

Title: Quantum Gravity - Review (PHYS 638) - Lecture 6

Date: Feb 01, 2010 10:00 AM

URL: <http://pirsa.org/10020060>

Abstract:

Gauge-fixing à la Faddeev-Popov

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$S[\phi^A]$ is invariant under the action of a local gauge group G

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$\int \mathcal{D}\phi^A e^{-iS[\phi^A]}$ "overcounts" configurations

typically, set divergence since $\text{vol}(G) = \infty$

by example

$S(x, y) \in C^\infty(\mathbb{R}^2)$, invariant

by example

$S(x, y) \in C^\infty(\mathbb{R}^2)$, invariant
under $SO(2)$ -rotations

what is $W = \int$

by example

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what is $W = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{iS(x,y)}$

$= \int dx$

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$$\text{what is } W = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{iS(x,y)} = \int_0^{2\pi} d\varphi \int_0^{\infty} dr r e^{iS(r)}$$

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$$S(r, \varphi + \alpha)$$

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" φ " is unphysical gauge d.o.f.

original configuration space $\mathcal{C} = \mathbb{R}^2$

physical configuration space $\mathcal{C}_{\text{phys}}$

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original configuration space $\mathcal{C} = \mathbb{R}^2$

physical configuration space $\mathcal{C}_{\text{phys}} = \mathcal{C}/G = \mathbb{R}^2/SO(2) = \mathbb{R}^+$

equiv.
 $(r, \varphi) \simeq (r, \varphi')$ if $\exists \alpha$ s.t. $\varphi' = \varphi + \alpha$

φ) ?

$= \mathbb{R}^+$

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el.s of $e / SO(2)$ are equiv. classes $[(r, \varphi)]$

(r, φ)
?

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el.s of $\mathcal{E}/SO(2)$ are equiv. classes $[(r, \varphi)]$ $y \uparrow$ gauge orbit

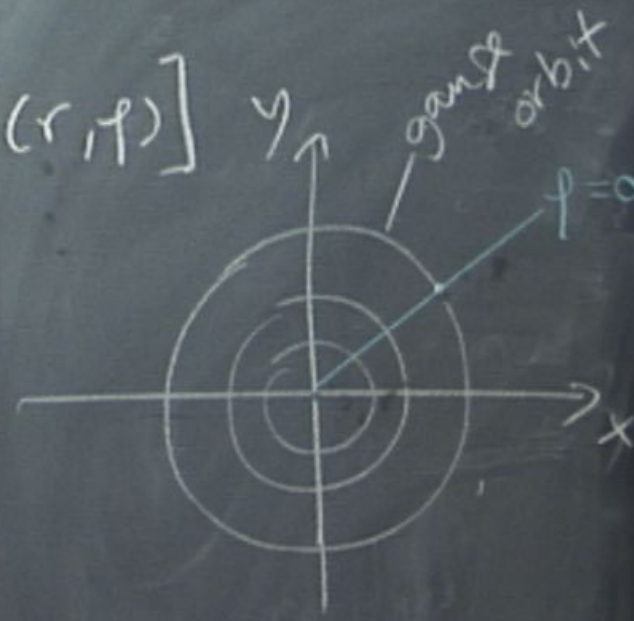


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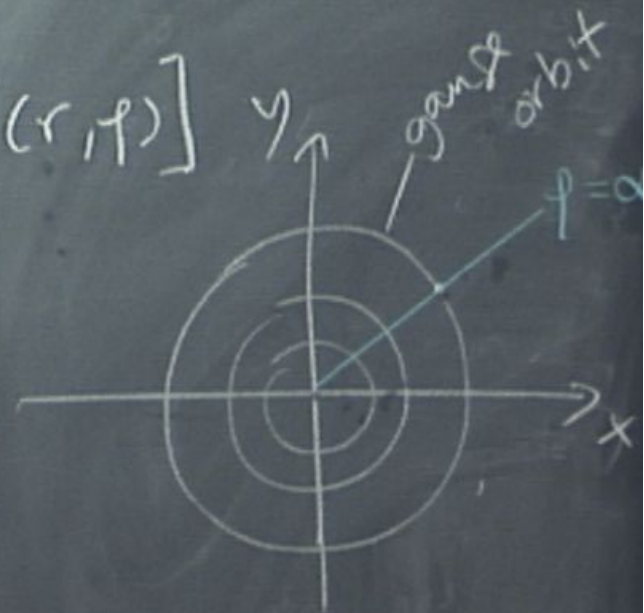
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? gauge orbits are points in \mathcal{L}_{phys}

