Title: A review of Spinfoams and Group Field Theory

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Abstract: We will review the definitions of spin foam models for quantum gravity and the recent advances in this field, such as the "graviton propagator", the definition of coherent states of geometry and the derivation of non-commutative field theories as describing the effective dynamics of matter coupled to quantum gravity. I will insist on the role of group field theories as providing a non-perturbative definition of spinfoams and their intricate relation with non-commutative geometry and matrix models.

Some Recent Progress in SpinFoam Models

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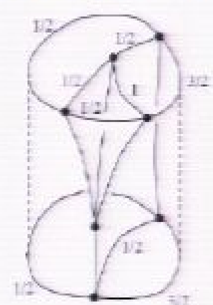
February 2010 in Waterloo at

Where do Spinfoams come from?

Three dual perspectives:

- State-sum Models for Topological BF Theory:
  $S[B, A] = \int_{\mathcal{M}} \mathrm{Tr}\, \mathrm{B} \wedge \mathrm{F[A]}$ with gauge connection A and field B
  No local degree of freedom $\Rightarrow$ the path integral discretized on a triangulation of $\mathcal{M}$ provides an exact quantization.
  $\rightarrow$ The Ponzano-Regge model for 3d quantum gravity (1968)
- Quantized Regge calculus with discrete lengths/areas.
- Histories of Spin Networks in Loop Quantum Gravity:
  Allows to compute transition amplitudes between spin network states as "sum-over-surfaces" [Reisenberger-Rovelli 96]
  $\rightarrow$ "bubbles" of space-time [Baez 99]



So what are Spinfoams?

Goal: A framework for a regularized path integral for gravity

How?

- Write general relativity as a BF gauge theory with nontrivial potential $S = \int B \wedge F[A] + V[B]$
- Discretize the path integral on a triangulation (or cellular decomposition) of the (4d) space-time manifold

What?

- A spinfoam model is a choice of amplitude for each triangulation defined as the product of local terms
- The dual to (3d) space triangulation is a spin network state
- This defines a probability amplitude to each history of evolving spin networks

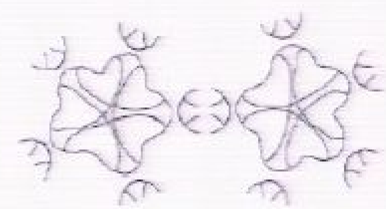
The Spinfoam Ansatz

The data: Consider the gauge group G, then dress up the triangulation:

- Triangle $\rightarrow$ representation (spin) j_t of G
- Tetrahedron $\rightarrow$ intertwiner $\mathcal{I}_T$ between its 4 triangles
- 4-Simplex  $\rightarrow$ evaluation of the boundary spin network $=$ contraction of its 5 tetrahedra

A local ansatz:

$$\mathcal{A}[\Delta] = \sum_{\{j_t\}} \underbrace{\prod_t A_t(j_t) \prod_T A_T(j_t, \mathcal{I}_T)}_{\text{Statistical weights}} \underbrace{\prod_\sigma A_\sigma(j_t, \mathcal{I}_T)}_{\text{Dynamics}}$$



The Standard Models

1. Ponzano-Regge : 3d gravity with G=SU(2) or SU(1,1)
   - Discretization of topological BF Theory in 3d
   - Amplitude for 3-simplex (tetrahedron) $= \{6j\}$
   - Related to Regge calculus $\{6j\} \sim \cos(S_R)$
   - Cosmological Constant Λ related to q-deformation $U_q(SU(2))$
2. SU(2) BF Theory : 4-simplex $= \{15j\}$, non-geometric
3. Barrett-Crane : constrained BF Theory with G=Spin(4)
   - Quantization of a single 4-simplex $\rightarrow \{10j\}$
   - No dynamics for intertwiners
   - Related to Regge calculus $\{10j\} \sim \text{bad terms} + \cos(S_R)$
4. EPR-FK : G=Spin(4) or Spin(3,1)
   - Uses LS coherent intertwiners
   - Good control on large spin asymptotics
5. *à la* Freidel-Starodubtsev : G=Spin(5) or Spin(4,1)
   - Based on the McDowell-Mansouri action for GR
   - Doesn't exist yet...

