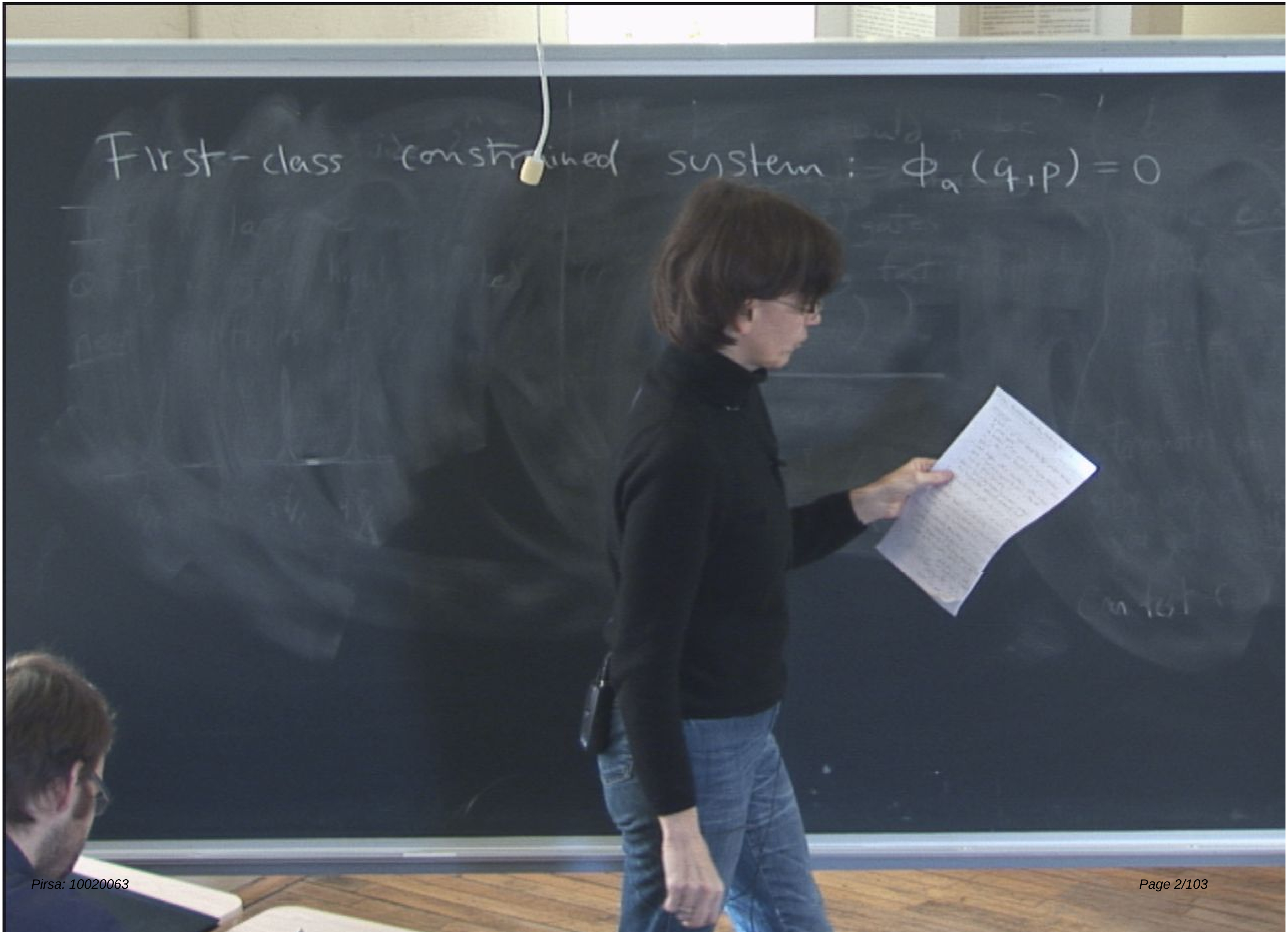
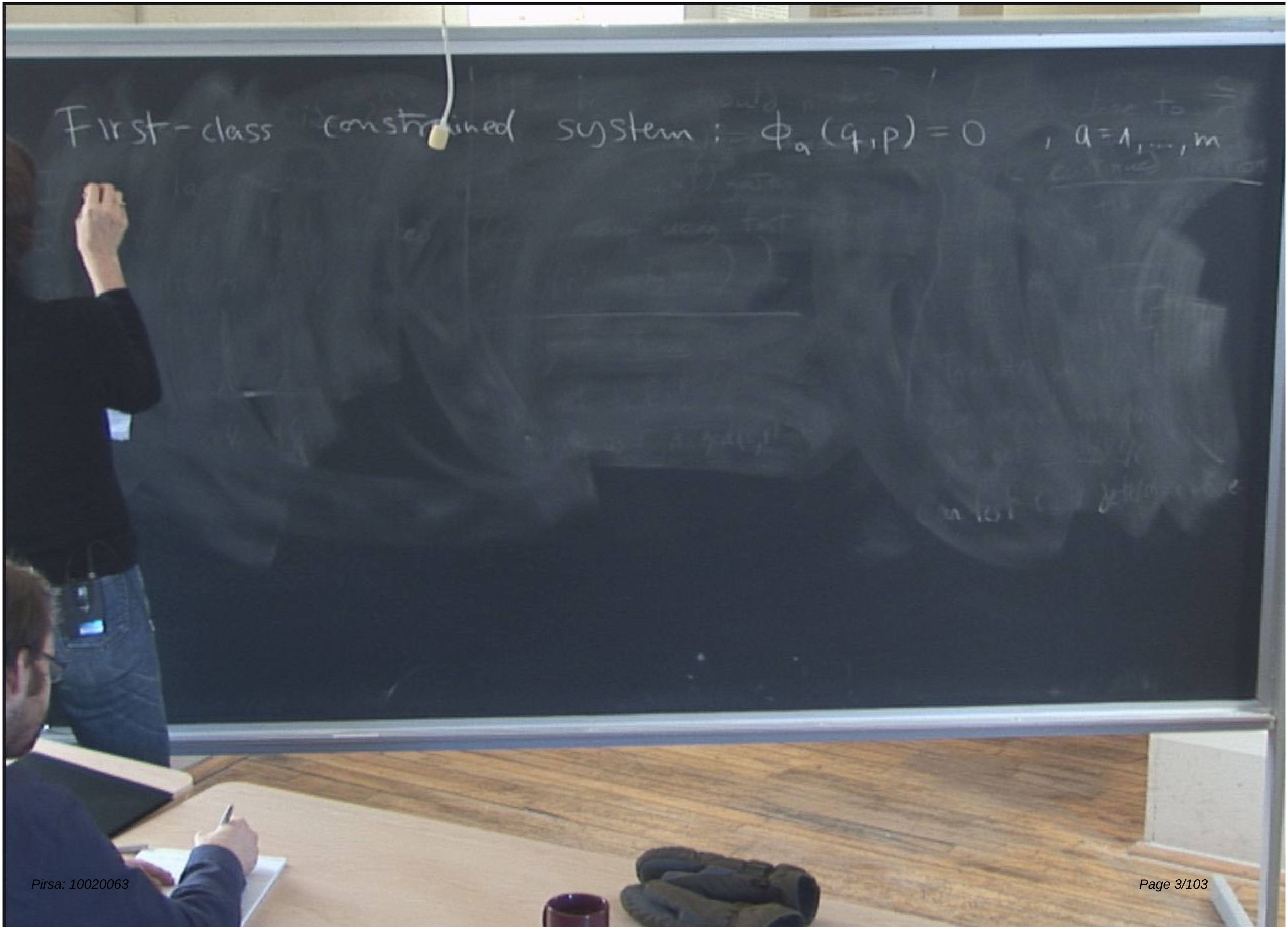
Title: Quantum Gravity - Review (PHYS 638) - Lecture 9

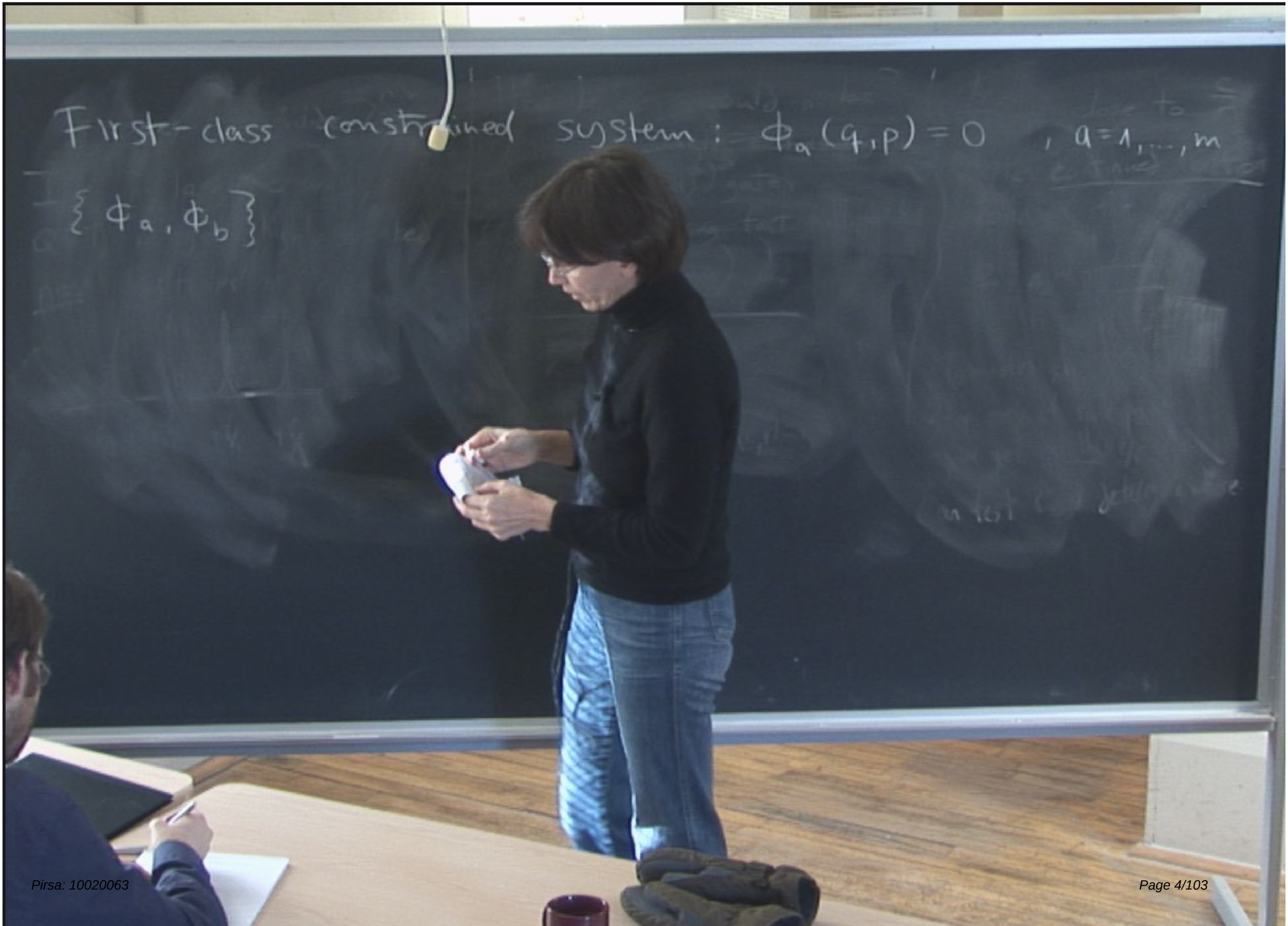
Date: Feb 04, 2010 10:00 AM

URL: http://pirsa.org/10020063

Abstract:


First-class constrained system : $\phi_a(q,p) = 0$


First-class constrained system : $\phi_a(q,p)=0$, $a=1,\dots,m$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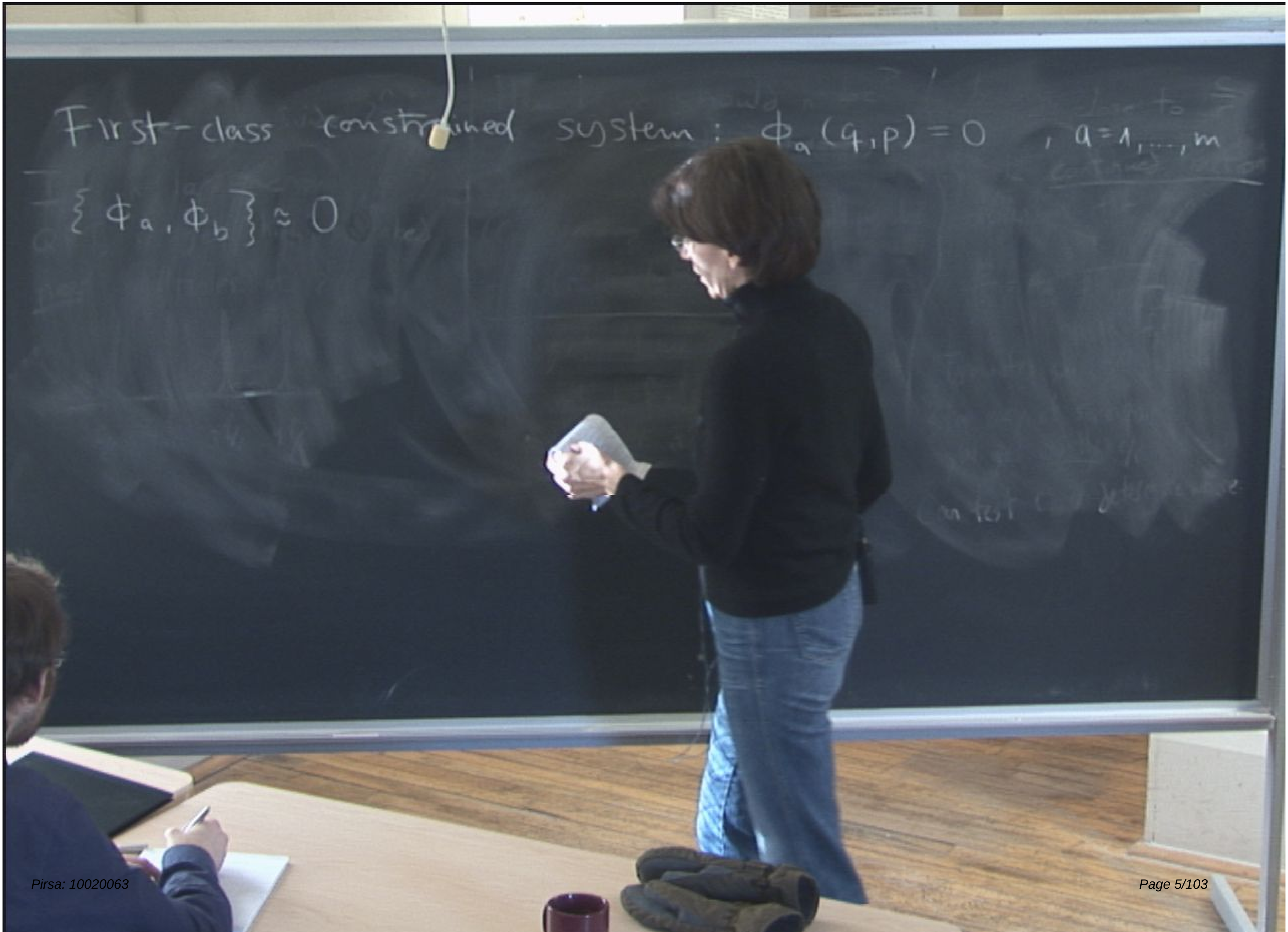
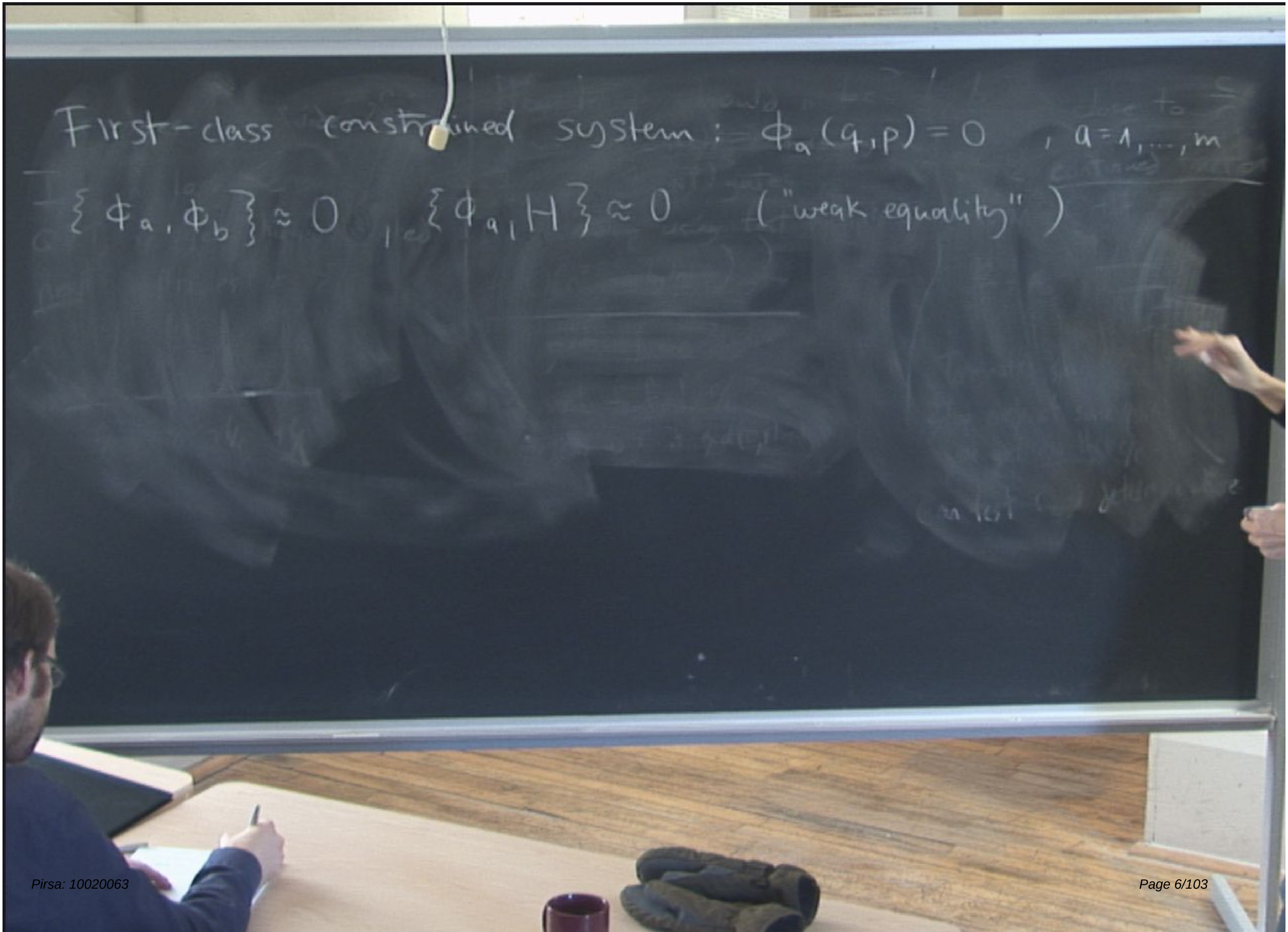
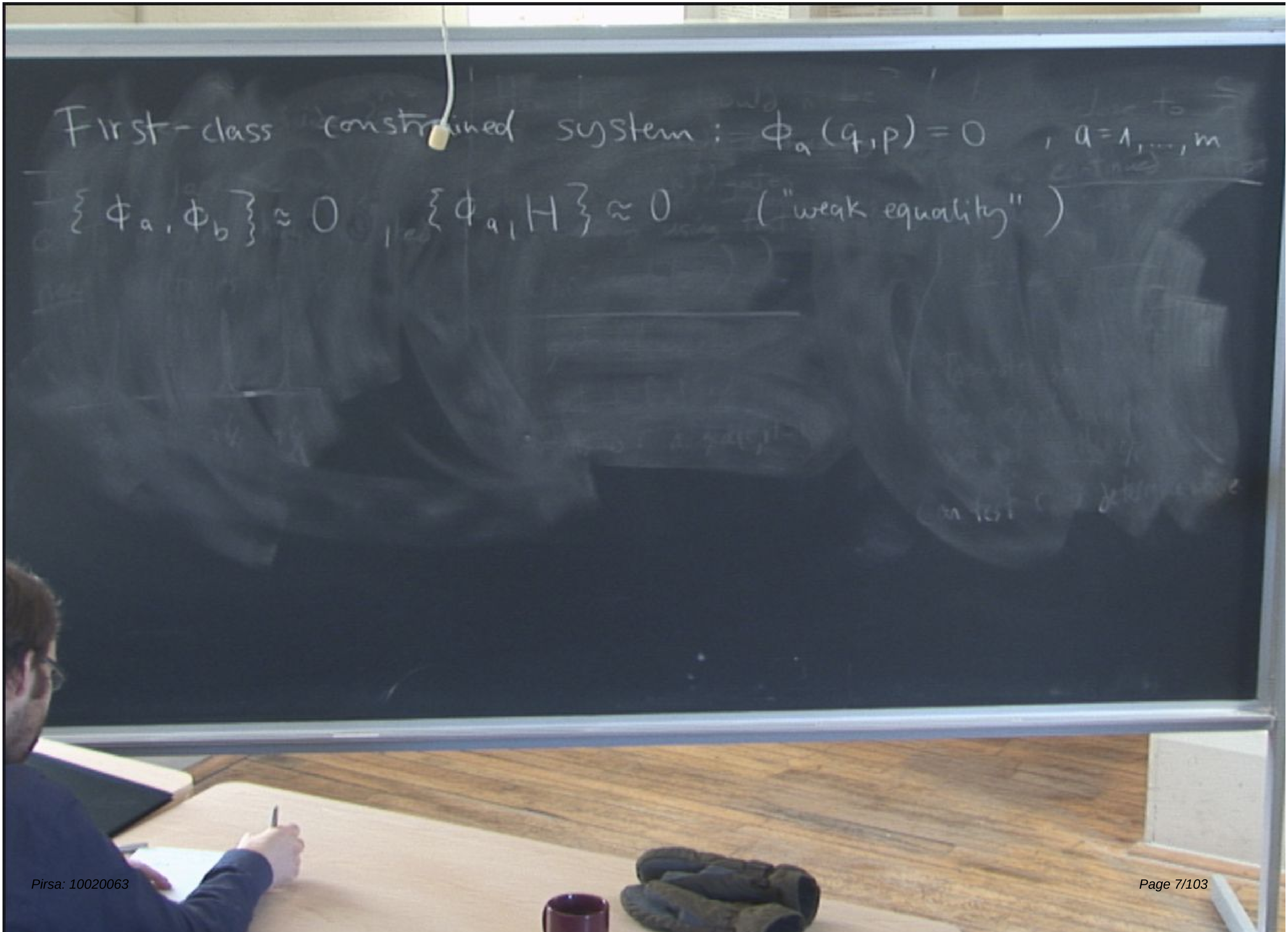
$\{\phi_a, \phi_b\}$


First-class constrained system: $\phi_a(q,p)=0$, $a=1,\dots,m$
$\{\phi_a,\phi_b\}\approx 0$


First-class constrained system: $\phi_a(q,p) = 0$, $a = 1, \ldots, m$
$\{\phi_a, \phi_b\} \approx 0$, $\{\phi_a, H\} \approx 0$ ("weak equality")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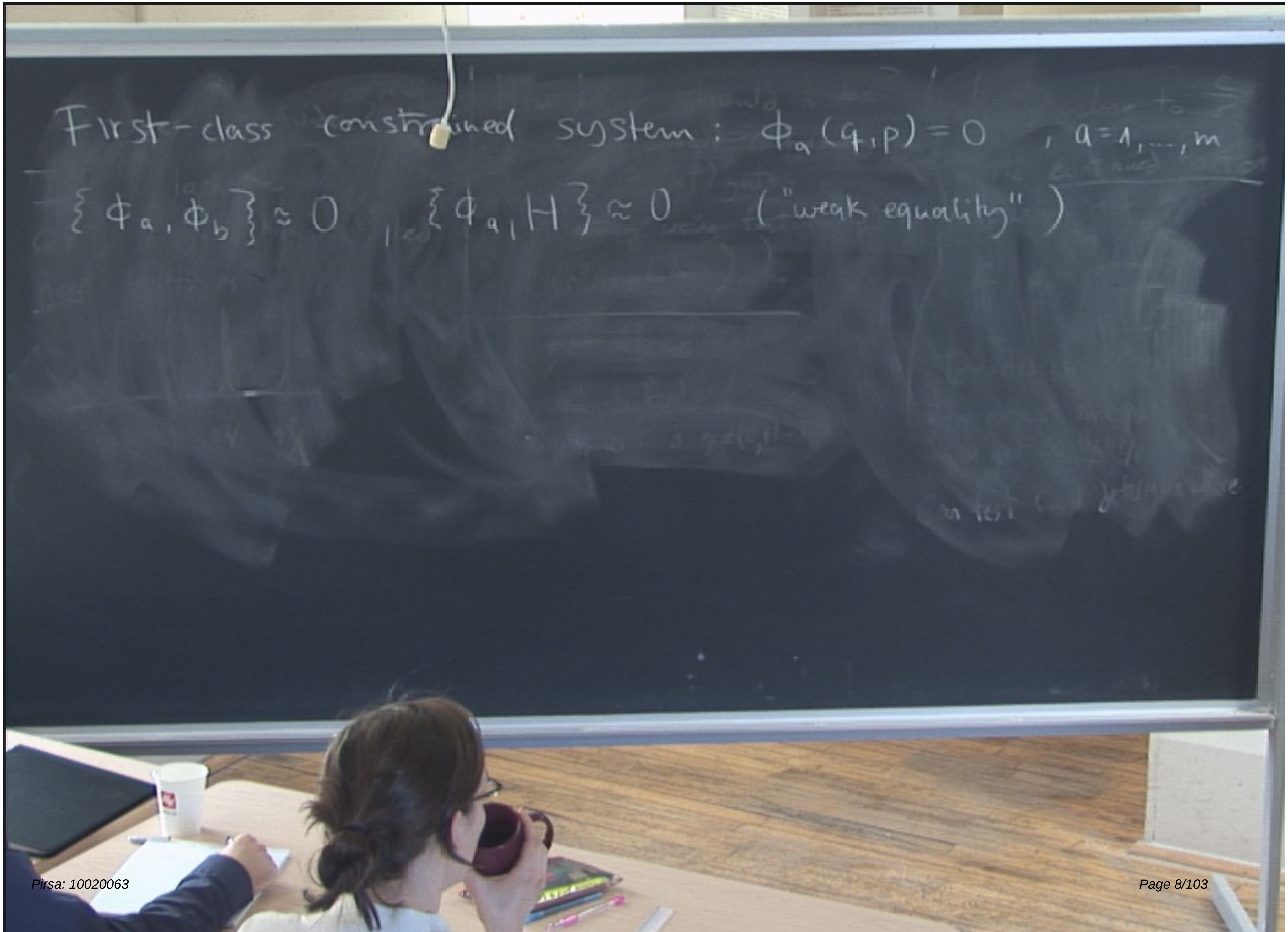
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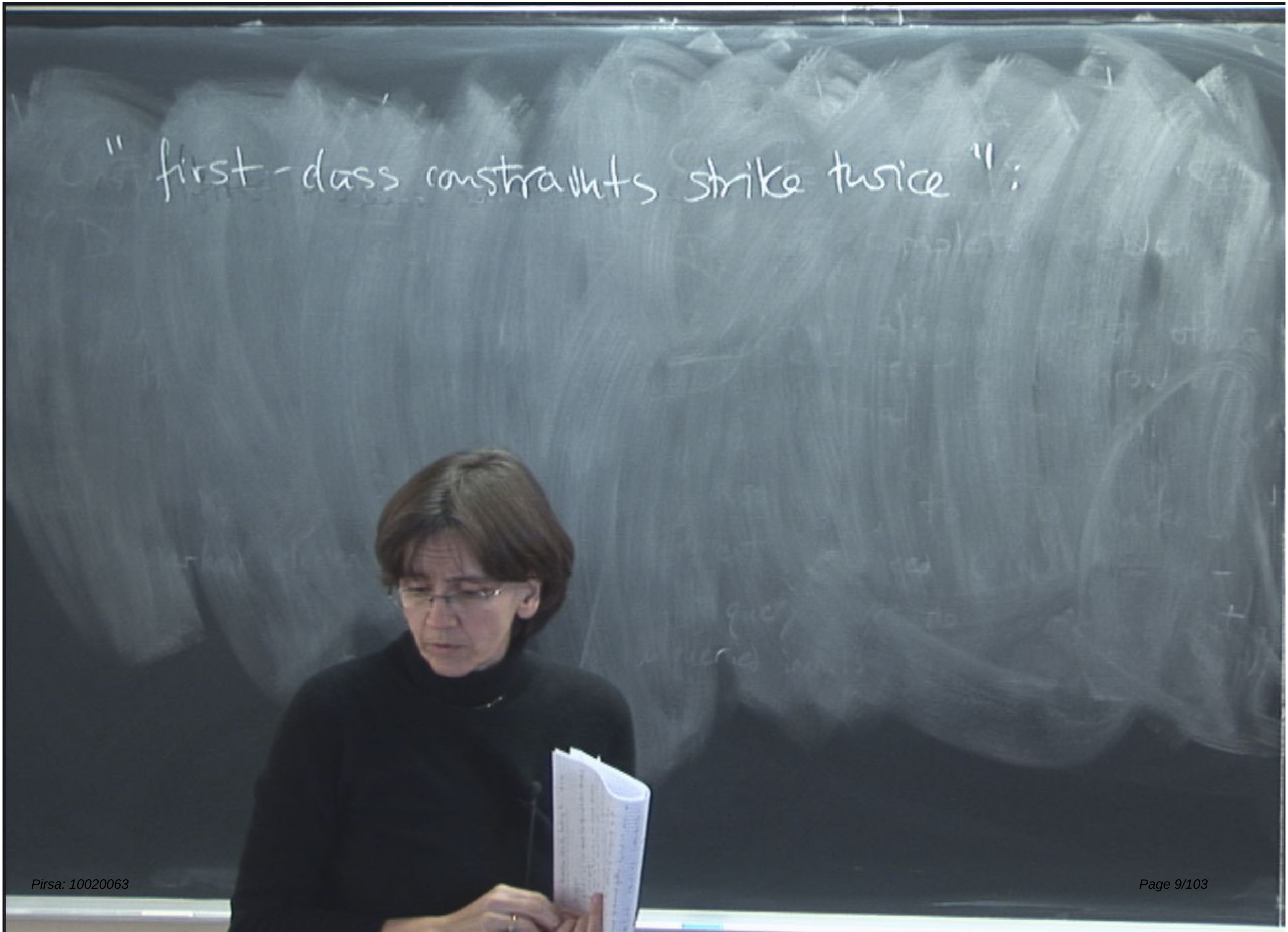
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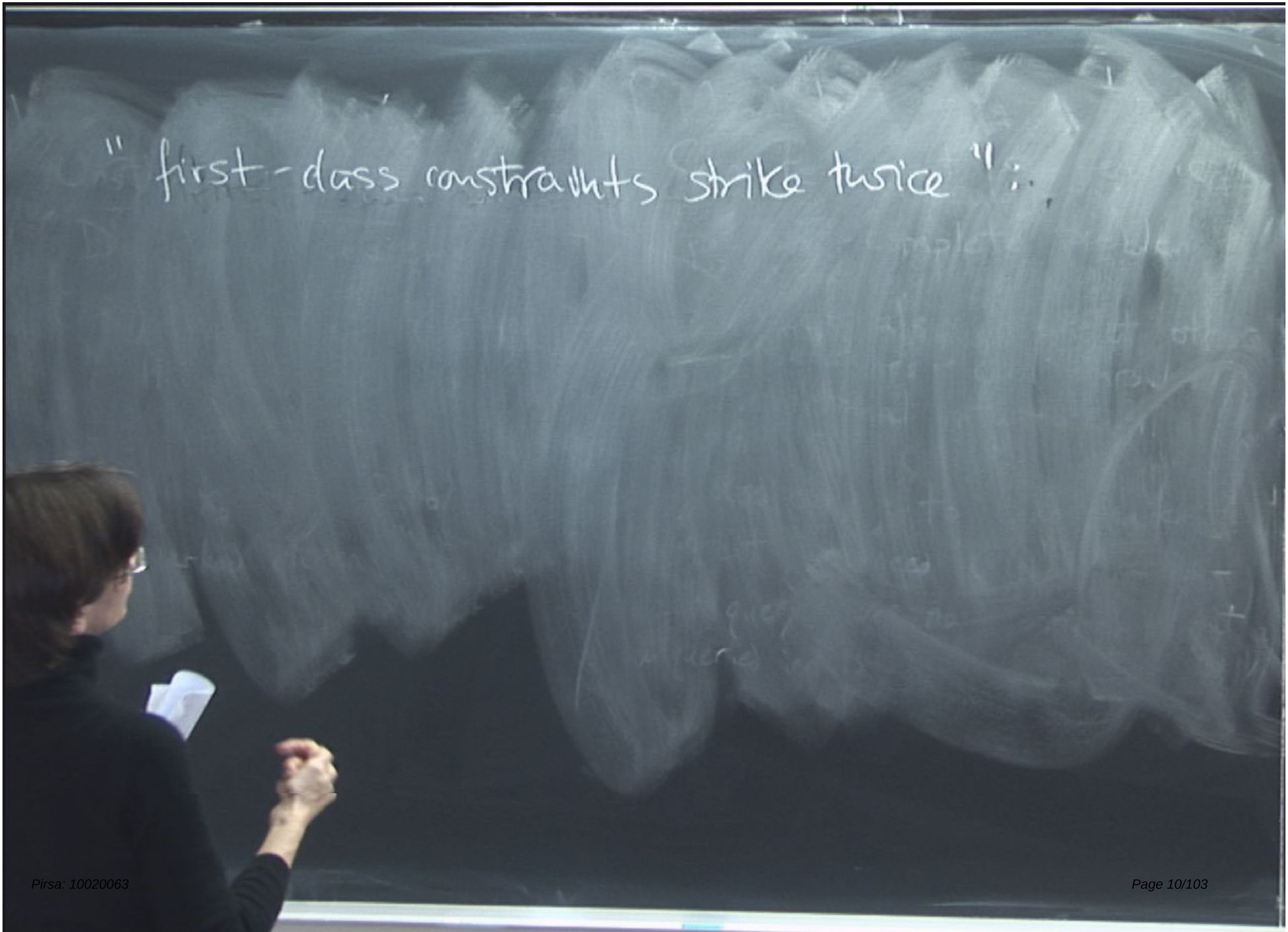
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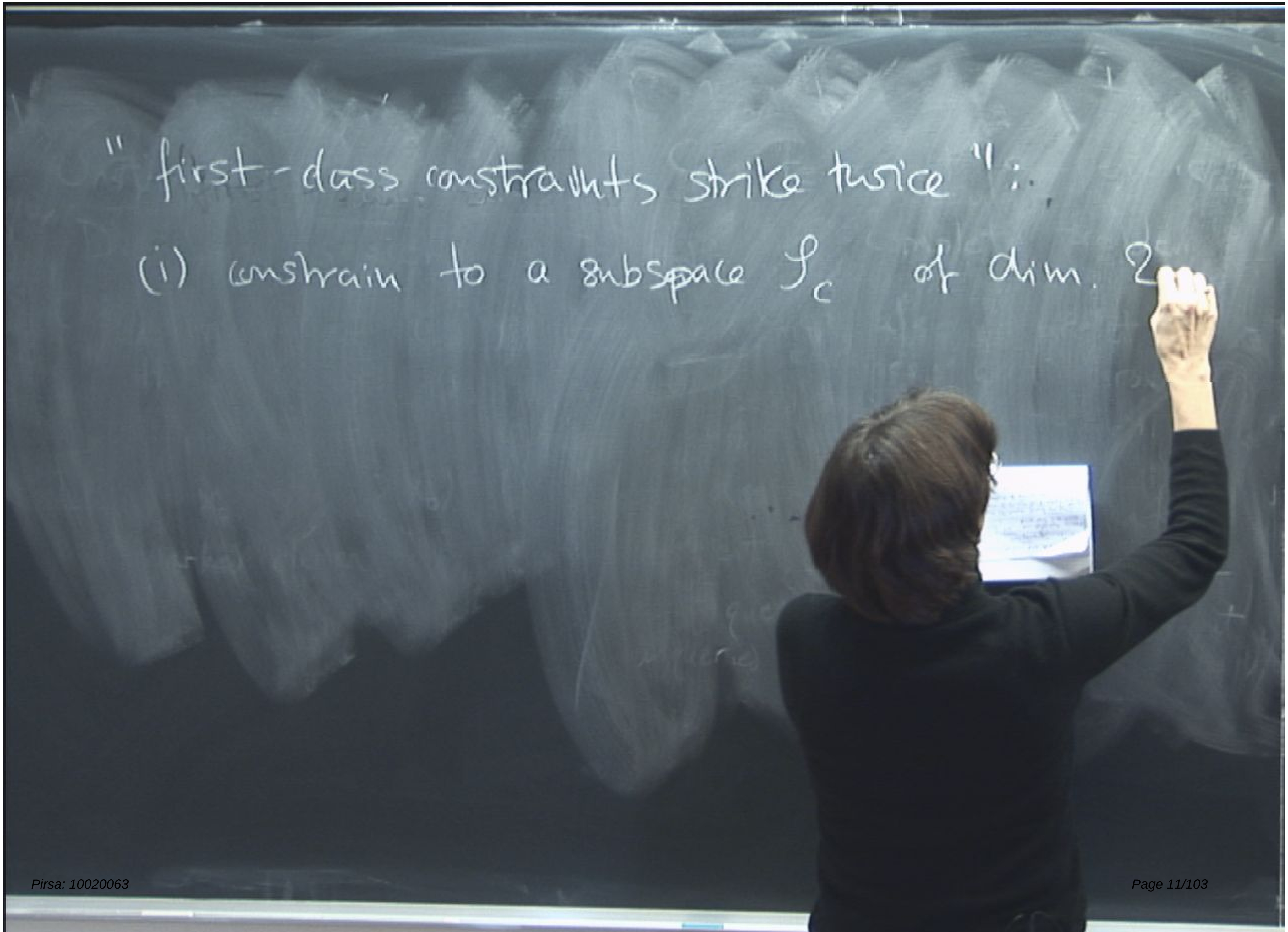
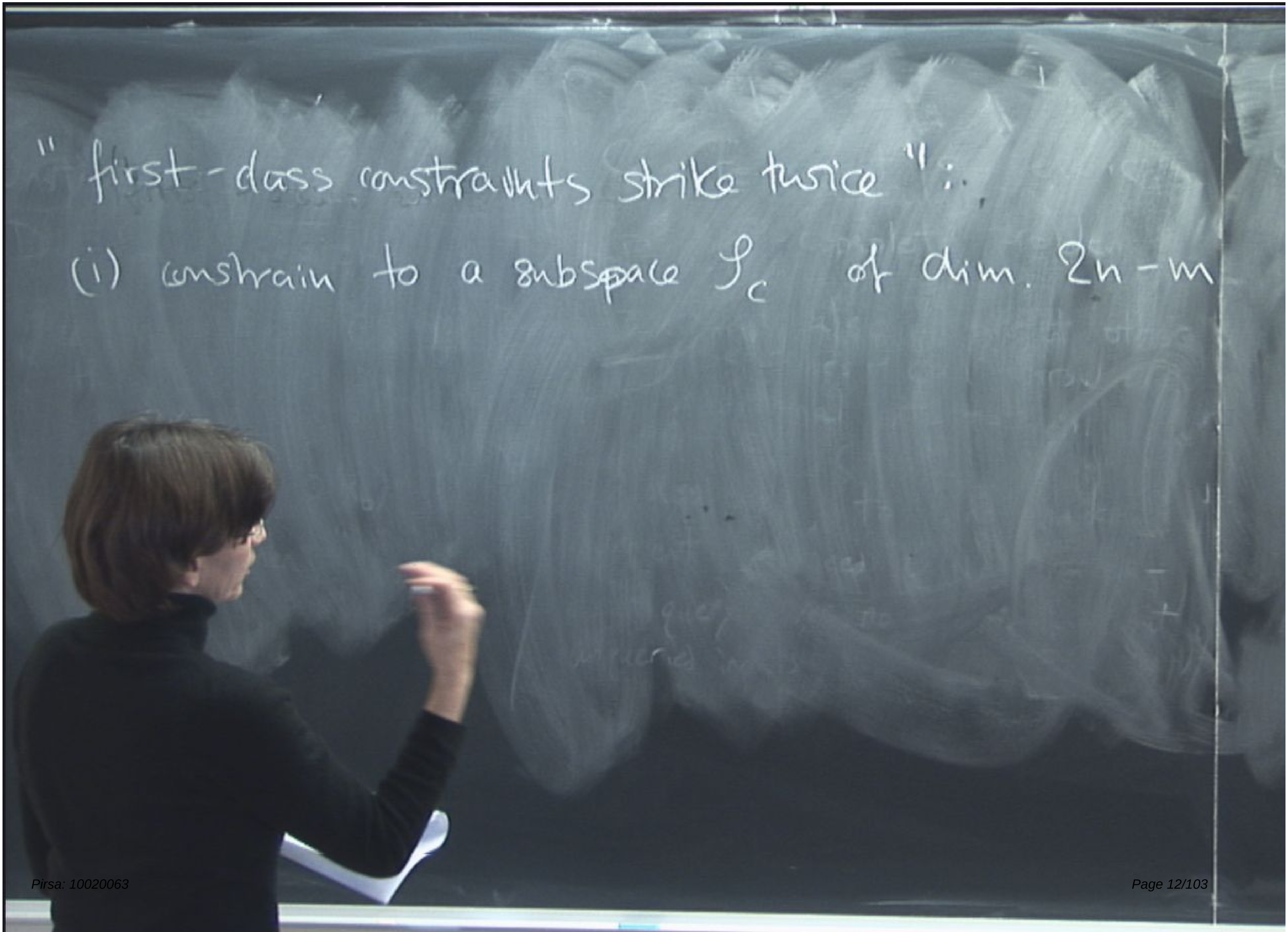

