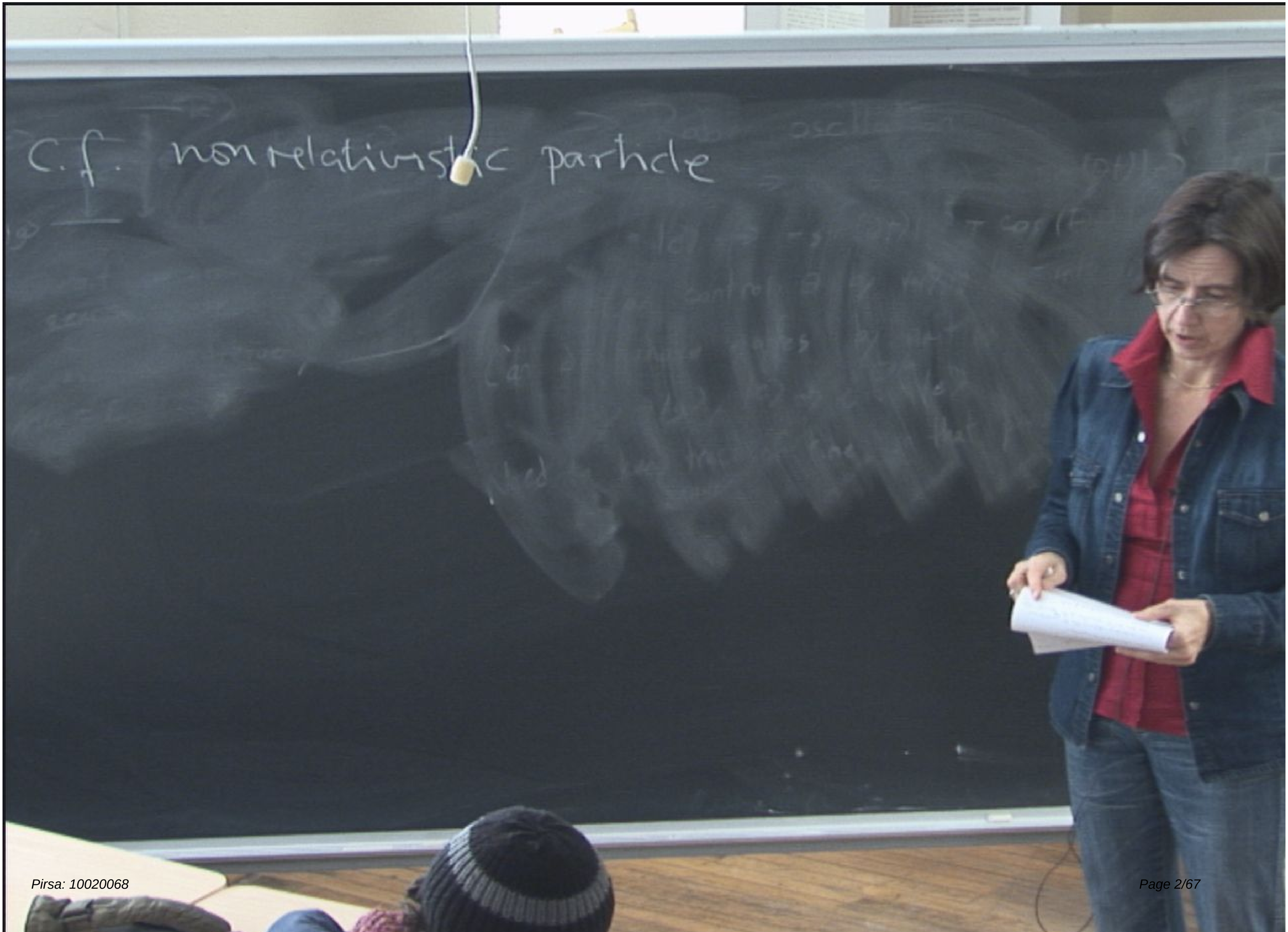
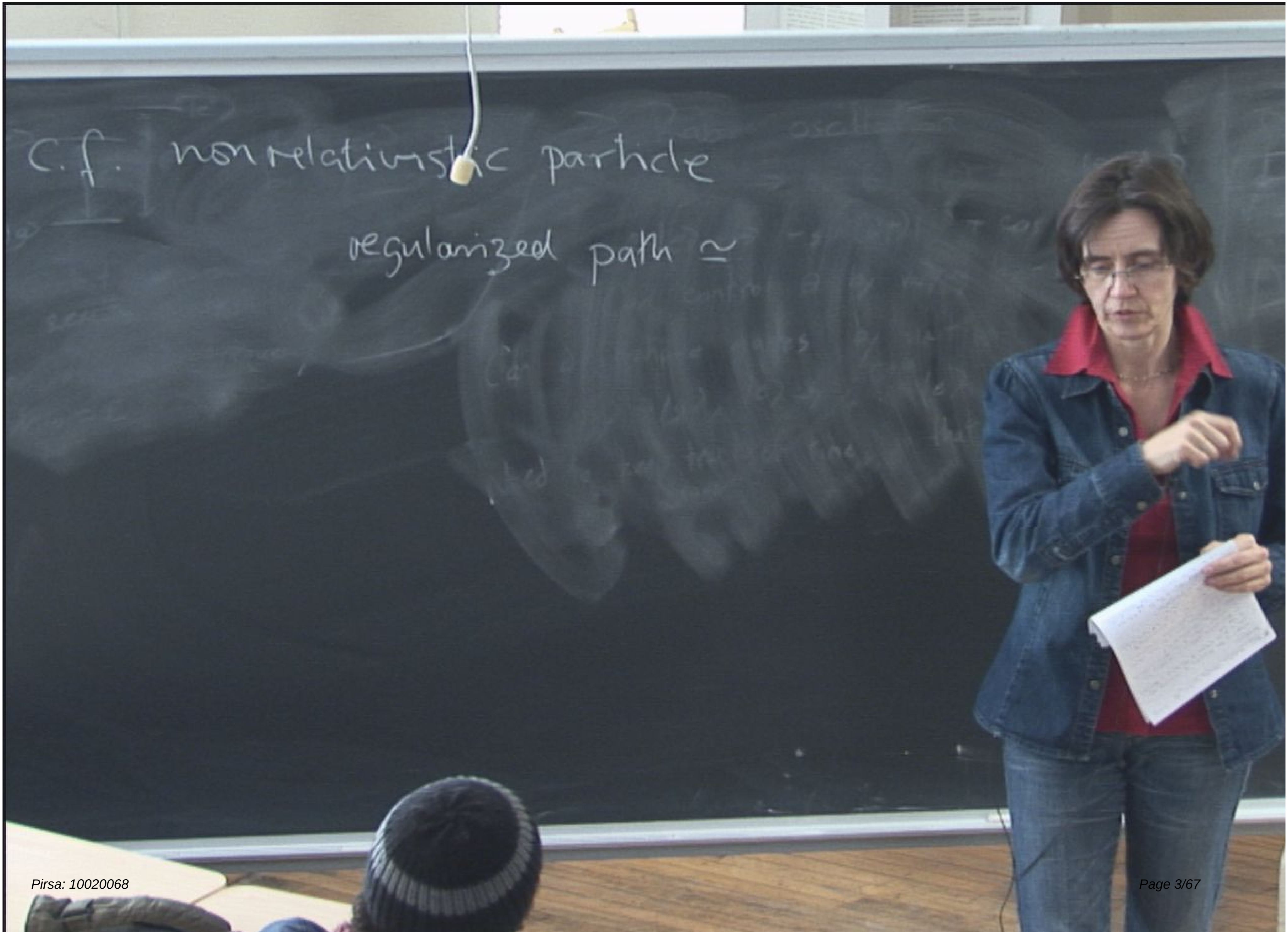
Title: Quantum Gravity - Review (PHYS 638) - Lecture 14

Date: Feb 11, 2010 10:00 AM

URL: http://pirsa.org/10020068

Abstract:


c.f. non relativistic particle


c.f. nonrelativistic particle
regularized path ≃

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regularized path $\simeq$ finite set of data

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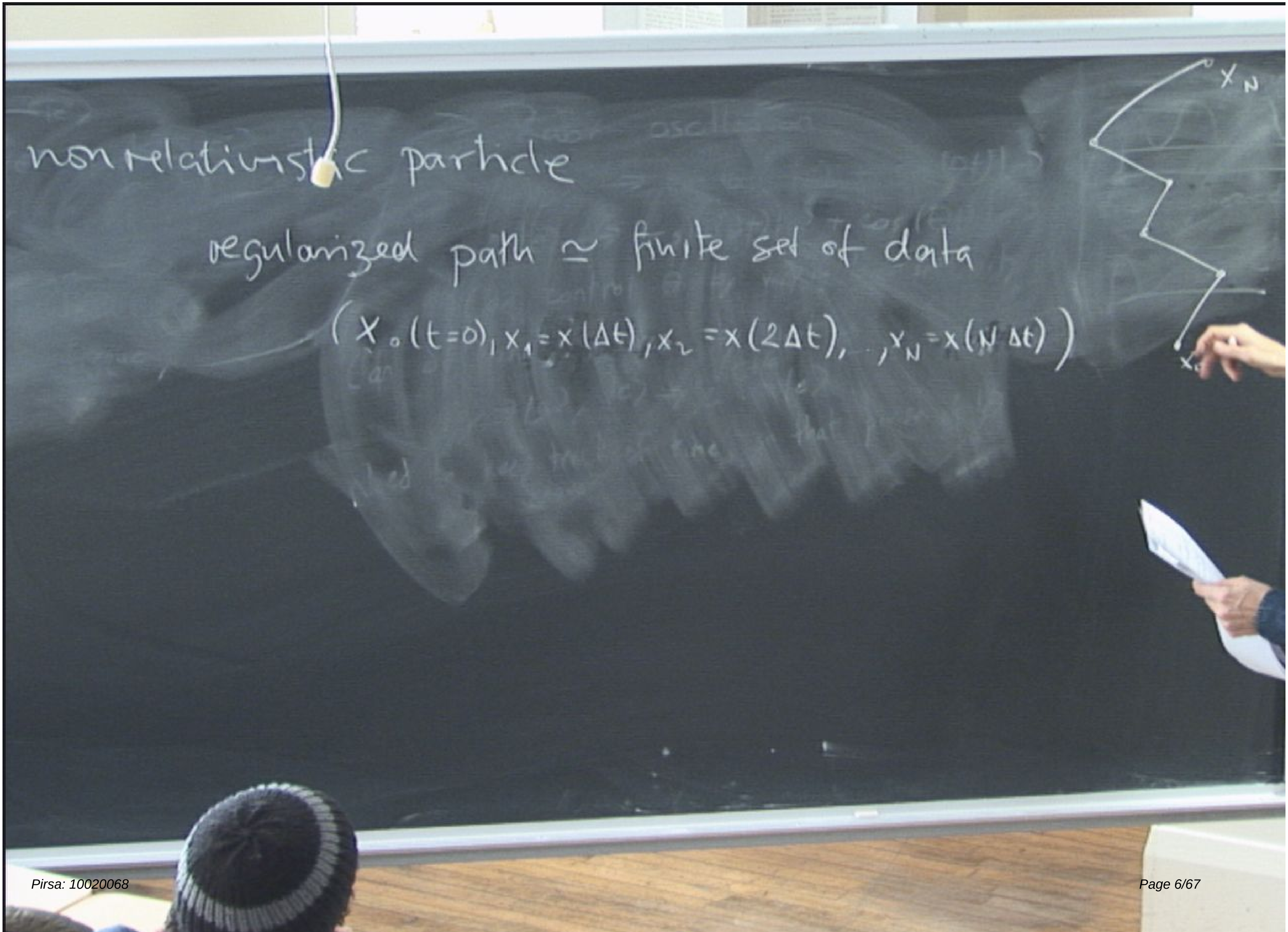
$$\left(x_0(t=0),\ x_1 = x(\Delta t),\ x_2 = x(2\Delta t),\ \ ,\ x_N = x(N\Delta t) \right)$$


