Title: Switching boxes connections in operational theories and its consequence on causality

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Abstract: How can we describe a device that takes two unknown operational boxes and conditionally on some input variable connects them in different orders? In order to answer this question, I will introduce maps from transformations to transformations within operational probabilistic theories with purification, and show their characterisation in terms of operational circuits. I will then proceed exploring the hierarchy of maps on maps. A particular family of maps in the hierarchy are the ones whose output is in the set of transformations. These maps can be fully characterised by their correspondence with channels with memory, and it is exactly the family of transformations that can be implemented through operational circuits. I will then show the problems that arise in defining the remainder of the hierarchy, and the reason why we cannot avoid considering its elements. The main consequence of admitting the generalised transformations as possible within the operational theory is that we cannot describe them in terms of simple causal connection of transformations in a circuit with a fixed causal structure. In quantum theory, we can understand such higher order transformations in terms of superpositions of circuits with different causal structures. The problem whether computations exploiting higher-order transformations can be efficiently simulated by a conventional circuital computer is posed.

Switching boxes' connections in operational theories and its consequence on causality

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Perimeter Institute, Waterloo, February 2 2010

In collaboration with

- G. M. D'Ariano

- G. Chiribella

Outline

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- The operational framework - causality
- Maps from transformations to transformations: non causal theories

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  - Causal representation
- Combs and their realisation
- Maps from combs to combs

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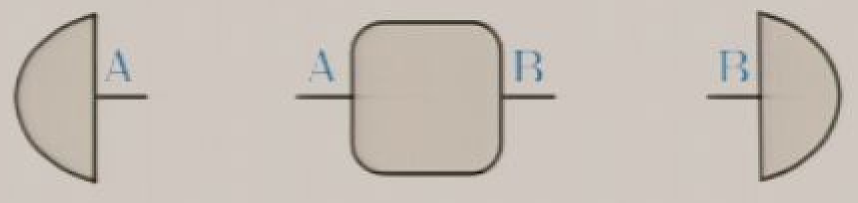
- Maps from transformations to transformations: non causal theories
  - Causal representation
- Combs and their realisation
- Maps from combs to combs
  - Problems
  - The switch map
- Higher-order maps and non-causal theories without causal interpretation

Operational Framework

States, transformations, effects
Systems



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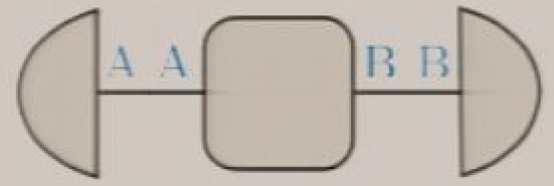

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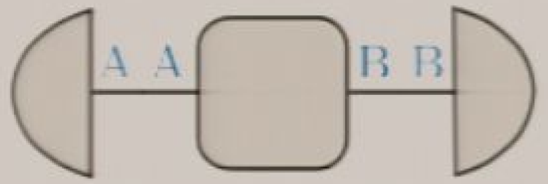

Sequential composition

Operational Framework

Outcomes

States, transformations, effects
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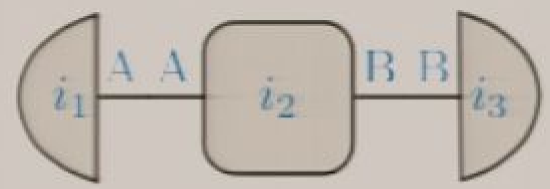
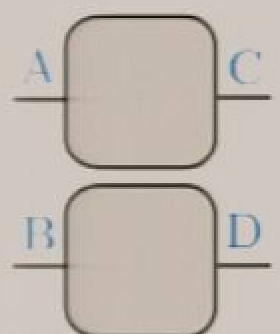
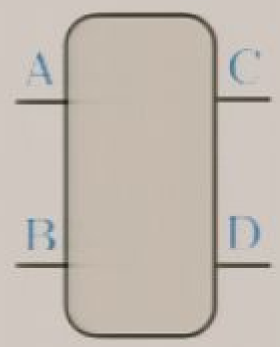


Sequential composition

Parallel composition





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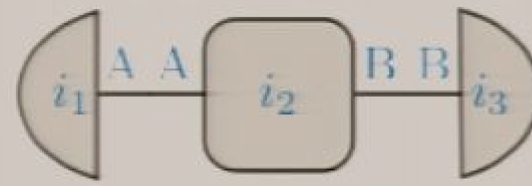
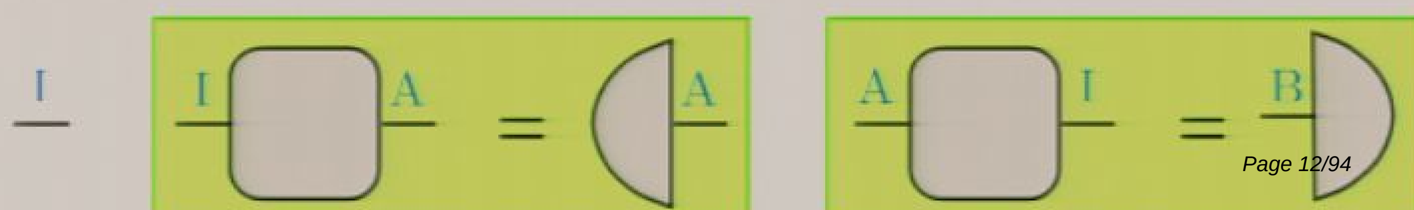


Sequential composition

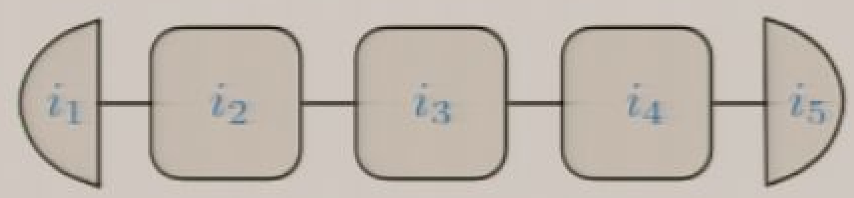
Parallel composition

Trivial system



Probabilistic theories

Every closed chain is a probability



$$= p(i_1, i_2, i_3, i_4, i_5)$$

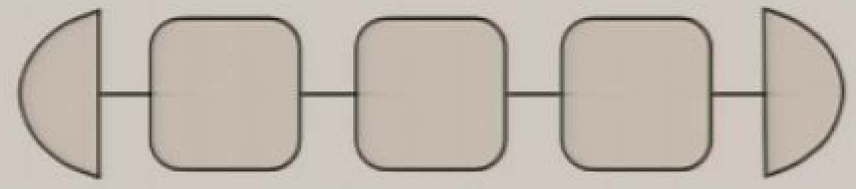
$$0 \leq p \leq 1$$

The chain for a full test is deterministic

Causality

Causality

- Causality



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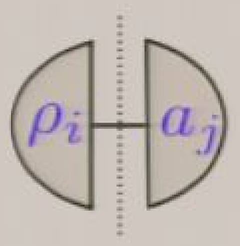

$p(\rho_i$

Causality

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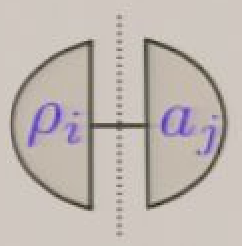

$$p(\rho_i) = p_a(\rho_i) := \sum_j (a_j|\rho_i)$$

Causality

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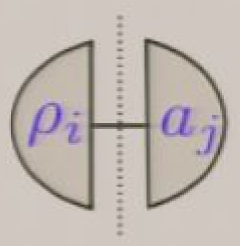

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$$\sum_j (a_j| = \sum_k (b_k| = (e|$$

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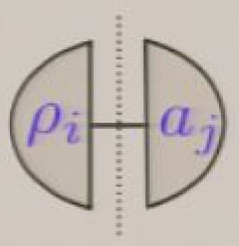

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- Prior probabilities for preparation events

- Normalised states are a base for the cone

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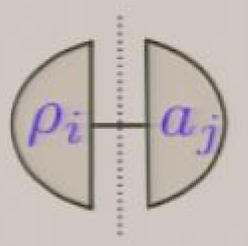

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- **Uniqueness of the deterministic effect** $\sum_j (a_j| = \sum_k (b_k| = (e|$

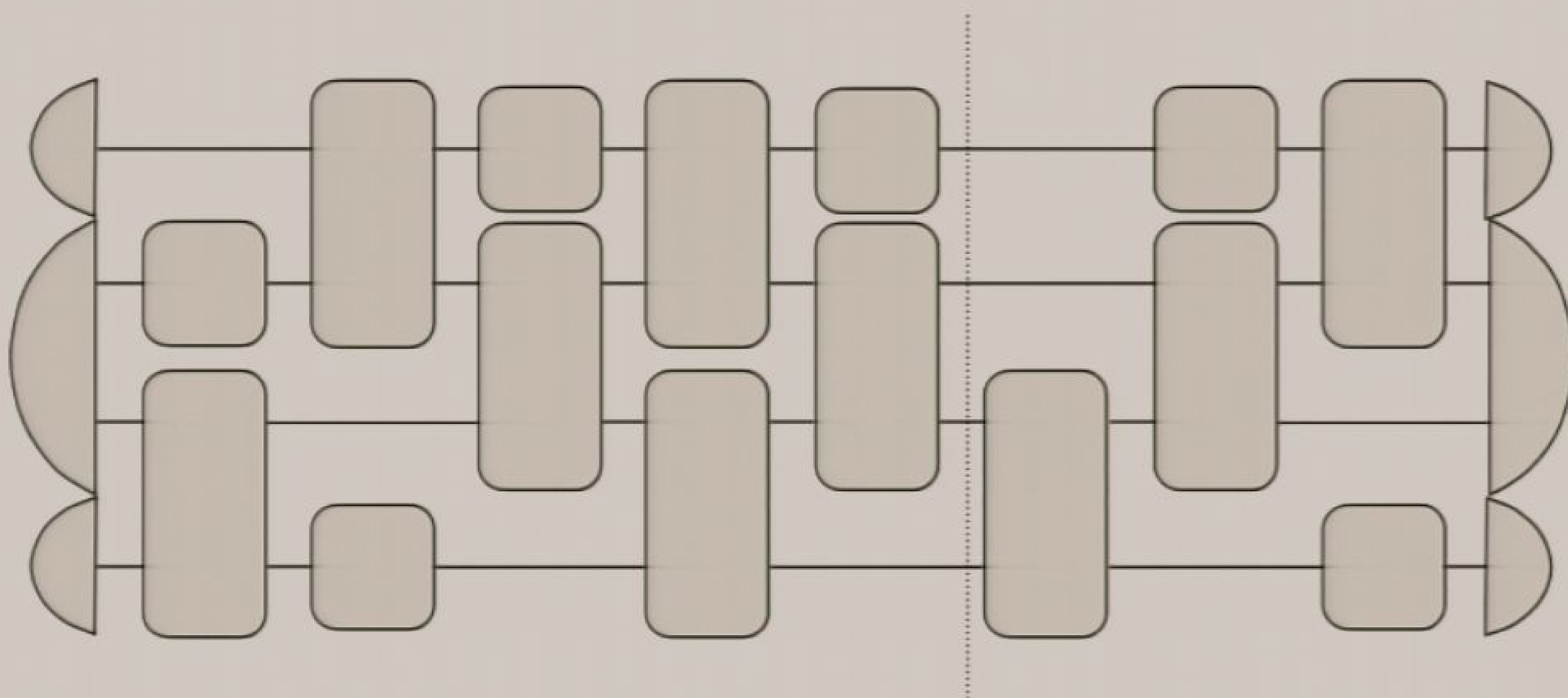
- Prior probabilities for preparation events

- Normalised states are a base for the cone

- Unrestricted conditioning

- Deterministic transformations

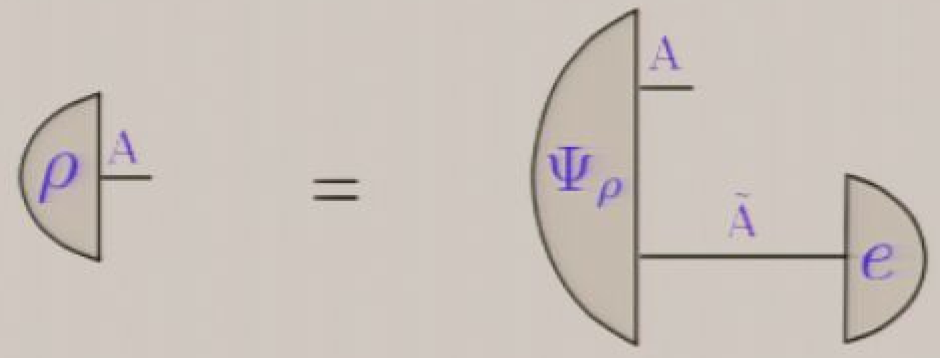
About causality



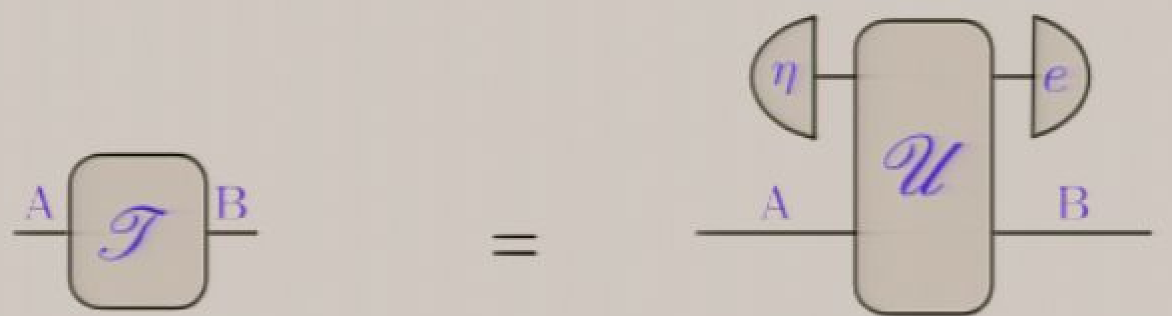
No causality ⇒ probability of outcomes on lhs may depend on boxes on the rhs

Purification

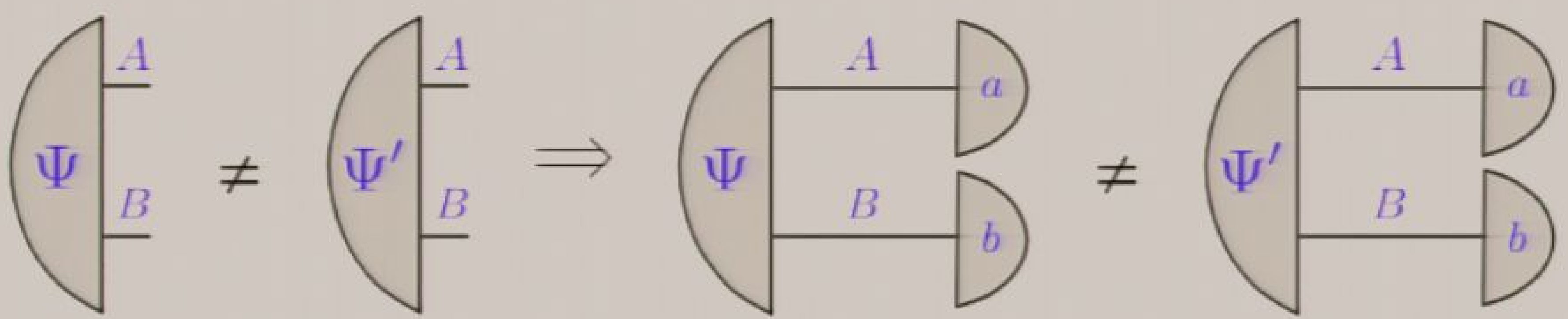
- For any system type A there exists a **conjugate** system type Ã such that



