Title: Lecture 4: Line Operators in N=2 Gauge Theories: wallcrossing, relation to Hitchin systems

Date: Feb 25, 2010 11:15 AM

URL: http://pirsa.org/10020076

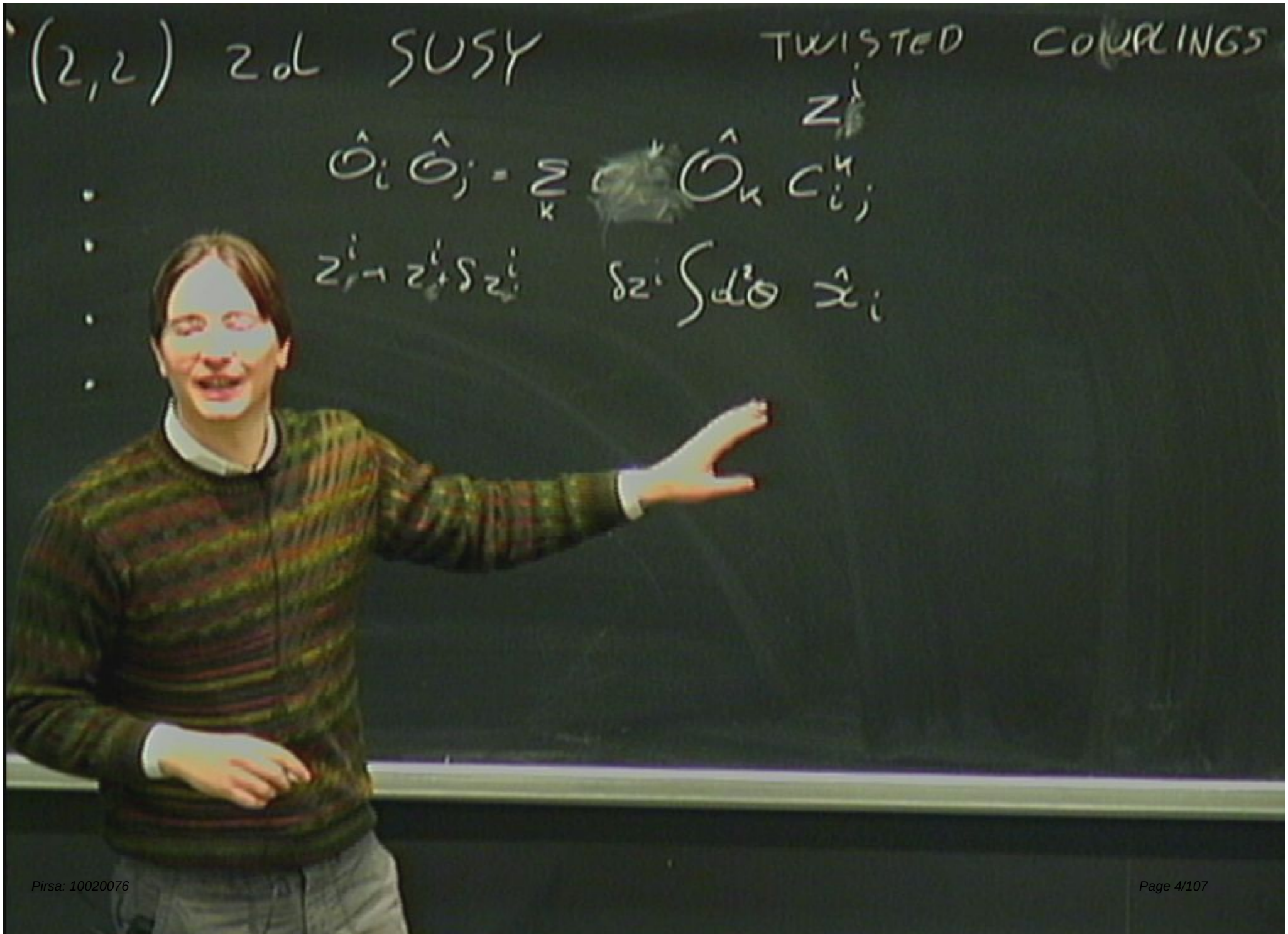
Abstract:

(2,2) 2d SUSY

(2,2) 2d SUSY

TWISTED COUPLINGS

z_i


(2,2) 2d SUSY
TWISTED COUPLINGS
$\hat{O}_i \hat{O}_j = \sum_k \hat{O}_k C^k_{ij}$
$z^i \to z^i + \delta z^i$
$\delta z^i \int d^2\theta \, \hat{\Sigma}_i$

(2,2) 2d SUSY

TWISTED COUPLINGS

z^i

$$\hat{O}_i \hat{O}_j = \sum_k \hat{O}_k \, C^k_{ij}(z^i)$$

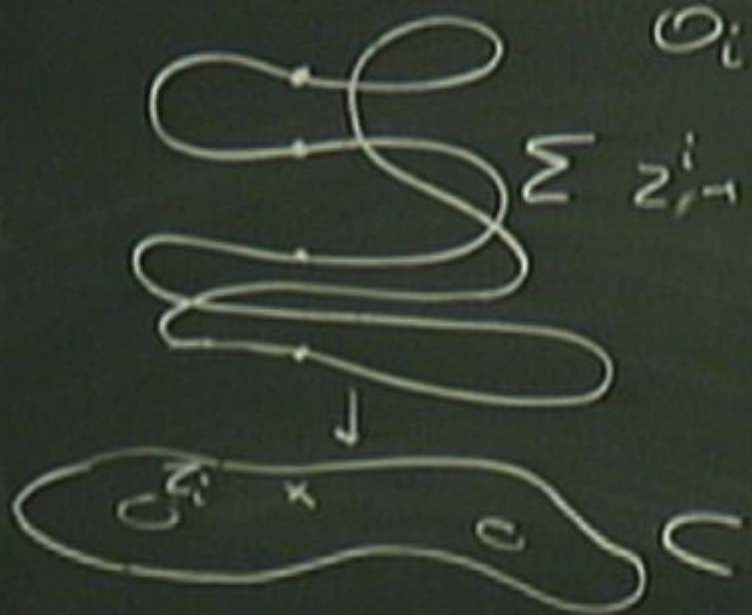
$$z^i \to z^i + \delta z^i \qquad \delta z^i \int d^2\theta \, \hat{x}_i$$

(2,2) 2d SUSY

TWISTED COUPLING

z^i

$\hat{O}_i \hat{O}_j = \sum_k \hat{O}_k \, c^k_{ij}(z^i)$



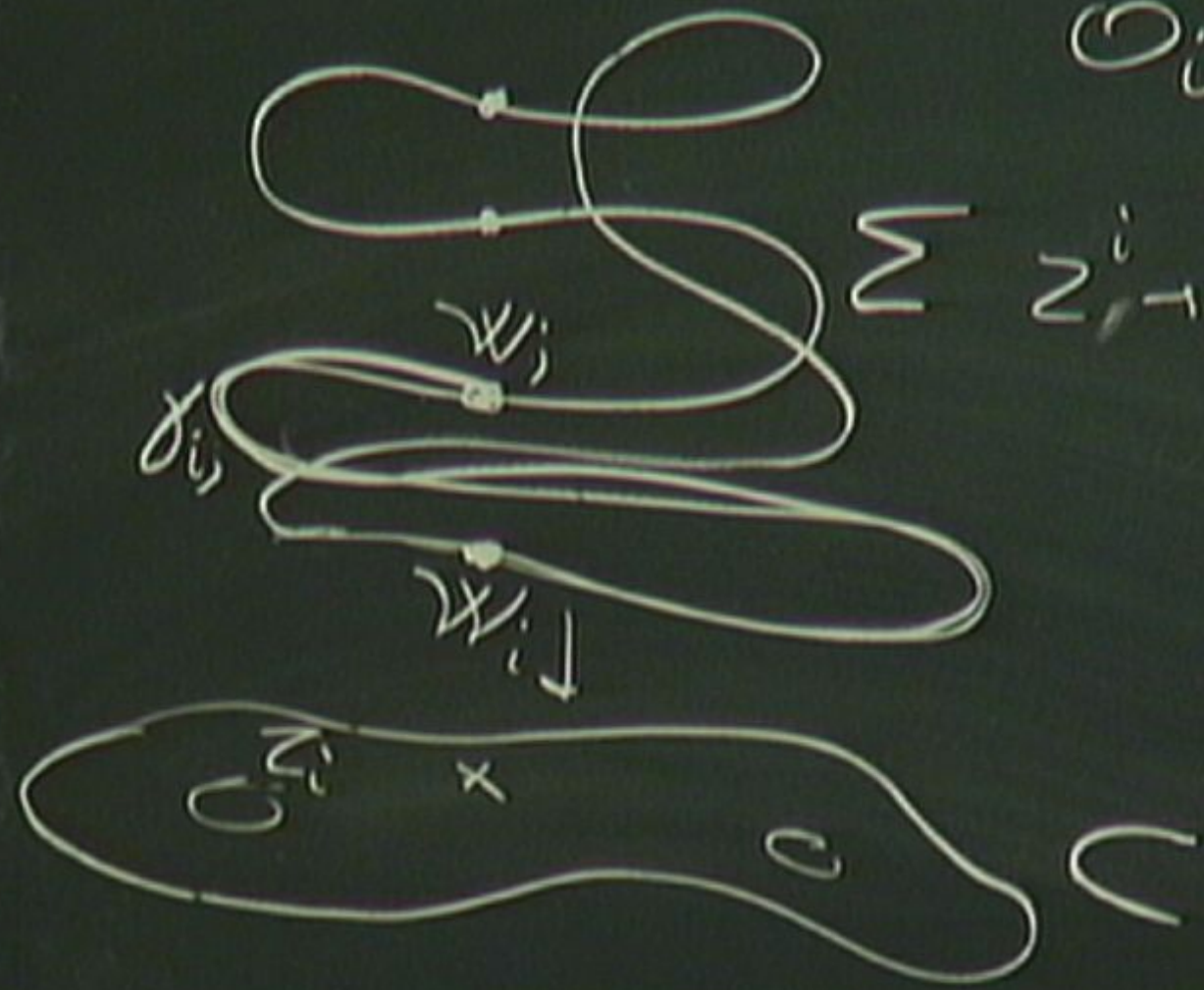
$z^i \to z^i + \delta z^i \qquad \delta z^i \int d^2\theta \, \hat{x}_i$

$x_i = \langle \hat{x}_i \rangle \qquad \lambda = x_i \, dz^i$ ON Σ

$$M_{ij} = |x_i - x_j|$$

$(2,2)$ 2d SU

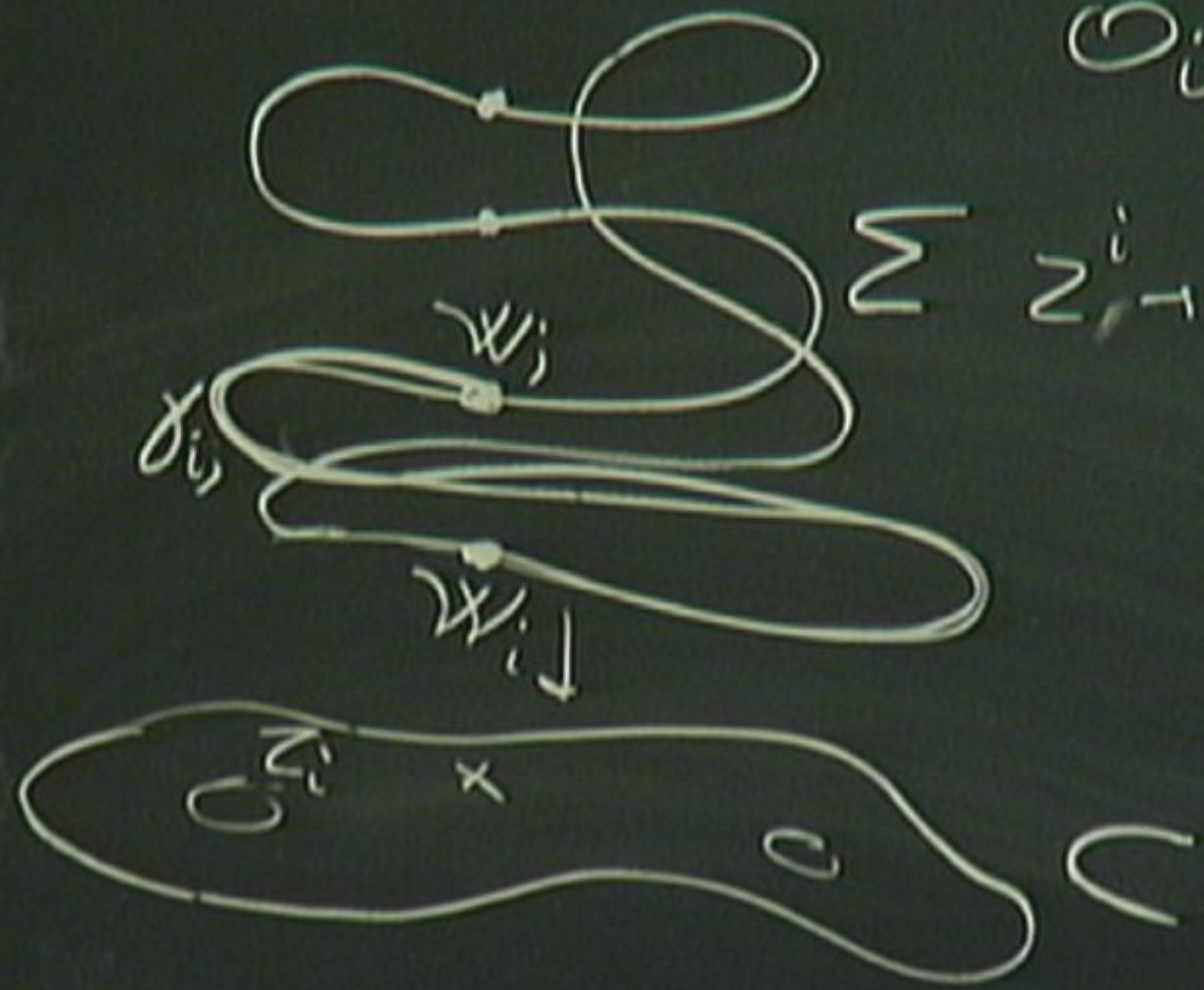
$\hat{\sigma}_i \hat{\sigma}$



$$M_{ij} = |w_i - w_j| = |z_{ij}| \qquad z_{ij} = \oint_{\gamma_{ij}} \lambda$$

$(2,2)$ 2d SU

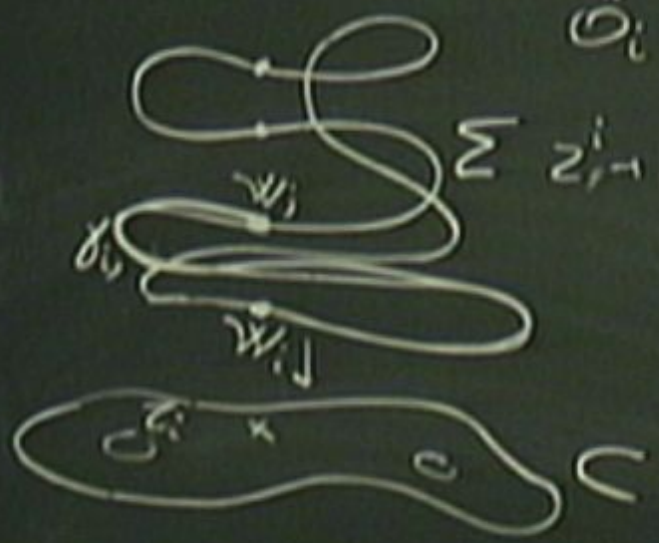
$\hat{\sigma}_i \hat{\sigma}$



(2,2) 2d SUSY

TWISTED COUPLINGS

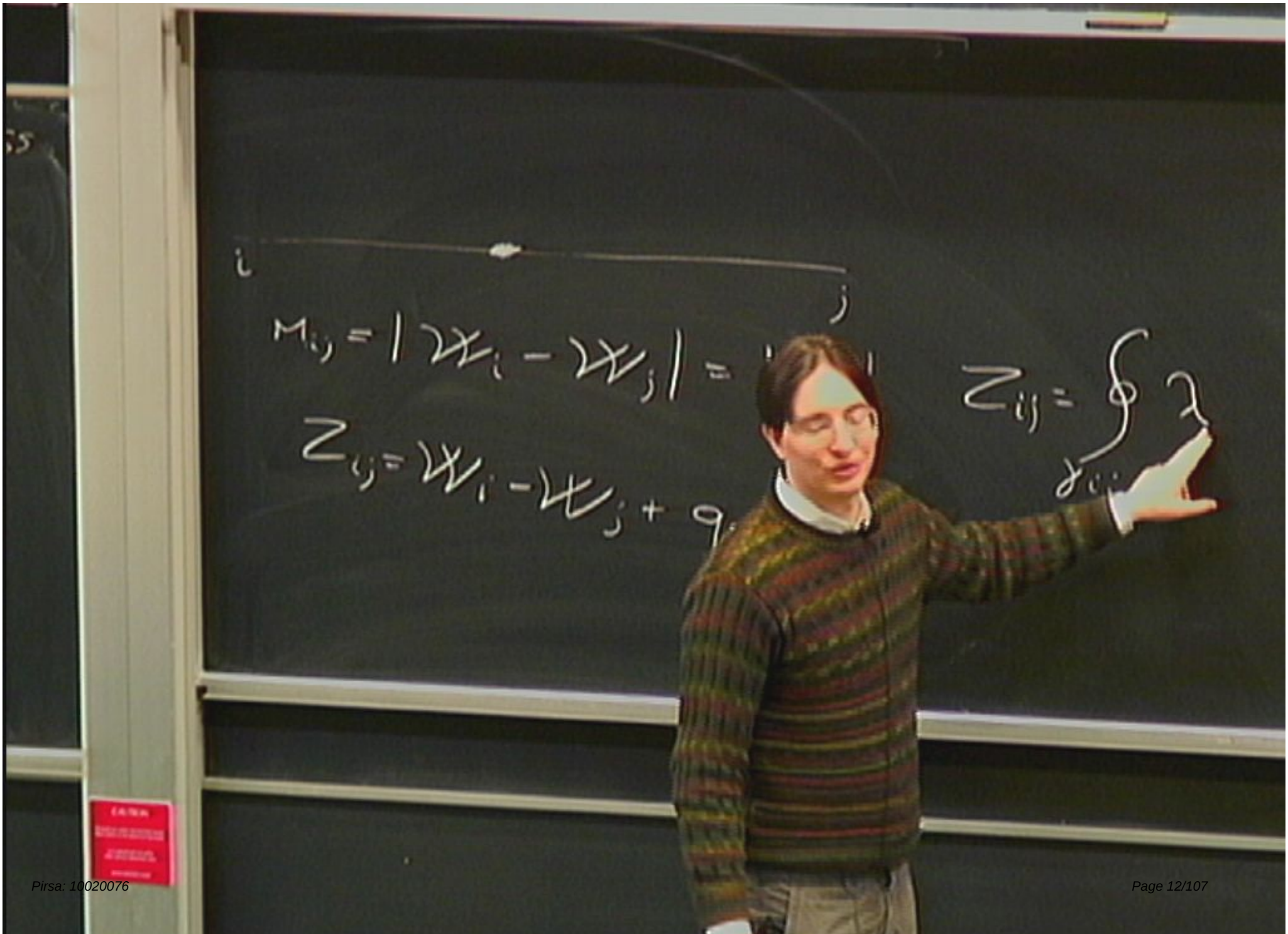
$$\hat{\mathcal{O}}_i \hat{\mathcal{O}}_j = \sum_k \hat{\mathcal{O}}_k \, C_{ij}^{k}(z^i, \eta)$$

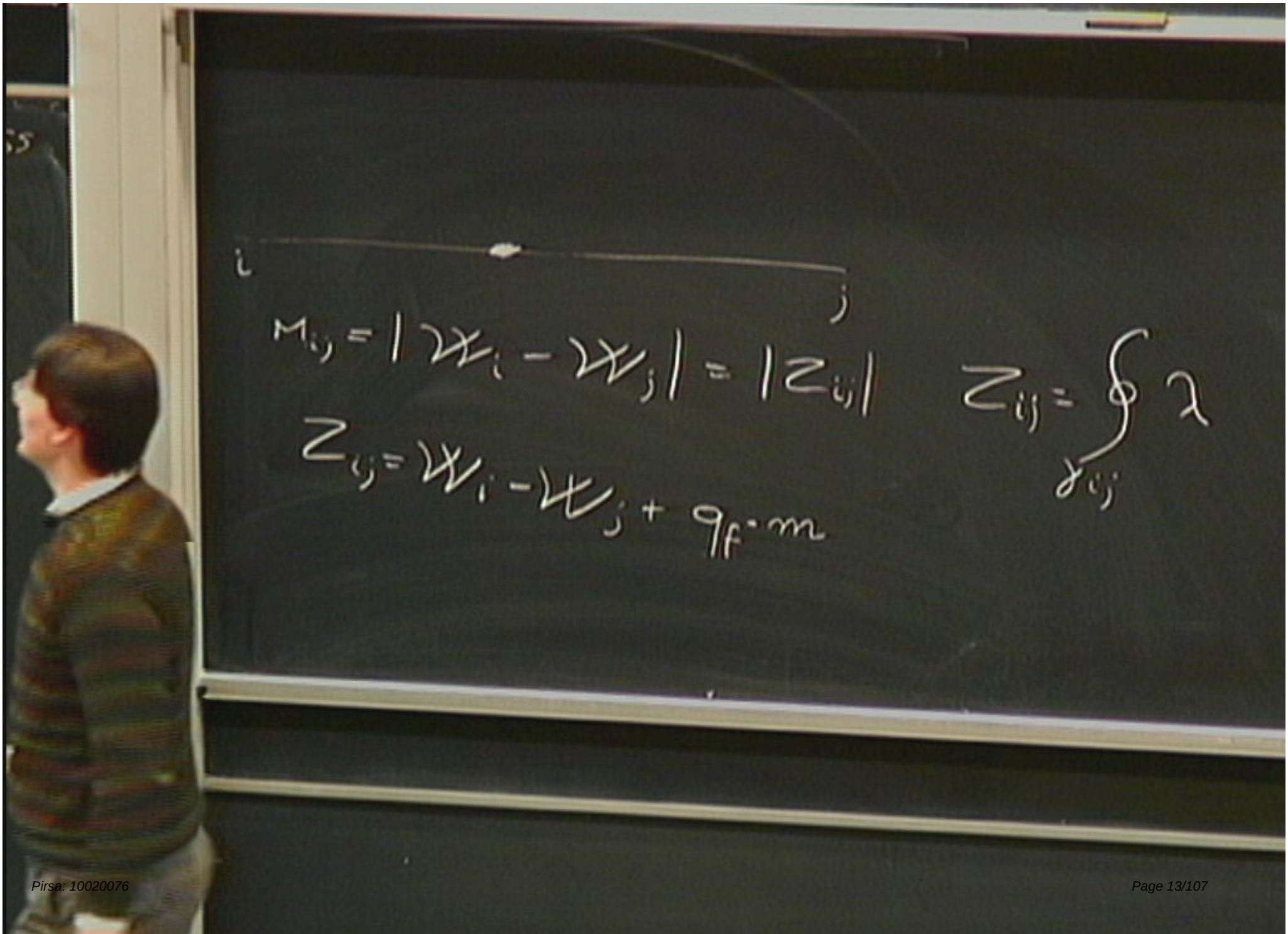


Σ $\quad z^i \to z^i + \delta z^i \qquad \delta z^i \int d^2\theta \, \hat{x}_i$

$x_i = \langle \hat{x}_i \rangle \qquad \lambda = x_i dz^i$ ON Σ

FLAVOR SYMMETRIES $\Rightarrow m$

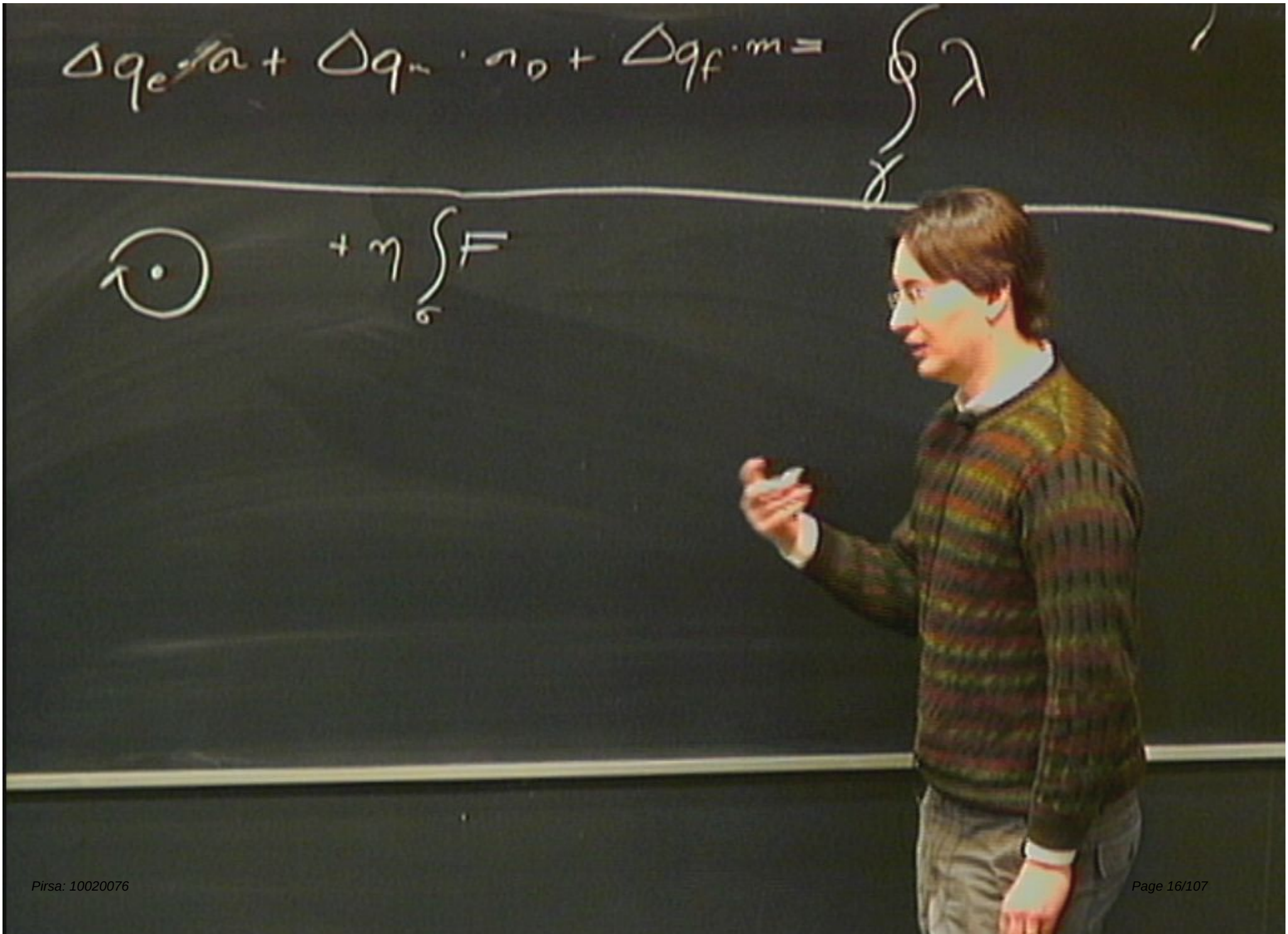

i
j
$M_{ij} = |w_i - w_j| =$
$Z_{ij} = w_i - w_j + q$
$Z_{ij} = \oint_{\gamma_{ij}} \lambda$

