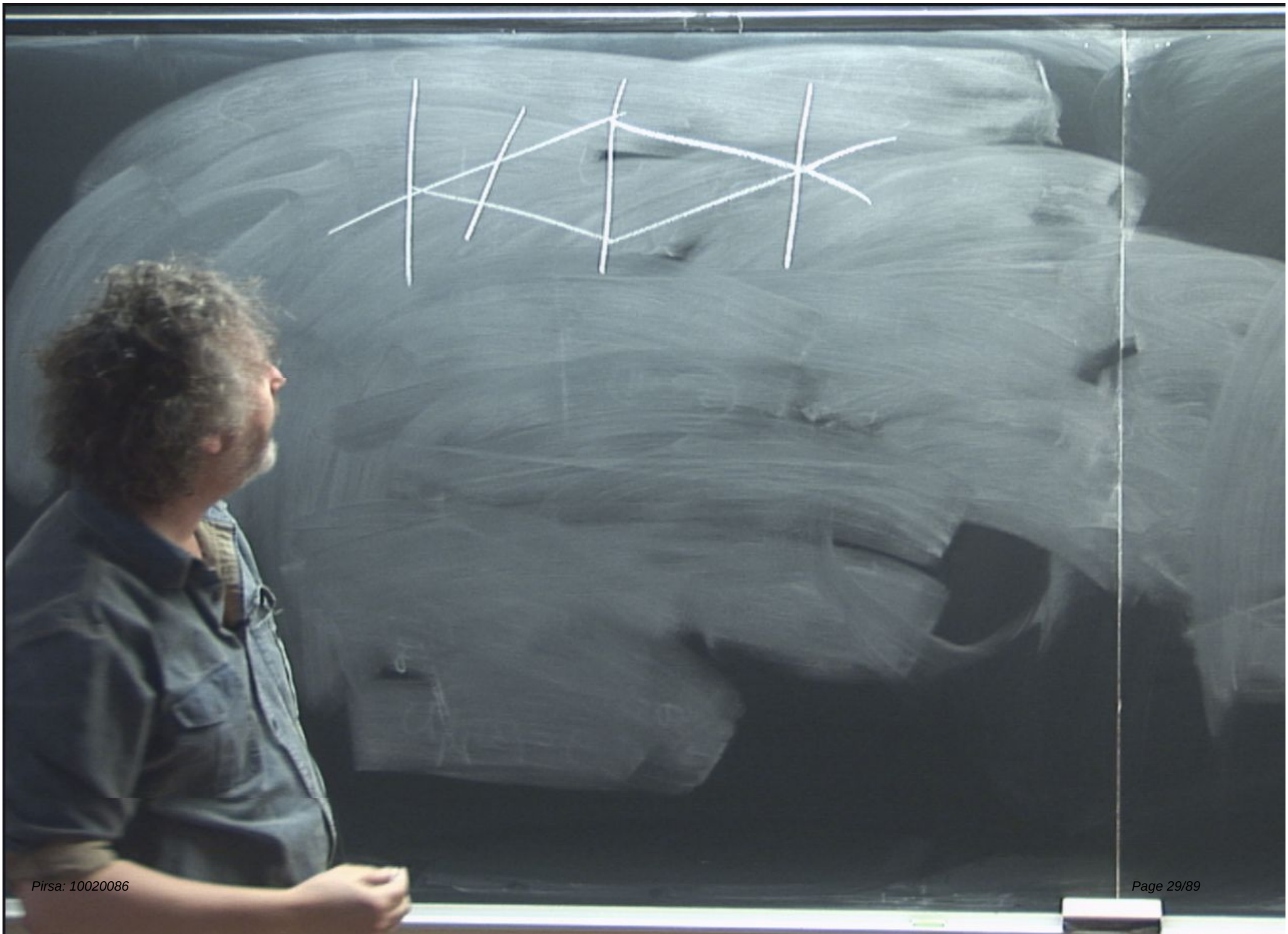
Transitions: $\Omega_i \in \left\{ |0\rangle\langle 0|,\ |0\rangle\langle 1|,\ |1\rangle\langle 0|,\ |1\rangle\langle 1| \right\}$

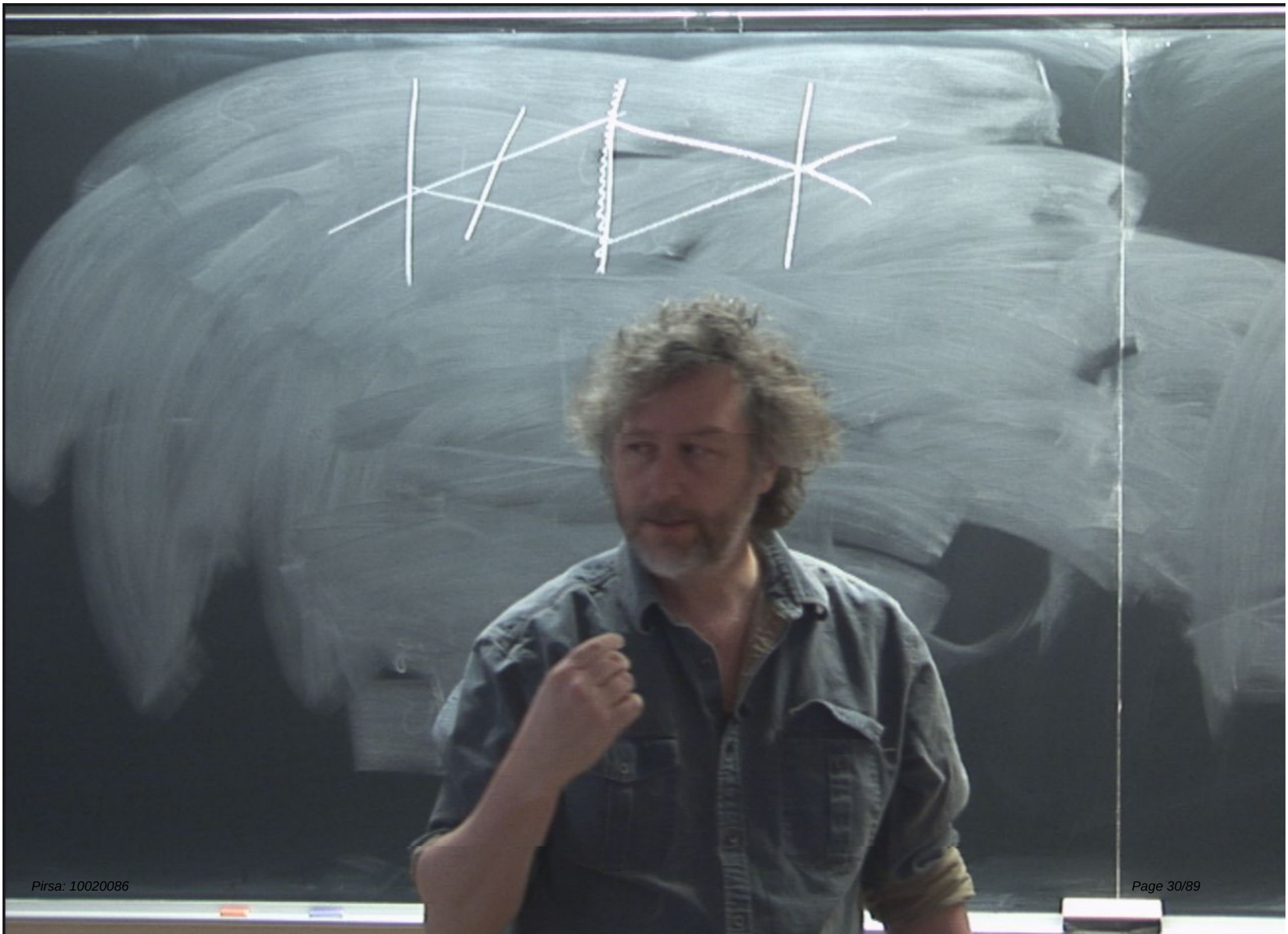
$$\left\{ \frac{(\mathbb{1}+\sigma_z)}{2},\ \sigma_+,\ \sigma_-,\ \frac{(\mathbb{1}-\sigma_z)}{2} \right\}$$



