Title: Lecture 5: Surface operators in N=2 theories: Generalized SW geometry, 2d-4d wallcrossing

Date: Feb 26, 2010 11:15 AM

URL: http://pirsa.org/10020077

Abstract:

4d $\mathcal{N}=2$ SCFT $T_{g,n}$

$$Z_{S^4}[T_{g,n}](\tau_{uv})$$

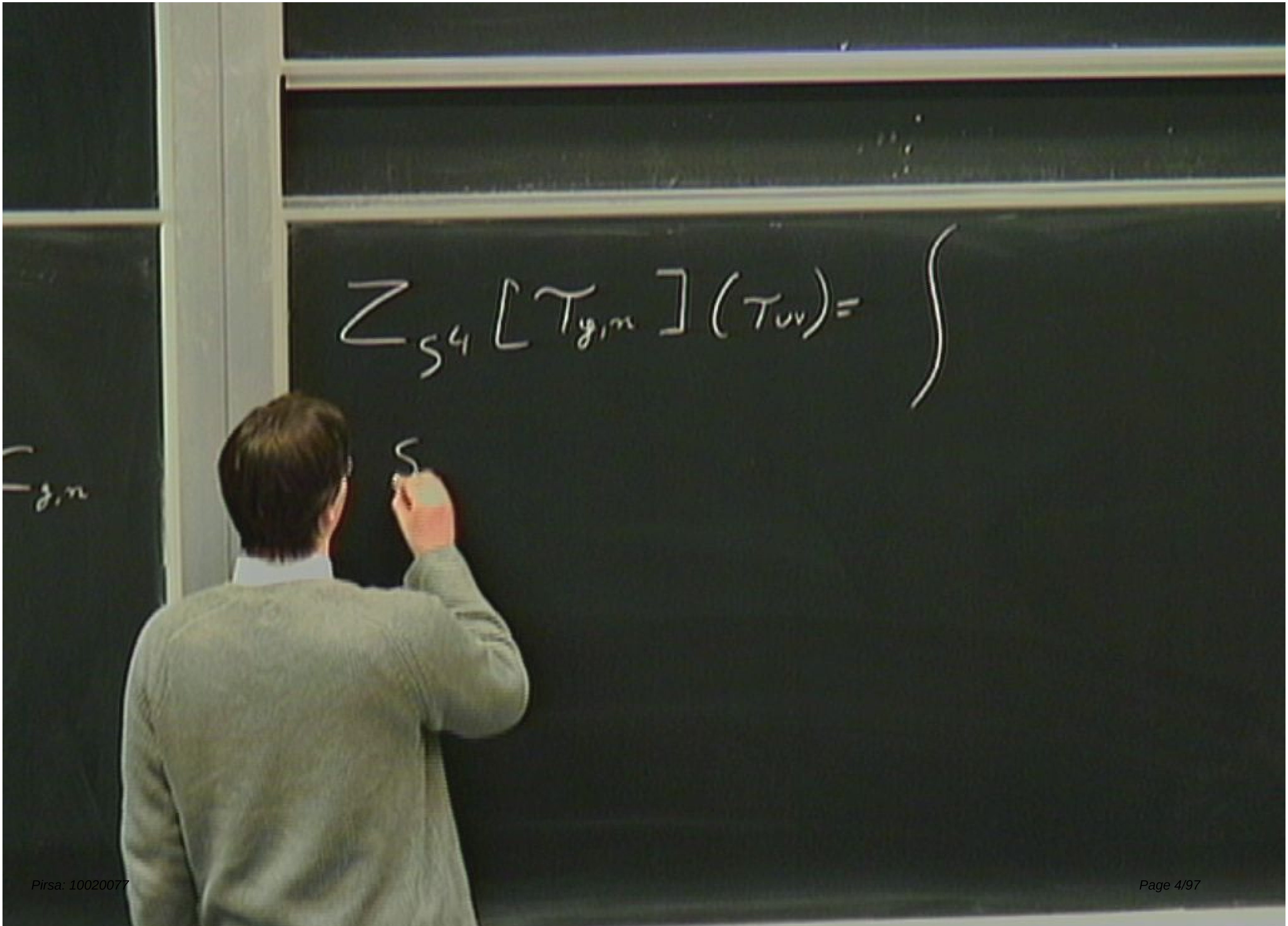
$C_{g,n}$

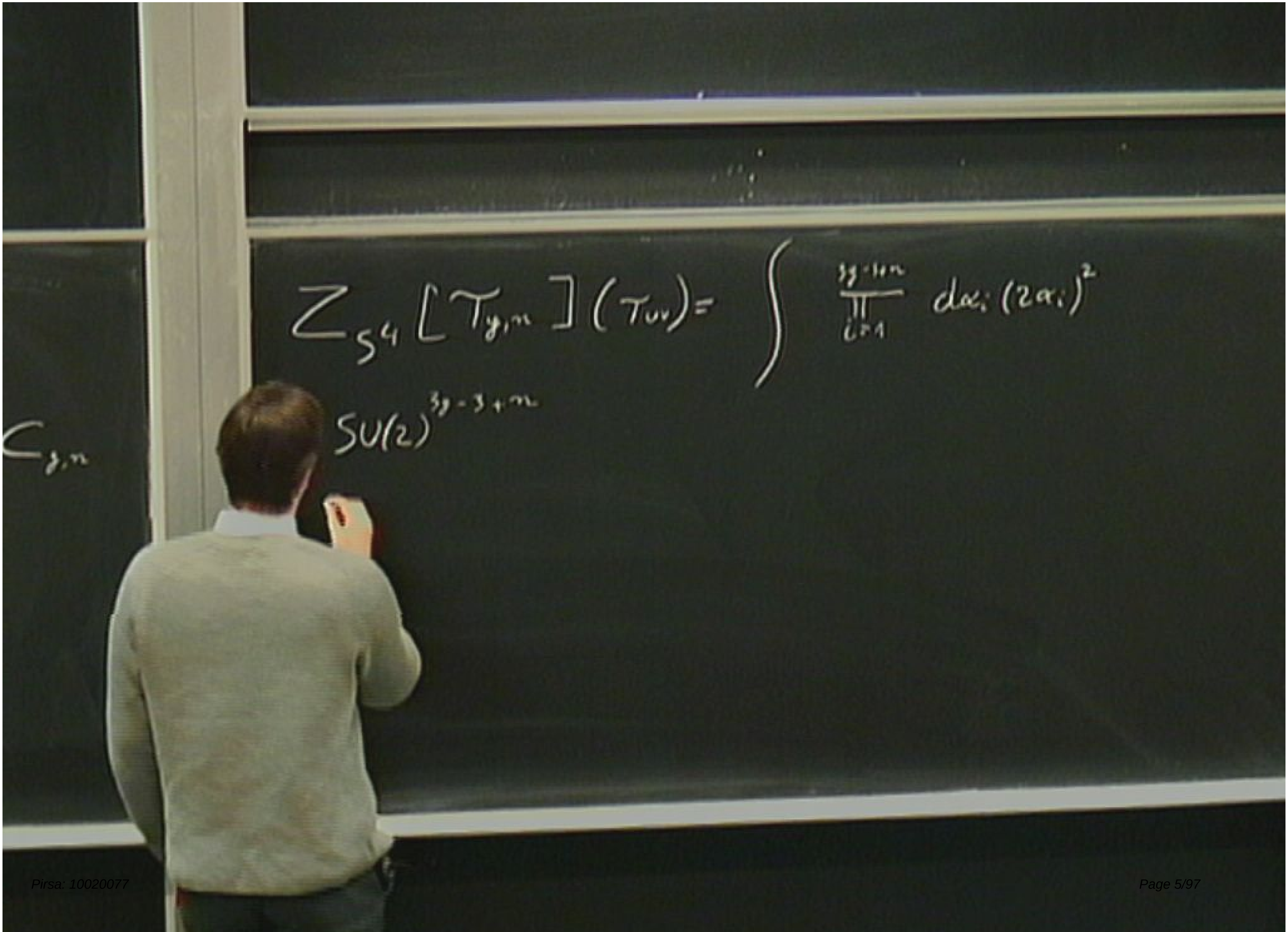
4d $\mathcal{N}=2$ SCFT $T_{g,n}$

$Z_{S^4}[T_{g,n}](\tau_{uv})$

$C_{g,n}$

PESTUN LOCALIZATION


$Z_{S^4}[T_{g,n}](\tau_{UV}) = \int$


$Z_{S^4}[T_{g,n}](\tau_{uv}) = \int \prod_{i=1}^{3g-3+n} da_i\,(2a_i)^2$
$SU(2)^{3g-3+n}$
$C_{g,n}$

$$Z_{S^4}[T_{g,n}](\tau_{UV}) = \int_{SU(2)^{3g-3+n}} \prod_{i=1}^{3g-3+n} da_i \, (2a_i)^2$$

$C_{g,n}$

$$\phi_i = \begin{pmatrix} a_i & \\ & -a_i \end{pmatrix}$$

$$Z_{S^4}[T_{g,n}](\tau_{UV}) = \int_{SU(2)^{3g-3+n}} \prod_{i=1}^{3g-3+n} d\alpha_i \,(2\alpha_i)^2$$

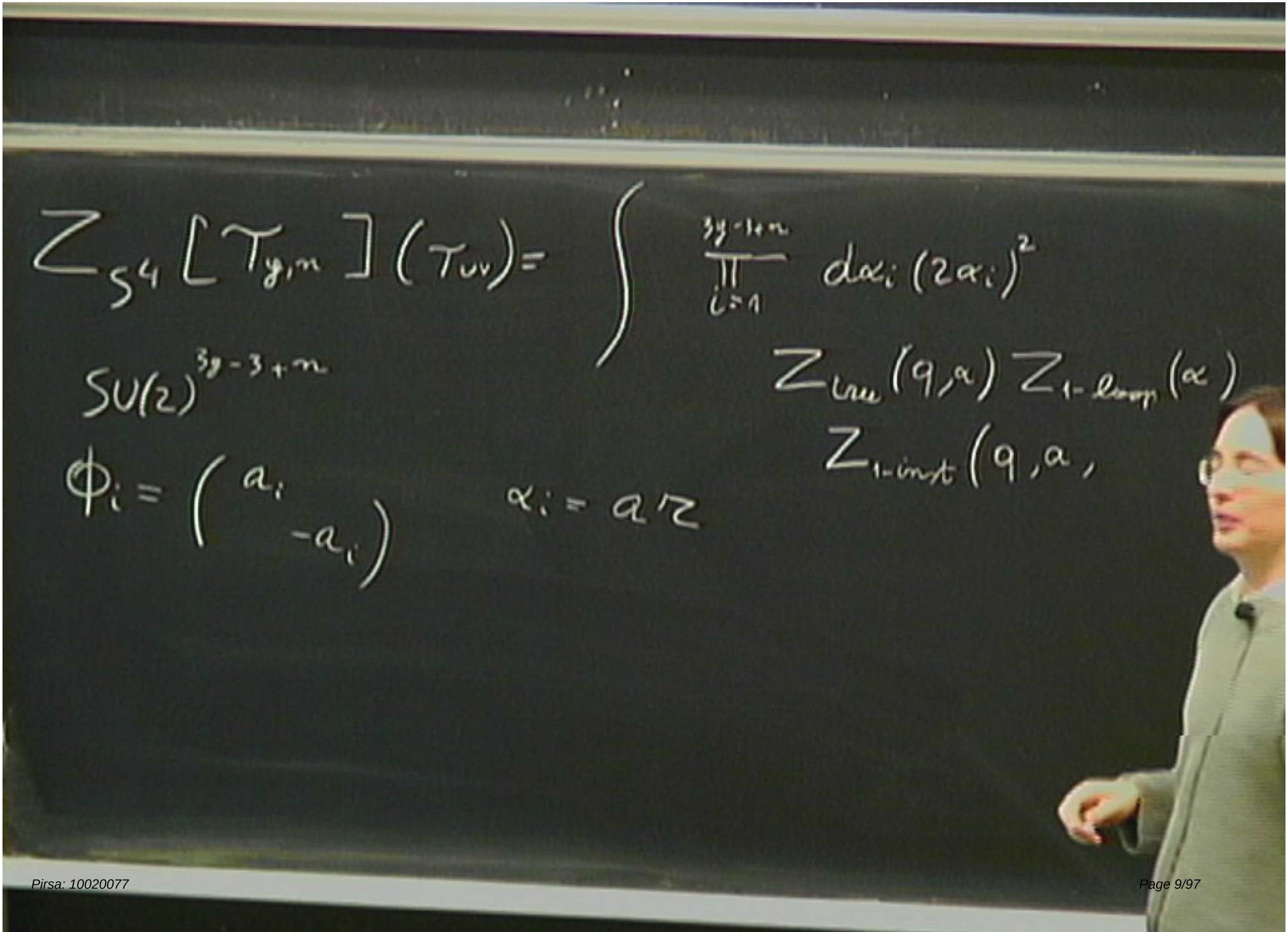
$$\phi_i = \begin{pmatrix} a_i & \\ & -a_i \end{pmatrix} \qquad \alpha_i = a r$$

$$Z_{S^4}[T_{g,n}](\tau_{UV}) = \int \prod_{i=1}^{3g-3+n} da_i\,(2a_i)^2$$

$$SU(2)^{3g-3+n} \qquad Z_{\text{tree}}(q,a)\, Z_{1\text{-loop}}(a)$$

$$Z$$

$$\phi_i = \begin{pmatrix} a_i & \\ & -a_i \end{pmatrix} \qquad a_i = a\tau$$


$Z_{S^4}[T_{g,n}](\tau_{UV}) = \int \prod_{i=1}^{3g-3+n} d\alpha_i (2\alpha_i)^2$
$Z_{tree}(q,\alpha) Z_{1-loop}(\alpha)$
$Z_{1-inst}(q, a,$
$SU(2)^{3g-3+n}$
$\phi_i = \begin{pmatrix} a_i & \\ & -a_i \end{pmatrix}$
$\alpha_i = a r$

$$Z_{S^4}[T_{g,n}](\tau_{uv}) = \int \prod_{i=1}^{3g-3+n} da_i \, (2a_i)^2 \; Z_{tree}(q \bar{q} a) \, Z_{1\text{-}loop}(a) \, Z_{inst}(q, a, \tfrac{1}{2}, \tfrac{1}{2}) \, Z_{anti\text{-}inst}(\bar{q}, a, \tfrac{1}{2}, \tfrac{1}{2})$$

$SU(2)^{3g-3+n}$

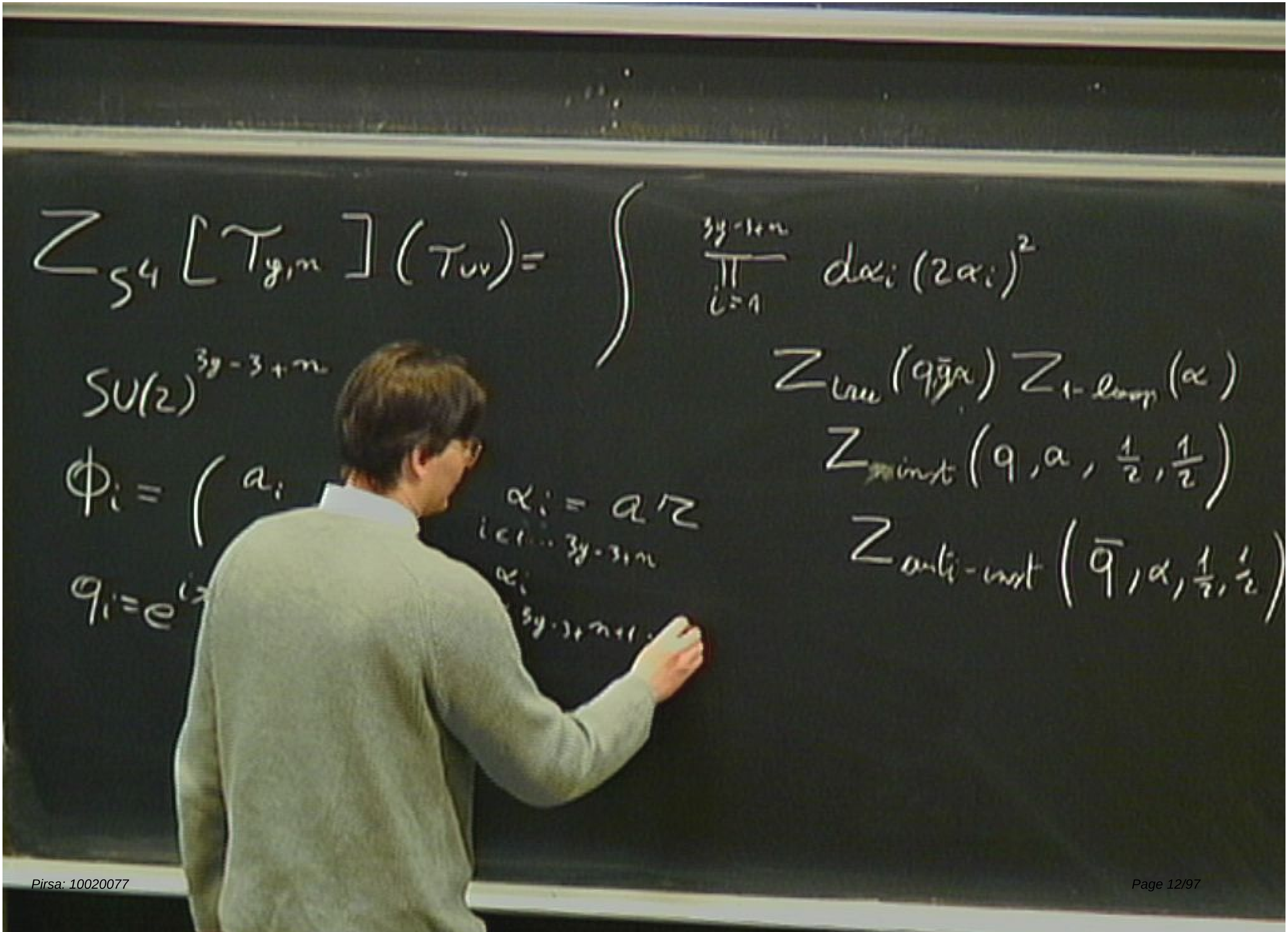
$$\phi_i = \begin{pmatrix} a_i & \\ & -a_i \end{pmatrix} \qquad a_i = a r$$

$$Z_{S^4}[T_{g,n}](\tau_{uv}) = \int \prod_{i=1}^{3g-3+n} d\alpha_i \, (2\alpha_i)^2 \, Z_{tree}(q\bar{q}\alpha) \, Z_{1-loop}(\alpha) \, Z_{inst}(q, a, \tfrac{1}{2}, \tfrac{1}{2}) \, Z_{anti-inst}(\bar{q}, \alpha, \tfrac{1}{2}, \tfrac{1}{2})$$

$$SU(2)^{3g-3+n}$$

$$\phi_i = \begin{pmatrix} a_i & \\ & -a_i \end{pmatrix} \qquad \alpha_i = a$$

$$q_i = e^{i\pi\tau_i}$$


$Z_{S^4}[T_{g,n}](\tau_{uv}) = \int \prod_{i=1}^{3g-3+n} da_i (2a_i)^2$
$Z_{tree}(q\bar{q}a)\, Z_{1-loop}(a)$
$Z_{inst}(q, a, \frac{1}{2}, \frac{1}{2})$
$Z_{anti-inst}(\bar{q}, a, \frac{1}{2}, \frac{1}{2})$
$SU(2)^{3g-3+n}$
$\phi_i = (a_i$
$a_i = a/2$
$i \in 1 \ldots 3g-3+n$
$q_i = e^{i}$

$$\alpha_i = a\tau$$

$$i \in 1 \ldots 3g-3+n$$

$$\alpha_i \equiv m\tau$$

$$i \in 3g-3+n+1 \ldots 3g-3+2n$$

$$Z_{\text{tree}} \sim \prod q_i^{\alpha_i^2} \quad q_i^{\alpha_i^2}$$

$$Z_{\text{tree}} \sim \prod q_i^{\alpha_i^2} \quad \bar{q}_i^{\alpha_i^2}$$

$$Z_{\text{1-loop}} =$$

$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \quad q_i^{\alpha_i^2}$$

$$Z_{1\text{-loop}} = \frac{\prod Z_{vect}(2\alpha_i)}{\prod Z_{hyp}}$$

$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \qquad q_i^{\alpha_i^2}$$

$$Z_{1\text{-loop}} = \frac{\prod_{i=1}^{3g-3+n} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+n} Z_{hyp}(}$$

$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \quad q_i^{\alpha_i^2}$$

$$Z_{1\text{-}loop} = \frac{\prod_{i=1}^{3g-3+n} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+n} Z_{hyper}(\alpha_i, \ldots}$$

4d $\mathcal{N}=2$ SCFT $T_{g,n}$

$$Z_{S^4}[T_{g,n}](\tau_{UV})$$

$C_{g,n}$

PESTUN LOCALIZATION

$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \qquad q_i^{\alpha_i^2}$$

$$Z_{1\text{-}loop} = \frac{\prod_{i=1}^{3g-3+n} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+n} Z_{hyp}(\alpha_i, \alpha_j, \alpha_k)}$$