Some Main Results

We have been trying to work hard in the past five years...

- The Graviton Propagator : To recover the standard perturbative of quantum general relativity... with improvements hopefully!
- Using Coherent Intertwiners : tool for semi-classical expansion and central objects in constructing "new" spinfoam models
- Coupling Matter to Spinfoams: deriving non-commutative field theory describing the effective dynamics of matter coupled to the quantum geometry (DSR)
  $\longrightarrow$ Experimental signature of deformed Poincaré symmetry?
- Group Field Theory : Generating Spinfoam amplitudes as Feynman diagrams
  $\longrightarrow$the non-perturbative definition of spinfoam models

Other Cool Results

- Better understanding of the relation to (area-)Regge calculus [Dittrich,Freidel,Ryan,Speziale 07-08]
  $\rightarrow$ Discrete Lagrangians for spinfoam models [Conrady,Freidel 08 Bonzom,EL 08]
- Explicit link with the canonical LQG framework [Engle,Livine,Pereira,Rovelli 07 Freidel,Speziale 10]
- Spinfoam models for supergravity [EL,Ryan 07] and for BF+strings [Baez,Perez 06 Fairbairn,Perez,Noui 07,08,09]
- Recursion relations and symmetries of spinfoam amplitudes [Bonzom,Dupuis,EL,Speziale 09]
- Reconstruction of gravity from BF theory [Krasnov 08 Freidel 08 Smolin,Speziale 09]
- I'm sure that I have forgotten other projects...

The Graviton Propagator

An original proposal by Rovelli '05:
Compute correlations between areas in spinfoam models and relate them to the graviton propagator of perturbative GR.
$\Rightarrow$ A first real test for Spinfoam models !

Since then, a real task force between CPT (Marseille), ENS Lyon and University of Western Ontario

[Alesci, Bianchi, Bonzom, Christensen, Dupuis, EL, Khavkine, Magliaro, Modesto, Perini, Speziale]

And deeply intertwined with progress on the asymptotics of spinfoam amplitudes, with PI and the Nottingham group

[also Barrett, Conrady, Dowdall, Fairbairn, Freidel, Gomes, Hellmann]

Computing correlations between geometric observables ...

The general setting: Consider a triangulation Δ with boundary

- Choose a spinfoam model and define a suitable semi-classical physical boundary state $\psi(\{j_t, \mathcal{I}_T\}_{t,T\in\partial\Delta})$
- Choose two $\triangle$ in $\partial\Delta$ and define the correlation

$$\langle \mathcal{O}(j_a)\mathcal{O}(j_b)\rangle = \frac{1}{Z}\sum_{j_t,\mathcal{I}_T} \mathcal{O}(j_a)\mathcal{O}(j_b)\psi(j_{\partial\Delta},\mathcal{I}_{\partial\Delta})\mathcal{A}_\Delta(j_t,\mathcal{I}_T)$$

- Gauge fix the spinfoam amplitude $\mathcal{A}_\Delta$
- Sum on all Δ compatible with fixed boundary structure $\partial\Delta$
- This defines the graviton propagator $\langle h_{\alpha\beta}h_{\gamma\delta}\rangle$

... Computing correlations between geometric observables

The actual setting: Having fun with a 4-simplex...

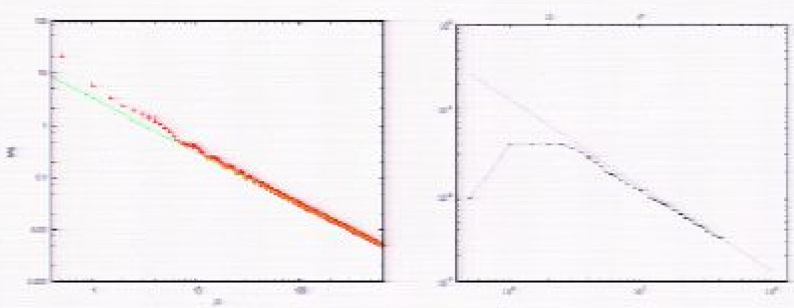
- Choose two $\triangle$ on the 4-simplex and take $\mathcal{O}(j) = j(j+1)$
- Mainly study the Barrett-Crane model (or Ponzano-Regge model in 3d), simpler because no intertwiner dynamics...

