Title: Nonclassical correlations from random measurements

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Abstract: In this talk, I will demonstrate that correlations inconsistent with any locally causal description can be a generic feature of measurements on entangled quantum states. Specifically, spatially-separated parties who perform local measurements on a maximally-entangled state using randomly chosen measurement bases can, with significant probability, generate nonclassical correlations that violate a Bell inequality. For n parties using a Greenberger-Horne-Zeilinger state, this probability of violation rapidly tends to unity as the number of parties increases. Moreover, even with both a randomly chosen two-qubit pure state and randomly chosen measurement bases, a violation can be found about 10% of the time. Amongst other applications, our work provides a feasible alternative for the demonstration of Bell inequality violation without a shared reference frame.

Phys. Rev. Lett. **104**, 050401 / arXiv:0909.0990

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Quantum Foundations Seminar,
Perimeter Institute for Theoretical Physics, 4th February 2010

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Bell inequality violation without a shared reference frame

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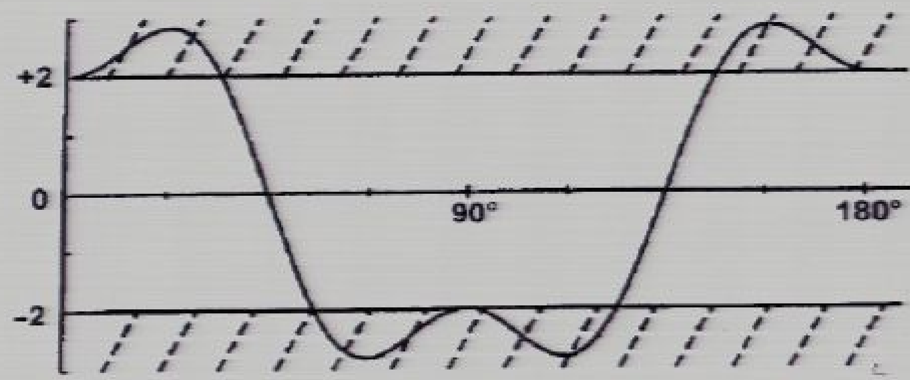
Outline

1. **Overview**
   - Demonstration of Bell inequality violation
   - The problem
2. **What is known about the problem?**
   - Trivial solutions?
   - Non-trivial solutions
3. **Random measurements**
   - Preliminaries
   - Examples
   - Nonclassical correlations from GHZ state
   - Improving the chance of finding a Bell violation
   - Random Bell violation for other quantum states
4. **Concluding remarks**

Motivation

Motivation - Why random measurements?



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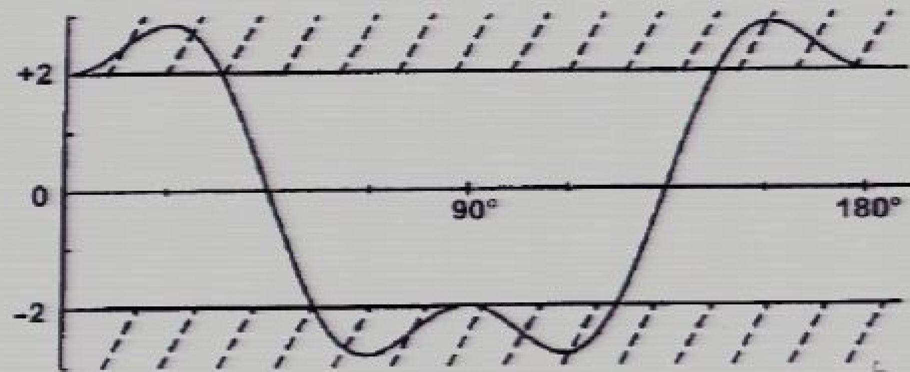
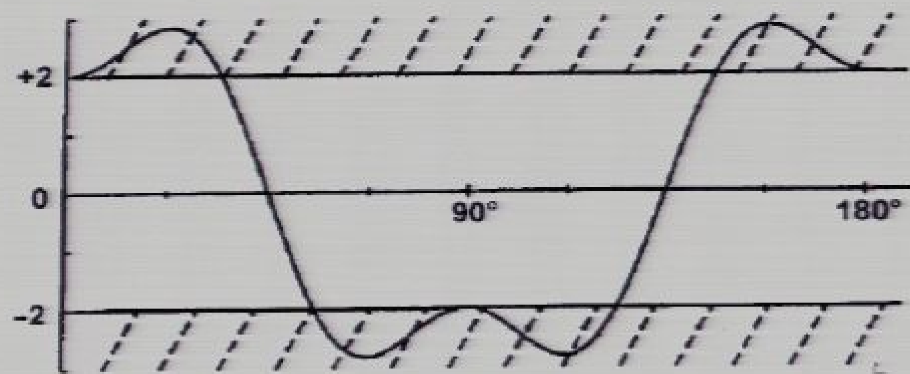


- What is the chance of finding a nonclassical correlation if measurement bases are chosen randomly?
- Perhaps very rare (in the classical limit of large number of particles)??
- Role of reference frames in Bell violation.
- Reduction of technical requirements for the demonstration of Bell violation.

Motivation - Why random measurements? Why reference frame?