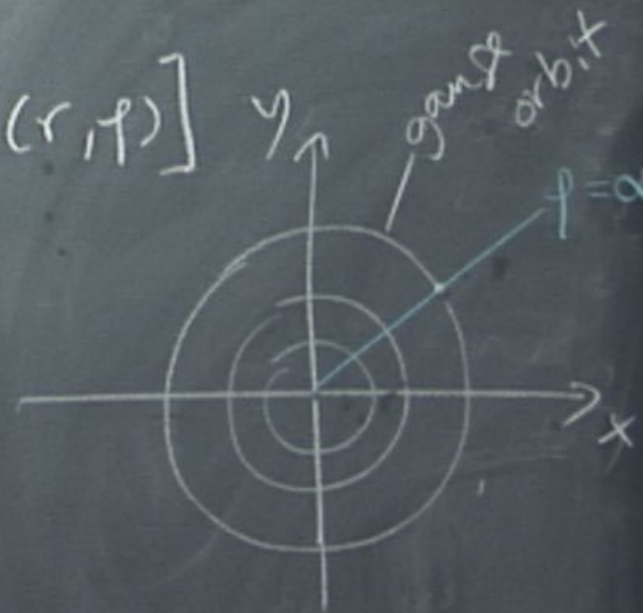


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prescription: insert "1" into PJ