non relativistic particle
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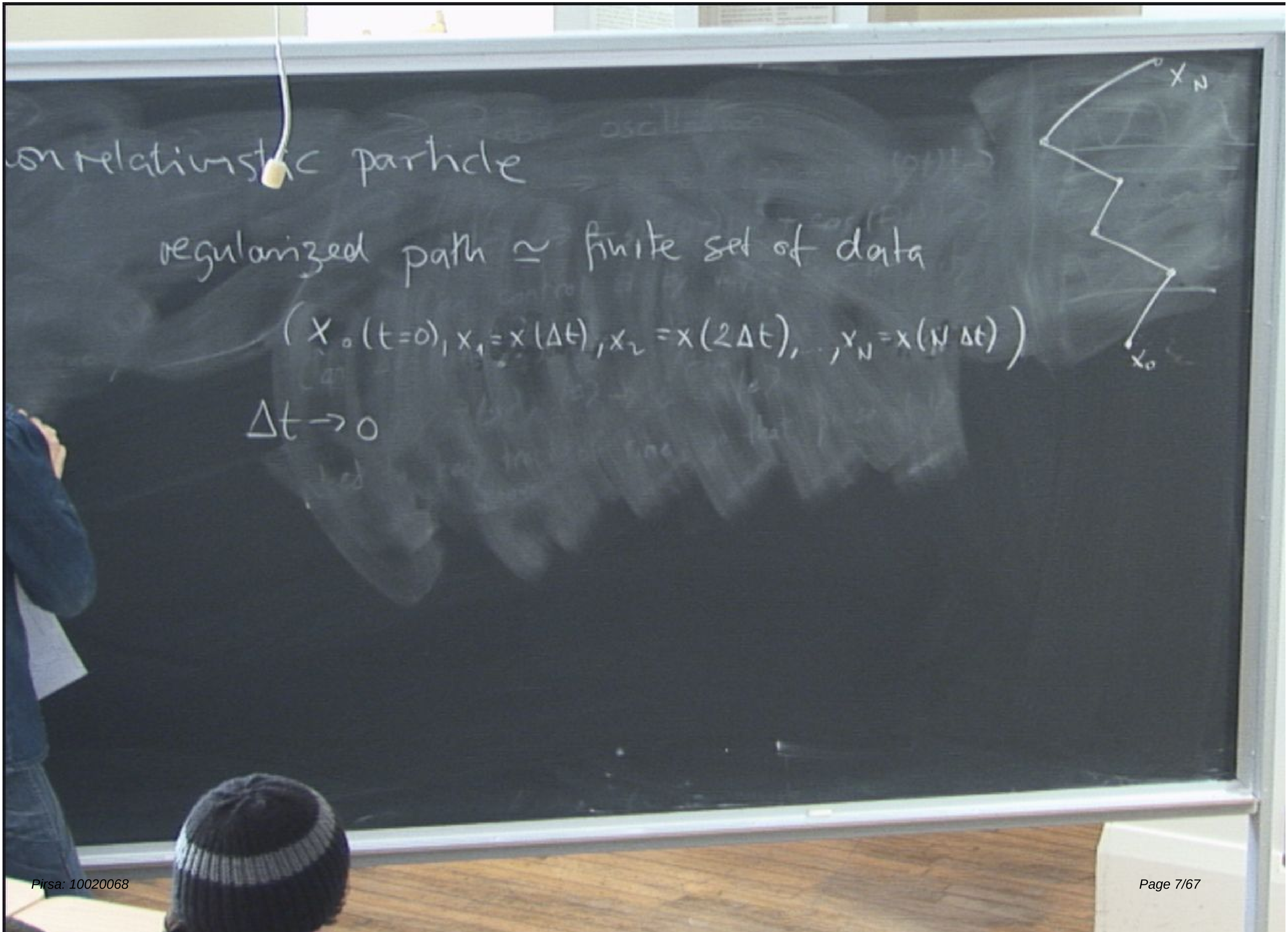
x_N



non relativistic particle
regularized path ≃ finite set of data
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$\Delta t \to 0$
x_N
x_0

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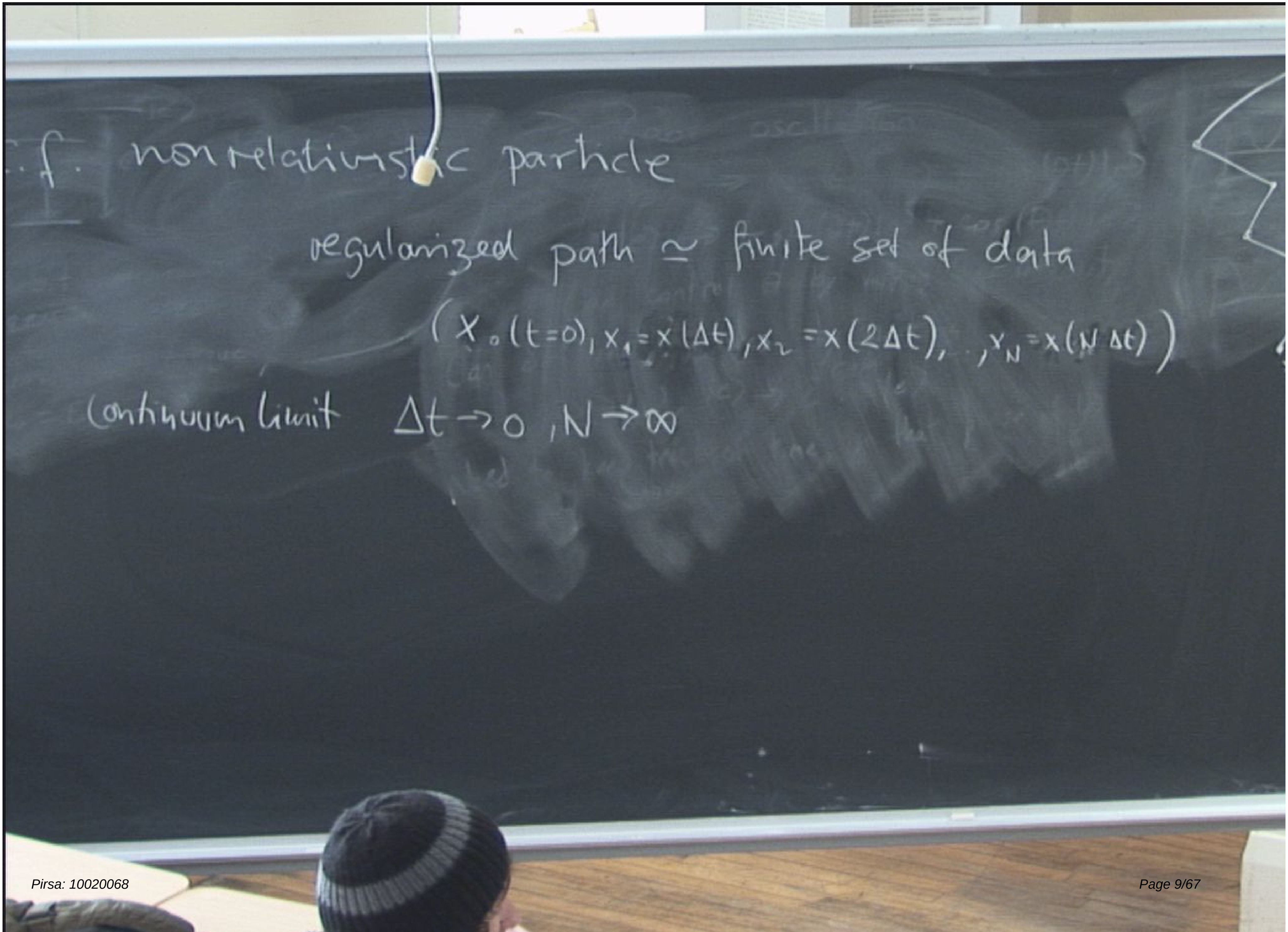
continuum limit $\Delta t \to 0,\ N \to \infty$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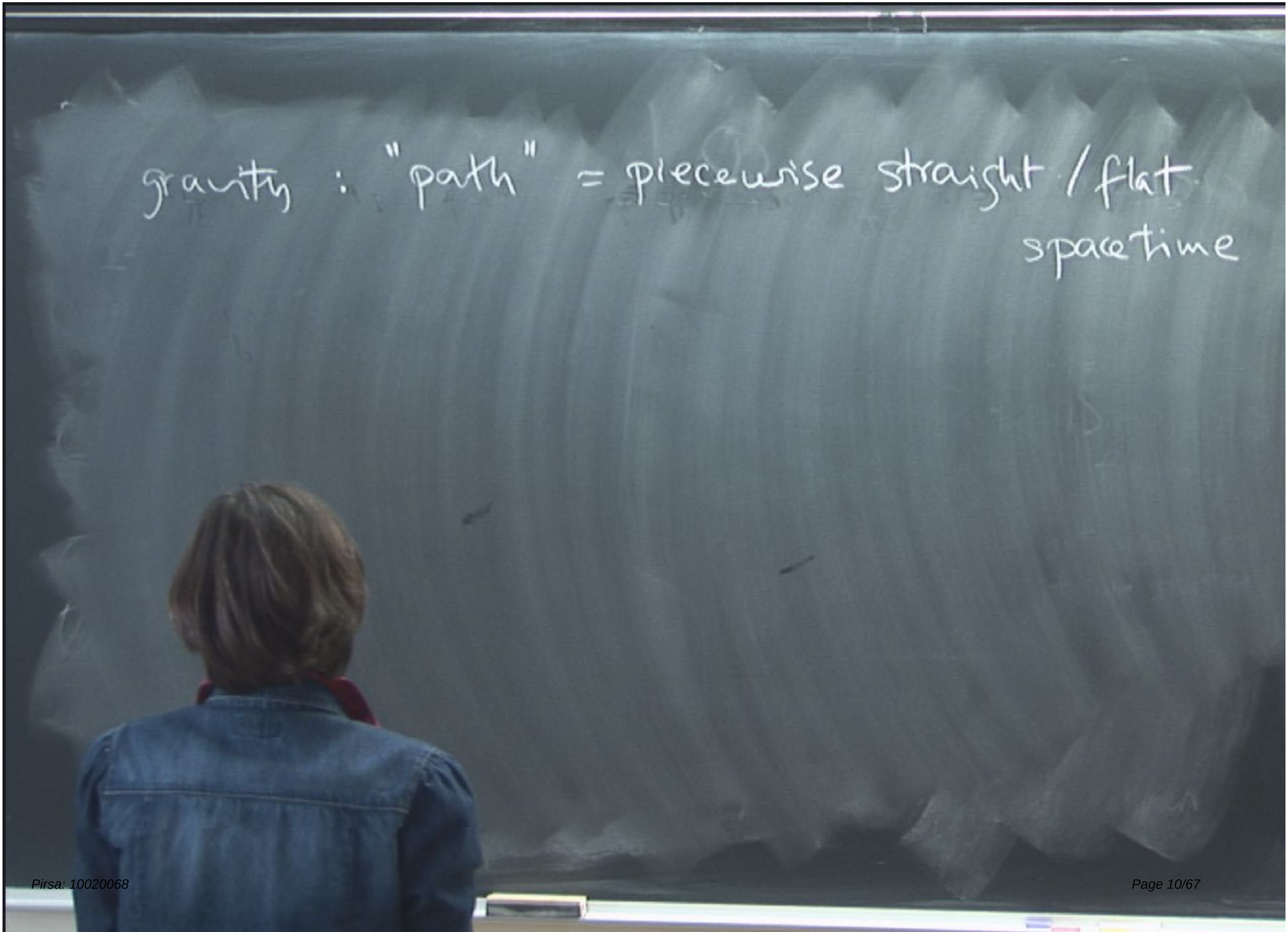
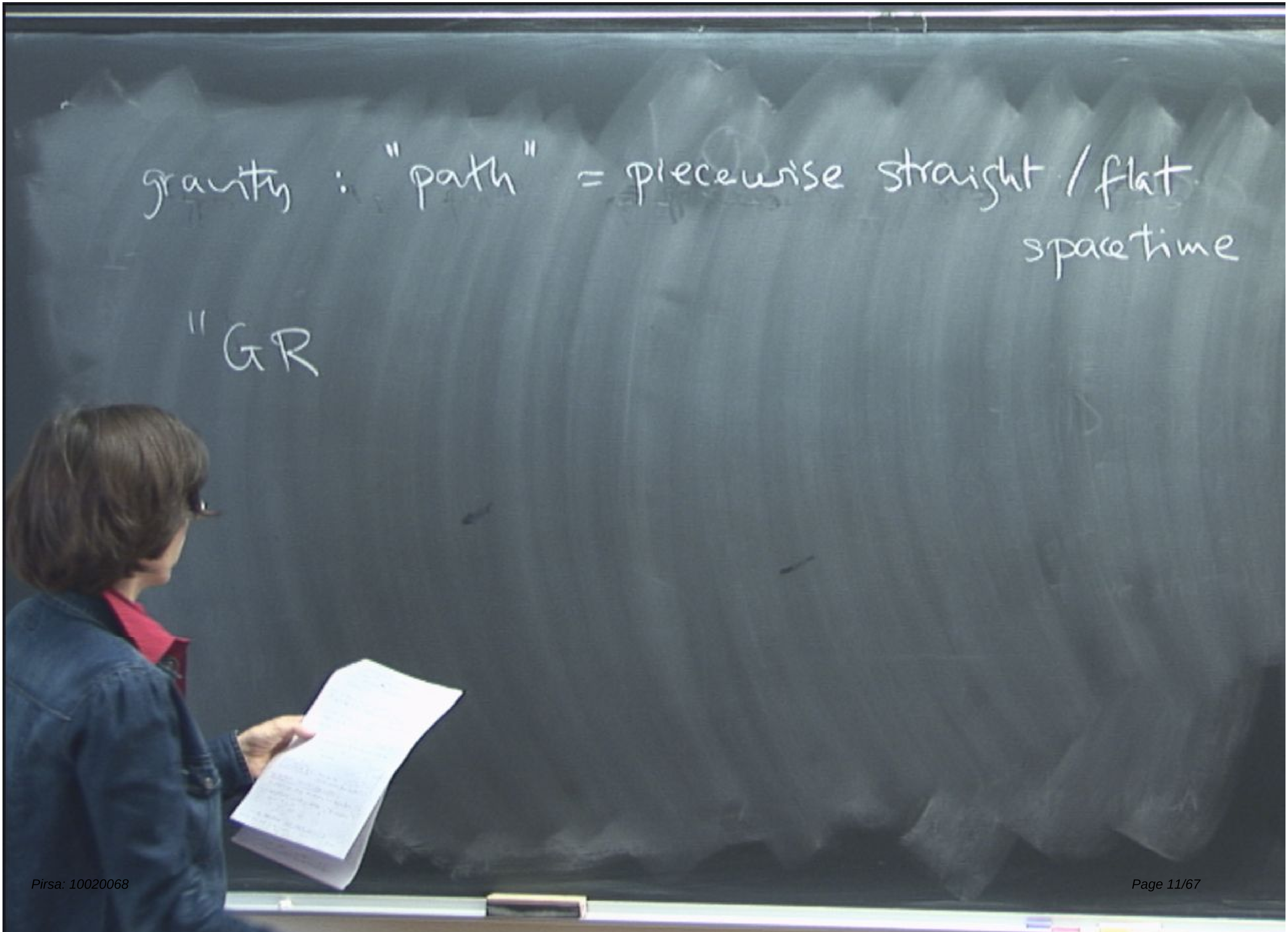

gravity : "path" = piecewise straight / flat
spacetime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"GR without coordinates" (Regge 1961)

simplicial approximation :