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Notations

CHSH inequality

$$S_{\text{CHSH}} \equiv |E(1,1) + E(1,2) + E(2,1) - E(2,2)|$$

- Two parties (Alice & Bob) sharing a bipartite system are allowed to perform two local 2-outcome measurements.
- Alice (Bob) can perform measurement s_a (s_b) $= 1, 2$ which produces two possible outcomes o_a (o_b) $= \pm 1$.
- Joint (conditional) probability: $P(o_a, o_b | s_a, s_b)$.
- Correlation function/ Correlator : $E(s_a, s_b) \equiv \sum_{o_a, o_b = \pm 1} o_a o_b P(o_a, o_b | s_a, s_b)$
- There exist quantum states and local Hermitian observables corresponding to $s_a = 1, 2$ and $s_b = 1, 2$ such that CHSH inequality is violated, i.e., $S_{\text{CHSH}} > 2$
- For spin singlet state $(|0\rangle|1\rangle - |1\rangle|0\rangle)/\sqrt{2}$

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Demonstration of quantum nonlocality - entanglement

- Experimentally, Bell inequality, in particular CHSH violation has been demonstrated quite convincingly using various entangled states,

$$\rho_{\text{Ent}} \neq \sum_i p_i \rho_i^{(1)} \otimes \rho_i^{(2)} \otimes \cdots \otimes \rho_i^{(n)}$$

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- Entanglement by itself does not (immediately) lead to demonstration of nonclassical correlations via Bell inequality violation.
- It is not clear if one can always derive nonclassical correlations from arbitrary (mixed) entangled state!
- For entangled states that can exhibit quantum nonlocality, we need appropriate choice of measurements to demonstrate Bell inequality violation.
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$$\mathcal{S}_{\text{CHSH}} = \left| \sum_{s_a, s_b = 1}^{2} (-1)^{s_a s_b + 1} E(s_a, s_b) \right| \leq 2$$

if $s_a = 1: \quad \hat{x},\ s_a = 2: \quad \hat{z},$
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A hypothetical scenario ...

Why don't we first establish a shared reference frame?

- First sacrifice some pairs of shared singlet states to establish a shared reference frame?

- Use two-way communication to establish a shared Cartesian reference frame?

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Some other options ...

- Consider preparing entangled states that are encoded in some decoherence free subspace. Requires very complicated state prepartion!
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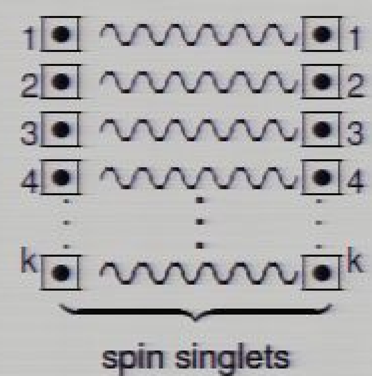
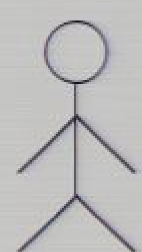
Schematic view...

Perfectly-Correlated Inc. Ltd.





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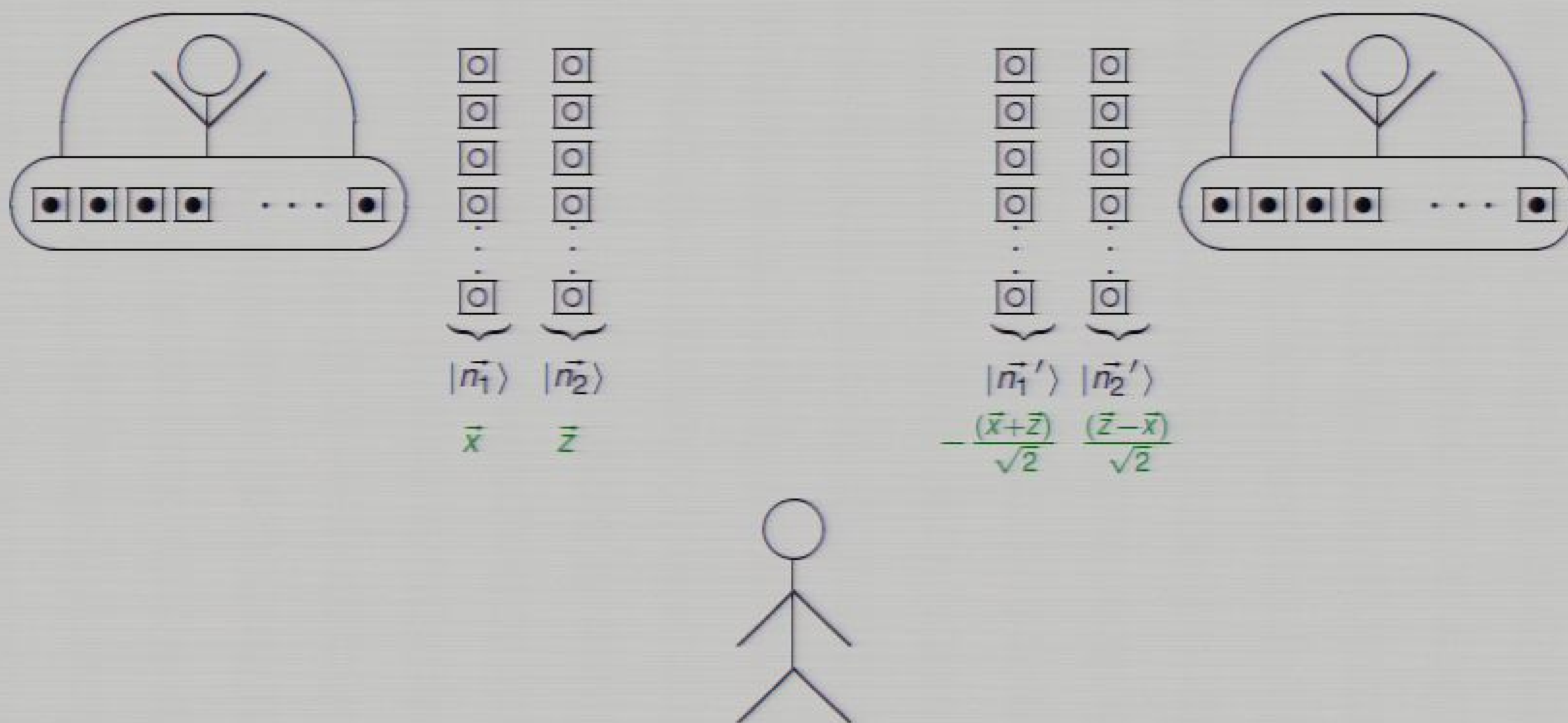