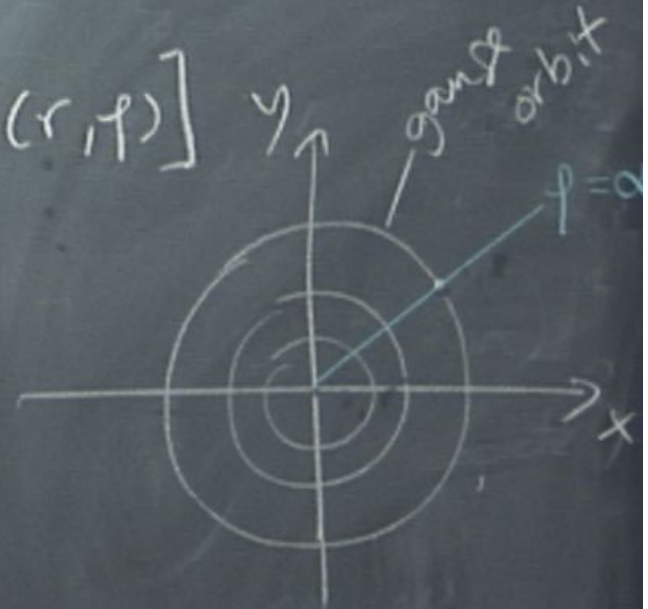


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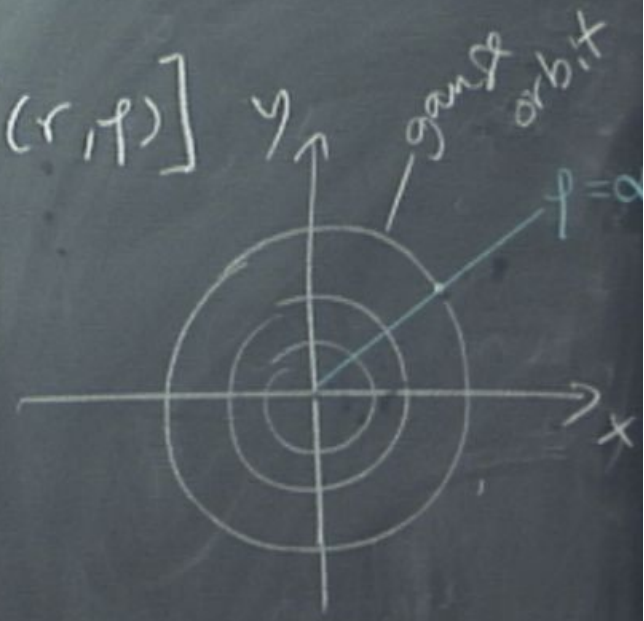
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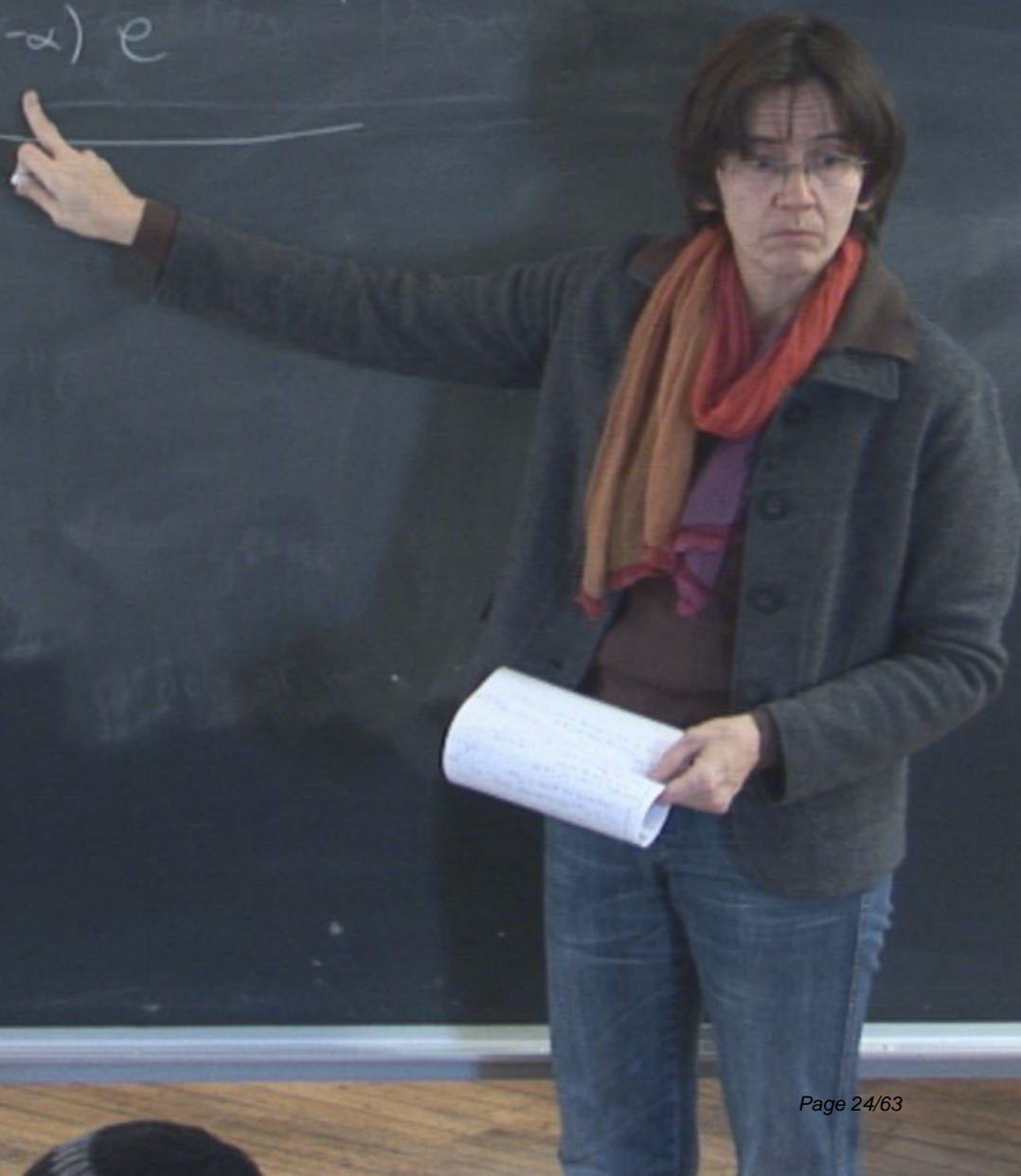
prescription: insert "1" into PI
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$$\text{here } 1 = \int_0^{2\pi} d\alpha \delta(\varphi - \alpha)$$



$$W = \int_0^{2\pi} d\alpha \underbrace{\int_0^{\infty} dr r \int_0^{2\pi} d\varphi \delta(\varphi - \alpha) e^{iS(r, \varphi)}}_{W_\alpha}$$

