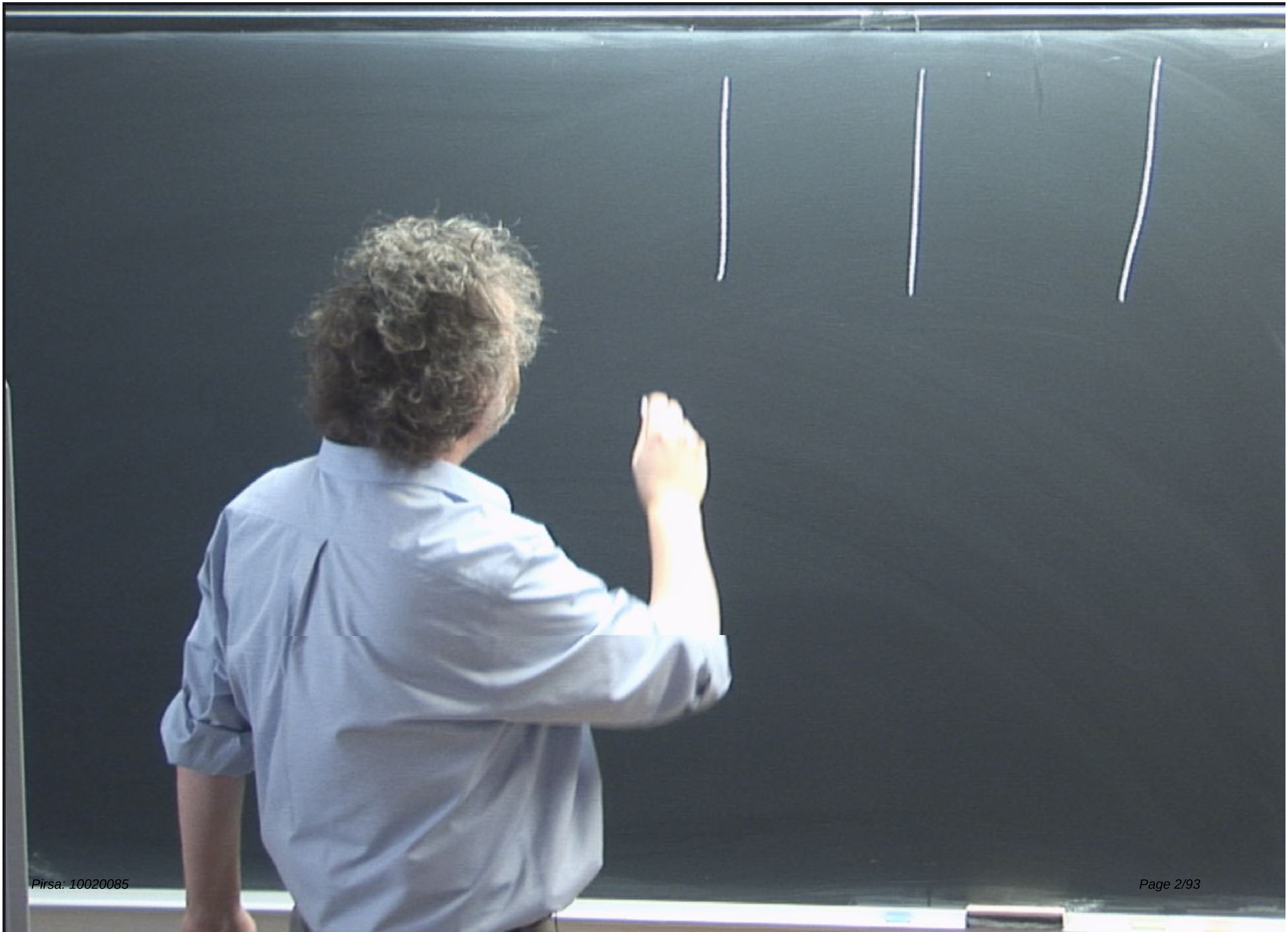
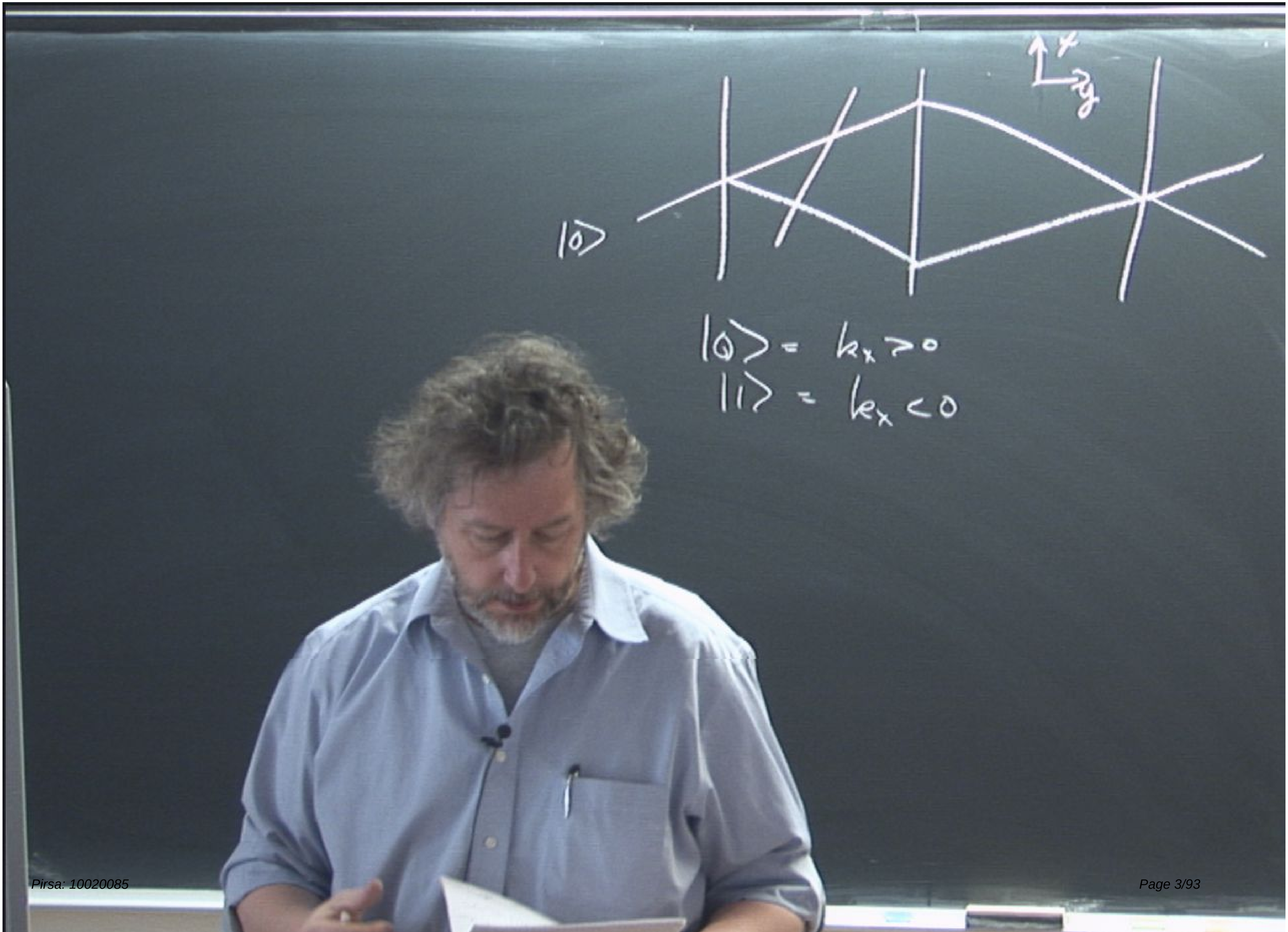
Title: Explorations in Quantum Info. (PHYS 641) - Lecture 2

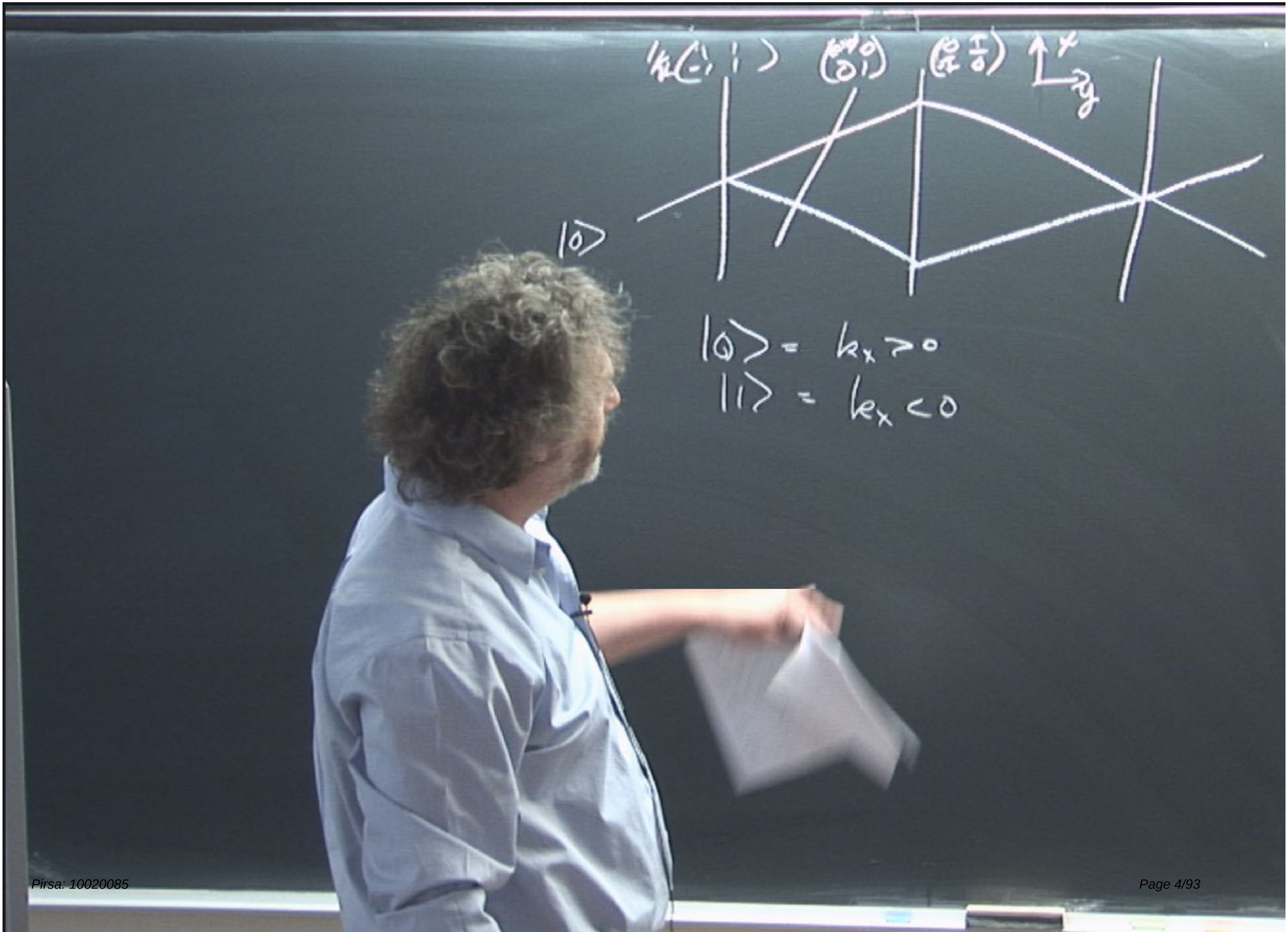
Date: Feb 17, 2010 09:00 AM

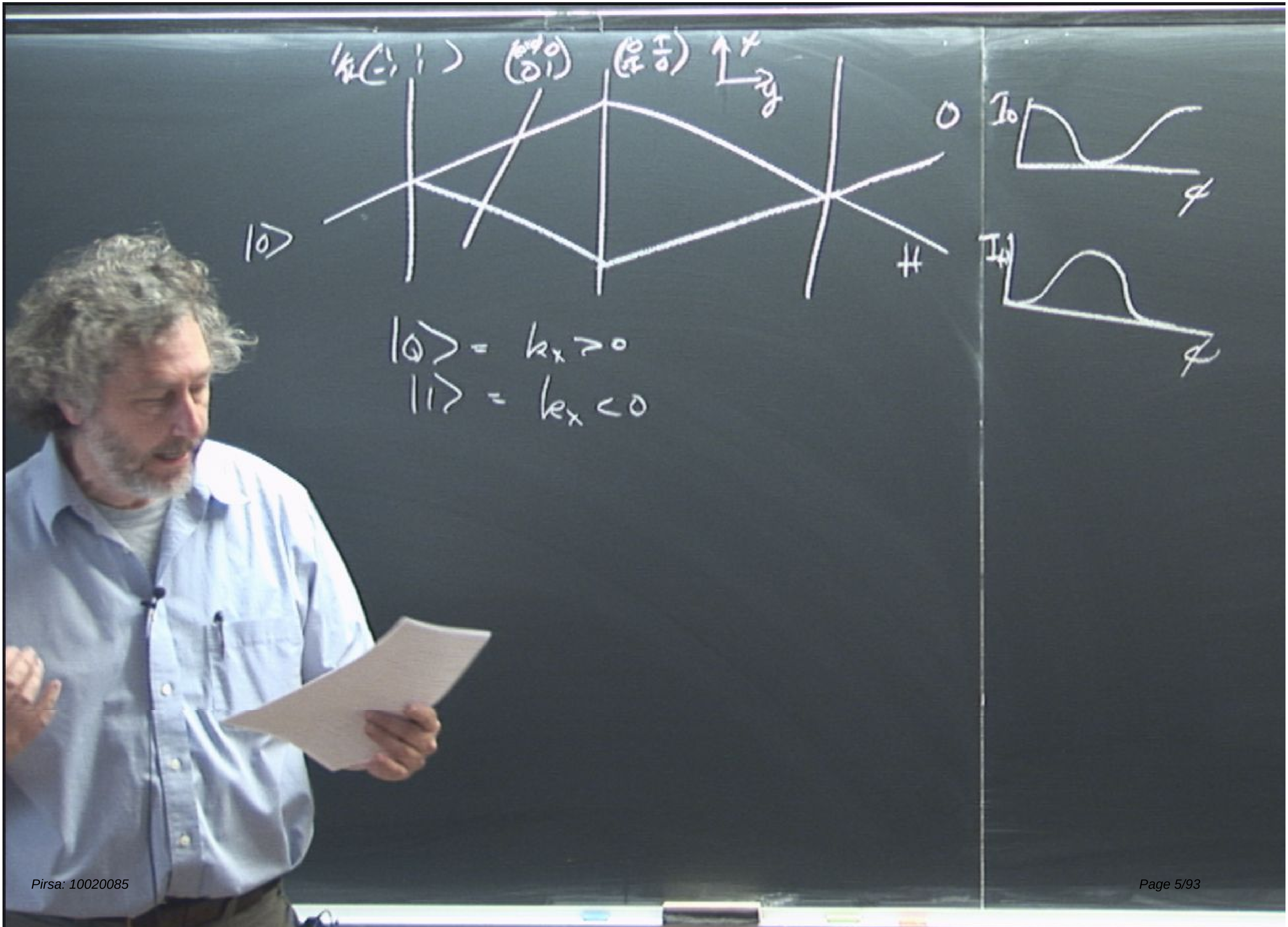
URL: http://www.pirsa.org/10020085

Abstract:

