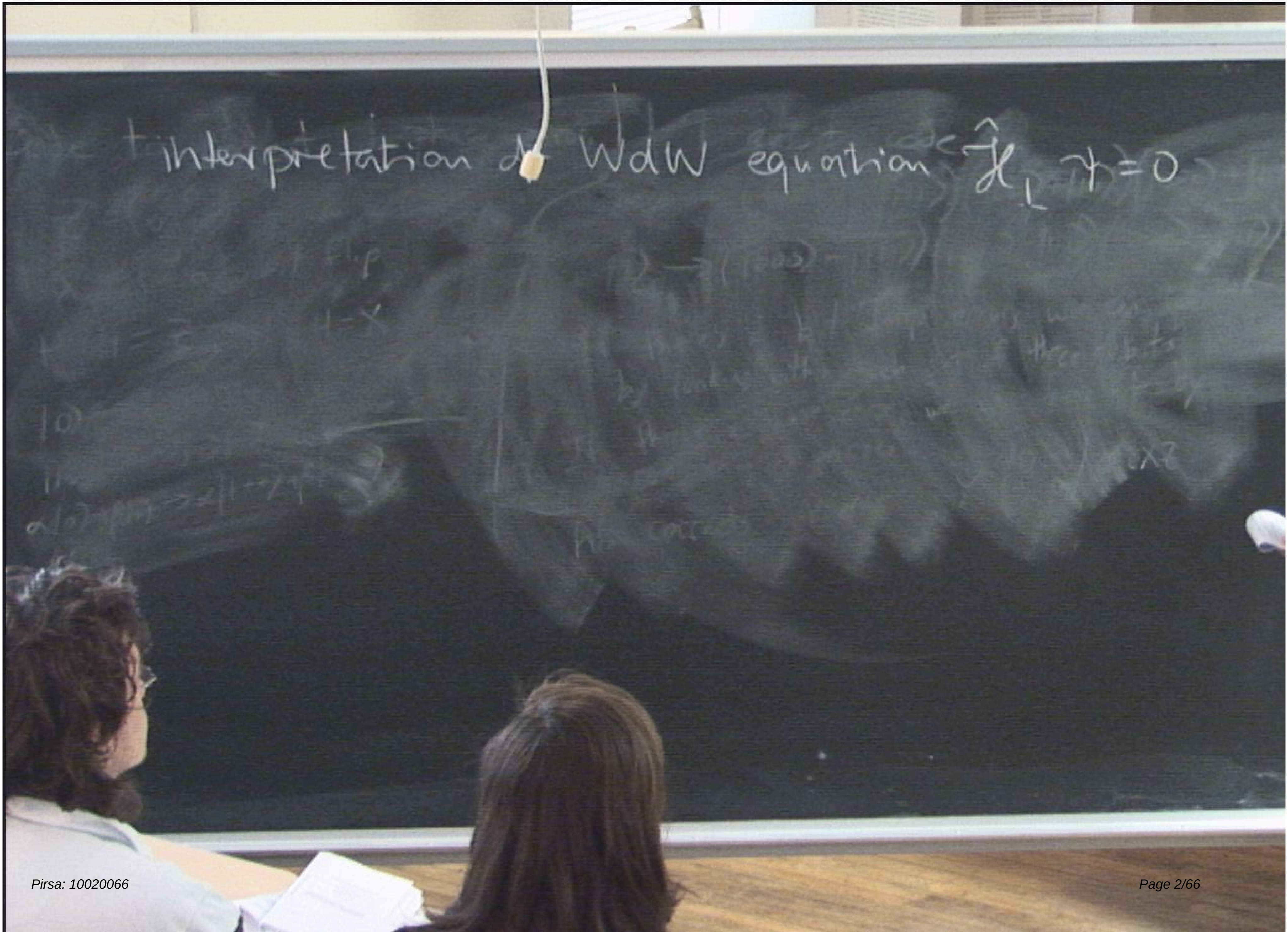
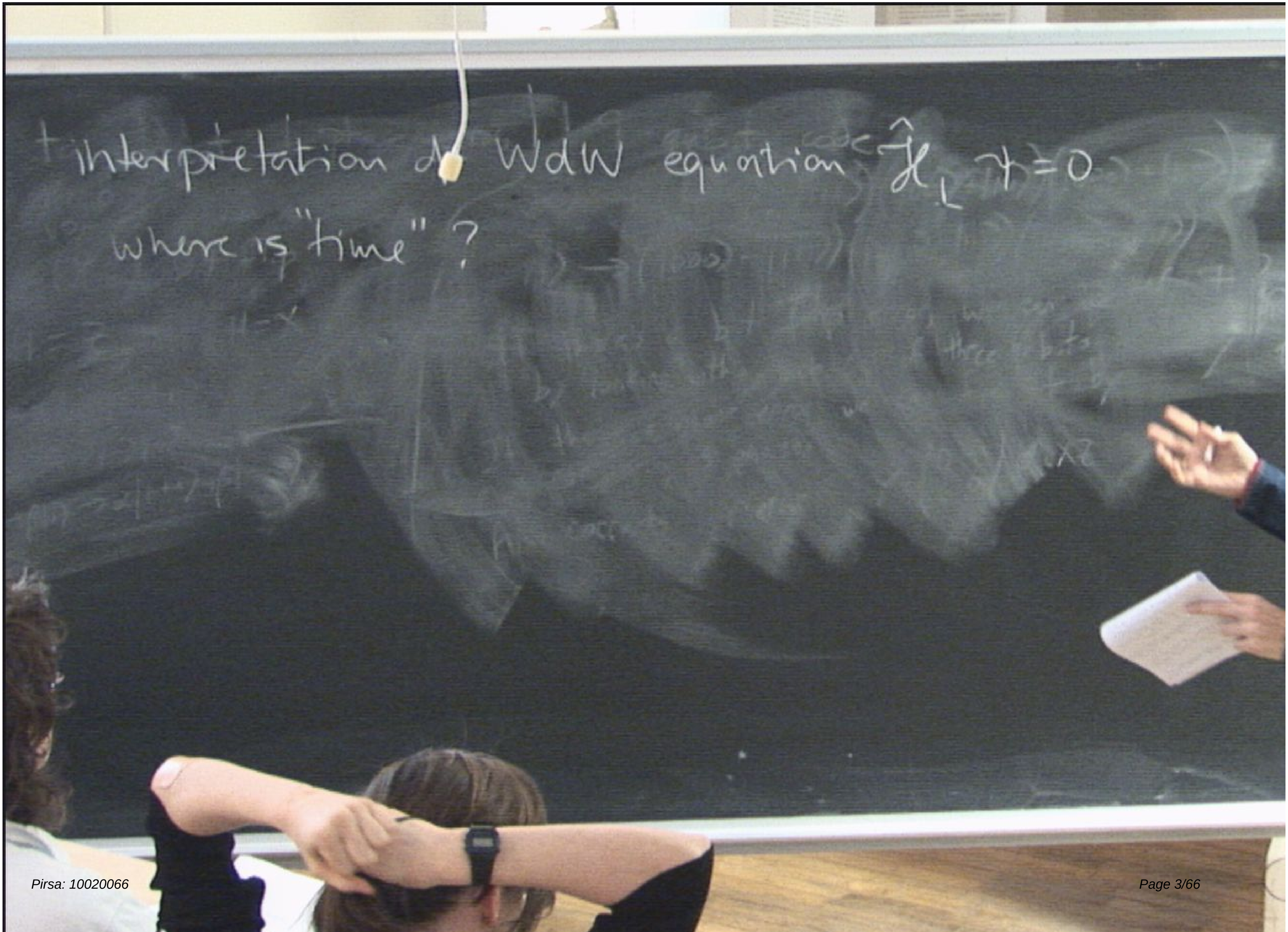
Title: Quantum Gravity - Review (PHYS 638) - Lecture 12

Date: Feb 09, 2010 10:00 AM

URL: http://pirsa.org/10020066

Abstract:


interpretation of WdW equation $\hat{\mathcal{H}}_L \psi = 0$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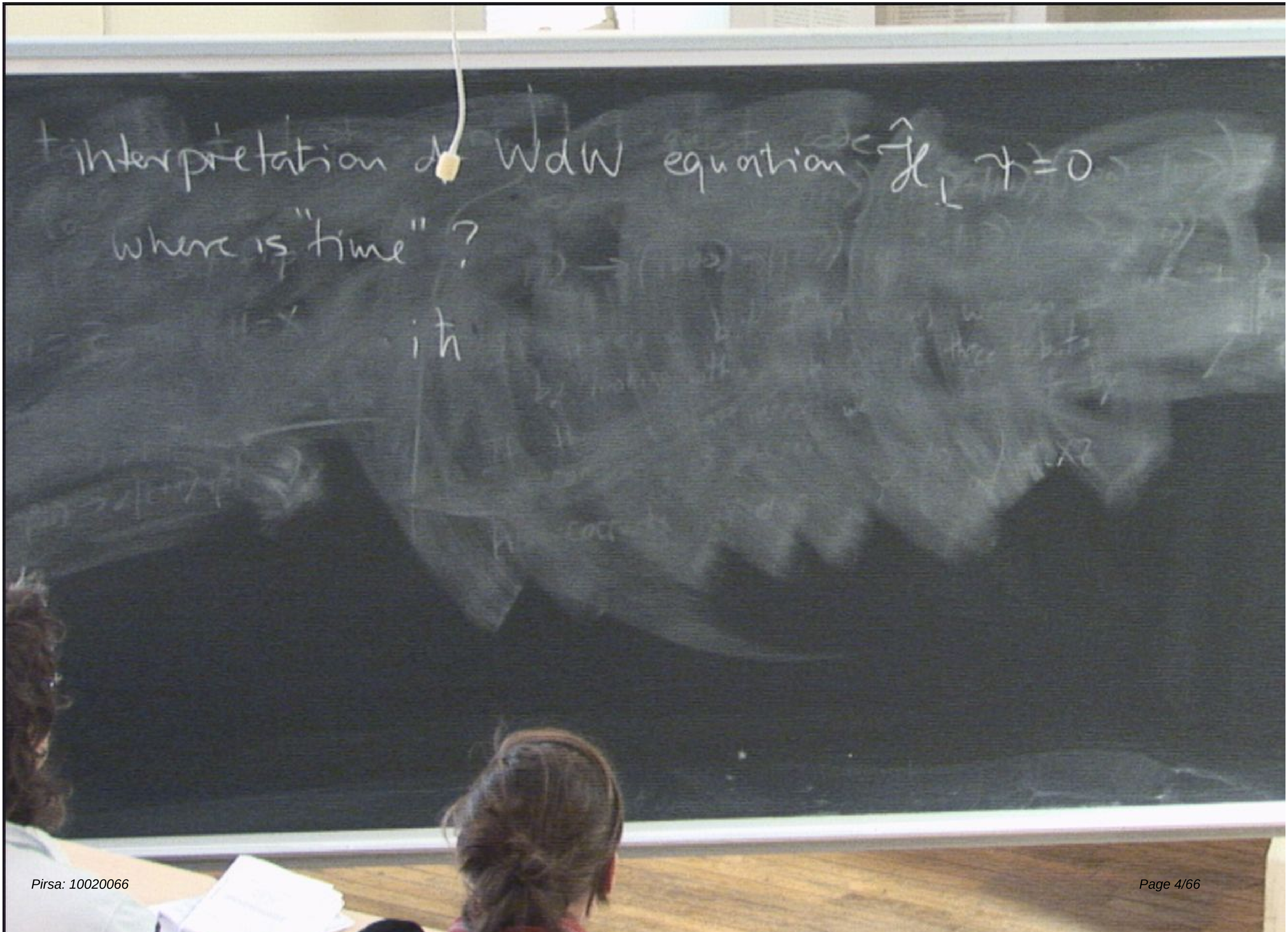
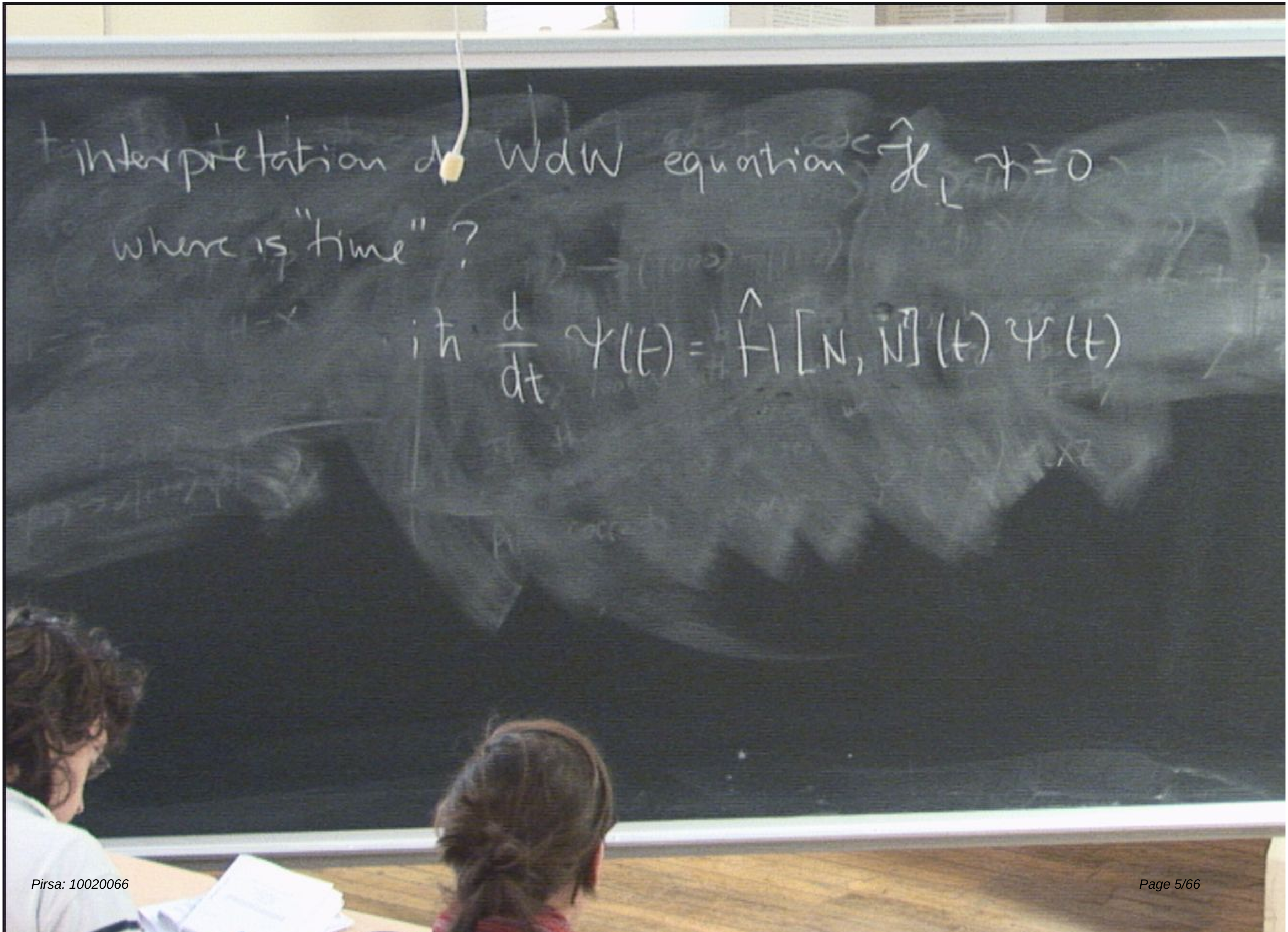
where is "time" ?