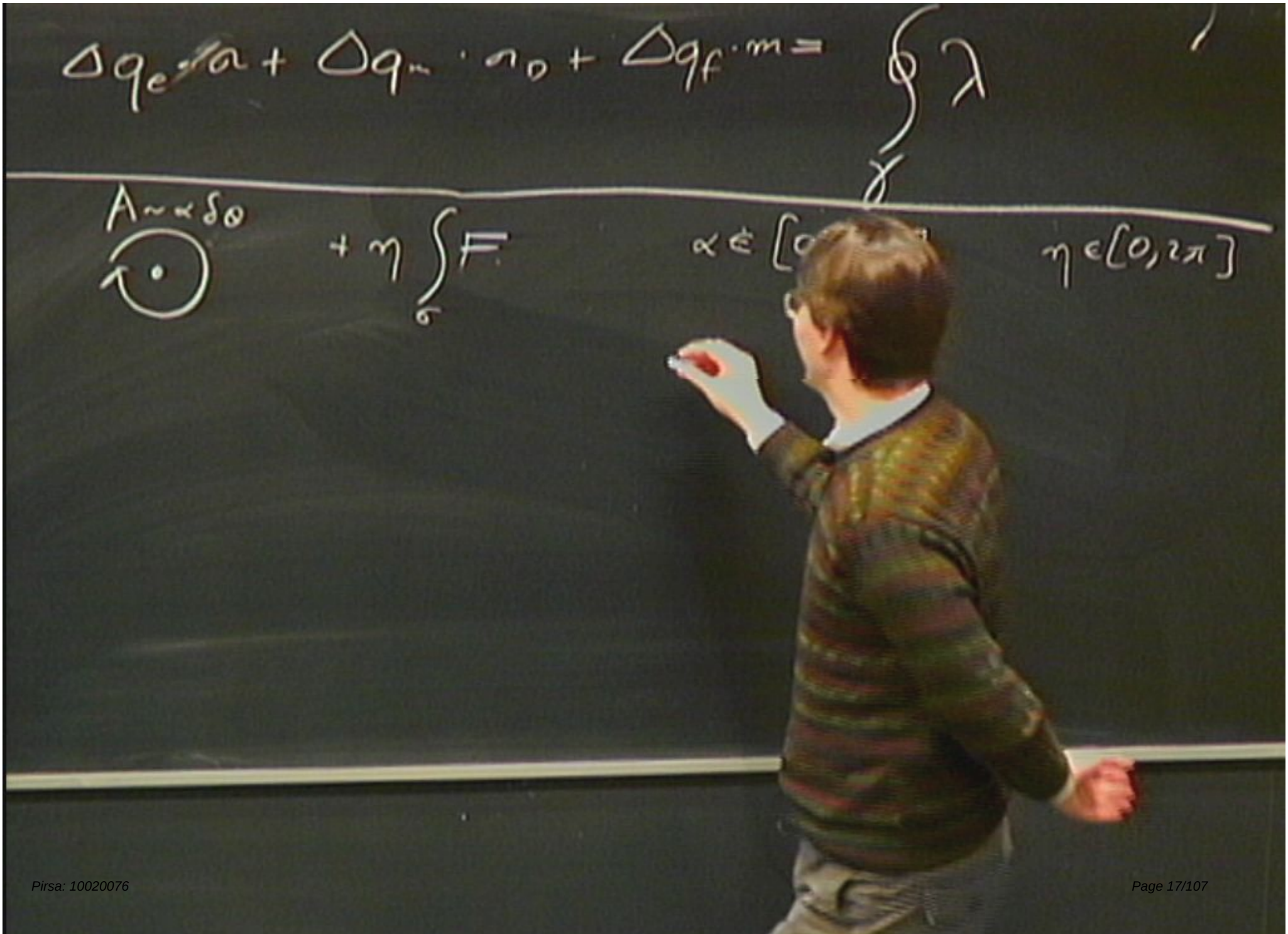

i
j
$M_{ij} = |w_i - w_j| = |Z_{ij}|$
$Z_{ij} = \oint_{\gamma_{ij}} \lambda$
$Z_{ij} = w_i - w_j + q_f \cdot m$

$$M_{ij} = |\mathcal{W}_i - \mathcal{W}_j| = |Z_{ij}| \qquad Z_{ij} = \oint_{\gamma_{ij}} \lambda$$

$$Z_{ij} = \mathcal{W}_i - \mathcal{W}_j + q_f \cdot m + q_e \cdot a + q_m \cdot a_D$$

$$\Delta q_e \cdot a + \Delta q_n \cdot n_D + \Delta q_f \cdot m = \oint_\gamma \lambda$$


$\Delta q_e \cdot a + \Delta q_m \cdot n_D + \Delta q_f \cdot m = \oint_\gamma \lambda$
$+ \eta \int_\sigma F$


$\Delta q_e \cdot a + \Delta q_m \cdot n_D + \Delta q_f \cdot m = \oint_\gamma \lambda$
$A \sim \alpha \delta_\Theta$
$+ \eta \int_\sigma F$
$\alpha \in [0$
$\eta \in [0, 2\pi]$

$$\Delta q_e \cdot a + \Delta q_m \cdot n_D + \Delta q_f \cdot m = \oint_\gamma \lambda$$

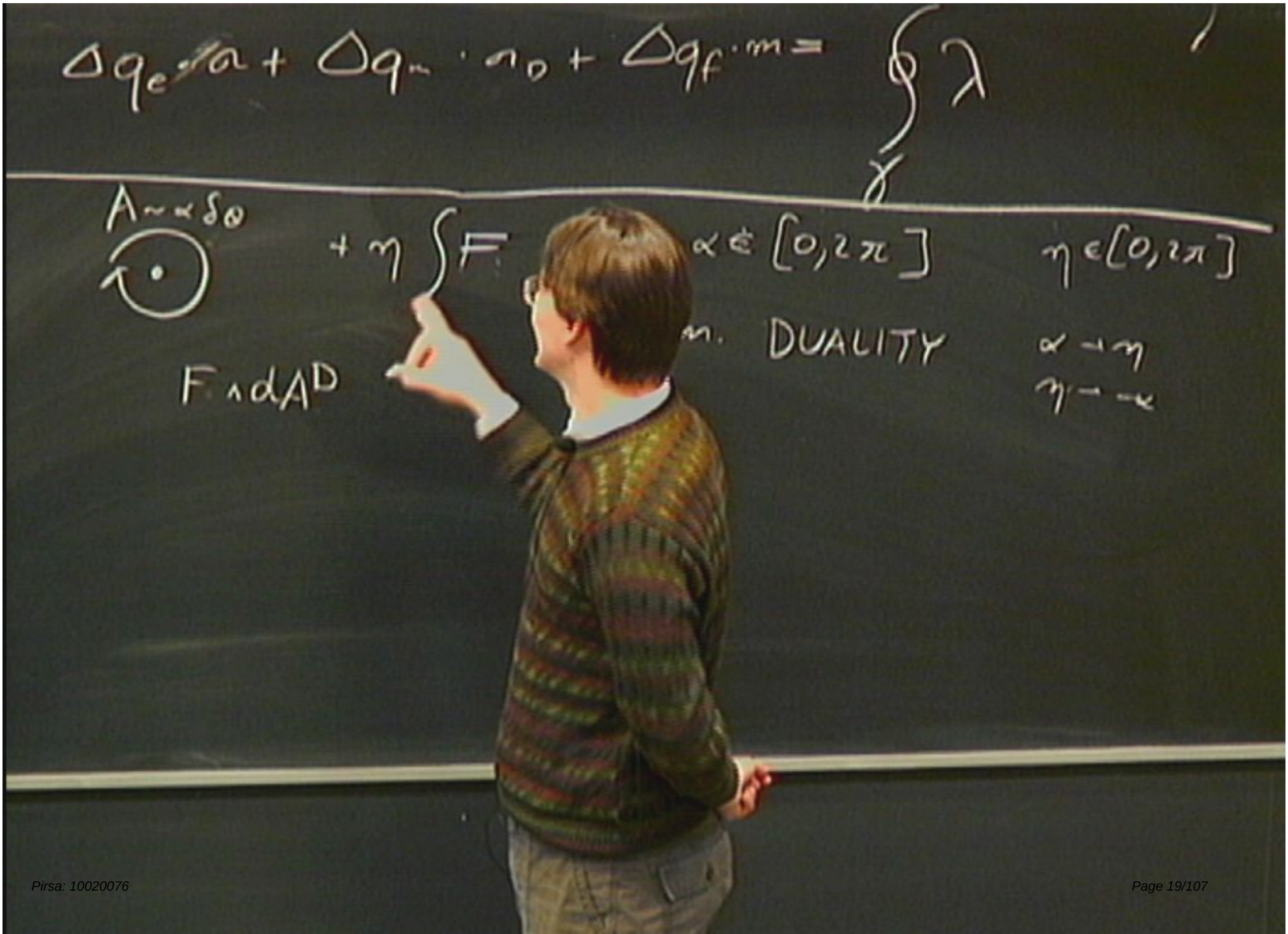
$A \sim \alpha \delta_\odot$

$$+ \eta \int_\sigma F$$

$\alpha \in [0, 2\pi]$ $\quad$ $\eta \in [0, 2\pi]$

E.m. DUALITY $\quad$ $\alpha \to \eta$

$\eta \to -\alpha$


$\Delta q_e \cdot a + \Delta q_m \cdot n_D + \Delta q_f \cdot m = \oint_\gamma \lambda$
$A \sim \alpha \delta_\theta$
$+ \eta \int F$
$\alpha \in [0, 2\pi]$
$\eta \in [0, 2\pi]$
DUALITY
$\alpha \to \eta$
$\eta \to -\alpha$
$F \wedge dA^D$

$$\Delta q_e \cdot a + \Delta q_m \cdot n_D + \Delta q_f \cdot m = \oint_\gamma \lambda$$

$A \sim \alpha \delta_\theta$

$$+ \eta \int_\sigma F$$

$F \wedge dA^D$

$\alpha \in [0, 2\pi]$

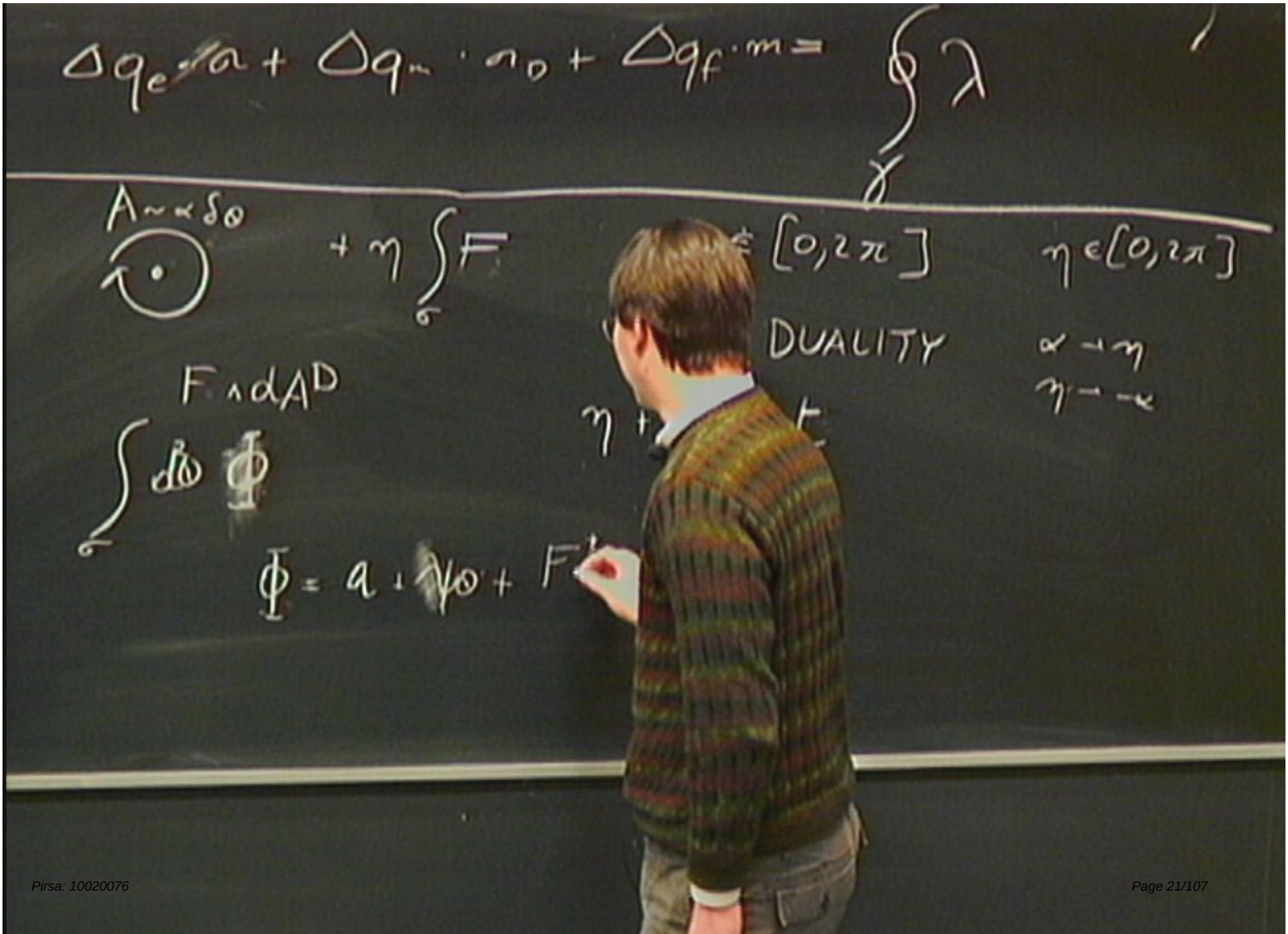
$\eta \in$

E.m. DUALITY

$\alpha \to$

$\eta \to$

$\eta + \alpha\tau = t$


$\Delta q_e \cdot a + \Delta q_m \cdot a_D + \Delta q_f \cdot m =$
$\oint \lambda$
γ
$A \sim \alpha \delta_\theta$
$+ \eta \int F$
$\in [0, 2\pi]$
$\eta \in [0, 2\pi]$
DUALITY
$\alpha \to \eta$
$\eta \to -\alpha$
$F \wedge dA^D$
$\int d\theta \, \Phi$
$\Phi = a + \lambda\psi + F$

$$\Delta q_e \cdot a + \Delta q_m \cdot n_D + \Delta q_f \cdot m = \oint_\gamma \lambda$$

$$A \sim \alpha \delta_\theta$$

$$+ \eta \int_\sigma F$$

$\alpha \in [0, 2\pi]$ $\quad$ $\eta \in [0, 2\pi]$

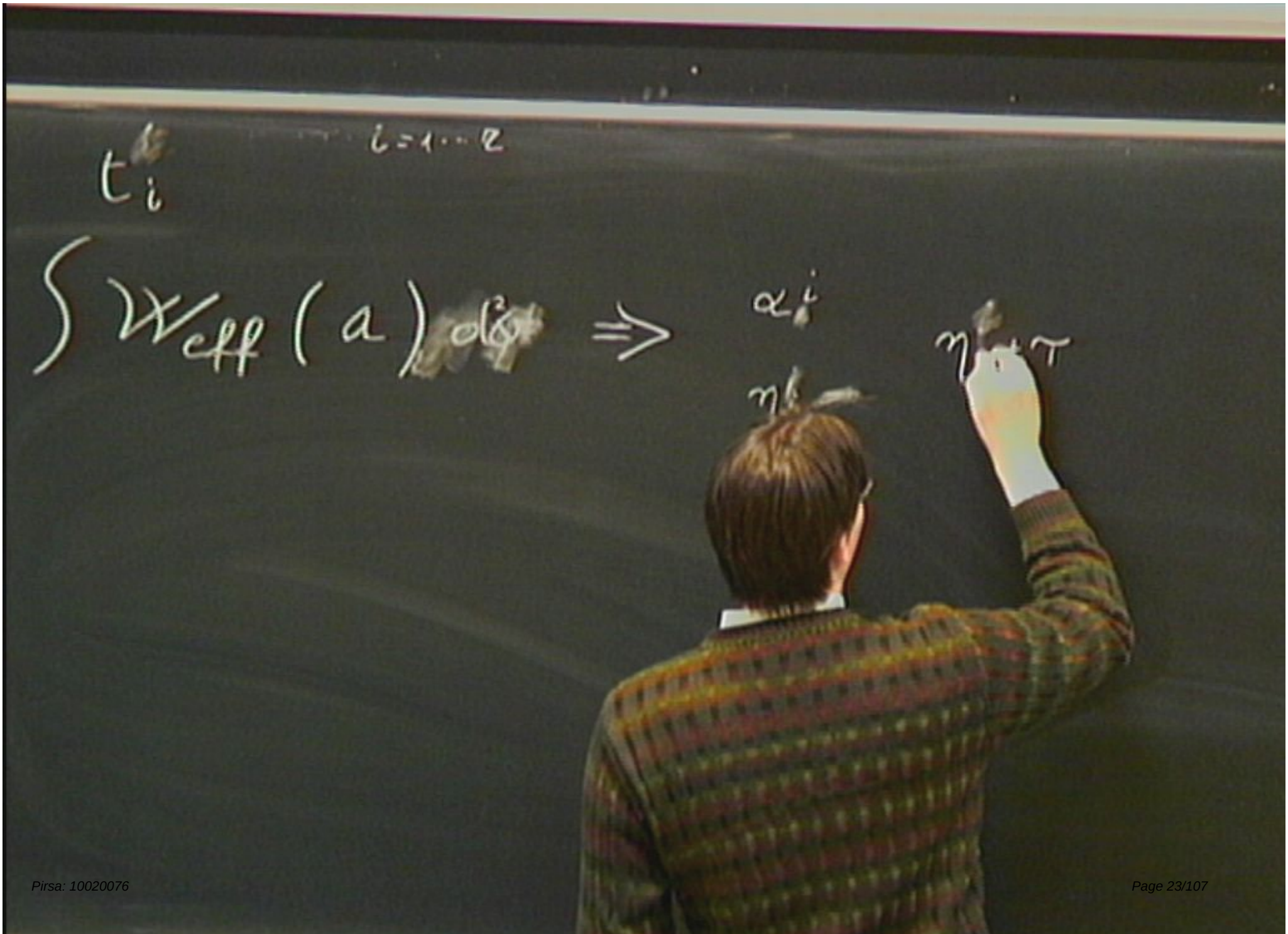
E.m. DUALITY $\quad$ $\alpha \to \eta$, $\eta \to -\alpha$

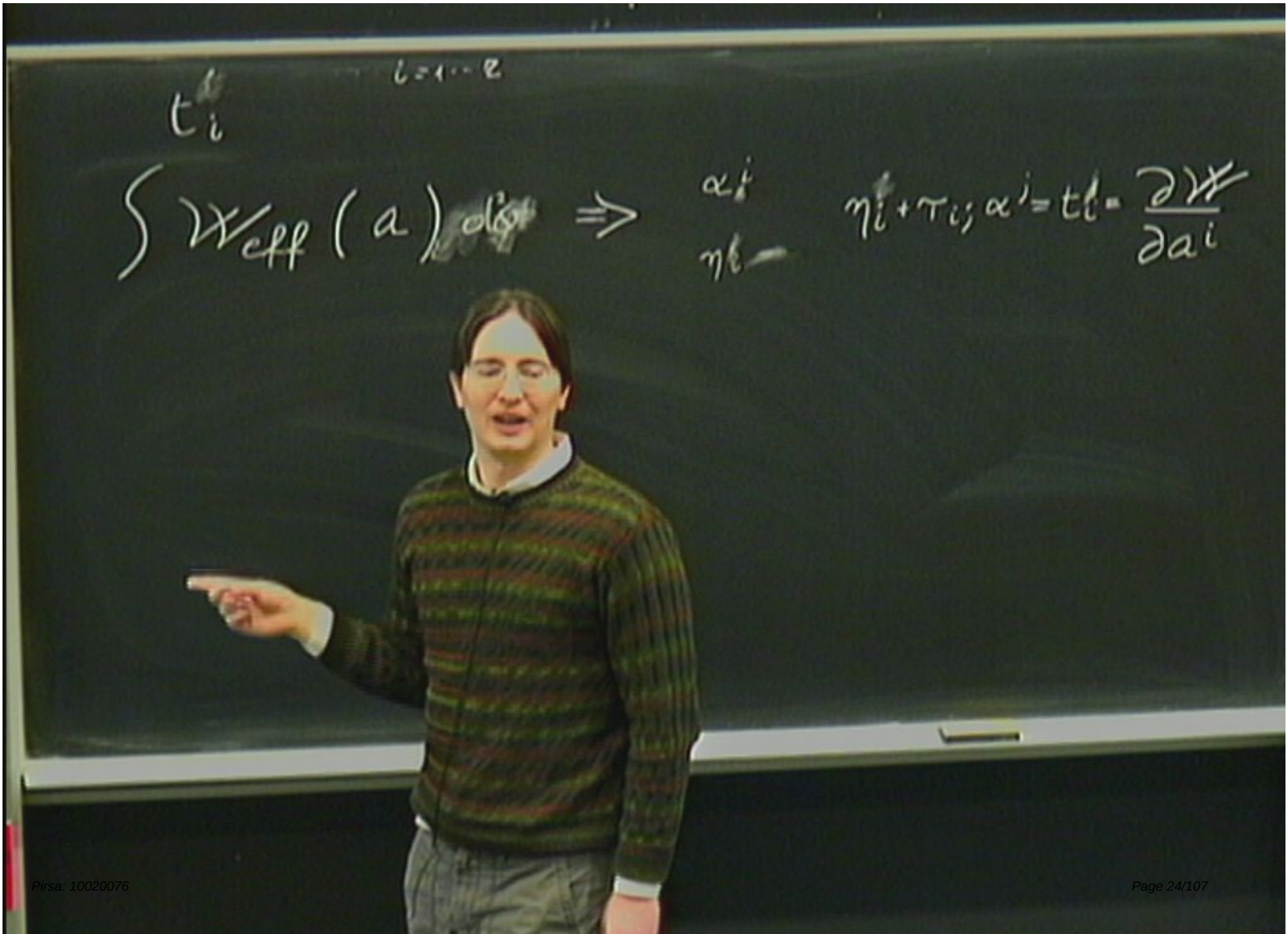
$$F \wedge dA^D$$

$$\eta + \alpha \tau = t$$

$$t \int_\sigma d\theta \, \Phi$$

$$\Phi = a + \lambda \theta + F' \theta\theta + \dots$$


t_i
$i = 1 \cdots 2$
$\int W_{eff}(a)$
$\Rightarrow$
α_i


t_i^{ℓ}
$i = 1 \ldots e$
$\int W_{eff}(a)\, d^2\phi \Rightarrow$
α_i
η_i
$\eta_i + \tau_{ij}\alpha^j = t_i = \frac{\partial W}{\partial a^i}$

$i = 1 \cdots 2$

t_i

$$\int W_{eff}(a)\, d^2\theta \Rightarrow \quad \alpha^i \quad \eta^i + \tau_{ij}\alpha^j = t_i = \frac{\partial W}{\partial a^i}$$

η^i

$$\int \frac{\partial W}{\partial a^i}\left(F_i + * \tau_{ij} F^j\right) + c.c$$

$i = 1 \cdots g$

t_i

$$\int W_{eff}(a)\, d^2\theta \Rightarrow \quad \alpha_i \quad \eta_i \qquad \eta_i + \tau_{ij}\alpha^j = t_i = \frac{\partial W}{\partial a^i}$$

$$\int \frac{\partial W}{\partial a^i}\left(F_i + *\tau_{ij}F^j\right) + c.c \qquad \oint_\gamma \frac{\partial t^i}{\partial z}\, dz$$

