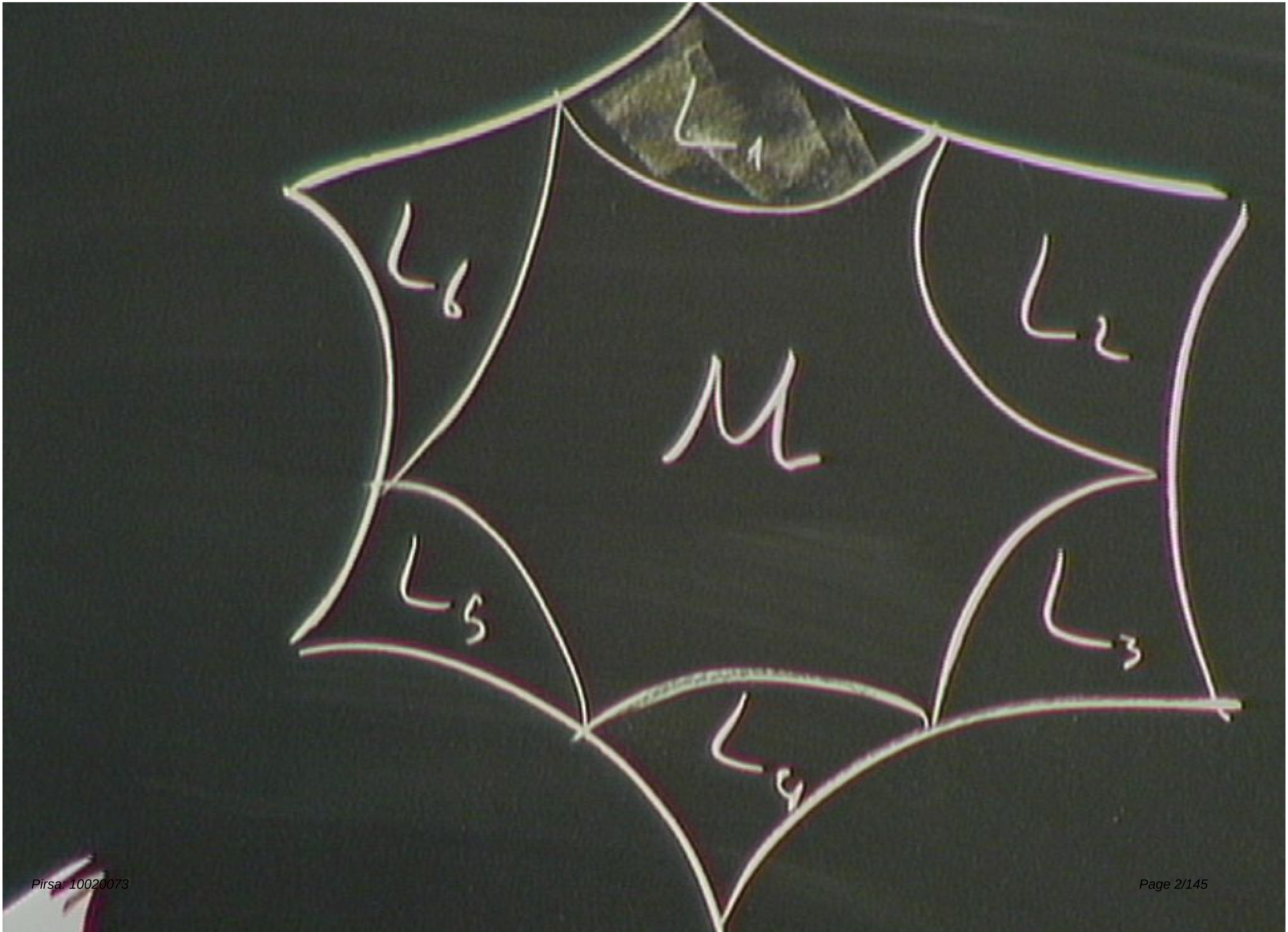
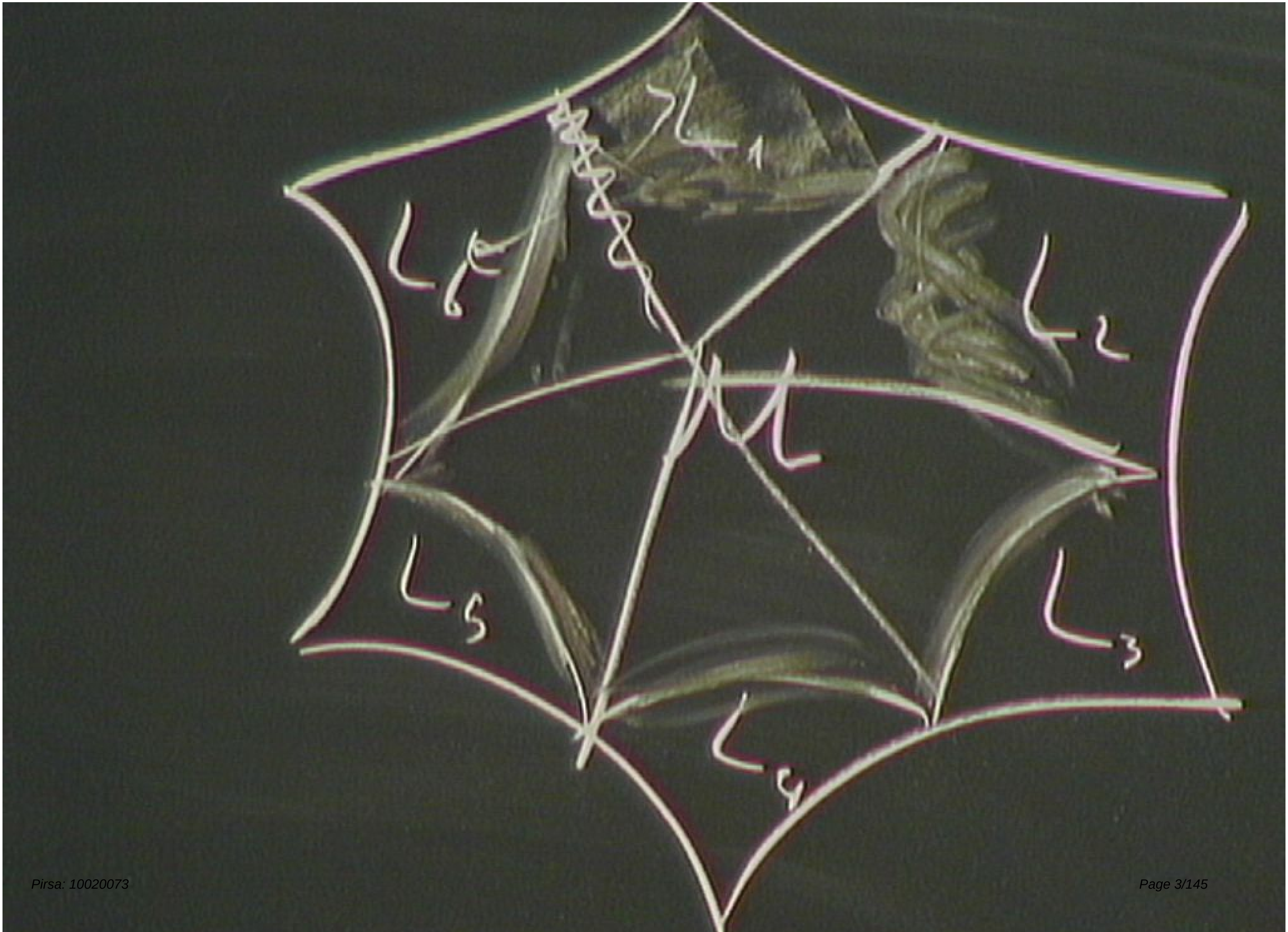
Title: Lecture 1: S-duality

Date: Feb 22, 2010 11:15 AM

URL: http://pirsa.org/10020073

Abstract:


L_1
L_2
L_3
L_4
L_5
L_6
M



L_1
L_2
L_3
L_4
L_5
L_6
M

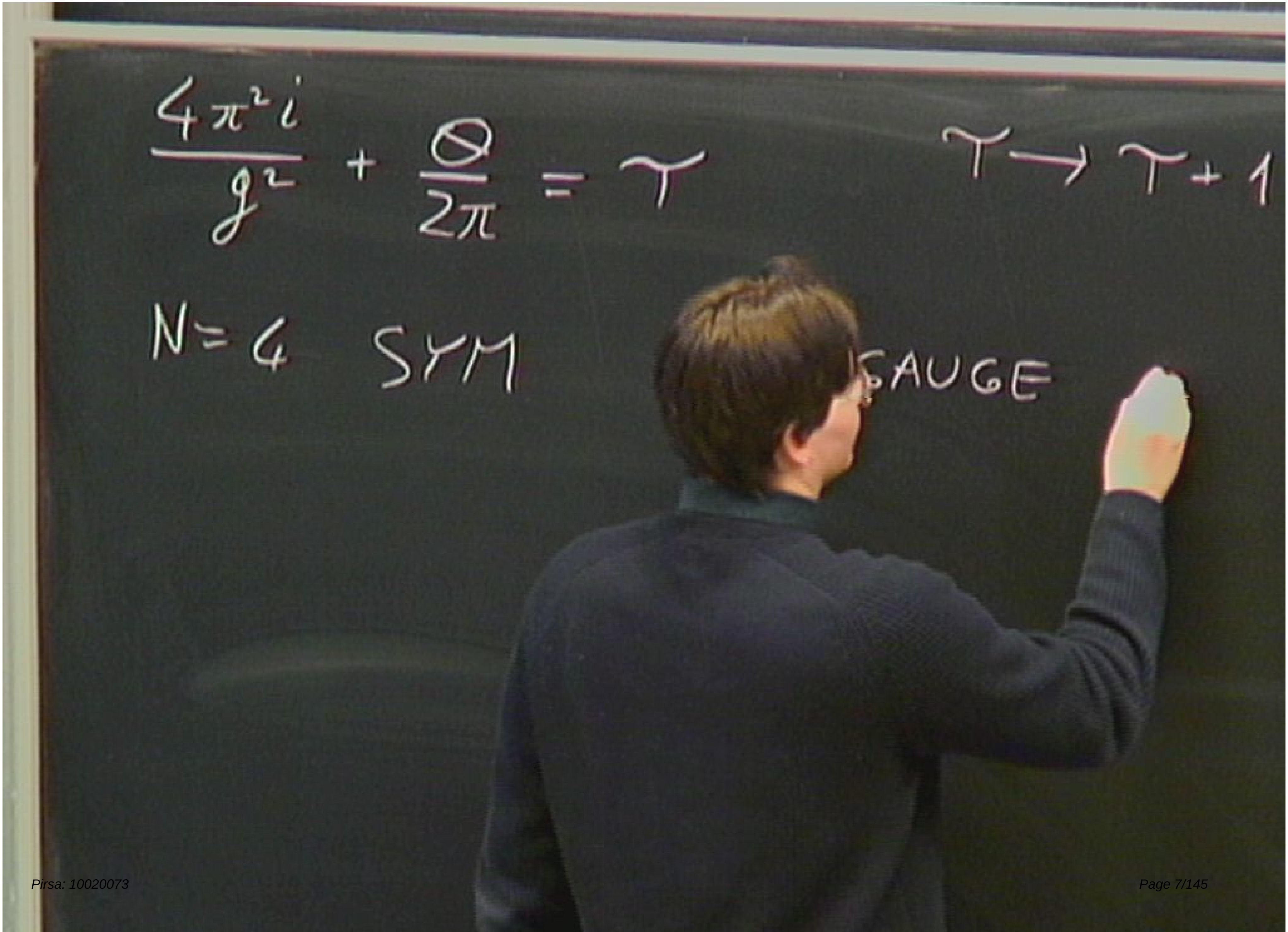
$$\frac{1}{g^2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \theta\, \mathrm{Tr}\, F \wedge F$$

$$\frac{1}{g^2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \Theta\, \mathrm{Tr}\, F \wedge F$$

$$\Theta \to \Theta + 2\pi$$

$$\frac{4\pi^2 i}{g^2} + \frac{\theta}{2\pi} = \tau$$

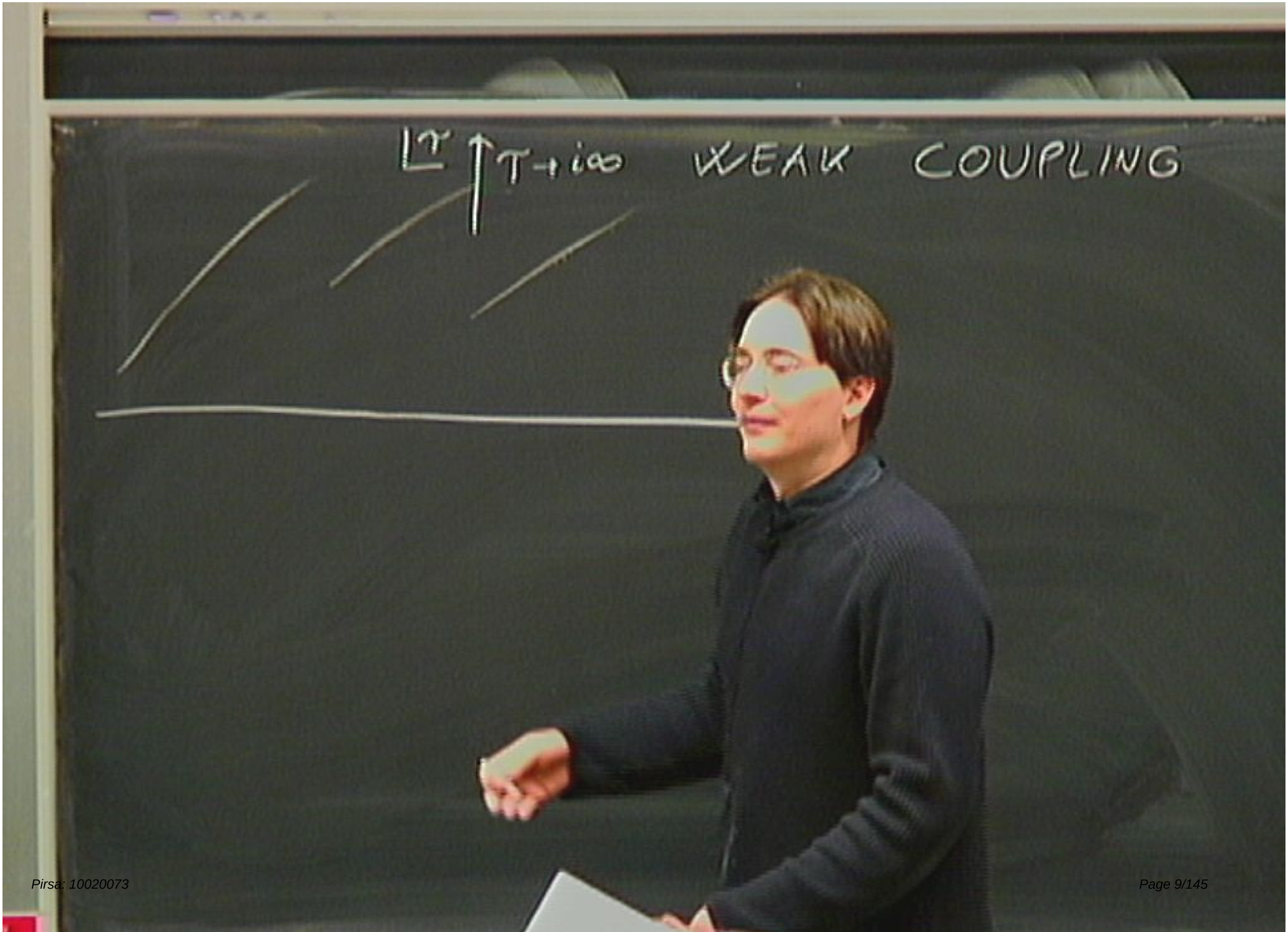
$N =$

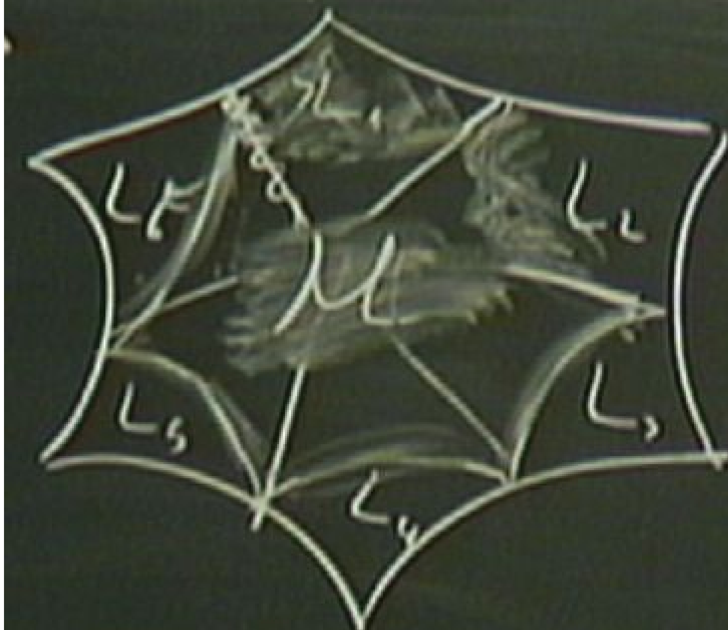

$\frac{4\pi^2 i}{g^2} + \frac{\theta}{2\pi} = \tau$
$\tau \to \tau + 1$
$N=4$ SYM
GAUGE

$$\frac{4\pi^2 i}{g^2} + \frac{\theta}{2\pi} = \tau \qquad \tau \to \tau + 1$$

$N=4$ SYM ADE GAUGE GROUP

S:


WEAK COUPLING



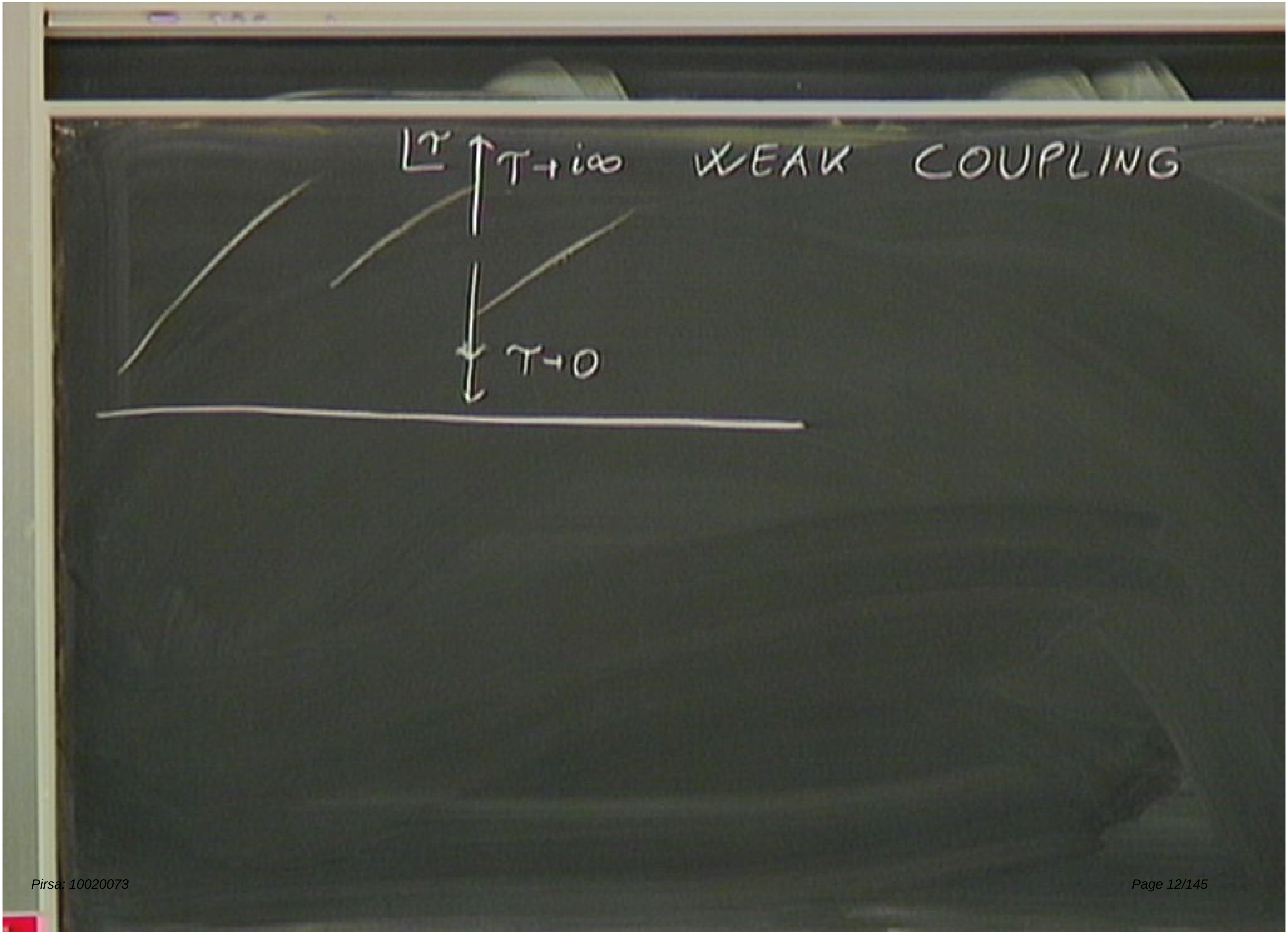
$$S = \int \frac{1}{g^2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \theta\, \mathrm{Tr}\, F \wedge F + \cdots$$

$$\theta \to \theta + 2\pi$$

$$\int e^{iS}$$

L^{τ} $\uparrow \tau \to i\infty$ WEAK COUPLING

$\downarrow \tau \to 0$


$\mathrm{Im}\,\tau$
$\tau \to i\infty$ WEAK COUPLING
$\tau \to 0$

$$\frac{4\pi^2 i}{g^2} + \frac{\Theta}{2\pi} = \tau \qquad \tau \to \tau + 1$$

$N=4$ SYM ADE GAUGE GROUP

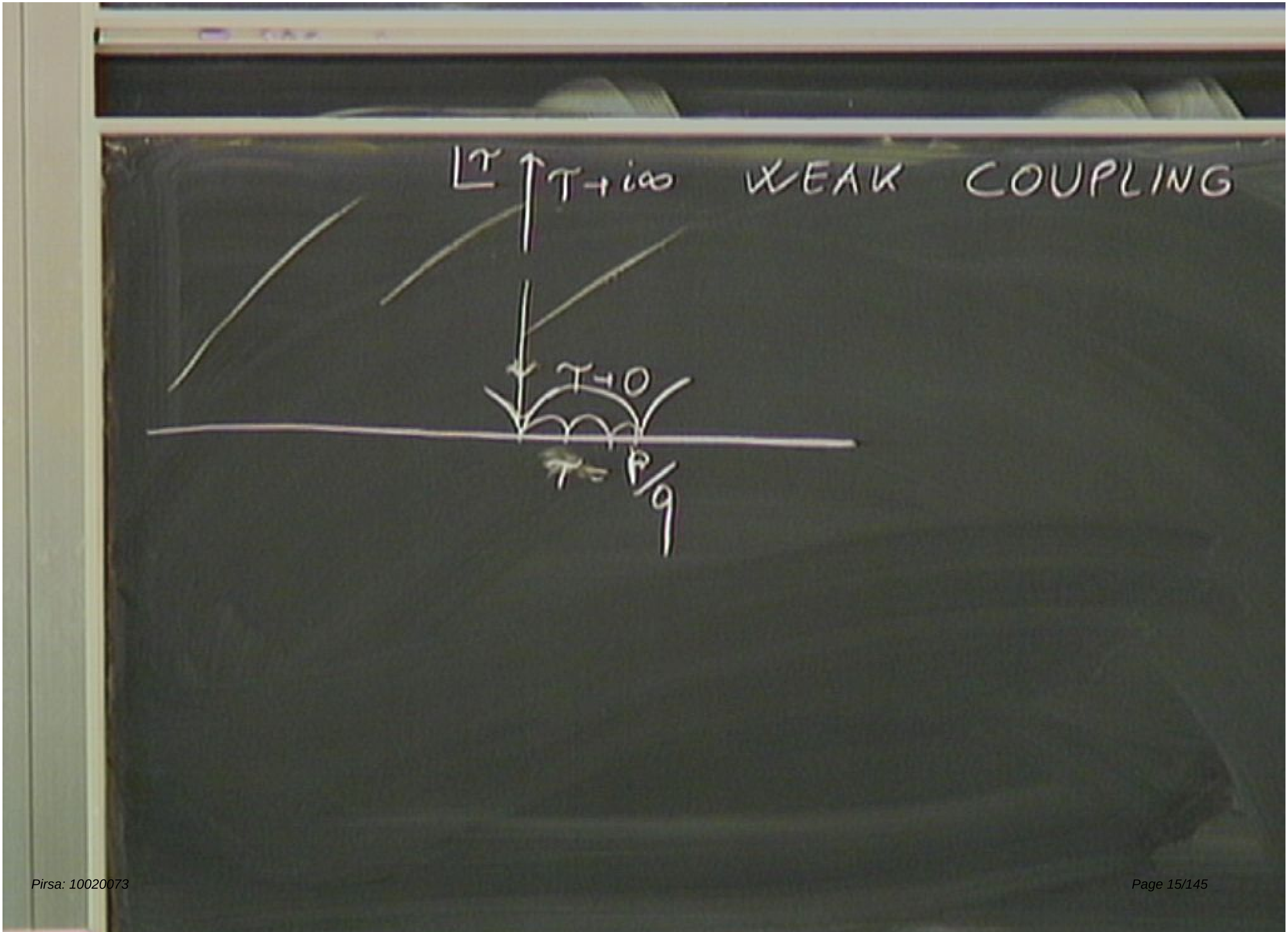
$$S: \quad \tau \to -\frac{1}{\tau}$$

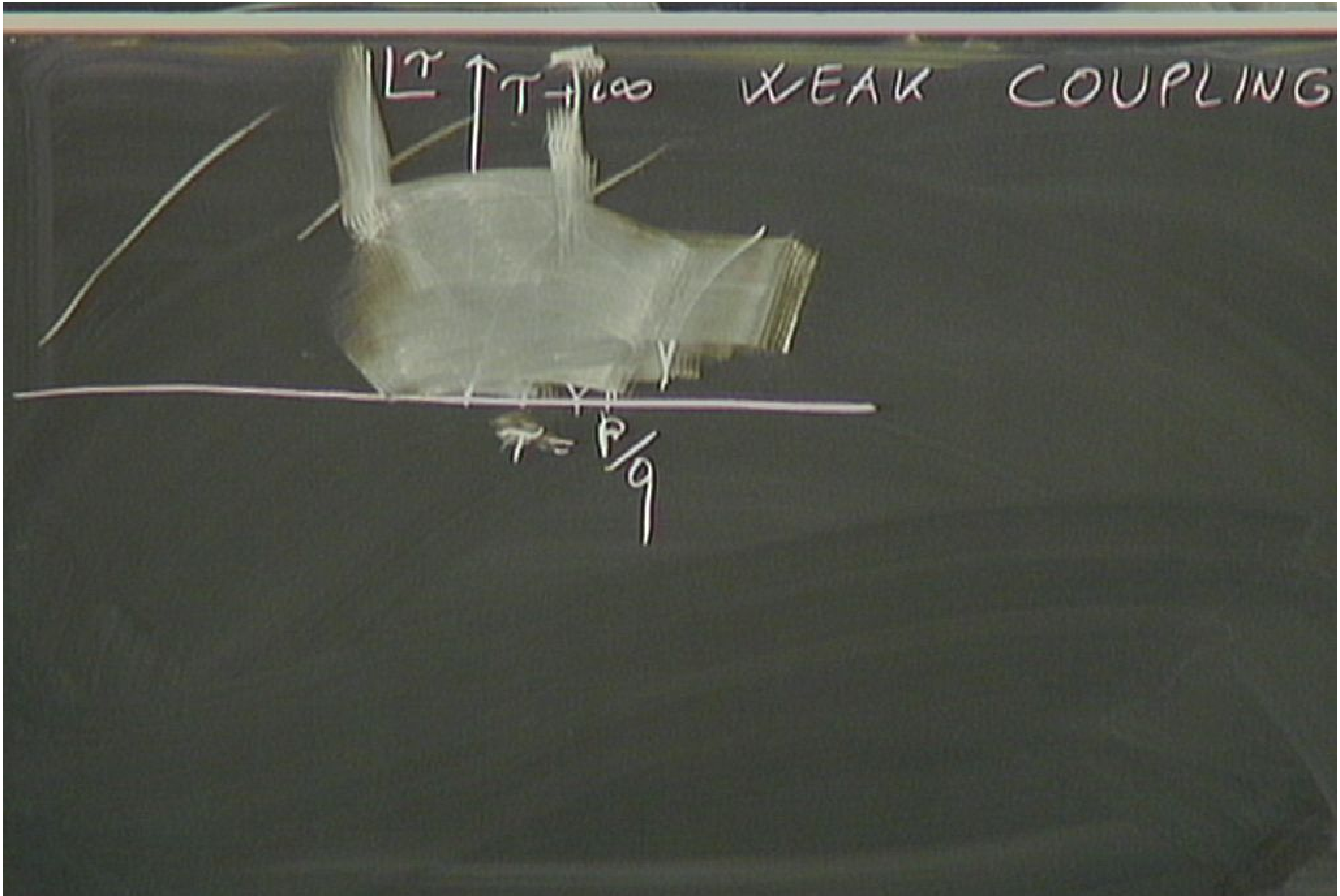
$$\frac{4\pi^2 i}{g^2} + \frac{\theta}{2\pi} = \tau \qquad T: \tau \to \tau + 1$$

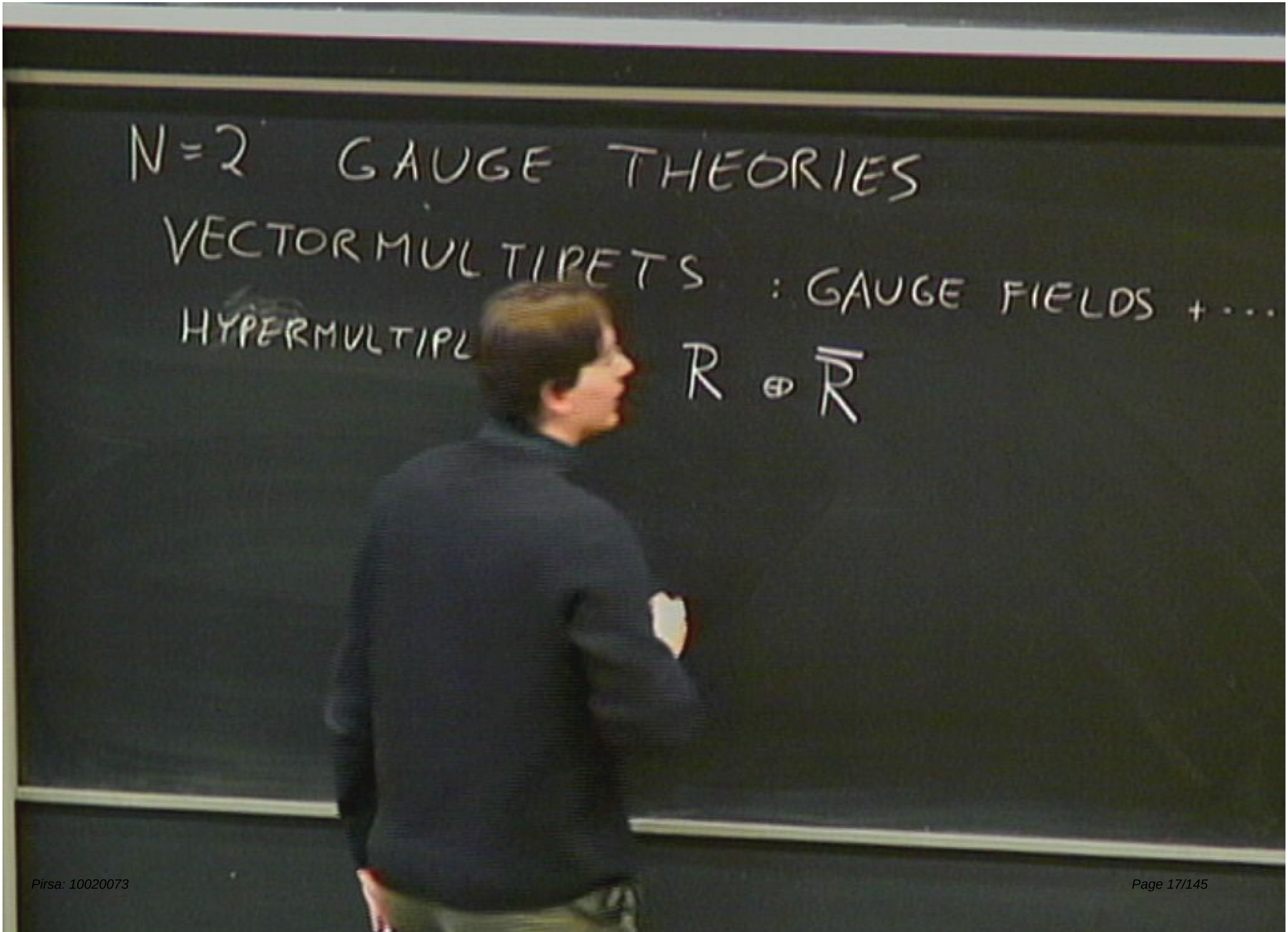
N=4 SYM ADE GAUGE GROUP

$$S: \tau \to -\frac{1}{\tau}$$

$$SL(2,\mathbb{Z}) \qquad \tau \to \frac{a\tau + b}{c\tau + d}$$


τ → i∞
WEAK COUPLING
τ → 0
τ = p/q


L^τ
$\tau \to i\infty$
WEAK COUPLING
$\tau = p/q$


N=2 GAUGE THEORIES
VECTORMULTIPETS : GAUGE FIELDS + ...
HYPERMULTIPL
$R \oplus \bar{R}$

N=2 GAUGE THEORIES

VECTORMULTIPETS : GAUGE FIELDS + ...

HYPERMULTIPLETS : $R \oplus \overline{R}$

N=2 GAUGE THEORIES

VECTORMULTIPETS : GAUGE FIELDS + . . .

HYPERMULTIPLETS : $R \oplus \overline{R}$

GAUGE COUPLINGS τ_i

$N=2$ GAUGE THEORIES

VECTORMULTIPETS : GAUGE FIELDS + ...

HYPERMULTIPLETS : $R \oplus \bar{R}$

GAUGE COUPLINGS τ_i

$R = \oplus \mathbb{C}^{n_i}$ $\quad \prod U(n_i)$

N=2 GAUGE THEORIES

VECTORMULTIPETS : GAUGE FIELDS + ...

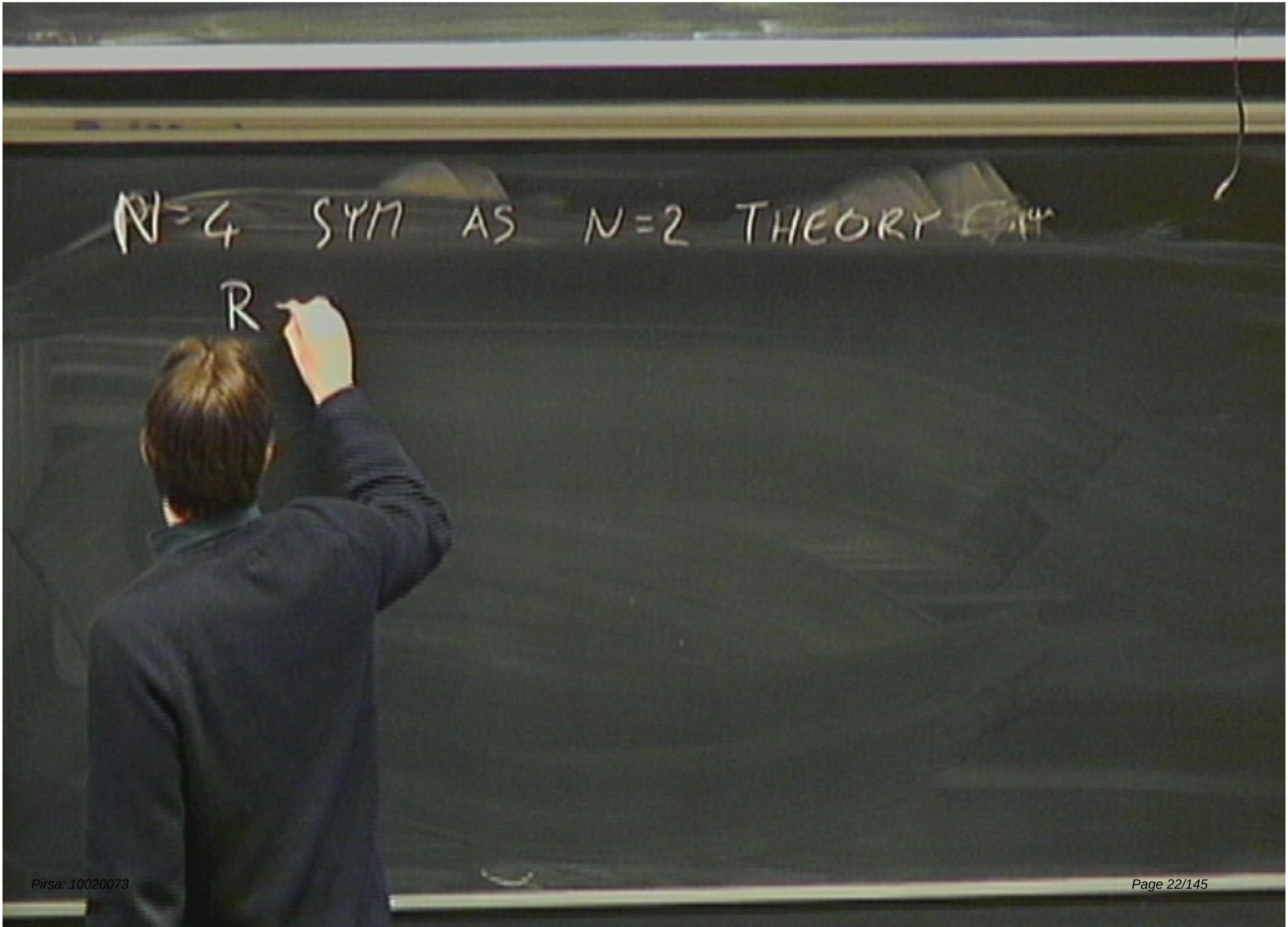
HYPERMULTIPLETS : $R \oplus \overline{R}$

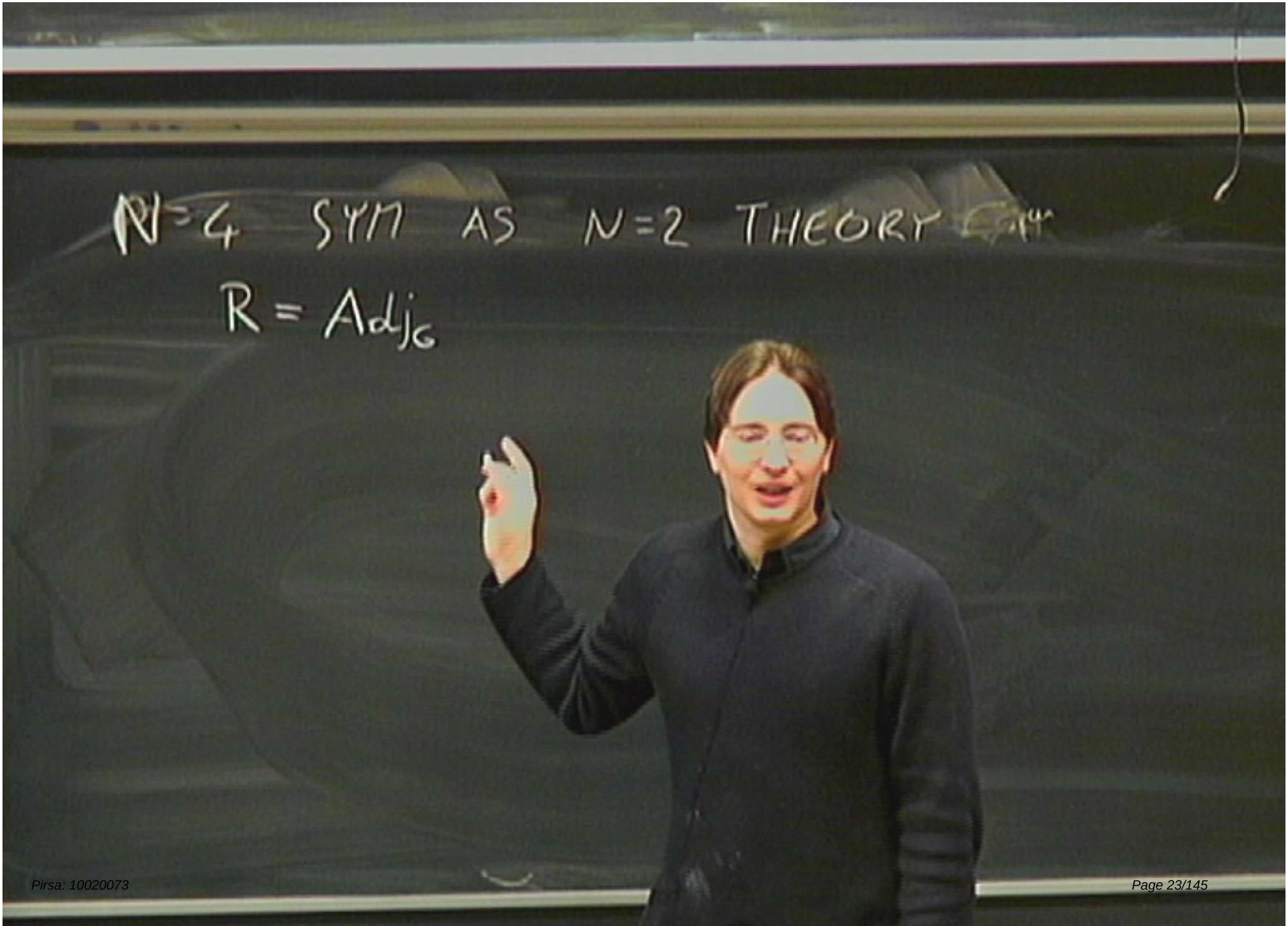
GAUGE COUPLINGS τ_i

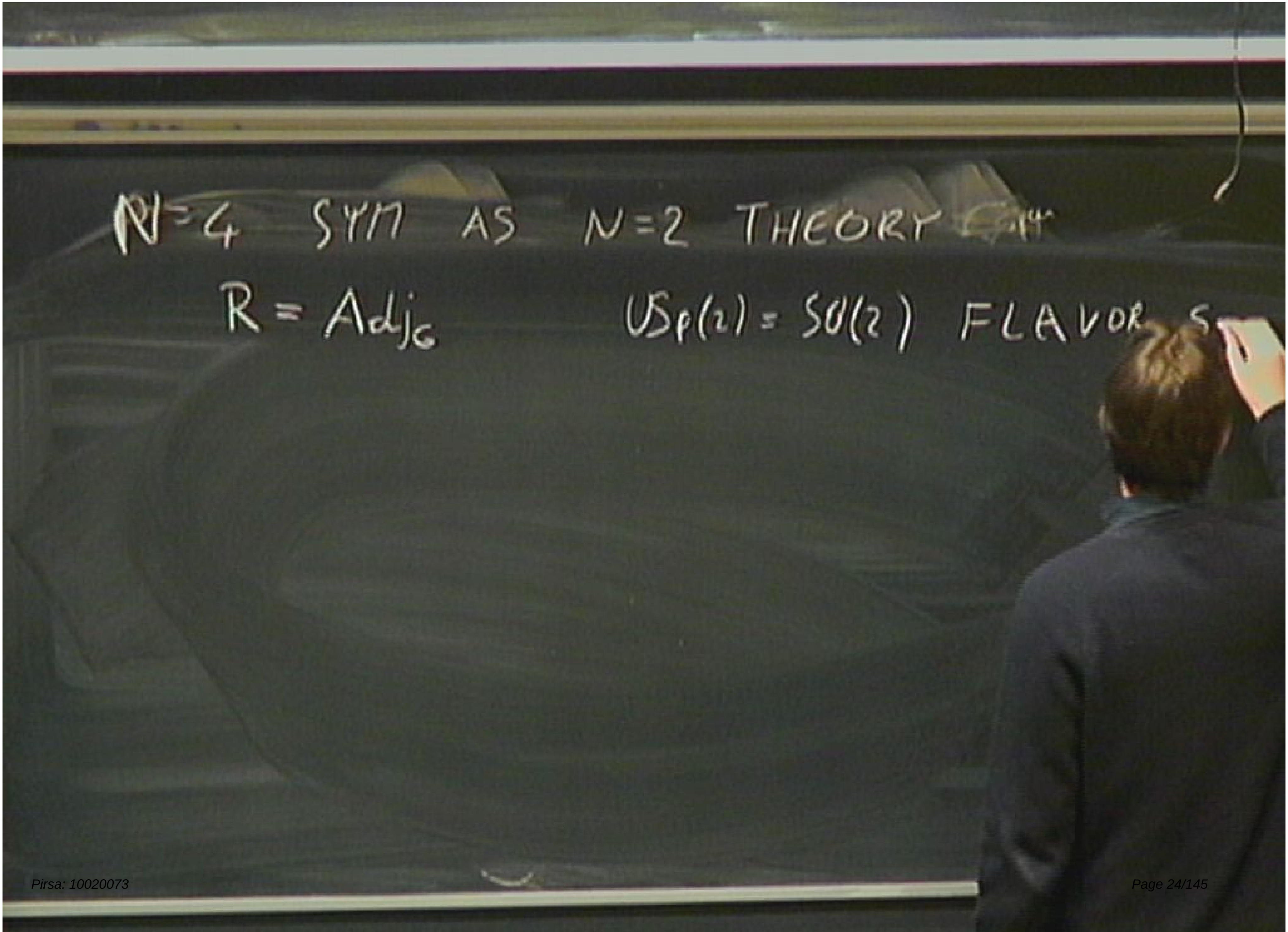
$R = \oplus\, r_i^{n_i}$ $\qquad \prod U(n_i)$