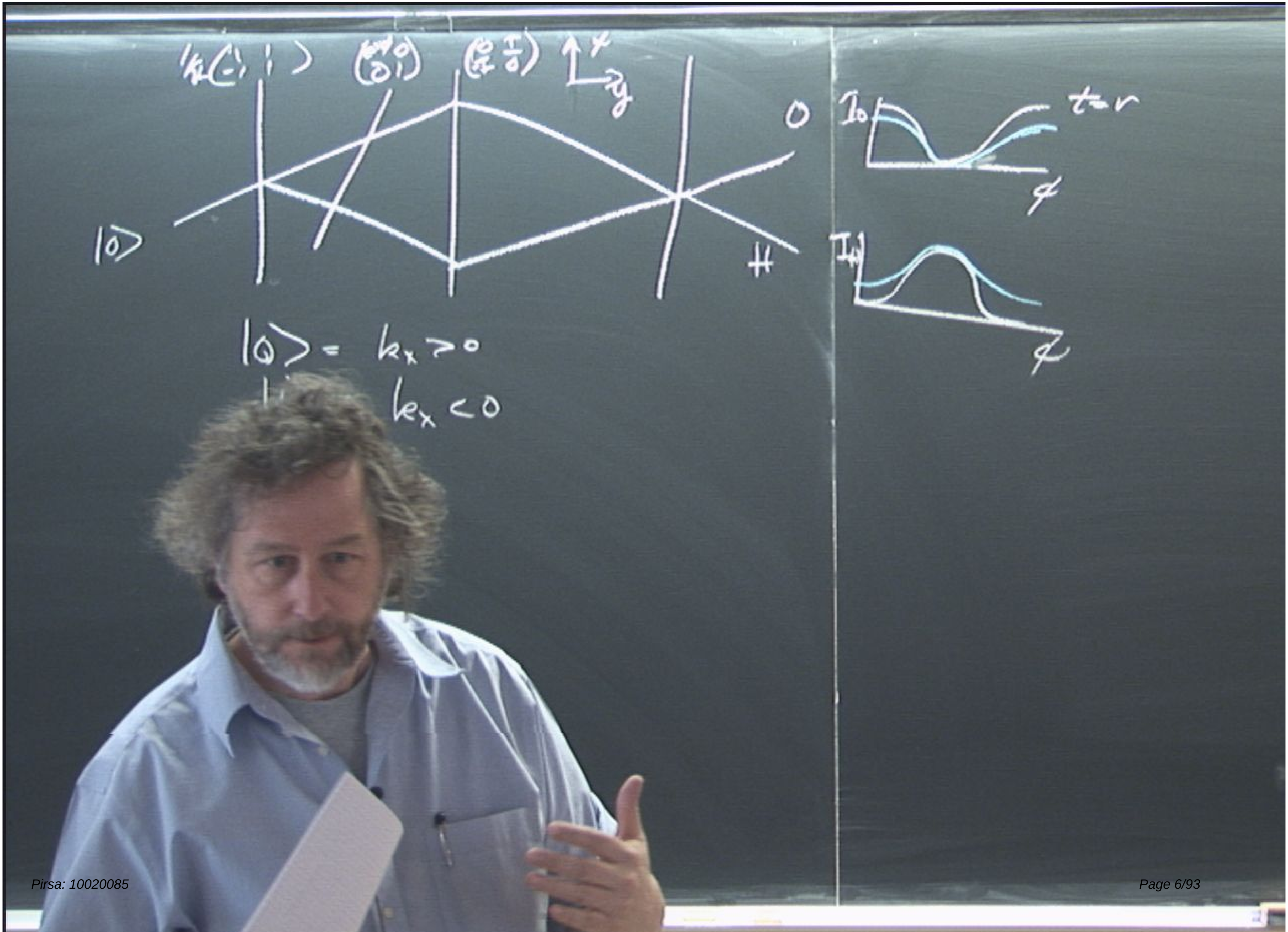


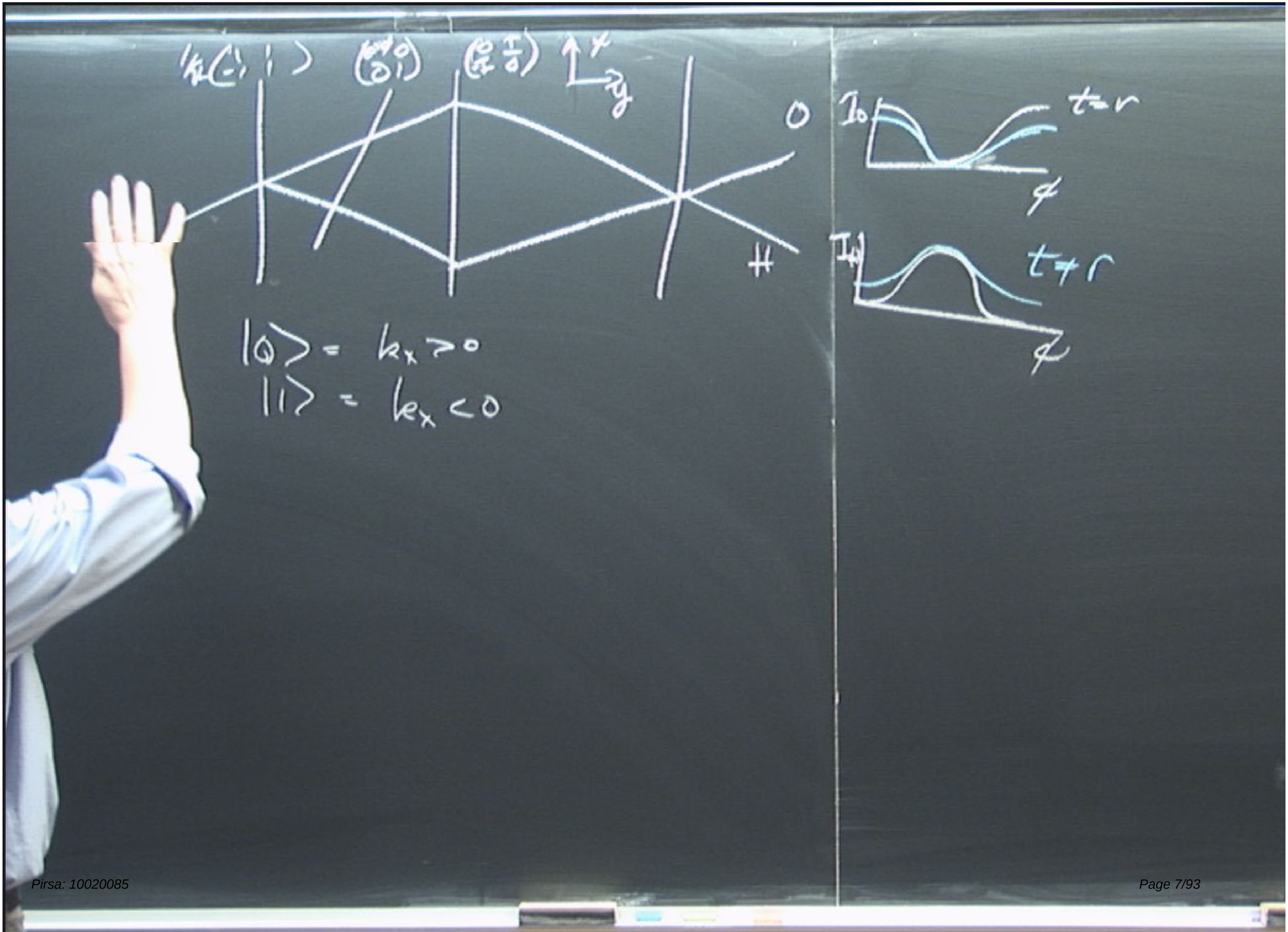

x
y
|0>
|0> = k_x > 0
|1> = k_x < 0


|0>
|0> = kx > 0
|1> = kx < 0

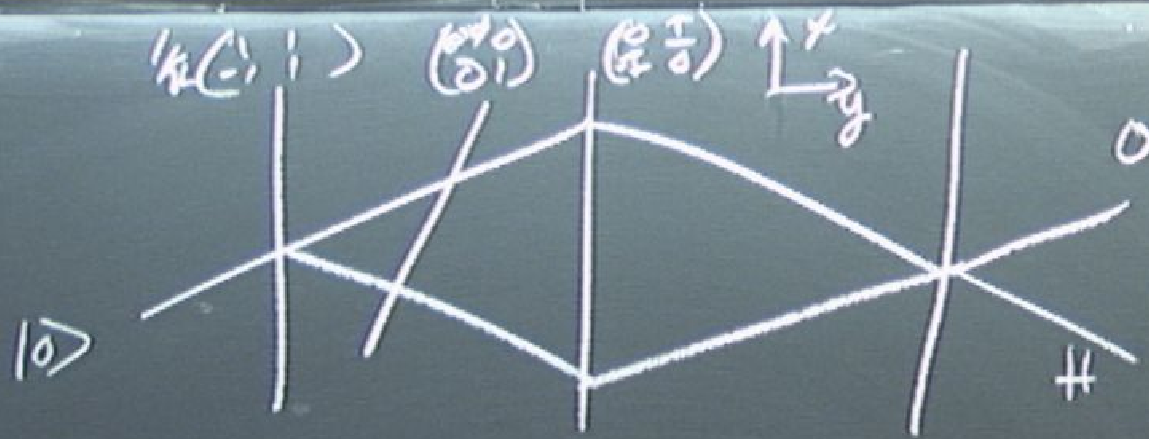


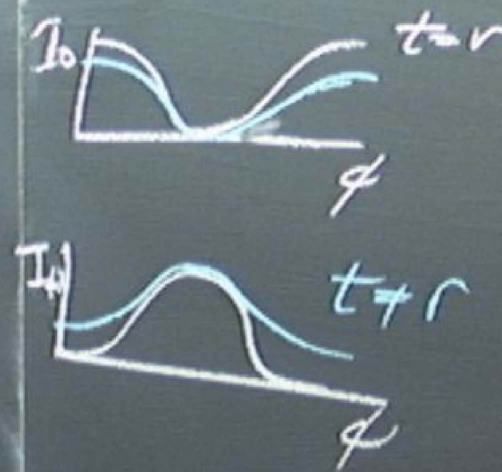
|0>
|0> = kx > 0
|1> = kx < 0
I0
φ


|0⟩
0
H
t=r
|0⟩ = k_x > 0
k_x < 0



|0> = k_x > 0
|1> = k_x < 0
t=r
t≠r

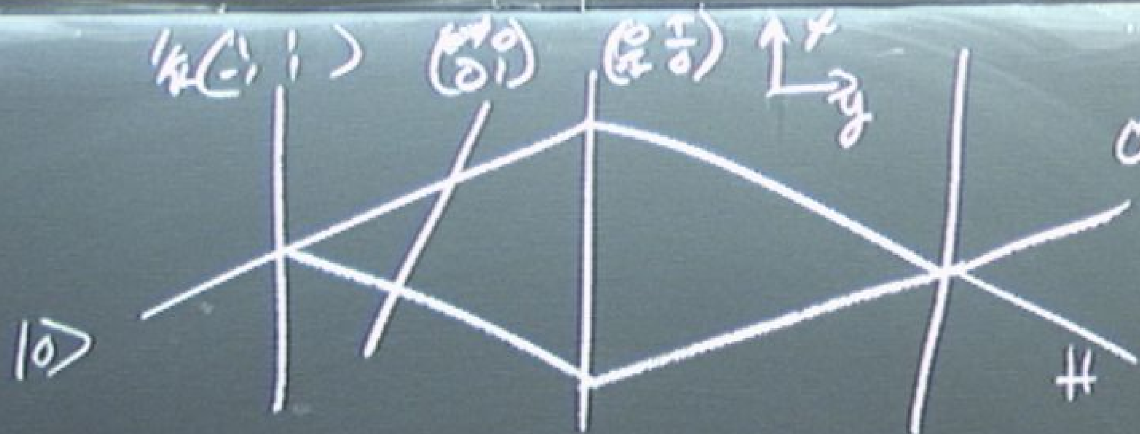


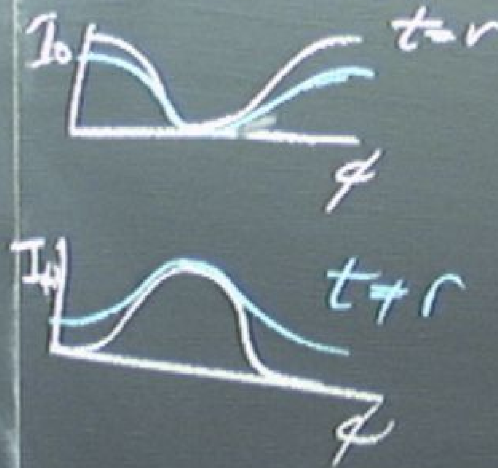


$|0\rangle = k_x > 0$

$|1\rangle = k_x < 0$

Describe as a map



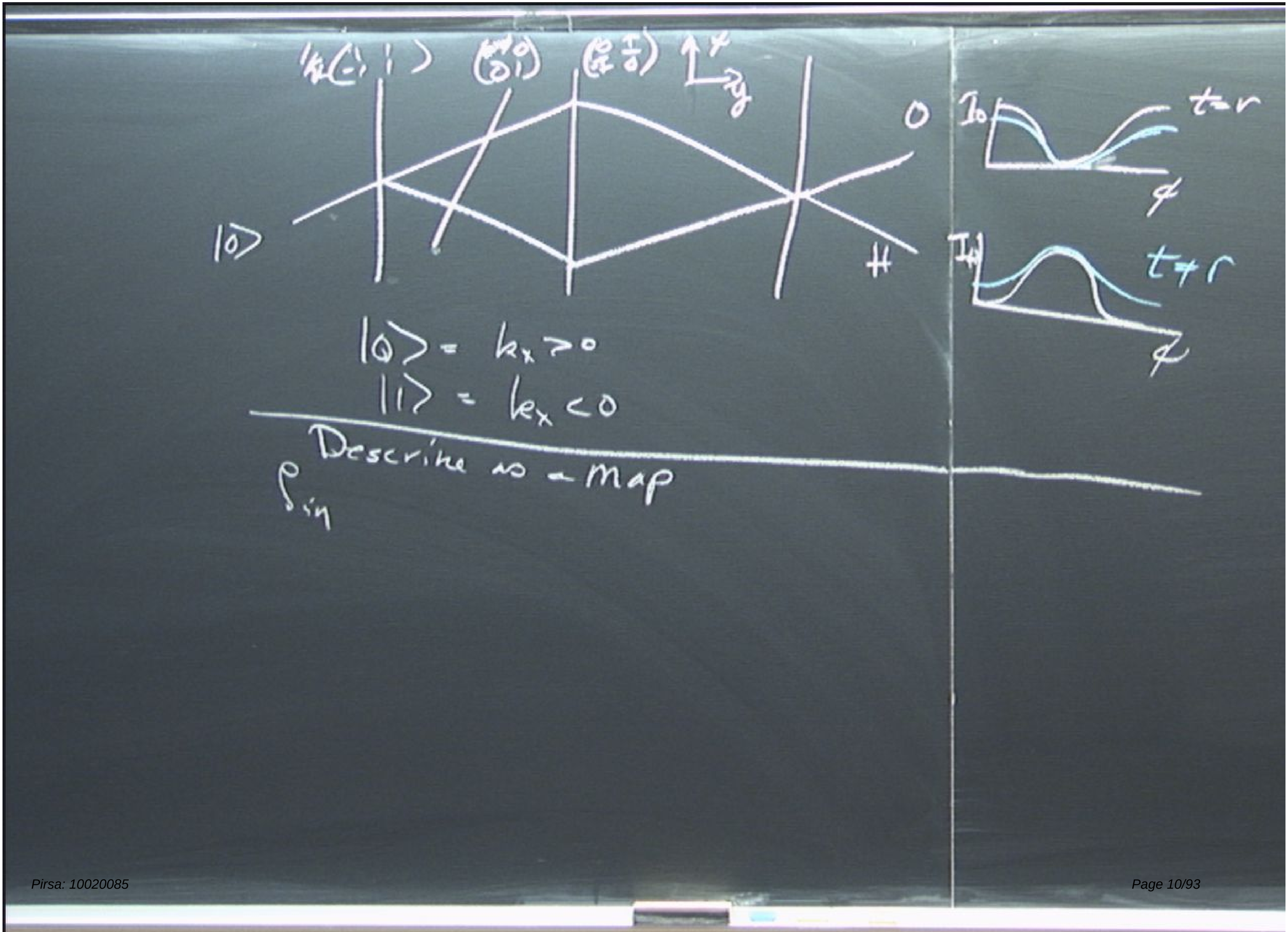


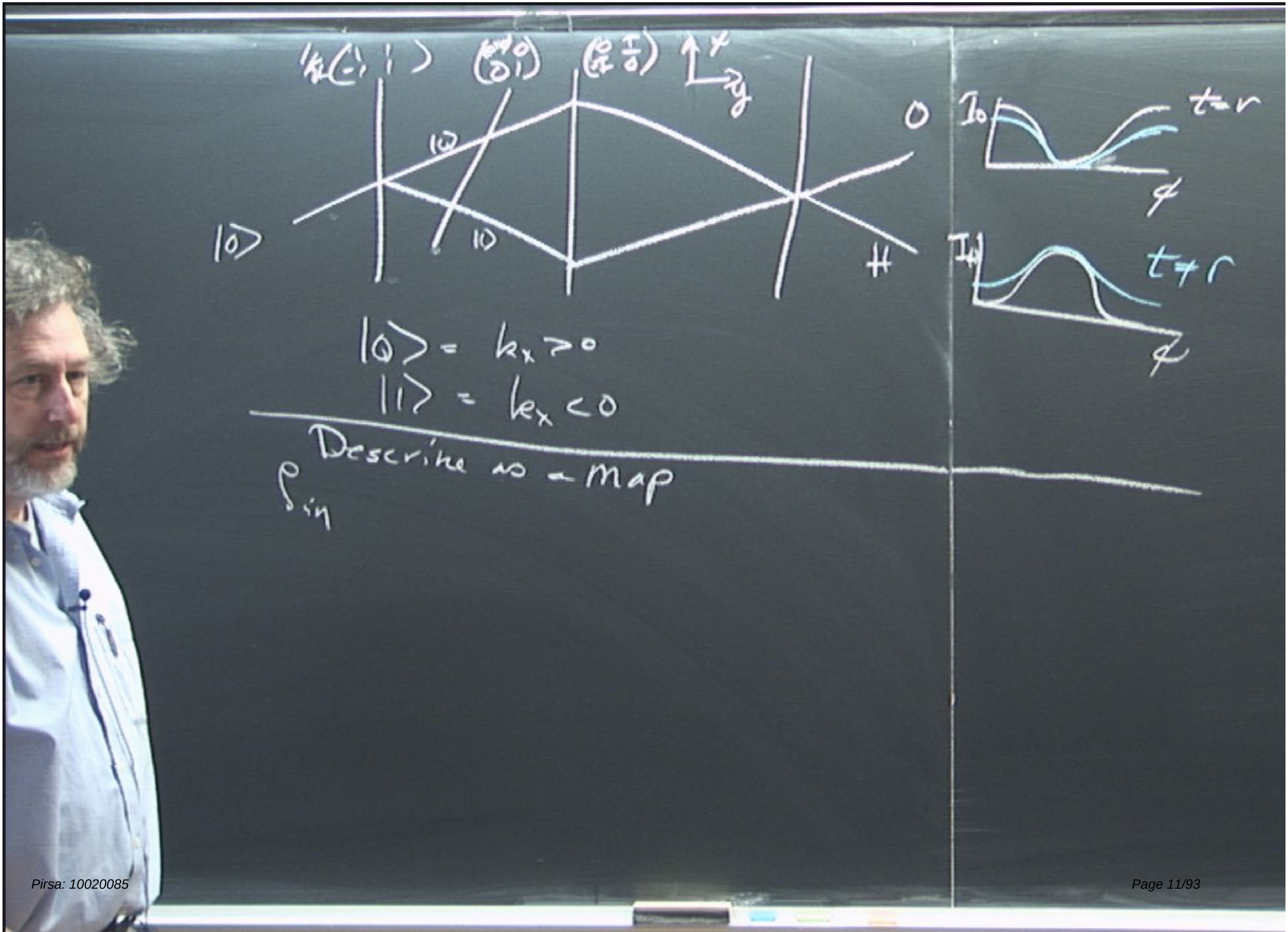
$|0\rangle = k_x > 0$

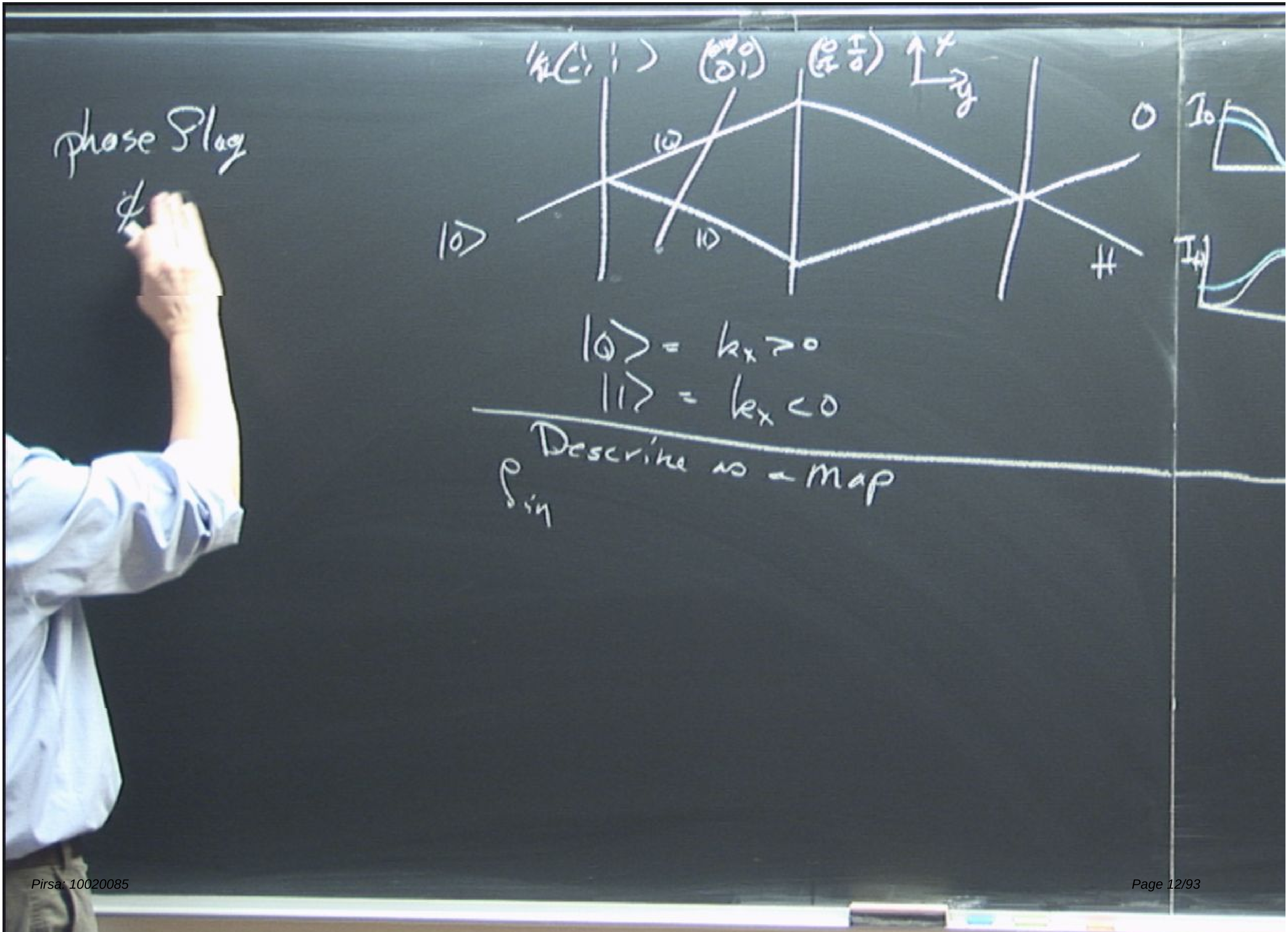
$|1\rangle = k_x < 0$

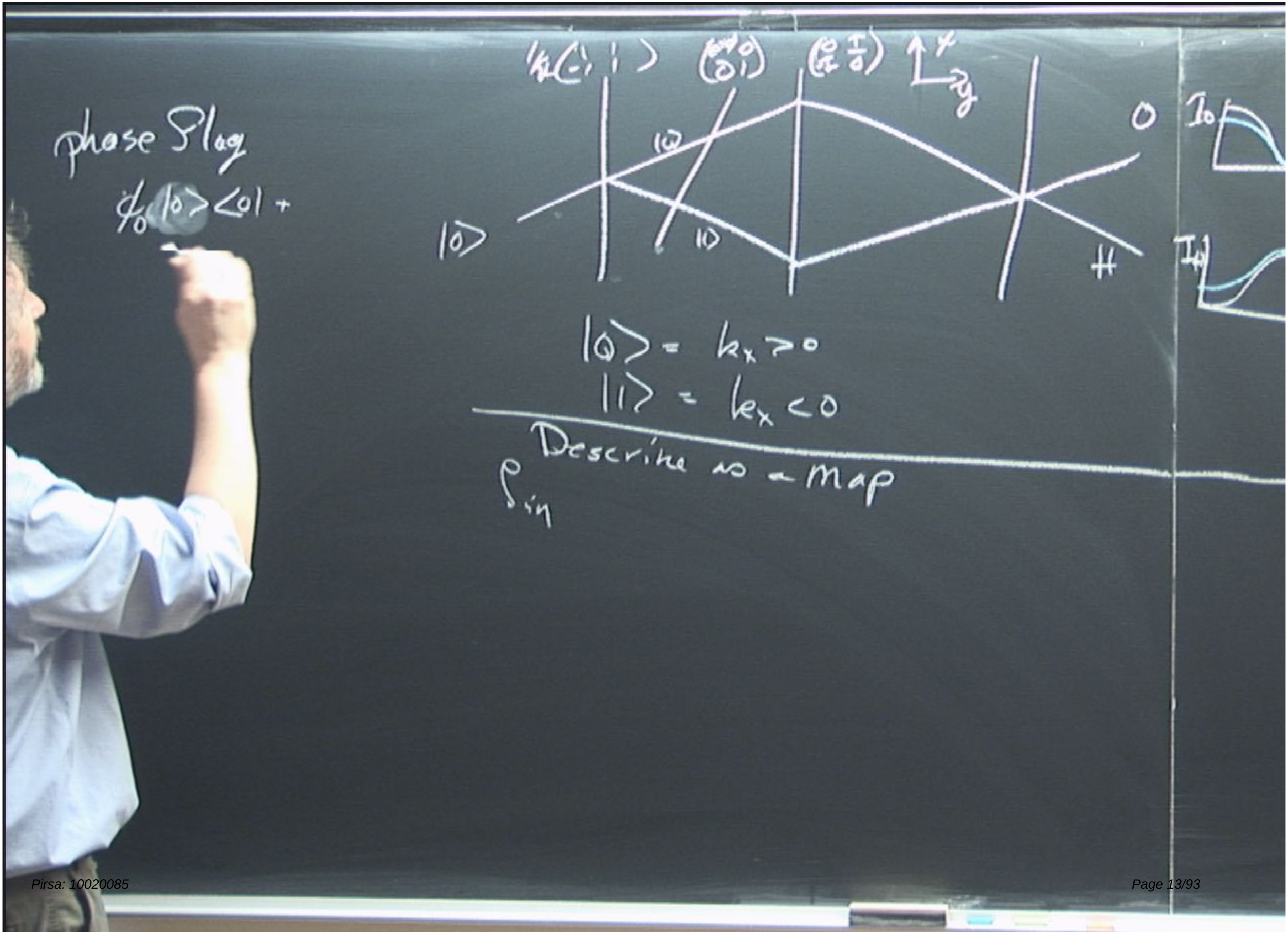
Describe as a map

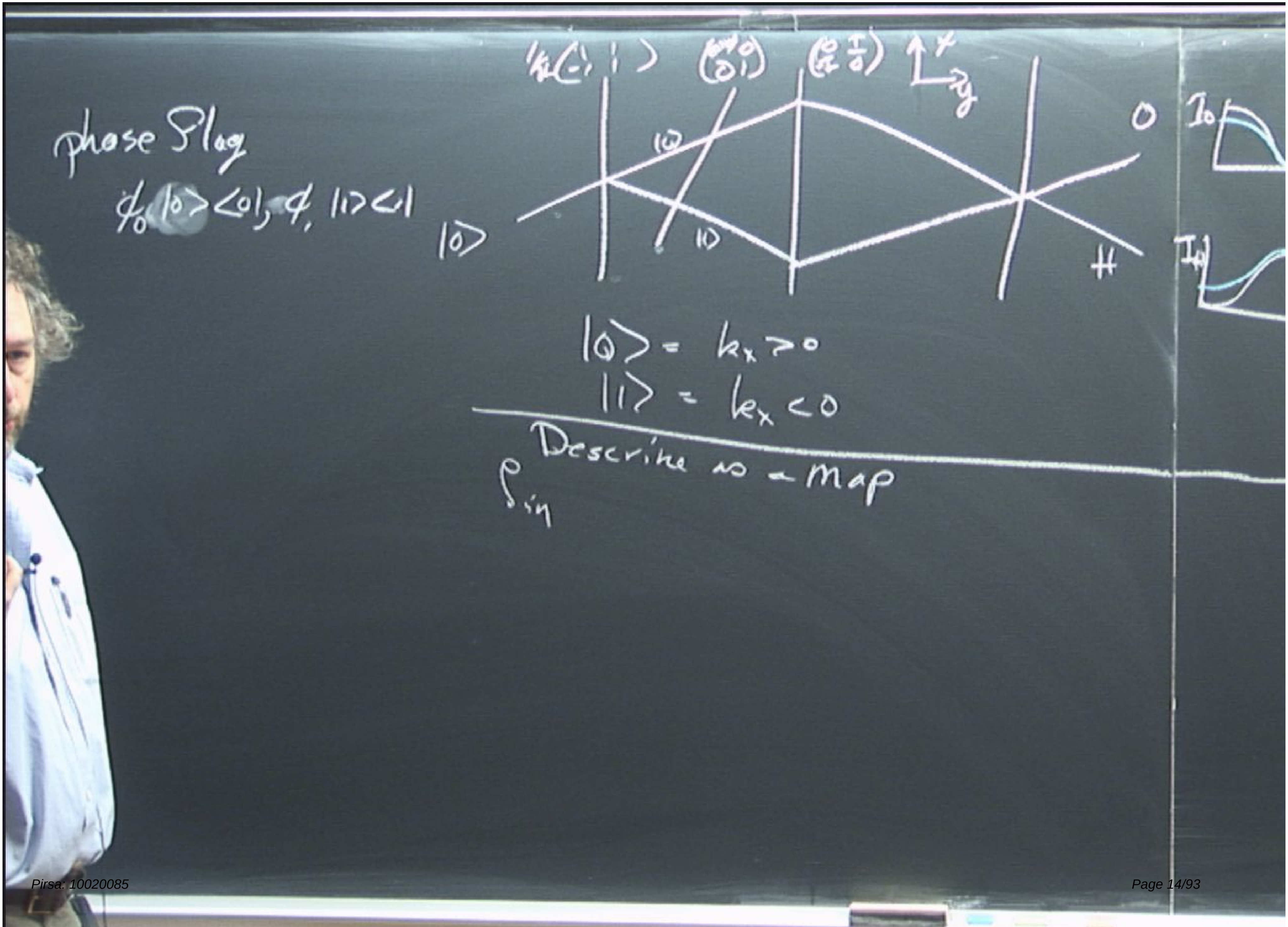
ρ_{in}

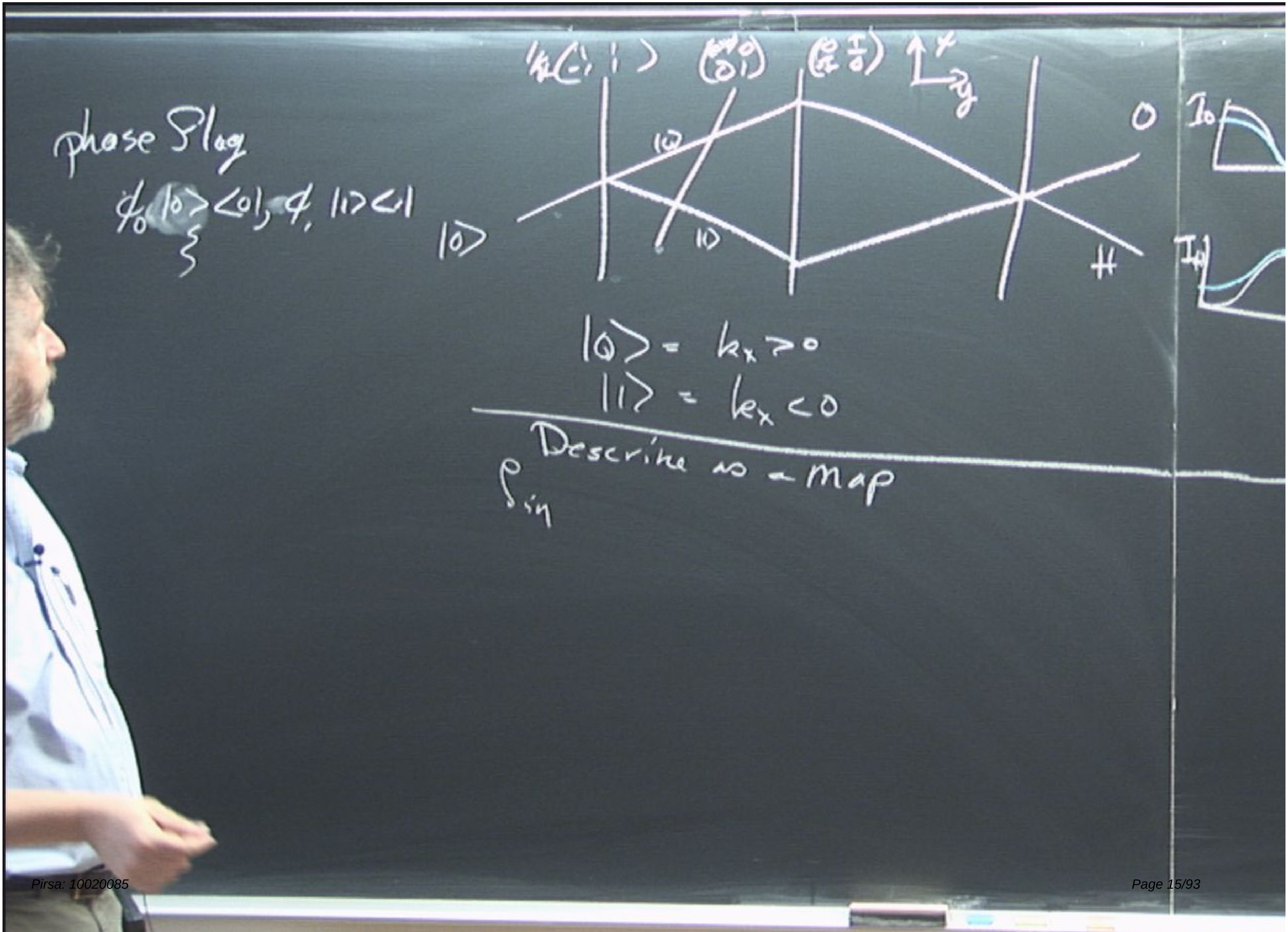

|0> = kx > 0
|1> = kx < 0
Describe as a map
ρin
t=r
t≠r
|0>

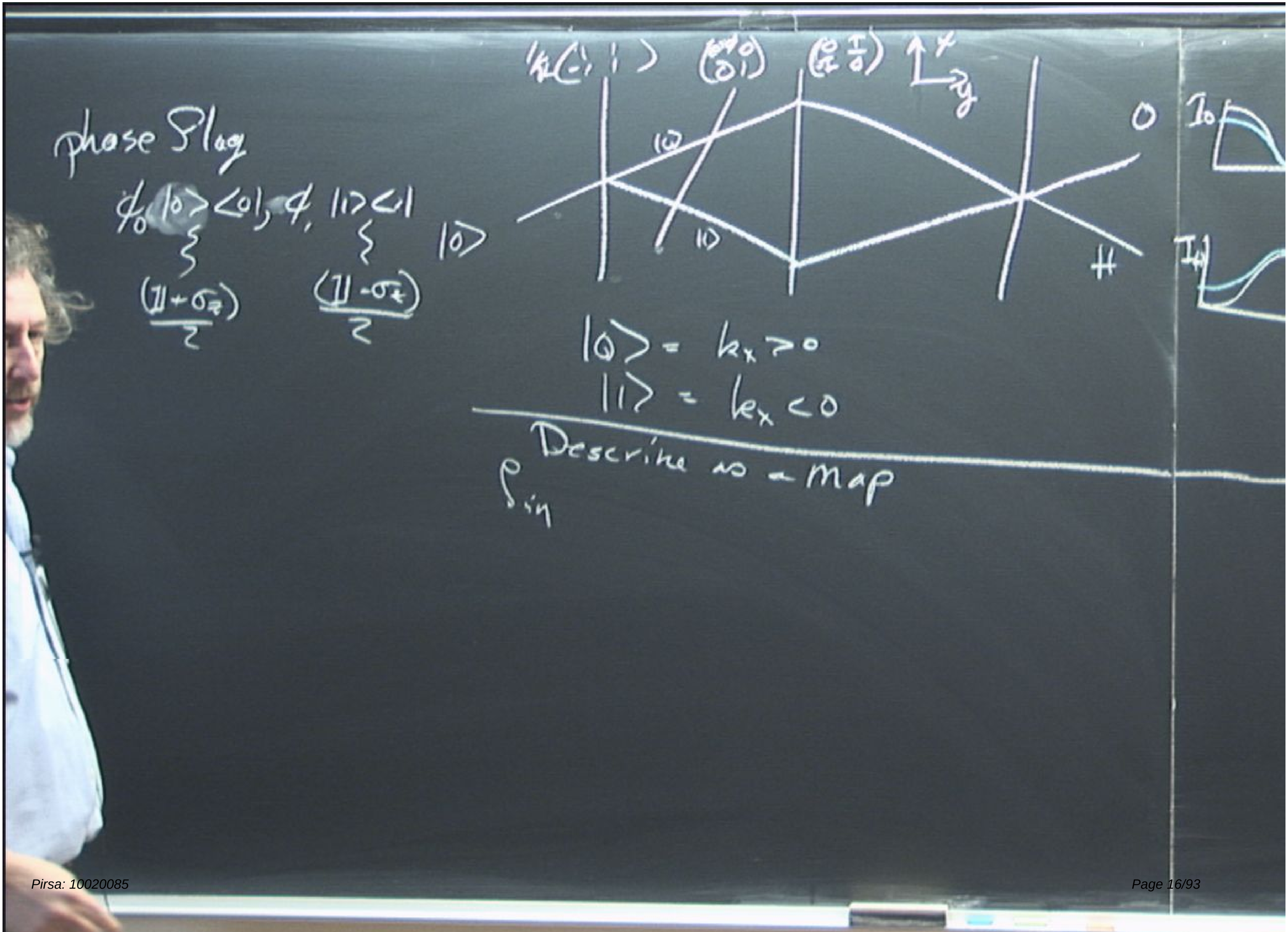

|0⟩ = k_x > 0
|1⟩ = k_x < 0
Describe as a map
ρ_in
t=r
t≠r

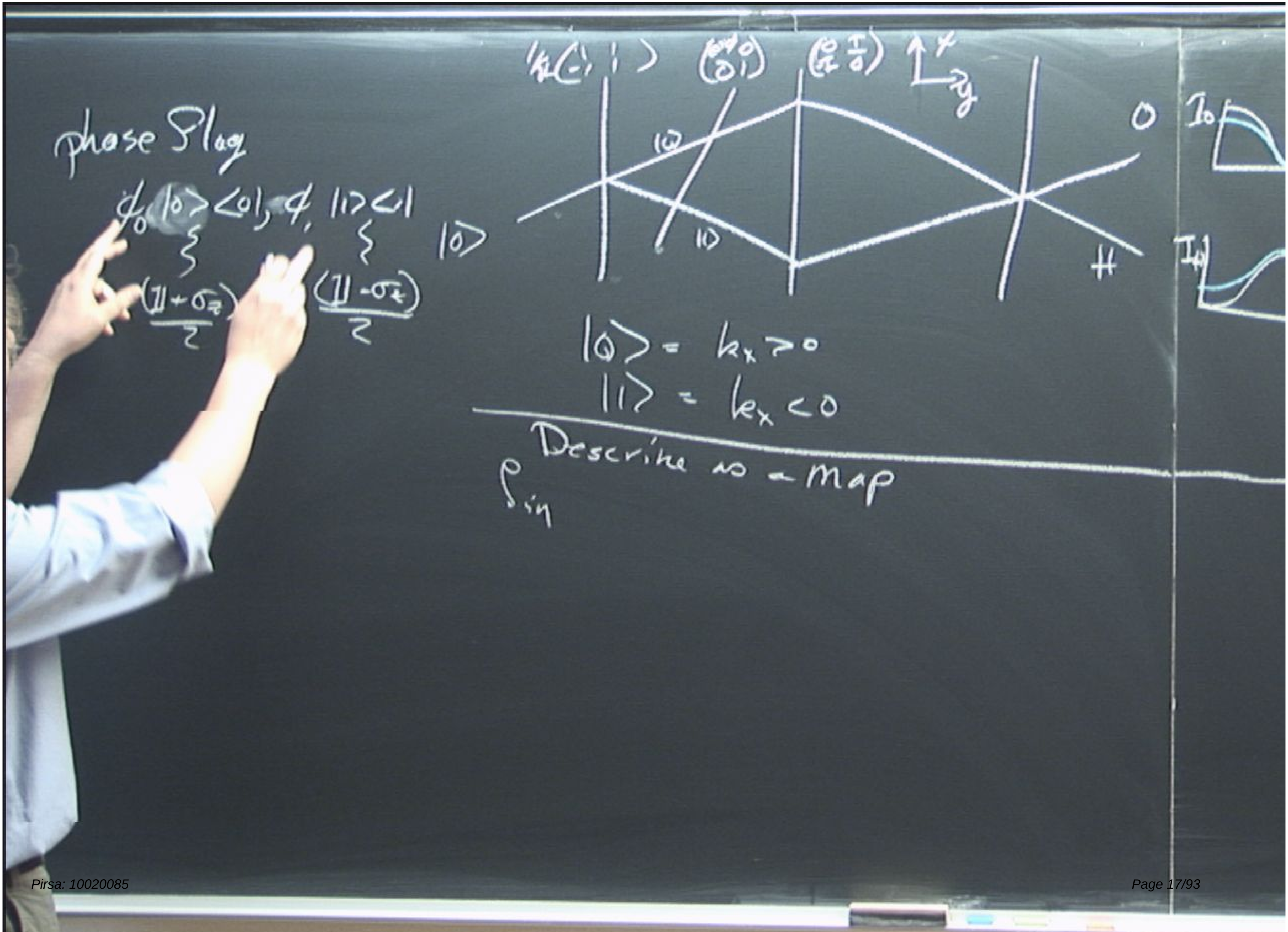

phase flag
|0>
|1>
H
|0> = kx > 0
|1> = kx < 0
Describe as a map
ρin

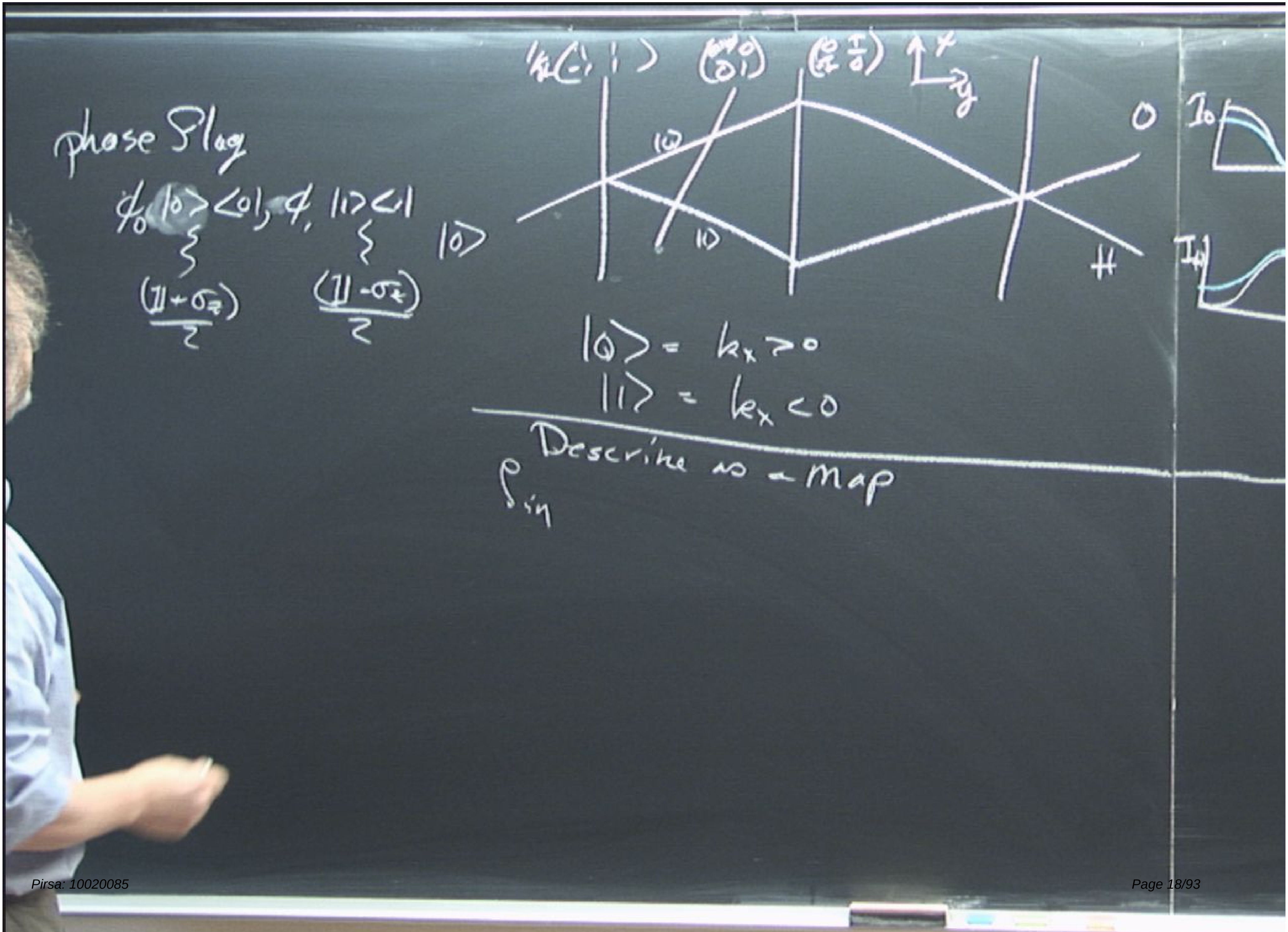

phase flag
|0> = kx > 0
|1> = kx < 0
Describe as a map

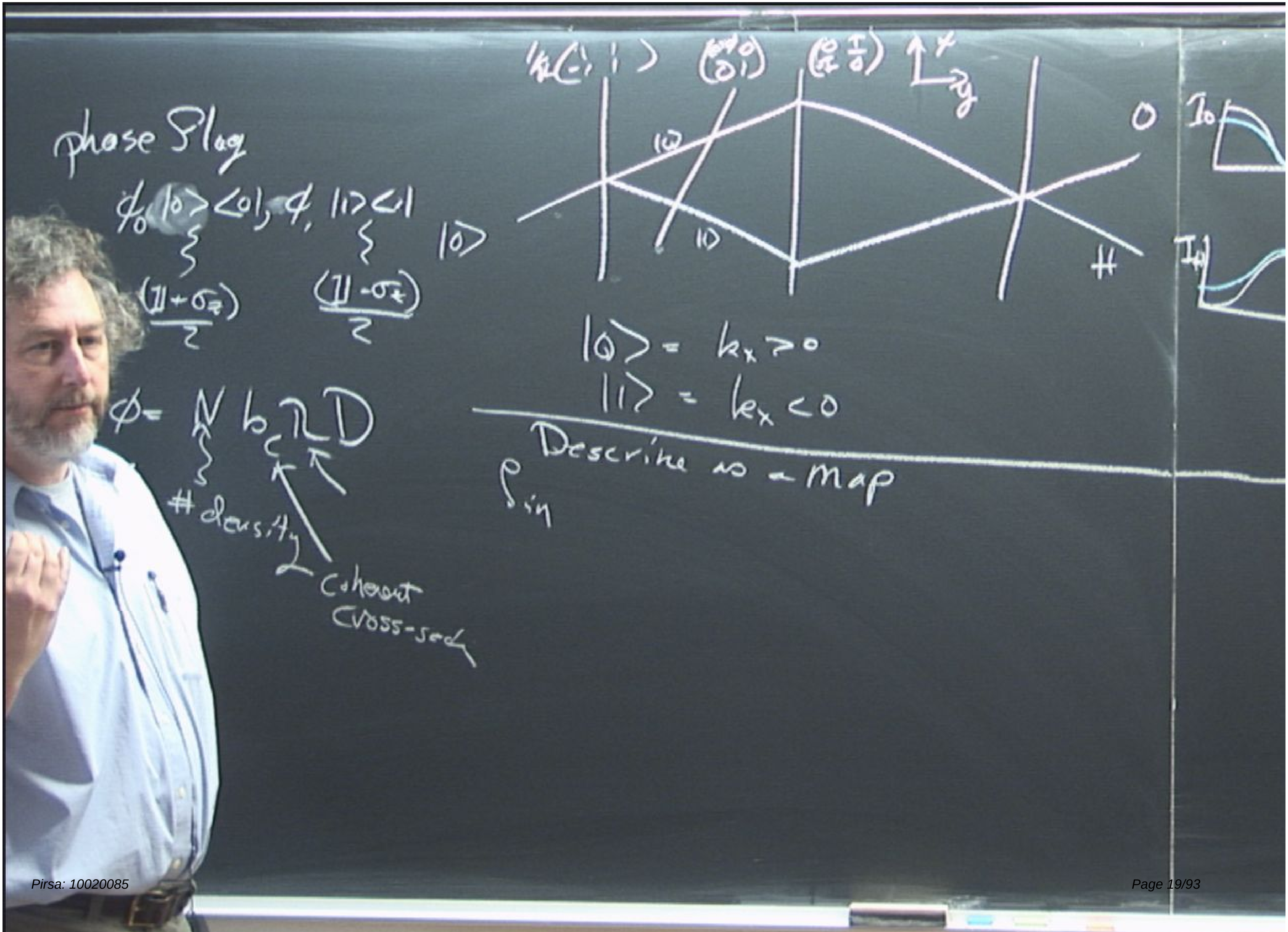

phase flag
|0⟩ = kx > 0
|1⟩ = kx < 0
Describe as a map

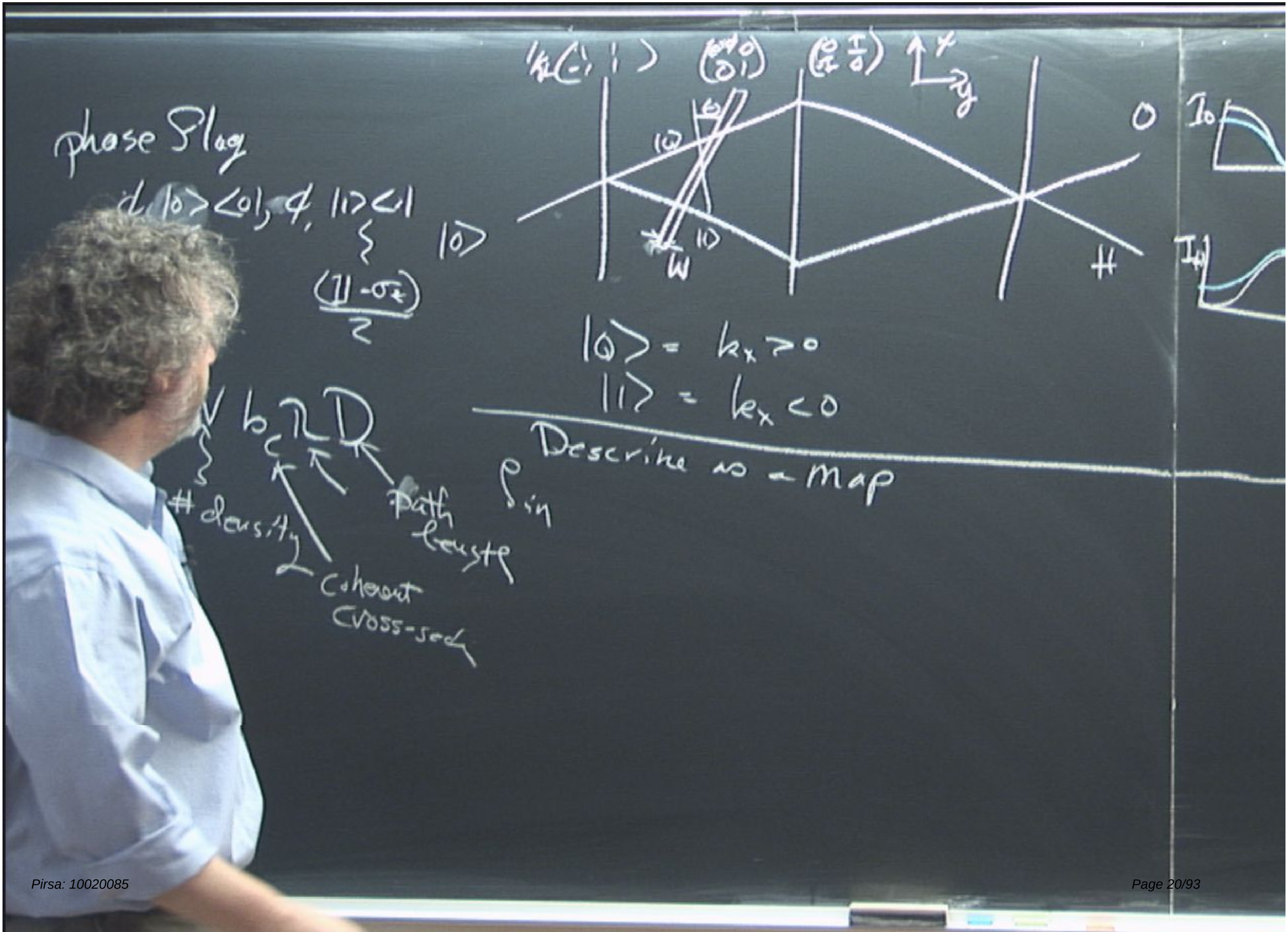

phase flag
|0> = kx > 0
|1> = kx < 0
Describe as a map
ρin

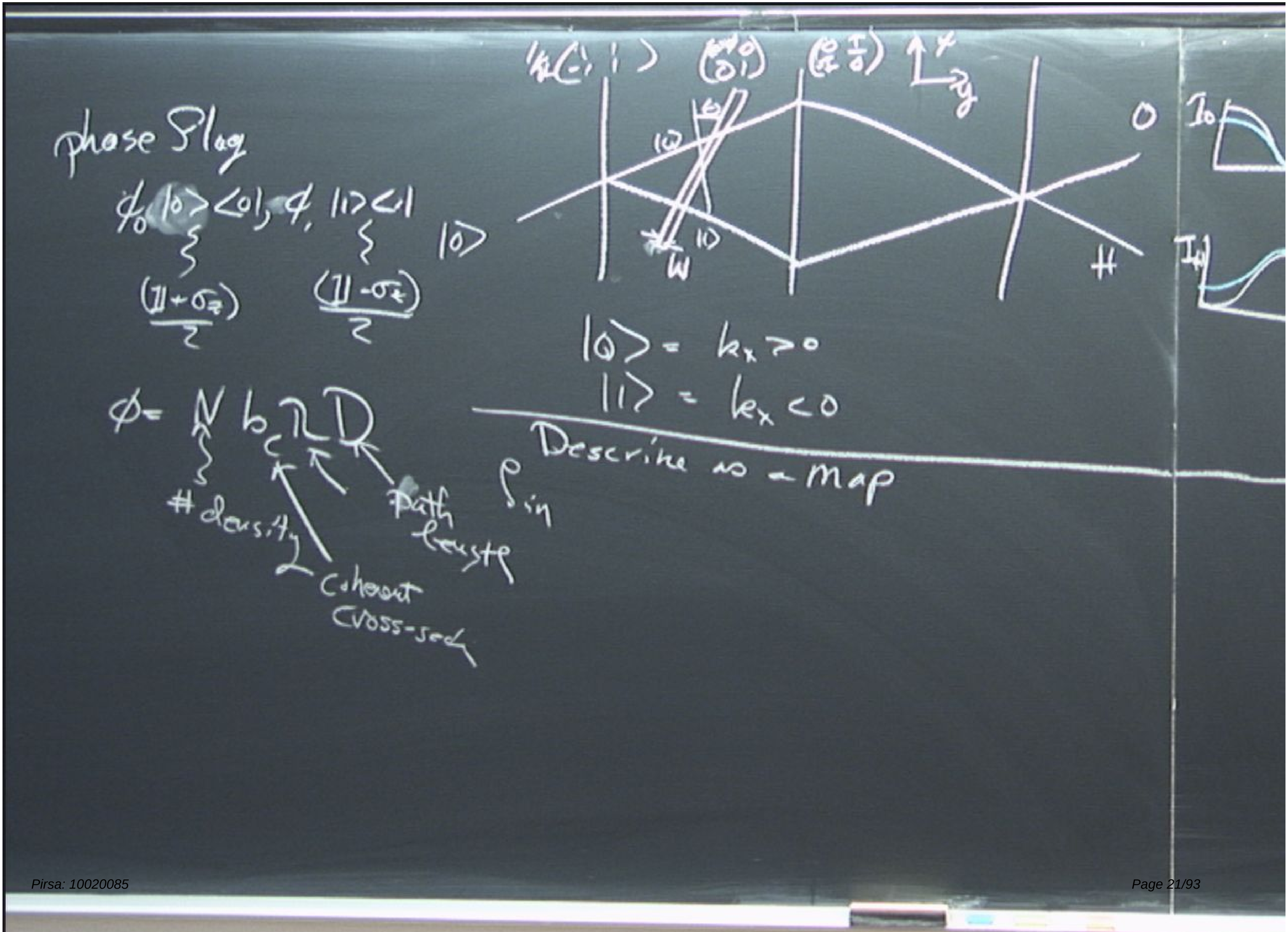

phase flag
|0> = kx > 0
|1> = kx < 0
Describe as a map
ρin

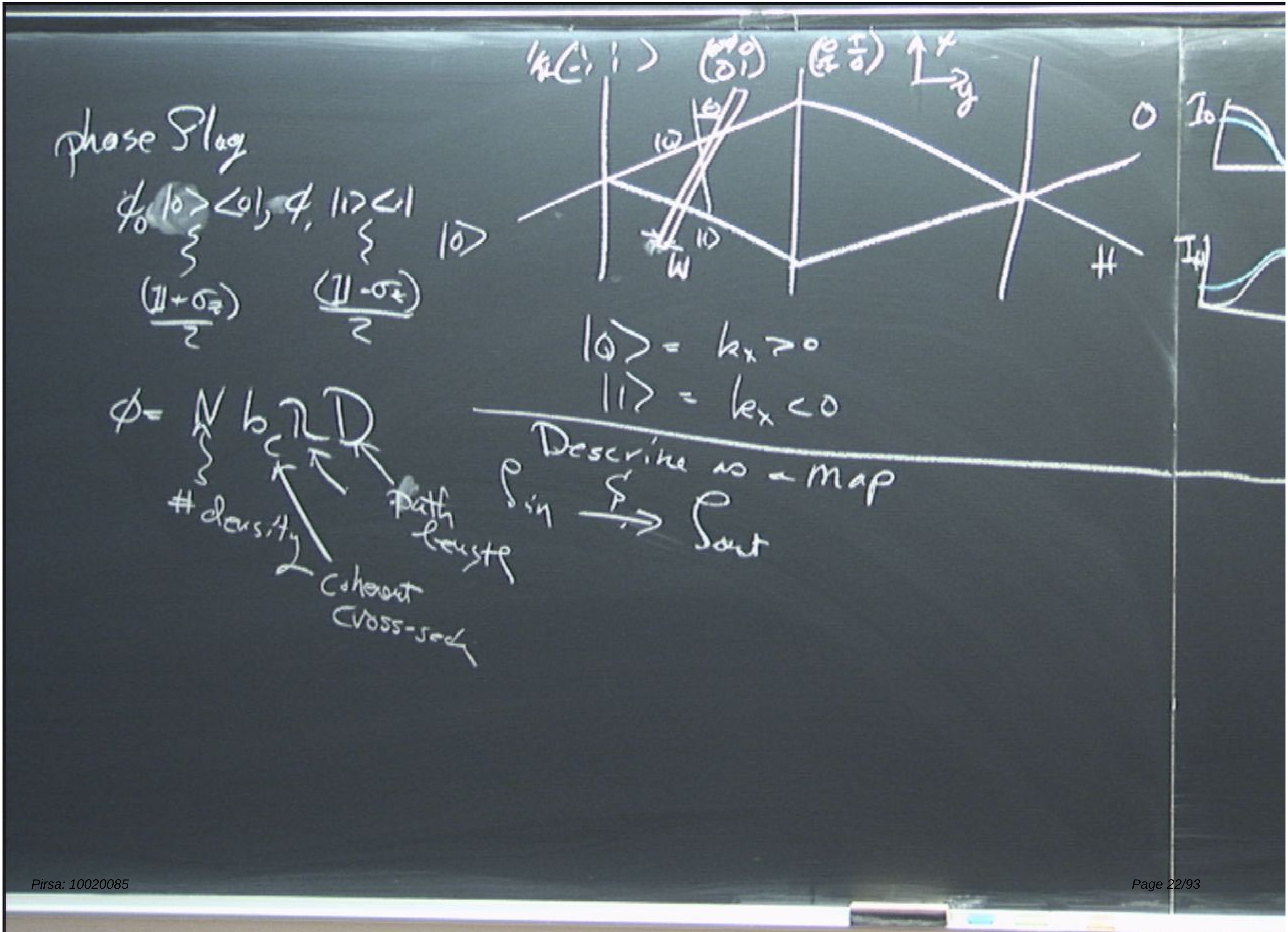

phase flag
|0⟩<0|
|1⟩<1|
|0⟩
0
|0⟩ = kx > 0
|1⟩ = kx < 0
Describe as a map

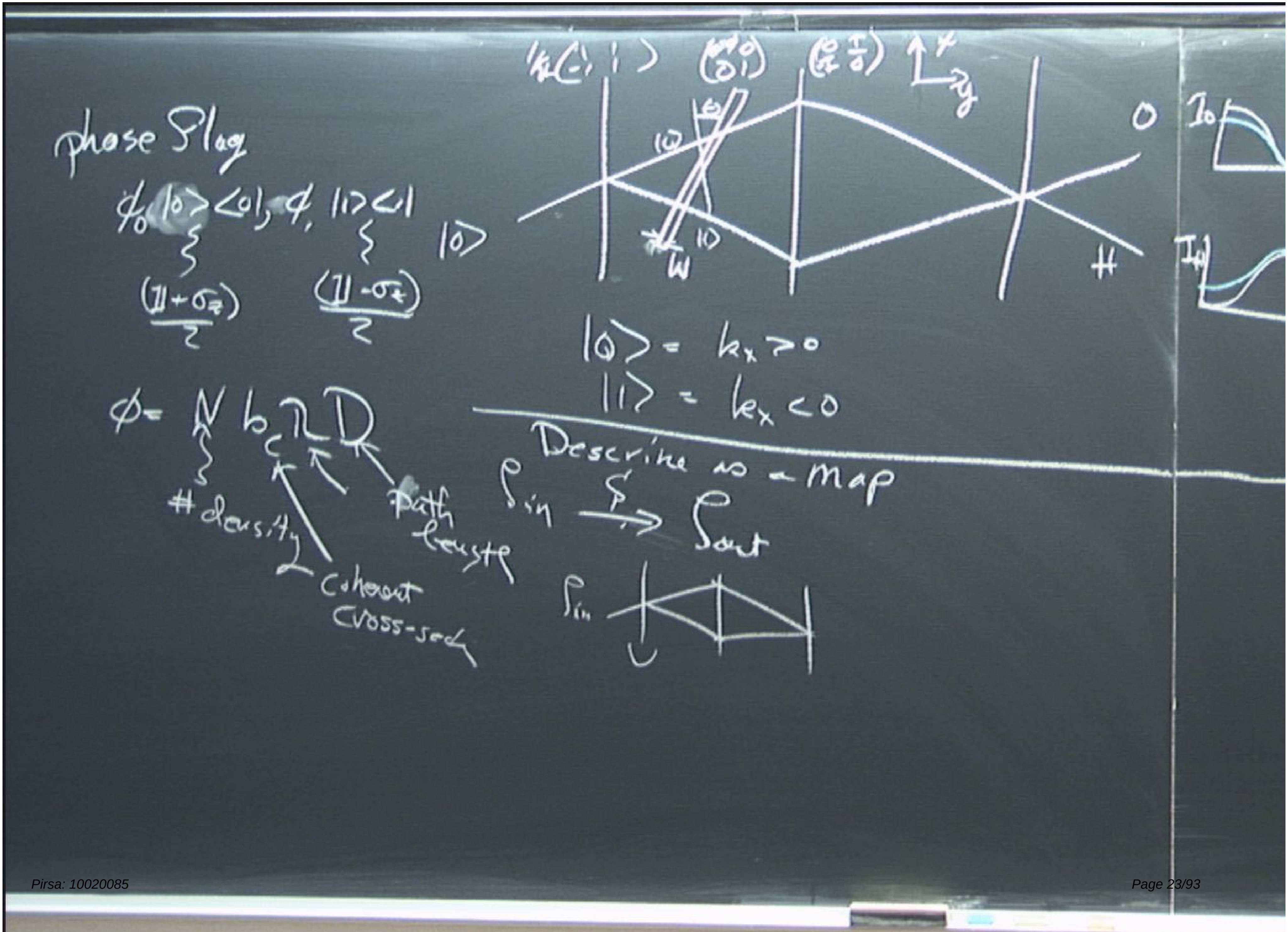

phase flag
|0><0|, |1><1|
(𝟙 + σz)/2
(𝟙 − σz)/2
|0>
|0> = kx > 0
|1> = kx < 0
Describe as a map
ρin
0
1