$$S_{Pd} = S_P$$

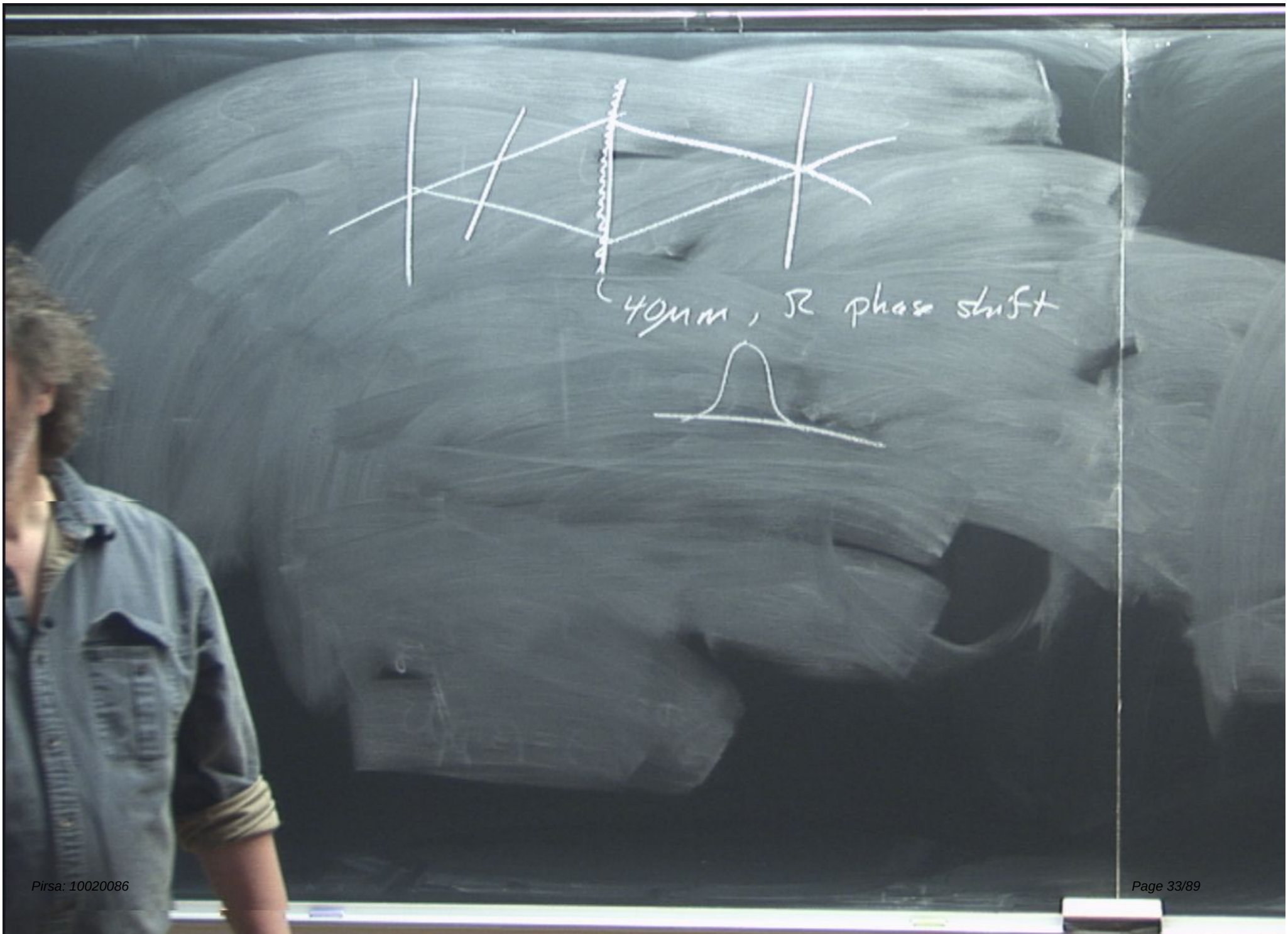
$$U_{Pd}(z) = e^{-i\xi' z \sigma_z/2}$$

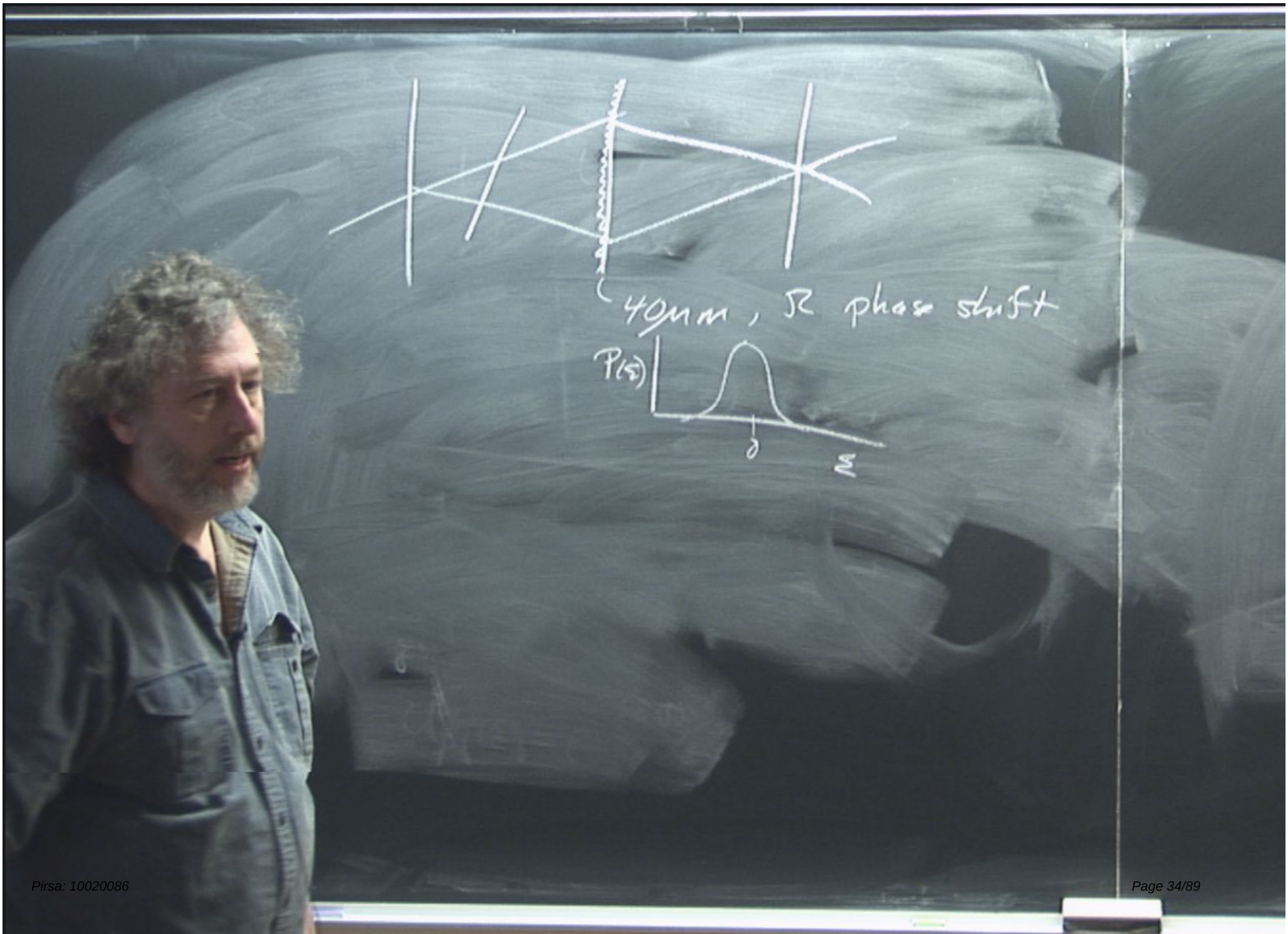


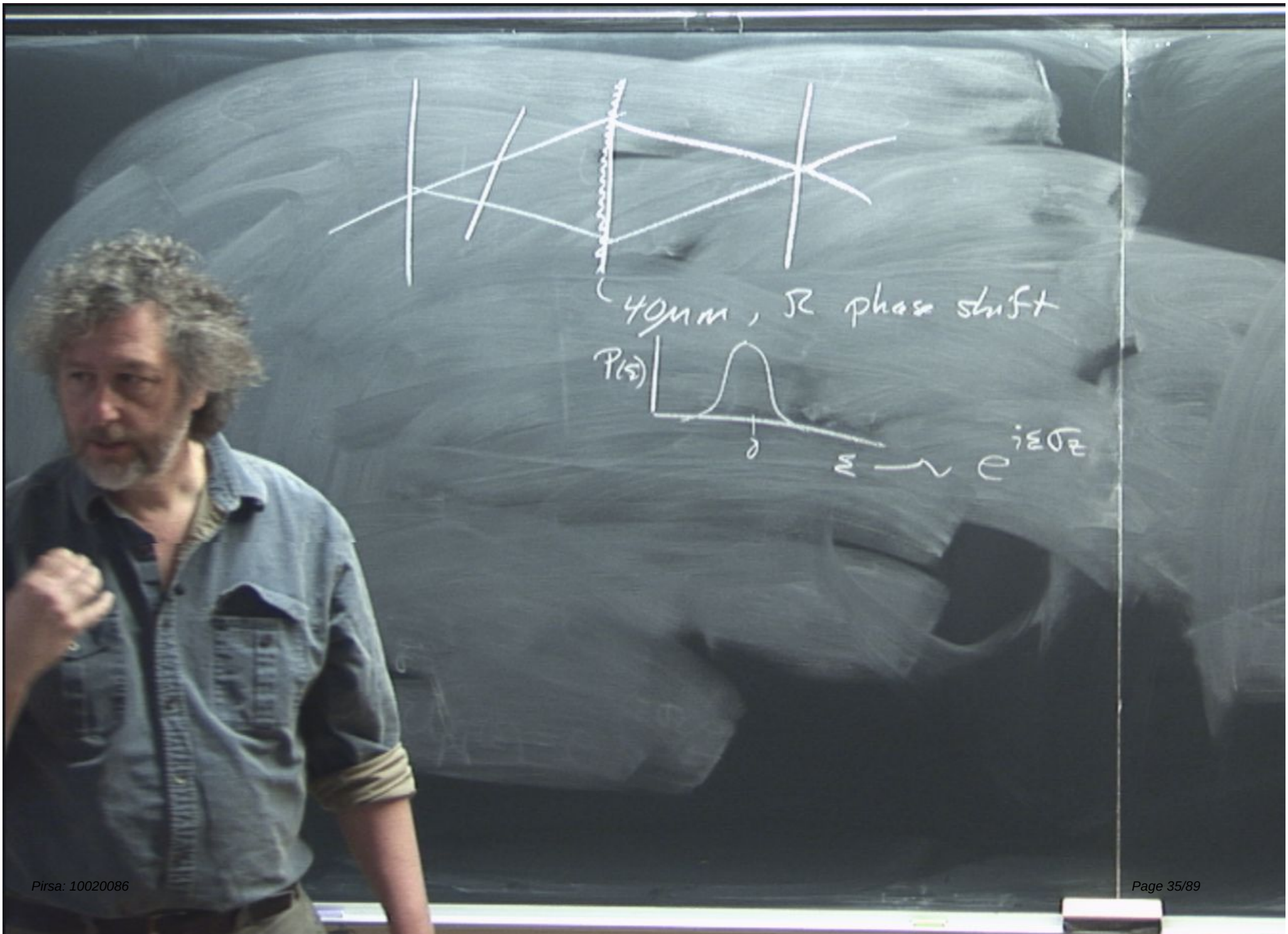


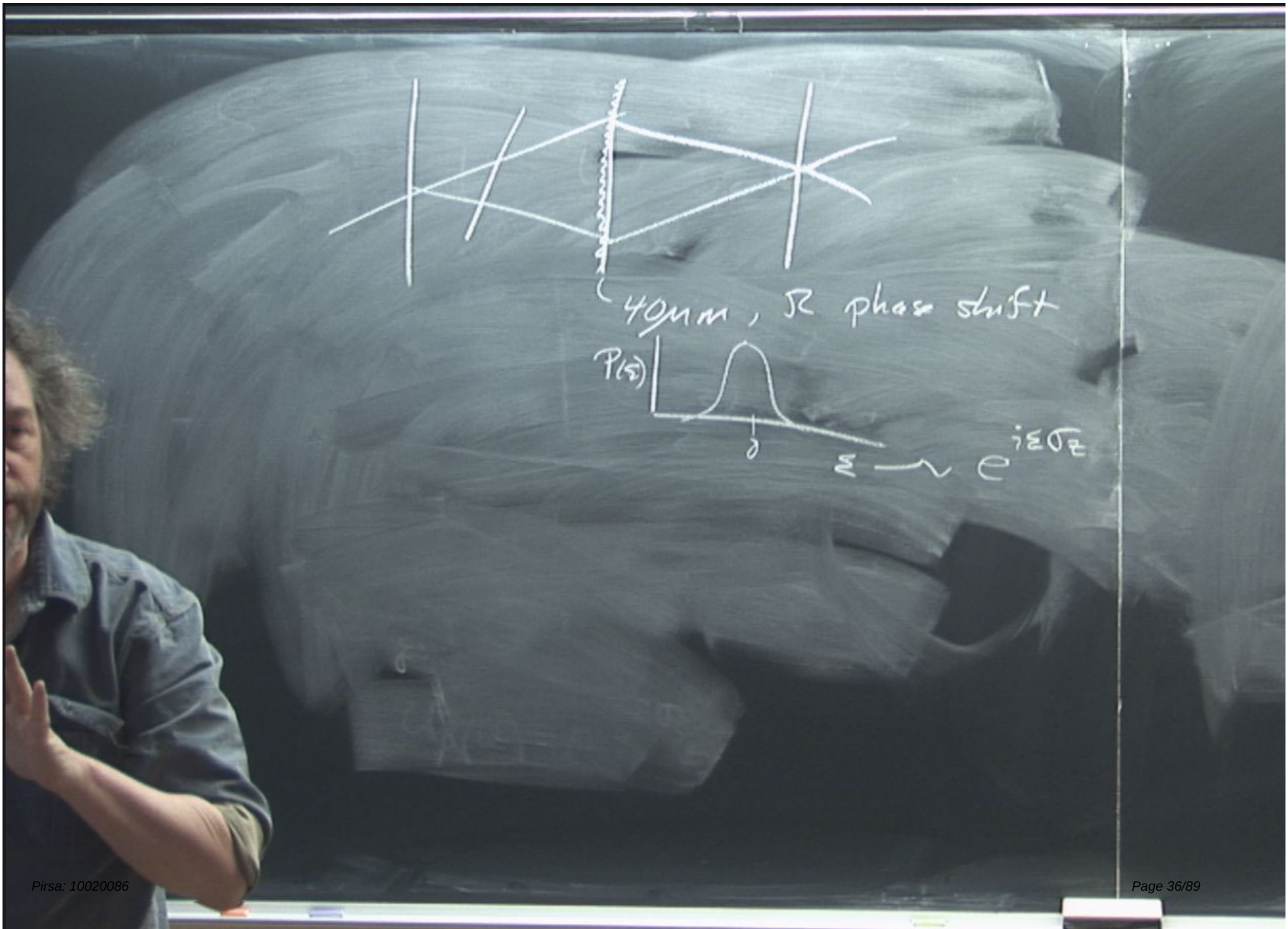
40μm, π phase shift

40μm, π phase shift

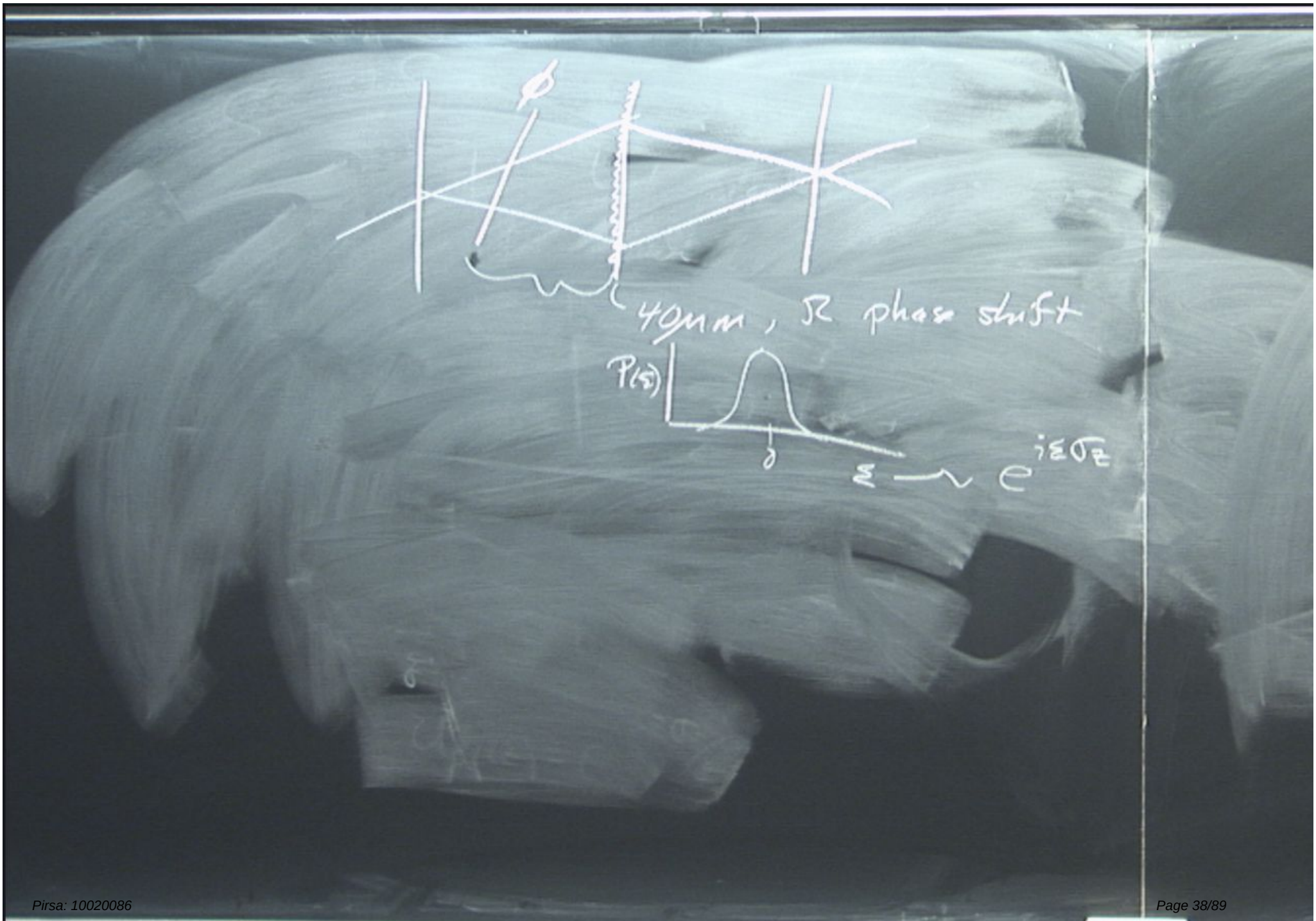

40μm, π phase shift


40μm, π phase shift
P(δ)
δ


40μm, R phase shift
P(δ)
δ
ε ~ e^{iεσz}


40μm, π phase shift
P(ε)
0
ε ~ e^{iεσz}

H/W – add phase noise to all blocks.


40μm, π phase shift
P(ε)
ε ~ e^{iεσz}