IF r_i ARE REAL $Sp(2n_i)$

" " " PSEUDOREAL $SO(2n_i)$


N=4 SYM AS N=2 THEORY
R

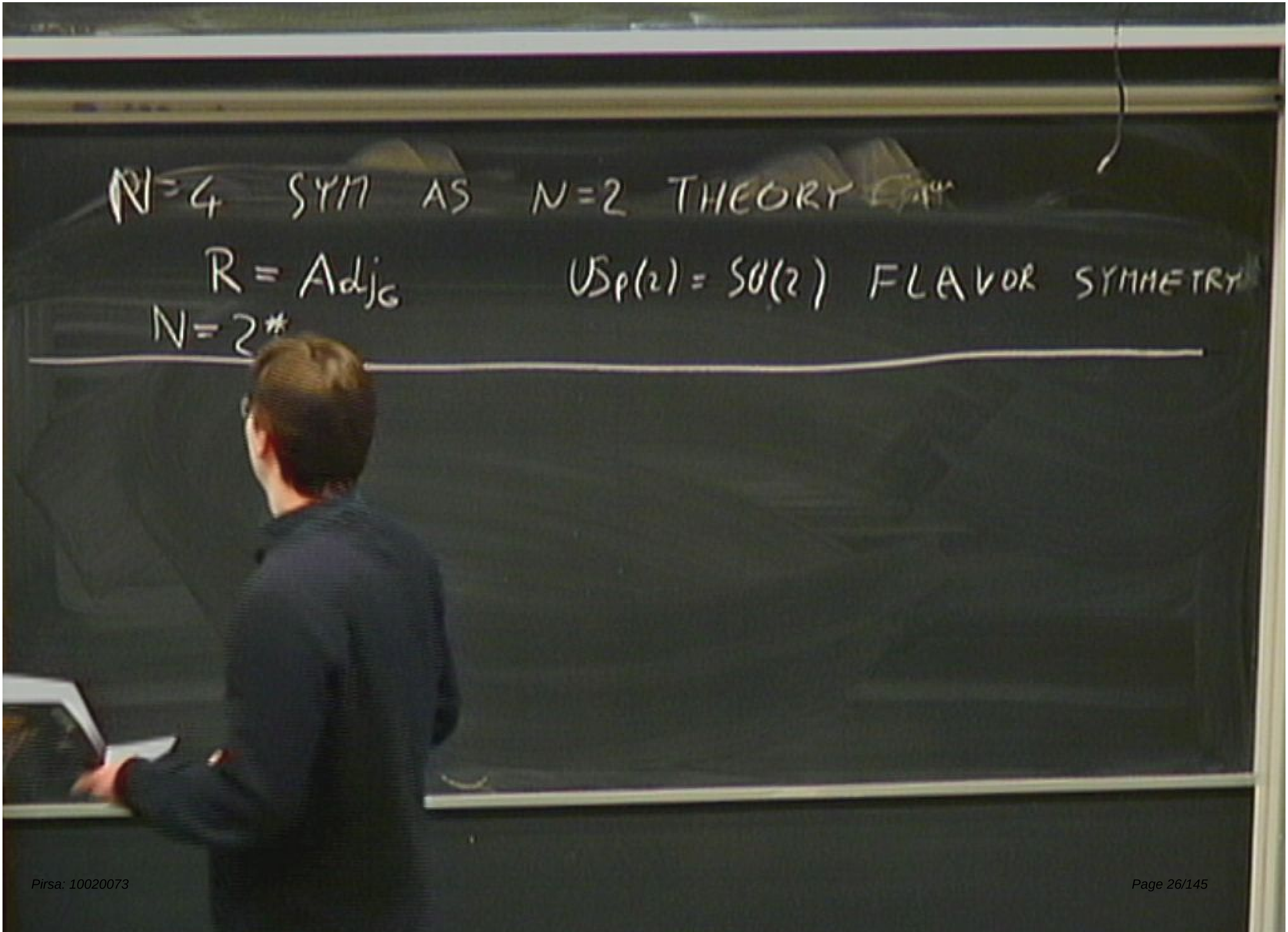

N=4 SYM AS N=2 THEORY
R = Adj_G

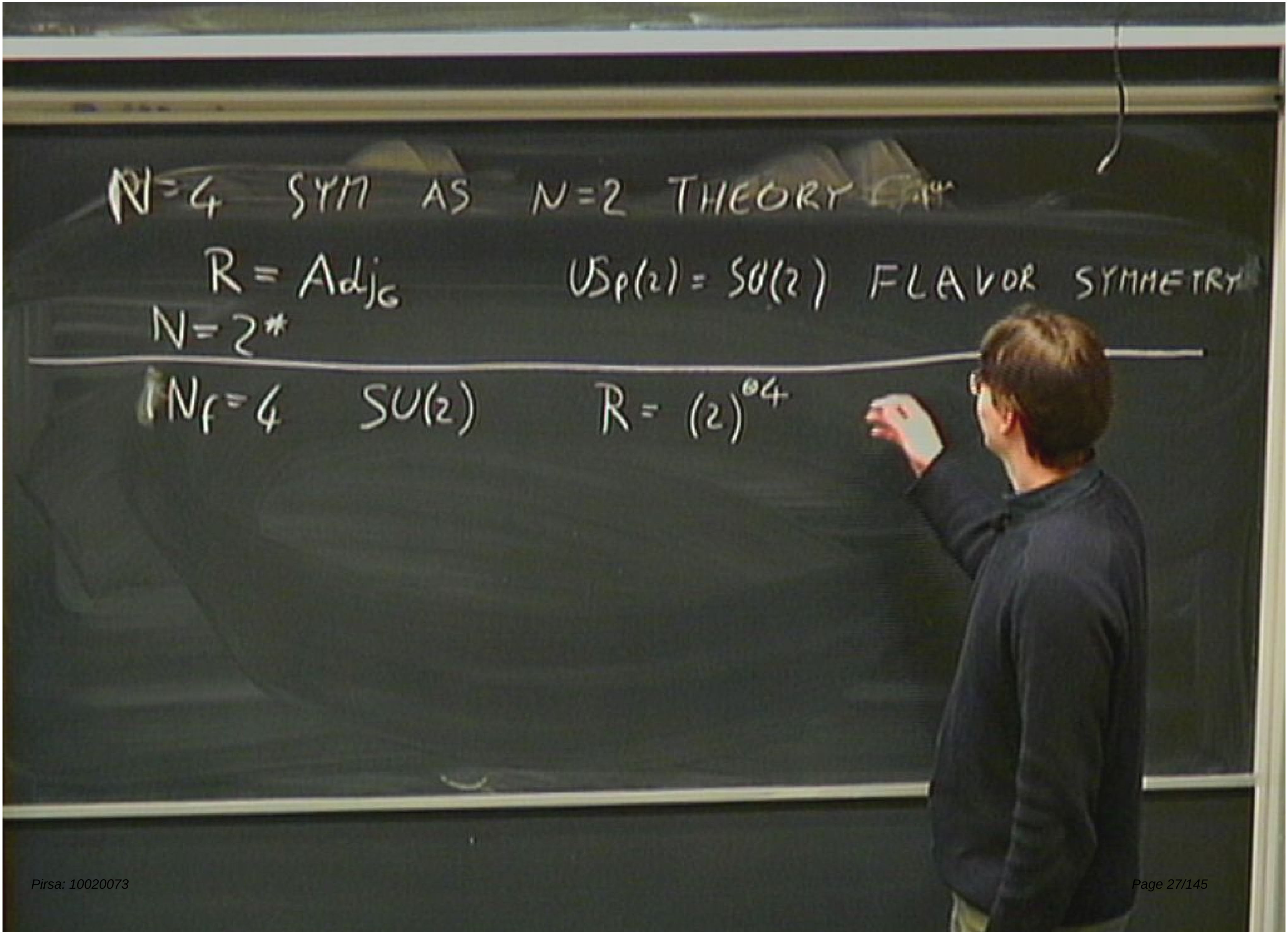

N=4 SYM AS N=2 THEORY
R = Adj_G
USp(2) = SU(2) FLAVOR S

$N=4$ SYM AS $N=2$ THEORY

$R = Adj_G$ $\qquad$ $USp(2) = SU(2)$ FLAVOR SYMMETRY

$N=2^*$


N=4 SYM AS N=2 THEORY
R = Adj_G
USp(2) = SU(2) FLAVOR SYMMETRY
N=2*


N=4 SYM AS N=2 THEORY
R = Adj_G
USp(2) = SU(2) FLAVOR SYMMETRY
N=2*
N_f=4 SU(2)
R = (2)^⊗4

$\mathcal{N}=4$ SYM AS $N=2$ THEORY

$R = \mathrm{Adj}_G$ $\qquad$ $USp(2) = SU(2)$ FLAVOR SYMMETRY

$N=2^*$

$N_f = 4$ $\quad$ $SU(2)$ $\qquad$ $R = (2)^{\oplus 4}$ $\qquad$ $SO(8)$ FLAVOR SYMMETRY

$\mathcal{N}=4$ SYM AS $\mathcal{N}=2$ THEORY

$R = \mathrm{Adj}_G$ $\quad$ $USp(2) = SU(2)$ FLAVOR SYMMETRY

$\mathcal{N}=2^*$

$N_f = 4$ $SU(2)$ $\quad$ $R = (2)^{\oplus 4}$ $\quad$ $SO(8)$ FLAVOR SYMMETRY

$\tau = \frac{8\pi^2 i}{g^2} + \frac{\Theta}{\pi}$ $\quad$ $SL(2,\mathbb{Z})$ S-DUALITY GROUP

$\mathcal{N}=4$ SYM AS $\mathcal{N}=2$ THEORY [illegible]

$R = \mathrm{Adj}_G$ $\qquad$ $USp(2) = SU(2)$ FLAVOR SYMMETRY

$\mathcal{N}=2^*$

$N_f = 4 \quad SU(2)$ $\qquad$ $R = (2)^{\oplus 4}$ $\qquad$ $SO(8)$ FLAVOR SYMMETRY

$\tau = \frac{8\pi^2 i}{g^2} + \frac{\theta}{\pi}$ $\qquad$ $SL(2,\mathbb{Z})$ S-DUALITY GROUP

$S: \tau \to -\frac{1}{\tau}$

$T: \tau \to \tau + 1$

$\mathcal{N}=4$ SYM AS $N=2$ THEORY

$R = \mathrm{Adj}_G$ $\quad USp(2) = SU(2)$ FLAVOR SYMMETRY

$N = 2^*$

$N_f = 4 \quad SU(2) \qquad R = (2)^{\oplus 4} \qquad SO(8)$ FLAVOR SYMMETRY

$$\tau = \frac{8\pi^2 i}{g^2} + \frac{\theta}{\pi} \qquad SL(2,\mathbb{Z}) \text{ S-DUALITY GROUP}$$

$$S: \tau \to -\frac{1}{\tau}$$

$$T: \tau \to \tau + 1$$

$\mathcal{N}=4$ SYM AS $\mathcal{N}=2$ THEORY

$R = \mathrm{Adj}_G$ $\quad USp(2) = SU(2)$ FLAVOR SYMMETRY

$\mathcal{N}=2^*$

$N_f = 4 \quad SU(2) \qquad R = (2)^{\otimes 4} \qquad SO(8)$ FLAVOR SYMMETRY

$$\tau = \frac{8\pi^2 i}{g^2} + \frac{\theta}{\pi}$$

$SL(2,\mathbb{Z})$ S-DUALITY GROUP

$$S: \tau \to -\frac{1}{\tau}$$

$T: \tau \to \tau + 1$ ALSO DO REFLECTION

$\mathcal{N}=4$ SYM AS $\mathcal{N}=2$ THEORY

$R = \mathrm{Adj}_G$ $\qquad$ $USp(2) = SU(2)$ FLAVOR SYMMETRY

$\mathcal{N}=2^*$

$N_f = 4$ $\quad SU(2)$ $\qquad R = (2)^{\oplus 4}$ $\qquad SO(8)$ FLAVOR SYMMETRY

$$\tau = \frac{8\pi i}{g^2} + \frac{\theta}{\pi}$$

$SL(2,\mathbb{Z})$ S-DUALITY GROUP

$$S: \tau \to -\frac{1}{\tau}$$

$$T: \tau \to \tau + 1$$

ALSO DO REFLECTION $8_s \leftrightarrow 8_c$

$\mathcal{N}=4$ SYM AS $N=2$ THEORY [illegible]

$R = Adj_G$ $\quad USp(2) = SU(2)$ FLAVOR SYMMETRY

$N=2^*$

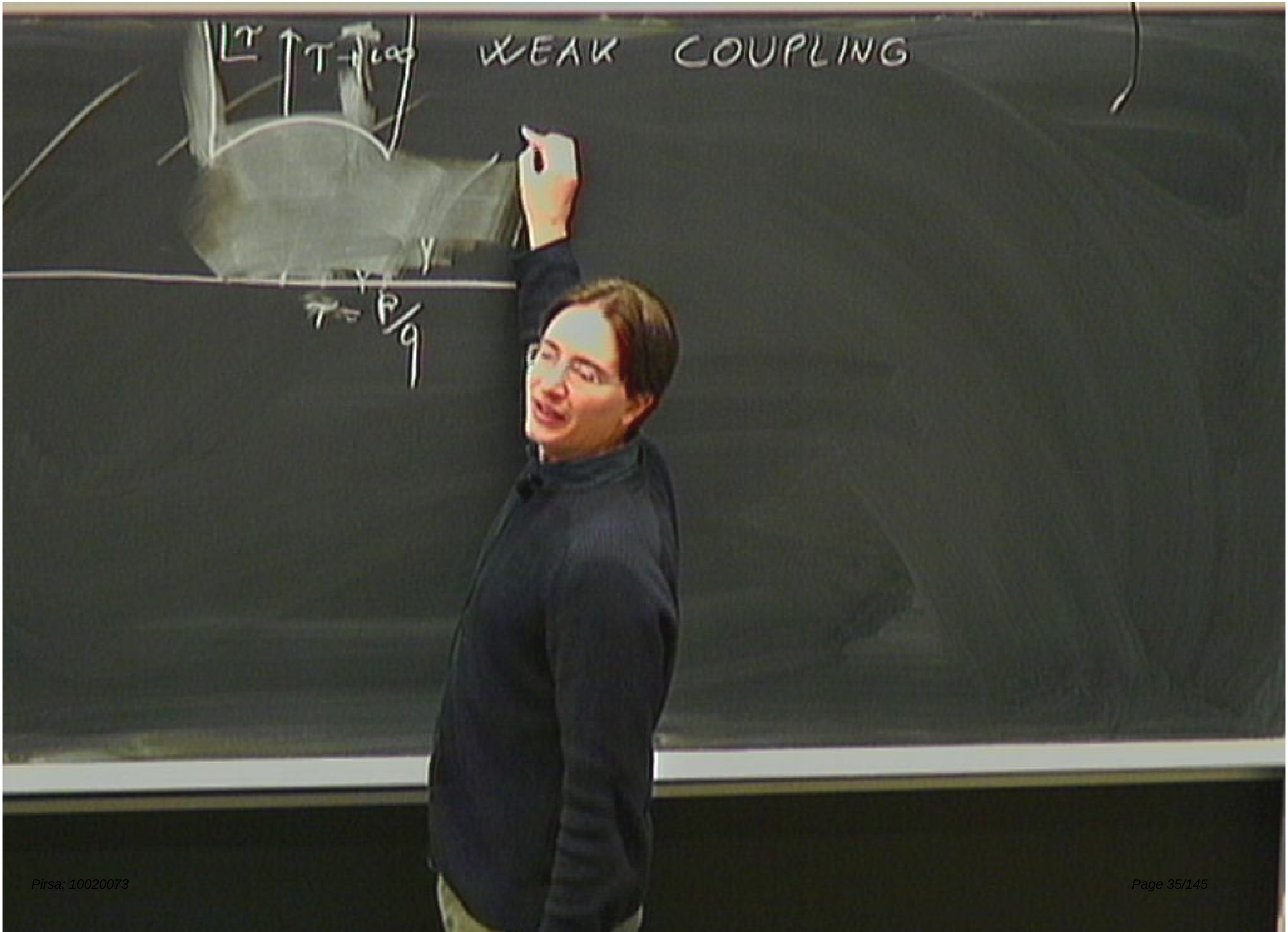
$N_f = 4 \quad SU(2) \qquad R = (2)^{\oplus 4} \qquad SO(8)$ FLAVOR SYMMETRY

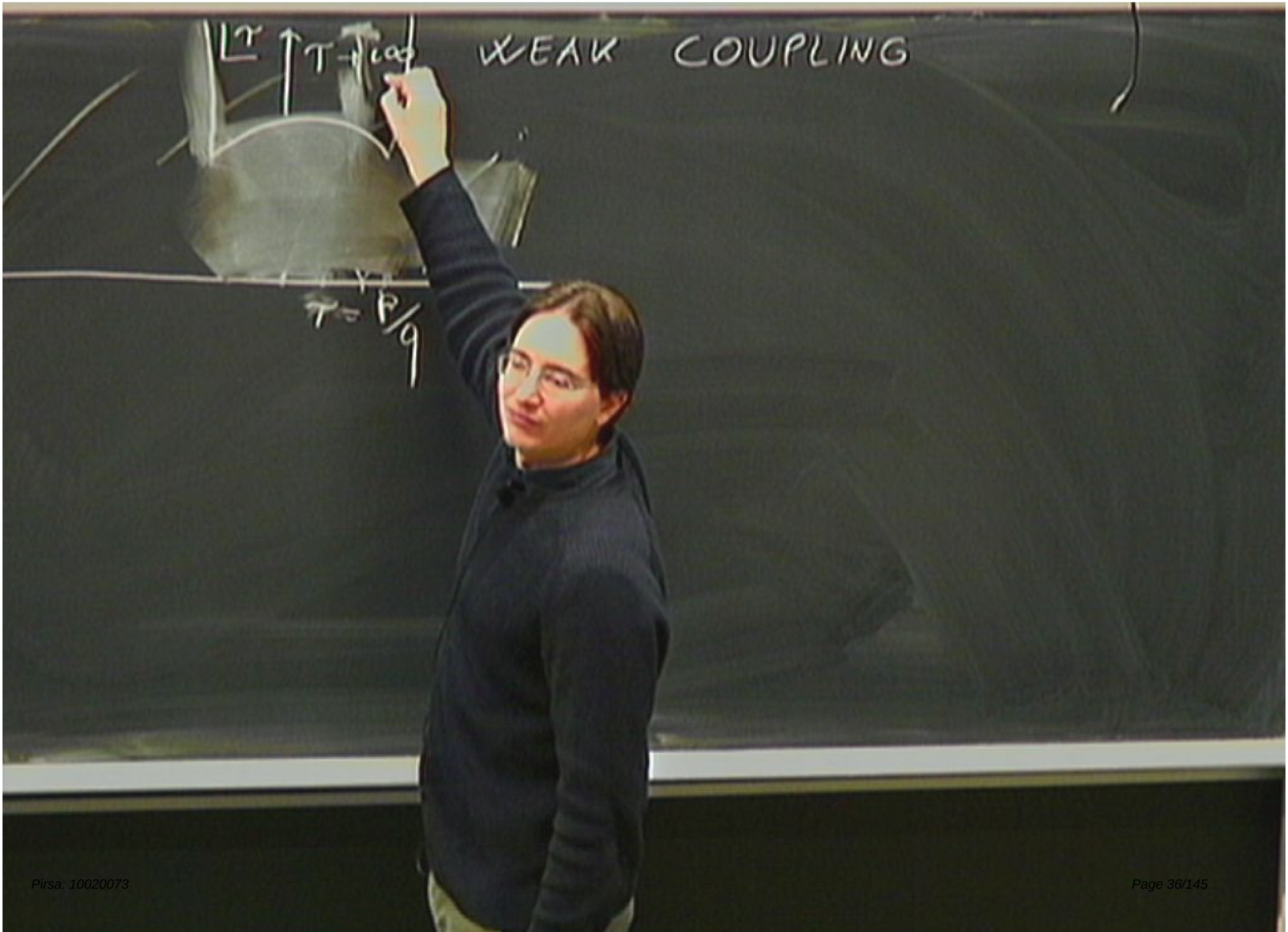
$$\tau = \frac{8\pi^2 i}{g^2} + \frac{\theta}{\pi}$$

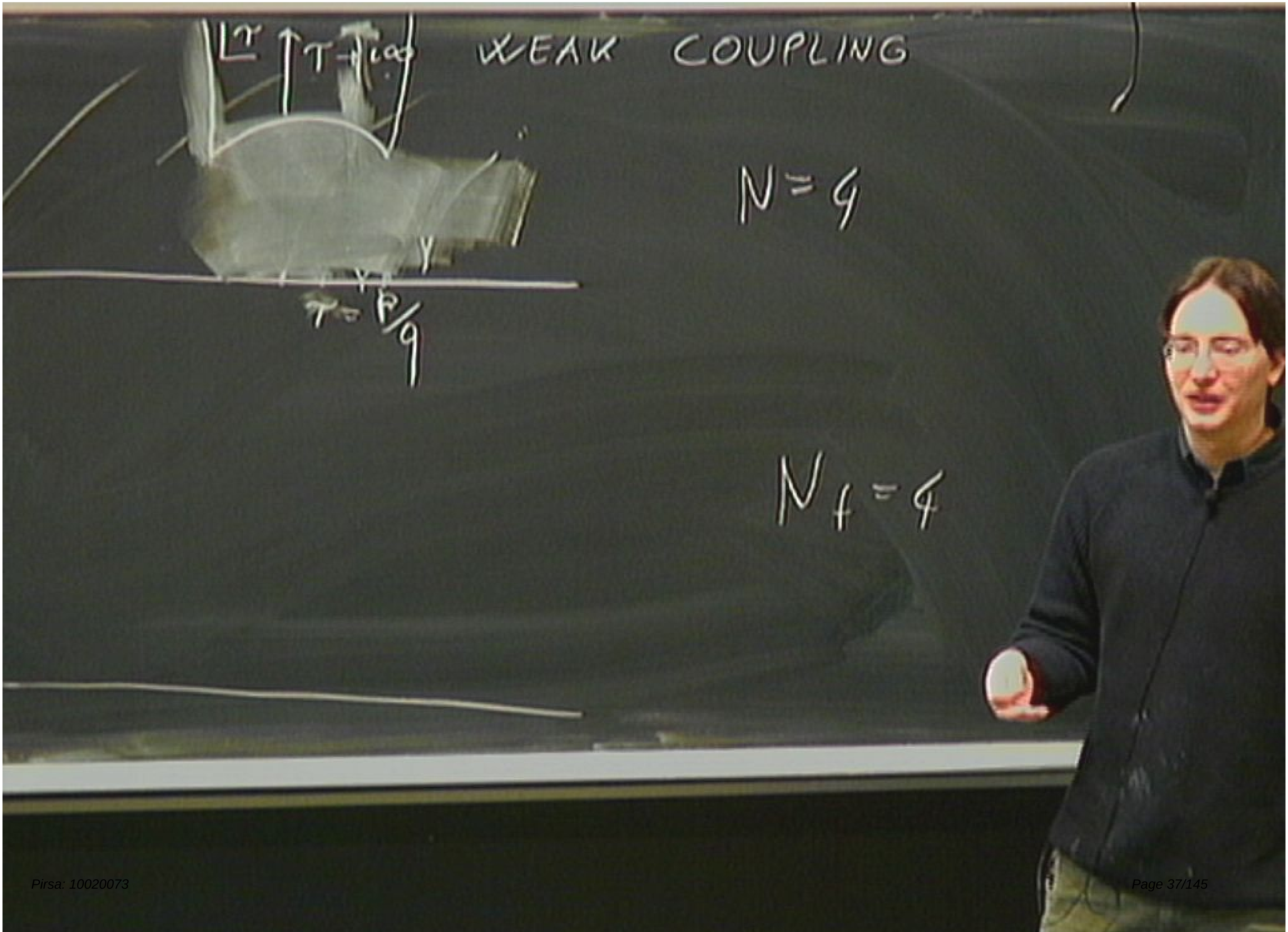
$SL(2,\mathbb{Z})$ S-DUALITY GROUP

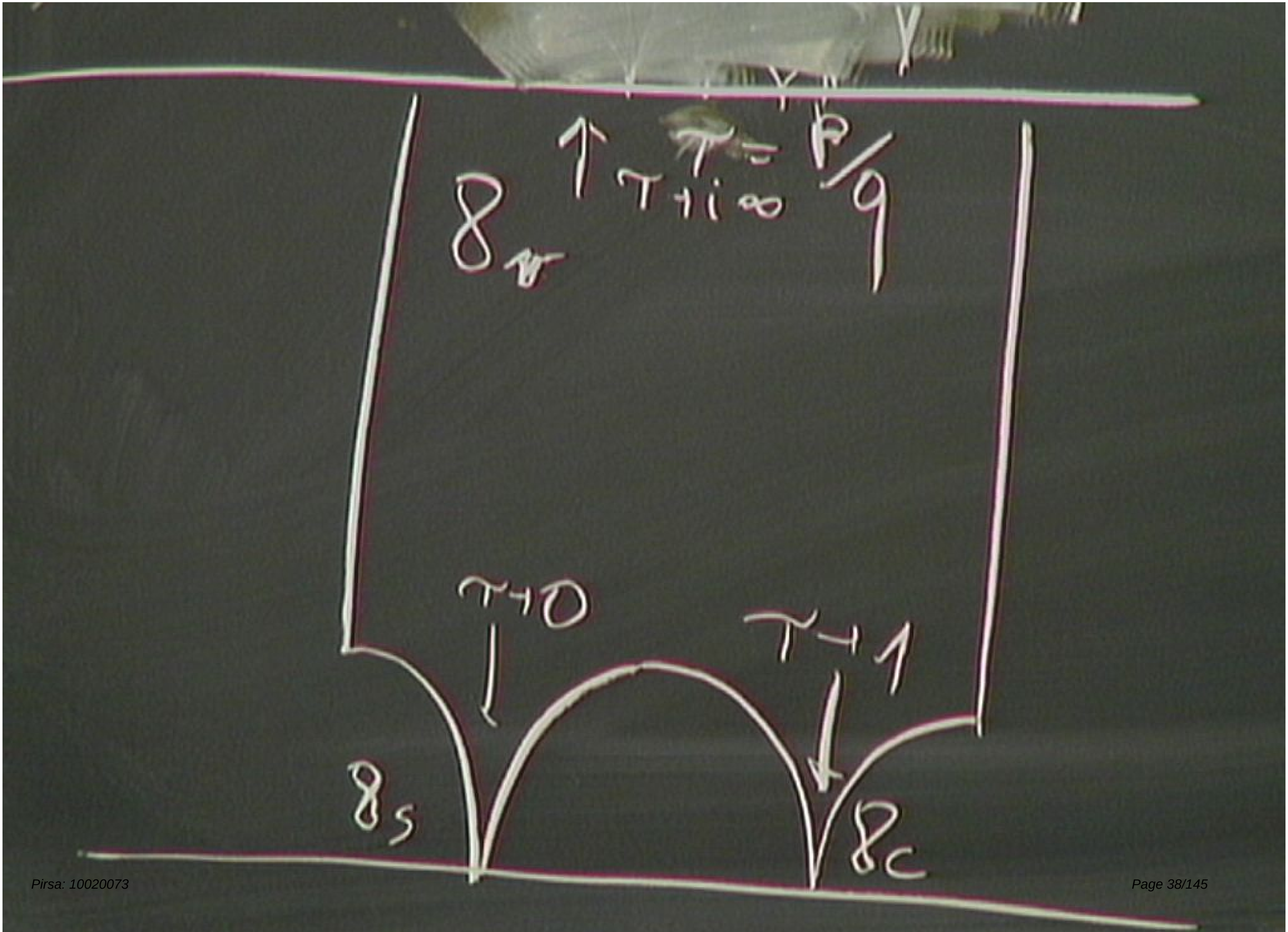
$S: \tau \to -\frac{1}{\tau} \qquad 8_v \leftrightarrow 8_s$

$T: \tau \to \tau + 1$ ALSO DO REFLECTION $8_s \leftrightarrow 8_c$


WEAK COUPLING


WEAK COUPLING

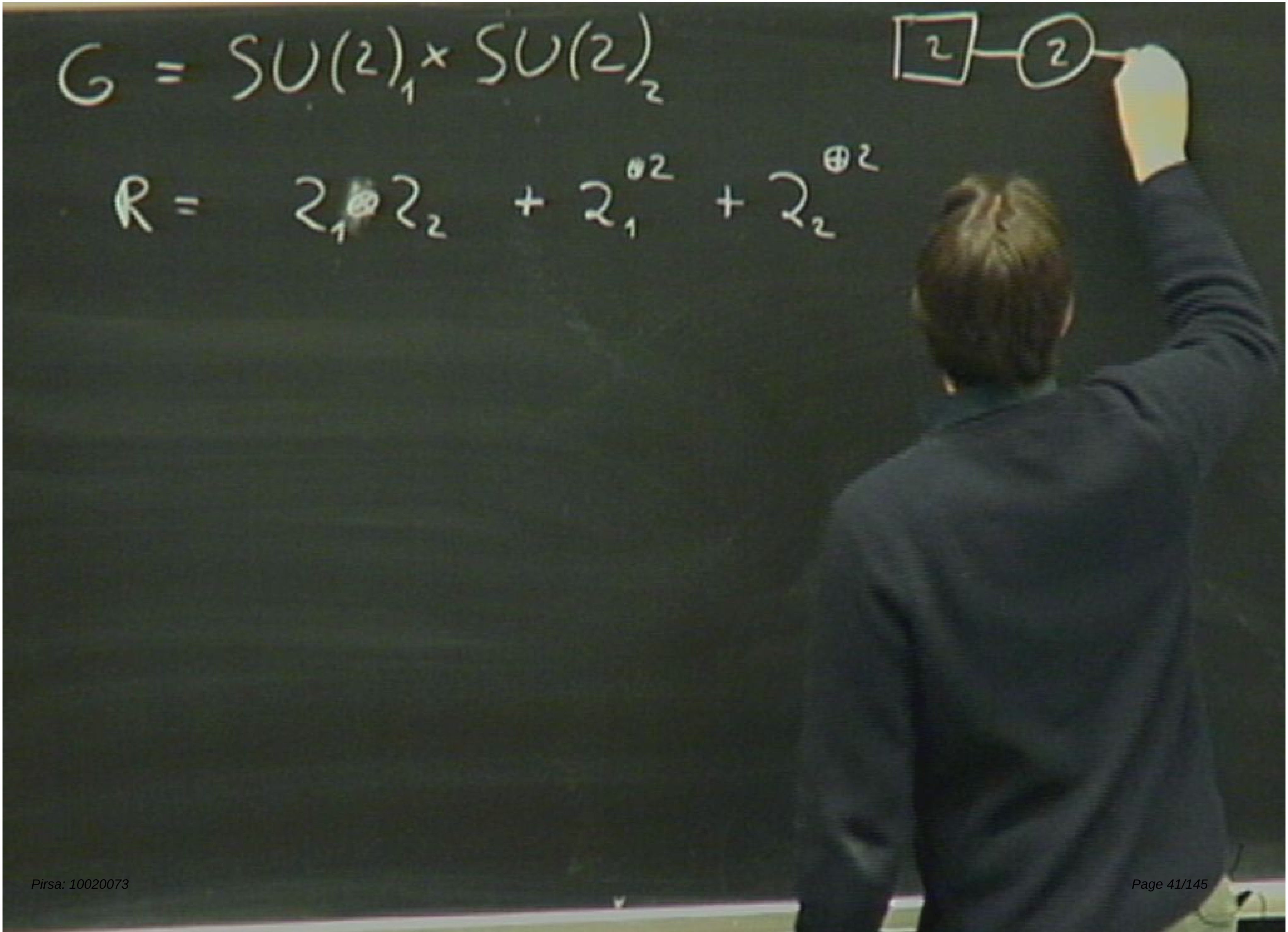

WEAK COUPLING
N = 4
N_f = 4
τ = p/q

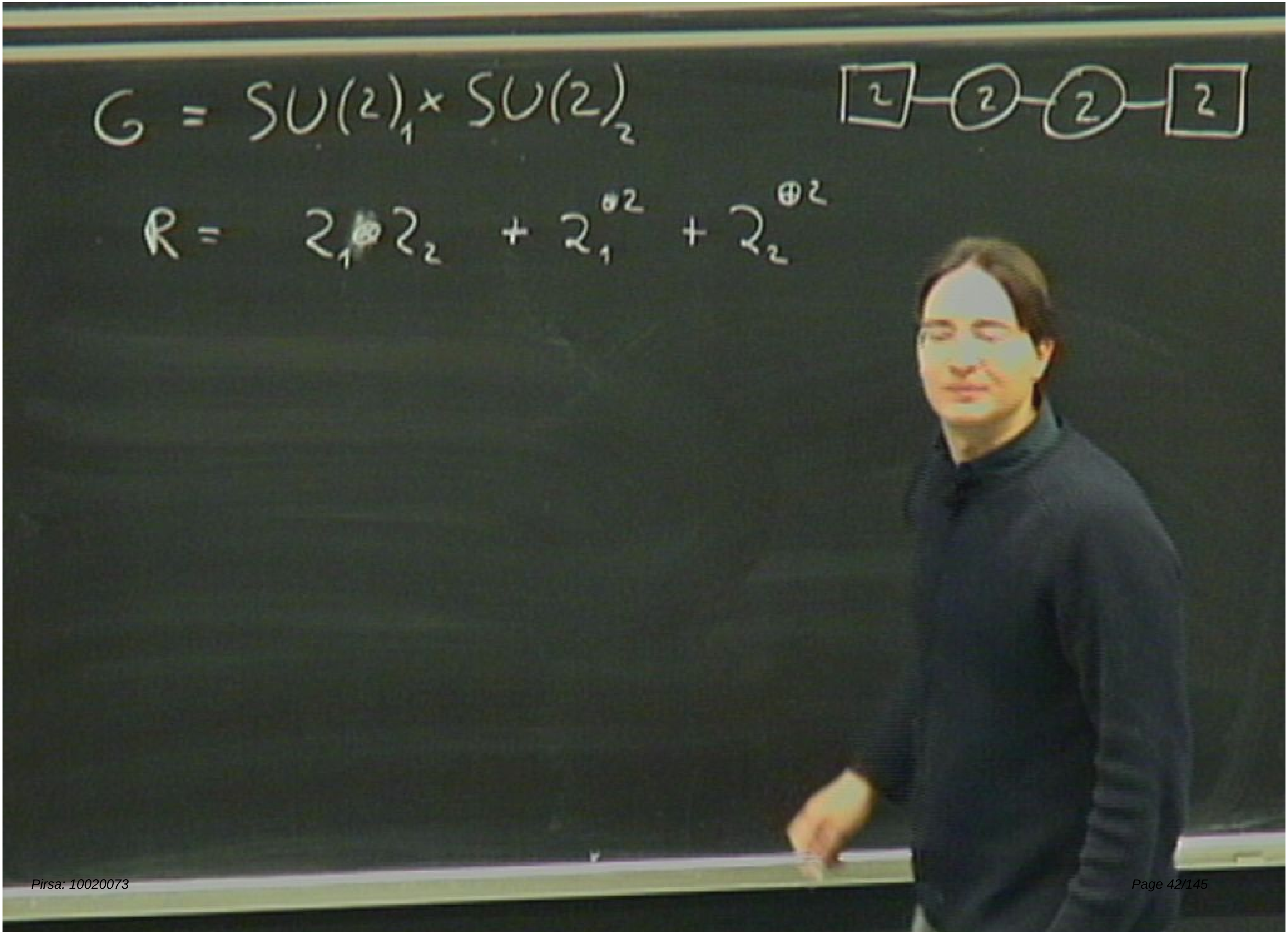

τ = p/q
τ → i∞
8
τ → 0
τ → 1
8s
8c

$G = SU(2) \times SU(2)$

$$G = SU(2)_1 \times SU(2)_2$$

$$R = 2_1 \otimes 2_2 + 2^{\oplus 2} + 2_2^{\oplus 2}$$

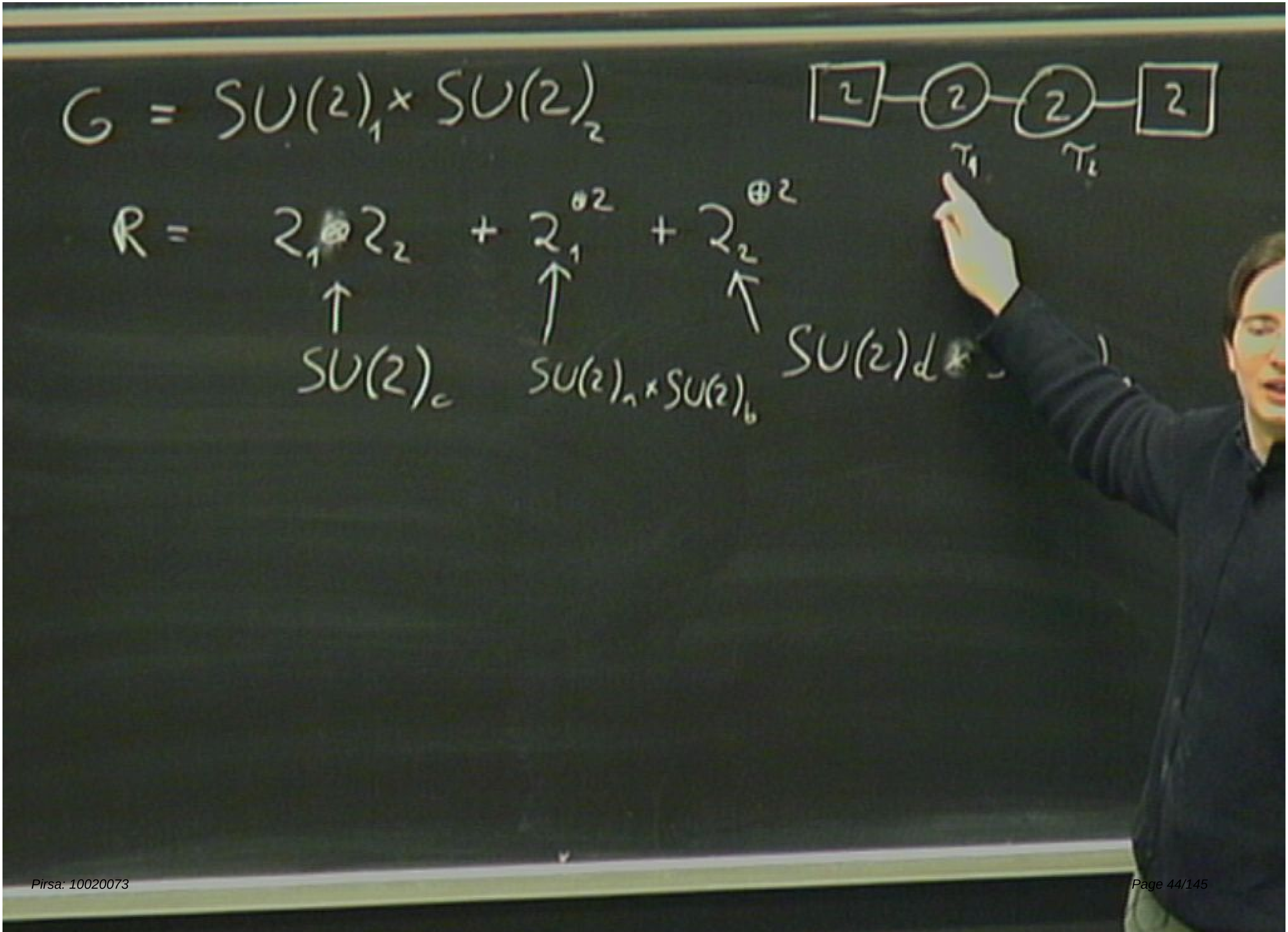

$G = SU(2)_1 \times SU(2)_2$
$R = 2_1 \otimes 2_2 + 2_1^{\oplus 2} + 2_2^{\oplus 2}$
2
2


$G = SU(2)_1 \times SU(2)_2$
$R = 2_1 \otimes 2_2 + 2_1^{\oplus 2} + 2_2^{\oplus 2}$
2
2
2
2

$$G = SU(2)_1 \times SU(2)_2$$

$$\boxed{2} - (2) - (2) - \boxed{2}$$

$$R = \underbrace{2_1 \otimes 2_2}_{SU(2)_c} + 2_1^{\oplus 2} + 2_2^{\oplus 2}$$


$G = SU(2)_1 \times SU(2)_2$
2
2
2
2
τ_1
τ_2
$R = 2_1 \otimes 2_2 + 2_1^{\oplus 2} + 2_2^{\oplus 2}$
$SU(2)_c$
$SU(2)_a \times SU(2)_b$
$SU(2)d$

$$SO(4) \times SO(4) \subset SO(8)$$

$$8_v =$$

$$SO(4) \times SO(4) \subset SO(8)$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$$N_f = 4$$

$$8_v = 2_a \times 2_b + 2_c \times 2_d$$

$$SO(4) \times SO(4) \subset SO(8)$$

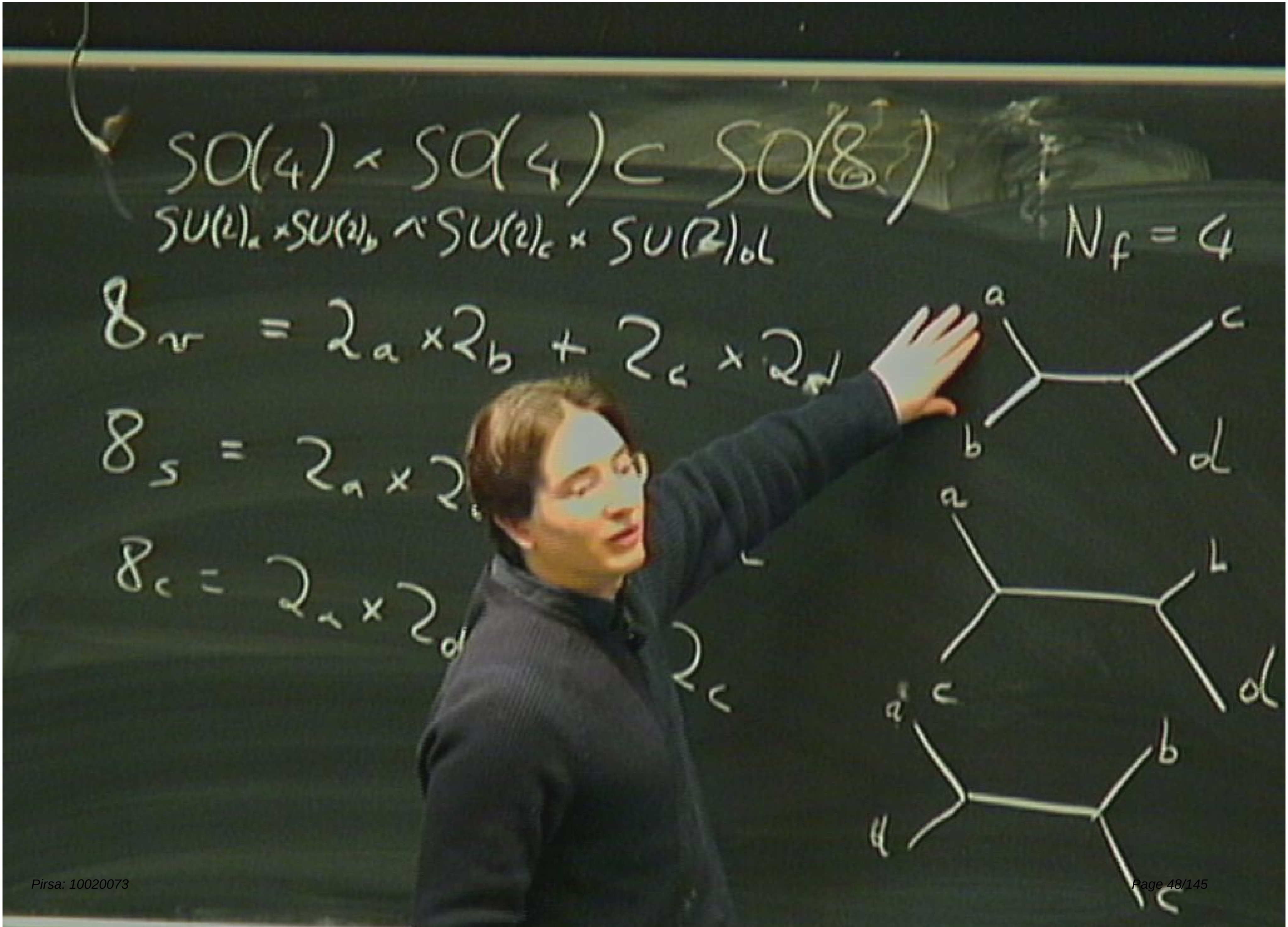
$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$N_f = 4$

$$8_v = 2_a \times 2_b + 2_c \times 2_d$$

$$8_s = 2_a \times 2_c + 2_b \times 2_d$$

$$8_c = 2_a \times 2_d + 2_b \times 2_c$$

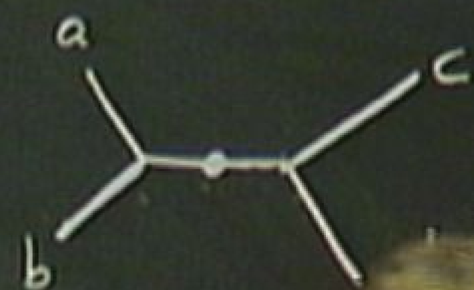

SO(4) × SO(4) ⊂ SO(8)
SU(2)a × SU(2)b × SU(2)c × SU(2)d
Nf = 4
8v = 2a × 2b + 2c × 2d
8s = 2a × 2
8c = 2a × 2d
2c

$$SO(4) \times SO(4) \subset SO(8)$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