- Consequence: all channels and tests have a reversible extension



Local discriminability



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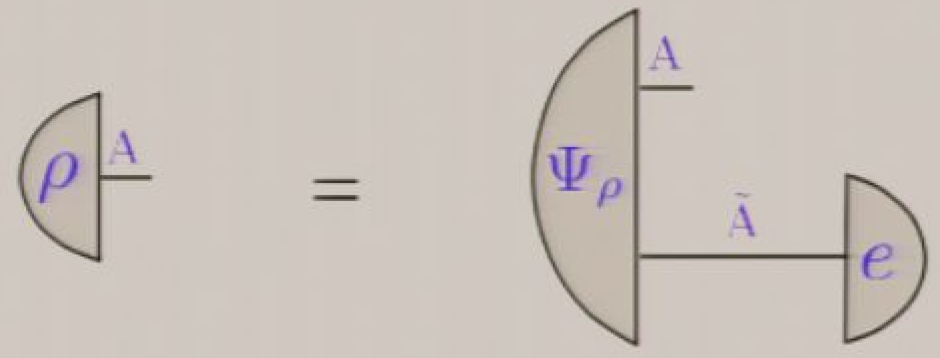

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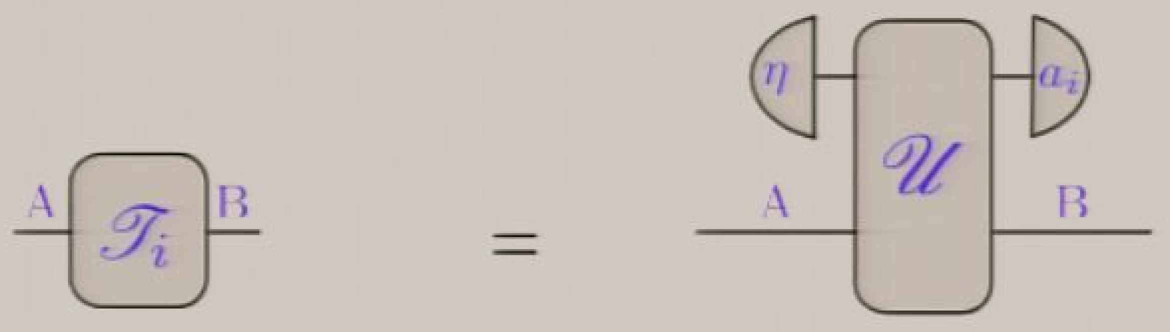

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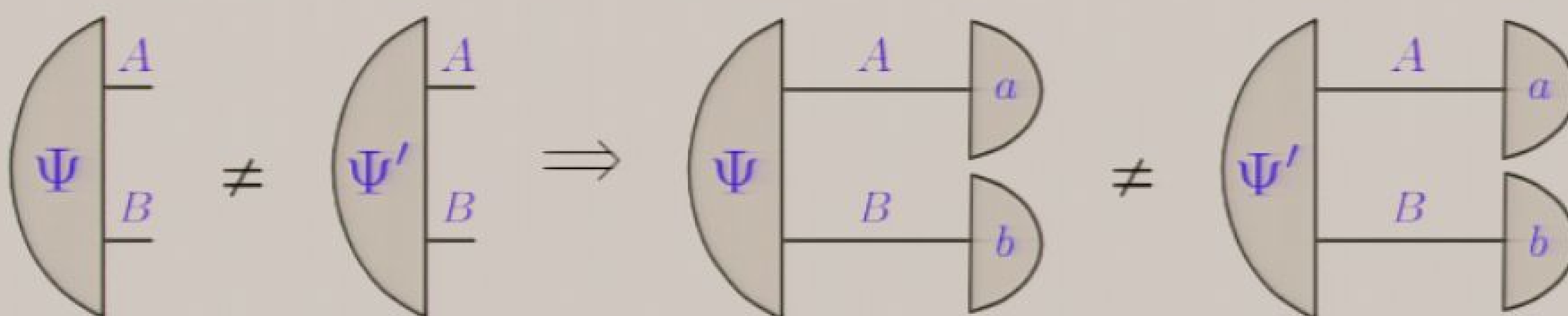
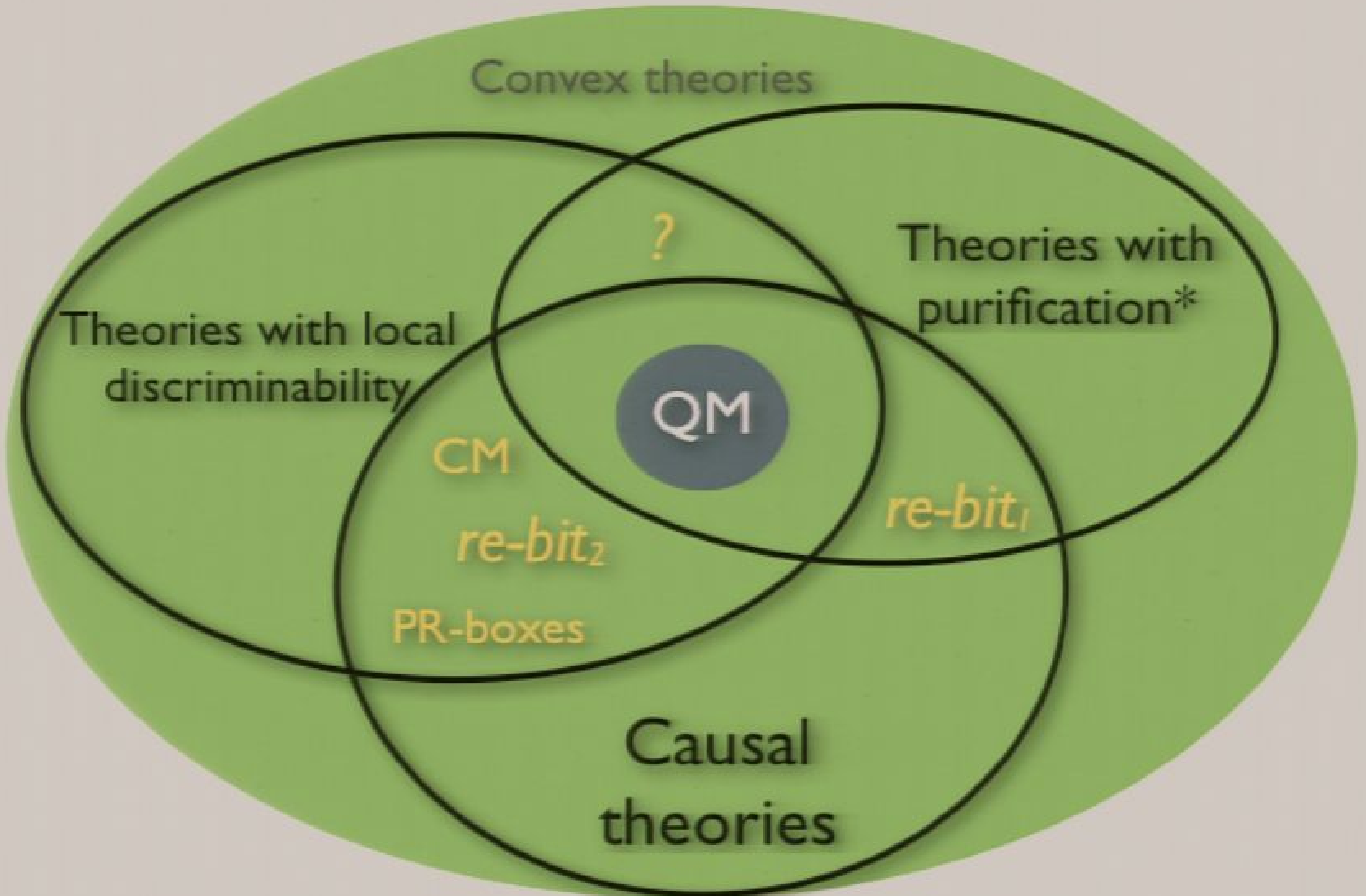


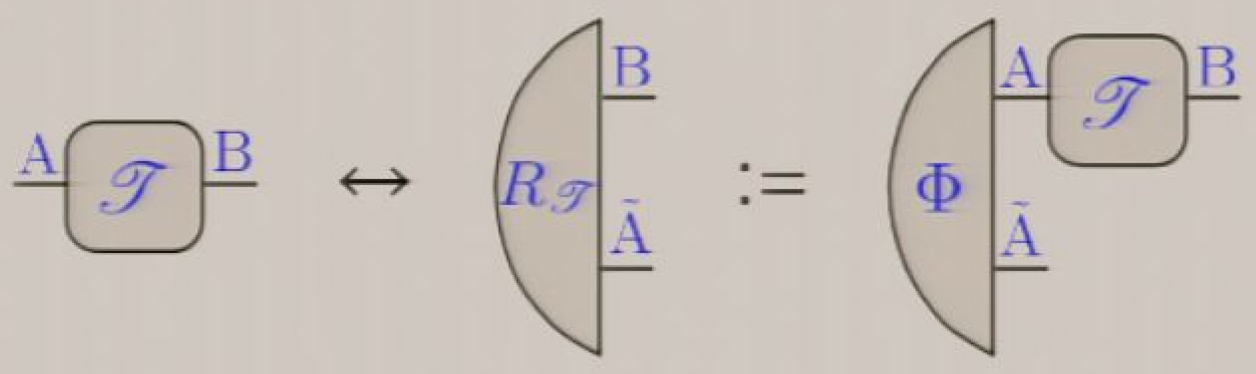

Convex theories
?
Theories with purification*
Theories with local discriminability
QM
CM
re-bit₂
re-bit₁
PR-boxes
Causal theories

Choi-Jamiołkowski correspondence

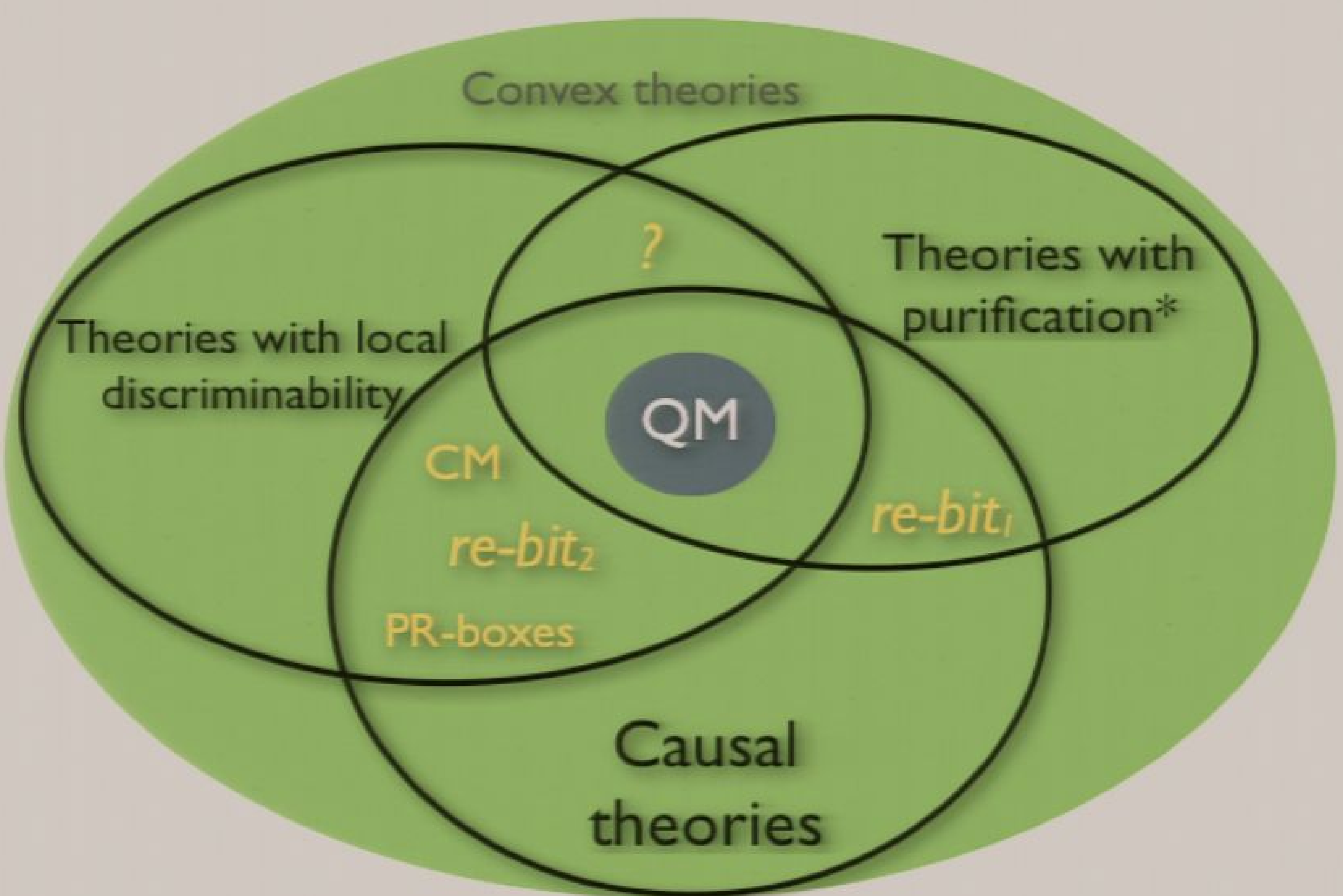
- Correspo

Choi-Jamiołkowski correspondence

- Correspondence between bipartite states BÃ and transformations A→B



Choi-Jamiołkowski correspondence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Choi-Jamiołkowski correspondence

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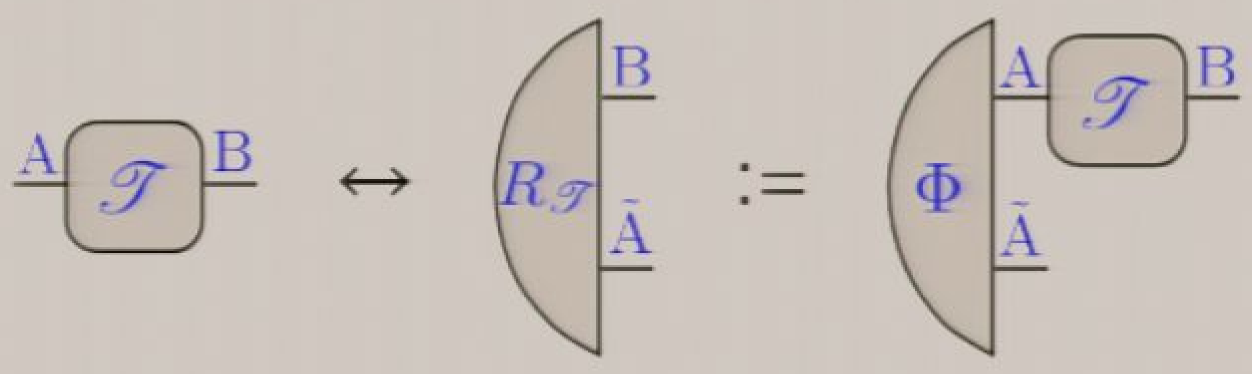
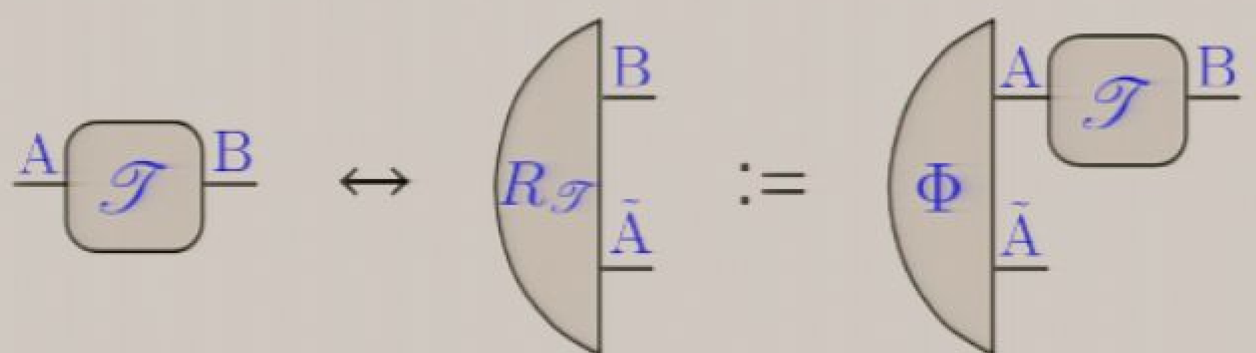
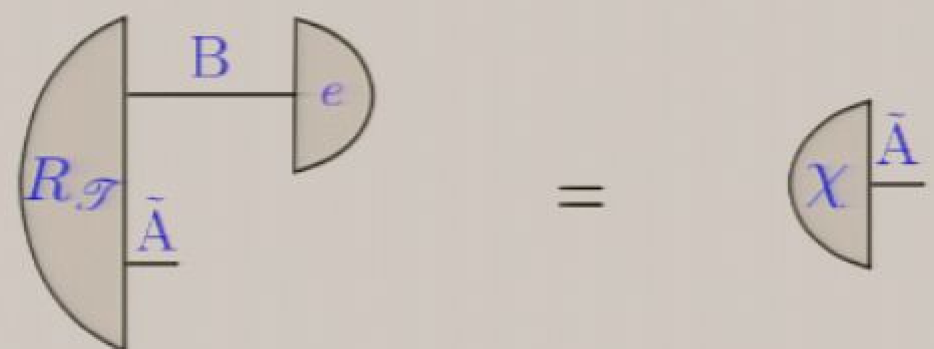