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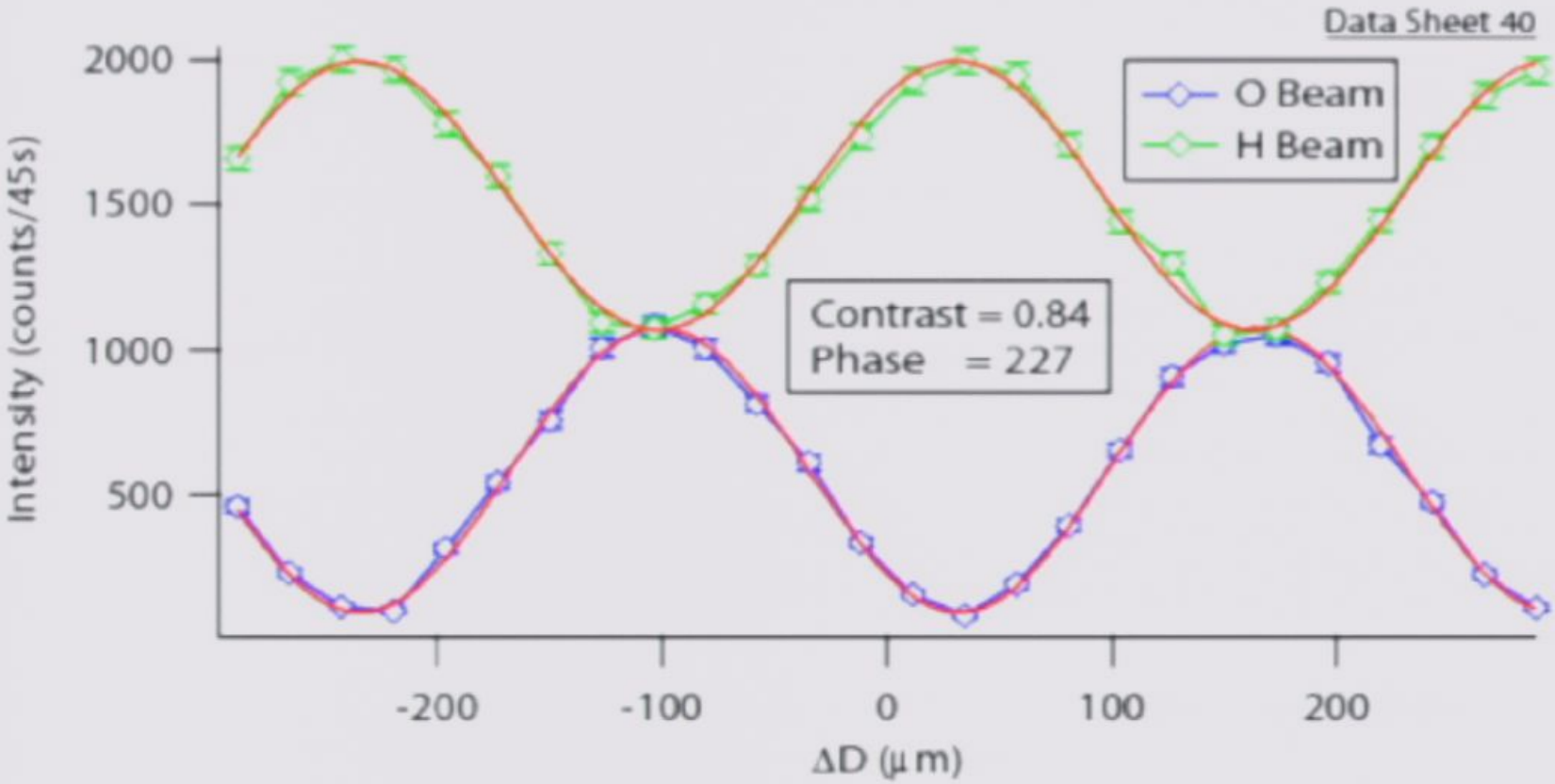
#2.nb



- Problem 24: estimate the transmission coefficient and the loss for the experimental NI setup.

eventh experiment: Describe the super-operator for the general map of Experiment 10


```
S[a_, r_] :=
  Table[Integrate[nd[x, r] * Tr[PauliMatrix[i] . Ublade . Uphase[0, a * x] . Um
```

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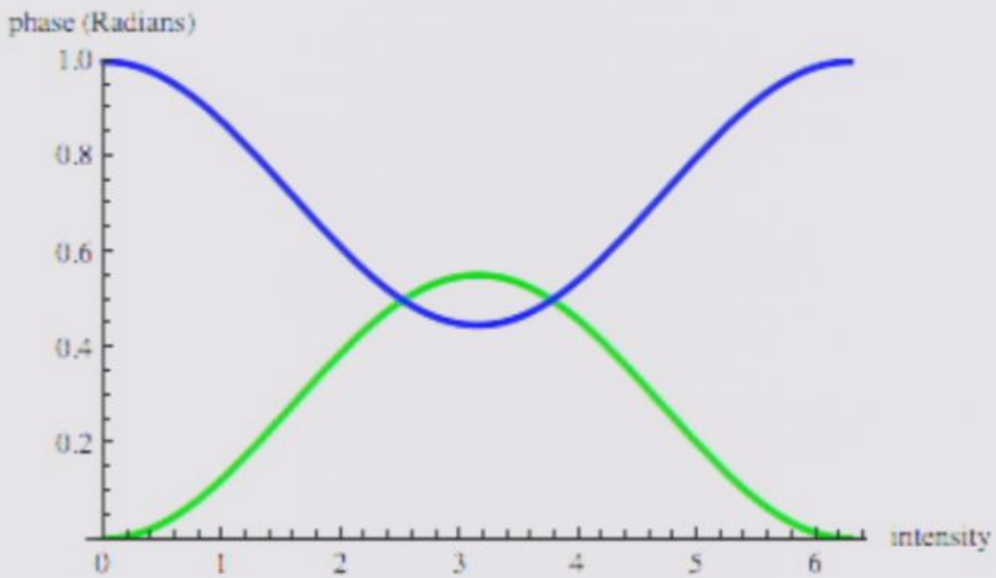
#2.nb

r

T



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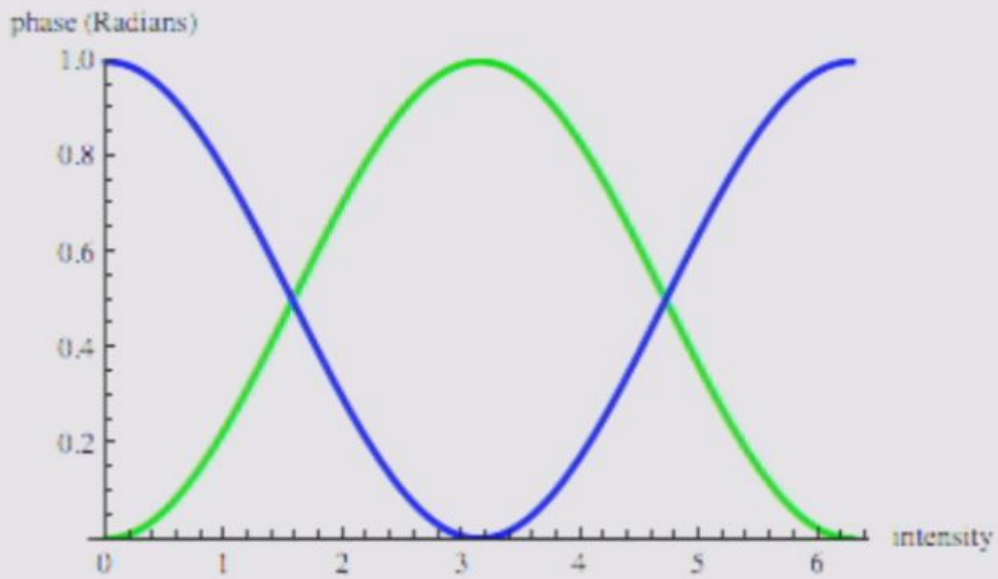