Correlations from randomly chosen measurement bases

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Violation of CHSH

- In the CHSH scenario, for given quantum state ρ

$$p^{\rho}_{\mathrm{CHSH}} = \frac{1}{(4\pi)^4}\iiiint f^{\mathrm{CHSH}}(\{\Omega^{[k]}_{s_k}\}_{k=1}^{2}) \prod_{\substack{k=1,2\\ s_k=1,2}} \mathrm{d}\Omega^{[k]}_{s_k}$$

- $f^{\mathrm{CHSH}}(\{\Omega^{[k]}_{s_k}\}_{k=1}^{2})$ - gives 1 if the given choice of local measurements violates the CHSH inequality and 0 otherwise.
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Violation of CHSH by two-qubit maximally entangled state

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Violation of MABK inequalities by GHZ state

- How does this chance of finding a Bell inequality violation scale with the number of parties n?
- What is the chance of finding an n-partite Mermin-Ardehali-Belinskiĭ-Klyshko (MABK) inequality violation using randomly chosen spin observables and an n-partite GHZ state?
- $f_n^{\text{MABK}}(\{\Omega_{s_k}^{[k]}\}_{k=1}^{n})$ - gives 1 if the given choice of local measurements violates the n-partite MABK inequality and 0 otherwise.
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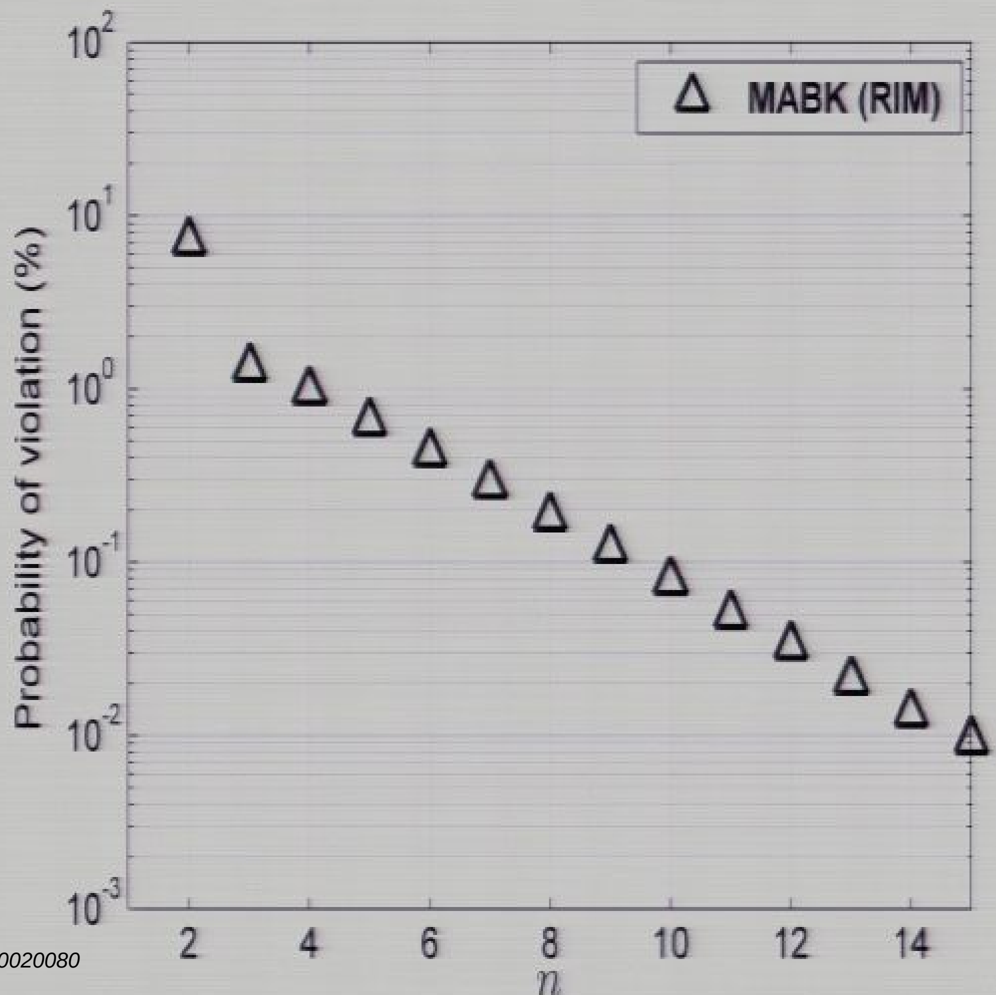
Violation of MABK inequalities by GHZ state - Random Isotropic Measurements



n	$p_n^{\vert GHZ_n \rangle}$ (%)
2	7.08
3	1.33
4	0.97
5	0.64
6	0.43
7	0.28
10	0.08
12	0.03
15	0.009