$I \leftrightarrow (i, j, k)$

$$Z_{1\text{-loop}} = \frac{\prod_{i=1}^{3g-3+n} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+n} Z_{hyper}(\alpha_i, \alpha_j, \alpha_k)}$$

$$I \leftrightarrow (i, j, k)$$

$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \quad q_i^{\alpha_i^2}$$

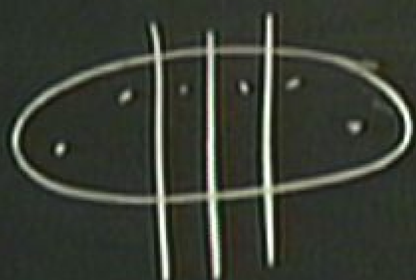
$$Z_{inst} = \sum \prod q_i^{n_i} R$$

$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \; \bar{q}_i^{\alpha_i^2}$$

$$Z_{1\text{-}loop} = \frac{\prod_{i=1}^{3g-3+m} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+m} Z_{hyp}(\alpha_i, \alpha_j, \alpha_k)}$$

$$I \leftrightarrow ((i,j),k)$$

$$Z_{inst} = \sum \prod q_i^{n_i} \, \mathcal{R}_{\{n_i\}}(\alpha_i)$$

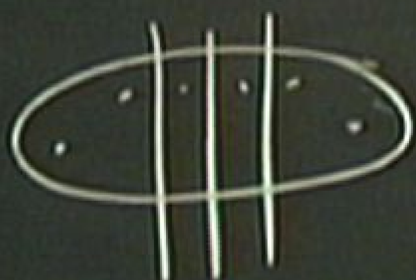


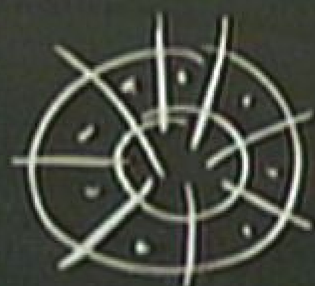
$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \quad q_i^{\alpha_i^2}$$

$$Z_{1\text{-}loop} = \frac{\prod_{i=1}^{3g-3+n} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+n} Z_{hyp}(\alpha_i, \alpha_j, \alpha_k)}$$

$$I \leftrightarrow (i,j,k)$$

$$Z_{inst} = \sum \prod q_i^{n_i} \mathcal{R}_{\{n_i\}}(\alpha_i)$$



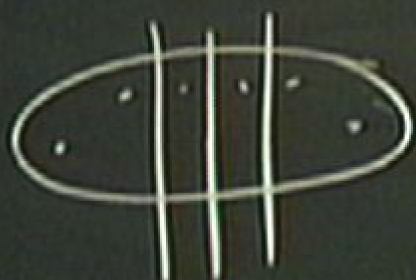


$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \quad q_i^{\alpha_i^2}$$

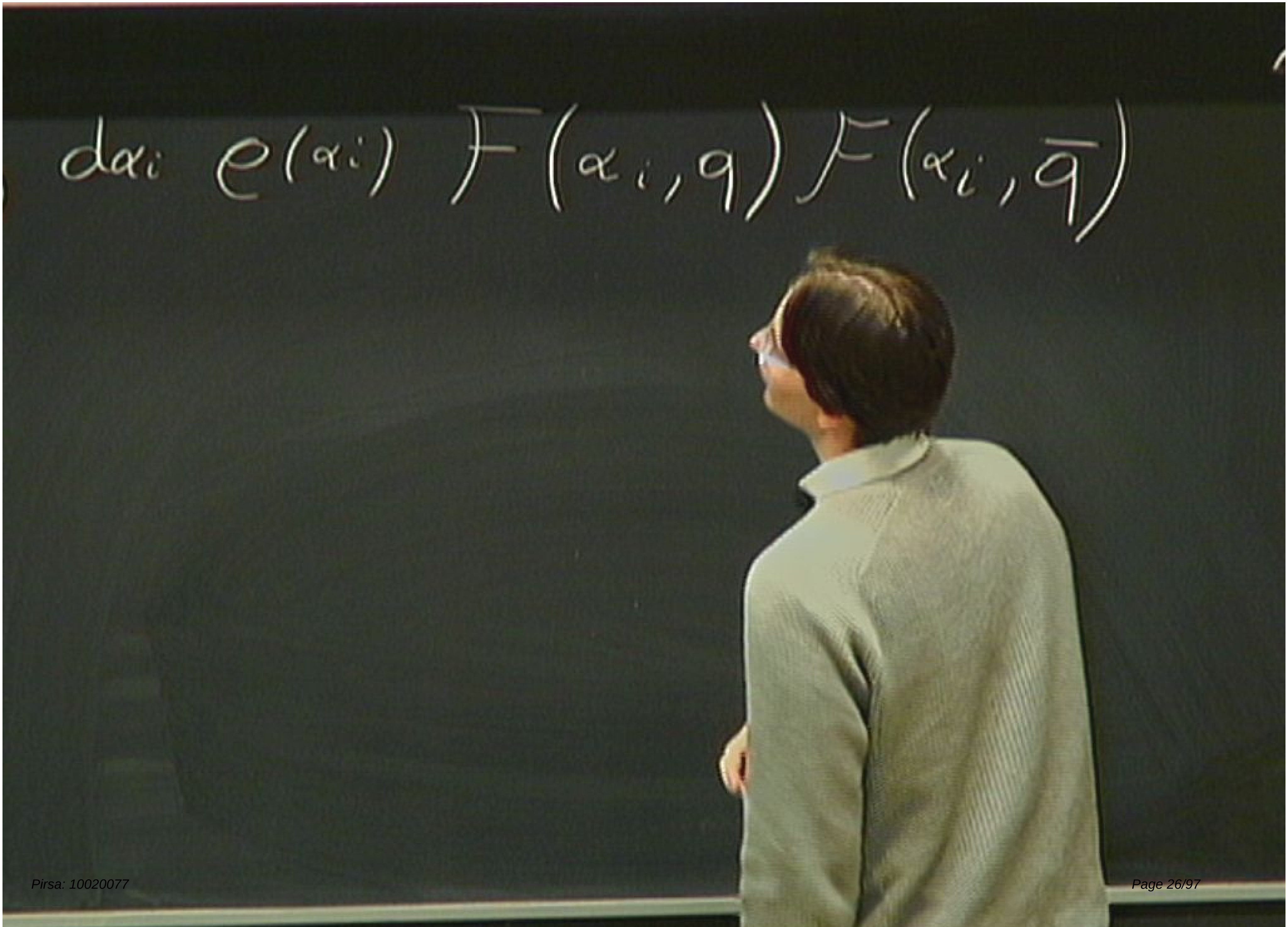
$$Z_{1\text{-}loop} = \frac{\prod_{i=1}^{3g-3+m} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+m} Z_{hyper}(\alpha_i, \alpha_j, \alpha_k)}$$

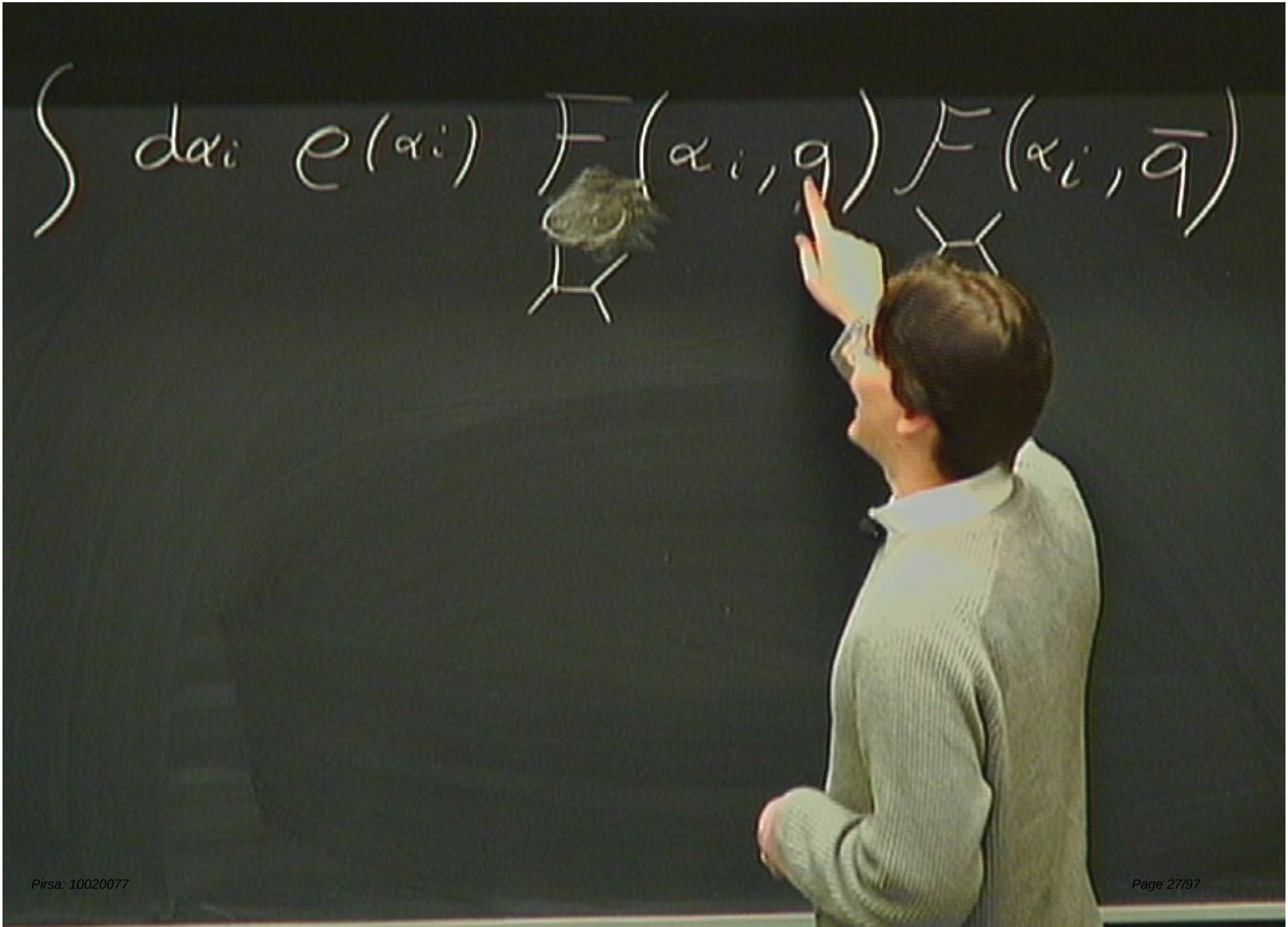
$$I \leftrightarrow ((i,j),k)$$

$$Z_{inst} = \sum \prod q_i^{n_i} \mathcal{R}_{\{n_i\}}(\alpha_i)$$



$$\prod U(2)$$


$d\alpha_i \;\; \rho(\alpha_i) \;\; \mathcal{F}(\alpha_i, q)\, \mathcal{F}(\alpha_i, \bar{q})$


$\int d\alpha_i \; e(\alpha_i) \; F(\alpha_i, q) \, F(\alpha_i, \bar{q})$

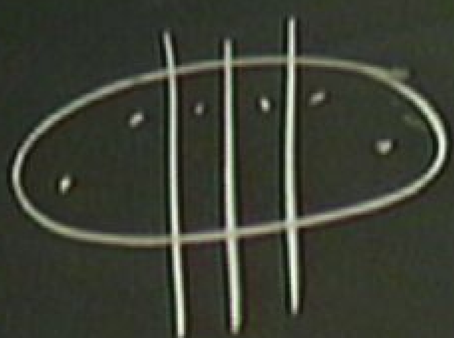
$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \quad q_i^{\alpha_i^2}$$

$$Z_{1\text{-}loop} = \frac{\prod_{i=1}^{3g-3+n} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+n} Z_{hyp}(\alpha_i, \alpha_j, \alpha_k)}$$

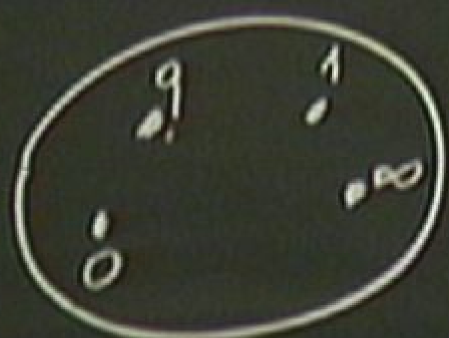
$I = 1$

$I \leftrightarrow (i,j,k)$

$$Z_{inst} = \sum \prod q_i^{n_i} \, R_{\{n_i\}}(\alpha_i)$$



$\prod U(2)$

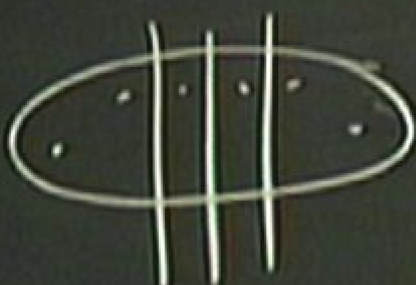


$$Z_{tree} \sim \prod q_i^{\alpha_i^2} \quad q_i^{\alpha_i^2}$$

$$Z_{1\text{-}loop} = \frac{\prod_{i=1}^{3g-3+n} Z_{vect}(2\alpha_i)}{\prod_{I=1}^{2g-2+n} Z_{hyper}(\alpha_i, \alpha_j, \alpha_k)}$$

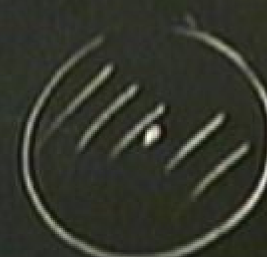
$$I \leftrightarrow (i,j,k)$$

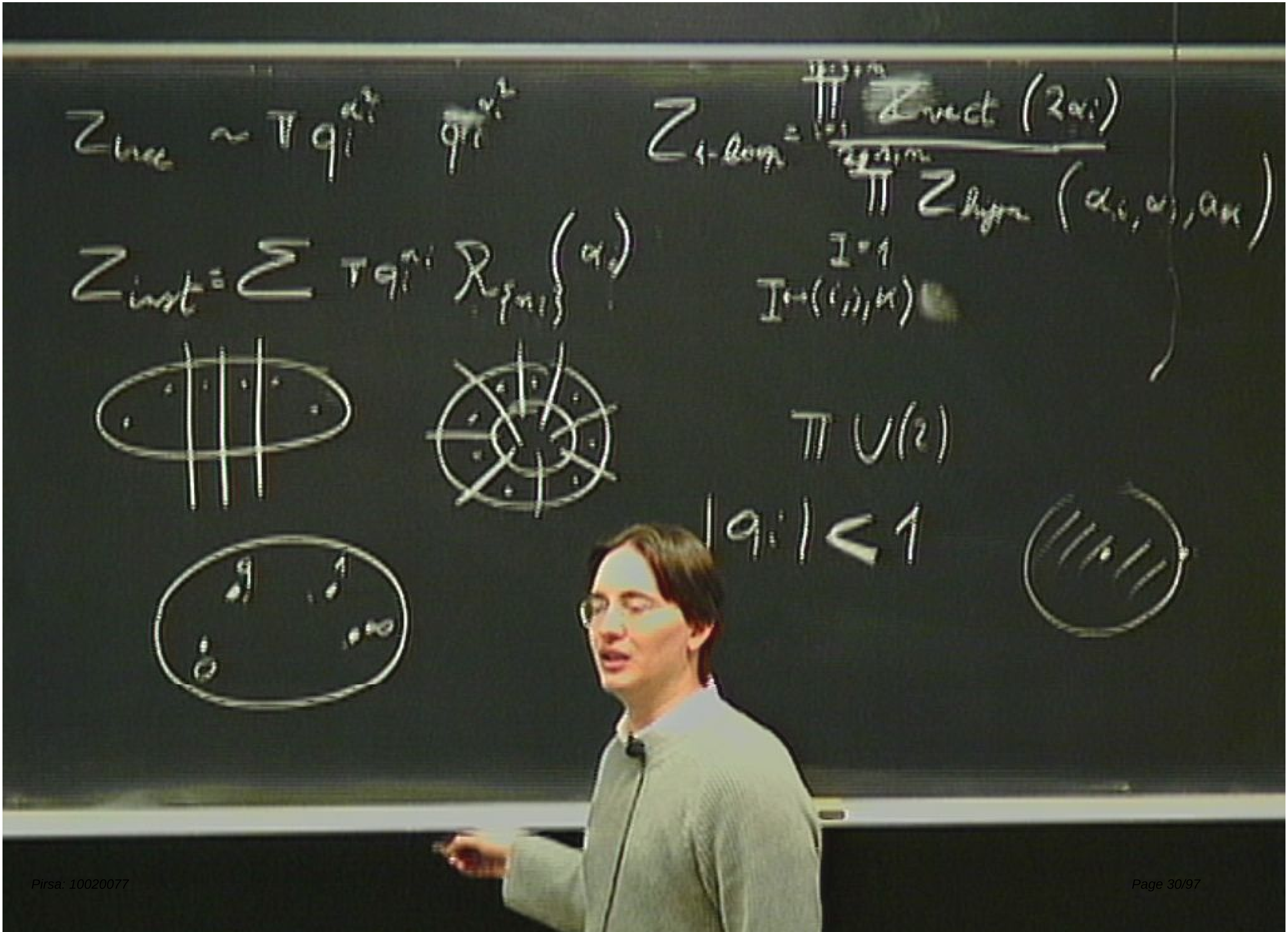
$$Z_{inst} = \sum \prod q_i^{n_i} \, R_{\{n_i\}}(\alpha_i)$$



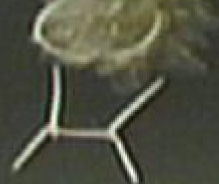
$$\prod U(2)$$

$$|q_i| < 1$$

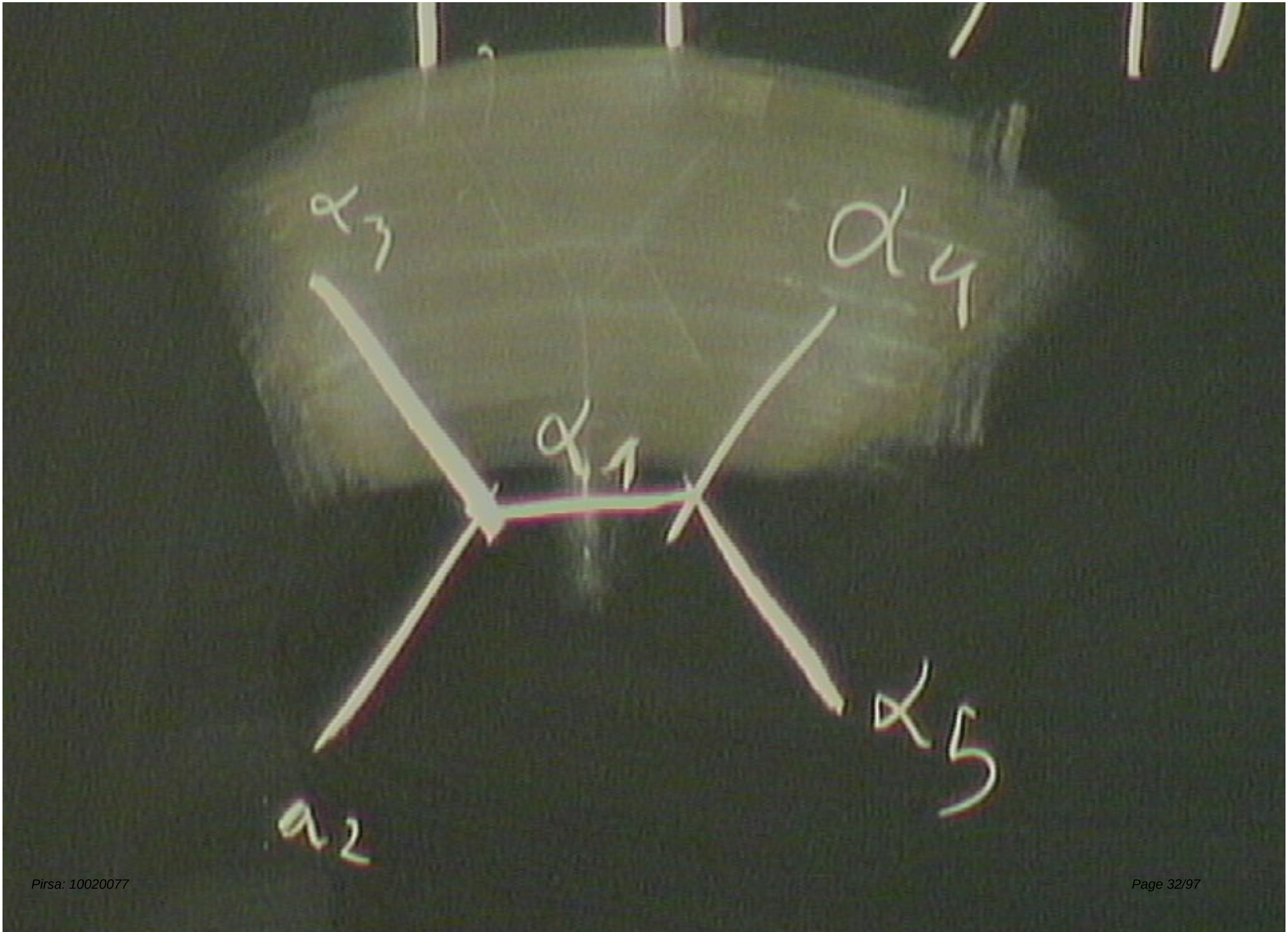


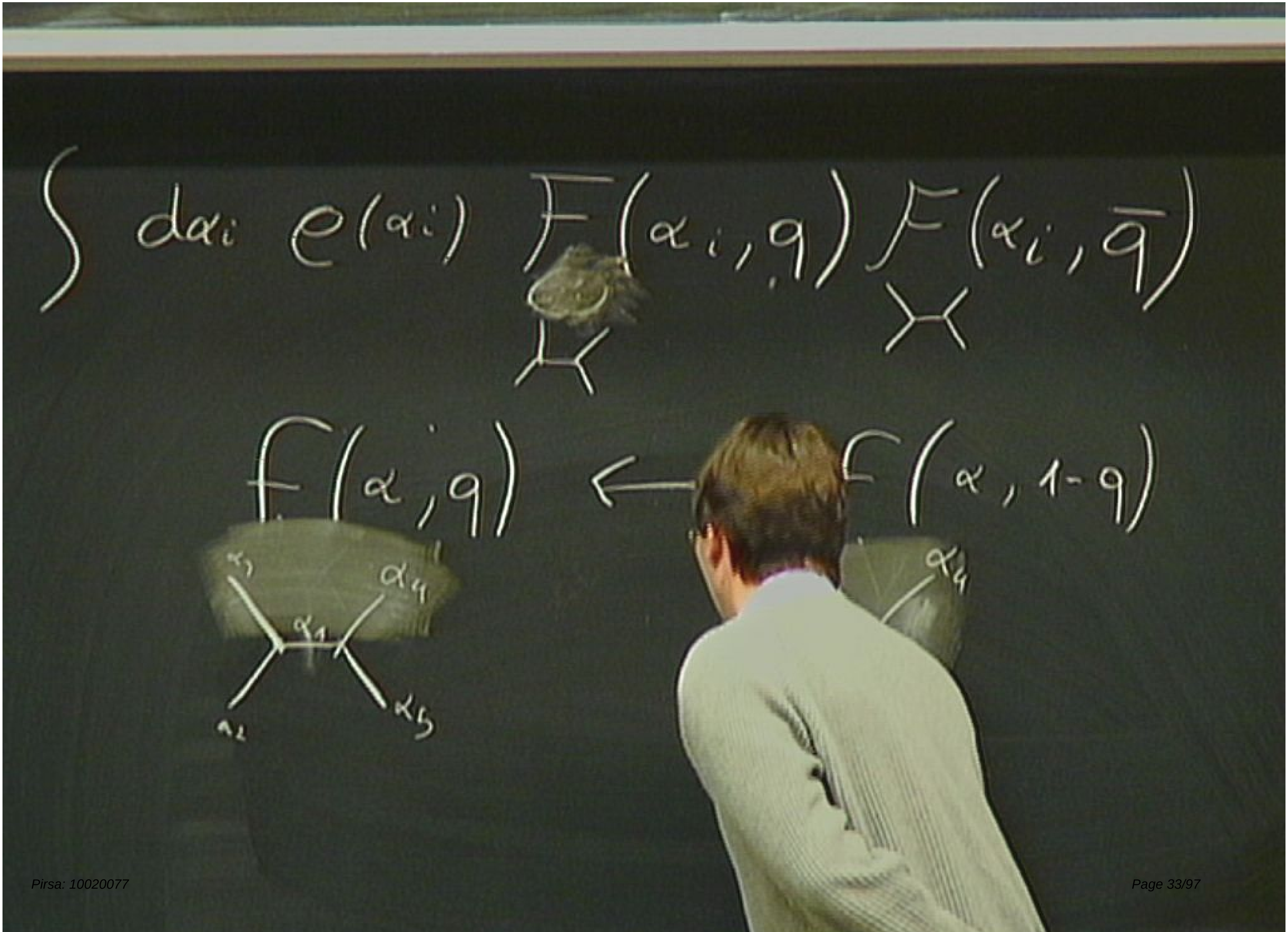

Z_tree ~ Π q_i^{a_i^2}
Z_inst = Σ Π q_i^{n_i}
Z_vect (2a_i)
Z_hyper
I = 1
Π U(2)
|q_i| < 1