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$$(M, g_{\mu\nu}(x)) \simeq (T, \{l$$

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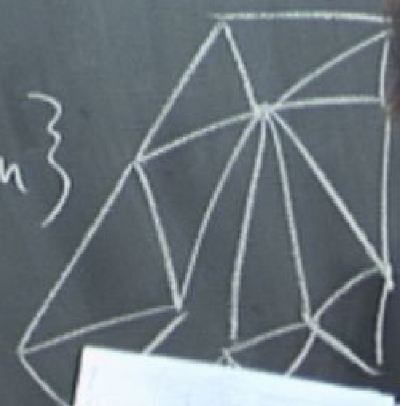
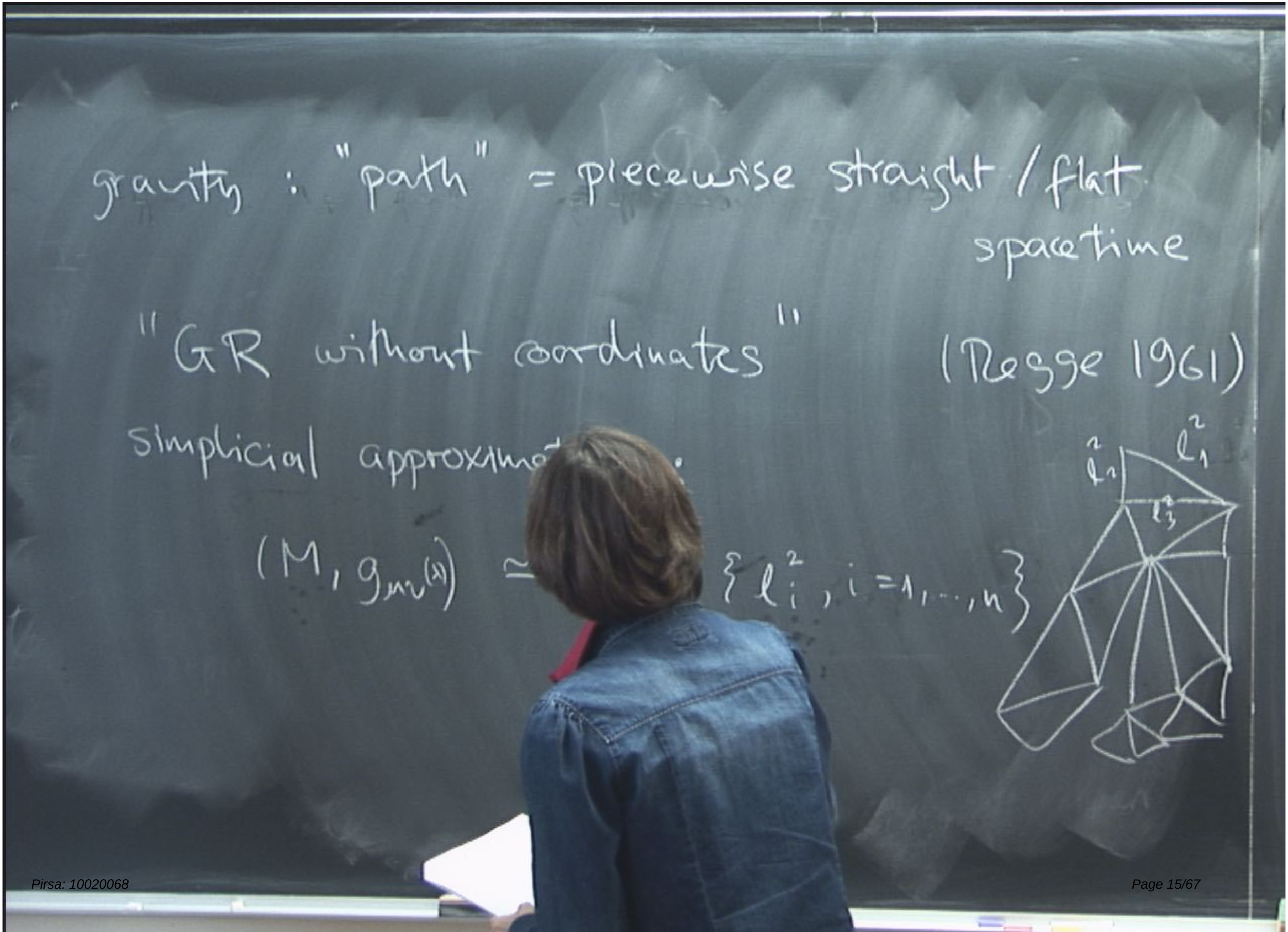
$$(M, g_{\mu\nu}(x)) \simeq (T, \{l_i^2, i=1,\dots,n\})$$




gravity : "path" = piecewise straight / flat spacetime
"GR without coordinates" (Regge 1961)
simplicial approxim
$(M, g_{\mu\nu}(x)) \simeq$
$\{ l_i^2 , i = 1, \ldots, n \}$
l_1^2
l_1^2
l_3^2

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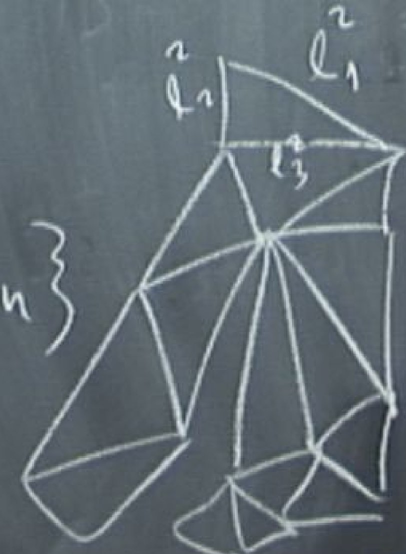
$\uparrow$ triangulation



geometric d.o.f.

1) connectivity of T (how building blocks are glued together)

2)

gravity : "path" = piecewise straight / flat spacetime

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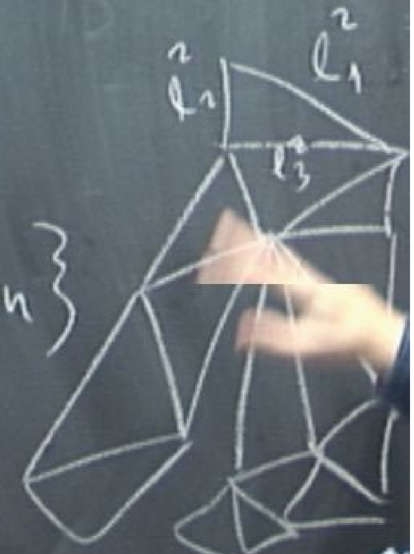
triangulation



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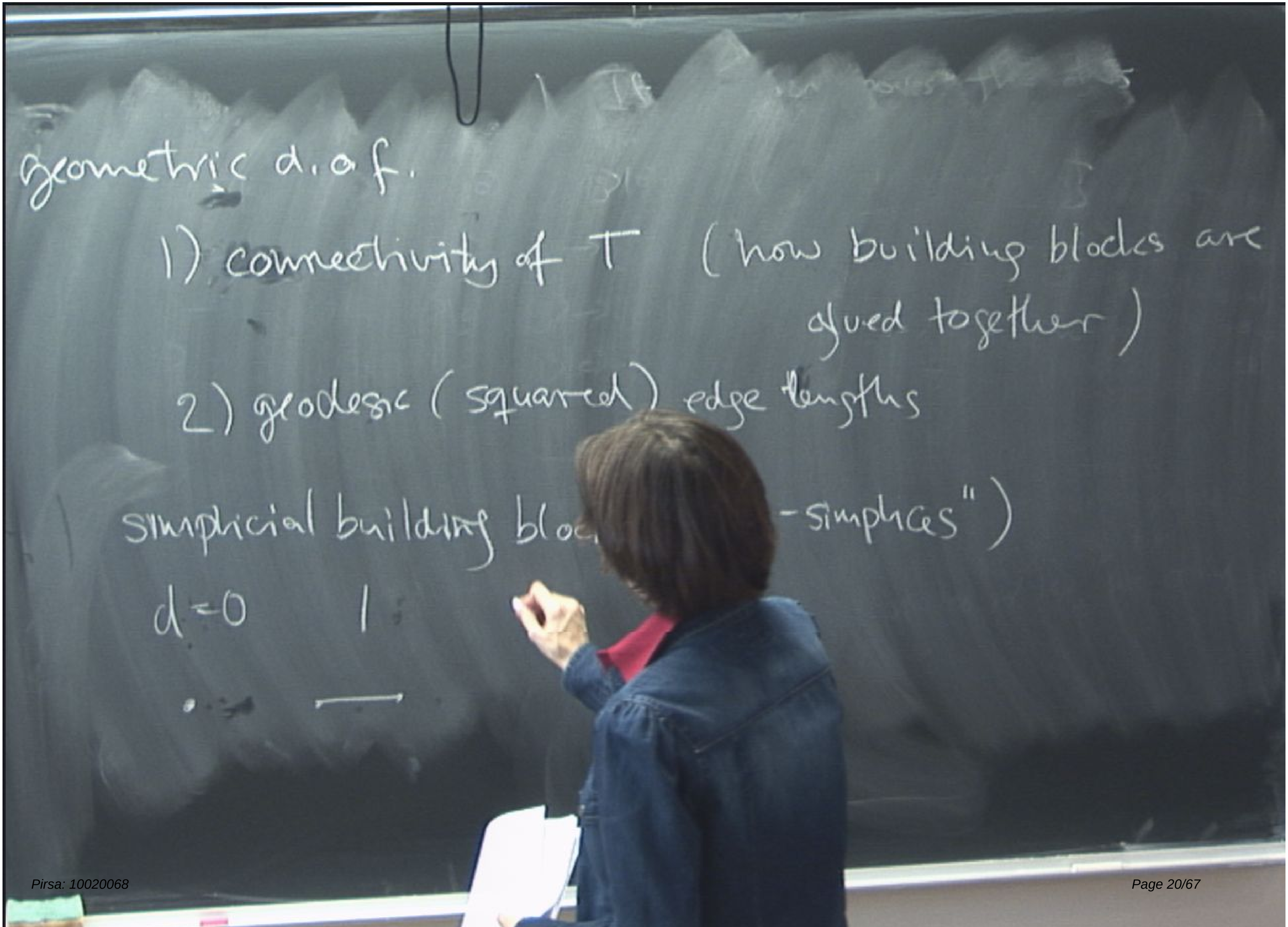
1) connectivity of T (how building blocks are glued together)

2) geodesic (squared) edge lengths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simplicial building blo
-simplices")
d=0
1

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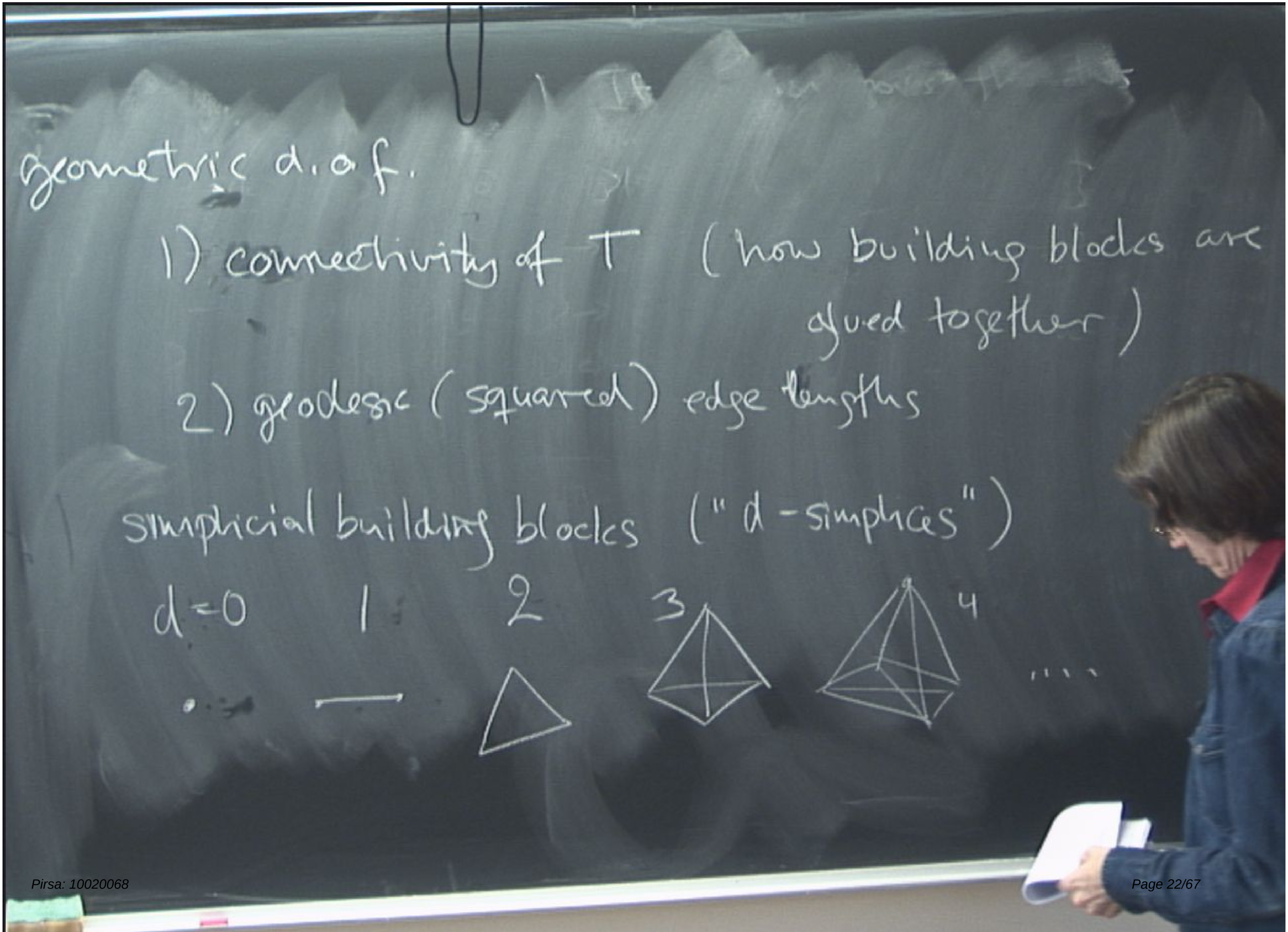
simplicial building blocks ("d-simpl

d=0 1 2 3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d=0 1 2 3 4 ....