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W_α "gauge-fixed integral"

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general gauge-fixing $g(r, \varphi) = 0$

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general gauge-fixing $g(r, \varphi) = 0$, good gauge: the sub
 $g=0$ cuts each gauge orbit exactly once

$$N = \int_0^{2\pi} d\alpha \int_0^{\infty} dr r \int_0^{2\pi} d\varphi \delta(\varphi - \alpha) e^{iS(r,p)}$$

2π
 $= \text{vol}(SO(2))$

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 (by going to $\tilde{p} = p - \alpha$)

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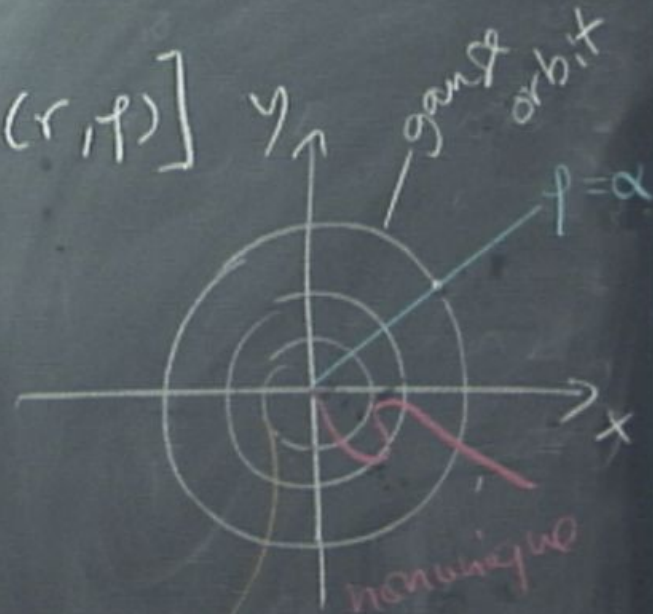
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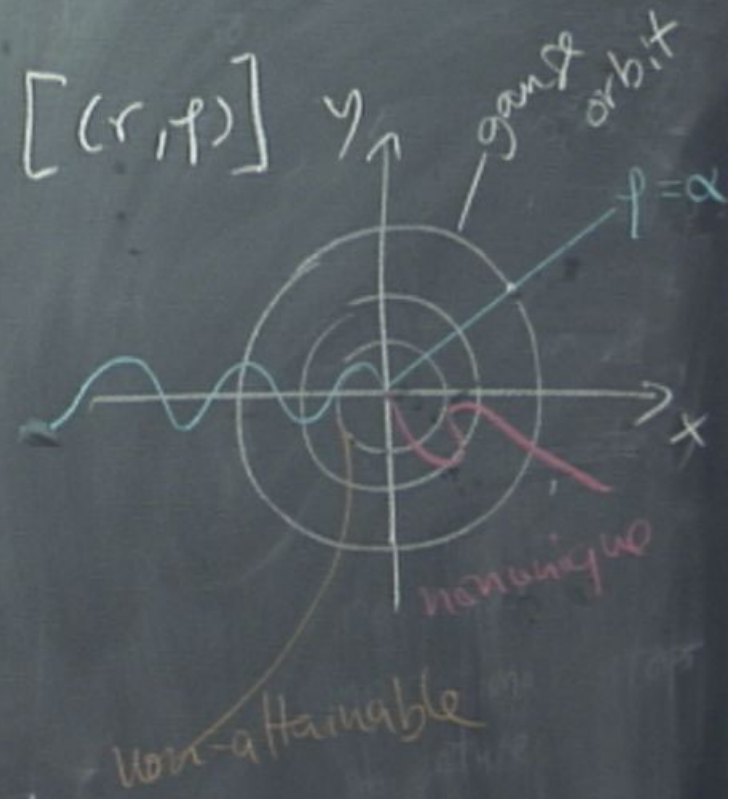
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$$\int d\alpha \delta(g(r, \varphi - \alpha)) = \int d\alpha \frac{1}{\left| \frac{\partial g}{\partial \varphi} \right|_{g=0}} \delta(\varphi - \alpha)$$