Labeling and equivalent Bell inequalities I

- Should make use the verifier's freedom in labeling the measurement settings and/or outcomes:

Run	Alice		Bob	
k	↑	→	↙	↖
1	↑		↙	
2		→	↗	
⋮	⋮	⋮	⋮	⋮

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$s_a =$ ↑
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$s_b = 1$ ↙
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k	$\uparrow$	$\rightarrow$	$\swarrow$	$\nwarrow$
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2		$\rightarrow$	$\nearrow$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- $S_{\text{CHSH}} = |E(1,1) + E(1,2) + E(2,1) - E(2,2)|$
  $E(s_a, s_b) = P(o_a = o_b | s_a, s_b) - P(o_a \neq o_b | s_a, s_b)$

-
  $s_a = \quad \uparrow$
  $s_a = \quad \rightarrow$
  $s_b = 1 \quad \swarrow$
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Labeling and equivalent Bell inequalities II

- Testing the measurement results against

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- For any given choice of measurement, at most one of these equivalent CHSH inequalities can be violated.
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$$p_{\mathrm{CHSH}}^{|\Psi^-\rangle} = \frac{\pi-3}{2} \approx$$

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\begin{aligned}
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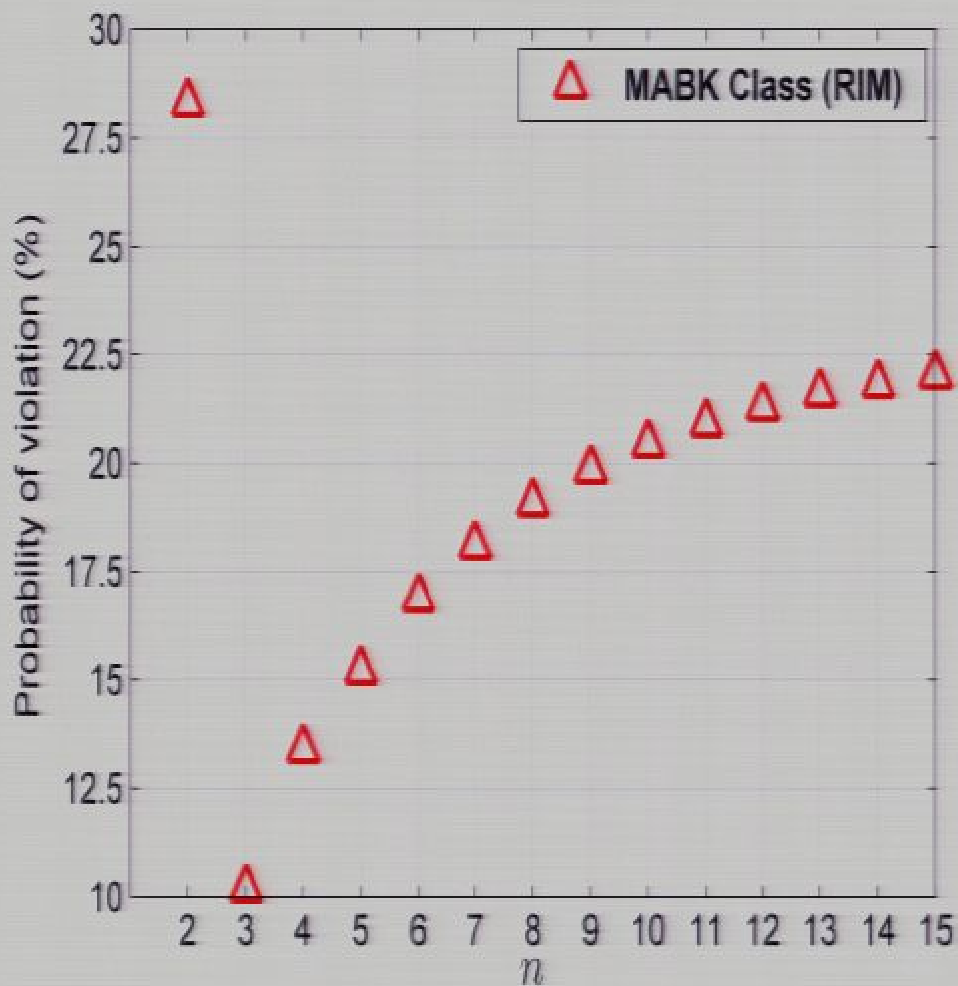
Violation of MABK inequalities by GHZ state



n	Single	All (2^{n+1})
2	7.08	28.3
3	1.33	10.2
4	0.97	13.4
5	0.64	15.2
6	0.43	16.9
7	0.28	18.1
10	0.08	20.4
12	0.03	21.3
15	0.009	22.0

Bell violation of randomly generated correlation from GHZ state I

- What is the chance that the correlation (measurement statistics) derived from $2 \times n$ randomly chosen spin observables $(\{\Omega_{s_k}^{[k]}\})$on $|\mathrm{GHZ}_n\rangle$ is nonclassical, i.e., inconsistent with a locally causal description?