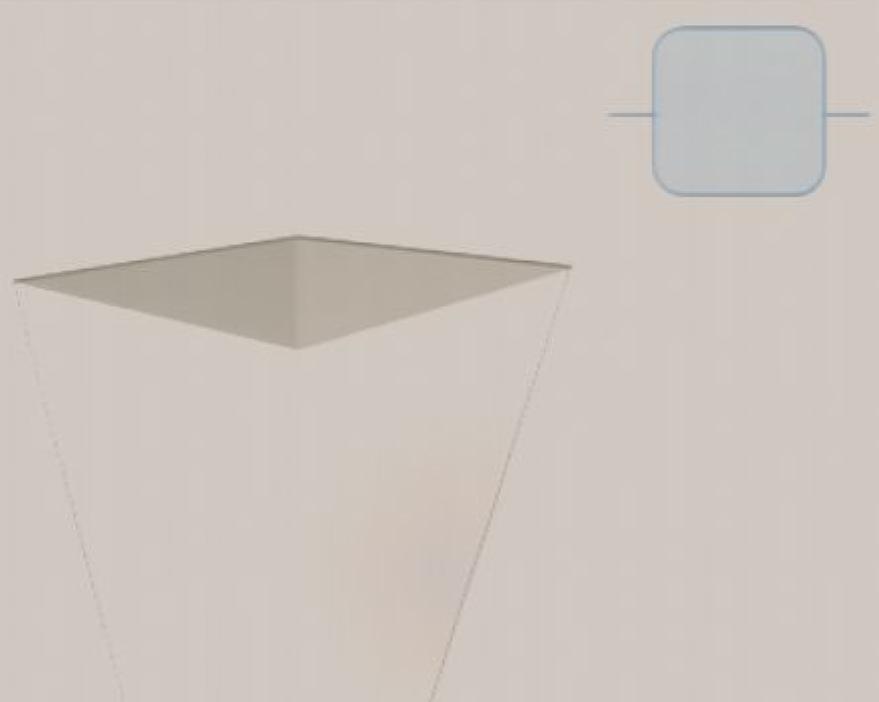
Choi-Jamiołkowski correspondence

- Correspondence between bipartite states BÃ and transformations A→B
  - Deterministic transformations are in correspondence with some states





Transformations as states: supermaps



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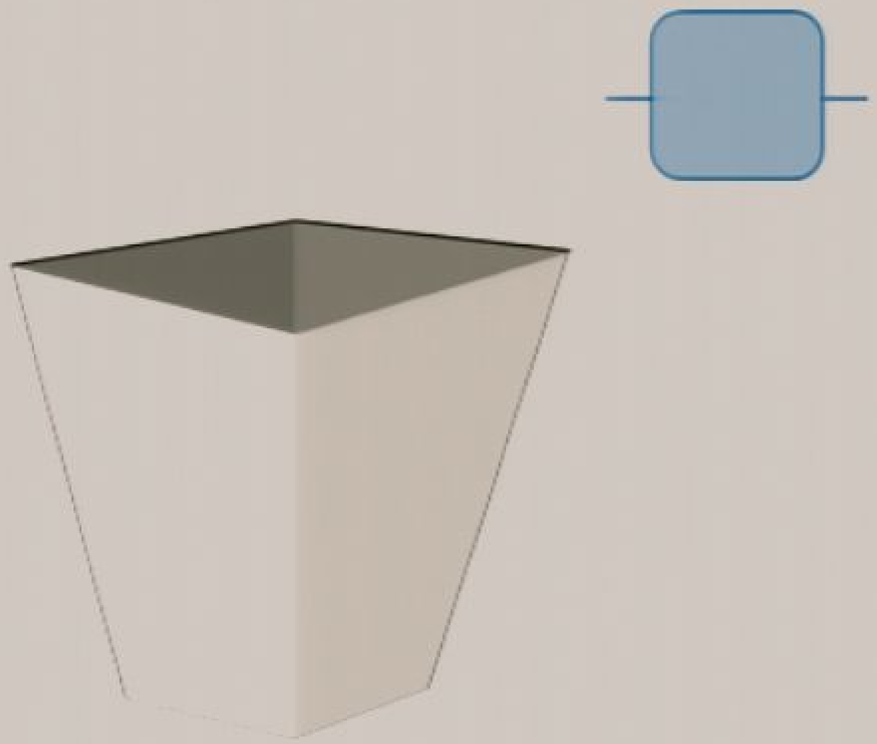

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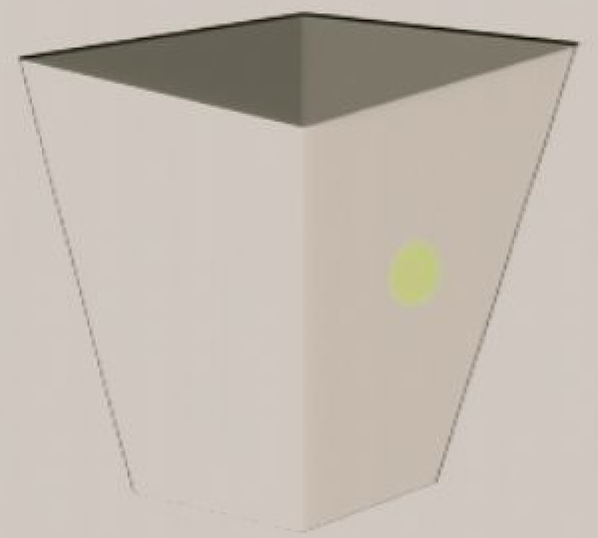

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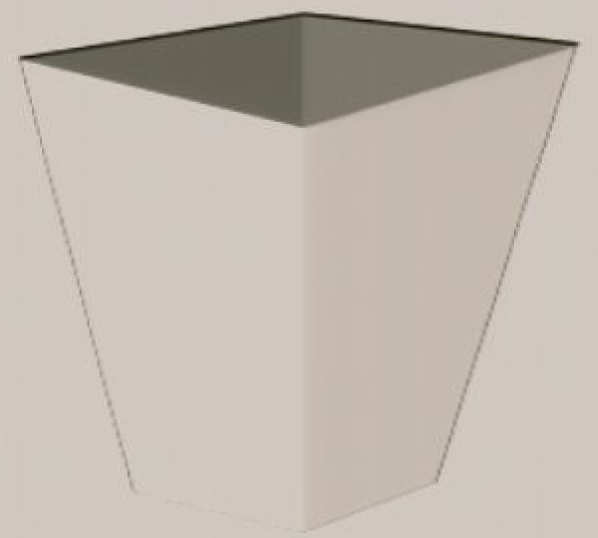

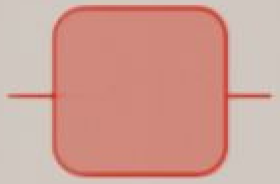

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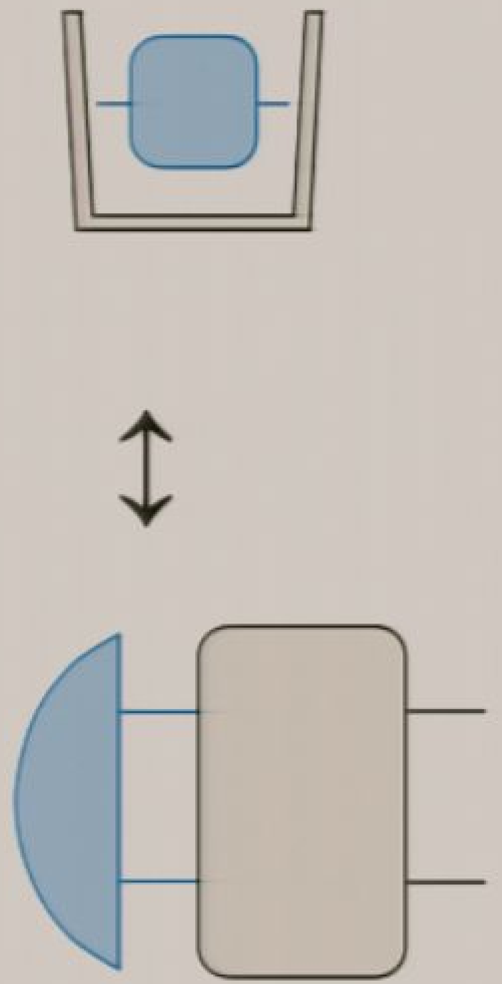

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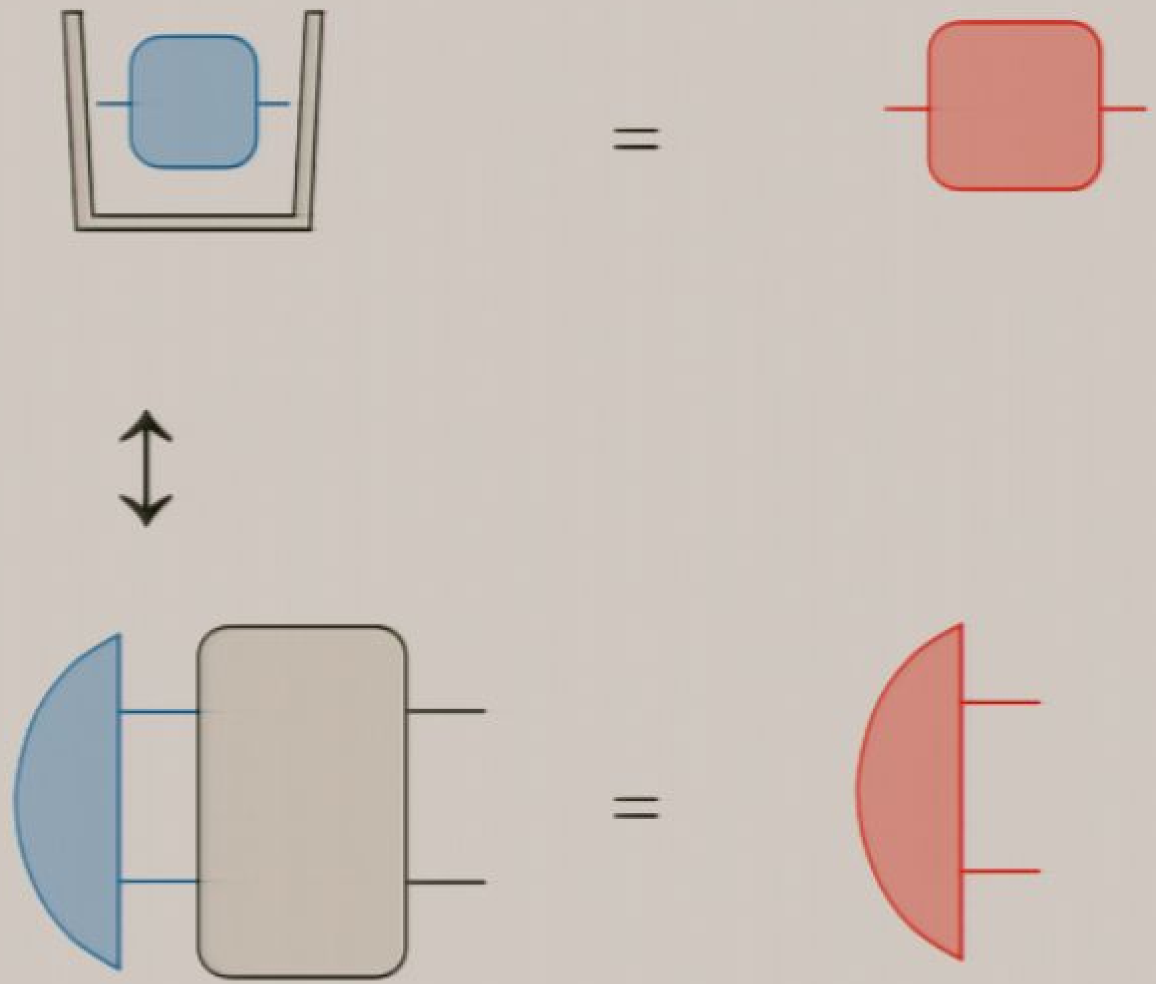


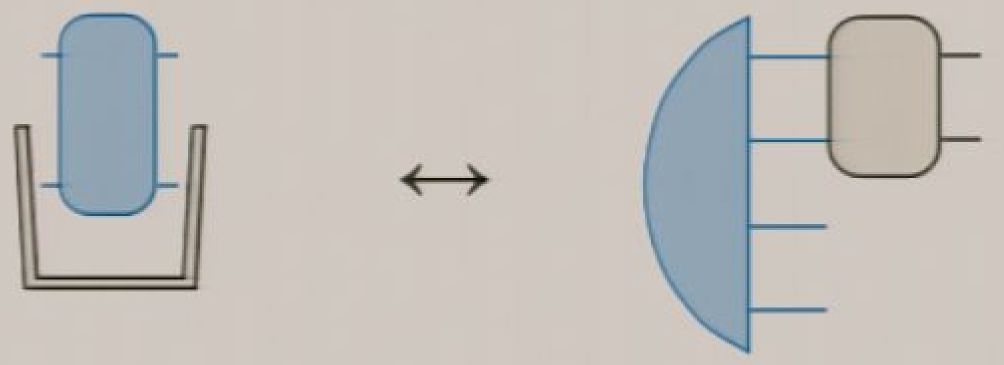
Transformations as states: supermaps

Admissibility conditions

Transformations as states: supermaps

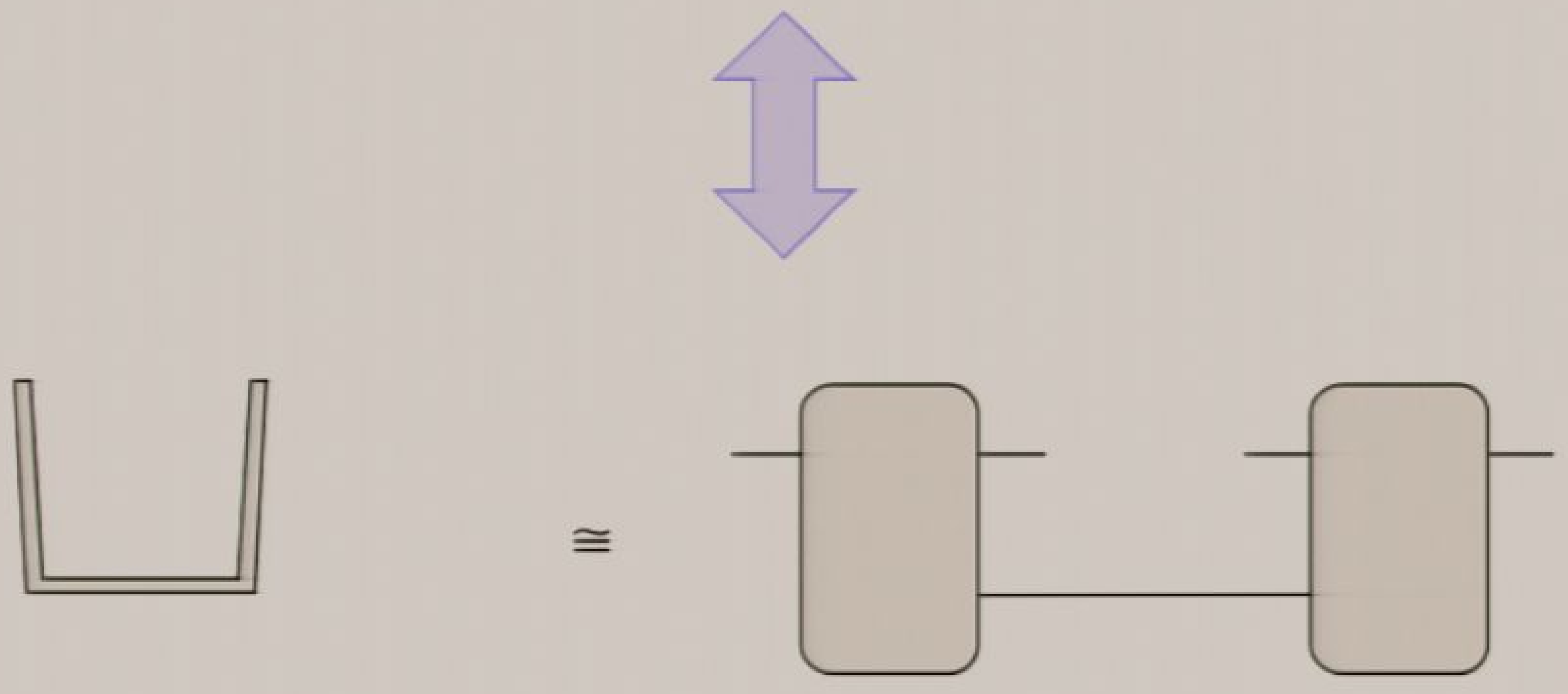
Admissibility conditions

- Linear → preservation of convex combinations (probabilities)
- Completely Positive