$$\langle \mathcal{O}(j_a)\mathcal{O}(j_b)\rangle = \frac{1}{Z}\sum_{j_t} \mathcal{O}(j_a)\mathcal{O}(j_b)\psi(j_t)\{10j\}$$

- Take Gaussian ansatz $\psi(j_t) = \prod_t e^{-\beta\frac{(j_t-j_0)^2}{j_0}} e^{i\theta_0 j_t}$ peaked on flat (equilateral) 4-simplex (or improved Bessel ansatz)
- Physical state? Turns out to fix the width β in 3d [EL,Speziale 07]
- More complex triangulations? Renormalization of spinfoams?
- Use coherent intertwiners and EPR-FK spinfoams?

Some actual results

A simplified setting but we get actual analytical and numerical results ! [EL,Speziale 06 +Christensen,Khavkine 07,09 +Bonzom,Smerlak 08]



- Leading order in $1/j_0 \Rightarrow$ Newton's law
- Regularized correlations at small scale $j_0 \to 0$ with dynamic minimal scale

- Compute quantum corrections with dependence on measure
- Probing the asymptotics of the $\{10j\}$-symbol (and $\{6j\}$ too!)
- Testing the tensorial structure of graviton propagator $\to$ asymptotical ansatz for new spinfoam amplitudes [Alesci 08-09]

Coherent Intertwiners for Tetrahedra

A simple observation: the standard intertwiner basis used in LQG and SF, labeled by a internal spin, is not suited to semi-classical analysis.

Coherent Intertwiners: Consider 4-valent intertwiners, dual to tetrahedra, between $j_1, .., j_4$ which give the area of the triangles, and build the averaged tensor product of four SU(2) coherent states

$$\int_{\mathrm{SU}(2)} dg\, g \triangleright \otimes_{i=1}^{4} |j_i, \hat{n}_i\rangle$$

These are semi-classical states approximating classical tetrahedra for large spins j's, with the $\hat{n}_i$ giving the normals to the triangles.

[EL,Speziale 07]

Using Coherent Intertwiners

- Used in spinfoam asymptotics and graviton calcultations
- Used to build the new EPR-FK spinfoam models [Freidel,Krasnov 07 EL,Speziale 07 Conrady,Freidel 08 Engle,Pereira 08]
  → To solve in a coherent way the simplicity constraints (turning BF into gravity) and resolve the "no intertwiner dynamics" issue of the Barrett-Crane model: it helps both for the relation between spinfoams and canonical LQG and for the issue of coupling between 4-simplices.

and Going Further

- Refined into holomorphic intertwiners $|j_1,..j_4,Z\rangle$ [Conrady,Freidel 09 Freidel,Krasnov,EL 09]
  $\rightarrow$ Coherent intertwiners use 4+8=12 labels for the semi-classical states, but a tetrahedron is characterized by only 6 numbers. The extra 6 labels are S0(3) rotations plus the closure constraints. Together they form a SL(2,C) transformations, which allow to reduce the labels to a single complex number Z .

Coupling particles to the Ponzano-Regge model

3d Quantum gravity: topological and completely flat

$$Z_{\mathcal{M}} = \int [dBdA] e^{\int B\wedge F[A]} = \int [dA]\delta(F[A])$$

$$\rightarrow Z_\Delta = \int [dg_t] \prod_e \delta(\prod_{e\in t} g_t) = \sum_{\{j_e\}} \prod_e (2j_e+1) \prod_T \{6j\}$$

Particles are topological defects : the mass m becomes a deficit angle $\theta = \frac{m}{\kappa}$.