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$$= \left| \frac{\partial g}{\partial p} \right|_{g=0}^{-1} \underbrace{\int d\alpha \delta(p - \alpha)}_1$$

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$$1 = \underbrace{\left| \frac{\partial g}{\partial p} \right|_{g=0}}_{\text{Faddeev-Popov determinant}} \int d\alpha \delta(g(r, p - \alpha)) \quad 1$$

Faddeev-Popov
determinant

$$\Delta_g(r, p)$$

$$\int d\alpha \delta(g(r, p - \alpha)) = \int d\alpha \frac{1}{\left| \frac{\partial g}{\partial p} \right|_{g=0}} \delta(p - \alpha) = \dots$$

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Faddeev-Popov
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$$\Delta_g(r, p)$$

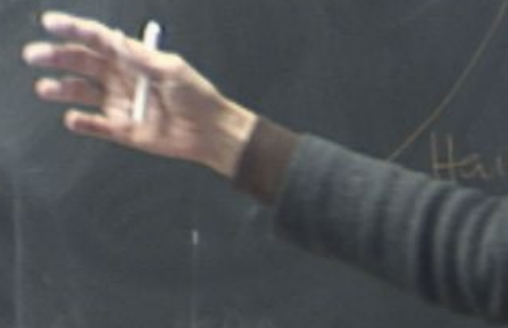
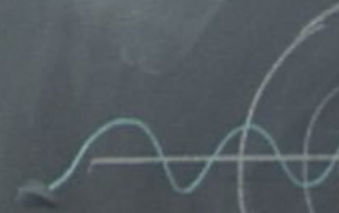
$$W = \int d\alpha \int dr r \int dt \left| \frac{\partial g}{\partial \alpha} \right|_{g=g_0} \delta(g(r, t) - \alpha) e^{iS(r, t)}$$



Non-Abelian

$$W = \int d\alpha \int dr r \int dt \frac{|\dot{y}|}{|g|} \delta(g(r, t - \alpha)) e^{iS(r, t)}$$

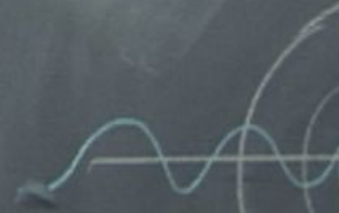
$$W_\alpha, W_\alpha = W_{\alpha'}$$



$$W = \int d\alpha \int dr r \int dt \frac{|Dg|}{|Sg|_{g=0}} \delta(g(r, t - \alpha)) e^{iS(r, t)}$$

$$W_\alpha, W_\alpha = W_\alpha$$

Crucial: gauge-invariance of the measure on G



non-gauge

$$W = \int d\alpha \int dr r \int dt \underbrace{\left(\frac{Dg}{Dg_0} \right) \delta(g(r, t - \alpha))}_{\substack{W_\alpha, \\ W_\alpha = W_{\alpha'}}} e^{iS(r, t)}$$

Crucial: gauge-invariance of the measure on G

→ E. Mottola, hep-th/9502109

$$W = \int d\alpha \int dr r \int dt \underbrace{\left(\frac{2g}{5\pi |g_{\alpha 0}} \right)}_{\substack{W_{\alpha} \\ W_{\alpha} = W_{\alpha'}}} \delta(g(r, t - \alpha)) e^{iS(r, t)}$$

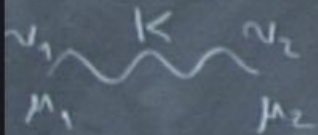
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propagator in harmonic gauge

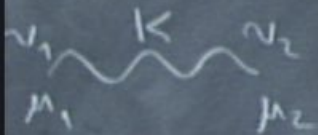


propagator in harmonic gauge



$$D_{\mu_1\nu_1\mu_2\nu_2}(k) = \frac{1}{2} \frac{\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}}{k^2 + i\epsilon}$$

propagator in harmonic gauge



$$D_{\mu_1 \nu_1, \mu_2 \nu_2}(k)$$

$$= \frac{1}{2}$$

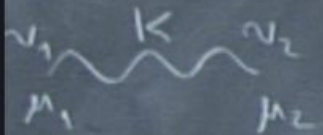
$$\frac{+\eta_{\mu_1 \nu_2} \eta_{\nu_1 \mu_2} - \eta_{\mu_1 \nu_1} \eta_{\mu_2 \nu_2}}{+i\epsilon}$$