interpretation of WdW equation $\hat{\mathcal{H}}_\perp \Psi = 0$
where is "time"?
$i\hbar$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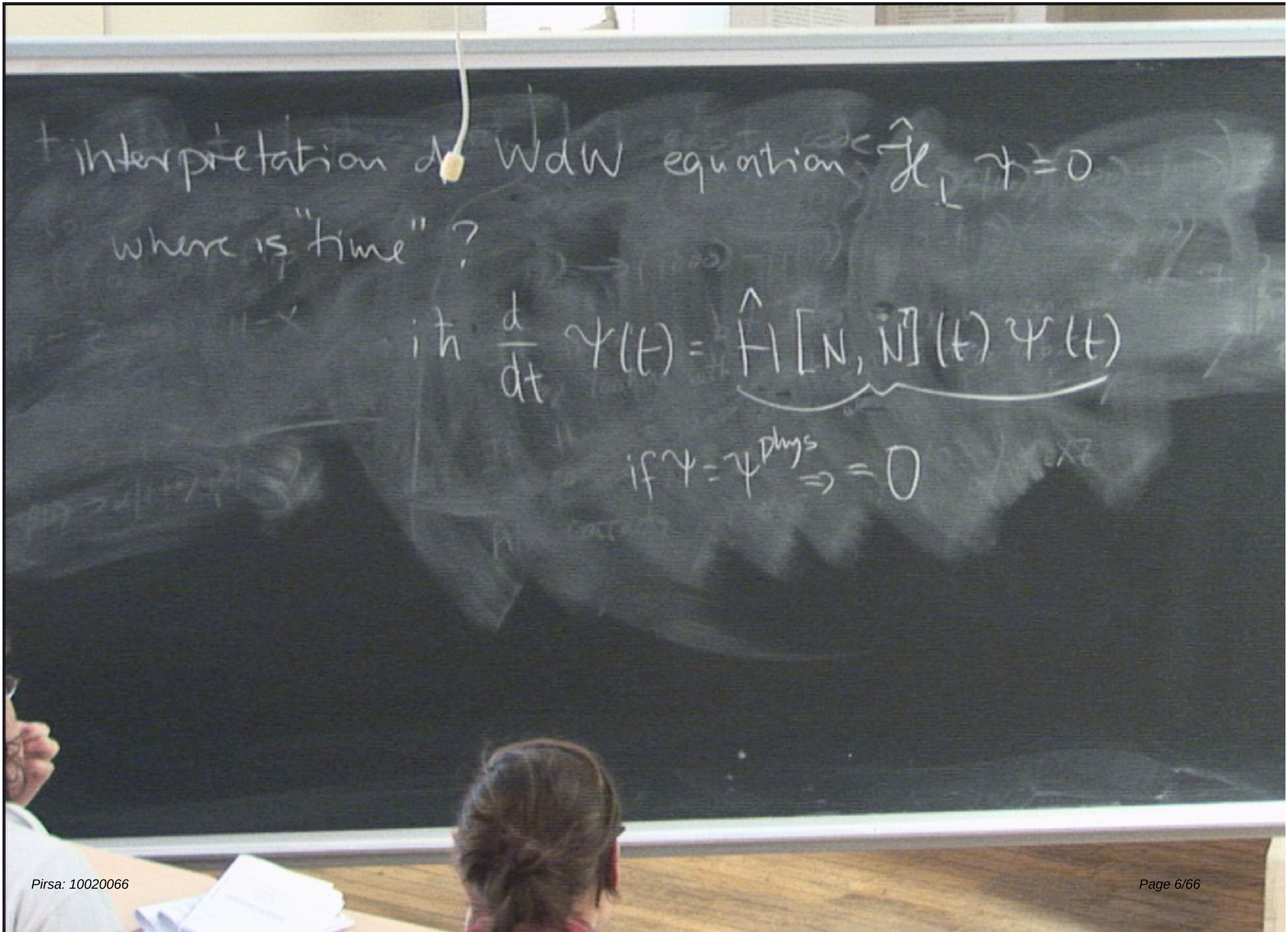
$i\hbar \frac{d}{dt} \Psi(t) = \hat{H}[N, \vec{N}](t)\, \Psi(t)$



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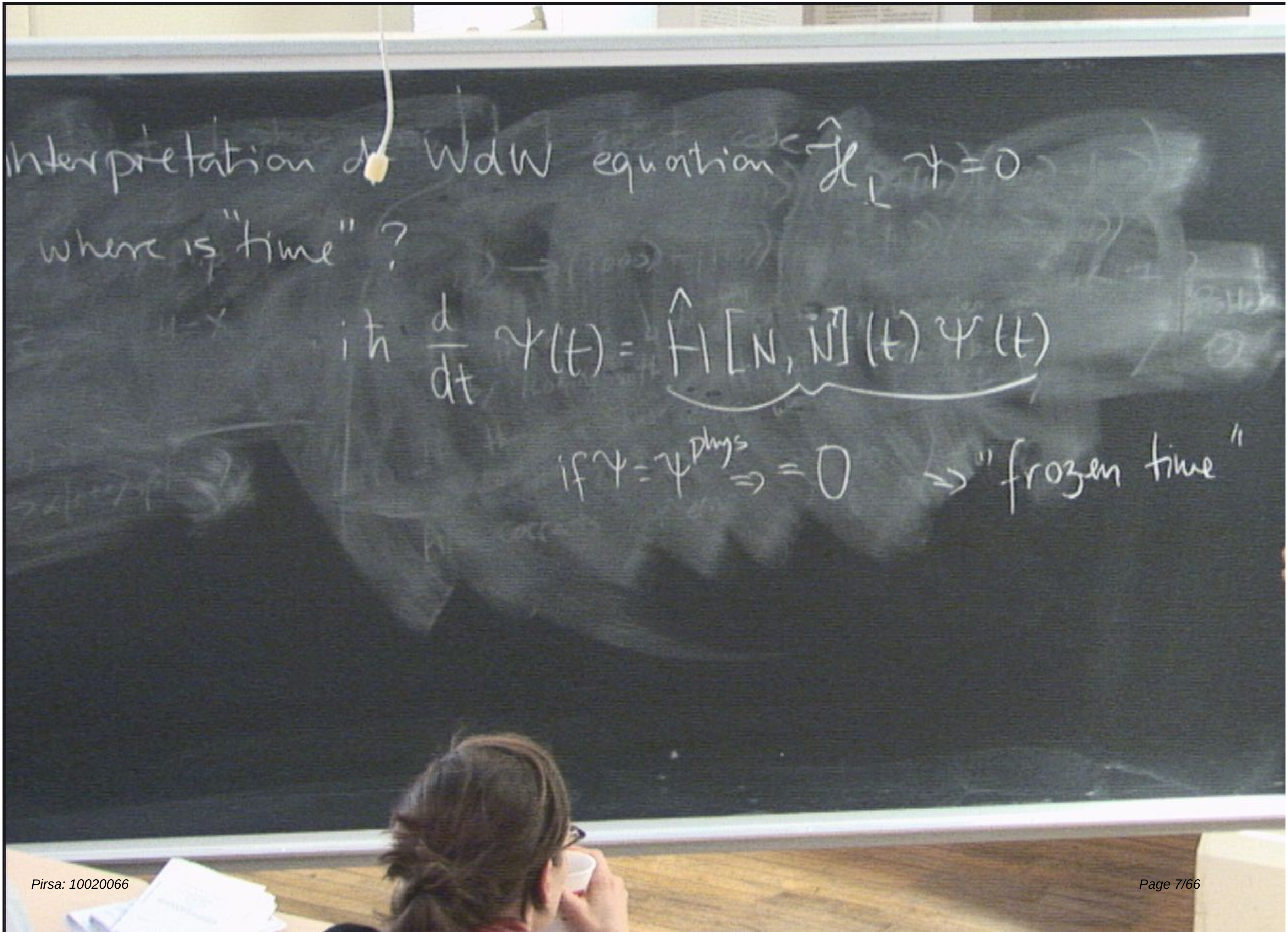
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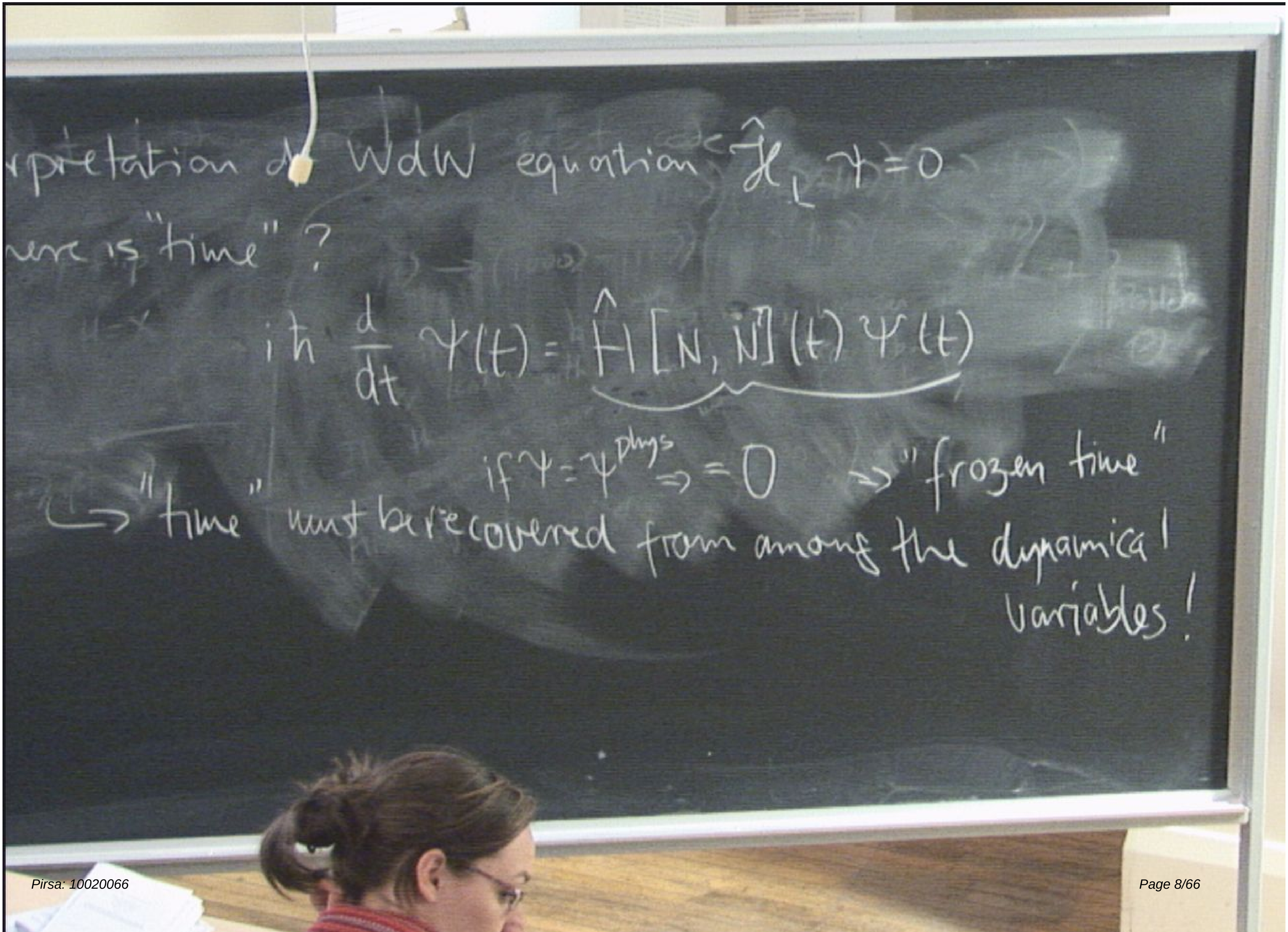
if $\Psi = \Psi^{phys} \Rightarrow = 0$ $\Rightarrow$ "frozen time"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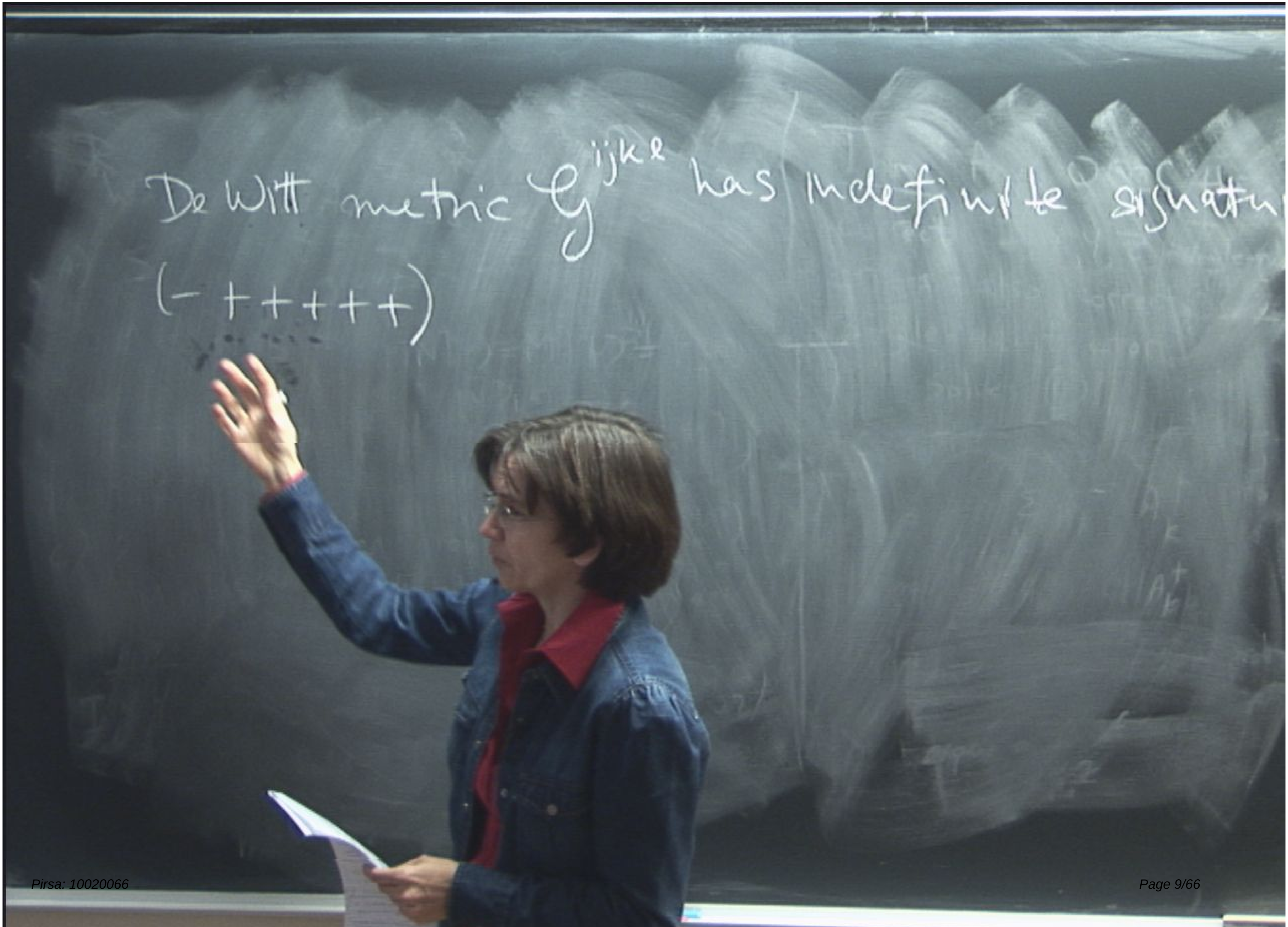
here is "time"?
$i\hbar \frac{d}{dt} \Psi(t) = \underbrace{\hat{H}[N, \vec{N}](t)} \Psi(t)$
if $\Psi = \Psi^{phys} \Rightarrow = 0$ ⇒ "frozen time"
↳ "time" must be recovered from among the dynamical variables!


De Witt metric G^ijkl has indefinite signatu
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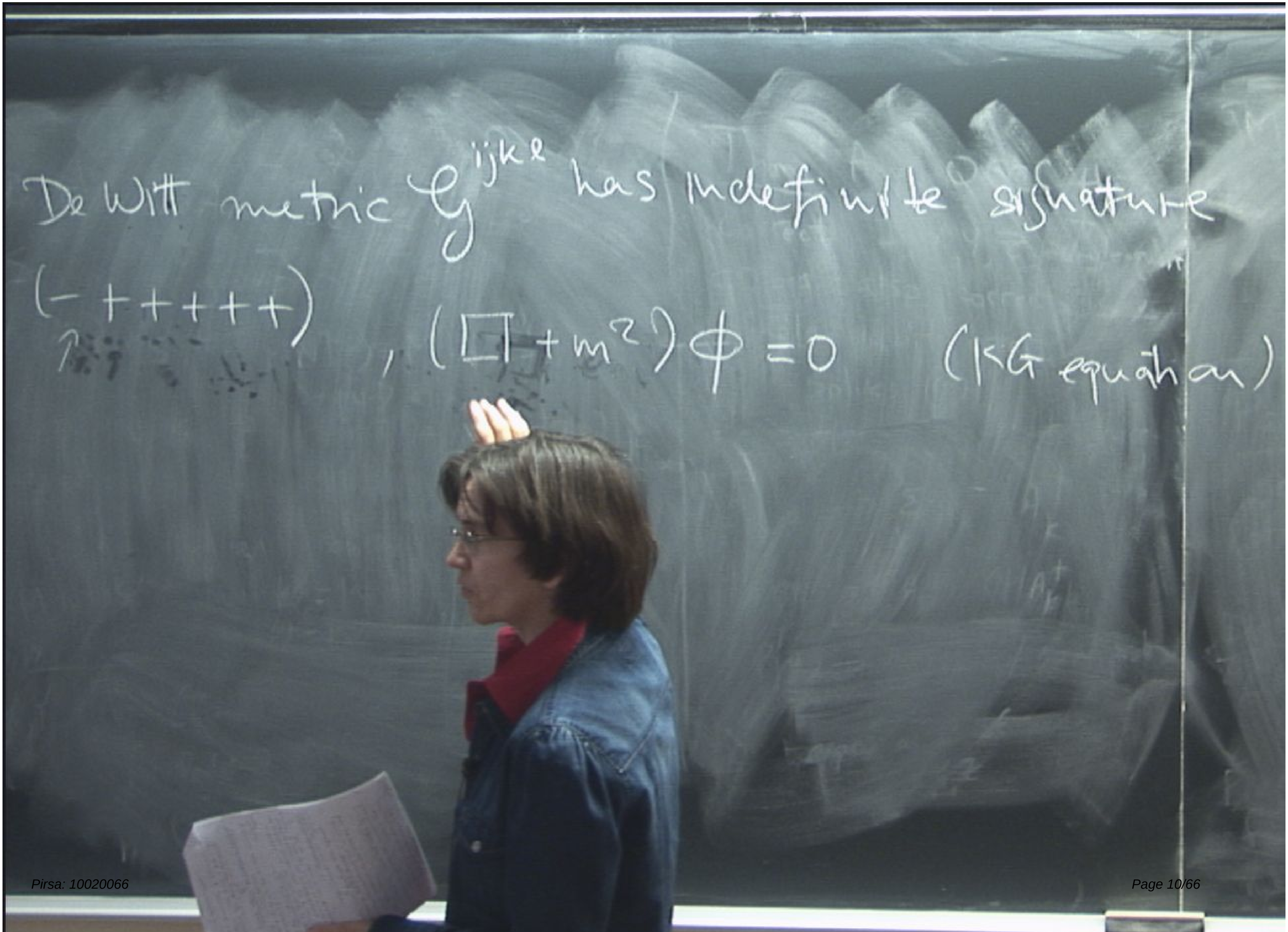
De Witt metric $G^{ijkl}$ has indefinite signature
$(-+++++)$
$, \; (\Box + m^2)\phi = 0$ (KG equation)

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Isham, gr/qc-9210011

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Isham, gr/qc - 9210011

$\Rightarrow$ stuck?

