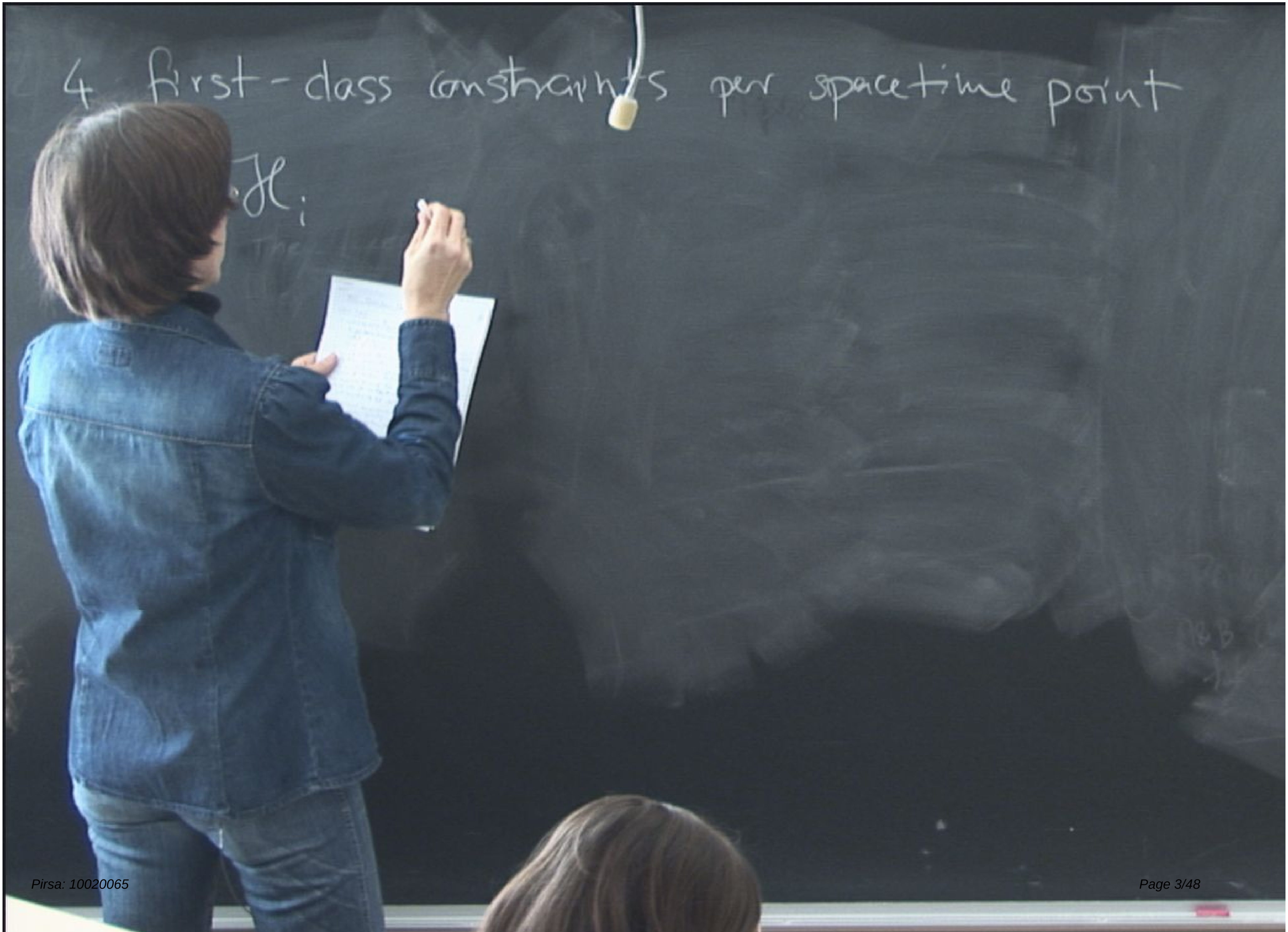
Title: Quantum Gravity - Review (PHYS 638) - Lecture 11

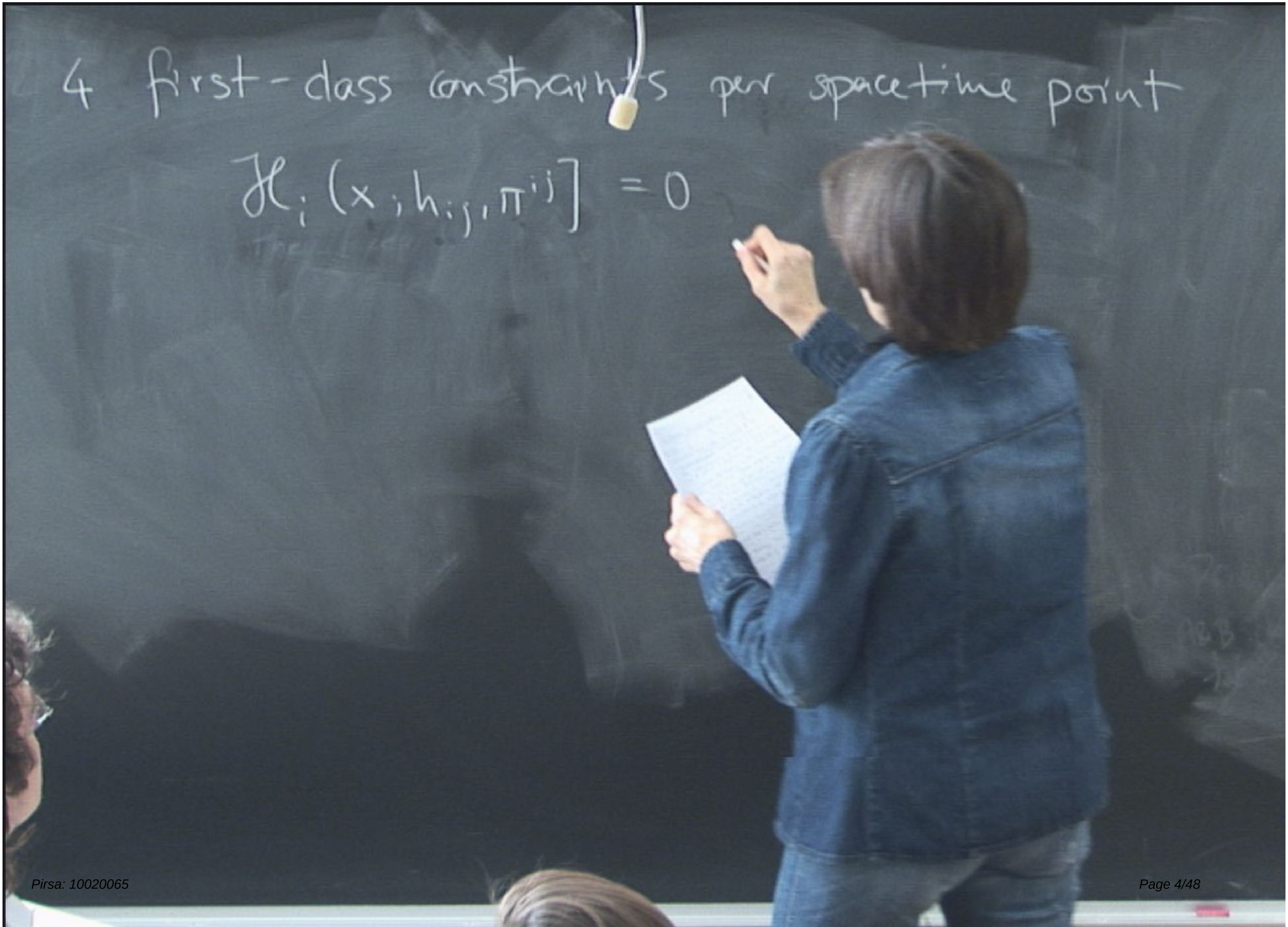
Date: Feb 08, 2010 10:00 AM

URL: http://pirsa.org/10020065

Abstract:

perimeter scholars
international


4 first-class constraints per spacetime point
$\mathcal{H}_i$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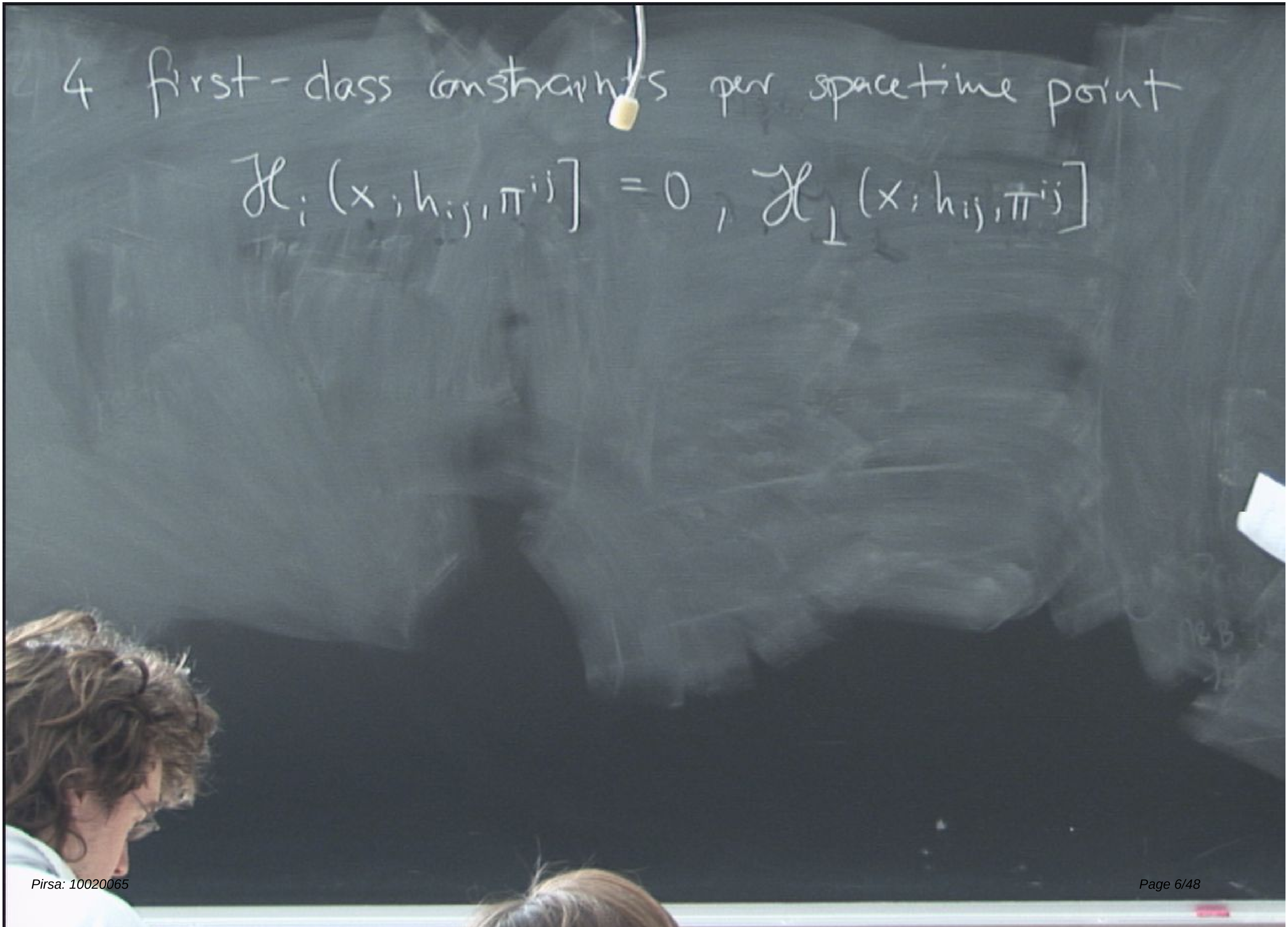
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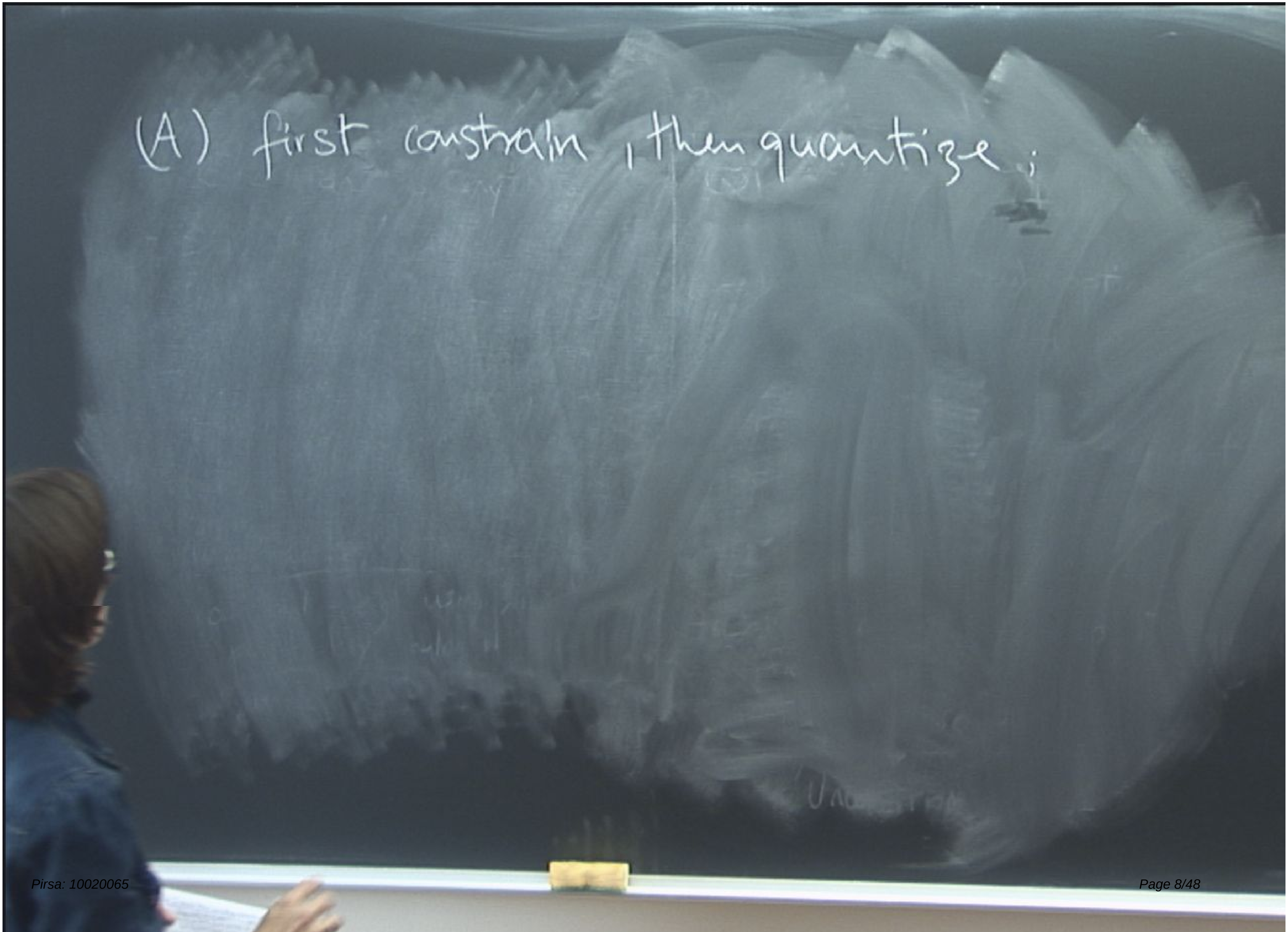
$$\mathcal{H}_i(x; h_{ij}, \pi^{ij}] = 0, \quad \mathcal{H}_\perp(x; h_{ij}, \pi^{ij}] = 0$$