Realisation theorem

Admissibility conditions



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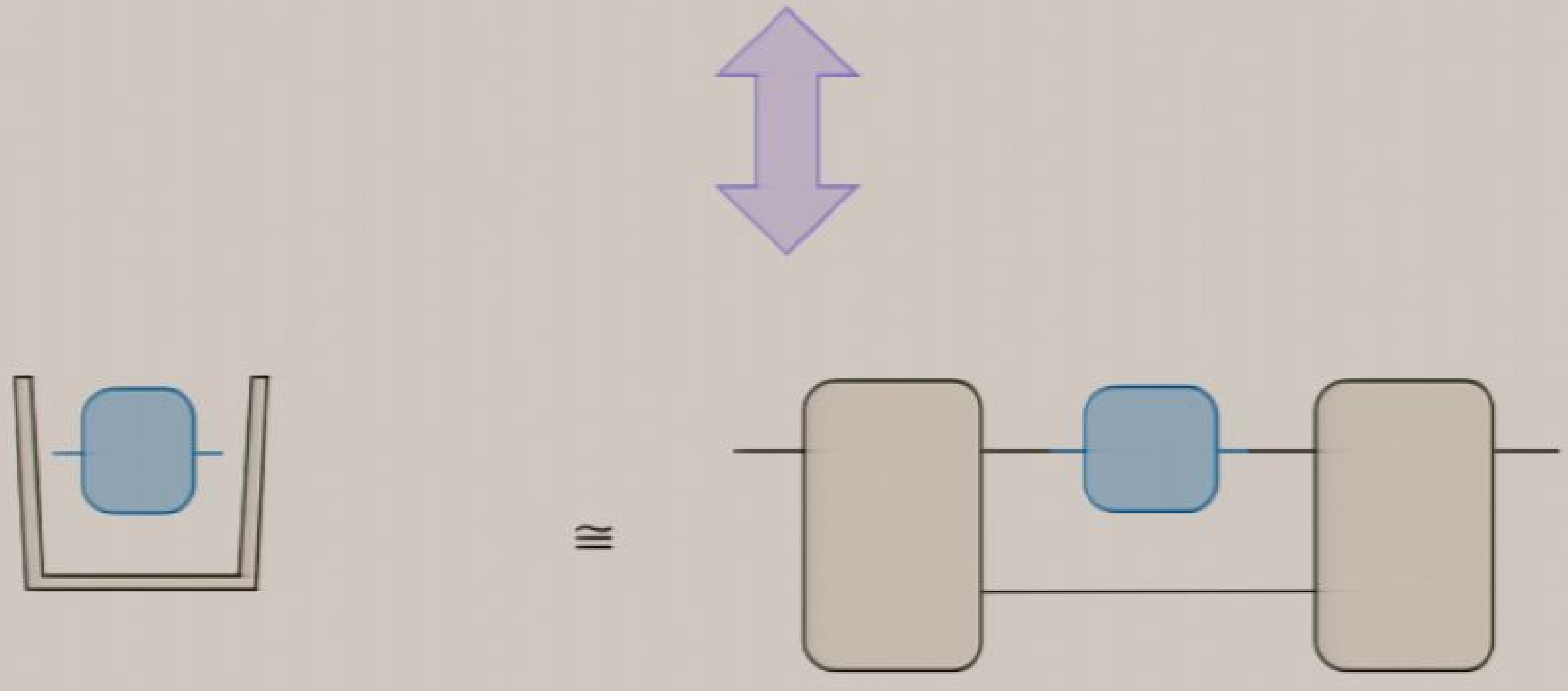

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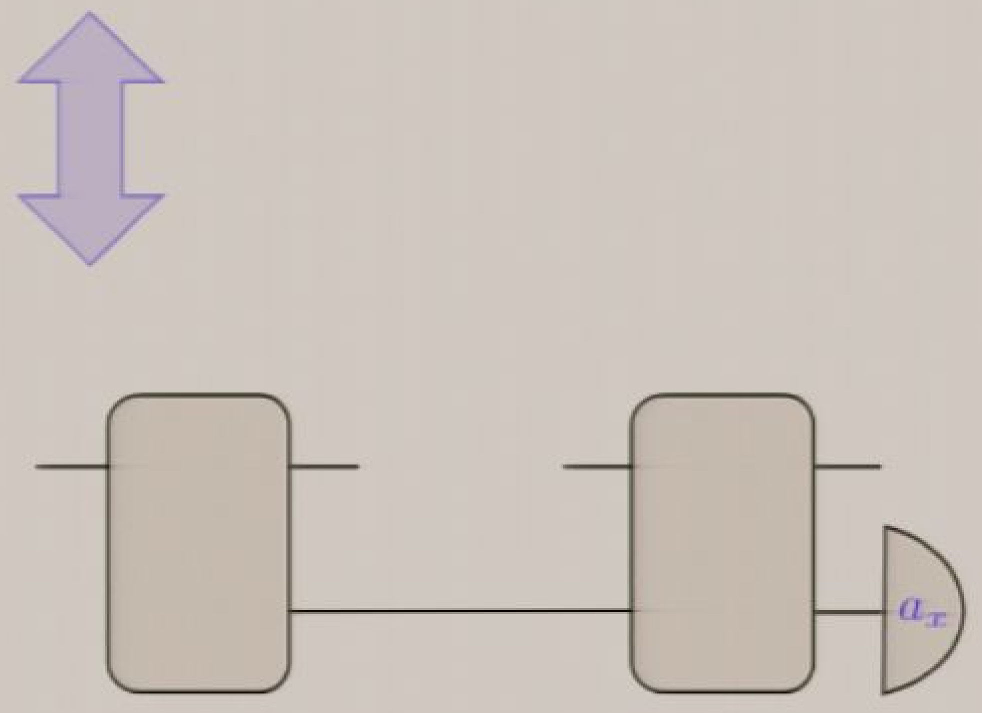

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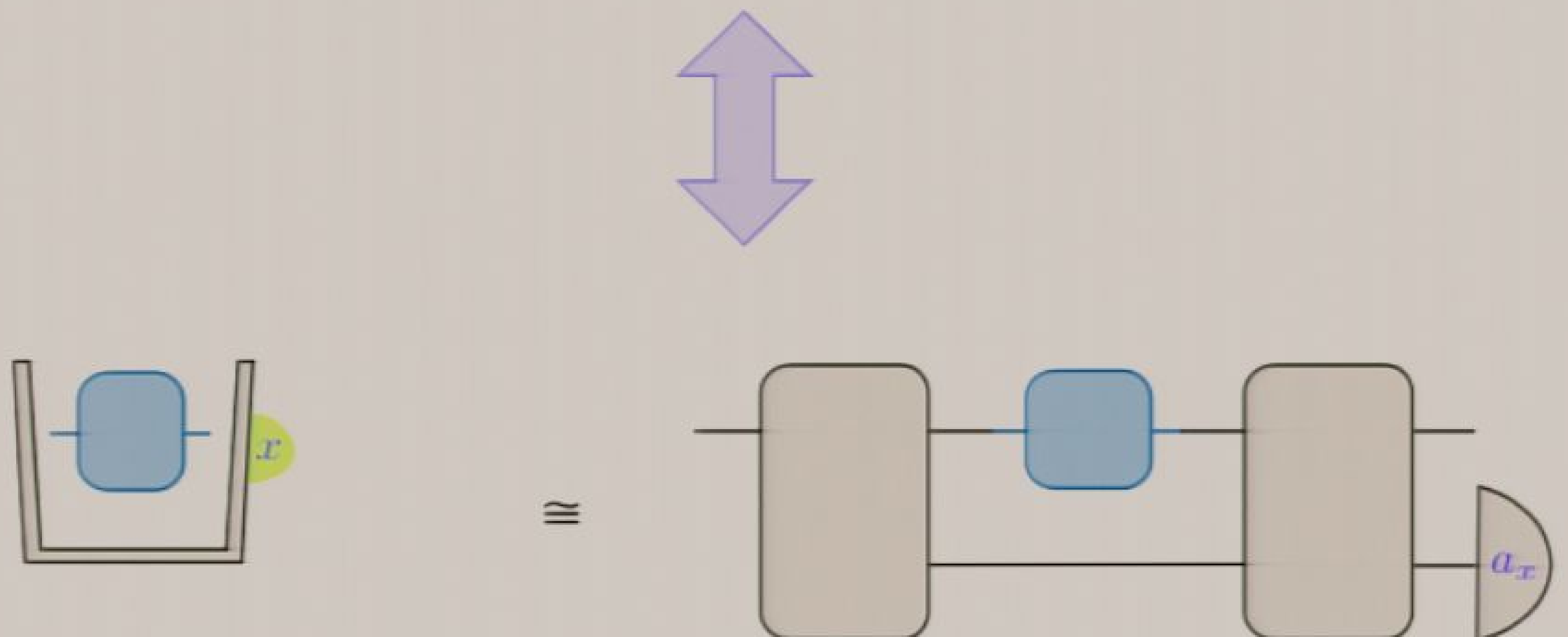

Testers

- A measurement on a transformation provides probabilities as an output
- A probability is a transformation on the trivial system I
- Realisation theorem: a tester is a collection of supermaps with the following implementation

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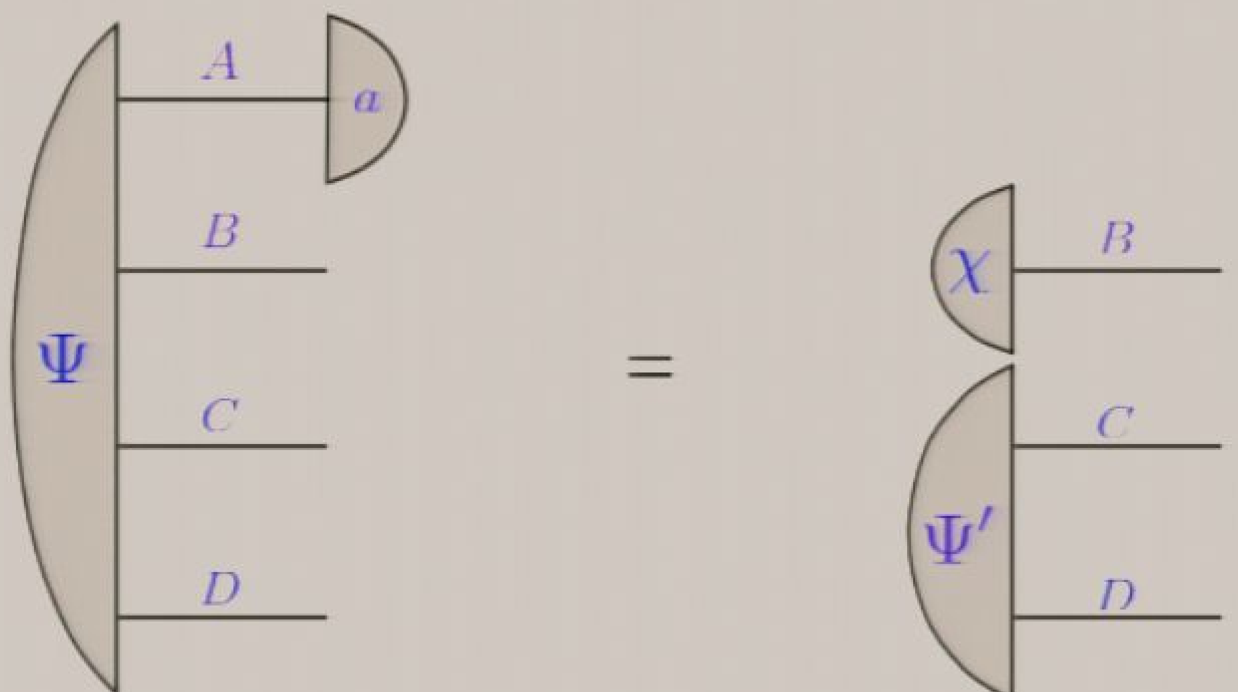
Supermaps and Choi-Jamiołkowski

- Supermaps are in correspondence with states
  - Deterministic supermaps are in correspondence with **some** states
  - The cones coincide



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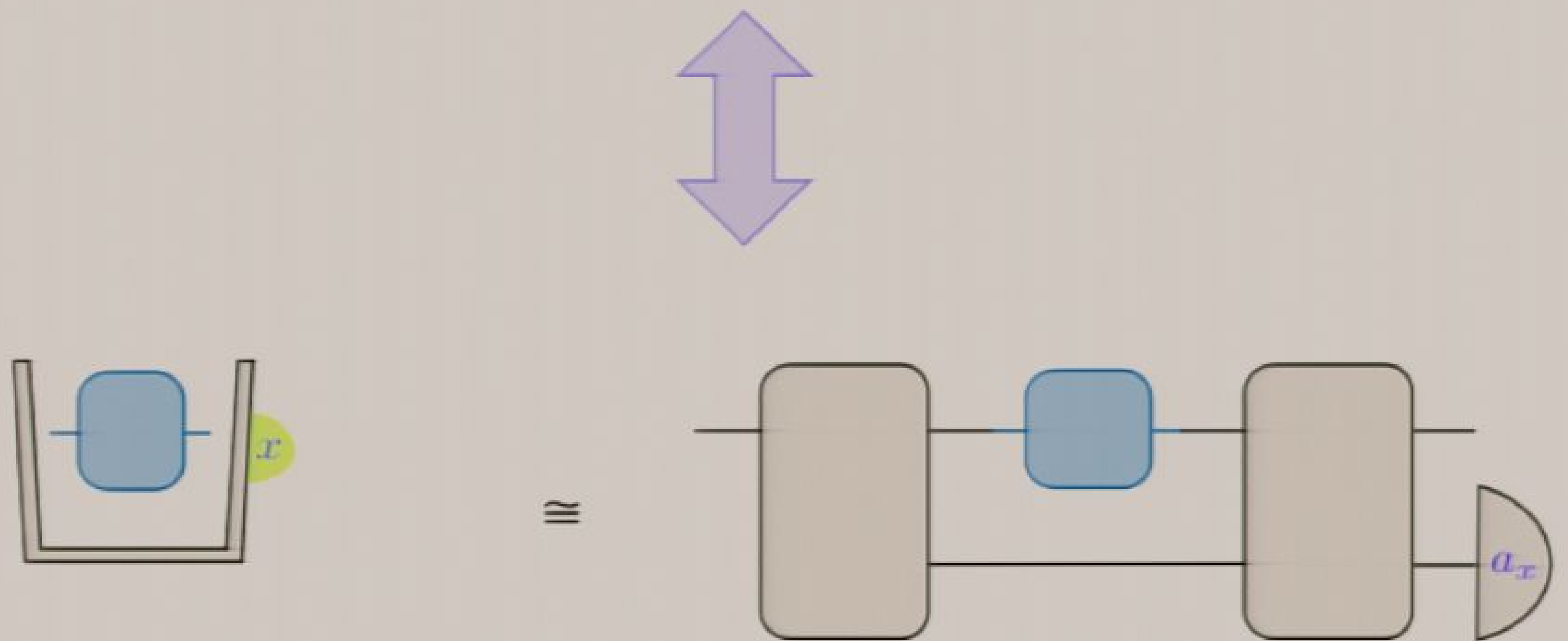

