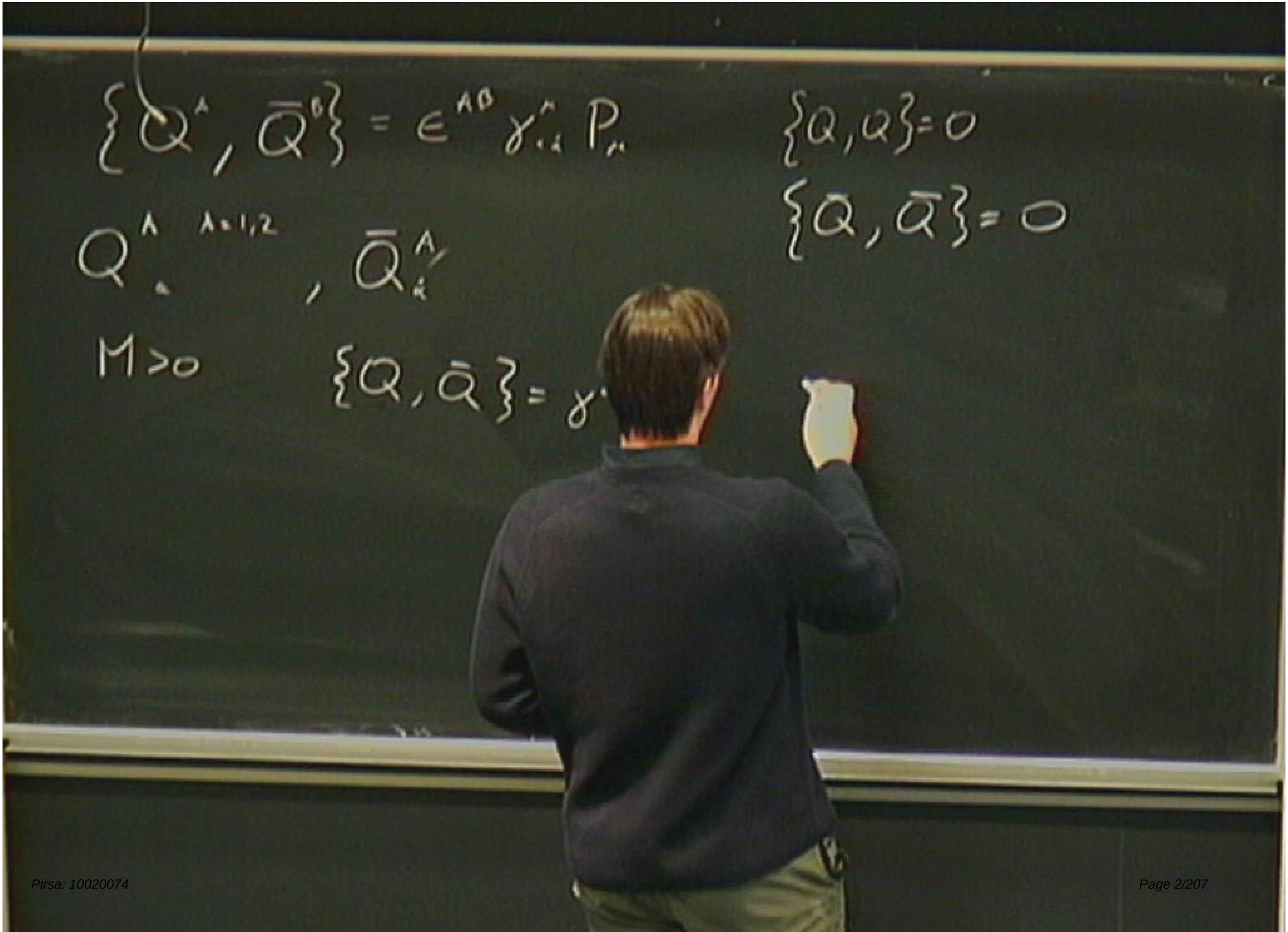
Title: Lecture 2: SW solution of gauge theory with matter

Date: Feb 23, 2010 11:15 AM

URL: http://pirsa.org/10020074

Abstract:



$\{Q^A, \bar{Q}^B\} = \epsilon^{AB} \gamma^{\mu}_{\alpha\dot{\alpha}} P_{\mu}$

$\{Q, Q\} = 0$

$\{\bar{Q}, \bar{Q}\} = 0$

Q^{A}_{α} $A=1,2$, $\bar{Q}^{A}_{\dot{\alpha}}$

$M > 0$ $\{Q, \bar{Q}\} = \gamma$

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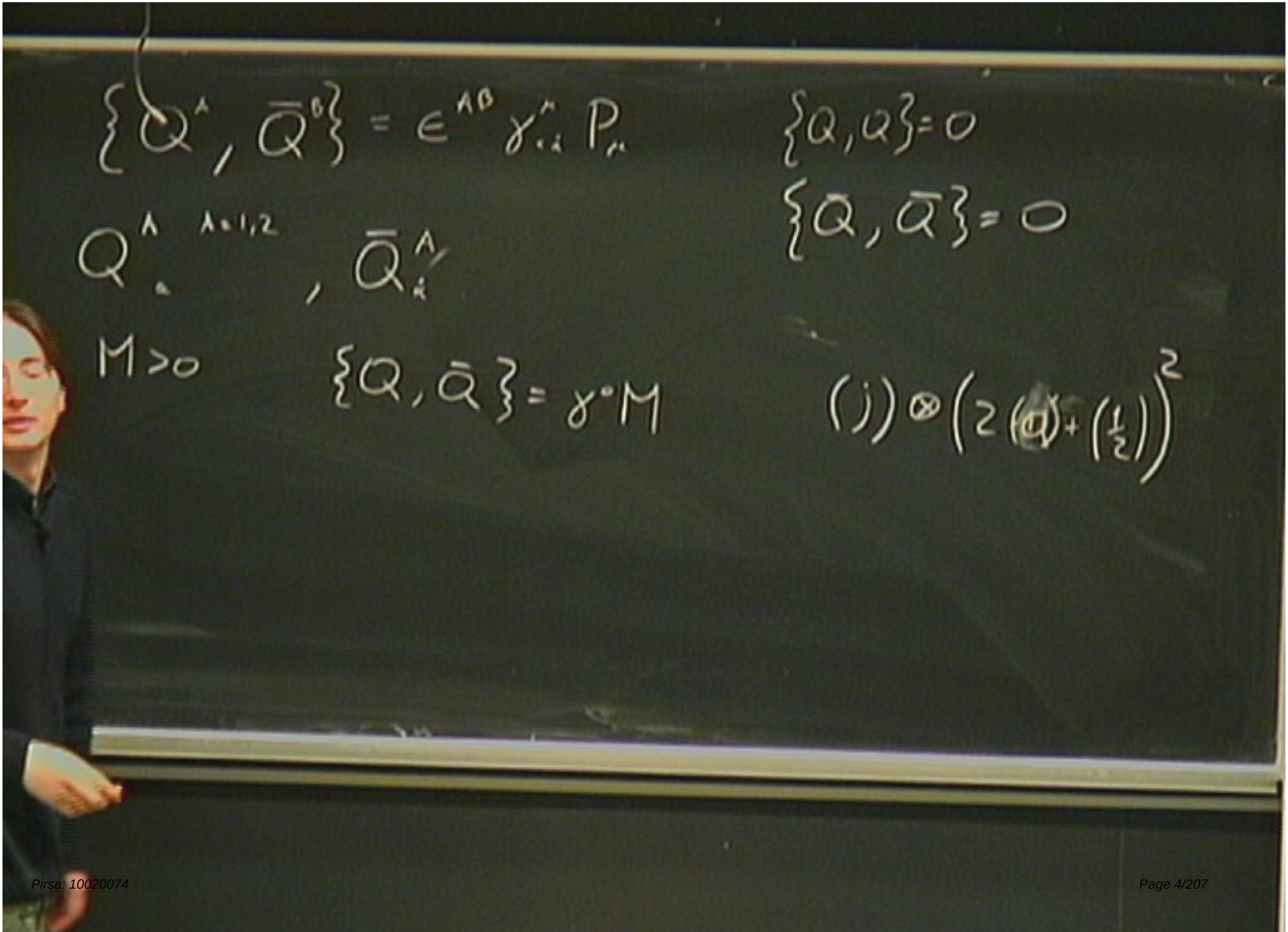

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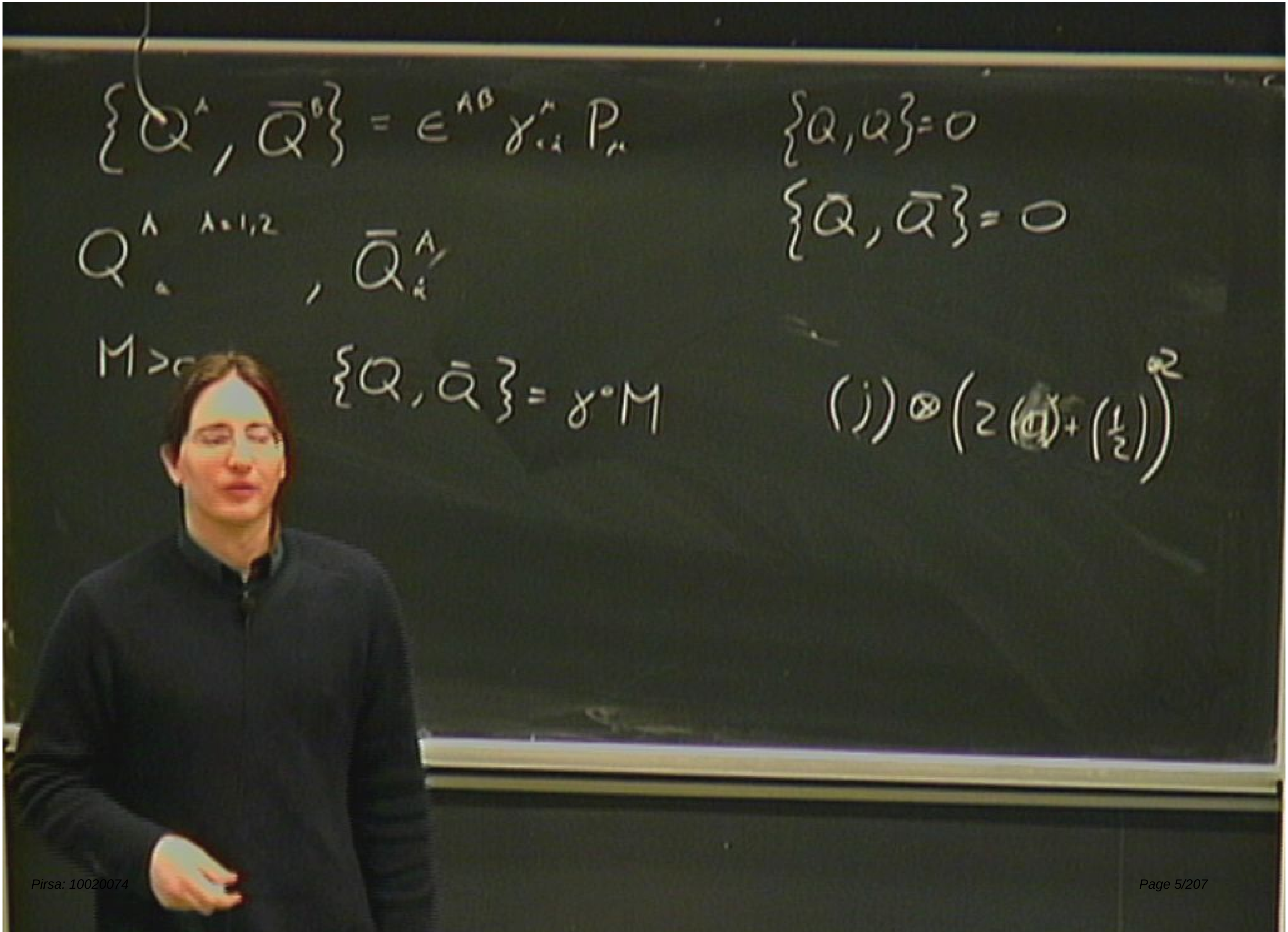
$$(j) \otimes \left(2(0) + \left(\tfrac{1}{2}\right)\right)^2$$



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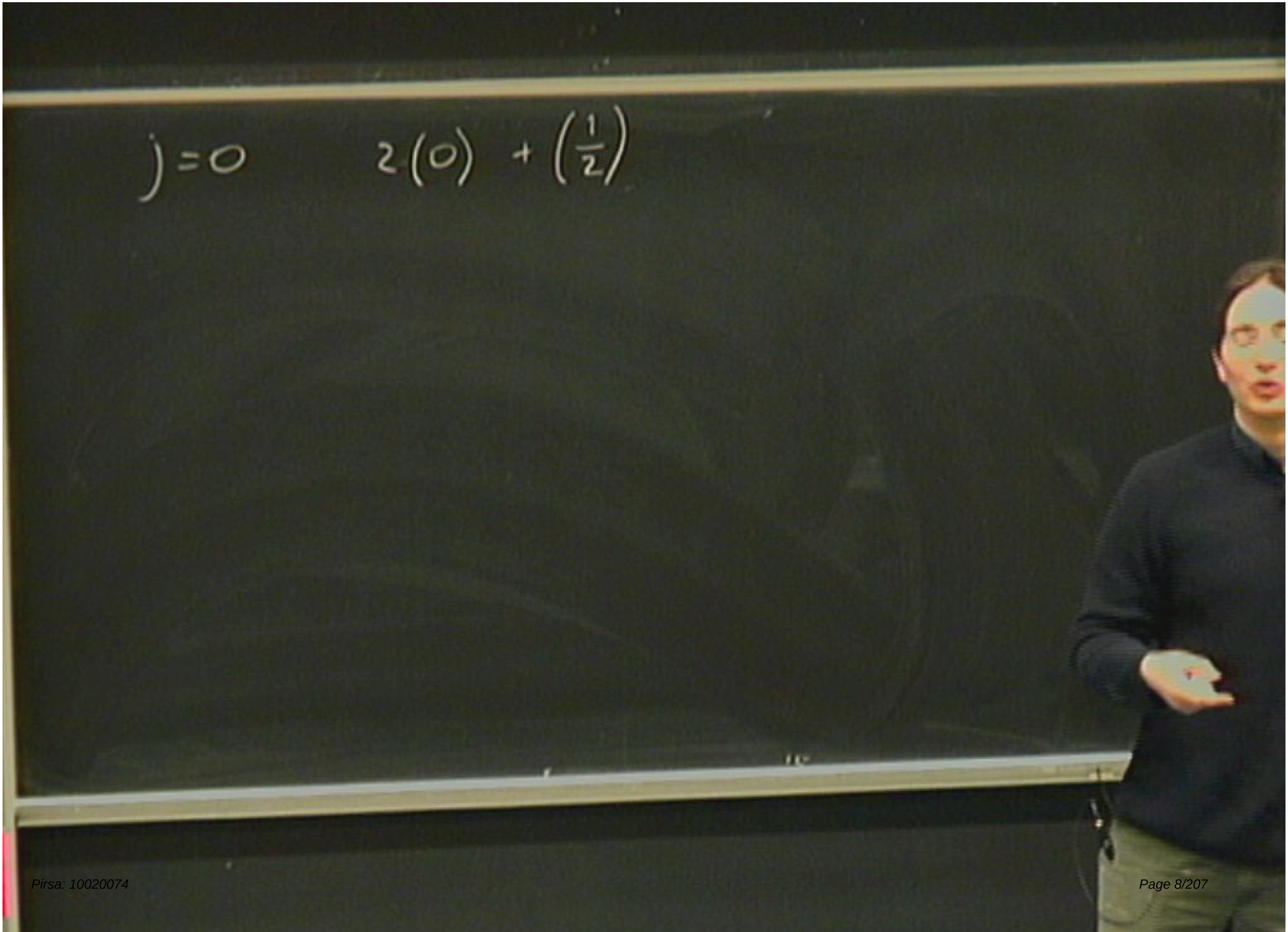

$j=0$ $\qquad 2(0)+\left(\frac{1}{2}\right)$

$$j=0 \qquad 2(0) + \left(\tfrac{1}{2}\right) \otimes \left(R \oplus \bar{R}\right)$$

$$j=\tfrac{1}{2} \qquad (1) + 2\left(\tfrac{1}{2}\right) + (0)$$

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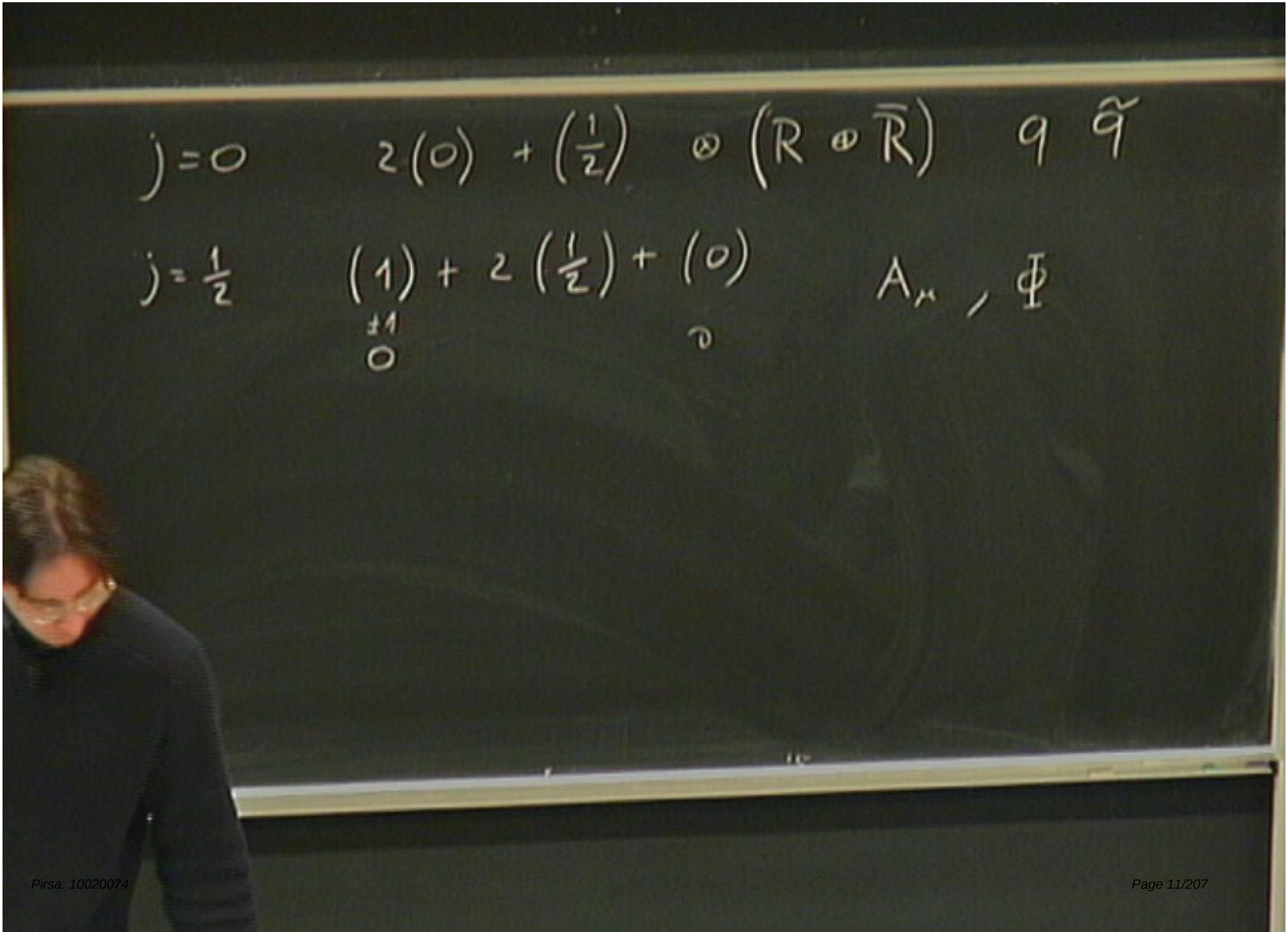

$j=0 \qquad 2(0) + \left(\tfrac{1}{2}\right) \otimes \left(R \oplus \bar{R}\right) \qquad q \quad \tilde{q}$

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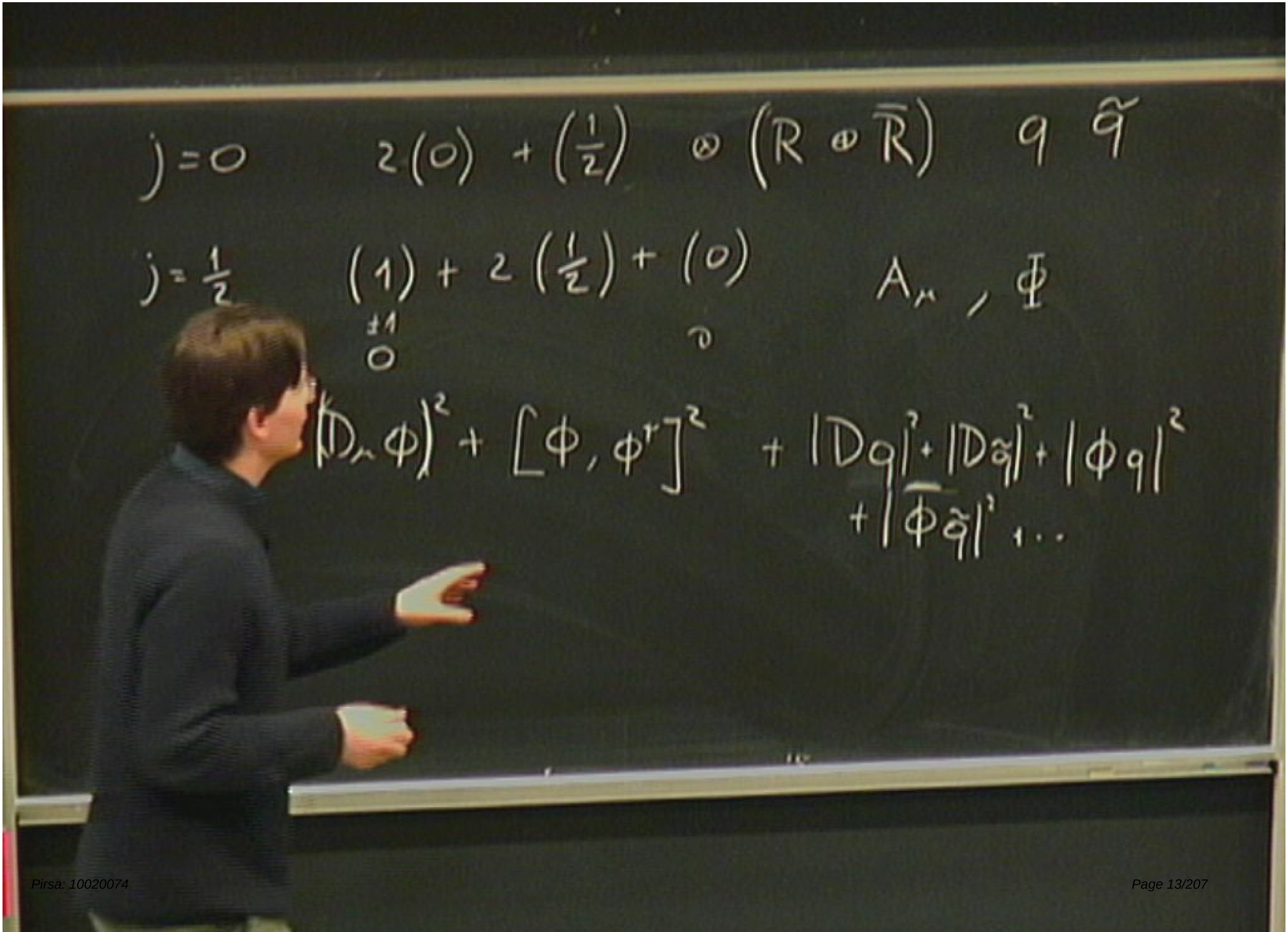

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$$|D_\mu \phi|^2 + [\phi, \phi^\dagger]^2 + |Dq|^2 + |D\tilde{q}|^2 + |\phi q|^2 + |\bar{\phi}\tilde{q}|^2 + \dots$$

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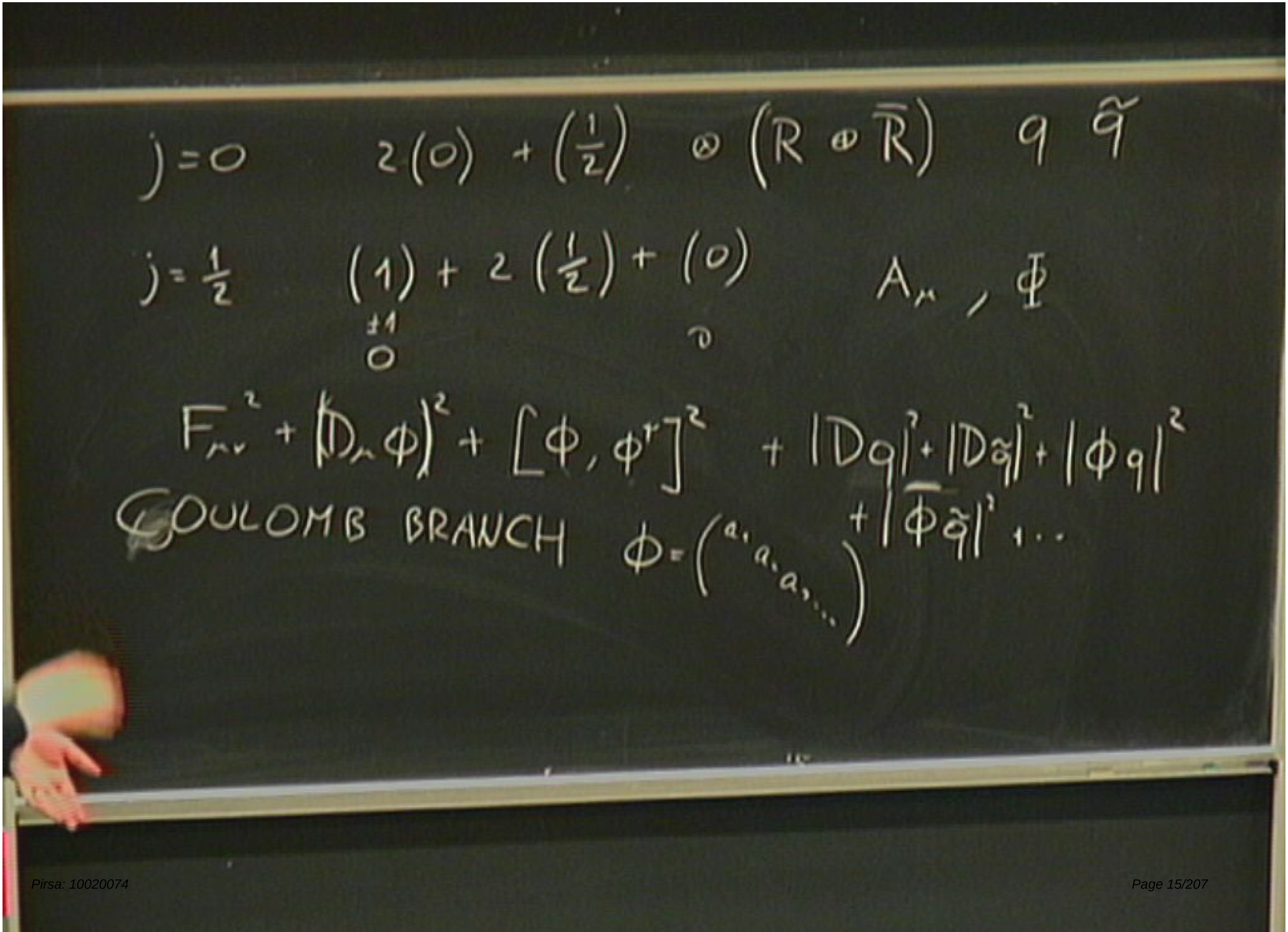

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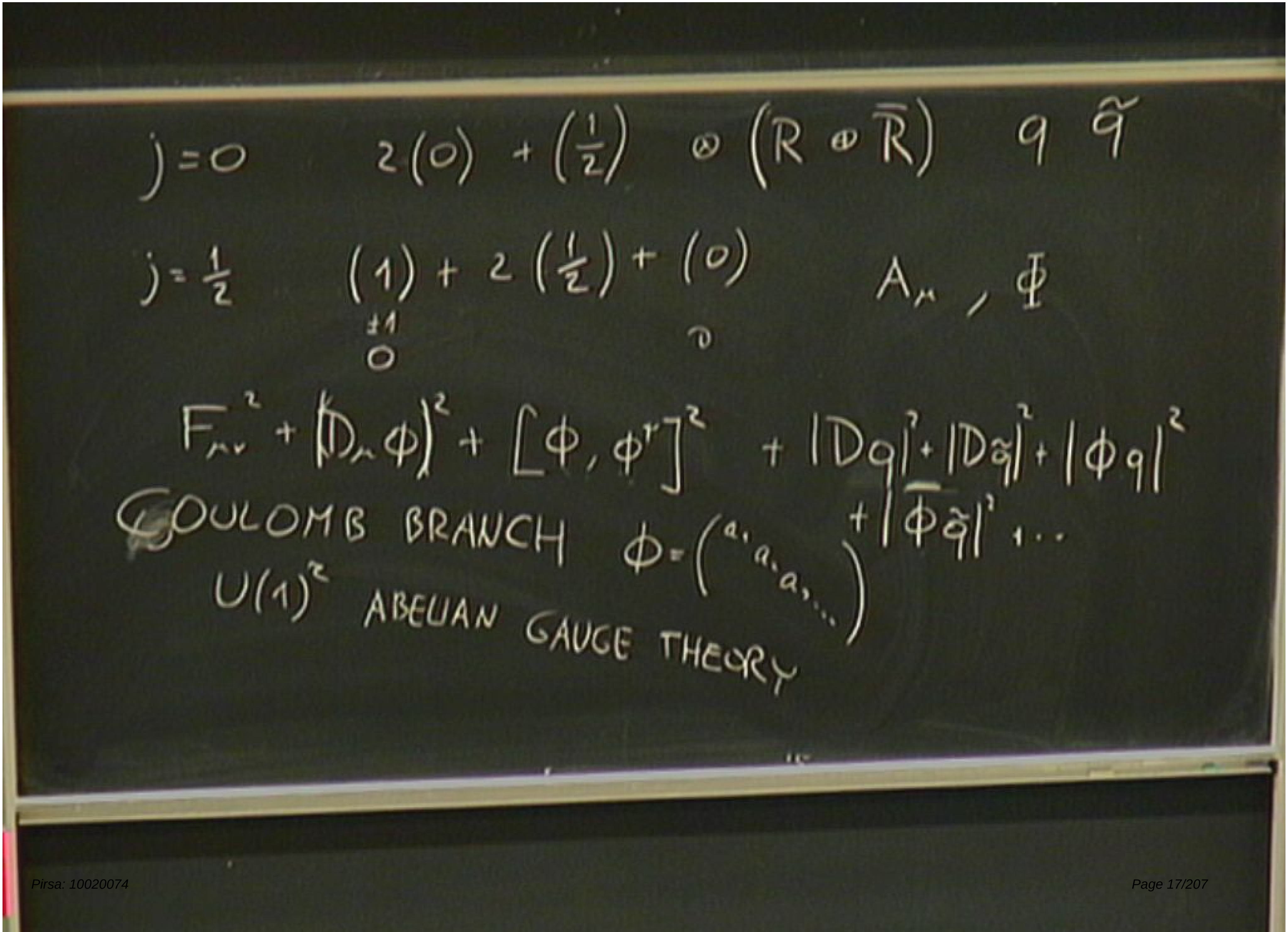

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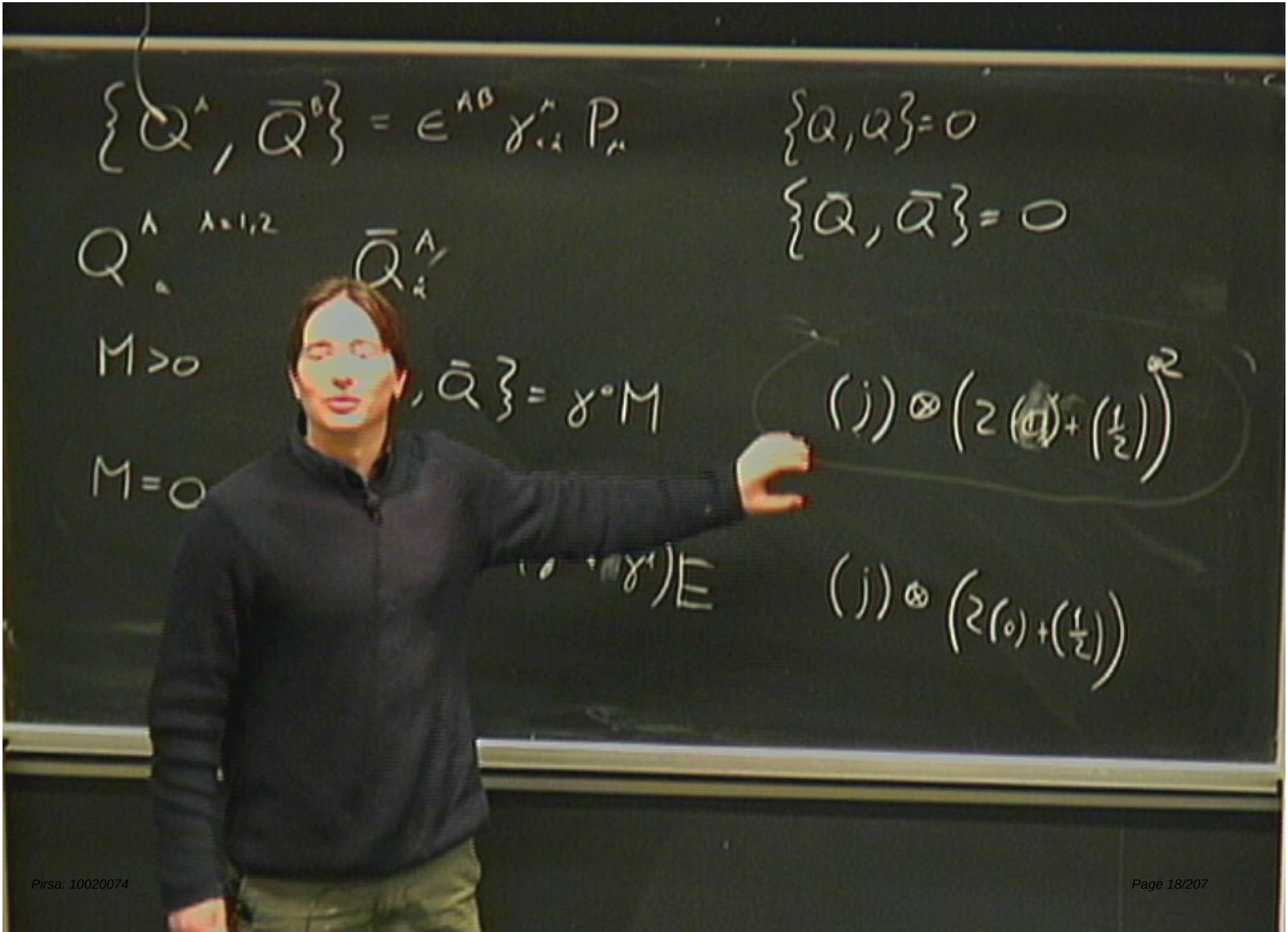
$U(1)^r$ ABELIAN GAUGE THEORY



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Q^A_α $A = 1,2$ $\bar{Q}^A_{\dot{\alpha}}$

$M > 0$ $, \bar{Q}\} = \gamma^0 M$

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$\{Q, Q\} =$ GAUGE TRANSF. PARAMETER ϕ

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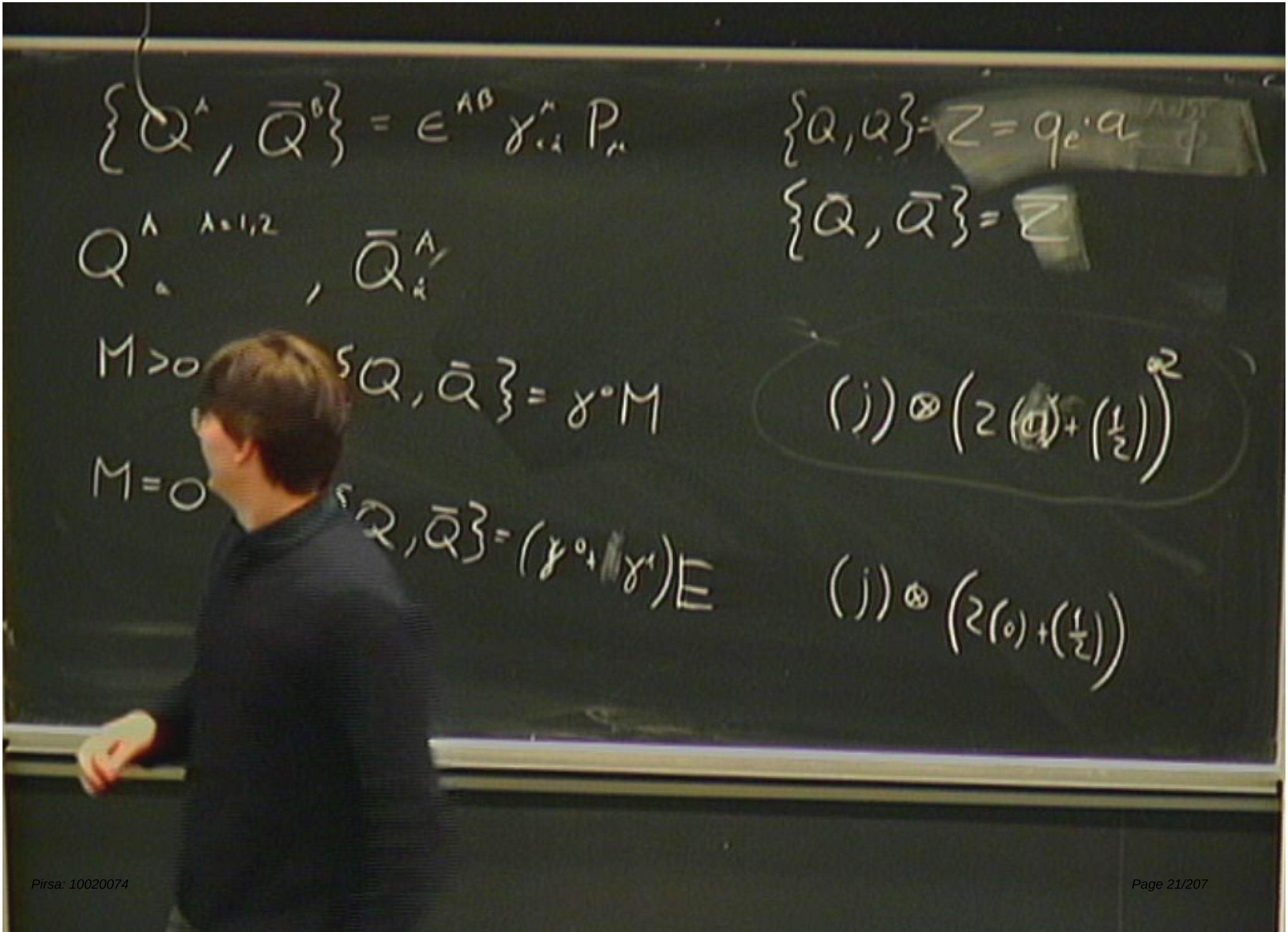

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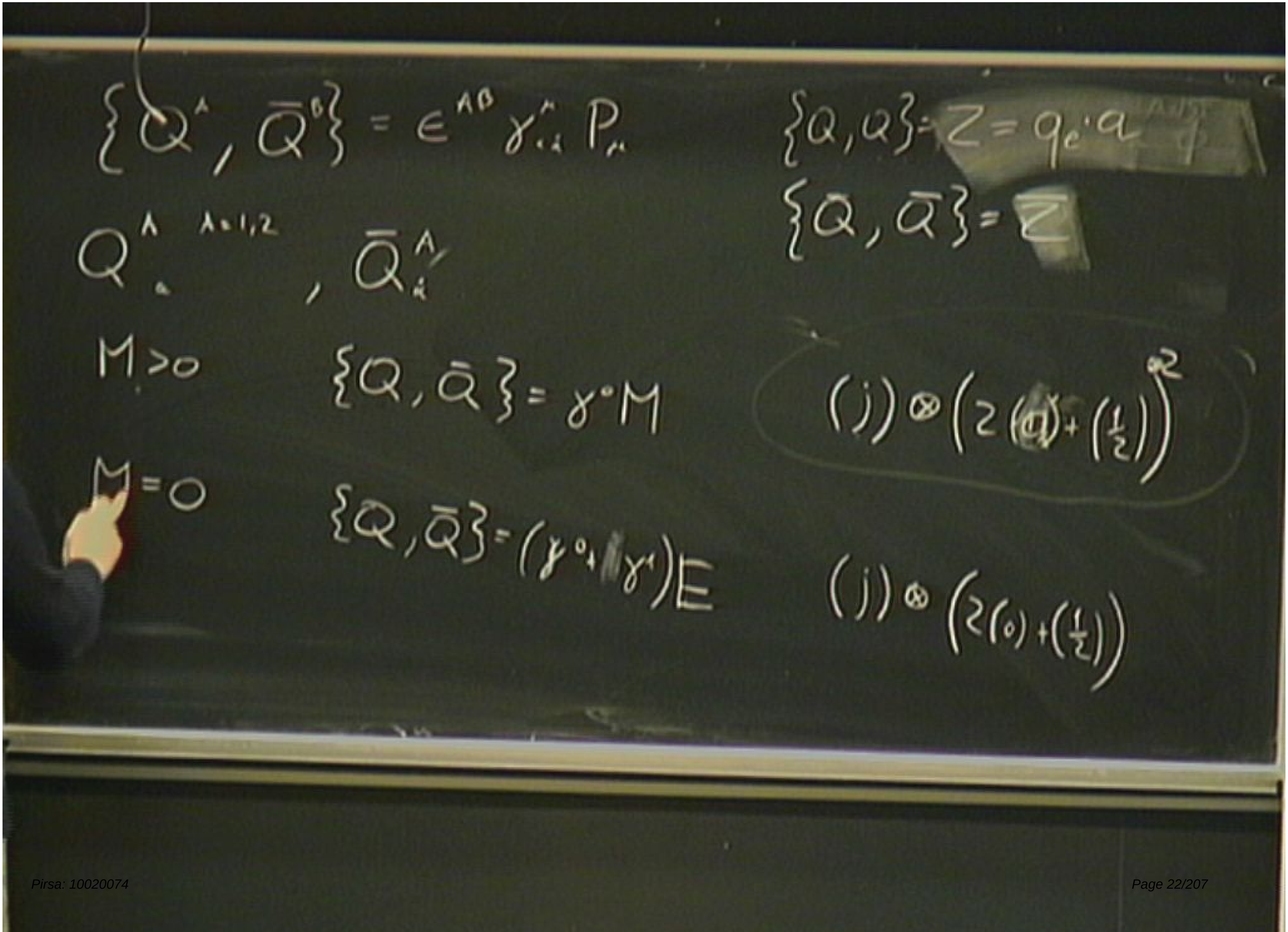

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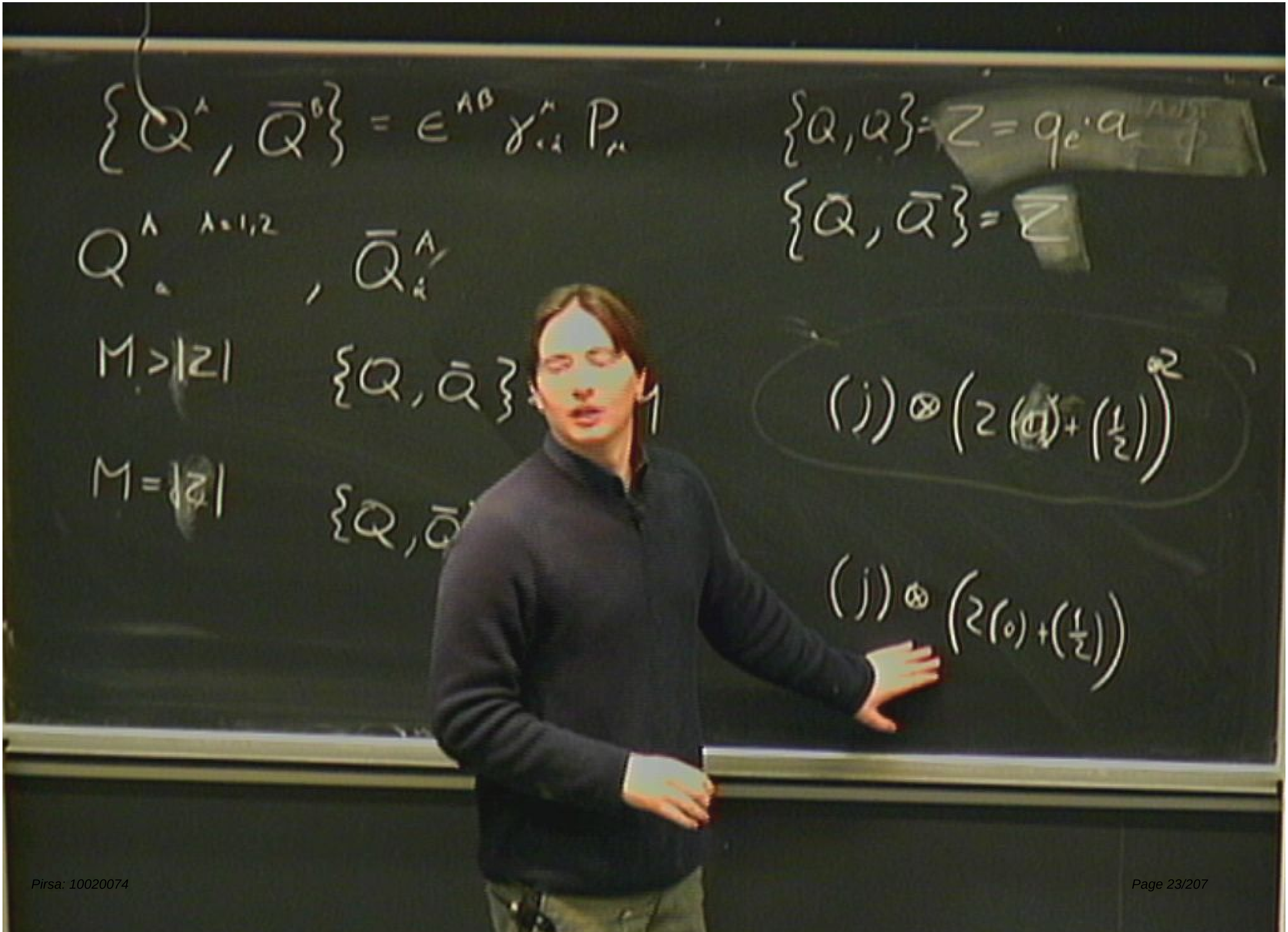

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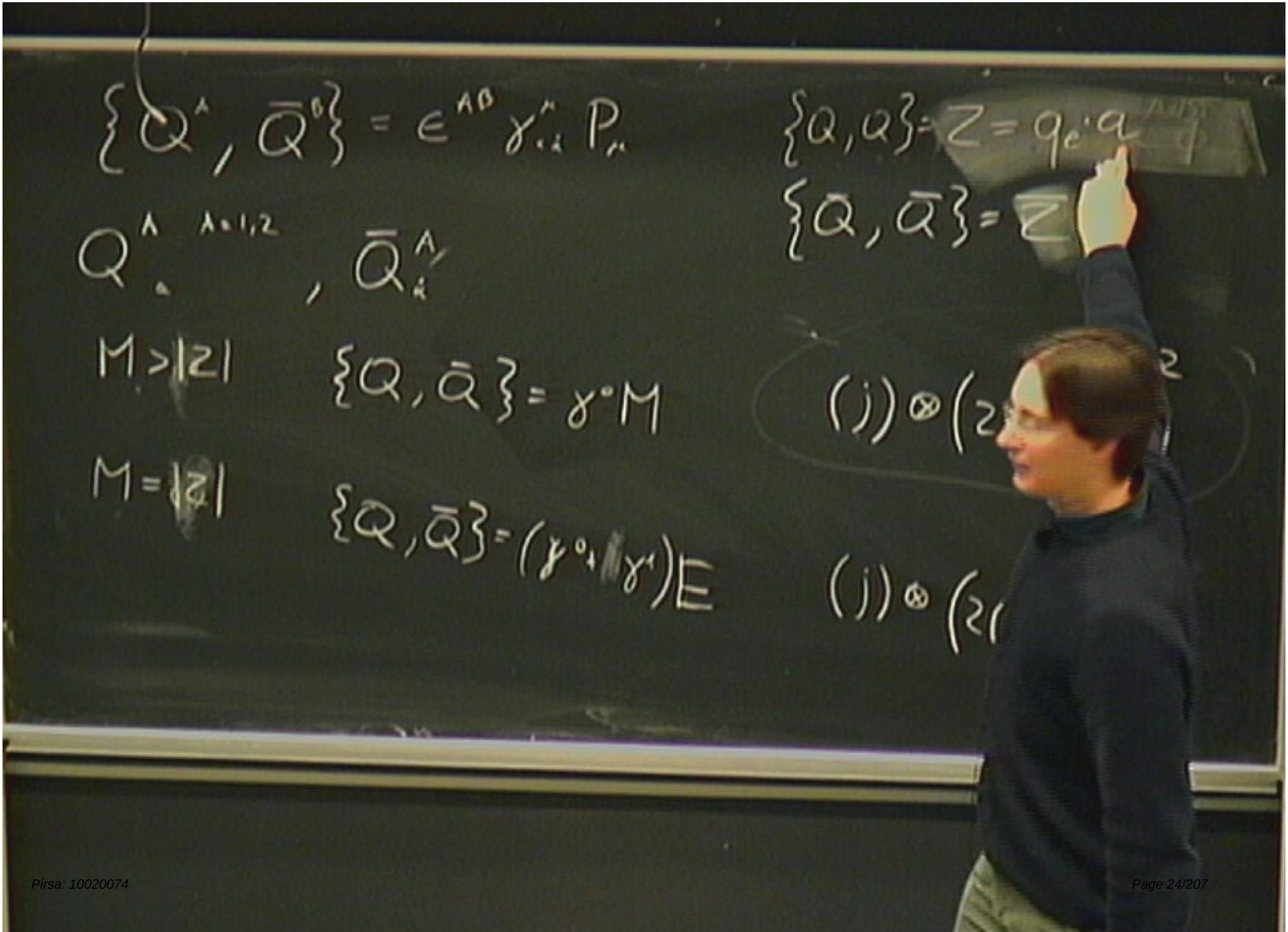

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$(j) \otimes (2$

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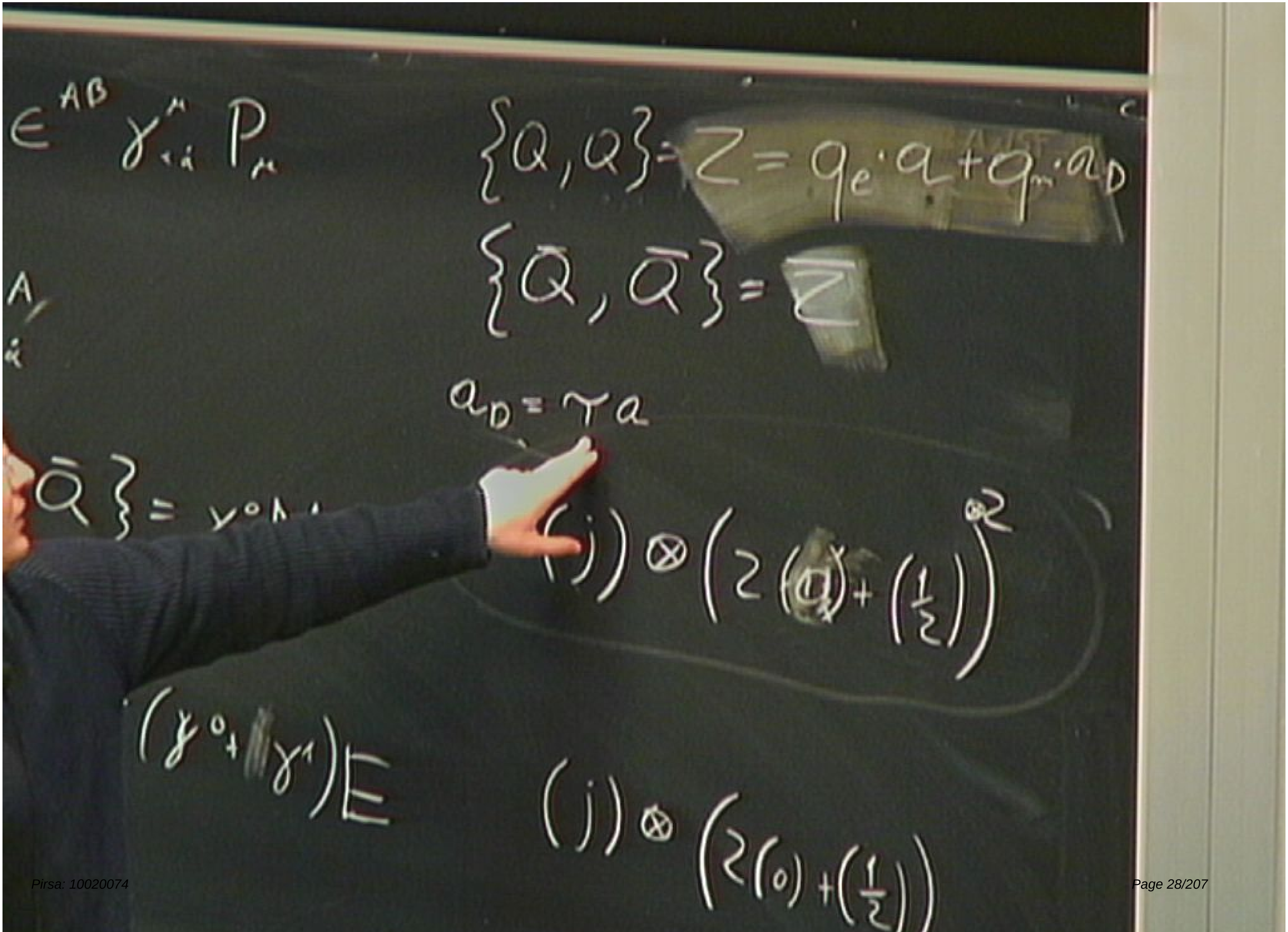
$U(1)^2$ ABELIAN GAUGE THEORY

$$F_{ij} = \epsilon_{ijk} D_k \phi$$



$\epsilon^{AB}\gamma^{\mu}_{\alpha\dot\alpha}P_{\mu}$

$\{Q,Q\} = Z = q_e \cdot a + q_m \cdot a_D$

$\{\bar Q,\bar Q\} = \bar Z$

A

$\dot\alpha$

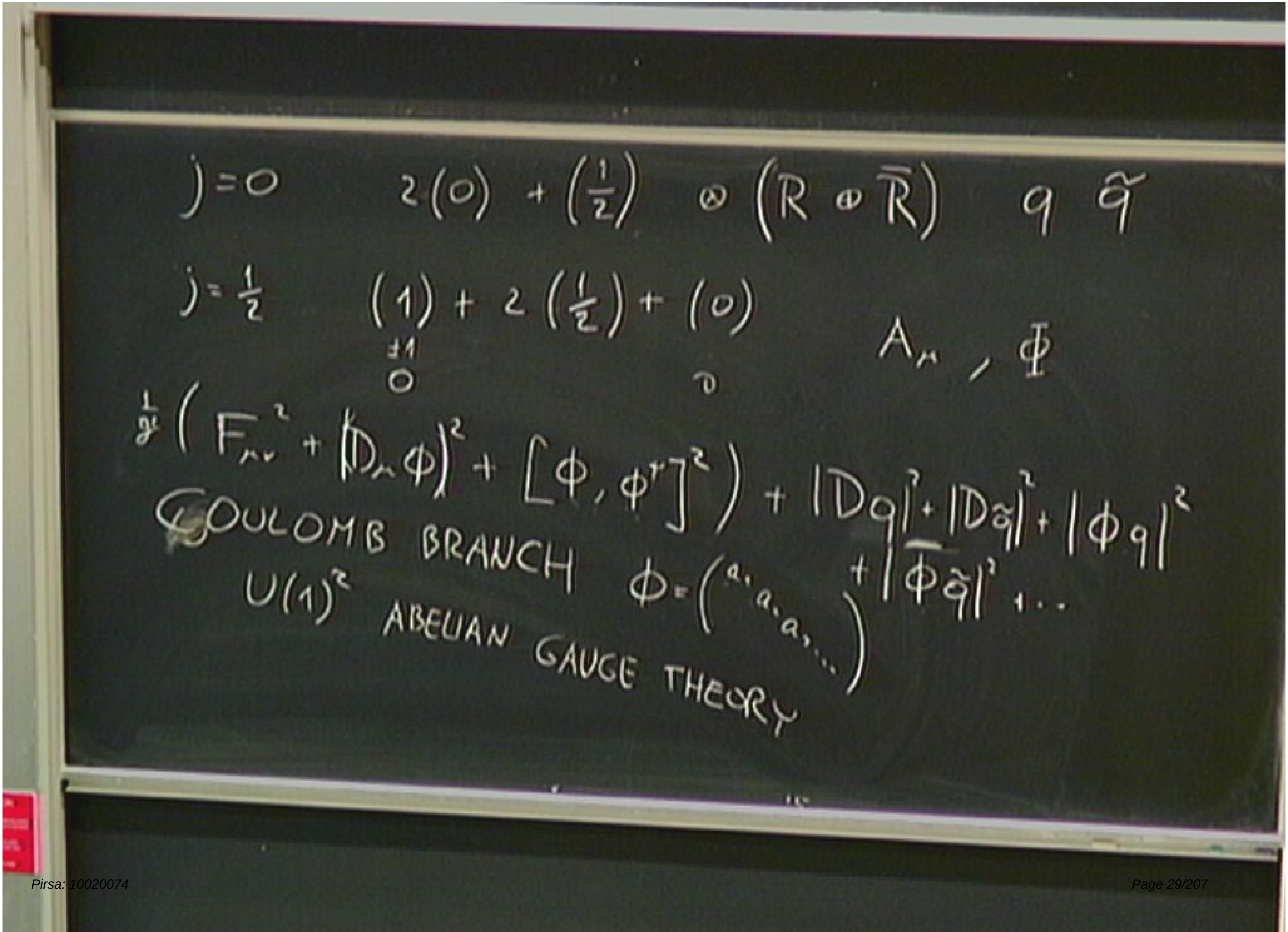
$a_D = \tau a$

$\bar Q\} = \gamma^0$

$(j)\otimes\left(2(0)+\left(\frac{1}{2}\right)\right)^{\otimes 2}$

$(\gamma^0 + \gamma^1)E$

$(j)\otimes\left(2(0)+\left(\frac{1}{2}\right)\right)$



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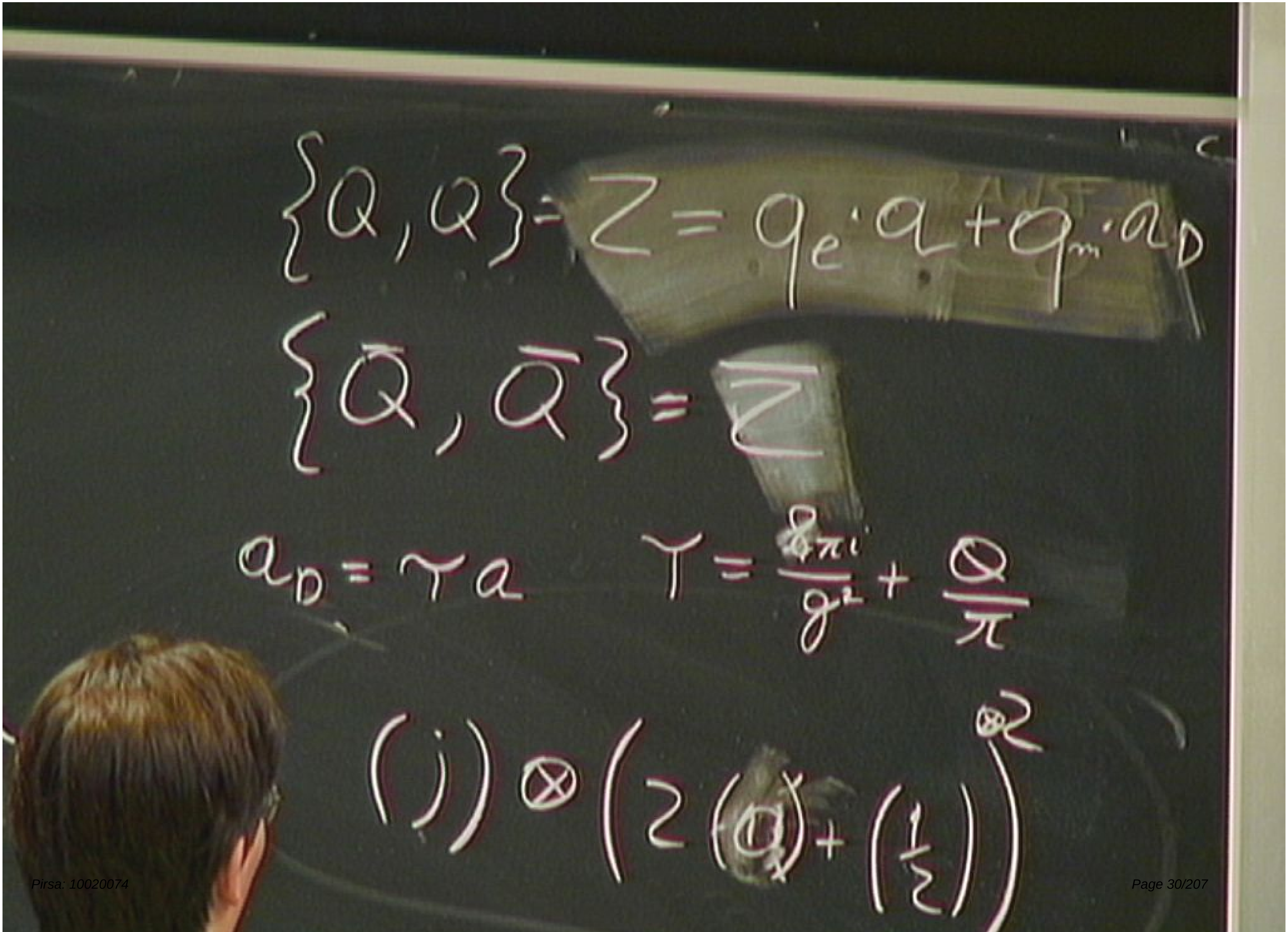
$U(1)^2$ ABELIAN GAUGE THEORY



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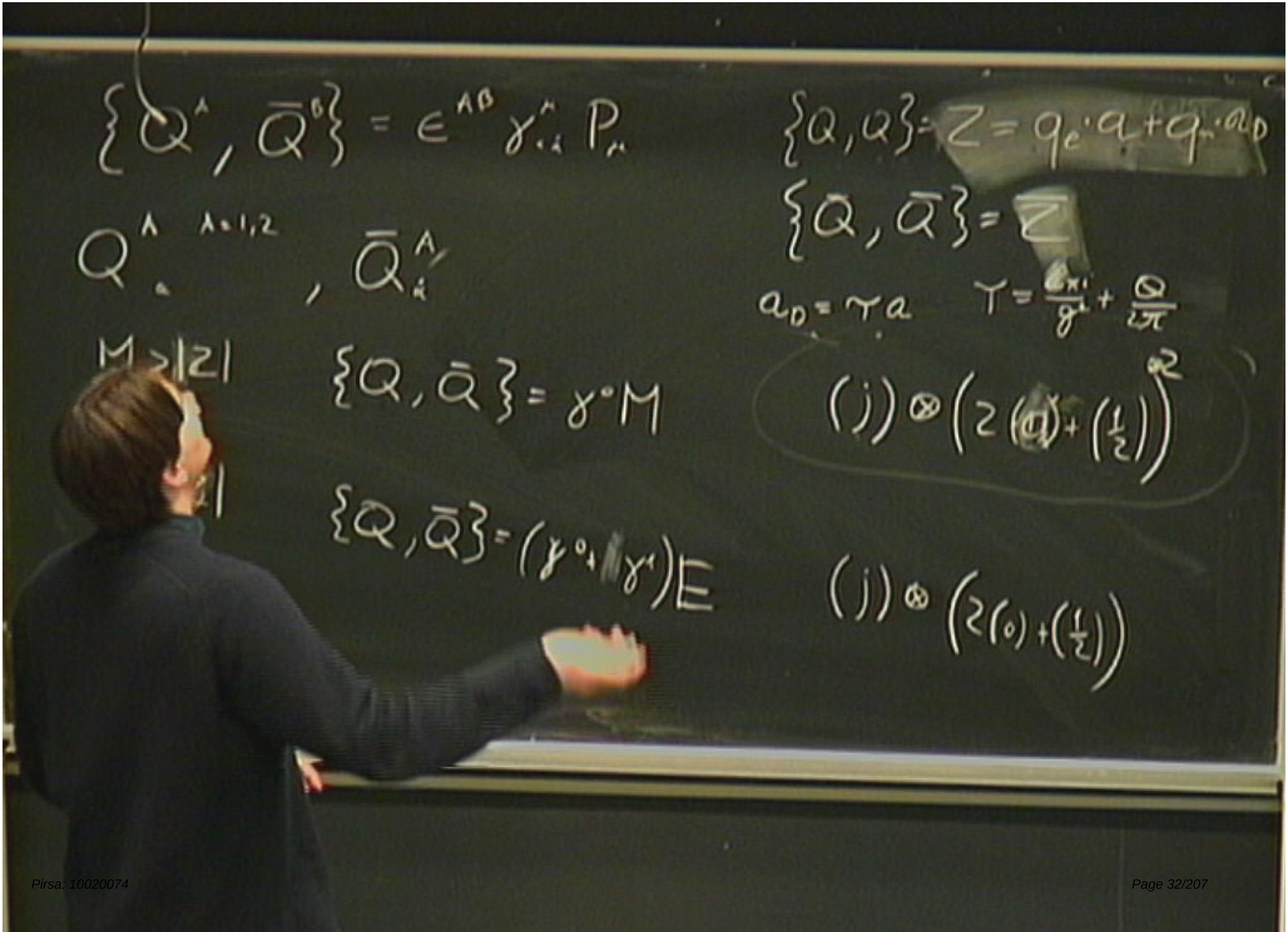

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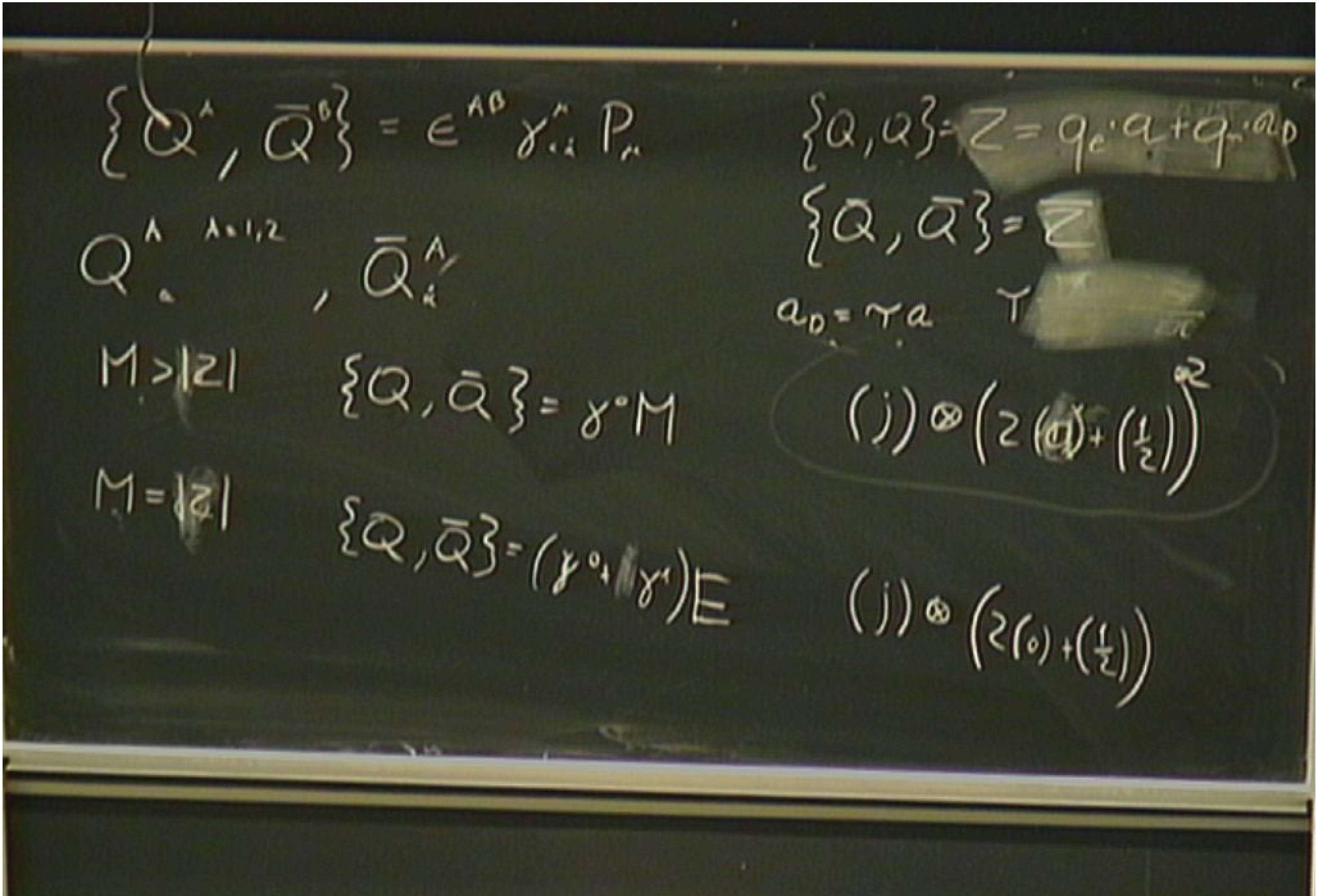

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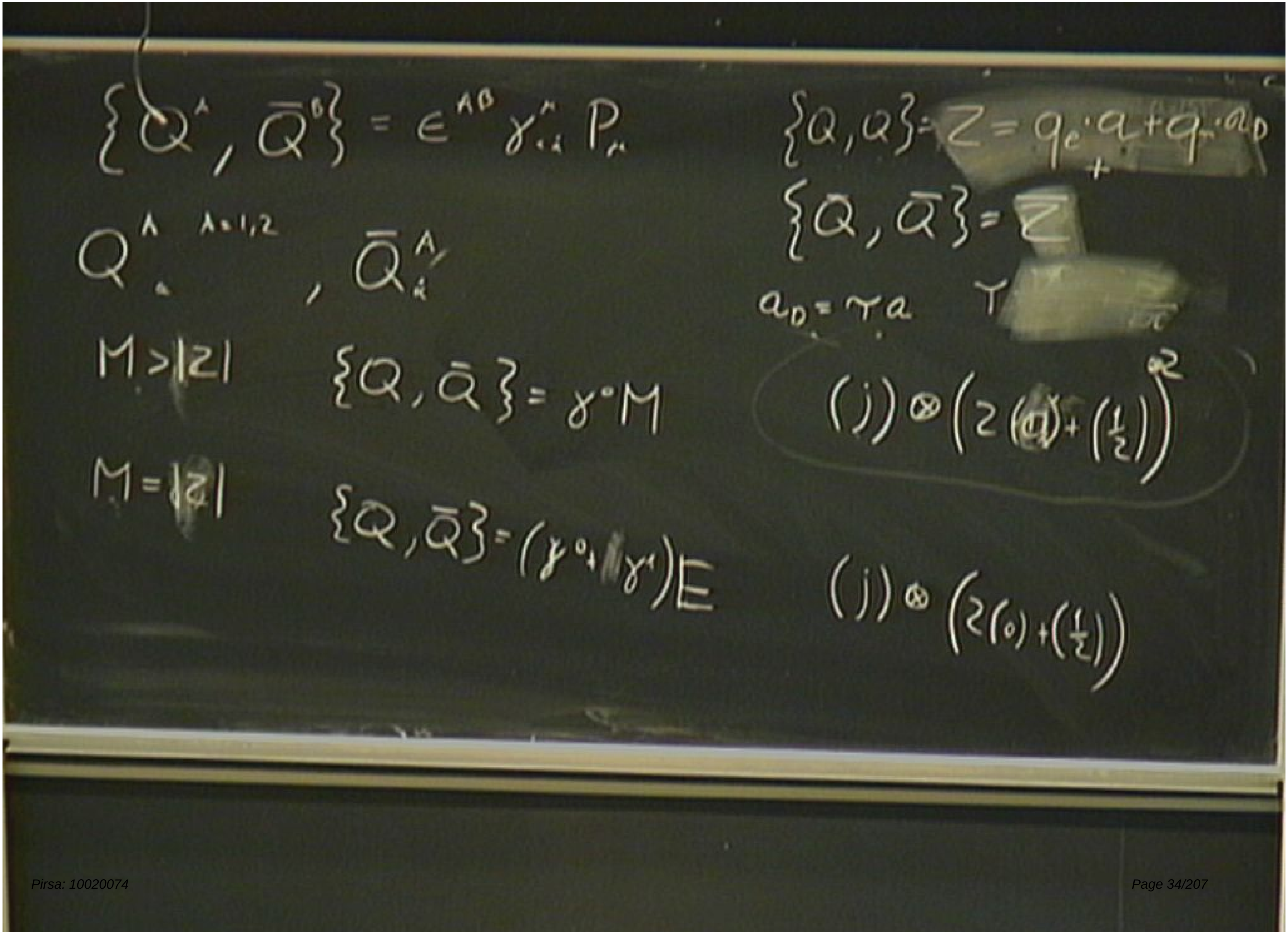

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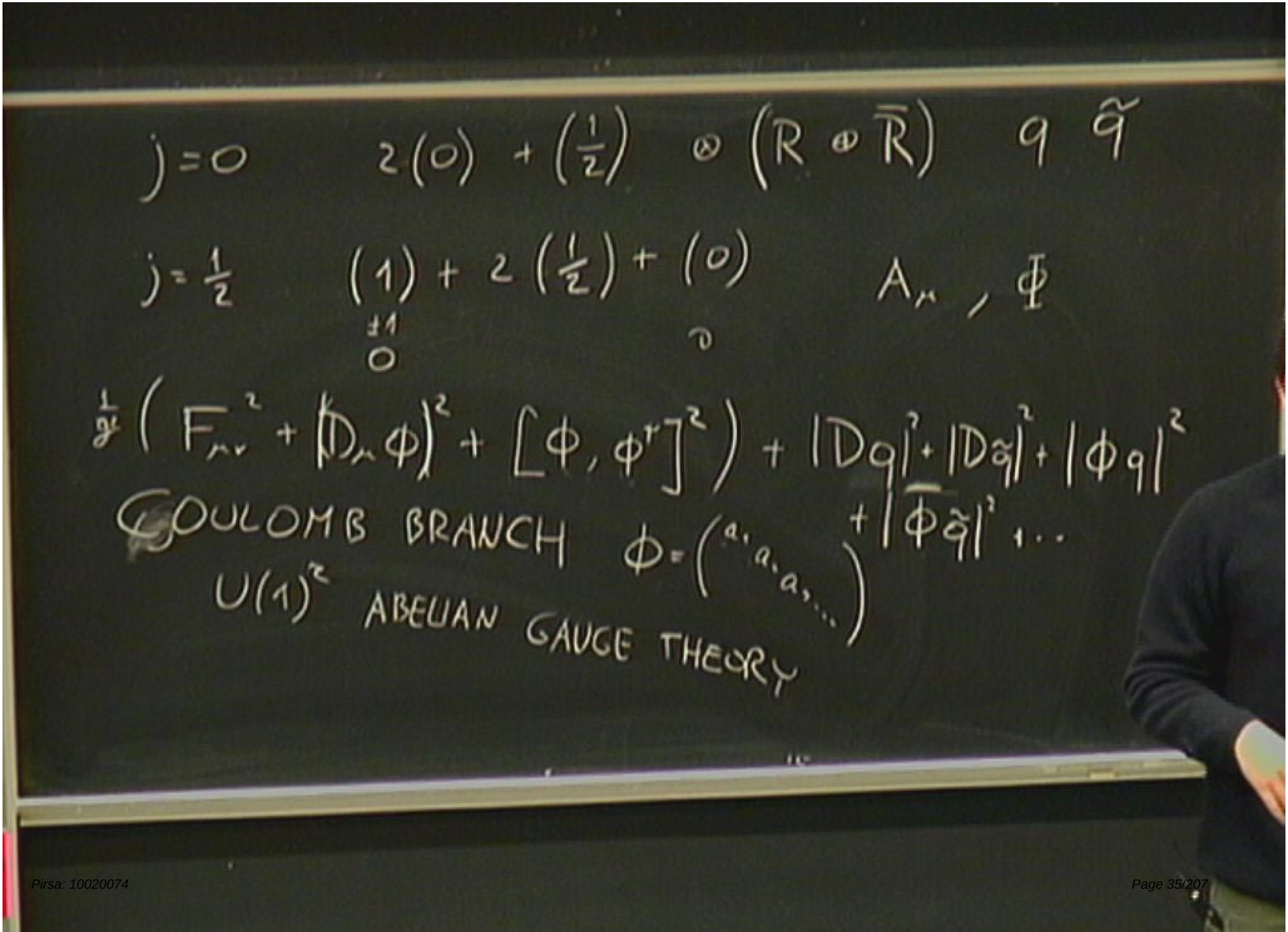