" first-class constraints strike twice ":
(i) constrain to a subspace $\mathcal{S}_c$ of dim. 2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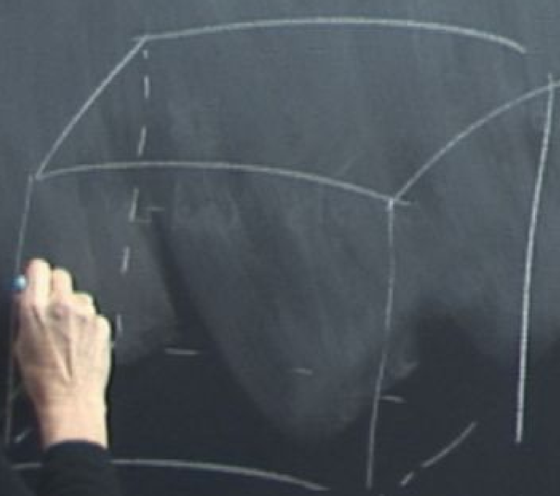

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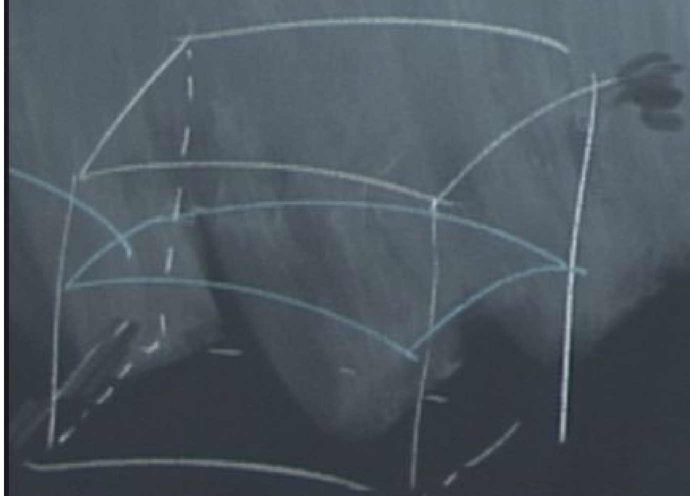

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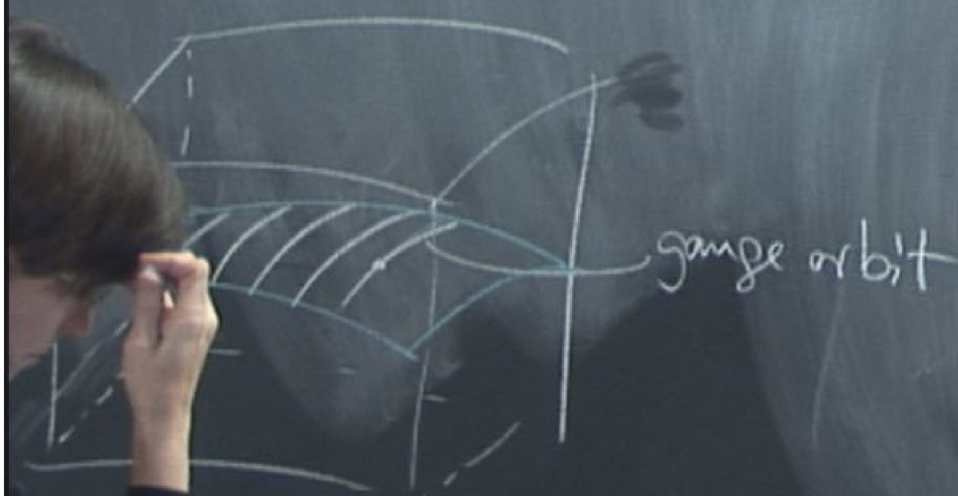

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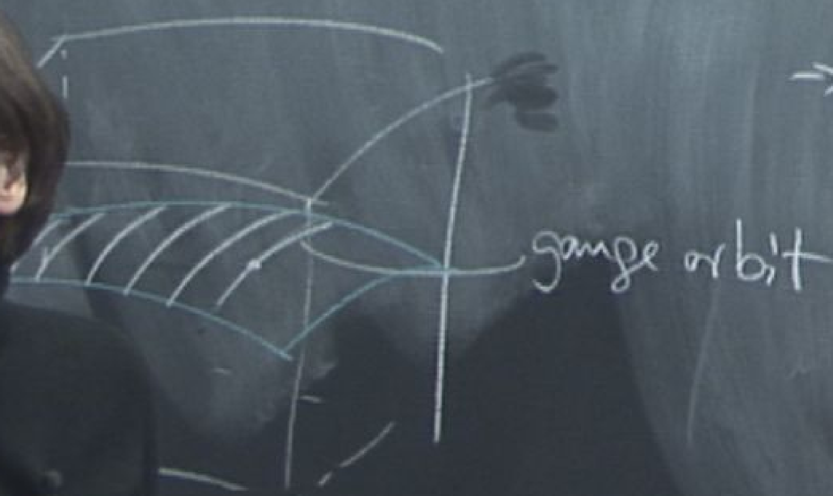
→ points



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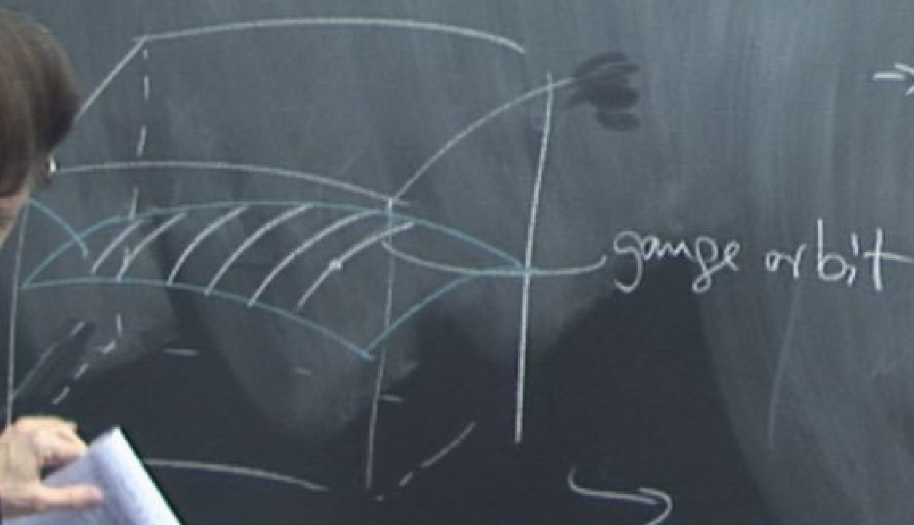
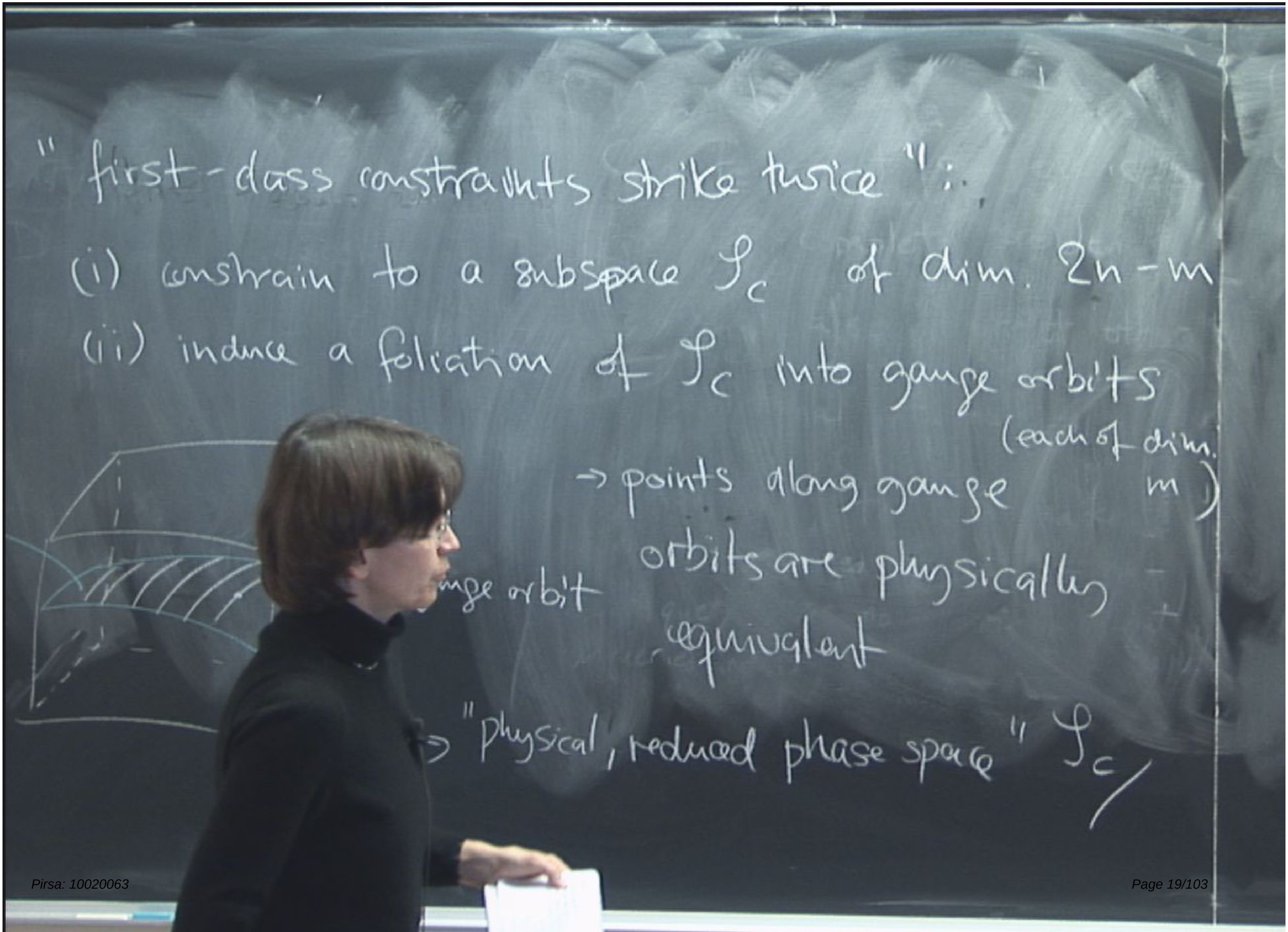


→ points along gauge orbits are physically equivalent


" first-class constraints strike twice ":
(i) constrain to a subspace S_c of dim. 2n - m
(ii) induce a foliation of S_c into gauge orbits
(each of dim. m)
→ points along gauge orbits are physically equivalent
orbit
⇒ "physical, reduced phase space" S_c/

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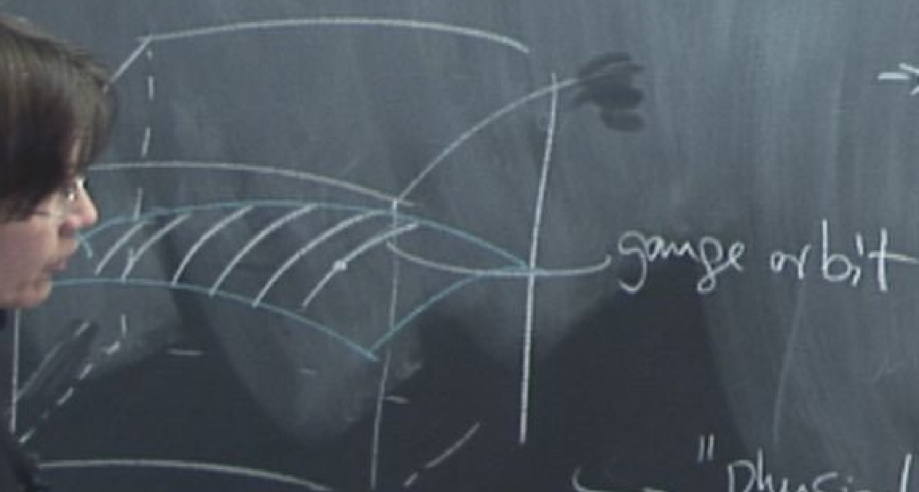

↪ "physical, reduced phase space" $\mathcal{S}_c/\sim$

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↪ "physical, reduced phase space" $\mathcal{S}_c/\sim$ $=: \mathcal{S}_{phys}$

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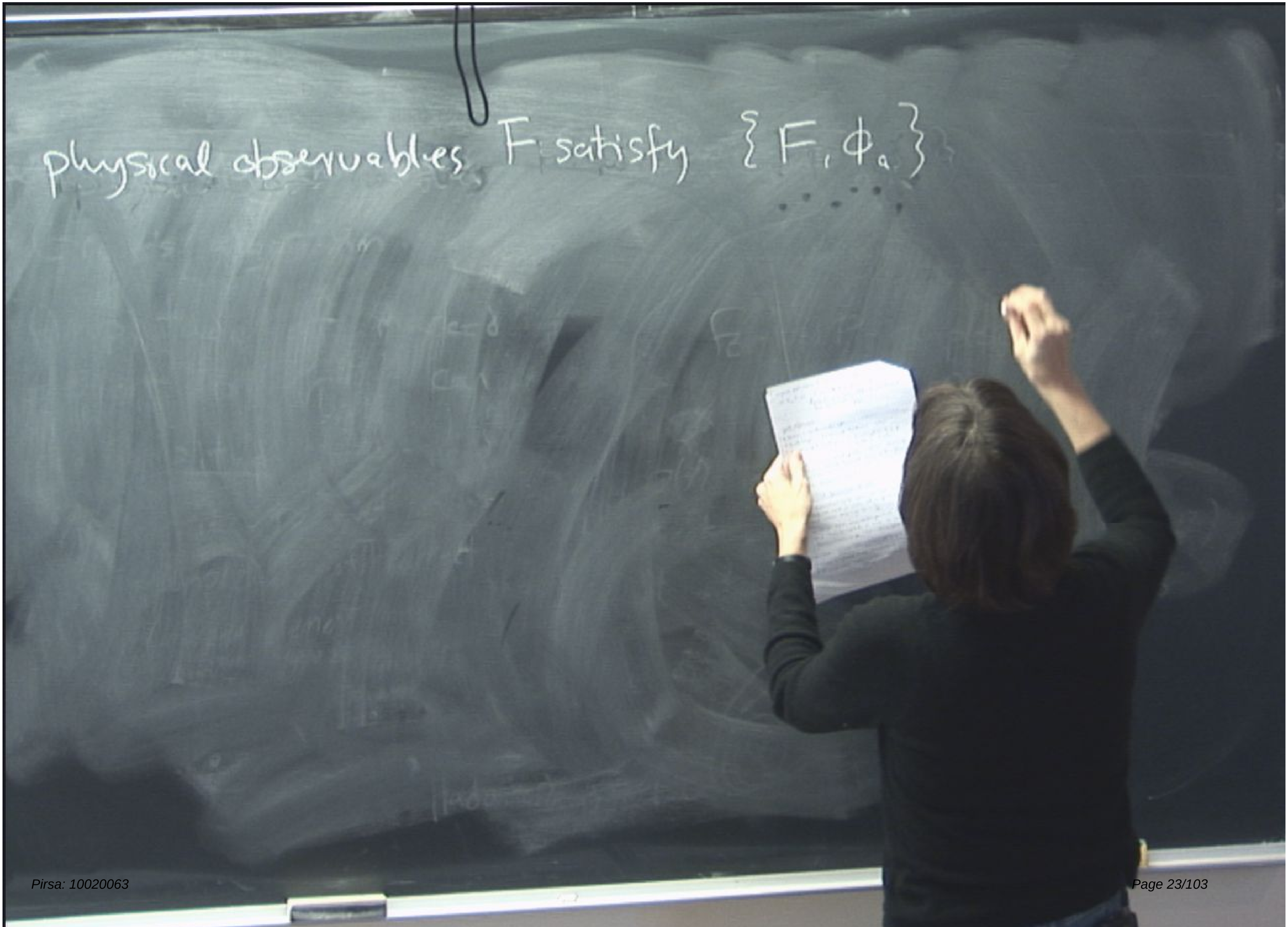
physical observables F satisfy { F, φa }

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ref. P. Dirac : Lectures on QM (1964)

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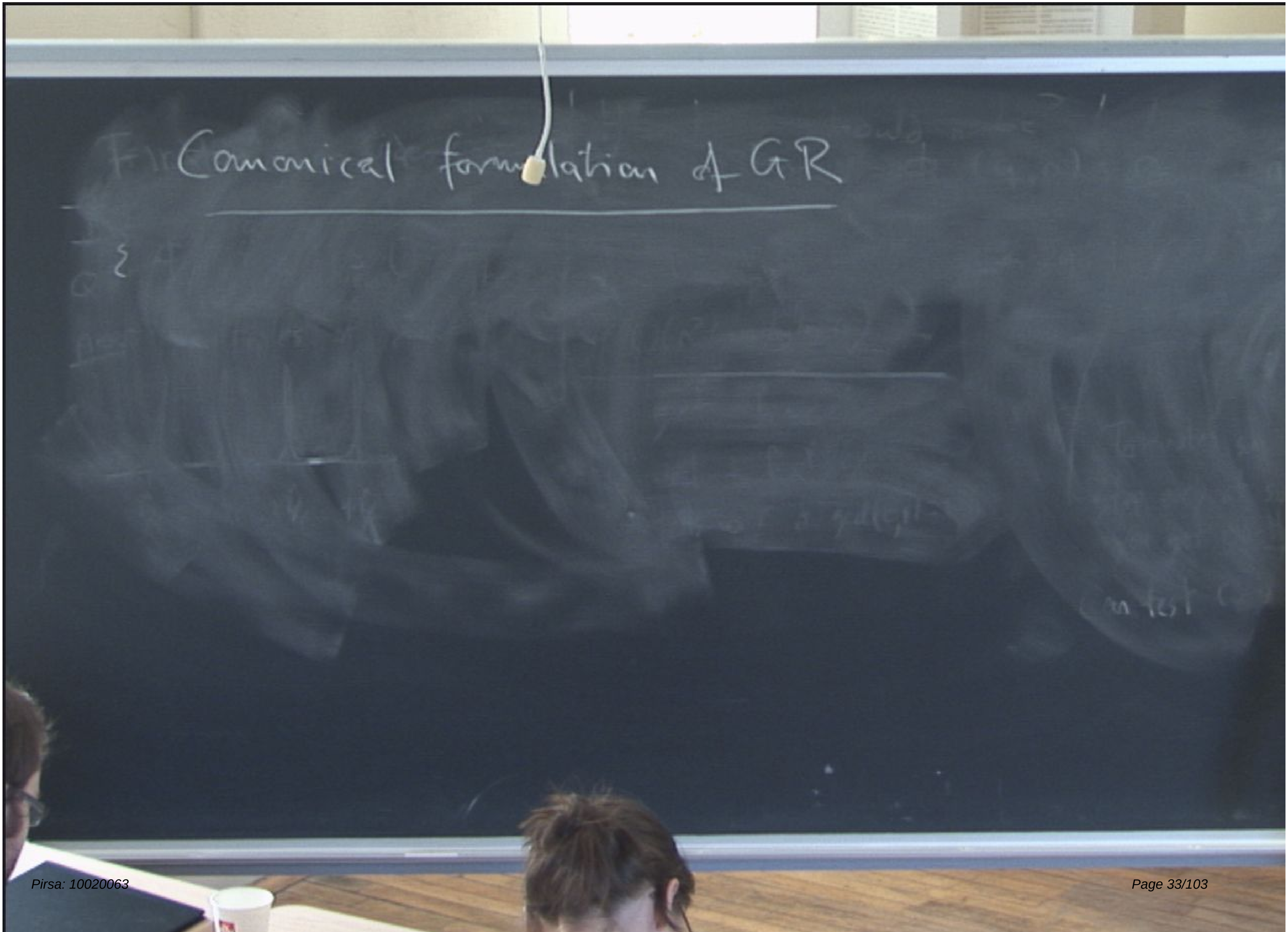
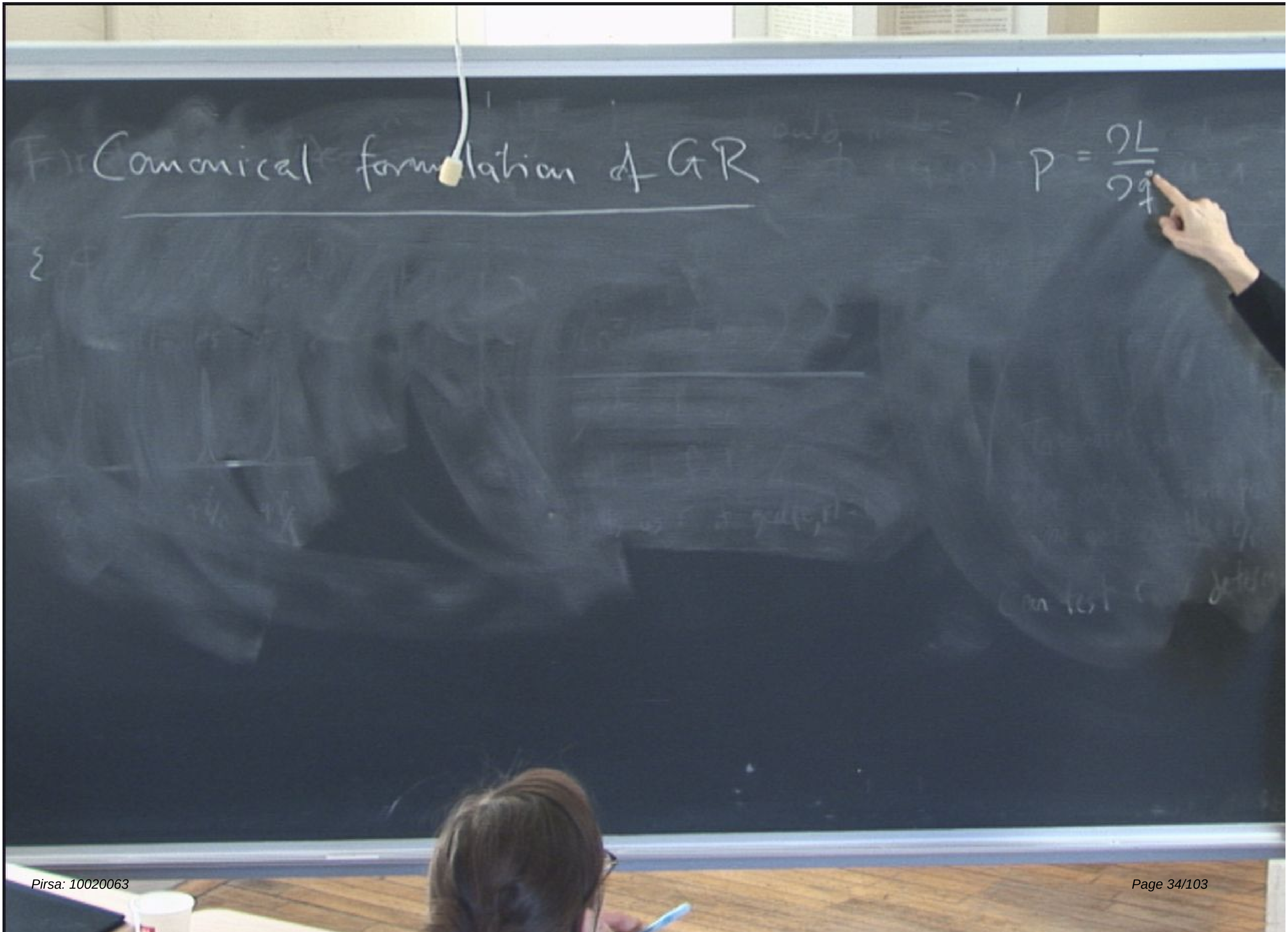
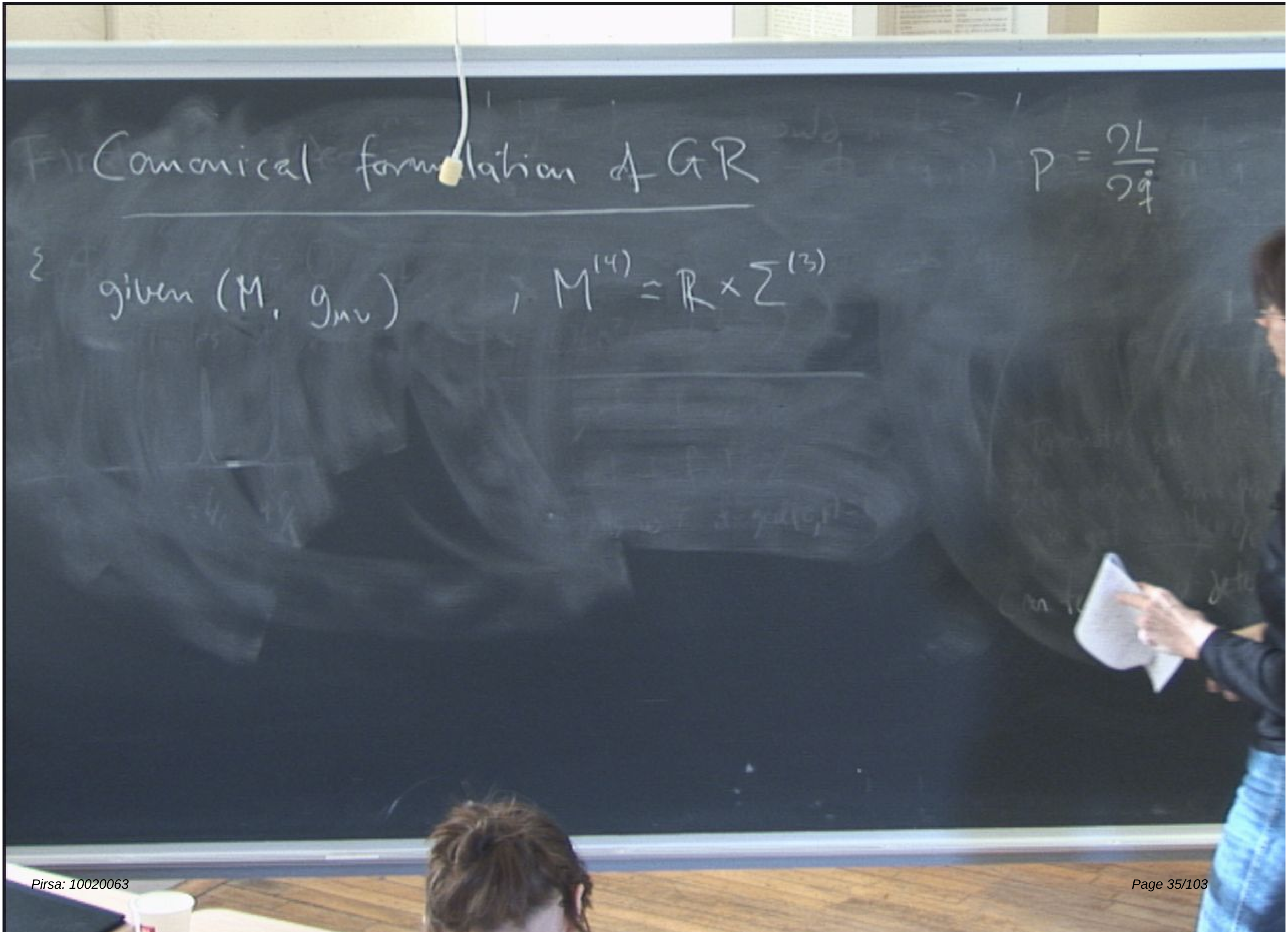
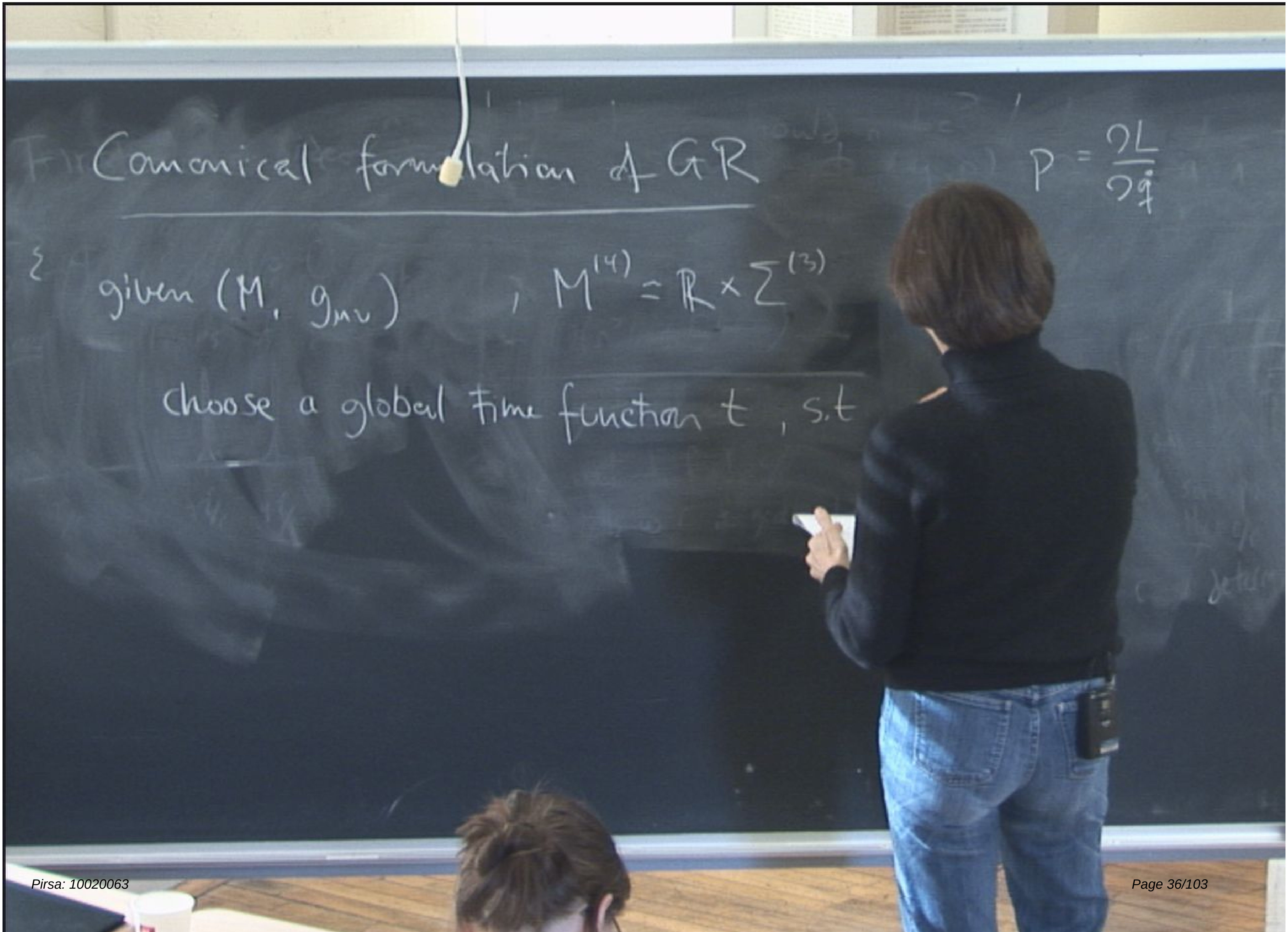
M. Henneaux & C. Teitelboim : Quantization of gauge systems (1994)