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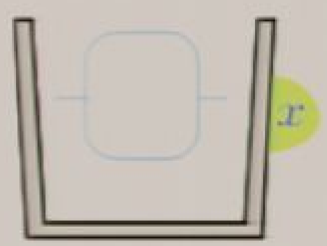

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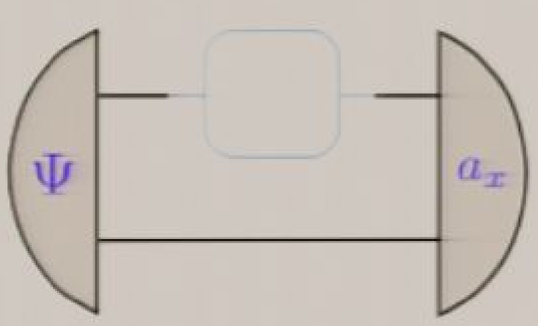

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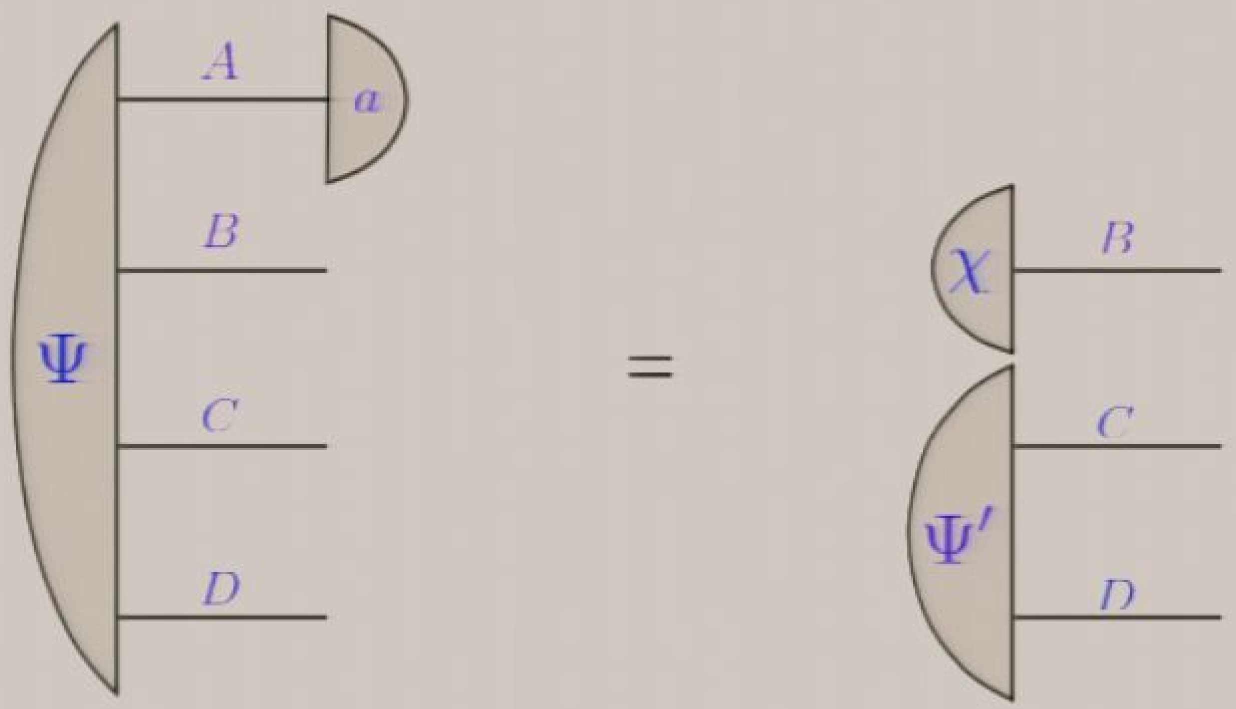
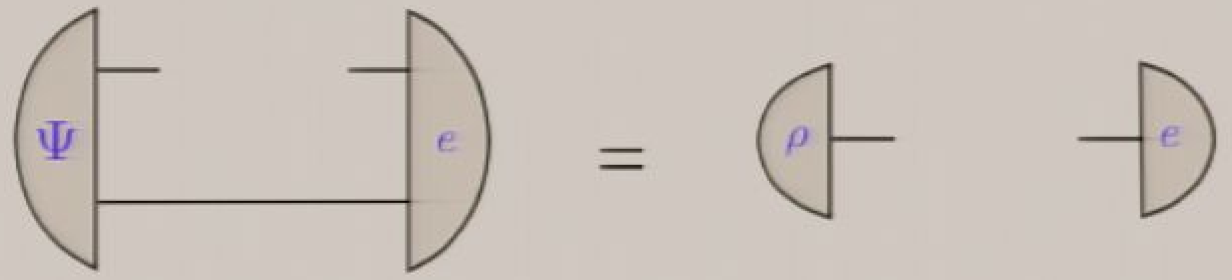


Second-order theory

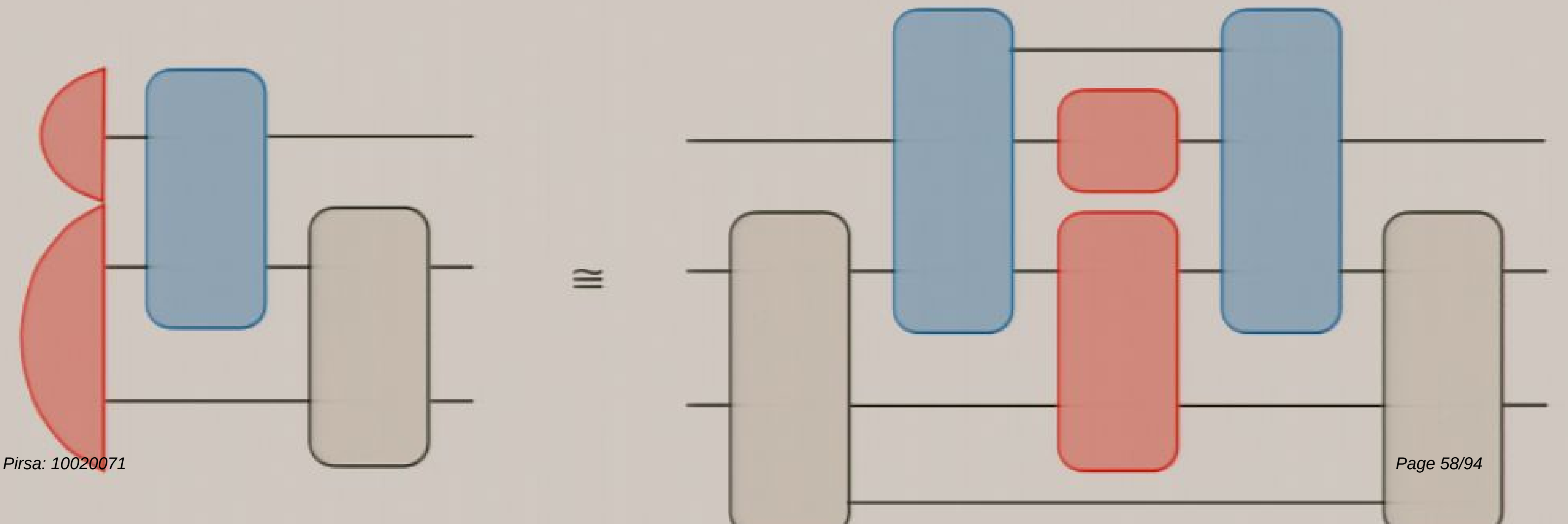
- System types are $A \to B$
- States of the system $A \to B$ are transformations from A to B
- Transformations $(A \to B) \to (C \to D)$ are supermaps
- Effects are testers
  - Deterministic effect $\leftrightarrow$ deterministic tester

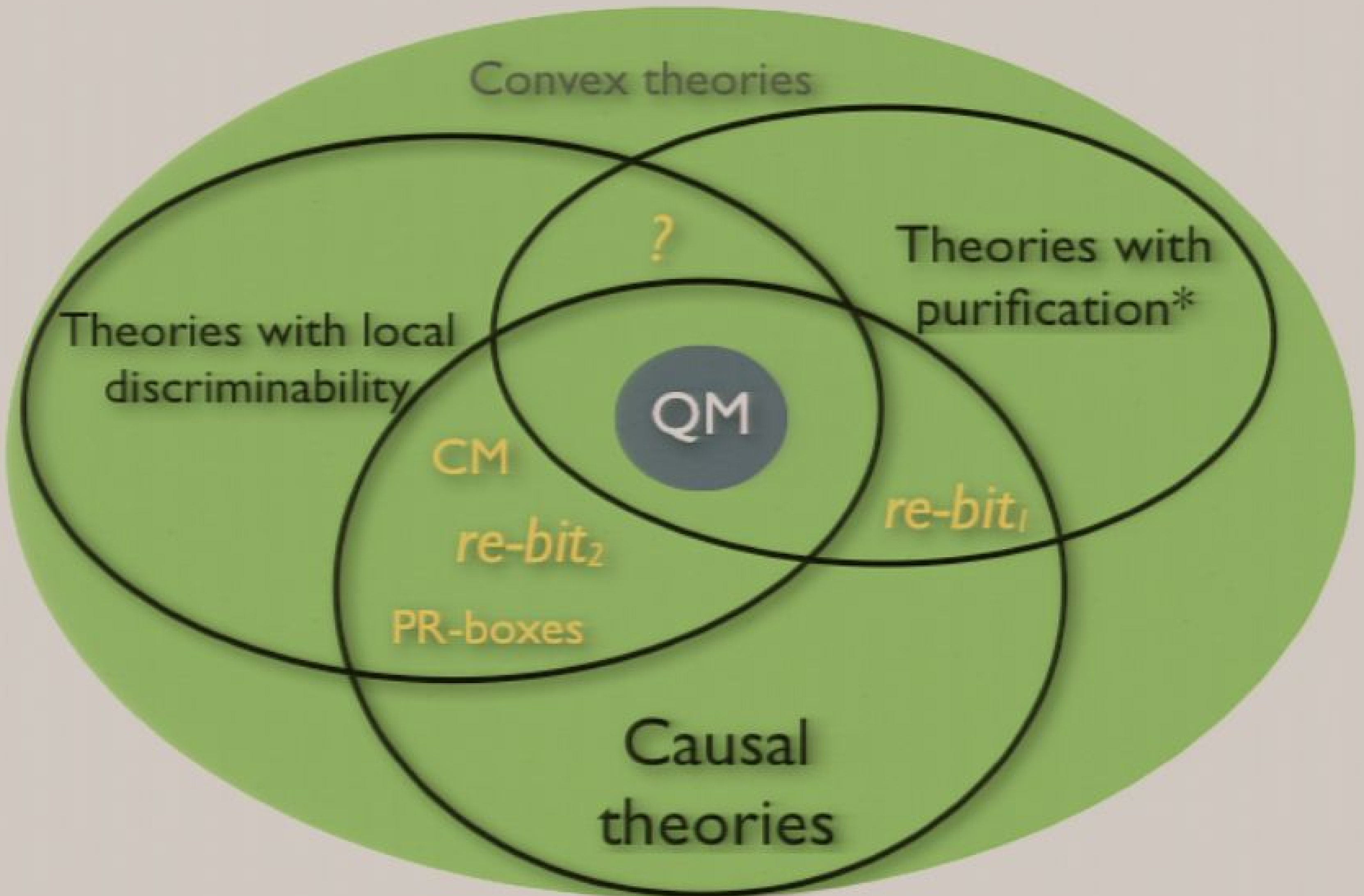
Second order theory

- The second order theory is **non causal**



- Circuits of a second order theory are perfectly simulated in a **causal** theory

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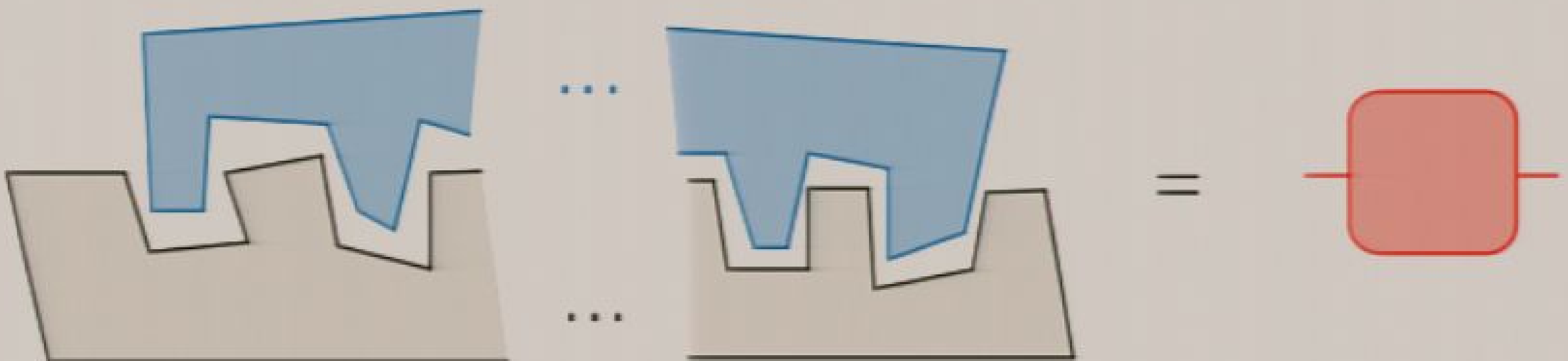
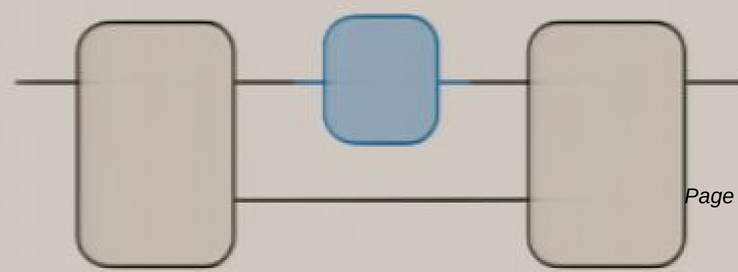
The hierarchy of combs

- Consider the following recursively defined hierarchy of transformations
  - **1-Combs:** transformations in a causal theory with purification
  - **N-Combs:** transformations from N-1-combs to 1-Combs