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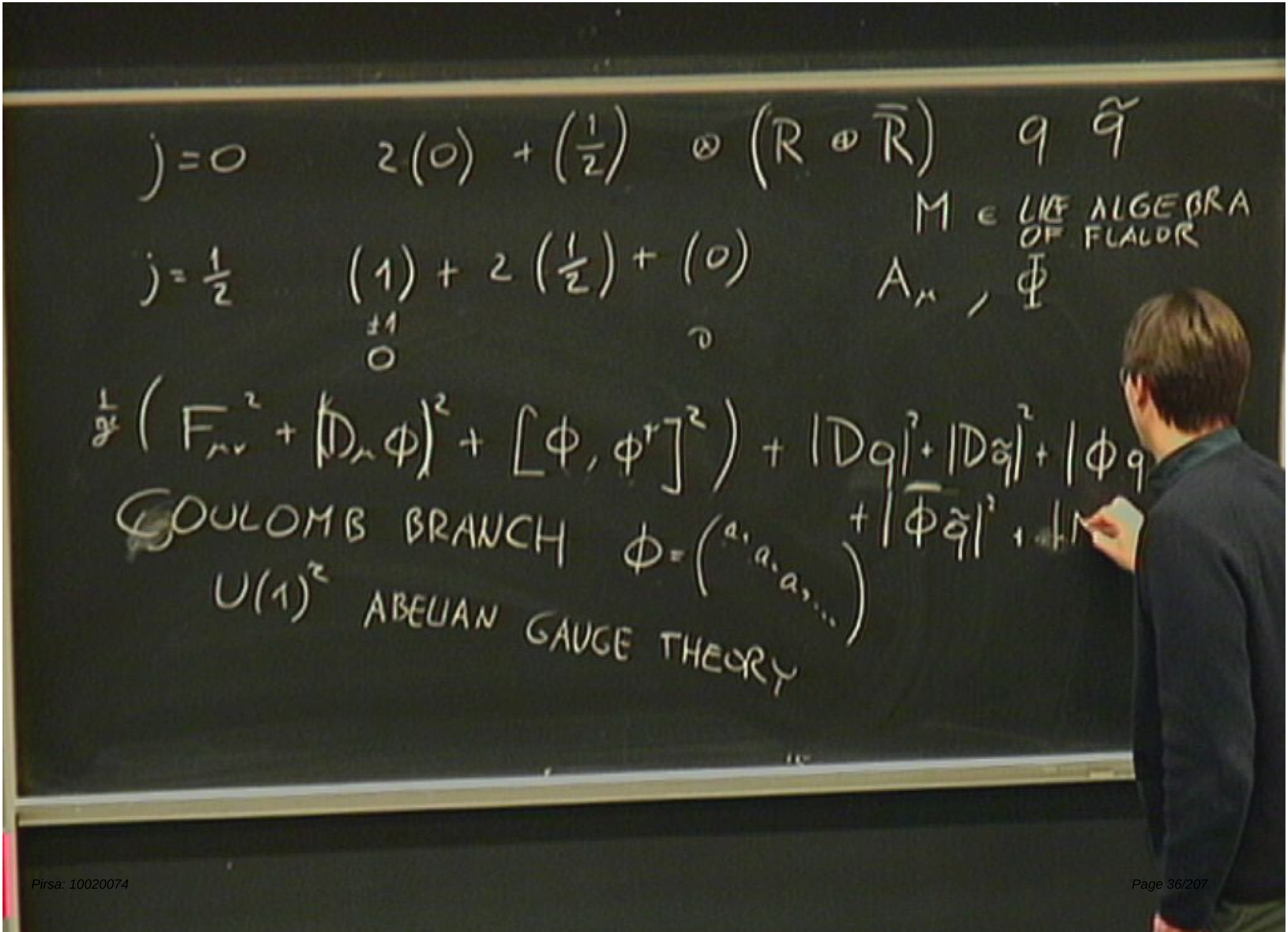
COULOMB BRANCH $\phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & \ddots \end{pmatrix}$

$U(1)^2$ ABELIAN GAUGE THEORY



$j=0 \qquad 2(0) + \left(\tfrac{1}{2}\right) \otimes \left(R \oplus \bar{R}\right) \qquad q \quad \tilde{q}$

$M \in$ ~~LIE~~ LIE ALGEBRA OF FLAVOR

$j=\tfrac{1}{2} \qquad \underset{\substack{\pm 1\\ 0}}{(1)} + 2\left(\tfrac{1}{2}\right) + \underset{0}{(0)} \qquad A_\mu \,,\ \Phi$

$$\frac{1}{g^2}\left(F_{\mu\nu}^2 + |D_\mu \phi|^2 + [\phi, \phi^\dagger]^2 \right) + |Dq|^2 + |D\tilde{q}|^2 + |\phi q$$

$$+ |\bar{\phi}\tilde{q}|^2 + |M$$

COULOMB BRANCH $\phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & \ddots \end{pmatrix}$

$U(1)^r$ ABELIAN GAUGE THEORY

$j=0 \qquad 2(0) + \left(\tfrac{1}{2}\right) \otimes \left(R \oplus \bar{R}\right) \qquad q \quad \tilde{q}$

$M \in$ LIE ALGEBRA OF FLAVOR

$j=\tfrac{1}{2} \qquad \underset{\substack{\pm 1 \\ 0}}{(1)} + 2\left(\tfrac{1}{2}\right) + \underset{0}{(0)} \qquad A_\mu \, , \; \Phi$

$$\frac{1}{g^2}\left(F_{\mu\nu}^2 + |D_\mu \phi|^2 + [\phi, \phi^\dagger]^2\right) + |Dq|^2 + |D\tilde{q}|^2 + |\phi q|^2 + |\bar{\phi}\tilde{q}|^2 + |Mq|^2$$

COULOMB BRANCH $\phi = \begin{pmatrix} a, & & & \\ & a, & & \\ & & a, & \\ & & & \ddots \end{pmatrix}$

$U(1)^2$ ABELIAN GAUGE THEORY

$$\{Q^A, \bar{Q}^B\} = \epsilon^{AB} \gamma^{\mu}_{\alpha\dot{\alpha}} P_{\mu}$$

$$\{Q, Q\} = Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m$$

$$Q^A_{\alpha} \quad A=1,2 \quad , \quad \bar{Q}^A_{\dot{\alpha}}$$

$$\{\bar{Q}, \bar{Q}\} = \bar{Z}$$

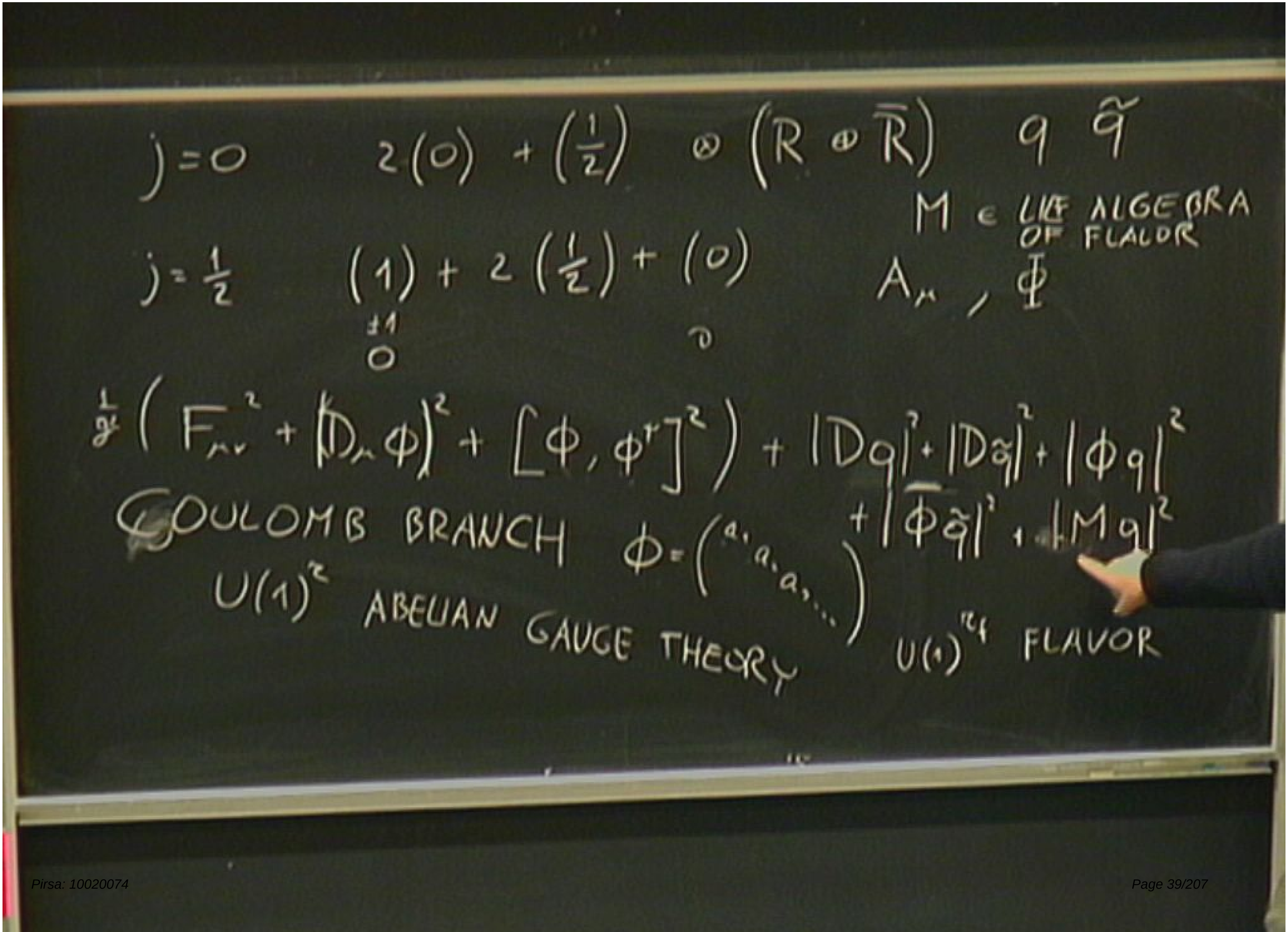
$$a_D = \tau a \qquad \tau$$

$$M > |Z| \qquad \{Q, \bar{Q}\} = \gamma^0 M$$

$$(j) \otimes \left(2(0) + \left(\tfrac{1}{2}\right)\right)^{\otimes 2}$$

$$M = |Z| \qquad \{Q, \bar{Q}\} = (\gamma^0 + \gamma^1) E$$

$$(j) \otimes \left(2(0) + \left(\tfrac{1}{2}\right)\right)$$



$j=0 \qquad 2(0) + \left(\tfrac{1}{2}\right) \otimes \left(R \oplus \bar{R}\right) \qquad q \quad \tilde{q}$

M ∈ LIE ALGEBRA OF FLAVOR

$j=\tfrac{1}{2} \qquad \underset{\substack{\pm 1\\ 0}}{(1)} + 2\left(\tfrac{1}{2}\right) + \underset{0}{(0)} \qquad A_\mu \,,\ \Phi$

$$\frac{1}{g^2}\left(F_{\mu\nu}^2 + |D_\mu\phi|^2 + [\phi,\phi^\dagger]^2\right) + |Dq|^2 + |D\tilde{q}|^2 + |\phi q|^2 + |\bar{\phi}\tilde{q}|^2 + |Mq|^2$$

COULOMB BRANCH $\phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & \ddots \end{pmatrix}$

$U(1)^2$ ABELIAN GAUGE THEORY

$U(1)^{N_f}$ FLAVOR

$$\{Q^A, \bar{Q}^B\} = \epsilon^{AB} \gamma^{\mu}_{\alpha\dot{\alpha}} P_{\mu}$$

$$Q^A_{\alpha} \quad A=1,2 \quad , \quad \bar{Q}^A_{\dot{\alpha}}$$

$$M > |Z| \qquad \{Q, \bar{Q}\} = \gamma^0 M$$

$$M = |Z| \qquad \{Q, \bar{Q}\} = (\gamma^0 + \gamma^1) E$$

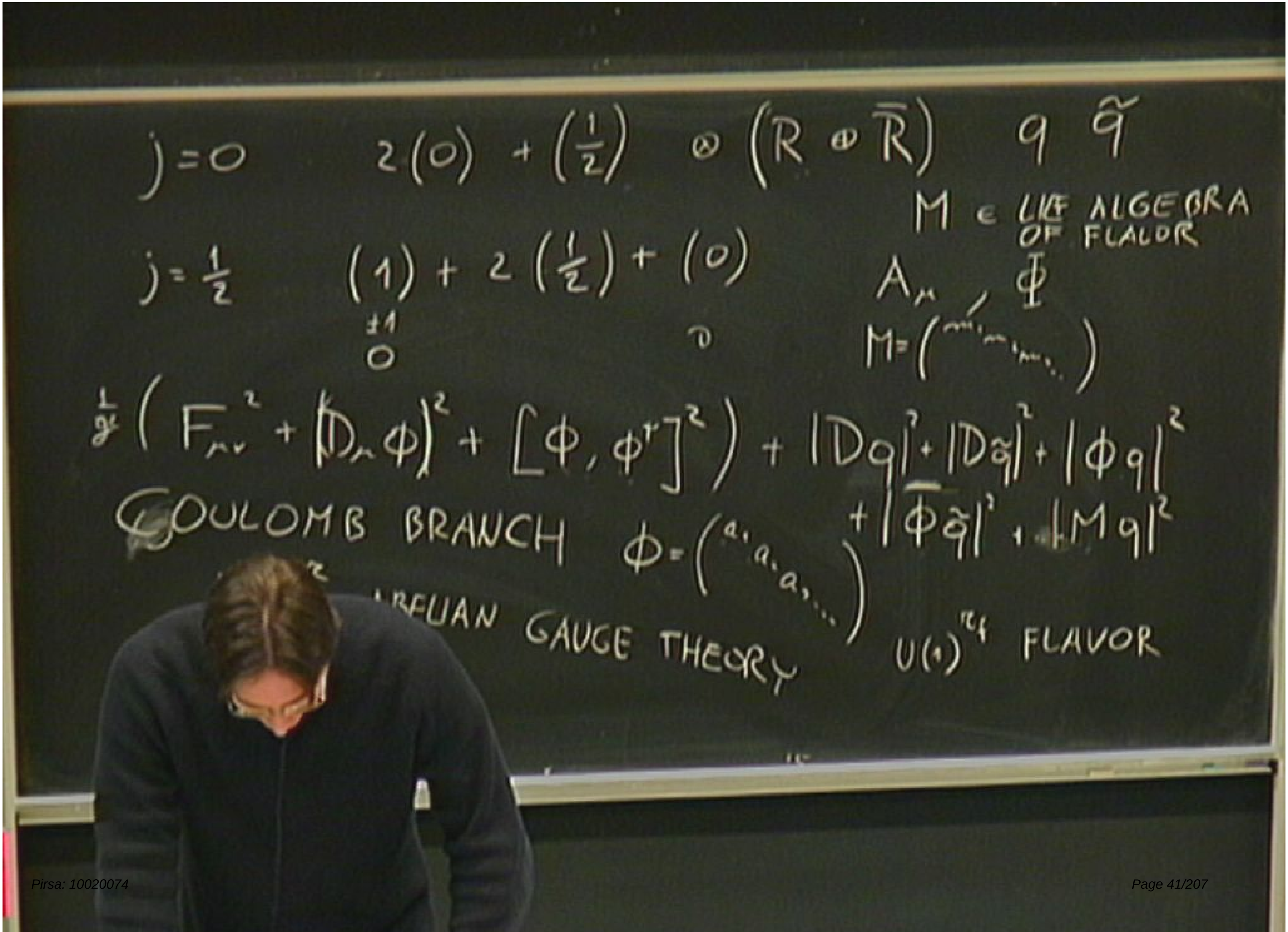
$$\{Q, Q\} = Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m$$

$$\{\bar{Q}, \bar{Q}\} = \bar{Z}$$

$$a_D = \tau a \qquad \tau$$

$$(j) \otimes \left(2(0) + \left(\tfrac{1}{2}\right)\right)^{\otimes 2}$$

$$(j) \otimes \left(2(0) + \left(\tfrac{1}{2}\right)\right)$$



$j=0 \qquad 2(0) + \left(\tfrac{1}{2}\right) \otimes \left(R \oplus \bar{R}\right) \qquad q \quad \tilde{q}$

$M \in$ LIE ALGEBRA OF FLAVOR

$j=\tfrac{1}{2} \qquad (1) + 2\left(\tfrac{1}{2}\right) + (0) \qquad A_\mu \,,\; \Phi$

$\pm 1 \quad 0 \qquad\qquad 0$

$M = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \\ & & & \ddots \end{pmatrix}$

$$\frac{1}{g^2}\left(F_{\mu\nu}^2 + (D_\mu \phi)^2 + [\phi, \phi^\dagger]^2 \right) + |Dq|^2 + |D\tilde{q}|^2 + |\phi q|^2 + |\bar{\Phi}\tilde{q}|^2 + |Mq|^2$$

COULOMB BRANCH $\quad \phi = \begin{pmatrix} a_1 & & \\ & a_2 & \\ & & a_3 \\ & & & \ddots \end{pmatrix}$

BELIAN GAUGE THEORY $\qquad U(1)^{n_f}$ FLAVOR

$j = 0 \qquad 2(0) + \left(\tfrac{1}{2}\right) \otimes \left(R \oplus \bar{R}\right) \qquad q \quad \tilde{q}$

$M \in$ LIE ALGEBRA OF FLAVOR

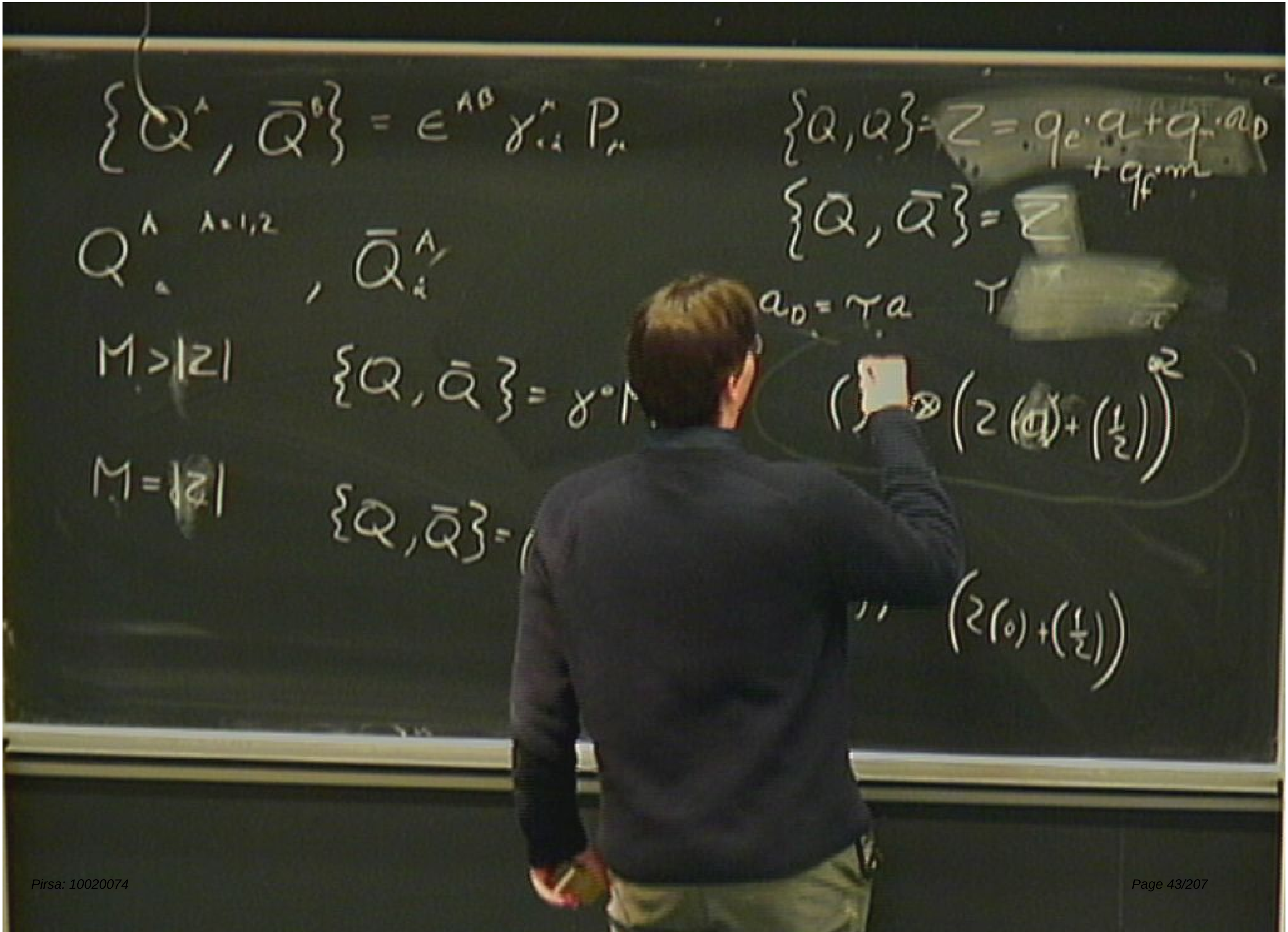
$j = \tfrac{1}{2} \qquad \underset{\substack{\pm 1 \\ 0}}{(1)} + 2\left(\tfrac{1}{2}\right) + \underset{0}{(0)} \qquad A_\mu \,,\; \Phi$

$M = \begin{pmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & \ddots \end{pmatrix}$

$$\frac{1}{g^2}\left(F_{\mu\nu}^2 + |D_\mu \phi|^2 + [\phi, \phi^\dagger]^2 \right) + |Dq|^2 + |D\tilde{q}|^2 + |\phi q|^2 + |\bar{\phi}\tilde{q}|^2 + |Mq|^2$$

COULOMB BRANCH $\quad \phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & \ddots \end{pmatrix}$

$U(1)^2$ ABELIAN GAUGE THEORY $\qquad U(1)^{n_f}$ FLAVOR



$\{Q^A, \bar{Q}^B\} = \epsilon^{AB} \gamma^\mu_{\alpha\dot\alpha} P_\mu$

Q^A_α $A=1,2$, $\bar{Q}^A_{\dot\alpha}$

$M > |Z|$ $\{Q, \bar{Q}\} = \gamma^0 M$

$M = |Z|$ $\{Q, \bar{Q}\} = ($

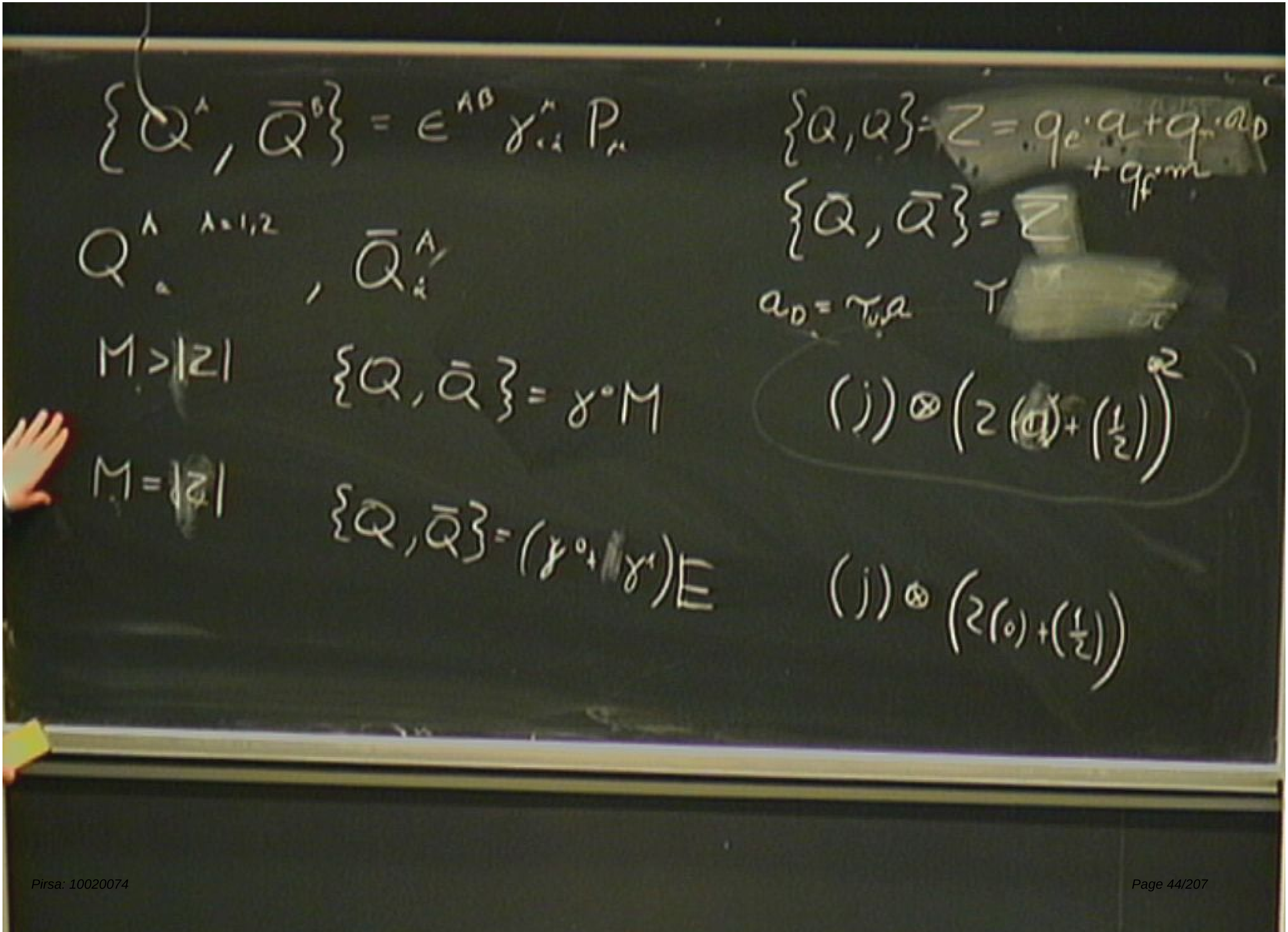
$\{Q, Q\} = Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m$

$\{\bar{Q}, \bar{Q}\} = \bar{Z}$

$a_D = \tau a$ τ

$(\quad) \otimes \left(2(0) + \left(\tfrac{1}{2}\right)\right)^{\otimes 2}$

$'' \quad \left(2(0) + \left(\tfrac{1}{2}\right)\right)$



$$\{Q^A, \bar{Q}^B\} = \epsilon^{AB} \gamma^\mu_{\alpha\dot{\alpha}} P_\mu$$

$$Q^A_\alpha \quad A=1,2 \quad , \quad \bar{Q}^A_{\dot{\alpha}}$$

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$M = |Z|$ $\qquad \{Q, \bar{Q}\} = (\gamma^0 + \gamma^1) E$

$$\{Q, Q\} = Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m$$

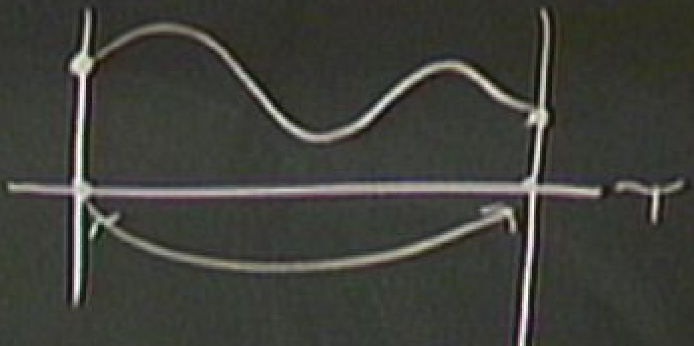
$$\{\bar{Q}, \bar{Q}\} = \bar{Z}$$

$a_D = \tau_U a$ $\quad Y$

$$(j) \otimes \left(2(0) + \left(\tfrac{1}{2}\right)\right)^{\otimes 2}$$

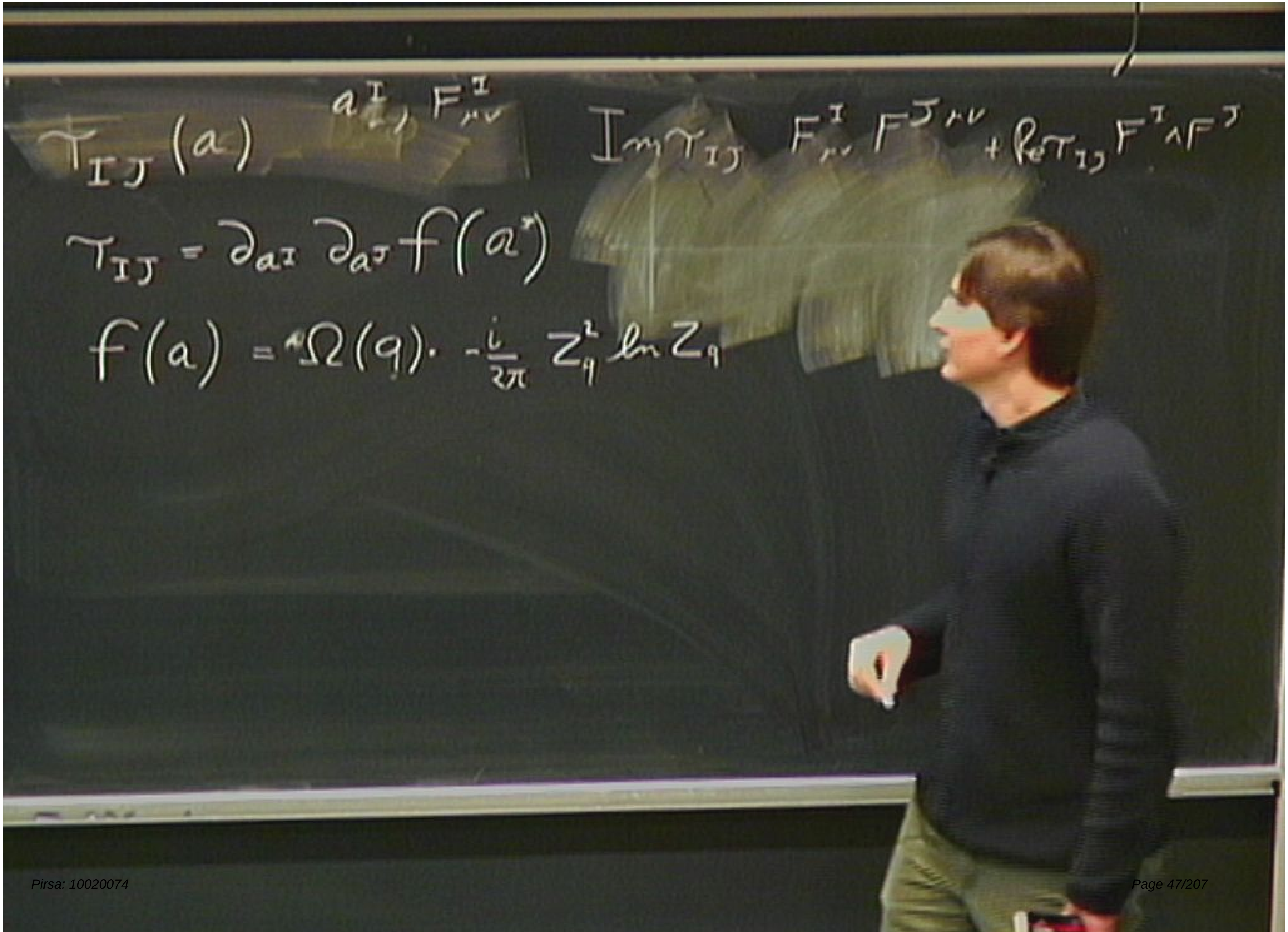
$$(j) \otimes \left(2(0) + \left(\tfrac{1}{2}\right)\right)$$

$$F_{ij} = \epsilon_{ijk} D_k \phi$$



$\tau_{IJ}(a)$

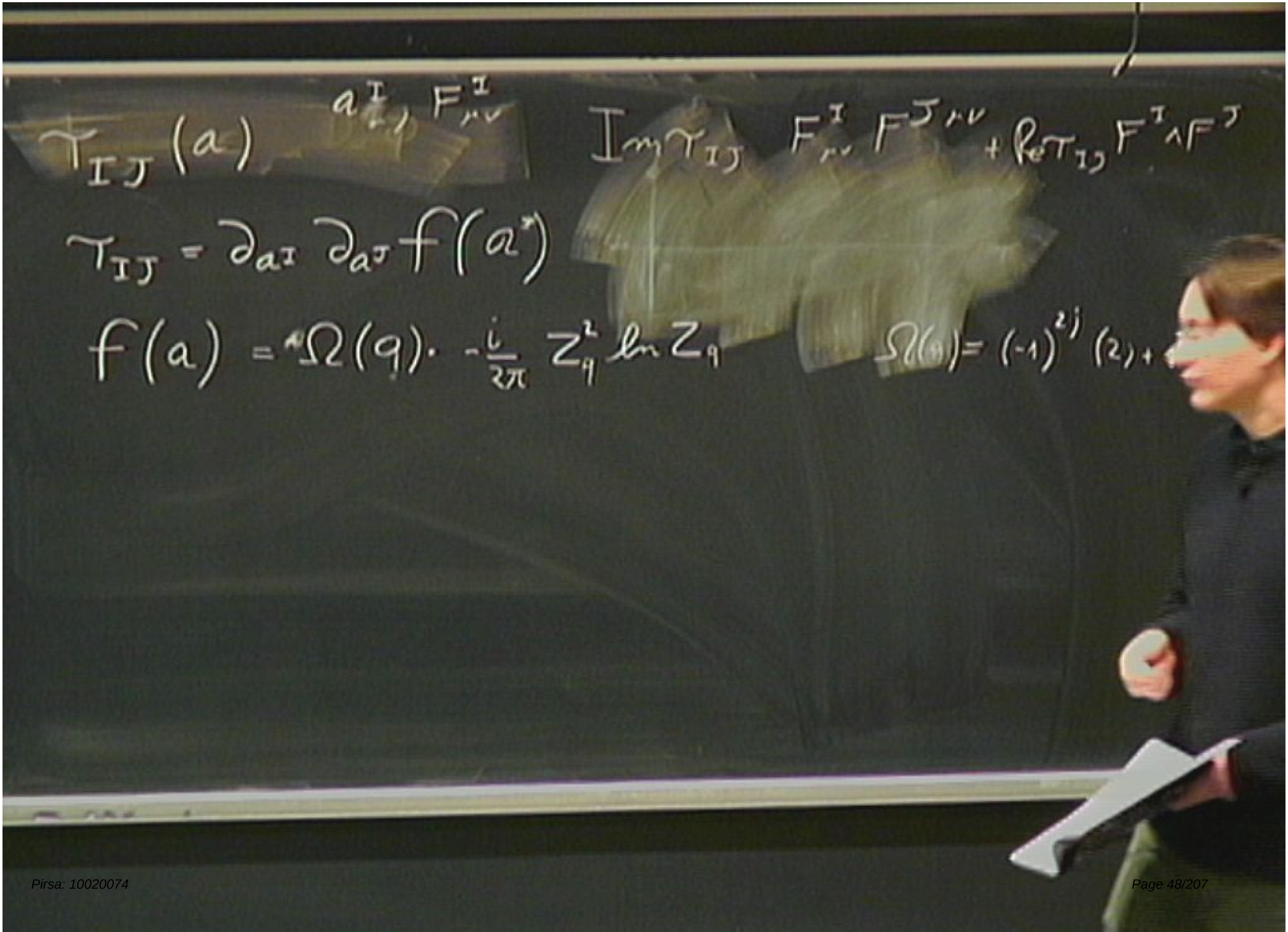
$\mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$



$\tau_{IJ}(a)$ $\quad a^I,\ F^I_{\mu\nu}$ $\quad \mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\,\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$

$$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a^\alpha)$$

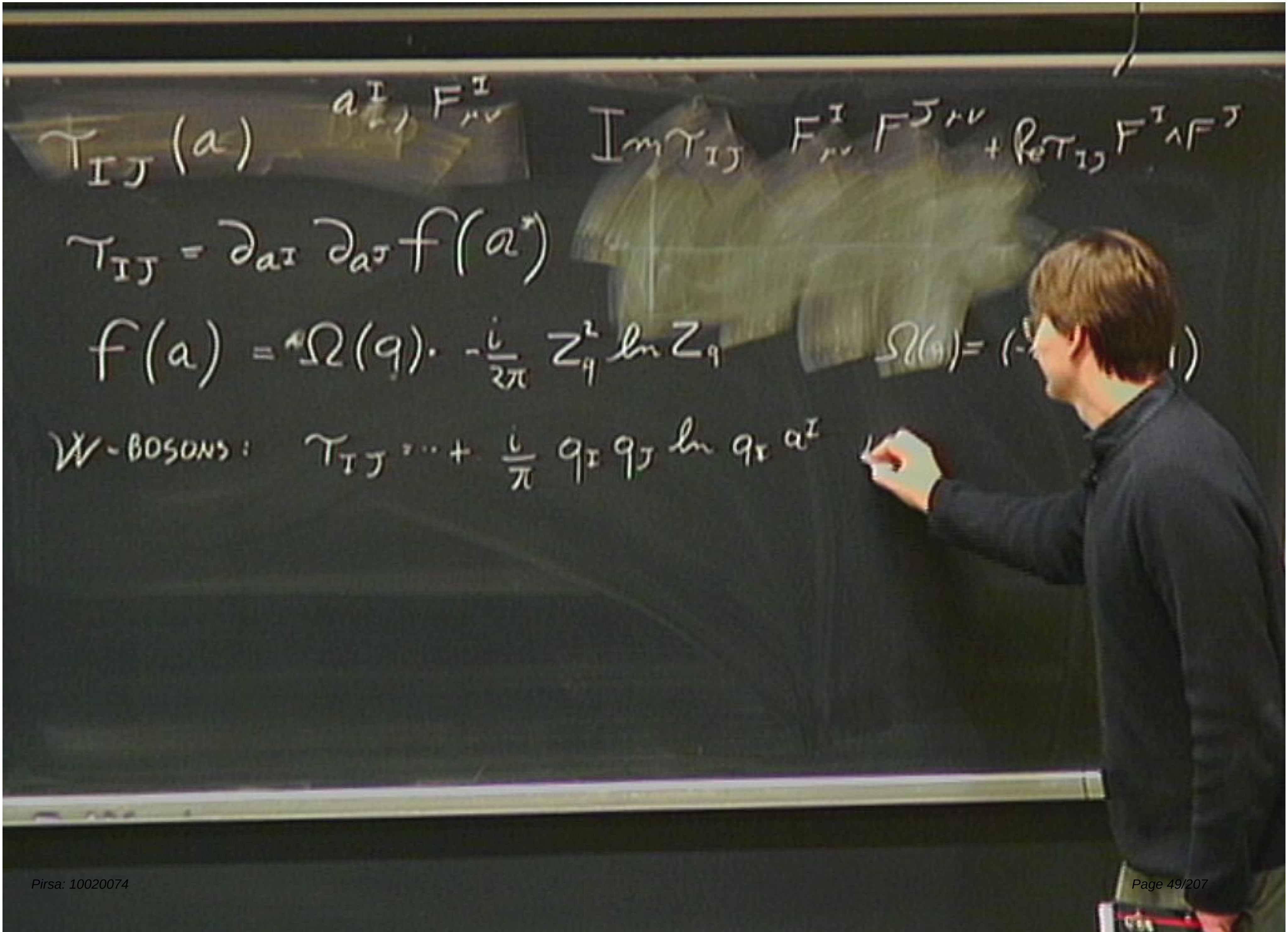
$$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q$$



$\tau_{IJ}(a)$ $\quad a^I_{\ldots},\ F^I_{\mu\nu}$ $\quad \mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\,\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$

$$\tau_{IJ} = \partial_{a^I}\partial_{a^J} f(a^I)$$

$$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q \qquad \Omega(q) = (-1)^{2j}(2j+$$

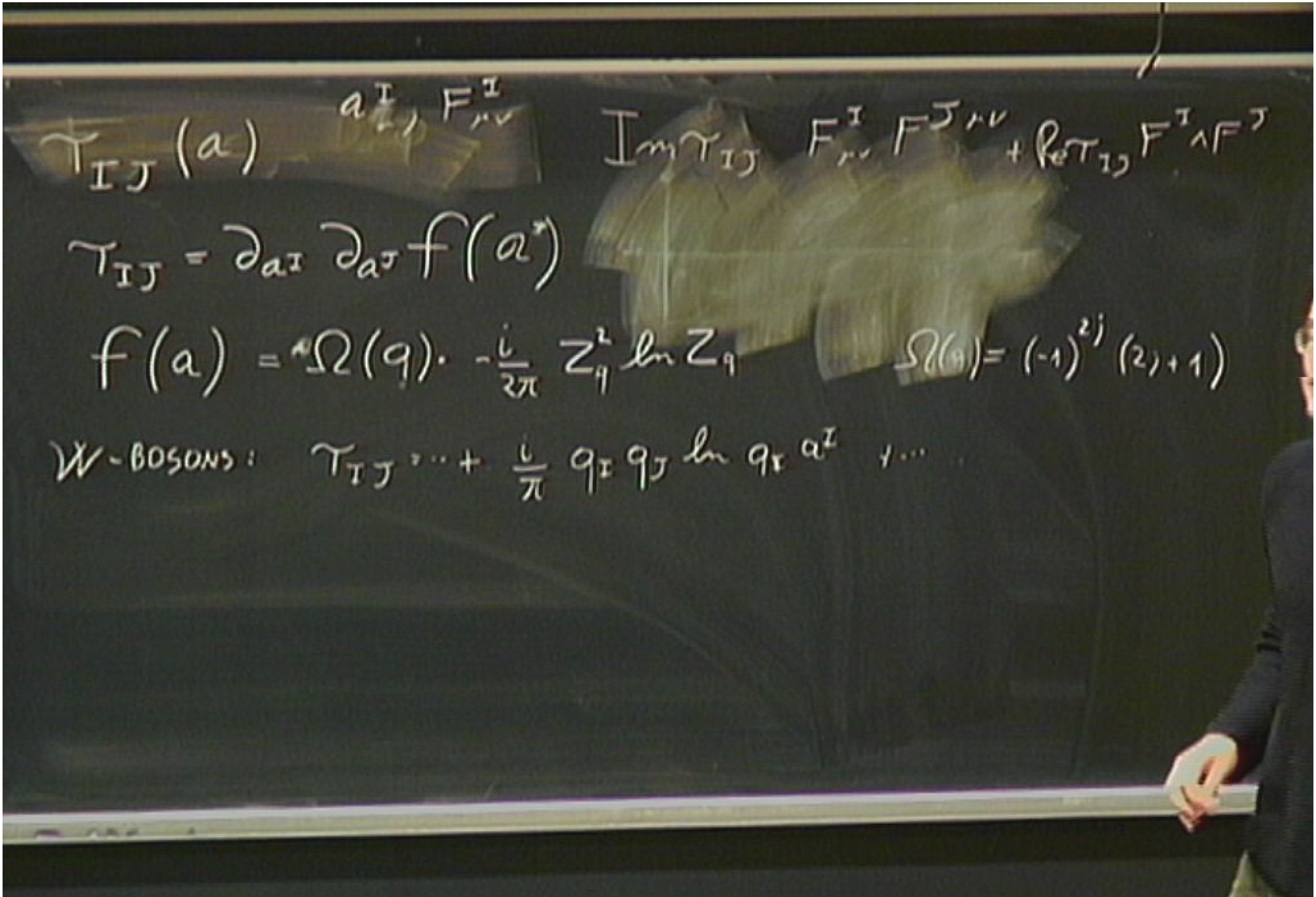


$\tau_{IJ}(a)$ a^I, $F^I_{\mu\nu}$ $\text{Im}\,\tau_{IJ}\,F^I_{\mu\nu}F^{J\mu\nu} + \text{Re}\,\tau_{IJ}\,F^I \wedge F^J$

$\tau_{IJ} = \partial_{a^I}\partial_{a^J} f(a^I)$

$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q$ $\Omega(q) = (-\quad)$

W-BOSONS: $\tau_{IJ} = \cdots + \frac{i}{\pi} q_I q_J \ln q_I a^I$

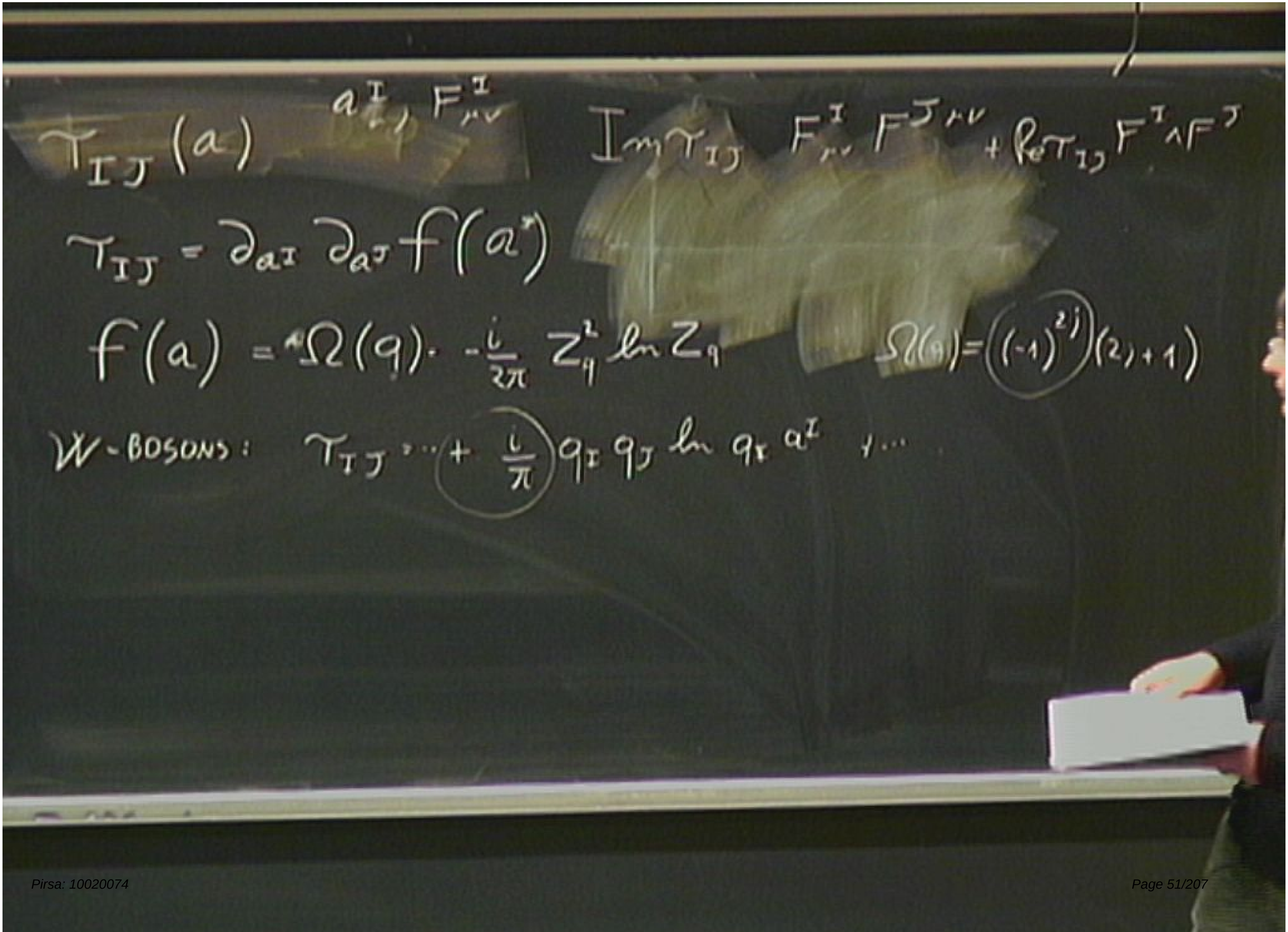


$\tau_{IJ}(a)$ $\quad a^I, F^I_{\mu\nu}$ $\quad \mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\,\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$

$$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a^I)$$

$$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q \qquad \Omega(q) = (-1)^{2j}(2j+1)$$

W-BOSONS: $\tau_{IJ} = \cdots + \frac{i}{\pi} q_I q_J \ln q_I a^I + \cdots$

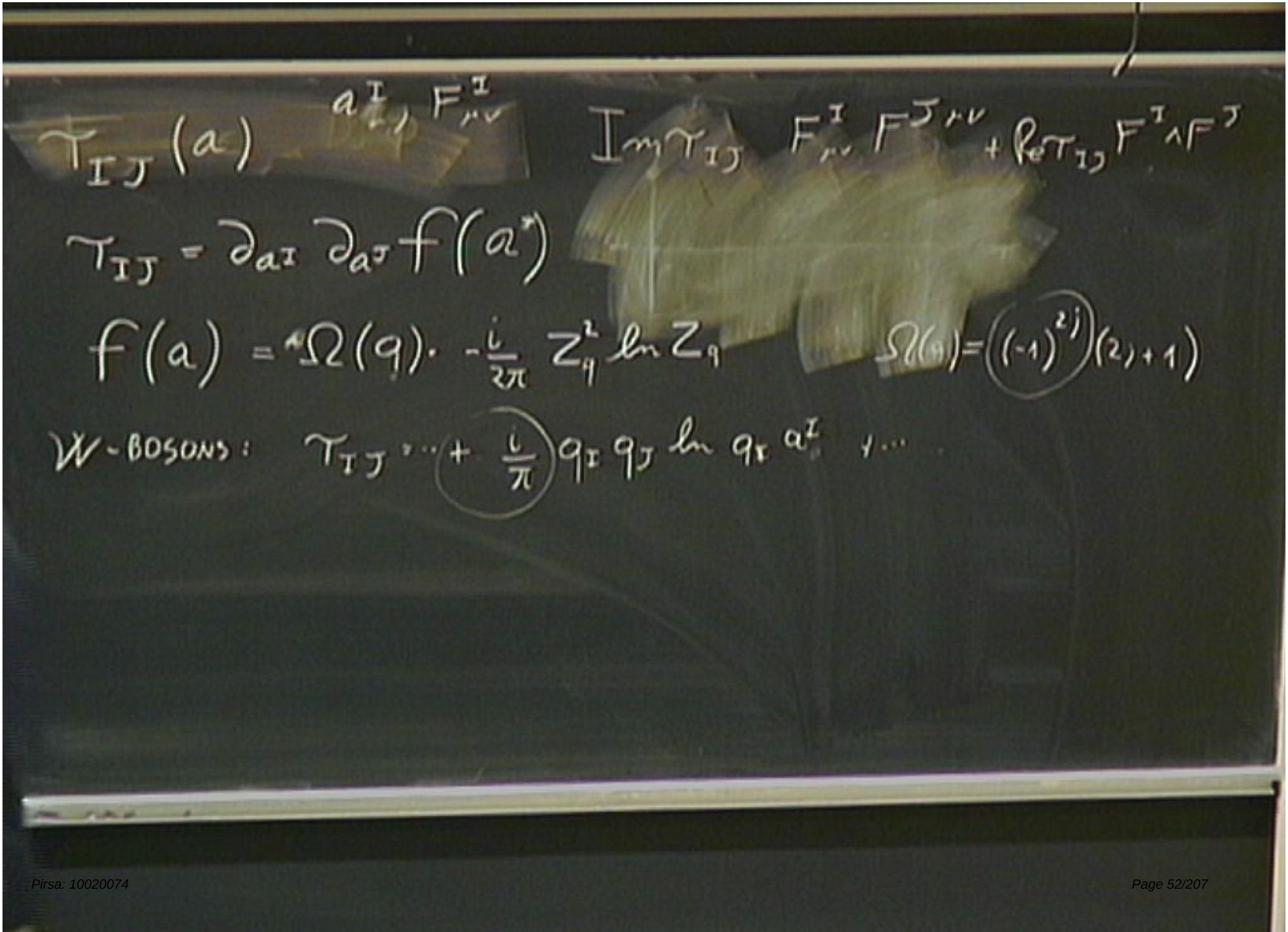


$\tau_{IJ}(a)$ a^I, $F^I_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\,\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$

$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a^n)$

$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q$ $\Omega(q) = \boxed{(-1)^{2j}}(2j+1)$

W-BOSONS: $\tau_{IJ} = \cdots \boxed{+ \frac{i}{\pi}} q_I q_J \ln q_I a^I + \cdots$

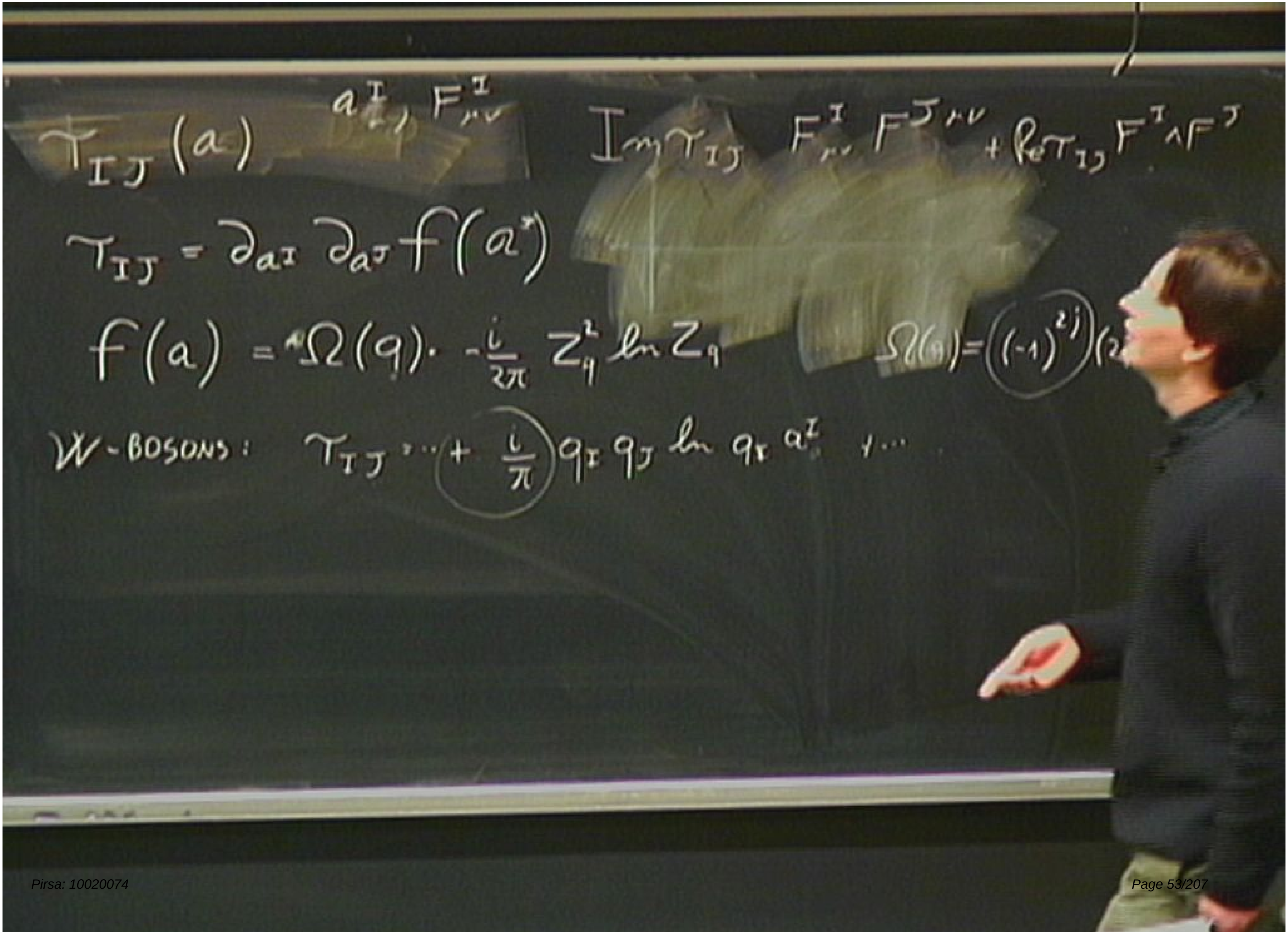


$\tau_{IJ}(a)$ $a^I, F^I_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ} F^I_{\mu\nu} F^{J\mu\nu} + \mathrm{Re}\,\tau_{IJ} F^I \wedge F^J$

$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a^*)$

$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q$ $\Omega(q) = \boxed{(-1)^{2j}}(2j+1)$

W-BOSONS: $\tau_{IJ} = \cdots \boxed{+ \frac{i}{\pi}} q_I q_J \ln q_I a^I + \cdots$



$\tau_{IJ}(a)$ $\quad a^I,\ F^I_{\mu\nu}$ $\quad \mathrm{Im}\,\tau_{IJ}\ F^I_{\mu\nu} F^{J\,\mu\nu} + \mathrm{Re}\,\tau_{IJ} F^I \wedge F^J$

$$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a^I)$$

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W-BOSONS: $\tau_{IJ} = \dots + \frac{i}{\pi} q_I q_J \ln q_I a^I + \dots$

$\tau_{IJ}(a)$ $a^I, F^I_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\,\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$

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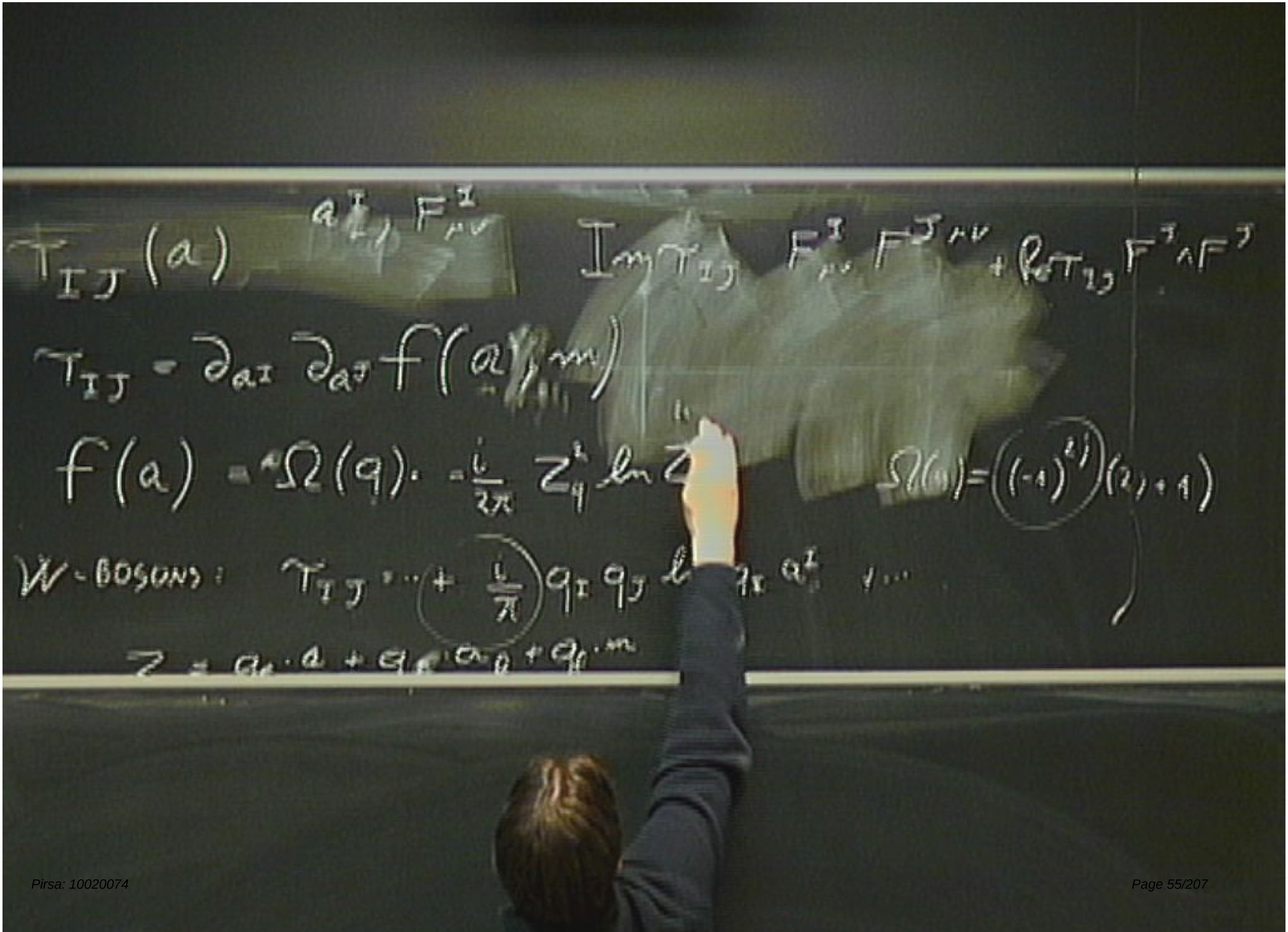
W-BOSONS: $\tau_{IJ} = \dots \boxed{+ \frac{i}{\pi}} q_I q_J \ln q_I a^I + \dots$



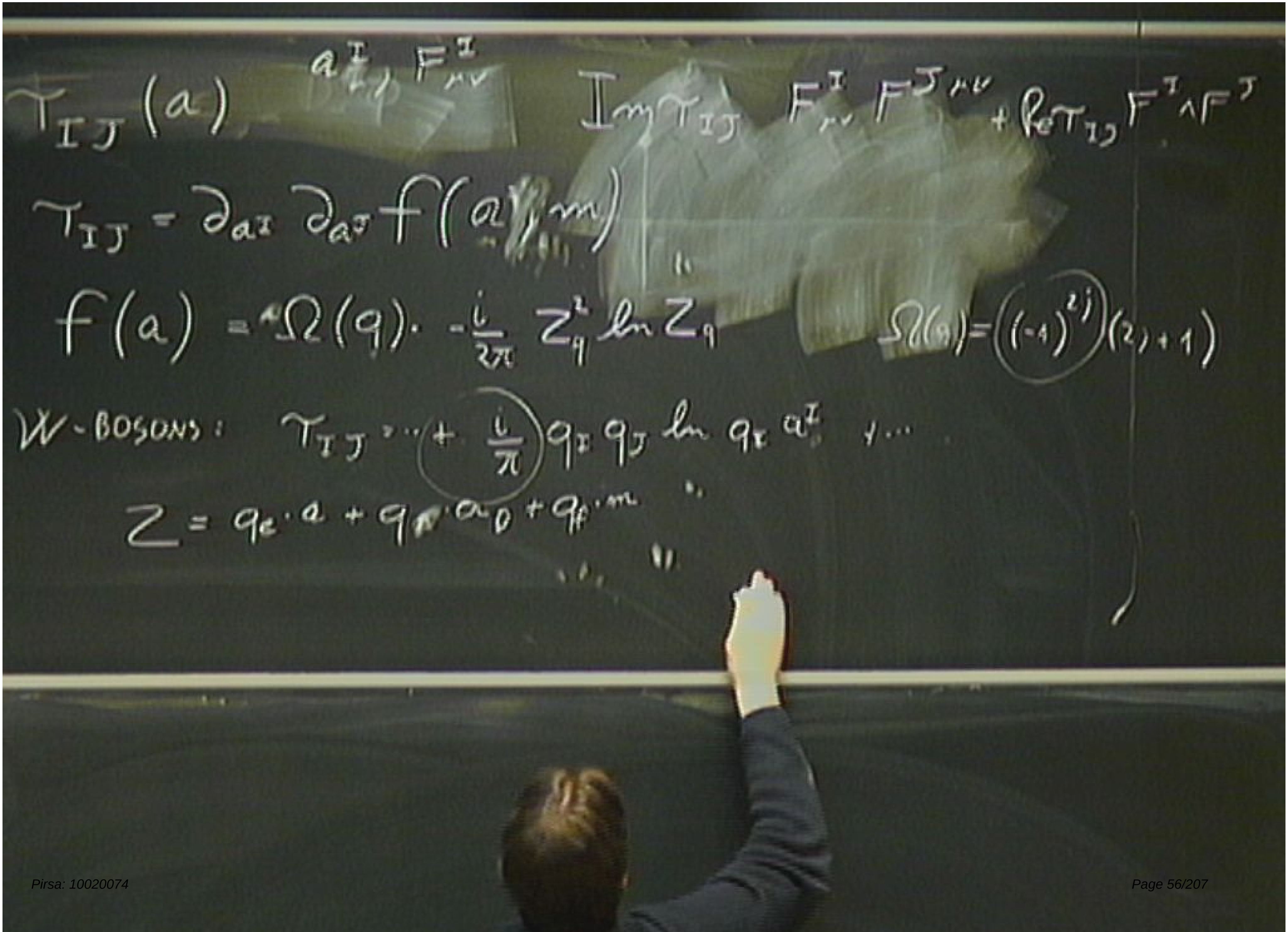
$\tau_{IJ}(a)$ $a^I_\mu, F^I_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$

$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a, m)$

$f(a) = \Omega(q) \cdot \frac{i}{2\pi} Z_q^2 \ln Z$ $\Omega(q) = \left((-1)^{2j}\right)(2j+1)$

W-BOSONS: $\tau_{IJ} = \dots + \frac{i}{\pi} q_I q_J \ell \; q_K a^K, \dots$

$Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m$



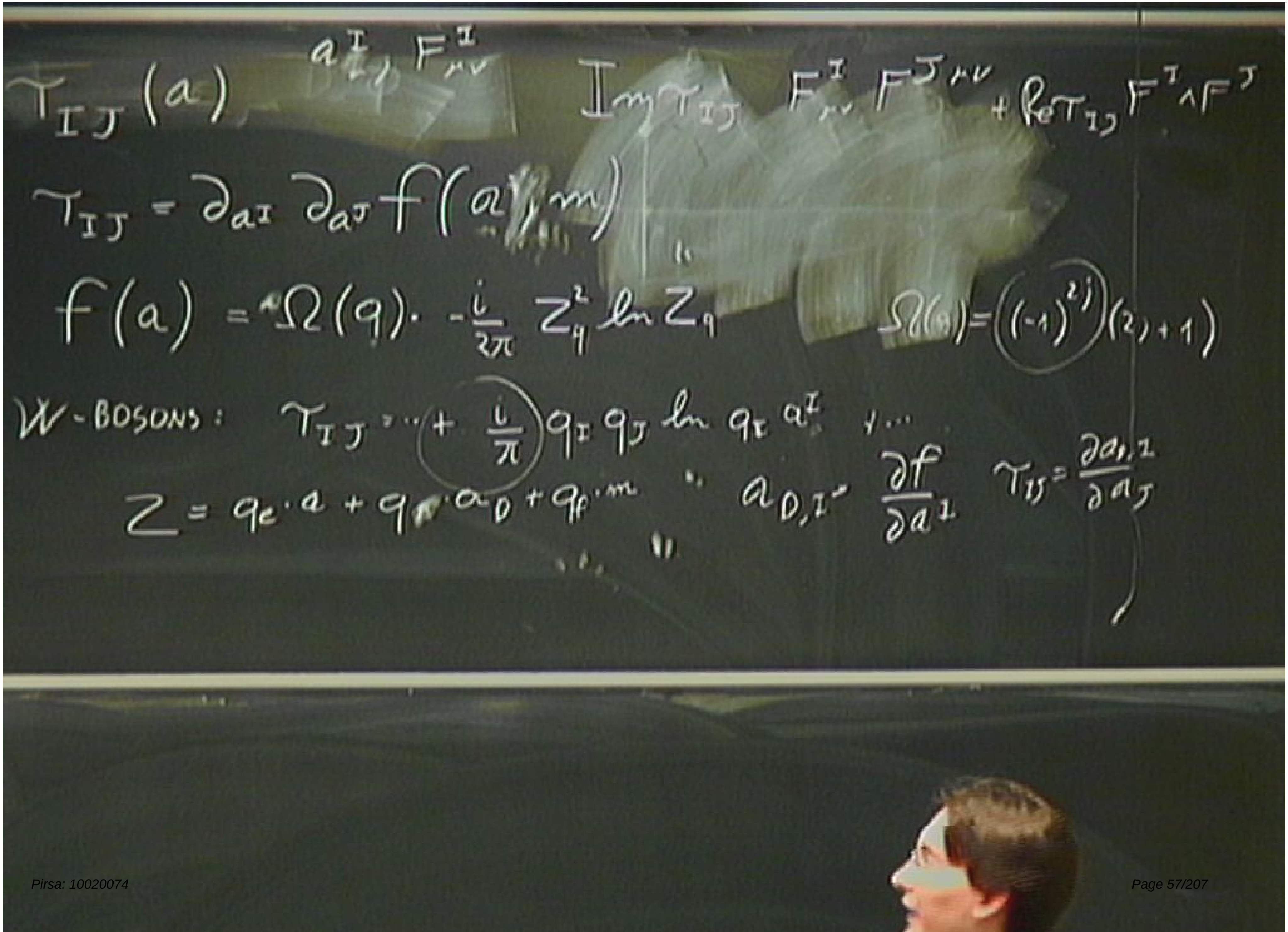
$\tau_{IJ}(a)$ a^I_{μ}, $F^I_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ} F^I_{\mu\nu} F^{J\mu\nu} + \mathrm{Re}\,\tau_{IJ} F^I \wedge F^J$

$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a, m)$

$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q$ $\Omega(q) = ((-1)^{2j})(2j+1)$

W-BOSONS: $\tau_{IJ} = \dots + \frac{i}{\pi} q_I q_J \ln q_I a^I + \dots$

$Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m$



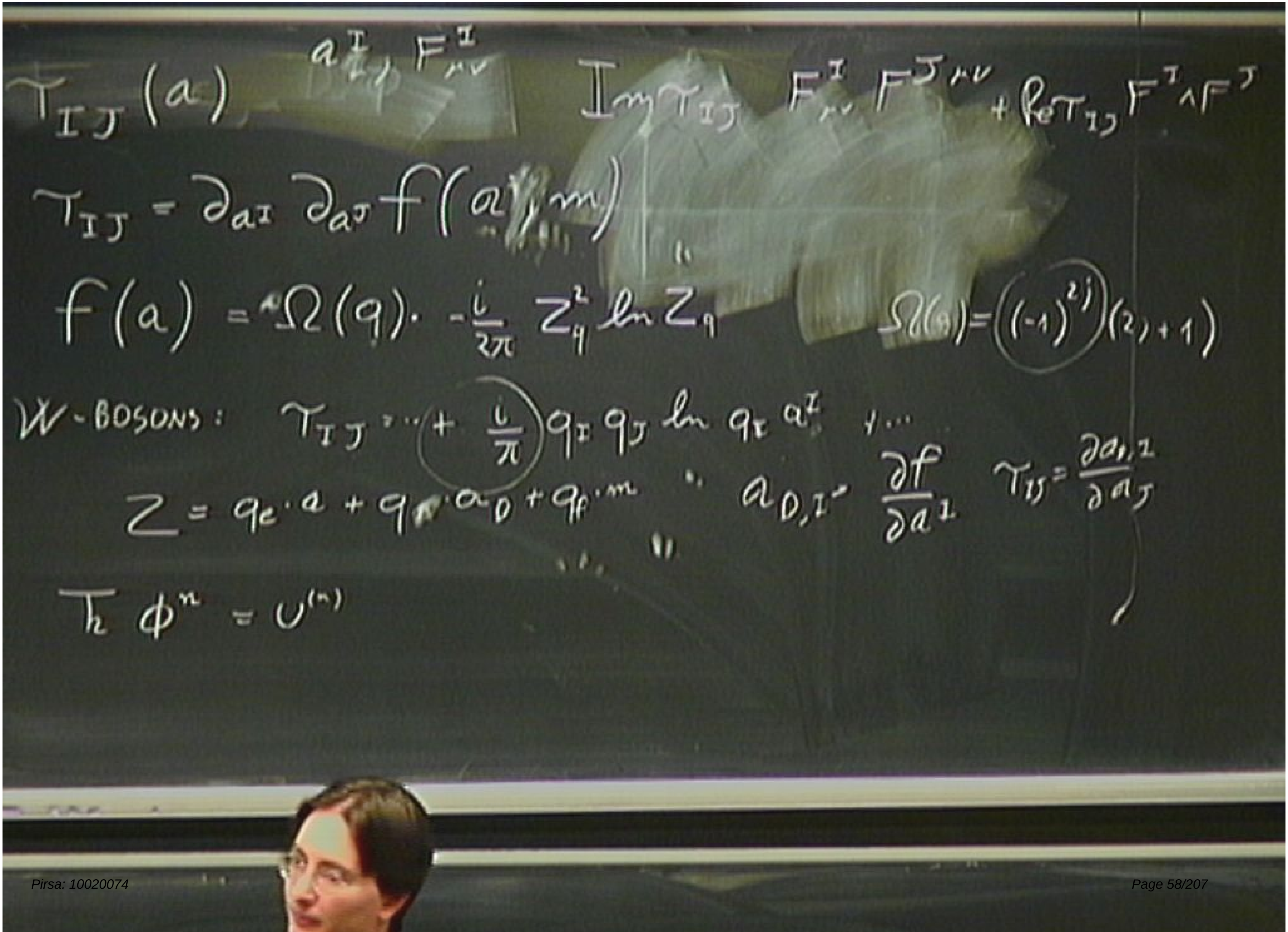
$\tau_{IJ}(a)$ a^I, $F^I_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ} F^I_{\mu\nu} F^{J\mu\nu} + \mathrm{Re}\,\tau_{IJ} F^I \wedge F^J$

$$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a^I, m)$$

$$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q \qquad \Omega(q) = \boxed{(-1)^{2j}}(2j+1)$$

W-BOSONS: $\tau_{IJ} = \dots \boxed{+ \frac{i}{\pi}} q_I q_J \ln q_I a^I + \dots$

$$Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m \qquad a_{D,I} = \frac{\partial f}{\partial a^I} \qquad \tau_{IJ} = \frac{\partial a_{D,I}}{\partial a_J}$$



$\tau_{IJ}(a)$ a^I, $F^I_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ} F^I_{\mu\nu} F^{J\mu\nu} + \mathrm{Re}\,\tau_{IJ} F^I \wedge F^J$

$$\tau_{IJ} = \partial_{a^I} \partial_{a^J} f(a, m)$$

$$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q \qquad \Omega(q) = \boxed{(-1)^{2j}}(2j+1)$$

W-BOSONS: $\tau_{IJ} = \cdots \boxed{+ \frac{i}{\pi}} q_I q_J \ln q_I a^I + \cdots$

$$Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m \qquad a_{D,I} = \frac{\partial f}{\partial a^I} \qquad \tau_{IJ} = \frac{\partial a_{D,I}}{\partial a_J}$$

$$\mathrm{Tr}\, \phi^n = u^{(n)}$$

$\tau_{IJ}(a)$ a^I_{i}, $F^I_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\,\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$

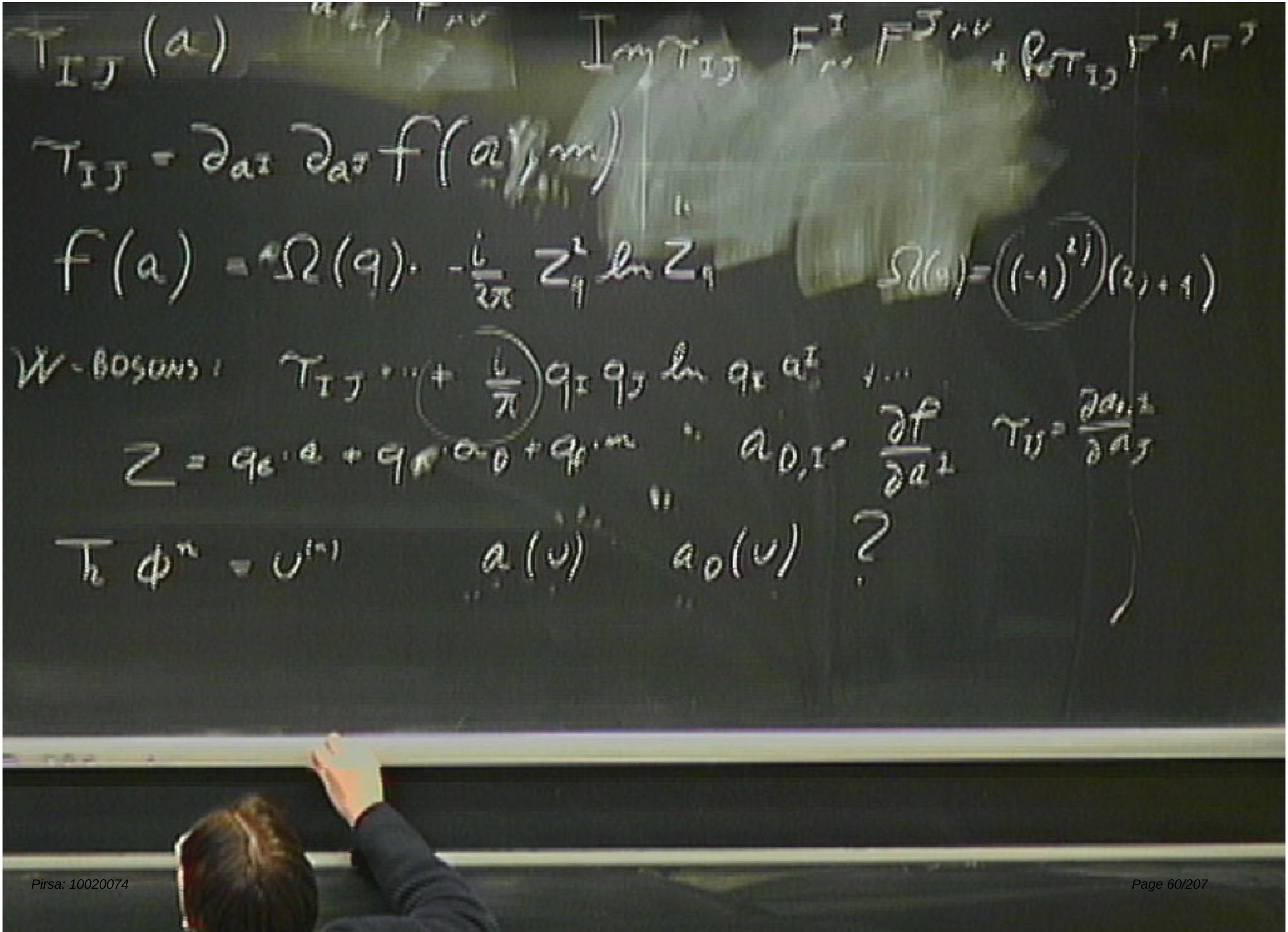
$\tau_{IJ} = \partial_{a^I}\partial_{a^J} f(a^I, m)$

$f(a) = \Omega(q)\cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q$ $\Omega(q) = \boxed{(-1)^{2j}}(2j+1)$

W-BOSONS: $\tau_{IJ} = \cdots \boxed{+ \frac{i}{\pi}} q_I q_J \ln q_I a^I + \cdots$

$Z = q_e\cdot a + q_m\cdot a_D + q_f\cdot m$; $a_{D,I} = \frac{\partial f}{\partial a^I}$ $\tau_{IJ} = \frac{\partial a_{D,I}}{\partial a_J}$

$\mathrm{Tr}\,\phi^n = u^{(n)}$ $a(u)$ $a_D(u)$



$\tau_{IJ}(a)$ $a^I, F_{\mu\nu}$ $\mathrm{Im}\,\tau_{IJ}\, F^I_{\mu\nu} F^{J\mu\nu} + \mathrm{Re}\,\tau_{IJ}\, F^I \wedge F^J$

$\tau_{IJ} = \partial_{a^I}\partial_{a^J} f(a^I, m)$

$f(a) = \Omega(q) \cdot -\frac{i}{2\pi} Z_q^2 \ln Z_q$ $\Omega(q) = ((-1)^{2j})(2j+1)$

W-BOSONS: $\tau_{IJ} + \dots + \frac{i}{\pi} q_I q_J \ln q_K a^K + \dots$

$Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m$; $a_{D,I} = \frac{\partial f}{\partial a^I}$ $\tau_{IJ} = \frac{\partial a_{D,I}}{\partial a^J}$

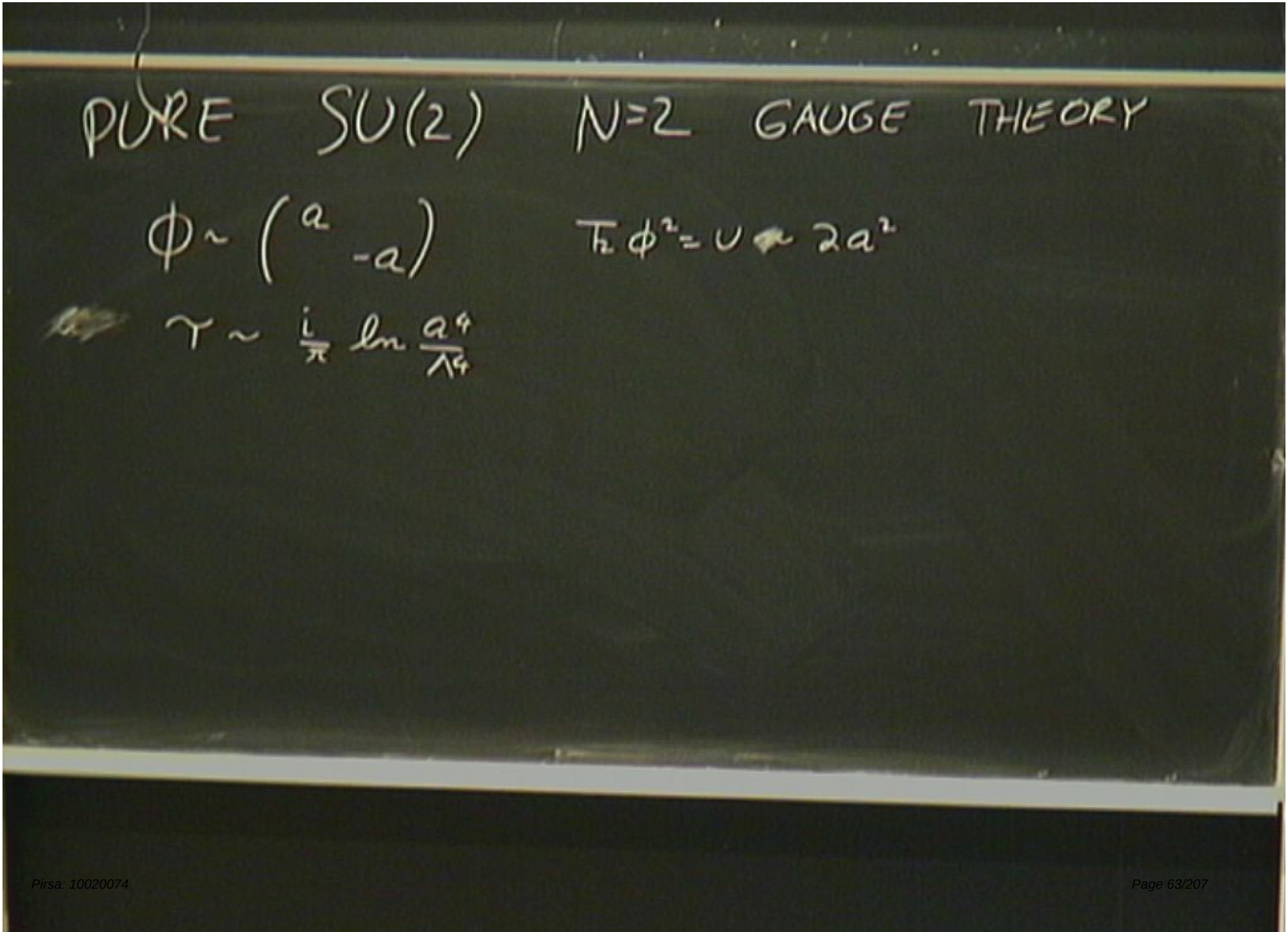
$\mathrm{Tr}\,\phi^n = u^{(n)}$ $a(u)$ $a_D(u)$?