Canonical formulation of GR


Canonical formulation of GR
p = ∂L/∂q̇



Canonical formulation of GR
$p = \frac{\partial L}{\partial \dot{q}}$
given $(M, g_{\mu\nu})$, $M^{(4)} = \mathbb{R} \times \Sigma^{(3)}$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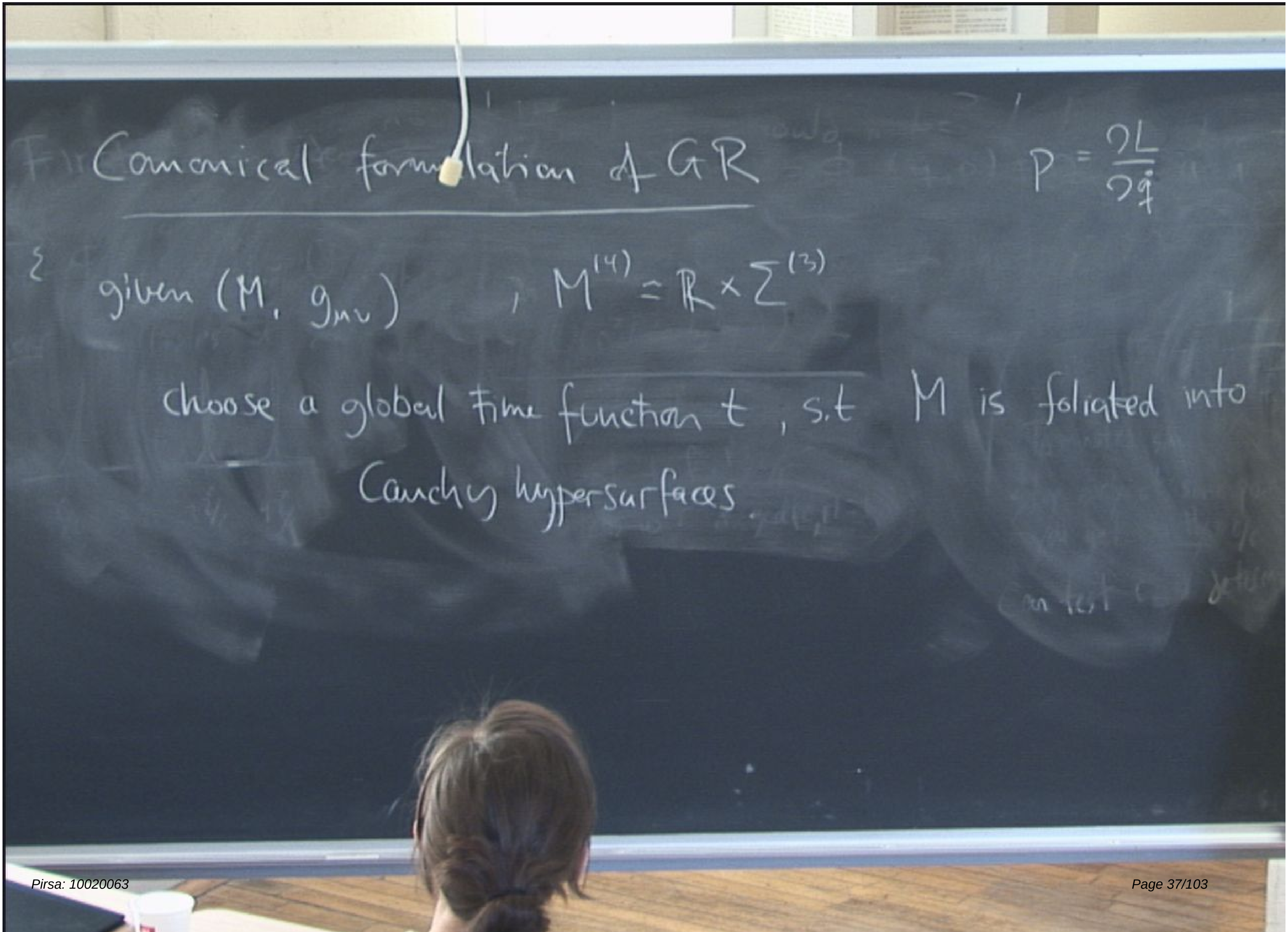
choose a global time function t , s.t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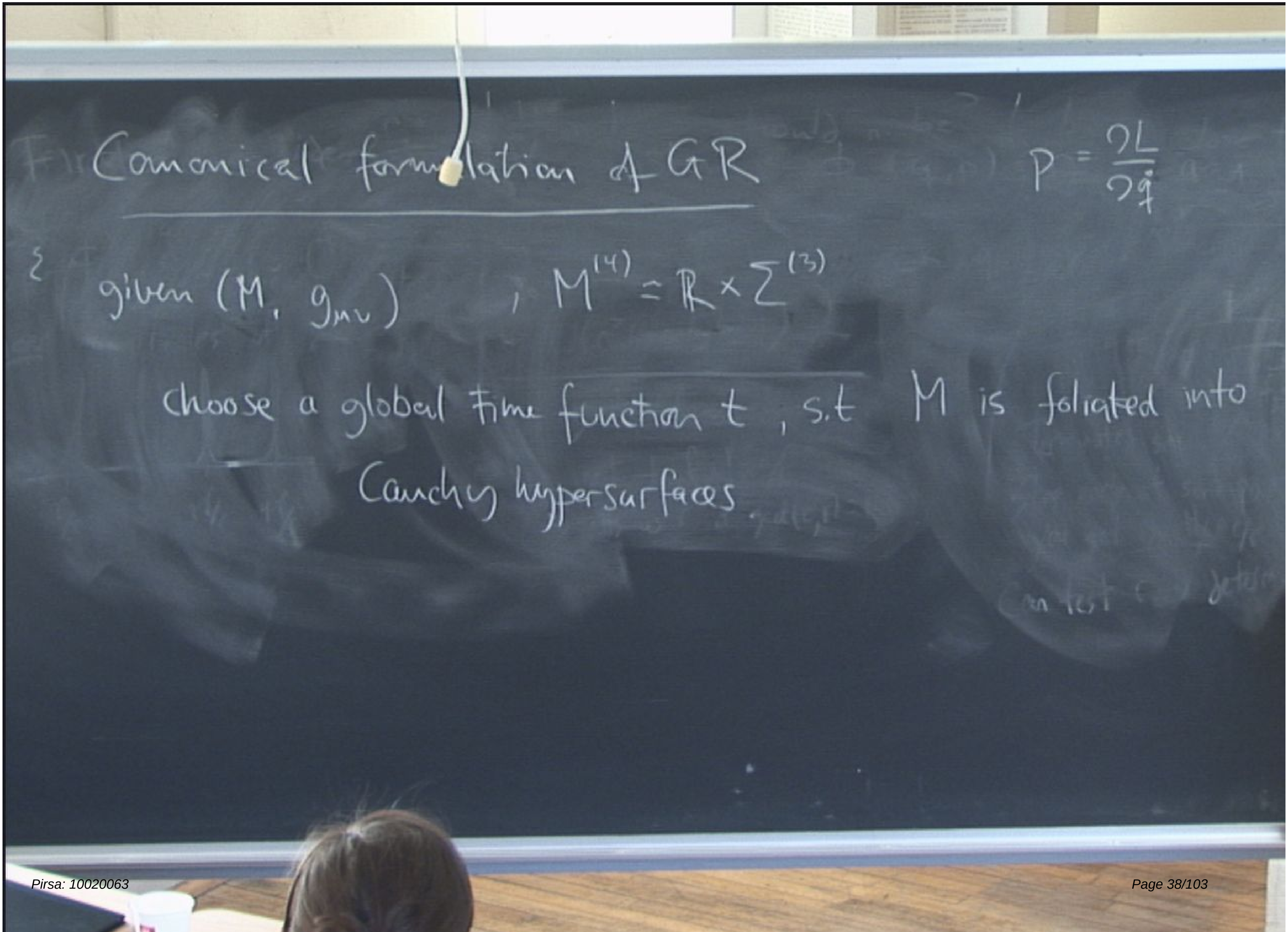
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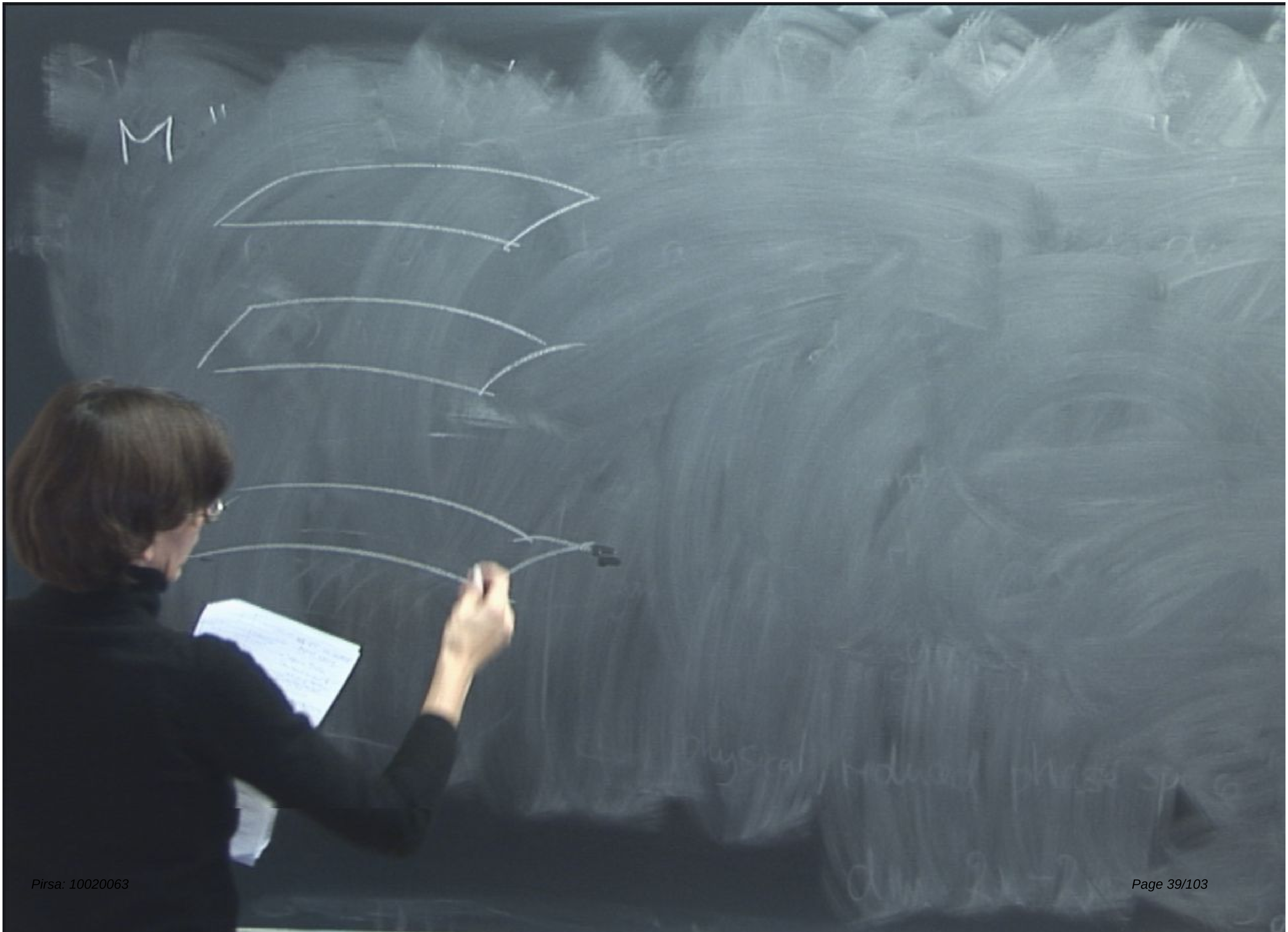
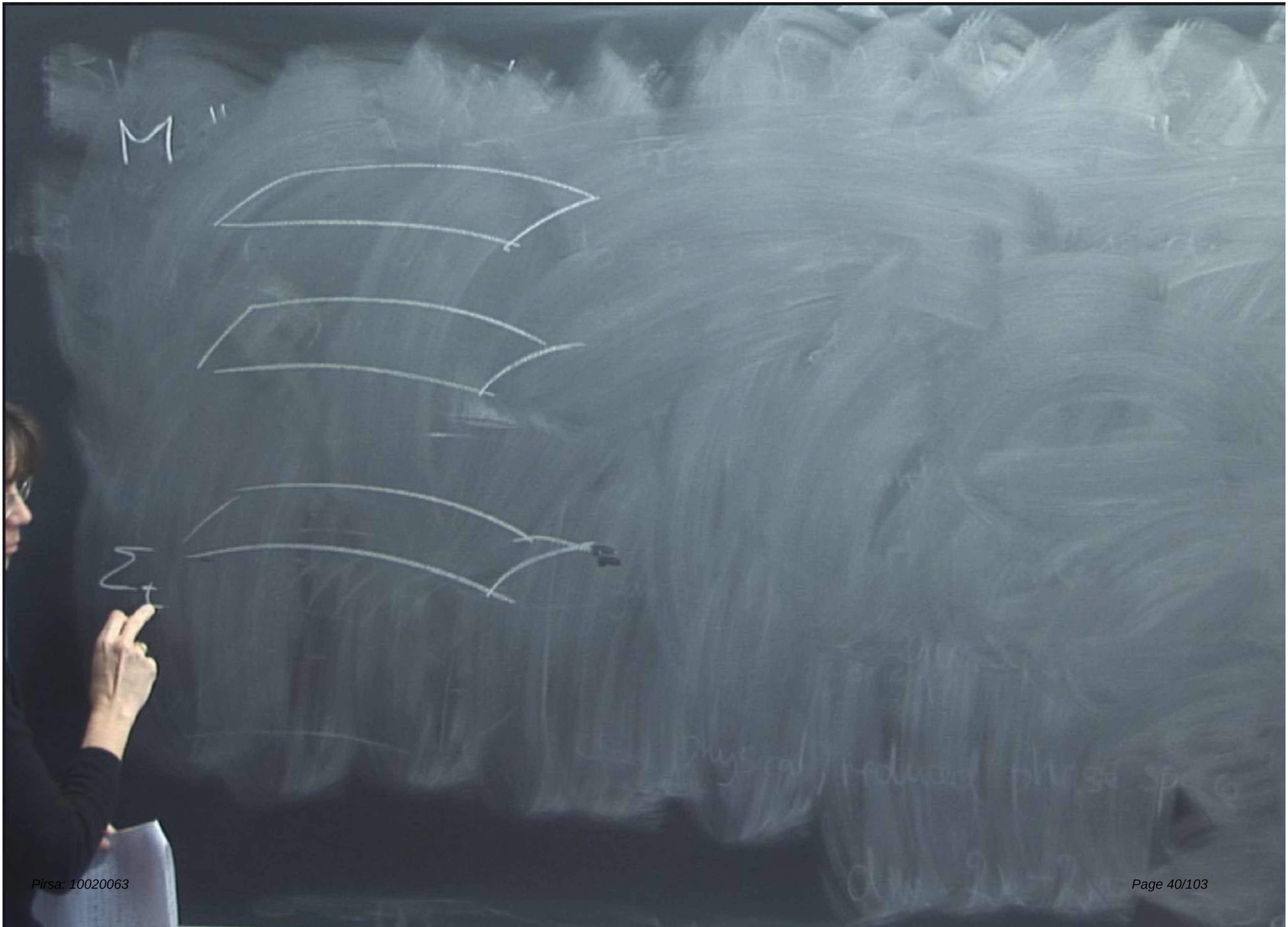
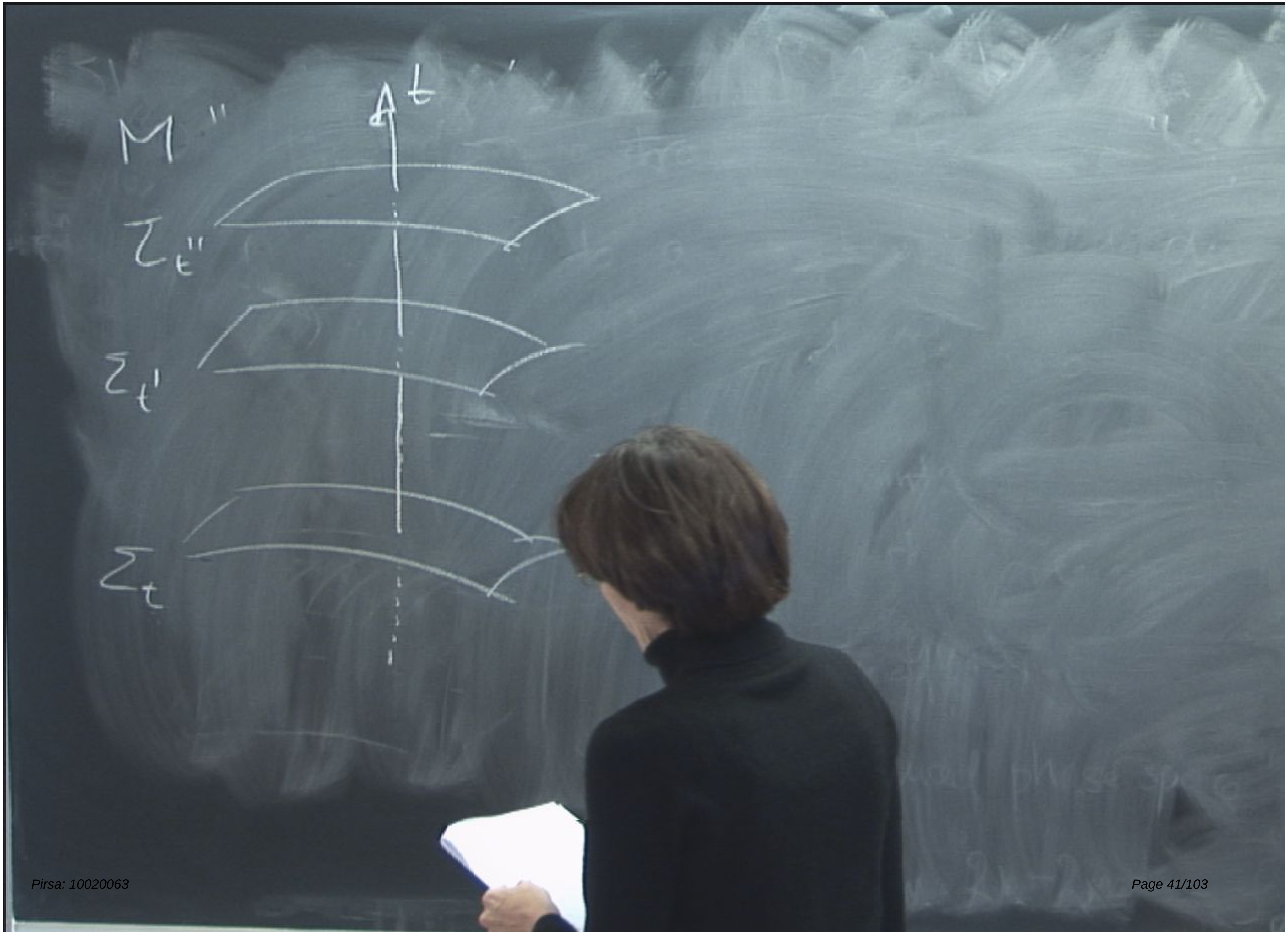
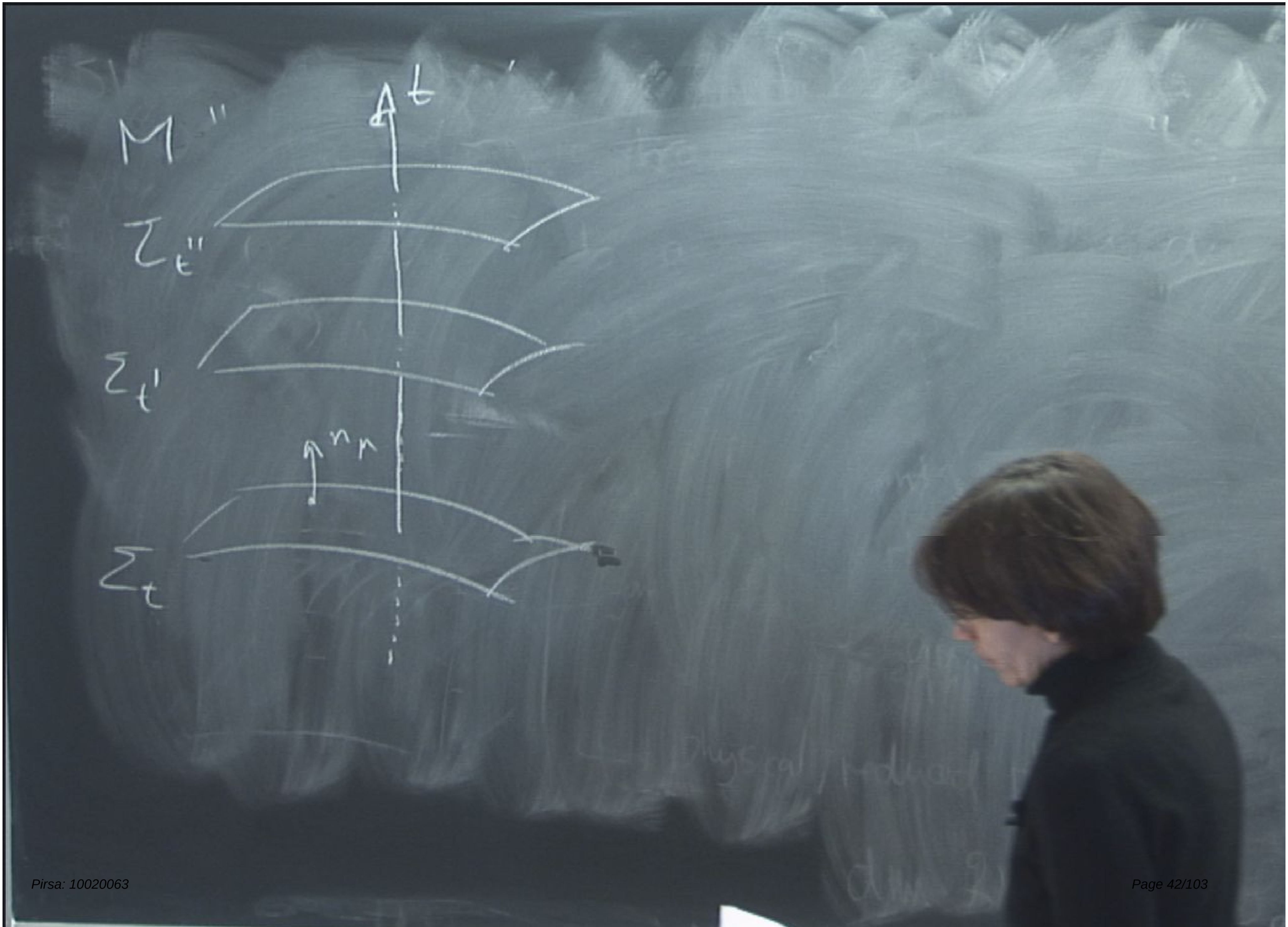
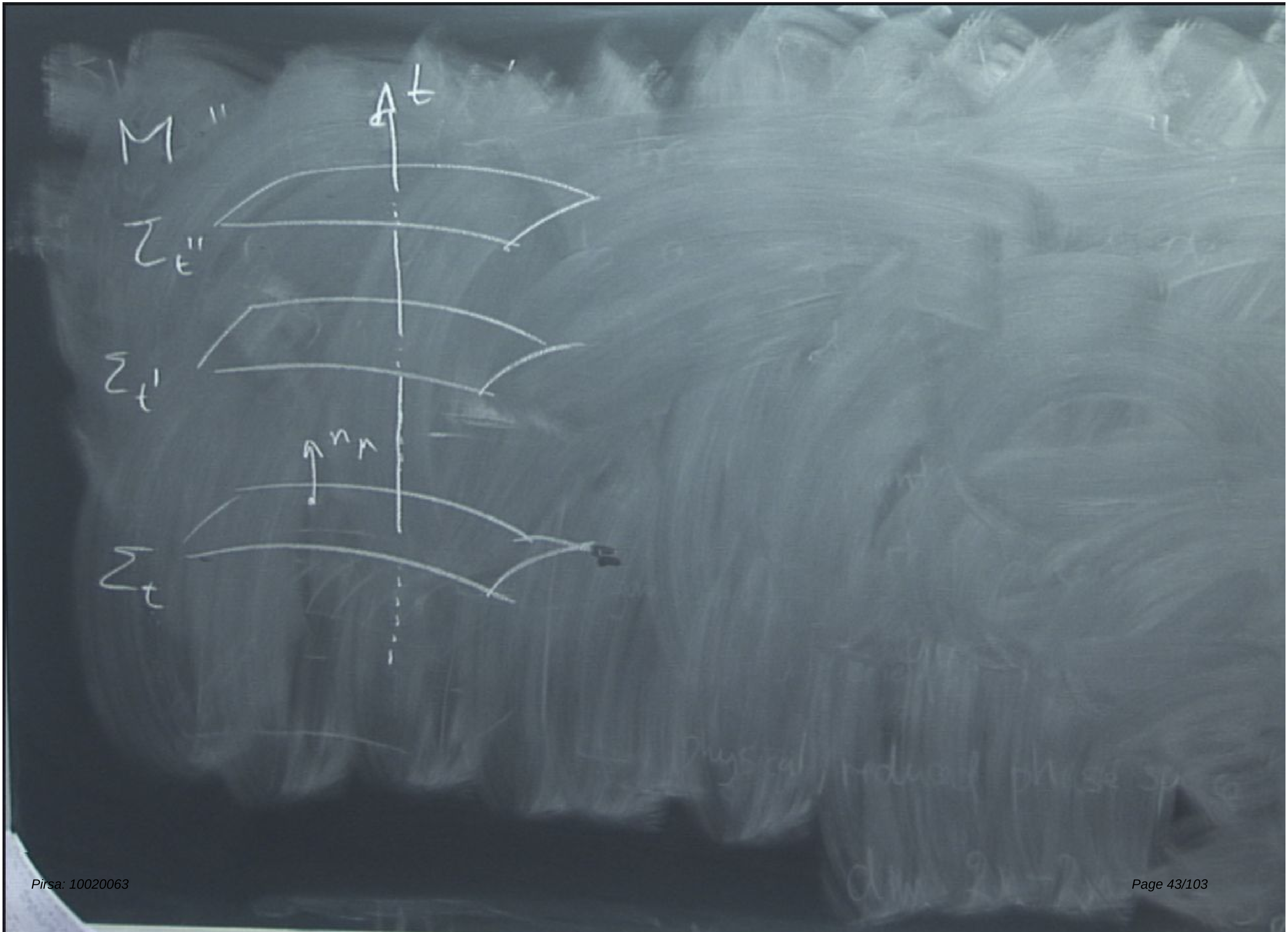
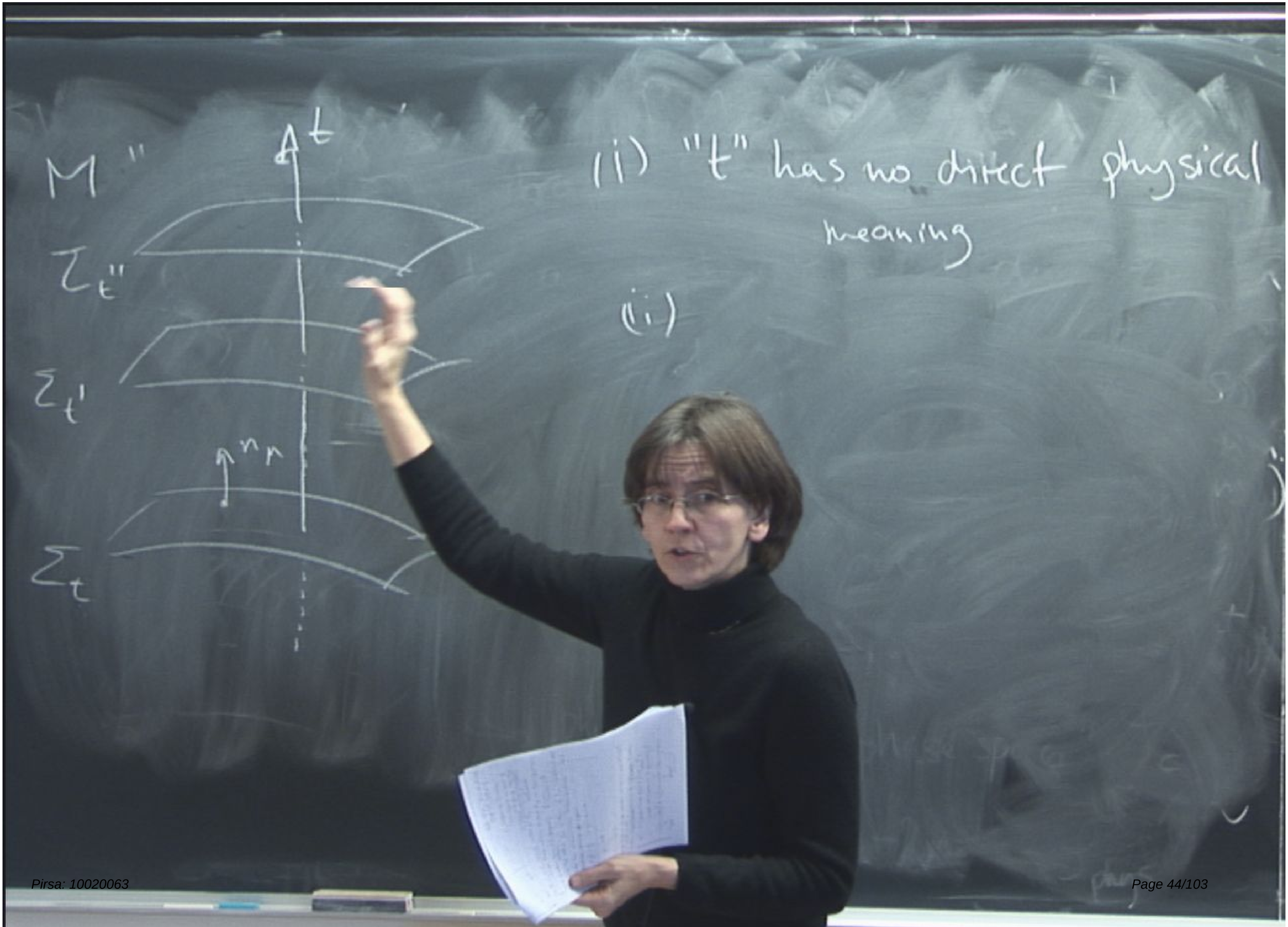