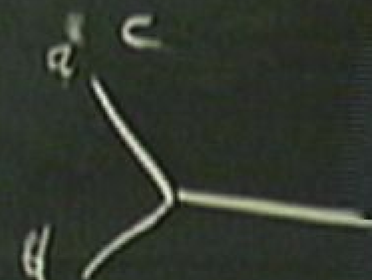
$N_f = 4$

$$8_v = 2_a \times 2_b + 2_c \times 2_d$$

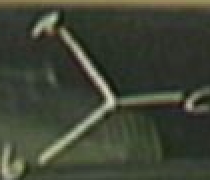


$$8_s = 2_a \times 2_c + 2_b \times 2_d$$

$$8_c = 2_a \times 2_d + 2_b \times 2_c$$



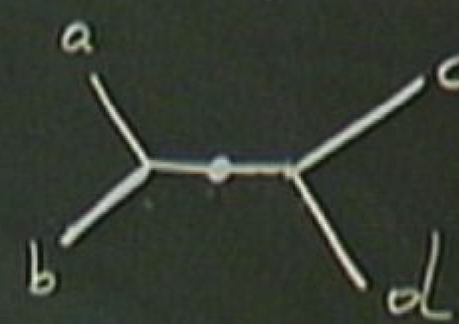
$$SO(4) \times SO(4) \subset SO(8)$$



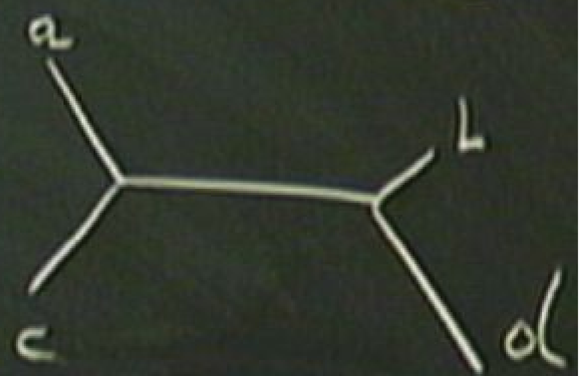
$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$$N_f = 4$$

$$8_v = 2_a \times 2_b + 2_c \times 2_d$$

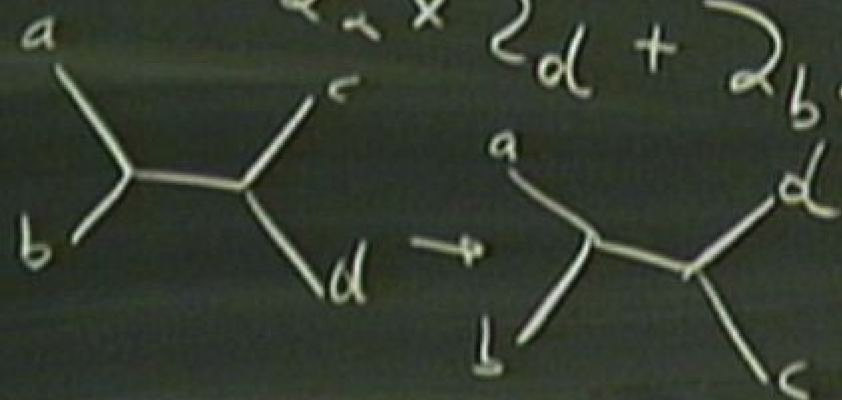


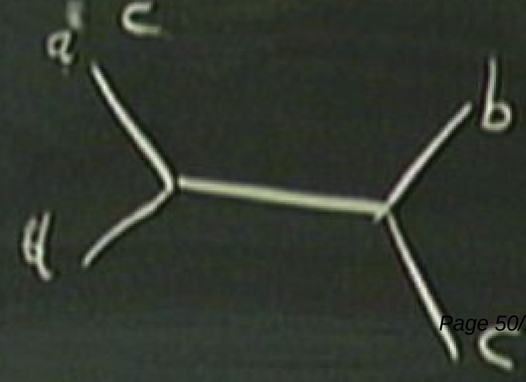
$$8_s = 2_a \times 2_c + 2_b \times 2_d$$



$$8_c = 2_a \times 2_d + 2_b \times 2_c$$

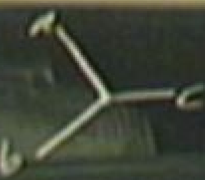
T:





$$SO(4)\times SO(4)\subset SO(8)$$

$$SU(2)_a\times SU(2)_b\times SU(2)_c\times SU(2)_d$$

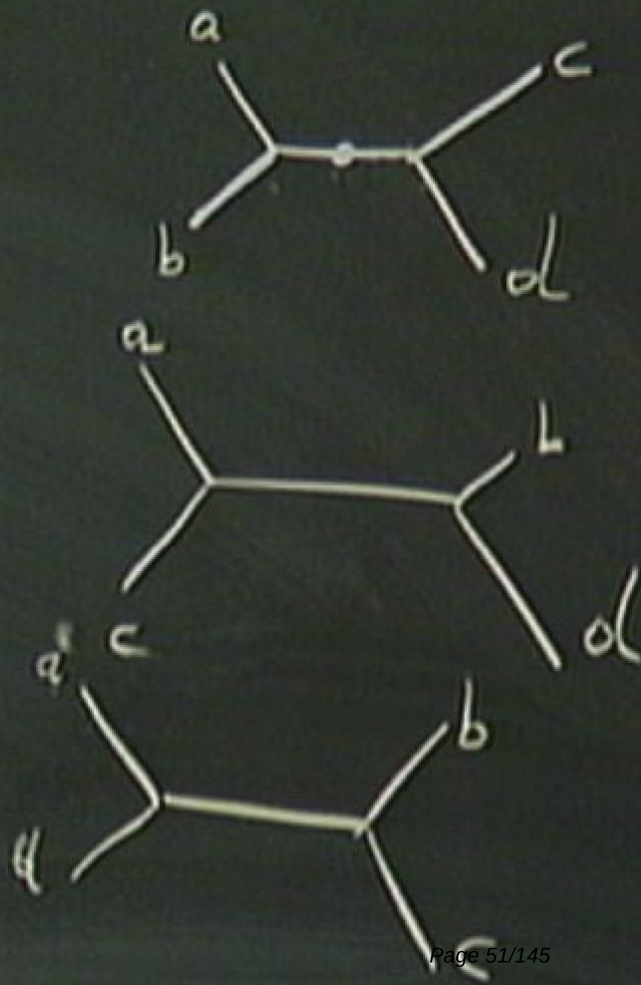


$$N_f = 4$$

$$8_v = 2_a\times 2_b + 2_c\times 2_d$$

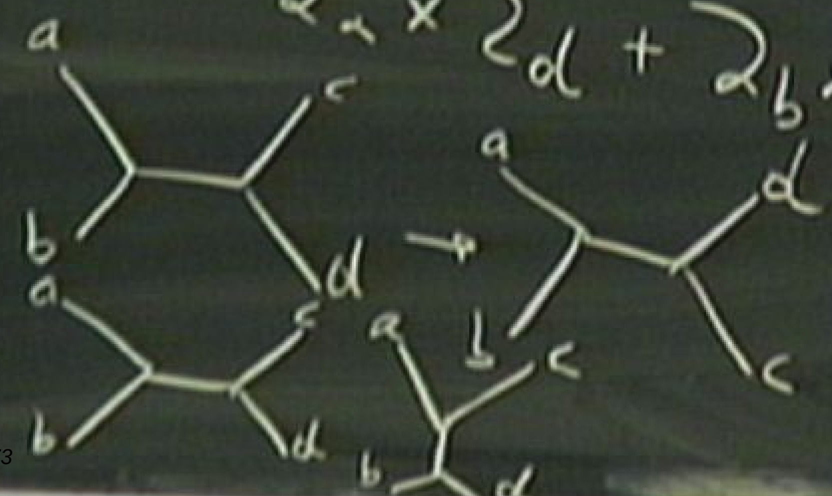
$$8_s = 2_a\times 2_c + 2_b\times 2_d$$

$$8_c = 2_a\times 2_d + 2_b\times 2_c$$



T:

S:



$\mathcal{N}=4$ SYM AS $N=2$ THEORY 

$R = \mathrm{Adj}_G$ $\quad USp(2) = SU(2)$ FLAVOR SYMMETRY

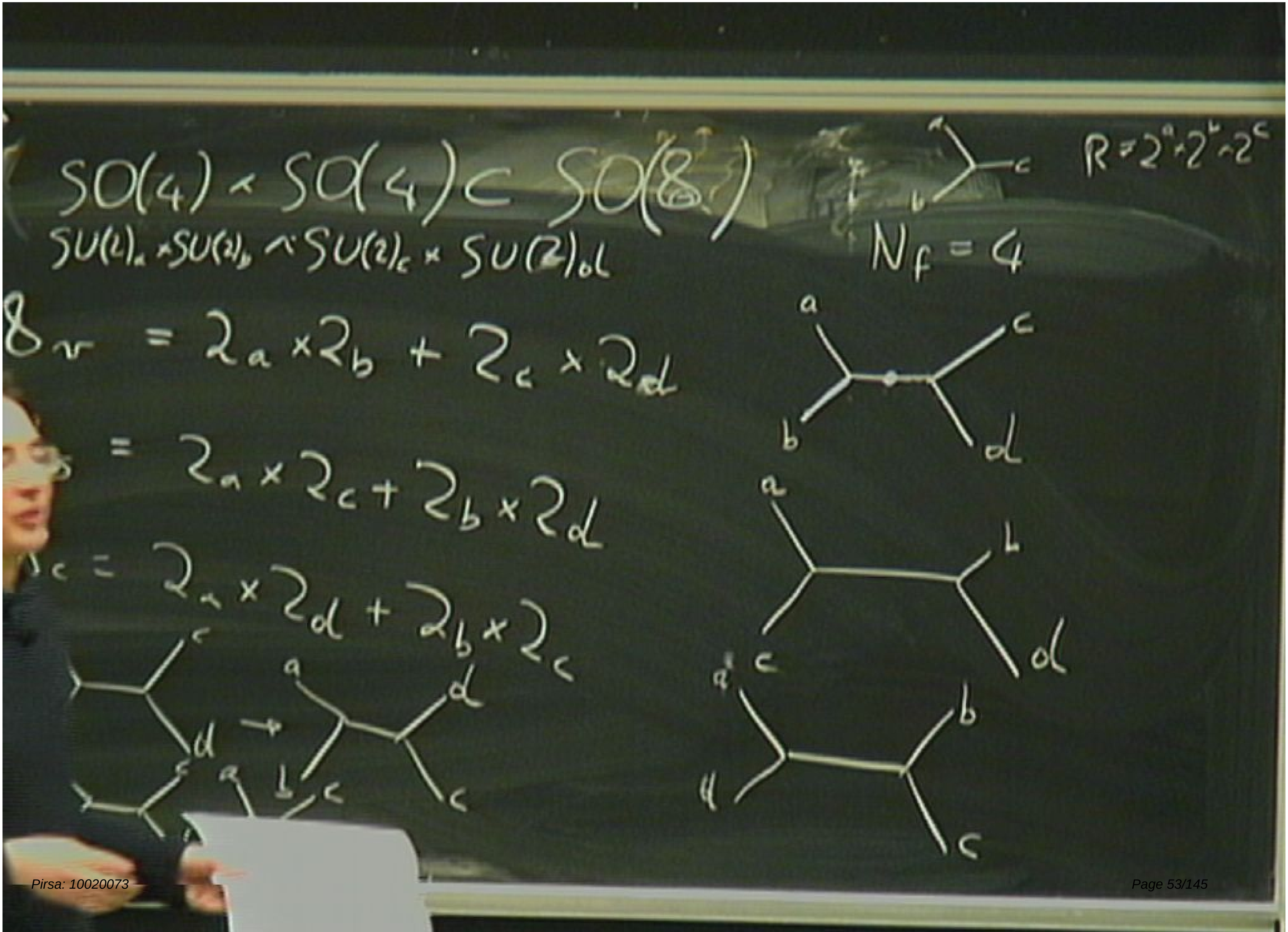
$N=2^*$

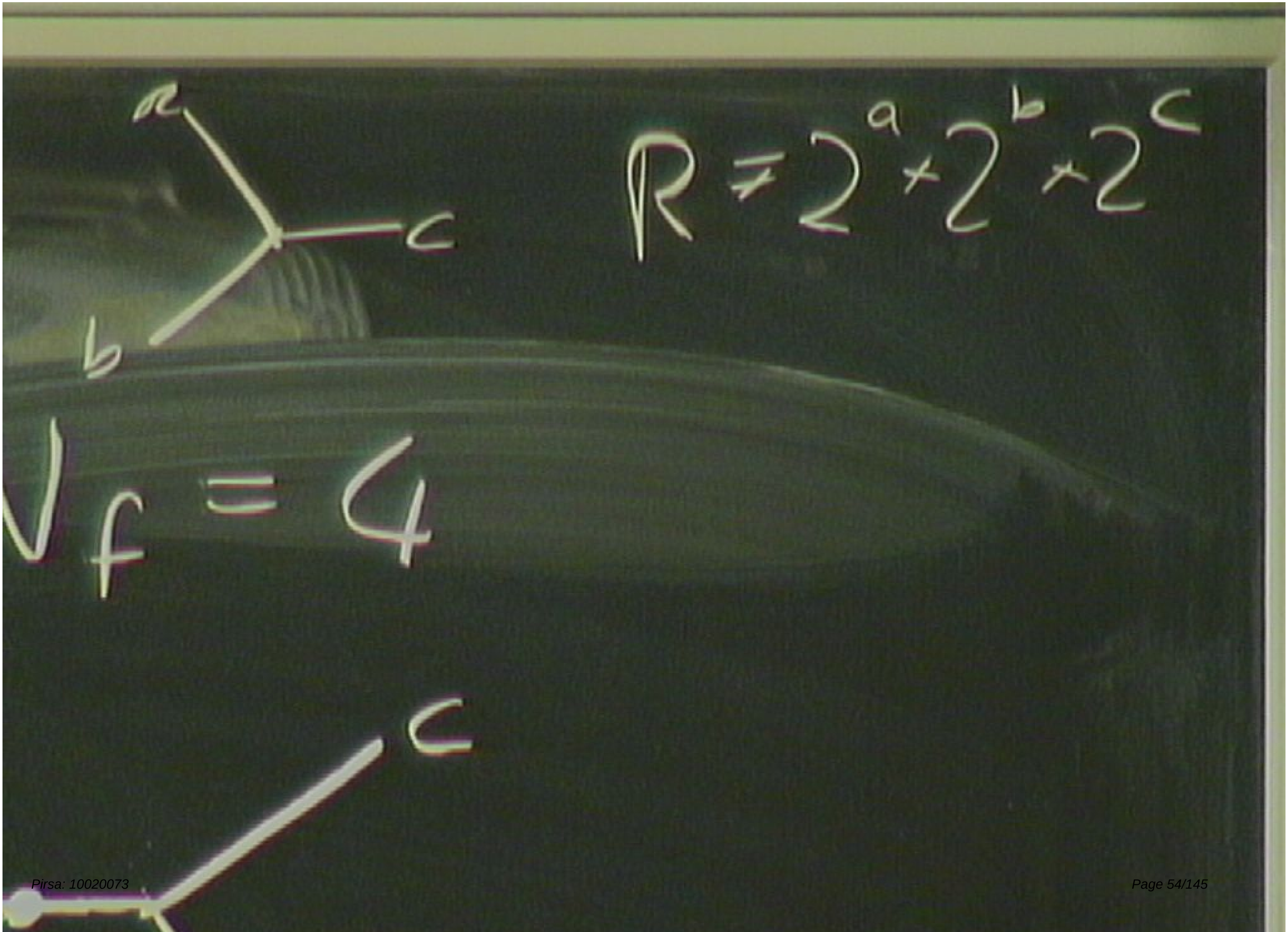
$N_f = 4 \quad SU(2) \quad R = (2)^{\otimes 4} \quad SO(8)$ FLAVOR SYMMETRY

$\tau = \frac{8\pi^2 i}{g^2} + \frac{\theta}{\pi} \quad SL(2,\mathbb{Z})$ S-DUALITY GROUP

$S: \tau \to -\frac{1}{\tau} \quad 8_v \leftrightarrow 8_s$

$T: \tau \to \tau + 1$ ALSO DO REFLECTION $8_s \leftrightarrow 8_c$


SO(4) × SO(4) ⊂ SO(8)
SU(2)a × SU(2)b × SU(2)c × SU(2)d
R = 2^a × 2^b × 2^c
Nf = 4
8v = 2a × 2b + 2c × 2d
= 2a × 2c + 2b × 2d
c = 2a × 2d + 2b × 2c
a
b
c
d


a
c
b
$R \neq 2^a + 2^b + 2^c$
$N_f = 4$
c

$\mathcal{N}=4$ SYM AS $\mathcal{N}=2$ THEORY 

$R = \mathrm{Adj}_G$ $\quad$ $USp(2) = SU(2)$ FLAVOR SYMMETRY

$\mathcal{N}=2^*$

$N_f = 4$ $\quad$ $SU(2)$ $\quad$ $R = (2)^{\otimes 4}$ $\quad$ $SO(8)$ FLAVOR SYMMETRY

$\tau = \frac{8\pi^2 i}{g^2} + \frac{\theta}{\pi}$ $\quad$ $SL(2,\mathbb{Z})$ S-DUALITY GROUP

$S: \tau \to -\frac{1}{\tau}$ $\quad$ $8_v \leftrightarrow 8_s$

$T: \tau \to \tau + 1$ $\quad$ ALSO DO REFLECTION $\quad$ $8_s \leftrightarrow 8_c$

$SO(4)\times SO(4)\subset SO(8)$

$SU(2)_a\times SU(2)_b\times SU(2)_c\times SU(2)_d$

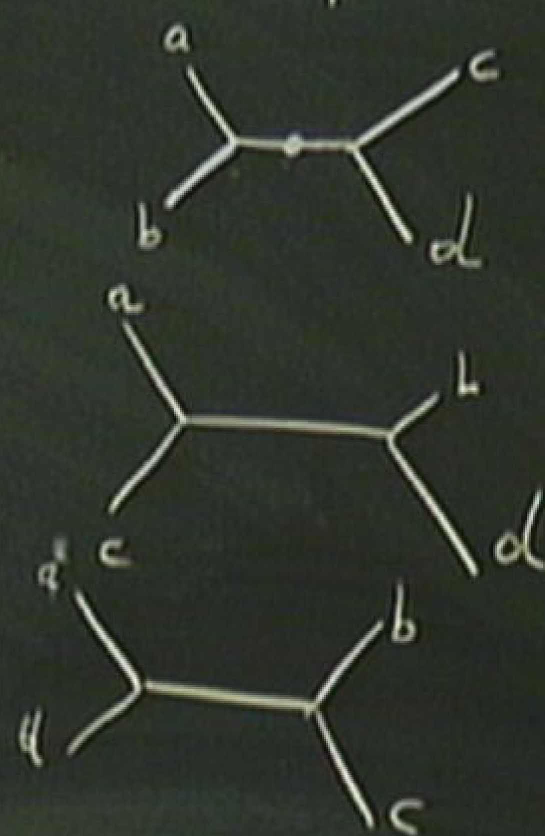
$R = 2^a\times 2^b\times 2^c$

$N_f = 4$

$$8_v = 2_a\times 2_b + 2_c\times 2_d$$

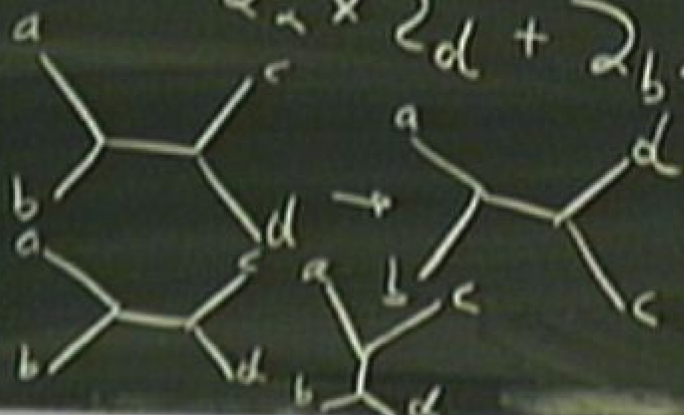
$$8_s = 2_a\times 2_c + 2_b\times 2_d$$

$$8_c = 2_a\times 2_d + 2_b\times 2_c$$



T:

S:



$$SO(4) \times SO(4) \subset SO(8)$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$R = 2^a \times 2^b \times 2^c$

$N_f = 4$

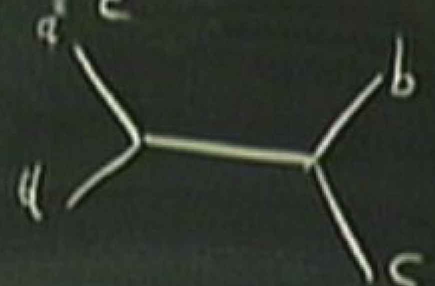
$$8_v = 2_a \times 2_b + 2_c \times 2_d$$

$\tau \to i\infty$

$$8_s = 2_a \times 2_c + 2_b \times 2_d$$

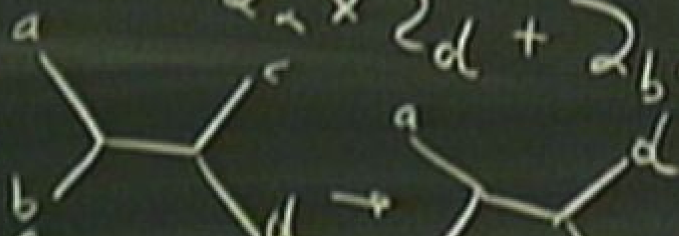
$$8_c = 2_a \times 2_d + 2_b \times 2_c$$

τ

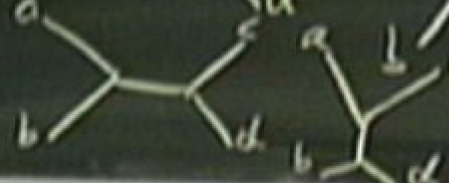


$\tau = 1$

T:



S:



$SO(4) \times SO(4) \subset SO(8)$

$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$

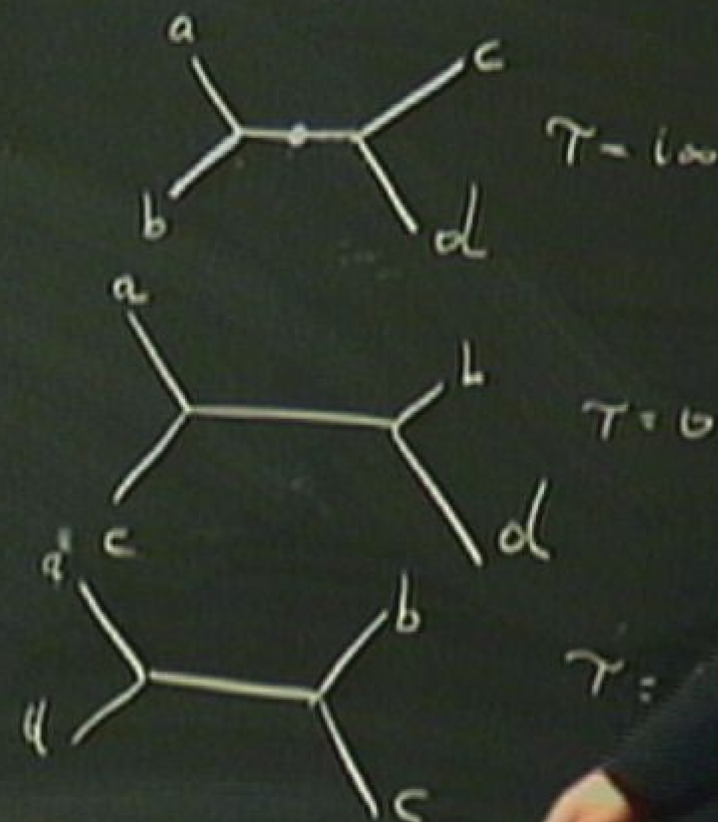
$R = 2^a \times 2^b \times 2^c$

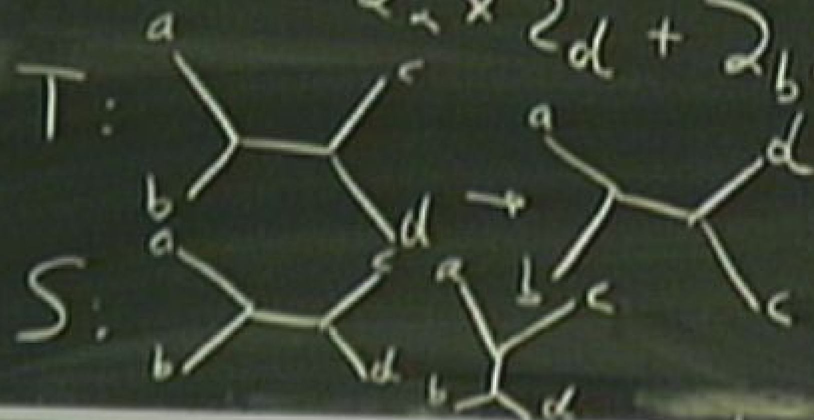
$N_f = 4$

$8_v = 2_a \times 2_b + 2_c \times 2_d$

$8_s = 2_a \times 2_c + 2_b \times 2_d$

$8_c = 2_a \times 2_d + 2_b \times 2_c$





$$SO(4) \times SO(4) \subset SO(8)$$

$$SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$$

$R = 2^a \times 2^b \times 2^c$

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$$8_v = 2_a \times 2_b + 2_c \times 2_d$$

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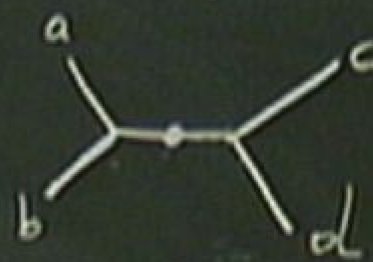
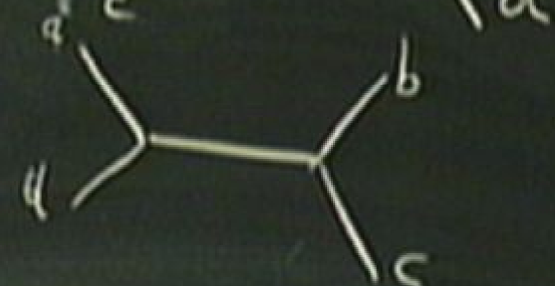
$$8_c = 2_a \times 2_d + 2_b \times 2_c$$



$\tau \to i\infty$

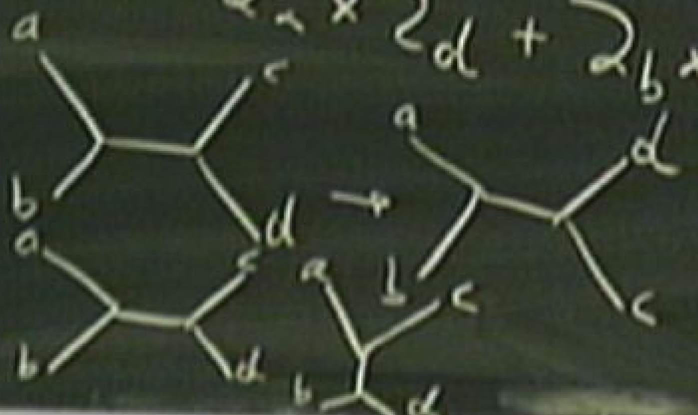
$\tau = 0$

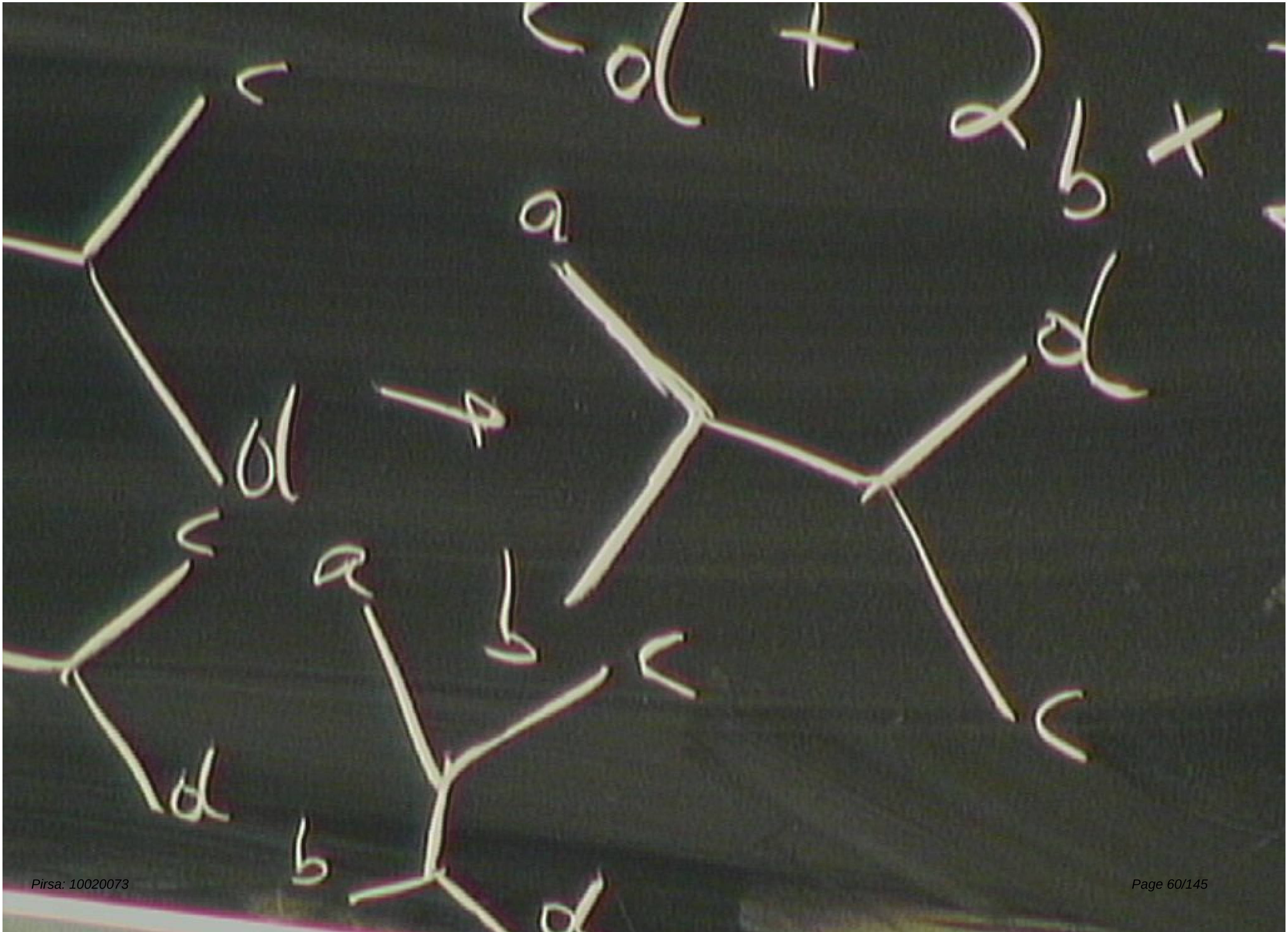


$\tau = 1$

T:

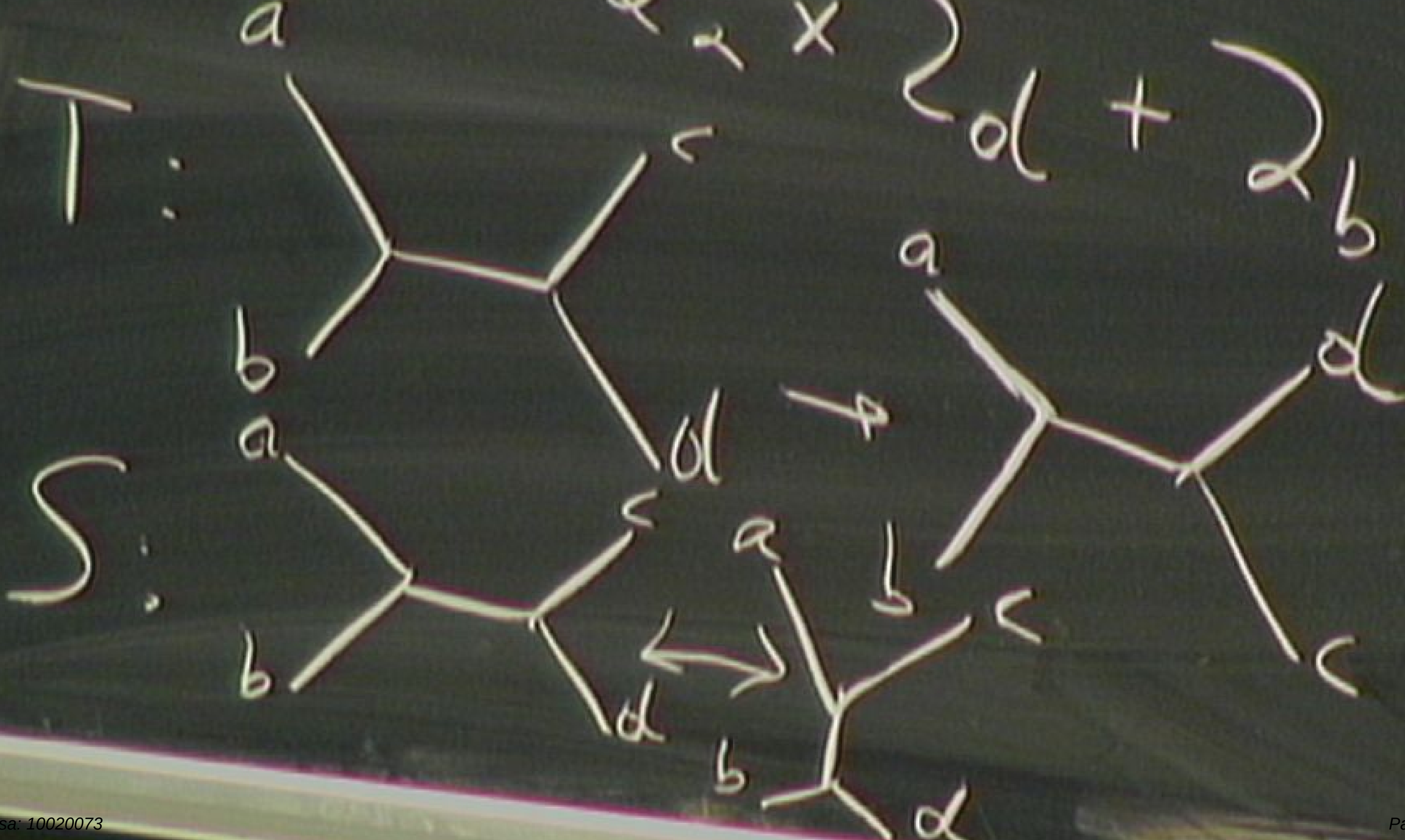
S:



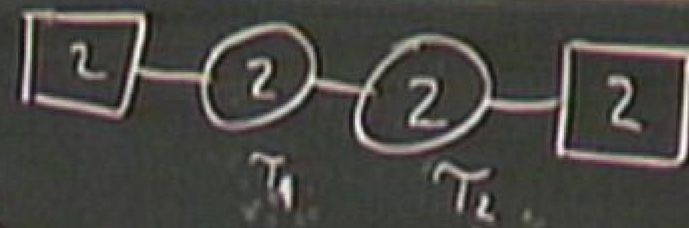

d +
2 b +
a
d
d
+
c
a
b
c
c
d
b
d

$\ldots_a \times 2_c + 2_b \times 2 \ldots$

$8_c = 2_a \times 2_d + 2_b \times 2 \ldots$



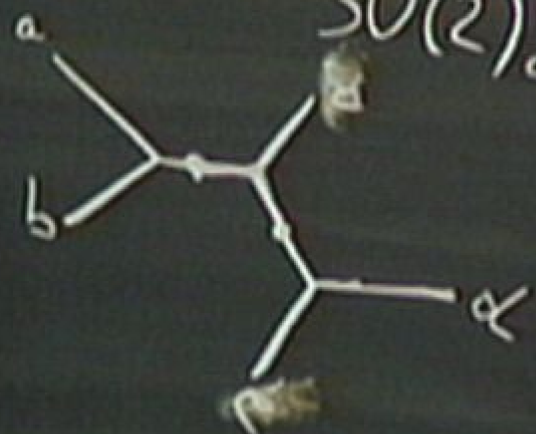
$$G = SU(2)_1 \times SU(2)_2$$



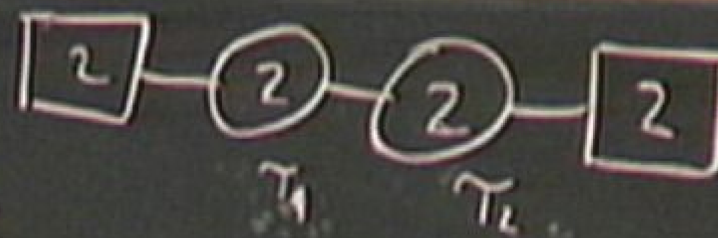
$$R = 2_1 \otimes 2_2 + 2_1^{\oplus 2} + 2_2^{\oplus 2}$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \nwarrow$$

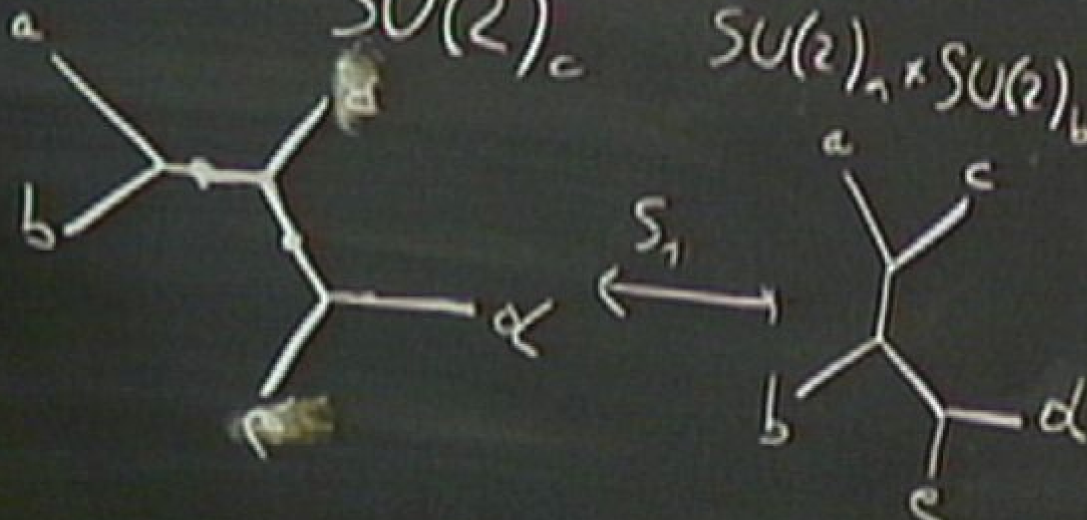
$$SU(2)_c \qquad SU(2)_a \times SU(2)_b \qquad SU(2)_d \times SU(2)_e$$



$$G = SU(2)_1 \times SU(2)_2$$

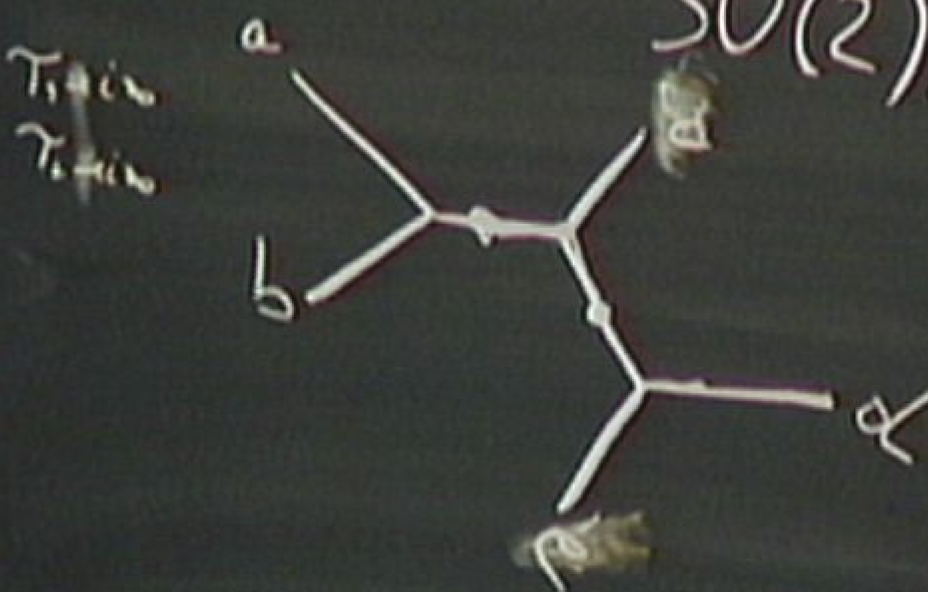


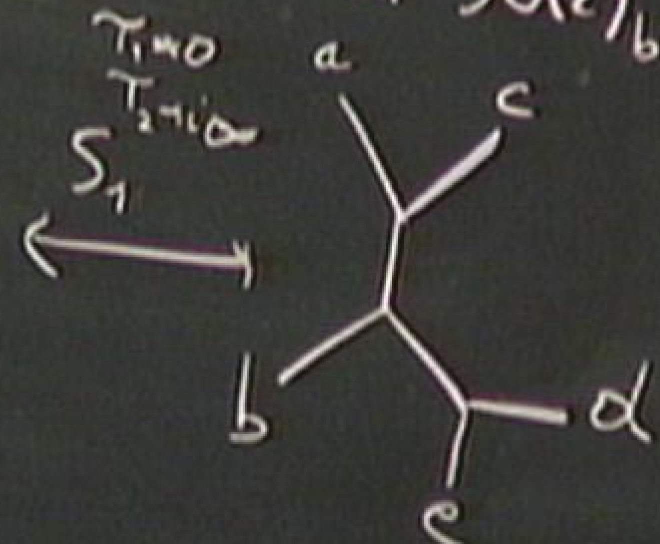
$$R = \underset{\substack{\uparrow \\ SU(2)_c}}{2_1 \otimes 2_2} + \underset{\substack{\uparrow \\ SU(2)_a \times SU(2)_b}}{2_1^{\oplus 2}} + \underset{\substack{\uparrow \\ SU(2)_d \times SU(2)_e}}{2_2^{\oplus 2}}$$

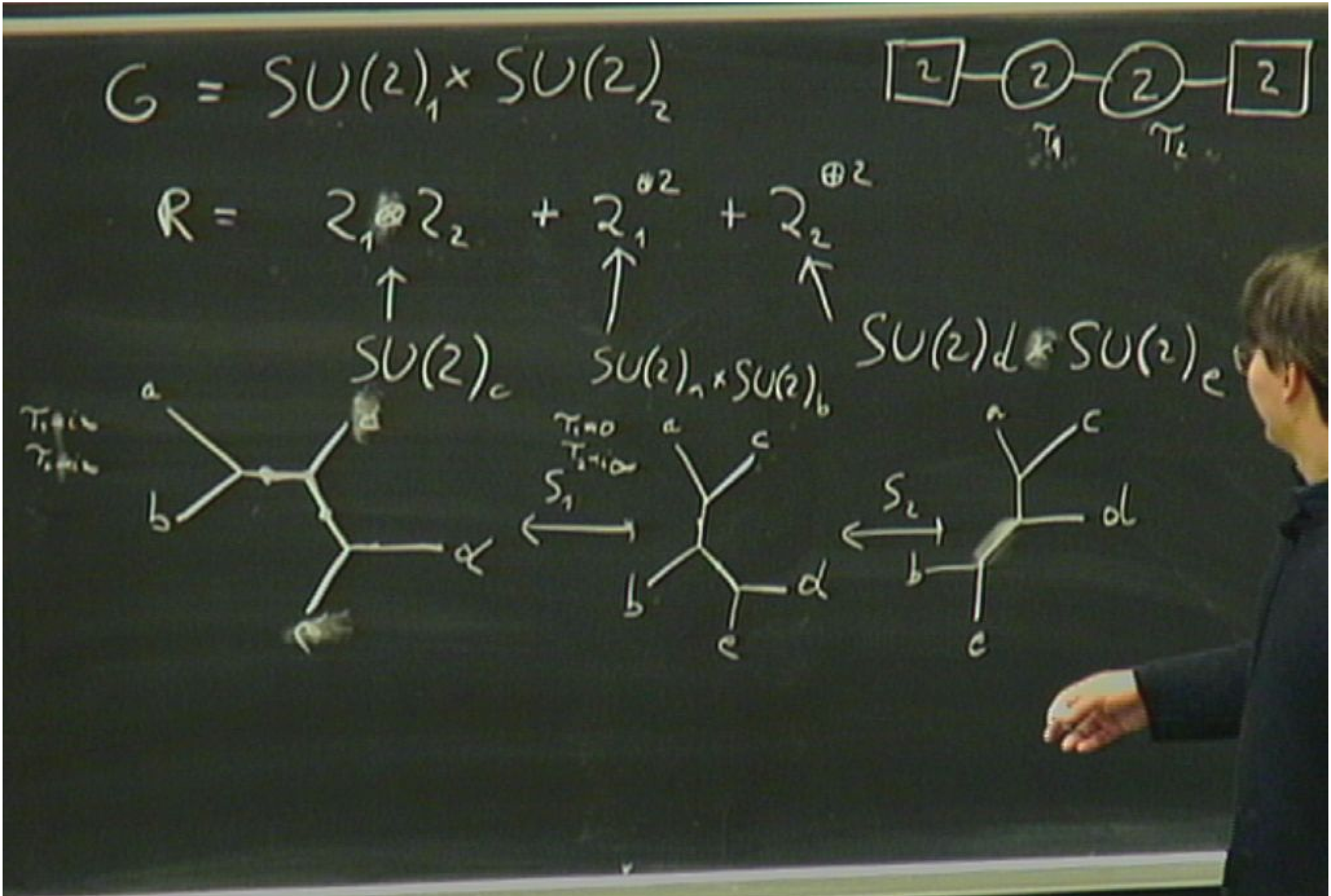


$$R = 2_1 \otimes 2_2 + 2_1^{\oplus 2} + 2_2^{\oplus 2}$$

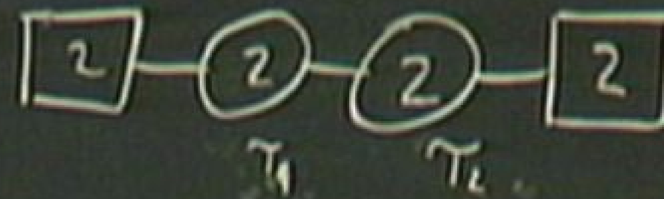
$SU(2)_c$ $\qquad$ $SU(2)_a \times SU(2)_b$ $\qquad$ $SU(2)$