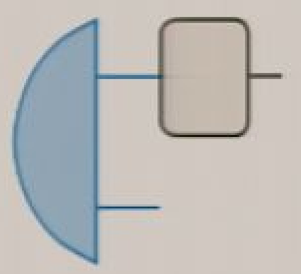


- Example: 2-Combs are supermaps



The hierarchy of combs

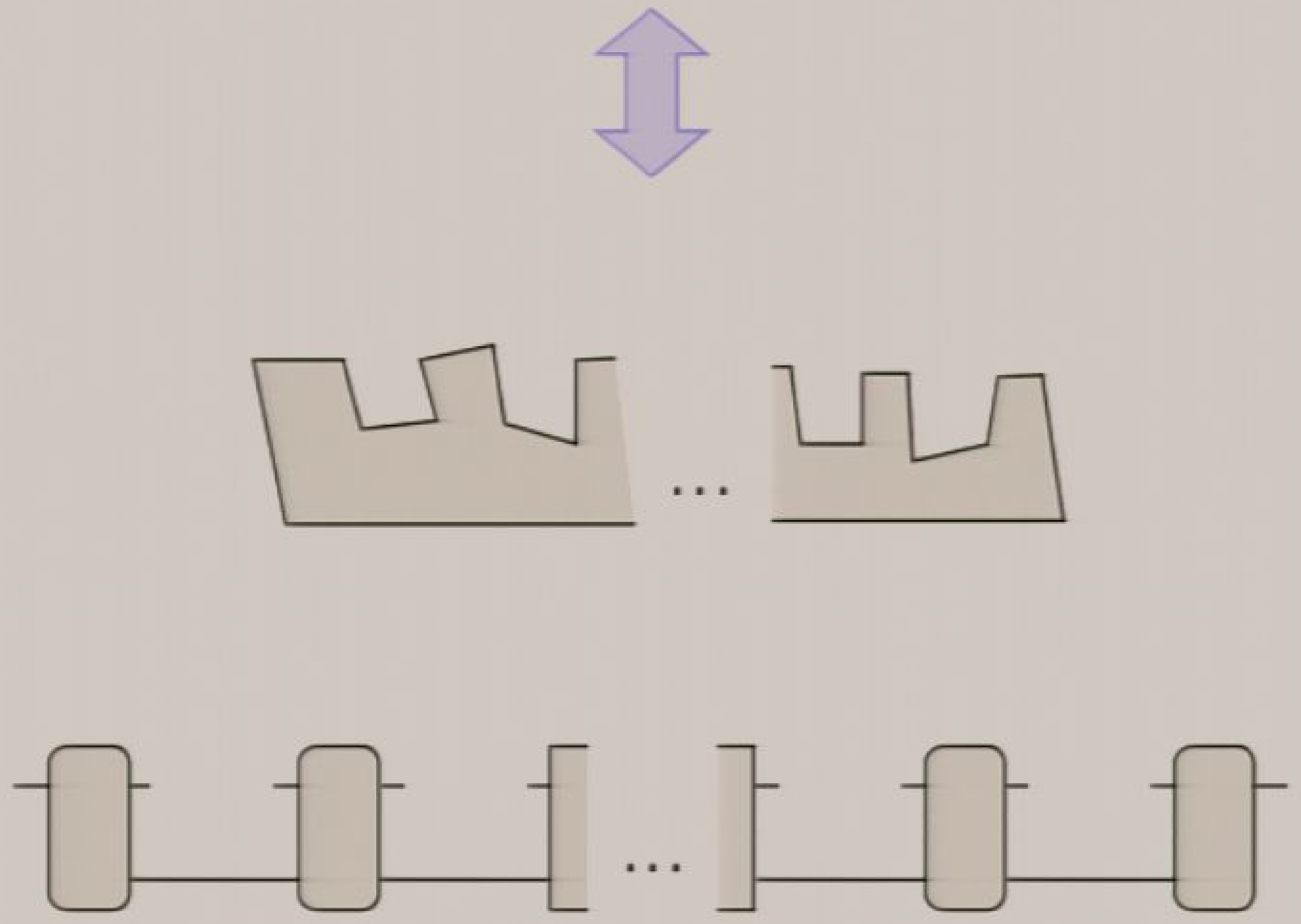
- 1-Combs are in correspondence with states
- If N-1-combs are in correspondence with states, N-combs are in correspondence with transformations, hence with states themselves
- Admissibility conditions:
  - Linear
  - CP



  - Deterministic → Deterministic are mapped to Deterministic (recursive)

Realisation Theorem

Admissibility conditions



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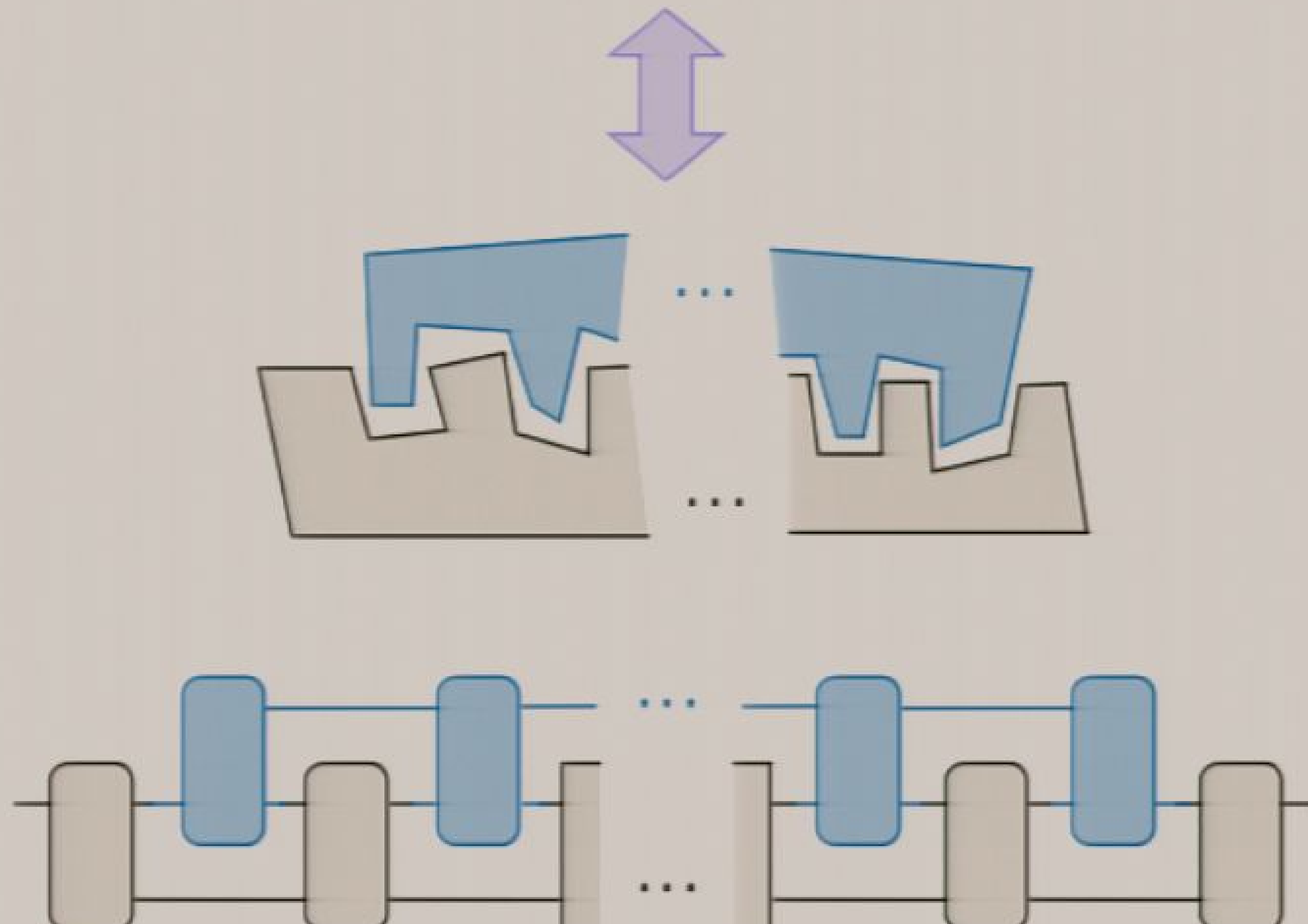

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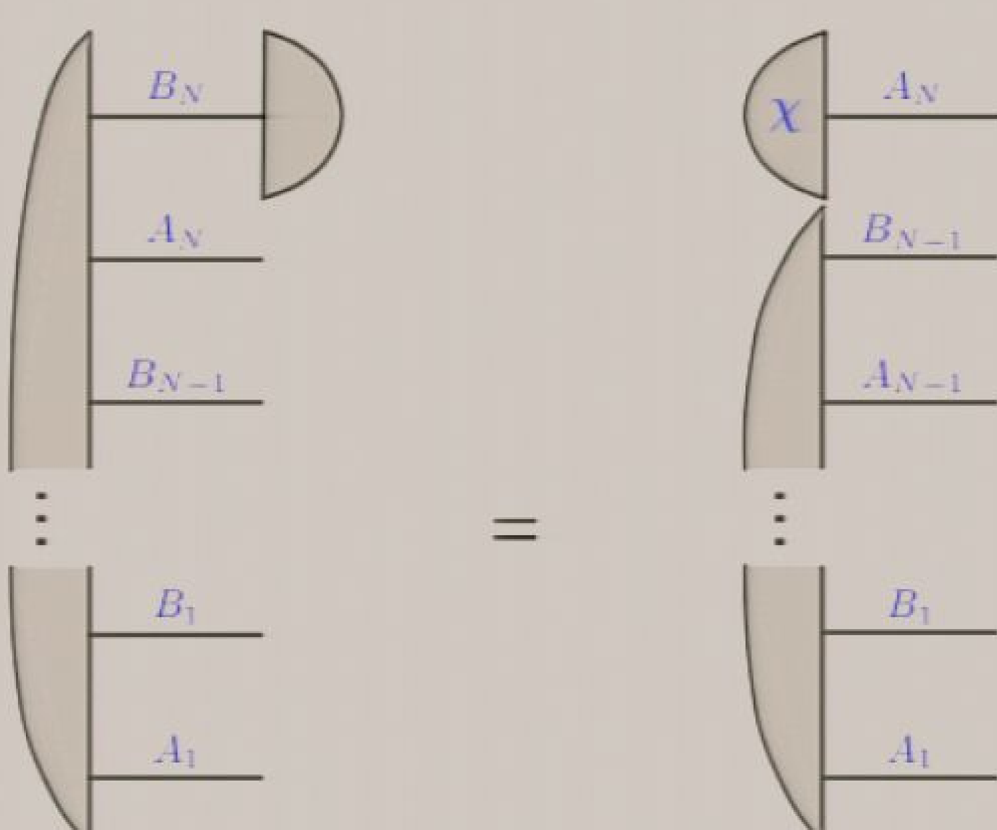
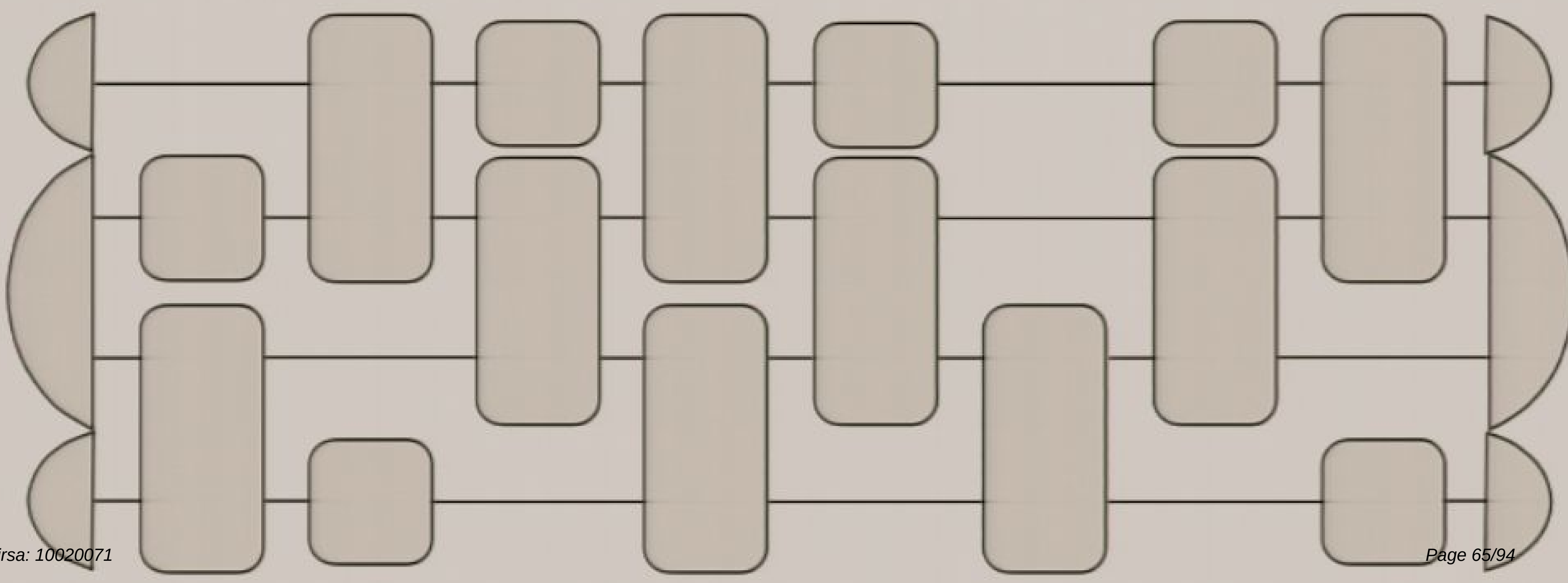
  - The cones coincide



Combs and circuits

- Every comb transformation is perfectly simulated by a causal circuit

- An input/output device (computer?) describable as a circuit performs a comb transformation



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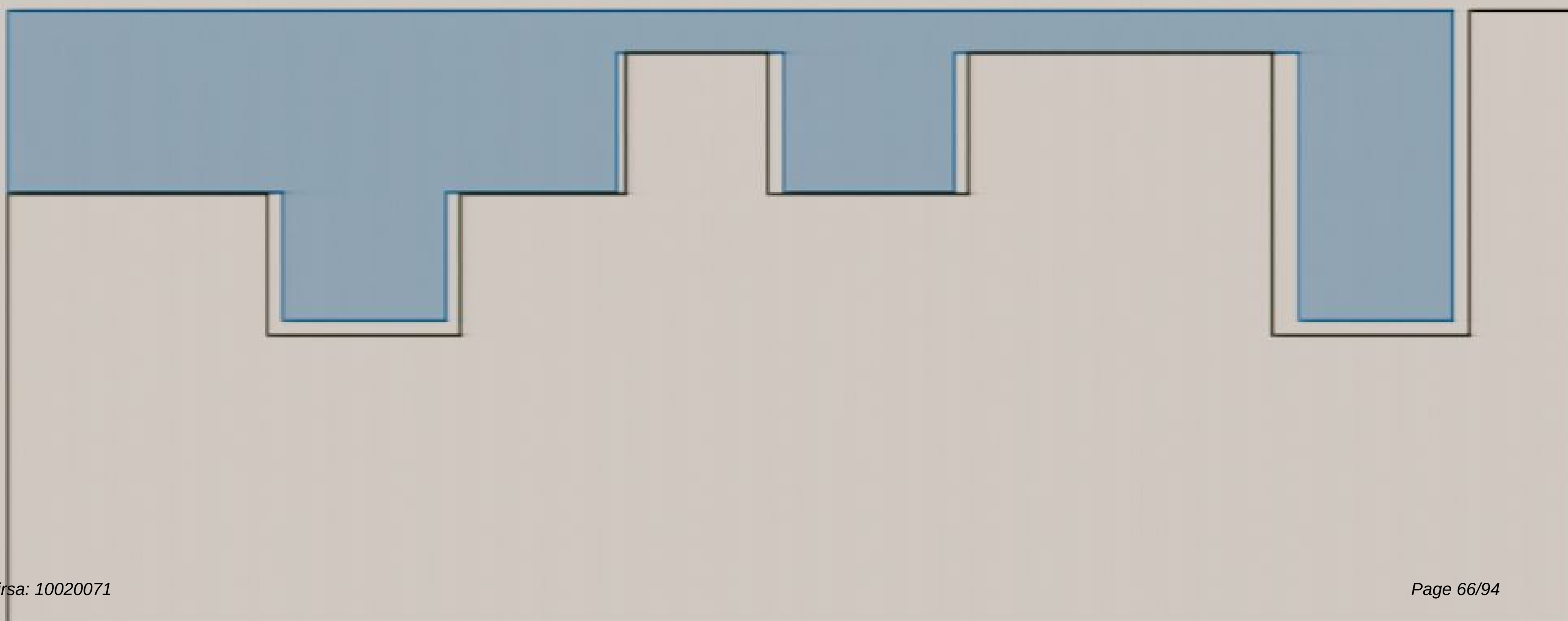