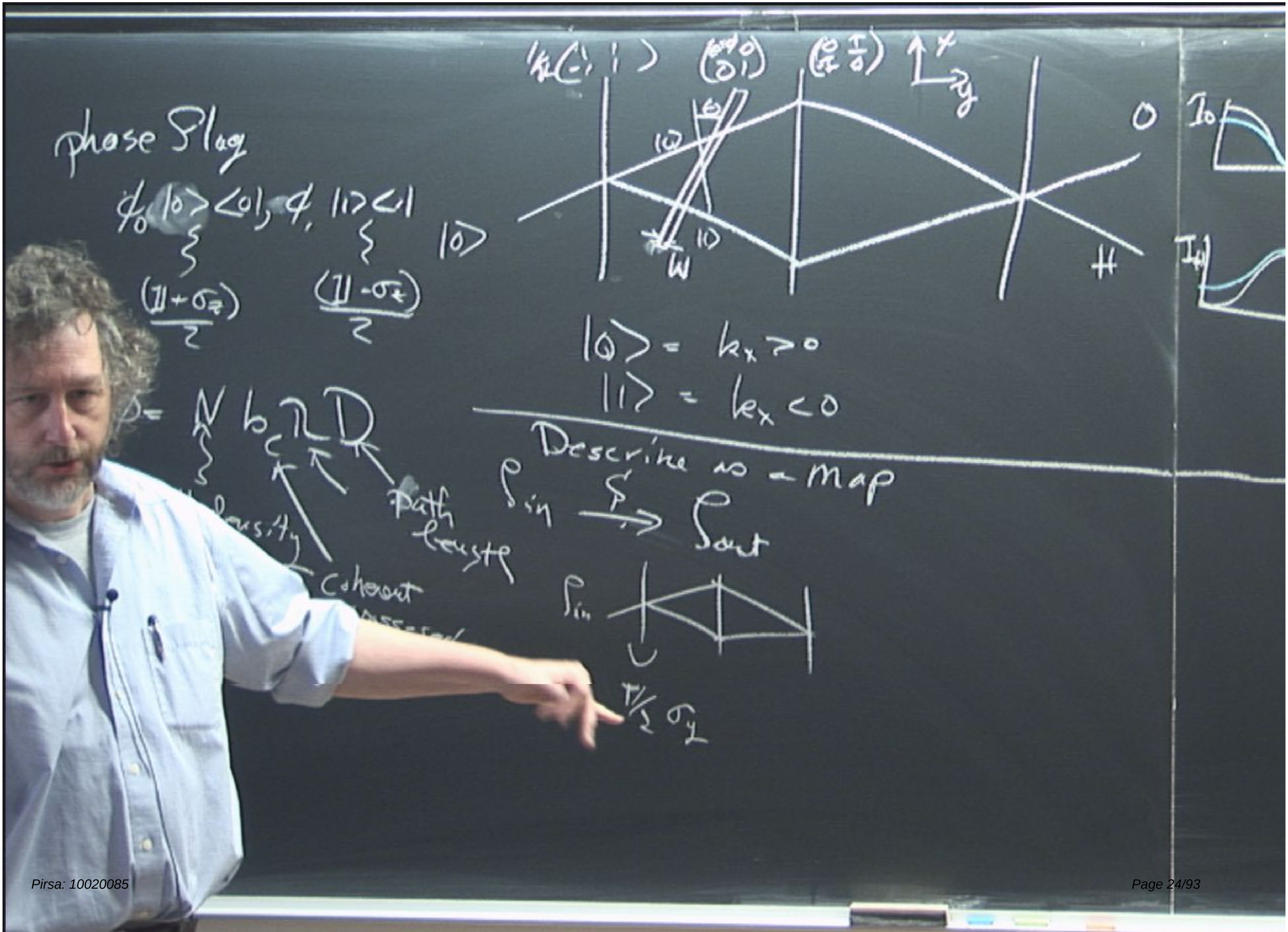

phase flag
|0> = kx > 0
|1> = kx < 0
Describe as a map
density
coherent
cross-sect

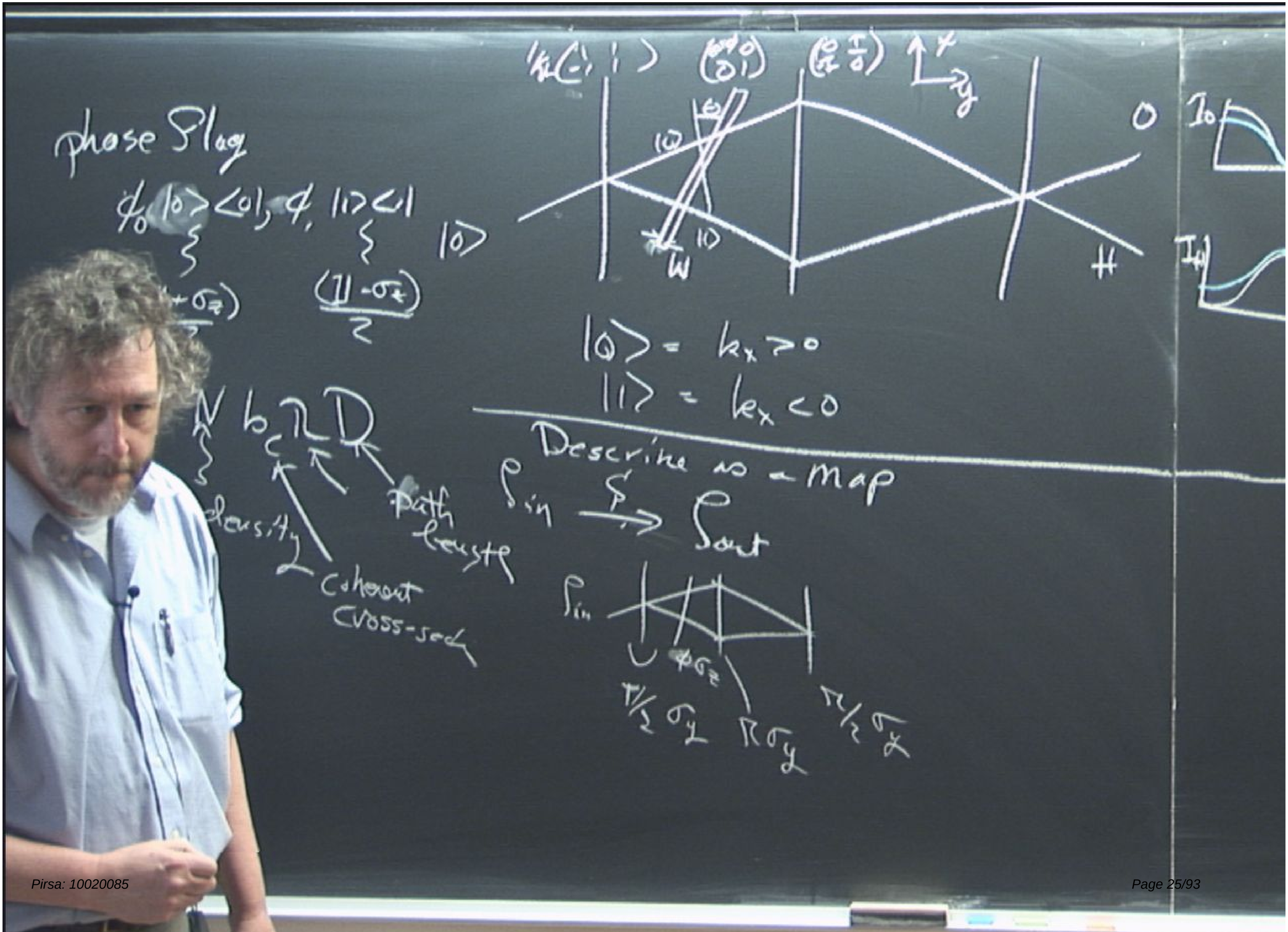

phase flag
|0⟩⟨0|, |1⟩⟨1|
(𝟙 - σ_z)/2
|0⟩
|0⟩ = k_x > 0
|1⟩ = k_x < 0
Describe as a map
ρ_in
density
path length
coherent cross-sect
0
H

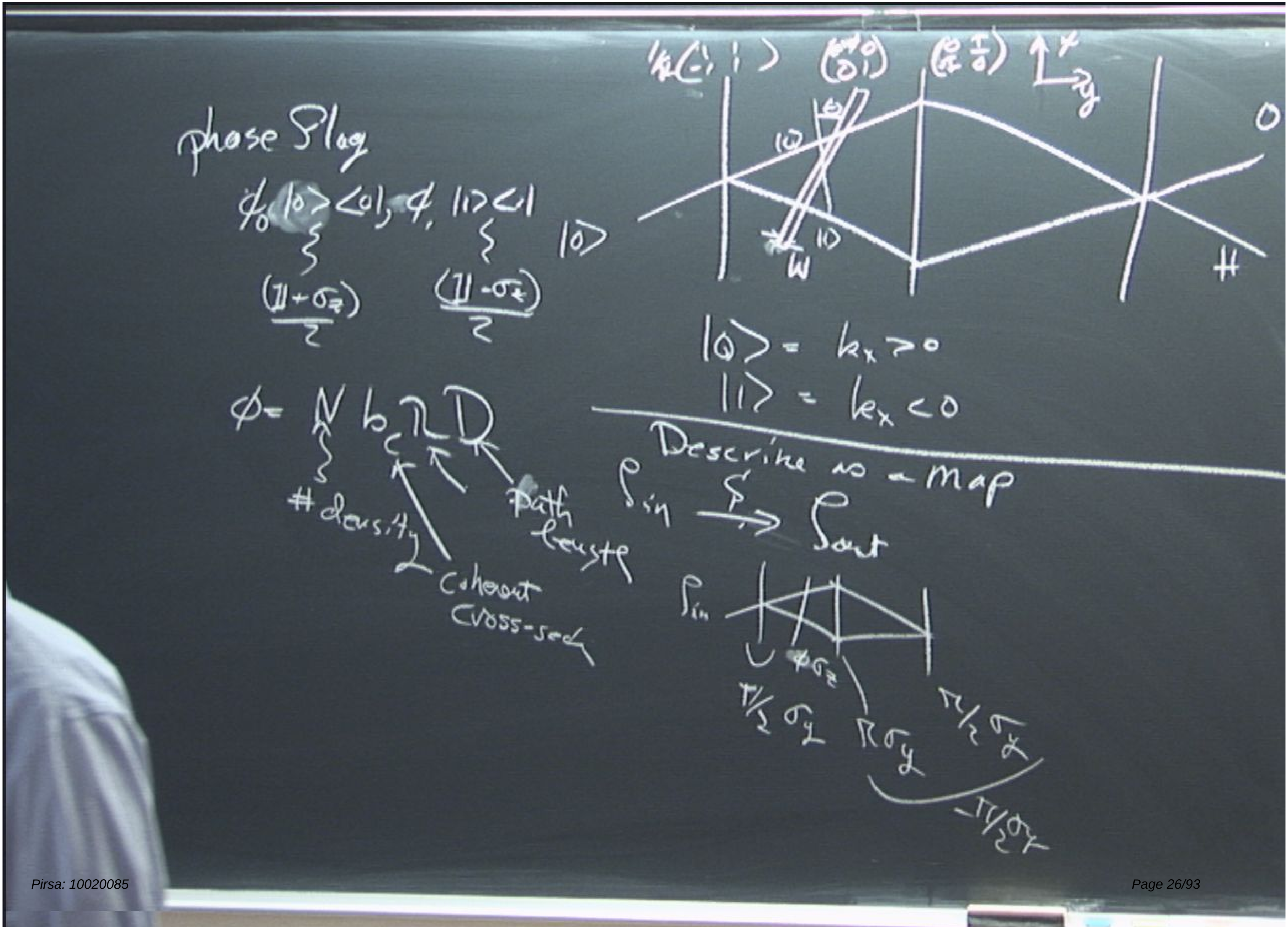

phase flag
|0>
|0> = kx > 0
|1> = kx < 0
Describe as a map
density
path length
coherent cross-sect

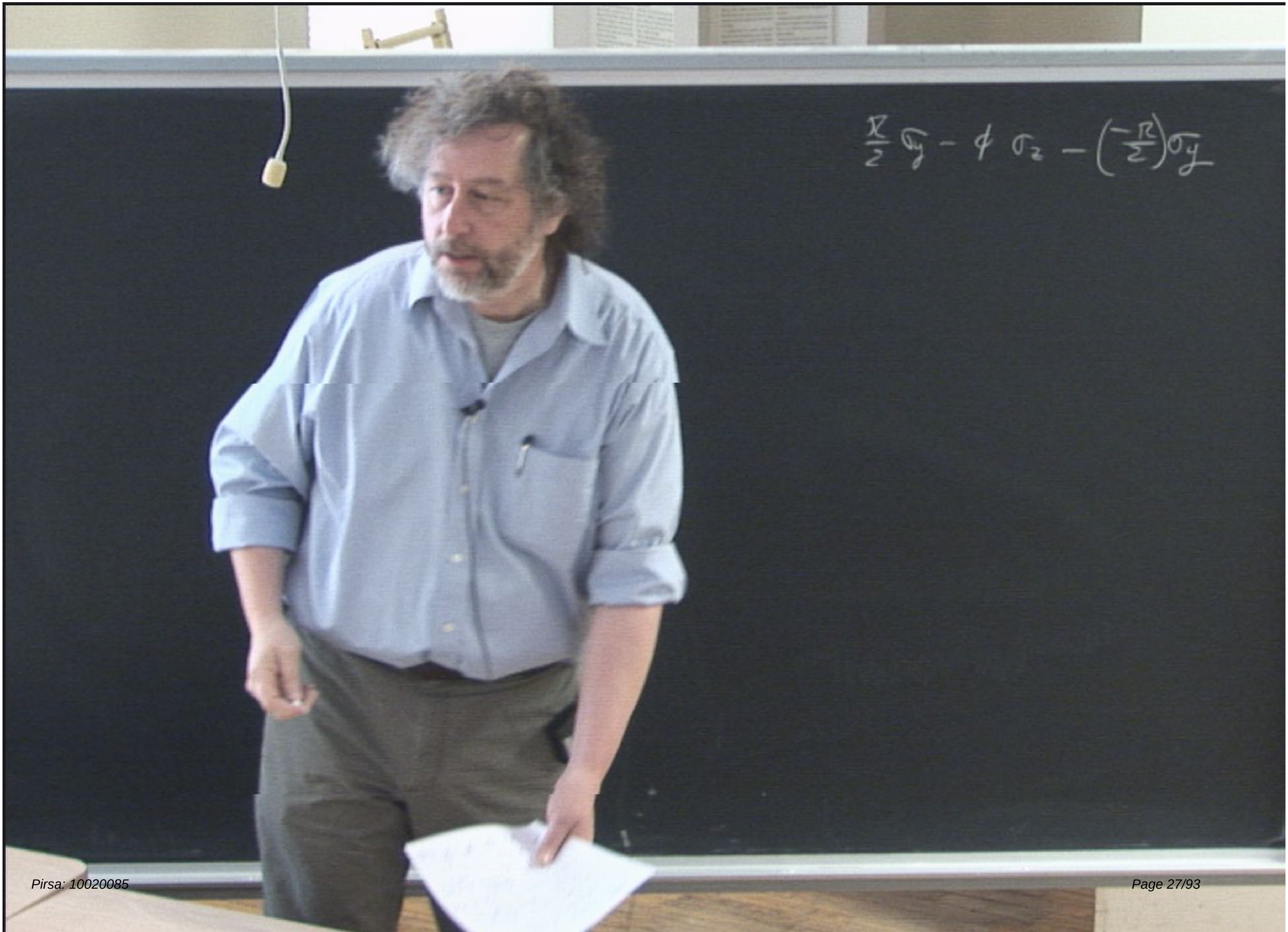

phase flag
|0⟩ = k_x > 0
|1⟩ = k_x < 0
Describe as a map
ρ_in → ρ_out
density
coherent cross-sect
path length


phase flag
|0⟩
|0⟩ = kx > 0
|1⟩ = kx < 0
Describe as a map
density
path length
coherent cross-sect

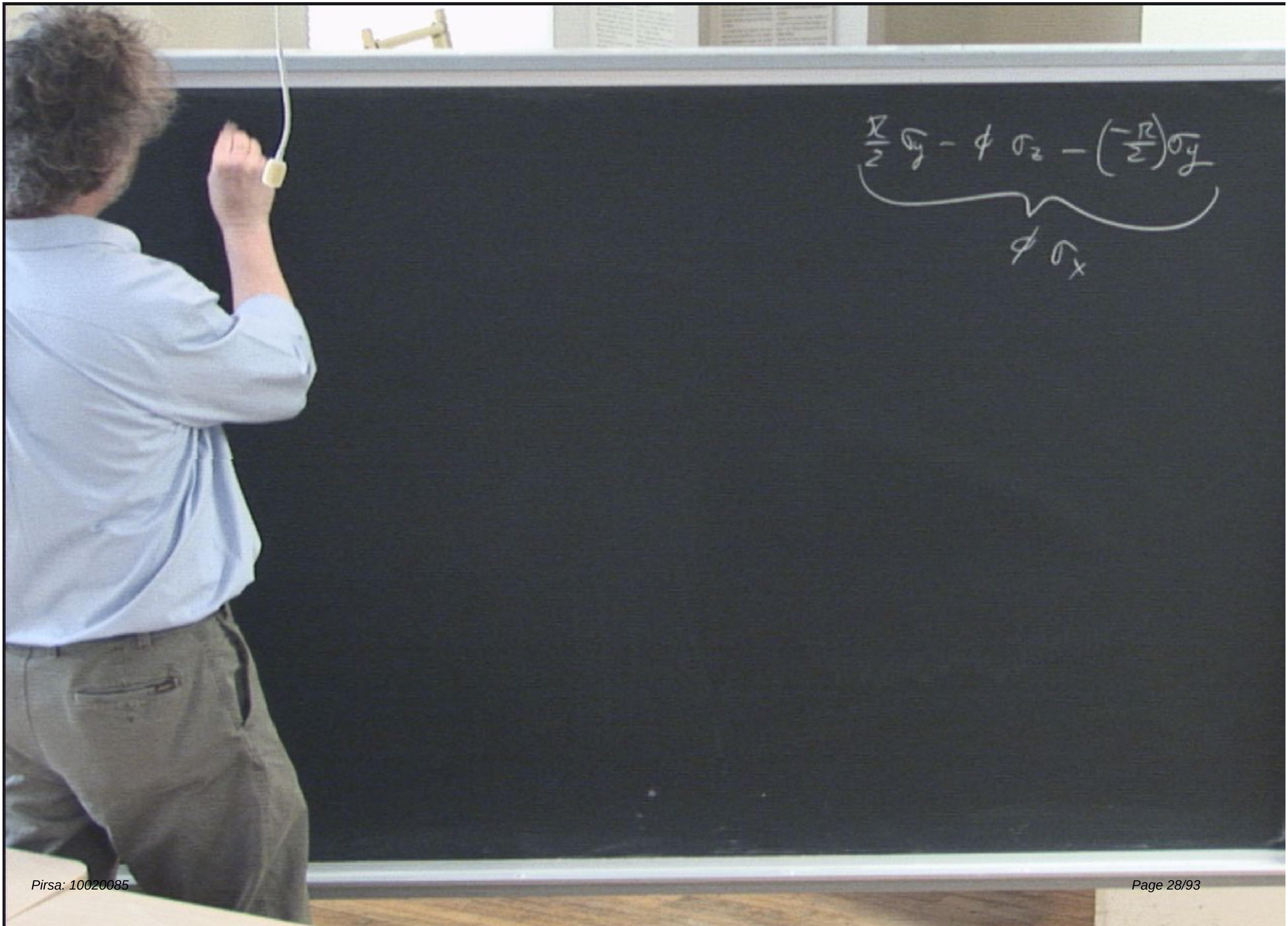

phase flag
|0> = kx > 0
|1> = kx < 0
Describe as a map
path
coherent

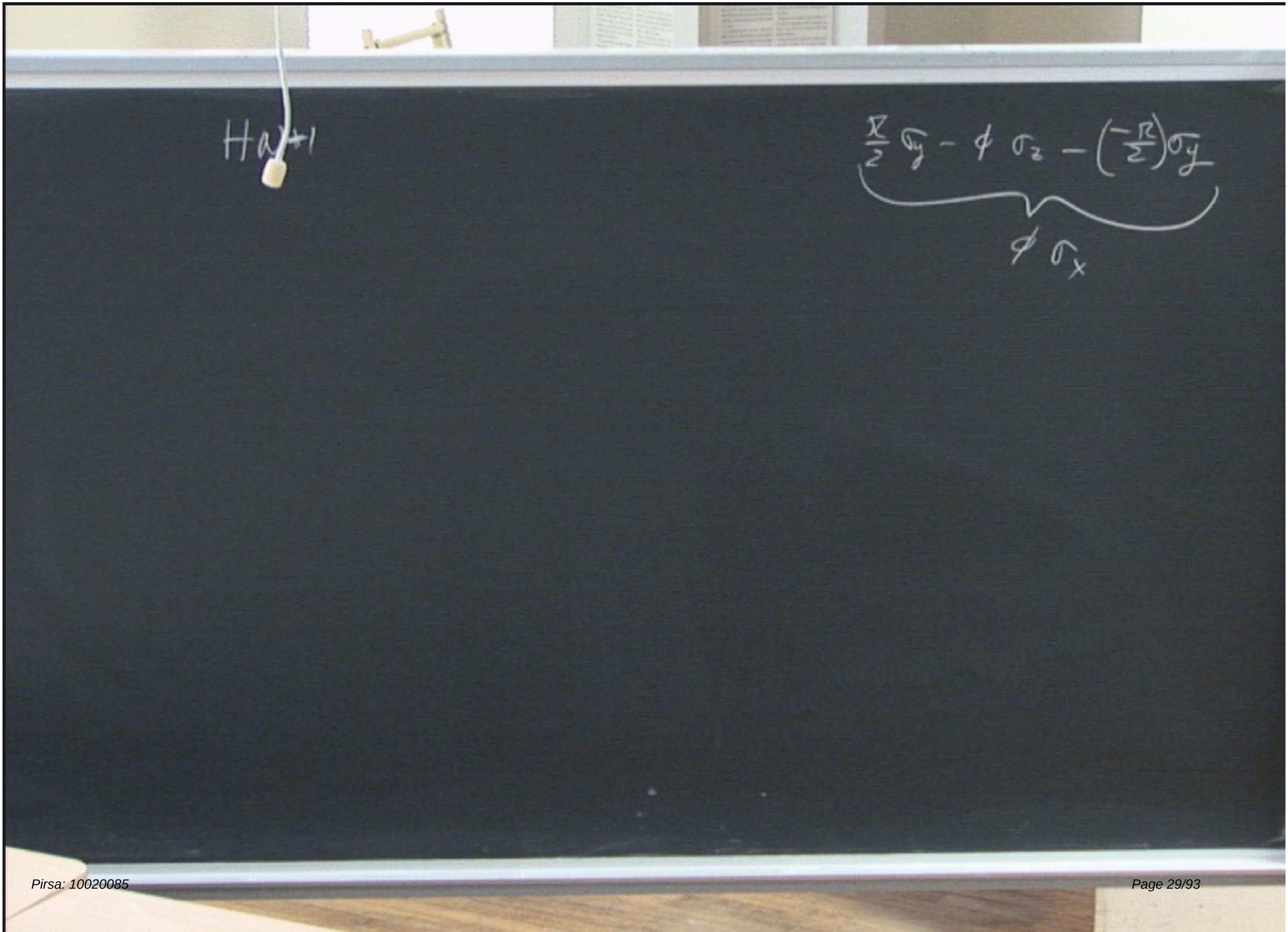

phase flag
|0>
|0> = k_x > 0
|1> = k_x < 0
Describe as a map
ρ_in
ρ_out
density
path length
coherent
cross-sec

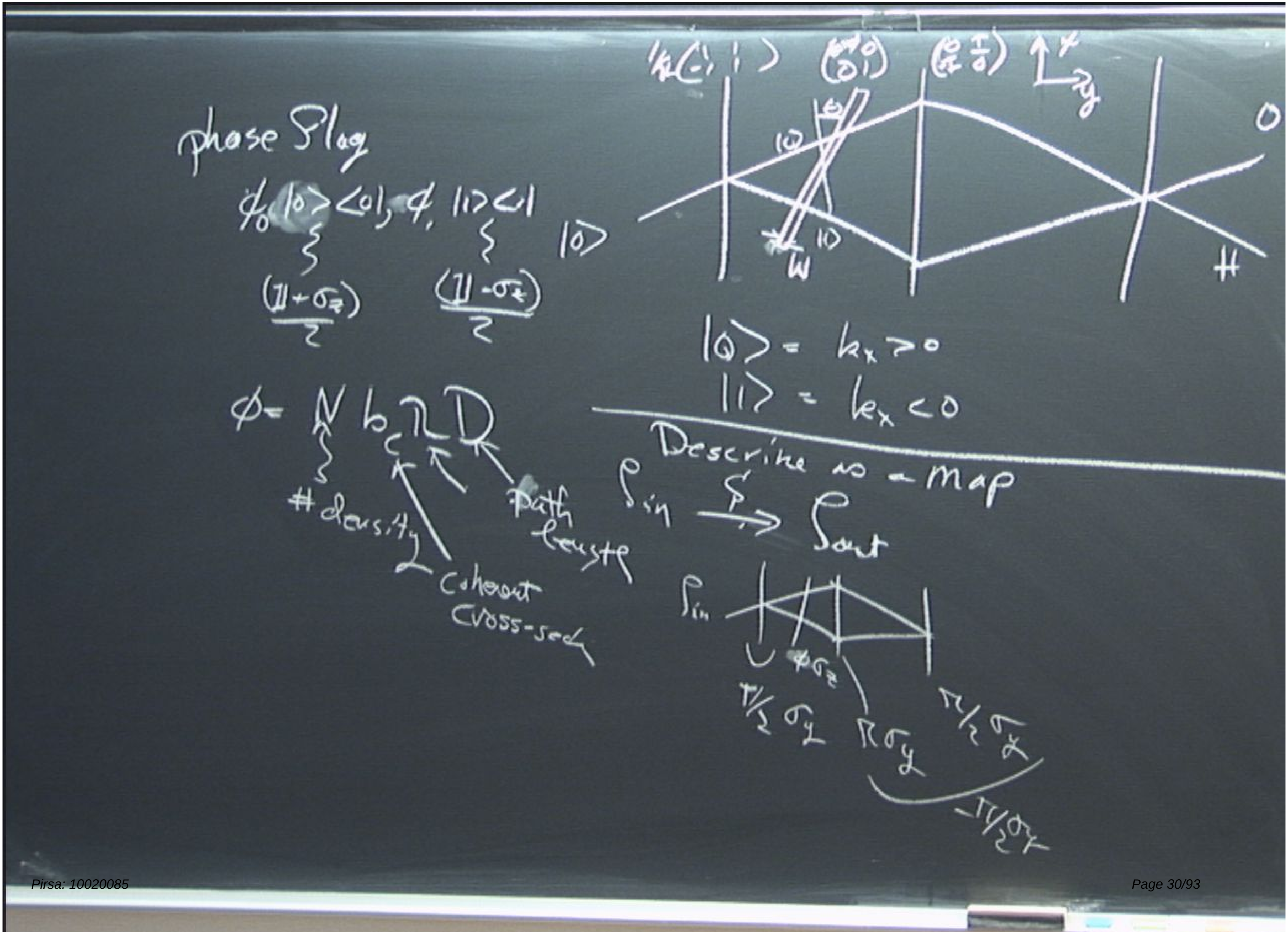

phase flag
|0⟩ = kx > 0
|1⟩ = kx < 0
Describe as a map
ρin → ρout
density
path length
coherent cross-sect

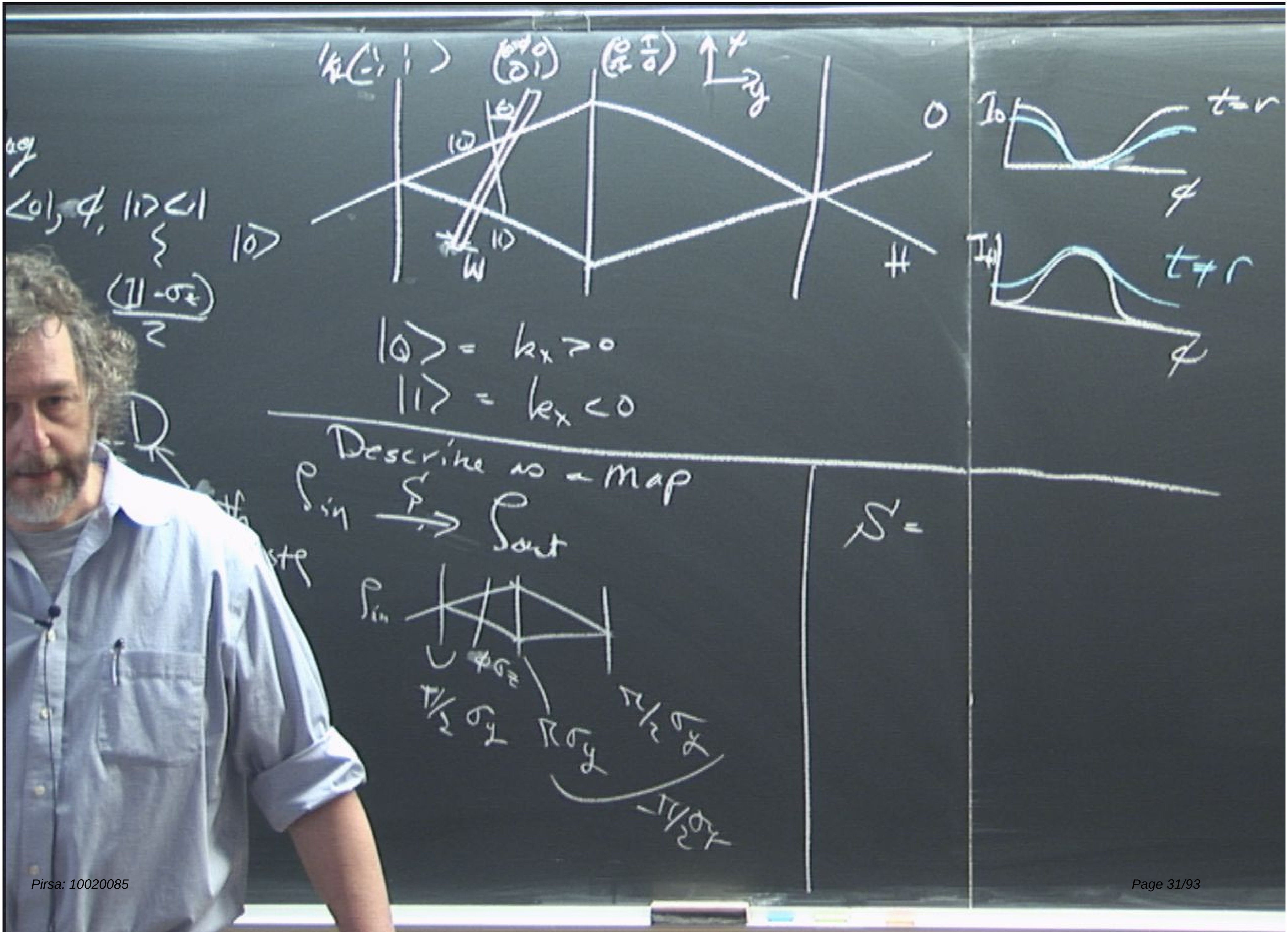


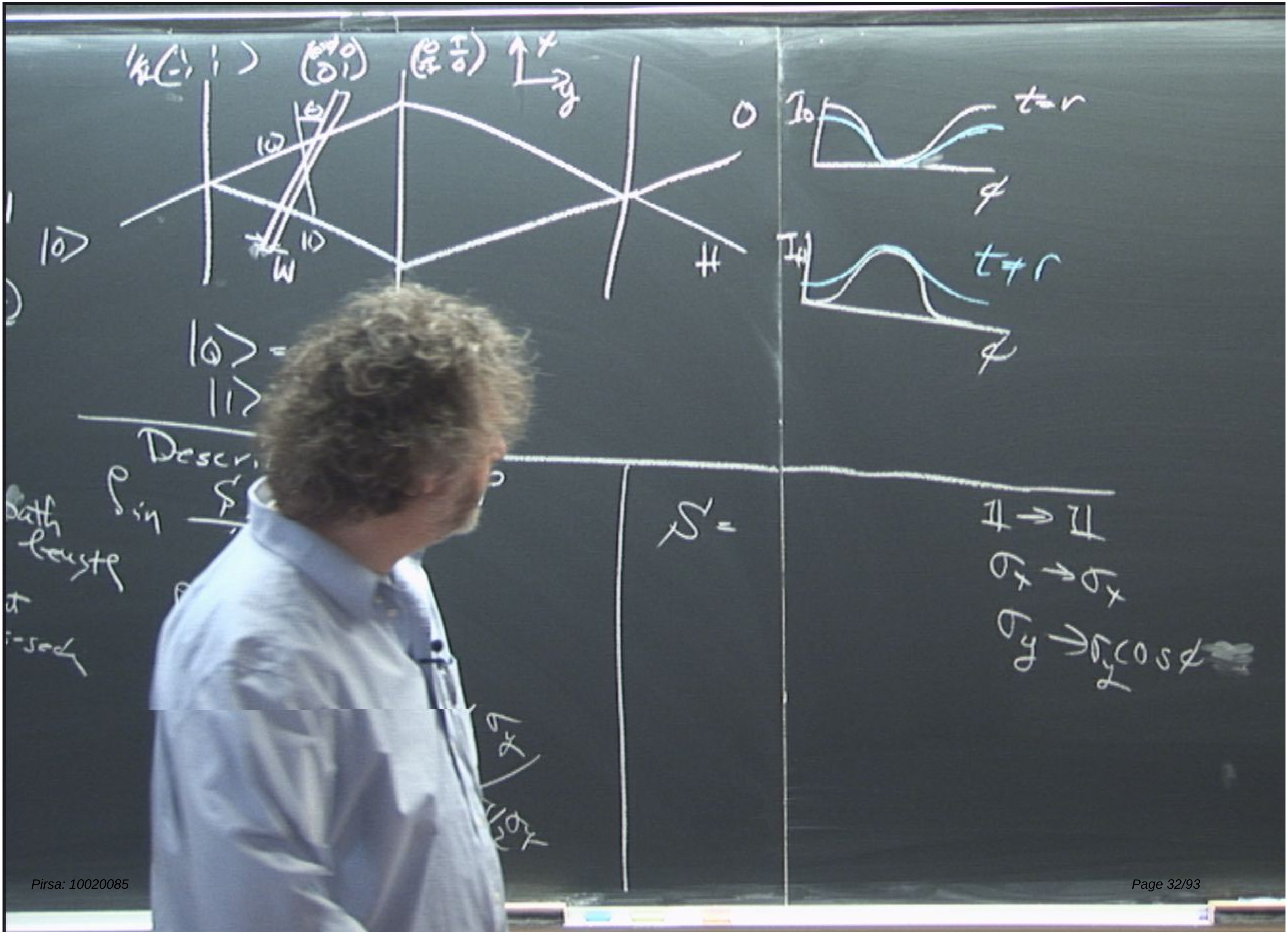
$\frac{R}{2}\sigma_y - \phi\,\sigma_z - \left(-\frac{R}{\Sigma}\right)\sigma_y$