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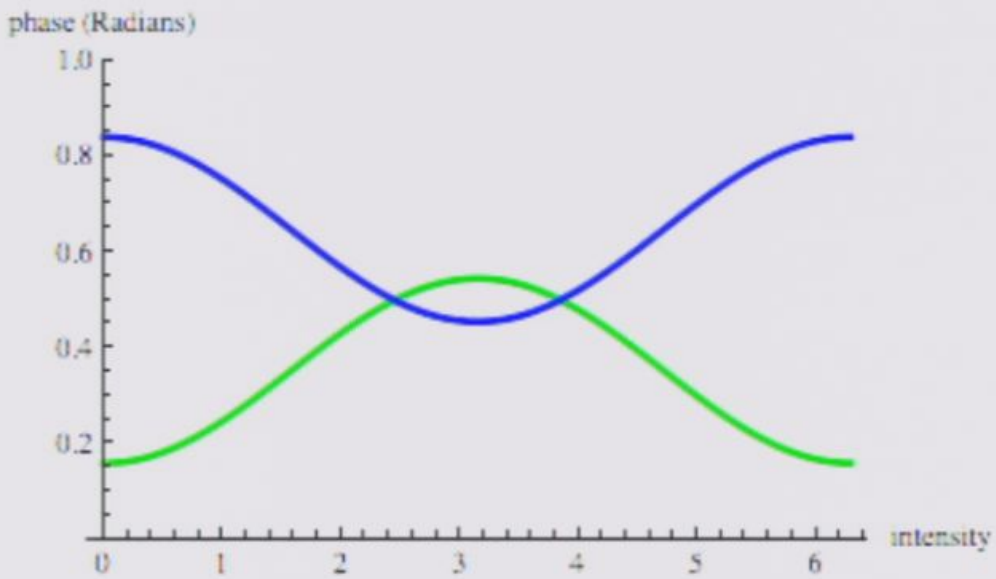
r

T



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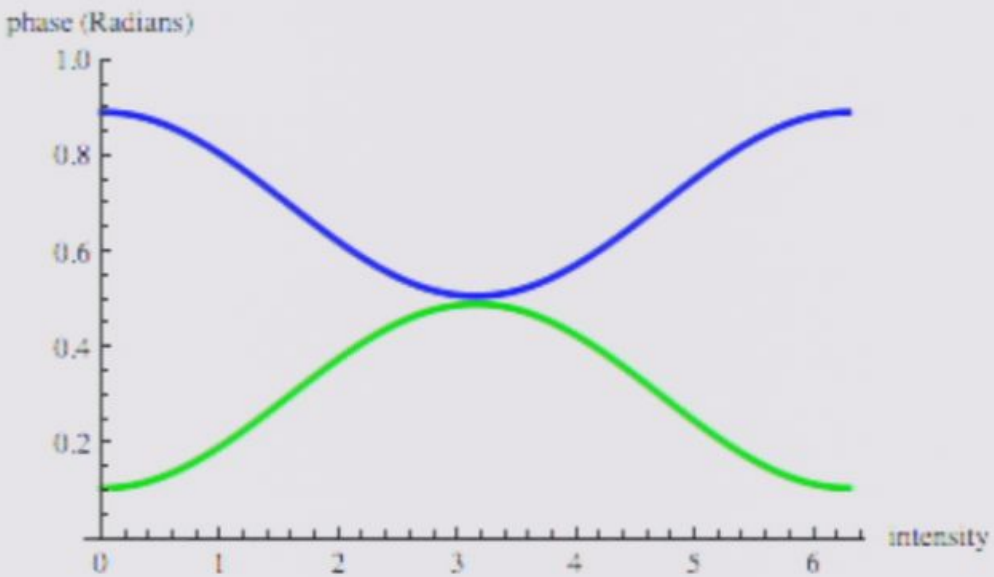
#2.nb

r

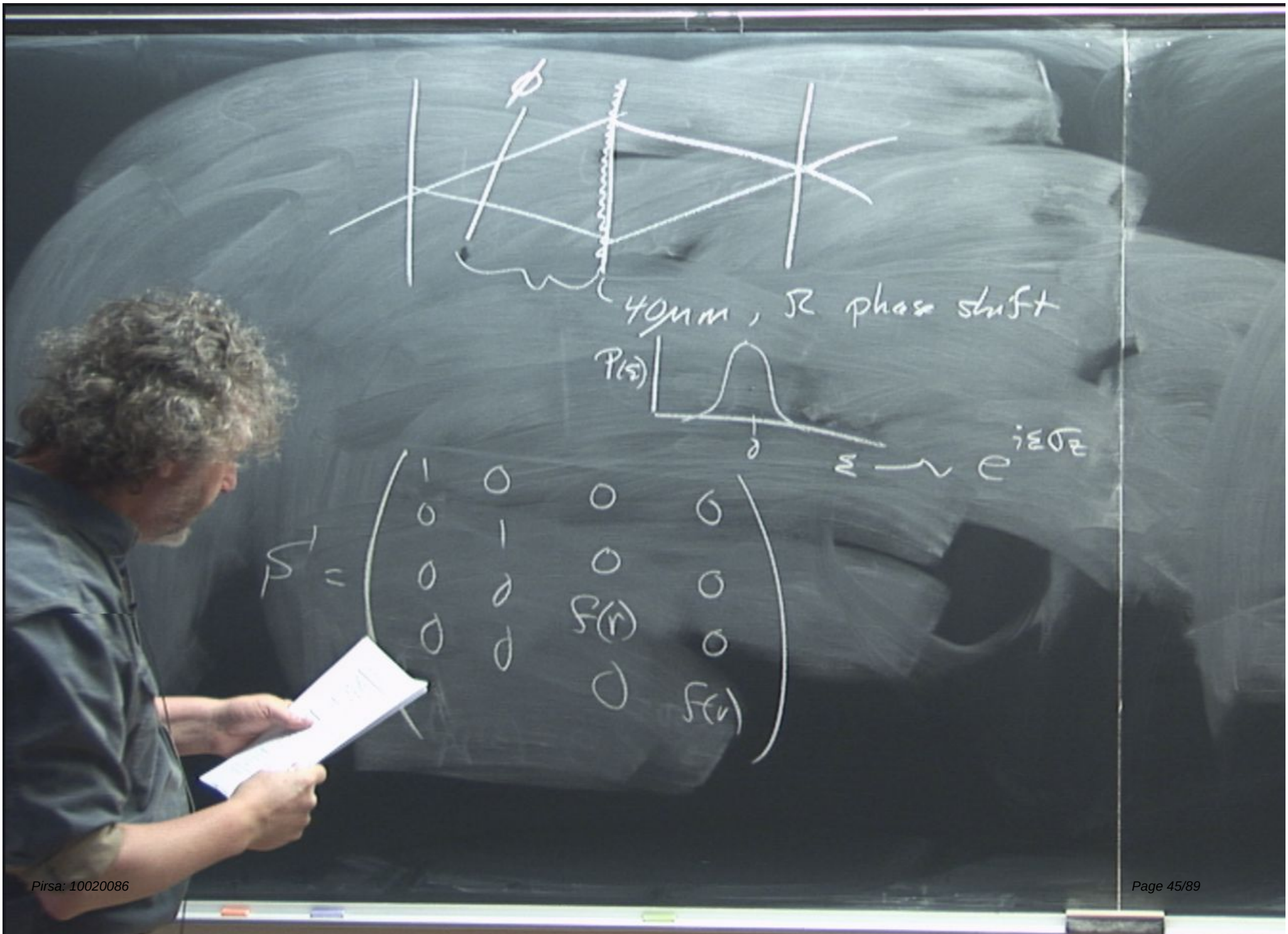
T

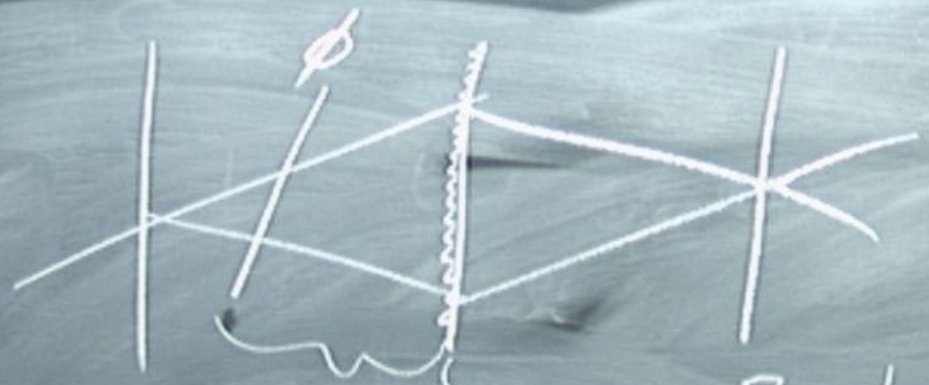


40nm, $\Re$ phase shift

$\varepsilon \sim e^{i\varepsilon\sigma_z}$

$$S' = \begin{pmatrix} 1 & 0 & & \\ 0 & 1 & & \\ & & 0 & 0 \\ & & 0 & 0 \end{pmatrix}$$


ϕ
40μm, π phase shift
$P(\varepsilon)$
$\varepsilon \sim e^{i\varepsilon\sigma_z}$
$S' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & f(r) & 0 \\ 0 & 0 & 0 & f(r) \end{pmatrix}$



40μm, Ω phase shift

$\varepsilon \sim e^{i\varepsilon\sigma_z}$

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & f(r) & 0 \\ 0 & 0 & 0 & f(r) \end{pmatrix}$$

$f(r) < 1$

$\sim \mathrm{Exp}$

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$\mathbb{1}$	$(\mathbb{1} + \sigma_z)$
σ_x	σ_+
σ_y	σ_-
σ_z	$\frac{(\mathbb{1} - \sigma_z)}{2}$

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$$S_T = T\, S_p\, T^\dagger$$

$\mathbb{1}$ $\quad$ $(\mathbb{1} + \sigma_z)$

σ_x $\quad$ σ_+

σ_y $\quad$ σ_-

σ_z

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & +1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$$S_T = T\, S_p\, T^{-1}$$

$\mathbb{1}$	$(\mathbb{1} + \sigma_z)$
σ_x	σ_+
σ_y	σ_-
σ_z	$\frac{(\mathbb{1} - \sigma_z)}{2}$

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$$S_T = T\, S_p\, T^{-1}$$

Choi Matrix

$\mathbb{1}$	$(\mathbb{1} + \sigma_z)$
σ_x	σ_+
σ_y	σ_-
σ_z	$\frac{(\mathbb{1} - \sigma_z)}{2}$

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$$S_T = T\, S_p\, T^{-1}$$

Choi' Matrix

$\mathbb{1}$	$(\mathbb{1} + \sigma_z)/2$
σ_x	σ_+
σ_y	σ_-
σ_z	$\frac{(\mathbb{1} - \sigma_z)}{2}$

$$S_T = \begin{pmatrix} (A) & (C) \\ (B) & (D) \end{pmatrix}$$

$$S_C = \begin{pmatrix} (A) & (B) & (C) & (D) \end{pmatrix}$$

$$\rho_{out} = \sum_{k} K_k \rho_{in} K_k^{\dagger} ;$$

H/W – add phase noise to all blocks.