PURE SU(2) $N=2$ GAUGE THEORY

PURE SU(2) N=2 GAUGE THEORY

$\phi \sim \begin{pmatrix} a & \\ & -a \end{pmatrix}$

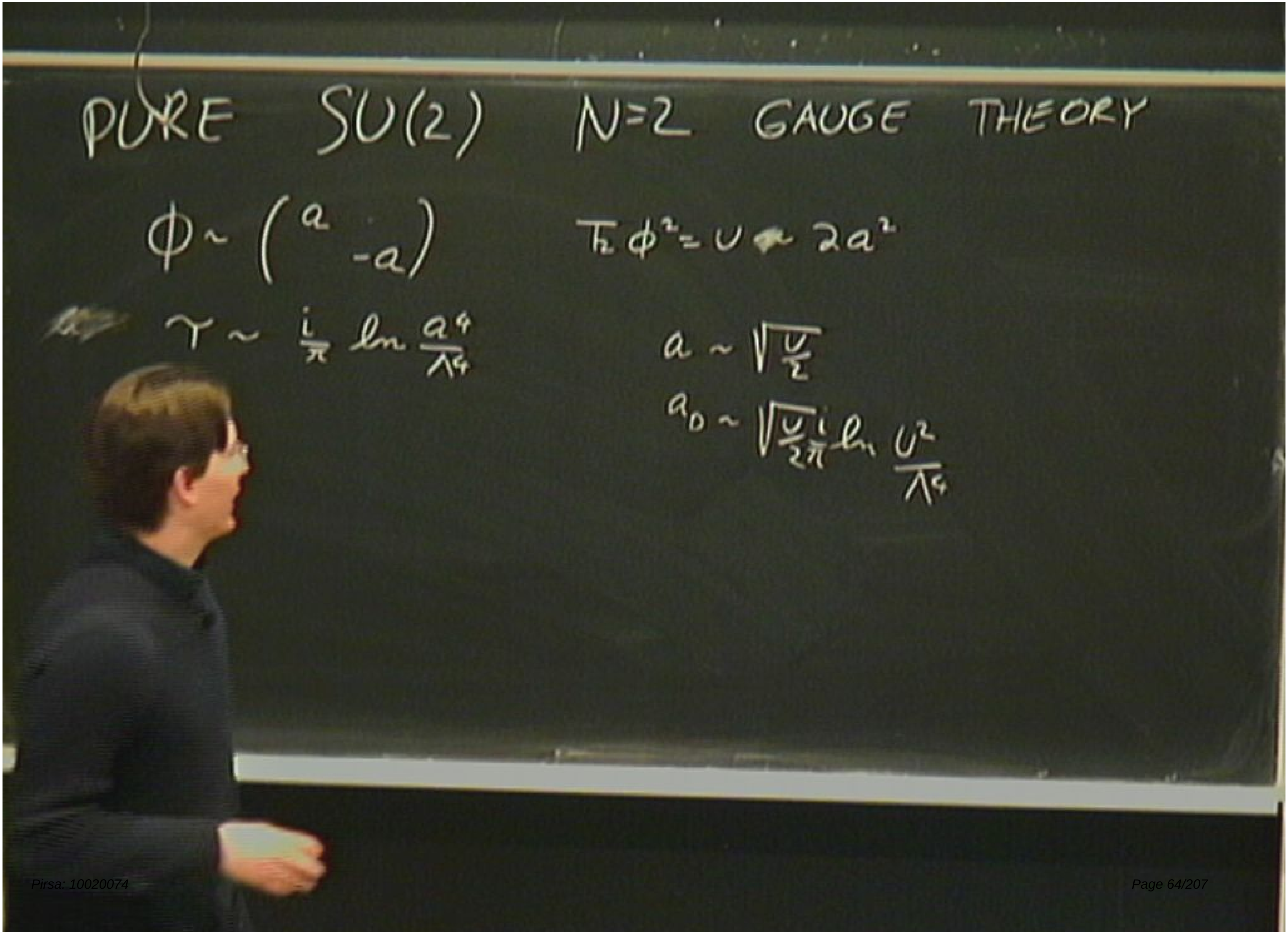
$\mathrm{Tr}\,\phi^2 = u \sim 2a^2$



PURE SU(2) N=2 GAUGE THEORY

$$\phi \sim \begin{pmatrix} a & \\ & -a \end{pmatrix} \qquad \mathrm{Tr}\,\phi^2 = u \quad 2a^2$$

$$\tau \sim \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4}$$



PURE SU(2) N=2 GAUGE THEORY

$$\phi \sim \begin{pmatrix} a & \\ & -a \end{pmatrix} \qquad \mathrm{Tr}\,\phi^2 = u \quad 2a^2$$

$$\tau \sim \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4}$$

$$a \sim \sqrt{\frac{u}{2}}$$

$$a_D \sim \sqrt{\frac{u}{2}}\,\frac{i}{\pi} \ln \frac{u^2}{\Lambda^4}$$

PURE SU(2) N=2 GAUGE THEORY

$\Phi \sim \begin{pmatrix} a & \\ & -a \end{pmatrix}$

$\mathrm{Tr}\,\phi^2 = u = 2a^2$

$\tau \sim \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4}$

$a = \sqrt{\frac{u}{2}} + ?$

$a_D = \sqrt{\frac{u}{2}}\,\frac{i}{\pi} \ln \frac{u^2}{\Lambda^4} + ?$

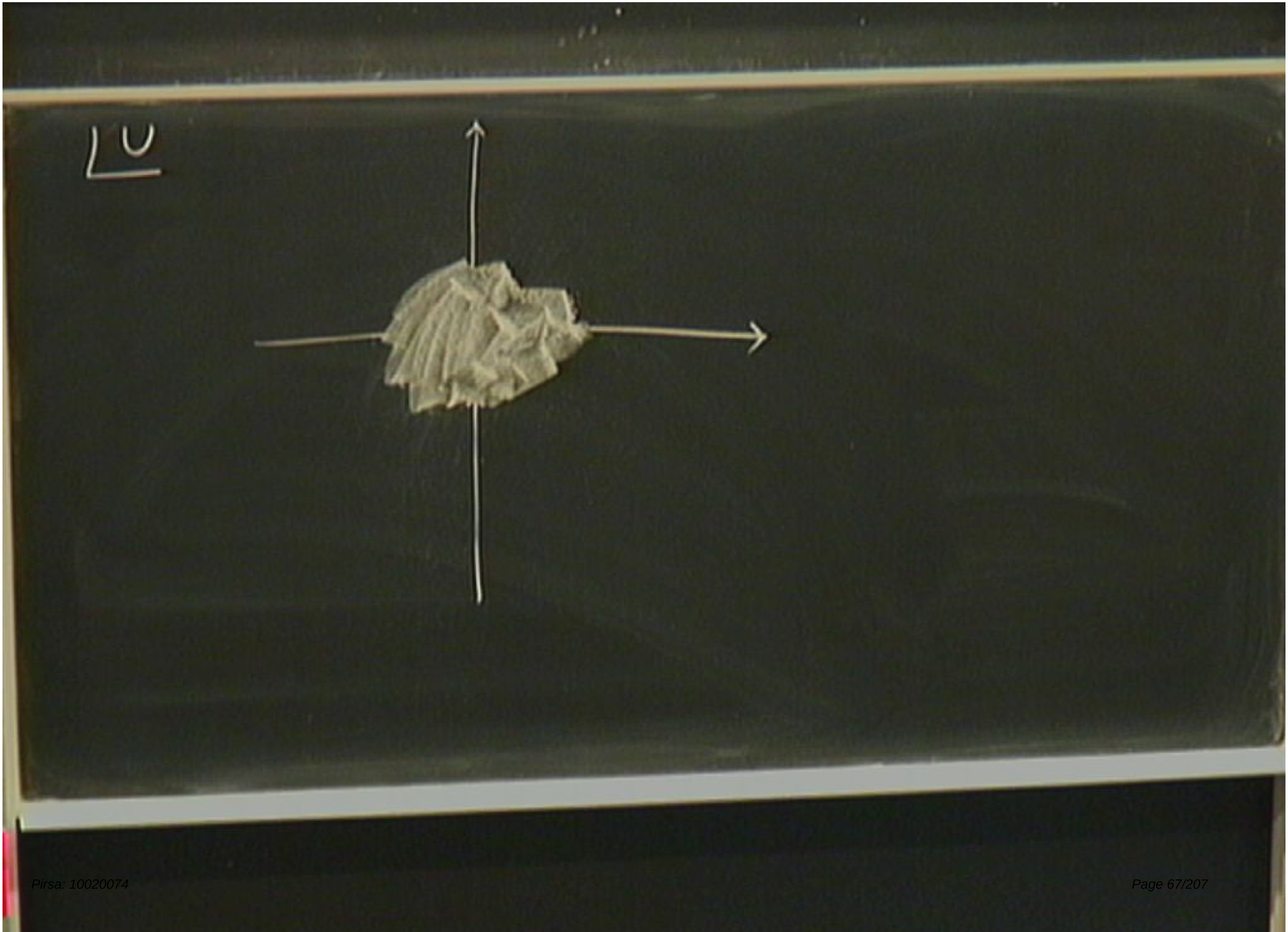
PURE SU(2) N=2 GAUGE THEORY

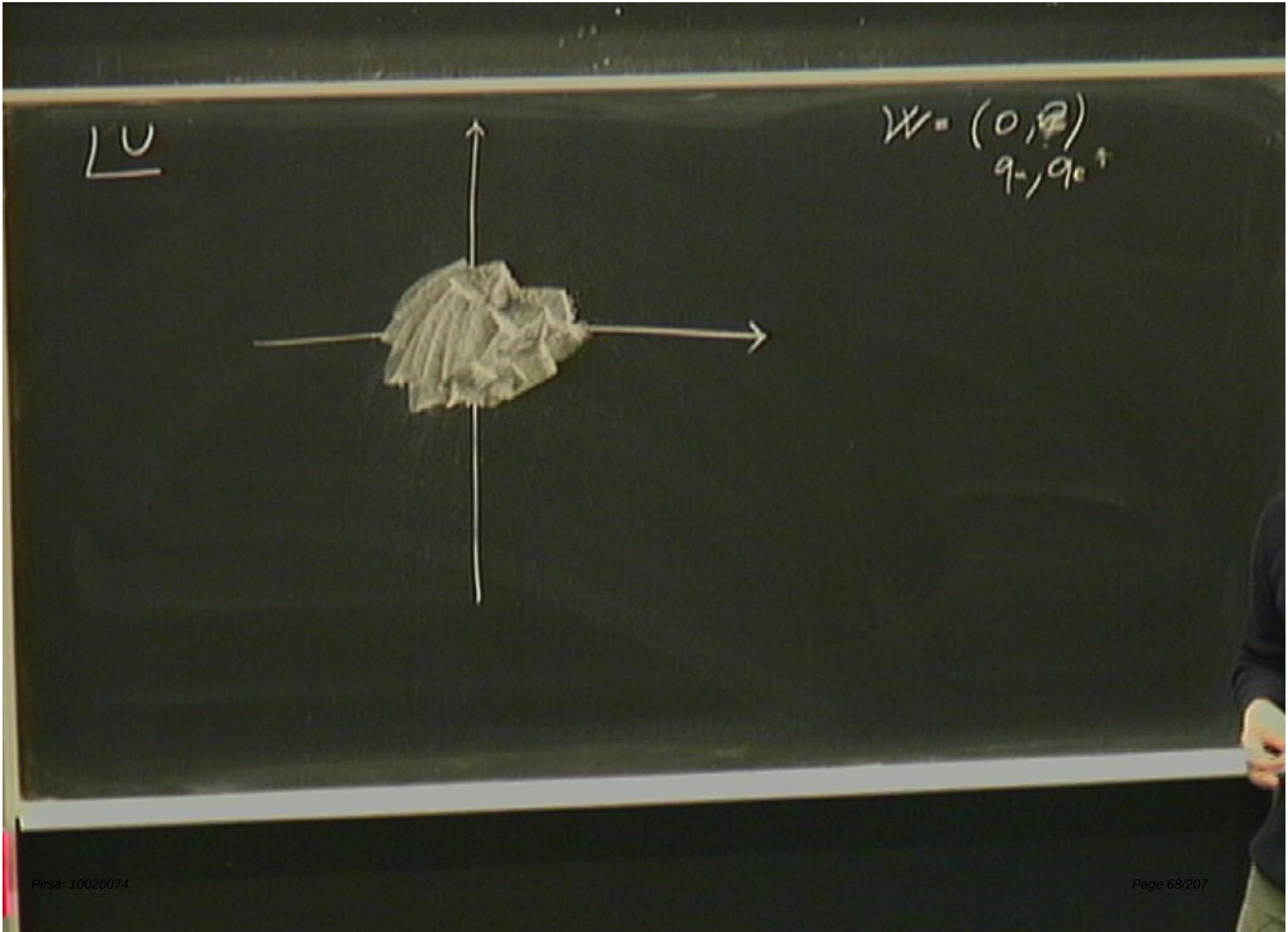
$\phi \sim \begin{pmatrix} a & \\ & -a \end{pmatrix}$ $\quad \mathrm{Tr}\,\phi^2 = u = 2a^2$

$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} + \cdots$

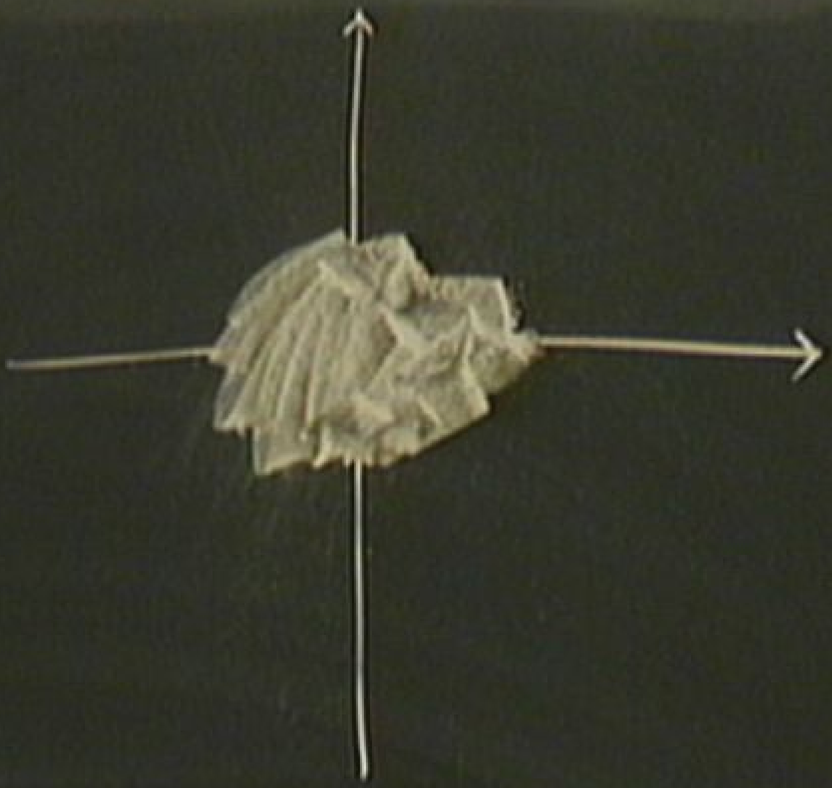
$a = \sqrt{\frac{u}{2}} + ?$

$a_D = \sqrt{\frac{u}{2}}\frac{i}{\pi} \ln \frac{u^2}{\Lambda^4} + ?$


L^U


LU
$W = (0, \varphi)$
$q_-, q_0^\dagger$

L^U

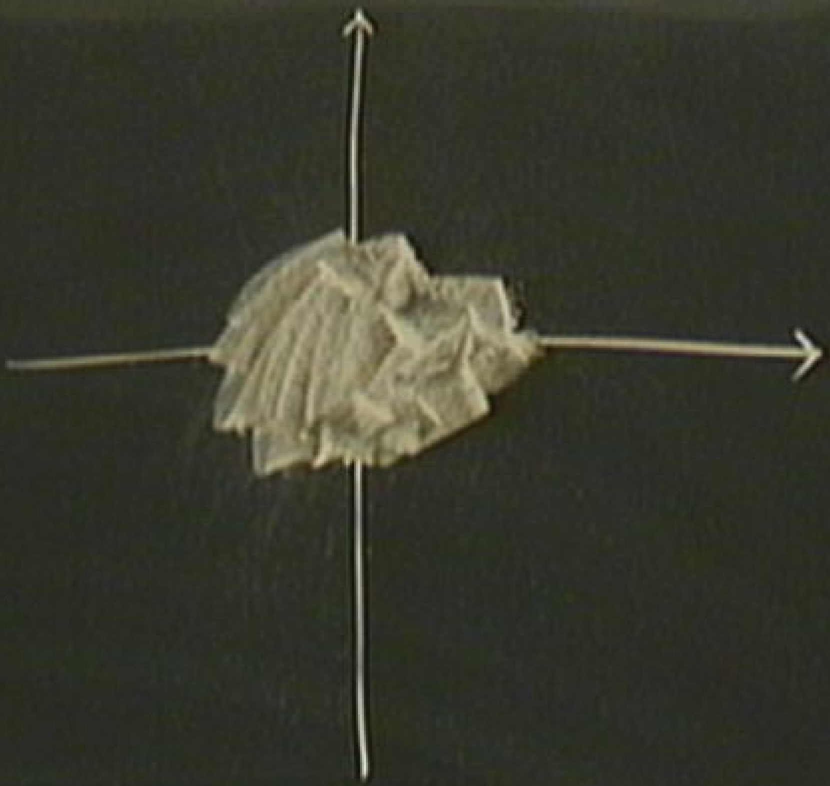


$\mathcal{W} = (0, 2)$

$q_-, q_0^\dagger$

$\pm(1, 2m) \quad m \in \mathbb{Z}$

LU



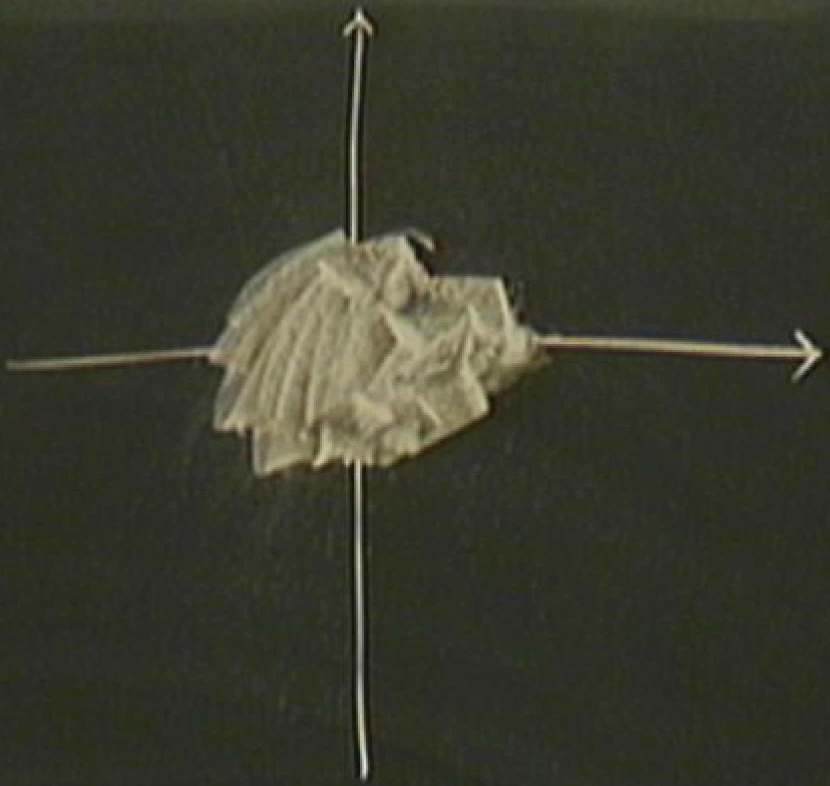
$W = (0, 2)$

q_m, q_e

$\pm(1, 2m) \quad m \in \mathbb{Z}$

$M_m = | a_D + 2m a |$

LU



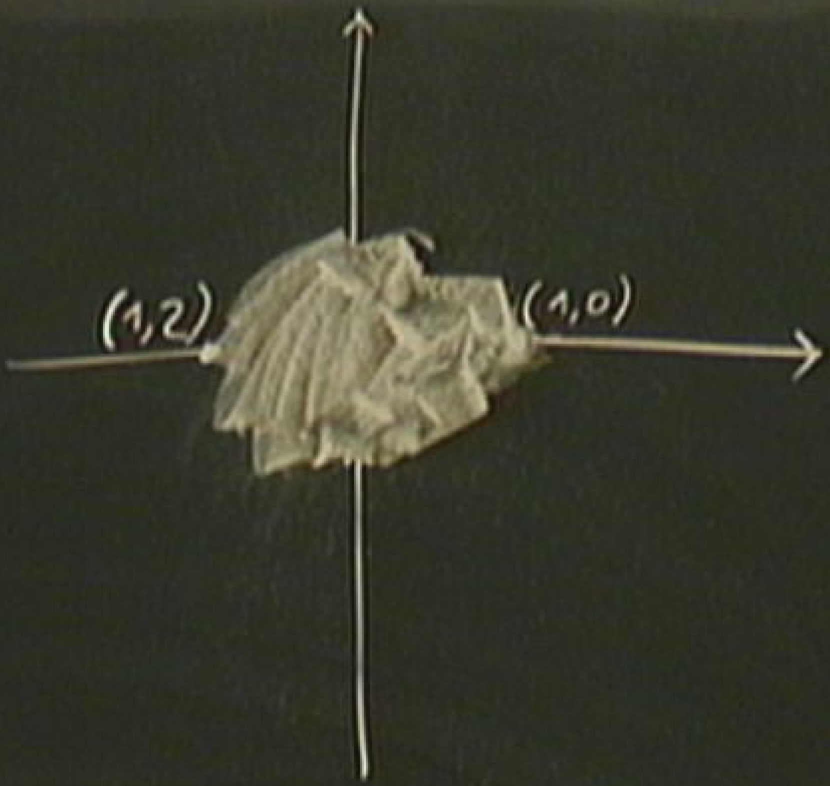
$\mathcal{W} = (0, 2)$

q_m, q_e

$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = |a_D + 2n\, a|$

LU



$W = (0, 2)$

q_m, q_e

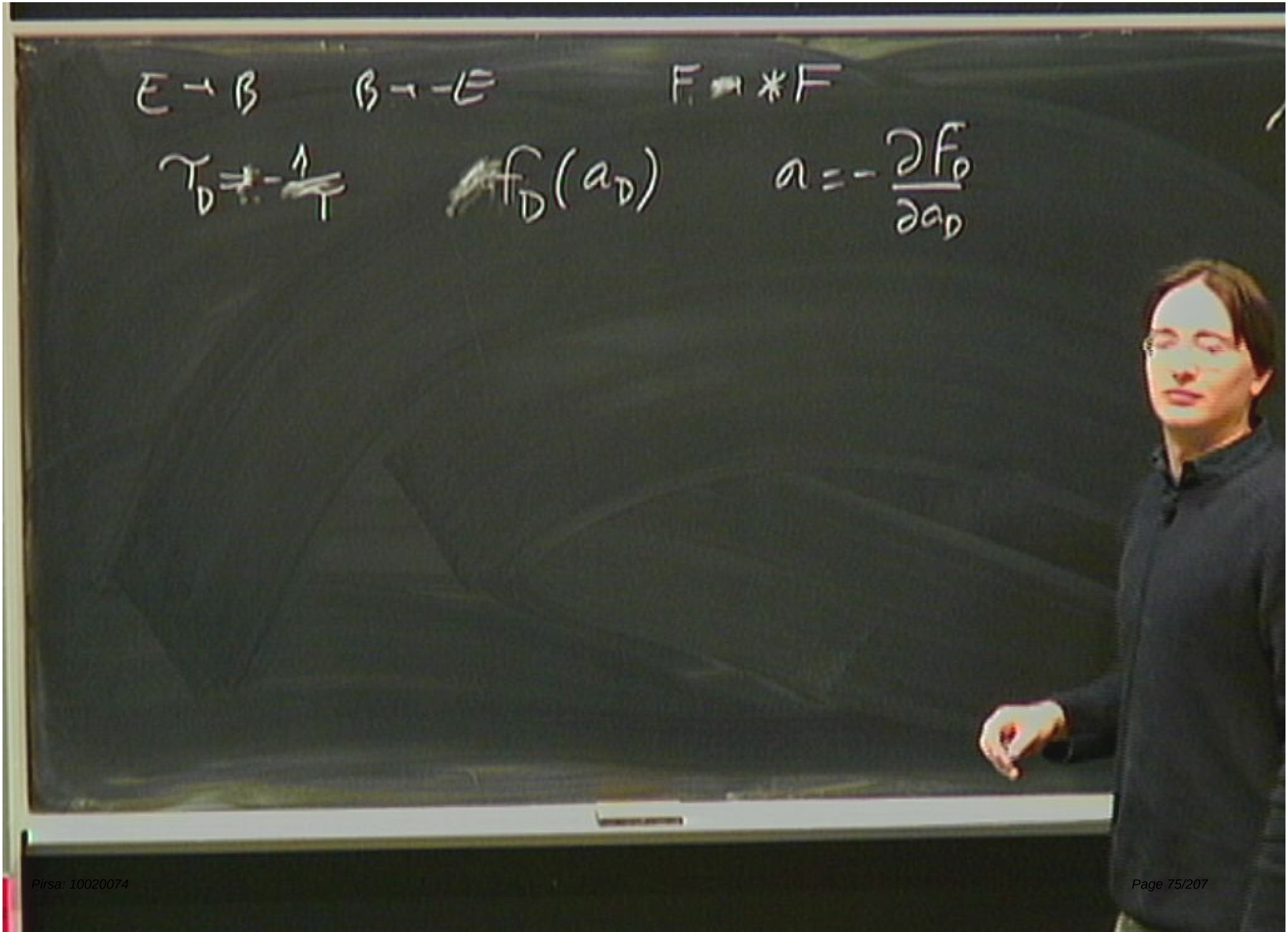
$\pm(1, \pm 2m) \quad m \in \mathbb{Z}$

$M_m = | a_0 + 2m a |$

$E \to B \qquad B \to -E \qquad F \to {*F}$

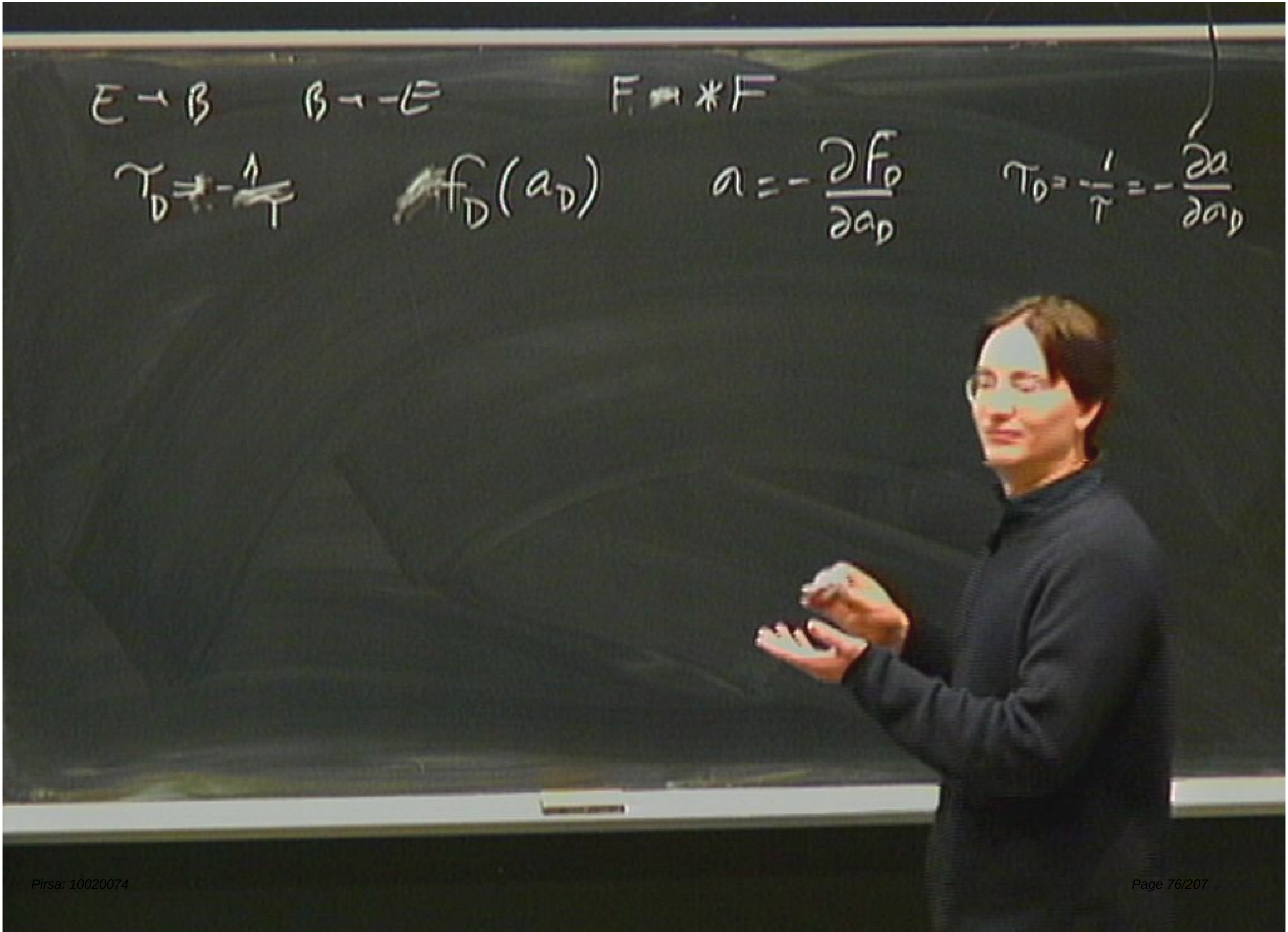
$E \to B \qquad B \to -E \qquad F \to *F$

$\tau \to -\frac{1}{\tau}$



$E \to B \quad B \to -E \quad F \to *F$

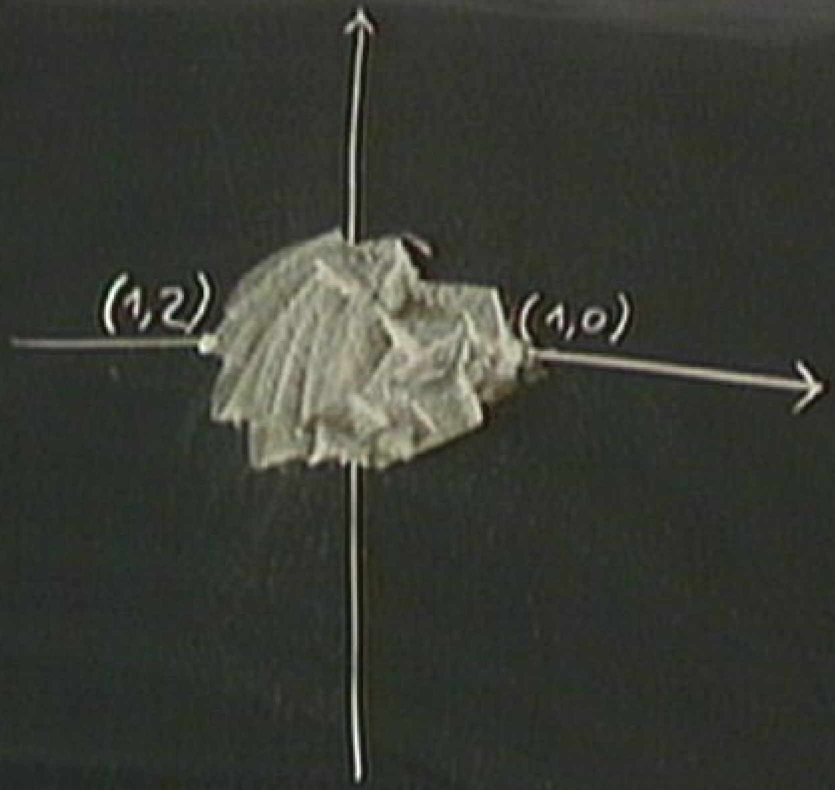
$\tau_D = -\frac{1}{\tau} \quad f_D(a_D) \quad a = -\frac{\partial f_D}{\partial a_D}$



$E \to B \qquad B \to -E \qquad F \to *F$

$\tau_D = -\frac{1}{\tau} \qquad f_D(a_D) \qquad a = -\frac{\partial f_D}{\partial a_D} \qquad \tau_D = -\frac{1}{\tau} = -\frac{\partial a}{\partial a_D}$

LU



$W = (0,2)$

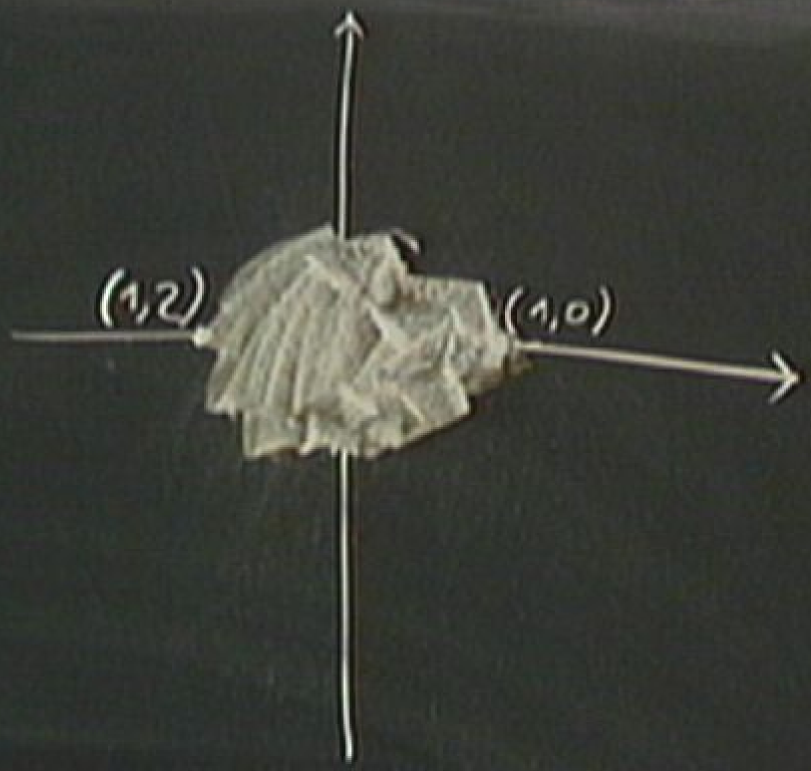
q_m, q_e

$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = |a_0 + 2n\,a|$

$a_0 \to 0$

LU



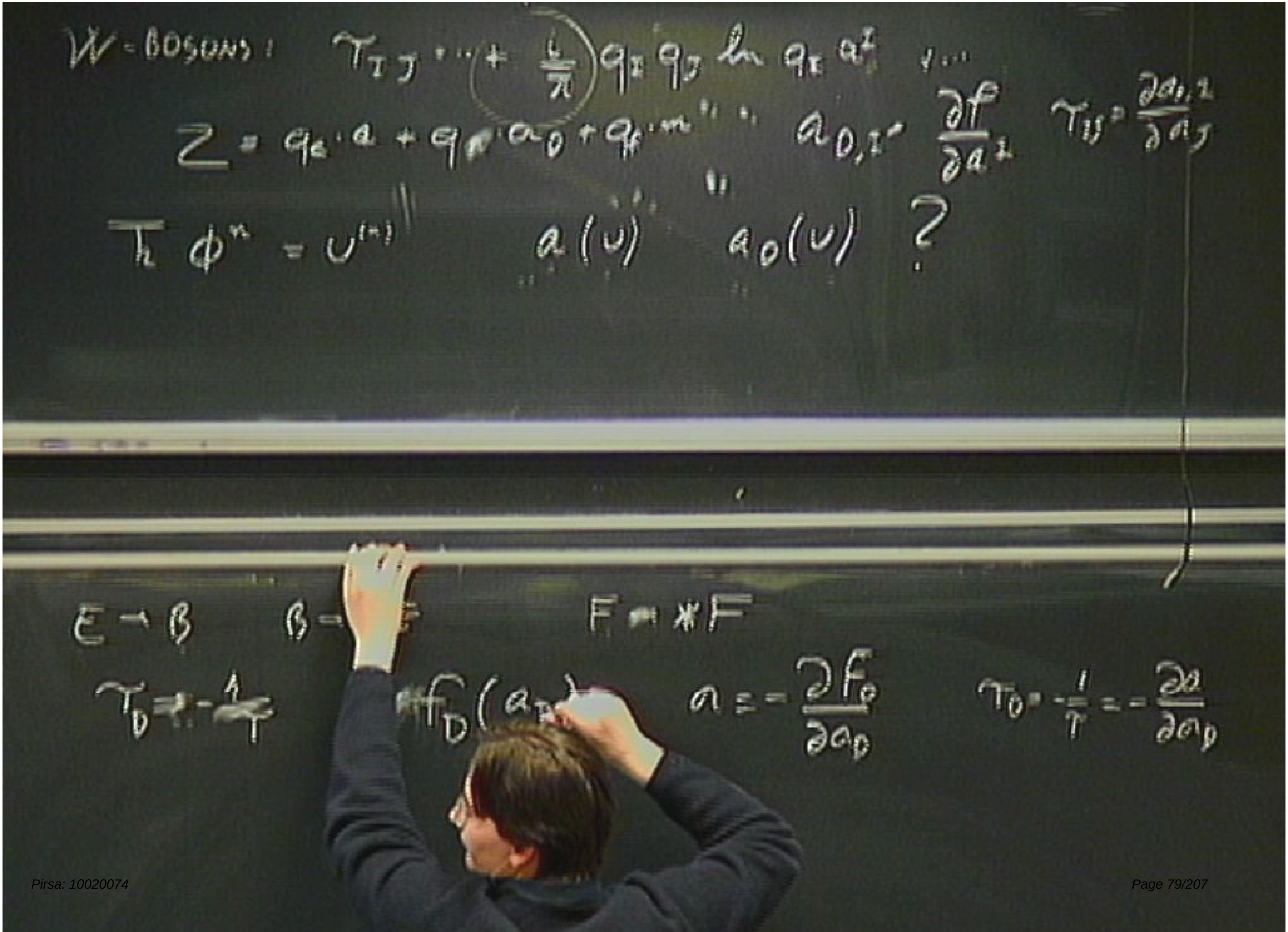
$W = (0,2)$

$q_m, q_e \uparrow$

$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = |a_D + 2n\, a|$

$a_D \to 0 \quad \tau_D \sim -\frac{i}{2\pi} \log$



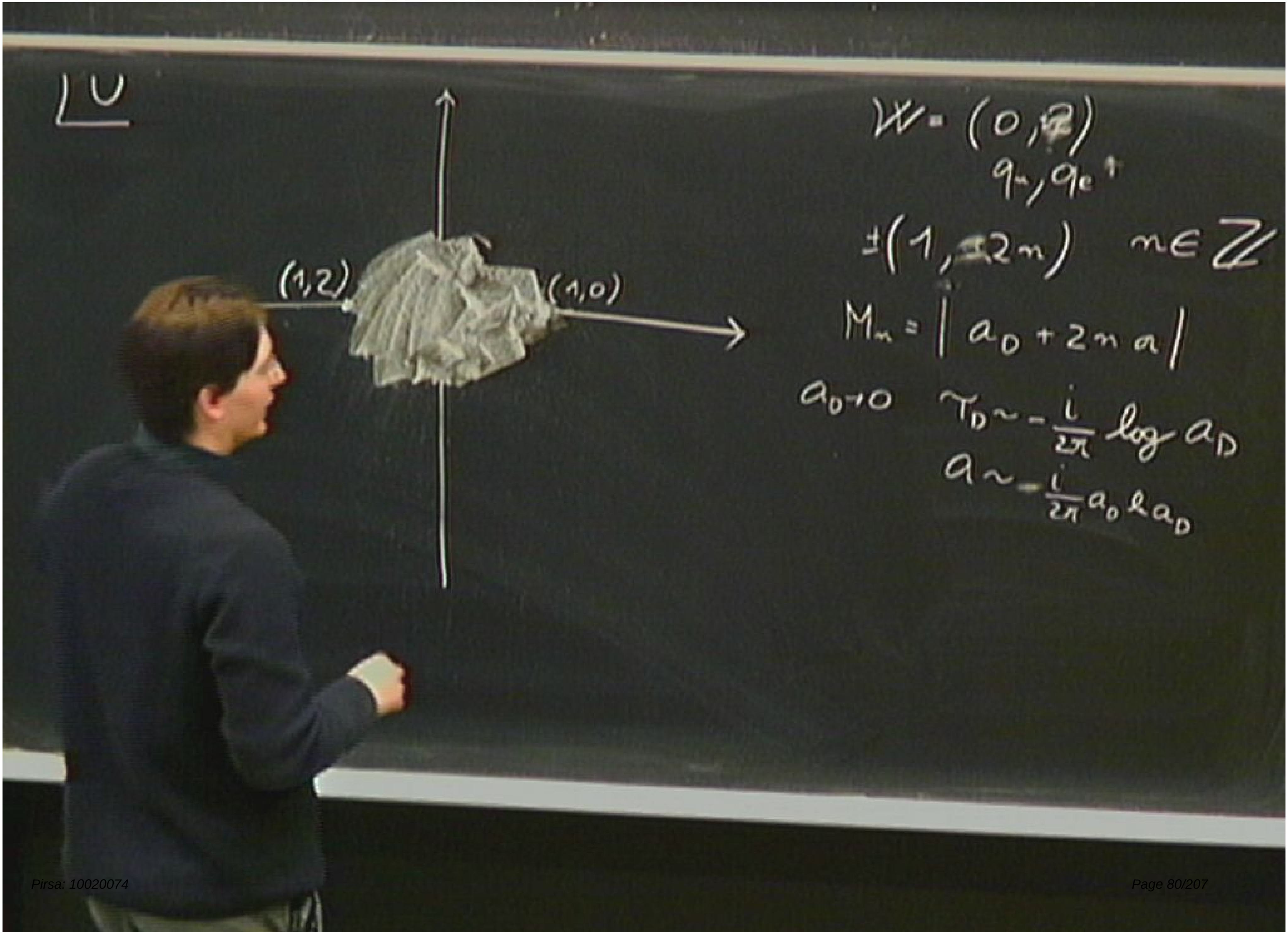
W-BOSONS: $\tau_{IJ} = \cdots + \frac{i}{\pi} q_I q_J \ln q_K a^2 + \cdots$

$Z = q_e \cdot a + q_m \cdot a_D + q_f \cdot m$ $a_{D,I} = \frac{\partial F}{\partial a^I}$ $\tau_{IJ} = \frac{\partial a_{D,I}}{\partial a_J}$

$\mathrm{Tr}\,\phi^n = u^{(n)}$ $a(u)$ $a_D(u)$?

$E \to B$ $B \to$ $F \to {*F}$

$\tau_D = -\frac{1}{\tau}$ $F_D(a_D)$ $a = -\frac{\partial F_D}{\partial a_D}$ $\tau_D = -\frac{1}{\tau} = -\frac{\partial a}{\partial a_D}$



$\mathcal{W} = (0, 2)$

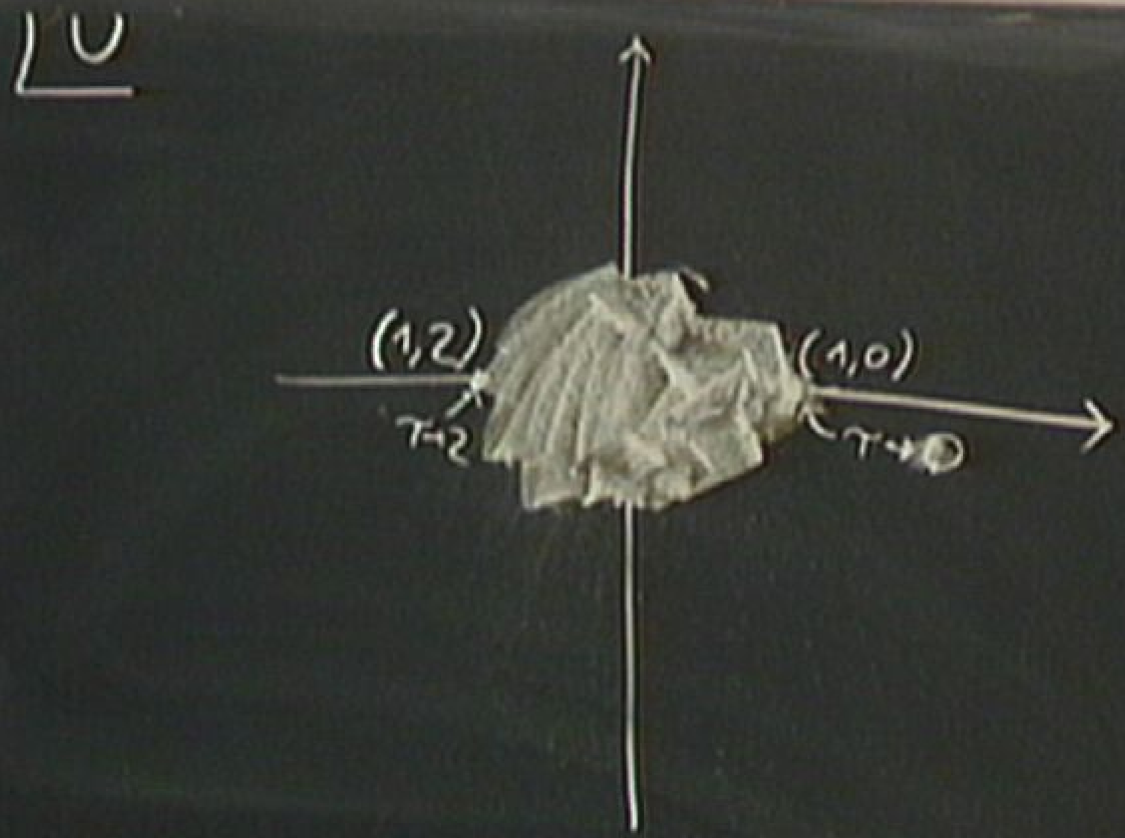
q_m, q_e ↑

$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = | a_D + 2n\, a |$

$a_D \to 0 \quad \tau_D \sim -\frac{i}{2\pi} \log a_D$

$a \sim -\frac{i}{2\pi} a_D\, l\, a_D$



$W = (0,2)$

q_m, q_e ?

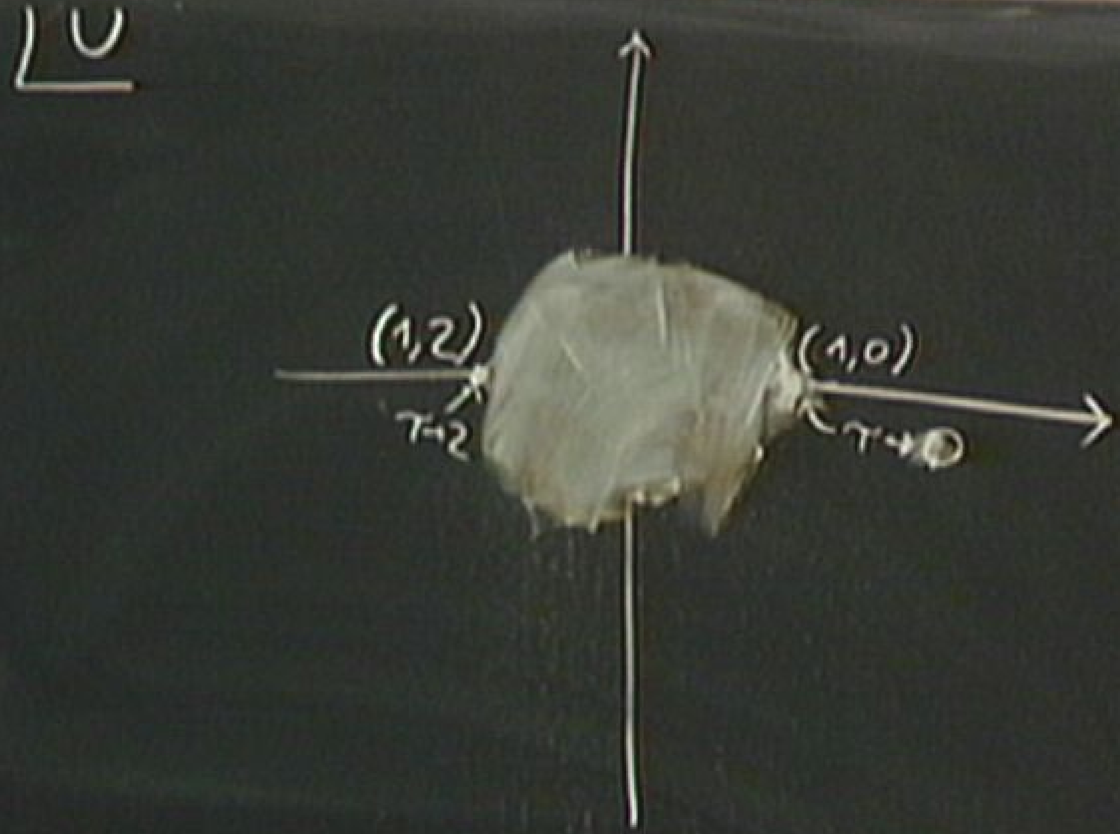
$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = |a_D + 2n a|$

$a_D \to 0 \quad \frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi} \log a_D$

$a \sim -\frac{i}{2\pi} a_D \ln a_D$

$a_D + 2a \to 0 \quad -\frac{1}{\tau - 2} \sim -\frac{i}{2\pi} \ln a_{D+2a}$



$$\mathbb{W} = (0,2)$$

$$q_m, q_e$$

$$\pm(1, 2n) \quad n \in \mathbb{Z}$$

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$$a_D \to 0 \quad -\frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi} \log a_D$$

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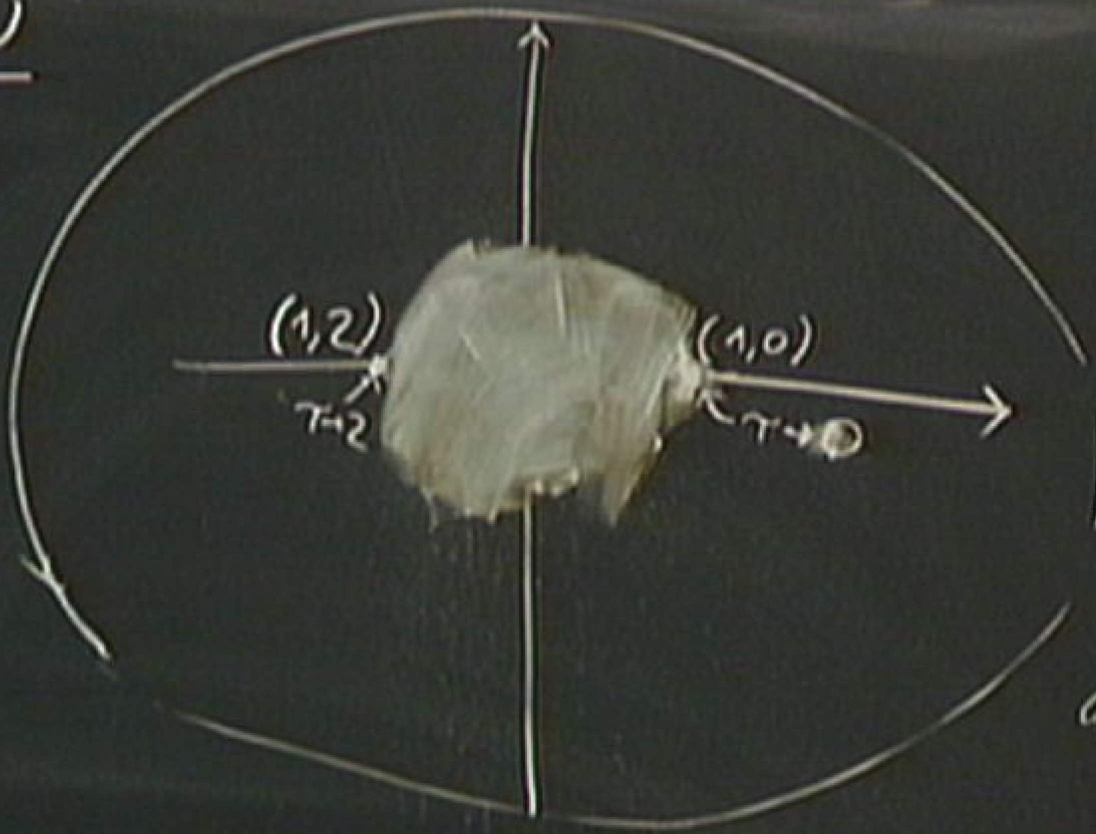
LU



$W = (0, 2)$

q_m, q_e

$\pm(1, 2n) \quad n\in\mathbb{Z}$

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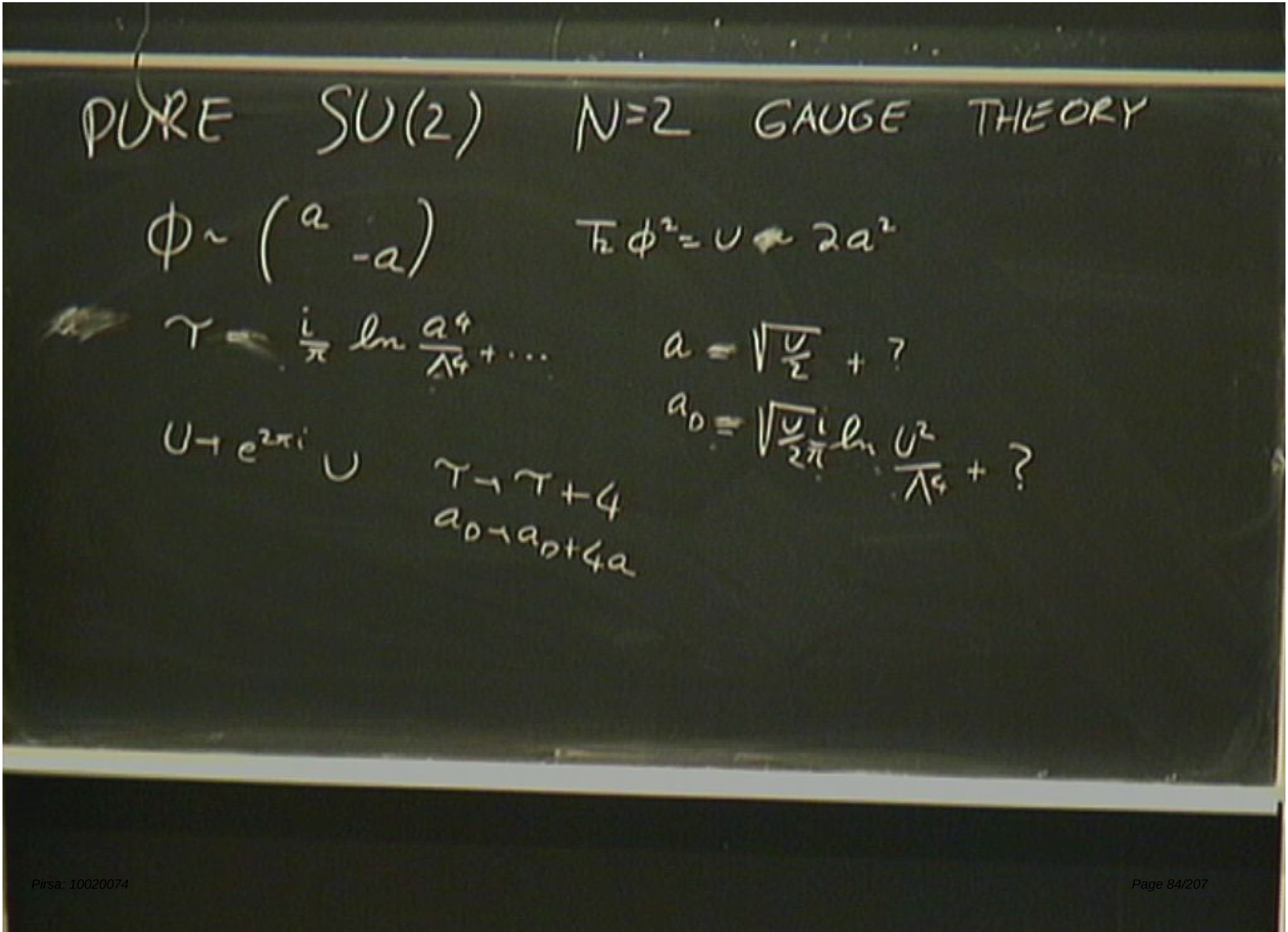
$a_D\to0 \quad -\frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi}\log a_D$

$a \sim -\frac{i}{2\pi} a_D \ln a_D$

$a_D+2a\to0 \quad -\frac{1}{\tau-2} \sim -\frac{i}{2\pi}\ln a_{D+2a}$



PURE SU(2) N=2 GAUGE THEORY

$\phi \sim \begin{pmatrix} a & \\ & -a \end{pmatrix}$

$\mathrm{Tr}\,\phi^2 = u = 2a^2$

$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} + \cdots$

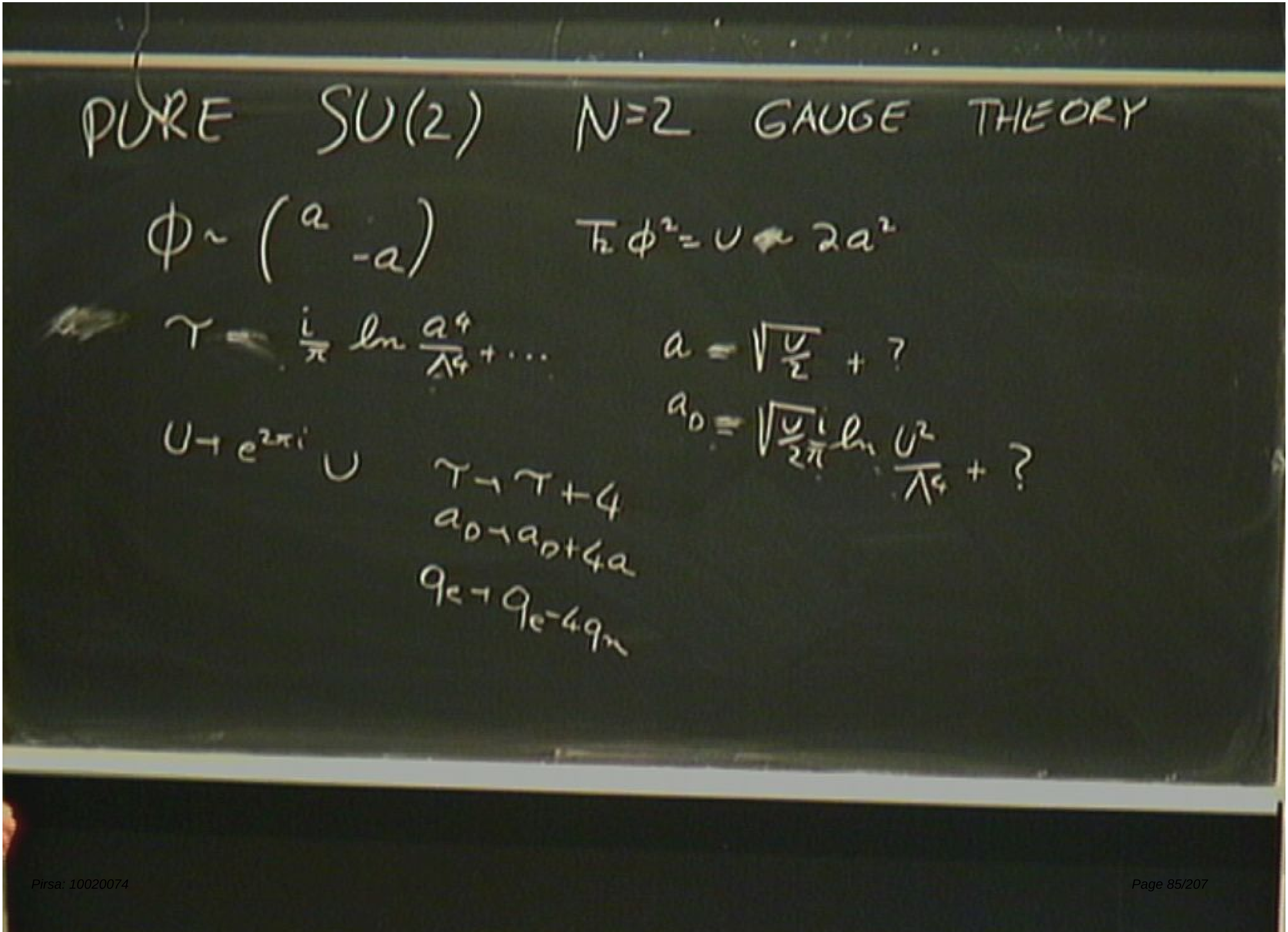
$a = \sqrt{\frac{u}{2}} + ?$

$a_D = \sqrt{\frac{u}{2}} \frac{i}{\pi} \ln \frac{u^2}{\Lambda^4} + ?$

$u \to e^{2\pi i} u$

$\tau \to \tau + 4$

$a_D \to a_D + 4a$



PURE SU(2) N=2 GAUGE THEORY

$\phi \sim \begin{pmatrix} a & \\ & -a \end{pmatrix}$ $\mathrm{Tr}\,\phi^2 = u = 2a^2$

$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} + \dots$

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$a_D = \sqrt{\frac{u}{2}}\frac{i}{\pi} \ln \frac{u^2}{\Lambda^4} + ?$

$u \to e^{2\pi i} u$

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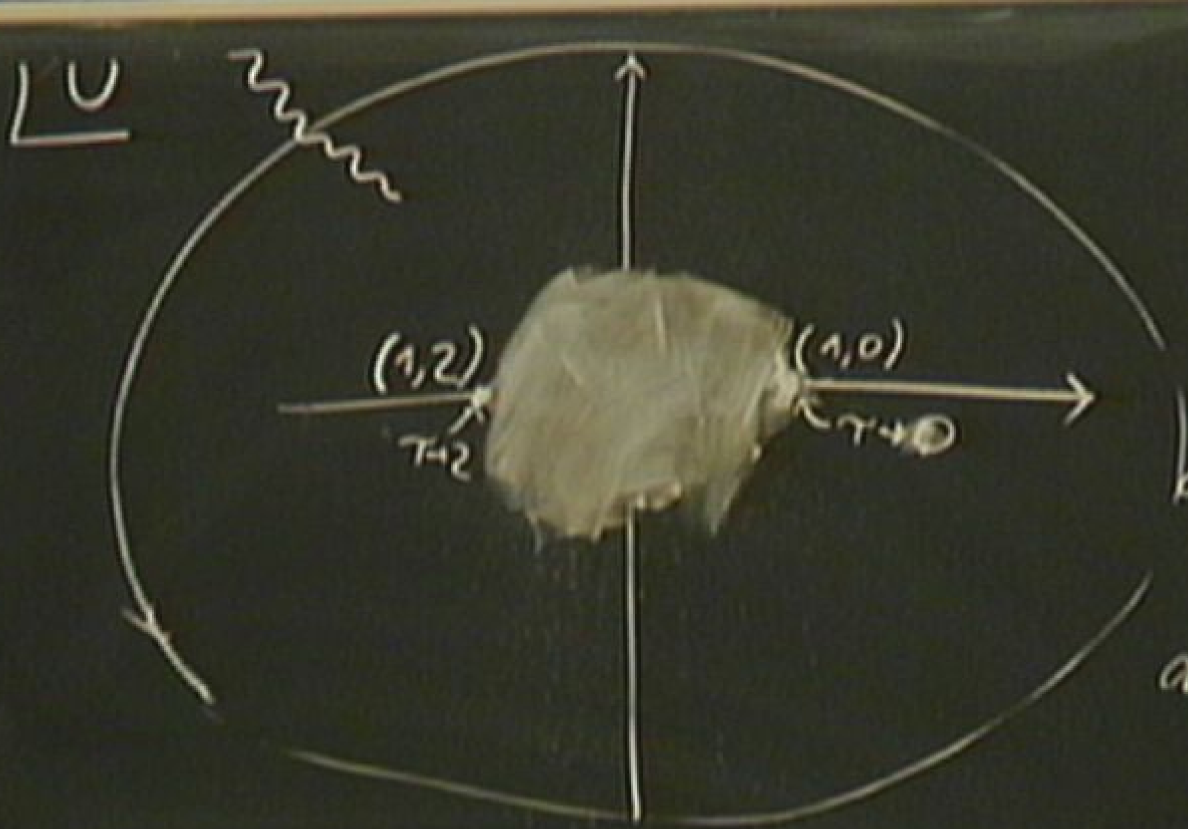
$a_D \to a_D + 4a$

$q_e \to q_e - 4q_m$



$W = (0,2)$

$q_m, q_e \uparrow$

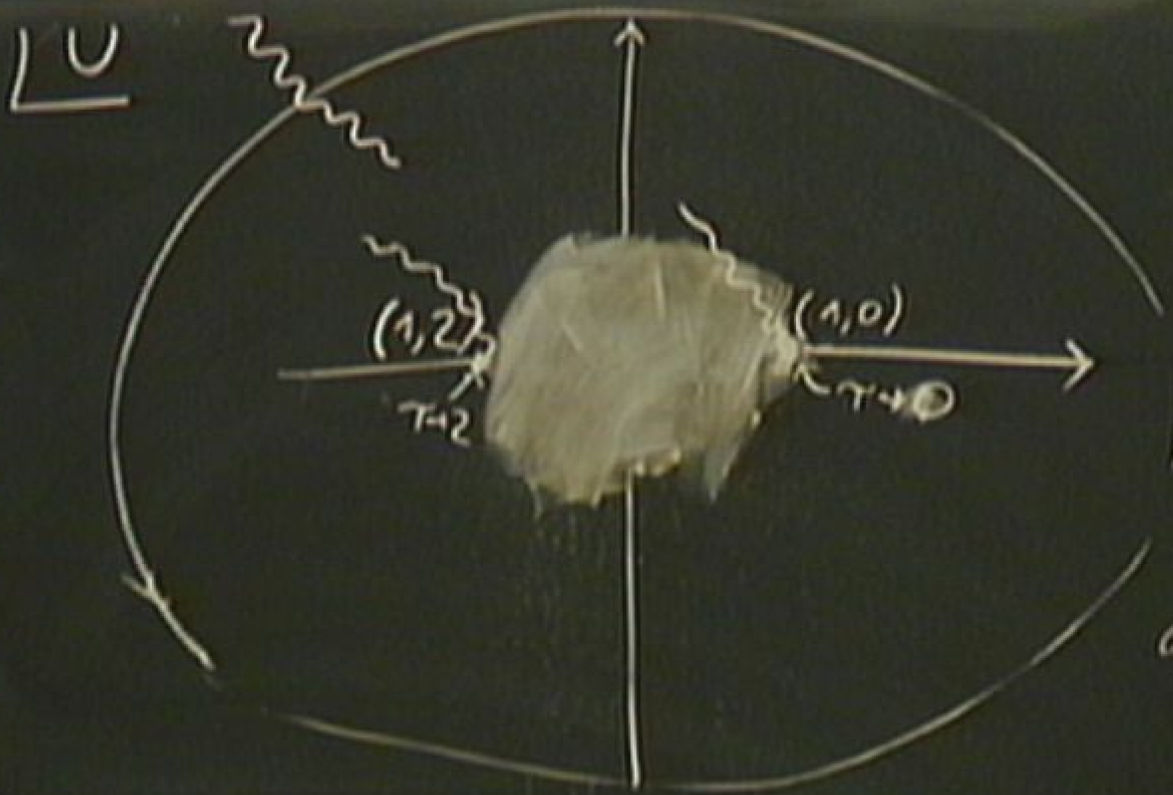
$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = |a_D + 2n\, a|$

$a_D \to 0 \quad -\frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi} \log a_D$

$a \sim -\frac{i}{2\pi} a_D \ell a_D$

$a_D + 2a \to 0 \quad -\frac{1}{\tau - 2} \sim -\frac{i}{2\pi} \ell a_{D+2a}$



$W = (0, 2)$

q_m, q_e

$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = | a_D + 2n\, a |$

$a_D \to 0 \quad \frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi} \log a_D$

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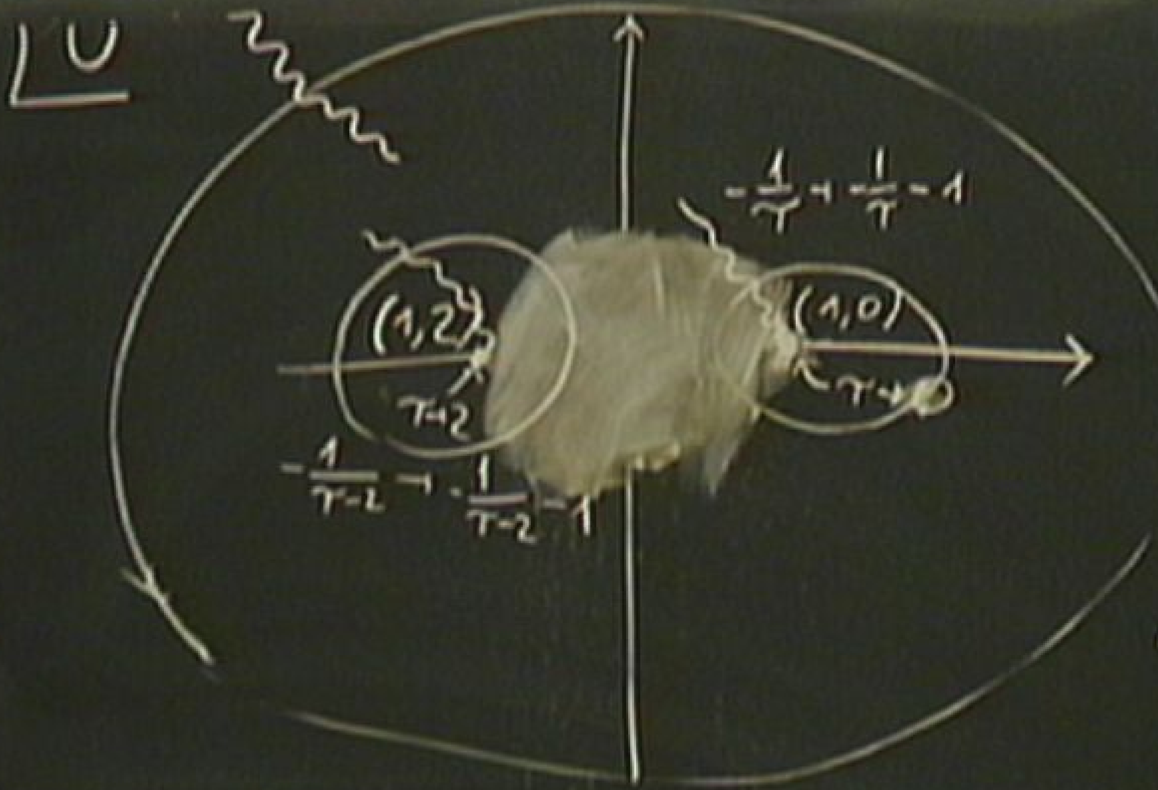

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$W = (0, 2)$

q_m, q_e

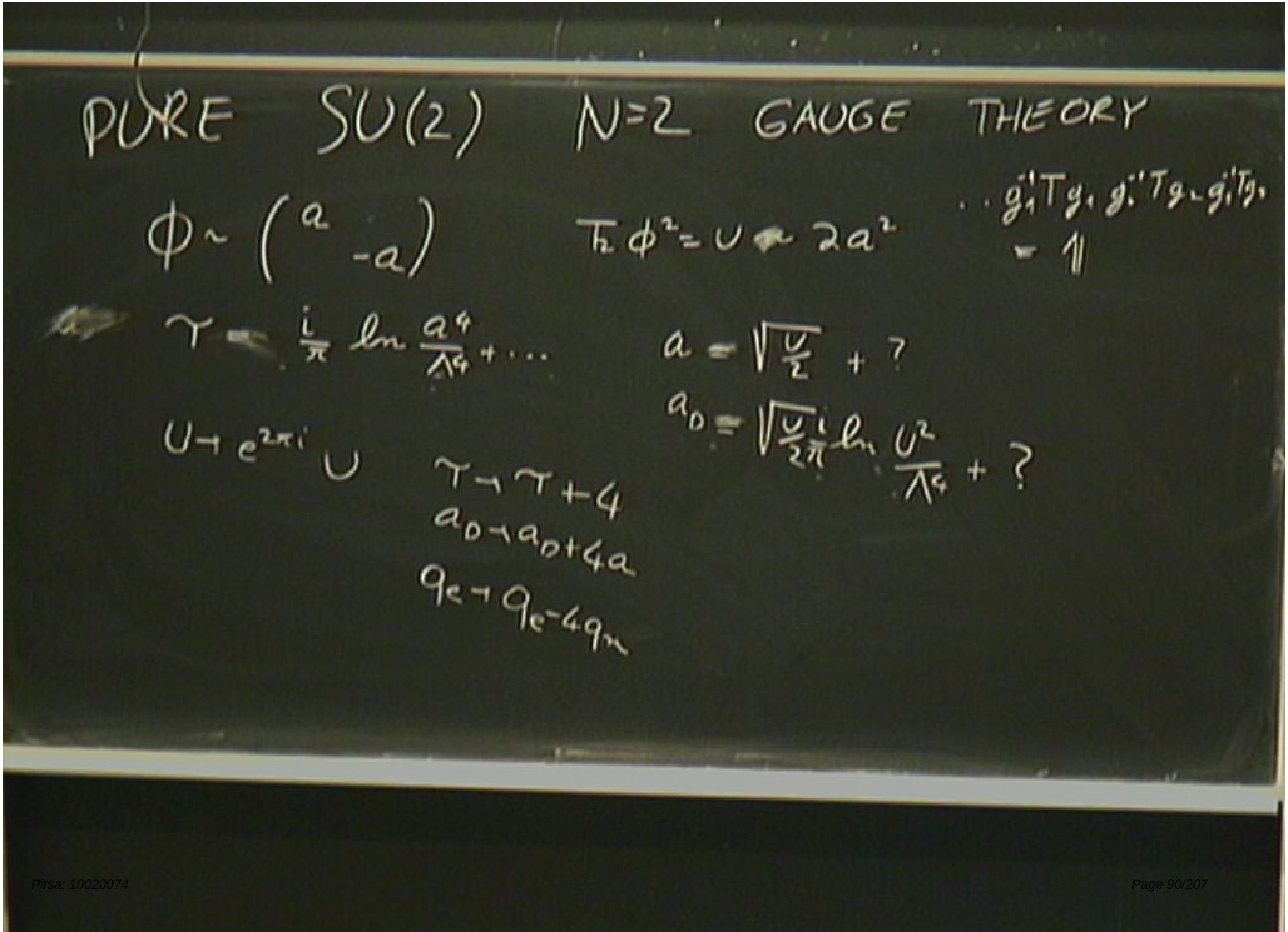
$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = |a_D + 2n a|$

$a_D \to 0 \quad \frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi} \log a_D$