$t^{\pm}$ $\quad \pm = 1 \cdots g$

$$\int W_{eff}(a)\, d^2\theta \Rightarrow \quad \alpha^{\pm} \quad \eta^{\pm} \qquad \eta^{\pm}_{I} + \tau_{Ij}\alpha^{j} = t^{\pm}_{I} = \frac{\partial W}{\partial a^{I}}$$

$$\int_{\sigma} \frac{\partial W}{\partial a^{i}}\left(F^{+}_{I} + *\tau_{Ij}F^{J}\right) + c.c \qquad \oint_{\gamma} \frac{\partial t^{+}_{I}}{\partial z^{i}} dz^{i}$$

$$\mathbb{E}^{I} \, T_{IJ} \, \alpha^{J} = t_I = \frac{\partial \mathcal{W}}{\partial a^I}$$

$$\oint_\gamma \frac{\partial t_I}{\partial z^i} dz^i = \Delta t_I = \Delta \eta_I + \Delta \alpha^J T_{IJ}$$

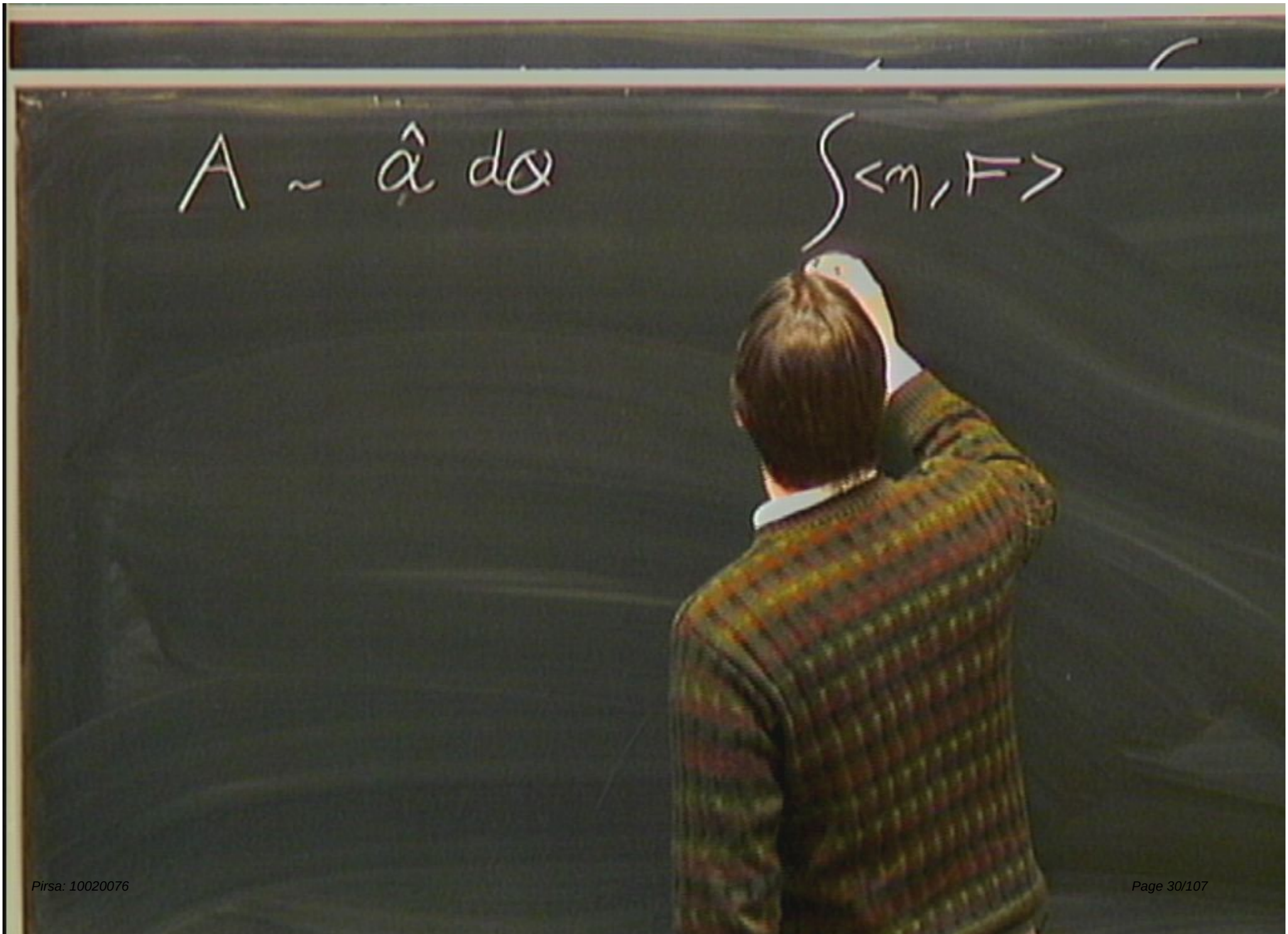
$$= 2\pi n_I + T_{IJ} \, 2\pi m_J$$

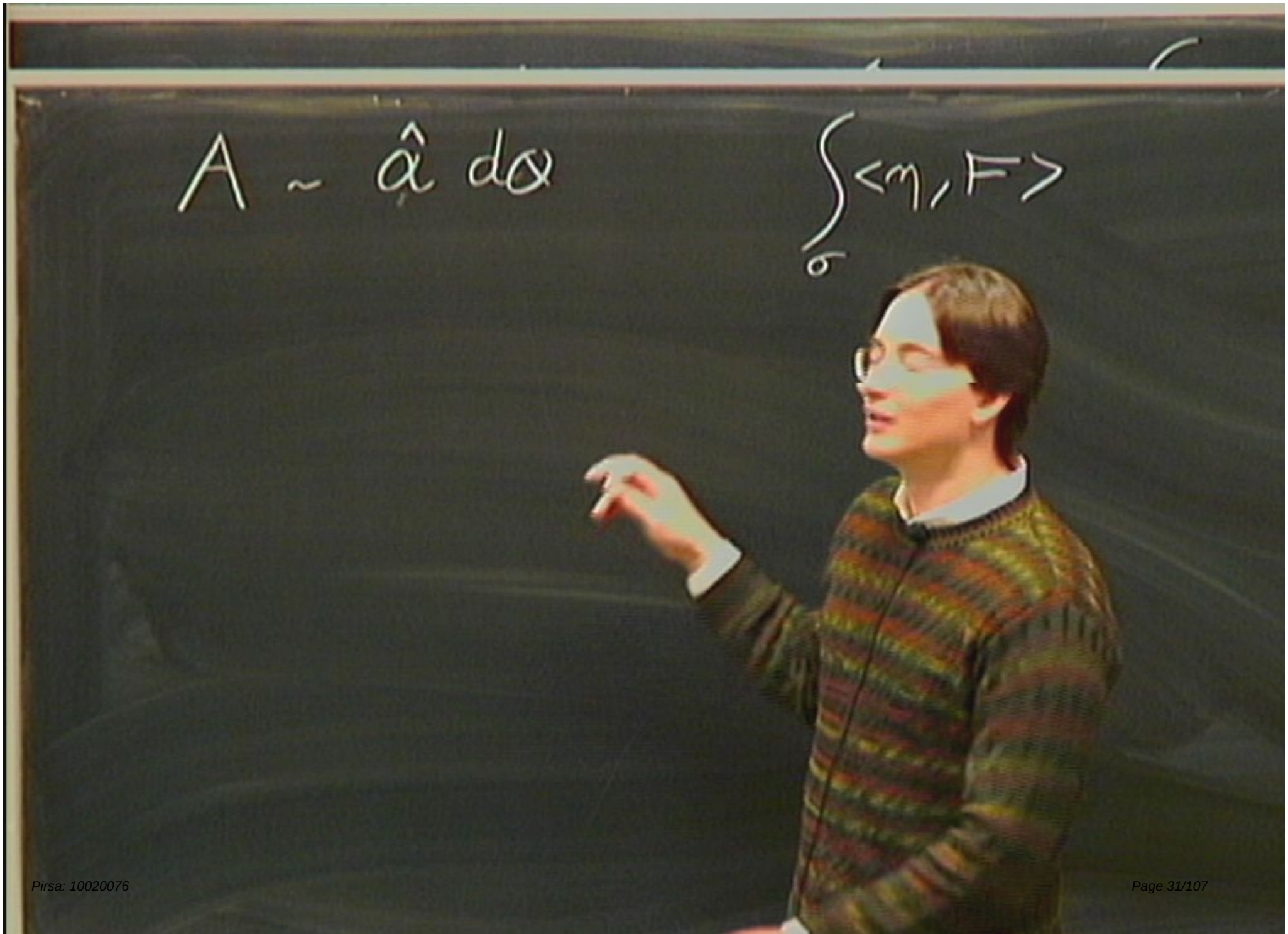
$t_{\pm}$ $\quad \pm = 1 \cdots g$

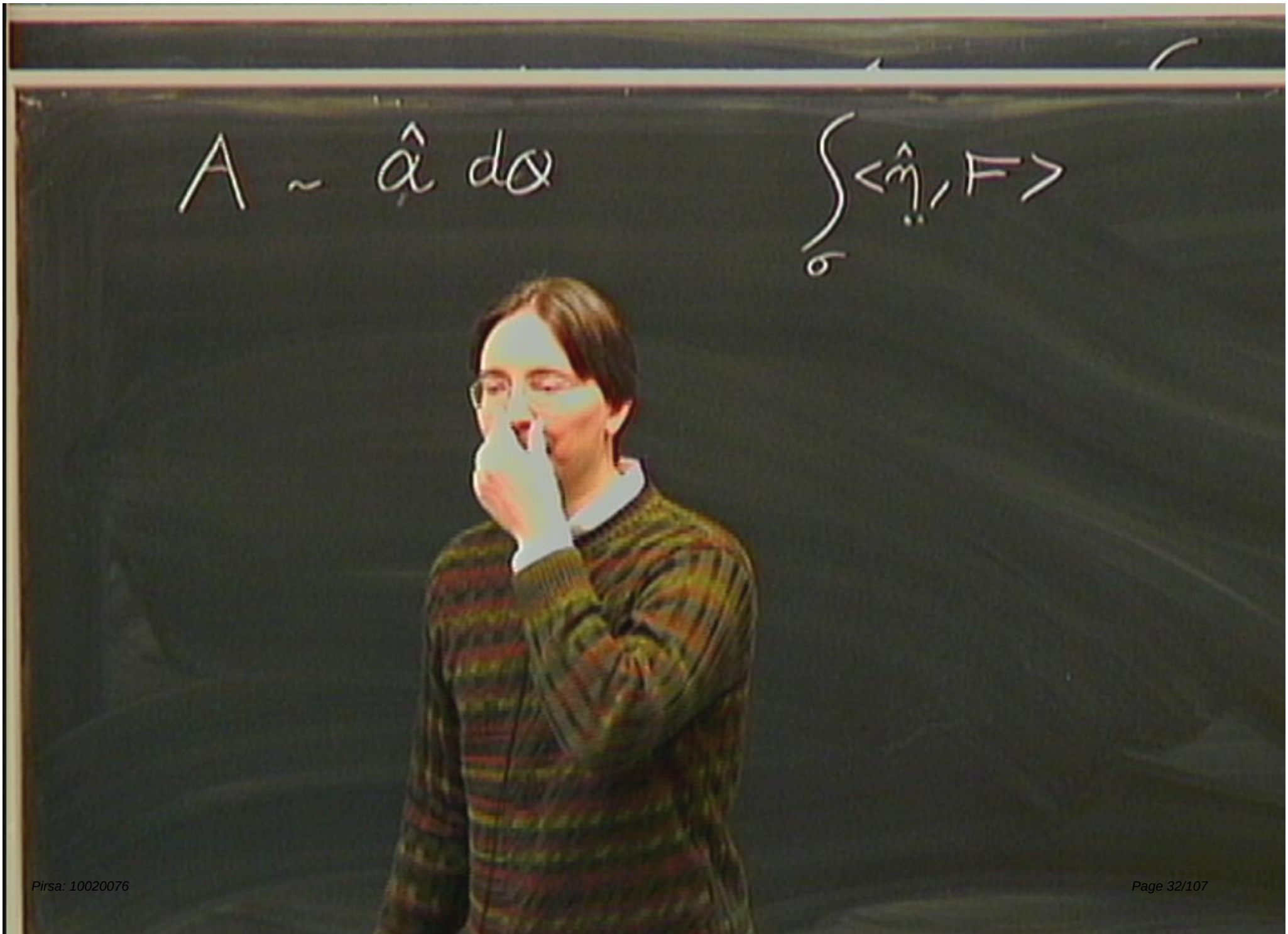
$$\int \mathcal{W}_{eff}(a)\, d^2\theta \Rightarrow \quad \alpha^{\pm} \quad \eta_{\pm} \qquad \eta_I + \tau_{IJ}\alpha^J = t_I = \frac{\partial \mathcal{W}}{\partial a^I}$$

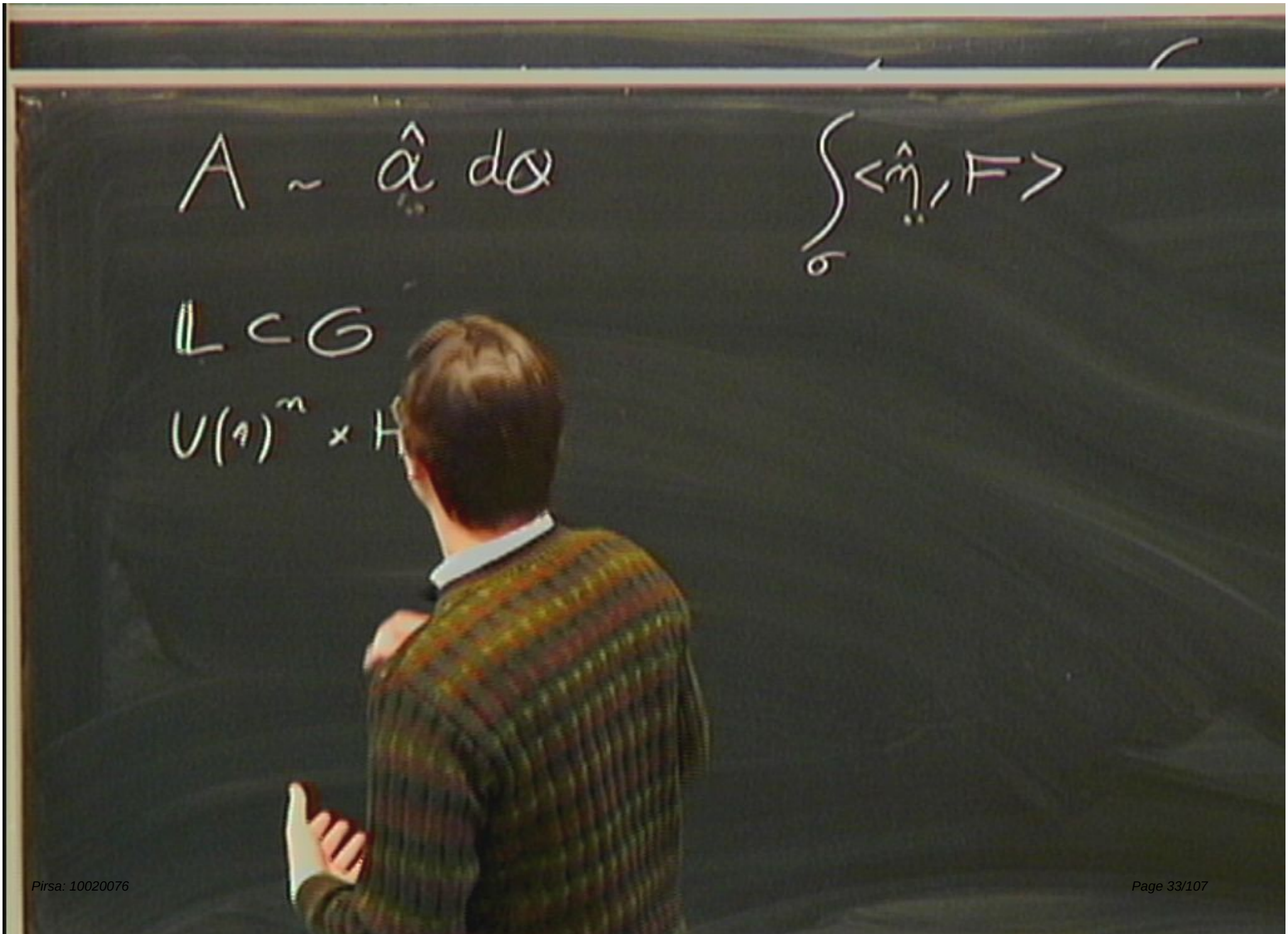
$$\int_{\sigma} \frac{\partial \mathcal{W}}{\partial a^i}\left(F_I + * \tau_{IJ} F^J\right) + c.c \qquad \oint_{\gamma} \frac{\partial t_I}{\partial z^i} dz^i = \Delta t_I = \Delta \eta_I + \Delta \alpha^J \tau_{IJ}$$

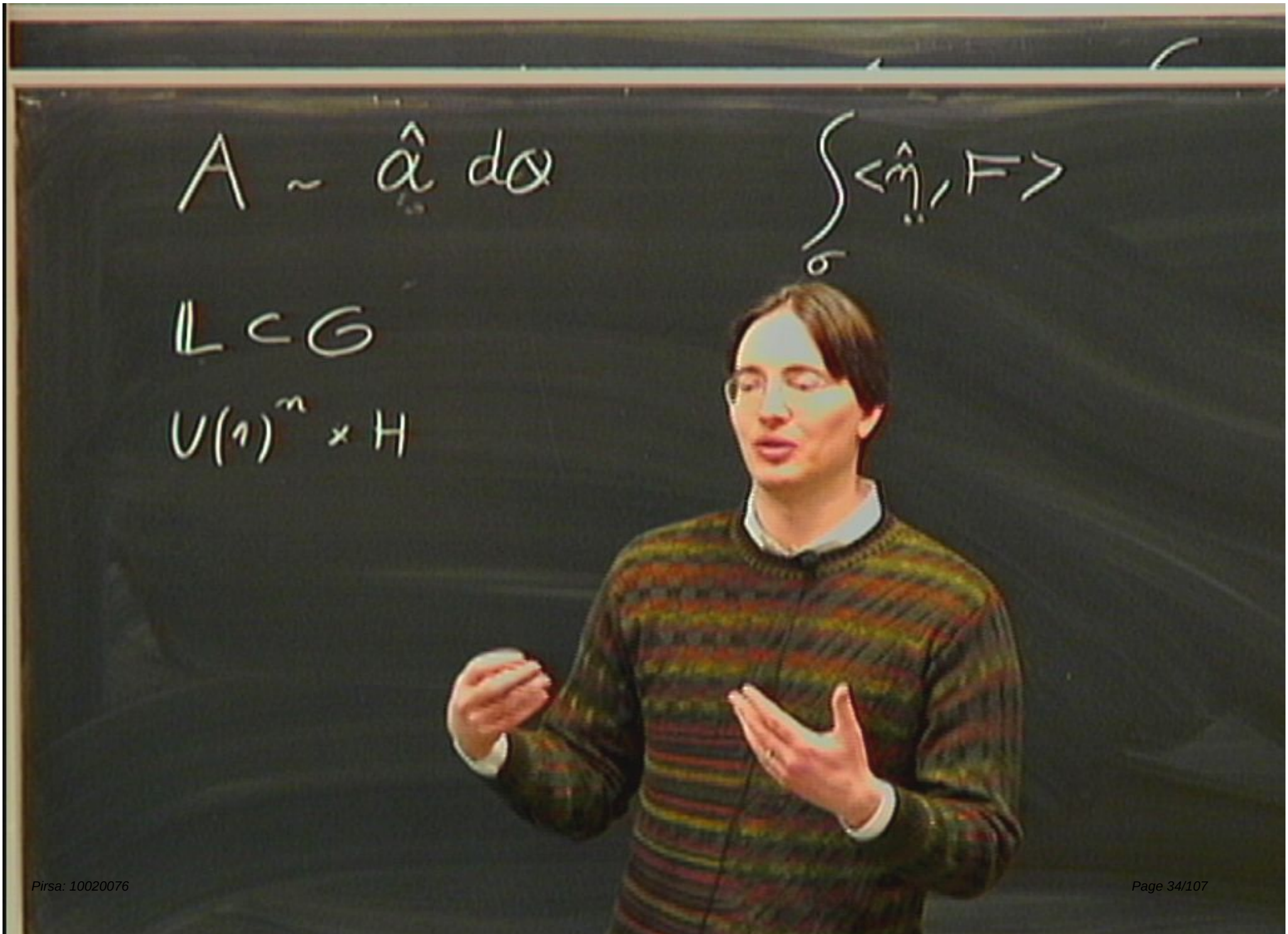
$$\oint_{\gamma} \frac{\partial \lambda}{\partial a_I} = 2\pi n_I + \tau_{IJ}\, 2\pi n_J$$


A ~ â dα
∫<η, F>


A ~ â dα
∫σ <η, F>


A ~ â dα
∫σ <η̂, F>


A ~ â dα
∫σ <η̂, F>
L ⊂ G
U(1)^n × H


A ~ $\hat{a}$ dα
$\int_\sigma \langle \hat{\eta}, F \rangle$
$\mathbb{L} \subset G$
$U(1)^n \times H$

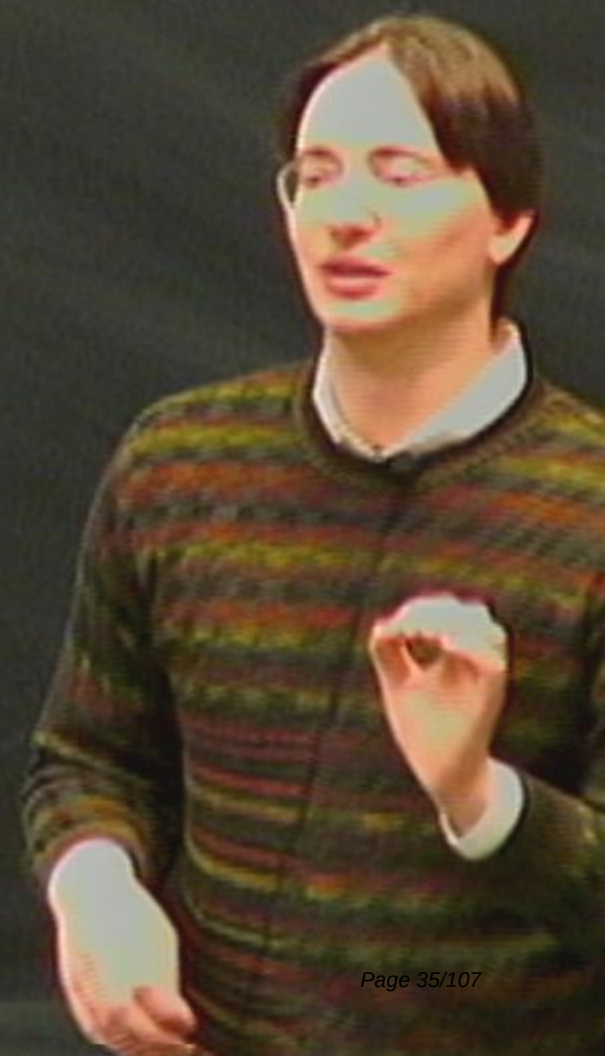
$$A \sim \hat{a}\, d\alpha$$

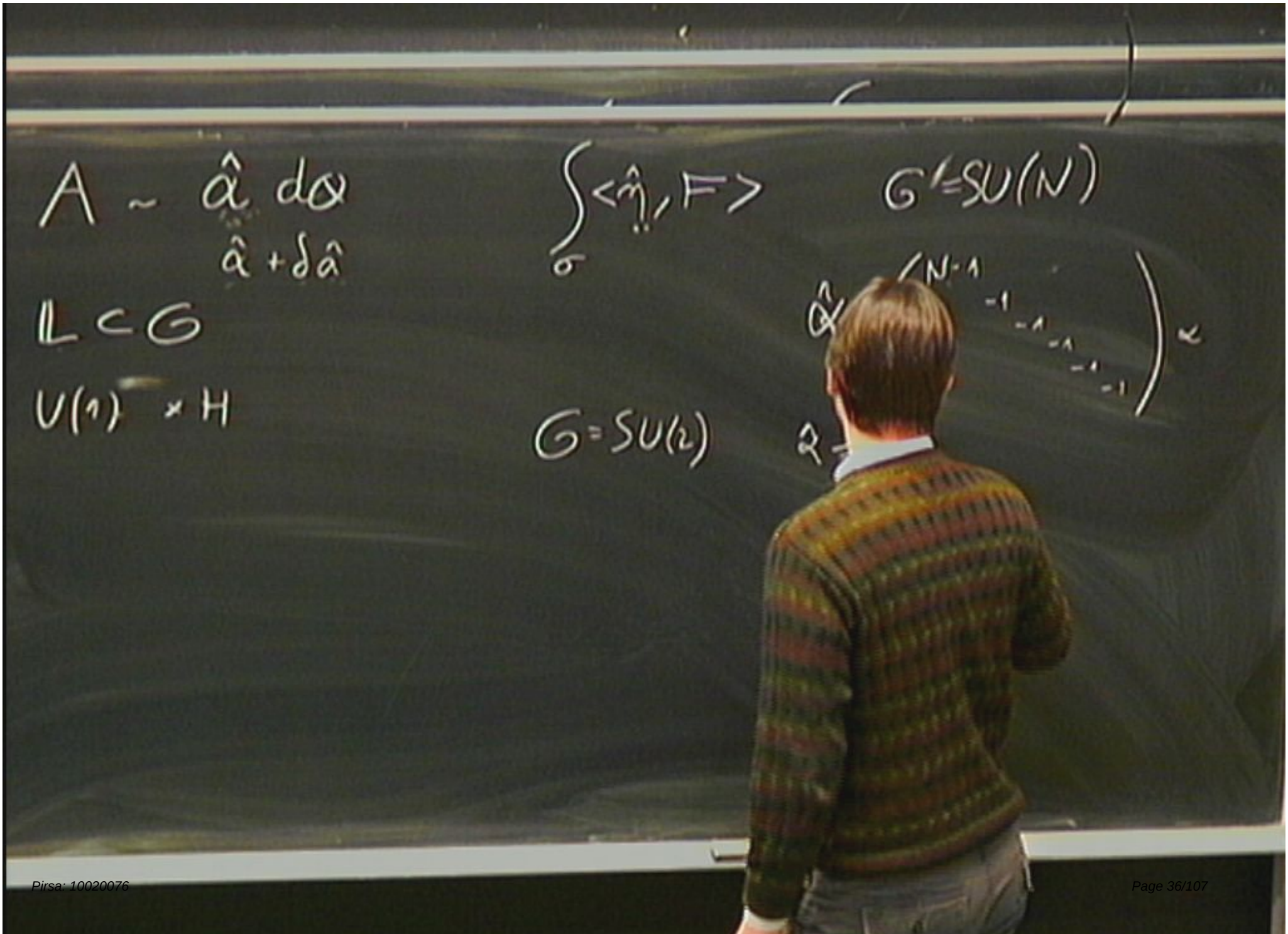
$$\hat{a} + \delta\hat{a}$$

$$\int_\sigma \langle \hat{\eta}, F \rangle$$

$$\mathbb{L} \subset G$$

$$U(1) \times H$$




A ~ â dα
â + δâ
L ⊂ G
U(1) × H
∫_σ <η̂, F>
G' = SU(N)
G = SU(2)