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Isham, gr/qc-9210011

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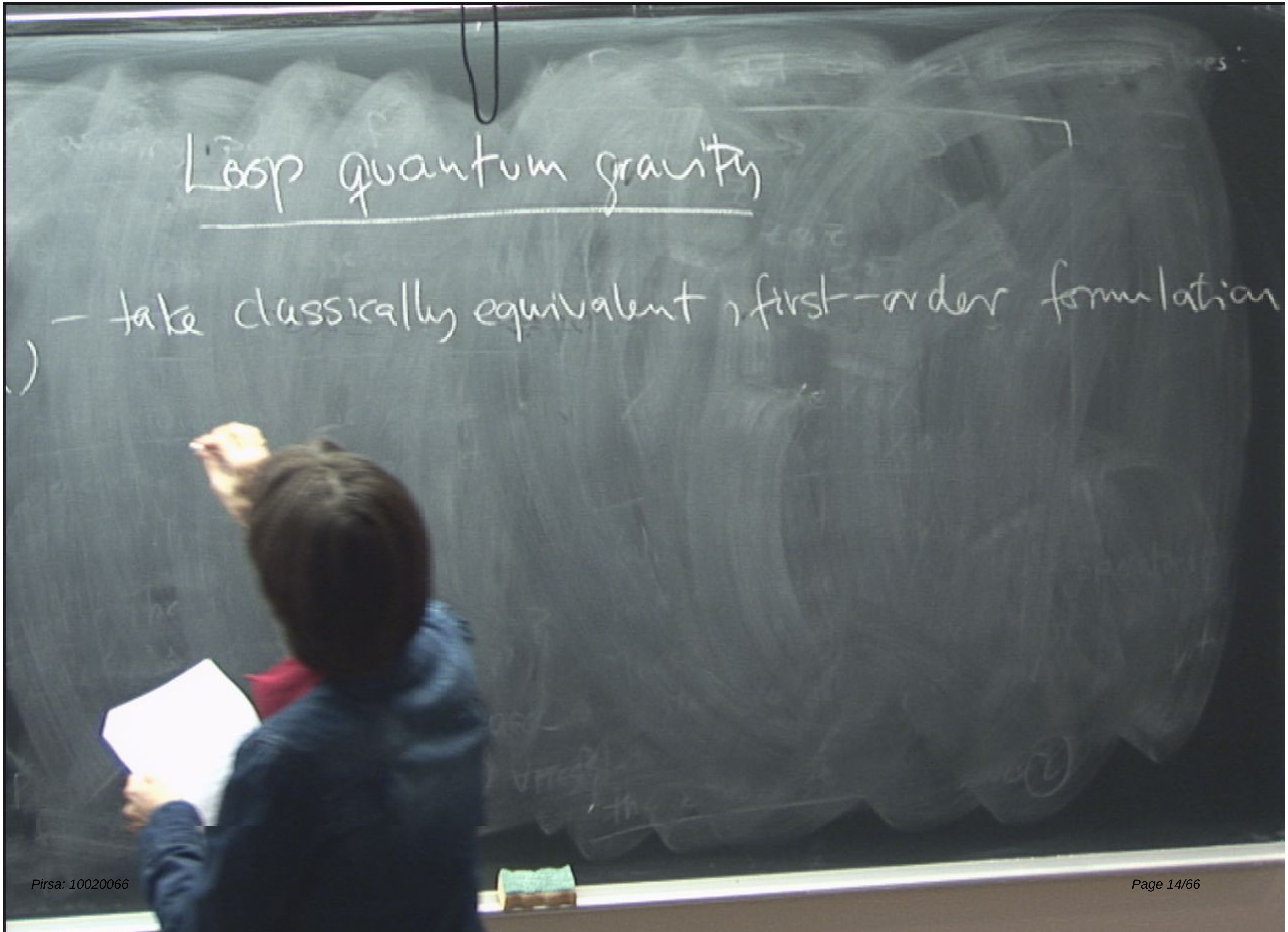
"new connection variables" (Ashtekar 1986)


Loop quantum gravity
– take classically equivalent, first-order formulation

# Loop quantum gravity

- take classically equivalent, first-order formulation of GR in terms of orthonormal frame (tetrad) fields $e_\mu{}^A(x)$, $\mu = 0,\dots,3$, $A = 0,\dots,3$ internal $so(3,1)$-index; and a spin connection $\omega_\mu{}^A{}_B(x)$, considered as independent variables.

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$$g_{\mu\nu} = \eta_{AB}\, e_\mu{}^A e_\nu{}^B$$

internal Minkowski metric

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internal Minkowski metric

perform a 3+1 split and partially fix the gauge (s.t. $so(3,1)$

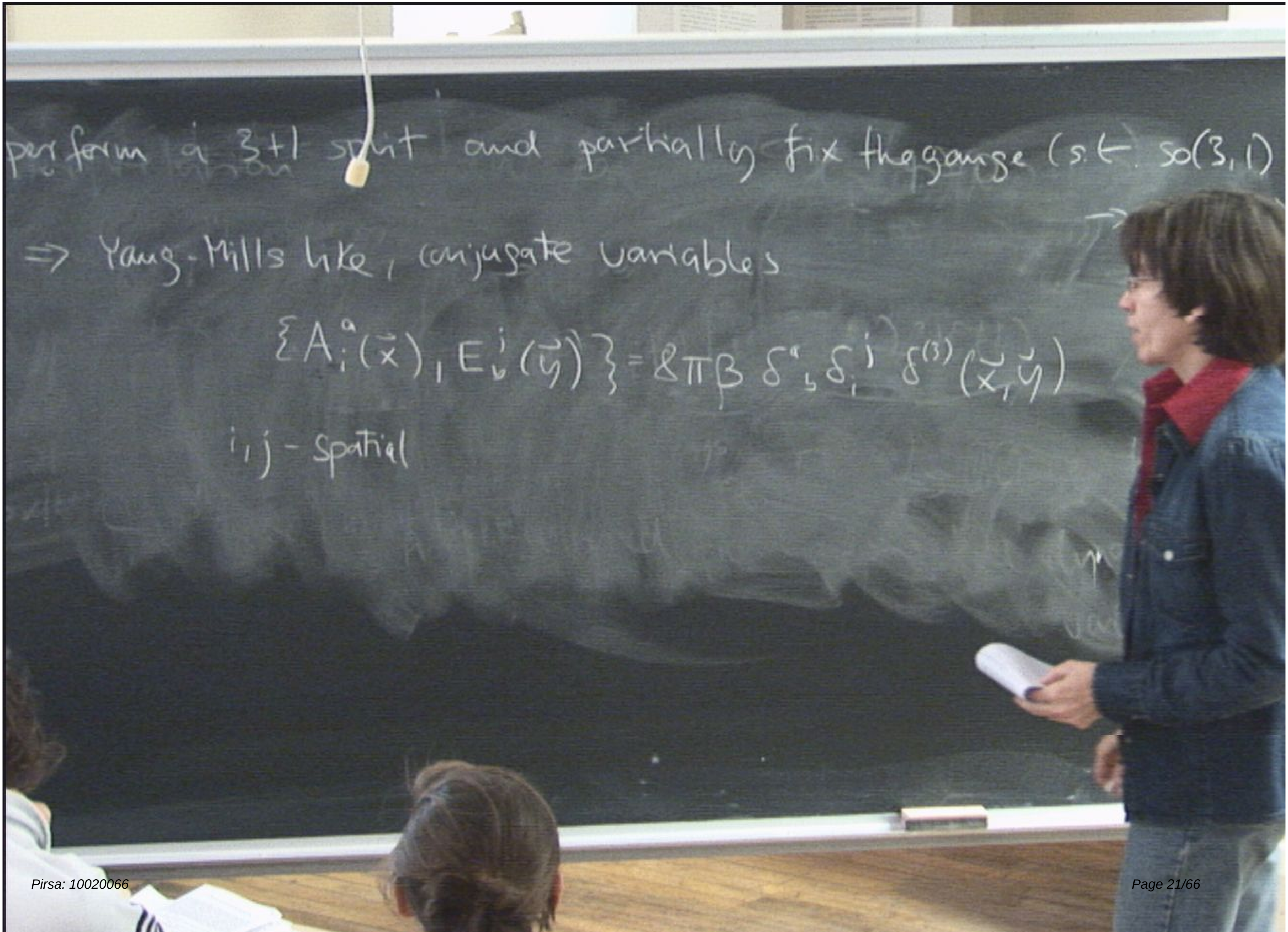
$\rightarrow so(3)$

$\Rightarrow$

perform a 3+1 split and partially fix the gauge (s.t. so(3,1) $\to$ so(3))

$\Rightarrow$ Yang-Mills like, conjugate variables

$$\{A_i^a(\vec{x}), E_b^j(\vec{y})\} = 8\pi\beta\, \delta^a_b\, \delta_i^j\, \delta^{(3)}(\vec{x}, \vec{y})$$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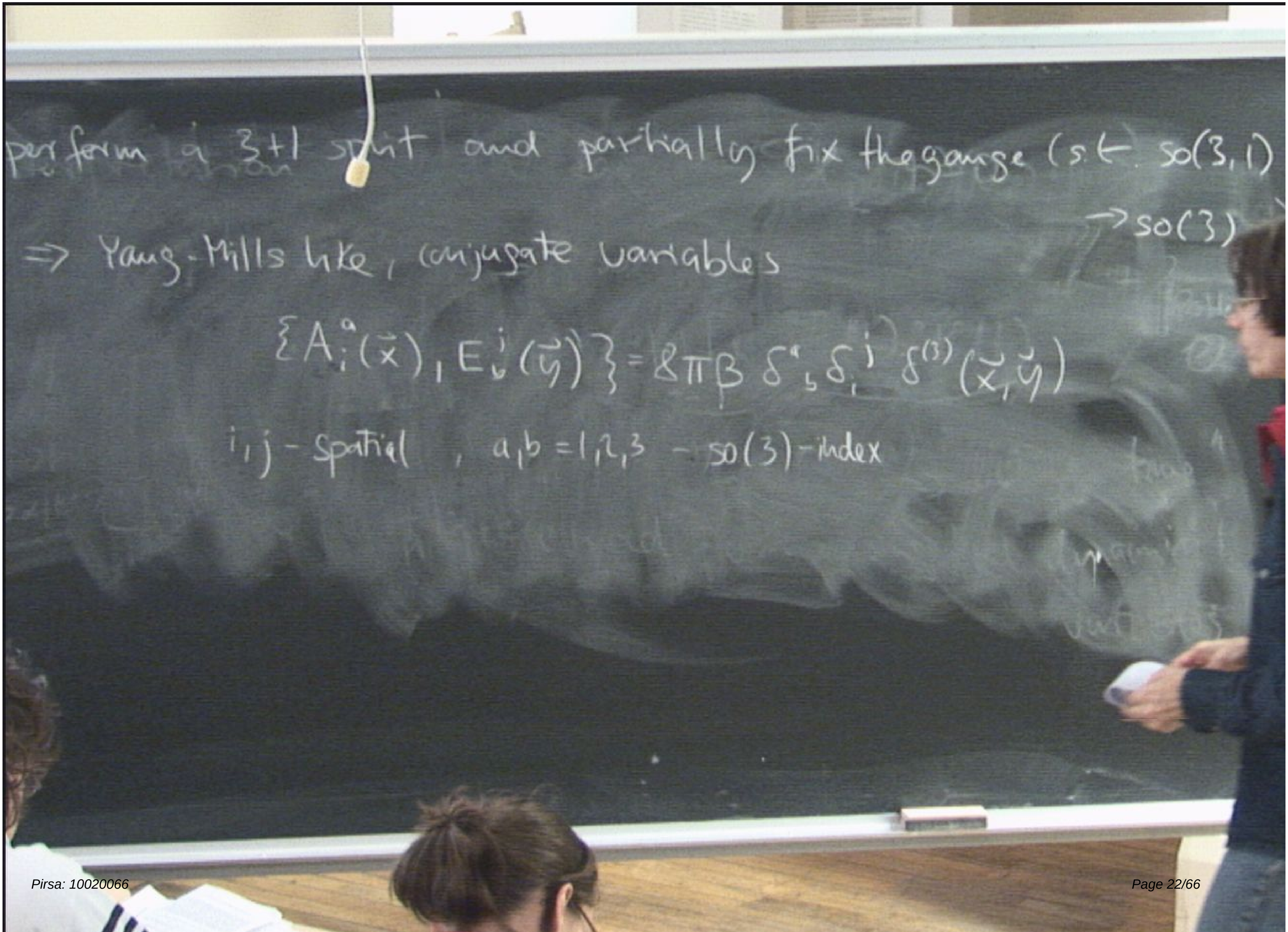
i, j - spatial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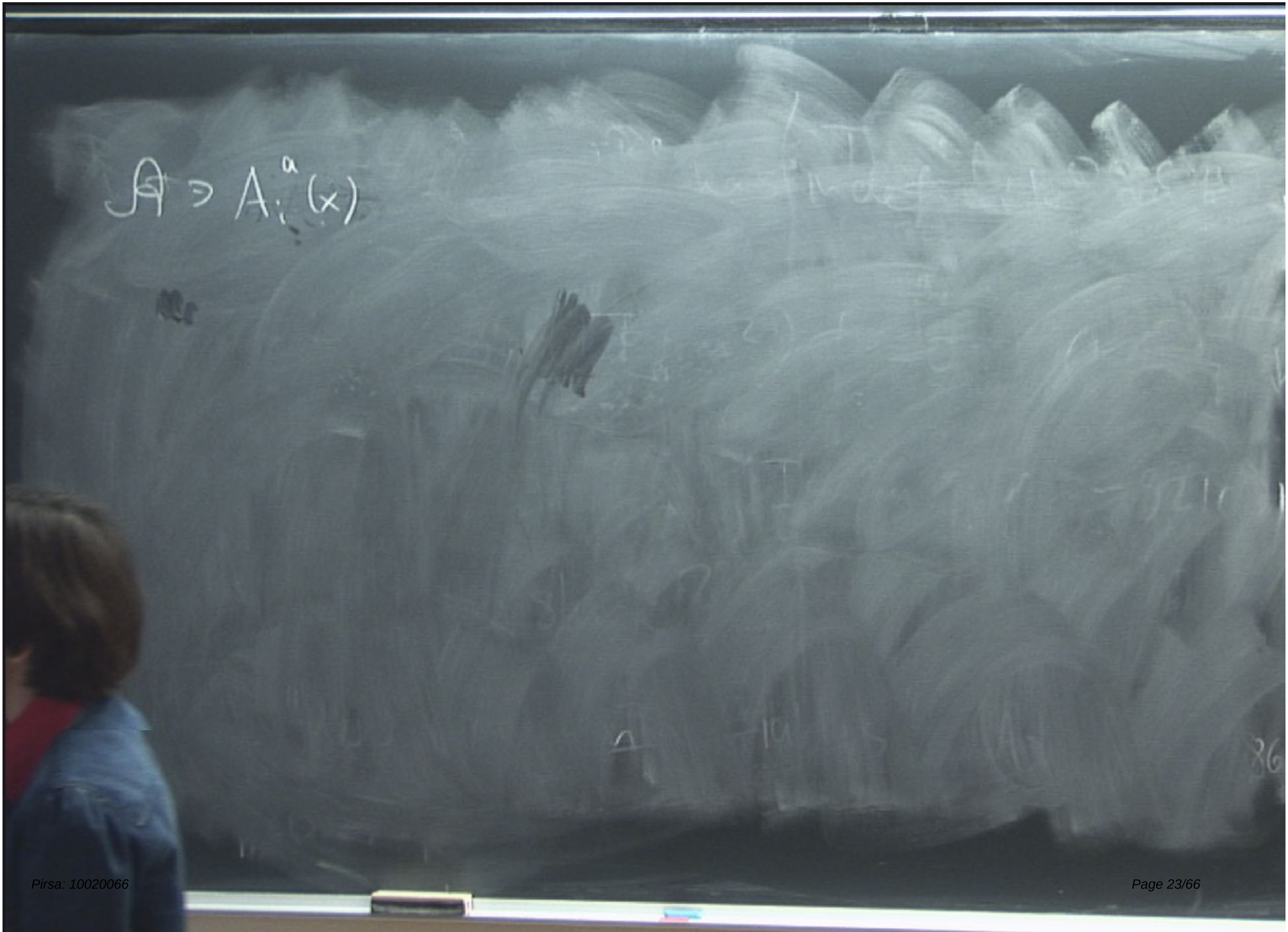
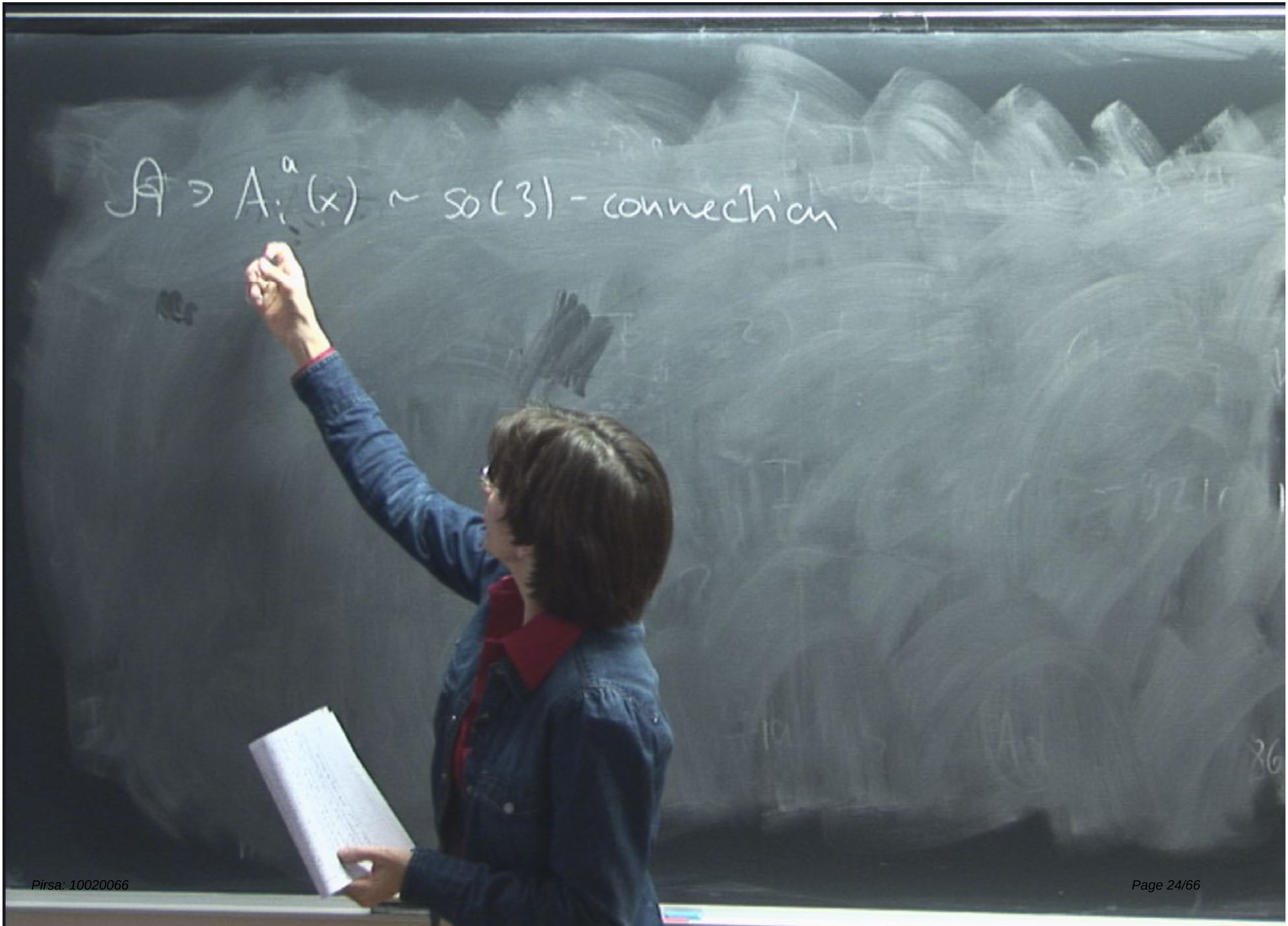
i, j - spatial , a, b = 1,2,3 - so(3)-index