M

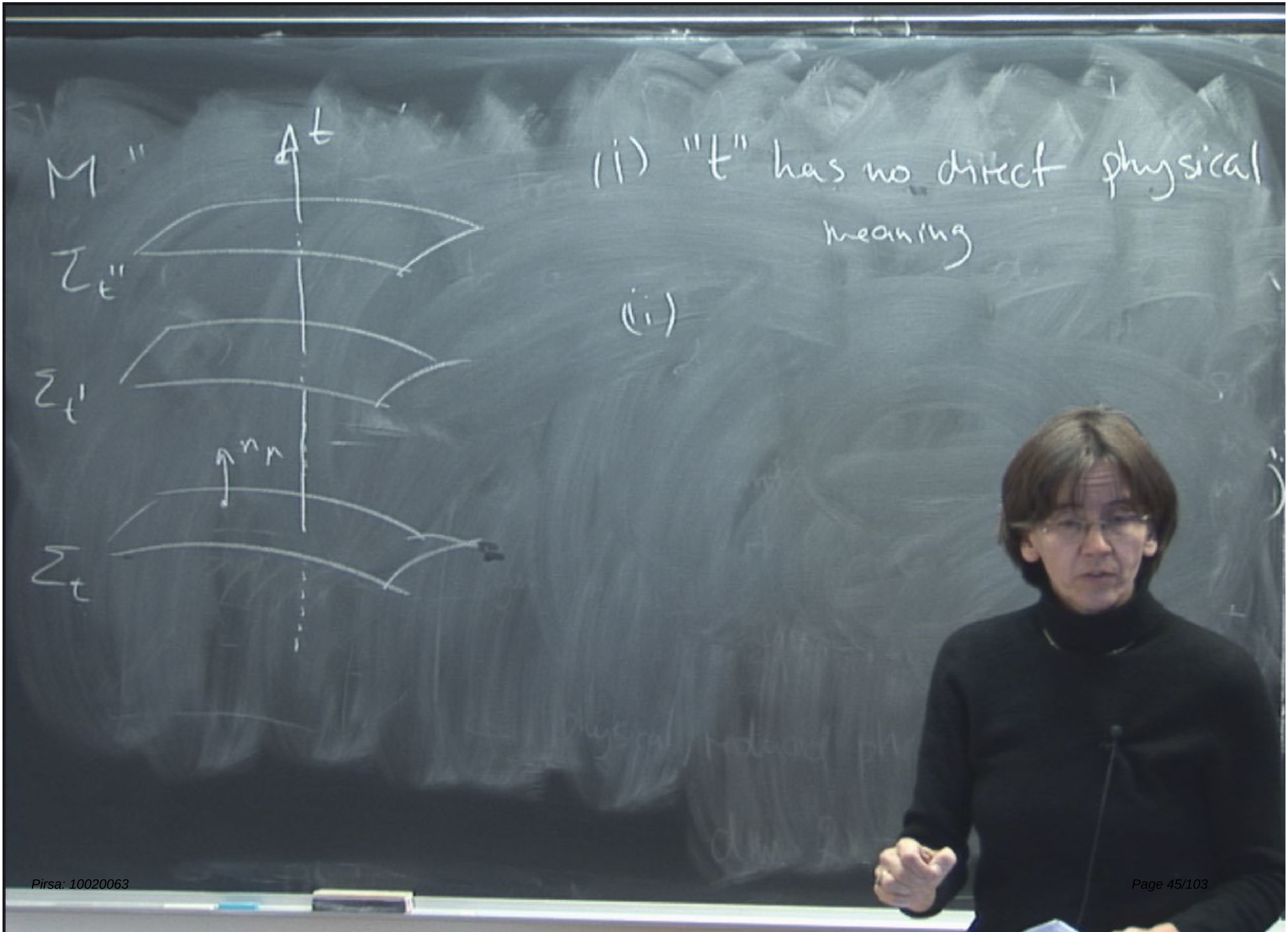

M
Σ_t

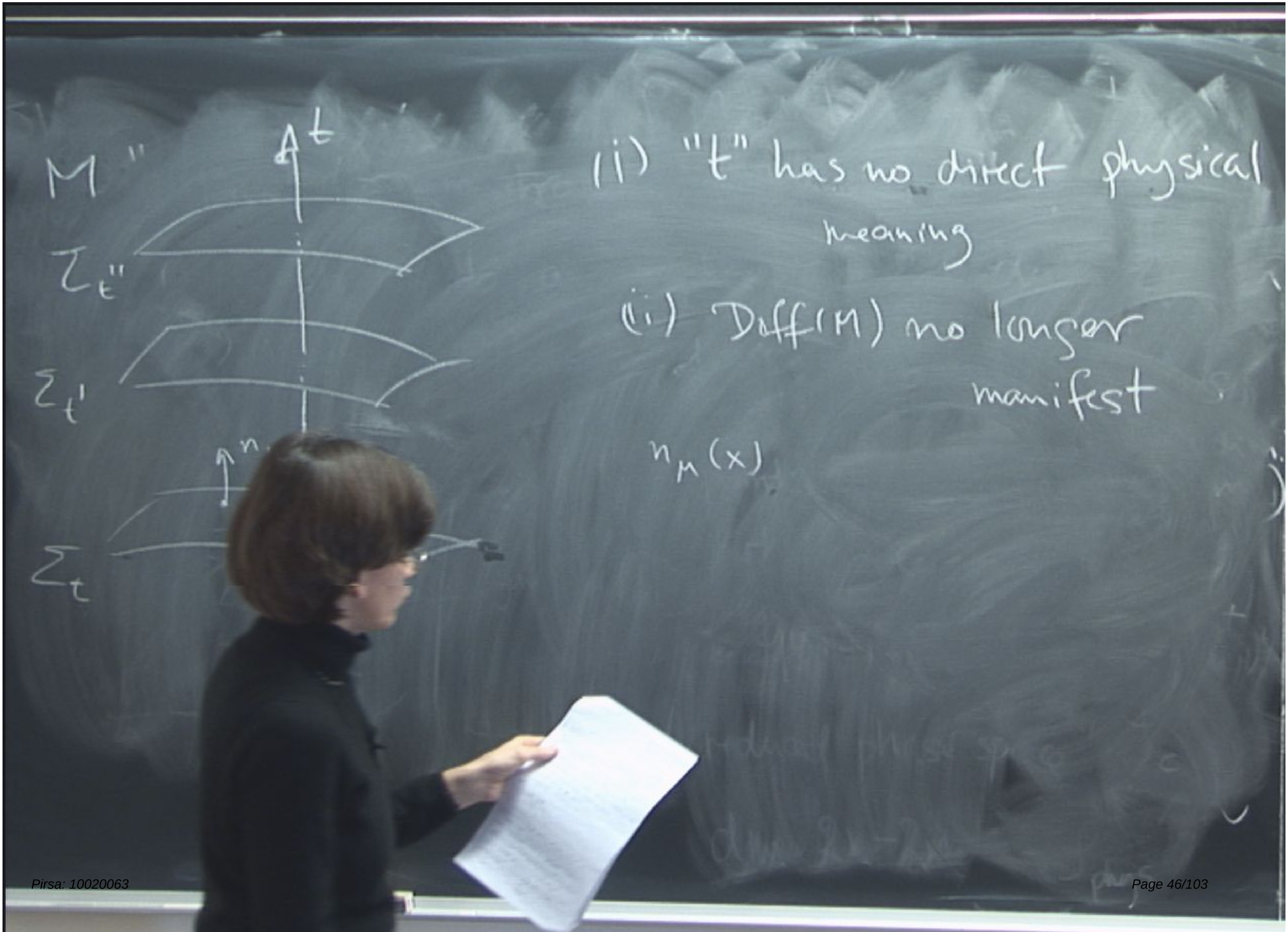

M
t
$\Sigma_{t''}$
$\Sigma_{t'}$
Σ_t

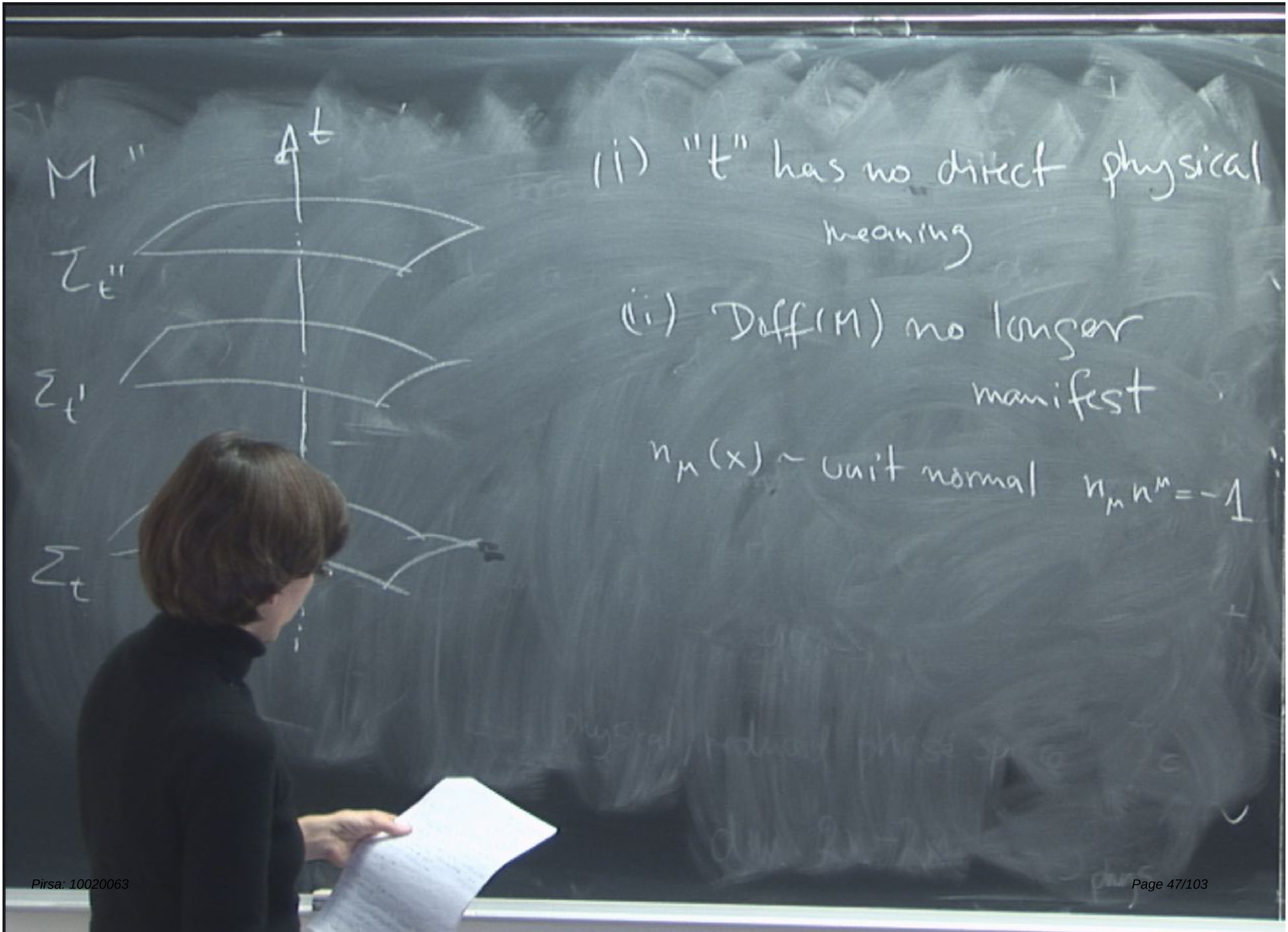

M
t
$\Sigma_{t''}$
$\Sigma_{t'}$
n_μ
Σ_t

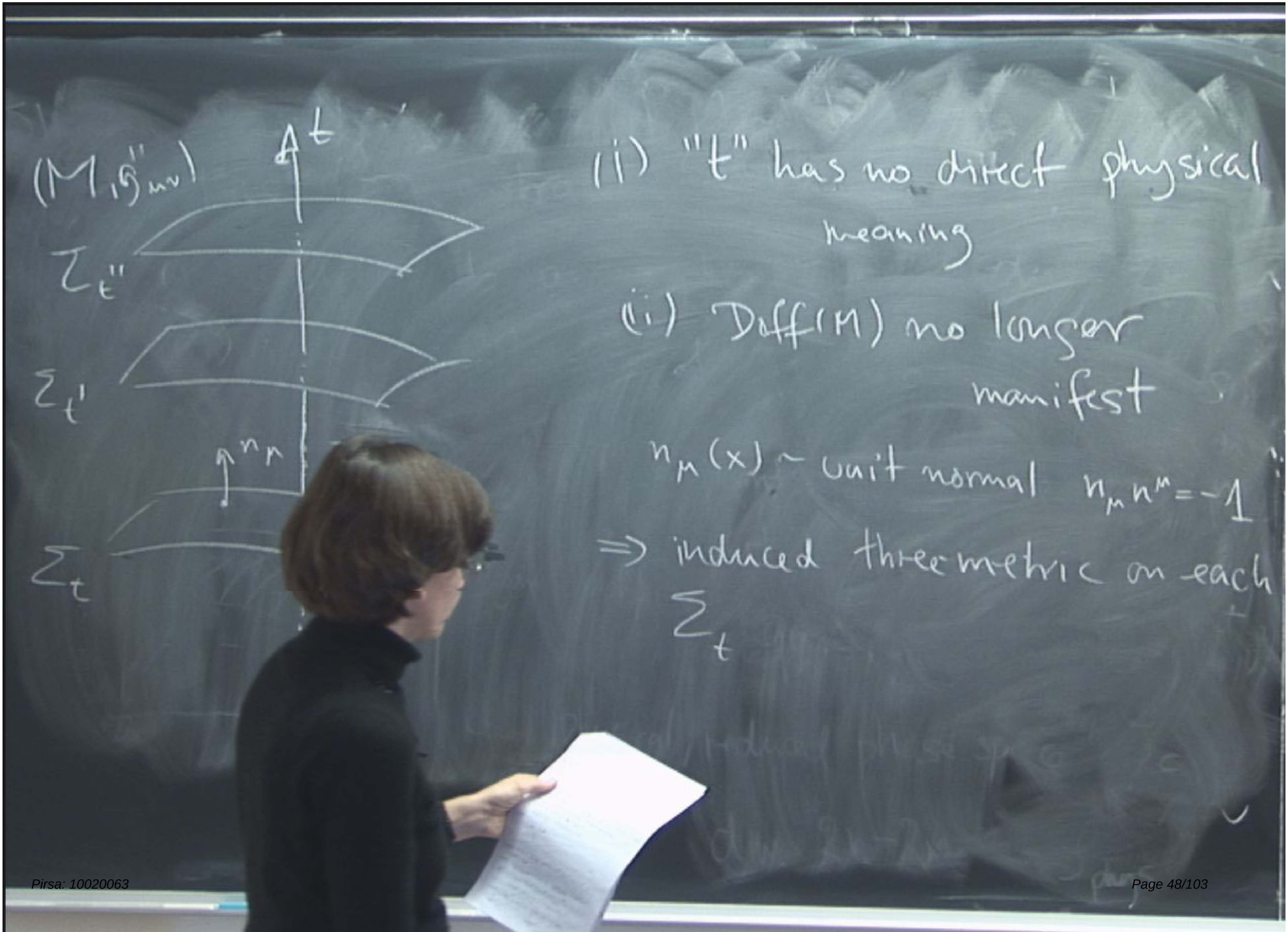

M
t
Σ_{t''}
Σ_{t'}
n_μ
Σ_t

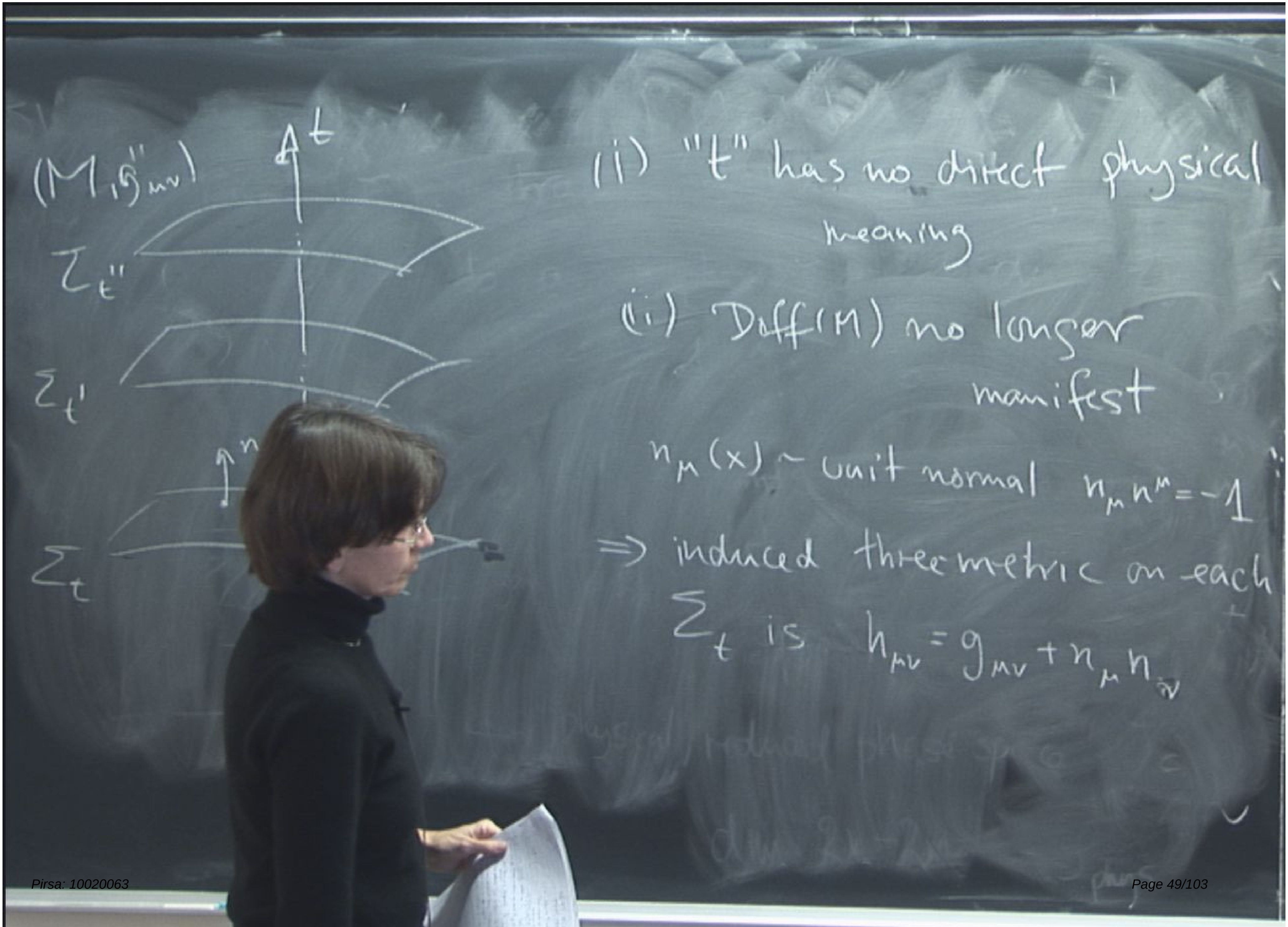

M
t
(i) "t" has no direct physical meaning
(ii)
Σ_t''
Σ_t'
Σ_t
n_μ

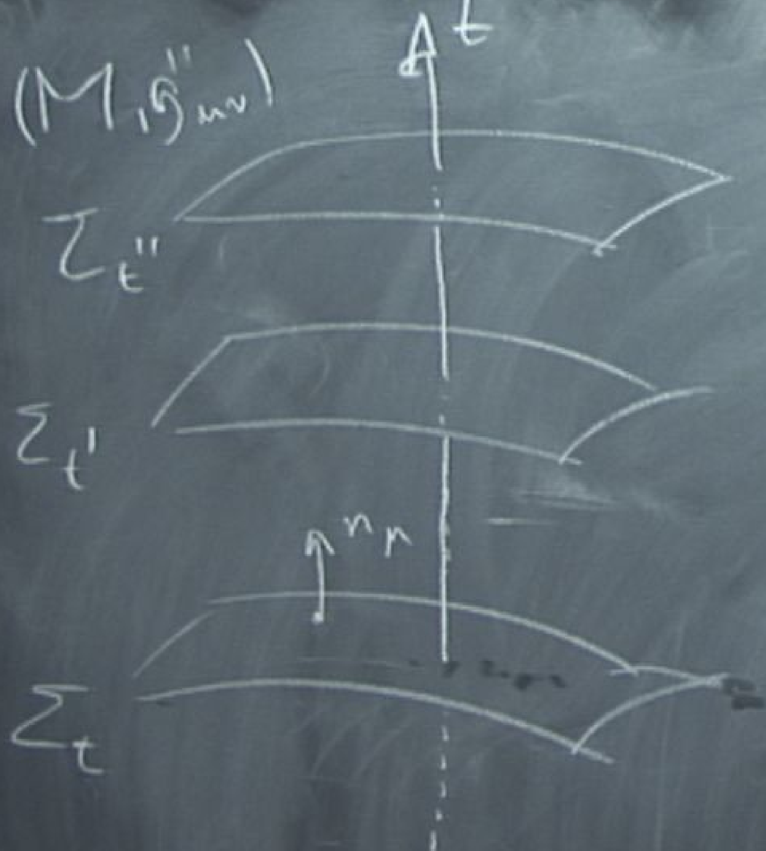

M
t
Σt''
Σt'
Σt
(i) "t" has no direct physical meaning
(ii)


M
t
$\Sigma_{t''}$
$\Sigma_{t'}$
Σ_t
n
(i) "t" has no direct physical meaning
(ii) Diff(M) no longer manifest
$n_\mu(x)$


"M"
t
Σ_{t''}
Σ_{t'}
Σ_t
(i) "t" has no direct physical meaning
(ii) Diff(M) no longer manifest
n_μ(x) ~ unit normal n_μ n^μ = -1


(M, g_μν)
t
Σ_t''
Σ_t'
n_μ
Σ_t
(i) "t" has no direct physical meaning
(ii) Diff(M) no longer manifest
n_μ(x) ~ unit normal n_μ n^μ = -1
⇒ induced three-metric on each Σ_t


$(M, g_{\mu\nu})$
t
$\Sigma_{t''}$
$\Sigma_{t'}$
n
Σ_t
(i) "t" has no direct physical meaning
(ii) Diff(M) no longer manifest
$n_\mu(x)$ ~ unit normal $n_\mu n^\mu = -1$
$\Rightarrow$ induced three-metric on each Σ_t is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$



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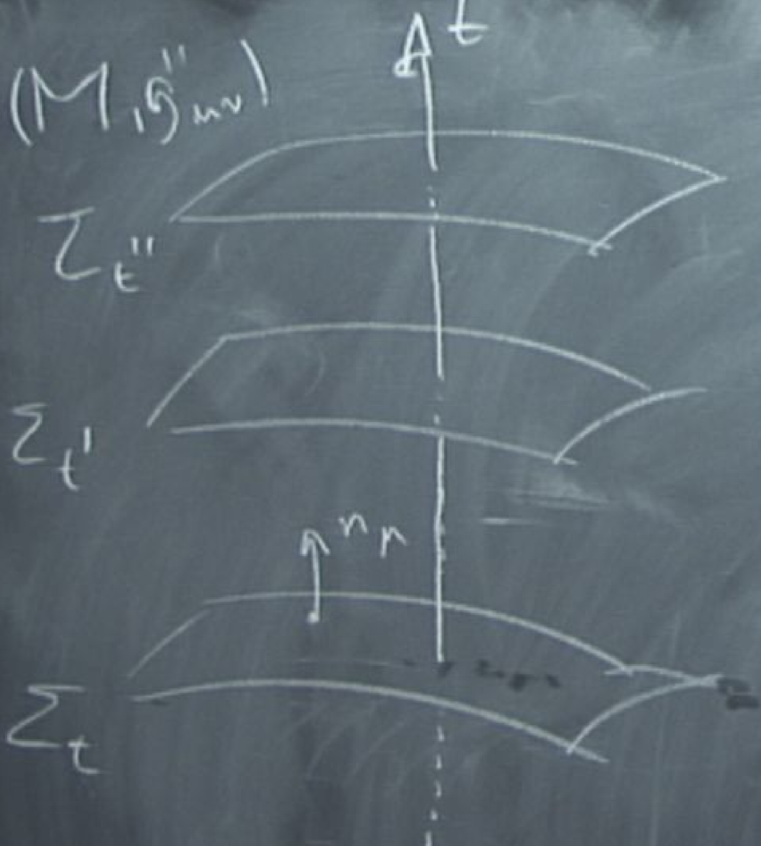

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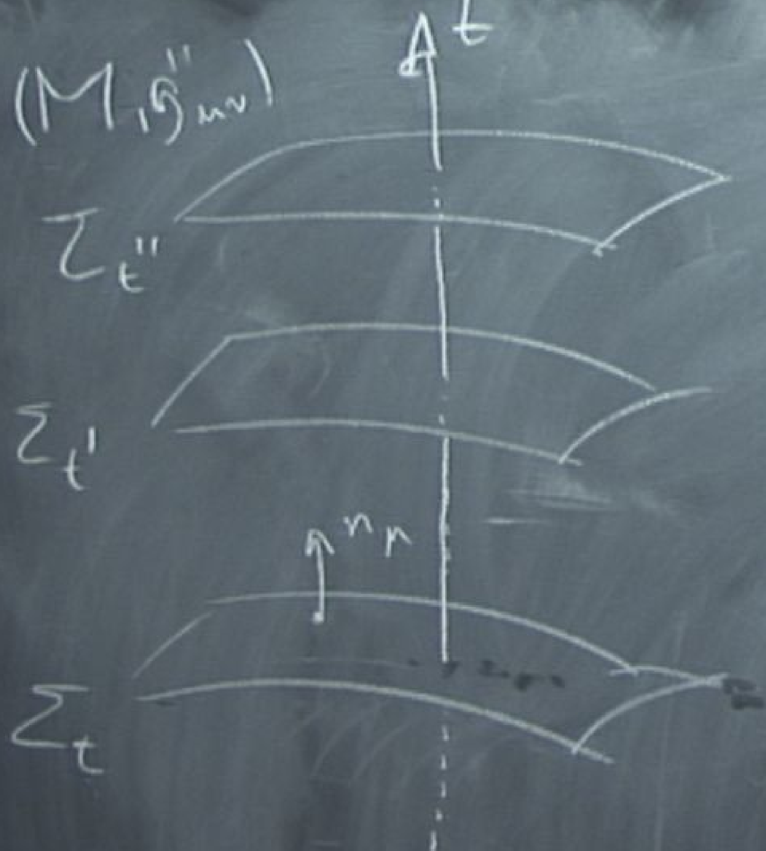

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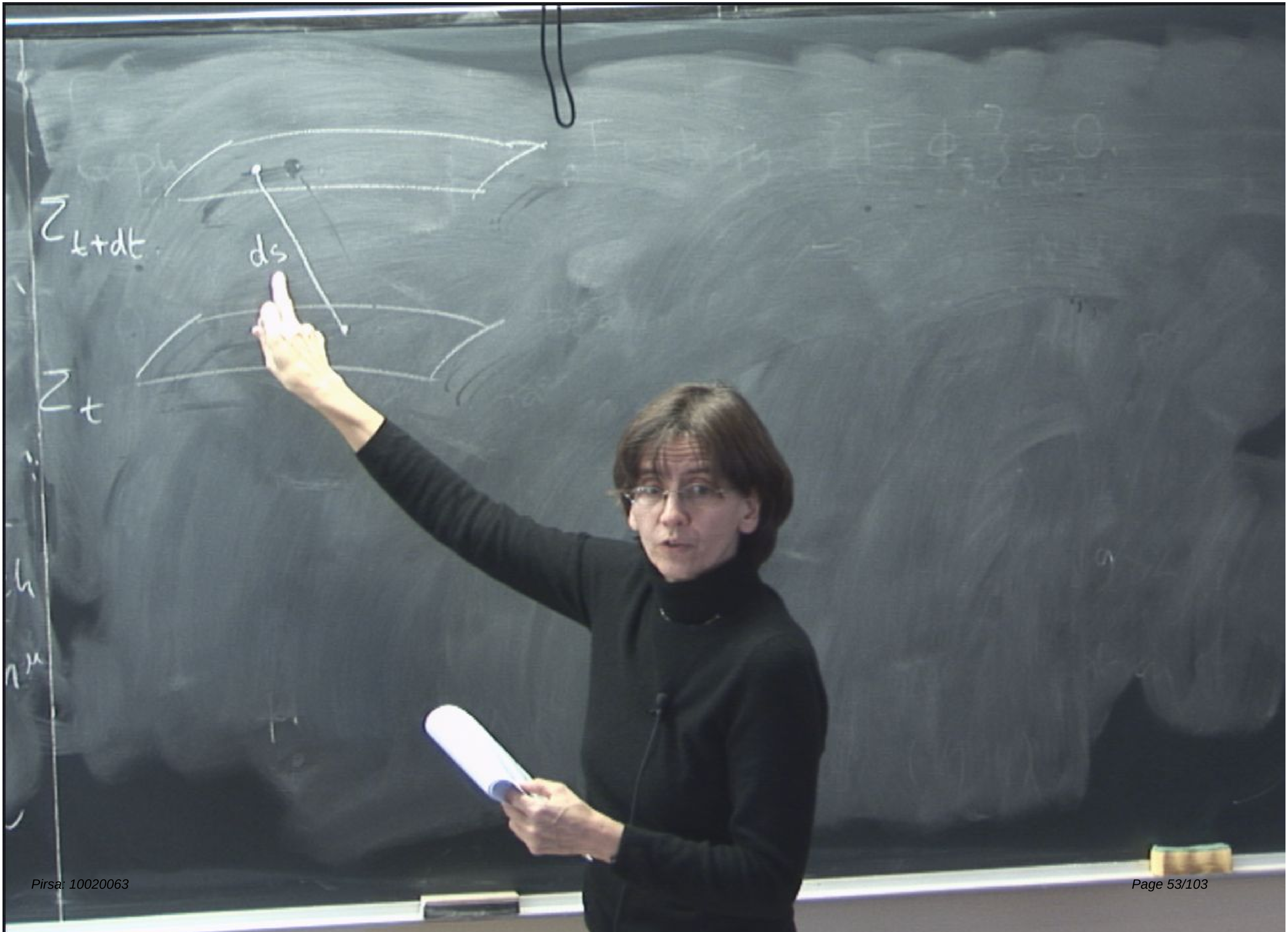
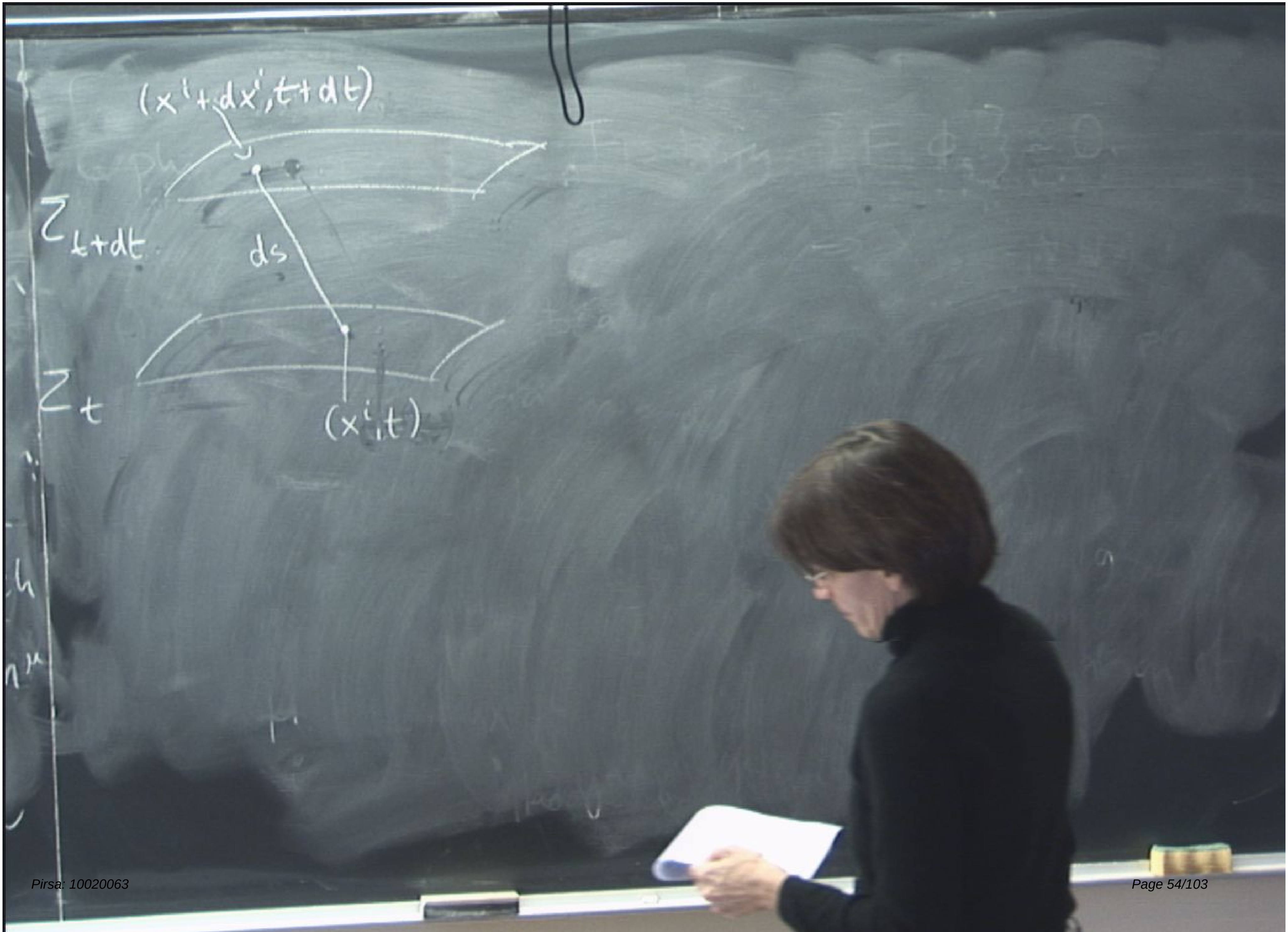
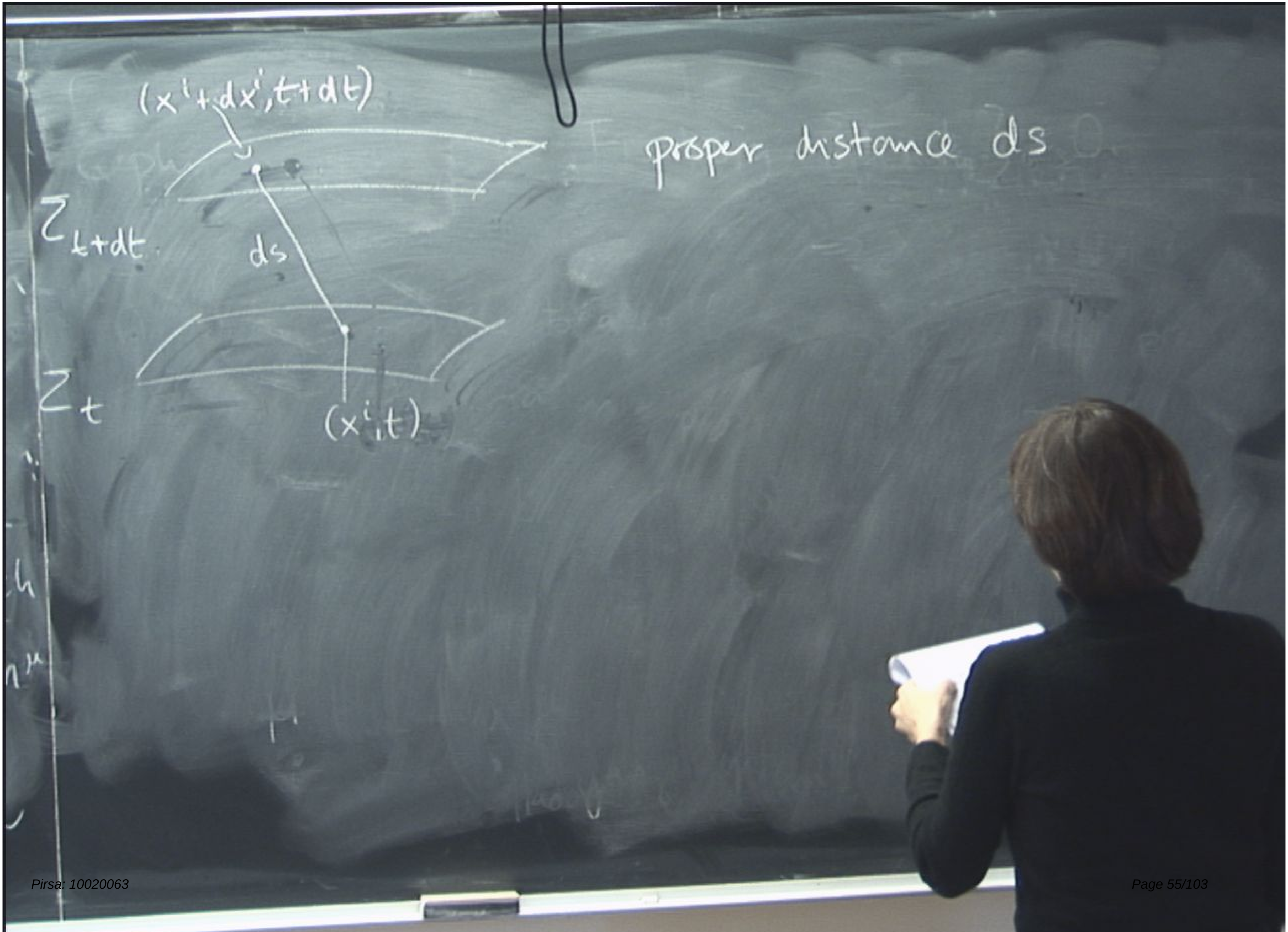
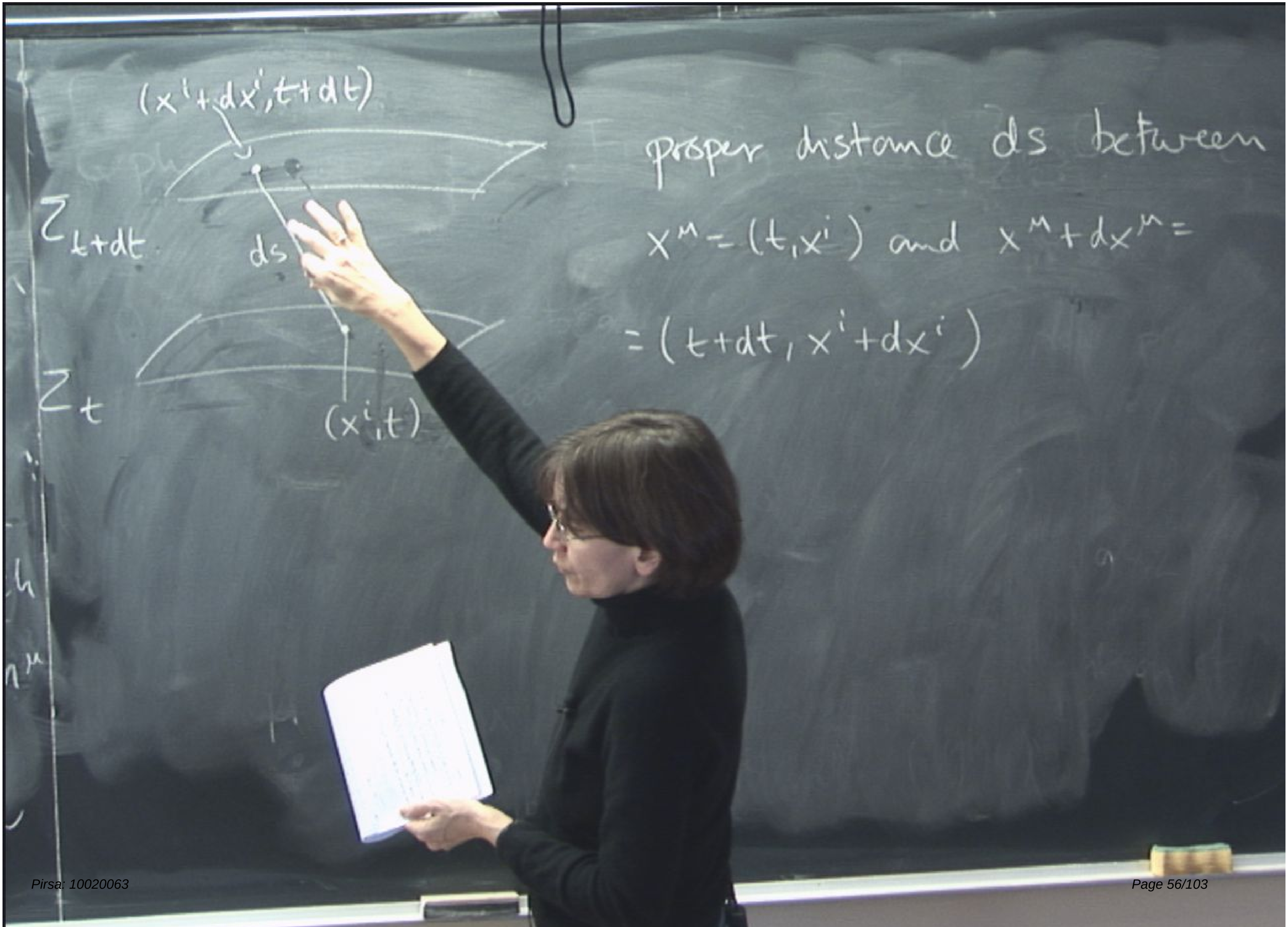
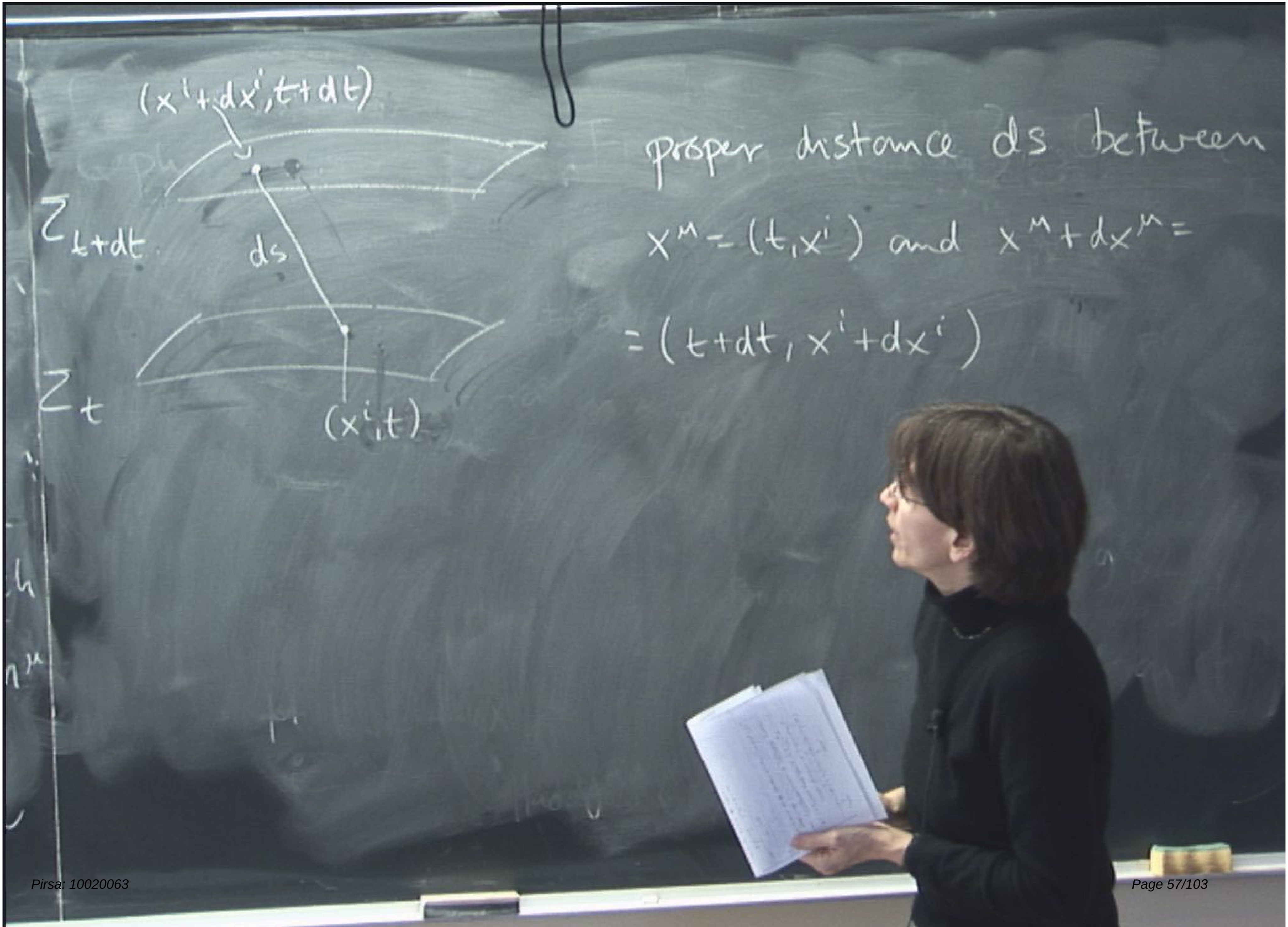


Σ_{t+dt}
ds
Σ_t

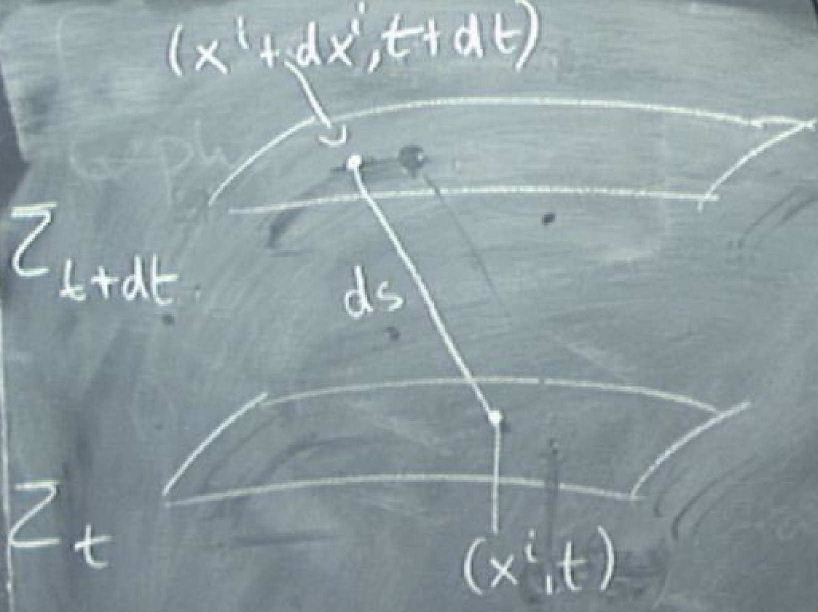

(x^i + dx^i, t+dt)
Σ_{t+dt}
ds
Σ_t
(x^i, t)


(x^i + dx^i, t+dt)
proper distance ds
Σ_{t+dt}
ds
Σ_t
(x^i, t)


(x^i + dx^i, t + dt)
Σ_{t+dt}
ds
Σ_t
(x^i, t)
proper distance ds between
x^μ = (t, x^i) and x^μ + dx^μ =
= (t + dt, x^i + dx^i)