ν



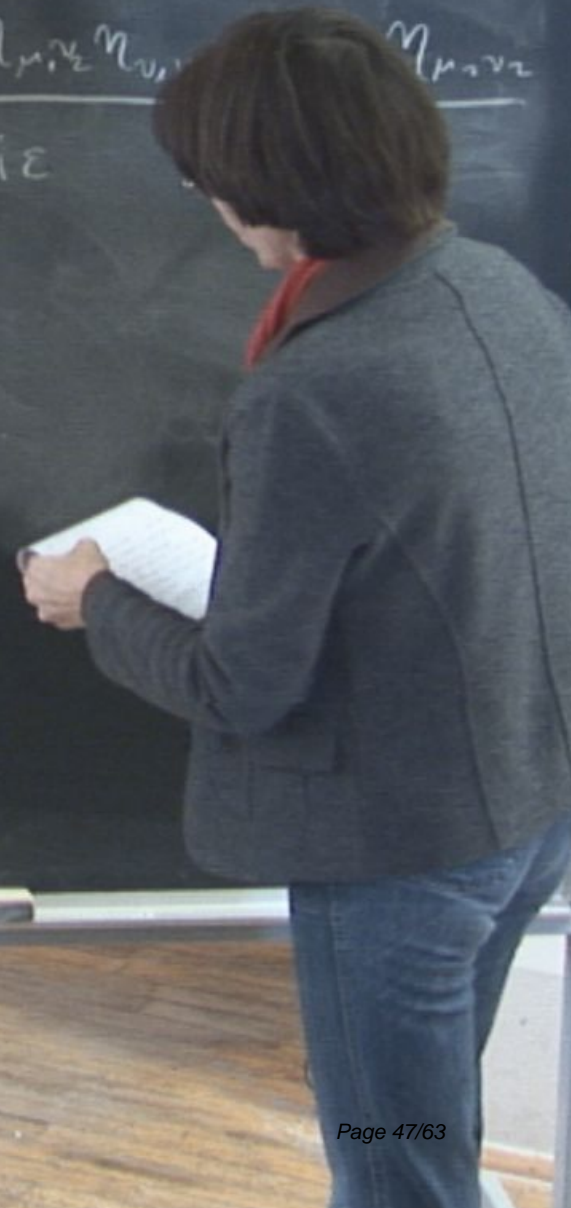
propagator in harmonic gauge



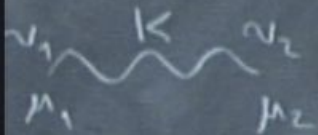
$$D_{\mu_1 \nu, \mu_2 \nu} (k) = \frac{1}{2} \frac{\eta_{\mu_1 \mu_2} \eta_{\nu \nu} + \eta_{\mu_1 \nu} \eta_{\nu \mu_2}}{k^2 + i\epsilon}$$



$$\gamma^{\mu_1 \nu, \mu_2 \nu, \mu_3 \nu} (k_1, k_2, k_3) = k_1 \cdot k_2 \eta$$



propagator in harmonic gauge

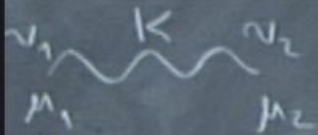


$$D_{\mu_1 \nu_1, \mu_2 \nu_2}(K) = \frac{1}{2} \frac{\eta_{\mu_1 \mu_2} \eta_{\nu_1 \nu_2} + \eta_{\mu_1 \nu_2} \eta_{\nu_1 \mu_2} - \eta_{\mu_1 \nu_1} \eta_{\mu_2 \nu_2}}{K^2 + i\epsilon}$$



$$\mathcal{V}^{\mu_1 \nu_1, \mu_2 \nu_2, \mu_3 \nu_3}(K_1, K_2, K_3) = K_1 K_2 \eta^{\mu_1 \nu_1} \eta^{\mu_2 \nu_2} \eta^{\mu_3 \nu_3} + K_1^{\mu_3} K_2^{\nu_3} \eta^{\mu_1 \mu_2} \eta^{\nu_1 \nu_2} + \text{many more terms}$$

propagator in harmonic gauge



$$D_{\mu_1\nu_1\mu_2\nu_2}(k) = \frac{1}{2} \frac{\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}}{k^2 + i\epsilon}$$