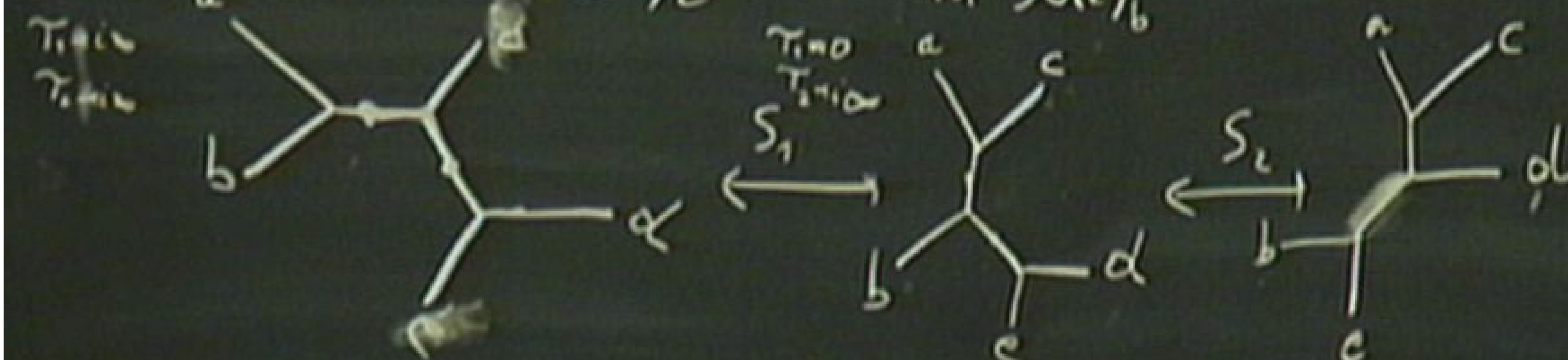

G = SU(2)_1 × SU(2)_2
R = 2_1 ⊗ 2_2 + 2_1^{⊕2} + 2_2^{⊕2}
SU(2)_c
SU(2)_a × SU(2)_b
SU(2)_d × SU(2)_e
S_1
S_2

$$G = SU(2)_1 \times SU(2)_2$$



$$R = 2_1 \otimes 2_2 + 2_1^{\oplus 2} + 2_2^{\oplus 2}$$

$SU(2)_c$ $\quad$ $SU(2)_a \times SU(2)_b$ $\quad$ $SU(2)_d \times SU(2)_e$



$G = SU(2)_1 \times SU(2)_2$

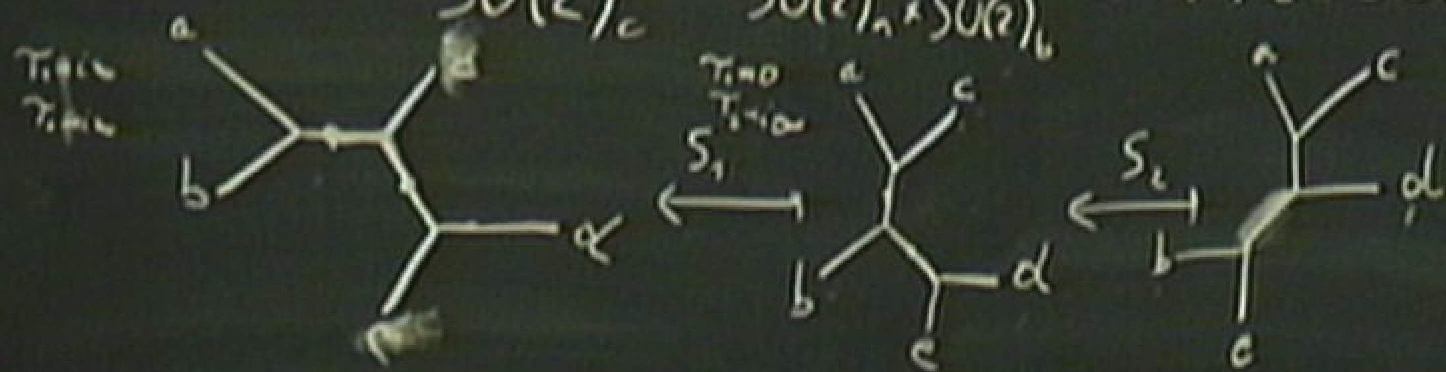
$\boxed{2} - (2) - (2) - \boxed{2}$

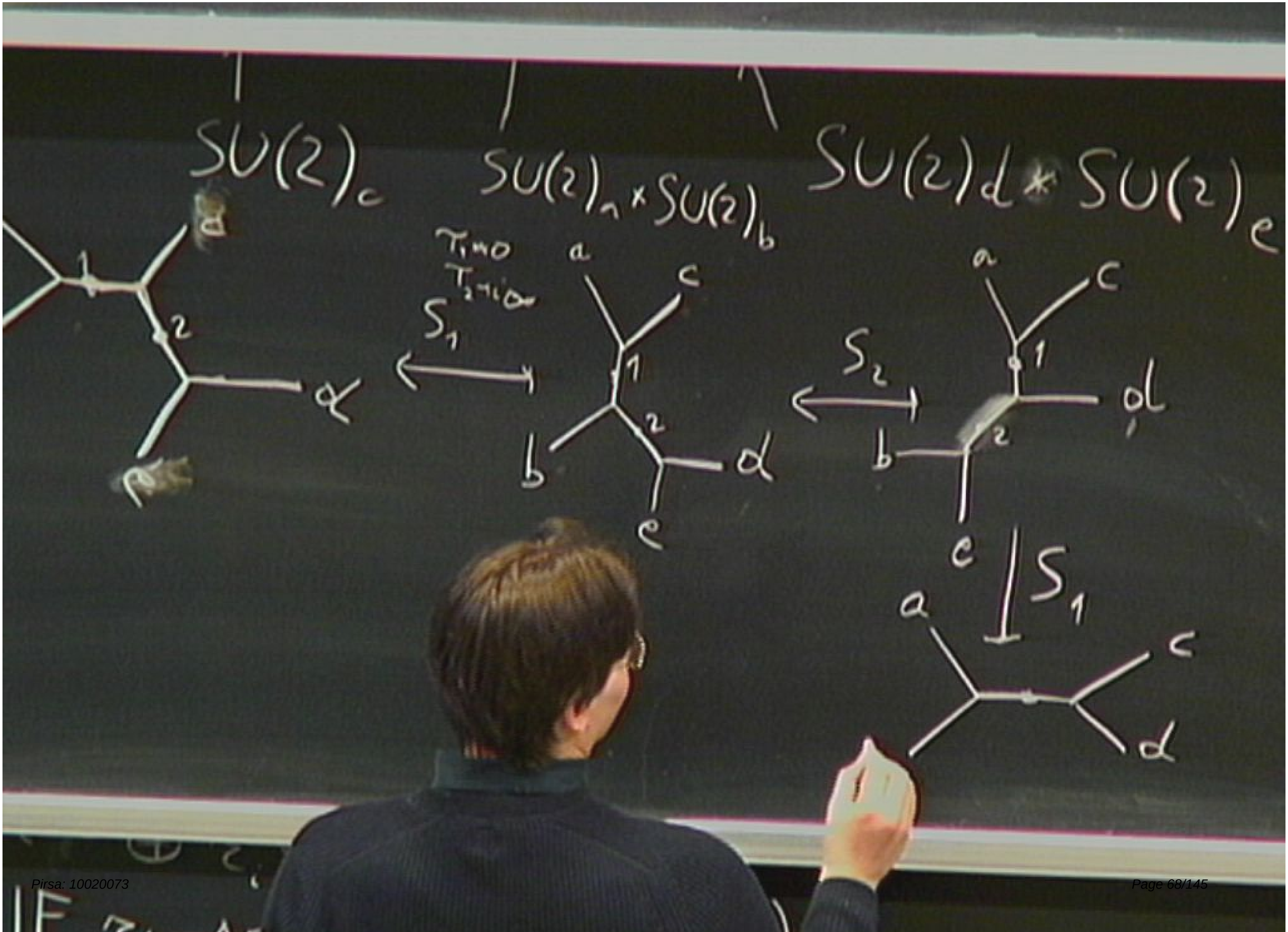
$\tau_1 \quad \tau_2$

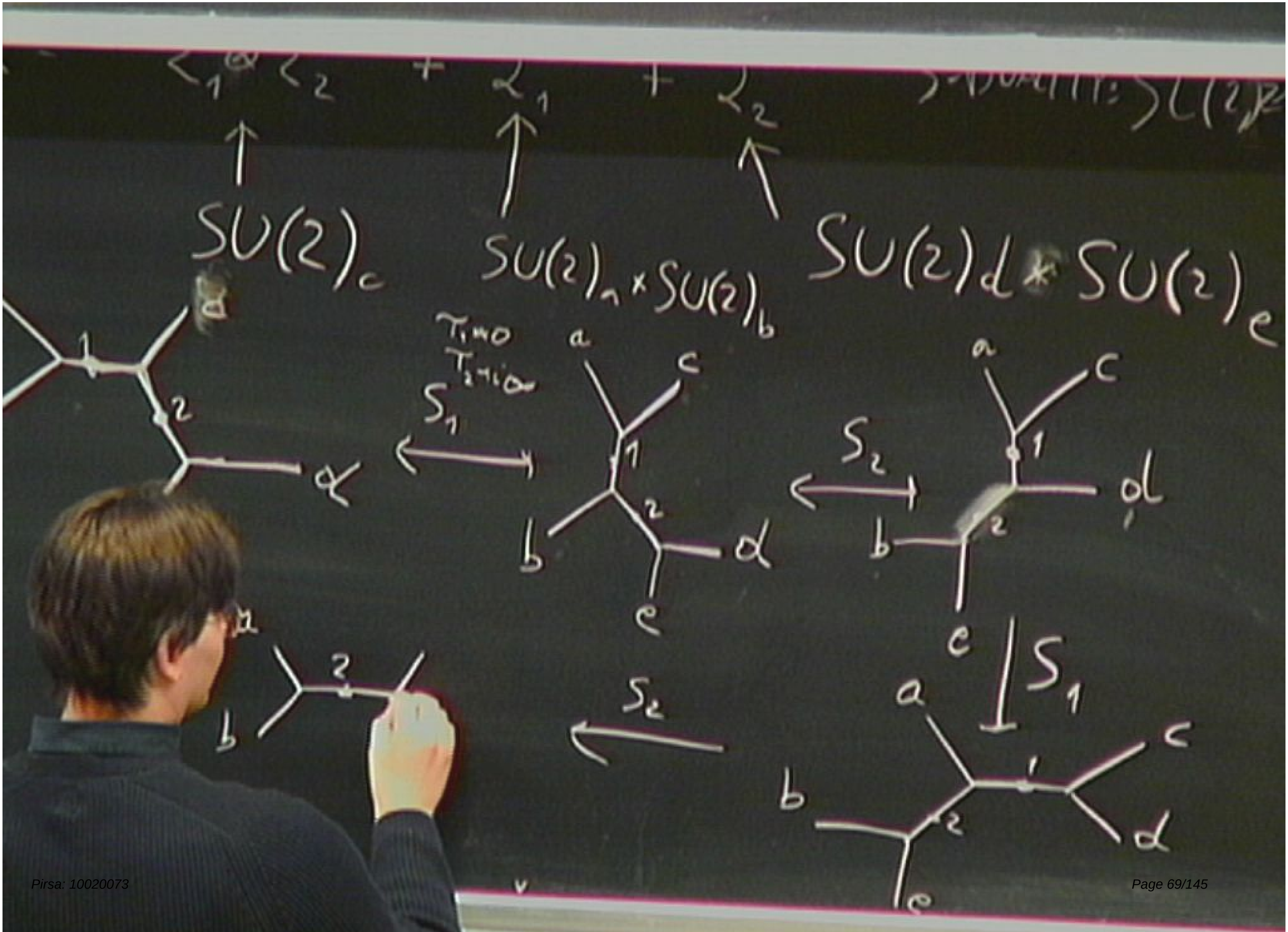
$R = 2_1 \otimes 2_2 + 2_1^{\oplus 2} + 2_2^{\oplus 2}$

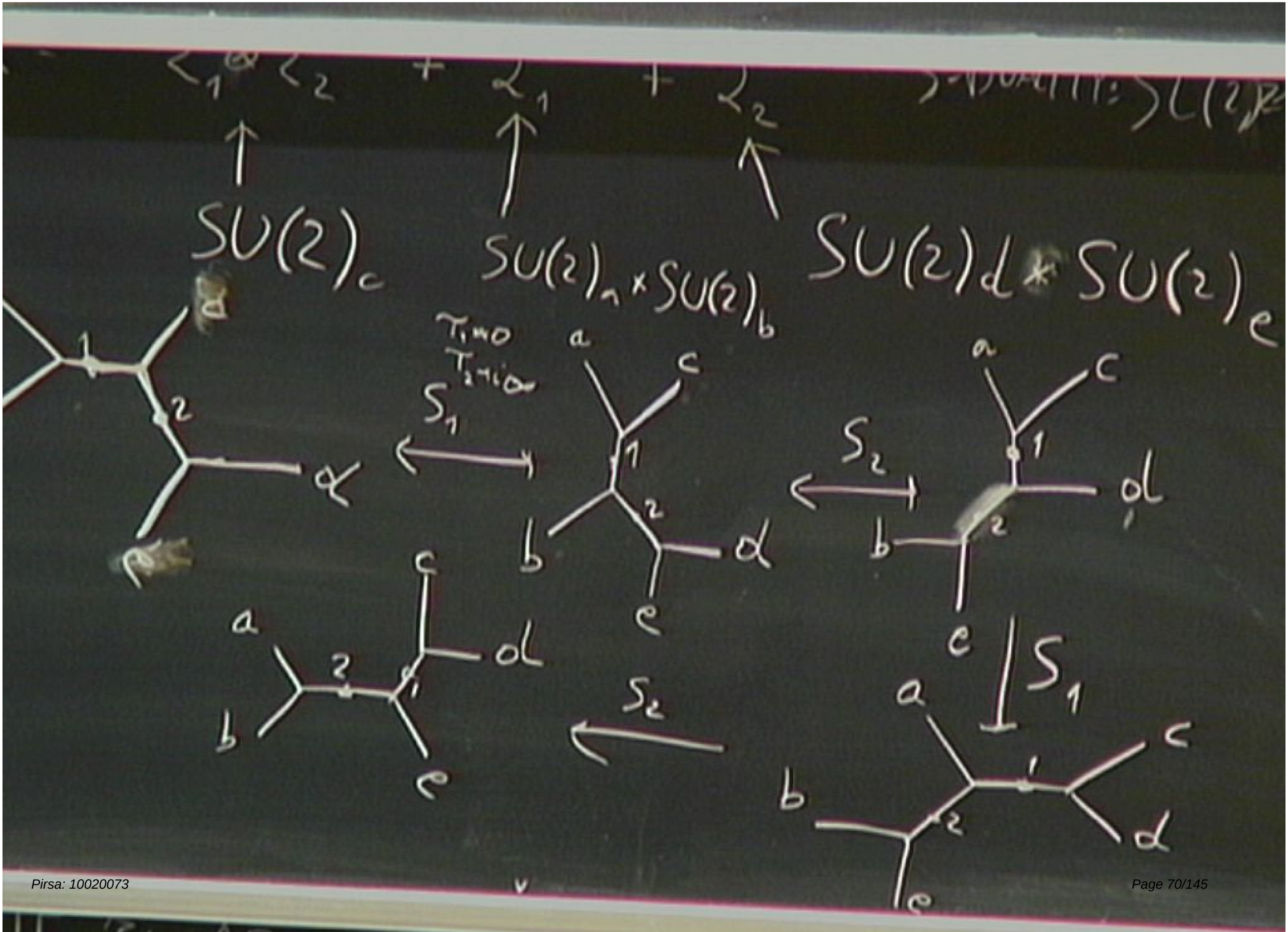
S-DUALITY: $SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z})$?

$SU(2)_c \qquad SU(2)_a \times SU(2)_b \qquad SU(2)_d \times SU(2)_e$




SU(2)c
SU(2)a × SU(2)b
SU(2)d * SU(2)e
S1
S2
S1
a
b
c
d
e
1
2

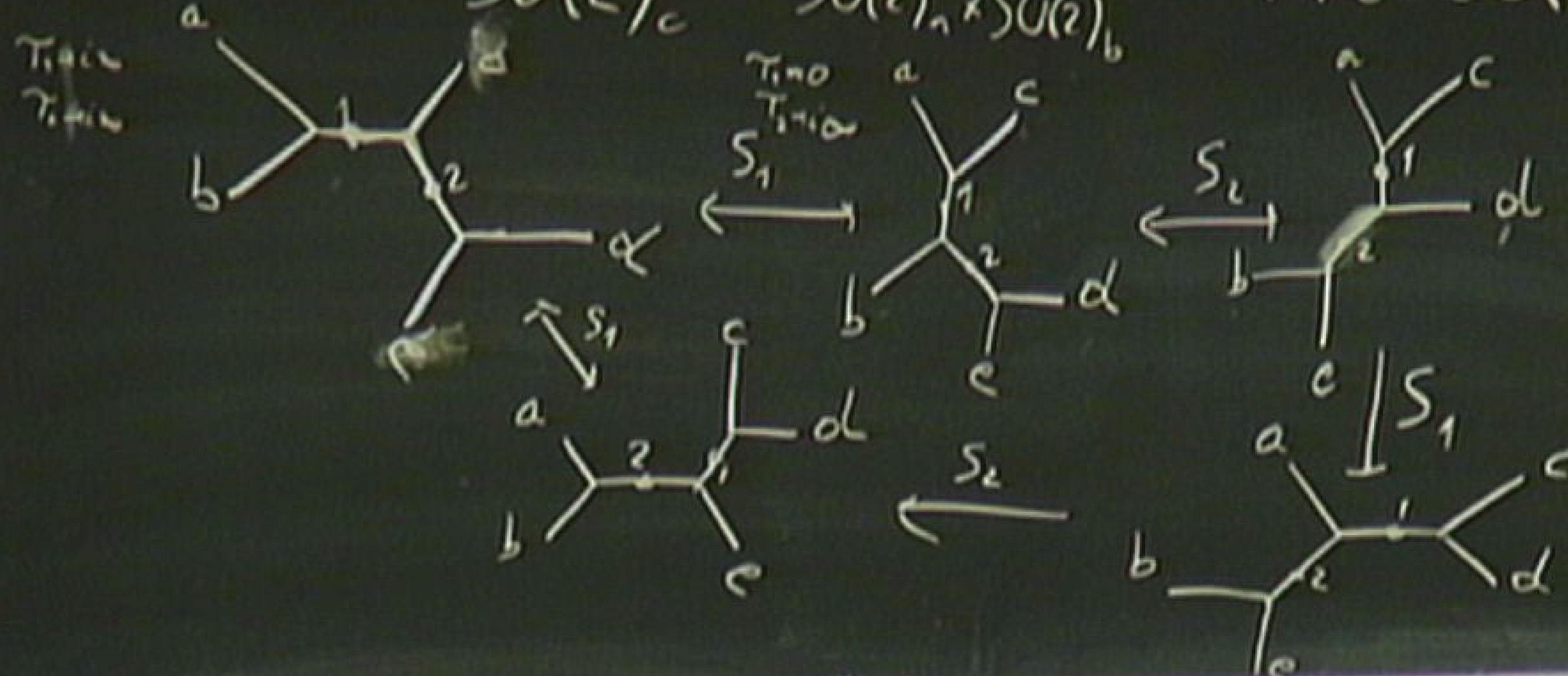

SU(2)c
SU(2)a × SU(2)b
SU(2)d × SU(2)e
S1
S2
S2
S1



SU(2)c
SU(2)a × SU(2)b
SU(2)d * SU(2)e
S1
S2
a
b
c
d
e
1
2

$R = \epsilon_1 \otimes \epsilon_2 + \alpha_1 + \alpha_2$

$SU(2)_c \qquad SU(2)_a \times SU(2)_b \qquad SU(2)_d \times SU(2)_e$

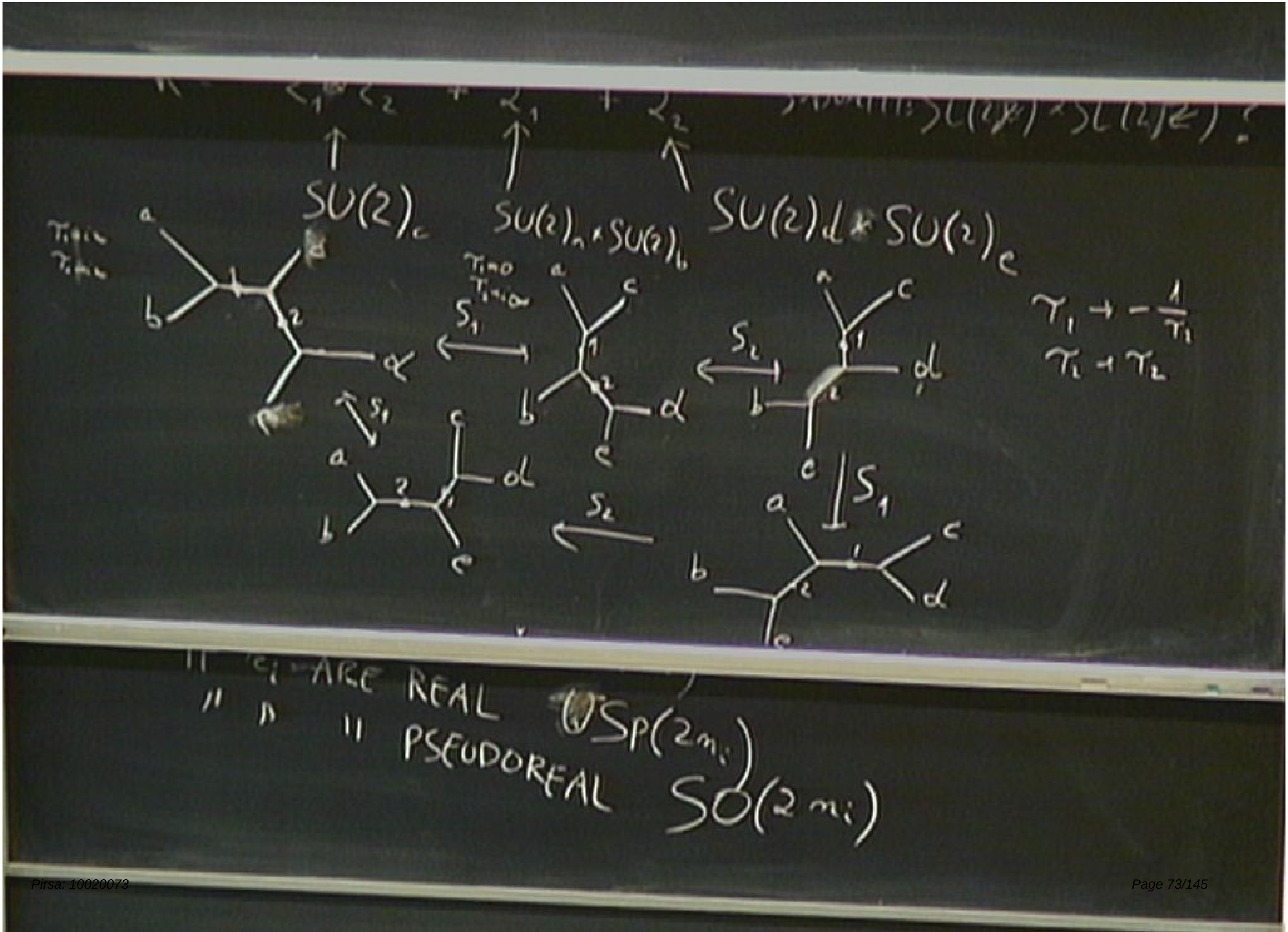


'' c_i ARE REAL $Sp(2n_i)$

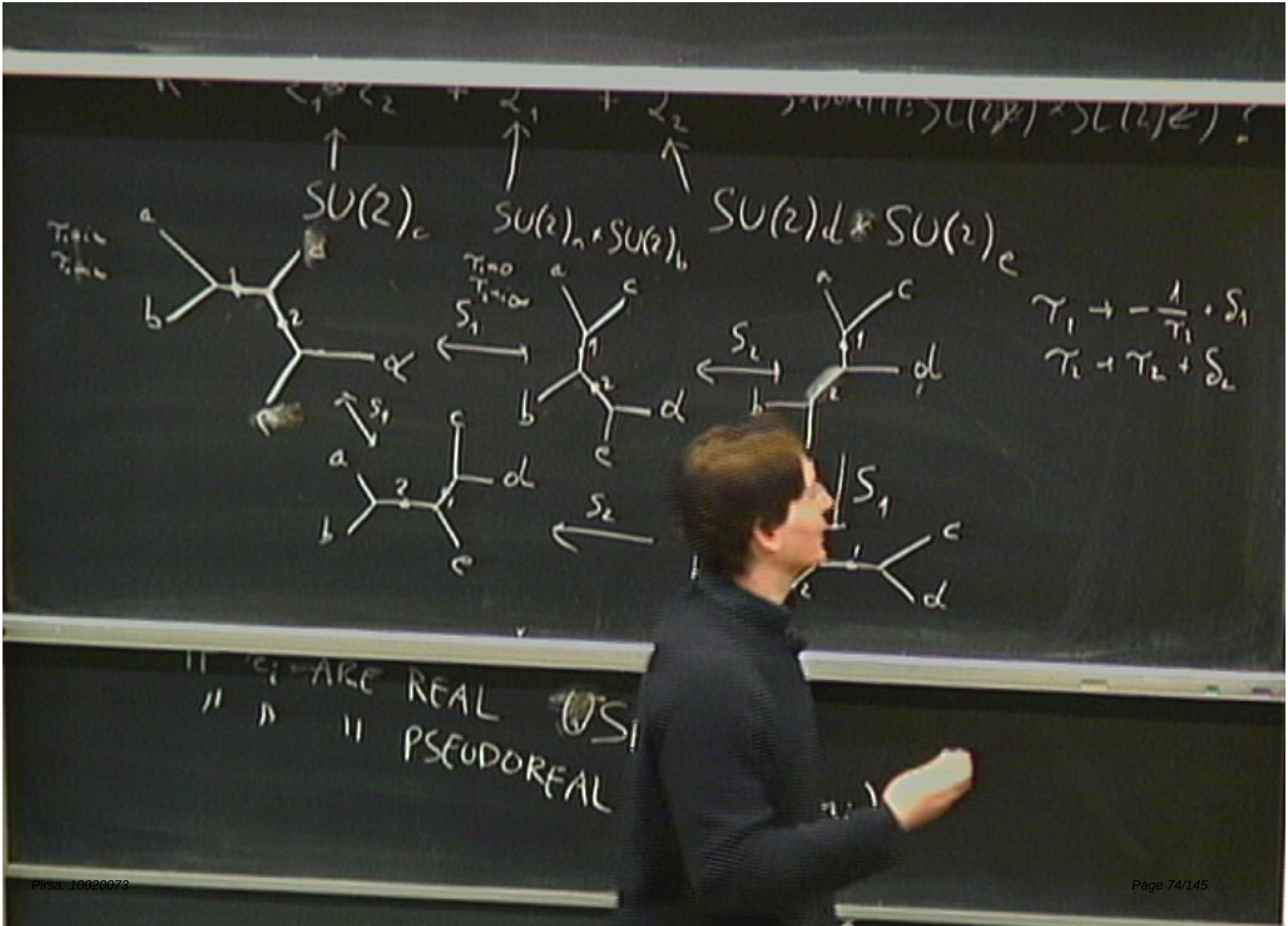
'' '' '' PSEUDOREAL $SO(2n_i)$

$$\tau_1 \rightarrow -\frac{1}{\tau_2}$$

$$\tau_2 \rightarrow \tau_2$$



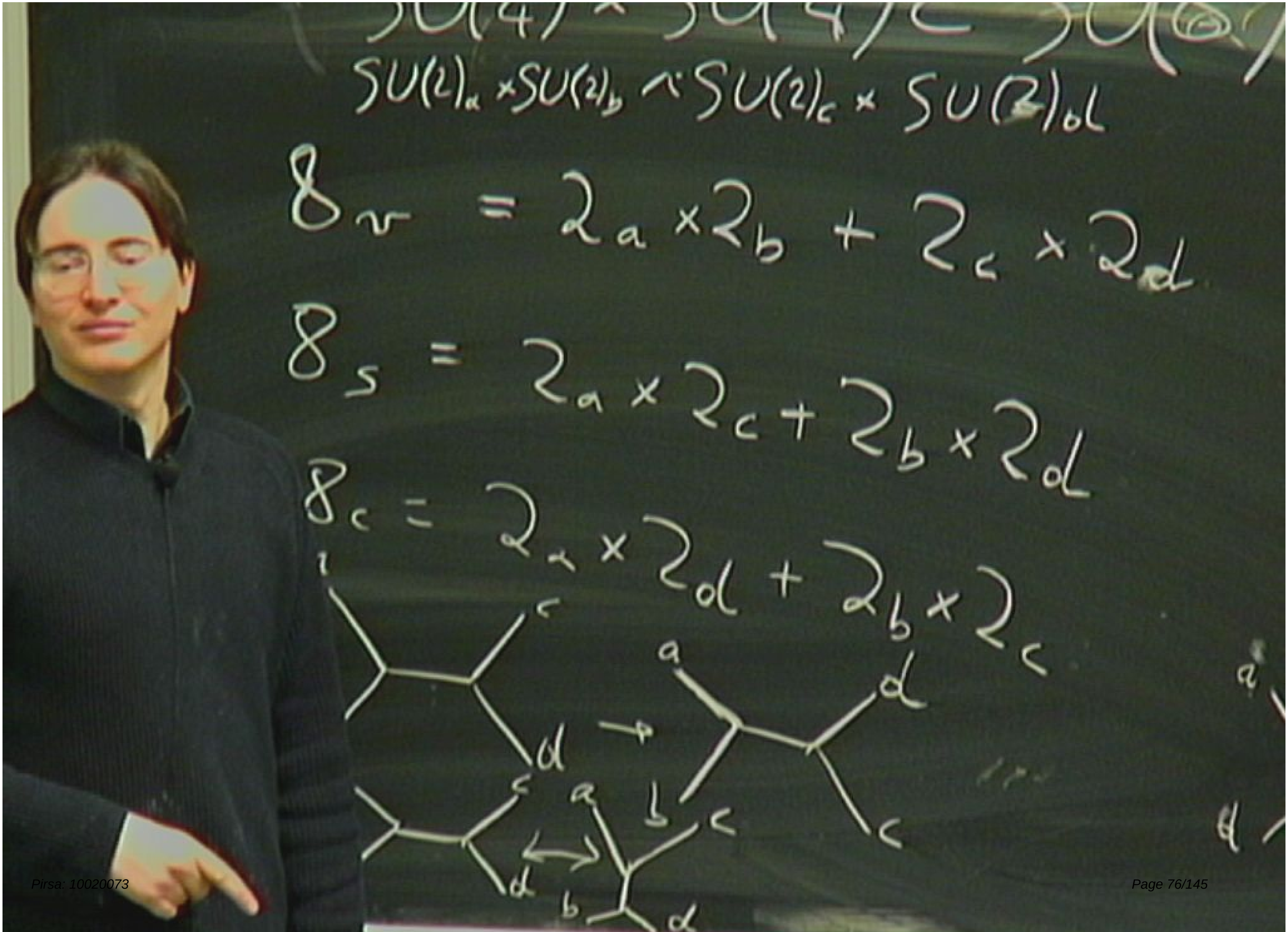
SU(2)_c
SU(2)_a × SU(2)_b
SU(2)_d × SU(2)_e
S_1
S_2
S_1
S_2
S_1
τ_1 → −1/τ_1
τ_2 → τ_2
a b c d e
ARE REAL
PSEUDOREAL
Sp(2n_i)
SO(2n_i)


SU(2)c
SU(2)a × SU(2)b
SU(2)d × SU(2)e
S1
S2
τ1 → − 1/τ1 + S1
τ2 → τ2 + S2
ARE REAL
PSEUDOREAL

e

$$\tau_1 \to -\frac{1}{\tau_2} + \delta_1$$

$$\tau_2 \to \tau_2 + \delta_2$$


SU(2)a × SU(2)b ⊂ SU(2)c × SU(2)d
8v = 2a × 2b + 2c × 2d
8s = 2a × 2c + 2b × 2d
8c = 2a × 2d + 2b × 2c

$$SU(4)\times SU(4)\subset SU(8)$$

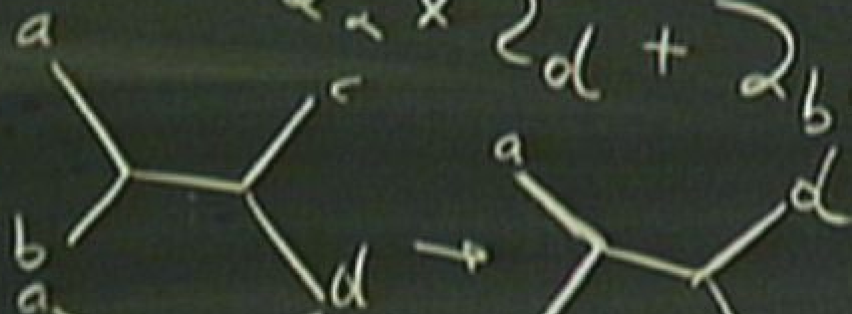
$$SU(2)_a\times SU(2)_b \times SU(2)_c\times SU(2)_d$$

$$8_v = 2_a\times 2_b + 2_c\times 2_d$$

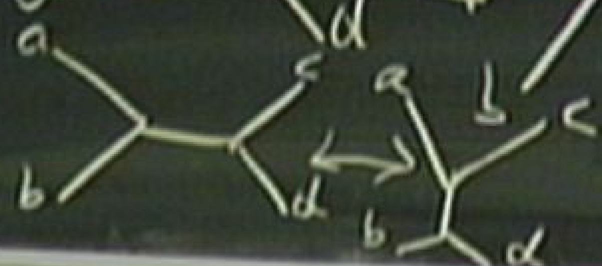
$$8_s = 2_a\times 2_c + 2_b\times 2_d$$

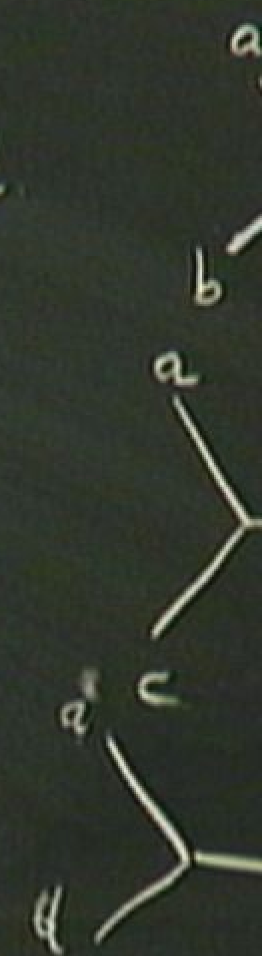
$$8_c = 2_a\times 2_d + 2_b\times 2_c$$

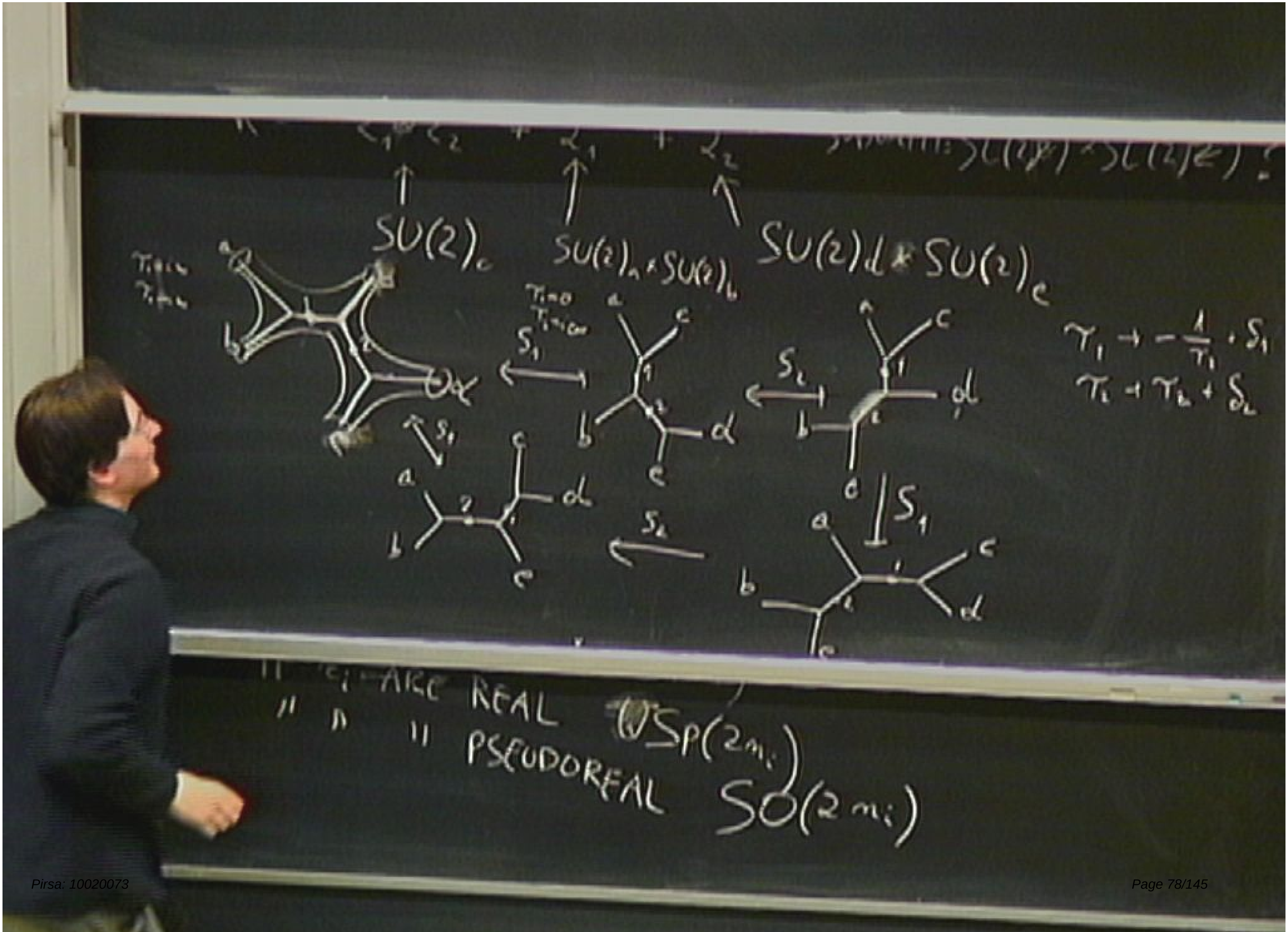
T:

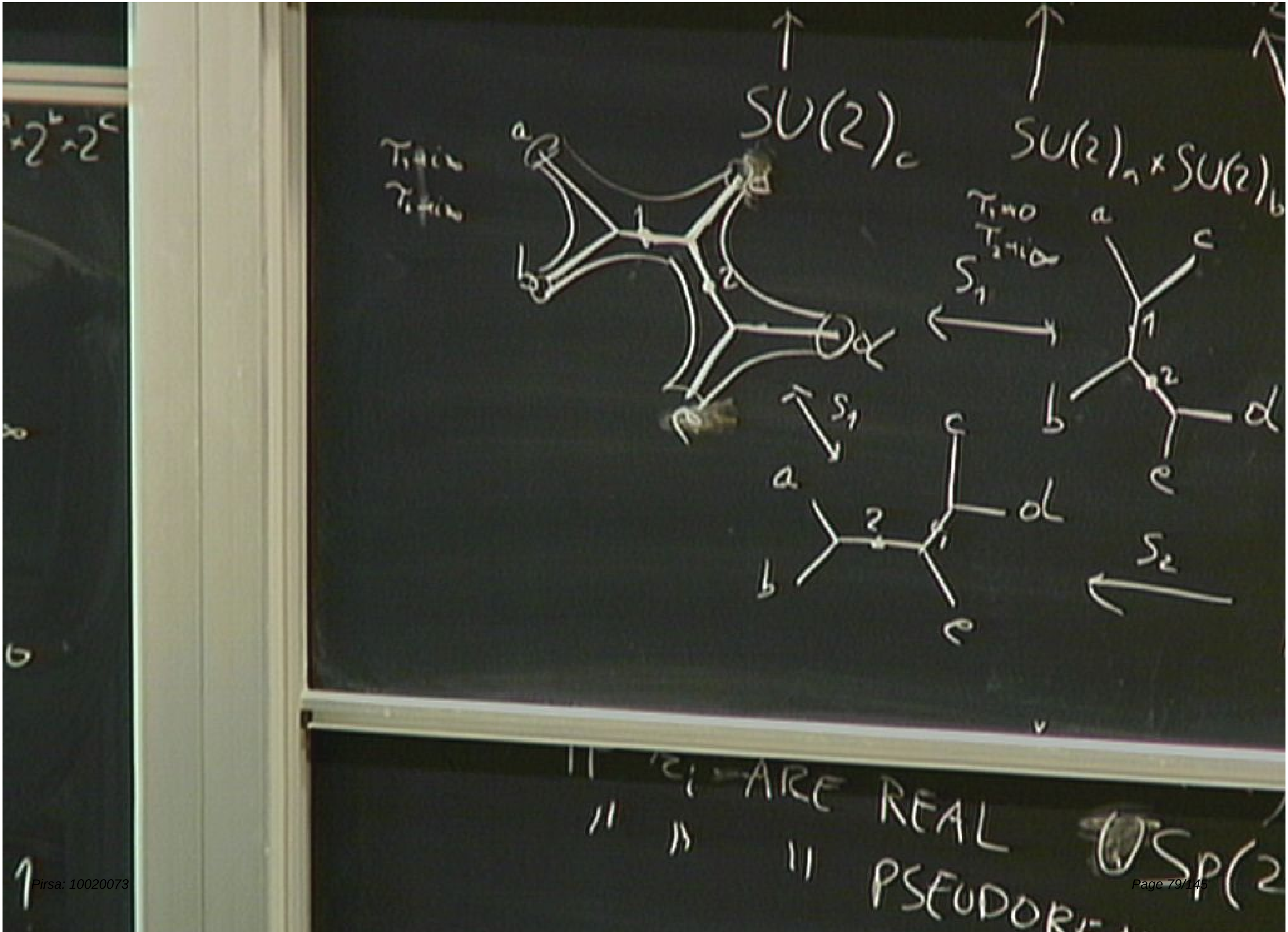


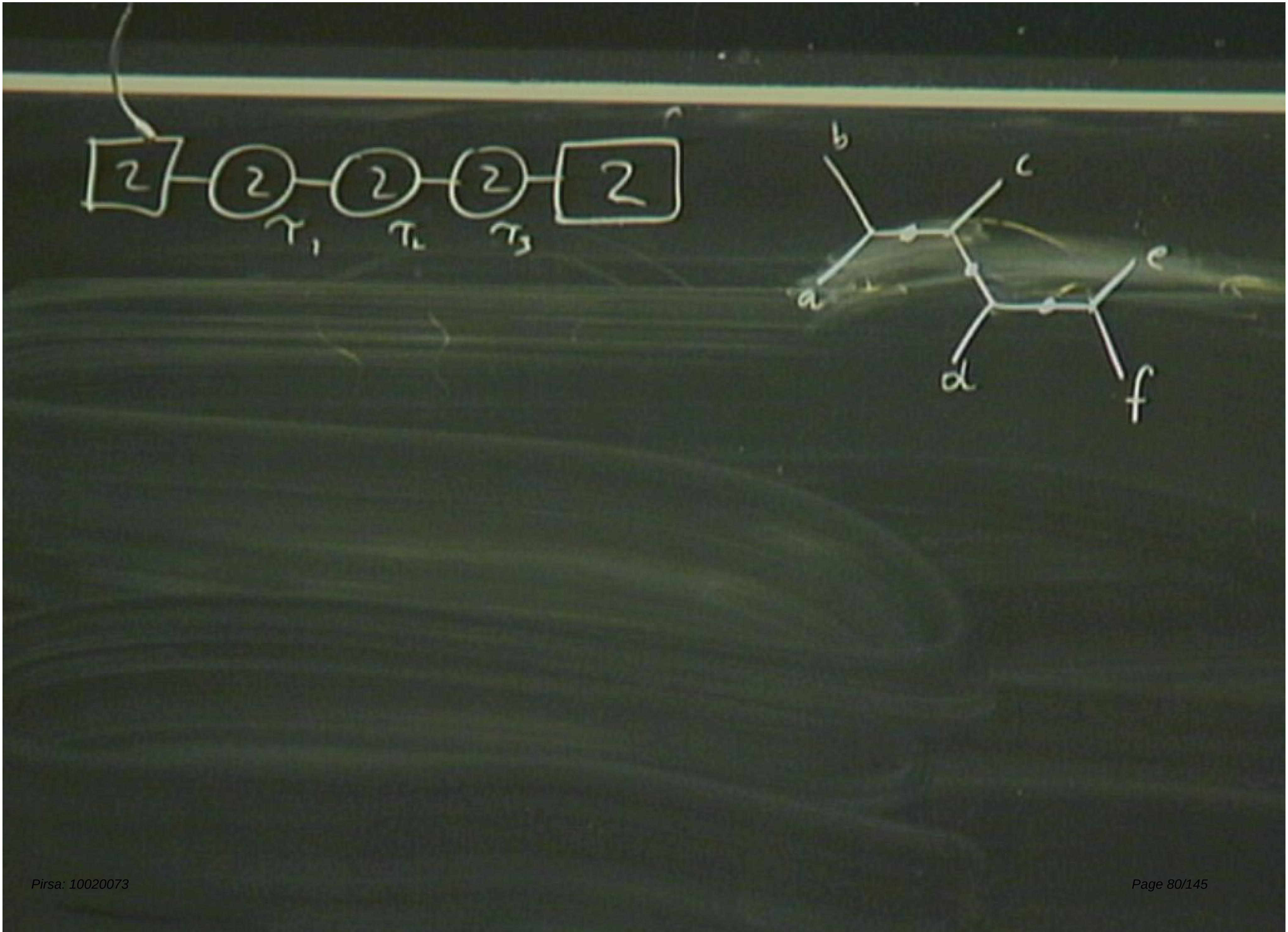
S:

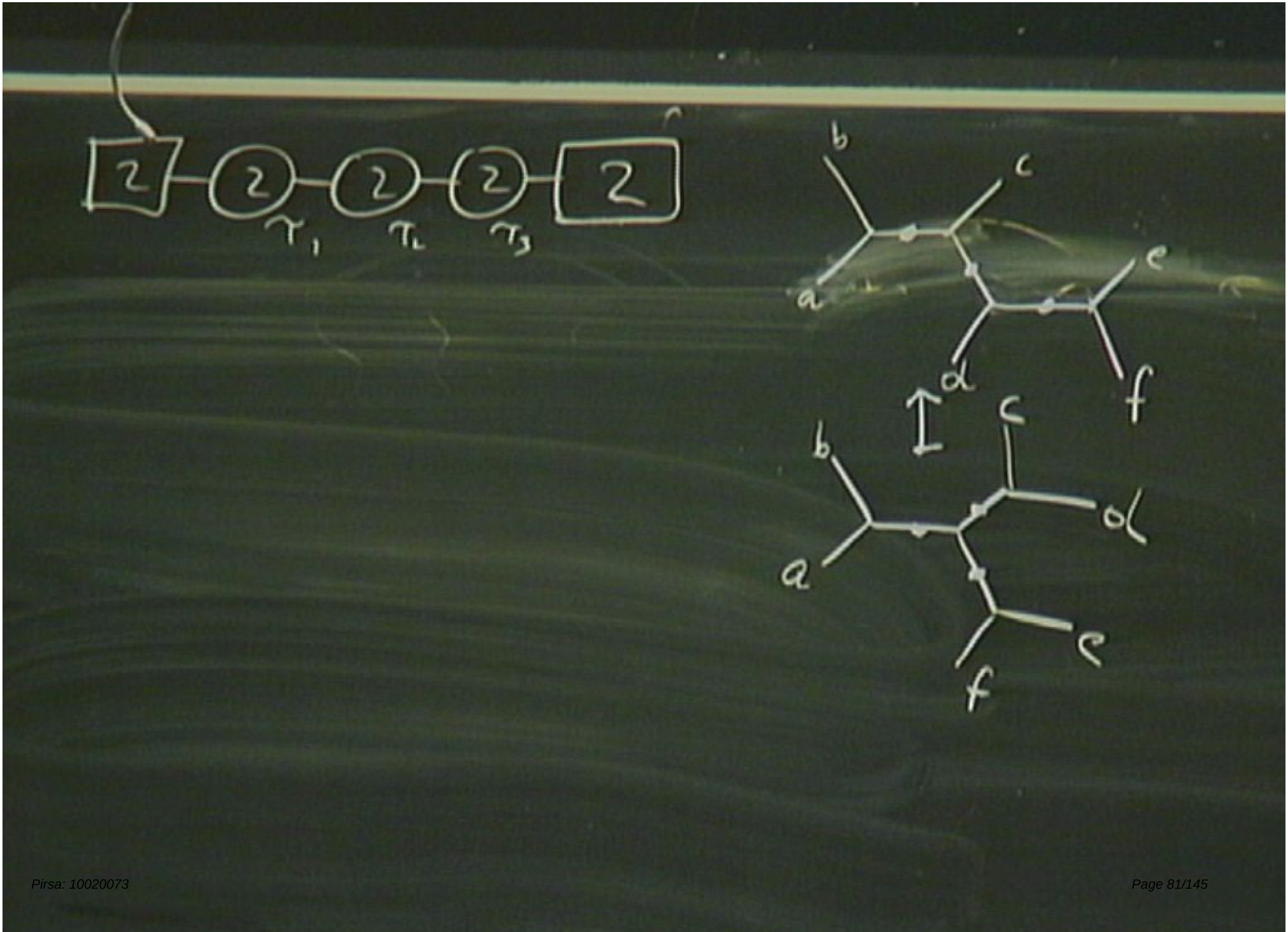


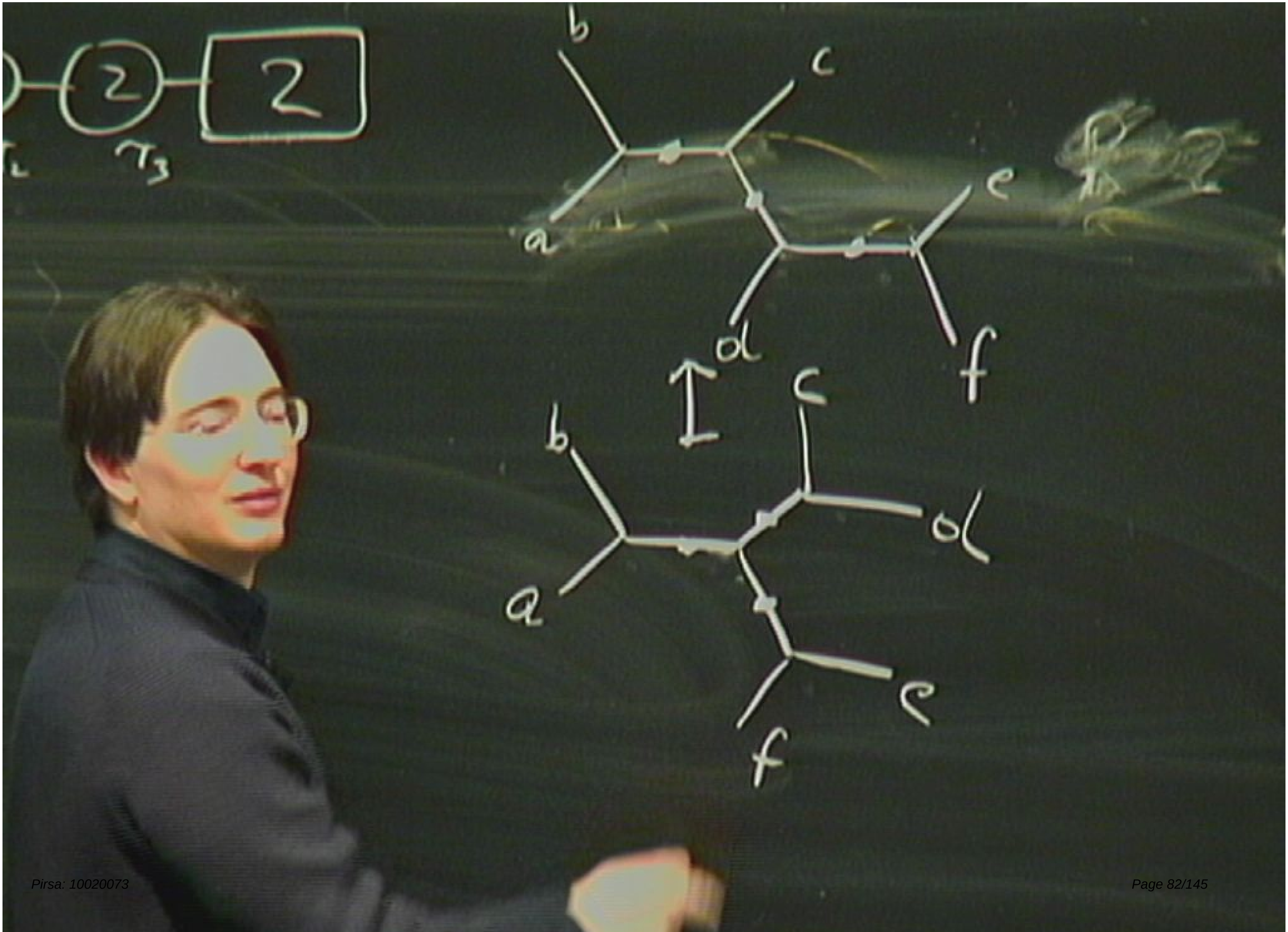


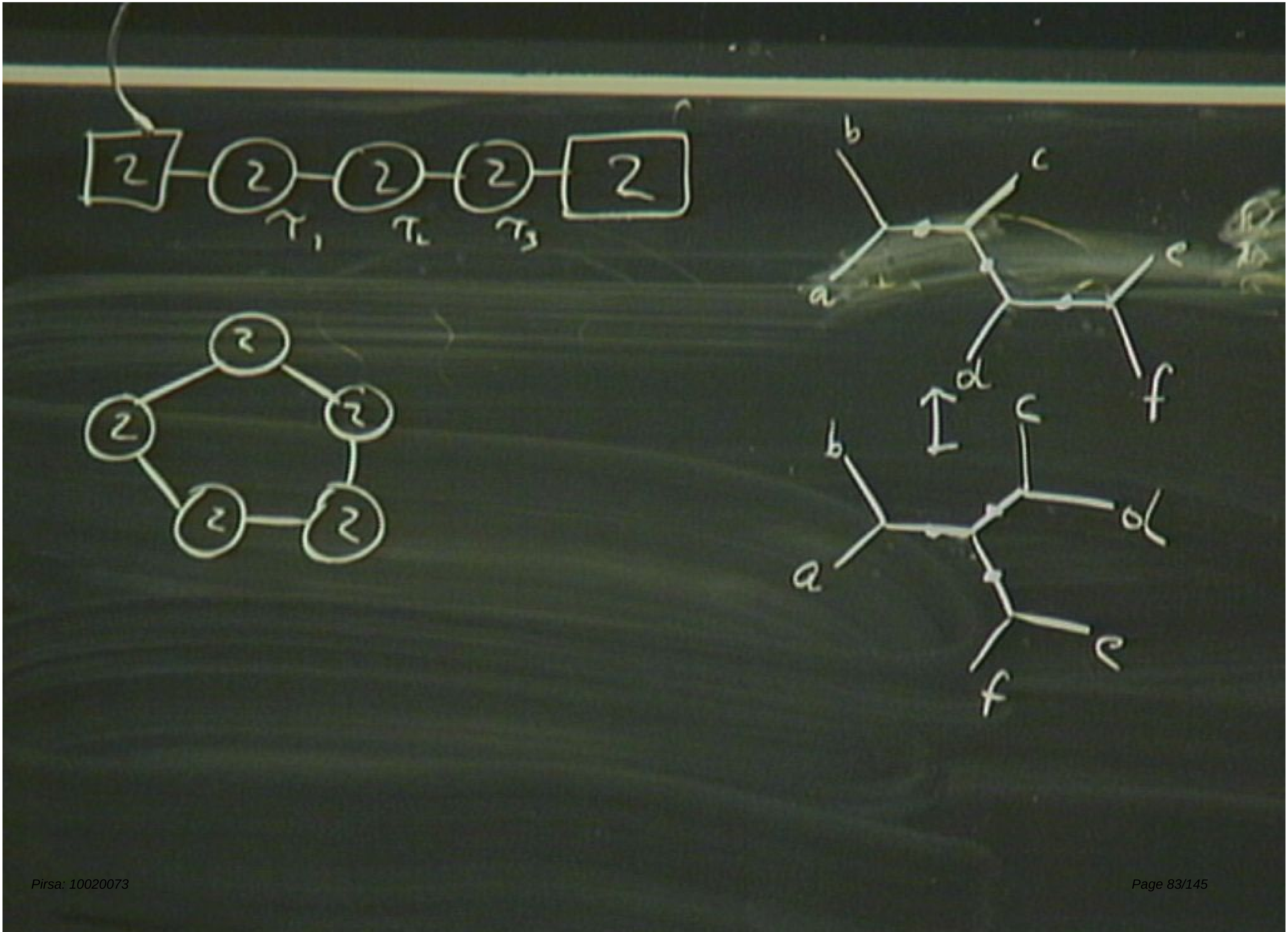

SU(2)c
SU(2)a × SU(2)b
SU(2)d × SU(2)e
S1
S2
τ1 → − 1/τ1 + S1
τ2 → τ2 + S2
" " " ARE REAL
" " " PSEUDOREAL
Sp(2mi)
SO(2mi)


SU(2)c
SU(2)a × SU(2)b
S1
S2
a b c d e
ARE REAL
PSEUDORE

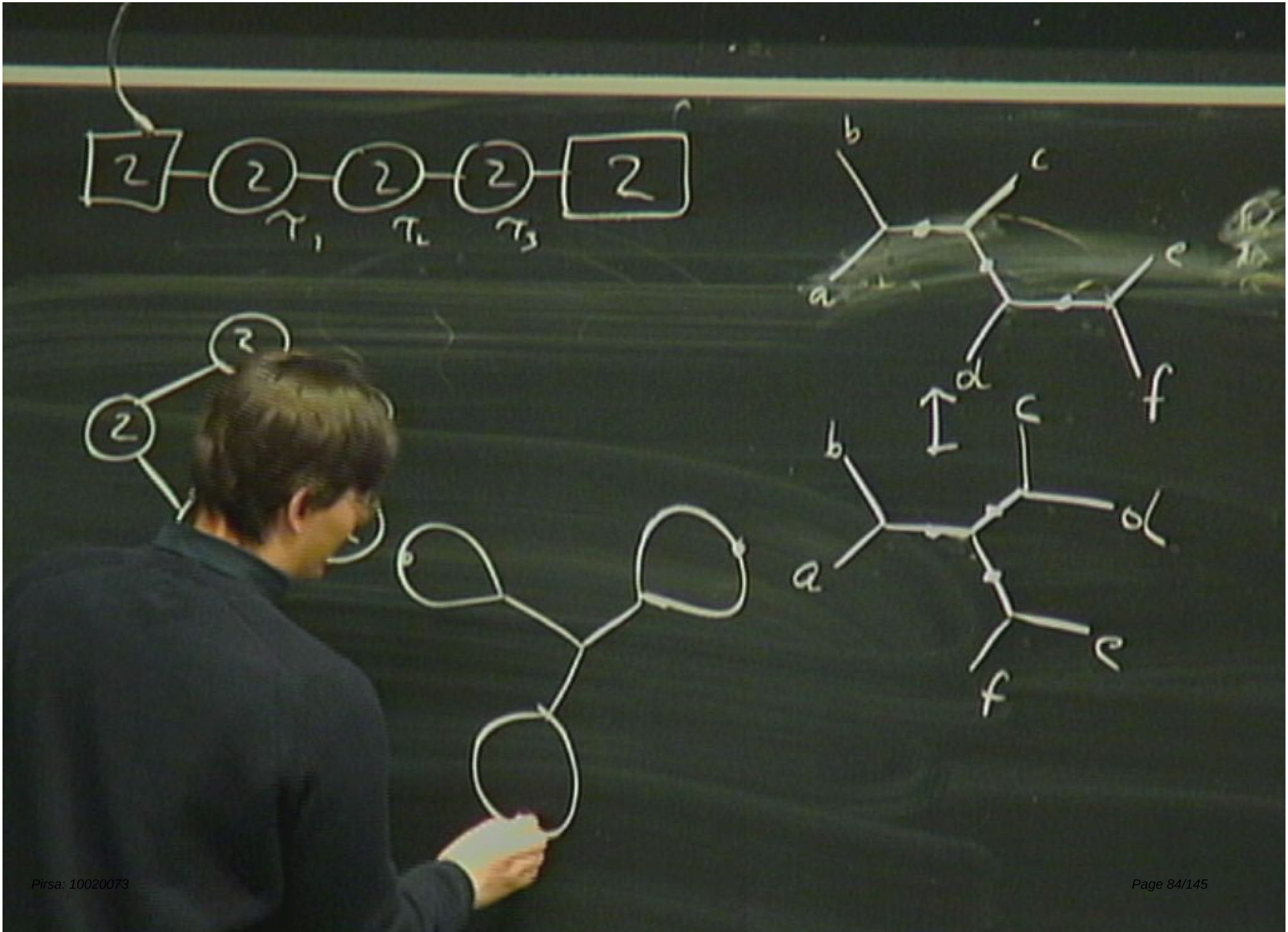

2
2
2
2
2
τ1
τ2
τ3
a
b
c
d
e
f

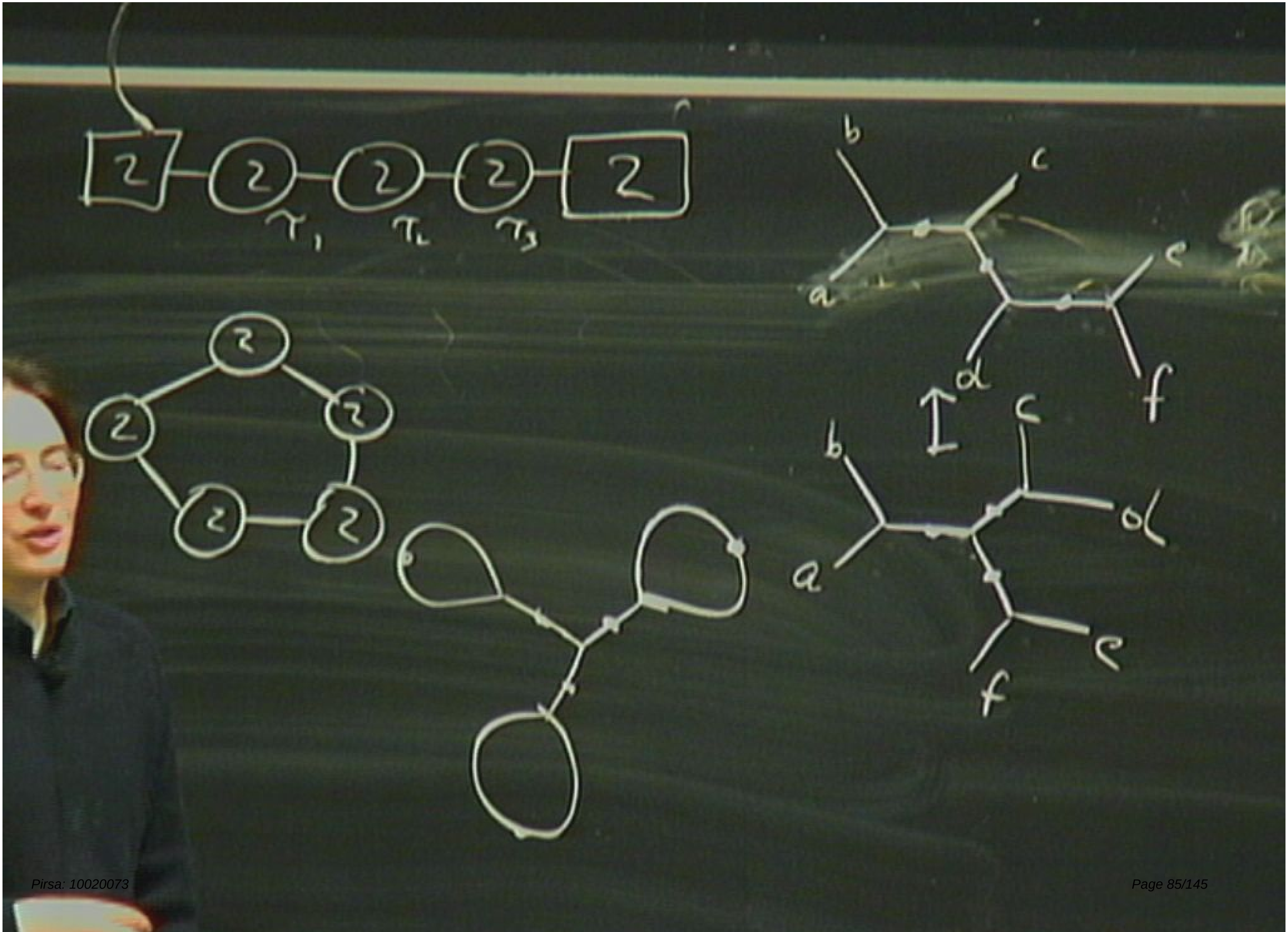

2
2
2
2
2
τ1
τ2
τ3
b
c
a
e
d
f
b
c
d
a
e
f

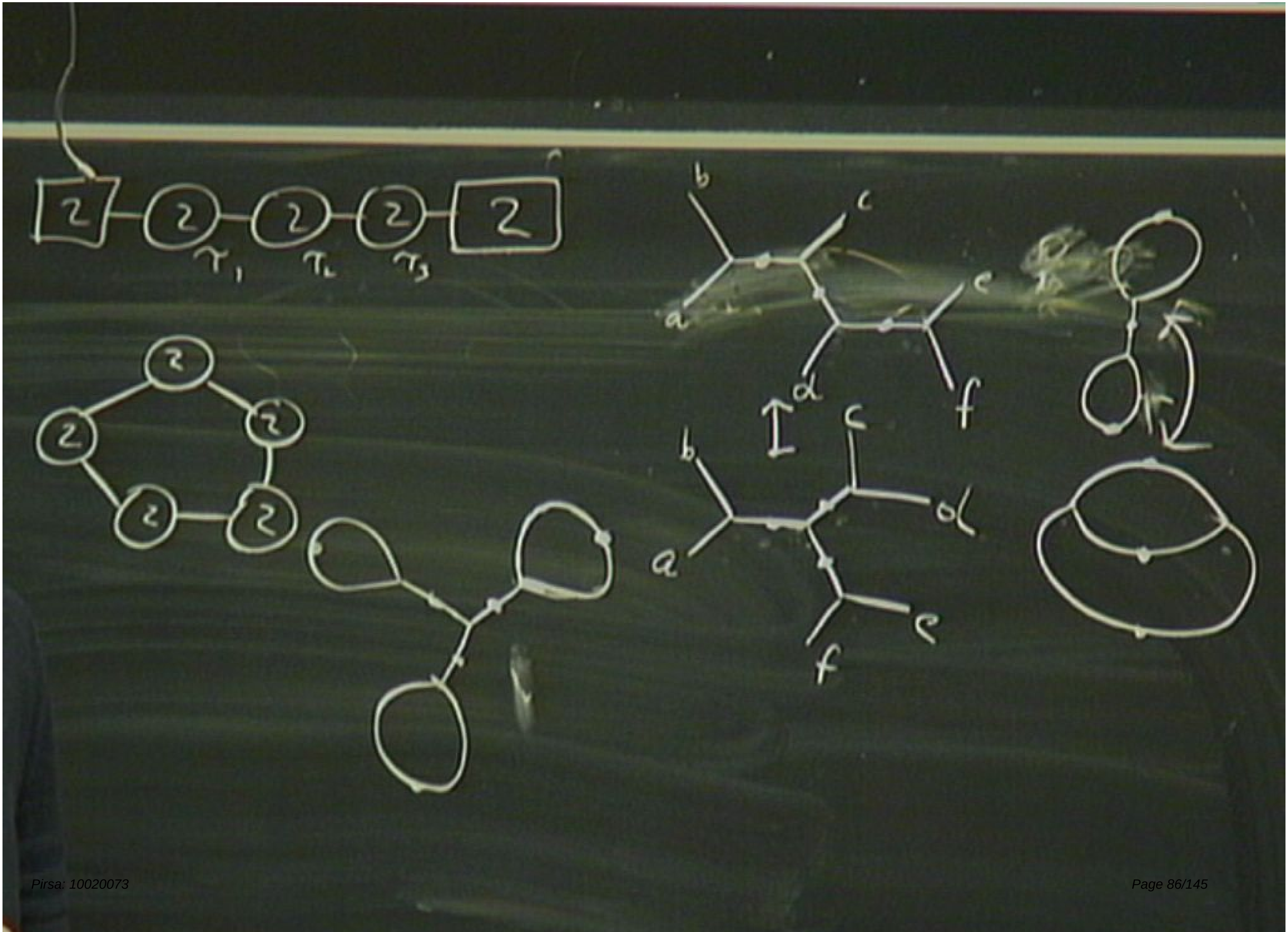

2
2
b
c
a
e
d
f
b
c
a
d
e
f

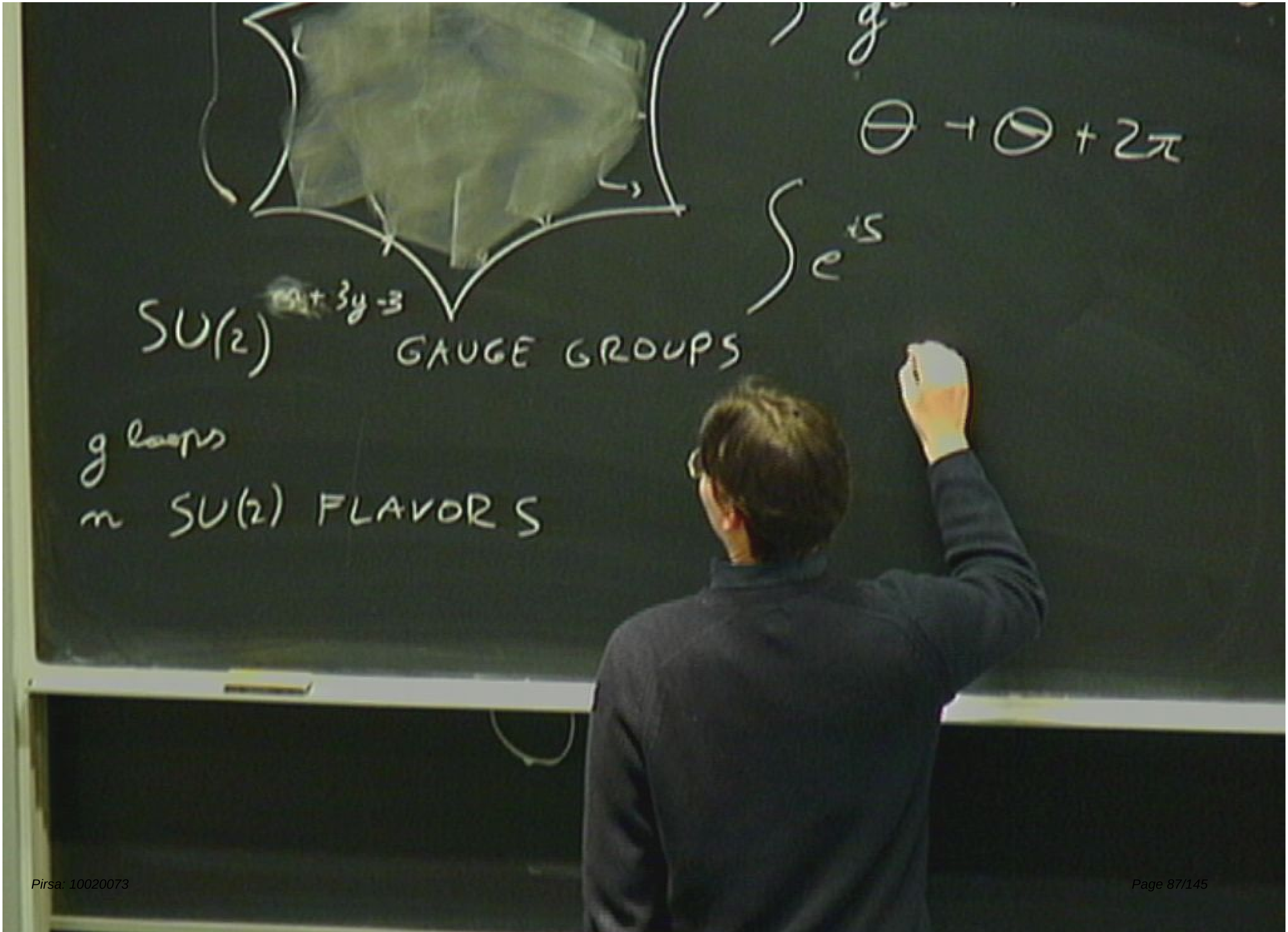


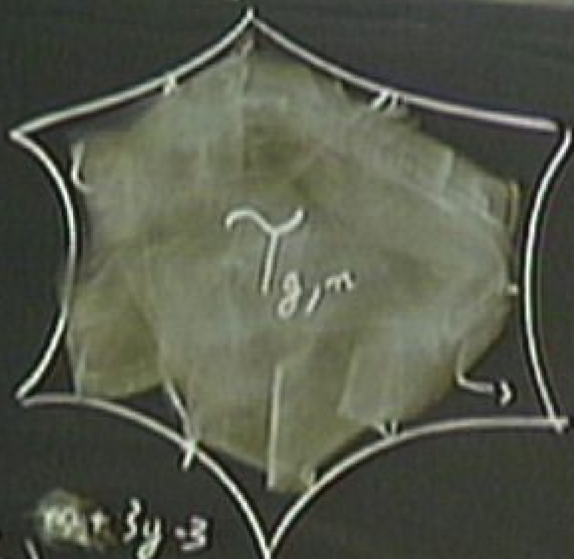
2 — 2 — 2 — 2 — 2
τ1 τ2 τ3
b c a e d f


2
2
2
2
2
τ1
τ2
τ3
b
c
a
e
d
f
I
b
c
a
d
e
f
2
2


2
2
2
2
2
τ1
τ2
τ3
b
c
a
e
d
f
I
c
b
d
a
e
f


2 — 2 — 2 — 2 — 2
τ_1 τ_2 τ_3
b c a e d f
b c a d e f
2 2 2 2 2


θ → θ + 2π
∫ e^{iS}
SU(2)^{…+3g-3}
GAUGE GROUPS
g loops
n SU(2) FLAVORS



$$S = \int \frac{1}{g^2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \theta\, \mathrm{Tr}\, F \wedge F + \cdots$$

$$\theta \to \theta + 2\pi$$

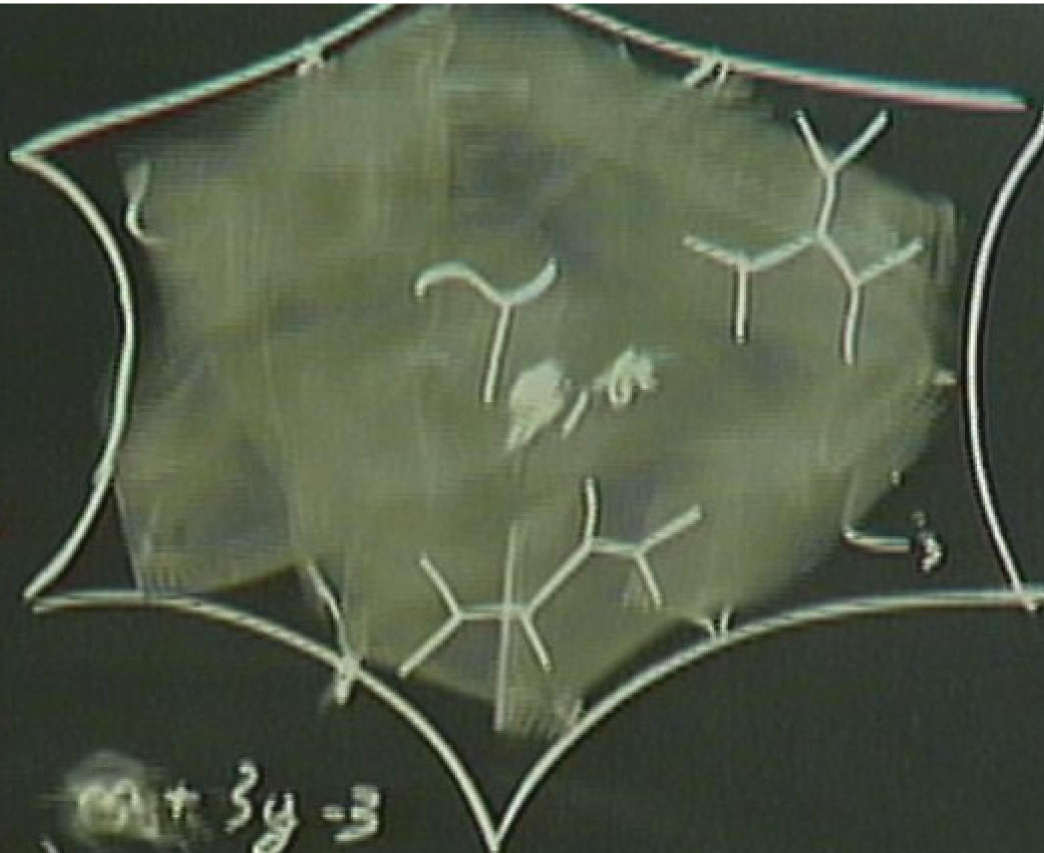
$$\int e^{iS}$$

$SU(2)^{m+3g-3}$ GAUGE GROUPS

$m+2g-2$ TRIFUNDAMENTALS

g loops

m $SU(2)$ FLAVORS



$SU(2)^{n+3g-3}$ GAUGE GROUPS

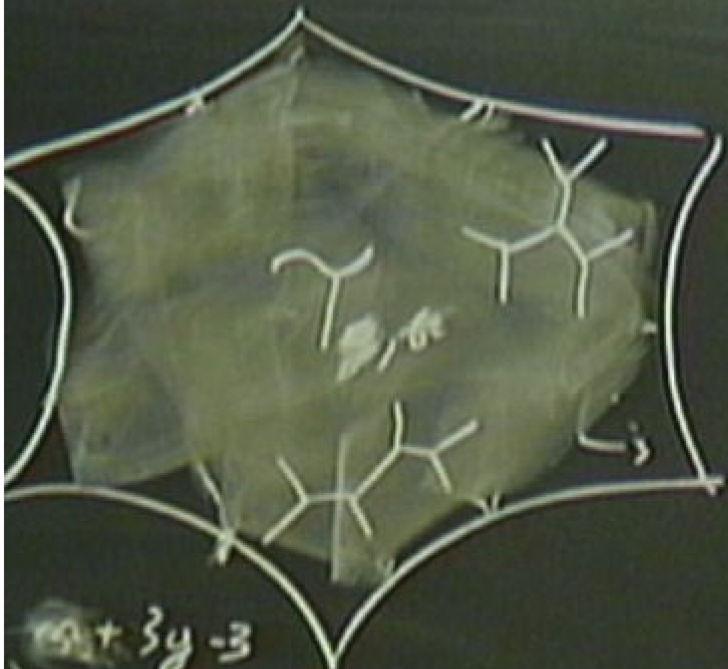
g loops

n $SU(2)$ FLAVOR

$T_{g,m}$

LAGRANGIAN DESCRIPTIONS $\equiv$

$\tau = 1$



$$S = \int \frac{1}{g^2} \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} + \theta \operatorname{Tr} F \wedge F + \dots$$

$$\theta \to \theta + 2\pi$$

$$\int e^{iS}$$

$n + 3g - 3$

1) GAUGE GROUPS

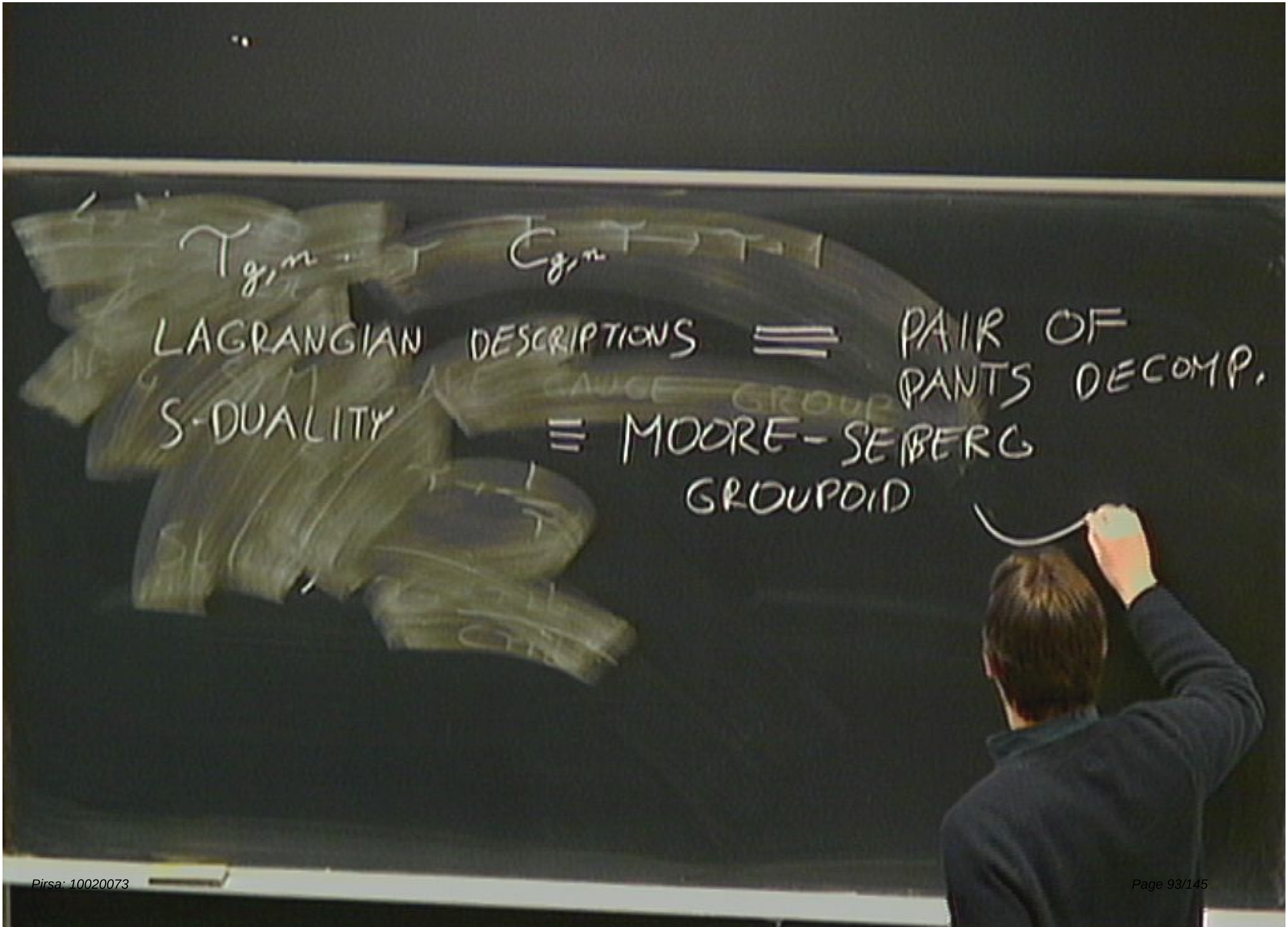
$n + 2g - 2$ TRIFUNDAMENTAL

2) FLAVORS

$T_{g,n} \to C_{g,n}$

LAGRANGIAN DESCRIPTIONS $\equiv$ PAIR OF PANTS DECOMP.

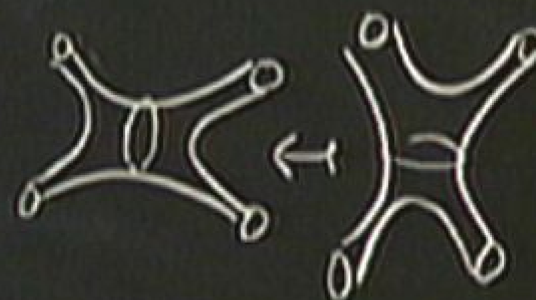
S-DUA

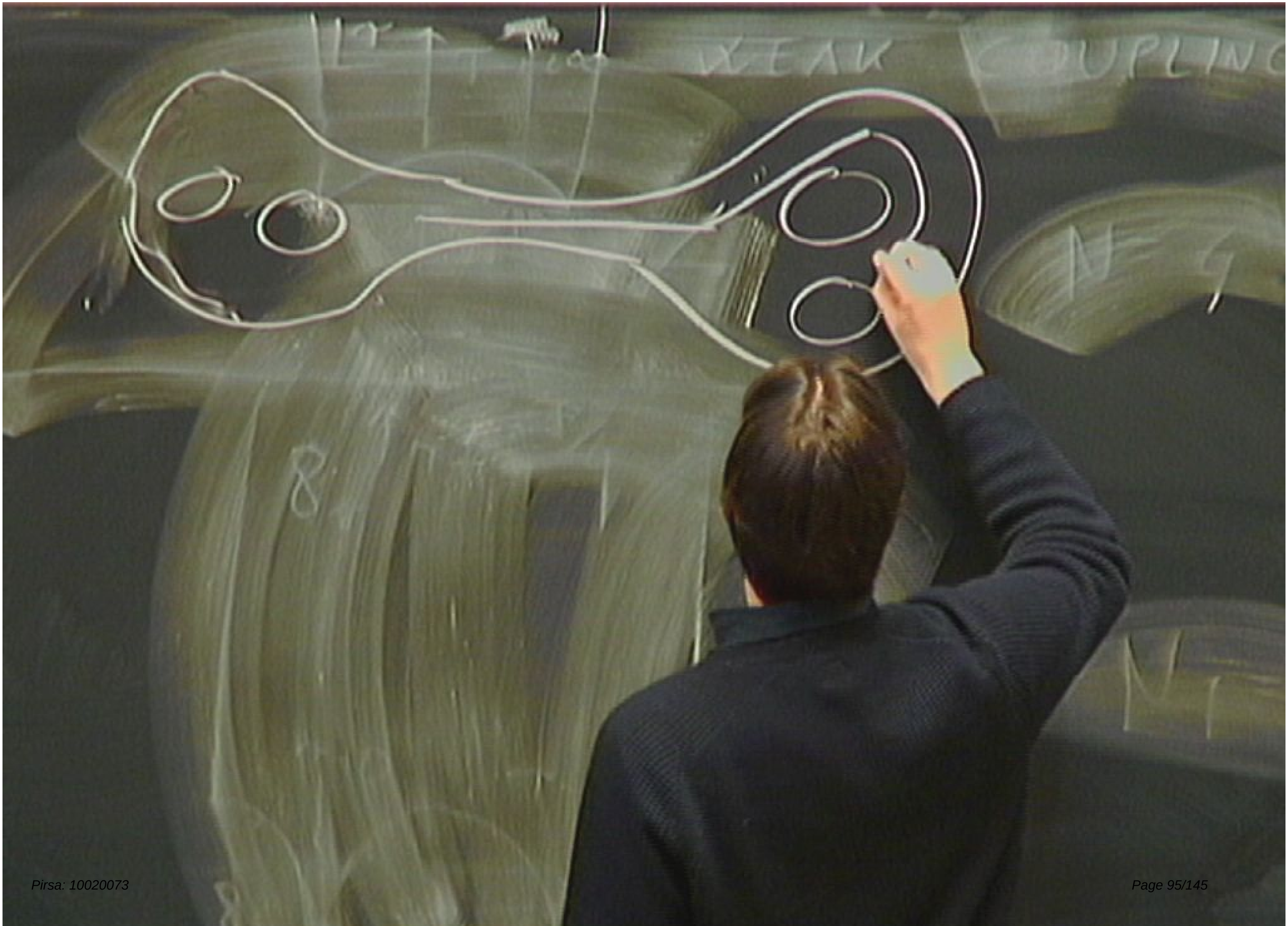

$T_{g,n} \to C_{g,n}$
LAGRANGIAN DESCRIPTIONS ≡ PAIR OF PANTS DECOMP.
S-DUALITY ≡ MOORE-SEIBERG GROUPOID

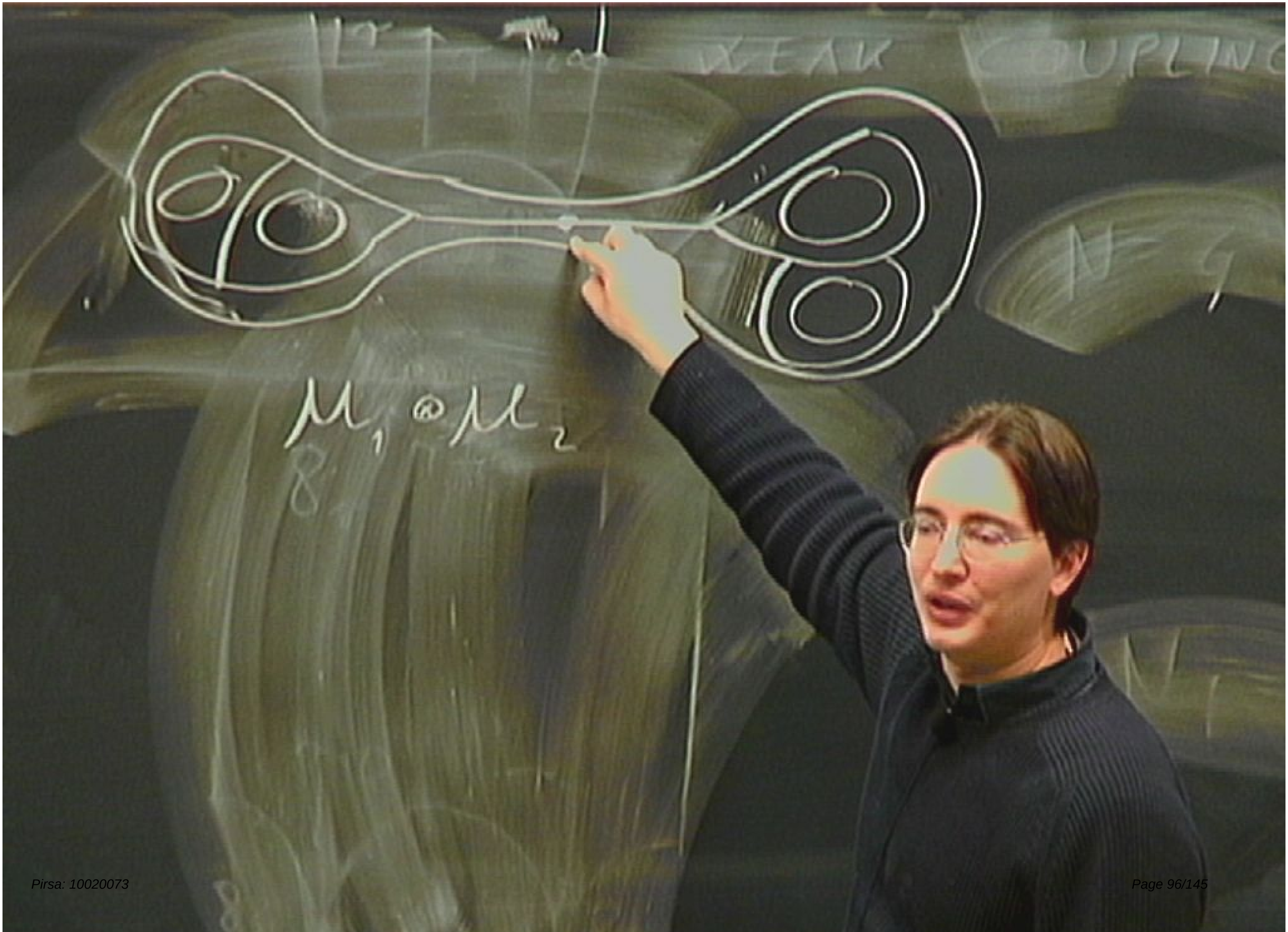
$T_{g,n}$ $C_{g,n}$

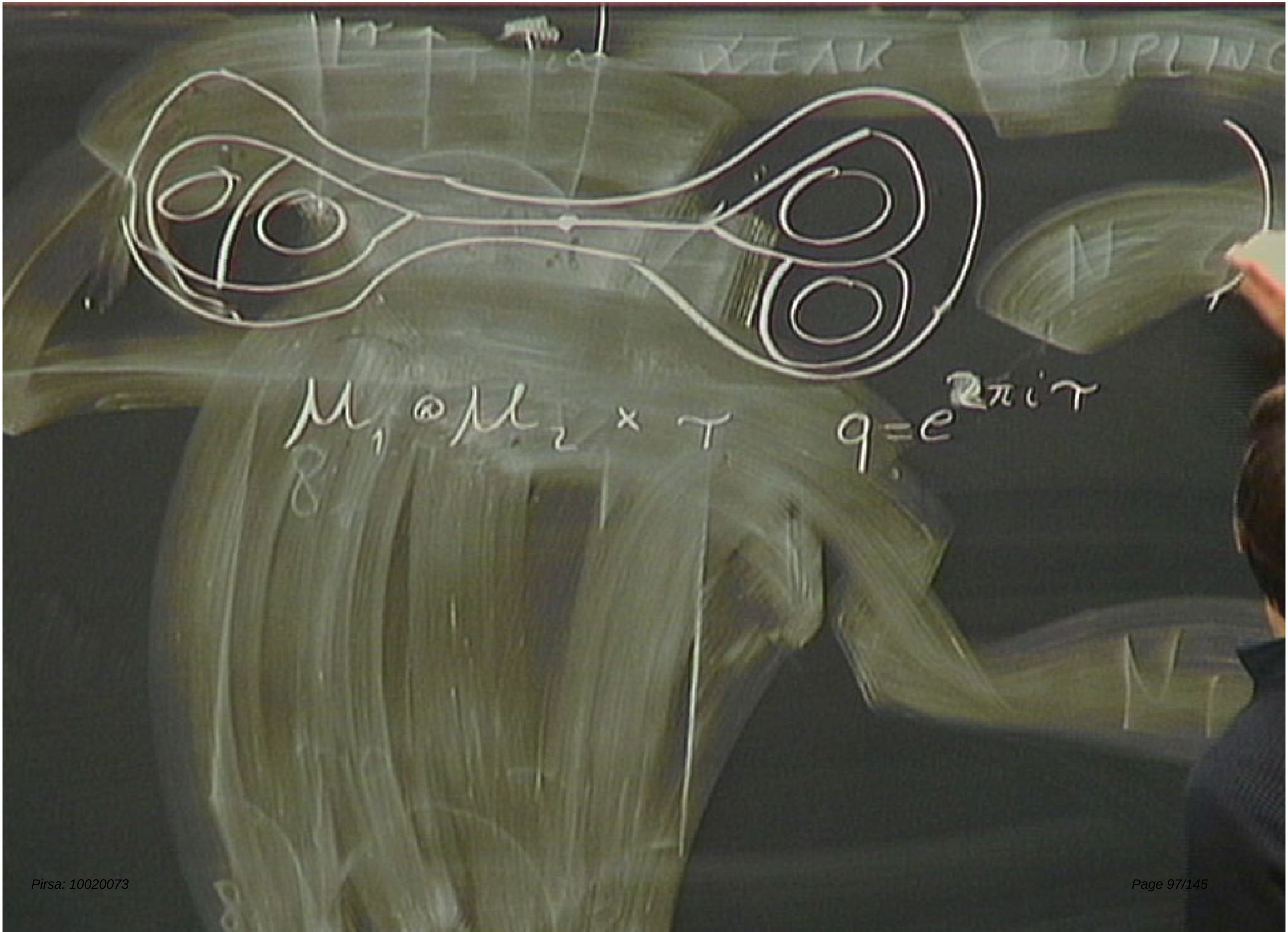
LAGRANGIAN DESCRIPTIONS $\equiv$ PAIR OF PANTS DECOMP.