QM example of non-commutativity   RL, PRD41 (1990) 3785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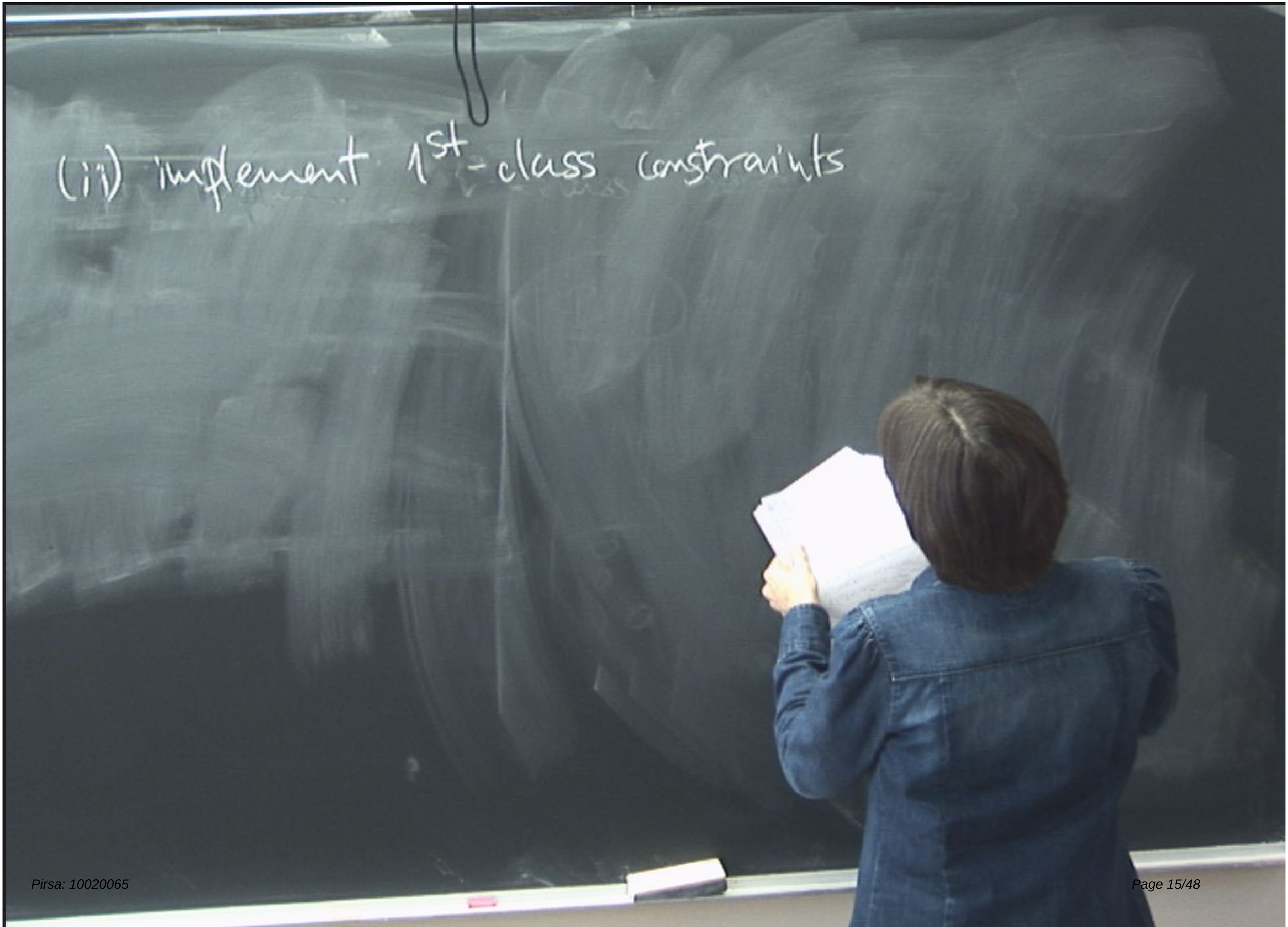
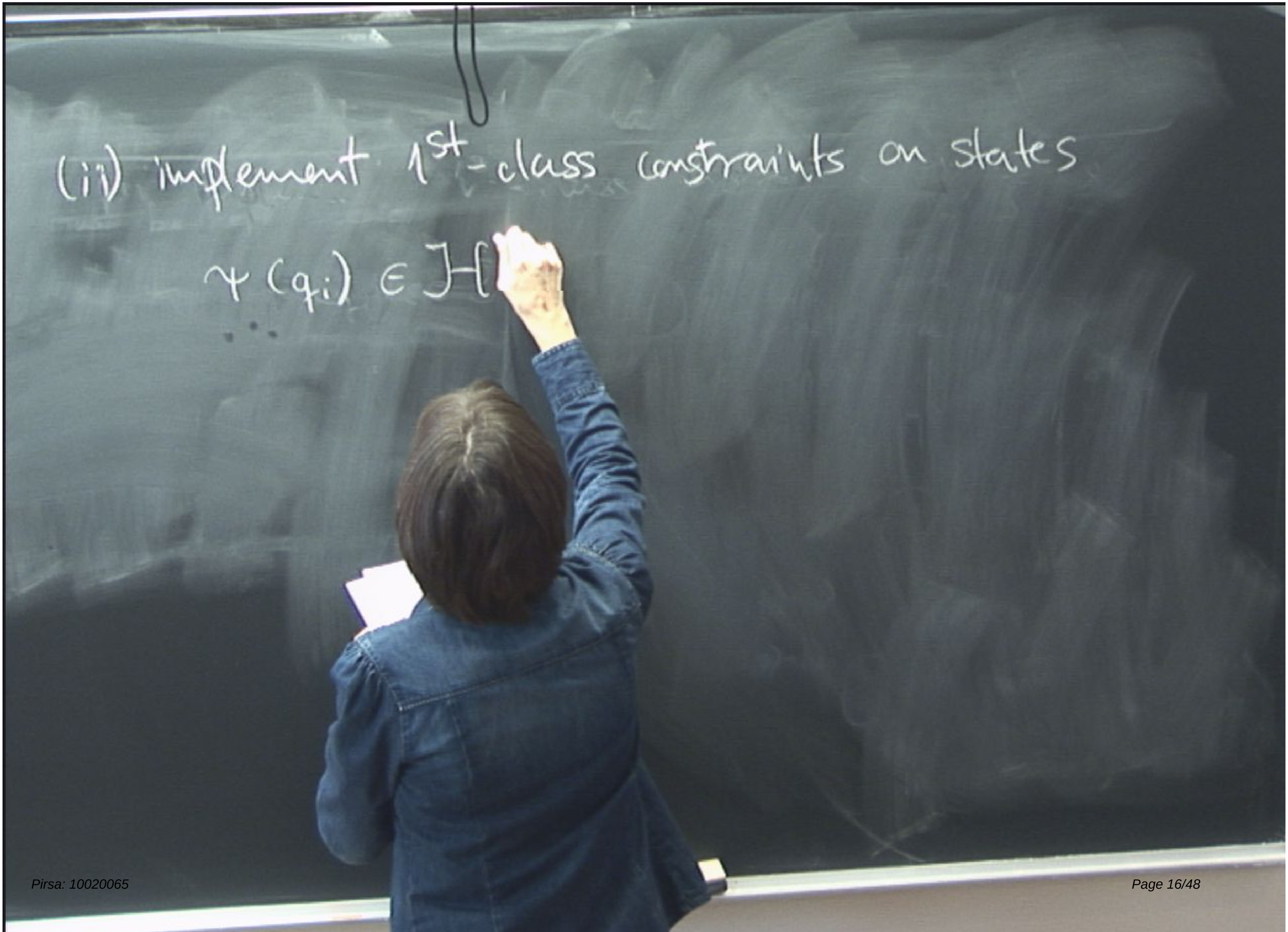