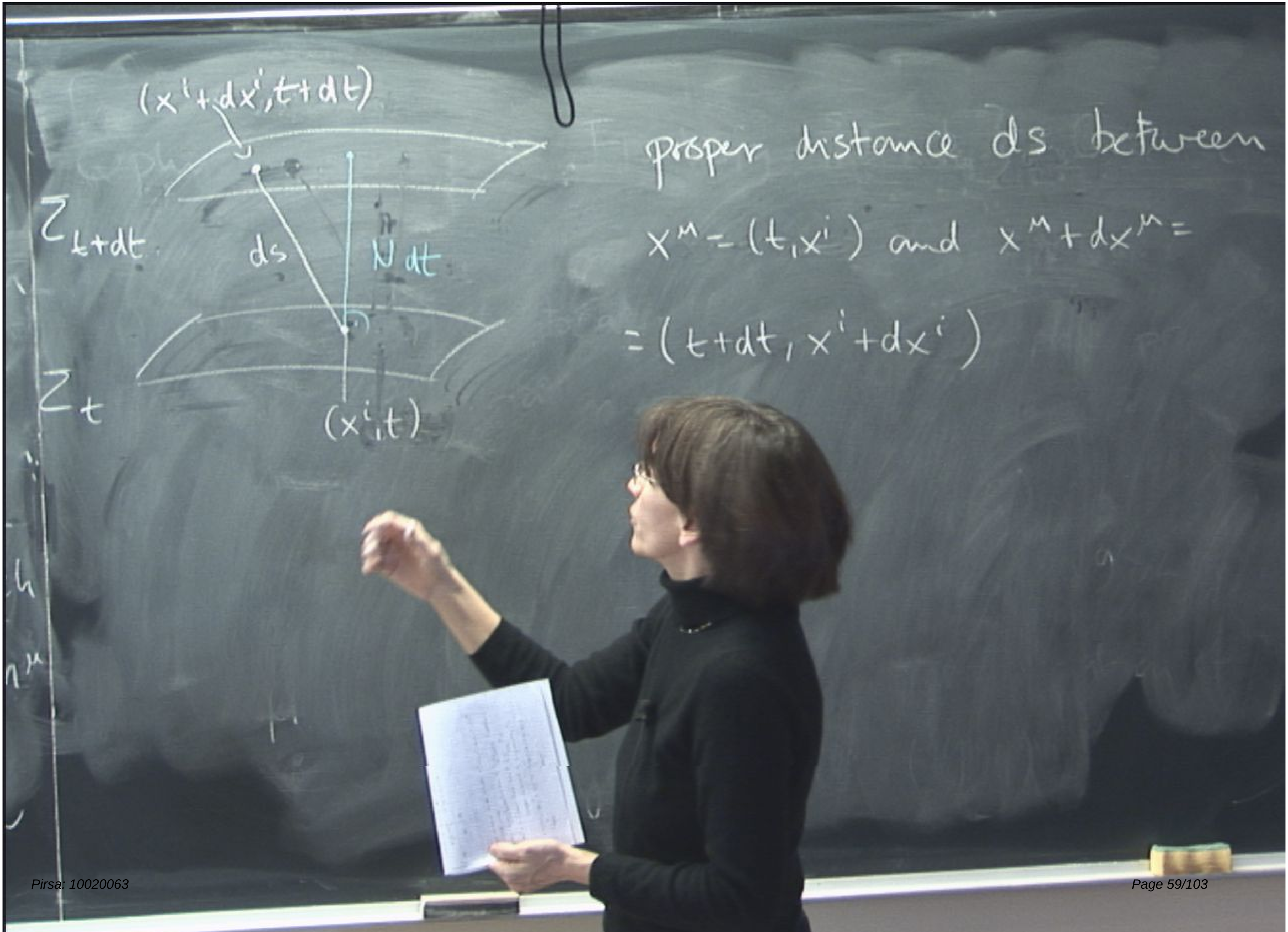
$(x^i + dx^i, t+dt)$
Σ_{t+dt}
ds
Σ_t
(x^i, t)
proper distance ds between
$x^\mu = (t, x^i)$ and $x^\mu + dx^\mu =$
$= (t+dt, x^i + dx^i)$



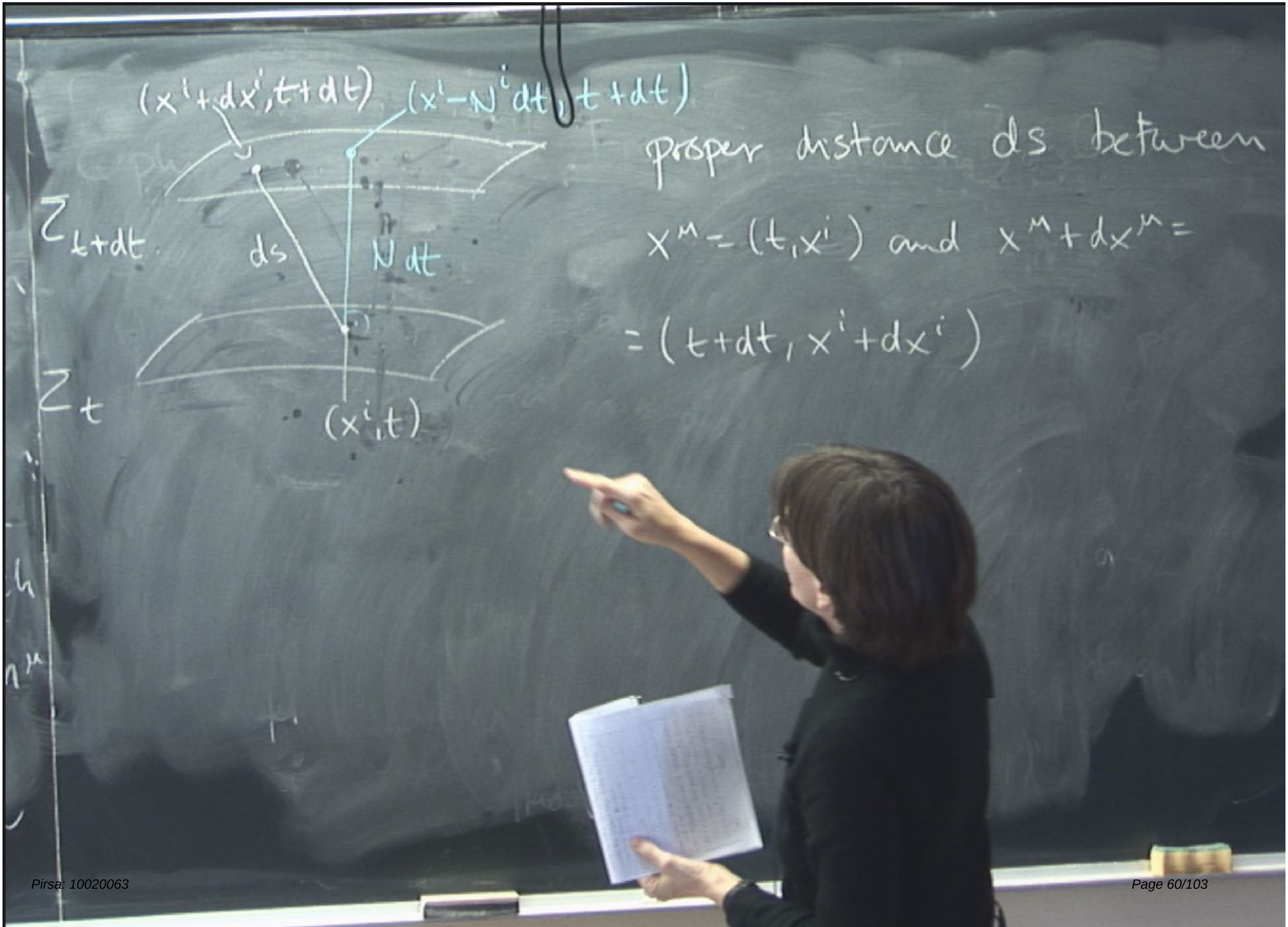
proper distance ds between

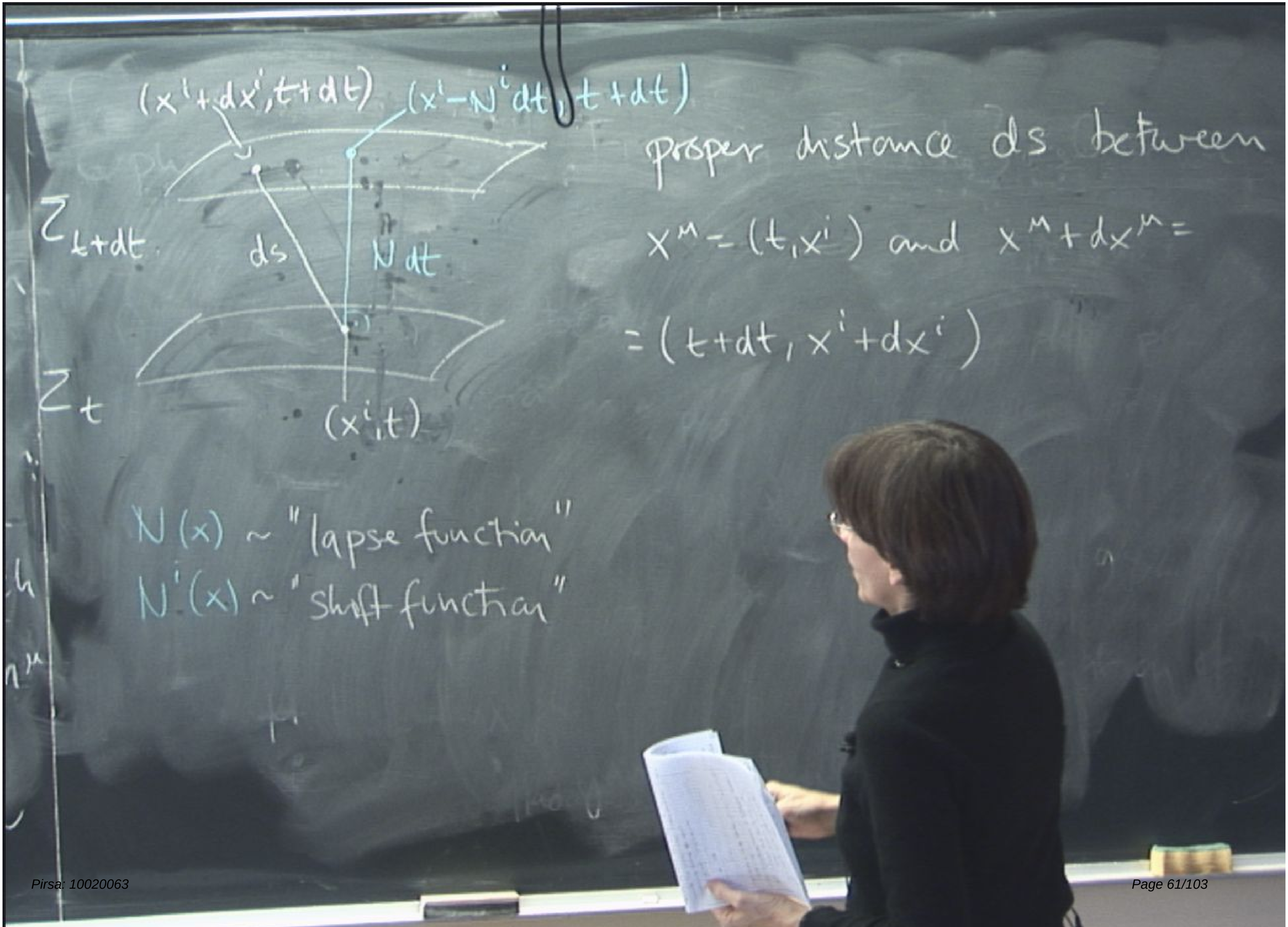
$x^\mu = (t, x^i)$ and $x^\mu + dx^\mu =$

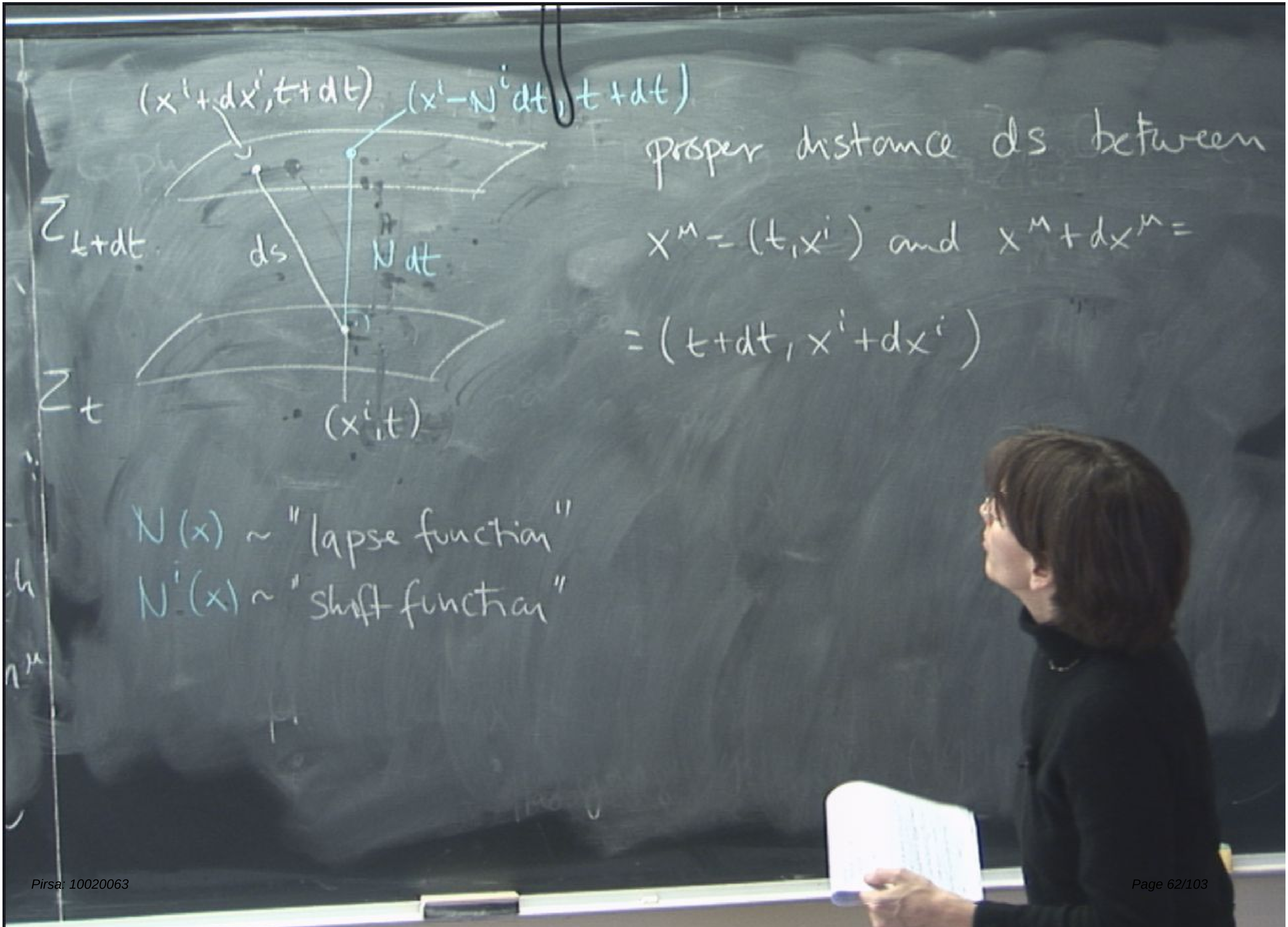
$= (t+dt, x^i+dx^i)$

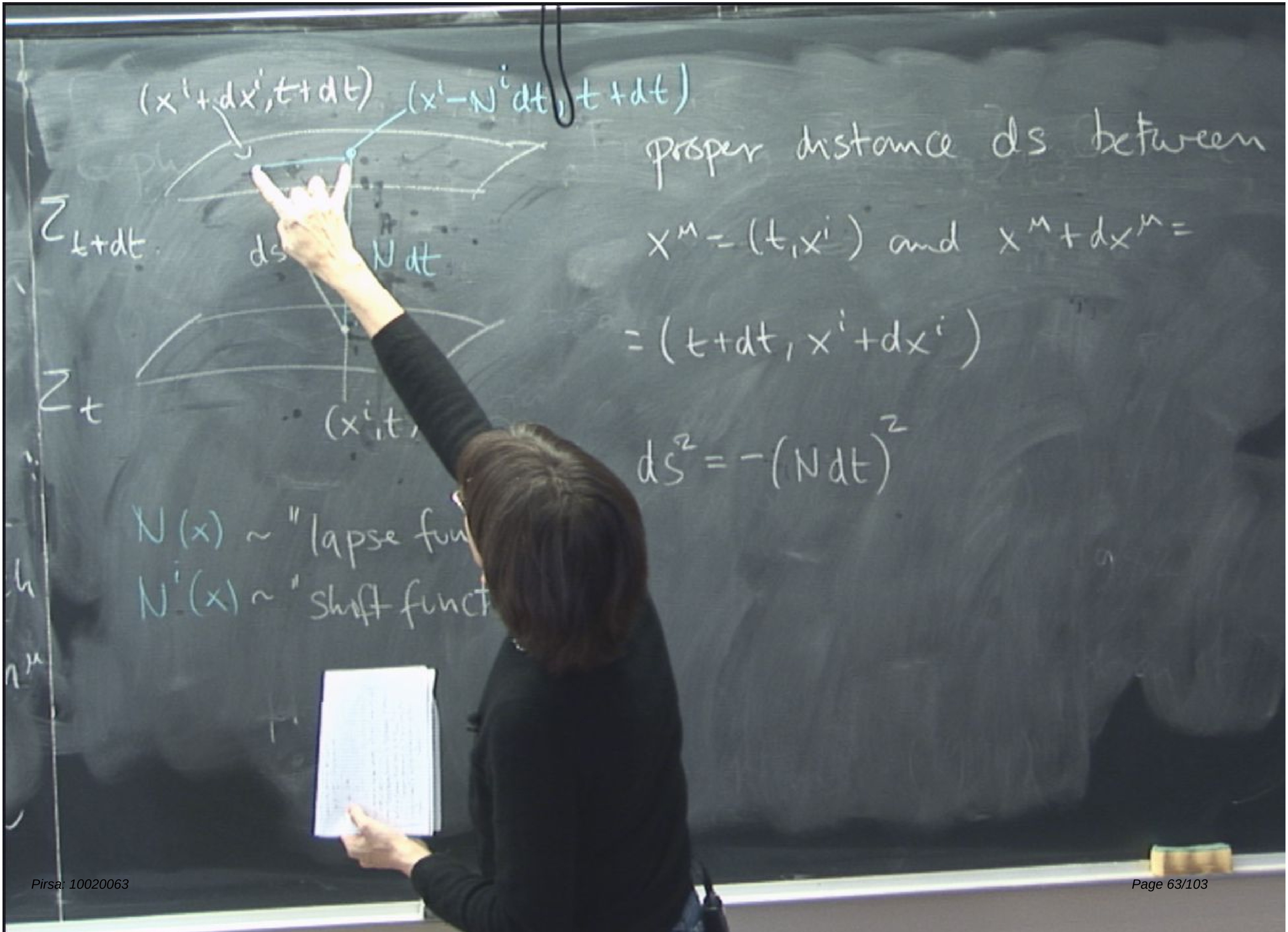


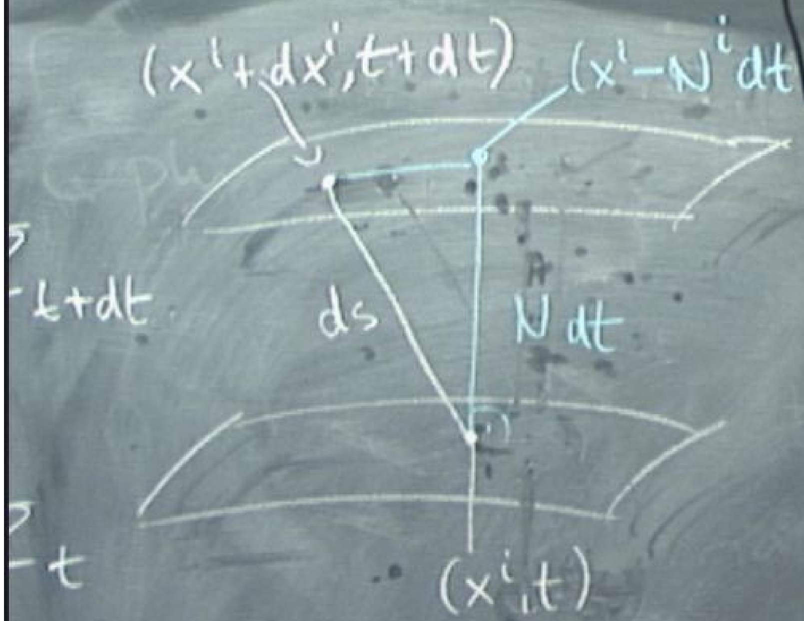
(x^i + dx^i, t + dt)
Σ t+dt
ds
N dt
Σ t
(x^i, t)
proper distance ds between
x^μ = (t, x^i) and x^μ + dx^μ =
= (t + dt, x^i + dx^i)


(x^i+dx^i, t+dt)
(x^i-N^i dt, t+dt)
Σ_{t+dt}
ds
N dt
Σ_t
(x^i, t)
proper distance ds between
x^μ = (t, x^i) and x^μ + dx^μ =
= (t+dt, x^i+dx^i)


(x^i + dx^i, t+dt)
(x^i − N^i dt, t+dt)
Σ t+dt
ds
N dt
Σ t
(x^i, t)
proper distance ds between
x^μ = (t, x^i) and x^μ + dx^μ =
= (t+dt, x^i + dx^i)
N(x) ~ "lapse function"
N^i(x) ~ "shift function"


(x^i + dx^i, t + dt)
(x^i − N^i dt, t + dt)
proper distance ds between
x^μ = (t, x^i) and x^μ + dx^μ =
= (t + dt, x^i + dx^i)
Σ_{t+dt}
ds
N dt
Σ_t
(x^i, t)
N(x) ~ "lapse function"
N^i(x) ~ "shift function"


(x^i+dx^i, t+dt)
(x^i−N^i dt, t+dt)
proper distance ds between
x^μ = (t, x^i) and x^μ + dx^μ =
= (t+dt, x^i+dx^i)
ds² = −(N dt)²
Σ t+dt
ds
N dt
Σ t
(x^i, t)
N(x) ~ "lapse fun
N^i(x) ~ "shift funct



proper distance ds between

$x^\mu = (t, x^i)$ and $x^\mu + dx^\mu =$

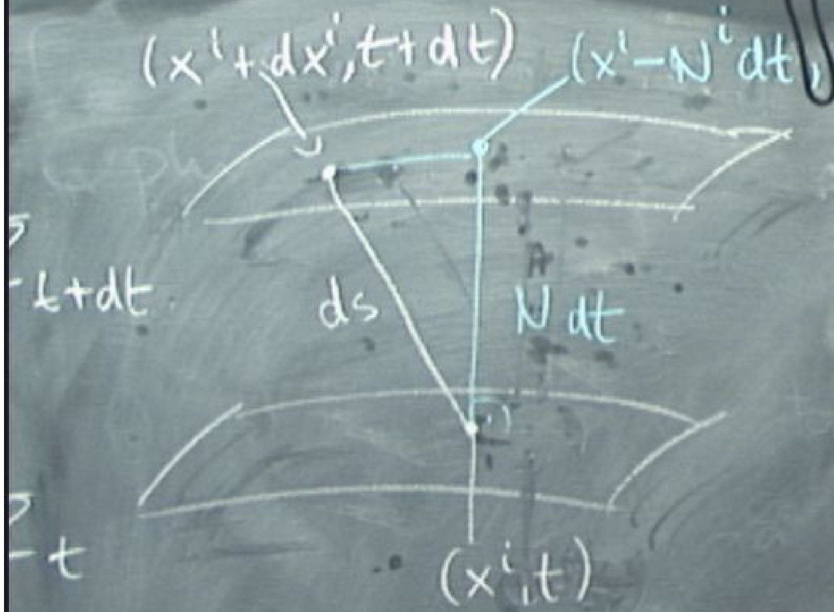
$= (t+dt, x^i + dx^i)$

$$ds^2 = -(N\,dt)^2 + h_{ij}\,(dx^i + N^i dt)\,(dx^j + N^j dt)$$

$N(x)$ ~ "lapse function"

$N^i(x)$ ~ "shift function"

$dx^i + N^i dt$



proper distance ds between

$x^\mu = (t, x^i)$ and $x^\mu + dx^\mu =$

$= (t+dt, x^i + dx^i)$

$$ds^2 = -(N dt)^2 + h_{ij}\,(dx^i + N^i dt)\cdot(dx^j + N^j dt)$$

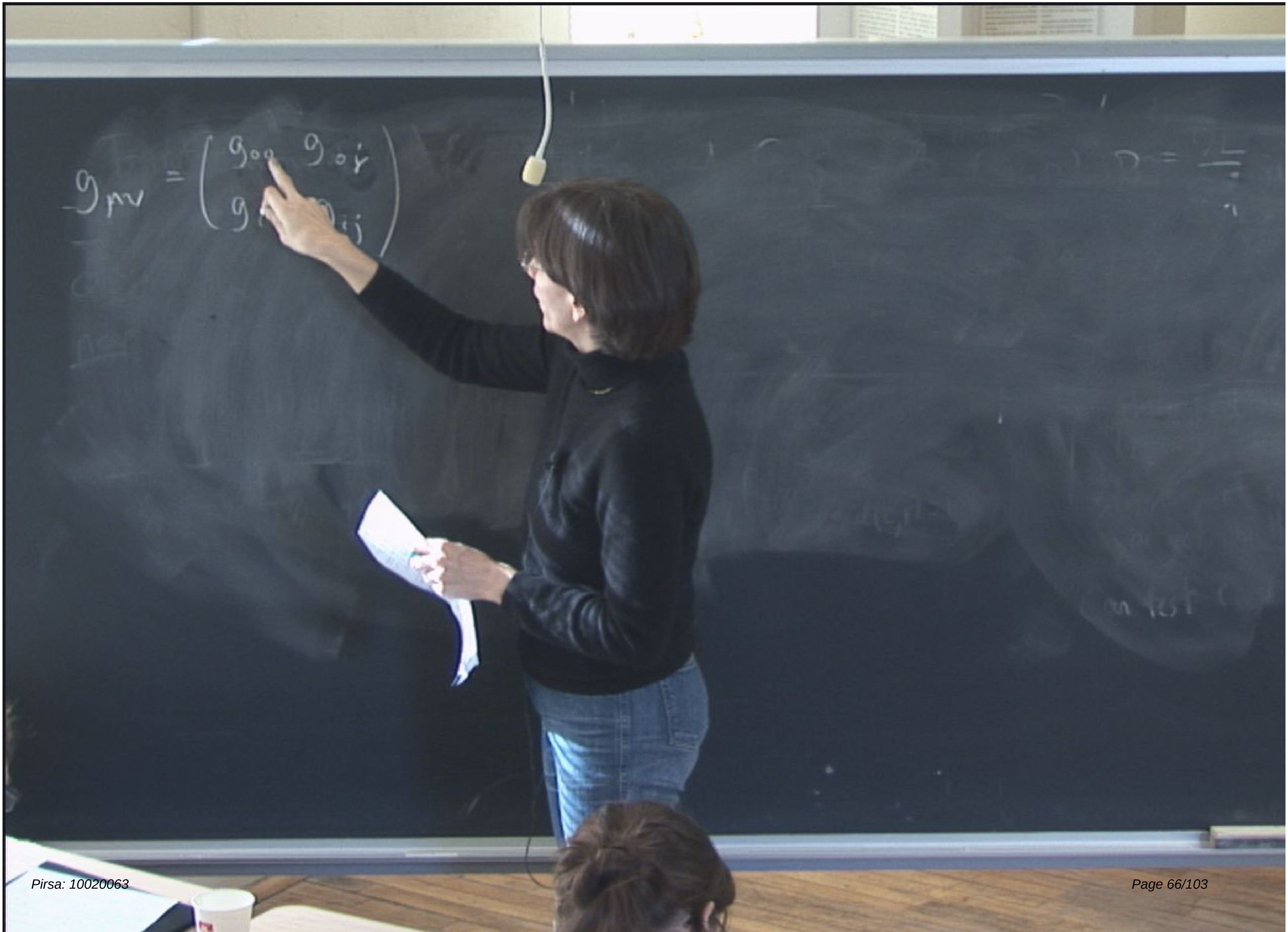
$N(x)$ ~ "lapse function"

$N^i(x)$ ~ "shift function"

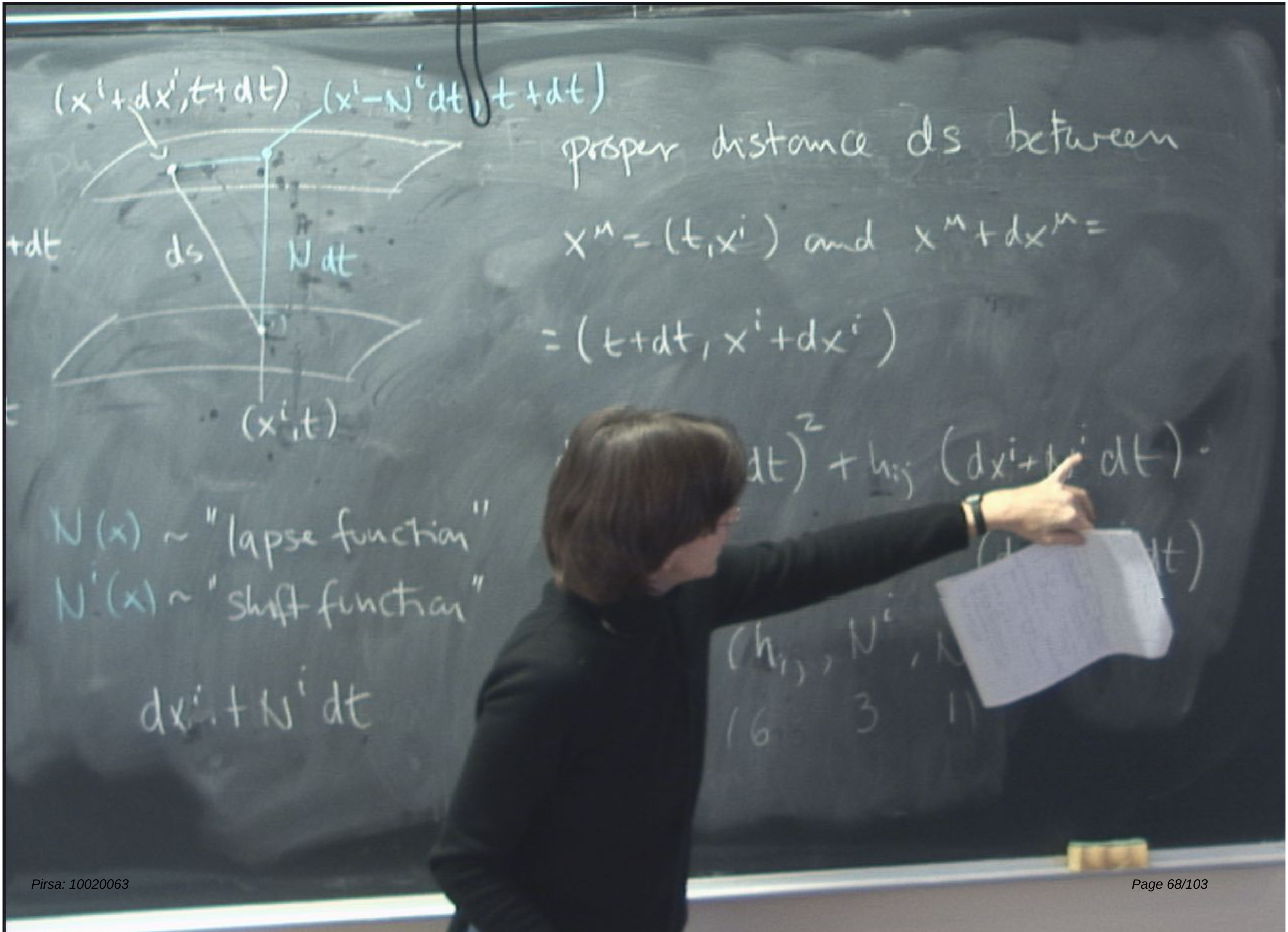
$dx^i + N^i dt$

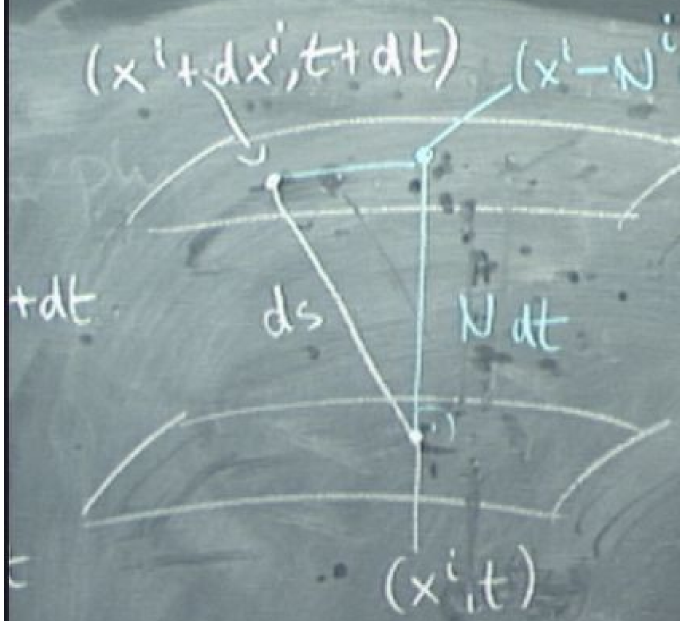
$$g_{\mu\nu} \mapsto (h_{ij}, N^i, N)$$

(10) (6 3 1)



$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N & N^k \\ & \end{pmatrix}$$


(x^i + dx^i, t+dt)
(x^i − N^i dt, t+dt)
ds
N dt
(x^i, t)
N (x) ~ "lapse function"
N^i (x) ~ "shift function"
dx^i + N^i dt
proper distance ds between
x^μ = (t, x^i) and x^μ + dx^μ =
= (t+dt, x^i + dx^i)



proper distance ds between

$x^\mu = (t, x^i)$ and $x^\mu + dx^\mu =$

$= (t+dt, x^i + dx^i)$

$$ds^2 = -(N\,dt)^2 + h_{ij}\,(dx^i + N^i dt)\,(dx^j + N^j dt)$$

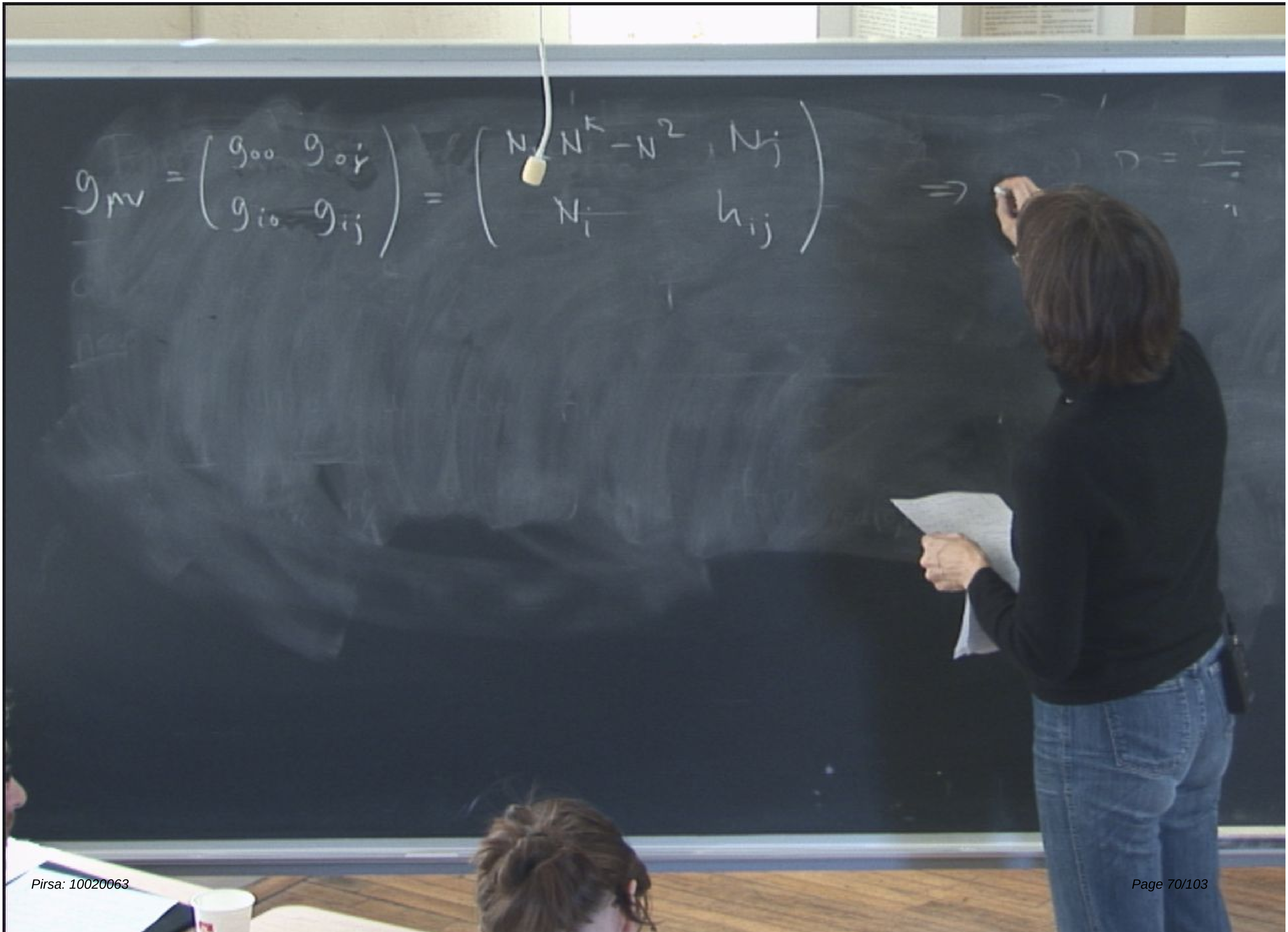
N(x) ~ "lapse function"