$$\delta(g) \rightarrow \delta_\theta(g) \text{ on-shell or } \mathcal{P}(g) = (p(g)^2 - \sin^2\theta)^{-1} \text{ off-shell}$$

We insert a Feynman diagram Γ into the spinfoam model:

$$Z_\Delta[\Gamma] = \int [dg_t] \prod_{e\in\Gamma} \mathcal{P}(g_e) \prod_{e\notin\Gamma} \delta(g_e) = \sum_{\{j_e\}} \prod_{e\in\Gamma} e^{-i\theta j_e} \prod_{e\notin\Gamma} (2j_e+1) \prod_T \{6j\}$$

Effective Non-Commutative Quantum Field Theory

We prove that the spinfoam amplitudes are Feynman diagrams of a Non-Commutative QFT: $Z_\Delta[\Gamma] = I[\Gamma]$ [Freidel,EL 05]

The momentum space is our gauge group SU(2):

$$S[\phi] = \int dg\, \phi(g^{-1})(p(g)^2 - \sin^2\theta)\phi(g) + \sum_n \alpha_n \int [dg]^n \phi(g_1)..\phi(g_n)\delta(g_1..g_n)$$

- Momentum space is curved $\Rightarrow$ coordinate space is non-commutative
- Momentum space is bounded $\Rightarrow$ there is a minimal length κ^{-1}
- $\delta(g_1..g_n)$ is the conservation of momentum

A group Fourier transform

We introduce a Fourier transform between SU(2) and $su(2) \sim \mathbb{R}^3$

$$\hat{\phi}(\vec{x}) = \int dg\, \phi(g)\, e^{\kappa \mathrm{Tr} xg} = \int dg\, \phi(g)\, e^{i\vec{x}\cdot\vec{p}(g)}$$

with $x = \vec{x}\cdot\vec{\sigma}$ and $\vec{p} = -i\kappa \mathrm{Tr} g\vec{\sigma}$.

This defines a $\star$-product dual to the convolution product on SU(2),

$$e^{\kappa \mathrm{Tr} x g_1} \star e^{\kappa \mathrm{Tr} x g_2} = e^{\kappa \mathrm{Tr} x g_1 g_2}$$

and a fuzzy δ-distribution: $\tilde{\delta}(x) \sim \frac{J_1(\kappa|x|)}{|x|}$

The NCQFT action can be written in coordinate space:

$$S[\hat{\phi}] = \int dx\, \hat{\phi}(x)(\Delta + \kappa \sin^2\theta)\hat{\phi}(x) + \sum_n \alpha_n \int [dx]^n\, \hat{\phi}^{\star n}(x)$$

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A deformed Poincaré symmetry

We have derived a NCQFT describing the effective dynamics of a (scalar) matter field coupled to 3d quantum gravity.

- It is invariant under the quantum double DSU(2), which is identified to a deformed Poincaré symmetry:

$$\phi(g) \underset{\Lambda}{\rightarrow} \phi(\Lambda g \Lambda^{-1}) \quad \phi(g) \underset{x}{\rightarrow} e^{\kappa \mathrm{Tr} x g} \phi(g)$$

- It has a non-trivial co-product:

$$\phi(g_1) \otimes \phi(g_2) \rightarrow e^{\kappa \mathrm{Tr} x g_1 g_2} \phi(g_1) \otimes \phi(g_2)$$

- which translates into a modified addition of momenta:

$$\vec{p}(g_1) \oplus \vec{p}(g_2) = \vec{p}(g_1 g_2) \neq \vec{p}(g_1) + \vec{p}(g_2)$$

- It has a non-trivial braiding: $(g_1, g_2) \rightarrow (g_1 g_2 g_1^{-1}, g_1)$

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And in four space-time dimensions?

It all works well in 3d, but can it work in 4d too?

- We don't have the spinfoam model for 4d quantum gravity
- Particles are not simply topological defects in 4d

But some notions still hold..