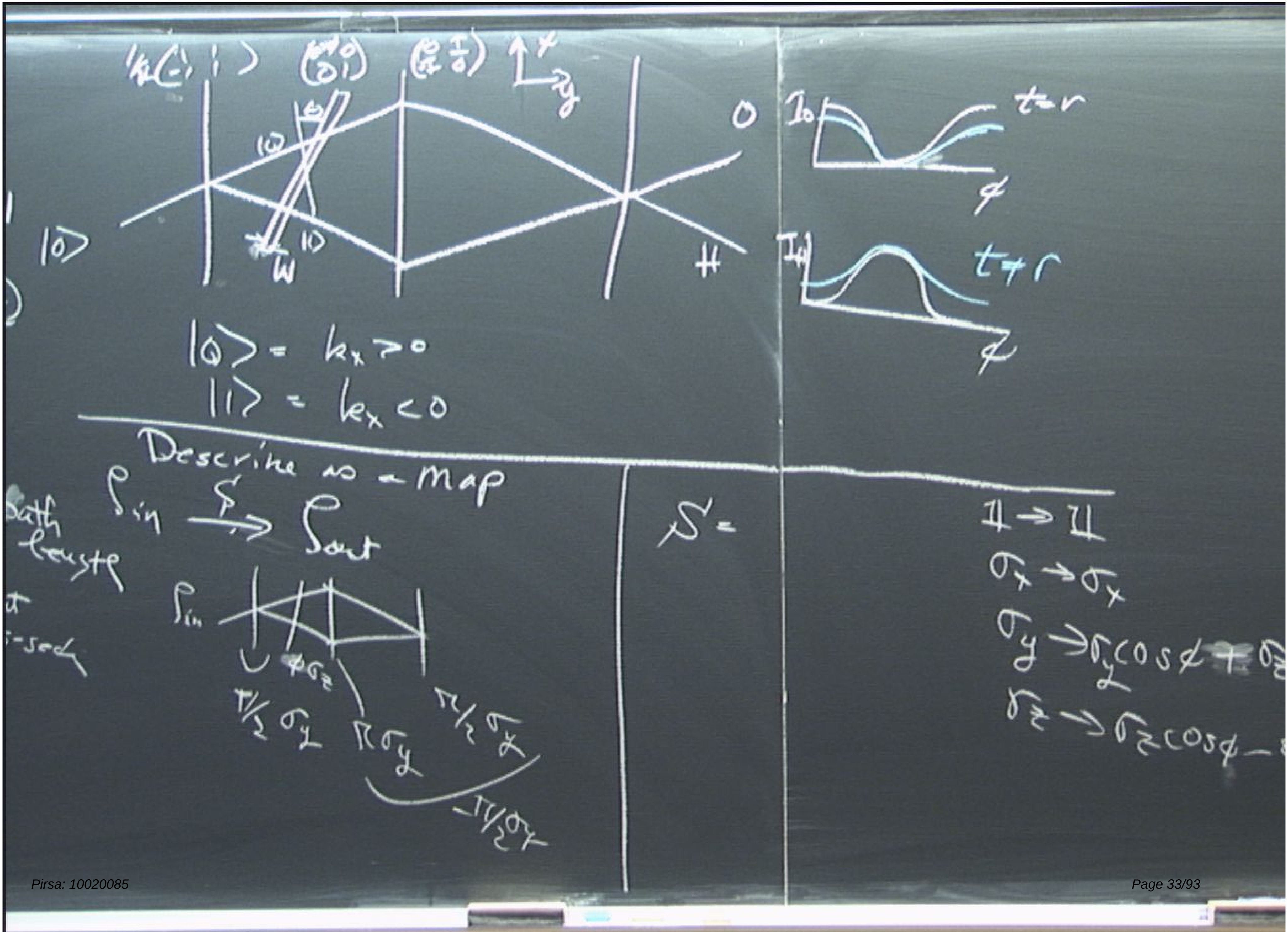

$\frac{\pi}{2}\sigma_y - \phi\,\sigma_z - \left(-\frac{\pi}{2}\right)\sigma_y$
$\phi\,\sigma_x$

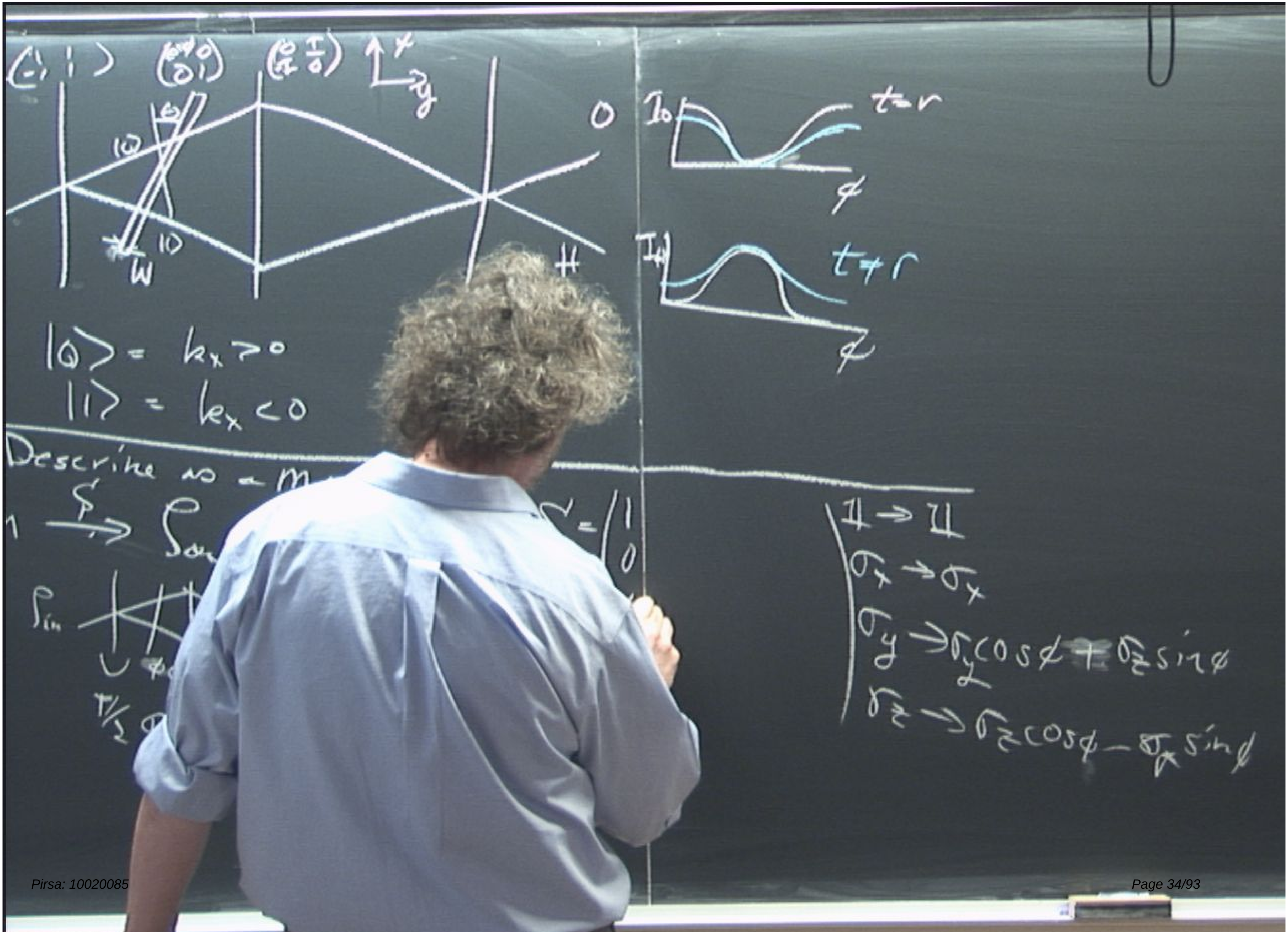

$\frac{\pi}{2}\sigma_y - \phi\,\sigma_z - \left(-\frac{\pi}{2}\right)\sigma_y$
$\phi\,\sigma_x$

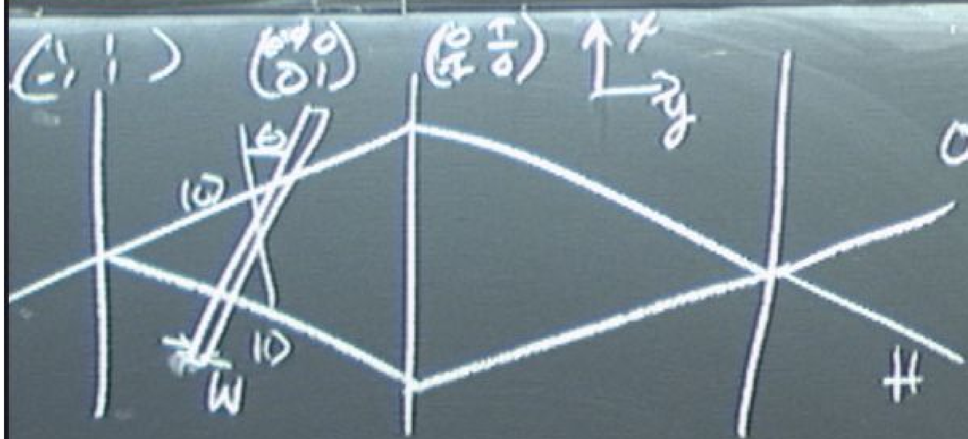

phase flag
|0⟩⟨0|, |1⟩⟨1|
(𝟙+σz)/2   (𝟙−σz)/2
|0⟩
|0⟩ = kx > 0
|1⟩ = kx < 0
Describe as a map
ρin → ρout
density
Coherent cross-sect.
path length
ρin
π/2 σy   π σy   π/2 σx
−π/2 σx

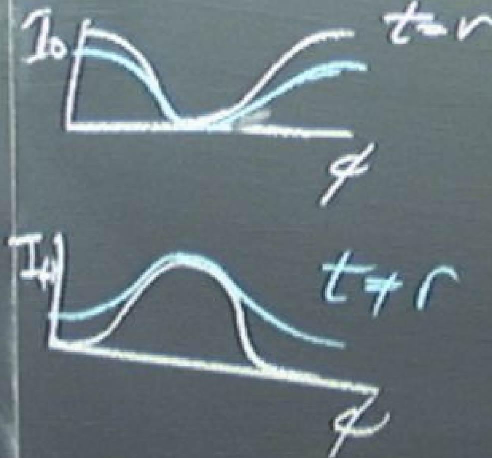

|0> = kx > 0
|1> = kx < 0
Describe as a map
ρin → ρout
S =


t=r
t≠r
Descr
S =
σx → σx
σy → σy cos φ


|0> = kx > 0
|1> = kx < 0
Describe as a map
ρin → ρout
S =
1 → 1
σx → σx
σy → σy cos φ
σz → σz cos φ
t = r
t ≠ r


|0> = k_x > 0
|1> = k_x < 0
Describe as a m
t = r
t ≠ r
𝟙 → 𝟙
σ_x → σ_x
σ_y → σ_y cos φ + σ_z sin φ
σ_z → σ_z cos φ − σ_x sin φ



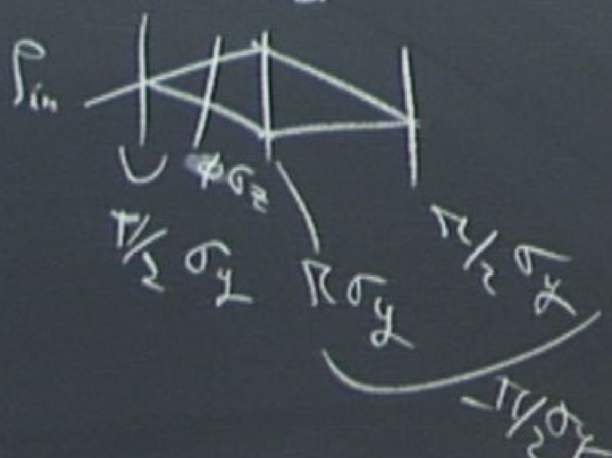


$|0\rangle = k_x > 0$

$|1\rangle = k_x < 0$

Describe as a map

$\rho_{in} \xrightarrow{S} \rho_{out}$



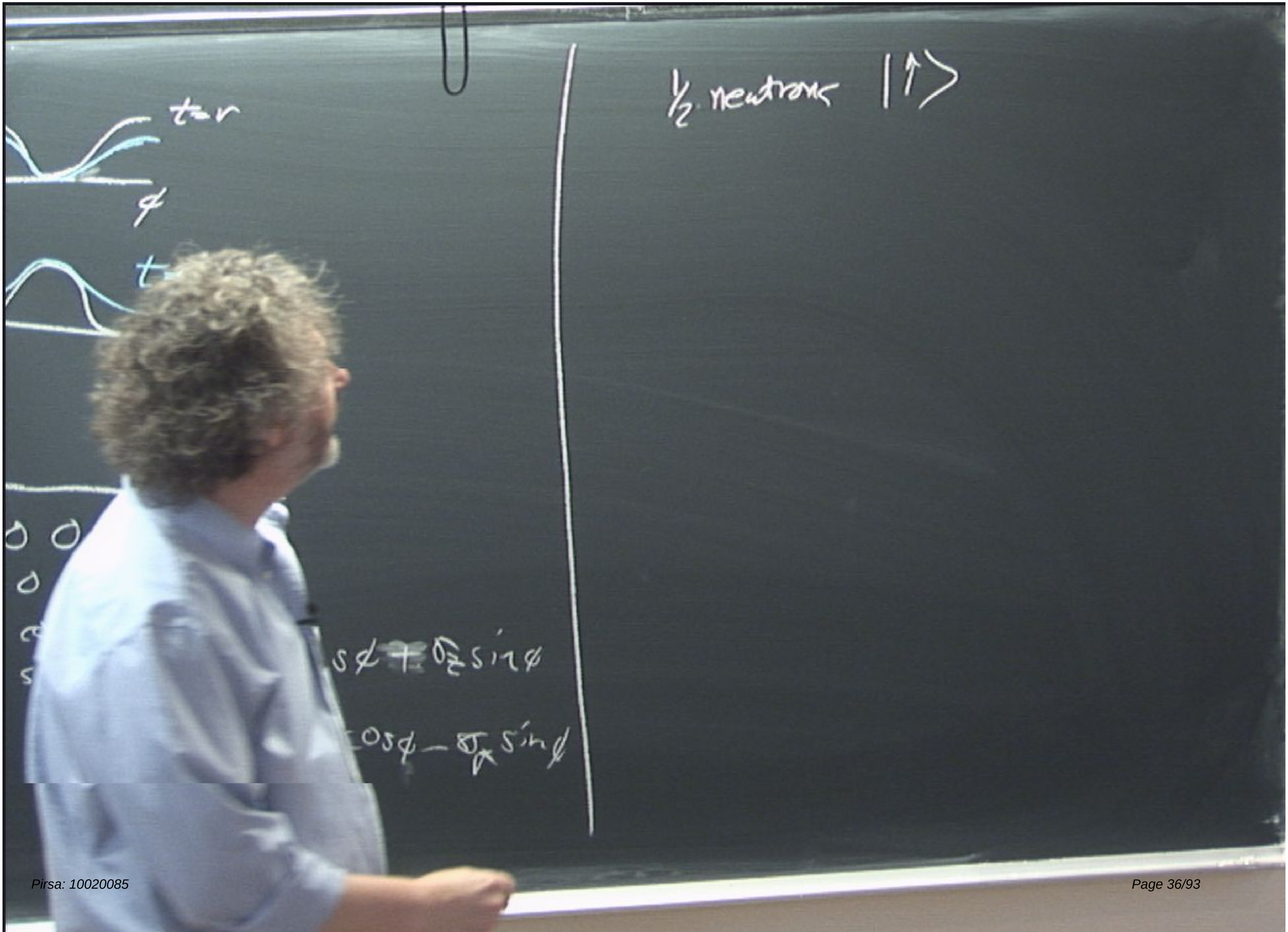
$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & \sin\phi & \cos\phi \end{pmatrix}$$

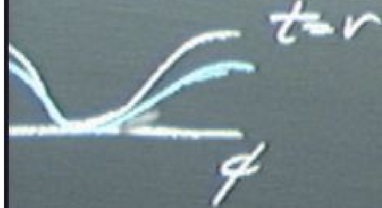
$\mathbb{1} \to \mathbb{1}$

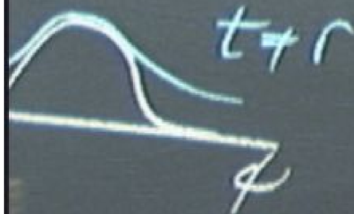
$\sigma_x \to \sigma_x$

$\sigma_y \to \sigma_y \cos\phi + \sigma_z \sin\phi$

$\sigma_z \to \sigma_z \cos\phi - \sigma_x \sin\phi$


½ neutrons |↑>

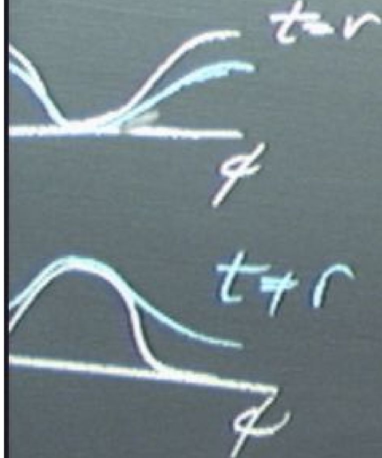




0	$\mathbb{1} \to \mathbb{1}$
	$\sigma_x \to \sigma_x$
φ	$\sigma_y \to \sigma_y \cos\phi + \sigma_z \sin\phi$
	$\sigma_z \to \sigma_z \cos\phi - \sigma_x \sin\phi$

½ neutrons $|\uparrow\rangle$ — $+\alpha\,\sigma_z$

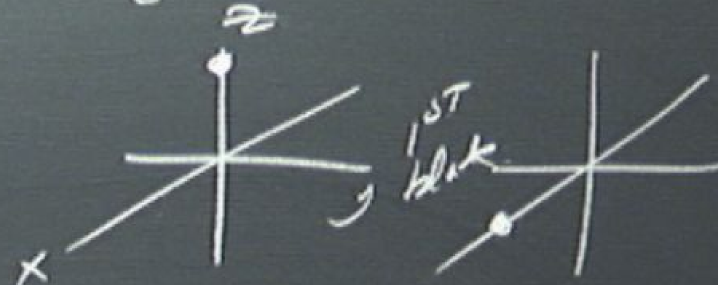
½ neutrons $|\downarrow\rangle$ — $-\alpha\,\sigma_z$

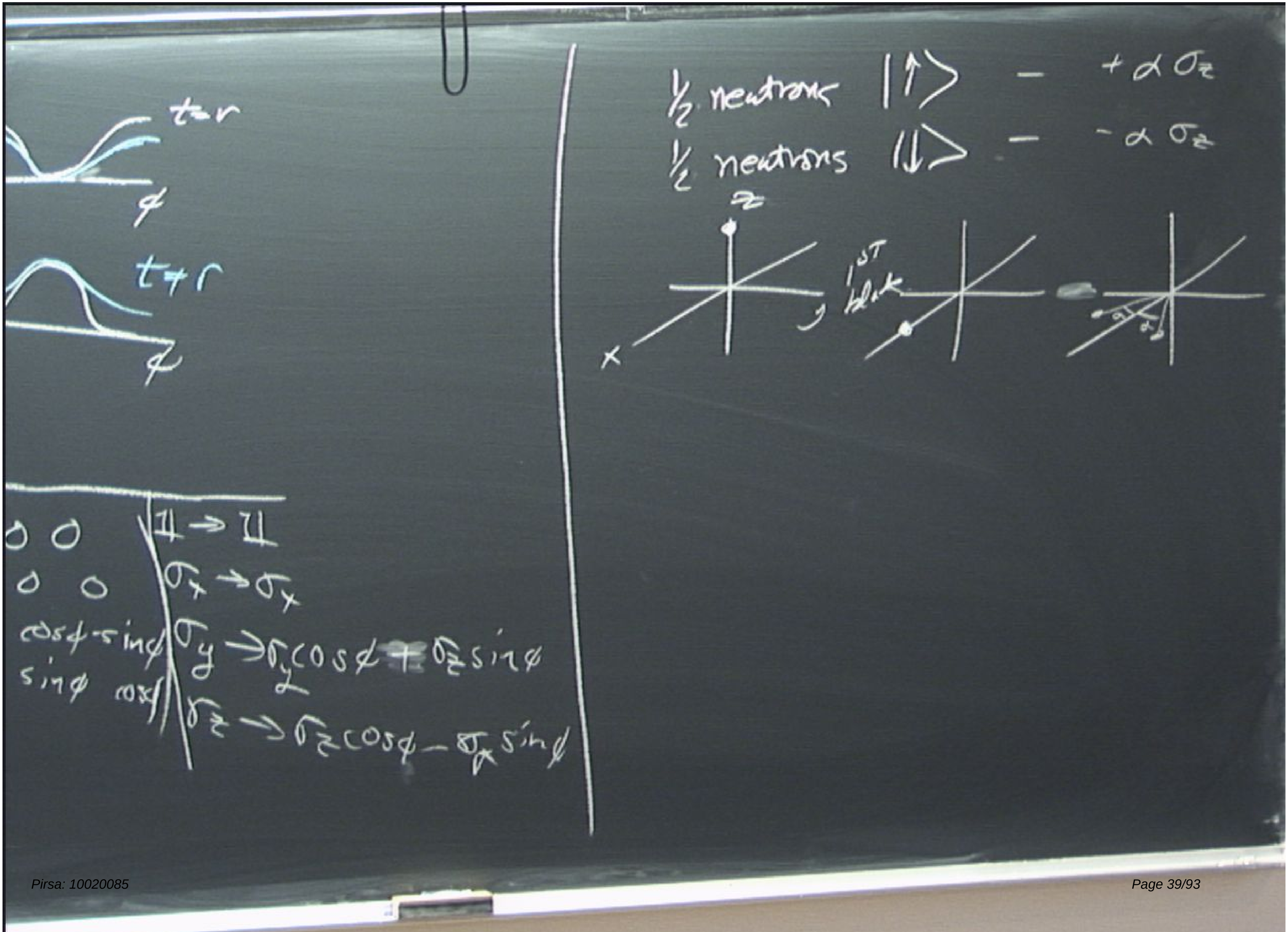


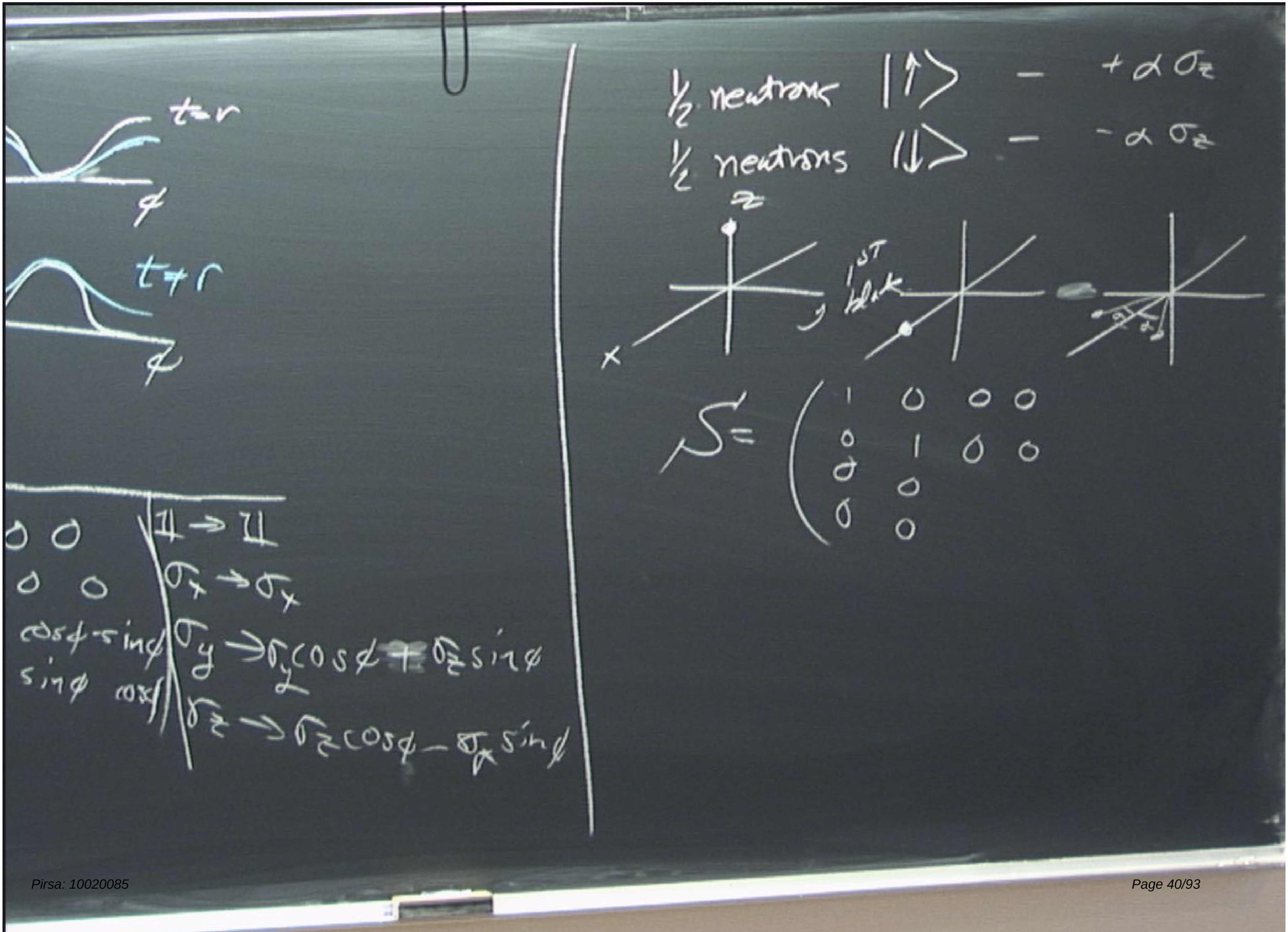
0	0	$\mathbb{1} \to \mathbb{1}$
0	0	$\sigma_x \to \sigma_x$
$\cos\phi$	$-\sin\phi$	$\sigma_y \to \sigma_y \cos\phi + \sigma_z \sin\phi$
$\sin\phi$	$\cos\phi$	$\sigma_z \to \sigma_z \cos\phi - \sigma_x \sin\phi$

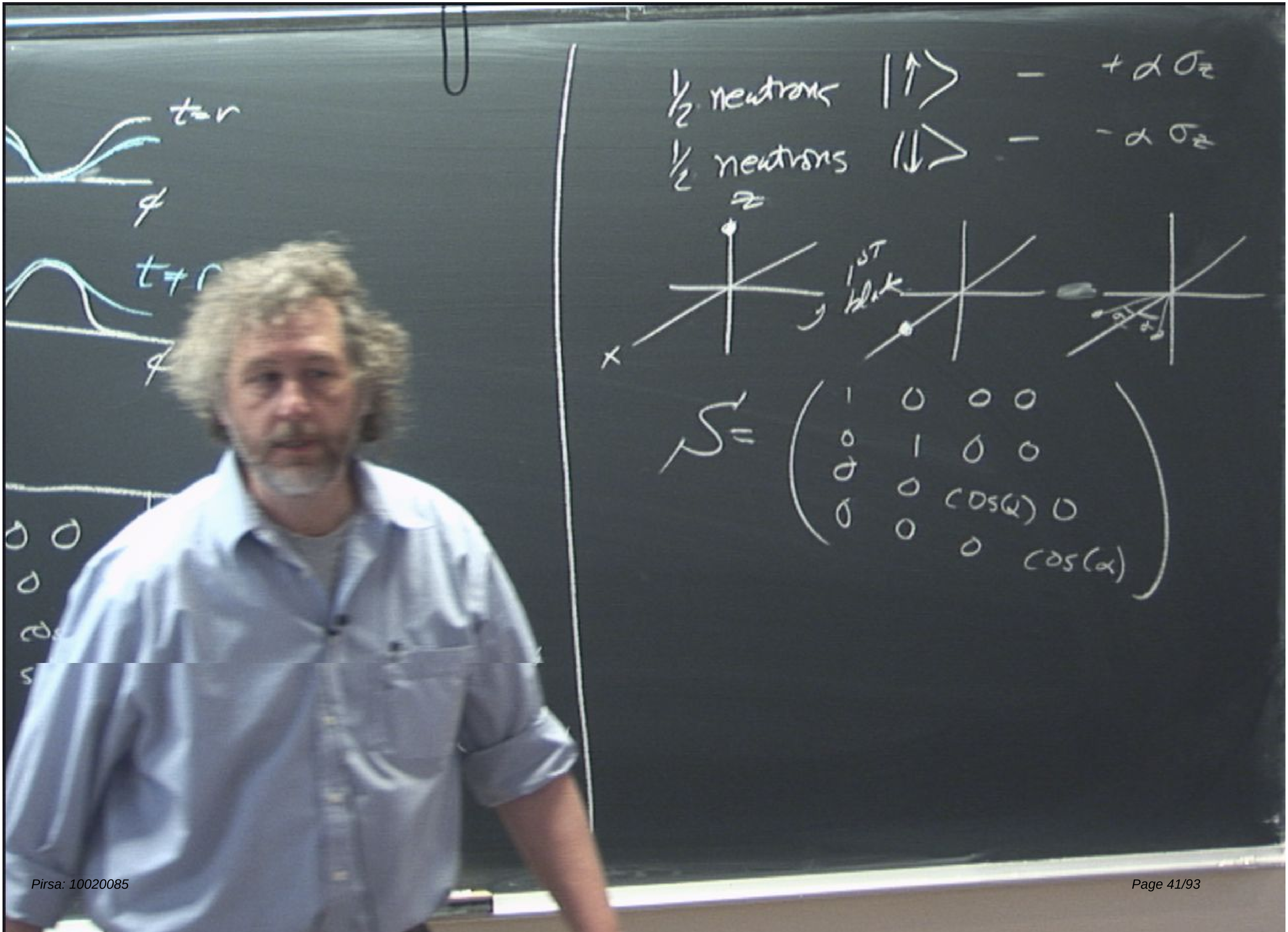
½ neutrons $|\uparrow\rangle$ — $+\alpha\sigma_z$

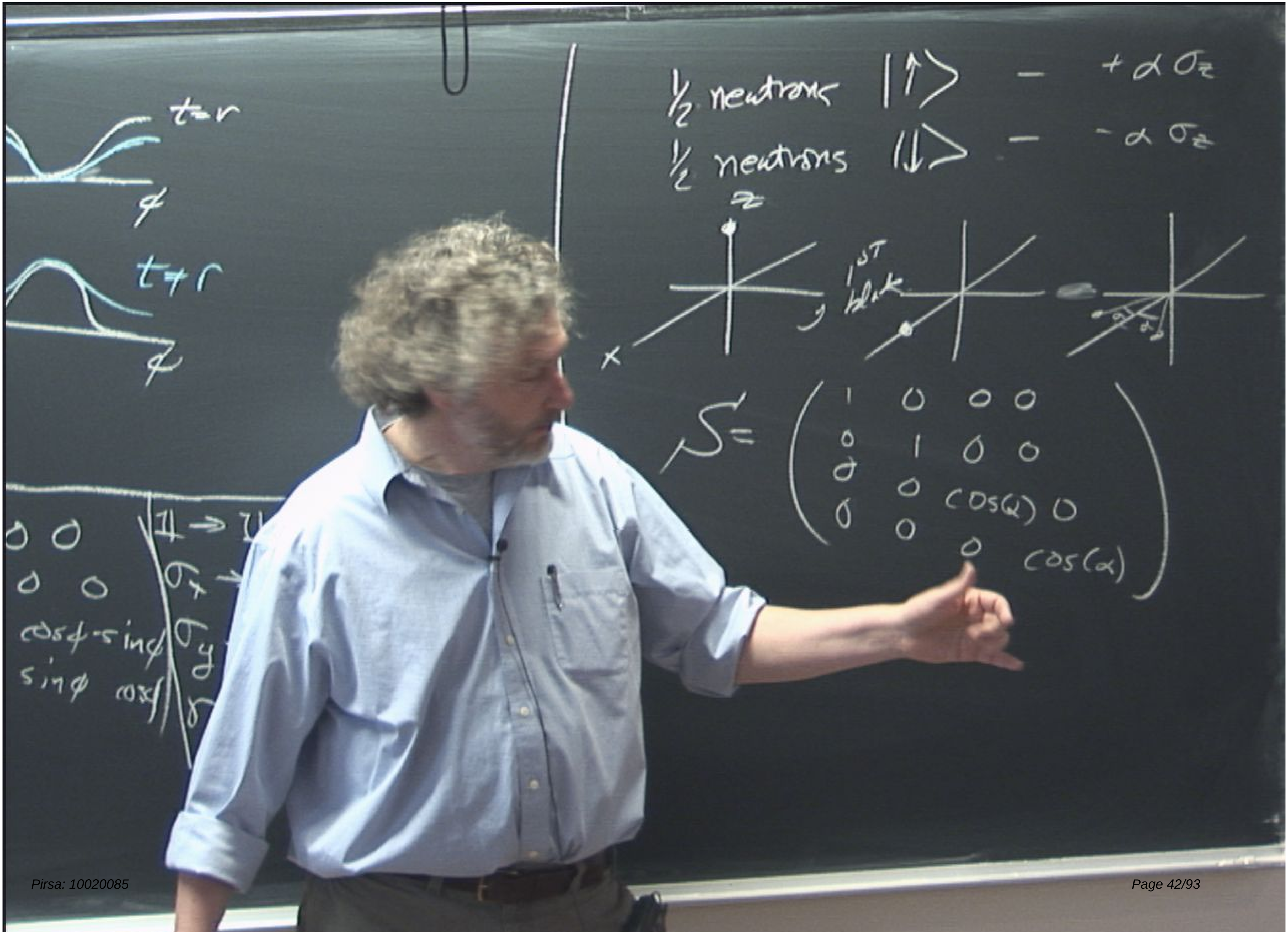
½ neutrons $|\downarrow\rangle$ — $-\alpha\sigma_z$



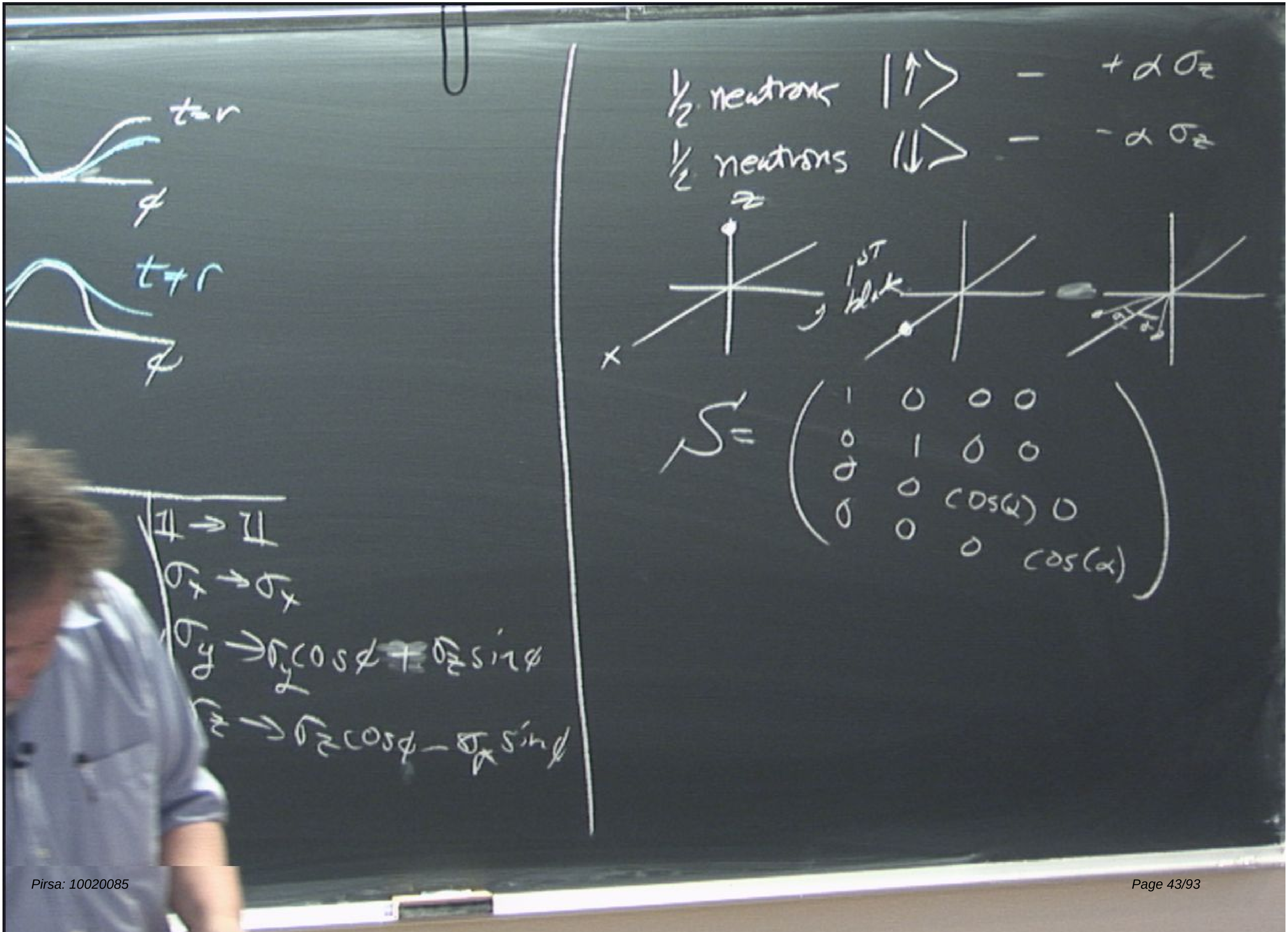

t=r
t≠r
½ neutrons |↑⟩ — +α σz
½ neutrons |↓⟩ — −α σz
z
x
y
1st block
I → I
σx → σx
cos φ −sin φ σy → σy cos φ + σz sin φ
sin φ cos φ σz → σz cos φ − σy sin φ

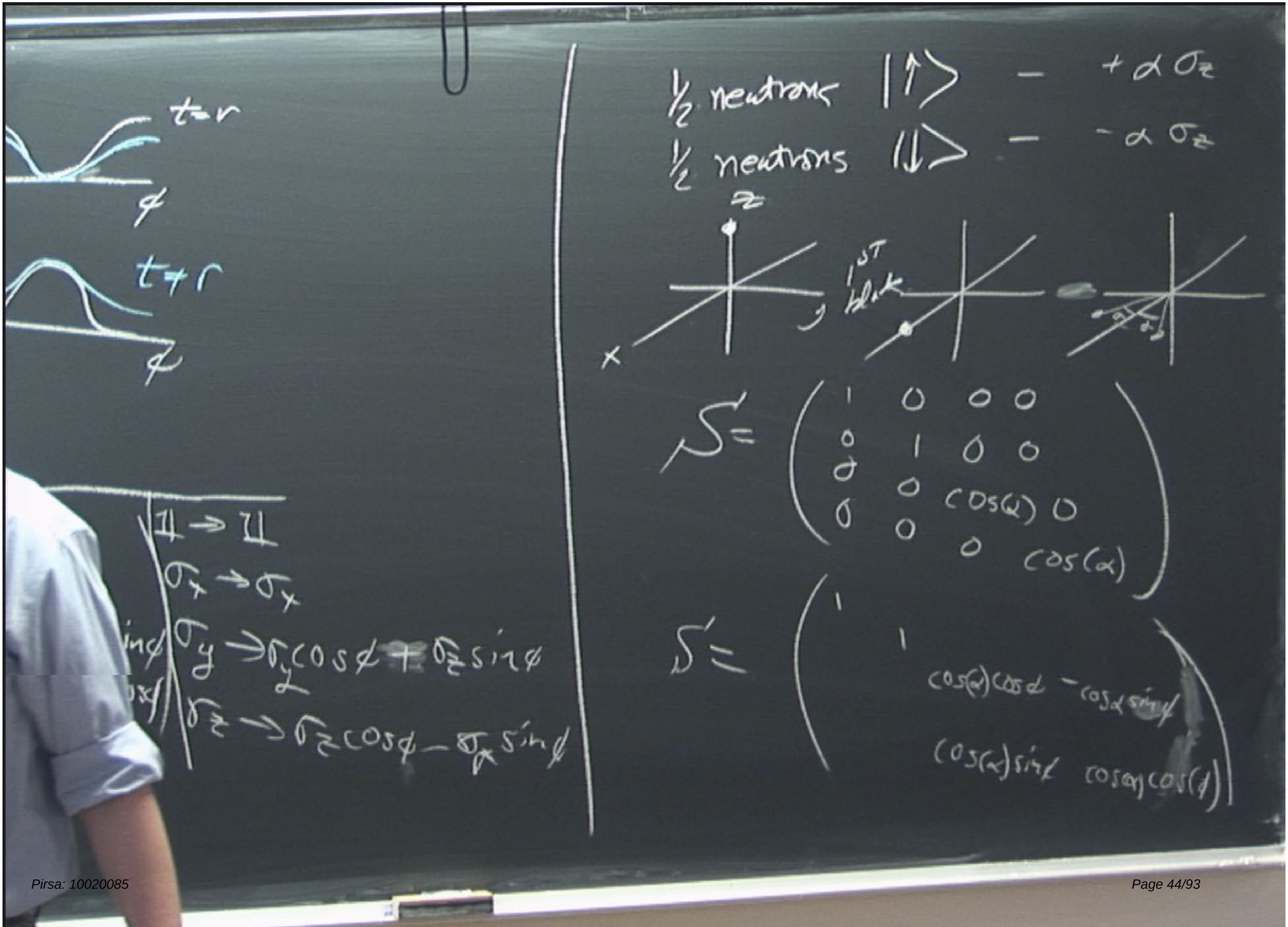

t=r
t≠r
1/2 neutrons |↑> — +α σz
1/2 neutrons |↓> — −α σz
1st block
S=
σx → σx
σy → σy cos φ + σz sin φ
σz → σz cos φ − σx sin φ