$$A \sim \hat{a}\, d\alpha$$

$$\hat{a} + \delta\hat{a}$$

$$\mathbb{L} \subset G$$

$$U(1) \times H$$

$$\int_\sigma \langle \hat{\eta}, F \rangle$$

$$G' = SU(N)$$

$$\hat{\alpha} = \begin{pmatrix} N-1 & & & & & \\ & -1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix}$$

$$G = SU(2) \qquad \hat{\alpha} = \begin{pmatrix} \alpha & \\ & -\alpha \end{pmatrix}$$

$$A \sim \hat{a}\, d\alpha$$

$$\hat{a} + \delta\hat{a}$$

$$\int_\sigma \langle \hat{\eta}, F \rangle$$

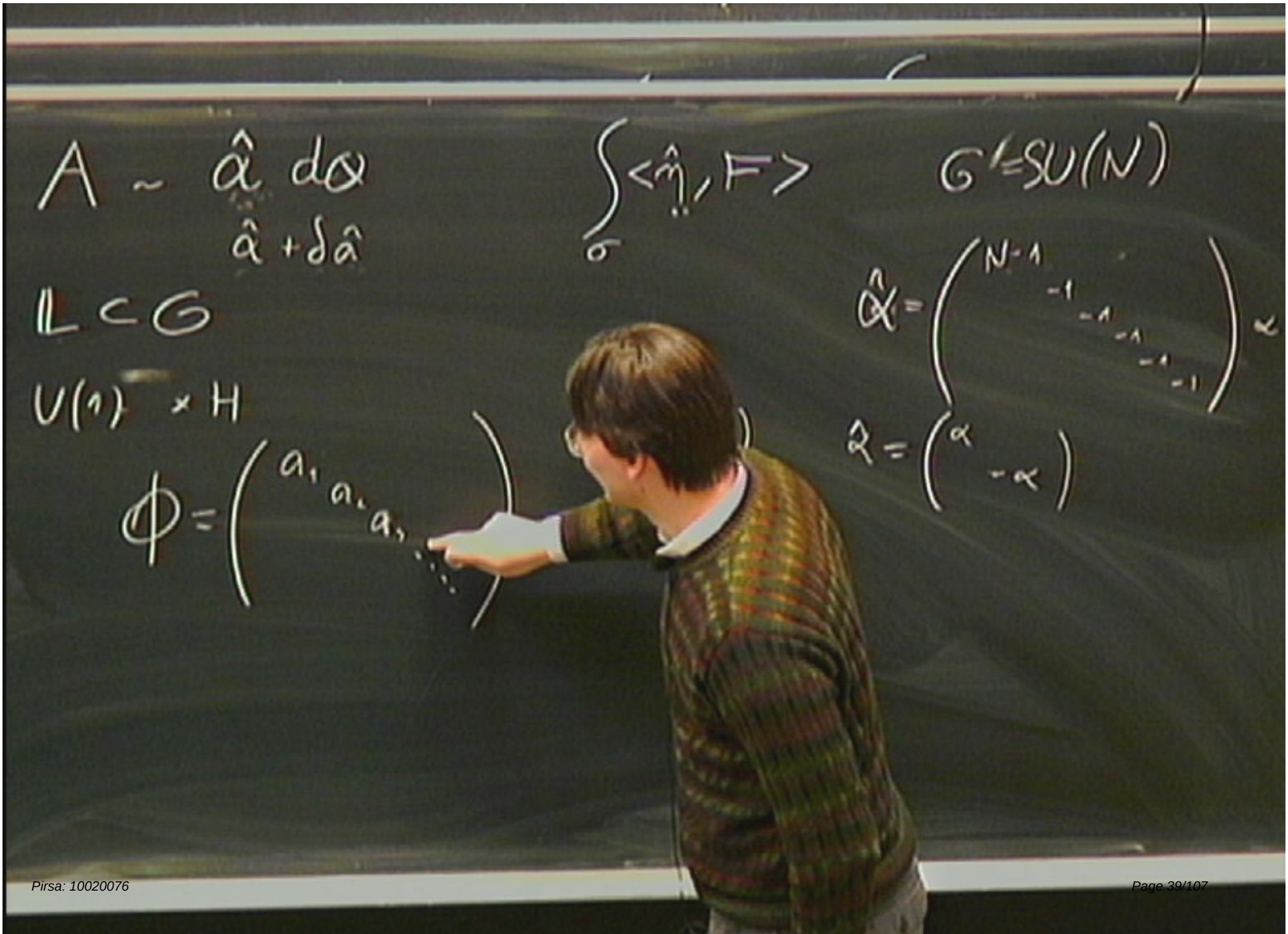
$$G' = SU(N)$$

$$\mathbb{L} \subset G$$

$$\hat{\alpha} = \begin{pmatrix} N-1 & & & & & \\ & -1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix} \alpha$$

$$U(1) \times H$$

$$G = SU(2) \qquad \hat{\alpha} = \begin{pmatrix} \alpha & \\ & -\alpha \end{pmatrix}$$


A ~ â dα
â + δâ
∫_σ ⟨η̂, F⟩
G' = SU(N)
L ⊂ G
U(1) × H
φ = (a₁ a₂ a₃ ...)
α̂ = (N-1 -1 -1 -1 -1 -1) α
R̂ = (α -α)

$$A \sim \hat{a}\, d\alpha$$

$$\hat{a} + \delta\hat{a}$$

$$\int_\sigma \langle \hat{\eta}, F \rangle$$

$$G' = SU(N)$$

$$\mathbb{L} \subset G$$

$$U(1) \times H$$

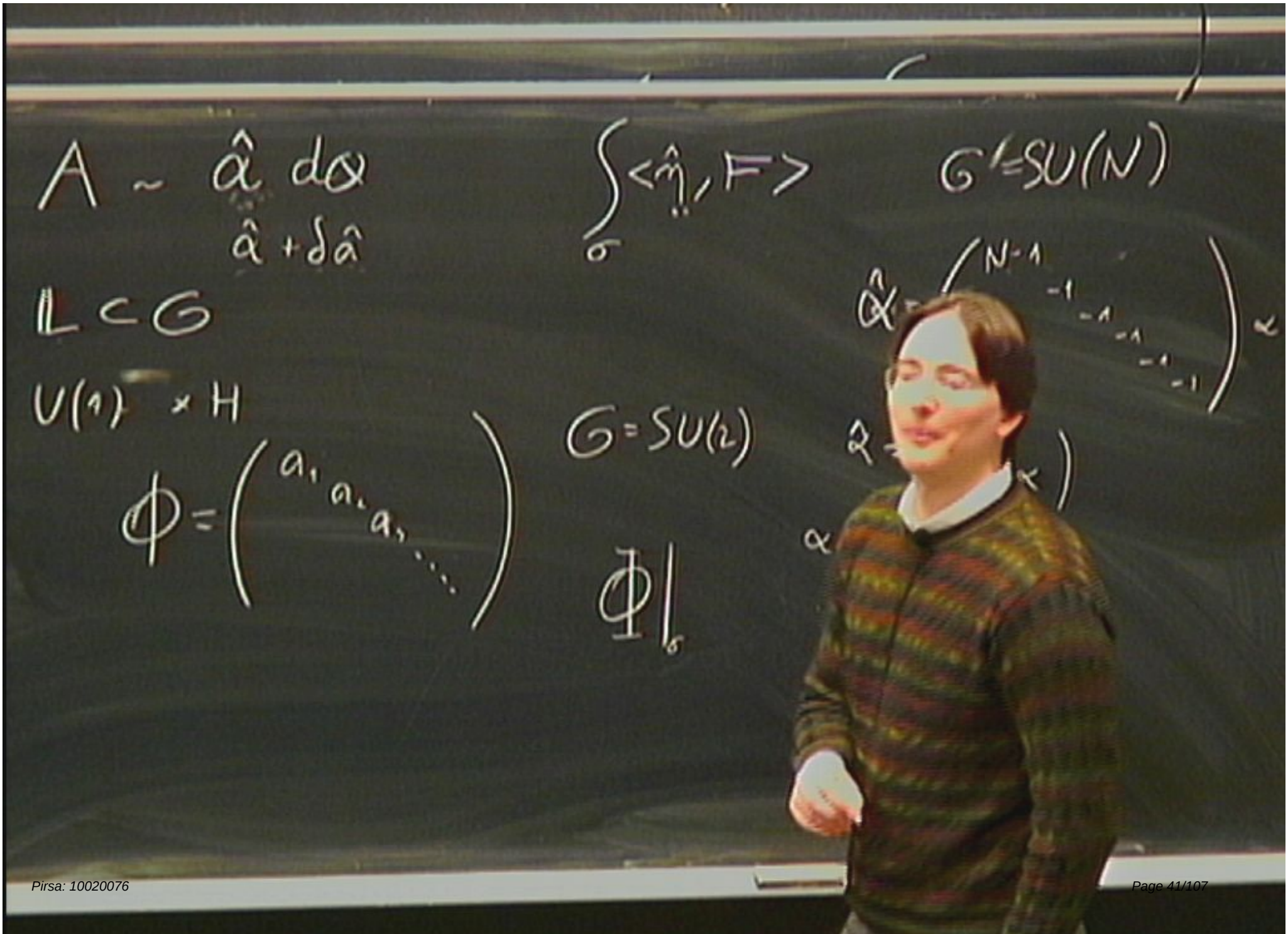
$$\hat{\alpha} = \begin{pmatrix} N-1 & & & & & & \\ & -1 & & & & & \\ & & -1 & & & & \\ & & & -1 & & & \\ & & & & -1 & & \\ & & & & & -1 & \\ & & & & & & -1 \end{pmatrix} \alpha$$

$$\phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & \ddots \end{pmatrix}$$

$$G = SU(2)$$

$$\hat{\alpha} = \begin{pmatrix} \alpha & \\ & -\alpha \end{pmatrix}$$

$$\alpha_{IR} = \pm \alpha_{UV}$$


A ~ â dα
â + δâ
∫_σ <η̂, F>
G' = SU(N)
L ⊂ G
U(1) × H
G = SU(2)
Φ = (a₁ a₂ a₃ ...)
Φ|_σ

$$A \sim \hat{a}\, d\alpha$$

$$\hat{a} + \delta\hat{a}$$

$$\int_\sigma \langle \hat{\eta}, F \rangle$$

$$G' = SU(N)$$

$$\mathbb{L} \subset G$$

$$U(1) \times H$$

$$\hat{\alpha} = \begin{pmatrix} N-1 & & & & & & \\ & -1 & & & & & \\ & & -1 & & & & \\ & & & -1 & & & \\ & & & & -1 & & \\ & & & & & -1 & \\ & & & & & & -1 \end{pmatrix} \alpha$$

$$G = SU(2) \qquad \hat{\alpha} = \begin{pmatrix} \alpha & \\ & -\alpha \end{pmatrix}$$

$$\Phi = \begin{pmatrix} a_1 & & & & \\ & a_2 & & & \\ & & a_3 & & \\ & & & \ddots & \\ & & & & \ddots \end{pmatrix} \approx \Phi\big|_\sigma$$

$$\alpha_{IR} = \pm \alpha_{UV}$$

$$\begin{pmatrix} a & \\ & -a \end{pmatrix}$$

$$A \sim \hat{a}\, d\alpha$$

$$\hat{a} + \delta\hat{a}$$

$$\int_\sigma \langle \hat{\eta}, F \rangle$$

$$G' = SU(N)$$

$$\mathbb{L} \subset G$$

$$\hat{\alpha} = \begin{pmatrix} N-1 & & & & & & \\ & -1 & & & & & \\ & & -1 & & & & \\ & & & -1 & & & \\ & & & & -1 & & \\ & & & & & -1 & \\ & & & & & & -1 \end{pmatrix} \alpha$$

$$U(1) \times H$$

$$G = SU(2)$$

$$\hat{\alpha} = \begin{pmatrix} \alpha & \\ & -\alpha \end{pmatrix}$$

$$\Phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & \ddots \end{pmatrix}$$

$$\alpha_{IR} = \pm \alpha_{UV}$$

$$\Phi\big|_\sigma \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$\begin{pmatrix} a & \\ & -a \end{pmatrix}$$

$$\hat{\alpha} = a|_\sigma$$

$$\int W_{eff}(a)\, d^2\theta \Rightarrow$$

$$\int \frac{\partial W}{\partial a^i}\left(F_I + * \tau_{Ij} F^J\right) + c.c.$$

$$\hat{x}^2 = \phi_2(t)$$

$$\lambda = \hat{x} > dt$$

$$\lambda^2 = \phi_2(t)\, dt^2$$

$$\int_\gamma \frac{\partial \lambda}{\partial a_I} =$$

$A \sim a\, dQ$

$\hat{a} + \delta\hat{a}$

$\mathbb{L} \subset G$

$U(1) \times H$

$\Phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & \ddots \end{pmatrix}$

$\hat{x} = a|_\sigma$

$t = \alpha_{UV} \cdot H$

$G = SU(2)$

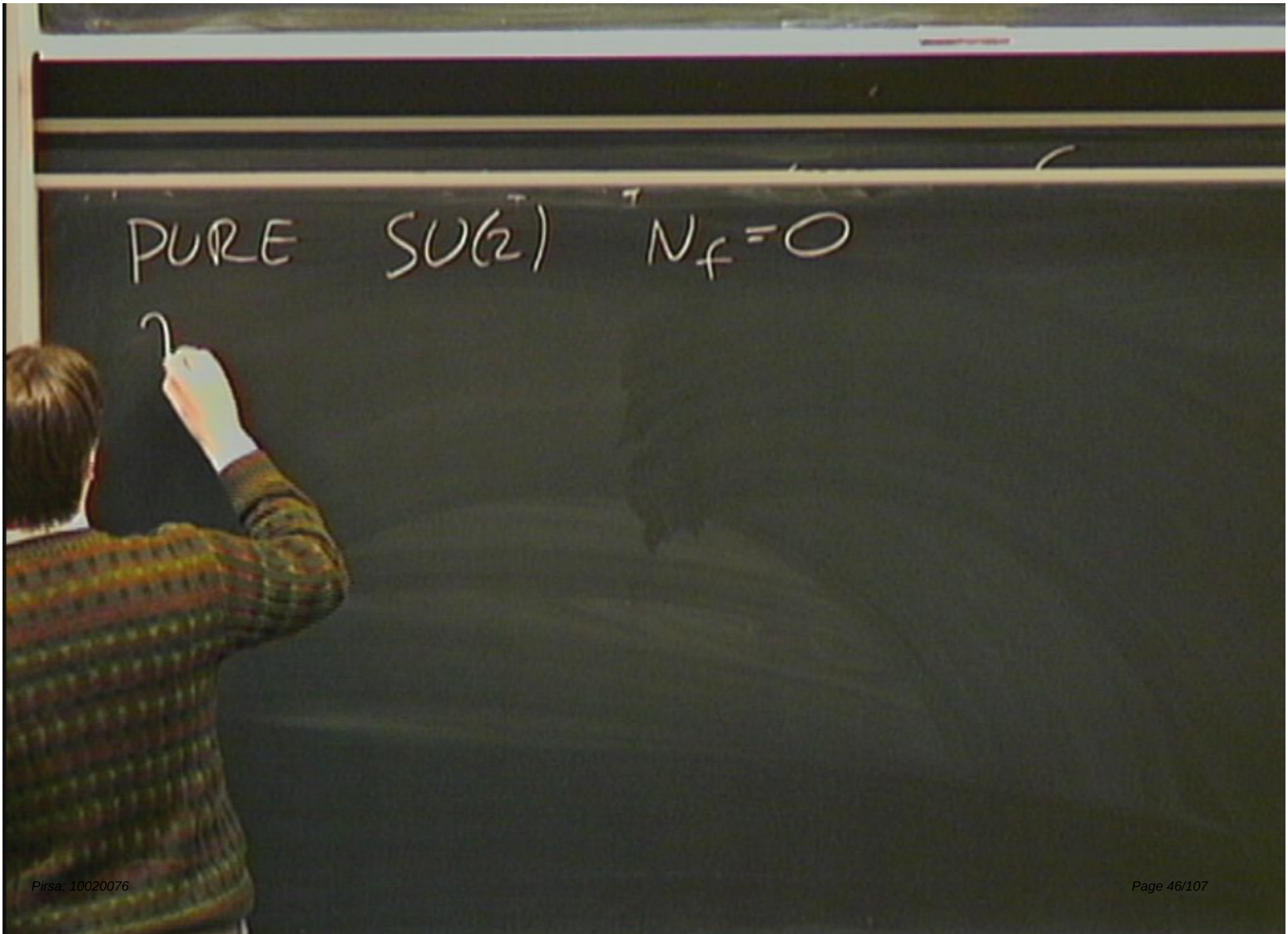
$\Phi|_\sigma \begin{pmatrix} & \\ & -1 \end{pmatrix}$

$\hat{\alpha} = \begin{pmatrix} N-1 & & & & & & \\ & -1 & & & & & \\ & & -1 & & & & \\ & & & -1 & & & \\ & & & & -1 & & \\ & & & & & -1 & \\ & & & & & & -1 \end{pmatrix}$

$\alpha = \begin{pmatrix} \alpha & \\ & -\alpha \end{pmatrix}$

$\alpha_{IR} = \pm \alpha_{UV}$

$\begin{pmatrix} a & \\ & -a \end{pmatrix}$


PURE SU(2) $N_f = 0$

PURE SU(2) $N_f = 0$

$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$$

PURE SU(2) $N_f = 0$

$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$$

$u \gg \Lambda^2$

$$\mathrm{Tr}\,\phi^2 = U = 2a^2$$

$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$$

$\mathrm{Tr}\,\phi^2$ $\quad 2a^2$

$u \gg \Lambda^2$

$$\lambda^2 = a^2 dt^2 = 2u\, dt^2$$

$$A \sim \hat{a}\, d\alpha$$

$$\hat{a} + \delta\hat{a}$$

$$\int_\sigma \langle \hat{\eta}, F \rangle$$

$$G' = SU(N)$$

$$\mathbb{L} \subset G$$

$$t = \hat{\alpha} T_{\mu\nu} + \hat{\eta}$$

$$\hat{\alpha} = \begin{pmatrix} N-1 & & & & & \\ & -1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & 1 \end{pmatrix} \alpha$$

$$U(1) \times H$$

$$G = SU(2)$$

$$\hat{\alpha} = \begin{pmatrix} \alpha & \\ & -\alpha \end{pmatrix}$$

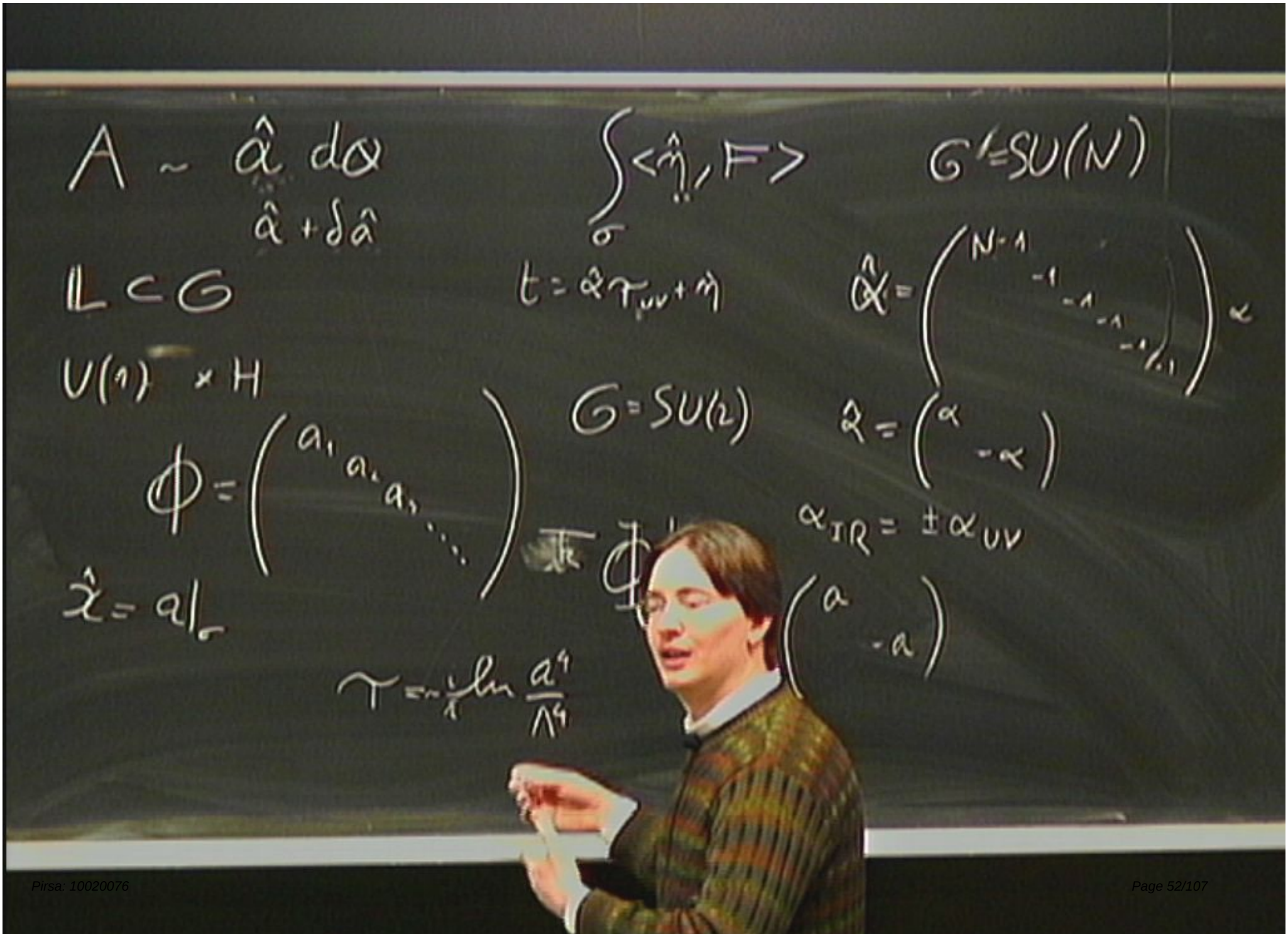
$$\Phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & \ddots \end{pmatrix}$$

$$\alpha_{IR} = \pm \alpha_{UV}$$

$$\Phi\big|_\sigma \begin{pmatrix} & \\ & -1 \end{pmatrix}$$

$$\begin{pmatrix} a & \\ & -a \end{pmatrix}$$

$$\hat{\alpha} = a|_\sigma$$


A ~ â dα
â + δâ
∫_σ <η̂, F>
G' = SU(N)
L ⊂ G
t = α̂ τ_UV + η
α̂ = (N-1, -1, -1, -1, -1) α
U(1) × H
G = SU(2)
α̂ = (α, -α)
Φ = (a_1, a_2, a_3, ...)
α_IR = ± α_UV
â = a|_σ
(a, -a)
τ = -1/4 ln a^4/Λ^4