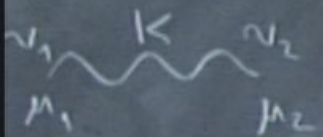


$$\mathcal{G}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}(k_1, k_2, k_3) = k_1 \cdot k_2 \eta^{\mu_1\nu_1} \eta^{\mu_2\nu_2} \eta^{\mu_3\nu_3} +$$

$$+ k_1^{\mu_3} k_2^{\nu_3} \eta^{\mu_1\mu_2} \eta^{\nu_1\nu_2} + \text{many more terms}$$

(DeWitt 1967) (~100)

propagator in harmonic gauge



$$D_{\mu_1\nu_1\mu_2\nu_2}(k) = \frac{1}{2} \frac{\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}}{k^2 + i\epsilon}$$

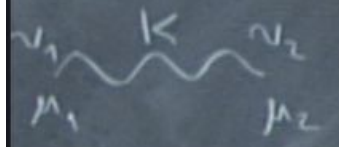


$$\mathcal{V}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}(k_1, k_2, k_3) = k_1 \cdot k_2 \eta^{\mu_1\nu_1} \eta^{\mu_2\nu_2} \eta^{\mu_3\nu_3} +$$

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(DeWitt 1967) (~100)

propagator in harmonic gauge



$$D_{\mu_1 \nu_1, \mu_2 \nu_2}(k) = \frac{1}{2} \frac{\eta_{\mu_1 \mu_2} \eta_{\nu_1 \nu_2} + \eta_{\mu_1 \nu_2} \eta_{\nu_1 \mu_2} - \eta_{\mu_1 \nu_1} \eta_{\mu_2 \nu_2}}{k^2 + i\epsilon}$$



$$\int \eta^{\mu_1 \nu_1} \eta^{\mu_2 \nu_2} \eta^{\mu_3 \nu_3}(k_1, k_2, k_3) = k_1 \cdot k_2 \eta^{\mu_1 \nu_1} \eta^{\mu_2 \nu_2} \eta^{\mu_3 \nu_3} +$$

$$+ k_1^{\mu_3} k_2^{\nu_3} \eta^{\mu_1 \mu_2} \eta^{\nu_1 \nu_2} + \text{many more terms}$$

fzfzf

(DeWitt 1967) (~100)

Is gravity perturbatively renormalizable?

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Is gravity perturbatively renormalizable? ~~Yes~~ N.O.

$$[G_N] = -2, \quad G_N \approx$$

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$$M, \Lambda \gg m, E$$

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$$M, \Lambda \gg m, E$$

$$M(1 + G_N)$$

Is gravity perturbatively renormalizable? ~~Yes~~ N.O.

$$[G_N] = -2, \quad G_N = \frac{1}{m_{\text{Pl}}^2}$$

$$M, \Lambda \Rightarrow m, E$$

$$M(1 + G_N \Lambda^2)$$

Is gravity perturbatively renormalizable? ~~NO~~ N.O.

$$[G_N] = -2, \quad G_N = \frac{1}{m_{\text{Pl}}^2}$$

$$M, \Lambda \Rightarrow m, E$$

$$M (1 + G_N \Lambda^2 + G_N^2 \Lambda^4 + \dots)$$

quadr. divergence

Is gravity perturbatively renormalizable? ~~Yes~~ N.O.

$$[G_N] = -2, \quad G_N = \frac{1}{m_{\text{Pl}}^2}$$

$$M, \Lambda \rightarrow m, E$$

$$M (1 + G_N \Lambda^2 + G_N^2 \Lambda^4 + \dots)$$

quadr. divergence

divergence gets worse at higher orders

pure quantum gravity is one-loop finite on-shell

't Hooft & Veltman (1974)

no longer true at two loops : Goroff & Sagnotti (1986)

van de Ven (1992)

pure quantum gravity is one-loop finite on-shell

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