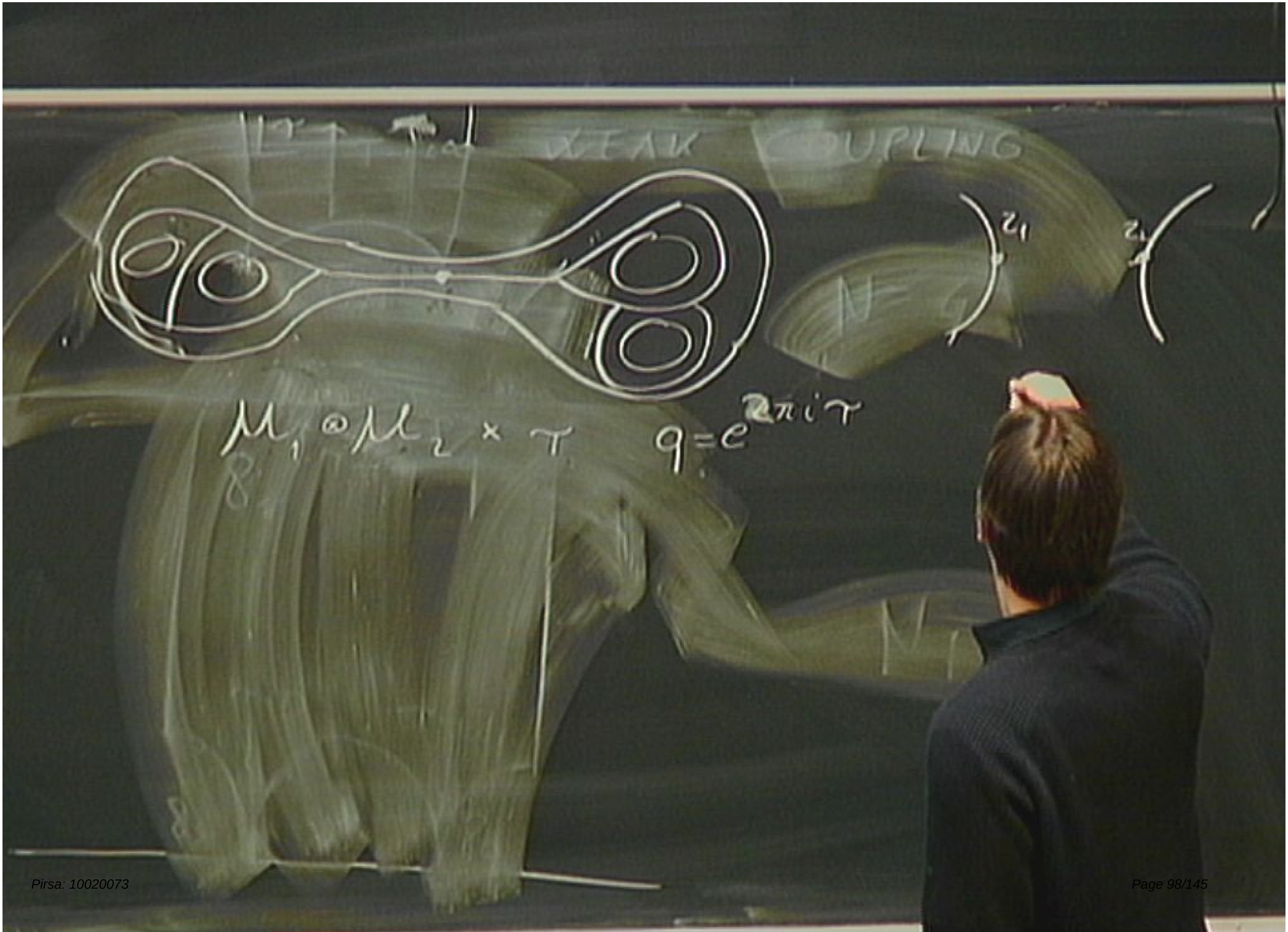
S-DUALITY $\equiv$ MOORE-SEIBERG GROUPOID

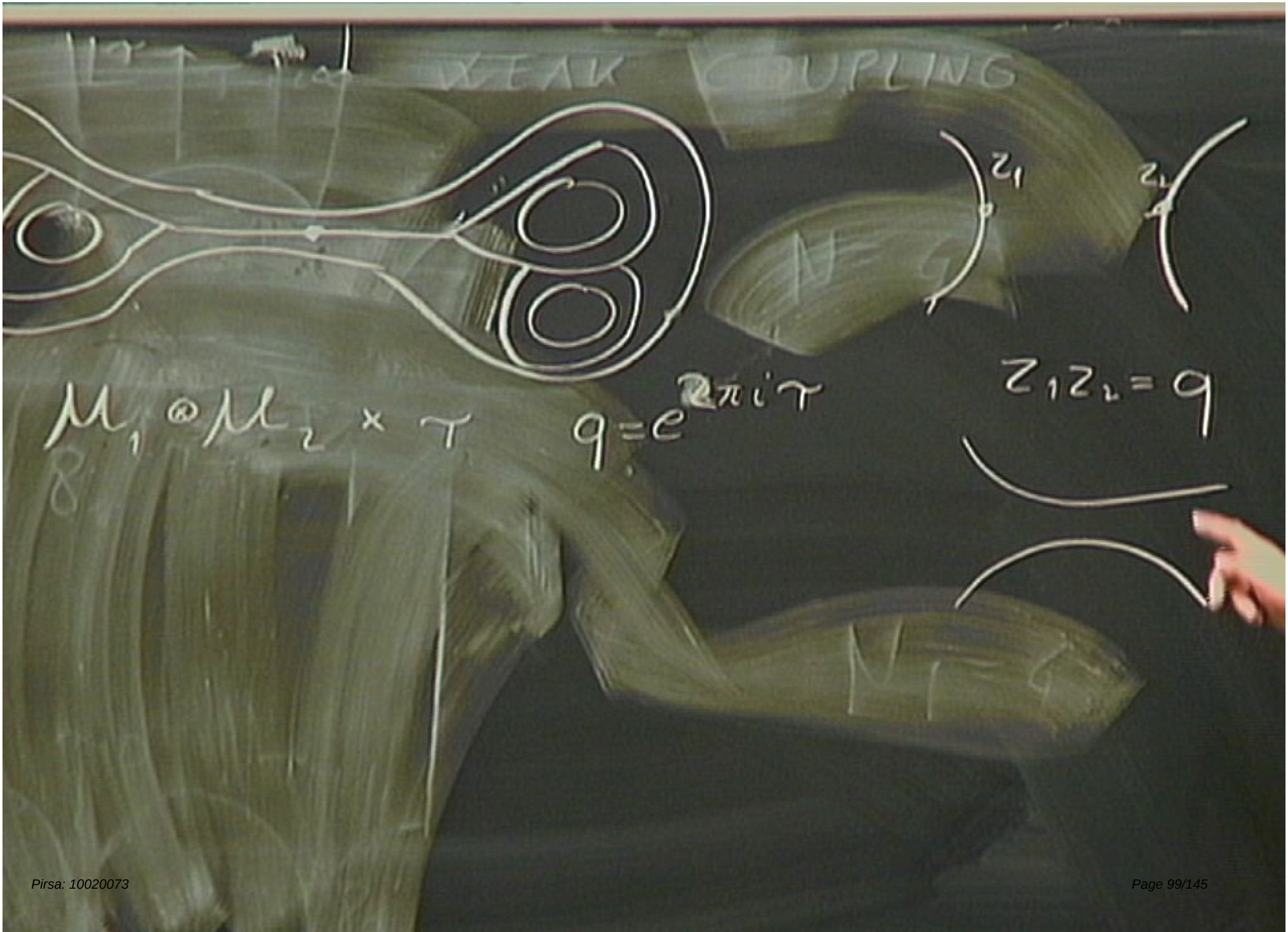


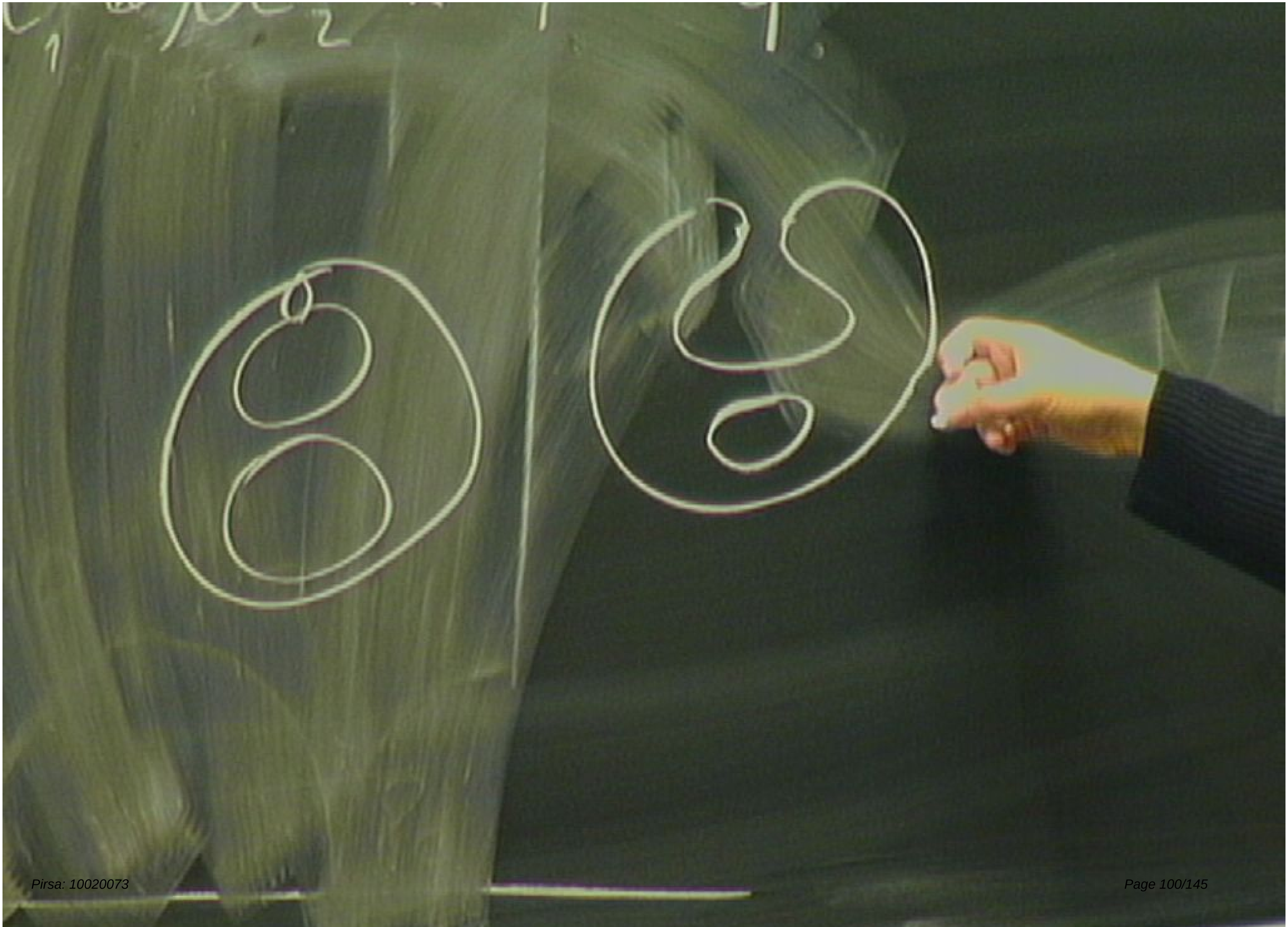

WEAK COUPLING

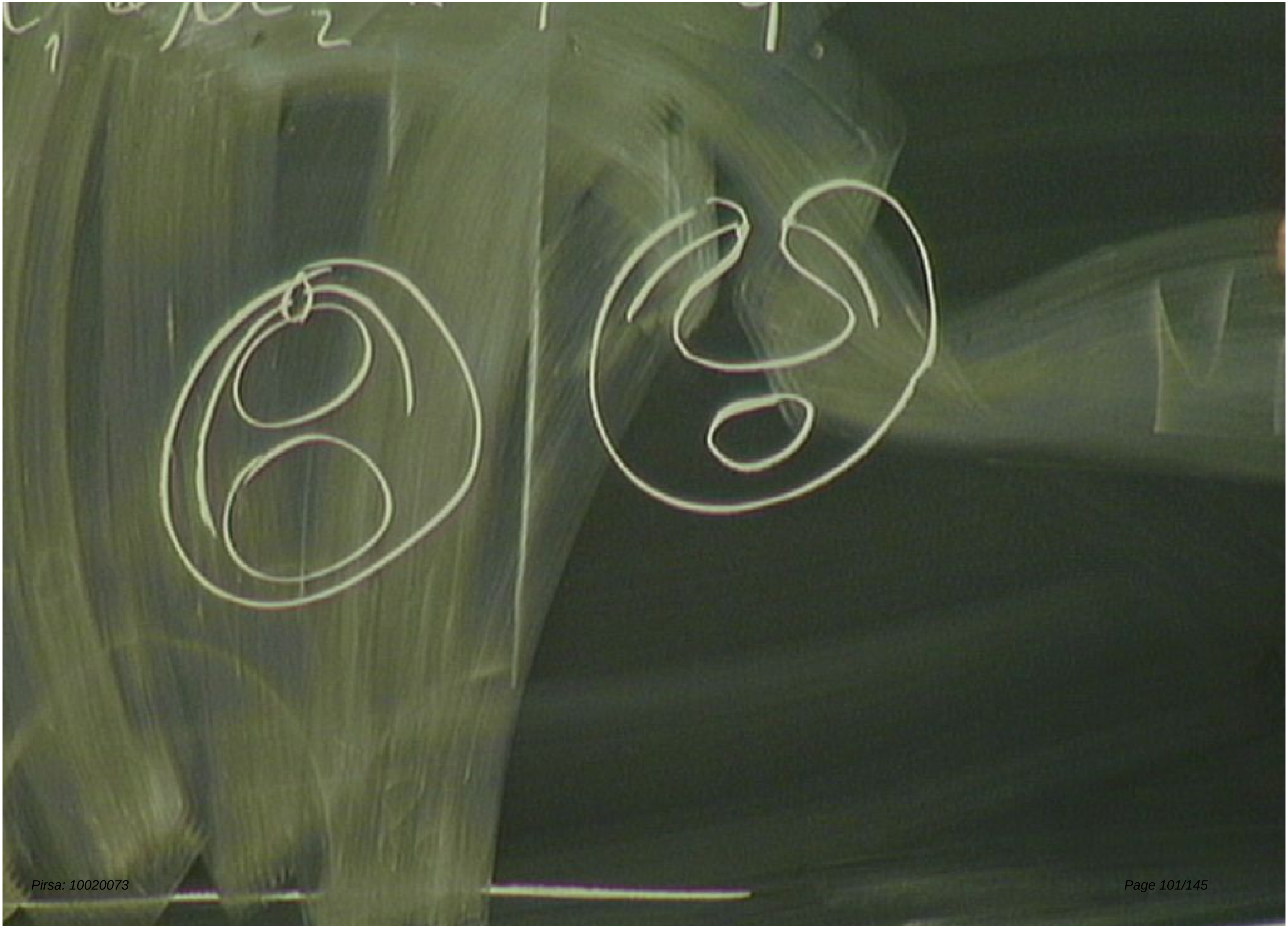

WEAK COUPLING
$M_1 \otimes M_2$


WEAK
$\mathcal{M}_1 \otimes \mathcal{M}_2 \times \tau$
$q = e^{2\pi i \tau}$


WEAK COUPLING
z_1
z_2
$\mathcal{M}_1 \otimes \mathcal{M}_2 \times \tau$
$q = e^{2\pi i \tau}$


WEAK COUPLING
z_1
z_2
$M_1 \otimes M_2 \times T$
$q = e^{2\pi i \tau}$
$z_1 z_2 = q$

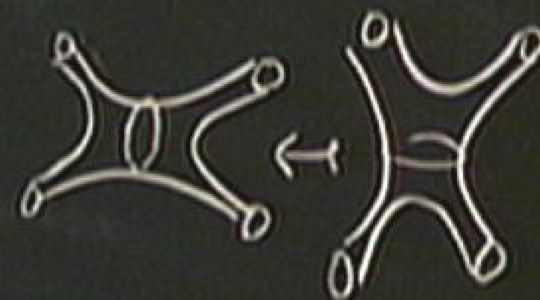




$T_{g,n}$ $C_{g,n}$

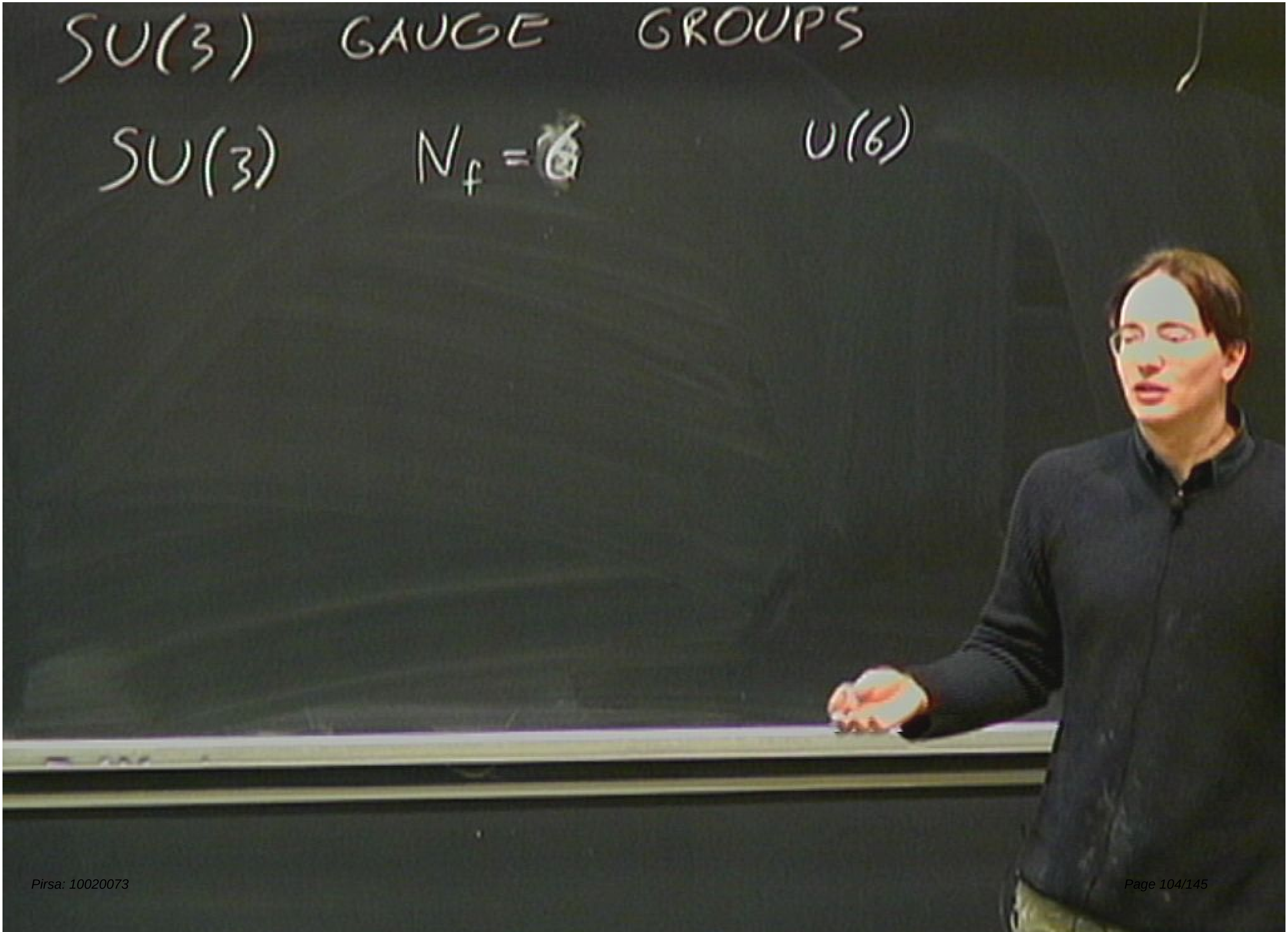
LAGRANGIAN DESCRIPTIONS $\equiv$ PAIR OF PANTS DECOMP.

S-DUALITY $\equiv$ MOORE-SEIBERG GROUPOID



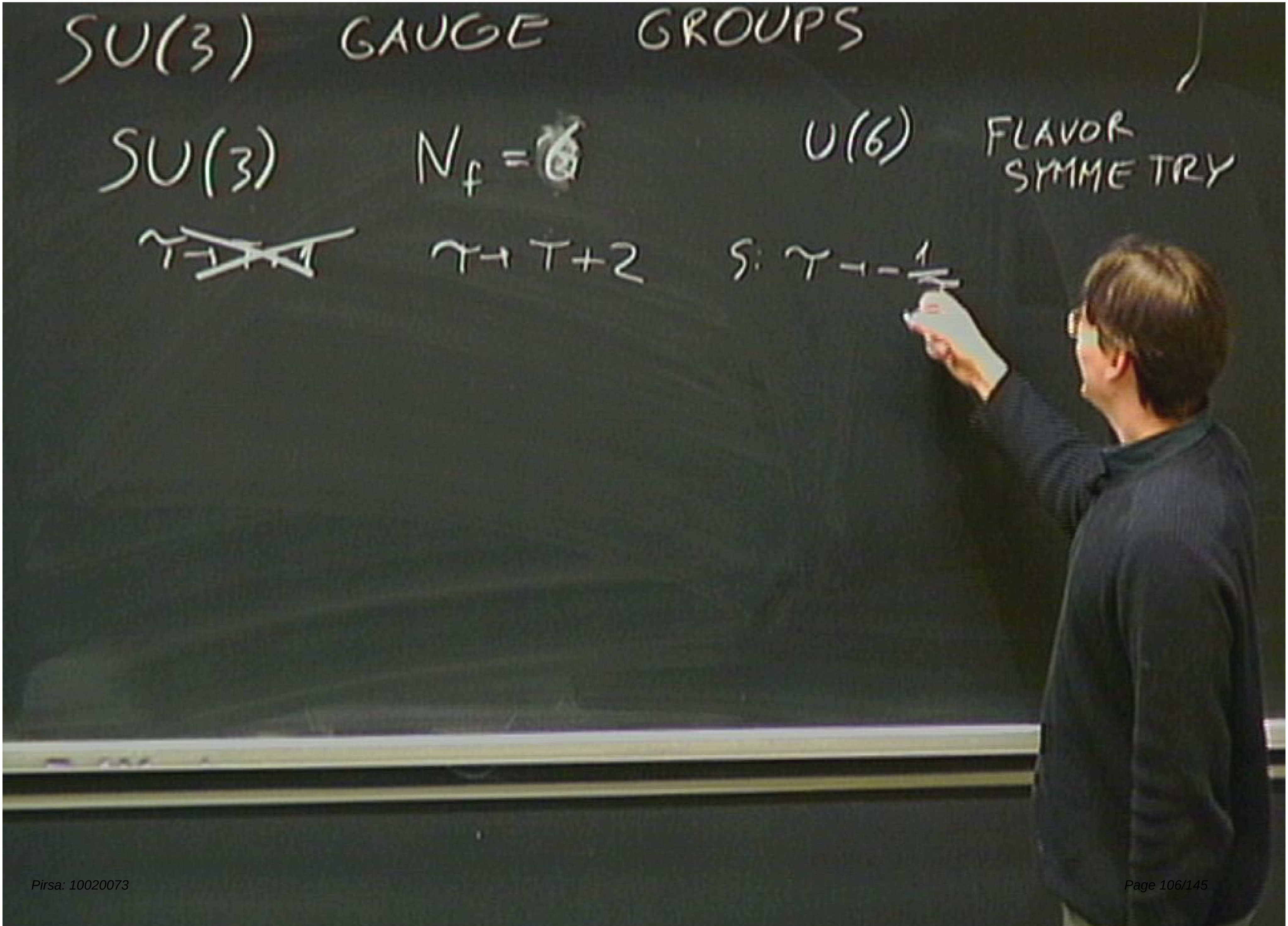
PARAMETER SPACE $\equiv$ $\mathcal{M}_{g,n}$

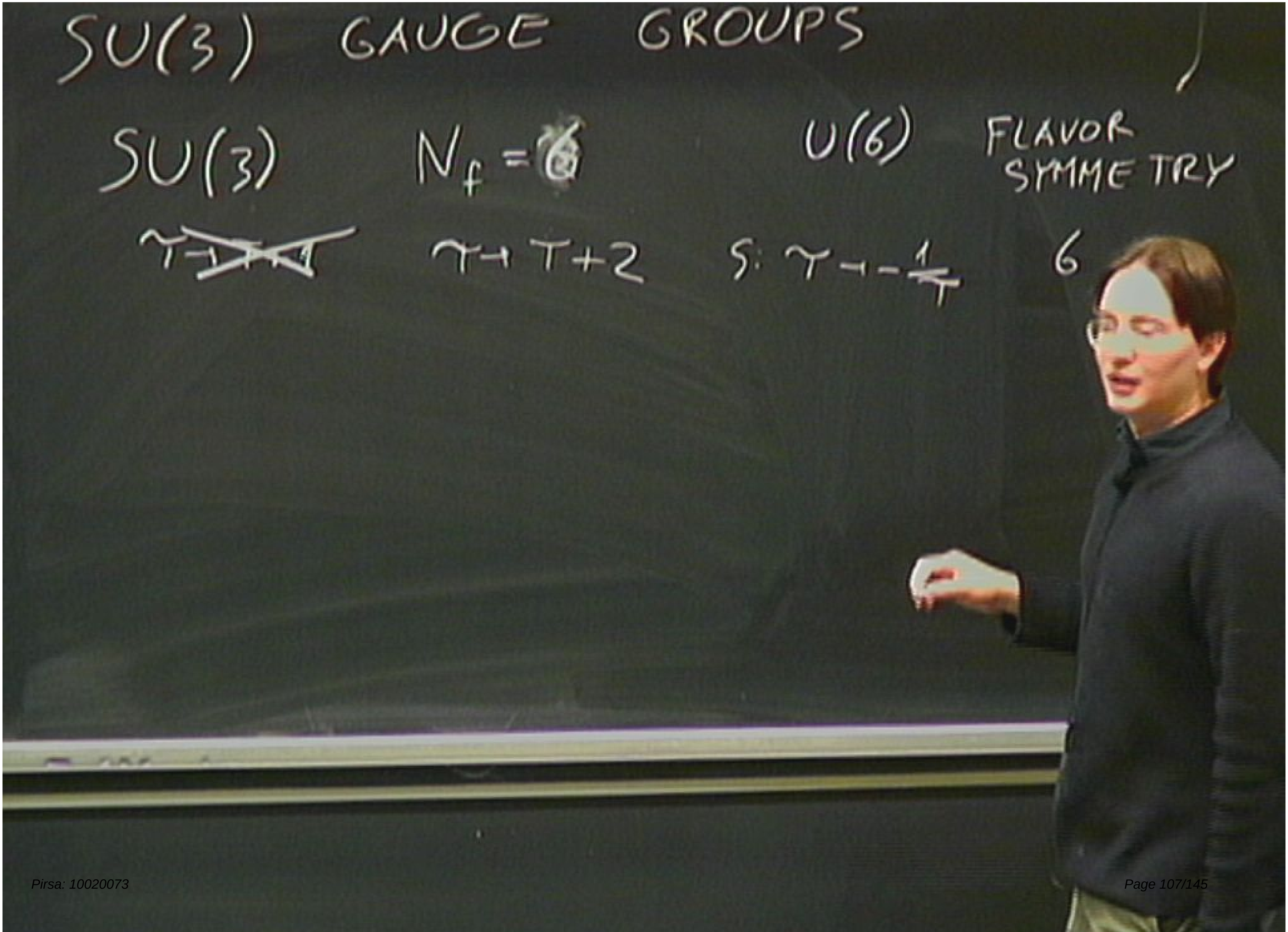
SU(3) GAUGE GROUPS


SU(3) GAUGE GROUPS
SU(3)
N_f = 6
U(6)

SU(3) GAUGE GROUPS

$SU(3)$ $\quad N_f = 6$ $\quad U(6)$ FLAVOR SYMMETRY


SU(3) GAUGE GROUPS
SU(3)
N_f = 6
U(6) FLAVOR SYMMETRY
T → T+2
S: T → -1/


SU(3) GAUGE GROUPS
SU(3)
$N_f = 6$
U(6) FLAVOR SYMMETRY
~~τ→τ+1~~
τ→τ+2
S: τ→ −1/τ
6

SU(3) GAUGE GROUPS

SU(3) $N_f = 6$ U(6) FLAVOR SYMMETRY

$\cancel{\tau \to \tau + 1}$ $\tau \to \tau + 2$ S: $\tau \to -\frac{1}{\tau}$ $6^+ \to 6^v$

SU(3) GAUGE GROUPS

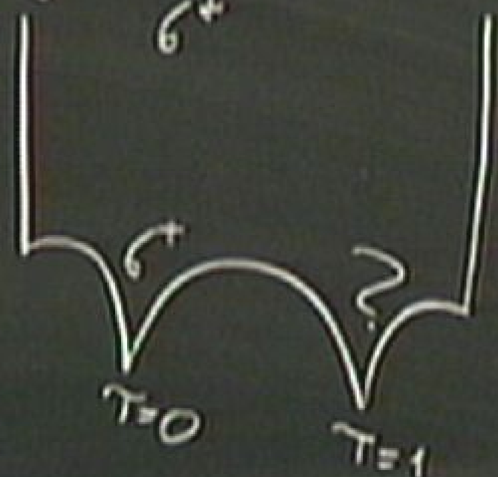
SU(3) $\quad N_f = 6 \quad$ U(6) FLAVOR SYMMETRY

$\cancel{\tau \to T+1} \quad \tau \to T+2 \quad S: \tau \to -\frac{1}{\tau} \quad 6^+ \to 6^*$

SU(3) GAUGE GROUPS

SU(3) $N_f = 6$ $\qquad U(6) = SU(6)\times U(1)$ FLAVOR SYMMETRY

$\cancel{\tau \to \tau+1}$ $\qquad \tau \to \tau+2$ $\qquad S: \tau \to -\frac{1}{\tau}$ $\qquad 6 \xrightarrow{+} 6^*$

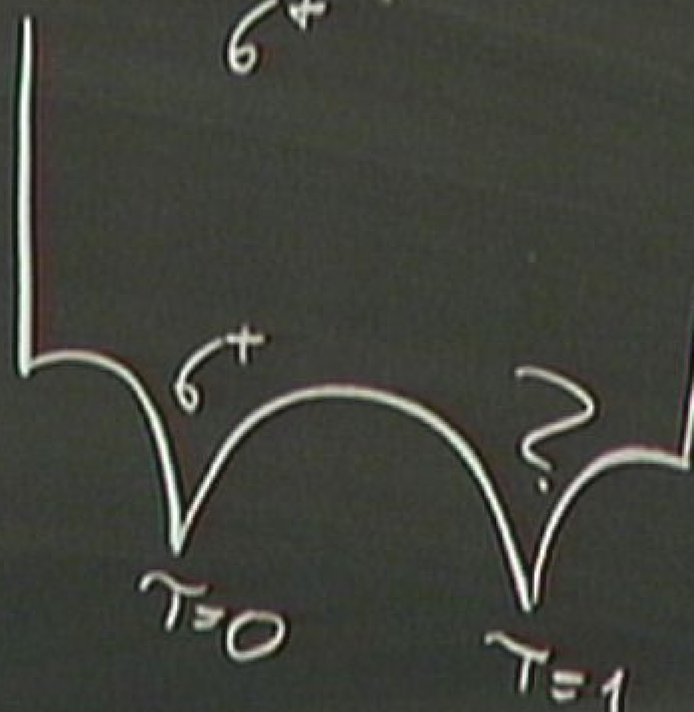


SU(3) GAUGE GROUPS

$SU(3)$ $\quad N_f = 6$ $\quad U(6)$

$\cancel{\tau \to \tau + 1}$ $\quad \tau \to \tau + 2$ $\quad S: \tau \to -\ldots$

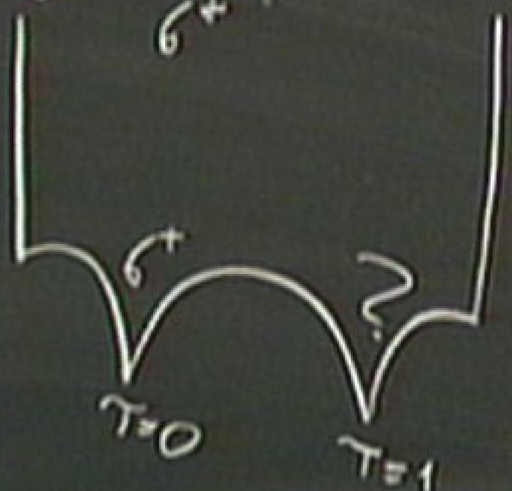
6^+



SU(3) GAUGE GROUPS

$SU(3) \quad N_f = 6$ $\qquad U(6) = SU(6) \times U(1)$ FLAVOR SYMMETRY

$\cancel{\tau \to \tau + 1}$ $\quad 6^+ \quad \tau \to \tau + 2$ $\qquad S: \tau \to -\frac{1}{\tau}$ $\qquad 6^+ \to 6^*$



? $\quad SU(2)$ GAUGE FIELD

$R=2$ $+ \; E_6$ SCFT

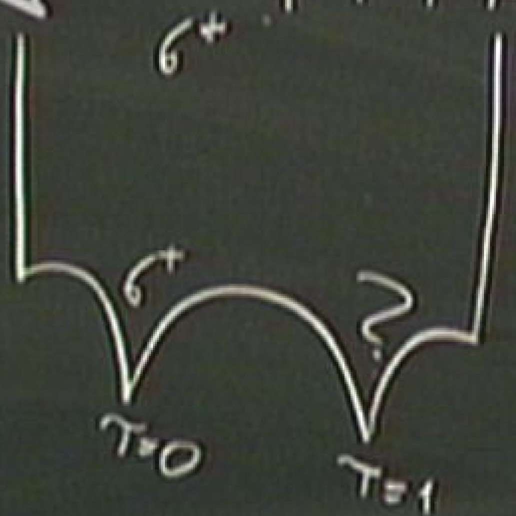
SU(3) GAUGE GROUPS

SU(3) $N_f = 6$

$U(6) = SU(6) \times U(1))$ FLAVOR SYMMETRY

$\cancel{\tau \to \tau + 1}$ $\tau \to \tau + 2$

$S: \tau \to -\frac{1}{\tau}$ $6^+ \to 6^-$



? SU(2) GAUGE FIELD

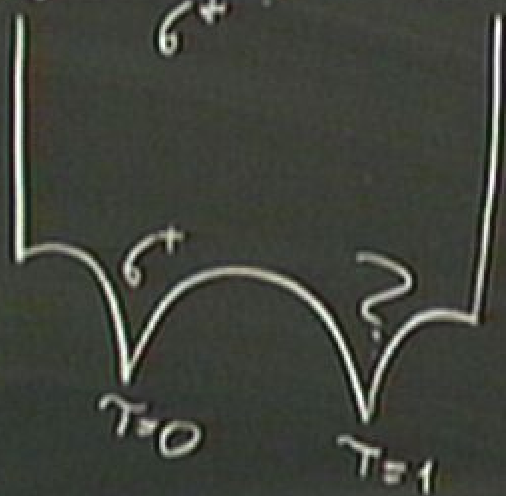
$R = 2$ + E_6 SCFT

$SU(2) \times SU(6) \subset E_6$

$SU(3)$ GAUGE GROUPS

$SU(3)$ $\quad N_f = 6$ $\quad U(6) = SU(6) \times U(1)$ FLAVOR SYMMETRY

$\cancel{\tau \to \tau + 1}$ $\quad 6^{+}\ \tau \to \tau + 2$ $\quad S: \tau \to -\frac{1}{\tau}$ $\quad 6^{+} \to 6^{*}$



? $SU(2)$ GAUGE FIELD

$R = 2$ $\quad + \quad E_6$ SCFT

$SU(2) \times SU(6) \subset E_6$

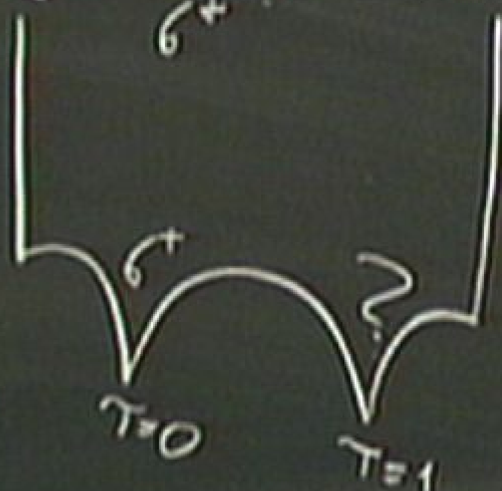
SU(3) GAUGE GROUPS

$SU(3)$ $N_f = 6$

$U(6) = SU(6) \times U(1)$ FLAVOR SYMMETRY

$\cancel{\tau \to \tau + 1}$ 6^+ $\tau \to \tau + 2$

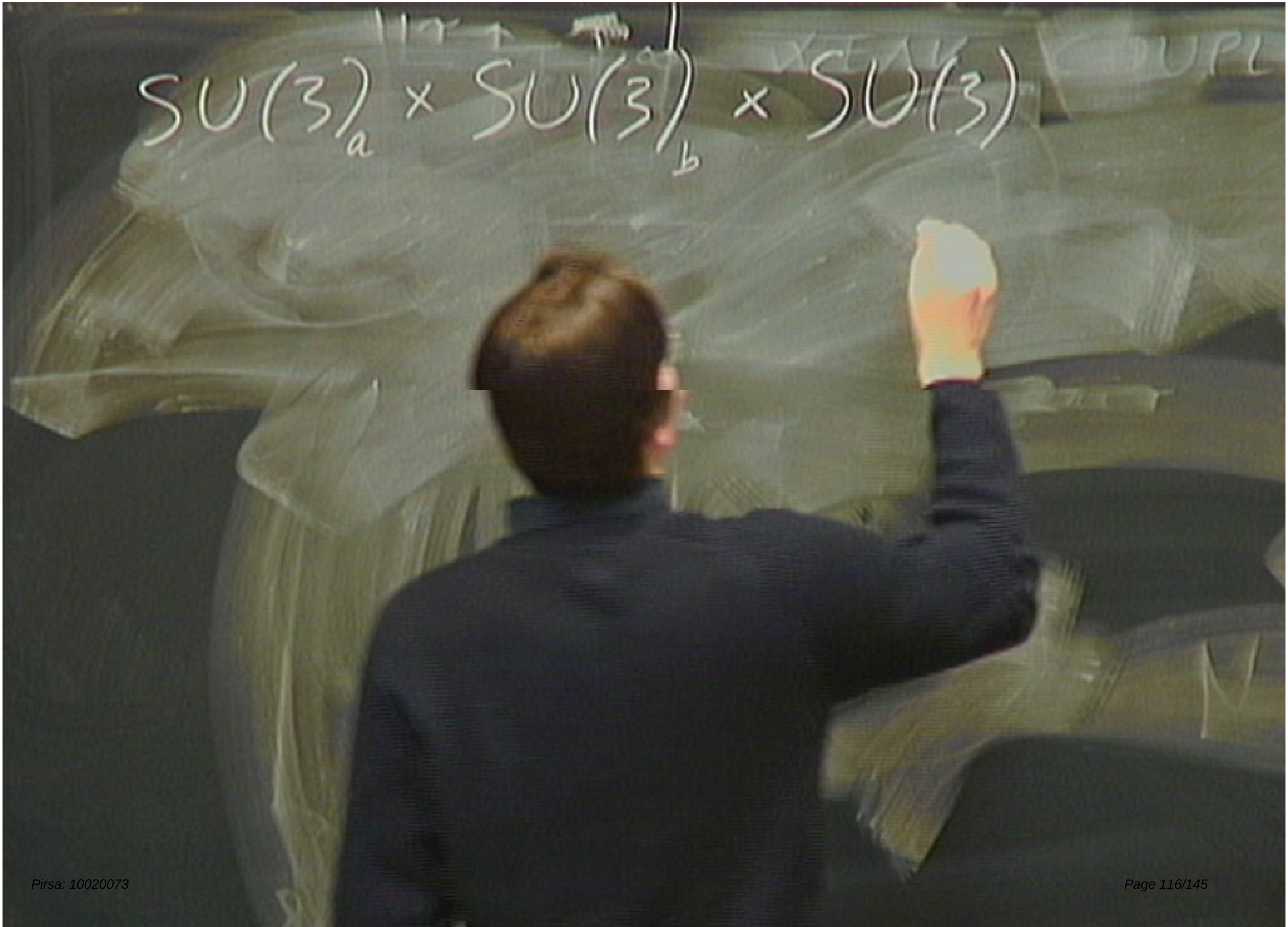
$S: \tau \to -\frac{1}{\tau}$ $6^+ \to 6^-$