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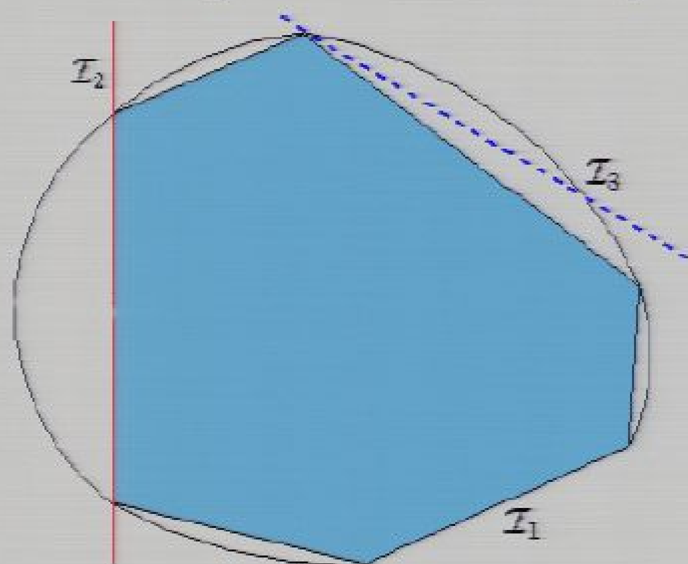

Bell violation of randomly generated correlation from GHZ state II

- For small n, possible to test against the complete set of Bell inequalities (for $n = 3$, test against the complete set of Bell inequalities obtained by Śliwa[†])
- In general, can use linear programming to test if a given correlation admits a locally causal description.
- Input to linear program scales exponentially with n, making it intractable to determine the probability of violation reliably for larger values of n.

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Bell violation of randomly generated correlation from GHZ state III

- Consider an extensive subset of Bell inequalities to gain insight to the behavior for larger values of n.
- The set of 2^{2^n} Werner-Wolf-Żukowski-Brukner (WWZB) Bell inequalities (n-party correlators).
- Violation of the WWZB inequalities $\Rightarrow$ correlation is nonclassical
- Probability of violating WWZB inequalities lower bounds probability that a correlation is nonclassical
- Amounts to testing if a given correlation satisfies a single nonlinear inequality:

$$\sum_{k_1,\ldots,k_n=\pm 1} \left| \sum_{s_1,\ldots,s_n=1}^{2} \prod_{j=1}^{n} k_j^{s_j-1} E(s_1,\ldots,s_n) \right| \leq 2^n$$

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Bell violation of randomly generated correlation from GHZ state IV

n	2	3	4	5	6
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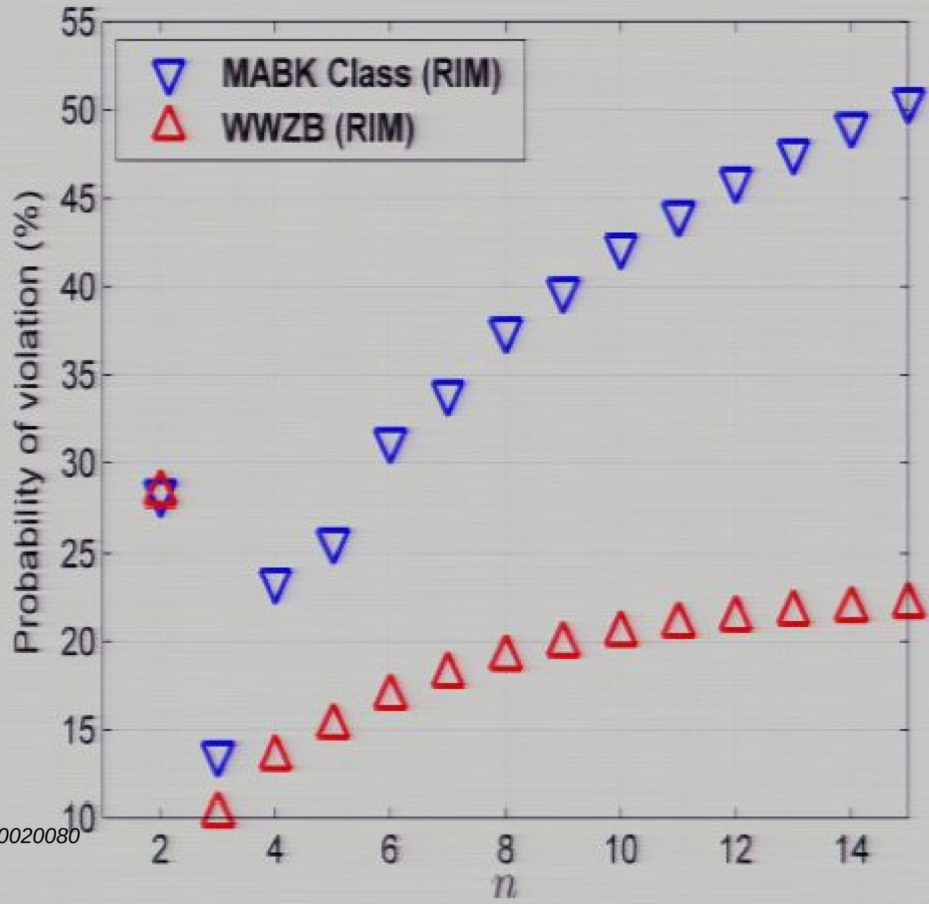

n	MABK (%)	WWZB (%)
2	28.3	28.3
3	10.2	13.6
4	13.4	23.4
5	15.2	25.7
6	16.9	31.2
7	18.1	33.9
10	20.4	42.3
12	21.3	46.0
15	22.0	50.6