PURE SU(2) $N_f = 0$

$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$$

$\mathrm{Tr}\,\phi^2 = u = 2a^2$

$u \gg \Lambda^2$

$$\lambda^2 = a^2 dt^2 = 2u\,dt^2$$

PURE SU(2) $N_f = 0$

$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$$

$\mathrm{Tr}\,\phi^2 = u = 2a^2$

$u \gg \Lambda^2$

$$\lambda^2 = a^2 dt^2 = 2u\,dt^2$$

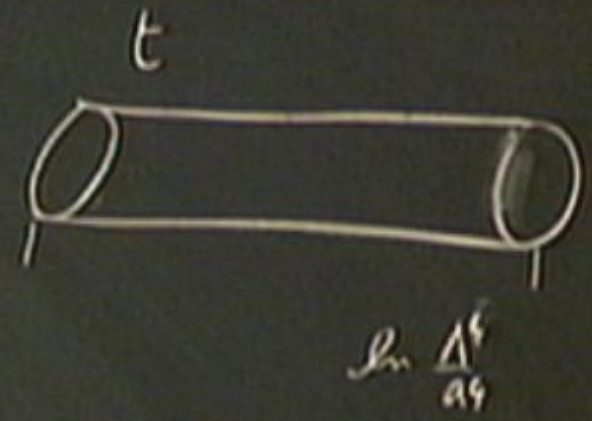
PURE $SU(2)$ $N_f = 0$

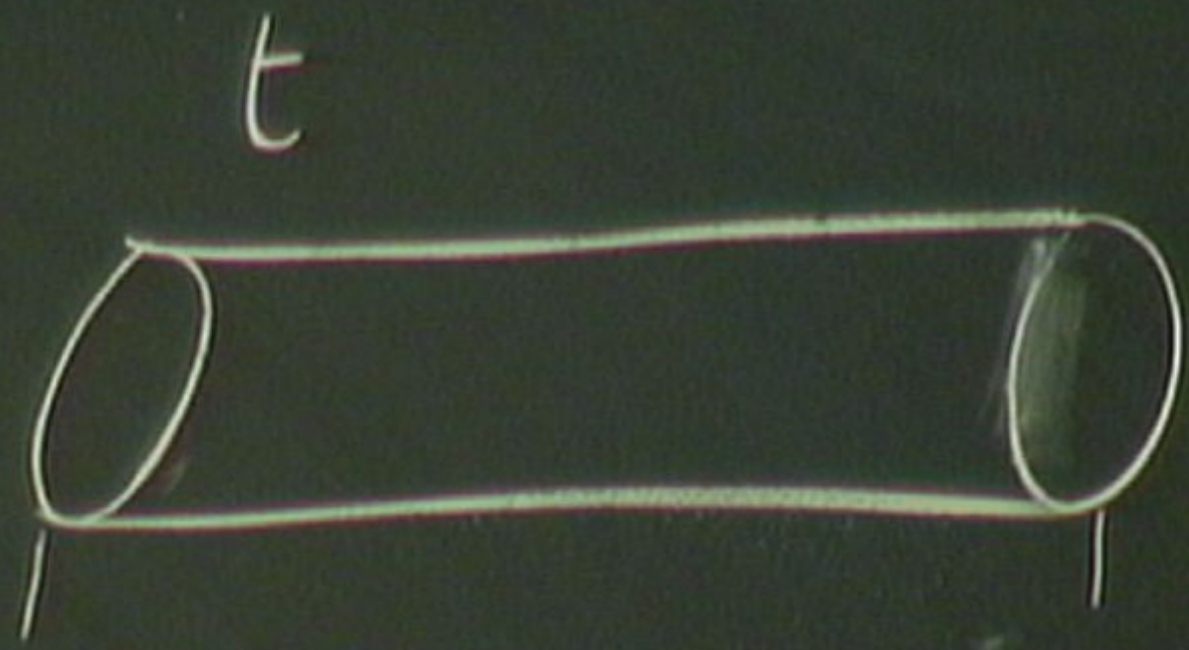
$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$$

$\mathrm{Tr}\,\phi^2 = u = 2a^2$

$u \gg \Lambda^2$

$$\lambda^2 = a^2 dt^2 = 2u\,dt^2$$

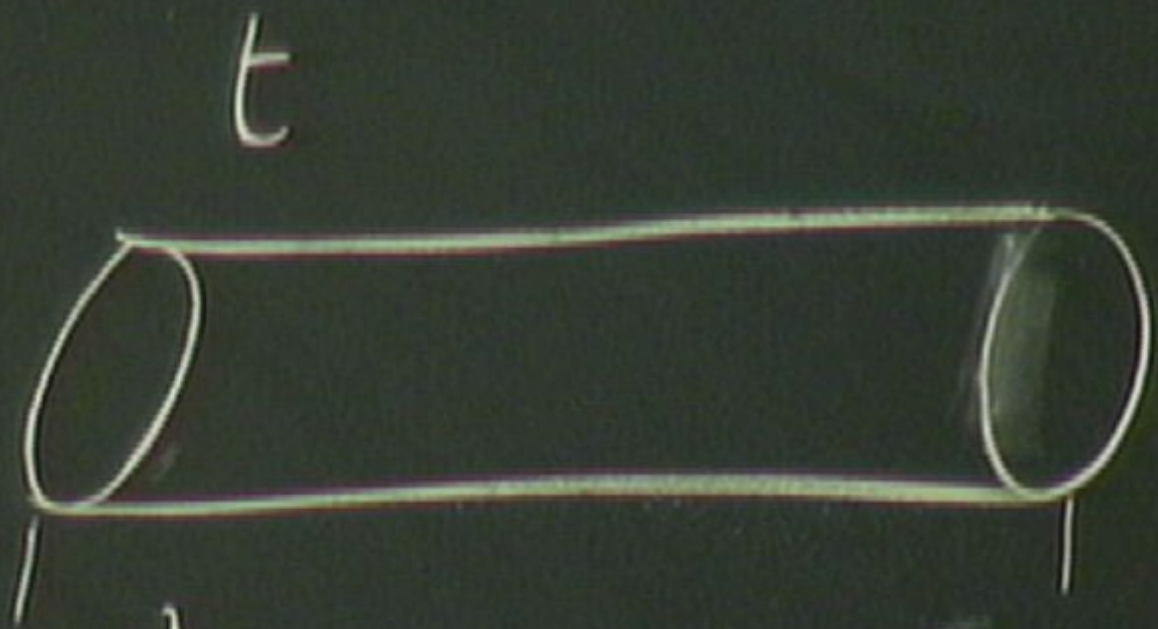




$$\ln \frac{\Lambda^4}{a_4}$$

$$\mathrm{Tr}\,\phi^2 = U = 2a^2$$

$$z = e^{2\pi i t}$$



$$-\ln\frac{\Lambda^2}{a^2} \qquad +\ln\frac{\Lambda^2}{a^2}$$

$\tilde{z} = a|_\sigma$ $\qquad \Phi|_\sigma \begin{pmatrix} a & \\ & -a \end{pmatrix}$

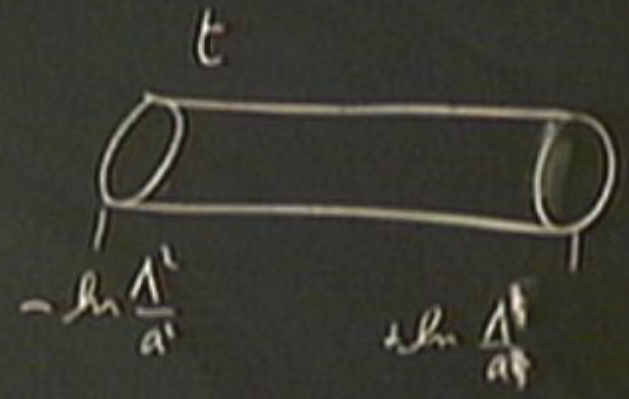
$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4}$

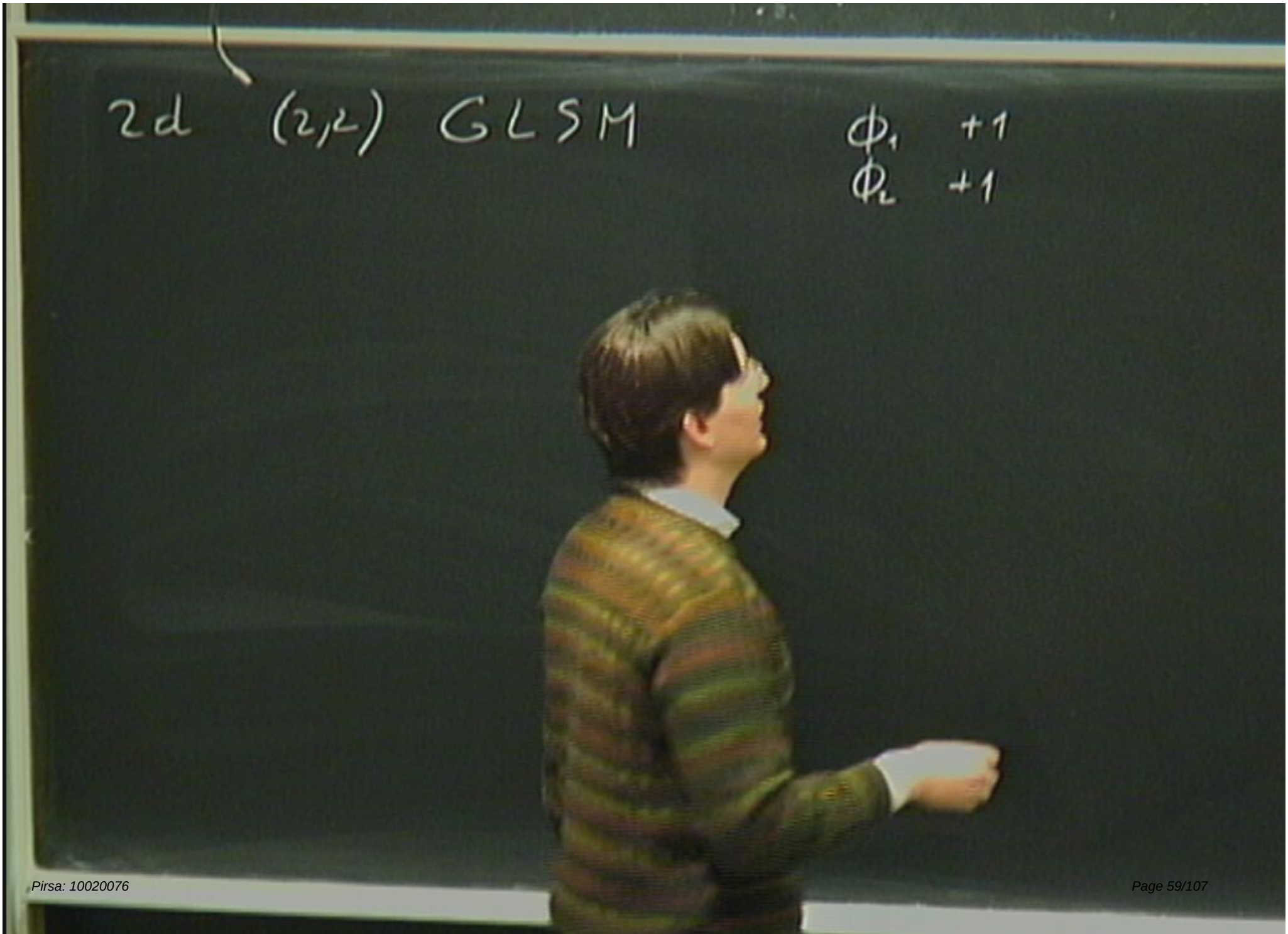
PURE $SU(2)$ $N_f = 0$

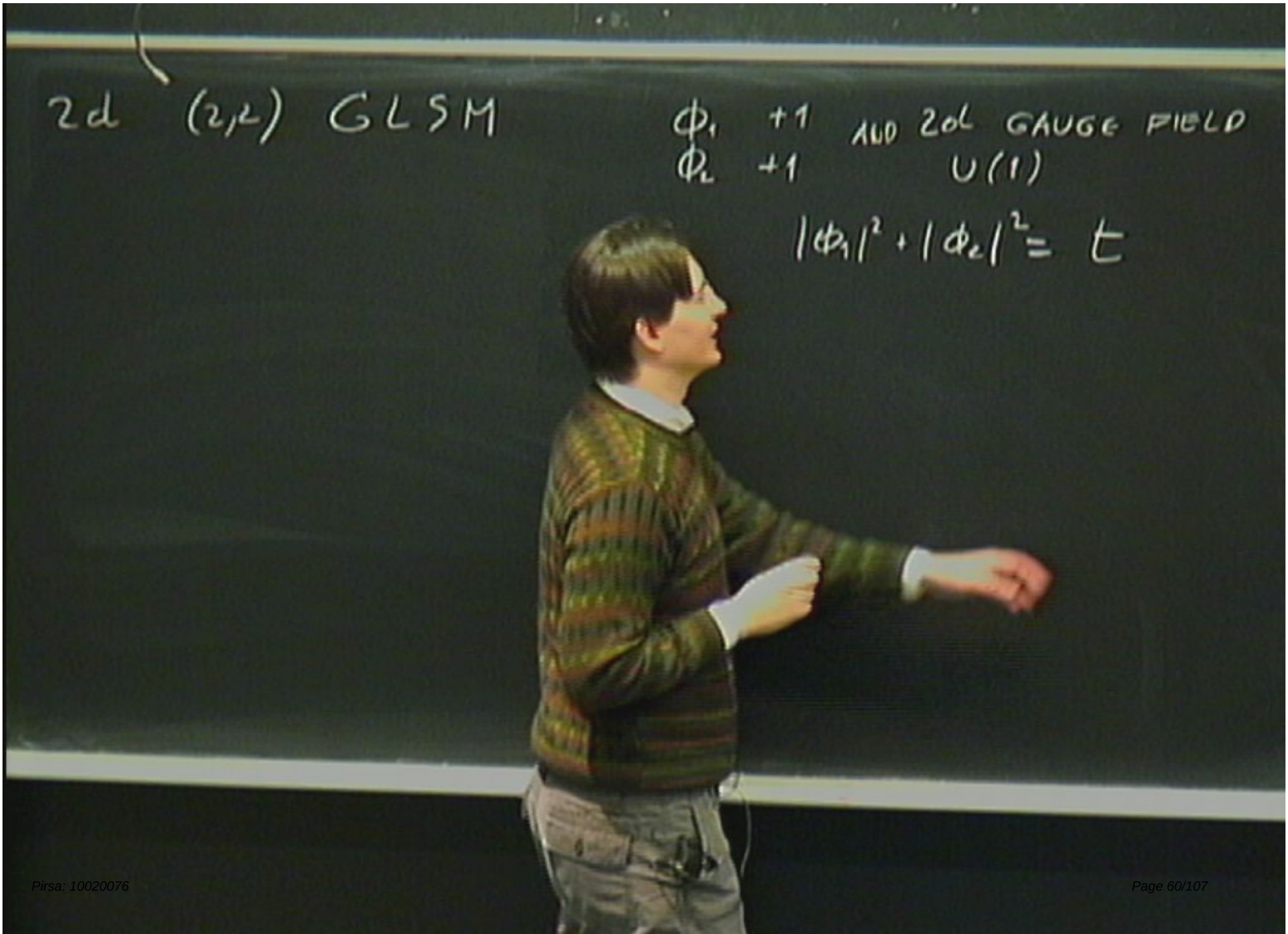
$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2 \right) \left(\frac{dz}{z} \right)^2 \qquad z \cdot e^{2\pi i t} \qquad \mathrm{Tr}\,\phi^2 = u = 2a^2$$

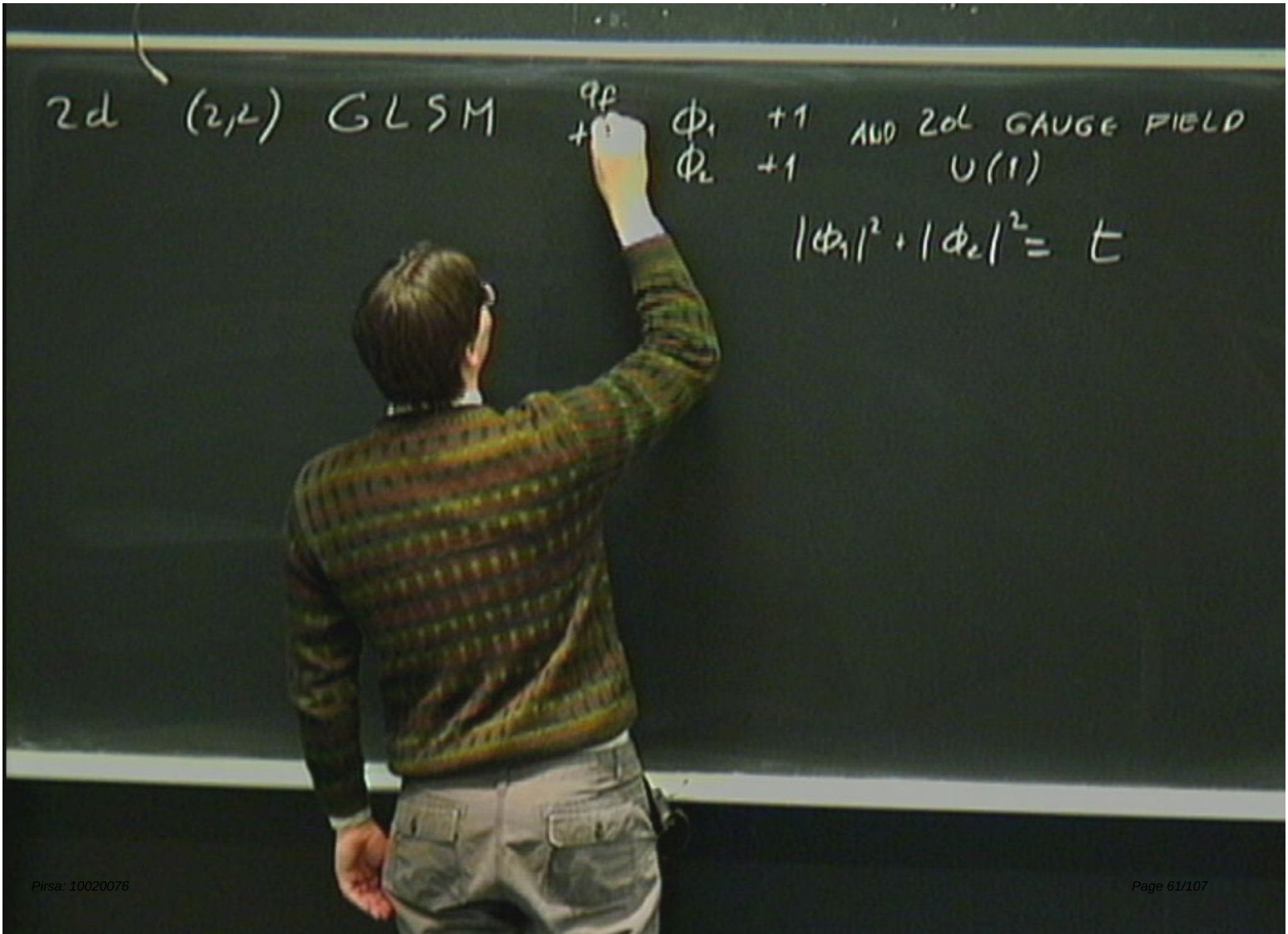
$u \gg \Lambda^2$

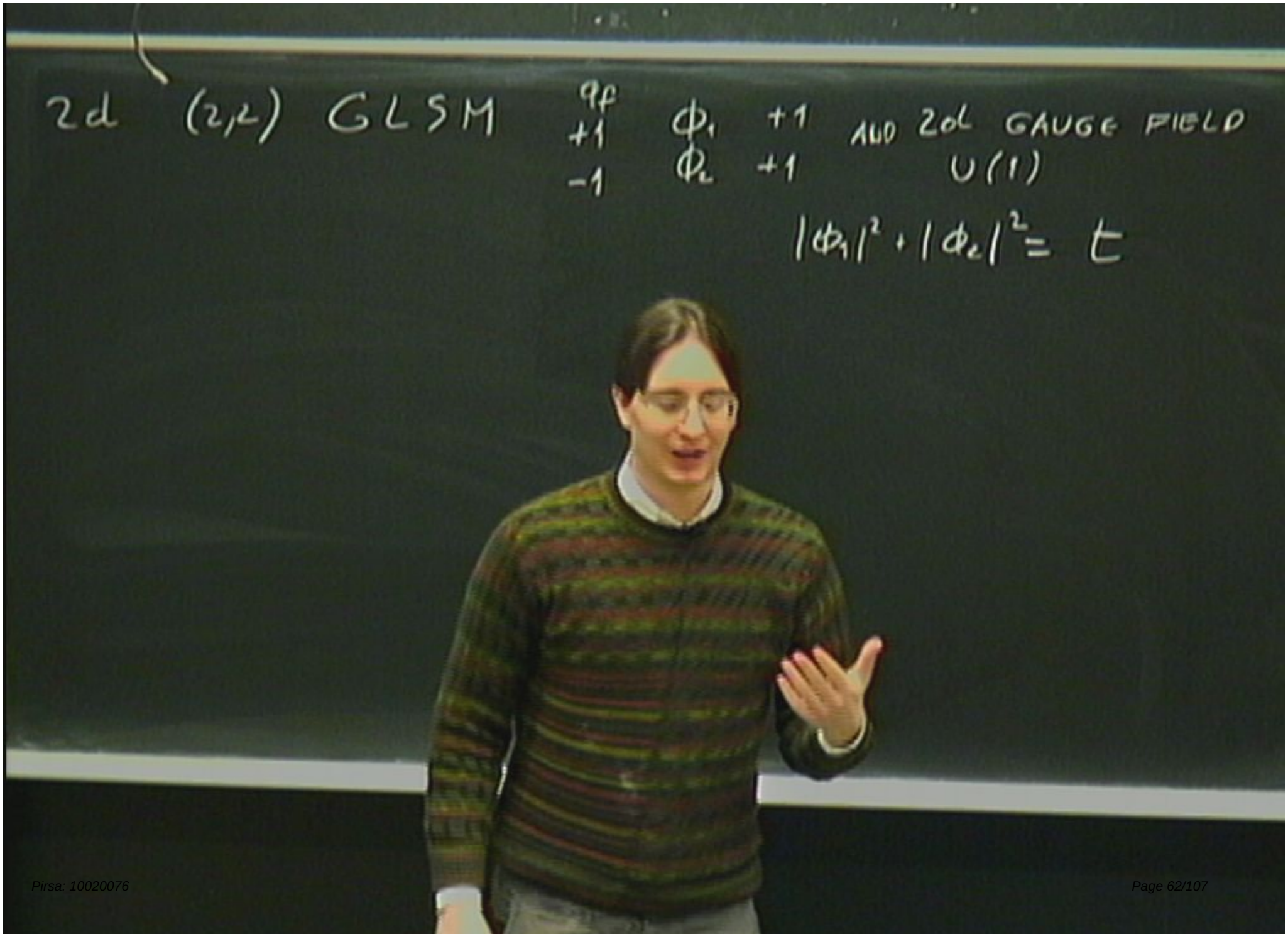
$$\lambda^2 = a^2 dt^2 = 2u\, dt^2$$




2d (2,2) GLSM
ϕ_1 +1
ϕ_2 +1


2d (2,2) GLSM
ϕ_1 +1
ϕ_2 +1
AND 2d GAUGE FIELD
U(1)
$|\phi_1|^2 + |\phi_2|^2 = t$


2d (2,2) GLSM
qf
+1 ϕ_1 +1
ϕ_2 +1
AND 2d GAUGE FIELD
U(1)
$|\phi_1|^2 + |\phi_2|^2 = t$


2d (2,2) GLSM
qf
+1 Φ_1 +1
-1 Φ_2 +1
AND 2d GAUGE FIELD
U(1)
$|\Phi_1|^2 + |\Phi_2|^2 = t$

2d (2,2) GLSM

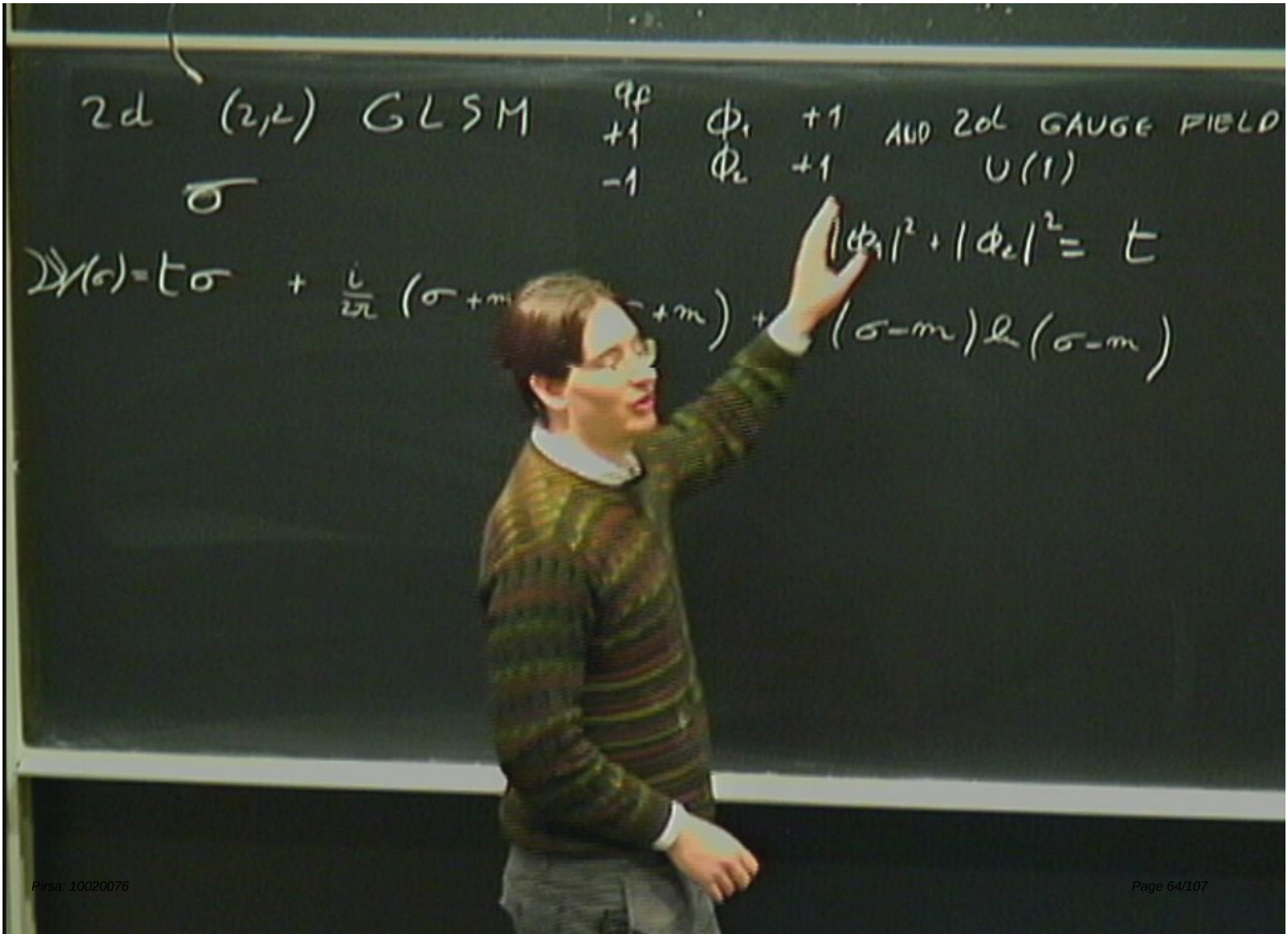
	qf		
Φ_1	+1	+1	
Φ_2	−1	+1	

AND 2d GAUGE FIELD $U(1)$

σ

$t\sigma$

$$|\Phi_1|^2 + |\Phi_2|^2 = t$$


2d (2,2) GLSM
σ
qf
+1 Φ1 +1
-1 Φ2 +1
AND 2d GAUGE FIELD
U(1)
|Φ1|² + |Φ2|² = t
+ i/2π (σ + m
+ m) +
(σ − m) ln (σ − m)