$a \sim -\frac{i}{2\pi} a_D \ell a_D$

$a_D + 2a \to 0 \quad -\frac{1}{\tau-2} \sim -\frac{i}{2\pi} \ell a_D + 2a$



PURE SU(2) N=2 GAUGE THEORY

$\Phi \sim \begin{pmatrix} a & \\ & -a \end{pmatrix}$

$\mathrm{Tr}\,\phi^2 = U = 2a^2$

$\cdots g_1^{-1} T g_1 \, g_2^{-1} T g_2 \, g_3^{-1} T g_3$

$= \mathbb{1}$

$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} + \cdots$

$a = \sqrt{\frac{U}{2}} + ?$

$a_D = \sqrt{\frac{U}{2}} \frac{i}{\pi} \ln \frac{U^2}{\Lambda^4} + ?$

$U \to e^{2\pi i} U$

$\tau \to \tau + 4$

$a_D \to a_D + 4a$

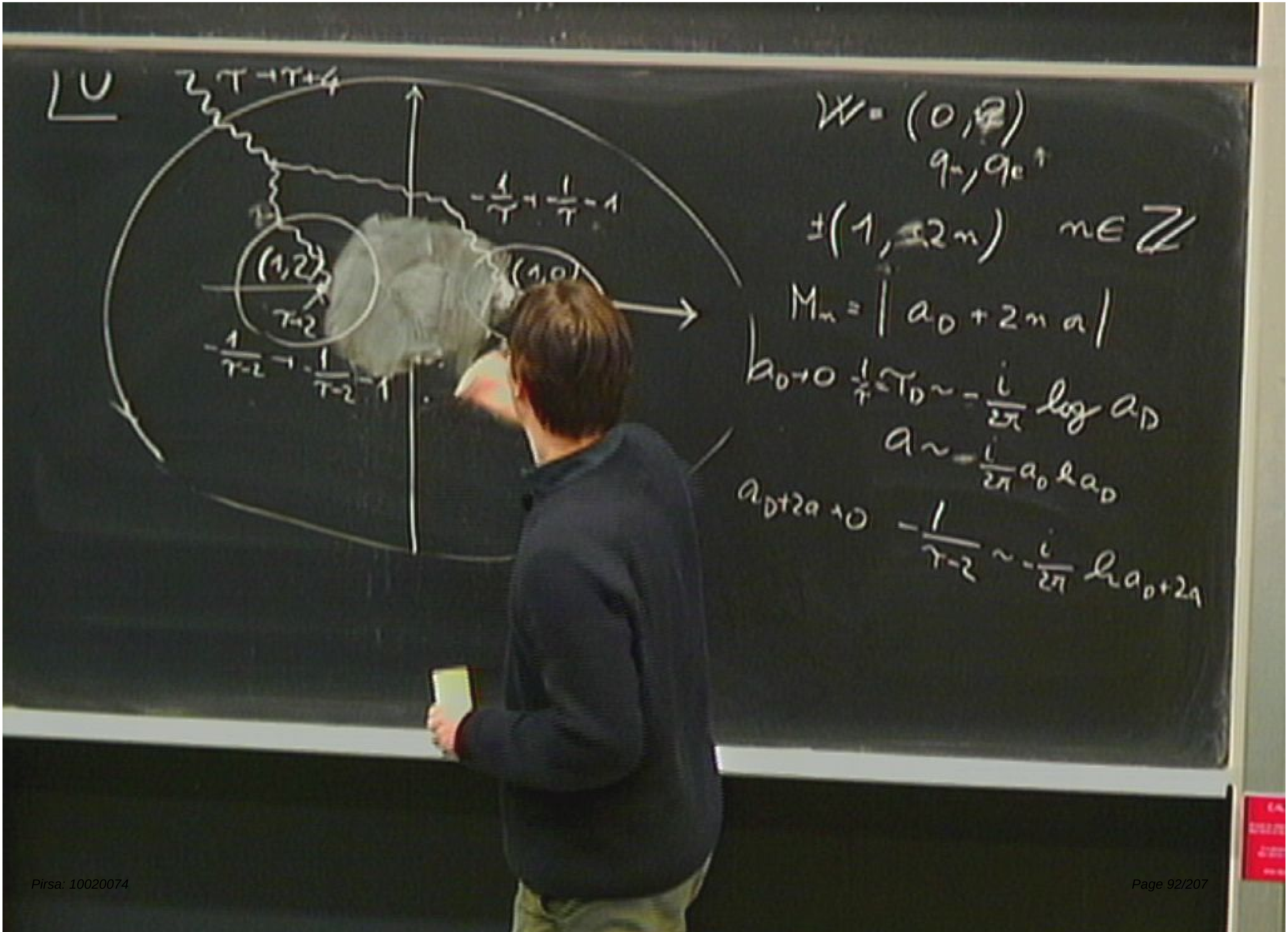
$q_e \to q_e - 4q_m$

$E \to B \quad B \to -E \quad F \to *F$

$\tau_D = -\frac{1}{\tau} \quad f_D(a_D) \quad a = -\frac{\partial f_D}{\partial a_D} \quad \tau_D = -\frac{1}{\tau} = -\frac{\partial a}{\partial a_D}$

$\tau(u)$

$a(u), a_D(u)$



$W = (0, 2)$

q_m, q_e

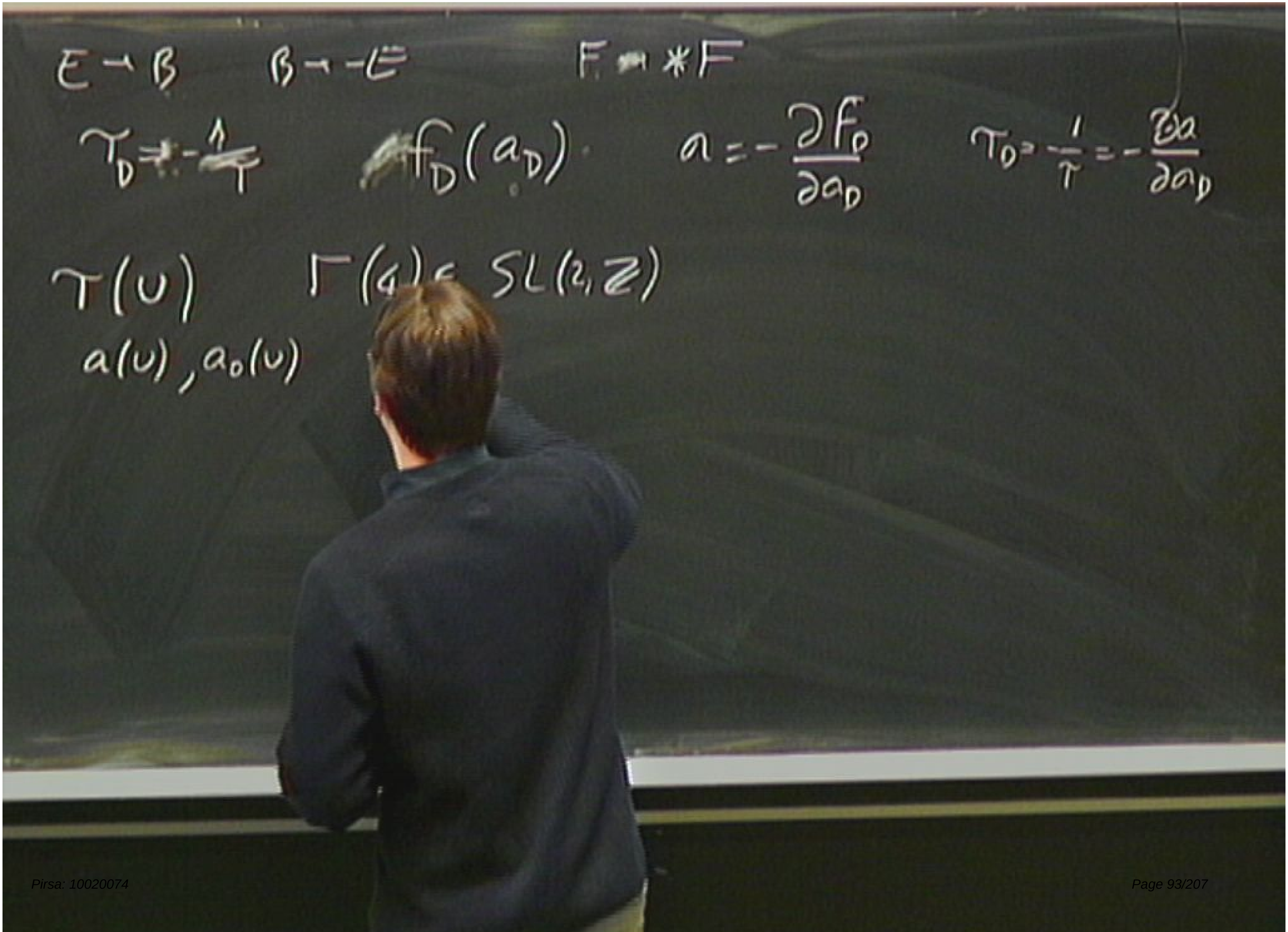
$\pm(1, 2m) \quad m \in \mathbb{Z}$

$M_m = | a_D + 2m\, a |$

$a_D \to 0 \quad \frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi} \log a_D$

$a \sim -\frac{i}{2\pi} a_D \ln a_D$

$a_D + 2a \to 0 \quad -\frac{1}{\tau - 2} \sim -\frac{i}{2\pi} \ln a_{D+2a}$

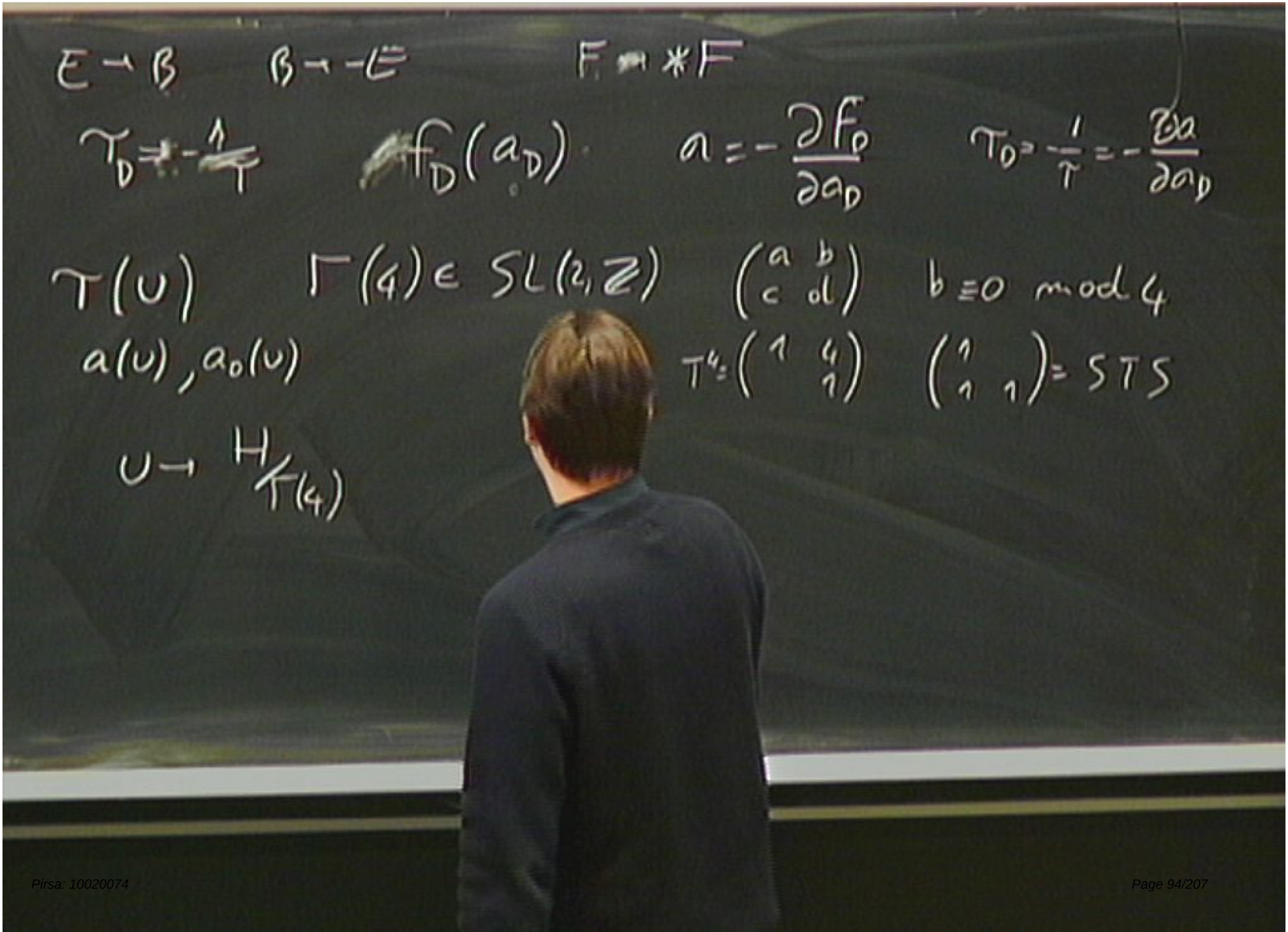


$E \to B \qquad B \to -E \qquad F \to {*F}$

$\tau_D = -\frac{1}{\tau} \qquad f_D(a_D) \qquad a = -\frac{\partial F_D}{\partial a_D} \qquad \tau_D = -\frac{1}{\tau} = -\frac{\partial a}{\partial a_D}$

$\tau(u) \qquad \Gamma(4) \in SL(2,\mathbb{Z})$

$a(u), a_D(u)$



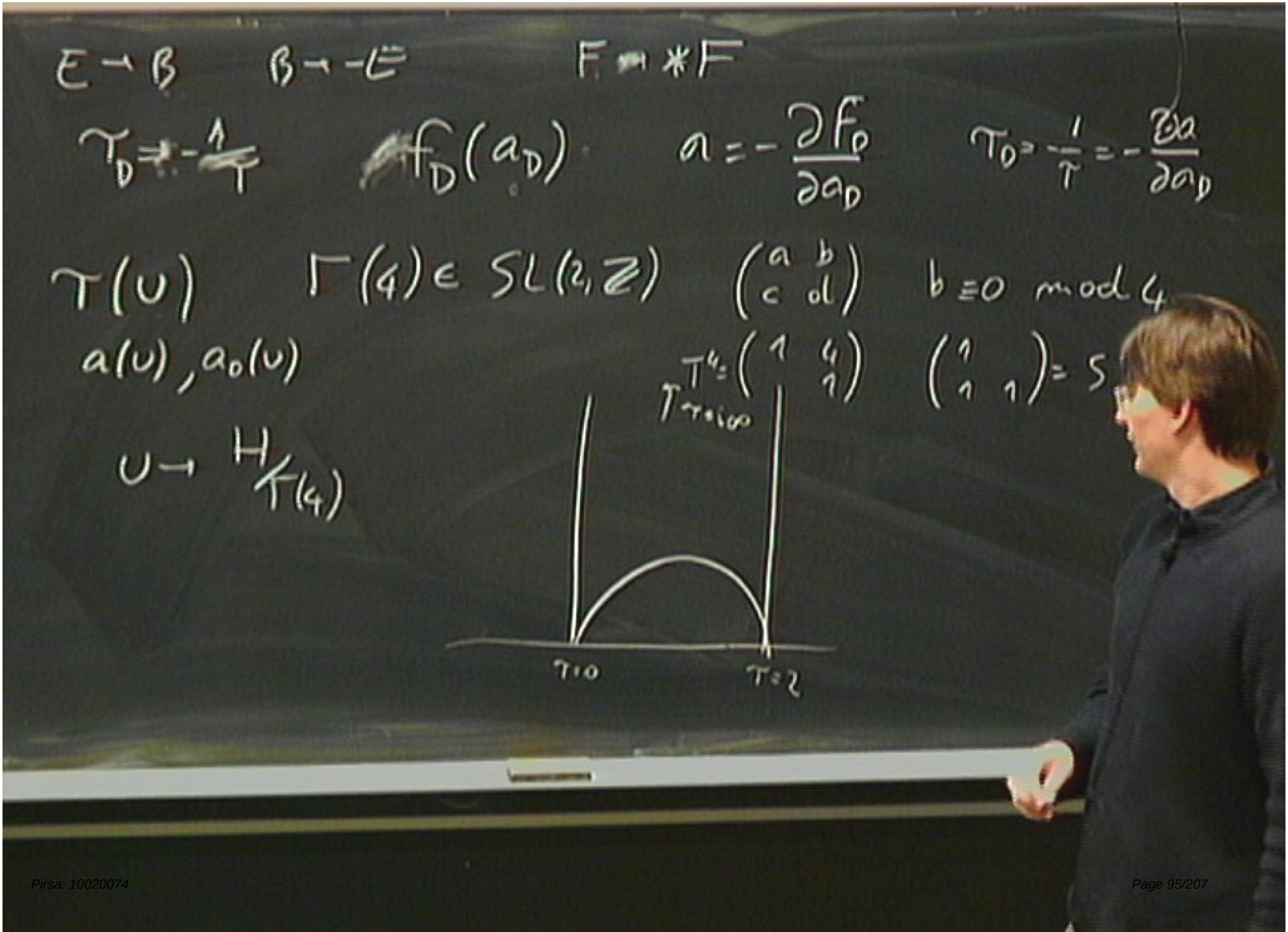
$E \to B$ $B \to -E$ $F \to *F$

$\tau_D = -\frac{1}{\tau}$ $f_D(a_D)$ $a = -\frac{\partial F_D}{\partial a_D}$ $\tau_D = -\frac{1}{\tau} = -\frac{\partial a}{\partial a_D}$

$\tau(u)$ $\Gamma(4) \in SL(2,\mathbb{Z})$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $b \equiv 0 \mod 4$

$a(u), a_D(u)$ $T^4 = \begin{pmatrix} 1 & 4 \\ & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & \\ 1 & 1 \end{pmatrix} = STS$

$u \to H/\Gamma(4)$



$E \to B \quad B \to -E \qquad F \to *F$

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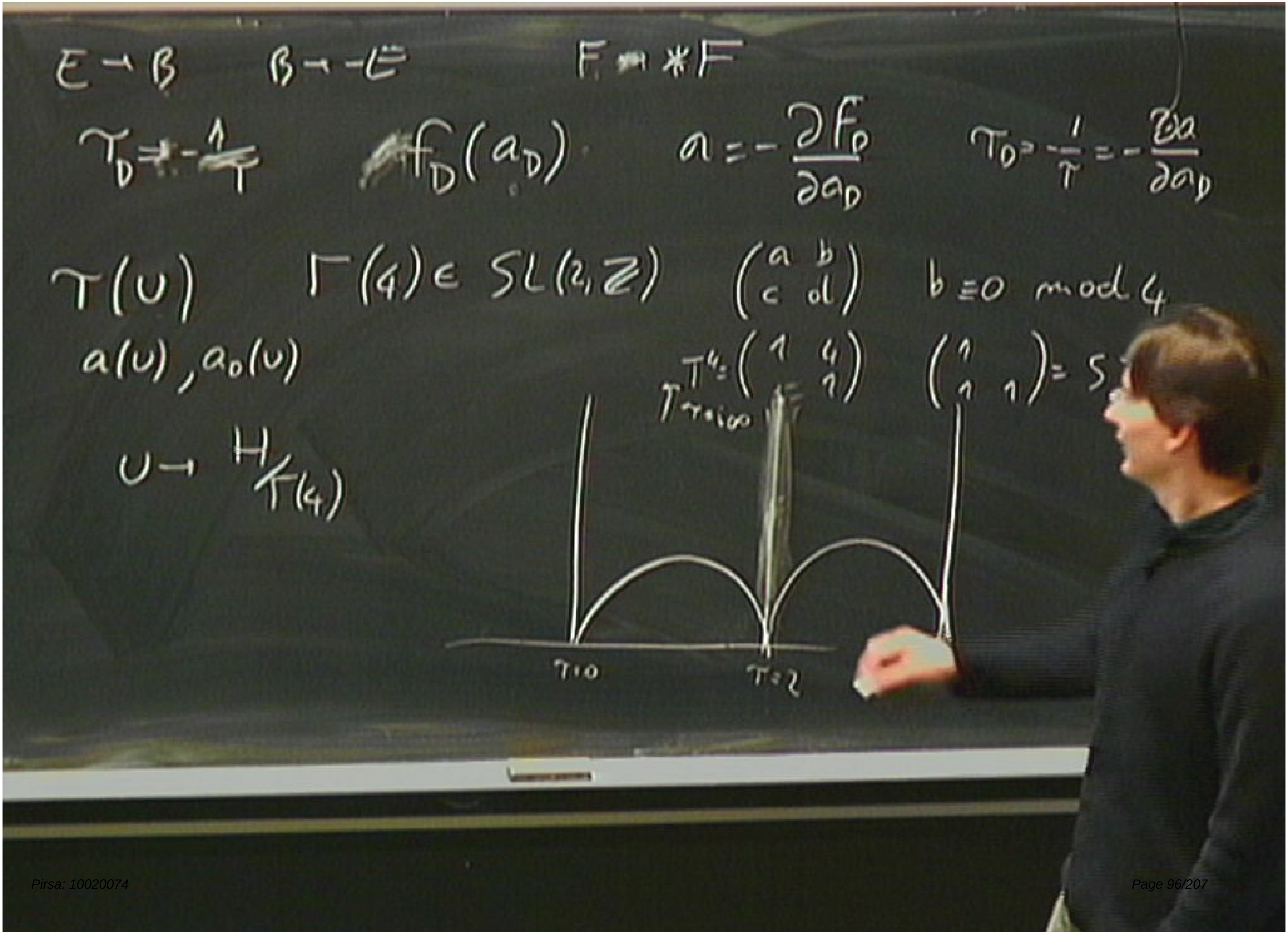
$a(u), a_D(u) \qquad T^4 = \begin{pmatrix} 1 & 4 \\ & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & \\ 1 & 1 \end{pmatrix} = S$

$u \to H/\Gamma(4)$



$E \to B \qquad B \to -E \qquad F \quad *F$

$\tau_D = -\frac{1}{\tau} \qquad f_D(a_D) \qquad a = -\frac{\partial F_D}{\partial a_D} \qquad \tau_D = -\frac{1}{\tau} = -\frac{\partial a}{\partial a_D}$

$\tau(u) \qquad \Gamma(4) \in SL(2,\mathbb{Z}) \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad b \equiv 0 \mod 4$

$a(u), a_D(u)$

$T^4 = \begin{pmatrix} 1 & 4 \\ & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & \\ 1 & 1 \end{pmatrix} = S$

$T_{\tau \to i\infty}$

$u \to H/\Gamma(4)$

$\tau = 0 \qquad \tau = 2$

$E \to B \qquad B \to -E \qquad F \to *F$

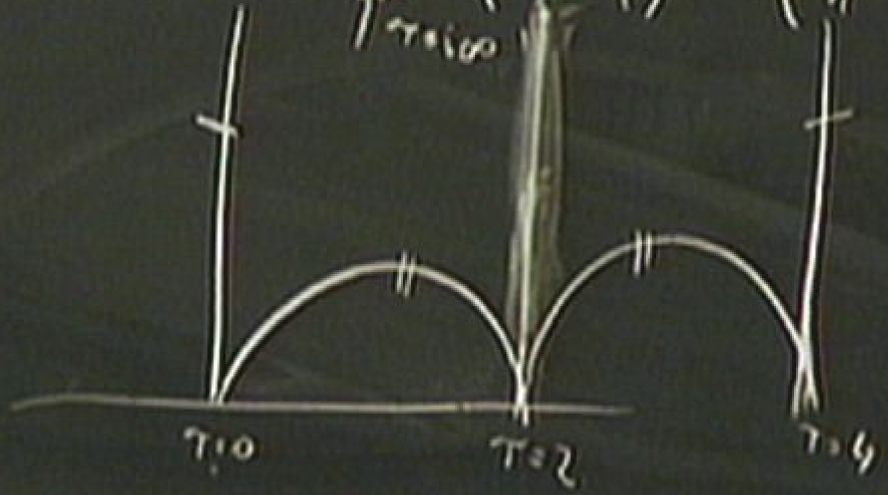
$\tau_D = -\frac{1}{\tau} \qquad f_D(a_D) \qquad a = -\frac{\partial F_D}{\partial a_D} \qquad \tau_D = -\frac{1}{\tau} = -\frac{\partial a}{\partial a_D}$

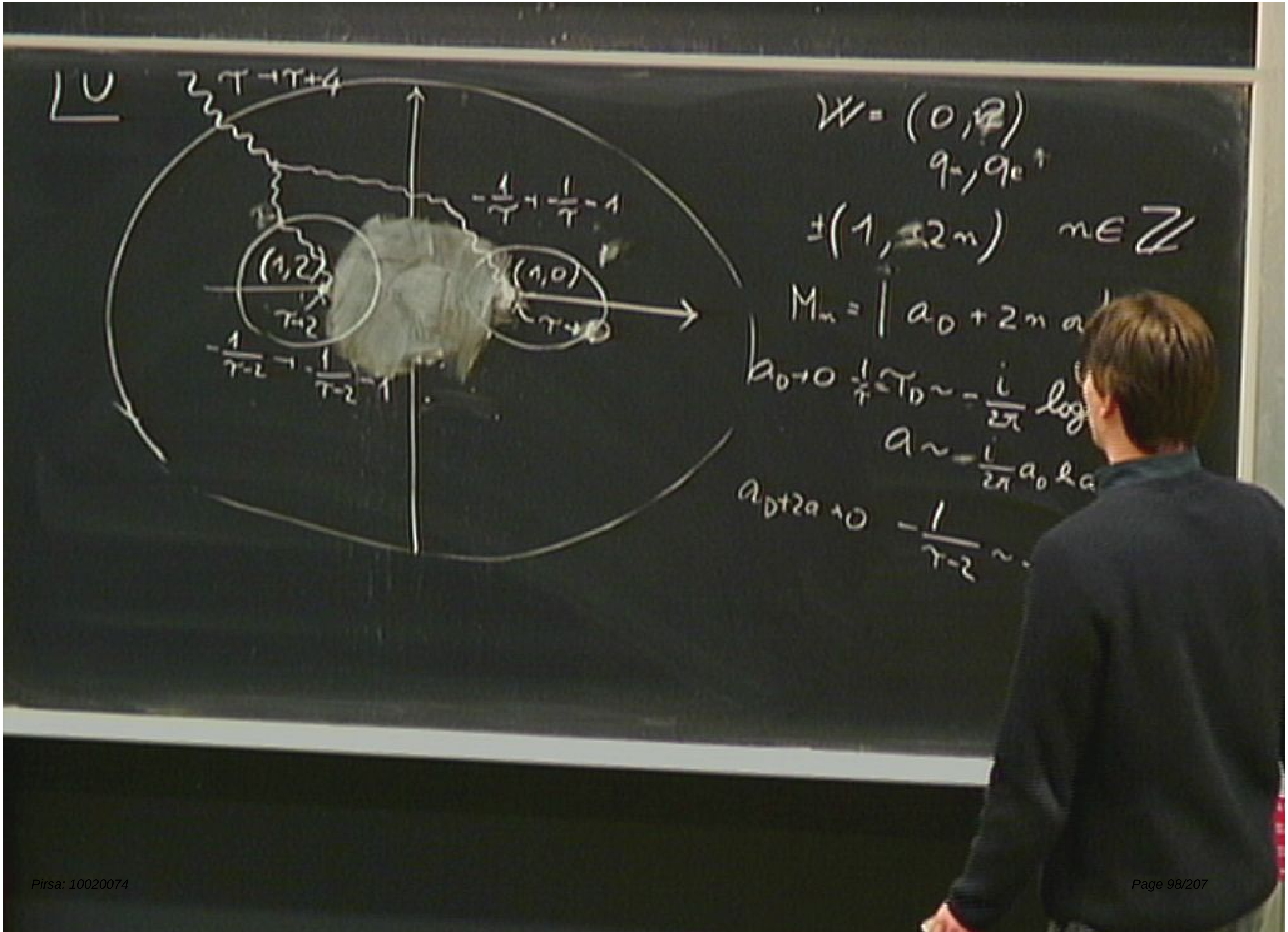
$\tau(u) \qquad \Gamma(4) \in SL(2,\mathbb{Z}) \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad b \equiv 0 \mod 4$

$a(u), a_D(u) \qquad T^4 = \begin{pmatrix} 1 & 4 \\ & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & \\ 1 & 1 \end{pmatrix} = STS$

$T_{\tau = i\infty}$

$u \to H/\Gamma(4)$





$W = (0, 2)$

$q_m, q_e \uparrow$

$\pm(1, 2n) \quad n \in \mathbb{Z}$

$M_n = | a_D + 2n a$

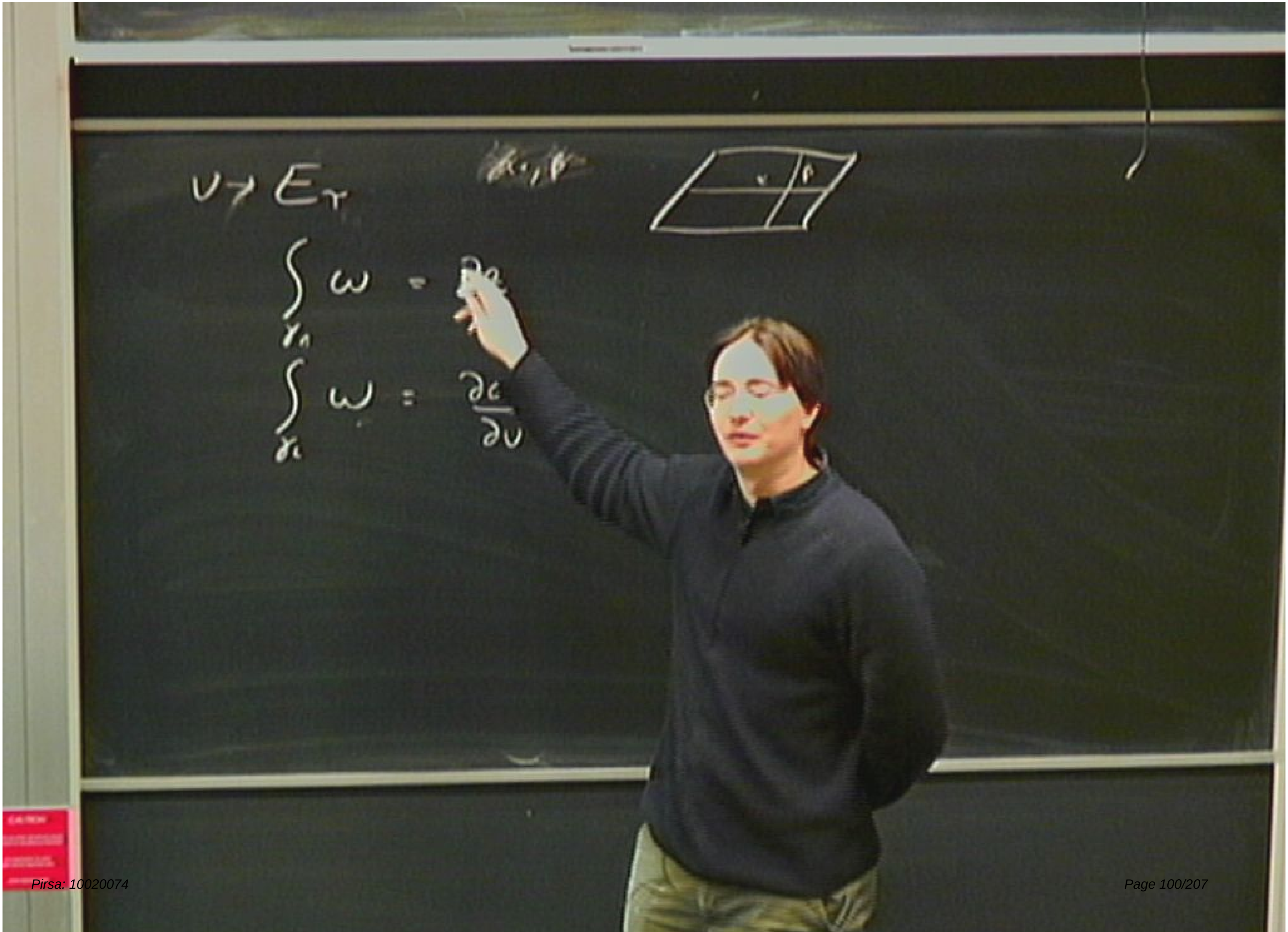
$a_D \to 0 \quad \frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi} \log$

$a \sim -\frac{i}{2\pi} a_D \log$

$a_D + 2a \to 0 \quad -\frac{1}{\tau-2} \sim$

$v \to E_\tau$

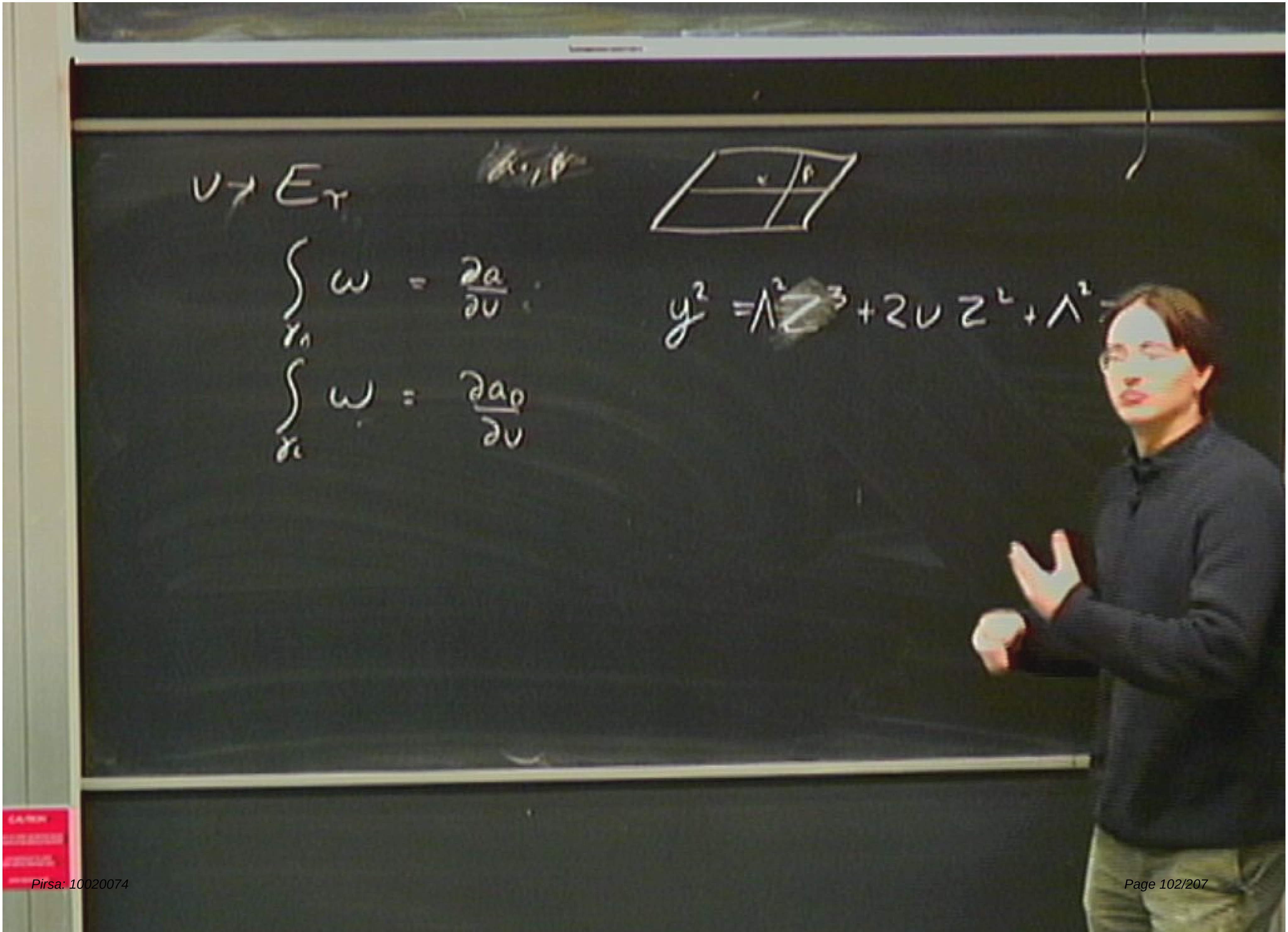
[illegible]


$v \to E_\tau$
α, β
$\int_{\gamma_1} \omega = \partial$
$\int_{\gamma_2} \omega = \frac{\partial c}{\partial v}$
α
β

$v \to E_\tau$

$$\int_{\gamma_\alpha} \omega = \frac{\partial a}{\partial v}$$

$$\int_{\gamma_\beta} \omega = \frac{\partial a_\beta}{\partial v}$$



$v \to E_\tau$

$$\int_{\gamma_1} \omega = \frac{\partial a}{\partial v}$$

$$\int_{\gamma_2} \omega = \frac{\partial a_D}{\partial v}$$

$$y^2 = \Lambda^2 z^3 + 2v z^2 + \Lambda^2$$

$u \to E_\tau$

$$\int_{\gamma_n} \omega = \frac{\partial a}{\partial u}$$

$$\int_{\gamma_t} \omega = \frac{\partial a_D}{\partial u}$$

$$y^2 = \Lambda^2 z^3 + 2u\, z^2 + \Lambda^2 z$$

$$\omega = \frac{dz}{y}$$

$v \to E_\tau$

α, β

$$\int_{\gamma_n} \omega = \frac{\partial a}{\partial v}$$

$$\int_{\gamma_c} \omega = \frac{\partial a_D}{\partial v}$$

$$y^2 = \Lambda^2 z^3 + 2v z^2 + \Lambda^2 z$$

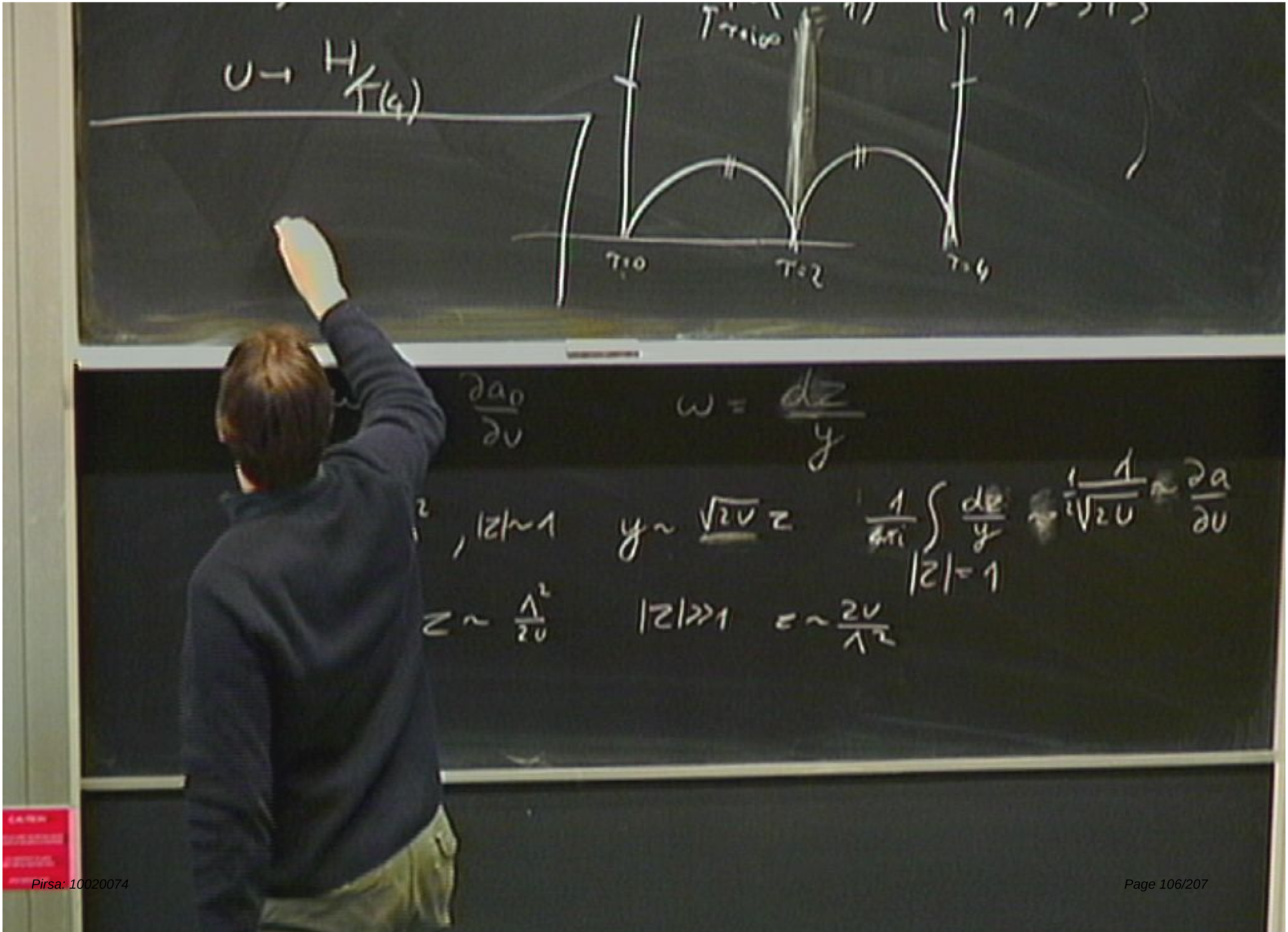
$$\omega = \frac{dz}{y}$$

$u \to E_\tau$

$$\int_{\gamma_1} \omega = \frac{\partial a}{\partial u} \qquad y^2 = \Lambda^2 z^3 + 2u\, z^2 + \Lambda^2 z$$

$$\int_{\gamma_2} \omega = \frac{\partial a_D}{\partial u} \qquad \omega = \frac{dz}{y}$$

$$u \gg \Lambda^2 \,,\ |z| \sim 1 \qquad y \sim \underline{\sqrt{2u}}\, z \qquad \frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{y} \sim \frac{1}{\sqrt{2u}} \sim \frac{\partial a}{\partial u}$$



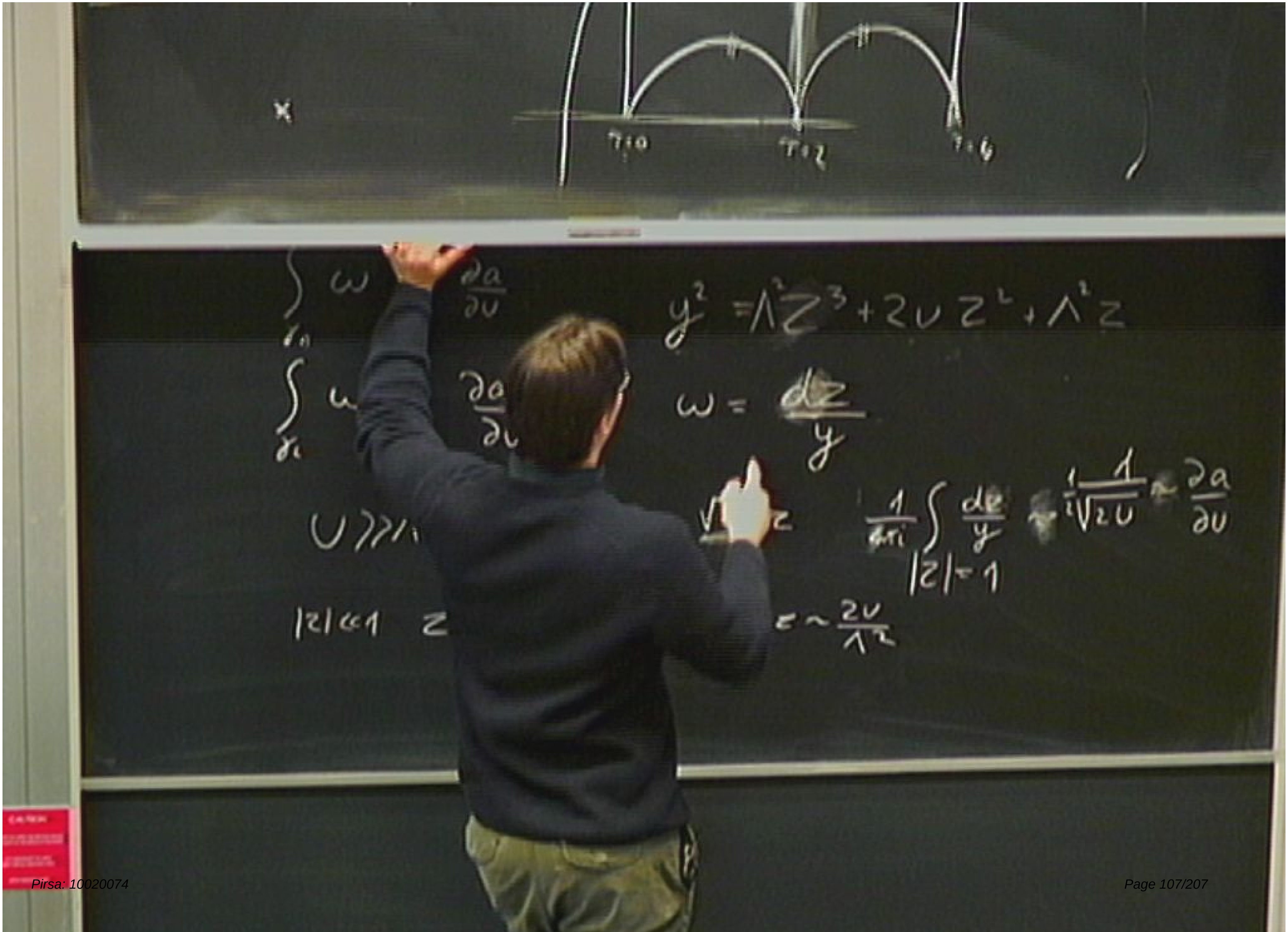
$U \to H/\Gamma(4)$

$\frac{\partial a_0}{\partial u}$

$\omega = \frac{dz}{y}$

$|z| \sim 1 \qquad y \sim \sqrt{2u}\, z \qquad \frac{1}{2\pi i}\oint_{|z|=1} \frac{dz}{y} \sim \frac{1}{\sqrt{2u}} \sim \frac{\partial a}{\partial u}$

$z \sim \frac{\Lambda^2}{2u} \qquad |z| \gg 1 \qquad z \sim \frac{2u}{\Lambda^2}$



$$\int_{\gamma_1} \omega \quad \frac{\partial a}{\partial u}$$

$$y^2 = \Lambda^2 z^3 + 2u\, z^2 + \Lambda^2 z$$

$$\int_{\gamma_1} \omega \quad \frac{\partial a}{\partial u}$$

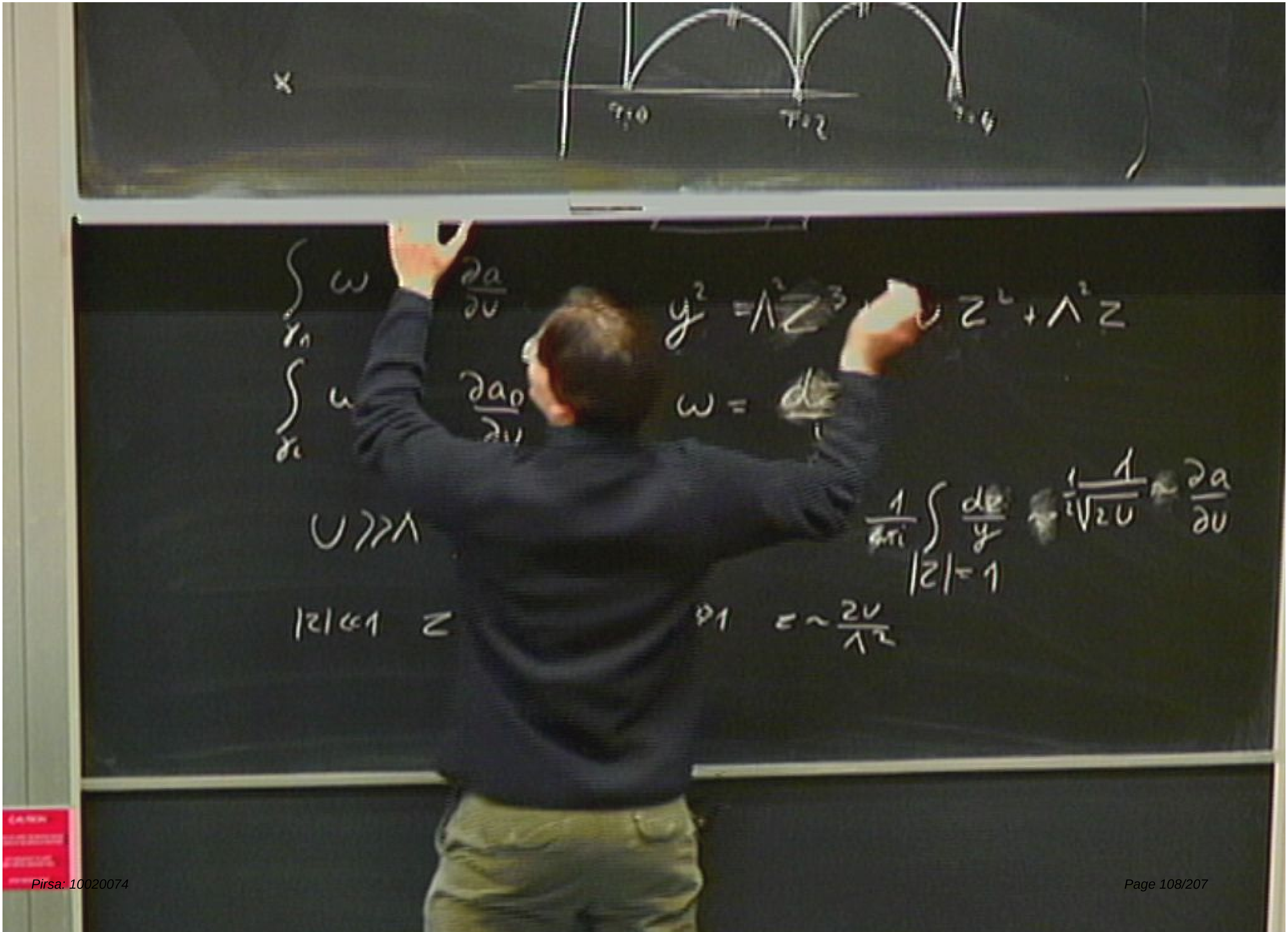
$$\omega = \frac{dz}{y}$$

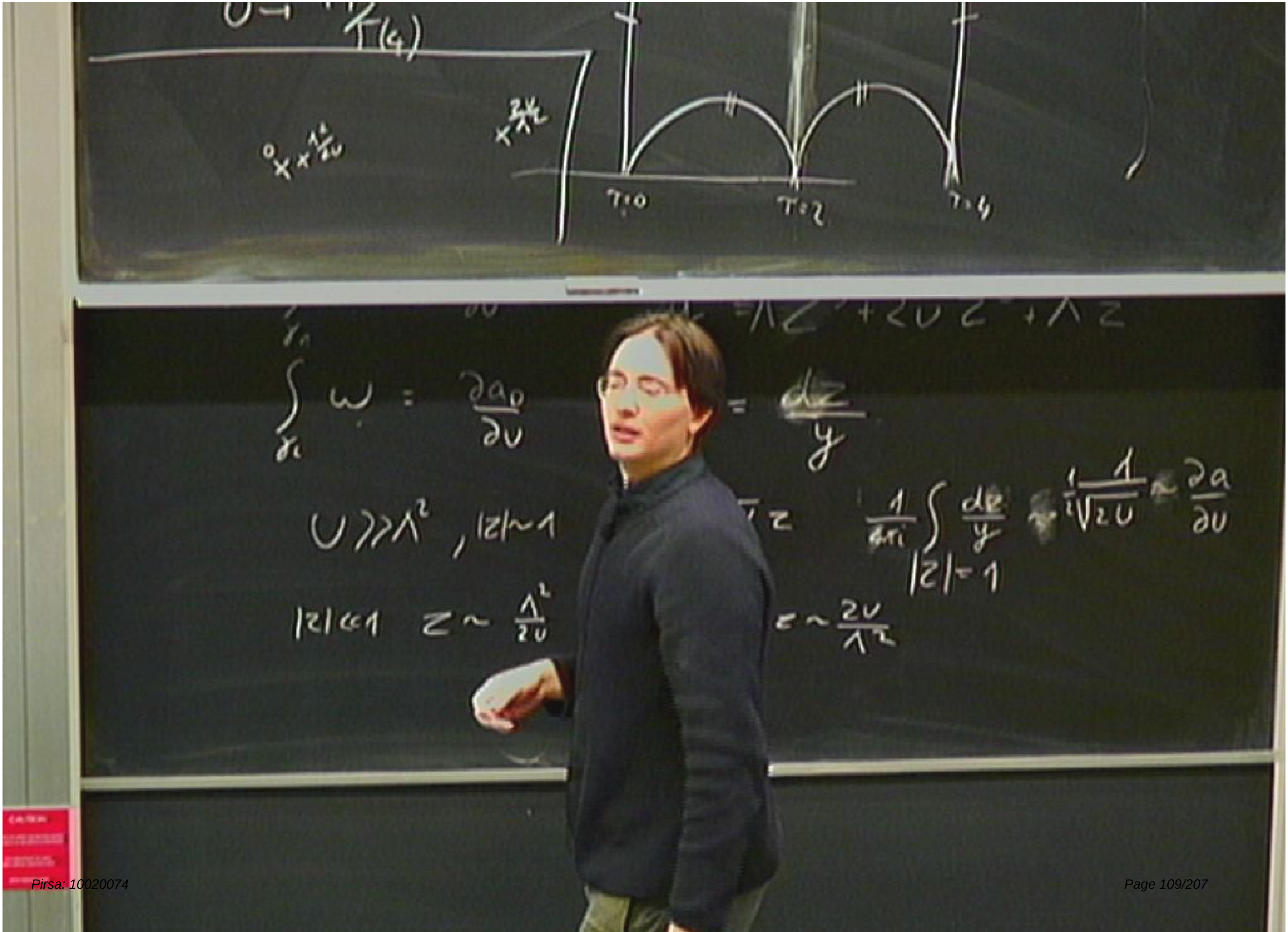
$u >> \Lambda$

$$\frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{y} \approx \frac{1}{i\sqrt{2u}} \approx \frac{\partial a}{\partial u}$$

$|z| \ll 1 \quad z$

$$z \sim \frac{2u}{\Lambda^2}$$

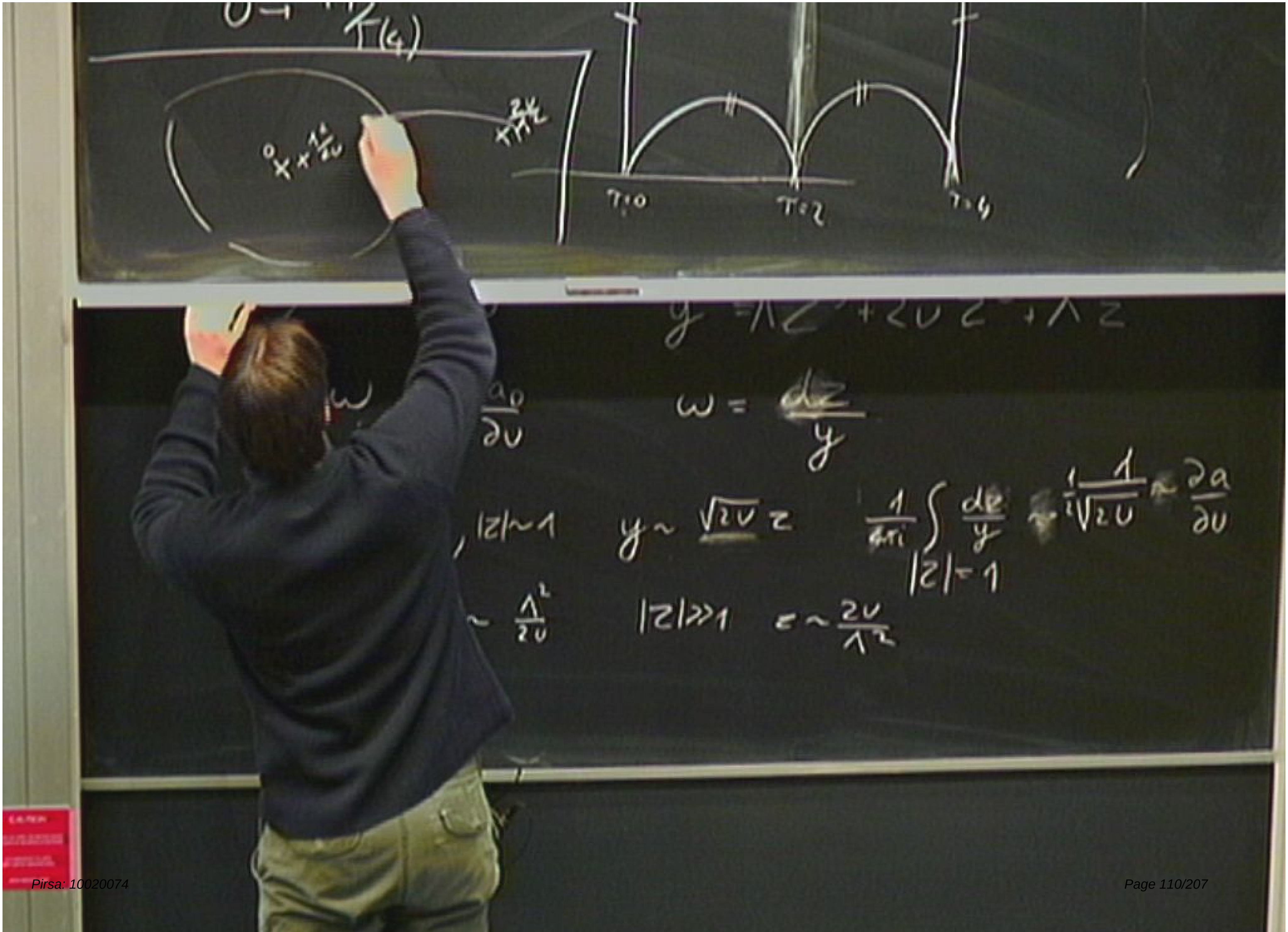

x
τ=0
τ=2
τ=∞
$\int_{\gamma_1} \omega$
$\frac{\partial a}{\partial u}$
$y^2 = \Lambda^2 z^3 + u z^2 + \Lambda^2 z$
$\int_{\gamma_2} \omega$
$\frac{\partial a_D}{\partial u}$
$\omega = \frac{dz}{y}$
$u \gg \Lambda$
$\frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{y} \approx \frac{1}{\sqrt{2u}} \approx \frac{\partial a}{\partial u}$
$|z| \ll 1$ z
$\gg 1$ $z \sim \frac{2u}{\Lambda^2}$



$\int_{\gamma_1} \omega = \frac{\partial a_0}{\partial u}$ $= \frac{dz}{y}$

$u \gg \Lambda^2$, $|z| \sim 1$ z $\frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{y} \approx \frac{1}{\sqrt{2u}} \approx \frac{\partial a}{\partial u}$

$|z| \ll 1$ $z \sim \frac{\Lambda^2}{2u}$ $z \sim \frac{2u}{\Lambda^2}$

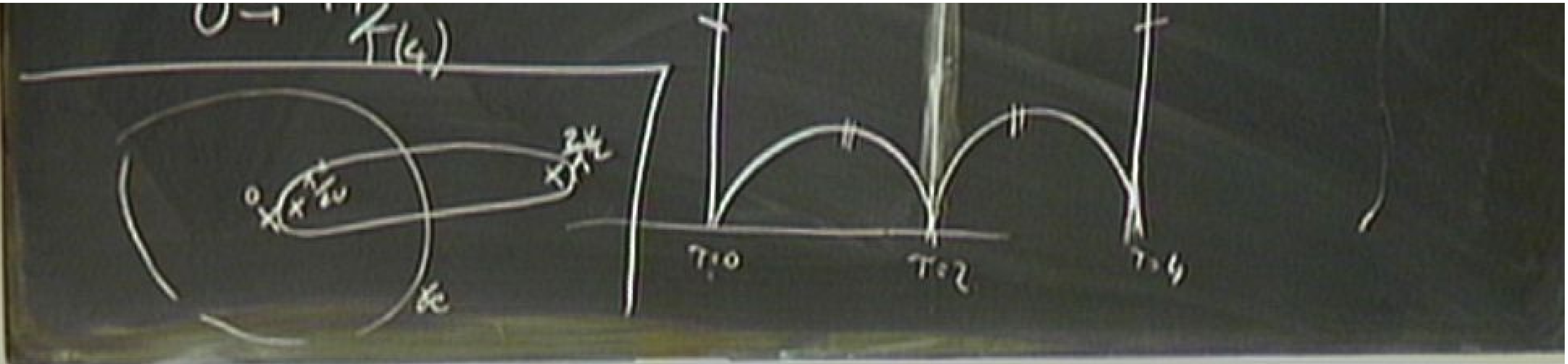


$y = \Lambda z + 2u z + \Lambda z$

ω $\frac{a_0}{\partial u}$ $\omega = \frac{dz}{y}$

$|z| \sim 1$ $y \sim \underline{\sqrt{2u}}\, z$ $\frac{1}{2\pi i}\oint_{|z|=1} \frac{dz}{y} \sim \frac{1}{\sqrt{2u}} \sim \frac{\partial a}{\partial u}$

$\sim \frac{\Lambda^2}{2u}$ $|z| \gg 1$ $z \sim \frac{2u}{\Lambda^2}$

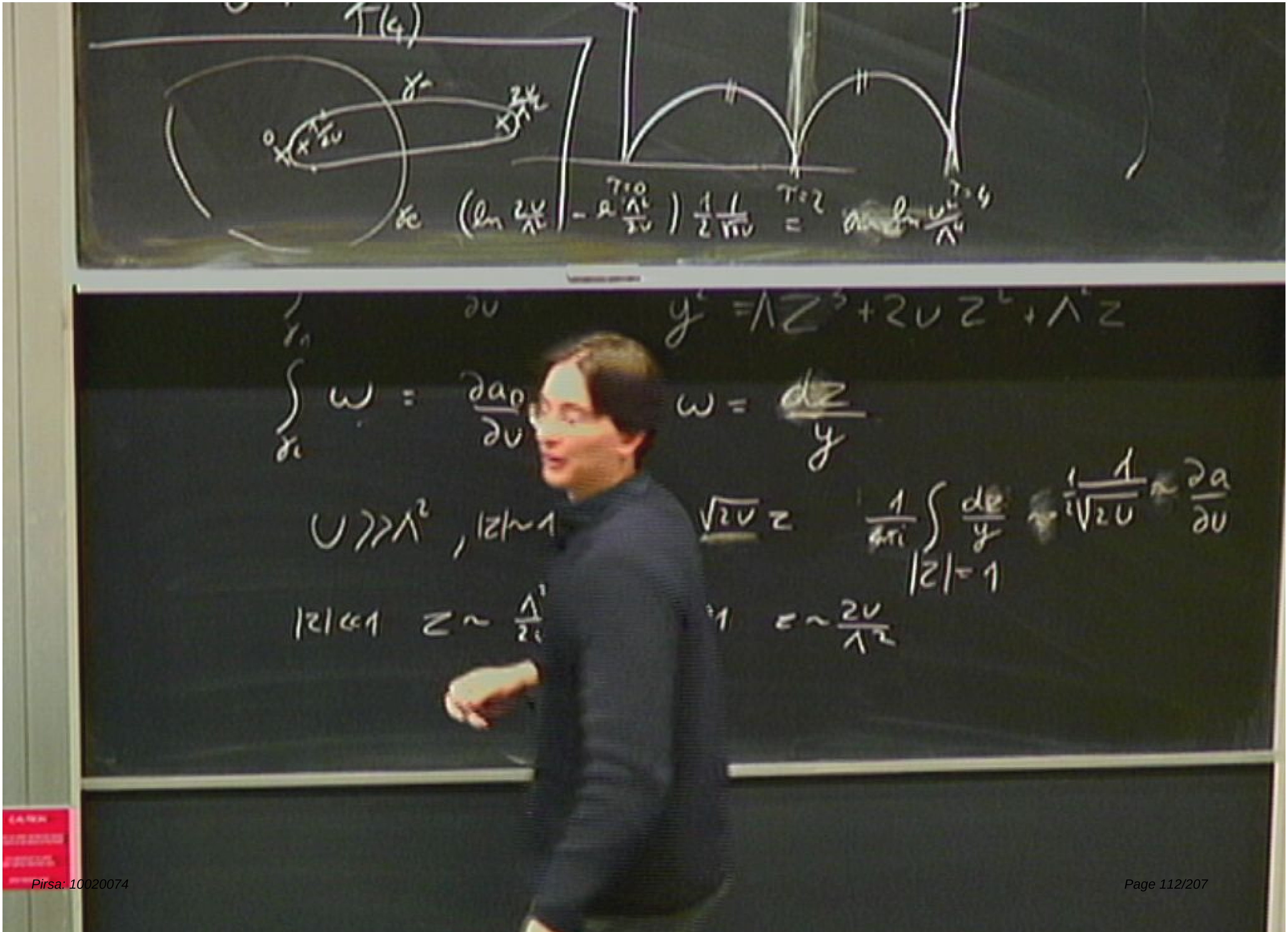


$y = \Lambda z \quad + 2u z \quad + \Lambda z$

$$\int_{\gamma_1} \omega = \frac{\partial a_D}{\partial u} \qquad \omega = \frac{dz}{y}$$

$$u \gg \Lambda^2, \ |z| \sim 1 \qquad y \sim \sqrt{2u}\, z \qquad \frac{1}{4\pi i} \int_{|z|=1} \frac{dz}{y} \sim \frac{1}{\sqrt{2u}} \sim \frac{\partial a}{\partial u}$$

$$|z| \ll 1 \quad z \sim \frac{\Lambda^2}{2u} \qquad |z| \gg 1 \quad z \sim \frac{2u}{\Lambda^2}$$



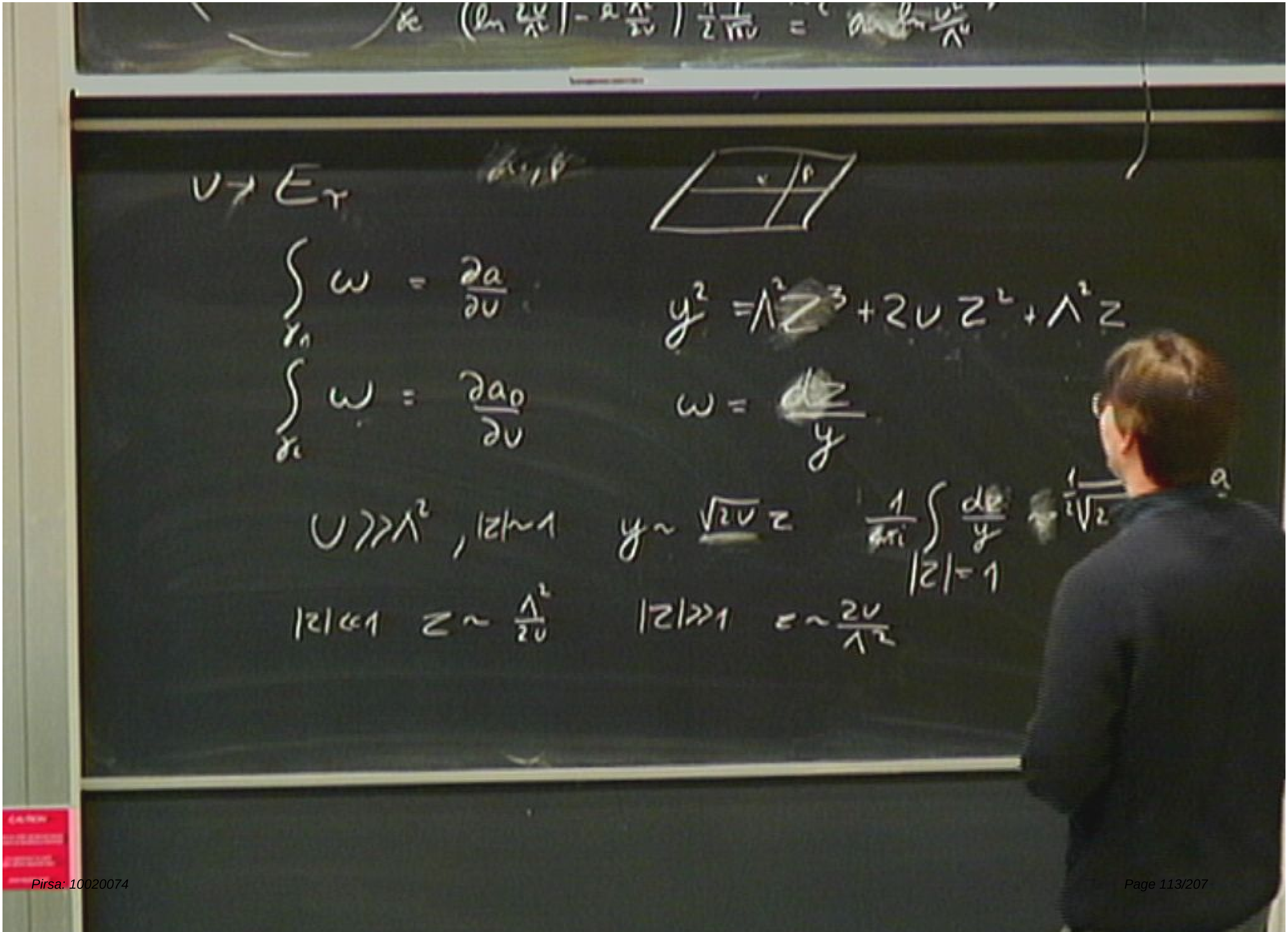
$\left(\ln \frac{2u}{\Lambda^2} - \frac{\Lambda^2}{2u}\right)\frac{1}{2}\frac{1}{\pi u}$ $\ln \frac{u^2}{\Lambda^4}$

$y^2 = \Lambda z^3 + 2u z^2 + \Lambda^2 z$

$\int_{\gamma_1}^{\gamma_2} \omega = \frac{\partial a_D}{\partial u}$ $\omega = \frac{dz}{y}$

$u \gg \Lambda^2$, $|z| \sim 1$ $\sqrt{2u}\, z$ $\frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{y} \approx \frac{1}{\sqrt{2u}} \approx \frac{\partial a}{\partial u}$

$|z| \ll 1$ $z \sim \frac{\Lambda^2}{2u}$ $z \sim \frac{2u}{\Lambda^2}$



$\left(\ln \frac{2v}{\Lambda^2}\right) - 2\frac{\Lambda^2}{2v}\Big) \frac{1}{2}\frac{1}{\sqrt{2v}} = \quad a \ln \frac{v^2}{\Lambda^4}$

$v \to E_\tau$

α, β

$$\int_{\gamma_1} \omega = \frac{\partial a}{\partial v}$$

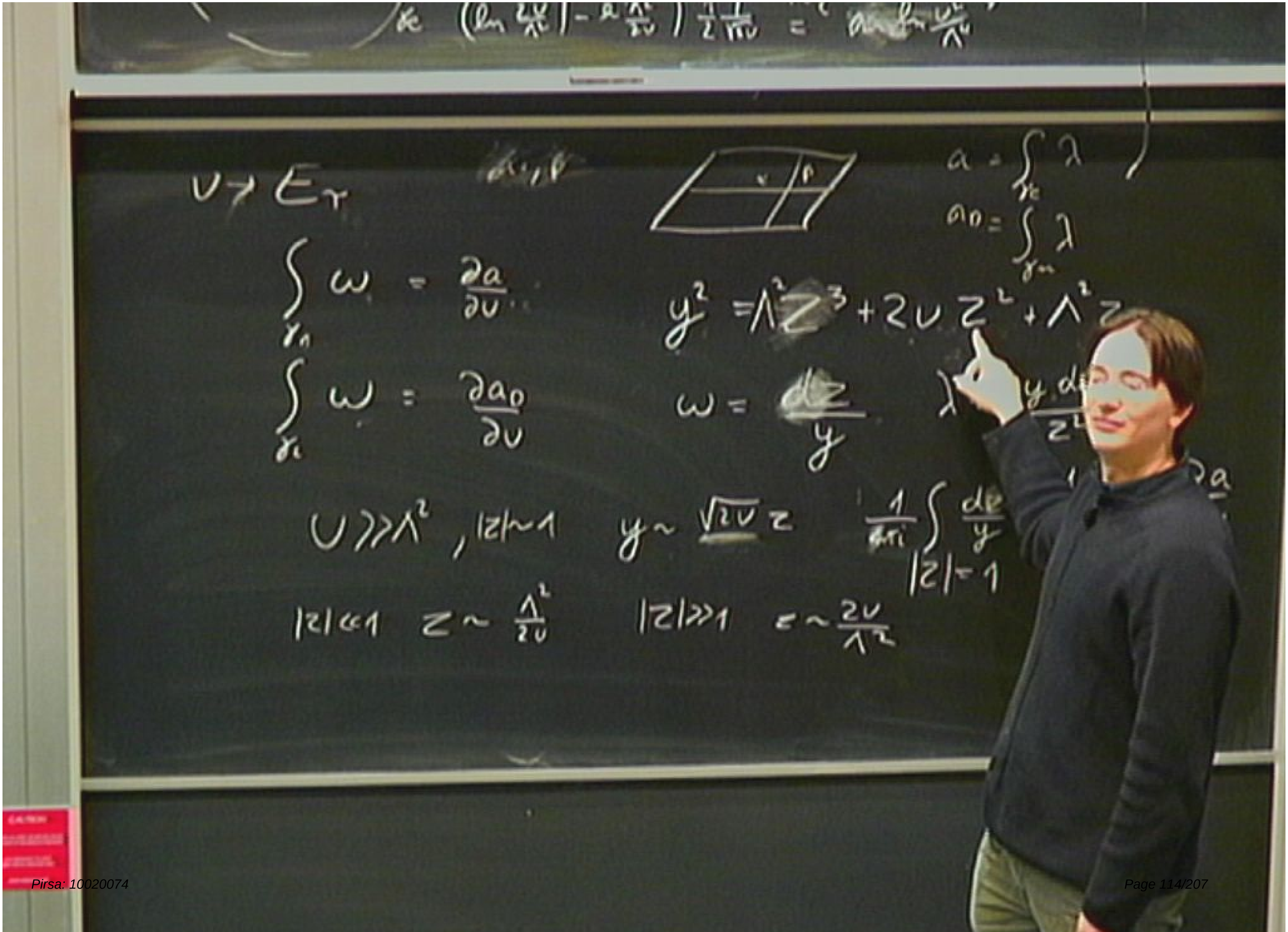
$$y^2 = \Lambda^2 z^3 + 2v\, z^2 + \Lambda^2 z$$

$$\int_{\gamma_2} \omega = \frac{\partial a_D}{\partial v}$$

$$\omega = \frac{dz}{y}$$

$v \gg \Lambda^2$, $|z| \sim 1$ $\quad y \sim \sqrt{2v}\, z \quad \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{y} \sim \frac{1}{i\sqrt{2}} \quad a$

$|z| \ll 1 \quad z \sim \frac{\Lambda^2}{2v} \qquad |z| \gg 1 \quad z \sim \frac{2v}{\Lambda^2}$



$\left(\ln \left|\frac{2u}{\Lambda^2}\right| - 2\frac{\Lambda^2}{2u}\right)\frac{1}{2}\frac{1}{\sqrt{2u}} = a \ln\frac{u^2}{\Lambda^4}$

$u \to E_\tau$

$a = \int_{\gamma_e} \lambda$

$a_D = \int_{\gamma_m} \lambda$

$\int_{\gamma_m} \omega = \frac{\partial a}{\partial u}$

$\int_{\gamma_e} \omega = \frac{\partial a_D}{\partial u}$

$y^2 = \Lambda^2 z^3 + 2u\, z^2 + \Lambda^2 z$

$\omega = \frac{dz}{y}$

$\lambda = \frac{y\, dz}{z^2}$

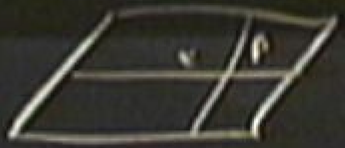
$u \gg \Lambda^2, \ |z| \sim 1 \quad y \sim \sqrt{2u}\, z$

$\frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{y}$

$|z| \ll 1 \quad z \sim \frac{\Lambda^2}{2u} \qquad |z| \gg 1 \quad z \sim \frac{2u}{\Lambda^2}$

$\left(\ln \frac{2u}{\Lambda^2}\right| - 2\frac{\Lambda^2}{2u}\Big) \frac{1}{2}\frac{1}{\sqrt{2u}} = \ln\frac{u^2}{\Lambda^4}$

$u \to E_\tau$ α, β



$a = \int_{\gamma_\alpha} \lambda$

$a_D = \int_{\gamma_\beta} \lambda$

$$\int_{\gamma_\alpha} \omega = \frac{\partial a}{\partial u}$$

$$\int_{\gamma_\beta} \omega = \frac{\partial a_D}{\partial u}$$

$$y^2 = \Lambda^2 z^3 + 2u\, z^2 + \Lambda^2 z$$

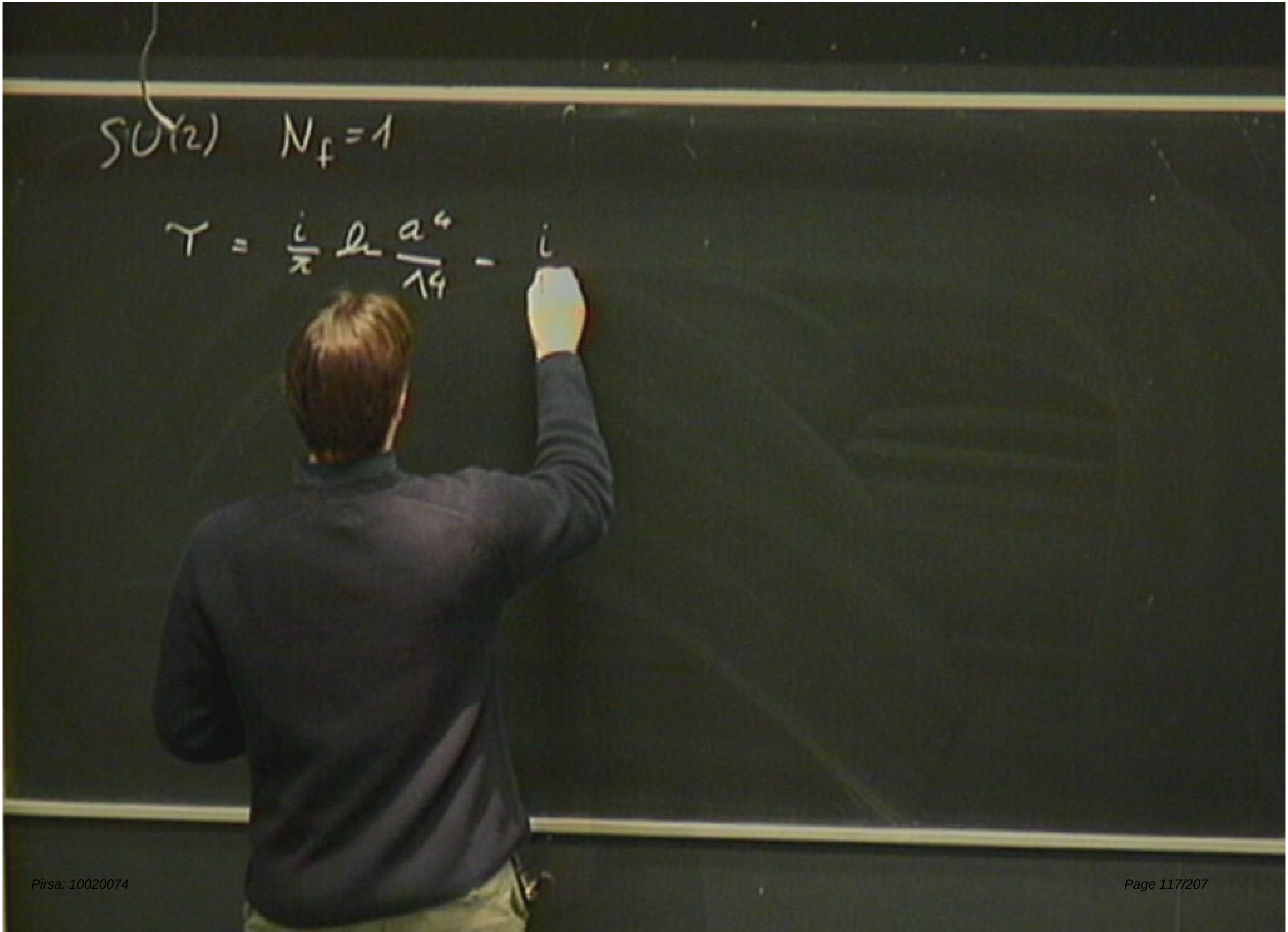
$$\omega = \frac{dz}{y} \qquad \boxed{\lambda = \frac{y\, dz}{z^2}}$$

$u \gg \Lambda^2$, $|z| \sim 1$ $\quad y \sim \sqrt{2u}\, z$ $\quad \frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{y} = \frac{1}{\sqrt{2u}} = \frac{\partial a}{\partial u}$

$|z| \ll 1 \quad z \sim \frac{\Lambda^2}{2u} \qquad |z| \gg 1 \quad z \sim \frac{2u}{\Lambda^2}$