(ii) implement 1st-class constraints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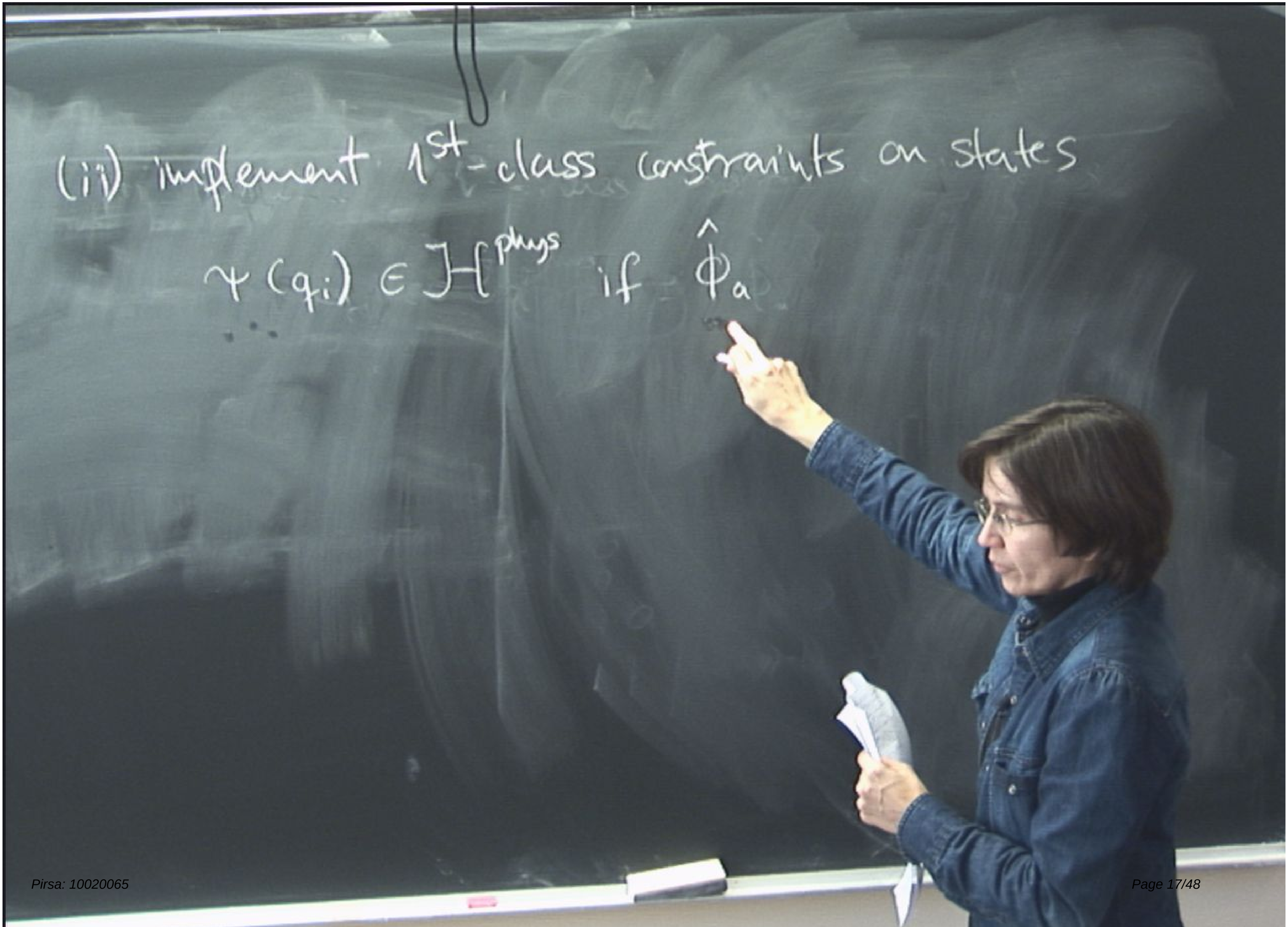
ψ(q_i) ∈ H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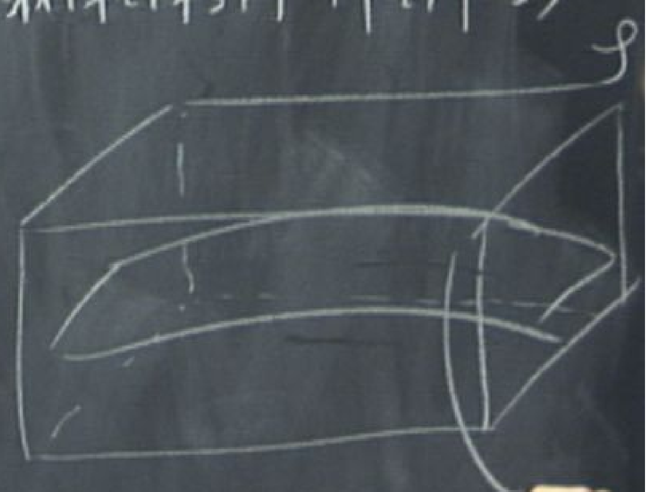

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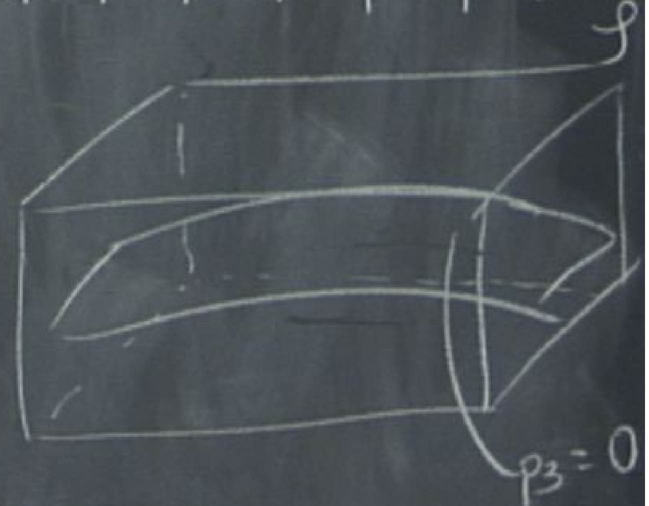