$\mathcal{A} \ni A_i^a(x)$


$\mathcal{A} \ni A_i^a(x) \sim so(3)$ - connection

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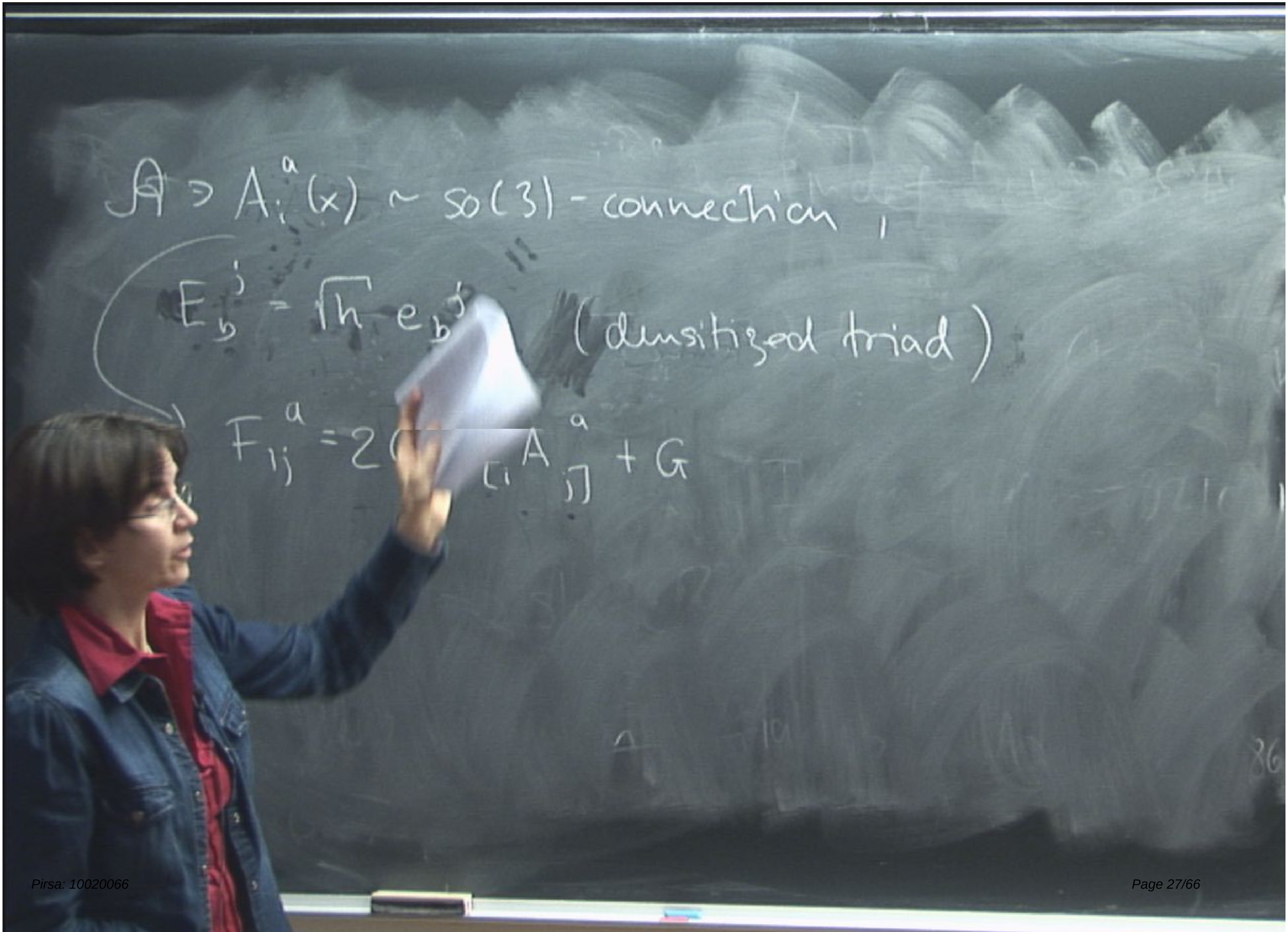
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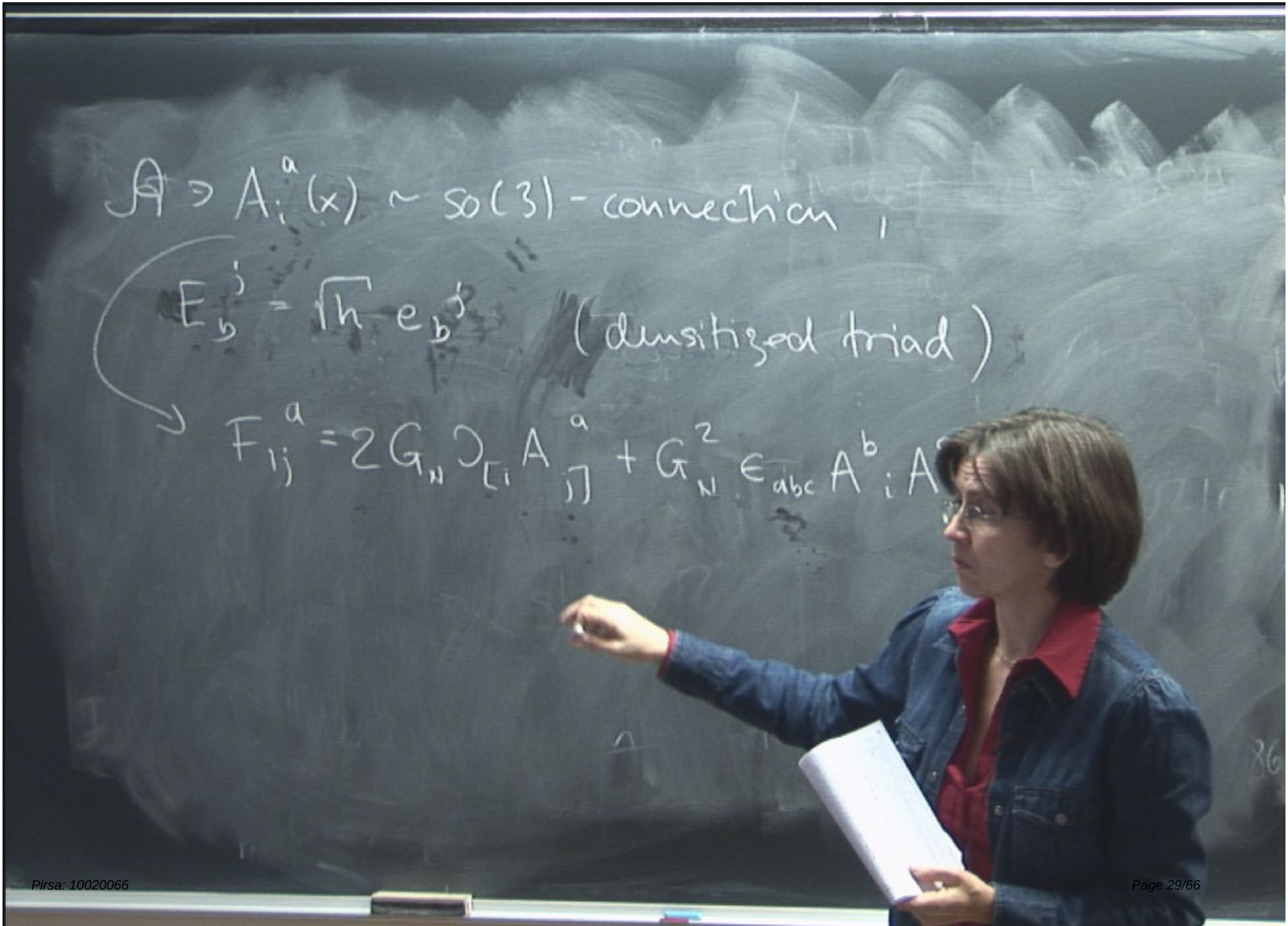
$$F_{ij}^a = 2 G_N \partial_{[i} A_{j]}^a + G_N^2 \epsilon_{abc} A^b_i A^c_j$$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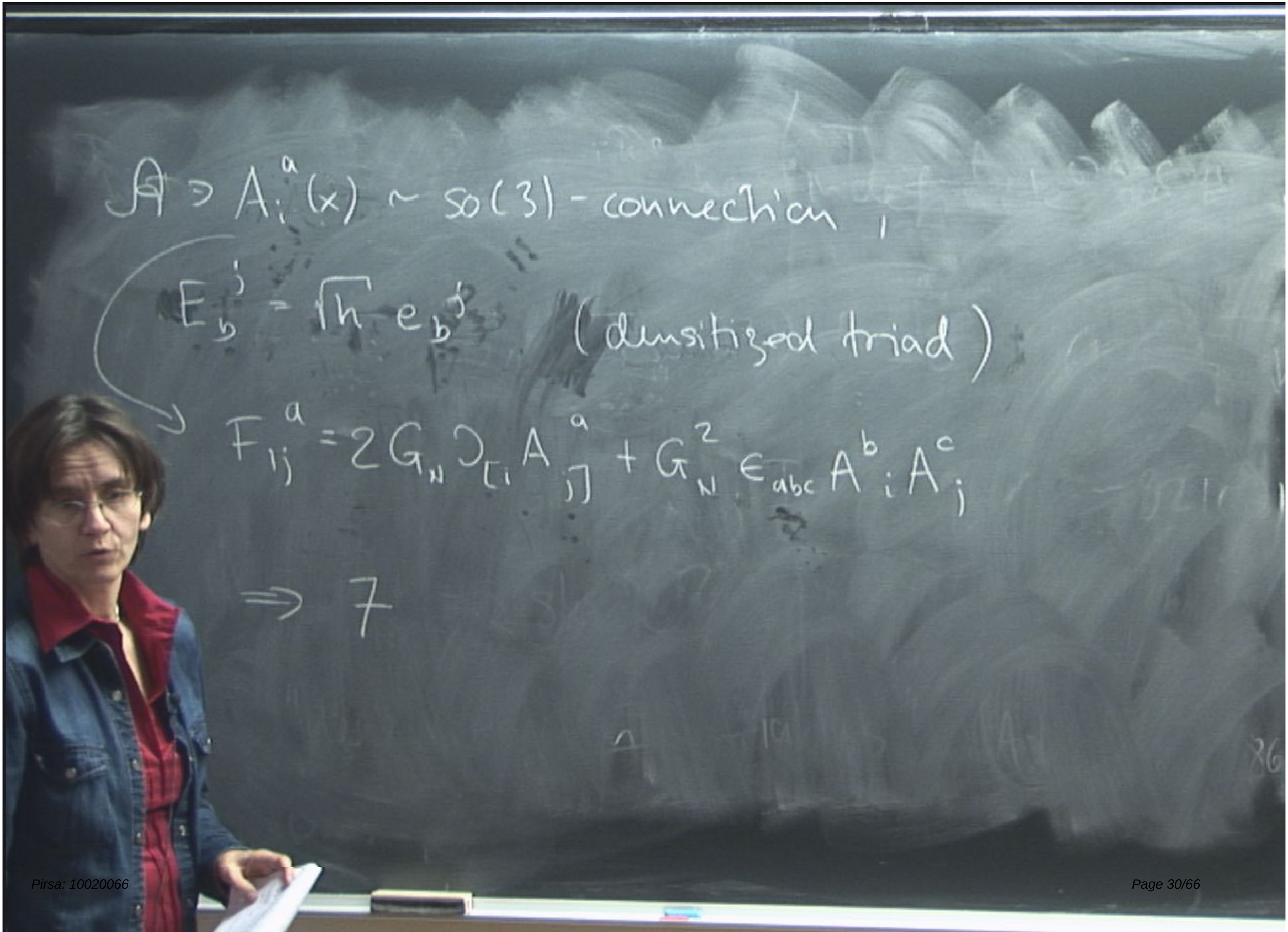
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$\Rightarrow$ 7