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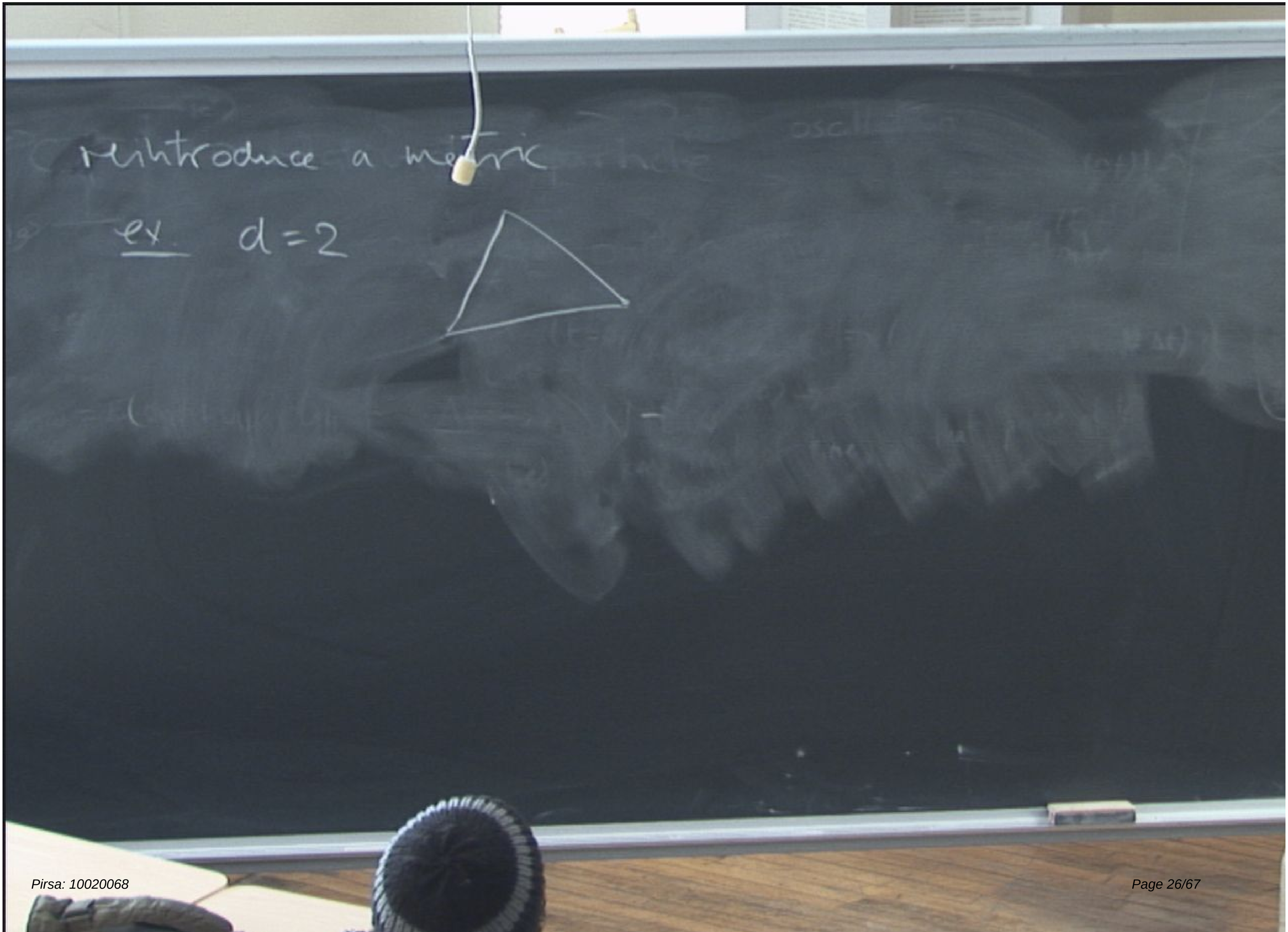
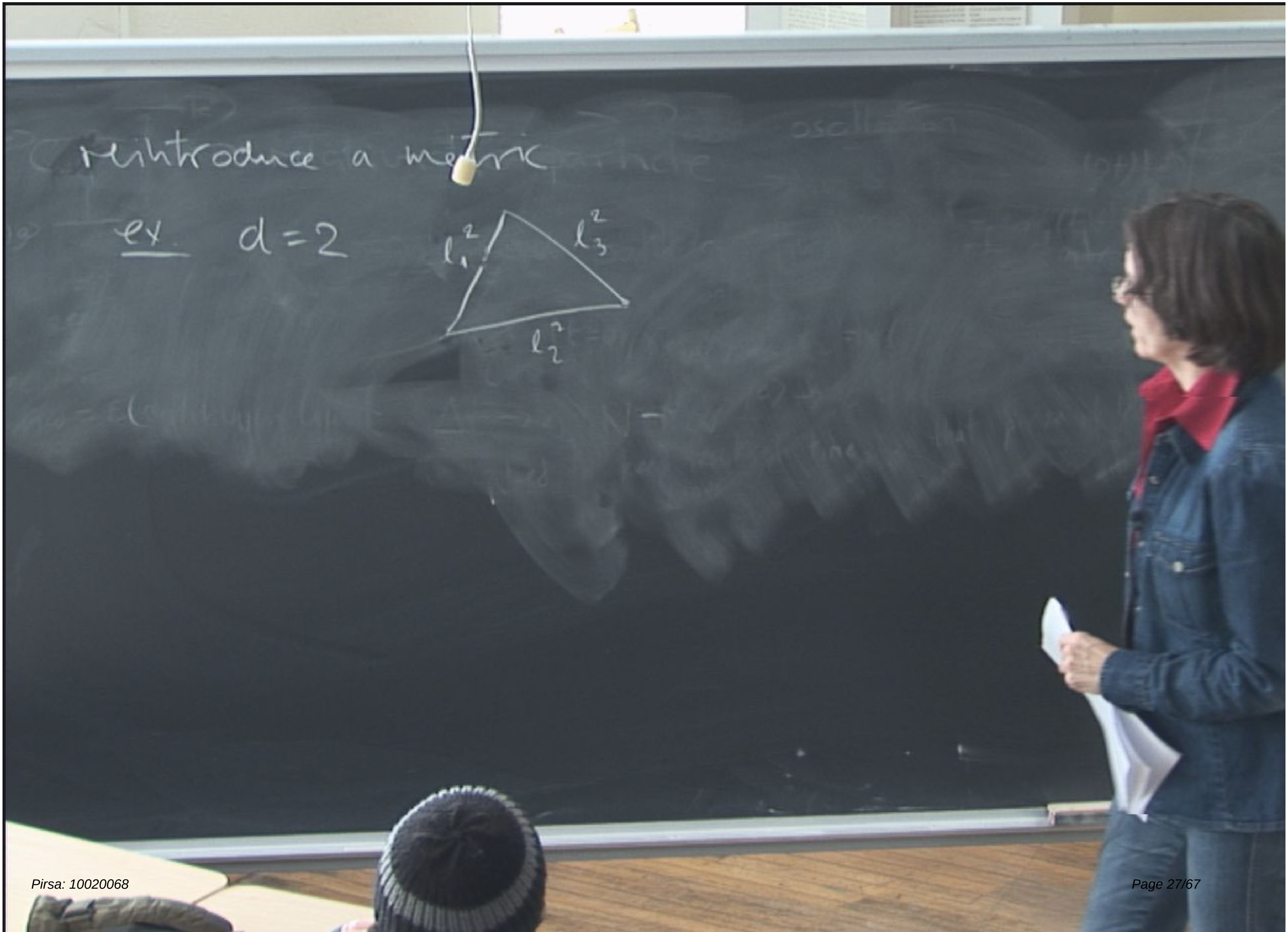

reintroduce a metric
ex. d=2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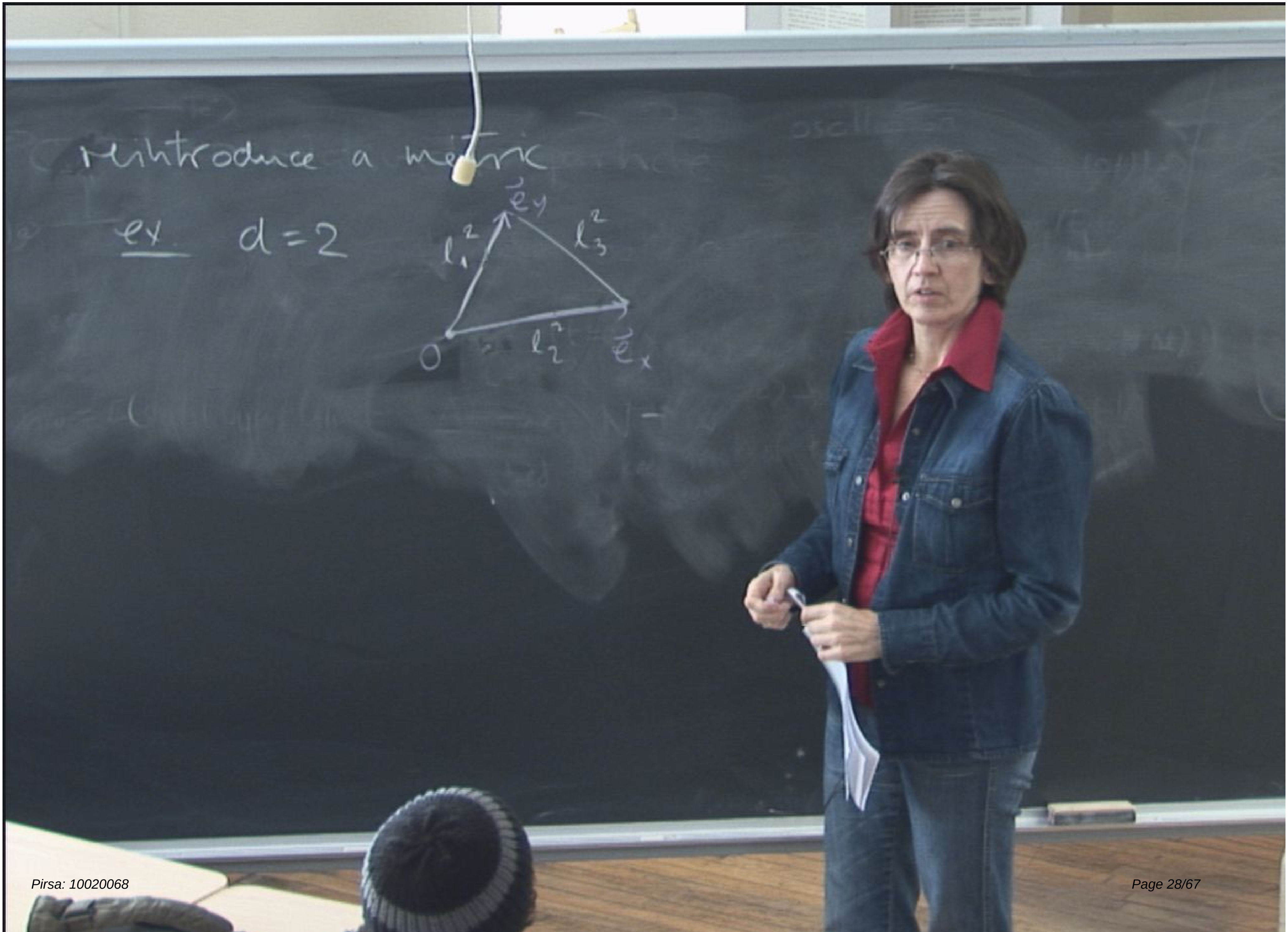
ex. d=2
l1^2
l3^2
l2^2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$\vec{e}_y$
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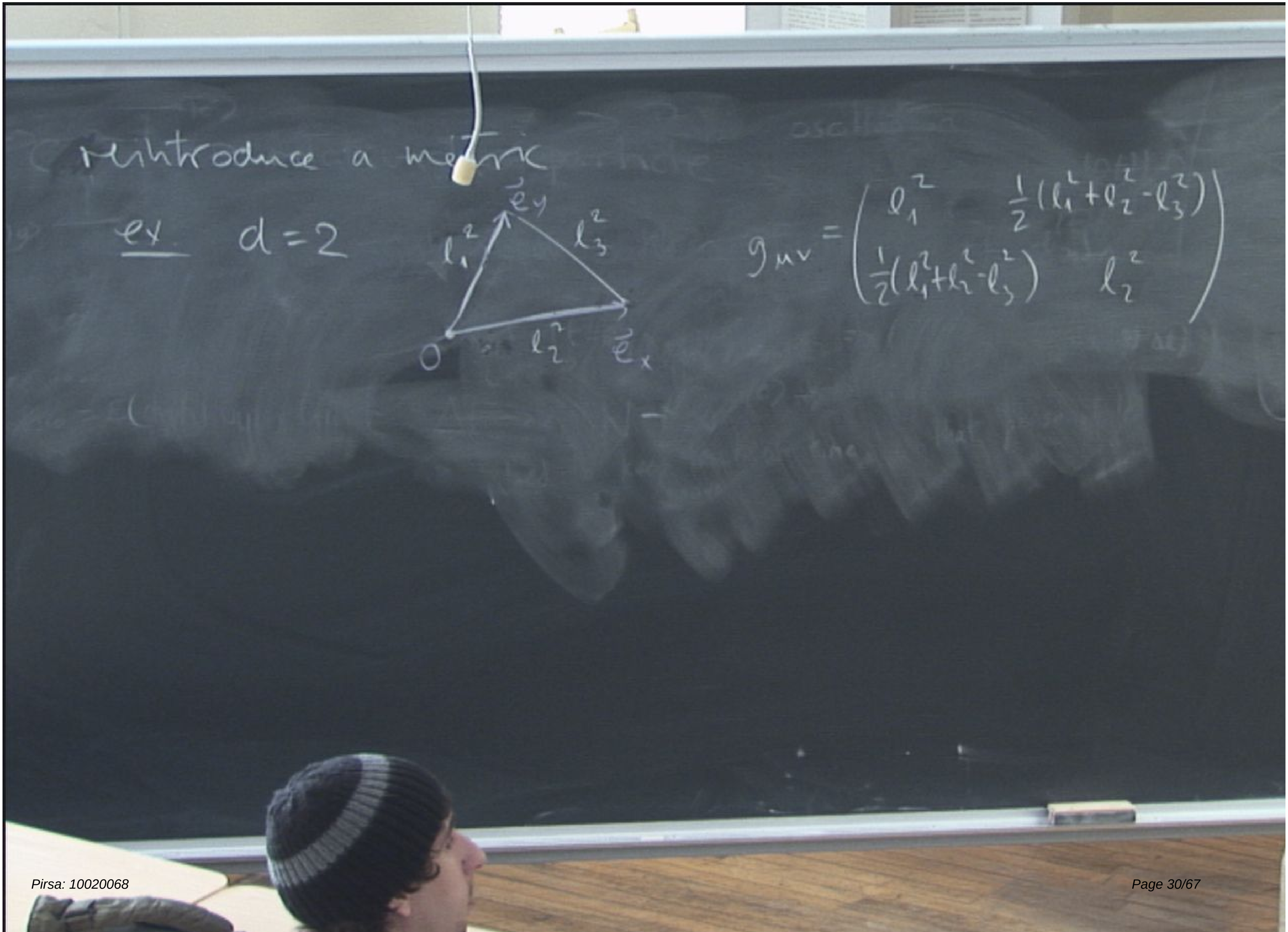
l_2^2
O
$\vec{e}_x$