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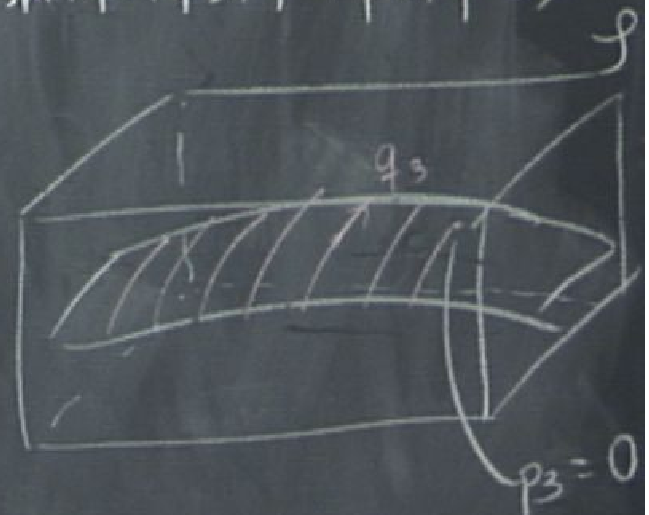
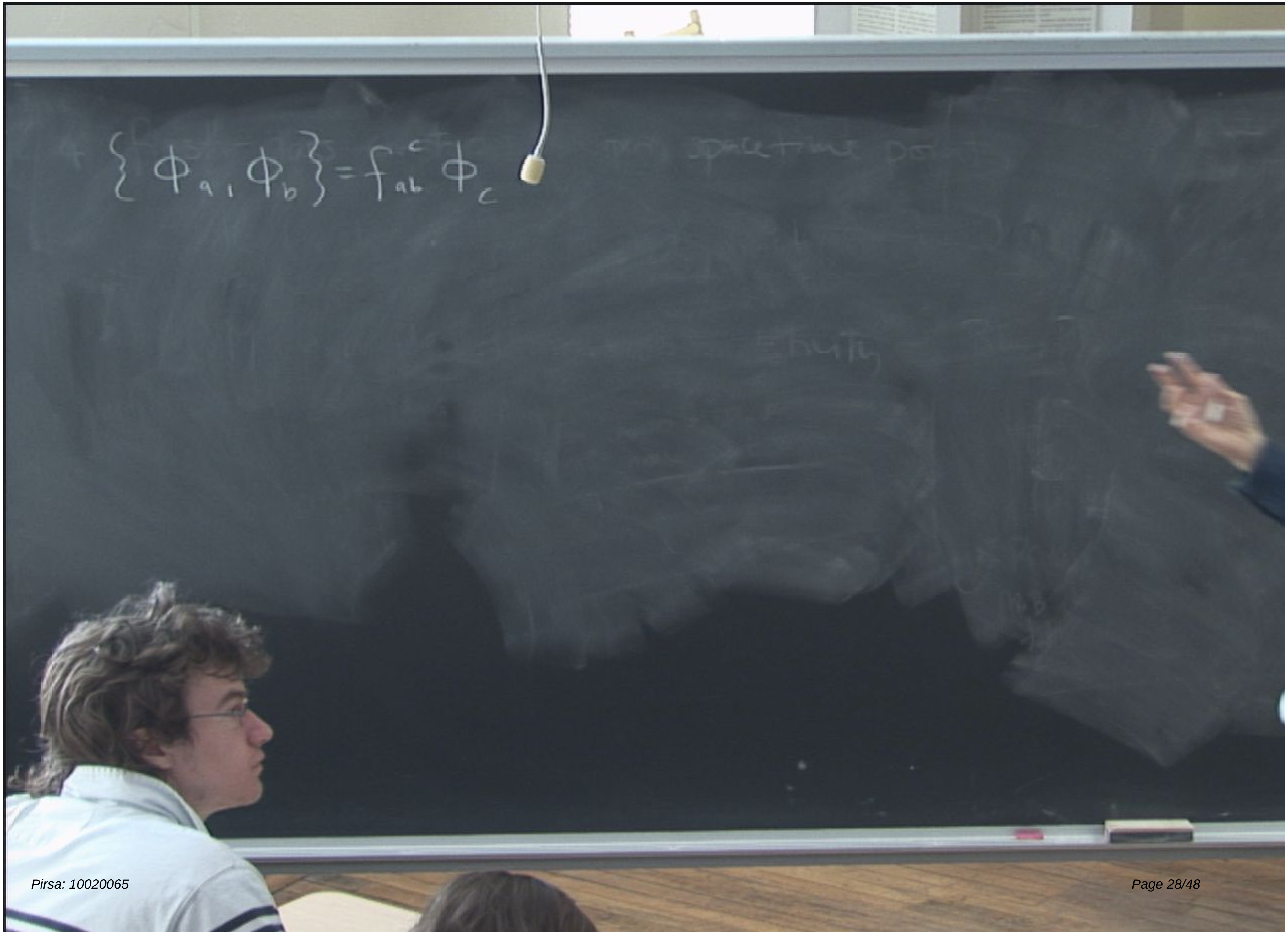
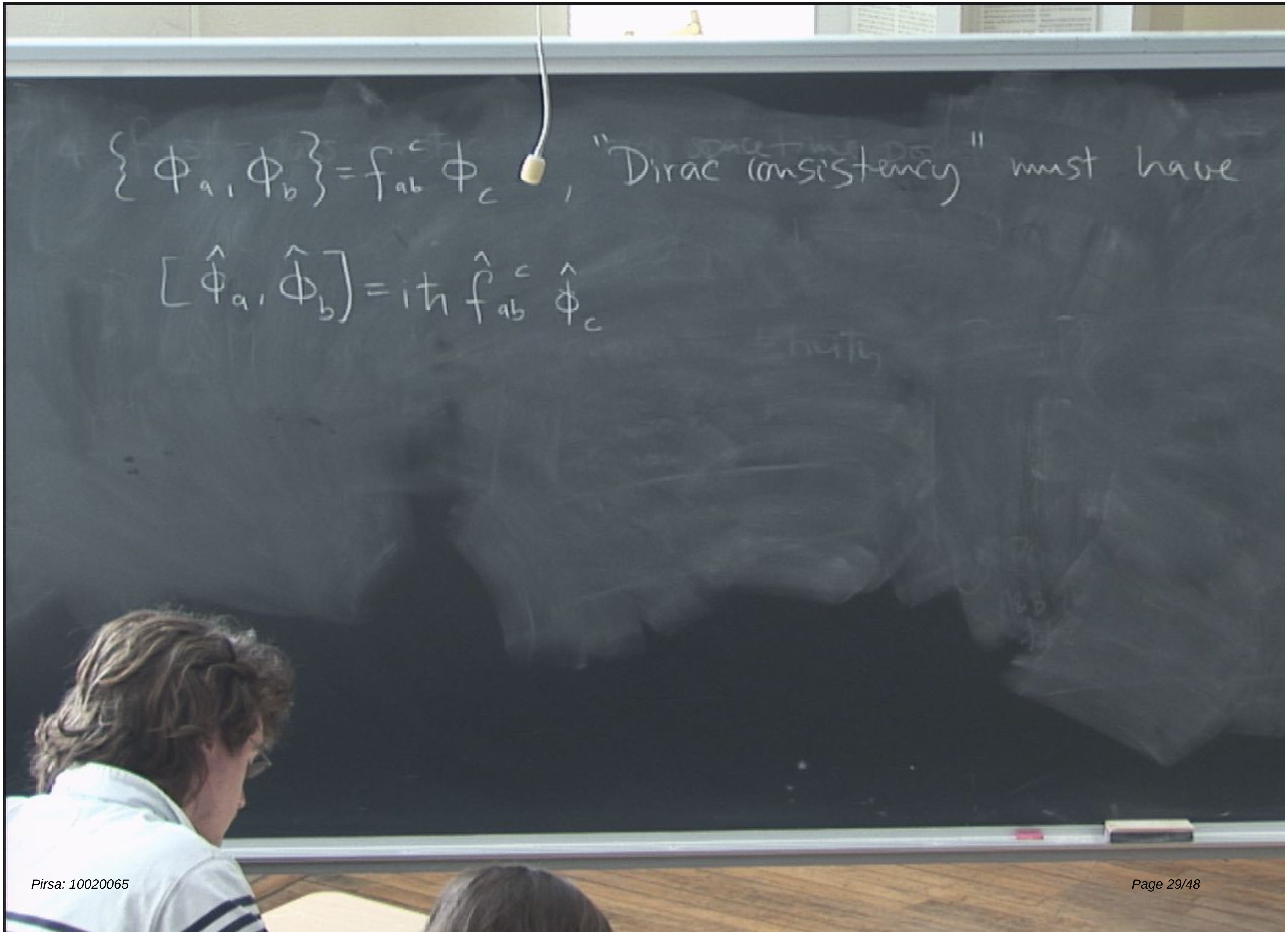