$$\int d\alpha_i \; e(\alpha_i) \; F(\alpha_i, q) \; F(\alpha_i, \bar{q})$$

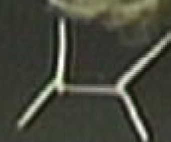


$$f(\alpha, q) \longleftrightarrow f(\alpha, 1-q)$$

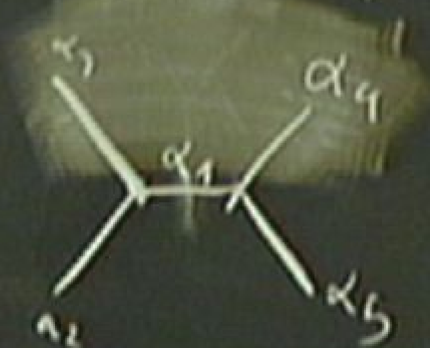

α3
α4
α1
α5
a2

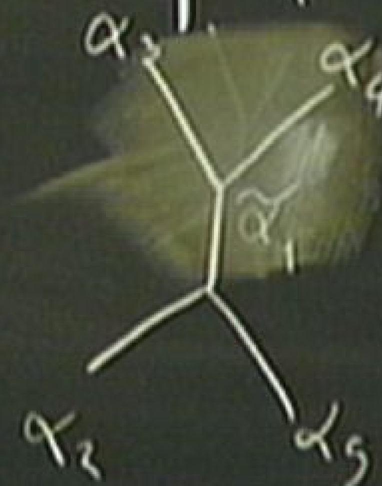

$\int d\alpha_i \, e(\alpha_i) \, F(\alpha_i, q) \, F(\alpha_i, \bar{q})$
$f(\alpha, q) \leftarrow f(\alpha, 1-q)$
α_1
α_4
α_2
α_5

$$\int d\alpha_i \; e(\alpha_i) \; F(\alpha_i, q) \, F(\alpha_i, \bar{q})$$

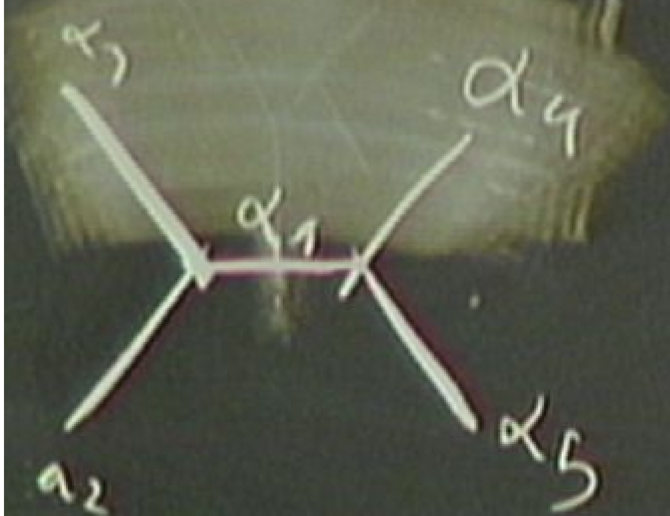
 

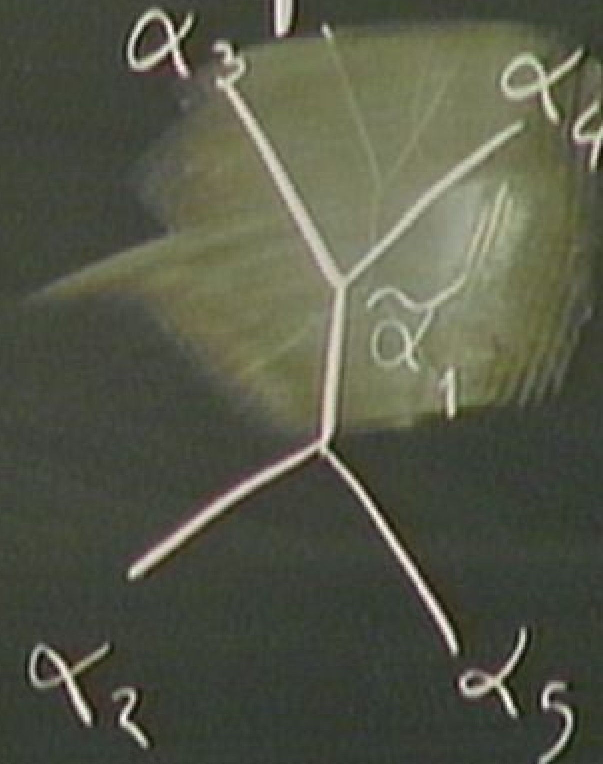
$$f(\alpha, q) \longleftrightarrow f(\tilde{\alpha}, 1-q)$$



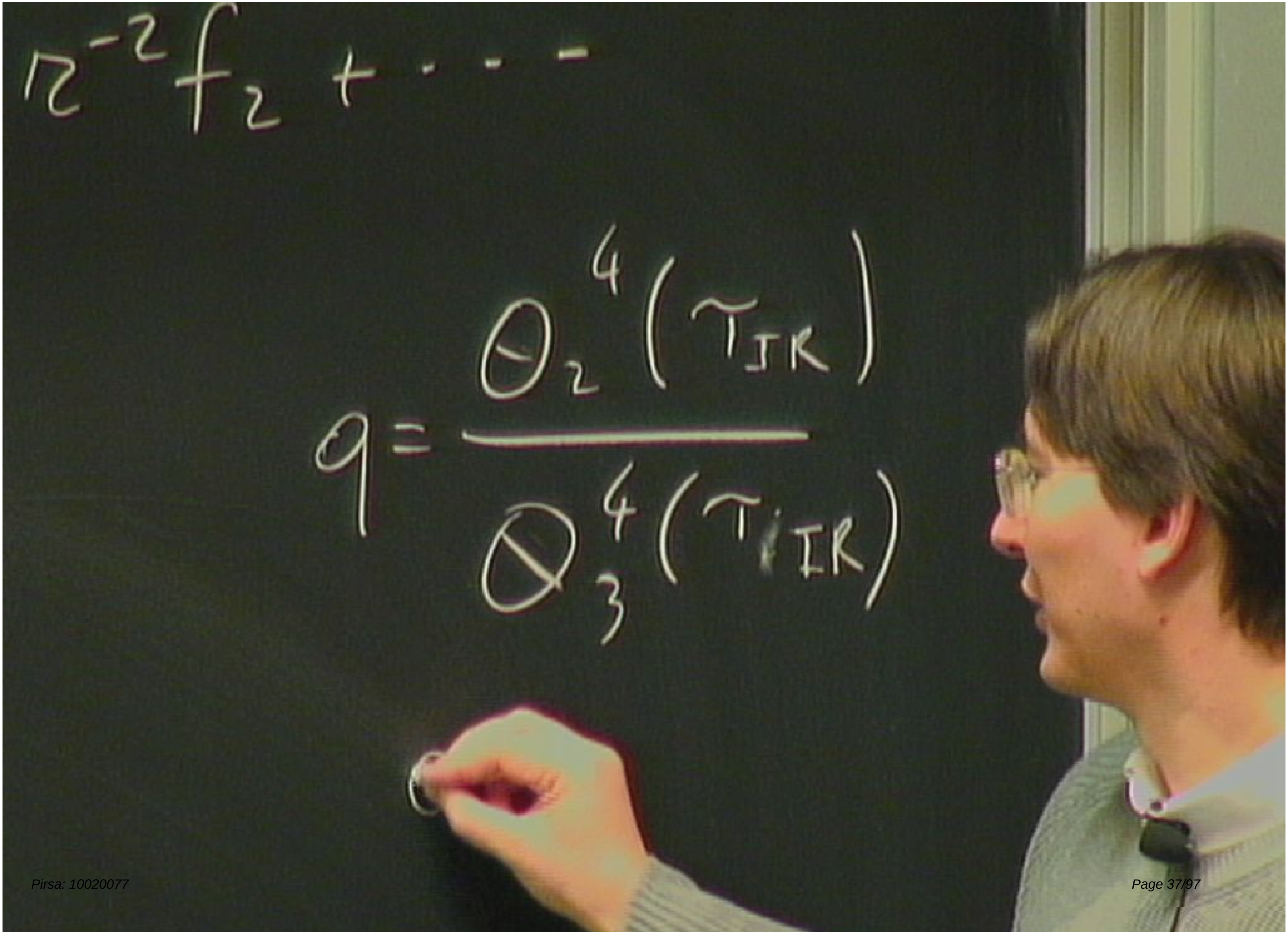


$$f(\alpha; q) \longleftrightarrow f(\tilde{\alpha}$$





$$f(ra) \xrightarrow{r\to\infty} e^{r^2 f_0 + f_1 + r^{-2} f_2 + \cdots}$$


$r^{-2} f_2 + \cdots$
$q = \dfrac{\theta_2^4(\tau_{IR})}{\theta_3^4(\tau_{IR})}$

$$q = \frac{\theta_2^4(\tau_{IR})}{\theta_3^4(\tau_{IR})}$$

$$q \to 1 - q$$

$$q \to 1/q$$

$$f(ra) \xrightarrow{r \to \infty} e^{r^2 f_0 + f_1 + r^{-2} f_2 + \cdots}$$

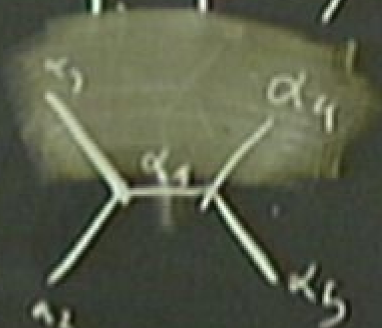
$$q = \frac{\Theta_2^4(\tau_{JK})}{\Theta_3^4(\tau_{JK})}$$

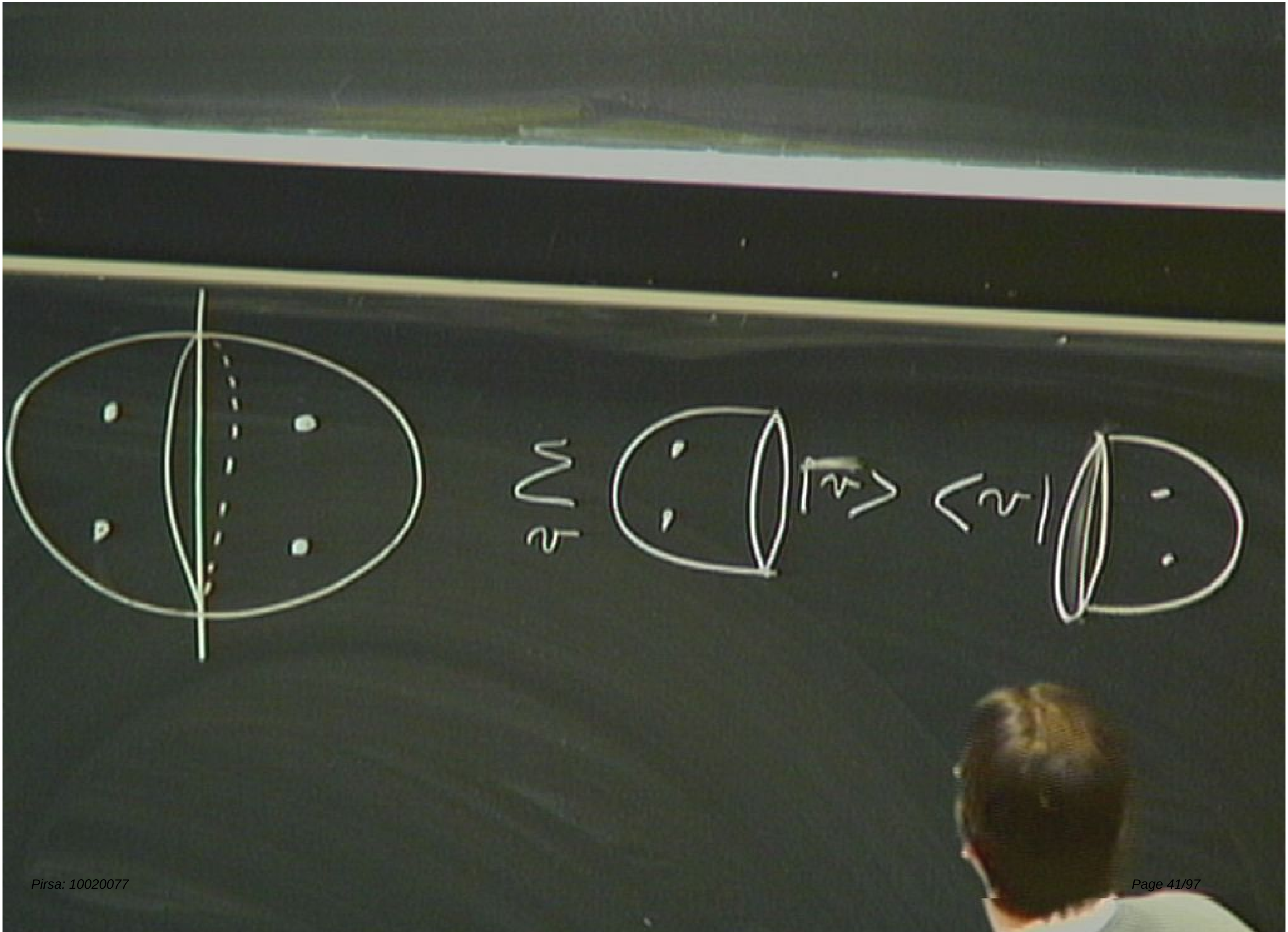
$q \to 1 - q$

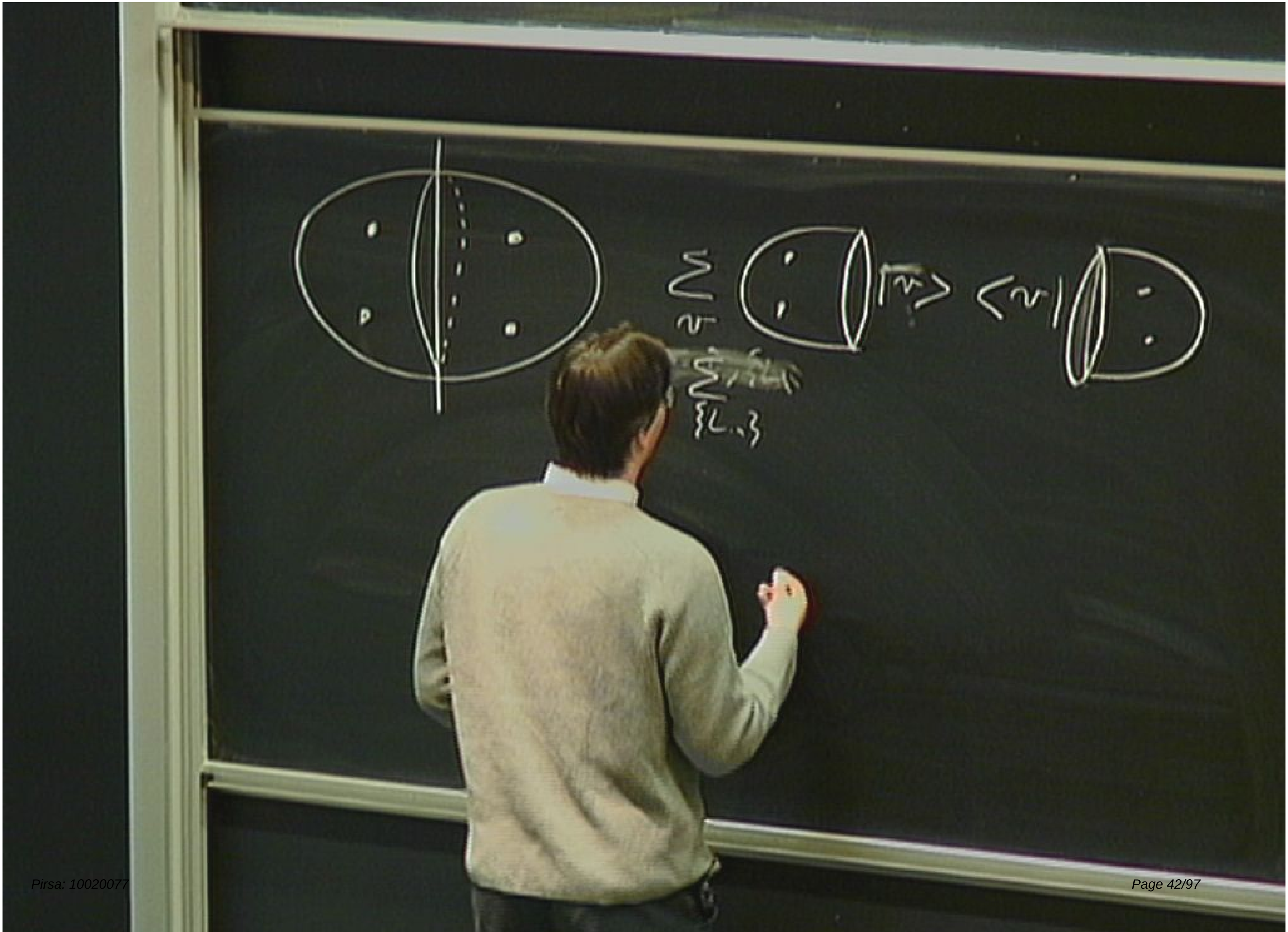
$q \to 1/q$

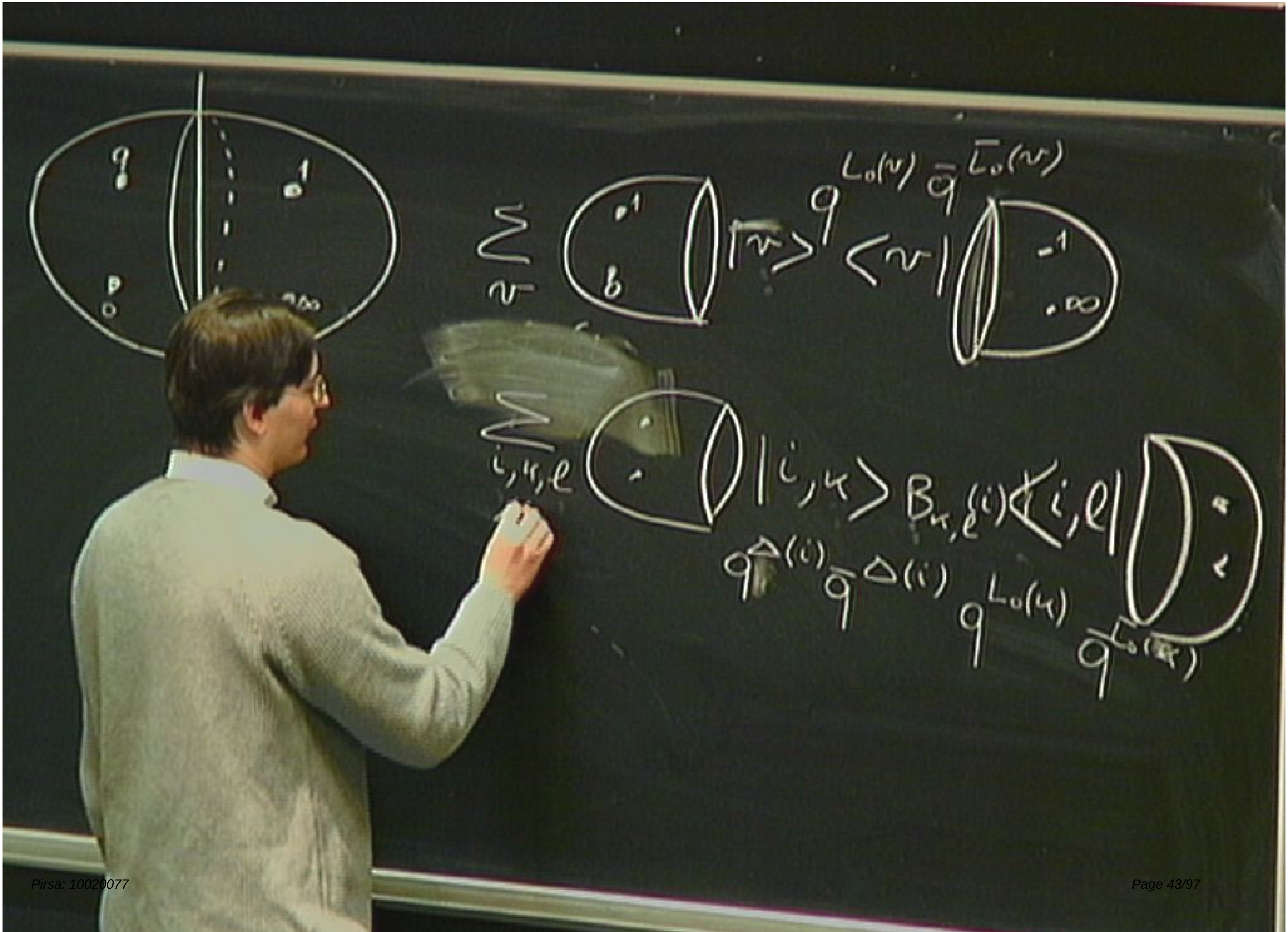
$$\int d\alpha_i \, e(\alpha_i) \, F(\alpha_i, q) \, F(\alpha_i, \bar{q})$$