2d (2,2) GLSM

	qf			
σ	$+1$	Φ_1	$+1$	AND 2d GAUGE FIELD
	-1	Φ_2	$+1$	$U(1)$

$$|\Phi_1|^2 + |\Phi_2|^2 = t$$

$$\mathcal{W}(\sigma) = t\sigma + \frac{i}{2\pi}(\sigma+m)\ln(\sigma+m) + \frac{i}{2\pi}(\sigma-m)\ln(\sigma-m)$$

2d (2,2) GLSM

	qf		
Φ_1	+1	+1	AND 2d GAUGE FIELD
Φ_2	+1	−1	$U(1)$

σ

$$|\Phi_1|^2 + |\Phi_2|^2 = t$$

$$W(\sigma) = t\sigma + \frac{i}{2\pi}(\sigma+m)\ln(\sigma+m) + \frac{i}{2\pi}(\sigma-m)\ln(\sigma-m)$$

$$\frac{\partial W}{\partial \sigma} = 0 \qquad 2\pi i t = \ln \sigma+m + \ln(\sigma-m)$$

$$\sigma^2 - m^2 = e^{2\pi i t}$$

2d (2,2) GLSM

	qf		
Φ_1	+1	+1	AND 2d GAUGE FIELD
Φ_2	+1	−1	$U(1)$

σ

$$|\Phi_1|^2 + |\Phi_2|^2 = t$$

$$W(\sigma) = t\sigma + \frac{i}{2\pi}(\sigma+m)\ln(\sigma+m) + \frac{i}{2\pi}(\sigma-m)\ln(\sigma-m)$$

$$\frac{\partial W}{\partial \sigma} = 0 \qquad 2\pi i t = \ln \sigma+m + \ln(\sigma-m)$$

$$\sigma^2 - m^2 = e^{2\pi i t}$$

2d (2,2) GLSM

	qf		
Φ_1	$+1$	$+1$	AND 2d GAUGE FIELD
Φ_2	$+1$	-1	$U(1)$

σ

$$|\Phi_1|^2 + |\Phi_2|^2 = t$$

$$\mathcal{W}(\sigma) = t\sigma + \frac{i}{2\pi}(\sigma+m)\ln(\sigma+m) + \frac{i}{2\pi}(\sigma-m)\ln(\sigma-m)$$

$$\frac{\partial \mathcal{W}}{\partial \sigma} = 0 \qquad 2\pi i t = \ln \sigma+m + \ln(\sigma-m)$$

$$\sigma^2 - m^2 = e^{2\pi i t}$$

$$\sigma = \hat{x} \qquad m = a$$

$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$$

$z = e$

$u \gg \Lambda^2$

$$\lambda^2 = a^2 dt^2 = 2u\, dt^2$$

$$\lambda^2 = \left(e^{2\pi i t} + 2u\right)$$

$\hat{x} = a|_\sigma$

$\Phi|_\sigma$

$\begin{pmatrix} a & \\ & -a \end{pmatrix}$

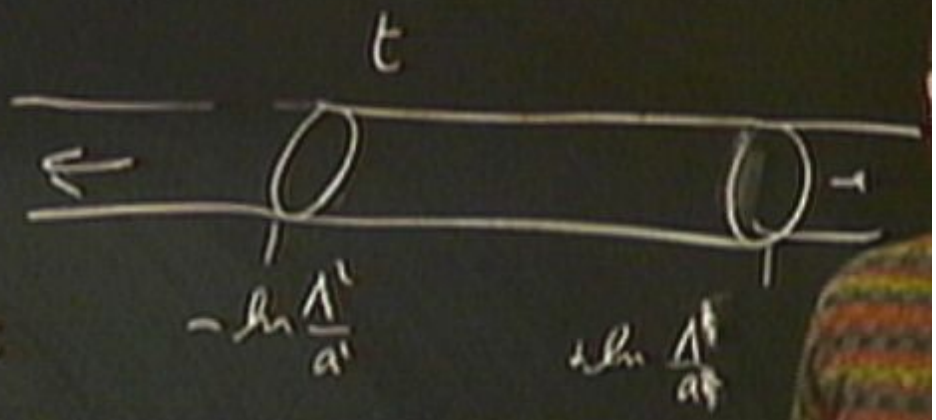
$\tau = -\frac{1}{4}\ln\frac{a^4}{\Lambda^4}$

$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$ $\quad z = e^{2\pi i t}$ $\quad \mathrm{Tr}\,\phi^2 = u = 2a^2$

$u \gg \Lambda^2$

$\lambda^2 = a^2 dt^2 = 2u\,dt^2$

$\lambda^2 = \left(e^{2\pi i t} + 2u\right) + e^{-2\pi i t}$

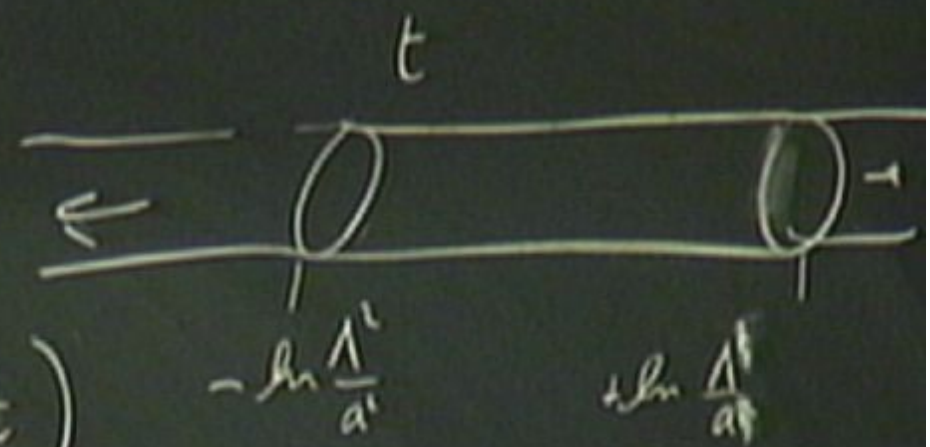


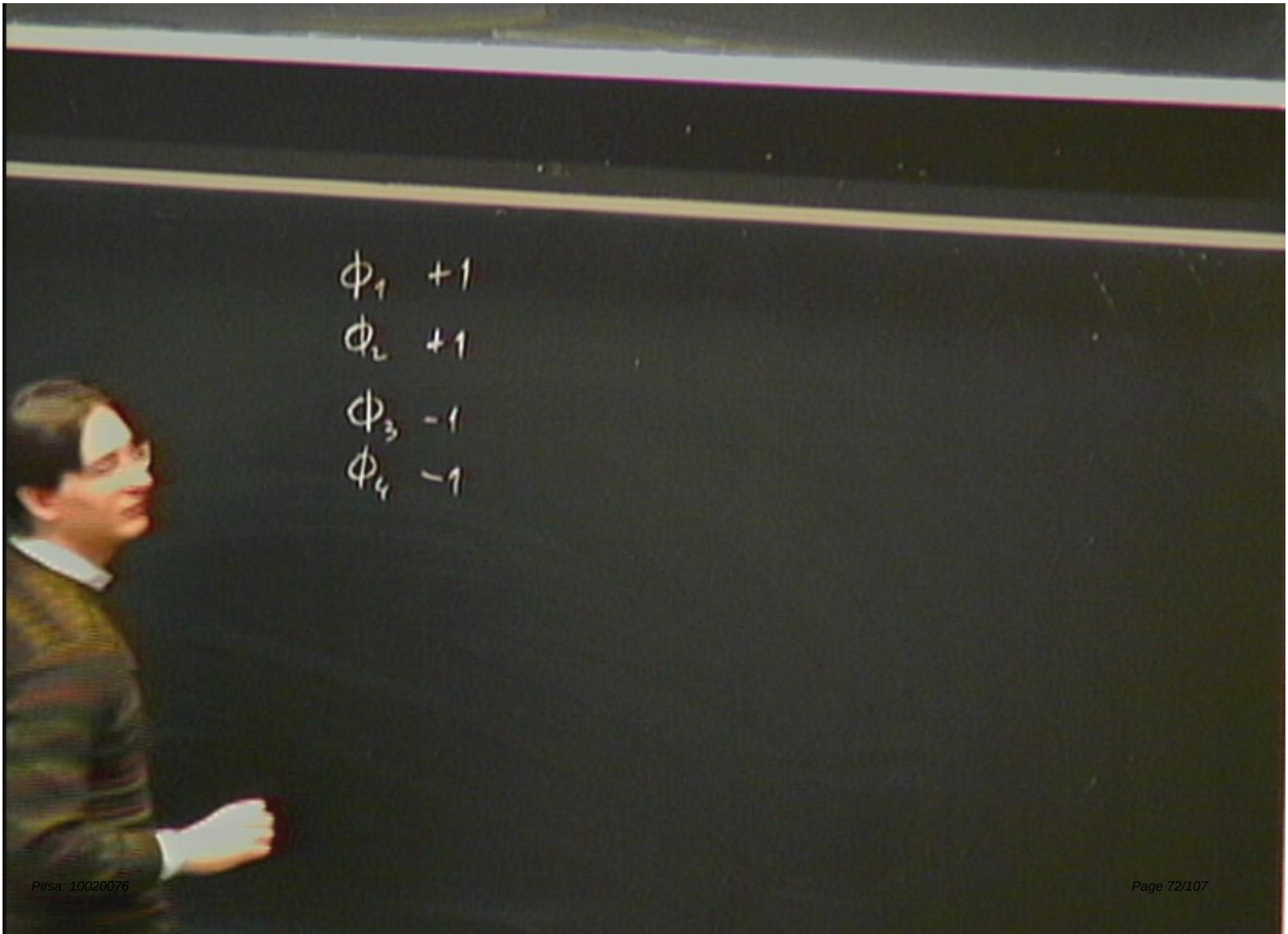
$$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2 \qquad z = e^{2\pi i t} \qquad \mathrm{Tr}\,\phi^2 = u = 2a^2$$

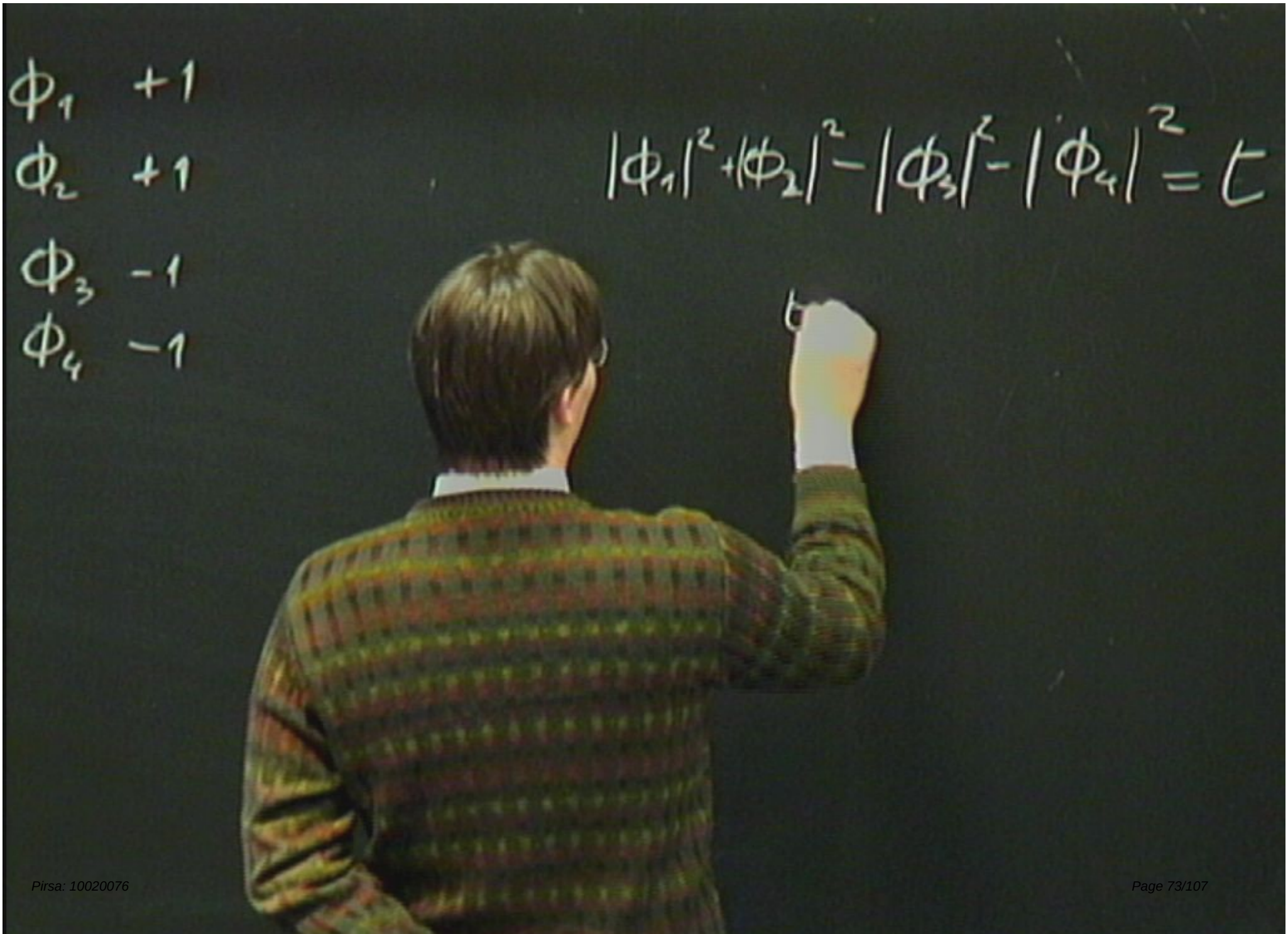
$u \gg \Lambda^2$

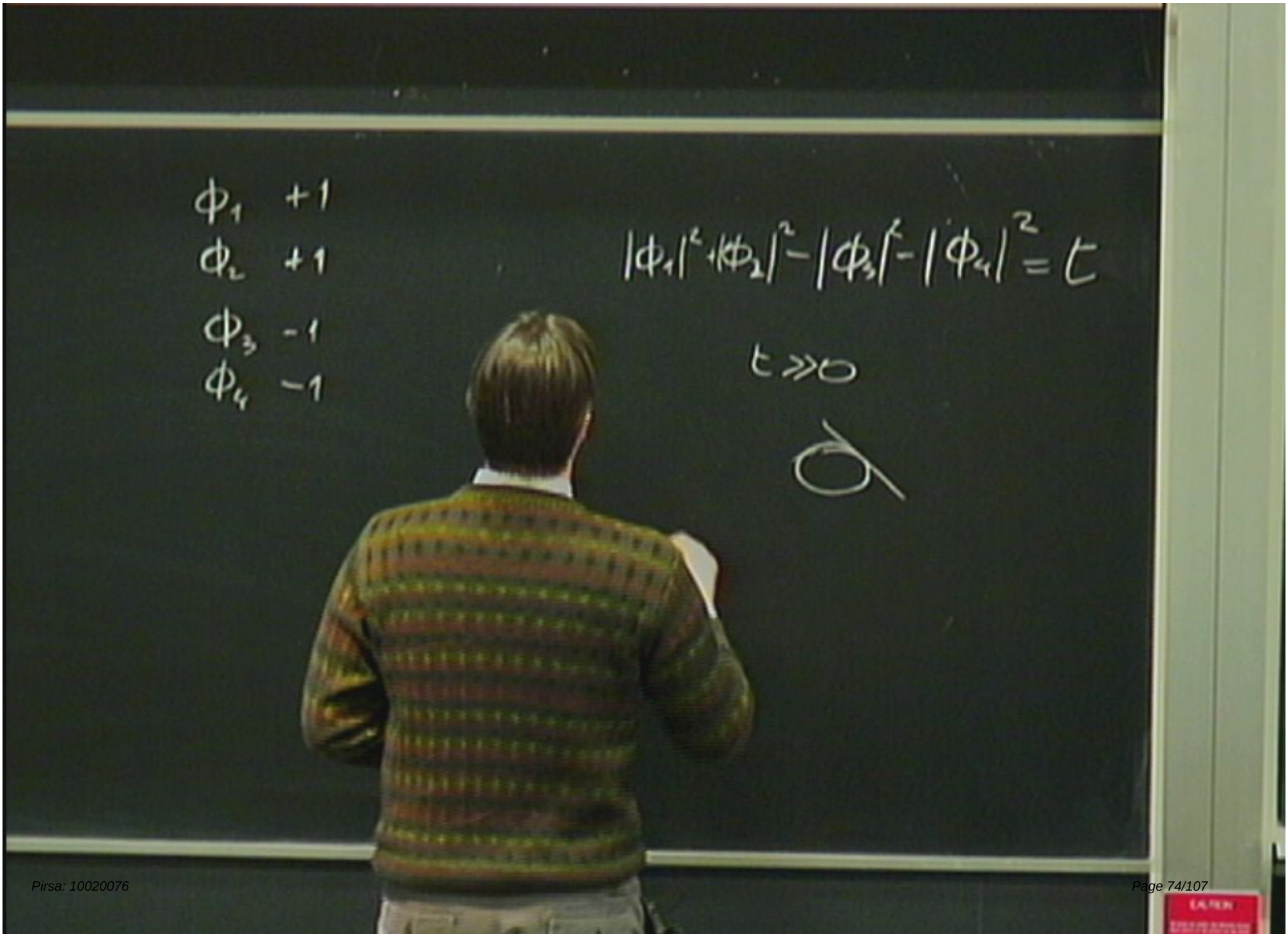
$$\lambda^2 = a^2 dt^2 = 2u\, dt^2$$

$$\lambda^2 = \left(e^{2\pi i t} + 2u + e^{-2\pi i t}\right)$$




ϕ_1 +1
ϕ_2 +1
ϕ_3 -1
ϕ_4 -1


Φ_1 +1
Φ_2 +1
Φ_3 −1
Φ_4 −1
$|\Phi_1|^2+|\Phi_2|^2-|\Phi_3|^2-|\Phi_4|^2=t$
t


ϕ_1 +1
ϕ_2 +1
ϕ_3 -1
ϕ_4 -1
$|\phi_1|^2+|\phi_2|^2-|\phi_3|^2-|\phi_4|^2=t$
$t \gg 0$

$\Phi_1 \quad +1$

$\Phi_2 \quad +1$

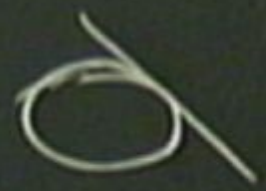
$\Phi_3 \quad -1$

$\Phi_4 \quad -1$

$$|\Phi_1|^2 + |\Phi_2|^2 - |\Phi_3|^2 - |\Phi_4|^2 = t$$

$t \ll 0$

$t \gg 0$



$\phi_1 \quad +1$

$\phi_2 \quad +1$

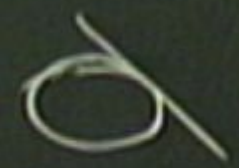
$\phi_3 \quad -1$

$\phi_4 \quad -1$

$$|\phi_1|^2 + |\phi_2|^2 - |\phi_3|^2 - |\phi_4|^2 = t$$

$t \ll 0$

$t \gg 0$



$$\frac{(\sigma - m_1)(\sigma - m_2)}{(\sigma - m_3)(\sigma - m_4)} = e^{2\pi i t} = z$$

$\phi_1 \quad +1$

$\phi_2 \quad +1$

$\phi_3 \quad -1$

$\phi_4 \quad -1$

$$|\phi_1|^2 + |\phi_2|^2 - |\phi_3|^2 - |\phi_4|^2 = t$$

$t \ll 0$ $\qquad$ $t \gg 0$

$$\frac{(\sigma - m_1)(\sigma - m_2)}{(\sigma - m_3)(\sigma - m_4)} = e^{\pi i t} = z$$

$$(\sigma - m_1)(\sigma - m_2) = z(\sigma - m_3)(\sigma - m_4)$$

$$(z-1)\sigma$$

$\Phi_3 \quad -1$

$\quad -1$

$t \ll 0 \qquad t \gg 0$

$$\frac{(\sigma - m_1)(\sigma - m_2)}{(\sigma - m_3)(\sigma - m_4)} = e^{\omega i t} = z$$

$$= z(\sigma - m_3)(\sigma - m_4)$$

$$1)\ \sigma^2 + (c_1 z + c_2)\sigma + (c_3 z + c_4) = 0$$

$$\begin{cases} \sigma^2 - m^2 = e^{2\pi i t} \\ \sigma = \hat{x} \\ m = a \end{cases}$$

$$\sigma = \hat{x} - \frac{1}{2}\frac{c_1 z + c_2}{z-1} \qquad x^2 = \frac{P_2(z)}{(z-1)^2} \qquad \lambda^2 = \frac{P_2(z)}{z^2(z-1)^2}\, dz^2$$

$$\frac{(\sigma - m_1)(\sigma - m_2)}{(\sigma - m_3)(\sigma - m_4)} = e^{2\pi i t} = z$$

$$(\sigma - m_1)(\sigma - m_2) = z(\sigma - m_3)(\sigma - m_4)$$

$$(z-1)\sigma^2 + (c_1 z + c_2)\sigma + (c_3 z + c_4) = 0$$

$$\sigma^2 - m^2 = e^{2\pi i t}$$

$$\sigma = \hat{x}$$

$$m = a$$

$$\sigma = \hat{x} - \frac{1}{2}\frac{c_1 z + c_2}{z-1}$$

$$x^2 = \frac{P_2(z)}{(z-1)^2} \qquad \lambda^2 = \frac{P_2(z)}{z^2(z-1)^2}\,dz^2$$

$$\frac{(\sigma - m_1)(\sigma - m_2)}{(\sigma - m_3)(\sigma - m_4)} = e^{2\pi i t} = z$$

$$(\sigma - m_1)(\sigma - m_2) = z(\sigma - m_3)(\sigma - m_4)$$

$$(z-1)\sigma^2 + (c_1 z + c_2)\sigma + (c_3 z + c_4) = 0$$

$\hat{x} = a|_\sigma$

$\Phi|_\sigma$

$\alpha_{IR} = \pm \alpha_{UV}$

$\begin{pmatrix} a & \\ & -a \end{pmatrix}$

$\tau = -\frac{1}{4}\ln\frac{a^4}{\Lambda^4}$

$\lambda^2 = \left(\frac{\Lambda^2}{z} + 2u + z\Lambda^2\right)\left(\frac{dz}{z}\right)^2$