$\{\Phi_a, \Phi_b\} = f_{ab}^{\ \ c}\,\Phi_c$


$\{\Phi_a, \Phi_b\} = f_{ab}{}^c \Phi_c$, "Dirac consistency" must have
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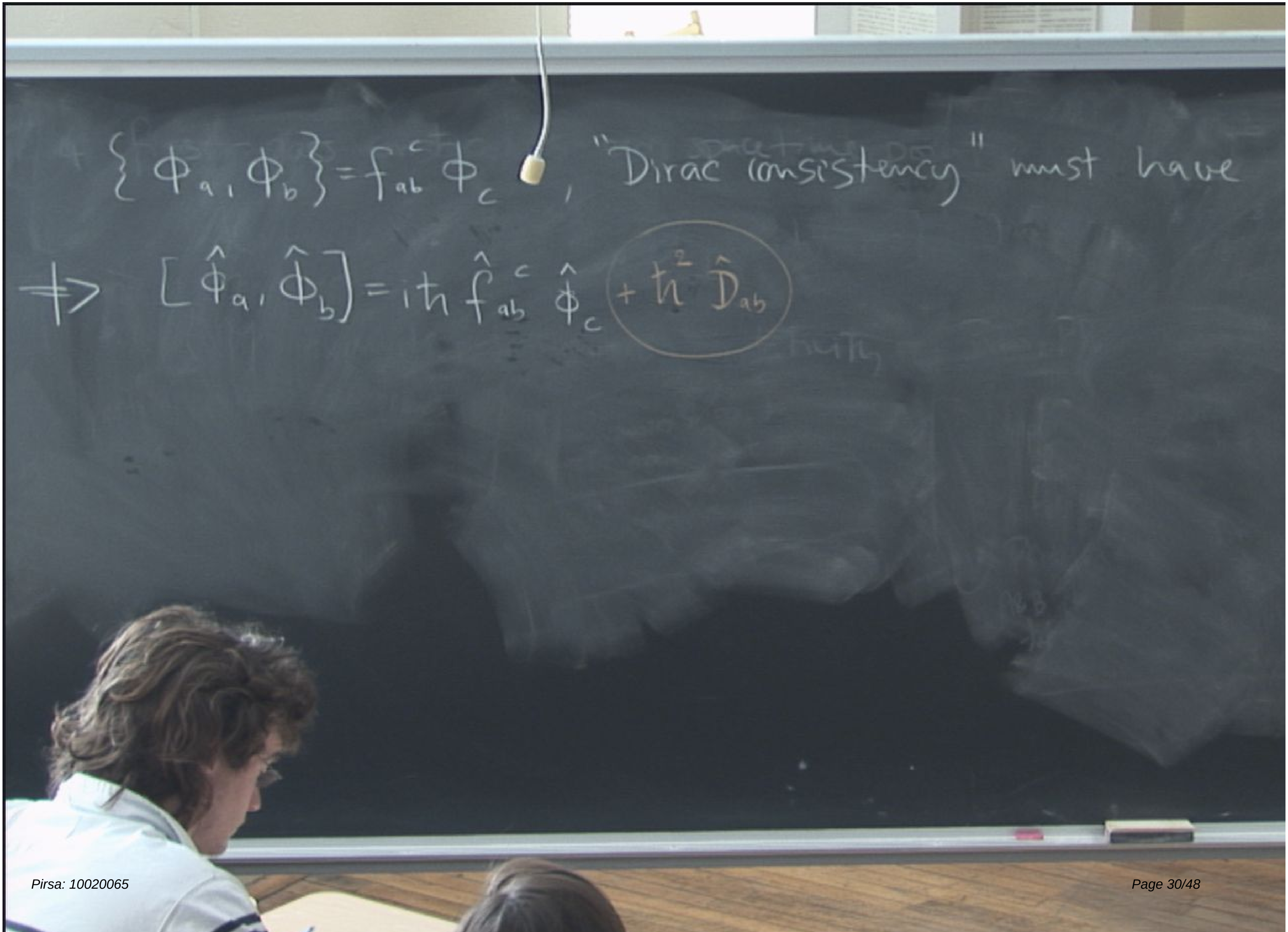
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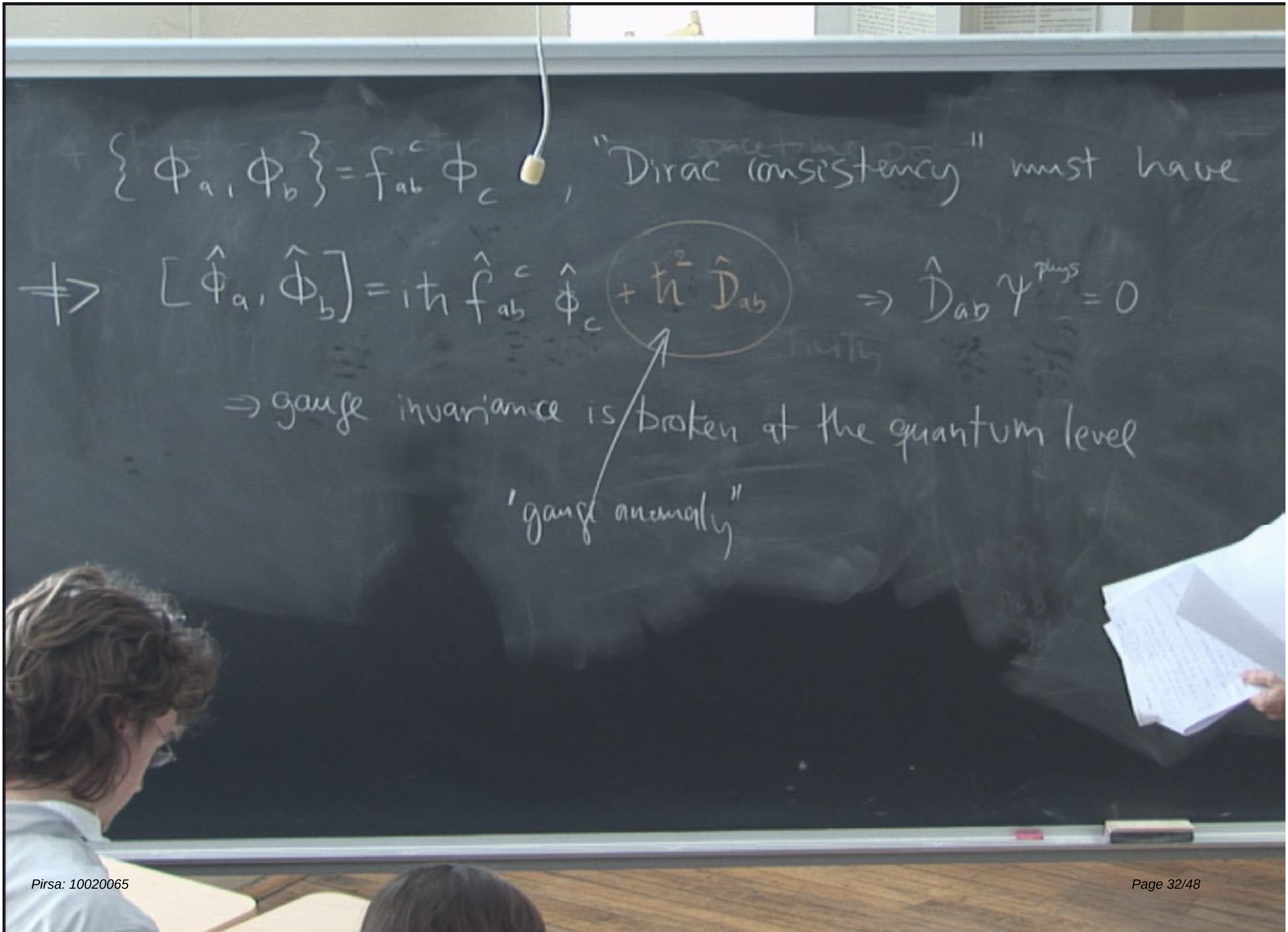
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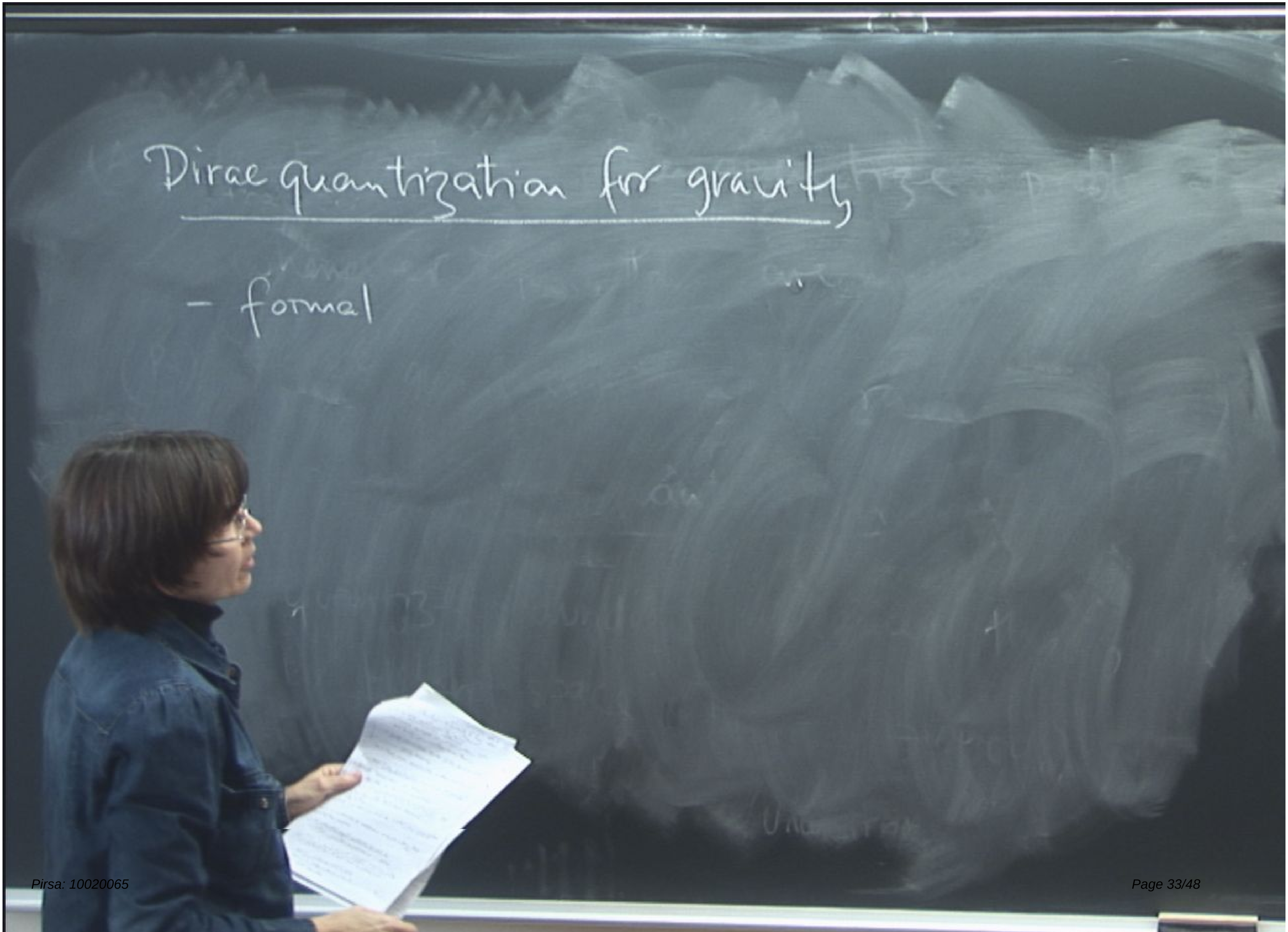