Combs and circuits

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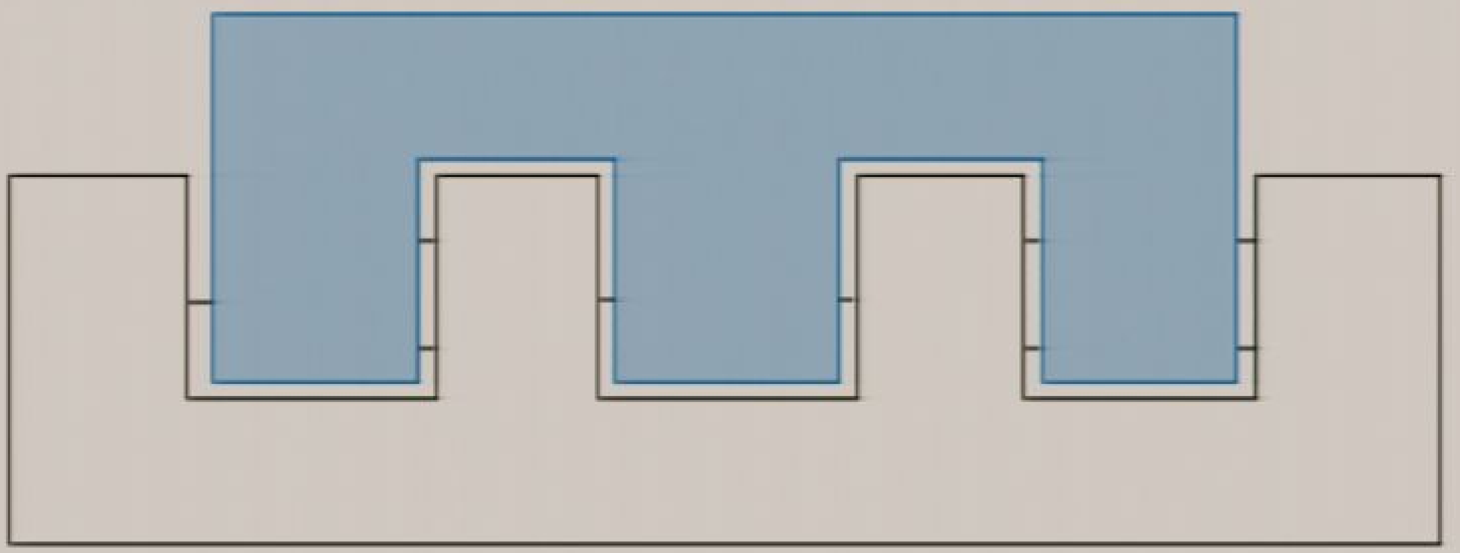
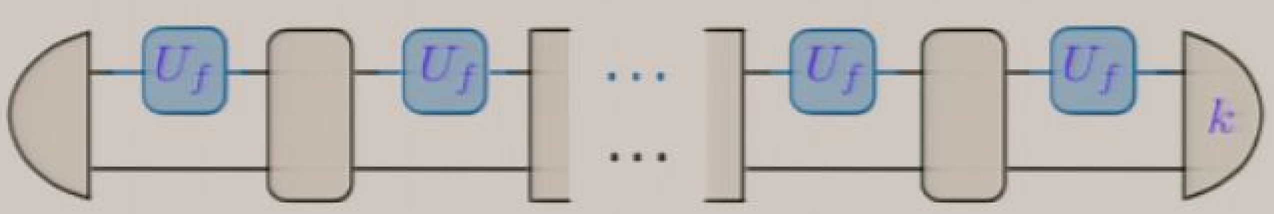


Computation and tester optimisation

- Optimising a computation is equivalent to optimising a tester

- Example: quantum computation



$U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$ $\qquad f \in X = \bigcup_i X_i \qquad X_i \cap X_j = \emptyset$

$$k = i \quad \text{iff} \quad f \in X_i$$

Higher-order maps

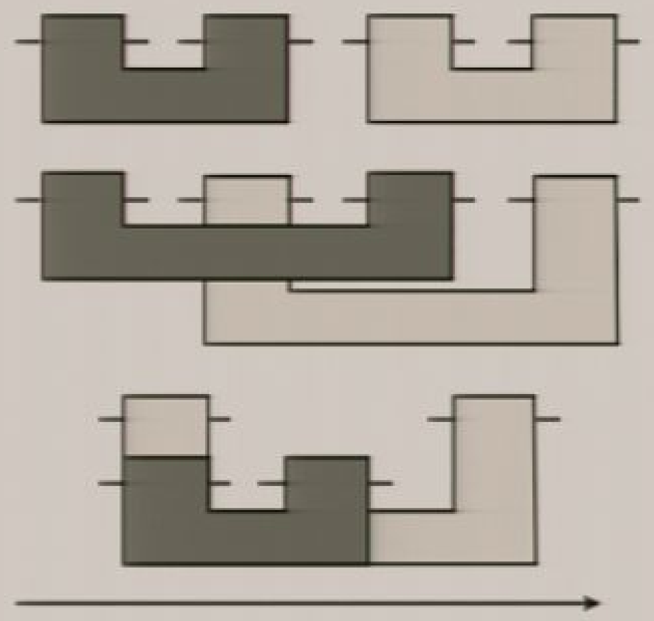
- We want to define maps from N-combs x to M-combs g_x
  - $g_x(y)$ is a transformation for any (M-1)-comb y
- We use an "Uncurrying" procedure

$$h(x, y) := g_x(y)$$

- A map from N-combs x to M-combs g_x is equivalent to a map from couples (x,y) to transformations (1-combs)

Higher-order maps

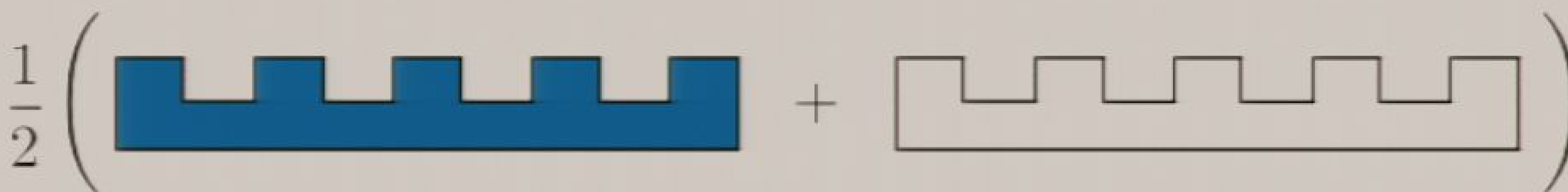
- What is a couple of combs?
    - Fix a global causal ordering: get a comb

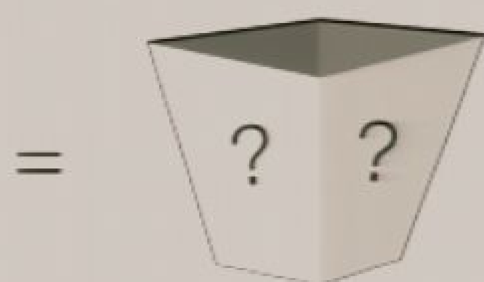


    - All combs on such combs are legitimate maps on the couples

Higher-order maps

- Consider convex combinations of maps of two different kinds





Higher-order maps and Choi-Jamiołkowski

- Combs are in correspondence with states
- Higher order maps are in correspondence with transformations
  - Deterministic higher order maps are in correspondence with some states
  - The cones coincide

Higher-order maps

- All we know is that deterministic higher-order maps are in correspondence with some class of multipartite states

- From purification we know that the purification of such states is still a "legitimate" higher-order map

  - Higher-order maps are not only combs

  - Higher-order maps are not only convex combinations of combs having different causal structures

  - Are such legitimate maps physical?

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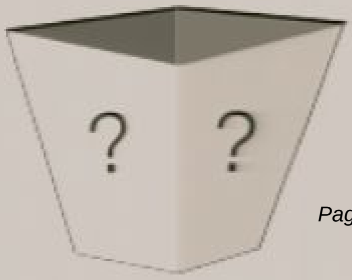
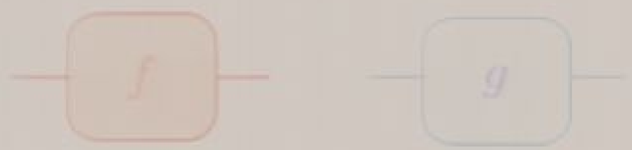
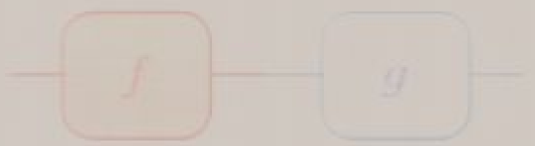