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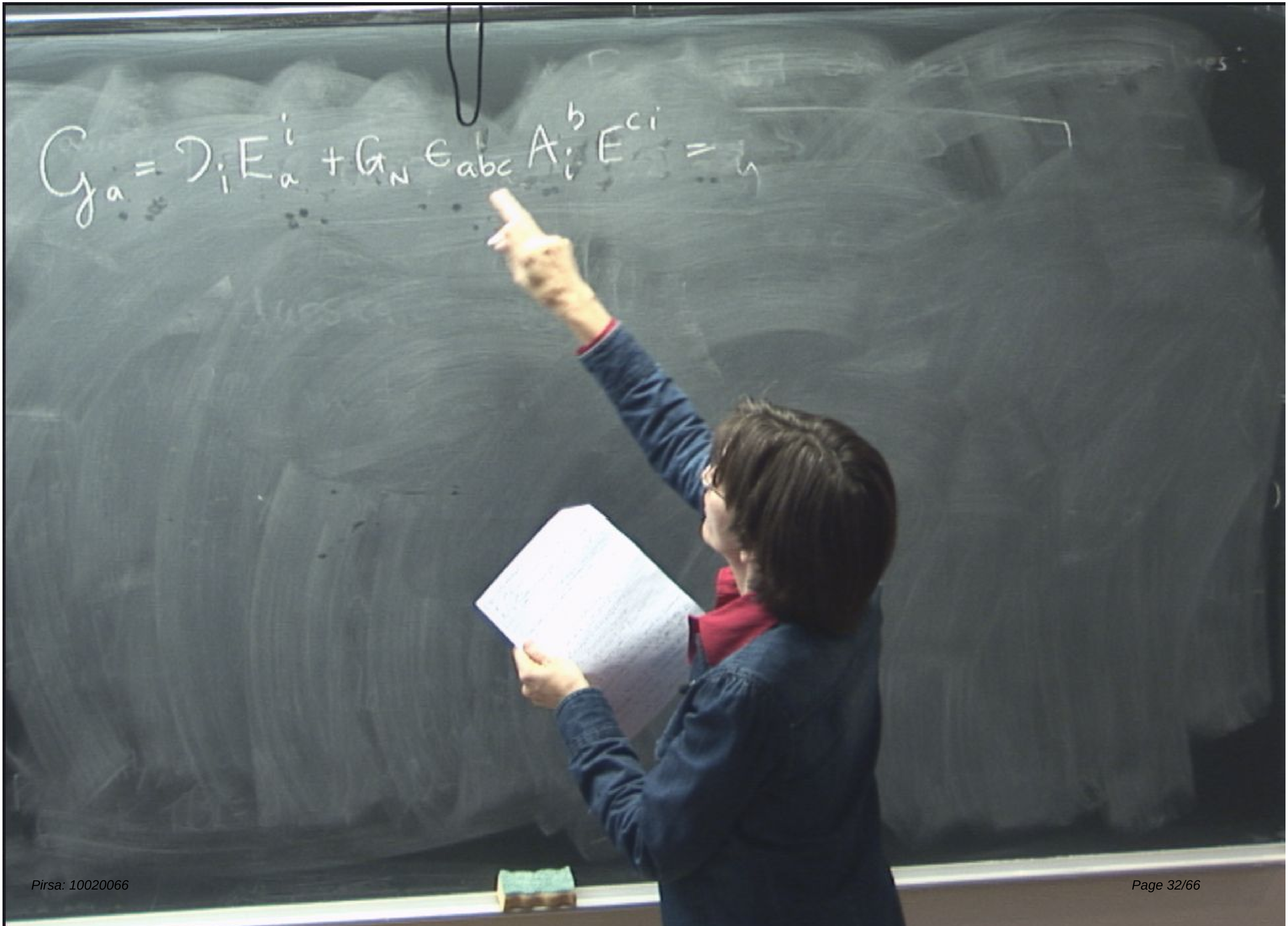
$\Rightarrow 7 \times \infty^3$ first-class constraints


$G_a = D_i E^i_a + G_N \epsilon_{abc} A^b_i E^{ci} =$

$$\mathcal{G}_a = D_i E^i_a + G_N \epsilon_{abc} A^b_i E^{ci} = \mathcal{D}_i E^i_a = 0$$

"Gauss constraints"

$$G_a = D_i E^i_a + G_N \epsilon_{abc} A^b_i E^{ci} = \mathcal{D}_i E^i_a = 0$$ "Gauss constraints"

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$$\mathcal{H}_\perp = \frac{1}{2} \frac{\epsilon^{abc} F_{ijc}}{\sqrt{|\det E^i_a|}} E^i_a E^j_b - \frac{\beta^2+1}{\beta^2 \sqrt{|\det E^a_i|}} E^i_{[a} E^j_{b]} \cdot (G_N A^a_i - \Gamma_i^{\ a}) \cdot (G_N A^b_j - \Gamma_j^{\ b})$$

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$\Gamma = \Gamma(E) \sim so(3)$ - connection

- perform a 3+1 split and partially fix the gauge (s.t. so(3

→ so(3)

⇒ Yang-Mills like, conjugate variables

$$\{A_i^a(\vec{x}), E_b^j(\vec{y})\} = 8\pi\beta\, \delta^a_b\, \delta_i^j\, \delta^{(3)}(\vec{x}, \vec{y})$$

$i, j$ - spatial, $a, b = 1,2,3$ - so(3)-index

$\beta$ — Immirzi-Barbero param

$$\mathcal{G}_a = D_i E^i_a + G_N \epsilon_{abc} A^b_i E^{ci} = \mathcal{D}_i E^i_a = 0 \qquad \text{"Gauss constraints"}$$

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$\beta = 1$

$\Gamma = \Gamma(E) \sim so(3)$-connection

$$\mathcal{G}_a = \mathcal{D}_i E^i_a + G_N \epsilon_{abc} A^b_i E^{ci} = \mathcal{D}_i E^i_a = 0$$ "Gauss constraints"

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PB algebra closes

$\beta = 1$

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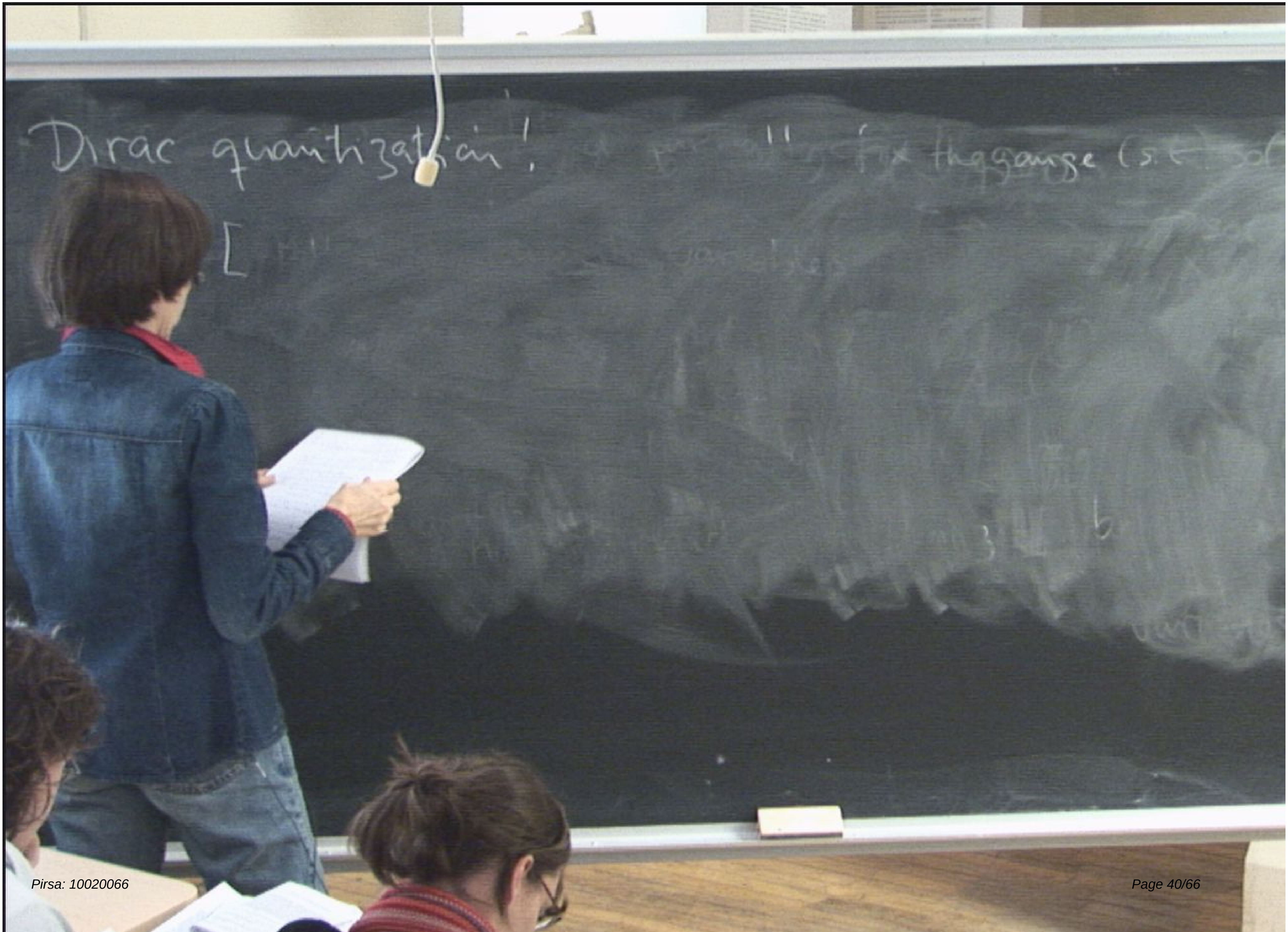
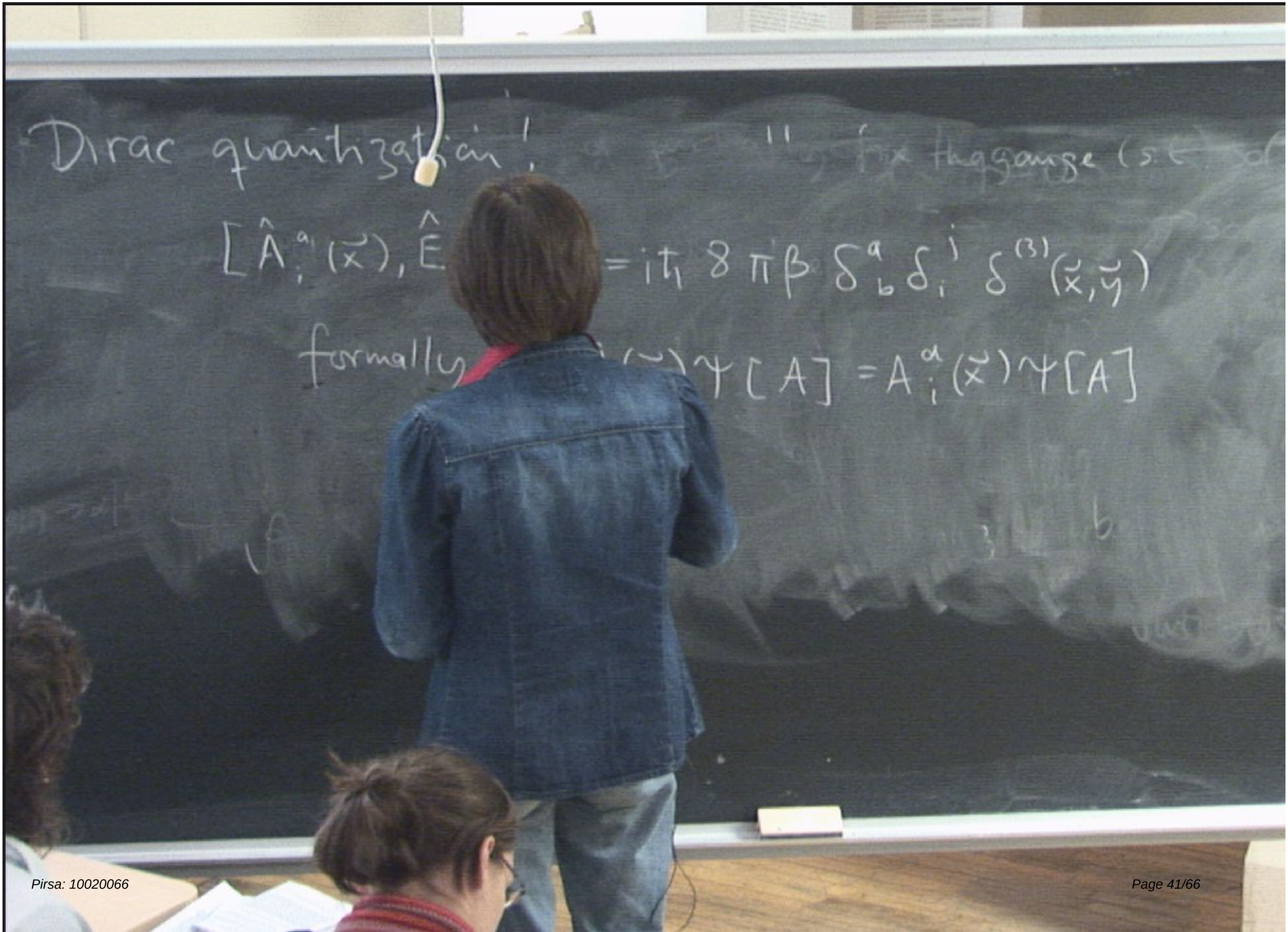
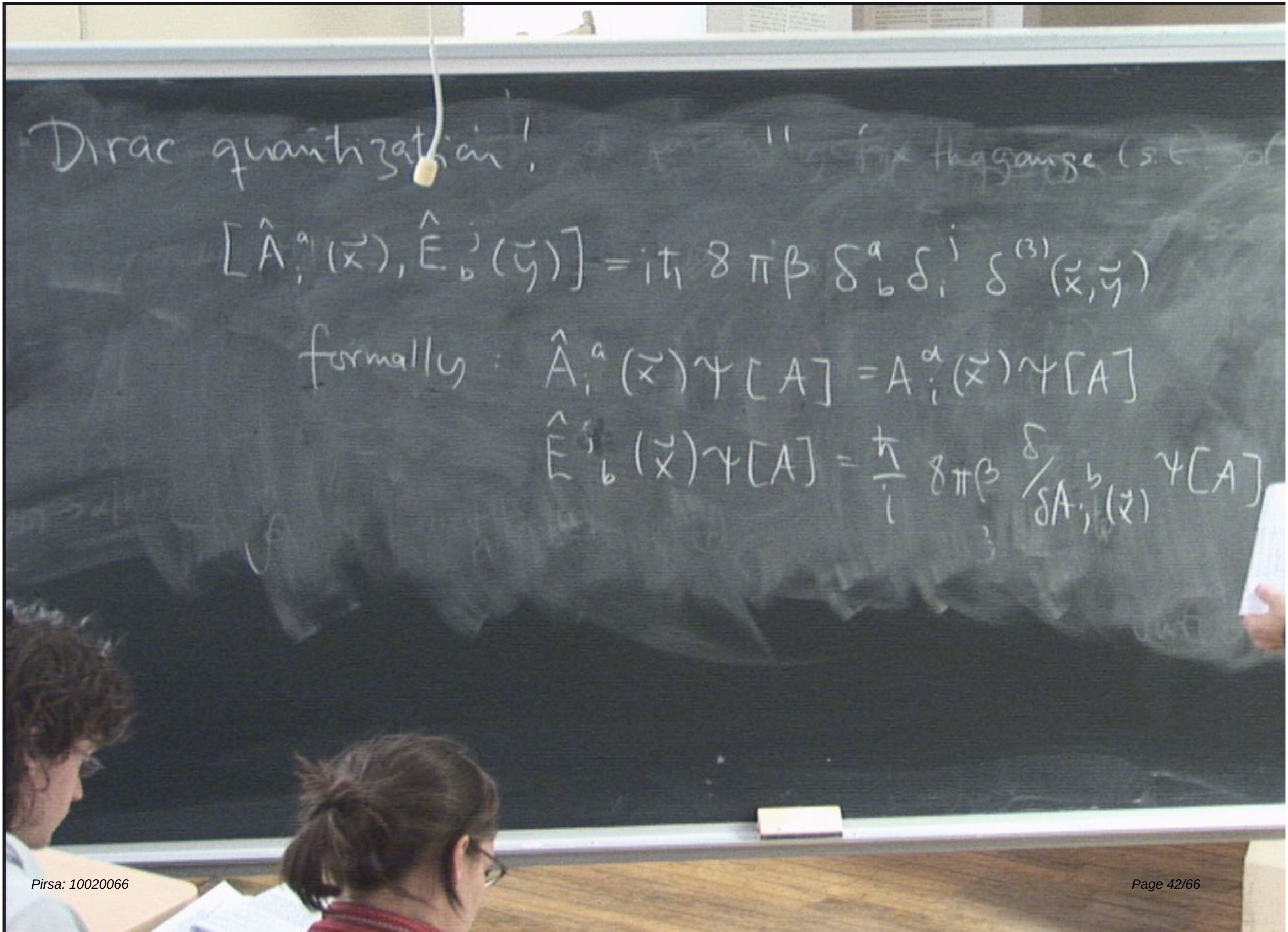

Dirac quantization!


Dirac quantization!
fix the gauge (s.t
$[\hat{A}^a_i(\vec{x}), \hat{E}$
$= i\hbar\, 8\pi\beta\, \delta^a_b\, \delta_i^j\, \delta^{(3)}(\vec{x},\vec{y})$
formally
$\Psi[A] = A^a_i(\vec{x})\Psi[A]$


Dirac quantization!
$[\hat{A}_i^a(\vec{x}), \hat{E}_b^j(\vec{y})] = i\hbar\, 8\pi\beta\, \delta^a_b\, \delta_i^j\, \delta^{(3)}(\vec{x},\vec{y})$
formally: $\hat{A}_i^a(\vec{x})\Psi[A] = A_i^a(\vec{x})\Psi[A]$
$\hat{E}^j_b(\vec{x})\Psi[A] = \frac{\hbar}{i}\, 8\pi\beta \frac{\delta}{\delta A_j^b(\vec{x})}\Psi[A]$

Dirac quantization!