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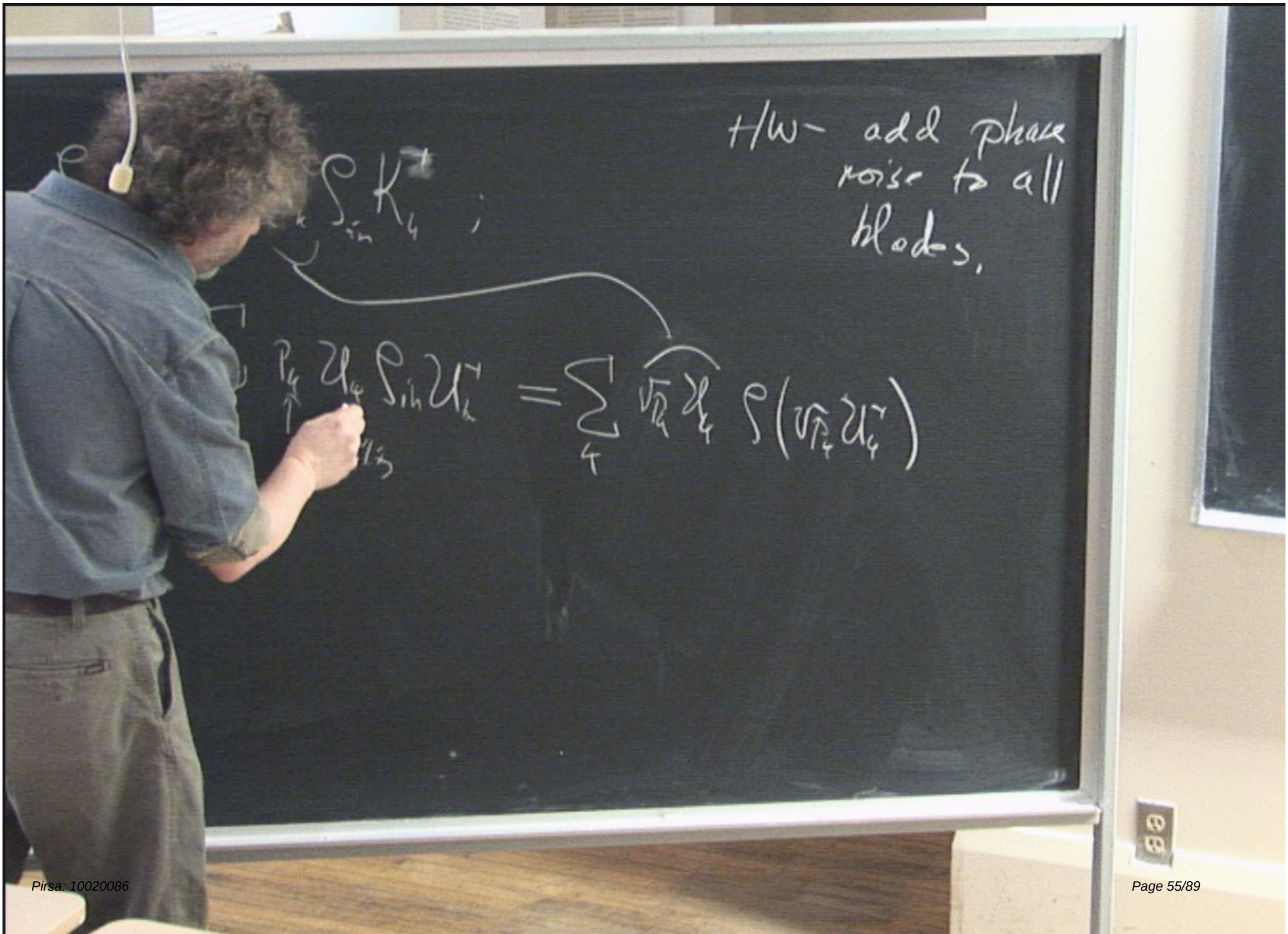
H/W – add phase noise to all blocks.

$$\rho_{out} = \sum_{k} K_k \rho_{in} K_k^\dagger ;$$

$$\rho_{out} = \sum_{k} p_k U_k \rho_{in} U_k^\dagger$$

probability

H/W – add phase noise to all blocks.


H/W - add phase noise to all blocks.

$$\rho_{out} = \sum_{k}^{D} K_k \rho_{in} K_k^\dagger ;$$

H/W – add phase noise to all blocks.

$$\rho_{out} = \sum_{k} \underset{\substack{\uparrow \\ \text{Probability}}}{p_k} \underline{\underline{U_k}} \rho_{in} U_k^\dagger = \sum_{k} \widehat{\sqrt{p_k}\, U_k}\, \rho \left(\sqrt{p_k}\, U_k^\dagger \right)$$

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#2.nb

```
SupOpTrans = BasisTrans . S[0, .5] . Inverse[BasisTrans];
```

we calculate the Choi matrix

```
ChoiS = Choi[SupOpTrans];
```

Kraus operators are given by the eigenvectors of the Choi matrix

```
KrausOps = √2 Eigenvectors[ChoiS];
```

probabilities associated with each Kraus operator are given by the eigenvalues of the Choi matrix

```
KrausProbs = 1/2 Eigenvalues[ChoiS];
```

```
MatrixForm[KrausProbs]
```

$$\begin{pmatrix} 0.941248 \\ 0.058751 \\ 0. \\ 0. \end{pmatrix}$$

```
MatrixForm[KrausOps]
```

$$\begin{pmatrix} 1. & 0. & 0. & 1. \\ 0. & 1. & 1. & 0. \\ 1. & 0. & 0. & -1. \\ 0. & 1. & -1. & 0. \end{pmatrix}$$

that this is for the overall dynamcis of the system.

e picture of the dynamics is that with

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$$S_T = T\, S_P\, T^{-1}$$

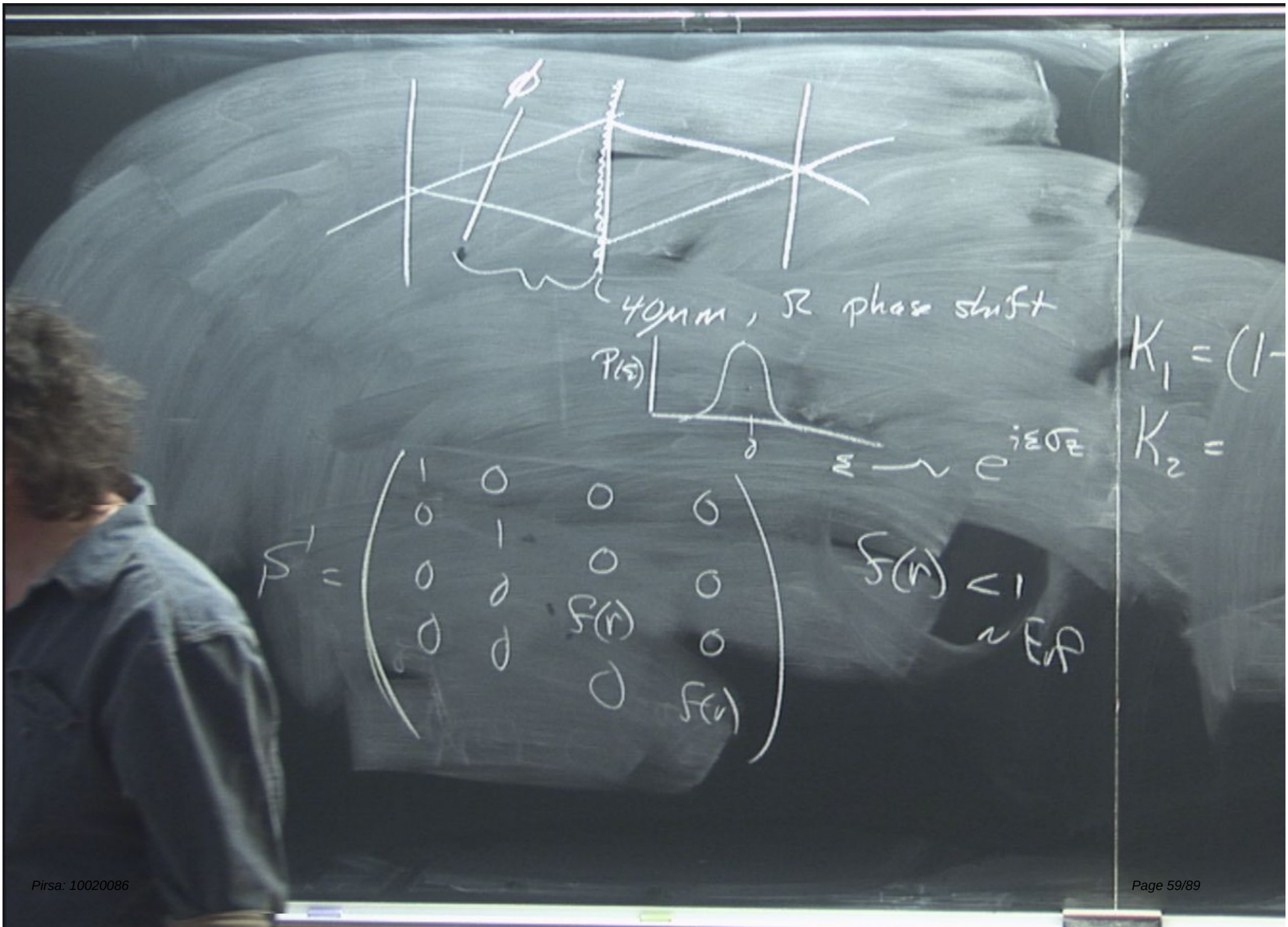
Choi Matrix

$\mathbb{1}$	$(\mathbb{1} + \sigma_z)$
σ_x	σ_+
σ_y	σ_-
σ_z	$\frac{(\mathbb{1} - \sigma_z)}{2}$

$(1-P)$	$\mathbb{1}$
P	σ_x

$$S_T = \begin{pmatrix} (A) & (C) \\ (B) & (D) \end{pmatrix}$$

$$S_C = \begin{pmatrix} (A) & (B) & (C) & (D) \end{pmatrix}$$


40μm, π phase shift
P(ε)
ε ~ e^{iεσ_z}
S' = (1 0 0 0; 0 1 0 0; 0 0 f(r) 0; 0 0 0 f(r))
f(r) < 1
~ Exp
K_1 = (1-
K_2 =

40μm, Ω phase shift

$\varepsilon \sim e^{i\varepsilon\sigma_z}$

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & f(r) & 0 \\ 0 & 0 & 0 & f(r) \end{pmatrix}$$

$f(r) < 1$

$\sim \exp$

$T =$

$K_1 = \sqrt{1-P}$ $\mathbb{1}$

$K_2 = \sqrt{P}$ σ

40μm, Ω phase shift

$\varepsilon \sim e^{i\varepsilon\sigma_z}$

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & f(r) & 0 \\ 0 & 0 & 0 & f(r) \end{pmatrix}$$

$f(r) < 1$

$\sim \mathrm{Exp}$

$T =$

$K_1 = \sqrt{(1-P)}\ \mathbb{1}$

$K_2 = \sqrt{P}\ \sigma$

$P = f(r)$

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$$= \sqrt{(1-P)}\ \mathbb{1}$$

$$= \sqrt{P}\ \sigma_x$$

$$P = f(r)$$

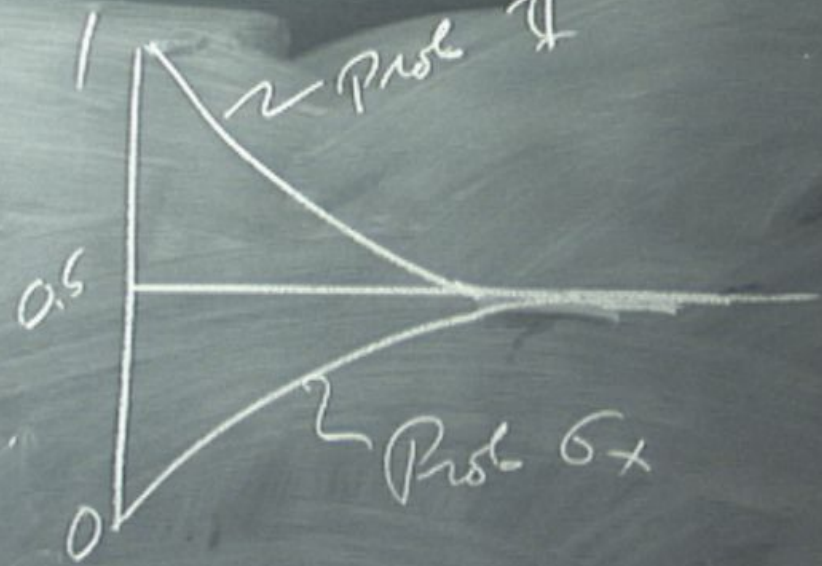
$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$$= \sqrt{(1-P)}\ \mathbb{1}$$