$\Rightarrow$ gauge invariance is broken at the quantum level
"gauge anomaly"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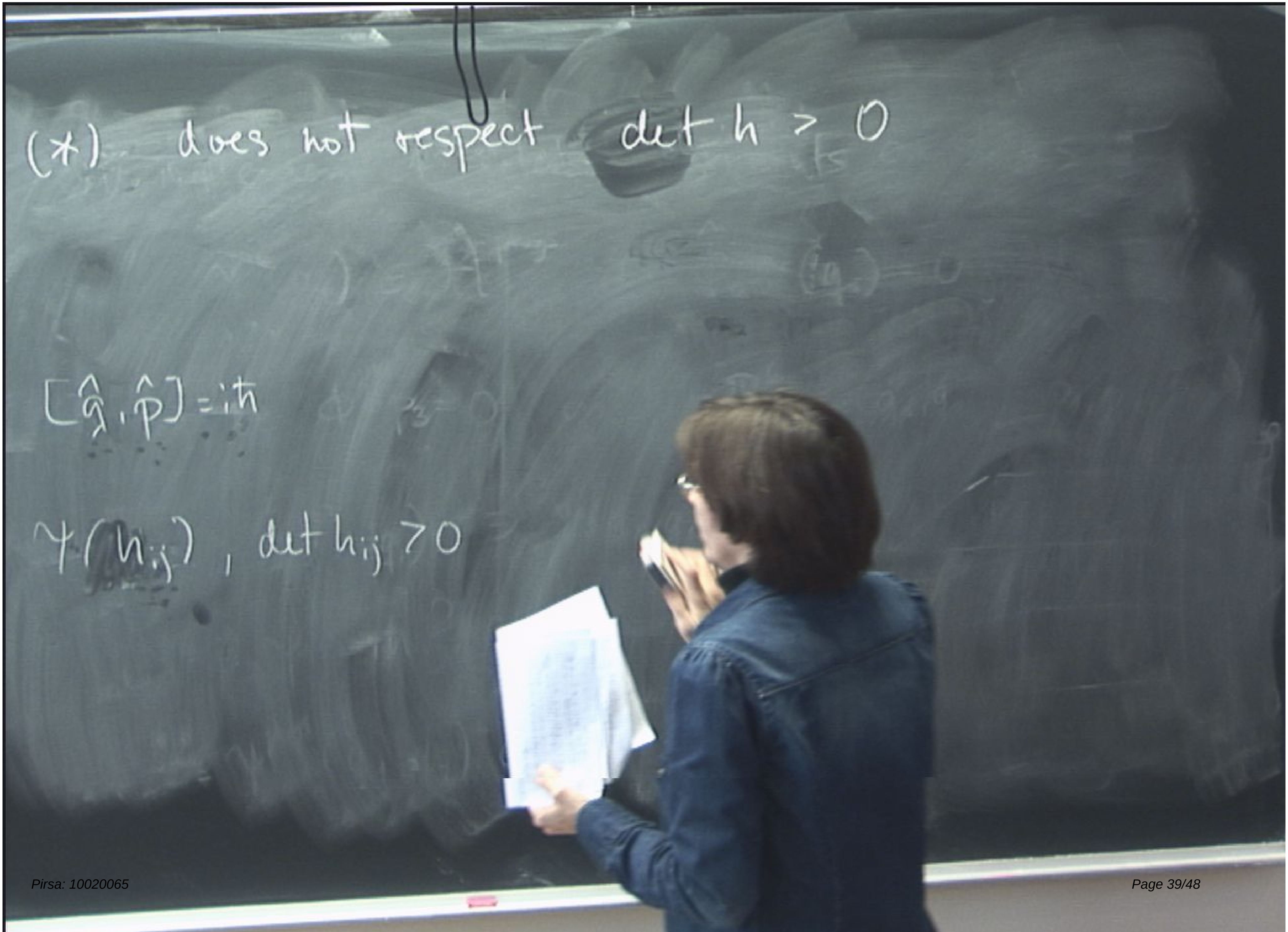
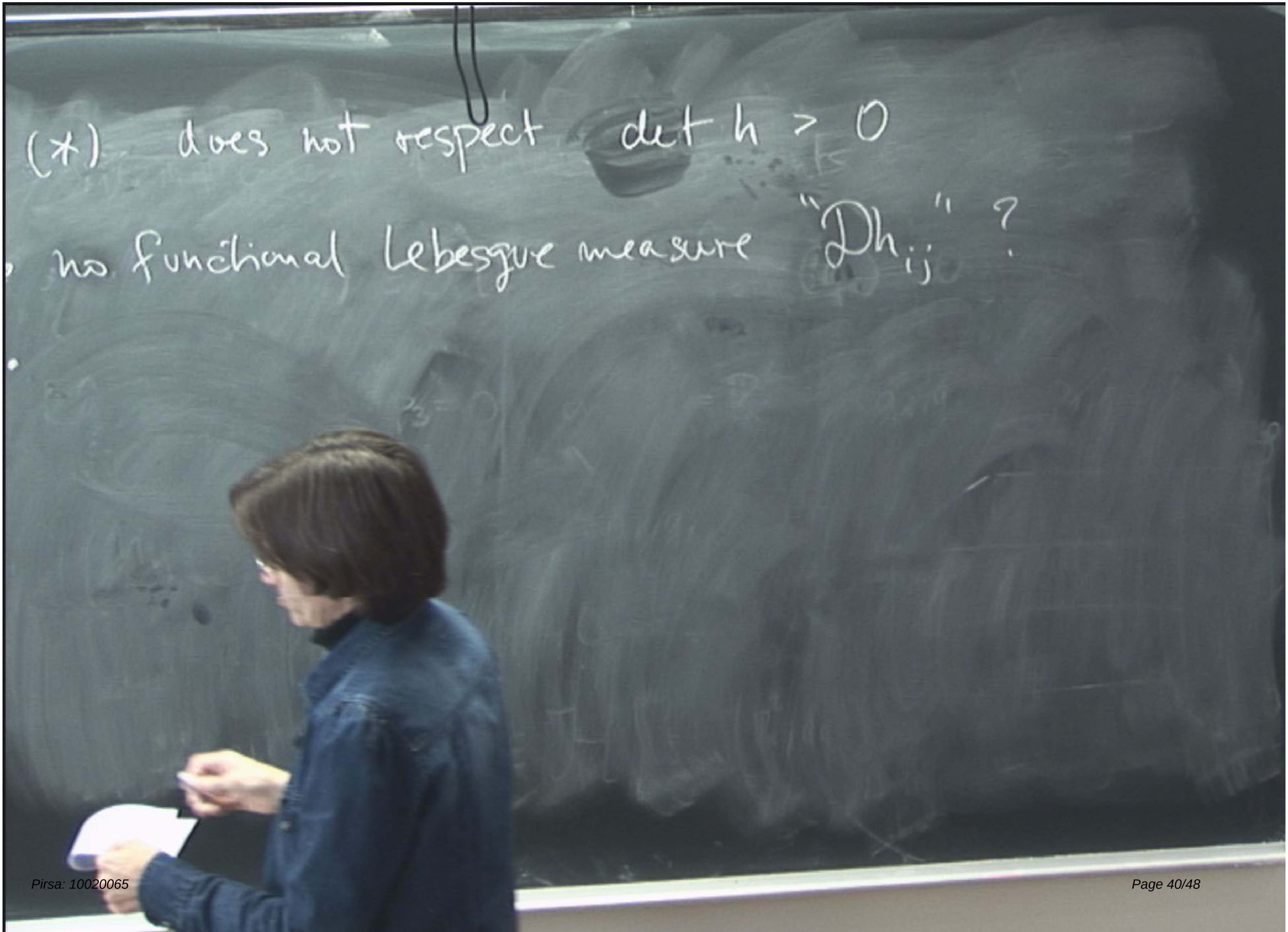
- quantize $\mathcal{H}_\mu = (\mathcal{H}_\perp, \mathcal{H}_i)$ and demand $\hat{\mathcal{H}}_\mu \Psi^{phys} = 0$
- find a scalar product on Ψ^{phys} $\hookrightarrow \mathcal{H}^{phys}$