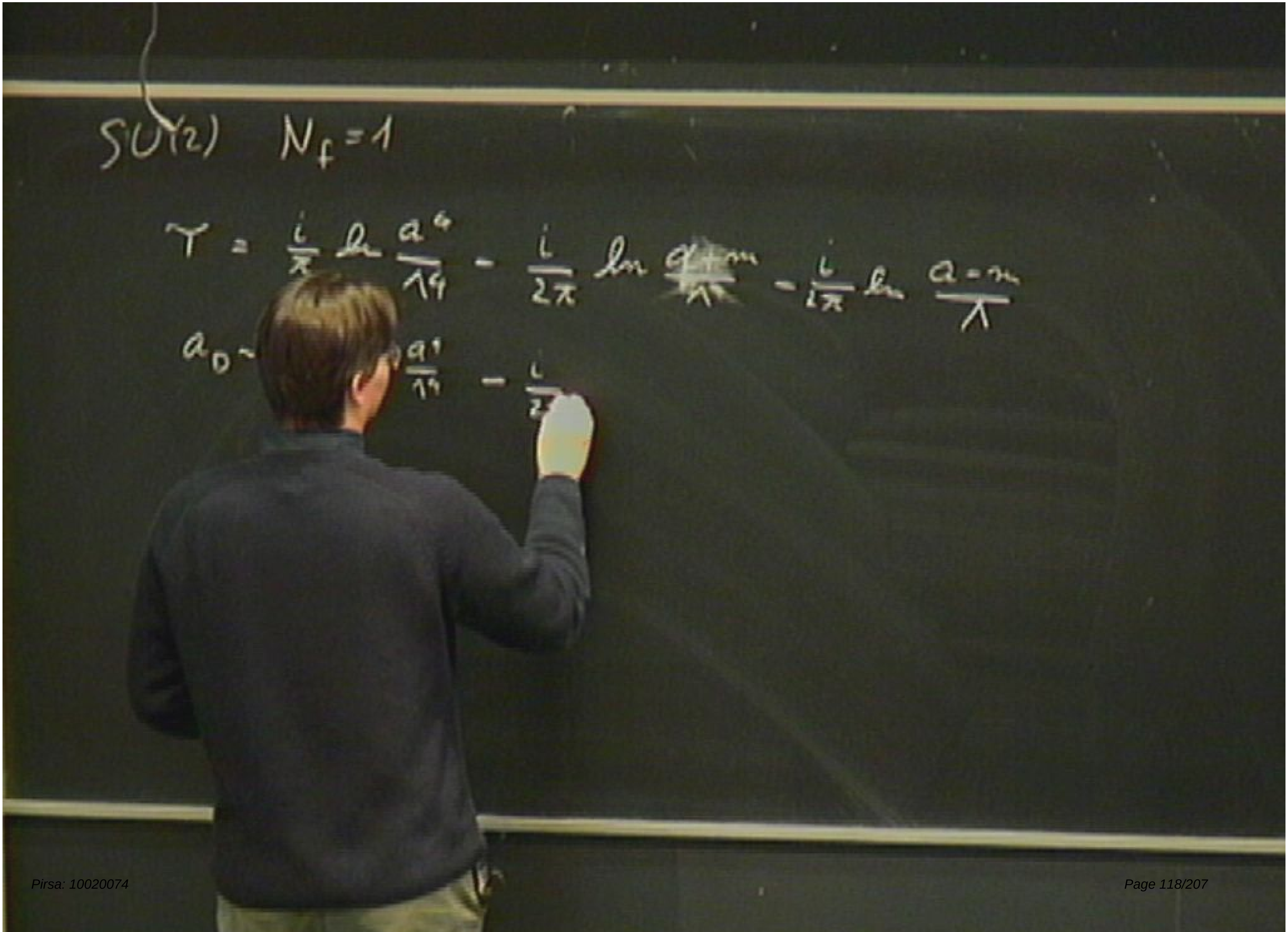
$SU(2) \quad N_f = 1$

Y



$SU(2) \quad N_f = 1$

$$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - i$$



SU(2) $N_f = 1$

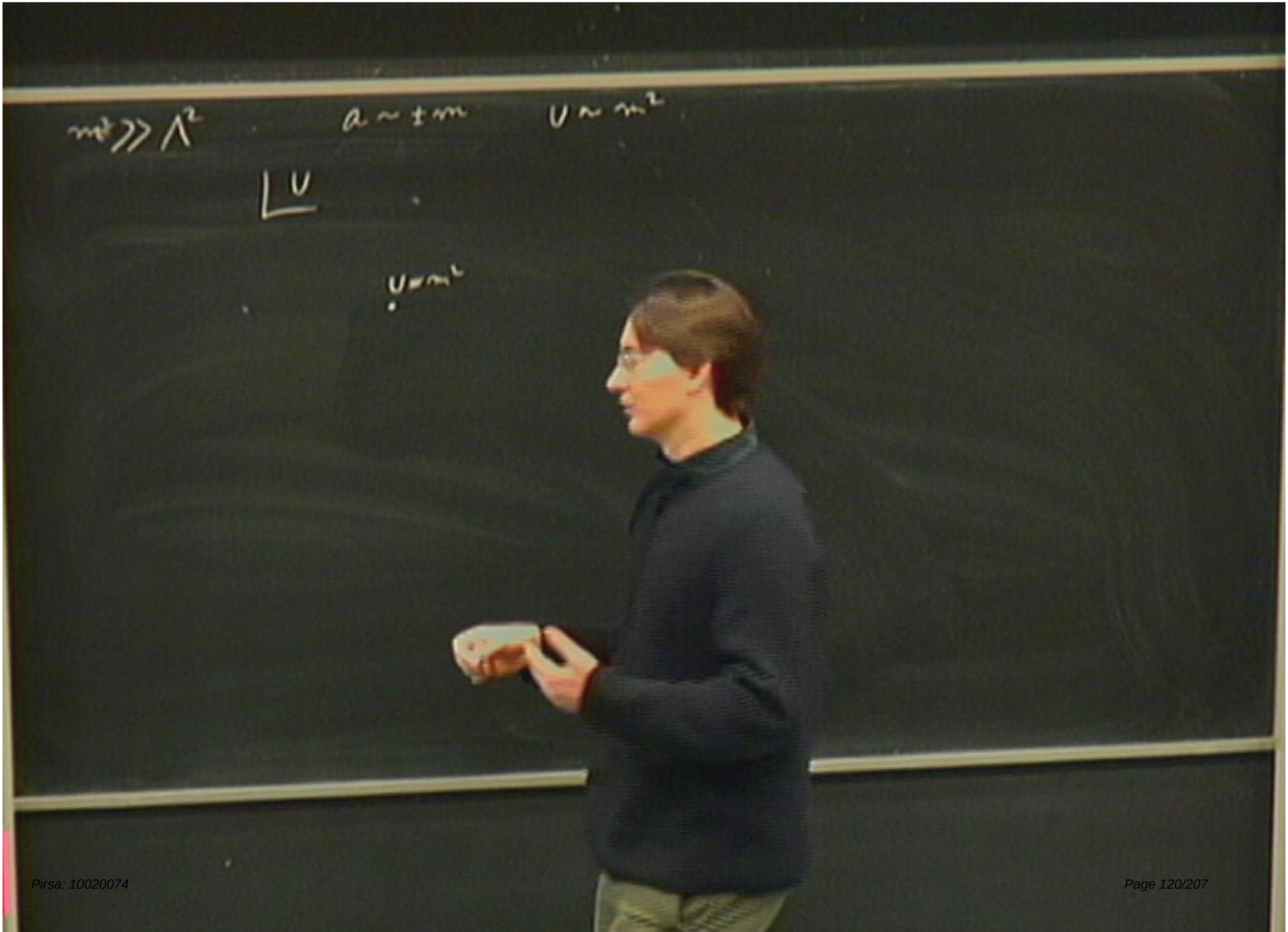
$$\Upsilon = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

$$a_D \sim \ldots \frac{a^4}{\Lambda^4} - \frac{i}{2\ldots}$$

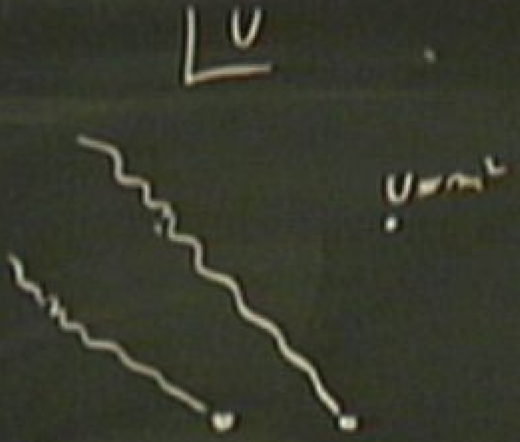
$SU(2) \quad N_f = 1$

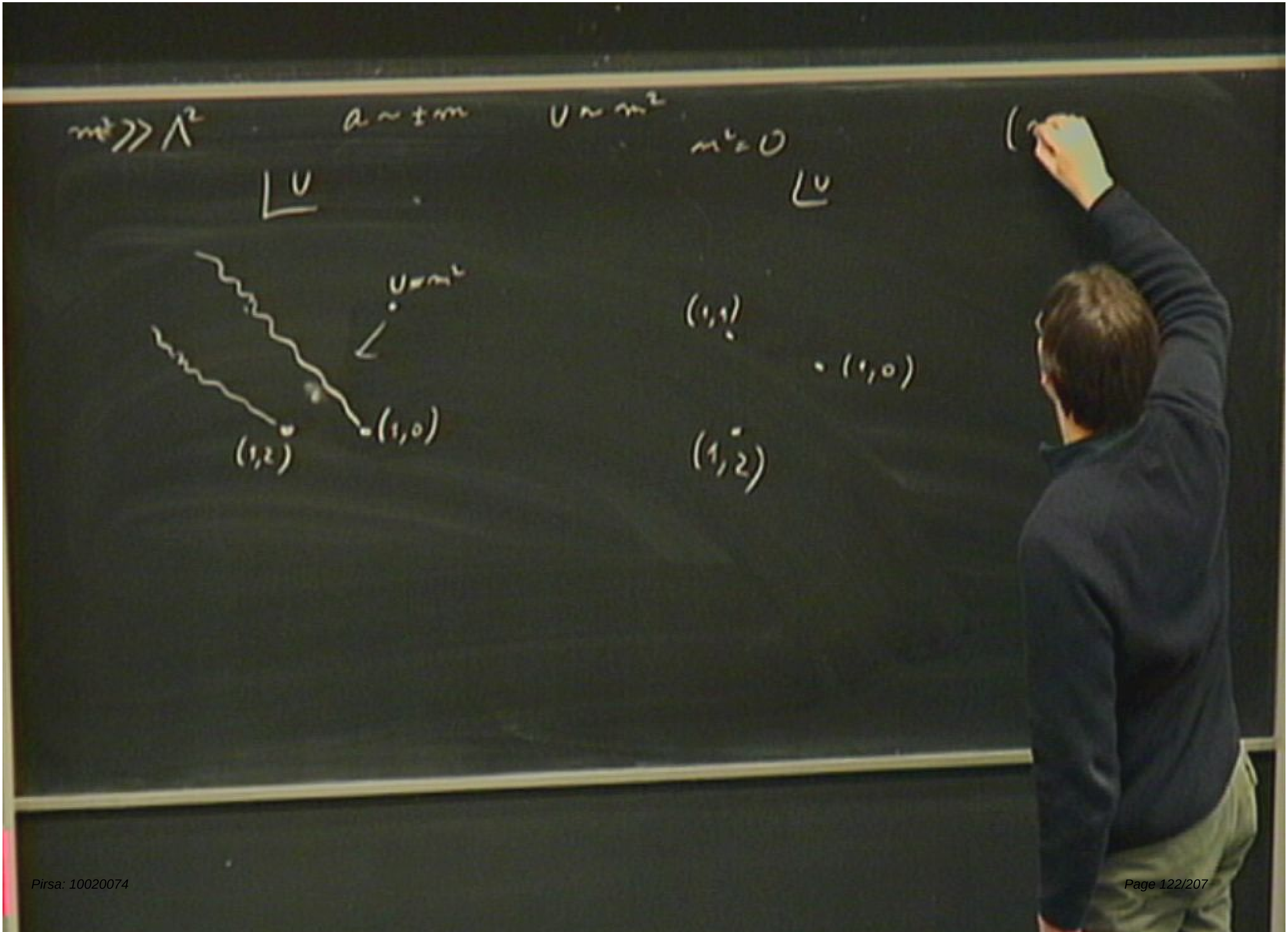
$$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

$$a_D \sim \frac{i}{\pi} a \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} (a+m) \ln \frac{(a+m)}{\Lambda} - \frac{i}{2\pi} (a-m) \ln \frac{a-m}{\Lambda}$$

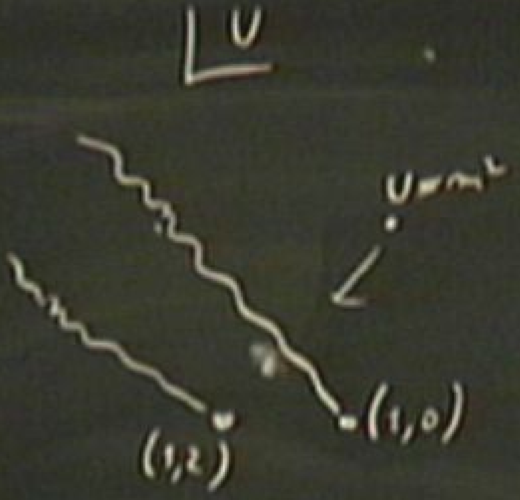

$m^2 \gg \Lambda^2$
$a \sim \pm m$
$U \sim m^2$
U
$U = m^2$

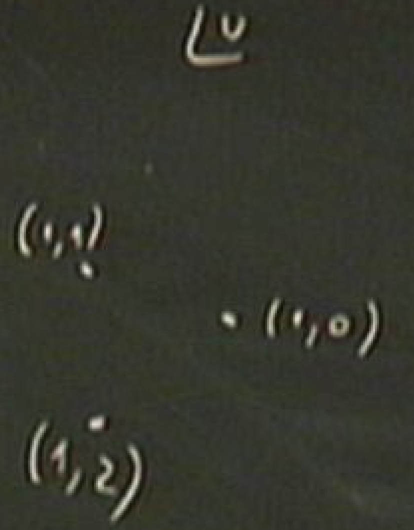
$m^2 \gg \Lambda^2$ $a \sim \pm m$ $U \sim m^2$

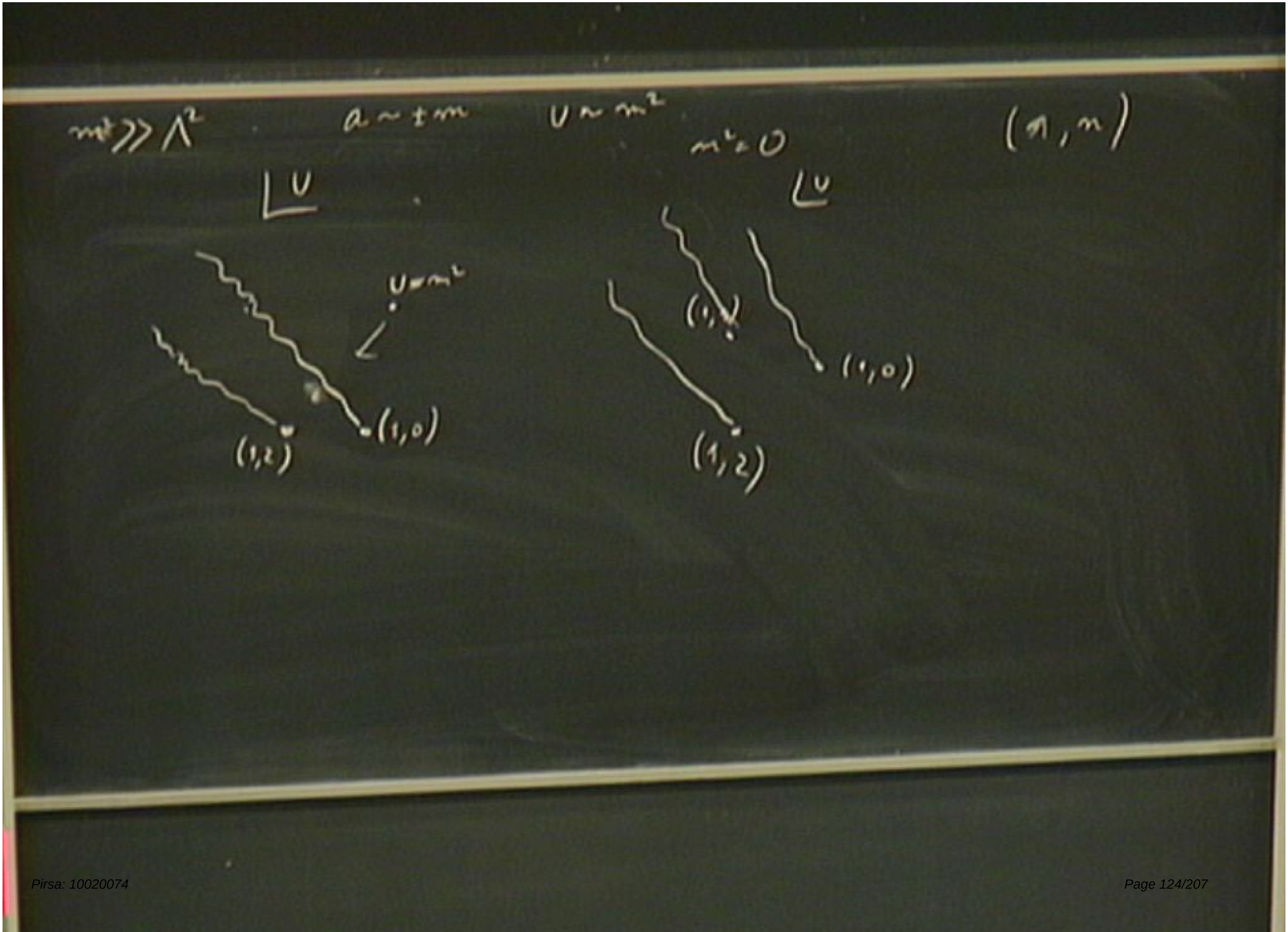


$m^2 \gg \Lambda^2$
$a \sim \pm m$
$u \sim m^2$
$m^2 = 0$
(1
u
$u = m^2$
(1,2)
(1,0)
u
(1,1)
(1,0)
(1,2)

$m^2 \gg \Lambda^2$ $a \sim \pm m$ $U \sim m^2$ $m^2 \approx 0$ (n, m)

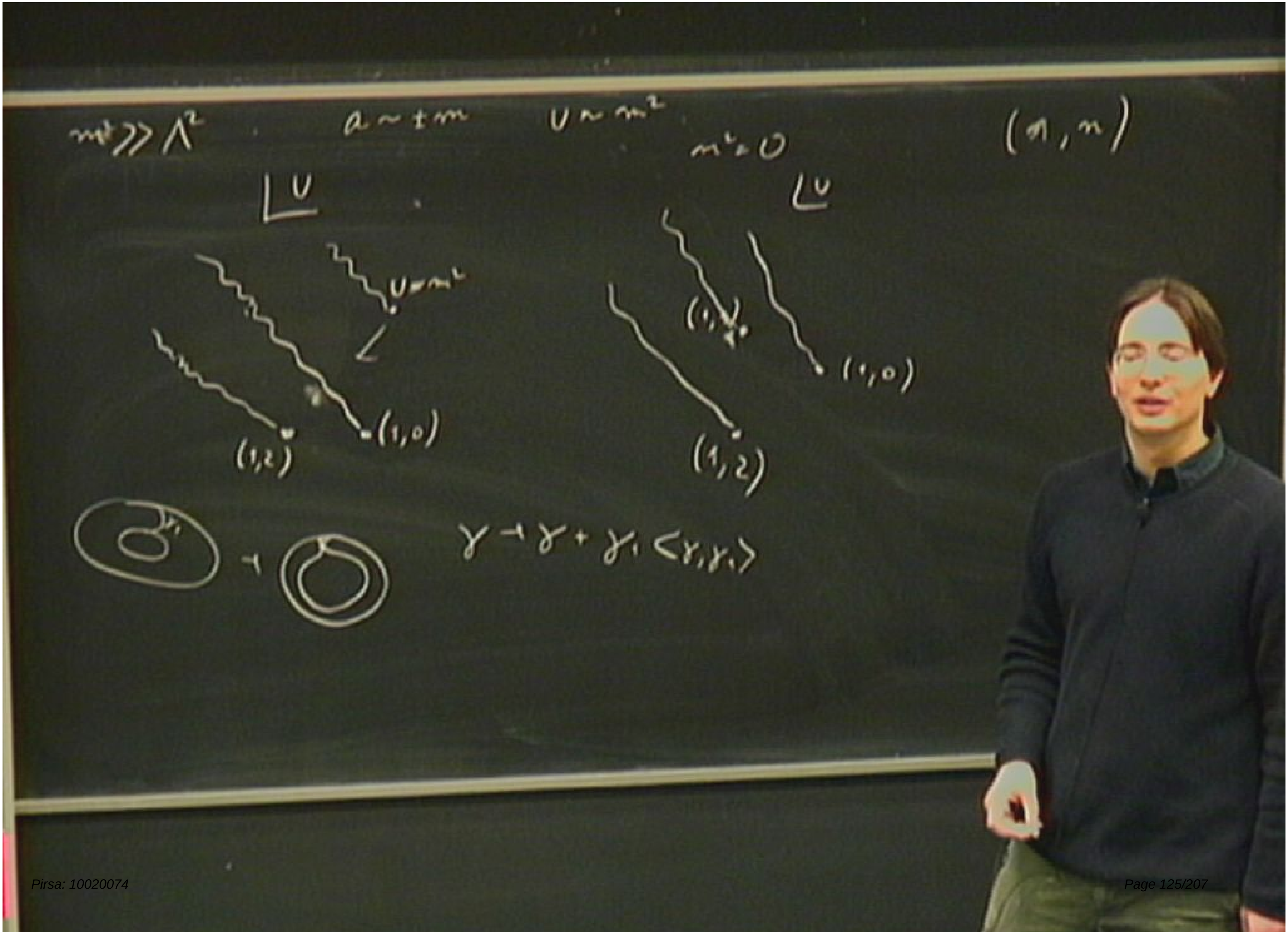






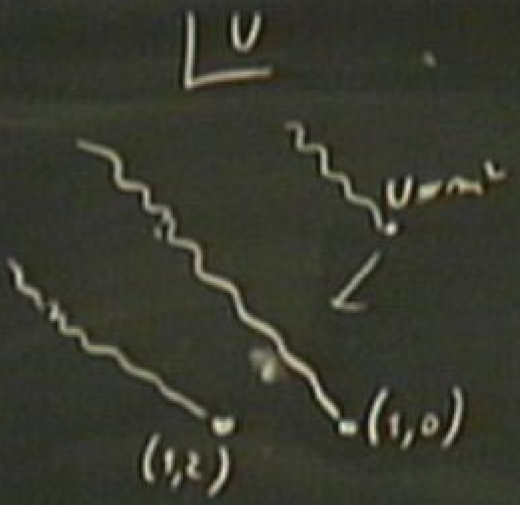
$m^2 \gg \Lambda^2$ $a \sim \pm m$ $U \sim m^2$ (m, n)

$m^2 \approx 0$

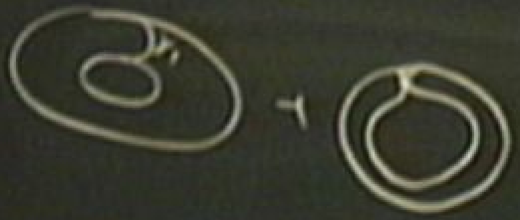


$$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$$

$m^2 \gg \Lambda^2$ $\quad a \sim \pm m \quad u \sim m^2$ $\quad (n, m)$



$m^2 = 0$



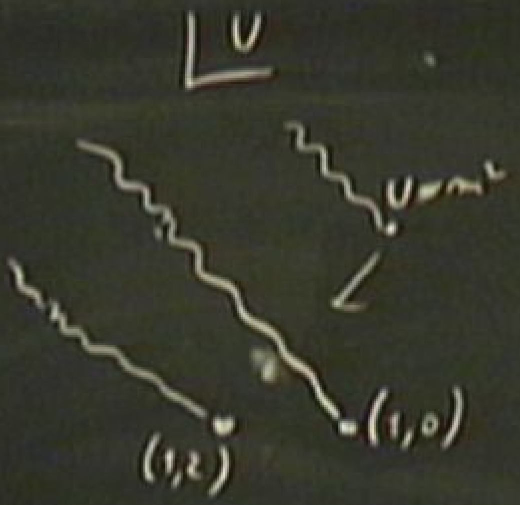
$$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$$

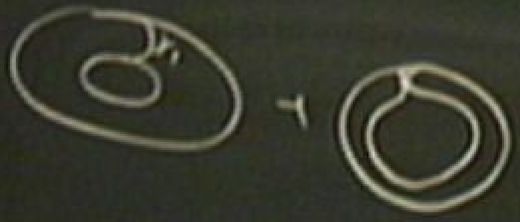
$$q \to q + q_1 \langle q, q_1 \rangle$$

$$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$$

$m^2 \gg \Lambda^2$ $\quad a \sim \pm m$ $\quad U \sim m^2$ $\quad (n, m)$

$m^2 \approx 0$





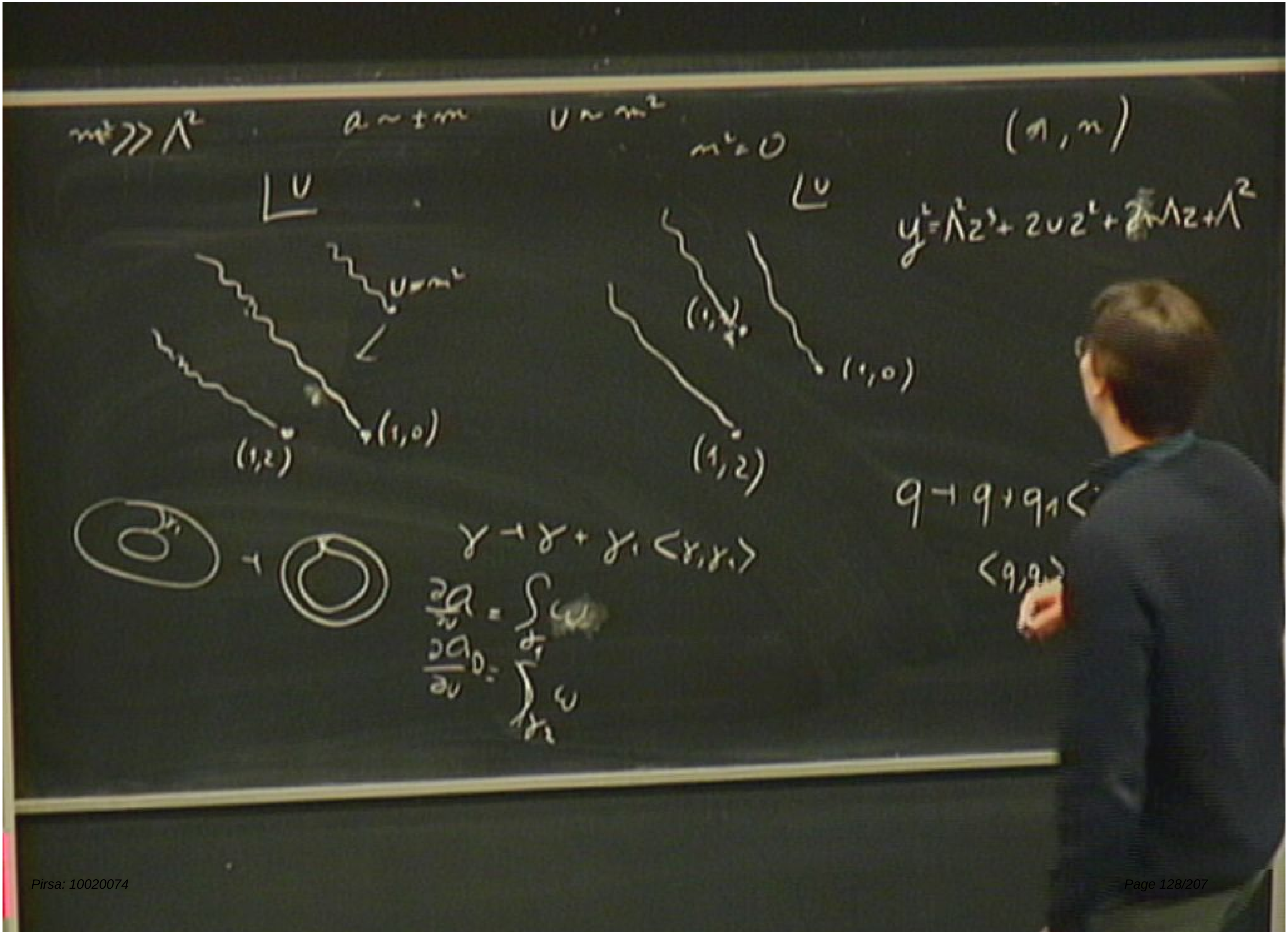
$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$

$\frac{\partial a}{\partial v} = \int_{\gamma_1} \omega$

$\frac{\partial a_D}{\partial v} = \int_{\gamma_2} \omega$

$q \to q + q_1 \langle q, q_1 \rangle$

$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$



$m^2 \gg \Lambda^2$ $a \sim \pm m$ $u \sim m^2$ (m,n)

$m^2 = 0$

$y^2 = \Lambda^2 z^3 + 2uz^2 + m\Lambda z + \Lambda^2$

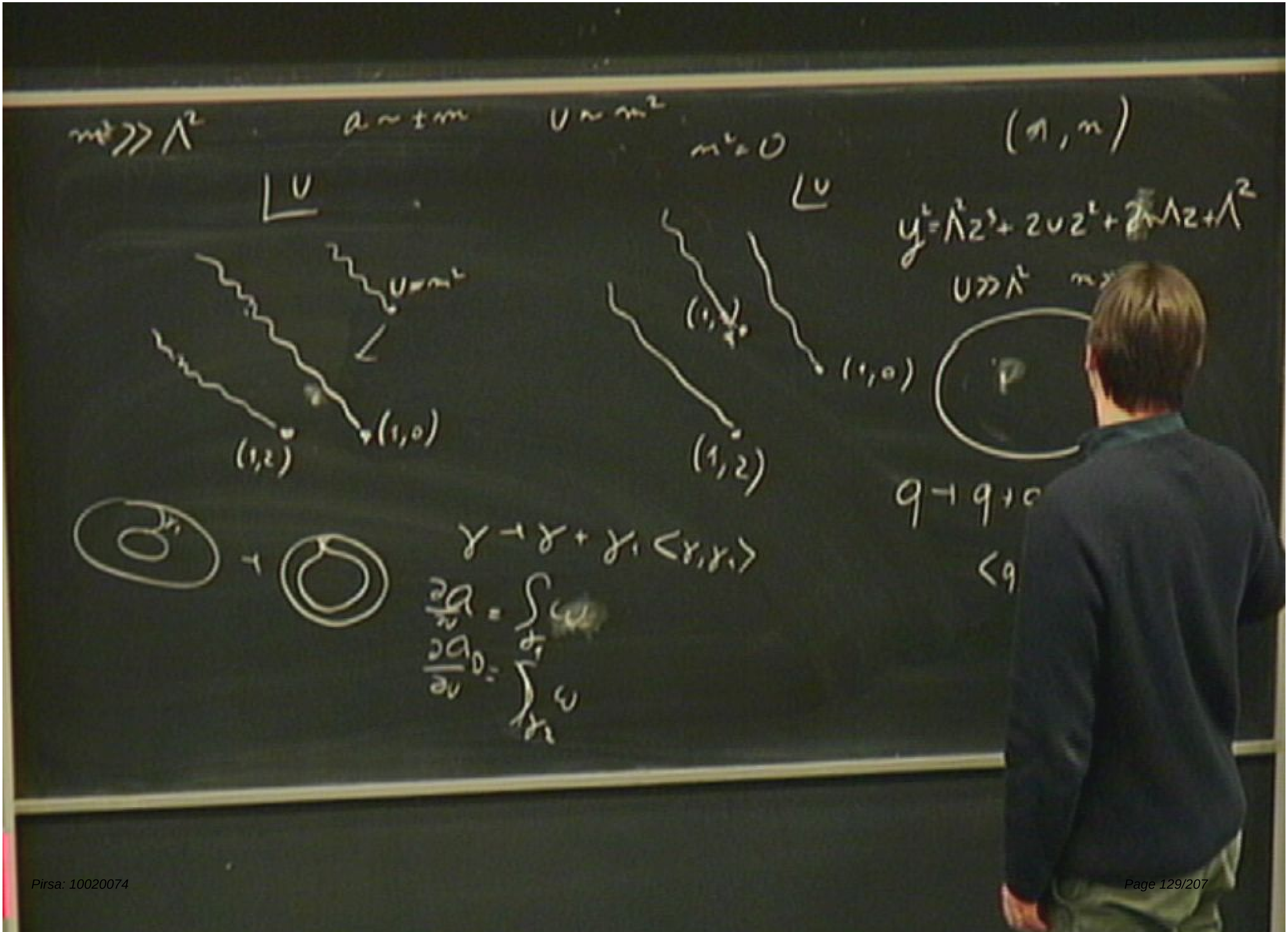
$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$

$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$

$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$

$q \to q + q_1 \langle$

$\langle q, q_1 \rangle$



$m^2 \gg \Lambda^2$ $a \sim \pm m$ $u \sim m^2$

$m^2 = 0$

(n, m)

$y^2 = \Lambda^2 z^3 + 2uz^2 + 2m\Lambda z + \Lambda^2$

$u \gg \Lambda^2$ m

$q \to q + c$

$\langle q$

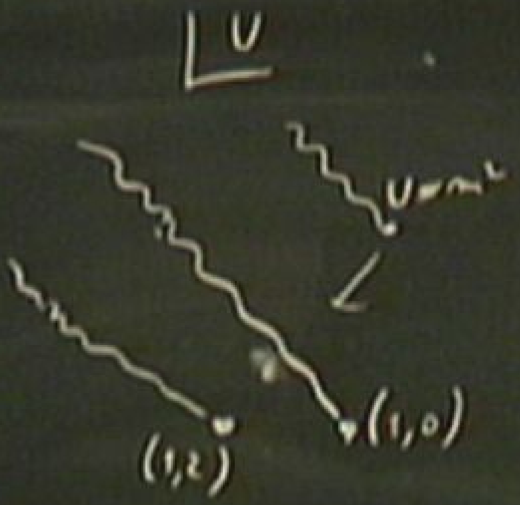
$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$

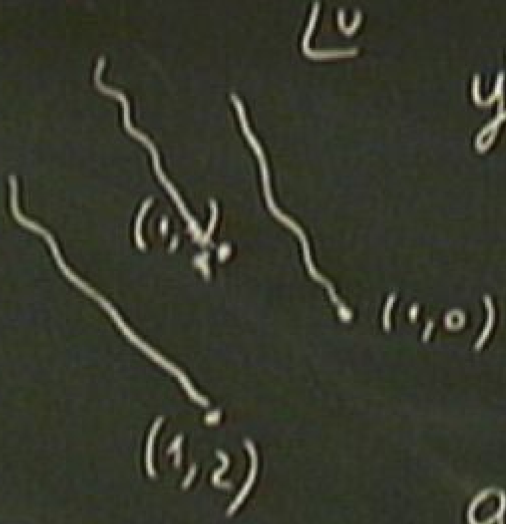
$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$

$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$

$m^2 \gg \Lambda^2$ $a \sim \pm m$ $u \sim m^2$ (n, m) $\frac{dz}{y}$

$m^2 = 0$





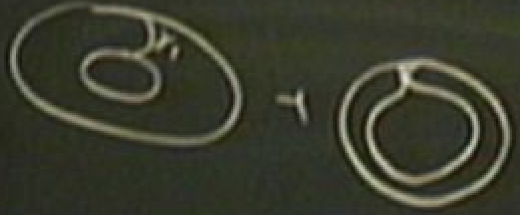
$y^2 = \Lambda^2 z^3 + 2uz^2 + 2m\Lambda z + \Lambda^2$

$u \gg \Lambda^2$ $m \gg \Lambda$ ×

$|z| = 1$

$q \to q + q_1 \langle q, q_1 \rangle$

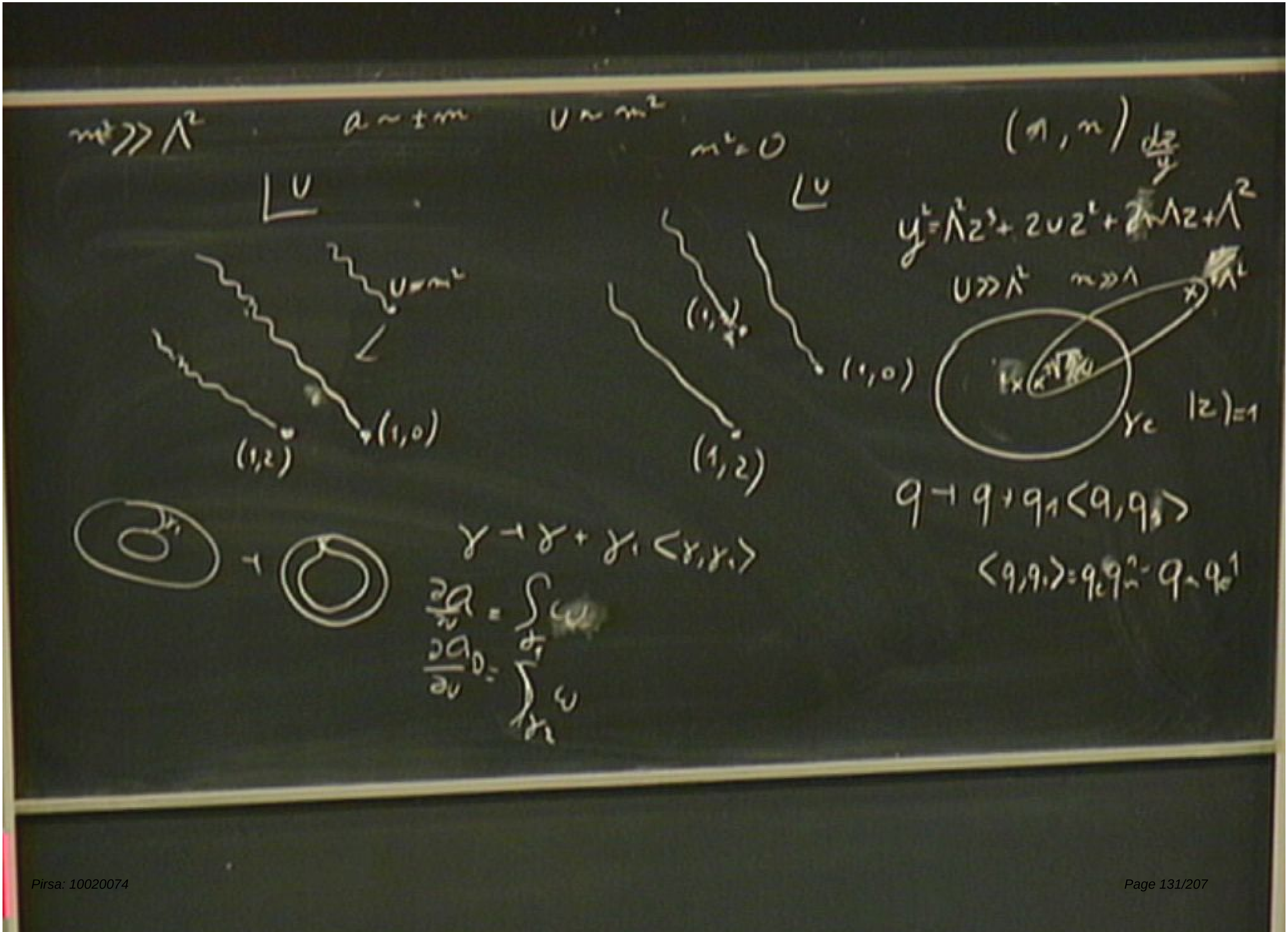
$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$



$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$

$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$

$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$



$m^2 \gg \Lambda^2$ $a \sim \pm m$ $u \sim m^2$

$m^2 = 0$

(n, m) $\frac{dz}{y}$

$y^2 = \Lambda^2 z^3 + 2uz^2 + 2m\Lambda z + \Lambda^2$

$u \gg \Lambda^2$ $m \gg \Lambda$

$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$

$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$

$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$

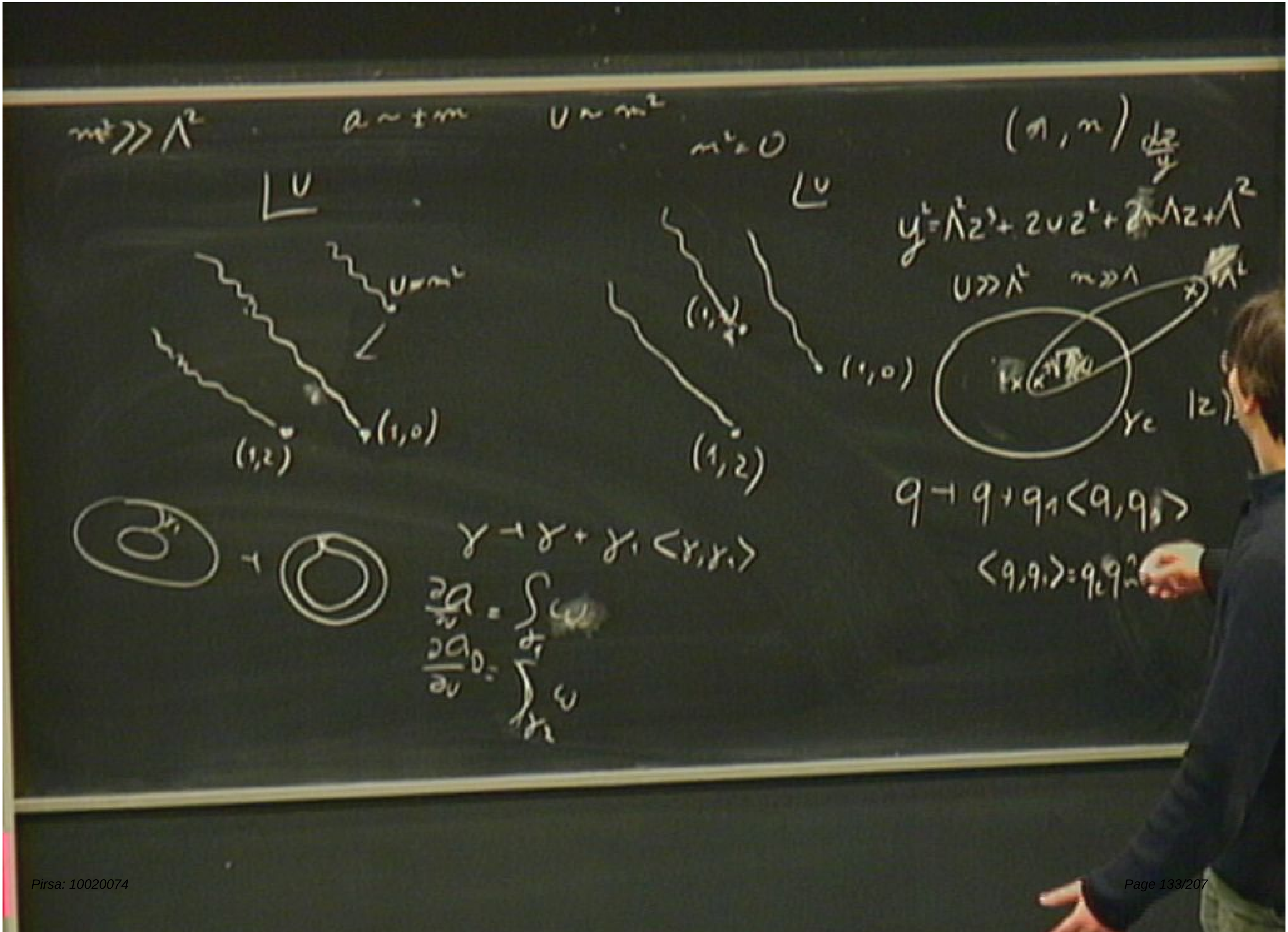
$q \to q + q_1 \langle q, q_1 \rangle$

$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$

$SU(2) \quad N_f = 1$

$$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

$$a_D \sim \frac{i}{\pi} a \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} (a+m) \ln \frac{(a+m)}{\Lambda} - \frac{i}{2\pi} (a-m) \ln \frac{a-m}{\Lambda}$$

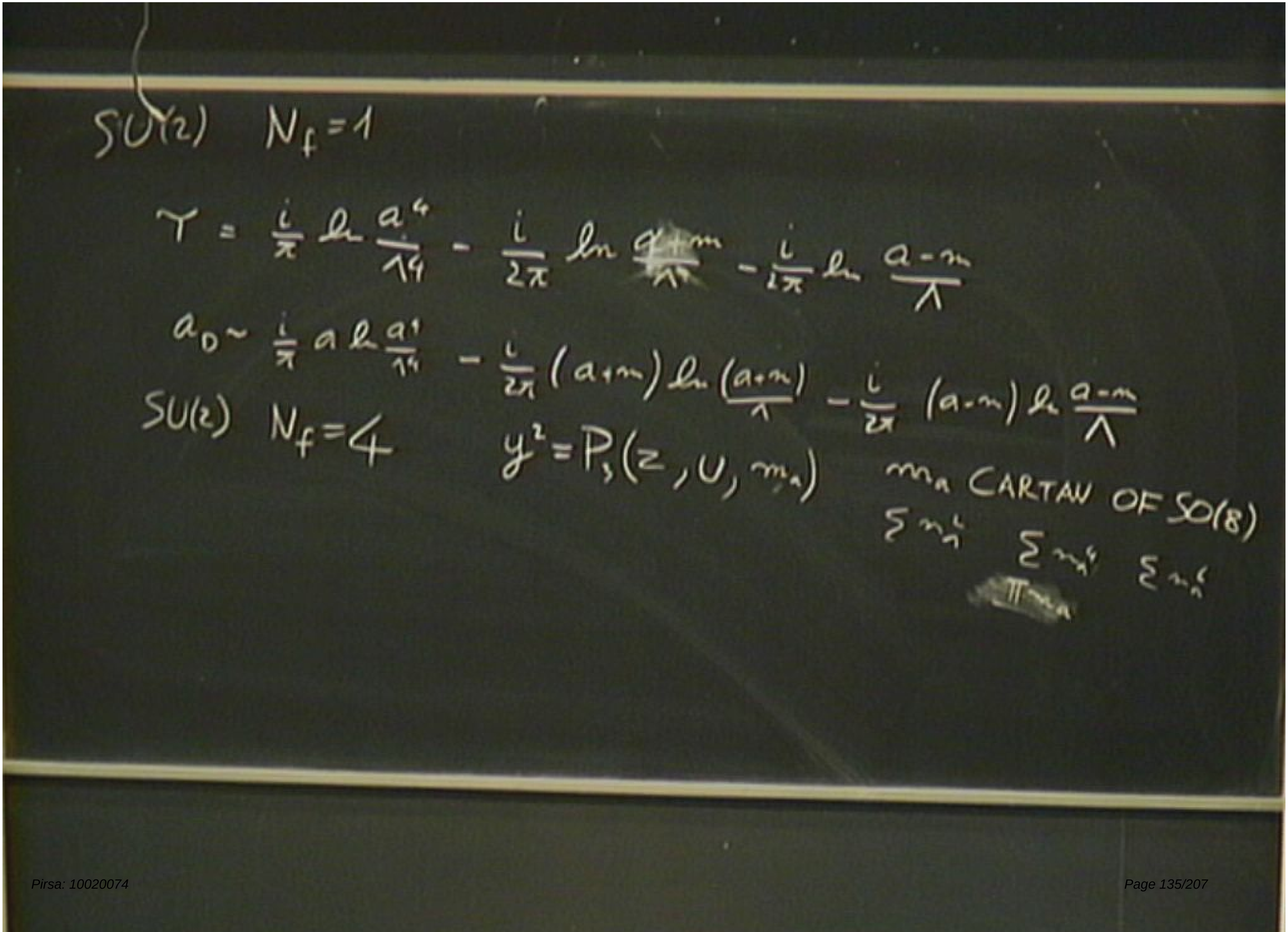

$m^2 \gg \Lambda^2$
$a \sim \pm m$
$u \sim m^2$
$m^2 \approx 0$
(n, m)
$\frac{dz}{y}$
u
$u = m^2$
$(1,2)$
$(1,0)$
$(1,2)$
$(1,0)$
$y^2 = \Lambda^2 z^3 + 2uz^2 + \ldots \Lambda z + \Lambda^2$
$u \gg \Lambda^2 \quad m \gg \Lambda$
γ_c
$|z|$
γ_1
$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$
$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$
$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$
$q \to q + q_1 \langle q, q_1 \rangle$
$\langle q, q_1 \rangle = q_e q_m \ldots$

$SU(2)$ $N_f = 1$

$$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

$$a_D \sim \frac{i}{\pi} a \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} (a+m) \ln \left(\frac{a+m}{\Lambda}\right) - \frac{i}{2\pi} (a-m) \ln \frac{a-m}{\Lambda}$$

$SU(2)$ $N_f = 4$ $y^2 = P_3(z, U, m_a)$ m_a CARTAN OF $SO(8)$



$SU(2) \quad N_f = 1$

$$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

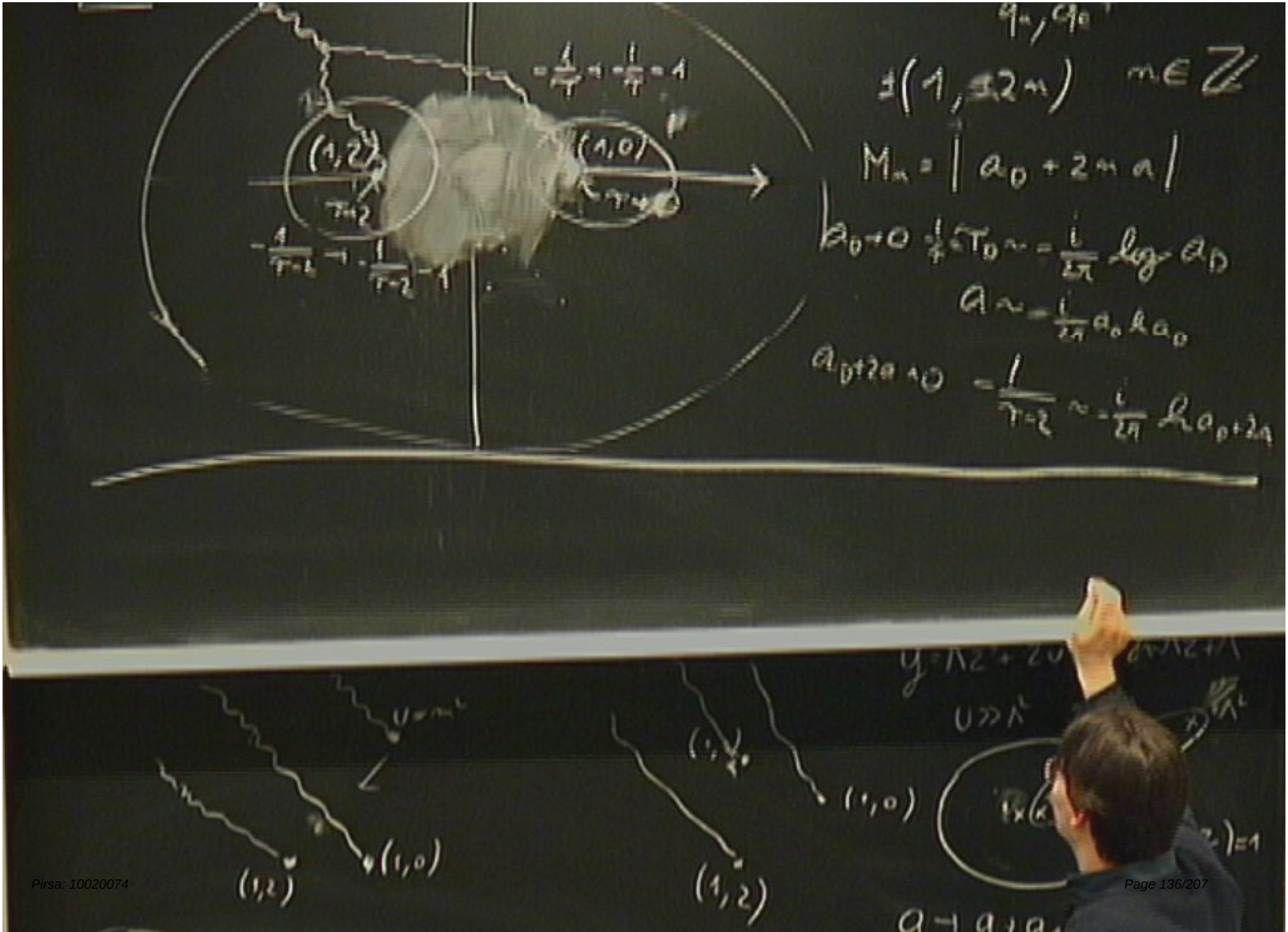
$$a_D \sim \frac{i}{\pi} a \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi}(a+m) \ln \frac{(a+m)}{\Lambda} - \frac{i}{2\pi}(a-m) \ln \frac{a-m}{\Lambda}$$

$SU(2) \quad N_f = 4 \qquad y^2 = P_3(z, U, m_a)$

m_a CARTAN OF $SO(8)$

$\sum m_a^2 \quad \sum m_a^4 \quad \sum m_a^6$

$\prod m_a$



q_m, q_0

$\pm(1, \pm 2m) \quad m \in \mathbb{Z}$

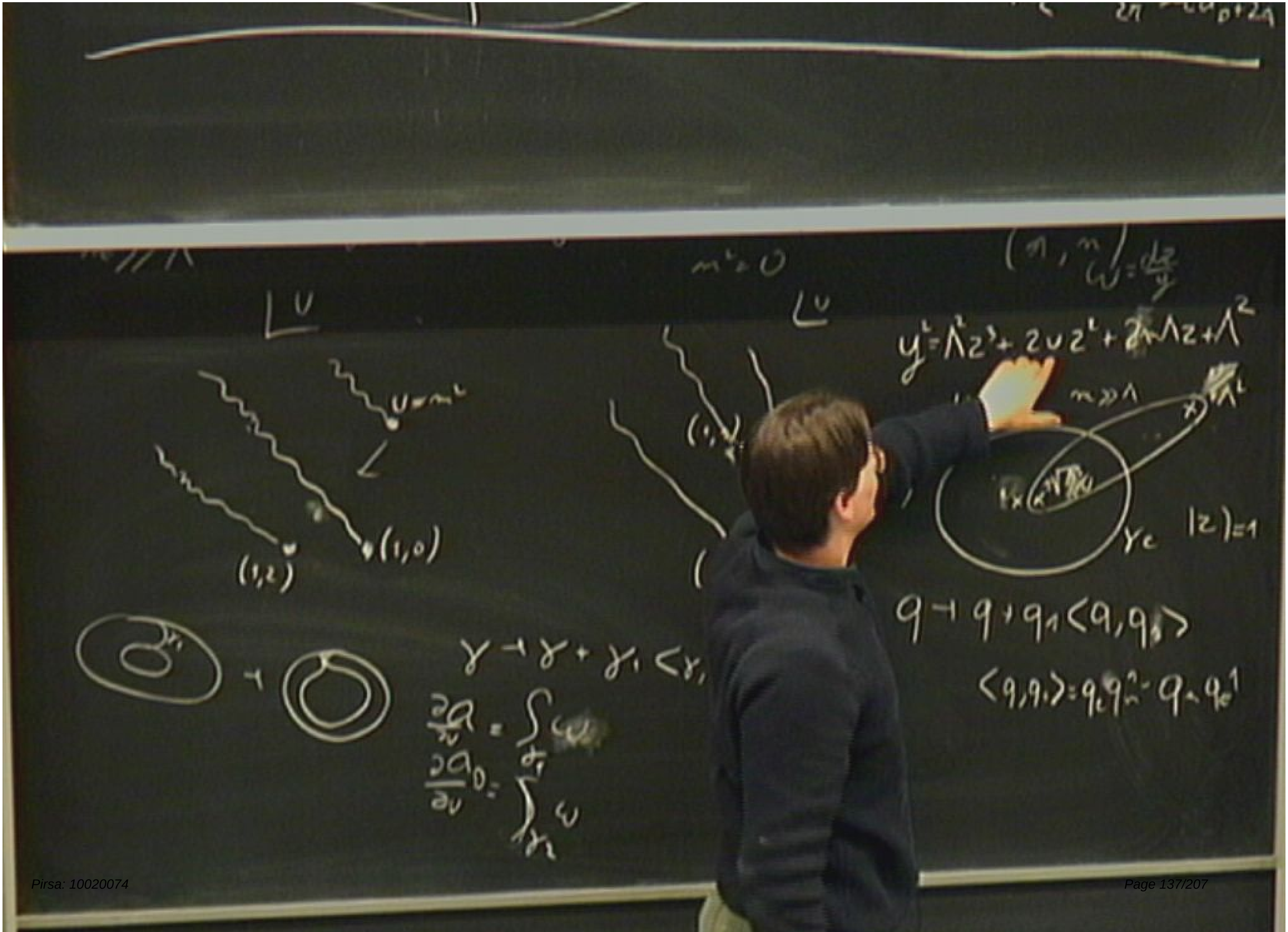
$M_m = |a_D + 2m\, a|$

$a_D \to 0 \quad -\frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi} \log a_D$

$a \sim -\frac{i}{2\pi} a_D \log a_D$

$a_D + 2a \to 0 \quad -\frac{1}{\tau - 2} \sim -\frac{i}{2\pi} \log a_{D+2a}$

$y = \Lambda z^2 + 2u \ldots \Lambda z + \Lambda$



(n, m)

$\omega = \frac{dz}{y}$

$m^2 \approx 0$

$u \approx m^2$

$(1,0)$

$(1,2)$

$y^2 = \Lambda^2 z^3 + 2 u z^2 + 2 m \Lambda z + \Lambda^2$

$m \gg \Lambda$

$\gamma_c \qquad |z| = 1$

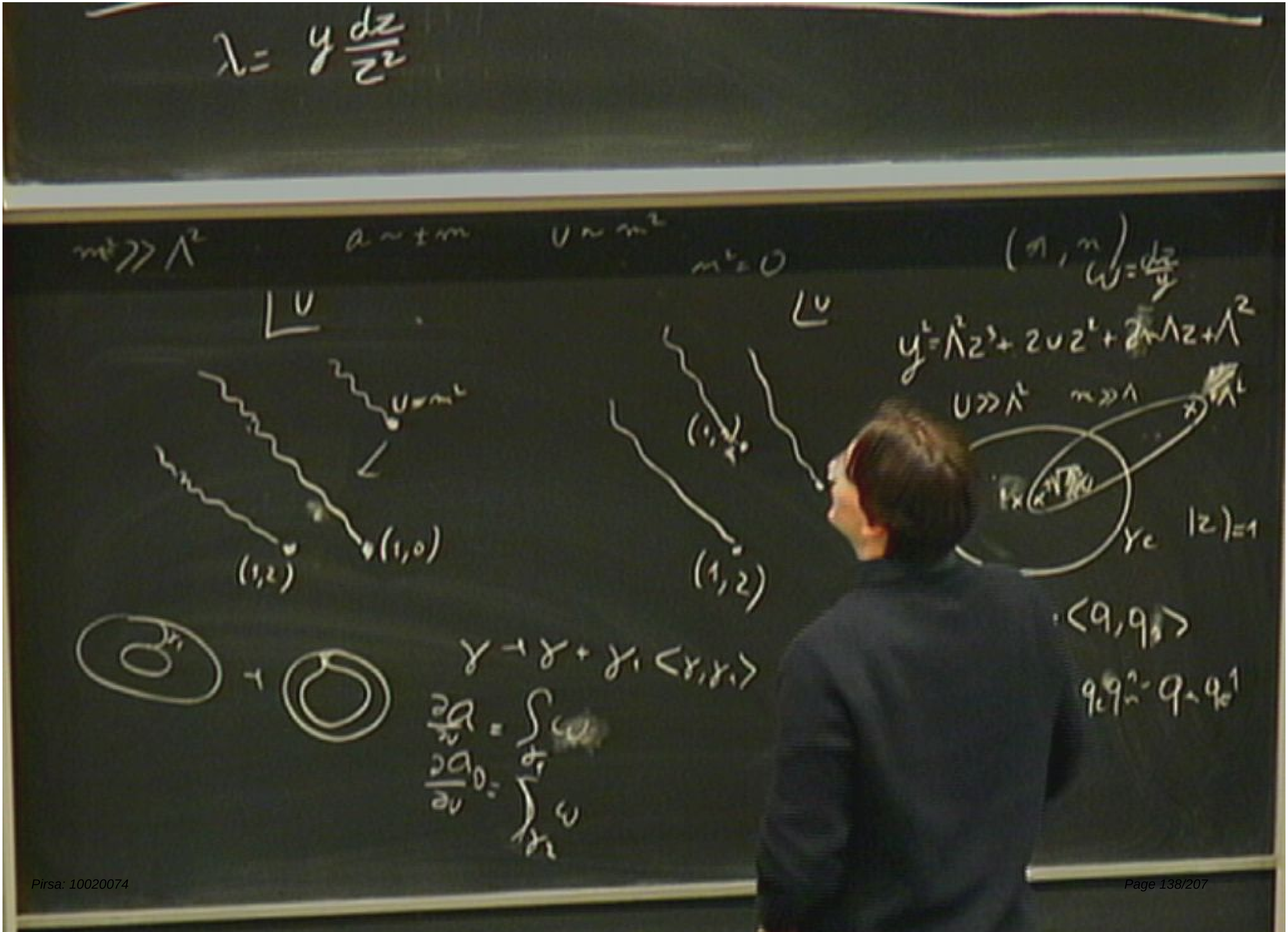
$q \to q + q_1 \langle q, q_1 \rangle$

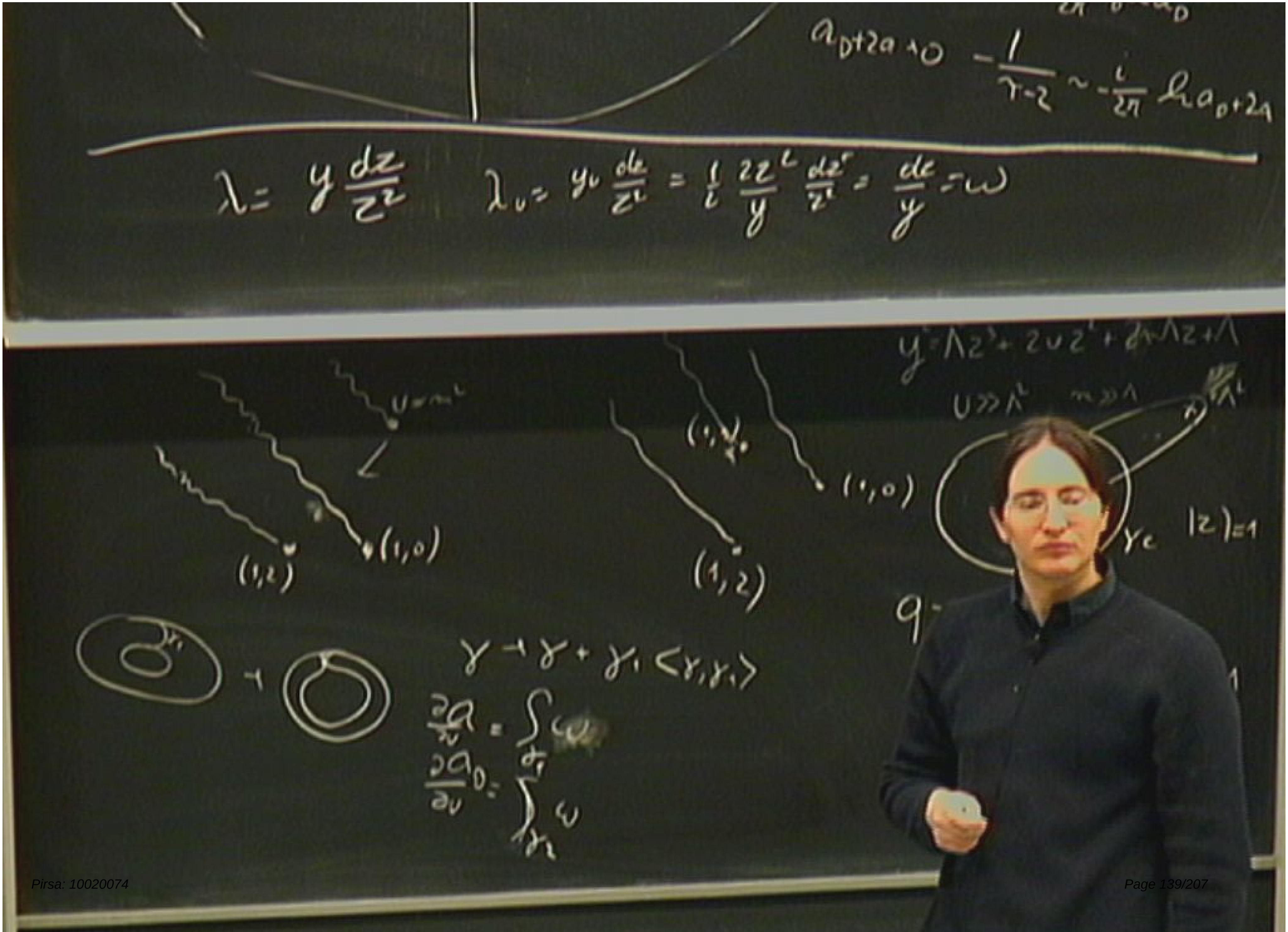
$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$

$\gamma \to \gamma + \gamma_1 \langle \gamma,$

$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$

$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$


$\lambda = y \frac{dz}{z^2}$
$m^2 >> \Lambda^2$
$a \sim \pm m$
$u \sim m^2$
$m^2 = 0$
$(1, n)$
$\omega = \frac{dz}{y}$
$y^2 = \Lambda^2 z^3 + 2 u z^2 + 2 m \Lambda z + \Lambda^2$
$u >> \Lambda^2$
$m >> \Lambda$
$u = m^2$
$(1,2)$
$(1,0)$
$(1,2)$
γ_c
$|z| = 1$
$\langle q, q_b \rangle$
$q_c q_m^n \sim q_m q_c^1$
$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$
$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$
$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$
γ_1



$$a_D + 2a \to 0 \quad -\frac{1}{\tau - 2} \sim -\frac{i}{2\pi} \ln a_D + 2a$$

$$\lambda = y\frac{dz}{z^2} \qquad \lambda_u = y_u\frac{dz}{z^2} = \frac{1}{2}\frac{2z^2}{y}\frac{dz}{z^2} = \frac{dz}{y} = \omega$$

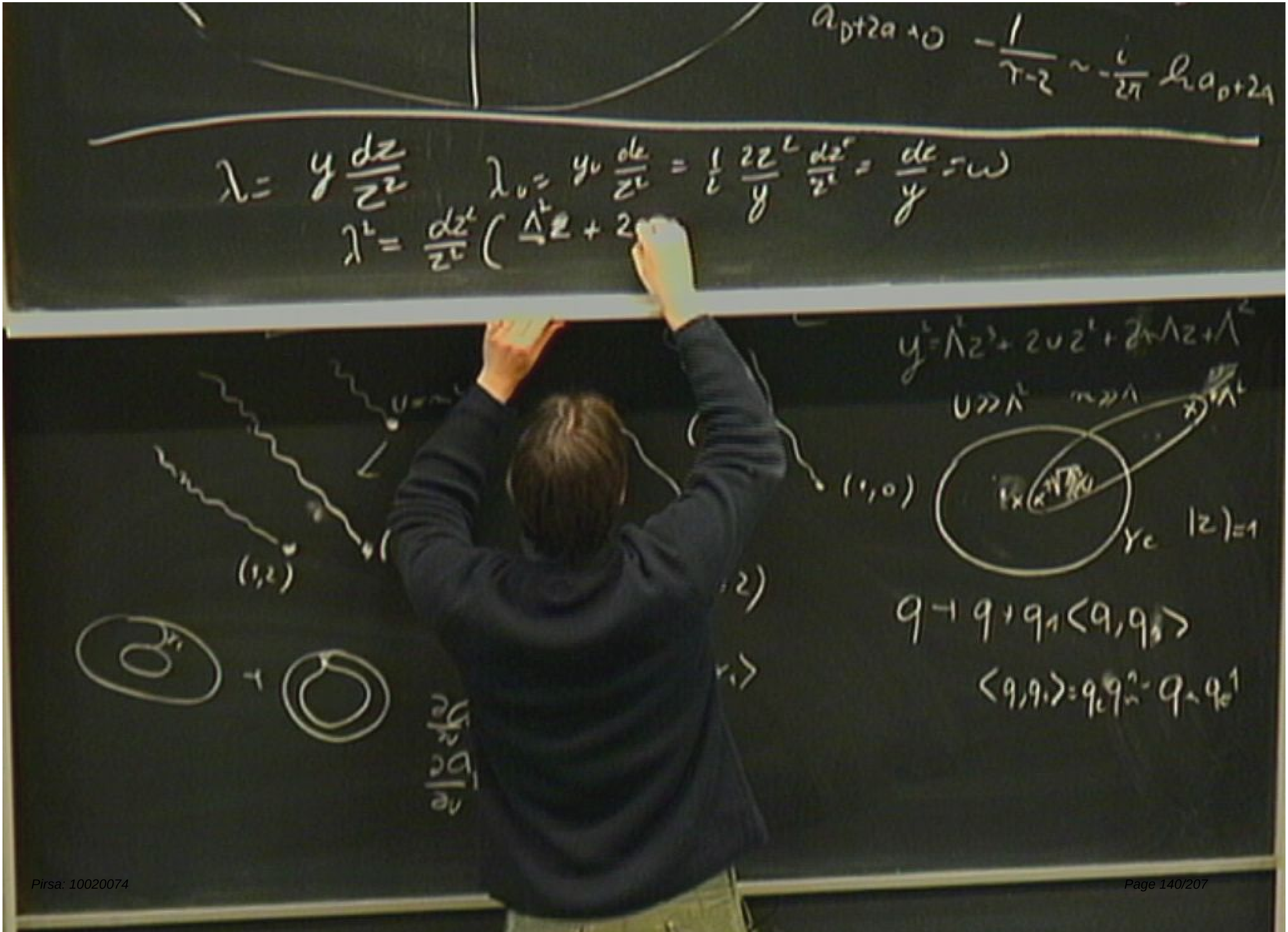
$$y^2 = \Lambda z^3 + 2uz^2 + 2m\Lambda z + \Lambda$$

$u \gg \Lambda^2 \quad m \gg \Lambda$

$$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$$

$$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$$

$$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$$



$a_D + 2a \to 0 \quad -\frac{1}{\tau - 2} \sim -\frac{i}{2\pi} \ln a_{D+2a}$

$\lambda = y \frac{dz}{z^2} \quad \lambda_u = y_u \frac{dz}{z^2} = \frac{1}{2} \frac{2z^2}{y} \frac{dz}{z^2} = \frac{dz}{y} = \omega$

$\lambda^2 = \frac{dz^2}{z^2} (\Lambda^2 z + 2$

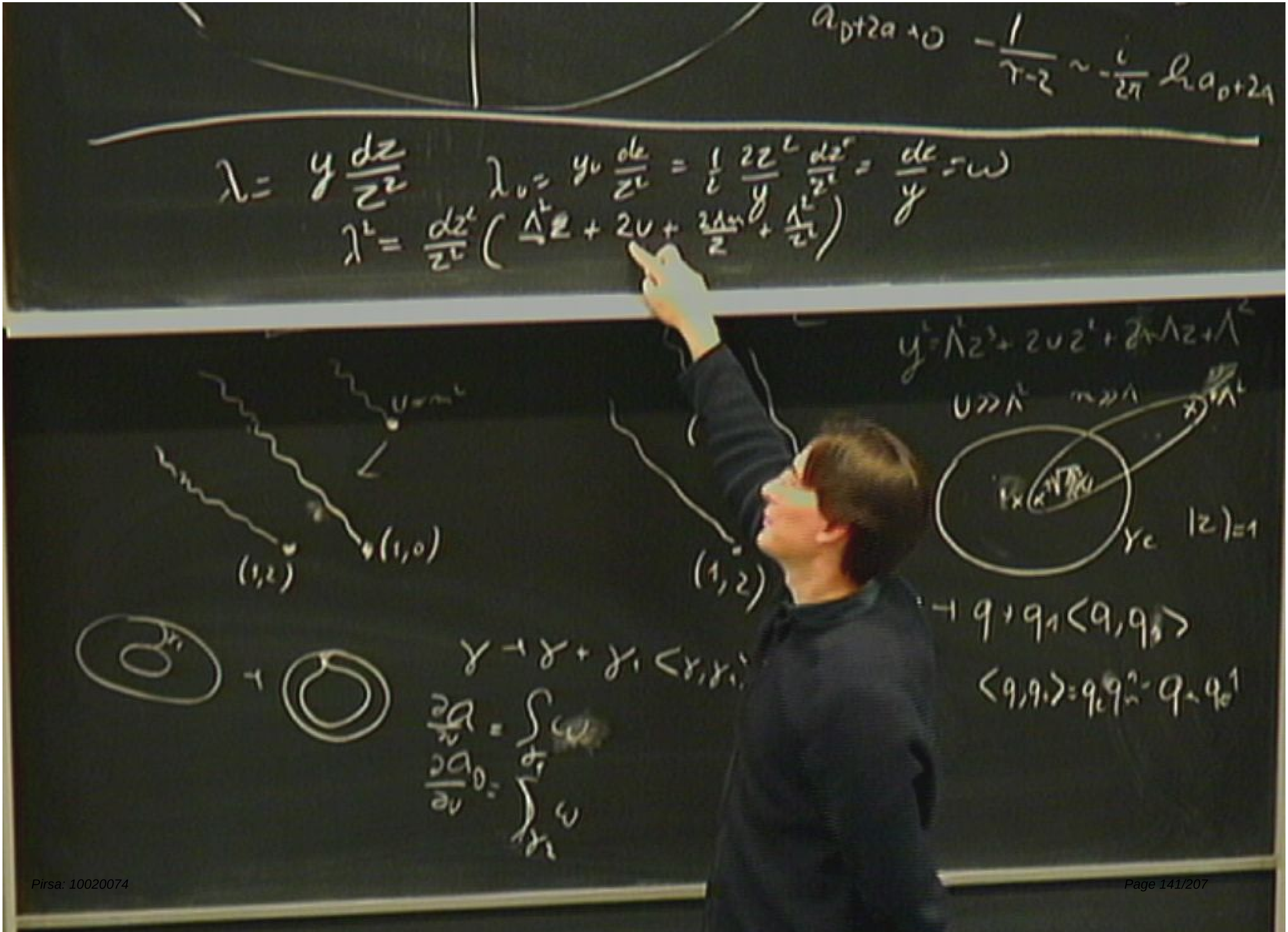
$y^2 = \Lambda^2 z^3 + 2u z^2 + 2m\Lambda z + \Lambda^2$

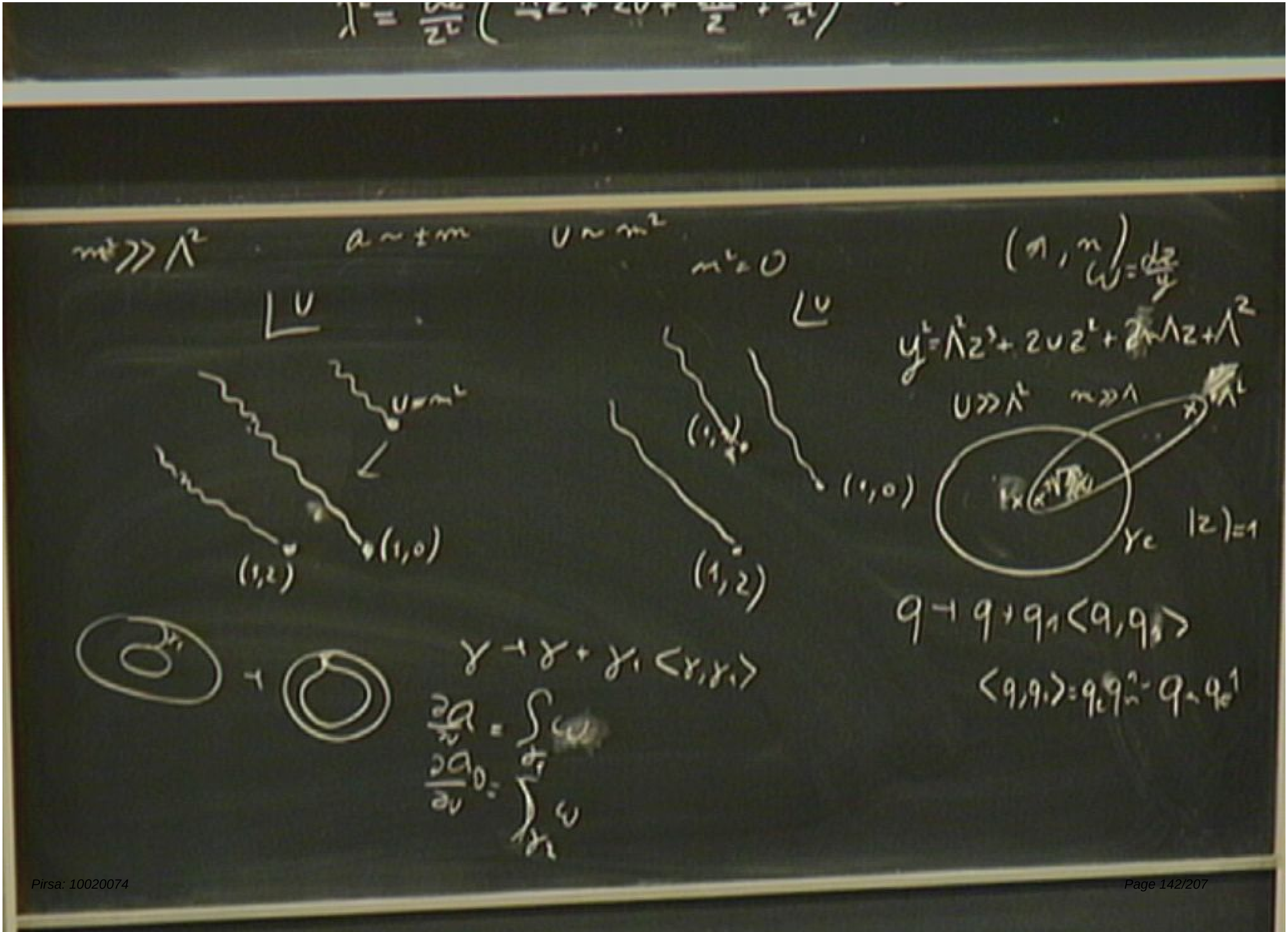
$u \gg \Lambda^2 \quad m \gg \Lambda$

$|z| = 1$

$q \to q + q_1 \langle q, q_1 \rangle$

$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$


$a_D + 2a \to 0$
$-\frac{1}{\tau - 2} \sim -\frac{i}{2\pi} \ln a_D + 2a$
$\lambda = y \frac{dz}{z^2}$
$\lambda_u = y_u \frac{dz}{z^2} = \frac{1}{2} \frac{2z^2}{y} \frac{dz}{z^2} = \frac{dz}{y} = \omega$
$\lambda^2 = \frac{dz^2}{z^2} \left(\Lambda^2 z + 2u + \frac{2\Lambda m}{z} + \frac{\Lambda^2}{z^2} \right)$
$y^2 = \Lambda^2 z^3 + 2u z^2 + 2m\Lambda z + \Lambda^2$
$u \gg \Lambda^2$
$m \gg \Lambda$
$u = m^2$
$(1,2)$
$(1,0)$
$(1,2)$
γ_c
$|z| = 1$
$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$
$q \to q + q_1 \langle q, q_1 \rangle$
$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$
$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$
$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$