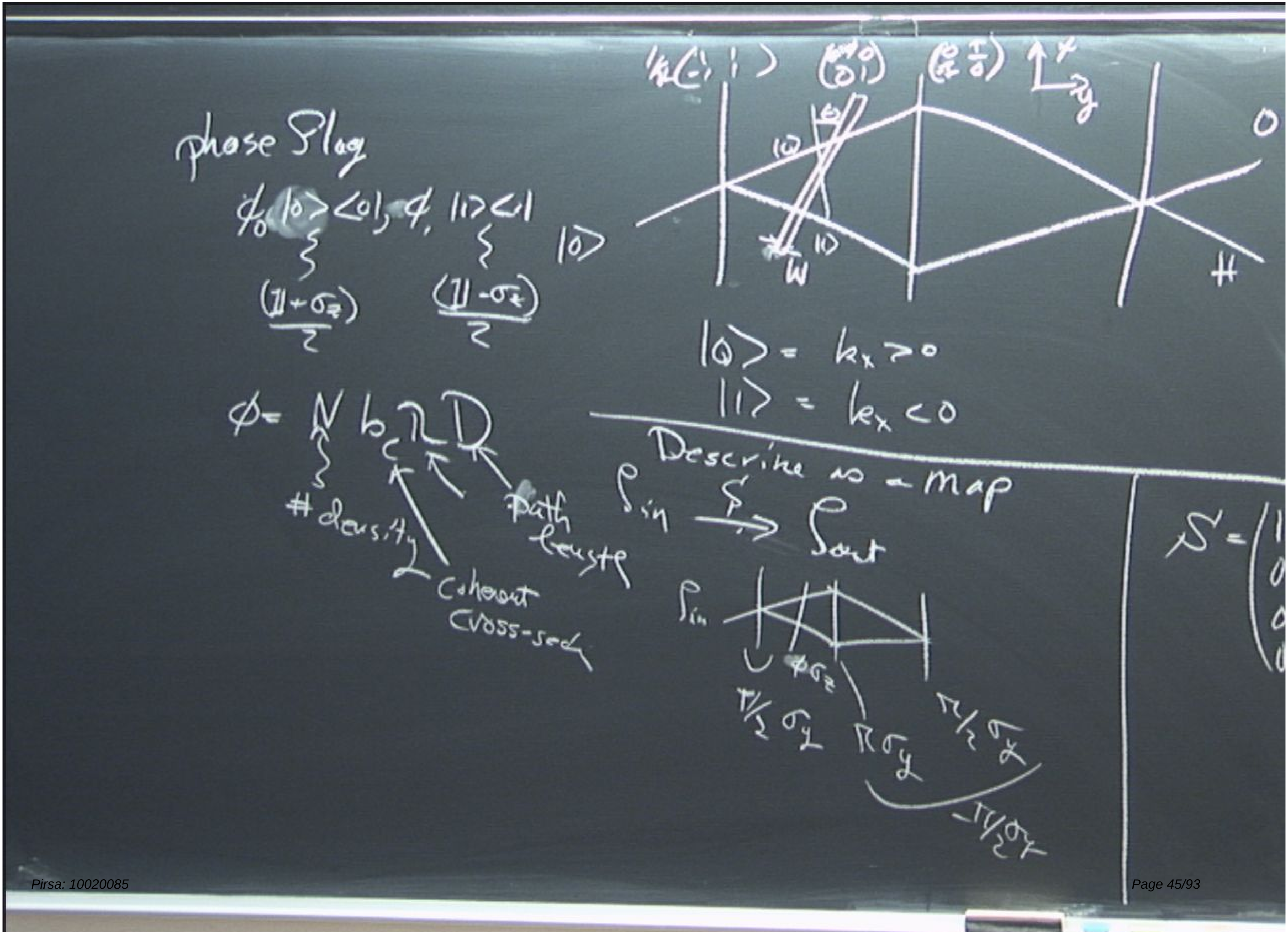

t=r
t≠r
½ neutrons |↑> — +α σz
½ neutrons |↓> — −α σz
1st blok
S=
cos(α)

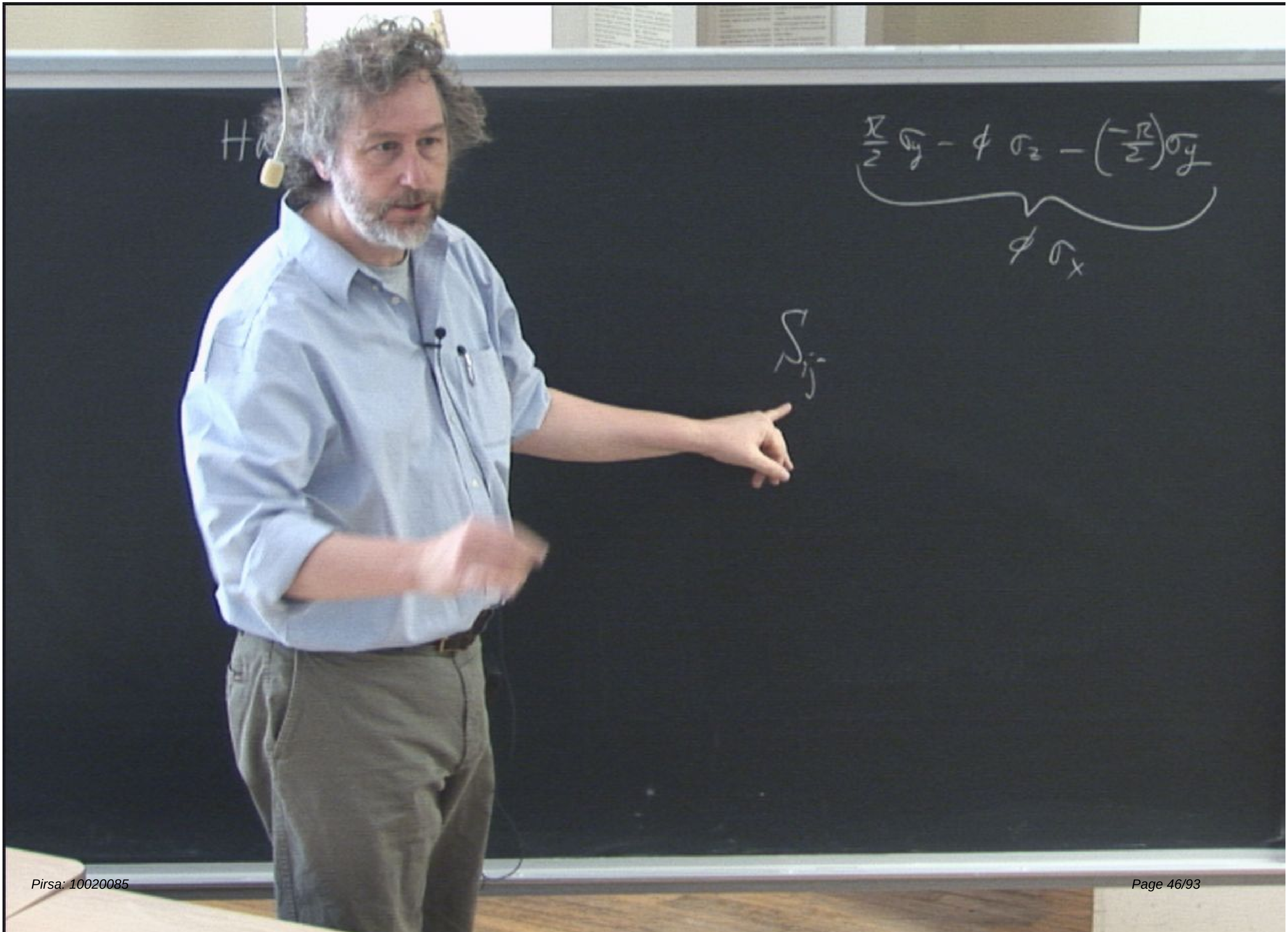


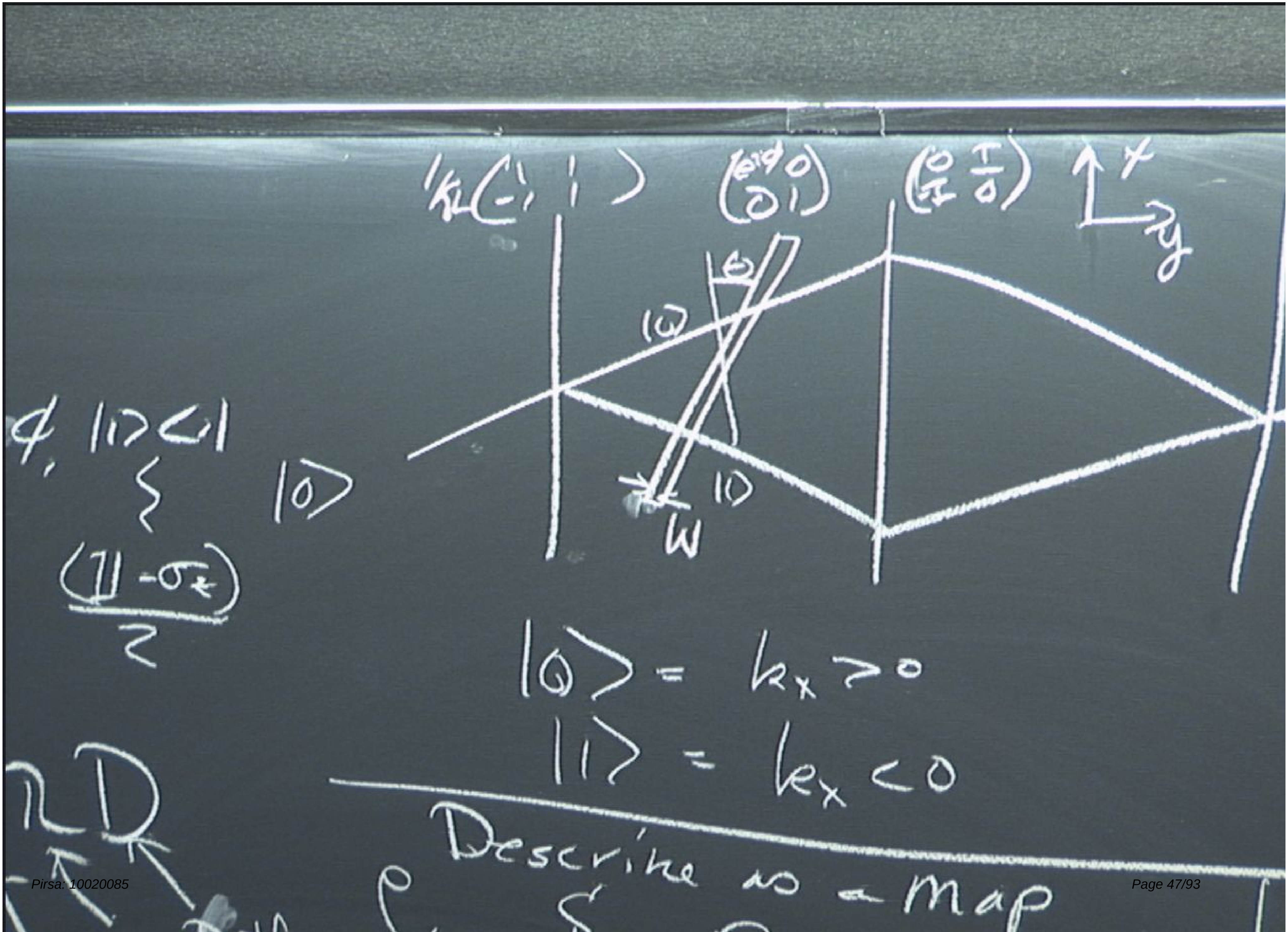
t=r
t≠r
1/2 neutrons |↑> — +α σz
1/2 neutrons |↓> — −α σz
z
1st block
cos(α)

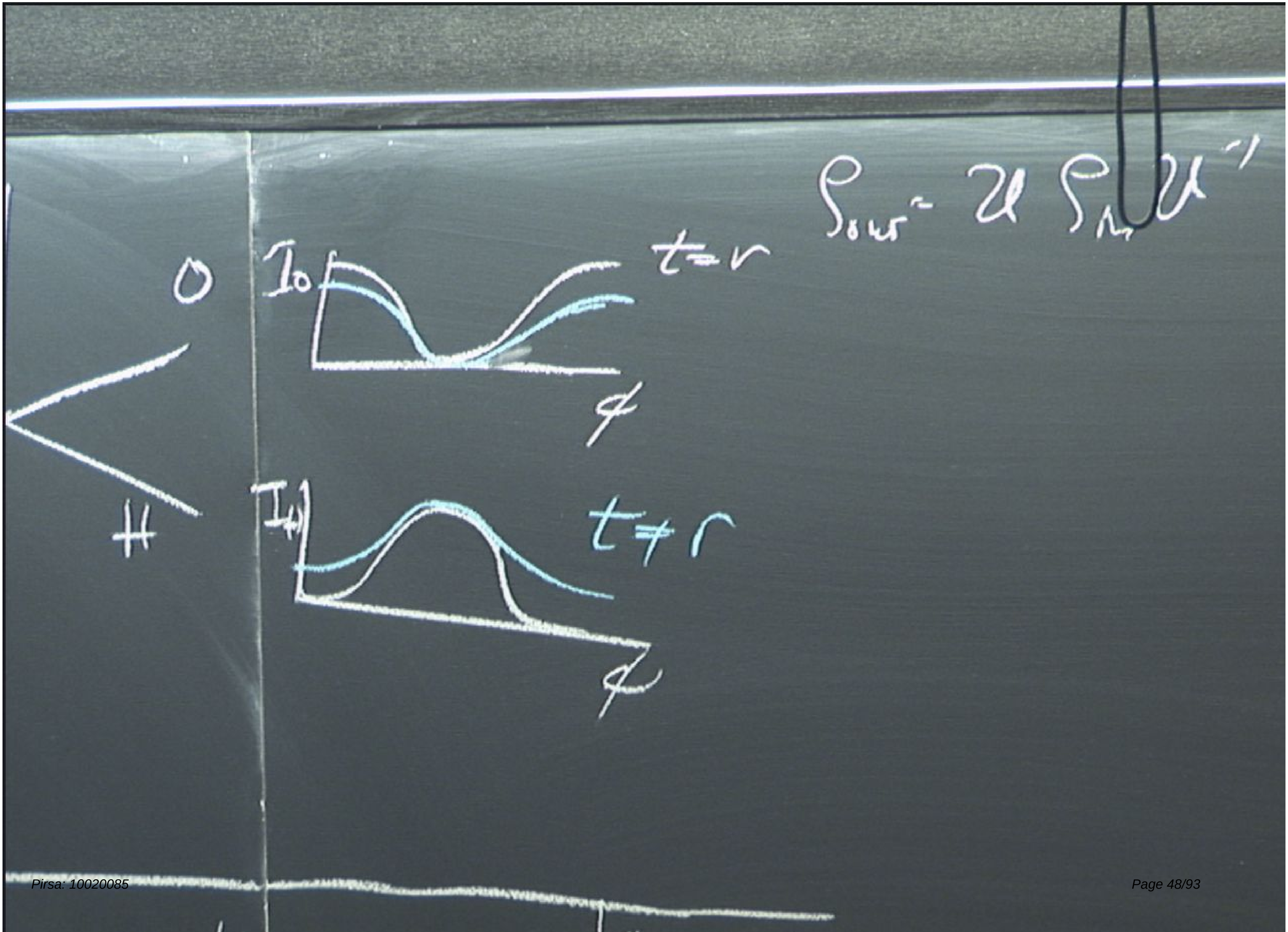

t=r
t≠r
1/2 neutrons |↑⟩ – +α σz
1/2 neutrons |↓⟩ – −α σz
1st block
S=
cos(α)
𝟙 → 𝟙
σx → σx
σy → σy cos φ + σz sin φ
σz → σz cos φ − σx sin φ

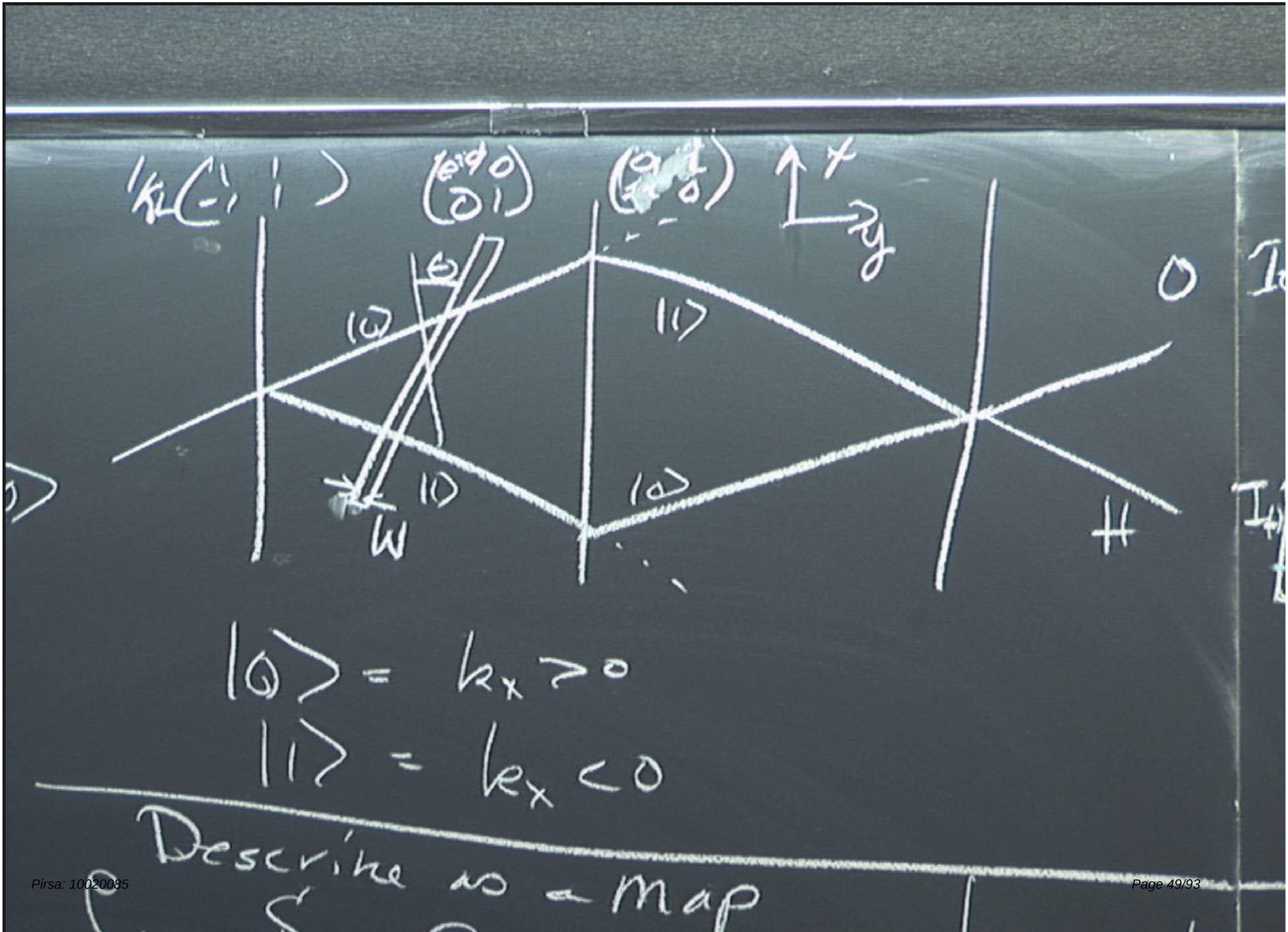

t=r
t≠r
1/2 neutrons |↑⟩ — +α σz
1/2 neutrons |↓⟩ — −α σz
z
x
1st block
𝟙 → 𝟙
σx → σx
σy → σy cos φ + σz sin φ
σz → σz cos φ − σx sin φ
S =
cos(α)
S =
cos(α) cos φ − cos α sin φ
cos(α) sin φ cos(α) cos(φ)


phase flag
|0⟩
|0⟩ = kx > 0
|1⟩ = kx < 0
Describe as a map
ρin → ρout
ρin
density
path length
coherent cross-sect
S =


$\frac{R}{2}\sigma_y - \phi\,\sigma_z - \left(-\frac{R}{2}\right)\sigma_y$
$\phi\,\sigma_x$
$S_{ij} =$


|0⟩ = k_x > 0
|1⟩ = k_x < 0
Describe as a map
|0⟩
|1⟩
W
x
y


ρout = U ρin U^-1
t=r
I0
O
H
I(t)
t≠r


|0> = k_x > 0
|1> = k_x < 0
Describe as a map

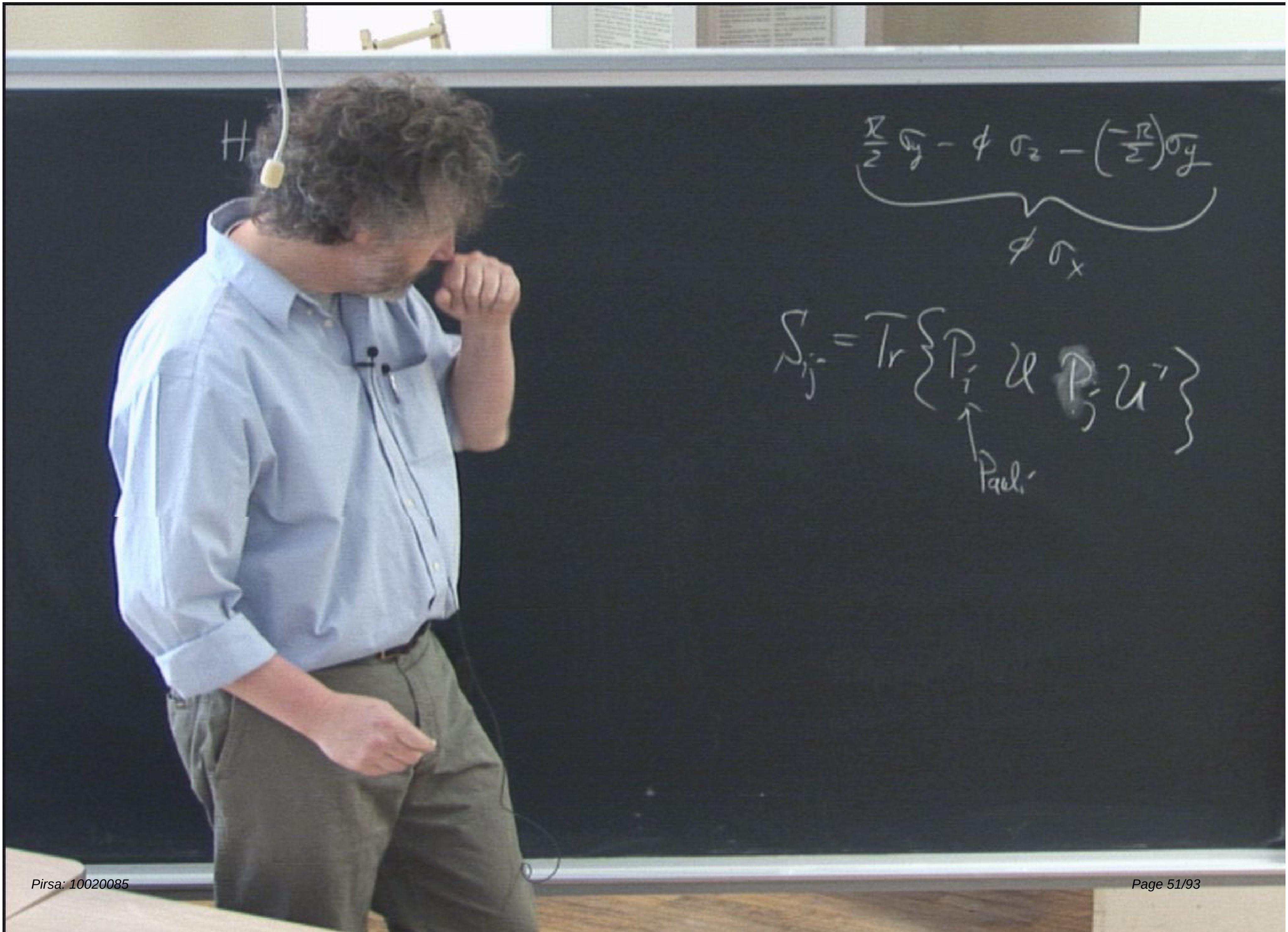
$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & \sin\phi & \cos\phi \end{pmatrix}$$

$$\mathbb{1} \to \mathbb{1}$$

$$\sigma_x \to \sigma_x$$

$$\sigma_y \to \sigma_y \cos\phi + \sigma_z \sin\phi$$

$$\sigma_z \to \sigma_z \cos\phi - \sigma_y \sin\phi$$


H
$\phi\,\sigma_x$
$S_{ij} = \mathrm{Tr}\{P_i\, U\, P_j\, U^{-1}\}$
Pauli

Ha#1

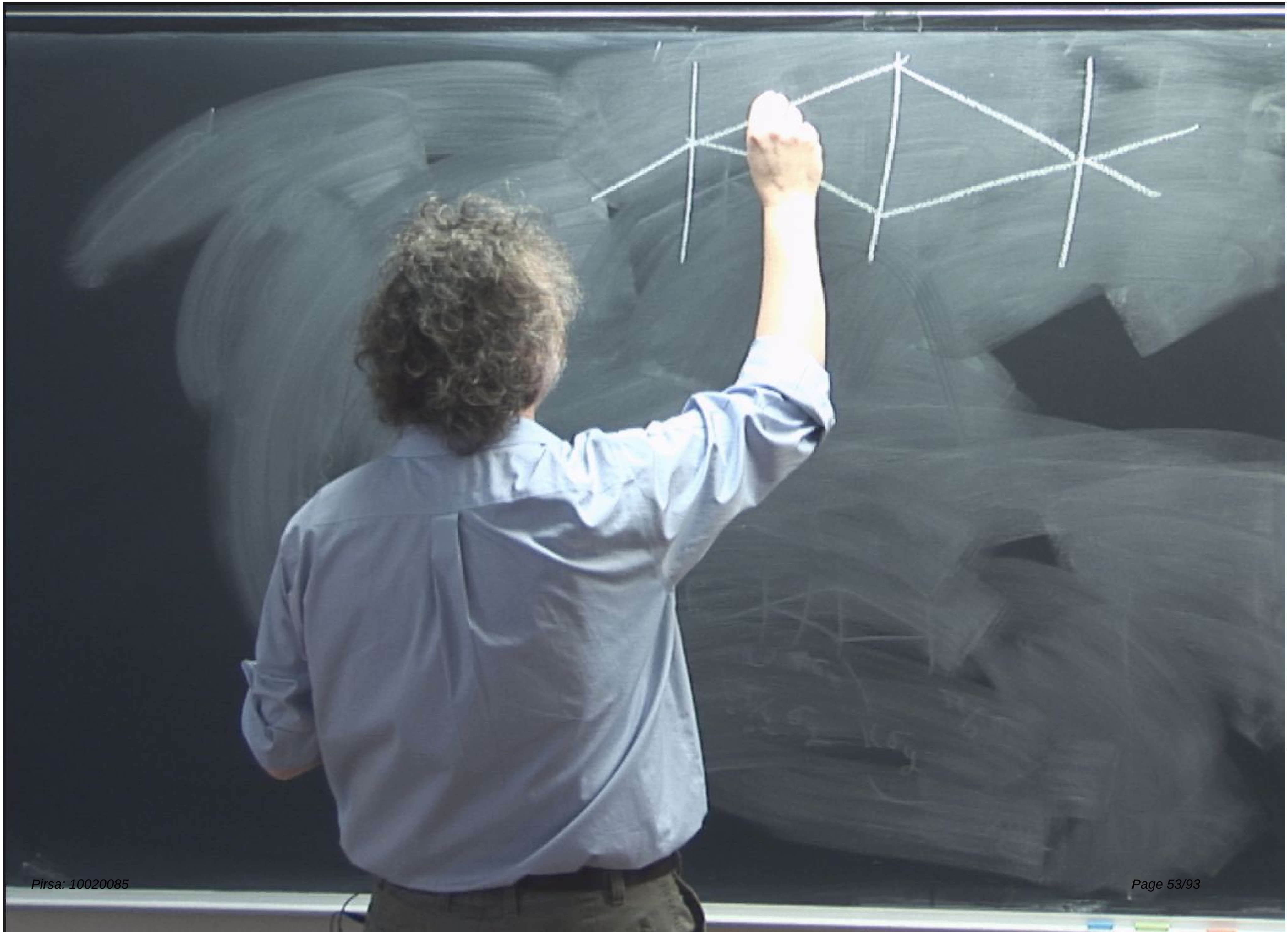
$$S_{ij} = Tr\left\{P_i \sum_k P_k U_k P_j U_k^{-1}\right\} \quad \underbrace{\frac{\pi}{2}\sigma_y - \phi\,\sigma_z - \left(-\frac{\pi}{2}\right)\sigma_y}_{\phi\,\sigma_x}$$

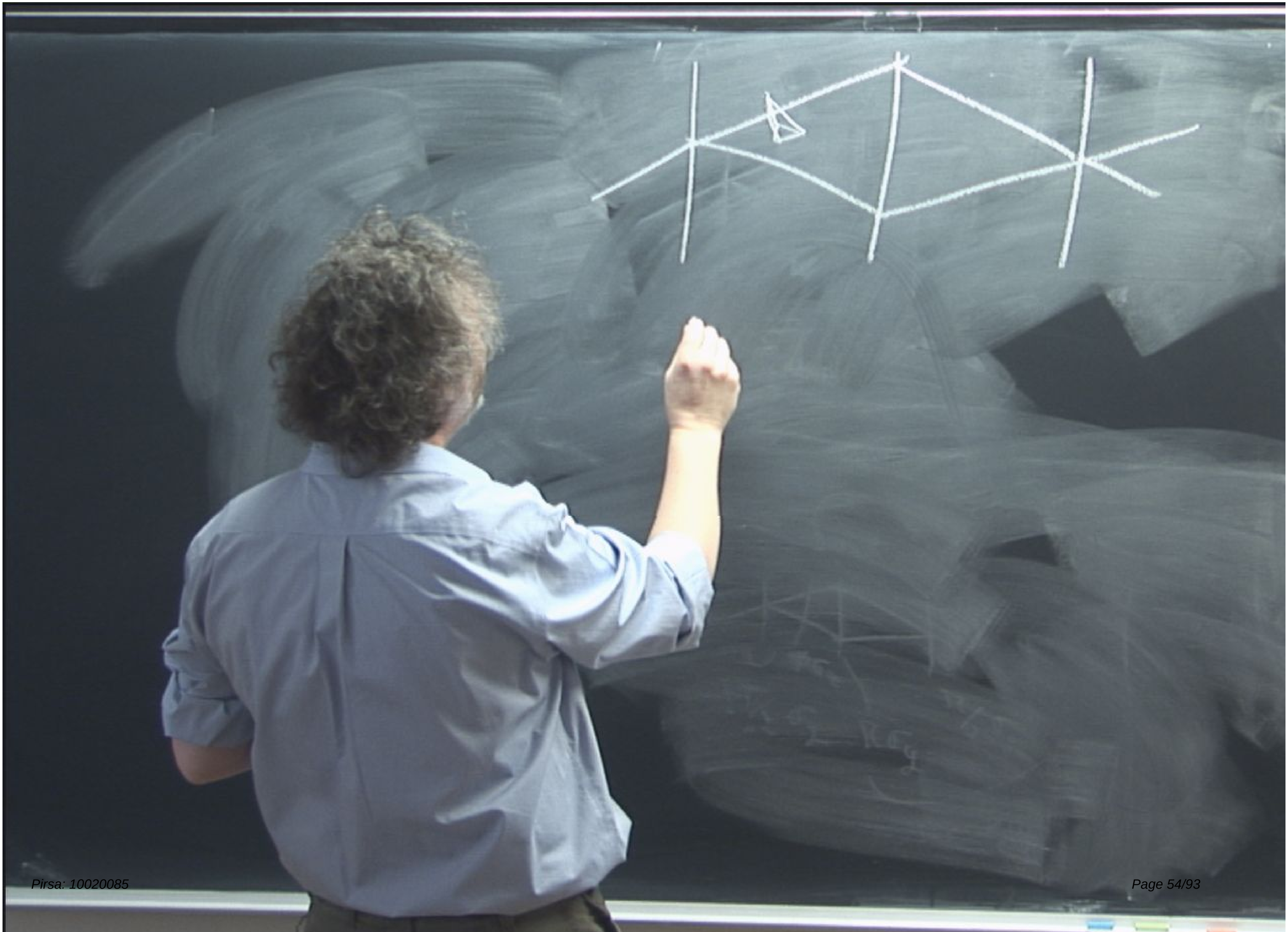
Probability

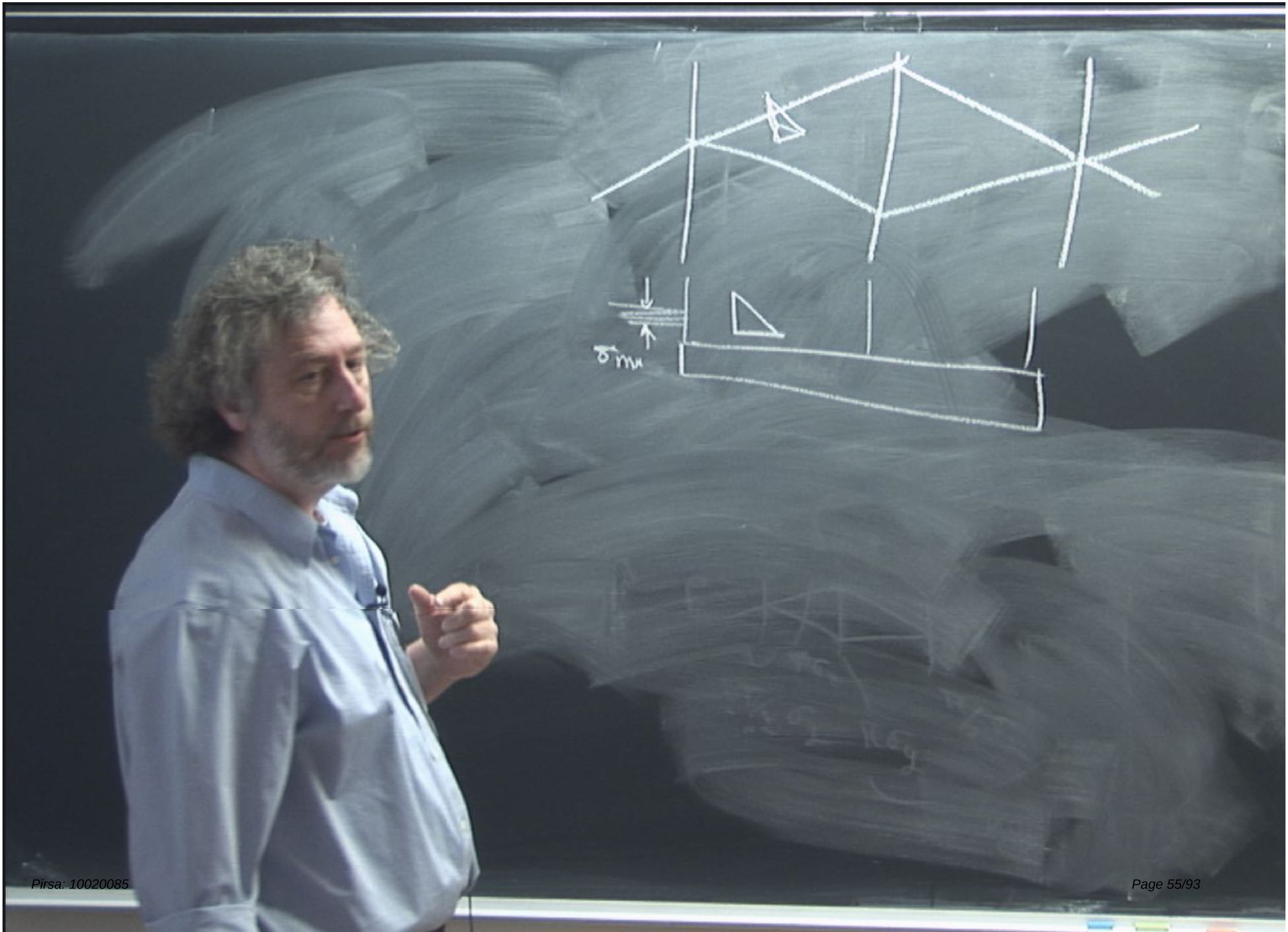
$$\sum_k P_k = 1$$

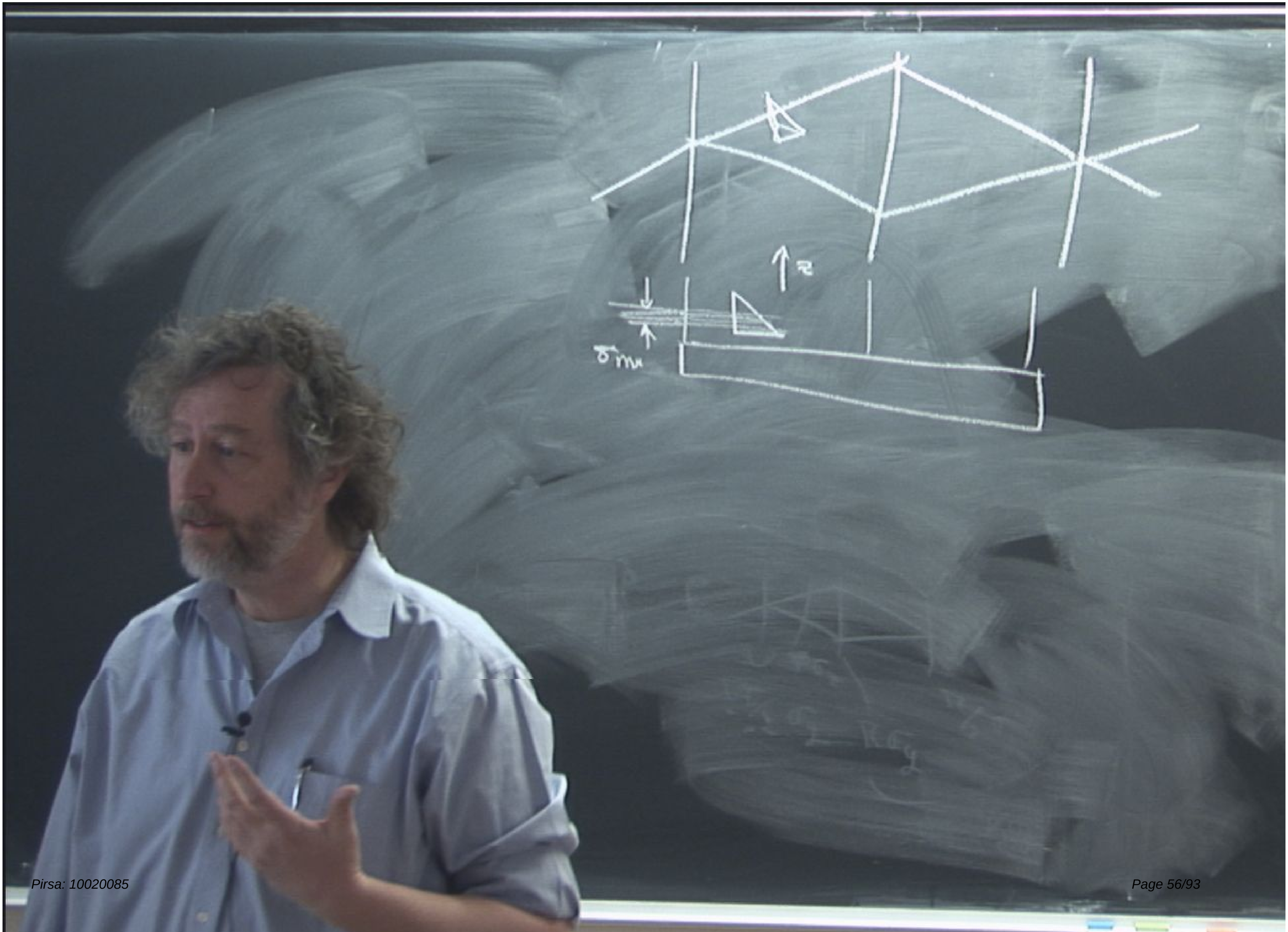
$$S_{ij} = Tr\left\{P_i\, U P_j U^{-1}\right\}$$