reintroduce a metric

ex $d=2$

$$g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2 + l_2^2 - l_3^2) \\ \frac{1}{2}(l_1^2 + l_2^2 - l_3^2) & l_2^2 \end{pmatrix}$$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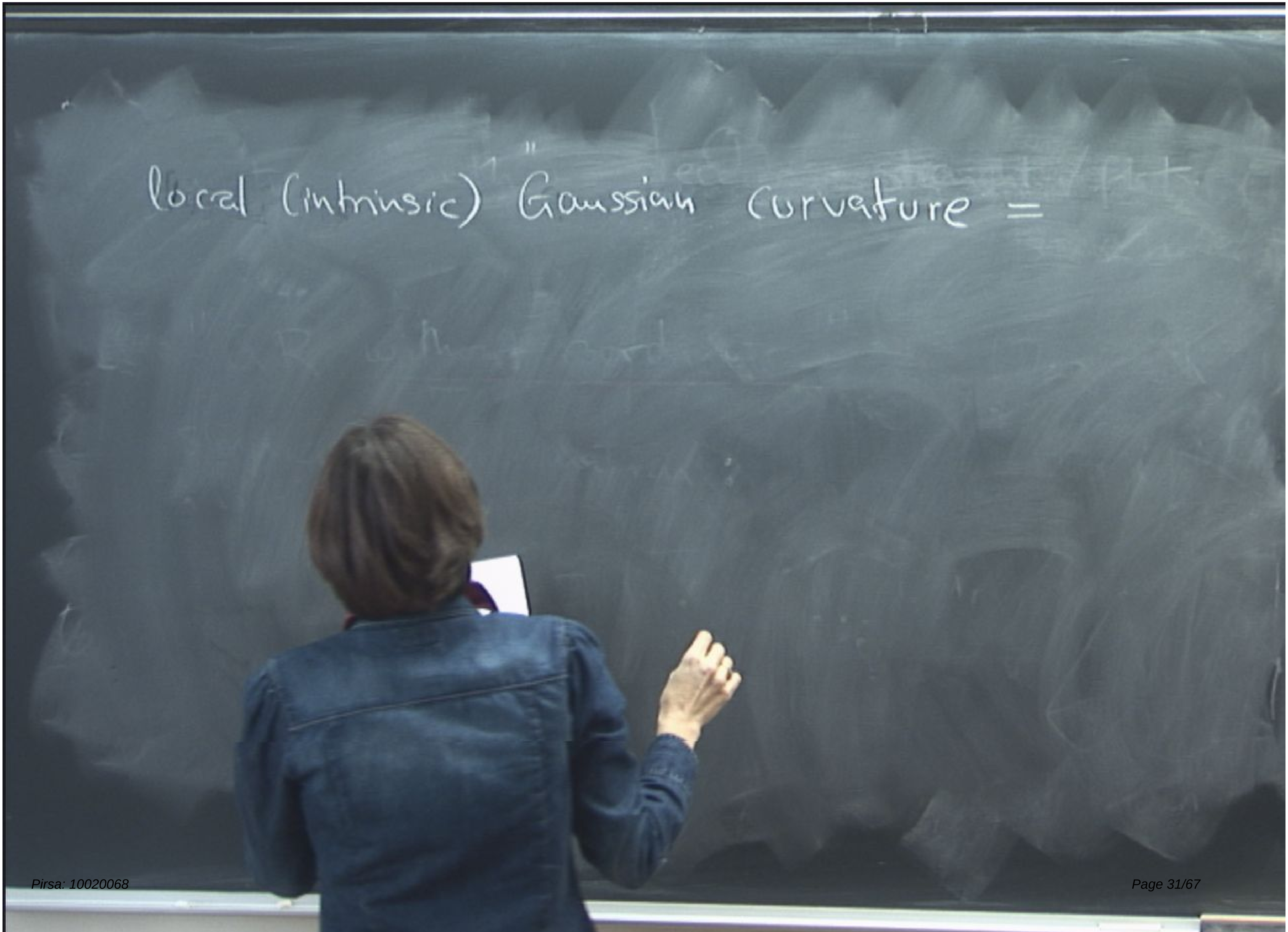
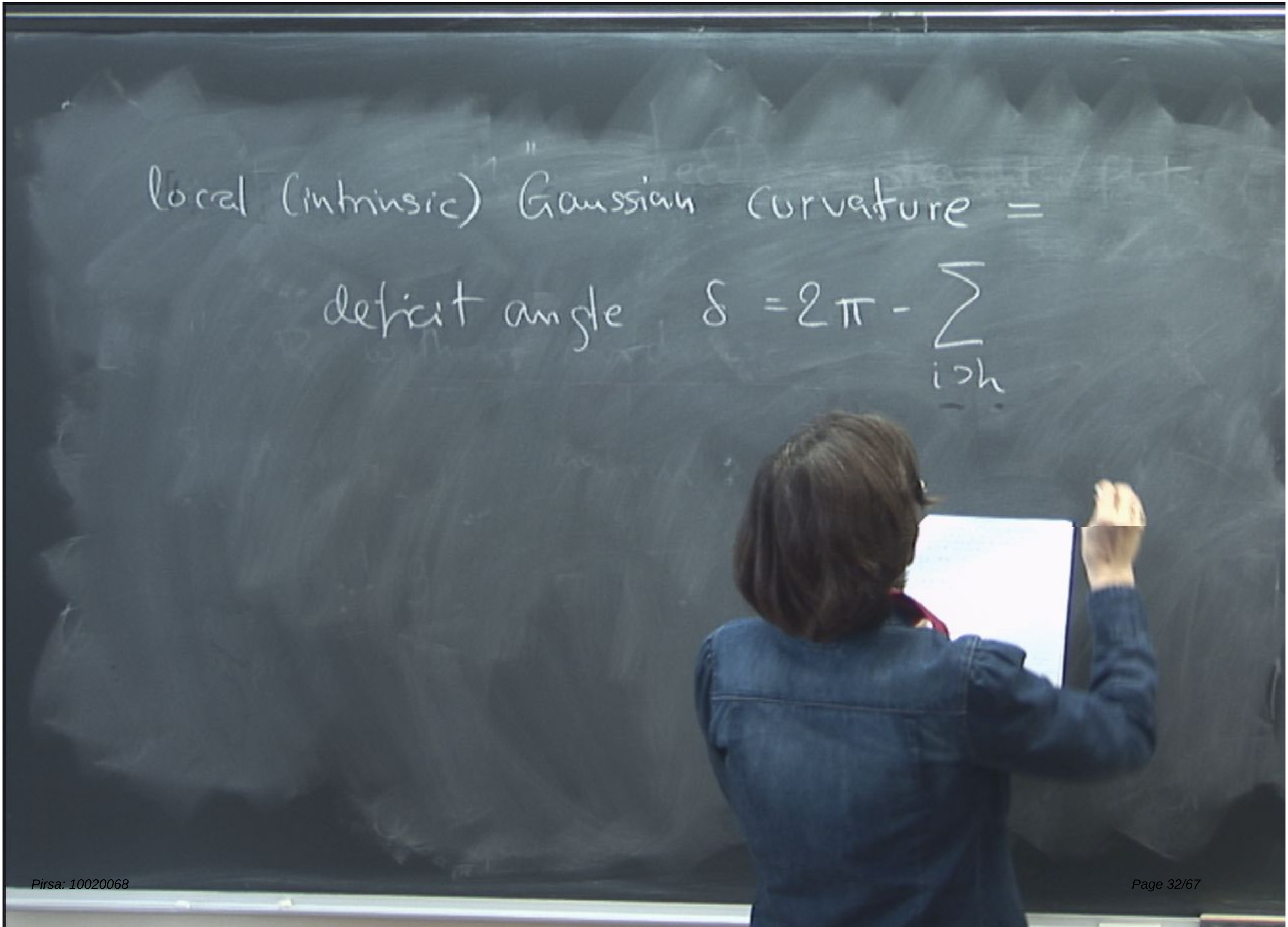
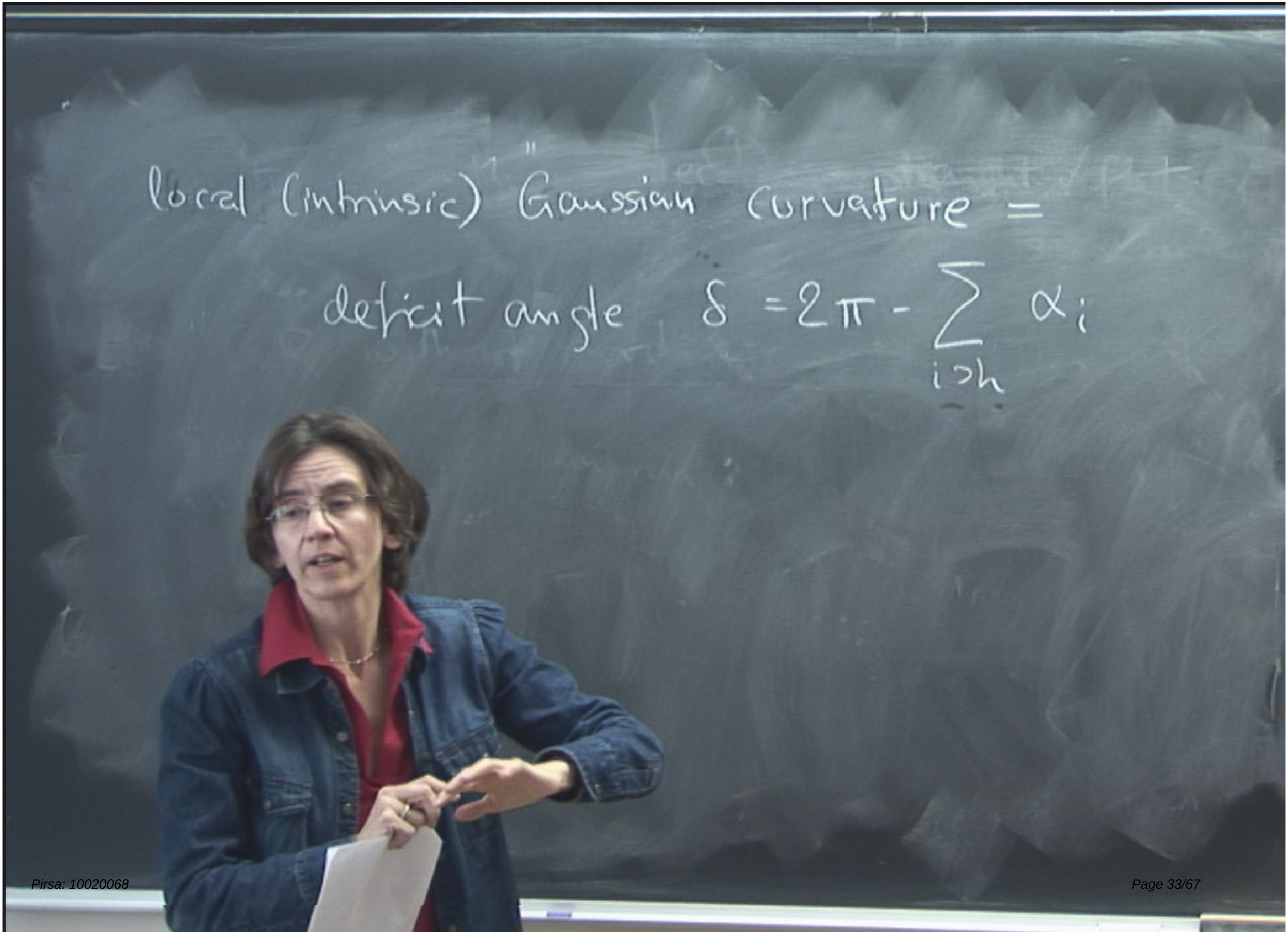