$$= \sqrt{P}\ \sigma_x$$

} unit operators,

$$P = f(r)$$



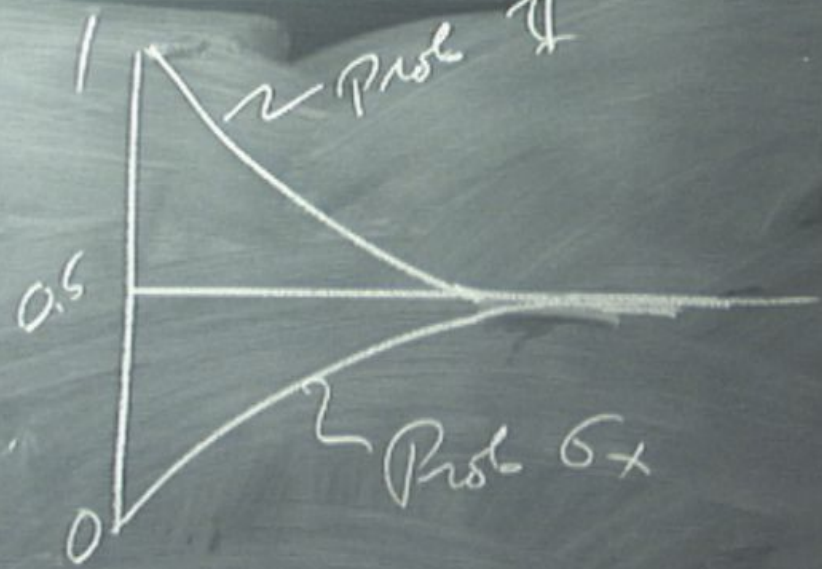
$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$$= \sqrt{(1-P)}\ \mathbb{1}$$

$$= \sqrt{P}\ \sigma_x$$

} jump operators,

$$P = f(r)$$



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#2.nb

that this is the standard dephasing picture, T2 is the rate of decoherence and t is the time decoherence acts for.

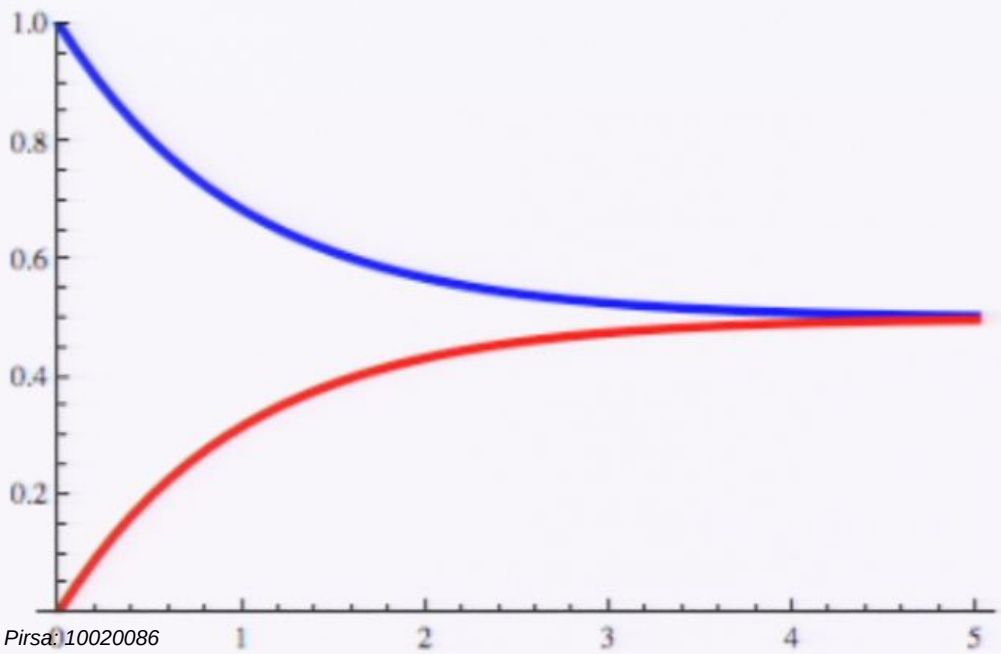
0 then only the first operator is present and it is proportional to the identity.

> T2 then both operators are present with equal probability.

```
KX[1, t_, T_] := Sqrt[(0.5 + 0.5 Exp[-t/T])] PauliMatrix[0];

KX[2, t_, T_] := Sqrt[(0.5 - 0.5 Exp[-t/T])] PauliMatrix[1];

Show[Plot[0.5 + 0.5 Exp[-t], {t, 0, 5},
   {PlotRange → {0, 1}, PlotStyle → {RGBColor[0, 0, 1], Thickness[0.01]}}],
 Plot[0.5 - 0.5 Exp[-t], {t, 0, 5},
   {PlotRange → {0, 1}, PlotStyle → {RGBColor[1, 0, 0], Thickness[0.01]}}]]
```



recall that the above operators should be read as the square-root of a probability times a jump operator. Clearly

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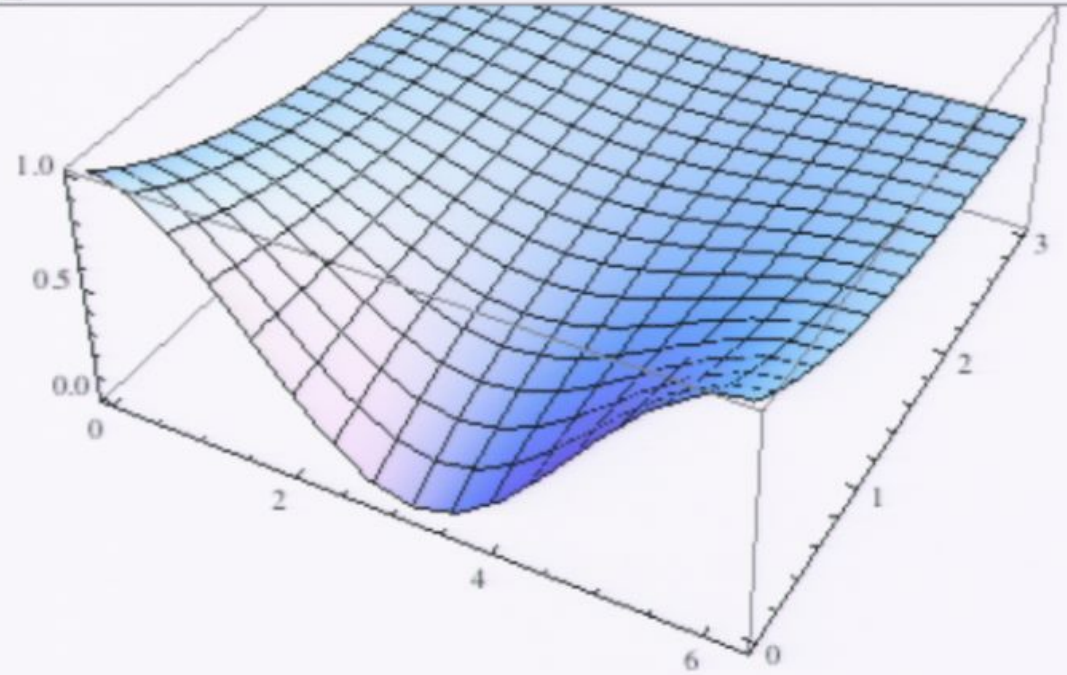
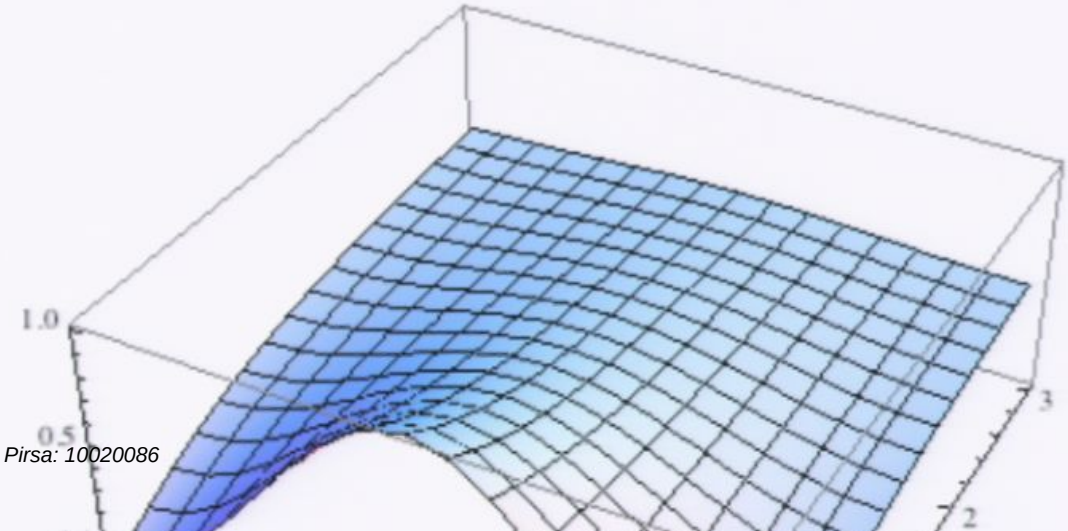



```
Plot3D[M13H[a, t, 1], {a, 0, 2 π}, {t, 0, 3}]
```



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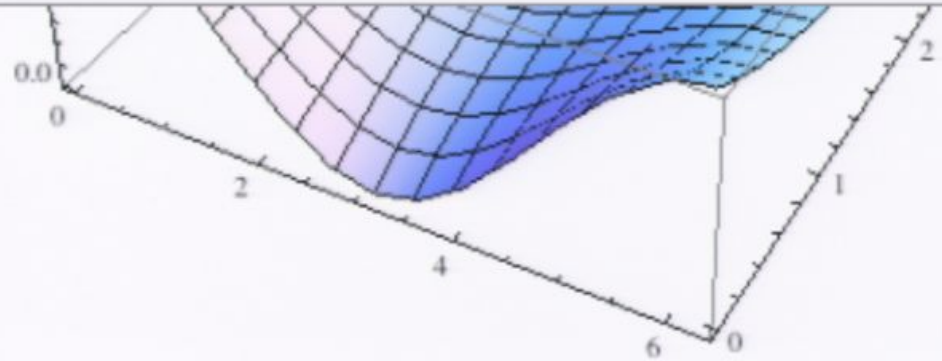
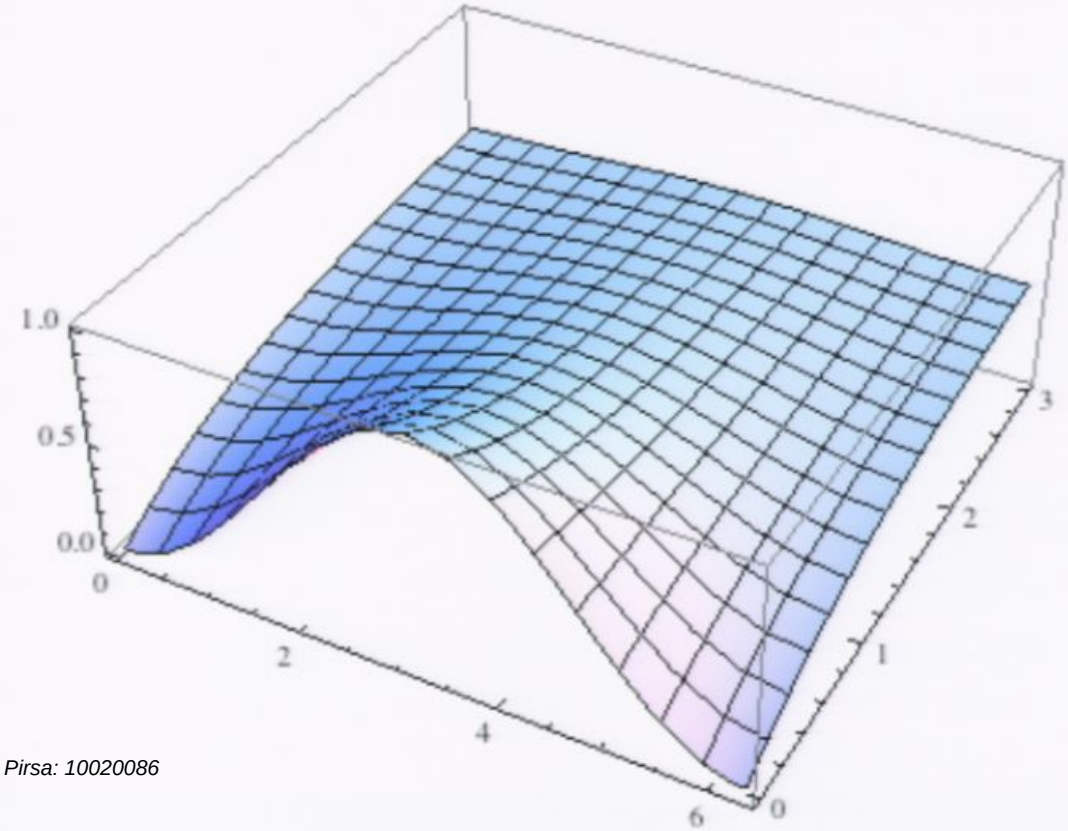
#2.nb




```
Plot3D[M13H[a, t, 1], {a, 0, 2 π}, {t, 0, 3}]
```