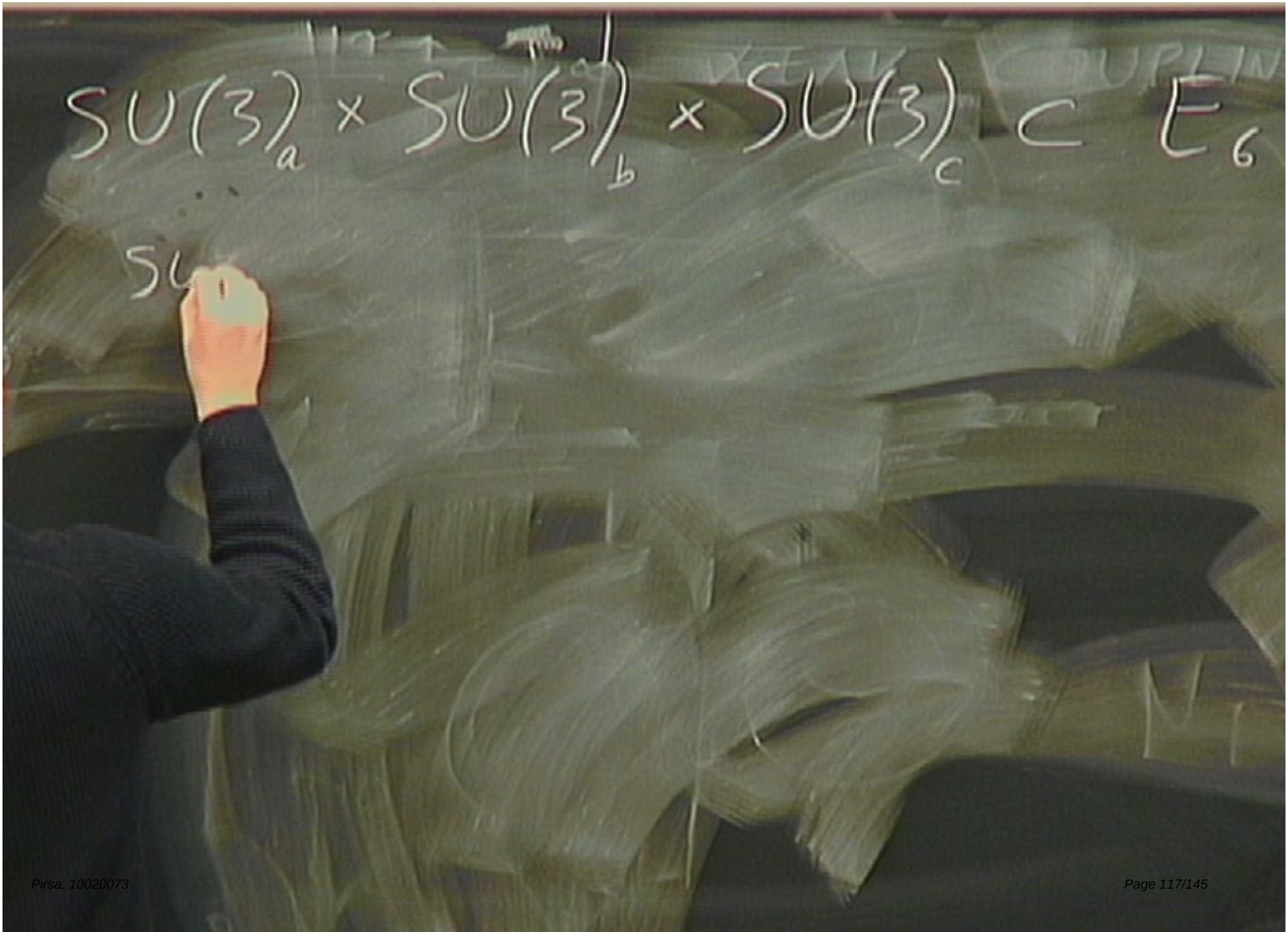


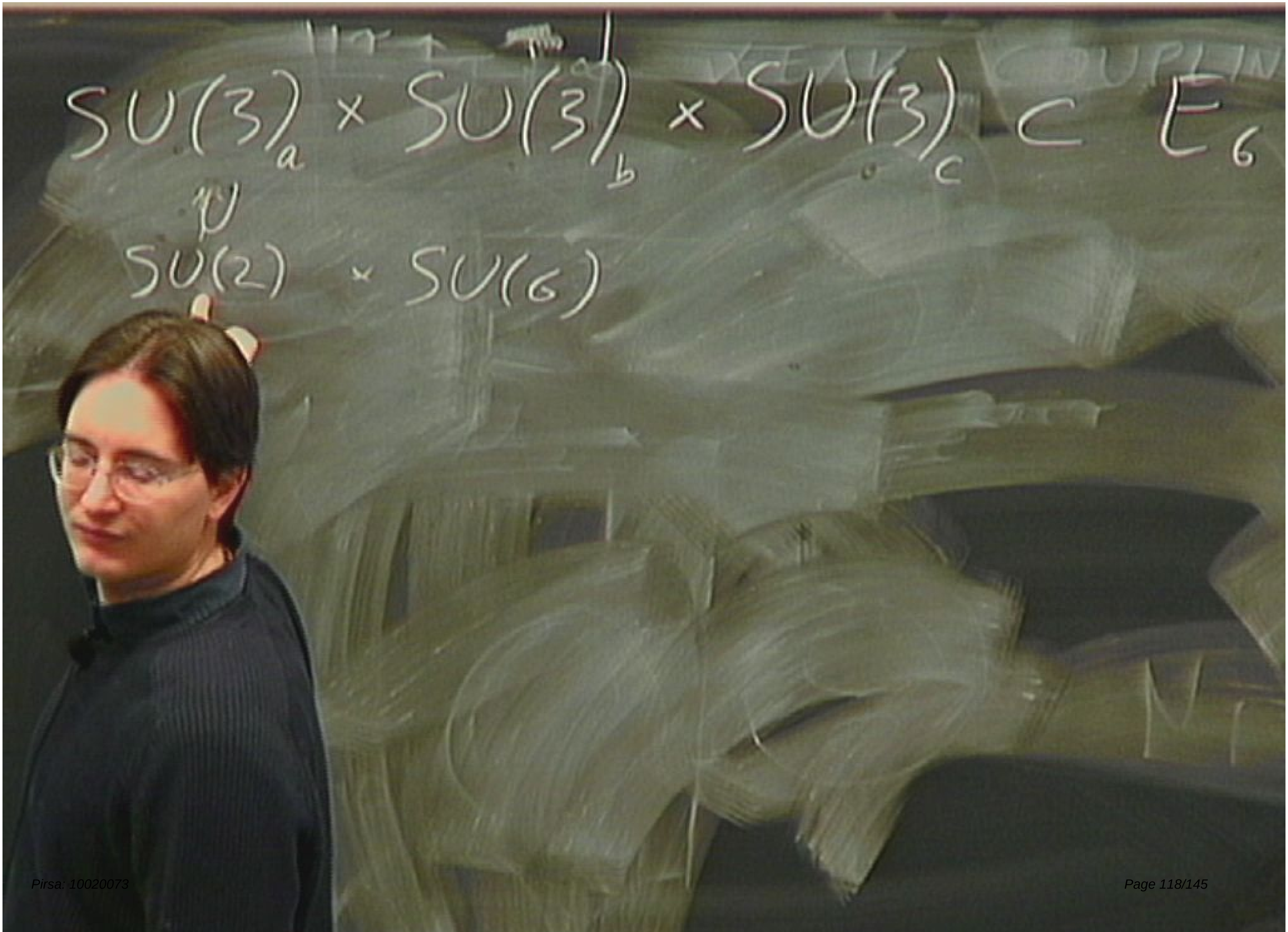
? $SU(2)$ GAUGE FIELD

$R=2$ + E_6 SCFT

$SU(2) \times SU(6) \subset E_6$


$SU(3)_a \times SU(3)_b \times SU(3)$


$SU(3)_a \times SU(3)_b \times SU(3)_c \subset E_6$
SU


$SU(3)_a \times SU(3)_b \times SU(3)_c \subset E_6$
$SU(2) \times SU(6)$

$$SU(3)_a \times SU(3)_b \times SU(3)_c \subset E_6$$

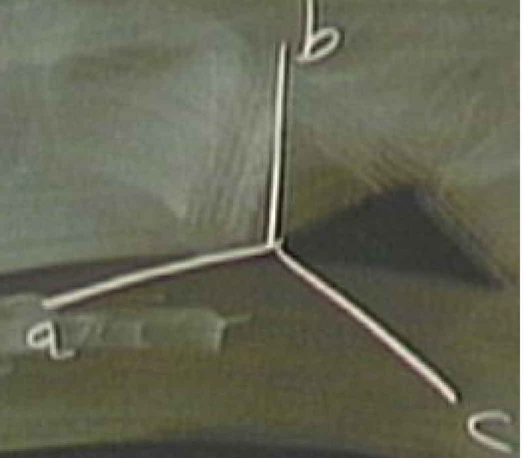
$$\cup$$

$$SU(2) \times SU(6)$$

$$SU(3)_a \times SU(3)_b \times SU(3)_c \subset E_6$$

$$\cup$$

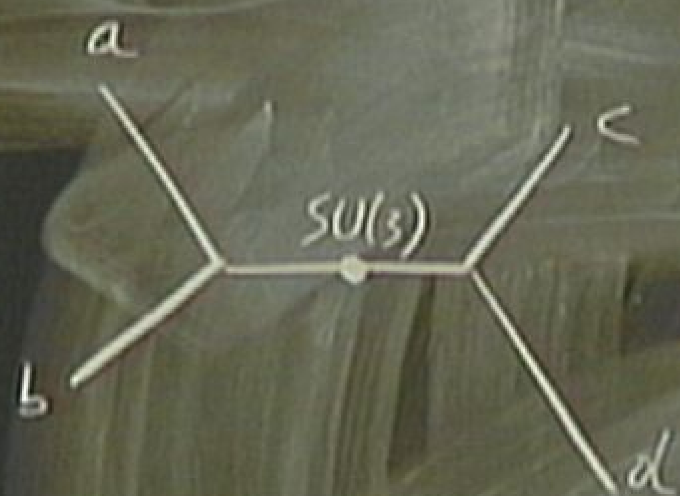
$$SU(2) \times SU(6)$$

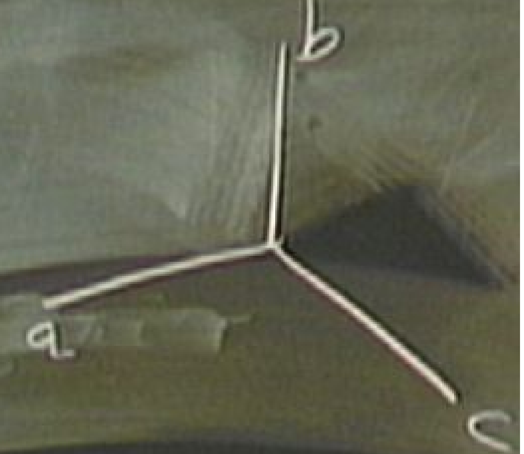


$$SU(3)_a \times SU(3)_b \times SU(3)_c \subset E_6$$

$$\cup$$

$$SU(2) \times SU(6)$$

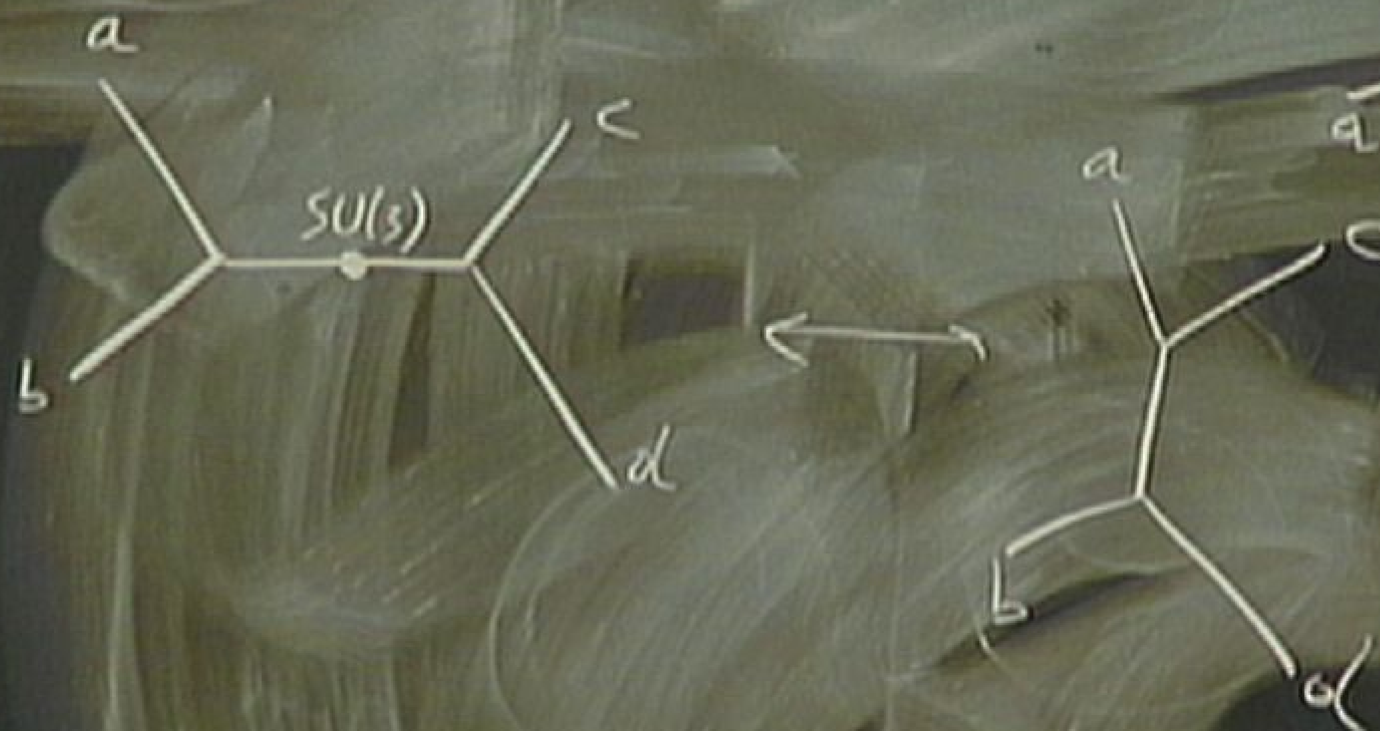


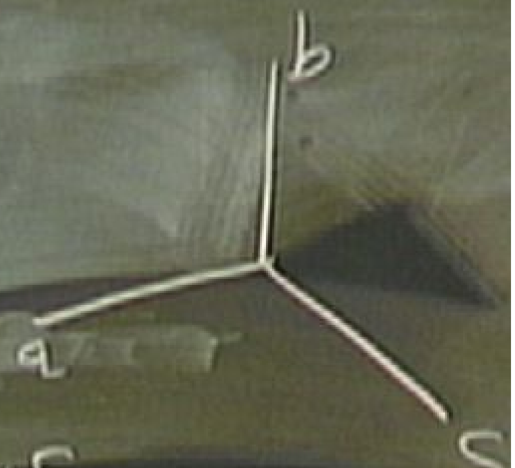


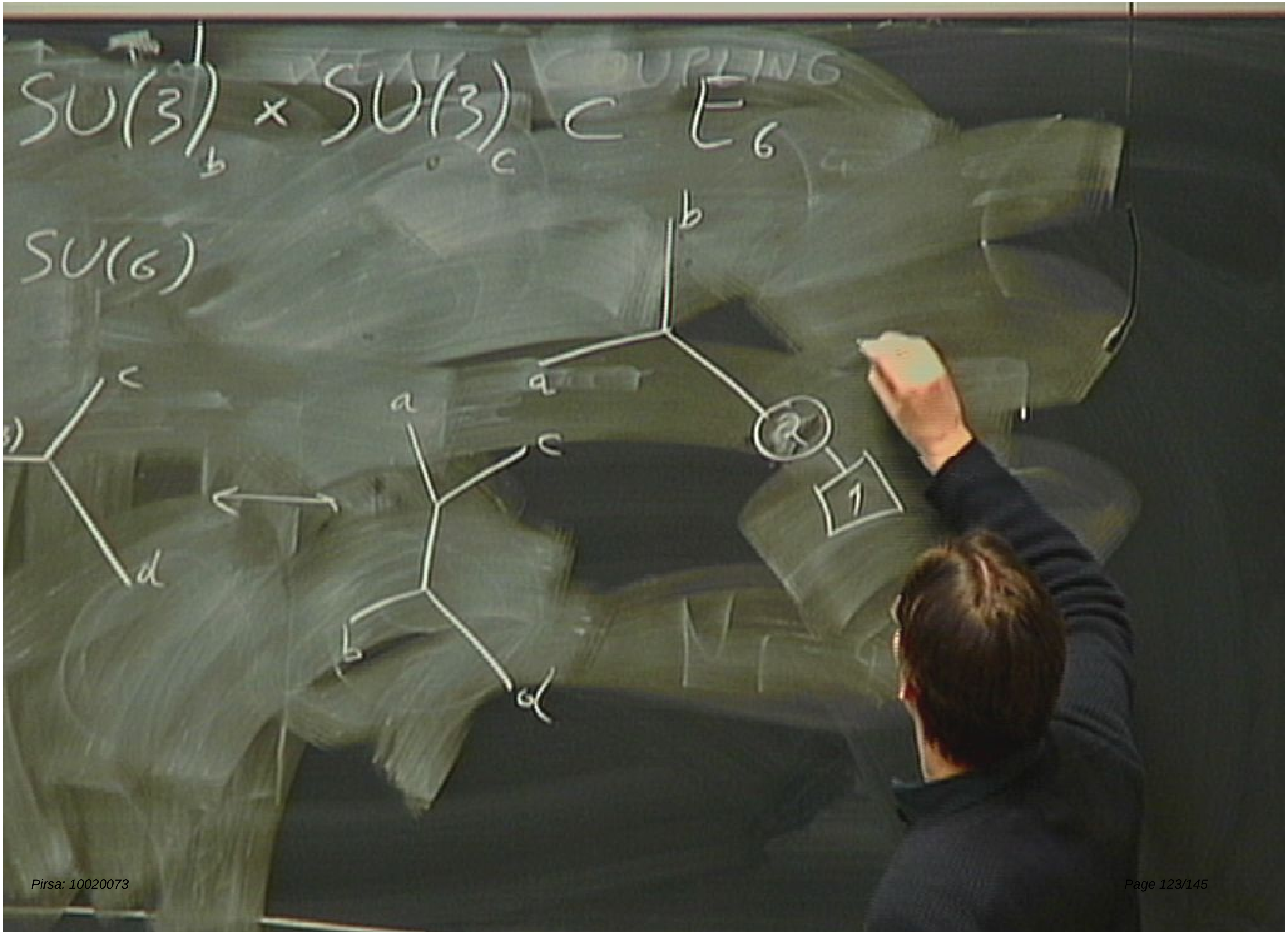
$$SU(3)_a \times SU(3)_b \times SU(3)_c \subset E_6$$

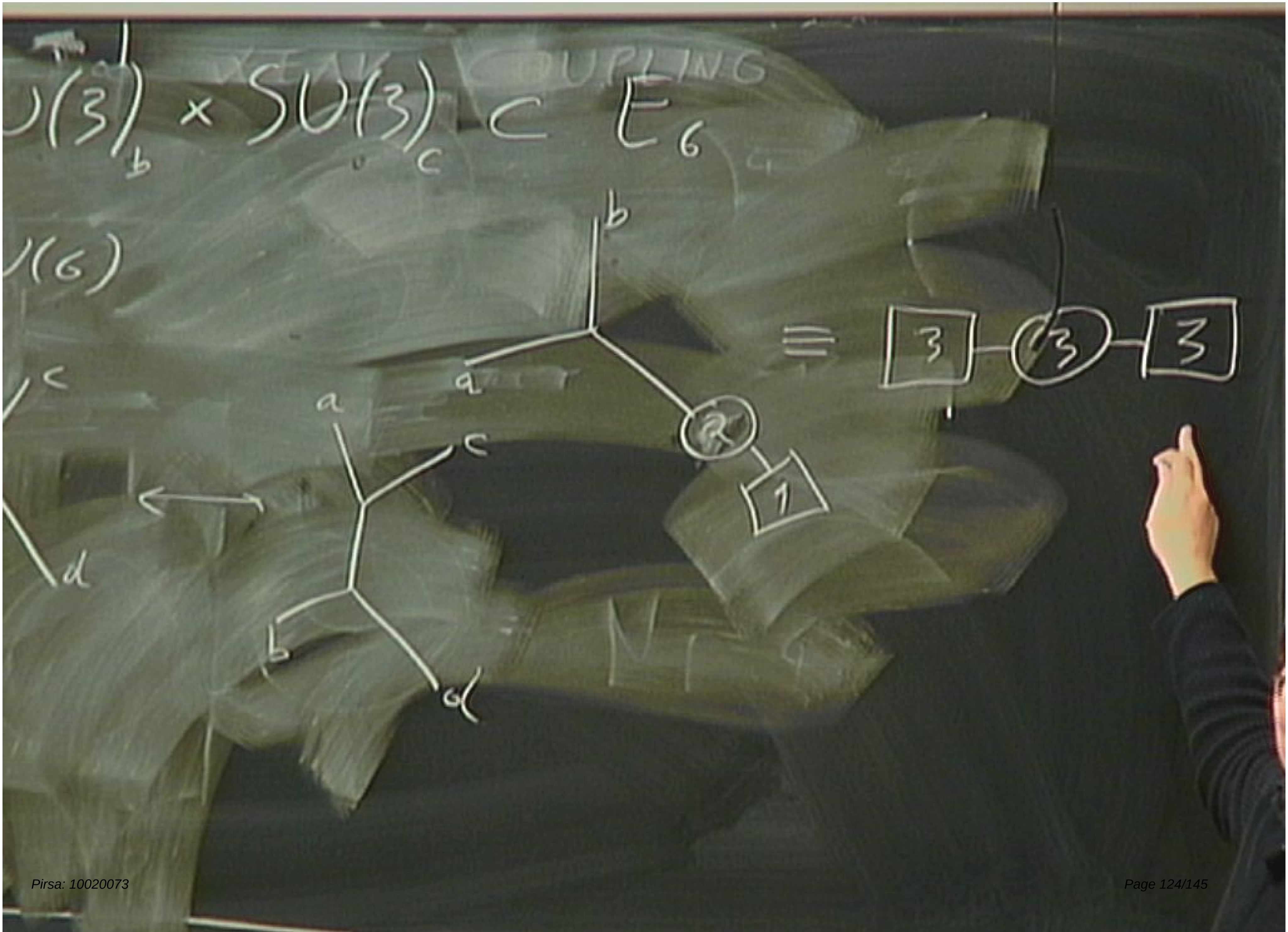
$$\Downarrow$$

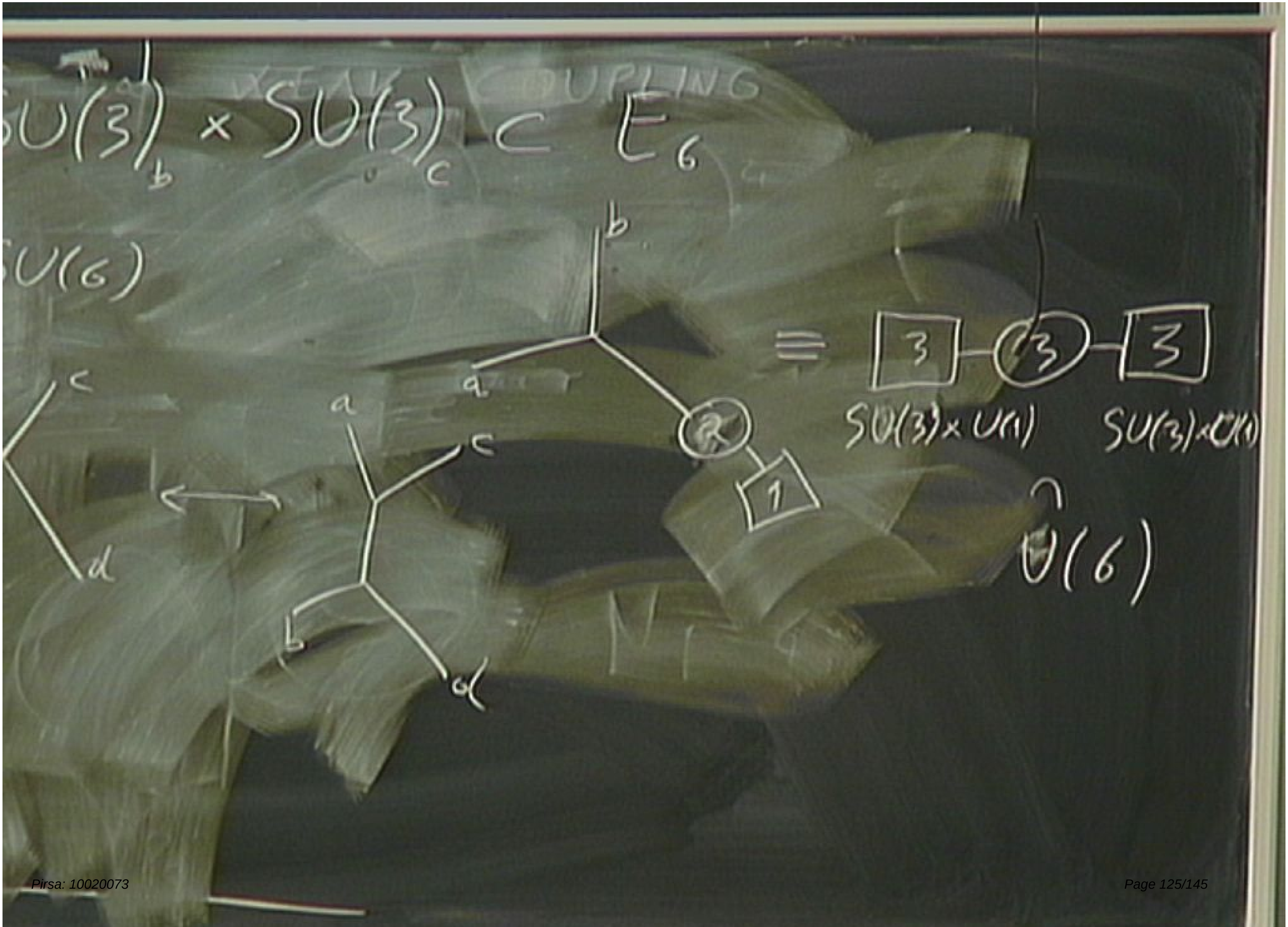
$$SU(2) \times SU(6)$$

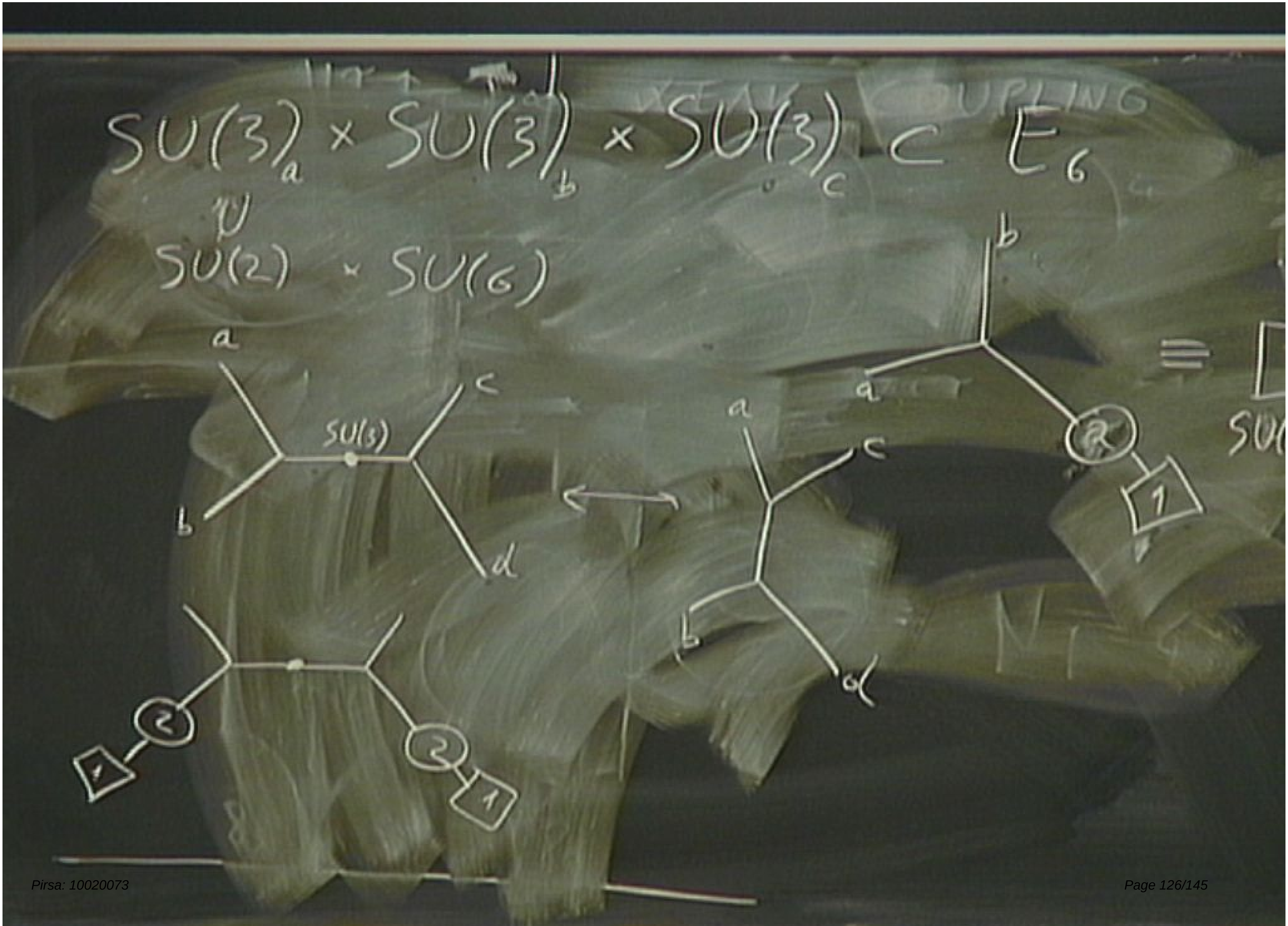


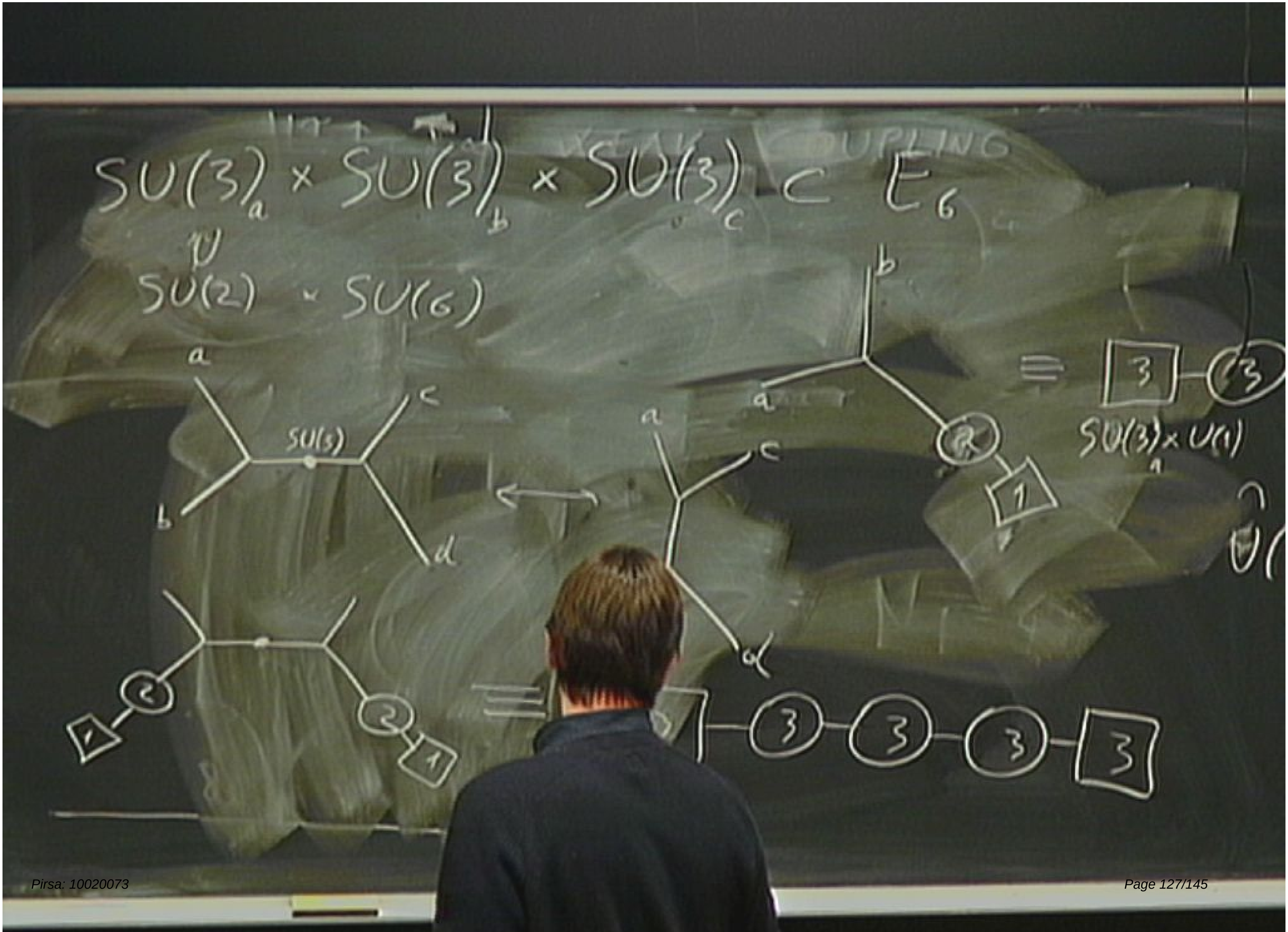


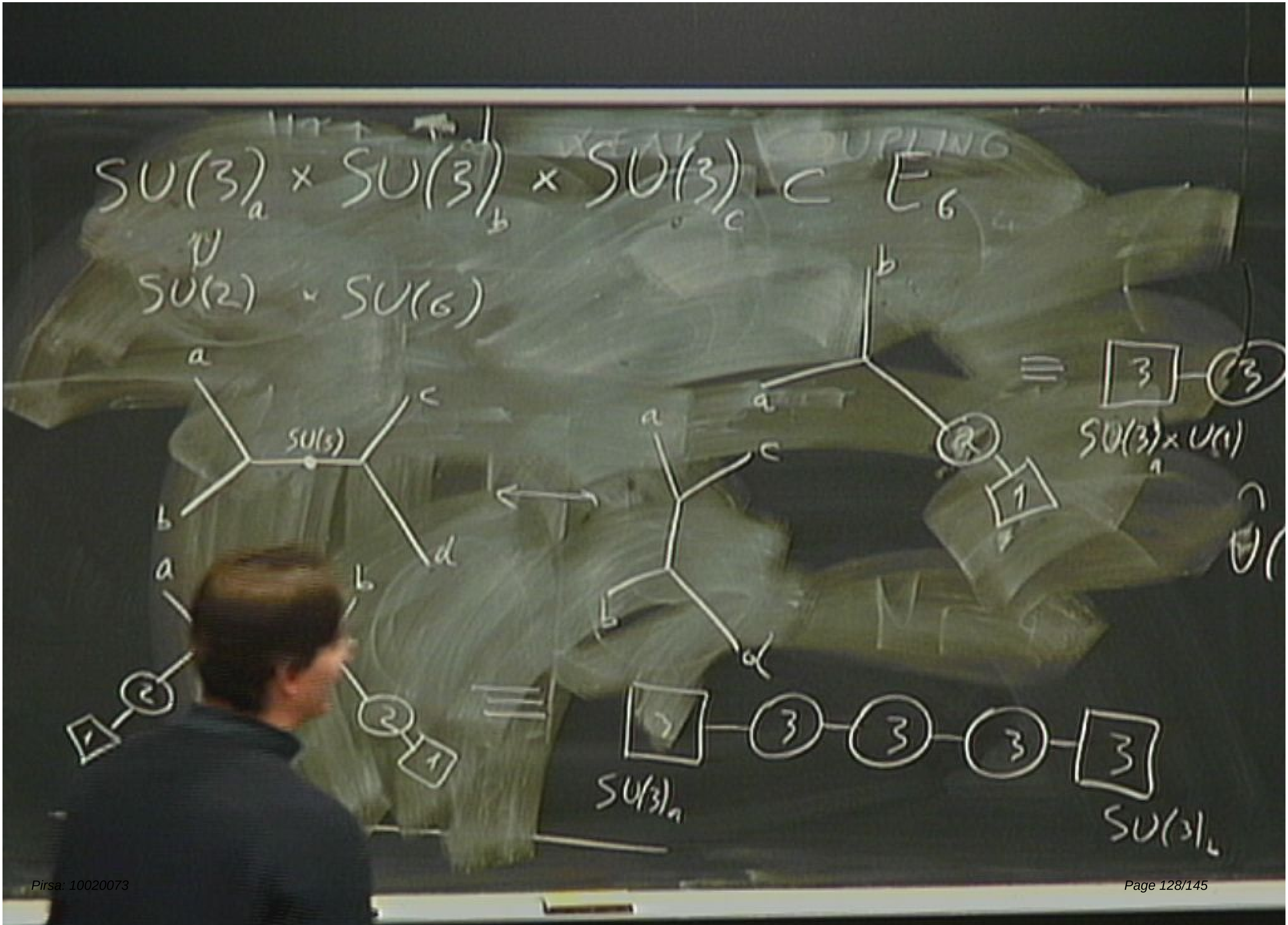

SU(3)_b × SU(3)_c ⊂ E_6
SU(6)
b
a
c
d
1

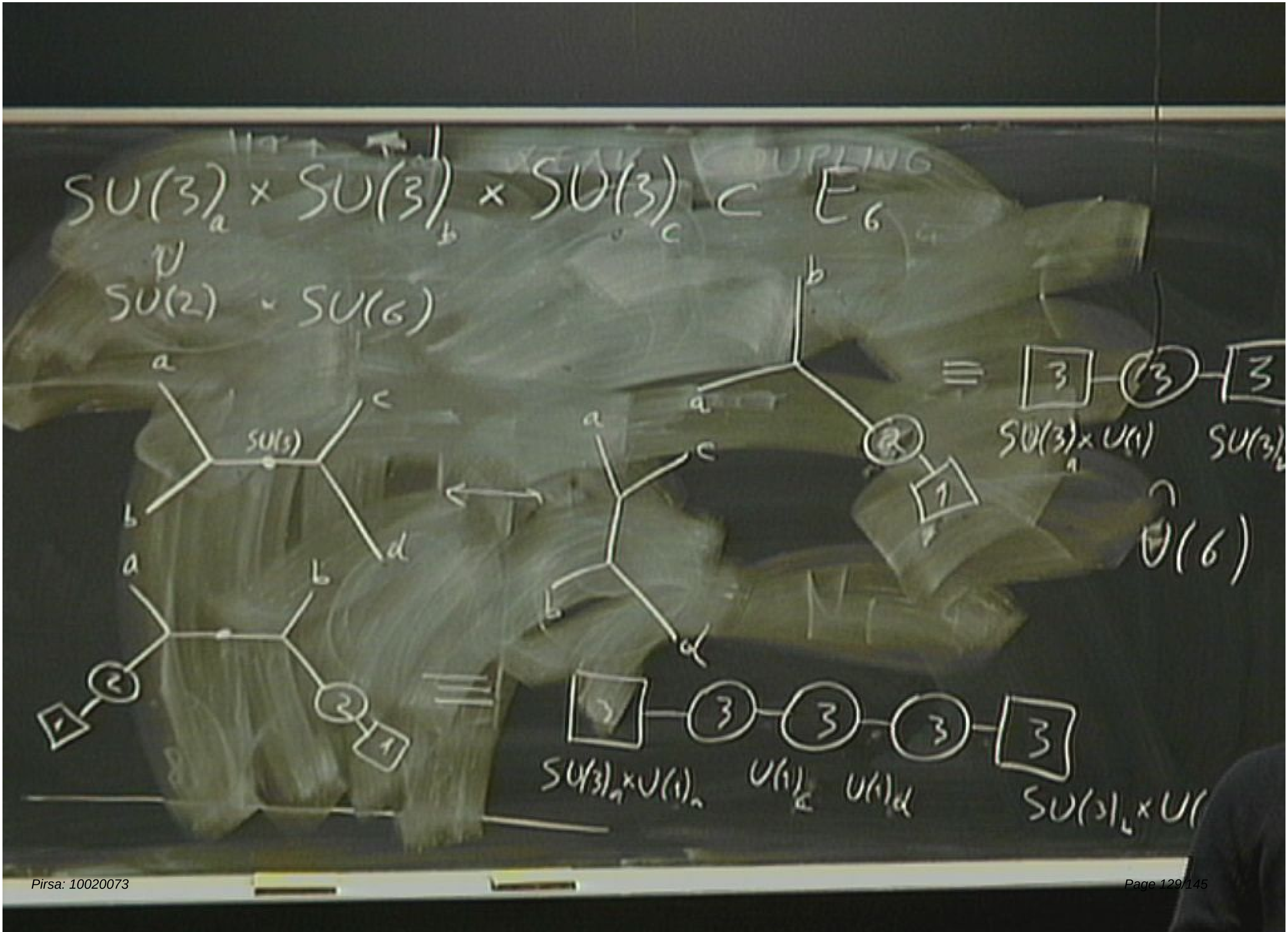

U(3)b × SU(3)c ⊂ E6
U(6)
≡ 3 — 3 — 3
1
a b c d


WEAK COUPLING
SU(3)b × SU(3)c ⊂ E6
SU(6)
SU(3)×U(1)
SU(3)×U(1)
U(6)

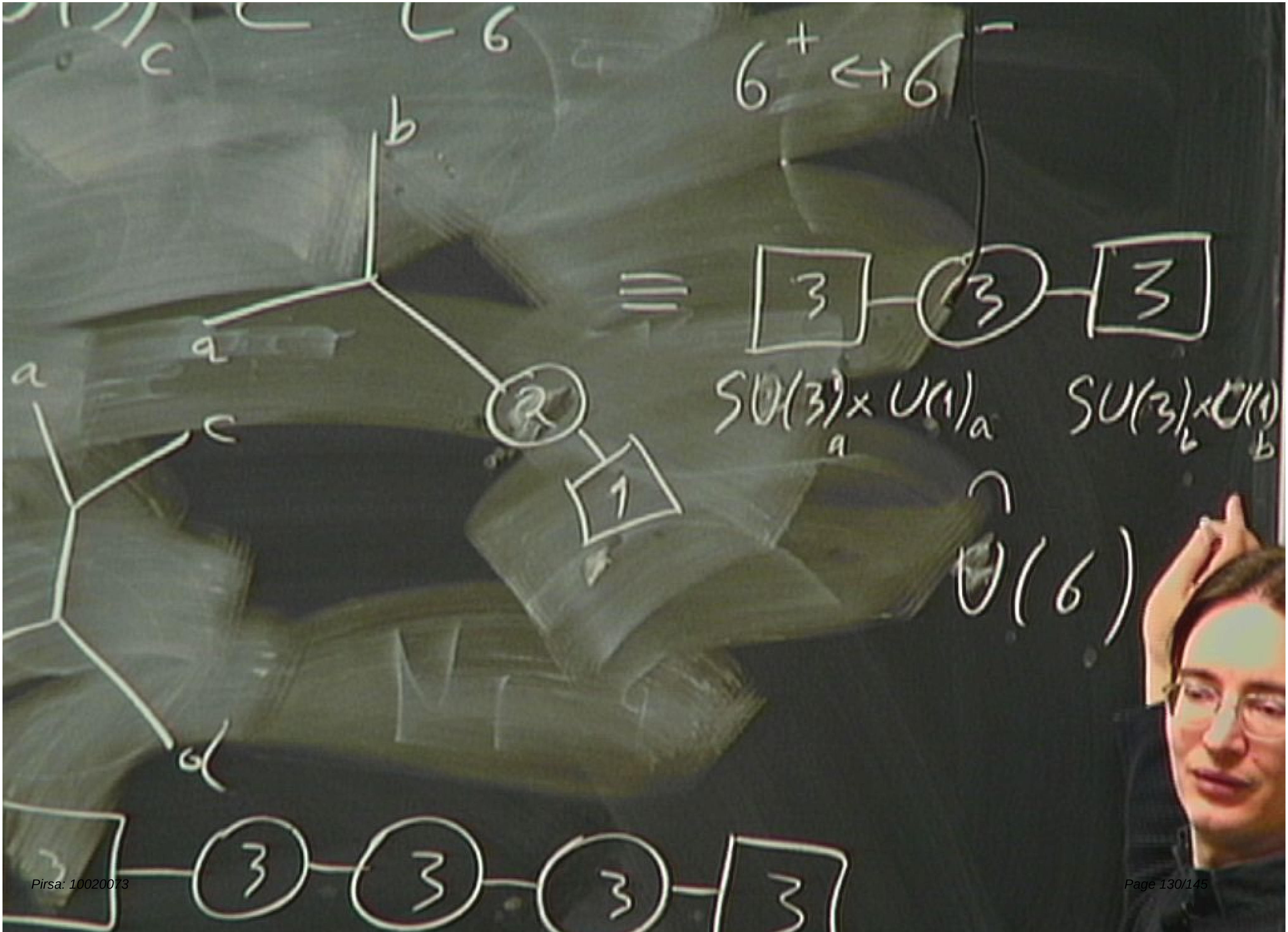

$SU(3)_a \times SU(3)_b \times SU(3)_c \subset E_6$
$\cup$
$SU(2) \times SU(6)$
a
b
c
d
SU(3)
1
2
≡