$[\hat{q}, \hat{p}] = i\hbar$


(*) does not respect det h > 0
$[\hat{q}, \hat{p}] = i\hbar$
$\Psi(h_{ij})$, det $h_{ij} > 0$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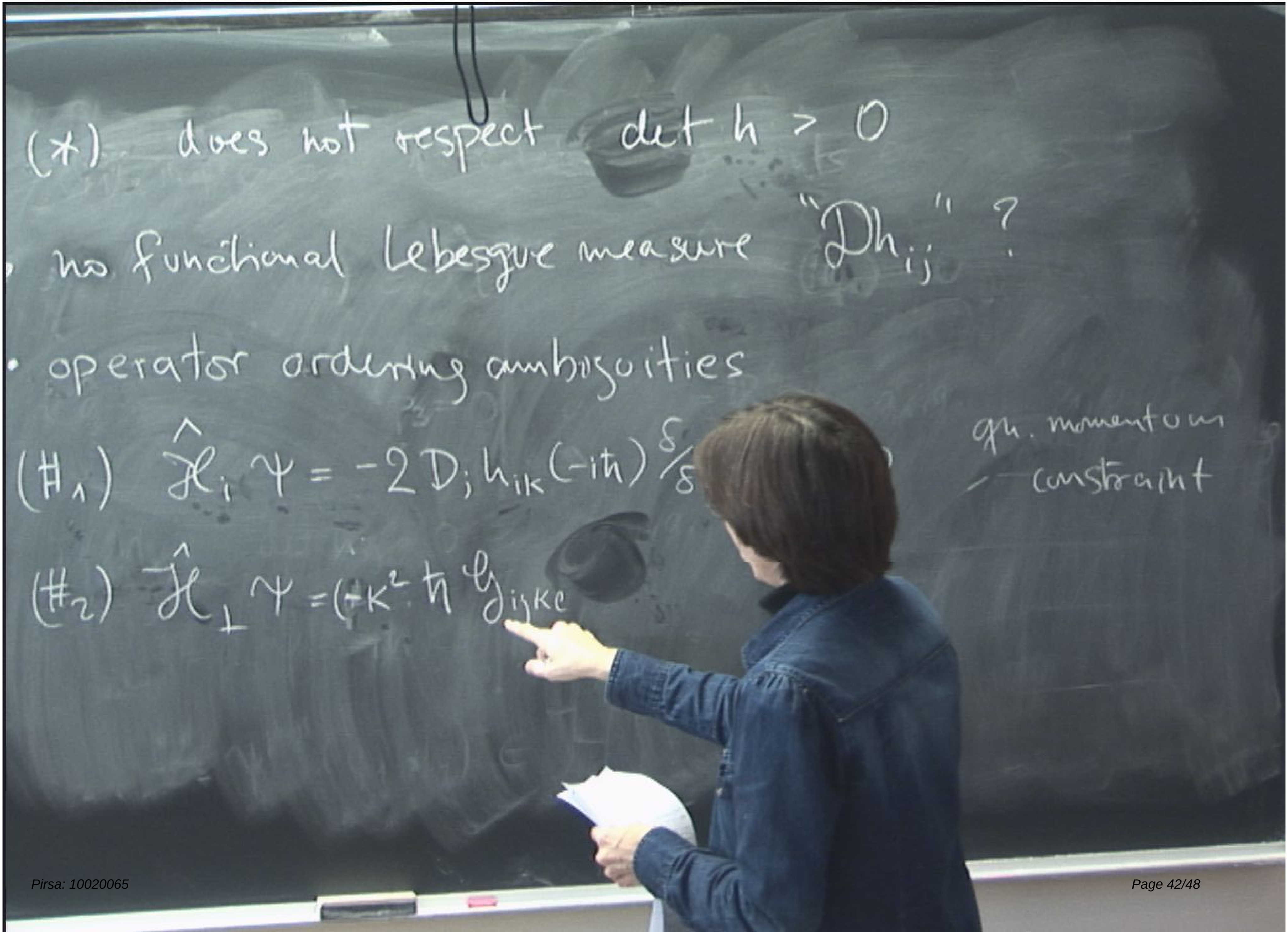
($\#_1$)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• no functional Lebesgue measure "$\mathcal{D}h_{ij}$" ?
• operator ordering ambiguities
$(\#_1)$ $\hat{\mathcal{H}}_i \Psi = -2 D_j h_{ik} (-i\hbar) \frac{\delta}{\delta}$
qu. momentum constraint
$(\#_2)$ $\hat{\mathcal{H}}_\perp \Psi = (-\kappa^2 \hbar \, G_{ijkl}$

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"Wheeler-DeWitt equation(s)"

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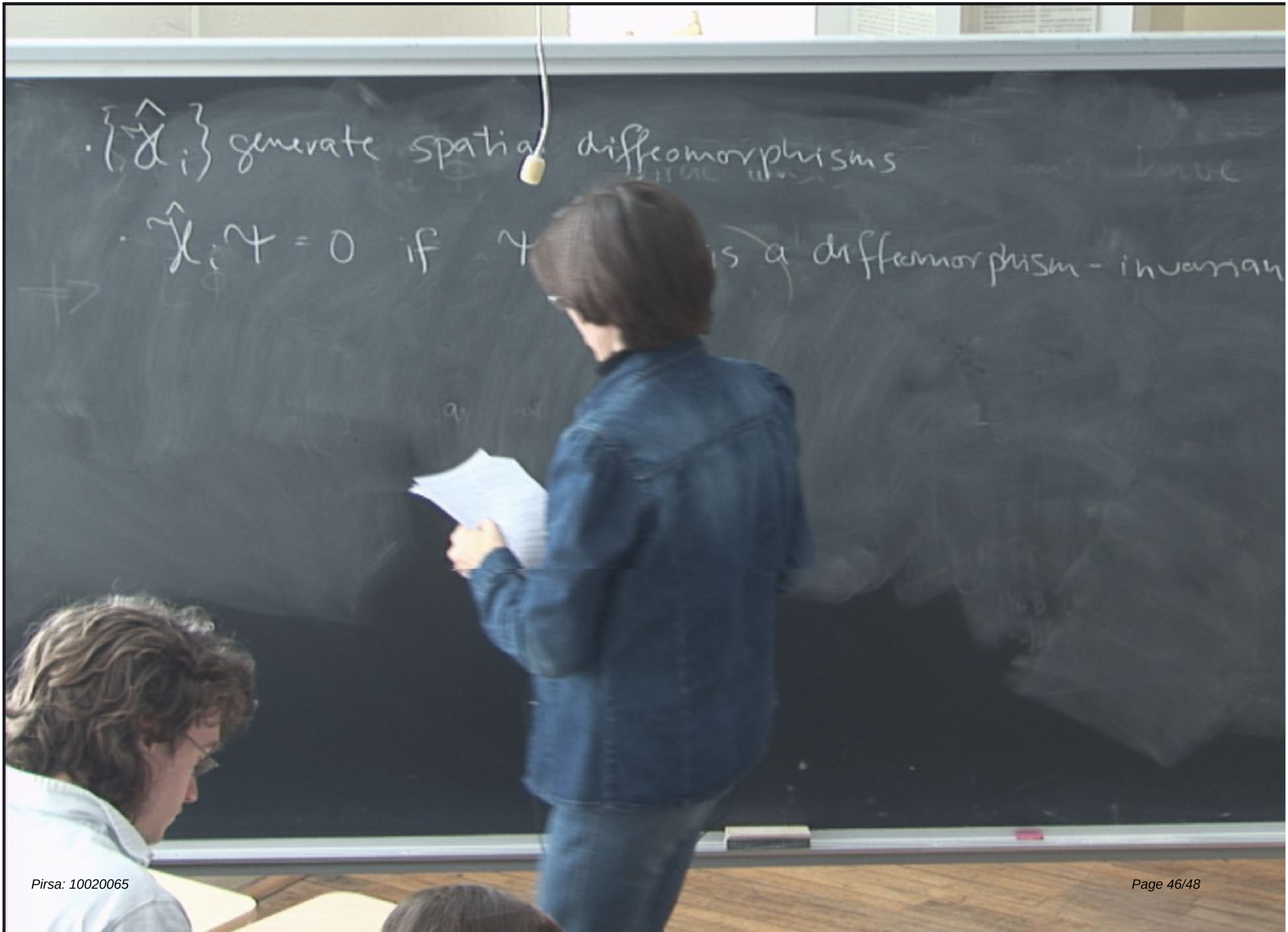
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$\cdot\ \hat{\chi}_i \Psi = 0$ if $\Psi[h_{ij}]$ is a diffeomorphism-invariant functional of h_{ij} $\Leftrightarrow$ Ψ is a functional on superspace

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$(\#_2)$ must be regularized / renormalized – this cannot be
rated from factor-ordering issues