$m^2 \gg \Lambda^2$ $a \sim \pm m$ $u \sim m^2$

$m^2 = 0$

(n, m) $\omega = \frac{dz}{y}$

$y^2 = \Lambda^2 z^3 + 2uz^2 + 2m\Lambda z + \Lambda^2$

$u \gg \Lambda^2$ $m \gg \Lambda$

$|z| = 1$

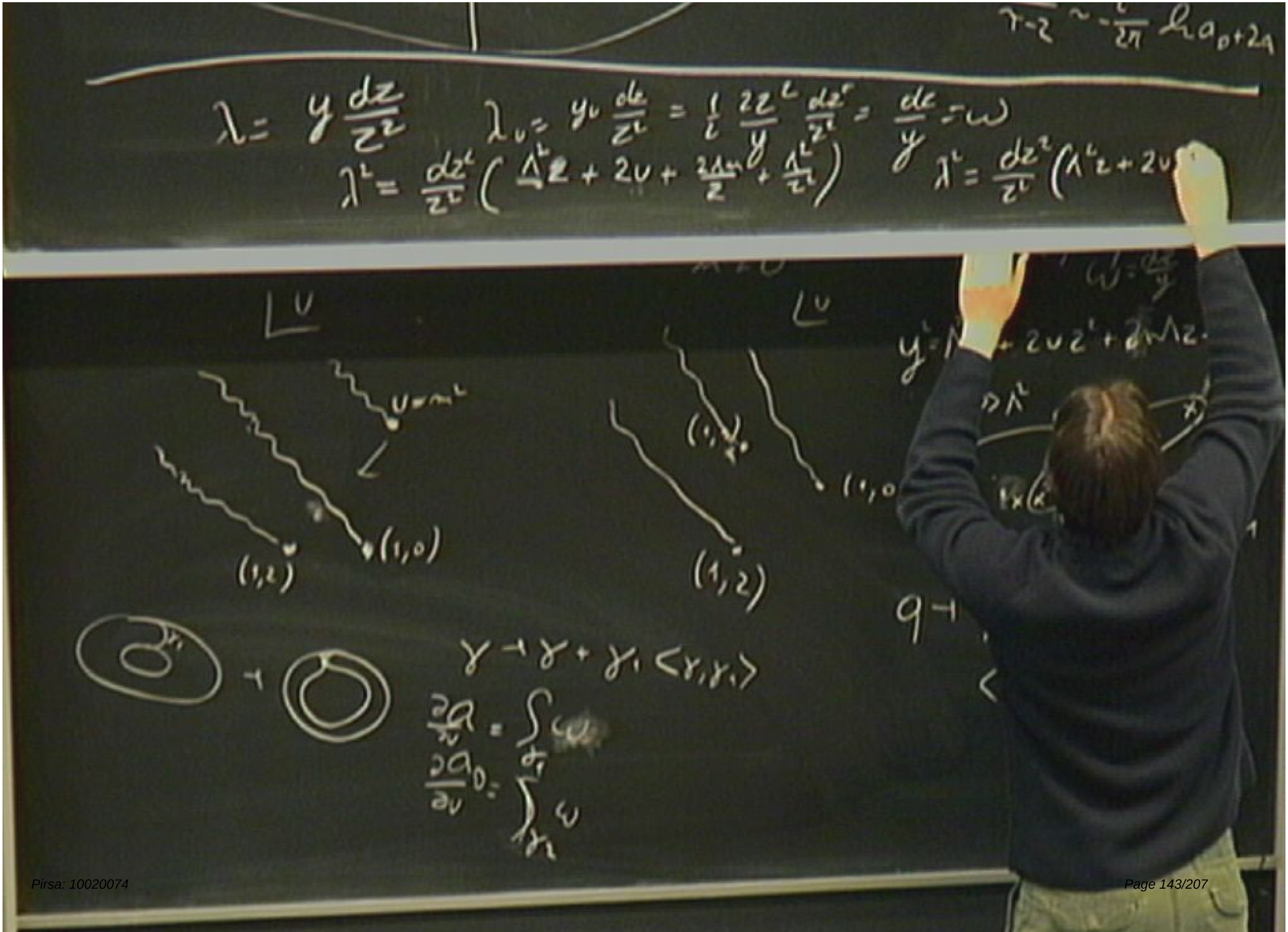
$q \to q + q_1 \langle q, q_1 \rangle$

$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$

$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$

$\frac{\partial a}{\partial v} = \int_{\gamma_1} \omega$

$\frac{\partial a_D}{\partial v} = \int_{\gamma_2} \omega$



$\tau - 2 \sim -\frac{i}{2\pi} \ln a_D + 2a$

$\lambda = y \frac{dz}{z^2}$ $\lambda_u = y_u \frac{dz}{z^2} = \frac{1}{2} \frac{2z^2}{y} \frac{dz}{z^2} = \frac{dz}{y} = \omega$

$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{2\Lambda m}{z} + \frac{\Lambda^2}{z^2} \right)$ $\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u \right.$

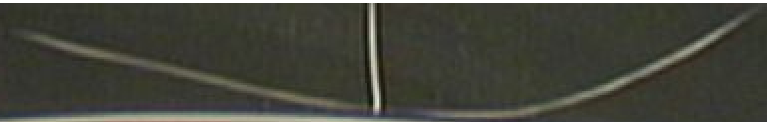
$\omega = \frac{dz}{y}$

$y^2 = \Lambda^2 \ldots + 2uz^2 + 2m\Lambda z$

$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$

$\frac{\partial a}{\partial u} = \int_{\gamma_1} \omega$

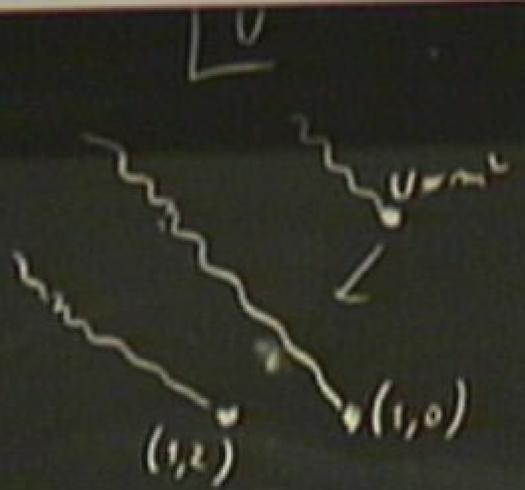
$\frac{\partial a_D}{\partial u} = \int_{\gamma_2} \omega$

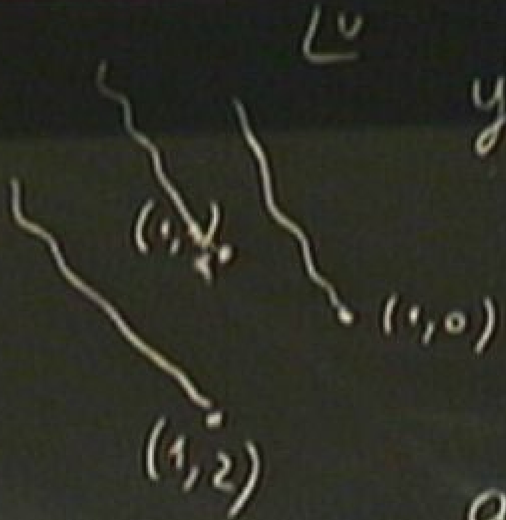


$$\frac{1}{\tau-2} \sim -\frac{i}{2\pi}\ln a_D + 2a$$

$$\lambda = y\frac{dz}{z^2} \qquad \lambda_u = y_u\frac{dz}{z^2} = \frac{1}{2}\frac{2z^2}{y}\frac{dz}{z^2} = \frac{dz}{y} = \omega$$

$$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{2\Lambda m}{z} + \frac{\Lambda^2}{z^2}\right) \qquad \lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{\Lambda^2}{z}\right)$$

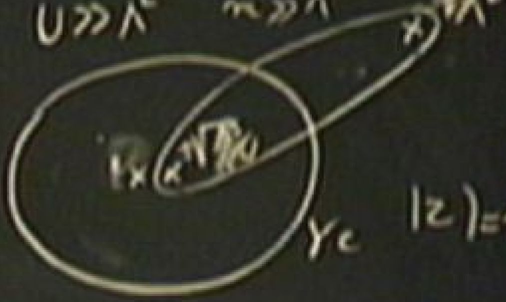




$\omega = \frac{dz}{y}$

$$y^2 = \Lambda^2 z^3 + 2uz^2 + 2m\Lambda z + \Lambda$$

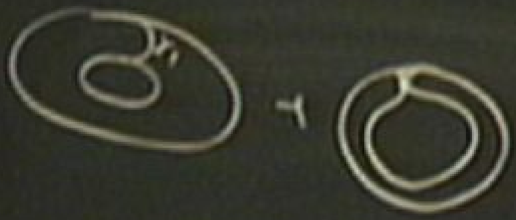
$u \gg \Lambda^2 \quad m \gg \Lambda$



$|z| = 1$

$$q \to q + q_1\langle q, q_1\rangle$$

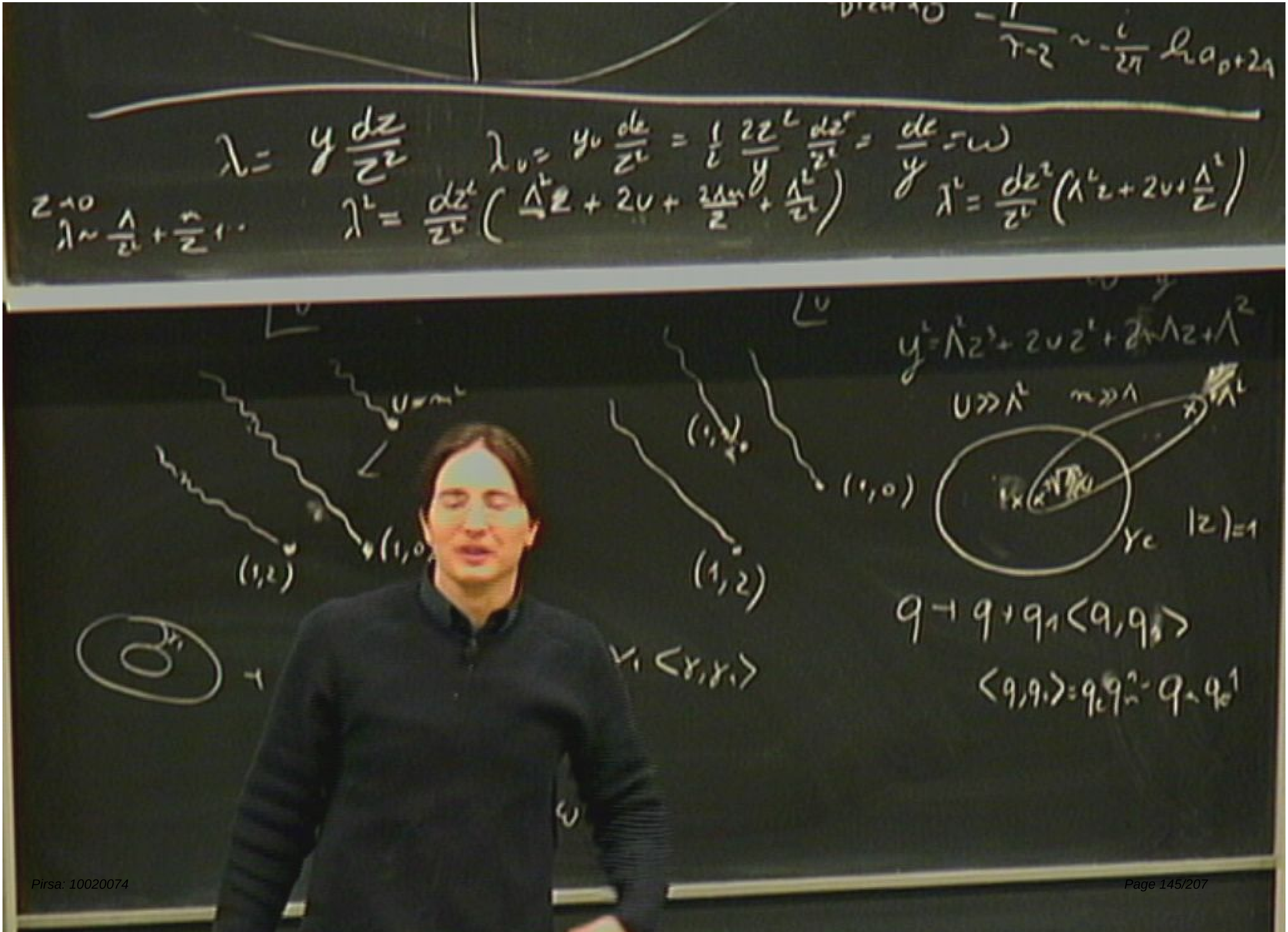
$$\langle q, q_1\rangle = q_c q_1^{\,n} \cdots q_1 q_c^{1}$$



$$\gamma \to \gamma + \gamma_1\langle\gamma,\gamma_1\rangle$$

$$\frac{\partial a}{\partial u} = \int_{\gamma_1}\omega$$

$$\frac{\partial a_D}{\partial u} = \int_{\gamma_2}\omega$$



$-\frac{1}{\lambda - z} \sim -\frac{i}{2\pi} \ln a_0 + 2a$

$\lambda = y \frac{dz}{z^2}$ $\lambda_u = y_u \frac{dz}{z^2} = \frac{1}{2} \frac{2z^2}{y} \frac{dz^2}{z^2} = \frac{dz}{y} = \omega$

$z \to 0$

$\lambda \sim \frac{\Lambda}{z^2} + \frac{m}{z} + \cdots$

$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{2\Lambda m}{z} + \frac{\Lambda^2}{z^2}\right)$

$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{\Lambda^2}{z}\right)$

$y^2 = \Lambda^2 z^3 + 2u z^2 + 2m\Lambda z + \Lambda^2$

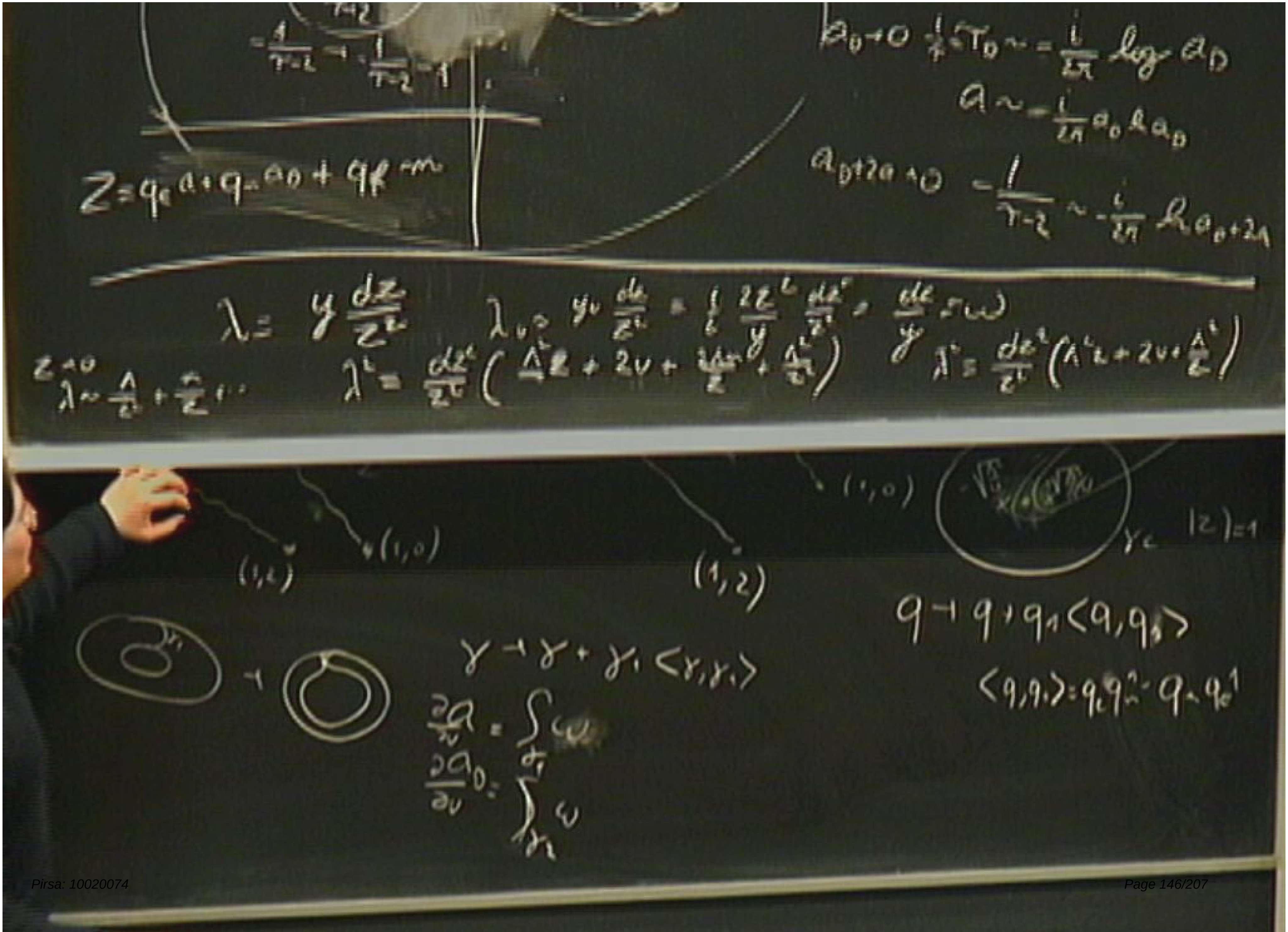
$u \gg \Lambda^2$ $m \gg \Lambda$

$|z| = 1$

$q \to q + q_1 \langle q, q_1 \rangle$

$\langle q, q_1 \rangle = q_e q_m^1 - q_m q_e^1$

$\gamma \to \gamma + \gamma_1 \langle \gamma, \gamma_1 \rangle$



$= \frac{1}{\tau-2} \to \frac{1}{\tau-2} - 1$

$Z = q_e a + q_m a_D + q_f m$

$a_D \to 0 \quad \frac{1}{?}\tau_D \sim -\frac{i}{2\pi}\log a_D$

$a \sim -\frac{i}{2\pi} a_D \log a_D$

$a_D + 2a \to 0 \quad -\frac{1}{\tau-2} \sim -\frac{i}{2\pi}\log a_{D+2a}$

$\lambda = y\frac{dz}{z^2} \quad \lambda_u \sim y_u \frac{dz}{z^2} = \frac{1}{2}\frac{2z^2}{y}\frac{dz}{z^2} = \frac{dz}{y} = \omega$

$z \to 0$

$\lambda \sim \frac{\Lambda}{z^2} + \frac{u}{z} + \cdots$

$\lambda^2 = \frac{dz^2}{z^4}\left(\Lambda^2 z + 2u + \frac{?}{z} + \frac{\Lambda^2}{z^2}\right)$

$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{\Lambda^2}{z}\right)$

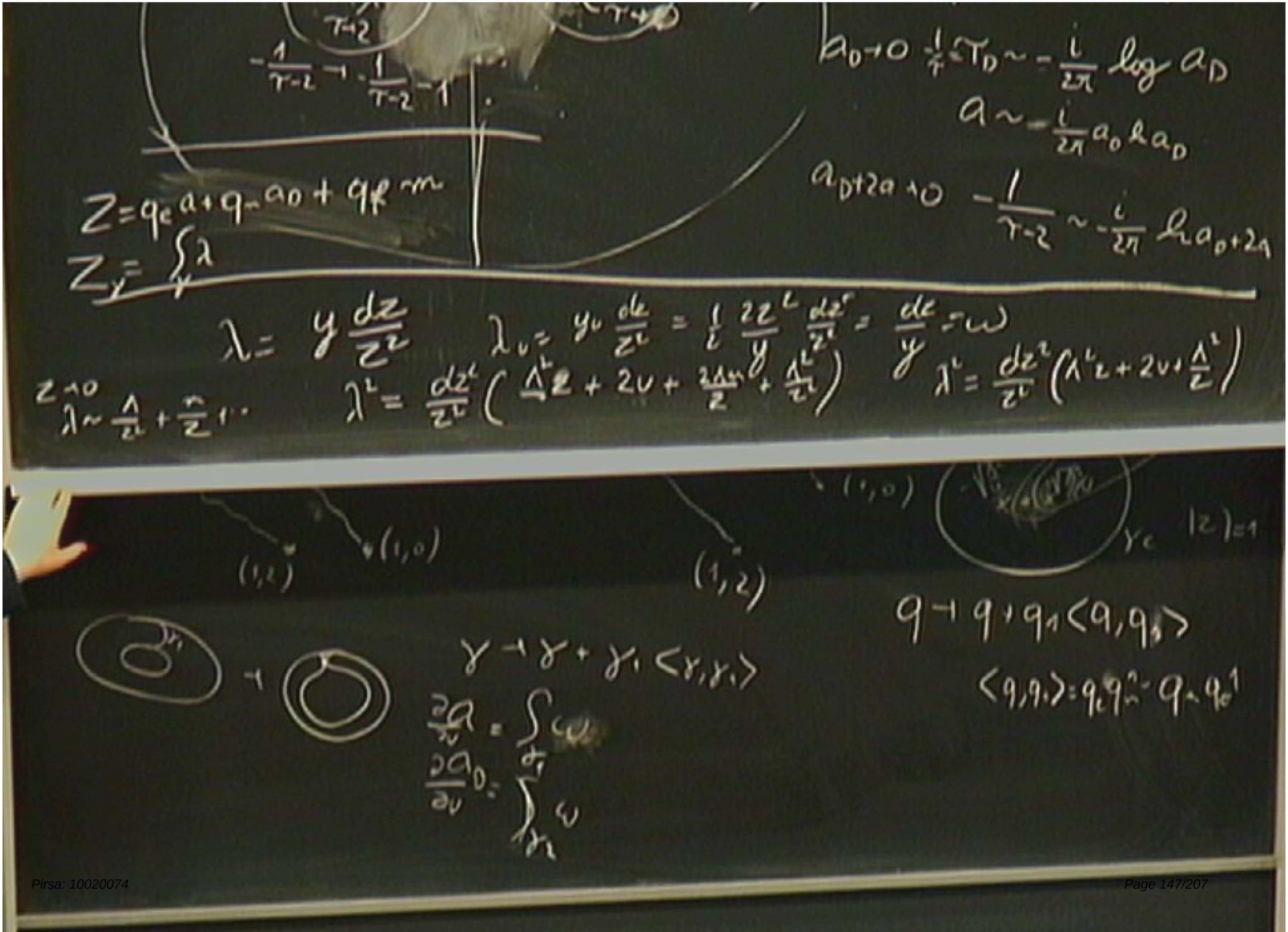
$q \to q + q_1\langle q, q_1\rangle$

$\langle q, q_1\rangle = q_e q_m^1 - q_m q_e^1$

$\gamma \to \gamma + \gamma_1\langle\gamma,\gamma_1\rangle$

$\frac{\partial a}{\partial u} = \int_{\gamma_1}\omega$

$\frac{\partial a_D}{\partial u} = \int_{\gamma_2}\omega$



$$-\frac{1}{\tau-2} \to -\frac{1}{\tau-2}-1$$

$$Z = q_e a + q_m a_D + q_f m$$

$$Z_\gamma = \oint_\gamma \lambda$$

$$a_D \to 0 \quad \frac{1}{\tau} = \tau_D \sim -\frac{i}{2\pi}\log a_D$$

$$a \sim -\frac{i}{2\pi} a_D \ln a_D$$

$$a_D + 2a \to 0 \quad -\frac{1}{\tau-2} \sim -\frac{i}{2\pi}\ln a_{D+2a}$$

$$\lambda = y\frac{dz}{z^2} \qquad \lambda_u = y_u\frac{dz}{z^2} = \frac{1}{2}\frac{2z^2}{y}\frac{dz}{z^2} = \frac{dz}{y} = \omega$$

$z \to 0$

$$\lambda \sim \frac{\Lambda}{z^2} + \frac{m}{z} + \cdots$$

$$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{2\Lambda m}{z} + \frac{\Lambda^2}{z^2}\right)$$

$$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{\Lambda^2}{z}\right)$$

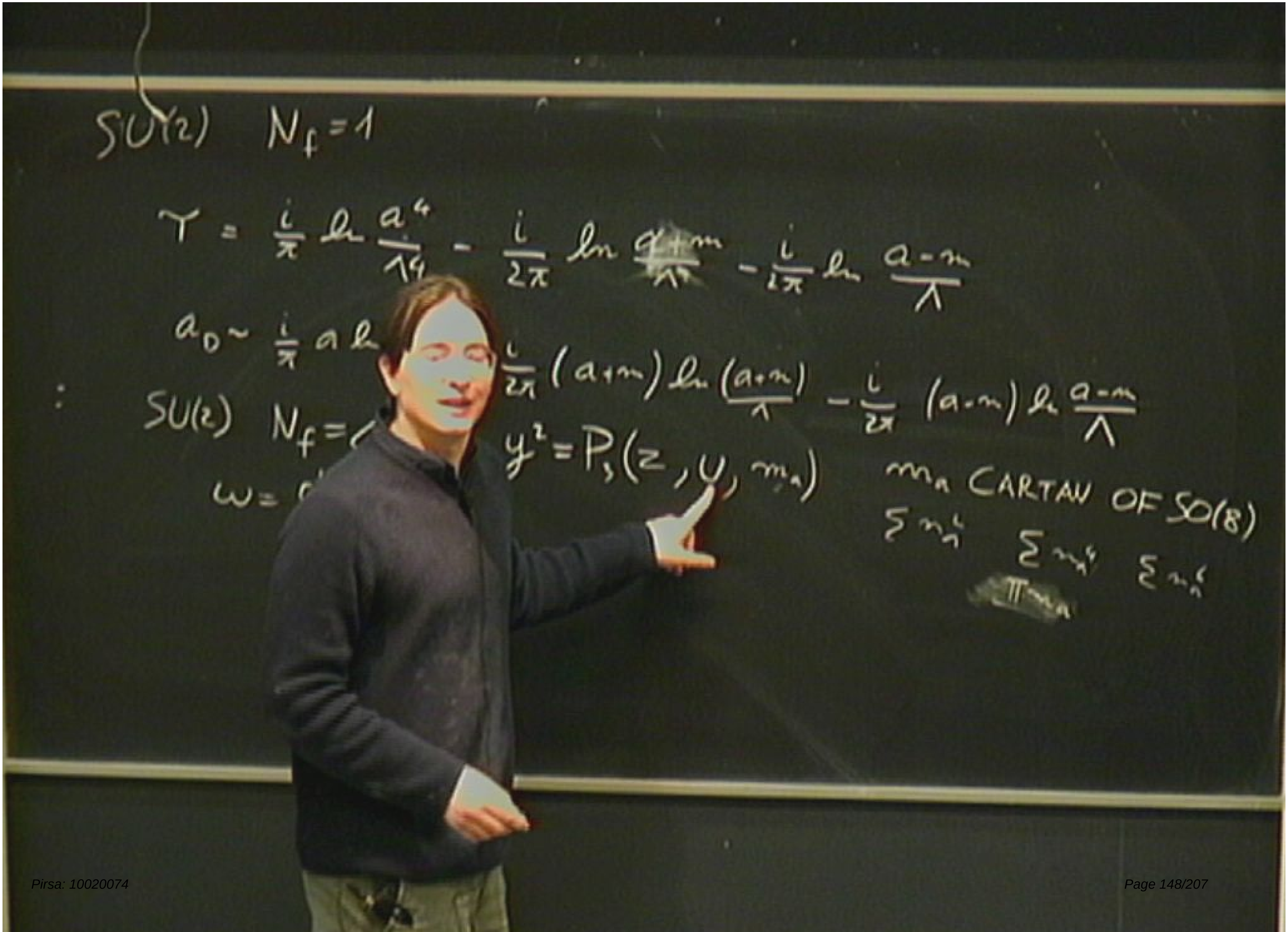
$$\gamma \to \gamma + \gamma_1\langle\gamma,\gamma_1\rangle$$

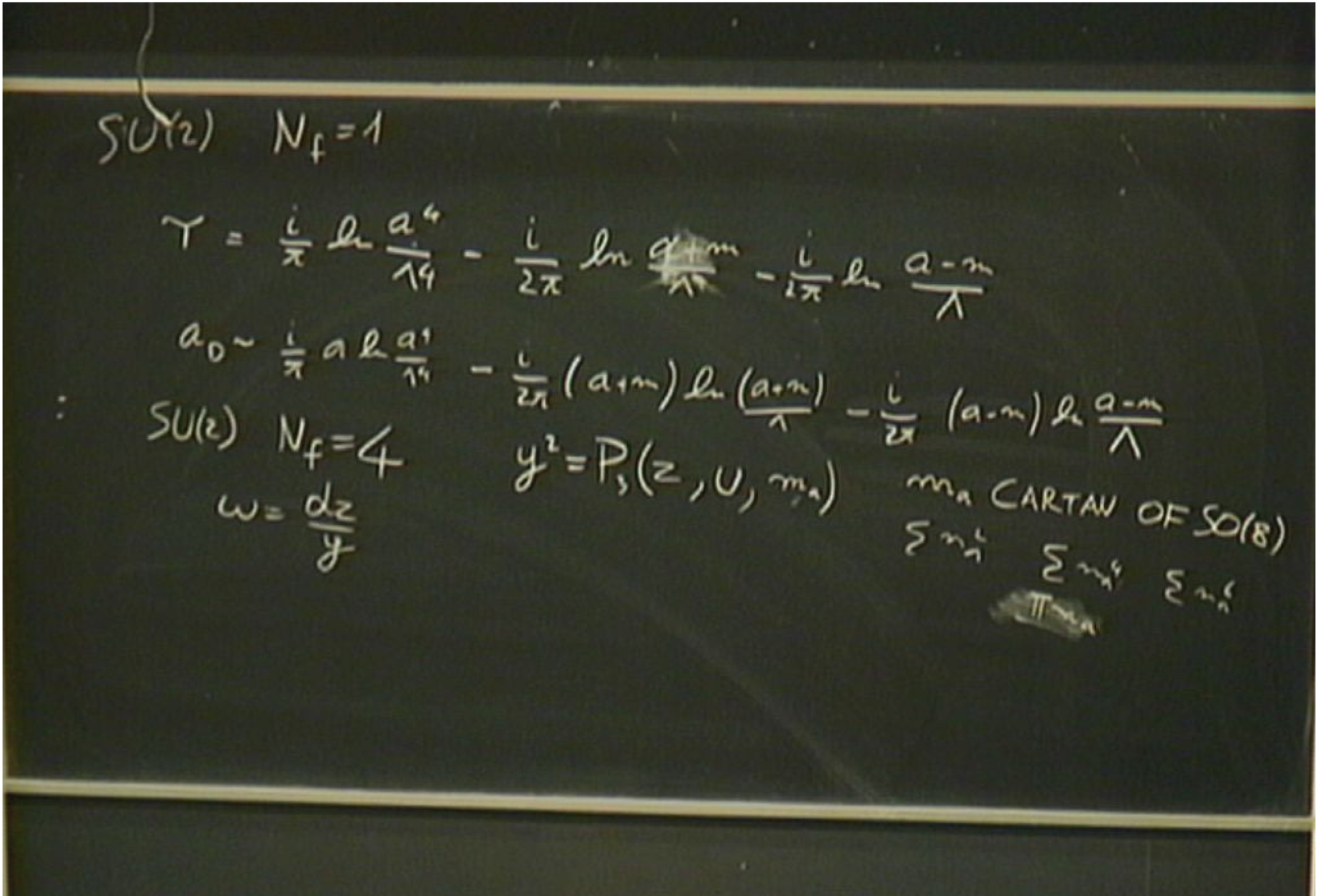
$$\frac{\partial a}{\partial u} = \oint_{\gamma_1}\omega$$

$$\frac{\partial a_D}{\partial u} = \oint_{\gamma_2}\omega$$

$$q \to q + q_1\langle q, q_1\rangle$$

$$\langle q, q_1\rangle = q_e q_m^1 - q_m q_e^1$$





SU(2) $N_f = 1$

$$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

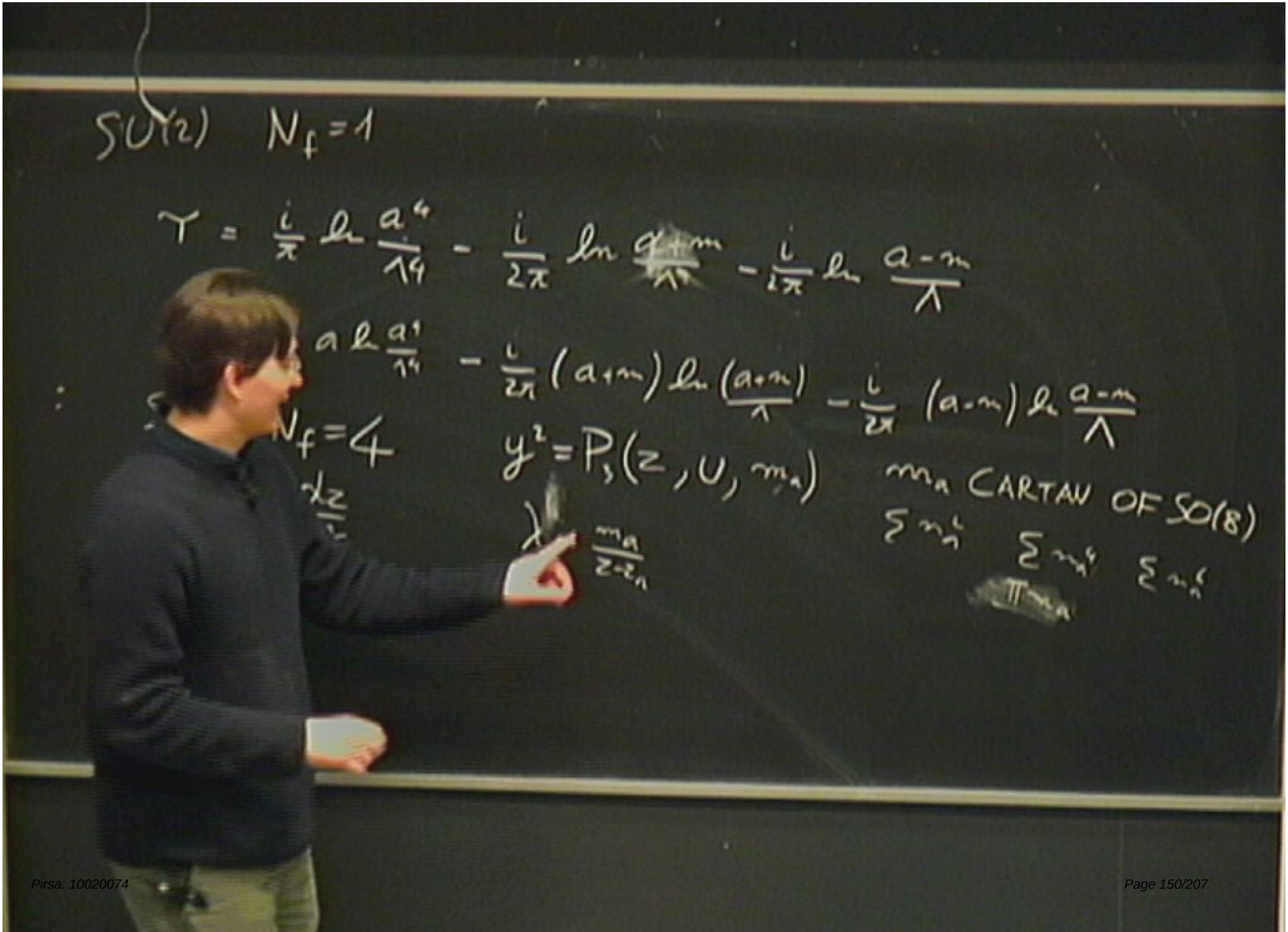
$$a_D \sim \frac{i}{\pi} a \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi}(a+m) \ln \frac{(a+m)}{\Lambda} - \frac{i}{2\pi}(a-m) \ln \frac{a-m}{\Lambda}$$

SU(2) $N_f = 4$ $\quad y^2 = P_3(z, u, m_a)$ $\quad m_a$ CARTAN OF SO(8)

$$\omega = \frac{dz}{y}$$

$\sum m_a^2 \quad \sum m_a^4 \quad \sum m_a^6$

$\prod m_a$



SU(2) $N_f = 1$

$$\Upsilon = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

$$a \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} (a+m) \ln \left(\frac{a+m}{\Lambda}\right) - \frac{i}{2\pi} (a-m) \ln \frac{a-m}{\Lambda}$$

$N_f = 4$ $\quad y^2 = P_3(z, U, m_a)$ $\quad m_a$ CARTAN OF SO(8)

dz

$\lambda = \frac{m_a}{z-z_a}$ $\quad \Sigma m_a^2 \quad \Sigma m_a^4 \quad \Sigma m_a^6$

Πm_a

$SU(2)\quad N_f = 1$

$$\tau = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

$$a_D \sim \frac{i}{\pi} a \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} (a+m) \ln \frac{(a+m)}{\Lambda} - \frac{i}{2\pi} (a-m) \ln \frac{a-m}{\Lambda}$$

$SU(2)\quad N_f = 4$

$$y^2 = P_3(z, U, m_a)$$

m_a CARTAN OF $SO(8)$

$$\omega = \frac{dz}{y}$$

$$\lambda \sim \frac{m_a}{z - z_a}$$

$$\lambda^8 + P_1 \lambda^6 + P_2' \lambda^4 + \dots$$

$\sum m_a^2 \quad \sum m_a^4 \quad \sum m_a^6 \quad \prod m_a$

$SU(2) \quad N_f = 1$

$$\Upsilon = \frac{i}{\pi} \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi} \ln \frac{a+m}{\Lambda} - \frac{i}{2\pi} \ln \frac{a-m}{\Lambda}$$

$$a_D \sim \frac{i}{\pi} a \ln \frac{a^4}{\Lambda^4} - \frac{i}{2\pi}(a+m) \ln \frac{(a+m)}{\Lambda} - \frac{i}{2\pi}(a-m) \ln \frac{a-m}{\Lambda}$$

:

$SU(2) \quad N_f = 4$

$$y^2 = P_3(z, U, m_a) \qquad m_a \text{ CARTAN OF } SO(8)$$

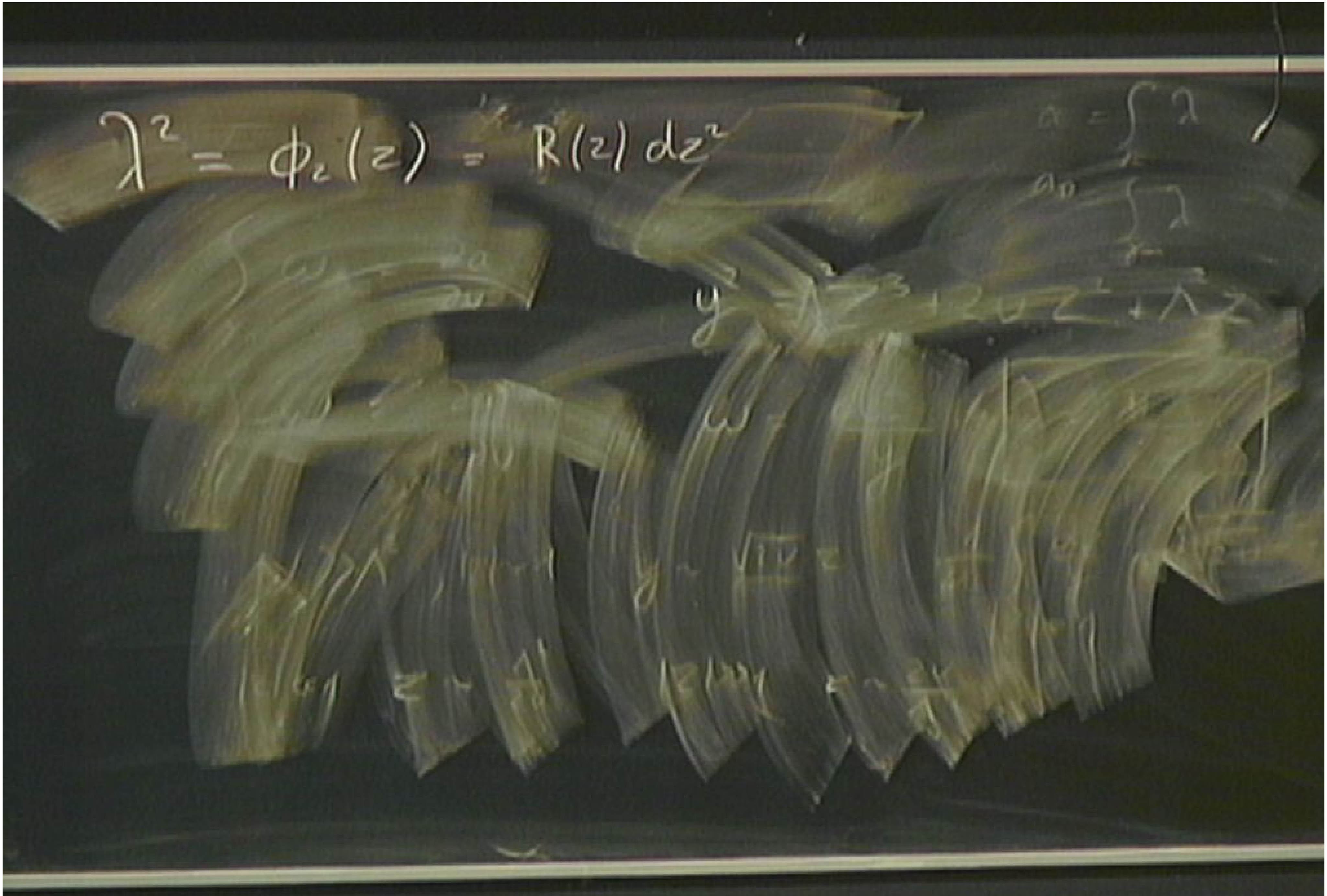
$$\omega = \frac{dz}{y}$$

$$\lambda \sim \frac{m_a}{z - z_n}$$

$\sum m_n^2 \quad \sum m_n^4 \quad \sum m_n^6$

$\prod m_n$

$$\lambda^8 + P_1 \lambda^6 + P_2' \lambda^4 + \dots = 0$$

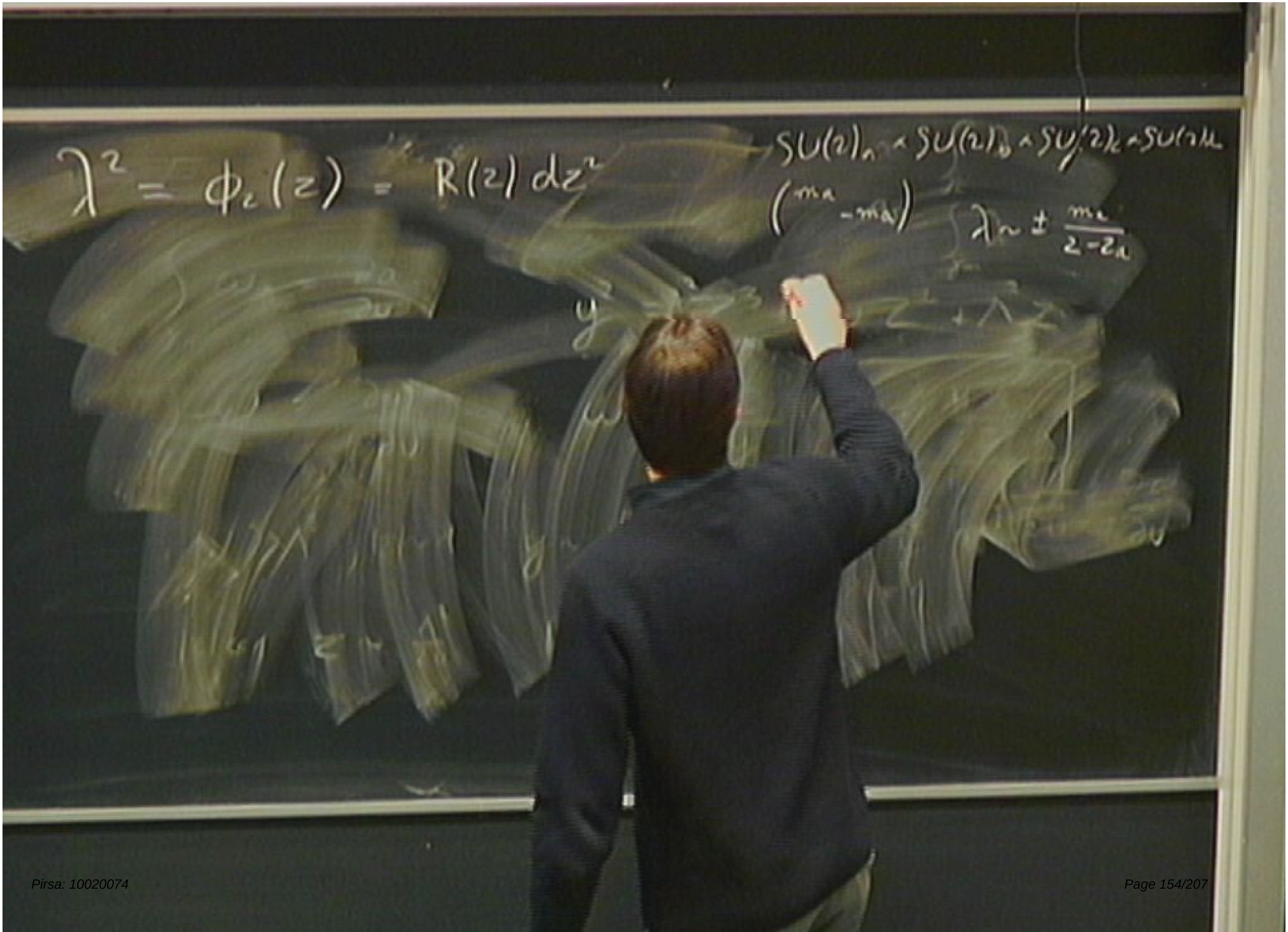


$$\lambda^2 = \phi_2(z) = R(z)\, dz^2$$

$$a = \int \lambda$$

$$a_D = \int \lambda$$

$$y^2 = z^3 + 2u z^2 + \Lambda z$$



$\lambda^2 = \phi_2(z) = R(z)\,dz^2$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$

$\lambda \sim \pm \frac{m_a}{z - z_a}$

$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

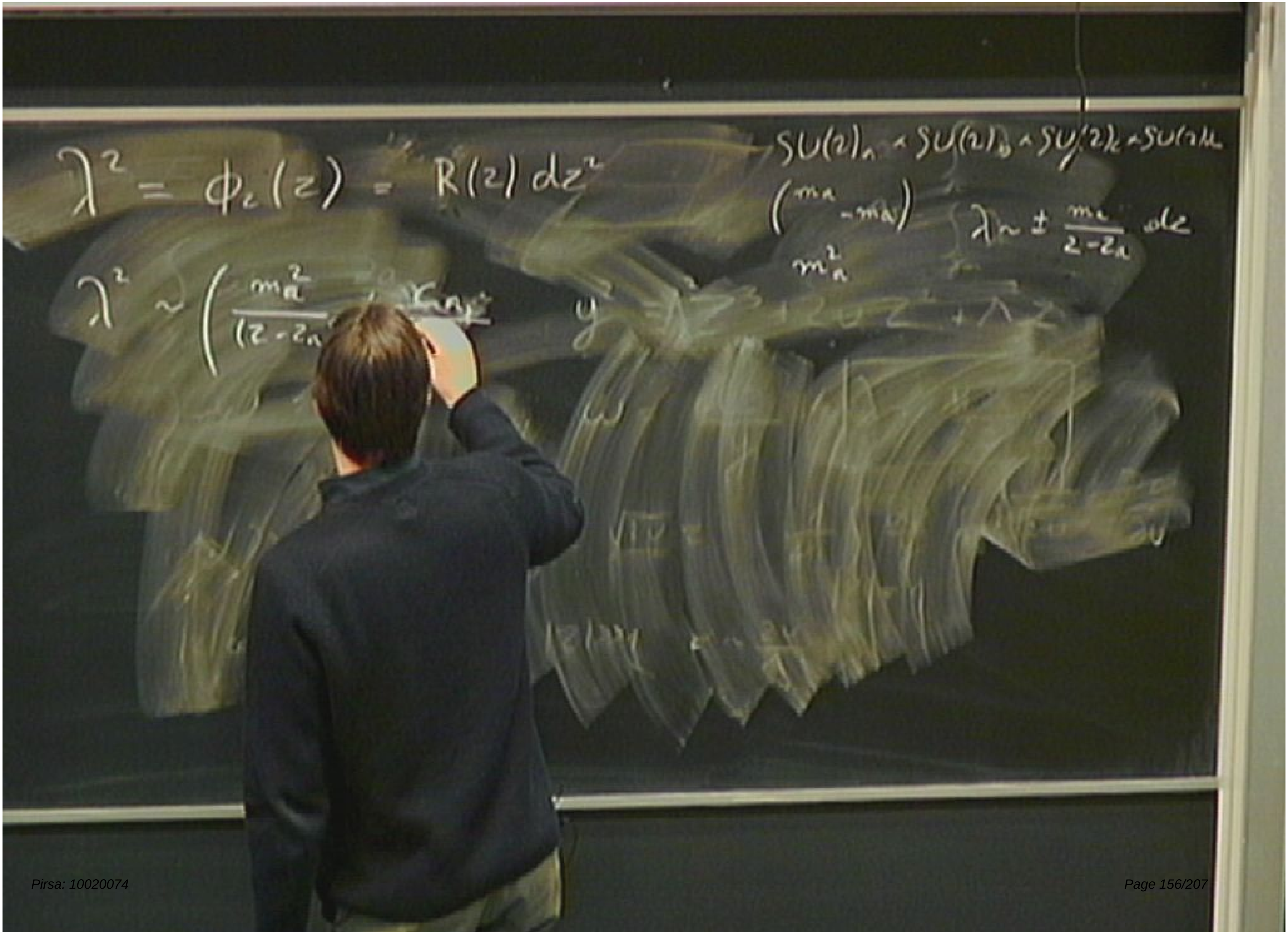
$$\lambda^2 \sim \frac{m_a^2}{(z-z_a)^2}\,dz^2$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$$

$$\lambda \sim \pm \frac{m_a}{z-z_a}\,dz$$

$$m_a^2$$



$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$$

$$\lambda \sim \pm \frac{m_a}{z - z_a}\,dz$$

$$m_a^2$$

$$\lambda^2 \sim \left(\frac{m_a^2}{(z - z_a} \right.$$

$$\lambda^2 = \phi_2(z) = R(z)\, dz^2$$

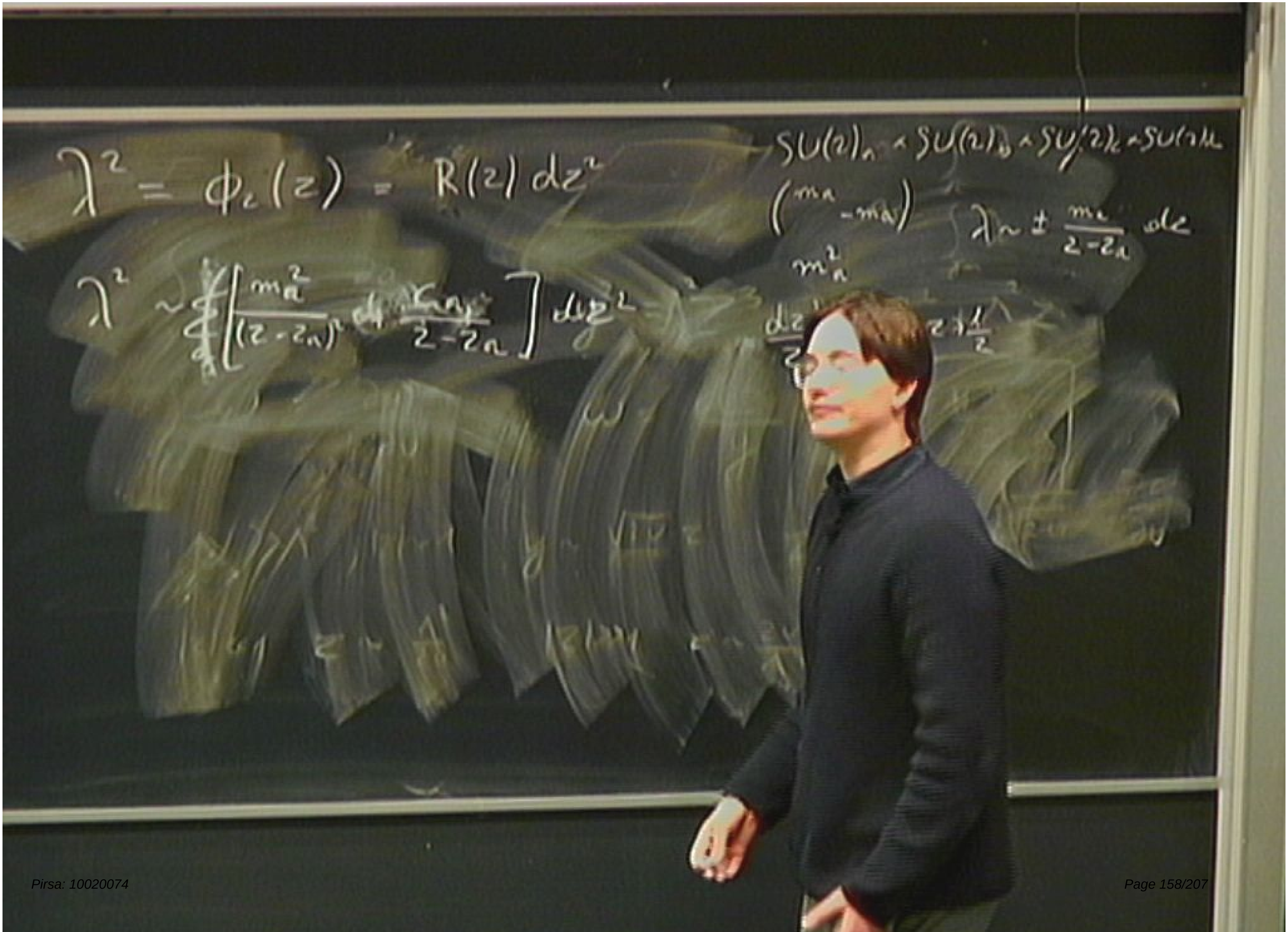
$$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{\text{[illegible]}}{z-z_a}\right] dz^2$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$$

$$m_a^2$$

$$\lambda \sim \pm \frac{m_a}{z-z_a}\, dz$$



$\lambda^2 = \phi_2(z) = R(z)\, dz^2$

$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{\text{[illegible]}}{z-z_a}\right] dz^2$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

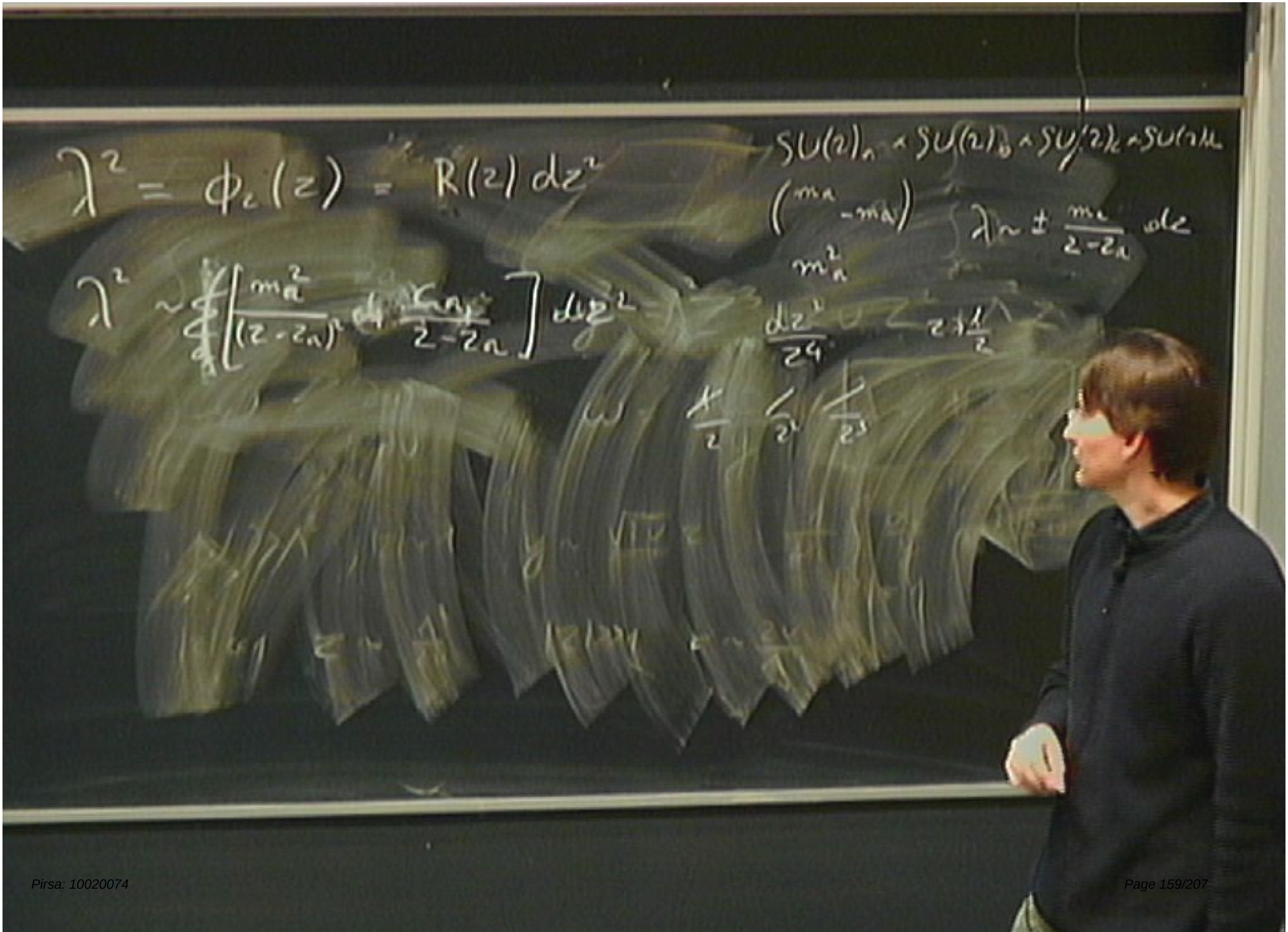
$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$

$\lambda \sim \pm \frac{m_a}{z-z_a}\, dz$

m_a^2

$\frac{dz}{z}$

$z + \frac{1}{2}$



$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

$$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{\text{[illegible]}}{z-z_a}\right] dz^2$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

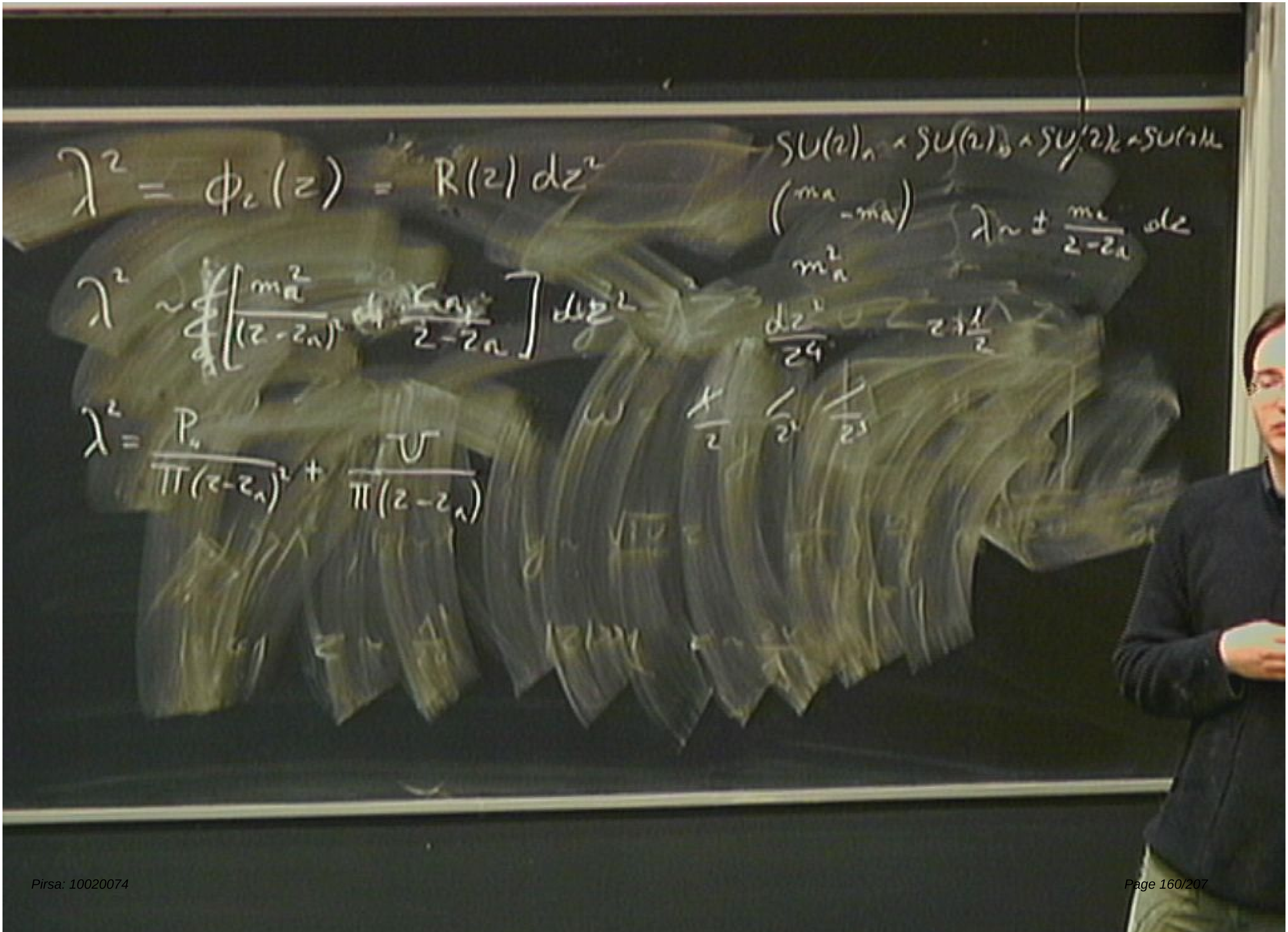
$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$$

$$\lambda \sim \pm \frac{m_a}{z-z_a}\,dz$$

$$m_a^2$$

$$\frac{dz^2}{z^4} \qquad z \to \frac{1}{z}$$

$$\frac{1}{z} \quad \frac{1}{z^2} \quad \frac{1}{z^3}$$



$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

$$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{[illegible]}{z-z_a}\right] dz^2$$

$$\lambda^2 = \frac{P_a}{\pi(z-z_a)^2} + \frac{U}{\pi(z-z_a)}$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$$

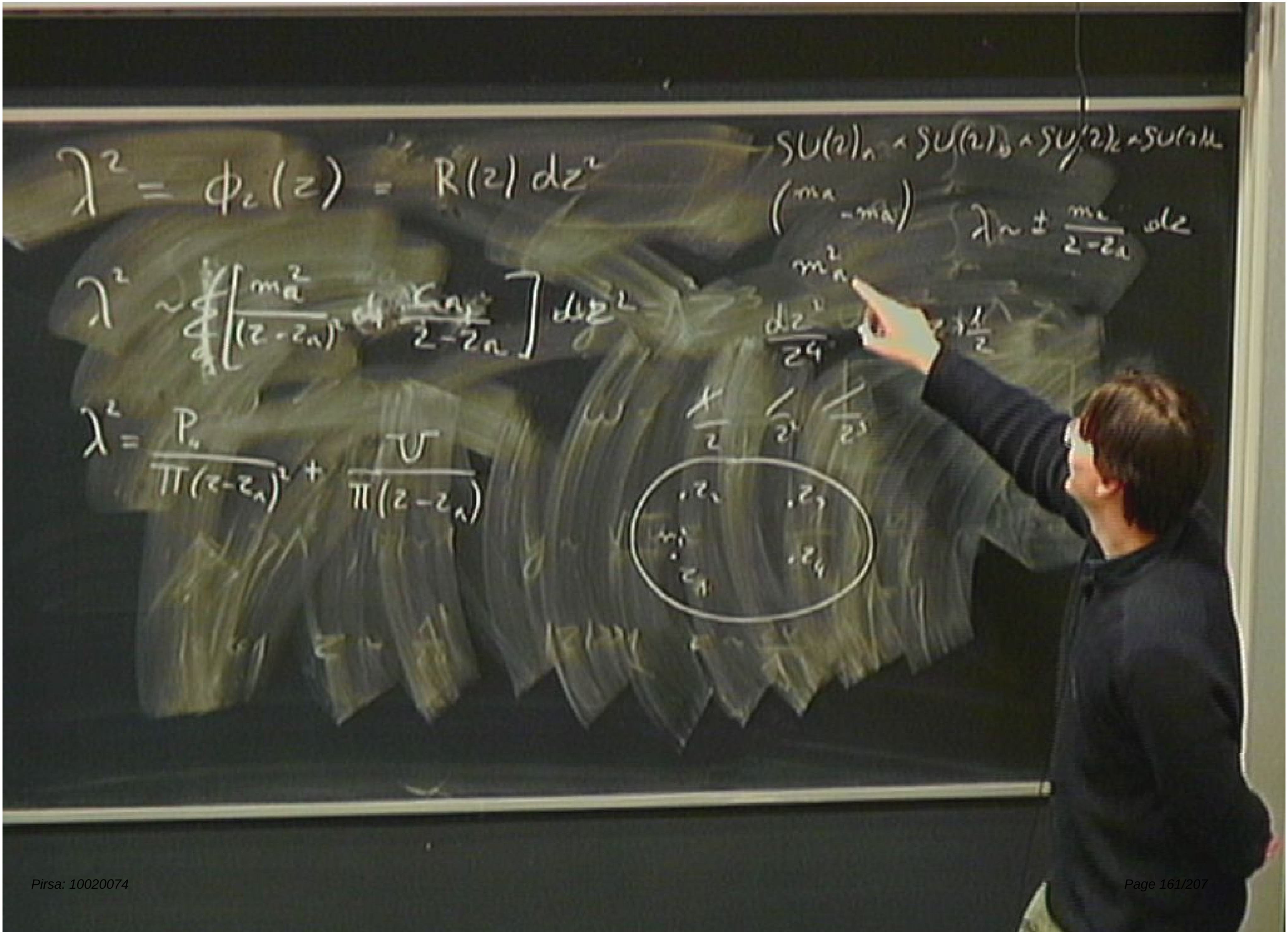
$$\lambda \sim \pm \frac{m_a}{z-z_a}\,dz$$

$$m_a^2$$

$$\frac{dz^2}{z^4}$$

$$z \to \frac{1}{z}$$

$$\frac{1}{z} \quad \frac{1}{z^2} \quad \frac{1}{z^3}$$



$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

$$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{c_a}{z-z_a}\right] dz^2$$

$$\lambda^2 = \frac{P_4}{\prod(z-z_a)^2} + \frac{U}{\prod(z-z_a)}$$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

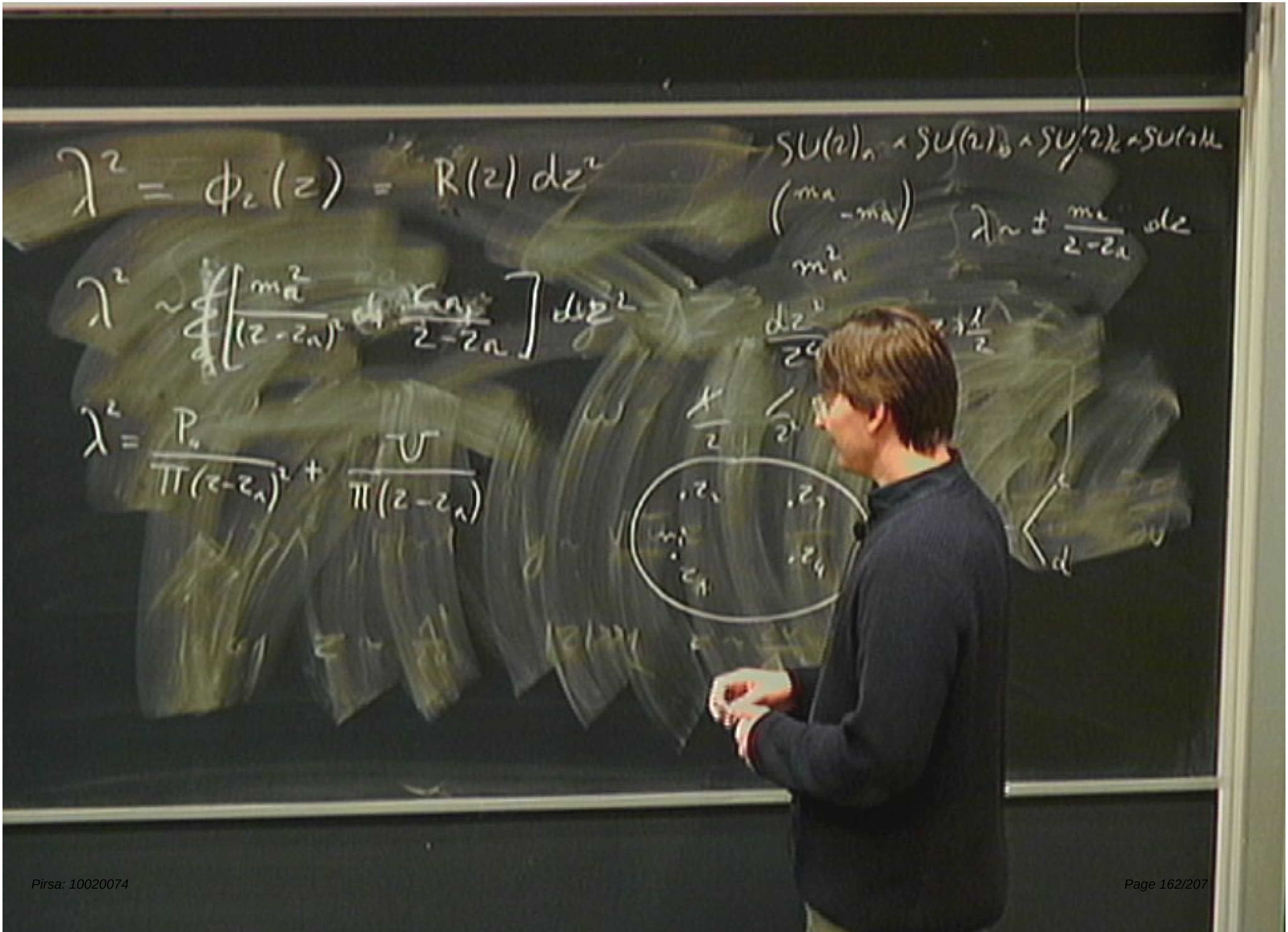
$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix} \qquad \lambda \sim \pm \frac{m_a}{z-z_a}\,dz$$

m_a^2

$\frac{dz^2}{z^4}$ $\frac{1}{z}$

$\frac{1}{z}$ $\frac{1}{z^2}$ $\frac{1}{z^3}$

z_2 z_3 m_a^2 z_a z_4



$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

$$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{[illegible]}{z-z_a}\right] dz^2$$

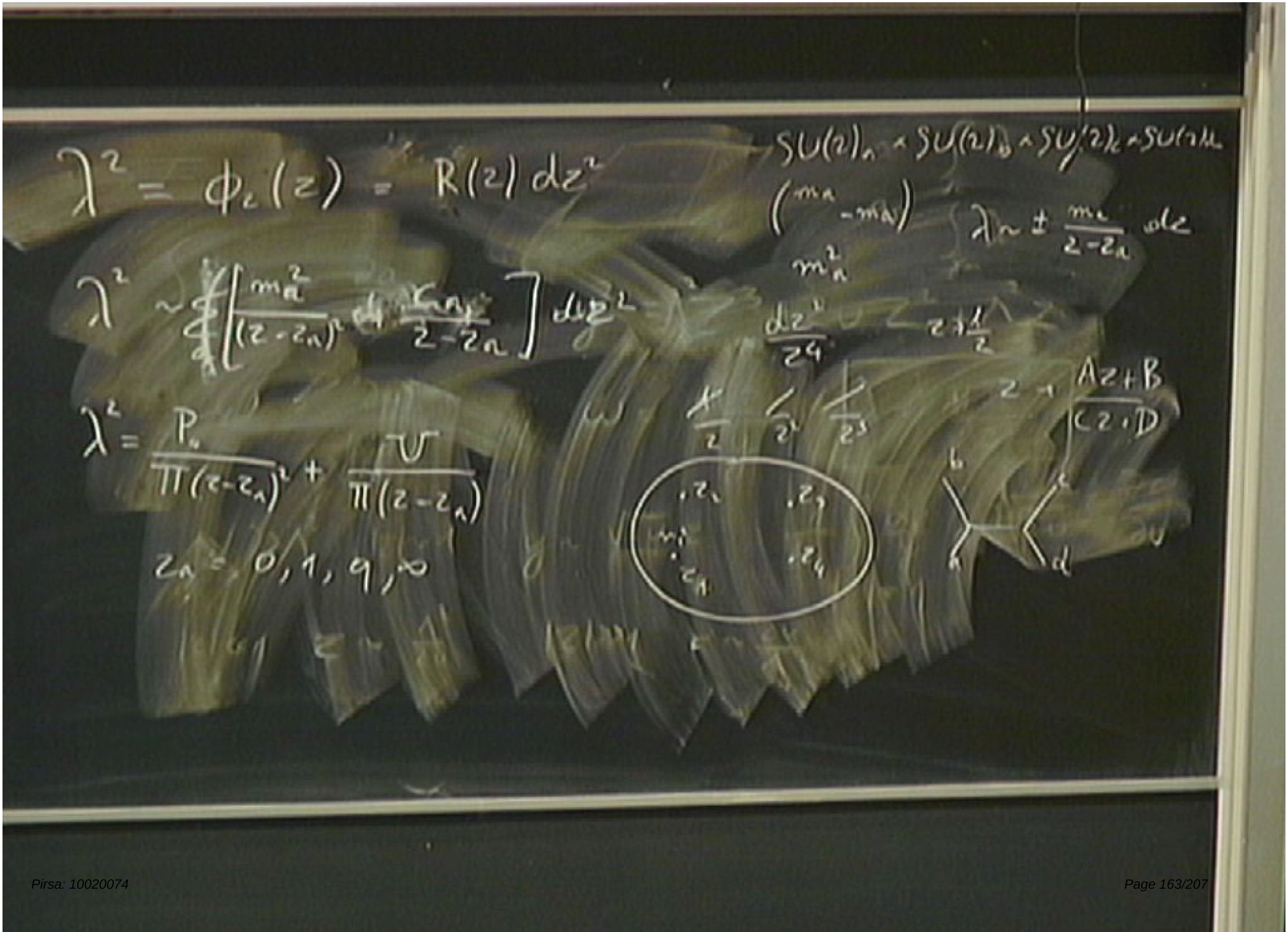
$$\lambda^2 = \frac{P_a}{\prod(z-z_a)^2} + \frac{U}{\prod(z-z_a)}$$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix} \qquad \lambda \sim \pm \frac{m_a}{z-z_a}\,dz$$

m_a^2

$\frac{dz^2}{z^4}$



$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

$$\lambda^2 \sim \sum \left[\frac{m_a^2}{(z-z_a)^2} + \frac{c_a}{z-z_a}\right] dz^2$$

$$\lambda^2 = \frac{P_4}{\prod(z-z_a)^2} + \frac{u}{\prod(z-z_a)}$$

$$z_a = 0, 1, q, \infty$$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

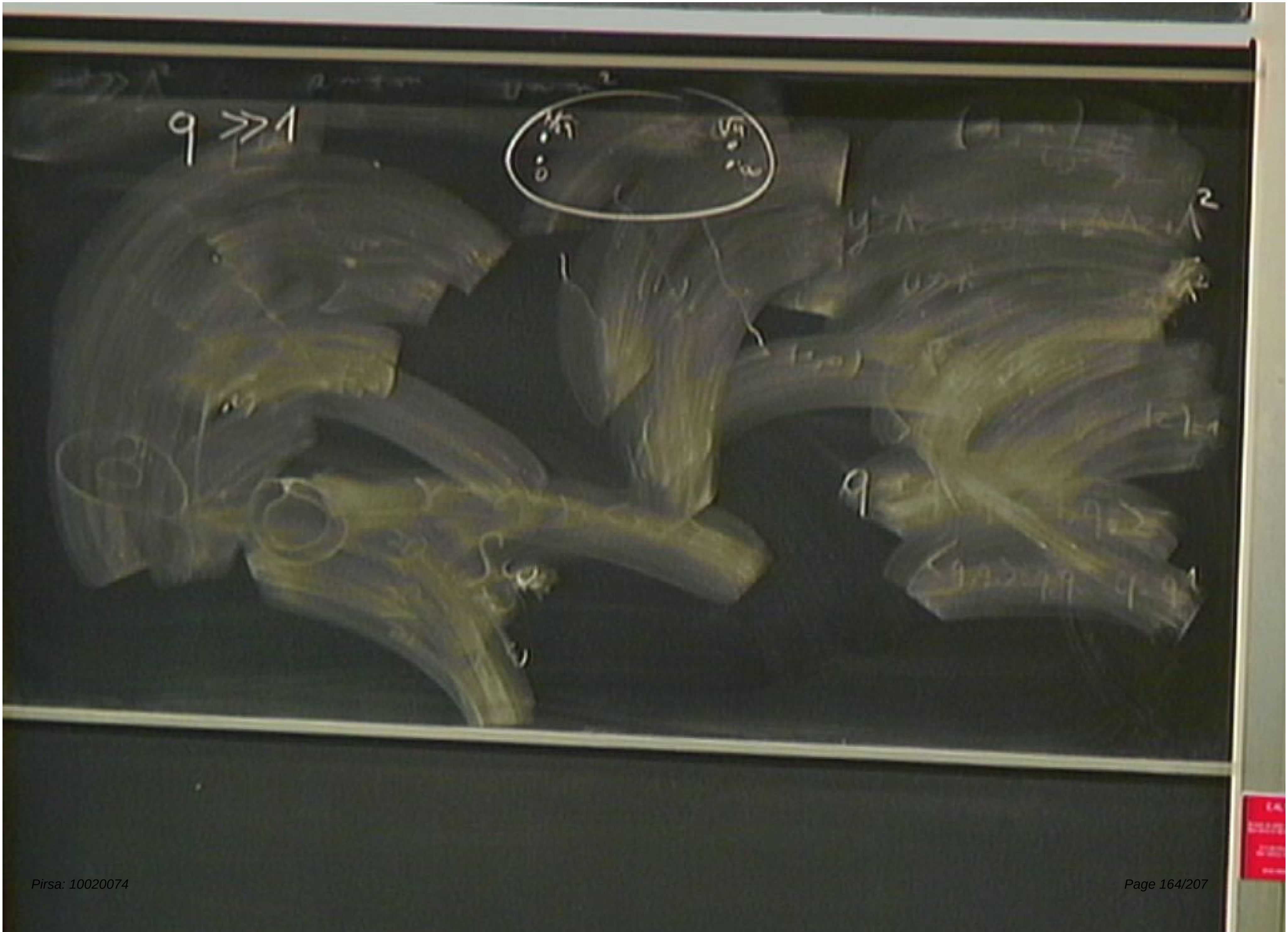
$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$ $\quad \lambda \sim \pm \frac{m_a}{z-z_a}\,dz$

m_a^2

$\frac{dz^2}{z^4}$ $\quad z \to \frac{1}{z}$

$\frac{x}{z}$ $\quad \frac{1}{z^2}$ $\quad \frac{1}{z^3}$

$z \to \frac{Az+B}{Cz+D}$


g ≫ 1
g

$g \gg 1$

$\lambda^2 \simeq \frac{v}{z^2}$

Λ^2

g

$$\lambda^2 = \phi_2(z) = R(z)\, dz^2$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times$$