Classification: open problem

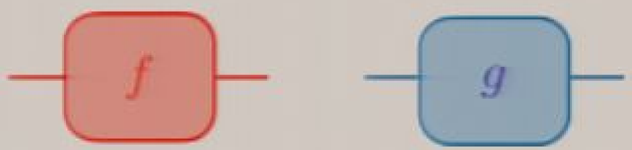
The switch primitive



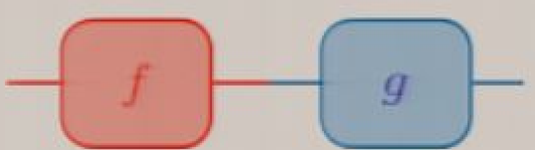


The switch primitive

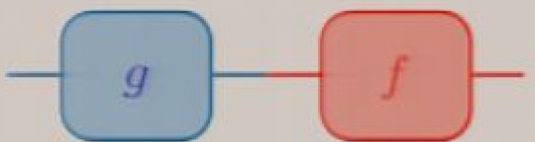
- Input: x=0,1



  - If x=1, then do

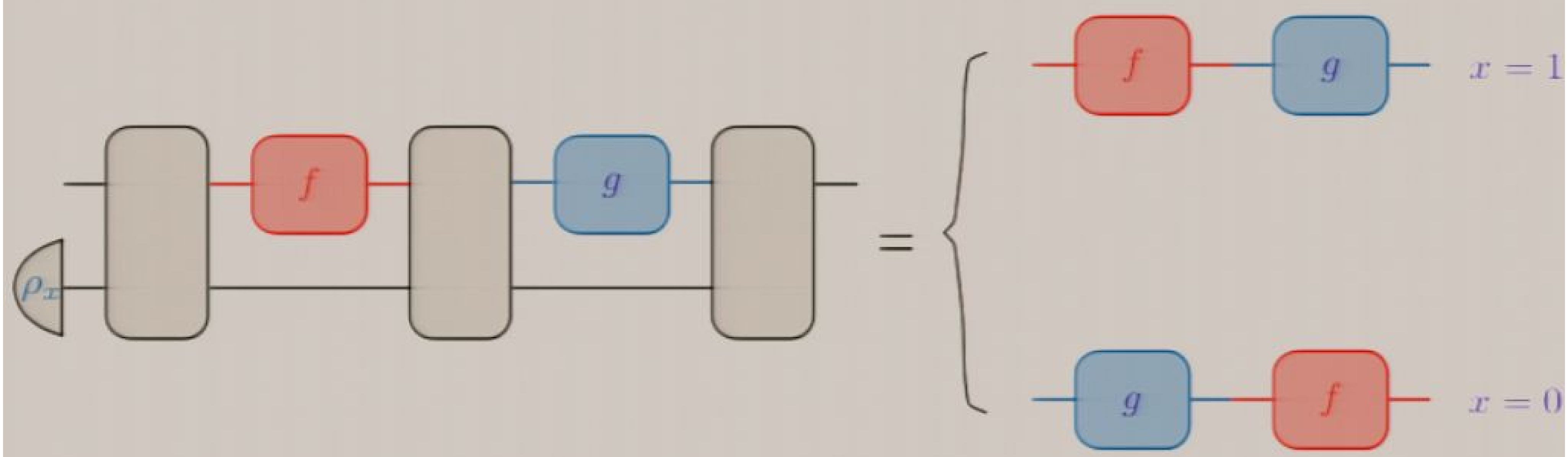


  - If x=0, then do

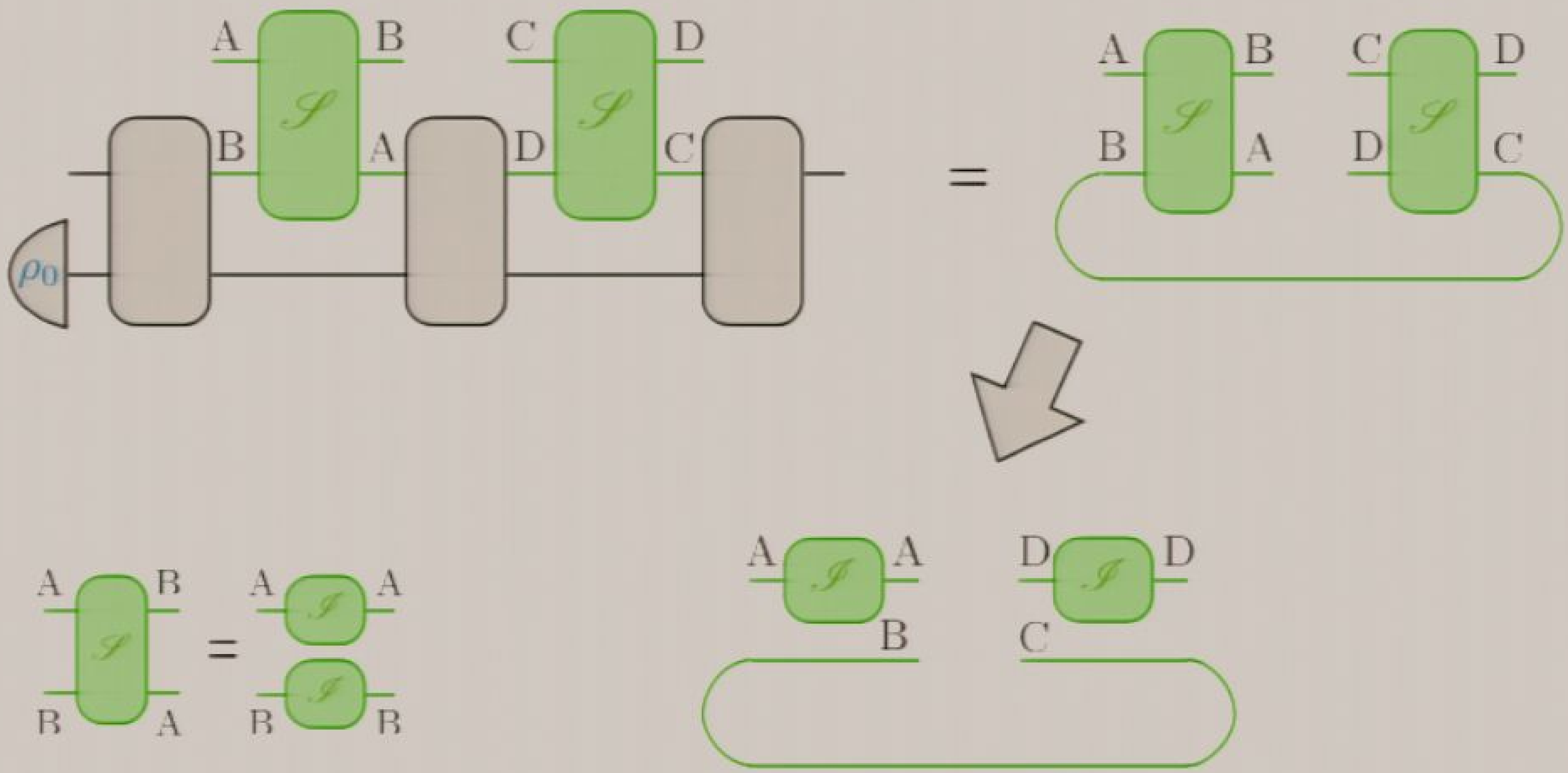


No-switch theorem

Suppose a circuit exists that performs the SWITCH

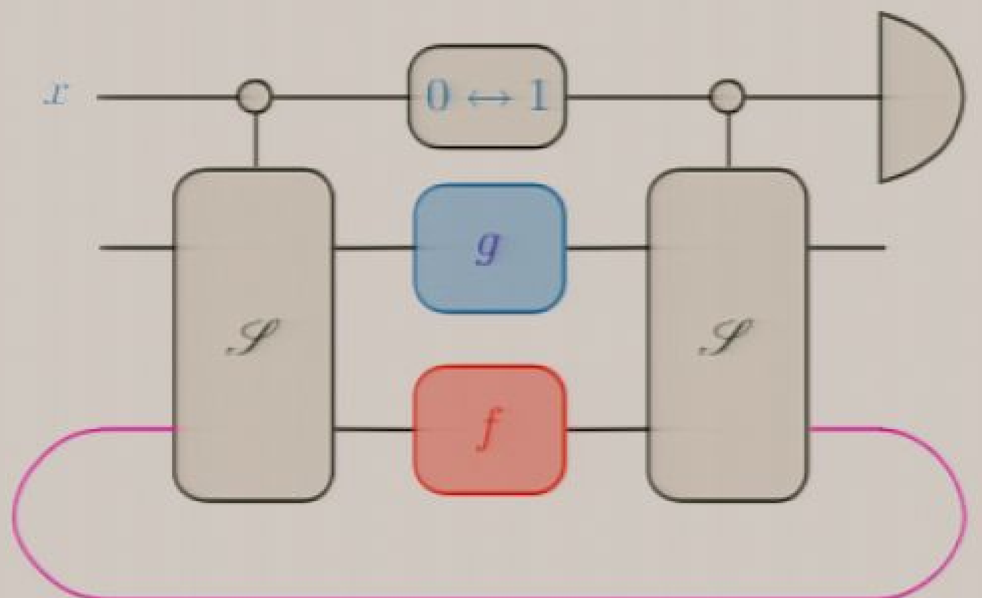


No-switch theorem



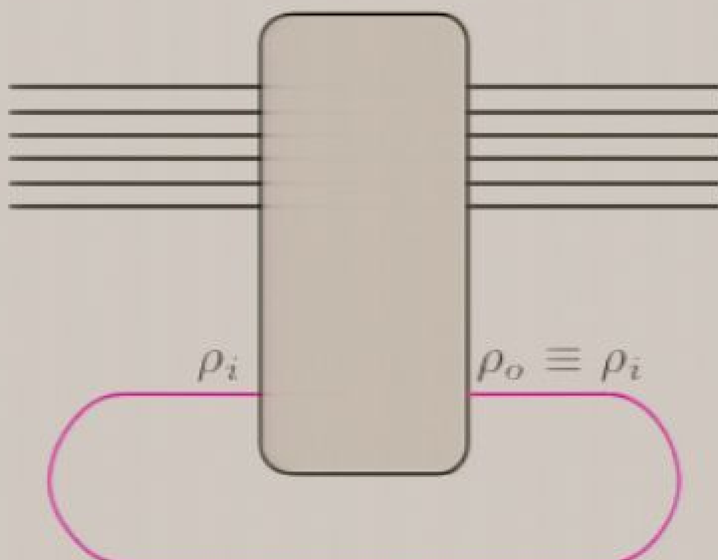
Equivalence of switch and time loops

- If we have access to a time loop we can make a circuit for the SWITCH



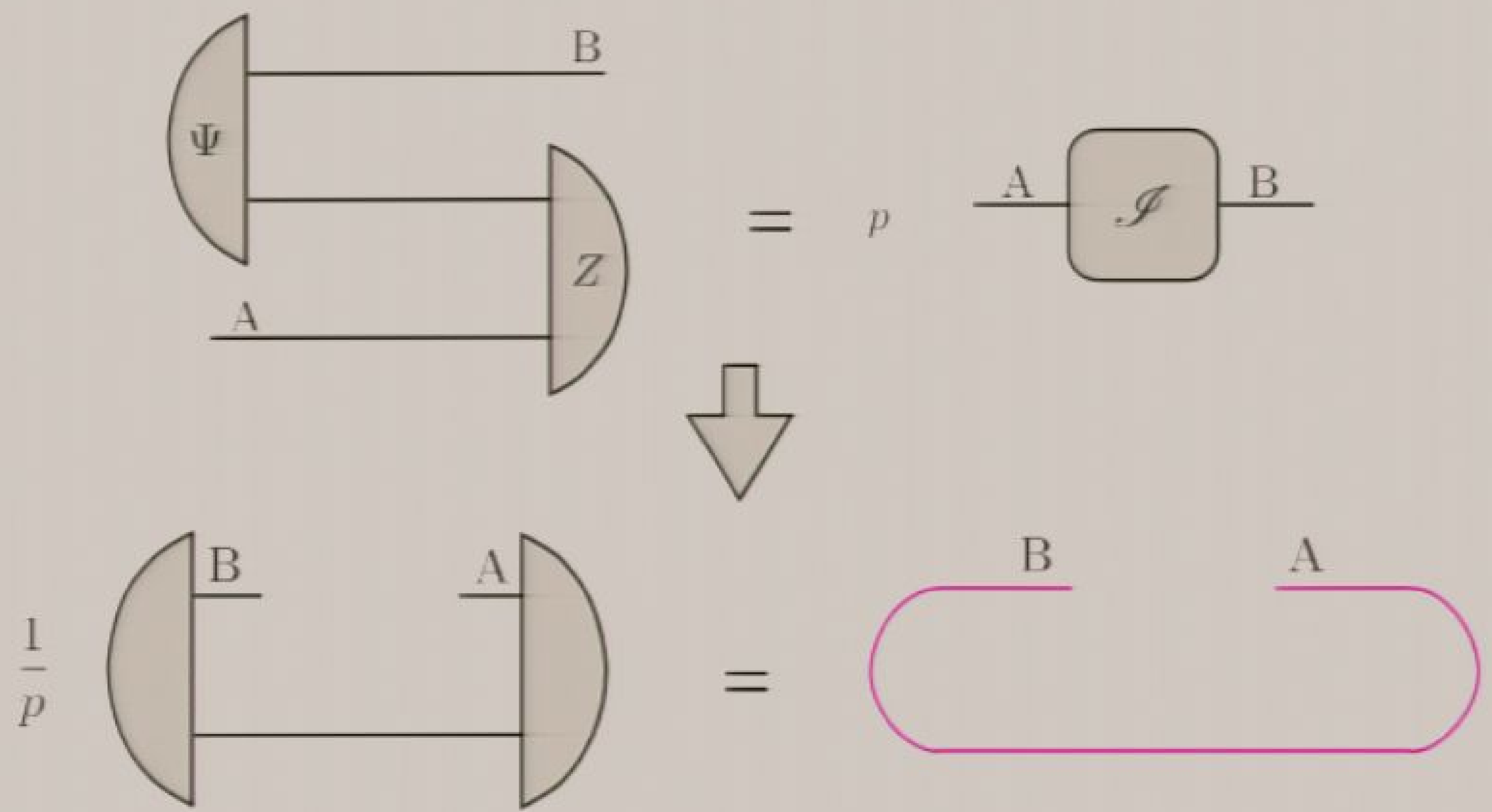
The Deutsch model of time loops

- It is a quantum theoretical model
- It can be extended to general operational probabilistic theories
- Drawback: non-linearity (and wrong result in Q.T.)



Teleportation from the future

- Drawback: non-causal effect (1/p) Z



Way out: Operational representation of the oracle

- The resource: transformations f

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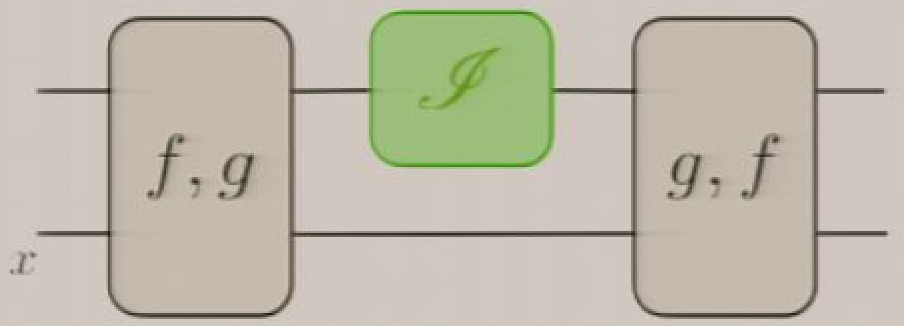

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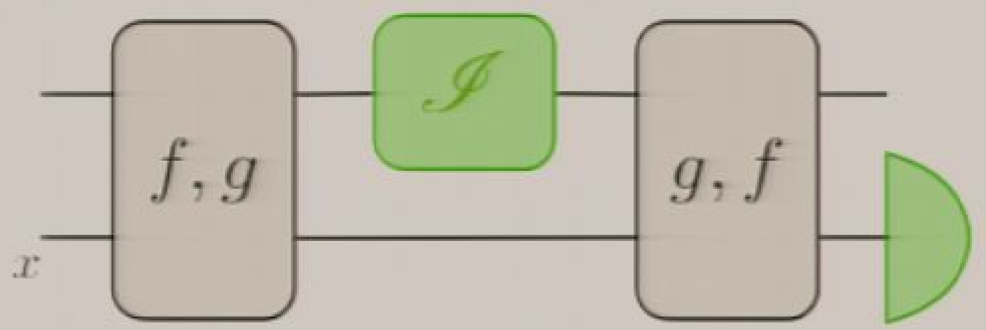

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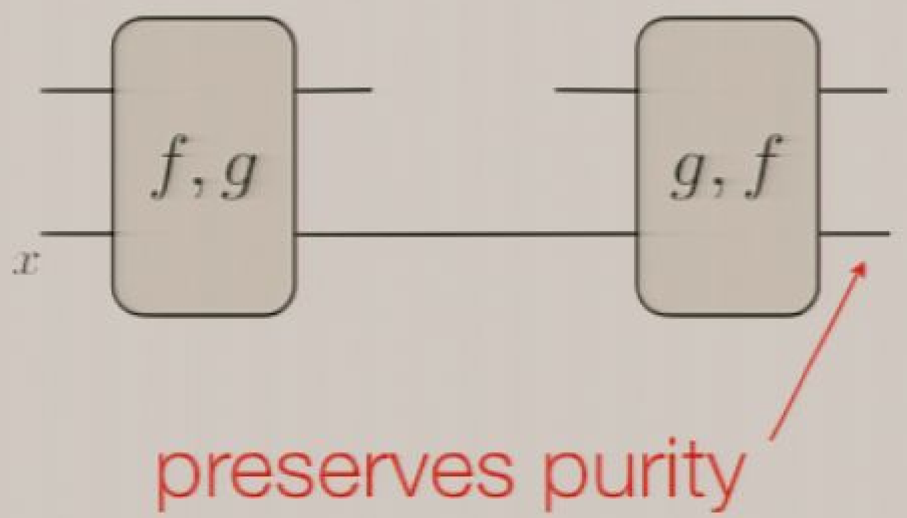

Is it really a way out?

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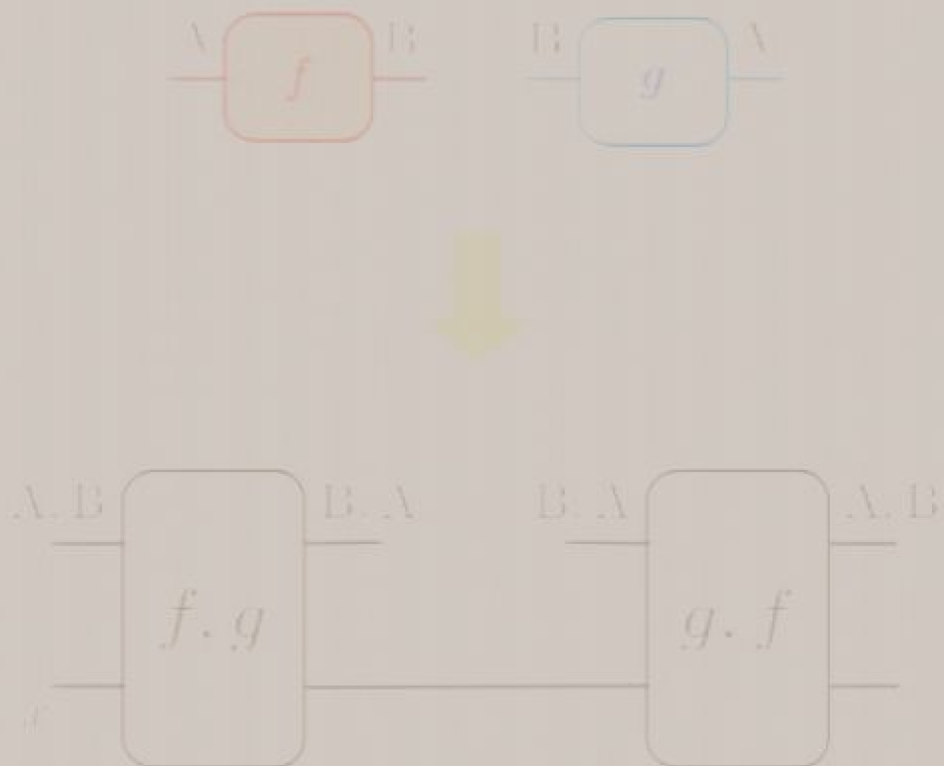
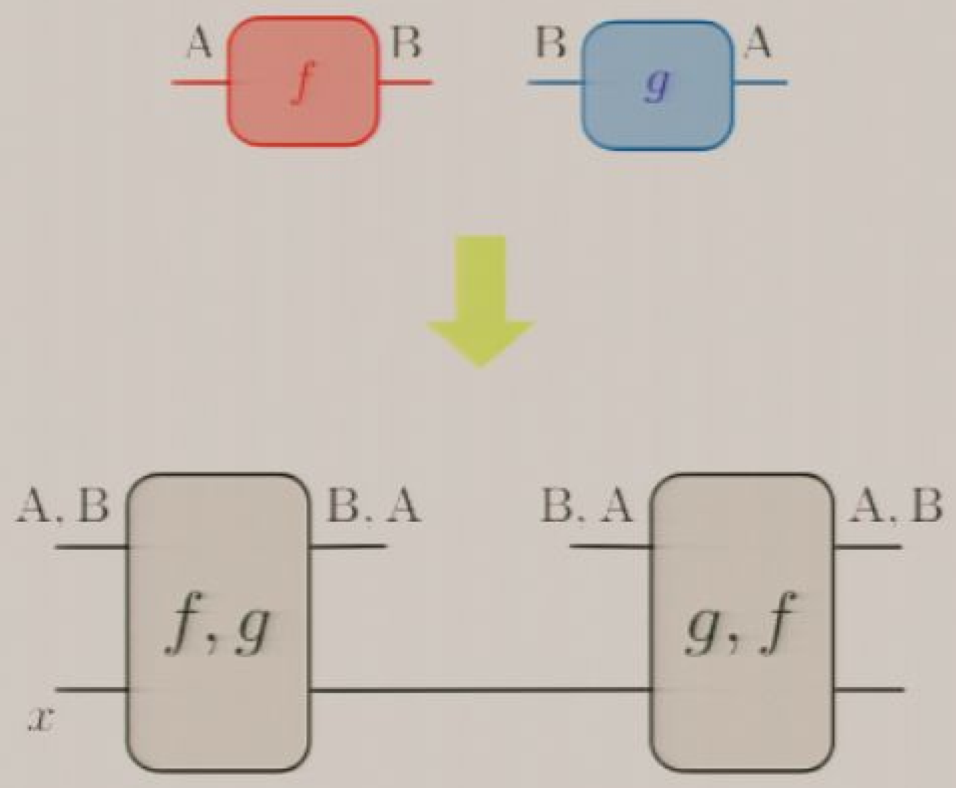
- What is the oracle



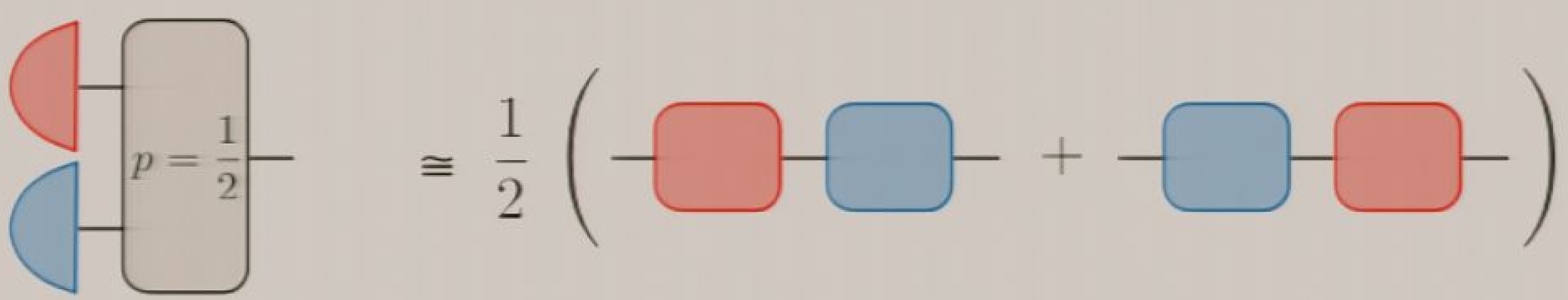
Is it really a way out?

- What is the oracle if the systems are different?



Higher order theory

- Higher order circuits **cannot** be perfectly simulated in a causal theory
  - We lack an operational box for convex combinations of circuits



  - Or - even worse - their purification
  - We need to redefine couples of combs by oracles instead of juxtaposition

Conclusions

- There is something important missing in a general operational theory
  - We cannot describe an elementary primitive like the switch
- If we want to recover the switch in a circuit model:
  - Conditioning
    - Convex combination of different causal networks (and purification)
  - Oracle as a comb (what kind?)
- Conjecture: switch - controlled permutation - are the only primitives we need

Thank you for your attention