SU(3)a × SU(3)b × SU(3)c ⊂ E6
SU(2) × SU(6)
SU(5)
SU(3)×U(1)
a
b
c
d
3
1
2

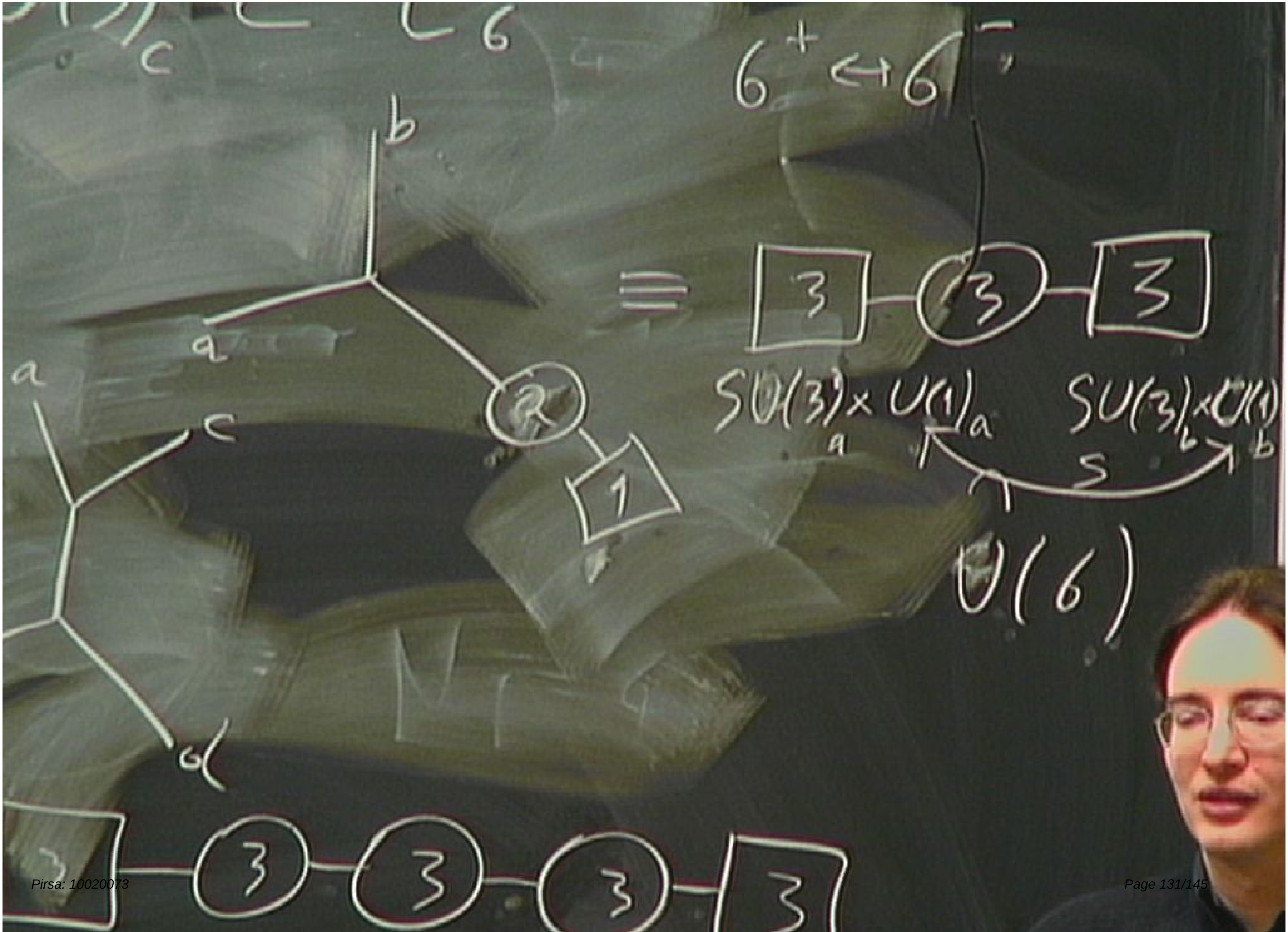

SU(3)a × SU(3)b × SU(3)c ⊂ E6
SU(2) × SU(6)
SU(3)
SU(3) × U(1)
SU(3)a
SU(3)c



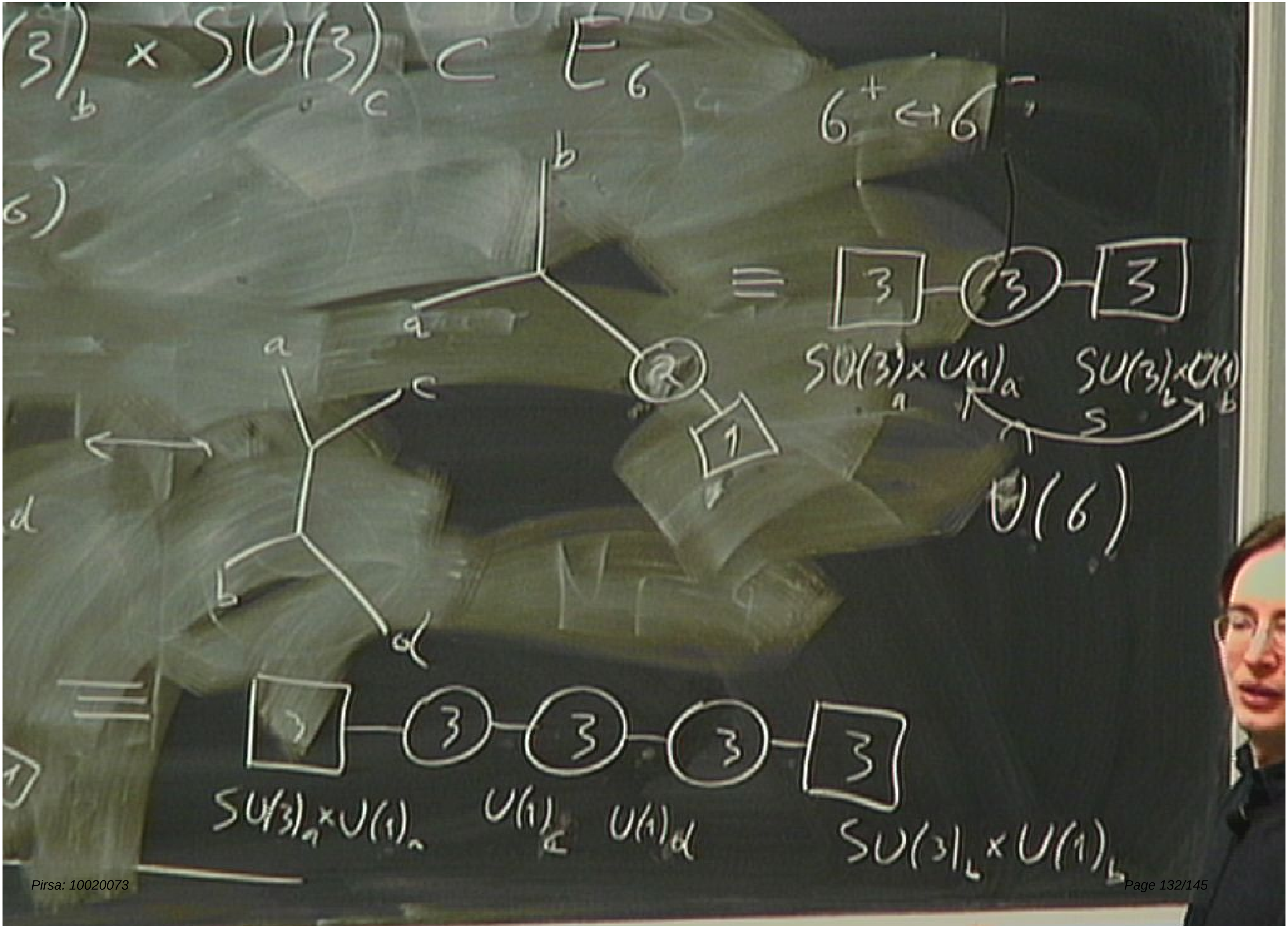
SU(3)a × SU(3)b × SU(3)c ⊂ E6
SU(2) × SU(6)
SU(3)
SU(3)×U(1)
U(6)
SU(3)a×U(1)a
U(1)c
U(1)d
SU(3)b×U(



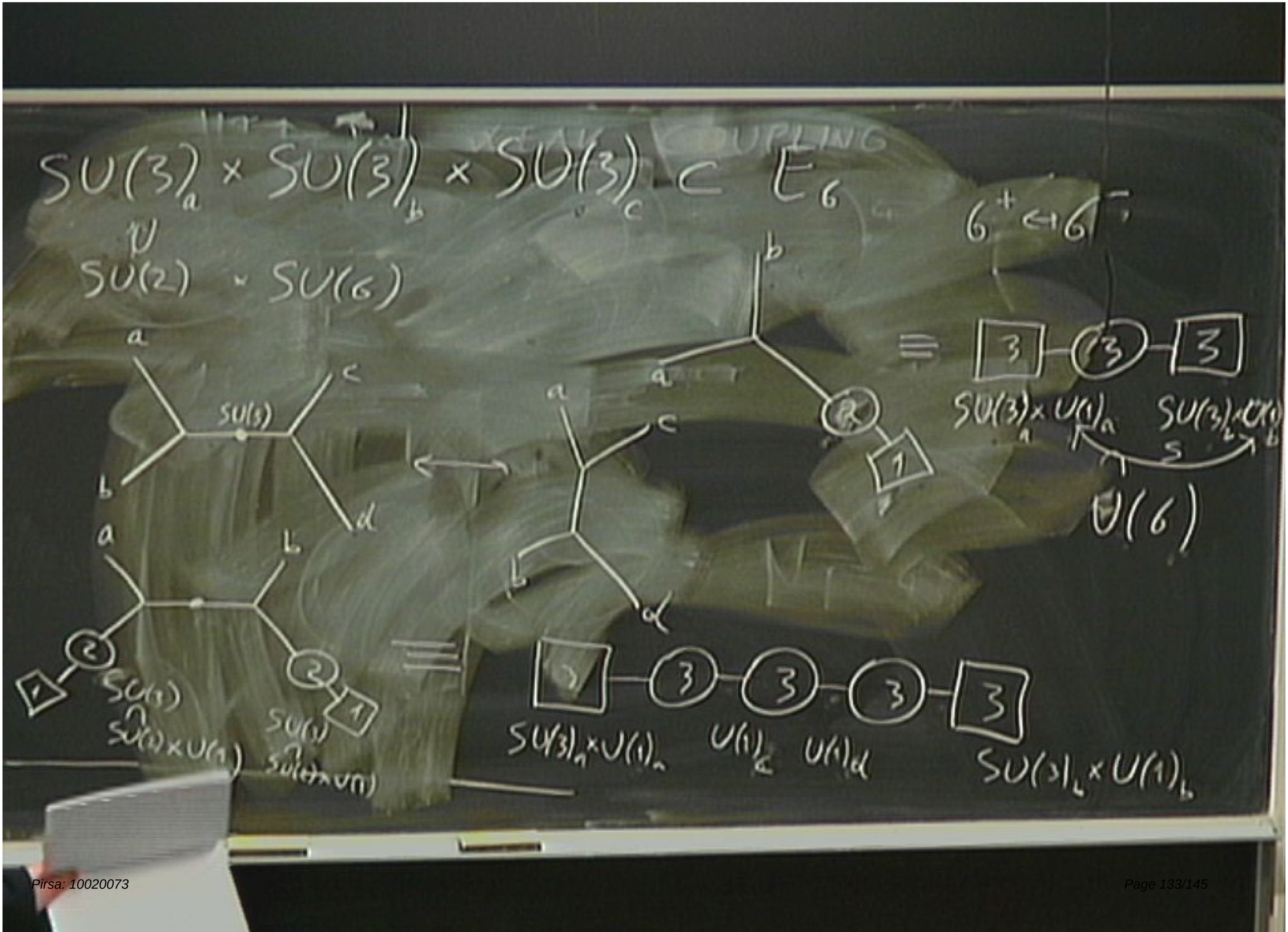
6+ ↔ 6−
≡
3
3
3
SU(3) × U(1)
a
SU(3) × U(1)
b
c
d
1
U(6)

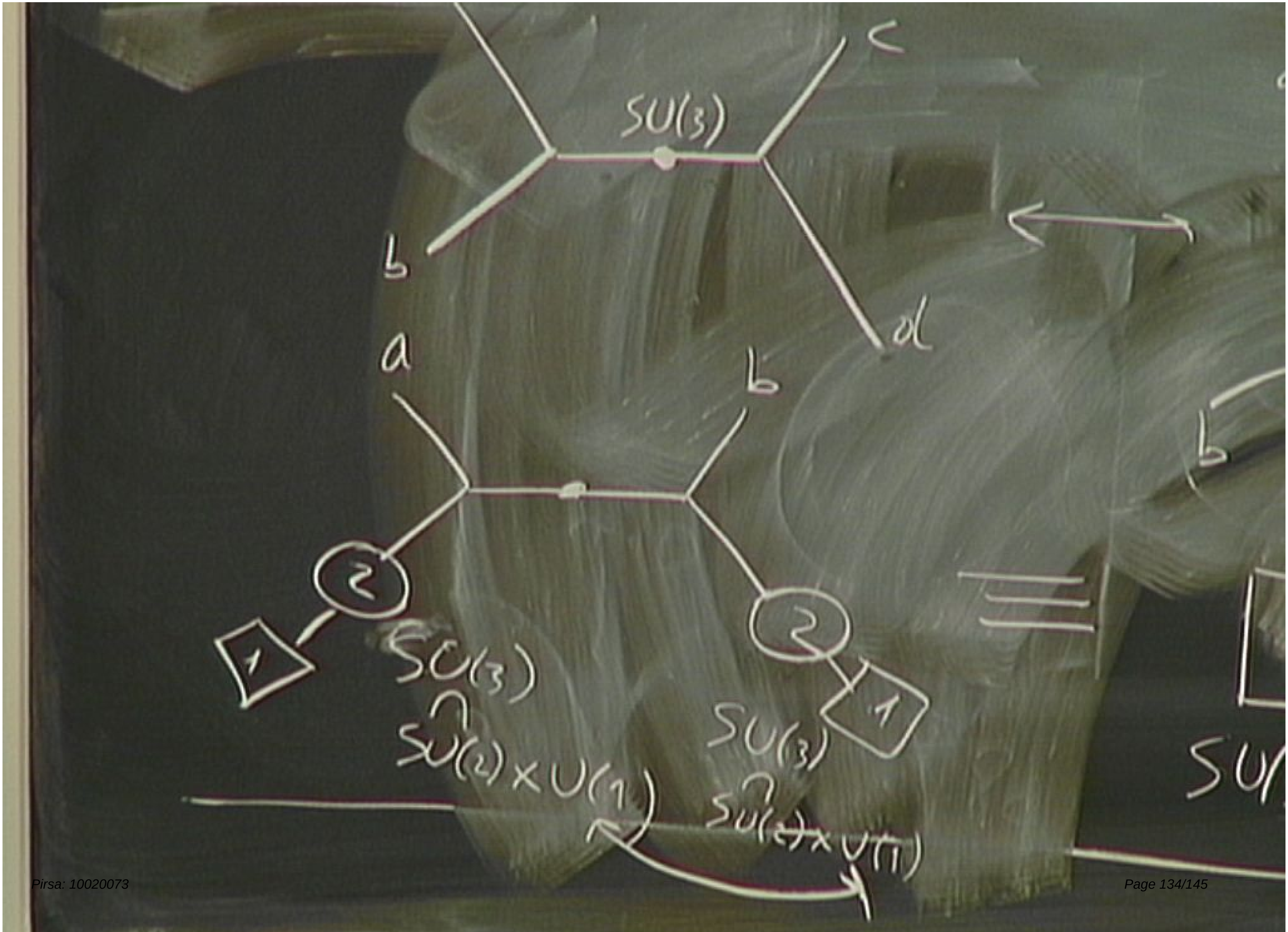


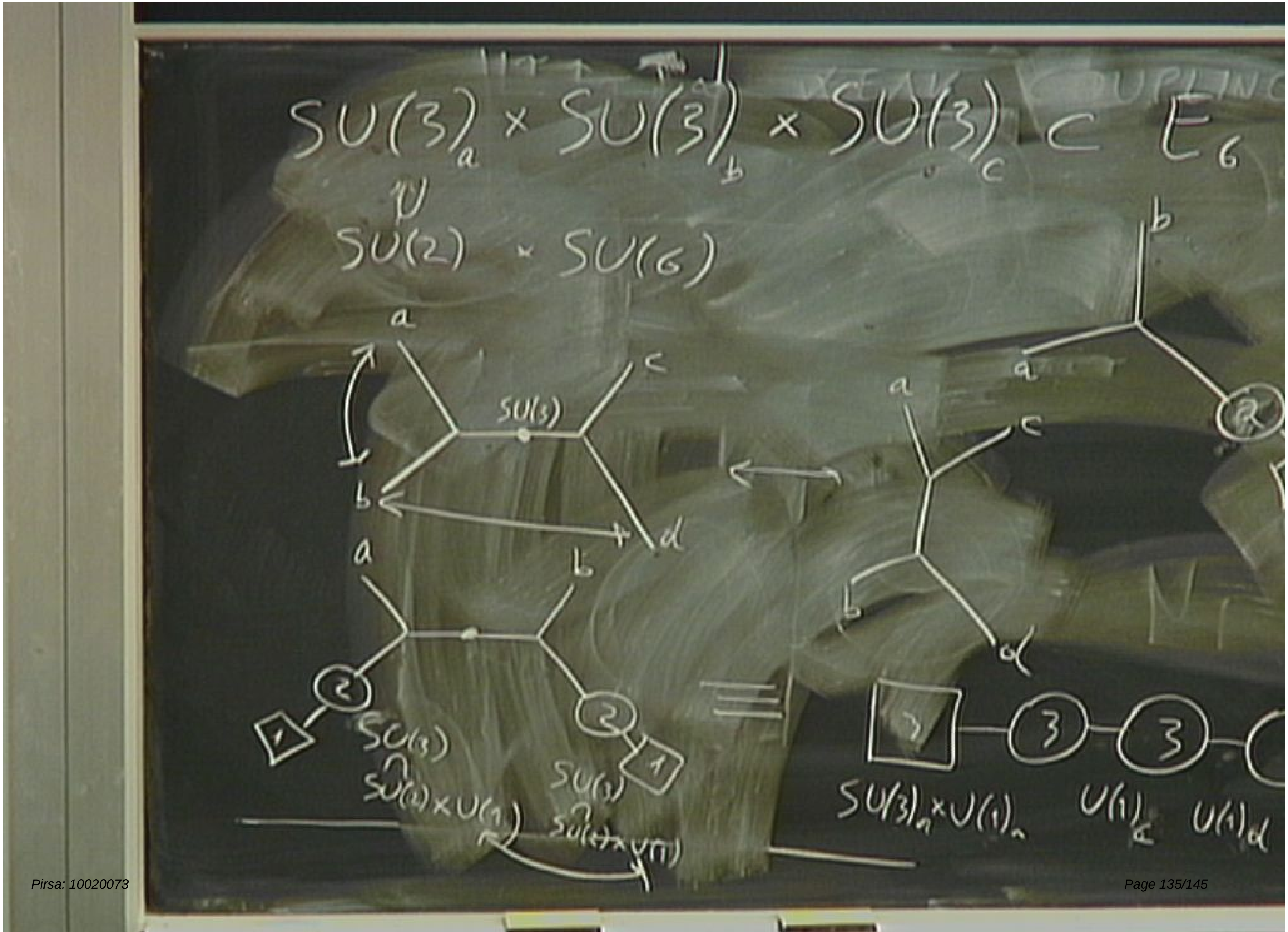
$6^+ \leftrightarrow 6^-$
$SU(3) \times U(1)_a$
$SU(3) \times U(1)_b$
$U(6)$

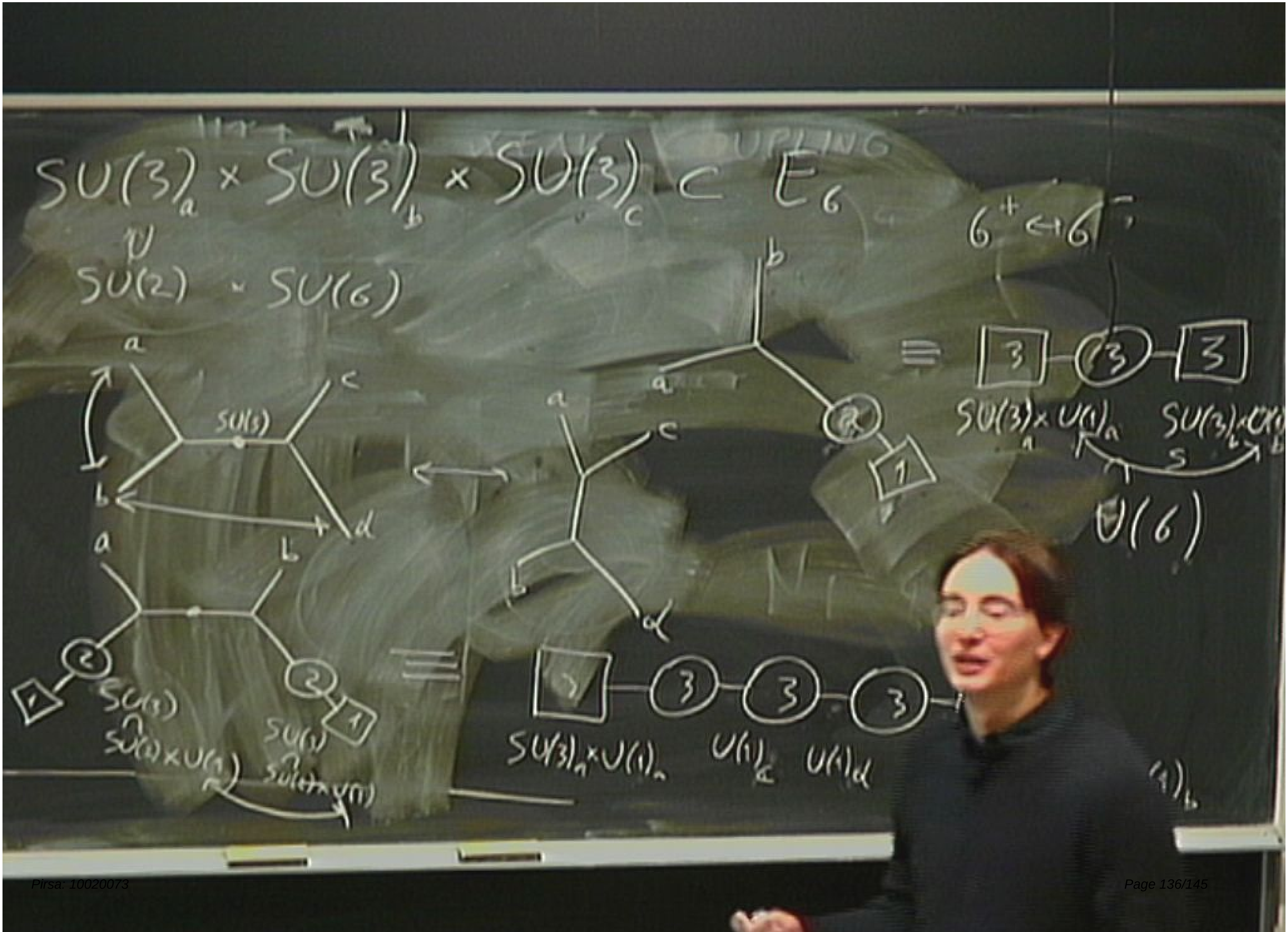


$\times SU(3)_c \subset E_6$
$6^+ \leftrightarrow 6^-$
$SU(3)\times U(1)_a$
$SU(3)_b\times U(1)_b$
$U(6)$
$SU(3)_a\times U(1)_a$
$U(1)_c$
$U(1)_d$
$SU(3)_b\times U(1)_b$


WEAK COUPLING
SU(3)a × SU(3)b × SU(3)c ⊂ E6
SU(2) × SU(6)
6+ ↔ 6−
SU(3)
SU(3)a×U(1)a
SU(3)b×U(1)b
U(6)
SU(3)a×U(1)a
U(1)c
U(1)d
SU(3)b×U(1)b


c
SU(3)
b
d
a
b
2
1
SU(3)
∩
SU(2)×U(1)
2
1
SU(3)
∩
SU(2)×U(1)
b

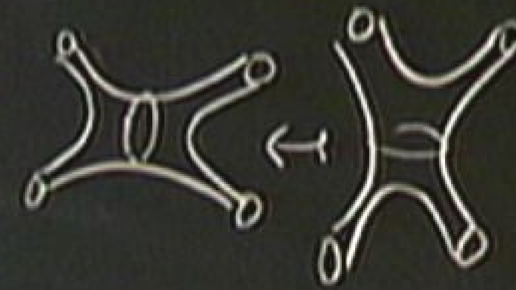

SU(3)a × SU(3)b × SU(3)c ⊂ E6
SU(2) × SU(6)
a
c
SU(3)
b
d
a
b
2
2
1
1
SU(3)
SU(2)×U(1)
SU(3)
SU(2)×U(1)
b
a
a
c
b
d
3
3
SU(3)a×U(1)a
U(1)c
U(1)d



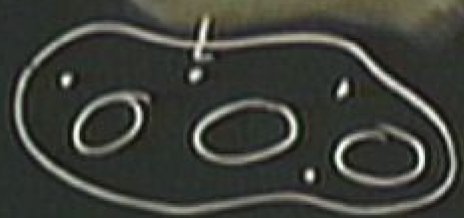
SU(3)a × SU(3)b × SU(3)c ⊂ E6
SU(2) × SU(6)
6+ ↔ 6−
SU(3)
SU(3)a×U(1)a
U(6)
SU(3)×U(1)
SU(3)a×U(1)a
U(1)c
U(1)d

LAGRANGIAN DESCRIPTIONS ≡ PAIR OF PANTS DECOMP.

S-DUALITY ≡ MOORE-SEIBERG GROUPOID



≡ $\mathcal{M}_{g,n}$

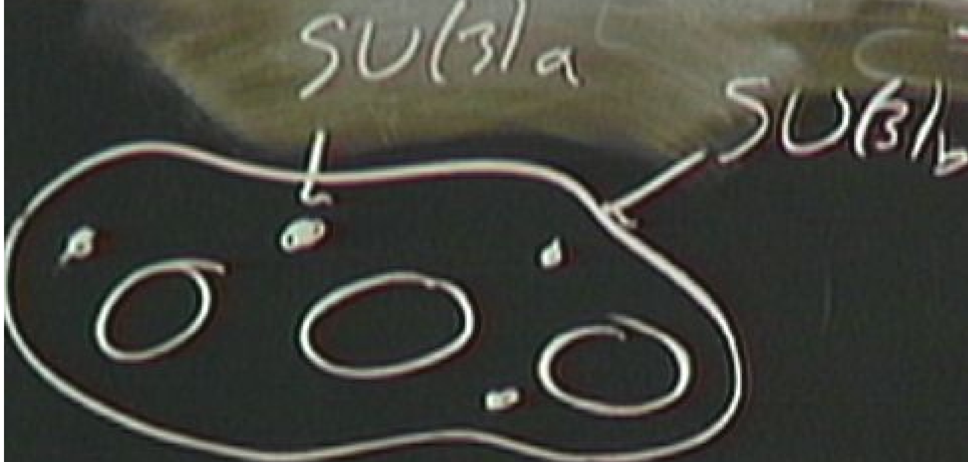


GAUGE COUPLINGS τ_i

$R = \oplus \tau_i^{n_i}$ $\quad \prod U(n_i)$

IF τ_i ARE REAL $\prod Sp(2n_i)$

" " " PSEUDOREAL $SO(2n_i)$



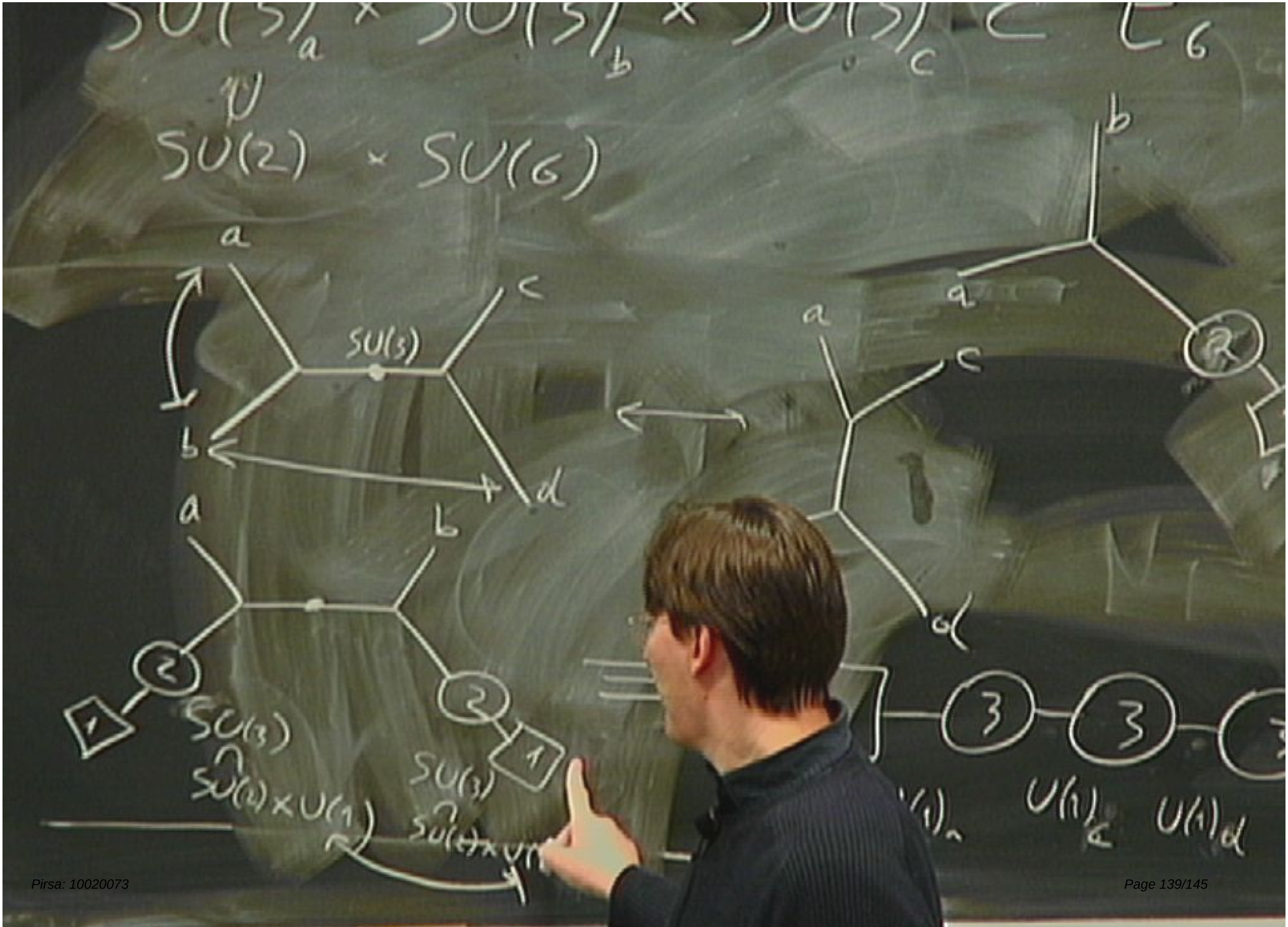
$\equiv \mathcal{M}_{g,n}$

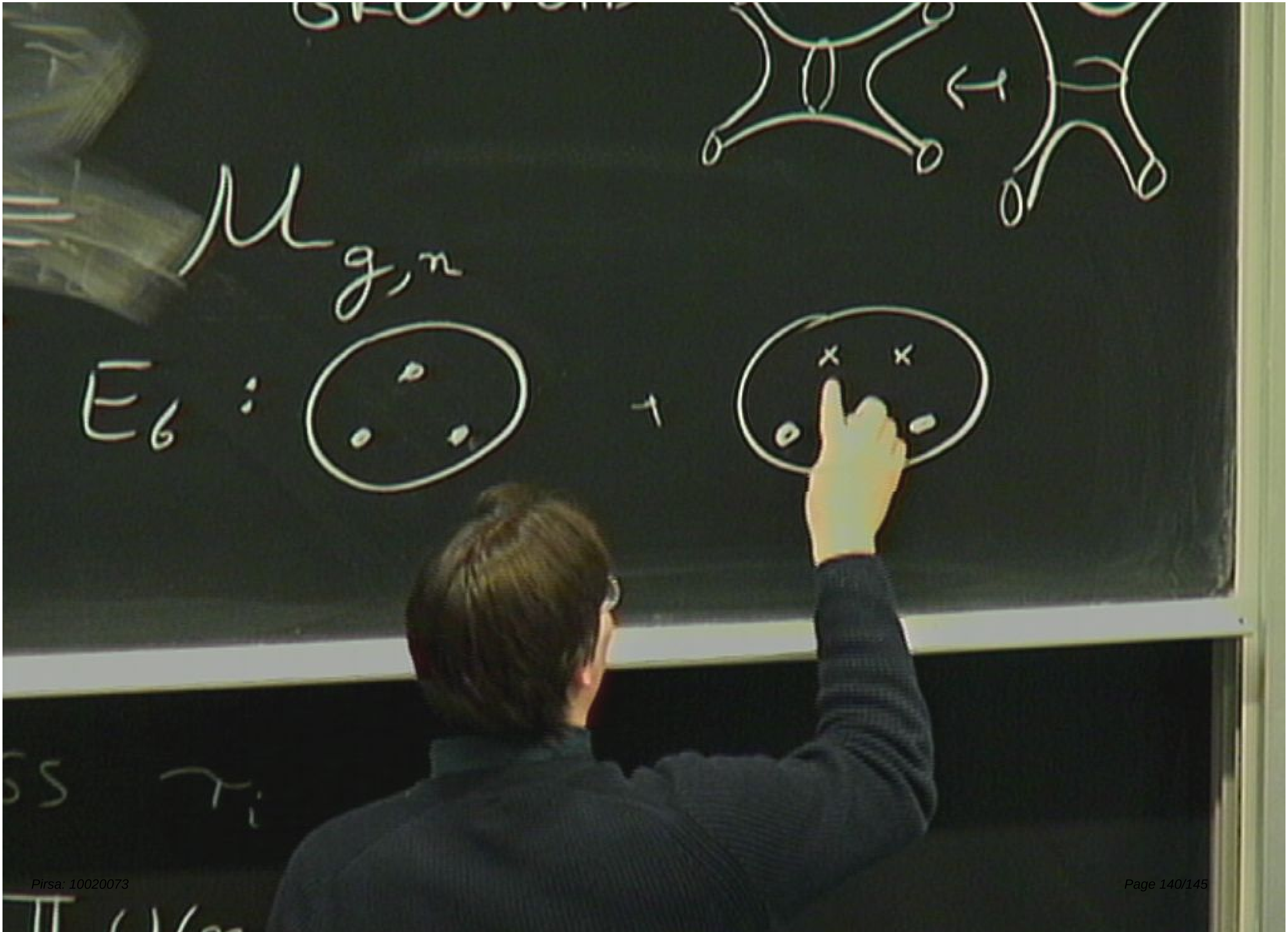
E_6 :

GAUGE COUPLINGS τ_i

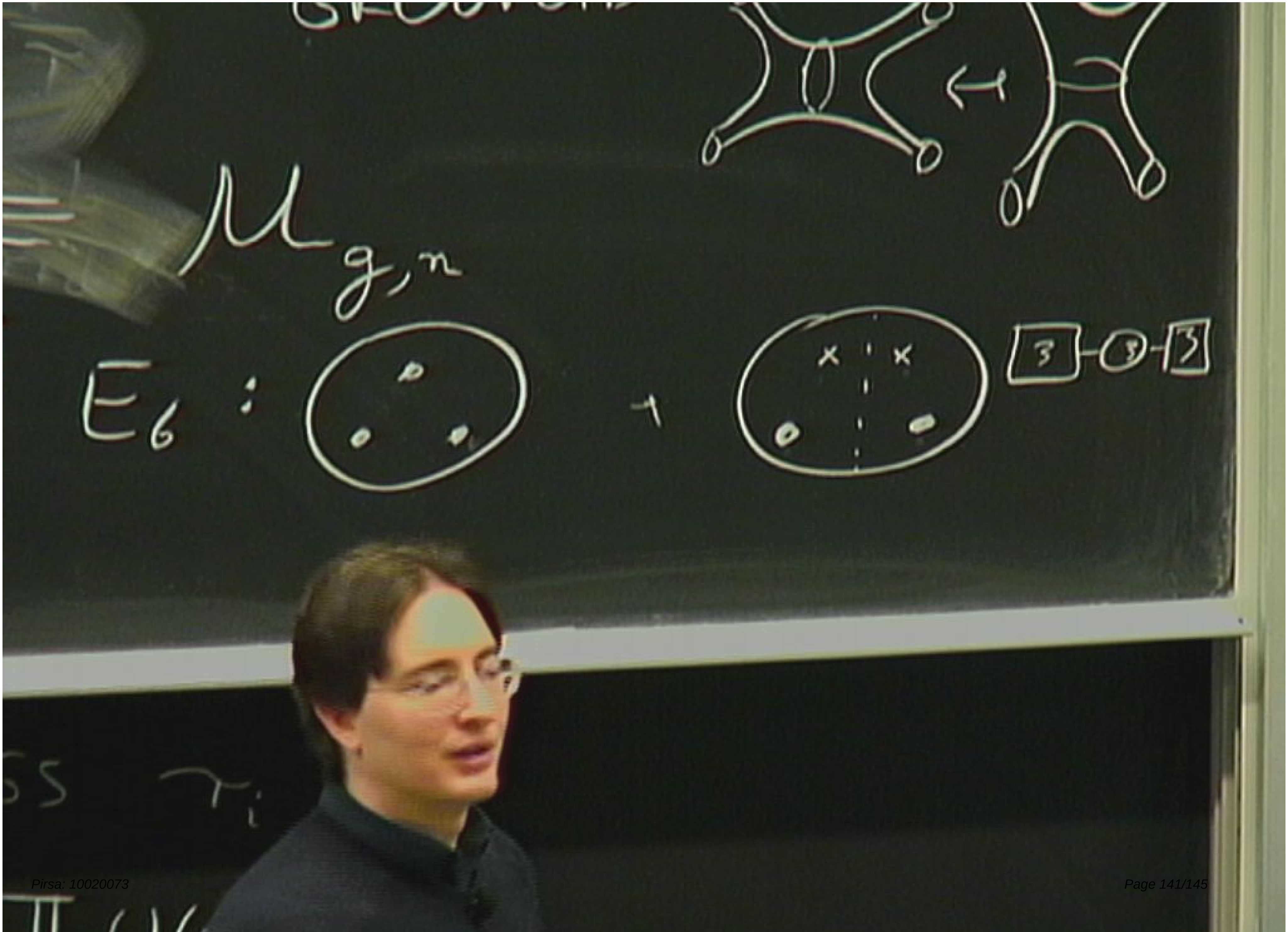
$R = \oplus\, z_i^{n_i} \qquad \prod U(n_i)$

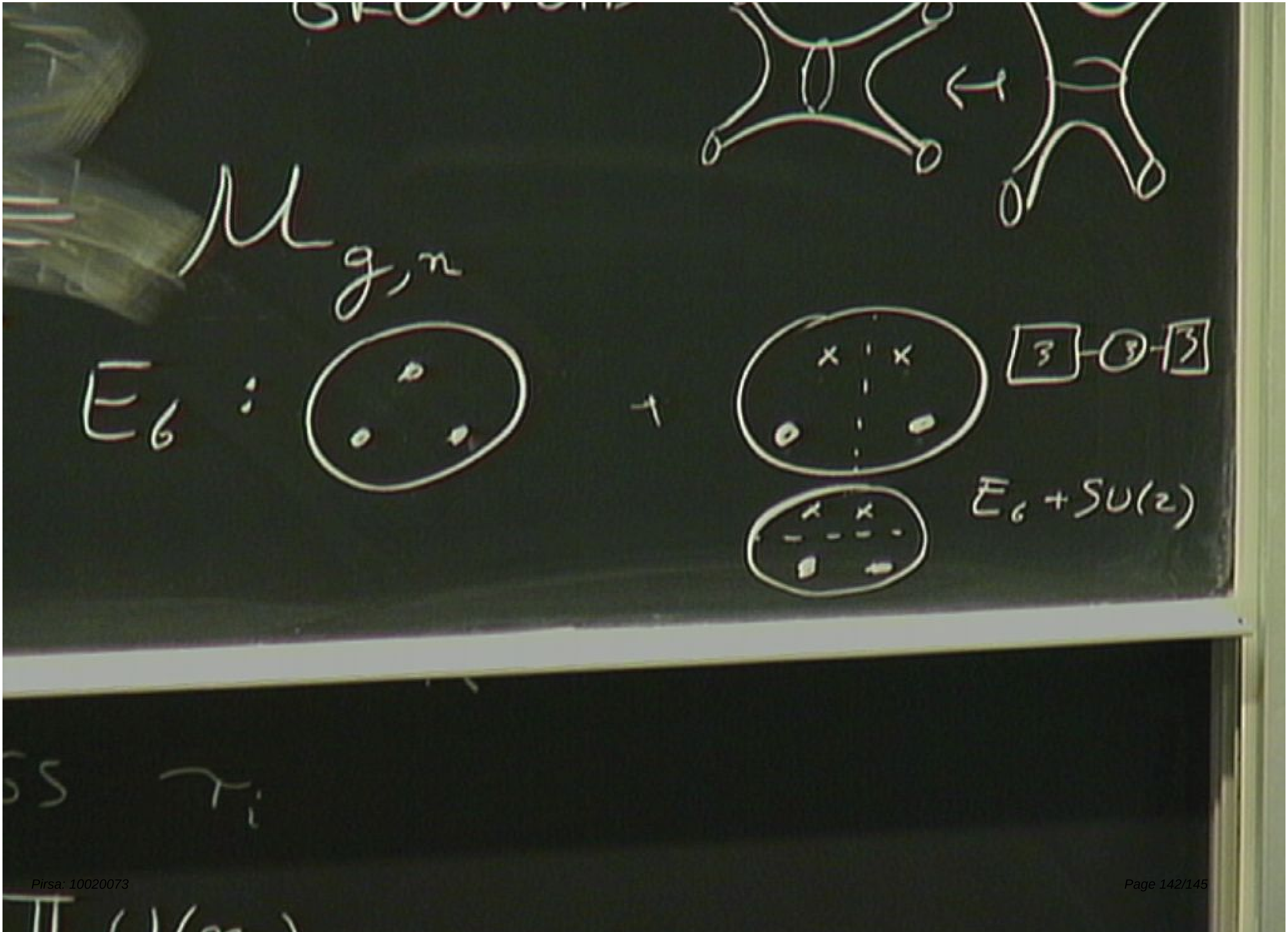
IF z_i ARE REAL


SU(5)a × SU(5)b × SU(5)c ⊂ E6
SU(2) × SU(6)
SU(3)
SU(3)
SU(2)×U(1)
SU(3)
U(1)c
U(1)d



$\mathcal{M}_{g,n}$
E_6 :

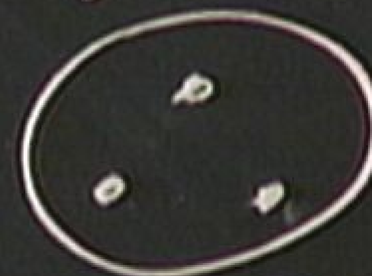

M g,n
E6 :
3 3 3


M_{g,n}
E_6 :
3
3
3
E_6 + SU(2)
T_i

GROUPOI

$\mathcal{M}_{g,n}$

E_6 :



$T_{g,\{n_1,n_2\}}$

n_1 SU(3)

n_2 U(1)

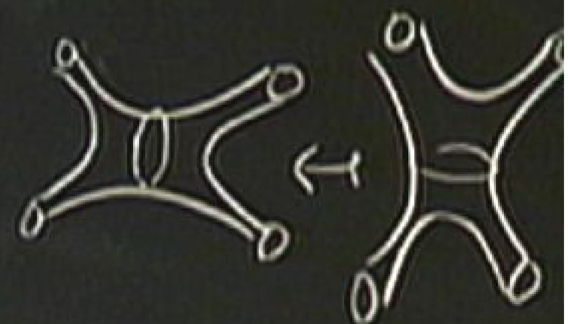
GAUGE COUPLINGS τ_i

$R = \oplus z_i^{n_i}$

$T_{g,n}$ $C_{g,n}$

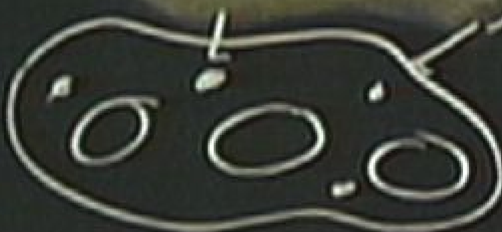
LAGRANGIAN DESCRIPTIONS $\equiv$ PAIR OF PANTS DECOMP.

S-DUALITY $\equiv$ MOORE-SEIBERG GROUPOID



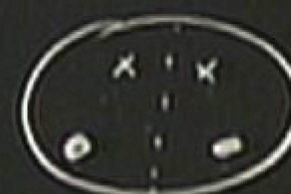
$\equiv \mathcal{M}_{g,n}$

$SU(3)_a$ $SU(3)_b$



E_6 :

+



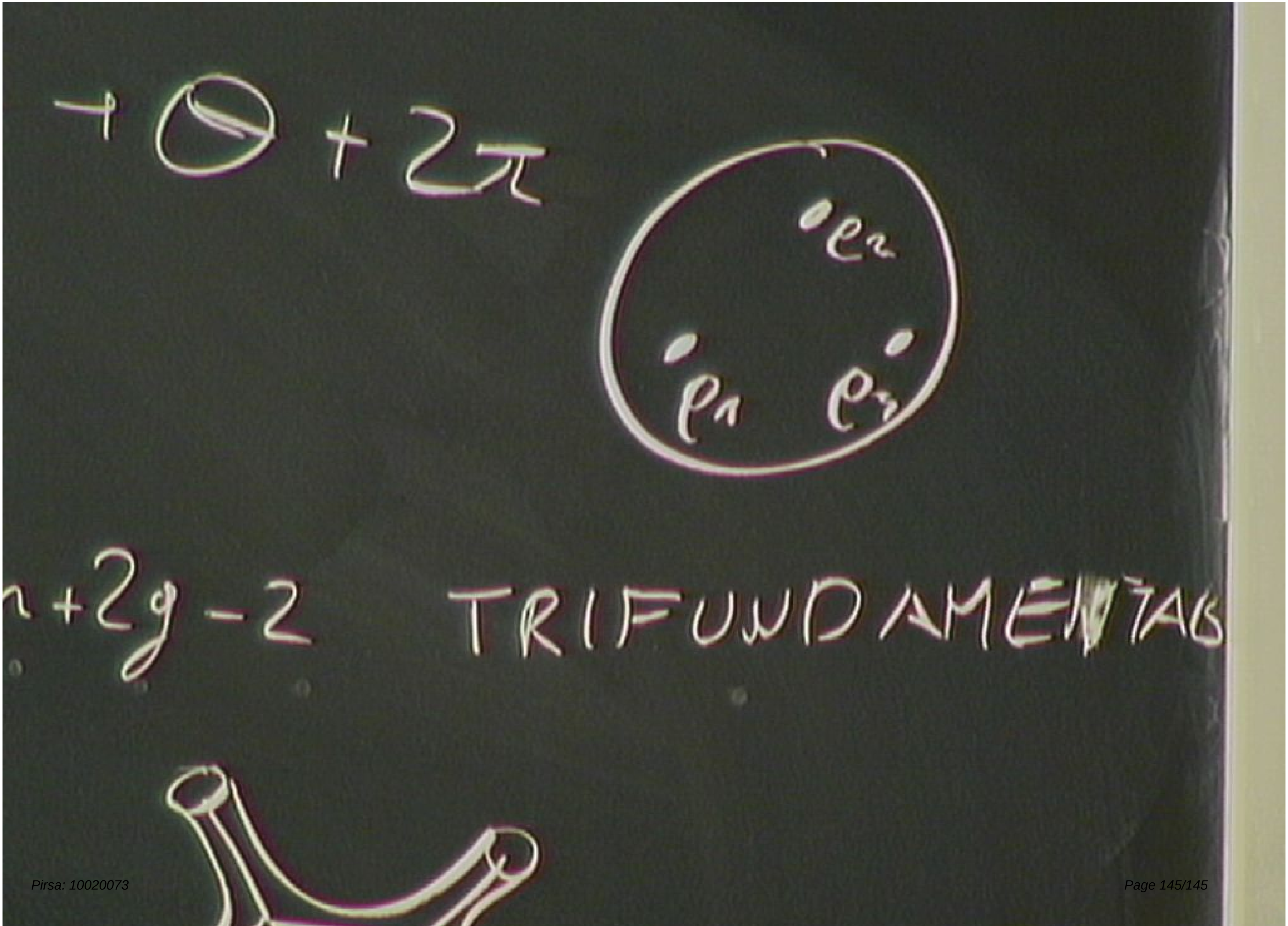
$T_{g,\{n_1,n_2\}}$ n_1 $SU(3)$ n_2 $U(1)$

$E_6 + SU(2)$

GE COUPLINGS τ_i

$\oplus\ z_i^{n_i}$ $\prod U(n_i)$

-ARE


+ Θ + 2π
e2
e1
e3
+2g − 2
TRIFUNDAMENTAL