$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$$

$$\lambda \sim \pm \frac{m_a}{z - z_a}$$

$$m_a^2$$

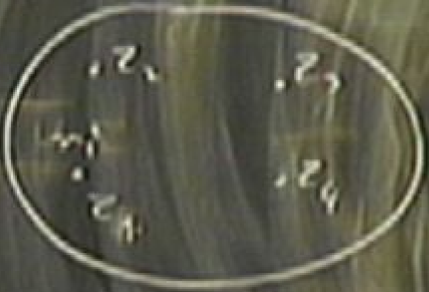
$$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{c_a}{z - z_a} \right] dz^2$$

$$\frac{dz^2}{z^4} \qquad z \to \frac{1}{z}$$

$$\omega \qquad \frac{x}{z} \qquad \frac{1}{z^2} \qquad \frac{1}{z^3} \qquad z \to \frac{Az+}{(z+}$$

$$\lambda^2 = \frac{P_4}{\Pi(z-z_a)^2} + \frac{U}{\Pi(z-z_a)}$$

$$z_a = 0, 1, q, \infty$$



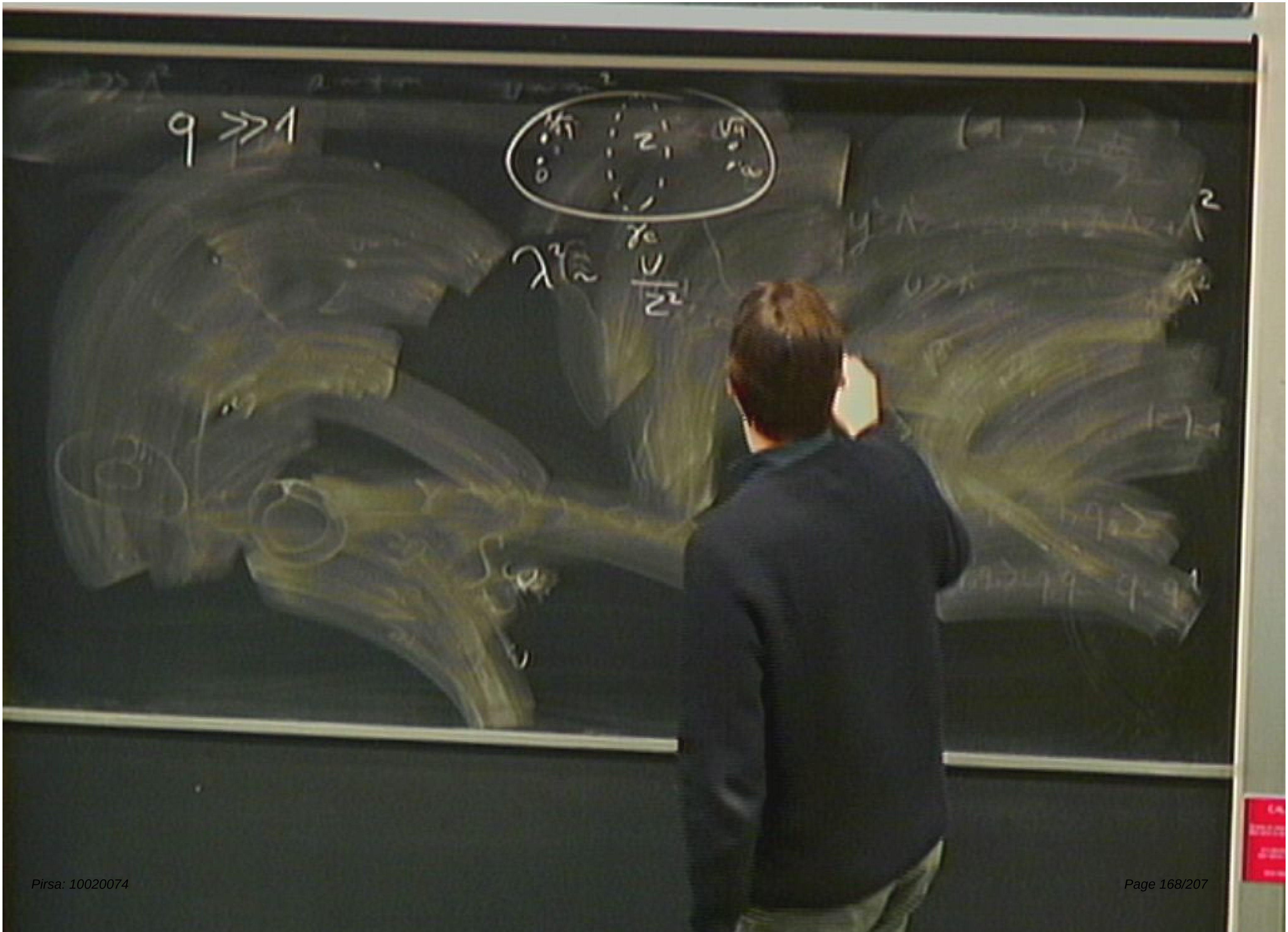
$q \gg 1$

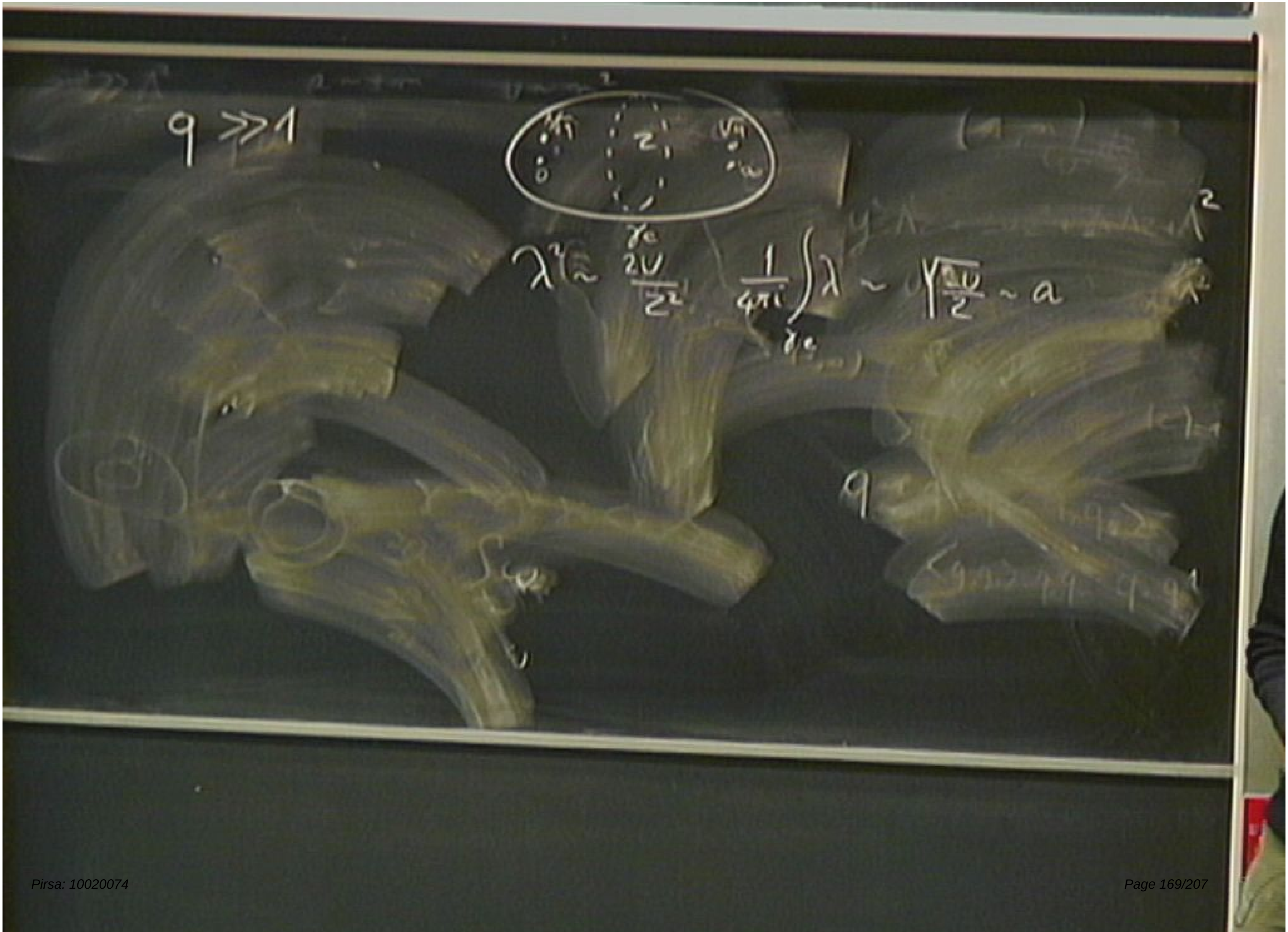
z

$\lambda^2 \tilde{\;}\; \frac{v}{z^2}$

Λ^2

q

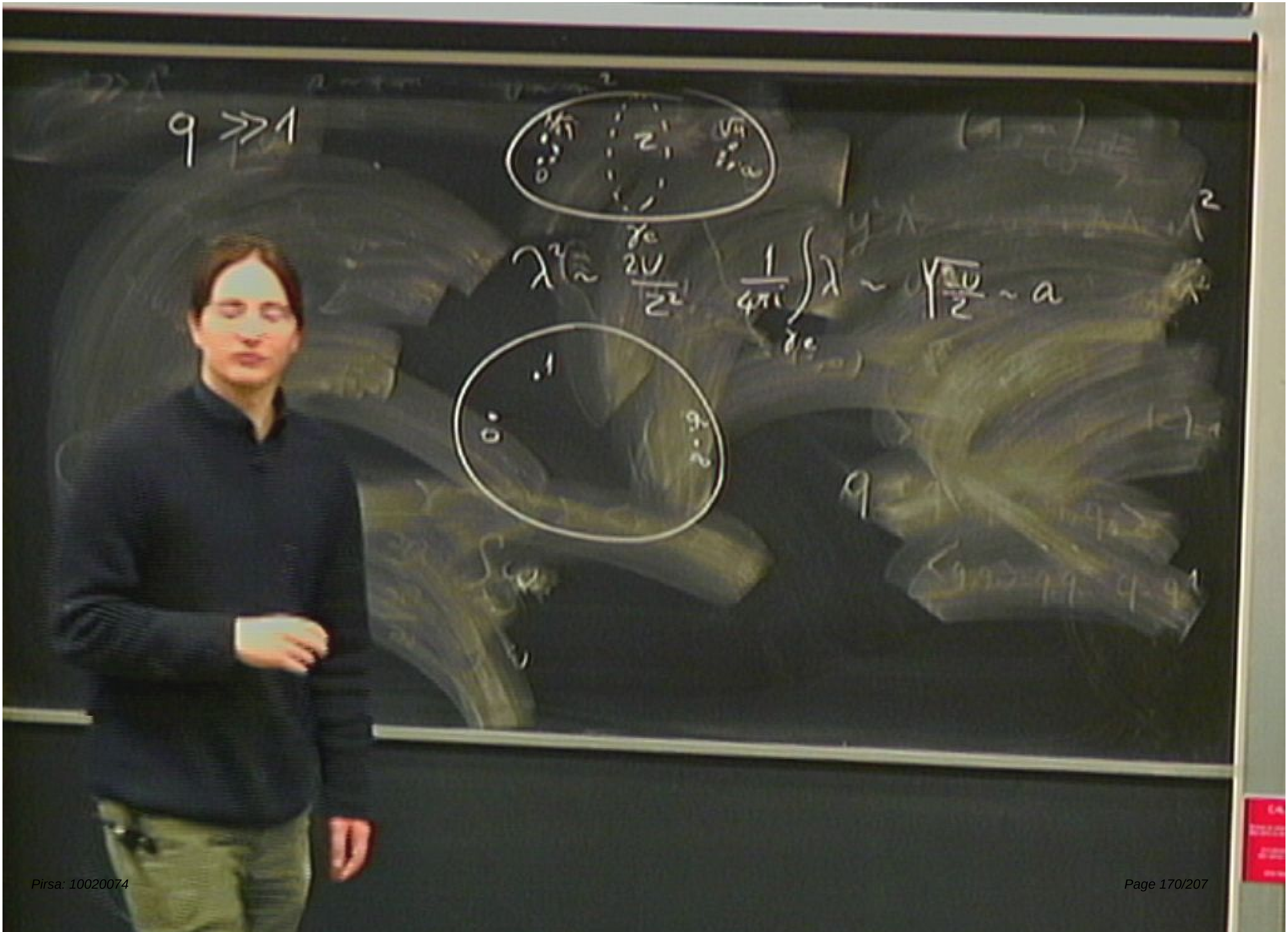

$g \gg 1$
z
γ_c
$\lambda^2 \gamma_c \approx \frac{v}{z^2}$



$q \gg 1$

$$\lambda^2 \simeq \frac{2u}{z^2} \qquad \frac{1}{4\pi i}\int_{\gamma_e} \lambda \sim \sqrt{\frac{2u}{2}} \sim a$$

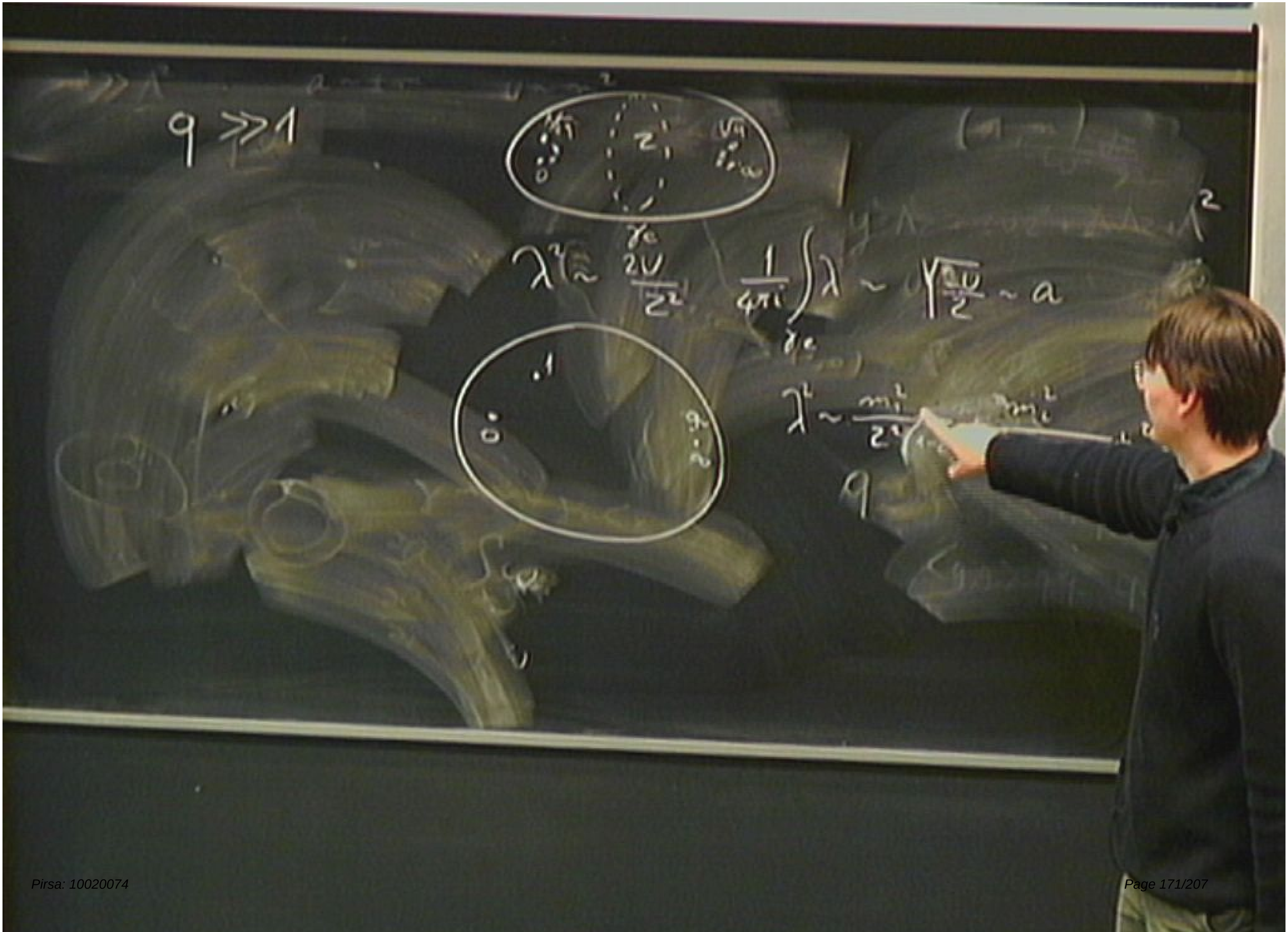
q

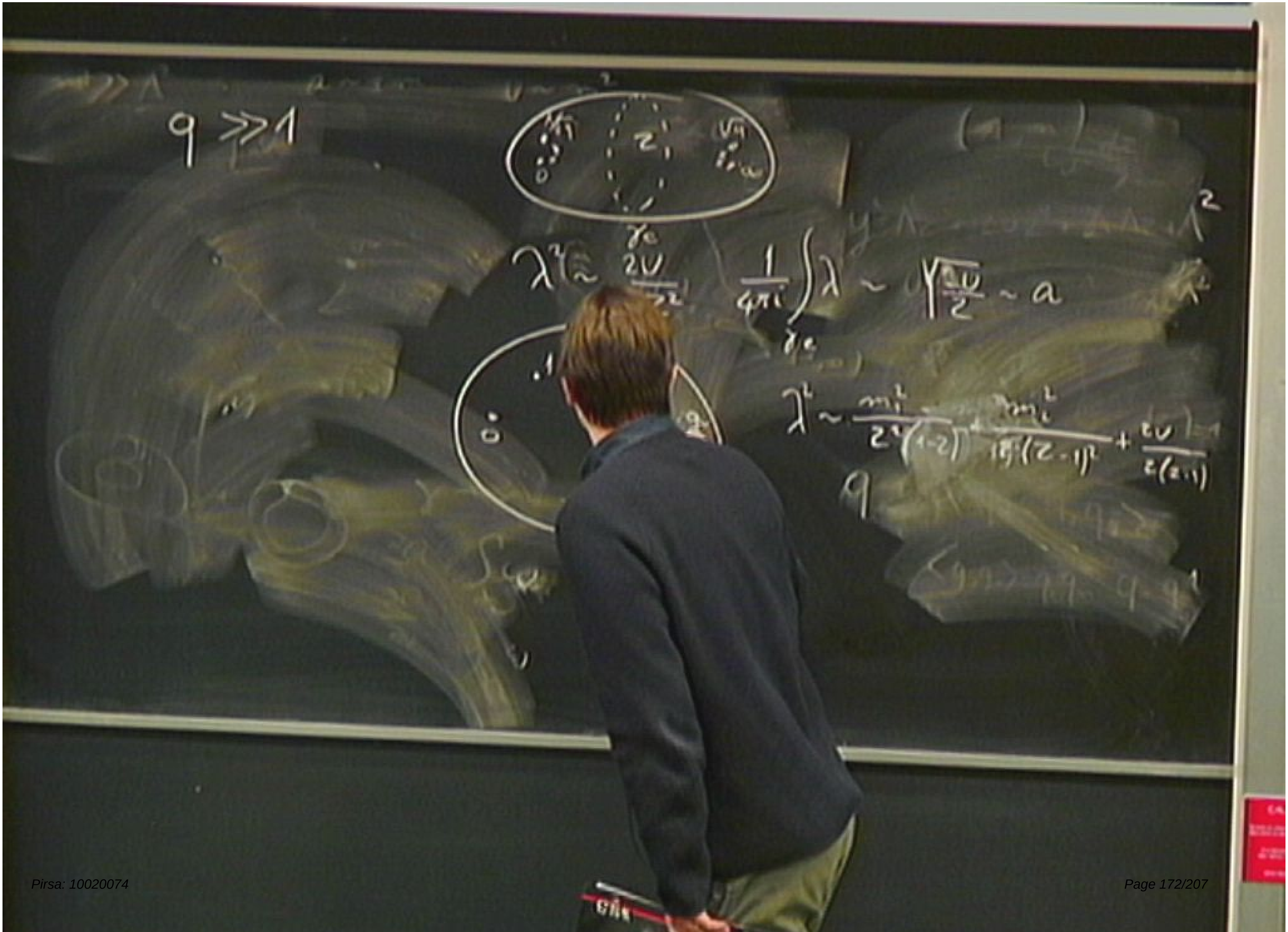


$q \gg 1$

γ_e

$\lambda^{\gamma_e} \approx \frac{2V}{z^2}$ $\quad \frac{1}{4\pi i}\int_{\gamma_e} \lambda \sim \sqrt{\frac{2V}{z}} \sim a$


q ≫ 1
z
λ² ~ 2V/z²
1/4πi ∮ λ ~ √(2V)/z ~ a
γ_e
λ² ~ m_i²/z²
q
λ²



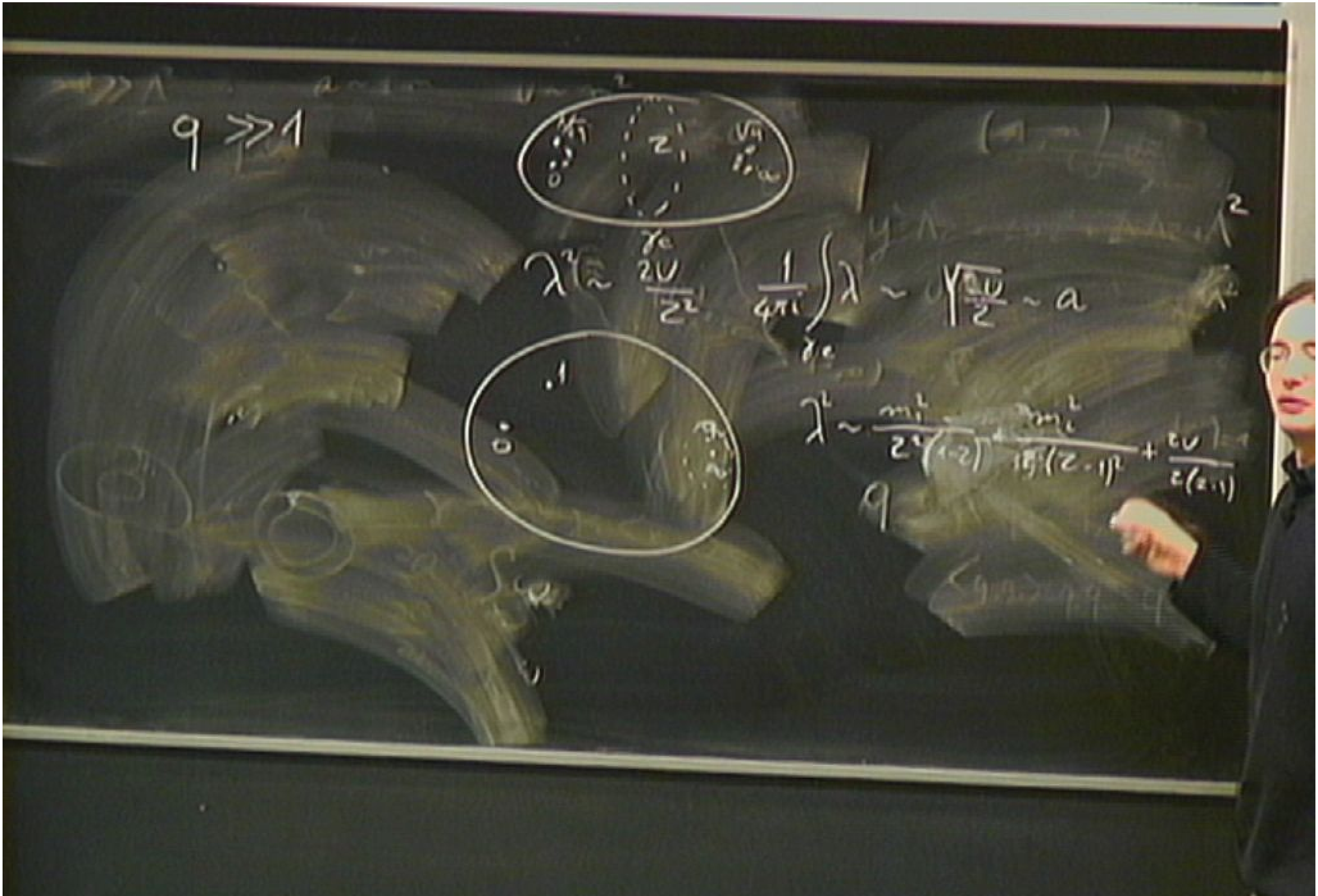
$q \gg 1$

$\lambda^2 \sim \frac{2v}{z^2}$

$\frac{1}{4\pi i}\oint_{\gamma_e} \lambda \sim \sqrt{\frac{2v}{2}} \sim a$

$\lambda^2 \sim \frac{m_1^2}{z^4(1-z)} + \frac{m_2^2}{(z-1)^2} + \frac{2v}{z(z-1)}$

q

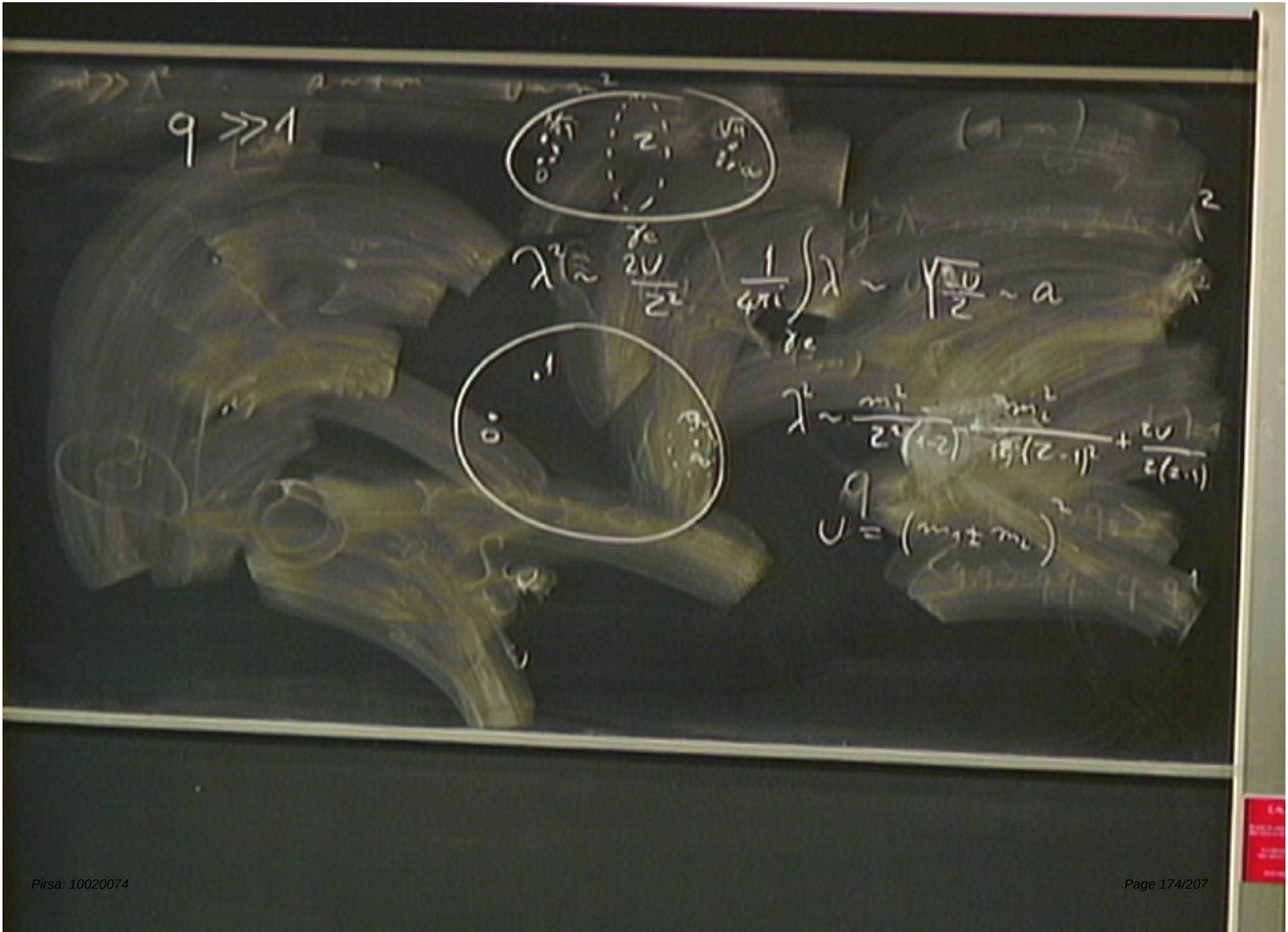


$q \gg 1$

$\lambda^2 \approx \frac{2v}{z^2}$

$\frac{1}{4\pi i}\int_{\gamma_e} \lambda \sim \sqrt{\frac{2v}{z}} \sim a$

$\lambda^2 \sim \frac{m_1^2}{z^2(z-2)} + \frac{m_2^2}{(z-1)^2} + \frac{2v}{z(z-1)}$

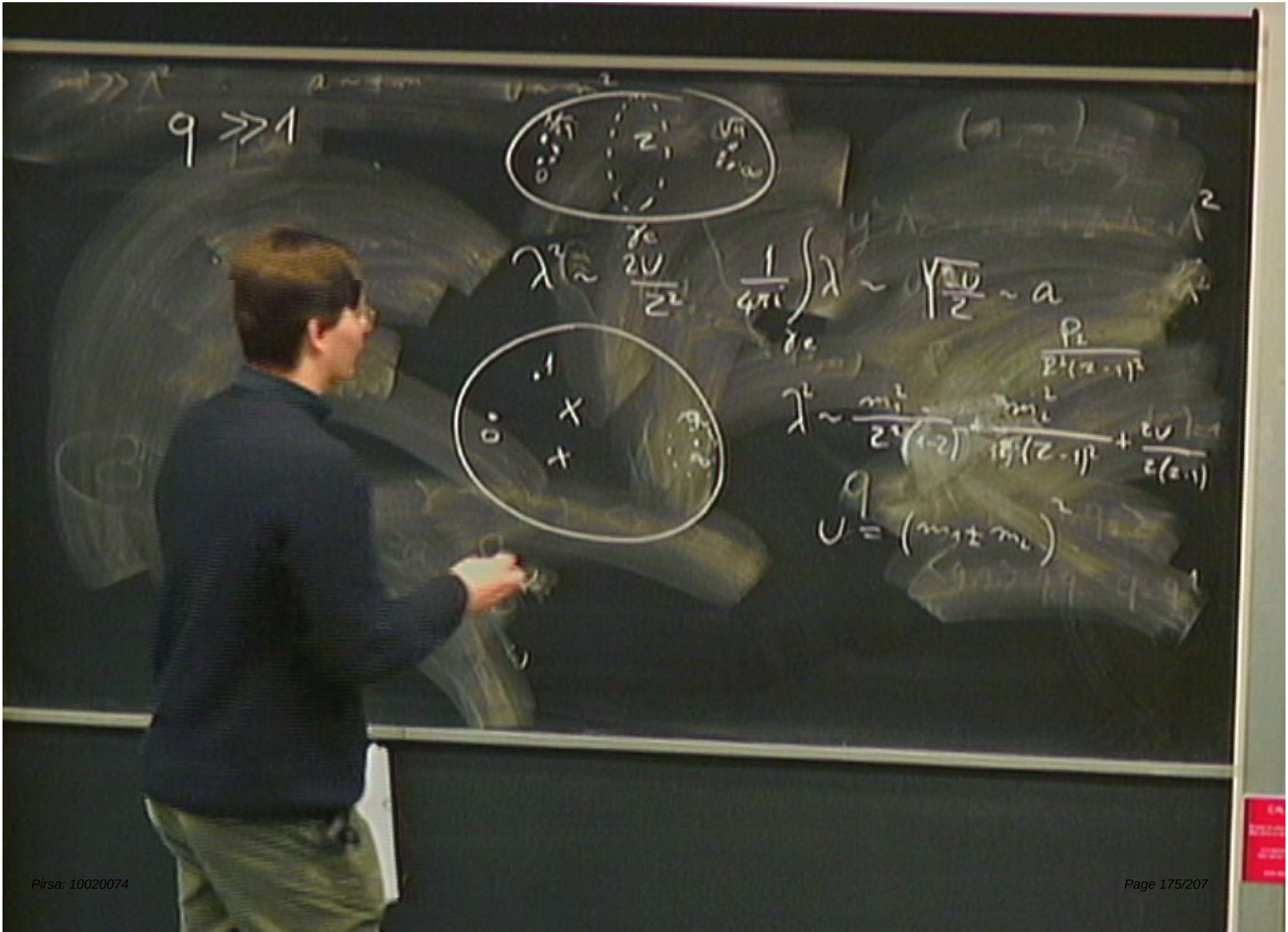


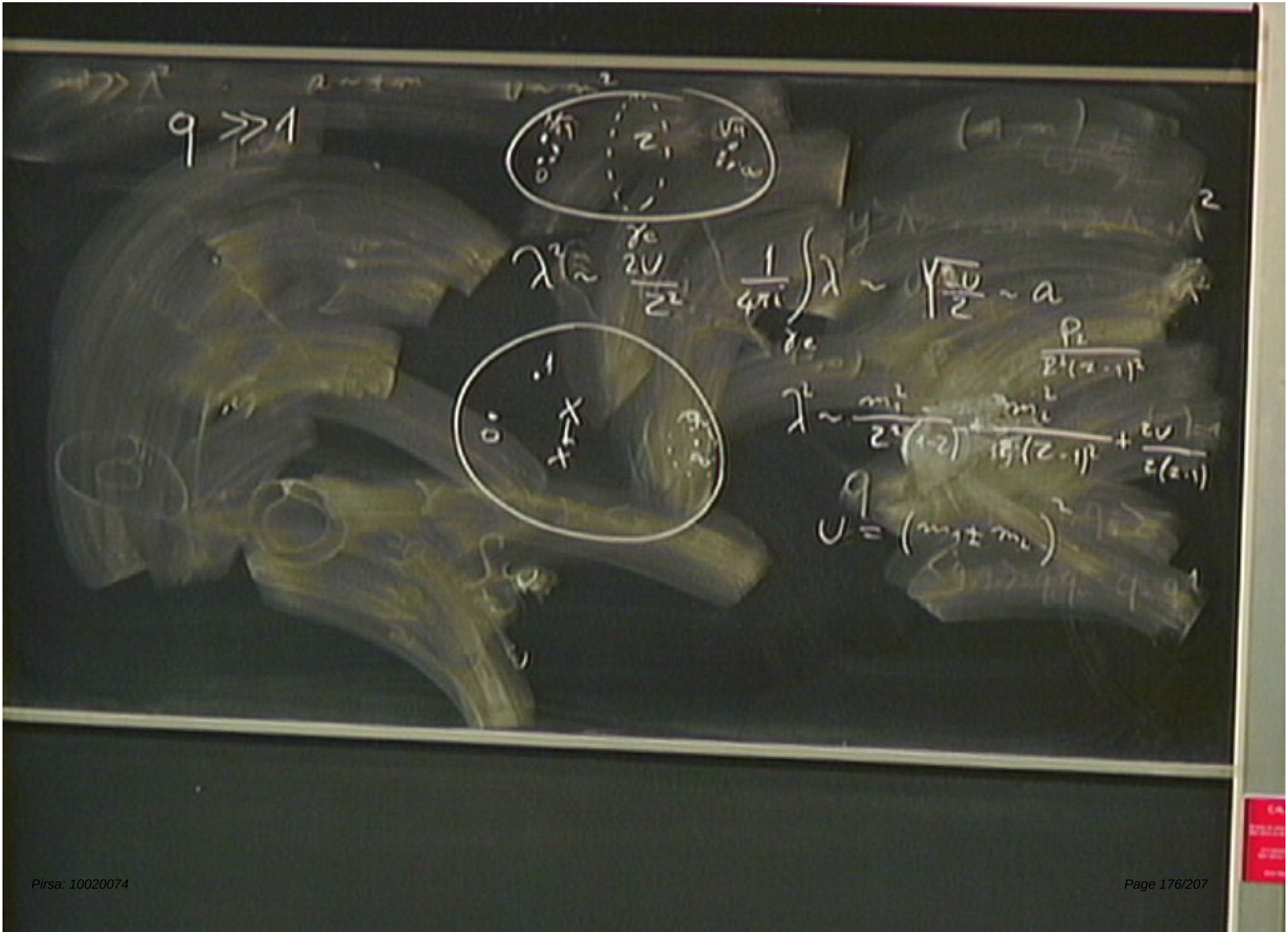
$q \gg 1$

$$\lambda^2 \simeq \frac{2U}{z^2} \qquad \frac{1}{4\pi i}\oint_{\gamma_e} \lambda \sim \sqrt{\frac{2U}{z}} \sim a$$

$$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} + \frac{2U}{z(z-1)}$$

$$U \overset{q}{=} (m_1 \pm m_2)^2$$


$q \gg 1$
$\lambda^2 \approx \frac{2v}{z^2}$
γ_e
$\frac{1}{4\pi i}\int_{\gamma_e} \lambda \sim \sqrt{\frac{2v}{z}} \sim a$
$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} + \frac{2v}{z(z-1)}$
$\frac{P_2}{z^2(z-1)^2}$
$v = (m_1 \pm m_2)^2$
λ^2

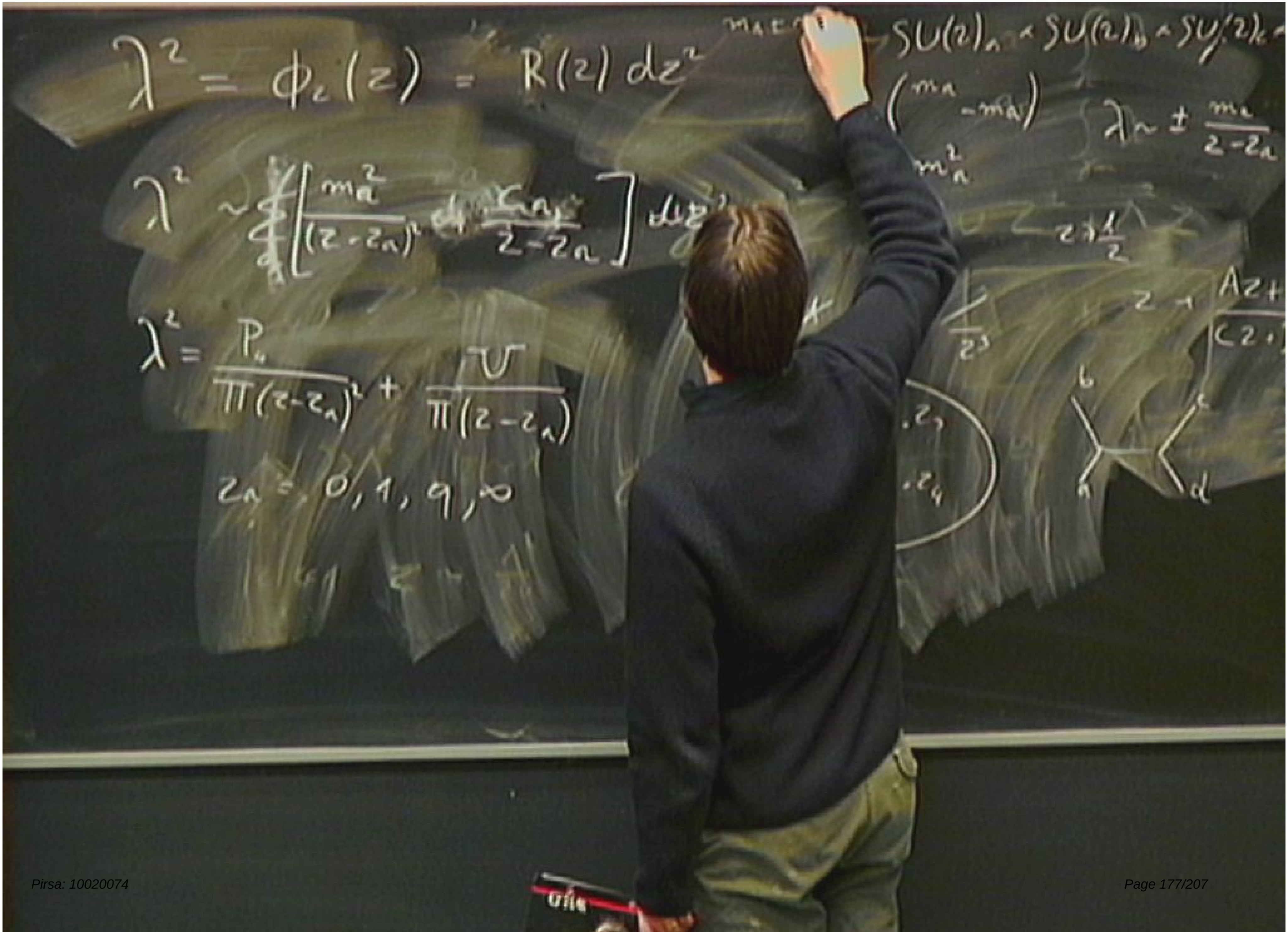


$q \gg 1$

$$\lambda^2 \simeq \frac{2u}{z^2} \qquad \frac{1}{4\pi i}\oint_{\gamma_e} \lambda \sim \sqrt{\frac{2u}{2}} \sim a$$

$$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} + \frac{2u}{z(z-1)}$$

$$\overset{q}{u} = (m_1 \pm m_2)^2$$



$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

$$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{c_a}{z-z_a}\right] dz^2$$

$$\lambda^2 = \frac{P_4}{\prod(z-z_a)^2} + \frac{u}{\prod(z-z_a)}$$

$$z_a = 0, 1, q, \infty$$

$m_a \in \mathbb{C} \to SU(2)_a \times SU(2)_b \times SU(2)_c \times$

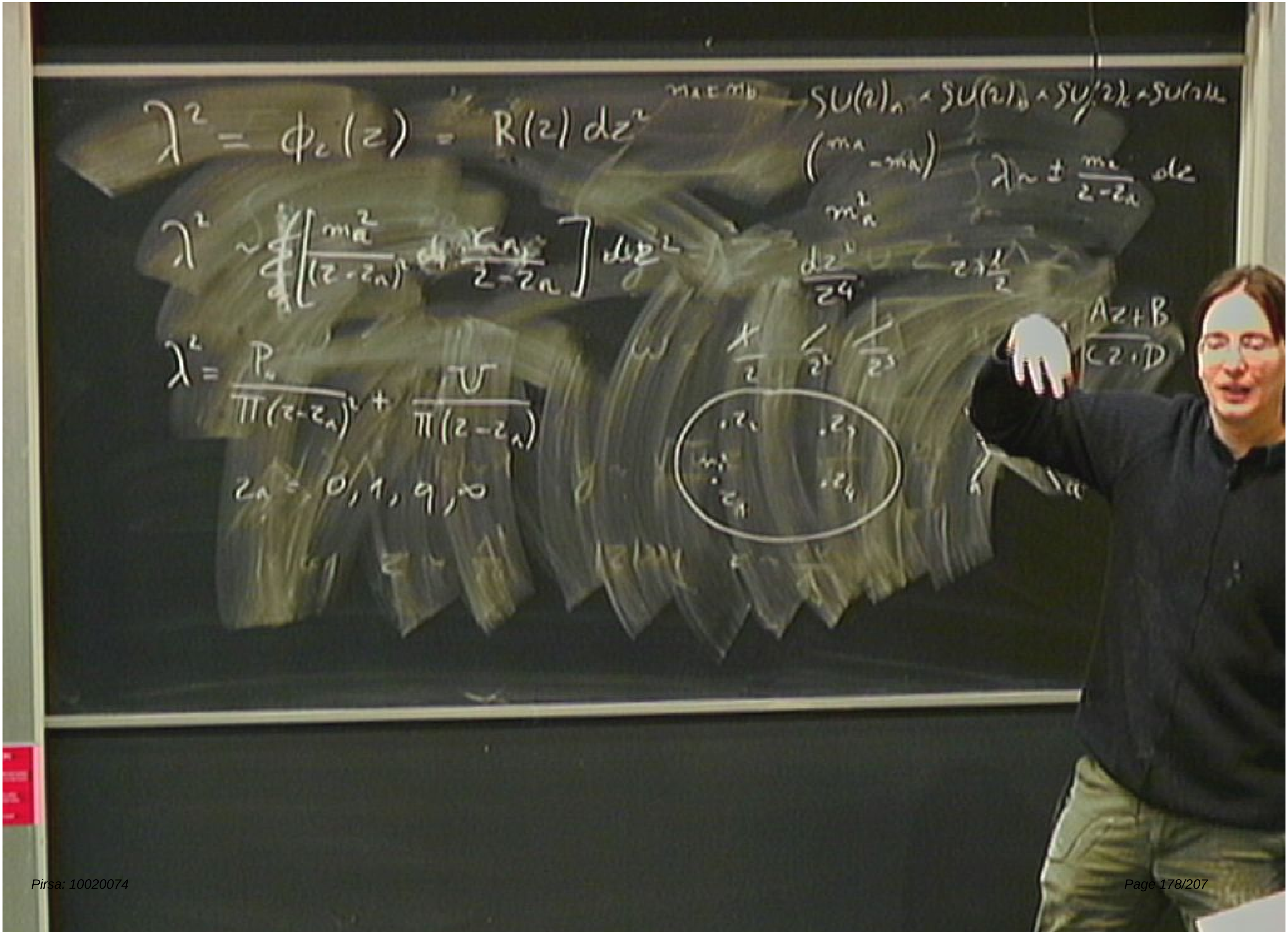
$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$ $\quad \lambda \sim \pm \frac{m_a}{z-z_a}$

m_a^2

$z \to \frac{1}{z}$

$\frac{1}{z^3}$

$z \to \frac{Az+}{Cz+}$



$\lambda^2 = \phi_2(z) = R(z)\, dz^2$

$\lambda^2 \sim \sum \left[\frac{m_a^2}{(z-z_a)^2} + \frac{[illegible]}{z-z_a} \right] dz^2$

$\lambda^2 = \frac{P_a}{\pi(z-z_a)^2} + \frac{U}{\pi(z-z_a)}$

$z_a = 0, 1, q, \infty$

$m_a \neq m_b$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$

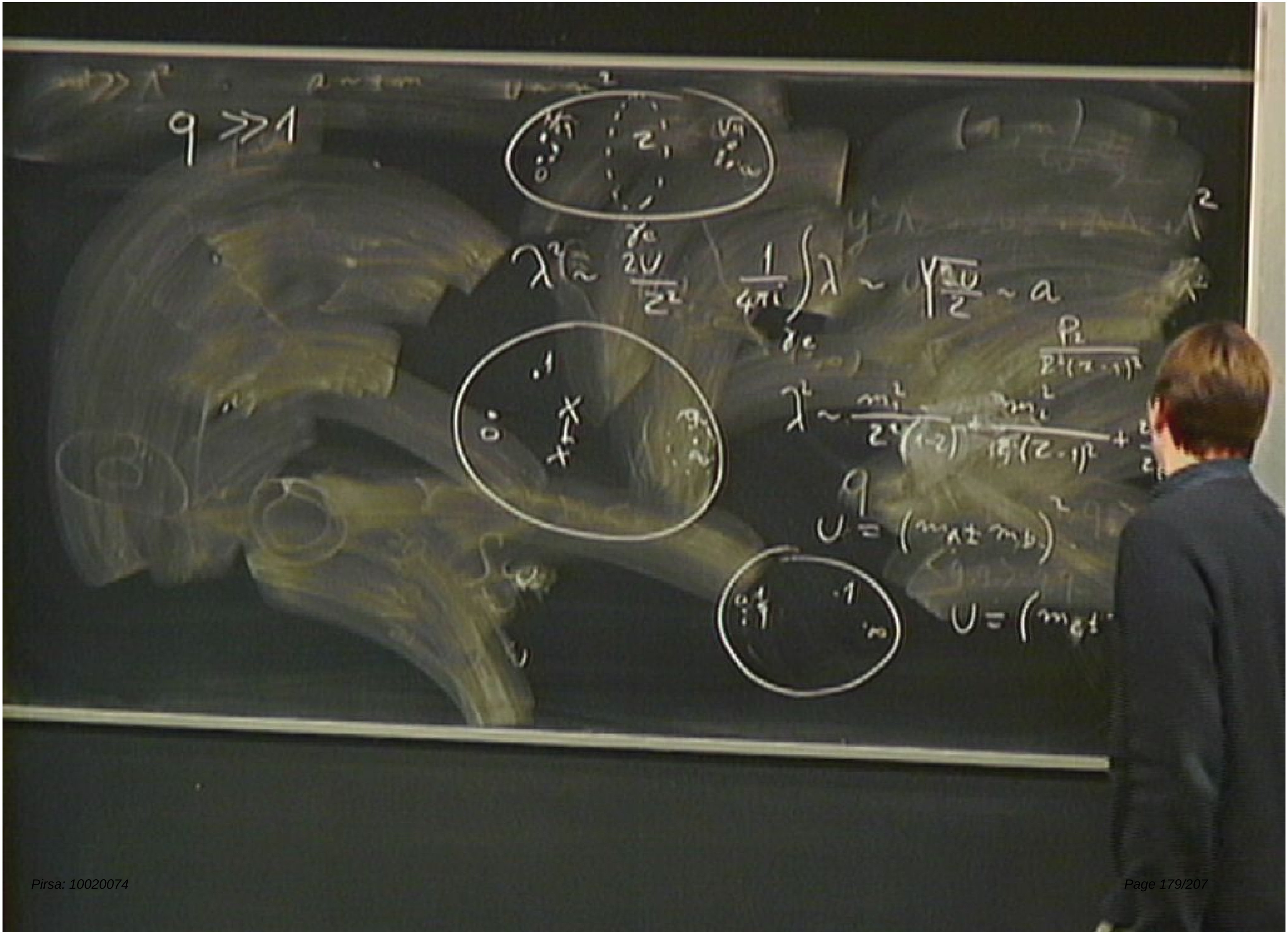
$\lambda \sim \pm \frac{m_a}{z-z_a}\, dz$

m_a^2

$\frac{dz^2}{z^4}$ $z \to \frac{1}{z}$

$\frac{A}{z}$ $\frac{}{z^2}$ $\frac{}{z^3}$

$\frac{Az+B}{Cz+D}$



$q \gg 1$

$\lambda^2 \sim \frac{2U}{z^2}$

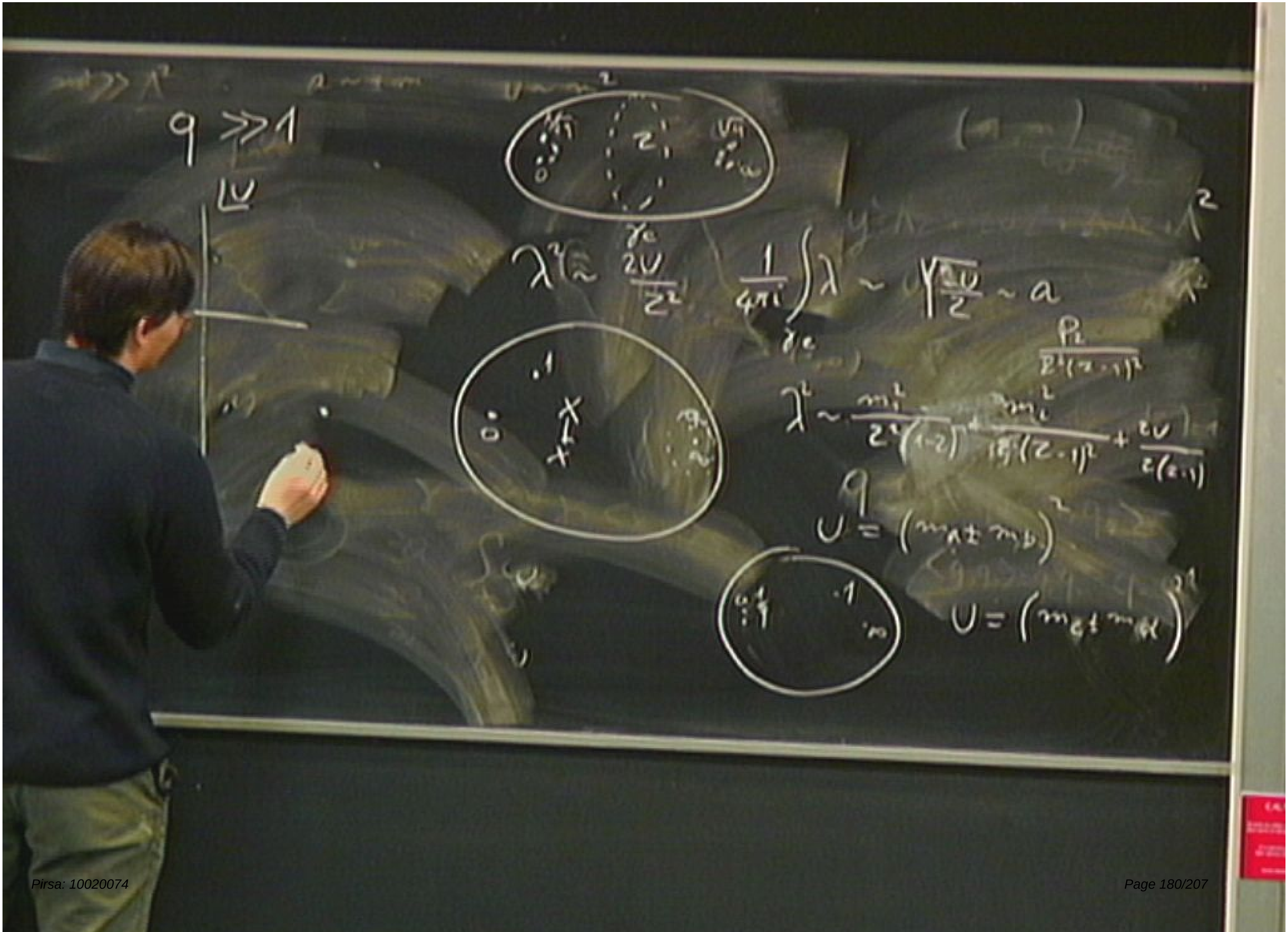
$\frac{1}{4\pi i}\oint_{\gamma_e} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$

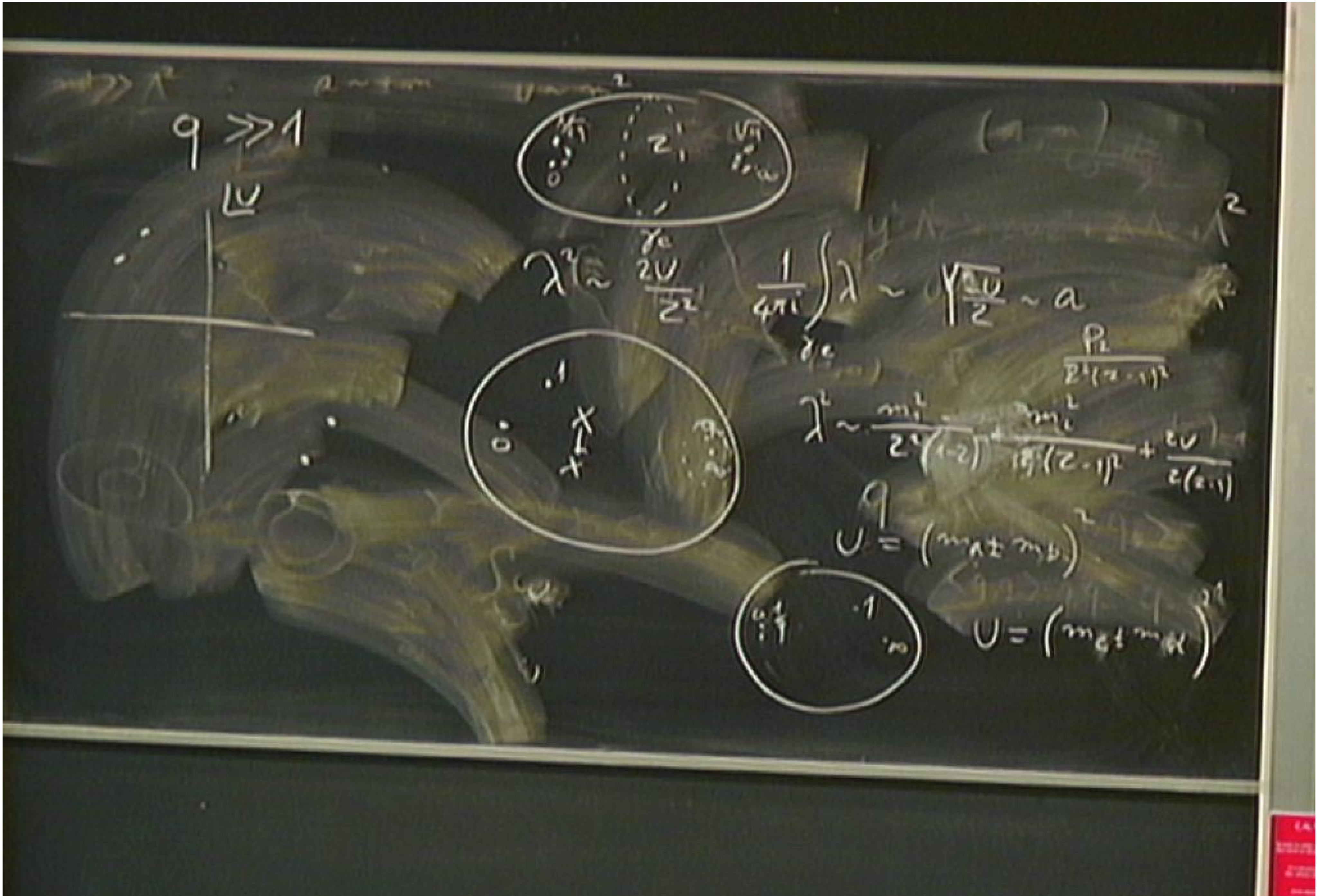
$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} + \dots$

q

$U = (m_A \pm m_B)^2$

$U = (m_C \pm$


q >> 1
λ² ~ 2U/z²
1/(4πi) ∮ λ ~ √(2U/z) ~ a
λ² ~ m₁²/(z²(z-1)) + m₂²/((z-1)²) + 2U/(z(z-1))
U = (m_A ± m_B)²
U = (m_C ± m_D)²



$q \gg 1$

$\lambda^2 \approx \frac{2v}{z^2}$

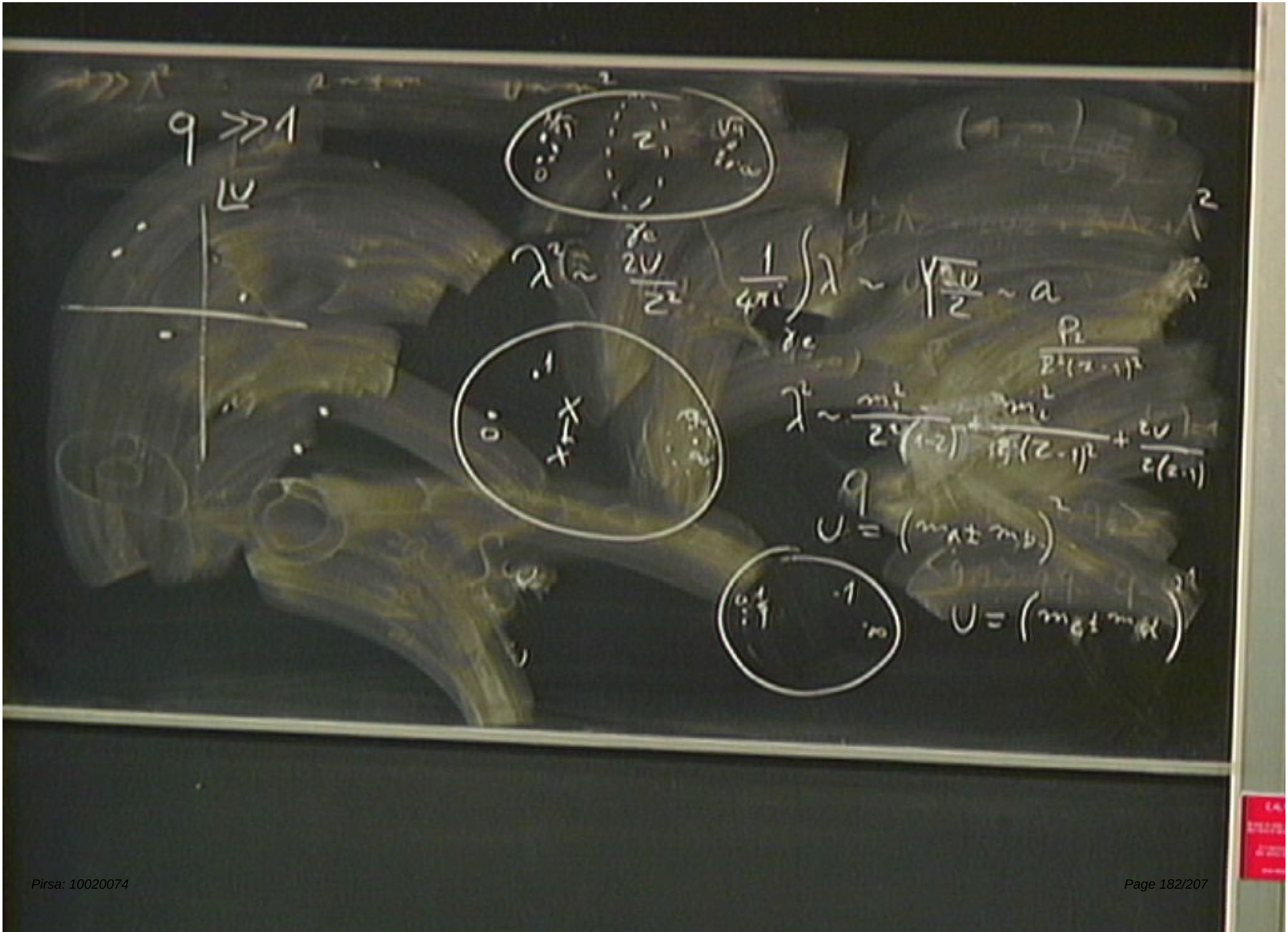
$\frac{1}{4\pi i}\int_{\gamma_e} \lambda \sim \sqrt{\frac{2v}{z}} \sim a$

$\frac{P_2}{z^2(z-1)^2}$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{z^2(z-1)^2} + \frac{2v}{z(z-1)}$

$v = (m_A \pm m_B)^2$

$v = (m_C \pm m_D)^2$



$q \gg 1$

$\lambda^2 \simeq \frac{2U}{z^2}$

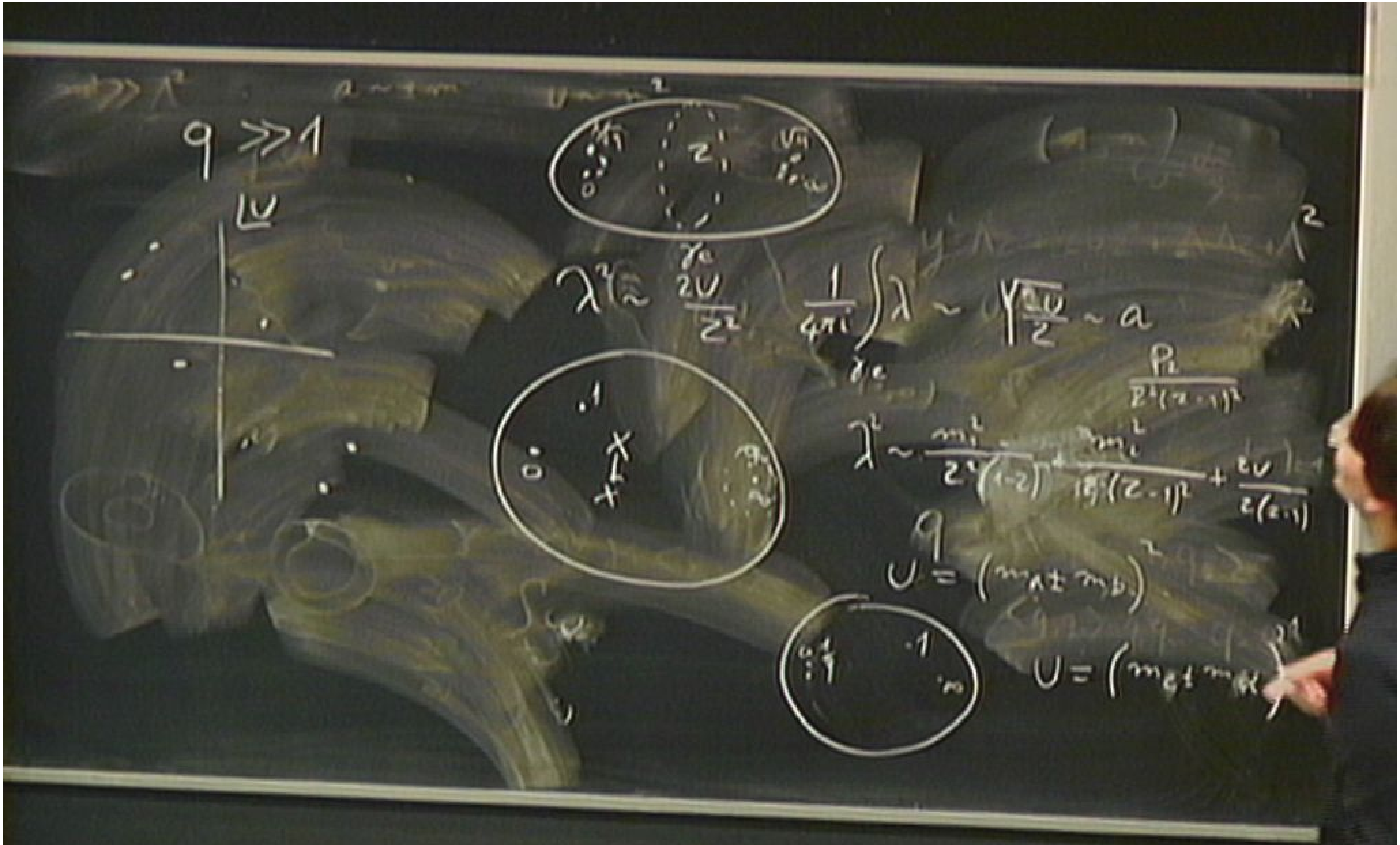
$\frac{1}{4\pi i}\oint_{\gamma_e} \lambda \sim \sqrt{\frac{2U}{z}} \sim a$

$\frac{P_2}{z^2(z-1)^2}$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{z^2(z-1)^2} + \frac{2U}{z(z-1)}$

$U = (m_A \pm m_B)^2$

$U = (m_C \pm m_D)^2$



$q \gg 1$

$\lambda^2 \sim \frac{2U}{z^2}$

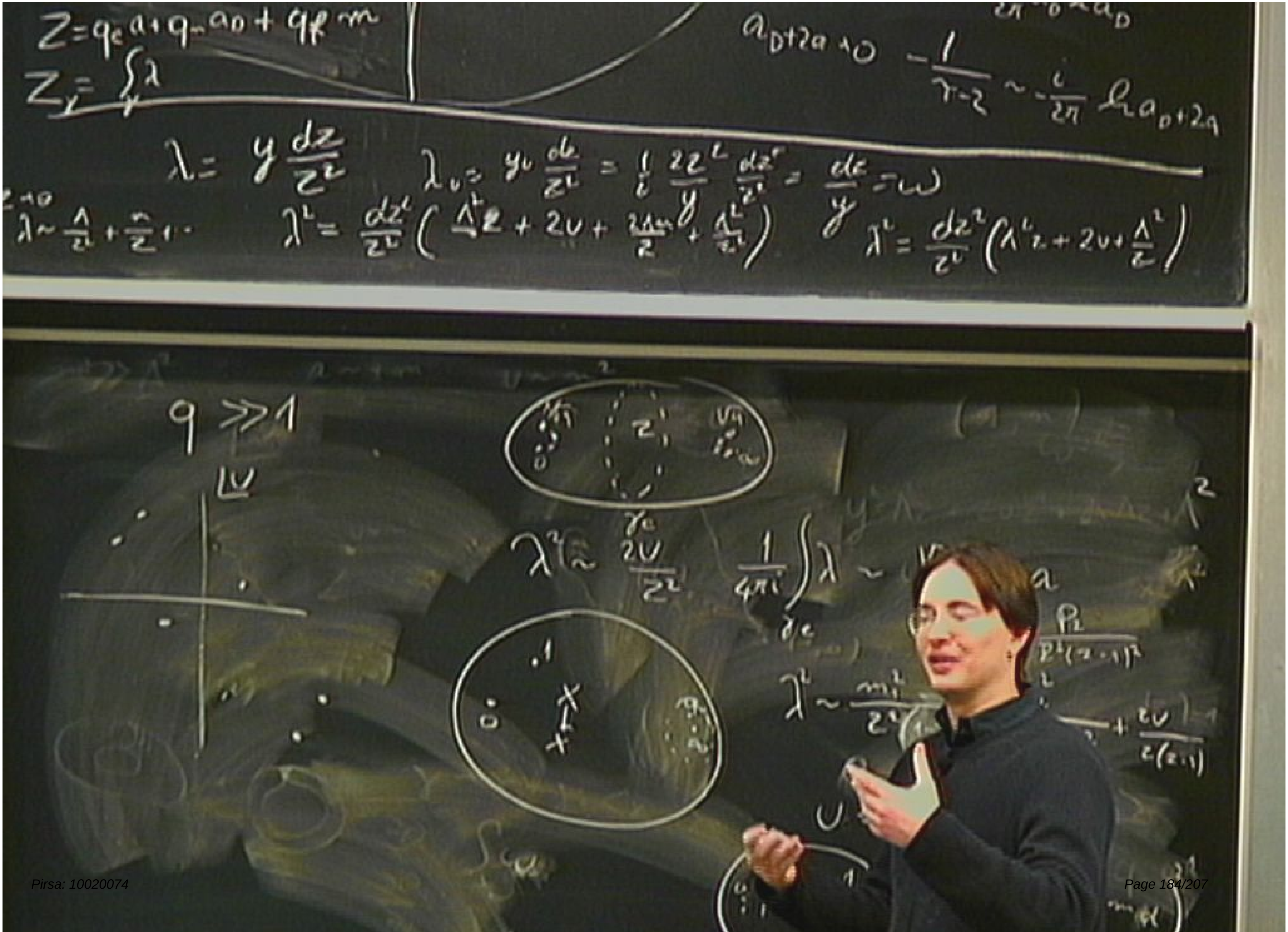
$\frac{1}{4\pi i}\int_{\gamma_c} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$

$\frac{P_2}{z^2(z-1)^2}$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} + \frac{2U}{z(z-1)}$

$U = (m_a \pm m_b)^2$

$U = (m_c \pm m_d)^2$



$Z = q_e a + q_m a_D + q_f m$

$Z_\gamma = \oint_\gamma \lambda$

$a_D + 2a \to 0 \quad -\frac{1}{\tau - 2} \sim -\frac{i}{2\pi} \ln a_D + 2a$

$\lambda = y \frac{dz}{z^2} \quad \lambda_u = y_u \frac{dz}{z^2} = \frac{1}{2} \frac{2z^2}{y} \frac{dz}{z^2} = \frac{dz}{y} = \omega$

$z \to 0$

$\lambda \sim \frac{\Lambda}{z^2} + \frac{m}{z} + \cdots$

$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{2\Lambda m}{z} + \frac{\Lambda^2}{z^2}\right)$

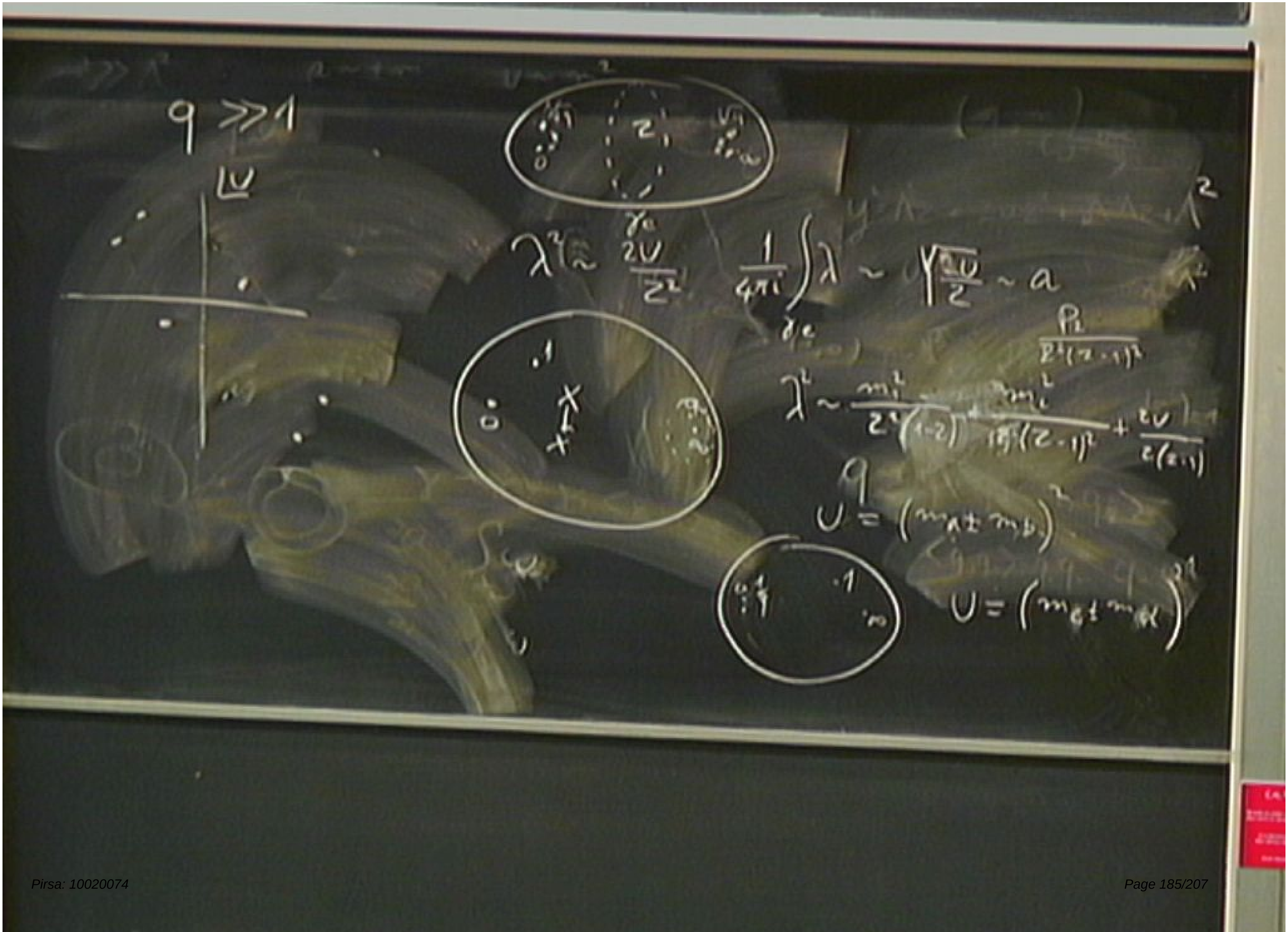
$\lambda^2 = \frac{dz^2}{z^2}\left(\Lambda^2 z + 2u + \frac{\Lambda^2}{z}\right)$

$q \gg 1$

$\lambda^2 \approx \frac{2u}{z^2}$

$\frac{1}{4\pi i}\oint_{\gamma_e} \lambda \sim a$

$\lambda^2 \sim \frac{m_f^2}{z^2} \quad + \frac{2u}{z(z-1)}$



$q \gg 1$

$\lambda^2 \approx \frac{2v}{z^2}$

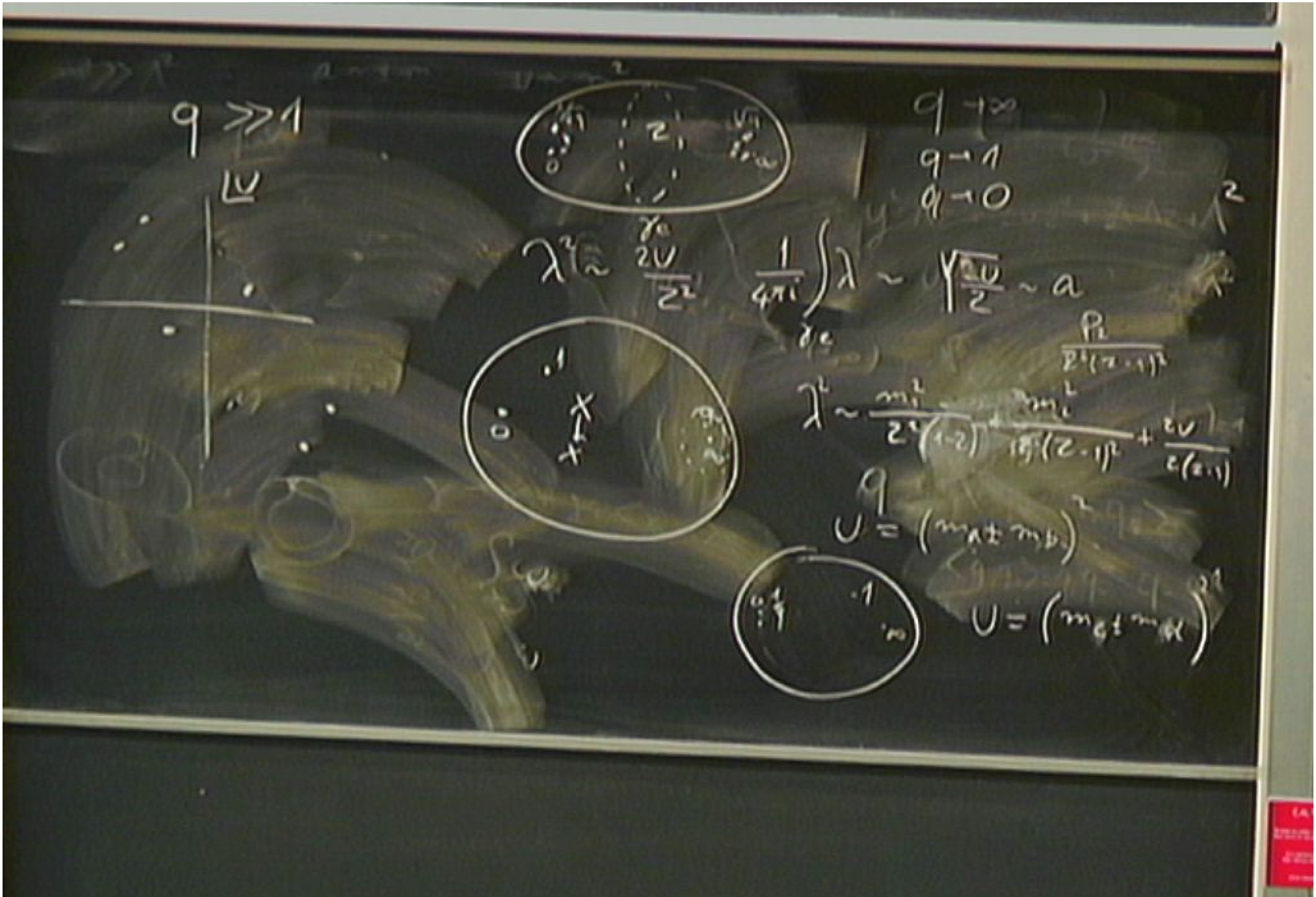
$\frac{1}{4\pi i}\oint_{\gamma_a} \lambda \sim \sqrt{\frac{2v}{z}} \sim a$