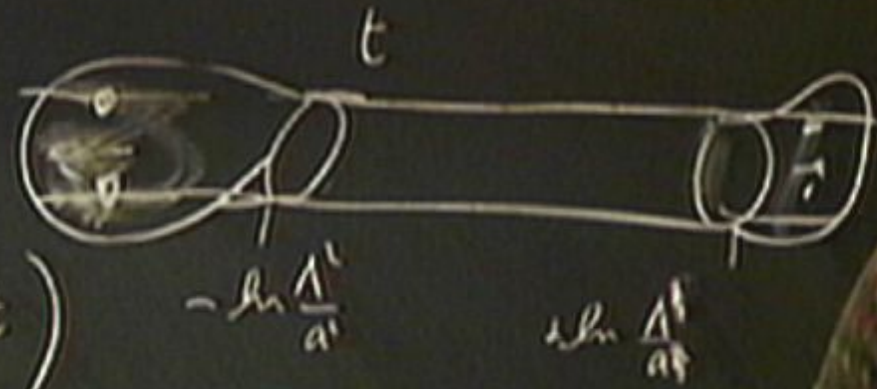
$z = e^{2\pi i t}$

$\mathrm{Tr}\,\phi^2 = u = 2a^2$

$u \gg \Lambda^2$

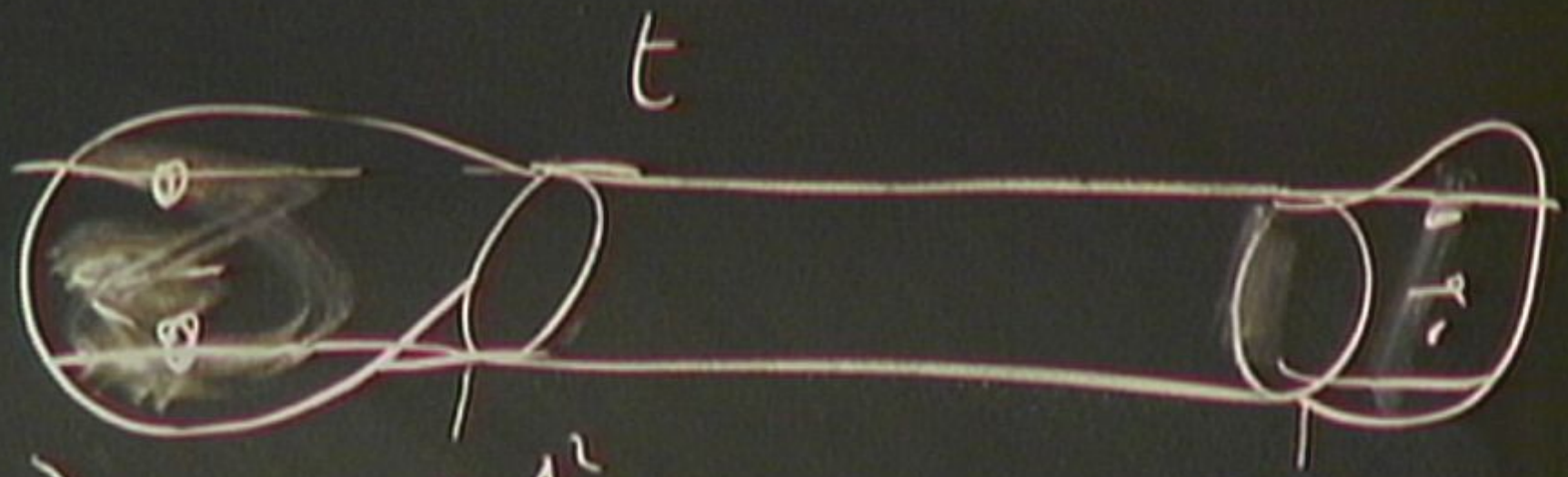
$\lambda^2 = a^2 dt^2 = 2u\,dt^2$

$\lambda^2 = \left(e^{2\pi i t} + 2u + e^{-2\pi i t}\right)$



$$z = e^{2\pi i t}$$

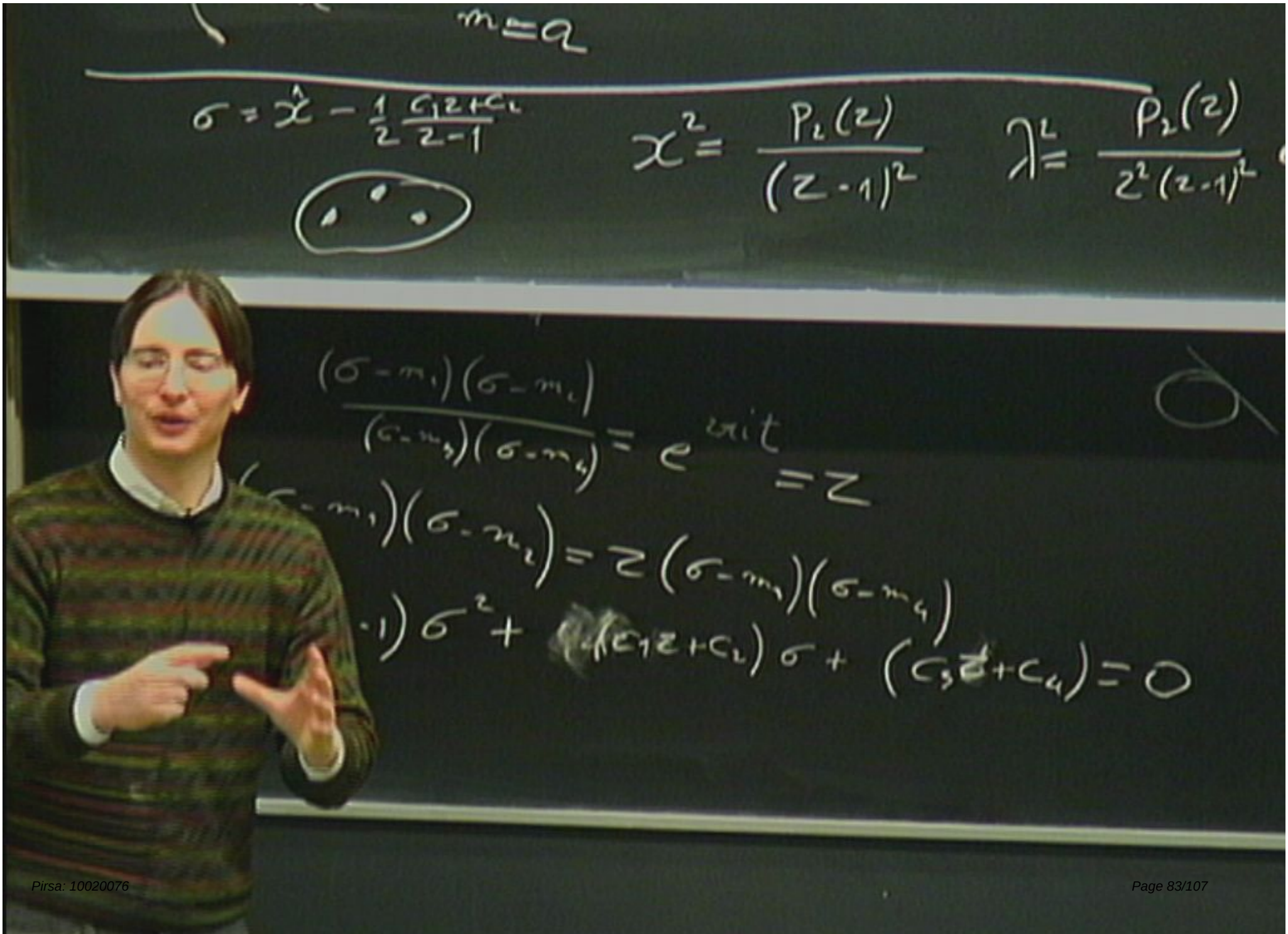
$$\mathrm{Tr}\,\phi^2 = v = 2a^2$$



$$\pi i t)$$

$$-\ln\frac{\Lambda^2}{a^2}$$

$$+\ln\frac{\Lambda^4}{a^4}$$


m=a
σ = x̂ − 1/2 (c1 z + c2)/(z − 1)
x² = P2(z)/(z − 1)²
λ² = P2(z)/z²(z − 1)²
(σ − m1)(σ − m2)/(σ − m3)(σ − m4) = e^{2πit} = z
(σ − m1)(σ − m2) = z(σ − m3)(σ − m4)
(z − 1) σ² + (c1 z + c2) σ + (c3 z + c4) = 0

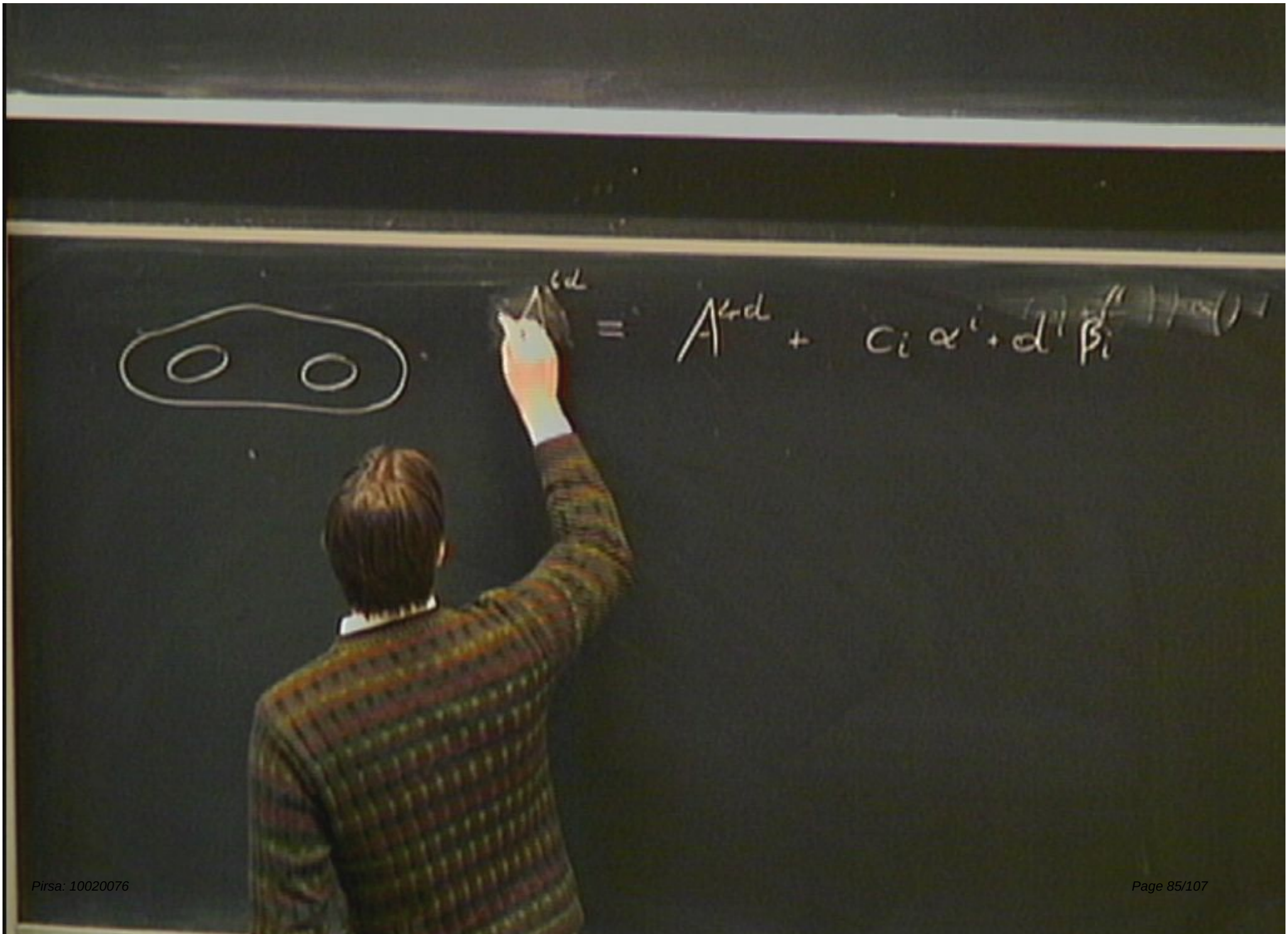
$$\sigma = x - \frac{1}{2}\frac{c_1 z + c_2}{z-1}$$

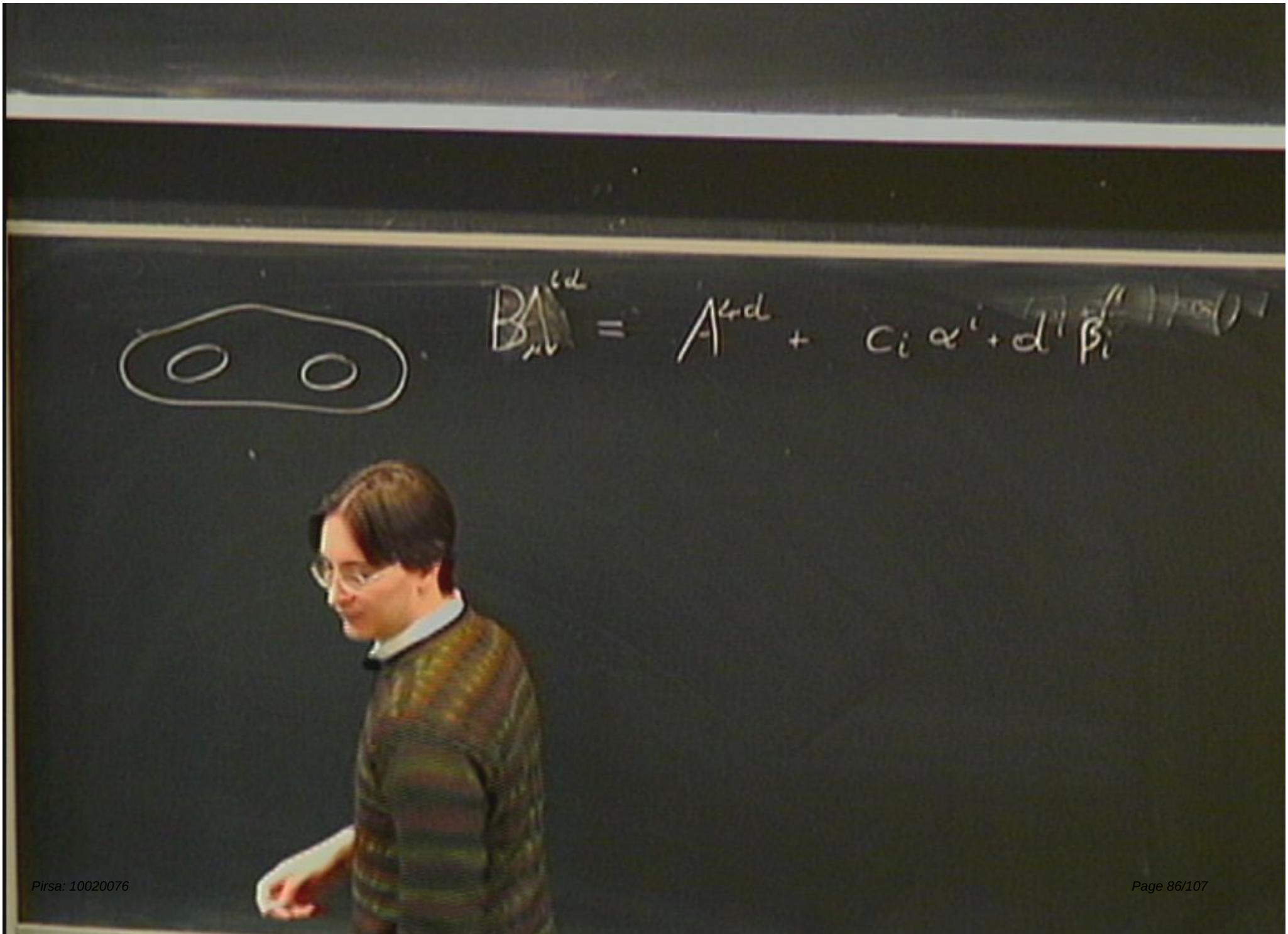
$$x^2 = \frac{P_2(z)}{(z-1)^2} \qquad \lambda^2 = \frac{P_2(z)}{z^2(z-1)^2}$$

$$\frac{(\sigma - m_1)(\sigma - m_2)}{(\sigma - m_3)(\sigma - m_4)} = e^{2\pi i t} = z$$

$$(\sigma - m_1)(\sigma - m_2) = z(\sigma - m_3)(\sigma - m_4)$$

$$(z-1)\sigma^2 + (c_1 z + c_2)\sigma + (c_3 z + c_4) = 0$$


$A^{c+d} = A^{c+d} + c_i \alpha^i + d^i \beta_i$


$B_{\mu\nu}^{cd} = A^{cd} + c_i \alpha^i + d^i \beta_i$

$$B_{\mu\nu}^{4d} = B_{\mu\nu}^{4-d} + A_i \alpha^i + A^i \beta_i$$

$$B_{\mu\nu}^{4d} = B_{\mu\nu}^{4+d} + A_i \alpha^i + A^i \beta_i$$

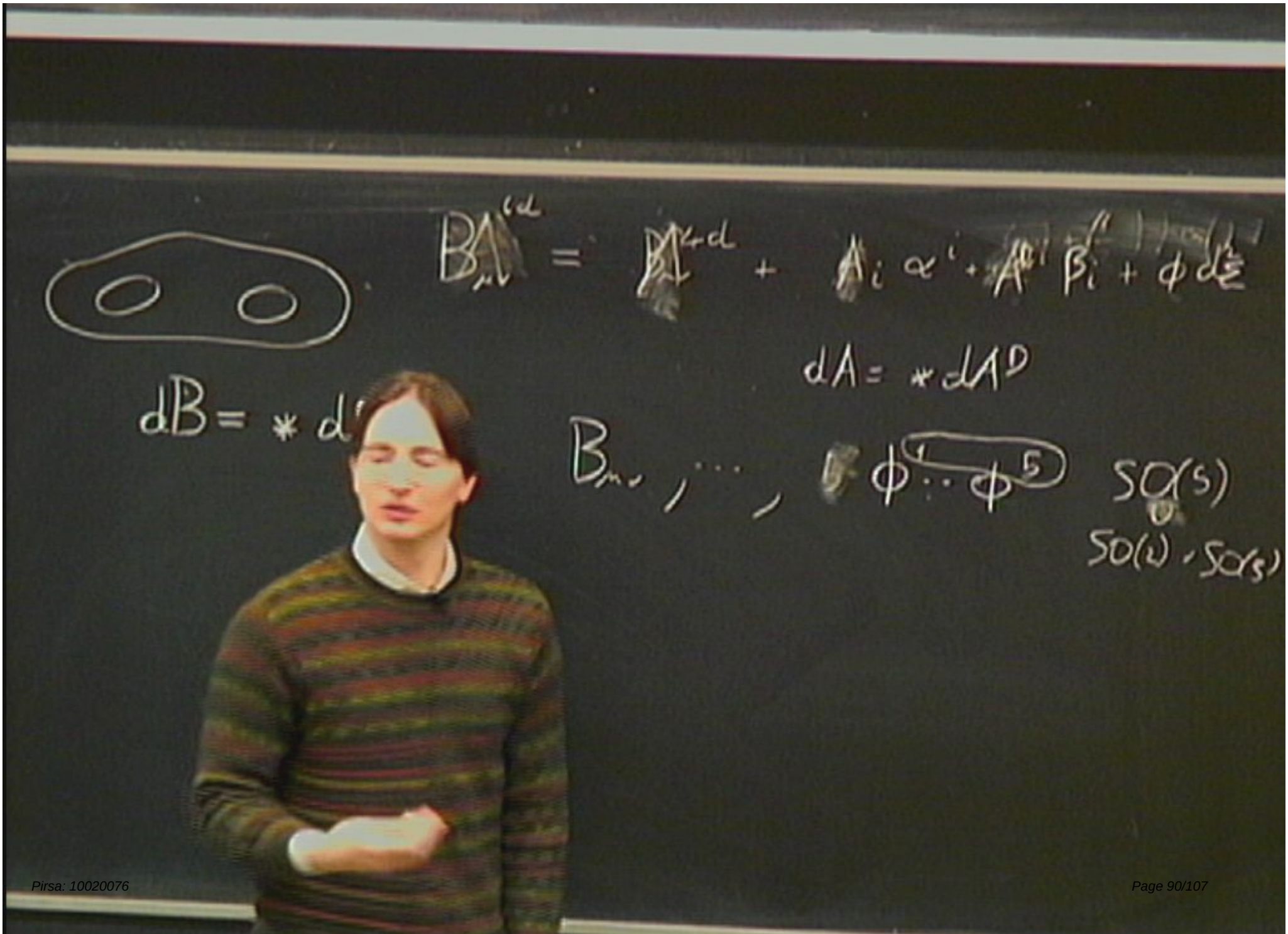
$$dB = * dB$$

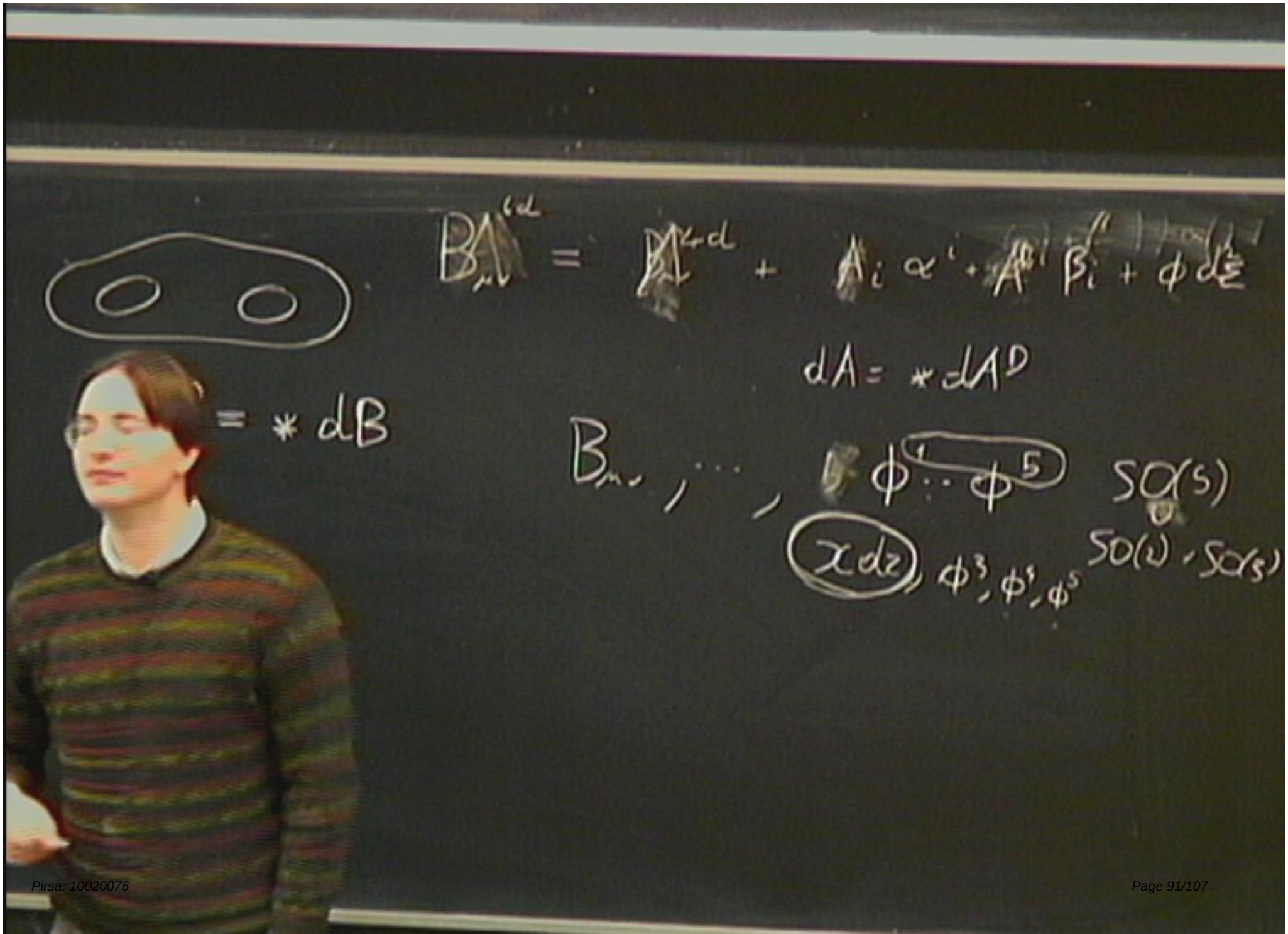
$$B_{\mu\nu}^{6d} = B_{\mu\nu}^{4d} + A_i \alpha^i + A^D_i \beta^i + \phi\, d\xi$$

$$dA = * dA^D$$

$$dB = * dB$$

$$B_{\mu\nu}, \ldots, \phi^1 \ldots \phi^5$$


dA = * dA^D
SO(5)
SO(4) × SO(5)


$B_{\mu\nu}^{6d} = B_{\mu\nu}^{4d} + A_i \alpha^i + A^i \beta_i + \phi\, d\xi$
$dA = * dA^D$
$= * dB$
$B_{\mu\nu}, \ldots, \phi^1 \ldots \phi^5$
$SO(5)$
$SO(2) \cdot SO(5)$
$x\, dz, \phi^3, \phi^4, \phi^5$

T_{IJ}

$$B_{\mu\nu}^{6d} = B_{\mu\nu}^{4d} + A_i \alpha^i + A^i \beta_i + \phi\, d\Sigma$$

$$dA = * dA^D$$

$$dB = * dB$$

$$B_{\mu\nu}, \ldots, \phi^1 \ldots \phi^5 \qquad SO(5)$$

$$x\,dz, \phi^3, \phi^4, \phi^5 \qquad SO(2) \cdot SO(5)$$

T_{IIB}

$$B_{\mu\nu}^{cl} = B_{\mu\nu}^{cl} + A_i \alpha^i + A^D_i \beta^i + \phi\, d\tilde{z}$$

$$dA = * dA^D$$

$$dB = * dB$$

$$B_{\mu\nu}, \ldots, \phi^1 \ldots \phi^5 \qquad SO(5)$$

MS $\quad x^2 = \phi_2(z)$

$$\Sigma \subset T^*C \qquad x\,dz, \phi^3, \phi^4, \phi^5 \qquad SO(2) \cdot SO(5)$$

T_{IJ}

$$B_{\mu\nu}^{cl} = B^{cl} + A_i \alpha^i + A^D \beta_i + \phi\, d\tilde{z}$$

$$dA = * dA^D$$

$$dB = * dB$$

$$B_{\mu\nu}, \ldots, \phi^1 \ldots \phi^5 \qquad SO(5)$$

MS $x^2 = \phi_2(z)$ $\qquad \Sigma \subset T^*C \qquad x\,dz, \phi^3, \phi^4, \phi^5 \qquad SO(2) \cdot SO(5)$

$x^2 = 0$

2MS ON $C \subset T^*C$

T_{IIB}

$$B_{\mu\nu}^{6d} = A^{4d} + A_i \alpha^i + A^D \beta_i + \phi\, d\xi$$

$$dA = * dA^D$$

$$dB = * dB$$

$$B_{\mu\nu}, \ldots, \phi^1 \ldots \phi^5 \quad SO(5)$$

MS $x^2 = \phi_2(z)$ $\quad \Sigma \subset T^*C$ $\quad x\,dz, \phi^3, \phi^4, \phi^5$ $\quad SO(2) \cdot SO(5)$

$x^2 = 0$

2MS ON $C \subset T^*C$

(2,0) 6d THEORY A

T_{IIB}

$$B_{\mu\nu}^{6d} = A^{4d} + A_i \alpha^i + A^D \beta_i + \phi\, d\xi$$

$$dA = * dA^D$$

$$dB = * dB$$

$$B_{\mu\nu}, \ldots, \phi^1 \ldots \phi^5 \qquad SO(5)$$

$$SO(1) \cdot SO(5)$$

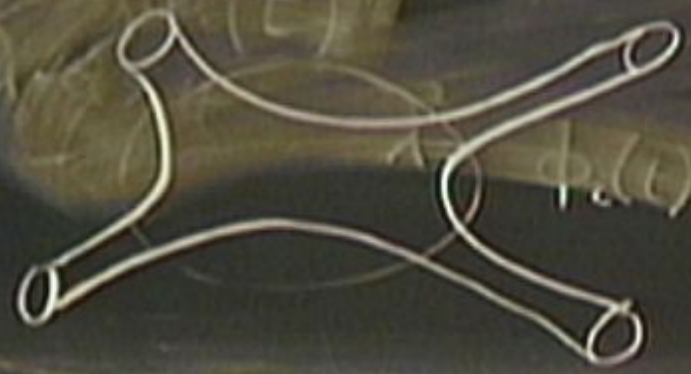
M5 $\quad x^2 = \phi_2(z) \qquad \Sigma \subset T^*C \qquad x\,dz, \phi^3, \phi^4, \phi^5$