- the correspondence between Feynman diagrams and observables of a topological spinfoam model [Baratin,Freidel 06]
- NCQFT can be written as theories of a curved group manifold
- Particles are almost topological defects from the viewpoint of BF theory [Freidel,Kowalski-Glikman,Starodubtsev 06]

$\Rightarrow$ It will work using group field theories ! [Fairbairn,EL 07]
$\Rightarrow$ We will derive DSR with a κ-deformed Poincaré symmetry from 4d spinfoam models [Girelli,EL,Oriti 09]

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Generating spinfoams as Feynman diagrams

Inspired from Matrix Models generating 2d triangulations:

$$S[M] = \underbrace{\frac{1}{2}\mathrm{Tr}M^2}_{\text{gluing }\triangle\text{s}} - \underbrace{\frac{\lambda}{3}\mathrm{Tr}M^3}_{\text{generates }\triangle\text{s}}$$

n-**d Group Field Theories**:

- Interaction term of GFTs represents *n*-simplex
- Propagator gives the gluing of *n*-simplices
- Feynman diagrams are *n*d (pseudo)triangulations and their evaluation gives the relevant spinfoam amplitudes
- Allows a rigorous and systematic definition of spinfoam amplitudes and provides a non-perturbative definition of the sum over triangulations.

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2d Group Field Theory

2d Group Field Theory for $\mathrm{SU}(2)$ BF theory:
Take a gauge-invariant field $\varphi(g_1, g_2) = \varphi(g_1 g, g_2 g)$ and define:

$$S[\varphi] = \frac{1}{2}\int [dg]^2\, \varphi(g_1, g_2)\varphi(g_2, g_1) - \frac{\lambda}{3!}\int [dg]^3\, \varphi(g_1, g_2)\varphi(g_2, g_3)\varphi(g_3, g$$

Gets written in term of gauge-fixed field $\varphi(g_1, g_2) = \phi(g_1 g_2^{-1})$:

$$S[\phi] = \frac{1}{2}\int [dg]\, \phi(g^{-1})\phi(g) - \frac{\lambda}{3!}\int [dg]^3\, \phi(g_1)\phi(g_2)\phi(g_3)\, \delta(g_1 g_2 g_3)$$

Decompose field in SU(2) irreps $\phi(g) = \sum_j d_j \mathrm{Tr}\phi^j D^j(g)$ with $d_j \times d_j$ matrices ϕ^j and we recover (decoupled) matrix models:

$$S[\phi] = \sum_j d_j \left[\frac{1}{2}\mathrm{Tr}(\phi^j)^2 - \frac{\lambda}{3!}\mathrm{Tr}(\phi^j)^3\right]$$

3d Group Field Theory

3d Boulatov's Group Field Theory for Ponzano-Regge:
Take a gauge-invariant field $\varphi(g_1, g_2, g_3) = \varphi(g_1 g, g_2 g, g_3 g)$:

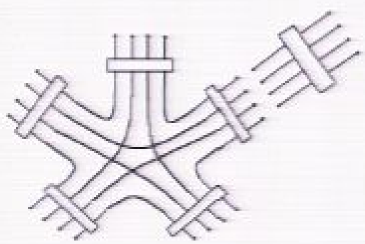
$$S[\varphi] = \frac{1}{2}\int [dg]^3\, \varphi(g_1, g_2, g_3)\varphi(g_3, g_2, g_1) - \frac{\lambda}{4!}\int [dg]^6\, \varphi(g_1, g_2, g_3)\varphi(g_3, g_4, g_5)\varphi(g_5, g_2, g_6)\varphi(g_6, g_4, g_1)$$

The interaction term defines a tetrahedron and this GFT generates spinfoam amplitudes for SU(2) BF theory.
Decompose field in irreps using Peter-Weyl theorem
$\varphi(g_1, g_2, g_3) = \sum_{j_i} \varphi^{j_1 j_2 j_3}_{m_1 m_2 m_3} \prod_i D^{j_i}_{m_i n_i}(g_i) \mathcal{I}^{j_1 j_2 j_3}_{n_1 n_2 n_3}$

$$S[\varphi] = \frac{1}{2}\sum |\varphi|^2 - \frac{\lambda}{4!}\sum \varphi\varphi\varphi\varphi\, \{6j\}$$

4d Group Field Theory

We can play the same game in 4d using a gauge-invariant field over G^4.