$\frac{P_2}{z^2(z-1)^2}$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{z(z-1)^2} + \frac{2v}{z(z-1)}$

$v = (m_a \pm m_b)^2$

$v = (m_c \pm m_d)^2$



$q \gg 1$

$q \to \infty$

$q \to 1$

$q \to 0$

$\lambda^2 \sim \frac{2U}{z^2}$

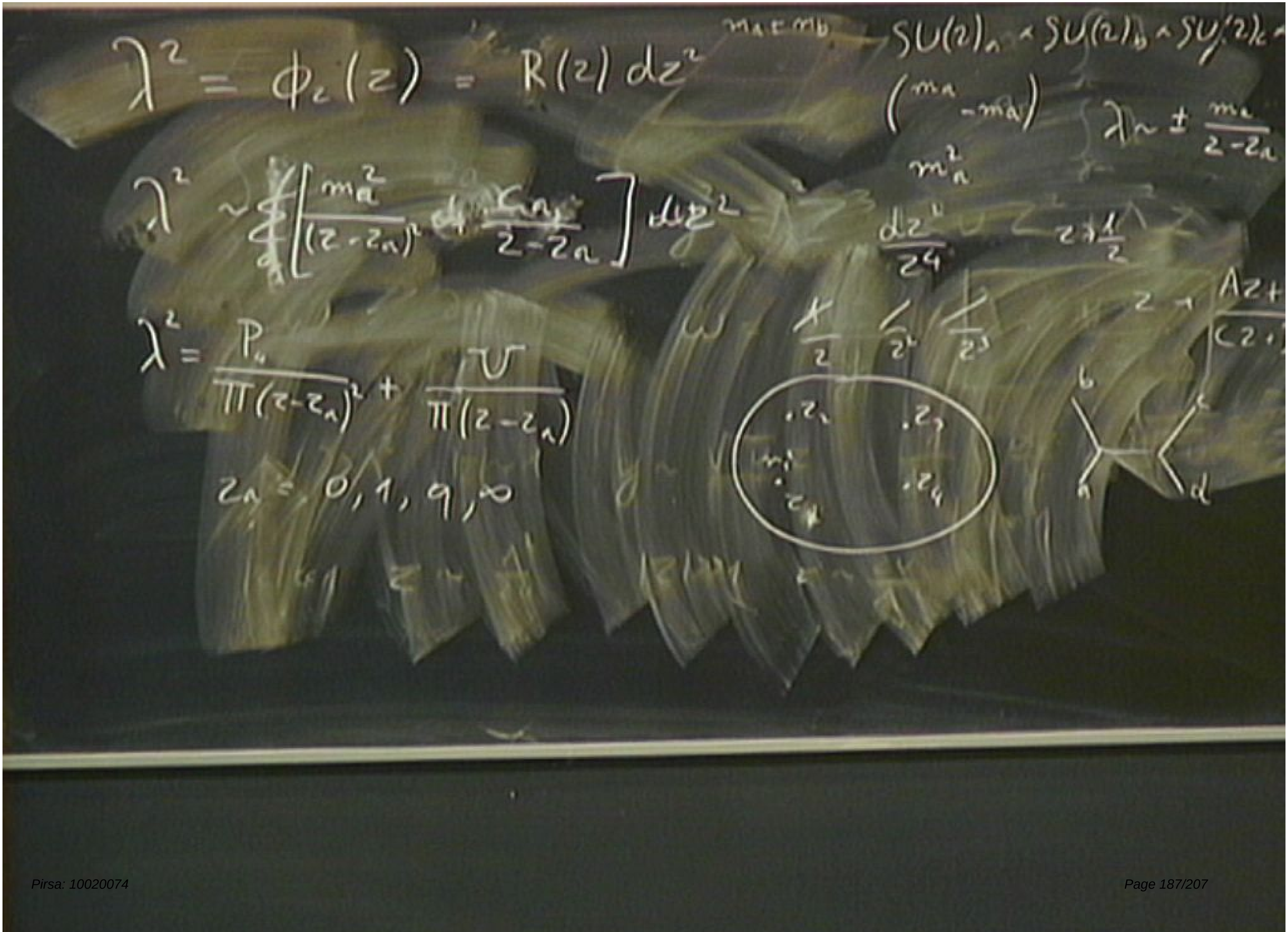
$\frac{1}{4\pi i}\oint_{\gamma_0} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{z(z-1)^2} + \frac{2U}{z(z-1)}$

$\frac{P_2}{z^2(z-1)^2}$

$U = (m_a \pm m_b)^2$

$U = (m_c \pm m_d)^2$



$m_a \pm m_b$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times$

$\lambda^2 = \phi_2(z) = R(z)\,dz^2$

$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$

$\lambda \sim \pm \frac{m_a}{z-z_a}$

m_a^2

$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{c_a}{z-z_a}\right] dz^2$

$\frac{dz^2}{z^4}$

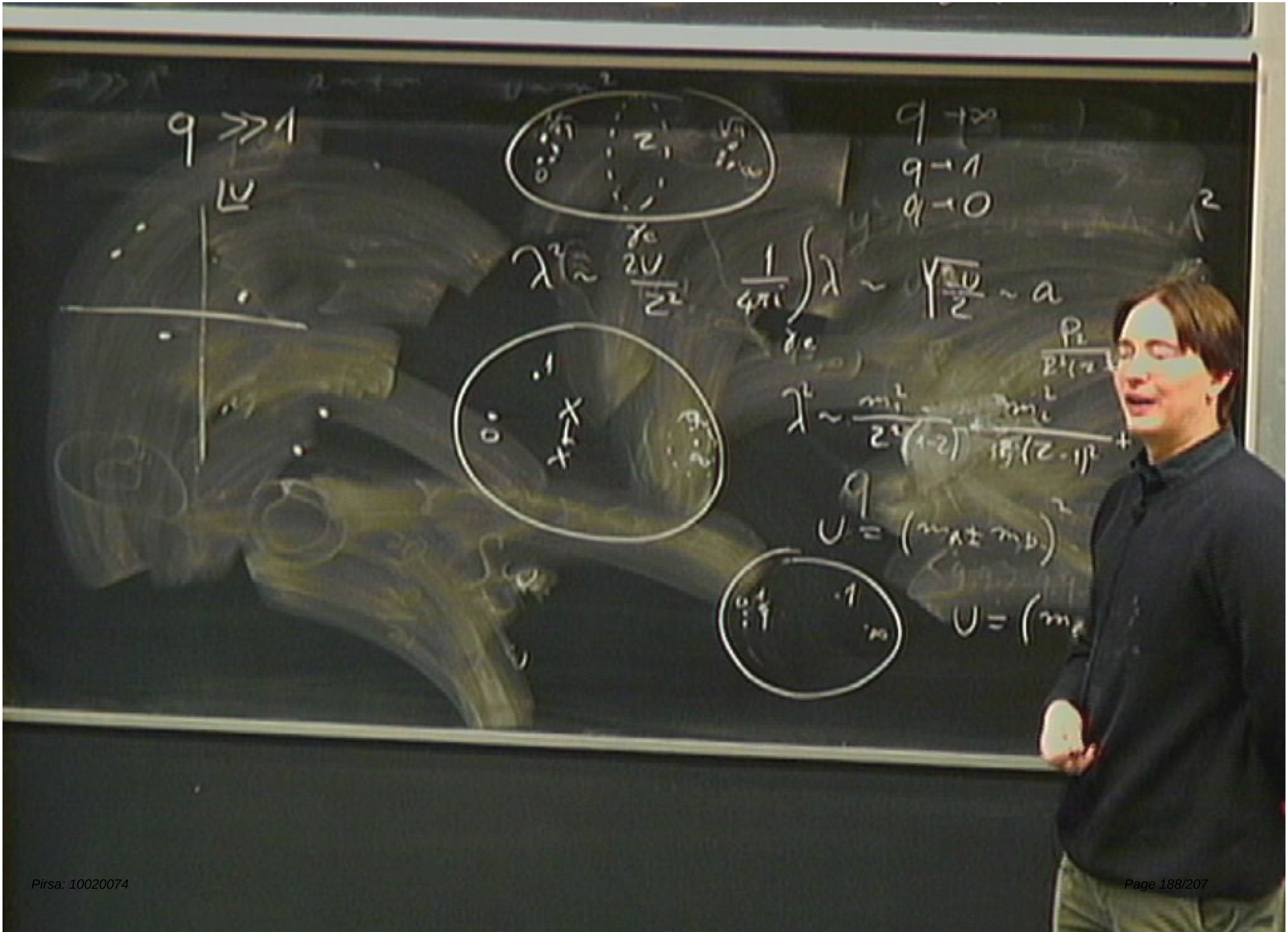
$z \to \frac{1}{z}$

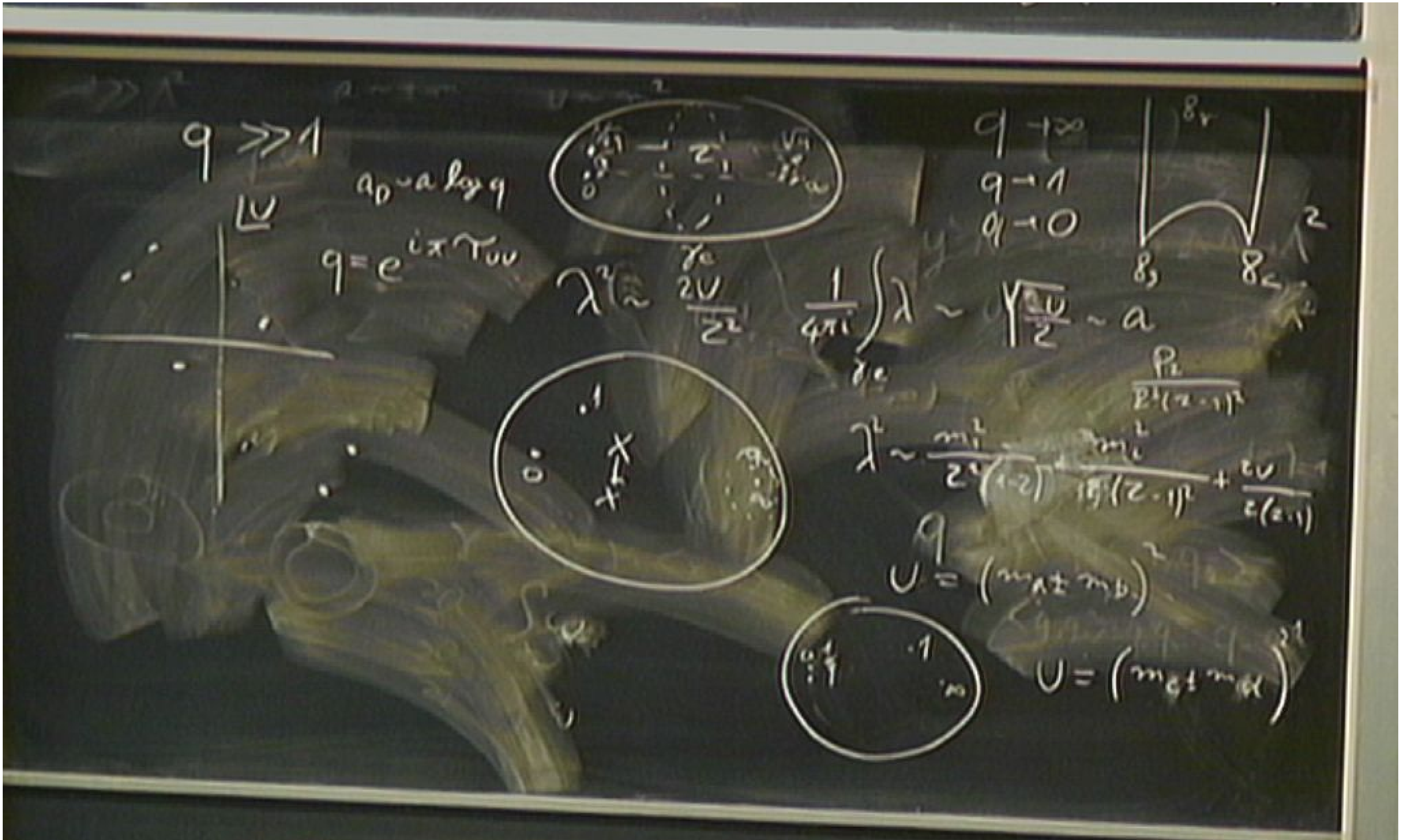
$\frac{x}{z}$ $\frac{1}{z^2}$ $\frac{1}{z^3}$

$z \to \frac{Az+}{(z+}$

$\lambda^2 = \frac{P_4}{\Pi(z-z_a)^2} + \frac{v}{\Pi(z-z_a)}$

$z_a = 0, 1, q, \infty$


q ≫ 1
q → ∞
q → 1
q → 0
$\lambda^2 \sim \frac{2U}{z^2}$
$\frac{1}{4\pi i}\oint_{\gamma_e} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$
$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} +$
$U = (m_A \pm m_B)^2$
$U = (m$



$q \gg 1$

$a_D \sim a \log q$

$q = e^{i\pi\tau_{UV}}$

$\lambda^2 \simeq \frac{2U}{z^2}$

$\frac{1}{4\pi i}\int_{\gamma_e} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$

$q \to \infty$

$q \to 1$

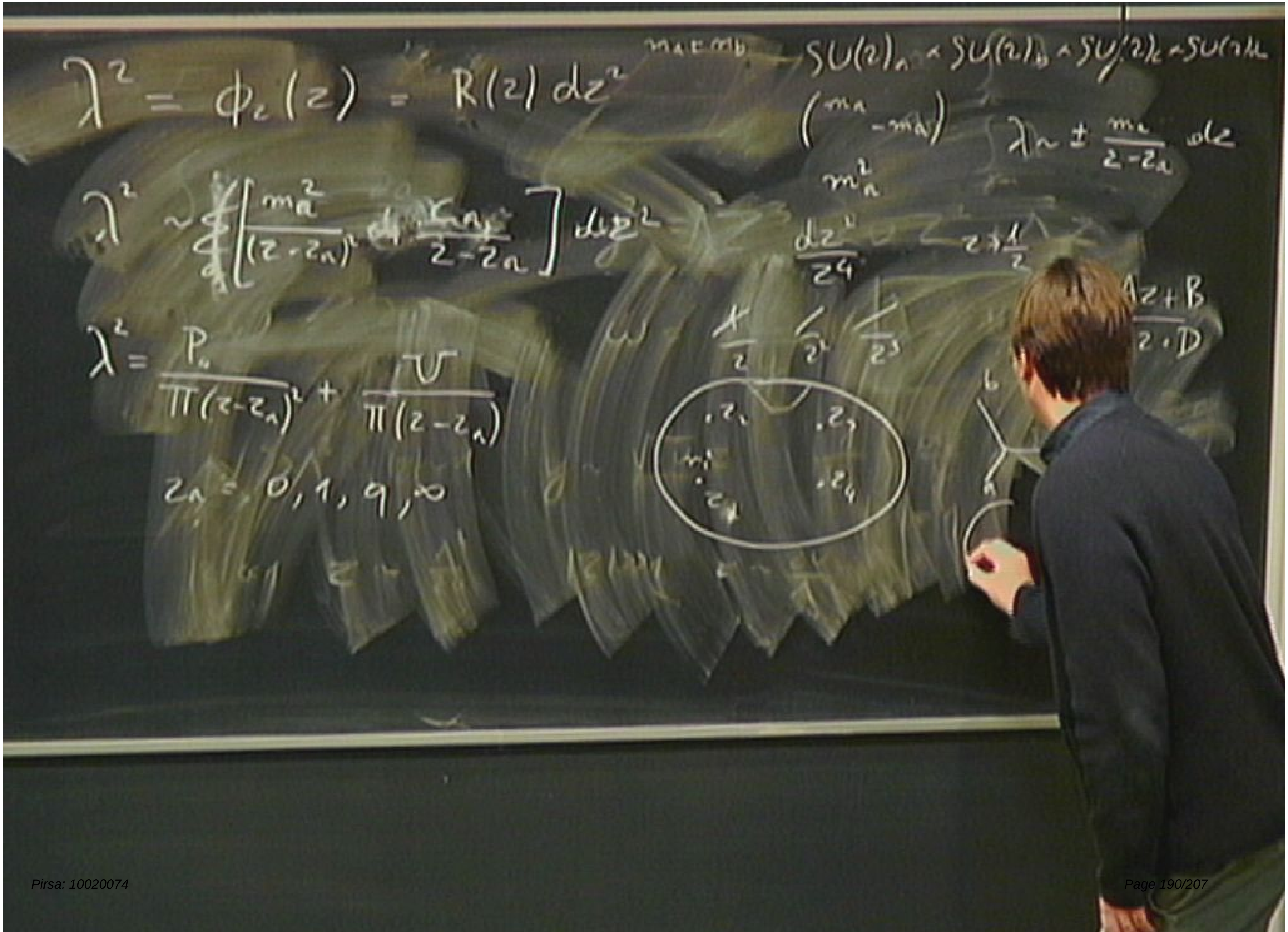
$q \to 0$

$\frac{P_2}{z^2(z-1)^2}$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} + \frac{2U}{z(z-1)}$

$U \overset{q}{=} (m_A \pm m_B)^2$

$U = (m_C \pm m_D)^2$



$$\lambda^2 = \phi_2(z) = R(z)\,dz^2$$

$m_a = m_b$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

$$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$$

$$\lambda \sim \pm \frac{m_a}{z - z_a}\,dz$$

m_a^2

$$\lambda^2 \sim \left[\frac{m_a^2}{(z - z_a)^2} + \frac{c_a}{z - z_a}\right] dz^2$$

$$\frac{dz^2}{z^4} \qquad z \to \frac{1}{z}$$

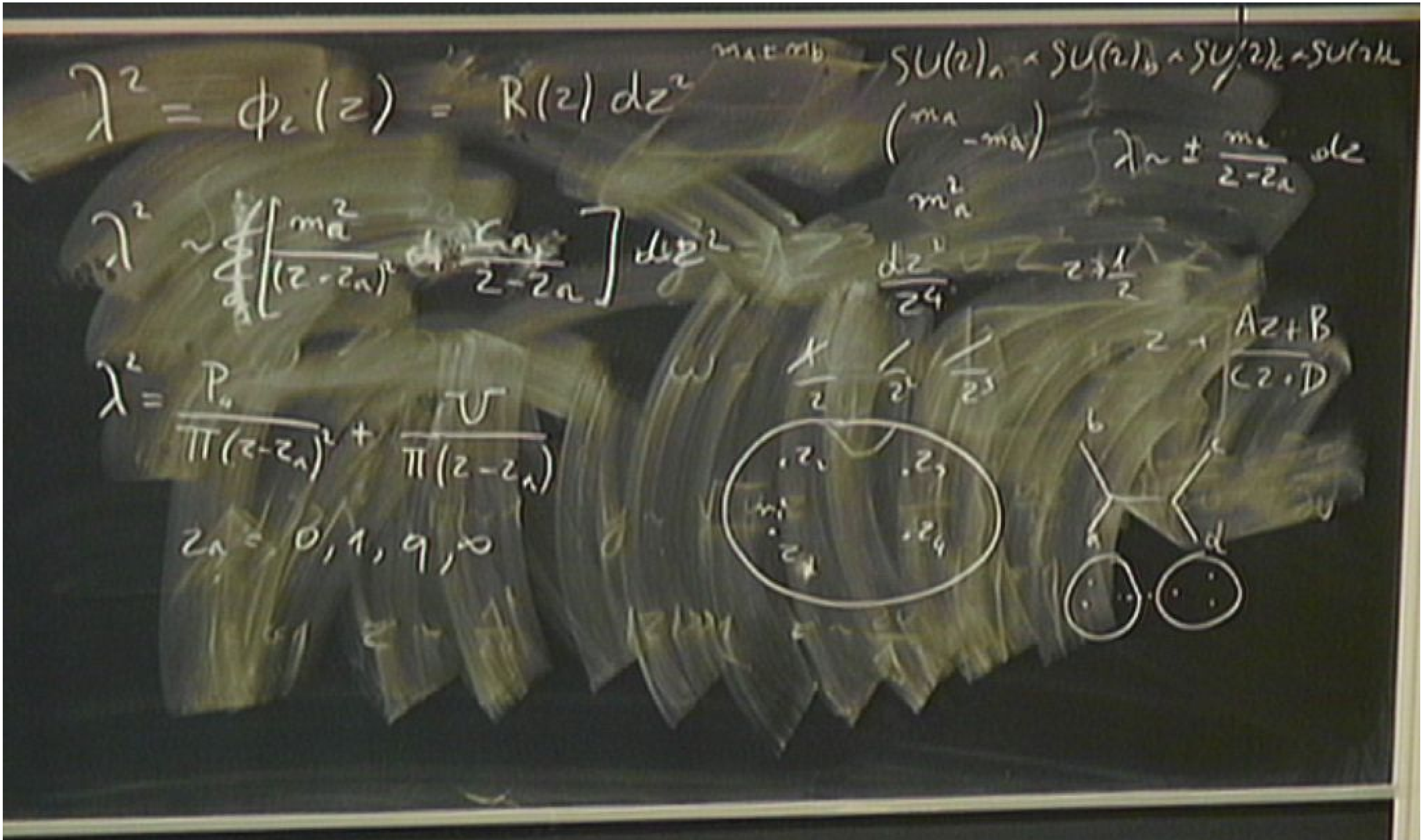
$$\frac{A}{z} \quad \frac{}{z^2} \quad \frac{}{z^3} \qquad \frac{Az + B}{z + D}$$

$$\lambda^2 = \frac{P_4}{\Pi(z - z_a)^2} + \frac{U}{\Pi(z - z_a)}$$

$z_a = 0, 1, q, \infty$

$z_1 \quad z_2 \quad z_3 \quad z_4$

$b \quad a$



$m_a = m_b$ $SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

$\lambda^2 = \phi_2(z) = R(z)\,dz^2$

$\begin{pmatrix} m_a & \\ & -m_a \end{pmatrix}$ $\lambda \sim \pm \frac{m_a}{z-z_a}\,dz$

m_a^2

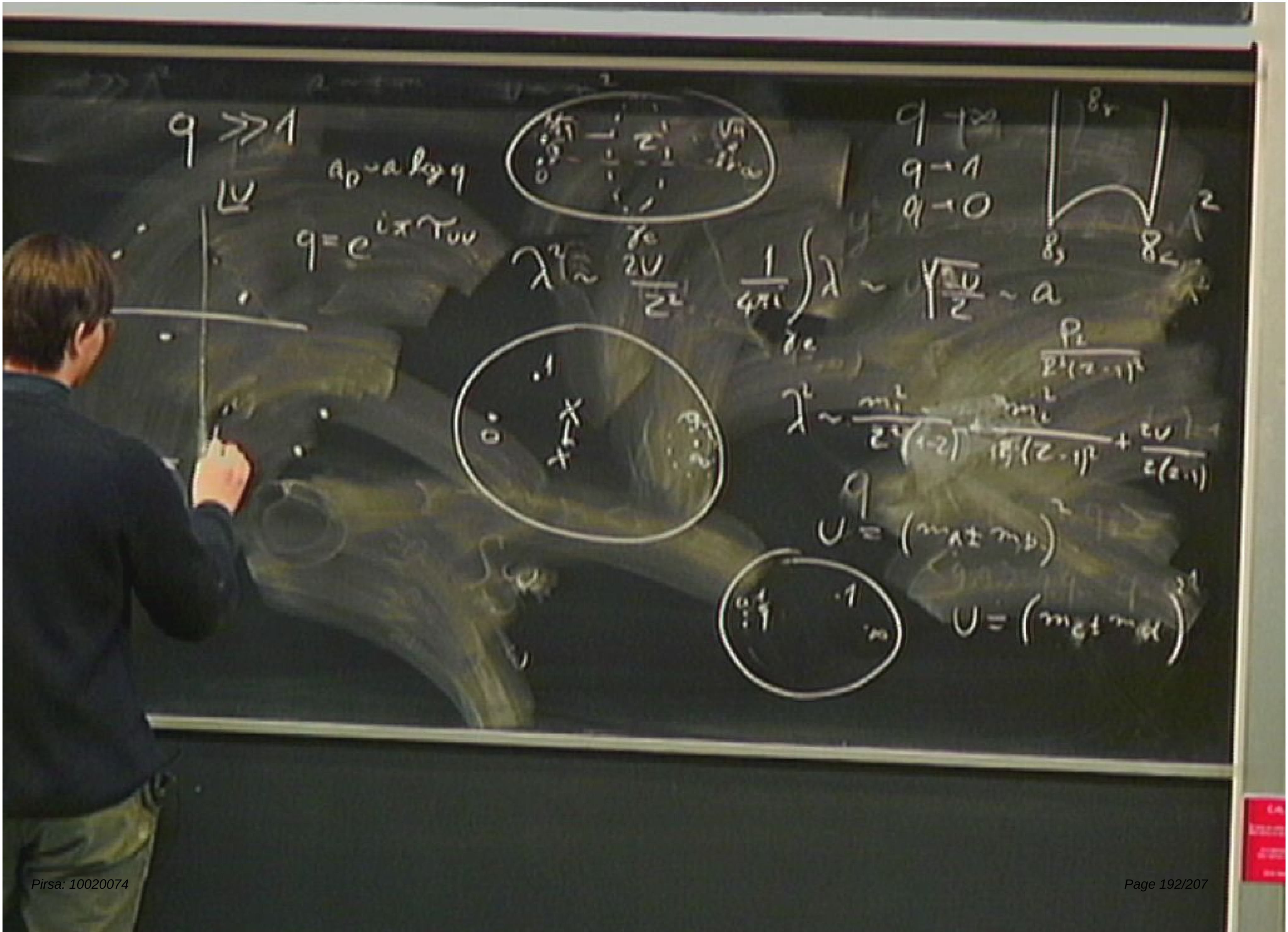
$\lambda^2 \sim \left[\frac{m_a^2}{(z-z_a)^2} + \frac{c_a}{z-z_a}\right] dz^2$

$\frac{dz^2}{z^4}$ $z \to \frac{1}{z}$

$\frac{A}{z}$ $\frac{1}{z^2}$ $\frac{1}{z^3}$ $z \to \frac{Az+B}{Cz+D}$

$\lambda^2 = \frac{P_a}{\prod(z-z_a)^2} + \frac{U}{\prod(z-z_a)}$

$z_a = 0, 1, q, \infty$



$q \gg 1$

$a_D \sim a \log q$

$q = e^{i\pi\tau_{UV}}$

$q \to \infty$

$q \to 1$

$q \to 0$

$\lambda^2 \simeq \frac{2U}{z^2}$

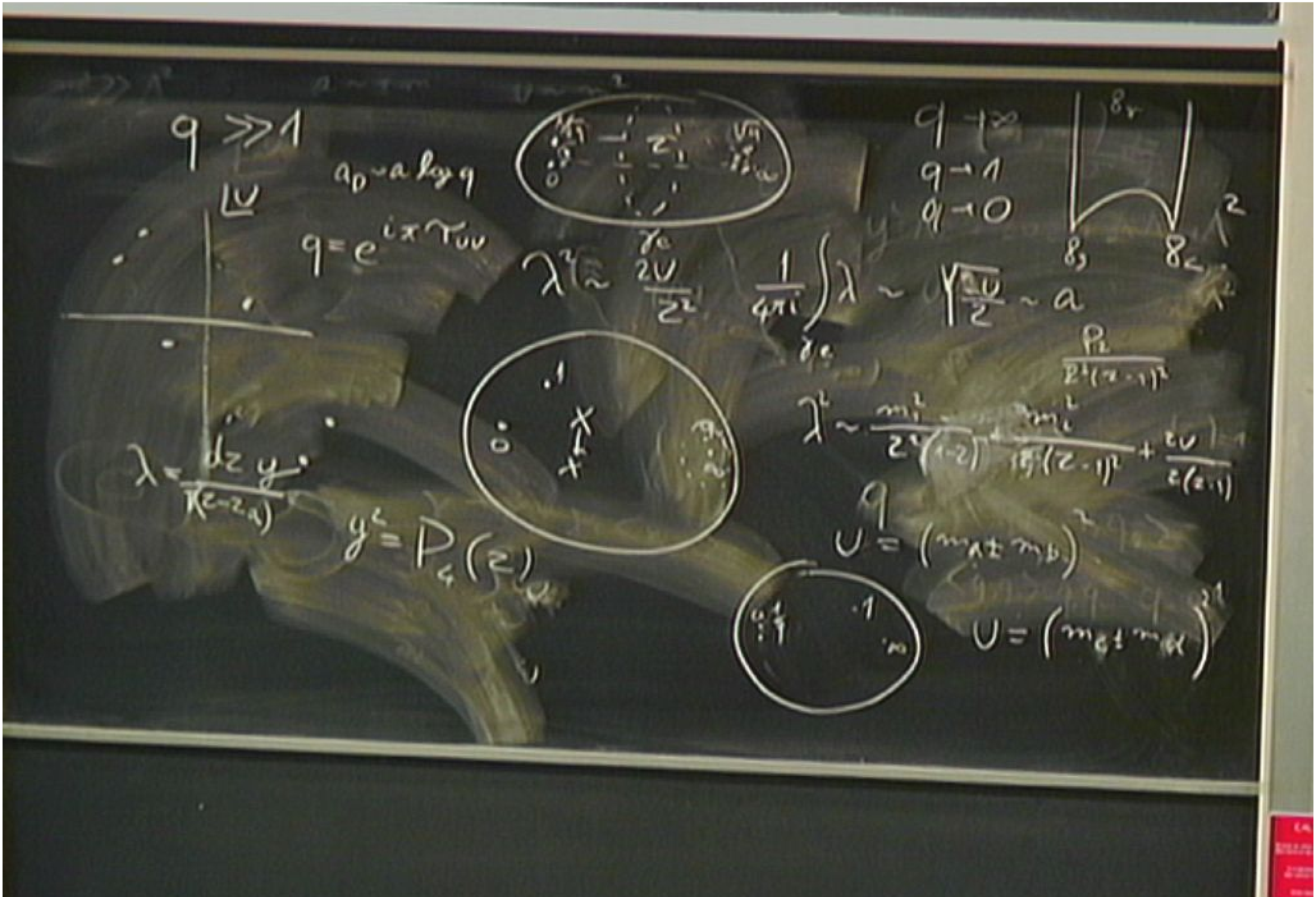
$\frac{1}{4\pi i}\oint_{\gamma_c} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$

$\frac{P_2}{z^2(z-1)^2}$

$\lambda^2 \sim \frac{m_1^2}{z^2(z-2)} + \frac{m_2^2}{(z-1)^2} + \frac{2U}{z(z-1)}$

$U = (m_A \pm m_B)^2$

$U = (m_C \pm m_D)^2$



$q \gg 1$

$a_D \sim a \log q$

LV

$q = e^{i\pi\tau_{UV}}$

$\lambda = \frac{dz\, y}{z(z-z_a)}$

$y^2 = P_4(z)$

$\lambda^2 \approx \frac{2U}{z^2}$

$\frac{1}{4\pi i}\int_{\gamma_c} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$

$q \to \infty$

$q \to 1$

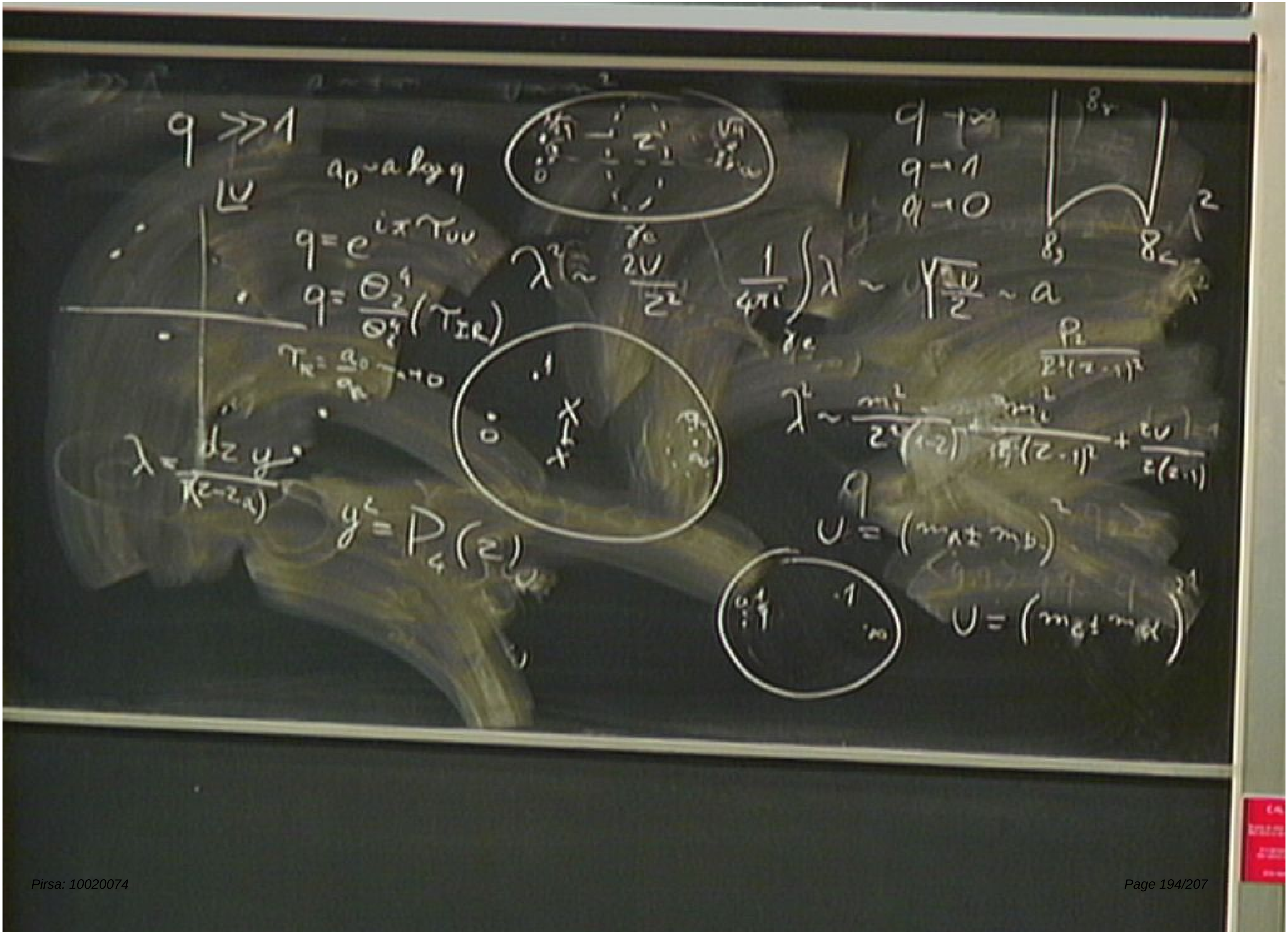
$q \to 0$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} + \frac{2U}{z(z-1)}$

$\frac{P_2}{z^2(z-1)^2}$

$U = (m_A \pm m_B)^2$

$U = (m_C \pm m_D)^2$



$q \gg 1$

$a_D \sim a \log q$

UV

$q = e^{i\pi \tau_{UV}}$

$q = \frac{\Theta_2^4}{\Theta_3^4}(\tau_{IR})$

$\tau_{IR} = \frac{a_D}{a} \to 0$

$\lambda = \frac{dz\, y}{z(z-z_a)}$

$y^2 = P_4(z)$

$\lambda^2 \approx \frac{2U}{z^2}$

$\frac{1}{4\pi i}\int_{\gamma_a} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$

$q \to \infty$

$q \to 1$

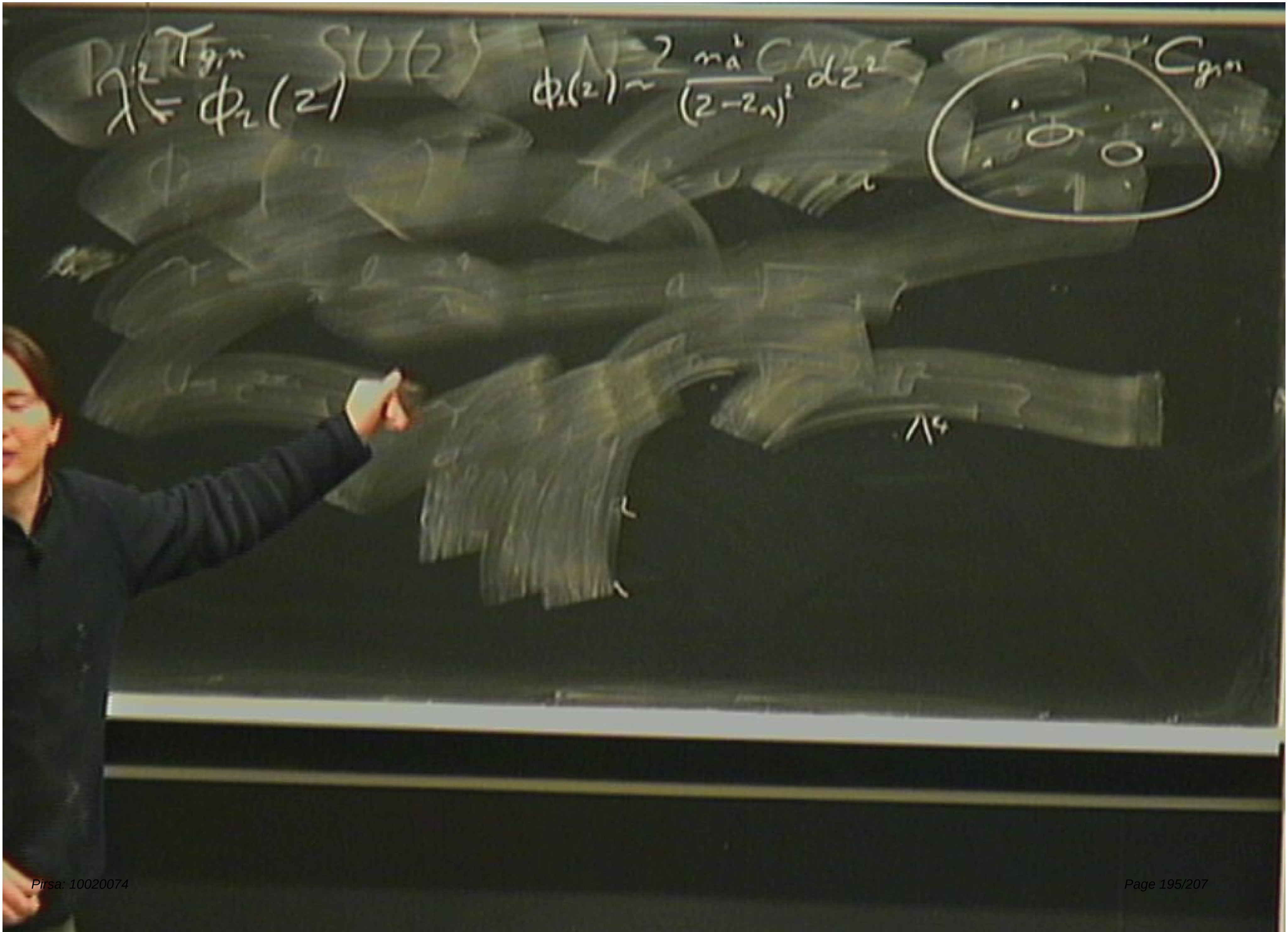
$q \to 0$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{z(z-1)^2} + \frac{2U}{z(z-1)}$

$\frac{P_2}{z^2(z-1)^2}$

$U = (m_a \pm m_b)^2$

$U = (m_c \pm m_d)^2$

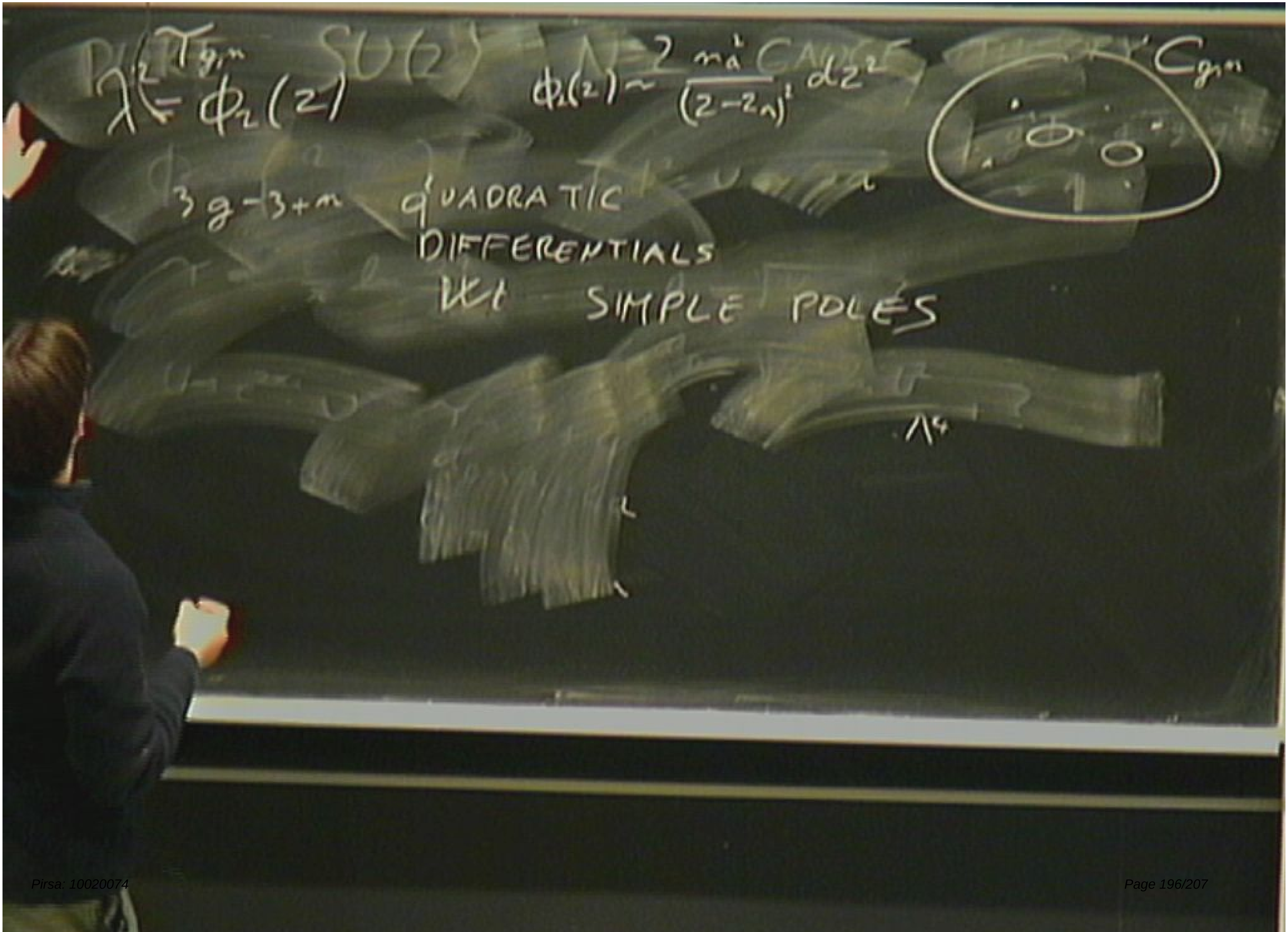


$T_{g,n}$ SU(2) $\mathcal{N}=2$ GAUGE THEORY $C_{g,n}$

$\lambda^2 = \phi_2(z)$

$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$

Λ^4



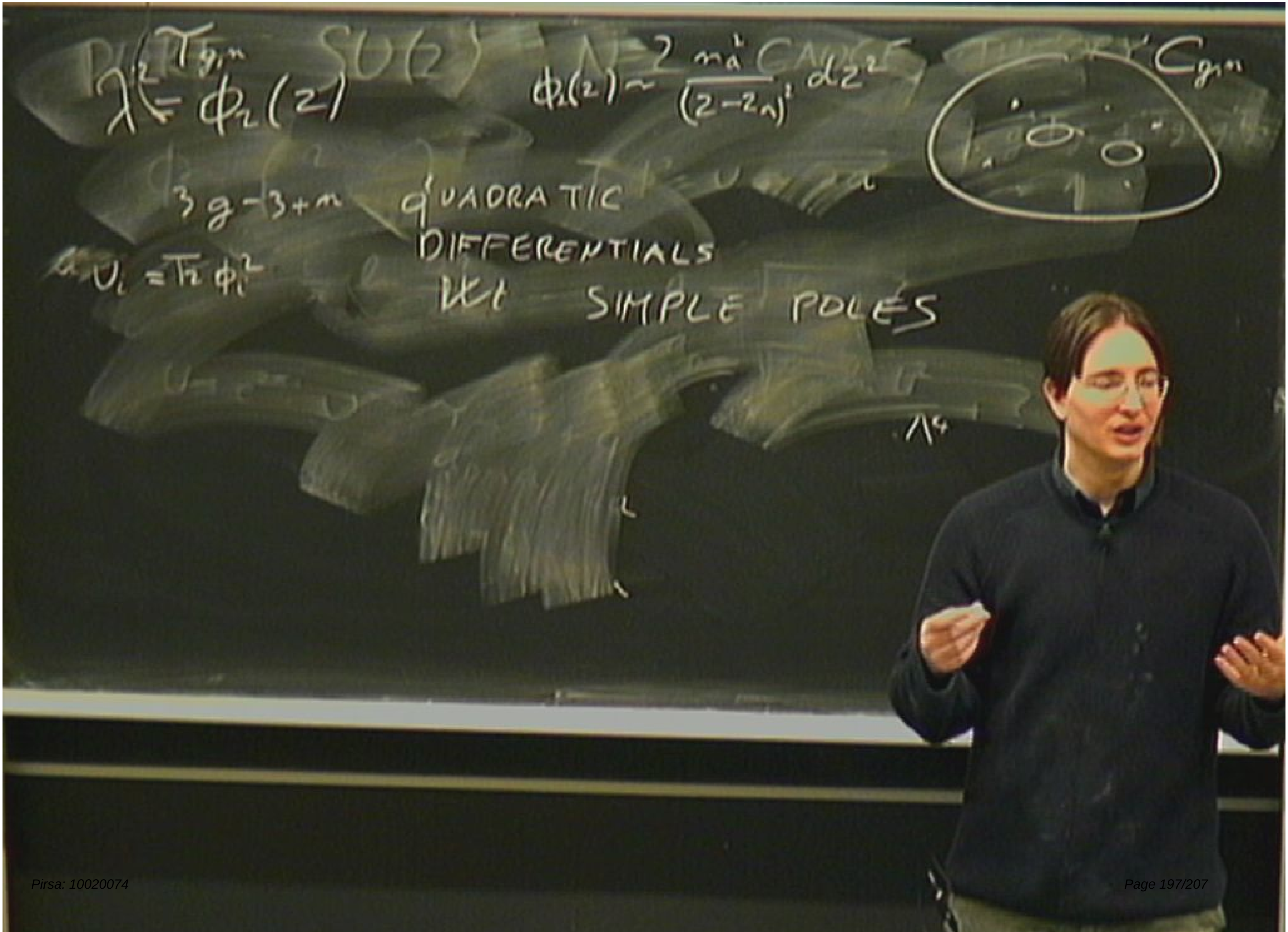
$T_{g,n}$ SU(2) $\mathcal{N}=2$ GAUGE THEORY $C_{g,n}$

$\lambda^2 = \phi_2(z)$

$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$

$3g-3+n$ QUADRATIC DIFFERENTIALS W/ SIMPLE POLES

Λ^4


$T_{g,n}$
$\lambda^2 = \phi_2(z)$
$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$
$C_{g,n}$
$3g-3+n$ QUADRATIC
DIFFERENTIALS
W/ SIMPLE POLES
$u_i = \mathrm{Tr}\,\phi_i^2$
Λ^4

$T_{g,n}$ SU(2) $\mathcal{N}=2$ GAUGE THEORY $C_{g,n}$

$\lambda^2 = \phi_2(z)$

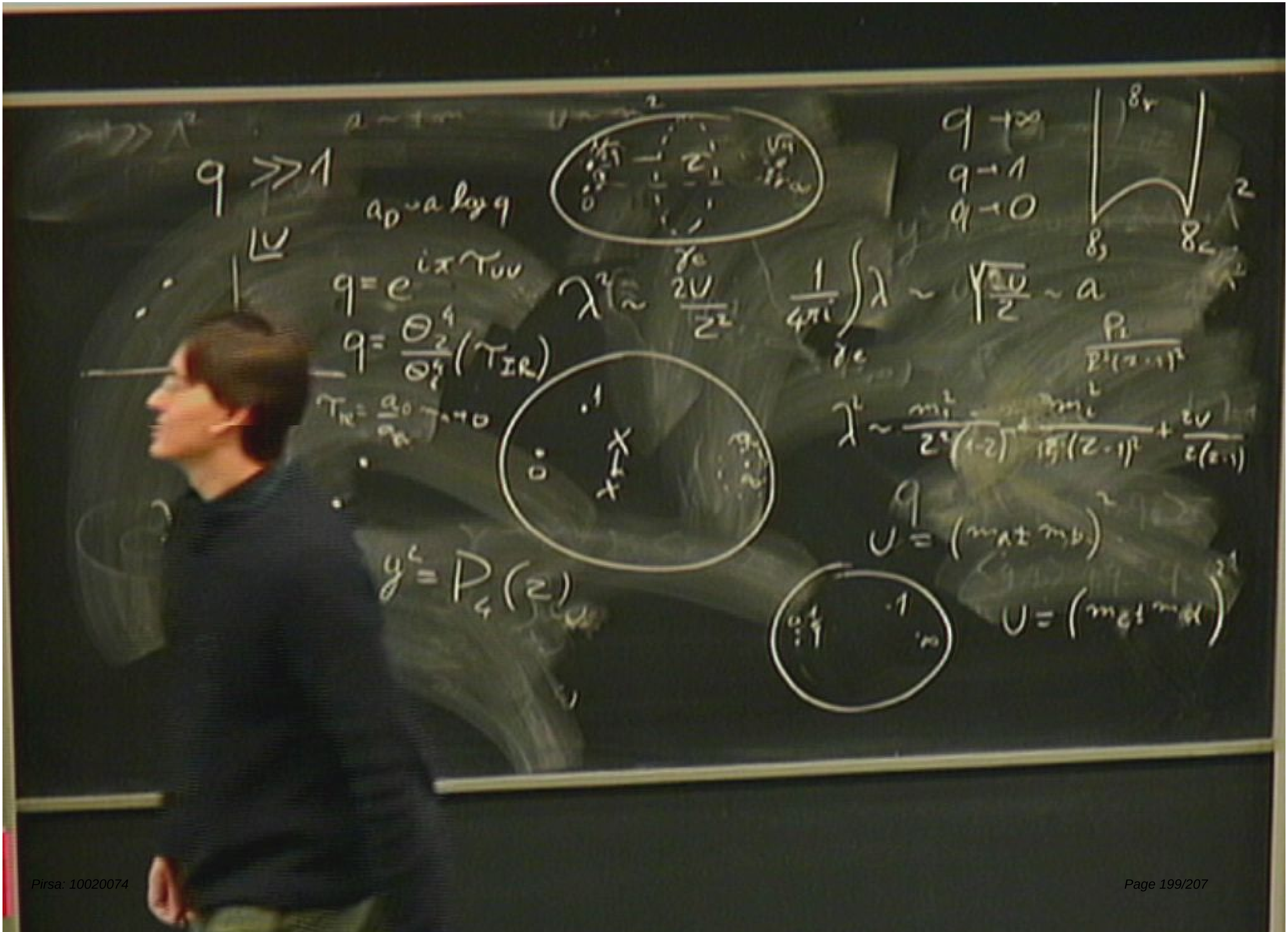
$$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2}\, dz^2$$

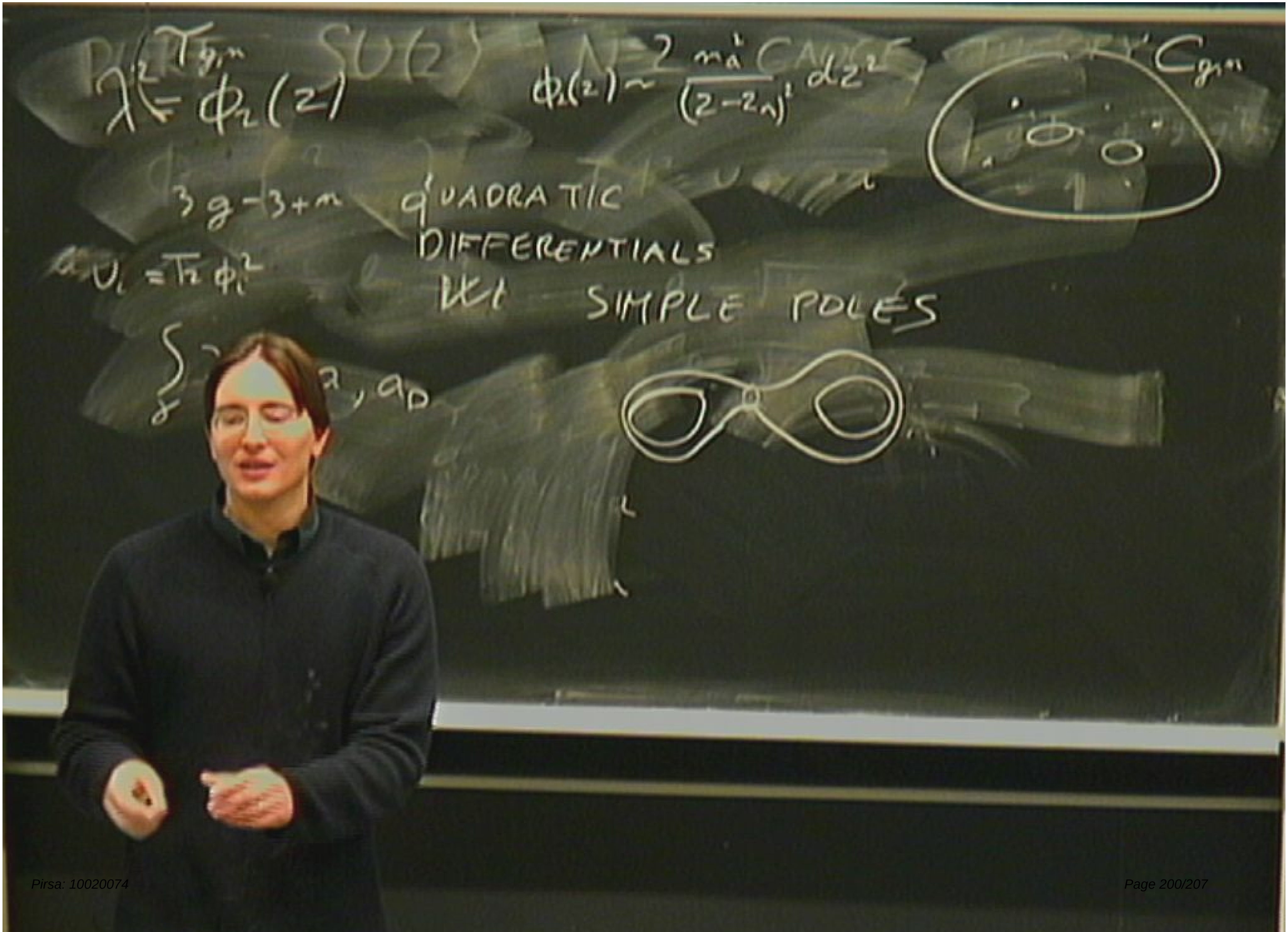
$3g-3+m$ QUADRATIC DIFFERENTIALS

$u_i = \mathrm{Tr}\, \phi_i^2$

w/ SIMPLE POLES

$$\oint_\gamma \lambda = a, a_D$$


q >> 1
a_D ~ a log q
q = e^{iπτ_UV}
q = θ_2^4/θ_4^4 (τ_IR)
λ^2 ~ 2U/z^2
1/4πi ∫ λ ~ √(2U/2) ~ a
q → ∞
q → 1
q → 0
λ^2 ~ m_1^2/z^2(1-z) + m_2^2/(z-1)^2 + 2U/z(z-1)
U = (m_a ± m_b)^2
U = (m_c ± m_d)^2
y^2 = P_4(z)



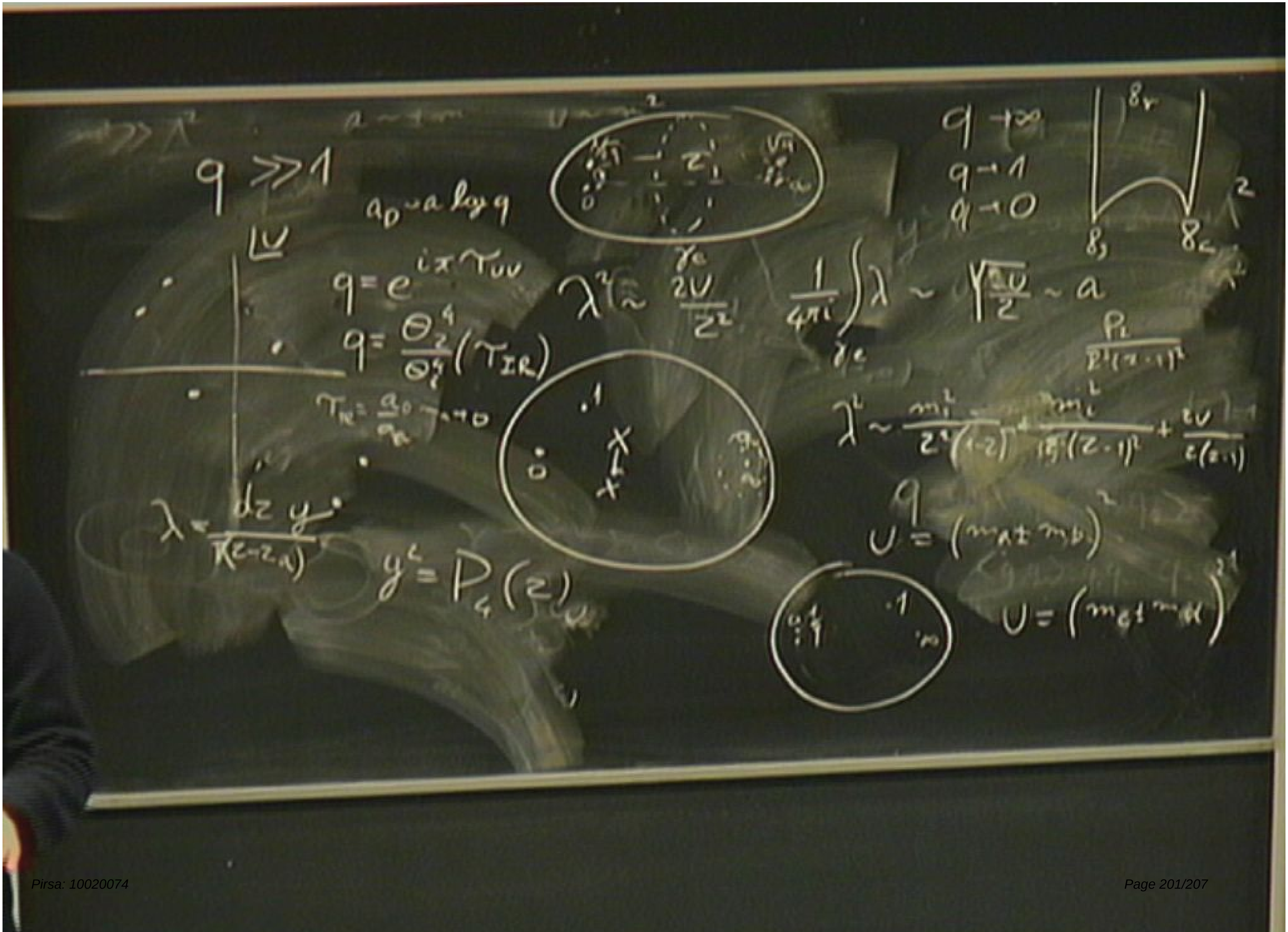
$T_{g,m}$

$\lambda^2 = \phi_2(z)$ $\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$ $C_{g,m}$

$3g-3+m$ QUADRATIC DIFFERENTIALS W/ SIMPLE POLES

$u_i = \mathrm{Tr}\,\phi_i^2$

$\int_\gamma \lambda$ [illegible] a, a_D



$q \gg 1$

$a_D \sim a \log q$

UV

$q = e^{i\pi\tau_{UV}}$

$q = \frac{\theta_2^4}{\theta_3^4}(\tau_{IR})$

$\tau_{IR} = \frac{a_D}{a} \to 0$

$\lambda = \frac{dz\, y}{\prod(z-z_i)}$

$y^2 = P_4(z)$

$\lambda^2 \sim \frac{2U}{z^2}$

γ_e

$\frac{1}{4\pi i}\oint_{\gamma_e} \lambda \sim \sqrt{\frac{2U}{2}} \sim a$

$q \to \infty$

$q \to 1$

$q \to 0$

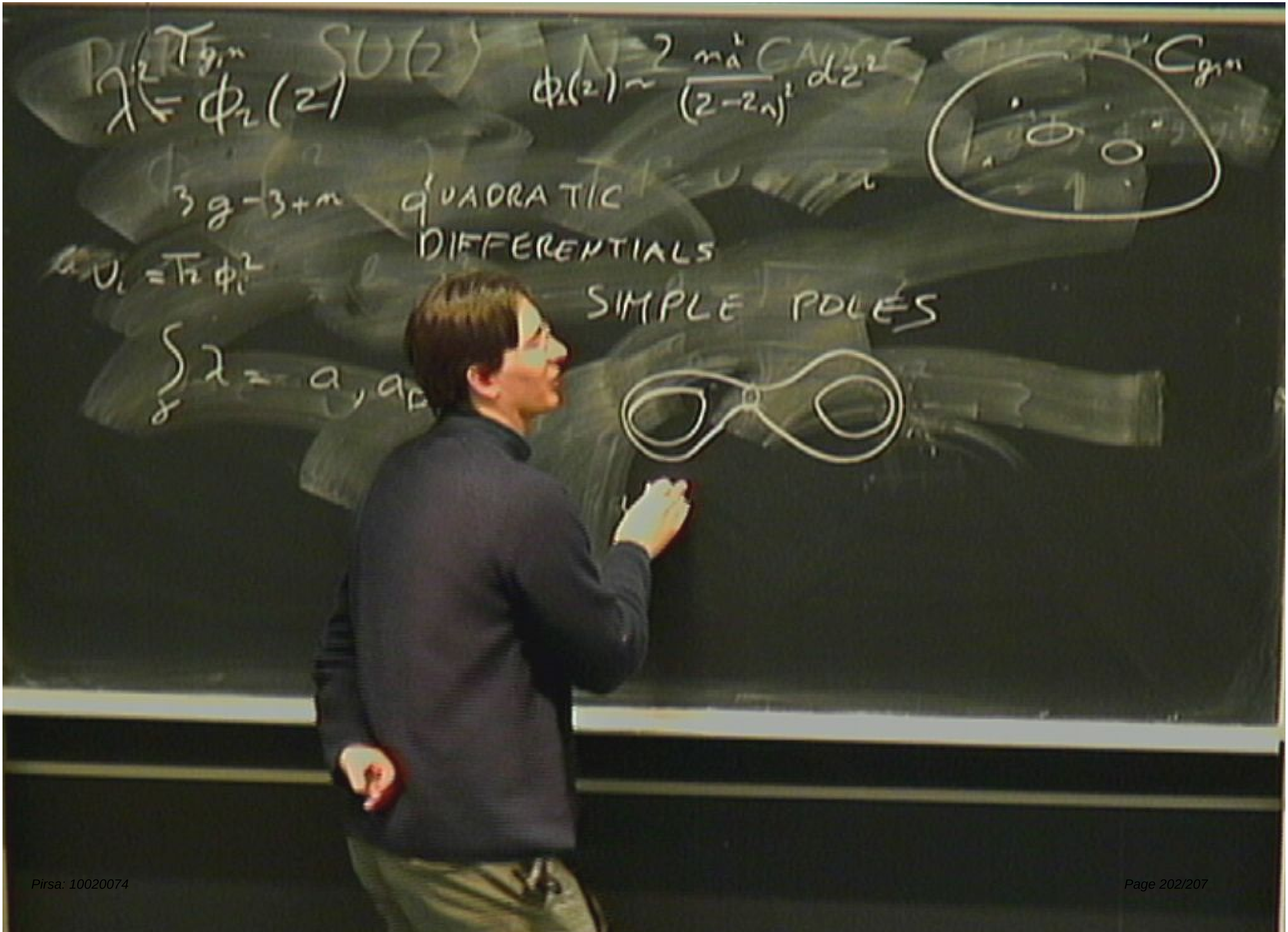
8_v 8_s 8_c

$\frac{P_2}{z^2(z-1)^2}$

$\lambda^2 \sim \frac{m_1^2}{z^2(1-z)} + \frac{m_2^2}{(z-1)^2} + \frac{2U}{z(z-1)}$

$U = (m_a \pm m_b)^2$

$U = (m_c \pm m_d)^2$



$\lambda^2 = \phi_2(z)$

$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$

$C_{g,n}$

$3g-3+m$ QUADRATIC DIFFERENTIALS

$u_i = \mathrm{Tr}\,\phi_i^2$

SIMPLE POLES

$\oint_\gamma \lambda = a, a_D$

$T_{g,n}$ SU(2) $\mathcal{N}=2$ GAUGE THEORY $C_{g,n}$

$\lambda^2 = \phi_2(z)$

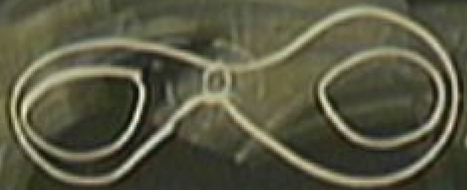
$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2}\, dz^2$

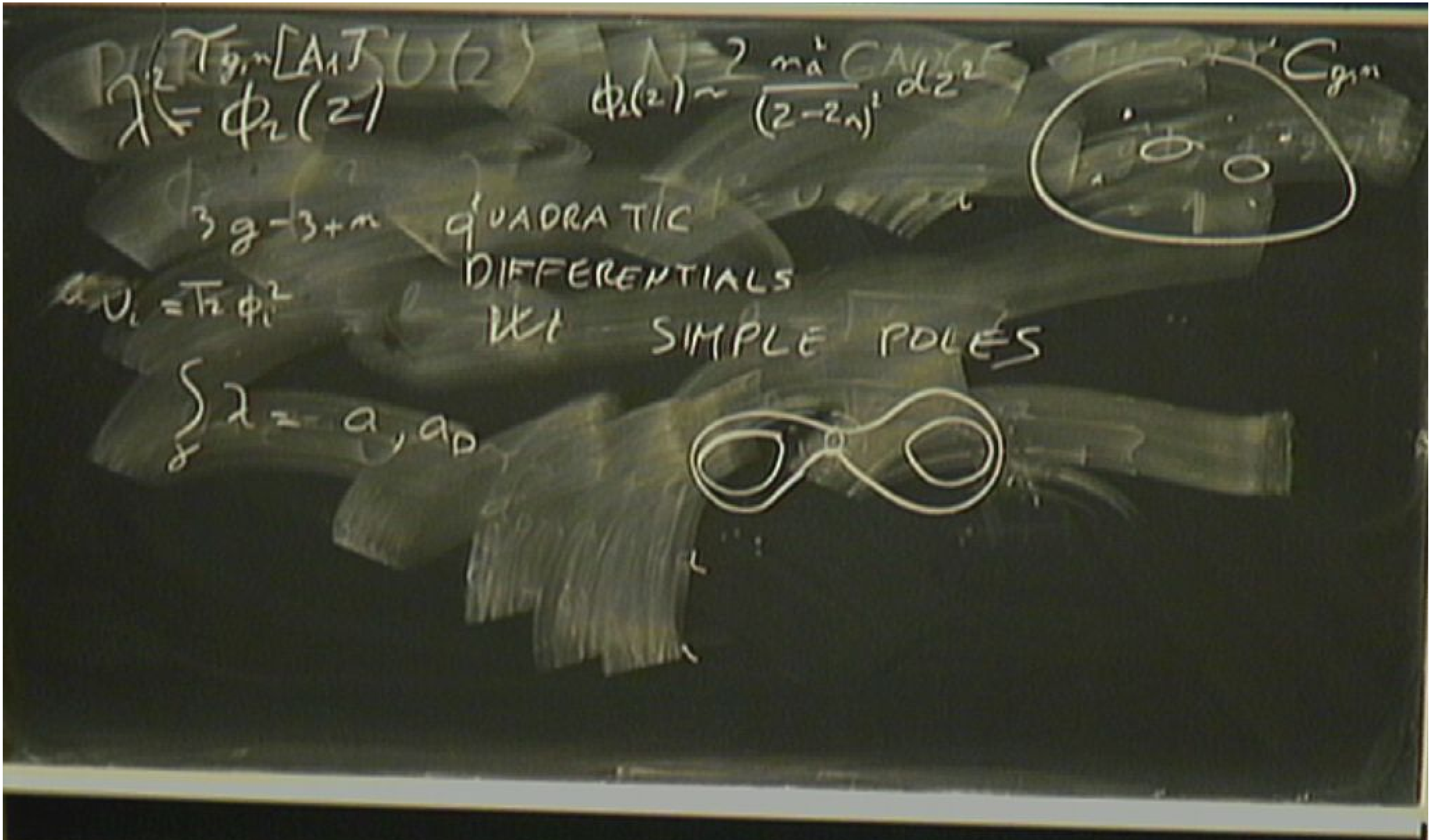
3g-3+n QUADRATIC DIFFERENTIALS

$u_i = \mathrm{Tr}\,\phi_i^2$

W/ SIMPLE POLES

$\oint_\gamma \lambda = a, a_D$





$T_{g,n}[A_1]$ SU(2) $\mathcal{N}=2$ GAUGE THEORY $C_{g,n}$

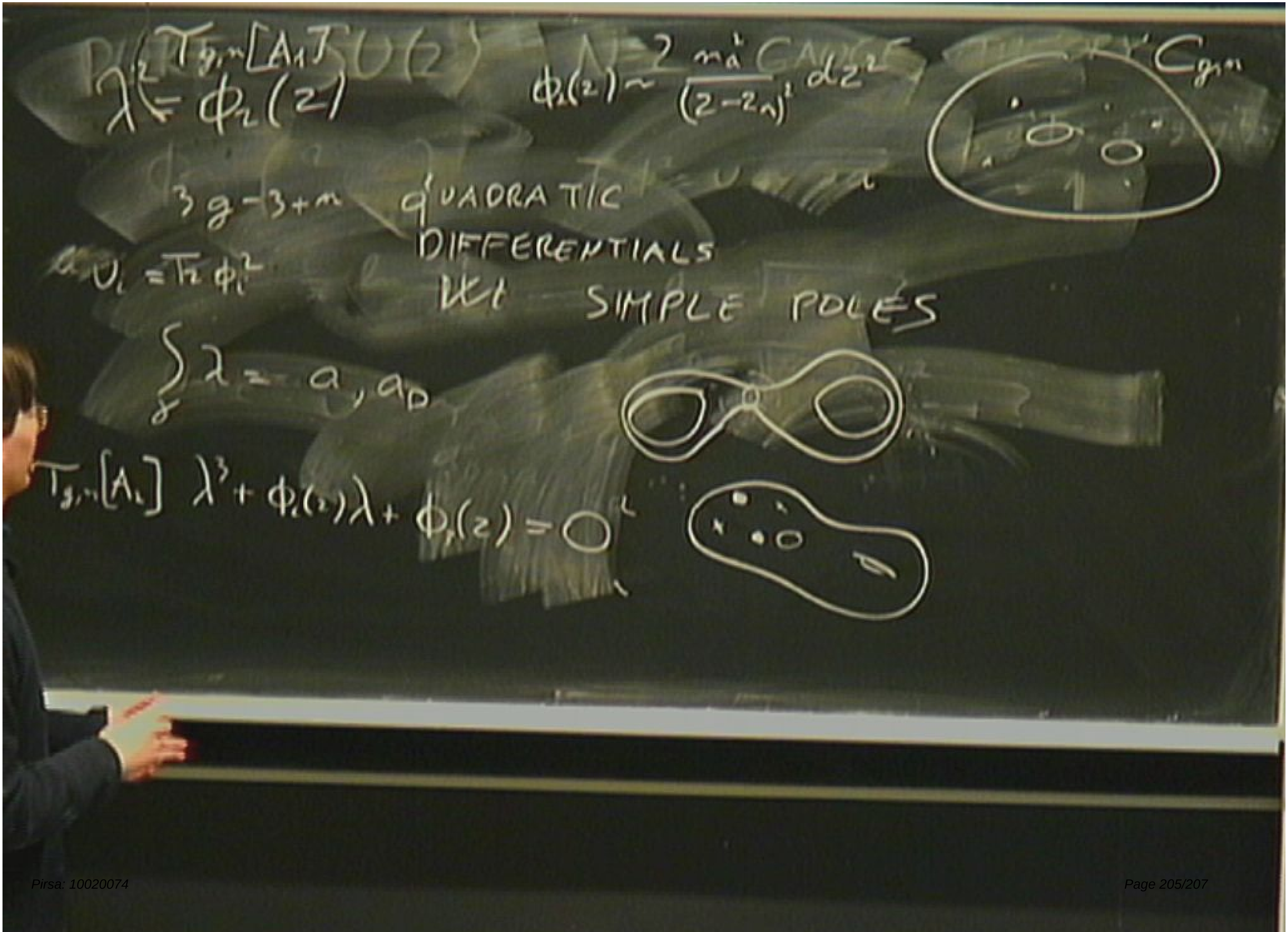
$\lambda^2 = \phi_2(z)$

$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$

$3g-3+n$ QUADRATIC DIFFERENTIALS W/ SIMPLE POLES

$u_i = \mathrm{Tr}\, \phi_i^2$

$\oint_\gamma \lambda = a, a_D$



$T_{g,n}[A_1]$

$\lambda^2 = \phi_2(z)$

$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$

$C_{g,n}$

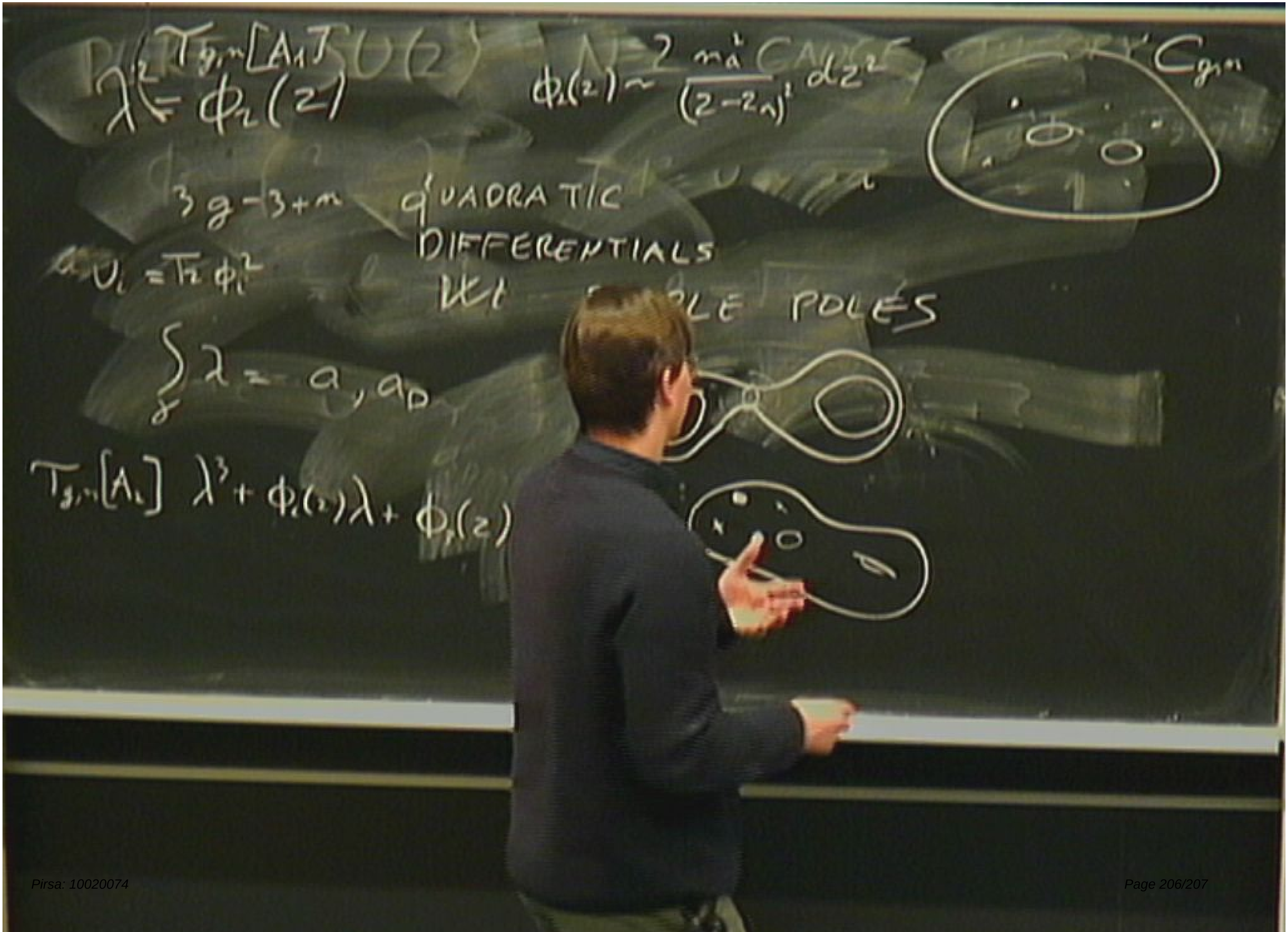
$3g-3+n$ QUADRATIC DIFFERENTIALS

$u_i = \mathrm{Tr}\,\phi_i^2$

Wt SIMPLE POLES

$\oint_\gamma \lambda = a, a_D$

$T_{g,n}[A_2]$ $\lambda^3 + \phi_2(z)\lambda + \phi_3(z) = 0$



$T_{g,m}[A_1]$

$\lambda^2 = \phi_2(z)$

$\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$

$C_{g,m}$

$3g-3+m$ QUADRATIC DIFFERENTIALS Wt LE POLES

$u_i = \mathrm{Tr}\, \phi_i^2$

$\int_\gamma \lambda = a, a_D$

$T_{g,m}[A_2]$ $\lambda^3 + \phi_2(z)\lambda + \phi_3(z)$

$(T_{g,n}[A_1])$ $U(2)$ $N=2$ GAUGE $C_{g,n}$

$\lambda^2 = \phi_2(z)$ $\phi_2(z) \sim \frac{m_a^2}{(z-z_a)^2} dz^2$

$3g-3+m$ QUADRATIC DIFFERENTIALS

$u_i = \mathrm{Tr}\, \phi_i^2$ W/ SIMPLE POLES

$\oint_\gamma \lambda = a, a_D$

$T_{g,n}[A_2]$ $\lambda^3 + \phi_2(z)\lambda + \phi_3(z) = 0$