Orthogonal spin measurements

- Optimal violations typically found when the pair of local measurement directions are orthogonal, i.e., $\Omega_1^{[k]} \cdot \Omega_2^{[k]} = 0$.
- Consider random sampling of measurement directions subjected to $\Omega_1^{[k]} \cdot \Omega_2^{[k]} = 0$: random orthogonal measurements (ROM).
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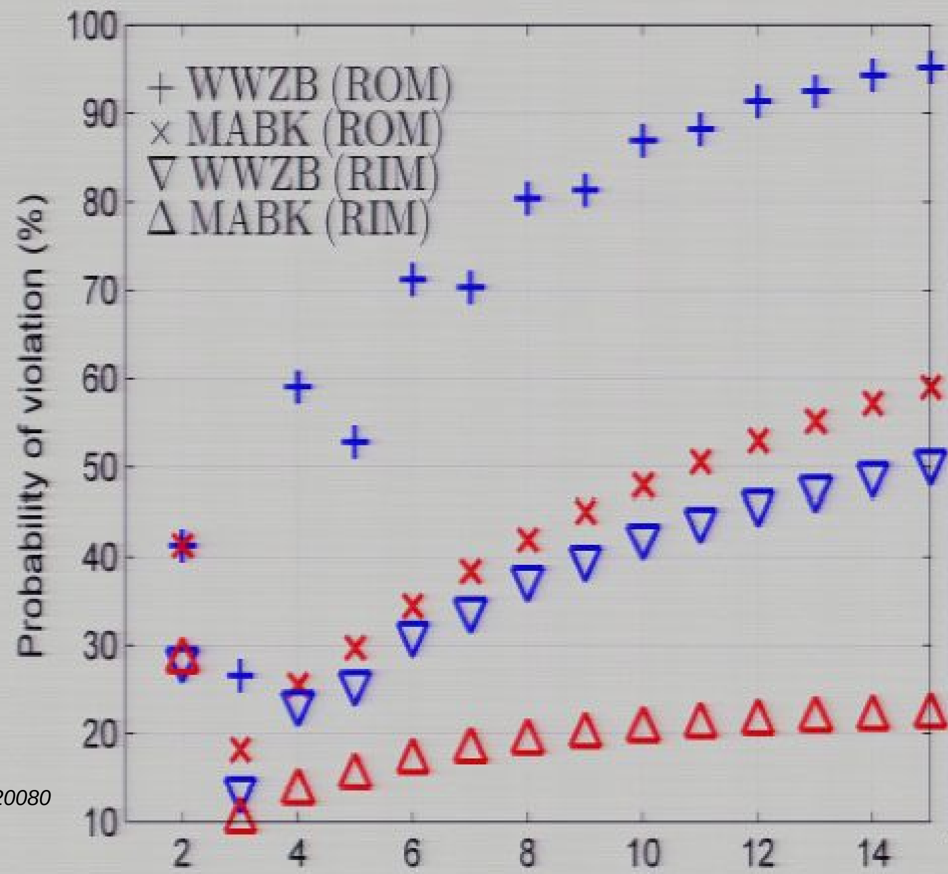
- For $n = 2$, if Alice and Bob only choose among orthogonal pair of spin observables, without relabeling, chance of violation: 7.08%

Nonclassical correlations from GHZ state - RIM and ROM

n	2	3	4	5	6
RIM	28.3185%	74.6899%	94.2345%	99.5941%	99.96%
ROM	41.2982%	96.2073%	99.9758%	99.9999%	100.00%



n	MABK (%)	WWZB (%)
2	41.3	41.3
3	18.2	26.6
4	25.6	59.0
5	29.7	52.8
6	34.4	71.2
7	38.4	70.3
10	48.0	86.9
12	53.0	91.4
15	59.0	95.2

Random Bell violation vs entanglement of pure two qubit state

- Two qubit pure state

$$|\Psi\rangle = \cos\theta|0\rangle_1|0\rangle_2 + \sin\theta|1\rangle_1|1\rangle_2$$

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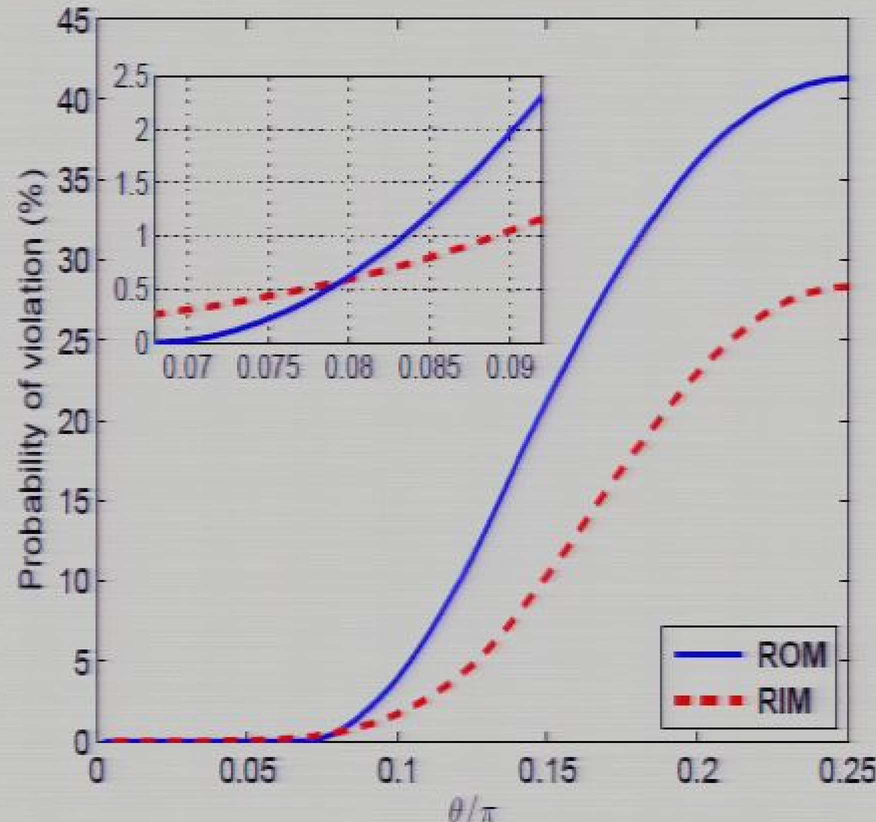