- The combinatorial structure of interaction reproduces a 4-simplex
- A trivial propagator glues the 4-simplices together

- Boulatov-Ooguri GFT generates SU(2) BF theory and the $\{15j\}$-symbol
- Any spinfoam model can be written as such a GFT with the interaction term given by the Fourier transform of the 4-simplex amplitude [Reisenberger-Rovelli]

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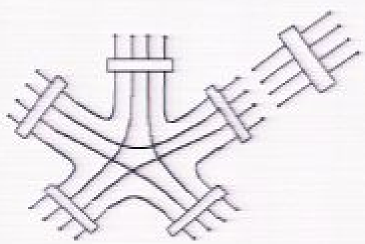

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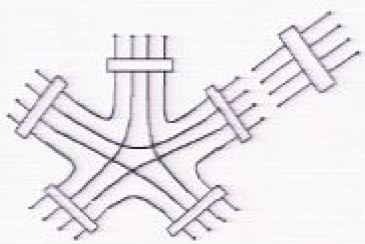
$$
\begin{aligned}
S[\varphi] \;=\; & \frac{1}{2}\int [dg]^3\, \varphi(g_1, g_2, g_3)\varphi(g_3, g_2, g_1) \\
& -\frac{\lambda}{4!}\int [dg]^6\, \varphi(g_1, g_2, g_3)\varphi(g_3, g_4, g_5)\varphi(g_5, g_2, g_6)\varphi(g_6, g_4, g_1)
\end{aligned}
$$

The interaction term defines a tetrahedron and this GFT generates spinfoam amplitudes for SU(2) BF theory.
Decompose field in irreps using Peter-Weyl theorem
$\varphi(g_1, g_2, g_3) = \sum_{j_i} \varphi^{j_1 j_2 j_3}_{m_1 m_2 m_3} \prod_i D^{j_i}_{m_i n_i}(g_i) \mathcal{I}^{j_1 j_2 j_3}_{n_1 n_2 n_3}$

$$S[\varphi] = \frac{1}{2}\sum |\varphi|^2 - \frac{\lambda}{4!}\sum \varphi\varphi\varphi\varphi\,\{6j\}$$

4d Group Field Theory

We can play the same game in 4d using a gauge-invariant field over G^4.



- The combinatorial structure of interaction reproduces a 4-simplex
- A trivial propagator glues the 4-simplices together

- Boulatov-Ooguri GFT generates SU(2) BF theory and the $\{15j\}$-symbol
- Any spinfoam model can be written as such a GFT with the interaction term given by the Fourier transform of the 4-simplex amplitude [Reisenberger-Rovelli]

The Interplay between Group field theory and NCQFT

$$\begin{tikzcd}
\text{Spinfoam Amplitudes for Geometry+Matter} \arrow[r, phantom, "="] & \text{Feynman diagrams of Effective NCQFT} \\
\text{Group Field Theory} \arrow[u] \arrow[r, phantom, "\overset{?}{=}"] & \text{NCQFT} \arrow[u]
\end{tikzcd}$$

NCQFT as Group Field Theories

Remember using SU(2) as the momentum manifold :

$$S[\phi] = \int dg\, \phi(g^{-1})\mathcal{K}(g)\phi(g) + \alpha_n \int [dg]^n\, \phi(g_1)..\phi(g_n)\delta(g_1..g_n)$$

and compare to 2d GFT:

$$S[\phi] = \frac{1}{2}\int [dg]\, \phi(g^{-1})\phi(g) - \frac{\lambda}{3!}\int [dg]^3\, \phi(g_1)\phi(g_2)\phi(g_3)\, \delta(g_1 g_2 g_3)$$

It's the same type of momentum conservation. The only difference is the trivial propagator , which ensures the consistent gluing of simplices.