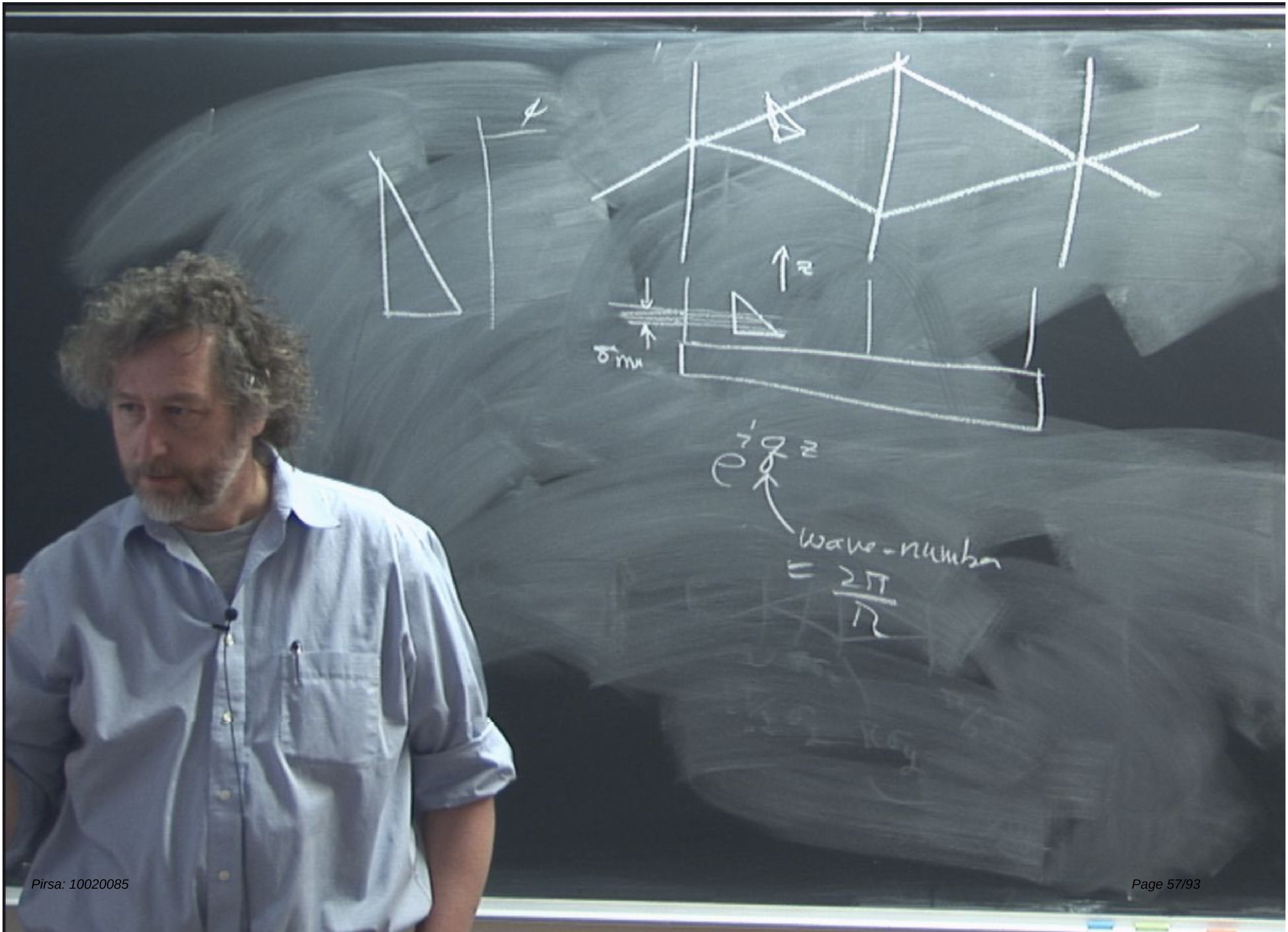
Pauli

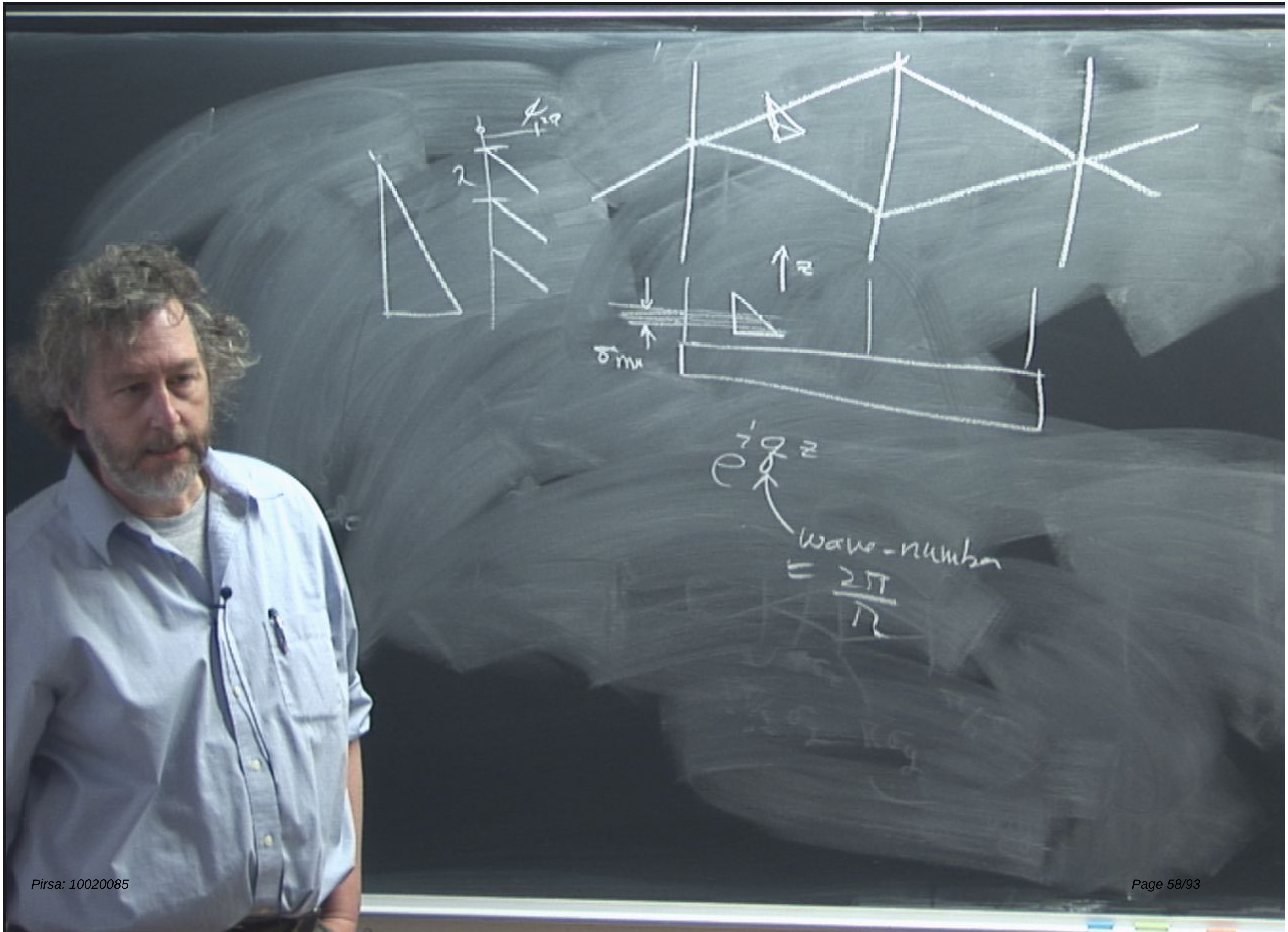


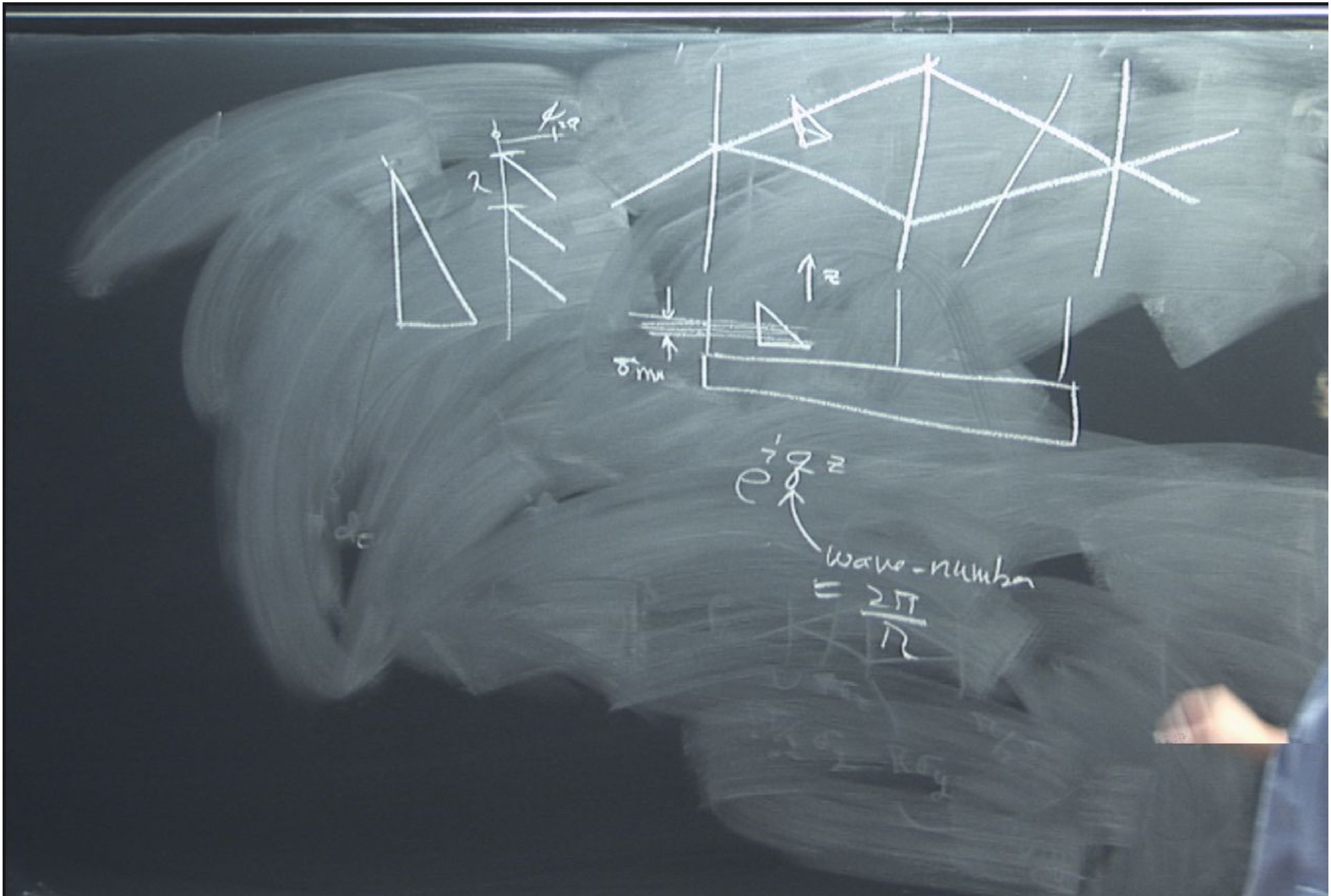


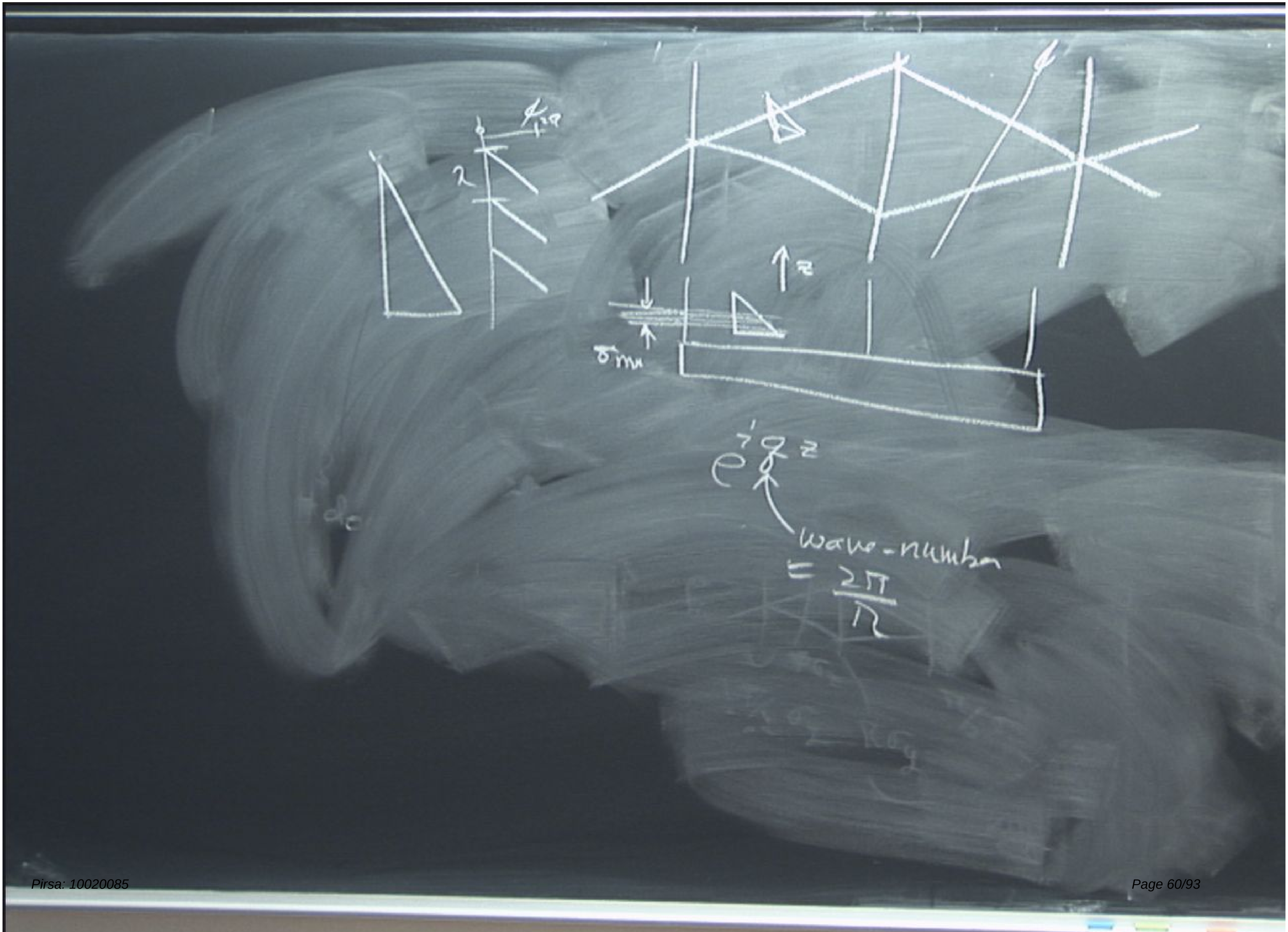


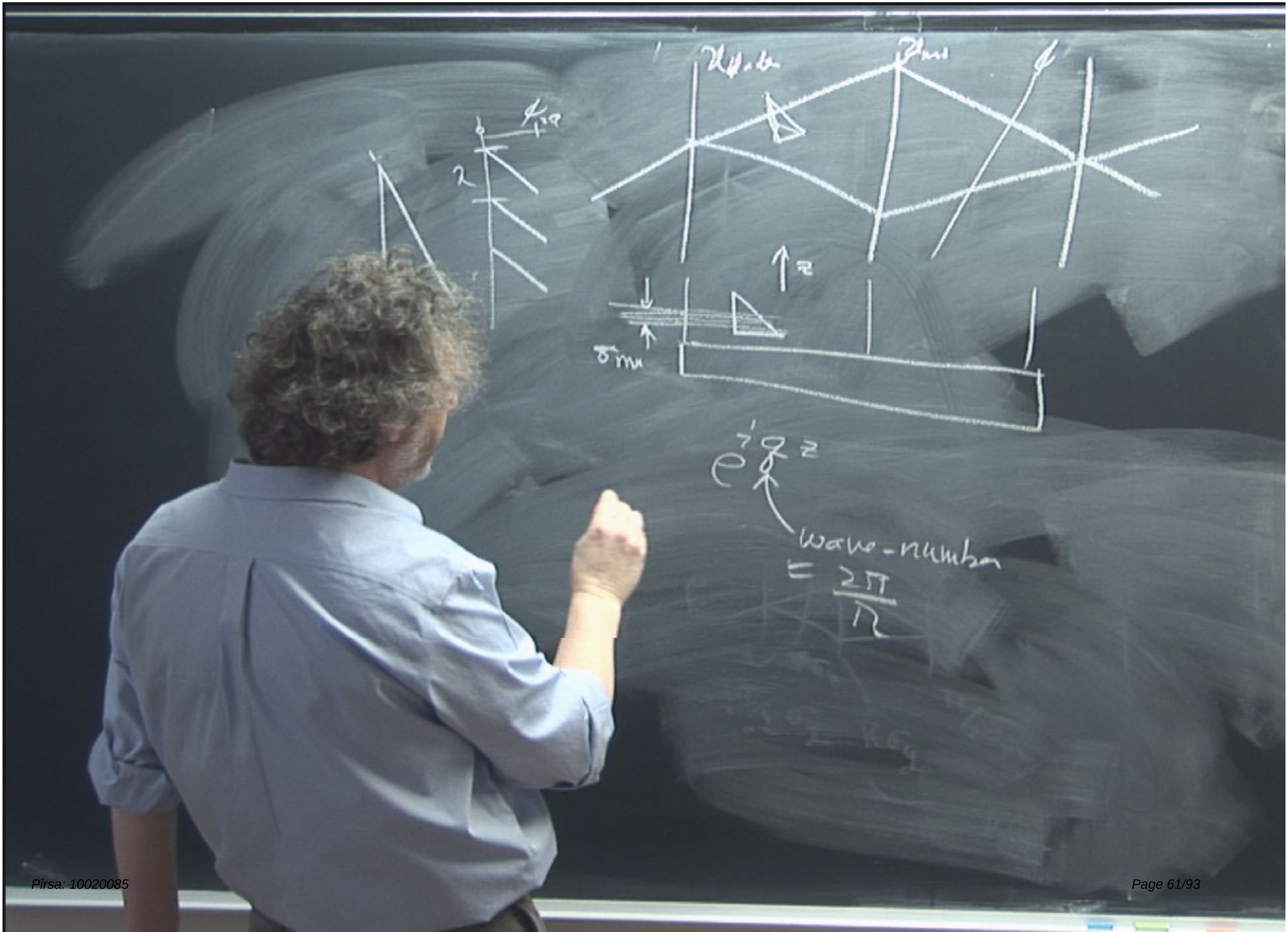


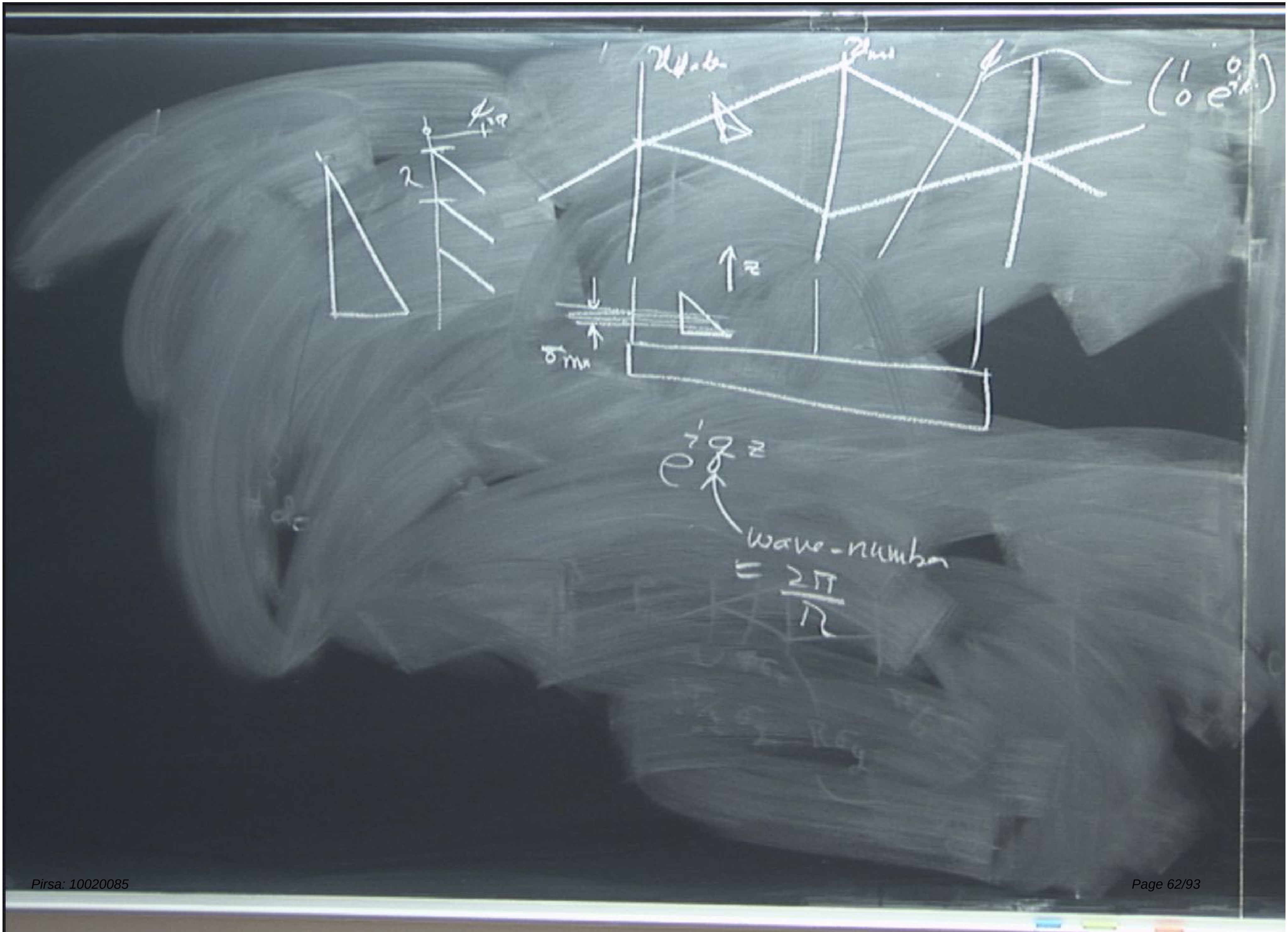

wave-number
= 2π


wave-number


wave-number


λ
z
e^{iqz}
wave-number
= 2π/λ


wave-number
= 2π/λ


wave-number
= 2π/λ

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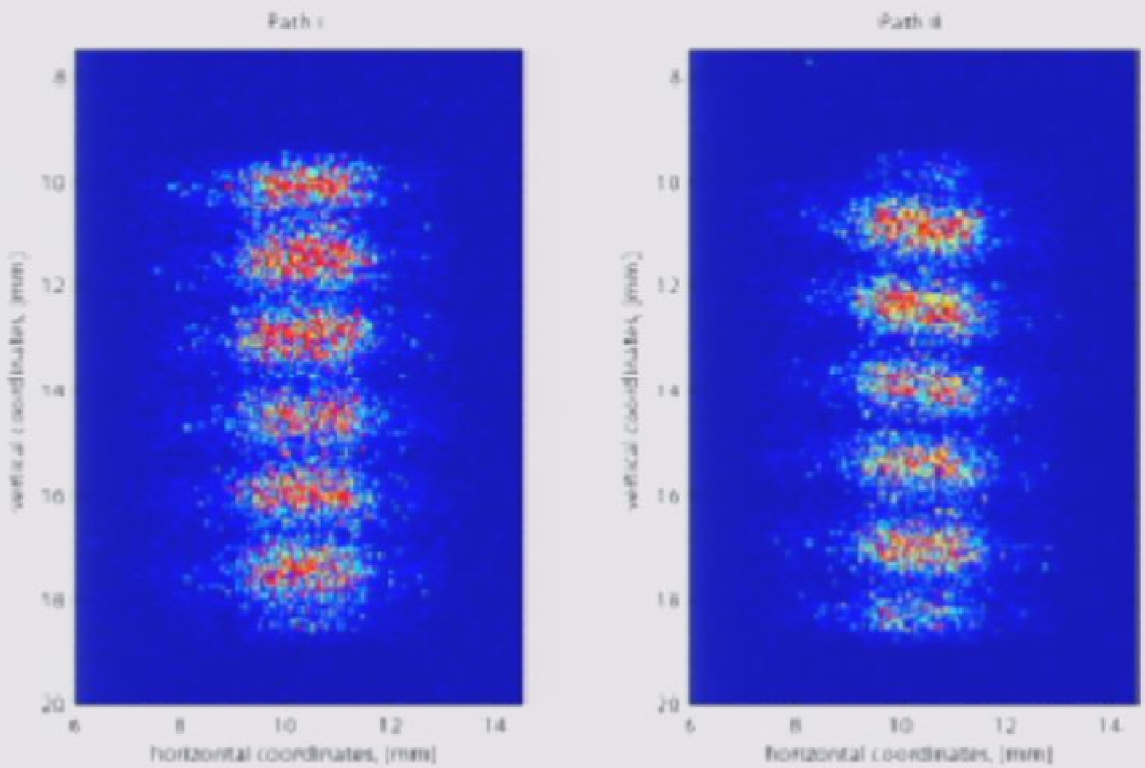
#2.nb

ce that as the position sensitive detector is moved from one path to the other that the fringes are complementary.



;

- Problem 20: Why are the fringes tilted, and why are the O and H fringes complementary. How can you show from this that the state is an incoherent distribution of pure states?

ghth Experiment: Refocus the phase distribution by placing a wedge in each path

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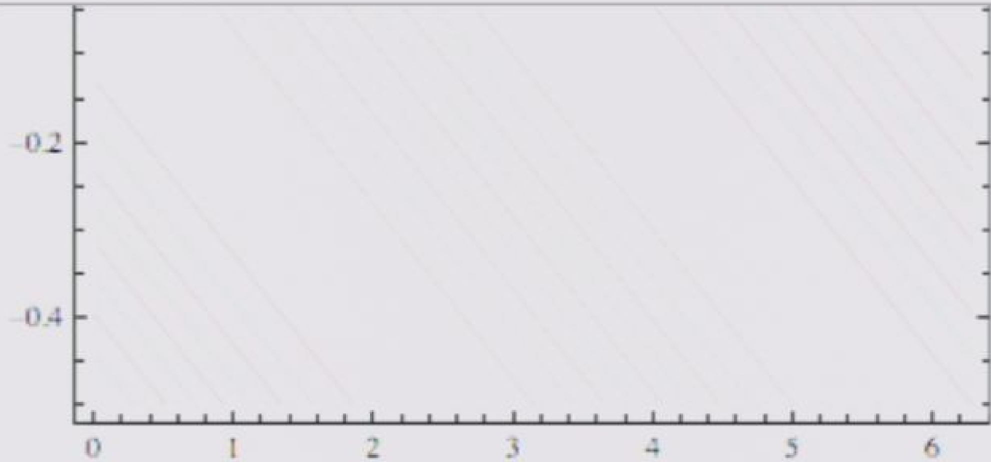
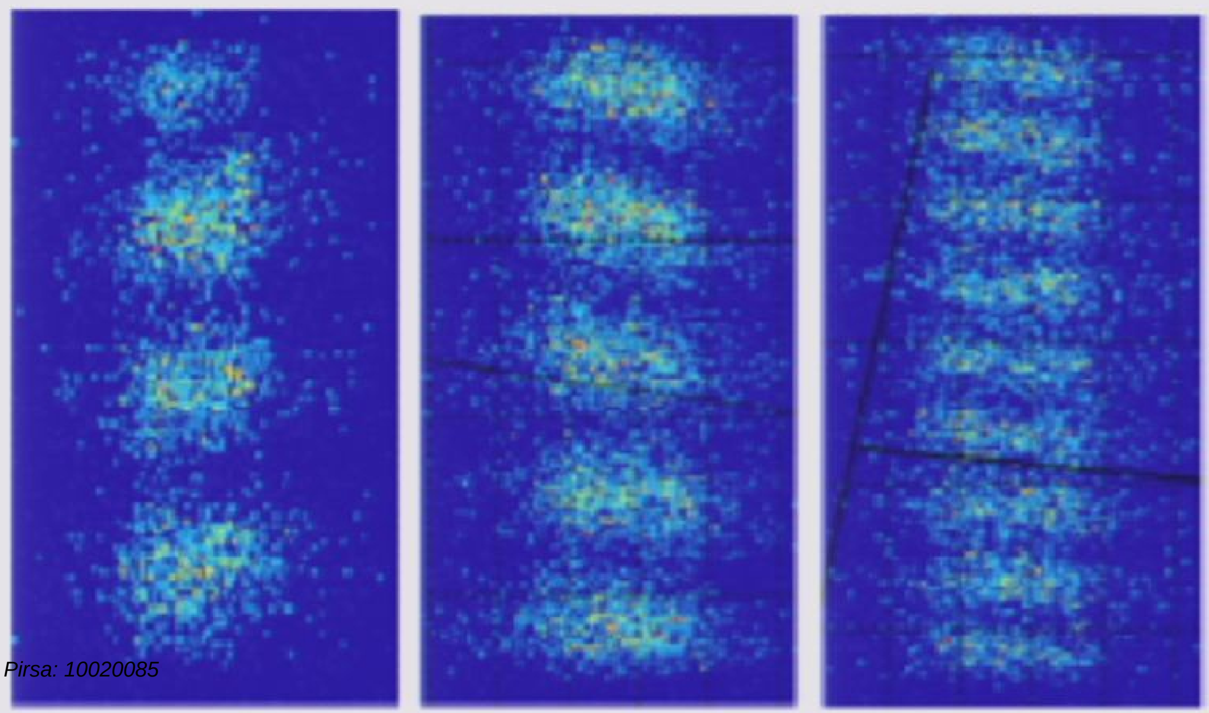
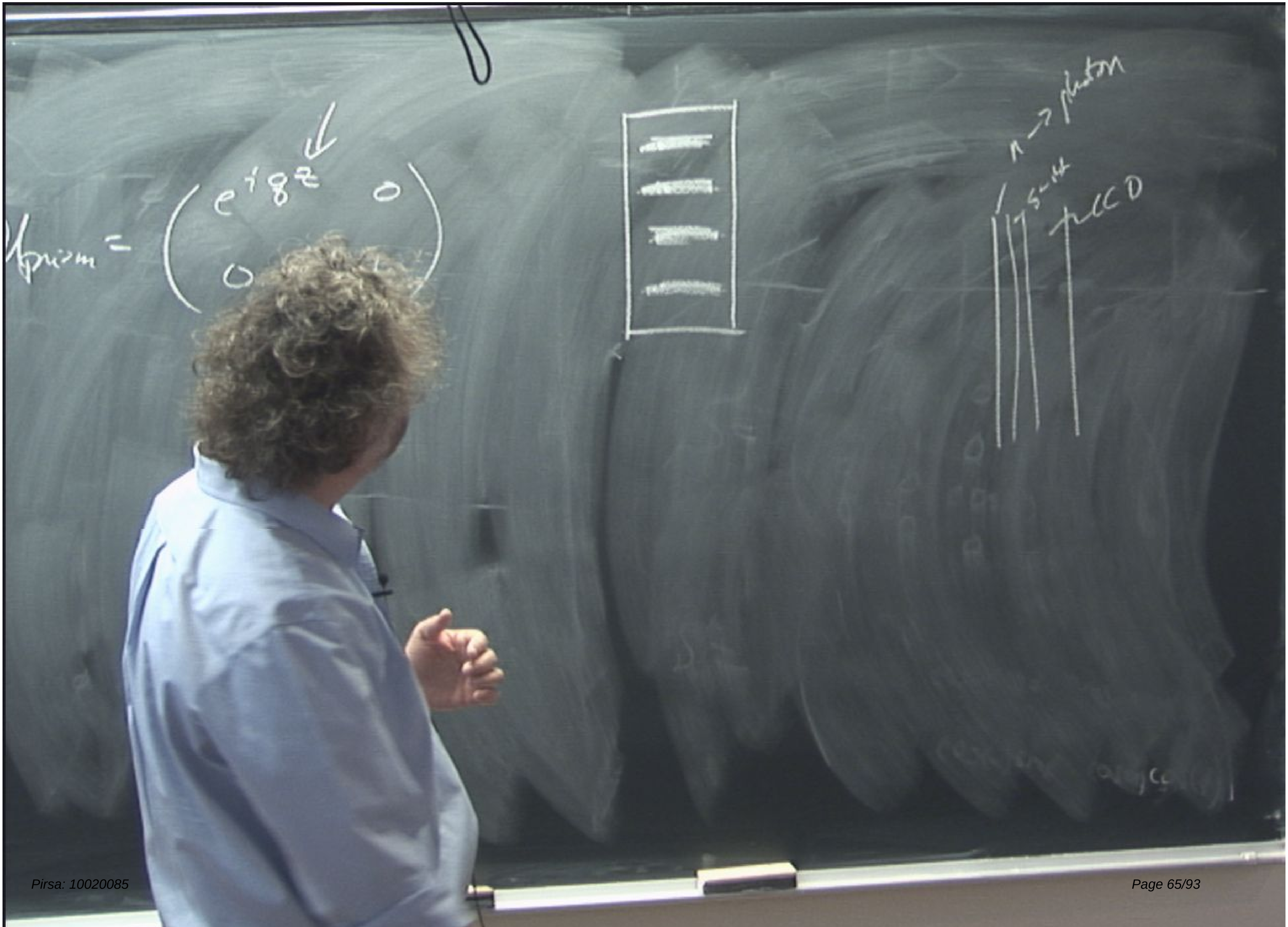
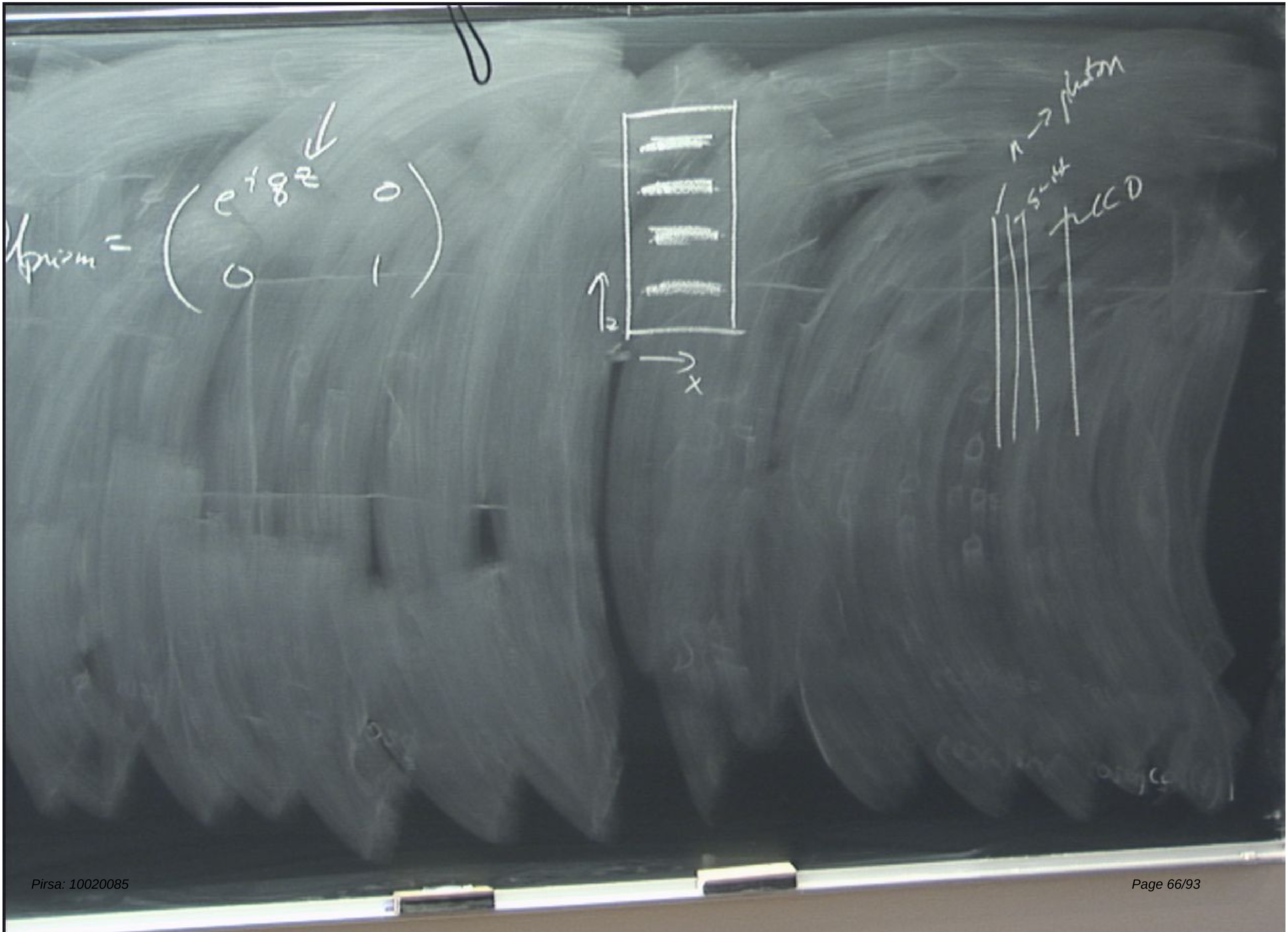
#2.nb