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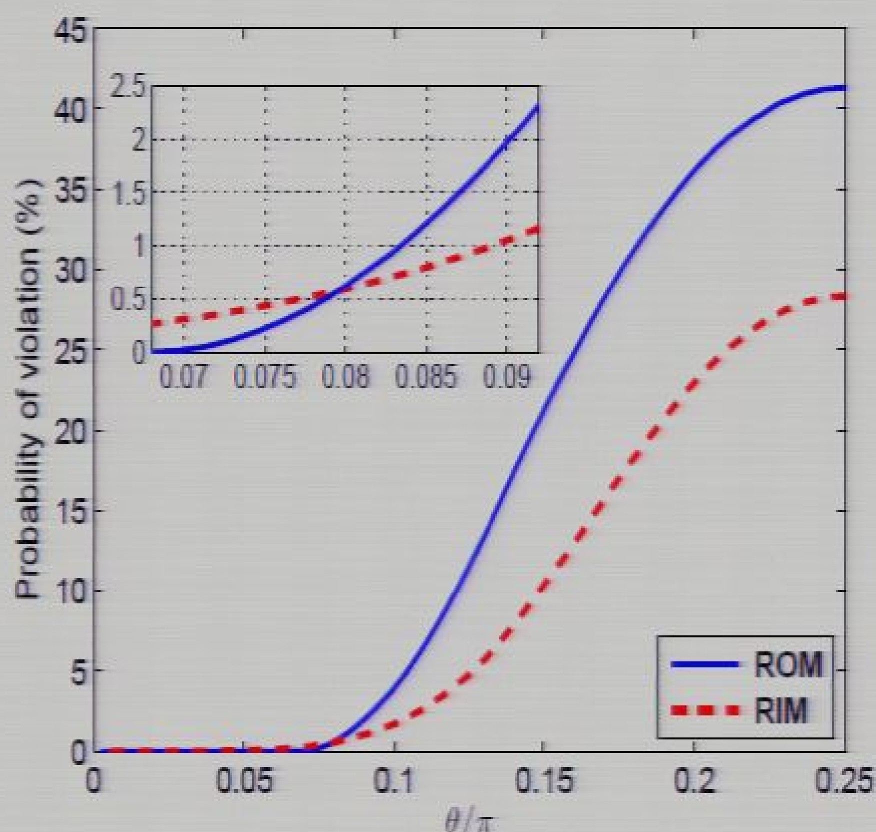

- Sample two-qubit pure state randomly according to the SU(4) Haar measure.
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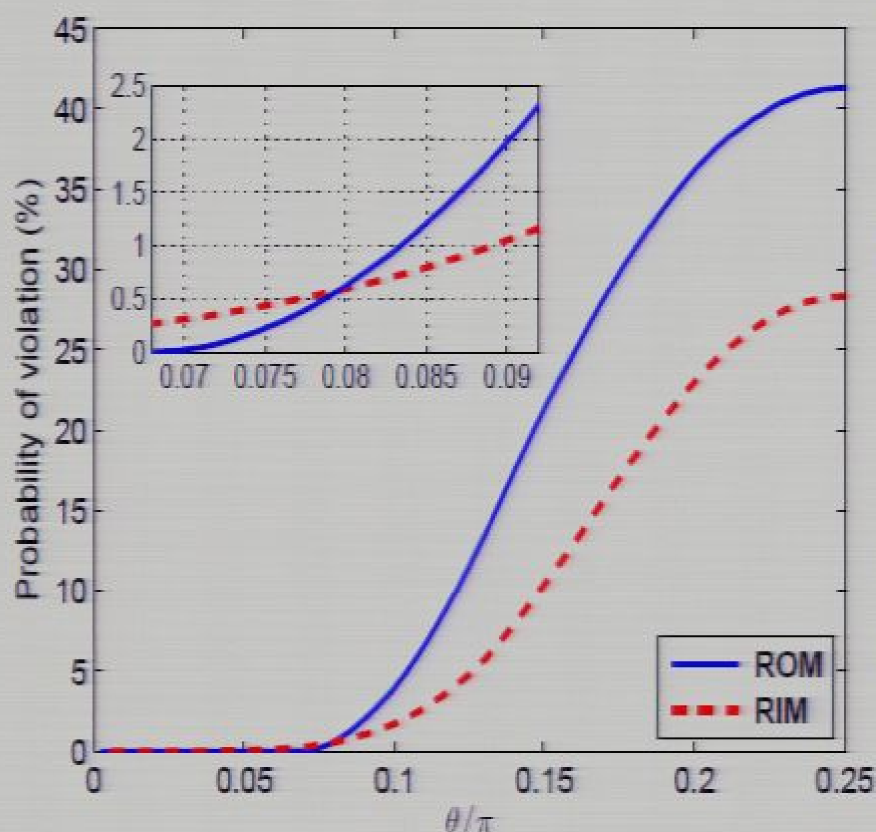