local (intrinsic) Gaussian curvature =


local (intrinsic) Gaussian curvature =
deficit angle δ = 2π - Σ
i⊃h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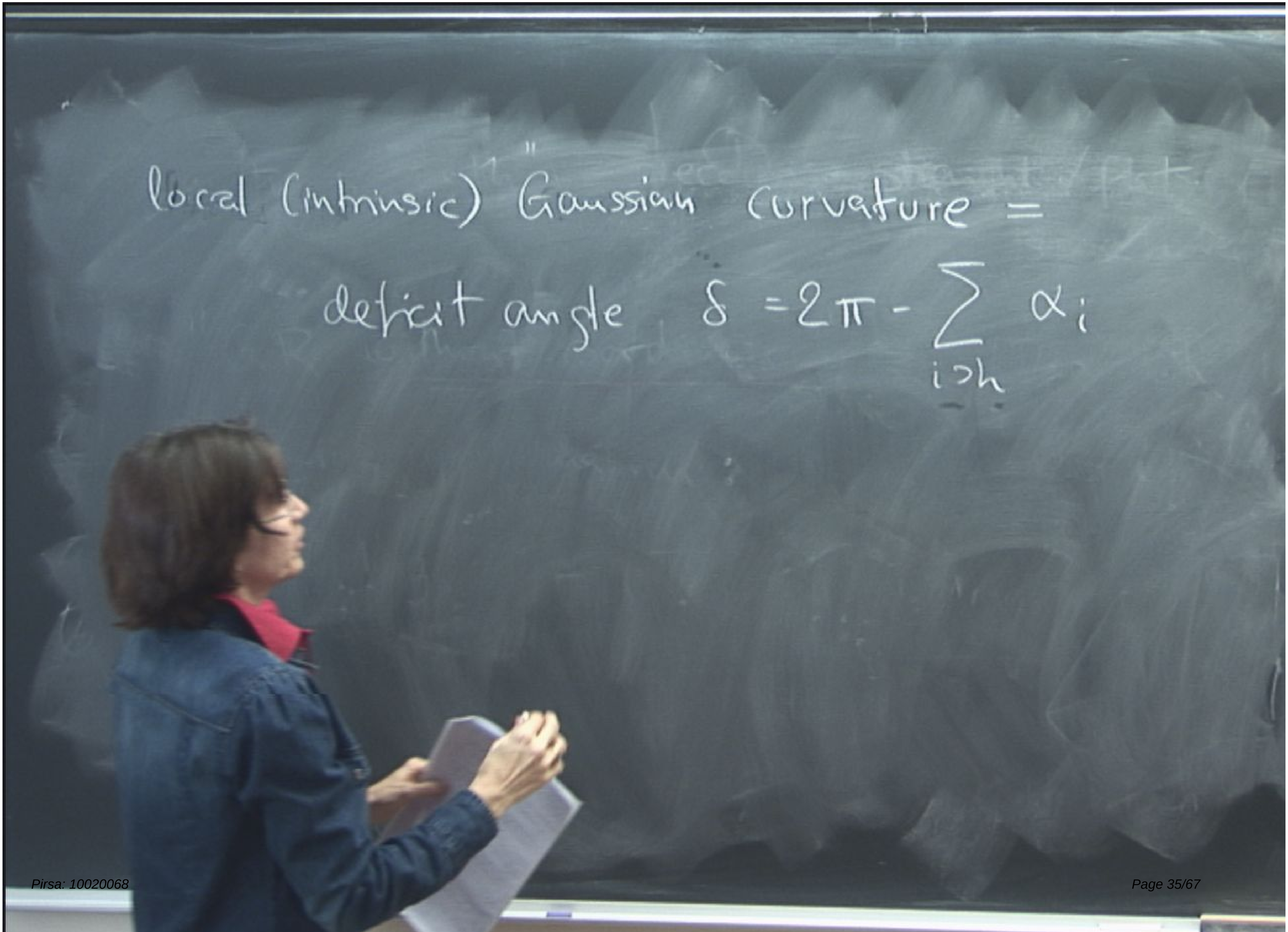
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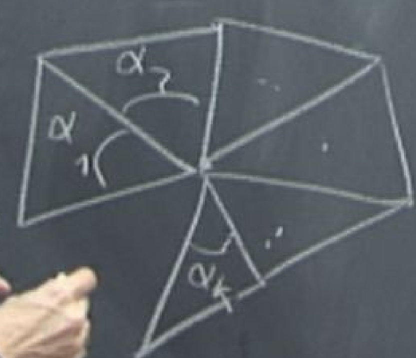

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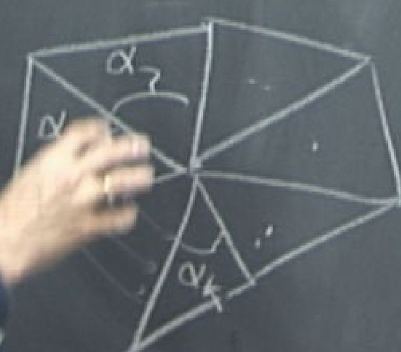

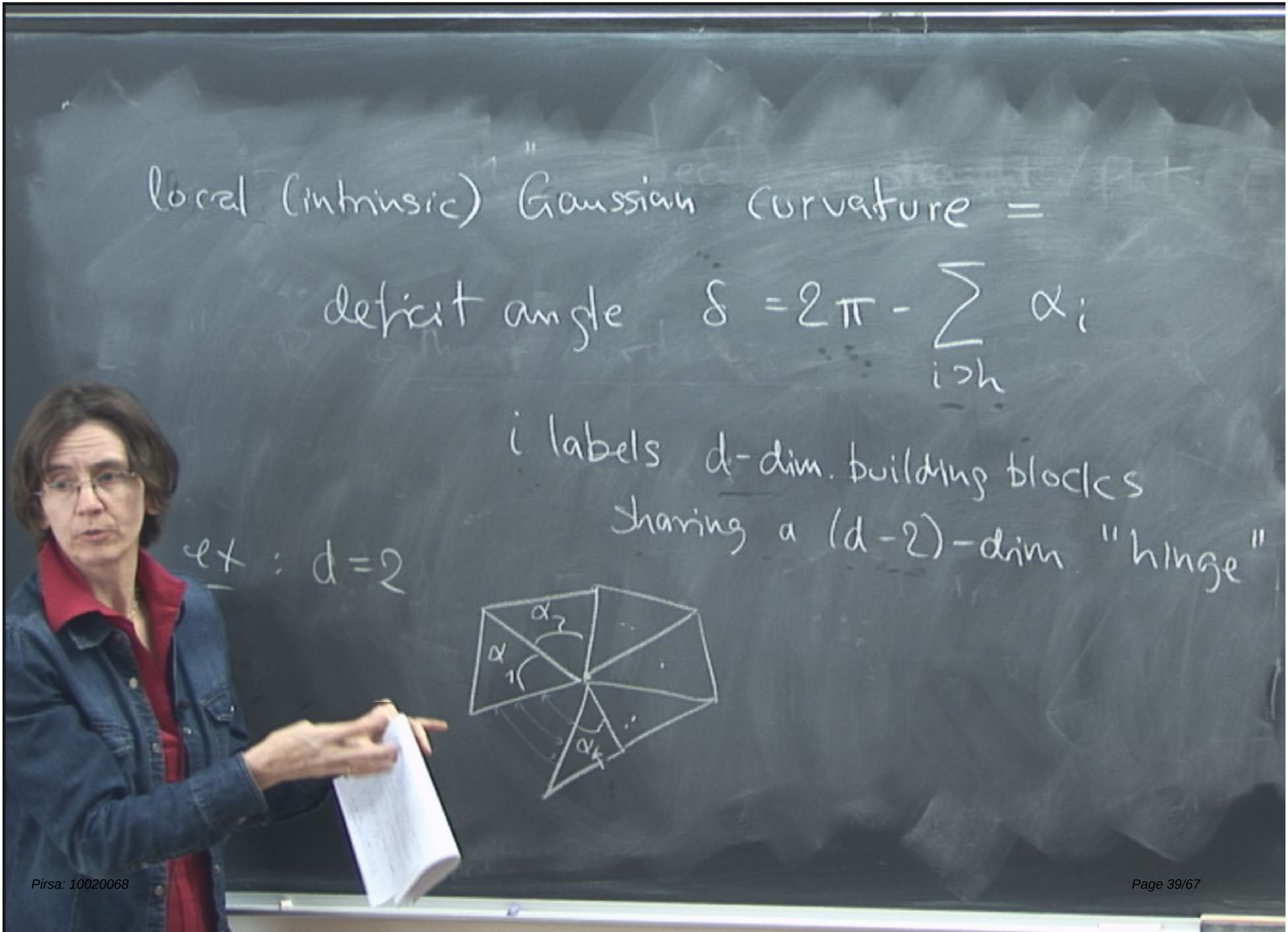
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ex: d=2
α_2
α_1
α_k

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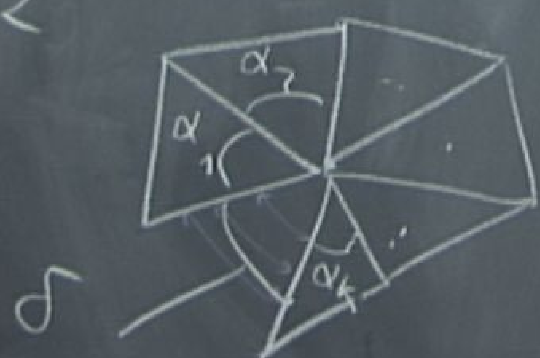

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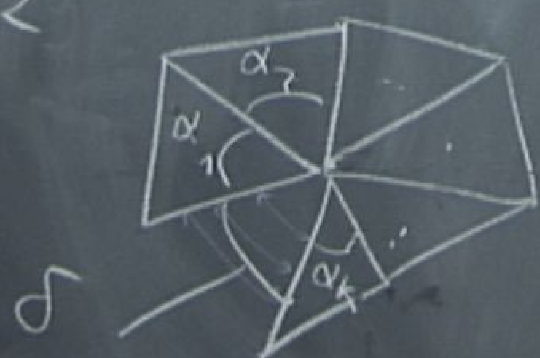

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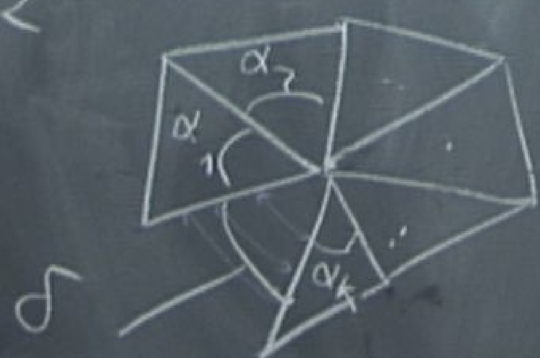

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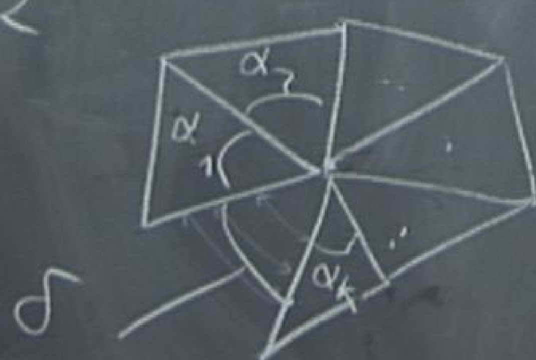

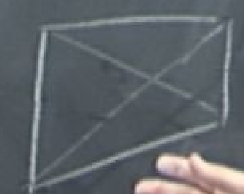

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simplicial building blocks ("d-simplices")