$$f(\alpha, q) \longleftrightarrow f(\tilde{\alpha}, 1-q)$$




$\sum_v$
$|v\rangle$
$\langle v|$

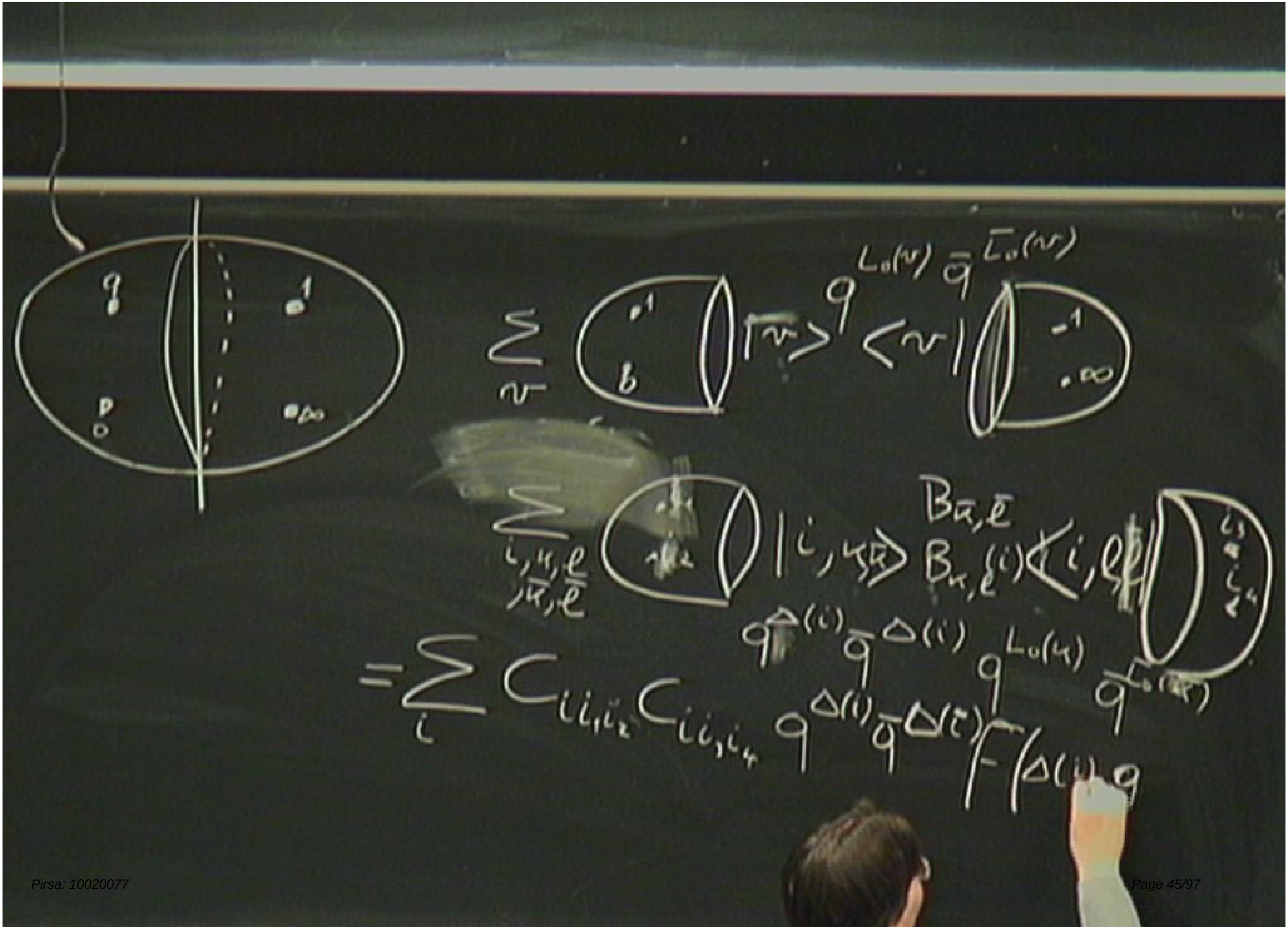


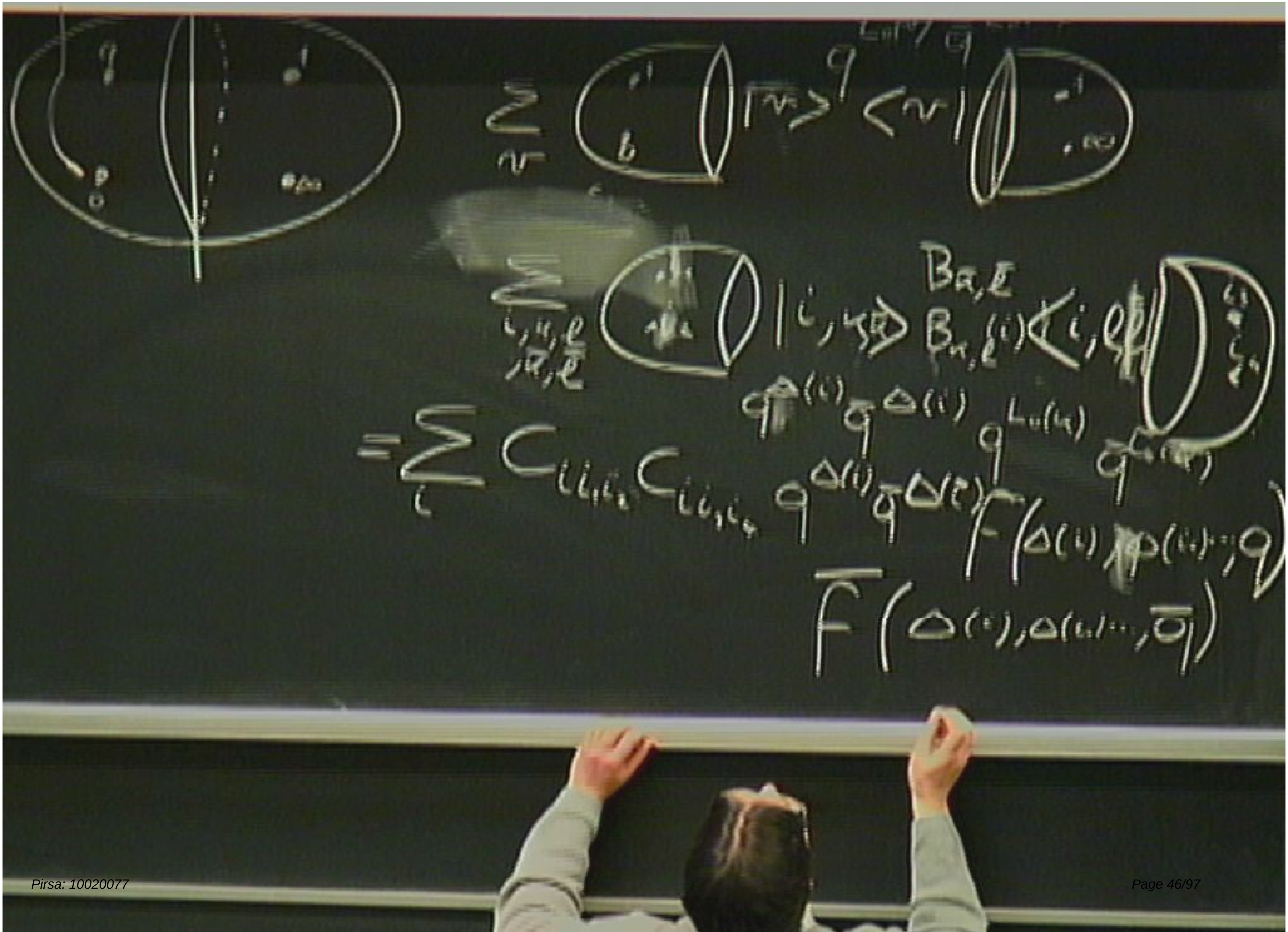


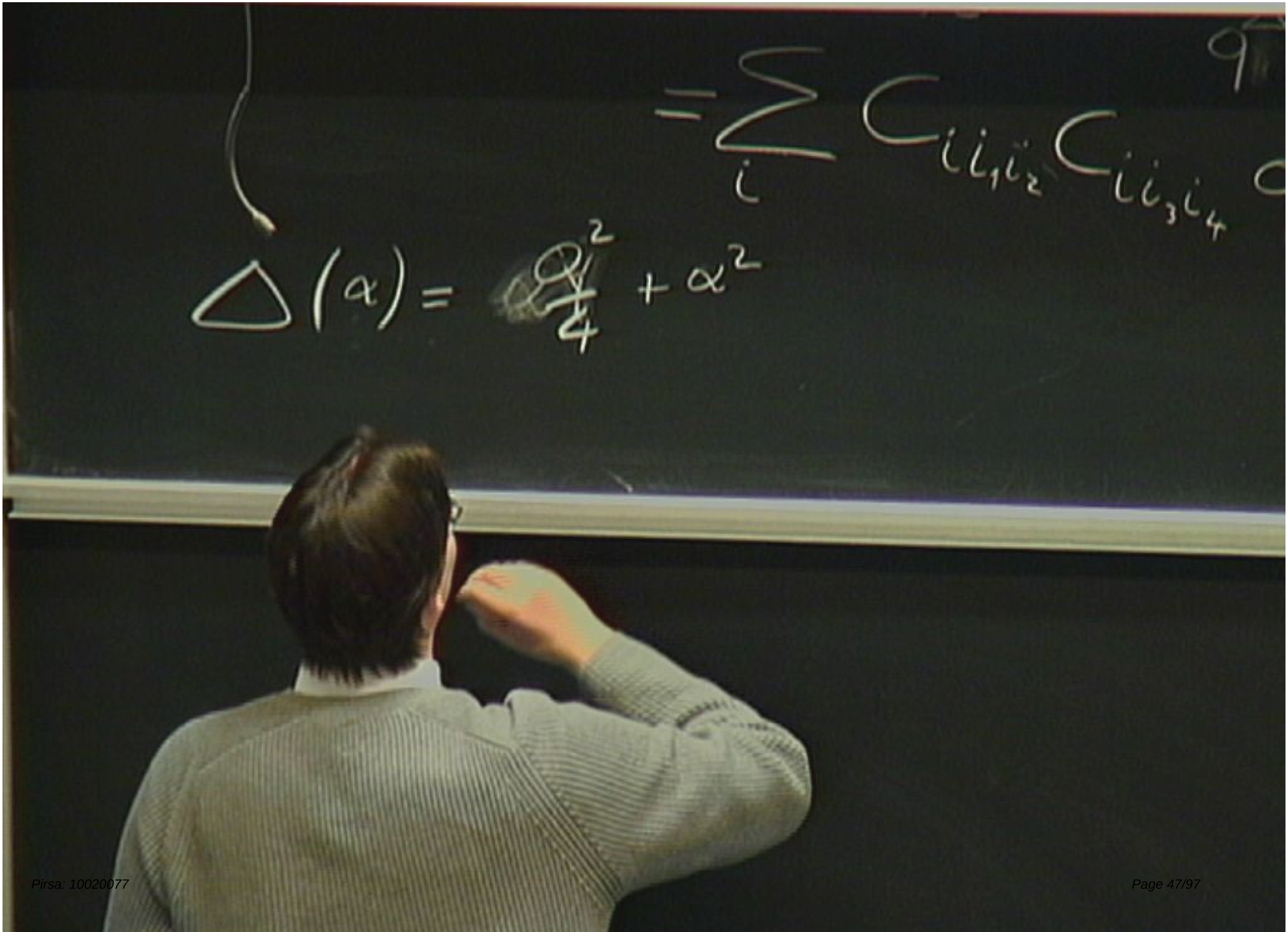
$$\sum_{v} \; \cdot 1 \;\; b \; |v\rangle \, q^{L_0(v)} \, \bar{q}^{\bar{L}_0(v)} \, \langle v| \; \cdot -1 \;\; \cdot \infty$$

$$\sum_{\substack{i,k,\ell \\ \bar{k},\bar{\ell}}} \; |i,k,\bar{k}\rangle \; B_{\bar{k},\bar{\ell}} \, B_{k,\ell}(i) \, \langle i,\ell,\bar{\ell}|$$

$$q^{\Delta(i)} \, \bar{q}^{\Delta(i)} \quad q^{L_0(k)} \quad \bar{q}^{L_0(\bar{k})}$$


q
1
0
∞
$\sum_{v} |v\rangle\, q^{L_0(v)}\, \bar{q}^{\bar{L}_0(v)} \langle v|$
$\sum_{i,k,\ell,\bar{k},\bar{\ell}} |i,k,\bar{k}\rangle\, B_{k,\ell}(i)\, B_{\bar{k},\bar{\ell}} \langle i,\ell,\bar{\ell}|$
$q^{\Delta(i)}\, \bar{q}^{\bar{\Delta}(i)}\, q^{L_0(k)}\, \bar{q}^{\bar{L}_0(\bar{k})}$
$= \sum_{i} C_{ii_1i_2} C_{ii_3i_4}\, q^{\Delta(i)} \bar{q}^{\bar{\Delta}(i)} F(\Delta(i), q$

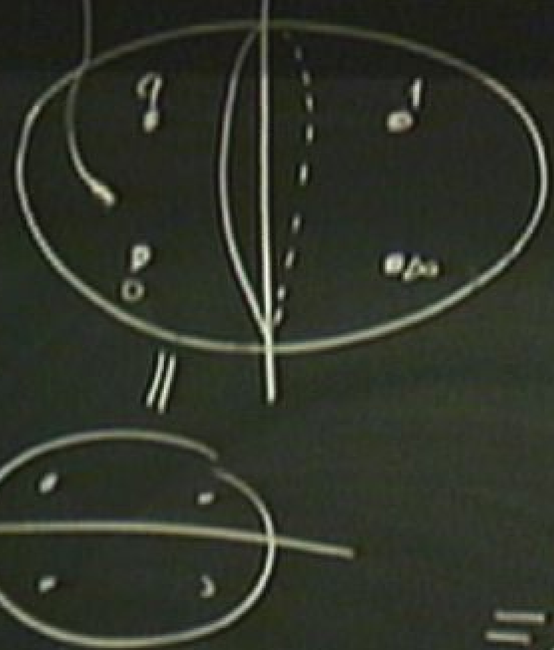


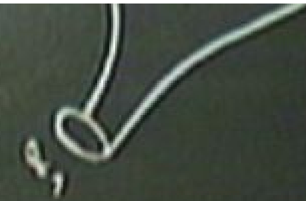

$= \sum_i C_{ii_1i_2} C_{ii_3i_4}$
$\Delta(\alpha) = \frac{\alpha^2}{4} + \alpha^2$

$$= \sum_i C_{i i_1 i_2} C_{i i_3 i_4}$$

$$\Delta(\alpha) = \frac{Q^2}{4} + \alpha^2$$

$$Q = b + \frac{1}{b} \qquad c = 1 + 6Q^2$$





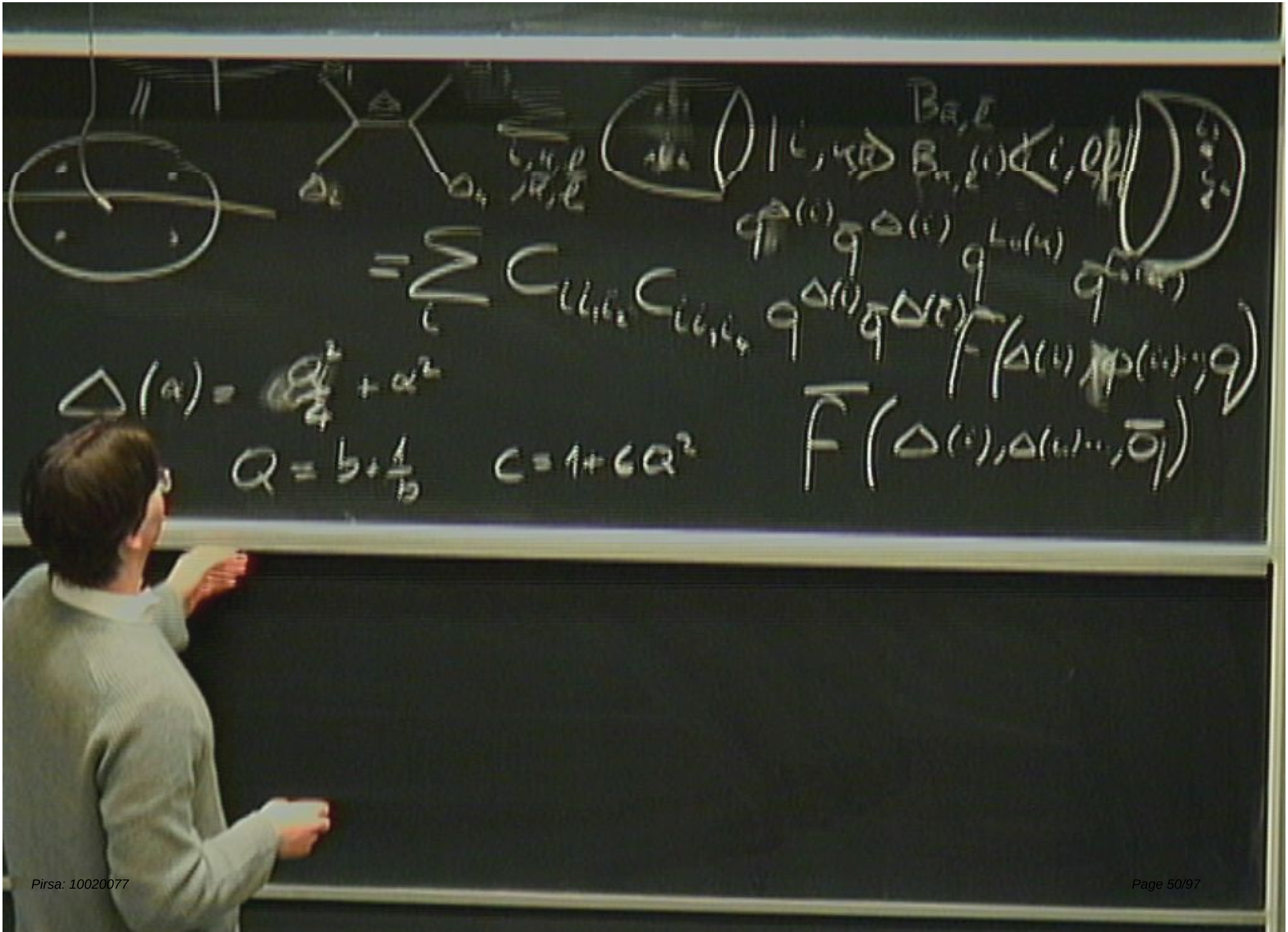
$$= \sum_{v} |v\rangle\, q^{L_0(v)}\, \bar{q}^{\bar{L}_0(v)} \langle v|$$

$$= \sum_{\substack{i,k,\ell \\ \bar{k},\bar{\ell}}} |i,k,\bar{k}\rangle\, B_{k,\ell}(i)\, B_{\bar{k},\bar{\ell}} \langle i,\ell,\bar{\ell}|\; q^{\Delta(i)}\, \bar{q}^{\Delta(i)}\, q^{L_0(k)}\, \bar{q}^{\bar{L}_0(\bar{k})}$$

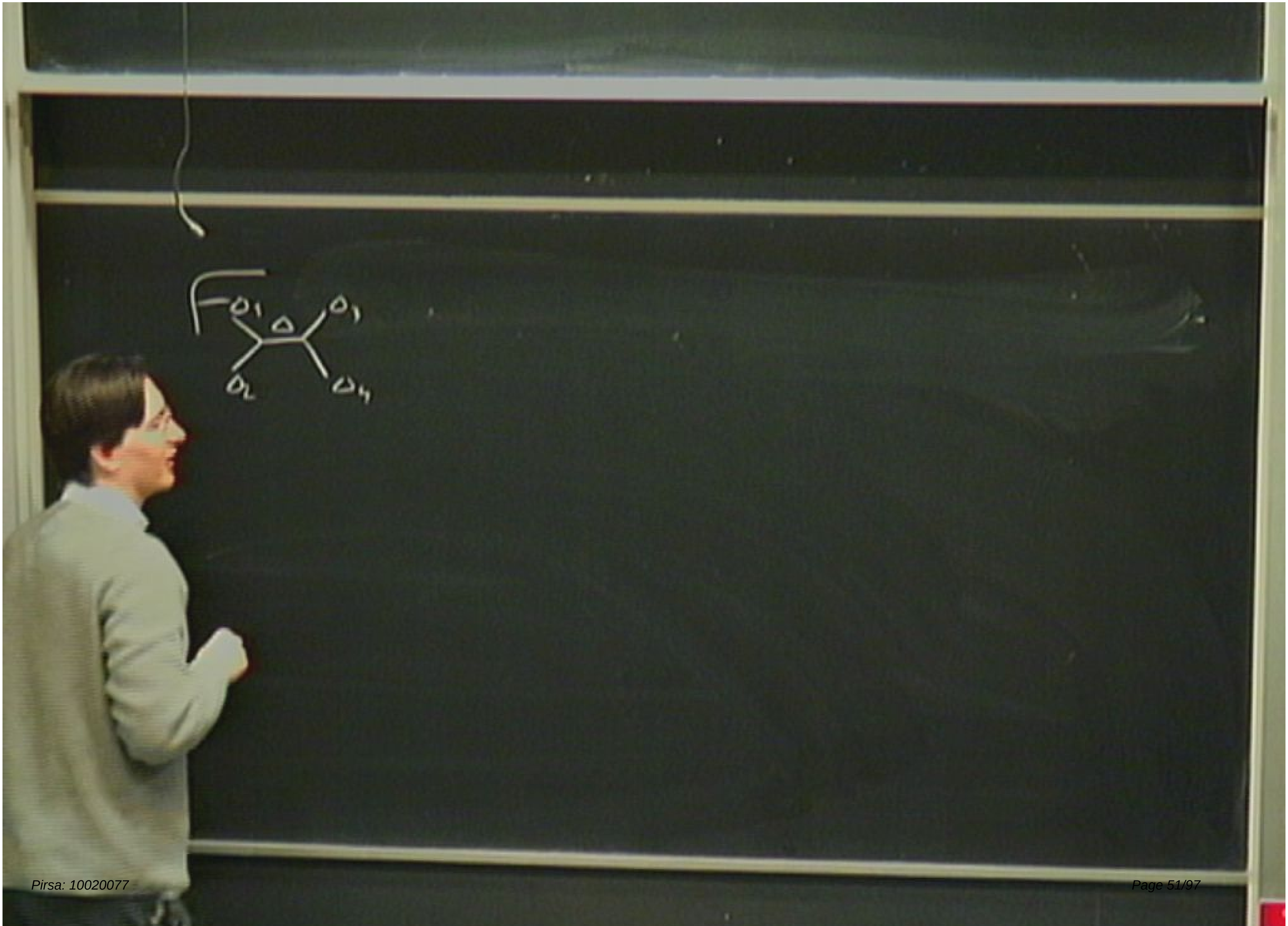
$$= \sum_{i} C_{i i_1 i_2} C_{i i_3 i_4}\, q^{\Delta(i)}\, \bar{q}^{\Delta(i)}\, F(\Delta(i), \Delta(i_1) \cdots ; q)\; \bar{F}(\Delta(i), \Delta(i_1) \cdots, \bar{q})$$