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Summary

- We investigated the probability of finding a nonclassical, Bell-inequality-violating correlation from random measurements
- With GHZ state, the probability of finding a nonclassical correlation rapidly $\to 1$ as n increases.
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Outlook

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- What about mixed states?
- Robust against noise?
- What about other inequalities that detect other kinds of multipartite entanglement?
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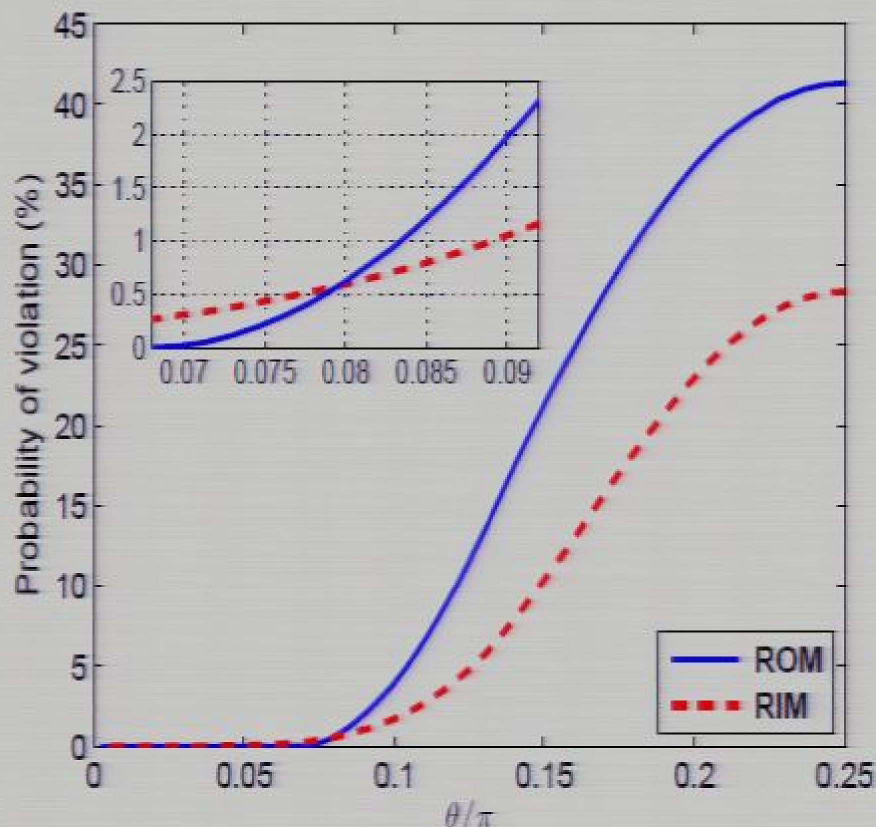

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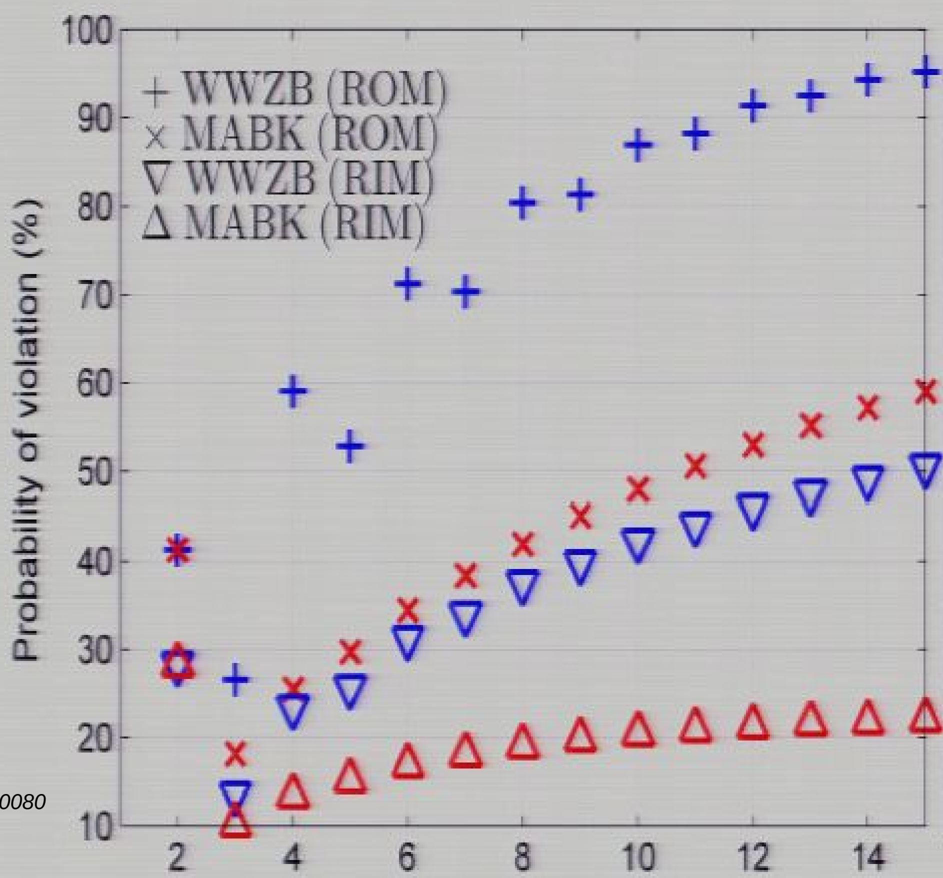

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