From Group field theory to NCQFT: 2d variations

Starting from the 3d GFT for the Ponzano-Regge model for 3d quantum gravity, we define 2d variations $\varphi(g_1, g_2, g_3) = \psi(g_1 g_3^{-1})$ to reduce it to a 2d GFT:

$$
\begin{aligned}
S[\varphi] \;&=\; \frac{1}{2}\int [dg]^3\, \varphi(g_1, g_2, g_3)\varphi(g_3, g_2, g_1) \\
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$$
\rightarrow S[\varphi = \psi] = \frac{1}{2}\int [dg]\, \psi(g^{-1})\psi(g) - \frac{\lambda}{4!}\int [dg]^4\, \psi(g_1)..\psi(g_4)\, \delta(g_1..g_4)
$$

From Group field theory to NCQFT: classical solutions

We identify a class of "flat" classical solutions to the 3d GFT :

$$\varphi_f(g_1, g_2, g_3) \equiv \sqrt{\frac{3!}{\lambda}} \int dg \delta(g_1 g) f(g_2 g) \delta(g_3 g) \text{ with } \int f^2 = 1$$

and we define the effective dynamics of 2d variations around such classical field configurations:

$$S_{eff}[\psi] = S[\varphi_f + \psi] - S[\varphi_f]$$

This leads to a non-trivial propagator!

$$S_{eff}[\psi] = \frac{1}{2} \int [dg]\, \psi(g^{-1}) \mathcal{K}_f(g) \psi(g) - \frac{\mu_f}{3!} \int [dg]^3 \prod_i^3 \psi(g_i)\, \delta(g_1..g_3) - \frac{\lambda}{4!} \int [dg]^4 \prod_i^4 \psi(g_i)\, \delta(g_1..g_4)$$

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A new class of matrix models

We decompose this effective NCQFT into SU(2) representations and we get a matrix model with

- the kinetic term $\mathcal{K}(g)$ coupling matrices of different sizes
- but the whole action still invariant under the deformed Poincaré symmetry.

$\Rightarrow$ a new family of matrix models .
Can we solve them using standard matrix model techniques? It would open an approach to solving this class of NCQFT and the GFTs.

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What happened?

We have closed the diagram and identified a phase of the 3d GFT which is the NCQFT describing the effective dynamics of matter fields coupled to the quantum geometry. How did we do that?

- Looking at certain 2d group field variations around non-trivial classical solutions
- Matter in pure quantum gravity? GFT are summing over all geometries and topologies, matter is represented through some non-trivial topology&geometry configuration
- Classical solutions to the GFT provide non-trivial geometry backgrounds for gravity

Deriving 4d DSR from GFT

We follow the same steps as in 3d: [Girelli,EL,Oriti 09]

1. Start with the GFT for topological BF theory with group SO(4,1) used in the Freidel-Starodubtsev spinfoam approach (McDowell-Mansouri)
2. Write the DSR field theory with a κ-deformed Poincaré symmetry in term of a momentum space AN_3 and identify AN_3 as a subgroup of SO(4,1)
3. Find "flat" classical solutions of the GFT and study 2d variations localized on AN_3 around them
4. DSR is a specific phase of the 4d GFT
5. generic variations? dynamical localization? spinfoam for gravity?

Group Field Theories as NCQFT

Some lessons and directions for the future of spinfoams:

- The group manifold for GFT is the momentum representation.
- We should encounter the same problems with GFTs as with NCQFTs...Braiding of GFTs? Or do GFTs help to solve them?
- Matter is already in the GFT! But can we get gauge fields and fermions?
- A deformed Poincaré invariance for 3d GFT [Girelli,EL 10]
- Already a lot to understand about GFTs at the classical level...

What can we do?

We could try to..

- Classify good classical solutions of the 4d GFTs (use coherent intertwiners?) and compute some mean field approximation
- What are the (quantum) symmetries of the GFTs?
- Compute NCQFT correlations using matrix model techniques
- Renormalize GFTs using NCQFT techniques
- Integrable Structures in GFTs following matrix model results?

E xploit the interplay between GFT for spinfoams, NCQFT and matrix models!

What can we do?

Or is it time for a coffee break?