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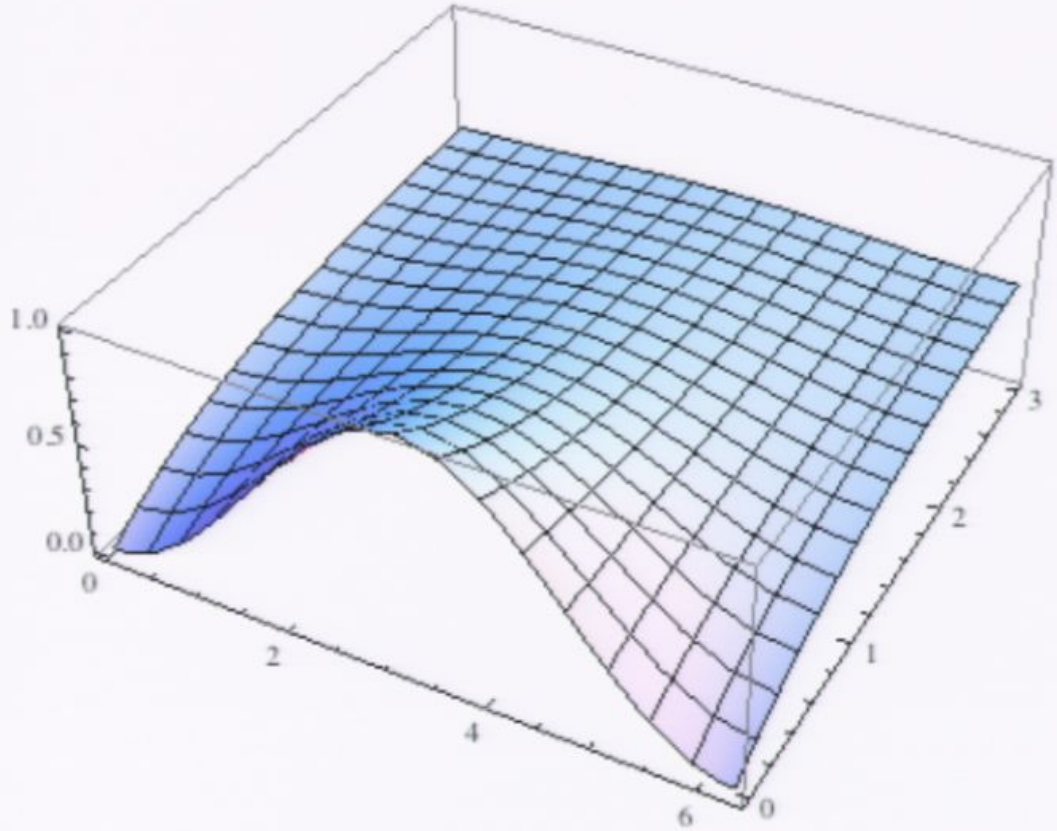
#2.nb

```
Plot3D[M13H[a, t, 1], {a, 0, 2 π}, {t, 0, 3}]
```



$SU(2)$

$\mathcal{H} = \omega \sigma_z$

$SU(2)$

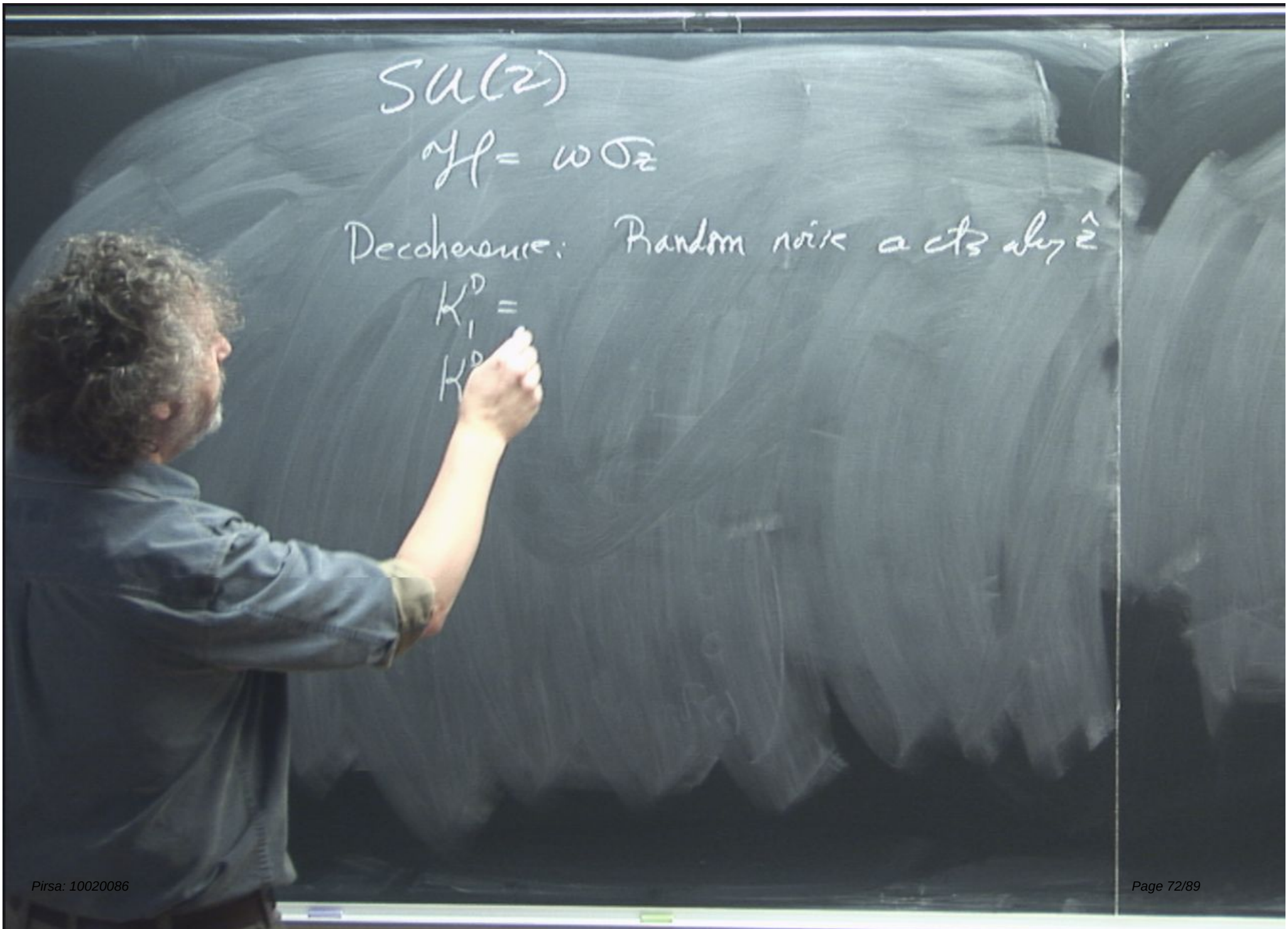
$\mathcal{H} = \omega \sigma_z$

Decoherence:

$SU(2)$

$\mathcal{H} = \omega \sigma_z$

Decoherence: Random noise acts along $\hat{z}$


SU(2)
$\mathcal{H} = \omega \sigma_z$
Decoherence: Random noise acts along $\hat{z}$
$K_1^D =$
K_1^D

$SU(2)$

$\mathcal{H} = \omega \sigma_z$

Decoherence: Random noise acts along $\hat{z}$

$K_1^D = \mathbb{1}$

$K_2^D =$

$SU(2)$

$\mathcal{H} = \omega \sigma_z$

Decoherence: Random noise acts along $\hat{z}$

T_2

$$K_1^D = \left(0.5 + 0.5 e^{-t/T_2}\right)^{1/2} \mathbb{1}$$

$$K_2^D = \left(0.5 - 0.5 e^{-t/T_2}\right)^{1/2} \sigma_z$$

$SU(2)$

$\mathcal{H} = \omega \sigma_z$

Decoherence: Random noise along $\hat{z}$

T_2

$$K_1^D = \left(0.5 + 0.5 e^{-t/T_2}\right)^{1/2} \mathbb{1}$$

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$SU(2)$

$\mathcal{H} = \omega \sigma_z$

Decoherence: Random noise acts along $\hat{z}$

T_2 = rate

$$K_1^D = \left(0.5 + 0.5 e^{-t/T_2}\right)^{1/2} \mathbb{1}$$

$$K_2^D = \left(0.5 - 0.5 e^{-t/T_2}\right)^{1/2} \sigma_z$$

$SU(2)$

$$\mathcal{H} = \omega \sigma_z$$

Decoherence: Random noise acts along $\hat{z}$

$$K_1^D = \left(0.5 + 0.5 e^{-t/T_2}\right)^{1/2} \mathbb{1}$$

$$K_2^D = \left(0.5 - 0.5 e^{-t/T_2}\right)^{1/2} \sigma_z$$

T_2 = rate of loss of coherence

$SU(2)$

$\mathcal{H} = \omega \sigma_z \quad ; \quad \mathcal{U} = e^{-i\omega \sigma_z t/2}$

Decoherence: Random noise acts along $\hat{z}$

$$K_1^D = \left(0.5 + 0.5 e^{-t/T_2}\right)^{1/2} \mathbb{1}$$

T_2 = rate of loss of coherence

$$K_2^D = \left(0.5 - 0.5 e^{-t/T_2}\right)^{1/2} \sigma_z$$

$SU(2)$

$\mathcal{H} = \underline{\omega}\,\sigma_z \quad ; \quad \mathcal{U} = e^{-i\omega\sigma_z t/2}$