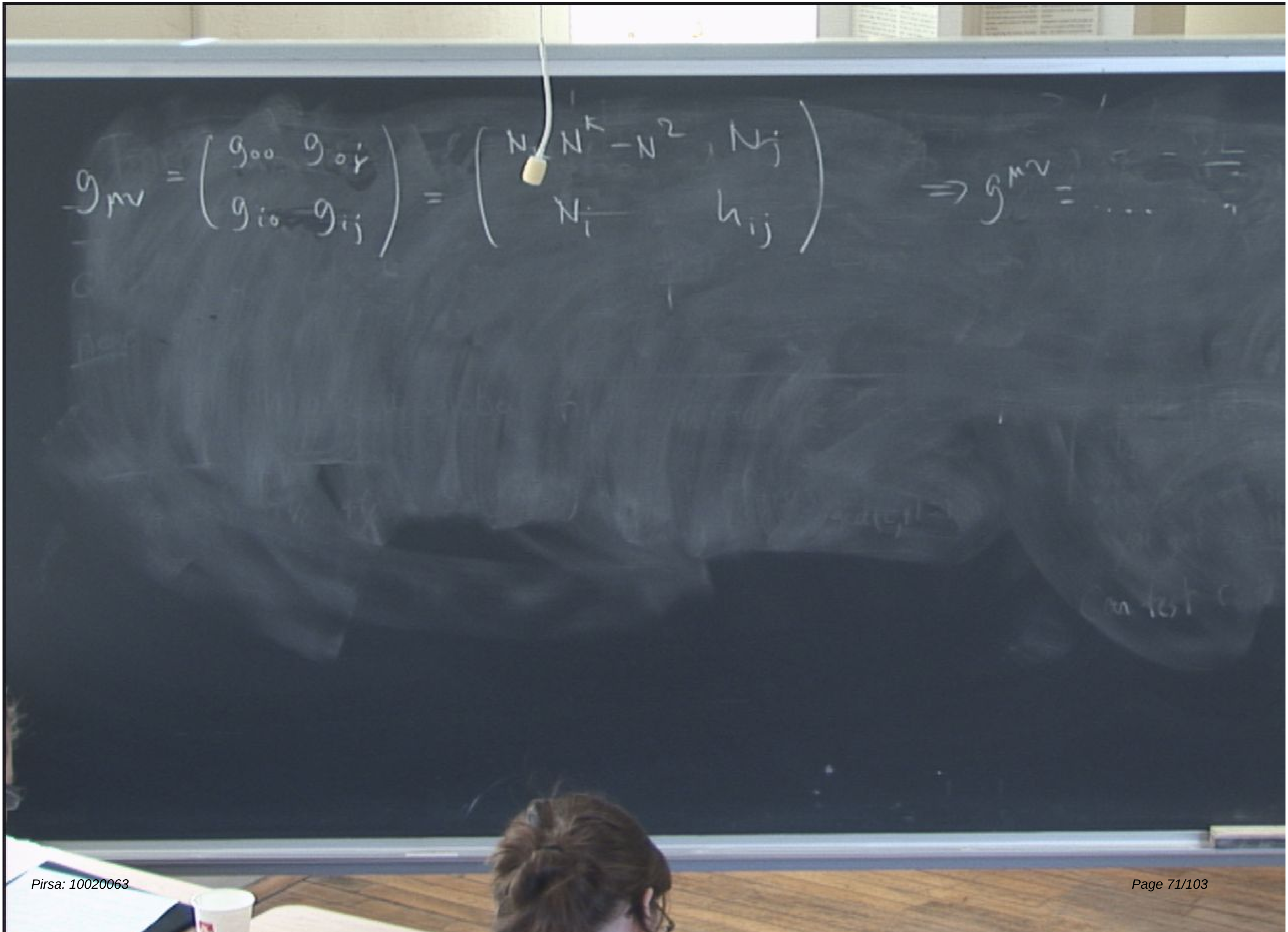
$N^i(x)$ ~ "shift function"

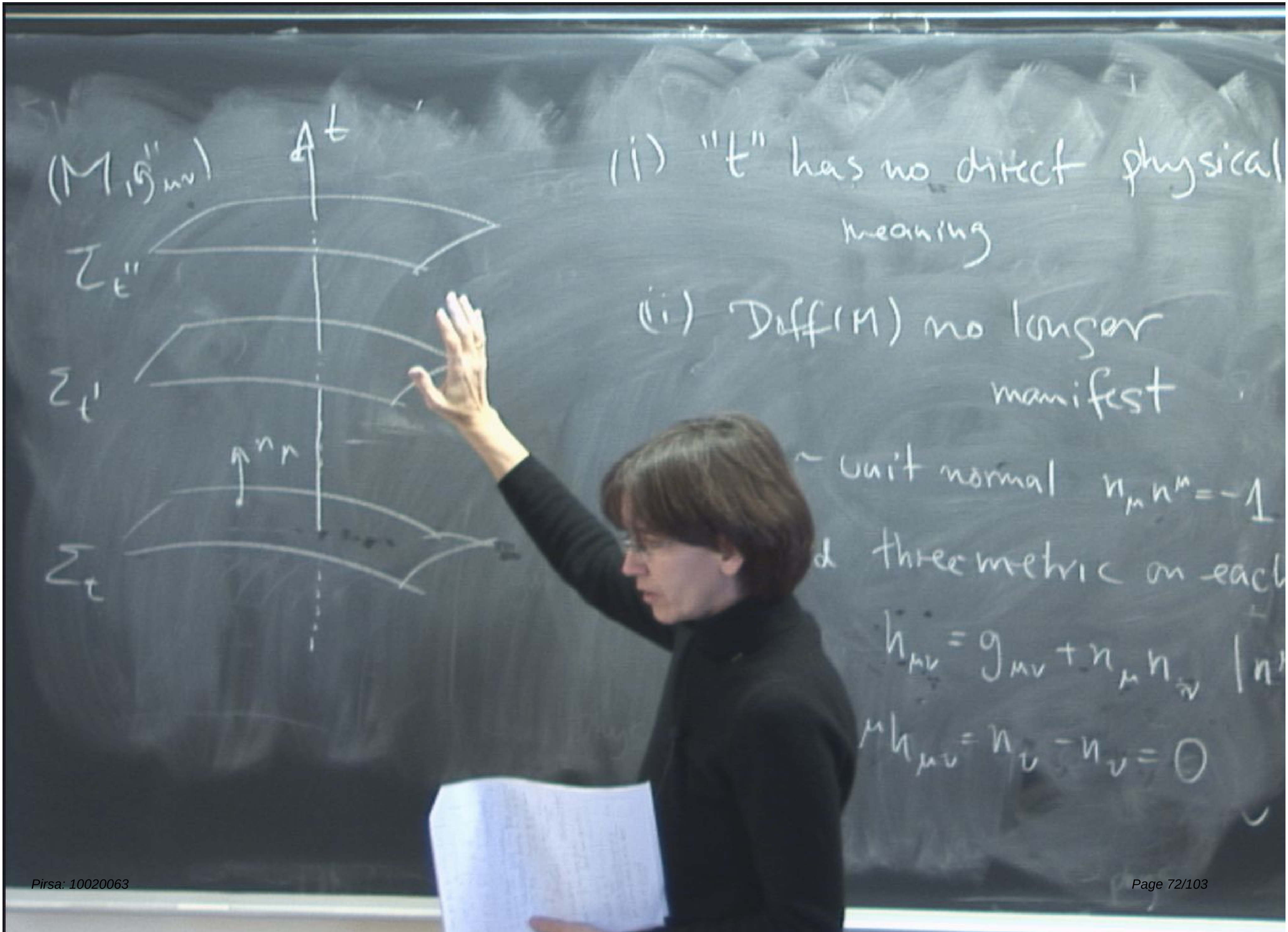
$dx^i + N^i dt$

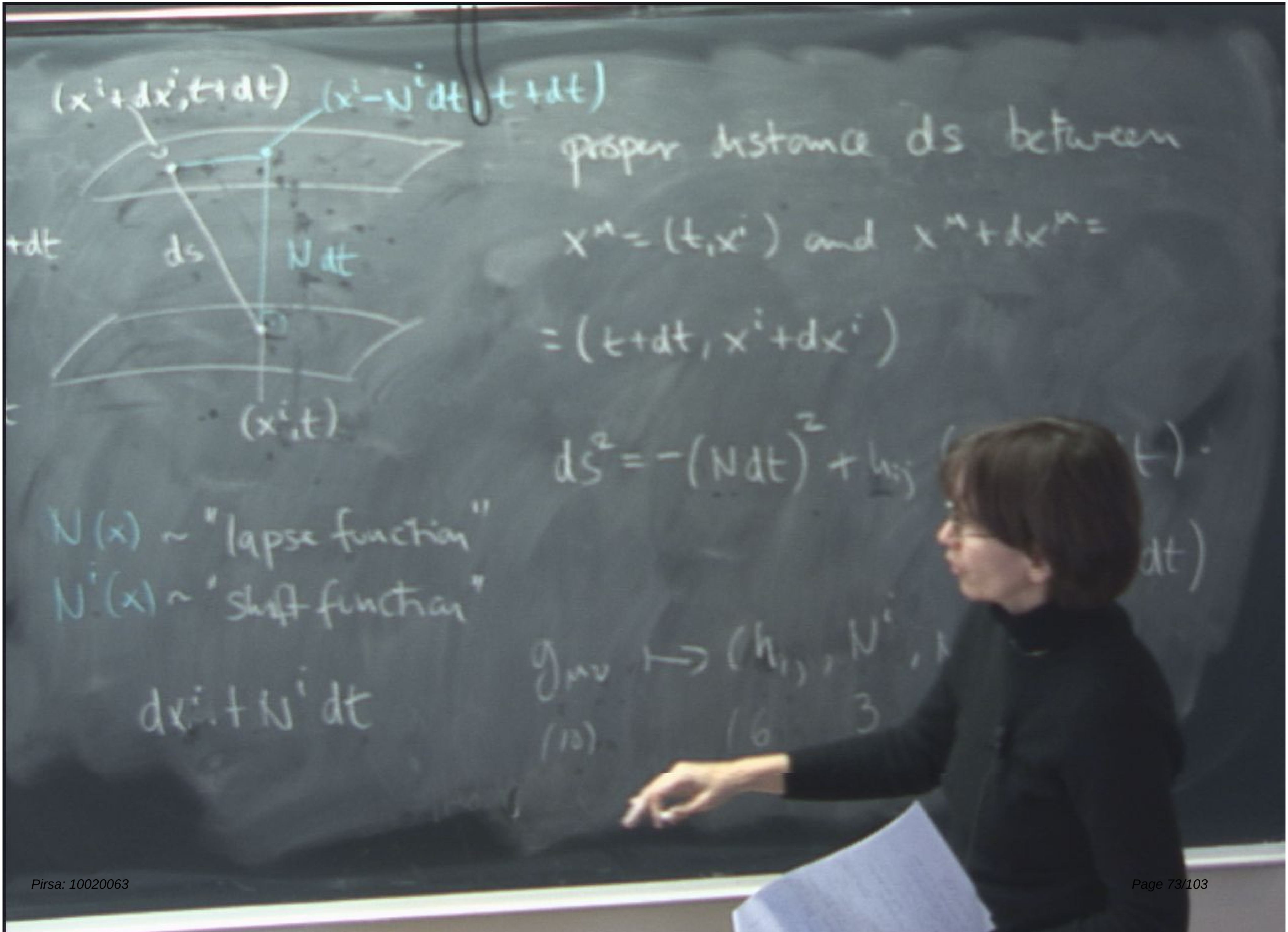
$$g_{\mu\nu} \mapsto (h_{ij},\ N^i,\ N)$$

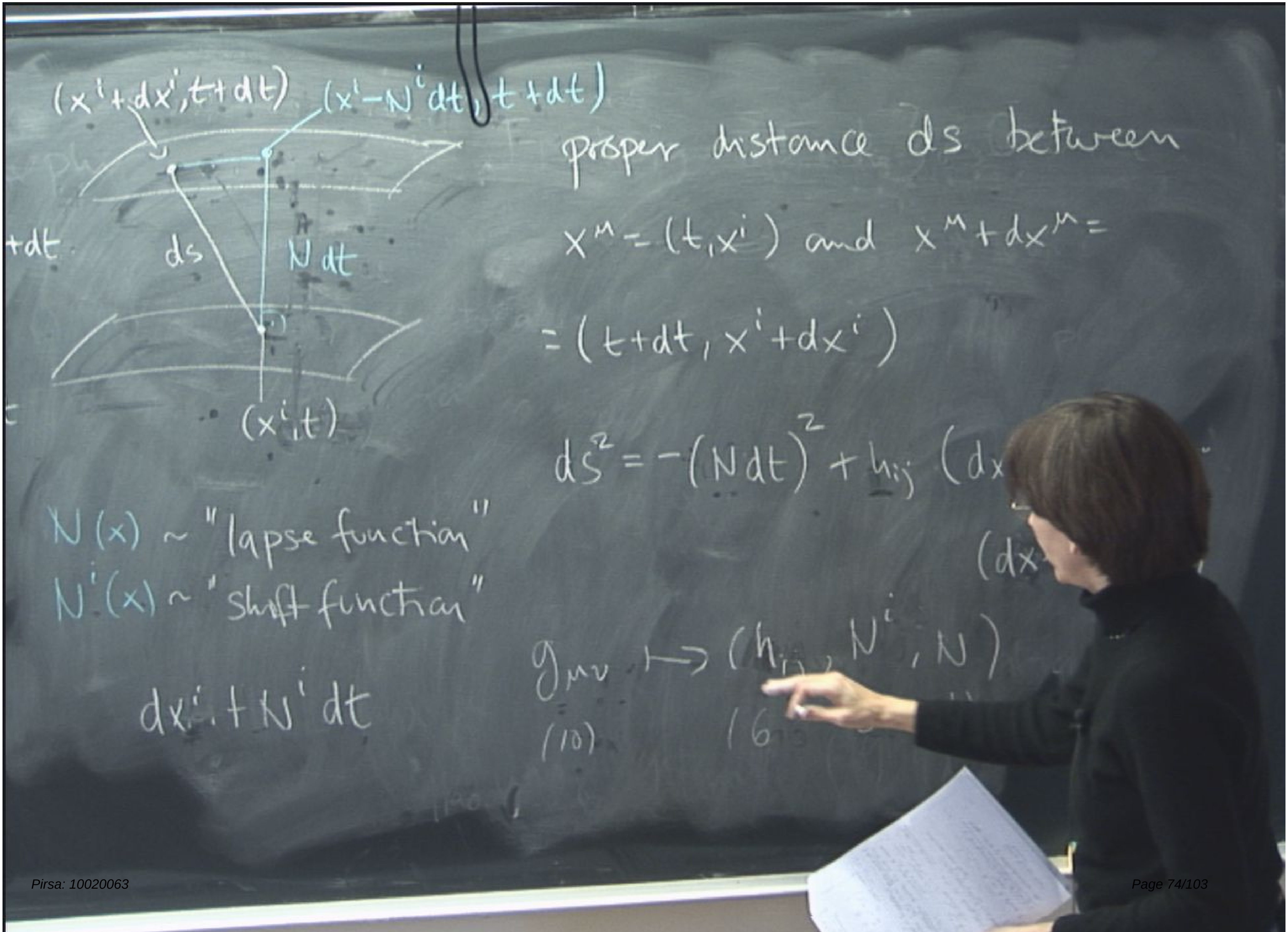
(10) (6 3 1

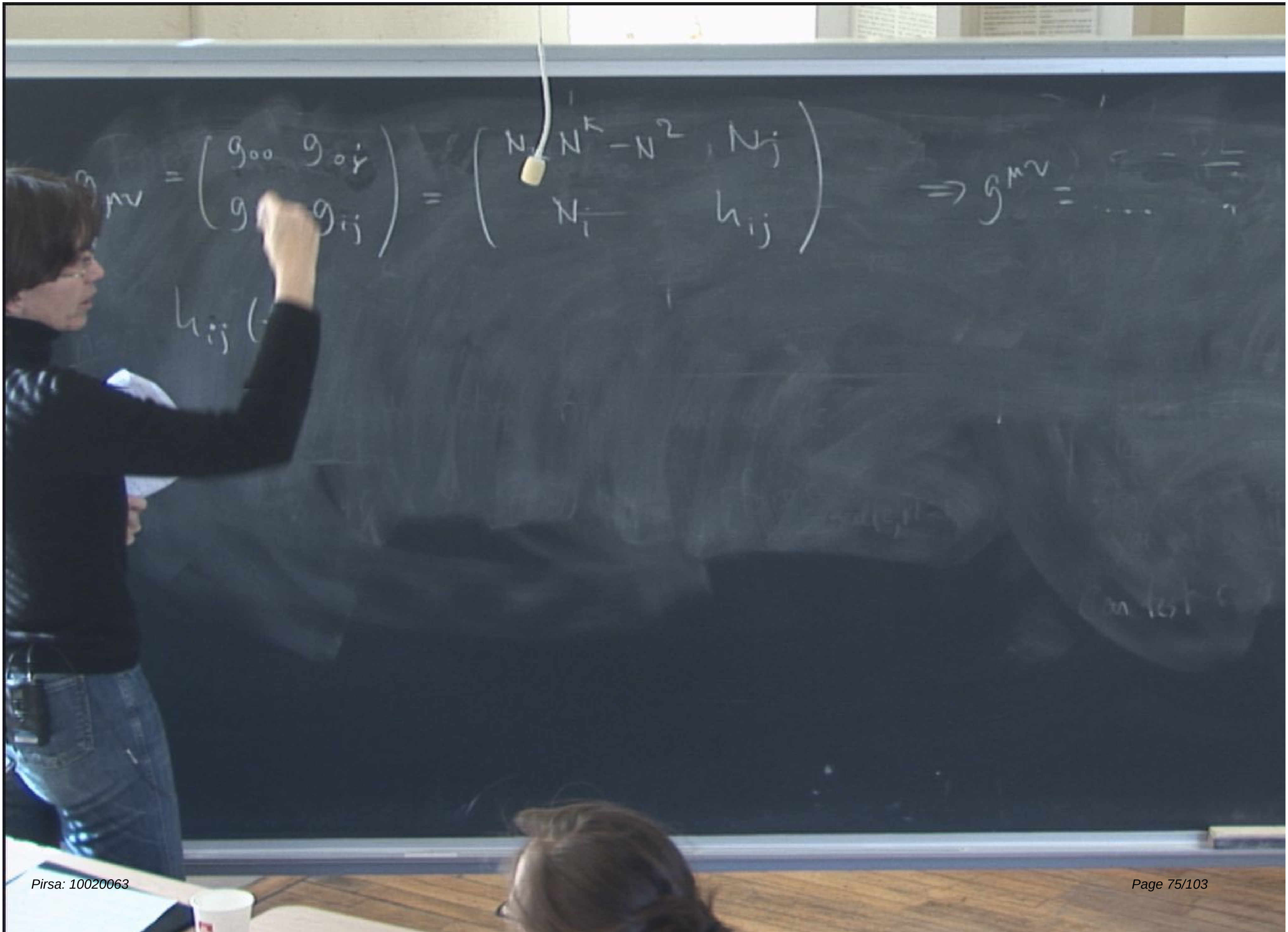

$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow$


$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow g^{\mu\nu} = \ldots$

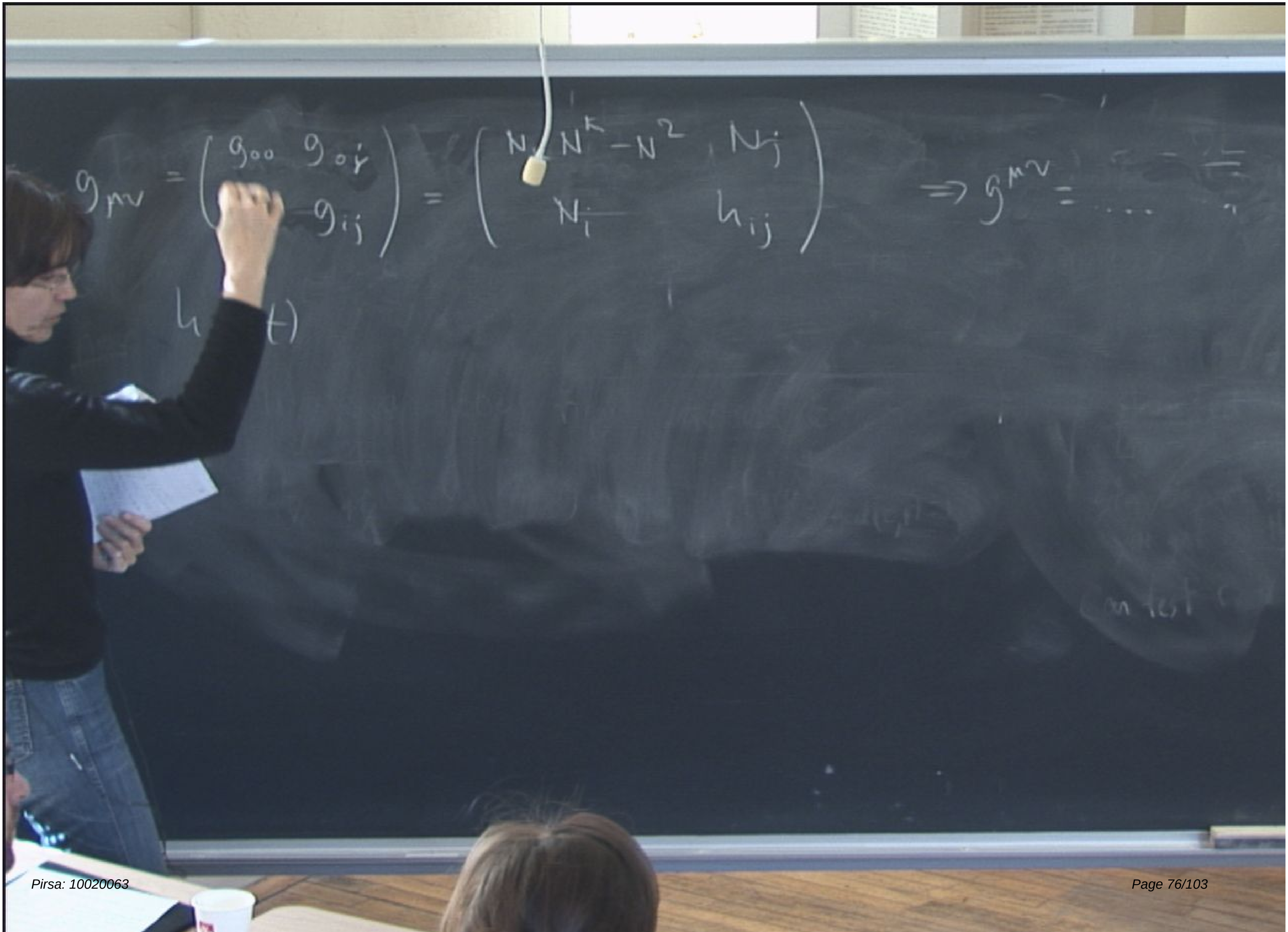

(M, g_μν)
t
Σ_t''
Σ_t'
n_μ
Σ_t
(i) "t" has no direct physical
meaning
(ii) Diff(M) no longer
manifest
unit normal n_μ n^μ = -1
three metric on each
h_μν = g_μν + n_μ n_ν
h_μν = n_ν - n_ν = 0


(x^i+dx^i, t+dt)
(x^i−N^i dt, t+dt)
ds
N dt
(x^i, t)
proper distance ds between
x^μ = (t, x^i) and x^μ+dx^μ =
= (t+dt, x^i+dx^i)
ds² = −(N dt)² + h_ij
N(x) ~ "lapse function"
N^i(x) ~ "shift function"
dx^i + N^i dt
g_μν ↦ (h_ij, N^i,
(10)
(6
3

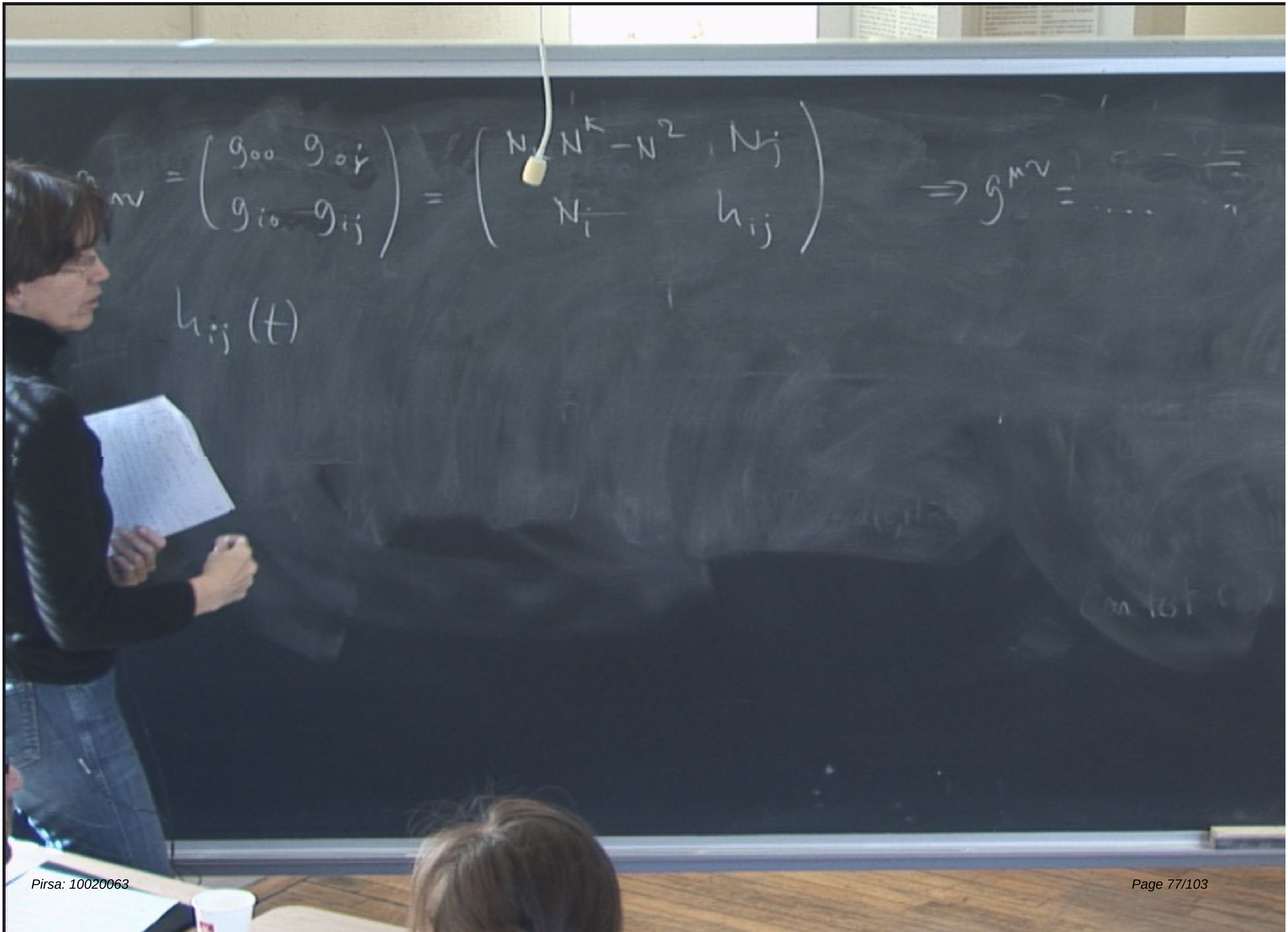

(x^i + dx^i, t+dt)
(x^i − N^i dt, t+dt)
+dt
ds
N dt
(x^i, t)
proper distance ds between
x^μ = (t, x^i) and x^μ + dx^μ =
= (t+dt, x^i + dx^i)
ds^2 = −(N dt)^2 + h_ij (dx
(dx
N(x) ~ "lapse function"
N^i(x) ~ "shift function"
dx^i + N^i dt
g_μν ↦ (h_ij, N^i, N)
(10)
(6



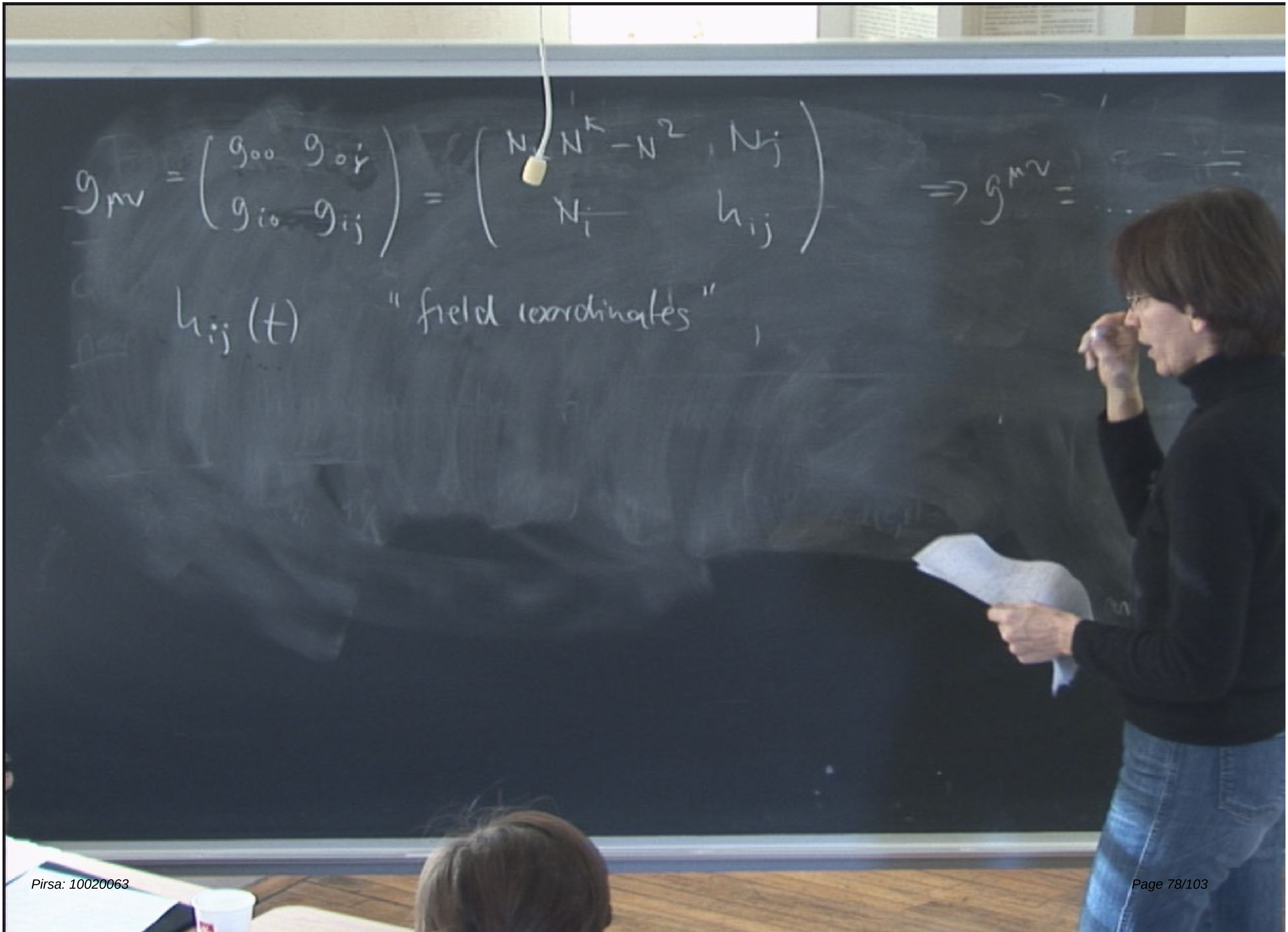
$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow g^{\mu\nu} = \ldots$
h_{ij} (

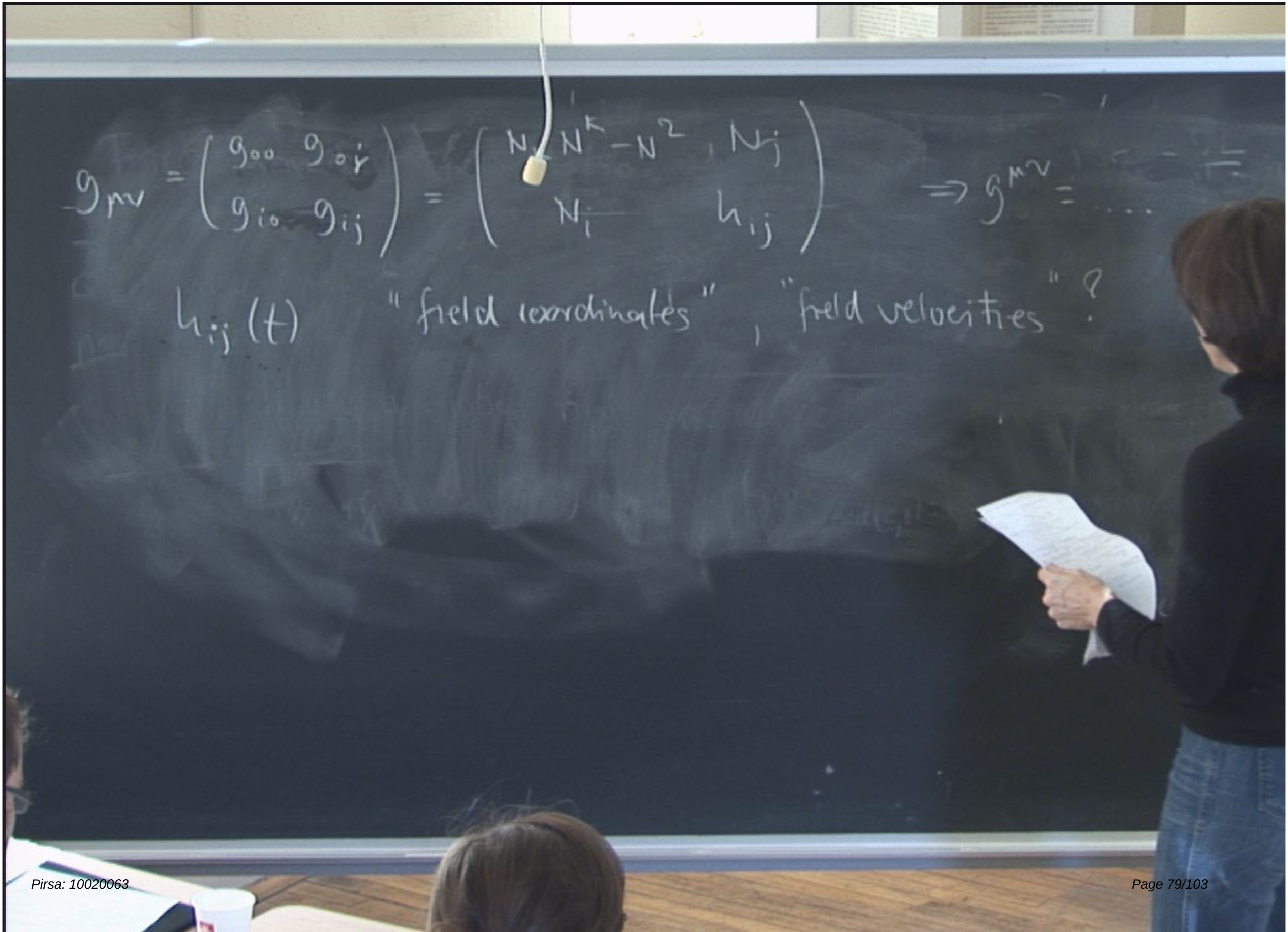


$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow g^{\mu\nu} = \dots$
h (t)