$$\Delta(\alpha) = \frac{Q^2}{4} + \alpha^2$$

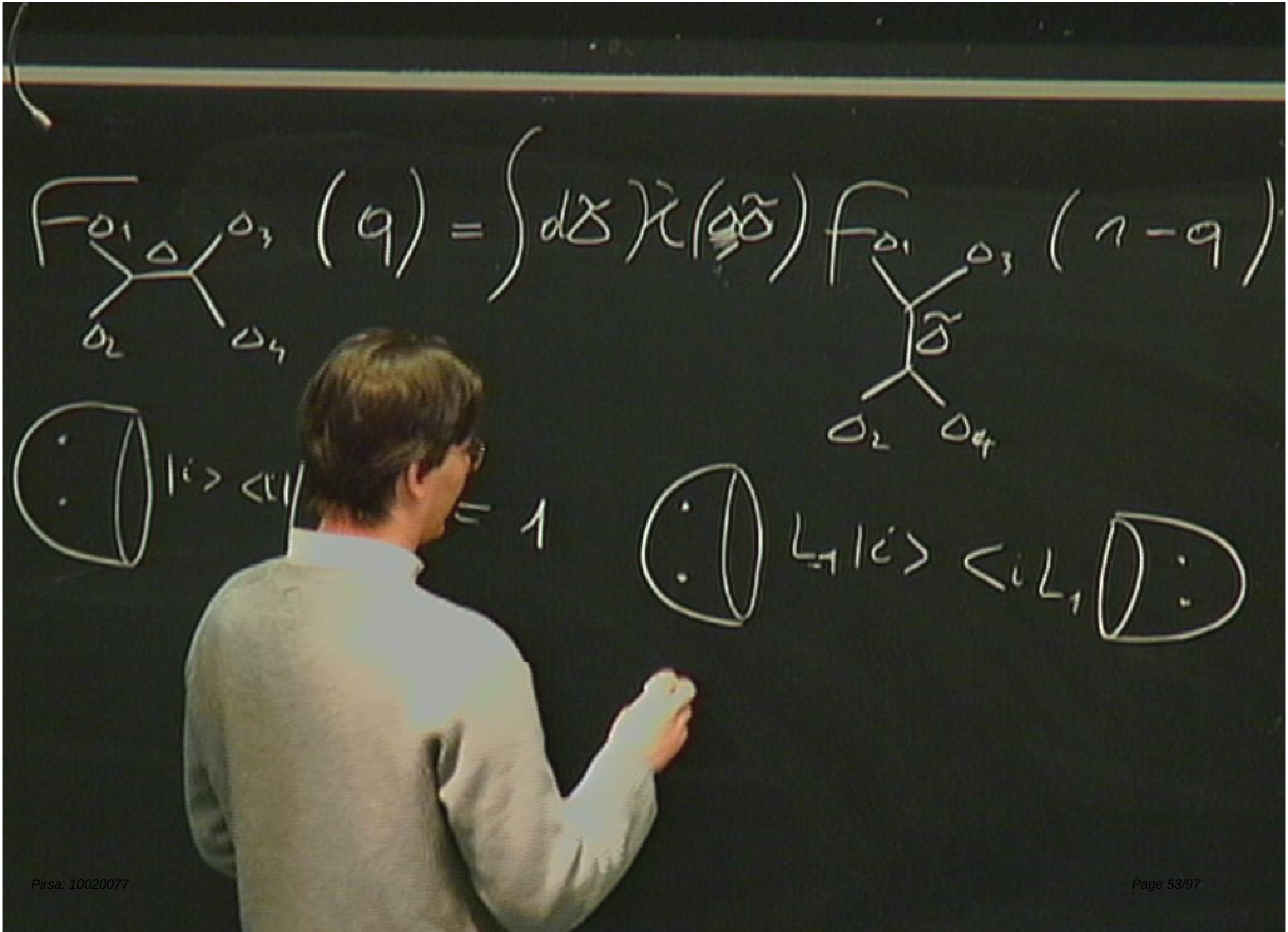
$$Q = b + \frac{1}{b} \qquad c = 1 + 6Q^2$$

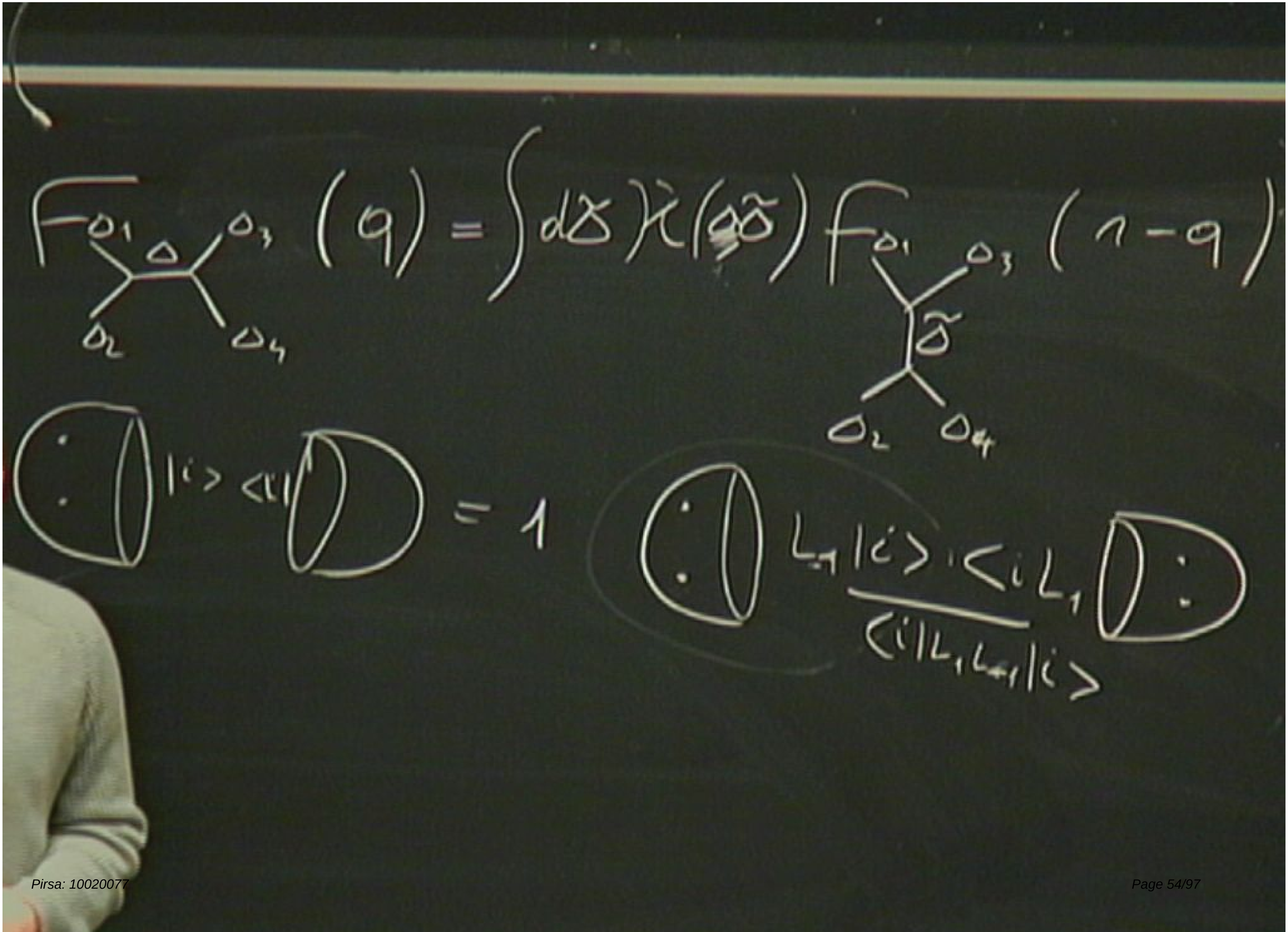


Q = b + 1/b
c = 1 + 6Q²



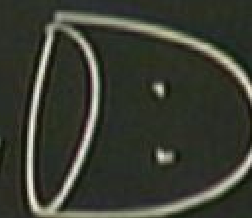
$$F_{\begin{smallmatrix}\Delta_1 & & \Delta_3\\ & \Delta & \\ \Delta_2 & & \Delta_4\end{smallmatrix}}(q) = \int d\Delta \, K(\Delta\tilde{\Delta})\, F_{\begin{smallmatrix}\Delta_1 & \Delta_3\\ \tilde{\Delta} & \\ \Delta_2 & \Delta_4\end{smallmatrix}}(1-q)$$


F (q) = ∫ dΔ̃ K(ΔΔ̃) F (1-q)
O1 O3 Δ O2 O4
O1 O3 Δ̃ O2 O4
|i> <i| = 1
L-1 |i> <i L1

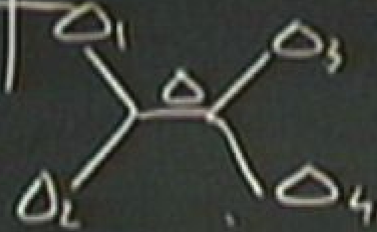

$F_{O_1 O_2 \Delta O_3 O_4}(q) = \int d\tilde{\Delta}\, K(\Delta\tilde{\Delta})\, F_{O_1 O_3 \tilde{\Delta} O_2 O_4}(1-q)$
$|i\rangle\langle i| = 1$
$L_{-1}|i\rangle \, \langle i L_1$
$\langle i|L_1 L_{-1}|i\rangle$

$$F^{\Delta}_{O_1 O_2,\, O_3 O_4}(q) = \int d\tilde{\Delta}\, \mathcal{K}(q\tilde{\Delta})\, F^{\tilde{\Delta}}_{O_1 O_3,\, O_2 O_4}(1-q)$$

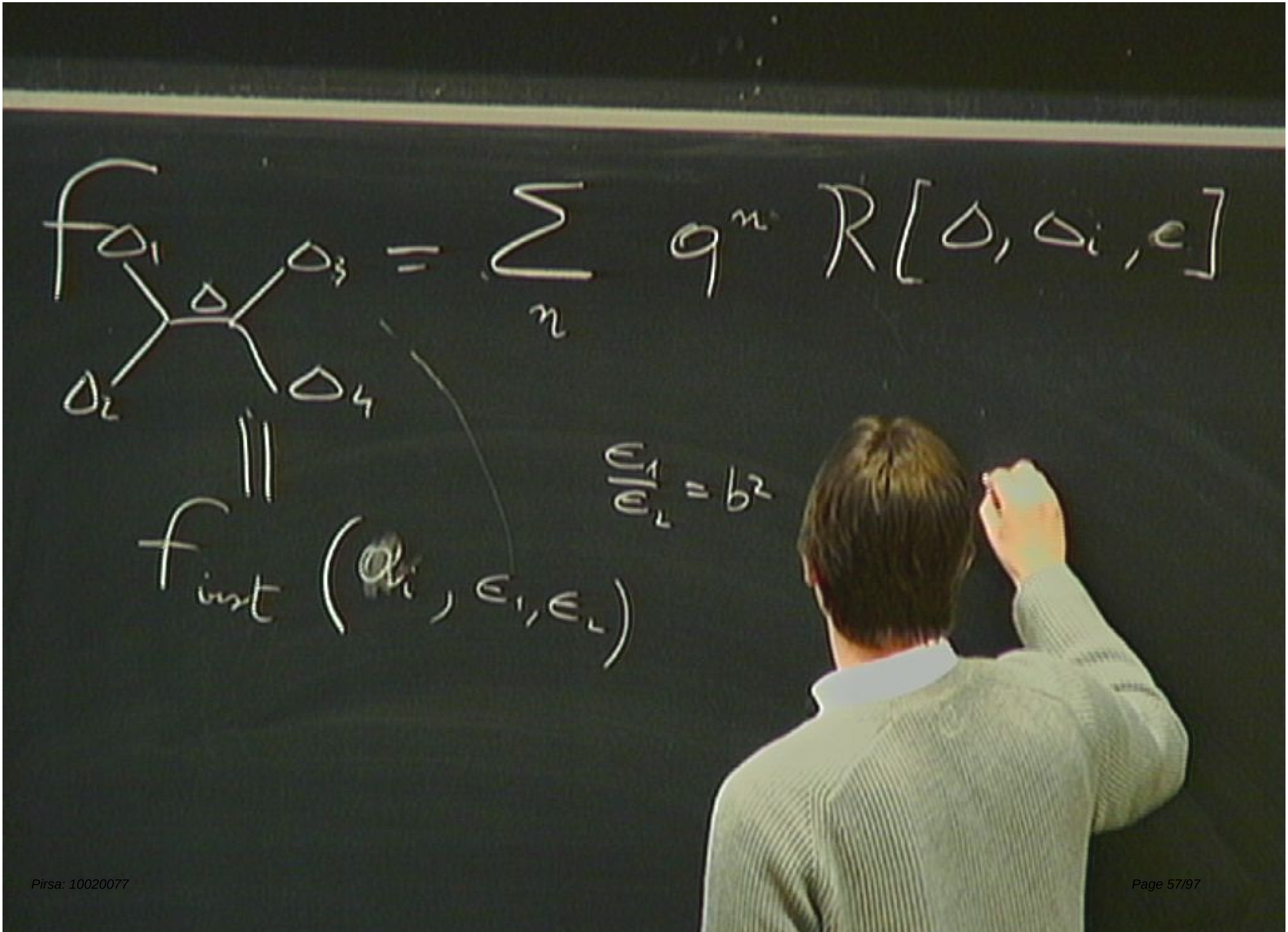
 $|i\rangle\langle i|$  $= 1$

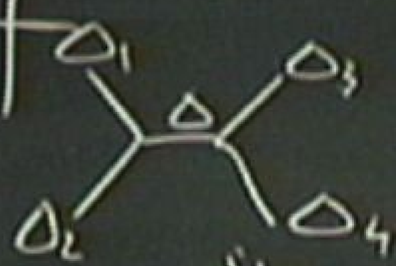
 $L_{-1}|i\rangle \dfrac{\langle i\, L_1}{\langle i|L_1 L_{-1}|i\rangle}$ 

$$\frac{(\Delta_1+\Delta_2-\Delta)(\Delta_3+\Delta_4-\Delta)}{2\Delta}$$

f 

$$= \sum_n q^n \, R[\Delta, \Delta_i, c]$$


Δ_1 Δ_3 Δ Δ_2 Δ_4
$= \sum_n q^n R[\Delta, \Delta_i, \epsilon]$
$f_{inst}(\Delta_i, \epsilon_1, \epsilon_2)$
$\frac{\epsilon_1}{\epsilon_2} = b^2$

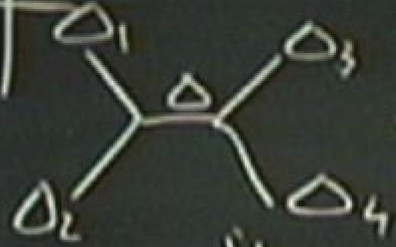
f 

$$= \sum_n q^n \, R[\Delta, \Delta_i, c]$$

$$\|$$

$$f_{inst}(a_i, \epsilon_1, \epsilon_2)$$

$$\frac{\epsilon_1}{\epsilon_2} = b^2 \qquad \alpha_i = \frac{a_i}{\sqrt{\epsilon_1 \epsilon_2}} \qquad \Delta_i = \frac{\alpha^2}{4} - q^2$$

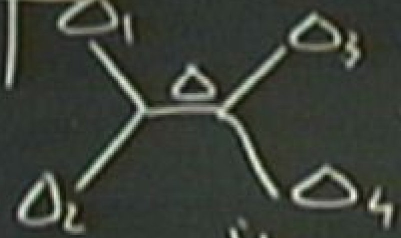
f 

$$= \sum_n q^n \, R[\Delta, \Delta_i, \epsilon]$$

$$\| $$

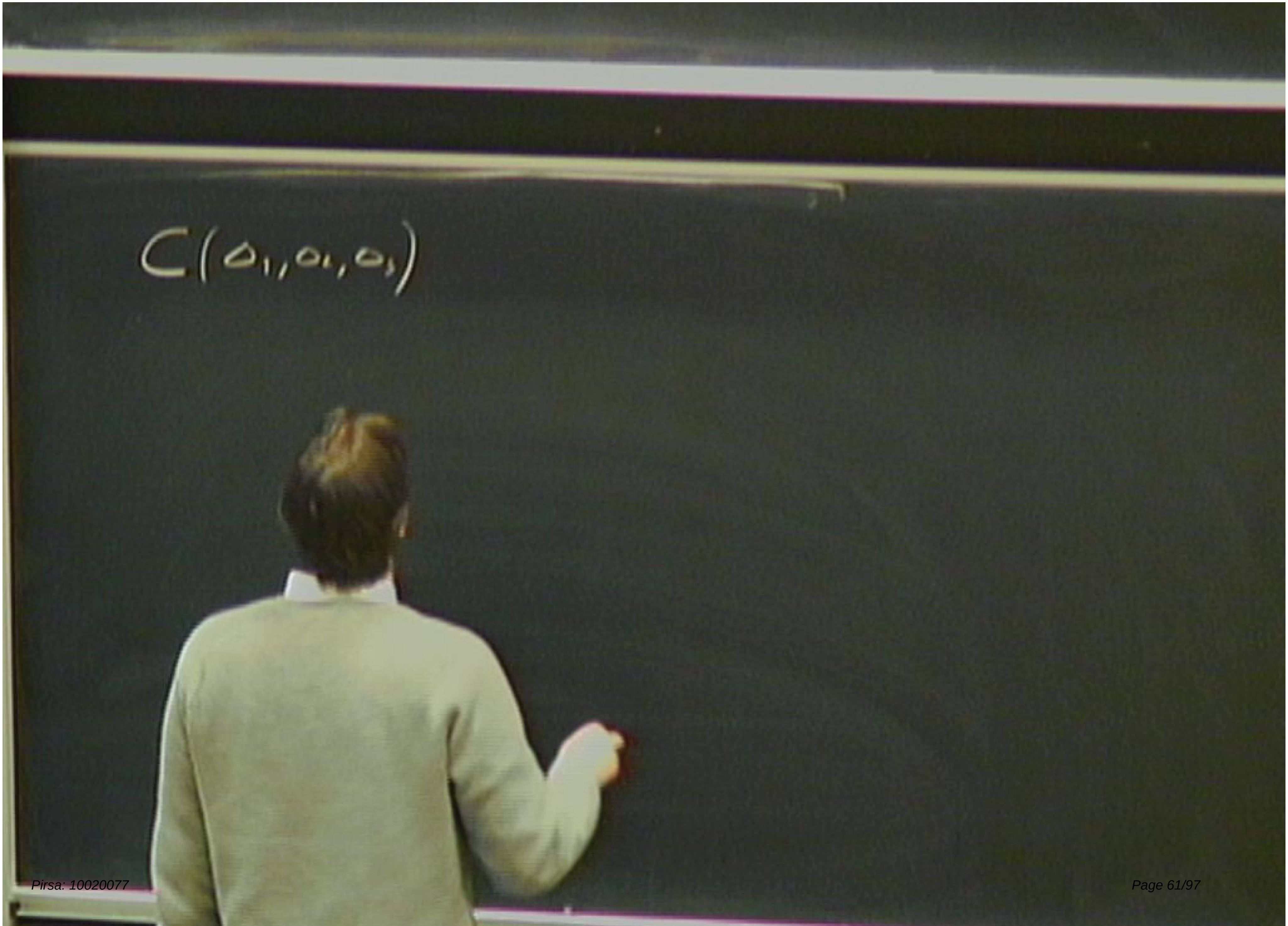
$$f_{inst}(a_i, \epsilon_1, \epsilon_2)$$

$$\frac{\epsilon_1}{\epsilon_2} = b^2 \qquad \alpha_i = \frac{a_i}{\sqrt{\epsilon_1 \epsilon_2}} \qquad \Delta_i = \frac{\alpha^2}{4} - q^2$$

f  $= \sum_n q^n R[\Delta, \Delta_i, \epsilon]$

$$\| \quad f_{inst}(\alpha_i, \epsilon_1, \epsilon_2)$$

$$\frac{\epsilon_1}{\epsilon_2} = b^2 \qquad \alpha_i = \frac{a_i}{\sqrt{\epsilon_1 \epsilon_2}} \qquad \Delta_i = \frac{\alpha^2}{4} - q^2$$