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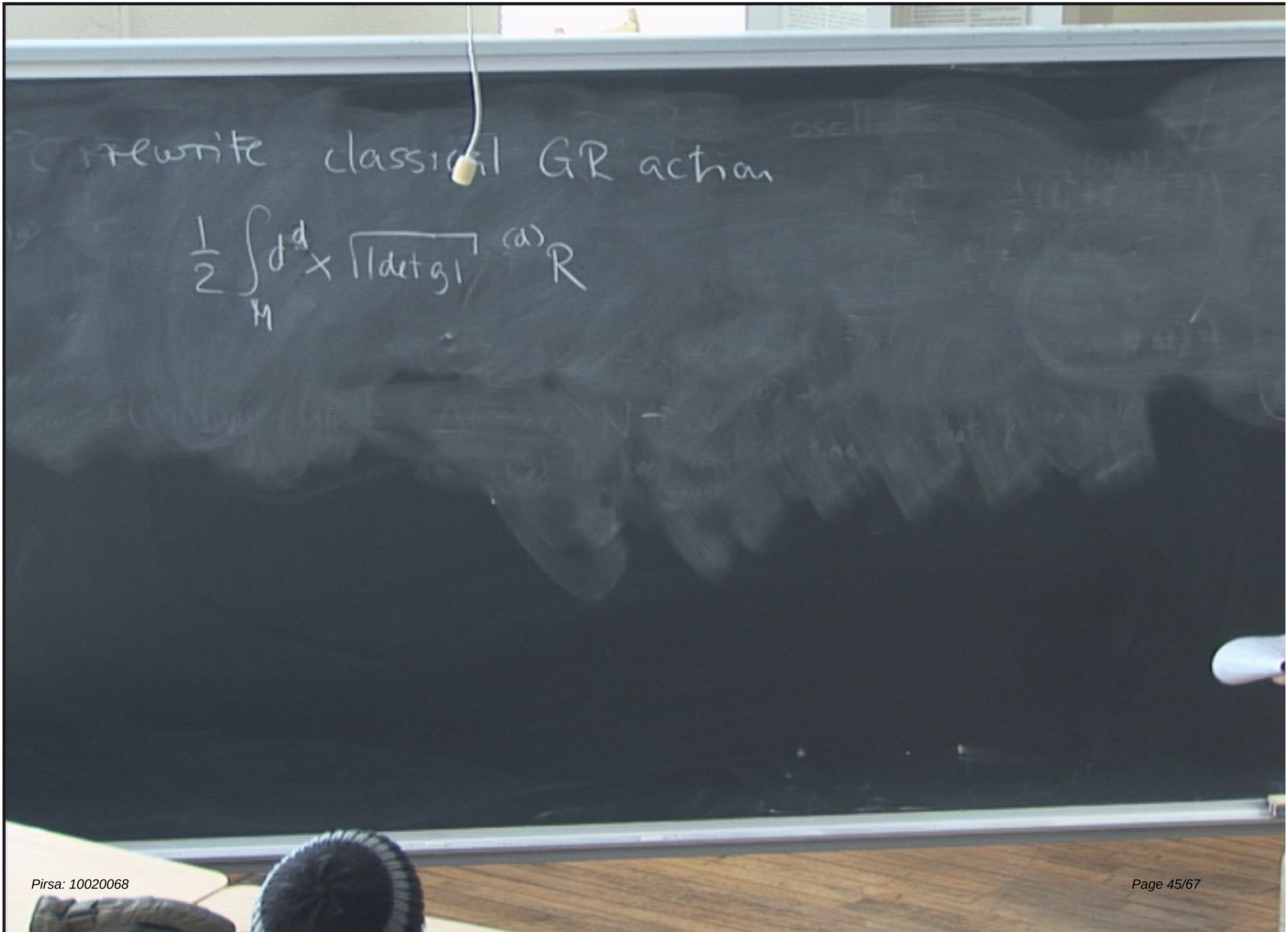
rewrite classical GR action
$\frac{1}{2}\int_M d^d x \sqrt{|\det g|}\; {}^{(d)}R$

rewrite classical GR action

$$\frac{1}{2}\int_M d^dx \sqrt{|\det g|}\; {}^{(d)}R \longrightarrow \sum_{i \in T} V$$

rewrite classical GR action

$$\frac{1}{2}\int_M d^d x \sqrt{|\det g|}\ {}^{(d)}R \longrightarrow \sum_{i \in T} \mathrm{vol}\left(i^{th}\ (d-2)\text{-simplex}\right) \cdot \delta_i$$

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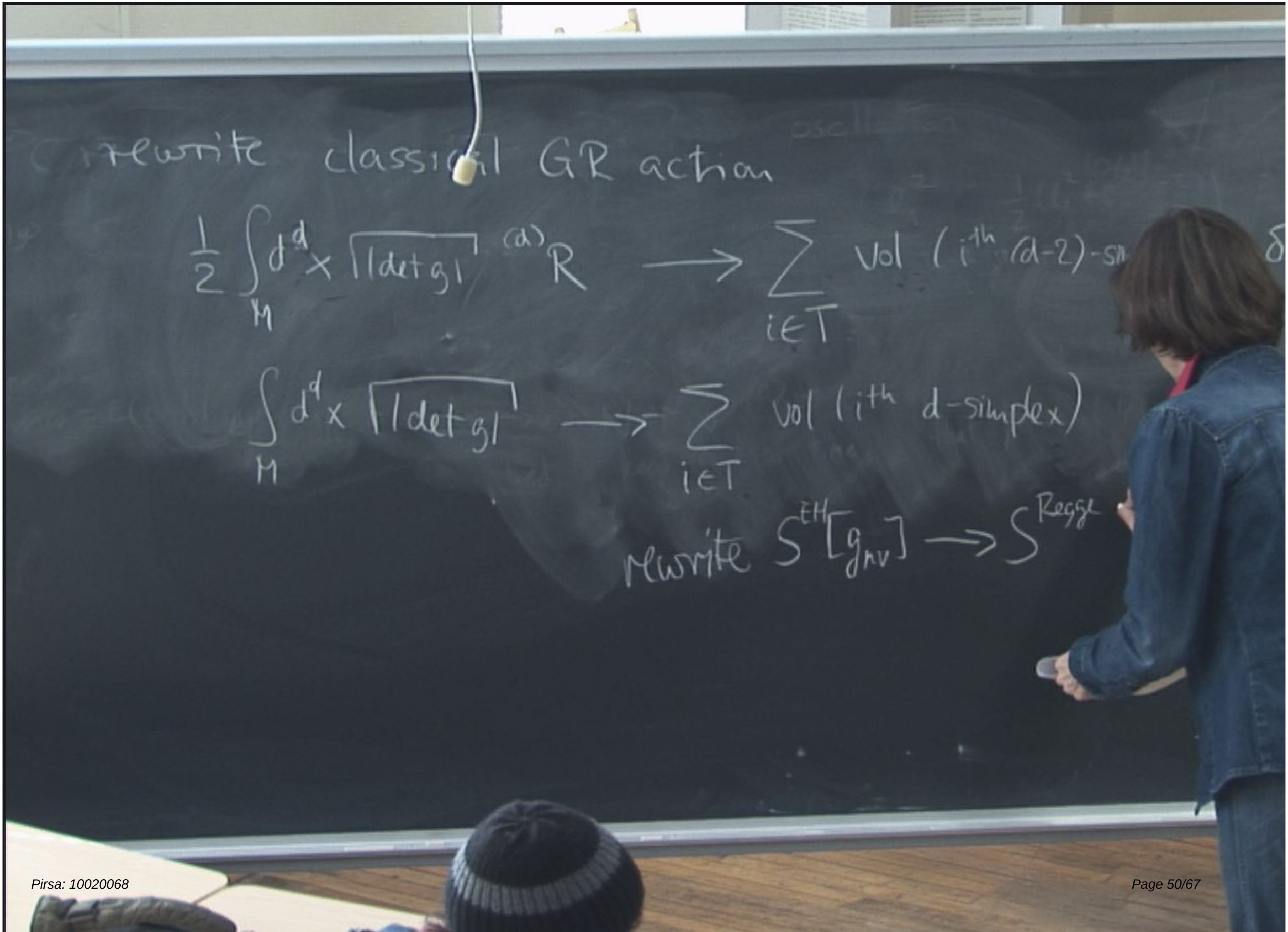

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$\int_M d^dx \sqrt{|\det g|} \longrightarrow \sum_{i\in T} \text{vol}(i^{th}\ d\text{-simplex})$
rewrite $S^{EH}[g_{\mu\nu}] \longrightarrow S^{Regge}$

rewrite classical GR action

$$\frac{1}{2}\int_M d^d x \sqrt{|\det g|}\ {}^{(d)}R \longrightarrow \sum_{i \in T} \mathrm{vol}\,(i^{th}\ (d-2)\text{-simplex}) \cdot \delta$$

$$\int_M d^d x \sqrt{|\det g|} \longrightarrow \sum_{i \in T} \mathrm{vol}\,(i^{th}\ d\text{-simplex})$$

rewrite $S^{EH}[g_{\mu\nu}] \longrightarrow S^{Regge}[T, \{l_i^2\}]$

(a) Quantum Regge Calculus (QRC)

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fix T, integrate over all l_i

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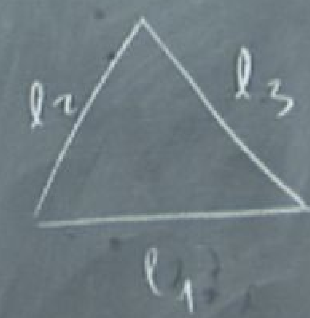

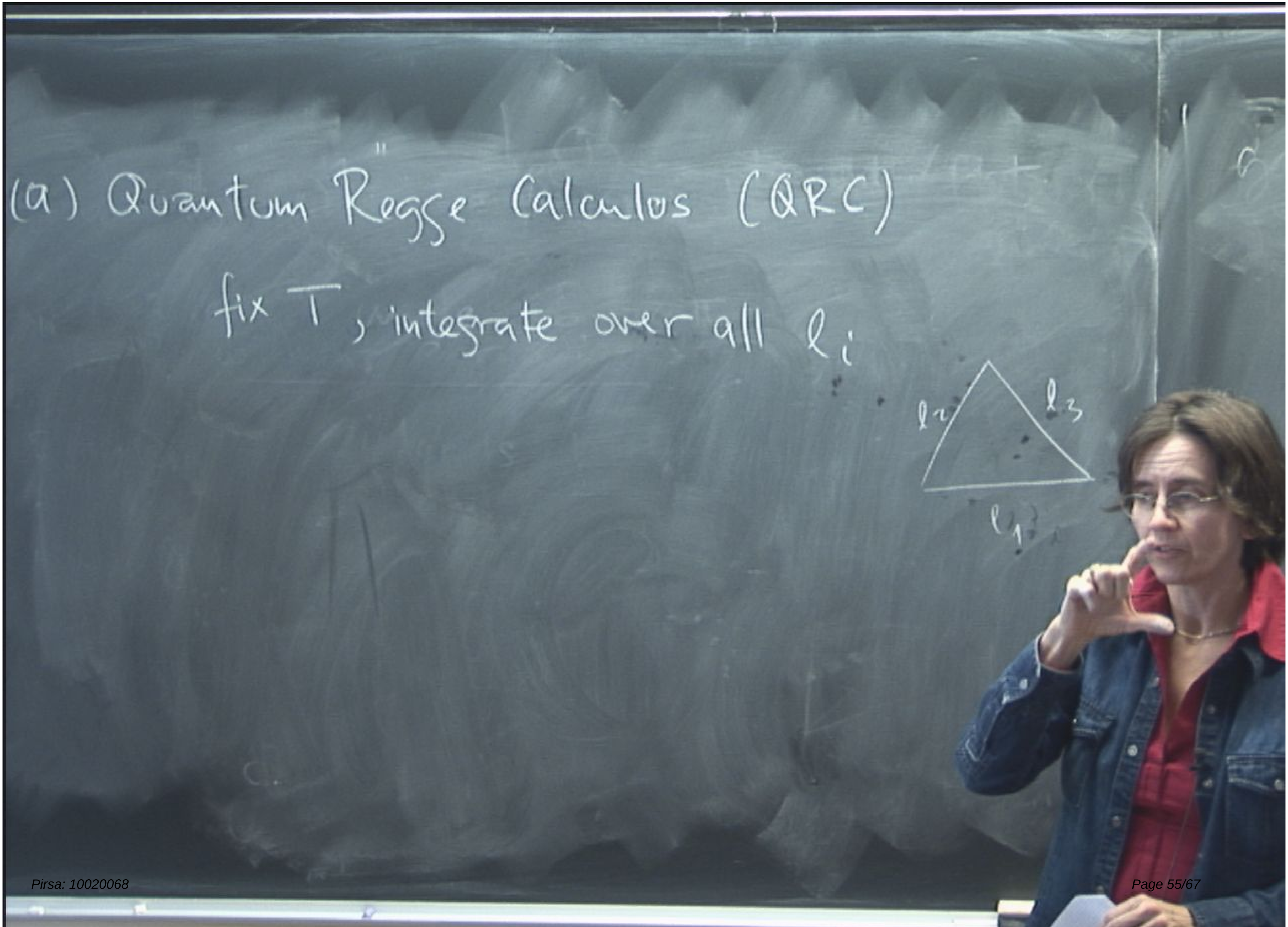
(a) Quantum Regge Calculus (QRC)
fix T, integrate over all l_i
l_2
l_3
l_1

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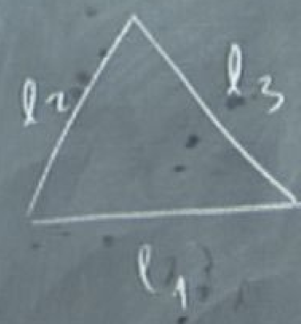

(b) Dynamical Triangulation

(a) Quantum Regge Calculus (QRC)

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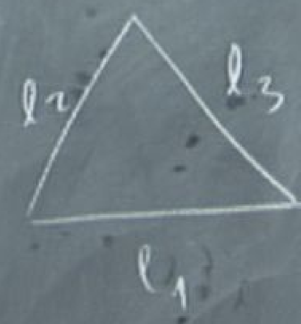

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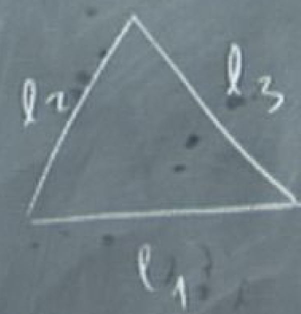
(b) Dynamical Triangulation (DT)

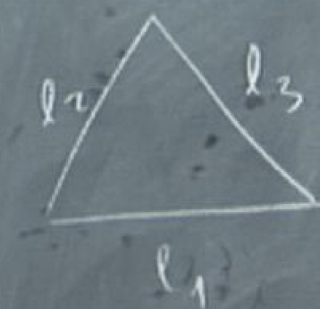


fix $l_i^2 = \pm a^2$, sum over all T

(a) "Quantum Regge Calculus (QRC)