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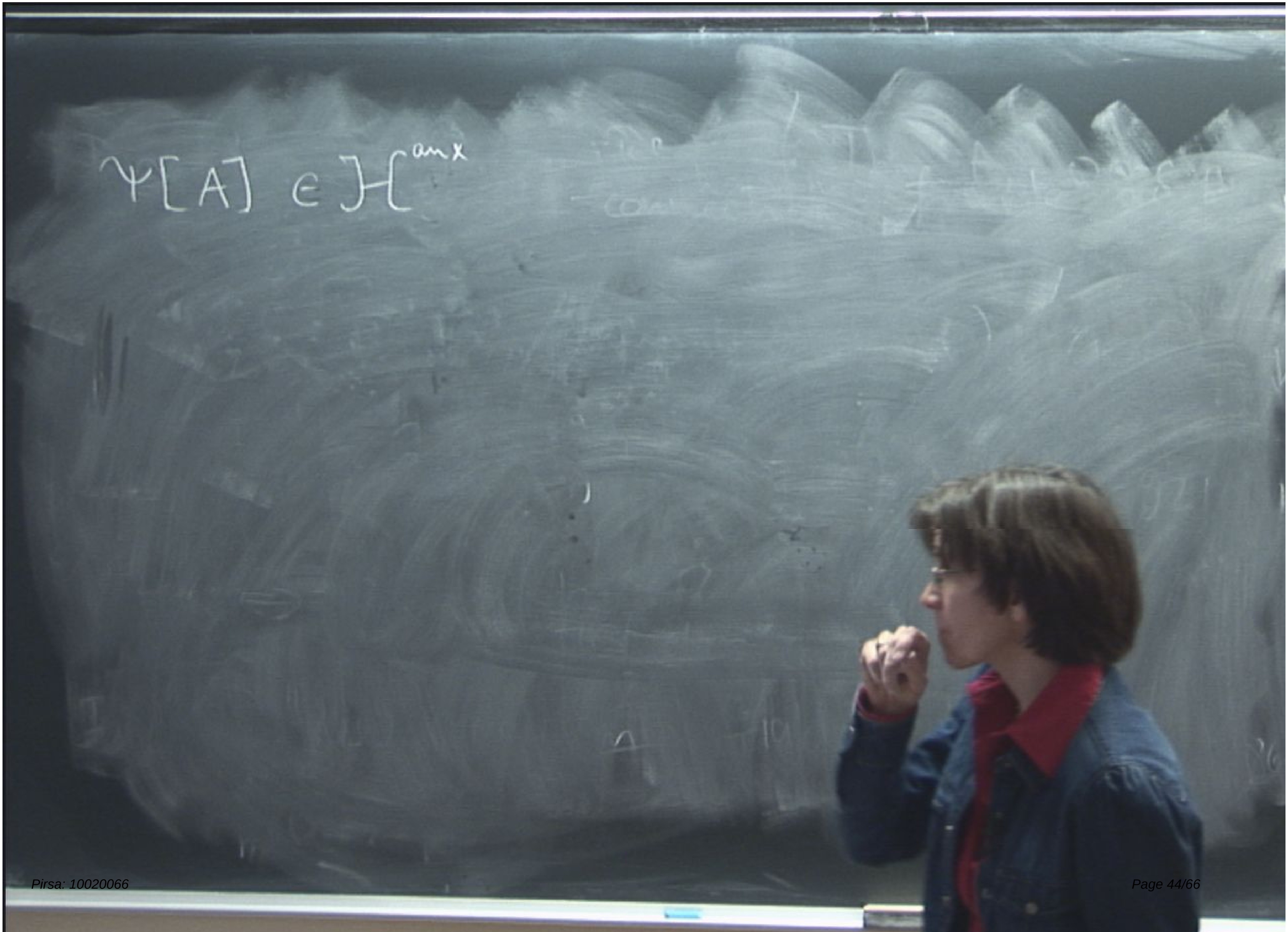
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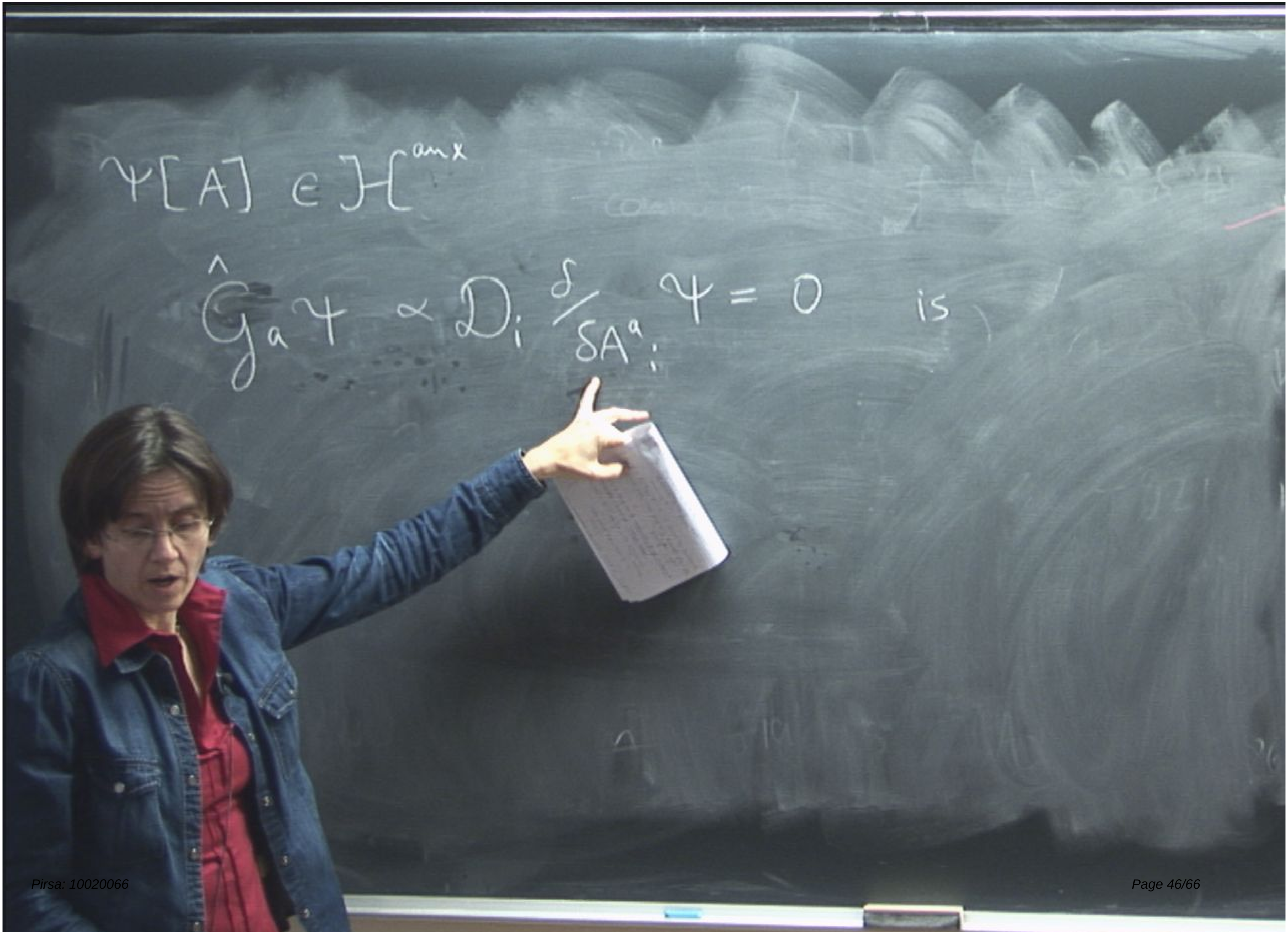
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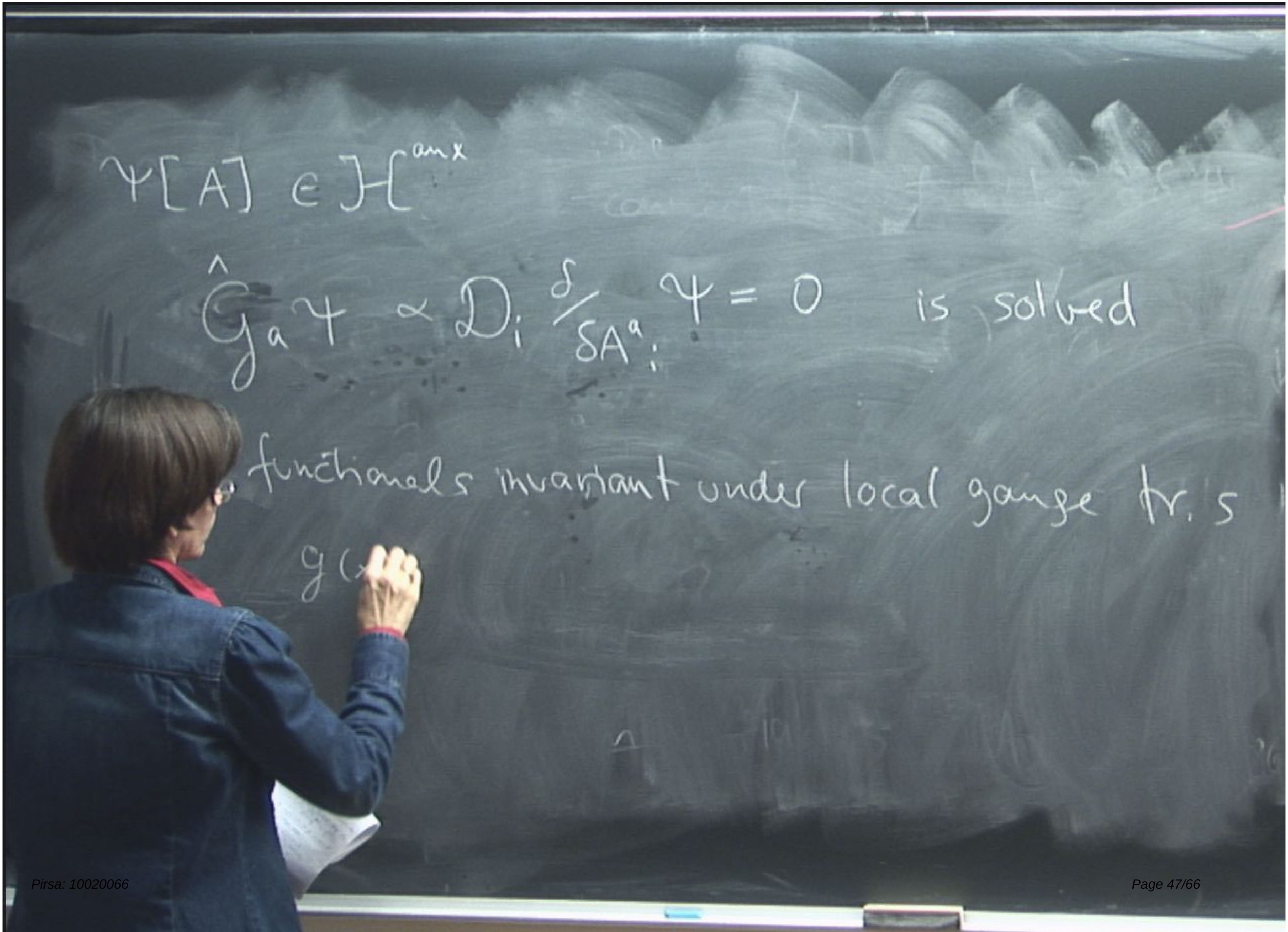
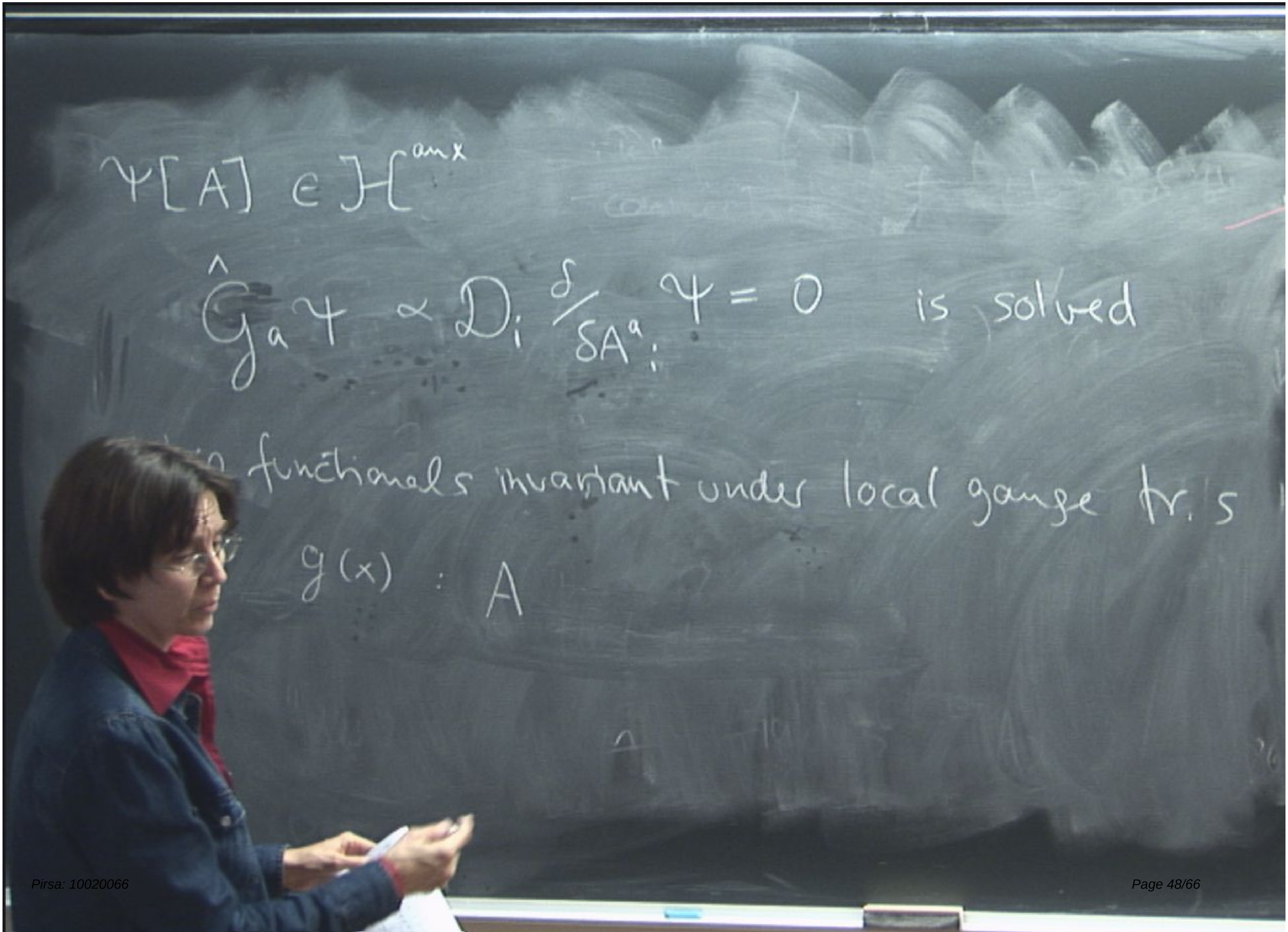
$\hat{G}_a \Psi \propto \mathcal{D}_i \frac{\delta}{\delta A^a_i} \Psi = 0$ is


$\Psi[A] \in \mathcal{H}^{aux}$
$\hat{G}_a \Psi \propto D_i \frac{\delta}{\delta A^a_i} \Psi = 0$ is solved
functionals invariant under local gauge tr.'s
$g(x$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$g(x)$ : $A$

$\Psi[A] \in \mathcal{H}^{\mathrm{aux}}$

$$\hat{\mathcal{G}}_a \Psi \propto \mathcal{D}_i \frac{\delta}{\delta A^a_i} \Psi = 0 \quad \text{is solved}$$

by functionals invariant under local gauge tr.'s

$\mathcal{G}$

$\ni g(x) : \quad A_i \longrightarrow A^g_i = g^{-1} A_i g - g^{-1} \partial_i g$

$\Psi[A] \in \mathcal{H}^{\mathrm{aux}}$

$\hat{\mathcal{G}}_a \Psi \propto \mathcal{D}_i \frac{\delta}{\delta A^a_i} \Psi = 0$ is solved

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$\mathcal{G}^{SO(3)} \ni g(x) : A_i \longrightarrow A_i^g = g^{-1} A_i g - g^{-1} \partial_i g$

$\Psi[A] \in \mathcal{H}^{aux}$

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$\mathcal{G}^{SO(3)} \ni g(x) : A_i \to A_i^g = g^{-1} A_i g - g^{-1} \partial_i g$