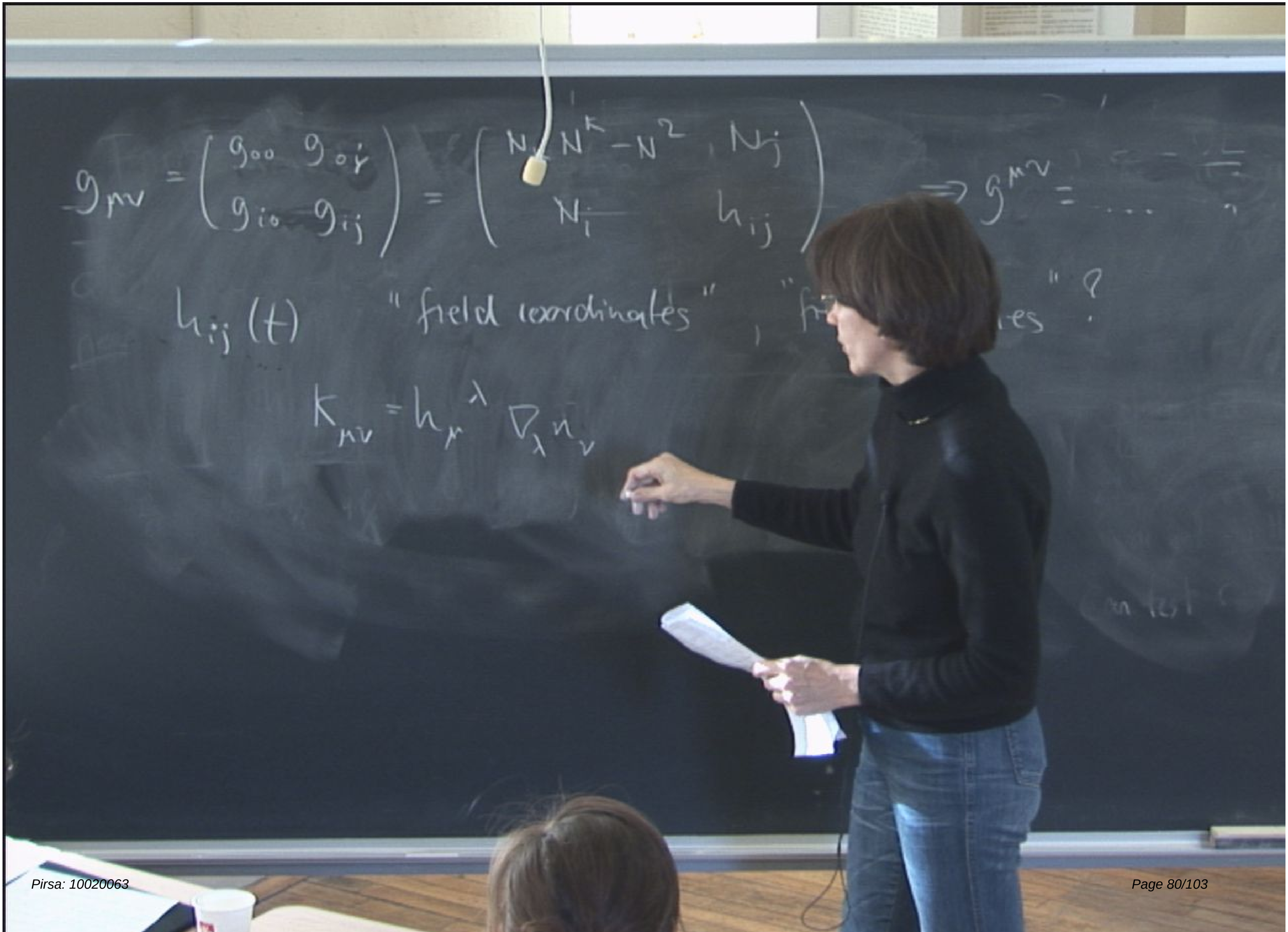


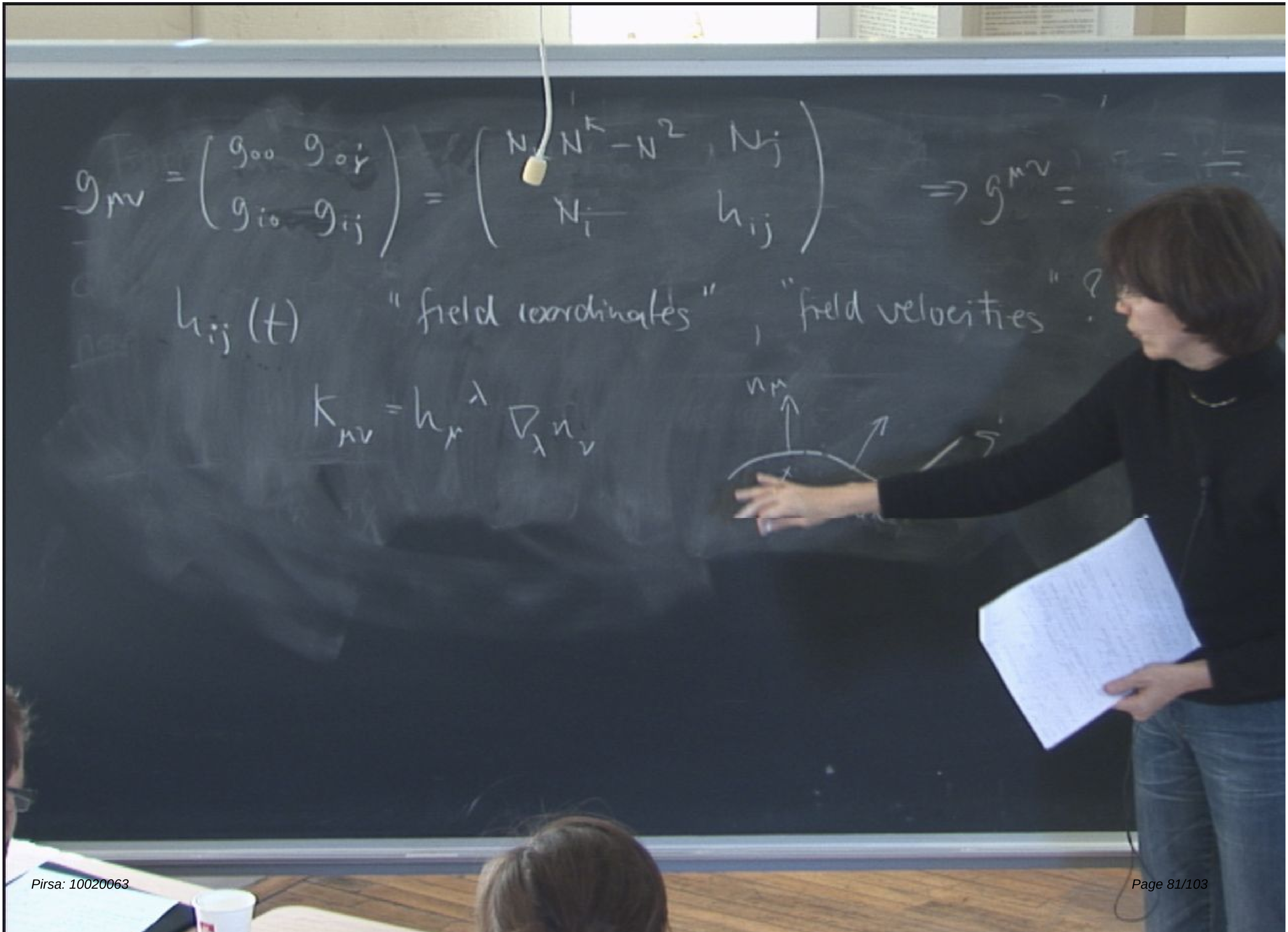
= (g00 g0j / gi0 gij) = (N_k N^k − N^2   Nj / Ni   hij)
⇒ g^{μν} = ...
hij (t)

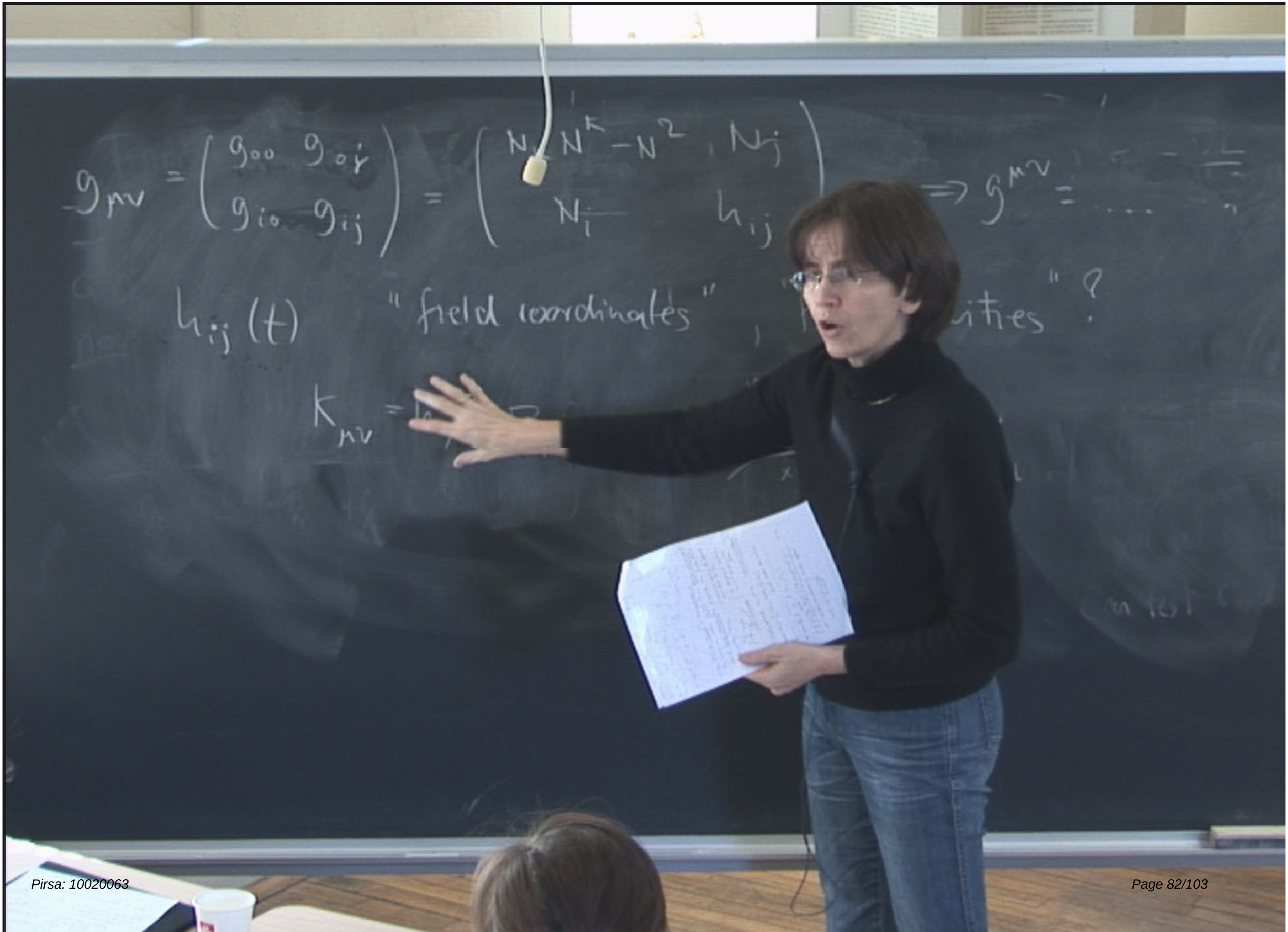

$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow g^{\mu\nu} = \dots$
$h_{ij}(t)$ "field coordinates"

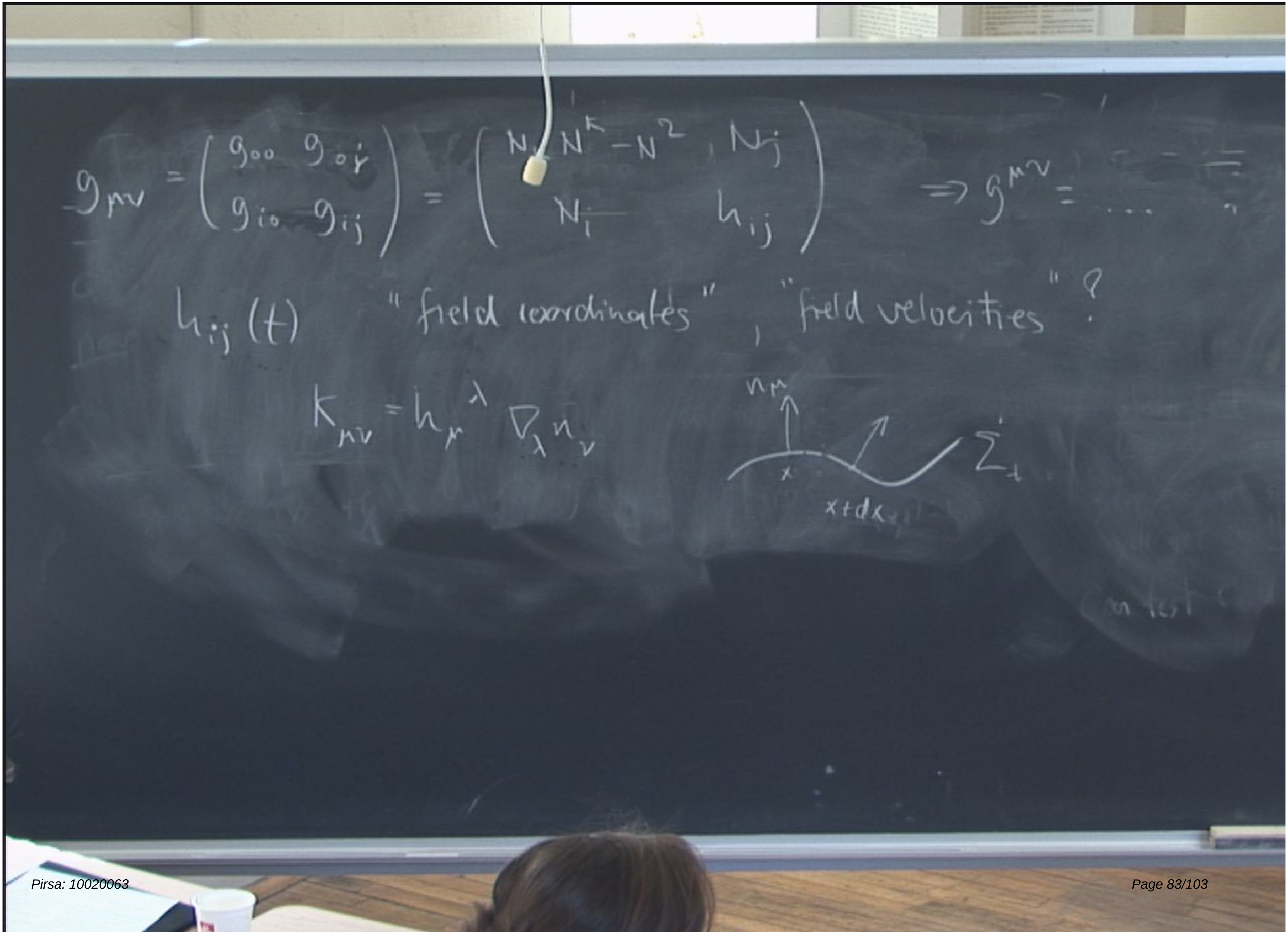


g_μν = (g_00 g_0j ; g_i0 g_ij) = (N_k N^k − N^2 , N_j ; N_i h_ij) ⇒ g^μν = ...
h_ij (t) "field coordinates" , "field velocities" ?


"field coordinates"


g_{μν} = (g_{00} g_{0j} / g_{i0} g_{ij}) = (N_k N^k − N^2 , N_j / N_i h_{ij}) ⇒ g^{μν} =
h_{ij}(t) "field coordinates" , "field velocities" ?
K_{μν} = h_μ^λ ∇_λ n_ν
n_μ

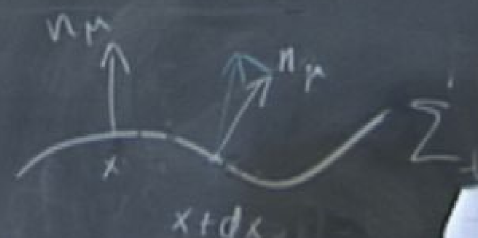

" field coordinates "

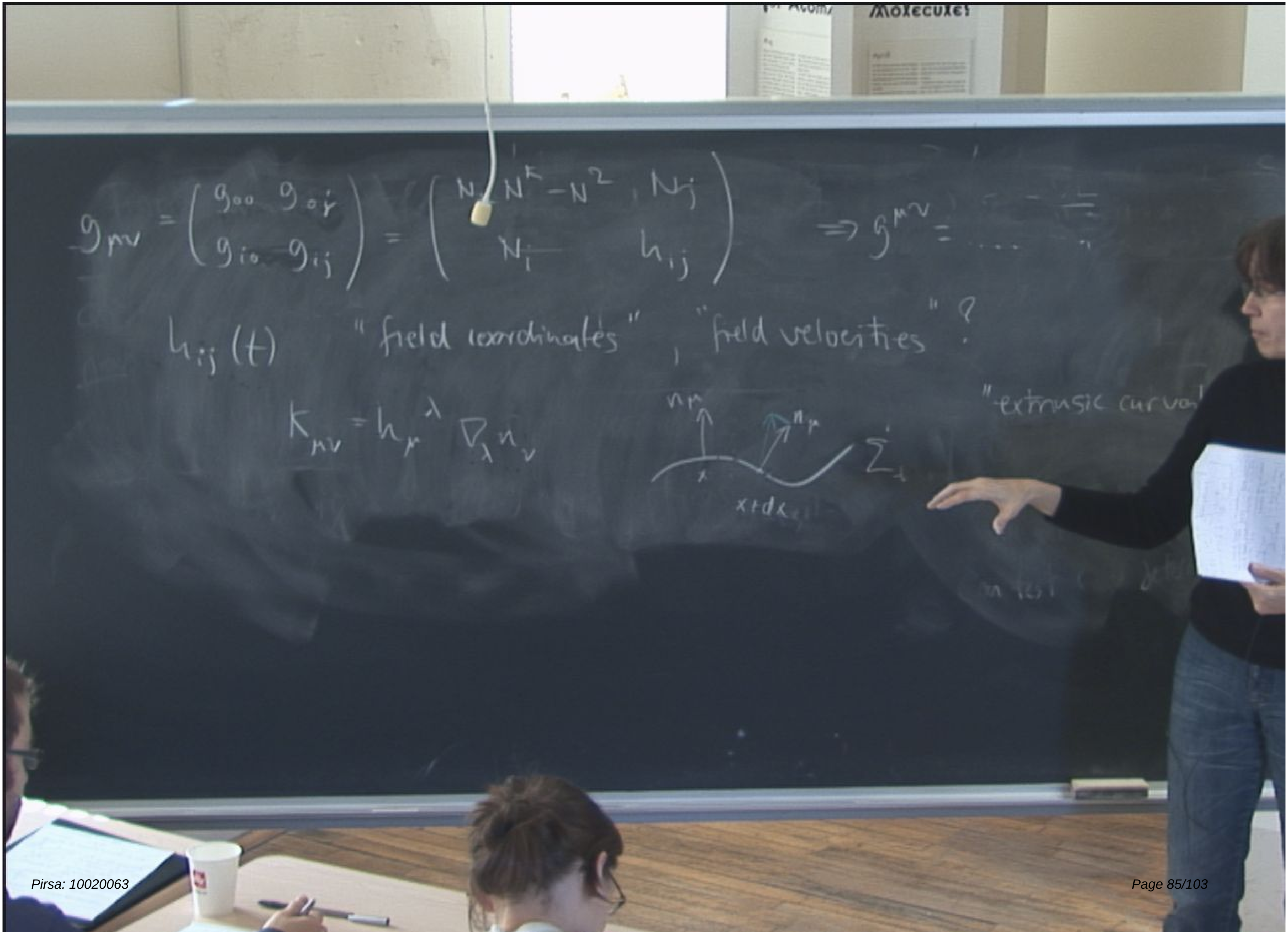

$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow g^{\mu\nu} = \ldots$
$h_{ij}(t)$ "field coordinates", "field velocities"?
$K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$
n_μ
x
$x + dx$
Σ_t

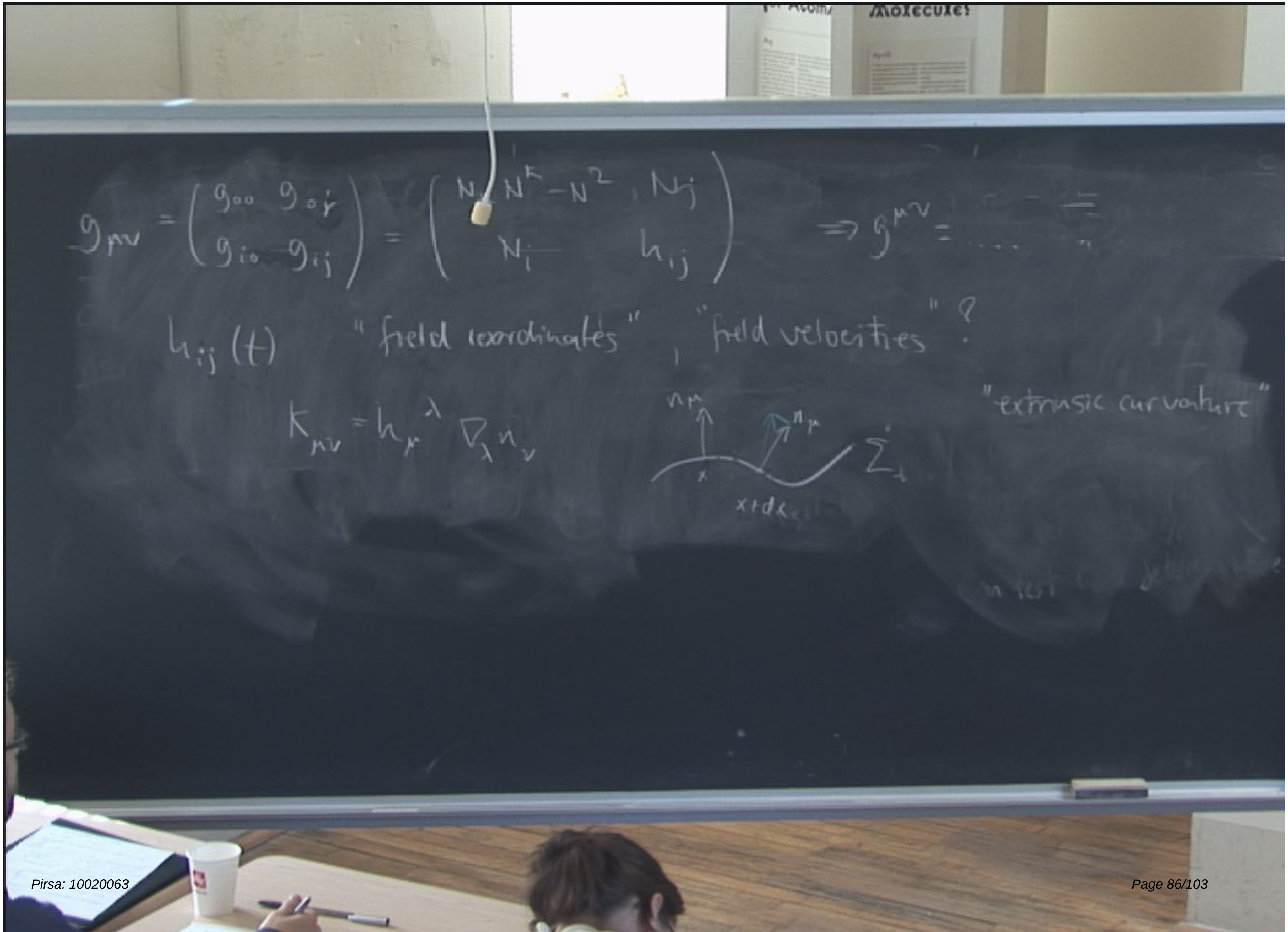
$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow g^{\mu\nu} = \ldots$$

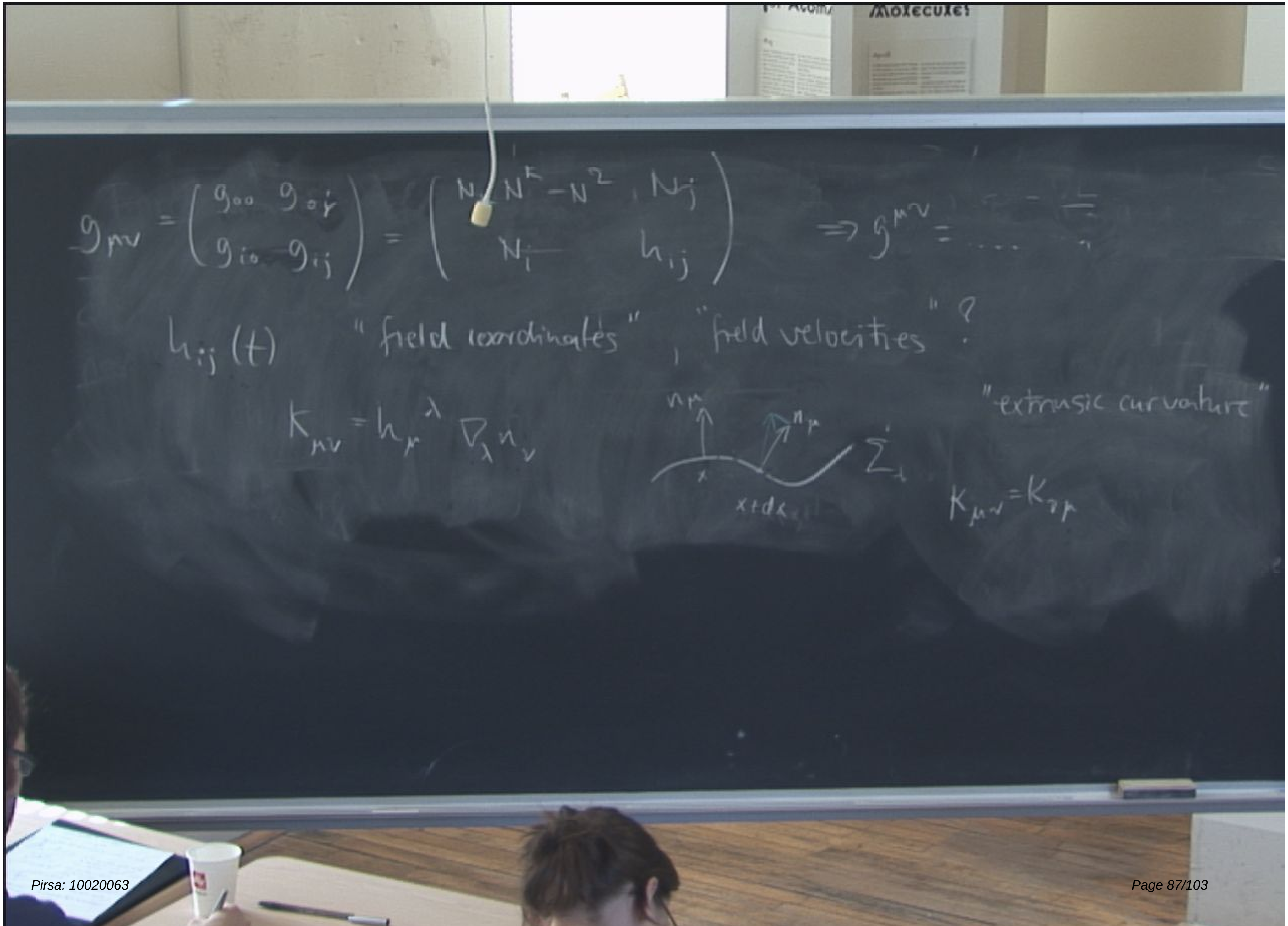
$h_{ij}(t)$ "field coordinates", "field velocities"?

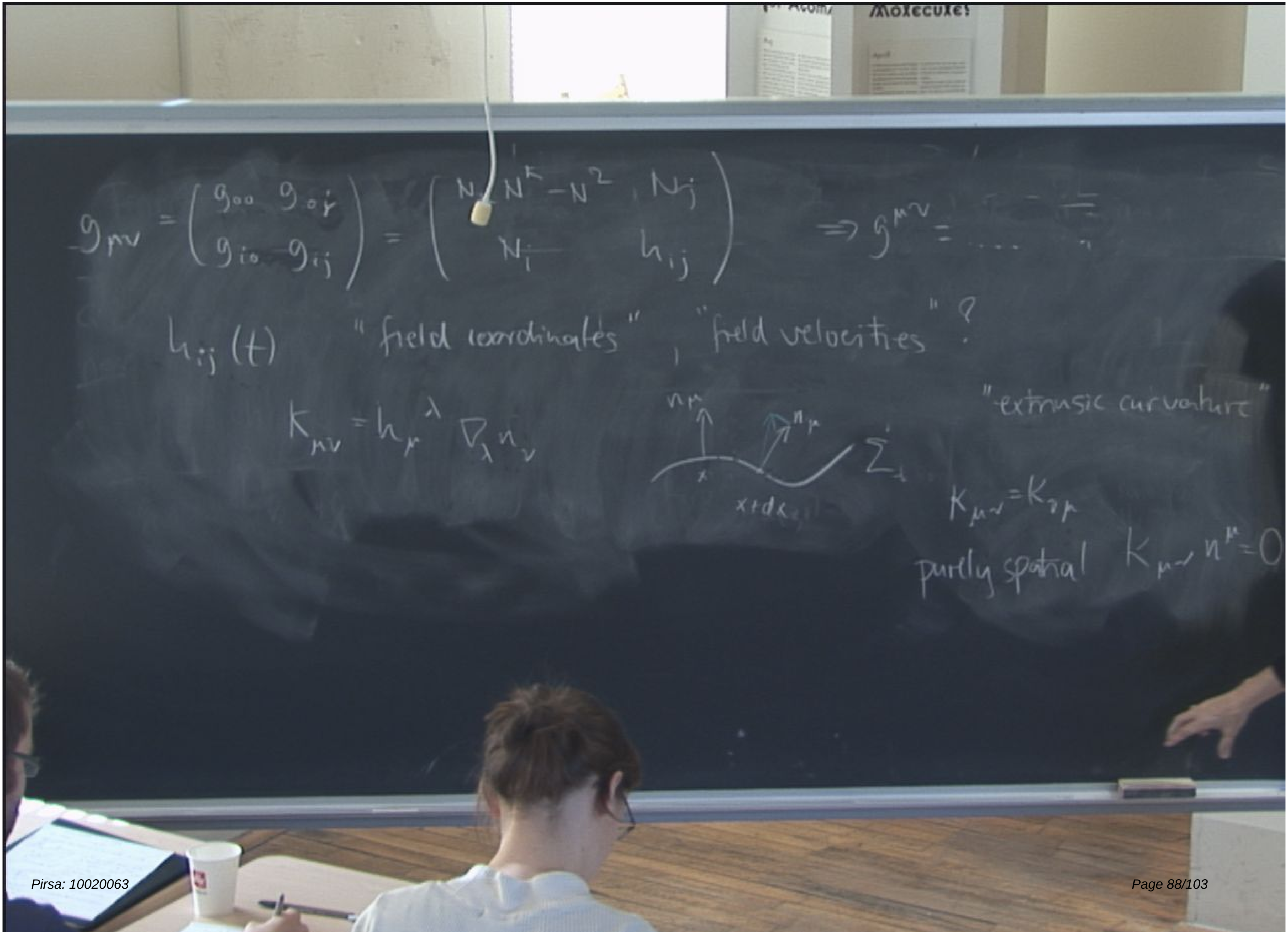
$$K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$$

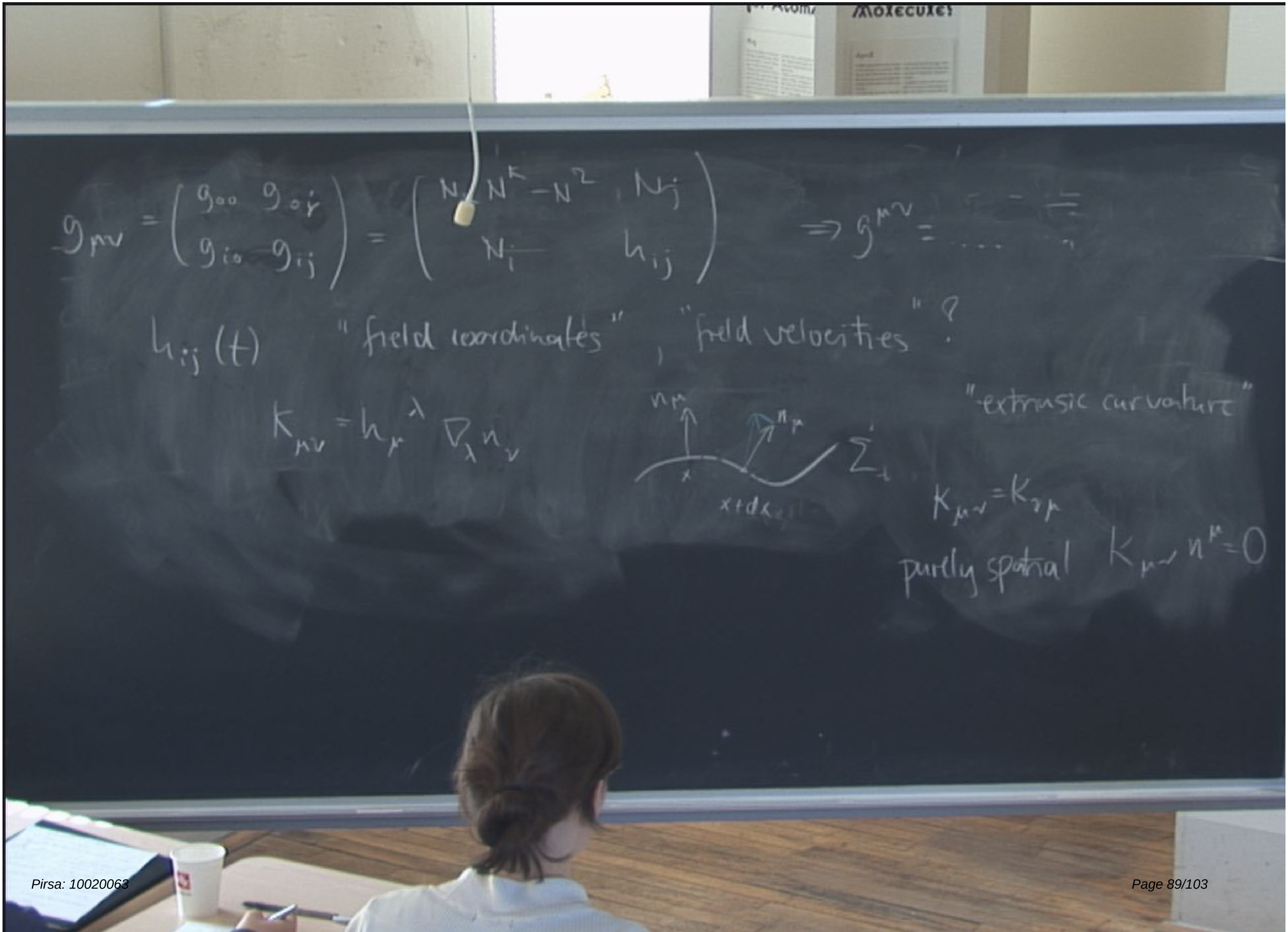


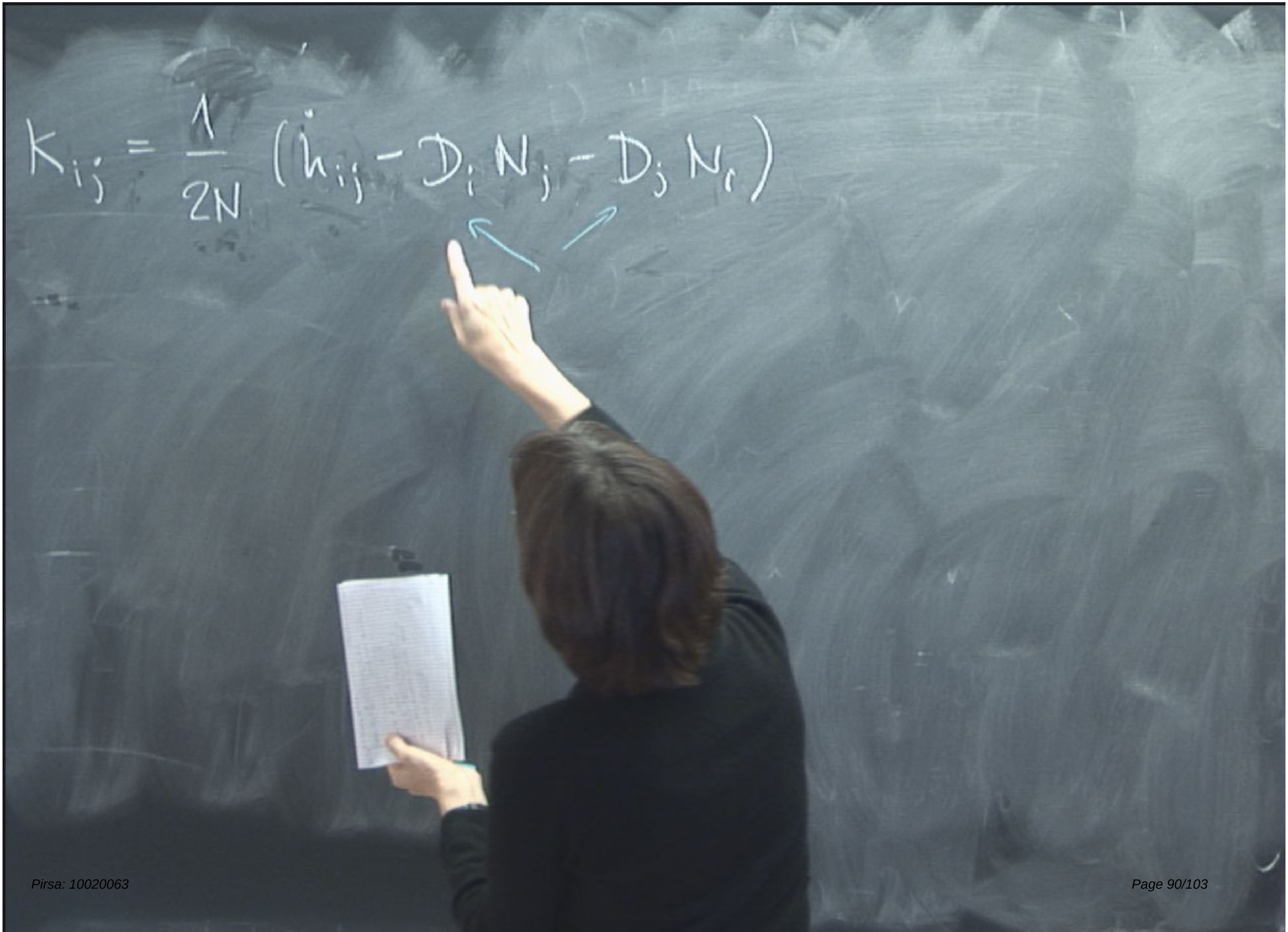

"field coordinates"
"field velocities"?