Decoherence: Random noise acts along $\hat{z}$

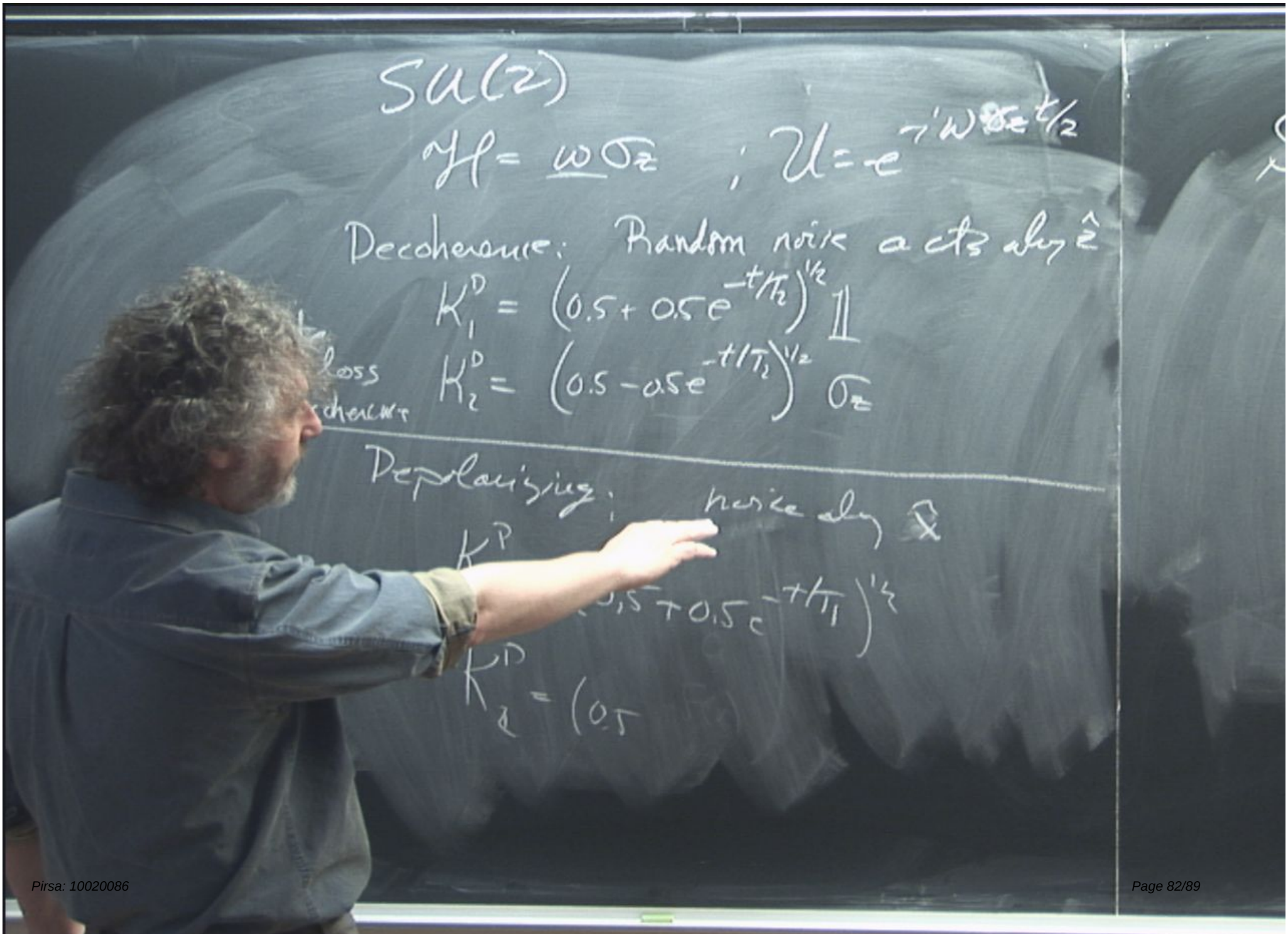
$$K_1^D = \left(0.5 + 0.5\,e^{-t/T_2}\right)^{1/2} \mathbb{1}$$

$$K_2^D = \left(0.5 - 0.5\,e^{-t/T_2}\right)^{1/2} \sigma_z$$

$= T_2 =$ rate of loss of coherence

$$S_{ij}^{D} = \mathrm{Tr}\left\{ P_i \sum_{k=1}^{2} K_k^{D}(t)\, \mathcal{U}(t)\, P_j\, \mathcal{U}(-t)\, K_k^{D\dagger}(t) \right\}$$

$$S_{ij}^{D} = \mathrm{Tr}\left\{ P_i \sum_{k=1}^{2} K_k^{D}(t)\, \mathcal{U}(t)\, P_j\, \mathcal{U}(-t)\, K_k^{D\dagger}(t) \right\}$$


SU(2)
$\mathcal{H} = \omega \sigma_z$; $\mathcal{U} = e^{-i\omega \sigma_z t/2}$
Decoherence: Random noise acts along $\hat{z}$
$K_1^D = (0.5 + 0.5 e^{-t/T_2})^{1/2} \mathbb{1}$
$K_2^D = (0.5 - 0.5 e^{-t/T_2})^{1/2} \sigma_z$
loss
Depolarizing: noise along $\hat{x}$
K^P
$(0.5 + 0.5 e^{-t/T_1})^{1/2}$
$K_2^D = (0.5$

$SU(2)$

$\mathcal{H} = \underline{\omega}\,\sigma_z \quad ; \quad \mathcal{U} = e^{-i\omega\sigma_z t/2}$

Decoherence: Random noise acts along $\hat{z}$

$$K_1^D = \left(0.5 + 0.5\,e^{-t/T_2}\right)^{1/2} \mathbb{1}$$

$$K_2^D = \left(0.5 - 0.5\,e^{-t/T_2}\right)^{1/2} \sigma_z$$

$= T_2 =$ rate of loss of coherence

Depolarizing: noise along $\hat{x}$

$$K_1^D = \left(0.5 + 0.5\,e^{-t/T_1}\right)^{1/2} \mathbb{1}$$

$$K_2^D = \left(0.5 - 0.5\,e^{-t/T_1}\right)^{1/2} \sigma_x$$

$SU(2)$

$$\mathcal{H} = \underline{\omega}\,\sigma_z \quad ; \quad \mathcal{U} = e^{-i\omega\sigma_z t/2}$$

Decoherence: Random noise acts along $\hat{z}$

$$K_1^D = \left(0.5 + 0.5\,e^{-t/T_2}\right)^{1/2} \mathbb{1}$$

$$K_2^D = \left(0.5 - 0.5\,e^{-t/T_2}\right)^{1/2} \sigma_z$$

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Depolarizing: noise along $\hat{x}$

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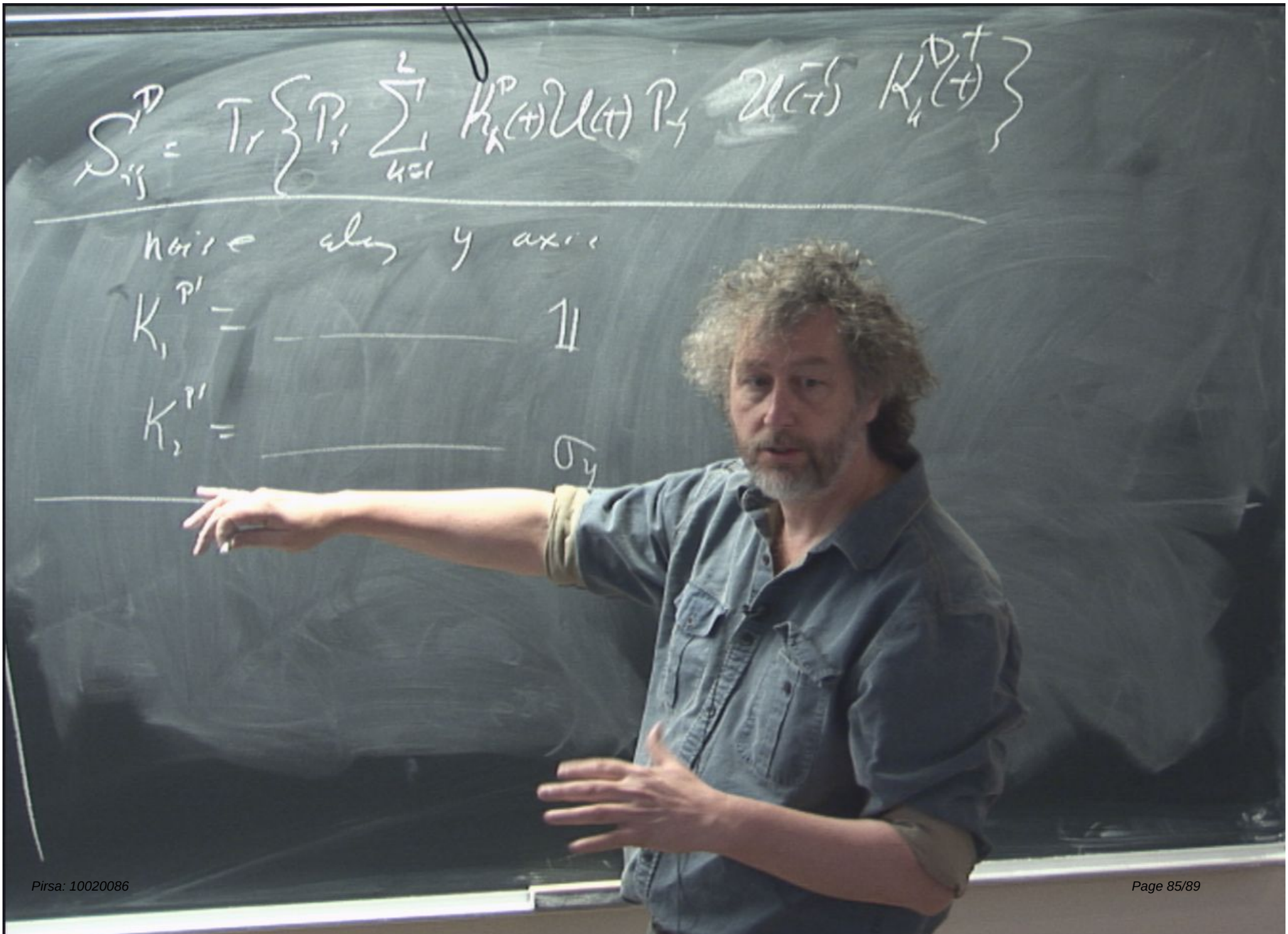
$S_{ij}^{D} = \mathrm{Tr}\left\{ P_i \sum_{k=1}^{2} K_k^{D}(t)\, \mathcal{U}(t)\, P_j\, \mathcal{U}(-t)\, K_k^{D\dagger}(t) \right\}$
noise along y axis
$K_1^{D'} = \mathbb{1}$
$K_2^{D'} = \sigma_y$

$$S_{ij}^{D} = \mathrm{Tr}\left\{ P_i \sum_{k=1}^{2} K_k^{D}(t)\, \mathcal{U}(t)\, P_j\, \mathcal{U}(-t)\, K_k^{D\dagger}(t) \right\}$$

noise along y axis

$K_1^{D'} = _____ \; \mathbb{1}$

$K_2^{D'} = _____ \; \sigma_y$

noise acts along $x \pm y$

$$S_{ij}^{D} = \mathrm{Tr}\left\{ P_i \sum_{k=1}^{L} K_k^{D}(t)\, \mathcal{U}(t)\, P_j\, \mathcal{U}(-t)\, K_k^{D\dagger}(t) \right\}$$

noise along y axis

$K_1^{D'} =$ _____ $\mathbb{1}$

$K_2^{D'} =$ _____ σ_y

noise acts along x & y

$K_1 =$ $\mathbb{1}$

$K_2 =$ σ_x

$K_3 =$ σ_y

$K_4 =$ σ_z

$$S_{ij}^{D} = \mathrm{Tr}\left\{ P_i \sum_{k=1}^{2} K_k^{D}(t)\, \mathcal{U}(t)\, P_j\, \mathcal{U}(-t)\, K_k^{D\dagger}(t) \right\}$$

noise along y axis

$K_1^{D'} =$ _____ $\mathbb{1}$

$K_2^{D'} =$ _____ σ_y

noise acts along $x \pm y$

$K_1 =$ $\mathbb{1}$

$K_2 =$ σ_x

$K_3 =$ σ_y

$K_4 =$ σ_z

$$S_{ij}^{D} = \mathrm{Tr}\left\{ P_i \sum_{k=1}^{2} K_k^{D}(t)\, \mathcal{U}(t)\, P_j\, \mathcal{U}^{\dagger}(t)\, K_k^{D\dagger}(t) \right\}$$

noise along y axis

$K_1^{D'} = _____ \; \mathbb{1}$

$K_2^{D'} = _____ \; \sigma_y$

noise acts along x & y

$K_1 = \sqrt{1-3P} \quad \mathbb{1}$

$K_2 = \sqrt{P} \quad \sigma_x$

$K_3 = \sqrt{P} \quad \sigma_y$

$K_4 = \sqrt{P} \quad \sigma_z$