C(Δ1, Δ2, Δ3)

$$f_{\Delta_1 \Delta_2}{}^{\Delta_3}_{\Delta_4} = \sum_n q^n \, R[\Delta, \Delta_i, \epsilon]$$

$$\|$$

$$f_{inst}(\alpha_i, \epsilon_1, \epsilon_2)$$

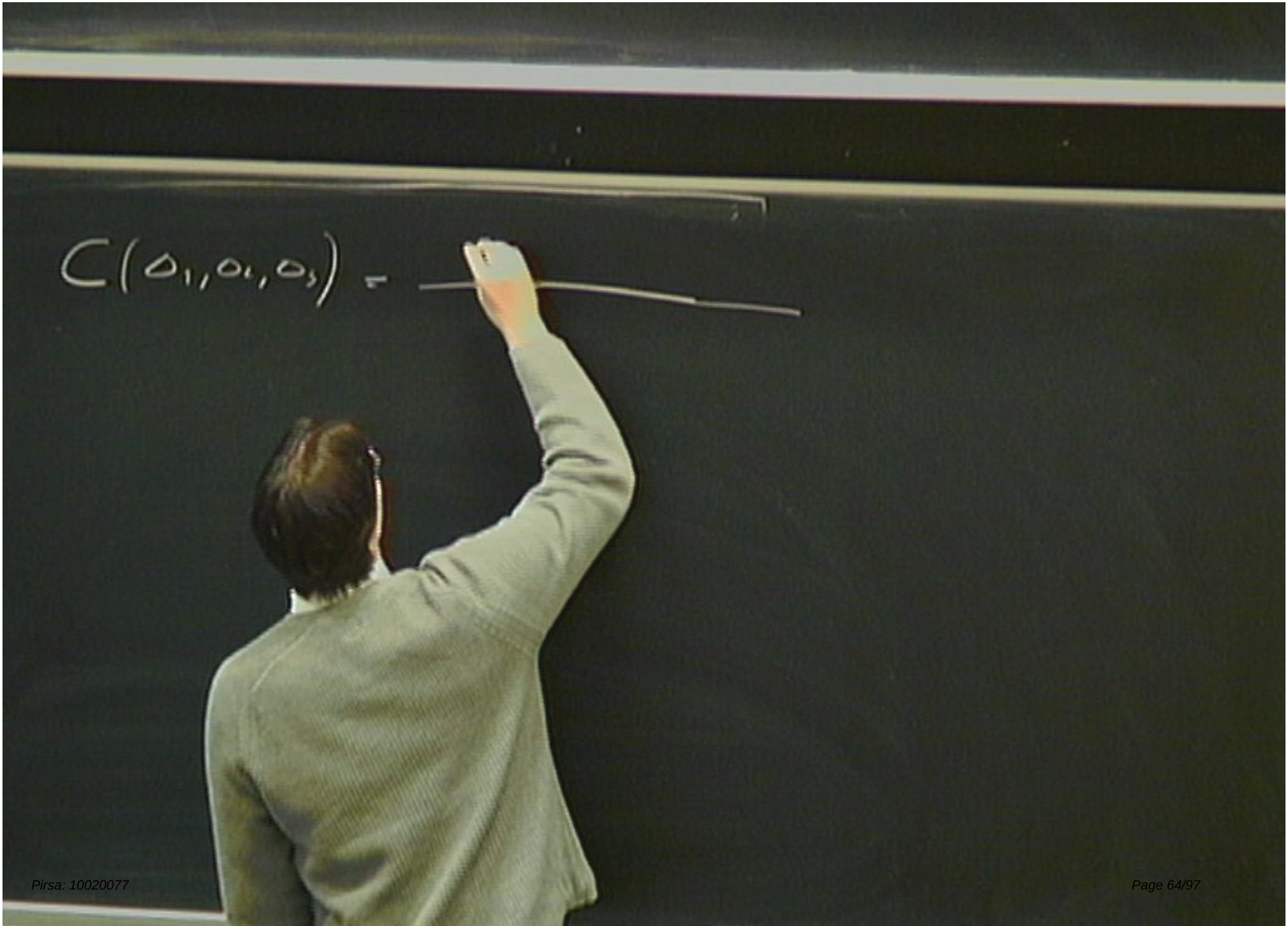
$$\frac{\epsilon_1}{\epsilon_2} = b^2 \qquad \alpha_i = \frac{a_i}{\sqrt{\epsilon_1 \epsilon_2}} \qquad \Delta_i = \frac{\alpha^2}{4} - a_i^2$$

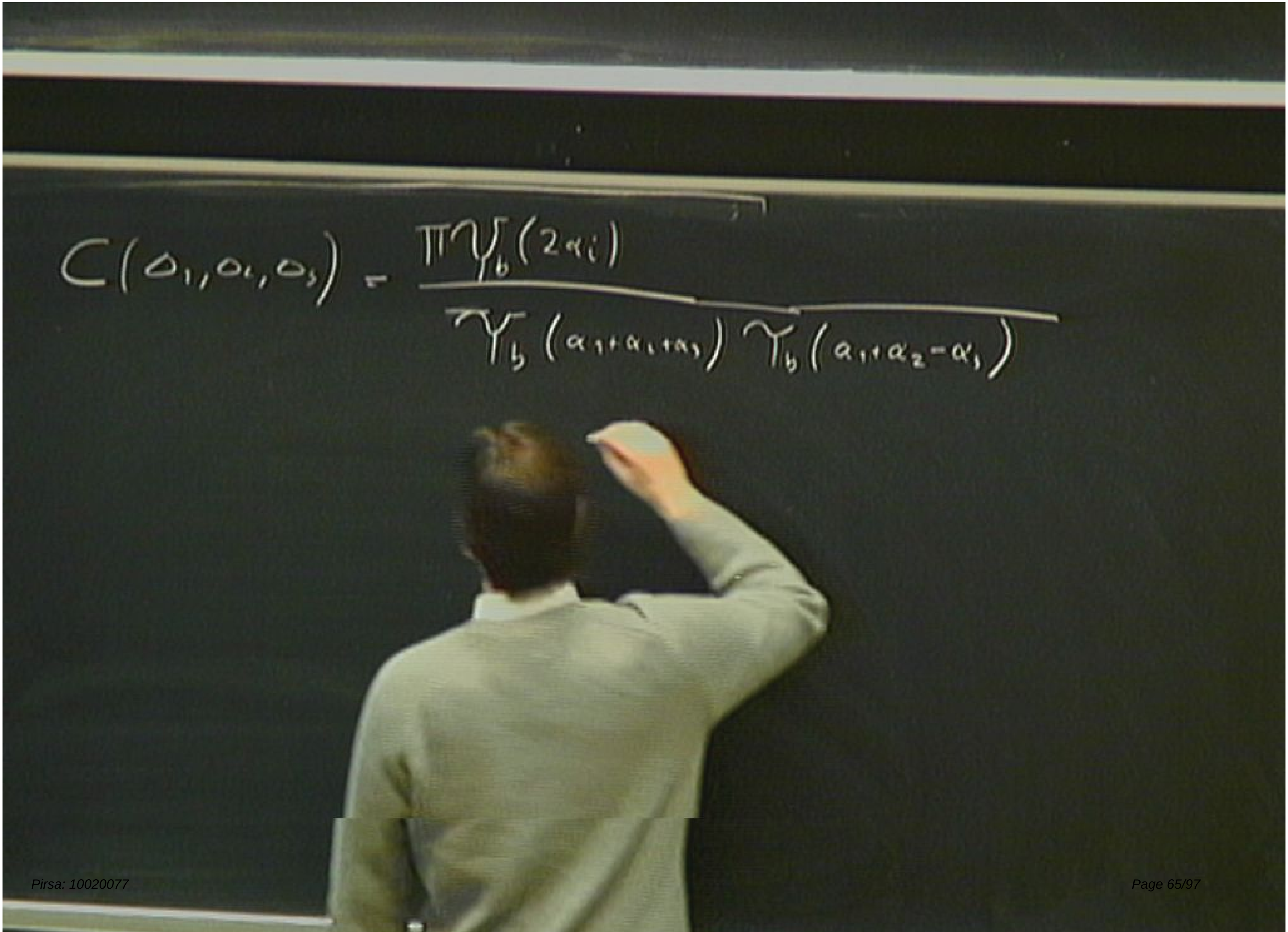
Δ_2

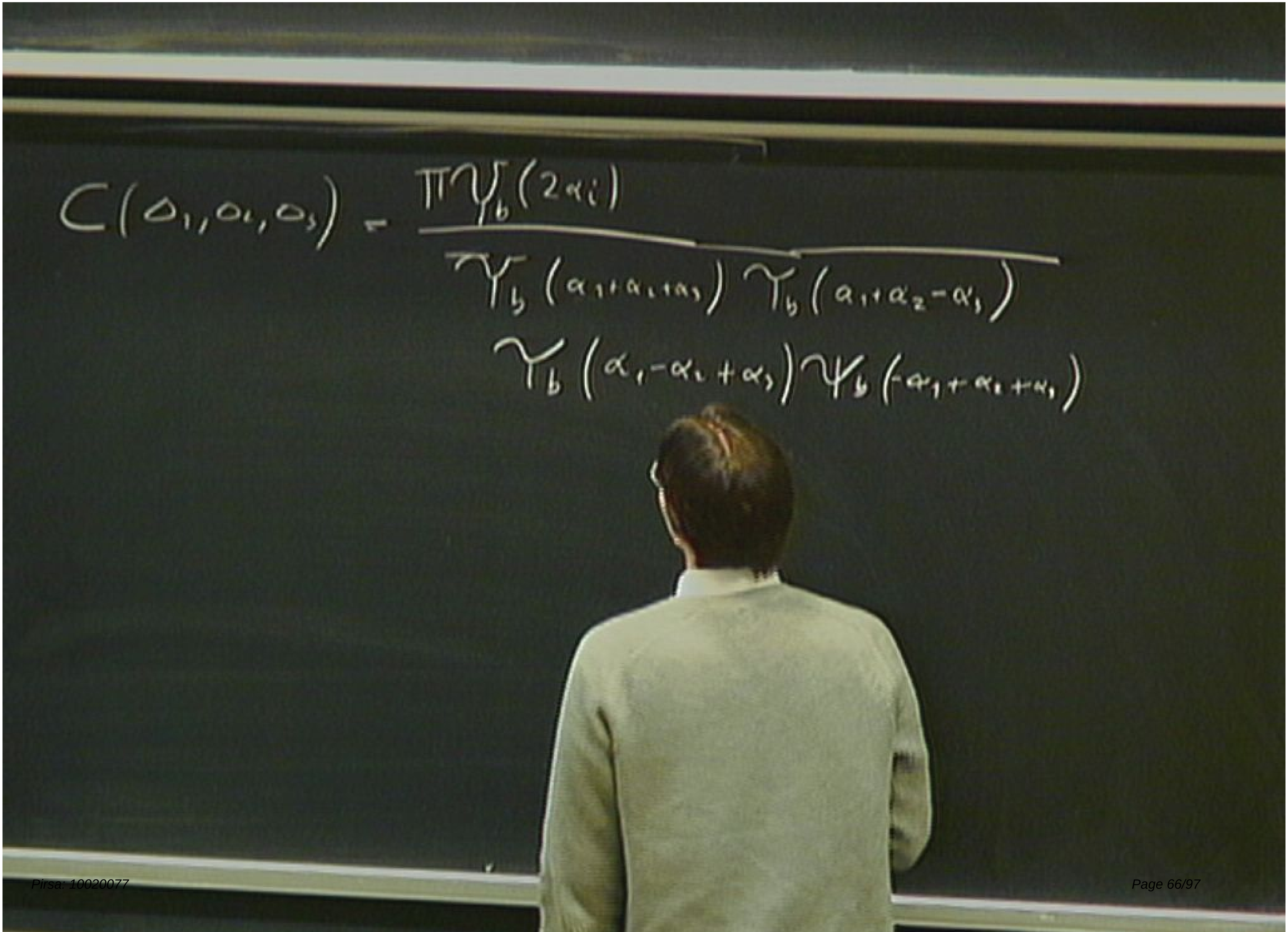
$$\frac{\epsilon_1}{\epsilon_2} = b^2 \qquad \alpha_i = \frac{a_i}{\sqrt{\epsilon_1\epsilon_2}} \qquad \Delta_i = \frac{Q^2}{4} - \alpha_i^2$$

$(\alpha_i, \epsilon_1, \epsilon_2)$

$$V = e^{(\frac{Q}{2} + i\alpha)\varphi}$$


$C(O_1, O_2, O_3) =$


$C(\alpha_1,\alpha_2,\alpha_3) = \frac{\prod \Upsilon_b(2\alpha_i)}{\Upsilon_b(\alpha_1+\alpha_2+\alpha_3)\,\Upsilon_b(\alpha_1+\alpha_2-\alpha_3)}$



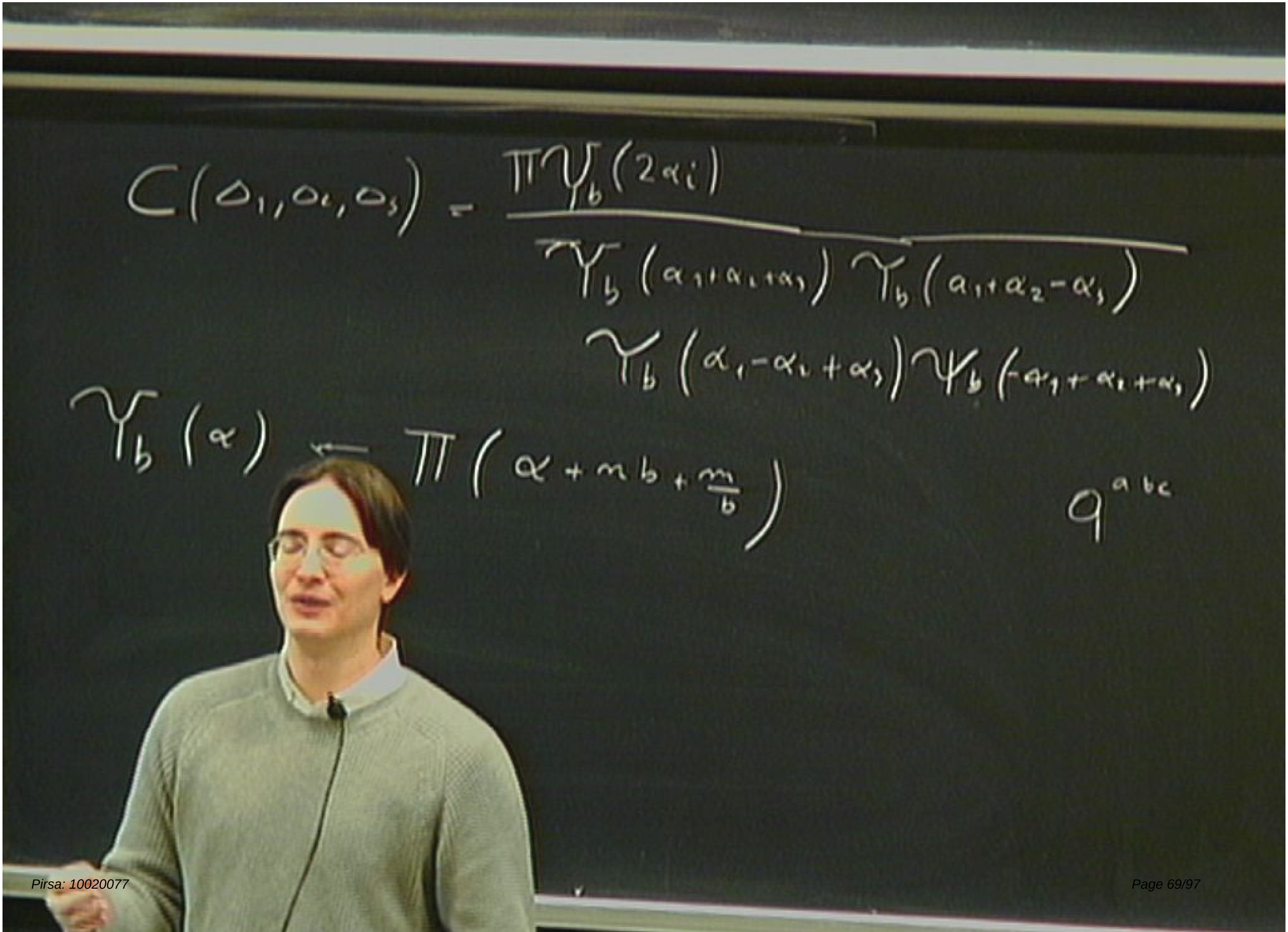
$C(\alpha_1,\alpha_2,\alpha_3) = \frac{\prod \Upsilon_b(2\alpha_i)}{\Upsilon_b(\alpha_1+\alpha_2+\alpha_3)\,\Upsilon_b(\alpha_1+\alpha_2-\alpha_3)\,\Upsilon_b(\alpha_1-\alpha_2+\alpha_3)\,\Upsilon_b(-\alpha_1+\alpha_2+\alpha_3)}$

$$C(\alpha_1,\alpha_2,\alpha_3) = \frac{\prod \Upsilon_b(2\alpha_i)}{\Upsilon_b(\alpha_1+\alpha_2+\alpha_3)\,\Upsilon_b(\alpha_1+\alpha_2-\alpha_3)\,\Upsilon_b(\alpha_1-\alpha_2+\alpha_3)\,\Upsilon_b(-\alpha_1+\alpha_2+\alpha_3)}$$

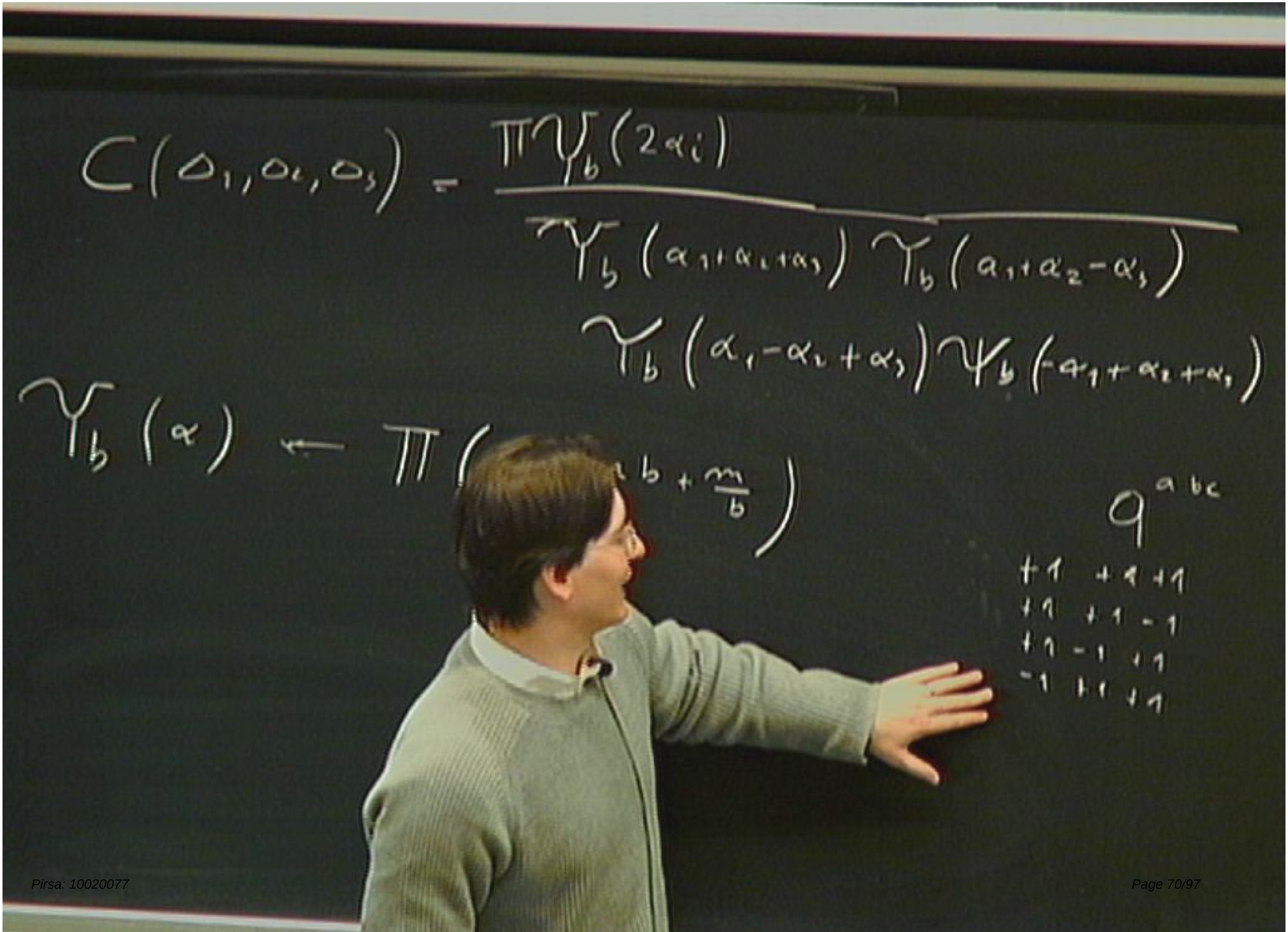
$$\Upsilon_b(\alpha) \leftarrow \prod\left(\alpha + mb + \frac{n}{b}\right)$$

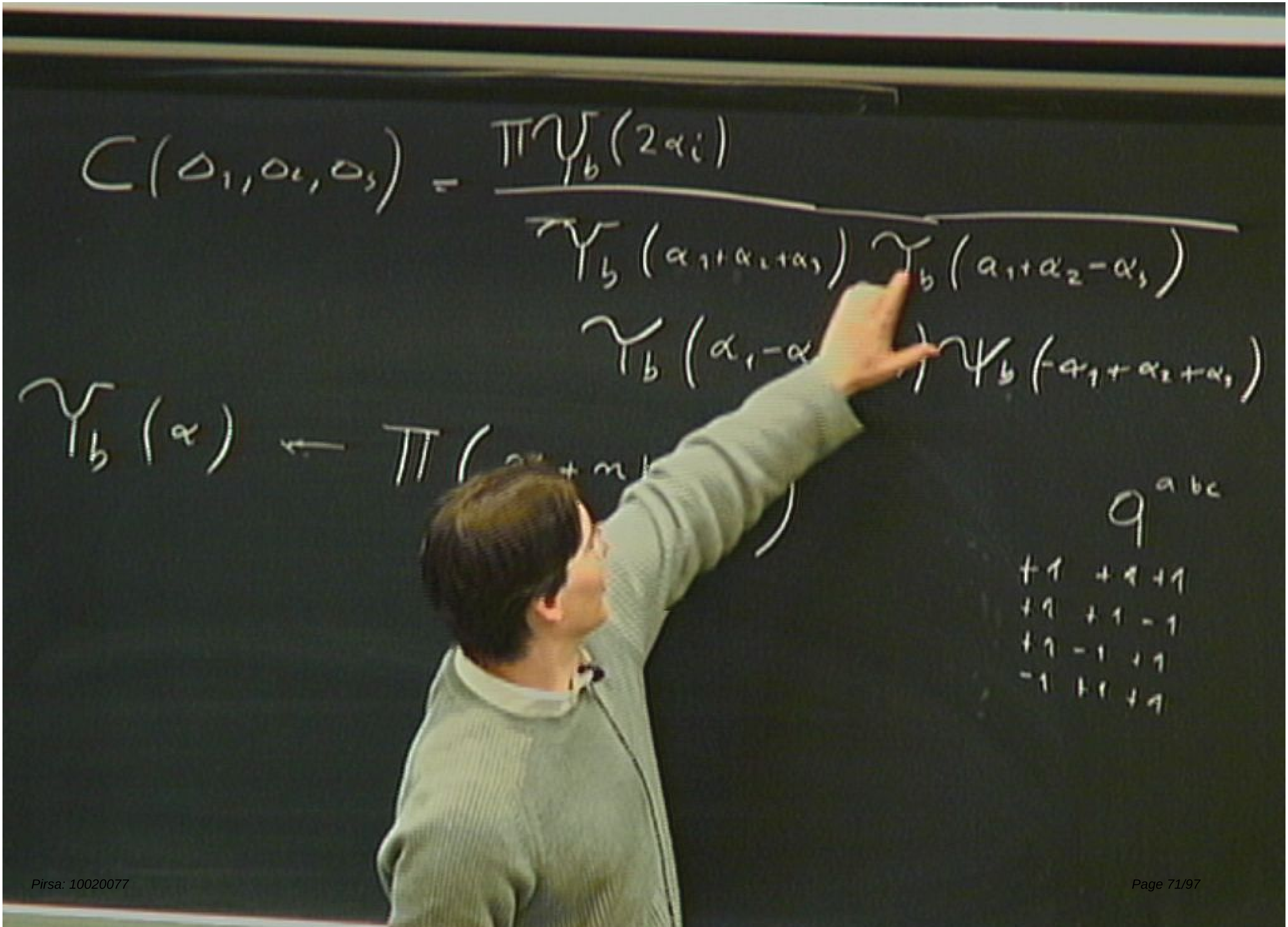
$$C(\alpha_1, \alpha_2, \alpha_3) = \frac{\prod \Upsilon_b(2\alpha_i)}{\Upsilon_b(\alpha_1+\alpha_2+\alpha_3)\,\Upsilon_b(\alpha_1+\alpha_2-\alpha_3)\,\Upsilon_b(\alpha_1-\alpha_2+\alpha_3)\,\Upsilon_b(-\alpha_1+\alpha_2+\alpha_3)}$$