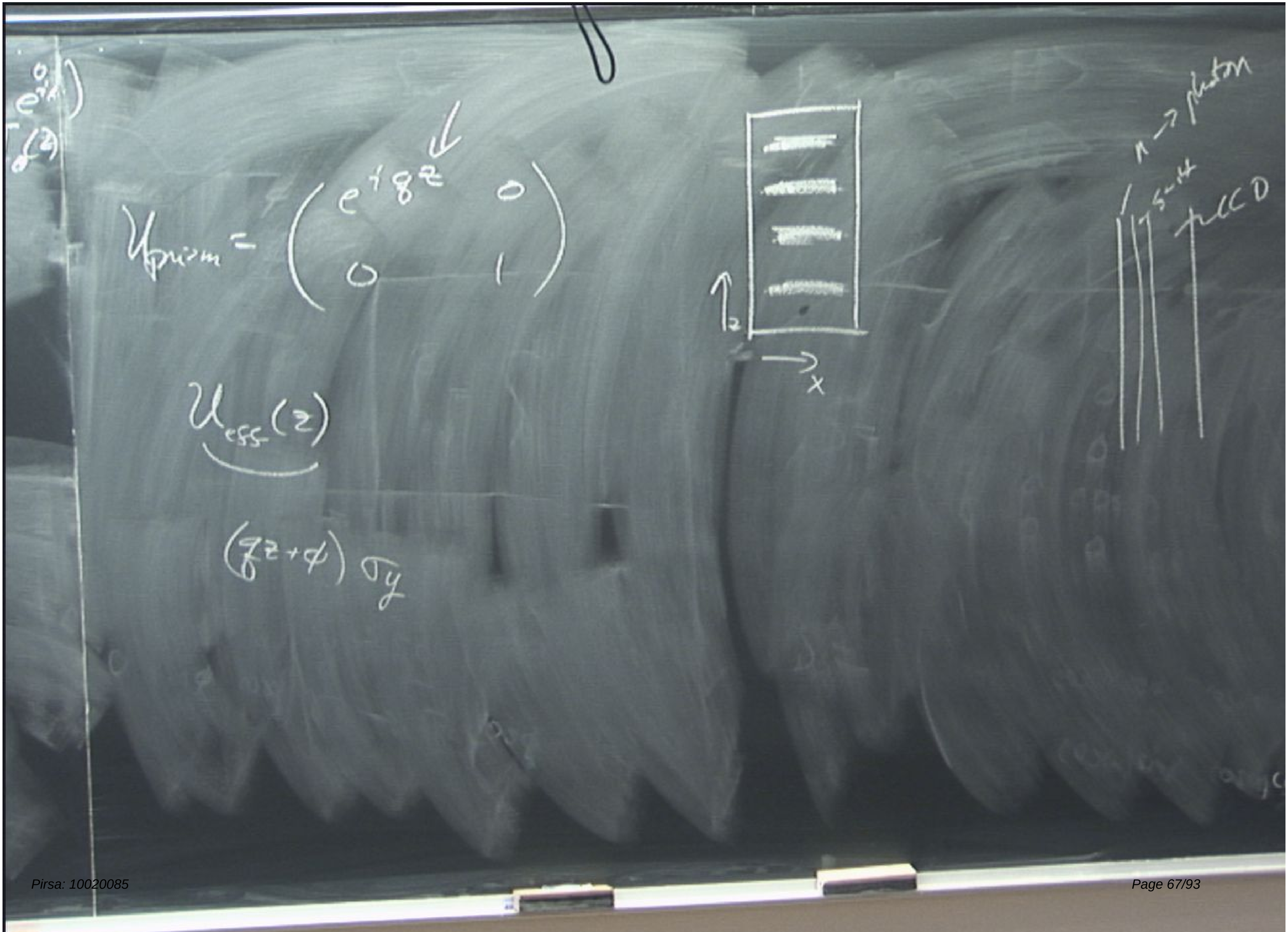




;


photon
CCD


U_prism = (e^{igz} 0 / 0 1)
n → photon
CCD
z
x


U_prism = (e^{iqz} 0 ; 0 1)
U_ess(z)
(qz+φ) σ_y
z
x
n → photon
CCD

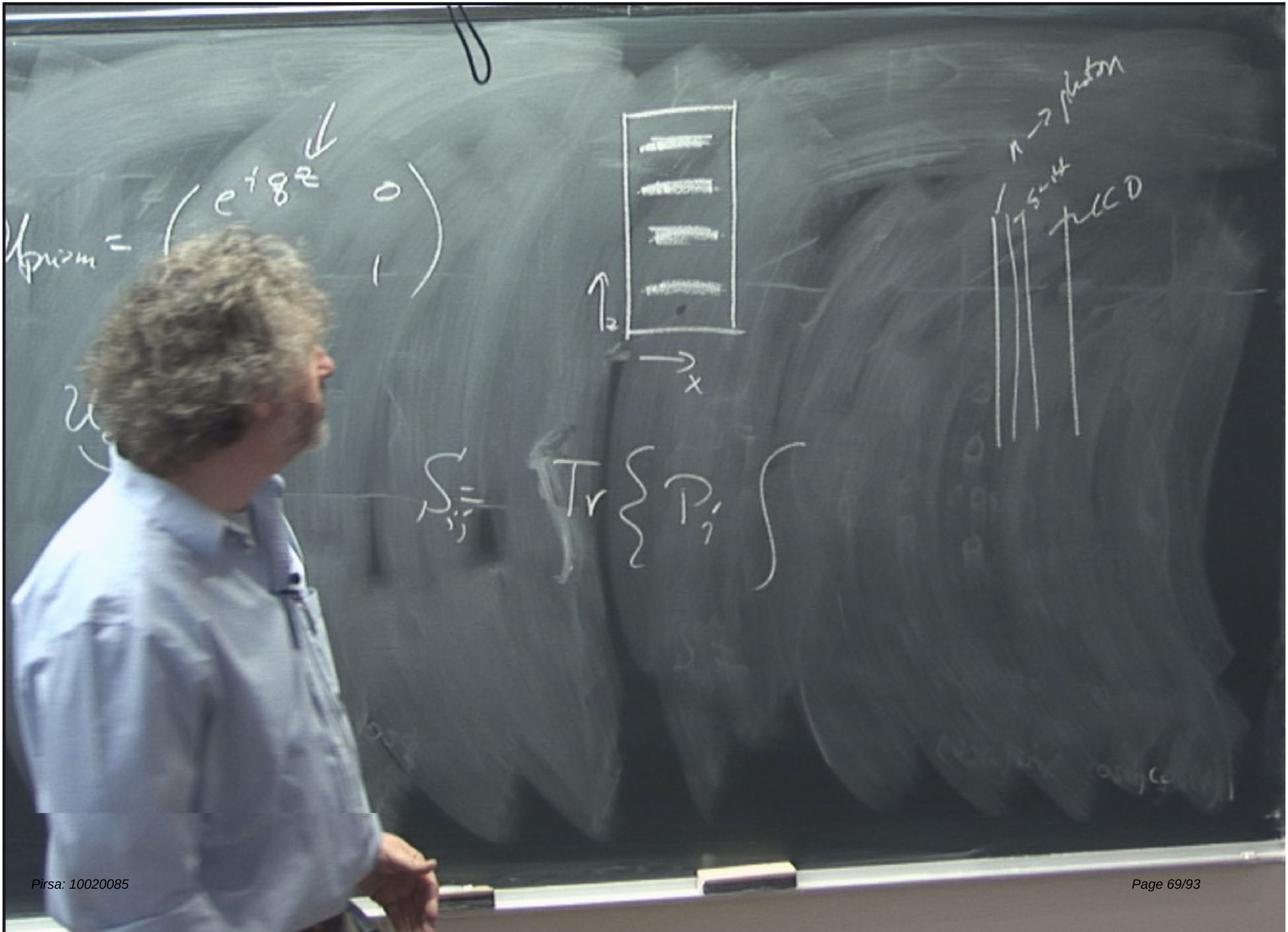
H a = 1

$$S_{ij} = Tr\left\{P_i \sum_k p_k U_k P_j U_k^{-1}\right\} \qquad \underbrace{\frac{\pi}{2}\sigma_y - \phi\,\sigma_z - \left(-\frac{\pi}{2}\right)\sigma_y}_{\phi\,\sigma_x}$$

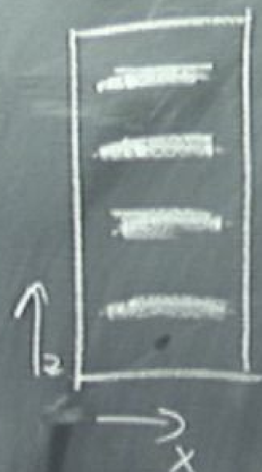
probability

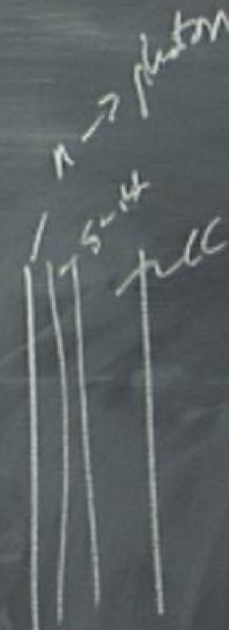
$$\sum_k{}' p_k = 1 \qquad S_{ij} = Tr\left\{P_i\, U\, P_j\, U^{-1}\right\}$$

Pauli


photon
CCD

$$U_{prism} = \begin{pmatrix} e^{iqz} & 0 \\ 0 & 1 \end{pmatrix}$$





$U_{eff}(z)$

$(qz+\phi)\,\sigma_y$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \int_{\Omega} dz\, U_{eff}(z)\, P_j\, U^{\dagger}_{eff}(z) \right\}$$

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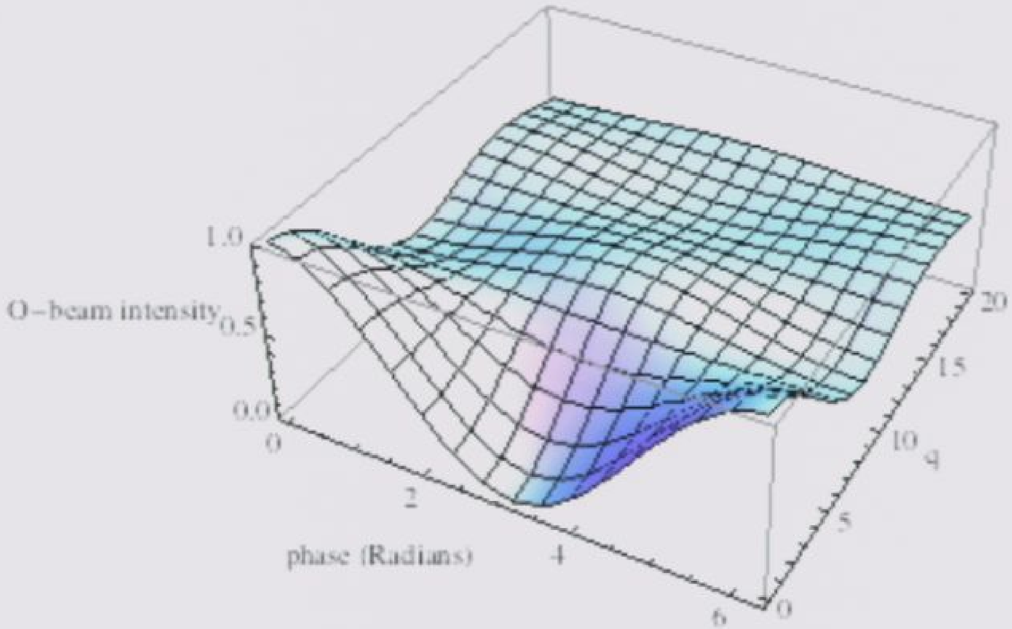
#2.nb

that the q dependence is a sinc function (the Fourier transform of a TopHat function).

```
Plot3D[M6O[q, a], {a, 0, 2 π}, {q, 0, 20},
 {AxesLabel → { "phase (Radians)", "q", "O-beam intensity"}, PlotRange → {0, 1}}]
```




```
M6H[q_,  a_] := Integrate[Tr[Ezm . res6[q, z, a]], {z, -0.5, 0.5}]
```

```
M6H[q, a]
```

$$\frac{0.5\,q + 0.5\,\mathrm{Sin}[a - 0.5\,q] - 0.5\,\mathrm{Sin}[a + 0.5\,q]}{q}$$

```
Plot3D[M6H[q, a], {a, 0, 2 π}, {q, 0, 20},
 {AxesLabel → { "phase (Radians)", "q", "O-beam intensity"}, PlotRange → {0, 1}}]
```

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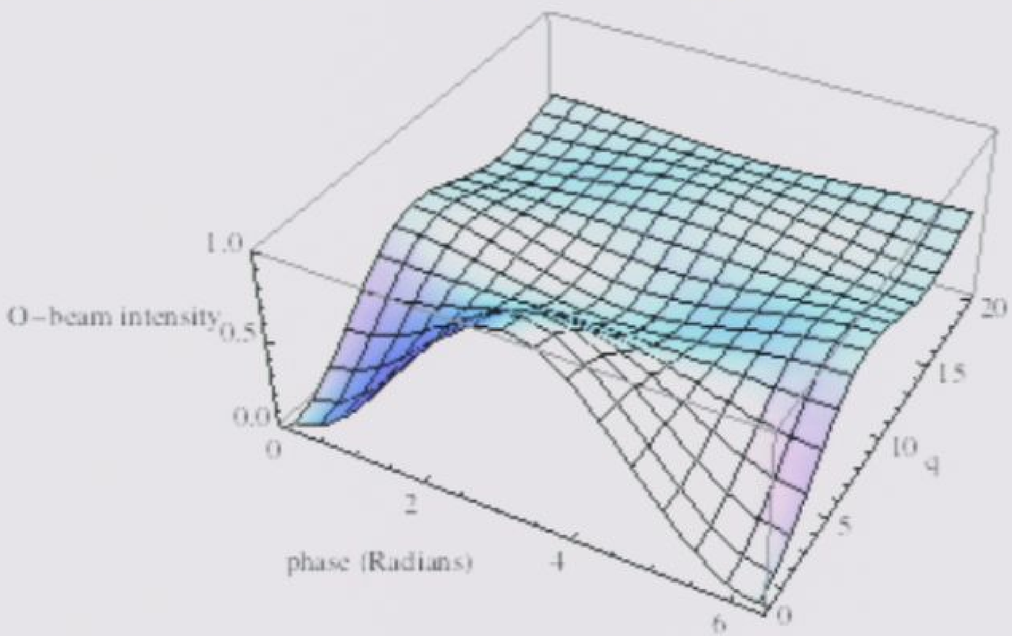
#2.nb

$$\frac{0.5\,q + 0.5\,\mathrm{Sin}[a - 0.5\,q] - 0.5\,\mathrm{Sin}[a + 0.5\,q]}{q}$$

```
Plot3D[M6H[q, a], {a, 0, 2 π}, {q, 0, 20},
 {AxesLabel → { "phase (Radians)", "q", "O-beam intensity"}, PlotRange → {0, 1}}]
```



see that as expected all neutrons are detected at one beam or the other, but as the wave-number of the prism is
ased the contrast vanishes.

```
Simplify[Refine[TrigReduce[M6H[q, a] + M6O[q, a]]], Element[{a, q}, Reals]]
```

$$\frac{1.\,q + 0.\,\mathrm{Sin}[a - 0.5\,q] + 0.\,\mathrm{Sin}[a + 0.5\,q]}{q}$$

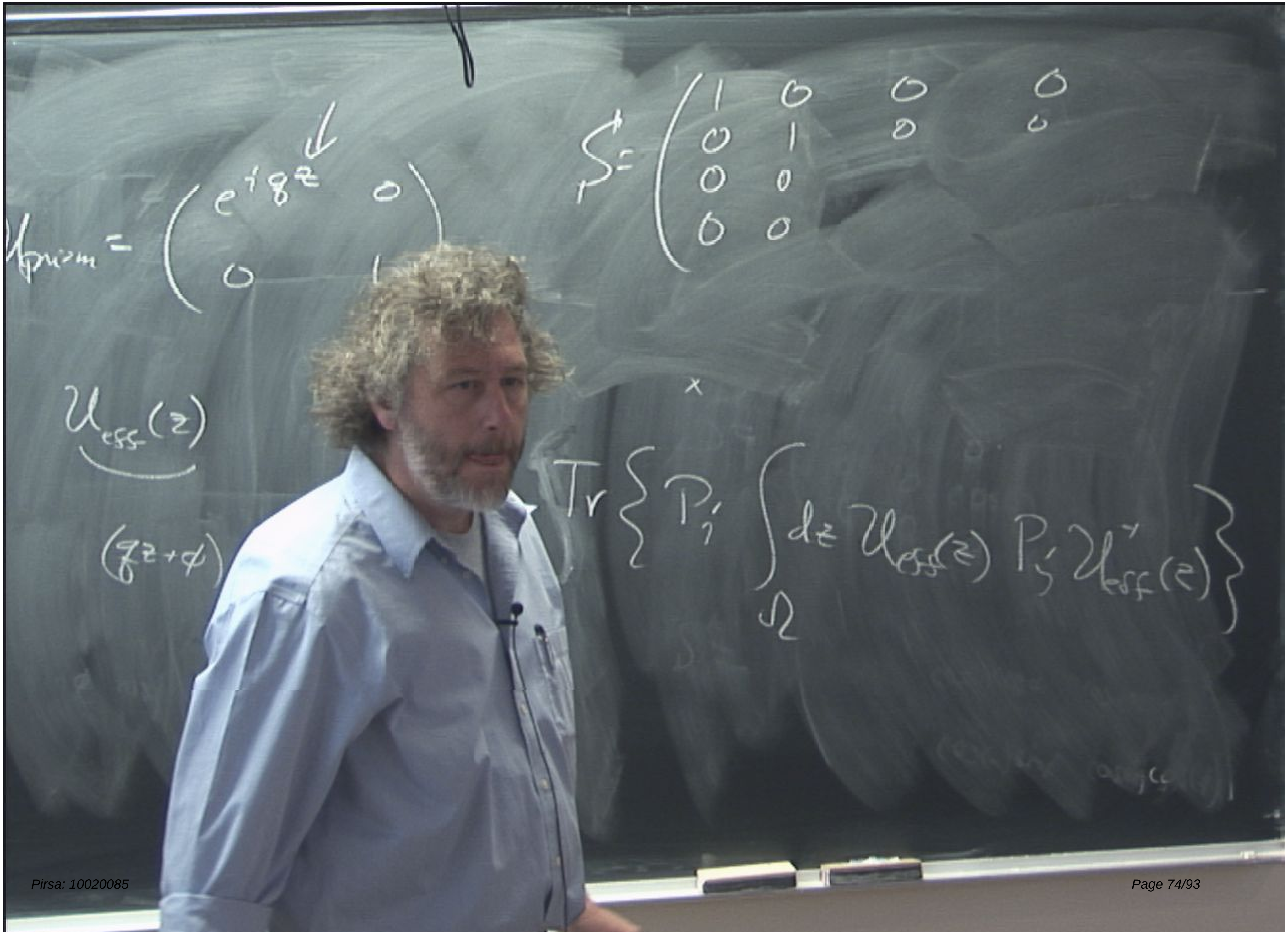
$$U_{prism} = \begin{pmatrix} e^{iqz} & 0 \\ 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

$U_{eff}(z)$

$(qz+\phi)\,\sigma_y$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \int_{\Omega} dz\, U_{eff}(z)\, P_j\, U_{eff}^{-1}(z) \right\}$$


U_prism = (e^{iqz} 0 ; 0 1)
S = (1 0 0 0 ; 0 1 0 0 ; 0 0 ; 0 0)
U_eff(z)
(qz+φ)
Tr{ P_i ∫_Ω dz U_eff(z) P_j U†_eff(z) }

$$U_{prism} = \begin{pmatrix} e^{iqz} & 0 \\ 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 & 0 & & 0 \\ 0 & 1 & 0 & & 0 \\ 0 & 0 & \cos(\phi)\frac{\sin(.58)}{8} & & \\ 0 & 0 & & & \end{pmatrix}$$

$U_{eff}(z)$

$(qz+\phi)\,\sigma_x$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \int_\Omega dz\, U_{eff}(z)\, P_j\, U_{eff}^{\dagger}(z) \right\}$$

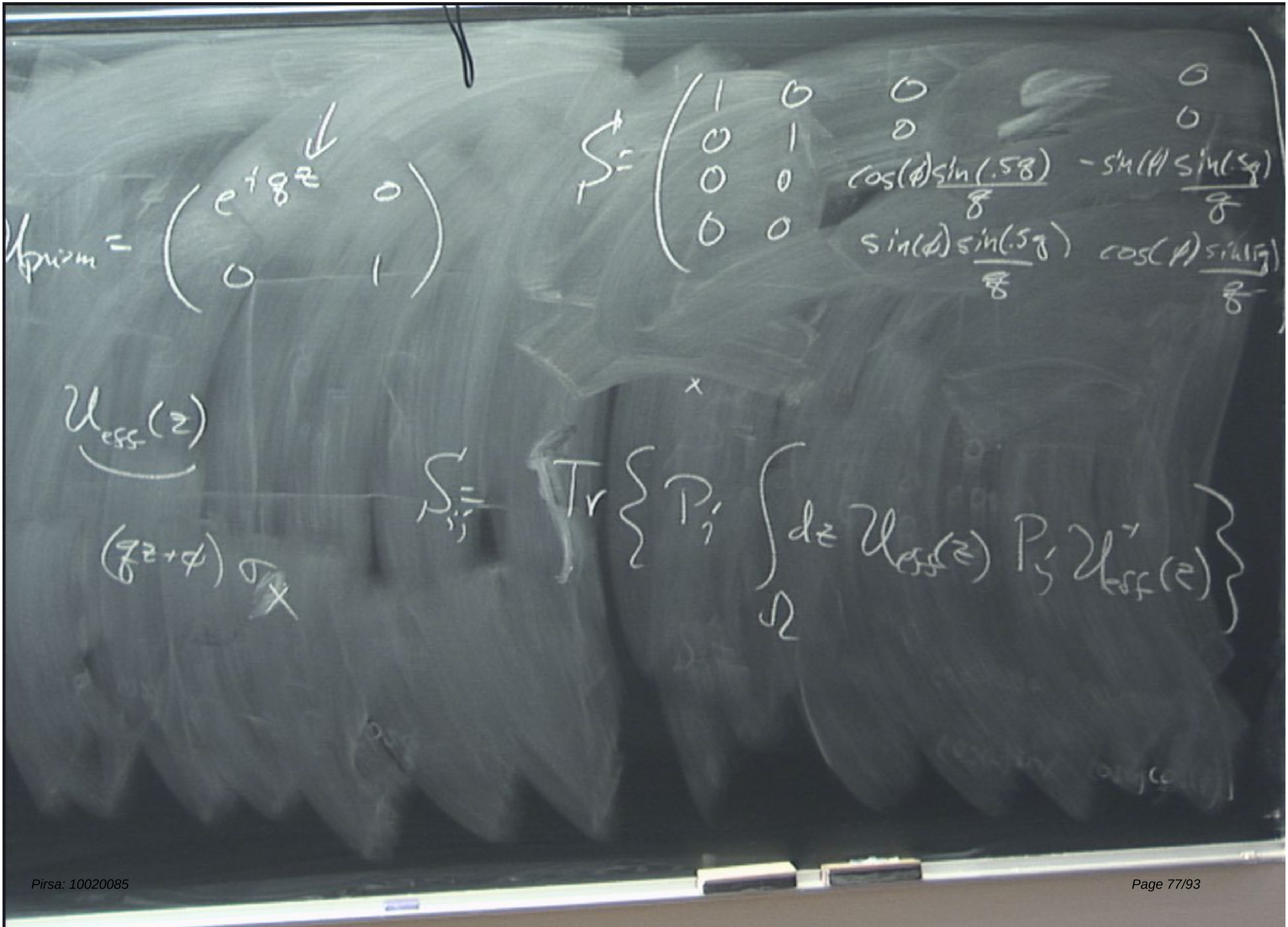
$$U_{prism} = \begin{pmatrix} e^{iqz} & 0 \\ 0 & 1 \end{pmatrix}$$

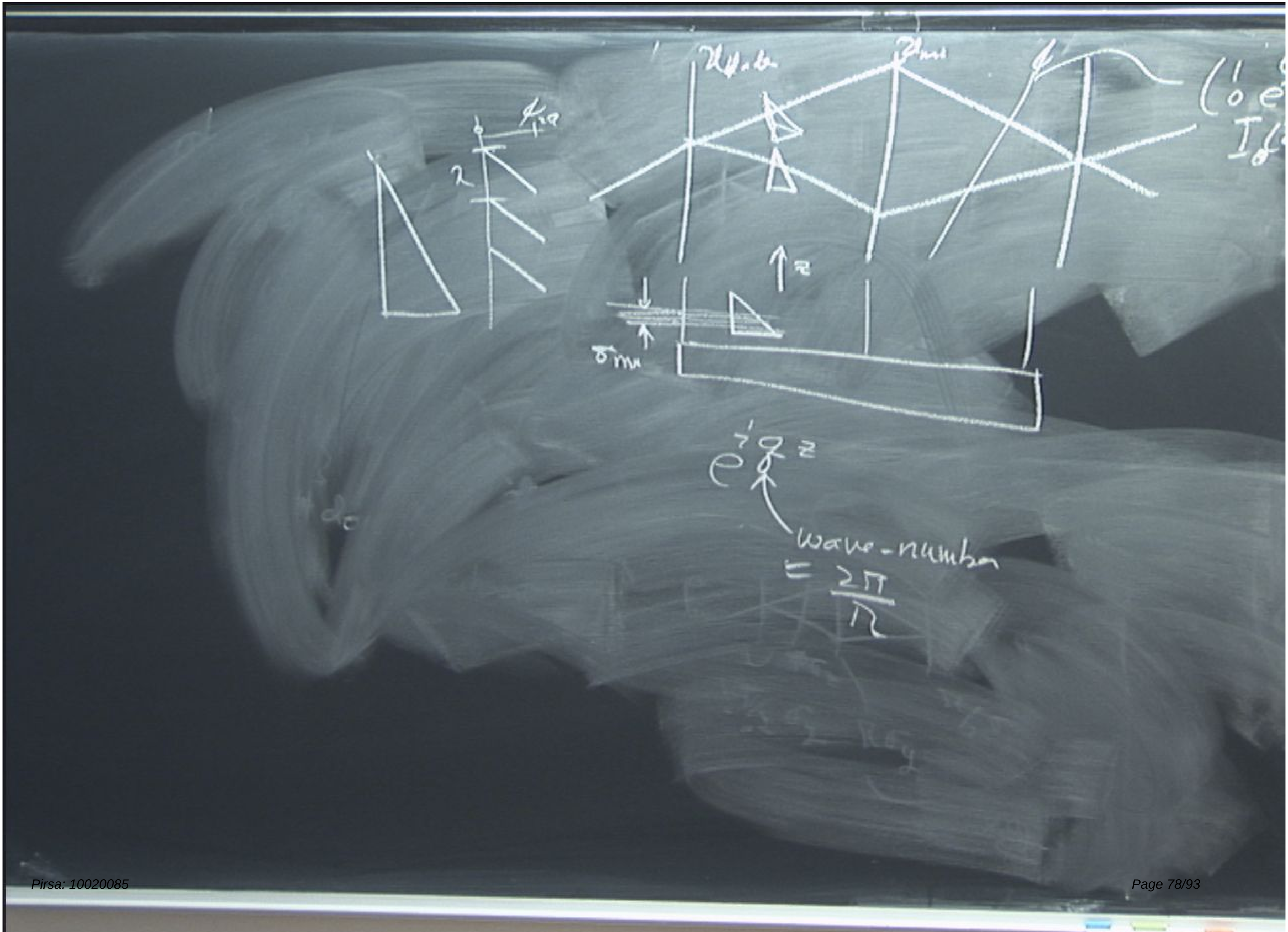
$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\phi)\frac{\sin(.5q)}{q} & -\sin(\phi)\frac{\sin(.5q)}{q} \\ 0 & 0 & \sin(\phi)\frac{\sin(.5q)}{q} & \cos(\phi)\frac{\sin(.5q)}{q} \end{pmatrix}$$

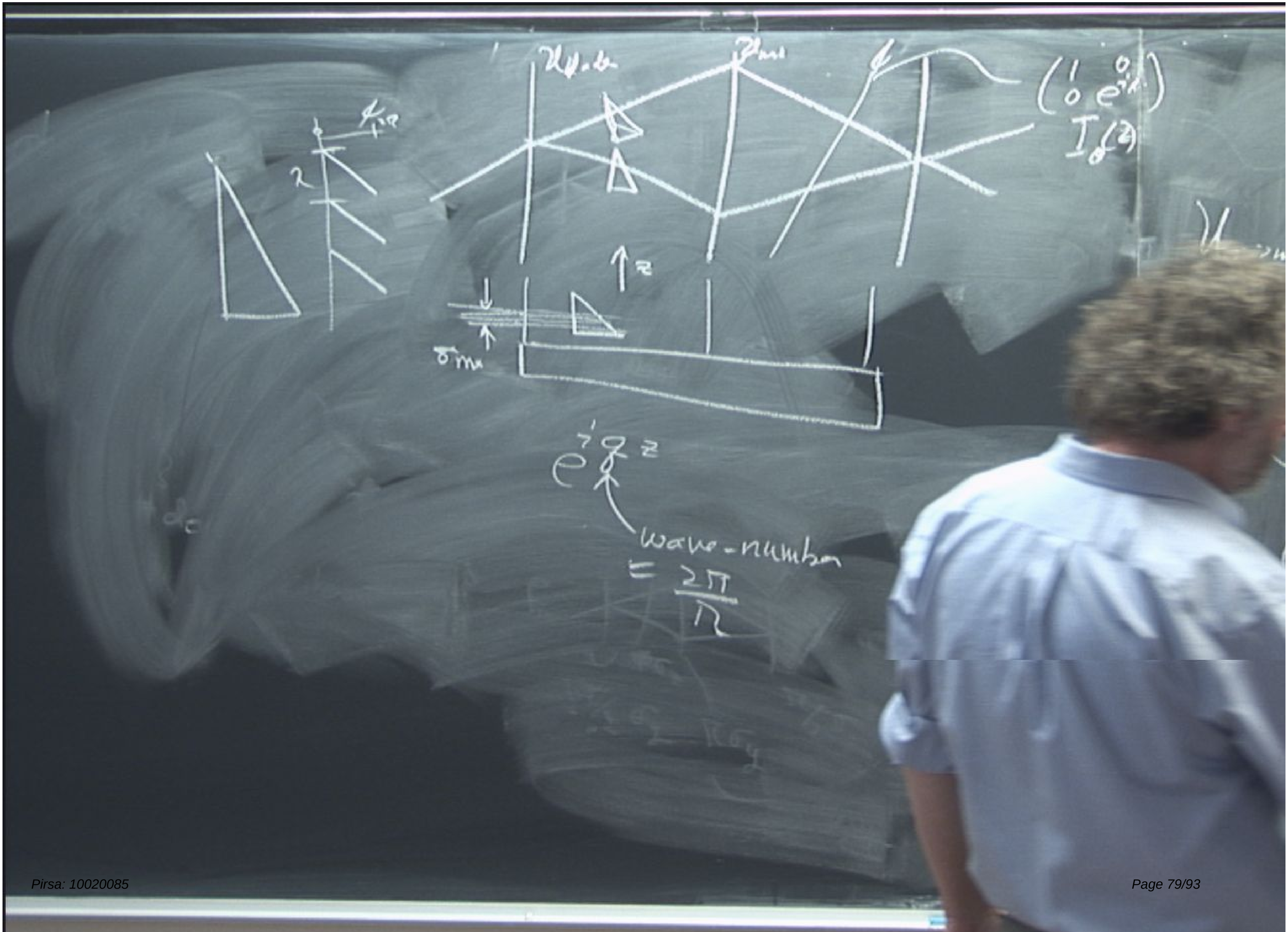
$U_{eff}(z)$

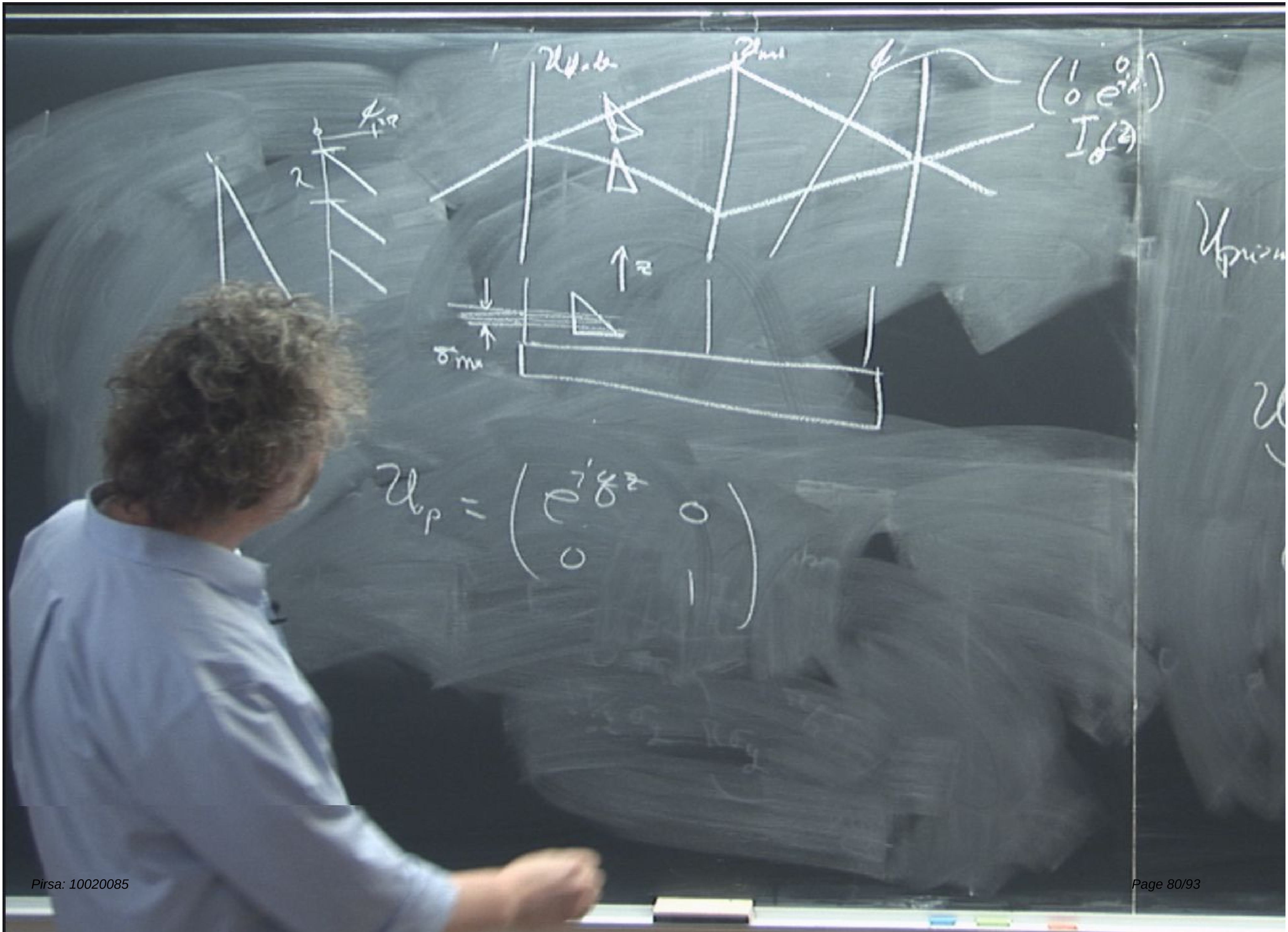
$(qz+\phi)\,\sigma_x$

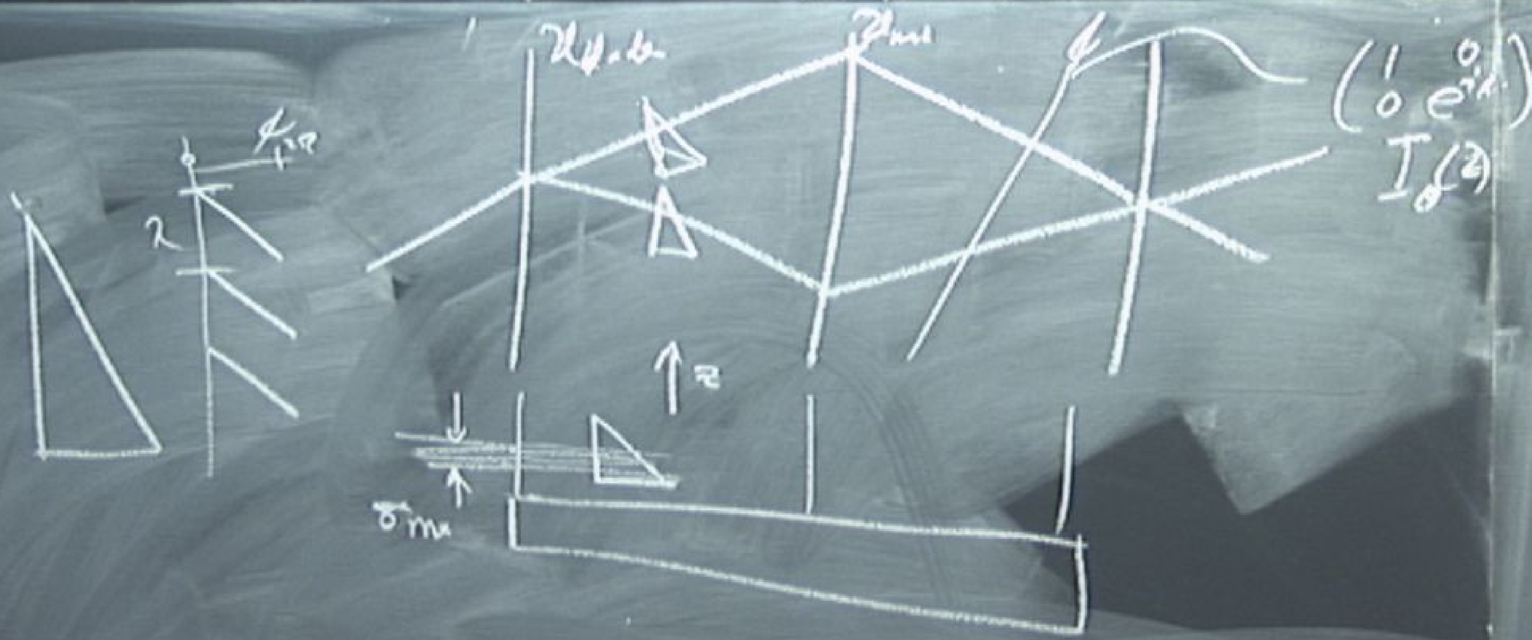
$$S_{ij} = \mathrm{Tr}\left\{ P_i \int_{\Omega} dz\, U_{eff}(z)\, P_j\, U_{eff}^{\dagger}(z) \right\}$$


U_prism = (e^{iqz} 0 ; 0 1)
S = (1 0 0 0 ; 0 1 0 0 ; 0 0 cos(φ) sin(.5q)/q -sin(φ) sin(.5q)/q ; 0 0 sin(φ) sin(.5q)/q cos(φ) sin(.5q)/q)
U_eff(z)
(qz+φ) σ_x
S_ij = Tr{ P_i ∫_Ω dz U_eff(z) P_j U_eff^{-1}(z) }