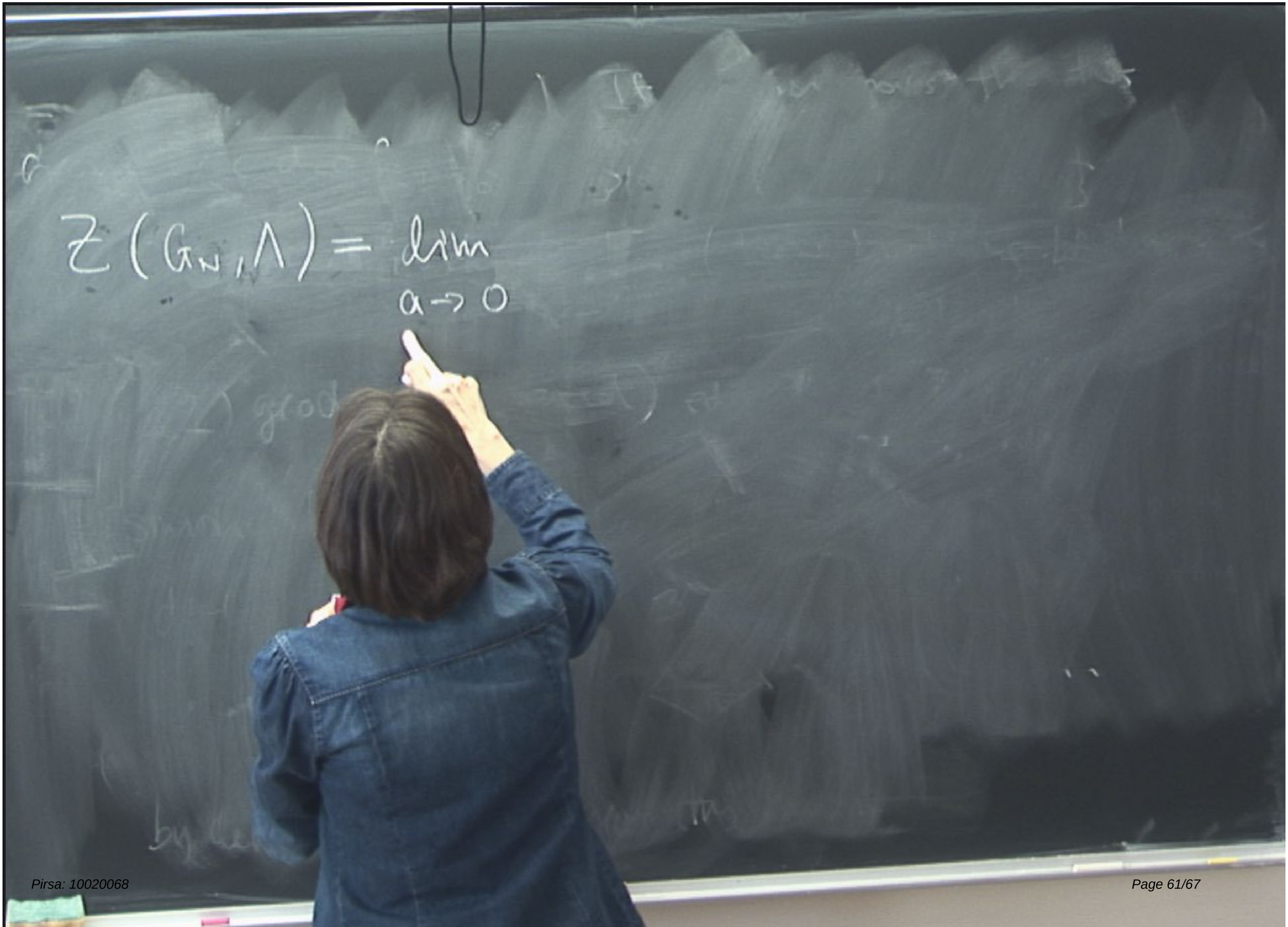
fix T, integrate over all ℓ_i

(b) Dynamical Triangulation (DT)

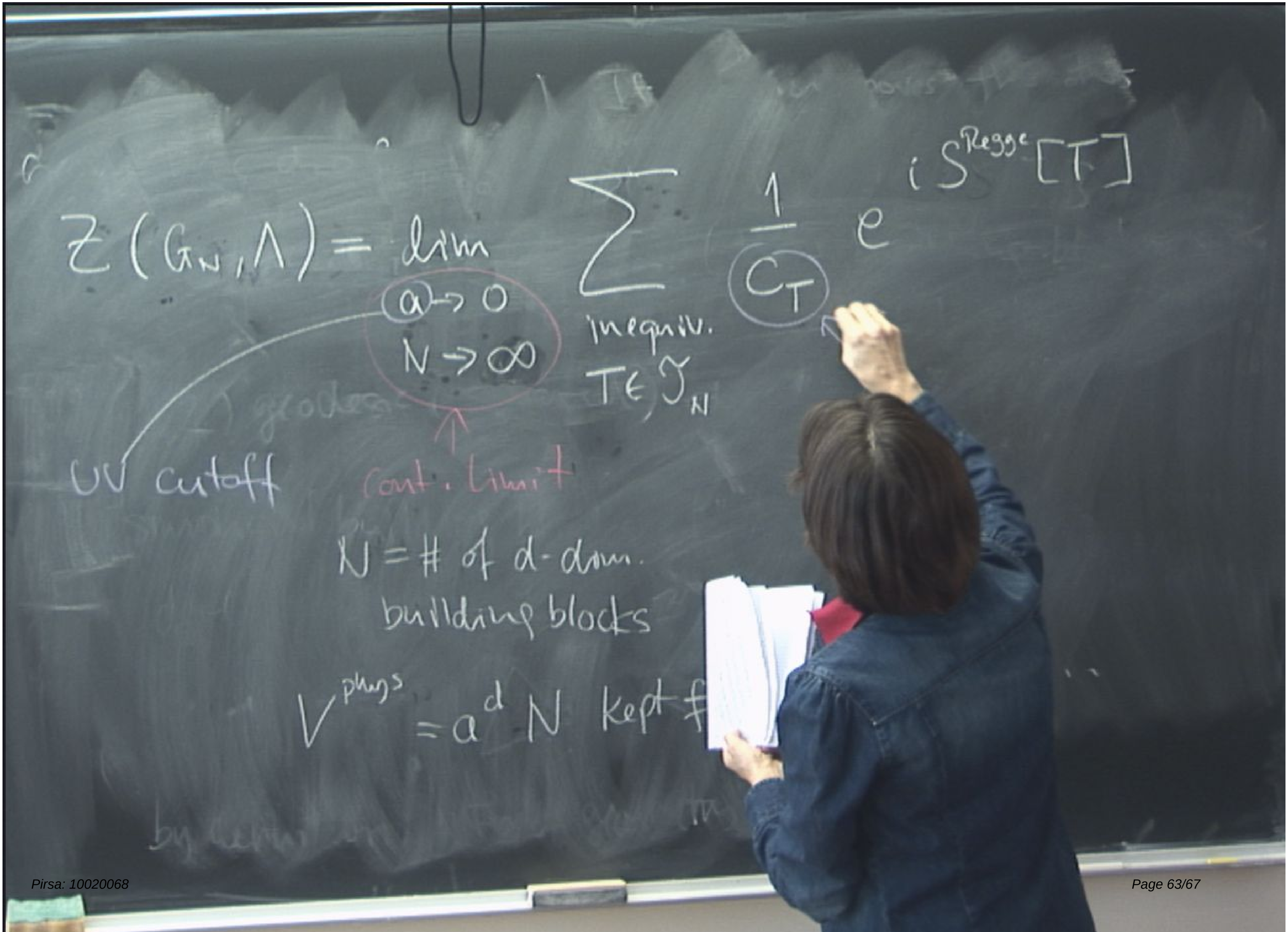


fix $\ell_i^2 = \pm a^2$, sum over all T

$$Z(G_N, \Lambda) =$$


$Z(G_N, \Lambda) = \lim_{a \to 0}$

$$Z(G_N, \Lambda) = \lim_{\substack{a \to 0 \\ N \to \infty}} \sum_{\substack{\text{inequiv.} \\ T \in \mathcal{T}_N}} \frac{1}{C_T} e^{i S^{\text{Regge}}[T]}$$


Z(G_N, Λ) = lim ∑ 1/C_T e^{i S^Regge [T]}
a → 0
N → ∞
inequiv.
T ∈ 𝒯_N
UV cutoff
cont. limit
N = # of d-dim.
building blocks
V^phys = a^d N kept

$$Z(G_N,\Lambda) = \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\substack{\text{inequiv.}\\ T\in \mathcal{T}_N}} \frac{1}{C_T}\, e^{\,i S^{Regge}[T]}$$

UV cutoff

cont. limit

C_T = |autom. group of T|

N = # of d-dim. building blocks

$V^{phys} = a^d N$ kept finite

$$Z(G_N,\Lambda) = \lim_{\substack{a \to 0 \\ N \to \infty}} \sum_{\substack{\text{inequiv.} \\ T \in \mathcal{T}_N}} \frac{1}{C_T}\, e^{i S^{\text{Regge}}[T]}$$

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UV cutoff

cont. limit

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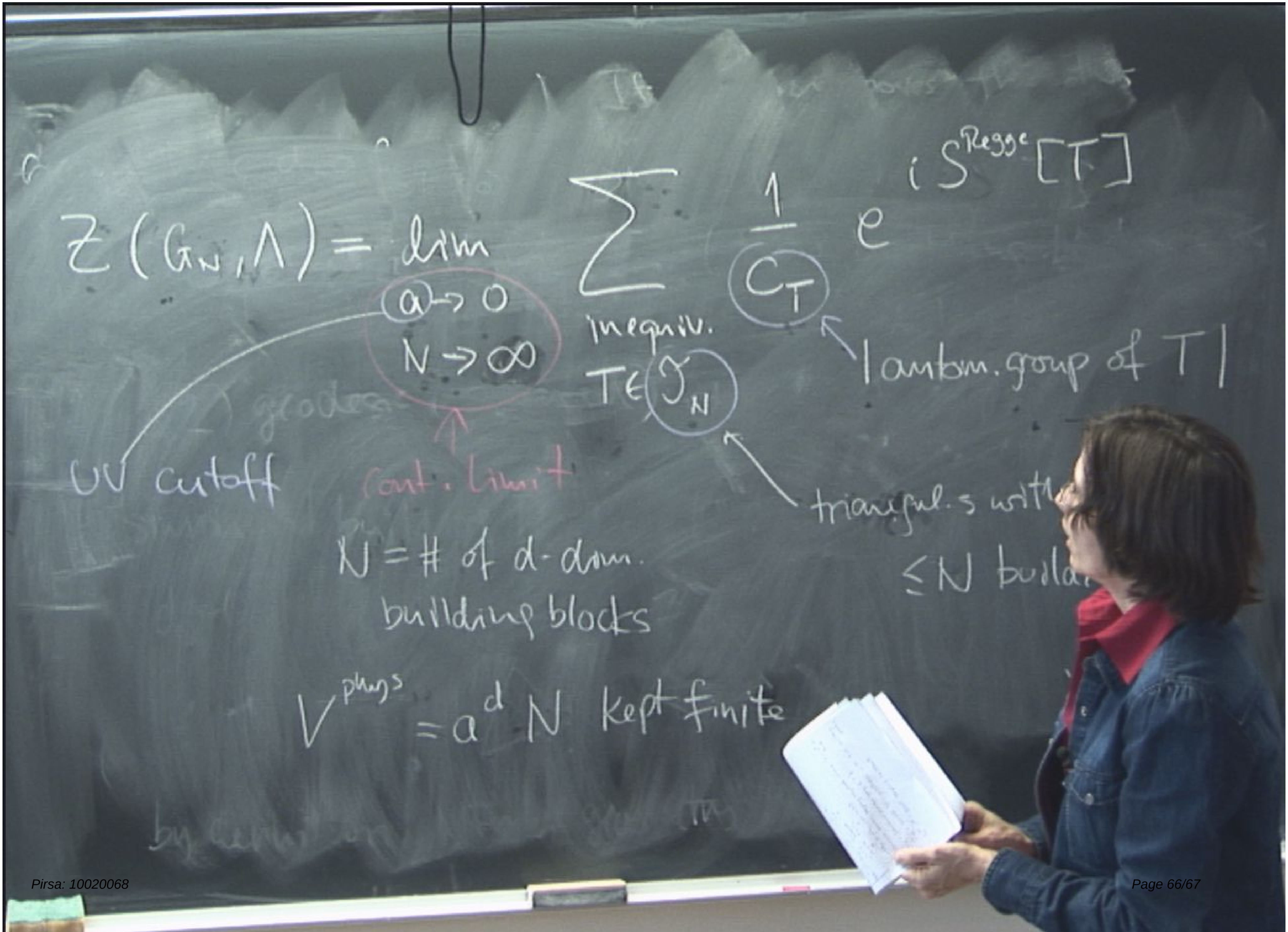
$Z(G_N, \Lambda) = \lim_{\substack{a \to 0 \\ N \to \infty}} \sum_{\substack{\text{inequiv.} \\ T \in \mathcal{T}_N}} \frac{1}{C_T} e^{i S^{Regge}[T]}$
|autom. group of T|
UV cutoff
cont. limit
triangul.s with
≤N build
N = # of d-dim.
building blocks
$V^{phys} = a^d N$ kept finite

$$Z(G_N,\Lambda) = \lim_{\substack{a \to 0 \\ N \to \infty}} \sum_{\substack{\text{inequiv.} \\ T \in \mathcal{T}_N}} \frac{1}{C_T}\, e^{\,i S^{Regge}[T]}$$

UV cutoff (a)

cont. limit ($a \to 0$, $N \to \infty$)

C_T = |autom. group of T|

$\mathcal{T}_N$ = triangul.s with $\leq N$ building bl.

N = # of d-dim. building blocks

$V^{phys} = a^d N$ kept finite