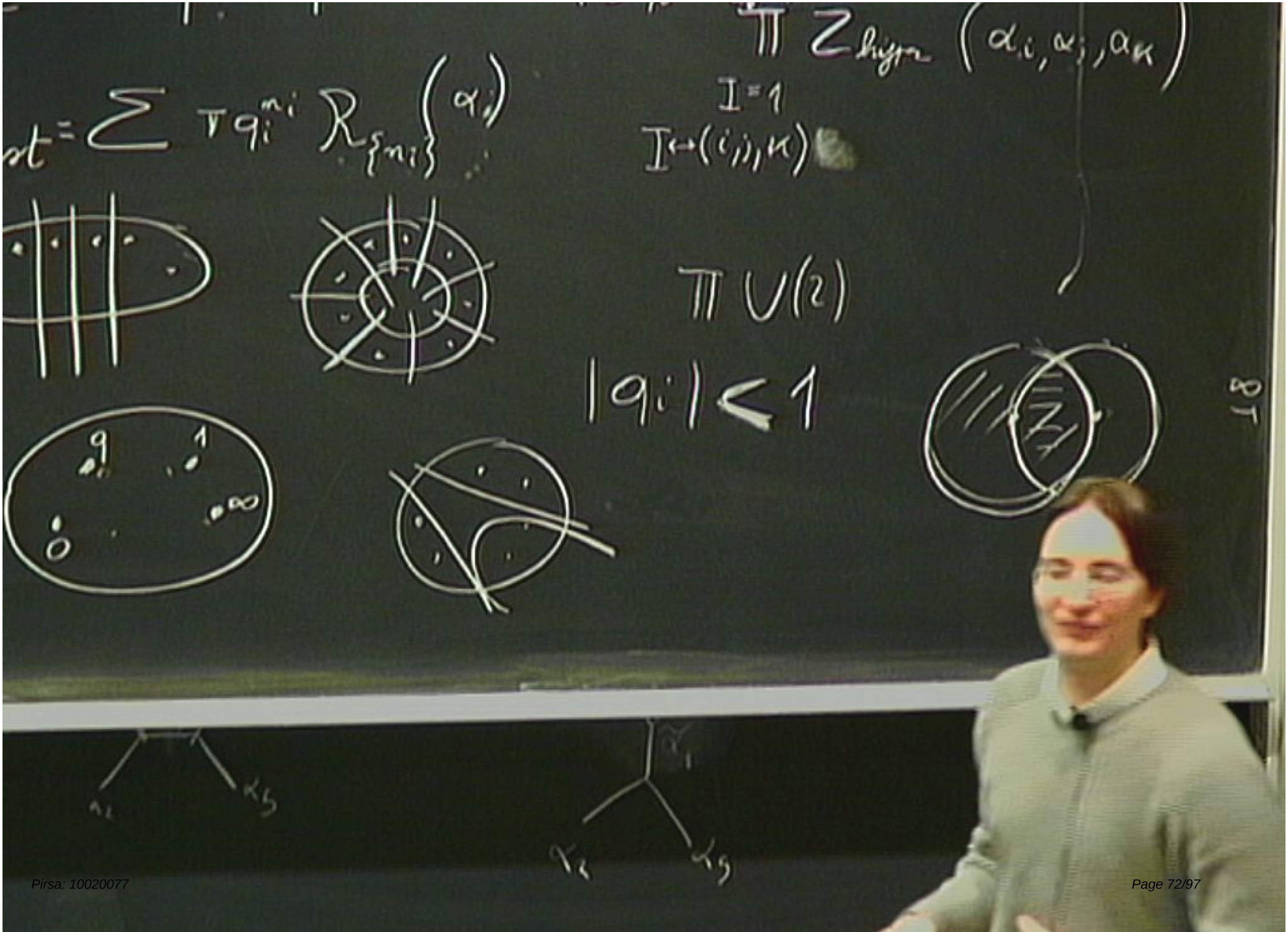
$$\Upsilon_b(\alpha) \leftarrow \prod \left(\alpha + m b + \frac{n}{b}\right)$$



$C(\alpha_1, \alpha_2, \alpha_3) = \frac{\prod \Upsilon_b(2\alpha_i)}{\Upsilon_b(\alpha_1+\alpha_2+\alpha_3)\,\Upsilon_b(\alpha_1+\alpha_2-\alpha_3)\,\Upsilon_b(\alpha_1-\alpha_2+\alpha_3)\,\Upsilon_b(-\alpha_1+\alpha_2+\alpha_3)}$
$\Upsilon_b(\alpha) \leftarrow \prod\left(\alpha + mb + \frac{n}{b}\right)$
q^{abc}


$C(\alpha_1,\alpha_2,\alpha_3) = \frac{\prod \Upsilon_b(2\alpha_i)}{\Upsilon_b(\alpha_1+\alpha_2+\alpha_3)\,\Upsilon_b(\alpha_1+\alpha_2-\alpha_3)\,\Upsilon_b(\alpha_1-\alpha_2+\alpha_3)\,\Upsilon_b(-\alpha_1+\alpha_2+\alpha_3)}$
$\Upsilon_b(\alpha) \leftarrow \prod$
q^{abc}
+1 +1 +1
+1 +1 −1
+1 −1 +1
−1 +1 +1


$C(\alpha_1,\alpha_2,\alpha_3) = \frac{\prod \Upsilon_b(2\alpha_i)}{\Upsilon_b(\alpha_1+\alpha_2+\alpha_3)\,\Upsilon_b(\alpha_1+\alpha_2-\alpha_3)}$
$\Upsilon_b(\alpha_1-\alpha$ $)\,\Upsilon_b(-\alpha_1+\alpha_2+\alpha_3)$
$\Upsilon_b(\alpha) \leftarrow \prod ($
q^{abc}
+1 +1 +1
+1 +1 −1
+1 −1 +1
−1 +1 +1


∏ Z_hyper (α_i, α_j, α_k)
I=1
I↔(i,j,k)
∏ U(2)
|q_i| < 1

$$= \int \prod_{i=1}^{3g-3+n} d\alpha_i \, (2\alpha_i)^2$$

$$Z_{tree}(q,\bar{q},\alpha) \, Z_{1\text{-}loop}(\alpha)$$

$$Z_{inst}\left(q, \alpha, \tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$Z_{anti\text{-}inst}\left(\bar{q}, \alpha, \tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$\chi_R(\alpha)$$

$\alpha_i = a'\tau$

$\ldots 3g-3+n$

$= m\tau$

$3g-3+n+1 \ldots 3g-3+2n$

$$\langle T_{uv} \rangle = \int \prod_{i=1}^{3g-3+n} d\alpha_i \, (2\alpha_i)^2$$

$$Z_{tree}(q,\bar{q},\alpha) \, Z_{1\text{-}loop}(\alpha)$$

$$Z_{inst}\left(q, \alpha, \tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$Z_{anti\text{-}inst}\left(\bar{q}, \alpha, \tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$\chi_R(\alpha)$$

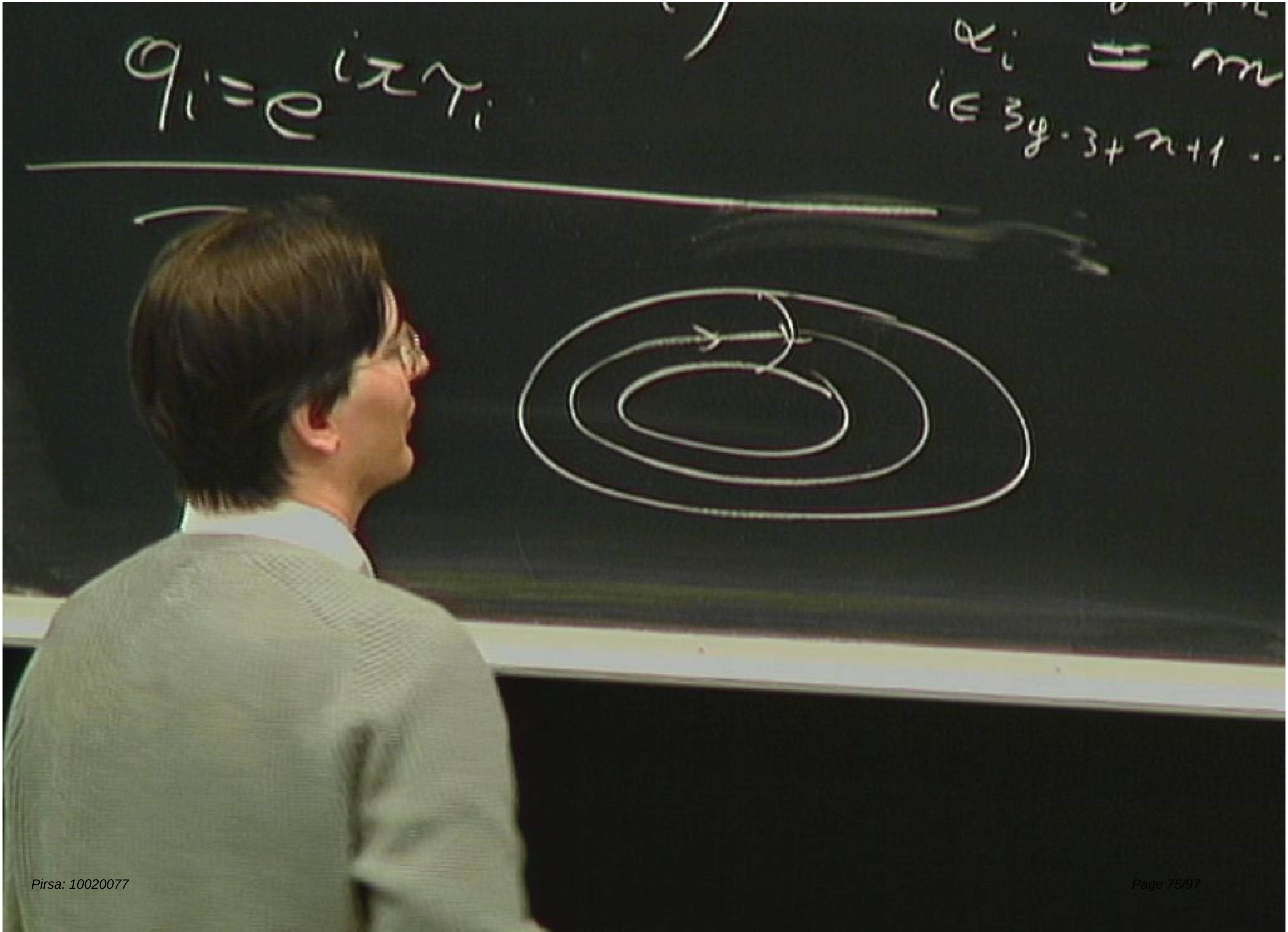
$$e^{i\pi\alpha} + e^{-i\pi\alpha}$$

$\alpha_i = a\,\mathbb{Z}$

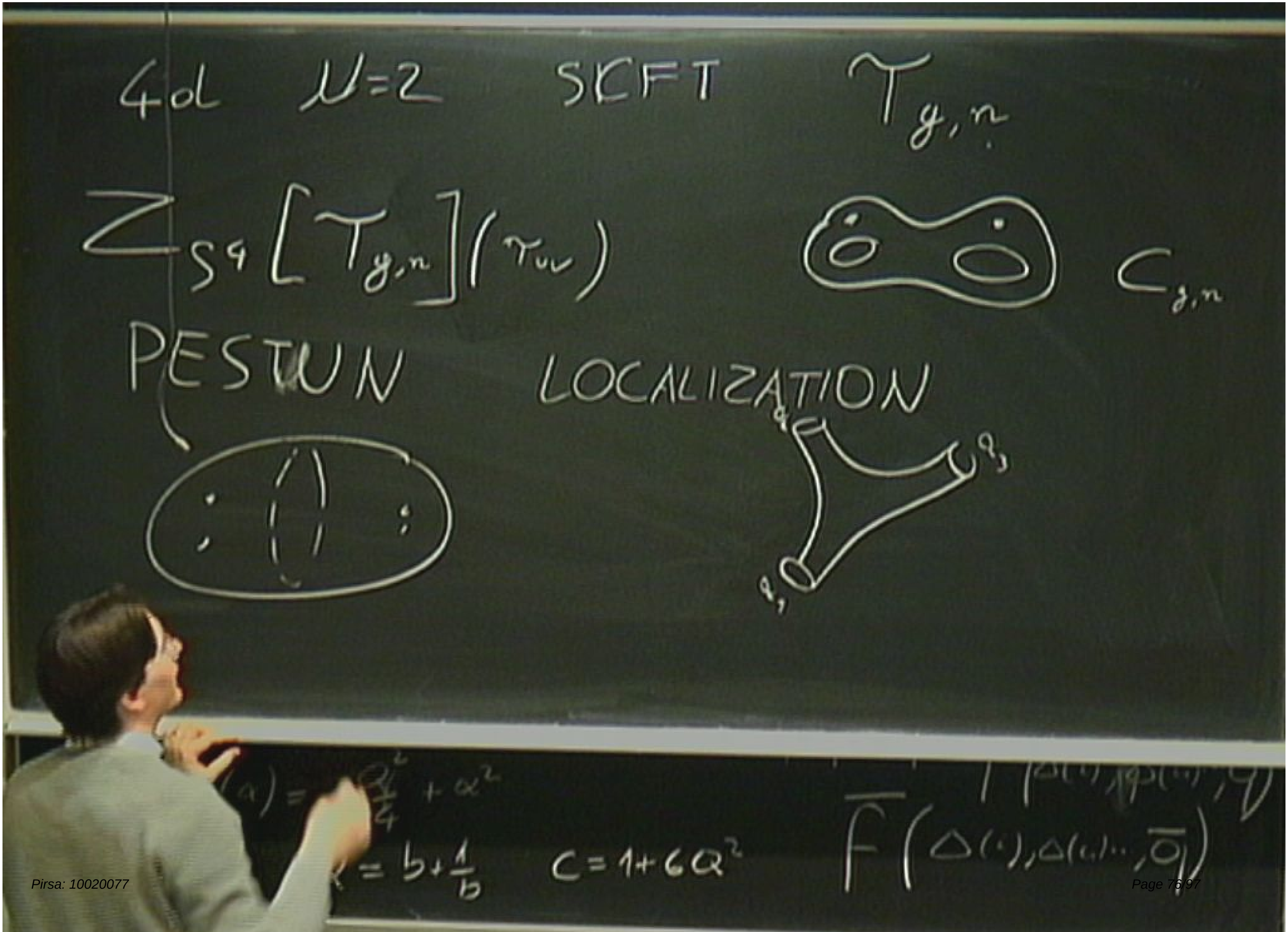
$i \in 1 \ldots 3g-3+n$

$\alpha_i = m\,\mathbb{Z}$

$i \in 3g-3+n+1 \ldots 3g-3+2n$



$q_i = e^{i\pi\gamma_i}$



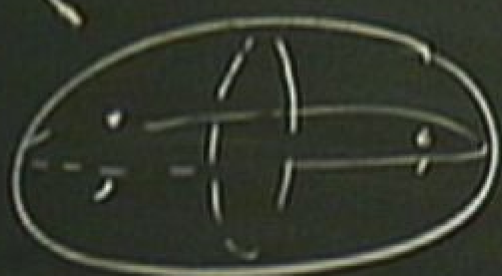
4d $\mathcal{N}=2$ SCFT $T_{g,n}$
$Z_{S^4}[T_{g,n}](\tau_{UV})$
$C_{g,n}$
PESTUN LOCALIZATION
q_1
q_2
q_3
$= b + \frac{1}{b}$
$c = 1 + 6Q^2$
$\bar{F}(\Delta(\cdot),\Delta(\cdot)\ldots,\bar{q})$