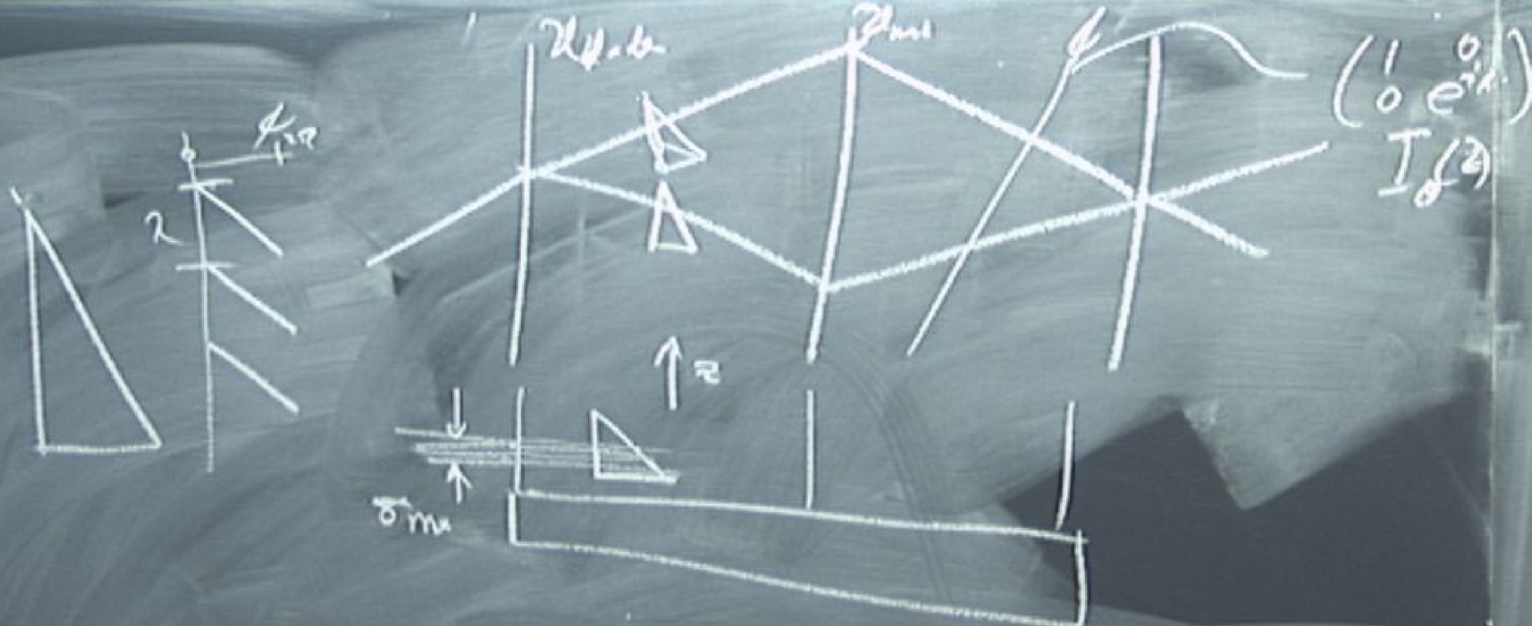

e^{iqz}
wave-number
$= \frac{2\pi}{\lambda}$


wave-number


$U_p = \begin{pmatrix} e^{i\gamma z} & 0 \\ 0 & 1 \end{pmatrix}$

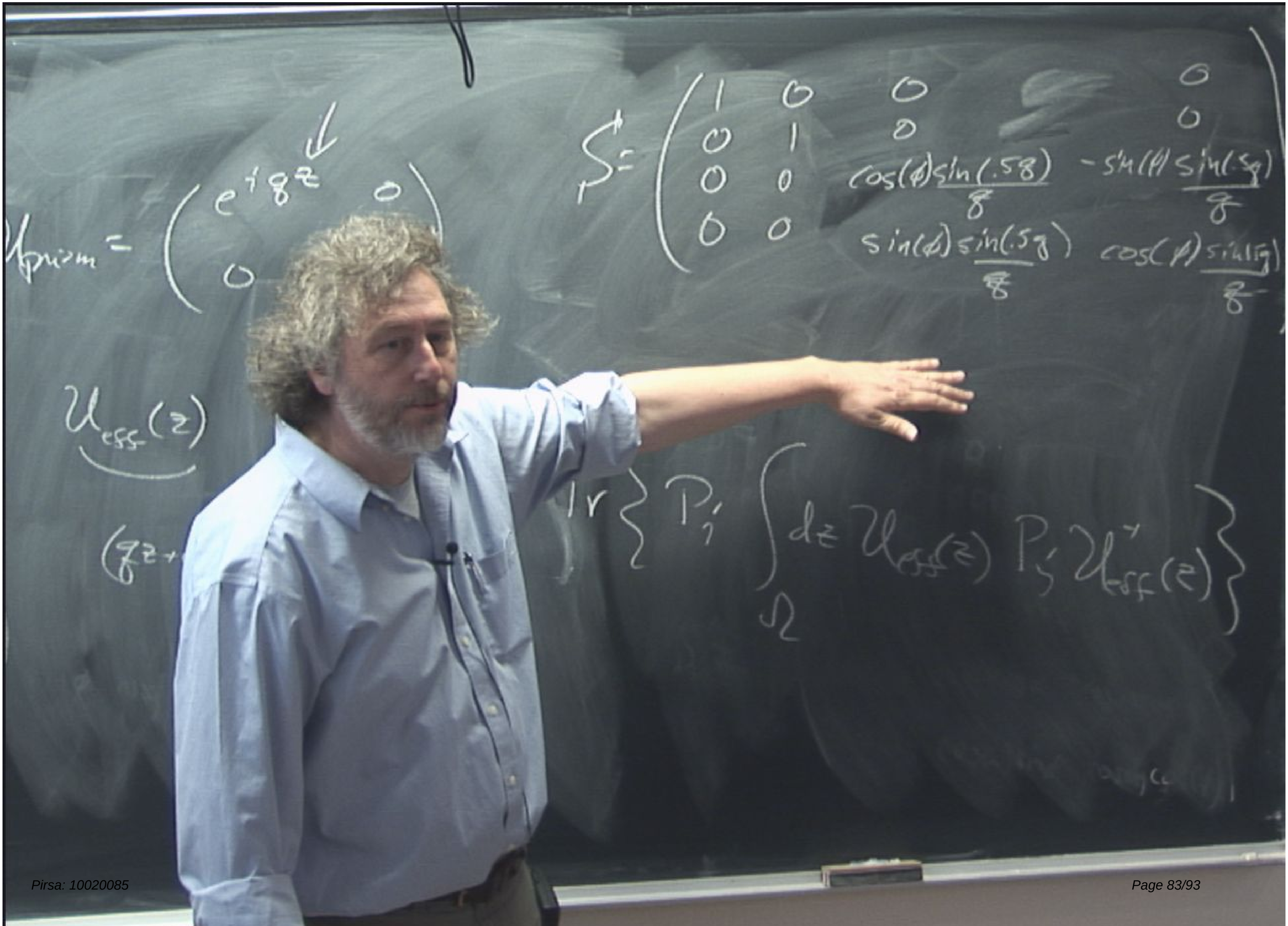


$$U_p = \begin{pmatrix} e^{igz} & 0 \\ 0 & 1 \end{pmatrix} ; \quad U_{down} = \begin{pmatrix} 1 & 0 \\ 0 & e^{ig'z} \end{pmatrix}$$



$$U_p = \begin{pmatrix} e^{igz} & 0 \\ 0 & 1 \end{pmatrix}; \quad U_{down} = \begin{pmatrix} 1 & 0 \\ 0 & e^{ig'z} \end{pmatrix}$$

$$U_{comb} = e^{igz} \begin{pmatrix} 1 & 0 \\ 0 & e^{i(g'-g)z} \end{pmatrix}$$


U_prism = (e^{iqz} 0 ; 0)
S = (1 0 0 0 ; 0 1 0 0 ; 0 0 cos(φ)sin(.5g)/g -sin(φ)sin(.5g)/g ; 0 0 sin(φ)sin(.5g)/g cos(φ)sin(.5g)/g)
U_eff(z)
Tr { P_i ∫_Ω dz U_eff(z) P_j U†_eff(z) }

$$U_{prism} = \begin{pmatrix} e^{iqz} & 0 \\ 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\phi)\frac{\sin(.5g)}{g} & -\sin(\phi)\frac{\sin(.5g)}{g} \\ 0 & 0 & \sin(\phi)\frac{\sin(.5g)}{g} & \cos(\phi)\frac{\sin(.5g)}{g} \end{pmatrix}$$

$U_{eff}(z)$

$(gz+\phi)\,\sigma_x$

$$S_{ij} = \mathrm{Tr}\left\{ P_i \int_{\Omega} dz\, U_{eff}(z)\, P_j\, U_{eff}^{-1}(z) \right\}$$

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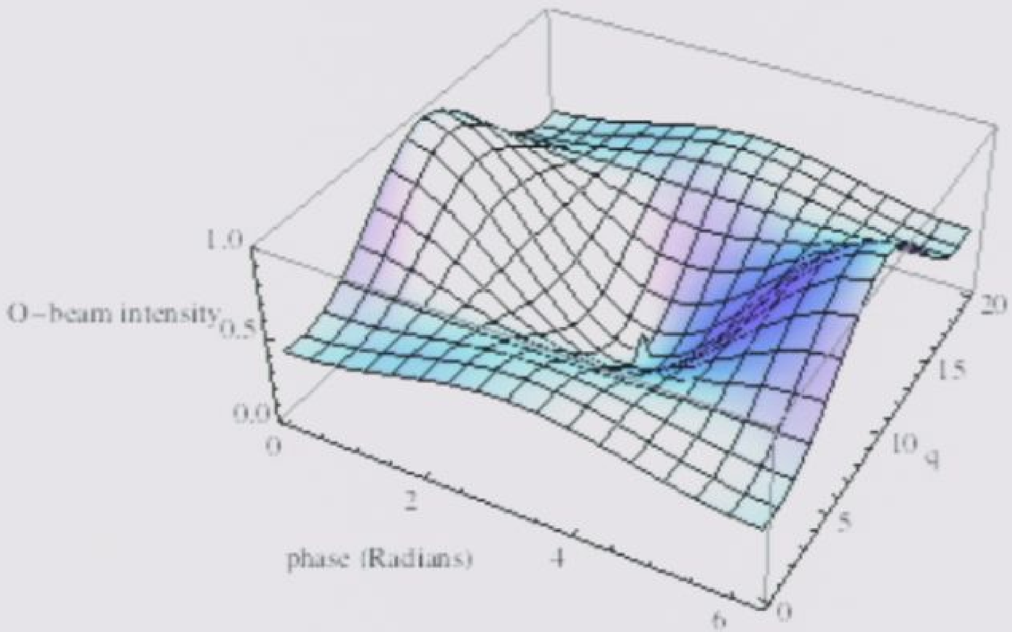
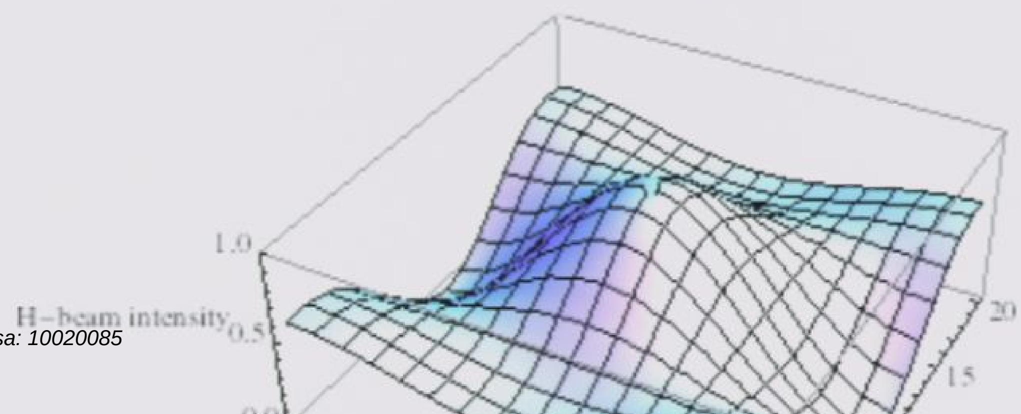
#2.nb

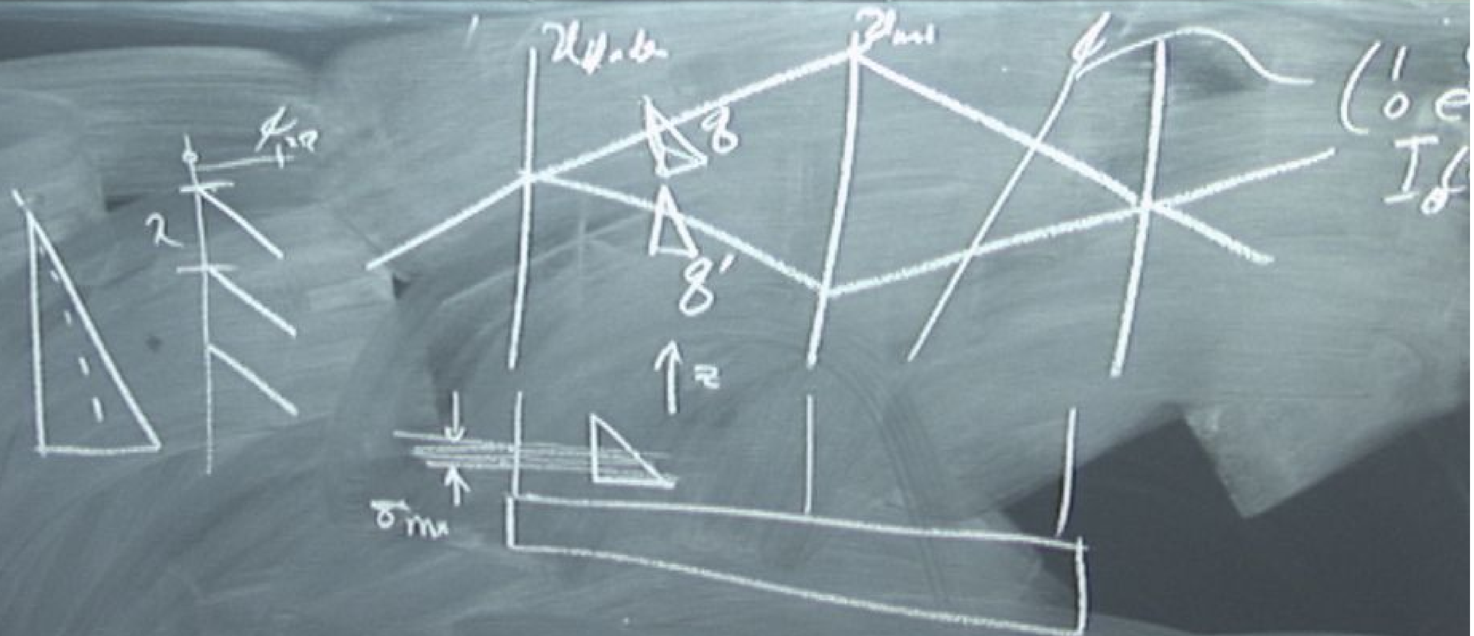
```
{AxesLabel → {"phase (Radians)", "q", "O-beam intensity"}, PlotRange → {0, 1}}]
```




```
M8H[qup_, qdown_,  a_] := Integrate[Tr[Ezm . res8[qup, qdown, z, a]], {z, -0.5, 0.5}]
```

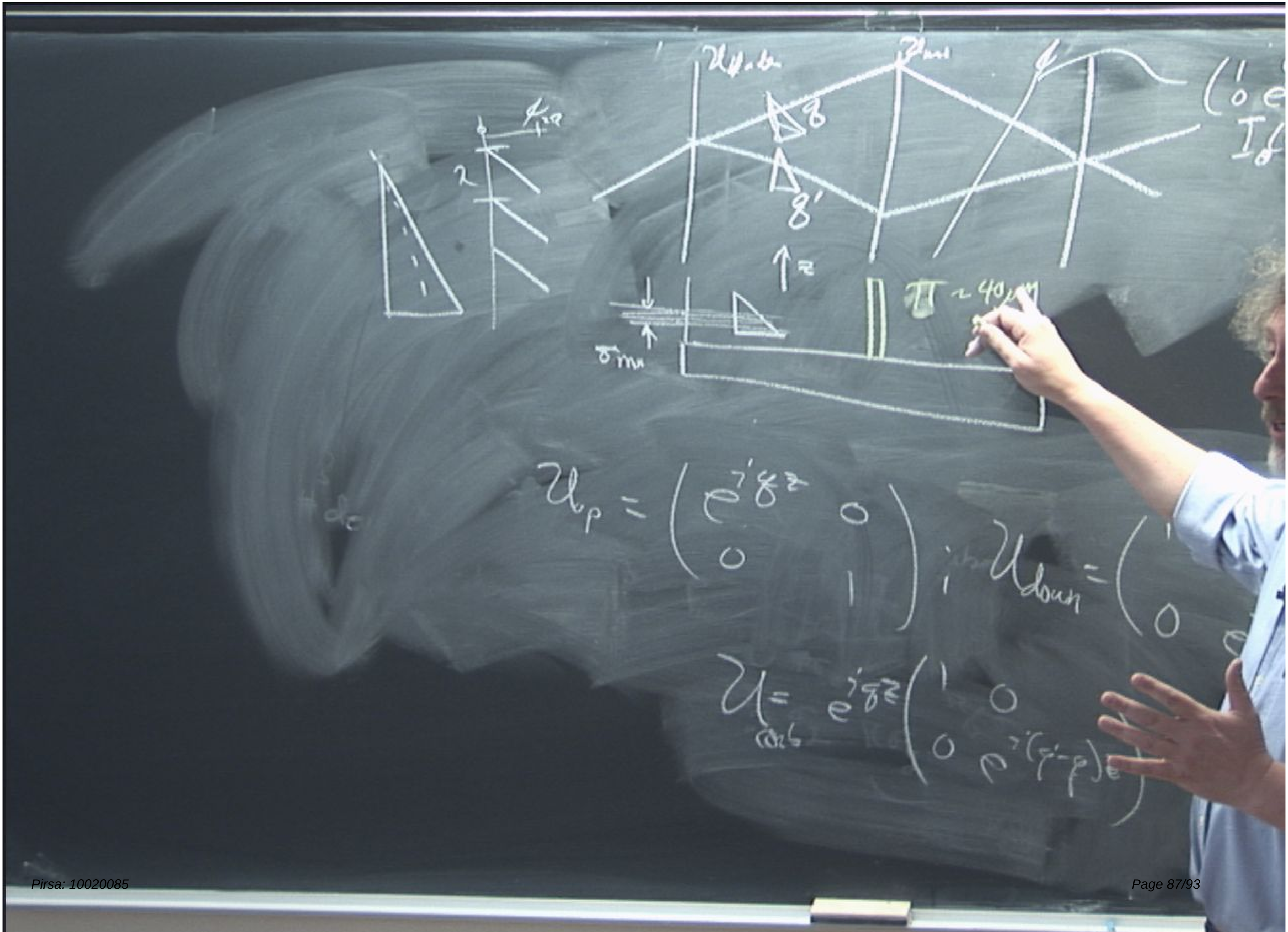
```
Plot3D[M8H[10, q, a], {a, 0, 2π}, {q, 0, 20},
 {AxesLabel → {"phase (Radians)", "q", "H-beam intensity"}, PlotRange → {0, 1}}]
```

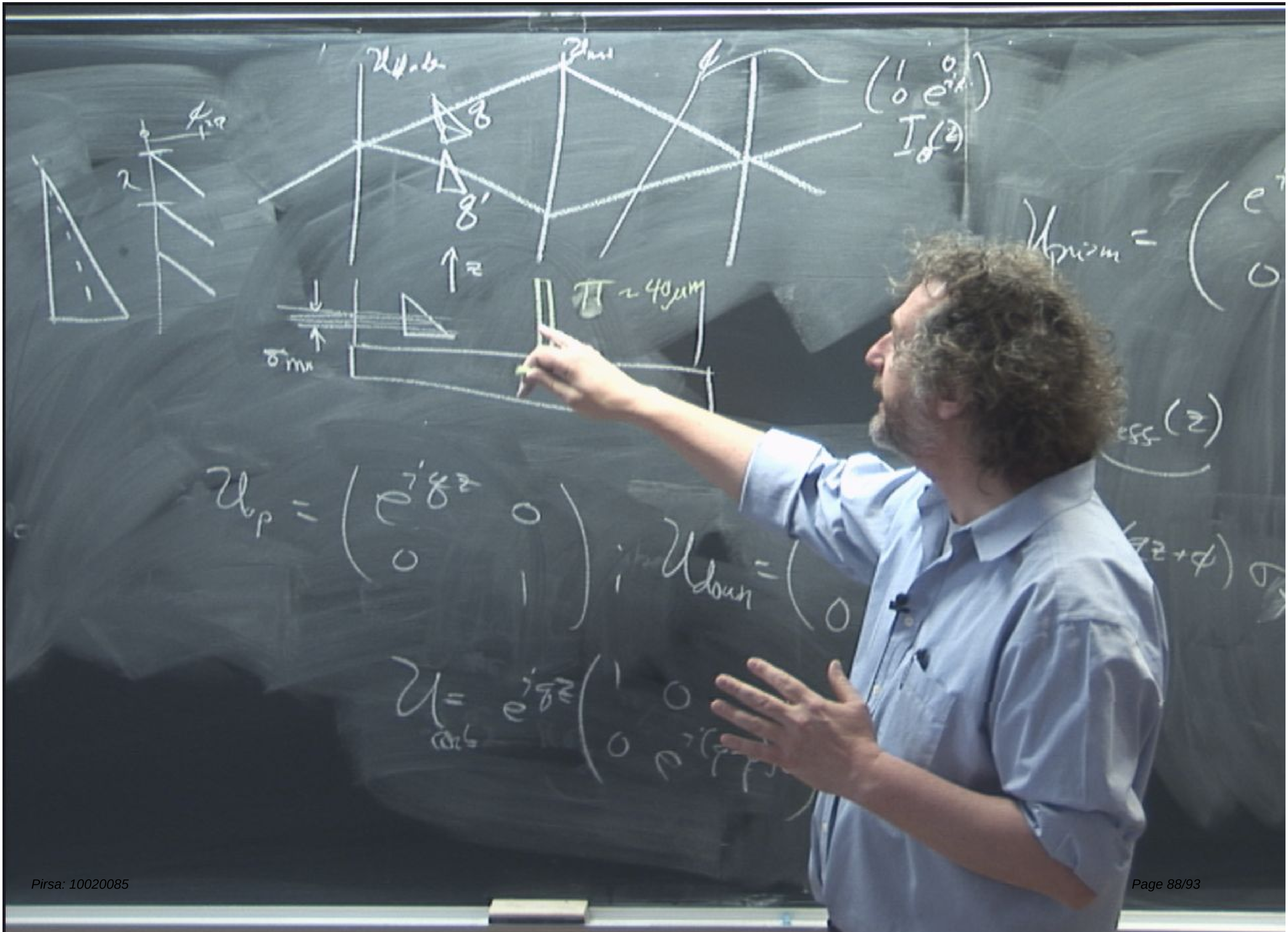


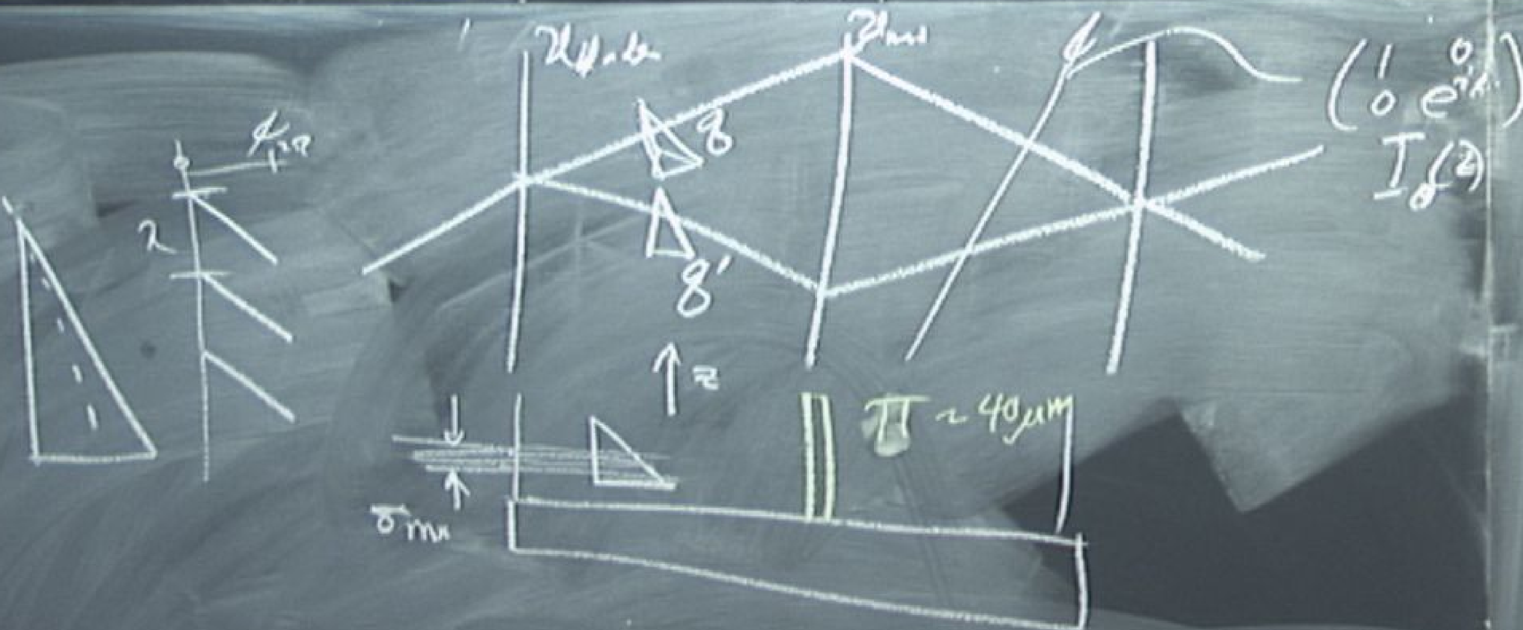


$$U_p = \begin{pmatrix} e^{igz} & 0 \\ 0 & 1 \end{pmatrix}; \quad U_{down} = \begin{pmatrix} 1 & 0 \\ 0 & e^{ig} \end{pmatrix}$$

$$U_{comb} = e^{igz}\begin{pmatrix} 1 & 0 \\ 0 & e^{i(g'-g)z} \end{pmatrix}$$







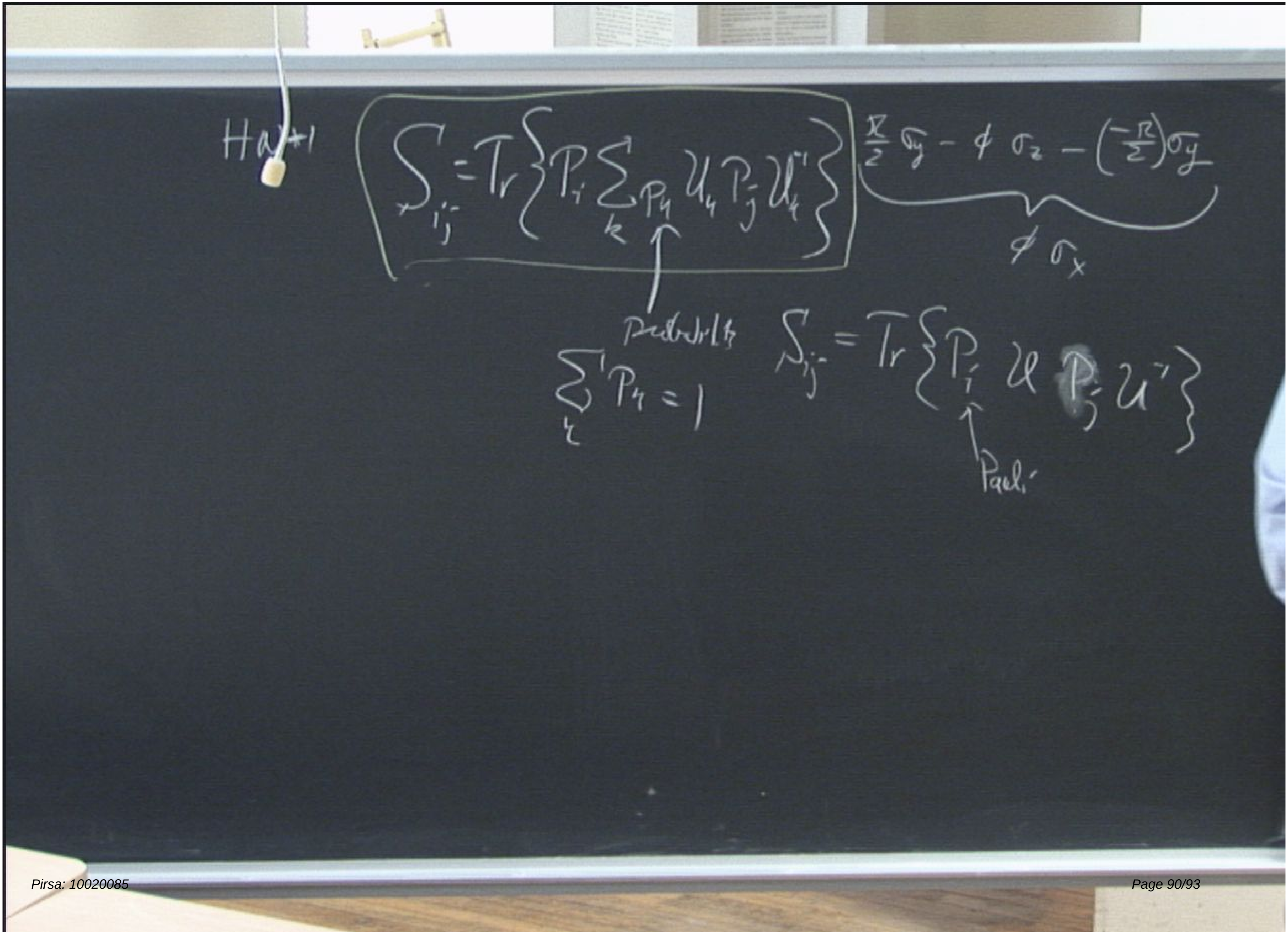
$$U_{up} = \begin{pmatrix} e^{igz} & 0 \\ 0 & 1 \end{pmatrix} ; \quad U_{down} = \begin{pmatrix} 1 & 0 \\ 0 & e^{ig'z} \end{pmatrix}$$

$$U_{comb} = e^{igz} \begin{pmatrix} 1 & 0 \\ 0 & e^{i(g'-g)z} \end{pmatrix}$$

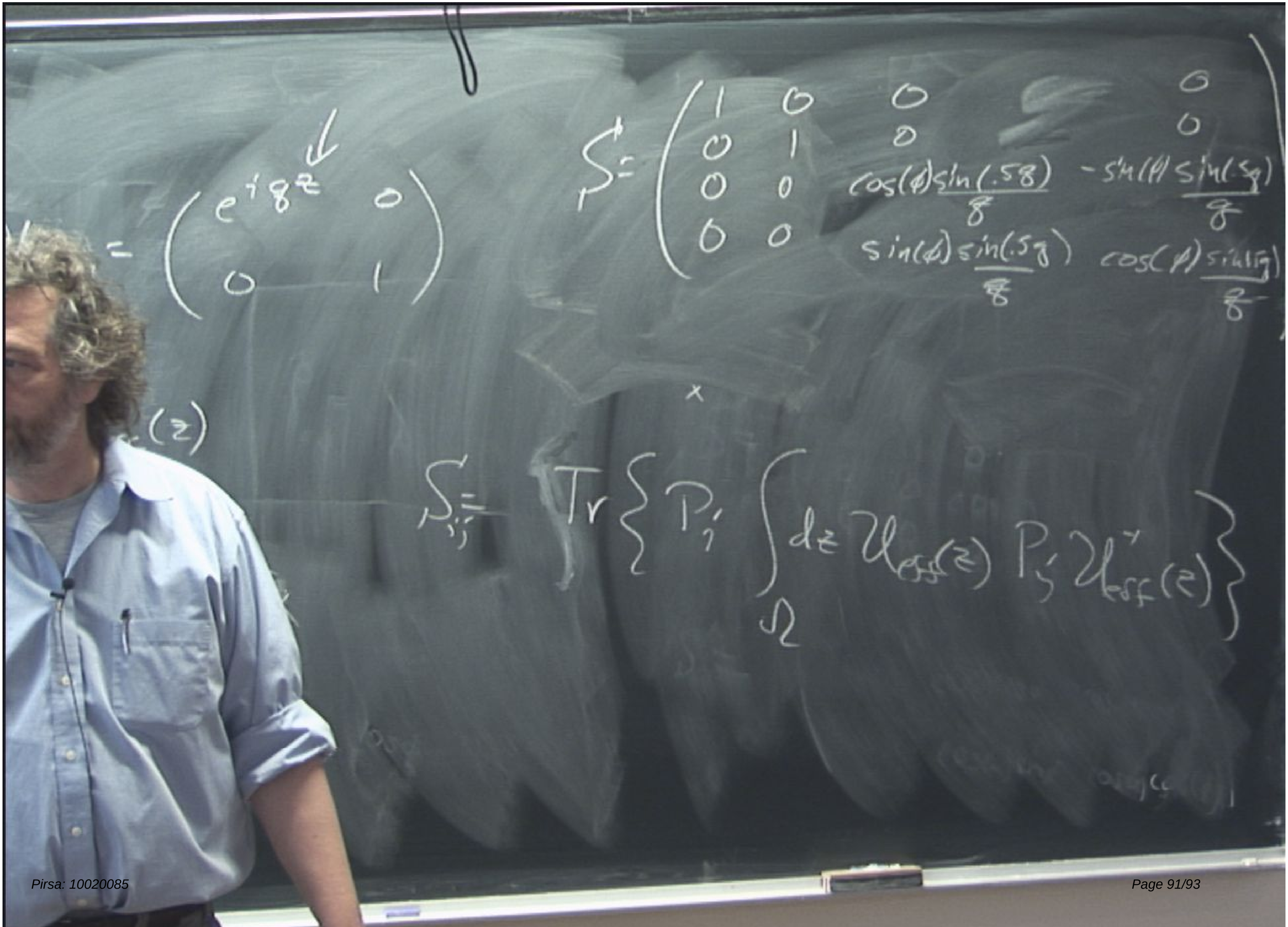
$U_{prism} = \Big(e \;\; 0$

$\underline{U_{eff}(z)}$

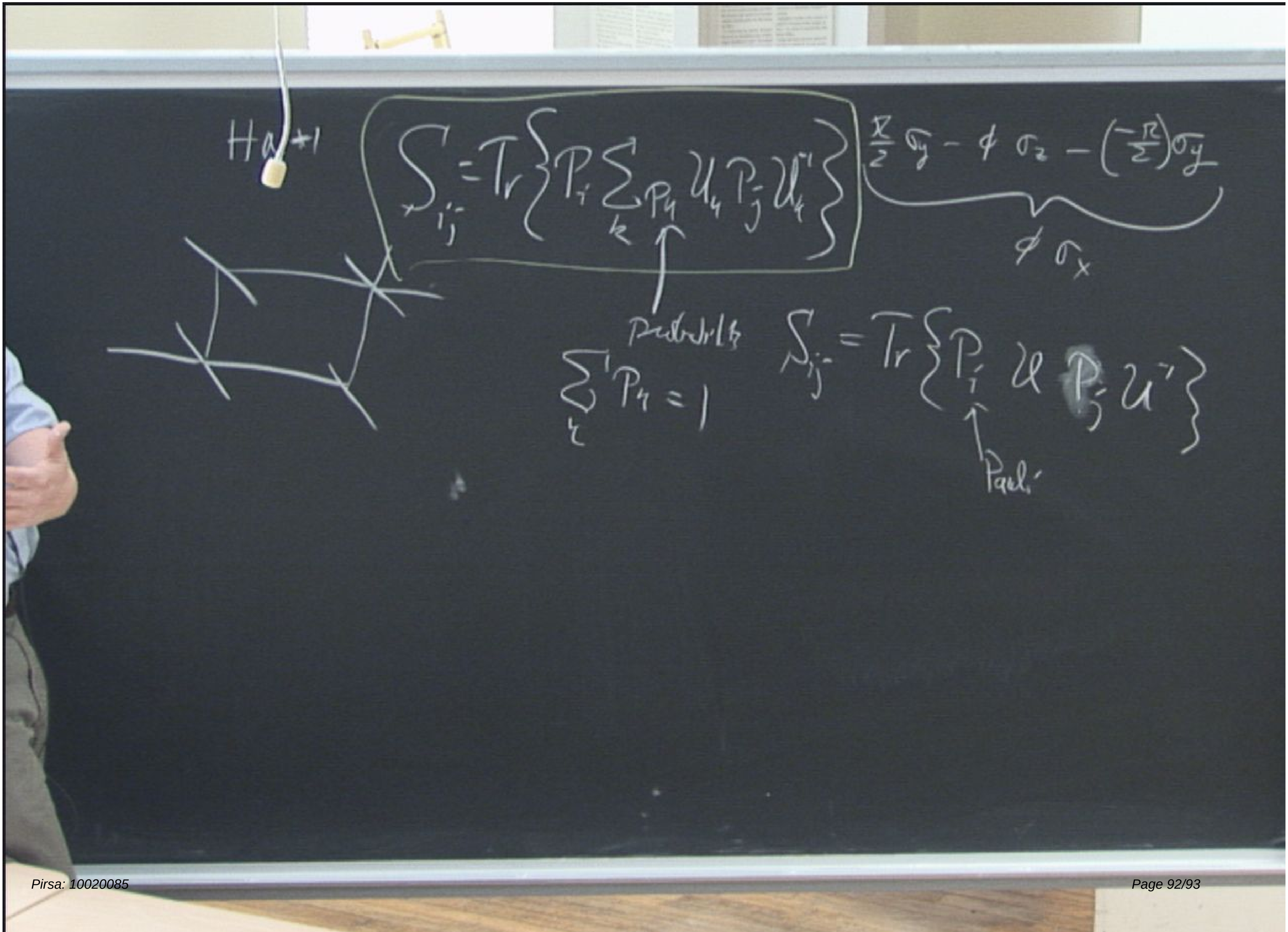
$(gz+\phi)\,\sigma$



S_ij = Tr{P_i Σ_k p_k U_k P_j U_k^-1}
π/2 σ_y - φ σ_z - (-π/2) σ_y
φ σ_x
Σ_k p_k = 1
S_ij = Tr{P_i U P_j U^-1}
Pauli



$\begin{pmatrix} e^{iqz} & 0 \\ 0 & 1 \end{pmatrix}$
$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\phi)\frac{\sin(.5q)}{q} & -\sin(\phi)\frac{\sin(.5q)}{q} \\ 0 & 0 & \sin(\phi)\frac{\sin(.5q)}{q} & \cos(\phi)\frac{\sin(.5q)}{q} \end{pmatrix}$
$S_{ij} = \mathrm{Tr}\left\{ P_i \int_\Omega dz\, \mathcal{U}_{eff}(z)\, P_j\, \mathcal{U}^{\dagger}_{eff}(z) \right\}$


S_ij = Tr{P_i Σ_k P_k U_k P_j U_k^-1}
Σ_k P_k = 1
S_ij = Tr{P_i U P_j U^-1}
Pauli
φ σ_x

Ha=1

$$S_{i_j} = Tr\left\{P_i \sum_k P_k U_k P_j U_k^{-1}\right\} \quad \frac{\pi}{2}\sigma_y - \phi\,\sigma_z - \left(-\frac{\pi}{2}\right)\sigma_y$$

$$\underbrace{\frac{\pi}{2}\sigma_y - \phi\,\sigma_z - \left(-\frac{\pi}{2}\right)\sigma_y}_{\phi\,\sigma_x}$$

probability (P_k)

$$\sum_k P_k = 1$$

$$S_{i_j} = Tr\left\{P_i\, U\, P_j\, U^{-1}\right\}$$

Pauli (P_i)