$\Psi[A] \in \mathcal{H}^{\text{aux}}$

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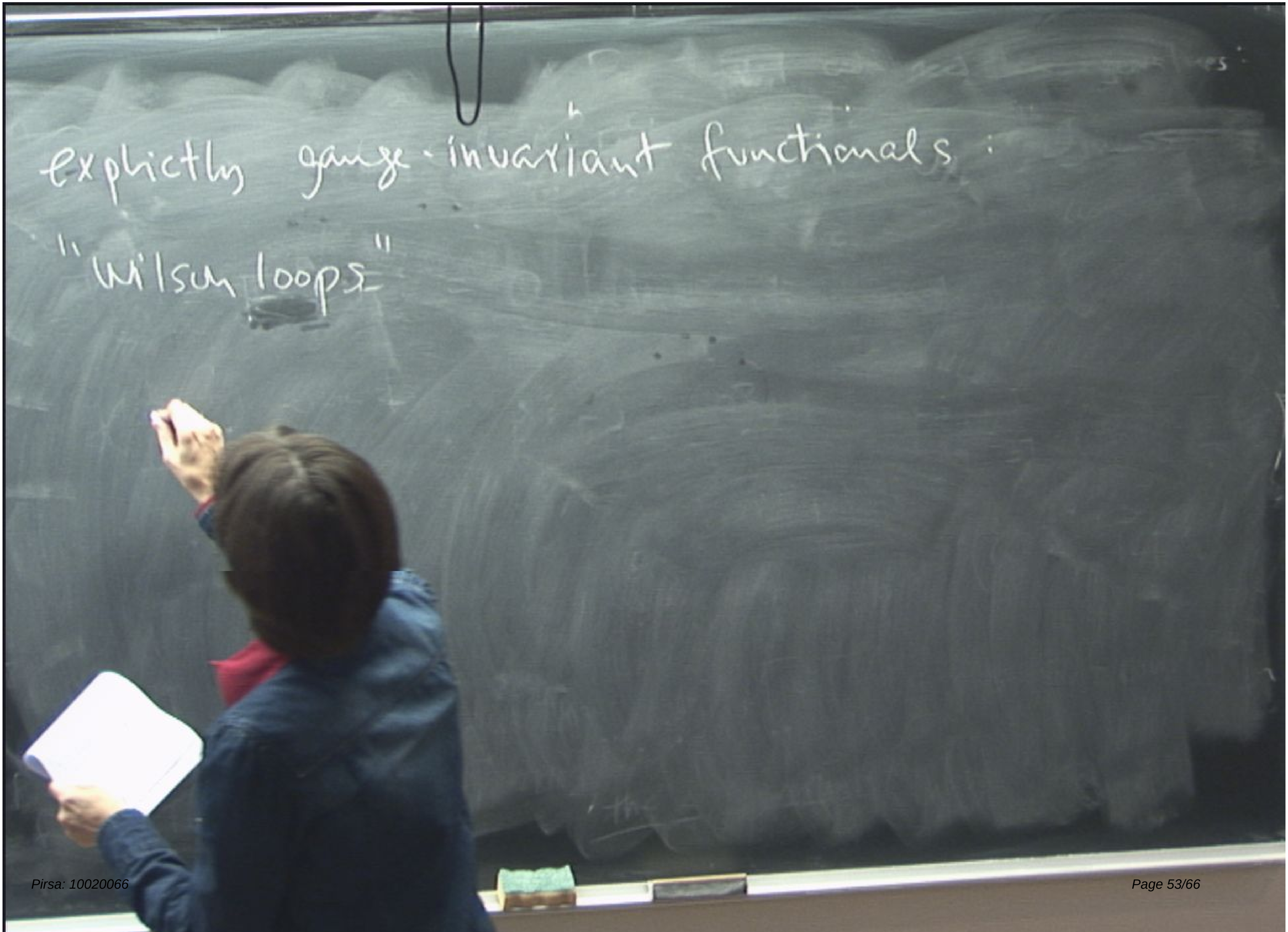
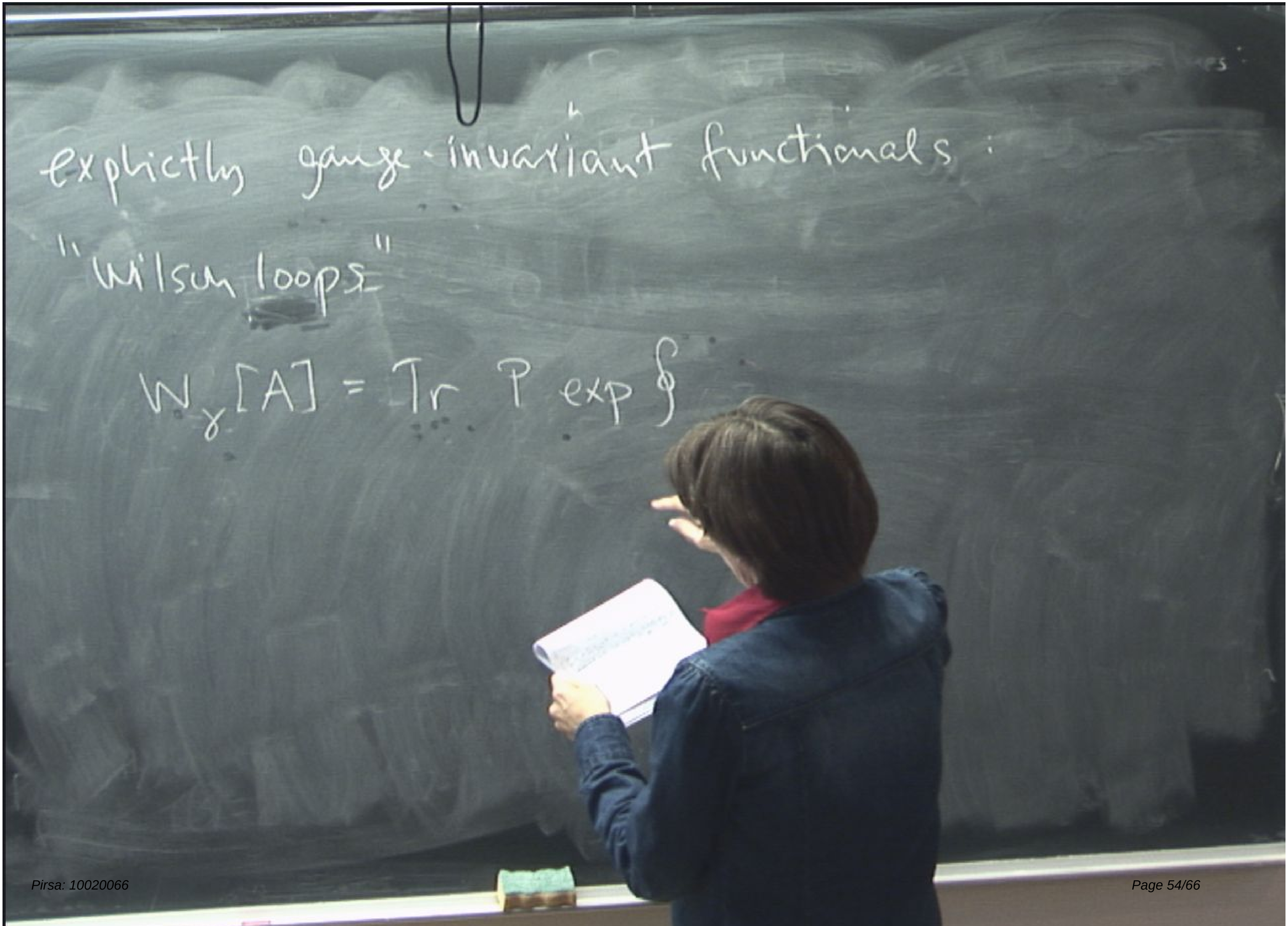
$\mathcal{G}^{SO(3)}$

$\downarrow$

$\mathcal{G}^{SU(2)}$

$\ni g(x) : A_i \longrightarrow A^g_i = g^{-1} A_i g - g^{-1} \partial_i g$


explictly gauge-invariant functionals :
"Wilson loops"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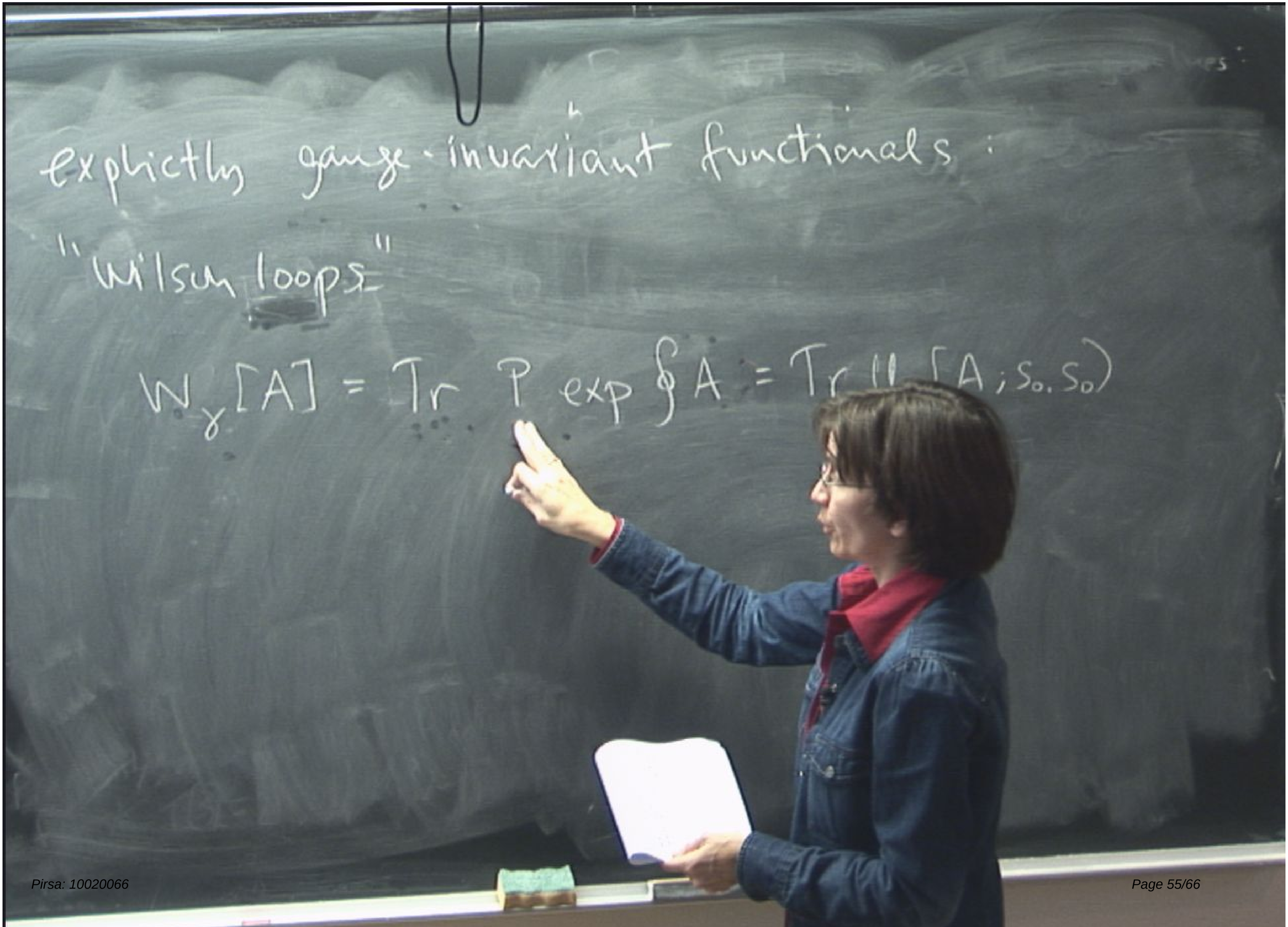
$W_\gamma[A] = \mathrm{Tr}\ P \exp \oint$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explicitly gauge-invariant functionals:

"Wilson loops"

holonomy

$$W_\gamma[A] = \mathrm{Tr}\ P \exp \oint_\gamma A = \mathrm{Tr}\ U_\gamma[A; s_0, s_0)$$

"path-ordered exponential of A along $\gamma$"

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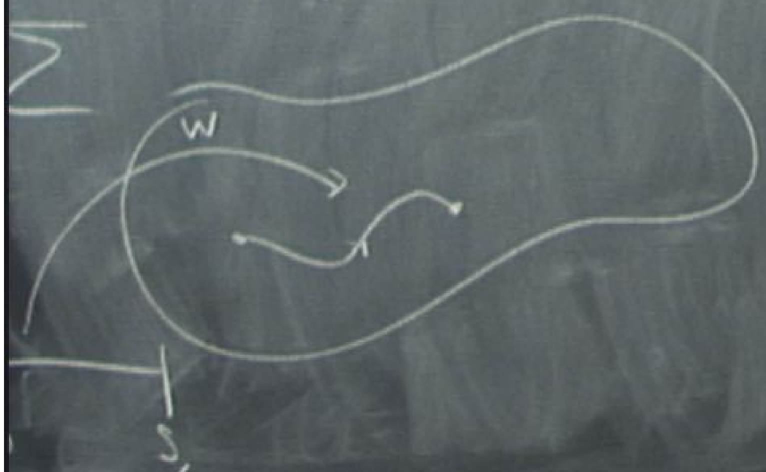

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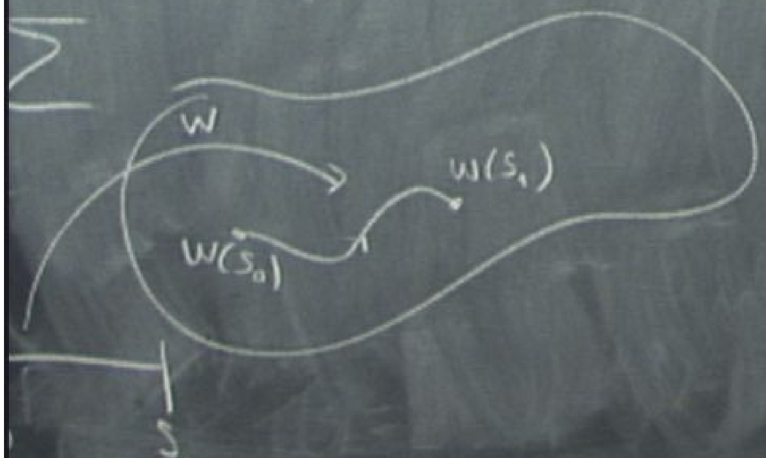
"path-ordered exponential of A along $\gamma$"

path in $\Sigma$

$w: [s_1, s_2]$



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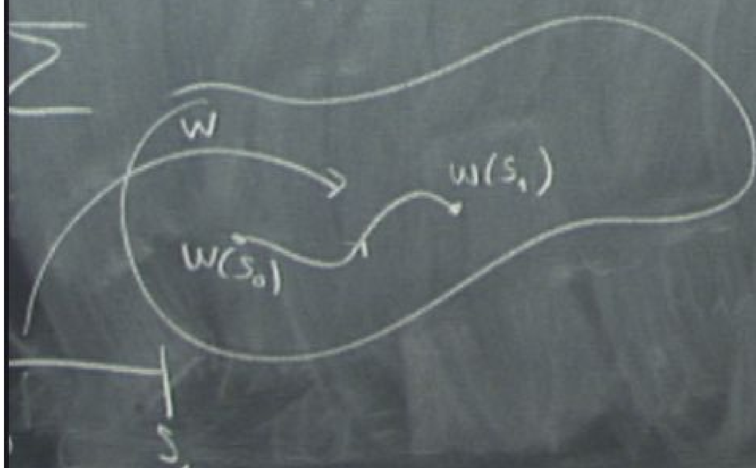
"Wilson loops"

holonomy

$$W_\gamma[A] = \mathrm{Tr}\ P \exp \oint_\gamma A = \mathrm{Tr}\, U_\gamma[A; s_0, s_0)$$

"path-ordered exponential of A along $\gamma$"




path in $\Sigma$

$w: [s_1, s_2] \to \Sigma, \quad s \mapsto w^\mu(s)$

explicitly gauge-invariant functionals:

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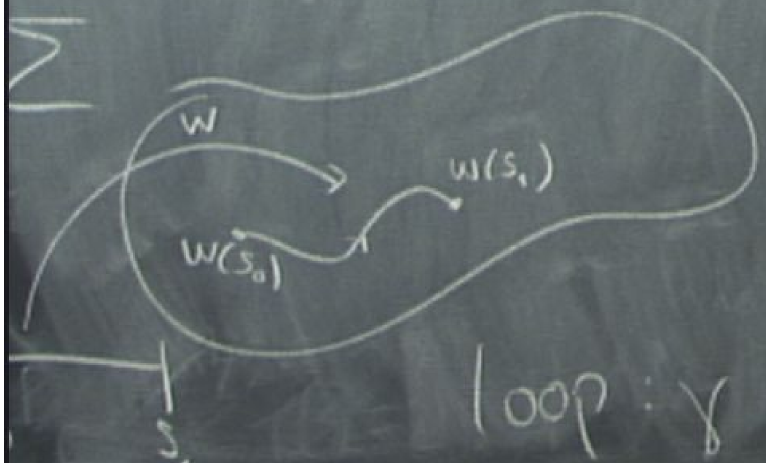


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loop: $\gamma : [0,1] \to \Sigma, \quad s \mapsto \gamma^\mu(s), \quad \gamma(0) = \gamma(1)$