$x^2 = 0$ $\quad$ 2M5 ON $C \subset T^*C$

$\boxed{(2,0)\ 6d\ \text{THEORY}\ A_1}$ ON C

$$\frac{\partial}{\partial a^i}\left(F_I + *\tau_{IJ}F^J\right) + c.c$$

$$\oint_\gamma \frac{\partial t_I}{\partial z^i} dz^i = \Delta t_I = \Delta\phi_I + \Delta v^J T_{IJ}$$



MS $x^2 = \phi_2(z)$ $\quad \Sigma \subset T^*C$ $\quad x\,dz$, ϕ^3, ϕ^5, ϕ^7 $\quad SO(2) \cdot SO(5)$

$x^2 = 0$ $\quad$ 2MS ON $C \subset T^*C$

$\boxed{(2,0) \text{ 6d THEORY } A_1}$ ON $C_{g,n}$

$$\frac{\partial}{\partial a^i}\left(F_I + *\tau_{IJ}F^J\right) + c.c$$

$$\oint_\gamma \frac{\partial t_I}{\partial z^i} dz^i = \Delta t_I = \Delta\phi_I + \Delta v^J T_{IJ}$$

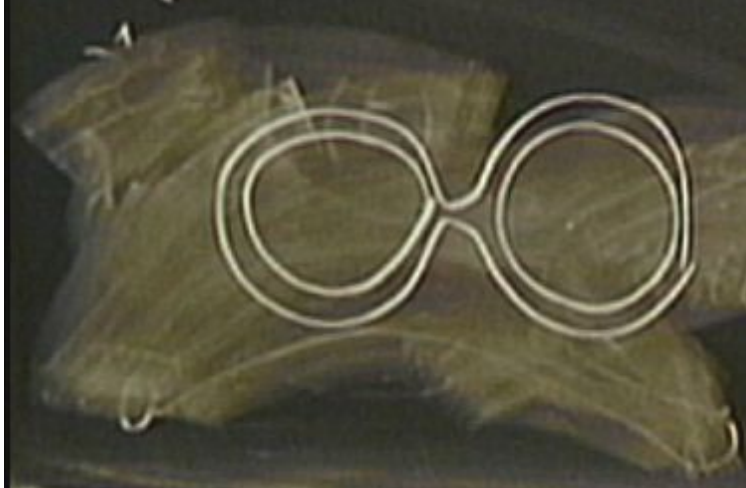
M5 $x^2 = \phi_i(z)$ $\Sigma \subset T^*C$ $x\,dz$, ϕ^3, ϕ^5, ϕ^6 $SO(2) \cdot SO(5)$

$x^2 = 0$ 2M5 ON $C \subset T^*C$

$\boxed{(2,0) \text{ 6d THEORY } A_1}$ ON $C_{g,n}$

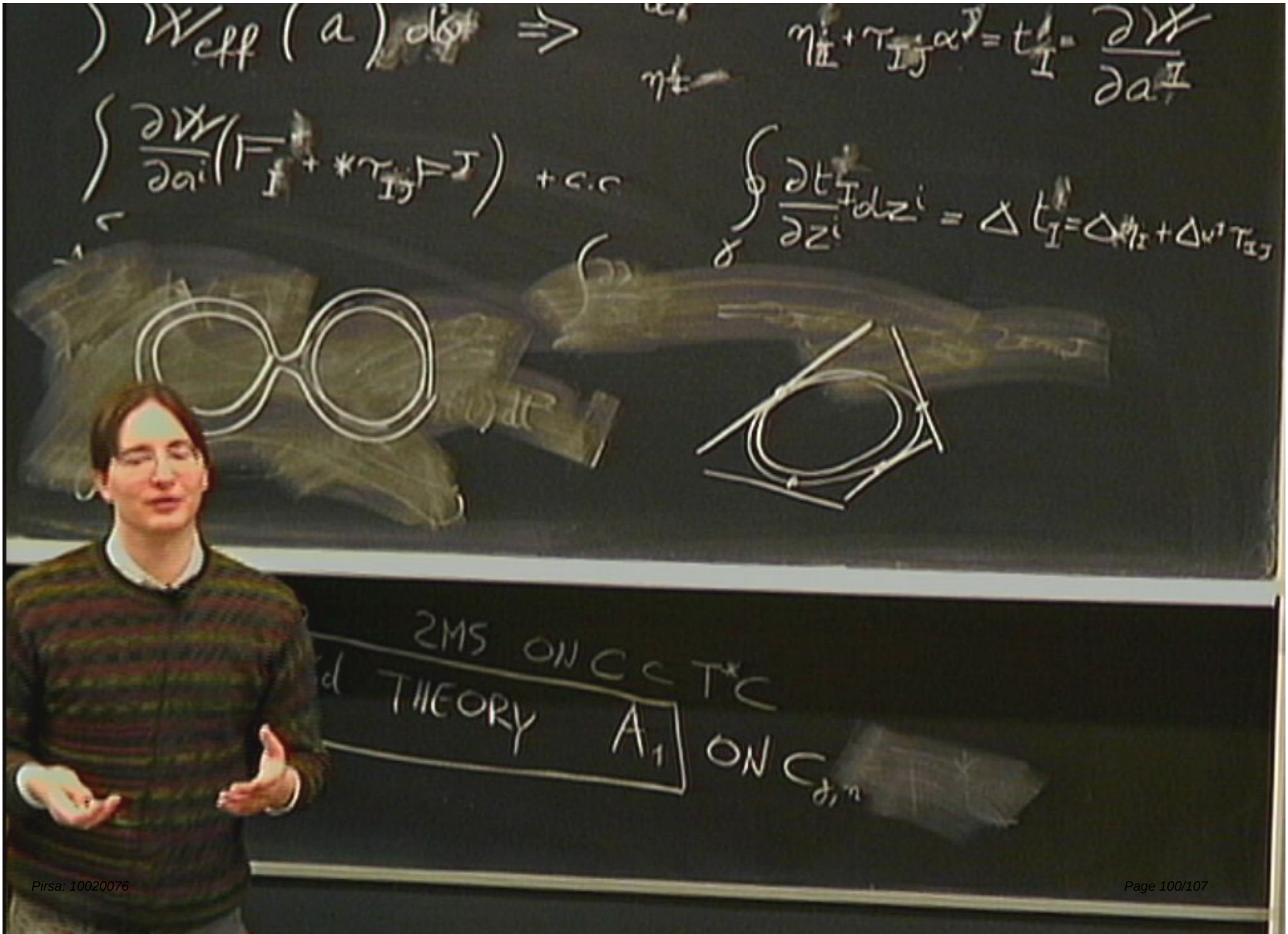
$$\mathcal{W}_{eff}(a) \Rightarrow \eta_I + \tau_{IJ} a^J = t_I = \frac{\partial \mathcal{W}}{\partial a^I}$$

$$\int \frac{\partial \mathcal{W}}{\partial a^i}\left(F_I + *\tau_{IJ} F^J\right) + c.c \qquad \oint_\gamma \frac{\partial t_I}{\partial z^i} dz^i = \Delta t_I = \Delta\eta_I + \Delta n^J \tau_{IJ}$$



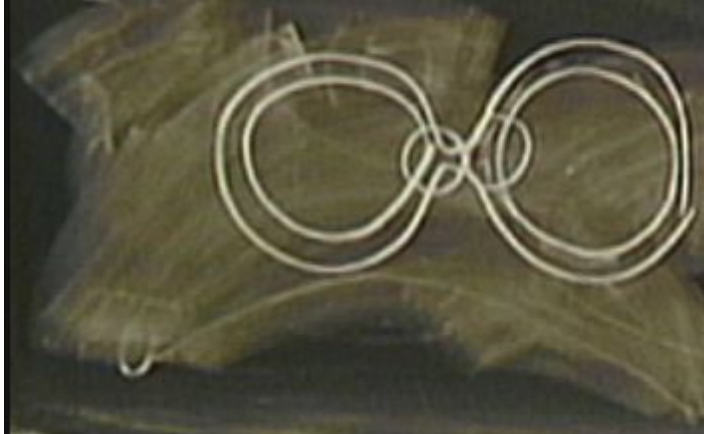
$\mathcal{X}^2 = 0$ 2M5 ON $C \subset T^*C$

$\boxed{(2,0)\ 6d\ \text{THEORY}\ A_1}$ ON $C_{\gamma,n}$


2M5 ON C ⊂ T*C
THEORY
A1
ON C

$$\mathcal{W}_{eff}(a)\, d\theta \Rightarrow \qquad \eta_I + \tau_{IJ} a^J = t_I = \frac{\partial \mathcal{W}}{\partial a^I}$$

$$\int_C \frac{\partial \mathcal{W}}{\partial a^i}\left(F_I + *\tau_{IJ} F^J\right) + c.c \qquad \oint_\gamma \frac{\partial t_I}{\partial z^i} dz^i = \Delta t_I = \Delta\eta_I + \Delta n^J \tau_{IJ}$$

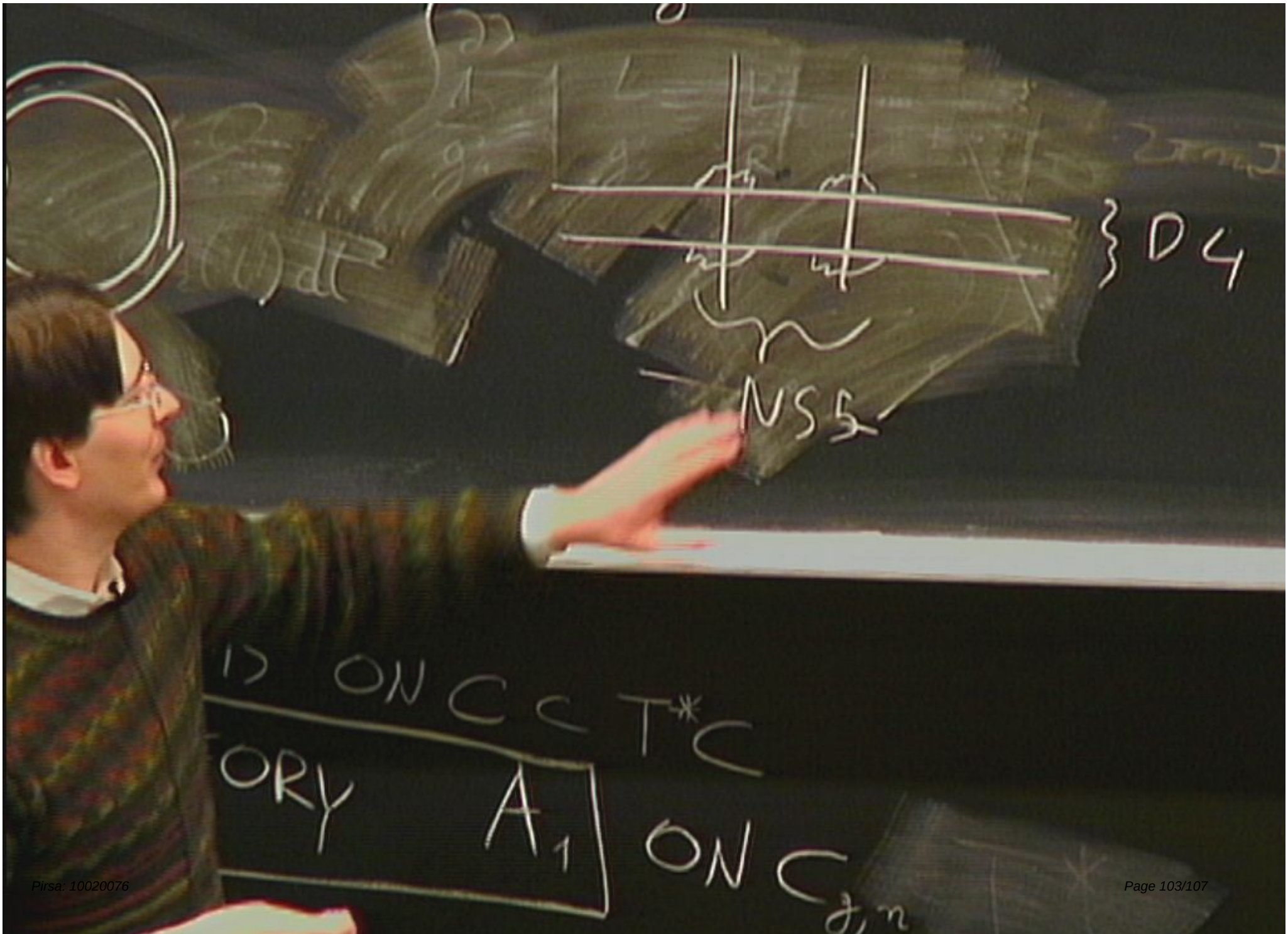


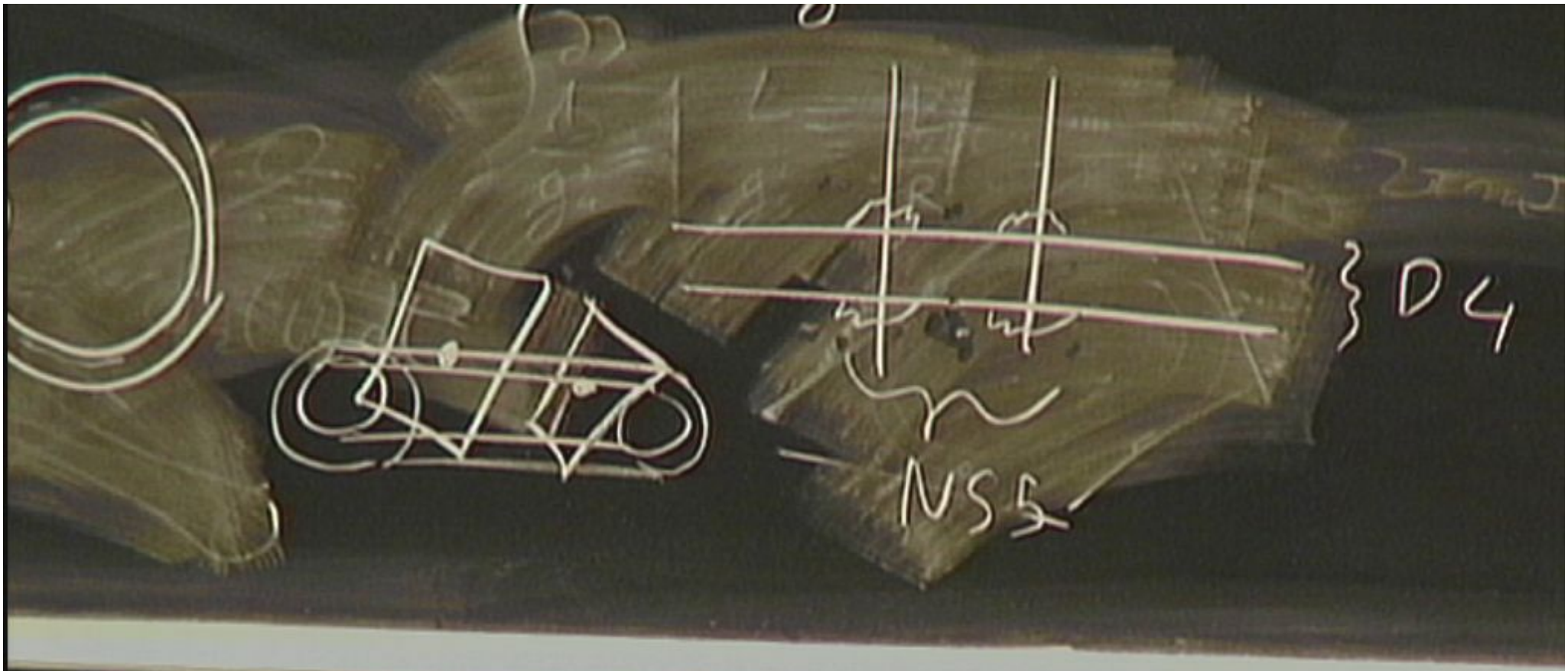
$$\frac{1}{g_s^2} = \frac{L}{g_s} = \frac{L}{R}$$

$x^2 = 0$ 2M5 ON $C \subset T^*C$

$\boxed{(2,0)\ 6d\ \text{THEORY}\ A_1}$ ON $C_{g,n}$

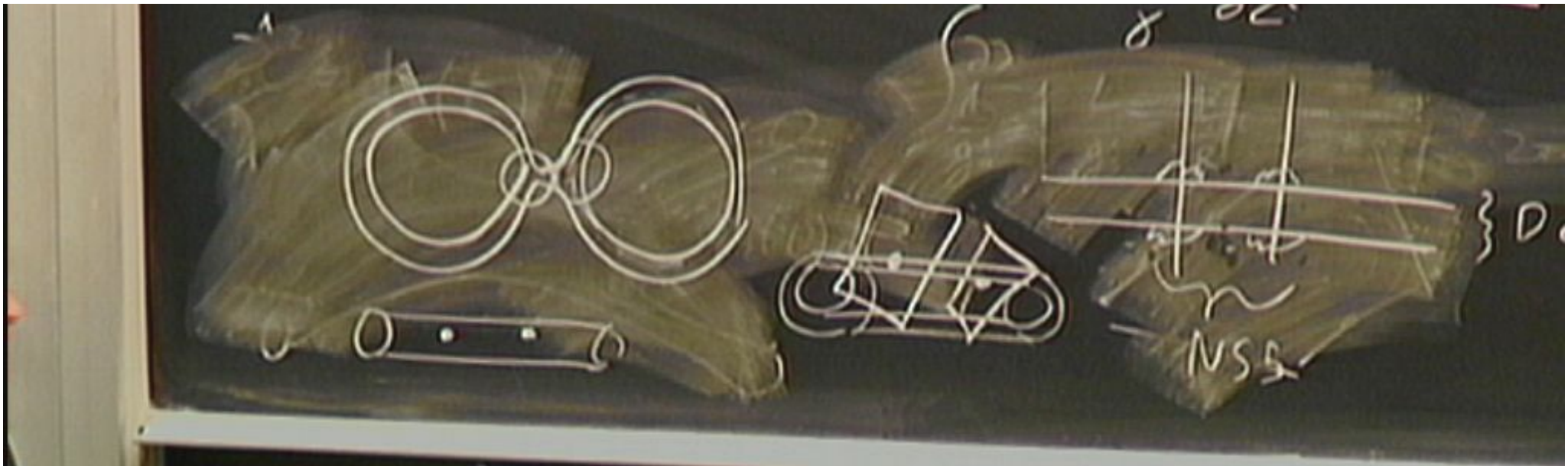
D4
NS5
T*C
A1
ON


D4
NS5
ON
T*C
A1
ON



2MS ON $C \subset T^*C$

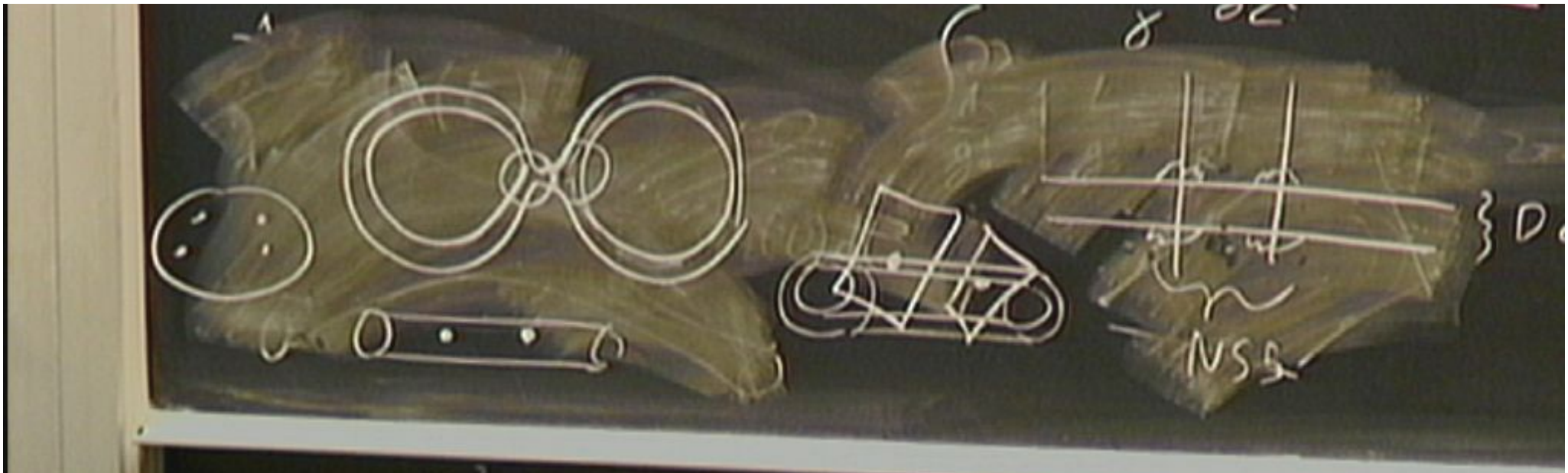
THEORY $\boxed{A_1}$ ON $C_{g,n}$



$x^2 = 0$

2M5 ON $C \subset T^*C$

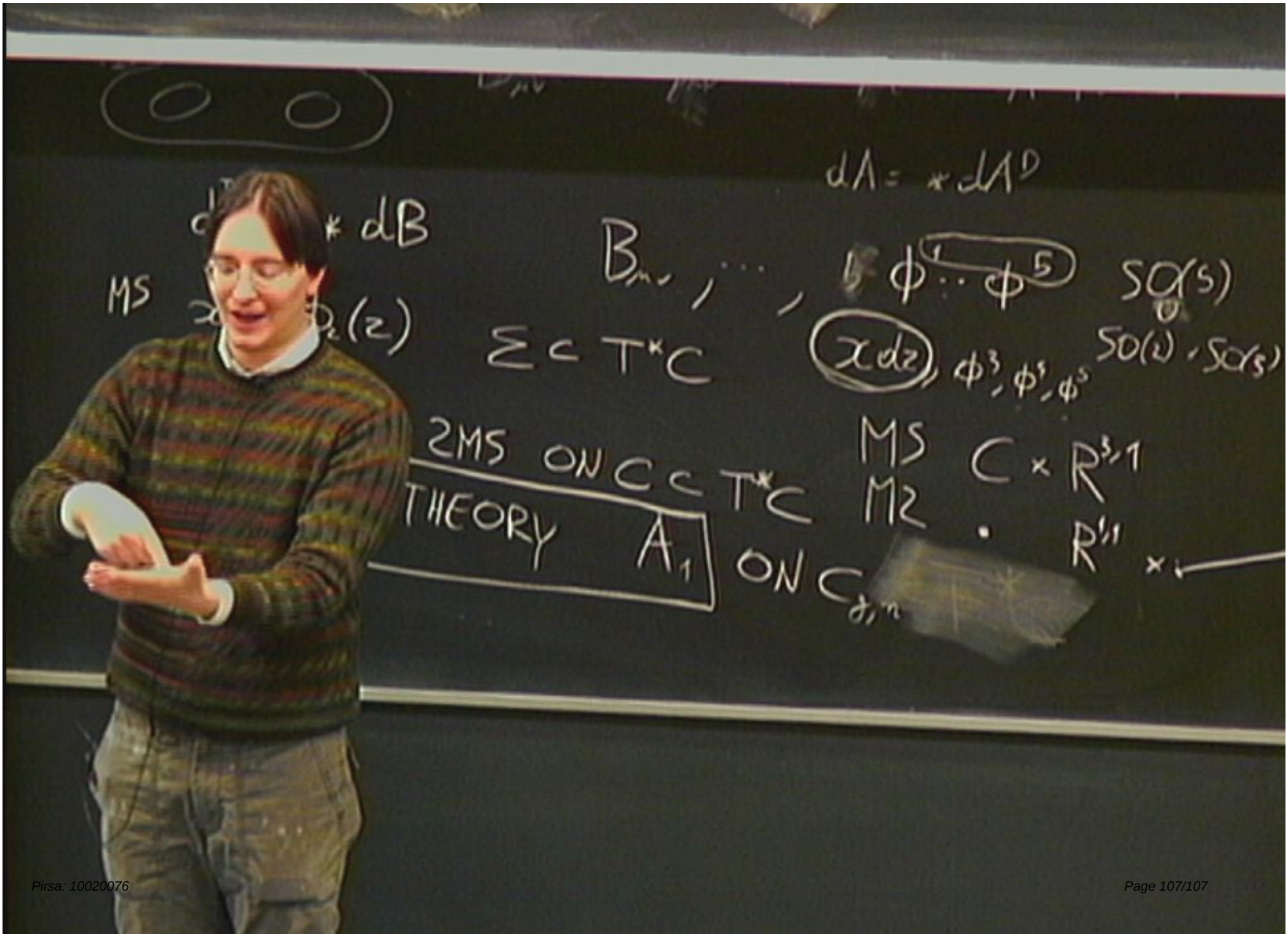
(2,0) 6d THEORY A_1 ON $C_{\gamma,n}$



$x^2 = 0$

2M5 ON $C \subset T^*C$

$\boxed{(2,0) \text{ 6d THEORY } A_1}$ ON $C_{\gamma,n}$


dΛ = * dΛ^D
* dB
MS
Σ ⊂ T*C
SO(5)
SO(2) × SO(5)
2MS ON C ⊂ T*C
MS C × R^{3,1}
M2
THEORY A1 ON C_{g,n}