4d $\mathcal{N}=2$ SCFT $\quad T_{g,n}$

$Z_{S^4}[T_{g,n}](\tau_{uv})$

$C_{g,n}$

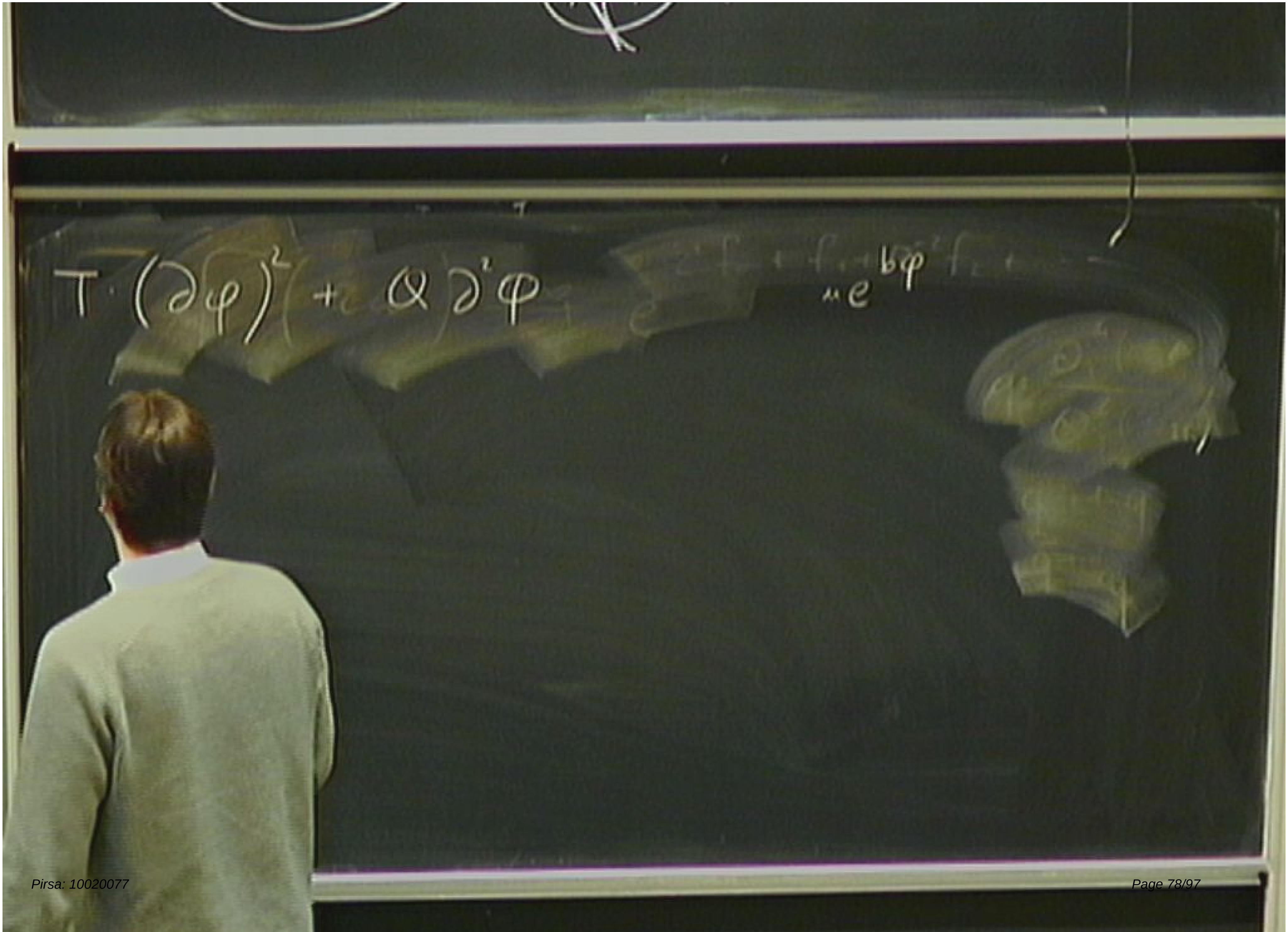
PESTUN LOCALIZATION

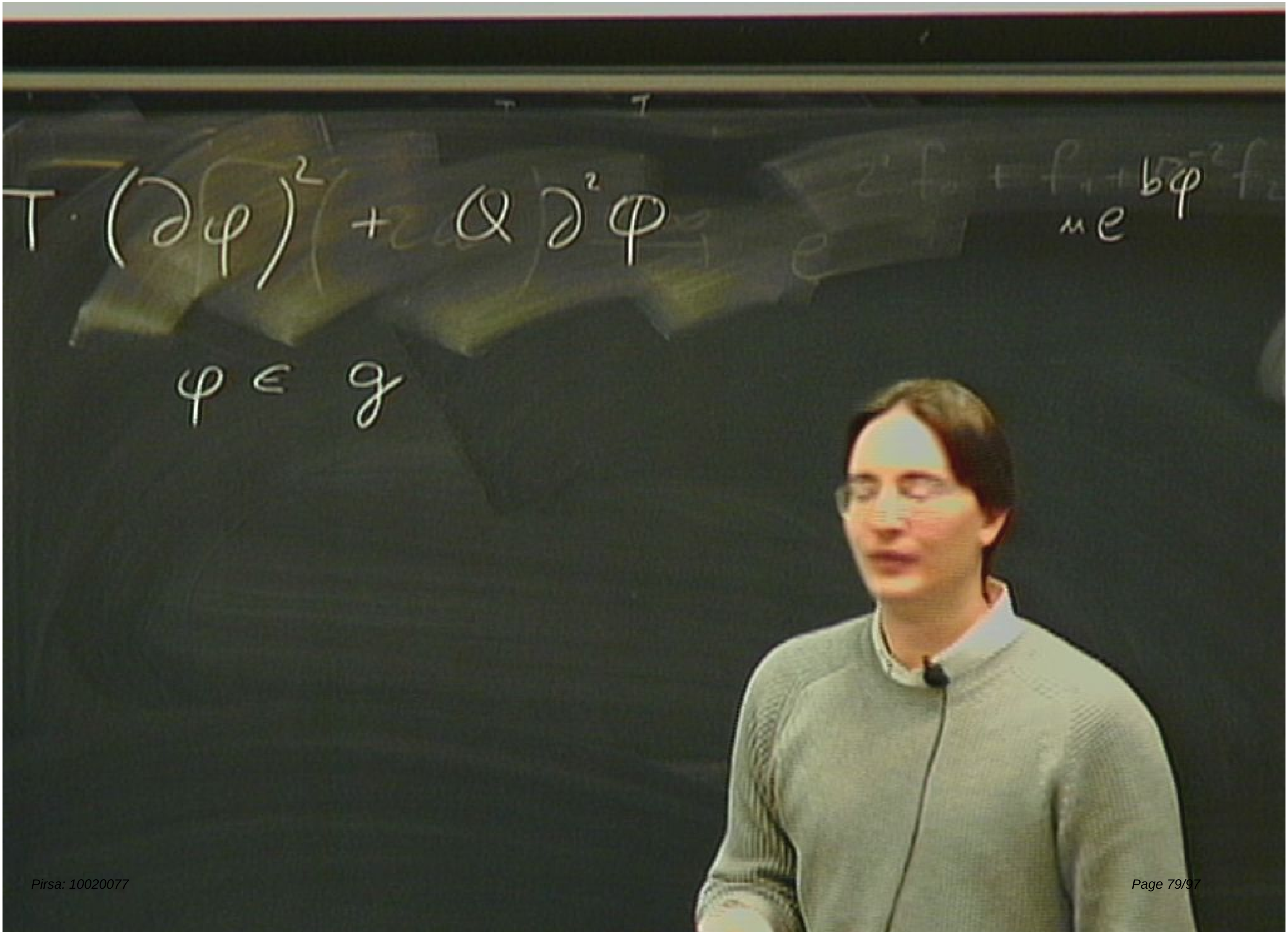


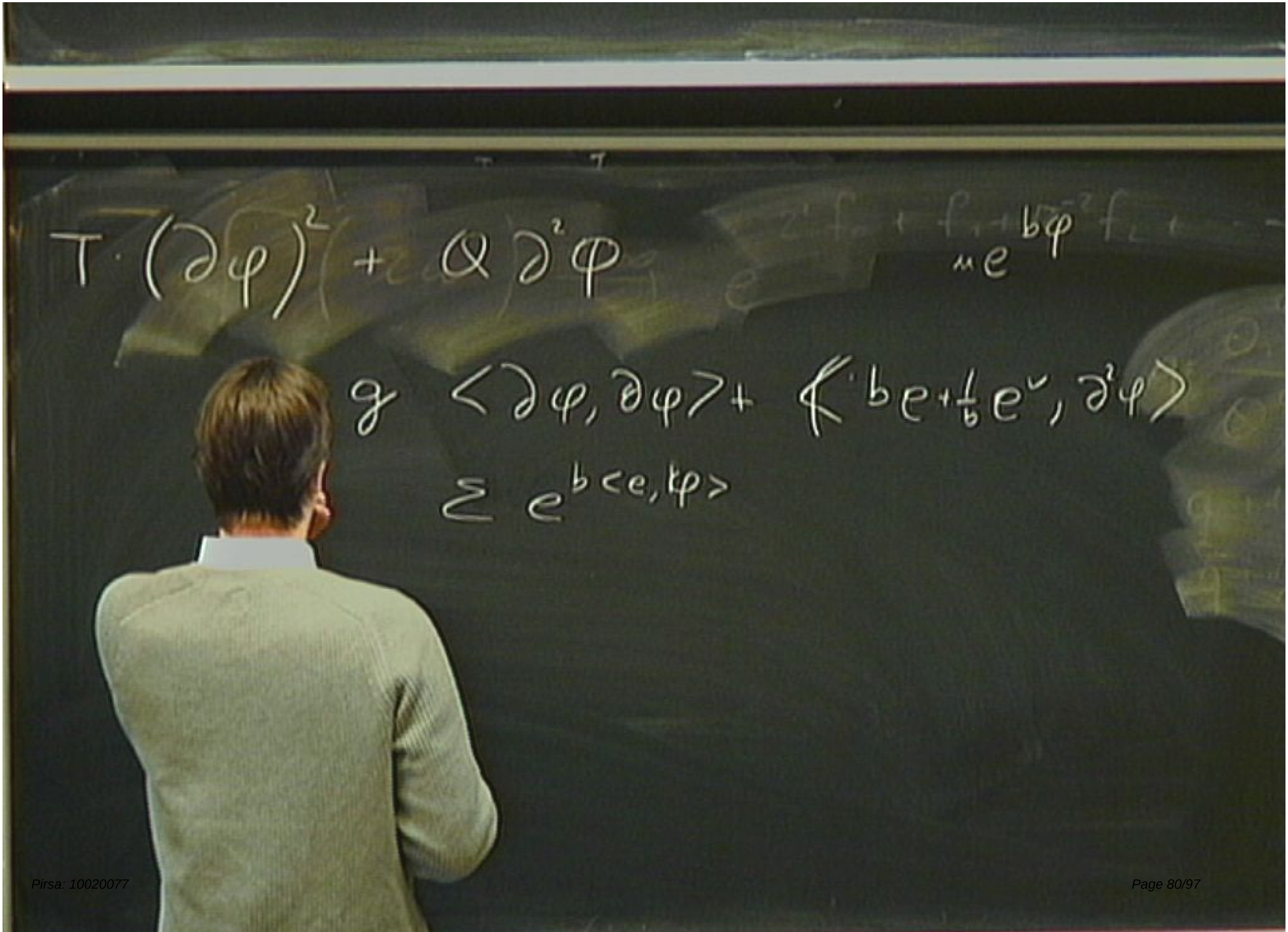
$\Delta(\alpha) = \frac{Q^2}{4} + \alpha^2$

$Q = b + \frac{1}{b} \qquad c = 1 + 6Q^2$

$\overline{F}(\Delta(\cdot), \Delta(\cdot) \ldots, \overline{q})$


$T\cdot(\partial\varphi)^2 + Q\,\partial^2\varphi$
$\mu\, e^{b\varphi}$


T·(∂φ)² + Q ∂²φ
μ e^{bφ}
φ ∈ g


$T \cdot (\partial\varphi)^2 + Q\,\partial^2\varphi$
$\mu e^{b\varphi}$
$g\ \langle \partial\varphi, \partial\varphi \rangle + \langle \rho\, b e + \frac{1}{b} e^{\vee}, \partial^2\varphi \rangle$
$\Sigma\, e^{b\langle e, k\varphi \rangle}$

$$T\cdot(\partial\varphi)^2 + Q\,\partial^2\varphi \qquad \mu e^{b\varphi}$$

$$\varphi\in\mathfrak{g} \quad \langle\partial\varphi,\partial\varphi\rangle + \langle b\rho + \tfrac{1}{b}\rho^\vee, \partial^2\varphi\rangle$$

$$\sum e^{b\langle e_i, \varphi\rangle}$$

$$T \cdot (\partial\varphi)^2 + Q\,\partial^2\varphi \qquad \mu e^{b\varphi}$$

$$\varphi \in \mathfrak{g} \quad \langle \partial\varphi, \partial\varphi \rangle + \langle\!\langle b\rho + \tfrac{1}{b}\rho^\vee, \partial^2\varphi \rangle$$

$$\sum e^{b\langle e_i, \varphi\rangle}$$

$$C^{(i)}(\partial\varphi) + \cdots = W^{(i)}$$

$AdS_7 \times S_4$

$R^6 \to AdS_2 \times S^4$

$AdS_7 \times S_4$

$R^6 \to \frac{AdS_2}{\Gamma} \times S^4$

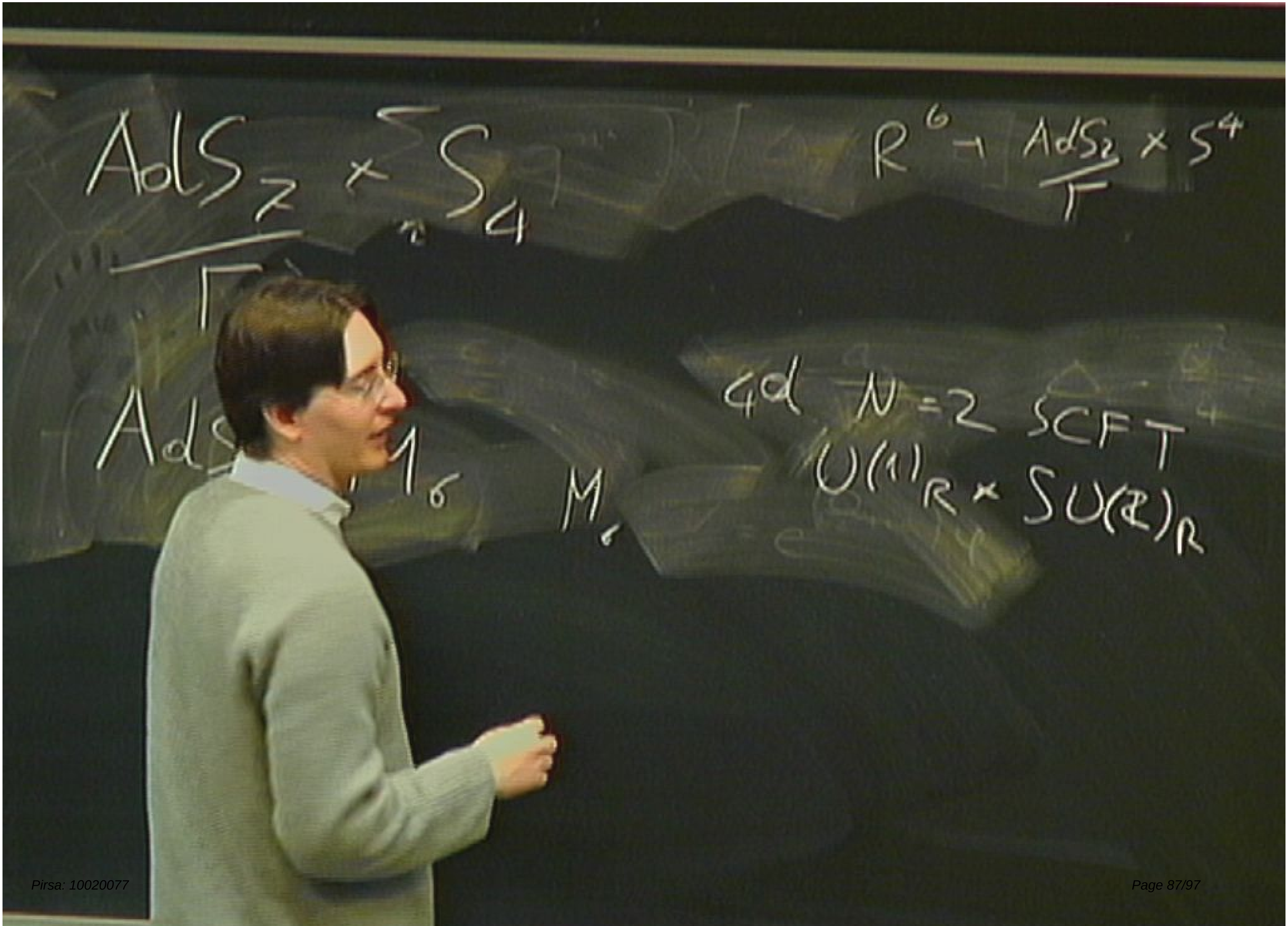
$$\frac{AdS_2 \times S_4}{\Gamma_4}$$

$$R^6 \to \frac{AdS_2}{\Gamma} \times S^4$$

$$\frac{AdS_2 \times S_4}{\Gamma}$$

$$R^6 \to \frac{AdS_2}{\Gamma} \times S^4$$

$$AdS_5 \times M_6$$


AdS2 × Σ4 / Γ
R6 → AdS2 / Γ × S4
4d N=2 SCFT
U(1)R × SU(2)R
M6

$$\frac{AdS_7 \times S_4}{\Gamma}$$

$\mathbb{R}^6 \to \frac{AdS_2}{\Gamma} \times S^4$

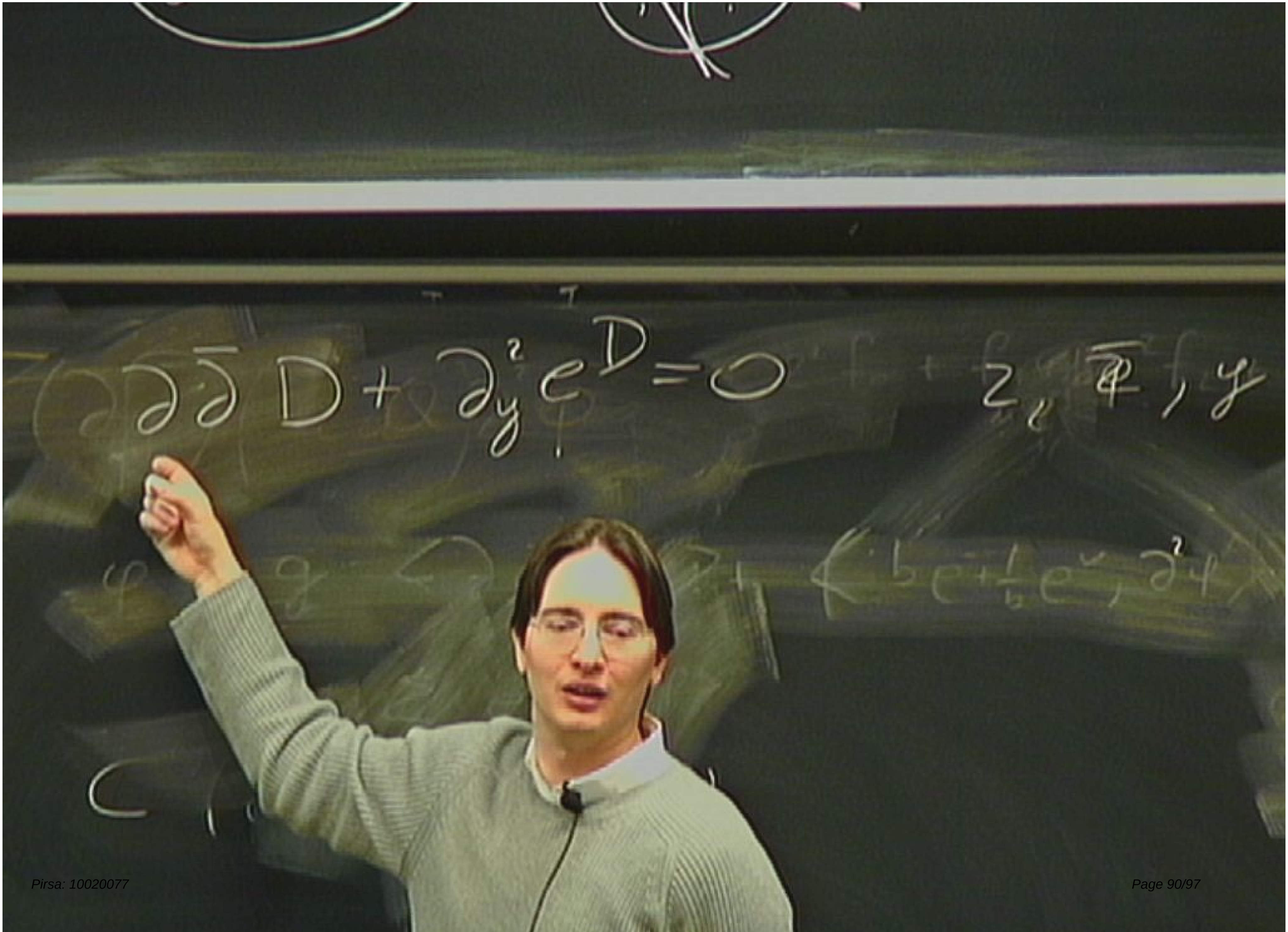
$AdS_5 \times M_6$

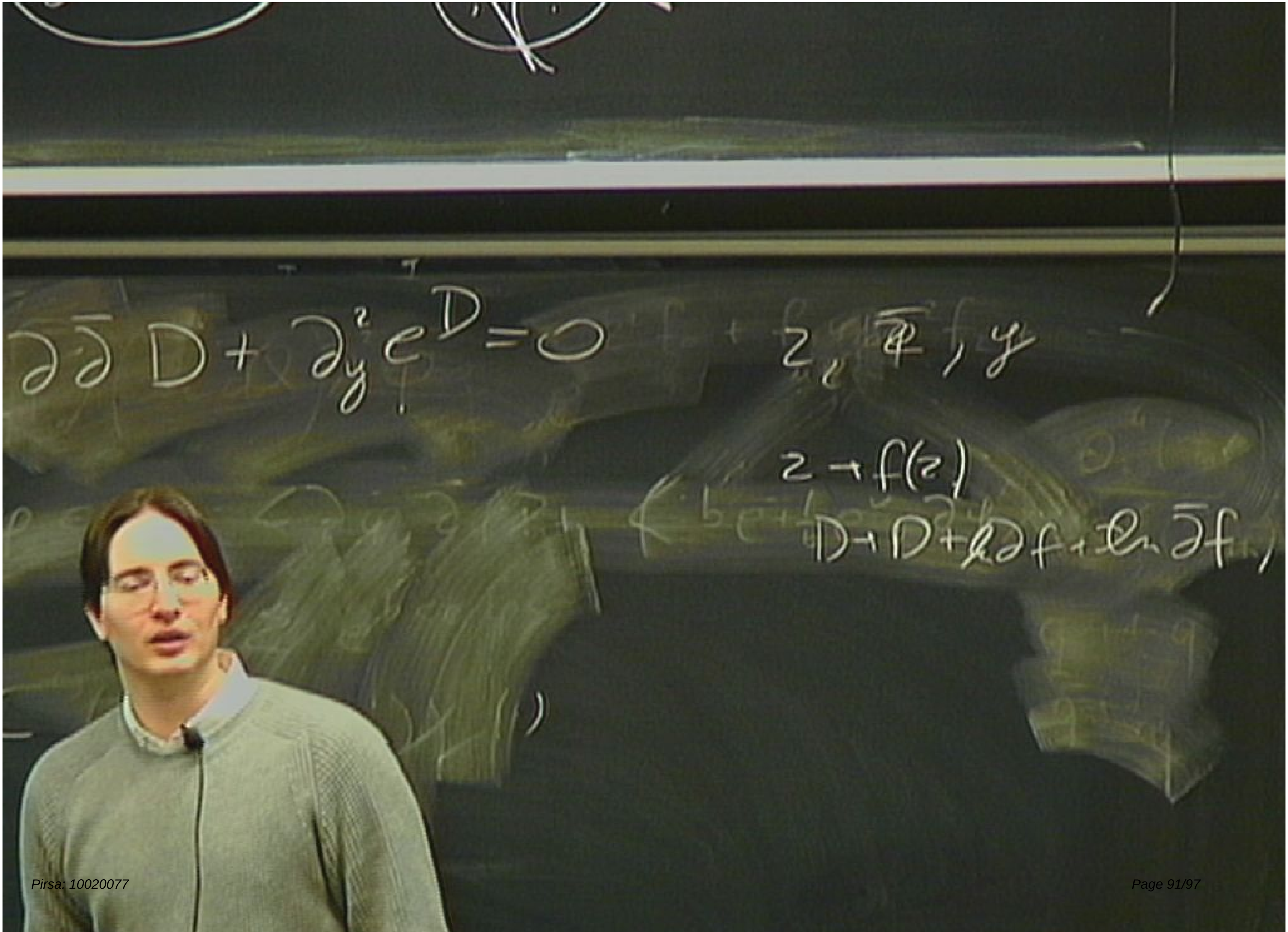
$$\begin{array}{c} M_6 \longleftrightarrow S_1 \times S_2 \\ \downarrow \\ B_3 \end{array}$$

4d $\mathcal{N}=2$ SCFT

$U(1)_R \times SU(2)_R$

$$\partial\bar{\partial} D + \partial_y^2 e^D = 0$$


$\partial\bar{\partial} D + \partial_y^2 e^D = 0$
$z, \bar{z}, y$


∂∂̄ D + ∂y² e^D = 0
z, z̄, y
z → f(z)

$$\partial\bar{\partial} D + \partial_y^2 e^D = 0$$

$z, \bar{z}, y$

$$D = (1-y^2)e^{\phi}$$

$z \to f(z)$

$$D \to D + \ln \partial f + \ln \bar{\partial} \bar{f}$$

$$\partial\bar{\partial}\phi + e^{\phi} = 0$$

$$\partial\bar{\partial} D + \partial_y^2 e^{D} = 0$$

$z, \bar{z}, y$

$$D = (1-y^2)e^{\phi}$$

$z \to f(z)$

$D \to D + \ln\partial f + \ln\bar{\partial}\bar{f}$

$$\partial\bar{\partial}\phi + e^{\phi} = 0$$

$$\partial\bar{\partial} D + \partial_y^2 e^D = 0$$

$z, \bar{z}, y$

$$D = (1-y^2)\,e^{\phi}$$

$z \to f(z)$

$D \to D + \ln \partial f + \ln \bar{\partial} f$

$$\partial\bar{\partial}\phi + e^{\phi} = 0$$

$B_3 =$

$$\partial\bar{\partial} D + \partial_y^2 e^D = 0$$

$z, \bar{z}, y$

$$D = (1-y^2)\, e^{\phi}$$

$$\partial\bar{\partial}\phi + e^{\phi} = 0$$

$z \to f(z)$

$D \to D + \ln \partial f + \ln \bar{\partial} f$

$B_3 =$ 

$$\partial\bar{\partial} D + \partial_y^2 e^D = 0 \qquad z, \bar{z}, y$$

$$D = (1-y^2)e^{\phi} \qquad z \to f(z)$$

$$D \to D + \ln \partial f + \ln \bar{\partial} \bar{f}$$

$$\partial\bar{\partial}\phi + e^{\phi} = 0$$

$\mathcal{B}_3 =$

$$\partial\bar\partial D + \partial_y^2 e^D = 0$$

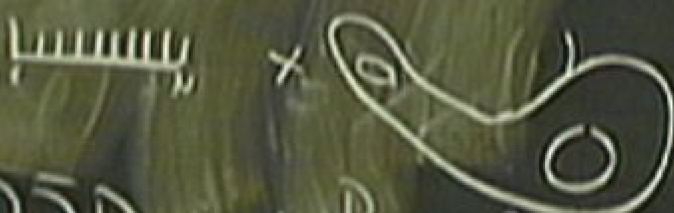
$z, \bar z, y$

$$D = (1-y^2)e^\phi$$

$z \to f(z)$

$D \to D + \ln \partial f + \ln \bar\partial \bar f$

$$\partial\bar\partial\phi + e^\phi = 0$$

$\mathbb{B}_3 =$ 

$$\partial\bar\partial D_i + e^{D_{i+1}} + e^{D_{i-1}} - 2e^{D_i} = 0$$