$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow g^{\mu\nu} = \dots$
$h_{ij}(t)$ "field coordinates", "field velocities"?
$K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$
n^μ
n_μ
x
$x+dx$
Σ_t
"extrinsic curvature"


g_μν = (g_00 g_0j / g_i0 g_ij) = (N_k N^k − N^2 N_j / N_i h_ij) ⇒ g^μν = ...
h_ij(t) "field coordinates", "field velocities" ?
K_μν = h_μ^λ ∇_λ n_ν
n^μ
n_μ
x
x+dx
Σ_t
"extrinsic curvature"
K_μν = K_νμ


"field coordinates", "field velocities"?
"extrinsic curvature"
purely spatial


$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix} \Rightarrow g^{\mu\nu} = \ldots$
$h_{ij}(t)$ "field coordinates", "field velocities"?
$K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$
"extrinsic curvature"
$K_{\mu\nu} = K_{\nu\mu}$
purely spatial $K_{\mu\nu} n^\mu = 0$


$K_{ij} = \frac{1}{2N} \left(\dot{h}_{ij} - D_i N_j - D_j N_i \right)$

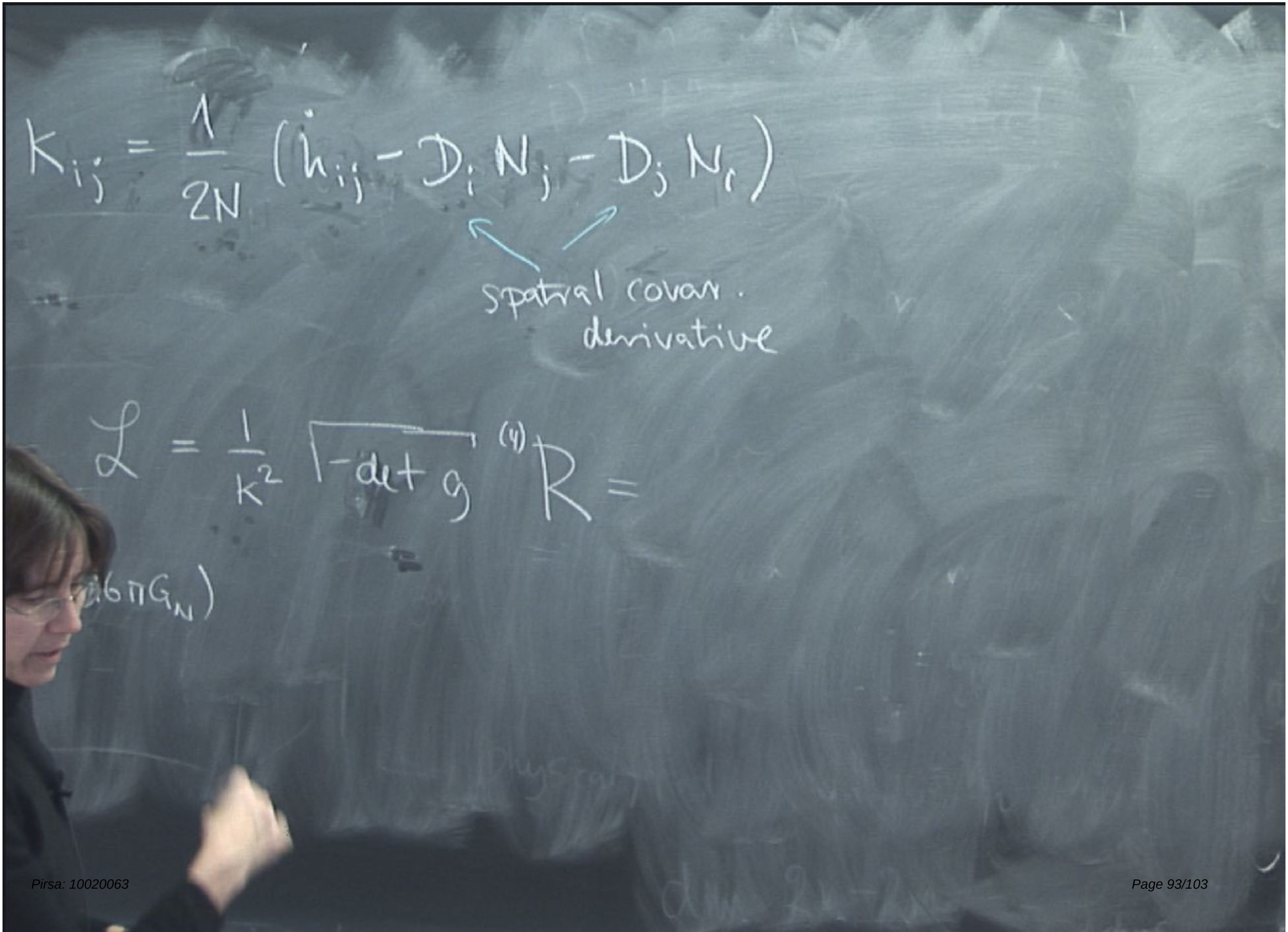
$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$$

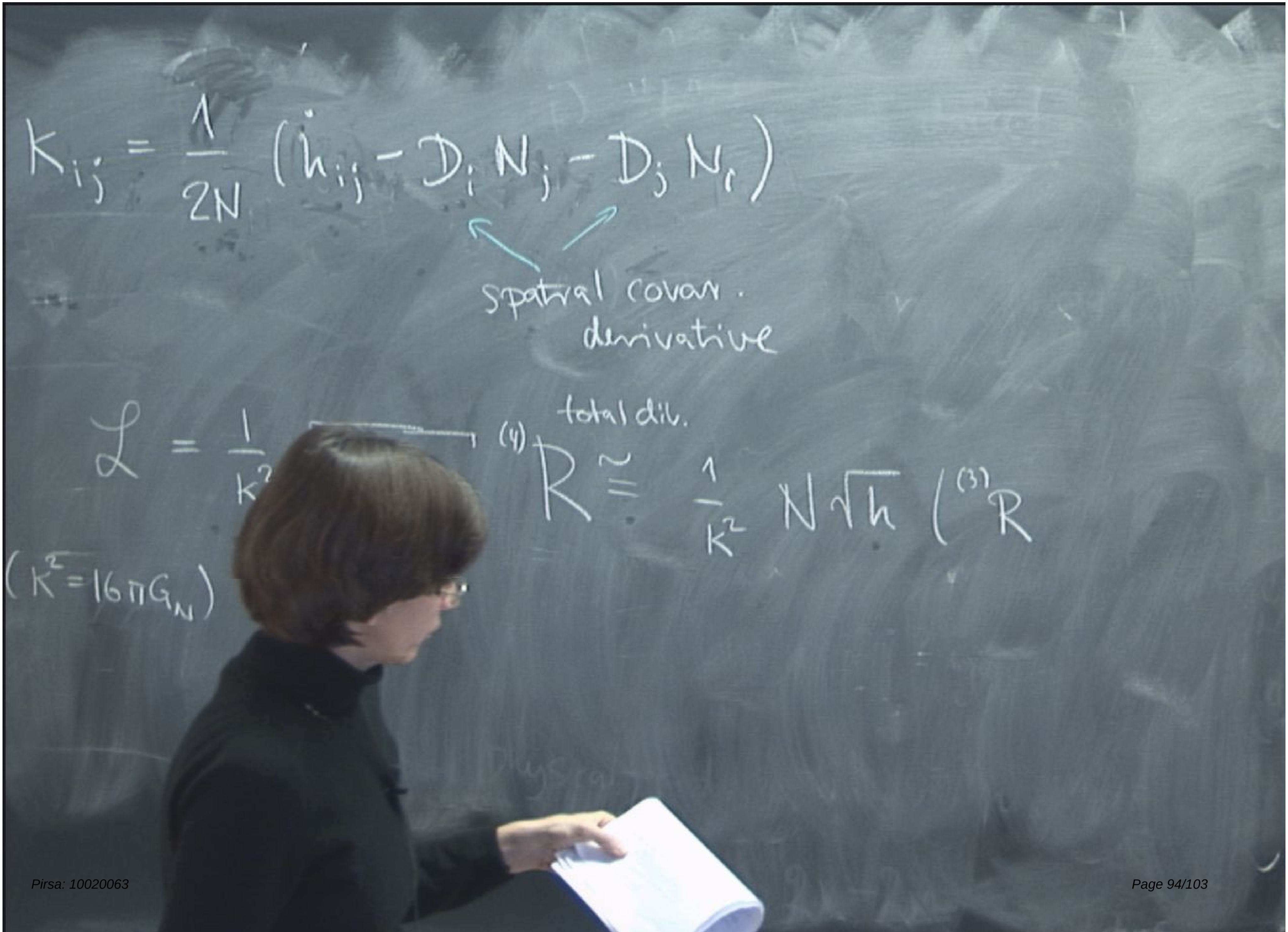
spatial covar.
derivative

$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$$

spatial covar. derivative

$$\mathcal{L} = \frac{1}{k^2}$$


$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i)$
spatial covar. derivative
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\ {}^{(4)}R =$
$16\pi G_N)$


$K_{ij} = \frac{1}{2N} (\dot{h}_{ij} - D_i N_j - D_j N_i)$
spatial covar. derivative
total div.
$\mathcal{L} = \frac{1}{\kappa^2}$
$^{(4)}R \cong \frac{1}{\kappa^2} N \sqrt{h} \, (^{(3)}R$
$(\kappa^2 = 16\pi G_N)$

$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$$

spatial covar. derivative

$$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\;{}^{(4)}R \cong \frac{1}{\kappa^2} N\sqrt{h}\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right)$$

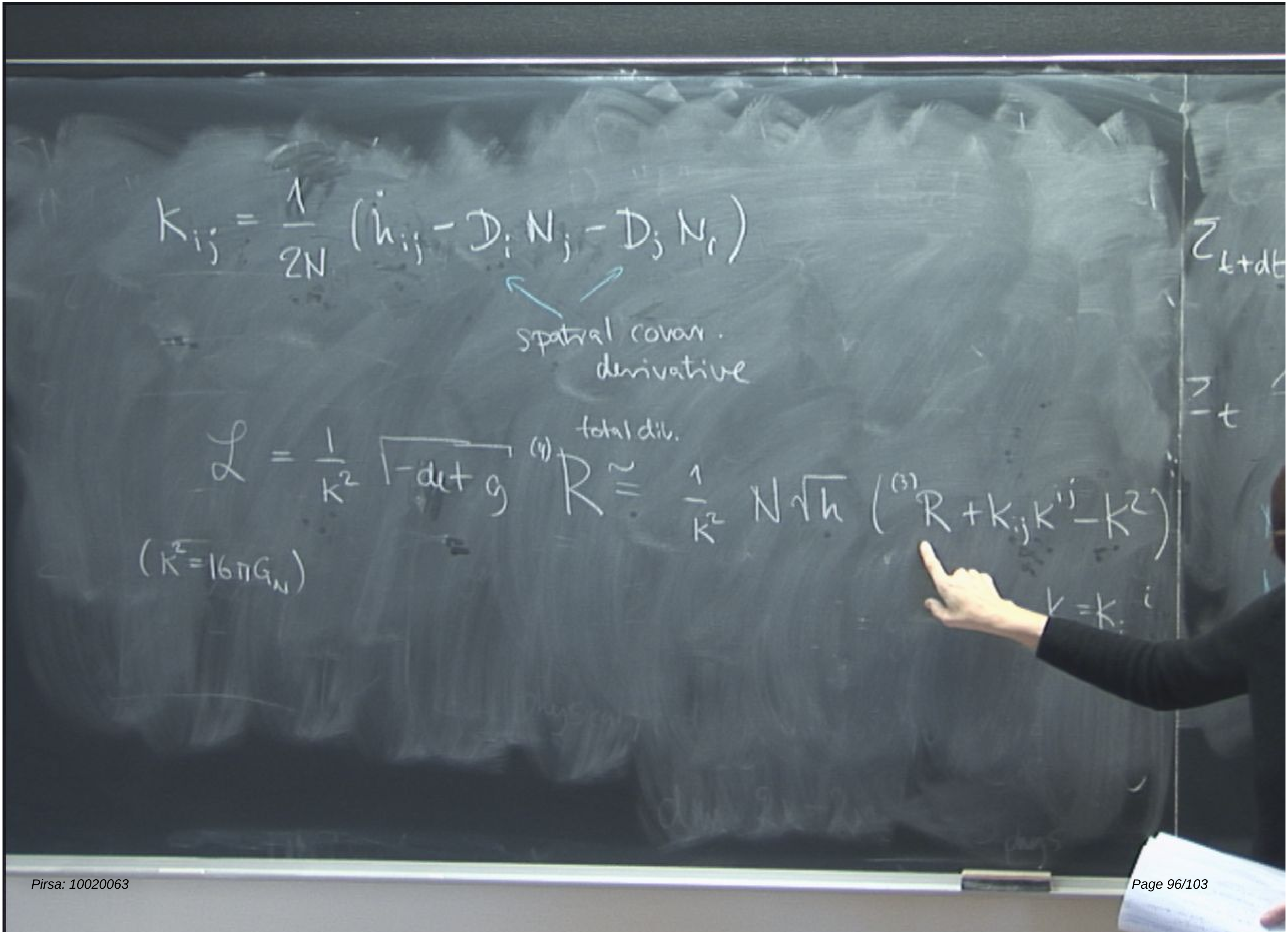
total div.

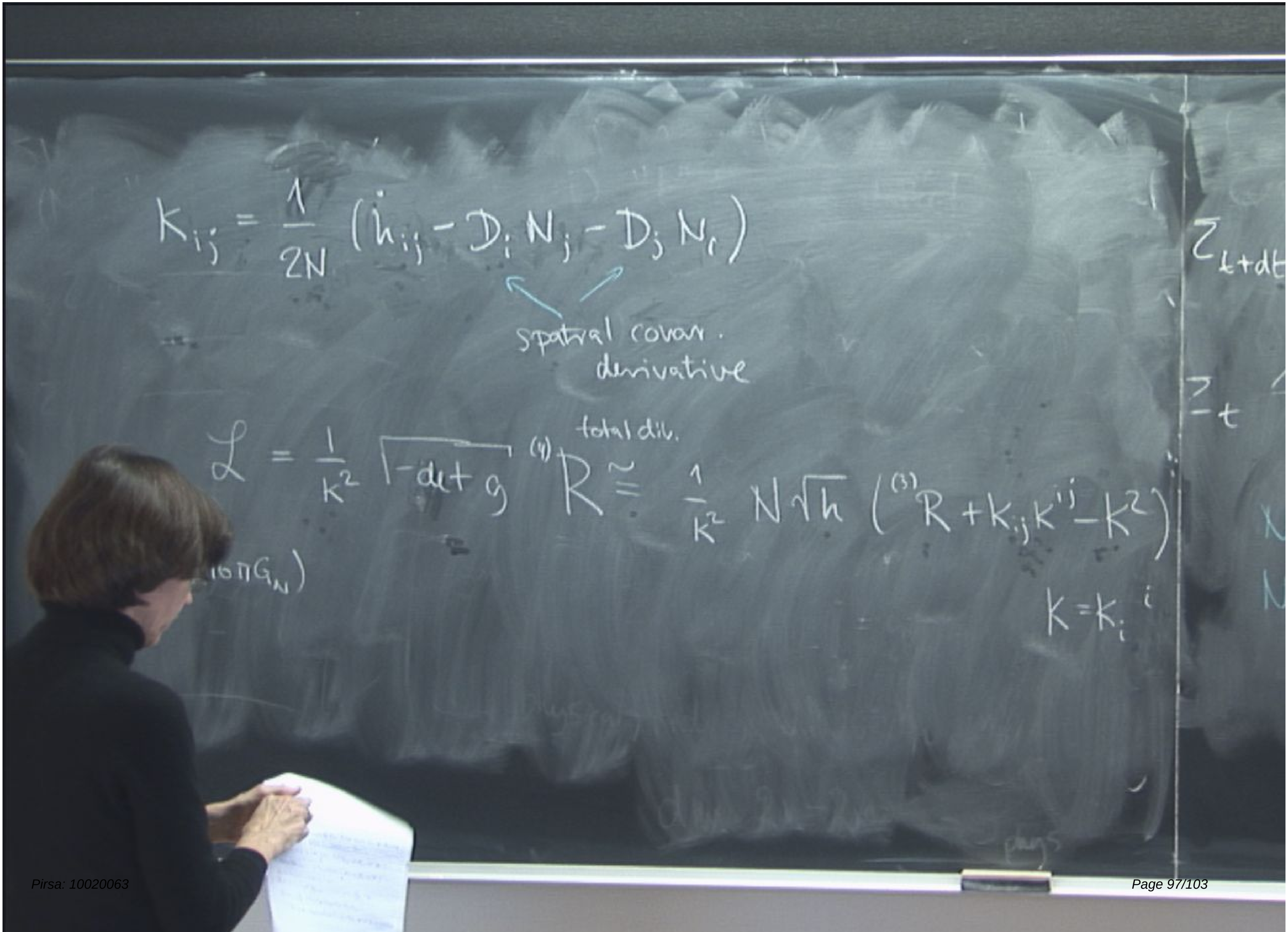
$(\kappa^2 = 16\pi G_N)$

$K = K_i{}^i$

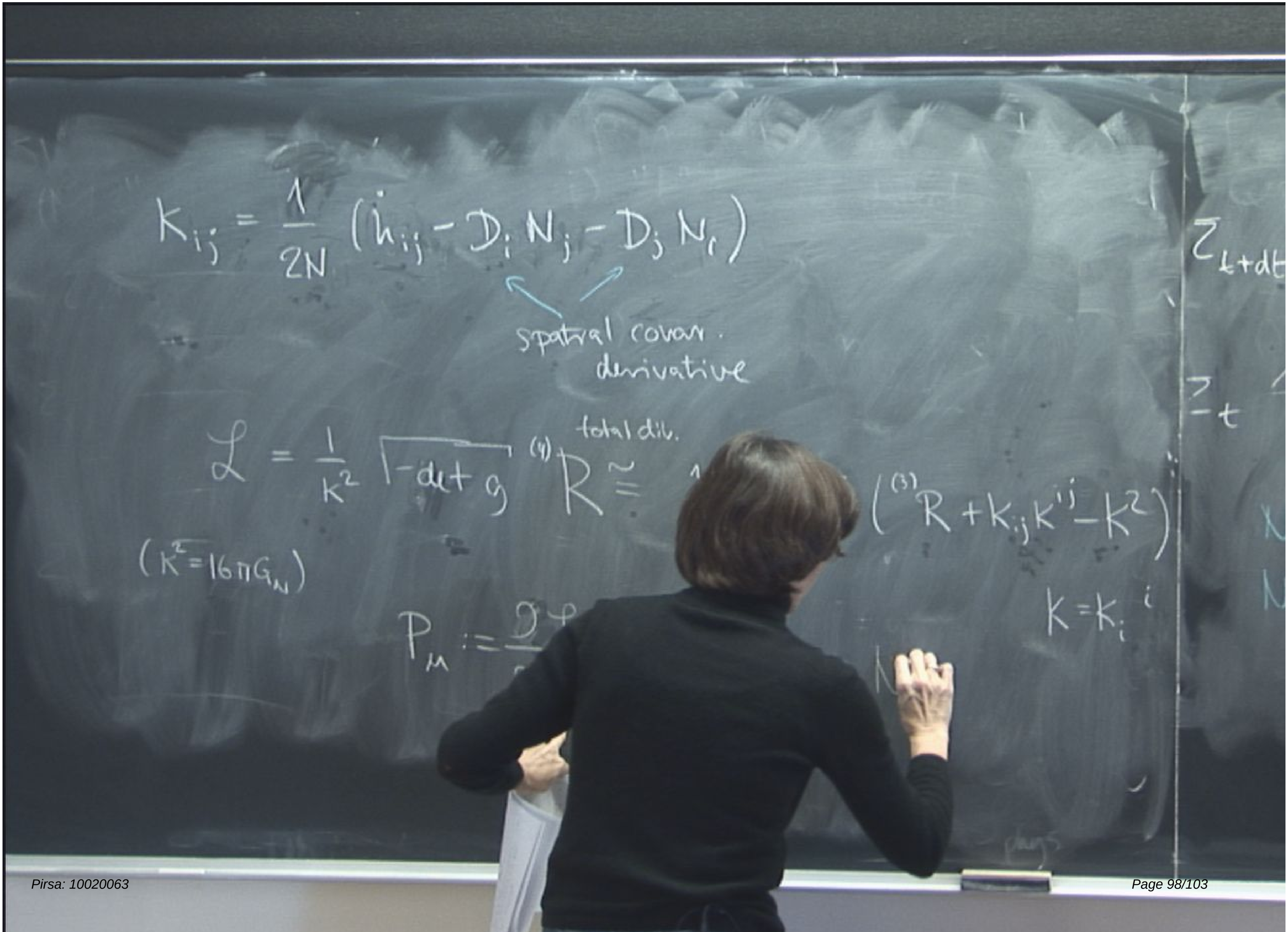
Σ_{t+dt}

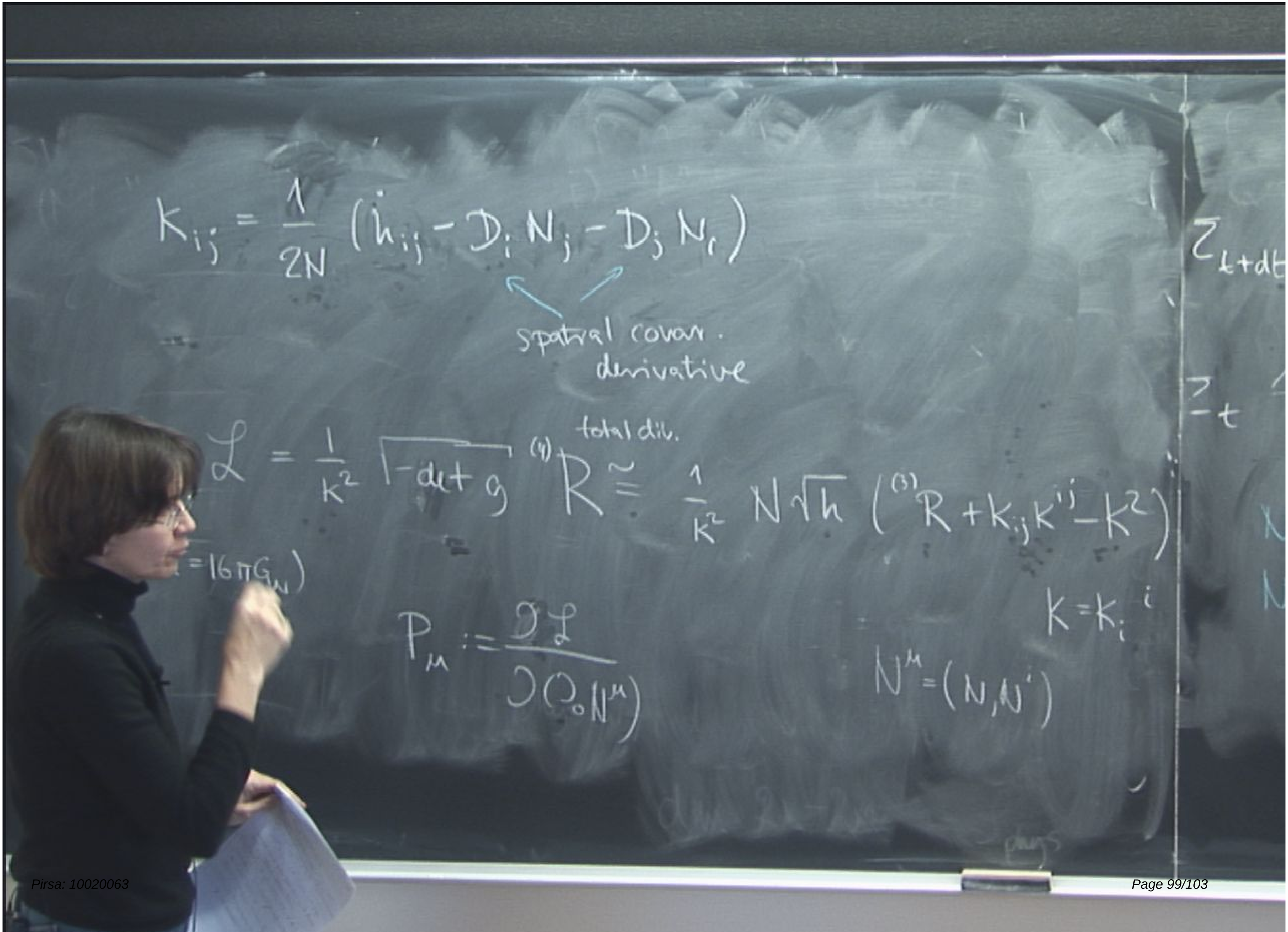
Σ_t

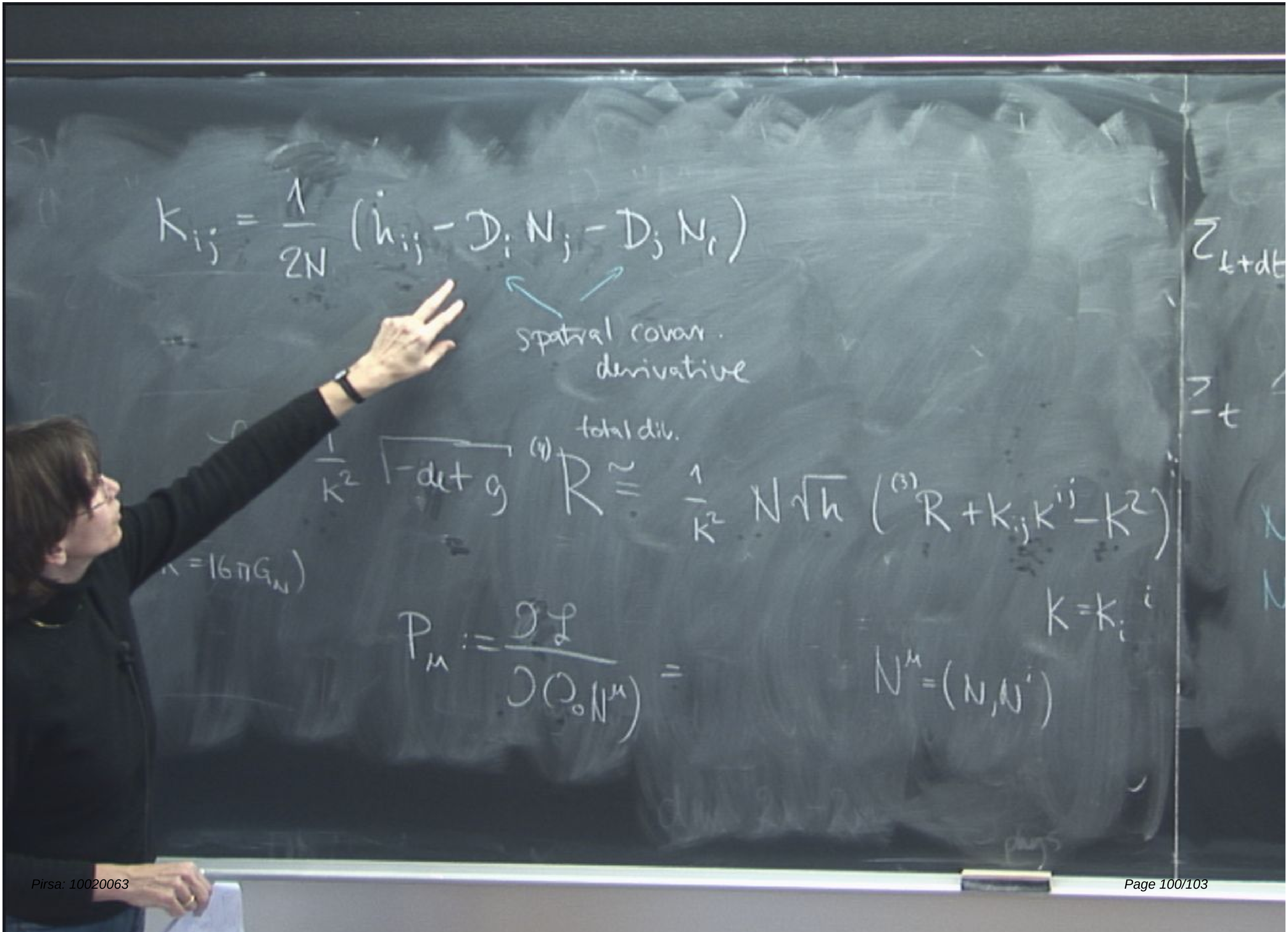

$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i)$
spatial covar. derivative
total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\,{}^{(4)}R \simeq \frac{1}{\kappa^2} N\sqrt{h}\,({}^{(3)}R + K_{ij}K^{ij} - K^2)$
$(\kappa^2 = 16\pi G_N)$
$K = K_i^{\ i}$
Σ_{t+dt}
Σ_t



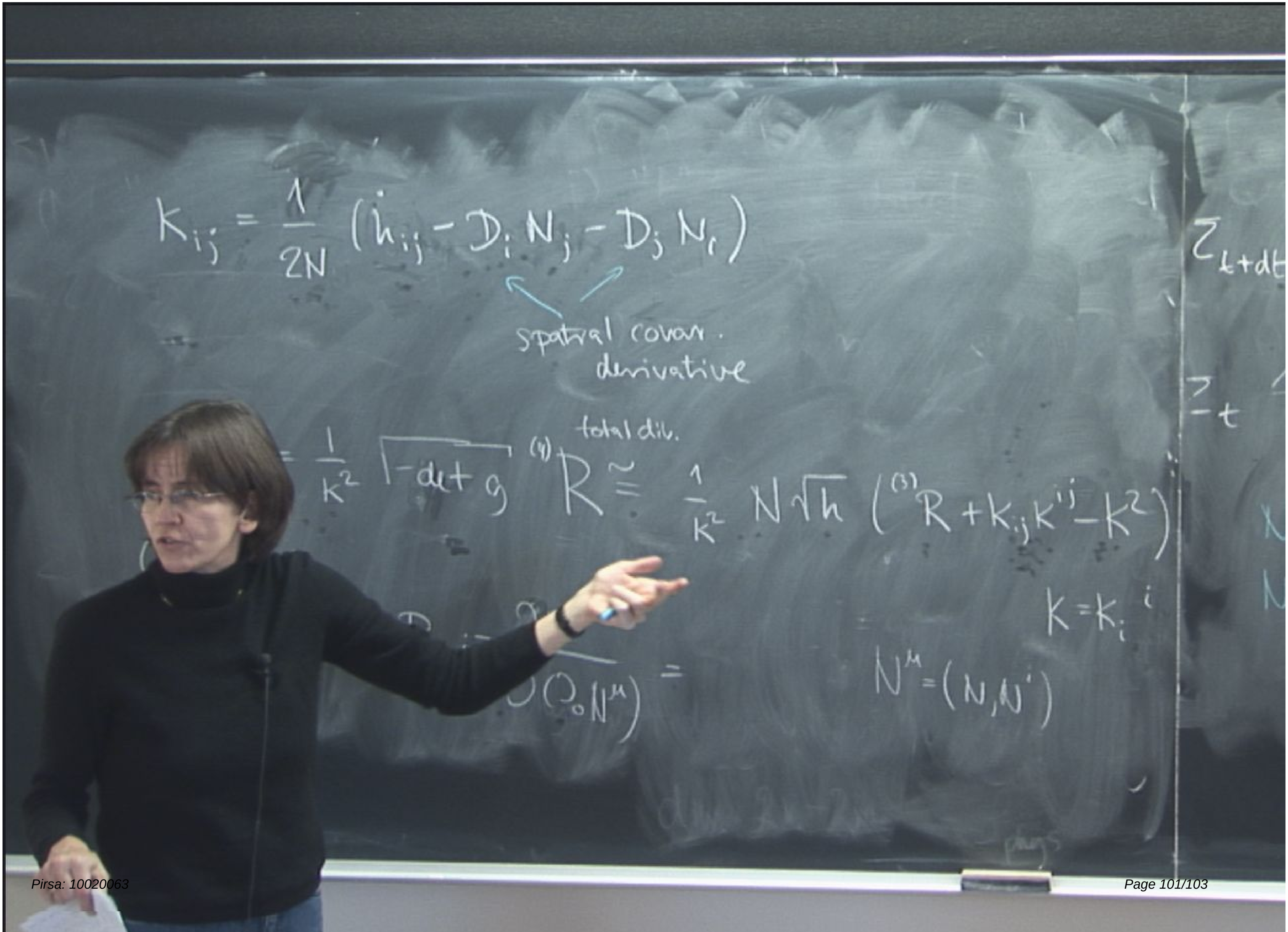
$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i)$
spatial covar. derivative
total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\,{}^{(4)}R \cong \frac{1}{\kappa^2} N\sqrt{h}\,({}^{(3)}R + K_{ij}K^{ij} - K^2)$
$16\pi G_N)$
$K = K_i{}^i$
Σ_{t+dt}
Σ_t

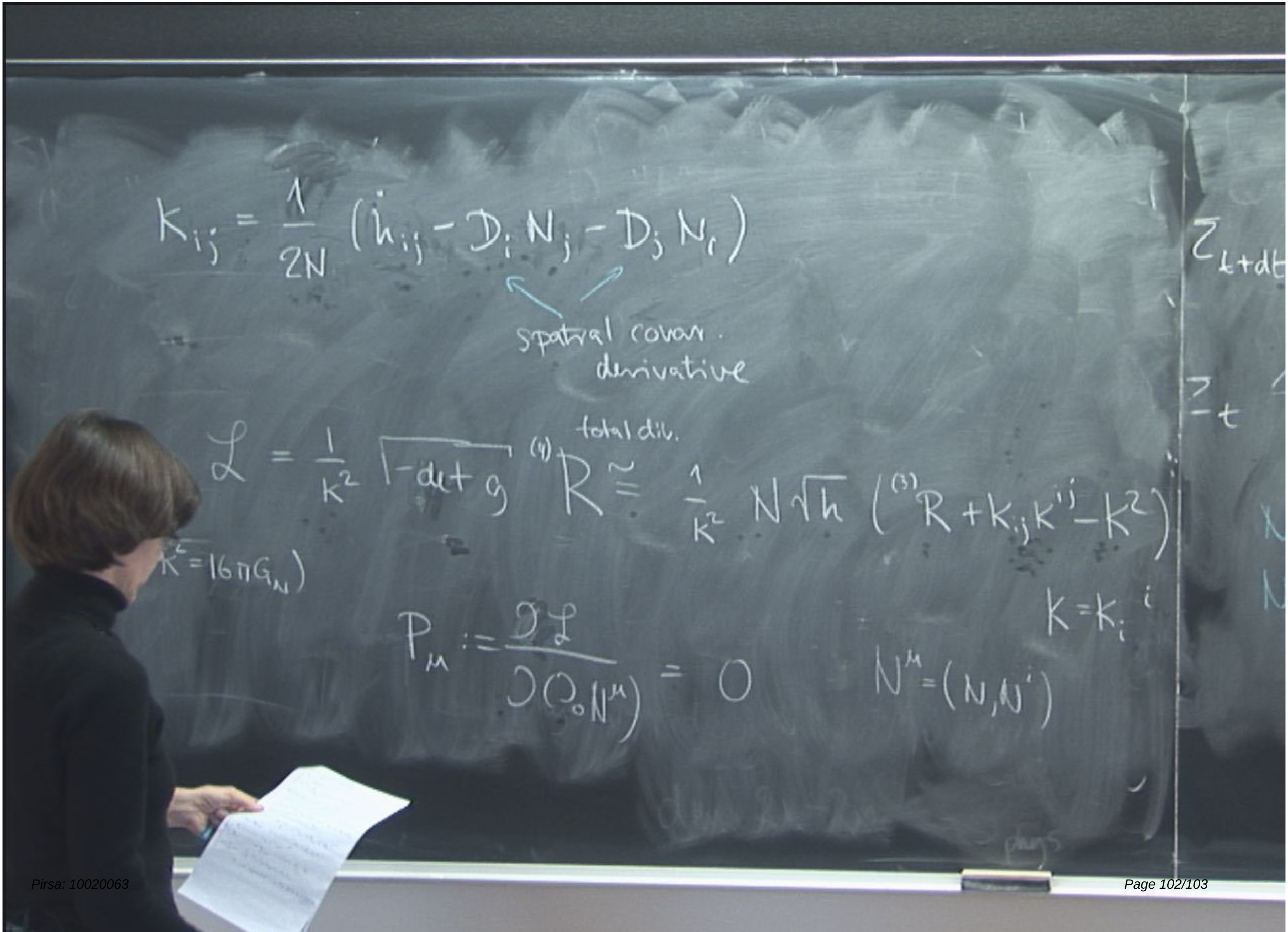

$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i)$
spatial covar. derivative
total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\ {}^{(4)}R \cong$
$({}^{(3)}R + K_{ij}K^{ij} - K^2)$
$(\kappa^2 = 16\pi G_N)$
$K = K_i{}^i$
Σ_{t+dt}
Σ_t


K_ij = 1/2N (ḣ_ij − D_i N_j − D_j N_i)
spatial covar. derivative
total div.
L = 1/κ² √(−det g) (4)R ≅ 1/κ² N √h ((3)R + K_ij K^ij − K²)
= 16πG_N)
P_μ := ∂L / ∂(∂_0 N^μ)
K = K_i^i
N^μ = (N, N^i)
Σ_t+dt
Σ_t



$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i)$
spatial covar.
derivative
total div.
$\frac{1}{\kappa^2}\sqrt{-\det g}\;{}^{(4)}R \cong \frac{1}{\kappa^2} N\sqrt{h}\,({}^{(3)}R + K_{ij}K^{ij} - K^2)$
$\kappa = 16\pi G_N)$
$K = K_i{}^i$
$P_\mu := \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} =$
$N^\mu = (N, N^i)$
Σ_{t+dt}
Σ_t


K_ij = 1/2N (ḣ_ij − D_i N_j − D_j N_i)
spatial covar. derivative
total div.
1/κ² √−det g ⁽⁴⁾R ≅ 1/κ² N √h (⁽³⁾R + K_ij K^ij − K²)
K = K_i^i
N^μ = (N, N^i)
Σ_t+dt
Σ_t



K_ij = 1/2N (ḣ_ij − D_i N_j − D_j N_i)
spatial covar. derivative
L = 1/κ² √(−det g) ⁽⁴⁾R ≅ 1/κ² N √h (⁽³⁾R + K_ij K^ij − K²)
total div.
(κ² = 16πG_N)
K = K_i^i
P_μ := ∂L / ∂(∂_0 N^μ) = 0
N^μ = (N, N^i)
Σ_t+dt
Σ_t

$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$$

spatial covar. derivative

$$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\;{}^{(4)}R \overset{\text{total div.}}{\simeq} \frac{1}{\kappa^2} N\sqrt{h}\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right)$$

$(\kappa^2 = 16\pi G_N)$

$K = K_i{}^i$

$$P_\mu := \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} = 0 \qquad N^\mu = (N, N^i)$$

$$\pi_{ij} := \frac{\partial \mathcal{L}}{\partial(\partial_0 h_{ij})} = -\frac{1}{\kappa^2}\sqrt{h}\left(K^{ij} - K h^{ij}\right)$$

Σ_{t+dt}

Σ_t