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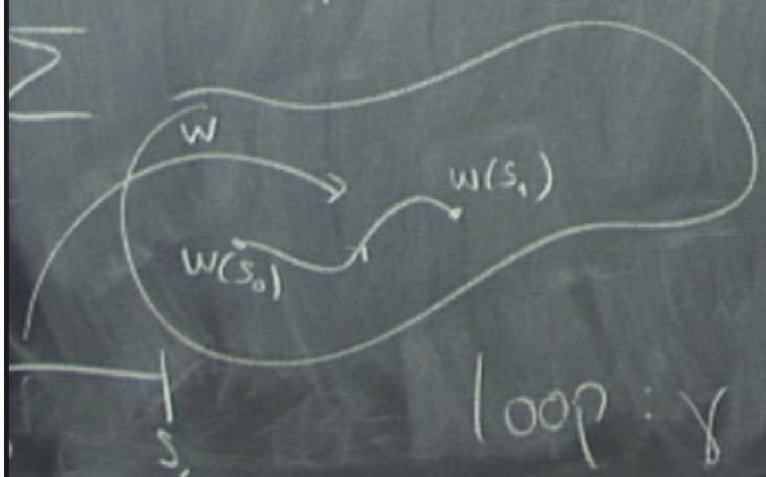


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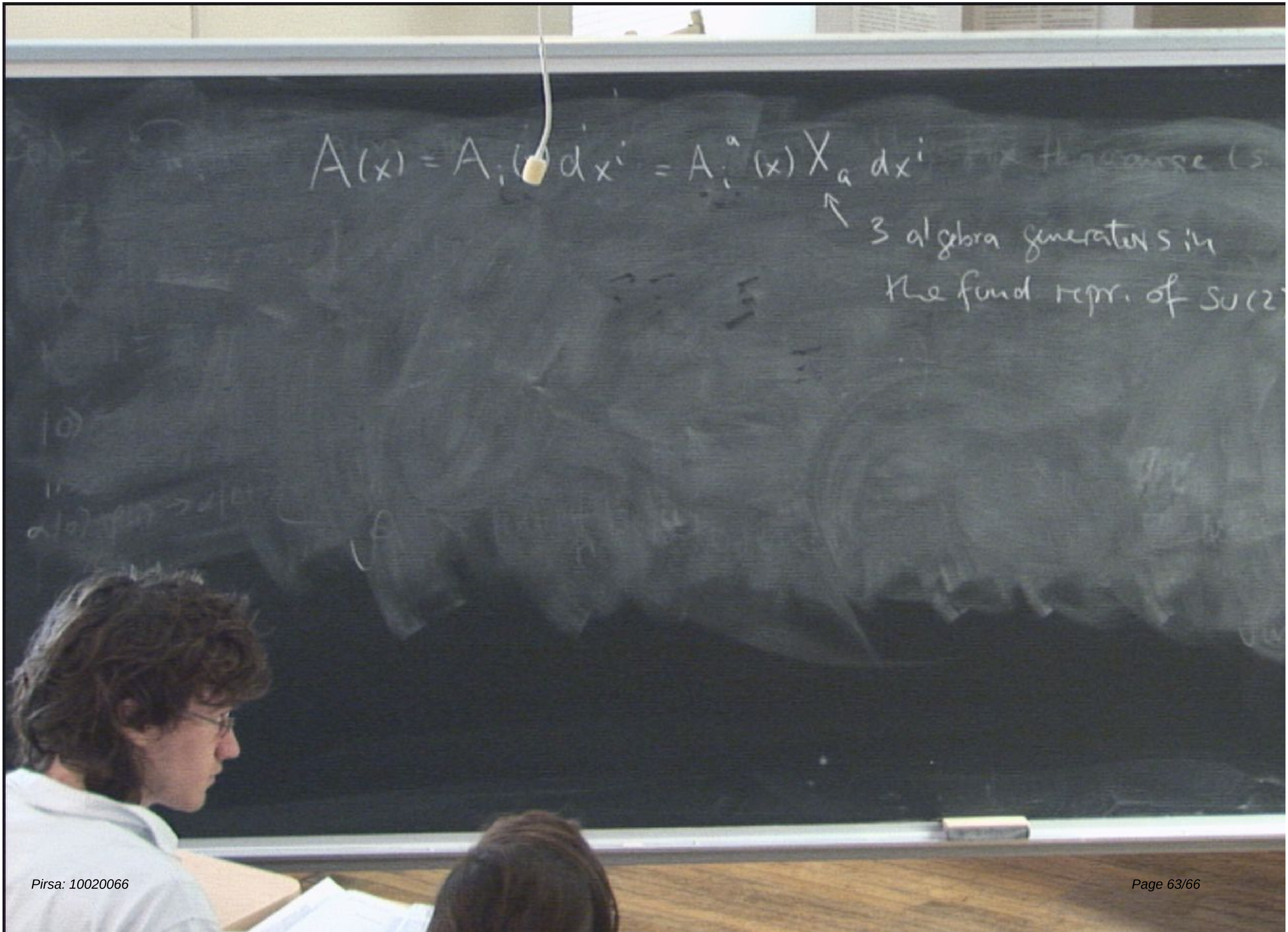
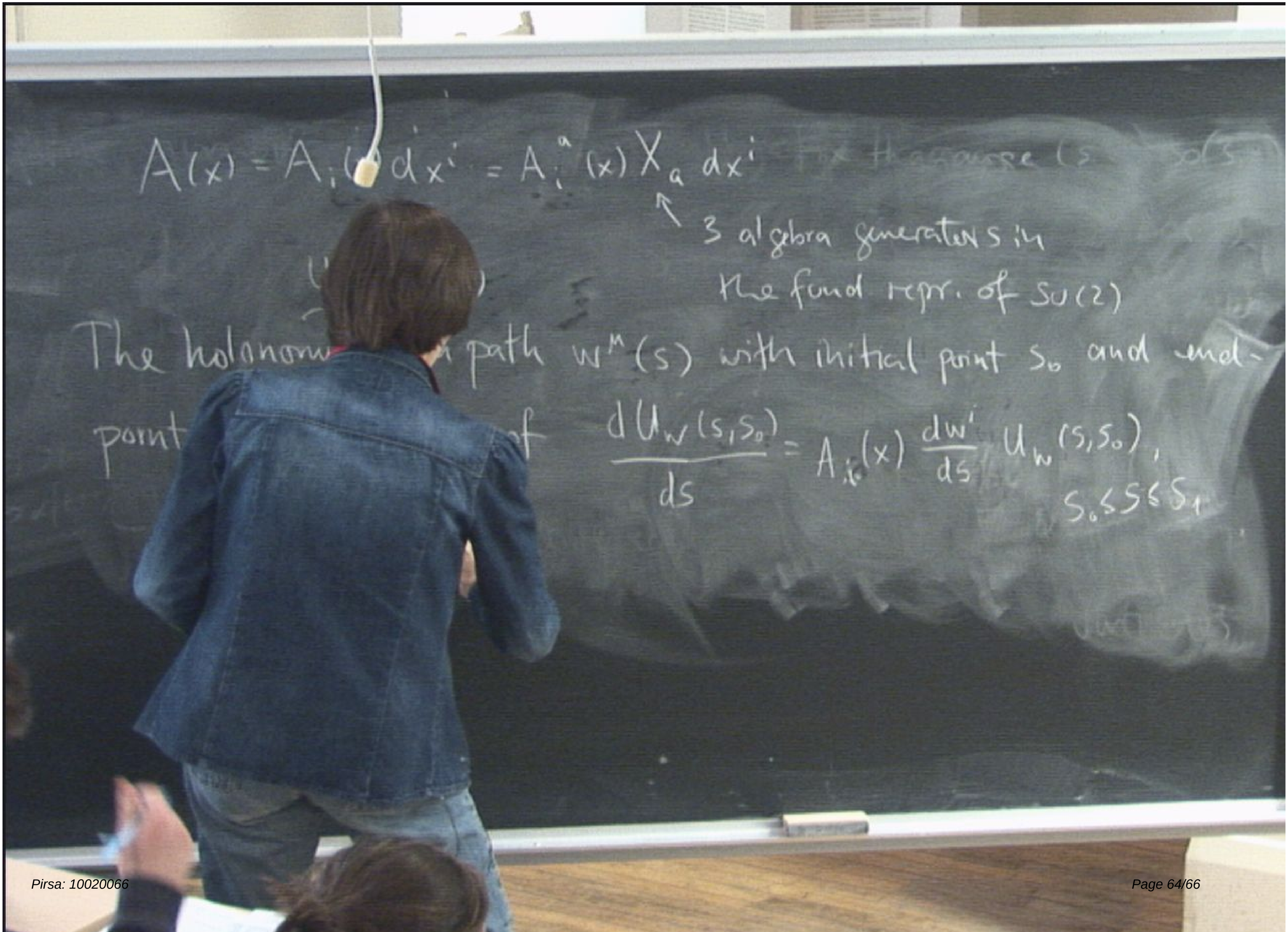

A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i
3 algebra generators in
the fund repr. of SU(2)


$A(x) = A_i(x)\, dx^i = A_i^a(x)\, X_a\, dx^i$
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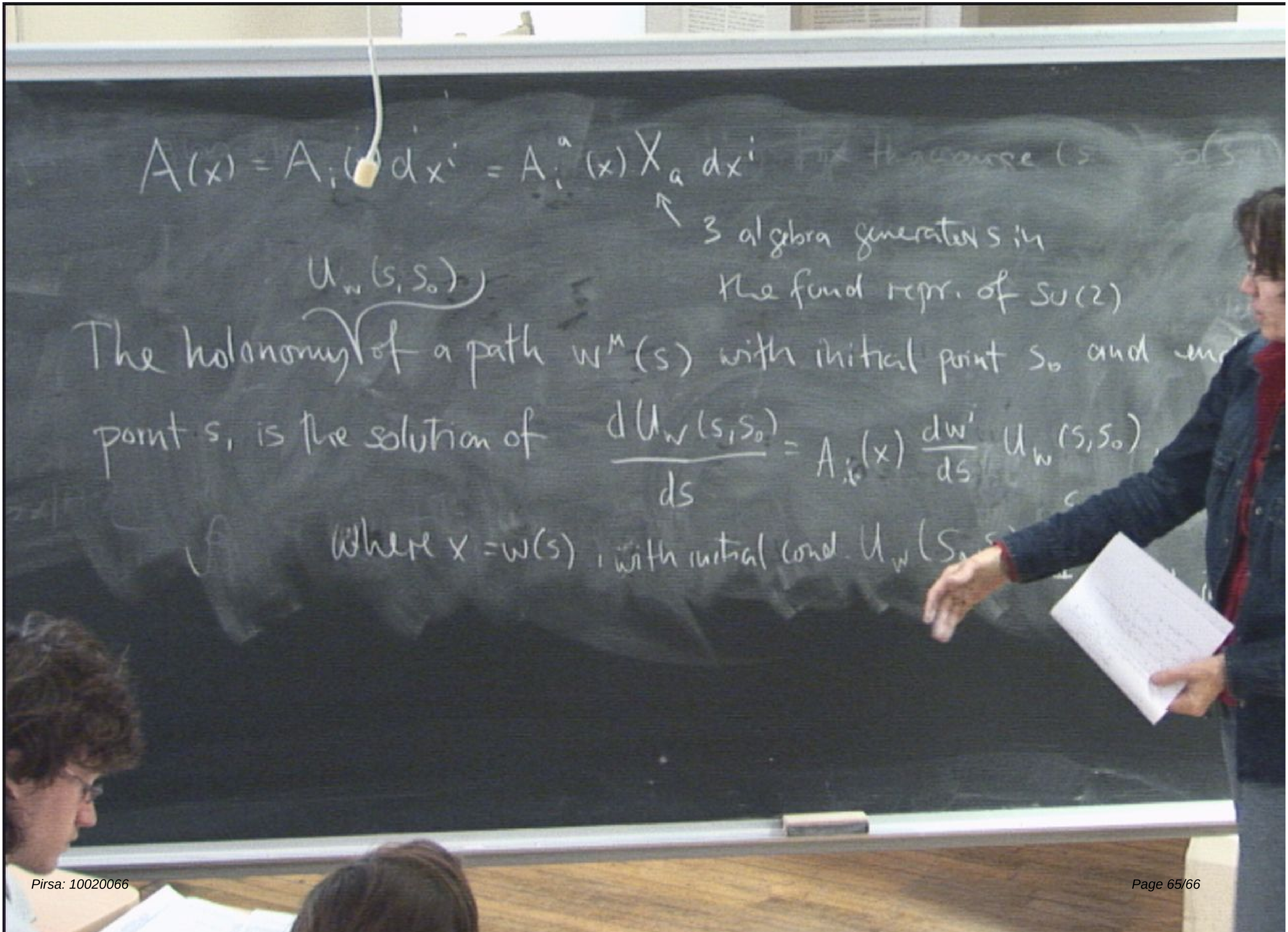
The holonom path $w^\mu(s)$ with initial point $s_0$ and end-
point
$\frac{dU_w(s,s_0)}{ds} = A_i(x) \frac{dw^i}{ds} U_w(s,s_0)$,
$s_0 \leq s \leq s_1$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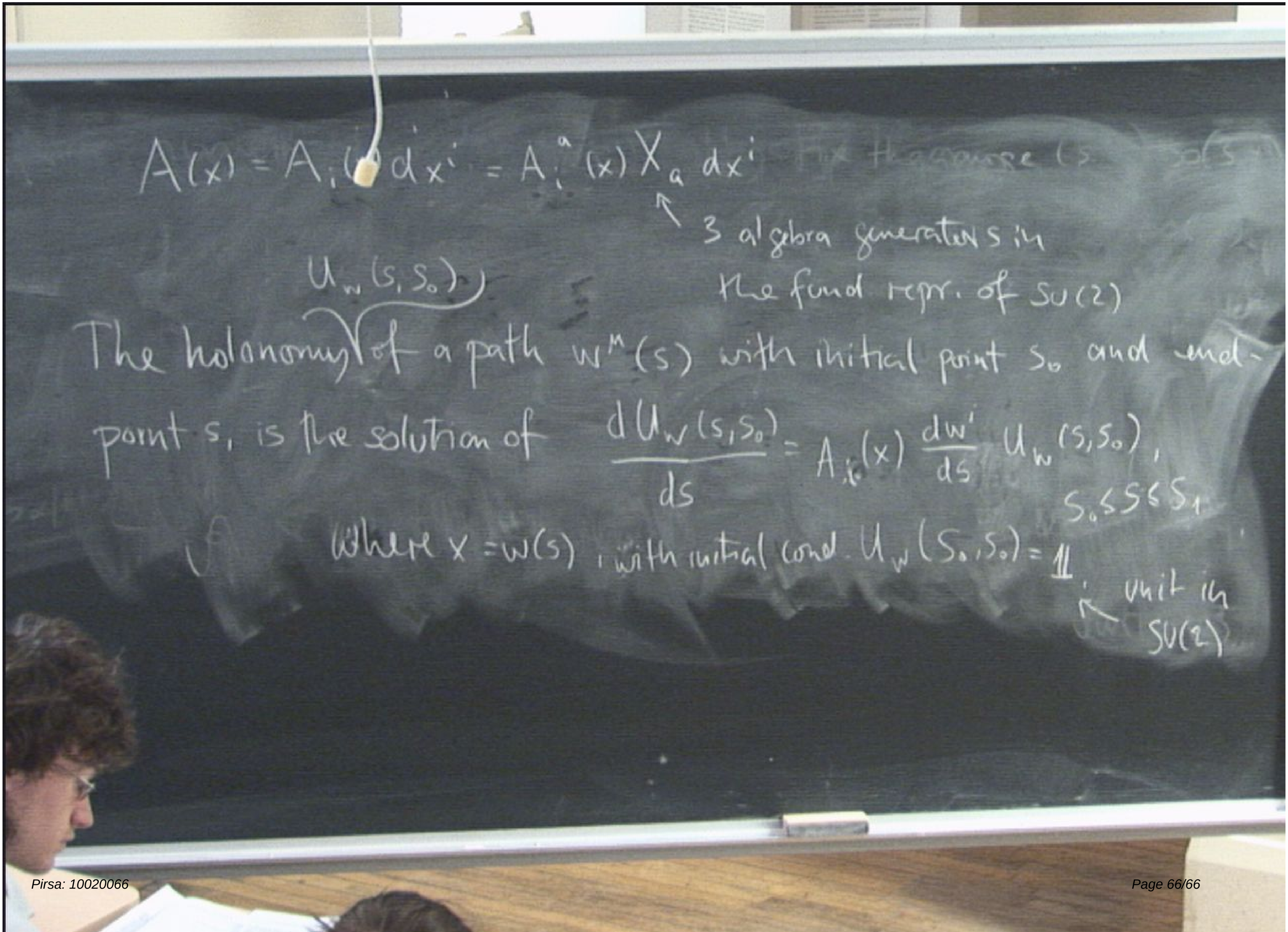
$U_w(s, s_0)$
The holonomy of a path $w^\mu(s)$ with initial point $s_0$ and end
point $s_1$ is the solution of $\frac{dU_w(s,s_0)}{ds} = A_i(x) \frac{dw^i}{ds} U_w(s,s_0)$
Where $x = w(s)$, with initial cond. $U_w(S_0, S$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Where $x = w(s)$, with initial cond. $U_w(s_0, s_0) = \mathbb{1}$
unit in SU(2)